WEAPON-TARGET ALLOCATION AND SCHEDULING FOR AIR DEFENSE WITH TIME VARYING HIT PROBABILITIES

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

TANER GÜLEZ

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
INDUSTRIAL ENGINEERING

JUNE 2007

Approval of the Graduate School of Natural and Applied Sciences

Prof. Dr. Canan ÖZGEN
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science

Prof. Dr. Çağlar GÜVEN
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Prof. Dr. Levent Kandiller
Co-Supervisor

Prof. Dr. Nur Evin ÖZDEMİREL
Supervisor

Examining Committee Members

Prof. Dr. Ömer KIRCA	(METU, IE)	
Prof. Dr. Nur Evin ÖZDEMİREL	(METU, IE)	
Prof. Dr. Levent KANDİLLER	(METU, IE)	
Assoc. Prof. Dr. Sedef MERAL	(METU, IE)	
Dr. Hüseyin YAVUZ	(ASELSAN)	

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name:
Taner GÜLEZ

Signature :

ABSTRACT

WEAPON-TARGET ALLOCATION AND SCHEDULING FOR AIR DEFENSE WITH TIME VARYING HIT PROBABILITIES

GÜLEZ, Taner

M.S., Department of Industrial Engineering

Supervisor: Prof. Dr. Nur Evin ÖZDEMİREL

Co-Supervisor: Prof. Dr. Levent KANDİLLER

June 2007, 164 pages

In this thesis, mathematical modeling and heuristic approaches are developed for surface-to-air weapon-target allocation problem with time varying single shot hit probabilities (SSHP) against linearly approaching threats. First, a nonlinear mathematical model for the problem is formulated to maximize sum of the weighted survival probabilities of assets to be defended. Next, nonlinear objective function and constraints are linearized. Time varying SSHP values are approximated with appropriate closed forms and adapted to the linear model obtained. This model is tested on different scenarios and results are compared with those of the original nonlinear model. It is observed that the linear model is solved much faster than the nonlinear model and produces reasonably good solutions. It is inferred from the solutions of both models that engagements should be made as late as possible, when the threats are closer to the weapons, to have SSHP values higher. A construction heuristic is developed based on this scheme. An improvement heuristic that uses the solution of the construction heuristic is also proposed. Finally, all methods are tested on forty defense scenarios. Two fastest solution methods, the linear model and the construction heuristic, are compared on a large scenario and proposed as appropriate solution techniques for the weapon-target allocation problems.

Keywords: Air Defense, Weapon-Target Allocation, Scheduling, Time Varying Single Shot Hit Probability, Linearization.

ÖZ

ZAMANA BAĞLI DEĞİŞEN VURMA OLASILIKLARIYLA HAVA SAVUNMASI İÇİN SİLAH-TEHDİT TAHSİSİ VE ÇİZELGELEMESİ

GÜLEZ, Taner

Y. Lisans, Endüstri Mühendisliği Bölümü

Tez Yöneticisi: Prof. Dr. Nur Evin ÖZDEMİREL

Ortak Tez Yöneticisi: Prof. Dr. Levent KANDİLLER

Haziran 2007, 164 Sayfa

Bu tezde zamana göre değişken tek atışta vurma olasılıklarına (TAVO) sahip hava savunma silahlarının doğrusal yaklaşan tehditlere tahsisi için matematiksel programlama ve sezgisel çözüm yöntemleri geliştirilmiştir. İlk olarak, savunulan tesislerin toplam hayatta kalma şansını maksimize etmek için doğrusal olmayan bir matematiksel model formülasyonu yapılmıştır. Daha sonra, bu modelin amaç fonksiyonu ve kısıtları doğrusallaştırılmıştır. Zamana bağlı TAVO değerlerine uygun bir kapalı form yaklaşımı bulunmuş ve bu yaklaşım doğrusal modele uyarlanmıştır. Doğrusal model değişik senaryolarla test edilmiş ve sonuçlar doğrusal olmayan orijinal modelin sonuçları ile karşılaştırılmıştır. Doğrusal modelin doğrusal olmayan modele göre çok daha hızlı çözüldüğü ve uygun sonuçlar verdiği gözlenmiştir. Model çözümlerinden, tehditlerin silahlara yaklaşması ve böylece TAVO değerlerinin yükseltilmesi için angajmanların mümkün olduğunca geç çizelgelenmesi gerektiği sonucuna varılmıştır. Bu gözlem temel alınarak, problem için bir sezgisel yapılandırma yöntemi geliştirilmiştir. Bu sezgisel yöntemin çıktılarını kullanan ve eldeki çözümü iyileştirmeyi hedefleyen ikinci bir sezgisel yöntem de önerilmiştir. Son olarak tüm çözüm yöntemleri kırk senaryo üzerinde test edilmiştir. En hızlı iki çözüm yöntemi olan doğrusal model ve sezgisel yapılandırma, büyük bir senaryo için karşılaştırılmış ve silah-tehdit tahsis problemi için en uygun teknikler olarak önerilmiştir.

Anahtar Kelimeler: Hava Savunması, Silah-Tehdit Tahsisi, Çizelgeleme, Zamana Bağlı Tek Atışta Vurma Olasılıkları, Doğrusallaştırma

ACKNOWLEDGMENTS

I would like to express my gratitude and sincere thanks to my supervisors, Prof. Dr. Levent Kandiller and Prof. Dr. Nur Evin Özdemirel for their patience, guidance and encouragement. Without their hard work, understanding and support, it would be impossible to complete this thesis. Being their thesis student was an invaluable experience.

I am also thankful to respected committee members, Prof. Dr. Ömer Kırca and Assoc. Prof. Dr. Sedef Meral for accepting to participate in committee and their valuable comments and suggestions.

I am indebted to Dr. Hüseyin Yavuz, my manager and committee member, for his support, consideration, valuable comments and suggestion for this work. I am grateful to him for his help in and out of the text.

I would like to express my appreciation to my manager İsmet Atalar and my colleagues Mehmet Kayhan, İnci Yüksel, Aybeniz Yiğit and Özgün Töreyen for their motivation and friendship.

I would like to thank to my family for all their support for the hard times I had during the completion of this work.

Finally, I must express my great thanks to Özlem Bayri, for her love, patience, support and being with me whenever I need a hand.

To Özlem

TABLE OF CONTENTS

ABSTRACT iv
ÖZ vi
ACKNOWLEDGMENT viii
LIST OF FIGURES xiv
1.INTRODUCTION 1
2.LITERATURE SURVEY 4
2.1.Linear and Nonlinear Modeling Approaches 4
2.2.Heuristic Approaches 8
2.3.Most Relevant Studies 9
3.PROBLEM FORMULATION 13
3.1.Problem Definition 13
3.2.Assumptions of a Scenario 15
3.3.Problem Formulation 18
3.3.1.Nonlinearity in Constraints 20
3.3.2.Nonlinearity in Objective Function 22
3.3.3.Nonlinearity Due to Time Varying Hit Probabilities 29
3.3.4.Final Formulation 32
3.4.Verification of the Mathematical Model 33
3.4.1.Solutions of Scenarios with Varying Hit Probabilities 38
3.4.2.Solutions of Scenarios with Fixed Hit Probabilities 40
4.HEURISTIC SOLUTION 43
4.1.Construction Heuristic 44
4.2.Improvement Heuristic 51
5.EXPERIMENTAL RESULTS 55
5.1.Experimental Settings 55
5.2.Experimental Results 59

6.CONCLUSION 71
REFERENCES 75
APPENDIX A 78
APPENDIX B 80
APPENDIX C 86

LIST OF TABLES

Table 1 – Points of Inflection for Different Number of Threats 26

Table 2 – Best points for $a_k^{|I_k|}$ graphs in cases of two and four segments 27

Table 3 – Probability Values for Scenarios... 34

Table 4 – Parameters of Approximated SSHP Functions 34

Table 5 – SSHP Values over Time Obtained With Fitted Curves 36

Table 6 – Fixed SSHP Values Used In Scenarios 37

Table 7 – Coefficients of the Approximation to SSHP Function.......................... 37

Table 8 – Solutions with Varying SSHP Values 39

Table 9 – Solutions with Fixed SSHP Values.. 41

Table 10 – Initial Engageability Time Windows 49

Table 11 – Threats Used In the Example 49

Table 12 – Weapons Used In the Example 49

Table 13 – Construction Heuristic Output for the Example................................. 51

Table 14 – Improvement Heuristic Output for the Example................................ 54

Table 15 – Scenarios of Size 3x3x2 56

Table 16 – Scenarios of Size 6x6x4 57

Table 17 – Objective Function Values and Deviations from Upper Bound for Scenarios of Size 2x2x1 60

Table 18 – Objective Function Values and Deviations from Upper Bound for Scenarios of Size 3x3x2 61

Table 19 – Objective Function Values and Deviations from Upper Bound for Scenarios of Size 6x6x4 62

Table 20 – Solution Times For Scenarios of Size 2x2x1 (Seconds)..................... 67

Table 21 – Solution Times for Scenarios of Size 3x3x2 (seconds)....................... 67

Table 22 – Solution Times for Scenarios of Size 6x6x4 (seconds)....................... 68

Table 23 – Percent Deviation from Upper Bound and Time for Linear Model with 0.2% Gap 69
Table 24 –Linear Modeling and Construction Heuristic Outputs for Scenario Size Of 36x36x24 with Varying SSHP Values 70
Table 25 –Linear Modeling and Construction Heuristic Outputs for Scenario Size Of 36x36x24 with Fixed SSHP Values 70
Table 26 – Results of Curve Fitting Software for (1-SSHP) Values 80

LIST OF FIGURES

Figure 1 – Complete Picture of the Environment 14
Figure 2 – Threat Is Attacking Across From the Asset and Weapon 16
Figure 3 – Threat Is Approaching and Descending To Leave Ammunitions 16
Figure 4 – $a_k^{|I_k|}$ graphs for different values of $|I_k|$ 24
Figure 5 – $a_k^{|I_k|}$ graph for $|I_k| = 5$ 25
Figure 6 – Line Drawn To b_k Axis from Inflection Point 26
Figure 7 – Change in SSHP Values With Respect To Distance 29
Figure 8 – Change in SSHP Values With Respect To Time 30
Figure 9 – Graph Of $(1 - p_{jir})$ With Respect To Time 31
Figure 10 – Four Different SSHP Values and Their Approximations 35
Figure 11 – Flowchart of the Construction Heuristic 47
Figure 12 – Example Scenario for the Construction Heuristic 48
Figure 13 – Flowchart of the Improvement Heuristic 53
Figure 14 – Short Notation for Scenarios 55
Figure 15 – Average Percent Deviation of the Methods 63
Figure 16 – Maximum Percent Deviation of the Methods 64
Figure 17 – Percent Deviation of Models for Scenarios 66
Figure 18 – Generic Concave Curve 79

CHAPTER 1

INTRODUCTION

The history of war started with stones and sticks and continued with swords, axes, javelins and arrows. When horses and elephants started to appear in battlefields, they provided advantage to armies. After then, gunpowder and cannons were high technology equipment and had great impact on victories like conquest of Istanbul by Mehmet the Conqueror. However, these equipments were successful provided that knowledge and tactic were involved in using them.

Today technology still contributes to battlefield activities with newly developed weapons, intelligence and surveillance equipment, logistic vehicles and so on. Although technology has greatly changed since the early ages, the question remains the same: "How to use weapons and other equipment *effectively* in warfare?"

Since the invention of airplanes, air-to-surface attack has been greatly effective. In order to survive airplane attacks, air defense weapons have been developed. Today there are many different kinds of planes used in attack, and as many different types of weapons for defense against them. Again, the question is how to use these defensive weapons against the enemy air threats in the most competent way.

In warfare environment, effective usage of air defense weapons with awareness of the combat situation (weapon readiness, danger imposed by the enemy, asset vulnerability) and immediate reaction to enemy are crucial. The motivation behind this thesis is the need for a decision tool, which takes environmental weapon and threat related- characteristics into account, and suggests an effective course of action for air defense in a complex attack environment. From defense planning perspective, the complexity is mainly due to large number of weapon-threat combinations to be considered for potential engagements as well as the instantaneous decision-making. For example, if there are 10 weapons and 10 threats in a scenario, there are 100 possible combinations just for a single shot. Moreover, air attacks do not take a long time; if the range of an artillery weapon is reckoned, the attack-defense process lasts less than a minute. Therefore, engagement solutions for battlefield management are needed instantly.

In a surface-to-air defense, there are assets with no defense capability against air threats. Some of the assets possess high importance and these are natural targets of the threats. The purpose of this thesis is to offer a method to achieve maximum overall survivability of assets considering their importance or value. We propose a mixed integer programming formulation to assign weapons to the threats and schedule rounds fired at threats.

In literature, vast majority of the sources uses fixed single shot hit probability values against threats, which remain constant through a scenario. The contribution to literature intended by this thesis is to suggest a solution scheme for air defense weapons with time varying hit probabilities.

Besides its combinatorial nature, our mixed integer formulation is nonlinear in objective function, in constraints and due to time varying hit probabilities. We develop techniques to linearize all these and obtain a form that can be solved in reasonable time. We also propose a heuristic solution approach for large scenarios. One possible use of our work is in the weapon and ammunition requirement planning by means of solving a large number of plausible scenarios. Its use in real-time defense planning to provide decision support for command and control centers may also be considered as well as off-line training exercises.

The thesis is organized is as follows. In Chapter 2, a brief overview of literature is given. Since the weapon target allocation problems have been studied to great extend, the most relevant work are mentioned only. Two main studies to which our work is closely related are presented in more detail. Chapter 3 contains problem definition, assumptions and the nonlinear model formulation of the problem. Linearization of the nonlinear model and verification of this linear model are included. Construction and improvement heuristics developed for the problem are given in Chapter 4. In Chapter 5, five different solution methods are tested on forty scenarios. Their solution quality and computation times are compared and discussed. Finally, Chapter 6 concludes the thesis by discussing the proposed methods for weapon-target allocation problem and suggests future work.

CHAPTER 2

LITERATURE SURVEY

Weapon target assignment (WTA) problems have been investigated as a class of resource allocation problems and therefore numerous articles can be found in literature. Among the solution approaches taken in these articles, vast majority are linear and nonlinear modeling. Heuristic approaches also have a significant share in solution procedures. WTA problems are usually solved for military purposes regardless of the weapon/target platform. However, WTA approach to advertising and economics are also present in literature. With this wide area of application, we need to focus on the part of the literature that is close to our work. For a comprehensive set of classification criteria one can refer to Matlin (1968). Although this work is relatively old and about the missile allocation problem, it provides good insight for classification purposes. In this chapter, we summarize the literature we find relevant in three sections. Section 2.1 reviews linear and nonlinear modeling approaches to different types of WTA problems. Section 2.2 is about the heuristic approaches to WTA problems with some novel applications. Two sources that are closely related to our work are discussed in Section 2.3.

2.1. Linear and Nonlinear Modeling Approaches

WTA problems usually have probabilistic character and thus they are nonlinearly modeled. It is clear that nonlinearity results in complex and time consuming solution procedures. Even for the simplest forms of WTA problems, Lloyd and Witsenhausen (1986) prove that the problem is NP-Complete. Hence, fast procedures providing near optimal solutions are considered acceptable. We review relevant studies in chronological order and give objective functions when they are similar to the one we use, since constraint sets are more or less the same, basically weapon and munitions availability constraints.

Den Broeder et al. (1959) investigate the WTA problem with identical weapons. Their purpose is to assign m missiles to n targets in order to neutralize at least k targets. In the first version of the problem, all targets have the same value but each target has a different probability of getting killed, independent of the missile type attacking. In the second version, it is assumed that the value of each target is known. This second version is also referred to as the Flood's Assignment Problem.

Lemus and David (1963) study the threats carrying different types of weapons to attack an assemblage of targets each of which has a certain worth. Probability of a target being hit by each attacker is known. The aim is to maximize the expected value of the targets destroyed by assigning highly reliable and powerful weapons to the targets. A nonlinear objective function is given and it is simplified down to the transportation problem. Lagrange Multipliers method is applied to solve this problem.

Day (1966) considers a three-stage WTA problem and tries to allocate a given number of weapons to the enemy with the aim of maximizing the expected weighted damage to the target system. Each element of this target system is indexed according to its strategic value. Priority indexes are used to assign specific types of weapons to specific target elements.

The three early studies mentioned above make use of nonlinear models. Passy (1971) also analyzes two different classes of nonlinear assignment problems and shows that one class of problem can be reformulated as a geometric program, while the second can be transformed into a complementary geometric program. Moreover, an algorithm for weapon assignment problems is developed and applied to an example. The objective function of the first problem is

$$\min \sum_{j=1}^{j=n} a_j \prod_{i=1}^{i=m} \left(1 - p_{ij}\right)^{x_{ij}}$$

where a_j is the value of target j, p_{ij} is the probability that weapon i hits target j, and x_{ij} is the binary assignment variable. Objective function of the second problem, which is equivalent to the one above, is

$$\min \sum_{j=1}^{j=n} a_j \prod_{i=1}^{i=m} \left(1 - p_{ij} x_{ij}\right)$$

Both problems minimize the expected value of surviving enemy targets. The first problem is handled by applying the logarithmic transformation, converting it to general polynomial optimization problem with n^2 variables, and then solving the dual where n is the total variable number. The second problem is obtained by replacing $\left(1 - p_{ij}\right)^{x_{ij}}$ with $\left(1 - p_{ij} x_{ij}\right)$ and defining new constraints in the linear program. A case of four weapons and four targets is investigated.

Multi-stage version of the WTA problem, where the offense launches a number of rounds aimed at assets of the defense, is investigated by Hosein and Athans (1990). In each stage of the engagement, the defense observes the outcomes of the assignments made in the previous stage before assigning a subset of the remaining weapons in the present stage. Under suitable assumptions, as the number of targets approaches infinity, the problem can be treated as a deterministic one in which the number of targets that survive over a stage equals its expected value. This result can be used to provide lower bounds on the optimal cost for problems with a finite number of targets.

Kwon et al. (1999) consider a WTA problem with the objective of minimizing the overall firing cost. The problem is formulated as a nonlinear integer programming model. Lagrangean relaxation followed by a branch-and-bound scheme is applied to the problem after linearizing the nonlinear constraints.

An extensive study to develop exact methods for solving WTA problems is carried out by Ahuja et al. (2003). They try to assign n weapons to m targets such that the total expected survival value of the enemies after the engagement is minimum. The main aim of their work is to deliver an exact method for the solution of the problem. They propose several approaches such as linear programming, integer programming, network flow based lower bounding methods, network flow based construction heuristic, as well as a very large-scale neighborhood search algorithm.

Malcolm (2004) investigates two classes of static defensive resource allocation problems: "target-value based" weapon target allocation and "asset-value based" weapon allocation problem. Target-value based problem is converted to the transportation problem. Asset-value based problem, on the other hand, is more difficult than the target value based problem. It maximizes the following objective function

$$\max \sum_{k}^{K} W_k \prod_{i \in G_k} \left[1 - \pi_i \prod_{j=1}^{M} \left(1 - p_{(i,j)}\right)^{x_{(i,j)}} \right]$$

where W_k is the value of asset k, G_k is subset of targets aimed at k, π_i is probability that target i destroys the asset it aims at, $p_{(i,j)}$ is probability that weapon j eliminates target i, and $x_{(i,j)}$ is the binary decision variable indicating the engagement. Malcolm considers a case of five threats and five defensive resources for the target-based problem. The asset-based problem is formulated, but its solution is left for future work.

Rosenberger et al. (2005) extend the basic WTA problem by allowing for multiple target assignments per platform, subject to the number of weapons available and their effectiveness. They formulate the problem as a linear integer programming and investigate two solution methods. The first method is a greedy approach based on the sequential application of the auction algorithm that was generalized for assigning n resources to m targets. The second method is built on a branch-and-bound framework that enumerates feasible tours of resources (a process that can become computationally intensive with increasing number of sources and targets but finds an optimal solution).

2.2. Heuristic Approaches

As a heuristic approach, neural network based optimization in WTA is examined by Wacholder (1989). He focuses on the solution of the static problem to combine fast solutions of successive static problems to reach good solutions for the dynamic problem. Wacholder approaches the problem aiming to minimize the total leakage between defense layers. He specifies the single shot kill probability for each weapon platform-target pair. In addition, this probability depends on the relative geometry of the weapon and the target. If the velocity and the direction are given, the probability is assumed to be calculated.

Lee et al. (2002) proposed an immunity-based ant colony optimization with the fitness function of

$$\min \sum_{i=1}^{T} EDV(i) \times PK(i),$$

where *EDV(i)* is the expected damaging value of target *i* and *PK(i)* is the probability of missing target *i*. It is computed as

$$PK(i) = 1 - \prod_{j=1}^{W} \left(1 - K_{ij}\right)^{X_{ij}},$$

where K_{ij} is probability of killing a target and X_{ij} is the binary decision variable of engagement. An assumption Lee et al. make is that all weapons must be assigned to targets. They assume that kill probabilities are known for all weapon-target pairs.

Lee et al. (2003) contributed to the related heuristic literature with an evolutionary algorithm. In this study, they use quality improvement process of an offspring, which is called “eugenics”. They use their exact algorithm’s objective function described above as the fitness function in their genetic algorithm.

Blodgett et al. (2003) study application of the tabu search heuristic to the WTA problem. The problem is to construct defense plans to guide the allocation and scheduling of different types of defense weapons against anti-ship missiles, subject to various physical and operational constraints. The approach first constructs a feasible base scheme and then improves it by using tabu search heuristic methods.

In all these sources, except Wacholder (1989), the single shot kill probabilities are assumed constant during the attack scenario. In Wacholder's work, variability of these probabilities is acknowledged to unknown enemy flight trajectories. No solution procedure is given for this case, but several problem types are attached assuming constant probabilities. In our work, we allow these probabilities to vary over time as the air threat approaches its land target during the course of the engagement.

2.3. Most Relevant Studies

So far, we have introduced related work done in the literature. Our study makes direct use of two previous studies, Karasakal (2004) and Metler and Preston (1989), therefore we review them here in more detail.

