A STUDY ON PRESERVICE ELEMENTARY MATHEMATICS TEACHERS' MATHEMATICAL PROBLEM SOLVING BELIEFS

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I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

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ABSTRACT

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This study analyzes the kinds of beliefs pre-service elementary mathematics teachers hold about mathematical problem solving, and investigates whether, or not, gender and university attended have any significant effect on their problem solving beliefs. The sample of the present study consisted of 244 senior undergraduate students studying in Elementary Mathematics Teacher Education programs at 5 different universities located in Ankara, Bolu, and Samsun. Data were collected in spring semester of 2005-2006 academic years. Participants completed a survey composed of three parts as demographic information sheet, questionnaire items, and non-routine mathematics problems.

The results of the study showed that in general the pre-service elementary mathematics teachers indicated positive beliefs about mathematical problem solving. However, they still had several traditional beliefs related to the importance of computational skills in mathematics education, and following predetermined sequence of steps while solving problems. Moreover, a number of pre-service teachers appeared to highly value problems that are directly related to the mathematics curriculum, and do not require spending too much time. Also, it was found that although the pre-service teachers theoretically appreciated the importance and role of the technology while solving problems, this belief was not apparent in their comments about non-routine problems. In addition to these, the present study indicated that female and male pre-service teachers did not differ in terms of their beliefs about mathematical problem solving. However, the pre-service teachers' beliefs showed significant difference when the universities attended was concerned.

Keywords: Mathematical Problem Solving, Pre-service Elementary Mathematics Teachers, Beliefs, Teacher Education, Mathematic Education

ÖZ

İLKÖĞRETİM MATEMATİK ÖĞRETMEN ADAYLARININ MATEMATİKSEL PROBLEM ÇÖZMEYE YÖNELİK İNANIŞLARI

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Bu çalışmada ilköğretim matematik öğretmen adaylarının problem çözme ile ilgili inanışları incelenmiş ve cinsiyet ile üniversitenin öğretmen adaylarının problem çözme inanışları üzerinde etkisi olup olmadığı araştırılmıştır. Araştırmanın örneklemi 2005-2006 eğitim yılı bahar döneminde Ankara, Bolu ve Samsun illerindeki 5 üniversitenin ilköğretim matematik öğretmenliği bölümlerinde okuyan 244 öğretmen adayıdır. Veriler araştırmacı tarafından geliştirilen bir anket aracılığıyla toplanmıştır. Anket, kişisel bilgileri, matematiğe yönelik inanışları ve rutin olmayan matematik problem yorumlarını edinmeye yönelik üç bölümden oluşmaktadır.

Araştırmanın sonucunda genel olarak ilköğretim matematik öğretmen adaylarının problem çözme ile ilgili pozitif görüşlere sahip oldukları ancak hâlâ hesaplama becerilerinin önemi ve problem çözerken önceden belirlenmiş adımları takip etmenin gerekliliği gibi bazı gelenekçi görüşlere sahip oldukları saptanmıştır.

Ayrıca bazı öğretmen adaylarının çok zaman harcamayı gerektirmeyen ve direkt matematik müfredatı ile ilgili olan problemlere oldukça değer verdikleri belirlenmiş, öğretmen adaylarının problem çözerken teknoloji kullanmanın önemi ve değeri hakkındaki inanışlarının ise sadece teorik oldukları bulunmuştur. Bunların yanında öğretmen adaylarının problem çözme inanışlarının cinsiyete bağlı olarak farklılık göstermediği ancak devam ettikleri üniversiteler bazında önemli farklılık gösterdiği saptanmıştır.

Anahtar Kelimeler: Matematiksel Problem Çözme, İlköğretim Matematik Öğretmen Adayları, İnanışlar, Öğretmen Eğitimi, Matematik Eğitimi

To My Parents

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CHAPTER 1

INTRODUCTION

1.1. Background of the Study

The term ‘problem’ may have different meanings depending on one’s perspective. In daily life, problem is explained as any situation for which a solution is needed, and for which a direct way of solution is not known (Polya, 1962). From mathematical perspective, problem is defined as something to be found or shown and the way to find or show it is not immediately obvious by the current knowledge or information available (Grouws, 1996). To a teacher of mathematics, problem is an engaging question for which students have no readily available set of mathematical steps to solve, but have the necessary factual and procedural knowledge to do so (Schoenfeld, 1989).

A mathematics problem can be a routine or a non-routine one. Routine problem is the one which is practical in nature, containing at least one of the four arithmetic operations or ratio (Altun, 2001), whereas non-routine problem is the one mostly concerned with developing students’ mathematical reasoning, and fostering the understanding that mathematics is a creative subject matter (Polya, 1966).

It is also important to differentiate between a mathematics problem and an exercise. An exercise is “designed to check whether a student can correctly use a recently introduced term or symbol of the mathematical vocabulary” (Polya, 1953, p.126). Therefore, the student can do the exercise if he or she understands the introduced idea. However, a problem can not be solved basically by “the mere application of existing knowledge” (Frensch & Funke, 1995, p.5). Also, while doing exercises, students are expected to come up with a correct answer which is usually agreed upon beforehand. However, while solving problems, there might be no solution to the problem, or on the contrary, there can be more than one correct solution to the same problem (Lester, 1980). While solving a problem, the critical point is not reaching to a solution but trying to “figure out a way to work it” (Henderson & Pingry, 1953, p. 248). Moreover, doing exercises demands no invention or challenge (Polya, 1953) whereas solving problems poses curiosity and enthusiasm together with a challenge to students’ intelligence.

The National Council of Teachers of Mathematics (NCTM, 2000) explains several characteristics of good mathematics problems to be the ones that contain clear and unambiguous wording, related to the real world, engage and interest students, not readily solvable by using a previously taught algorithm, promote active involvement of students, allow multiple approaches and solutions, and connect to other mathematical concepts and to other disciplines. In this aspect, problem solving is not just solving a mathematics problem. However, it is “dealing effectively with novel situations and creating flexible, workable, elegant solutions” (Gail, 1996, p. 255). Problem solving involves much more than “simple recall of facts or application of well-learned procedures” (Lester, 1994, p. 668). It is a process by which students experience the power and usefulness of mathematics in the world around them, as well as being a method of inquiry and application (NCTM, 1989).

Problem solving is an important component of mathematics education, because it mainly encompasses skills and functions which are important part of everyday life (NCTM, 1980) by which students can “perform effectively when situations are unpredictable and task demands challenge” (Resnick, 1987, p.18). Actually, problem solving is more than a vehicle for teaching and reinforcing mathematical knowledge, and helping to meet everyday challenges; it is also a skill which can enhance logical thinking aspect of mathematics (Taplin, 1988). Polya (1973) states that if education is unable to contribute to the development of the intelligence, then it is obviously incomplete; yet intelligence is essentially the ability to solve problems both of everyday and personal problems. Moreover, while students are solving problems, they experience a range of emotions associated with various stages in the solution process and feel themselves as mathematicians (Taplin, 1988). As a result, it is also possible to conclude that "being able to solve mathematics problems contribute to an appreciation for the power and beauty of mathematics" (NCTM, 1989, p.77).

Problem solving has been used in school mathematics for several reasons. Stanic and Kilpatrick (1989) identify three general themes that have characterized the role of problem solving in school mathematics; problem solving as a context, problem solving as a skill, and problem solving as an art. The former one indicates that problem solving has been used as justification for teaching mathematics; that is, in order to persuade students of the value of mathematics, that the content is related to real world problem solving experiences (Stanic & Kilpatrick, 1989). Problem solving has also been used to motivate students, to get their interest in a specific mathematical topic or algorithm by providing real world examples of its use, as well as providing a fun activity often used as a reward or break from routine studies (Stanic & Kilpatrick, 1989). However, the most widespread aim of solving problems has been reinforcing skills and concepts that have been taught directly. Besides these roles of problem solving, Polya (1953) suggested that problem solving could be introduced as a practical art, like playing piano or swimming, as an act of inquiry and discovery to develop students' abilities to become skillful problem solvers and independent thinkers.

In summary, problem solving has been given value from kindergarten to high school as a goal for mental development, as a skill to be taught, and as a method of teaching in mathematics education (Brown, 2003; Manuel, 1998; Schoenfeld, 1989; Lester, 1981; Polya, 1953). Especially for the last three decades, problem solving has been promoted “not an isolated part of the mathematics curriculum”, but as “an integral part of all mathematics learning” (NCTM, 2000, p.52) in many countries such as England, Canada, Brazil, China, Japan, Italy, Portugal, Malaysia, Ireland, Sweden, Singapore, and the United States. In other words, teaching problem solving as a separate skill or as a separate topic has shifted to infusing problem solving throughout the curriculum to develop both conceptual understanding and basic skills (Stanic & Kilpatrick, 1989). Currently, in the new Turkish mathematics curriculum problem solving is emphasized as an integral part of the mathematics curriculum, and as one of the vital common basic skills that students need to demonstrate for all subject matters (Millî Eğitim Bakanlığı Talim ve Terbiye Kurulu Başkanlığı, 2005).

The intense emphasis given to problem solving instruction in the reform movements is due to its role in not only for success in daily life, but also for the future of societies and improvement in the work force (Brown, 2003) in the 21st century. With this current view of problem solving, an ideal mathematics classroom where problem solving approach takes such an integral place includes interactions between students and teacher as well as mathematical dialogue and consensus between students (Van Zoest, Jones, & Thornton, 1994). Teachers provide just enough information to establish background of the problem, whereas students clarify, interpret, and attempt to construct one or more solution processes (Manuel, 1998). Moreover, teachers’ role is guiding, coaching, asking insightful questions and sharing in the process of solving problems (Lester, 1994). Therefore, it is expected that problem solving approach to mathematics instruction will provide a vehicle for students to construct their own ideas about mathematics, and to take responsibility for their own learning (Grouws, 1996).

In order for these innovations in the mathematics curricula to take place in classrooms, it is very essential that both teachers and students believe in the importance and role of problem solving in mathematics instruction. Research showed that problem solving instruction is most effective when students sense two things; “that the teacher regards problem solving as an important activity and that the teacher actively engages in solving problems as a regular part of mathematics instruction" (Lester, 1980, p.43). Therefore, it is mainly teachers who are the key component of the implementation of these educational changes.

Beliefs have a considerable effect on individuals' actions. Hersh (1986) indicated that "one's conception of what mathematics is affects one's conception of how it should be presented and one's manner of presenting it is an indication of what one believes to be the most essential in it" (p. 13). Therefore, teachers' beliefs play a crucial role in changing the ways teaching takes place. As teachers' beliefs determine the nature of the classroom environment that the teacher creates, that environment, in turn, shapes students' beliefs about the nature of mathematics (Wilkins & Brand, 2004; Frykholm, 2003; Ball, 1998; Grouws, 1996; Schoenfeld, 1992; Peterson, Fennema, Carpenter, & Loef, 1989). Therefore, due to the fact that teachers' beliefs, knowledge and decisions have a close relation with students' beliefs, attitudes and performance in mathematics, it becomes highly important to know these beliefs and be aware of their effects on classroom practices.

Lastly, addressing the beliefs of pre-service teachers is critical for improving mathematics teaching. An important goal of teacher education programs is "to help pre-service teachers develop beliefs and dispositions that are consistent with current educational reform" (Wilkins & Brand, 2004, p.226). Therefore, it is vital to examine pre-service teachers' hindering beliefs related to mathematical problem solving, and offer opportunities to challenge those beliefs.

1.2. Purpose of the Study

The present study was conducted with pre-service teachers studying in elementary mathematics teacher education program of five universities located in Ankara, Bolu and Samsun during the academic year of 2005-2006.

There were two main areas of investigation in this study. The first one was to explore the kinds of beliefs the pre-service elementary mathematics teachers have toward problem solving in mathematics. It was measured descriptively by the pre-service teachers’ responses to questionnaire items and their interpretations about several mathematics problems. Another area of investigation was to determine whether, or not, gender and university attended have any significant effect on the pre-service elementary mathematics teachers’ problem solving beliefs. It was measured by the pre-service teachers’ responses to the questionnaire items through several inferential statistics.

1.3. Research Questions

The general purpose of the present study was to investigate pre-service elementary mathematics teachers’ beliefs about mathematical problem solving. In more detail, the present study attempted to respond to the following research questions:

1. What are the pre-service elementary mathematics teachers’ beliefs about mathematical problem solving?

1.1. What are the pre-service elementary mathematics teachers’ beliefs about the importance of understanding why a solution to a mathematics problem works?

1.2. What are the pre-service elementary mathematics teachers’ beliefs about mathematics problems that cannot be solved by following a predetermined sequence of steps?

1.3. What are the pre-service elementary mathematics teachers’ beliefs about time consuming mathematics problems?

1.4. What are the pre-service elementary mathematics teachers’ beliefs about mathematics problems that have more than one way of solution?

1.5. What are the pre-service elementary mathematics teachers’ beliefs about the kind of mathematics instruction emphasized by the principles of new curriculum?

1.6. What are the pre-service elementary mathematics teachers' beliefs about the usage of technologic equipments while solving mathematics problems?

1.7. What are the pre-service elementary mathematics teachers' beliefs about non-routine mathematic problems that are emphasized in the new curriculum?

2. What is the effect of gender and university attended on the pre-service elementary mathematics teachers' beliefs about mathematical problem solving?

1.4. Significance of the Study

Mathematics is "not something which is passively learned, but is something which people do" (Dilworth, 1966, p.91), and problem solving "lies at the heart of doing mathematics" (Lester, 1980, p.29) mainly because of the fact that it provides students with a meaningful and powerful means of developing their own understanding in mathematics (Toluk & Olkun, 2002). For nearly three decades, there have been attempts all around the world to make problem solving "the focus of school mathematics" (NCTM, 1980, p.1) rather than being an isolated part of the mathematics curriculum. Currently, in the new Turkish mathematics curriculum, problem solving is viewed as an integral part of mathematics education as well as being one of the vital common basic skills that students need to demonstrate for all subject matters (Millî Eğitim Bakanlığı Talim ve Terbiye Kurulu Başkanlığı, 2005).

It is stated that "problem solving instruction is most effective when students sense two things; that the teacher regards problem solving as an important activity, and that the teacher actively engages in solving problems as a regular part of mathematics instruction" (Lester, 1980, p.43). That is, what teachers believe is very critical to how we improve problem solving practices in our mathematics education. Research showed that teachers' beliefs and preferences about how to teach mathematics play a significant role in how teachers teach mathematics in classroom environments (Peterson, Fennema, Carpenter, & Loef, 1989; Grouws, 1996; Frykholm, 2003; Wilkins & Brand, 2004).

As the emphasis given to problem solving is that widespread, it becomes vitally important to have an understanding of what a mathematical problem is, and the place of problem solving in mathematics teaching and learning. However, Lester (1994) indicated that "although conference reports, curriculum guides, and textbooks insist that problem solving has become central to instruction at every level", it is evident that teachers do not have adequate knowledge about what problem solving is, and that there is a need to examine more about what beliefs teachers hold about mathematical problem solving (p.660). It is also reported that while research has examined some of the factors relate problem solving and students' abilities to solve problems, little has been conducted to show how teachers view problem solving (Ford, 1994; Brown, 2003). Furthermore, studies in Turkey have emphasized the lack of research activities in teacher training institutions to be one of the most critical problems in Turkish education (Altun, 1996; Cakıroğlu & Cakıroğlu, 2003). However, very little research has been conducted related to pre-service teachers, especially related to their beliefs about mathematical problem solving.

This study provides insight into how Turkish pre-service elementary mathematics teachers view problem solving in mathematics education, and examines several factors such as gender and teacher training program to be having any influence on pre-service teachers' beliefs about mathematical problem solving. Exploring these points will lead future developments of mathematical problem solving training in teacher education programs. Moreover, learning more about pre-service teachers' beliefs will guide us in choosing and implementing better professional development for both pre-service and in-service teachers in future.

1.5. Assumptions and Limitations

It is assumed that the participating pre-service elementary mathematics teachers gave careful attention on each item in the instrument, and their responses were honest and based on their personal beliefs and feelings. Also, it is assumed that their beliefs could be measured through several survey questions.

The nature of this study is limited to the data collected from 244 pre-service elementary mathematics teachers studying at five universities, located in three cities, whereas there are 23 universities offering elementary mathematics education program in Turkey. Therefore, the study may be limited in its application to a more generalized population of pre-service elementary mathematics teachers.

Another limitation is that the results of the present study were based on quantitative data collected from participants through a questionnaire. Therefore, the study was limited by the representation of the items on the survey. Interviews might have been conducted to gather more detailed information from the respondents.

1.6. Definitions

The following terms have been used throughout the present study, and defined below for clarity in their application to this study.

Problem is a situation where something is to be found or shown and the way to find or show it is not immediately obvious (Grouws, 1996).

Problem solving is engaging in a task for which the solution method is not known in advance (NCTM, 2000).

Routine problem is a problem which is practical in nature and containing at least one of the four arithmetic operations or ratio (Altun, 2001).

Non-routine problem is a problem that requires some degree of independence, judgment, originality, and creativity such as planning, guessing, estimating, forming conjectures, and looking for patterns (Lester, 1994).

Belief is the collection of cognitive concepts that develop gradually and which hold varying degrees of influence over one's actions (Abelson, 1979).

CHAPTER 2

LITERATURE REVIEW

This chapter deals with the definition of concepts and terms related to mathematical problem solving as well as the importance of problem solving in mathematics education. It also explains several approaches to problem solving, and the role of technology in problem solving. Then, it refers to the previous research studies conducted abroad and in Turkey related to how teachers view problem solving and how their beliefs influence their classroom practices.

2.1. The Nature of Mathematical Problem Solving

2.1.1. What is Problem and Problem Solving?

A problem is typically defined as "a situation where something is to be found or shown and the way to find or show it is not immediately obvious" (Grouws, 1996, p.72). That is, "the situation is unfamiliar in some sense to the individual and a clear path from the problem conditions to the solution is not apparent" (Grouws, 1996, p.72) by the mere application of existing knowledge (Frensch & Funke, 1995). Therefore, a problem can be stated as "a situation for which one does not have a ready solution" for it (Henderson & Pingry, 1953, p.248). In his book of *Mathematical Discovery,* Polya indicates that "to have a problem means: to search consciously for some action appropriate to attain a clearly conceived, but not immediately attainable aim" (1962, p.117).

What may be a problem for one individual may not be a problem for another. Lester (1980) proposes that whether or not a situation is a problem for an individual is determined by the individual's reaction to it. He claims that in order for a situation to be a problem for an individual, first the person must "be aware of the situation" and "be interested in solving it", then the person should "be unable to proceed directly to a solution" so that he must "make deliberate attempt to find it" (Lester, 1980, p. 30). Besides this, a problem for a particular individual today may not be a problem for him tomorrow (Henderson & Pingry, 1953, p.229). For a task to require problem solving again, "novel elements or new circumstances must be introduced or the level of challenge must be raised" (Martinez, 1998, p.606).

From a broad aspect, if a problem is described as an unknown entity in some context for which solving it has some social, cultural, or intellectual value (Jonassen, 2004), then problem solving can be defined as "any goal directed sequence of cognitive operations" (Anderson, 1980, p.257) directed at finding that unknown. The term problem solving has taken on different meanings at different points in education over time. At one extreme, problem solving is taken to include "situations that require little more than recall of a procedure or applications of a skill" (Grouws, 1996, p.71). At the other end of the continuum, problem solving is taken so broadly that it is synonymous with mathematical thinking (Grouws, 1996).

Problem solving according to the *Principles and Standards for School Mathematics* published by the National Council of Teachers of Mathematics (NCTM) means "getting involved in a task for which there is no immediate answer" (2000, p.9). That is, the solution is not known in advance. For instance, if we know exactly how to get from point A to point B, then reaching point B does not involve problem solving. As we solve problems every time we achieve something without having known beforehand how to do so, in order to solve a problem we need to coordinate our "knowledge, previous experience, intuition, various analytical abilities" (Lester, 1980, p.32) and "mathematical reasoning" (Willoughby,1985, p.91). From this point of view, problem solving also can be defined as "dealing effectively with novel situations and creating flexible, workable, elegant solutions" (Gail, 1996, p.255). In addition to the question of what a problem is and what it means to solve a problem is the question of why problem solving is important in mathematics education.

2.1.2. Learning Mathematics and Problem Solving

2.1.2.1. The Importance of Problem Solving in Mathematics Education

Problem solving has been given value as a goal for mental development, as a skill to be taught, and as a method of teaching in mathematics education (Brown, 2003; Giganti, 2004; Jonassen, 2004; Lester, 1981; Manuel, 1998; Martinez, 1998; Naussbaum, 1997; Polya, 1953; Schoenfeld, 1989; & Willoughby, 1985). Especially for the last three decades, problem solving has been promoted to take place in mathematics classes from kindergarten to high school in many countries such as "Brazil, China, Japan, Italy, Portugal, Sweden, the United Kingdom" (Lester, 1994) and the United States (NCTM, 2000). The reason of this intense emphasis given to problem solving instruction recently is due to the characteristics and necessity of problem solving not only for success in daily life, but also for the future of societies and improvement in the work force (Brown, 2003).

Problem solving has existed since the first human being realized a need to find shelter and food or to escape from the predators (Brown, 2003). As human society developed and advanced, due to the unpredictable contingencies and dangerous uncertainties, new problems revealed and caused the need for new ways of solving problems. Meanwhile, mathematics evolved in response to these needs and the development of mathematics offered more opportunities to accomplish harder problems (Brown, 2003). That is why, for mathematicians, doing mathematics is considered as solving problems (Schoenfeld, 1989) and those who were better able solve problems have been found more successful throughout history (Jonassen, 2004).

Problems provide "an environment for students to reflect on their conceptions about the nature of mathematics and develop a relational understanding of mathematics" (Skemp, 1978, p.9) which is stated by Shroeder and Lester (1989) as the most important role of problem solving in mathematics. To understand mathematics is essentially to see how things fit together in mathematics. When students make rote memorization, they cannot see the connections and how things fit together (Manuel, 1998). In particular, a person's understanding increases as one "relates a given mathematical idea to a greater variety of contexts, as one relates a given problem to a greater number of the mathematical ideas implicit in it, or as one constructs relationships among the various mathematical ideas embedded in a problem" (Shroeder & Lester, 1989, p.37).

Problems create cognitive conflict by directing students to think about their present concepts about mathematics. As students are working through mathematical problems, "they confirm or redefine their conceptual knowledge, relearn mathematics content and become more open to alternative ways of learning mathematics" (Steele& Widman, 1997, p.190). That is, solving problems helps students see mathematics as a dynamic discipline in which they have the opportunity to organize their ideas, engage in mathematical discussions, and defend their conjectures (Manuel, 1998). Moreover, by reflecting on their solutions, students use a variety of mathematical skills, develop a deeper insight into the structure of mathematics, and gain a disposition toward generalizing which also helps them to acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that serve them well outside the mathematics classroom (NCTM, 2000).

Dealing with new and unfamiliar situations and resolving the difficulties that such situations frequently pose is the essence of problem solving (Brown, 2003).

Thus, problem solving involves much more than “the simple recall of facts or the application of well-learned procedures” (Lester, 1994, p.668). First of all, problem solver needs to be aware of the current activity and the overall goal, and the effectiveness of those strategies (Martinez, 1998). Also, problem solver needs to have some degree of creativity and originality (Polya, 1953).

Good problem solvers recognize what they know and do not know, what they are good at and not so good at (NCTM, 2000).That is, good problem solvers are “aware of their strengths and weaknesses as problem solvers” (Lester, 1994, p.665). Hence, they can use their time and energy in a better manner by making plans more carefully and taking time to check their progress periodically (NCTM, 2000). Also, good problem solvers can analyze situations carefully in mathematical terms (NCTM, 2000) as “their knowledge is well connected and composed of rich schemata” (Lester, 1994, p. 665). Instead of focusing on “surface features”, good problem solvers tend to focus their attention on “structural features” of problems to “monitor and regulate their problem solving efforts” and hence, “obtain elegant solutions” to problems (Lester, 1994, p.665).

Lester described a similarity between learning how to solve problems and learning how to play baseball. He states that “just as one can not expect to become a good baseball player if one never plays baseball, a student cannot expect to become a good problem solver without trying to solve problems” (1981, p.44). Like Lester, Willoughby (1985) assimilated problem solving to bicycle riding as both activity requires lots of practice. Although it is strongly advised to “make problem solving an integral part of school mathematics” (NCTM, 2000, p.52) and highly recommended to practice it as much as possible, research show that some teachers consider the main goal of mathematics as mainly performing computation and, therefore, postpone problem solving until students master their facts or pass all timed tests (Capraro, 2001). Consequently, in 2000, NCTM reemphasized the need of the practice of solving problems and argued that “the essential component steps of the problem -setting up, organizing, discourse, drawing a picture, connecting to the real world- do not need to be postponed until students can do twenty workbook pages of the same kind of operation" (p.9).

2.1.2.2. Approaches to Problem Solving Instruction

There are different approaches for teaching mathematical problem solving based on the role given to it by curriculum developers, textbook writers, and classroom teachers. It is important to understand the characteristics of these approaches because of the fact that the way problems are used in mathematics education, and the emphasis given to problem solving in mathematics curriculum dramatically change over time. Therefore, the way one approach to problem solving can give clue about whether or not the person has a traditional view or reformist view in mathematics education.

One of the most well known distinctions made between these approaches was presented in a paper written by Larry Hatfield in 1978. Hatfield (1978) defined three basic approaches to problem solving instruction such as "teaching about problem solving", "teaching for problem solving", and "teaching via problem solving", which was later reemphasized by Schroeder and Lester in 1989.

Teaching about Problem Solving

This approach involves teaching about how problems are solved. In order to solve a problem, a teacher who teaches about problem solving, first selects a problem solving model, and then basically follows the steps introduced in it. In another words, "the teacher demonstrates how to solve a certain problem and directs the students' attention to salient procedures and strategies that enhance the solution of the problem" (Lester, 1980, p.41). Hence, when students are taught about problem solving, they are expected to solve problems by following the same procedures their teacher exhibited.

In mathematics, the most well known and taught model of problem solving is Polya's model of problem solving. In 1945, George Polya wrote *How to Solve it*, in which several interdependent steps are described for solving mathematics problems as; understanding the problem, devising a plan, carrying out the plan, and looking back. According to Polya, in order to solve a problem, one should follow the steps in the same order. In the first step which is understanding the problem, several questions are asked such as "What is the unknown?, What are the data?, What is the condition?, Is it possible to satisfy the condition?, Is the condition sufficient to determine the unknown?" to understand the problem (Polya, 1973, p.7). During the next step which is called as devising a plan, possible connections between the data and the unknown are found to develop a plan for the solution (Polya, 1973). In the third step that is carrying out the plan, the steps in the prepared plan is followed to come up with a solution of the problem (Polya, 1973). In the last step called looking back, the solutions obtained are examined and the problem is extended by using the result obtained, or the method used, for generating another problem (Polya, 1973).

Although "Descartes in the 1600s in his *Geometry* and Dewey in the early 1900s in his *How We Think* had each listed the same sets of steps for solving problems", Polya has been given credit for making these steps essential in mathematics education while solving problems(Brown, 2003, p.21). Polya, other than introducing these four steps for solving a problem, also emphasized a number of heuristics, also called as strategies, to use in devising and carrying out plans in solving problem (Schroeder & Lester, 1989). Some of these strategies include draw a picture, try and adjust, look for a pattern, make a table or chart, make an organized list, work backward, logical reasoning, try a simpler problem, and write an equation or open sentence. These problem solving strategies are believed to help students in choosing the path that seems to result in some progress toward the goal (Martinez, 2000). Moreover, as they are content free, they can be applied across many different situations (Martinez, 2000), thus, improve students' performance on reasonably wide range of problems (Grouws, 1996). However, using problem solving strategies does not guarantee that a solution will be found if it exists, indeed these strategies merely "increase the probability that a solution is found" (Frensch & Funke, 1995, p.12).

