SIMULATION OF HUMAN GAIT USING COMPUTED TORQUE CONTROL METHOD

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ABSTRACT

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The present study is concerned with simulation of the gait cycle including both the single (stance and swing phases) and double support phases. The inverse dynamic analysis and the gait control under disturbing features are investigated.

The single support phase foot-flat and heel-off periods, the swing phase and the double support phase are all modelled by separate linkage systems. Nominal variations of the joint angles are determined by considering the kinematic constraints and desired trajectories for the hip and ankle joints.

Dynamic equations are obtained and feedback control is used in the form of computed torque control as the human body also uses a kind of feedback control to stabilize its motion during the gait. The control stiffness coefficients of the feedback

control algorithm are determined depending on the amount of tracking error. In the double support phase, the torque redundancy problem of the system is solved by weighted least squares optimization method.

Results are presented and compared with three dimensional gait analysis.

Keywords: Gait Simulation, Single Support Phase, Double Support Phase, Computed Torque Control

ÖZ

HESAPLANAN TORK DENETİM YÖNTEMİYLE İNSAN YÜRÜYÜŞÜNÜN BENZETİMİ

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Bu çalışma, tek destek (duruş ve salınım evreleri) ve çift destek evrelerinin ikisini birden kapsayan bir yürüyüş çevriminin benzetimiyle ilgilidir. Evrik dinamik analiz ve bozucu etkiler altında yürüyüş denetimi incelenmektedir.

Tek destek evresi tüm basma ve topuk kalkış bölümleri,salınım evresi ve çift destek evresi ayrı eklem sistemleri kullanılarak modellenmişlerdir. Eklem açılarının nominal değişimleri, kinematik kısıtlamalar ile kalça ve ayak bileğinin izlenmesi istenen yörüngeleri göz önüne alınarak belirlenmişlerdir.

Dinamik denklemler elde edilmiş ve insan vücudu da yürüyüş sırasında hareketini dengelemek için bir çeşit geribeslemeli denetim kullandığından, geribeslemeli denetim hesaplanan tork yöntemiyle kullanılmıştır. Geribeslemeli denetim algoritmasının denetim sertliği katsayıları, izleme hatası miktarına göre ayarlanarak uygulanmıştır.Sistemin çift destek evresindeki tork hesaplanmasındaki belirsizlik problemi, ağırlıklı en küçük kareler optimizasyon yöntemiyle çözülmüştür.

Sonuçlar üç boyutlu yürüme analiziyle karşılaştırmalı olarak sunulmuştur.

Anahtar Kelimeler: Yürüyüş Benzetimi, Tek Destek Evresi, Çift Destek Evresi, Hesaplanan Tork Denetim Yöntemi

to my parents

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INTRODUCTION

1.1 Analysis and Simulation of Gait

Gait analysis is a systematic study of human walking. The applications are divided into two main categories: clinical gait analysis and scientific gait analysis. Clinical gait analysis has the aim of helping individual patients directly, whereas scientific gait analysis improves our understanding of gait.

Clinical gait analysis involves performing gait analysis on a single person, with the aim of benefiting that person directly. Documentation of the current status or planning of treatment or the monitoring of the progress over a period of time can be done. The results of the analysis may be used to identify the areas where the treatment is ineffective, or it may define an end point for stopping treatment when progress appears to have ceased. The shape of ground reaction force curve is sometimes used to monitor the progress in rehabilitation.

Scientific gait analysis improves our understanding of gait, either normal or pathological. It includes clinical research and fundamental research. Clinical research is research involving patients, which is not necessarily expected to benefit those patients directly, but which will hopefully benefit other patients in the future. An example to the clinical research is the lower limb prosthetics design studies. It may be used to examine the effects of changing the design of a prosthetic limb, such as by altering the mass distribution or using knee joints with different types of braking mechanisms.

Fundamental research aims to further our knowledge, especially about human performance and physiology. The study of human performance covers all aspects of human activity including sporting activities. The ability to estimate the forces generated by the muscles by using gait analysis has proved to be valuable in the study of muscle physiology [1].

Beside scientific gait analysis, another tool for improving our understanding of gait is the simulation of the walking process. Gait simulation is especially attractive to the pre-test hyphotheses regarding the method of treatment, rehabilitation and orthotic and prosthetic applications. This, however requires a succesful simulation of the normal gait, and such an undertaking is far from being easy. Not only the mechanics of gait is complex, but it also involves precise and coordinated controls.

With the enhanced computing facilities, gait simulation has recently gained popularity. Although gait analysis methodology seems to have been well established, gait simulation is a continuing research area, and several simplified models and controls are being tested for their performance.

1.2 Description and Phases of Gait

Human walking is a process of locomotion in which the moving body is supported by one leg and then the other. As the moving body passes over the supporting leg, the other leg is swinging forward in preparation for its next support phase. One foot or the other is always on the ground, and during that period, when the support of the body is transferred from the trailing to the lading leg, there is a brief period when both feet are on the ground. As a person walks faster, these periods of double support become smaller and smaller fractions of walking until, eventually as the person starts to run, they disappear altogether and are replaced by brief periods when neither foot is on the ground. The cyclic alternations of the support function of each leg and the existence of a transfer period when both feet are on the ground, are essential features of the process known as walking. The word gait describes the manner or style of walking rather then the walking process itself [2].

The gait cycle is defined as the time interval between two successive occurrences of one of the repetitive events of walking. Although any event could be chosen to define the beginning of a gait cycle, it is usually convenient to use the heel contact of one foot. If it is decided to start with the right foot, as shown in Figure 1.1, then the cycle will last until the next heel contact by the right foot. The left foot goes through exactly the same series of events as the right, but displaced in time by half a cycle. Each leg in turn has a swing phase, when it moves towards through the air, and a stance phase, when the foot is on the ground and the body passes over the top of it.

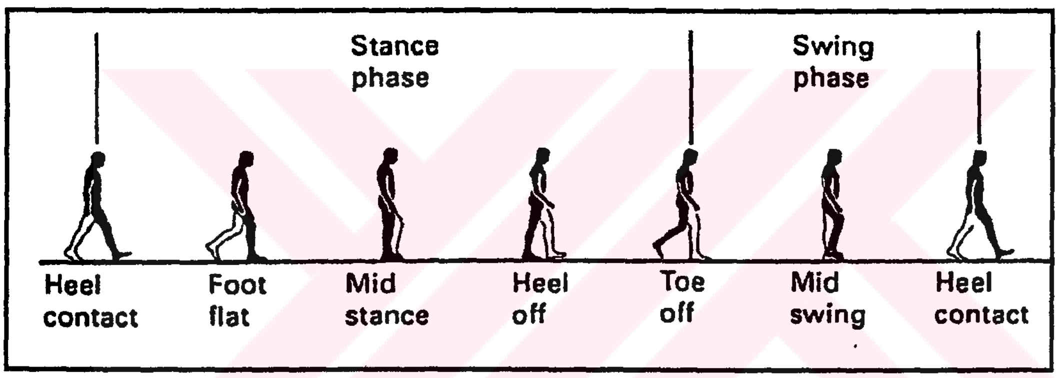

Figure 1.1 Positions of the legs during a single gait cycle from right heel contact to right heel contact

The following major events are used to divide the gait cycle into convenient periods:

1. Heel contact
2. Foot-flat
3. Mid stance
4. Heel-off
5. Toe off
6. Mid swing

The stance phase, which is also called the support phase or contact phase, lasts from heel contact to toe off. The swing phase lasts from toe off to the next heel contact. The duration of the complete gait cycle is known as the stride time, it is divided into the stance time and the swing time.

Figure 1.2 shows the timings of heel contact and toe off for both feet during a single gait cycle. Heel contact by the right foot occurs while the left foot is still on the ground, and there is a period of double support between heel contact on the right and toe off on the left. During the swing phase on the left side, only the right foot is on the ground, giving a period of right single support, which ends with heel contact by the left foot. There is then another period of double support, until toe off of the right side. Left single support corresponds to the right swing phase, and the cycle ends with the next heel contact on the right.

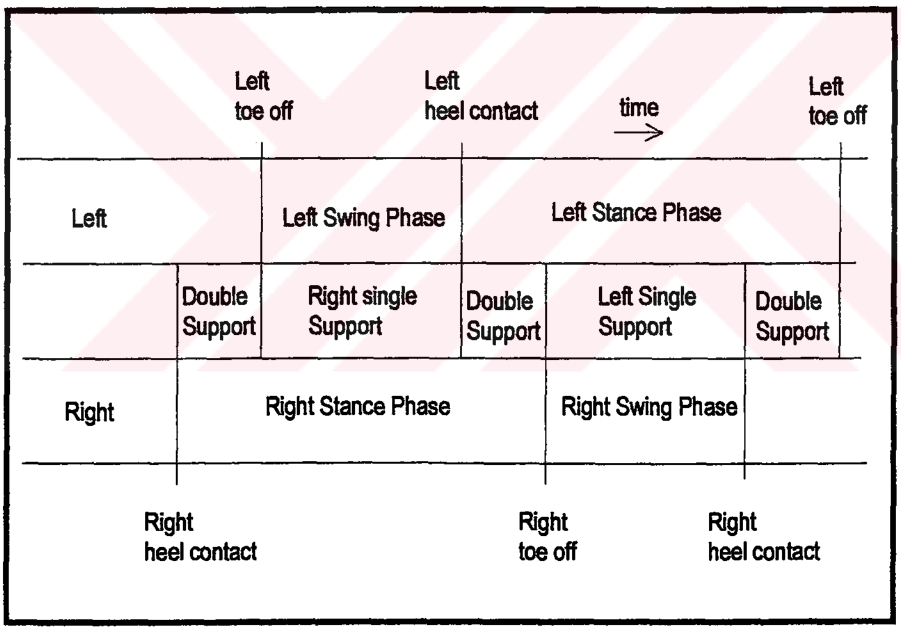

Figure 1.2 Timing of single and double support during a single gait cycle from right heel contact to right heel contact

In each gait cycle, there are thus two periods of double support and two periods of single support. The stance phase usually lasts for about 60 per cent of the cycle, the swing phase for about 40 per cent, and each period of double support for about 10 per cent. However this varies with the speed of walking, the swing phase becoming proportionally longer, and the stance phase and double support phases shorter, as the speed increases [1].

During the gait, the upper body moves forwards throughout the gait cycle. Its velocity varies a little, being fastest during the double support phases and slowest in the middle of the stance and swing phases. The trunk twists about a vertical axis, the pelvis and shoulder girdle rotating in opposite directions. The arms swing out of phase with the legs, so that as the left leg and the left side of the pelvis move forwards, so do the right arm and the right side of the shoulder girdle. The whole trunk rises and falls twice during the cycle, through a total range of about 50mm, being lowest during double support, and highest around mid stance or mid swing.

There are six optimizations used to minimize the excursions of the center of gravity of the body. These are called the ' determinants of gait'. The six determinants of gait are as follows [1,3]:

1. Pelvic rotation: The first determinant of the gait is the way in which the pelvis twists about a vertical axis during the gait cycle, bringing the hip joint forwards as the hip flexes, and backwards as it extends. This means that for a given stride length, less flexion and extension of the hip is required, since a proportion of the stride length comes from the forward and backward movement of the hip joint rather than the angular movement of the leg. The reduction in the range of hip flexion and extension leads to a reduction in the vertical movement of the trunk.

2. Pelvic tilt: The second determinant of gait is the way in which the pelvis tilts from side to side, so that when the hip of the stance phase leg is at its highest point, the pelvis is inclined, to lower the hip of the swing phase leg. The

height of the trunk depends not on the height of one or the other hip joint but on the average of the two of them, so the pelvic tilt reduces the total vertical excursion of the trunk.

3. Knee flexion in stance phase: The third determinant is the stance phase flexion of the knee. As the femur passes from flexion of the hip into extension, the hip joint would rise and then fall if the leg remained straight. However, flexion of the knee shortens the leg in the middle of this movement, reducing the height of the apex of the curve.

4. Ankle mechanism: Complementary to the way in which the apex of the curve is reduced by shortening the leg in the middle of the movement from hip flexion to extension, the beginning of the curve is elevated by lengthening the leg at the time of heel contact. This is achieved by the fourth determinant of gait- the ankle mechanism. Because the heel sticks out behind the ankle joint, it effectively lengthens the leg during the period between the heel contact and foot-flat.

5. Foot mechanism: In the same way that the heel lengthens the leg at the start of the stance phase, the forefoot lengthens it at the end, as the fifth determinant.

6. Lateral displacement of the body: The sixth determinant is concerned with side to side movement of the center of gravity. By keeping the walking base narrow, little lateral movement is needed to preserve balance during walking[1].

The global gait parameters used to describe the placement of feet on the ground are as follows. The stride length is the distance between two successive placements of the same foot. It consists of two step lengths, left and right, each of which is the distance by which the named foot moves forward in front of the other one. The walking base is the side to side distance between the line of two feet. The toe out is the angle in degrees between the direction of progression and a reference line on the sole of the foot. The cadence is the number of steps taken in a given time, the usual units being steps per minute. The velocity of walking is the distance covered by the whole body in a given time, in a particular direction.

There also some anatomical terms used to describe the body movements during gait. These are abduction, adduction, extension, flexion, dorsiflexion, and plantar flexion. Abduct is defined as to draw a limb away from a position near or parallel to the center line, abduction is the act of abducting a limb. Adduction is the act of drawing a limb toward the centerline of the body. Extension is the motion of a joint when it is being straightened. Flexion is the motion of a joint when it is being bent. Dorsiflexion is the flexion of a joint toward the back of the part (especially used in ankle joint). Plantar flexion is the flexion of the ankle in the direction of the sole of the foot [4].

1.3 Gait Analysis Methods

A complete gait analysis process requires measurement, description, analysis and assessment steps. Several measurement devices can do measurement of a data and the description of a data can have many forms such as pen recorder curves, plots of body coordinates or stick diagrams (Figure 1.3). In the analysis step, any mathematical operation can be performed on a set of data to present them in another form or combination of data from several sources can be done to produce a variable that is not directly measurable. From the analyzed data, information may be extracted to assist in the assessment stage [4].

In the measurement step, the direct measurement devices generally used are electrogoniometers, electromyography, accelerometers and force platforms. The three principle types of electrogoniometers (used for measuring the angle of a joint during walking) are based on the potentiometer, the flexible strain gauge, and the resistance of a tube filled with mercury. Electromyography (EMG) is the measurement of the electrical activity of a contracting muscle by using electrodes. The force platform, which is also known as force plate, is used to measure the ground reaction force as a subject walks across it.

The imaging measurement techniques generally used are television/computer systems, active marker systems and optoelectronic scanners. In the television/computer systems reflective markers are fixed to the subject's limbs, either close to the joint centers, or fixed to the limb segment in such a way so as to identify its position and orientation. In the active marker systems, light emitting diodes are used and the camera contains a device that measures the centroid of all the light falling on it. The optoelectronic scanners system is made up of very high quality prismatic reflectors and three scanners [1].

The analysis starts with smoothing of the raw data. The converted coordinate data from imaging measurement devices are called raw data. They contain additive noise from many sources: electronic noise from optoelectronic devices, spatial precision of the TV scan or film digitizing system, or human error in film digitizing. Smoothing and fitting of data can be made by curve-fitting techniques or digital filtering. From the smooth data, calculation of angles, velocity and acceleration can be made [4].

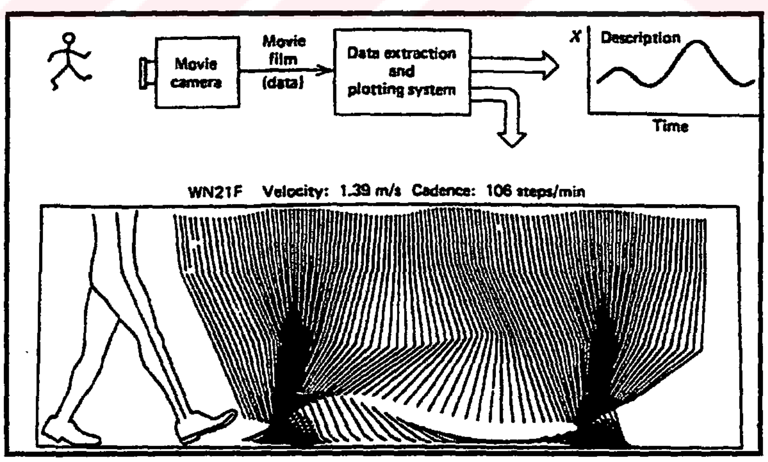

Figure 1.3 Flow of data from a camera system and plotting of data in two different forms

1.4 Previous Studies on Simulation of Gait

Modelling of the human gait problem has been the interests of researchers for many years. Regardless of the complexity of the models employed, the equations of motion are derived and the direct and inverse problem is then solved. The gait simulation is generally taken up in two separate parts, single support phase and double support phase.

In one of the studies, the problem of developing a model to describe and study human gait was undertaken using Lagrangian mechanics [5]. The approach is that of the initial value problem, ie. starting with the initial conditions which are the limb angles and velocities, and the system inputs which are the applied joint moments, the system response is found. Seven segments were used to model the human body with complete three segment lower limbs, and the head and trunk as one segment. There were six joints, two each at the hips, knees, and ankles. Due to the differences between the mathematical model and the human body, the calculated joint moments were not exactly what are required for the link segment model to progress through a desirable walking cycle. So the joint moments were adjusted manually to attain a desirable gait[5].

Amirouche, Ider and Trimble modelled the gait with a support phase and a swing phase. Without the double support phase, the ankle joint of the stance leg was taken as stationary during the swing, the knee joint of the support leg taken as stiff (means the knee angle does not change) and the upper body was taken as vertical through the analysis. Athough the model is presented for gait simulation, its application was limited to an inverse dynamics problem [6].

Various studies about the single support phase have been presented. The effects of swing leg dynamics, stance knee flexion-extension, foot and knee interactions on the gait are closely observed. The high unstability of the system is clearly seen from the results obtained by changing any of the gait parameters [7]. Berme and Pandy simulated the single support phase, by integrating the equations of motion for link angular velocities and displacements, assuming the input moments are known [8]. Also predictions on the optimum trajectory of the swing limb have been done. Chou, Song and Draganich [9] developed an algorithm to predict the minimum energy consumption trajectory of the swing limb. A multistage optimization method was applied to generate the optimum trajectory of the swing ankle which minimized the mechanical energy required to generate the moments of the joints of the lower extremities during the single support phase of gait. The results of this study supported the hypothesis that human gait is energy efficient [9]. Another swing phase study is on the linear approximations for swing leg motion during gait, in which the time-variant and time-invariant approximations are compared [10].

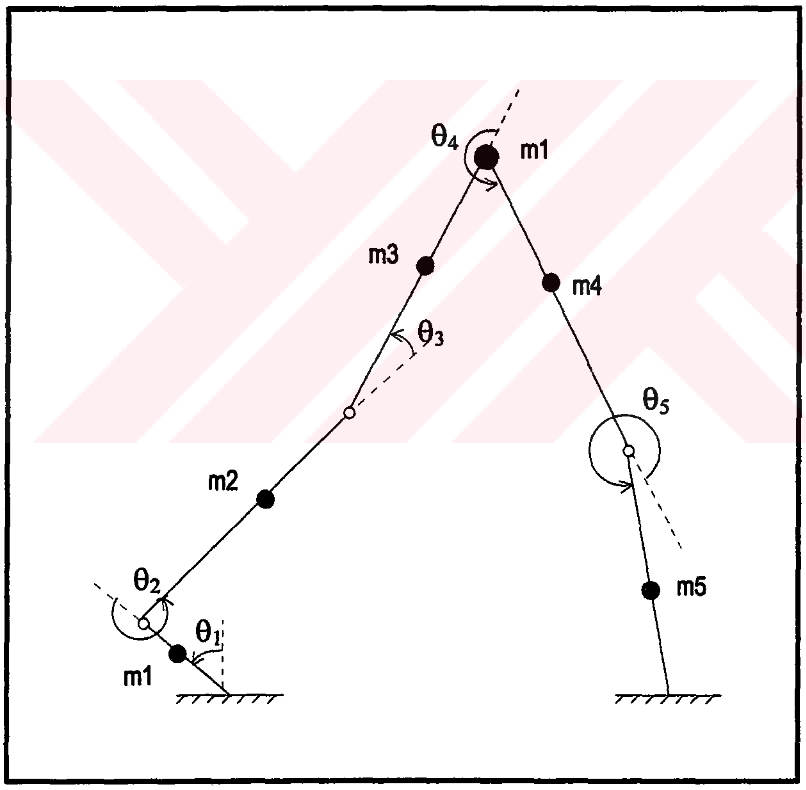

Figure 1.4 Schematic representation of the model for double support [8]

The double support phase simulations are not as clear as the single support phase simulations because of the redundancy in the system. Berme and Pandy used a five link planar model for double support as shown in Figure 1.4 [8].

Using the kinematic constraint equations and applied moments, the coordinates and ground reaction forces were found. The applied joint moments were chosen on the bases of trial and error. The input moments for the single and double support phases are as in Table 1.1 [8].

Table 1.1 Input joint moments applied to the planar models for single and double support [8]

Model	Joint No.	Time, t(s)	Moment (Nm)
Stance knee flexion-extension	1	0< t < 0,27	241.t
	4	0< t < 0,27	900.t
	5	0< t < 0,27	-100.t
Foot and knee interaction	1	-	0,0
	2	0,27< t < 0,43	200.(t-0,27)+65,0
	3	0,27< t < 0,43	14,0
	4		0,0
	5	0,38< t < 0,43	-5,0
Double Support	1	-	0,0
	2	0,43< t < 0,58	-500.(t-0,43)+80,0
	3	0,43< t < 0,58	20,0
	4	-	0,0
	5	0,43< t < 0,58	-1,0

Ju and Mansour used a nonlinear feedback controller in their simulation, to correct abnormal limb trajectories [11]. Given a set of desired trajectories for the independent angular accelerations, velocities and displacements, the nonlinear feedback controller was used to generate a set of joint moments that would produce a gait pattern that was close to the desired trajectories. The desired trajectories were specified from experimentally measured kinematic data. The two dimensional model was made up of seven body links representing the combined head, arms, and torso, and the left and right thighs, shanks and feet (Figure 1.5). Singular value decomposition method was used to overcome the redundancy problem [11].

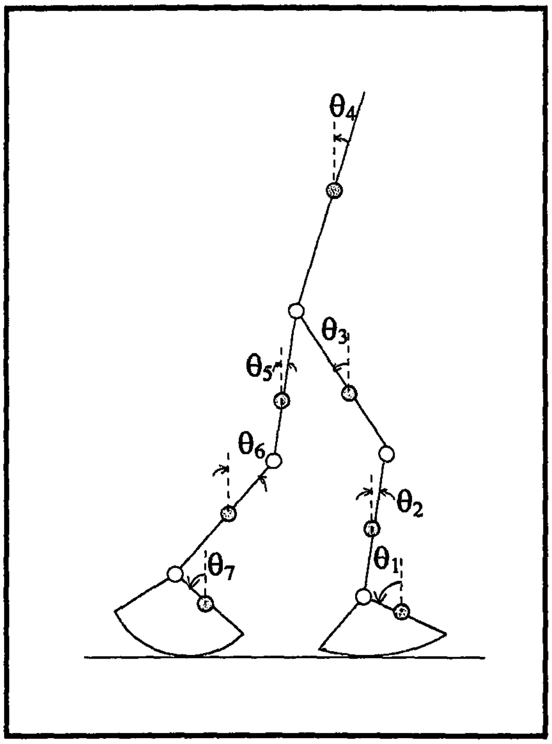

Figure 1.5 Configuration of the two dimensional model, using feet with curved surfaces [11]

Planar models mostly use three-dimensional experimental data. The investigation of the movements of skin markers located on the pelvis shows that projections can be misleading. Even small rotations about the vertical axes can appear as horizontal translations in a planar projection. Two-dimensional representation of three-dimensional pelvic motion may lead to wrong results especially if the simulations use desired trajectories [12].

Gait simulation studies become extremely important when the design and control of a lower limb prosthesis is of concern. Prosthetic studies aim exact simulation of human gait by using different approaches. Passive prostheses use spring-damper systems, pneumatic dampers or polycentric mechanisms as knee joint, nonlinear springs (SACH- solid ankle cushioned heel) as heel [13, 14, 15, 16]. Changes in the alignment of knee and ankle joints are made to perfectly duplicate human gait [17]. Active prostheses use different control methods for duplication of gait. Some examples of control mechanisms are adaptive control and artificial reflex control [18, 19, 20]. The analysis of moments and forces applied to the joints are also important for both the prosthetic studies and some pathology studies [21, 22].

1.5 Purpose and Scope of the Present Study

The aim of the present study is the simulation of gait including both the single and double support phases of the swing and the stance leg, i.e the complete gait cycle. Feedback control is used in the form of computed torque control as the human body also uses a kind of feedback control to stabilize its position during the gait.

Chapter 2 presents the gait models. Three separate planar models are used for the complete cycle, two models for single support phase and one for the double support phase. Specification of the desired trajectories that are used in the control algorithm is also presented in this chapter.

In chapter 3, the dynamic equations and control with computed torque method is presented. The Lagrangian formulation of single and double support models are given separately. The weighted least squares optimization method used for the double support phase is presented in detail. Because of the redundancy problem in the double support phase, a special technique is used to solve for the moments. Properly selected weighting constants are used for the calculation of the moments, which is a technique different from those used in the previous studies.

In chapter 4, the inverse dynamics results and gait control results under disturbing effects are given. Discussion of the sources of errors and evaluation of the results is presented.

In chapter 5 the conclusion of this study is given.

CHAPTER 2

MODELLING OF HUMAN GAIT

In this chapter the physical modelling of human gait is presented. Three different linkage systems are proposed representing different phases of gait. The following assumptions are done for the simplicity of the models:

i) The models are two dimensional.
ii) The swinging foot is rigidly connected to the shank.
iii) HAT (head arms and trunk) is not rotating throughout the gait.
iv) The stance leg is pivoted to the ground from its ankle joint, toe and heel during different periods of gait.

2.1 Single Support Phase

The first phase of the gait cycle is the single support phase, which begins at the instant of toe-off and ends at the instant of heel strike of the contralateral leg. In an attempt to model the single support phase, one step is considered in which the left leg is in swing phase while the right leg is in contact with ground. Gait cycle is represented by a similar single support phase in which the roles of the two legs interchange together with two double support phases.

Two models are proposed for the single support phase corresponding to two stages. The first stage of single support covers almost 60 per cent of the swing phase of the swing leg and the foot-flat period of the stance leg. The second stage covers the period subsequent to the heel-off of the stance leg while the other leg continues swinging. The gait pattern is described by prescribing the hip joint and the ankle joint trajectories. In what follows, the models for the two stages of the single support phase are described.

2.1.1 Stance Leg Foot-flat Period (Model 1)

The stance leg foot-flat period of the single support phase is modelled by introducing some assumptions. The stance leg (right leg) ankle is taken as connected to the ground by a revolute joint and the swing leg (left leg) foot is taken as connected to the shank rigidly. The degree of freedom of this model is four, and schematically represented as in Figure 2.1. The two hip angles (θ_{hl} , θ_{hr}) and the two knee angles (θ_{kl} , θ_{kr}) are chosen as the generalized coordinates.

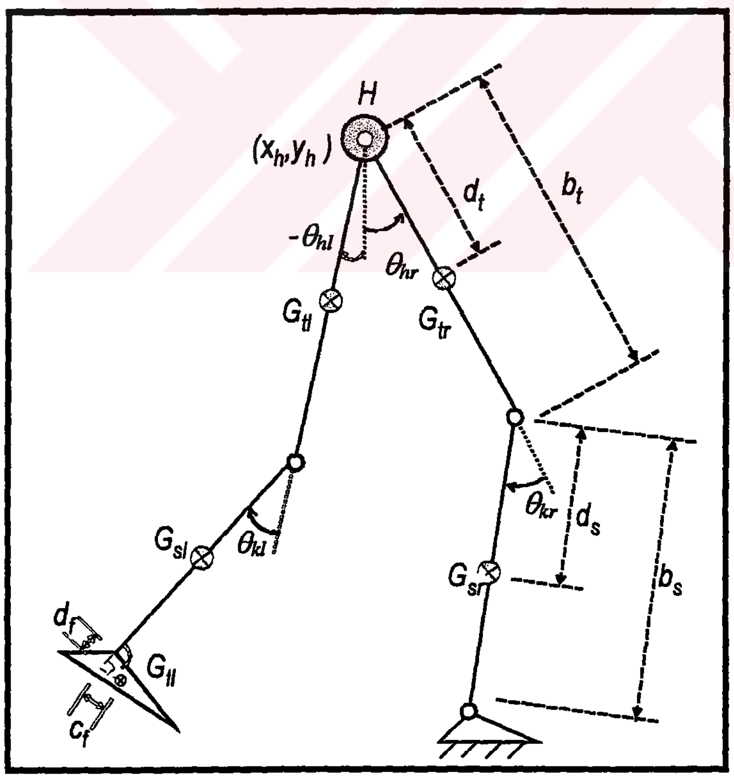

Figure 2.1 Model 1: The stance leg foot-flat period of single support phase

The hip joint angle is the flexion angle of the thigh and it is measured from the vertical. The knee joint angle is the flexion angle of the shank and is measured from the extension of the thigh.

The limb lengths for each leg are defined such that; b_t is the length of thigh, b_s is the length of shank, d_t is the distance of mass center of thigh from the proximal end, and d_s is the distance of mass center of shank from the proximal end. For the swinging leg, d_f and c_f define the position of mass center of the foot. They are

perpendicular to each other such that; d_f is the vertical and c_f is the horizontal length from the ankle to the mass center of the foot.

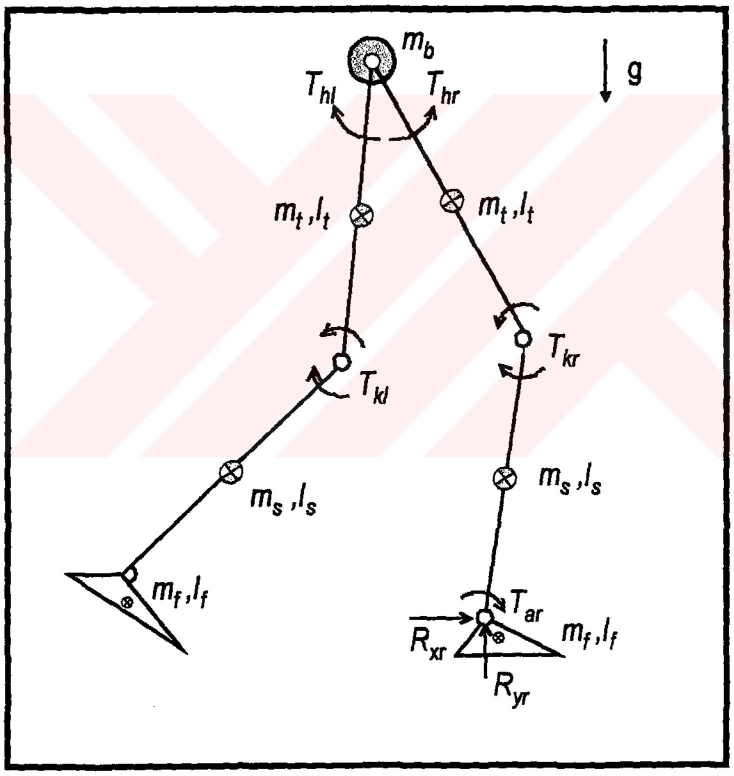

Figure 2.2 Forces and moments acting on model 1

Inertial properties of each body segment are represented by their mass and centroidal mass moment of inertia (m_s, I_s, m_t, I_t, m_f, I_f). The mass of HAT (head, arms and trunk) is lumped at the hip joint. The model is driven by four moments, as shown in Figure 2.2. Flexion moments (T_{kr}, T_{kl}) are exerted on the knee joints and dorsiflexion moment (T_{ar}) is exerted on the ankle joint. The moment exerted on the hip joint is taken as equal in magnitude and opposite in direction on each leg (T_{hr} and T_{hl}), because HAT is assumed to be not rotating throughout the gait. Hence on the stance leg, flexion moment is taken as positive whereas on the swing leg, extension moment is taken as positive. This assumption is verified to an appreciable extent, in view of the hip joint moments obtained from the gait analysis of a normal subject, as shown in Figure 2.3. The ankle joint reactions R_{xr} and R_{yr} correspond to the ground reaction forces of the stance leg.

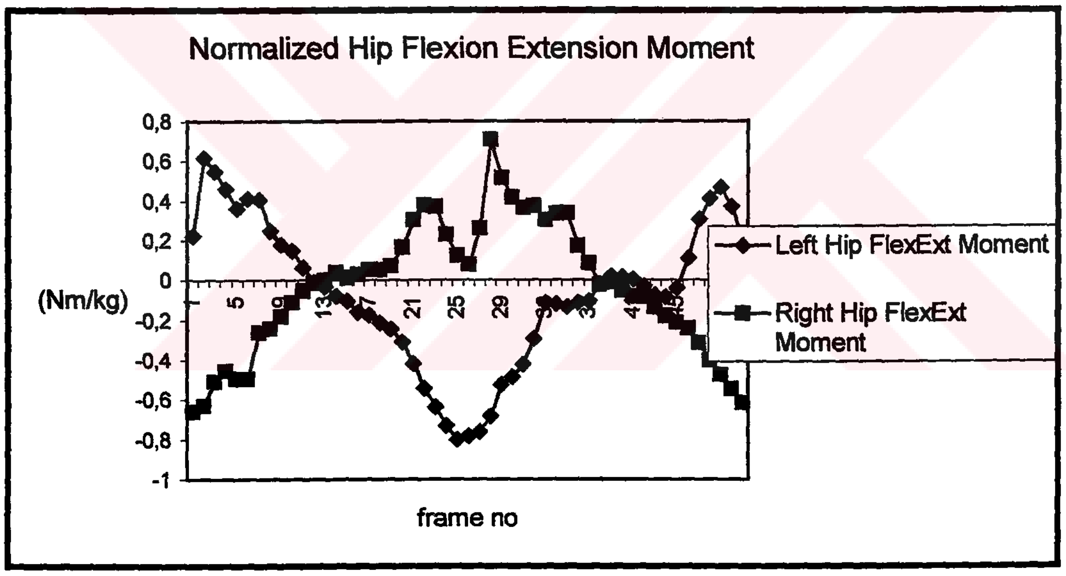

Figure 2.3 Comparison of experimental left and right hip flexion extension moments[23]

2.1.2 Stance Leg Heel-off Period (Model 2)

Stance leg heel-off period occupies almost 40 per cent of the single support phase. The stance leg foot begins rotating around its toe and the other leg continues its swing phase. In this period the swing leg can still be modelled as in the stance leg foot-flat period but the stance leg model changes. As seen in Figure 2.4 the stance foot is connected to the ground by a revolute joint from its toe. This brings the system an additional degree of freedom and makes it five.

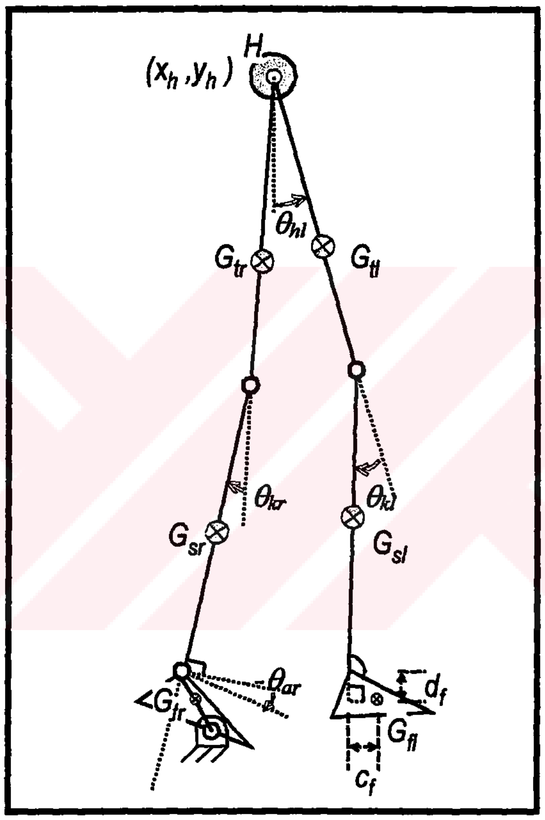

Figure 2.4 Model 2: The stance leg heel-off period of single support phase

The hip joint angles and knee joint angles are defined as the same as in stance leg foot-flat period. In addition to these angles the stance ankle angle is chosen as the fifth generalized coordinate. The ankle joint angle θ_{ar} is measured from the perpendicular to the extension of shank, to a line parallel to the sole of foot. Dorsiflexion direction is taken as positive (Figure 2.5).

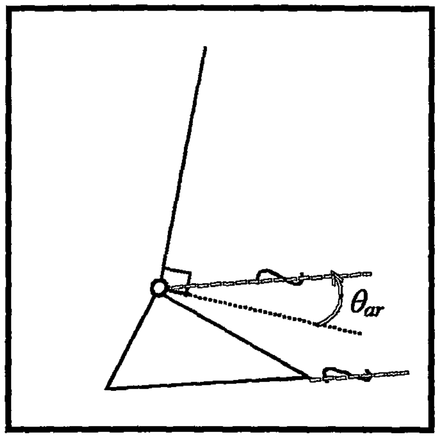

Figure 2.5 Detailed model of ankle angle

The detailed model of the foot is shown in Figure 2.6. The length of the link between the ankle and toe of the right leg is b_{fr}. The mass centre of the foot is at the midpoint of this link. The angle between the base of the foot and the link is α_{fr}. This angle remains constant throughout the gait as it is only related with the geometry of the foot.

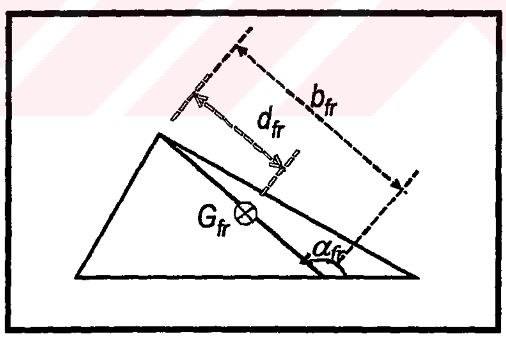

Figure 2.6 The detailed model of the right foot

Moments and external forces exerted on the stance leg heel-off period model are as shown in Figure 2.7.

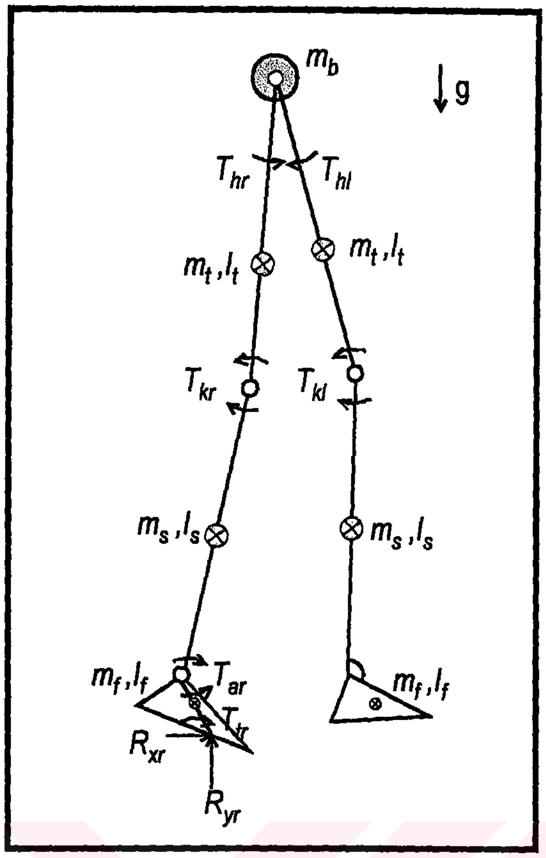

Figure 2.7 Forces and moments acting on model 2

The direction of the moments for this period is exactly the same as the stance leg foot-flat period. Flexion moments are exerted on the knee joints, dorsiflexion moment on the ankle joint, and moments of same magnitude but opposite directions on the swing and stance legs from the hip joint. Ground reaction forces R_{xr} and R_{yr} act on the distal end of the ankle-toe link. Other than these, an additional moment acts on the foot from its toe. This moment is T_{tr} and positive direction is chosen as clockwise.

2.2 Double Support Phase (Model 3)

This phase begins with the heel strike of the swing leg. During double support, one leg rotates around its heel while the other continues rotating around its toe. In a gait cycle (starting with toe off and ending with the second toe off of the same leg), the double support phase occurs twice, with the right and left foot exchanging their roles and each occupy almost 10 % of the cycle. The model of this phase is shown in Figure 2.8.

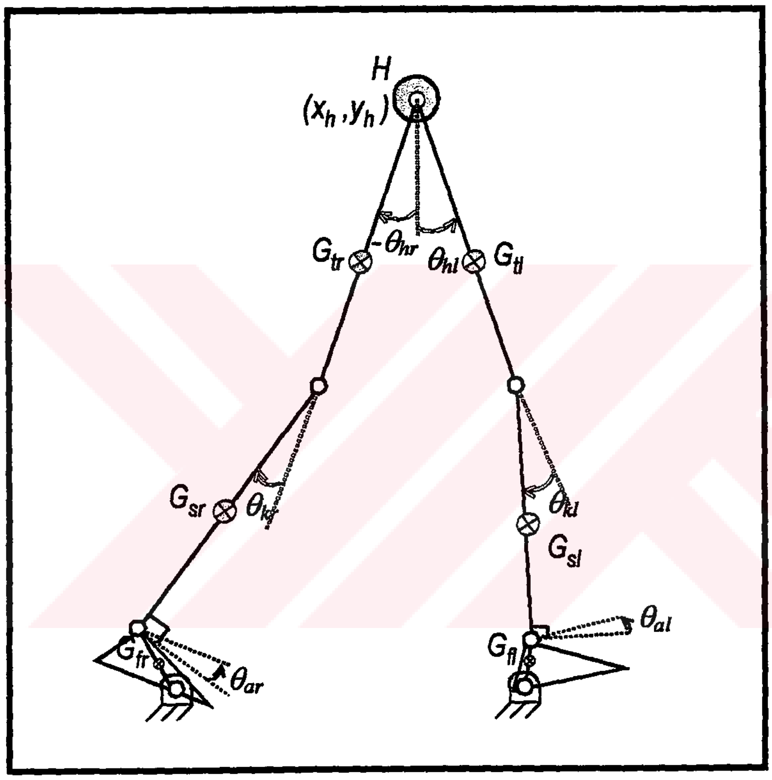

Figure 2.8 Model 3: The double support phase

In this model both feet are connected to the ground by revolute joints. One foot is connected from its toe while the other is connected from its heel. Unlike the single support phase, the left ankle joint now has a freedom The left ankle joint angle (θ_{al}) is measured in the dorsiflexion direction. The degree of freedom of the double support phase model is four.

The detailed model of the left foot is shown in Figure 2.9. The length of the link between the ankle and heel of the left leg is b_{fl}. The mass center is at the midpoint of this link. The angle between the base of the foot and the link is α_{fl}. As in the previous phase, this angle remains constant throughout the phase as it is only related with the geometry of the foot.

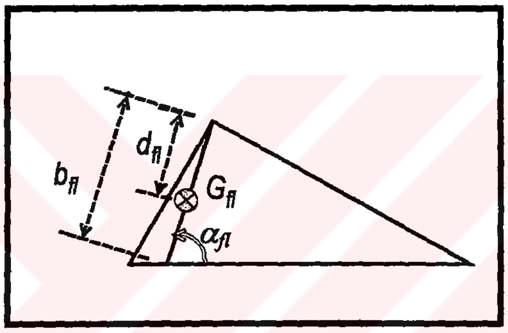

Figure 2.9 The detailed model of the left foot

The moments and forces acting on the double support model are shown in Figure 2.10.