Karasakal (2004) has studied the air defense problem of a naval task group as the Missile Allocation Problem (MAP). Ships of the task group are assumed to be stationary, hence they are similar to land targets. Among his formulations, MAP1 is the model that checks whether the desired minimum level of hit probability against each threat is attainable. The only nonlinear constraint is

$$1-\prod_{\{j\in M|(i,j)\in V\}}\left(1-p_{ij}\right)^{x_{ij}}\geq h_i \qquad \text{for all } i\in N \tag{2.1}$$

where M is the set of weapons, N is the set of threats, V is the set of achievable engagements, p_{ij} is probability of killing threat j by weapon i, h_i is the minimum level of success desired against threat i, and x_{ij} is the binary decision variable that indicates the engagement of weapon i to threat j. Linearization is done by taking the natural logarithm of both sides of (2.1). The resulting constraint is

$$\sum_{\{j\in M|(i,j)\in V\}} a_{ij}x_{ij}\geq b_i \qquad \text{for all } i\in N \tag{2.2}$$

where $a_{ij} = \lfloor -\beta \ln(1 - p_{ij}) \rfloor$, $b_i = \lfloor -\beta \ln(1 - h_i) \rfloor$ and β is a large number scaling the constraint . MAP1 is modified in two directions. One of them, MAP1.1 focuses on minimizing the total ammunition used, while the other, MAP1.2, tries to find the minimum divergence from b_i in (2.2) by introducing an "excess" e_i to be minimized.

$$\sum_{\{j \in M | (i,j) \in V\}} a_{ij} x_{ij} + e_i \geq b_i \text{ for all } i \in N$$

Lagrangean relaxation method is proposed for solving these formulations. To extend MAP 1, time dimension is added as small discrete intervals. Engagement scheduling and ammunition inventory constraints are introduced in MAP2. Shoot-Look-Shoot (SLS) tactic is used, which means that consecutive engagements on a threat should not overlap in time. The objective of MAP2 is to maximize the no-leaker probability, i.e.

$$\max \prod_{i \in N} \left(1 - \prod_{\{k \in K\}\{j \in M | (i,j) \in V\}} (1 - p_{ij})^{x_{ijk}} \right)$$

Here index k indicates the k^{th} discrete time slot, and p_{ij} probability of hit of the weapon i against threat j. Binary decision variable x_{ijk} indicates that an engagement (a single shot) is scheduled to being in time slot k. The same linearization technique as above and some additional operations are proposed for linearly solving the problem.

Karasakal also formulates the problem with continuous time dimension and names it as MAP3. Continuous time scheduling constraints are brought in for engagements on a threat which should not overlap according to the SLS tactic. In order to calculate the time spent for each engagement correctly, head-to-head collision is assumed. The formulation of head-to-head collision time in terms of velocity of threat, detection distance and setup time of weapon causes a nonlinearity in the model. This nonlinearity is resolved by introducing new variables and linking constraints. The objective function is similar to that of

MAP2 so that its linearization is already handled. A heuristic approach is developed for MAP3 and validated against implicit enumeration results.

In our modeling, we make use of MAP3 except for the objective function. We use a different objective function, but our constraints are the same as Karasakal's MAP3. We cope with the nonlinearity in the objective function and in the collision constraint in a similar manner as Karasakal does. However, there is a fundamental difference. As the single shot kill probabilities, p_{ij} are used in the MAP3 objective function, i.e. they are not time-variant but constant over the entire planning horizon. In our case, we allow time-variant probabilities, which introduce a third source of nonlinearity.

The second source we make use of is Metler and Preston (1989). In this work, allocating defense resources against ballistic missile threats is modeled. Two heuristic solution approaches are described, based on a greedy scheme and on linear programming. The objective function used in this source is

$$\max \sum_{k} w_k \prod_{j \in J_k} \left[1 - \prod_{i} \prod_{t} \left(1 - p_{ijt} \right)^{x_{ijt}} \right] \tag{2.3}$$

where w_k is the value of the asset k, J_k is the set of threats aiming at asset k, p_{ijt} is the probability of killing threat j by weapon i at time t. They assume that kill probability of the threats is equal to unity. In other words, if any threat leaks through the defense layers, the targeted asset is hit with certainty. A heuristic approach to solve this problem formulation begins with determining an index of defendability and ranking the assets according to this index. This part is called the *asset selection* algorithm. Then, starting with the first asset, the *shot algorithm*, which is based on a rule of marginal objective increase in each engagement and avoiding constraint violation, is applied to the shots. The result is fed back to the asset selection algorithm. When an asset loses its defendability property according to the asset selection algorithm, it is dropped from consideration for additional defensive shots. The heuristic runs until resource constraints are violated or all assets are dropped.

In their second approach, Metler and Preston linearize (2.3) in a manner that resembles to Karasakal's linearization, however some additional operations are done.

Our objective function is similar to Metler and Preston's, however we have assume that enemy may not succeed every time it leaks. In other words, there is also a probability that the threat hits the asset. We linearize the objective function in a similar manner, which will be explained in detail in Chapter 3.

CHAPTER 3

PROBLEM FORMULATION

3.1. Problem Definition

In warfare, surface-to-air defense has three main elements: assets, threats and weapons. Assets are of several types such as

- command and control (C^2) centers,
- ally radars,
- arsenal buildings, and
- communication centers.

Assets usually have no self-defense capability, but sometimes weapons are also considered as assets. Assets usually differ in terms of importance. Thus, while some assets are critical in value and need to be protected better, others may not be cared as much.

The main aim of air defense is to defend the assets by using weapons to neutralize the threats. Threats are generally airplanes flying at very high speeds. They send rockets to or drop bombs onto assets. Of the wide variety of weapons, we will be dealing with artillery type of weapons in this study. During the engagement period, radars supply information to C^2 center on velocity, position and type of threats. C^2 center checks the weapon availability, decides on the best engagement strategy and sends engagement orders to weapons. If the weapon accepts the order, it prepares to fire. This preparation includes

- loading ammunition,
- turning the muzzle,
- watching and tracking the threat, and
- engaging and firing.

This engagement period as a whole is called *weapon setup time*, which may be different for each weapon. A picture of the environment is given in Figure 1

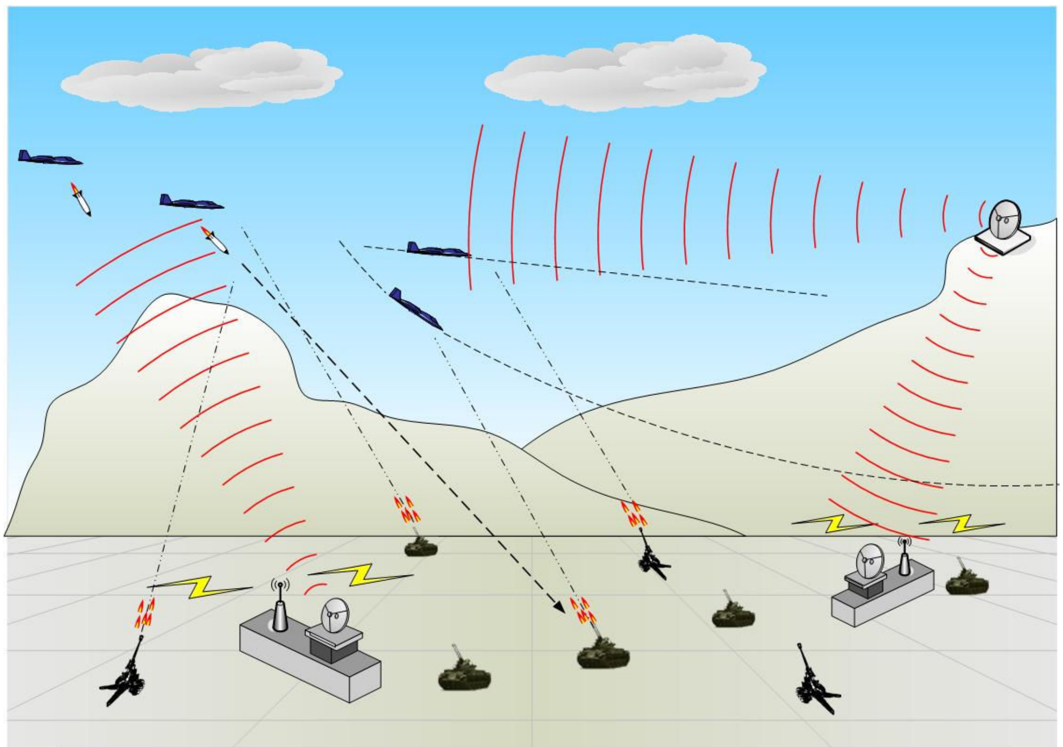

Figure 1 – Complete Picture of the Environment

In this environment, C^2 center has to decide on which weapon should engage which threat, how many rounds should be fired at each threat, and exactly when these rounds should be fired. We intend to develop an approach for making these decisions so as to maximize the weighted sum of the survival probabilities of assets. Here, the weights reflect the value or importance of assets. In doing this, we need to consider the ammunition availability and the number of engagement opportunities, given the range of weapons and time of flight of threats.

3.2. Assumptions of a Scenario

Assumptions of the problem are classified according the elements of the warfare environment. First, assumptions about threats are given, followed by assumptions on weapons and other assumptions.

Assumptions about threats:

- Threats are usually airplanes. However, if necessary, ammunition can also be regarded as threats.
- Threats make linear movement with constant speed so that their velocity does not change during the attack.
- A threat can attack one or more assets in a given scenario.
- The probability that a threat kills the asset it is aiming at is constant.
- A threat may attack in two ways:
    - Shoot at a distance: Threats do not come near assets. They send rockets from a distance and try to stay clear of danger (Figure 2). In this case the threat is the rocket itself and weapons try to intercept the rockets.
    - Descend and drop bombs: Planes that fly at high altitudes descend rapidly as they get close to assets, drop their bombs, and head off (Figure 3). In this case, the probability that a threat kills the asset is an aggregation of the hit probabilities of individual bombs. Weapons try to hit planes.

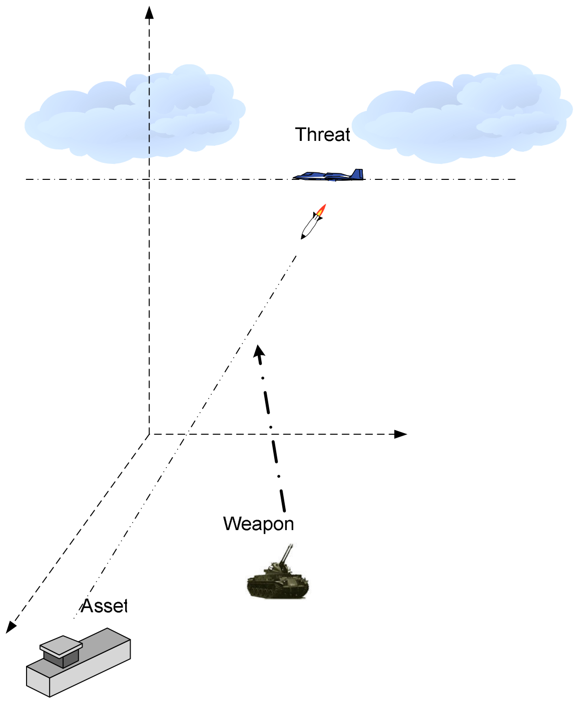

Figure 2 – Threat Is Attacking Across From the Asset and Weapon

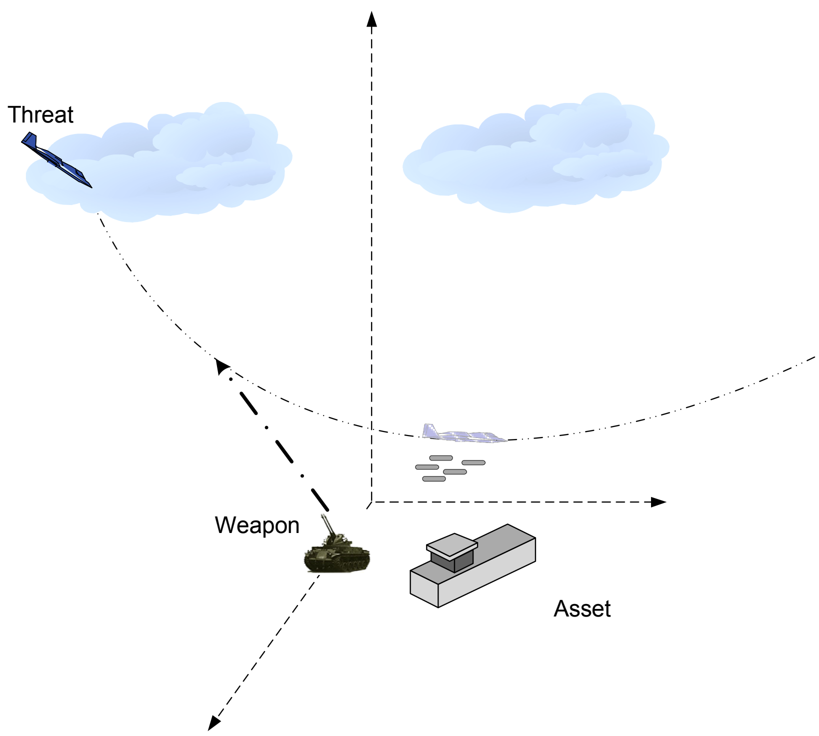

Figure 3 – Threat Is Approaching and Descending To Leave Ammunitions

Assumptions about weapons:

- Weapons have specific minimum and maximum target ranges.
- Weapons have variable single shot hit probabilities based on the position of the threat. Since velocities of threats are invariant during the attack and weapons are immobile, these probabilities change in time as the threat moves. These time-varying probabilities are calculated by Yüksel (2007) considering the random and systematical errors in the weapon systems and will be used in this study.
- Each weapon has its own constant setup time.
- Weapons are not dedicated to defending a specific asset. They can engage any threat if the engagement is feasible. This implies that area defense is assumed.
- Weapons can also be defined as assets.
- A weapon can engage multiple threats simultaneously.

Other assumptions related to the scenario:

- Assets are immobile and vulnerable to attack.
- Radars supply information correctly without any time delay through the communication system so that we know the initial detection position of each threat.
- In the decision process of engagements, Shoot-Look-Shoot (SLS) tactic is assumed. In this tactic, after a shot is fired at a threat, C^2 center "looks" to see whether the threat survives or not and then have weapons fire another round only if it is not neutralized. This tactic requires that engagements on a threat should not overlap. In other words, there must be at most one engagement to an enemy at a time in a given scenario.

3.3. Problem Formulation

In an attack scenario, assume that there are J weapons indexed as $j = 1,\ldots,J$ defending K assets, indexed as $k = 1,\ldots,K$ against I threats indexed as $i = 1,\ldots,I$. We use the following notation.

w_k : Importance or value of asset k.

q_{ik} : Kill probability of threat i against asset k.

I_k : The set of threats aiming at asset k. If a weapon is considered as an asset, I_k can also be defined for it.

s_i : Maximum number of rounds that can be fired at threat i until the threat reaches its target, according to the SLS tactic. Each of these rounds on a threat is indexed as r such that $r = 1, 2, ..., s_i$ where s_i is found by assuming the fastest weapon fires all rounds at a threat, and the threat is detected at the farthest point in weapon's range allowing as many rounds as possible.

d_j : Number of rounds of ammunition available on weapon j.

p_{jir} : Single Shot Hit Probability (SSHP) of weapon j against threat i for the r^{th} round.

$\left[Q_{ji}, R_{ji}\right]$: Engageability time window of threat i and weapon j due to weapon's range. The earliest engagement must not start earlier than Q_{ji}. The latest engagement must end no later than R_{ji}.

Δ_j : Constant set up time of weapon j.

M : A very large number.

The decisions include assignment of a weapon for each shot to be fired at a threat and scheduling of shots. Thus, we define three sets of decision variables.

$$x_{jir} = \begin{cases} 1, & \text{if weapon } j \text{ is assigned to target } i \text{ for the } r^{th} \text{ shot} \\ 0, & \text{otherwise} \end{cases}$$

t_{ir} : Beginning time of engagement for the r^{th} shot at threat i according to the SLS tactic.

$E\Delta_{jir}$: Time-to-engagement of weapon j to threat i for the r^{th} shot. According to the SLS tactic, the next shot at threat i can be fired only after the r^{th} shot reaches to threat in this duration.

Our mixed integer programming formulation is as follows.

$$\max \sum_{k=1}^{K} w_k \prod_{i \in I_k} \left(1 - q_{ik} \left[\prod_{j=1}^{J} \prod_{r=1}^{s_i} \left(1 - p_{jir}\right)^{x_{jir}} \right] \right) \tag{3.1}$$

s.t.

$$\sum_{i=1}^{I} \sum_{r=1}^{s_i} x_{jir} \le d_j \qquad \forall j \tag{3.2}$$

$$\sum_{i=1}^{J} x_{ji1} \le 1 \qquad \forall i \tag{3.3}$$

$$\sum_{j=1}^{J} x_{ji,r+1} \le \sum_{j=1}^{J} x_{jir} \qquad \forall i, r = 1,...,s_i - 1 \tag{3.4}$$

$$M\left(1 - \sum_{j=1}^{J} x_{ji,r+1}\right) + t_{i,r+1} \ge t_{ir} + \sum_{j=1}^{J} x_{jir} E\Delta_{jir} \quad \forall i,\ r = 1,...,s_i - 1 \tag{3.5}$$

$$\sum_{j=1}^{J} x_{jir} Q_{ji} \le t_{ir} \le \sum_{j=1}^{J} x_{jir} \left(R_{ji} - E\Delta_{jir}\right) \qquad \forall i,\ r = 1,...,s_i \tag{3.6}$$

$$t_{ir} \ge 0 \qquad \forall i, \forall r = 1,...,s_i \tag{3.7}$$

$$x_{jir} \in \{0,1\} \qquad \forall j, \forall i, \forall r = 1,...,s_i \tag{3.8}$$

(3.1) is the objective function to maximize the overall survivability of the assets considering their value, SSHP of the weapons and kill probabilities of the threats. Malcolm (2004) gives a similar objective function but he does not consider rounds of ammunition. Constraint (3.2) states that the number of round fired from weapon j cannot exceed the number of available on it. (3.3) and (3.4) express that the first shot on a target must be made at the beginning and shots should be in order afterwards. Constraint (3.5) assures that, after firing the r^{th} shot at threat i, we are allowed to fire the $(r+1)^{th}$ shot at the same threat after a certain time which is not less than time-to-engagement. If r^{th} shot is the last shot, then all binary variables for shot $r+1$ and beginning time of engagement for shot $r+1$ are zero, and use of the big number M makes the constraint feasible. Finally (3.6) ensures that beginning time of the r^{th} shot is within the engageability window.

Three points in the mathematical model cause it to be nonlinear. These are:

1. Multiplication of the terms $E\Delta_{jir}$ and x_{jir} in constraints (3.5) and (3.6).
2. Having x_{jir} in the exponent and multiplication of the terms in the objective function.
3. Time varying SSHP values.

Each of these sources of nonlinearity and their linearization are presented in the following sections.

3.3.1. Nonlinearity in Constraints

In (3.5) and (3.6), multiplication of the variables $E\Delta_{jir}$ and x_{jir} cause nonlinearity because $E\Delta_{jir}$ is defined as a function of t_{ir}. In order to eliminate this, we define the following parameters.

D_{ji} : Initial detection distance of target i to weapon j.

v_i : Velocity of threat i.

v_j : Velocity of round fired from weapon j.

Since threats are approaching linearly with constant speed, we should be able to express $E\Delta_{jir}$ as follows, assuming a head-to-head collision course.

$$E\Delta_{jir} = \frac{D_{ji} - v_i\left(\Delta_j + t_{ir}\right)}{v_j + v_i} + \Delta_j$$

Note that the $E\Delta_{jir}$ calculation given above is an oversimplification of the true engagement time. It assumes that the threat and the round fired are on a path for head-to-head collision. This is not true in general. For a better approximation, deflection angles need to be considered, which is beyond our scope.

We can interpret $\sum_{j=1}^{J} x_{jir} E\Delta_{jir}$ in constraints (3.5) and (3.6) as follows.

$$\sum_{j=1}^{J} x_{jir} E\Delta_{jir} = \sum_{j=1}^{J} x_{jir} \left(\frac{D_{ji} - v_i \left(\Delta_j + t_{ir} \right)}{v_j + v_i} + \Delta_j \right)$$

$$= \sum_{j=1}^{J} \left(\frac{D_{ji} - v_i \Delta_j}{v_j + v_i} \right) x_{jir} - \sum_{j=1}^{J} \left(\frac{v_i}{v_j + v_i} \right) x_{jir} t_{ir} + \sum_{j=1}^{J} \Delta_j x_{jir} \quad \forall i, \forall r = 1, \ldots, s_i$$

Here, multiplication of variables x_{jir} and t_{jir} creates nonlinearity. Hence, if we define a variable $y_{jir} = x_{jir} t_{ir}$, we can ensure $y_{jir} = 0$ when $x_{jir} = 0$, and $y_{jir} = t_{ir}$ whenever $x_{jir} = 1$. Let $\beta_{ji} = \frac{D_{ji} - v_i \Delta_j}{v_j + v_i}$ and $\gamma_{ji} = \frac{v_i}{v_j + v_i}$. Then, (3.5) and (3.6) respectively become

$$M\left(1 - \sum_{j=1}^{J} x_{ji,r+1}\right) + t_{i,r+1} \geq t_{ir} + \sum_{j=1}^{J} \beta_{ji} x_{jir} - \sum_{j=1}^{J} \gamma_{ji} y_{jir} + \sum_{j=1}^{J} \Delta_j x_{jir} \qquad \forall i, r = 1, \ldots, s_i - 1 \tag{3.9}$$

$$\sum_{j=1}^{J} x_{jir} Q_{ji} \leq t_{ir} \leq \sum_{j=1}^{J} x_{jir} R_{ji} - \sum_{j=1}^{J} \beta_{ji} x_{jir} + \sum_{j=1}^{J} \gamma_{ji} y_{jir} - \sum_{j=1}^{J} \Delta_j x_{jir} \qquad \forall i, r = 1, \ldots, s_i - 1 \tag{3.10}$$

We need the following additional constraints.

$$y_{jir} \leq M x_{jir} \qquad \forall j, \forall i, r = 1, \ldots, s_i \tag{3.11}$$

$$y_{jir} \leq t_{ir} \qquad \forall j, \forall i, r = 1, \ldots, s_i \tag{3.12}$$

$$y_{jir} \geq 0 \qquad \forall j, \forall i, r = 1,...,s_i \tag{3.13}$$

This resolves the nonlinearity induced by $E\Delta_{jir}$ term. Usage of y_{jir} is effective if we can enforce this variable to be equal to t_{jir} when $x_{jir} = 1$. To make the "less than or equal to" constraint in (3.12) binding, we have to add some weight for y_{jir} in the objective function so that the formulation tries to increase its value. However, this perturbed value should be as small as possible to avoid introducing bias to the original objective function and affecting values of x_{jir} variables.