In mathematics, there are problem solving models other than Polya's model. For instance, Lester developed a problem solving model containing "six distinct but interrelated stages such as problem awareness, problem comprehension, goal analysis, plan development, plan implementation, and procedures and solution evaluation" (Lester, 1980, p.33). For the stage problem awareness, the problem solver is expected to be aware of an existing problem, realize difficulty in the given situation and show willingness for solving it. For comprehension stage, the problem solver is expected to make the problem meaningful for him or her by internalizing it. During the third stage, goal analysis, some sub-goals can be determined for better analyzing the structure of the problem. During the next stage, plan development, an appropriate plan is developed for solving the problem. For the fifth stage which is called plan implementation, the steps in the plan is tried out. Finally, in the procedures and solution evaluation stage, the appropriateness of the decisions and the solutions is questioned. Actually, the sixth step involves the evaluation of all decisions made during the problem solving process.

Although teaching about problem solving is one of the most widespread approaches preferred by teachers and textbook writers, it has a very big limitation such that problem solving is regarded as a topic to be added to the curriculum, as an isolated unit of mathematics, not as a context in which mathematics is learned and applied (Schroeder & Lester, 1989).

Teaching for Problem Solving

Teaching for problem solving involves applying the knowledge gained during the lesson in order to solve problems. That is, the purpose of learning mathematics is to solve problems. A teacher who teaches for problem solving "concentrates on ways in which the mathematics being taught can be applied in the solution of both routine and non-routine problems" (Schroeder & Lester, 1989, p.32).

Routine problems are the problems which are practical in nature and containing at least one of the four arithmetic operations or ratio (Altun, 2001). Therefore, solving routine problems depends mostly on knowing arithmetic operations and knowing what arithmetic to do in the first place. Polya (1966) indicated that routine problems can be useful and necessary "if administered at the right time in the right dose", and discouraged the usage of "overdoses of routine problems" (p.126).

Unlike routine problems, non-routine problems are mostly concerned with developing students' mathematical reasoning power and fostering the understanding that mathematics is a creative subject matter (Polya, 1966). Non-routine problems require higher order thinking skills and investment of time (London, 1993). It is indicated that solving a sequence of non-routine problems "gives students experience with additional problem solving skills" such as finding a pattern and generalizing, developing algorithms or procedures, generating and organizing data, manipulating symbols and numbers, and reducing a problem to an easier equivalent problem (London, 1993, p.5). Also, it is found that students that solve several non-routine problems "demonstrate a mathematical maturity" (London, 1993, p.5); that is, they begin to "act like mathematicians" (London, 1993, p.11).

When taught for problem solving, students are given many opportunities to apply the concepts and structures they study in mathematics lessons to solve both routine and non-routine problems. Further, the teacher who teaches for problem solving is very concerned about "students' ability to transfer what they have learned from one problem context to others" (Schroeder & Lester, 1989, p.32).

This approach directly relates the process of learning mathematics to the practice of doing mathematics. So, at this point, a distinction should be made between solving problems and doing exercises as both are considered to be vehicles for practicing mathematics. First of all, an exercise is “designed to check whether a student can correctly use a recently introduced term or symbol of the mathematical vocabulary” (Polya, 1953, p.126), therefore the student can do the exercise if he understands the introduced idea. However, a problem can not be solved basically by “the mere application of existing knowledge” (Frensch & Funke, 1995, p.5). That is, only the pure knowledge is not enough. Also, while doing exercises students are expected to use the given information, so they are expected to come up with a correct answer which is usually agreed upon beforehand. However, while solving problems, there might be no solution to the problem, or on the contrary, more than one correct solution can exist (Lester, 1980). The critical point is not reaching to a solution but trying to “figure out a way to work it” (Henderson & Pingry, 1953, p.248). Moreover, doing exercises demands no invention or challenge (Polya, 1953) whereas solving problems poses curiosity and enthusiasm together with a challenge to students’ intelligence.

Schroeder and Lester (1989) pointed out some important shortcoming arising from teaching for problem solving when it is interpreted narrowly as:

> Problem solving is viewed as an activity that students engage in only after the introduction of a new concept or following work on a computational skill or algorithm. Often a sample story problem is given as a model for solving other, very similar problems, and solutions of these problems can be obtained simply by following the same pattern established. Therefore, when students are taught in this way, they often simply pick out the number in each problem and apply the given operations to them without regard for the problem’s context. Furthermore, a side effect is that students come to believe that all mathematics problems can be solved quickly and relatively effortlessly without any need to understand how the mathematics they are using relates to real situations (p.34).

Teaching via Problem Solving

In teaching via problem solving, “problems are valued not only as a purpose for learning mathematics but also as a primary means of doing so” (Schroeder &

Lester, 1989, p.33). That is, problems are used as "a vehicle to introduce and study the mathematical content" (Manuel, 1998, p.634). Schroeder and Lester explained the environment of a mathematics class where students are taught via problem solving as "the teaching of a mathematical topic begins with a problem situation that embodies key aspects of the topic, and mathematical techniques are developed as reasonable responses to reasonable problems" (1989, p.33). Consequently, students that are learning mathematics via problem solving, mainly study a specific mathematical idea through discussion of particular problems, generally non-routine ones, by being constantly asked to "present their ideas, propose possible approaches, communicate their arguments, and evaluate their solutions" (Manuel, 1998, p.636). Therefore, in this approach "learning and understanding are enhanced by students being intimately involved with problems and ideas, and by struggling to come to grips with mathematical concepts" (Holton, Anderson, Thomas, & Fletcher, 1999, p.351) where problem solving is used as an umbrella under which all other mathematical concepts and skills are taught (Capraro, 2001).

In addition to introducing and studying the mathematical content through solving problems, another fundamental idea in this approach is that the goal of learning mathematics is considered as "to transform certain non-routine problems into routine ones" (Schroeder & Lester, 1989, p.33). Schroeder and Lester explained that "the learning of mathematics in this way can be viewed as a movement from the concrete to the abstract" (p.33). By the concrete, they meant "a real world problem that serves as an instance of mathematical concept or technique", whereas by the abstract, they meant "a symbolic representation of a class of problems and techniques for operating with these symbols" (p.33).

Teaching via problem solving is actually the approach suggested by NCTM in their publication of *Principles and Standards for School Mathematics*, written in 2000. NCTM (2000) proposed that "students can learn about, and deepen their understanding of, mathematical concepts by working through carefully selected problems that allow applications of mathematics to their contexts, and these well-chosen problems can be particularly valuable in developing or deepening students' understanding of important mathematical ideas" (p.54).

The critical point in this approach is that, curriculum developers, textbook writers, or classroom teachers that want to teach the content via problem solving should be very careful while selecting the suitable problems in order to cover the intended content (Manuel, 1998). First of all, the selected problems should be appropriate for the students' grade level, knowledge, skills and understandings (Henderson & Pingry, 1953). Next, the problems should be appealing to students' interest and "meaningful from the students' viewpoint" (Polya, 1953, p.127). Moreover, the mathematical idea introduced in the problems should be parallel to the idea in the intended content matter. In addition, the selected problems should be creating a class environment where students have opportunity for discussing their ideas, and questioning relevancy. Furthermore, after solving a problem, the next problems introduced should be different than the previously illustrated one, that is, they should not be solvable by applying the preceding ideas or previously followed procedures in the same manner.

In conclusion, although in theory there are differences among the individual's and groups' conceptions of how to integrate problem solving in teaching mathematics, as Schroeder and Lester (1989) stated in practice these three approaches "overlap and occur in various combinations and sequences, thus, it is probably counterproductive to argue in favor of one or more of these types of teaching or against the others" (p.33).

2.1.3. Technology and Mathematical Problem Solving

One of the main aims of this study was to explore pre-service elementary mathematics teachers' beliefs about technology usage in mathematics instruction while solving mathematical problems. That is why, it is essential to understand the role and importance of technology in mathematics teaching.

Technology is an essential tool in teaching and learning mathematics (NCTM, 2000) which enhances productivity, communication, research, problem-solving, and decision-making (Niess, 2005), consequently assisting students in their understanding and appreciation of mathematics. In the *Principles and Standards for School Mathematics*, it was stated that students can learn more mathematics more deeply with the appropriate and responsible use of technology" (NCTM, 2000).

Jurdak (2004) examined the role of technological tools, especially computers, as facilitators in problem solving in mathematics education, and concluded that technology can serve as a power for building bridges between abstract mathematics and problem solving in real life. Both calculators and computers were found to be reshaping the mathematical landscape, allowing students to work at higher levels of generalization and abstraction (NCTM, 2000), consequently resulting in a deeper mathematical understanding (Mathematical Association of America, 1991).

Mathematical Sciences Education Board (MSEB, 1989) found that the proper use of calculators can "enhance children's understanding and mastery of mathematics", especially in arithmetic (p.47), and that calculators allow "the growth of a realistic and productive number sense in each child" (p.48). MSEB (1989) observed that the students who used calculators learn traditional arithmetic as well as those who do not use calculators, and demonstrate better problem solving skills and much better attitudes towards mathematics.

Similarly, Mathematical Association of America (MAA, 1991) emphasized that "given carefully designed instructions, computers can aid in visualizing abstract concepts and create new environments which extend reality"; therefore "divorcing mathematics from technology" will result in limiting students' mathematical power (p.6). So, it was recommended that prospective mathematics teachers should "use calculators and computers to pose problems, explore patterns, test conjectures, conduct simulations, and organize and represent data" (p.7).

Several studies have been conducted to understand how technology is used in classroom environments (Becker & Ravitz, 1999; Sheingold & Hadley, 1990), which beliefs teachers hold towards teaching and learning with technology (Turner & Chauvot, 1995; Cooney & Wilson, 1995), how technology could support learners (Adiguzel & Akpinar, 2004; Fey, 1989), and how to integrate technology into the curriculum (Ely, 1990). For instance, Sheingold and Hadley (1990) used a survey to discover the ways in which teachers in USA use computers in their classrooms, how their teaching changed, and the kinds of barriers experienced while integrating computers into their teaching. Data were gathered over 600 teachers in grades 4 through 12 who were comfortable with computer technology, devoting their own time to learning how to use computers, using computers for many purposes including demonstrating an idea, instruction, word processing, and promoting student-generated products, as well as presenting more complex materials to their students, and fostering more independence in the classroom. At the end of the study, the teachers reported that they changed from being the sole provider of information and knowledge in the classroom to sharing that role with students and providing more complex materials. Also, the students were found to be working with increasing independence as a result of computer usage. Therefore, it is concluded that technologies can help teachers to teach differently as well as providing more complex kinds of tasks for students to engage. Furthermore, it is proposed that to achieve these professional developments, teachers need adequate time and support while experimenting with technology, and designing and implementing good technology -based activities within their curricula.

In order to analyze the status of technology use in elementary and secondary schools, Becker and Ravitz (1999) conducted several survey studies in 1989 and 1997 in U.S.A. The study in 1989 showed that very few teachers and students were major technology users due to the lack of adequate access to technology. Unlike the conditions in the study of 1989, the study in 1997 took place in schools working with consistent access to information. In the study of 1997, the value of technology in education was appreciated and adequate support for how to implement instructional changes together with adequate computer software and telecommunication resources for students use were provided. Under these conditions, teachers were observed to be more willing to discuss subjects even if they lack expertise, allow multiple simultaneous activities occurring during class time, appreciate long and complex projects for students to undertake, and give students greater choice in their tasks and the materials and resources they use.

Turner and Chauvot (1995) conducted a longitudinal study to conceptualize the belief structures of pre-service teachers with regard to technology. Several interviews, observations and examinations were administered to two pre-service through a four quarter sequence which consisted of two courses in mathematics education, student teaching, and a post student teaching seminar. During these courses, graphing calculators and computers were used as investigative tools and several activities took place which included the use of technology as an integrated approach to learning mathematics. Teachers hold various beliefs such as "success in technology results from a prerequisite knowledge of mathematics"(p.5), "once the mathematical knowledge was obtained by paper and pencil skills, technology can be used for further mathematical investigation" (p.5), "technology should be used only in the upper level classes" (p.6), "technology is an alternative method of teaching, so it can be replaced with methods such as group work, manipulative, and peer teaching", and "technology can be used as a demonstrative tool" (p.7). It was stated that the belief structures of these pre-service teachers would play a crucial role in determining how and when these pre-service teachers would use technology in their future classrooms.

Cooney and Wilson (1995) investigated secondary pre-service teachers' beliefs about mathematics through a teacher education program that promoted the NCTM standards and the use of technology. There was a considerable emphasis on different teaching methods and daily opportunities for the teachers to engage in activities including an extensive use of technology. Previously, these pre-service teachers believed that it was non sense to spend time using the computer, whereas toward the end of the program they believed that technology can fundamentally change the teaching of mathematics, so emphases should be given to the technology usage.

Fey (1989) studied the impact of applying electronic information technology in creation of new environments in mathematics education, and listed several ways in which computer-based representations of mathematical ideas are unique and valuable for instruction and problem solving in mathematics such as “computer representations of mathematical ideas and procedures can be made dynamic in ways that no text or chalkboard diagram can; the computer makes it possible to offer individual students an environment for work with representations that are flexible, but at the same time, constrained to give corrective feedback to each individual user whenever appropriate; the electronic representation plays a role in helping move students from concrete thinking about an idea or procedure to an ultimately more powerful abstract symbolic form; the versatility of computer graphics has made it possible to give entirely new kinds of representations for mathematics representation that can be created by each computer user to suit particular purposes; and the machine accuracy of computer generated numerical, graphic, and symbolic representations make those computer representations available as powerful new tools for actually solving problems” (p.255).

Adiguzel and Akpinar (2004) designed and implemented a computer software, LaborScale, which was beginning with the concrete representations and reaching the symbolic representations by using visual components supported by audio, to improve seventh grade students' word problem solving skills through computer-based multiple representations including graphic, symbolic, and audio representations. Students from both public and private elementary schools which had computer laboratories were administered pretest and posttests while studying work and pool problems in their classes. It was found that seventh grade students’ performance on work and pool problems increased significantly through the application of this computer representation which assisted students with the transition from concrete experiences to abstract mathematical ideas, with the practice of skills, and with the process of problem solving.

Ely (1990) proposed that in order for effective technology in-service programs to be successful, conditions should support the overall implementation of educational technology. Through the carefully examinations of several conditions, eight factors were detected that influenced the effective implementation of educational technology. These factors were dissatisfaction with the status quo, knowledge and skills, resources, rewards and incentives, commitment, leadership, time, and participation. For example, the factor dissatisfaction with the status quo suggested that there must be a reason for members of the system to want to implement technology. Also, in order to implement the use of any type of educational technology, teachers must feel confident in its operation and their own ability to integrate it into daily classroom practices. It was recommended that both hardware and software resources should be available, individuals at all levels of the system must participate in the innovation, and in order to encourage the implementation of innovations rewards and incentives can be used. While examining the factors influencing the diffusion, adoption, and implementation of technology in education, Ely (1990) found the time factor to be the most emphasized one in almost all studies. Teachers believed that computers created more work for them, and even the accomplished technology-using teachers rated the lack of time as one of the biggest obstacle to technology utilization in schools. Furthermore, it was stated that individuals should be given the opportunity to plan and participate in decisions concerning the innovation and its implementation.

Mathematical Sciences Education Board (MSEB) in 1989 suggested that "priorities for mathematics education must change the ways technology is used in mathematics" (p.63). NCTM (2000) recommended mathematics teachers to redesign the mathematics they teach, investigate technological tools for learning mathematics, and consider how they can create an atmosphere where technology is used as a tool in students' learning mathematics. Also, it was stated that as all students regardless of their access to technology, deserve life opportunities arising from a quality education, "it is important not to wait for high access to technology, nor to pursue it to the exclusion of developing better models for its use" (Coppola, 2003, p.55).

MAA (1991) stated that the mathematical preparation of teachers must include experiences in which they use technology such as calculators and computers as "tools to present mathematical ideas and construct different representations of mathematical concepts, and to develop and use alternative strategies for solving problems" (p.7). Furthermore, Cooney and Wilson (1995) suggested that recognition of belief structures of teachers are also of considerable importance when developing teacher education programs that promote reflection and adaptive teaching.

2.1.4. The Role of Problem Solving

2.1.4.1. Problem Solving in the World

As the improvements in the workplace, economy, business, industries, aeronautics, and politics have become more competitive in the world, the necessity of setting higher standards in the teaching and learning arena also becomes unavoidable. Many countries in the world such as England, Canada, Brazil, China, Japan, Italy, Portugal, Malaysia, Ireland, Sweden, Singapore, and the United States have been investigating and discussing extensively the necessity of a mathematical reform that meet the requirements of the 21st century. As a result, new mathematics curricula have been developed "to give students deeper understanding of the basic mathematical concepts and to stimulate them to do creative and independent thinking with these concepts" (Dilworth, 1966, p.92).

Since the early 1980s, in the United States, the NCTM has investigated ways to improve educational practices, and "provide a framework to develop effective curricula, instructional strategies, and assessment tools" (Alba, 2001, p.5). To do this, the *Curriculum and Evaluation Standards for School Mathematics* (1989), the *Professional Standards for Teaching Mathematics* (1991), *Assessment Standards for School Mathematics* (1995), and the *Principles and Standards for School Mathematics* (2000) were developed to reform mathematics education at K-12 in the United States. The *Principles and Standards for School Mathematics* (2000) consists of two parts as Principles and Standards. The Principles are "features necessary for high quality mathematics education" (Alba, 2001, p.6) such as equity, curriculum, teaching, learning, assessment, and technology. They are developed to reflect basic perspectives on which educators base their educational decisions (NCTM, 2000). On the other hand, the Standards are comprehensive set of goals for mathematics instruction such as Content Standards which are number sense, algebra, geometry, measurement, data analysis & probability, and Process Standards which are problem solving, reasoning and proof, communication, connections, and representation (NCTM, 2000).

In U.S.A, according to this new pedagogical perspective, teachers are responsible of selecting suitable curricular materials, using appropriate instructional tools and techniques (NCTM, 2000), guiding instruction, and creating an intellectual environment where students learn to think mathematically (Steele & Widman, 1997). To do this, teachers are expected to refresh their professional knowledge, both of mathematical content and of pedagogy (NCTM, 2000). On the other hand, students are expected to be active in the learning process, construct their own knowledge (Steele, & Widman, 1997), take control of their own learning, learn mathematical concepts with understanding, and use technologies that broaden and deepen their understanding of mathematics (NCTM, 2000). It is proposed that when students understand mathematics properly, they can use their knowledge more flexibly (NCTM, 2000). Therefore, it is claimed that this meaningful learning will enable students to deal with novel problems and settings that they have not encountered before, and at the same time by working through such problems, they can learn about and deepen their understanding of mathematical concepts (NCTM, 2000).

In Japan, since 1989, there have been several changes in mathematics curriculum by making mathematical problem solving the back bone of mathematics instruction (Nobuhiko, 1996). Japanese students are well known by their relatively high scores on various international tests of mathematics achievement carried by the International Association for the Evaluation of Educational Achievement (IEA) such as the First International Mathematics Study (FIMS) in 1964 and the Second International Mathematics Study (SIMS) in 1980. However, these exams showed that the reason of Japanese students' high scores were due to their success in solving questions involving computation with numbers, not mathematical problems requiring a full understanding of their content for solution (Toshio, 1996). Research studies also found that "Japanese teachers place a lot of emphasis on doing calculations" and not taking the ability to solve problems into consideration (Yoshishige, 1996, p.153). Therefore, there have several changes made in Japanese mathematics curriculum aiming to developing "the ability and attitude of children to make generalizations, to make guess or hypotheses, to formulate and solve problems, to revise or improve findings, to make connections among things, and to use calculators while solving real world problems" (Yoshishige, 1996, p.154). Moreover, in order to decrease the emphasis given to paper and pencil computation, using estimation and calculators were recommended as computational alternatives in a mathematical problem solving context (Nobuhiko, 1996).

In China, after 1990 the national education has become a top priority promoting the reform and development of the education system especially related to mathematical problem solving (Linrong, 2005). The emphasis in mathematics education shifted from exercise doing to problem solving (Dianzhou, 1996; Guoqing, 1996). It was stated that in mathematics education, "through self-exploration and cooperation, students were expected to solve challenging and comprehensive problems with applications close to real life scenarios, develop their problem solving abilities", and deepen their understanding of all aspects of mathematics (Linrong, 2005, p.6) because of the fact that "solving problems not only can be used to examine students for their grasp of mathematical concepts, but it can also develop their logical thinking and cultivate their thinking abilities" (Xiaoming, 1996, p.221). It was also stated that "the selected problems are not necessarily required to closely follow the teaching material" (Xiaoming, 1996, p.217). In order to draw attention to the mathematical problem solving, nationwide seminars were held in China (Dianzhou, 1996), and "innovation in the mathematics problems of the entrance examinations was being promoted as an important way to emphasize mathematical problem solving" (Dianzhou, 1996, p.97).

2.1.4.2. Problem Solving in Turkey

The scientific and technologic developments achieved in the world, the progress taken in educational sciences, the need for increasing and deepening the quality in national education, the need for providing eight year basic education entirely, the need for providing conceptual understanding not only among different topics in the same courses but also among different subject matters, and the Turkish students' performance in PISA, TIMMS, and PIRLS exams have shown the vital necessity of changing the mathematics teaching and learning in Turkey.

Due to these necessities and according to research studies, national and international reports, experiences of teachers and academicians, curricula of other countries, and the current national curriculum, the Ministry of National Education in Turkey have made some changes in the mathematics curriculum for schools in 2005. The new mathematics curriculum is fundamentally based on the idea that students will be provided environments where they can investigate, discover, solve problems, share and discuss their solutions.

According to this new perspective of learning and teaching, students are expected to be mentally and physically active and responsible about their own learning during the learning process. They are expected to question the new information, think critically, work cooperatively, discover, discuss, solve problems, form their own problems, and as a result construct their own learning. In addition, while solving problems students are expected to develop their own strategies and use them while solving their daily problems. Similarly, the role of teachers in the classroom environment was announced to be a guide in instruction, providing activities related to the topic, questioning students' responses, giving motivation and encouragement, and assessing students' performance fairly. To do this, teachers are expected to improve their knowledge both in their profession and the subject matter. They need to share their knowledge with their colleagues, use computers and internet as well as calculators in their instruction.

The principle of the new mathematics curricula is that every child can learn mathematics, and the aim of the new mathematics curriculum is to raise individuals that are capable of using mathematics in their daily lives, solving mathematical problems, sharing their ideas and solutions with their peers, explaining and defending their ideas, constructing rich mathematical concepts, relating it with other subject matters, enjoying mathematics and having self confidence in mathematical applications (Millî Eğitim Bakanlığı Talim ve Terbiye Kurulu Başkanlığı, 2005).

Problem solving is placed as an integral part of the new mathematics curriculum. It was emphasized as one of the vital common basic skills that students need to demonstrate for all subject matters, similar to other common skills such as using Turkish effectively, thinking critically, thinking creatively, communicating, investigating, solving problems, making decisions, using technology, and being active. It was stated that problem solving should not be perceived as an algorithm or step by step procedures. Instead, problem is defined as a situation where the way of solution is unclear, and it needs students to use their previous knowledge and intelligence to solve.

Ministry of National Education (2005) in Turkey made several suggestions about the role of teachers and students in mathematics teaching and learning. According to these suggestions, teachers should select problems which are interesting and useful for their students, teachers are expected to value different ways of solutions to the same problems, and give more importance to students' solution ways and strategies instead of merely focusing on the right answers. To do this, teacher should observe how students solve the problem, which variables are used, how these variables are represented, which strategies are developed and how these strategies help the student while finding the solution. Similarly, students are expected to understand the problem, make plans, use different problem solving strategies, carry out the plan, and check their solutions, as well as posing their own mathematical problems. Furthermore, it is recommended that sometimes problems can be asked just to measure whether students understand the problem, whether there are any missing variables or extra variables in the problem, and which strategies are suitable for that problem. That is, students might be asked only a part of a problem without completely solving and reaching a solution.

2.2. Problem Solving and Teachers

2.2.1. Teachers' Beliefs and the Factors Affecting Their Beliefs

Beliefs are "psychologically held understandings, premises, or propositions about the world that are felt to be true" (Richardson, 1996, p.103, as cited in Op't Eynde, De Corte, & Verschaffel, 2002). As being the personal views, assumptions, and values (Ernest, 1989), beliefs indicate the decisions individuals make throughout their lives (Dewey, 1933). Therefore, by being at the heart of one's actions (Margaret, 2001), beliefs can explain one's certain behaviors (Alba, 2001).

Beliefs are also explained as what one believes to be true, regardless of the fact that others agree or not, and regardless of the fact that others know it to be true or not (Op't Eynde, De Corte, & Verschaffel, 2002). "As persons whose daily task is to understand and interpret the rapid flow of events in a classroom, and to make decisions and act on their interpretations, all teachers obviously rely on their knowledge and beliefs" (Peterson, Fennema, Carpenter, & Loef, 1989, p.3) with which they define their work (Nespor, 1987).

As beliefs have a considerable effect on individuals' actions, teachers' beliefs play a crucial role in changing the ways teaching takes place. Hersh (1986) indicated that "one's conception of what mathematics is affects one's conception of how it should be presented and one's manner of presenting it is an indication of what one believes to be the most essential in it" (p.13). Basically, teachers possess beliefs about "their profession, their students, how learning takes place, and the subject areas they teach" (Margaret, 2001, p.4).

To better understand teachers' practices, research has been done about the factors affecting their beliefs (Alba, 2001; Grouws, 1996; Hart, 2002; Lloyd & Frykholm, 2000; Quinn, 1997; Seaman, Szydlik, & Beam, 2005; Simon & Schifter, 1991). For instance, Grouws (1996) investigated possible factors affecting teachers' beliefs about how teaching should take place and concluded that "school goals, classroom climate, the physical setting including availability of instructional equipment and materials, school policies and curriculum guides, administrators, and teachers' colleagues" (p.82) significantly affected teachers' beliefs. A similar study was performed by Alba (2001) who found that "peer interaction, collegial support, teachers' kindergarten through 12th grade experiences as students, their teacher education programs, college methods instructors, and their own classroom practices as teachers" influenced teachers' beliefs towards their subject matter (p.31).

Quinn (1997) made a research on pre-service elementary and secondary school teachers about how mathematical methods courses affected their knowledge in mathematics and their attitudes and beliefs about how to teach mathematics. The participants were 47 pre-service teachers at the University of Nevada, Las Vegas. The philosophy of the mathematics methods courses given in the study was highly consistent with the recommendations of NCTM regarding the use of cooperative learning, problem solving and technology. Following the completion of these methods courses, pre-service teachers' knowledge, assumptions, and feelings about mathematics as well as their beliefs about their role as teachers in the classroom changed significantly. It was concluded that "if reform in mathematics education is to be successful, then teachers of mathematics must have an adequate knowledge of meaningful mathematical content" by taking these methods courses (p.113). Similar to Quinn's study (1997), Wilkins & Brand (2004) made research on 89 preservice teachers in order to evaluate the effectiveness of a methods course. Findings from the study suggested a positive relationship between participating in the mathematics methods course and change in teacher beliefs and attitudes. Also, Emenaker (1996) examined the impact a problem-solving based mathematics content course for preservice elementary education teachers had on challenging the beliefs they held with respect to mathematics and themselves as doers of mathematics, and observed significant positive changes for four of the five beliefs.

A further study was performed by Hart (2002) who conducted a study about pre-service teachers participating in an alternative certification program for teaching in an urban setting. This study investigated the relationship between taking a mathematics methods course, changing teachers' beliefs to be more consistent with the current reform movement in mathematics education, and changing teachers' self-efficacy. It was proposed that changing teachers beliefs would take time. For three semesters, the pre-service teachers took methods courses in which concepts were introduced through problem situations and discussions. Before and after the program, pre-service teachers completed a Mathematics Beliefs Instrument composed of three parts; the first part measured the consistency of a person's beliefs about mathematics teaching and learning with the NCTM Standards (1989), the second part measured beliefs about teaching and learning mathematics in general, whereas the third part measured pre-service teachers' perception of their effectiveness as a mathematics teacher and learner. The findings from the study suggested that after participating in the mathematics methods courses, pre-service teachers changed their beliefs in a way that was more consistent with mathematics education reform proposed by NCTM, and changed their sense of self-efficacy in a positive way. Similar to Hart (2002), Cooney & Wilson (1995) found that the beliefs about mathematics held by pre-service teachers considerably changed after participating in a teacher education program that promoted the NCTM standards and the use of technology.