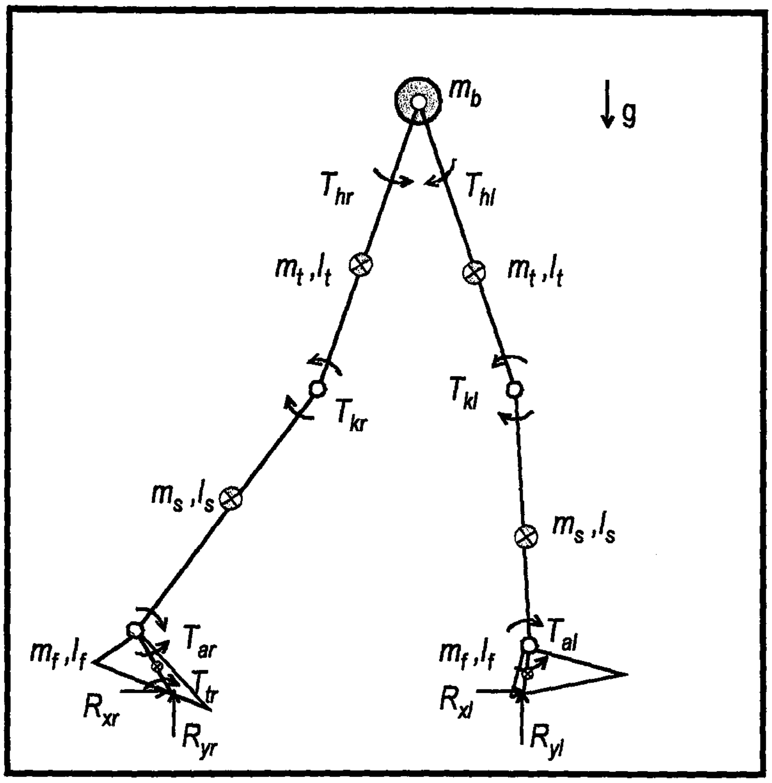

Figure 2.10 Forces and moments acting on model 3

During the double support phase, knee flexion moments (T_{kl} and T_{kr}), ankle dorsi-flexion moments (T_{al} and T_{ar}) and flexion and extension moments (T_{hl} and T_{hr}) continues acting on the knee, ankle and hip joints, respectively. Also, on the right toe, the clockwise moment T_{tr} acts. Unlike the other models, ground reaction forces act on both the left heel (R_{xl} and R_{yl}) and on the right toe (R_{xr} and R_{yr}).

As seen in Figure 2.10, there are six moments acting on the four degree of freedom system This redundancy problem will be handled in Chapter 3, by introducing a specially developed procedure for this purpose.

2.3 Kinematic Constraints

The normal gait pattern is described by the hip joint trajectory for the complete gait cycle, and by the ankle joint trajectory of the swing leg. Knowing the x and y coordinates of the hip and ankle joints at any instant of time, the corresponding hip and knee angles can be calculated by utilizing the kinematic constraint equations. With reference to Figure 2.11, the following loop closure equations are written:

$$x_h = x_a - b_s \sin(\theta_h - \theta_k) - b_t \sin(\theta_h) \quad (2.1)$$

$$y_h = y_a + b_s \cos(\theta_h - \theta_k) + b_t \cos(\theta_h) \quad (2.2)$$

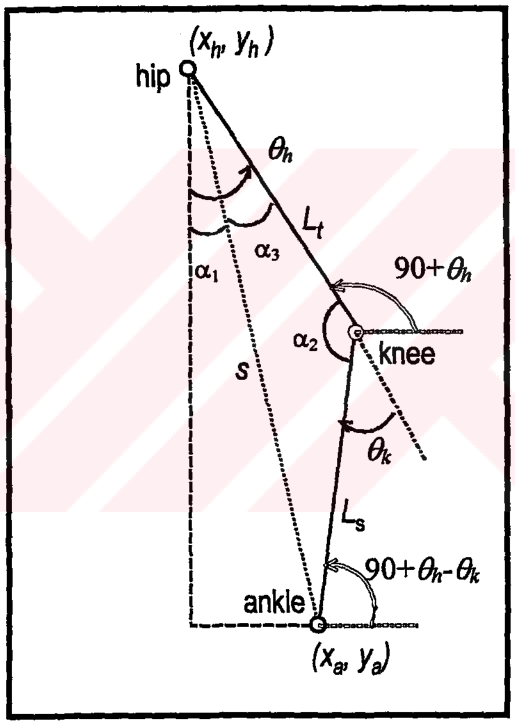

Figure 2.11 Hip-knee-ankle triangle

The following algorithm yields closed form solution of nonlinear equations (1) and (2) for θ_h and θ_k:

when $x_h < x_a$ or $x_h = x_a$:

$\alpha_1 = \arctan[(x_a - x_h) / (y_h - y_a)]$

$s = [(y_h - y_a)^2 + (x_a - x_h)^2]^{0.5}$

$\alpha_2 = \arccos[(b_t^2 + b_s^2 - s^2) / (2\ b_t\ b_s)]$

$\theta_k = 180 - \alpha_2$

$\alpha_3 = \arcsin[b_s\ \sin(\alpha_2) / s]$

$\theta_h = \alpha_1 + \alpha_3$

when $x_h > x_a$:

$\alpha_1 = \arctan[-(x_a - x_h) / (y_h - y_a)]$

$s = [(y_h - y_a)^2 + (-x_a + x_h)^2]^{0.5}$

$\alpha_2 = \arccos[(b_t^2 + b_s^2 - s^2) / (2\ b_t\ b_s)]$

$\theta_k = 180 - \alpha_2$

$\alpha_3 = \arcsin[b_s\ \sin(\alpha_2) / s]$

$\theta_h = -(\alpha_1 - \alpha_3)$

Differentiating equations (2.1) and (2.2) twice yields the velocity and acceleration constraint equations which are linear in terms of hip and knee angular velocities and accelerations.

$$\begin{bmatrix} \dot{\theta}_h \\ \dot{\theta}_k \end{bmatrix} = \begin{bmatrix} -b_s \cos(\theta_h - \theta_k) - b_t \cos(\theta_h) & b_s \cos(\theta_h - \theta_k) \\ -b_s \sin(\theta_h - \theta_k) - b_t \sin(\theta_h) & b_s \sin(\theta_h - \theta_k) \end{bmatrix}^{-1} \begin{bmatrix} \dot{x}_h - \dot{x}_a \\ \dot{y}_h - \dot{y}_a \end{bmatrix} \qquad (2.3)$$

and

$$\begin{bmatrix} \ddot{\theta}_h \\ \ddot{\theta}_k \end{bmatrix} = \begin{bmatrix} -b_s\cos(\theta_h-\theta_k)-b_t\cos(\theta_h) & b_t\cos(\theta_h) \\ -b_s\sin(\theta_h-\theta_k)-b_t\sin(\theta_h) & b_t\sin(\theta_h) \end{bmatrix}^{-1}$$

$$\cdot \begin{bmatrix} \ddot{x}_h-\ddot{x}_a-b_s\sin(\dot{\theta}_h-\dot{\theta}_k)^2-b_t\sin(\theta_h)\dot{\theta}_h{}^2 \\ \ddot{y}_h-\ddot{y}_a+b_s\cos(\dot{\theta}_h-\dot{\theta}_k)^2+b_t\cos(\theta_h)\dot{\theta}_h{}^2 \end{bmatrix} \tag{2.4}$$

The above equations yield the hip and knee angles as well as their first and second derivatives for the swing leg for which both hip and ankle trajectories are specified.

As mentioned in Chapter 1, desired gait pattern is indicated by a smooth hip joint trajectory and foot clearance off the ground during swing phase. The control strategy adapted in this study aims at satisfying these requirements, and therefore a desired hip trajectory for the complete gait cycle and a desired ankle joint trajectory for the swing leg are imposed.

A desired hip joint trajectory is obtained by fitting a 10^{th} order and a 4^{th} order polynomial time function to the experimentally obtained hip joint y and x-coordinates during a normal gait, respectively. The data used was taken from a GCD (Gait Cycle Data) output data file of the VICON Clinical Manager [23]. The y-coordinate equation is derived by using a curve fit program "Jandel Scientific- Table Curve", which uses optimization techniques and selects the most appropriate equation among 8100 equation types. The x-coordinate equation is derived by using a nonlinear regression analysis program "Sherrod Software- NLREG". A higher order polynomial function is not needed for the x-coordinate as the movement along the x-coordinate is almost a linear function of time. As a result, the following functions are obtained. Both coordinates are expressed in meters.

$$x_h = 0.512 + 1.634\ t - 2.864\ t^2 + 4.963\ t^3 - 2.691\ t^4 \tag{2.5}$$

$$y_h = 0.875 - 0.070\ t + 2.044\ t^2 - 12.672\ t^3 + 49.070\ t^4 - 97.253\ t^5 + 51.117\ t^6 + 69.554\ t^7 - 36.687\ t^8 - 77.433\ t^9 + 51.981\ t^{10} \tag{2.6}$$

Hip joint velocity and acceleration expressions are readily obtained by taking the first and second derivatives of these equations.

A desired ankle joint trajectory of the swing leg is obtained similarly. While obtaining the swing ankle joint equations, foot clearance is an important constraint. A 10^{th} order polynomial time function for the x-direction and a 4^{th} order polynomial time function for the y-direction are obtained (in meters) by using the same curve fitting programs;

$$x_{al} = 0.194 + 0.353\ t - 1.139\ t^2 + 76.676\ t^3 - 410.295\ t^4 + 943.255\ t^5$$
$$- 659.986\ t^6 - 666.734\ t^7 + 587.180\ t^8 + 908.954\ t^9 - 804.316\ t^{10} \qquad (2.7)$$

$$y_{al} = -0.472 + 7.892\ t - 28.961\ t^2 + 40.887\ t^3 - 19.994\ t^4 \qquad (2.8)$$

The equations (2.1-2.4) can also be used for the stance leg with the following modifications:

i) Stance leg pivoting around ankle (Model 1):

In this case the right ankle is modelled as connected to the ground by a revolute joint. Hence the x and y coordinates of the right ankle are constant throughout the single support foot-flat period.

$$x_{ar}=x_{0r}$$
$$y_{ar}=y_{0r}$$

where x_{0r} and y_{0r} are determined by taking the average of the experimental ankle joint coordinates through the foot-flat phase

ii) Stance leg pivoting around toe (Model 2-Model 3):

In this case, the foot is connected to the ground from its toe by a revolute joint so the ankle joint is following a circular path of radius b_{fr}. To obtain the trajectory of the ankle joint , first a time function polynomial of the x-coordinate is obtained (in meters). "Sherrod Software-NLREG" program is used for this purpose. A special polynomial having the first and second derivatives zero at the beginning of the heel-off period is used, as during the foot-flat period the velocity and acceleration of the ankle joint is zero.

$$x_{ar} = x_{0r} + 3.831\ (t - 0.401)^{3} - 17.127\ (t - 0.401)^{4} + 44.817\ (t - 0.401)^{5} \quad (2.9)$$

The ankle joint y-coordinate equation is a function of x_{ar}. By this way a circular path of radius b_{fr} is obtained. The link length b_{fr} is adjusted by comparing the experimental y-coordinate data with the function y_{ar} and considering the ankle position in foot-flat period.

$$y_{ar} = (b_{fr}^{\ 2} - (b_{fr} \cdot \cos(\phi) - (x_{ar} - x_{0r}))^{2})^{0.5} \quad (2.10)$$

where $\phi = 180 - \alpha_{fr}$, which is the angle between the ground and the ankle-toe link at the beginning of the heel-off period

The velocity and acceleration equations of the y-coordinate are obtained by taking the first and second derivatives of the y-coordinate equation.

iii) Stance leg pivoting around heel (Model 3):

In the double support phase, the left leg completes its swing phase and starts rotating around its heel. The left foot is connected to the ground by a revolute joint, hence the left ankle joint follows a circular trajectory of radius b_{fl}.

The 10^{th} order polynomial time function used for the x-coordinate of swing leg can still be used, as the fit was made by using the whole experimental data from the beginning of the left leg swing till the end of the double support. The first and second derivatives of this function define the x-direction velocity and acceleration. The y-coordinate equation is a function of x_{al}.

$$y_{al} = (b_{fl}{}^{2} - (x_{al} - x_{0l})^{2})^{0.5} \tag{2.13}$$

where $x_{0l} = -b_{fl}\cos(\alpha_{fl}+\varphi)+x_{ael}$

$\varphi = \sin^{-1}(y_{ael}/b_{fl})$

x_{0l} is the x-coordinate of the connection point of the ankle-heel link to the ground. x_{ael} and y_{ael} are the x and y-coordinates of the left ankle at the end of swing phase respectively and φ is the angle between the ground and the base of foot at the instant of heel-contact.

The link length b_{fl} is adjusted such that the function y_{al} is close to the experimental trajectory. The first and second derivatives of equation 2.13 give the velocity and acceleration functions of the left ankle joint movement in y-coordinate,respectively.

For each model, suitable combinations of the above equations are used for the right and the left leg. Once the required equations are defined, the angles θ_{hr}, θ_{hl}, θ_{kr}, θ_{kl} can be calculated as shown before, in Figure 2.11. The ankle angles are calculated by using the following equations (Figure 2.12).

$$x_a - x_g - b_f\cos(\theta_h - \theta_k + \theta_a + \alpha_f) = 0 \tag{2.14}$$

$$y_a - b_f\sin(\theta_h - \theta_k + \theta_a + \alpha_f) = 0 \tag{2.15}$$

where $x_g = x_{0l}$ for the left leg and $x_g = x_{0r} + b_{fr}\cos(\phi)$ for the right leg

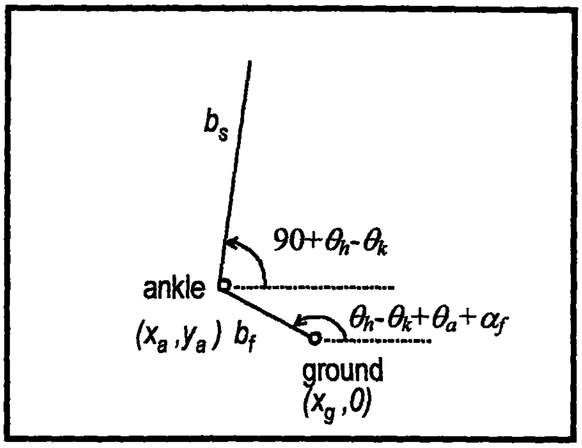

Figure 2.12 Ankle joint

2.4 Model Parameters

During the modelling of human gait, once the physical models and their kinematic constraints are obtained, the model parameters are defined. The masses and centroidal inertias of the links are defined by using the related anthropometric equations [4]. Table 2.1 shows the anthropometric data for a typical human body.

Table 2.1 Anthropometric data for a typical human body

	foot	shank	thigh
Segment weight / Total body weight	0,0145	0,0465	0,1
Distance of center of mass from distal end / Segment length	0,5	0,567	0,347
Inertia (kg.m^2)	m_f (0,475 b_f)	m_s (0,475 b_s)	m_t (0,475 b_t)

Taking the subject's mass as 80 kg[23], the segment masses are calculated as follows:

m_f= 1.16 kg

m_s = 3.72 kg

m_t = 8 kg

$m_b = 80 - 2\,(m_f - m_s - m_b) = 54.24$ kg

In order to determine the mass center distances and the mass moment of inertias, first the limb lengths are to be determined. Determining the limb lengths is a special and difficult process. The anthropometric equations can not be used because of the kinematic constraints of the model. During the human gait the heel and toe of the feet are not actually connected to the ground, the limb lengths calculated with the anthropometric equations may not satisfy the model's constraint equations. So in order to make a complete simulation, longer links are used in order to follow the trajectory and avoid the break off at the knee joint.

The link lengths are determined by fine tuning the anthropometric data so as to satisfy the kinematic constraints. The approach used during tuning was finding the closest link lengths to the anthropometric equations, while avoiding the break off and matching the experimental and modelled joint trajectories. For example, using a thigh length of b_t=0.43 m instead of 0.438 m (a decrease of 8 mm) would make the modelled and experimental hip joint and knee joint angles closer in the single support phase but at the beginning of the double support phase, this would cause a break off at the left knee joint. When t = 0.590 s., the hip joint coordinates are x_h =1.172 m y_h =0.879 m and the left ankle joint coordinates are x_{al} =1.426 m y_{al} =0.078 m Calculating s (Figure 2.11) at this point gives s =0.8403 m, but b_t+b_s=0.84 m, so a break off at the knee joint occurs. Avoiding this break off by increasing the link lengths takes the modelled angles considerably away from the experimental values.

The link lengths obtained via tuning are then used together with the anthropometric equations [4] to yield the mass center positions and mass moment of inertias of the limbs. The location of the pivots (x_{0r}, y_{0r}) are determined by matching the coordinate system of the model with the one used in [23]. The following values are thus obtained as the remaining model parameters:

$b_t = 0.438$ m $\quad b_s = 0.41$ m

$d_t = 0.789$ m $\quad d_s = 0.177$ m

$b_{fr} = 0.160$ m $\quad b_{fl} = 0.078$ m

$c_f = 0.067$ m $\quad d_f = 0.035$ m

$\alpha_{fl} = 70$ deg. $\quad \alpha_{fr} = 150$ deg.

$I_t = 0.160$ kg.m^2 $\quad I_s = 0.057$ kg.m^2

$I_f = 0.019$ kg.m^2

$x_{0r} = 0.85$ m $\quad y_{0r} = 0.08$ m

$x_{ael} = 1.426$ m $\quad y_{ael} = 0.078$ m

After determining the model parameters, the joint angles θ_{hr}, θ_{kr}, θ_{hl}, θ_{kl} can be calculated. The Figures 2.13-2.24 clearly demonstrate the limitations of the present model. The differences between the measured and the modelled joint angles are primarily due to the two-dimensional nature of the model. As discussed in Chapter 1, there are six optimizations used to smooth the hip joint trajectory and three of them are related to out-of plane motions. The measured angles are, therefore, projections of the three dimensional motion. The present model tries to give the same trajectory with only sagittal plane motions and this causes the joint angles to deviate from the measured values. Another reason for the apparent discrepancy is due to the rigid foot pivoted on the ground. In reality, foot-ground interaction is a contact between ground and deformable foot of quite irregular shape. Instead of pivoting the foot around the heel, rolling cylinder constraint was also tried but this too did not bring any improvement. The Figures 2.13-2.16 show the experimental and modelled angles for the single support phase foot-flat period (model 1). The time interval for this period is as given below:

$0.189\,\text{s.} \leq t < 0.401\,\text{s.}$

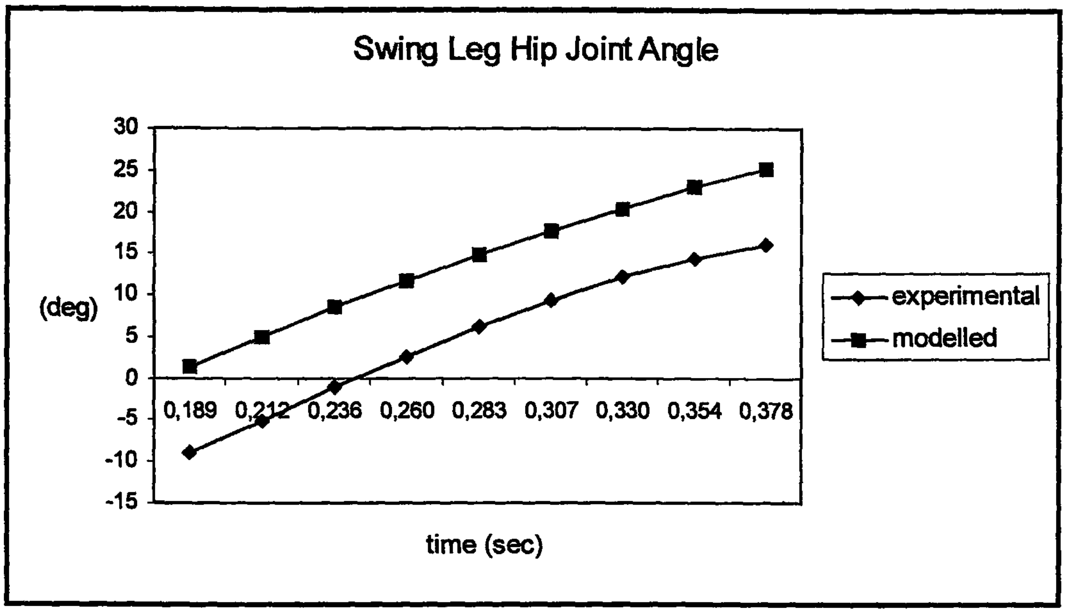

Figure 2.13 Swing Leg Hip Joint Angle in Foot-flat Period

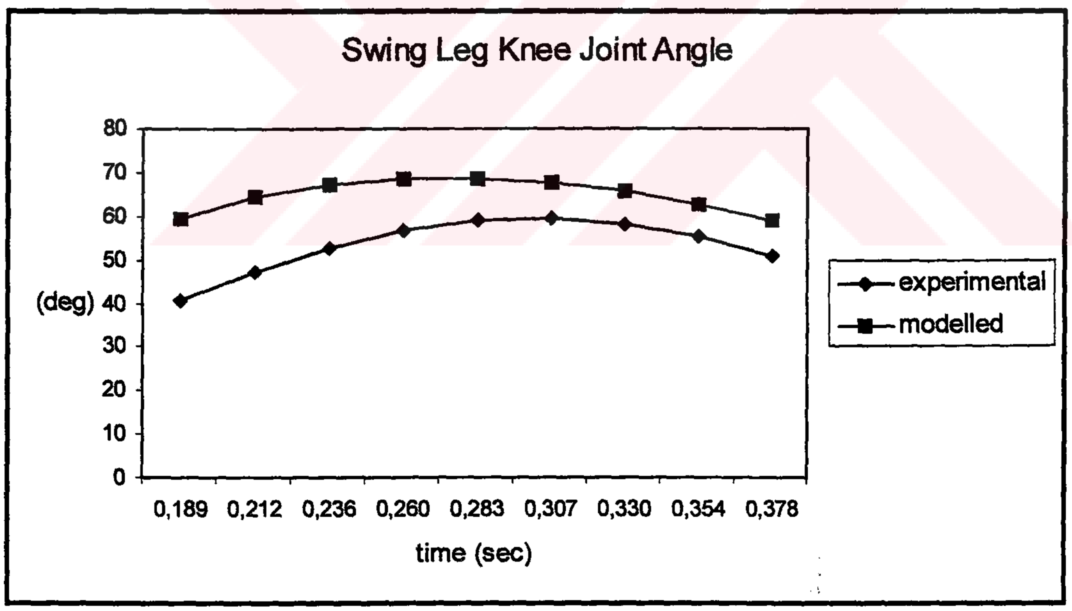

Figure 2.14 Swing Leg Knee Joint Angle in Foot-flat Period

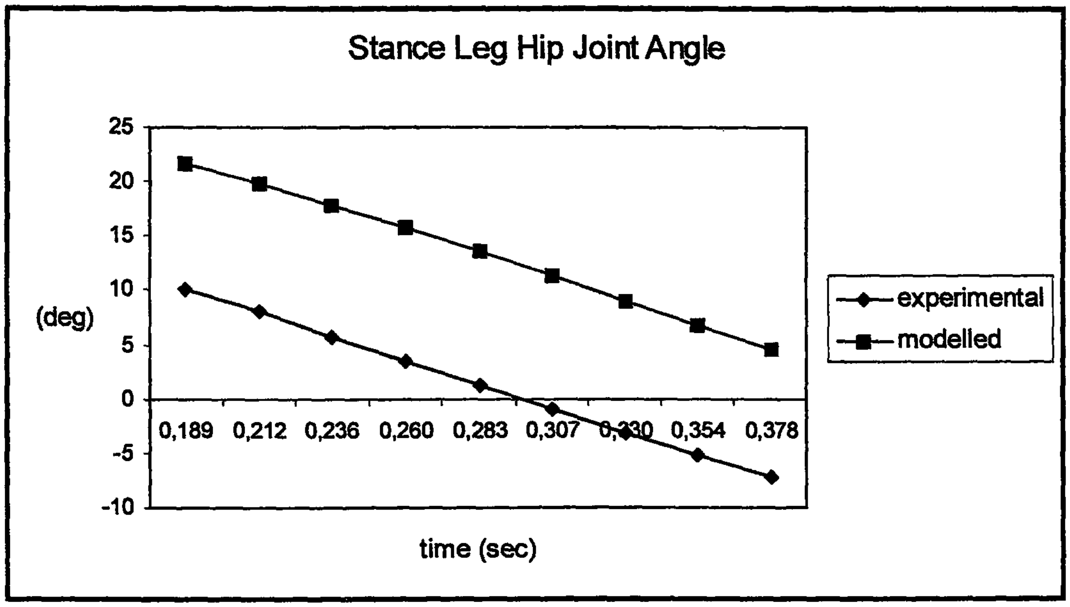

Figure 2.15 Stance Leg Hip Joint Angle in Foot-flat Period

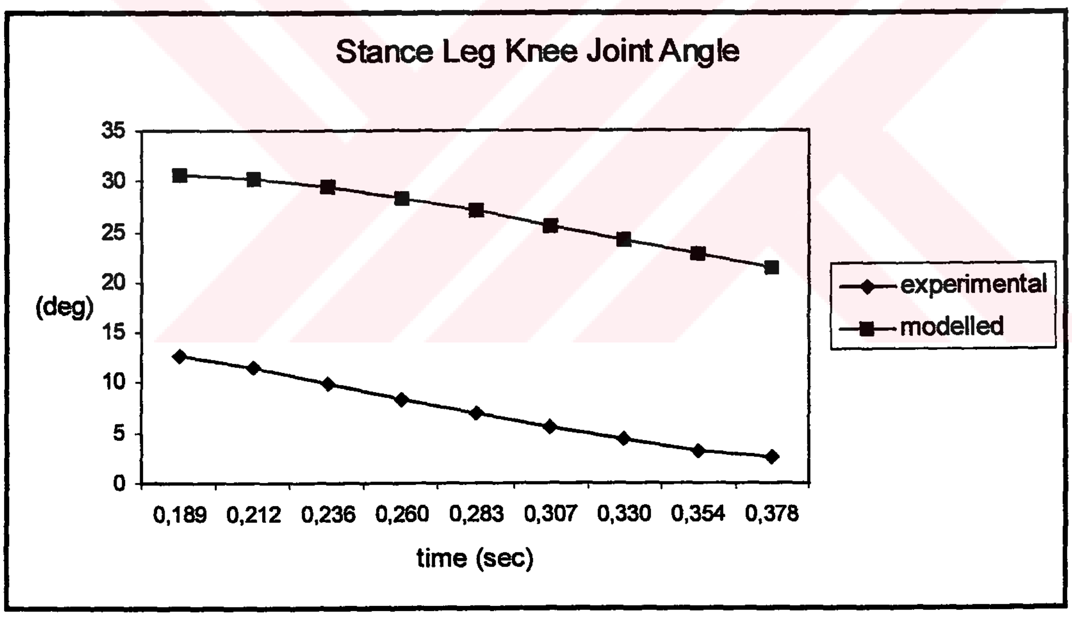

Figure 2.16 Stance Leg Knee Joint Angle in Foot-flat Period

For the single support heel-off period:

$0.401\,\text{s.} \leq t < 0.590\,\text{s.}$

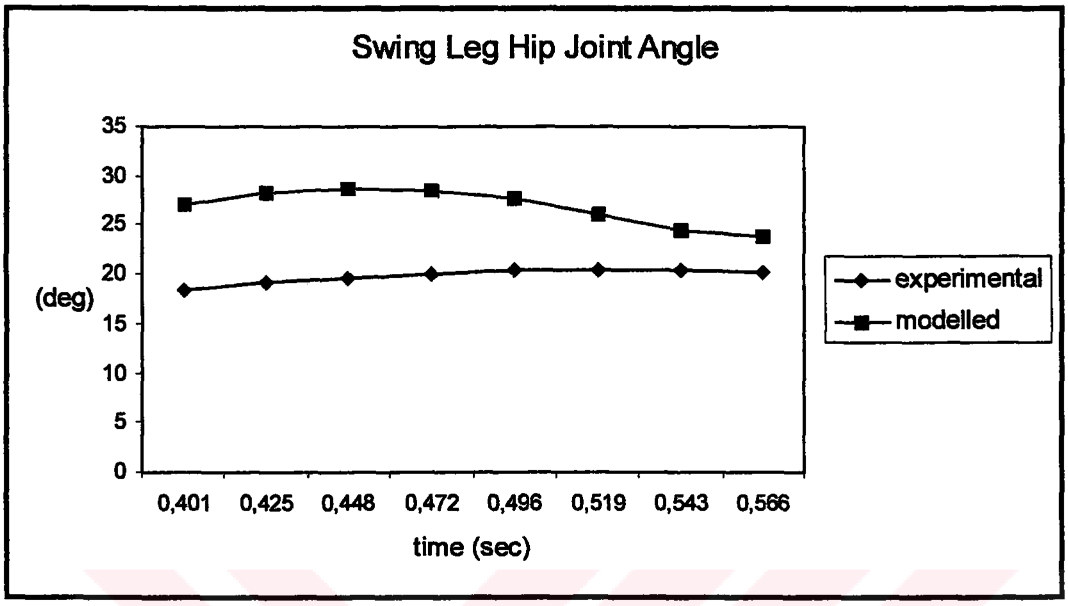

Figure 2.17 Swing Leg Hip Joint Angle in Heel-off Period

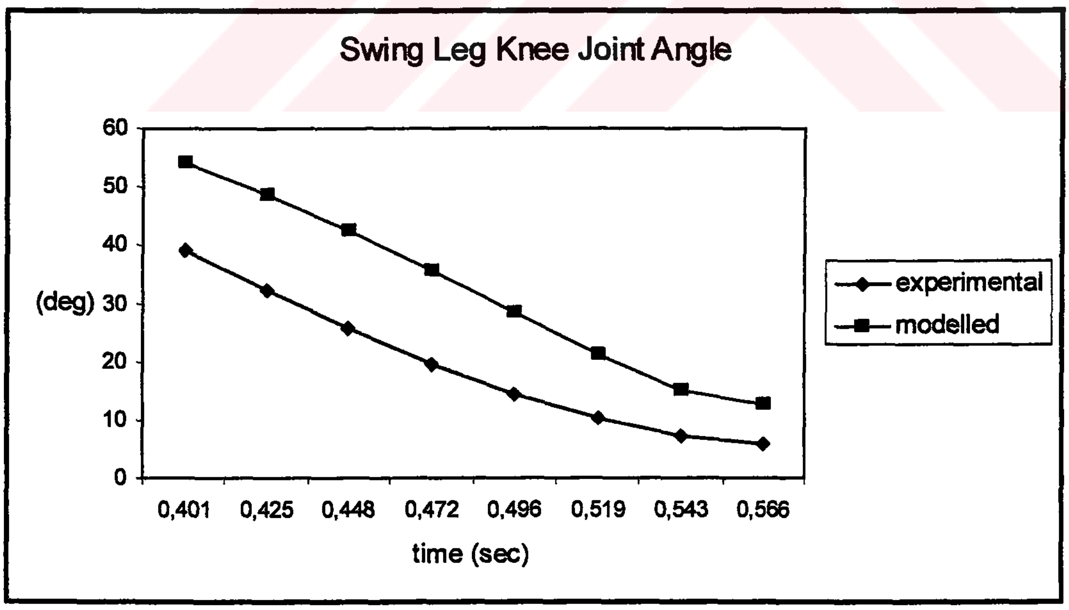

Figure 2.18 Swing Leg Knee Joint Angle in Heel-off Period

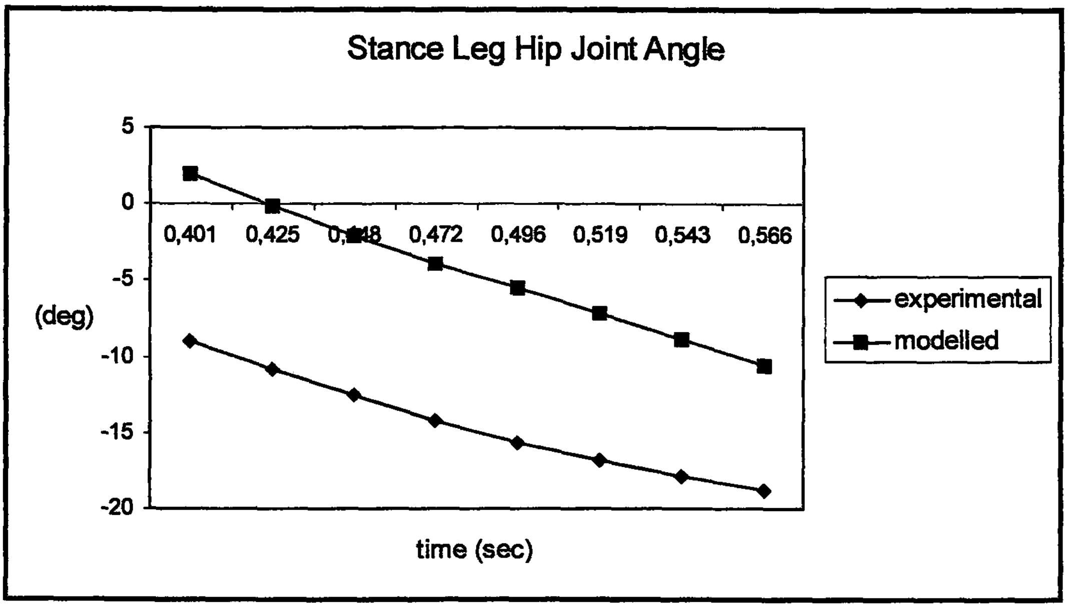

Figure 2.19 Stance Leg Hip Joint Angle in Heel-off Period

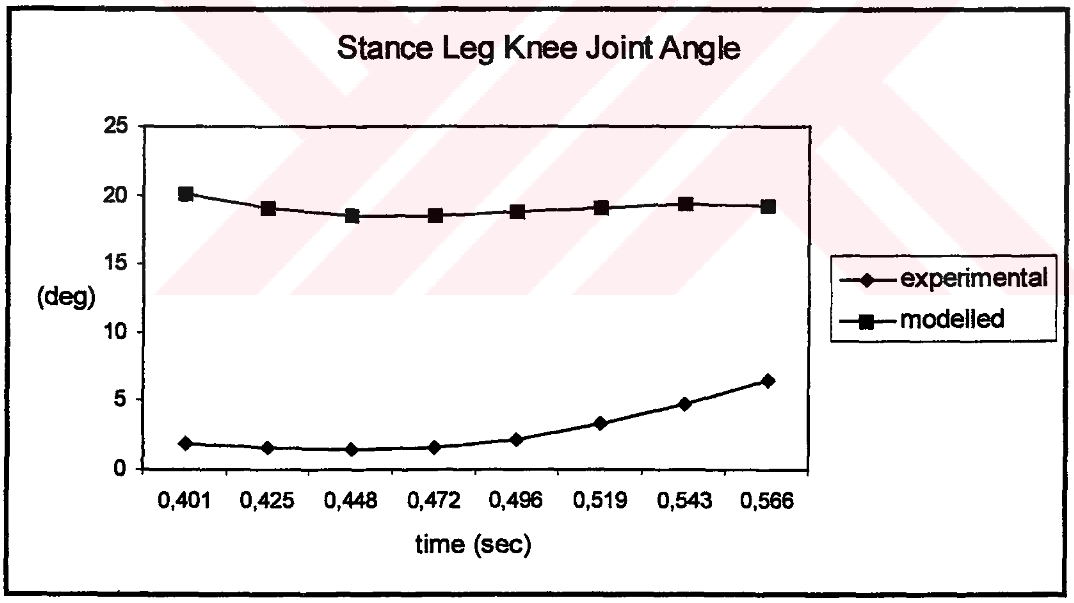

Figure 2.20 Stance Leg Knee Joint Angle in Heel-off Period

For the double support phase:

$0.590\,\text{s.} \le t < 0.684\,\text{s.}$

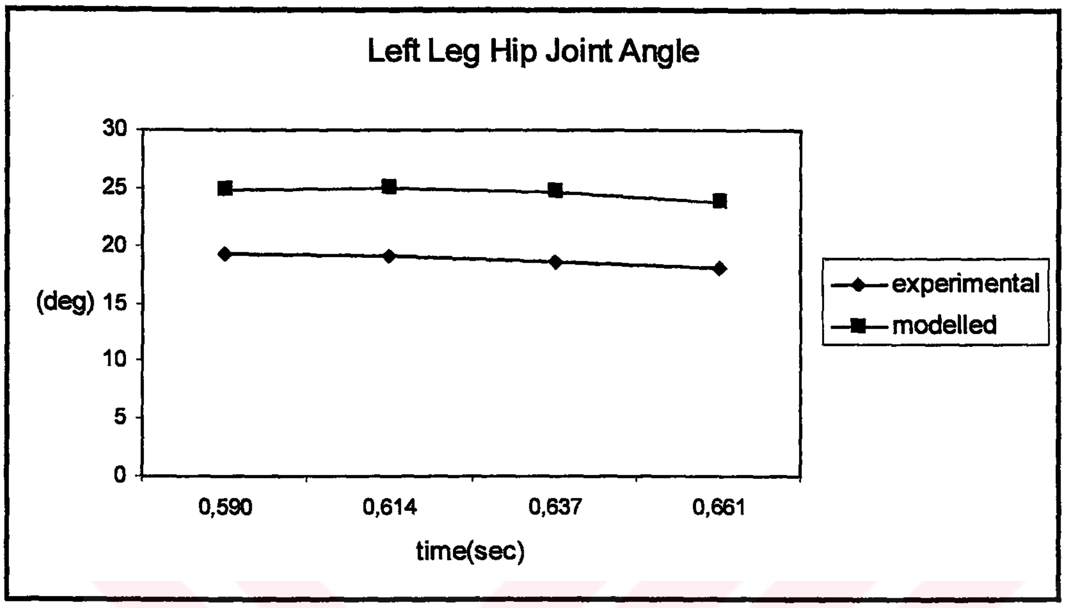

Figure 2.21 Left Leg Hip Joint Angle in Double Support Phase

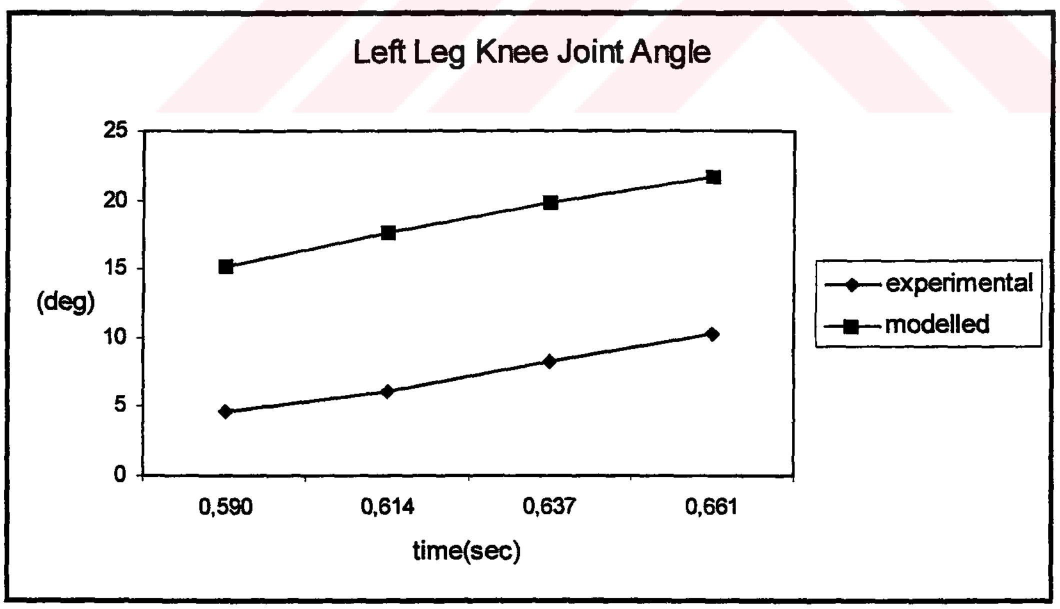

Figure 2.22 Left Leg Knee Joint Angle in Double Support Phase

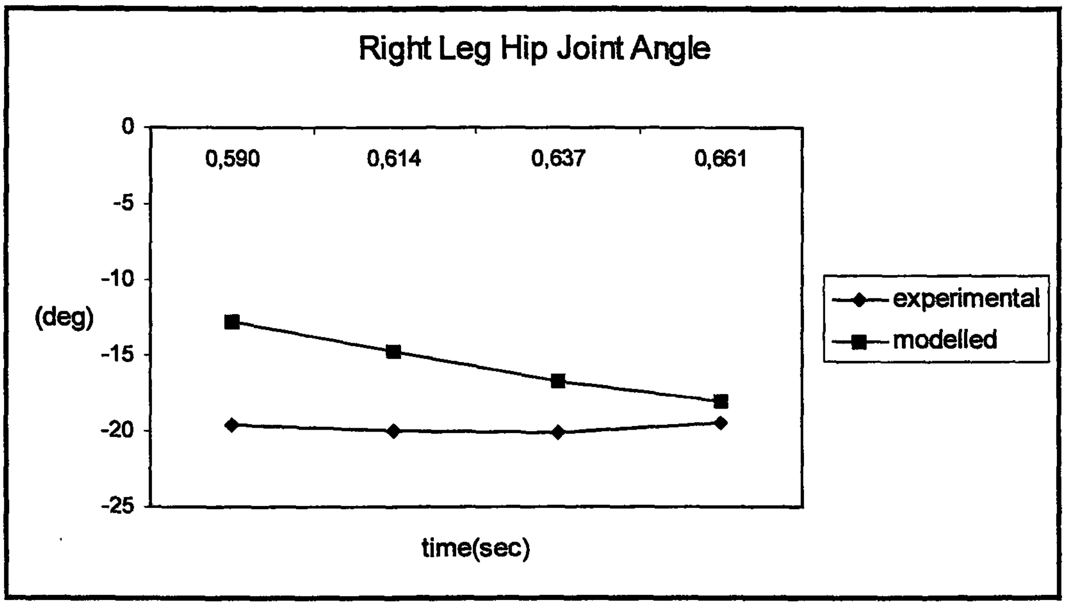

Figure 2.23 Right Leg Hip Joint Angle in Double Support Phase

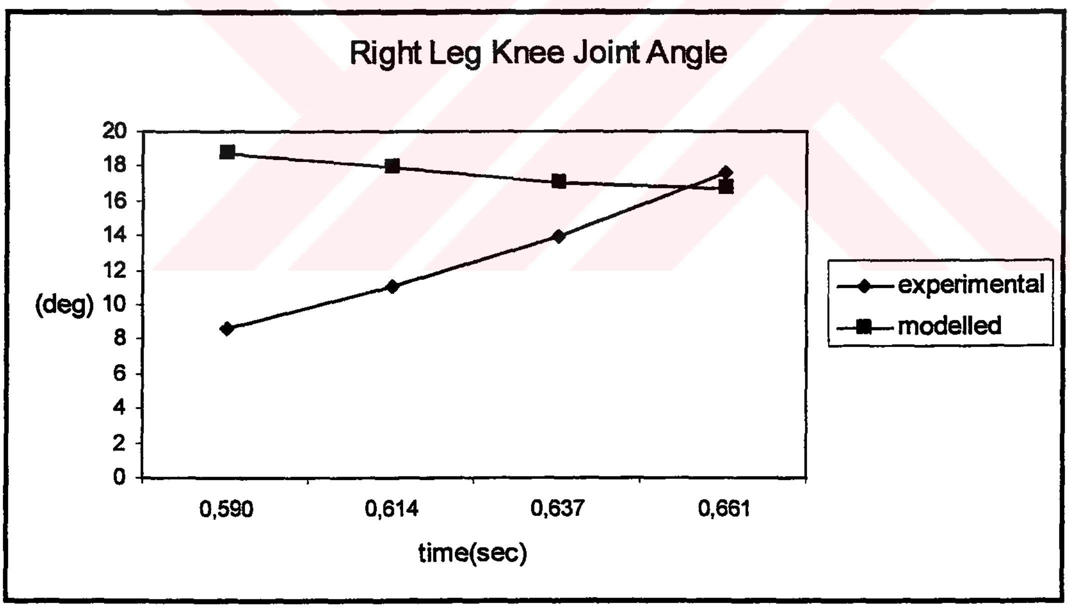

Figure 2.24 Right Leg Knee Joint Angle in Double Support Phase

CHAPTER 3

DYNAMIC EQUATIONS AND CONTROL WITH COMPUTED TORQUE METHOD

In this chapter the dynamic equations of the models described in Chapter 2 are derived, and computed torque method is utilized in order to model the feedback control activity necessary to stabilize the human gait.