Then, our objective function becomes

$$\max \sum_{k=1}^{K} w_k \prod_{i \in I_k} \left(1 - q_{ik} \left[\prod_{j=1}^{J} \prod_{r=1}^{s_i} \left(1 - p_{jir}\right)^{x_{jir}} \right] \right) + \sum_{j=1}^{J} \sum_{i=1}^{I} \sum_{r=1}^{s_i} \varepsilon y_{jir} \tag{3.14}$$

The parameter ε is scenario dependent and tuning of it will be investigated later.

3.3.2. Nonlinearity in Objective Function

Another nonlinearity in our model is due to having x_{jir} in the exponent and multiplication of the terms in the objective function given by (3.1). In order to handle that, we can adapt the approach of Metler and Preston (1999) for linearization. If we introduce constraint (3.16) together with a new decision variable set, a_{ik}, then our objective function can be stated as in (3.15).

$$\max \sum_{k=1}^{K} w_k \prod_{i \in I_k} a_{ik} \tag{3.15}$$

$$a_{ik} \leq 1 - q_{ik} \left[\prod_{j=1}^{J} \prod_{r=1}^{s_i} \left(1 - p_{jir}\right)^{x_{jir}} \right] \qquad \forall k, i \in I_k \tag{3.16}$$

Here a_{ik} is the survivability probability of asset k, against threat i. Our objective function and constraint (3.16) are still nonlinear. At this point, we can make another substitution that will have important effect on our formulation. This will change the meaning of the objective function for the sake of linearity. If we represent the survivability probability not in terms of threat-asset pair, but only based on each asset, we can linearize the objective function. Thus, replacing a_{ik} values with the lower bound $a_k = \min_{i \in I_k} \{a_{ik}\}$, we have

$$\max \sum_{k=1}^{K} w_k \prod_{i \in I_k} a_k = \sum_{k=1}^{K} w_k a_k^{|I_k|} \tag{3.17}$$

$$a_k \le a_{ik} \le 1 - q_{ik} \left[\prod_{j=1}^{J} \prod_{r=1}^{s_i} \left(1 - p_{jir}\right)^{x_{jir}} \right] \quad \forall k, i \in I_k \tag{3.18}$$

Note that using the lower bound a_k instead of the individual a_{ik} values changes the meaning of the objective function as maximizing the minimum weighted survival probability. In their work, Metler and Preston assumed $q_{ik} = 1$, that is if any threat leaks through the defense, it will certainly hit the asset. Consequently, if possible, each target will get at least one engagement on it and the probability of no-leaker will be greater than zero. In our formulation, since $0 < q_{ik} \le 1$, for small q_{ik} value of a threat, the model may choose not to assign any weapon to it. Then, no-leaker probability might be zero for a scenario.

Nevertheless, this new formulation might give us several clues about the solution of the problem and may provide some insights. Continuing with our investigation, the right hand side of (3.18) can be linearized by taking logarithms of both sides.

$$\ln\left(1 - a_k\right) \ge \ln\left(1 - a_{ik}\right) \ge \ln\left(q_{ik}\right) + \sum_{j=1}^{J} \sum_{r=1}^{s_i} x_{jir} \ln\left(1 - p_{jir}\right) \quad \forall k, i \in I_k \tag{3.19}$$

If we let $\ln\left(1 - a_k\right) = -b_k$ then

$$a_k^{|I_k|} = \left(1 - e^{-b_k}\right)^{|I_k|} \tag{3.20}$$

Here, b_k is not a variable but a value greater than or equal to zero. b_k will be used for making approximation to (3.17). The graph of $a_k^{|I_k|}$ for different values of $|I_k|$ is given in Figure 4. We can make piecewise linear approximation to a curve in

Figure 4 for a specific value of $|I_k|$. For example, when $|I_k| = 5$ for asset k, piecewise linear approximation can be done as in Figure 5.

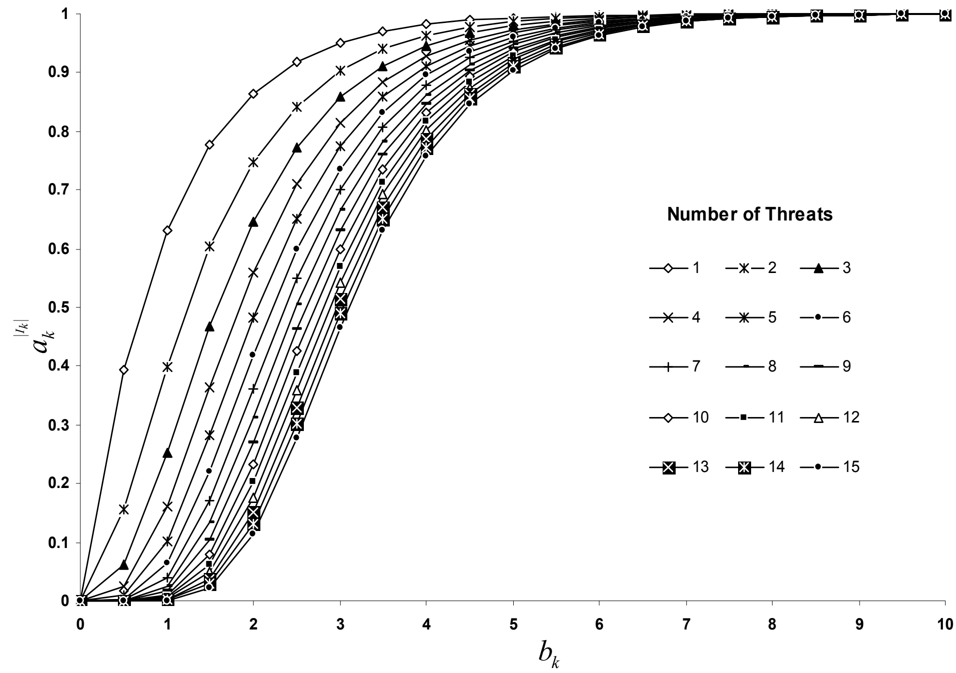

Figure 4 – $a_k^{|I_k|}$ graphs for different values of $|I_k|$

In Figure 5, we have zone related $Z_{\alpha k}$ variables, where α represents the segment number and $c_{\alpha k}$ stands for the slope of the α^{th} segment. Since $c_{1k} \geq c_{2k} \geq c_{3k}$, maximizing $a_k^{|I_k|}$ is equivalent to maximizing $\sum_{\alpha=1}^{3} c_{\alpha k} Z_{\alpha k}$. If we have Υ segments, indexed as $\alpha = 1,...,\Upsilon$, the objective function (3.16) can be rewritten as

$$\max \sum_{k=1}^{K} w_k \sum_{\alpha=1}^{\Upsilon} c_{\alpha k} Z_{\alpha k} \tag{3.21}$$

with the additional constraint

$$\sum_{j=1}^{J} \sum_{r=1}^{s_i} \left[\ln\left(1 - p_{jir}\right) \right] x_{jir} + \sum_{\alpha=1}^{\Upsilon} Z_{\alpha k} \leq -\ln\left(q_{ik}\right) \quad \forall k, \forall i \tag{3.22}$$

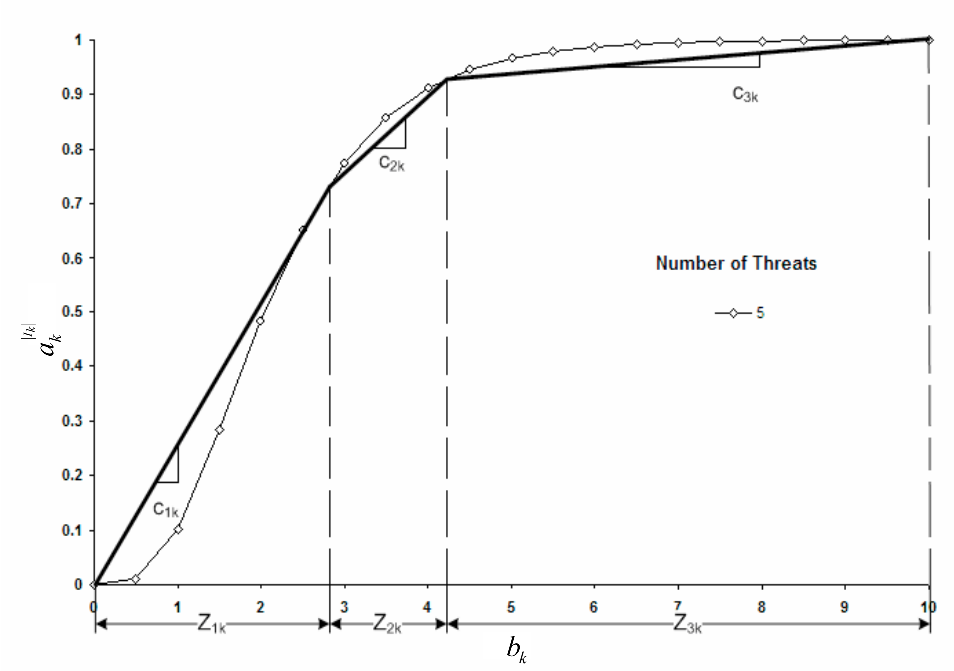

Figure 5 – $a_k^{|I_k|}$ graph for $|I_k| = 5$

To sum up, after all these operations, we have a tradeoff between the objective function and linearity. We have obtained a linear objective function at the expense of changing the meaning of the objective. The curve in Figure 5 consists of convex and concave parts. The first line segment which corresponds to the convex part passes through the origin. Instead, if we draw this line from the point of inflection to b_k axis as in Figure 6, the curve can be treated as a concave graph.

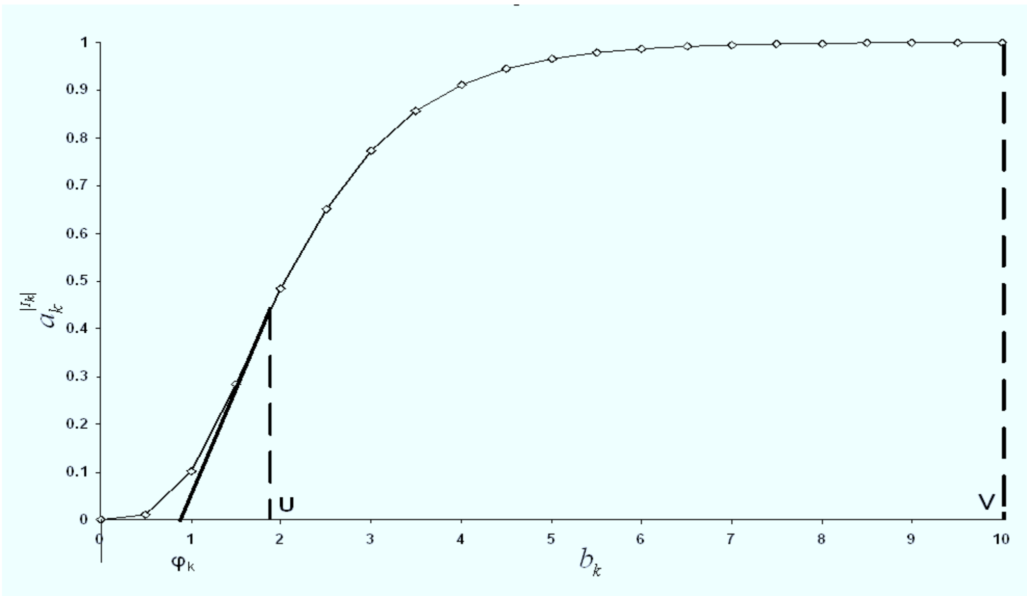

Figure 6 – Line Drawn To b_k Axis from Inflection Point

The method of optimally segmenting a concave graph into two parts is given in Appendix A. Inflection points for different number of threats are given in Table 1

Table 1 – Points of Inflection for Different Number of Threats

# of Enemies	**Concavity Begins (b_k)**	**Concavity Begins ($a_k^{\|I_k\|}$)**	**Slope of the segment**	**Intersection with b_k-axis (φ_k)**
1	0.00	0.0000	0.0000	0.0000
2	0.70	0.2534	0.5000	0.1931
3	1.10	0.2969	0.4444	0.4319
4	1.39	0.3180	0.4219	0.6363
5	1.61	0.3279	0.4096	0.8094
6	1.80	0.3382	0.4019	0.9584
7	1.95	0.3455	0.3965	1.0787
8	2.08	0.3438	0.3927	1.2044
9	2.20	0.3475	0.3897	1.3083
10	2.31	0.3516	0.3874	1.4026
11	2.40	0.3513	0.3855	1.4888
12	2.49	0.3540	0.3840	1.5682
13	2.57	0.3552	0.3827	1.6419
14	2.64	0.3547	0.3816	1.7105
15	2.71	0.3560	0.3806	1.7747

In Figure 6, suppose the function is concave for $u \le b_k \le v$, and let λ_k be the variable representing the b_k value at the point of segmentation. In the same manner, $f(\lambda_k) = a_k^{|I_k|} = \left(1-e^{-\lambda_k}\right)^{|I_k|}$. Then,

$$f'(\lambda_k) = |I_k| e^{-\lambda_k} \left(1-e^{-\lambda_k}\right)^{|I_k|-1}, \tag{3.23}$$

and

$$\alpha = \frac{\left(1-e^{-v}\right)^{|I_k|} - \left(1-e^{-u}\right)^{|I_k|}}{(v-u)} . \tag{3.24}$$

Using the main result in Appendix A, we get

$$e^{-\lambda_k} \left(1-e^{-\lambda_k}\right)^{|I_k|-1} = \frac{\left(1-e^{-v}\right)^{|I_k|} - \left(1-e^{-u}\right)^{|I_k|}}{(v-u)}. \tag{3.25}$$

Table 2 – Best points for $a_k^{|I_k|}$ graphs in cases of two and four segments

# of Threats	**Concavity Begins (b_k)**	**Concavity Begins ($a_k^{\|I_k\|}$)**	**2 Pieces**	**4 Pieces**		
			λ_k	λ_k^1	λ_k^2	λ_k^3
1	0.00	0.0000	2.7726	1.0843	2.7726	5.3549
2	0.70	0.2534	3.6879	2.0036	3.6879	6.2091
3	1.10	0.2969	4.1195	2.4453	4.1195	6.6080
4	1.39	0.3180	4.4142	2.7496	4.4141	6.8790
5	1.61	0.3279	4.6343	2.9781	4.6343	7.0810
6	1.80	0.3382	4.8172	3.1686	4.8172	7.2482
7	1.95	0.3455	4.9643	3.3221	4.9643	7.3827
8	2.08	0.3438	5.0909	3.4544	5.0909	7.4981
9	2.20	0.3475	5.2048	3.5738	5.2048	7.6019
10	2.31	0.3516	5.3077	3.6817	5.3077	7.6954
11	2.40	0.3513	5.3953	3.7735	5.3953	7.7750
12	2.49	0.3540	5.4792	3.8618	5.4792	7.8512
13	2.57	0.3552	5.5547	3.9411	5.5547	7.9197
14	2.64	0.3547	5.6223	4.0121	5.6223	7.9810
15	2.71	0.3560	5.6876	4.0809	5.6876	8.0401

We cannot obtain a closed form expression for λ_k, and we need to apply numerical techniques to obtain the optimal points. Using quasi-Newton search with starting point of zero, we get the values in Table 2. In this table, we have the best λ_k points for two and four pieced linear approximation.

Setting $c_{\alpha k}$ as the slope and $U_{\alpha k}$ as the upper bound of the α^{th} segment out of Υ segments in an appropriate linear piecewise approximation, we can rewrite the approximated objective function as

$$\max \sum_{k=1}^{K} w_k \sum_{\alpha=1}^{\Upsilon} c_{\alpha k} Z_{\alpha k} + \sum_{j=1}^{J} \sum_{i=1}^{I} \sum_{r=1}^{s_i} \varepsilon y_{jir} \tag{3.26}$$

We need to add (3.22) to our constraint set. Yet, since our piecewise approximation does not start at origin for $|I_k| \geq 2$, we need to add the first part of b_k axis. In addition, this modification helps us to make assignments that provide high hit probabilities. If no assignment is made, the constraint is satisfied since $\ln(q_{ik}) \leq 0$. Thus, we modify (3.22) as

$$\sum_{j=1}^{J} \sum_{r=1}^{s_i} \left[\ln\left(1 - p_{jir}\right) \right] x_{jir} + \sum_{\alpha=1}^{\Upsilon} Z_{\alpha k} + \leq -\left(\ln\left(q_{ik}\right) + \varphi_k \right) \quad \forall\, k, \forall i \tag{3.27}$$

$$0 \leq Y_{\alpha k} \leq U_{\alpha k} \qquad \forall k, \alpha = 1, 2, ..., \Upsilon \tag{3.28}$$

where φ_k is the point that the first piece of the approximation touches the abscissa of $a_k^{|I_k|}$ graph for asset k.

By altering the meaning of the objective function as maximizing the minimum survivability of each asset, we achieve to have an approximated linear objective function where the original nonlinear objective function means maximizing the overall weighted survivability of all assets. In (3.27), as q_{ik} gets higher for threat i, model will try to make more engagements on it in order to satisfy the constraint. As the number of attackers to asset k decreases, φ_k will decrease and when $I_k = 1$, constraint will work only for one asset with $\varphi_k = 0$. On the other hand, when I_k increases, (3.28) brings the constant term φ_k and the threat with the highest q_{ik} will be the most influential threat in group I_k.

3.3.3. Nonlinearity Due to Time Varying Hit Probabilities

The final point left in our discussion is the time varying SSHP values in the objective function. Each weapon has a different SSHP against each threat when it fires the r^{th} shot at that threat. As stated in Section 3.2, these probability values are derived by Yüksel (2007) from error analysis considering the random and systematical errors in weapon systems. Results of these calculations show that distance between the weapon and the threat is the most influential factor on SSHP. A sample SSHP as a function of the distance between the weapon and its target can be seen in Figure 7.

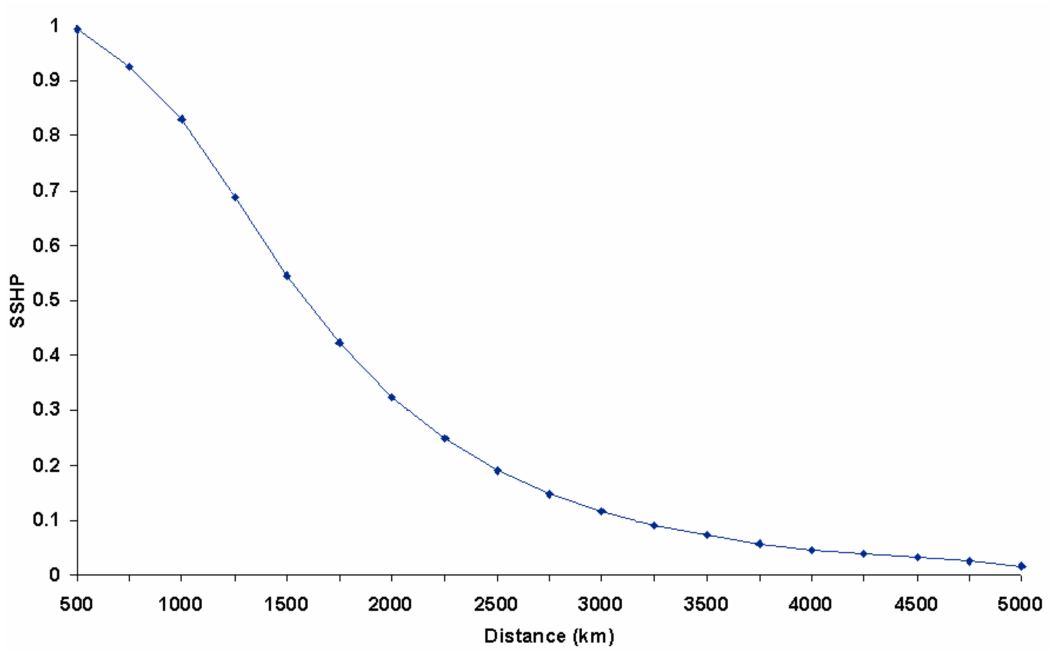

Figure 7 – Change in SSHP Values With Respect To Distance

Since it is assumed that velocity of a threat does not change and the distance between a weapon and a threat decreases at a constant rate, we can plot SSHP of a weapon against a threat with respect to time as in Figure 8.

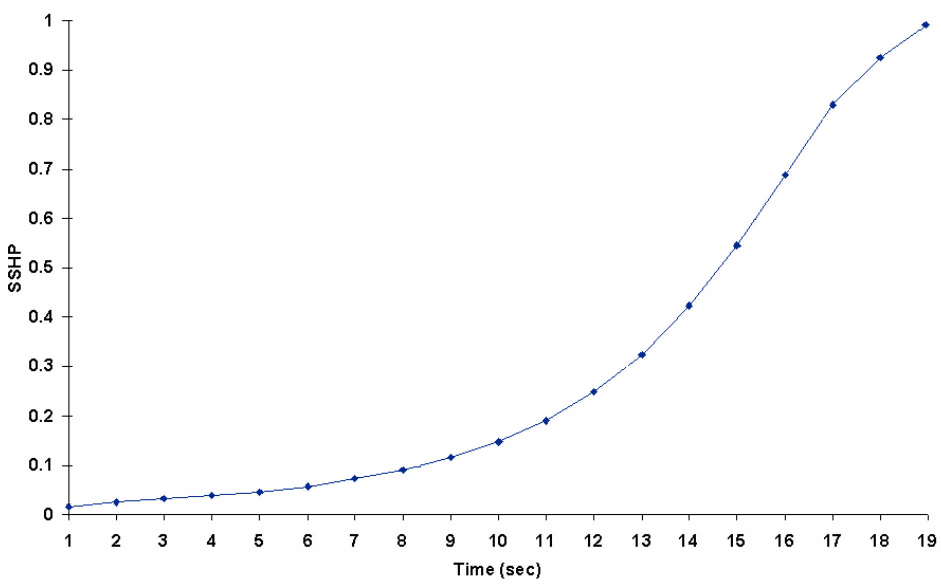

Figure 8 – Change in SSHP Values With Respect To Time

The expression we need to deal with in our formulation is the natural logarithm of probability values in (3.27). If we find a good substitute for $\left(1-p_{jir}\right)$ curve (Figure 9) such that when we take its natural logarithm it gives linear terms, then we can use it as an approximation.