Lloyd and Frykholm (2000) examined pre-service teachers' conceptions about the nature of mathematics and their future classroom practices. In this study, 50 pre-service teachers engaged in reform oriented activities during their teacher education coursework. It was stated that pre-service teachers brought to the class their past experiences as learners, their beliefs about the nature of mathematics, and their emerging projections of themselves as future classroom teachers. After working in cooperative groups, discussing multiple solution strategies, and studying mathematics through relevant problem situations outlined in these activities, prospective teachers more deeply understood the subject matter and significantly changed their previous conceptions about mathematics and how to teach mathematics.

Seaman, Szydlik, & Beam (2005) replicated a study done by Collier's in 1972 that focused on the beliefs of pre-service elementary mathematics teachers about both the nature of and the teaching of mathematics. The study aimed to find out whether pre-service teachers' beliefs in 1998 were different than their counterparts' beliefs in 1968 both at the University of Wisconsin Oshkosh. It was stated that there were differences in pre-service teachers' beliefs due to the changes in the culture of schooling with respect to the school curriculum and pedagogy, and the changes offered by restructured educational reform in mathematics education.

After understanding the factors influencing teachers' beliefs and practices, it is also critically important to know how a classroom culture develops through the interactions of teachers and students. Grouws (1996) explains that each mathematics classroom forms its own culture according to the unique knowledge, beliefs, and values that participants bring to the classroom. For instance, the students bring "views of what one does in mathematics class, judgments about how good they are at mathematics, and feeling about how well they like mathematics", whereas the teacher brings to the class "a view of mathematics, routines for teaching the class, expectations about what should be accomplished in the class, personal experience with learning mathematics, and either a like or dislike for the discipline" (Grouws, 1996, p. 84). When all these kinds of beliefs are combined together, the classroom culture develops and reflects all the shared meanings and beliefs that teacher and students bring to the classroom.

Beliefs develop highly gradually and do not change easily (Abelson, 1979). Especially, central beliefs which are more grounded and held more strongly are "less open to rational criticism or change compared to peripheral beliefs which are more open to examination and possible change" (Turner & Chauvot, 1995, p.4). So modifying or changing these strongly held beliefs will have more far reaching consequences than changing the others (Op't Eynde, De Corte, & Verschaffel, 2002). Hollifield (2000) and Anderson (1995) suggested that if reformers want to improve the content and methodology used in teaching, they need to give their attention to previously formulated beliefs and dispositions of teachers and students. Hollifield (2000) emphasized that "supplying new curricula, incentives, or regulations" are not sufficient to change teaching practices as long as "teachers do not understand or do not agree with the goals and strategies" proposed by these innovations (p.22). In addition to understanding and agreeing with the new ideas, in order for teachers to willingly change their beliefs, they need to experience cognitive conflicts associated with their current state of teaching, decide to change, make a commitment to change, construct a vision to change, and reflect on their instructional practices (Brosnan et al., 1996; Wood, Cob, & Yackel, 1991).Due to the fact that teachers play a major role in the lives of today's students and tomorrow's adults (Brown, 2003), and long lasting instructional changes only result from essential modifications in what teacher's believe, know, and practice (Putnam, Wheaten, Prawat, & Remillard, 1992) , it becomes vitally important to understand teachers' beliefs and the factors influencing these beliefs and how these beliefs affect their classroom practices.

2.2.2. Teachers' Beliefs about Mathematics and Problem Solving

Many researchers (Brosnan, 1996; Emenaker, 1996; Ford, 1994; Frykholm, 2003; Hollifield, 2000; Lerman, 1983; McKnight, 1987; Nathan & Koedinger, 2000; Peterson, Fennema, Carpenter, & Loef, 1989; Raymond, 1997; Schoenfeld, 1991) have given their emphases on teachers' beliefs in order to better understand what teachers believe about their profession as well as their subject matter.

Lerman (1983) differentiated teachers' beliefs as the knowledge centered and problem centered. Teachers who viewed mathematics as knowledge centered believed that "mathematics is an accumulated body of hierarchical knowledge" and solving problems is a final experience based on previously acquired knowledge (p.62). On the contrary, teachers who viewed mathematics as problem centered believed that "mathematics is composed of hypothesis making, justification, generalization and searching for new problems", and problem solving is a means for learning mathematics (p.62). Similar to Lerman, Hollifield (2000) distinguished teachers as the ones believing "mathematics is a static set of concepts and procedures", and the ones believing "mathematics is a mental process of constructing hypothesis, proofs, and refutations" to solve problems (2000, p.21).

Some studies related to mathematics teachers (Brosnan, 1996; Nathan & Koedinger, 2000; Raymond, 1997) showed that mathematics teachers had the following beliefs about their subject matter: answers are more important than processes; students must master computational skills before they can solve problems; the teacher has to be in charge of the learning, and spending time on problem solving is wasteful. Similarly, some teachers believed that "the basic computational skills were the most essential component of mathematics curriculum" (Frykholm, 2003, p.135) as students were tested mostly about these skills. Moreover, both McKnight (1987) and Schoenfeld (1991) found that mathematics teachers as well as students hold the following beliefs: mathematics is passed to students from above (the teacher) for memorization; mathematics is a solitary activity; school mathematics has little to do with the real world; proof has nothing to do with mathematical discovery or invention; there is one correct answer to any problem; there is one correct way to solve any problem, and all problems can be solved in 5 minutes or less (McKnight,1987; Schoenfeld, 1991 as cited in Becker, 1996, p.26).

Emenaker (1996) studied the impacts of a problem solving based mathematics methods course on pre-service elementary teachers' beliefs about mathematics and how to teach mathematics. The pre-service teachers at Indiana University in Bloomington were given a Likert style survey prepared by Kloosterman and Stage in 1992 that categorized beliefs at five scales as time, memory, step, understand and several. Before the methods course, the pre-service teachers hold the following beliefs: "if a math problem takes more than 5-10 minutes, it is impossible to solve; math is mostly memorization; all problems can be solved using a step by step algorithm or a single equation; only geniuses are capable of creating or understanding formulas and equations; there is only one correct way to solve any problem" (p.79). During the course, the pre-service teachers worked in groups and experienced alternate methods of solutions to the same problem. Many of the problems posed in the course were not solvable by the mere application of a previously memorized formula or procedures, so they had to re-consider situations, discover many concepts and re-derive formula on their own. From the start to the end of the semester, pre-service teachers' beliefs changed significantly at four categories such that they believed: "understanding concepts in mathematics is more important than memorizing procedures; many mathematics problems can be solved without having to rely on memorized step by step procedures; it is reasonable to expect people of average mathematical ability to discover some mathematical concepts on their own; there is more than one way to solve a problem and some problems have more than one correct answer" (p.80). The pre-service teachers only conserved their beliefs about time as they considered just homework problems from the textbooks while responding to the items related to this category.

Peterson, Fennema, Carpenter and Loef (1989) examined relationships among first grade teachers' pedagogical content beliefs in addition and subtraction, together with teachers' pedagogical content knowledge, and students' achievement in mathematics. The study took place in Madison, Wisconsin. The results of the study showed that teachers with a less cognitively based perspective believed that "children receive knowledge" (p.6), "skills should be taught in isolation from understanding and problem solving", "formal mathematics should be the basis for sequencing topics for instruction", and "instruction should be organized to facilitate teachers' presentation of knowledge" (p.7), whereas, teachers with a more cognitively based perspective believed that "children construct their own knowledge", "skills should be taught in relation to understanding and problem solving" (p.6), "children's natural development of mathematical ideas should provide the basis for sequencing topics for instruction", "mathematics instruction should facilitate children's construction of knowledge" (p.7). Furthermore, when the teachers with a less cognitively based perspective and teachers with a more cognitively based perspective's view of the roles of teacher and learner were examined, it was found that unlike the teachers with a more cognitively based perspective who viewed the teacher and the learner "as actively engaged with one another in construction of mathematical knowledge and understanding", the teachers with a less cognitively based perspective viewed the teacher's role as "one organizing and presenting mathematical knowledge and the child's role as one receiving and presenting mathematical knowledge presented by the teacher" (p.25).

Also, Ford (1994) tried to discover what teachers believe about the nature of mathematical problem solving, attributions about the causes of students' performance in problem solving, and beliefs about the teaching and learning of problem solving in mathematics. To analyze these beliefs, interviews were conducted with ten 5th grade teachers in a large rural school district of South Carolina as well as their students. The following conclusions were drawn: "students and teachers believe that problem solving is primarily an application of computational skills"; "students and teachers reported that their judgments about successful problem solving were based on right answers"; "students' and teachers' attributions about the causes of success and failure affect learning in problem solving"; "teachers focused on right answers and strongly discouraged the use of calculators for problem solving"; and "teachers tended to overestimate students' ability to do problems involving computation and underestimate students' ability to do reasoning problems" (p.320). Additionally, students' beliefs were found to be consistent with the beliefs held by their teachers. Especially the students of teachers that strongly discouraged the use of calculators for problem solving believed that "using calculators in problem solving is cheating" (p.319).

2.2.3. The Impact of Teachers' Beliefs on their Classroom Practices and Students

Research suggests that teachers' beliefs are importantly linked to teachers' classroom practices and, consequently to students' learning in mathematics (Ball, 1998; Frykholm, 2003; Grouws, 1996; Lloyd & Wilson, 1998; Peterson, Fennema, Carpenter, & Loef, 1989; Thompson, 1984; Wilkins & Brand, 2004). Even, the improvements obtained in students' achievements are half to half attributed to the changes in teacher's beliefs and classroom practices (Sparks, 1999) such as "the ways in which they present the subject matter, the kinds of task they set, assessment methods, procedures and criteria" (Mason, 2003, p.83). Therefore, due to the fact that teachers' beliefs, knowledge, judgments, and decisions have a close relation with students' beliefs, attitudes and performance in mathematics, it becomes highly important to know these beliefs and be aware of their effects on classroom practices.

Brown (2003) indicated that teachers' beliefs had significant impact on their students' attitudes toward and beliefs about mathematics and problem solving, as well as their students' performance in mathematics and problem solving. For instance, Karp (1991) found that teachers' beliefs affected their students attitudes in a way that students of teachers with negative mathematical beliefs showed "a learned helplessness response by passively receiving information" (p.267), whereas students of teachers with positive mathematical beliefs "explored and discovered mathematical meanings and interrelationships" (p.268). Furthermore, Carter and Noreood (1997) found that those teachers who believed that problem solving, hard work, and understanding were key components in mathematics had students who held the same beliefs.

Peterson, Fennema, Carpenter, and Loef (1989) examined the first grade mathematics teachers' pedagogical content beliefs and classroom practices while teaching addition and subtraction. Among these mathematics teachers, some demonstrated a more cognitively based perspective whereas the others demonstrated a less cognitively based perspective for mathematics. The results showed that the ones with a more cognitively based perspective made extensive use of word problems while introducing a new subject matter, tried to make the learnt concepts relevant to students' lives to enhance their understanding, demonstrated greater knowledge of word problem types and greater knowledge of their students' problem-solving strategies, and evaluated their students improvements by observing them in problem situations rather than by merely relying on tests and formal assessments. Compared to the teachers with a more cognitively based perspective, teachers with a less cognitively based perspective did not use word problems early in the year to teach addition and subtraction, used word problems after introducing the facts, solved problems by following step by step algorithmic procedures and by focusing on key words in the problems. At the end of the semester, students of more cognitively based teachers scored higher on word problem-solving achievement than did students of a less cognitively based teachers, and students from both types of classes did equally well on number facts although students of less cognitively based teachers were heavily exposed to number facts .

Brown (2003) stated that if teachers want to integrate problem solving in their mathematics lessons, then they need to believe in the importance of problem solving in mathematics education. Like Brown, Grouws (1996) indicated that if teachers believe it is vital for students to develop connections between ideas through problem solving, then these teachers prepared their lessons and planed instructional activities including "explorations of situations, hypothesis generation, problem posing, multiple solutions and solution methods and arguments followed by justification and verifications" (p.80). Also it is concluded that if teachers believed in the essence of problem solving, it positively influenced what students learned and how they performed in these classroom activities (Grouws, 1996).

Students' beliefs about mathematics education are "situated in, and determined by, the context in which they participate as well as by their individual psychological needs, desires and goals" (Op't Eynde, De Corte, & Verschaffel, 2002, p.27). There were also studies into students' conceptions of a problem. For example, Frank (1988) conducted a study with 27 mathematically talented middle school students to investigate their beliefs about mathematics and how these beliefs influence their problem solving practices. She used a questionnaire, interviews and observations. She found that students believed that mathematical problems must be solvable quickly in a few steps and that mathematical problems were routine tasks which could be done by the application of known algorithms. They perceived non-routine problems as "extra credit" tasks. Students believed that if a problem could not be solved in less than 5 to 10 minutes, either something was wrong with them or the problem. The goal of doing mathematics was to obtain "right answers." Students focused entirely on answers which to them were either completely right or completely wrong.

Another study was conducted by Garafola (1989) who examined students' beliefs about the nature of mathematics and found the following beliefs; "the difficulty of a mathematics problem is due to the size and quantity of the numbers, all problems can be solved by performing one arithmetical operation, in rare cases two; the operation to be performed is determined by the keywords of the problem, usually introduced in the last sentence or in the question, thus it is not necessary to read the whole text of the problem; the decision to check what has been done depends on how much time is available" (p.503). Also, it was found that "students believed normal homework and test problems should be solved in a few minutes, and if not they should not waste time on them, as they would never find the solution" (Mason, 2003, p.74).

An additional study was conducted by Schoenfeld (1989) who analyzed students' beliefs and behaviors in mathematical problem solving. He administered questionnaires to a group of over 200 high school students. The result showed that most students believed that mathematics was memorization of facts and mastery of skills and had little relevance to daily life. In addition, it was found that these students viewed problem solving as part of what was done in mathematics classroom instead of as a fundamental and necessary part of everyday life. Moreover, Grouws (1996) mentioned about some students beliefs about mathematics and problem solving as: "the daily mathematics work was composed of doing endless sets of routine exercises; there is one way to do every mathematics task and that there is always one answer" (p.85); "mathematics is mainly memorizing, "if one understands mathematics, then one should be able to do problems quickly; there is always a rule to follow in solving mathematics problems" (p.86).

Similar to other studies, Spangler (1992) found that one of the common beliefs among students was that a mathematical problem has only one correct answer. Students were not prepared to accept that a problem could have different answers, all being correct. They indicated that they preferred one method to multiple methods for solving a problem because they did not have to remember much. Students admitted that they could obtain the correct answer to a problem without understanding what they were doing. Students rarely checked to see if their answers made sense in the context of the given problem. They verified their answers with the teacher or by checking the text and they are not inclined to look for multiple solutions or to generalize their results.

Finally, Kroll and Miller (1993) reported that an important difference between successful and unsuccessful problem solvers lied in their beliefs about problem solving, about themselves as problem solvers, and about ways they approach to problem solving. For example, it was found that students, who developed the belief that all mathematics problems could be solved quickly and directly, directly gave up solving the problem when they did not immediately know how to solve a problem. Also, these students were found to be viewing themselves as incompetent problem solvers. Furthermore, the students who believed that there is just one right way to solve any mathematics problem were observed to be depending on the teacher and answer keys for verification of their solutions.

In conclusion, teachers' beliefs about what counts as a mathematical context and what they find interesting and important are found to be strongly influencing the situations they are sensitive to, and whether or not they engage in these situations (Op't Eynde, De Corte, & Verschaffel, 2002). Unless teachers regard problem solving as an important part of mathematics education and regularly engage in such activities (Lester, 1980), it is expected that students will "fail to appreciate the excitement and insight that can come from solving a problem" (NCTM, 2000, p.258). Therefore, in order to make up the shortages in our education system and counteract negative dispositions, we need to examine the belief structures of our mathematics teachers, and start the changes by changing the negative beliefs teachers hold toward mathematics and how to teach mathematics.

2.3. Research Studies in Turkey related to Problem Solving and Teachers' Beliefs

When literature was examined with respect to Turkish publications related to mathematical problem solving, and teachers' beliefs about problem solving, very little research study was found about these issues most of which were conducted in the last three years. Moreover, research showed that nearly all of these studies were related to elementary students' problem solving abilities. For instance, Soylu and Soylu (2006) conducted a study on elementary students in order to determine students' difficulties and errors in problem solving. The subjects of the study consist of 2nd grade students in 13 classes who attend to a primary school in Erzurum. Research data were obtained through the answers that students gave to tests consisting of 10 exercises and 10 essay problems that require the same process of solution, and the interviews conducted with the subjects. It was found that the students did not have difficulty in answering the exercises that required procedural knowledge related to addition-subtraction-multiplication, whereas they had a difficulty in solving the problems that required conceptual and operational knowledge.

Another study was conducted by Karataş and Güven (2004) examining and discussing students' sufficiency and weakness in problem solving process. In the study, four word problems were prepared and implemented to a sample of five students at 8^{th} grade by using clinical interviews. The data showed that although students could explain problems by using variable in representation stage, they failed in defining problem correctly, as well as in writing an equation and reaching a correct answer.

An additional study was carried out by Adigüzel and Akpinar (2004) who studied the effect of technology usage on elementary students' problem solving skills. The study showed that 7^{th} grade students' word problem-solving skills especially related to work and pool problems improved through computer-based multiple representations including graphic, symbolic, and audio representations. Moreover, Toluk and Olkun (2002) investigated how elementary school mathematics textbooks approach to problem solving. The results of the study showed that elementary mathematics textbooks displayed the traditional view of problem solving where mathematical concepts and skills are considered as prerequisites for problem solving. They also found that in these textbook, first mathematical concepts are taught, then applied for solving word problems.

A further study was carried out by Korkmaz, Gür and Ersoy (2006) to examine what mathematics and elementary prospective teachers do in problem posing process, and to determine the misunderstandings that they have in this process. The findings showed that first of all prospective teachers did not know the difference between problem and exercise. They defined problems to be the exercises solved at the end of lesson in order to practice the introduced idea. Next, they thought that there is only one solution to any problem, and believed that textbooks are sufficient in developing students' problem posing abilities. Also it was found that the pre-service teachers were not appreciating open ended mathematics problems to be asked in mathematics instruction.

2.4. The Need for more Research on Problem Solving

Problem solving has been given value from kindergarten to high school as a goal for mental development, as a skill to be taught, and as a method of teaching in mathematics education (Brown, 2003; Giganti, 2004; Jonassen, 2004; Lester, 1981; Manuel, 1998; Martinez, 1998; Naussbaum, 1997; Polya, 1953; Schoenfeld, 1989; Willoughby, 1985) all around the world, and currently in new Turkish mathematics curriculum it was emphasized as a common skill for all subject matters. As the emphasis given to problem solving increases, it becomes vitally important to learn more about problem solving and how to implement it in mathematics classrooms.

Research showed that teachers' beliefs and preferences about how to teach mathematics play a significant role in how teachers teach in the classroom environments (Ball, 1998; Frykholm, 2003; Grouws, 1996; Lloyd & Wilson, 1998; Peterson, Fennema, Carpenter, & Loef, 1989; Thompson, 1984; Wilkins & Brand, 2004). Teachers' actions in the classroom and the observable effects of those actions can be better understood if their thought processes are better understood. Therefore, it is also vitally important to learn more about the belief structures of teachers, and how they value problem solving in mathematics education.

Nevertheless, when teachers were asked to explain problem solving in their own words, it is found that “teachers have different conceptions of what constitutes problem solving” (Grouws, 1996, p.89). Similarly, Lester (1994) stated that “although conference reports, curriculum guides, and textbooks insist that problem solving has become central to instruction at every level”, it is evident that teachers do not have adequate knowledge about what problem solving is, and there is a need to examine more about what beliefs teachers hold about mathematical problem solving (p.660). Similarly, Grouws (1996) reported teachers’ beliefs in teaching mathematical problem solving as a much neglected area of research. These findings when taken all together suggest that there is clearly a need for further research in the area of what a mathematical problem solving is, and teachers’ beliefs about the importance and role of problem solving in mathematics education.

Especially in Turkey, after the current innovation made in the mathematics curriculum, it becomes extremely important to understand what teachers know and believe about these intended changes. Research showed that in Turkey very few studies have been conducted about how pre-service teachers view mathematical problem solving. However, it is important to study pre-service teachers’ beliefs, to give us insight into possible changes that could be made in pre-service education program. Moreover, learning more about pre-service teachers’ beliefs will guide us “in choosing and implementing professional development programs for both pre-service and in-service teachers” (Brown, 2003, p.13). Therefore, it is vital to be aware of pre-service teachers' hindering beliefs related to mathematical problem solving, and offer opportunities to challenge those beliefs.

CHAPTER 3

METHODOLOGY

The research design and procedures used in this study are explained in this chapter. This chapter has six main parts. The first part explains the overall research design and variables of the study; the second part explains participants of the study; the third part explains the content of the instrument; the fourth part explains the construction and development processes of the instrument; the fifth part explains data collection procedures and the last part explains the analyses of the data.

3.1. Research Design

The main area of investigation in the present study was to explore the kinds of beliefs the pre-service elementary mathematics teachers have towards mathematical problem solving, and investigate whether, or not, gender and university attended have a significant effect on their problem solving beliefs.

It was a survey study designed to collect information from pre-service elementary mathematics teachers on their beliefs about mathematical problem solving by direct administration of a survey which was prepared by the researcher. It was a survey study, because a survey design mainly provides “a quantitative or numeric description of trends, attitudes, or opinions of a population by studying a sample of that population” (Creswell, 2003, p.153).

The sample of the present study consisted of 244 senior elementary mathematics students studying at five different universities in three different cities in Turkey. Participants were presented with a questionnaire having both 5 point likert type items and open-ended questions. The likert type items were asked to evaluate the pre-service teachers beliefs on several topics related to mathematical problem solving. The open-ended type questions were asked in order to "allow more individualized responses" to investigate whether, or not, the pre-service teachers' assessment of mathematics problems were in line with their problem solving beliefs (Fraenkel & Wallen, 1996, p.374). During the spring semester of 2006, the instrument was administered to the participants in their classroom settings within 25 minutes, and SPSS software program was used for data analysis.

This study had two independent variables (IVs) and one dependent variable (DV). The independent variables were gender and university, whereas the dependent variable was the mean scores of pre-service elementary teachers' beliefs on mathematical problem solving.

Independent Variables:

1. Gender: It was a categorical variable with two levels (1 = male, 2 = female).
2. University: It was a categorical variable with five levels (1 = University A, 2 = University B, 3 = University C, 4 =University D, and 5 = University E).

Dependent Variable:

Mean Scores of Mathematical Problem Solving Beliefs Scale: It was a continues variable with a minimum value of 1, and a maximum value of 5. (1= Strongly Disagree, 2= Disagree, 3= Neutral, 4= Agree, 5= Strongly Agree; The higher the score, the stronger beliefs pre-service elementary mathematics teachers have toward mathematical problem solving)

A summary of the overall research design is presented in the Table 3.1 below.

Table 3.1 Overall Research Design

1. Research Design	Survey Study (Cross-sectional Survey)
2. Sampling	Convenience Sampling
3. Variables	Independent Variables: Gender, University
	Dependent Variable: The mean scores of pre-service elementary teachers' beliefs on mathematical problem solving
4. Instrument	*'Belief Survey of Pre-service Mathematics Teachers on Mathematical Problem Solving'* constructed by combining four previously implemented belief instruments
5. Data collection procedure	Direct administration of the survey to 244 pre-service elementary mathematics teachers at five universities in their classroom settings within 25 minutes
6. Data analysis procedure	Descriptive statistics and two way ANOVA

3.2. Sample of the Study

The target population of the present study was all pre-service teachers studying in Elementary Mathematics Education department in Turkey. There were 23 universities offering this program in Turkey. As it would be difficult to reach all these pre-service elementary mathematics teachers, a convenience sampling method was preferred.

At first, only the pre-service elementary mathematics teachers in Ankara participated in the study as it was an accessible sample for the researcher. Later, in order to reach more participants and obtain more information from different cities, the pre-service elementary mathematics teachers in Bolu and Samsun were included in the study, still forming a convenient sample. Therefore, the sample of the present study consisted of 244 senior pre-service elementary mathematics teachers studying at Elementary Mathematics Teacher Education programs at 5 different universities located in Ankara, Bolu, and Samsun in 2005-2006 spring semesters.

Table 3.2 shows the number of senior pre-service elementary mathematics teachers in these five universities, and the number of pre-service teachers participated voluntarily in this study. There were totally 443 senior pre-service elementary mathematics teachers in these five universities. At University A, among 33 senior pre-service elementary mathematics teachers, 31 pre-service teachers contributed. Similarly, 49 pre-service teachers at University B; 56 pre-service teachers at University E; 38 pre-service teachers at University D; and 70 pre-service teachers at University C participated. So, totally 244 senior pre-service elementary mathematics teachers volunteered in this study.

Table 3.2 Number of Senior Pre-service Elementary Mathematics Teachers

University Attended	Number of pre-service teachers	Number of participants	Percentage of participants
University A	33	31	94
University B	70	49	70
University C	100	70	70
University D	60	38	63
University E	80	56	70
Total	343	244	71.4

Actually, it was expected to reach approximately 80 to 100% of these 343 pre-service teachers. However, as absenteeism was not taken for most of the courses in the last year, and as senior pre-service teachers were in mood of graduating, many of them were not attending their lessons. By the help of the instructors in these universities, the study was administered to the participants both during their lessons, mid-terms and finals. Then, the participation reached hardly above 70%.

All of the participants were chosen to be 4th year pre-service elementary mathematics teachers so that they have sufficient background in their subject area and pedagogy. The pre-service elementary mathematics teachers at University B, University C, University D and University E had similar undergraduate coursework. However, the pre-service elementary mathematics teachers at University A had an undergraduate course work with several differences. For example, the number of courses taken during the undergraduate study was different such that University A students had to complete 25 courses in total; 10 of which related to mathematics and 15 related to pedagogy, whereas the students in the other universities had to complete 30 courses in total; 13 courses related to mathematics and 17 courses related to pedagogy. Also, some courses were taken in different semesters or in different years. However, in the last year at spring semester, all the pre-service teachers were taking the same courses which were left to graduate.

The following tables present the courses related to mathematics and pedagogy that were offered during the undergraduate elementary mathematics teacher education program. Table 3.3 illustrates the courses at University B, University C, University D and University E, whereas Table 3.4 shows the courses at University A.

Table 3.3 The Undergraduate Courses for Universities

	Fall Semester	Spring Semester
First Year	Calculus-I	Calculus II
	Discrete Mathematics	Geometry
	Intro. to Teaching Profession	School Experience I
Second Year	Calculus III	Calculus IV
	Linear Algebra I	Linear Algebra II
	Computer	Inst. Planning and Evaluation
	Development and Learning	
Third Year	Statistics and Probability I	Statistics and Probability II
	Introduction to Algebra	Elementary Number Theory
	Lab. App. in Science I	Lab. App. in Science II
	Analytic Geometry	Classroom Management
	Inst. Tech.& Material Devel.	Special Teaching Methods I
Fourth Year	Computer Assisted Math. Edu.	Textbook Analy. in Math. Edu.
	Methods of Science Teaching	Counseling
	School Experience II	Practice Teaching
	Special Teaching Methods II	

Table 3.4 The Undergraduate Courses for University A

	First Semester	Second Semester
First Year	Calculus with Analytic Geo.	Discrete Mathematics
	Fundamentals of Mathematics	Calculus II
	Intro. to Teaching Profession	School Experience I
Second Year	Analytic Geometry	Basic Algebraic Structures
	Elementary Geometry	Intro. to Differential Equations
	Development and Learning	Inst. Planning and Evaluation
		Computer Applications in Edu.
Third Year	Basic Linear Algebra	Probability and Statistics
	Inst. Dev.&Media in Math Edu.	Lab. Applications in Science II
	Laboratory App. in Science I	Meth. of Science Mat Teaching
		Classroom Management
Fourth Year	School Experience II	Practice Teaching in Ele. Edu.
	Methods of Math. Teaching	Textbook Analysis in Math Edu
		Guidance

Table 3.5 shows the gender and university distributions of the participants. Out of 244 pre-service teachers, 113 were males, and 131 were females. In each university the number of participants was nearly half males and half females. The

number of the overall participation according to the universities from the highest to the lowest is as follows; 70 pre-service teachers from University C (41.4% males and 58.6% females), 56 pre-service teachers from University E (48.2% males and 51.5% females), 49 pre-service teachers from University B (55.1% males and 44.9% females), 38 pre-service teachers from University D (39.5% males and 60.5% females), and 31 pre-service teachers from University A (48.4% males and 51.6% females).