3.1 Dynamic Equations

Lagrange equations are utilized to derive the equations of motion for both the single support and double support phases. For this purpose it is first necessary to write down expressions for the kinetic and potential energies of the system. This is achieved by considering the body in a floating state as shown in Figure 3.1, and each phase of the gait is then described by introducing appropriate constraints.

For the system shown in Figure 3.1, the vectors of generalized coordinates are $q_r \in R^3$, $q_l \in R^3$, $q_b \in R^2$; where subscript r refers to right leg, l refers to left leg and b refers to HAT (Head-Arms-Trunk). More specifically:

$$q_r = \begin{bmatrix} \theta_{hr} \\ \theta_{kr} \\ \theta_{ar} \end{bmatrix} \qquad q_l = \begin{bmatrix} \theta_{hl} \\ \theta_{kl} \\ \theta_{al} \end{bmatrix} \qquad q_b = \begin{bmatrix} x_h \\ y_h \end{bmatrix}$$

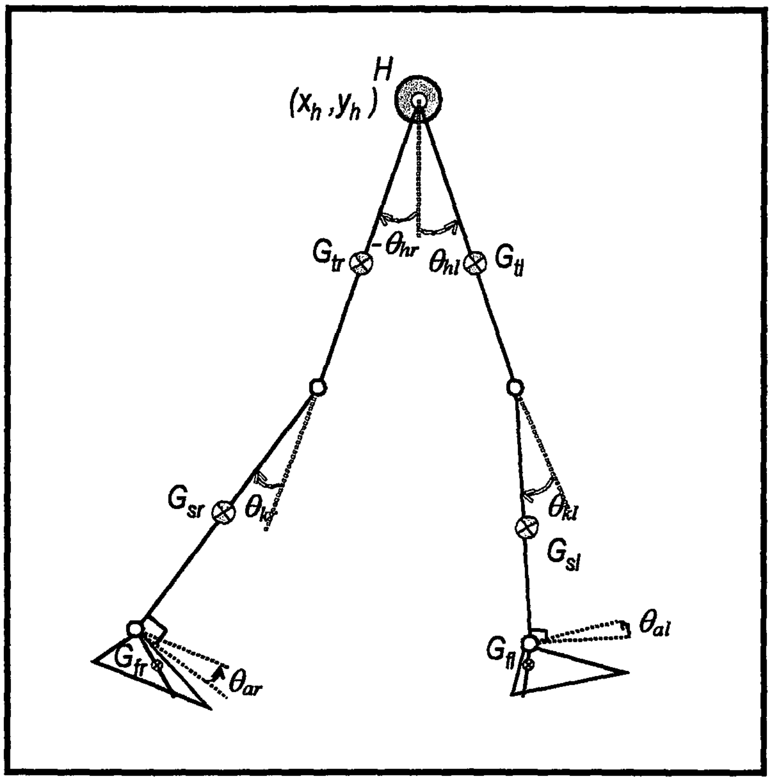

Figure 3.1 Model in a floating state

The total kinetic and potential energies of the system are

$$K = K_r + K_l + K_b \tag{3.1}$$

$$U = U_r + U_l + U_b \tag{3.2}$$

where;

$$K_r = K_r(\dot{q}_r, \dot{q}_b; q_r)$$

$$K_l = K_l(\dot{q}_l, \dot{q}_b; q_l)$$

$$K_b = K_b(\dot{q}_b) = \frac{1}{2} m_b \dot{q}_b^T \dot{q}_b = \frac{1}{2} m_b (\dot{x}_h{}^2 + \dot{y}_h{}^2)$$

and

$$U_r = U_r(q_r, q_b)$$

$$U_l = U_l(q_l, q_b)$$

$$U_b = U_b(q_b) = m_b g\, y_h$$

where g is the gravitational acceleration

The expressions for K_r , K_l , U_r and U_l require x and y coordinates for the mass centers of the thigh, shank and foot of both legs. For the right leg:

$$x_{Gtr} = x_h + d_t \sin(\theta_{hr}) \tag{3.3.a}$$
$$y_{Gtr} = y_h - d_t \cos(\theta_{hr}) \tag{3.3.b}$$

$$x_{Gsr} = x_h + b_t \sin(\theta_{hr}) + d_s \sin(\theta_{hr} - \theta_{kr}) \tag{3.4.a}$$
$$y_{Gsr} = y_h - b_t \cos(\theta_{hr}) - d_s \cos(\theta_{hr} - \theta_{kr}) \tag{3.4.b}$$

$$x_{Gfr} = x_h + b_t \sin(\theta_{hr}) + b_s \sin(\theta_{hr} - \theta_{kr}) - d_{fr} \cos(\theta_{hr} - \theta_{kr} + \theta_{ar} + \alpha_{fr}) \tag{3.5.a}$$
$$y_{Gfr} = y_h - b_t \cos(\theta_{hr}) - b_s \cos(\theta_{hr} - \theta_{kr}) - d_{fr} \sin(\theta_{hr} - \theta_{kr} + \theta_{ar} + \alpha_{fr}) \tag{3.5.b}$$

Differentiation of the above expressions with respect to time yields the velocity components of the corresponding mass centers. The kinetic and potential energies are then given as:

$$\begin{aligned} K_r = m_t \frac{(\dot{x}_{Gtr}{}^2 + \dot{y}_{Gtr}{}^2)}{2} + m_s \frac{(\dot{x}_{Gsr}{}^2 + \dot{y}_{Gsr}{}^2)}{2} + m_f \frac{(\dot{x}_{Gfr}{}^2 + \dot{y}_{Gfr}{}^2)}{2} \\ + I_t \frac{\dot{\theta}_{hr}{}^2}{2} + I_s \frac{(\dot{\theta}_{hr} - \dot{\theta}_{kr})^2}{2} + I_f \frac{(\dot{\theta}_{hr} - \dot{\theta}_{kr} + \dot{\theta}_{ar})^2}{2} \end{aligned} \tag{3.6}$$

$$U_r = g\,(m_t y_{Gtr} + m_s y_{Gsr} + m_f y_{Gfr}) \tag{3.7}$$

Note that corresponding expressions for the left leg can readily be obtained when subscript r is replaced by subscript l.

3.1.1 Lagrange Equations for the Single Support Phase

During single support phase, it is assumed that the left leg is swinging and the foot of the leg is not moving relative to the shank. Therefore for the swing leg shown in Figure 3.2 θ_{al} is taken to be a constant (θ_{al}=0), i.e. q_l becomes a member of R^2 as

$$q_l = \begin{bmatrix} \theta_{hl} \\ \theta_{kl} \end{bmatrix}$$

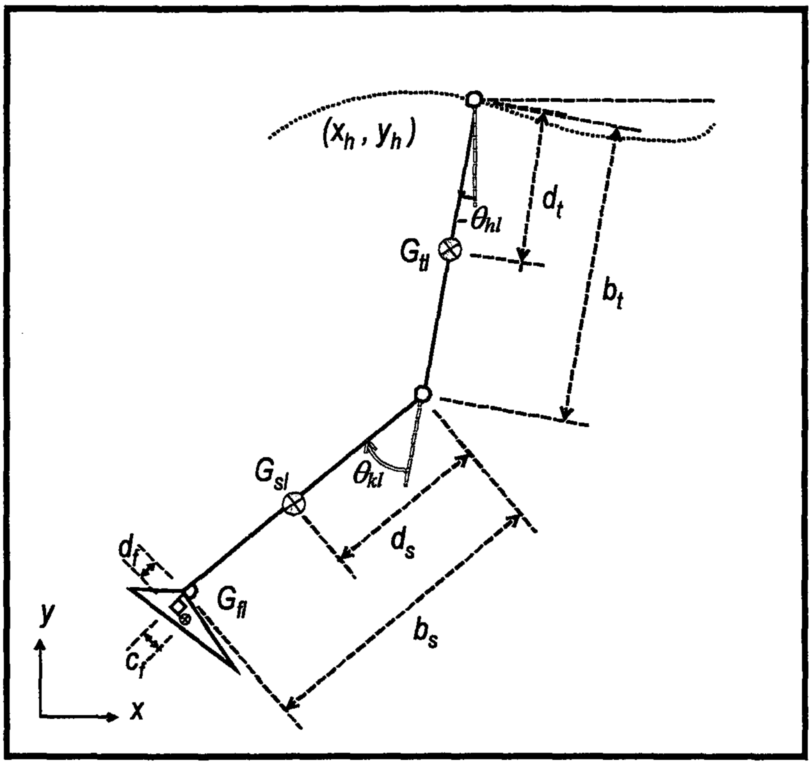

Figure 3.2 Swing leg model

The coordinates of the mass center of the swinging foot are

$$x_{Gfl} = x_h + b_t \sin(\theta_{hl}) + (b_s + d_f)\sin(\theta_{hl} - \theta_{kl}) + c_f \cos(\theta_{hl} - \theta_{kl}) \tag{3.8.a}$$

$$y_{Gfl} = y_h - b_t \cos(\theta_{hl}) - (b_s + d_f)\cos(\theta_{hl} - \theta_{kl}) + c_f \sin(\theta_{hl} - \theta_{kl}) \tag{3.8.b}$$

As described previously, during single support the stance leg undergoes two periods, i.e. foot-flat and heel-off periods.

i) Foot-flat Period

It is assumed that the right leg is pivoting about the ankle as shown in Figure3.3. So, θ_{ar} is no longer a generalized coordinate since $\theta_{ar} = \theta_{kr} - \theta_{hr}$. In this way q_r also becomes a member of R^2 as

$$q_r = \begin{bmatrix} \theta_{hr} \\ \theta_{kr} \end{bmatrix}$$

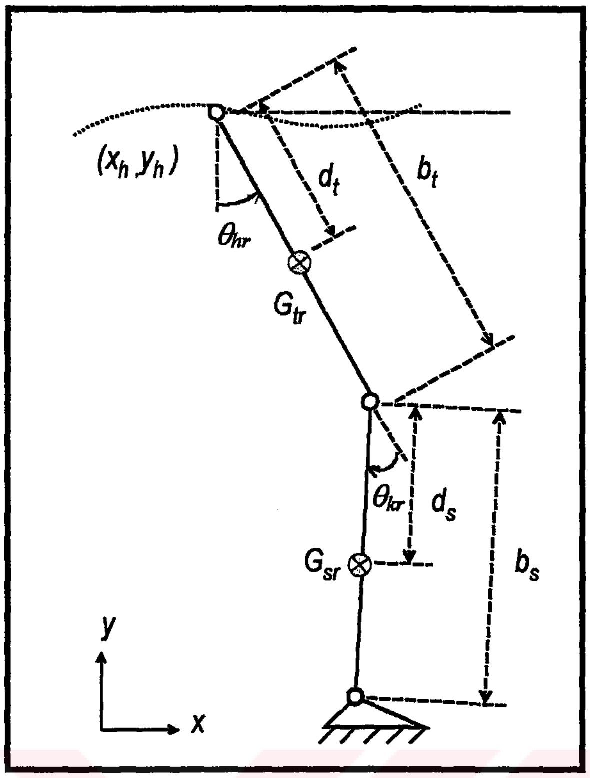

Figure 3.3 Foot-flat leg model

Since the ankle is fixed, the following constraint equations arise

$$x_h + \mathrm{f}_x(\mathrm{q}_r) = x_{0r} \tag{3.9}$$

$$y_h + \mathrm{f}_y(\mathrm{q}_r) = y_{0r} \tag{3.10}$$

where

$$\mathrm{f}_x(\mathrm{q}_r) = b_s \sin(\theta_{hr} - \theta_{kr}) + b_t \sin(\theta_{hr})$$

$$\mathrm{f}_y(\mathrm{q}_r) = -b_s \cos(\theta_{hr} - \theta_{kr}) - b_t \cos(\theta_{hr})$$

Note that with reference to equations (3.5.a) and (3.5.b) these constraints dictate the following conditions

$\dot{x}_{Gfr} = 0$ and $\dot{y}_{Gfr} = 0$

The virtual variations of these constraint equations are

$$\delta x_h + \left(\frac{\partial \mathrm{f_x}}{\partial \mathrm{q}_r}\right)^{\mathrm{T}} \delta \mathrm{q}_r = 0 \tag{3.11}$$

$$\delta y_h + \left(\frac{\partial \mathrm{f_y}}{\partial \mathrm{q}_r}\right)^{\mathrm{T}} \delta \mathrm{q}_r = 0 \tag{3.12}$$

Introducing the Lagrange multipliers λ_x and λ_y the constraints lead to the following constraint forces:

$$\mathrm{Q_b}' = \begin{bmatrix} \lambda_x \\ \lambda_y \end{bmatrix} \qquad \mathrm{Q}_r' = \begin{bmatrix} \dfrac{\partial \mathrm{f_x}}{\partial \mathrm{q}_r} & \dfrac{\partial \mathrm{f_y}}{\partial \mathrm{q}_r} \end{bmatrix} \begin{bmatrix} \lambda_x \\ \lambda_y \end{bmatrix}$$

Note that $\lambda_x = R_{xr}$, $\lambda_y = R_{yr}$, which are the ground reaction forces on the right foot, corresponding to the fixed ankle joint.

The active generalized forces are determined by considering the virtual work expression as follows:

$$\begin{aligned} \delta \mathrm{W} &= \mathrm{Q}_r^{\mathrm{T}} \delta \mathrm{q}_r + \mathrm{Q}_l^{\mathrm{T}} \delta \mathrm{q}_l \\ &= T_{hr}\, \delta\theta_{hr} + T_{kr}\, \delta\theta_{kr} + T_{kl}\, \delta\theta_{kl} - T_{hr} \delta\theta_{hl} + T_{ar}\, (\delta\theta_{kr} - \delta\theta_{hr}) \end{aligned} \tag{3.13}$$

where T_{hr}, T_{kr}, T_{ar}, T_{hl}, T_{kl} are actual joint torques. Hence it is obtained that

$$\mathrm{Q}_r = \begin{bmatrix} T_{hr} - T_{ar} \\ T_{kr} + T_{ar} \end{bmatrix} \qquad \mathrm{Q}_l = \begin{bmatrix} -T_{hr} \\ T_{kl} \end{bmatrix} \qquad \mathrm{Q_b} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

And the constrained Lagrange's equations become:

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{\theta}_{hr}}\right)-\frac{\partial K}{\partial \theta_{hr}}+\frac{\partial U}{\partial \theta_{hr}}=T_{hr}-T_{ar}+\left(\frac{\partial f_x}{\partial \theta_{hr}}\right)\lambda_x+\left(\frac{\partial f_y}{\partial \theta_{hr}}\right)\lambda_y \tag{3.14}$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{\theta}_{kr}}\right)-\frac{\partial K}{\partial \theta_{kr}}+\frac{\partial U}{\partial \theta_{kr}}=T_{kr}+T_{ar}+\left(\frac{\partial f_x}{\partial \theta_{kr}}\right)\lambda_x+\left(\frac{\partial f_y}{\partial \theta_{kr}}\right)\lambda_y \tag{3.15}$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{\theta}_{hl}}\right)-\frac{\partial K}{\partial \theta_{hl}}+\frac{\partial U}{\partial \theta_{hl}}=-T_{hr} \tag{3.16}$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{\theta}_{kl}}\right)-\frac{\partial K}{\partial \theta_{kl}}+\frac{\partial U}{\partial \theta_{kl}}=T_{kl} \tag{3.17}$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}_h}\right)-\frac{\partial K}{\partial x_h}+\frac{\partial U}{\partial x_h}=\lambda_x \tag{3.18}$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{y}_h}\right)-\frac{\partial K}{\partial y_h}+\frac{\partial U}{\partial y_h}=\lambda_y \tag{3.19}$$

The left-hand sides of the equations (3.14-3.19) are processed by Mathematica and the expressions obtained are presented in Appendix A.1.

Upon elimination of x_h , y_h and the Lagrange multipliers, the Lagrange's equations can be written in a more compact form as follows:

$$M\ddot{q}+b=Q \tag{3.20}$$

where

$$q=\begin{bmatrix}\theta_{hr}\\ \theta_{kr}\\ \theta_{hl}\\ \theta_{kl}\end{bmatrix} \qquad Q=\begin{bmatrix}T_{hr}-T_{ar}\\ T_{kr}+T_{ar}\\ -T_{hr}\\ T_{kl}\end{bmatrix}$$

ii) Heel-off Period

In this period, the right foot is pivoting about the toe, which is assumed to be fixed on the ground, as shown in Figure 3.4. Since, θ_{ar} is now included, q_r is in R^3.

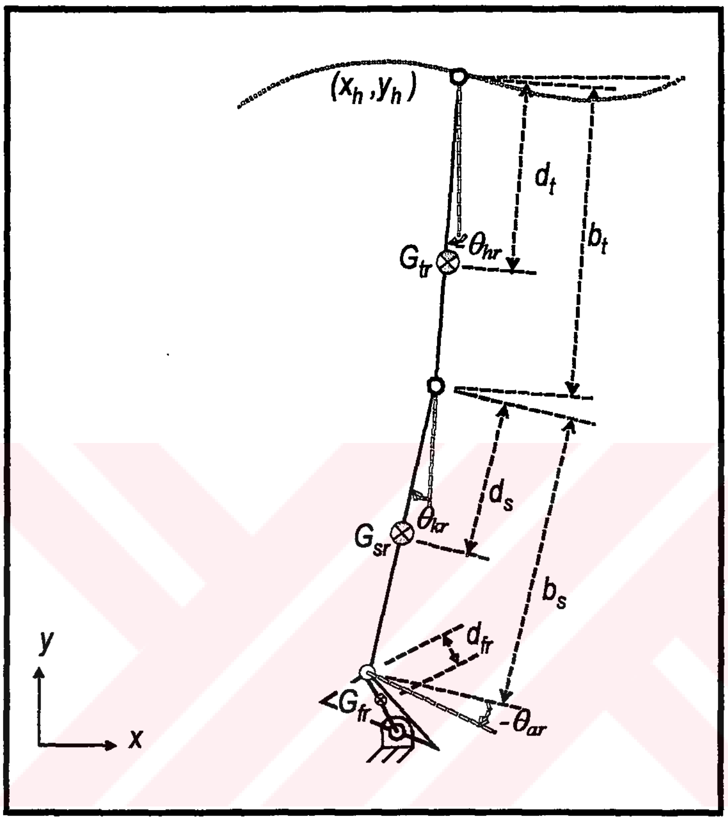

Figure 3.4 Heel-off leg model

The constraint equations for this case are

$$x_h + f_x(q_r) = x_{0r} + b_{fr}\cos(180 - \alpha_{fr}) \qquad (3.21)$$

$$y_h + f_y(q_r) = 0 \qquad (3.22)$$

where

$$f_x(q_r) = b_s \sin(\theta_{hr} - \theta_{kr}) + b_t \sin(\theta_{hr}) - b_{fr}\cos(\alpha_{fr} + \theta_{hr} - \theta_{kr} + \theta_{ar})$$

$$f_y(q_r) = -b_s \cos(\theta_{hr} - \theta_{kr}) - b_t \cos(\theta_{hr}) - b_{fr}\sin(\alpha_{fr} + \theta_{hr} - \theta_{kr} + \theta_{ar})$$

The virtual variations of these constraint equations and the constraint forces are found by following the same procedure as in the foot-flat period (equations (3.11) and (3.12)). Note that again $\lambda_x = R_{xr}$, $\lambda_y = R_{yr}$, which are the ground reaction forces on the right foot, corresponding to the fixed toe joint.

The active generalized forces are determined by considering the virtual work equation as follows:

$$\begin{aligned}\delta W = T_{hr}\,\delta\theta_{hr} + T_{kr}\,\delta\theta_{kr} + T_{kl}\,\delta\theta_{kl} - T_{hr}\delta\theta_{hl} + T_{ar}\,\delta\theta_{ar} \\ + T_{tr}\,(\delta\theta_{kr} - \delta\theta_{hr} - \delta\theta_{ar})\end{aligned} \tag{3.23}$$

where T_{hr} , T_{kr} , T_{ar}, T_{hl} , T_{kl} , T_{tr} are actual joint torques. Hence it is obtained that

$$Q_r = \begin{bmatrix} T_{hr} - T_{tr} \\ T_{kr} + T_{tr} \\ T_{ar} - T_{tr} \end{bmatrix} \qquad Q_l = \begin{bmatrix} -T_{hr} \\ T_{kl} \end{bmatrix} \qquad Q_b = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

And the constrained Lagrange's equations become:

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{\theta}_{hr}}\right) - \frac{\partial K}{\partial \theta_{hr}} + \frac{\partial U}{\partial \theta_{hr}} = T_{hr} - T_{tr} + \left(\frac{\partial f_x}{\partial \theta_{hr}}\right)\lambda_x + \left(\frac{\partial f_y}{\partial \theta_{hr}}\right)\lambda_y \tag{3.24}$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{\theta}_{kr}}\right) - \frac{\partial K}{\partial \theta_{kr}} + \frac{\partial U}{\partial \theta_{kr}} = T_{kr} + T_{tr} + \left(\frac{\partial f_x}{\partial \theta_{kr}}\right)\lambda_x + \left(\frac{\partial f_y}{\partial \theta_{kr}}\right)\lambda_y \tag{3.25}$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{\theta}_{ar}}\right) - \frac{\partial K}{\partial \theta_{ar}} + \frac{\partial U}{\partial \theta_{ar}} = T_{ar} - T_{tr} + \left(\frac{\partial f_x}{\partial \theta_{ar}}\right)\lambda_x + \left(\frac{\partial f_y}{\partial \theta_{ar}}\right)\lambda_y \tag{3.26}$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{\theta}_{hl}}\right) - \frac{\partial K}{\partial \theta_{hl}} + \frac{\partial U}{\partial \theta_{hl}} = -T_{hr} \tag{3.27}$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{\theta}_{kl}}\right) - \frac{\partial K}{\partial \theta_{kl}} + \frac{\partial U}{\partial \theta_{kl}} = T_{kl} \tag{3.28}$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}_h}\right)-\frac{\partial K}{\partial x_h}+\frac{\partial U}{\partial x_h}=\lambda_x \tag{3.29}$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{y}_h}\right)-\frac{\partial K}{\partial y_h}+\frac{\partial U}{\partial y_h}=\lambda_y \tag{3.30}$$

The left-hand sides of these equations are presented in Appendix A.2. Upon elimination of x_h , y_h and the Lagrange multipliers, the final form of the Lagrange's equations will be

$$M\ddot{q}+b=Q \tag{3.31}$$

where

$$q=\begin{bmatrix}\theta_{hr}\\ \theta_{kr}\\ \theta_{ar}\\ \theta_{hl}\\ \theta_{kl}\end{bmatrix} \qquad Q=\begin{bmatrix}T_{hr}-T_{tr}\\ T_{kr}+T_{tr}\\ T_{ar}-T_{tr}\\ -T_{hr}\\ T_{kl}\end{bmatrix}$$

3.1.2 Lagrange Equations for the Double Support Phase

In this phase, the left leg is assumed to be in the heel contact state while the right leg is still in the heel-off state. As the left leg gets into the foot-flat state, the right leg lifts off the ground and the next swing phase of the gait begins and this corresponds to the termination of the double support phase.

Both q_l and q_r are in R^3 in this phase. Since both feet are in contact with the ground, there will be four constraint equations as (equations 3.32-3.35)

$$x_h+f_x(q_r)=x_{0r}+b_{fr}\cos(180-\alpha_{fr}) \tag{3.32}$$

$$y_h+f_y(q_r)=0 \tag{3.33}$$

where,

$$f_x(q_r) = b_s \sin(\theta_{hr} - \theta_{kr}) + b_t \sin(\theta_{hr}) - b_{fr} \cos(\alpha_{fr} + \theta_{hr} - \theta_{kr} + \theta_{ar})$$
$$f_y(q_r) = -b_s \cos(\theta_{hr} - \theta_{kr}) - b_t \cos(\theta_{hr}) - b_{fr} \sin(\alpha_{fr} + \theta_{hr} - \theta_{kr} + \theta_{ar})$$

and

$$x_h + g_x(q_l) = x_{0l} \tag{3.34}$$
$$y_h + g_y(q_l) = 0 \tag{3.35}$$

where

$$g_x(q_r) = b_s \sin(\theta_{hl} - \theta_{kl}) + b_t \sin(\theta_{hl}) - b_{fl} \cos(\alpha_{fl} + \theta_{hl} - \theta_{kl} + \theta_{al})$$
$$g_y(q_l) = -b_s \cos(\theta_{hl} - \theta_{kl}) - b_t \cos(\theta_{hl}) - b_{fl} \sin(\alpha_{fl} + \theta_{hl} - \theta_{kl} + \theta_{al})$$

With the Lagrange multipliers λ_x , λ_y for the first set, and μ_x , μ_y for the second set, the constraint forces are obtained as follows:

$$Q_b' = \begin{bmatrix} \lambda_x + \mu_x \\ \lambda_y + \mu_y \end{bmatrix} \qquad Q_r' = \begin{bmatrix} \dfrac{\partial f_x}{\partial q_r} & \dfrac{\partial f_y}{\partial q_r} \end{bmatrix} \begin{bmatrix} \lambda_x \\ \lambda_y \end{bmatrix} \qquad Q_l' = \begin{bmatrix} \dfrac{\partial g_x}{\partial q_l} & \dfrac{\partial g_y}{\partial q_l} \end{bmatrix} \begin{bmatrix} \mu_x \\ \mu_y \end{bmatrix}$$

Again note that $\lambda_x = R_{xr}$, $\lambda_y = R_{yr}$, $\mu_x = R_{xl}$, $\mu_y = R_{yl}$ are the ground reaction forces at the toe of the right leg and at the heel of the left leg, respectively.

The active generalized forces can be obtained as described below. The virtual work equation can be written as

$$\begin{aligned} \delta W &= Q_r^T \delta q_r + Q_l^T \delta q_l \\ &= T_{hr}\, \delta\theta_{hr} + T_{kr}\, \delta\theta_{kr} + T_{kl}\, \delta\theta_{kl} - T_{hr}\delta\theta_{hl} + T_{ar}\, \delta\theta_{ar} + T_{al}\delta\theta_{al} \\ &\qquad + T_{tr}\, (\delta\theta_{kr} - \delta\theta_{hr} - \delta\theta_{ar}) \end{aligned} \tag{3.36}$$

Hence,

$$Q_r = \begin{bmatrix} T_{hr} - T_{tr} \\ T_{kr} + T_{tr} \\ T_{ar} - T_{tr} \end{bmatrix} \qquad Q_l = \begin{bmatrix} -T_{hr} \\ T_{kl} \\ T_{al} \end{bmatrix} \qquad Q_b = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Then, the constrained Lagrange's equations can be written as

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{\theta}_{hr}}\right)-\frac{\partial K}{\partial \theta_{hr}}+\frac{\partial U}{\partial \theta_{hr}}=T_{hr}-T_{tr}+\left(\frac{\partial f_x}{\partial \theta_{hr}}\right)\lambda_x+\left(\frac{\partial f_y}{\partial \theta_{hr}}\right)\lambda_y \tag{3.37}$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{\theta}_{kr}}\right)-\frac{\partial K}{\partial \theta_{kr}}+\frac{\partial U}{\partial \theta_{kr}}=T_{kr}+T_{tr}+\left(\frac{\partial f_x}{\partial \theta_{kr}}\right)\lambda_x+\left(\frac{\partial f_y}{\partial \theta_{kr}}\right)\lambda_y \tag{3.38}$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{\theta}_{ar}}\right)-\frac{\partial K}{\partial \theta_{ar}}+\frac{\partial U}{\partial \theta_{ar}}=T_{ar}-T_{tr}+\left(\frac{\partial f_x}{\partial \theta_{ar}}\right)\lambda_x+\left(\frac{\partial f_y}{\partial \theta_{ar}}\right)\lambda_y \tag{3.39}$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{\theta}_{hl}}\right)-\frac{\partial K}{\partial \theta_{hl}}+\frac{\partial U}{\partial \theta_{hl}}=-T_{hr}+\left(\frac{\partial g_x}{\partial \theta_{hl}}\right)\mu_x+\left(\frac{\partial g_y}{\partial \theta_{hl}}\right)\mu_y \tag{3.40}$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{\theta}_{kl}}\right)-\frac{\partial K}{\partial \theta_{kl}}+\frac{\partial U}{\partial \theta_{kl}}=T_{kl}+\left(\frac{\partial g_x}{\partial \theta_{kl}}\right)\mu_x+\left(\frac{\partial g_y}{\partial \theta_{kl}}\right)\mu_y \tag{3.41}$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{\theta}_{al}}\right)-\frac{\partial K}{\partial \theta_{al}}+\frac{\partial U}{\partial \theta_{al}}=T_{al}+\left(\frac{\partial g_x}{\partial \theta_{al}}\right)\mu_x+\left(\frac{\partial g_y}{\partial \theta_{al}}\right)\mu_y \tag{3.42}$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{x}_h}\right)-\frac{\partial K}{\partial x_h}+\frac{\partial U}{\partial x_h}=\lambda_x+\mu_x \tag{3.43}$$

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{y}_h}\right)-\frac{\partial K}{\partial y_h}+\frac{\partial U}{\partial y_h}=\lambda_y+\mu_y \tag{3.44}$$

Upon elimination of x_h, y_h and the Lagrange multipliers λ_x, λ_y; the Lagrange equations can be written in the following compact form:

$$M\ddot{q}+b=Q+\Phi^T\mu \tag{3.45}$$

$$\phi(q)=e \tag{3.46}$$

The relevant quantities in these equations are defined as follows:

$$q = [\theta_{hl} \quad \theta_{kl} \quad \theta_{hr} \quad \theta_{kr} \quad \theta_{al} \quad \theta_{ar}]^T \qquad \mu = [\mu_x \quad \mu_y]^T$$

$$Q = [Q_1 \quad Q_2 \quad Q_3 \quad Q_4 \quad Q_5 \quad Q_6]^T$$

$$Q_1 = -T_{hr}$$
$$Q_2 = T_{kl}$$
$$Q_3 = T_{hr} - T_{tr}$$
$$Q_4 = T_{kr} + T_{tr}$$
$$Q_5 = T_{al}$$
$$Q_6 = T_{ar} - T_{tr}$$

$$\phi = \begin{bmatrix} \phi_x \\ \phi_y \end{bmatrix} = \begin{bmatrix} g_x(q_l) - f_x(q_r) \\ g_y(q_l) - f_y(q_r) \end{bmatrix}$$

$$e = \begin{bmatrix} x_{0l} - x_{0r} - b_{fr}\cos(180 - \alpha_{fr}) \\ 0 \end{bmatrix}$$

$$\Phi = \frac{\partial \phi}{\partial q} = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & \Phi_{16} \\ \Phi_{21} & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} & \Phi_{26} \end{bmatrix}$$

The elements of Φ are as given below:

$$\Phi_{11} = b_t\cos(\theta_{hl}) + b_s\cos(\theta_{hl} - \theta_{kl}) + b_{fl}\sin(\alpha_{fl} + \theta_{hl} - \theta_{kl} + \theta_{al})$$
$$\Phi_{12} = -b_s\cos(\theta_{hl} - \theta_{kl}) - b_{fl}\sin(\alpha_{fl} + \theta_{hl} - \theta_{kl} + \theta_{al})$$
$$\Phi_{13} = -b_t\cos(\theta_{hr}) - b_s\cos(\theta_{hr} - \theta_{kr}) - b_{fr}\sin(\alpha_{fr} + \theta_{hr} - \theta_{kr} + \theta_{ar})$$
$$\Phi_{14} = b_s\cos(\theta_{hr} - \theta_{kr}) + b_{fr}\sin(\alpha_{fr} + \theta_{hr} - \theta_{kr} + \theta_{ar})$$
$$\Phi_{15} = b_{fl}\sin(\alpha_{fl} + \theta_{hl} - \theta_{kl} + \theta_{al})$$
$$\Phi_{16} = -b_{fr}\sin(\alpha_{fr} + \theta_{hr} - \theta_{kr} + \theta_{ar})$$

$$\Phi_{21} = b_t \sin(\theta_{hl}) + b_s \sin(\theta_{hl} - \theta_{kl}) - b_{fl} \cos(\alpha_{fl} + \theta_{hl} - \theta_{kl} + \theta_{al})$$
$$\Phi_{22} = -b_s \sin(\theta_{hl} - \theta_{kl}) + b_{fl} \cos(\alpha_{fl} + \theta_{hl} - \theta_{kl} + \theta_{al})$$
$$\Phi_{23} = -b_t \sin(\theta_{hr}) - b_s \sin(\theta_{hr} - \theta_{kr}) + b_{fr} \cos(\alpha_{fr} + \theta_{hr} - \theta_{kr} + \theta_{ar})$$
$$\Phi_{24} = b_s \sin(\theta_{hr} - \theta_{kr}) - b_{fr} \cos(\alpha_{fr} + \theta_{hr} - \theta_{kr} + \theta_{ar})$$
$$\Phi_{25} = -b_{fl} \cos(\alpha_{fl} + \theta_{hl} - \theta_{kl} + \theta_{al})$$
$$\Phi_{26} = b_{fr} \cos(\alpha_{fr} + \theta_{hr} - \theta_{kr} + \theta_{ar})$$

As noticed, due to the constraints, two elements of q are excess coordinates. Let q be separated as

$$q = \begin{bmatrix} p \\ s \end{bmatrix}$$

where

$$p = \begin{bmatrix} \theta_{hl} \\ \theta_{kl} \\ \theta_{hr} \\ \theta_{kr} \end{bmatrix} \qquad s = \begin{bmatrix} \theta_{al} \\ \theta_{ar} \end{bmatrix}$$

The elements of s are taken as the excess coordinates. Then, s and μ can be eliminated as described below:

Equation 3.45 can be written as

$$\begin{bmatrix} M_{pp} & M_{ps} \\ M_{sp} & M_{ss} \end{bmatrix} \begin{bmatrix} \ddot{p} \\ \ddot{s} \end{bmatrix} + \begin{bmatrix} b_p \\ b_s \end{bmatrix} = \begin{bmatrix} Q_p \\ Q_s \end{bmatrix} + \begin{bmatrix} \Phi_p{}^T \\ \Phi_s{}^T \end{bmatrix} \mu \tag{3.47}$$

where

$$Q_p = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{bmatrix} \qquad Q_s = \begin{bmatrix} Q_5 \\ Q_6 \end{bmatrix}$$

From the second line of equation (3.47), μ can be solved as

$$\mu = (\Phi_s^{-1})^T (M_{sp}\ddot{p} + M_{ss}\ddot{s} + b_s - Q_s) \tag{3.48}$$

On the other hand, s and its derivatives can be solved from the constraint equation (3.46) as

$$s = f(p) \tag{3.49}$$

$$\dot{s} = -(\Phi_s^{-1}\Phi_p)\dot{p} \tag{3.50}$$

$$\ddot{s} = -(\Phi_s^{-1}\Phi_p)\ddot{p} - d \tag{3.51}$$

where

$$d = \Phi_s^{-1}(\dot{\Phi}_p\dot{p} + \dot{\Phi}_s\dot{s})$$

$$\Phi_p = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} \\ \Phi_{21} & \Phi_{22} & \Phi_{23} & \Phi_{24} \end{bmatrix} \qquad \Phi_s = \begin{bmatrix} \Phi_{15} & \Phi_{16} \\ \Phi_{25} & \Phi_{26} \end{bmatrix}$$

After necessary substitutions into the first line of equation (3.47), the following equation is obtained:

$$H\ddot{p} + \beta = E\,Q \tag{3.52}$$

where

$$H = \left[M_{pp} - M_{ps}\psi - \psi^T M_{sp} + \psi^T M_{ss}\psi\right]$$

$$\beta = \left[b_p - \psi^T b_s - (M_{ps} - \psi^T M_{ss})d\right]$$

$$E = \left[I \quad -\psi^T\right]$$

$$\psi = \Phi_s^{-1}\Phi_p$$

I is 4×4 identity matrix

3.2 Gait Control

Walking is a method of locomotion involving the use of the two legs, alternately, to provide both support and propulsion. The human body applies feedback control in order to achieve satisfactory gait. Without feedback control, it would be impossible to stabilize the body in the upright position.

It is possible to model this control in several ways [10,24]. The feedback control used in this study is in the form of computed torque control [25], which finds application in the field of robotics as well. The application of this control will be investigated separately for the single and the double support phases.

3.2.1 Single Support Phase

Both in the foot-flat and heel-off periods, the Lagrange's equations are in the following form:

$$M\ddot{q} + b = Q \tag{3.53}$$

It is assumed that the body generates Q as follows:

$$Q = M\alpha + b \tag{3.54}$$

Then, equation (3.53) becomes:

$$\ddot{q} = \alpha \tag{3.55}$$

Here, α is the vector of control accelerations to be applied at the joints. The body can be assumed to generate α according to the PD (proportional + derivative) control law as

$$\alpha = \ddot{q}^* + C(\dot{q}^* - \dot{q}) + K(q^* - q) \tag{3.56}$$

where q^* denotes the reference (or ideal) gait

In the foot - flat period, $q \in R^4$. So,

$$K = \mathrm{diag}\begin{bmatrix}\omega_1^2 & \omega_2^2 & \omega_3^2 & \omega_4^2\end{bmatrix}$$

$$C = 2\xi\,\mathrm{diag}\begin{bmatrix}\omega_1 & \omega_2 & \omega_3 & \omega_4\end{bmatrix}$$

And in the heel - off period, $q \in R^5$. So,

$$K = \mathrm{diag}\begin{bmatrix}\omega_1{}^2 & \omega_2{}^2 & \omega_3{}^2 & \omega_4{}^2 & \omega_5{}^2\end{bmatrix}$$

$$C = 2\xi\,\mathrm{diag}\begin{bmatrix}\omega_1 & \omega_2 & \omega_3 & \omega_4 & \omega_5\end{bmatrix}$$

K and C are diagonal matrices as each joint is assumed to be controlled according to its own tracking error. Also it is reasonable to assume that the human responses are fast and non-oscillatory. So it is proper to take $\xi=1$ for the closed loop system to be critically damped. Substituting $\xi=1$ into equation (3.56) leads to the following error equation:

$$\ddot{\varepsilon} + C\dot{\varepsilon} + K\varepsilon = 0 \qquad (3.57)$$

where $\varepsilon = q^* - q$

This equation indicates that even if $\varepsilon(t_0) \neq 0$, $\varepsilon(t)$ will become practically zero a short while after t_0.

3.2.2 Double Support Phase

In this phase, the degree of freedom of the system is 4 while there are still 6 torques to be applied as control inputs. So, the human body most probably makes some kind of an optimization in deciding how to apply them.

Here, it is assumed that the body determines them in such a way that the following performance function is minimized

$$J = T^T \hat{W}\, T \qquad (3.58)$$

where

$$T = \begin{bmatrix}T_{hr} & T_{kl} & T_{al} & T_{kr} & T_{ar} & T_{tr}\end{bmatrix}$$

and $\hat{W}$ is a properly selected diagonal weighting matrix, i.e.

$$\hat{W} = \mathrm{diag}\begin{bmatrix}W_1 & W_2 & W_3 & W_4 & W_5 & W_6\end{bmatrix}$$

The elements of the weighting matrix can be selected as appropriate to the specific needs of the problem in hand.

The actual torque vector T can be related to the generalized torque vector Q as follows

$$T = AQ \tag{3.59}$$

where

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ -1 & 0 & -1 & 0 & 0 & 1 \\ -1 & 0 & -1 & 0 & 0 & 0 \end{bmatrix}$$

So with $W = A^T \hat{W} A$, the performace function becomes

$$J = Q^T W Q \tag{3.60}$$

On the other hand, as in the single support phase, it will be assumed that the body generates the control acceleration vector α so that ;

$\ddot{p} = \alpha$

According to equation (3.52), this necessitates the following equation to be satisfied by the joint torques:

$$H\alpha + \beta = EQ \tag{3.61}$$

Taking this equation as a constraint on Q, the performance function can be augmented as

$$\hat{J} = Q^T W Q + \gamma^T [H\alpha + \beta - EQ] \tag{3.62}$$

where γ is the vector of associated Lagrange multiplier s,

$\hat{J}$ is minimized if $\partial \hat{J} / \partial Q = 0$. That is,

$$WQ - E^T \gamma = 0 \tag{3.63}$$

Hence, Q is expressed as

$$Q = W^{-1} E^T \gamma \tag{3.64}$$

Inserting this expression into equation(3.52), γ is found as

$$\gamma = (EW^{-1}E^{T})^{-1}(H\alpha + \beta) \tag{3.65}$$

Hence, the optimal Q vector is found as

$$Q = B(H\alpha + \beta)$$
$$\text{where } B = W^{-1}E^{T}(EW^{-1}E^{T})^{-1} \tag{3.66}$$

With this Q, the dynamic equations (3.52) reduce to

$$\ddot{p} = \alpha$$

Again, α can be assumed to be generated according to the PD control law as

$$\alpha = \ddot{p}^{*} + C(\dot{p}^{*} - \dot{p}) + K(p^{*} - p) \tag{3.67}$$

Which leads to the following error equation:

$$\ddot{\varepsilon} + C\dot{\varepsilon} + K\varepsilon = 0 \tag{3.68}$$

where

$$\varepsilon = p^{*} - p$$
$$K = \mathrm{diag}\begin{bmatrix}\omega_1^2 & \omega_2^2 & \omega_3^2 & \omega_4^2\end{bmatrix}$$
$$C = 2\xi\,\mathrm{diag}\begin{bmatrix}\omega_1 & \omega_2 & \omega_3 & \omega_4\end{bmatrix}$$

With the assumption of critically damped closed loop system, ξ is again taken as unity.

3.2.3 Determination of the Control Stiffness Coefficients

As noticed above, the error equation has the same form both in the single support and in the double support phases. It can be repeated here for the k'th joint with ξ=1 as

$$\ddot{\varepsilon}_k + 2\omega_k\dot{\varepsilon}_k + \omega_k^{2}\varepsilon_k = 0 \tag{3.69}$$

In this equation, it would be very simplistic to assume that ω_k is constant. It is most probable that the human body applies a stiffening control such that $\omega_k \to \omega_k^0$, which is a low value, when ε_k gets larger so that the resulting torques do not exceed the physical capacity and $\omega_k \to \omega_k^*$, which is a high value, when ε_k approaches zero so that the robustness is increased against perturbations once the desired gait is established. In other words, it can be assumed that the body applies some kind of scheduling on ω_k depending on the magnitude of ε_k. In this work, this scheduling is modelled as follows:

$$\omega_k = \omega_k^0 + (\omega_k^* - \omega_k^0) e^{-\sigma_k \left| \varepsilon_k / \varepsilon_k^0 \right|} \tag{3.70}$$

where ε_k^0 is the maximum expected error.