When we apply curve fitting algorithms to $\left(1-p_{jir}\right)$, the best fitting curves include additive terms. However, additive terms in logarithm are not useful for substitution. Even though it is an inferior approximation to $\left(1-p_{jir}\right)$, we consider

$$\left(\frac{1-p_{ji}}{p_{ji}^{\psi_{ji}t_{jir}}}\right) \tag{3.29}$$

where p_{ji} is a constant close to SSHP values at time zero and ψ_{ji} is another constant equalizing (3.29) to $\left(1-p_{jir}\right)$ at the end of the attack duration. This is an inferior approximation as it is convex, but we have tried about 250 closed forms with a curve-fitting software and could not find a more suitable form (Results of this software for 251 closed forms of functions can be found in Appendix B).

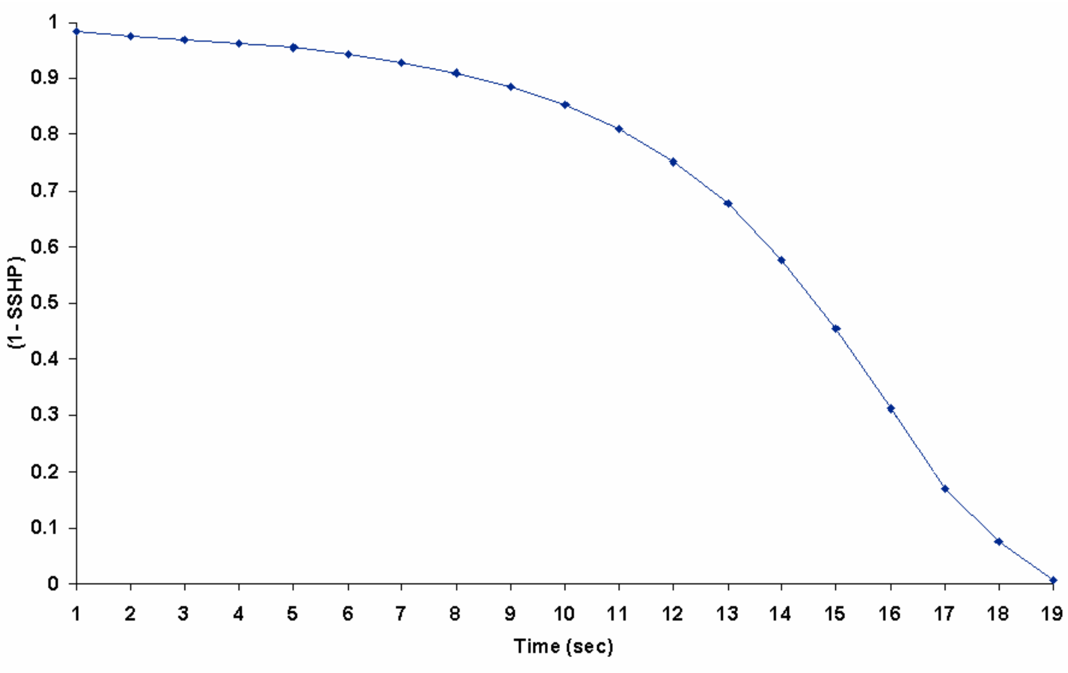

Figure 9 – Graph Of $\left(1-p_{jir}\right)$ With Respect To Time

Logarithm of (3.29) can easily be taken resulting in linear terms. Putting (3.29) into (3.27) instead of $\left(1-p_{jir}\right)$, we have

$$\sum_{j=1}^{J}\sum_{r=1}^{s_i}\left[\ln\left(1-p_{ji}\right)\right]x_{jir}-\sum_{j=1}^{J}\sum_{r=1}^{s_i}\left[\psi_{ji}\ln\left(p_{ji}\right)\right]x_{jir}t_{jir}+\sum_{\alpha=1}^{\Upsilon}Z_{\alpha k} \le -\left(\ln\left(q_{ik}\right)+\varphi_k\right) \quad \forall k,\ \forall i \tag{3.30}$$

In this new constraint, the term $x_{jir}t_{ir}$ has been linearized by introducing new variable y_{jir}. Then, constraint (3.30) becomes

$$\sum_{j=1}^{J}\sum_{r=1}^{s_i}\left[\ln\left(1-p_{ji}\right)\right]x_{jir}-\sum_{j=1}^{J}\sum_{r=1}^{s_i}\left[\psi_{ji}\ln\left(p_{ji}\right)\right]y_{jir}+\sum_{\alpha=1}^{\Upsilon}Z_{\alpha k} \le -\left(\ln\left(q_{ik}\right)+\varphi_k\right) \quad \forall k,\ \forall i \tag{3.31}$$

Note that the t_{ir} is changed to t_{jir} since time of firing is an important result from a specific weapon viewpoint. This change has been reflected in the linear model by replacing t_{ir} with $\sum_{j=1}^{J} t_{jir}$. This is a valid change because there can be only one weapon firing round *r* at enemy i. As a result, there will be only one t_{jir} that is greater than zero.

3.3.4. Final Formulation

Our final formulation is given as

$$\max \sum_{k=1}^{K} w_k \sum_{\alpha=1}^{\Upsilon} c_{\alpha k} Z_{\alpha k} + \sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{r=1}^{s_i} \varepsilon y_{jir} \tag{3.26}$$

s.t.

$$\sum_{j=1}^{J}\sum_{r=1}^{s_i}\left[\ln\left(1-p_{ji}\right)\right]x_{jir}-\sum_{j=1}^{J}\sum_{r=1}^{s_i}\left[\psi_{ji}\ln\left(p_{ji}\right)\right]y_{jir}+\sum_{\alpha=1}^{\Upsilon}Z_{\alpha k} \le -\left(\ln\left(q_{ik}\right)+\varphi_k\right) \quad \forall k,\ \forall i \tag{3.31}$$

$$\sum_{i=1}^{I}\sum_{r=1}^{s_i} x_{jir} \le d_j \qquad \forall j \tag{3.2}$$

$$\sum_{j=1}^{J} x_{ji1} \le 1 \qquad \forall i \tag{3.3}$$

$$\sum_{j=1}^{J} x_{ji,r+1} \le \sum_{j=1}^{J} x_{jir} \qquad \forall i, r=1,\ldots,s_i-1 \tag{3.4}$$

$$M\left(1-\sum_{j=1}^{J}x_{ji,r+1}\right)+\sum_{j=1}^{J}t_{ji,r+1}\geq\sum_{j=1}^{J}t_{jir}+\sum_{j=1}^{J}\beta_{ji}x_{jir}-\sum_{j=1}^{J}\gamma_{ji}y_{jir} +\sum_{j=1}^{J}\Delta_{j}x_{jir} \qquad \forall i,r=1,...,s_i-1 \tag{3.9}$$

$$\sum_{j=1}^{J}x_{jir}Q_{ji}\leq\sum_{j=1}^{J}t_{jir}\leq\sum_{j=1}^{J}x_{jir}R_{ji}-\sum_{j=1}^{J}\beta_{ji}x_{jir}+\sum_{j=1}^{J}\gamma_{ji}y_{jir}-\sum_{j=1}^{J}\Delta_{j}x_{jir} \qquad \forall i,r=1,...,s_i-1 \tag{3.10}$$

$$y_{jir}\leq Mx_{jir} \qquad \forall i,\forall j,r=1,...,s_i \tag{3.11}$$

$$y_{jir}\leq t_{jir} \qquad \forall i,\forall j,r=1,...,s_i \tag{3.12}$$

$$y_{jir}\geq 0 \qquad \forall i,\forall j,r=1,...,s_i \tag{3.13}$$

$$t_{jir}\geq 0 \qquad \forall i,r=1,...,s_i \tag{3.7}$$

$$x_{jir}\in\{0,1\} \qquad \forall i,\forall j,r=1,...,s_i \tag{3.8}$$

$$0\leq Z_{\alpha k}\leq U_{\alpha k} \qquad \forall k,\alpha=1,2,...,\Upsilon \tag{3.28}$$

3.4. Verification of the Mathematical Model

In this section, we try to verify the solutions of our model in its final form. It has several approximations as explained in previous sections. Although it is not desirable to have so many approximations in a modeling approach, our model might still give us some insight about the problem and its solution characteristics.

While solving the model, taking $\varepsilon=0$ has led unbinding y_{jir} values in some solutions, thus we have taken the smallest possible ε value, which is 10^{-8}. Scenarios should cover possible combinations of SSHP and q_{ik} values. This leads us to have eight different scenarios given in Table 3.

Assume that there are one asset, two weapons and two threats in a scenario. Four different (1-p_{jir}) curves for weapon and threat combinations are considered as given in Figure 10. Curves fitted according to (3.29) are also presented in Figure 10. Parameters of these approximated SSHP functions can be seen in Table 4

Table 3 – Probability Values for Scenarios

Scenario	**SSHP Values**	q_{ik} **Values**
1	$p_{11} = p_{12} = p_{21} = p_{22}$	$q_{11} = q_{21}$
2	$p_{11} > p_{12} = p_{21} = p_{22}$	$q_{11} = q_{21}$
3	$p_{11} > p_{12} > p_{21} = p_{22}$	$q_{11} = q_{21}$
4	$p_{11} > p_{12} > p_{21} > p_{22}$	$q_{11} = q_{21}$
5	$p_{11} = p_{12} = p_{21} = p_{22}$	$q_{11} > q_{21}$
6	$p_{11} > p_{12} = p_{21} = p_{22}$	$q_{11} > q_{21}$
7	$p_{11} > p_{12} > p_{21} = p_{22}$	$q_{11} > q_{21}$
8	$p_{11} > p_{12} > p_{21} > p_{22}$	$q_{11} > q_{21}$

Table 4 – Parameters of Approximated SSHP Functions

	$\boldsymbol{p_{ji}}$	$\boldsymbol{\psi_{ji}}$
Lowest	0.04843	-0.06360
Medium	0.13249	-0.14191
High	0.19990	-0.17791
Highest	0.22113	-0.33266

Note that p_{ji} values are not legitimate set of probabilities but they are parameters of (3.29).

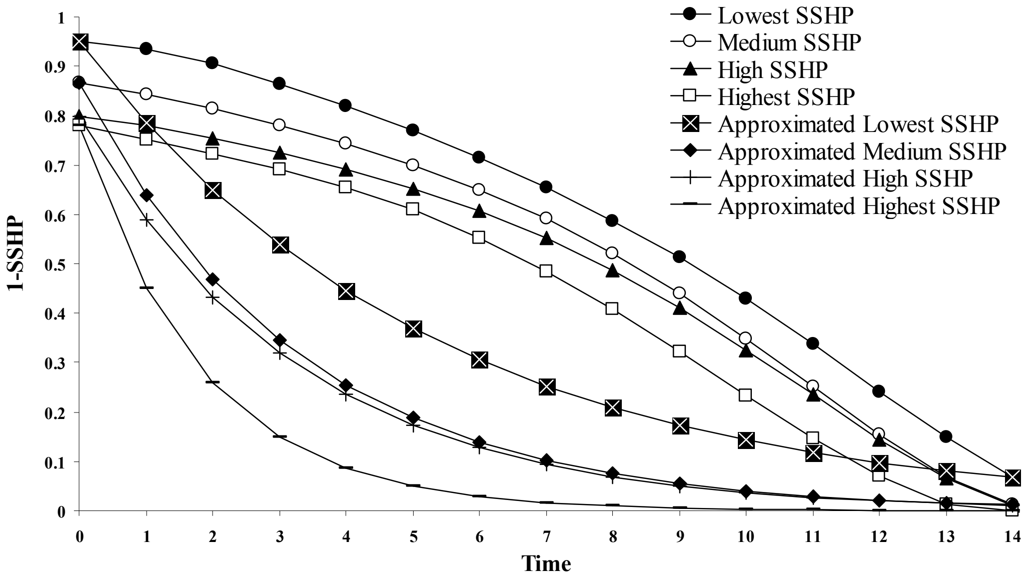

Figure 10 – Four Different SSHP Values and Their Approximations

We assume that engageabilty interval is $\left[Q_{ji}, R_{ji}\right] = \left[0,14\right]$ and both weapons have three rounds of ammunition in their inventory. In addition, kill probabilities of threats are taken as 0.5 and 0.8. Speed of threats is 300 m/s and that of rounds is 1400 m/s.

Our verification experiments include the following.

- Scenarios solved by our linearized model
    - Model 1: The one with linearized objective function and SSHP values approximated by linearizing fitted curves given in
    - Figure 10.
    - Model 2: The one with linearized objective function and SSHP values fixed at their average values.
- Scenarios solved by BARON solver of GAMS

- Model 3: The one with original objective function and nonlinear fitted curves (fifth degree polynomials) for original SSHP functions. In this case probability values become decision variables depending on time of shot.

- Model 4: The one with original objective function and SSHP values fixed at their average values.

To fix the SSHPs at their average values for Models 2 and 4, numerical values of these functions are computed at discrete points in time as seen in Table 5, and averages are found. Then, we fixed the SSHP values by rounding these averages as seen in Table 5. Since the difference between medium and high average probability values is small in Table 5, we have increased the values of fixed probabilities and used 0.4, 0.5, 0.6 and 0.7 for lowest, medium, high and highest probabilities, respectively.

Table 5 – SSHP Values over Time Obtained With Fitted Curves

Time	Lowest	Medium	High	Highest
0	0.0484	0.1325	0.1999	0.2211
1	0.0687	0.1573	0.2214	0.2460
2	0.0963	0.1859	0.2462	0.2748
3	0.1323	0.2187	0.2750	0.3085
4	0.1772	0.2565	0.3086	0.3480
5	0.2300	0.3003	0.3481	0.3944
6	0.2878	0.3516	0.3945	0.4490
7	0.3491	0.4106	0.4491	0.5131
8	0.4152	0.4798	0.5132	0.5876
9	0.4884	0.5599	0.5877	0.6725
10	0.5698	0.6506	0.6725	0.7650
11	0.6603	0.7493	0.7651	0.8581
12	0.7574	0.8484	0.8581	0.9374
13	0.8540	0.9329	0.9374	0.9855
14	0.9358	0.9844	0.9855	0.9993
Average	**0.4047**	**0.4812**	**0.5175**	**0.5707**

Table 6 – Fixed SSHP Values Used In Scenarios

Scenario	**SSHP Values**	q_{ik} **Values**
1	$p_{11} = p_{12} = p_{21} = p_{22} = 0.50$	$q_{11} = q_{21} = 0.50$
2	$p_{11} = 0.70 > p_{12} = p_{21} = p_{22} = 0.50$	$q_{11} = q_{21} = 0.50$
3	$p_{11} = 0.70 > p_{12} = 0.60 > p_{21} = p_{22} = 0.50$	$q_{11} = q_{21} = 0.50$
4	$p_{11} = 0.70 > p_{12} = 0.60 > p_{21} = 0.50 > p_{22} = 0.40$	$q_{11} = q_{21} = 0.50$
5	$p_{11} = p_{12} = p_{21} = p_{22} = 0.50$	$q_{11} = 0.80 > q_{21} = 0.50$
6	$p_{11} 0.70 > p_{12} = p_{21} = p_{22} = 0.50$	$q_{11} = 0.80 > q_{21} = 0.50$
7	$p_{11} = 0.70 > p_{12} = 0.60 > p_{21} = p_{22} = 0.50$	$q_{11} = 0.80 > q_{21} = 0.50$
8	$p_{11} = 0.70 > p_{12} = 0.60 > p_{21} = 0.50 > p_{22} = 0.40$	$q_{11} = 0.80 > q_{21} = 0.50$

We need closed form expressions for SSHP values for Model 3. Since BARON is a non-linear programming solver, we can nonlinearly approximate the probability values and resulting approximation can be used in the original model. We use fifth order polynomial approximations. Coefficients of these polynomial approximations are given in Table 7.

Table 7 – Coefficients of the Approximation to SSHP Function

Case	x^5	x^4	x^3	x^2	x^1	x^0
Lowest	4.11E-06	-0.0001366	0.00162	-0.011380	-0.00525	0.9498
Medium	6.40E-06	-0.0001728	0.00145	-0.007245	-0.01600	0.8665
High	6.00E-06	-0.0001609	0.00133	-0.006649	-0.01328	0.7992
Highest	5.46E-06	-0.0001194	0.00070	-0.003866	-0.02123	0.7789

3.4.1. Solutions of Scenarios with Varying Hit Probabilities

Solutions of Models 1 and 3 with varying SSHPs are presented in Table 8. For the solutions obtained from Model 1, objective function values are re-calculated by using the fifth order polynomials fitted for the SSHP functions. Solution time of BARON is limited to six hours. The solution reported at termination may not be the global optimal. Our observations are given below.

- If SSHP functions of weapons are the same and expected hit probabilities of threats are equal to each other, original and linearized models yield the same results. Thus, usage of linearized model is better since the solution time is within one second whereas the solution of the original model with BARON is obtained in more than six hours.

- The largest deviation of the linear model from the original model is 2.69% and the average deviation is 1.22%.

- Overall, the differences between the two solutions for different scenarios are considerably small, which means the linearized model produces reasonably good solutions.

Table 8 – Solutions with Varying SSHP Values

Scenario	Linearized Model Assignments W*→ T‡, Time	Linearized Model Obj. Func.	Original Model Assignments W→ T, Time	Original Model Obj. Func.	Deviation Numeric	Deviation Percent
1	1→1, 5.3050 1→1, 8.7806 1→1, 11.6428 2→2, 5.3050 2→2, 8.7806 2→2, 11.6428	0.94202	1→2, 5.3050 1→2, 8.7806 1→2, 11.6429 2→1, 5.3050 2→1, 8.7806 2→1, 11.6429	0.94202	0.0000	0.00
2	1→1, 5.3050 1→1, 8.7806 1→1, 11.6428 2→2, 5.3050 2→2, 8.7806 2→2, 11.6428	0.96105	1→1, 8.7806 1→2, 8.7806 1→2, 11.6429 2→1, 11.6429 2→2, 1.0847 2→2, 5.3050	0.94399	0.0171	1.78
3	1→1, 8.7806 1→1, 11.6428 1→2, 5.3050 2→1, 5.3050 2→2, 8.7806 2→2, 11.6428	0.96153	1→1, 11.6429 1→2, 8.7806 1→2, 11.6429 2→1, 8.7806 2→2, 1.0847 2→2, 5.3050	0.95658	0.0050	0.51
4	1→1, 11.6428 1→2, 8.7806 1→2, 11.6428 2→1, 5.3050 2→1, 8.7806 2→2, 5.3050	0.95689	1→2, 5.3050 1→2, 8.7806 1→2, 11.6429 2→1, 5.3050 2→1, 8.7806 2→1, 11.6429	0.94722	0.0097	1.01
5	1→1, 8.7806 1→1, 11.6428 1→2, 11.6428 2→1, 5.3050 2→2, 5.3050 2→2, 8.7806	0.92489	1→1, 11.6429 1→2, 5.3050 1→2, 11.6429 2→1, 5.3050 2→1, 8.7806 2→2, 8.7806	0.92489	0.0000	0.00
6	1→1, 5.3050 1→1, 8.7806 1→1, 11.6428 2→2, 5.3050 2→2, 8.7806 2→2, 11.6428	0.95533	1→1, 8.7806 1→2, 1.0847 1→2, 5.3050 2→1, 11.6429 2→2, 8.7806 2→2, 11.6429	0.92961	0.0257	2.69
7	1→1, 5.3050 1→1, 8.7806 1→1, 11.6428 2→2, 5.3050 2→2, 8.7806 2→2, 11.6428	0.95533	1→1, 8.7806 1→2, 8.7806 1→2, 11.6429 2→1, 11.6429 2→2, 1.0847 2→2, 5.3050	0.93431	0.0210	2.20
8	1→1, 11.6428 1→2, 8.7806 1→2, 11.6428 2→1, 5.3050 2→1, 8.7806 2→2, 5.3050	0.94803	1→2, 5.3050 1→2, 8.7806 1→2, 11.6429 2→1, 5.3050 2→1, 8.7806 2→1, 11.6429	0.93320	0.0148	1.56

*W: Weapon, ‡T: Threat

3.4.2. Solutions of Scenarios with Fixed Hit Probabilities

We have solved the same scenarios with the fixed probabilities in order to eliminate the nonlinearity due to time varying SSHP values, and to isolate the effect of linearization of the objective function and constraints. Solutions of Models 2 and 4 with fixed SSHP values are presented in Table 9.

- If probability values of weapons are equal and expected hit probabilities of threats are also equal to each other, original and linearized models give the same results. Thus, usage of linearized model is better since the solution time is less than one second whereas the solution of the original model takes about three seconds.

- When all SSHPs are different from each other (hit probabilities of threats are equal), the largest deviation observed is 0.28%. The average deviation is 0.07%.

- Overall, the differences between the two solutions for different scenarios are considerably small, which means the linearized model produces reasonably good solutions. The original model can be used in this case due to its less time requirement.

As far as the solution times are concerned, the linearized model is superior to the original model with its solution time of less than one second with varying and fixed probabilities, whereas the original model is solved in about three seconds with fixed probabilities and over six hours for varying probabilities.

At the first glance, it is clear that the model tries to make engagements as late as possible considering the SLS tactic. This is logical because as the threats get closer to weapons in time, the SSHP values increase. This is an important insight, which we will use in developing our heuristic solution procedure.