Table 3.5 University and Gender Distributions of the Participants

	Male		Female		Total	
	n	%	n	%	n	%
University A	15	48.4	16	51.6	31	12.7
University B	27	55.1	22	44.9	49	20.1
University C	29	41.4	41	58.6	70	28.7
University D	15	39.5	23	60.5	38	15.6
University E	27	48.2	29	51.8	56	22.9
Total	113	46.3	131	53.7	244	100

3.3. Data Collection Instrument

In the present study, '*Belief Survey of Pre-service Mathematics Teachers on Mathematical Problem Solving*' was administered as the data collection instrument. It was constructed by the researcher by making use of four different instruments in the related field with some modifications in the light of the review of literature.

It consisted of three parts as follows: (1) Demographic Information; (2) Beliefs Related to Mathematical Problems; and (3) The Belief Survey on Mathematical Problem Solving. In the demographic information sheet, there were several questions related to participants' personal characteristics such as gender, university they attended, university grade level, grade point average (G.P.A), whether they have taken courses related to problem solving, whether they have been interested in problem solving, whether they have completed certain pedagogical courses such as School Experience and Methods of Mathematics Teaching, and whether they have finished their must courses related to mathematics.

In the second part of the questionnaire, there were five mathematics problems chosen from Turkish and foreign mathematics textbooks in a way that they were all valuable for elementary mathematics education and consistent with the current reform movement in mathematics education. The participants were asked to evaluate each problem as either poor, average or strong according to the educational value of the given problems regarding their appropriateness in elementary mathematics education, and then explain their reasons. Participants were also asked to make additional comments and interpretations at the end of this part.

In the last part of the questionnaire, there were 39 items related to several beliefs about problem solving in mathematics education. These items were asked in a 5 point Likert type in order to evaluate the pre-service teachers' beliefs about mathematical problem solving. Among these items, 22 of them were positively stated and 17 of them were negatively stated. Participants were asked to indicate their agreements or disagreements with these statements on a five-point Likert scale ranging from 5 to 1; 5indicating 'strongly agree', 4 indicating 'agree', 3 indicating 'neutral', 2 indicating 'disagree', and 1 indicating 'strongly disagree'.

It took nearly 2 minutes to fill in personal information, 10 minutes to interpret mathematics problems, and 15 minutes to fill in questionnaire items, so it took approximately 25 minutes of participants to fill in the questionnaire.

3.3.1.1. Construction of the Instrument

There were several steps followed during the construction and development of the instrument used in this study. First of all, the questionnaire items were developed by combining and modifying four previously implemented instruments which were found during the literature review. Following the preparation of the questionnaire items, a number of mathematics problems were added in the instrument to investigate whether pre-service teachers' assessment of mathematics problems fits in with their problem solving beliefs determined during the questionnaire items. Another reason of adding these mathematics problems was that the mere application of the questionnaire items might not provide an adequate insight into students' beliefs, only it might give a baseline data. So, besides the questionnaire items, mathematics problems were added in the instrument to provide some more information about students' belief structures on mathematical problem solving. After the construction of the questionnaire items and addition of mathematical problems, the instrument was translated into Turkish by the researcher, and edited by an expert of Turkish language. As a last step for the construction of the instrument, a number of demographic information questions were listed to gather more descriptive information about participants' characteristics.

3.3.1.1.1. Literature Review

Before preparing the questionnaire, a substantial literature review was carried out. First of all, several databases such as EBSCOhost, ERIC, and Digital Dissertations and Theses were searched to find out the recent studies conducted on mathematical problem solving and teachers' beliefs. Next, ULAKBIM was explored to examine the Turkish publications on this subject. Then, several libraries were searched for books, periodicals, articles, and theses together with e-books, e-periodicals and e-theses to find more detailed information about what has been done about mathematical problem solving and what has been used as data collection instrument. During this literature review, no instrument was found in Turkish, and only a few English instruments that were specially designed for assessing the belief structure of teachers on mathematical problem solving were found. Some of these instruments were derivations of each other.

3.3.1.1.2. Preparation of the Questionnaire Items

The questionnaire items of the present study were mainly formed by combining several parts of four previously implemented belief questionnaires. Instrument 1, *Indiana Mathematics Belief Scales* (IMBS), was constructed by Kloosterman and Stage (1992) specifically to measure beliefs held by pre-service elementary teachers about problem solving. It was a 30-item questionnaire on a likert scale of 5 possible responses ranging from strongly agree to strongly disagree. The items were stated in one of five categories such as; (1) Time Consuming Problems are Worthwhile: "I can solve time-consuming problems", (2) Not Always Step by Step: "There are word problems that can not be solved using simple, step-by-step procedures", (3) Understanding Important: "Understanding concepts is important in mathematics", (4) Word Problems: "Word problems are important in mathematics", and (5) Effort Pays: "Effort can increase mathematical ability".

Instrument 2 was developed by Emenaker in 1996, also at Indiana University. It aimed to assess the impact a problem-solving based mathematics content course on pre-service elementary education teachers' beliefs held with respect to mathematics and themselves as doers of mathematics. Participants were asked to indicate the extent to which they agree or disagree on a 5-point likert scale. It consisted of 41 items, and part of the instrument is based on survey questions from Kloosterman and Stage (1992) or Schoenfeld (1989), and some questions were developed specifically. The items were expressed in one of five categories such as;

(1) Time: “If a math problem takes more than 5 - 10 minutes, it is impossible to solve”, (2) Memory: “Math is mostly memorization”, (3) Step: “All problems can be solved using a step-by-step algorithm or a single equation”, (4) Understand: “Only geniuses are capable of creating or understanding formulas and equations”, and (5) Several: “There is only one correct way to solve any problem”.

Instrument 3, *The Standards Belief Instrument* (SBI), was prepared by Zollman and Mason (1992) in order to measure the consistency of teachers’ beliefs about mathematics teaching and learning with the NCTM Standards. The instrument consisted of 16 questions based on the NCTM Standards (1989) from the NCTM's publication *Curriculum and Evaluation Standards for School Mathematics*. The items were expressed in a way that they are either nearly direct quotes or inverse of direct quotes from the Standards.

Instrument 4, *Mathematics Beliefs Instrument* (MBI), was developed by Hart in 2002 as an instrument for evaluating the effectiveness of teacher education programs in promoting teacher beliefs and attitudes that are consistent with the underlying philosophy of current reform efforts in mathematics education. Survey contained 30 items in total distributed into three parts. Part A was a 16 item questionnaire on 2-point likert scale which is a form of the *SBI* in order to determine how consistent an individual’s beliefs are with respect to the philosophy of the NCTM. Part B was a 12 item questionnaire on 4-point likert scale adapted from the *Problem-Solving Project* (Schoenfeld, 1989) to assess the change in teachers’ beliefs about teaching and learning mathematics within and outside the school setting. Part C was a two item questionnaire on 4-point likert scale to measure teachers’ perception of their effectiveness as a mathematics teacher and learner.

The questionnaire items in this study were first formed by selecting several categories from Instrument 1 (IMBS) and Instrument 2. From the first instrument, three categories were chosen to be appropriate for this study, which were “Understanding Important”, “Not Always Step by Step”, and “Time Consuming Problems Worthwhile”. Similarly, from the second instrument, three categories were selected, which were “Several”, “Step” and “Time”. Among the selected categories in these two instruments, two categories were so similar to each other; “Step” with “Not Always Step by Step”, and “Time” with “Time Consuming Problems Worthwhile”. Out of 30 items in the first instrument, 13 items were selected without making any changes, and similarly out of 41 items in the second instrument, 5 items were selected, forming 18 items in total. The other categories in these instruments were not included in the study, because they were not directly related to mathematical problem solving beliefs; instead most of them were reflecting beliefs about mathematics in general.

After the combination of these two instruments, additional literature review was carried out, and some items were added in the questionnaire reflecting beliefs about the kinds of mathematics teaching and learning taking place, and the usage of technologic equipments in a problem solving environment. First, some research was done on belief studies to analyze the kinds of teacher and student beliefs recorded previously. For example, Ford (1994) examined what beliefs teachers held about problem solving in mathematics and to what extend these beliefs were reflected in their students. In the light of her results, two negatively stated items were added in the questionnaire; such that “problem solving is primarily the application of computational skills in mathematics”, and “using calculators while solving problem is a kind of cheating” (p.319).

Several items were added in the questionnaire from Instrument 3 (SBI) and Instrument 4 (MBI). The main reason of using SBI and MBI was that both of these instruments were measuring the consistency of teachers’ beliefs about mathematics teaching and learning with the NCTM Standards. Also, the items in these instruments were similar and consistent with the previously added items such as the importance of understanding a problem, whether problems are solved by following a simple step by step procedure, and whether there can be several ways to solve a mathematics problem. Besides, some items were selected to measure beliefs about the kinds of mathematics instruction taking place while solving problems, for instance whether students should share their problem solving approaches with each other, and whether problem solving should be a separate part of the mathematics curriculum or permeate the entire program.

As a last step of the preparation of the questionnaire, some items were added in the questionnaire related to the necessity of technology usage in mathematics education, especially while solving problems. Most of these items were written from the technology principle of the NCTM published at the *Principles and Standards for School Mathematics* in 2000, as well as the statements in SBI and Ford's (1994) study. So, when all these studies were put together, 39 items were developed for the questionnaire related to several beliefs about problem solving in mathematics education, 22 of which were positively stated and 17 were negatively stated.

3.3.1.1.3. Addition of Mathematics Problems

To investigate pre-service teachers' mathematical problem solving beliefs intensively and determine whether their assessment of mathematics problems fits in with their problem solving beliefs illustrated in the questionnaire items, five mathematics problems were added in the data collection instruments. These problems were selected in a way that they were representing the kinds of student learning being advocated in the current mathematics reform efforts in the world, especially in Turkish education.

In the new Turkish curriculum, problem solving has been introduced as an integral part of mathematics education as well as a common skill in all kinds of subject areas. Problems were expected to be chosen in a way that they were part of daily life experiences, related to the mathematical content and activities covered in their lessons, interesting and challenging such that they can not be solved by mere application of mathematical knowledge or formula (M.E.B., 2005). Also, problems were expected to be relating different mathematical concepts with each other, as well as with other subject matters.

3.3.1.1.4. Translation of the Instrument

As a last step of the construction of the instrument, both questionnaire items and mathematics problems were translated into Turkish by the researcher. Afterwards, they were edited on clarity and grammar by an expert of Turkish language and literature. During this redaction process, it was agreed that some items needed retranslation and a few changes on word order, vocabulary, clause types, conjunctions, active-passive form, and punctuation. Next, the Turkish version of the instrument was given to five colleagues having mathematics background, some of which had graduate degrees. They were requested to evaluate the translated items and problems in terms of the content and clarity. Finally, in the light of these criticisms, the instrument was revised and necessary changes were made on the unclear instructions and mathematical vocabulary.

3.3.1.2. Development of the Instrument

After the construction of the instrument, several revisions and corrections were made in order to develop and finalize the instrument and to improve its reliability. For example, feedbacks were taken from experts, and two pilot studies were performed one by one, on 2nd and 3rd grade pre-service elementary mathematics teachers, until the clarity and reliability of the instrument was found to be satisfactory.

3.3.1.2.1. Expert Opinion

As soon as the translation process was completed, the first draft of the instrument was given to one academician and one research assistant working at Department of Elementary Mathematics Education, together with one research assistant working at Secondary Science and Mathematics Education at METU, so as to evaluate the instrument critically regarding the construct validity and clarity.

Their main criticism focused on some unclear items, especially double-barreled ones, and possible corrections were recommended. According to these feedbacks, revisions were made on the questionnaire items. Furthermore, some of the mathematics problems were found to be puzzle type, however as these kinds of problems were also given attention and value in the new curriculum, they were not extracted from the instrument.

3.3.1.2.2. Pilot Study 1

Pilot testing is important in a survey study to establish the construct validity of an instrument, which means whether the items measure the construct they were intended to measure, and to ensure that the instructions, questions, format, and scale items are clear (Creswell, 2003, p.158). In the present study, two pilot testing were put into practice. The samples of both pilot studies were chosen to be pre-service elementary mathematics teachers so as to be similar and representative to the potential respondents.

The first pilot study was carried out in the first week of March 2006. It was administered to 29 elementary mathematics education students studying in the second year at University A. Nearly 65% of them were females while 35% of them were male. The participation was voluntary, and the instrument was directly administered to the participants during one of their Instructional Planning and

Evaluation lesson with the permission of their instructor. The questionnaire consisted of 39 items, and five problems for interpretations. It took nearly half an hour of participants to fill out the entire instrument.

For the statistical analyses of the internal consistency, Cronbach alpha coefficient (α) was computed. Ideally, the Cronbach alpha coefficient of a scale should be "above 0.70" (Pallant, 2001, p.85). For the first pilot study, it was calculated as 0.78 indicating satisfactory reliability and internal consistency between items. However, the measured coefficient was not indicating very high reliability.

According to the outcomes of the first pilot study, several corrections were made on the instrument. For instance, one item was extracted from the questionnaire. The item was stated as "Problem solving should be a separate, distinct part of the mathematics curriculum". However, the second grade pre-service teachers interpreted this item in an inconsistent way. As this decreased the overall reliability, it was extracted from the instrument.

Several changes were made also on the part for mathematics problems. For instance, there was an explanation part that gave direction to the participant on which bases to evaluate the given problems, such as considering multiple ways, possible strategies, time, and technology usage while solving these problems, as well as daily life relations, mathematical value, and clarity of the problems. Although there were many criteria, the participants generally took only one of them, and interpreted all the problems on the same base. So, in order for this instruction to be clearer and less restrictive for interpretations, it was shortened and written in general such that participant were asked to evaluate the educational value of given problems for their appropriateness in elementary mathematics education in general. Next, some participants left the interpretation part empty as they found it difficult to express their ideas written.

In order to take a general evaluation to these problems, and get data from such participants, the participants were asked first to evaluate problems as either poor, average or strong according to the educational value, and then indicate their reason for this evaluation. Lastly, some of the participants recommended to add a chess board picture near the related problem so that anybody that have not seen a chess board would not have difficulty in solving the problem. Although it was given that a chess board has a shape of 8 x 8 tiles, the picture was also added near the problem.

3.3.1.2.3. Pilot Study 2

Although for the first pilot study, the Cronbach alpha coefficient and split-half coefficient were found to be indicating satisfactory reliability, in order to increase consistency and develop the instrument in a better way, second pilot study was conducted.

The second pilot study was carried out at the end of March with 23 elementary mathematics education students studying in the third year at University A. Nearly 70% of them were female while 30% of them were male. The instrument was administered to the participants during one of their Textbook Analysis lesson with the permission of their instructor. It took nearly 25 minutes of participants to fill out the entire instrument which consisted of 40 items, and five mathematics problems.

For the internal consistency estimates, Cronbach alpha coefficient and split-half coefficients were calculated. Before computing split-half internal consistency reliability measure, two underlying assumptions of split-half method were checked: (a) the halves must have almost equal standard deviations and (b) the halves must be alike in content. It was found that the two halves had similar standard deviations (SD for first half= 6.4 and SD for second half= 6.8) and since the all items in the questionnaire were orderly distributed, and measured the beliefs about mathematical problem solving, it was assumed that the two halves are identical with respect to content.

For the split-half coefficient, the Equal-length Spearmen-Brown was determined as both halves included equal number of items. The Equal-length Spearmen-Brown coefficient was calculated as 0.88, and the Cronbach alpha coefficient, α, was computed as 0.87. They were both indicating high internal consistency between items.

When the coefficients were examined for each item one by one, similar to the first pilot study, one item was extracted from the questionnaire due to the fact that it was decreasing the overall reliability. The item was as follows: 'Increased emphases should be given to the use of key words (clue words) to determine which operation to use in problem solving'. The third grade pre-service teachers interpreted this item in an inconsistent way. As this decreased the overall reliability, it was extracted from the instrument.

3.3.1.2.4. Internal Consistency Reliability Measures

Internal consistency refers "the degree to which the items that make up the scale hang together", that is whether they all "measure the same underlying construct" (Pallant, 2001, p.85). In research studies, one of the most commonly used internal consistency indicators is Cronbach's alpha coefficient, α. Values of this coefficient range from 0 to 1, with higher values indicating greater reliability (Green, Salkind & Akey, 2000).

In the present study, the overall reliability of the items in the instrument was calculated as 0.87 which indicates high consistency between instrument items. As the reliability of a scale indicates "how free it is from random error" (Pallant, 2000, p.6), a reliability coefficient of 0.87 means that 87% of the variance depends on true variance in the construct measured, and 13% depends on error variance.

3.4. Data Collection Procedure

The final draft of the instrument was administered to 244 senior elementary mathematics education students studying at University A, University B, University C, University D and University E, in their classroom settings. Prior to the implementation of the data collection instrument, the permission of the related instructors were taken via submitting the sample instrument and a summary of the purpose of the study.

The respondents were explained the purpose of the study before answering the questions. Pre-service teachers were informed that participation was voluntarily and it would not result negatively if they do not want to contribute to the study. In addition, it was declared that all their responses would be kept completely confidential and would only be used for the study. Each administration took approximately 25 minutes. Although the instrument was directly administered and collected from the participants only once in a time, the data collection procedure took about two months to reach a sufficient number of participants. The total response rate was seventy one percent (N = 244).

3.5. Data Analysis Procedure

In the present study, a number of descriptive and inferential statistics were conducted by using SPSS software program. First, the demographic information was analyzed by using frequencies, percentages, mean and standard deviations. Then, each questionnaire item was analyzed by using its frequency, percentage, mean and standard deviation. The responses to questionnaire items were assigned a numeric value from 1 to 5 with 1 the least favorable response and 5 with the most favorable. For the items whose wording indicated a negative belief, the scale was reversed. The scores for the 39 items were summed to give a total belief score for each participant, 195 indicating the most favorable beliefs whereas 39 represented the least.

Next, the participants' views about the given non-routine mathematics problems were analyzed by scanning through all response categories indicated as poor, average and strong for each problem, summarizing each response under common themes, and using frequencies of each theme for each response category. Afterwards, a two way ANOVA was performed on the mean belief scores to determine the significance of the differences that could exist among participants due to the differences in gender, universities attended, and the interaction of gender and university attended. Finally, post hoc test was performed to see which university differed within the whole group.

CHAPTER 4

RESULTS

This chapter presents the results obtained from the data analysis. In the first part, results regarding the demographic information of the participants were presented. Afterwards, results of descriptive and inferential statistics were reported based on the research questions. For research question 1, both the participants' responses to the questionnaire items and their interpretations about several non-routine mathematics problems were analyzed. In order to explore the participants' beliefs about mathematical problem solving, research question 1 was partitioned into seven sub-questions, then several descriptive statistics such as frequencies, percentages, mean and standard deviations were reported. For research question 2, in order to explore the effect of gender and university on the participants' beliefs about mathematical problem solving, results of ANOVA and post hoc test were reported.

4.1. Findings Regarding the Demographic Information

The demographic information of the participants was gathered from demographic information sheets so that based on this information, an insight about the data would be provided. The data presented are (a) gender of the participants, (b) university attended, (c) their grade point averages, (d) whether they have taken any courses related to problem solving, (e) whether they have been interested in problem solving other than taking courses, (f) whether they have completed their courses related to pedagogy such as School Experiences, and Methods of Mathematics Teaching, and (g) whether they have completed their courses related to mathematics. The percentages and frequencies associated with each variable were summarized in the following tables from Table 4.1 to Table 4.5 respectively.

There were 131 females (53.7%) and 113 males (46.3%) in the sample of the study, giving a total of 244 participants. Male participants and female participants had relatively equal group sizes (Table 4.1).

Table 4.1 Participants' Demographic Data

Variable	Frequency (N)	Percentage (%)	Mean for GPA Score (M)	Std. Deviation for GPA Score (SD)
Gender				
Male	113	46.3	2.60	0.38
Female	131	53.7	2.96	0.48
University Attended				
University A	31	12.7	2.91	0.42
University B	49	20.1	2.65	0.45
University C	70	28.7	2.88	0.46
University D	38	15.6	2.75	0.38
University E	56	23.0	2.71	0.31
Total	244	100	2.78	0.43

Approximately 30 percent of the participants were attending to University C, and the remaining was attending to University E (23%), University B (20%), University D (15%), and University A (12%) (Table 4.1)

The mean and standard deviations of the GPA scores were similar for each university, ranging from 2.91 to 2.65, with a mean of 2.78 and a standard deviation of 0.43 (Table 4.1).

In question 5 (Appendix A), the participants were asked to state whether they have taken any courses about mathematical problem solving. As represented in Table 4.2, approximately 60% of the participants (N=145) did not take any course related to problem solving, whereas 40% of the participants have taken courses related to problem solving.

Table 4.2 Whether Participants Took Courses Related to Problem Solving

	Frequency	Percentage
Taken	99	40.6
Not Taken	145	59.4
Total	244	100.0

In addition, as a follow-up question, the participants that took courses related to problem solving were asked to write down which courses they took. 69 participants reported 'Methods of Mathematics Teaching', 20 participants reported 'Problem Solving', and 10 participants in total reported some other courses such as 'Textbook Analysis in Mathematics Education', 'Instructional Technology & Material Development', 'Active Learning' and 'Differential Equation' as the courses they took about problem solving.

In question 6 (Appendix A), the participants were asked to state whether they were interested in mathematical problem solving other than taking courses. Approximately 40% of the participants (N= 99) stated that they were interested in problem solving, whereas 60% indicated that they were not interested in problem solving other than taking courses (Table 4.3).

Table 4.3 Interested in Mathematical Problem Solving

	Frequency	Percentage
Interested	99	40.6
Not Interested	145	59.4
Total	244	100.0

In addition, as a follow-up question, the participants were asked to write down in which ways they were interested in problem solving. Their responses included statements such as solving mathematical problems in textbooks and trying different strategies while solving mathematics problems (N= 26), making researches about problem solving in the internet (N= 11), reading books (N= 10), solving mathematical puzzles (N= 8), while giving private lessons (N= 7) and while getting prepared for exams such as KPSS (Kamu Personel Seçme Sınavı) and LES (Lisansüstü Eğitim Sınavı) (N= 6).

The participants were asked about whether they took several pedagogy courses such as School Experiences, and Methods of Mathematics Teaching which might have influenced their beliefs about problem solving. Almost all of the participants took these courses, except for the course 'Practice Teaching in Elementary Education' which is usually taken during the last semester of the four year undergraduate education. So, it was acceptable that half of the participants were still taking this course (Table 4.4).

Table 4.4 Courses Taken Related to Pedagogy

	Frequency			Percentage		
	Taken	Still Taking	Not Taken	Taken	Still Taking	Not Taken
School Experience I	244	0	0	100.0	0	0
School Experience II	237	0	7	83.6	13.5	2.9
Practice Teac. in Ele. Edu.	115	120	7	47.1	49.2	2.9
Meth. of Math. Teaching	232	0	12	95.1	0.4	4.5

Lastly, when the participants were asked about whether they completed their courses about mathematics, it was found that more than 75% of the participants (N=184) completed all of their mathematics courses (Table 4.5).

Table 4.5 Whether Participants Completed Their Courses Related to Mathematics

	Frequency	Percentage
Completed	184	75.4
Not Completed	60	24.6
Total	244	100.0

4.2. Results of the Study Regarding the Research Questions

4.2.1. Research Question 1

What are the pre-service elementary mathematics teachers' beliefs about mathematical problem solving?

The first research question aimed to investigate the kinds of beliefs the participants had about mathematical problem solving. In order to explore this question, both the participants' responses to the questionnaire items and their interpretations about several non-routine mathematics problems were analyzed.

The questionnaire items were grouped into six categories as follows; beliefs about (1) the importance of understanding why a solution to a mathematics problem works (Understanding), (2) mathematics problems that cannot be solved by following a predetermined sequence of steps (Step by Step Solutions), (3) time consuming mathematics problems (Time), (4) mathematics problems that have more than one way of solution (Multiple Solutions), (5) the kind of mathematics instruction emphasized by the principles of new curriculum (Instruction), and (6) the usage of technologic equipments while solving mathematics problems (Technology). Therefore, the responses given to the questionnaire items were analyzed under six categories by forming six sub-research questions for each category.

In addition to these questions, one more sub-research question was addressed for the participants' interpretations about several non-routine, daily life mathematics problems. Therefore, the first research question was partitioned into seven sub-research questions in total.

4.2.1.1. Beliefs about the Importance of Understanding

In the present study, it was aimed to examine the participants' responses to several questionnaire items related to the importance of understanding why a solution to a mathematics problem works. There were two negatively stated items (Items 1 and 12) and four positively stated items (Items 6, 18, 24, and 29) related to this category. In Appendix C, descriptive statistics of these questionnaire items were reported.

While analyzing the questionnaire results, negatively stated items were reversed in scoring. Therefore, for negatively stated items, a higher mean indicates participants disagree with the statements, and a lower mean indicates participants agree with the statements. On the other hand, for positively stated items, a higher mean indicates participants agree with the statements, and a lower mean indicates participants disagree with the statements. Moreover, minimum possible mean score is 1, whereas maximum possible mean score is 5.

Approximately three fourth of the participants (with the mean of 3.96) indicated their disagreement (overall responses of strongly disagree and disagree) to the idea that it is not important to understand why a mathematical procedure works as long as it gives a correct answer (Item 1). Likewise, 93% of the participants (with the mean of 4.45) reported that if a person does not understand why an answer to a mathematics problem is correct, then he has not really solved the problem (Item 6).

More than three fourth of the participants (with the mean of 3.93) stated their disagreement to the idea that it does not really matter if you understand a mathematics problem as long as you get the right answer (Item 12). Similarly, 96% of the participants (with the mean of 4.44) thought that in addition to getting a right answer in mathematics, it is also important to understand why the answer is correct (Item 29).

Almost 90% of the participants (with the mean of 4.30 and 4.14 respectively) appreciated a demonstration of good reasoning rather than merely finding a correct answer (Item 24), and supported the idea of spending time for investigating why a solution to a mathematics problem works (Item 18). Moreover, none of the participants reported strong disagreement for these two items.

4.2.1.2. Beliefs about Following Predetermined Sequence of Steps

In the present study, it was aimed to explore the participants' responses to several questionnaire items related to mathematics problems that cannot be solved by following a predetermined sequence of steps. There were four negatively stated items (Items 2, 13, 25, and 34) and four positively stated items (Items 7, 19, 37, and 30) related to this category. In Appendix C, descriptive statistics of these questionnaire items were reported.

Approximately half of the participants (with the mean of 3.17 and 3.33 respectively) indicated their disagreement to the idea that mathematics problems are solved by following a step-by-step procedure (Item 2 and 34). Similarly, three fourth of the participants (with the mean of 3.75) stated that some problems can not be solved by just following a predetermined sequence of steps (Item 37). However, although it is a positively stated item, 60% of the pre-service teachers (with the mean of 2.52) expressed negative belief to item 7. These pre-service teachers were against the idea that mathematicians rarely have step-by-step procedures to solve mathematical problems.

For items 13 and 30, pre-service teachers had no strong belief such that their responses were distributed among agreement, disagreement or neutral. For instance, for item 30 that expressed the uselessness of memorizing steps while learning to solve problems, 34% of the participants reported their agreement, 36% of the participants reported their disagreement, and 30% of the participants reported indecision. Similarly, for item 13 that was the opposite of item 30, 37% of the pre-service teachers reported their agreement, 43% of the pre-service teachers reported their disagreement, and 20% of the pre-service teachers reported indecision. On the other hand, for item 19, almost 70% of the participants (with the mean of 3.76) showed agreement, and indicated that it is possible to solve problems without remembering formulas. Lastly, half of the participants (with the mean of 3.20) disagreed that pre-service teachers should be taught the correct procedure to solve mathematics problems (Item 25).