The values taken for ω_k^0, ω_k^* and σ_k and the simulation results are given in Chapter 4.

CHAPTER 4

RESULTS AND DISCUSSION

4.1 Range of Application of the Dynamic Model

The dynamic model described in the previous chapter can be used for the following applications:

i) Forward Dynamic Analysis:

For the single support phase the equations 3.20 and 3.31, and for the double support phase the equation 3.52 can be integrated for specified joint torques to simulate the resulting gait. This requires a suitable integration scheme such as Runge-Kutta method. The specified joint torques can correspond to those of a normal or a disabled person, thus enabling to simulate both normal and pathological gait. Note that such an analysis needs a correlation to be established between the joint torques and the underlying pathology, such as changes in joint torques for a disabled person wearing prosthetic or assistive orthotic devices.

ii) Inverse Dynamic Analysis:

The same equations can also be used to calculate the joint torques necessary to obtain a desired gait. It should, however, be noted that the number of joint torques exceeds the degrees of freedom of the model for the double support phase, and this requires some kind of optimization, such as the scheme presented in section 3.2.2, to be used. Such an analysis may help to propose rehabilitation or surgical treatments necessary to approach the normal gait.

iii) Gait Control:

The computed torque control law presented in this work can be tested for its effectiveness to stabilize gait under various disturbing features.

In this study results are presented for the third application. However, if there are no disturbing features in the system, the computed torque gait control algorithm reduces to the inverse dynamic solution. This is another advantage of modelling the inherent feed back control action of a human body by means of computed torque method. Therefore results for the second application are also presented by introducing zero initial errors and zero disturbances in the control algorithm.

4.2 Results

The gait models presented in this study are developed by using the Gait Cycle Data (GCD) output file of the VICON Clinical Manager [23], which presents gait analysis data for a male subject of 80 kg mass and 1.85 m height. Not only the hip and ankle trajectories representing the "desired gait", but also switching times between different phases and stages of the gait as well as the model parameters and initial conditions are all tuned by utilizing the data of this standard gait analysis. The same data can also be used to demonstrate how successfully the proposed models describe the actual gait. For this purpose results are presented for both the modelled and experimentally measured joint angle trajectories (section 4.2.1), and for the joint moments (inverse dynamic analysis) obtained from the model and yielded by the gait analysis (section 4.2.2). Finally results are presented to demonstrate the effectiveness of the computed torque control strategy as the feedback controller to stabilize the gait under some disturbing features (section 4.2.3).

4.2.1 Joint Angle Trajectories (Kinematic Comparison)

Figure 4.1 shows the stick diagrams for the model and for the standard gait analysis. Although the modelled and experimental stick diagrams seem quite consistent at the first sight, there are in fact considerable deviations in the joint angle trajectories.

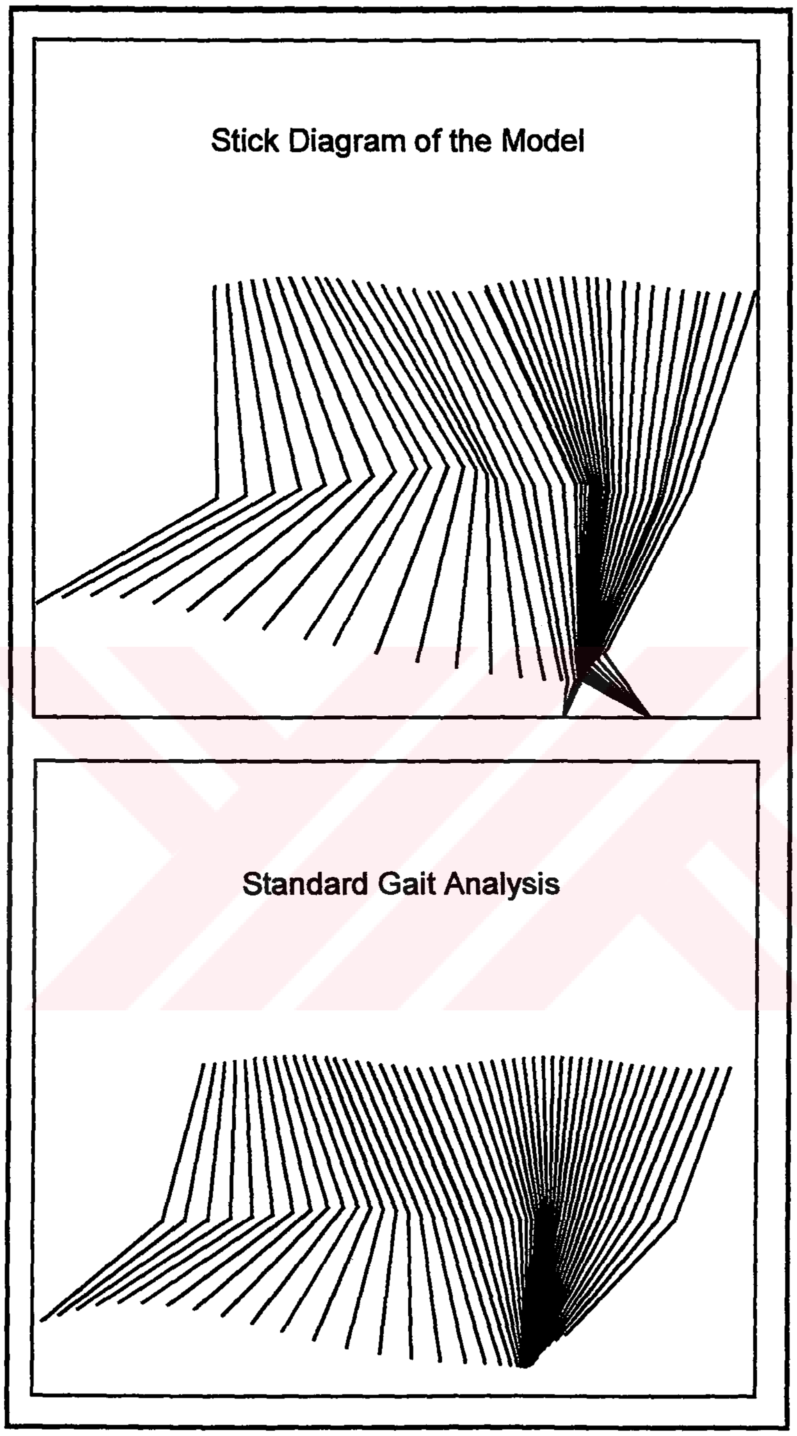

Figure 4.1 Stick diagrams for the model and standard gait analysis

Comparisons of the modelled and measured joint trajectories have already been presented in Chapter 2. Figures 4.2-4.5 present the same for the complete gait cycle by combining the results from different models. It is clear that the proposed models realize the desired gait pattern by considerably different joint angle trajectories, and as discussed in Chapter 2, this is believed to be primarily due to the planar nature of the models representing an actually three-dimensional kinematics.

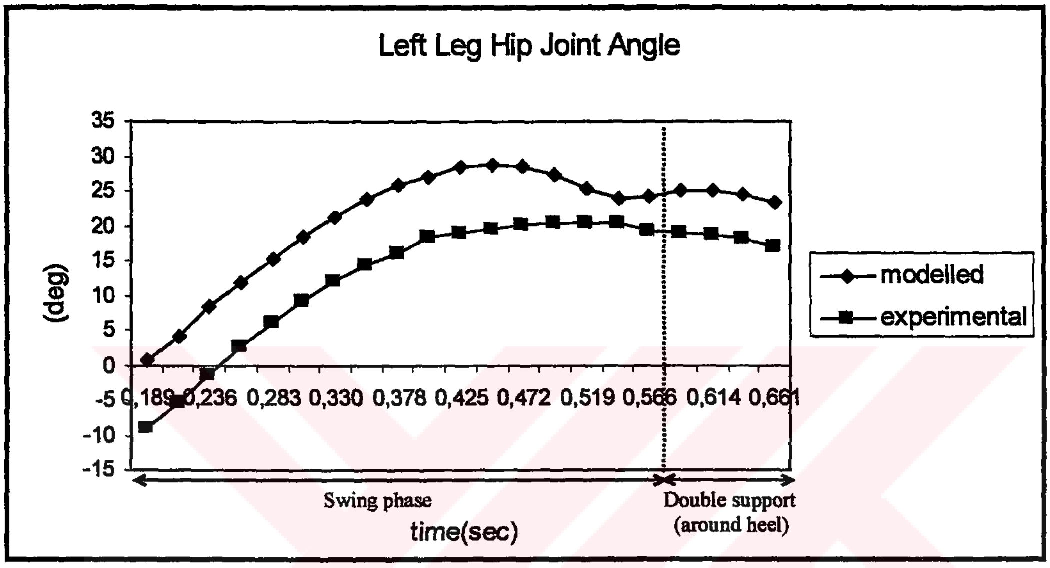

Figure 4.2 Left leg hip joint angle

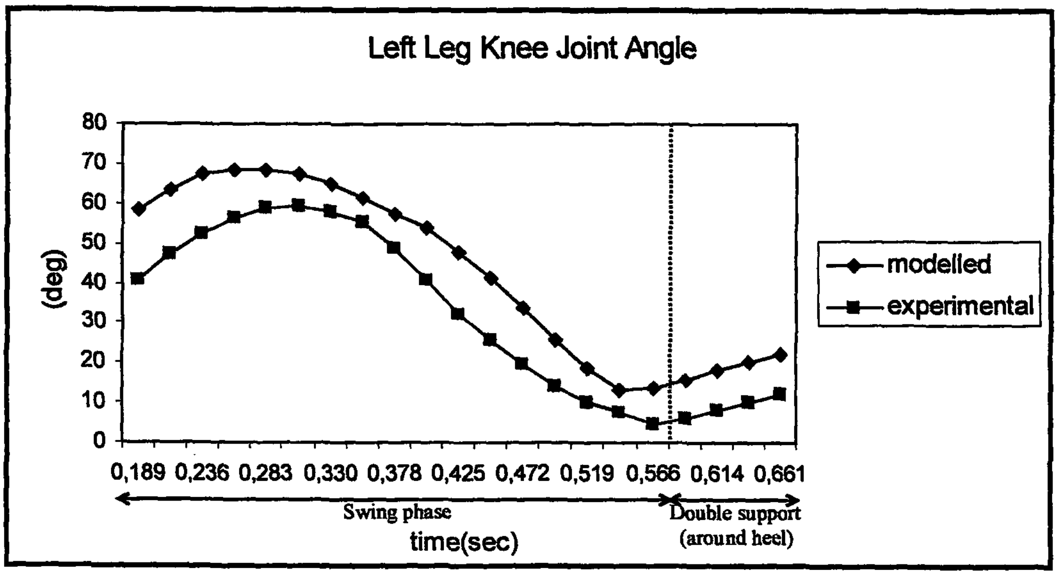

Figure 4.3 Left leg knee joint angle

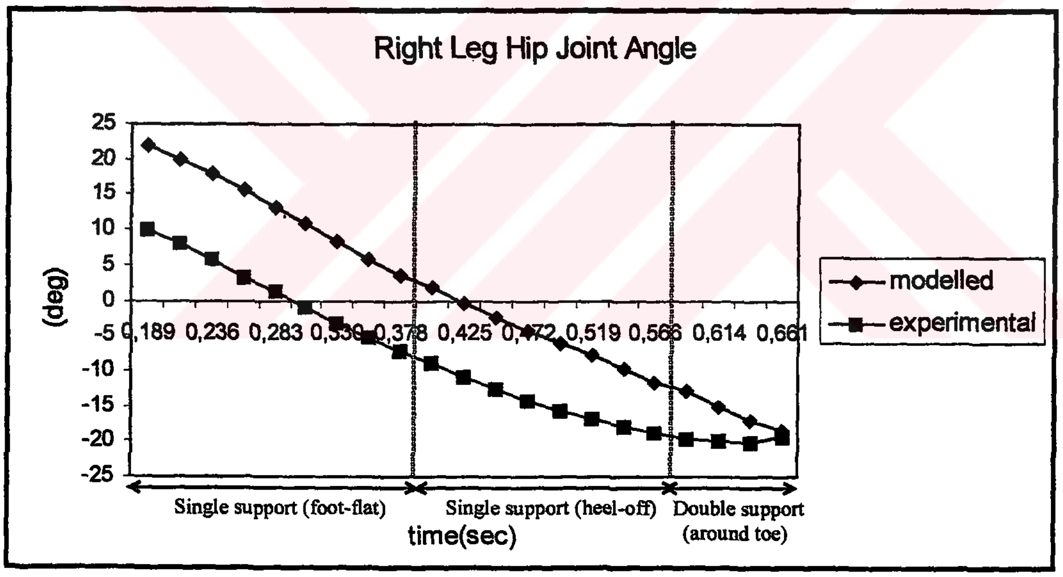

Figure 4.4 Right leg hip joint angle

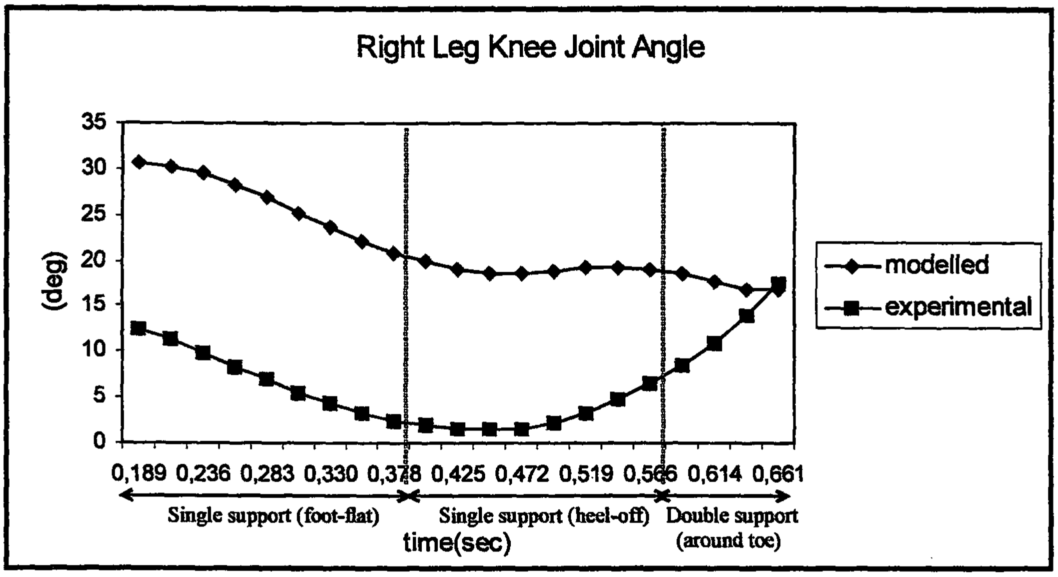

Figure 4.5 Right leg knee joint angle

4.2.2 Joint Moments (Kinetic Comparison)

Figures 4.6-4.11 display the joint moments calculated from the inverse dynamic analysis of the proposed models. For the double support phase, the inverse dynamic analysis requires properly selected weighting factors. Exhaustive tests have been conducted with different weighting factors and the following values are chosen based on the closeness of model solution and gait analysis results:

$$\hat{W} = \mathrm{diag}\begin{bmatrix}100 & 80 & 1500 & 5 & 0.1 & 10\end{bmatrix}$$

Note that there exists no toe moment in the standard gait analysis, therefore Fig. 4.9 does not contain a gait analysis result for comparison. The other figures demonstrate that the joint torques calculated from the proposed models deviate from those yielded by the gait analysis especially during double support phase. The unrealistically high joint moments encountered during double support phase can be attributed to the modelling strategy. Instant switching from one model to the other, each possessing different kinematic structures, cause abrupt changes in joint moments. Differences during other phases are reasonably small and these can be attributed to both differences in joint angle trajectories as well as inaccuracies inherent in the standard gait analysis methodology in estimating the joint moments. Finally the simulated vertical ground reaction force is presented for the complete stance phase together with the force plate measurement in Fig. 4.12. Here, unlike joint moments, there is no inaccuracy problem related to the directly measured ground reaction force, and therefore any deviation is to be attributed to the modelling errors. In general the expected double humped " M " shaped vertical ground reaction curve is obtained from the model. However, the double support phase again shows some abrupt changes in vertical ground reaction force. It should also be noted that force plates with high sampling rates also show some ripples during the initiation of double support due to the impact of the foot with ground, but not as much as estimated by the model.

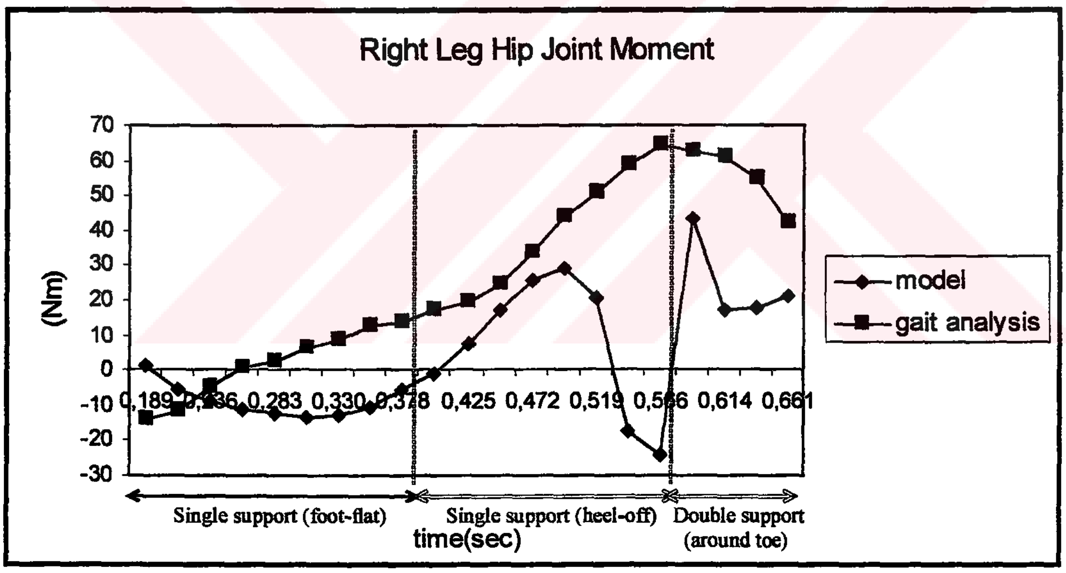

Figure 4.6 Right leg hip joint moment

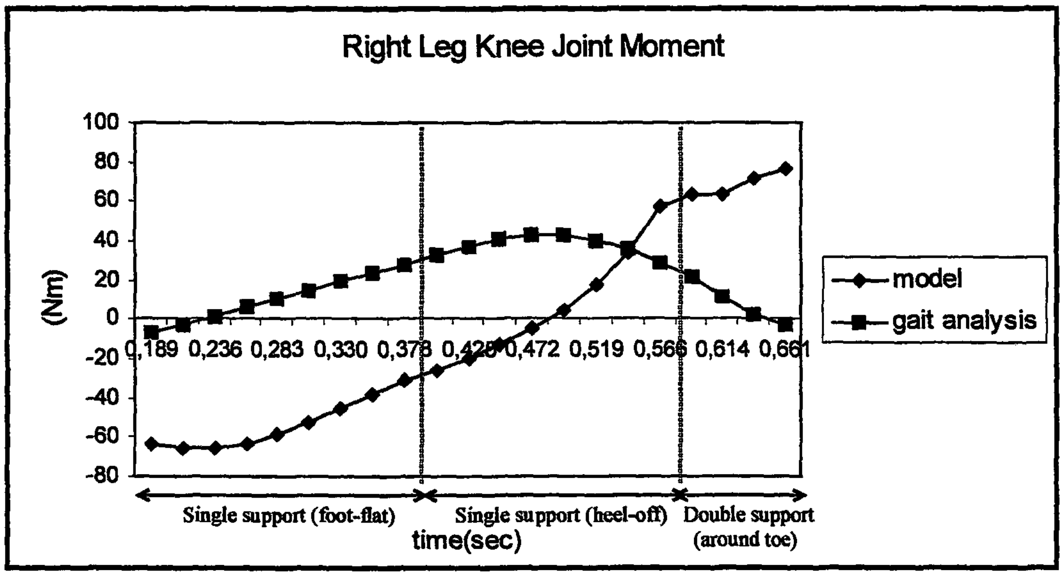

Figure 4.7 Right Leg Knee Joint Moment

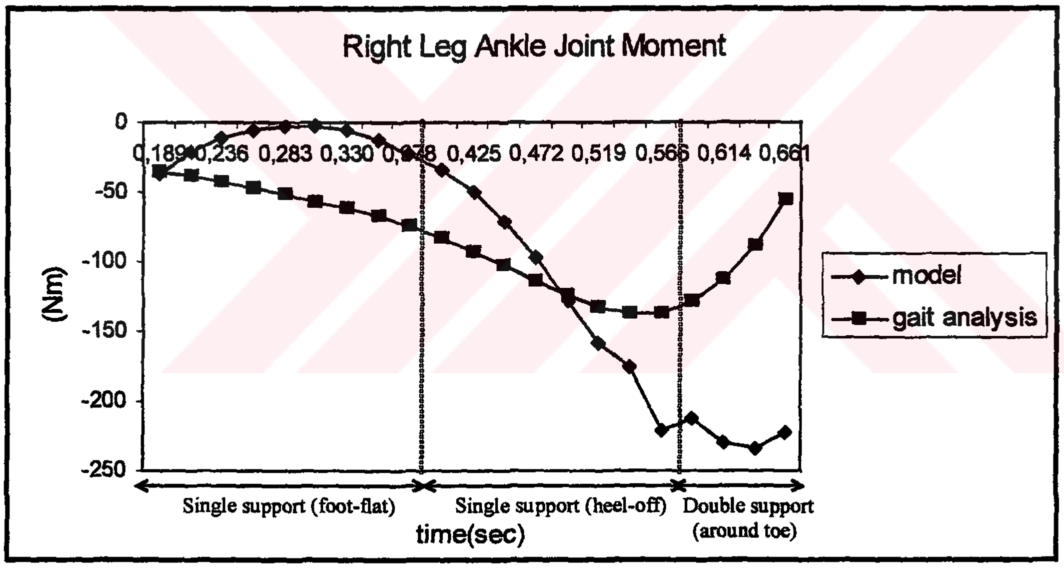

Figure 4.8 Right leg ankle joint moment

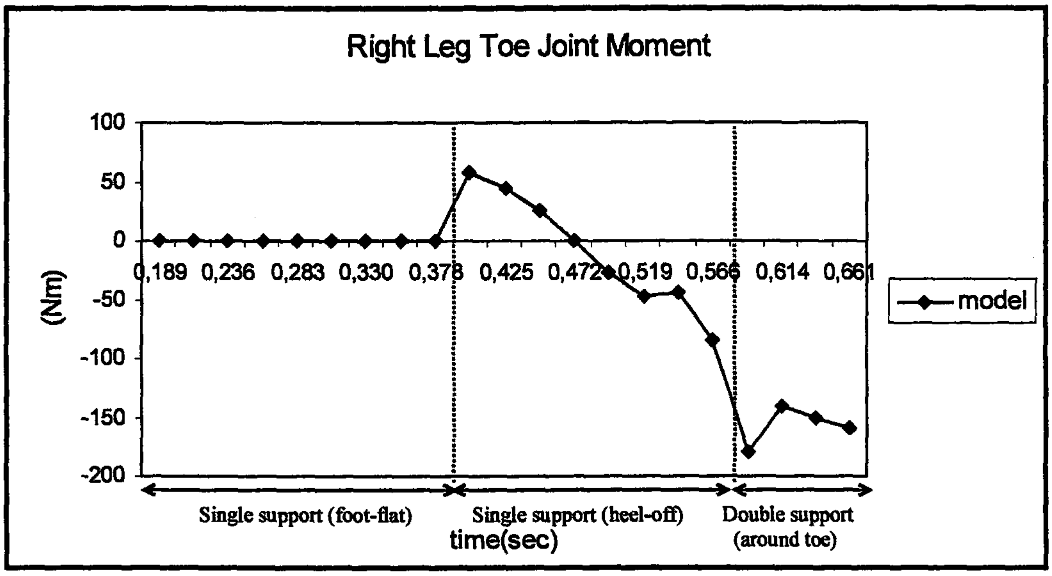

Figure 4.9 Right leg toe joint moment

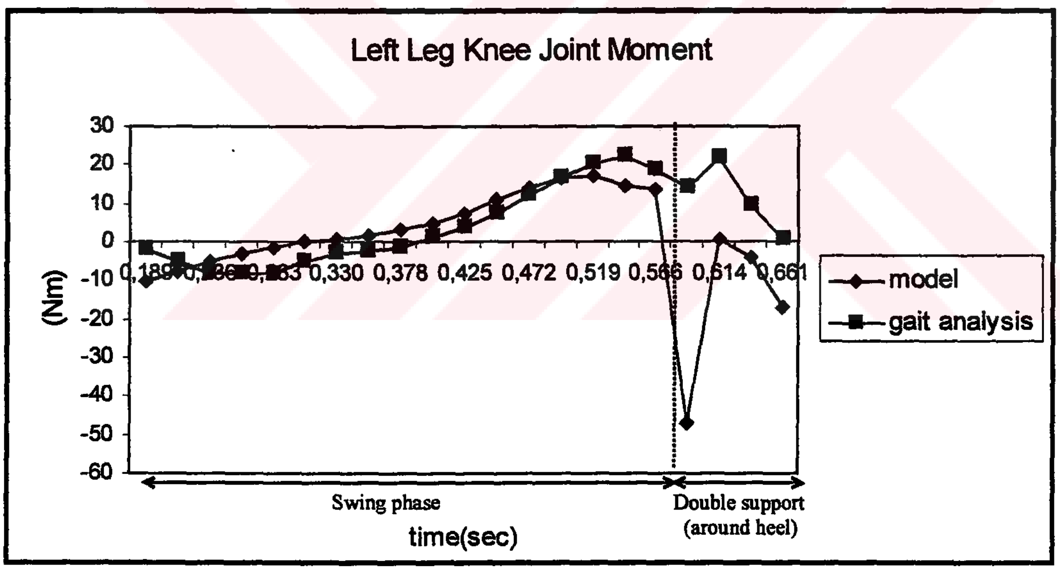

Figure 4.10 Left leg knee joint moment

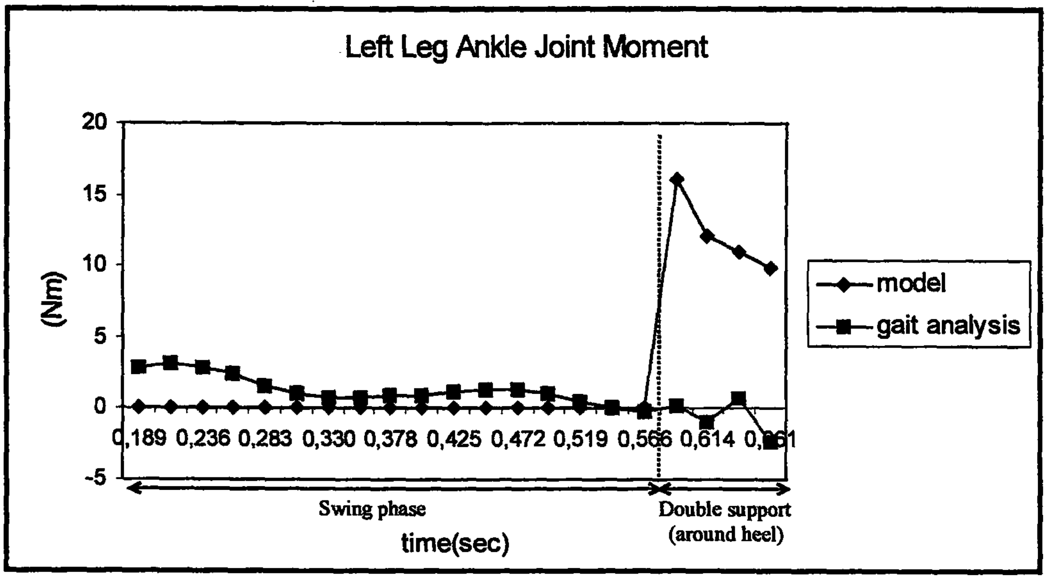

Figure 4.11 Left leg ankle joint moment

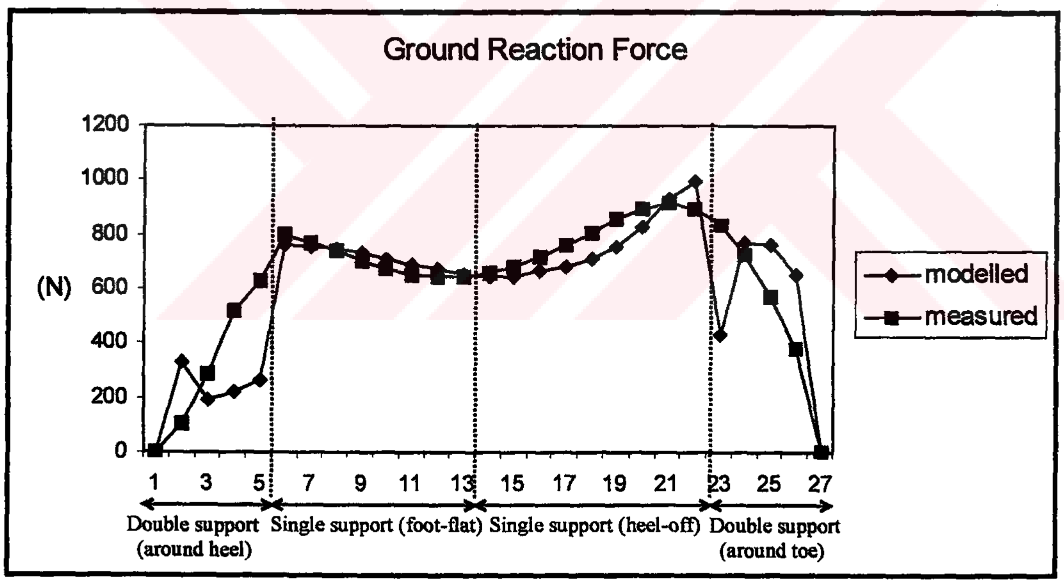

Figure 4.12 Vertical ground reaction force

4.2.3 Disturbed Gait Control

Effectiveness of the computed torque control proposed in this study is tested by introducing disturbances in the form of deviation from the desired trajectories. Such a deviation may be due to the initiation of the gait from a posture which does not conform with the desired trajectory, or may be due to the subject being stumbled during gait. To simulate both, two disturbances are artificially introduced at the beginning and somewhere in the middle of the gait cycle.

As the error differential equation is now excited by these disturbances, suitable lower and upper bounds for the control stiffnesses (ω_k^0 and ω_k^*) as well as the suitable gain scheduling constant (σ_k) need to be chosen. Again several trials have been conducted and the following values are chosen based on the magnitudes of the resulting corrective efforts (joint moments) and corresponding fast convergence to the desired trajectories:

For foot-flat period:

$$q=\begin{bmatrix}\theta_{hr}\\ \theta_{kr}\\ \theta_{hl}\\ \theta_{kl}\end{bmatrix} \qquad \omega^0=\begin{bmatrix}10\\ 10\\ 15\\ 15\end{bmatrix} \qquad \omega^*=\begin{bmatrix}100\\ 100\\ 100\\ 100\end{bmatrix} \qquad \sigma=\begin{bmatrix}4\\ 4\\ 4\\ 4\end{bmatrix}$$

For heel-off period:

$$q=\begin{bmatrix}\theta_{hr}\\ \theta_{kr}\\ \theta_{ar}\\ \theta_{hl}\\ \theta_{kl}\end{bmatrix} \qquad \omega^0=\begin{bmatrix}10\\ 10\\ 10\\ 15\\ 15\end{bmatrix} \qquad \omega^*=\begin{bmatrix}100\\ 100\\ 20\\ 100\\ 100\end{bmatrix} \qquad \sigma=\begin{bmatrix}4\\ 4\\ 4\\ 4\\ 4\end{bmatrix}$$

For double support phase:

$$q = \begin{bmatrix} \theta_{hr} \\ \theta_{kr} \\ \theta_{hl} \\ \theta_{kl} \end{bmatrix} \qquad \omega^0 = \begin{bmatrix} 10 \\ 10 \\ 15 \\ 15 \end{bmatrix} \qquad \omega^* = \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \end{bmatrix} \qquad \sigma = \begin{bmatrix} 4 \\ 4 \\ 4 \\ 4 \end{bmatrix}$$

All through the anaysis the control sampling time is 0.005 sec. and Euler integration method is used for numerical integration.

Figures 4.13-4.16 display the desired and controlled joint angle trajectories. Computed torque control strategy proposed in this study seems to be effective in correcting the gait pattern against the two disturbances introduced artificially. Despite the considerable disturbance given especially at the initiation of gait cycle, the feedback controller designed is able to introduce corrective action within one cycle.

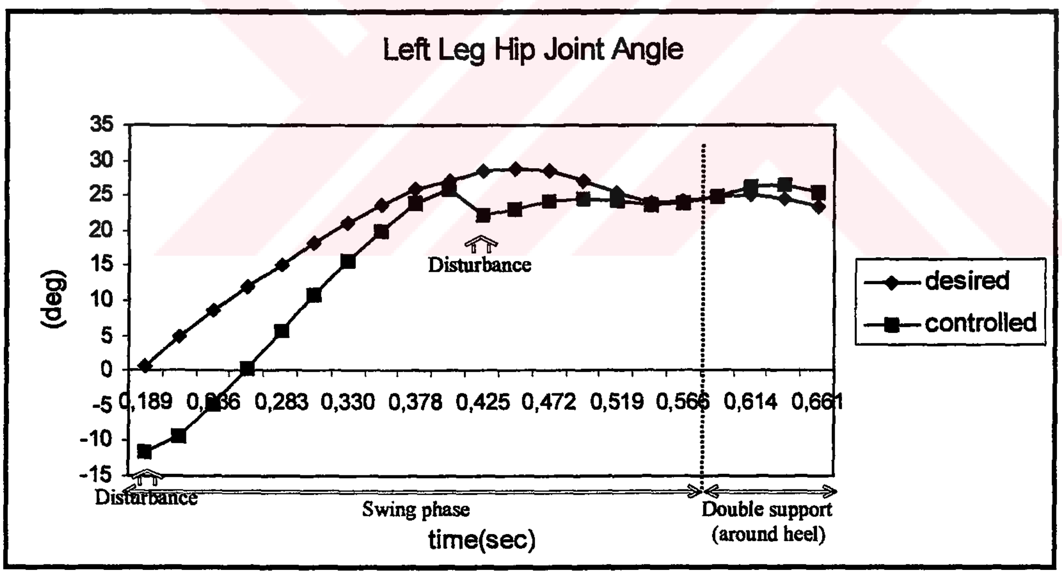

Figure 4.13 Left leg hip joint angle (disturbed)

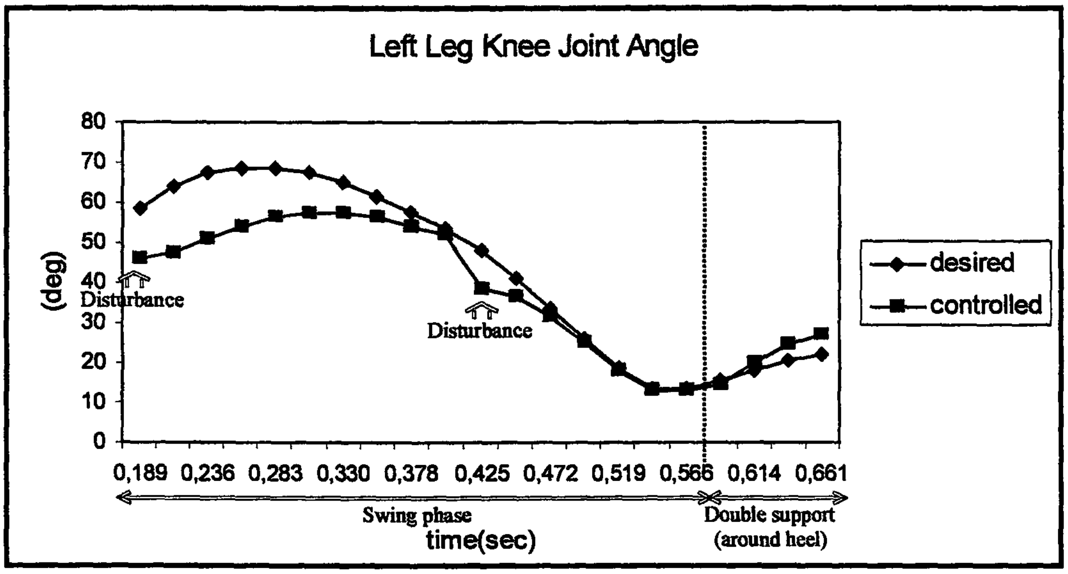

Figure 4.14 Left leg knee joint angle (disturbed)

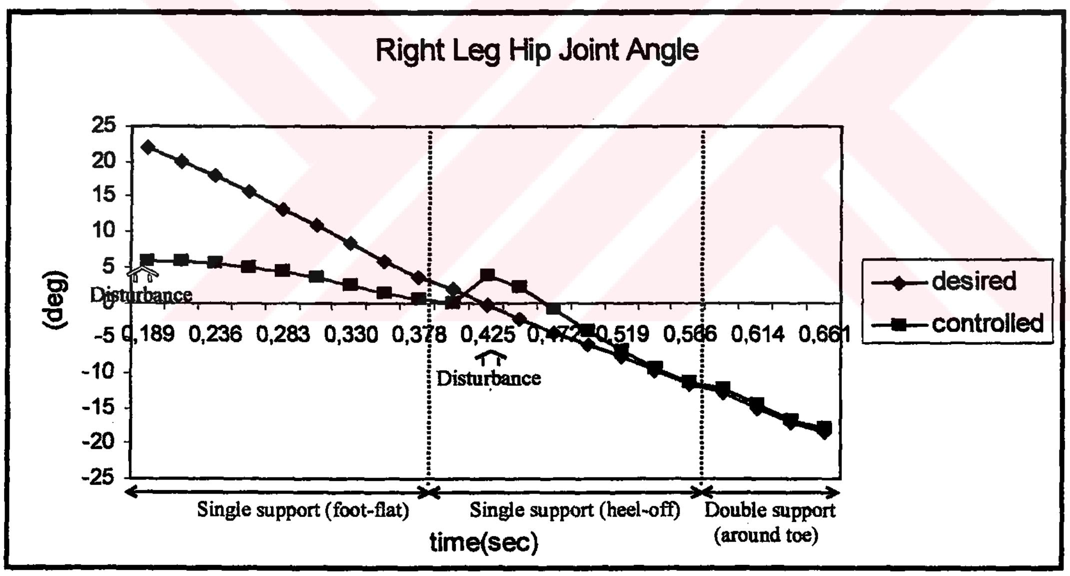

Figure 4.15 Right leg hip joint angle (disturbed)

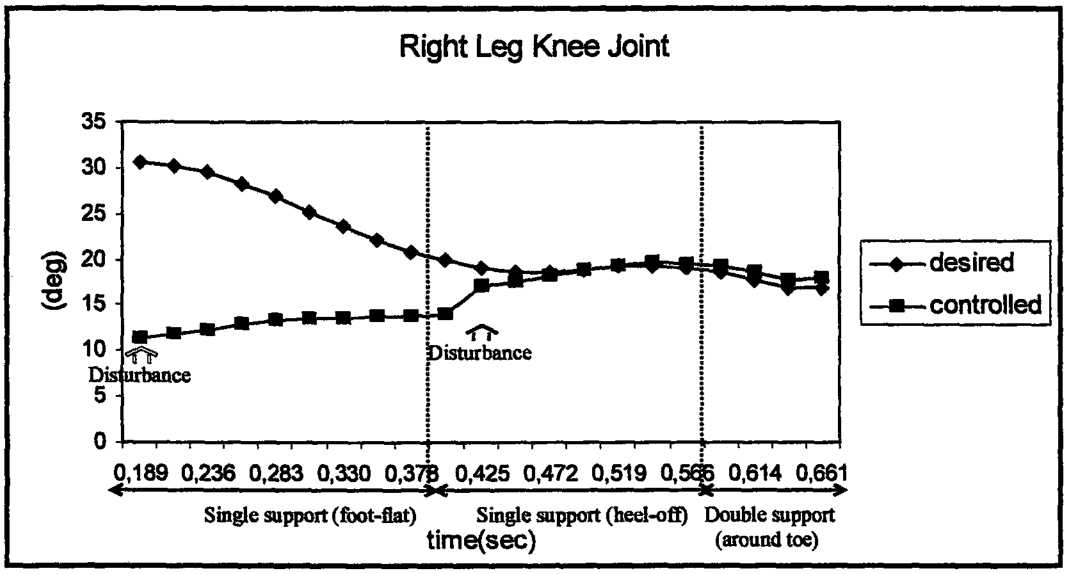

Figure 4.16 Right leg knee joint angle (disturbed)

Figures 4.17-4.22 show the joint moments necessary to stabilize the gait against the disturbances. Although the effect of the disturbance at the initiation of the cycle can be corrected by corrective efforts (joint moments) of reasonable magnitudes, unrealistically high joint moments are involved as the corrective action against the disturbance in the middle of the gait (i.e. stumbling). This is probably due to the sudden stop and movement of the legs at the instant of this disturbance. It is of course possible to reduce the magnitudes of the corrective efforts by changing the control stiffnesses, but then the effectiveness of the feedback controller is lost. It is believed that the primary cause for apparently unrealistic joint moment magnitudes involved is the discrete modelling strategy (i.e. instantly switching from one model to the other which have different kinematic topologies).