Table 9 – Solutions with Fixed SSHP Values

Scenario	Linearized Model Assignments W*→ T‡, Time	Linearized Model Obj. Func.	Original Model Assignments W→ T, Time	Original Model Obj. Func.	Deviation Numeric	Deviation Percent
1	1 → 1, 5.3050 1 → 1, 8.7806 1 → 1, 11.6428 2 → 2, 5.3050 2 → 2, 8.7806 2 → 2, 11.6428	0.8789	1 → 1, 8.7806 1 → 1, 11.6428 1 → 2, 11.6428 2 → 1, 5.3050 2 → 2, 5.3050 2 → 2, 8.7806	0.8789	0.0000	0.00
2	1 → 1, 8.7806 1 → 1, 11.6428 1 → 2, 8.7806 2 → 2, 1.0847 2 → 2, 5.3050 2 → 2, 11.6428	0.9252	1 → 1, 8.7806 1 → 1, 11.6428 1 → 2, 11.6428 2 → 2, 1.0847 2 → 2, 5.3050 2 → 2, 8.7806	0.9252	0.0000	0.00
3	1 → 1, 5.3050 1 → 2, 5.3050 1 → 2, 11.6428 2 → 1, 8.7806 2 → 1, 11.6428 2 → 2, 8.7806	0.9240	1 → 1, 5.3050 1 → 1, 8.7806 1 → 1, 11.6428 2 → 2, 5.3050 2 → 2, 8.7806 2 → 2, 11.6428	0.9248	0.0008	0.09
4	1 → 1, 8.7806 1 → 1, 11.6428 1 → 2, 1.0847 2 → 2, 5.3050 2 → 2, 8.7806 2 → 2, 11.6428	0.9137	1 → 1, 8.7806 1 → 2, 5.3050 1 → 2, 11.6428 2 → 1, 5.3050 2 → 1, 11.6428 2 → 2, 8.7806	0.9163	0.0026	0.28
5	1 → 1, 5.3050 1 → 1, 8.7806 1 → 2, 5.3050 2 → 1, 11.6428 2 → 2, 8.7806 2 → 2, 11.6428	0.8438	1 → 1, 5.3050 1 → 2, 5.3050 1 → 2, 8.7806 2 → 1, 8.7806 2 → 1, 11.6428 2 → 2, 11.6428	0.8438	0.0000	0.00
6	1 → 1, 5.3050 1 → 1, 8.7806 1 → 1, 11.6428 2 → 2, 5.3050 2 → 2, 8.7806 2 → 2, 11.6428	0.9173	1 → 1, 5.3050 1 → 1, 8.7806 1 → 1, 11.6428 2 → 2, 5.3050 2 → 2, 8.7806 2 → 2, 11.6428	0.9173	0.0000	0.00
7	1 → 1, 8.7806 1 → 1, 11.6428 1 → 2, 5.3050 2 → 1, 5.3050 2 → 2, 8.7806 2 → 2, 11.6428	0.9158	1 → 1, 5.3050 1 → 1, 8.7806 1 → 1, 11.6428 2 → 2, 5.3050 2 → 2, 8.7806 2 → 2, 11.6428	0.9173	0.0014	0.16
8	1 → 1, 8.7806 1 → 1, 11.6428 1 → 2, 5.3050 2 → 1, 5.3050 2 → 2, 8.7806 2 → 2, 11.6428	0.8946	1 → 1, 8.7806 1 → 2, 5.3050 1 → 2, 11.6428 2 → 1, 5.3050 2 → 1, 11.6428 2 → 2, 8.7806	0.8949	0.0003	0.03

*W: Weapon, ‡T: Threat

In order to achieve highest possible kill probabilities, the models try to schedule shots according to the latest time weapons can fire. That is why the first engagements start at 1.0847 but not at zero. The result of BARON with the original model also verifies that we should make engagements as late as possible. However, note that this is valid when we have constant kill probability of threats regardless of their position. In real life situations, their kill probabilities also increase, as they get closer to their targets.

Another observation is that better survivability results are attained when more rounds are assigned to more dangerous (with higher q_{ik} values) threat(s). On the other hand, increasing the number of rounds assigned to the threat with a high value of survivability also increases the objective function value. One must consider the tradeoff between the two strategies.

CHAPTER 4

HEURISTIC SOLUTION

The original nonlinear mathematical model for our weapon-target assignment problem is investigated in the previous chapter. If the decision maker wishes to use precise probabilities and nonlinear characteristic of the problem is not to be sacrificed, solution time of the original model can be extremely long. Global optima may not be found even after several hours of computation. Linearized model runs relatively fast, but it requires effort to approximate the SSHP values and to determine the ε value used in the objective function. Furthermore, linearization alters the objective function from maximizing the overall survivability of all assets to maximizing the minimum survivability of an asset. We may also not be able to obtain the true optimum solution with linearized model as explained in the previous chapter. Hence, these two approaches have their own advantages and disadvantages.

Weapon-target assignment problems have short time windows and demand fast solutions. Therefore, a heuristic approach is developed in this chapter. First, we present a construction heuristic, and then introduce an improvement heuristic to enhance the solution. In our heuristic algorithms, we make use of the original nonlinear objective function and SSHP values.

4.1. Construction Heuristic

In order to have a good heuristic solution, we need a construction method that produces a solution as close to the optimal solution as possible. For our weapon-target assignment problem, we have developed a construction heuristic based on the ideas derived from the mathematical model solutions presented in the previous chapter.

Assume that there are weapons $(j = 1, \ldots, J)$ defending assets $(k = 1, \ldots, K)$ against threats $(i = 1, \ldots, I)$. We use the following additional notation in our heuristic algorithm.

w_k * : Risk of asset k.

UT_k : Set of threats aiming at asset *k*.

AD_k : Set of attack durations (time of flight) of threats aiming at asset k. i^{th} element of this set corresponds to "remaining" attack duration of the i^{th} threat in set UT_k. In order to refer the elements of this set, we use $AD_k(i)$ for i^{th} element of AD_k.

M : A period of time longer than the overall scenario duration.

TOF_{ji} : Time of flight of ammunition fired by weapon *j* at threat *i*.

$d(j)$: Number of rounds of ammunition that that weapon *j* has.

$AW(j)$: Set of available time windows of weapon j. Each element of this set has two components indicating starting and ending time of one engageability period for weapon j.

$FLAG - A(k)$: Situation indicator for asset k. If for any asset $FLAG - A(k) = 0$, there is still room to make assignments to protect this asset. If $FLAG - A(k) = 1$, no further assignments are possible to defend the asset.

$FLAG-T(i)$: Situation indicator for threat i. If for any threat $FLAG-T(i)=0$, there is still room to make assignments against it, we set $FLAG-T(i)=1$ otherwise.

$FLAG-W(j)$: Situation indicator for weapon j. If $FLAG-W(j)=0$, weapon j is still able to fire ammunition on threats. If $FLAG-T(i)=1$, it is out of ammunition so there is no possible engagement it can make.

The flowchart of the heuristic is shown in Figure 11 and steps are explained below.

Initialization: Set $w_k{*}=w_k\left(1-\prod_{i\in I_k}\left(1-q_{ik}\right)\right)$ and $FLAG-A(k)=0$ for all $k=1,2,...,K$, $FLAG-W(j)=0$ for all $j=1,2,...,J$ and $FLAG-T(i)=0$ for all $i=1,2,...,I$. Initialize set AC to include all assets. Calculate $AW(j)$ for all weapons from time zero to time *M*.

Step 0: Choose the assets whose $FLAG-A(k)<1$ and rank them according to their $w_k{*}$. If there are no such assets, STOP and terminate the program. Otherwise, choose the asset with the highest $w_k{*}$ (the asset under highest weighted risk), call it *a*.

Step 1: Determine the threats aiming at asset a and construct the sets AD_a and UT_a considering $FLAG-T(i)<1$.

Step 2: If $UT_a=\{\ \}$, set $FLAG-A(k)$=*1* and go to Step 0. Otherwise, choose from AD_a, the threat with the highest kill probability against *a*, and call it *b*.

Step 3: Check if there is any available weapon with $FLAG-W(j)<1$ and whether $AD_a(b)$ can be fitted into one of the elements of $AW(j)$, i.e. find the weapons with available times such that they can be assigned to threat *b* during its attack duration. If found, make a candidate weapons list with these weapons. Otherwise adjust, $AD_a(b) = \max_{j=1,..,J}\{$*latest feasible time in* $AW(j)\}_b$, make a candidate weapons list. If no candidate weapons are found, then set $FLAG\text{-}T(b) = 1$ and go to Step 2.

Step 4: Calculate the single shot hit probabilities (Yüksel, 2007) of candidate weapons against threat *b* considering their setup times for a possible shot. We assume each weapon fires at the threat when it is at the closest point to the weapon, i.e. the engagement takes place at the latest time according to SLS tactic. Here, the purpose is to use the maximum possible SSHP values. For calculation purposes, we need to check whether this assignment is the first assignment on threat b.

- If this is the first assignment on threat *b* set

$$P_{abj}' = q_{ba}\left(1 - p_{jb}\right) \qquad \forall j \in M .$$

- If this is *not* the first assignment, there is already P_{abj} accumulated on threat *b*, so set

$$P_{abj}' = \left(1 - P_{abj}\right)\left(1 - p_{jb}\right) \qquad \forall j \in M .$$

Step 5: Choose the weapon resulting in lowest P'_{abj}, call it c and set $P_{abc} = 1 - P'_{abc}$. Schedule a shot from weapon *c* to threat *b*. Update $AD_a(b)$ and $AW(c)$ with the setup time of the weapon, i.e. $AD_a(b) = AD_a(b) - TOF_{cb} - \Delta_c$ and $AW(c) = AW(c) - \Delta_c$. Also update $d(c)=d(c)\text{ - }1$ and if $d(c)=0$, set $FLAG - W(c) = 1$.

Step 6: Set $w_a{}^* = w_a\left(1 - \prod_{i \in I_k, j=1,\ldots,J} P_{aij}\right)$ and go to Step 0.

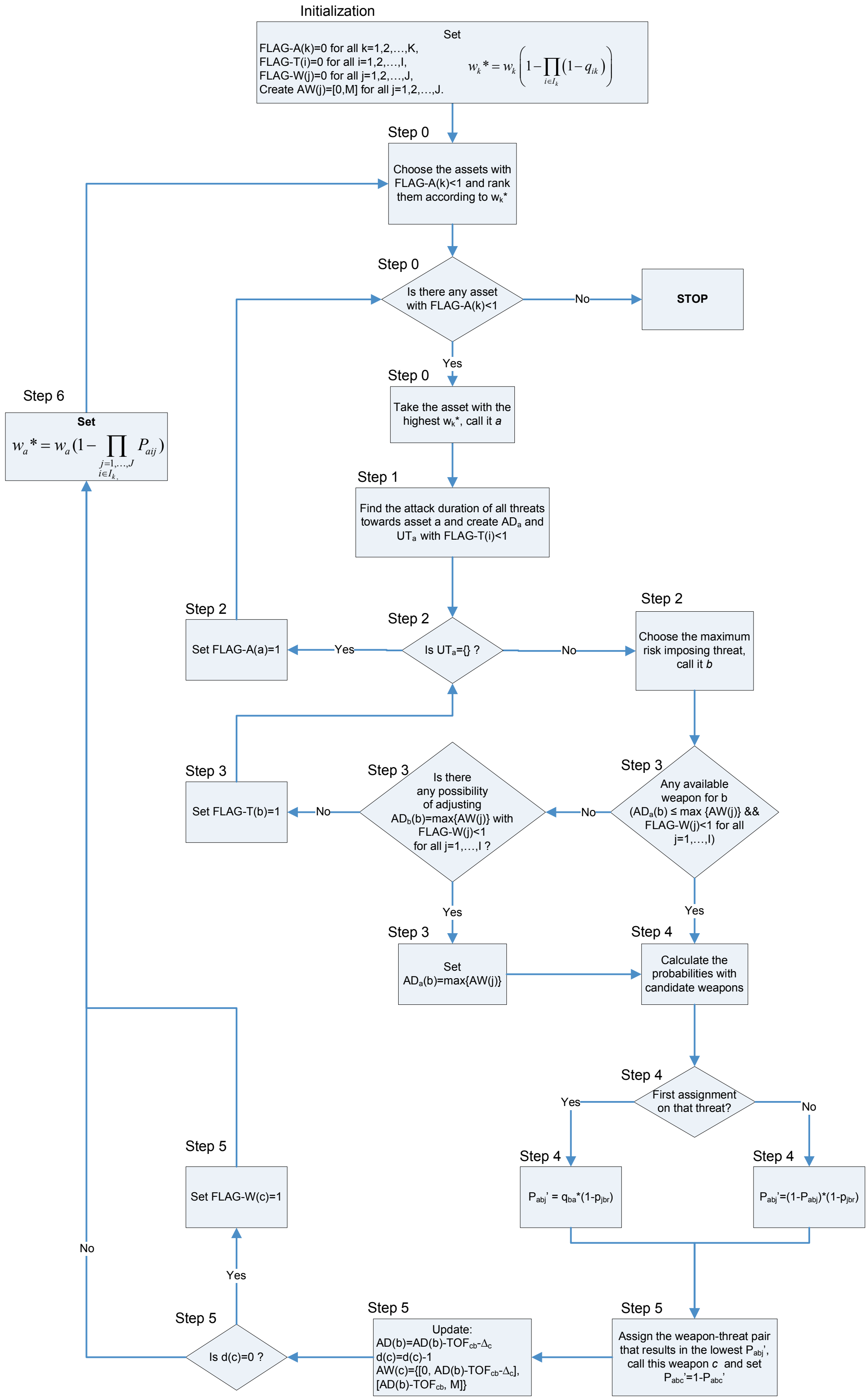

Figure 11 – Flowchart of the Construction Heuristic

As an example to illustrate how this heuristic works, assume that we have three weapons, three assets and four threats as in Figure 12. In this figure, Threat 4 is ammunition aiming at Asset 1. Other threats, which are airplanes and flying over, aim to bomb Asset 2.

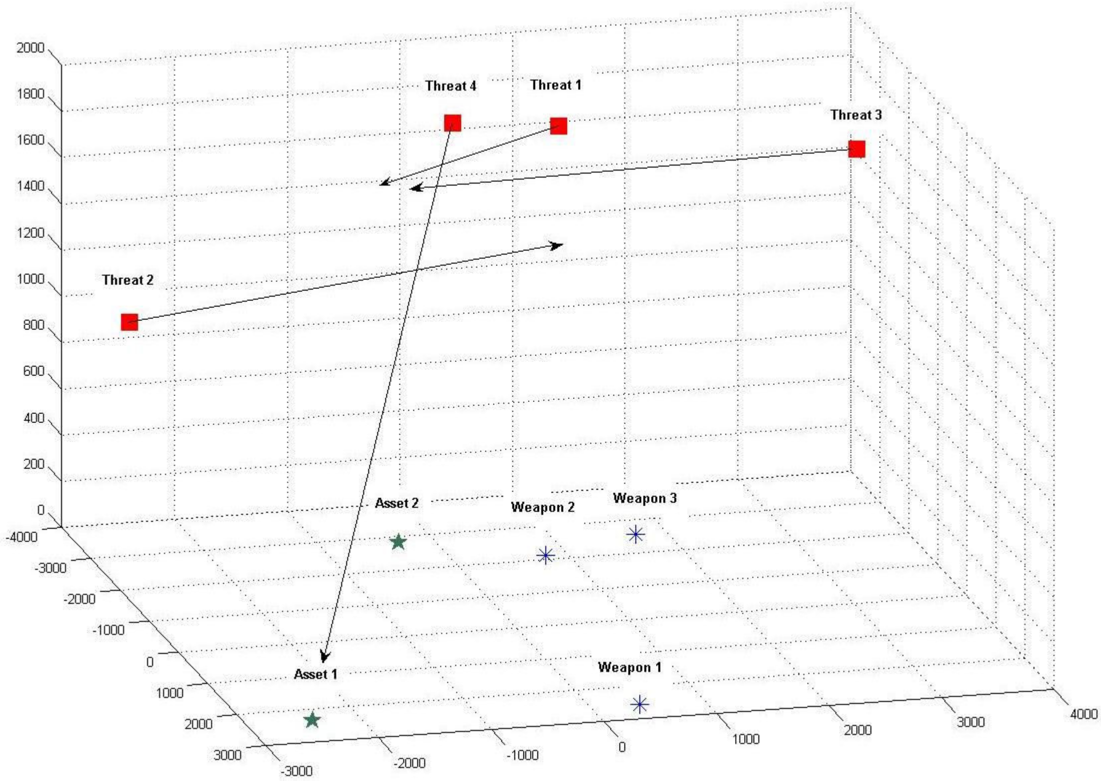

Figure 12 – Example Scenario for the Construction Heuristic

Weights of the assets are assumed to be 5 and 8 for Asset 1 and 2, respectively. Table 10 contains the initial engageability intervals of each weapon-threat pair. Engageability interval indicates the time window during which it is possible to attack the threat. It starts when the threat enters the weapon range and ends when the threat goes out of the weapon range or reaches the asset hence engagement becomes impossible. If both starting and ending times are zero, the threat never enters the weapon range or it is not possible to fire at that threat with that weapon.

Table 10 – Initial Engageability Time Windows

Weapon	Threat	Engagement Start	Engagement End
1	1	0.0000	0.0000
1	2	5.8966	18.626
1	3	0.0000	0.0000
1	4	0.0000	0.0000
2	1	0.0000	5.7227
2	2	0.0000	2.1161
2	3	0.0000	7.4786
2	4	0.0000	0.0000
3	1	0.0000	1.7097
3	2	0.0000	0.0000
3	3	0.0000	3.4014
3	4	0.0000	0.0000

Characteristics of threats and weapons are given in Tables 11 and 12, respectively, where X, Y, Z denote threat and weapon coordinates. Risk of each asset is calculated according to these tables.

Table 11 – Threats Used In the Example

Threat ID	Speed	X	Y	Z	Targeted Asset	Hit Probability	Attack Duration
1	774.83	-3305.2	1227.5	1680.2	1	0.38330	5.7227
2	825.48	-1735.1	-2983.5	1191.3	1	0.73503	18.626
3	884.92	-1502.6	3407.7	1749.8	1	0.34996	7.4786
4	475.57	-2379.6	48.461	1862.9	2	0.57632	9.2456

Table 12 – Weapons Used In the Example

Weapon ID	X	Y	Z	Number of Rounds	Setup Time (Seconds)
1	2571.9	432.44	0	19	2
2	-2111.0	806.43	0	8	2
3	-2551.7	1716.2	0	4	2

Initialization: For all weapons, available time is set to between [0,20], assuming M=20. All flags for all items are set to zero.

1st iteration: Risk of Asset 1 is 4.4689 and Asset 2 is 4.6106, so Asset 2 is chosen as the most risked asset (Step 0). Most threatening threat (and the only one) is Threat 4 for Asset 2 (Steps 1 and 2). Next, heuristic checks the attack duration of Threat 4 against the engageability intervals. Since the threat never gets close enough to any weapon, there is no possibility for an engagement on it (Step 3). Therefore, no engagements are scheduled on this threat and FLAG-T (4) = 1. Also, there is no other threat having FLAG-T< 1 and attacking Asset 2, thus FLAG-A (2) = 1 (Step 2) and risk of Asset 2 is set to zero. We cannot make any assignment to defend this asset.

2nd iteration: Heuristic ranks the remaining assets according to their updated risks. Newly chosen asset is Asset 1(Step 0) and Threat 2 is the most risk-imposing enemy for this asset (Steps 1 and 2). Since the latest time to shoot at this threat is 18.626, engagements ending at this point are considered. Weapons 2 and 3 are unable to make any engagements ending at 18.626, so their SSHP is zero, leaving only Weapon 1 (Step 3). As a result, Weapon 1 is scheduled to engage Threat 2 in [14.514, 18.626] with set up time of 2 seconds, ammunition time of flight of 2.112 seconds and a SSHP of 0.1165 (Step 4). Its inventory is reduced by one and its available times are updated as AW(1)= {[0, 16.626], [18.626 20]}. Latest attack duration of the threat is updated as 14.514, according to SLS tactic (Step 5).

After completion of the remaining steps, objective function value is found as 4.5155 and the result of the construction heuristic can be seen in Table 13. Threat 4 cannot be engaged by any weapon. Threat 3 has no engagements since heuristic gave more priority to other threats.

Table 13 – Construction Heuristic Output for the Example

Weapon	Threat	Engagement Starts	Engagement Ends	SSHP
2	1	2.7165	5.7227	0.5804
2	2	0.0890	4.8889	0.0508
1	2	4.8889	10.0199	0.0300
1	2	10.0199	14.5143	0.0679
1	2	14.5143	18.6260	0.1165

4.2. Improvement Heuristic

The construction heuristic yields a feasible set of assignments. Given a feasible solution set, we should look for possible ways to improve the objective function value. Improvement heuristic is based on changing the weapon assignments between threats. This is performed by choosing the weapon with the lowest hit probability against the least threatening threat attacking the lowest risk asset, and trying to assign it to the most threatening threat attacking the highest risk asset. Here, the risk for asset k is defined as

$$ARISK_k = w_k \left(1 - \prod_{i \in \bar{I}_k} \left[1 - q_{ik} \prod_{j=1}^{J} \prod_{r=1}^{s_i} \left(1 - p_{jir}\right)^{x_{jir}} \right] \right)$$

and risk for threat *i* attacking asset *k* is defined as

$$TRISK_i = q_{ik} \left[\prod_{j=1}^{J} \prod_{r=1}^{s_i} \left(1 - p_{jir}\right)^{x_{jir}} \right].$$

While searching for a place to insert a new assignment into another threat's assignment list, previously scheduled engagements on the threat are pushed backward and forward in order to create a gap if necessary. The flowchart is given in Figure 13 and explained below.

Initialization: Read initial solution *S* generated by the construction heuristic.

Step 0: Rank the assets in ascending order of $ARISK_k$. For each asset, find the threats attacking the asset and rank them in ascending order of $TRISK_i$.

Step 1: If all assets are considered, STOP and report the solution. Otherwise, choose the asset with the next lowest $ARISK_k$, call it *a*.

Step 2: If all the threats attacking asset *a* are tried, go to Step 1. Otherwise, find the threat attacking asset *a*, with the next lowest $TRISK_i$, call it *b*.

Step 3: If all weapons of all engagements on threat *b* are tried, go to Step 2. Otherwise, among the engagements on threat *b*, select the one that has the lowest contribution, P_{jbr}, call its weapon *c*. If the engagement duration for *c* is $[t_s, t_e]$, make weapon c available for time period $[t_s, t_s + \Delta_c]$.

Step 4: If all assets are considered, go to Step 3, otherwise, choose the asset having the next highest $ARISK_k$, call it *h*.

Step 5: If all threats are considered attacking asset *h*, go to Step 4. Otherwise, find the most dangerous threat for asset *h*, and call it *d*.