4.2.1.3. Beliefs about Time Consuming Mathematics Problems

In the present study, it was aimed to investigate the participants' responses to several questionnaire items related to time consuming mathematics problems. There were two negatively stated items (Items 8 and 20) and two positively stated items (Items 3 and 14) related to this category. In Appendix C, descriptive statistics of these questionnaire items were reported.

The majority of the participants (85%) indicated their disagreement (with the mean of 4.13) to the idea that if a solution to a mathematics problem takes a long time, it can not be completed (Item 8).

For item 3, which stated that time consuming problems are not bothering, pre-service teachers (with the mean of 3.10) had no strong belief such that their responses were distributed among agreement, disagreement or neutral.

Furthermore, almost 90% of the participants (with the mean of 4.21) supported the idea that hard mathematics problems can be solved if one just struggle for that (Item 14). Lastly, more than three quarter of the participants (with the mean of 3.99) were either neutral or in disagreement with the suggested relation between being good in math and solving mathematics problems quickly (Item 20).

4.2.1.4. Beliefs about Mathematics Problems Having Several Ways of Solution

It was also aimed to examine the participants' responses to several questionnaire items related to mathematics problems that have more than one way of solution. There were four negatively stated items (Items 9, 21, 31, and 38) and four positively stated items (Items 4, 15, 26, and 35) related to this category. In Appendix C, descriptive statistics of these questionnaire items were reported.

More than 90% of the pre-service teachers (with a mean of 4.54 and 4.30 respectively) disagreed with the idea that there is only one correct way to solve a mathematics problem (Item 9), and if a number of mathematicians were given a mathematical problem, they would all solve it in the same way (Item 21). In addition, 70% of the pre-service teachers (with the mean of 3.71) indicated that if a student is unable to solve a problem one way, there are usually other ways to get the correct answer (Item 26).

Approximately 93% of the pre-service teachers (with a mean of 4.42) stated that it is possible to get the correct answer to a mathematics problem using methods other than the teacher or the textbook uses (Item 4). Besides, 85% of the pre-service teachers (with the mean of 4.11) determined that if a student forgets how to solve a mathematics problem the way the teacher did, it is possible to develop different methods that will give the correct answer (Item15).

Almost all of the participants (with a mean of 4.57) determined good mathematics teachers to be the one showing students lots of ways for solving the same question (Item 35), and nearly three fourth of the participants (N=171, with the mean of 3.92) did not believe that hearing different ways to solve the same problem can confuse students' mind (Item 38). Lastly, for item 31 which stated good mathematics teachers to be the ones showing students the exact way to answer the math questions they will be tested on, participants had no strong belief such that their responses were distributed among agreement, disagreement or neutral. For instance, 37% of the participants reported their agreement, whereas 46% of the participants reported their disagreement, and 17% of the participants reported indecision.

4.2.1.5. Beliefs about the Kind of Mathematics Instruction

In this research, it was aimed to find out the participants' responses to several questionnaire items related to the kind of mathematics instruction emphasized by the principles of new curriculum. There were two negatively stated items (Items 16 and 27) and three positively stated items (Items 10, 22, and 32) related to this category. In Appendix C, descriptive statistics of these questionnaire items were reported.

The participants did not show very strong beliefs about the value of problem solving in the new mathematics curriculum. For instance, only 67% of the participants (with the mean of 3.79) stated their agreement with item 10 which proposed that problem solving is a process that should permeate the entire curriculum. Besides, although it was a negatively stated item, 80% of the participants (N=195) agreed with the idea (with a mean of 2.11) that problem solving is primarily the application of computational skills in mathematics education (Item 16).

However, the participants reflected very positive beliefs about the importance of problem solving in classroom environment. For example, the majority of them (94%) indicated (with a mean of 4.32) that students should share their problem solving thinking and approaches with other students (Item22). Furthermore, more than three quarters of the participants (with a mean of 4.09) were against the idea that it is better to tell or show students how to solve problems than to let them discover how on their own (Item 27). Lastly, 95% of the participants (N=231) proposed (with a mean of 4.46) that teachers should encourage students to write their own mathematical problems (Item 32).

4.2.1.6. Beliefs about Usage of Technologic Equipments

This research also aimed to identify the participants' responses to several questionnaire items related to the usage of technologic equipments while solving mathematics problems. There were three negatively stated items (Items 11, 28, and 36) and five positively stated items (Items 5, 17, 23, 33, and 39) related to this category. In Appendix C, descriptive statistics of these questionnaire items were reported.

The majority of participants stated that (with a mean of 4.41) teachers can create new learning environments for their students with the usage of technology (Item 23), and (with a mean of 4.15) it can give students greater choice in their tasks (Item 33). Moreover, 70% of the participants (with the mean of 3.90) indicated that students can learn more mathematics more deeply with the appropriate and responsible use of technology (Item 39); however, slightly more than 20% of the participants were neutral in this idea.

Over three fourth of the participants (with a mean of 4.07 and 4.06 respectively) disagreed with the idea that using technology is a waste of time while solving problems (Item 28), and it harms students' ability to learn mathematics (Item 36). In addition, three fourth of the participants (with the mean of 3.92) did not consider the usage of technology as a kind of cheating (Item 11).

Responses to items 17 and 5 clearly indicated that the majority of pre-service teachers supported the appropriate usage of technologic equipments while solving problems (with a mean of 4.05), and supported the availability of such materials to all students at all times (with a mean of 4.45).

4.2.1.7. Summary of Results related to Questionnaire Items

In general, it was found that the pre-service elementary mathematics teachers' responses to the questionnaire items were in line with the principles emphasized by the new mathematics curriculum in Turkey.

The pre-service teachers usually reflected positive beliefs about mathematical problem solving. Especially they indicated strong positive beliefs about the importance of understanding why a solution to a mathematics problem works, and the usefulness of using technologic equipments in mathematics education.

However, for several questionnaire items, especially the ones related to following a pre-determined sequence of steps while solving mathematical problems, they did not demonstrate strong beliefs. For these items, their responses were distributed among agreement, disagreement and neutral scales.

In addition, the pre-service teachers reflected negative beliefs for two questionnaire items. They believed that mathematicians often have step-by-step procedures to solve mathematical problems (Item 7), and problem solving is primarily the application of computational skills in mathematics (Item 16).

4.2.1.8. Beliefs about Non-routine Mathematic Problems

The participants were given five non-routine mathematics problems. These problems were different from many ordinary examples in textbooks or other materials in a way that they were non-routine problems, related to daily life, requiring mathematical reasoning and critical thinking,.

The participants were asked both to evaluate the value of these problems in elementary mathematics education as being poor, average or strong, and to explain the reason for their evaluations. The evaluations are analyzed by giving a summary of descriptive statistics such as frequencies and percentages. Besides, the explanations are analyzed by scanning through all responses given to each category (Poor, Average, and Strong), and looking for common themes, and then summarizing them under each related category for each problem.

Table 4.6 illustrates the frequencies and percentages associated with the participants' evaluations given to each problem. As illustrated by the data, approximately half of the participants evaluated Problem 1 as average and Problem 5 as poor, whereas they evaluated Problem 2, Problem 3 and Problem 4 as strong. When analyzed as a whole, approximately 40% of the participants evaluated these problems as strong, whereas 36% evaluated as average, and 23% evaluated as poor.

Table 4.6 Pre-service Teachers' Evaluations of Problems

	Poor		Average		Strong	
	f	%	f	%	f	%
Problem 1	21	8.6	118	48.4	105	43.0
Problem 2	76	31.1	53	21.7	115	47.1
Problem 3	38	15.6	86	35.2	120	49.2
Problem 4	43	17.6	91	37.3	110	45.1
Problem 5	108	44.3	93	38.1	43	17.6
Total	286	23.4	441	36.2	493	40.4

Below, there is the summary of participants' explanations given to each evaluation category (Poor, Average, and Strong) for each problem.

Problem 1: Serkan was studying the Romans in history and came across an ancient document about a great army that advanced upon Alexandria. He was unable to read the size of the army as two digits were smudged, but he knew it was "45_ _ 8" and that the attacking army was divided into 9 equal battalions, to cover the 9 different entrances to Alexandria.

What are the possible sizes for the attacking army?

Poor

The first problem is evaluated by 21 participants (8.6%) as poor. The participants' responses are summarized in Table 4.7.

Table 4.7 Comments related to the First Problem Stated as Poor

	Frequency
It is only about one subject area which is divisibility, its solution depends on only four operations	11
It is an easy question, it does not lead students to make interpretations	6
Students may not pay attention to this problem as it is written in a story type	4
It is a difficult problem	2

* The statements are not exact quotes of participants. They are composed expressions.

* The frequencies represent how often these composed expressions are mentioned by the participants

Average

The first problem is evaluated by 118 participants (48.4%) as average. The participants' responses are summarized in Table 4.8.

Table 4.8 Comments related to the First Problem Stated as Average

	Frequency
It has moderate difficulty level; it does not require high creativity and critical thinking	26
It is different from ordinary mathematics questions. It has an interesting and enjoyable context that may attract students' attention	19
The problem stem is too long	12
It is only about one subject area which is divisibility	9
A student that knows the related topic and four operations can solve this problem easily	7
It covers the desired content	7
It can not be solved by mere knowledge or memorization; it requires creativity	7
It connects mathematics with other subject areas	6
It has multiple solutions	5
It measures both verbal and mathematical skills	3

* The statements are not exact quotes of participants. They are composed expressions.

* The frequencies represent how often these composed expressions are mentioned by the participants

Strong

The first problem is evaluated by 105 participants (43%) as strong. The participants' responses are summarized in Table 4.9.

Table 4.9 Comments related to the First Problem Stated as Strong

	Frequency
It develops students' long term skills such as creativity, intelligence, mathematical reasoning and problem solving.	42
It is different from ordinary mathematics questions. It has an interesting and enjoyable context that may attract students' attention	30
It covers the desired content	26
It is related to daily life	9
It can not be solved by mere knowledge or memorization; it requires creativity	8
It develops both verbal and mathematical skills	7
It may provide permanent learning as it is linked with an interesting example in history.	7
It has multiple solutions	7
This kind of problems are suggested by the new mathematics curriculum	2

* The statements are not exact quotes of participants. They are composed expressions.

* The frequencies represent how often these composed expressions are mentioned by the participants

Problem 2: How many rectangles are there on an 8 x 8 chess board?

Poor

The second problem is evaluated by 76 participants (31.1%) as poor. The participants' responses are summarized in Table 4.10.

Table 4.10 Comments related to the Second Problem Stated as Poor

	Frequency
It is not appropriate for elementary level	22
It will take a long time to solve this problem	12
It is enough to know the answer of 8 x 8	12
It does not cover any objective in mathematics curriculum	6
It is only related to counting	5
It does not lead students to make interpretations, it can be solved by applying only one formula	3
First students should be given an easier example, then given this problem	3

* The statements are not exact quotes of participants. They are composed expressions.

* The frequencies represent how often these composed expressions are mentioned by the participants

Average

The second problem is evaluated by 53 participants (21.7%) as average. The participants' responses are summarized in Table 4.11.

Table 4.11 Comments related to the Second Problem Stated as Average

	Frequency
The problem is interesting as it relates mathematics with a daily life example. Chess can attract students' attention	6
It develops students' long term skills such as creativity, intelligence, mathematical reasoning and problem solving.	5
It will direct students to think in multiple ways	5
It will take a long time to solve this problem	5
It is a difficult problem	5
It is an ordinary permutation question	4
It does not cover any objective in mathematics curriculum	3
Students can develop their own formula and discover mathematics by these kinds of problems	3
It requires students to think about the relation between rectangle and square	3

* The statements are not exact quotes of participants. They are composed expressions.

* The frequencies represent how often these composed expressions are mentioned by the participants

Strong

The second problem is evaluated by 115 participants (47.1%) as strong. The participants' responses are summarized in Table 4.12.

Table 4.12 Comments related to the Second Problem Stated as Strong

	Frequency
It is a challenging problem. It requires high level of mathematical thinking	30
It relates mathematics with different mathematical concepts	27
It will direct students to think in multiple ways	23
Students can develop their own formula and discover mathematics by these kinds of problems	20
It leads students to think critically and make brain storming	15
It develops students' long term skills such as creativity, intelligence, mathematical reasoning and problem solving.	14
This kind of problems are suggested by the new mathematics curriculum	10
It will take a long time to solve this problem	6
The problem is interesting as it relates mathematics with a daily life example. Chess can attract students' attention	5

* The statements are not exact quotes of participants. They are composed expressions.

* The frequencies represent how often these composed expressions are mentioned by the participants

Problem 3: A man wants to take her fox, chicken and a bag of corn across the river in a canoe. The canoe can hold only one thing in addition to the man. If left alone, the fox would eat the chicken, or the chicken would eat the corn.

How can the man take everything across the river safely?

Poor

The third problem is evaluated by 38 participants (15.6%) as poor. The participants' responses are summarized in Table 4.13.

Table 4.13 Comments related to the Third Problem Stated as Poor

	Frequency
It does not involve any number; it is not related to mathematics education	21
It does not cover any topic in the curriculum	19
It is an ordinary problem	3
It will take a long time to solve this problem	2

* The statements are not exact quotes of participants. They are composed expressions.

* The frequencies represent how often these composed expressions are mentioned by the participants

Average

The third problem is evaluated by 86 participants (35.2%) as average. The participants' responses are summarized in Table 4.14.

Table 4.14 Comments related to the Third Problem Stated as Average

	Frequency
It is not related to mathematics education	14
It does not involve any number; however it will be very helpful for developing students' reasoning abilities	13
It will teach students to analyze multiple relations occurring at the same time, and think in multiple ways	8
It has moderate difficulty level; it does not require high creativity and critical thinking	7
It can develop students' problem solving skills	3
It is related to daily life	3
It is a nice problem for an average student; however it may not be a problem for a high achiever student	2

* The statements are not exact quotes of participants. They are composed expressions.

* The frequencies represent how often these composed expressions are mentioned by the participants

Strong

The third problem is evaluated by 120 participants (49.2%) as strong. The participants' responses are summarized in Table 4.15.

Table 4.15 Comments related to the Third Problem Stated as Strong

	Frequency
It requires thinking well, and develops students' long term skills such as creativity, intelligence, mathematical reasoning and problem solving.	78
It will teach students to analyze multiple relations occurring at the same time, and think in multiple ways	21
It is an interesting and enjoyable problem	19
It does not involve numbers and it is a nice problem to show that mathematics does not only mean struggling with numbers	14
It evaluates high level cognitive skills	9
It is related to daily life	4

* The statements are not exact quotes of participants. They are composed expressions.

* The frequencies represent how often these composed expressions are mentioned by the participants

Problem 4: Five women participated in a 10 km walk, but started at different times. At a certain time in the walk the following descriptions were true.

- Melek was at the halfway point.
- Filiz was 2 km ahead of Canan.
- Nuray was 3 km ahead of Sibel.
- Melek was 1 km behind Canan.
- Sibel was 3.5 km behind Filiz.

How far from the finish line was Nuray at that time?

Poor

The fourth problem is evaluated by 43 participants (17.6%) as poor. The participants' responses are summarized in Table 4.16.

Table 4.16 Comments related to the Fourth Problem Stated as Poor

	Frequency
It does not cover any topic in the curriculum	16
It is not appropriate for elementary level; it can confuse students' minds	9
It is an easy question, it does not lead students to make interpretations	8
It is only related to making calculations	3

* The statements are not exact quotes of participants. They are composed expressions.

* The frequencies represent how often these composed expressions are mentioned by the participants

Average

The fourth problem is evaluated by 91 participants (37.3%) as average. The participants' responses are summarized in Table 4.17.

Table 4.17 Comments related to the Fourth Problem Stated as Average

	Frequency
It is important to analyze the relationships between the given variables	15
Students can use drawings to visualize this problem	14
The problem stem is too long and complicated	6
It is an easy question, it does not lead students to make interpretations	6
It can not be solved by mere knowledge or memorization; it requires creativity and critical thinking	4
It is a non-routine problem	3

* The statements are not exact quotes of participants. They are composed expressions.

* The frequencies represent how often these composed expressions are mentioned by the participants

Strong

The fourth problem is evaluated by 110 participants (45.1%) as strong. The participants' responses are summarized in Table 4.18.

Table 4.18 Comments related to the Fourth Problem Stated as Strong

	Frequency
It is important to analyze the relationships between the given variables	27
It develops students' long term skills such as creativity, intelligence, mathematical reasoning and problem solving	23
It is very enjoyable and thought-provoking	15
It requires students to think in a multiple way	11
Students can use drawings to visualize this problem	9
It measures both verbal and mathematical skills	7
Students can create different strategies for this problem	6
It is related to daily life	6
It is a non-routine problem	5
Students can solve it step by step	5
It will take a long time to solve this problem	2

* The statements are not exact quotes of participants. They are composed expressions.

* The frequencies represent how often these composed expressions are mentioned by the participants

Problem 5: In 2000, Ankara had a population of 4,007,860 and covers an area of 25,978 square kilometers. Yalova had a population of 168,593 with an area of 847 square kilometers.

Which city was more densely populated?

Poor

The last problem is evaluated by 108 participants (44.3%) as poor. The participants' responses are summarized in Table 4.19.

Table 4.19 Comments related to the Fifth Problem Stated as Poor

	Frequency
It is an easy question, it does not lead students to make interpretations	46
It is only about one subject area which is ratio and proportion, its solution depends on only four operations	39
It will take a long time to reach a solution as the numbers are very big and complicated	6
It uses students as a calculator	3
It can be used in physic lessons while teaching density	3
The definition of population density should not be given, students should find it	2
The answer of this problem is not a whole number	2

* The statements are not exact quotes of participants. They are composed expressions.

* The frequencies represent how often these composed expressions are mentioned by the participants

Average

The last problem is evaluated by 93 participants (38.1%) as average. The participants' responses are summarized in Table 4.20.

Table 4.20 Comments related to the Fifth Problem Stated as Average

	Frequency
It is appropriate for elementary level	15
It is an easy question; it does not lead students to make interpretations. It can be solved with only a formula	13
It is only about one subject area which is ratio and proportion, its solution depends on only four operations	12
The numbers are very big and complicated. Either smaller numbers should be given or rounding should be allowed	6
It is important to analyze the relationships between the given variables	6
It connects mathematics with other subject areas	5
It is a routine problem	4
It measures both computation and reasoning skills	3
It is related to daily life	2

* The statements are not exact quotes of participants. They are composed expressions.

* The frequencies represent how often these composed expressions are mentioned by the participants

Strong

The last problem is evaluated by 43 participants (17.6%) as strong. The participants' responses are summarized in Table 4.21.

Table 4.21 Comments related to the Fifth Problem Stated as Strong

	Frequency
It connects mathematics with real life and other subject areas	19
It is important to analyze the relationships between the given variables	9
It will develop students' computation skills	6
It leads students to make interpretations	5
It is an interesting and enjoyable problem	5
It is appropriate for elementary level	3
Students can use estimation and rounding	2

* The statements are not exact quotes of participants. They are composed expressions.

* The frequencies represent how often these composed expressions are mentioned by the participants

4.2.1.9. Summary of Results related to Comments about Mathematics Problems

When the reasons of the pre-service teachers' evaluations of these problems as being poor were examined, it was found that they generally indicated the following beliefs such as; they are ordinary problems, they are not appropriate for elementary level, they cover only one subject area, they do not cover any objective in mathematics curriculum, they do not lead students to make interpretations, they do not involve numbers, they are asked in a story type, they do not involve operating with whole numbers, and it will take a long time to solve these problems.

On the other hand, when the reasons of the pre-service teachers' evaluations of these problems as being strong were examined, it was found that they generally indicated the following beliefs such as; they are different from ordinary mathematics questions as they have interesting and enjoyable context that may attract students' attention, they offer challenge, they require high level of mathematical thinking, they lead students to make interpretations, they cover the desired content, they connect mathematics with real life and with other subject areas, they develop students' long term skills such as creativity, intelligence, mathematical reasoning and problem solving, they develop both verbal and mathematical skills, they direct students to analyze the relationships between the given variables and think in multiple ways, and with these kinds of problems students can create different strategies, and develop their own formula, therefore discover mathematics.

Lastly, when the reasons of the pre-service teachers' evaluations of these problems as being average were examined, it was found that in general the pre-service teachers indicated similar beliefs that were presented both in poor and strong categories. In other words, the beliefs introduced in the average category were a combination of the beliefs presented in the other two categories.

4.2.1.10. Additional Interpretations

After the participants evaluated these mathematics problems as being poor, average or strong, and explained their reasons for these evaluations, they were asked to indicate their additional interpretation for these problems as in general.

Among 244 participants, only 30 participants (12.3%) indicated their additional interpretations. In general, the pre-service teachers' additional interpretations were about the importance of problem solving in mathematics education, and in daily life.

Besides, two pre-service teachers from two different universities reported their complaint about the insufficiency of their undergraduate courses. They stated that they completed all their must courses related to mathematics education; however, in order to teach more effectively, they should have taken more courses, especially related to solving and posing these kinds of problems.

4.2.2. Research Question 2

What is the effect of gender and university attended on the pre-service elementary mathematics teachers' beliefs about mathematical problem solving?

There were two independent variables (gender and university attended) and one dependent variable (belief). Gender and university attended were two categorical independent variables, whereas belief of the pre-service mathematics teachers was a continuous dependent variable.

Two-way between groups analysis of variance (ANOVA) was used to uncover the main and interaction effects of gender and university attended on the pre-service elementary mathematics teachers' beliefs about mathematical problem solving. That is, two-way ANOVA was used to evaluate the mean differences that may be produced by either of these two factors independently and by these two factors acting together on the pre-service teachers' problem solving beliefs.

4.2.2.1. Assumptions of ANOVA

Prior to using ANOVA for hypothesis testing, ANOVA assumptions were checked for violation. There were three assumptions to be satisfied;

1. "The observations within each sample must be independent" (Independent Observations),

2. "The populations from which the samples are selected must be normal" (Normality), and
3. "The populations from which the samples are selected must have equal variances" (Homogeneity of Variance) (Gravetter & Wallnau, 2003, p.484).

1. Independent Observations

Independent observations are a basic requirement for nearly all hypotheses testing procedure. As Green, Salkind & Akey (2000), Pallant (2001), and Gravetter & Wallnau (2003), indicate, the score obtained for one individual should not be influenced by the score obtained from any other individual.

In this study, the data were collected from five universities in their classroom settings. Each class was independent from the other. In addition, the instrument was directly administered and collected only once in a time. So, the responses of each participant were assumed to be not influenced from the responses of any other.

2. Normality

Similar to the assumption of independent observations, normality is also a basic requirement for many of the statistical techniques. Normal is described as "a symmetrical, bell shaped curve, which has the greatest frequency in the middle and relatively smaller frequencies toward either extreme" (Gravetter & Wallnau, 2003, p.49). In order to assess normality, a number of statistics are suggested such as skewness and kurtosis, histograms and test of normality (Pallant, 2001).

Skewness shows "the degree to which a variable's score fall at one or the other ends of the variable's scale" (Green, Salkind & Akey, 2000, p.122), and the skewness value provides an indication of "the symmetry of the distribution" (Pallant, 2001, p.53). Besides, kurtosis shows "the relative frequency of scores in both extremes of a distribution" (Green, Salkind & Akey, 2000, p.122), and it provides information about "the peakedness of the distribution" (Pallant, 2001, p.53). Pallant (2001) points out that if the distribution is perfectly normal, the skewness and kurtosis values would be 0.

As shown in Table 4.22, the mean belief scores of participants at University A, University B, University D, and University E had negative skewness value, whereas the mean belief scores of participants at University C had positive skewness value. It means that many participants at University C received low mean scores, whereas many participants at the other universities received high mean scores.

The mean belief scores of participants at University A and University B had kurtosis values below 0, indicating a distribution that is relatively flat; whereas the mean belief scores of participants at University C, University D, and University E had kurtosis values above 0, indicating a distribution clustered in the centre.

Table 4.22 Skewness and Kurtosis Values of Mean Belief Scores for Universities

	Skewness		Kurtosis	
	Statistic	Std. Error	Statistic	Std. Error
University A	-0.319	0.421	-0.807	0.821
University B	-0.079	0.340	-0.330	0.668
University C	0.975	0.287	0.289	0.566
University D	-0.866	0.383	0.312	0.750
University E	-0.055	0.319	0.618	0.628

As demonstrated in Table 4.23, the male participants' mean belief scores had positive skewness value, whereas the female participants' mean belief scores had negative skewness value. It means that many male participants received low mean scores, whereas many female participants received high mean scores. In addition, both male and female participants had kurtosis values below 0, indicating a distribution that is relatively flat.

Table 4.23 Skewness and Kurtosis Values of Mean Belief Scores for Gender

	Skewness		Kurtosis	
	Statistic	Std. Error	Statistic	Std. Error
Male	0.150	0.227	-0.538	0.451
Female	-0.139	0.212	-0.466	0.420

Therefore, the skewness and kurtosis values of the participants' mean beliefs both with respect to universities attended and gender did not indicate perfect normal distributions. However, Pallant (2001) also states that skewness and kurtosis will not "make a substantive difference in the analysis with large samples"; therefore, recommends inspecting the shape of the distribution by using histograms (p.54).

At Appendix D, the histograms and Normal Q-Q plots of the participants' mean belief scores both with respect to universities attended and gender were demonstrated. When the universities attended were concerned, for University A, University B, and University E, the mean belief scores appeared to be reasonably normally distributed. This was also supported by an inspection of the Normal Q-Q plots as they formed reasonably straight lines. For University C and University D, the mean belief scores appeared to be quite far from normal distribution. When the gender was concerned, both male and female participants' mean belief scores appeared to be reasonably normally distributed, and their Normal Q-Q plots formed reasonably straight lines.

To asses the normality of the distribution of scores, Tests of Normality, which gives the results of the Kolmogorov-Smirnov statistic, was also recommended by Pallant (2001). It was stated that a non-significant result, that is significance value of more than 0.05, indicates normality.

Participants' mean of belief scores with respect to gender and universities attended are shown in Table 4.24.

Table 4.24 Test of Normality

	Kolmogorov-Smirnov		
	Statistic	*df*	Sig.
University A			
Male	0.204	15	0.093
Female	0.118	16	0.200*
University B			
Male	0.088	27	0.200*
Female	0.109	22	0.200*
University C			
Male	0.186	29	0.011
Female	0.126	41	0.098
University D			
Male	0.146	15	0.200*
Female	0.222	23	0.005
University E			
Male	0.115	27	0.200*
Female	0.118	29	0.200*

* This is a lower bound of the true significance

As a summary, only the mean of belief scores for the male participants at University C and female participants at University D did not show normal distribution. The results of other groups suggested no violation of the assumption of normality. However, it is accepted that with large enough sample sizes, such as with 30 or more participants, the violation of normality assumption does not cause any major problem (Pallant, 2001, p.172; Gravetter & Wallnau, 2003, p.303). Therefore, as the sample size of male participants at University C was 30, it may not cause any problem.

3. Homogeneity of Variance

Homogeneity of variance means the populations from which the samples are selected must have the same variances (Green, Salkind & Akey, 2000). In order to determine whether or not homogeneity of variance assumption was satisfied, both Pallant (2001) and Gravetter & Wallnau (2003) recommended conducting Levene's test of equality of error variances. It was stated that a significant result denotes that the variance of the dependent variable across the groups is not equal (Pallant, 2001).

As shown in Table 4.25, the significance level was calculated as 0.293, which was a non-significant value. Therefore, from Levene's test, it was found that the homogeneity of variances assumption was not violated.

Table 4.25 Levene's Test of Equality of Error Variances

F	$df1$	$df2$	p
1,204	9	234	0.293

As a conclusion, when these three assumptions of ANOVA were checked for violation, it was found that they were all satisfied.

4.2.2.2. Descriptive Statistics of ANOVA

The mean scores and standard deviations of participants' beliefs with respect to gender and universities are summarized in Table 4.26. When the pattern of these values were examined, it was observed that the mean belief scores for females and males ranged from 3.89 to 3.82, indicating that there was small mean difference between females and males. However, the mean belief scores for universities ranged from 3.61 to 4.29, demonstrating possible effect of universities attended on participants' problem solving beliefs.