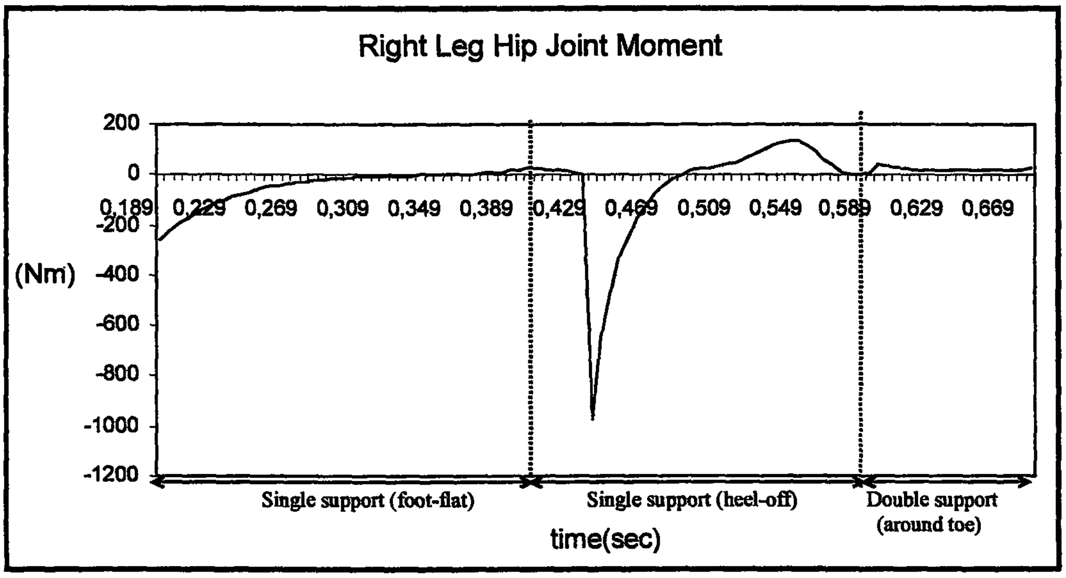

Figure 4.17 Right leg hip joint moment (disturbed)

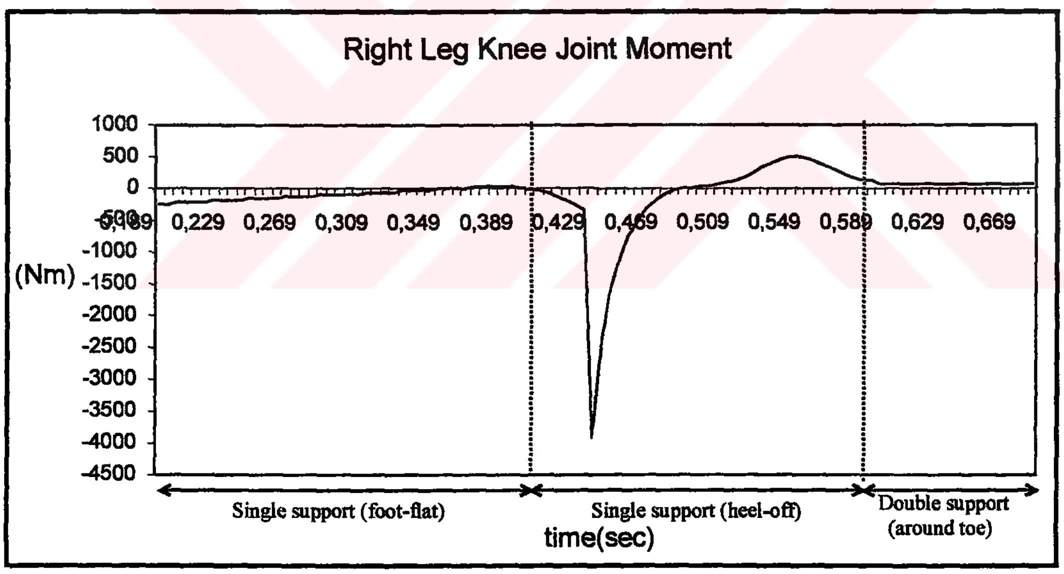

Figure 4.18 Right leg knee joint moment (disturbed)

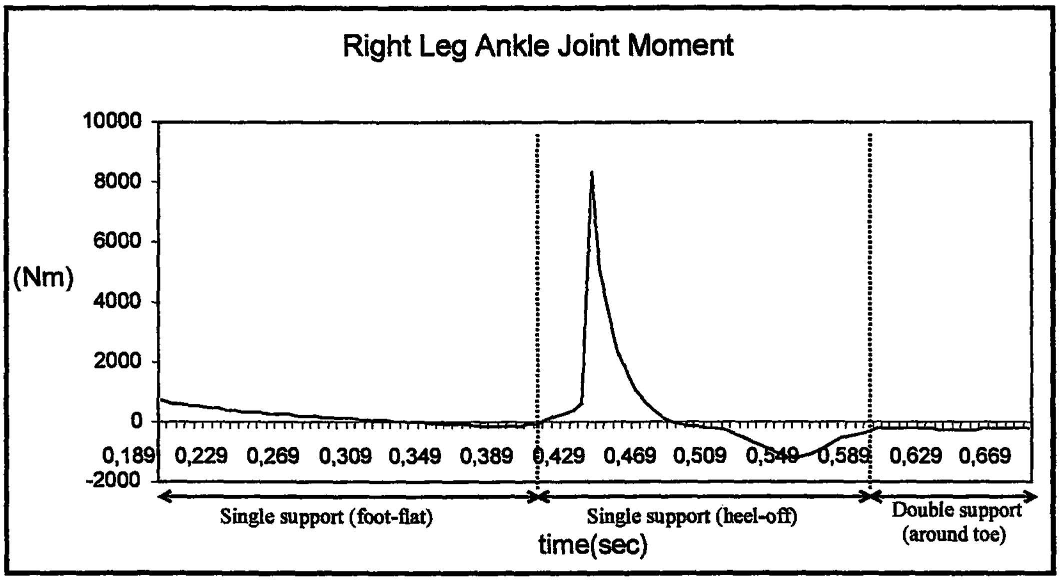

Figure 4.19 Right leg ankle joint moment (disturbed)

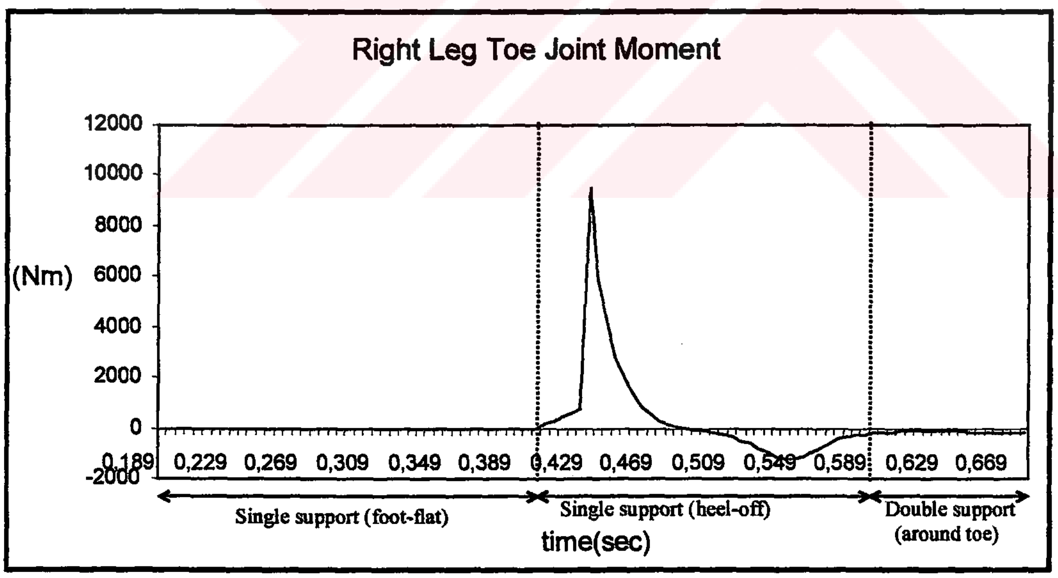

Figure 4.20 Right leg toe joint moment (disturbed)

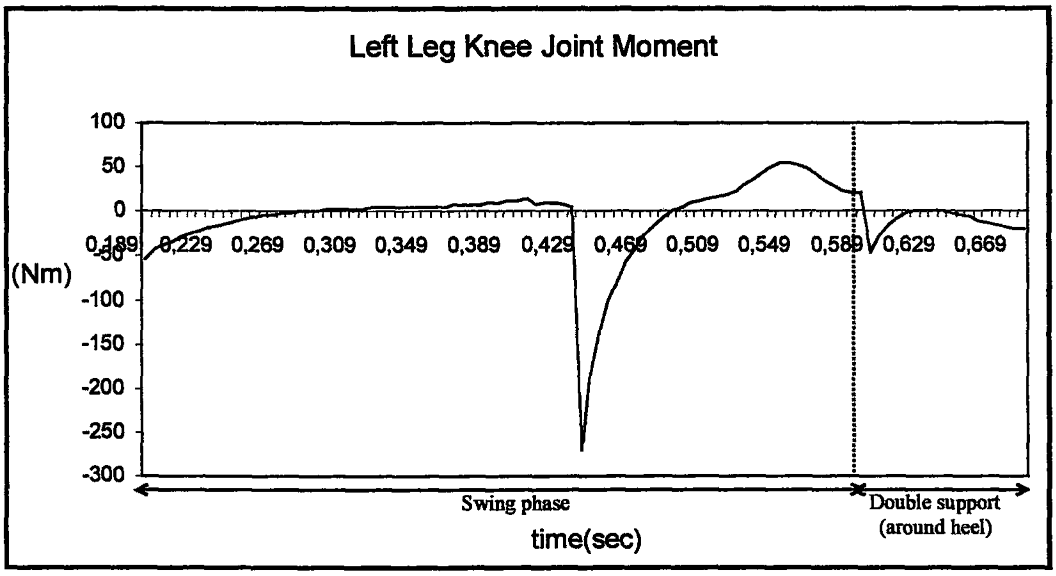

Figure 4.21 Left leg knee joint moment (disturbed)

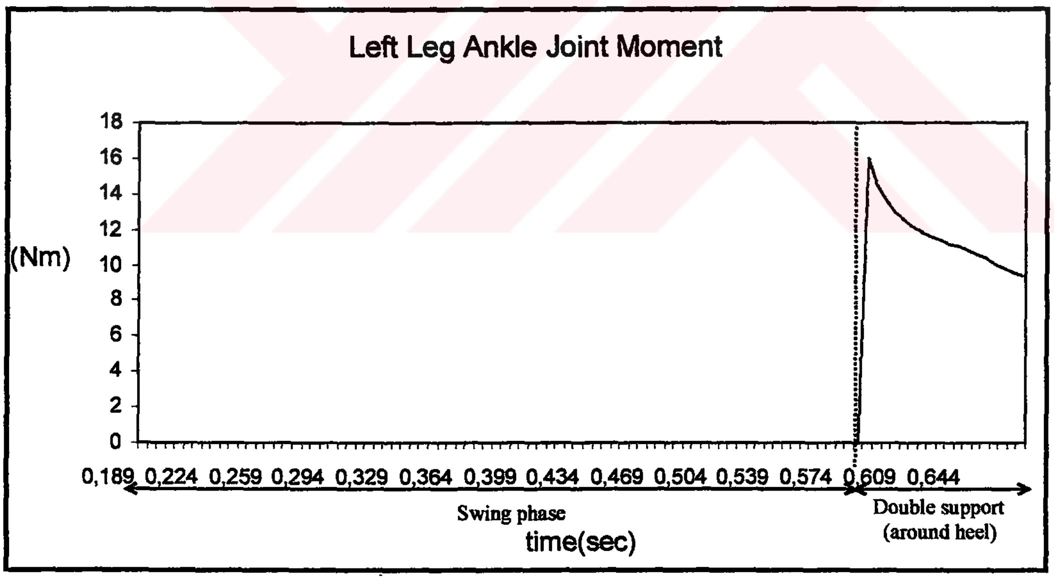

Figure 4.22 Left leg ankle joint moment (disturbed)

4.3 Discussion

The planar linkage model developed in this study to simulate the normal human gait has some drawbacks:

i) While the desired trajectories are being followed by the model, somewhat larger joint rotations occur. This is purely a geometrical problem and is mainly associated with the planar nature of the model. In fact the human gait is a three dimensional process. As mentioned in Chapter 1, there are six determinants of gait. Three of these determinants: the pelvic rotation, the pelvic tilt and lateral body displacement occur in the third dimension, providing balance throughout the gait. Without taking the third dimension into consideration, the effects of these out-of plane motions are ignored, and the required gait pattern is tried to be achieved by the planar movements. This limitation also entails the use of longer limb lengths, which further adds to the apparent deviations of joint angles. Furthermore, modelling the foot-ground interaction by a fixed pivot constraint is also responsible for discrepancies.

ii) The joint torques necessary to maintain the desired gait pattern is reasonably close to those yielded by gait analysis except at the double support phase. Abnormal joint moments encountered at the double support phase is due to the sudden switching from one kinematic model to another having completely different topological characteristics. The model for the double support phase is a closed loop linkage while the preceding model is an open loop one. Although displacements are continuous, higher order derivatives show abrubt changes and the number and nature of the joint torques change instantly. The present study is a first attempt to simulate the complete gait cycle. Previous research is limited to only a portion of the gait cycle, i.e. swing phase, single support or double support phases. Probably it is this inherent difficulty encountered in this study which directed previous researchers to study only one aspect of the gait in isolation from other phases.

Apart from the double support phase, joint torques also show some deviations from gait analysis results, but their magnitudes are at acceptable levels. This means that switching from model 1 to model 2 of the single support phase did not create discontinuity problems as much as it did for switching from single support to double support phases. This is probably due to the fact that both models 1 and 2 are open loop linkages differing only with the addition of foot rotation.

Deviations in joint torques during single support phase may be due to several factors. First of all calculations of the joint torques in the standard gait analysis methodology also involves considerable inaccuracies. After all, it is a well known drawback of standard gait methodology that, if continued upwards, it would give rise to an artificial moment at the top end of the head to satisfy the Newton-Euler formulation adapted. The extend of this inaccuracy depends on the measurement and data processing system used.

Another error arises from the fact that, in the human body, mass of the HAT is not lumped at the hip joint. The mass moment of inertia of the HAT around its mass center is 10.1 kg.m^2 for a person of 1.85 m height and 80 kg mass [4]. When the human gait is examined, continuous movement of the trunk and arms are observed, providing balance but in this model the trunk and the arms are assumed to be making no fore and aft movement.

One of the assumptions made during the modelling stage is taking the right and left hip joint moments equal in magnitude and opposite in direction. As seen in Figure 2.3 this assumption can be verified to some extend but it still causes error.

Finally the geometrical inconsistency between the modelled and measured joint angles is also reflected to the joint moments to some extend.

iii) The computed torque control introduced in this study is capable of regaining the desired gait pattern when it is disturbed. This, however, occurs at the expense of unrealistically high joint moments, especially for the stumbling disturbance introduced at the middle of the gait cycle.

These unrealistic joint moments may also be attributed to the modelling errors discussed before, rather then the computed torque control stategy adapted in this study. However gait control strategy also involves some features which may further be improved.

The gait control stage involves the determination of the control stiffness coefficients and the weighting constants. As mentioned in Section 3.2, the human body applies feedback control in order to achieve satisfactory gait. In this study, feedback control is used in the form of computed torque control. Although feedback control method is used before [11], there have been no studies made about the physical control stiffness values of human gait so the values used in this study are determined on the basis of trial and error. Different from the previous study [11], the control stiffness values are not taken as constant throughout the gait and are not the same for all joints. It is most probable that the human body applies a low stiffening value when the deviation from the desired trajectory is large, and stiffens the control when the deviation gets smaller. Application of a high stiffening value throughout the gait would cause the torques to reach undesirably high values and with the application of a small value, it would take too much time to reach the desired trajectory.

Using different control stiffness coefficients for different joints is another advantage of this study. It is probable that the human body also adjusts its control stiffness according to its need. During the test runs, it was observed that it took more time for the swing leg joint angles to reach the desired trajectory, when compared with the stance leg joint angles. So by increasing the lower stiffness boundary for the swing leg, the time needed is shortened.

The feedback control not only regulates the disturbed gait but also suppresses the errors of numerical integration. Enlargement of the deviations are prevented by the feedback control method.

The second step in the gait control stage is the determination of the weighting constants. In the double support phase, the degree of freedom of the system is four and there are six torques applied on the joints. Probably, the human body is applying some kind of optimization during the application of these torques. In this study, optimization is done by minimizing the performance function (3.58). The weighting constants used in this problem are determined according to the specific needs of the problem in hand. Determining six different constants as appropriate as possible is a very difficult problem. As no such studies on this problem were available in the literature, with trial and error, the most appropriate constants are tried to be found. Changing the weighting constants and the control stiffness coefficients changes the behaviour of the system completely. Hence the selection of these values should be handled together and solved as a different problem in the future studies.

When the results of the inverse dynamics analysis are inspected, the effects of the modelling and gait control stage assumptions are clearly seen. In Figures 4.2 to 4.5 the hip joint and knee joint angles of the gait and in Figures 4.6 to 4.11 the resulting joint torques are presented. The modelling stage and control stage assumptions lead to the deviations of the calculated torques from the experimental results. Although the results obtained are satisfactory to a great extend, the results of double support phase moments and ground reaction forces may be improved by changing the weighting constants.

When the results of the disturbed gait control are inspected, it is seen that the model reaches the desired trajectories in a short time (Figures 4.13 to 4.16). The foot-flat period starts from a different position and during the heel-off period a second stumbling disturbance is applied on the model. To shorten the response time, greater stiffening constants could have been used but in this case the step increases in the moments at the instant of stumbling would reach to very high values. To prevent the sharp increases and decreases in the moments, smaller control stiffness coefficients could be used but this would lengthen the response time.

The main problem in this study was to decide on the weighting constants and control stiffness coefficients. As mentioned before, there were no studies made on the determination of these constants so the values used in this study are found on the basis of trial and error. After making an enlarged analysis on the weighting constants and control stiffness coefficients, the results may be improved.

The foregoing discussion implies that modelling of the foot-ground interaction and confining the body segment motion into a two-dimensional space are the main causes of poor performance of the model.

In the model, the right foot link is taken as connected to the ground from the ankle joint in the foot-flat period and from the toe during the heel-off period and double support phase. The left foot is taken as rigidly connected to the left shank during the single support phase and to the ground from its heel during the double support phase. When experimental data for a human of 1.60 m height is examined [4], it is seen that during the foot-flat and heel-off periods, the center of pressure on the ground moves 110 mm and 39 mm forward respectively. During the double support phase the center of pressure of the right leg moves 20 mm and that of the left leg moves 84 mm forward on the ground. Connecting the links to the ground by a revolute joint makes this movement impossible and forces the ankle joint to move on a circular trajectory. This constraint obliges the desired ankle joint trajectory deviate from the experimental trajectory and hence the desired hip joint and knee joint angels change. To provide a continuous gait (without break off) the link lengths had to be increased, which again causes a change in the desired hip and knee joint trajectories. As a result of these, not only the angels but also their accelerations and velocities change. It can be clearly seen that modelling the stance legs fixed to the ground effects the trajectories and the moments and hence the ground reaction forces in both the single and double support phases. Appropriate modelling of foot-ground interaction is therefore necessary for the enhancement of gait simulation efforts.

CHAPTER 5

SUMMARY AND CONCLUSION

The aim of this study was to develop a simulation model of human gait and investigate the behaviours of the body under various disturbing features. Visual Basic 5.0 was used for the computer program of this study.

The first step was the physical modelling of human gait. Different linkage systems were proposed representing different phases of the gait. The desired trajectories for the joint angles were determined by using the experimental data [23] and by tuning the anthropometric data so as to satisfy the kinematic constraints. Computer programs Jandel Scientific-Table Curve and Sherrod Software-NLREG were used to determine the desired trajectories.

In the second step dynamic equations of the models were obtained by using Lagrange's equations. The derivation steps are handled by Mathematica 2.2 and Mathematica 3.0 computer programs.

The last step was the modelling of gait control. Feedback control was used in the form of computed torque control, to achieve satisfactory gait in both phases. Gain scheduling was applied on the control stiffness coefficients so as to achieve reasonable torques and short response time. The double support phase differs from the single support, as the degree of freedom of the model is 4 while there are 6 control torques, and the linkage system is closed loop. For the solution, the weighted least squares optimization method was introduced. By this method, the elements of the weighting matrix can be selected according to the problem.

Results demonstrated that the computed torque control can be a potentially effective feedback controller describing the actual controls applied by a human body. However, in some instances the necessary controls (joint torques) reached unrealistically high values. These problems were associated with unrealistic assumptions involved in modelling.

In order to improve the present study in view of approaching a more realistic gait simulation model, following future works are recommended:

1- A three-dimensional model can be developed to include the effects of the lateral displacement of the body, the pelvic rotation and the pelvic tilt.

2- Moveable toe and heel joints can be proposed for a better representation of the foot-ground interaction.

3- Head-Arms and Trunk (HAT) can be modelled as separate links and the right and left hip joint moments can be treated separately.

4- Limits for the applied torques can be introduced so as not to exceed the human capacity.

5- An optimization program that handles the calculation of the control stiffness coefficients and the elements of the weighting matrix together, can be generated. An algorithm that minimizes the differences between the calculated and experimental joint torques and ground reaction forces by an objective function can be used for this purpose.

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APPENDIX A-1

THE OPEN FORMS OF FOOT-FLAT PERIOD EQUATIONS

Left hand side of equation 3.14:

$b_t . g . m_f . \text{Sin}(\theta_{hr}) + b_t . g . m_s . \text{Sin}(\theta_{hr}) + d_t . g . m_t . \text{Sin}(\theta_{hr}) + b_s . g . m_f . \text{Sin}(\theta_{hr} - \theta_{kr}) + d_s . g . m_s . \text{Sin}(\theta_{hr} - \theta_{kr}) - 2 . b_s . b_t . m_f . \text{Sin}(\theta_{kr}) . \theta_{hr}' . \theta_{kr}' - 2 . b_t . d_s . m_s . \text{Sin}(\theta_{kr}) . \theta_{hr}' . \theta_{kr}' + b_s . b_t . m_f . \text{Sin}(\theta_{kr}) . \theta_{kr}'^2 + b_t . d_s . m_s . \text{Sin}(\theta_{kr}) . \theta_{kr}'^2 + I_s . \theta_{hr}'' + I_t . \theta_{hr}'' + b_s^2 . m_f . \theta_{hr}'' + b_t^2 . m_f . \theta_{hr}'' + b_t^2 . m_s . \theta_{hr}'' + d_s^2 . m_s . \theta_{hr}'' + d_t^2 . m_t . \theta_{hr}'' + 2 . b_s . b_t . m_f . \text{Cos}(\theta_{kr}) . \theta_{hr}'' + 2 . b_t . d_s . m_s . \text{Cos}(\theta_{kr}) . \theta_{hr}'' - I_s . \theta_{kr}'' - b_s^2 . m_f . \theta_{kr}'' - d_s^2 . m_s . \theta_{kr}'' - b_s . b_t . m_f . \text{Cos}(\theta_{kr}) . \theta_{kr}'' - b_t . d_s . m_s . \text{Cos}(\theta_{kr}) . \theta_{kr}'' + b_t . m_f . \text{Cos}(\theta_{hr}) . x_h'' + b_t . m_s . \text{Cos}(\theta_{hr}) . x_h'' + d_t . m_t . \text{Cos}(\theta_{hr}) . x_h'' + b_s . m_f . \text{Cos}(\theta_{hr} - \theta_{kr}) . x_h'' + d_s . m_s . \text{Cos}(\theta_{hr} - \theta_{kr}) . x_h'' + b_t . m_f . \text{Sin}(\theta_{hr}) . y_h'' + b_t . m_s . \text{Sin}(\theta_{hr}) . y_h'' + d_t . m_t . \text{Sin}(\theta_{hr}) . y_h'' + b_s . m_f . \text{Sin}(\theta_{hr} - \theta_{kr}) . y_h'' + d_s . m_s . \text{Sin}(\theta_{hr} - \theta_{kr}) . y_h''$

Left hand side of equation 3.15:

$-(b_s . g . m_f . \text{Sin}(\theta_{hr} - \theta_{kr})) - d_s . g . m_s . \text{Sin}(\theta_{hr} - \theta_{kr}) + b_s . b_t . m_f . \text{Sin}(\theta_{kr}) . \theta_{hr}'^2 + b_t . d_s . m_s . \text{Sin}(\theta_{kr}) . \theta_{hr}'^2 - I_s . \theta_{hr}'' - b_s^2 . m_f . \theta_{hr}'' - d_s^2 . m_s . \theta_{hr}'' - b_s . b_t . m_f . \text{Cos}(\theta_{kr}) . \theta_{hr}'' - b_t . d_s . m_s . \text{Cos}(\theta_{kr}) . \theta_{hr}'' + I_s . \theta_{kr}'' + b_s^2 . m_f . \theta_{kr}'' + d_s^2 . m_s . \theta_{kr}'' - b_s . m_f . \text{Cos}(\theta_{hr} - \theta_{kr}) . x_h'' - d_s . m_s . \text{Cos}(\theta_{hr} - \theta_{kr}) . x_h'' - b_s . m_f . \text{Sin}(\theta_{hr} - \theta_{kr}) . y_h'' - d_s . m_s . \text{Sin}(\theta_{hr} - \theta_{kr}) . y_h''$

Left hand side of equation 3.16:

$$-(c_f \cdot g \cdot m_f \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl})) + b_t \cdot g \cdot m_f \cdot \mathrm{Sin}(\theta_{hl}) + b_t \cdot g \cdot m_s \cdot \mathrm{Sin}(\theta_{hl}) + d_t \cdot g \cdot m_t \cdot \mathrm{Sin}(\theta_{hl}) + b_s \cdot g \cdot m_f \cdot \mathrm{Sin}(\theta_{hl} - \theta_{kl}) + d_f \cdot g \cdot m_f \cdot \mathrm{Sin}(\theta_{hl} - \theta_{kl}) + d_s \cdot g \cdot m_s \cdot \mathrm{Sin}(\theta_{hl} - \theta_{kl}) - 2 \cdot b_t \cdot c_f \cdot m_f \cdot \mathrm{Cos}(\theta_{kl}) \cdot \theta_{hl}' \cdot \theta_{kl}' - 2 \cdot b_s \cdot b_t \cdot m_f \cdot \mathrm{Sin}(\theta_{kl}) \cdot \theta_{hl}' \cdot \theta_{kl}' - 2 \cdot b_t \cdot d_f \cdot m_f \cdot \mathrm{Sin}(\theta_{kl}) \cdot \theta_{hl}' \cdot \theta_{kl}' - 2 \cdot b_t \cdot d_s \cdot m_s \cdot \mathrm{Sin}(\theta_{kl}) \cdot \theta_{hl}' \cdot \theta_{kl}' + b_t \cdot c_f \cdot m_f \cdot \mathrm{Cos}(\theta_{kl}) \cdot \theta_{kl}'^2 + b_s \cdot b_t \cdot m_f \cdot \mathrm{Sin}(\theta_{kl}) \cdot \theta_{kl}'^2 + b_t \cdot d_f \cdot m_f \cdot \mathrm{Sin}(\theta_{kl}) \cdot \theta_{kl}'^2 + b_t \cdot d_s \cdot m_s \cdot \mathrm{Sin}(\theta_{kl}) \cdot \theta_{kl}'^2 + I_f \cdot \theta_{hl}'' + I_s \cdot \theta_{hl}'' + I_t \cdot \theta_{hl}'' + b_s^2 \cdot m_f \cdot \theta_{hl}'' + b_t^2 \cdot m_f \cdot \theta_{hl}'' + c_f^2 \cdot m_f \cdot \theta_{hl}'' + 2 \cdot b_s \cdot d_f \cdot m_f \cdot \theta_{hl}'' + d_f^2 \cdot m_f \cdot \theta_{hl}'' + b_t^2 \cdot m_s \cdot \theta_{hl}'' + d_s^2 \cdot m_s \cdot \theta_{hl}'' + d_t^2 \cdot m_t \cdot \theta_{hl}'' + 2 \cdot b_s \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{kl}) \cdot \theta_{hl}'' + 2 \cdot b_t \cdot d_f \cdot m_f \cdot \mathrm{Cos}(\theta_{kl}) \cdot \theta_{hl}'' + 2 \cdot b_t \cdot d_s \cdot m_s \cdot \mathrm{Cos}(\theta_{kl}) \cdot \theta_{hl}'' - 2 \cdot b_t \cdot c_f \cdot m_f \cdot \mathrm{Sin}(\theta_{kl}) \cdot \theta_{hl}'' - I_f \cdot \theta_{kl}'' - I_s \cdot \theta_{kl}'' - b_s^2 \cdot m_f \cdot \theta_{kl}'' - c_f^2 \cdot m_f \cdot \theta_{kl}'' - 2 \cdot b_s \cdot d_f \cdot m_f \cdot \theta_{kl}'' - d_f^2 \cdot m_f \cdot \theta_{kl}'' - d_s^2 \cdot m_s \cdot \theta_{kl}'' - b_s \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{kl}) \cdot \theta_{kl}'' - b_t \cdot d_f \cdot m_f \cdot \mathrm{Cos}(\theta_{kl}) \cdot \theta_{kl}'' - b_t \cdot d_s \cdot m_s \cdot \mathrm{Cos}(\theta_{kl}) \cdot \theta_{kl}'' + b_t \cdot c_f \cdot m_f \cdot \mathrm{Sin}(\theta_{kl}) \cdot \theta_{kl}'' + b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hl}) \cdot x_h'' + b_t \cdot m_s \cdot \mathrm{Cos}(\theta_{hl}) \cdot x_h'' + d_t \cdot m_t \cdot \mathrm{Cos}(\theta_{hl}) \cdot x_h'' + b_s \cdot m_f \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot x_h'' + d_f \cdot m_f \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot x_h'' + d_s \cdot m_s \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot x_h'' + c_f \cdot m_f \cdot \mathrm{Sin}(\theta_{hl} - \theta_{kl}) \cdot x_h'' - c_f \cdot m_f \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot y_h'' + b_t \cdot m_f \cdot \mathrm{Sin}(\theta_{hl}) \cdot y_h'' + b_t \cdot m_s \cdot \mathrm{Sin}(\theta_{hl}) \cdot y_h'' + d_t \cdot m_t \cdot \mathrm{Sin}(\theta_{hl}) \cdot y_h'' + b_s \cdot m_f \cdot \mathrm{Sin}(\theta_{hl} - \theta_{kl}) \cdot y_h'' + d_f \cdot m_f \cdot \mathrm{Sin}(\theta_{hl} - \theta_{kl}) \cdot y_h'' + d_s \cdot m_s \cdot \mathrm{Sin}(\theta_{hl} - \theta_{kl}) \cdot y_h''$$

Left hand side of equation 3.17:

$$c_f \cdot g \cdot m_f \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) - b_s \cdot g \cdot m_f \cdot \mathrm{Sin}(\theta_{hl} - \theta_{kl}) - d_f \cdot g \cdot m_f \cdot \mathrm{Sin}(\theta_{hl} - \theta_{kl}) - d_s \cdot g \cdot m_s \cdot \mathrm{Sin}(\theta_{hl} - \theta_{kl}) + b_t \cdot c_f \cdot m_f \cdot \mathrm{Cos}(\theta_{kl}) \cdot \theta_{hl}'^2 + b_s \cdot b_t \cdot m_f \cdot \mathrm{Sin}(\theta_{kl}) \cdot \theta_{hl}'^2 + b_t \cdot d_f \cdot m_f \cdot \mathrm{Sin}(\theta_{kl}) \cdot \theta_{hl}'^2 + b_t \cdot d_s \cdot m_s \cdot \mathrm{Sin}(\theta_{kl}) \cdot \theta_{hl}'^2 - I_f \cdot \theta_{hl}'' - I_s \cdot \theta_{hl}'' - b_s^2 \cdot m_f \cdot \theta_{hl}'' - c_f^2 \cdot m_f \cdot \theta_{hl}'' - 2 \cdot b_s \cdot d_f \cdot m_f \cdot \theta_{hl}'' - d_f^2 \cdot m_f \cdot \theta_{hl}'' - d_s^2 \cdot m_s \cdot \theta_{hl}'' - b_s \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{kl}) \cdot \theta_{hl}'' - b_t \cdot d_f \cdot m_f \cdot \mathrm{Cos}(\theta_{kl}) \cdot \theta_{hl}'' - b_t \cdot d_s \cdot m_s \cdot \mathrm{Cos}(\theta_{kl}) \cdot \theta_{hl}'' + b_t \cdot c_f \cdot m_f \cdot \mathrm{Sin}(\theta_{kl}) \cdot \theta_{hl}'' + I_f \cdot \theta_{kl}'' + I_s \cdot \theta_{kl}'' + b_s^2 \cdot m_f \cdot \theta_{kl}'' + c_f^2 \cdot m_f \cdot \theta_{kl}'' + 2 \cdot b_s \cdot d_f \cdot m_f \cdot \theta_{kl}'' + d_f^2 \cdot m_f \cdot \theta_{kl}'' + d_s^2 \cdot m_s \cdot \theta_{kl}'' - b_s \cdot m_f \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot x_h'' - d_f \cdot m_f \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot x_h'' - d_s \cdot m_s \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot x_h'' - c_f \cdot m_f \cdot \mathrm{Sin}(\theta_{hl} - \theta_{kl}) \cdot x_h'' + c_f \cdot$$

$m_f . \text{Cos}(\theta_{hl} - \theta_{kl}) . \; y_h'' - \; b_s . m_f . \text{Sin}(\theta_{hl} - \theta_{kl}) . \; y_h'' - \; d_f . m_f . \text{Sin}(\theta_{hl} - \theta_{kl}) . \; y_h'' - d_s . m_s . \text{Sin}(\theta_{hl} - \theta_{kl}) . \; y_h''$

Left hand side of equation 3.18:

$$m_t . (-(d_t . \text{Sin}(\theta_{hl}) . \theta_{hl}'^2) + d_t .\text{Cos}(\theta_{hl}) . \theta_{hl}'' + x_h'') + m_t . (-(d_t . \text{Sin}(\theta_{hr}) . \theta_{hr}'^2) + d_t .\text{Cos}(\theta_{hr}) . \theta_{hr}'' + x_h'') + m_s . (-(b_t . \text{Sin}(\theta_{hl}) . \theta_{hl}'^2) - d_s . \text{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')^2 + b_t .\text{Cos}(\theta_{hl}) . \theta_{hl}'' + d_s .\text{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}'' - \theta_{kl}'') + x_h'') + m_f . (-(b_t . \text{Sin}(\theta_{hl}) . \theta_{hl}'^2) + c_f .\text{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')^2 - (b_s + d_f) . \text{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')^2 + b_t .\text{Cos}(\theta_{hl}) . \theta_{hl}'' + (b_s + d_f) .\text{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}'' - \theta_{kl}'') + c_f . \text{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}'' - \theta_{kl}'') + x_h'') + m_f . (-(b_t . \text{Sin}(\theta_{hr}) . \theta_{hr}'^2) - b_s . \text{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 + b_t .\text{Cos}(\theta_{hr}) . \theta_{hr}'' + b_s .\text{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}'' - \theta_{kr}'') + x_h'') + m_s . (-(b_t . \text{Sin}(\theta_{hr}) . \theta_{hr}'^2) - d_s . \text{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 + b_t .\text{Cos}(\theta_{hr}) . \theta_{hr}'' + d_s .\text{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}'' - \theta_{kr}'') + x_h'') + m_b . x_h''$$

Left hand side of equation 3.19:

$$2 . g . (m_f + m_s + m_t) + m_t . (d_t .\text{Cos}(\theta_{hl}) . \theta_{hl}'^2 + d_t . \text{Sin}(\theta_{hl}) . \theta_{hl}'' + y_h'') + m_t . (d_t .\text{Cos}(\theta_{hr}) . \theta_{hr}'^2 + d_t . \text{Sin}(\theta_{hr}) . \theta_{hr}'' + y_h'') + m_f . (b_t . \text{Cos}(\theta_{hl}) . \theta_{hl}'^2 + (b_s + d_f) .\text{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')^2 + c_f . \text{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')^2 + b_t . \text{Sin}(\theta_{hl}) . \theta_{hl}'' - c_f .\text{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}'' - \theta_{kl}'') + (b_s + d_f) . \text{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}'' - \theta_{kl}'') + y_h'') + m_s . (b_t .\text{Cos}(\theta_{hl}) . \theta_{hl}'^2 + d_s .\text{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')^2 + b_t . \text{Sin}(\theta_{hl}) . \theta_{hl}'' + d_s . \text{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}'' - \theta_{kl}'') + y_h'') + m_f . (b_t .\text{Cos}(\theta_{hr}) . \theta_{hr}'^2 + b_s .\text{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 + b_t . \text{Sin}(\theta_{hr}) . \theta_{hr}'' + b_s . \text{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}'' - \theta_{kr}'') + y_h'') + m_s . (b_t .\text{Cos}(\theta_{hr}) . \theta_{hr}'^2 + d_s .\text{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 + b_t . \text{Sin}(\theta_{hr}) . \theta_{hr}'' + d_s . \text{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}'' - \theta_{kr}'') + y_h'') + m_b .g$$

APPENDIX A-2

THE OPEN FORMS OF HEEL-OFF PERIOD EQUATIONS

Left hand side of equation 3.24:

$$
\begin{aligned}
&-(d_{fr} \cdot g \cdot m_f.\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})) + b_t \cdot g \cdot m_f \cdot \mathrm{Sin}(\theta_{hr}) + b_t \cdot g \cdot m_s \cdot \mathrm{Sin}(\theta_{hr}) + d_t \cdot \\
&g \cdot m_t \cdot \mathrm{Sin}(\theta_{hr}) + b_s \cdot g \cdot m_f \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + d_s \cdot g \cdot m_s \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + b_s \cdot d_{fr} \cdot \\
&m_f.\mathrm{Cos}(\alpha_{fr} + \theta_{ar}) \cdot \theta_{ar}'^2 + b_t \cdot d_{fr} \cdot m_f.\mathrm{Cos}(\alpha_{fr} + \theta_{ar} - \theta_{kr}) \cdot \theta_{ar}'^2 + 2 \cdot b_s \cdot d_{fr} \cdot \\
&m_f.\mathrm{Cos}(\alpha_{fr} + \theta_{ar}) \cdot \theta_{ar}' \cdot \theta_{hr}' + 2 \cdot b_t \cdot d_{fr} \cdot m_f.\mathrm{Cos}(\alpha_{fr} + \theta_{ar} - \theta_{kr}) \cdot \theta_{ar}' \cdot \theta_{hr}' - 2 \cdot b_s \cdot d_{fr} \cdot \\
&m_f.\mathrm{Cos}(\alpha_{fr} + \theta_{ar}) \cdot \theta_{ar}' \cdot \theta_{kr}' - 2 \cdot b_t \cdot d_{fr} \cdot m_f.\mathrm{Cos}(\alpha_{fr} + \theta_{ar} - \theta_{kr}) \cdot \theta_{ar}' \cdot \theta_{kr}' - 2 \cdot b_t \cdot d_{fr} \cdot \\
&m_f.\mathrm{Cos}(\alpha_{fr} + \theta_{ar} - \theta_{kr}) \cdot \theta_{hr}' \cdot \theta_{kr}' - 2 \cdot b_s \cdot b_t \cdot m_f \cdot \mathrm{Sin}(\theta_{kr}) \cdot \theta_{hr}' \cdot \theta_{kr}' - 2 \cdot b_t \cdot d_s \cdot m_s \cdot \\
&\mathrm{Sin}(\theta_{kr}) \cdot \theta_{hr}' \cdot \theta_{kr}' + b_t \cdot d_{fr} \cdot m_f.\mathrm{Cos}(\alpha_{fr} + \theta_{ar} - \theta_{kr}) \cdot \theta_{kr}'^2 + b_s \cdot b_t \cdot m_f \cdot \mathrm{Sin}(\theta_{kr}) \cdot \theta_{kr}'^2 \\
&+ b_t \cdot d_s \cdot m_s \cdot \mathrm{Sin}(\theta_{kr}) \cdot \theta_{kr}'^2 + I_f \cdot \theta_{ar}'' + d_{fr}^2 \cdot m_f \cdot \theta_{ar}'' + b_s \cdot d_{fr} \cdot m_f.\mathrm{Sin}(\alpha_{fr} + \theta_{ar}) \cdot \\
&\theta_{ar}'' + b_t \cdot d_{fr} \cdot m_f \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} - \theta_{kr}) \cdot \theta_{ar}'' + I_f \cdot \theta_{hr}'' + I_s \cdot \theta_{hr}'' + I_t \cdot \theta_{hr}'' + b_s^2 \cdot m_f \cdot \\
&\theta_{hr}'' + b_t^2 \cdot m_f \cdot \theta_{hr}'' + d_{fr}^2 \cdot m_f \cdot \theta_{hr}'' + b_t^2 \cdot m_s \cdot \theta_{hr}'' + d_s^2 \cdot m_s \cdot \theta_{hr}'' + d_t^2 \cdot m_t \cdot \theta_{hr}'' \\
&+ 2 \cdot b_s \cdot b_t \cdot m_f.\mathrm{Cos}(\theta_{kr}) \cdot \theta_{hr}'' + 2 \cdot b_t \cdot d_s \cdot m_s.\mathrm{Cos}(\theta_{kr}) \cdot \theta_{hr}'' + 2 \cdot b_s \cdot d_{fr} \cdot m_f \cdot \\
&\mathrm{Sin}(\alpha_{fr} + \theta_{ar}) \cdot \theta_{hr}'' + 2 \cdot b_t \cdot d_{fr} \cdot m_f \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} - \theta_{kr}) \cdot \theta_{hr}'' - I_f \cdot \theta_{kr}'' - I_s \cdot \theta_{kr}'' - b_s^2 \cdot \\
&m_f \cdot \theta_{kr}'' - d_{fr}^2 \cdot m_f \cdot \theta_{kr}'' - d_s^2 \cdot m_s \cdot \theta_{kr}'' - b_s \cdot b_t \cdot m_f.\mathrm{Cos}(\theta_{kr}) \cdot \theta_{kr}'' - b_t \cdot d_s \cdot \\
&m_s.\mathrm{Cos}(\theta_{kr}) \cdot \theta_{kr}'' - 2 \cdot b_s \cdot d_{fr} \cdot m_f \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar}) \cdot \theta_{kr}'' - b_t \cdot d_{fr} \cdot m_f \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} - \\
&\theta_{kr}) \cdot \theta_{kr}'' + b_t \cdot m_f.\mathrm{Cos}(\theta_{hr}) \cdot x_h'' + b_t \cdot m_s.\mathrm{Cos}(\theta_{hr}) \cdot x_h'' + d_t \cdot m_t.\mathrm{Cos}(\theta_{hr}) \cdot x_h'' \\
&+ b_s \cdot m_f.\mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot x_h'' + d_s \cdot m_s.\mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot x_h'' + d_{fr} \cdot m_f \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} \\
&+ \theta_{hr} - \theta_{kr}) \cdot x_h'' - d_{fr} \cdot m_f.\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot y_h'' + b_t \cdot m_f \cdot \mathrm{Sin}(\theta_{hr}) \cdot y_h'' + b_t \cdot \\
&m_s \cdot \mathrm{Sin}(\theta_{hr}) \cdot y_h'' + d_t \cdot m_t \cdot \mathrm{Sin}(\theta_{hr}) \cdot y_h'' + b_s \cdot m_f \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot y_h'' + d_s \cdot m_s \cdot \\
&\mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot y_h''
\end{aligned}
$$

Left hand side of equation 3.25:

$$d_{fr} \cdot g \cdot m_f \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_s \cdot g \cdot m_f \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - d_s \cdot g \cdot m_s \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr})$$
$$- b_s \cdot d_{fr} \cdot m_f \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar}) \cdot \theta_{ar}'^2 - 2 \cdot b_s \cdot d_{fr} \cdot m_f \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar}) \cdot \theta_{ar}' \cdot \theta_{hr}' + b_t \cdot d_{fr} \cdot$$
$$m_f \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} - \theta_{kr}) \cdot \theta_{hr}'^2 + b_s \cdot b_t \cdot m_f \cdot \mathrm{Sin}(\theta_{kr}) \cdot \theta_{hr}'^2 + b_t \cdot d_s \cdot m_s \cdot \mathrm{Sin}(\theta_{kr}) \cdot \theta_{hr}'^2$$
$$+ 2 \cdot b_s \cdot d_{fr} \cdot m_f \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar}) \cdot \theta_{ar}' \cdot \theta_{kr}' - I_f \cdot \theta_{ar}'' - d_{fr}^2 \cdot m_f \cdot \theta_{ar}'' - b_s \cdot d_{fr} \cdot m_f \cdot \mathrm{Sin}(\alpha_{fr}$$
$$+ \theta_{ar}) \cdot \theta_{ar}'' - I_f \cdot \theta_{hr}'' - I_s \cdot \theta_{hr}'' - b_s^2 \cdot m_f \cdot \theta_{hr}'' - d_{fr}^2 \cdot m_f \cdot \theta_{hr}'' - d_s^2 \cdot m_s \cdot \theta_{hr}'' - b_s \cdot b_t \cdot$$
$$m_f \cdot \mathrm{Cos}(\theta_{kr}) \cdot \theta_{hr}'' - b_t \cdot d_s \cdot m_s \cdot \mathrm{Cos}(\theta_{kr}) \cdot \theta_{hr}'' - 2 \cdot b_s \cdot d_{fr} \cdot m_f \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar}) \cdot \theta_{hr}'' - b_t \cdot$$
$$d_{fr} \cdot m_f \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} - \theta_{kr}) \cdot \theta_{hr}'' + I_f \cdot \theta_{kr}'' + I_s \cdot \theta_{kr}'' + b_s^2 \cdot m_f \cdot \theta_{kr}'' + d_{fr}^2 \cdot m_f \cdot \theta_{kr}'' +$$
$$d_s^2 \cdot m_s \cdot \theta_{kr}'' + 2 \cdot b_s \cdot d_{fr} \cdot m_f \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar}) \cdot \theta_{kr}'' - b_s \cdot m_f \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot x_h'' - d_s \cdot$$
$$m_s \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot x_h'' - d_{fr} \cdot m_f \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot x_h'' + d_{fr} \cdot m_f \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar}$$
$$+ \theta_{hr} - \theta_{kr}) \cdot y_h'' - b_s \cdot m_f \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot y_h'' - d_s \cdot m_s \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot y_h''$$