Step 6: For engagement set on threat *d*, check time interval $[t_s, t_s + \Delta_{c_l} + TOF]$ to see if the threat is unassigned. If yes, go to step 9, otherwise go Step 7.

Step 7: Search for any time interval to see if threat *d* is suitable for an assignment from weapon *c*. If yes, go to step 9, otherwise go to Step 8.

Step 8: Try to create an appropriate time interval by moving existing engagement(s) on threat *d* if possible. If this is possible go to step 9, otherwise go to step 5.

Step 9: Assign weapon *c* to threat *d*. With this new assignment, create the new solution S'. Calculate the objective function. If there is any increase in the objective function, set $S = S'$ and go to step 0, else discard S' and go to step 5.

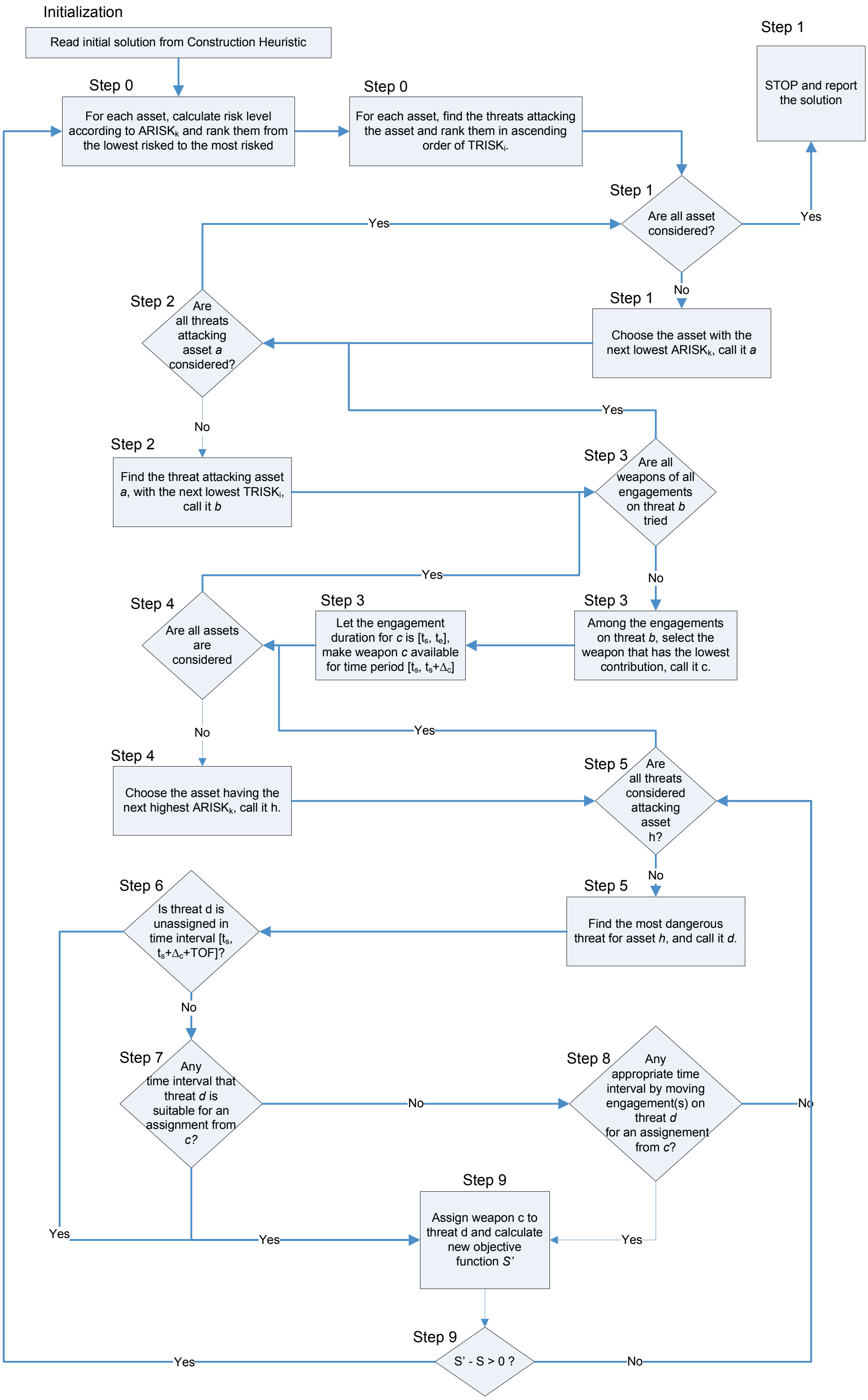

Figure 13 – Flowchart of the Improvement Heuristic

For the example given in the previous section, we have five engagements. If we run the improvement heuristic on this example, we will have the following steps.

Initialization: After construction heuristic, risk of Asset 1 is 2.5141 and risk of Asset 2 is 4.61056.

1st iteration: Asset 1 is chosen as the lowest and Asset 2 as the highest risk assets. Among the enemies approaching Asset 1, Threat 1 is chosen as the lowest risk imposing threat. Among the weapons attacking it, Weapon 2 has the lowest contribution. For Asset 2, there is only one threat but Weapon 2 cannot be considered for a candidate assignment.

2nd iteration: There is only one engagement on Threat 1. Thus next weapon to try as candidate is Weapon 1. However, since the threat does not enter into the range of Weapon 1, this engagement is not feasible.

After these iterations, engagements on Threat 2 will be considered for moving to defend Asset 2. After all threats on Asset 2 are examined and no changes are made, Asset 1 will be taken as the next highest risk asset. In other words, altering the engagements on threats attacking Asset 1 will be investigated. In these iterations, engagement on Threat 3, which was left unassigned by the construction heuristic, is considered. After remaining calculations, the new engagement set will be as in Table 14, and the improved objective function value is 4.7021. Note that Weapon 2, which is supposed to fire the first shot at Threat 2 according to the construction heuristic, now fires at Threat 3.

Table 14 – Improvement Heuristic Output for the Example

Weapon	**Threat**	**Engagement Starts**	**Engagement Ends**	**SSHP**
2	1	2.7165	5.7227	0.5804
2	3	4.8315	7.4786	0.2167
1	2	4.8889	10.0199	0.0300
1	2	10.0199	14.5143	0.0679
1	2	14.5143	18.6260	0.1165

CHAPTER 5

EXPERIMENTAL RESULTS

We have constructed a mixed integer linear model and developed heuristic approach for the weapon-target allocation problem. In this chapter, these approaches will be tested and compared on different scenarios of various sizes. In Section 5.1, settings of the experiments are given. In Section 5.2, results of this empirical study are presented and solutions are compared in terms of objective function value and computation time.

5.1. Experimental Settings

For testing purposes, the sample case defined in Chapter 3 (scenarios of size 1 asset, 2 weapons and 2 threats $-2\times2\times1$) and two larger cases derived from the sample are used. In Table 15, scenarios of size 2 assets, 3 weapons and 3 threats ($3\times3\times2$) are presented for 16 different scenarios. P_j indicates the single shot hit probability function of weapon j to hit any threat in the scenario. Whenever SSHP values differ for each weapon-threat pair regardless of the weapon, that scenario is marked as "all different". We use a three-digit code as in Figure 14 to refer to a particular scenario.

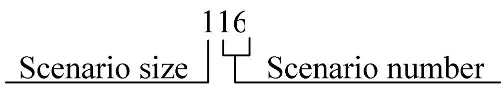

Figure 14 – Short Notation for Scenarios

If first digit is 1 it indicates scenarios of size $2 \times 2 \times 1$, while 2 and 3 stand for $3 \times 3 \times 2$ and $6 \times 6 \times 4$ scenarios respectively. Remaining digits indicate the scenario number.

Table 15 – Scenarios of Size 3x3x2

Scenario	q_{ik} **Values**	**SSHP Values**	w_k **Values**
201	$q_{11} = q_{21} = q_{32}$	$P_1 = P_2 = P_3$	$w_1 = w_2$
202	$q_{11} = q_{21} = q_{32}$	$P_1 = P_2 = P_3$	$w_1 > w_2$
203	$q_{11} = q_{21} = q_{32}$	*all different*	$w_1 = w_2$
204	$q_{11} = q_{21} = q_{32}$	*all different*	$w_1 > w_2$
205	$q_{11} = q_{21} = q_{32}$	$P_1 > P_2 = P_3$	$w_1 = w_2$
206	$q_{11} = q_{21} = q_{32}$	$P_1 > P_2 = P_3$	$w_1 > w_2$
207	$q_{11} = q_{21} = q_{32}$	$P_1 > P_2 > P_3$	$w_1 = w_2$
208	$q_{11} = q_{21} = q_{32}$	$P_1 > P_2 > P_3$	$w_1 > w_2$
209	$q_{11} = q_{21} < q_{32}$	$P_1 = P_2 = P_3$	$w_1 = w_2$
210	$q_{11} = q_{21} < q_{32}$	$P_1 = P_2 = P_3$	$w_1 > w_2$
211	$q_{11} = q_{21} < q_{32}$	*all different*	$w_1 = w_2$
212	$q_{11} = q_{21} < q_{32}$	*all different*	$w_1 > w_2$
213	$q_{11} = q_{21} < q_{32}$	$P_1 > P_2 = P_3$	$w_1 = w_2$
214	$q_{11} = q_{21} < q_{32}$	$P_1 > P_2 = P_3$	$w_1 > w_2$
215	$q_{11} = q_{21} < q_{32}$	$P_1 > P_2 > P_3$	$w_1 = w_2$
216	$q_{11} = q_{21} < q_{32}$	$P_1 > P_2 > P_3$	$w_1 > w_2$

In Table 16, scenarios with 4 assets, 6 weapons and 6 threats ($6 \times 6 \times 4$) are presented. Probability values and asset weights are obtained by duplicating the scenarios given in Table 15.

Table 16 – Scenarios of Size 6x6x4

	q_{ik} **Values**	**SSHP Values**	w_k **Values**
301	$q_{11}=q_{21}=q_{32}=q_{43}=q_{53}=q_{64}$	$P_1=P_2=P_3=P_4=P_5=P_6$	$w_1=w_2=w_3=w_4$
302	$q_{11}=q_{21}=q_{32}=q_{43}=q_{53}=q_{64}$	$P_1=P_2=P_3=P_4=P_5=P_6$	$w_1=w_3>w_2=w_4$
303	$q_{11}=q_{21}=q_{32}=q_{43}=q_{53}=q_{64}$	*all different*	$w_1=w_2=w_3=w_4$
304	$q_{11}=q_{21}=q_{32}=q_{43}=q_{53}=q_{64}$	*all different*	$w_1=w_3>w_2=w_4$
305	$q_{11}=q_{21}=q_{32}=q_{43}=q_{53}=q_{64}$	$P_1=P_4>P_2=P_3=P_5=P_6$	$w_1=w_2=w_3=w_4$
306	$q_{11}=q_{21}=q_{32}=q_{43}=q_{53}=q_{64}$	$P_1=P_4>P_2=P_3=P_5=P_6$	$w_1=w_3>w_2=w_4$
307	$q_{11}=q_{21}=q_{32}=q_{43}=q_{53}=q_{64}$	$P_1=P_4>P_2=P_5>P_3=P_6$	$w_1=w_2=w_3=w_4$
308	$q_{11}=q_{21}=q_{32}=q_{43}=q_{53}=q_{64}$	$P_1=P_4>P_2=P_5>P_3=P_6$	$w_1=w_3>w_2=w_4$
309	$q_{11}=q_{21}=q_{43}=q_{53}<q_{32}=q_{64}$	$P_1=P_2=P_3=P_4=P_5=P_6$	$w_1=w_2=w_3=w_4$
310	$q_{11}=q_{21}=q_{43}=q_{53}<q_{32}=q_{64}$	$P_1=P_2=P_3=P_4=P_5=P_6$	$w_1=w_3>w_2=w_4$
311	$q_{11}=q_{21}=q_{43}=q_{53}<q_{32}=q_{64}$	*all different*	$w_1=w_2=w_3=w_4$
312	$q_{11}=q_{21}=q_{43}=q_{53}<q_{32}=q_{64}$	*all different*	$w_1=w_3>w_2=w_4$
313	$q_{11}=q_{21}=q_{43}=q_{53}<q_{32}=q_{64}$	$P_1=P_4>P_2=P_3=P_5=P_6$	$w_1=w_2=w_3=w_4$
314	$q_{11}=q_{21}=q_{43}=q_{53}<q_{32}=q_{64}$	$P_1=P_4>P_2=P_3=P_5=P_6$	$w_1=w_3>w_2=w_4$
315	$q_{11}=q_{21}=q_{43}=q_{53}<q_{32}=q_{64}$	$P_1=P_4>P_2=P_5>P_3=P_6$	$w_1=w_2=w_3=w_4$
316	$q_{11}=q_{21}=q_{43}=q_{53}<q_{32}=q_{64}$	$P_1=P_4>P_2=P_5>P_3=P_6$	$w_1=w_3>w_2=w_4$

In all these scenarios, the number of rounds of ammunition available on a weapon is assumed to be three. All are solved with five different methods for comparison. Names and explanations of the methods are given below.

1. *Baron global:* While solving the original nonlinear model with BARON, it requires all decision variables to be bounded to guarantee global optimality. However, bounding the variables and letting solver to search the entire solution space takes long time. That is why we have to set a time limit of six hours. If the optimal solution is not found at this deadline, the best solution found in six hours is used.

2. *Baron local:* The BARON solution procedure starts with multiple local search attempts and it accepts the best solution among them as a starting point. If it has some or all variables unbounded, it tries to improve the solution by searching the local area around this starting point. In this case, however, BARON does not guarantee global optimality. This search with unbounded variables is much faster in reaching a solution than Baron global, but it has certainly inferior results.

3. *Linear model*: Linearized model given in Chapter 3 solved by CPLEX.

4. *Construction heuristic*: Heuristic given in Chapter 4.

5. *Improvement heuristic*: Heuristic given in Chapter 4 applied to the solution of the construction heuristic.

Since BARON could not find the global optimum for any scenario in six hours, we need a reasonable upper bound on objective function value to compare the quality of the results of these five approaches. The upper bound that BARON reports is the sum of all asset weights, assuming all assets survive. This is a trivial and loose bound; therefore, we propose a tighter upper bound.

In every scenario, any weapon can engage any threat. In other words, in each scenario, there is a possibility for each threat to be attacked by the most powerful weapon. In order to find an upper bound, we assume that each threat is attacked by the most powerful weapon against it. Additionally, attack duration of each threat is known, and the maximum number of engagements attainable in this duration according to SLS tactic can be calculated for each threat. Since we have limited number of rounds to fire from each weapon, assigning all available rounds of the most powerful weapon to each threat or the maximum number attainable in threat's attack duration (whichever is smaller) yields an upper bound.

Solution times of the models are also important. In Baron global, we set the solution time limit to 6 hours. In linear model, when the solution gets close to optimal by 0.5% gap, the solution process is terminated. Other three approaches have no such limitations. The computer used in solving the scenarios is a Pentium 4, 3.00 GHz, with 504 MB RAM desktop computer.

5.2. Experimental Results

In this section, experimental results are presented and discussed. Objective function values of five methods and their gaps with respect to the upper bound are presented in Tables 17, 18 and 19. For the solution of the linear model, the objective function value is recomputed using the original nonlinear objective function and the fifth order polynomial fit for the SSHP function (also used in BARON). Solution for each scenario can be found in Appendix C.

We expect the nonlinear model to give the best result. In small scenarios ($2\times2\times1$ and $3\times3\times2$) Baron global indeed yields the best outcomes. However, for scenarios of size $6\times6\times4$, the best solutions are found by the heuristic approach. In 14 of 16 scenarios, heuristic solution gives higher objective values than Baron global, which is better than linear model and Baron local solution approaches.

In heuristic approach, effectiveness of the improvement heuristic is observed usually to be quite small, sometimes insignificant. This indicates that the construction heuristic is the one that adds a value.

Table 17 – Objective Function Values and Deviations from Upper Bound for Scenarios of Size 2x2x1

Scenario No	Objective Function Value						Percent Deviation				
	Upper Bound	**Baron Global**	**Baron Local**	**Linear Model**	**Const. Heuristic**	**Impro. Heuristic**	**Baron Global**	**Baron Local**	**Linear Model**	**Const. Heuristic**	**Impro. Heuristic**
101	0.951093	0.942021	0.933371	0.942021	0.942022	0.942022	0.95	1.86	0.95	0.95	0.95
102	0.968073	0.961047	0.961047	0.94399	0.953609	0.953609	0.73	0.73	2.49	1.49	1.49
103	0.97407	0.961533	0.961047	0.956576	0.956577	0.956577	1.29	1.34	1.80	1.80	1.80
104	0.97407	0.956889	0.945583	0.947217	0.951941	0.951941	1.76	2.92	2.76	2.27	2.27
105	0.936605	0.924887	0.919152	0.924887	0.919154	0.919154	1.25	1.86	1.25	1.86	1.86
106	0.963773	0.955329	0.955329	0.92961	0.946973	0.946973	0.88	0.88	3.54	1.74	1.74
107	0.969743	0.955329	0.951037	0.934313	0.950492	0.950492	1.49	1.93	3.65	1.99	1.99
108	0.969743	0.948026	0.933879	0.933202	0.948026	0.948026	2.24	3.70	3.77	2.24	2.24
						Average:	**1.32**	**1.90**	**2.53**	**1.79**	**1.79**
						Max:	**2.24**	**3.70**	**3.77**	**2.27**	**2.27**

Table 18 – Objective Function Values and Deviations from Upper Bound for Scenarios of Size 3x3x2

Scenario No	Objective Function Value						Percent Deviation				
	Upper Bound	**Baron Global**	**Baron Local**	**Linear Model**	**Const. Heuristic**	**Impro. Heuristic**	**Baron Global**	**Baron Local**	**Linear Model**	**Const. Heuristic**	**Impro. Heuristic**
201	1.926333	1.912599	1.904149	1.912598	1.903615	1.903615	0.71	1.15	0.71	1.18	1.18
202	2.901574	2.883177	2.879389	2.883176	2.883178	2.883178	0.63	0.76	0.63	0.63	0.63
203	1.966638	1.951345	1.947661	1.948084	1.948621	1.948771	0.78	0.96	0.94	0.92	0.91
204	2.947919	2.927617	2.915017	2.920522	2.926943	2.926943	0.69	1.12	0.93	0.71	0.71
205	1.978007	1.954626	1.924681	1.944364	1.950021	1.950021	1.18	2.70	1.70	1.41	1.41
206	2.970658	2.937368	2.905358	2.929162	2.939428	2.939428	1.12	2.20	1.40	1.05	1.05
207	1.978007	1.957476	1.946796	1.948793	1.953180	1.953180	1.04	1.58	1.48	1.26	1.26
208	2.970658	2.944292	2.922946	2.933592	2.943256	2.943256	0.89	1.61	1.25	0.92	0.92
209	1.911477	1.894946	1.889293	1.894945	1.894947	1.894947	0.86	1.16	0.86	0.86	0.86
210	2.871862	2.849677	2.839219	2.847869	2.849679	2.849679	0.77	1.14	0.84	0.77	0.77
211	1.955406	1.937175	1.925973	1.931547	1.935632	1.935632	0.93	1.51	1.22	1.01	1.01
212	2.925457	2.901338	2.888927	2.887447	2.900874	2.900874	0.82	1.25	1.30	0.84	0.84
213	1.973598	1.945505	1.908077	1.935243	1.945508	1.945508	1.42	3.32	1.94	1.42	1.42
214	2.961839	2.919517	2.872845	2.91092	2.922021	2.922021	1.43	3.00	1.72	1.34	1.34
215	1.973598	1.949177	1.939503	1.939672	1.948951	1.948951	1.24	1.73	1.72	1.25	1.25
216	2.961839	2.927179	2.903884	2.91535	2.926947	2.926947	1.17	1.96	1.57	1.18	1.18
						Average:	**0.98**	**1.70**	**1.26**	**1.05**	**1.05**
						Max:	**1.43**	**3.32**	**1.94**	**1.42**	**1.42**

Table 19 – Objective Function Values and Deviations from Upper Bound for Scenarios of Size 6x6x4

Scenario No	Objective Function Value						Percent Deviation				
	Upper Bound	**Baron Global**	**Baron Local**	**Linear Model**	**Const. Heuristic**	**Impro. Heuristic**	**Baron Global**	**Baron Local**	**Linear Model**	**Const. Heuristic**	**Impro. Heuristic**
301	3.852667	3.825197	3.808097	3.816547	3.807231	3.807252	0.71	1.16	0.94	1.18	1.18
302	5.803147	5.766353	5.757766	5.757702	5.766357	5.766357	0.63	0.78	0.78	0.63	0.63
303	3.956014	3.935343	3.928757	3.841385	3.931997	3.932498	0.52	0.69	2.90	0.61	0.59
304	5.941316	5.906018	5.866369	5.772686	5.915191	5.915191	0.59	1.26	2.84	0.44	0.44
305	3.956014	3.888241	3.883875	3.860986	3.900042	3.900048	1.71	1.82	2.40	1.41	1.41
306	5.941316	5.859342	5.854562	5.82132	5.878856	5.878856	1.38	1.46	2.02	1.05	1.05
307	3.956014	3.896961	3.883107	3.860437	3.906359	3.906417	1.49	1.84	2.42	1.26	1.25
308	5.941316	5.863434	5.847805	5.818162	5.886513	5.886513	1.31	1.57	2.07	0.92	0.92
309	3.822955	3.789891	3.778768	3.764234	3.789894	3.789894	0.86	1.16	1.54	0.86	0.86
310	5.743723	5.699353	5.689142	5.648466	5.699358	5.699541	0.77	0.95	1.66	0.77	0.77
311	3.947195	3.920312	3.884818	3.826963	3.92335	3.92335	0.68	1.58	3.05	0.60	0.60
312	5.923678	5.887633	5.885464	5.748773	5.887013	5.891579	0.61	0.65	2.95	0.62	0.54
313	3.947195	3.880155	3.867694	3.841404	3.891016	3.891016	1.70	2.01	2.68	1.42	1.42
314	5.923678	5.829073	5.827279	5.759918	5.844042	5.844091	1.60	1.63	2.76	1.34	1.34
315	3.947195	3.879759	3.861642	3.831494	3.897902	3.897902	1.71	2.17	2.93	1.25	1.25
316	5.923678	5.836166	5.833085	5.775731	5.853895	5.854001	1.48	1.53	2.50	1.18	1.18
						Average:	**1.11**	**1.39**	**2.28**	**0.97**	**0.97**
						Max:	**1.71**	**2.17**	**3.05**	**1.42**	**1.42**

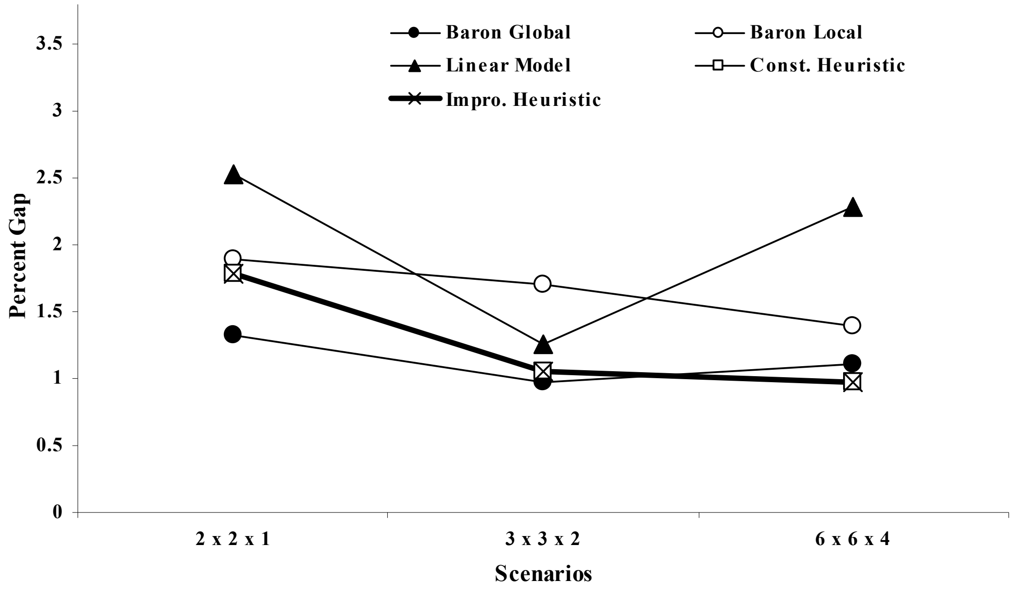

Figure 15 – Average Percent Deviation of the Methods

Average deviation from upper bound is a good measure to infer how well a method behaves. The graphical representation of the average gap values is given in Figure 15. Heuristic methods and Baron global tend to give smaller gaps whereas linear model shows an unstable pattern. Inferior outputs of linear model may be due to the setting to stop solver when 0.5% gap from the best possible is realized. This may also be caused by the linear approximations used.