Table 4.26 Belief Scores with respect to Gender and University

	Female			Male			Total		
	M	SD	N	M	SD	N	M	SD	N
University A	4.18	0.08	15	4.10	0.08	16	4.14	0.06	31
University B	4.05	0.07	27	3.80	0.06	22	3.91	0.05	49
University C	3.59	0.07	29	3.65	0.07	41	3.61	0.04	70
University D	3.84	0.08	15	3.87	0.10	23	3.85	0.06	38
University E	4.07	0.05	27	3.85	0.08	29	3.97	0.05	56
Total	3.89	0.04	113	3.82	0.03	131	3.86	0.39	244

In order to check whether or not these inspected mean differences are statistically significant, inferential statistics were conducted.

4.2.2.3. Inferential Statistics of ANOVA

A two-way ANOVA was conducted at the $p<0.05$ level of significance to explore the impact of gender and university attended on the participants' mathematical problem solving beliefs. Participants were studying at five different universities located in Ankara, Bolu and Samsun.

As presented in Table 4.27, the university attended produced a statistically significant main effect [$F(4, 234) = 15.35$, $p = 0.000$] on pre-service teachers' mathematical problem solving beliefs. On the other hand, the main effect for gender [$F(2, 234) = 3.55$, $p = 0.061$] did not reach a statistical significance. This means that males and females did not differ in terms of their problem solving beliefs; however, there was a significant difference in belief scores when the participants' universities were concerned.

Table 4.27 Two-way ANOVA regarding Gender and University

	Type III Sum of Squares	*df*	Mean Square	*F*	Sig.	Partial Eta Squared	Observed Power*
Gender	0.41	1	0.41	3.55	0.061	0.015	0.46
University Attended	7.24	4	1.81	15.35	**0.000**	0.208	1.00
Gender * University Attended	1.07	4	0.26	2.27	0.062	0.037	0.65
Error	27.59	234	0.11				
Total	3665.7	244					
Corrected Total	36.68	243					

*Computed using alpha = 0.05

Moreover, similar to gender main effect, the interaction effect of gender and university attended [$F(1, 234) = 2.27$, $p = 0.062$] did not reach statistical significance (Table 4.27). This indicates that there was no significant difference in the effect of university attended for males and females on their problem solving beliefs.

In order to check whether or not these calculated significances or non significances are practical and theoretical, it is recommended to check the effect sizes, also referred as “strength of associations” of these variables (Pallant, 2001, p.175).

There are a number of different effect size statistics, “the most common of which are partial eta squared” (Pallant, 2001, p.175). Partial eta squared values “ranges from 0 to 1”, and interpreted as “the proportion of variance of the dependent variable that is related to a particular main or interaction source, excluding the other main and interaction sources" (Green, Salkind & Akey, 2000, p.169). It is stated that the cutoff of effect size values are "0.01, 0.06 and 0.14" indicating small, medium and large effect sizes respectively (Cohen & Cohen, 1983).

As shown in Table 4.27, the partial eta squared values of the gender main effect, and the gender & university attended interaction effect were calculated as 0.015 and 0.037 respectively, both indicating small effect, whereas the partial eta squared value of the university attended main effect was calculated as 0.208, indicating large effect.

The power of a statistical test is explained as "the probability of reaching the correct decision" (Gravetter & Wallnau, 2003, p.250). As shown in Table 4.27, the power of the main effect of university attended was calculated as 1.00. Therefore, the decision of rejecting the null hypotheses was 100% correct, that is the university attended really significantly affect the participants' beliefs about mathematical problem solving.

4.2.2.4. Post Hoc Test

The ANOVA indicated that there were significant differences at participants' problem solving beliefs when the university attended was concerned. However, ANOVA did not identify where these differences occurred. Therefore, the analysis was continued with a post hoc test in order to make pair wise comparisons and determine exactly which universities were significantly different from each other.

There are a number of post hoc tests that can be used. The most commonly used and the one of the most cautious method for reducing the risk of a Type 1 error was recommended as Tukey's honesty significance difference (Pallant, 2001, p.175; Gravetter & Wallnau, 2003, p.403). It is important to protect against the possibility of an increased Type 1 error due to the large number of different pair wise comparisons being made (Pallant, 2001; Green, Salkind & Akey, 2000).

Tukey's HSD test was conducted at a significance level of 0.05 as a post hoc test. It computed a single value that "determines the minimum difference between treatment means that is necessary for significance" (Gravetter & Wallnau, 2003, p.402), referred as honesty significance difference or HSD.

As demonstrated in Table 4.28, multiple comparisons using the Tukey HSD test indicated that the mean belief score of participants at University C differ significantly from the mean belief score of participants at all the other universities. Also, the mean belief score of participants at University A differ significantly from the mean belief score of participants at University B, University C, and University D. In Appendix E, post hoc test multiple comparisons of universities were reported with mean differences, standard deviations, significance, and confidence intervals.

Table 4.28 Comparisons for Universities Attended

Universities Attended	University A	University B	University C	University D	University E
University A		**X**	**X**	**X**	
University B	**X**		**X**		
University C	**X**	**X**		**X**	**X**
University D	**X**		**X**		
University E			X		

CHAPTER 5

CONCLUSIONS, DISCUSSIONS, AND IMPLICATIONS

This chapter presents a summary of the study, discussions and conclusions of the major findings for each research question, and their implications for practice and for further research.

5.1. Summary of the Study

A problem is typically defined as a situation that is “unfamiliar in some sense to the individual and a clear path from the problem conditions to the solution is not apparent” (Grouws, 1996, p. 72) by the mere application of existing knowledge (Frensch & Funke, 1995). Problem solving has been given value as a goal for mental development, as a skill to be taught, and as a method of teaching in mathematics education (Giganti, 2004; Jonassen, 2004; Manuel, 1998; Schoenfeld, 1989; Willoughby, 1985; Lester, 1981) especially for the last three decades, and currently in our new education program.

In the new reform oriented Turkish mathematics curriculum, problem solving was placed as an integral part of the mathematics education, and emphasized as one of the vital common basic skills that students need to demonstrate for all subject matters (Millî Eğitim Bakanlığı Talim ve Terbiye Kurulu Başkanlığı, 2005). As being the key factors for implementing these new ideas and putting them in practice, teacher are announced to be the one guiding the instruction, using appropriate instructional tools and techniques, providing activities, giving motivation and encouragement, and assessing students' performance (Millî Eğitim Bakanlığı Talim ve Terbiye Kurulu Başkanlığı, 2005).

Research found that as one's beliefs are at the heart of one's actions (Margaret, 2001), and have considerable effect on the decisions made (Ernest, 1989), teachers' beliefs play a crucial role in changing the ways teaching takes place (Peterson, Fennema, Carpenter, & Loef, 1989; Margaret, 2001; Alba, 2001). As a result, due to the fact that long lasting instructional changes only result from essential modifications in what teacher's believe and practice (Putnam, Wheaten, Prawat, & Remillard, 1992), it becomes vitally important to understand teachers' beliefs and the factors influencing these beliefs.

The main area of investigation in the present study was to explore the kinds of beliefs the pre-service elementary mathematics teachers have towards mathematical problem solving, and investigate whether, or not, gender and university attended had significant effect on their problem solving beliefs.

In the present study, two main research questions were addressed;

1. What are the pre-service elementary mathematics teachers' beliefs about mathematical problem solving?

2. What is the effect of gender and university attended on the pre-service elementary mathematics teachers' beliefs about mathematical problem solving?

In order to explore the first research question, both the participants' responses to the questionnaire items and their interpretations about several non-routine mathematics problems were analyzed, and several descriptive statistics such as frequencies, percentages, mean and standard deviations were reported. On the other hand, so as to explore the second research question, the participants' responses to the questionnaire items were analyzed, and inferential statistics such as two way ANOVA and post hoc test were reported.

5.2. Major Findings and Discussions

5.2.1. Research Question 1

5.2.1.1. Beliefs about the Questionnaire Items

The belief scores of the pre-service elementary mathematics teachers in this study indicated that their beliefs were generally positive, in other words, in line with the new reform oriented Turkish curriculum. For instance, with this new mathematics curriculum, it was stated that problem solving is placed as an integral part of the mathematics program, and it should not be perceived as an algorithm or step by step procedures (Millî Eğitim Bakanlığı Talim ve Terbiye Kurulu Başkanlığı, 2005). Obviously, the majority of pre-service teachers appreciated problem solving as an integral part of the mathematics program, and stated that it is a process that should permeate the entire program. On the other hand, most of the pre-service teachers perceived problem solving as an algorithm, and identified problem solving as being primarily the application of computational skills. Moreover, when the items related to step by step procedures were concerned, it is possible to say that pre-service teachers slightly agreed with the idea that mathematics problems can be solved without having to rely on memorized step by step procedures. Especially, when asked about whether or not mathematicians have step by step procedures while solving problems, they appeared to be undecided.

Another principle of this new mathematics curriculum is that teachers are expected to appreciate students' solution ways and strategies more than their ability to find correct answers, value different ways of solutions to the same problems, and use computers and internet as well as calculators in their instruction (Millî Eğitim Bakanlığı Talim ve Terbiye Kurulu Başkanlığı, 2005). An examination of the items showed that almost all of the pre-service teachers appreciated a demonstration of good reasoning more than the students' ability to find correct answers, and strongly indicated that a person who does not understand why an answer to a mathematics problem is correct has not really solved the problem. Similarly, the majority of the pre-service teachers believed that different methods can be developed to solve a mathematics problem, and they classified good mathematics teachers as the ones showing students lots of ways to look at the same problem. In addition, when the items related to the usage of technology were examined, it was found that the pre-service teachers very strongly agreed with the appropriate usage of technologic equipments during their instruction, and believed that teachers can create new learning environments for their students with the use of technology.

The reform oriented mathematics program also indicated that students are expected to solve problems, think critically, work cooperatively, share their knowledge with their friends, discover, discuss, form their own problems, and as a result construct their own learning (Millî Eğitim Bakanlığı Talim ve Terbiye Kurulu Başkanlığı, 2005). Findings about the pre-service teachers' beliefs about the kinds of student behaviors also revealed that they were in accord with this new curriculum. For example, the majority of the pre-service teachers strongly agreed that students should share their problem solving thinking and approaches with other students. Moreover, the pre-service teachers disagreed with the idea of directly telling or showing students how to solve problems. Instead they preferred students to discover solution ways on their own. Findings of this section also revealed that the pre-service teachers supported the idea that teachers encourage students to write their own mathematical problems.

Another finding drawn from the questionnaire items is the pre-service teachers' belief about time consuming mathematics problems. A great deal of the pre-service teachers believed that hard mathematics problems can be done if one just hangs in there, and did not support the idea that to be good in math, one must be able to solve problems quickly.

When the research related to teachers' beliefs about problem solving were concerned, it was not possible to reach local research reports summarizing the case in Turkey. When we looked at the international studies it is possible to say that there are many studies that have differing findings some of which are parallel to the findings of the current study and others are not.

As an example, Hollifield (2000) found that after participating in NCTM standard-based workshops, teachers' perceptions of problem solving changed from viewing problem solving as "a distinct entity of mathematics" to "seeing its permeation throughout the mathematics curriculum"; that is they gave value to problem solving not as a distinct topic but as "a process that permeate the entire program" (NCTM, 1989, p.23). In addition, Frykholm (2003) stated that some teachers believed the basic computational skills to be "the most essential component of mathematics curriculum as students was tested mostly about these skills" (p.135).

Another study is reported by Brown (2003) who found that teachers think that "understanding problem is important; spending time on them may be beneficial" (p.115); and "mathematics is not an accumulation of facts, rules, and procedures" (p.116). Also, when the items related to the use of steps in solving problems in Brown's study were concerned, it was found that participants were "divided as to whether such steps existed for every problem, and whether if such steps existed, memorization of those steps would be necessary to solving problems" (p.115). Furthermore, Futch, Stephens, & James (1997) found that teachers in Georgia believed that "problem solving should permeate the entire program", "students should share their problem solving approaches with other students", and "a demonstration of good reasoning should be more highly regarded than a student's ability to find correct answers" (p.114).

On the other hand, both McKnight (1987) and Schoenfeld (1991) claimed that mathematics teachers believed that there is one correct way to solve any problem, and all problems can be solved in 5 minutes or less. Moreover, Ford (1994) indicated that teachers determined their judgments about successful problem solvers as the ones giving right answers; they strongly discouraged the use of calculators for problem solving, and even expressed using calculators in problem solving as a kind of cheating.

Another study is reported by Mason (2003) who found that from the first to the final year in high school, students become more and more convinced that “not all problems can be solved by applying routine procedures” (p.82). In addition to these studies, Brosnan (1996) and Raymond (1997) found that mathematics teachers believed answers to be more important than processes; spending time on problem solving is wasteful; and students should master computational skills before they can solve problems.

When the main findings of the questionnaire items in the present study are brought together, there are several questions that come to minds. For example, why the pre-service teachers viewed problem solving as a reason for practicing computation, and also somewhat supported following predetermined sequence of steps while solving problems, although their beliefs about the other questionnaire items were mostly in line with the new reform oriented mathematics curriculum? Also, another question is that why several studies supported the findings of the present study, whereas several studies did not support?

There can be possible explanations of these questions. One possible explanation of former questions is that although several changes has been made on the Turkish curriculum, students’ success is still measured by their scores taken on several exams, starting from elementary school until they get university, and even after university in order to gain a job. Therefore, as the mastery of computational skills still poses great importance in students’ lives, problem solving might have been considered as a reason for applying these computational skills. Moreover, a possible explanation of the pre-service teachers’ belief in following predetermined sequence of steps might arise from the methods lessons in which they are taught Polya’s model of problem solving. In mathematics, the most well known and taught model of problem solving is Polya’s model. In 1945, George Polya wrote How to Solve it, in which four steps were described for solving mathematics problems such as; understanding the problem, devising a plan, carrying out the plan, and looking back. The pre-service teachers might have understood following step-by-step procedures while solving mathematical problems as following Polya's four steps for solving mathematical problems, and that is why they supported this belief. However, by following step-by-step procedures, it was meant that following a set of memorized facts, rules, and procedures while solving problems and it was not a positive belief to be supported.

When the question about the previous research studies is considered, it might be explained by the impact of latest reform movement in mathematics education. That is, when the dates of the supporting studies and opposing studies are compared, it is obvious that most of the supporting studies were carried out after 2000, whereas most of the opposing studies were carried out before 2000. After their publications of *Standards-based Reform* (1989), *Professional Standards for Teaching Mathematics* (1991), and *Assessment Standards for School Mathematics* (1995), NCTM set up the latest reform movement in mathematics education by its publication *Principles and Standards for School Mathematics* in 2000. It clarified the previous reform messages, and aimed to "set forth a comprehensive and coherent set of learning goals, serve as a resource for teachers in examining and improving the quality of mathematics program, and guide the development of curriculum frameworks, assessment, and instructional materials" (NCTM, 2000, p.6). With this reform movement, many countries have made numerous instructional changes in their curricula, even it was stated that "most of the states have rewritten their frameworks to align with the new standards in language, grade level, and goals" (Herrera & Owens, 2001, p.90). Most probably, this wide speared vision of the new reform movement has affected teachers' views, assumptions, and values about teaching and learning; therefore, it affected their beliefs about their profession, their students, and how learning takes place.

5.2.1.2. Beliefs about the Mathematical Problems

In order to reveal more about the pre-service elementary mathematics teachers' beliefs about problem solving, they were given several non-routine mathematics problems, and asked both to evaluate the value of these problems in elementary mathematics education as being poor, average or strong, and to explain the reasons for their evaluations. Although, approximately half of the pre-service teachers evaluated the first problem as being average, and the last problem as being poor; when looked overall, the majority of the pre-service teachers evaluated these problems as being strong; that is, they believed that these problems possess high value in elementary mathematics education.

When the reasons of the pre-service teachers' evaluations of these problems as being poor were examined, it was found that they generally indicated the following beliefs such as; they are ordinary problems, they are not appropriate for elementary level, they cover only one subject area, they do not cover any objective in mathematics curriculum, they do not lead students to make interpretations, they do not involve numbers, they are asked in a story type, they do not involve operating with whole numbers, and it will take a long time to solve these problems. On the other hand, when the reasons of the pre-service teachers' evaluations of these problems as being strong were examined, it was found that they generally indicated the following beliefs such as; they are different from ordinary mathematics questions as they have interesting and enjoyable context that may attract students' attention, they offer challenge, they require high level of mathematical thinking, they lead students to make interpretations, they cover the desired content, they connect mathematics with real life and with other subject areas, they develop students' long term skills such as creativity, intelligence, mathematical reasoning and problem solving, they develop both verbal and mathematical skills, they direct students to analyze the relationships between the given variables and think in multiple ways, and with these kinds of problems students can create different strategies, and develop their own formula, therefore discover mathematics.

When we look at the literature, non routine problems are defined as the problems that demand creativity and originality from students, and contribute to their mental development process (Polya, 1966). It is stated that solving non routine problems requires organizing data, realizing the relationships between the given information, performing multi-step operations (Altun, 2001), and investing time (London, 1993). While selecting problems, it is suggested that the problems should be appropriate for the students' grade level, knowledge, skills and understandings (Henderson & Pingry, 1953), they should be appealing to students' interest and "meaningful from the students' viewpoint"(Polya, 1966, p.127). Furthermore, in our new mathematics program, it is also suggested to choose problems that are part of students' daily life experiences, and relating different mathematical concepts with each other, as well as with other subject matters (Millî Eğitim Bakanlığı Talim ve Terbiye Kurulu Başkanlığı, 2005). Lastly, Altun (2001) suggests asking problems that have different characteristics such as the ones including no number, the ones having no answer, and the ones having several answers. When compared with the major findings of this section, as the pre-service teachers' evaluations of these problems and their beliefs about these problems can be considered as reflection of their possible problem selections, it is possible to conclude that in general the pre-service teachers' beliefs were consistent with the theory, and expectations cited in the literature.

Another finding drawn from these problems is that a number of pre-service teachers tended to highly value the problems that are related to at least one topic or objective in the curriculum. For example, several pre-service teachers evaluated the first problem and the last problem as having high value in the mathematics education as they believed that these problems cover several topics in the elementary mathematics curriculum. On the other hand, they evaluated the other three problems as having low value in the mathematics education as they believed that these problems do not cover any topic in the elementary mathematics curriculum. Therefore, the pre-service teachers in this study appeared to have no strong belief in the value of asking problems that are not related to any topic in the curriculum as they believe that there is no use and means of asking such kind of problems. A possible explanation of this finding might be that these pre-service teachers view problem solving as an activity for practicing the introduced idea or algorithm.

In literature, Hatfield (1978) defined three basic approaches to problem solving instruction such as "teaching about problem solving", "teaching for problem solving", and "teaching via problem solving" based on the role given to problem solving by curriculum developers, textbook writers, and classroom teachers. Among these three approaches, it was stated that the teacher who teaches for problem solving is very concerned about applying the knowledge gained during the lesson in order to solve problems (Schroeder & Lester, 1989). In other words, the person that have "teaching for problem solving" approach views problem solving as "an activity that students engage in after the introduction of a new concept or following work on a computational skill or algorithm" (Schroeder & Lester, 1989, p.34). As a result, it is possible to conclude that a number of the pre-service teachers in this study had "teaching for problem solving" approach in mathematics education. However, the actual approach that is suggested by our Ministry of Education (2005) and NCTM (2000) is "teaching via problem solving" approach, with which "problems are valued not only as a purpose for learning mathematics but also as a primary means of doing so" (Schroeder & Lester, 1989, p.33); that is, the person that have "teaching via problem solving" approach uses problems as "a vehicle to introduce and study the mathematical content" (Manuel, 1998, p.634).

A further finding drawn from these problems is that for the same problem, a number of pre-service teachers made the same explanation; however, they evaluated the problem in a different way. For instance, for the third problem, a number of pre-service teachers indicated that the problem does not involve any number. Using the same reason, several pre-service teachers evaluated this problem as being poor and believed that if a problem does not include any number, it is not related to mathematics education. On the other hand, a number of pre-service teachers evaluated this problem as being average and believed that although the problem does not involve any number, it might be helpful for developing students' reasoning abilities. Furthermore, some pre-service teachers evaluated this problem as being strong and believed that such kind of problems show that mathematics does not only mean struggling with numbers. Although in literature it is suggested to ask problems that do not include any numbers (Altun, 2001) as long as they contribute to students' mental development process (Polya, 1966), the pre-service teachers in this study appeared to have no strong belief in the value of asking such kind of problems. This result can be considered as an important indicator of the pre-service teachers' belief in the importance of applying computational skills while solving problems. In the previous section, the pre-service teachers' beliefs about several questionnaire items were examined, and it was found that the pre-service teachers in this study perceived problem solving as an algorithm, and they identified problem solving as being primarily the application of computational skills. Steele (1997) indicated that "in traditional view, teachers see mathematics as being numbers and right answers; that is, they see mathematics as being able to memorizing facts and manipulating numbers (p.195). As a result, it is possible to conclude that the pre-service teachers hold several beliefs about problem solving and mathematics education which were corresponding to traditional views to some extend.

An additional finding drawn from these problems is that when the pre-service teachers' beliefs about these mathematics problems were compared with their beliefs illustrated in the questionnaire items, it was found that although the majority of beliefs pointed out in these two sections were consistent with each other, there were several points that were conflicting with each other. For example, when their beliefs about time consuming problems were examined from the questionnaire items, it was found that they were not bothered from time consuming problems, and they believed that hard mathematics problems can be done if one just hangs in there.

On the other hand, when their beliefs about these mathematics problems were examined, it was found that several pre-service teachers evaluated these non-routine problems as having low value in elementary mathematics education because they are time consuming problems. In addition, several pre-service teachers indicated their dislikes to the last problem as it included big and complicated numbers, therefore take a long time to come up with a solution.

Besides this, when their beliefs about the usage of technology were examined from the questionnaire items, it was found that the pre-service teachers strongly supported the appropriate usage of technologic equipments during mathematics instruction, and believed that teachers can create new learning environments for their students with the use of technology. However, when their beliefs about these mathematics problems were examined, it was found that the pre-service teachers did not mention about any kind of technology that can be used while solving these problems. Especially for the last problem, although the pre-service teachers realized that the numbers were big and complicated, they did not mention about the usage of calculators; yet, they stated that this problem uses students as a calculator. What is more, as a solution to handle these big numbers, they suggested using estimation and rounding techniques. In literature, it was found that teachers at all grade levels agreed to some extend that “calculators were useful for solving problems”; however, “they tended to disagree or to be undecided when asked if they allow students to use them”, and this inconsistency was explained by the possible “discrepancy between the perceived beliefs and the actual beliefs” (Zambo, 1994, p.15). Similarly, the inconsistencies obtained from these two sections can be explained by the differences between the pre-service teachers’ perceived beliefs and actual beliefs.

An additional finding drawn from these problems is that among the pre-service teachers that evaluated these problems as poor, some of them made irrelevant explanations about these problems. Such as, for the second problem, which asked the number of rectangles on an 8 x 8 chess board; a number of pre-service teachers indicated that in order solve this problem; it is enough to know the answer of 8 x 8. Also for the last problem, which gave the population and area of two cities and asked the more densely populated one; a number of pre-service teachers stated that this problem should be used in physic lesson while covering the "density" topic. These inconvenient explanations can be considered as reflections of these pre-service teachers' insufficient content knowledge, or another possible explanation might be that these pre-service teachers did not read the problems well and directly made interpretations without spending adequate time on them.

5.2.2. Research Question 2

5.2.2.1. Beliefs in terms of Gender and University Attended

In the present study, a two way ANOVA was conducted to explore the impact of gender and university attended on the participants' mathematical problem solving beliefs. It was found that male and female pre-service elementary mathematics teachers did not differ in terms of their problem solving beliefs; however, there was a significant difference in their belief scores when the universities attended were concerned. The order of universities from highest to lowest with respect to their mean belief scores was University A, University E University B, University D, and University C respectively. Post-hoc comparisons using the Tukey HSD test indicated that the mean belief score of participants at University C significantly differs from the mean belief score of participants at all the other universities. Also, the mean belief score of participants at University A significantly differs from the mean belief score of participants at University B, University C, and University D.

When we look at the literature, it was found that little research in mathematics education has explored gender issues with respect to teachers' beliefs. Among these researches, most of them could not distinguish between male and female teachers (Winocur, Schoen, & Sirowatka, 1989; Fennema, 1990). It was only Li (1999) who found that there were "differences between male and female teachers' beliefs about the nature of the subject, curriculum, and conceptions of their roles" and these beliefs together with their beliefs about male and female students appear to affect their behaviors, consequently affect their students' beliefs, behaviors and achievement (p.69). Therefore, it is possible to conclude that the findings in this study related to the effect of gender on the pre-service teachers' beliefs are parallel with the findings in literature. As a conclusion, the present study can be considered as a supporting study about gender equity.

When the literature was examined with respect to the impact of university attended on teachers' beliefs, it was found that little research has directly examined teachers' beliefs from this aspect, and found university attended to be a significant factor affecting teachers' beliefs about how teaching should take place (Grouws, 1996; Alba, 2001). However, there have been numerous studies that explicitly investigated factors affecting teachers' beliefs, and found the followings to be significantly affecting teachers' beliefs about how to teach their subject matter such as; mathematical method courses (Quinn,1997; Wilkins & Brand, 2004; Emenaker, 1996), performing reform oriented activities (Lloyd & Frykholm, 2000), participating in alternative certification programs (Hart, 2002; Cooney & Wilson, 1995), as well as school goals, classroom climate, availability of instructional equipment and materials, and school policies (Grouws,1996). Actually when all these factors are combined together, it is possible to conclude that the educational and physical settings are having influence on teachers' beliefs about their profession. If university is considered as a combination of these educational and physical settings, it is possible to conclude that the findings in this study related to the effect of university attended on the pre-service teachers' beliefs are parallel with the findings in literature.

When examined the pre-service teachers' educational opportunities about learning and practicing mathematical problem solving, it was found that almost all of the pre-service teachers in this study have taken Methods of Mathematics Teaching course in which they learnt concepts of methods and teaching strategies in elementary education such as expository, inquiry, discovery, demonstration, discussion, problem solving and cooperative learning. In addition, all of the pre-service teachers have taken School Experience courses in which they observed real classroom environments, the ways mathematics is taught, the ways students solve mathematical problems, as well as examining various teaching learning activities, materials and written sources. Moreover, half of the participants have taken, and the other half was still taking Practice Teaching in Elementary Education course in which they made field experience and teaching practice including class observation, planning and preparation for their own teaching, most probably including preparation of several mathematics problems. Other than taking these major courses, the pre-service teachers studying at University A and University D had further opportunity to take elective courses directly related to mathematical problem solving, whereas the pre-service teachers in the other universities were not offered such a course. In this course, the pre-service teachers learnt what problem and problem solving means, and how to integrate problem solving in their mathematics instructions.

As literature has indicated, taking these courses especially the one related to problem solving, most probably have influenced the pre-service teachers' beliefs about how to teach mathematics, and how to make problem solving as an integral part of their instruction. Yet, it is not only the courses that have affected the pre-service teachers' beliefs about mathematical problem solving. Because if it was so, it would be expected that University A and University D have the highest mean belief scores among the other universities as they additionally offered problem solving courses during their undergraduate study. However, although University A had the highest mean belief score among the other universities, the mean belief score of University D was only higher than the mean belief score of University C. In other words, although University E and University B did not offer problem solving courses, the pre-service teachers in these universities had higher mean belief scores than the pre-service teachers in University D. Therefore, other than taking courses, there were other factors that affected the pre-service teachers' beliefs about mathematical problem solving. A possible factor might be the pre-service teachers' interest about problem solving. When examined the demographic information of the pre-service teachers, it was found that almost half of the participants were interested in solving mathematical problems in textbooks and trying different strategies while solving these problems, as well as making researches about problem solving in the internet, reading books, and solving mathematical puzzles. Other than the participants' interest about problem solving, their universities goals and policies, classroom climate, instructors, as well as the universities physical settings such as their availability of instructional equipment and materials, library resources and technology usage might have all had influence on these pre-service teachers about what mathematical problem solving means and how it can be applied in mathematics instruction.

5.3. Conclusion

The aim of this study was to contribute in better understanding the kinds of beliefs the pre-service elementary mathematics teachers have toward mathematical problem solving, and then to investigate whether, or not, gender and university attended have any significant effect on their problem solving beliefs.