Left hand side of equation 3.26:

$$-(d_{fr} \cdot g \cdot m_f \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})) - b_s \cdot d_{fr} \cdot m_f \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar}) \cdot \theta_{hr}'^2 - b_t \cdot d_{fr} \cdot$$
$$m_f \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} - \theta_{kr}) \cdot \theta_{hr}'^2 + 2 \cdot b_s \cdot d_{fr} \cdot m_f \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar}) \cdot \theta_{hr}' \cdot \theta_{kr}' - b_s \cdot d_{fr} \cdot$$
$$m_f \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar}) \cdot \theta_{kr}'^2 + I_f \cdot \theta_{ar}'' + d_{fr}^2 \cdot m_f \cdot \theta_{ar}'' + I_f \cdot \theta_{hr}'' + d_{fr}^2 \cdot m_f \cdot \theta_{hr}'' + b_s \cdot d_{fr} \cdot$$
$$m_f \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar}) \cdot \theta_{hr}'' + b_t \cdot d_{fr} \cdot m_f \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} - \theta_{kr}) \cdot \theta_{hr}'' - I_f \cdot \theta_{kr}'' - d_{fr}^2 \cdot m_f \cdot \theta_{kr}'' -$$
$$b_s \cdot d_{fr} \cdot m_f \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar}) \cdot \theta_{kr}'' + d_{fr} \cdot m_f \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot x_h'' - d_{fr} \cdot$$
$$m_f \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot y_h''$$

Left hand side of equation 3.27:

$$-(c_f \cdot g \cdot m_f \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl})) + b_t \cdot g \cdot m_f \cdot \mathrm{Sin}(\theta_{hl}) + b_t \cdot g \cdot m_s \cdot \mathrm{Sin}(\theta_{hl}) + d_t \cdot g \cdot m_t \cdot$$
$$\mathrm{Sin}(\theta_{hl}) + b_s \cdot g \cdot m_f \cdot \mathrm{Sin}(\theta_{hl} - \theta_{kl}) + d_f \cdot g \cdot m_f \cdot \mathrm{Sin}(\theta_{hl} - \theta_{kl}) + d_s \cdot g \cdot m_s \cdot \mathrm{Sin}(\theta_{hl} - \theta_{kl}) -$$
$$2 \cdot b_t \cdot c_f \cdot m_f \cdot \mathrm{Cos}(\theta_{kl}) \cdot \theta_{hl}' \cdot \theta_{kl}' - 2 \cdot b_s \cdot b_t \cdot m_f \cdot \mathrm{Sin}(\theta_{kl}) \cdot \theta_{hl}' \cdot \theta_{kl}' - 2 \cdot b_t \cdot d_f \cdot m_f \cdot$$
$$\mathrm{Sin}(\theta_{kl}) \cdot \theta_{hl}' \cdot \theta_{kl}' - 2 \cdot b_t \cdot d_s \cdot m_s \cdot \mathrm{Sin}(\theta_{kl}) \cdot \theta_{hl}' \cdot \theta_{kl}' + b_t \cdot c_f \cdot m_f \cdot \mathrm{Cos}(\theta_{kl}) \cdot \theta_{kl}'^2 +$$
$$b_s \cdot b_t \cdot m_f \cdot \mathrm{Sin}(\theta_{kl}) \cdot \theta_{kl}'^2 + b_t \cdot d_f \cdot m_f \cdot \mathrm{Sin}(\theta_{kl}) \cdot \theta_{kl}'^2 + b_t \cdot d_s \cdot m_s \cdot \mathrm{Sin}(\theta_{kl}) \cdot \theta_{kl}'^2 +$$
$$I_f \cdot \theta_{hl}'' + I_s \cdot \theta_{hl}'' + I_t \cdot \theta_{hl}'' + b_s^2 \cdot m_f \cdot \theta_{hl}'' + b_t^2 \cdot m_f \cdot \theta_{hl}'' + c_f^2 \cdot m_f \cdot \theta_{hl}'' + 2 \cdot b_s \cdot d_f \cdot$$

$m_f . \theta_{hl}'' + d_f^2 . m_f . \theta_{hl}'' + b_t^2 . m_s . \theta_{hl}'' + d_s^2 . m_s . \theta_{hl}'' + d_t^2 . m_t . \theta_{hl}'' + 2 . b_s . b_t .$
$m_f .\mathrm{Cos}(\theta_{kl}) . \theta_{hl}'' + 2 . b_t . d_f . m_f .\mathrm{Cos}(\theta_{kl}) . \theta_{hl}'' + 2 . b_t . d_s . m_s .\mathrm{Cos}(\theta_{kl}) . \theta_{hl}'' - 2 . b_t .$
$c_f . m_f . \mathrm{Sin}(\theta_{kl}) . \theta_{hl}'' - I_f . \theta_{kl}'' - I_s . \theta_{kl}'' - b_s^2 . m_f . \theta_{kl}'' - c_f^2 . m_f . \theta_{kl}'' - 2 . b_s . d_f . m_f .$
$\theta_{kl}'' - d_f^2 . m_f . \theta_{kl}'' - d_s^2 . m_s . \theta_{kl}'' - b_s . b_t . m_f .\mathrm{Cos}(\theta_{kl}) . \theta_{kl}'' - b_t . d_f . m_f .\mathrm{Cos}(\theta_{kl}) .$
$\theta_{kl}'' - b_t . d_s . m_s .\mathrm{Cos}(\theta_{kl}) . \theta_{kl}'' + b_t . c_f . m_f . \mathrm{Sin}(\theta_{kl}) . \theta_{kl}'' + b_t . m_f .\mathrm{Cos}(\theta_{hl}) . x_h''$
$+ b_t . m_s .\mathrm{Cos}(\theta_{hl}) . x_h'' + d_t . m_t .\mathrm{Cos}(\theta_{hl}) . x_h'' + b_s . m_f .\mathrm{Cos}(\theta_{hl} - \theta_{kl}) . x_h'' + d_f .$
$m_f .\mathrm{Cos}(\theta_{hl} - \theta_{kl}) . x_h'' + d_s . m_s .\mathrm{Cos}(\theta_{hl} - \theta_{kl}) . x_h'' + c_f . m_f . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . x_h'' - c_f .$
$m_f .\mathrm{Cos}(\theta_{hl} - \theta_{kl}) . y_h'' + b_t . m_f . \mathrm{Sin}(\theta_{hl}) . y_h'' + b_t . m_s . \mathrm{Sin}(\theta_{hl}) . y_h'' + d_t . m_t .$
$\mathrm{Sin}(\theta_{hl}) . y_h'' + b_s . m_f . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . y_h'' + d_f . m_f . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . y_h'' + d_s . m_s .$
$\mathrm{Sin}(\theta_{hl} - \theta_{kl}) . y_h''$

Left hand side of equation 3.28:

$c_f . g . m_f .\mathrm{Cos}(\theta_{hl} - \theta_{kl}) - b_s . g . m_f . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) - d_f . g . m_f . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) - d_s . g . m_s .$
$\mathrm{Sin}(\theta_{hl} - \theta_{kl}) + b_t . c_f . m_f .\mathrm{Cos}(\theta_{kl}) . \theta_{hl}'^2 + b_s . b_t . m_f . \mathrm{Sin}(\theta_{kl}) . \theta_{hl}'^2 + b_t . d_f . m_f .$
$\mathrm{Sin}(\theta_{kl}) . \theta_{hl}'^2 + b_t . d_s . m_s . \mathrm{Sin}(\theta_{kl}) . \theta_{hl}'^2 - I_f . \theta_{hl}'' - I_s . \theta_{hl}'' - b_s^2 . m_f . \theta_{hl}'' - c_f^2 . m_f .$
$\theta_{hl}'' - 2 . b_s . d_f . m_f . \theta_{hl}'' - d_f^2 . m_f . \theta_{hl}'' - d_s^2 . m_s . \theta_{hl}'' - b_s . b_t . m_f .\mathrm{Cos}(\theta_{kl}) . \theta_{hl}'' -$
$b_t . d_f . m_f .\mathrm{Cos}(\theta_{kl}) . \theta_{hl}'' - b_t . d_s . m_s .\mathrm{Cos}(\theta_{kl}) . \theta_{hl}'' + b_t . c_f . m_f . \mathrm{Sin}(\theta_{kl}) . \theta_{hl}'' + I_f .$
$\theta_{kl}'' + I_s . \theta_{kl}'' + b_s^2 . m_f . \theta_{kl}'' + c_f^2 . m_f . \theta_{kl}'' + 2 . b_s . d_f . m_f . \theta_{kl}'' + d_f^2 . m_f . \theta_{kl}'' +$
$d_s^2 . m_s . \theta_{kl}'' - b_s . m_f .\mathrm{Cos}(\theta_{hl} - \theta_{kl}) . x_h'' - d_f . m_f .\mathrm{Cos}(\theta_{hl} - \theta_{kl}) . x_h'' - d_s . m_s .\mathrm{Cos}(\theta_{hl}$
$- \theta_{kl}) . x_h'' - c_f . m_f . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . x_h'' + c_f . m_f .\mathrm{Cos}(\theta_{hl} - \theta_{kl}) . y_h'' - b_s . m_f . \mathrm{Sin}(\theta_{hl} -$
$\theta_{kl}) . y_h'' - d_f . m_f . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . y_h'' - d_s . m_s . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . y_h''$

Left hand side of equation 3.29:

$m_t . (-(d_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}'^2) + d_t .\mathrm{Cos}(\theta_{hl}) . \theta_{hl}'' + x_h'') + m_t . (-(d_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}'^2) +$
$d_t .\mathrm{Cos}(\theta_{hr}) . \theta_{hr}'' + x_h'') + m_s . (-(b_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}'^2) - d_s . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' -$
$\theta_{kl}')^2 + b_t .\mathrm{Cos}(\theta_{hl}) . \theta_{hl}'' + d_s .\mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}'' - \theta_{kl}'') + x_h'') + m_f . (-(b_t . \mathrm{Sin}(\theta_{hl}) .$
$\theta_{hl}'^2) + c_f .\mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')^2 - (b_s + d_f) . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')^2 +$
$b_t .\mathrm{Cos}(\theta_{hl}) . \theta_{hl}'' + (b_s + d_f) .\mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}'' - \theta_{kl}'') + c_f . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}'' - \theta_{kl}'')$

$+\ x_h{}'') + m_s\,.\,(-(b_t\,.\,\mathrm{Sin}(\theta_{hr})\,.\ \theta_{hr}{}'^2) - d_s\,.\,\mathrm{Sin}(\theta_{hr}-\theta_{kr})\,.\,(\theta_{hr}{}' - \theta_{kr}{}')^2 + b_t\,.\mathrm{Cos}(\theta_{hr})\,.\,\theta_{hr}{}'' + d_s\,.\mathrm{Cos}(\theta_{hr}-\theta_{kr})\,.\,(\theta_{hr}{}'' - \theta_{kr}{}'') + x_h{}'') + m_f\,.\,(-(b_t\,.\,\mathrm{Sin}(\theta_{hr})\,.\,\theta_{hr}{}'^2) - b_s\,.\,\mathrm{Sin}(\theta_{hr}-\theta_{kr})\,.\,(\theta_{hr}{}' - \theta_{kr}{}')^2 + d_{fr}\,.\mathrm{Cos}(\alpha_{fr}+\theta_{ar}+\theta_{hr}-\theta_{kr})\,.\,(\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}')^2 + b_t\,.\mathrm{Cos}(\theta_{hr})\,.\,\theta_{hr}{}'' + b_s\,.\mathrm{Cos}(\theta_{hr}-\theta_{kr})\,.(\theta_{hr}{}'' - \theta_{kr}{}'') + d_{fr}\,.\,\mathrm{Sin}(\alpha_{fr}+\theta_{ar}+\theta_{hr}-\theta_{kr})\,.\,(\theta_{ar}{}'' + \theta_{hr}{}'' - \theta_{kr}{}'') + x_h{}'') + m_b\,.\,x_h{}''$

Left hand side of equation 3.30:

$2\,.\,g\,.\,(m_f + m_s + m_t) + m_t\,.\,(d_t\,.\mathrm{Cos}(\theta_{hl})\,.\,\theta_{hl}{}'^2 + d_t\,.\,\mathrm{Sin}(\theta_{hl})\,.\,\theta_{hl}{}'' + y_h{}'') + m_t\,.\,(d_t\,.\mathrm{Cos}(\theta_{hr})\,.\,\theta_{hr}{}'^2 + d_t\,.\,\mathrm{Sin}(\theta_{hr})\,.\,\theta_{hr}{}'' + y_h{}'') + m_f\,.\,(b_t\,.\mathrm{Cos}(\theta_{hl})\,.\,\theta_{hl}{}'^2 + (b_s + d_f)\,.\mathrm{Cos}(\theta_{hl}-\theta_{kl})\,.\,(\theta_{hl}{}' - \theta_{kl}{}')^2 + c_f\,.\,\mathrm{Sin}(\theta_{hl}-\theta_{kl})\,.\,(\theta_{hl}{}' - \theta_{kl}{}')^2 + b_t\,.\,\mathrm{Sin}(\theta_{hl})\,.\,\theta_{hl}{}'' - c_f\,.\mathrm{Cos}(\theta_{hl}-\theta_{kl})\,.\,(\theta_{hl}{}'' - \theta_{kl}{}'') + (b_s + d_f)\,.\,\mathrm{Sin}(\theta_{hl}-\theta_{kl})\,.\,(\theta_{hl}{}'' - \theta_{kl}{}'') + y_h{}'') + m_s\,.\,(b_t\,.\mathrm{Cos}(\theta_{hl})\,.\,\theta_{hl}{}'^2 + d_s\,.\mathrm{Cos}(\theta_{hl}-\theta_{kl})\,.\,(\theta_{hl}{}' - \theta_{kl}{}')^2 + b_t\,.\,\mathrm{Sin}(\theta_{hl})\,.\,\theta_{hl}{}'' + d_s\,.\,\mathrm{Sin}(\theta_{hl}-\theta_{kl})\,.\,(\theta_{hl}{}'' - \theta_{kl}{}'') + y_h{}'') + m_s\,.\,(b_t\,.\mathrm{Cos}(\theta_{hr})\,.\,\theta_{hr}{}'^2 + d_s\,.\mathrm{Cos}(\theta_{hr}-\theta_{kr})\,.\,(\theta_{hr}{}' - \theta_{kr}{}')^2 + b_t\,.\,\mathrm{Sin}(\theta_{hr})\,.\,\theta_{hr}{}'' + d_s\,.\,\mathrm{Sin}(\theta_{hr}-\theta_{kr})\,.\,(\theta_{hr}{}'' - \theta_{kr}{}'') + y_h{}'') + m_f\,.\,(b_t\,.\mathrm{Cos}(\theta_{hr})\,.\,\theta_{hr}{}'^2 + b_s\,.\mathrm{Cos}(\theta_{hr}-\theta_{kr})\,.\,(\theta_{hr}{}' - \theta_{kr}{}')^2 + d_{fr}\,.\,\mathrm{Sin}(\alpha_{fr}+\theta_{ar}+\theta_{hr}-\theta_{kr})\,.\,(\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}')^2 + b_t\,.\,\mathrm{Sin}(\theta_{hr})\,.\,\theta_{hr}{}'' + b_s\,.\,\mathrm{Sin}(\theta_{hr}-\theta_{kr})\,.\,(\theta_{hr}{}'' - \theta_{kr}{}'') - d_{fr}\,.\mathrm{Cos}(\alpha_{fr}+\theta_{ar}+\theta_{hr}-\theta_{kr})\,.\,(\theta_{ar}{}'' + \theta_{hr}{}'' - \theta_{kr}{}'') + y_h{}'') + m_b\,.\,g$

APPENDIX A-3

THE OPEN FORMS OF DOUBLE SUPPORT MATRICES

Open form of M_{pp}

$M_{pp}(1,1) = I_f + I_s + I_t + b_t^2 . m_f . \mathrm{Cos}(\theta_{hl})^2 + b_t^2 . m_s . \mathrm{Cos}(\theta_{hl})^2 + d_t^2 . m_t . \mathrm{Cos}(\theta_{hl})^2 + 2 . b_s . b_t . m_f . \mathrm{Cos}(\theta_{hl}) . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) + 2 . d_s . b_t . m_s . \mathrm{Cos}(\theta_{hl}) . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) + 2 . b_s^2 . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl})^2 + 2 . d_s^2 . m_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl})^2 + 2 . d_{fl} . b_s . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) + d_{fl}^2 . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl})^2 - 2 . b_s . b_t . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hl}) - 2 . d_s . b_t . m_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hl}) - 2 . d_{fl} . b_t . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hl}) + b_t^2 . m_f . \mathrm{Sin}(\theta_{hl})^2 + b_t^2 . m_s . \mathrm{Sin}(\theta_{hl})^2 + d_t^2 . m_t . \mathrm{Sin}(\theta_{hl})^2 + 2 . d_{fl} . b_t . m_f . \mathrm{Cos}(\theta_{hl}) . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) + 2 . d_{fl} . b_s . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) + d_{fl}^2 . m_f . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl})^2$

$M_{pp}(1,2) = -I_f - I_s - b_s . b_t . m_f . \mathrm{Cos}(\theta_{hl}) . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) - d_s . b_t . m_s . \mathrm{Cos}(\theta_{hl}) . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) - 2 . b_s^2 . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl})^2 - 2 . d_s^2 . m_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl})^2 - 2 . d_{fl} . b_s . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) - d_{fl}^2 . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl})^2 + b_s . b_t . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hl}) + d_s . b_t . m_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hl}) + d_{fl} . b_t . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hl}) - d_{fl} . b_t . m_f . \mathrm{Cos}(\theta_{hl}) . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) - 2 . d_{fl} . b_s . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) - d_{fl}^2 . m_f . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl})^2$

$$M_{pp}(1,3) = -(b_t^2 \cdot m_f \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Cos}(\theta_{hr})) - b_t^2 \cdot m_s \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Cos}(\theta_{hr}) - d_t \cdot b_t \cdot m_t \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Cos}(\theta_{hr}) - b_s \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) - d_s \cdot b_t \cdot m_s \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) - b_s \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) - b_s \cdot b_t \cdot m_s \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) - d_t \cdot b_s \cdot m_t \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) - b_s^2 \cdot m_f \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) - d_s \cdot b_s \cdot m_s \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) - b_{fr} \cdot b_s \cdot m_f \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_s \cdot b_{fr} \cdot m_s \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_{fl} \cdot b_{fr} \cdot m_f \cdot \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hl}) + b_{fr} \cdot b_t \cdot m_s \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hl}) + d_t \cdot b_{fr} \cdot m_t \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hl}) + b_s \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\theta_{hr}) + d_s \cdot b_t \cdot m_s \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\theta_{hr}) + d_{fl} \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\theta_{hr}) - b_t^2 \cdot m_f \cdot \mathrm{Sin}(\theta_{hl}) \cdot \mathrm{Sin}(\theta_{hr}) - b_t^2 \cdot m_s \cdot \mathrm{Sin}(\theta_{hl}) \cdot \mathrm{Sin}(\theta_{hr}) - d_t \cdot b_t \cdot m_t \cdot \mathrm{Sin}(\theta_{hl}) \cdot \mathrm{Sin}(\theta_{hr}) - d_{fl} \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) - d_{fl} \cdot b_s \cdot m_f \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) + b_s^2 \cdot m_f \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + d_s \cdot b_s \cdot m_s \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + d_{fl} \cdot b_s \cdot m_f \cdot \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - b_s \cdot b_t \cdot m_f \cdot \mathrm{Sin}(\theta_{hl}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - b_s \cdot b_t \cdot m_s \cdot \mathrm{Sin}(\theta_{hl}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - d_t \cdot b_s \cdot m_t \cdot \mathrm{Sin}(\theta_{hl}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - b_{fr} \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr} \cdot b_t \cdot m_s \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_t \cdot b_{fr} \cdot m_t \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr} \cdot b_s \cdot m_f \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_s \cdot b_{fr} \cdot m_s \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_{fl} \cdot b_{fr} \cdot m_f \cdot \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$$

$$M_{pp}(1,4) = b_s \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) + b_s \cdot b_t \cdot m_s \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) + d_t \cdot b_s \cdot m_t \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) + b_s^2 \cdot m_f \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) + d_s \cdot b_s \cdot m_s \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) + b_{fr} \cdot b_s \cdot m_f \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_s \cdot b_{fr} \cdot m_s \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_{fl} \cdot b_{fr} \cdot m_f \cdot \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr} \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hl}) - b_{fr} \cdot b_t \cdot m_s \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hl}) - d_t \cdot b_{fr} \cdot m_t \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hl}) + d_{fl} \cdot b_s \cdot m_f \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl}$$

$- \theta_{kl}) - b_s^2 . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - d_s . b_s . m_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - d_{fl} . b_s . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + b_s . b_t . m_f . \mathrm{Sin}(\theta_{hl}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + b_s . b_t . m_s . \mathrm{Sin}(\theta_{hl}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + d_t . b_s . m_t . \mathrm{Sin}(\theta_{hl}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + b_{fr} . b_t . m_f . \mathrm{Cos}(\theta_{hl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} . b_t . m_s . \mathrm{Cos}(\theta_{hl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_t . b_{fr} . m_t . \mathrm{Cos}(\theta_{hl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} . b_s . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_s . b_{fr} . m_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_{fl} . b_{fr} . m_f . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$

$M_{pp}(2,1) = -I_f - I_s - b_s . b_t . m_f . \mathrm{Cos}(\theta_{hl}) . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) - d_s . b_t . m_s . \mathrm{Cos}(\theta_{hl}) . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) - 2 . b_s^2 . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl})^2 - 2 . d_s^2 . m_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl})^2 - 2 . d_{fl} . b_s . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) - d_{fl}^2 . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl})^2 + b_s . b_t . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hl}) + d_s . b_t . m_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hl}) + d_{fl} . b_t . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hl}) - d_{fl} . b_t . m_f . \mathrm{Cos}(\theta_{hl}) . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) - 2 . d_{fl} . b_s . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) - d_{fl}^2 . m_f . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl})^2$

$M_{pp}(2,2) = I_f + I_s + 2 . b_s^2 . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl})^2 + 2 . d_s^2 . m_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl})^2 + 2 . d_{fl} . b_s . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) + d_{fl}^2 . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl})^2 + 2 . d_{fl} . b_s . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) + d_{fl}^2 . m_f . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl})^2$

$M_{pp}(2,3) = b_s . b_t . m_f . \mathrm{Cos}(\theta_{hr}) . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) + d_s . b_t . m_s . \mathrm{Cos}(\theta_{hr}) . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) + b_s^2 . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) + d_s . b_s . m_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) + b_{fr} . b_s . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_s . b_{fr} . m_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_{fl} . b_{fr} . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_s . b_t . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hr}) - d_s . b_t . m_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hr}) - d_{fl} . b_t . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hr}) + d_{fl} . b_t . m_f . \mathrm{Cos}(\theta_{hr}) .$

$Sin(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) + d_{fl} . b_s . m_f . Cos(\theta_{hr} - \theta_{kr}) . Sin(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) - b_s{}^2 . m_f . Cos(\theta_{hl} - \theta_{kl}) . Sin(\theta_{hr} - \theta_{kr}) - d_s . b_s . m_s . Cos(\theta_{hl} - \theta_{kl}) . Sin(\theta_{hr} - \theta_{kr}) - d_{fl} . b_s . m_f . Cos(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . Sin(\theta_{hr} - \theta_{kr}) + b_{fr} . b_s . m_f . Cos(\theta_{hl} - \theta_{kl}) . Sin(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_s . b_{fr} . m_s . Cos(\theta_{hl} - \theta_{kl}) . Sin(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_{fl} . b_{fr} . m_f . Sin(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . Sin(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$

$M_{pp}(2,4) = -(b_s{}^2 . m_f . Cos(\theta_{hl} - \theta_{kl}) . Cos(\theta_{hr} - \theta_{kr})) - d_s . b_s . m_s . Cos(\theta_{hl} - \theta_{kl}) . Cos(\theta_{hr} - \theta_{kr}) - b_{fr} . b_s . m_f . Cos(\theta_{hl} - \theta_{kl}) . Cos(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_s . b_{fr} . m_s . Cos(\theta_{hl} - \theta_{kl}) . Cos(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_{fl} . b_{fr} . m_f . Cos(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . Cos(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_{fl} . b_s . m_f . Cos(\theta_{hr} - \theta_{kr}) . Sin(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) + b_s{}^2 . m_f . Cos(\theta_{hl} - \theta_{kl}) . Sin(\theta_{hr} - \theta_{kr}) + d_s . b_s . m_s . Cos(\theta_{hl} - \theta_{kl}) . Sin(\theta_{hr} - \theta_{kr}) + d_{fl} . b_s . m_f . Cos(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . Sin(\theta_{hr} - \theta_{kr}) - b_{fr} . b_s . m_f . Cos(\theta_{hl} - \theta_{kl}) . Sin(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_s . b_{fr} . m_s . Cos(\theta_{hl} - \theta_{kl}) . Sin(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_{fl} . b_{fr} . m_f . Sin(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . Sin(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$

$M_{pp}(3,1) = -(b_t{}^2 . m_f . Cos(\theta_{hl}) . Cos(\theta_{hr})) - b_t{}^2 . m_s . Cos(\theta_{hl}) . Cos(\theta_{hr}) - d_t . b_t . m_t . Cos(\theta_{hl}) . Cos(\theta_{hr}) - b_s . b_t . m_f . Cos(\theta_{hr}) . Cos(\theta_{hl} - \theta_{kl}) - d_s . b_t . m_s . Cos(\theta_{hr}) . Cos(\theta_{hl} - \theta_{kl}) - b_s . b_t . m_f . Cos(\theta_{hl}) . Cos(\theta_{hr} - \theta_{kr}) - b_s . b_t . m_s . Cos(\theta_{hl}) . Cos(\theta_{hr} - \theta_{kr}) - d_t . b_s . m_t . Cos(\theta_{hl}) . Cos(\theta_{hr} - \theta_{kr}) - b_s{}^2 . m_f . Cos(\theta_{hl} - \theta_{kl}) . Cos(\theta_{hr} - \theta_{kr}) - d_s . b_s . m_s . Cos(\theta_{hl} - \theta_{kl}) . Cos(\theta_{hr} - \theta_{kr}) - b_{fr} . b_s . m_f . Cos(\theta_{hl} - \theta_{kl}) . Cos(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_s . b_{fr} . m_s . Cos(\theta_{hl} - \theta_{kl}) . Cos(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_{fl} . b_{fr} . m_f . Cos(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . Cos(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} . b_t . m_f . Cos(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . Sin(\theta_{hl}) + b_{fr} . b_t . m_s . Cos(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . Sin(\theta_{hl}) + d_t . b_{fr} . m_t . Cos(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . Sin(\theta_{hl}) + b_s . b_t . m_f . Cos(\theta_{hl} - \theta_{kl}) . Sin(\theta_{hr}) + d_s . b_t . m_s . Cos(\theta_{hl} - \theta_{kl}) . Sin(\theta_{hr}) + d_{fl} . b_t . m_f . Cos(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . Sin(\theta_{hr}) - b_t{}^2 . m_f . Sin(\theta_{hl}) . Sin(\theta_{hr}) - b_t{}^2 . m_s . Sin(\theta_{hl}) . Sin(\theta_{hr}) - d_t . b_t . m_t . Sin(\theta_{hl}) . Sin(\theta_{hr}) - d_{fl} . b_t . m_f . Cos(\theta_{hr}) . Sin(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) - d_{fl} . b_s . m_f . Cos(\theta_{hr} - \theta_{kr}) . Sin(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) + b_s{}^2 . m_f . Cos(\theta_{hl} - \theta_{kl}) . Sin(\theta_{hr} -$

$\theta_{kr}) + d_s \,.\, b_s \,.\, m_s \,.\, \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \,.\, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + d_{fl} \,.\, b_s \,.\, m_f \,.\, \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) \,.\, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - b_s \,.\, b_t \,.\, m_f \,.\, \mathrm{Sin}(\theta_{hl}) \,.\, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - b_s \,.\, b_t \,.\, m_s \,.\, \mathrm{Sin}(\theta_{hl}) \,.\, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - d_t \,.\, b_s \,.\, m_t \,.\, \mathrm{Sin}(\theta_{hl}) \,.\, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - b_{fr} \,.\, b_t \,.\, m_f \,.\, \mathrm{Cos}(\theta_{hl}) \,.\, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr} \,.\, b_t \,.\, m_s \,.\, \mathrm{Cos}(\theta_{hl}) \,.\, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_t \,.\, b_{fr} \,.\, m_t \,.\, \mathrm{Cos}(\theta_{hl}) \,.\, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr} \,.\, b_s \,.\, m_f \,.\, \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \,.\, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_s \,.\, b_{fr} \,.\, m_s \,.\, \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \,.\, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_{fl} \,.\, b_{fr} \,.\, m_f \,.\, \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) \,.\, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$

$M_{pp}(3,2) = b_s \,.\, b_t \,.\, m_f \,.\, \mathrm{Cos}(\theta_{hr}) \,.\, \mathrm{Cos}(\theta_{hl} - \theta_{kl}) + d_s \,.\, b_t \,.\, m_s \,.\, \mathrm{Cos}(\theta_{hr}) \,.\, \mathrm{Cos}(\theta_{hl} - \theta_{kl}) + b_s^{\,2} \,.\, m_f \,.\, \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \,.\, \mathrm{Cos}(\theta_{hr} - \theta_{kr}) + d_s \,.\, b_s \,.\, m_s \,.\, \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \,.\, \mathrm{Cos}(\theta_{hr} - \theta_{kr}) + b_{fr} \,.\, b_s \,.\, m_f \,.\, \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \,.\, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_s \,.\, b_{fr} \,.\, m_s \,.\, \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \,.\, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_{fl} \,.\, b_{fr} \,.\, m_f \,.\, \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) \,.\, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_s \,.\, b_t \,.\, m_f \,.\, \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \,.\, \mathrm{Sin}(\theta_{hr}) - d_s \,.\, b_t \,.\, m_s \,.\, \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \,.\, \mathrm{Sin}(\theta_{hr}) - d_{fl} \,.\, b_t \,.\, m_f \,.\, \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) \,.\, \mathrm{Sin}(\theta_{hr}) + d_{fl} \,.\, b_t \,.\, m_f \,.\, \mathrm{Cos}(\theta_{hr}) \,.\, \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) + d_{fl} \,.\, b_s \,.\, m_f \,.\, \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \,.\, \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) - b_s^{\,2} \,.\, m_f \,.\, \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \,.\, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - d_s \,.\, b_s \,.\, m_s \,.\, \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \,.\, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - d_{fl} \,.\, b_s \,.\, m_f \,.\, \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) \,.\, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + b_{fr} \,.\, b_s \,.\, m_f \,.\, \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \,.\, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_s \,.\, b_{fr} \,.\, m_s \,.\, \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \,.\, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_{fl} \,.\, b_{fr} \,.\, m_f \,.\, \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) \,.\, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$

$M_{pp}(3,3) = I_f + I_s + I_t + b_t^{\,2} \,.\, m_b \,.\, \mathrm{Cos}(\theta_{hr})^2 + b_t^{\,2} \,.\, m_f \,.\, \mathrm{Cos}(\theta_{hr})^2 + b_t^{\,2} \,.\, m_s \,.\, \mathrm{Cos}(\theta_{hr})^2 + b_t^{\,2} \,.\, m_t \,.\, \mathrm{Cos}(\theta_{hr})^2 + (-d_t + b_t)^2 \,.\, m_t \,.\, \mathrm{Cos}(\theta_{hr})^2 + 2 \,.\, b_s \,.\, b_t \,.\, m_b \,.\, \mathrm{Cos}(\theta_{hr}) \,.\, \mathrm{Cos}(\theta_{hr} - \theta_{kr}) + 2 \,.\, b_s \,.\, b_t \,.\, m_f \,.\, \mathrm{Cos}(\theta_{hr}) \,.\, \mathrm{Cos}(\theta_{hr} - \theta_{kr}) + 2 \,.\, b_s \,.\, b_t \,.\, m_s \,.\, \mathrm{Cos}(\theta_{hr}) \,.\, \mathrm{Cos}(\theta_{hr} - \theta_{kr}) + 2 \,.\, b_s \,.\, b_t \,.\, m_t \,.\, \mathrm{Cos}(\theta_{hr}) \,.\, \mathrm{Cos}(\theta_{hr} - \theta_{kr}) + 2 \,.\, b_s \,.\, (-d_t + b_t) \,.\, m_t \,.\, \mathrm{Cos}(\theta_{hr}) \,.\, \mathrm{Cos}(\theta_{hr} - \theta_{kr}) + b_s^{\,2} \,.\, m_b \,.\, \mathrm{Cos}(\theta_{hr} - \theta_{kr})^2 + b_s^{\,2} \,.\, m_f \,.\, \mathrm{Cos}(\theta_{hr} - \theta_{kr})^2 + b_s^{\,2} \,.\, m_s \,.\, \mathrm{Cos}(\theta_{hr} - \theta_{kr})^2 - d_s \,.\, (-d_s + b_s) \,.\, m_s \,.\, \mathrm{Cos}(\theta_{hr} - \theta_{kr})^2 + b_s \,.\, (-d_s + b_s) \,.\, m_s \,.\, \mathrm{Cos}(\theta_{hr} - \theta_{kr})^2 + 2 \,.\, b_s^{\,2} \,.\, m_t \,.\, \mathrm{Cos}(\theta_{hr} - \theta_{kr})^2 + b_{fr}^{\,2} \,.\, m_b \,.\, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + b_{fr}^{\,2} \,.\, m_f \,.\, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + d_{fr}^{\,2} \,.\, m_f \,.\, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} +$

$\theta_{hr} - \theta_{kr})^2 + 2 \, . \, b_{fr}^2 \, . \, m_s \, . \, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + 2 \, . \, b_{fr}^2 \, . \, m_t \, . \, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} +$
$\theta_{hr} - \theta_{kr})^2 - 2 \, . \, b_{fr} \, . \, b_t \, . \, m_b \, . \, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \, . \, \mathrm{Sin}(\theta_{hr}) - 2 \, . \, b_{fr} \, . \, b_t \, . \, m_f \, .$
$\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \, . \, \mathrm{Sin}(\theta_{hr}) - 2 \, . \, b_{fr} \, . \, b_t \, . \, m_s \, . \, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \, .$
$\mathrm{Sin}(\theta_{hr}) - 2 \, . \, b_{fr} \, . \, b_t \, . \, m_t \, . \, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \, . \, \mathrm{Sin}(\theta_{hr}) - 2 \, . \, b_{fr} \, . \, (-d_t + b_t) \, .$
$m_t \, . \, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \, . \, \mathrm{Sin}(\theta_{hr}) + b_t^2 \, . \, m_b \, . \, \mathrm{Sin}(\theta_{hr})^2 + b_t^2 \, . \, m_f \, . \, \mathrm{Sin}(\theta_{hr})^2 +$
$b_t^2 \, . \, m_s \, . \, \mathrm{Sin}(\theta_{hr})^2 + b_t^2 \, . \, m_t \, . \, \mathrm{Sin}(\theta_{hr})^2 + (-d_t + b_t)^2 \, . \, m_t \, . \, \mathrm{Sin}(\theta_{hr})^2 - 2 \, . \, b_{fr} \, . \, b_s \, .$
$m_b \, . \, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \, . \, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - 2 \, . \, b_{fr} \, . \, b_s \, . \, m_f \, . \, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} -$
$\theta_{kr}) \, . \, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + d_s \, . \, b_{fr} \, . \, m_s \, . \, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \, . \, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - 3 \, . \, b_{fr} \, .$
$b_s \, . \, m_s \, . \, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \, . \, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - b_{fr} \, . \, (-d_s + b_s) \, . \, m_s \, . \, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} +$
$\theta_{hr} - \theta_{kr}) \, . \, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - 4 \, . \, b_{fr} \, . \, b_s \, . \, m_t \, . \, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \, . \, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) +$
$2 \, . \, b_s \, . \, b_t \, . \, m_b \, . \, \mathrm{Sin}(\theta_{hr}) \, . \, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + 2 \, . \, b_s \, . \, b_t \, . \, m_f \, . \, \mathrm{Sin}(\theta_{hr}) \, . \, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + 2 \, .$
$b_s \, . \, b_t \, . \, m_s \, . \, \mathrm{Sin}(\theta_{hr}) \, . \, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + 2 \, . \, b_s \, . \, b_t \, . \, m_t \, . \, \mathrm{Sin}(\theta_{hr}) \, . \, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + 2 \, . \, b_s \, .$
$(-d_t + b_t) \, . \, m_t \, . \, \mathrm{Sin}(\theta_{hr}) \, . \, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + b_s^2 \, . \, m_b \, . \, \mathrm{Sin}(\theta_{hr} - \theta_{kr})^2 + b_s^2 \, . \, m_f \, . \, \mathrm{Sin}(\theta_{hr}$
$- \theta_{kr})^2 + b_s^2 \, . \, m_s \, . \, \mathrm{Sin}(\theta_{hr} - \theta_{kr})^2 - d_s \, . \, (-d_s + b_s) \, . \, m_s \, . \, \mathrm{Sin}(\theta_{hr} - \theta_{kr})^2 + b_s \, . \, (-d_s +$
$b_s). \, m_s \, . \, \mathrm{Sin}(\theta_{hr} - \theta_{kr})^2 + 2 \, . \, b_s^2 \, . \, m_t \, . \, \mathrm{Sin}(\theta_{hr} - \theta_{kr})^2 + 2 \, . \, b_{fr} \, . \, b_t \, . \, m_b \, . \, \mathrm{Cos}(\theta_{hr}) \, .$
$\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + 2 \, . \, b_{fr} \, . \, b_t \, . \, m_f \, . \, \mathrm{Cos}(\theta_{hr}) \, . \, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + 2 \, .$
$b_{fr} \, . \, b_t \, . \, m_s \, . \, \mathrm{Cos}(\theta_{hr}) \, . \, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + 2 \, . \, b_{fr} \, . \, b_t \, . \, m_t \, . \, \mathrm{Cos}(\theta_{hr}) \, . \, \mathrm{Sin}(\alpha_{fr}$
$+ \theta_{ar} + \theta_{hr} - \theta_{kr}) + 2 \, . \, b_{fr} \, . \, (-d_t + b_t) \, . \, m_t \, . \, \mathrm{Cos}(\theta_{hr}) \, . \, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + 2 \, .$
$b_{fr} \, . \, b_s \, . \, m_b \, . \, \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \, . \, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + 2 \, . \, b_{fr} \, . \, b_s \, . \, m_f . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \, .$
$\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_s \, . \, b_{fr} \, . \, m_s \, . \, \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \, . \, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + 3 \, .$
$b_{fr} \, . \, b_s \, . \, m_s \, . \, \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \, . \, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} \, . \, (-d_s + b_s) \, . \, m_s \, . \, \mathrm{Cos}(\theta_{hr} -$
$\theta_{kr}) \, . \, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + 4 \, . \, b_{fr} \, . \, b_s \, . \, m_t \, . \, \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \, . \, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} -$
$\theta_{kr}) + b_{fr}^2 \, . \, m_b \, . \, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + b_{fr}^2 \, . \, m_f \, . \, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 +$
$d_{fr}^2 \, . \, m_f \, . \, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + 2 \, . \, b_{fr}^2 \, . \, m_s \, . \, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + 2 \, .$
$b_{fr}^2 \, . \, m_t \, . \, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2$

$M_{pp}(3,4) = -I_f - I_s - b_s \, . \, b_t \, . \, m_b \, . \, \mathrm{Cos}(\theta_{hr}) \, . \, \mathrm{Cos}(\theta_{hr} - \theta_{kr}) - b_s \, . \, b_t \, . \, m_f \, . \, \mathrm{Cos}(\theta_{hr}) \, .$
$\mathrm{Cos}(\theta_{hr} - \theta_{kr}) - b_s \, . \, b_t \, . \, m_s \, . \, \mathrm{Cos}(\theta_{hr}) \, . \, \mathrm{Cos}(\theta_{hr} - \theta_{kr}) - b_s \, . \, b_t \, . \, m_t \, . \, \mathrm{Cos}(\theta_{hr}) \, . \, \mathrm{Cos}(\theta_{hr}$
$- \theta_{kr}) - b_s \, . \, (-d_t + b_t) \, . \, m_t \, . \, \mathrm{Cos}(\theta_{hr}) \, . \, \mathrm{Cos}(\theta_{hr} - \theta_{kr}) - b_s^2 \, . \, m_b \, . \, \mathrm{Cos}(\theta_{hr} - \theta_{kr})^2 - b_s^2 \, .$

$$m_f \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr})^2 - b_s^2 \cdot m_s \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr})^2 + d_s \cdot (-d_s + b_s) \cdot m_s \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr})^2 - b_s \cdot (-d_s + b_s) \cdot m_s \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr})^2 - 2 \cdot b_s^2 \cdot m_t \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr})^2 - b_{fr}^2 \cdot m_b \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - b_{fr}^2 \cdot m_f \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - d_{fr}^2 \cdot m_f \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - 2 \cdot b_{fr}^2 \cdot m_s \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - 2 \cdot b_{fr}^2 \cdot m_t \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + b_{fr} \cdot b_t \cdot m_b \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr}) + b_{fr} \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr}) + b_{fr} \cdot b_t \cdot m_s \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr}) + b_{fr} \cdot b_t \cdot m_t \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr}) + b_{fr} \cdot (-d_t + b_t) \cdot m_t \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr}) + 2 \cdot b_{fr} \cdot b_s \cdot m_b \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + 2 \cdot b_{fr} \cdot b_s \cdot m_f \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - d_s \cdot b_{fr} \cdot m_s \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + 3 \cdot b_{fr} \cdot b_s \cdot m_s \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + b_{fr} \cdot (-d_s + b_s) \cdot m_s \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + 4 \cdot b_{fr} \cdot b_s \cdot m_t \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - b_s \cdot b_t \cdot m_b \cdot \mathrm{Sin}(\theta_{hr}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - b_s \cdot b_t \cdot m_f \cdot \mathrm{Sin}(\theta_{hr}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - b_s \cdot b_t \cdot m_s \cdot \mathrm{Sin}(\theta_{hr}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - b_s \cdot b_t \cdot m_t \cdot \mathrm{Sin}(\theta_{hr}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - b_s \cdot (-d_t + b_t) \cdot m_t \cdot \mathrm{Sin}(\theta_{hr}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - b_s^2 \cdot m_b \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr})^2 - b_s^2 \cdot m_f \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr})^2 - b_s^2 \cdot m_s \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr})^2 + d_s \cdot (-d_s + b_s) \cdot m_s \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr})^2 - b_s \cdot (-d_s + b_s) \cdot m_s \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr})^2 - 2 \cdot b_s^2 \cdot m_t \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr})^2 - b_{fr} \cdot b_t \cdot m_b \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr} \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr} \cdot b_t \cdot m_s \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr} \cdot b_t \cdot m_t \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr} \cdot (-d_t + b_t) \cdot m_t \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - 2 \cdot b_{fr} \cdot b_s \cdot m_b \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - 2 \cdot b_{fr} \cdot b_s \cdot m_f \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_s \cdot b_{fr} \cdot m_s \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - 3 \cdot b_{fr} \cdot b_s \cdot m_s \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr} \cdot (-d_s + b_s) \cdot m_s \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - 4 \cdot b_{fr} \cdot b_s \cdot m_t \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr}^2 \cdot m_b \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - b_{fr}^2 \cdot m_f \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - d_{fr}^2 \cdot m_f \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - 2 \cdot b_{fr}^2 \cdot m_s \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - 2 \cdot b_{fr}^2 \cdot m_t \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2$$

$$M_{pp}(4,1) = b_s \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) + b_s \cdot b_t \cdot m_s \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) + d_t \cdot b_s \cdot m_t \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) + b_s^2 \cdot m_f \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr})$$