Maximum deviation from the upper bound for each scenario is given in Figure 16. As expected, the same pattern as in Figure 15 is observed. Baron global and heuristic methods have almost the same values for the first two scenario sizes. However, as in average deviation, when the scenario size gets larger, heuristic approach has smaller maximum deviation than Baron global does.

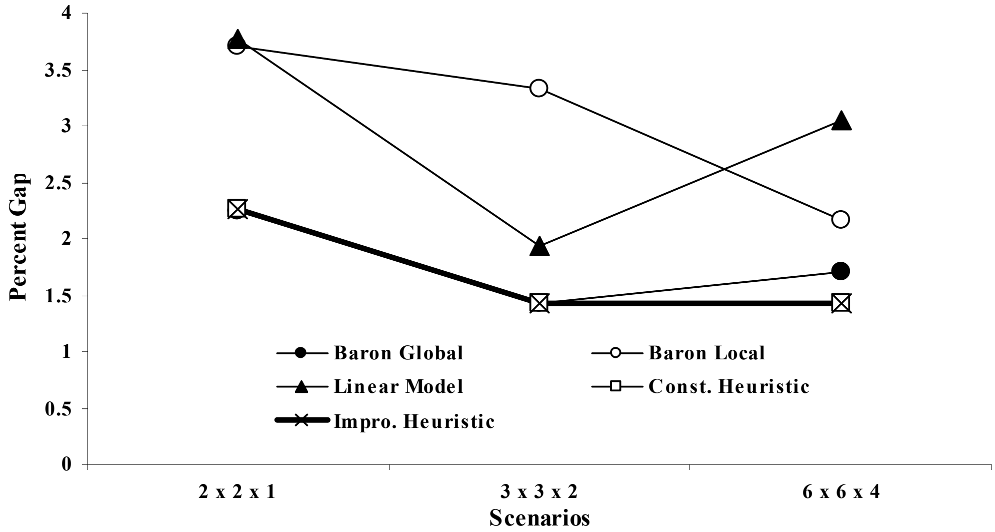

Figure 16 – Maximum Percent Deviation of the Methods

Figure 17 is the graphical representation of the gap values of individual scenarios. In scenarios of size $2 \times 2 \times 1$, Baron local and linear model have the most unpredictable behavior. For scenarios 102 and 106, Baron local has the best output, but for scenarios 101,104 and 108, it yields the worst results. In case of $3 \times 3 \times 2$ scenarios, Baron local is the worst method for almost every scenario, while linear model is the worst method for $6 \times 6 \times 4$ scenarios. Heuristic approaches form the worst (or tied for the worst) results for scenarios 103, 105, 201 and 301, although it has the best overall record throughout the testing process.

In even numbered scenarios, apart from Baron local results, the gap tends to decrease compared to previous odd numbered scenarios. For example, the gap is less for scenario 208 than that of 207. The only difference between these scenarios is in the weights of the assets. Since there is only one asset in scenarios of size $2 \times 2 \times 1$, we cannot observe this pattern. This shows that solution methods choose engagements and times better if the priorities of the assets are distinct from each other. When asset values are the same, construction heuristic yields bigger gaps than Baron global method for the scenarios of size $3 \times 3 \times 2$. With different asset weights, results of these scenarios get close. For larger scenarios, this trend disappears and construction heuristic dominates.

For scenarios of sizes $3\times3\times2$ and $6\times6\times4$, heuristic methods yields almost the same gaps for the same settings of scenarios. This shows that the solutions heuristics are consistent. It is an important inference since a good method should work consistently regardless of the scenario settings. None of the other methods produces similar gaps for different scenarios. This means that heuristic methods show consistent trends for scenarios of different size.

Apart from the objective value, solution speed is also an important factor. In Tables 20, 21 and 22, solution times are listed. The construction heuristic produces good solution quality as well as, it requires the shortest runtime among all. Improvement heuristic is slower than the construction heuristic since it runs in $O\left(n^5\right)$ and its contribution to solution is questionable when the solution time is taken into consideration. Another solution procedure that has short solution time is linear model. It is not affected by the increase in problem size as much as Baron local. Baron local has a behavior of showing a steep increase in solution time while problem size has increased. With an average of 175 seconds for $6\times6\times4$ scenarios, this approach is not of practical use.

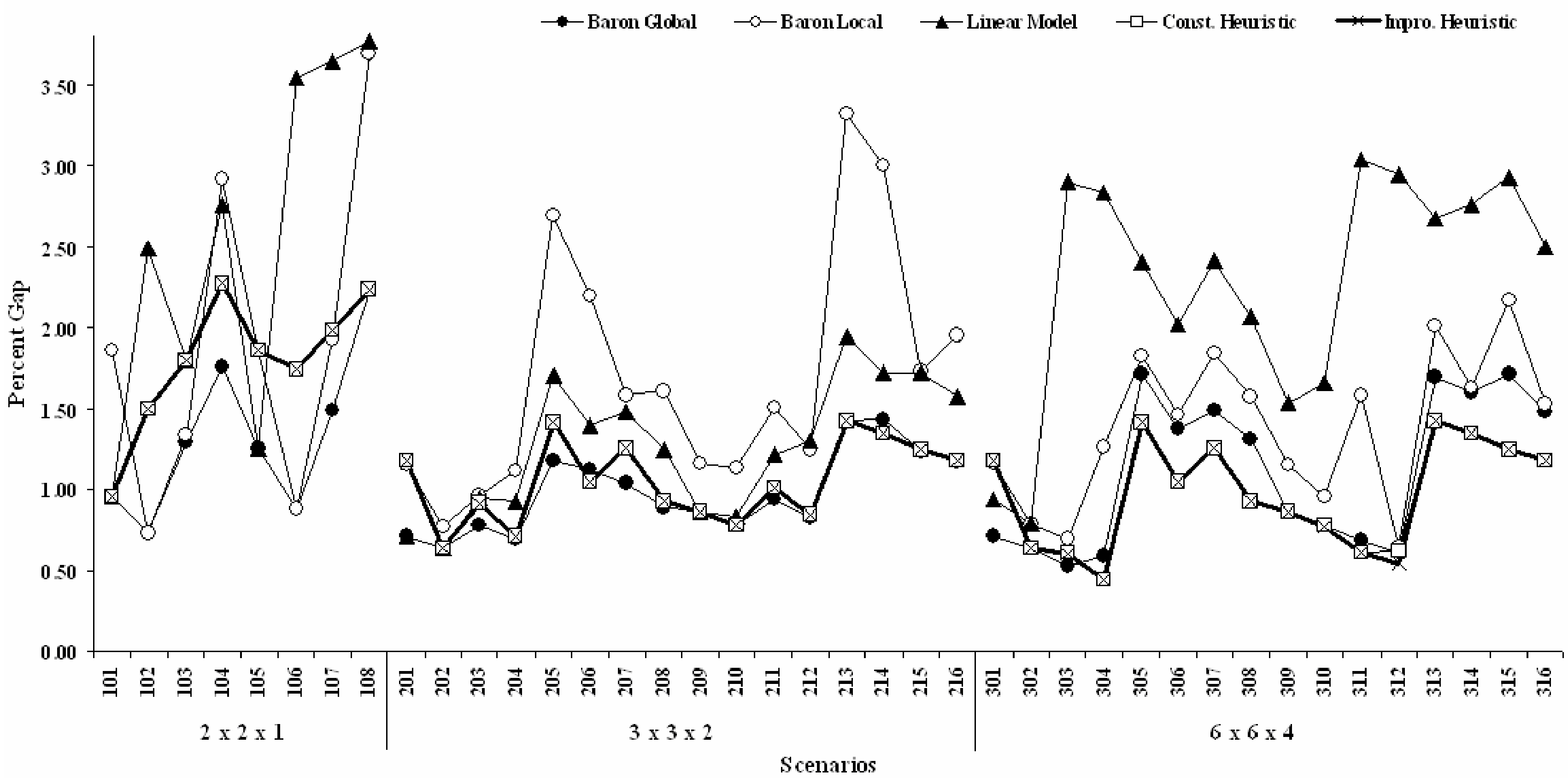

Figure 17 – Percent Deviation of Models for Scenarios

Table 20 – Solution Times For Scenarios of Size 2x2x1 (Seconds)

Scenario No	Baron Global	Baron Local	Linear Model	Const. Heuristic	Impro. Heuristic
101	21600	1.516	0.266	0.027	0.047
102	21600	1.453	0.204	0.028	0.107
103	21600	1.532	0.251	0.026	0.187
104	21600	1.500	0.157	0.027	0.138
105	21600	1.515	0.141	0.027	0.543
106	21600	1.421	0.235	0.028	0.133
107	21600	1.435	0.204	0.027	0.220
108	21600	1.427	0.188	0.028	0.967
Average	**21600**	**1.475**	**0.206**	**0.027**	**0.293**
Max	**21600**	**1.532**	**0.266**	**0.028**	**0.967**

Table 21 – Solution Times for Scenarios of Size 3x3x2 (seconds)

Scenario No	Baron Global	Baron Local	Linear Model	Const. Heuristic	Impro. Heuristic
201	21600	8.125	0.282	0.036	0.580
202	21600	8.469	0.235	0.055	0.887
203	21600	7.375	0.156	0.083	1.456
204	21600	6.734	0.188	0.084	1.163
205	21600	4.906	0.172	0.097	1.292
206	21600	5.141	0.188	0.106	0.654
207	21600	7.453	0.172	0.068	0.915
208	21600	7.484	0.172	0.107	1.045
209	21600	8.515	0.188	0.094	0.076
210	21600	8.266	0.141	0.127	0.461
211	21600	6.828	0.172	0.138	1.100
212	21600	7.377	0.109	0.116	1.170
213	21600	4.828	0.110	0.077	1.447
214	21600	4.750	0.172	0.056	0.375
215	21600	7.500	0.188	0.053	1.327
216	21600	8.904	0.298	0.073	0.015
Average	**21600**	**7.041**	**0.184**	**0.086**	**0.873**
Max	**21600**	**8.904**	**0.298**	**0.138**	**1.456**

Table 22 – Solution Times for Scenarios of Size 6x6x4 (seconds)

Scenario No	Baron Global	Baron Local	Linear Model	Const. Heuristic	Impro. Heuristic
301	21600	207.179	0.529	0.073	0.567
302	21600	204.785	0.764	0.043	1.682
303	21600	186.819	0.264	0.052	2.690
304	21600	208.874	0.265	0.053	1.046
305	21600	157.370	0.359	0.048	2.019
306	21600	155.995	0.452	0.047	2.612
307	21600	149.887	0.311	0.052	1.554
308	21600	152.146	0.359	0.048	2.090
309	21600	199.636	0.436	0.049	2.331
310	21600	199.105	0.436	0.076	1.761
311	21600	212.813	0.234	0.072	1.035
312	21600	180.723	0.265	0.063	2.441
313	21600	121.260	0.467	0.061	0.985
314	21600	144.295	0.483	0.057	2.786
315	21600	178.285	0.358	0.058	1.045
316	21600	148.523	0.421	0.058	2.365
Average:	**21600**	**175.481**	**0.400**	**0.057**	**1.813**
Max:	**21600**	**212.813**	**0.764**	**0.076**	**2.786**

To improve solution quality of linear model, we change the gap setting of the solver from 0.5% to 0.2%. We have the gap and time values as in Table 23 for $6 \times 6 \times 4$ scenarios.

Consequently, we are able to improve percent deviation from upper bound by changing the settings of linear model solver. Here, we are confronted with the question of “what is the best setting that maximizes our satisfaction in terms of solution quality and time?”. For example, in scenario 306, we can reduce the deviation while keeping the solution time almost the same; but in 313, reduction in deviation may not be worth the excessive solution time.

Table 23 – Percent Deviation from Upper Bound and Time for Linear Model with 0.2% Gap

Scenario	New Deviation	New Time	Old Deviation	Old Time
301	0.93	31.571	0.94	0.529
302	0.76	27.806	0.78	0.764
303	1.61	64.718	2.90	0.264
304	1.95	7.670	2.84	0.265
305	2.32	29.461	2.40	0.359
306	1.85	0.522	2.02	0.452
307	1.51	17.980	2.42	0.311
308	1.11	2.547	2.07	0.359
309	1.28	30.883	1.54	0.436
310	0.98	35.163	1.66	0.436
311	2.73	38.069	3.05	0.234
312	1.55	26.134	2.95	0.265
313	2.57	42.201	2.68	0.467
314	2.15	33.304	2.76	0.483
315	2.02	30.899	2.93	0.358
316	1.64	31.008	2.50	0.421
Average	**1.69**	**28.115**	**2.28**	**0.400**
Max	**2.73**	**64.718**	**3.05**	**0.764**

So far, the results of five different solution methods have been discussed for scenarios of size up to $6 \times 6 \times 4$. It has been shown that the construction heuristic based on “as late as possible engagement scheduling” works faster than any other method and yields acceptable solutions. For a larger scenario, linear model and construction heuristic are compared. A scenario consisting of 36 weapons, 36 threats and 24 assets has been solved by linear model and construction heuristic. In this scenario, all asset values have been chosen randomly between 1 and 10, each threat has equal probability of kill. Each weapon’s SSHP has been chosen randomly among four different SSHP curves used in previous scenarios. Results of these runs are given in Table 24. Time of the construction heuristic is still very small while maintaining a reasonable result. If a trade-off is considered, choosing linear model for a 1.3501 increase in the objective function may not be worth the additional solution time. A surprising result is that the construction heuristic, which was the best method in the previous scenarios, performed slightly worse than the linear modeling.

Table 24 –Linear Modeling and Construction Heuristic Outputs for Scenario Size Of 36x36x24 with Varying SSHP Values

Solution Method	Objective Value	Solution Time (seconds)
Linear Model	135.4950	147.0000
Construction Heuristic	134.1449	0.7048

If we remove the effect of linearization of SSHP function – if we use fixed probability values in linear model and construction heuristic – the resulting objective and time values will be as in Table 25. The objective value of the construction heuristic is now better than that of linear model. Linear model yields solution values smaller than the previous case. Since objective values of the two methods are almost the same in both cases, we can still vote for the construction heuristic over the linear model.

Table 25 –Linear Modeling and Construction Heuristic Outputs for Scenario Size Of 36x36x24 with Fixed SSHP Values

Solution Method	Objective Value	Solution Time (seconds)
Linear Model	131.8658	122.0320
Construction Heuristic	133.5935	0.7316

CHAPTER 6

CONCLUSION

In this thesis, we have developed mathematical modeling and heuristic approaches the for weapon-target allocation problem when the single shot kill probabilities of weapons against threats are time-variant. Assets are valued according to their importance and the sum of their weighted survival probabilities is to be maximized. The mathematical modeling approach starts with a nonlinear formulation. Linearizing the nonlinear objective function and constraints is one aspect of the linear modeling scheme. Approximating the time-variant single shot hit probabilities and embedding them in this partially linearized model is the second aspect. At the end, we have a linear model whose objective is to maximize the minimum survivability of the assets. Results produced by this model are reasonably close to the results of the original nonlinear model solved by BARON solver of GAMS. From both nonlinear and linear models, we infer that engagements should be scheduled as late as possible assuming the threat's kill probability is constant during the attack. The reason for this is that as threats get closer to weapons, SSHP values become higher, increasing the chance of neutralizing threats.

In the linear model, the objective is changed from minisum to minimax, and we approximate the nonlinear time varying SSHP values as well. We have also developed a heuristic approach for the original objective. It consists of a construction heuristic and improvement heuristic. Construction heuristic is based on assigning risk levels to assets considering the density of attack they are exposed to and the coverage provided by the defending weapons. Improvement heuristic uses the solution produced by the construction heuristic and tries to swap engagements from the highly defended assets to the least guarded ones.

After development of all these approaches, we have tested them on forty different scenarios and included solutions of Baron global and Baron local methods as well.

In the WTA problem literature, single shot hit probabilities are usually taken as constant parameters. What is newly introduced with this thesis is using time-variant single shot hit probability values in a linear formulation, which is the result of linearizing a nonlinear mathematical model. In our formulation, probability values are not parameters, but they are decision variables depending on the engagement times, which are also decision variables.

Among all the approaches we have considered in the experiments, the construction heuristic performs better than the others do, as the scenario size gets larger. Since solution time performance is important in WTA problems, we have compared the two fastest solution methods, linear model and construction heuristic, in a large scenario setting. Although the construction heuristic performs a little worse than the linear model for this problem in terms of the objective function value, its time performance is quite superior to the linear model. Overall, to have quick decision support against approaching enemy threats in a real warfare environment, the construction heuristic can be deployed in command and control centers. Having small deviations from the optimum in exchange for rapid solutions can be acceptable in surface-to-air defense is found desirable.

There is a tradeoff between assigning more rounds to a threat with high kill probability, and assigning more rounds to a threat that we are not very successful in neutralizing. Decision makers who are in charge of choosing the appropriate engagement strategy may suggest a tactic considering this tradeoff. A suggested tactic can be applied to the decision support models by defining some preference functions. This may constitute a possible future work issue.

Another future work that this thesis may address is that different flight trajectories and strategies corresponding to different single shot hit probabilities of weapons against threats can be considered. Thus, there may be different probability function approximations in a model. For several geographical locations, expected attack trajectories of threats are usually known. For such cases, hit probabilities can be pre-calculated using the method by Yüksel (2007). Then, piecewise linear-approximations or table lookup strategies can be developed to be used in the linear model.

In this thesis, SSHP and time of flight calculations are done by assuming head-to-head collisions. However, this may not be true in general. Better SSHP and time of flight calculations with prediction angles and varying ammunition speed can be studied in future to obtain better scheduling.

Another further research direction might be approaching the original nonlinear model or partially linearized model (linearized objective function and nonlinear constraints, or nonlinear objective function and linearized constraints) using different solution schemes. Different linear and nonlinear objective functions may also be tried. It would be more realistic to assume that kill probabilities of threats (q_{ik}) also vary in time as they approach their targets. Since these probabilities are taken as constant in this study, asset survivability is maximized by delaying the shots as much as possible, which is risky. If q_{ik} values differ over time, the defense may have to start firing at the threats earlier.

Additionally, we follow a static approach to make weapon-target allocation. In other words, engagement plan is constructed at the beginning of enemy attack and not changed during the whole period. However, if a success is realized at in early times of the attack, other engagements on the neutralized threat will remain idle. Thus, if engagement plan is reconstructed at that instance, it might differ and probably provide a better defense scheme. In addition, not only hitting an enemy, but also newly detected enemies, killed weapons and killed assets contributes to this dynamic version of the problem. As a result, allocation of weapons in an environment that changes in time might be an interesting future issue.

Our assumptions may be relaxed to allow tactics other than shoot-look-shoot, such as shoot-shoot-look. One may also restrict the weapons such that they cannot fire at multiple threats simultaneously. Although weapon setup times are considered in the heuristic approach, weapons are assumed to be able to fire at multiple threats at a given time to make the results comparable with the mathematical models.

Finally, developing and comparing different heuristic methods for the same problem or modified problems with differences mentioned above may be another future issue for researchers.

REFERENCES

Ahuja, R.K, Kumar, A., Jha, K., Orlin J. B. (2003), "Exact and Heuristic Methods for the Weapon Target Assignment Problem", MIT Sloan School of Management, Working Paper No: 4464-03

Blodgett et al. (2003), "A Tabu Search Heuristic for Resource Management In NavalWarfare", Journal of Heuristics, Issue: 9, pp. 145–169.

Day, R.H. (1966), "Allocating Weapons to Target Complexes by means of Nonlinear Programming ", Operations Research, Vol. 14 No. 6, pp. 992-1013.