To sum up, although in general the pre-service elementary mathematics teachers in this study indicated positive beliefs about mathematical problem solving, they had several moderate and negative beliefs. Such as, they appeared to give importance in understanding why a solution to a mathematics problem works, and appreciate developing different ways of solutions to the same problem. Also, they appreciated challenging problems that require mathematical thinking and reasoning abilities. On the other hand, they tended to believe that problem solving is primary the application of computational skills in mathematics education, and it is a matter of following a predetermined sequence of steps. Moreover, although the pre-service teachers theoretically value solving time consuming problems, their beliefs about several non-routine mathematics problems showed that they did not believe time consuming problems to have high value in mathematics education. Also, they did not give high value to problems that do not include numbers; however, they tended rate problems that are directly related to the mathematics curriculum. Finally, although the pre-service teachers theoretically appreciated using technologic equipments in mathematics education, they did not mention about any kind of technology usage while indicating their beliefs about several non-routine mathematics problems.

In conclusion, these findings revealed that the pre-service elementary mathematics teachers in this study gave importance to problem solving in mathematics education; however, they saw mathematics instruction and learning as focused on applying the knowledge gained during the lesson with the mastery of computational skills supported by problem solving. Moreover, they appreciated problems that require mathematical thinking and reasoning; however, they preferred the ones that directly cover the introduced idea, and do not require spending so much time.

In addition to these, the present study indicated that female and male pre-service elementary mathematics teachers did not differ in terms of their beliefs about mathematical problem solving. However, the pre-service teachers' beliefs showed significant difference when the universities attended was concerned, which could be related to the effectiveness of the courses taken, the pre-service teachers' availability to instructional equipments, materials, resources and technology, as well as university climate, goal and policies.

5.4. Internal and External Validity

Validity is defined as “the degree to which the inferences made based on the instrument are meaningful, useful, and appropriate” (Fraenkel & Wallen, 1996, p. 153). Therefore, it refers to “the correctness or credibility of a description, conclusion, explanation, or interpretation” (Maxwell, 1996, p.87). Maxwell indicates that “validity is a goal rather than a product; it is never something that can be proven or taken for granted” (Maxwell, 1996, p.86).

There are several threats to validity such as internal and external validity that can raise potential issues about the credibility of a study (Creswell, 2003). In a survey research, four main internal validity threats are stated to influence the outcome of the study such as; “mortality, location, instrumentation and instrument decay” (Fraenkel & Wallen, 1996, p.383).

Mortality threat is explained as “differential loss of subjects” in a longitudinal study (Maxwell, 1996, p.87). In the present study, the instrument was directly administered and collected from the participants only once in a time, in other words, the present study was a cross sectional study. Therefore, there was no mortality threat as there was no loss of subjects. Next, location threat is stated to occur if the collection of data is carried out in places that may affect participants’ responses (Maxwell, 1996), and the best method of control for a location threat is explained as holding location constant; that is, keeping it the same for all participants (Fraenkel & Wallen, 1996). In the present study, the instrument was administered to participants in their classroom settings. However, as the sample of the present study consisted of 244 pre-service elementary mathematics teachers studying at five different universities located at three different cities entirely, it was not applicable to bring them all together. Yet, the location was tried to keep constant at least for the participants studying at the same university.

Instrumentation threat is stated to occur if the measurement method changes during the intervention or evaluation period (Robson, Shannon, Goldenhar, & Hale, 2001). In the present study, the instrument was administered once to each pre-service teacher, and no change was made while administering it to others. Therefore, instrumentation is not a threat to internal validity in this study. Furthermore, instrument decay is stated to occur for example "in interview surveys if the interviewers get tired or they are rushed" (Fraenkel & Wallen, 1996, p.383). The instrument used in the present study was not a time consuming one; it took at most half an hour for participants to fill in the instrument. Therefore, instrumentation decay is also not expected to be a threat to internal validity in this study.

Besides internal validity, external validity is another threat to the credibility of a study. External validity is defined as the "extend to which the results of a study can be generalized from a sample to a population" (Fraenkel & Wallen, 1996, p.111). It is stated that "external validity threats arise when experimenters draw incorrect inferences from the sample data or other persons, other settings, and past or future situations" (Creswell, 2003, p.171). The target population of the present study was all pre-service teachers studying in Elementary Mathematics Teacher Education program in Turkey. There were 23 universities offering this program in 2005-2006 academic years in Turkey (Öğrenci Seçme ve Yerleştirme Merkezi, 2005). Although the researcher tried to contact with a number of these universities, because of procedural obstacles between the universities and time limitations in their lecture hours, the present study was implemented to only five of these universities, and the sample of the present study consisted of 244 senior undergraduate students studying at Elementary Mathematics Teacher Education program in five different universities in Ankara, Bolu and Samsun in 2005-2006 spring semesters. Yet, the sample of the present study still consisted of quite large number of participants, and it was representative of the intended population on at least to some degree.

5.5. Implications for Practice

The notion that teachers' pedagogical content beliefs affect their classroom actions and ultimately, affect students' classroom learning is widely accepted (Wilkins & Brand, 2004; Frykholm, 2003; Ball, 1998; Lloyd & Wilson, 1998; Pajares, 1992; Thompson, 1984). Therefore, investigating pre-service teachers' beliefs is important since these beliefs are expected to reflect these teachers' future classroom activities and performances. That is, the analysis of pre-service teachers' beliefs is essential if mathematics instruction and student learning are to improve. The present study revealed that although in general the pre-service elementary mathematics teachers indicated positive beliefs about mathematical problem solving, they presented several beliefs that were not in line with the theory of problem solving and with the principals of the current reform in mathematics education. Now, once these beliefs have been assessed, adequate educational interventions should be planned and implemented especially in elementary mathematics teacher education program as well as in elementary classroom settings in order to gradually challenge and change those irrelevant beliefs.

To start with teacher education is a central issue for any kind of change in education area. It is stated that "no reform of mathematics education is possible unless it begins with revitalization of undergraduate mathematics in both curriculum and teaching style" (MSEB, 1989, p.39). According to the new curriculum, problem solving is integral to mathematics and plays a major part in truly learning mathematics (Millî Eğitim Bakanlığı Talim ve Terbiye Kurulu Başkanlığı, 2005). If a goal of mathematics teacher education programs is to promote beliefs and attitudes that are consistent with the underlying current philosophy of mathematics education reform, then mathematical problem solving should be infused into all aspects of mathematics teacher training rather than presented as a separate stand alone topic covered in a methods course.

The results of the present study showed that the pre-service elementary mathematics teachers considered problem solving as being primarily the application of computational skills, and believed that it is somehow a matter of following predetermined sequence of steps. Steele (1997) stated that in traditional view, teachers see mathematics as being numbers and describe knowing mathematics as being able to memorize facts and manipulate numbers. These traditional views of the pre-service teachers are an indication of how they learnt mathematics content as well as learning the ways to teach mathematics. Wilkins and Brand (2004) indicated that an important measure of how well undergraduate courses are preparing future teachers is how well the programs help pre-service teachers develop beliefs consistent with current reform and develop positive beliefs about themselves as teachers and learners of mathematics. Therefore, mathematics teacher education program need to examine their undergraduate courses both related to mathematics content, and pedagogy with respect to whether, or not, they are highly emphasizing computational skills, memorizing formulas, definitions and theorems rather than emphasizing the development of problem solving skills such as mathematical thinking, realizing logical connections among variables, making generalizations and formulizations.

Another point to be examined is the kinds of mathematics problems emphasized in teacher education programs. The results of the present study revealed that several pre-service elementary mathematics teachers preferred problems that are directly related to the introduced idea, that involve operating with whole numbers, and do not require spending so much time. Moreover, some pre-service teachers did not appear to value problems that are asked in a story type, and include no number. These beliefs can be as a result of the kinds of problems posed to these pre-service teachers during their mathematics education, and the kinds of problems that were emphasized during their pedagogical development. NCTM (1989) suggested that teachers teach the way they are taught. As a result, these findings identify the need of underlying the importance of asking different kinds of mathematics problems especially challenging ones that require high level of mathematical thinking and spending big amount of time. Instructors can evaluate and modify their courses in terms of pre-service teacher beliefs, and textbook writers can examine their instructional products with respect to whether or not, they pose non-ordinary mathematics problems that add a new insight and experience to students' mathematical thinking and understanding, as well as relating mathematics with other disciplines and real world situations. Moreover, if the available resources are inadequate in term of offering different kinds of problems, pre-service teachers can develop their own mathematics problems in their classroom practices.

A further point to be examined is the way technology is introduced and practiced in teacher education program. The present study showed that although the pre-service teachers recognized the importance and role of the technology in mathematics education, they failed to associate technology with own teaching. NCTM (1989) stated that learning to teach is a process of integration. Teacher educators need to engage pre-service teachers in activities where they gain both theoretical and practical understanding of the place and the use of technologies in mathematics education. For instance, pre-service teachers can be offered to use the latest instructional technologies and media in order to prepare and develop instructional activities and materials such as worksheets, transparencies, slides, videotapes, and computer-based course materials for student needs. When pre-service teachers really experience how using technology can create new learning environments that are not feasible or not applicable in normal classroom settings, they can truly believe that the usage of such equipments can give them greater choice in their tasks. When these pre-service teachers become mathematics teachers, if they can not find these equipments in their schools, at least as a simple technology, they may offer their students the opportunity to use calculators while solving real world problems. Appropriate use of calculators can increase the amount and the quality of mathematics learning as well as decreasing the time and exaggerated emphasis given to computational skill.

An additional point to be examined is the differences among teacher education programs offered in different universities. The present study pointed out that the pre-service teachers' beliefs showed significant difference when the universities attended was concerned. In order to reduce the discrepancies among teacher education programs, the network of teacher educators can be extended and powered; that is, instructors can professionally interact with each other on a regular basis, and continue to collaborate in improving their teaching. Besides, instructors can perform a number of conferences, workshops, and staff developments in other universities in order to transfer their knowledge and experiences both to the other instructors and pre-service teachers. Engaging in these professional activities can be of great value both for teacher educators and pre-service teachers to challenge their knowledge and beliefs about mathematics and become aware of current trends in mathematics education.

Finally, the present study's findings have implications for policymakers as they try to find effective ways and means to support high level learning for all teachers and all students. Policy makers need to take measures to develop mathematics teachers' positive beliefs about problem solving, and then provide necessary support and services to ensure that these beliefs to come in practice elementary classrooms, as well as ensuring that teachers follow innovations in their fields, and maintain their professional development.

5.6. Recommendations for Further Research

The present study examined pre-service elementary mathematics teachers' beliefs about mathematical problem solving. A further study can be carried out by examining elementary mathematics teachers' beliefs about mathematical problem solving, which might give a better chance of understanding the place of problem solving in our mathematics education. This further study can be implemented to the pre-service teachers that attended the present study, also to question whether teachers are able to provide instruction that is consistent with their theoretical beliefs.

Also, in the present study the data were gathered only from participants' responses given to several questionnaire items. A further research can be carried out as a case study to see more detailed picture of how pre-service teachers view problem solving during a methods course, in which data can be gathered from various data sources such as observations, interviews, end-of-course questionnaires, and learner diaries.

Another further research can be carried out with elementary students to examine their mathematical problem solving skills, which might give a deeper understanding of how students are affected from their teachers' behaviors and current reform movements in mathematics education.

Lastly, besides examining beliefs about mathematical problem solving, a further study can be carried out about mathematical problem posing; what is known about problem posing and the kinds of problems asked during mathematics instruction.

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APPENDICES

APPENDIX A

THE INSTRUMENT (TURKISH)

İLKÖĞRETİM MATEMATİK ÖĞRETMEN ADAYLARININ MATEMATİKSEL PROBLEM ÇÖZME İNANIŞLARI

AÇIKLAMA:

Bu anketin amacı ilköğretim matematik öğretmen adaylarının matematiksel problem çözme hakkındaki inanışlarını araştırmaktır.

Ankete katılmak tercihe bağlıdır. Ankete katılırsanız sizinle ilgili kişisel bilgiler tamamen saklı tutulacaktır. Anketteki her bir maddeyi yanıtlamanız bu çalışma için çok faydalı olacaktır.

Katkılarınızdan dolayı şimdiden teşekkür ederim.

Fatma Kayan
ODTÜ İlköğretim Bölümü
Yüksek Lisans Öğrencisi

1. BÖLÜM : KİŞİSEL BİLGİLER

1. Cinsiyetiniz: Bay ☐ Bayan ☐

2. Devam ettiğiniz üniversite:

3. Sınıfınız: 1.sınıf ☐ 2.sınıf ☐ 3.sınıf ☐ 4.sınıf ☐

4. Genel not ortalamanız:

5. Problem çözme ile ilgili herhangi bir ders aldınız mı?

Aldım ☐ Almadım ☐

Aldıysanız, hangi dersleri aldınız?

6. Ders alma dışında problem çözme ile ilgilendiniz mi?

İlgilendim ☐ İlgilenmedim ☐

İlgilendiyseniz, ne şekilde ilgilendiniz?

7. Aşağıdaki dersleri aldınız mı?

	Aldım	Bu Dönem Alıyorum	Almadım
Okul Deneyimi I (School Experience I)			
Okul Deneyimi II (School Experience II)			
Öğretmenlik Uygulaması (Practice Teaching in Elementary Education)			
Özel Öğretim Yöntemleri II (Methods of Mathematics Teaching)			

8. Almak zorunda olduğunuz matematik içerikli bütün dersleri bitirdiniz mi?

Evet ☐ Hayır ☐

Cevabınız Hayır ise, hangi dersleri bitirmediniz?

2. BÖLÜM: PROBLEMLER HAKKINDA GÖRÜŞLER

Bu bölümdeki problemleri ilköğretim matematik eğitiminde kullanılabilirliği açısından eğitsel değerini göz önünde bulundurarak değerlendiriniz.

Problemleri çözmenize gerek yoktur.

1) Serkan, Roma tarihini araştırıken eski bir dökümanda büyük bir ordunun İskender'i yendiğini okur. Dökümanın bir sayfasında bu ordunun büyüklüğü ile ilgili "45_ _ 8" şeklinde okunabilen bir sayıya rastlar. Bu sayının kaç olabileceğini bulabilmesi için kullanabileceği tek bilgi, bu ordunun 9 farklı hücum noktasından eşit sayıda asker ile İskender'e saldırdığıdır.

Bu bilgiden yola çıkarak ordunun olası büyüklüklerini bulun.

Problemin matematik öğretimi açısından değeri:

Zayıf ☐ Orta ☐ Güçlü ☐

Lütfen nedenini açıklayınız.

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2) Bir satranç tahtasında kaç tane dikdörtgen vardır?
(Satranç tahtası 8 × 8 karelerden oluşur)

Problemin matematik öğretimi açısından değeri:

Zayıf ☐ Orta ☐ Güçlü ☐

Lütfen nedenini açıklayınız.

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3) Bir adam bir tilkiyi, bir tavuğu ve bir poşet mısırı nehrin karşısına kayık ile geçirmek ister. Ancak karşıya geçerken her seferinde yanına bunlardan sadece birini alabilir. Seçimini yaparken tilki ile tavuğu, tavuk ile de mısırı yalnız bırakmaması gerekmektedir; çünkü tilki tavuğu, tavuk da mısırı yiyecektir.

Bu durumda adam tilkiyi, tavuğu ve mısırı karşıya güvenle nasıl geçirebilir?

Problemin matematik öğretimi açısından değeri:

Zayıf ☐ Orta ☐ Güçlü ☐

Lütfen nedenini açıklayınız.

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4) Beş bayan farklı zamanlarda 10 km'lik bir yürüyüşe katılırlar. Yürüyüşün belirli bir anında hareketlerinin dondurulduğu varsayılırsa, aşağıdaki bilgileri kullanarak Nuray'ın bitiş noktasına uzaklığını bulun.

- Melek yolun yarısındadır.
- Filiz, Canan'dan 2 km öndedir.
- Nuray, Sibel'den 3 km öndedir.
- Melek, Canan'dan 1 km geridedir.
- Sibel, Filiz'den 3.5 km geridedir.

Problemin matematik öğretimi açısından değeri:

Zayıf ☐ Orta ☐ Güçlü ☐

Lütfen nedenini açıklayınız.

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5) 2000 yılında, Ankara'nın nüfusu 4.007.860 ve alanı ise 25.978 km^2 iken Yalova'nın nüfusu 168.593 ve alanı ise 847 km^2 idi.

Bu durumda 2000 yılında hangi şehrin nüfusu daha yoğundur?

(Nüfus Yoğunluğu: Birim alanda yaşayan insan sayısı)

Problemin matematik öğretimi açısından değeri:

Zayıf ☐ Orta ☐ Güçlü ☐

Lütfen nedenini açıklayınız.
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Genel olarak eklemek istedikleriniz için bu alanı kullanabilirsiniz.
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3. BÖLÜM: MATEMATİKSEL PROBLEM ÇÖZMEYE YÖNELİK İNANIŞLAR

Lütfen aşağıdaki her madde için düşüncenizi en iyi yansıtan tercihin karşısındaki rakamı işaretleyiniz.

Tamamen Katılıyorum:5, Katılıyorum:4, Tarafsızım:3, Katılmıyorum:2, Hiç Katılmıyorum:1

	Tamamen Katılıyorum	Katılıyorum	Tarafsızım	Katılmıyorum	Hiç Katılmıyorum
1. Matematiksel problem çözmede bir yöntemin kişiyi doğru cevaba ulaştırması, nasıl veya niye ulaştırdığından daha önemlidir.	(5)	(4)	(3)	(2)	(1)
2. Uygun çözüm yollarını bilmek bütün problemleri çözmek için yeterlidir.	(5)	(4)	(3)	(2)	(1)
3. Bir matematik probleminin çözümünün uzun zaman alması rahatsız edici değildir.	(5)	(4)	(3)	(2)	(1)
4. Bir problemi, öğretmenin kullandığı veya ders kitabında yer alanlar dışında yöntemler kullanarak çözmek mümkündür.	(5)	(4)	(3)	(2)	(1)
5. Matematik öğretiminde uygun teknolojik araçlar öğrenciler için her zaman erişilebilir olmalıdır.	(5)	(4)	(3)	(2)	(1)
6. Bir problemin çözümünün niye doğru olduğunu anlamayan kişi sonucu bulsa da aslında tam olarak o problemi çözmüş sayılmaz.	(5)	(4)	(3)	(2)	(1)
7. Matematikçiler problemleri çözerken önceden bilinen çözüm kalıplarını nadiren kullanırlar.	(5)	(4)	(3)	(2)	(1)
8. Bir problemin nasıl çözüleceğini anlamak uzun zaman alıyorsa o problem çözülemez.	(5)	(4)	(3)	(2)	(1)
9. Bir problemi çözmenin sadece bir doğru yöntemi vardır.	(5)	(4)	(3)	(2)	(1)
10. Problem çözme matematik müfredatının tamamına yansıtılmalıdır.	(5)	(4)	(3)	(2)	(1)

	Tamamen Katılıyorum	Katılıyorum	Tarafsızım	Katılmıyorum	Hiç Katılmıyorum
11. Problem çözerken teknolojik araçlar kullanmak bir tür hiledir.	(5)	(4)	(3)	(2)	(1)
12. Bir problemin çözümünü bulmak o problemi anlamaktan daha önemlidir.	(5)	(4)	(3)	(2)	(1)
13. Problem çözmeyi öğrenmek problemin çözümüne yönelik doğru yolları akılda tutmakla ilgilidir.	(5)	(4)	(3)	(2)	(1)
14. En zor matematik problemleri bile üzerinde ısrarla çalışıldığında çözülebilir.	(5)	(4)	(3)	(2)	(1)
15. Öğretmenin çözüm yöntemini unutan bir öğrenci aynı cevaba ulaşacak başka yöntemler geliştirebilir.	(5)	(4)	(3)	(2)	(1)
16. Problem çözme matematikte işlem becerileri ile doğrudan ilgilidir.	(5)	(4)	(3)	(2)	(1)
17. Teknolojik araçlar, problem çözmede faydalıdır.	(5)	(4)	(3)	(2)	(1)
18. Bir çözümü anlamaya çalışmak için kullanılan zaman çok iyi değerlendirilmiş bir zamandır.	(5)	(4)	(3)	(2)	(1)
19. İlgili formülleri hatırlamadan da problemler çözülebilir.	(5)	(4)	(3)	(2)	(1)
20. Matematikte iyi olmak, problemleri çabuk çözmeyi gerektirir.	(5)	(4)	(3)	(2)	(1)
21. Verilen herhangi bir problemin çözümünde tüm matematikçiler aynı yöntemi kullanır.	(5)	(4)	(3)	(2)	(1)
22. Öğrenciler, problem çözme yaklaşımlarını ve tekniklerini diğer öğrenciler ile paylaşmalıdır.	(5)	(4)	(3)	(2)	(1)
23. Öğretmenler, teknolojiyi kullanarak öğrencilerine yeni öğrenme ortamları oluşturmalıdır.	(5)	(4)	(3)	(2)	(1)

	Tamamen Katılıyorum	Katılıyorum	Tarafsızım	Katılmıyorum	Hiç Katılmıyorum
24. Bir çözümde öğrencinin mantıksal yaklaşımı, çözümün doğru olmasına kıyasla daha çok takdir edilmelidir.	(5)	(4)	(3)	(2)	(1)
25. Öğrencilerin matematik problemleri çözebilmeleri için çözüm yollarını önceden bilmesi gerekir.	(5)	(4)	(3)	(2)	(1)
26. Bir öğrenci, problemi bir yoldan çözemiyorsa başka bir çözüm yolu mutlaka bulabilir.	(5)	(4)	(3)	(2)	(1)
27. Öğrencilere problemlerin çözüm yollarını göstermek onların keşfetmesini beklemekten daha iyidir.	(5)	(4)	(3)	(2)	(1)
28. Problem çözerken teknolojiyi kullanmak zaman kaybıdır.	(5)	(4)	(3)	(2)	(1)
29. Bir matematik problemini çözerken doğru cevabı bulmanın yanında bu cevabın niye doğru olduğunu anlamak da önemlidir.	(5)	(4)	(3)	(2)	(1)
30. Çözüm yollarını akılda tutmak problem çözmede çok faydalı değildir.	(5)	(4)	(3)	(2)	(1)
31. Bir matematik öğretmeni, problemlerin çözümlerini tam olarak sınavda isteyeceği şekilde öğrencilere göstermelidir.	(5)	(4)	(3)	(2)	(1)
32. Matematik derslerinde öğrencilerin problem kurma becerileri geliştirilmelidir.	(5)	(4)	(3)	(2)	(1)
33. Teknolojiyi kullanmak öğrencilere çalışmalarında daha çok seçenek sunar.	(5)	(4)	(3)	(2)	(1)
34. Belirli bir çözüm yolunu kullanmadan bir matematik problemini çözmek mümkün değildir.	(5)	(4)	(3)	(2)	(1)
35. Bir matematik öğretmeni, öğrencilerine bir soruyu çözdürürken çok çeşitli yönlerden	(5)	(4)	(3)	(2)	(1)

	Tamamen Katılıyorum	Katılıyorum	Tarafsızım	Katılmıyorum	Hiç Katılmıyorum
bakabilmeyi de göstermelidir.					
36. Teknolojik araçlar, öğrencilerin matematik öğrenme becerilerine zarar verir.	(5)	(4)	(3)	(2)	(1)
37. Her matematiksel problem önceden bilinen bir çözüm yolu takip edilerek çözülemeyebilir.	(5)	(4)	(3)	(2)	(1)
38. Farklı çözüm yolları öğrenmek, öğrencilerin kafasını karıştırabilir.	(5)	(4)	(3)	(2)	(1)
39. Öğrenciler, uygun bir şekilde teknolojiyi kullanırlarsa matematiği daha derinlemesine anlayabilirler.	(5)	(4)	(3)	(2)	(1)

Teşekkür ederim.

APPENDIX B

THE INSTRUMENT (ENGLISH)

THE BELIEF SURVEY OF PRE-SERVICE MATHEMATICS TEACHERS ON MATHEMATICAL PROBLEM SOLVING

This survey is prepared to better understand the beliefs of pre-service elementary mathematics teachers hold toward problem solving in mathematics.

There is no penalty if you decide not to participate or to later withdraw from the study. Please be assured that your response will be kept absolutely confidential. The study will be most useful if you respond to every item in the survey, however you may choose not to answer one or more of them, without penalty.

Thank you in advance for your assistance in studying this survey.

Fatma Kayan
METU Elementary Education
Master Student

<u>PART I:</u> DEMOGRAPHIC INFORMATION SHEET

1. Gender: Male ☐ Female ☐

2. University Attended:

3. University Grade Level: 1st ☐ 2nd ☐ 3rd ☐ 4th ☐

4. What is your Grade Point Average (G.P.A)?

5. Are there any courses that you took related to problem solving?

Yes ☐ No ☐

If yes, what were they?

6. Have you been interested in problem solving other than taking courses?

Yes ☐ No ☐

If yes, how?

7. Have you taken the following courses?

	Already Taken	Taking Now	Not Taken Yet
School Experience I			
School Experience II			
Practice Teaching In Elementary Education			
Methods of Mathematics Teaching			

8. Did you finish your all must courses related to mathematics?

Yes ☐ No ☐

If not, which ones?

PART II: BELIEFS RELATED TO MATHEMATICAL PROBLEMS

Evaluate the value of given problems for their appropriateness in elementary mathematics education

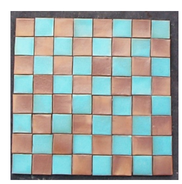
There is no need to solve these problems.

1) Serkan was studying the Romans in history and came across an ancient document about a great army that advanced upon Alexandria. He was unable to read the size of the army as two digits were smudged, but he knew it was "45_ _ 8" and that the attacking army was divided into 9 equal battalions, to cover the 9 different entrances to Alexandria.

What are the possible sizes for the attacking army?

The value of the problem with respect to mathematics education: Poor ☐ Average ☐ Strong ☐ Explain your reason please ....

2) How many rectangles are there on an 8 x 8 chess board?



The value of the problem with respect to mathematics education: Poor ☐ Average ☐ Strong ☐ Explain your reason please. ....

3) A man wants to take her fox, chicken and a bag of corn across the river in a canoe. The canoe can hold only one thing in addition to the man. If left alone, the fox would eat the chicken, or the chicken would eat the corn.

How can the man take everything across the river safely?

The value of the problem with respect to mathematics education:
Poor ☐ Average ☐ Strong ☐
Explain your reason please
....
....
....
....

4) Five women participated in a 10 km walk, but started at different times. At a certain time in the walk the following descriptions were true.

- ❖ Melek was at the halfway point.
- ❖ Filiz was 2 km ahead of Canan.
- ❖ Nuray was 3 km ahead of Sibel.
- ❖ Melek was 1 km behind Canan.
- ❖ Sibel was 3.5 km behind Filiz.

How far from the finish line was Nuray at that time?

The value of the problem with respect to mathematics education:
Poor ☐ Average ☐ Strong ☐
Explain your reason please
....
....
....
....

5) In 2000, Ankara had a population of 4.007.860 and covers an area of 25.978 square kilometers. Yalova had a population of 168.593 with an area of 847 square kilometers. Which city was more densely populated?

The value of the problem with respect to mathematics education: Poor ☐ Average ☐ Strong ☐ Explain your reason please ....

Use the given space for additional interpretations. ....

PART III: THE BELIEF SURVEY OF PRE-SERVICE MATHEMATICS TEACHERS ON MATHEMATICAL PROBLEM SOLVING

Please, provide your opinion for each item using the following scale by placing a tick on the response that best fits you.