$+ d_s . b_s . m_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) + b_{fr} . b_s . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_s . b_{fr} . m_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_{fl} . b_{fr} . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr} . b_t . m_f . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hl}) - b_{fr} . b_t . m_s . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hl}) - d_t . b_{fr} . m_t . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hl}) + d_{fl} . b_s . m_f . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) - b_s^{2} . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - d_s . b_s . m_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - d_{fl} . b_s . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + b_s . b_t . m_f . \mathrm{Sin}(\theta_{hl}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + b_s . b_t . m_s . \mathrm{Sin}(\theta_{hl}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + d_t . b_s . m_t . \mathrm{Sin}(\theta_{hl}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + b_{fr} . b_t . m_f . \mathrm{Cos}(\theta_{hl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} . b_t . m_s . \mathrm{Cos}(\theta_{hl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_t . b_{fr} . m_t . \mathrm{Cos}(\theta_{hl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} . b_s . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_s . b_{fr} . m_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_{fl} . b_{fr} . m_f . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$

$M_{pp}(4,2) = -(b_s^{2} . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\theta_{hr} - \theta_{kr})) - d_s . b_s . m_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) - b_{fr} . b_s . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_s . b_{fr} . m_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_{fl} . b_{fr} . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_{fl} . b_s . m_f . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) + b_s^{2} . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + d_s . b_s . m_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + d_{fl} . b_s . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - b_{fr} . b_s . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_s . b_{fr} . m_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_{fl} . b_{fr} . m_f . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$

$M_{pp}(4,3) = -I_f - I_s - b_s . b_t . m_b . \mathrm{Cos}(\theta_{hr}) . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) - b_s . b_t . m_f . \mathrm{Cos}(\theta_{hr}) . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) - b_s . b_t . m_s . \mathrm{Cos}(\theta_{hr}) . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) - b_s . b_t . m_t . \mathrm{Cos}(\theta_{hr}) . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) - b_s . (-d_t + b_t) . m_t . \mathrm{Cos}(\theta_{hr}) . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) - b_s^{2} . m_b . \mathrm{Cos}(\theta_{hr} - \theta_{kr})^{2} - b_s^{2} . m_f . \mathrm{Cos}(\theta_{hr} - \theta_{kr})^{2} - b_s^{2} . m_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr})^{2} + d_s . (-d_s + b_s) . m_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr})^{2} - b_s . (-d_s + b_s) . m_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr})^{2} - 2 . b_s^{2} . m_t . \mathrm{Cos}(\theta_{hr} - \theta_{kr})^{2} - b_{fr}^{2} . m_b .$

$\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - b_{fr}^2 . m_f . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - d_{fr}^2 . m_f . \mathrm{Cos}(\alpha_{fr}$
$+ \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - 2 . b_{fr}^2 . m_s . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - 2 . b_{fr}^2 . m_t . \mathrm{Cos}(\alpha_{fr} +$
$\theta_{ar} + \theta_{hr} - \theta_{kr})^2 + b_{fr} . b_t . m_b . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr}) + b_{fr} . b_t . m_f .$
$\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr}) + b_{fr} . b_t . m_s . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr})$
$+ b_{fr} . b_t . m_t . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr}) + b_{fr} . (-d_t + b_t) . m_t . \mathrm{Cos}(\alpha_{fr} +$
$\theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr}) + 2 . b_{fr} . b_s . m_b . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) +$
$2 . b_{fr} . b_s . m_f . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - d_s . b_{fr} . m_s . \mathrm{Cos}(\alpha_{fr} + \theta_{ar}$
$+ \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + 3 . b_{fr} . b_s . m_s . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) +$
$b_{fr} . (-d_s + b_s) . m_s . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + 4 . b_{fr} . b_s . m_t .$
$\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - b_s . b_t . m_b . \mathrm{Sin}(\theta_{hr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - b_s .$
$b_t . m_f . \mathrm{Sin}(\theta_{hr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - b_s . b_t . m_s . \mathrm{Sin}(\theta_{hr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - b_s . b_t . m_t .$
$\mathrm{Sin}(\theta_{hr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - b_s . (-d_t + b_t) . m_t . \mathrm{Sin}(\theta_{hr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - b_s^2 . m_b .$
$\mathrm{Sin}(\theta_{hr} - \theta_{kr})^2 - b_s^2 . m_f . \mathrm{Sin}(\theta_{hr} - \theta_{kr})^2 - b_s^2 . m_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr})^2 + d_s . (-d_s + b_s) .$
$m_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr})^2 - b_s . (-d_s + b_s) . m_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr})^2 - 2 . b_s^2 . m_t . \mathrm{Sin}(\theta_{hr} - \theta_{kr})^2$
$- b_{fr} . b_t . m_b . \mathrm{Cos}(\theta_{hr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr} . b_t . m_f . \mathrm{Cos}(\theta_{hr}) . \mathrm{Sin}(\alpha_{fr} +$
$\theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr} . b_t . m_s . \mathrm{Cos}(\theta_{hr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr} . b_t . m_t .$
$\mathrm{Cos}(\theta_{hr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr} . (-d_t + b_t) . m_t . \mathrm{Cos}(\theta_{hr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr}$
$- \theta_{kr}) - 2 . b_{fr} . b_s . m_b b . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - 2 . b_{fr} . b_s . m_f .$
$\mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_s . b_{fr} . m_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} +$
$\theta_{ar} + \theta_{hr} - \theta_{kr}) - 3 . b_{fr} . b_s . m_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr} . (-d_s +$
$b_s) . m_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - 4 . b_{fr} . b_s . m_t . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) .$
$\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr}^2 . m_b b . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - b_{fr}^2 . m_f . \mathrm{Sin}(\alpha_{fr}$
$+ \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - d_{fr}^2 . m_f . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - 2 . b_{fr}^2 . m_s . \mathrm{Sin}(\alpha_{fr} + \theta_{ar}$
$+ \theta_{hr} - \theta_{kr})^2 - 2 . b_{fr}^2 . m_t . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2$

$M_{pp}(4,4) = I_f + I_s + b_s^2 . m_b . \mathrm{Cos}(\theta_{hr} - \theta_{kr})^2 + b_s^2 . m_f . \mathrm{Cos}(\theta_{hr} - \theta_{kr})^2 + b_s^2 . m_s .$
$\mathrm{Cos}(\theta_{hr} - \theta_{kr})^2 - d_s . (-d_s + b_s) . m_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr})^2 + b_s . (-d_s + b_s) . m_s . \mathrm{Cos}(\theta_{hr} -$
$\theta_{kr})^2 + 2 . b_s^2 . m_t . \mathrm{Cos}(\theta_{hr} - \theta_{kr})^2 + b_{fr}^2 . m_b . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + b_{fr}^2 .$
$m_f . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + d_{fr}^2 . m_f . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + 2 . b_{fr}^2 . m_s .$

$\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + 2 .\ b_{fr}^2 .\ m_t . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - 2 .\ b_{fr} .\ b_s . m_b . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - 2 .\ b_{fr} .\ b_s . m_f . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + d_s .\ b_{fr} . m_s . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - 3 .\ b_{fr} . b_s . m_s . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - b_{fr} . (-d_s + b_s) . m_s . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - 4 .\ b_{fr} .\ b_s .\ m_t . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + b_s^2 . m_b . \mathrm{Sin}(\theta_{hr} - \theta_{kr})^2 + b_s^2 . m_f . \mathrm{Sin}(\theta_{hr} - \theta_{kr})^2 + b_s^2 . m_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr})^2 - d_s . (-d_s + b_s) . m_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr})^2 + b_s . (-d_s + b_s) . m_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr})^2 + 2 .\ b_s^2 .\ m_t . \mathrm{Sin}(\theta_{hr} - \theta_{kr})^2 + 2 .\ b_{fr} .\ b_s . m_b . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + 2 .\ b_{fr} . b_s . m_f . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_s .\ b_{fr} . m_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + 3 .\ b_{fr} .\ b_s . m_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} . (-d_s + b_s) . m_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + 4 .\ b_{fr} .\ b_s .\ m_t . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr}^2 . m_b . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + b_{fr}^2 . m_f . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + \mathrm{d}_{fr}^2 . m_f . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + 2 .\ b_{fr}^2 . m_s . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + 2 .\ b_{fr}^2 .\ m_t . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2$

$\mathrm{M_{ps}}(1,1) = I_f + d_{fl} .\ b_s . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) + d_{fl}^2 . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl})^2 - d_{fl} .\ b_t . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hl}) + d_{fl} . b_t . m_f . \mathrm{Cos}(\theta_{hl}) . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) + d_{fl} .\ b_s . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) + d_{fl}^2 . m_f . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl})^2$

Open form of $\mathrm{M_{ps}}$

$\mathrm{M_{ps}}(1,2) = -(b_{fr} .\ b_s . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})) - d_s .\ b_{fr} . m_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_{fl} .\ b_{fr} . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} .\ b_t . m_f . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hl}) + b_{fr} . b_t . m_s . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hl}) + d_t .\ b_{fr} .\ m_t . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hl}) - b_{fr} .\ b_t . m_f . \mathrm{Cos}(\theta_{hl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr} .\ b_t . m_s . \mathrm{Cos}(\theta_{hl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_t .\ b_{fr} .\ m_t . \mathrm{Cos}(\theta_{hl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr} .\ b_s .$

m_f . Cos(θ_{hl} - θ_{kl}) . Sin(α_{fr} + θ_{ar} + θ_{hr} - θ_{kr}) - d_s . b_{fr} . m_s . Cos(θ_{hl} - θ_{kl}) . Sin(α_{fr} + θ_{ar} + θ_{hr} - θ_{kr}) - d_{fl} . b_{fr} . m_f . Sin(α_{fl} + θ_{al} + θ_{hl} - θ_{kl}) . Sin(α_{fr} + θ_{ar} + θ_{hr} - θ_{kr})

M_{ps}(2,1) = -I_f - d_{fl} . b_s . m_f . Cos(θ_{hl} - θ_{kl}) . Cos(α_{fl} + θ_{al} + θ_{hl} - θ_{kl}) - $d_{fl}^{\,2}$. m_f . Cos(α_{fl} + θ_{al} + θ_{hl} - $\theta_{kl})^2$ - d_{fl} . b_s . m_f . Cos(θ_{hl} - θ_{kl}) . Sin(α_{fl} + θ_{al} + θ_{hl} - θ_{kl}) - $d_{fl}^{\,2}$. m_f . Sin(α_{fl} + θ_{al} + θ_{hl} - $\theta_{kl})^2$

M_{ps}(2,2) = b_{fr} . b_s . m_f . Cos(θ_{hl} - θ_{kl}) . Cos(α_{fr} + θ_{ar} + θ_{hr} - θ_{kr}) + d_s . b_{fr} . m_s . Cos(θ_{hl} - θ_{kl}) . Cos(α_{fr} + θ_{ar} + θ_{hr} - θ_{kr}) + d_{fl} . b_{fr} . m_f . Cos(α_{fl} + θ_{al} + θ_{hl} - θ_{kl}) . Cos(α_{fr} + θ_{ar} + θ_{hr} - θ_{kr}) + b_{fr} . b_s . m_f . Cos(θ_{hl} - θ_{kl}) . Sin(α_{fr} + θ_{ar} + θ_{hr} - θ_{kr}) + d_s . b_{fr} . m_s . Cos(θ_{hl} - θ_{kl}) . Sin(α_{fr} + θ_{ar} + θ_{hr} - θ_{kr}) + d_{fl} . b_{fr} . m_f . Sin(α_{fl} + θ_{al} + θ_{hl} - θ_{kl}) . Sin(α_{fr} + θ_{ar} + θ_{hr} - θ_{kr})

M_{ps}(3,1) = -(d_{fl} . b_{fr} . m_f . Cos(α_{fl} + θ_{al} + θ_{hl} - θ_{kl}) . Cos(α_{fr} + θ_{ar} + θ_{hr} - θ_{kr})) + d_{fl} . b_t . m_f . Cos(α_{fl} + θ_{al} + θ_{hl} - θ_{kl}) . Sin(θ_{hr}) - d_{fl} . b_t . m_f . Cos(θ_{hr}) . Sin(α_{fl} + θ_{al} + θ_{hl} - θ_{kl}) - d_{fl} . b_s . m_f . Cos(θ_{hr} - θ_{kr}) . Sin(α_{fl} + θ_{al} + θ_{hl} - θ_{kl}) + d_{fl} . b_s . m_f . Cos(α_{fl} + θ_{al} + θ_{hl} - θ_{kl}) . Sin(θ_{hr} - θ_{kr}) - d_{fl} . b_{fr} . m_f . Sin(α_{fl} + θ_{al} + θ_{hl} - θ_{kl}) . Sin(α_{fr} + θ_{ar} + θ_{hr} - θ_{kr})

M_{ps}(3,2) = I_f + $b_{fr}^{\,2}$. m_b . Cos(α_{fr} + θ_{ar} + θ_{hr} - $\theta_{kr})^2$ + $b_{fr}^{\,2}$. m_f . Cos(α_{fr} + θ_{ar} + θ_{hr} - $\theta_{kr})^2$ + $d_{fr}^{\,2}$. m_f . Cos(α_{fr} + θ_{ar} + θ_{hr} - $\theta_{kr})^2$ + 2 . $b_{fr}^{\,2}$. m_s . Cos(α_{fr} + θ_{ar} + θ_{hr} - $\theta_{kr})^2$ + 2 . $b_{fr}^{\,2}$. m_t . Cos(α_{fr} + θ_{ar} + θ_{hr} - $\theta_{kr})^2$ - b_{fr} . b_t . m_b . Cos(α_{fr} + θ_{ar} + θ_{hr} - θ_{kr}) . Sin(θ_{hr}) - b_{fr} . b_t . m_f . Cos(α_{fr} + θ_{ar} + θ_{hr} - θ_{kr}) . Sin(θ_{hr}) - b_{fr} . b_t . m_s . Cos(α_{fr} + θ_{ar} + θ_{hr} - θ_{kr}) . Sin(θ_{hr}) - b_{fr} . b_t . m_t . Cos(α_{fr} + θ_{ar} + θ_{hr} - θ_{kr}) . Sin(θ_{hr}) - b_{fr} . (-d_t + b_t) . m_t . Cos(α_{fr} + θ_{ar} + θ_{hr} - θ_{kr}) . Sin(θ_{hr}) - b_{fr} . b_s . m_b . Cos(α_{fr} + θ_{ar} + θ_{hr} - θ_{kr}) . Sin(θ_{hr} - θ_{kr}) - b_{fr} . b_s . m_f . Cos(α_{fr} + θ_{ar} + θ_{hr} - θ_{kr}) . Sin(θ_{hr} - θ_{kr}) - b_{fr} . b_s . m_s . Cos(α_{fr} + θ_{ar} + θ_{hr} - θ_{kr}) . Sin(θ_{hr} - θ_{kr}) - b_{fr} . (-d_s + b_s) . m_s . Cos(α_{fr} + θ_{ar} + θ_{hr}

$- \theta_{kr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - 2 .\ b_{fr} .\ b_s .\ m_t . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + b_{fr} .\ b_t . m_b . \mathrm{Cos}(\theta_{hr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} .\ b_t . m_f . \mathrm{Cos}(\theta_{hr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} .\ b_t . m_s . \mathrm{Cos}(\theta_{hr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} .\ b_t .\ m_t . \mathrm{Cos}(\theta_{hr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} . (-d_t + b_t) .\ m_t . \mathrm{Cos}(\theta_{hr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} .\ b_s . m_b . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} .\ b_s . m_f . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} .\ b_s . m_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} . (-d_s + b_s) . m_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + 2 .\ b_{fr} .\ b_s .\ m_t . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr}^{\ 2} . m_b . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + b_{fr}^{\ 2} . m_f .\ \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + d_{fr}^{\ 2} . m_f . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + 2 .\ b_{fr}^{\ 2} . m_s . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + 2 .\ b_{fr}^{\ 2} .\ m_t . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2$

$\mathrm{M_{ps}}(4,1) = d_{fl} .\ b_{fr} . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_{fl} .\ b_s . m_f . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) - d_{fl} .\ b_s . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + d_{fl} .\ b_{fr} . m_f . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$

$\mathrm{M_{ps}}(4,2) = -I_f - b_{fr}^{\ 2} . m_b . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - b_{fr}^{\ 2} . m_f . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - d_{fr}^{\ 2} . m_f . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - 2 .\ b_{fr}^{\ 2} . m_s . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - 2 .\ b_{fr}^{\ 2} .\ m_t . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + b_{fr} .\ b_s . m_b . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + b_{fr} .\ b_s . m_f . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + b_{fr} .\ b_s . m_s . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + b_{fr} . (-d_s + b_s) . m_s . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + 2 .\ b_{fr} .\ b_s .\ m_t . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - b_{fr} .\ b_s . m_b . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr} .\ b_s . m_f . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr} .\ b_s . m_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr} . (-d_s + b_s) . m_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - 2 .\ b_{fr} .\ b_s .\ m_t . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr}^{\ 2} . m_b . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - b_{fr}^{\ 2} . m_f . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - d_{fr}^{\ 2} . m_f . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - 2 .\ b_{fr}^{\ 2} . m_s . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - 2 .\ b_{fr}^{\ 2} .\ m_t . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2$

Open form of M_{sp}

$M_{sp}(1,1) = I_f + d_{fl} . b_s . m_f . \text{Cos}(\theta_{hl} - \theta_{kl}) . \text{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) + d_{fl}^2 . m_f . \text{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl})^2 - d_{fl} . b_t . m_f . \text{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \text{Sin}(\theta_{hl}) + d_{fl} . b_t . m_f . \text{Cos}(\theta_{hl}) . \text{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) + d_{fl} . b_s . m_f . \text{Cos}(\theta_{hl} - \theta_{kl}) . \text{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) + d_{fl}^2 . m_f . \text{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl})^2$

$M_{sp}(1,2) = -I_f - d_{fl} . b_s . m_f . \text{Cos}(\theta_{hl} - \theta_{kl}) . \text{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) - d_{fl}^2 . m_f . \text{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl})^2 - d_{fl} . b_s . m_f . \text{Cos}(\theta_{hl} - \theta_{kl}) . \text{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) - d_{fl}^2 . m_f . \text{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl})^2$

$M_{sp}(1,3) = -(d_{fl} . b_{fr} . m_f . \text{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \text{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})) + d_{fl} . b_t . m_f . \text{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \text{Sin}(\theta_{hr}) - d_{fl} . b_t . m_f . \text{Cos}(\theta_{hr}) . \text{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) - d_{fl} . b_s . m_f . \text{Cos}(\theta_{hr} - \theta_{kr}) . \text{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) + d_{fl} . b_s . m_f . \text{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \text{Sin}(\theta_{hr} - \theta_{kr}) - d_{fl} . b_{fr} . m_f . \text{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \text{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$

$M_{sp}(1,4) = d_{fl} . b_{fr} . m_f . \text{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \text{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_{fl} . b_s . m_f . \text{Cos}(\theta_{hr} - \theta_{kr}) . \text{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) - d_{fl} . b_s . m_f . \text{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \text{Sin}(\theta_{hr} - \theta_{kr}) + d_{fl} . b_{fr} . m_f . \text{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \text{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$

$M_{sp}(2,1) = -(b_{fr} . b_s . m_f . \text{Cos}(\theta_{hl} - \theta_{kl}) . \text{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})) - d_s . b_{fr} . m_s . \text{Cos}(\theta_{hl} - \theta_{kl}) . \text{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_{fl} . b_{fr} . m_f . \text{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \text{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} . b_t . m_f . \text{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \text{Sin}(\theta_{hl}) + b_{fr} . b_t . m_s . \text{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \text{Sin}(\theta_{hl}) + d_t . b_{fr} . m_t . \text{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \text{Sin}(\theta_{hl}) - b_{fr} . b_t . m_f . \text{Cos}(\theta_{hl}) . \text{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr} . b_t . m_s . \text{Cos}(\theta_{hl})$.

$\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_t \cdot b_{fr} \cdot m_t \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr} \cdot b_s \cdot m_f \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_s \cdot b_{fr} \cdot m_s \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_{fl} \cdot b_{fr} \cdot m_f \cdot \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$

$M_{sp}(2,2) = b_{fr} \cdot b_s \cdot m_f \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_s \cdot b_{fr} \cdot m_s \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_{fl} \cdot b_{fr} \cdot m_f \cdot \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} \cdot b_s \cdot m_f \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_s \cdot b_{fr} \cdot m_s \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_{fl} \cdot b_{fr} \cdot m_f \cdot \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$

$M_{sp}(2,3) = I_f + b_{fr}^2 \cdot m_b \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + b_{fr}^2 \cdot m_f \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + d_{fr}^2 \cdot m_f \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + 2 \cdot b_{fr}^2 \cdot m_s \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + 2 \cdot b_{fr}^2 \cdot m_t \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - b_{fr} \cdot b_t \cdot m_b \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr}) - b_{fr} \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr}) - b_{fr} \cdot b_t \cdot m_s \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr}) - b_{fr} \cdot b_t \cdot m_t \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr}) - b_{fr} \cdot (-d_t + b_t) \cdot m_t \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr}) - b_{fr} \cdot b_s \cdot m_b \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - b_{fr} \cdot b_s \cdot m_f \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + d_s \cdot b_{fr} \cdot m_s \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - 2 \cdot b_{fr} \cdot b_s \cdot m_s \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - 2 \cdot b_{fr} \cdot b_s \cdot m_t \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + b_{fr} \cdot b_t \cdot m_b \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} \cdot b_t \cdot m_s \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} \cdot b_t \cdot m_t \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} \cdot (-d_t + b_t) \cdot m_t \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} \cdot b_s \cdot m_b \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} \cdot b_s \cdot m_f \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - d_s \cdot b_{fr} \cdot m_s \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + 2 \cdot b_{fr} \cdot b_s \cdot m_s \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + 2 \cdot b_{fr} \cdot b_s \cdot m_t \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr}^2 \cdot m_b \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + b_{fr}^2 \cdot m_f \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + d_{fr}^2 \cdot m_f \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + 2 \cdot b_{fr}^2 \cdot m_s \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + 2 \cdot b_{fr}^2 \cdot m_t \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2$

$M_{sp}(2,4) = -I_f - b_{fr}^2 . m_b . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - b_{fr}^2 . m_f . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - d_{fr}^2 . m_f . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - 2 . b_{fr}^2 . m_s . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - 2 . b_{fr}^2 . m_t . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + b_{fr} . b_s . m_b . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + b_{fr} . b_s . m_f . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - d_s . b_{fr} . m_s . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + 2 . b_{fr} . b_s . m_s . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) + 2 . b_{fr} . b_s . m_t . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) - b_{fr} . b_s . m_b . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr} . b_s . m_f . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + d_s . b_{fr} . m_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - 2 . b_{fr} . b_s . m_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - 2 . b_{fr} . b_s . m_t . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_{fr}^2 . m_b . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - b_{fr}^2 . m_f . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - d_{fr}^2 . m_f . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - 2 . b_{fr}^2 . m_s . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 - 2 . b_{fr}^2 . m_t . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2$

Open form of M_{ss}

$M_{ss}(1,1) = I_f + d_{fl}^2 . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl})^2 + d_{fl}^2 . m_f . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl})^2$

$M_{ss}(1,2) = -(d_{fl} . b_{fr} . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})) - d_{fl} . b_{fr} . m_f . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$

$M_{ss}(2,1) = -(d_{fl} . b_{fr} . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})) - d_{fl} . b_{fr} . m_f . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$

$$\begin{aligned}
M_{ss}(2,2) = {} & I_f + b_{fr}^2 . m_b . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + b_{fr}^2 . m_f . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \\
& \theta_{kr})^2 + d_{fr}^2 . m_f . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + 2 . b_{fr}^2 . m_s . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 \\
& + 2 . b_{fr}^2 . m_t . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + b_{fr}^2 . m_b . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + \\
& b_{fr}^2 . m_f . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + d_{fr}^2 . m_f . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + 2 . b_{fr}^2 . \\
& m_s . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2 + 2 . b_{fr}^2 . m_t . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})^2
\end{aligned}$$

Open form of b_s

$$\begin{aligned}
b_s(1,1) = {} & -(g . d_{fl} . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl})) - d_{fl} . b_t . m_f . \mathrm{Cos}(\theta_{hl}) . \mathrm{Cos}(\alpha_{fl} + \\
& \theta_{al} + \theta_{hl} - \theta_{kl}) . \theta_{hl}'^2 - d_{fl} . b_t . m_f . \mathrm{Sin}(\theta_{hl}) . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \theta_{hl}'^2 + d_{fl} \\
& . b_t . m_f . \mathrm{Cos}(\theta_{hr}) . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \theta_{hr}'^2 + d_{fl} . b_t . m_f . \mathrm{Sin}(\theta_{hr}) . \mathrm{Sin}(\alpha_{fl} \\
& + \theta_{al} + \theta_{hl} - \theta_{kl}) . \theta_{hr}'^2 - d_{fl} . b_s . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . \\
& (\theta_{hl}' - \theta_{kl}')^2 - d_{fl} . b_s . m_f . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')^2 - \\
& (m_f . (2 . d_{fl} . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}') . (b_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}' - b_t . \\
& \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' - b_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') - d_{fl} . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' \\
& + \theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \\
& \theta_{hr}' - \theta_{kr}')) + 2 . d_{fl} . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}') . (b_t . \mathrm{Cos}(\theta_{hl}) . \\
& \theta_{hl}' - b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}' + b_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') + d_{fl} . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \\
& \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \\
& (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')))) / 2 + d_{fl} . m_f . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}') . (b_t . \\
& \mathrm{Sin}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' - b_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') - d_{fl} . \mathrm{Cos}(\alpha_{fl} + \\
& \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} \\
& + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + d_{fl} . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \\
& \theta_{kl}') . (b_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}' + b_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') + d_{fl} . \\
& \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \\
& \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + d_{fl} . b_s . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \\
& \theta_{kl}) . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 + d_{fl} . b_s . m_f . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hr} - \\
& \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 - d_{fl} . b_{fr} . m_f . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl})
\end{aligned}$$

$. (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 + d_{fl} . b_{fr} . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} -$
$\theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2$

$b_s(2,1) = g . (b_{fr} . m_b . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + b_{fr} . m_f . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) +$
$d_{fr} . m_f . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + 2 . b_{fr} . m_s . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + 2 . b_{fr} .$
$m_t . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})) + b_{fr} . b_t . m_f . \mathrm{Cos}(\theta_{hl}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) .$
$\theta_{hl}'^2 + b_{fr} . b_t . m_s . \mathrm{Cos}(\theta_{hl}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \theta_{hl}'^2 + d_t . b_{fr} . m_t .$
$\mathrm{Cos}(\theta_{hl}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \theta_{hl}'^2 + b_{fr} . b_t . m_f . \mathrm{Sin}(\theta_{hl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} +$
$\theta_{hr} - \theta_{kr}) . \theta_{hl}'^2 + b_{fr} . b_t . m_s . \mathrm{Sin}(\theta_{hl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \theta_{hl}'^2 + d_t . b_{fr} .$
$m_t . \mathrm{Sin}(\theta_{hl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \theta_{hl}'^2 - b_{fr} . b_t . m_b . \mathrm{Cos}(\theta_{hr}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar}$
$+ \theta_{hr} - \theta_{kr}) . \theta_{hr}'^2 - b_{fr} . b_t . m_f . \mathrm{Cos}(\theta_{hr}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \theta_{hr}'^2 - b_{fr} . b_t .$
$m_s . \mathrm{Cos}(\theta_{hr}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \theta_{hr}'^2 - b_{fr} . b_t . m_t . \mathrm{Cos}(\theta_{hr}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar}$
$+ \theta_{hr} - \theta_{kr}) . \theta_{hr}'^2 - b_{fr} . (-d_t + b_t) . m_t . \mathrm{Cos}(\theta_{hr}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \theta_{hr}'^2 -$
$b_{fr} . b_t . m_b . \mathrm{Sin}(\theta_{hr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \theta_{hr}'^2 - b_{fr} . b_t . m_f . \mathrm{Sin}(\theta_{hr}) .$
$\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \theta_{hr}'^2 - b_{fr} . b_t . m_s . \mathrm{Sin}(\theta_{hr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) .$
$\theta_{hr}'^2 - b_{fr} . b_t . m_t . \mathrm{Sin}(\theta_{hr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \theta_{hr}'^2 - b_{fr} . (-d_t + b_t) . m_t .$
$\mathrm{Sin}(\theta_{hr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \theta_{hr}'^2 + b_{fr} . b_s . m_f . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) .$
$\mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')^2 + d_s . b_{fr} . m_s . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) .$
$(\theta_{hl}' - \theta_{kl}')^2 + b_{fr} . b_s . m_f . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{hl}' - \theta_{kl}')^2 +$
$d_s . b_{fr} . m_s . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{hl}' - \theta_{kl}')^2 + d_{fl} . b_{fr} . m_f .$
$\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}')^2 - d_{fl} . b_{fr} .$
$m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}')^2 -(m_b .$
$(-2 . b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (-(b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' -$
$\theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}') - 2 . b_{fr} .$
$\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (-(b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} .$
$\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}'))) / 2 - (m_t . (-2 . b_{fr} .$
$\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (d_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) .$
$(\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}') - 2 .$
$b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (d_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}' - b_s . \mathrm{Cos}(\theta_{hr} -$

$\theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')))$
$/ 2 - (m_t . (-2 . b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (-((-d_t + b_t) . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}') - b_s .$
$\mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (\theta_{ar}' +$
$\theta_{hr}' - \theta_{kr}') - 2 . b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (-((-d_t + b_t) . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}') - b_s .$
$\mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (\theta_{ar}' +$
$\theta_{hr}' - \theta_{kr}'))) / 2 - (m_s . (-2 . b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (b_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}' - b_t .$
$\mathrm{Sin}(\theta_{hr}) . \theta_{hr}' - d_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} .$
$\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}') - 2 . b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar}$
$+ \theta_{hr} - \theta_{kr}) . (b_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}' + d_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')$
$- b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) .$
$(\theta_{ar}' + \theta_{hr}' - \theta_{kr}'))) / 2 - (m_f . (-2 . b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (b_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}' -$
$b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' - b_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') - d_{fl} . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) .$
$(\theta_{al}' + \theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) .$
$(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}') - 2 . b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (b_t .$
$\mathrm{Cos}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}' + b_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') + d_{fl} . \mathrm{Sin}(\alpha_{fl} +$
$\theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar}$
$+ \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}'))) / 2 - (m_s . (-2 . b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} +$
$\theta_{hr} - \theta_{kr}) . (-((-d_s + b_s) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')) + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) .$
$(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}') - 2 . b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (-((-d_s + b_s) .$
$\mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')) - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (\theta_{ar}' +$
$\theta_{hr}' - \theta_{kr}'))) / 2 - b_{fr} . b_s . m_b . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 -$
$b_{fr} . b_s . m_f . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 - b_{fr} . b_s . m_s .$
$\mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 - b_{fr} . (-d_s + b_s) . m_s . \mathrm{Cos}(\theta_{hr} -$
$\theta_{kr}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 - 2 . b_{fr} . b_s . m_t . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) .\mathrm{Cos}(\alpha_{fr}$
$+ \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 - b_{fr} . b_s . m_b . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) .$
$(\theta_{hr}' - \theta_{kr}')^2 - b_{fr} . b_s . m_f . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 - b_{fr}$
$. b_s . m_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 - b_{fr} . (-d_s + b_s) . m_s .$
$\mathrm{Sin}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 - 2 . b_{fr} . b_s . m_t . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) .$
$\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 - b_{fr} . m_b . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \quad (-(b_t .$
$\mathrm{Sin}(\theta_{hr}) . \theta_{hr}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' +$

$\theta_{hr}{}' - \theta_{kr}{}')) . (\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}') - b_{fr} . m_t . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (d_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}{}' - b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}{}' - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}{}' - \theta_{kr}{}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}')) . (\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}') - b_{fr} . m_t . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (-((-d_t + b_t) . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}{}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}{}' - \theta_{kr}{}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}')) . (\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}') - b_{fr} . m_s . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (b_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}{}' - b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}{}' - d_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}{}' - \theta_{kl}{}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}{}' - \theta_{kr}{}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}')) . (\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}') - b_{fr} . m_f . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (b_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}{}' - b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}{}' - b_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}{}' - \theta_{kl}{}') - d_{fl} . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}{}' + \theta_{hl}{}' - \theta_{kl}{}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}{}' - \theta_{kr}{}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}')) . (\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}') - b_{fr} . m_s . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (-((-d_s + b_s) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}{}' - \theta_{kr}{}')) + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}')) . (\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}')- b_{fr} . m_b . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (-(b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}{}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}{}' - \theta_{kr}{}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}')) . (\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}') - b_{fr} . m_t . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (d_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}{}' - b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}{}' - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}{}' - \theta_{kr}{}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}')) . (\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}') - b_{fr} . m_t . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (-((-d_t + b_t) . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}{}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}{}' - \theta_{kr}{}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}')) . (\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}') - b_{fr} . m_s . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (b_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}{}' - b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}{}' + d_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}{}' - \theta_{kl}{}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}{}' - \theta_{kr}{}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}')) . (\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}') - b_{fr} . m_f . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (b_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}{}' - b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}{}' + b_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}{}' - \theta_{kl}{}') + d_{fl} . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}{}' + \theta_{hl}{}' - \theta_{kl}{}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}{}' - \theta_{kr}{}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}')) . (\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}') - b_{fr} . m_s . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (-((-d_s + b_s) . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}{}' - \theta_{kr}{}')) - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}')) . (\theta_{ar}{}' + \theta_{hr}{}' - \theta_{kr}{}')$

Open form of b_p

$$
\begin{aligned}
& b_p(1,1) = g \cdot (d_t \cdot m_t \cdot \mathrm{Sin}(\theta_{hl}) + m_s \cdot (-(d_s \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl})) + b_t \cdot \mathrm{Sin}(\theta_{hl})) + m_f \cdot \\
& (-(b_s \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl})) - d_{fl} \cdot \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) + b_t \cdot \mathrm{Sin}(\theta_{hl}))) - b_s \cdot b_t \cdot m_f \cdot \\
& \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \theta_{hl}'^{2} - d_s \cdot b_t \cdot m_s \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \theta_{hl}'^{2} - d_{fl} \cdot \\
& b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) \cdot \theta_{hl}'^{2} - b_s \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \\
& \mathrm{Sin}(\theta_{hl}) \cdot \theta_{hl}'^{2} - d_s \cdot b_t \cdot m_s \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\theta_{hl}) \cdot \theta_{hl}'^{2} - d_{fl} \cdot b_t \cdot m_f \cdot \mathrm{Sin}(\theta_{hl}) \cdot \\
& \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) \cdot \theta_{hl}'^{2} + b_s \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \theta_{hr}'^{2} + d_s \cdot \\
& b_t \cdot m_s \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \theta_{hr}'^{2} + d_{fl} \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} \\
& - \theta_{kl}) \cdot \theta_{hr}'^{2} - b_t^{2} \cdot m_f \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Sin}(\theta_{hl}) \cdot \theta_{hr}'^{2} - b_t^{2} \cdot m_s \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Sin}(\theta_{hl}) \cdot \theta_{hr}'^{2} \\
& - d_t \cdot b_t \cdot m_t \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Sin}(\theta_{hl}) \cdot \theta_{hr}'^{2} + b_t^{2} \cdot m_f \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Sin}(\theta_{hr}) \cdot \theta_{hr}'^{2} + b_t^{2} \cdot \\
& m_s \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Sin}(\theta_{hr}) \cdot \theta_{hr}'^{2} + d_t \cdot b_t \cdot m_t \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Sin}(\theta_{hr}) \cdot \theta_{hr}'^{2} + b_s \cdot b_t \cdot m_f \cdot \\
& \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\theta_{hr}) \cdot \theta_{hr}'^{2} + d_s \cdot b_t \cdot m_s \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\theta_{hr}) \cdot \theta_{hr}'^{2} + d_{fl} \cdot \\
& b_t \cdot m_f \cdot \mathrm{Sin}(\theta_{hr}) \cdot \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) \cdot \theta_{hr}'^{2} - b_s \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Sin}(\theta_{hl} - \\
& \theta_{kl}) \cdot (\theta_{hl}' - \theta_{kl}')^{2} - d_s \cdot b_t \cdot m_s \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Sin}(\theta_{hl} - \theta_{kl}) \cdot (\theta_{hl}' - \theta_{kl}')^{2} - 2 \cdot b_s^{2} \cdot m_f \cdot \\
& \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\theta_{hl} - \theta_{kl}) \cdot (\theta_{hl}' - \theta_{kl}')^{2} - 2 \cdot d_s^{2} \cdot m_s \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\theta_{hl} - \\
& \theta_{kl}) \cdot (\theta_{hl}' - \theta_{kl}')^{2} - d_{fl} \cdot b_s \cdot m_f \cdot \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\theta_{hl} - \theta_{kl}) \cdot (\theta_{hl}' - \\
& \theta_{kl}')^{2} + b_s \cdot b_t \cdot m_f \cdot \mathrm{Sin}(\theta_{hl}) \cdot \mathrm{Sin}(\theta_{hl} - \theta_{kl}) \cdot (\theta_{hl}' - \theta_{kl}')^{2} + d_s \cdot b_t \cdot m_s \cdot \mathrm{Sin}(\theta_{hl}) \cdot \\
& \mathrm{Sin}(\theta_{hl} - \theta_{kl}) \cdot (\theta_{hl}' - \theta_{kl}')^{2} - d_{fl} \cdot b_s \cdot m_f \cdot \mathrm{Sin}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) \cdot \\
& (\theta_{hl}' - \theta_{kl}')^{2} + d_{fl} \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) \cdot (\theta_{al}' + \theta_{hl}' - \theta_{kl}')^{2} \\
& + d_{fl} \cdot b_s \cdot m_f \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) \cdot (\theta_{al}' + \theta_{hl}' - \theta_{kl}')^{2} - d_{fl} \cdot \\
& b_s \cdot m_f \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) \cdot (\theta_{al}' + \theta_{hl}' - \theta_{kl}')^{2} + d_{fl} \cdot b_t \cdot m_f \cdot \\
& \mathrm{Sin}(\theta_{hl}) \cdot \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) \cdot (\theta_{al}' + \theta_{hl}' - \theta_{kl}')^{2} -(m_t \cdot (2 \cdot d_t \cdot \mathrm{Cos}(\theta_{hl}) \cdot \theta_{hl}' \cdot \\
& (d_t \cdot \mathrm{Sin}(\theta_{hl}) \cdot \theta_{hl}' - b_t \cdot \mathrm{Sin}(\theta_{hr}) \cdot \theta_{hr}' - b_s \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot (\theta_{hr}' - \theta_{kr}') + b_{fr} \cdot \mathrm{Cos}(\alpha_{fr} \\
& + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) - 2 \cdot d_t \cdot \mathrm{Sin}(\theta_{hl}) \cdot \theta_{hl}' \cdot (d_t \cdot \mathrm{Cos}(\theta_{hl}) \cdot \theta_{hl}' - b_t \cdot \\
& \mathrm{Cos}(\theta_{hr}) \cdot \theta_{hr}' - b_s \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot (\theta_{hr}' - \theta_{kr}') - b_{fr} \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{ar}' + \\
& \theta_{hr}' - \theta_{kr}')))) / 2 - (m_s \cdot (2 \cdot (b_t \cdot \mathrm{Cos}(\theta_{hl}) \cdot \theta_{hl}' + d_s \cdot \mathrm{Sin}(\theta_{hl} - \theta_{kl}) \cdot (\theta_{hl}' - \theta_{kl}')) \cdot (b_t \cdot \\
& \mathrm{Sin}(\theta_{hl}) \cdot \theta_{hl}' - b_t \cdot \mathrm{Sin}(\theta_{hr}) \cdot \theta_{hr}' - d_s \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot (\theta_{hl}' - \theta_{kl}') - b_s \cdot \mathrm{Sin}(\theta_{hr} - \\
& \theta_{kr}) \cdot (\theta_{hr}' - \theta_{kr}') + b_{fr} \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + 2 \cdot (-(b_t \cdot \mathrm{Sin}(\theta_{hl}) \cdot
\end{aligned}
$$

$\theta_{hl}') - d_s . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')) . (b_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}' + d_s .$
$\mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} -$
$\theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')))) / 2 - (m_f . (2 . (b_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' + b_s . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}'$
$- \theta_{kl}') + d_{fl} . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}')) . (b_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}' - b_t .$
$\mathrm{Sin}(\theta_{hr}) . \theta_{hr}' - b_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') - d_{fl} . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}'$
$+ \theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' +$
$\theta_{hr}' - \theta_{kr}')) + 2 . (-(b_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}') - b_s . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') + d_{fl} . \mathrm{Cos}(\alpha_{fl}$
$+ \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}')) . (b_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}' + b_s .$
$\mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') + d_{fl} . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}') - b_s .$
$\mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')))) / 2 + d_t .$
$m_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' . (d_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' -$
$\theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + m_s . (b_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' +$
$d_s . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')) . (b_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' - d_s . \mathrm{Cos}(\theta_{hl}$
$- \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) .$
$(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + m_f . (b_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' + b_s . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') + d_{fl} .$
$\mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}')) . (b_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' -$
$b_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') - d_{fl} . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}') - b_s$
$. \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) - d_t . m_t$
$. \mathrm{Sin}(\theta_{hl}) . \theta_{hl}' . (d_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}' - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' -$
$\theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + m_s . (-(b_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}') -$
$d_s . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')) . (b_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}' + d_s .$
$\mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} -$
$\theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + m_f . (-(b_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}') - b_s . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') +$
$d_{fl} . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}')) . (b_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' - b_t .$
$\mathrm{Cos}(\theta_{hr}) . \theta_{hr}' + b_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') + d_{fl} . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}'$
$+ \theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' +$
$\theta_{hr}' - \theta_{kr}')) + b_s^2 . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 + d_s . b_s . m_s .$
$\mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 + d_{fl} . b_s . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) .$
$\mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 - b_s . b_t . m_f . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hl}) . (\theta_{hr}' - \theta_{kr}')^2 - b_s .$
$b_t . m_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hl}) . (\theta_{hr}' - \theta_{kr}')^2 - d_t . b_s . m_t . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hl}) .$