SA = Strongly Agree, **A =** Agree, **N =** Neutral, **D =** Disagree, **SD =** Strongly Disagree

		SA	A	N	D	SD
1.	It is not important to understand why a mathematical procedure works as long as it gives a correct answer.	(5)	(4)	(3)	(2)	(1)
2.	Any problem can be solved if you know the right steps to follow.	(5)	(4)	(3)	(2)	(1)
3.	Mathematics problems that take a long time are not bothering.	(5)	(4)	(3)	(2)	(1)
4.	It is possible to get the correct answer to a mathematics problem using methods other than the one the teacher or the textbook uses.	(5)	(4)	(3)	(2)	(1)
5.	Appropriate technologic equipments should be available to all students at all times.	(5)	(4)	(3)	(2)	(1)
6.	A person who does not understand why an answer to a mathematics problem is correct has not really solved the problem.	(5)	(4)	(3)	(2)	(1)
7.	Mathematicians seldom have step-by-step procedures to solve mathematical problems.	(5)	(4)	(3)	(2)	(1)
8.	Mathematics problems that take a long time to complete can not be solved.	(5)	(4)	(3)	(2)	(1)
9.	There is only one correct way to solve a mathematics problem.	(5)	(4)	(3)	(2)	(1)
10.	Problem solving is a process that should permeate the entire program.	(5)	(4)	(3)	(2)	(1)
11.	Using technologic equipments in problem solving is cheating.	(5)	(4)	(3)	(2)	(1)

	SA	A	N	D	SD
12. It does not really matter if you understand a mathematics problem if you can get the right answer.	(5)	(4)	(3)	(2)	(1)
13. Learning to do problems is mostly a matter of memorizing the right steps to follow.	(5)	(4)	(3)	(2)	(1)
14. Hard mathematics problems can be done if one just hang in there.	(5)	(4)	(3)	(2)	(1)
15. If a student forgets how to solve a mathematics problem the way the teacher did, it is possible to develop different methods that will give the correct answer.	(5)	(4)	(3)	(2)	(1)
16. Problem solving is primarily the application of computational skills in mathematics.	(5)	(4)	(3)	(2)	(1)
17. Technologic equipments are useful in solving problems.	(5)	(4)	(3)	(2)	(1)
18. Time used to investigate why a solution to a mathematics problem works is time well spent.	(5)	(4)	(3)	(2)	(1)
19. Problems can be solved without remembering formulas.	(5)	(4)	(3)	(2)	(1)
20. To be good in math, one must be able to solve problems quickly.	(5)	(4)	(3)	(2)	(1)
21. If a number of mathematicians were given a mathematical problem, they would all solve it in the same way.	(5)	(4)	(3)	(2)	(1)
22. Students should share their problem solving thinking and approaches with other students.	(5)	(4)	(3)	(2)	(1)
23. Teachers can create new learning environments for their students with the use of technology.	(5)	(4)	(3)	(2)	(1)
24. A demonstration of good reasoning should be regarded even more than students' ability to find correct answers.	(5)	(4)	(3)	(2)	(1)

		SA	A	N	D	SD
25.	To solve most mathematics problems, students should be taught the correct procedure.	(5)	(4)	(3)	(2)	(1)
26.	If a student is unable to solve a problem one way, there are usually other ways to get the correct answer.	(5)	(4)	(3)	(2)	(1)
27.	It is better to tell or show students how to solve problems than to let them discover how on their own.	(5)	(4)	(3)	(2)	(1)
28.	Using technology is a waste of time while solving problems.	(5)	(4)	(3)	(2)	(1)
29.	In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.	(5)	(4)	(3)	(2)	(1)
30.	Memorizing steps is not that useful for learning to solve problems.	(5)	(4)	(3)	(2)	(1)
31.	Good mathematics teachers show students the exact way to answer the math question they will be tested on.	(5)	(4)	(3)	(2)	(1)
32.	Teachers should encourage students to write their own mathematical problems.	(5)	(4)	(3)	(2)	(1)
33.	Using technology in solving problems can give students greater choice in their tasks.	(5)	(4)	(3)	(2)	(1)
34.	Without a step-by-step procedure, there is no way to solve a mathematics problem.	(5)	(4)	(3)	(2)	(1)
35.	Good mathematics teachers show students lots of ways to look at the same questions.	(5)	(4)	(3)	(2)	(1)
36.	Technologic equipments harm students' ability to learn mathematics.	(5)	(4)	(3)	(2)	(1)
37.	There are problems that just can not be solved by following a predetermined sequence of steps.	(5)	(4)	(3)	(2)	(1)

	SA	A	N	D	SD
38. Hearing different ways to solve the same problem can confuse students.	(5)	(4)	(3)	(2)	(1)
39. Students can learn more mathematics more deeply with the appropriate and responsible use of technology.	(5)	(4)	(3)	(2)	(1)

Thank you

APPENDIX C

RESULTS OF THE QUESTIONNAIRE ITEMS

Table 7.1 Results of the Questionnaire Items

ITEMS	Agree		Neutral		Disagree		Mean**	Stand .Dev.
	f	%	f	%	f	%	M	SD
1. *	28	11.4	28	11.5	188	77.1	3.96	1.118
2. *	89	36.5	35	14.3	120	49.2	3.17	1.198
3.	103	42.2	57	23.4	84	34.4	3.10	1.179
4.	228	93.4	10	4.1	6	2.4	4.42	0.741
5.	222	91.0	16	6.6	6	2.4	4.45	0.743
6.	228	93.4	5	2.0	11	4.5	4.45	0.813
7.	41	16.8	63	25.8	140	57.4	2.52	0.927
8. *	12	5.0	23	9.4	209	85.6	4.13	0.865
9. *	8	3.2	8	3.3	228	93.5	4.54	0.766
10.	163	66.8	41	16.8	40	16.4	3.79	1.177
11.*	21	8.6	38	15.6	185	75.9	3.92	0.946
12.*	34	13.9	15	6.1	195	79.9	3.93	1.102
13.*	91	37.3	48	19.7	105	43.0	3.09	1.284
14.	216	88.6	22	9.0	6	2.4	4.21	0.723
15.	218	85.7	15	6.1	20	8.2	4.11	0.945
16.*	195	80.0	25	10.2	24	9.8	2.11	0.848
17.	203	83.2	35	14.3	6	2.4	4.05	0.727

ITEMS	Agree		Neutral		Disagree		Mean**	Stand .Dev.
	f	%	f	%	f	%	M	SD
18.	208	85.2	30	12.3	6	2.5	4.14	0.716
19.	166	68.1	46	18.9	32	13.2	3.76	1.008
20.*	56	22.9	53	21.7	135	55.4	3.99	1.046
21.*	8	3.3	11	4.5	225	92.2	4.30	0.706
22.	229	93.8	12	4.9	3	1.2	4.32	0.625
23.	226	92.6	15	6.1	3	1.2	4.41	0.681
24.	219	89.7	20	8.2	5	2.0	4.30	0.707
25.*	75	30.7	45	18.4	124	50.8	3.20	1.237
26.	168	68.9	48	19.7	28	10.8	3.71	1.030
27.*	16	6.6	32	13.1	196	80.3	4.09	0.945
28.*	19	7.7	33	13.5	192	78.7	4.07	0.964
29.	235	96.3	6	2.5	3	1.2	4.44	0.629
30.	84	34.4	72	29.5	88	36.1	2.98	1.085
31.*	90	36.8	42	17.2	112	45.9	3.20	1.310
32.	231	94.7	10	4.1	3	1.2	4.46	0.656
33.	205	84.0	31	12.7	8	3.3	4.15	0.777
34.*	63	25.9	60	24.6	121	49.6	3.33	1.158
35.	229	93.9	10	4.1	5	2.0	4.57	0.673

ITEMS	Agree		Neutral		Disagree		Mean**	Stand .Dev.
	f	%	f	%	f	%	M	SD
36.*	13	5.3	44	18.0	187	76.7	4.06	0.901
37.	182	74.6	35	14.3	27	11.1	3.75	0.955
38.*	15	6.1	58	23.8	171	70.1	3.92	0.915
39.	171	70.1	53	21.7	20	8.2	3.90	0.982

* These items are negatively stated. Items reversed in scoring. Therefore, a higher mean indicates participants disagree with the statements.
** Minimum possible mean value is 1; maximum possible mean value is 5.

APPENDIX D

HISTOGRAMS AND NORMAL Q-Q PLOTS

FOR THE MEAN OF BELIEFS SCORES

A. Histograms and Normal Q-Q Plots for the Mean of Beliefs Scores with respect to Universities Attended

Figure 1 Histogram of the Mean of Belief Scores for University A

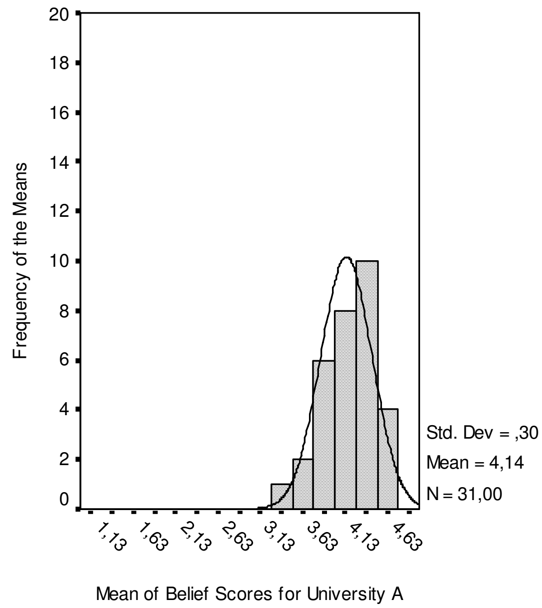


Figure 2 Histogram of the Mean of Belief Scores for University B

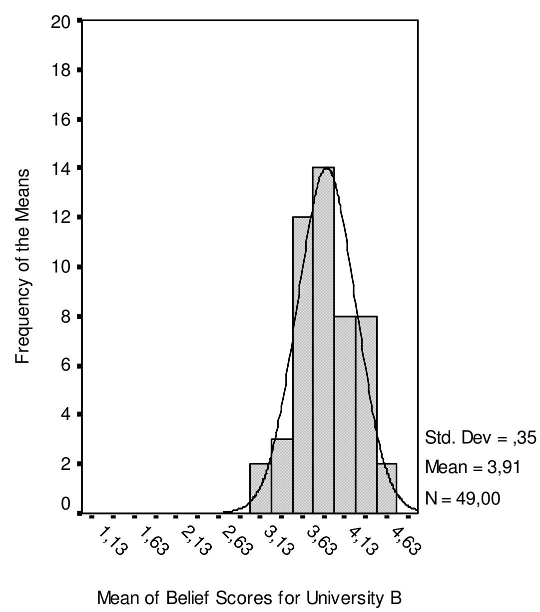


Figure 3 Histogram of the Mean of Belief Scores for University C

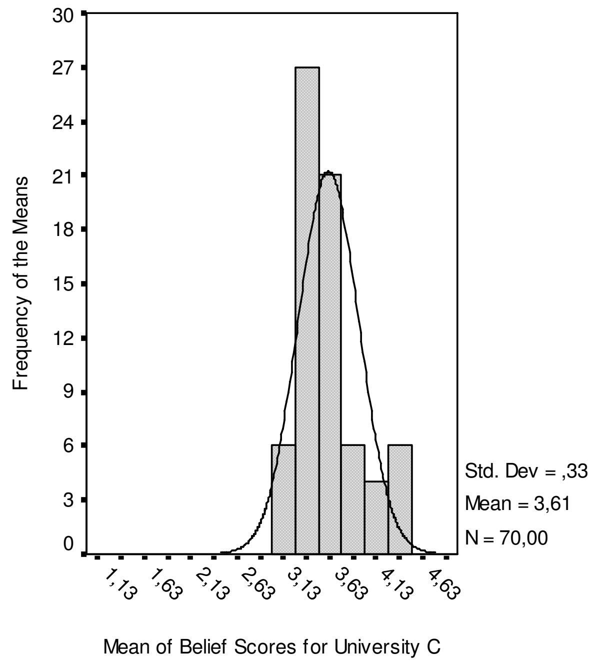


Figure 4 Histogram of the Mean of Belief Scores for University D

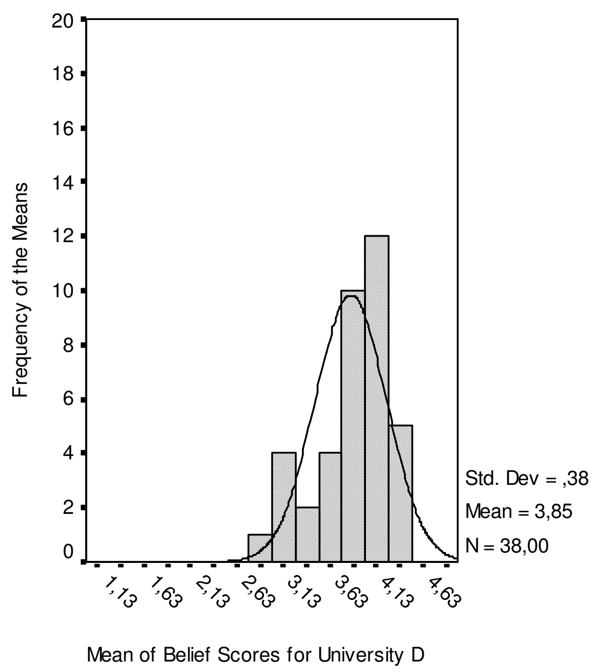


Figure 5 Histogram of the Mean of Belief Scores for University E

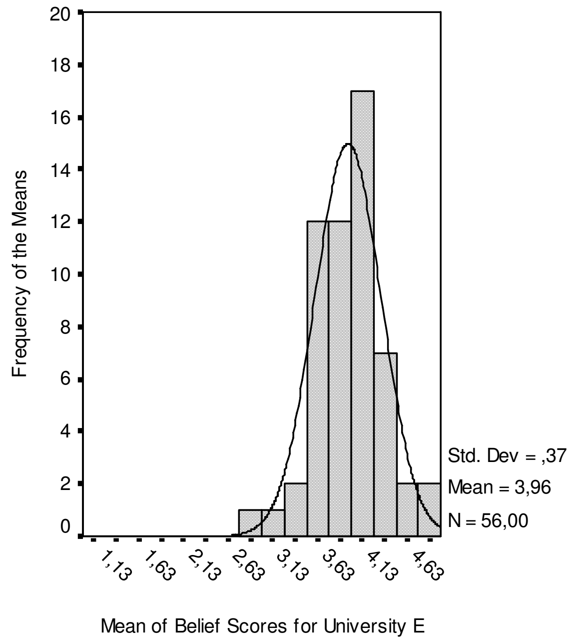


Figure 6 Normal Q-Q Plot of the Mean of Belief Scores for University A

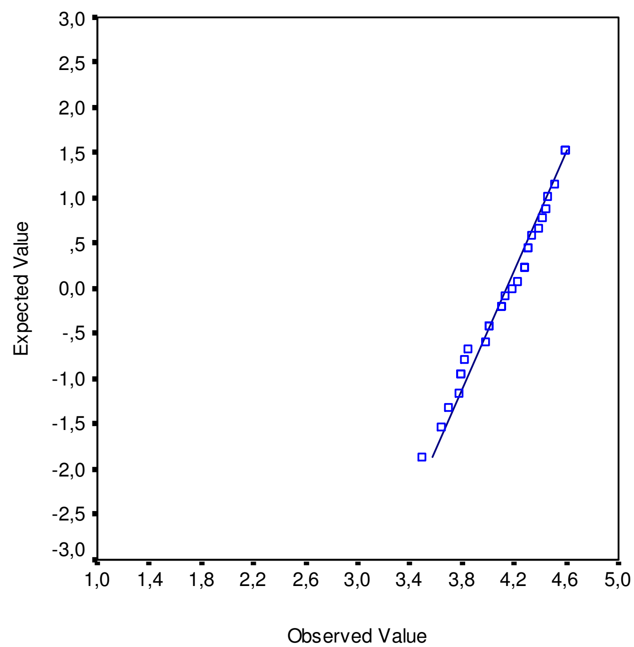


Figure 7 Normal Q-Q Plot of the Mean of Belief Scores for University B

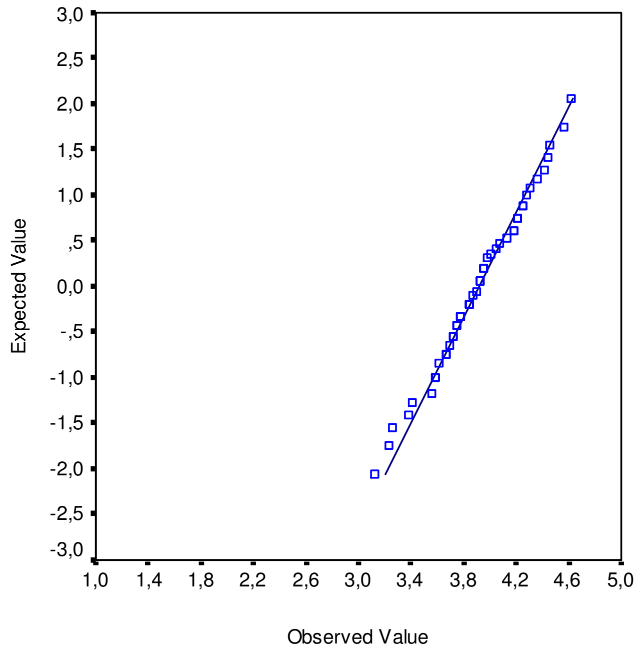


Figure 8 Normal Q-Q Plot of the Mean of Belief Scores for University C

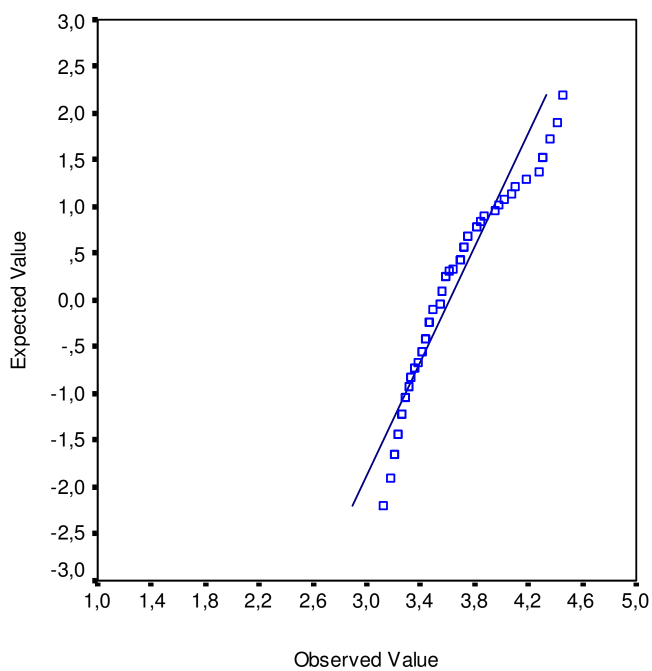


Figure 9 Normal Q-Q Plot of the Mean of Belief Scores for University D

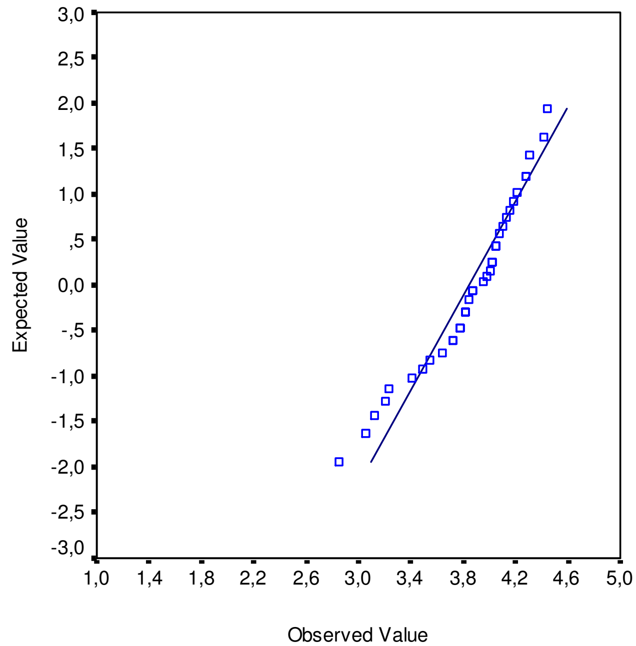
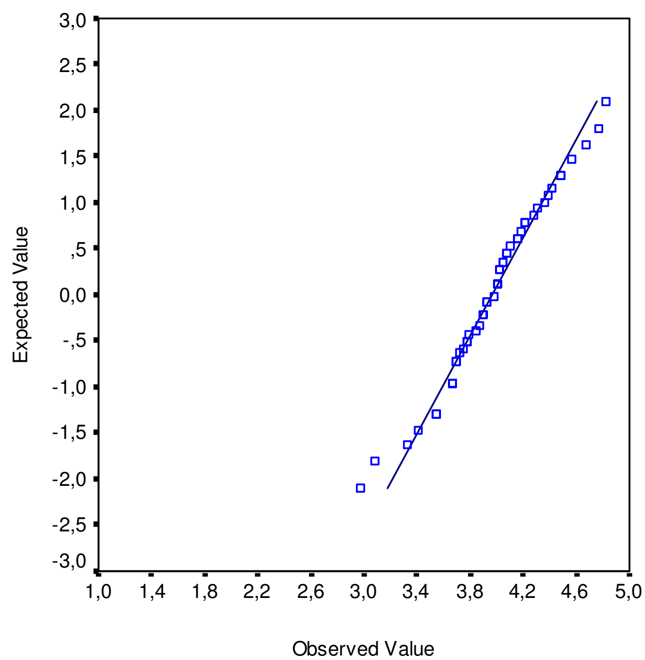


Figure 10 Normal Q-Q Plot of the Mean of Belief Scores for University E



B. Histograms and Normal Q-Q Plots for the Mean of Beliefs Scores with respect to Gender

Figure 11 Histogram of the Mean of Belief Scores for Male Participants

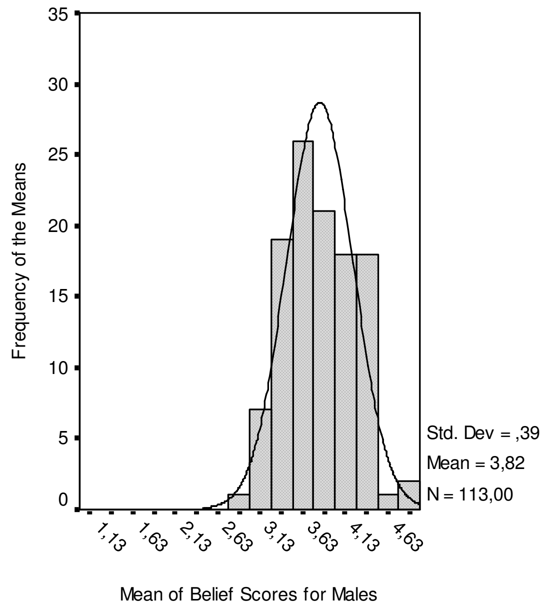


Figure 12 Histogram of the Mean of Belief Scores for Female Participants

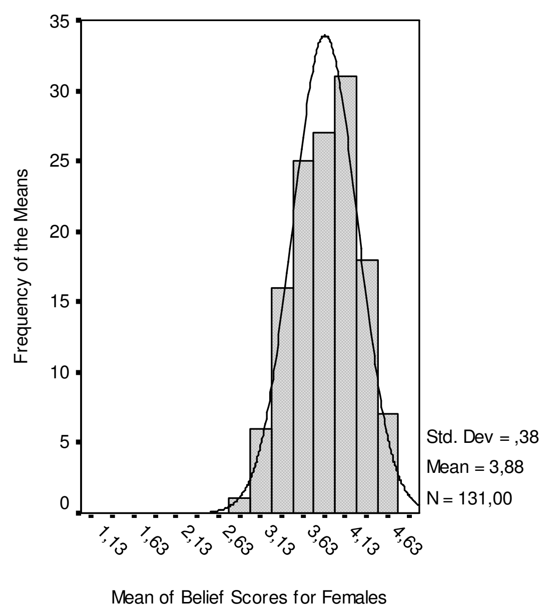


Figure 13 Normal Q-Q Plot of the Mean of Belief Scores for Male Participants

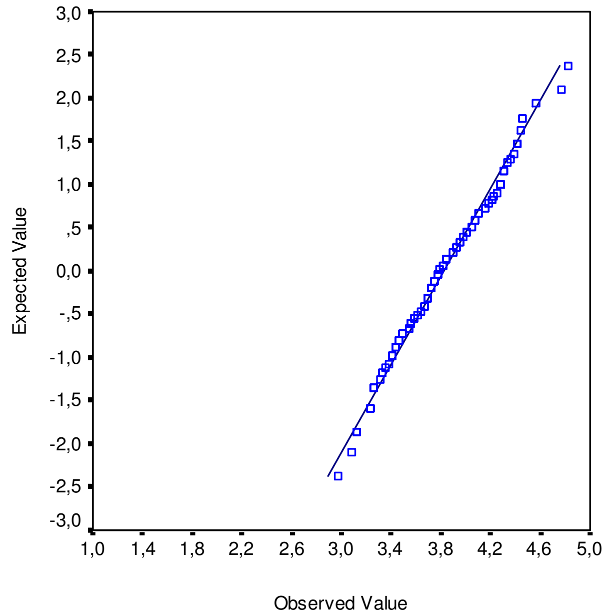
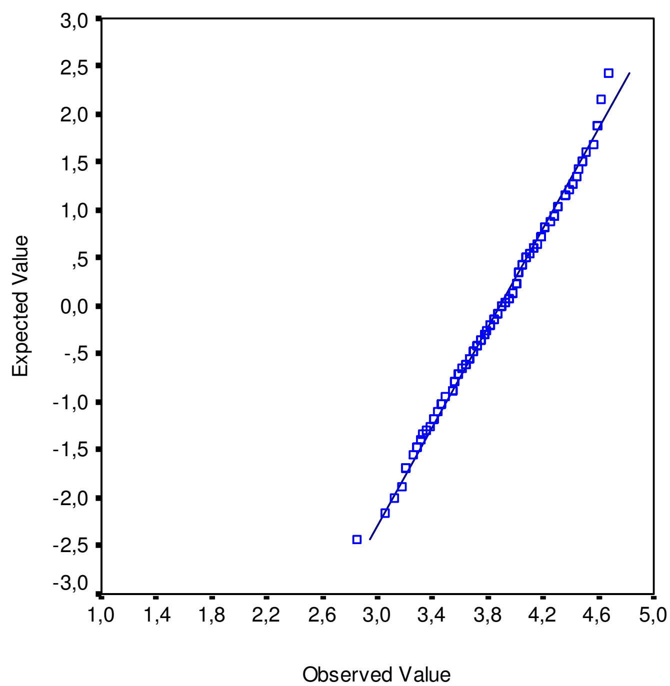


Figure 14 Normal Q-Q Plot of the Mean of Belief Scores for Female Participants



APPENDIX E

POST HOC TEST FOR UNIVERSITIES ATTENDED

Table 7.2 Multiple Comparisons for Universities Attended

Universities Attended[*] (A)	(B)	Mean Difference (A - B)	Std. Error	Sig.[**]	95% Confidence Interval Lower Bound	Upper Bound
1	2	0.227	0.078	**0.035**	0.010	0.443
	3	0.530	0.074	**0.000**	0.326	0.734
	4	0.290	0.083	**0.005**	0.062	0.519
	5	0.175	0.076	0.156	-0.036	0.386
2	1	-0.227	0.078	**0.035**	-0.443	-0.010
	3	0.303	0.063	**0.000**	0.127	0.478
	4	0.063	0.074	0.911	-0.140	0.267
	5	-0.052	0.067	0.937	-0.236	0.132
3	1	-0.530	0.074	**0.000**	-0.734	-0.326
	2	-0.303	0.063	**0.000**	-0.478	-0.127
	4	-0.239	0.069	**0.006**	-0.429	-0.049
	5	-0.355	0.061	**0.000**	-0.524	-0.186
4	1	-0.290	0.083	**0.005**	-0.519	-0.062
	2	-0.063	0.074	0.911	-0.267	0.140
	3	0.239	0.069	**0.006**	0.049	0.429
	5	-0.115	0.072	0.495	-0.314	0.082
5	1	-0.175	0.076	0.156	-0.386	0.036
	2	0.052	0.067	0.937	-0.132	0.236
	3	0.355	0.061	**0.000**	0.186	0.524
	4	0.115	0.072	0.495	-0.082	0.314

* 1 = University A, 2 = University B, 3 = University C, 4 = University D, 5 = University E
** The mean difference is significant at the 0.05 level.