$(\theta_{hr}' - \theta_{kr}')^2 + b_s . b_t . m_f . \text{Cos}(\theta_{hl}) . \text{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 + b_s . b_t . m_s . \text{Cos}(\theta_{hl}) . \text{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 + d_t . b_s . m_t . \text{Cos}(\theta_{hl}) . \text{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 + b_s^2 . m_f . \text{Cos}(\theta_{hl} - \theta_{kl}) . \text{Sin}(\theta_{hr} - \theta_{kr}) .(\theta_{hr}' - \theta_{kr}')^2 + d_s . b_s . m_s . \text{Cos}(\theta_{hl} - \theta_{kl}) . \text{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 + d_{fl} . b_s . m_f . \text{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \text{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 - b_{fr} . b_t . m_f . \text{Cos}(\theta_{hl}) . \text{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 - b_{fr} . b_t . m_s . \text{Cos}(\theta_{hl}) . \text{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 - d_t . b_{fr} . m_t . \text{Cos}(\theta_{hl}) . \text{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 - b_{fr} . b_s . m_f . \text{Cos}(\theta_{hl} - \theta_{kl}) . \text{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 - d_s . b_{fr} . m_s . \text{Cos}(\theta_{hl} - \theta_{kl}) . \text{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 - d_{fl} . b_{fr} . m_f . \text{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \text{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 + b_{fr} . b_s . m_f . \text{Cos}(\theta_{hl} - \theta_{kl}) . \text{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 + d_s . b_{fr} . m_s . \text{Cos}(\theta_{hl} - \theta_{kl}) . \text{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 + d_{fl} . b_{fr} . m_f . \text{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \text{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 - b_{fr} . b_t . m_f . \text{Sin}(\theta_{hl}) .\text{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 - b_{fr} . b_t . m_s . \text{Sin}(\theta_{hl}) .\text{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) .(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 - d_t . b_{fr} . m_t . \text{Sin}(\theta_{hl}) . \text{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2$

$b_p(2,1) = g . (d_s . m_s . \text{Cos}(\theta_{hl} - \theta_{kl}) + m_f . (b_s . \text{Cos}(\theta_{hl} - \theta_{kl}) + d_{fl} . \text{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}))) + b_s . b_t . m_f . \text{Cos}(\theta_{hl}) . \text{Cos}(\theta_{hl} - \theta_{kl}) . \theta_{hl}'^2 + d_s . b_t . m_s . \text{Cos}(\theta_{hl}) . \text{Cos}(\theta_{hl} - \theta_{kl}) . \theta_{hl}'^2 + d_{fl} . b_t . m_f . \text{Cos}(\theta_{hl}) . \text{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \theta_{hl}'^2 + b_s . b_t . m_f . \text{Cos}(\theta_{hl} - \theta_{kl}) . \text{Sin}(\theta_{hl}) . \theta_{hl}'^2 + d_s . b_t . m_s . \text{Cos}(\theta_{hl} - \theta_{kl}) . \text{Sin}(\theta_{hl}) . \theta_{hl}'^2 + d_{fl} . b_t . m_f . \text{Sin}(\theta_{hl}) . \text{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \theta_{hl}'^2 - b_s . b_t . m_f . \text{Cos}(\theta_{hr}) . \text{Cos}(\theta_{hl} - \theta_{kl}) . \theta_{hr}'^2 - d_s . b_t . m_s . \text{Cos}(\theta_{hr}) . \text{Cos}(\theta_{hl} - \theta_{kl}) . \theta_{hr}'^2 - d_{fl} . b_t . m_f . \text{Cos}(\theta_{hr}) . \text{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \theta_{hr}'^2 - b_s . b_t . m_f . \text{Cos}(\theta_{hl} - \theta_{kl}) . \text{Sin}(\theta_{hr}) . \theta_{hr}'^2 - d_s . b_t . m_s . \text{Cos}(\theta_{hl} - \theta_{kl}) . \text{Sin}(\theta_{hr}) . \theta_{hr}'^2 - d_{fl} . b_t . m_f . \text{Sin}(\theta_{hr}) . \text{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \theta_{hr}'^2 + 2 . b_s^2 . m_f . \text{Cos}(\theta_{hl} - \theta_{kl}) . \text{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')^2 + 2 . d_s^2 . m_s . \text{Cos}(\theta_{hl} - \theta_{kl}) . \text{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')^2 + d_{fl} . b_s . m_f . \text{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \text{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')^2 + d_{fl} . b_s . m_f . \text{Sin}(\theta_{hl} - \theta_{kl}) . \text{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')^2 - d_{fl} . b_s . m_f . \text{Cos}(\theta_{hl} - \theta_{kl}) . \text{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}')^2 + d_{fl} . b_s . m_f . \text{Cos}(\theta_{hl} - \theta_{kl}) . \text{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} -$

$\theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}')^2 -(m_s . (-2 . d_s . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') . (b_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}'$

$- b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' - d_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') +$

$b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + 2 . d_s . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') .$

$(b_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}' + d_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') - b_s .$

$\mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')))) / 2 -$

$(m_f . (2 . (-(b_s . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')) - d_{fl} . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' +$

$\theta_{hl}' - \theta_{kl}')) . (b_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' - b_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') -$

$d_{fl} . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') +$

$b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + 2 . (b_s . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')$

$- d_{fl} . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}')) . (b_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' - b_t .$

$\mathrm{Cos}(\theta_{hr}) . \theta_{hr}' + b_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') + d_{fl} . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}'$

$+ \theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' +$

$\theta_{hr}' - \theta_{kr}')))) / 2 -d_s . m_s . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') .(b_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}' - b_t .\mathrm{Sin}(\theta_{hr}) .$

$\theta_{hr}' - d_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} +$

$\theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + m_f . (-(b_s . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')) - d_{fl} .$

$\mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}')) . (b_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}'$

$- b_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') - d_{fl} . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}') -$

$b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + d_s .$

$m_s . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') . (b_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}' + d_s .$

$\mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} -$

$\theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + m_f . (b_s . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') - d_{fl} . \mathrm{Cos}(\alpha_{fl} + \theta_{al} +$

$\theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}')) . (b_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}' + b_s . \mathrm{Cos}(\theta_{hl} -$

$\theta_{kl}) . (\theta_{hl}' - \theta_{kl}') + d_{fl} . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Cos}(\theta_{hr} -$

$\theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) - b_s^2 . m_f . \mathrm{Cos}(\theta_{hl}$

$- \theta_{kl}) . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 - d_s . b_s . m_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' -$

$\theta_{kr}')^2 - d_{fl} . b_s . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 - b_s^2 .$

$m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 - d_s . b_s . m_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hr} -$

$\theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 - d_{fl} . b_s . m_f . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) .(\theta_{hr}' - \theta_{kr}')^2$

$+ b_{fr} . b_s . m_f . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 + d_s .$

$b_{fr} . m_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 + d_{fl} . b_{fr} . m_f .$

$\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) \cdot (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 - b_{fr} \cdot b_s \cdot$
$m_f \cdot \mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 - d_s \cdot b_{fr} \cdot m_s \cdot$
$\mathrm{Cos}(\theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 - d_{fl} \cdot b_{fr} \cdot m_f \cdot \mathrm{Cos}(\alpha_{fl} +$
$\theta_{al} + \theta_{hl} - \theta_{kl}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2$

$b_p(3,1) = g \cdot (d_{fr} \cdot m_f \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) + m_b \cdot (b_{fr} \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$
$- b_t \cdot \mathrm{Sin}(\theta_{hr}) - b_s \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr})) + m_f \cdot (b_{fr} \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_t \cdot \mathrm{Sin}(\theta_{hr})$
$- b_s \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr})) + m_s \cdot (b_{fr} \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_t \cdot \mathrm{Sin}(\theta_{hr}) - b_s \cdot \mathrm{Sin}(\theta_{hr} -$
$\theta_{kr})) + m_t \cdot (b_{fr} \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - b_t \cdot \mathrm{Sin}(\theta_{hr}) - b_s \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr})) + m_t \cdot$
$(b_{fr} \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - (-d_t + b_t) \cdot \mathrm{Sin}(\theta_{hr}) - b_s \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr})) + m_s \cdot (b_{fr} \cdot$
$\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) - (-d_s + b_s) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}))) + b_{fr} \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hl}) \cdot$
$\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \theta_{hl}'^2 + b_{fr} \cdot b_t \cdot m_s \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot$
$\theta_{hl}'^2 + d_t \cdot b_{fr} \cdot m_t \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \theta_{hl}'^2 + b_t^2 \cdot m_f \cdot \mathrm{Cos}(\theta_{hr}) \cdot$
$\mathrm{Sin}(\theta_{hl}) \cdot \theta_{hl}'^2 + b_t^2 \cdot m_s \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Sin}(\theta_{hl}) \cdot \theta_{hl}'^2 + d_t \cdot b_t \cdot m_t \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Sin}($
$\theta_{hl}) \cdot \theta_{hl}'^2 + b_s \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hl}) \cdot \theta_{hl}'^2 + b_s \cdot b_t \cdot m_s \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot$
$\mathrm{Sin}(\theta_{hl}) \cdot \theta_{hl}'^2 + d_t \cdot b_s \cdot m_t \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hl}) \cdot \theta_{hl}'^2 - b_t^2 \cdot m_f \cdot \mathrm{Cos}(\theta_{hl}) \cdot$
$\mathrm{Sin}(\theta_{hr}) \cdot \theta_{hl}'^2 - b_t^2 \cdot m_s \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Sin}(\theta_{hr}) \cdot \theta_{hl}'^2 - d_t \cdot b_t \cdot m_t \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Sin}(\theta_{hr}) \cdot$
$\theta_{hl}'^2 - b_s \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot \theta_{hl}'^2 - b_s \cdot b_t \cdot m_s \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Sin}(\theta_{hr} -$
$\theta_{kr}) \cdot \theta_{hl}'^2 - d_t \cdot b_s \cdot m_t \cdot \mathrm{Cos}(\theta_{hl}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot \theta_{hl}'^2 + b_{fr} \cdot b_t \cdot m_f \cdot \mathrm{Sin}(\theta_{hl}) \cdot$
$\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \theta_{hl}'^2 + b_{fr} \cdot b_t \cdot m_s \cdot \mathrm{Sin}(\theta_{hl}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot$
$\theta_{hl}'^2 + d_t \cdot b_{fr} \cdot m_t \cdot \mathrm{Sin}(\theta_{hl}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \theta_{hl}'^2 - b_{fr} \cdot b_t \cdot m_b \cdot$
$\mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \theta_{hr}'^2 - b_{fr} \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} +$
$\theta_{hr} - \theta_{kr}) \cdot \theta_{hr}'^2 - b_{fr} \cdot b_t \cdot m_s \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \theta_{hr}'^2 - b_{fr} \cdot b_t \cdot$
$m_t \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \theta_{hr}'^2 - b_{fr} \cdot (-d_t + b_t) \cdot m_t \cdot \mathrm{Cos}(\theta_{hr}) \cdot$
$\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \theta_{hr}'^2 - b_s \cdot b_t \cdot m_b \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr}) \cdot \theta_{hr}'^2 - b_s \cdot$
$b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr}) \cdot \theta_{hr}'^2 - b_s \cdot b_t \cdot m_s \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr}) \cdot \theta_{hr}'^2 -$
$b_s \cdot b_t \cdot m_t \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr}) \cdot \theta_{hr}'^2 - b_s \cdot (-d_t + b_t) \cdot m_t \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot$
$\mathrm{Sin}(\theta_{hr}) \cdot \theta_{hr}'^2 + b_s \cdot b_t \cdot m_b \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot \theta_{hr}'^2 + b_s \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hr}) \cdot$
$\mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot \theta_{hr}'^2 + b_s \cdot b_t \cdot m_s \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot \theta_{hr}'^2 + b_s \cdot b_t \cdot m_t \cdot$

$\mathrm{Cos}(\theta_{hr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . \theta_{hr}'^{2} + b_s . (-d_t + b_t) . m_t . \mathrm{Cos}(\theta_{hr}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . \theta_{hr}'^{2} -$
$b_{fr} . b_t . m_b . \mathrm{Sin}(\theta_{hr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \theta_{hr}'^{2} - b_{fr} . b_t . m_f . \mathrm{Sin}(\theta_{hr}) .$
$\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \theta_{hr}'^{2} - b_{fr} . b_t . m_s . \mathrm{Sin}(\theta_{hr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \theta_{hr}'^{2}$
$- b_{fr} . b_t . m_t . \mathrm{Sin}(\theta_{hr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \theta_{hr}'^{2} - b_{fr} . (-d_t + b_t) . m_t .$
$\mathrm{Sin}(\theta_{hr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \theta_{hr}'^{2} + b_s . b_t . m_f . \mathrm{Cos}(\theta_{hr}) . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) .$
$(\theta_{hl}' - \theta_{kl}')^2 + d_s . b_t . m_s . \mathrm{Cos}(\theta_{hr}) . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')^2 + b_s^2 . m_f . \mathrm{Cos}(\theta_{hr}$
$- \theta_{kr}) . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')^2 + d_s . b_s . m_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}'$
$- \theta_{kl}')^2 + b_{fr} . b_s . m_f . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')^2 + d_s .$
$b_{fr} . m_s . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')^2 - b_s . b_t . m_f . \mathrm{Sin}(\theta_{hr}) .$
$\mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')^2 - d_s . b_t . m_s . \mathrm{Sin}(\theta_{hr}) . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')^2 -$
$b_s^2 . m_f . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hl}' - \theta_{kl}')^2 - d_s . b_s . m_s . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) .$
$\mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hl}' - \theta_{kl}')^2 + b_{fr} . b_s . m_f . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) .$
$(\theta_{hl}' - \theta_{kl}')^2 + d_s . b_{fr} . m_s . \mathrm{Sin}(\theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{hl}' - \theta_{kl}')^2 -$
$d_{fl} . b_t . m_f . \mathrm{Cos}(\theta_{hr}) . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}')^2 - d_{fl} . b_s . m_f .$
$\mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}')^2 + d_{fl} . b_{fr} . m_f . \mathrm{Cos}(\alpha_{fr}$
$+ \theta_{ar} + \theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}')^2 - d_{fl} . b_t . m_f . \mathrm{Sin}(\theta_{hr}) .$
$\mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}')^2 - d_{fl} . b_s . m_f . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) .$
$\mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}')^2 - d_{fl} . b_{fr} . m_f . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . \mathrm{Sin}(\alpha_{fr} +$
$\theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}')^2 -(m_b . (2 . (b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' + b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) .$
$(\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (-(b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}') -$
$b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + 2 .$
$(-(b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) .$
$(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (-(b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} .$
$\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')))) / 2 - (m_t . (2 . (d_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}' - b_t .$
$\mathrm{Sin}(\theta_{hr}) . \theta_{hr}' - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' +$
$\theta_{hr}' - \theta_{kr}')) . (-(b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar}$
$+ \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + 2 . (b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' + b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')$
$- b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) .(d_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Cos}(\theta_{hr}) .$
$\theta_{hr}' - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' -$
$\theta_{kr}')))) / 2 - (m_t . (2 . ((-d_t + b_t) . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' + b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} .$

$\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (-(((-d_t + b_t) . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}') - b_s .$
$\mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + 2 .$
$(-(((-d_t + b_t) . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr}$
$- \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (-(((-d_t + b_t) . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' -$
$\theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')))) / 2 - (m_s . (2 . (b_t . \mathrm{Sin}(\theta_{hl}) .$
$\theta_{hl}' - b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' - d_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' -$
$\theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (-(b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}') - b_s .$
$\mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + 2 . (b_t .$
$\mathrm{Sin}(\theta_{hr}) . \theta_{hr}' + b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' +$
$\theta_{hr}' - \theta_{kr}')) . (b_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}' + d_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')$
$- b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')))) /$
$2 - (m_f . (2 . (b_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' - b_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') -$
$d_{fl} . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') +$
$b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (-(b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}') - b_s . \mathrm{Cos}(\theta_{hr}$
$- \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + 2 . (b_t .$
$\mathrm{Sin}(\theta_{hr}) . \theta_{hr}' + b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' +$
$\theta_{hr}' - \theta_{kr}')) . (b_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}' + b_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}')$
$+ d_{fl} . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') -$
$b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')))) / 2 - (m_s . (2 . ((-d_s + b_s) . \mathrm{Sin}(\theta_{hr} -$
$\theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (-(((-d_s + b_s) .$
$\mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')) - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + 2 .$
$(-(((-d_s + b_s) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')) + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' -$
$\theta_{kr}')) . (-(((-d_s + b_s) . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')) - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}'$
$+ \theta_{hr}' - \theta_{kr}')))) / 2+ m_b . (b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' + b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} .$
$\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (-(b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}') - b_s . \mathrm{Cos}(\theta_{hr} -$
$\theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + m_b . (-(b_t .$
$\mathrm{Sin}(\theta_{hr}) . \theta_{hr}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' +$
$\theta_{hr}' - \theta_{kr}')) . (-(b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar}$
$+ \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + m_t . (d_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' - b_s .$
$\mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (-(b_t .$

$\mathrm{Cos}(\theta_{hr}) . \theta_{hr}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' +$
$\theta_{hr}' - \theta_{kr}')) + m_s . (b_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' - d_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' -$
$\theta_{kl}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' -$
$\theta_{kr}')) . (-(b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr}$
$- \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + m_f . (b_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' - b_s . \mathrm{Cos}(\theta_{hl} -$
$\theta_{kl}) . (\theta_{hl}' - \theta_{kl}') - d_{fl} . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) .$
$(\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (-(b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}')$
$- b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) +$
$m_t . (b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' + b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) .$
$(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (d_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}' - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' -$
$\theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}'))+ m_t . ((-d_t + b_t) . \mathrm{Sin}(\theta_{hr}) .$
$\theta_{hr}' + b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' -$
$\theta_{kr}')) . (-((-d_t + b_t) . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} +$
$\theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + m_t . (-((-d_t + b_t) . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}') - b_s . \mathrm{Sin}(\theta_{hr} -$
$\theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (-((-d_t + b_t) .$
$\mathrm{Cos}(\theta_{hr}) . \theta_{hr}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' +$
$\theta_{hr}' - \theta_{kr}')) + m_s . (b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' + b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Cos}(\alpha_{fr} +$
$\theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (b_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}' + d_s .$
$\mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} -$
$\theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + m_f . (b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' + b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} .$
$\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (b_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}'$
$+ b_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') + d_{fl} . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}') -$
$b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + m_s .$
$((-d_s + b_s) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' -$
$\theta_{kr}')) . (-((-d_s + b_s) . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')) - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}'$
$+ \theta_{hr}' - \theta_{kr}'))+ m_s . (-((-d_s + b_s) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')) + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr}$
$- \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (-((-d_s + b_s) . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')) - b_{fr} . \mathrm{Sin}(\alpha_{fr} +$
$\theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) - b_{fr} . b_s . m_b . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} -$
$\theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 - b_{fr} . b_s . m_f . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) .(\theta_{hr}' - \theta_{kr}')^2$
$- b_{fr} . b_s . m_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 - b_{fr} . (-d_s + b_s) .$

$$
\begin{aligned}
& m_s \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{hr}' - \theta_{kr}')^2 - 2 \cdot b_{fr} \cdot b_s \cdot m_t \cdot \mathrm{Cos}(\theta_{hr} \\
& - \theta_{kr}) \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{hr}' - \theta_{kr}')^2 + b_s \cdot b_t \cdot m_b \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr}) \cdot \\
& (\theta_{hr}' - \theta_{kr}')^2 + b_s \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr}) \cdot (\theta_{hr}' - \theta_{kr}')^2 + b_s \cdot b_t \cdot m_s \cdot \\
& \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr}) \cdot (\theta_{hr}' - \theta_{kr}')^2 + b_s \cdot b_t \cdot m_t \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr}) \cdot (\theta_{hr}' - \\
& \theta_{kr}')^2 + b_s \cdot (-d_t + b_t) \cdot m_t \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\theta_{hr}) \cdot (\theta_{hr}' - \theta_{kr}')^2 - b_s \cdot b_t \cdot m_b \cdot \\
& \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot (\theta_{hr}' - \theta_{kr}')^2 - b_s \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot (\theta_{hr}' - \\
& \theta_{kr}')^2 - b_s \cdot b_t \cdot m_s \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot (\theta_{hr}' - \theta_{kr}')^2 - b_s \cdot b_t \cdot m_t \cdot \mathrm{Cos}(\theta_{hr}) \cdot \\
& \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot (\theta_{hr}' - \theta_{kr}')^2 - b_s \cdot (-d_t + b_t) \cdot m_t \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot (\theta_{hr}' - \theta_{kr}')^2 \\
& - b_{fr} \cdot b_s \cdot m_b \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{hr}' - \theta_{kr}')^2 - b_{fr} \cdot b_s \cdot m_f \cdot \\
& \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{hr}' - \theta_{kr}')^2 - b_{fr} \cdot b_s \cdot m_s \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot \\
& \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{hr}' - \theta_{kr}')^2 - b_{fr} \cdot (-d_s + b_s) \cdot m_s \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\alpha_{fr} + \\
& \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{hr}' - \theta_{kr}')^2 - 2 \cdot b_{fr} \cdot b_s \cdot m_t \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \\
& (\theta_{hr}' - \theta_{kr}')^2 + b_{fr} \cdot b_t \cdot m_b \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 + \\
& b_{fr} \cdot b_t \cdot m_f \cdot \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 + b_{fr} \cdot b_t \cdot m_s \cdot \\
& \mathrm{Cos}(\theta_{hr}) \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 + b_{fr} \cdot b_t \cdot m_t \cdot \mathrm{Cos}(\theta_{hr}) \cdot \\
& \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 + b_{fr} \cdot (-d_t + b_t) \cdot m_t \cdot \mathrm{Cos}(\theta_{hr}) \cdot \\
& \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 + b_{fr} \cdot b_s \cdot m_b \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Cos}(\alpha_{fr} \\
& + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 + b_{fr} \cdot b_s \cdot m_f \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} \\
& - \theta_{kr}) \cdot (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 + b_{fr} \cdot b_s \cdot m_s \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \\
& (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 + b_{fr} \cdot (-d_s + b_s) \cdot m_s \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot \\
& (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 + 2 \cdot b_{fr} \cdot b_s \cdot m_t \cdot \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{ar}' + \\
& \theta_{hr}' - \theta_{kr}')^2 + b_{fr} \cdot b_t \cdot m_b \cdot \mathrm{Sin}(\theta_{hr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 + \\
& b_{fr} \cdot b_t \cdot m_f \cdot \mathrm{Sin}(\theta_{hr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 + b_{fr} \cdot b_t \cdot m_s \cdot \\
& \mathrm{Sin}(\theta_{hr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 + b_{fr} \cdot b_t \cdot m_t \cdot \mathrm{Sin}(\theta_{hr}) \cdot \\
& \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 + b_{fr} \cdot (-d_t + b_t) \cdot m_t \cdot \mathrm{Sin}(\theta_{hr}) \cdot \mathrm{Sin}(\alpha_{fr} \\
& + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 + b_{fr} \cdot b_s \cdot m_b \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} \\
& - \theta_{kr}) \cdot (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 + b_{fr} \cdot b_s \cdot m_f \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{ar}' \\
& + \theta_{hr}' - \theta_{kr}')^2 + b_{fr} \cdot b_s \cdot m_s \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{ar}' + \theta_{hr}' - \\
& \theta_{kr}')^2 + b_{fr} \cdot (-d_s + b_s) \cdot m_s \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 \\
& + 2 \cdot b_{fr} \cdot b_s \cdot m_t \cdot \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \cdot \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \cdot (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2
\end{aligned}
$$

$$
\begin{aligned}
& b_p(4,1) = g \,.\, (-(d_{fr} \,.\, m_f \,.\, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})) + m_b \,.\, (-(b_{fr} \,.\, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \\
& \theta_{kr})) + b_s \,.\, \mathrm{Sin}(\theta_{hr} - \theta_{kr})) + m_f \,.\, (-(b_{fr} \,.\, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})) + b_s \,.\, \mathrm{Sin}(\theta_{hr} - \theta_{kr})) \\
& + m_s \,.\, (-(b_{fr} \,.\, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})) + b_s \,.\, \mathrm{Sin}(\theta_{hr} - \theta_{kr})) + 2 \,.\, m_t \,.\, (-(b_{fr} \,.\, \mathrm{Cos}(\alpha_{fr} \\
& + \theta_{ar} + \theta_{hr} - \theta_{kr})) + b_s \,.\, \mathrm{Sin}(\theta_{hr} - \theta_{kr})) + m_s \,.\, (-(b_{fr} \,.\, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})) + (-d_s + \\
& b_s) \,.\, \mathrm{Sin}(\theta_{hr} - \theta_{kr}))) - b_{fr} \,.\, b_t \,.\, m_f \,.\, \mathrm{Cos}(\theta_{hl}) \,.\, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \,.\, \theta_{hl}'^{2} - b_{fr} \,.\, \\
& b_t \,.\, m_s \,.\, \mathrm{Cos}(\theta_{hl}) \,.\, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \,.\, \theta_{hl}'^{2} - d_t \,.\, b_{fr} \,.\, m_t \,.\, \mathrm{Cos}(\theta_{hl}) \,.\, \mathrm{Cos}(\alpha_{fr} \\
& + \theta_{ar} + \theta_{hr} - \theta_{kr}) \,.\, \theta_{hl}'^{2} - b_s \,.\, b_t \,.\, m_f \,.\, \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \,.\, \mathrm{Sin}(\theta_{hl}) \,.\, \theta_{hl}'^{2} - b_s \,.\, b_t \,.\, m_s \,.\, \\
& \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \,.\, \mathrm{Sin}(\theta_{hl}) \,.\, \theta_{hl}'^{2} - d_t \,.\, b_s \,.\, m_t \,.\, \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \,.\, \mathrm{Sin}(\theta_{hl}) \,.\, \theta_{hl}'^{2} + b_s \,.\, b_t \,.\, \\
& m_f \,.\, \mathrm{Cos}(\theta_{hl}) \,.\, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \,.\, \theta_{hl}'^{2} + b_s \,.\, b_t \,.\, m_s \,.\, \mathrm{Cos}(\theta_{hl}) \,.\, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \,.\, \theta_{hl}'^{2} + d_t \,.\, \\
& b_s \,.\, m_t \,.\, \mathrm{Cos}(\theta_{hl}) \,.\, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \,.\, \theta_{hl}'^{2} - b_{fr} \,.\, b_t \,.\, m_f \,.\, \mathrm{Sin}(\theta_{hl}) \,.\, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \\
& \theta_{kr}) \,.\, \theta_{hl}'^{2} - b_{fr} \,.\, b_t \,.\, m_s \,.\, \mathrm{Sin}(\theta_{hl}) \,.\, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \,.\, \theta_{hl}'^{2} - d_t \,.\, b_{fr} \,.\, m_t \,.\, \\
& \mathrm{Sin}(\theta_{hl}) \,.\, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \,.\, \theta_{hl}'^{2} + b_{fr} \,.\, b_t \,.\, m_b \,.\, \mathrm{Cos}(\theta_{hr}) \,.\, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \\
& \theta_{hr} - \theta_{kr}) \,.\, \theta_{hr}'^{2} + b_{fr} \,.\, b_t \,.\, m_f \,.\, \mathrm{Cos}(\theta_{hr}) \,.\, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \,.\, \theta_{hr}'^{2} + b_{fr} \,.\, b_t \,.\, \\
& m_s \,.\, \mathrm{Cos}(\theta_{hr}) \,.\, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \,.\, \theta_{hr}'^{2} + b_{fr} \,.\, b_t \,.\, m_t \,.\, \mathrm{Cos}(\theta_{hr}) \,.\, \mathrm{Cos}(\alpha_{fr} + \\
& \theta_{ar} + \theta_{hr} - \theta_{kr}) \,.\, \theta_{hr}'^{2} + b_{fr} \,.\, (-d_t + b_t) \,.\, m_t \,.\, \mathrm{Cos}(\theta_{hr}) \,.\, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \,.\, \theta_{hr}'^{2} \\
& + b_s \,.\, b_t \,.\, m_b \,.\, \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \,.\, \mathrm{Sin}(\theta_{hr}) \,.\, \theta_{hr}'^{2} + b_s \,.\, b_t \,.\, m_f \,.\, \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \,.\, \mathrm{Sin}(\theta_{hr}) \,.\, \\
& \theta_{hr}'^{2} + b_s \,.\, b_t \,.\, m_s \,.\, \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \,.\, \mathrm{Sin}(\theta_{hr}) \,.\, \theta_{hr}'^{2} + b_s \,.\, b_t \,.\, m_t \,.\, \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \,.\, \\
& \mathrm{Sin}(\theta_{hr}) \,.\, \theta_{hr}'^{2} + b_s \,.\, (-d_t + b_t) \,.\, m_t \,.\, \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \,.\, \mathrm{Sin}(\theta_{hr}) \,.\, \theta_{hr}'^{2} - b_s \,.\, b_t \,.\, m_b \,.\, \\
& \mathrm{Cos}(\theta_{hr}) \,.\, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \,.\, \theta_{hr}'^{2} - b_s \,.\, b_t \,.\, m_f \,.\, \mathrm{Cos}(\theta_{hr}) \,.\, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \,.\, \theta_{hr}'^{2} - b_s \,.\, b_t \,.\, \\
& m_s \,.\, \mathrm{Cos}(\theta_{hr}) \,.\, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \,.\, \theta_{hr}'^{2} - b_s \,.\, b_t \,.\, m_t \,.\, \mathrm{Cos}(\theta_{hr}) \,.\, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \,.\, \theta_{hr}'^{2} - b_s \,.\, \\
& (-d_t + b_t) \,.\, m_t \,.\, \mathrm{Cos}(\theta_{hr}) \,.\, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \,.\, \theta_{hr}'^{2} + b_{fr} \,.\, b_t \,.\, m_b \,.\, \mathrm{Sin}(\theta_{hr}) \,.\, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} \\
& + \theta_{hr} - \theta_{kr}) \,.\, \theta_{hr}'^{2} + b_{fr} \,.\, b_t \,.\, m_f \,.\, \mathrm{Sin}(\theta_{hr}) \,.\, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \,.\, \theta_{hr}'^{2} + b_{fr} \,.\, b_t \,.\, \\
& m_s \,.\, \mathrm{Sin}(\theta_{hr}) \,.\, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \,.\, \theta_{hr}'^{2} + b_{fr} \,.\, b_t \,.\, m_t \,.\, \mathrm{Sin}(\theta_{hr}) \,.\, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} \\
& + \theta_{hr} - \theta_{kr}) \,.\, \theta_{hr}'^{2} + b_{fr} \,.\, (-d_t + b_t) \,.\, m_t \,.\, \mathrm{Sin}(\theta_{hr}) \,.\, \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \,.\, \theta_{hr}'^{2} - \\
& b_s^{2} \,.\, m_f \,.\, \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \,.\, \mathrm{Sin}(\theta_{hl} - \theta_{kl}) \,.\, (\theta_{hl}' - \theta_{kl}')^{2} - d_s \,.\, b_s \,.\, m_s \,.\, \mathrm{Cos}(\theta_{hr} - \theta_{kr}) \,.\, \\
& \mathrm{Sin}(\theta_{hl} - \theta_{kl}) \,.\, (\theta_{hl}' - \theta_{kl}')^{2} - b_{fr} \,.\, b_s \,.\, m_f \,.\, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \,.\, \mathrm{Sin}(\theta_{hl} - \theta_{kl}) \,.\, \\
& (\theta_{hl}' - \theta_{kl}')^{2} - d_s \,.\, b_{fr} \,.\, m_s \,.\, \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) \,.\, \mathrm{Sin}(\theta_{hl} - \theta_{kl}) \,.\, (\theta_{hl}' - \theta_{kl}')^{2} + \\
& b_s^{2} \,.\, m_f \,.\, \mathrm{Sin}(\theta_{hl} - \theta_{kl}) \,.\, \mathrm{Sin}(\theta_{hr} - \theta_{kr}) \,.\, (\theta_{hl}' - \theta_{kl}')^{2} + d_s \,.\, b_s \,.\, m_s \,.\, \mathrm{Sin}(\theta_{hl} - \theta_{kl}) \,.
\end{aligned}
$$

$\mathrm{Sin}(\theta_{hr} - \theta_{kr})$. $(\theta_{hl}' - \theta_{kl}')^2$ - b_{fr} . b_s . m_f . $\mathrm{Sin}(\theta_{hl} - \theta_{kl})$. $\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$. $(\theta_{hl}' - \theta_{kl}')^2$ - d_s . b_{fr} . m_s . $\mathrm{Sin}(\theta_{hl} - \theta_{kl})$. $\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$. $(\theta_{hl}' - \theta_{kl}')^2$ + d_{fl} . b_s . m_f . $\mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl})$. $\mathrm{Cos}(\theta_{hr} - \theta_{kr})$. $(\theta_{al}' + \theta_{hl}' - \theta_{kl}')^2$ - d_{fl} . b_{fr} . m_f . $\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$. $\mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl})$. $(\theta_{al}' + \theta_{hl}' - \theta_{kl}')^2$ + d_{fl} . b_s . m_f . $\mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl})$. $\mathrm{Sin}(\theta_{hr} - \theta_{kr})$. $(\theta_{al}' + \theta_{hl}' - \theta_{kl}')^2$ + d_{fl} . b_{fr} . m_f . $\mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl})$. $\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$. $(\theta_{al}' + \theta_{hl}' - \theta_{kl}')^2$ - (m_b . (2 . (-(b_s . $\mathrm{Sin}(\theta_{hr} - \theta_{kr})$. $(\theta_{hr}' - \theta_{kr}')$) + b_{fr} . $\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$. $(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')$) . (-(b_t . $\mathrm{Cos}(\theta_{hr})$. θ_{hr}') - b_s . $\mathrm{Cos}(\theta_{hr} - \theta_{kr})$. $(\theta_{hr}' - \theta_{kr}')$ - b_{fr} . $\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$. $(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')$) + 2 . (-(b_t . $\mathrm{Sin}(\theta_{hr})$. θ_{hr}') - b_s . $\mathrm{Sin}(\theta_{hr} - \theta_{kr})$. $(\theta_{hr}' - \theta_{kr}')$ + b_{fr} . $\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$. $(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')$) . (b_s . $\mathrm{Cos}(\theta_{hr} - \theta_{kr})$. $(\theta_{hr}' - \theta_{kr}')$ + b_{fr} . $\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$. $(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')$)))) / 2 - (m_t . (2 . (-(b_s . $\mathrm{Sin}(\theta_{hr} - \theta_{kr})$. $(\theta_{hr}' - \theta_{kr}')$) + b_{fr} . $\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$. $(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')$) . ($d_t$. $\mathrm{Cos}(\theta_{hl})$. θ_{hl}' - b_t . $\mathrm{Cos}(\theta_{hr})$. θ_{hr}' - b_s . $\mathrm{Cos}(\theta_{hr} - \theta_{kr})$. $(\theta_{hr}' - \theta_{kr}')$ - b_{fr} . $\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$. $(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')$) + 2 . ($d_t$. $\mathrm{Sin}(\theta_{hl})$. θ_{hl}' - b_t . $\mathrm{Sin}(\theta_{hr})$. θ_{hr}' - b_s . $\mathrm{Sin}(\theta_{hr} - \theta_{kr})$. $(\theta_{hr}' - \theta_{kr}')$ + b_{fr} . $\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$. $(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')$) . (b_s . $\mathrm{Cos}(\theta_{hr} - \theta_{kr})$. $(\theta_{hr}' - \theta_{kr}')$ + b_{fr} . $\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$. $(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')$)))) / 2- (m_t . (2 . (-(b_s . $\mathrm{Sin}(\theta_{hr} - \theta_{kr})$. $(\theta_{hr}' - \theta_{kr}')$) + b_{fr} . $\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$. $(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')$) . (-((-$d_t$ + b_t) . $\mathrm{Cos}(\theta_{hr})$. θ_{hr}') - b_s . $\mathrm{Cos}(\theta_{hr} - \theta_{kr})$. $(\theta_{hr}' - \theta_{kr}')$ - b_{fr} . $\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$. $(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')$) + 2 . (-((-$d_t$ + b_t) . $\mathrm{Sin}(\theta_{hr})$. θ_{hr}') - b_s . $\mathrm{Sin}(\theta_{hr} - \theta_{kr})$. $(\theta_{hr}' - \theta_{kr}')$ + b_{fr} . $\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$. $(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')$) . (b_s . $\mathrm{Cos}(\theta_{hr} - \theta_{kr})$. $(\theta_{hr}' - \theta_{kr}')$ + b_{fr} . $\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$. $(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')$)))) / 2 - ($m_s$. (2 . (-(b_s . $\mathrm{Sin}(\theta_{hr} - \theta_{kr})$. $(\theta_{hr}' - \theta_{kr}')$) + b_{fr} . $\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$. $(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')$) . (b_t . $\mathrm{Cos}(\theta_{hl})$. θ_{hl}' - b_t . $\mathrm{Cos}(\theta_{hr})$. θ_{hr}' + d_s . $\mathrm{Cos}(\theta_{hl} - \theta_{kl})$. $(\theta_{hl}' - \theta_{kl}')$ - b_s . $\mathrm{Cos}(\theta_{hr} - \theta_{kr})$. $(\theta_{hr}' - \theta_{kr}')$ - b_{fr} . $\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$. $(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')$) + 2 . (b_t . $\mathrm{Sin}(\theta_{hl})$. θ_{hl}' - b_t . $\mathrm{Sin}(\theta_{hr})$. θ_{hr}' - d_s . $\mathrm{Cos}(\theta_{hl} - \theta_{kl})$. $(\theta_{hl}' - \theta_{kl}')$ - b_s . $\mathrm{Sin}(\theta_{hr} - \theta_{kr})$. $(\theta_{hr}' - \theta_{kr}')$ + b_{fr} . $\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$. $(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')$) . (b_s . $\mathrm{Cos}(\theta_{hr} - \theta_{kr})$. $(\theta_{hr}' - \theta_{kr}')$ + b_{fr} . $\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$. $(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')$)))) / 2- ($m_f$. (2 . (-(b_s . $\mathrm{Sin}(\theta_{hr} - \theta_{kr})$. $(\theta_{hr}' - \theta_{kr}')$) + b_{fr} . $\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$. $(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')$) . (b_t . $\mathrm{Cos}(\theta_{hl})$. θ_{hl}' - b_t . $\mathrm{Cos}(\theta_{hr})$. θ_{hr}' + b_s . $\mathrm{Cos}(\theta_{hl} - \theta_{kl})$. $(\theta_{hl}' - \theta_{kl}')$ + d_{fl} . $\mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl})$. $(\theta_{al}' + \theta_{hl}' - \theta_{kl}')$ - b_s . $\mathrm{Cos}(\theta_{hr} - \theta_{kr})$. $(\theta_{hr}' - \theta_{kr}')$ - b_{fr} . $\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr})$. $(\theta_{ar}' + \theta_{hr}'$ -

$\theta_{kr}')) + 2 . (b_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' - b_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') -$
$d_{fl} . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} .$
$\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} .$
$\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')))) / 2 -(m_s . (2 . (-((-d_s + b_s) .\mathrm{Sin}(\theta_{hr} - \theta_{kr}) .$
$(\theta_{hr}' - \theta_{kr}')) + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (-((-d_s + b_s) . \mathrm{Cos}(\theta_{hr}$
$- \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')) - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + 2 . (-((-d_s +$
$b_s) . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')) + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) .$
$((-d_s + b_s) . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' -$
$\theta_{kr}')))) / 2 + m_b . (-(b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')) + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) .$
$(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (-(b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}') - b_s .\mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} .\mathrm{Sin}(\alpha_{fr}$
$+ \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + m_t . (-(b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')) + b_{fr} .$
$\mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (d_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}' -$
$b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + m_t .$
$(-(b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')) + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) .$
$(-((-d_t + b_t) . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr}$
$- \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + m_s . (-(b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')) + b_{fr} . \mathrm{Cos}(\alpha_{fr} +$
$\theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (b_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' -b_t .\mathrm{Cos}(\theta_{hr}) . \theta_{hr}' +d_s .\mathrm{Cos}(\theta_{hl}$
$- \theta_{kl}) . (\theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) .$
$(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + m_f . (-(b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')) + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} -$
$\theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (b_t . \mathrm{Cos}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Cos}(\theta_{hr}) . \theta_{hr}' + b_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) .$
$(\theta_{hl}' - \theta_{kl}') + d_{fl} . \mathrm{Sin}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) .$
$(\theta_{hr}' - \theta_{kr}') - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + m_s . (-((-d_s + b_s) .$
$\mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')) + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (-((-d_s$
$+ b_s) . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')) - b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}'))+$
$m_b . (-(b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} +$
$\theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} +$
$\theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + m_t . (d_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' - b_s .$
$\mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (b_s .$
$\mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + m_t . (-$
$((-d_t + b_t) . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr}$

$- \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} -$

$\theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + m_s . (b_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' - d_s .\mathrm{Cos}(\theta_{hl} -$

$\theta_{kl}) . (\theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}'$

$+ \theta_{hr}' - \theta_{kr}')) . (b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' +$

$\theta_{hr}' - \theta_{kr}')) + m_f . (b_t . \mathrm{Sin}(\theta_{hl}) . \theta_{hl}' - b_t . \mathrm{Sin}(\theta_{hr}) . \theta_{hr}' - b_s . \mathrm{Cos}(\theta_{hl} - \theta_{kl}) . (\theta_{hl}' -$

$\theta_{kl}') - d_{fl} . \mathrm{Cos}(\alpha_{fl} + \theta_{al} + \theta_{hl} - \theta_{kl}) . (\theta_{al}' + \theta_{hl}' - \theta_{kl}') - b_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' -$

$\theta_{kr}') + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . (b_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' -$

$\theta_{kr}') + b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) + m_s . (-((-d_s + b_s) . \mathrm{Sin}(\theta_{hr} -$

$\theta_{kr}) . (\theta_{hr}' - \theta_{kr}')) + b_{fr} . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')) . ((-d_s + b_s) .$

$\mathrm{Cos}(\theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}') + b_{fr} . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}'))+ b_{fr} . b_s .$

$m_b . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 + b_{fr} . b_s . m_f . \mathrm{Cos}(\theta_{hr} -$

$\theta_{kr}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 + b_{fr} . b_s . m_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Cos}(\alpha_{fr} +$

$\theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 + b_{fr} . (-d_s + b_s) . m_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr}$

$- \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 + 2 . b_{fr} . b_s . m_t . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{hr}' -$

$\theta_{kr}')^2 + b_{fr} . b_s . m_b . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 + b_{fr} . b_s .$

$m_f . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 +b_{fr} . b_s . m_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) .$

$\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 + b_{fr} . (-d_s + b_s) . m_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} +$

$\theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{hr}' - \theta_{kr}')^2 + 2 . b_{fr} . b_s . m_t .\mathrm{Sin}(\theta_{hr} - \theta_{kr}) .\mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} -\theta_{kr}) .$

$(\theta_{hr}' - \theta_{kr}')^2 - b_{fr} . b_s . m_b . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) .(\theta_{ar}' + \theta_{hr}' -\theta_{kr}')^2$

$- b_{fr} . b_s . m_f . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 - b_{fr} . b_s .$

$m_s . \mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 - b_{fr} . (-d_s + b_s) . m_s .$

$\mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 - 2 . b_{fr} . b_s . m_t .$

$\mathrm{Cos}(\theta_{hr} - \theta_{kr}) . \mathrm{Cos}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 - b_{fr} . b_s . m_b . \mathrm{Sin}(\theta_{hr} -$

$\theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 - b_{fr} . b_s .m_f . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) .\mathrm{Sin}(\alpha_{fr}$

$+ \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 - b_{fr} . b_s . m_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} -$

$\theta_{kr}) . (\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 - b_{fr} . (-d_s + b_s) . m_s . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) .$

$(\theta_{ar}' + \theta_{hr}' - \theta_{kr}')^2 - 2 . b_{fr} . b_s . m_t . \mathrm{Sin}(\theta_{hr} - \theta_{kr}) . \mathrm{Sin}(\alpha_{fr} + \theta_{ar} + \theta_{hr} - \theta_{kr}) . (\theta_{ar}' +$

$\theta_{hr}' - \theta_{kr}')^2$