Den Broeder, Jr. G.G., Ellinson, R.E., Emerling, L. (1959) "On Optimum Target Assignments", Operations Research, Vol. 7 No. 3, pp. 322-326.

Gülez, T., Kandiller, L., Özdemirel, N.E. (2007), "Değişken Vurma Olasılıklı Hava Savunma Silahlarının Grup Halinde Hava Savunma Stratejisinin Planlanması Ve Optimizasyonu", National Defense Applications Modeling and Simulation Conference 2007 Symposium, Ankara, Turkey

Hosein, P.A., Athans, M. (1990), "An Asymptotic Result For The Multistage Weapon-Target Allocation Problem", Proceedings of the 29th Conference on Decision and Control, Honolulu, Hawaii, USA, December 1990.

Karasakal,O. (2004), "Optimal Air Defense Strategies For A Naval Task Group", PhD. Thesis, Middle East Technical University, Ankara, Turkey.

Kwon, O., Kang, D., Lee, K., Park, S. (1999), "Lagrangean Relaxation Approach to the Targeting Problem", Naval Research Logistics, Vol.46 No.4, pp. 640-653.

Lee, Z-J., Su, S-F., and Lee, C-Y. (2002), "An Immunity-Based Ant Colony Optimization Algorithm for Solving Weapon–Target Assignment Problem", Applied Soft Computing Issue: 2, pp.39–47.

Lee, Z-J., Su, S-F., and Lee, C-Y. (2003), "Efficiently Solving General Weapon-Target Assignment Problem by Genetic Algorithms with Greedy Eugenics," Systems, Man and Cybernetics, Part B, IEEE Transactions on, Volume: 33, Issue: 1, pp.113-121.

Lemus, F., David, K.H. (1963), "An Optimum Allocation of Different Weapons to a Target Complex", Operations Research, Vol. 11 No. 5, pp. 787-794.

Lloyd, S.P. and H.S. Witsenhausen (1986), "Weapons Allocation is NP-Complete", Proceedings of the 1986 Summer Computer Simulation Conference, pp. 1054-1058.

Malcolm, W.P. (2004), "On the Character and Complexity of Certain Defensive Resource Allocation Problems", DSTO-TR-1570, DSTO Systems Sciences Laboratory, Australia, March 2004.

Matlin, S. (1970), "A Review of the Literature on the Missile-Allocation Problem", Operations Research, Vol. 18 No. 2, pp. 334-373.

Metler, W.A. and Preston, F.L. (1989), "Solutions to a Probabilistic Resource Allocation Problem", Proceedings of the 28th IEEE Conference on Decision and Control, Vol. 2, pp. 1606-1611.

Passy, U. (1971), "Nonlinear Assignment Problems Treated by Geometric Programming", Operations Research, Vol. 19 No. 7, pp. 1675-1690.

Rosenberger et al. (2005), "The Generalized Weapon Target Assignment Problem", 10th International Command and Control Research and Technology Symposium The Future of C^2 , McLean, Vancouver, June, 2005.

Wacholder, E. (1989), “A Neural Network-Based Optimization Algorithm for the Static Weapon-Target Assignment Problem”, ORSA Journal on Computing, Vol.1 No.4, pp. 232-246.

Yüksel, İ. (2007), “Single Shot Hit Probability Computation for Air Defense Based on Error Analysis”, M.Sc. Thesis, Middle East Technical University, Ankara, Turkey.

APPENDIX A

Optimally Linearizing a Concave Curve

For the concave part of the $a_k^{|I_k|}$ graph, to make a good linear approximation in two pieces, we need to minimize the areas remaining between the curve and the lines. In order to investigate how to make a good linear approximation to a generic concave graph, consider the generic concave curve as in Figure 2. Minimizing the areas $\widehat{SR}-|SR|$ and $\widehat{RU}-|RU|$ is equivalent to maximizing the shaded area defined by points *SRU*. If we divide this area into I and II, and maximize I + II, then we accomplish the approximation in the best way. Maximizing $\left[A(S\overset{\vartriangle}{P}R)-A(S\overset{\vartriangle}{P}Q)\right]+\left[A(Q\overset{\vartriangle}{T}U)-A(R\overset{\vartriangle}{T}U)\right]$ is equivalent to maximizing I + II. The segment $|PQ|$ can be expressed as αX where $\alpha=\dfrac{f(b)-f(a)}{b-a}$. Substituting this expression in the area calculations, we get the following set of equations.

$$I=\frac{1}{2}(x-a)\left(\left[f(x)-f(a)\right]-\left[\alpha x-f(a)\right]\right)=\frac{1}{2}(x-a)\left(f(x)-\alpha x\right)$$

$$II=\frac{1}{2}(b-x)\left(\left[f(b)-\alpha x\right]-\left[f(b)-f(x)\right]\right)=\frac{1}{2}(b-x)\left(f(x)-\alpha x\right)$$

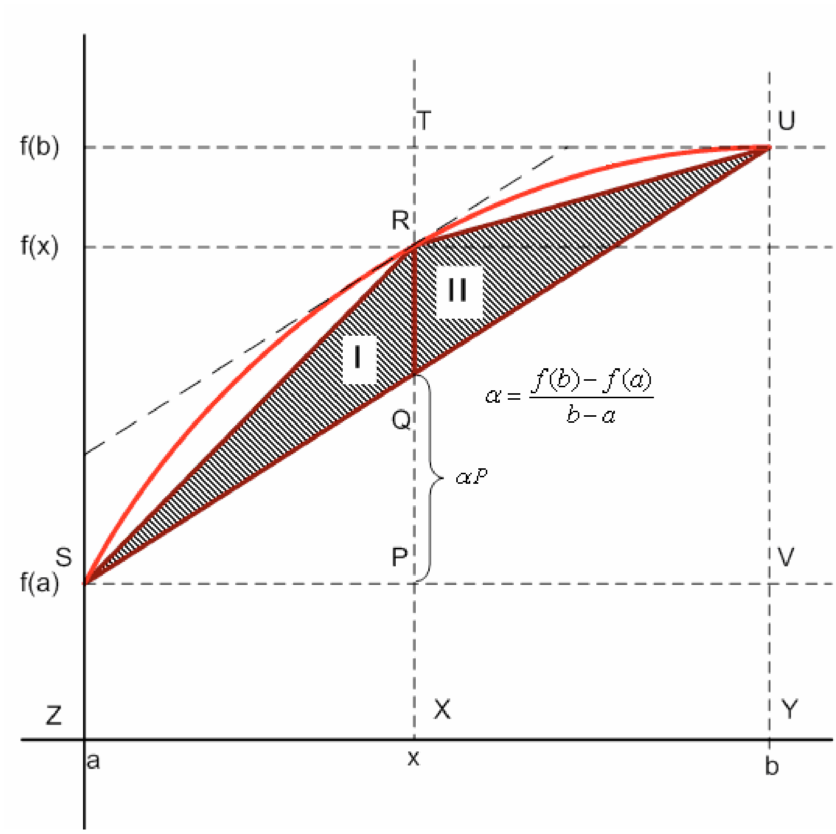

Figure 18 – Generic Concave Curve

$Max(I+II) \equiv Max\,2(I+II) = g(x)$

$$g(x) = \left[xf(x) - \alpha x^2 - af(x) + a\alpha x\right] + \left[bf(x) + \alpha x^2 - xf(x) + b\alpha x\right]$$

$$g(x) = (b-a)\left[f(x) - \alpha x\right] \qquad \text{(A.1)}$$

If we take the derivative of (A.1) and set it equal to zero, then we have the value of x where $g(x)$ is at its maximum. $\frac{dg(x)}{dx} = 0 = (b-a)\left[f'(x) - \alpha\right]$ and since $b \neq a$

$$f'(x) = \alpha \qquad \text{(A.2)}$$

As a result, when (A.2) is attained for an x value, optimal two-piece linearization is achieved.

APPENDIX B

Results of Curve Fitting Software for (1-SSHP) Functions

Table 26 – Results of Curve Fitting Software for (1-SSHP) Values

Rank	Model	Standard Error	Residual Sum	RSS	R^2
1	Tenth order polynomial	0.000000	0.000000	0.000000	1.000000
2	Ninth order polynomial	0.000000	0.000000	0.000000	1.000000
3	Eighth order polynomial	0.000000	0.000000	0.000000	1.000000
4	Seventh order polynomial	0.000000	0.000000	0.000000	1.000000
5	Sixth order polynomial	0.000000	0.000000	0.000000	1.000000
6	Fifth order polynomial	0.000000	0.000000	0.000000	1.000000
7	Fourth order polynomial	0.003870	0.000000	0.000135	0.999875
8	Fifth order logarithm	0.005582	0.000000	0.000249	0.999769
9	Fourth order logarithm	0.005486	0.000000	0.000271	0.999749
10	a*x^3+b*x^2+c*x+d	0.006114	0.000000	0.000374	0.999654
11	a+b*x*log(x)+c*x^1.5	0.006064	0.000000	0.000405	0.999625
12	a+b*x^1.5+c*log(x)^2	0.006212	0.000000	0.000425	0.999607
13	a+b*x^1.5+c*x^2	0.006254	0.000000	0.000430	0.999602
14	a+b*x^1.5+c*x^2*log(x)	0.006260	0.000000	0.000431	0.999601
15	a+b*x^1.5+c*x^2.5	0.006309	0.000000	0.000438	0.999595
16	a+b*x^1.5+c*x^3	0.006444	0.000000	0.000457	0.999577
17	a+b*x+c*x^1.5	0.006522	0.000000	0.000468	0.999567
18	a+b*x*log(x)+c*x^2	0.006525	0.000000	0.000468	0.999566
19	a+b*x*log(x)+c*x^2*log(x)	0.006553	0.000000	0.000472	0.999563
20	a+b*x^1.5+c*x^.5*log(x)	0.006568	0.000000	0.000475	0.999561
21	a+b*x*log(x)+c*x^2.5	0.006640	0.000000	0.000485	0.999551
22	a*x^2+b*x+c	0.006685	0.000000	0.000492	0.999545
23	a+b*x^2+c*x^2*log(x)	0.006769	0.000000	0.000504	0.999533
24	a+b*x^1.5+c*x^.5	0.006863	0.000000	0.000518	0.999520
25	a+b*x*log(x)+c*x^3	0.007004	0.000000	0.000540	0.999500
26	a+b*x^2+c*x^.5*log(x)	0.007098	0.000000	0.000554	0.999487
27	a+b*x^2+c*log(x)^2	0.007100	0.000000	0.000555	0.999486
28	a+b*x^1.5+c*log(x)	0.007284	0.000000	0.000584	0.999459

Table 26(continued) – Results of Curve Fitting Software for (1-SSHP) Values

Rank	Model	Standard Error	Residual Sum	RSS	R2
29	a+b*x*log(x)+c*log(x)^2	0.007485	0.000000	0.000616	0.999429
30	a+b*x^2+c*x^2.5	0.007679	0.000000	0.000649	0.999399
31	a+b*x^1.5+c/x^.5	0.007702	0.000000	0.000653	0.999396
32	a+b*x+c*x^2*log(x)	0.007950	0.000000	0.000695	0.999356
33	a+b*x^1.5+c/x	0.008050	0.000000	0.000713	0.999340
34	a+b*x+c*x^.5*log(x)	0.008133	0.000000	0.000728	0.999326
35	a+b*x^1.5+c*exp(-x)	0.008295	0.000000	0.000757	0.999299
36	a+b*x^1.5+c/x^1.5	0.008303	0.000000	0.000758	0.999298
37	a+b*x^2+c*x^.5	0.008339	0.000000	0.000765	0.999292
38	a+b*x^1.5+c/x^2	0.008473	0.000000	0.000790	0.999269
39	a+b*x^2*log(x)+c*log(x)^2	0.008611	0.000000	0.000816	0.999245
40	a+b*x^2+c*x^3	0.008653	0.000000	0.000824	0.999237
41	a+b*x^1.5+c*log(x)/x^2	0.008779	0.000000	0.000848	0.999215
42	a+b*x^1.5+c*exp(x)	0.008804	0.000000	0.000853	0.999210
43	a+b*x^1.5+c*log(x)/x	0.008981	0.000000	0.000887	0.999178
44	a+b*x+c*x^2.5	0.009054	0.000000	0.000902	0.999165
45	a+b*x^1.5	0.008698	0.000000	0.000908	0.999159
46	a+b*x^2*log(x)+c*x^.5*log(x)	0.009241	0.000000	0.000939	0.999130
47	a+b*x*log(x)+c*x^.5*log(x)	0.009353	0.000000	0.000962	0.999109
48	a+b*x+c*x*log(x)	0.009557	0.000000	0.001005	0.999070
49	a+b*x^2.5+c*log(x)^2	0.009959	0.000000	0.001091	0.998990
50	a+b*x*log(x)+c*x^.5	0.010470	0.000000	0.001206	0.998883
51	a+b*x^2.5+c*x^.5*log(x)	0.010799	0.000000	0.001283	0.998812
52	a+b*x^2+c*log(x)	0.010845	0.000000	0.001294	0.998802
53	a+b*x+c*x^3	0.011875	0.000000	0.001551	0.998563
54	a+b*x*log(x)+c*log(x)	0.011916	0.000000	0.001562	0.998554
55	a+b*x^2*log(x)+c*x^.5	0.012492	0.000000	0.001717	0.998410
56	Third order logarithm	0.013124	0.000000	0.001722	0.998405
57	a+b*x+c*log(x)^2	0.012664	0.000000	0.001764	0.998366
58	Fifth order inverse polynomial	0.015611	0.000000	0.001950	0.998194
59	a+b*x*log(x)+c/x^.5	0.013322	0.000000	0.001952	0.998192
60	a+b*x^2.5+c*x^3	0.013573	0.000000	0.002026	0.998123
61	a+b*x^2+c/x^.5	0.013575	0.000000	0.002027	0.998123
62	exp(-exp(a-b*x))	0.013224	-0.000086	0.002098	0.998057
63	a+b*x^3+c*log(x)^2	0.013846	0.000000	0.002109	0.998047
64	a+b*x+c*x^.5	0.014265	0.000000	0.002238	0.997927
65	log(a+b*x)	0.013765	0.000000	0.002274	0.997894
66	a+b*x*log(x)+c/x	0.014474	0.000000	0.002305	0.997866
67	a+b*x^2.5+c*x^.5	0.014686	0.000000	0.002373	0.997803
68	a+b*x*log(x)+c*exp(x)	0.014806	0.000000	0.002411	0.997767
69	a+b*x^3+c*x^.5*log(x)	0.014954	0.000000	0.002460	0.997722
70	a+b*x*log(x)+c*exp(-x)	0.015215	0.000000	0.002546	0.997642
71	a+b*x*log(x)+c/x^1.5	0.015314	0.000000	0.002580	0.997611
72	a+b*x*log(x)+c/x^2	0.015885	0.000000	0.002776	0.997430
73	a+b*x^2+c/x	0.016023	0.000000	0.002824	0.997385
74	a+b*x^2*log(x)+c*x^3	0.016347	0.000000	0.002939	0.997278
75	a+b*x^2+c*exp(x)	0.017019	0.000000	0.003186	0.997050

Table 26(continued) – Results of Curve Fitting Software for (1-SSHP) Values

Rank	Model	Standard Error	Residual Sum	RSS	R2
76	a+b*x*log(x)+c*log(x)/x^2	0.017548	0.000000	0.003387	0.996863
77	a+b*x*log(x)+c*log(x)/x	0.017570	0.000000	0.003396	0.996855
78	a+b*x^2+c*exp(-x)	0.017754	0.000000	0.003467	0.996789
79	a+b*x^2+c/x^1.5	0.017946	0.000000	0.003542	0.996719
80	a+b*x*log(x)	0.017390	0.000000	0.003629	0.996639
81	a+b*x^2*log(x)+c*log(x)	0.018302	0.000000	0.003685	0.996588
82	a+b*x^2+c/x^2	0.019334	0.000000	0.004112	0.996192
83	a+b*x+c*log(x)	0.019429	0.000000	0.004152	0.996155
84	a+b*x^3+c*x^.5	0.020941	0.000000	0.004824	0.995533
85	a+b*x^2.5+c*log(x)	0.021327	0.000000	0.005003	0.995367
86	a+b*x^2*log(x)+c*x^2.5	0.023820	0.000000	0.006242	0.994220
87	a+b*x^2*log(x)+c/x^.5	0.024129	0.000000	0.006404	0.994069
88	a+b*x^.5*log(x)+c*x^.5	0.024133	0.000000	0.006406	0.994067
89	a+b*x+c/x^.5	0.024301	0.000000	0.006496	0.993985
90	a+b*x^2+c*log(x)/x	0.024731	0.000000	0.006728	0.993770
91	a+b*x^2+c*log(x)/x^2	0.026743	0.000000	0.007867	0.992715
92	a+b*x^2	0.026109	0.000000	0.008180	0.992425
93	a+b*x^2.5+c/x^.5	0.027780	0.000000	0.008489	0.992139
94	a+b*x+c/x	0.028396	0.000000	0.008870	0.991786
95	a+b*x^.5*log(x)+c*log(x)	0.028835	0.000000	0.009146	0.991530
96	a+b*x^2*log(x)+c/x	0.029118	0.000000	0.009326	0.991363
97	a+b*x+c*exp(-x)	0.030646	0.000000	0.010331	0.990433
98	a+b*x^3+c*log(x)	0.031040	0.000000	0.010598	0.990185
99	a+b*x+c/x^1.5	0.031538	0.000000	0.010941	0.989868
100	a+b*x^2*log(x)+c*exp(-x)	0.032811	0.000000	0.011842	0.989033
101	a+b*x^2*log(x)+c/x^1.5	0.032910	0.000000	0.011914	0.988967
102	a+b*x^2.5+c/x	0.033202	0.000000	0.012126	0.988770
103	a+b*x^2*log(x)+c*exp(x)	0.033669	0.000000	0.012470	0.988452
104	a+b*x+c/x^2	0.033798	0.000000	0.012566	0.988363
105	a+b*x^.5+c*log(x)	0.034379	0.000000	0.013001	0.987960
106	a+b*x^.5*log(x)+c/x^.5	0.035104	0.000000	0.013555	0.987447
107	a+b*x+c*exp(x)	0.035434	0.000000	0.013812	0.987209
108	a+b*x^2*log(x)+c/x^2	0.035578	0.000000	0.013924	0.987105
109	a+b*x^2.5+c*exp(x)	0.035864	0.000000	0.014149	0.986898
110	a+b*x^2.5+c/x^1.5	0.037267	0.000000	0.015277	0.985852
111	a+b*x^2.5+c*exp(-x)	0.037281	0.000000	0.015289	0.985842
112	a*exp((-(x-b)^2)/(2*c^2))	0.037297	-0.042426	0.015301	0.985830
113	exp(a+b*x+c*x^2)	0.037297	-0.042426	0.015301	0.985830
114	Fourth order inverse polynomial	0.041700	0.000000	0.015650	0.985507
115	a+b*x+c*log(x)/x	0.039976	0.000000	0.017579	0.983721
116	a+b*x^2.5+c/x^2	0.040093	0.000000	0.017682	0.983625
117	a+b*x^.5*log(x)+c/x	0.040610	0.000000	0.018141	0.983200
118	a+b*x^3+c/x^.5	0.040643	0.000000	0.018170	0.983173
119	a+b*x^.5*log(x)+c*exp(-x)	0.043242	0.000000	0.020569	0.980952
120	a+b*x^.5+c/x^.5	0.043936	0.000000	0.021234	0.980336
121	a+b*x^.5*log(x)+c/x^1.5	0.044916	0.000000	0.022192	0.979449
122	a+b*x^2*log(x)+c*log(x)/x	0.046251	0.000000	0.023531	0.978209

Table 26(continued) – Results of Curve Fitting Software for (1-SSHP) Values

Rank	Model	Standard Error	Residual Sum	RSS	R2
123	a+b*x+c*log(x)/x^2	0.047172	0.000000	0.024478	0.977332
124	a*x+b	0.045167	0.000000	0.024480	0.977330
125	a*(x-b)	0.045167	0.000000	0.024480	0.977330
126	a+b*x^2*log(x)+c*log(x)/x^2	0.047327	0.000000	0.024639	0.977183
127	a+b*x^.5*log(x)+c/x^2	0.048055	0.000000	0.025402	0.976476
128	a+b*x^3+c/x	0.048574	0.000000	0.025953	0.975965
129	a+b*exp(x)+c*x^.5*log(x)	0.048750	0.000000	0.026142	0.975791
130	a+b*x^2*log(x)	0.047314	0.000000	0.026864	0.975122
131	Second order logarithm	0.049641	0.000000	0.027107	0.974897
132	a+b*log(x)^2+c*log(x)	0.049641	0.000000	0.027107	0.974897
133	a+b*x^.5*log(x)+c*log(x)^2	0.049808	0.000000	0.027290	0.974728
134	exp(-a*x^2)	0.047086	-0.195447	0.028822	0.973309
135	a+b*x^2.5+c*log(x)/x^2	0.051378	0.000000	0.029037	0.973110
136	a+b*x^2.5+c*log(x)/x	0.051380	0.000000	0.029038	0.973108
137	a+b*x^3+c*exp(x)	0.051887	0.000000	0.029614	0.972575
138	a+b*x^.5+c/x	0.052042	0.000000	0.029792	0.972411
139	a+b*log(x)^2+c/x^.5	0.052267	0.000000	0.030050	0.972171
140	a+b*exp(x)+c*log(x)^2	0.052694	0.000000	0.030544	0.971715
141	a+b*x^2.5	0.051821	0.000000	0.032225	0.970157
142	a+b*x^3+c/x^1.5	0.054407	0.000000	0.032562	0.969846
143	a+b*x^3+c*exp(-x)	0.054806	0.000000	0.033040	0.969402
144	a+b*x^.5+c*exp(-x)	0.055096	0.000000	0.033391	0.969078
145	1-exp(-a*b^x)	0.052997	-0.170936	0.033705	0.968787
146	a+b*x^.5*log(x)+c*log(x)/x	0.055391	0.000000	0.033750	0.968745
147	a+b*log(x)^2+c/x	0.056338	0.000000	0.034913	0.967668
148	a+b*log(x)^2+c*exp(-x)	0.057913	0.000000	0.036894	0.965834
149	a+b*x^.5+c/x^1.5	0.058382	0.000000	0.037493	0.965279
150	a+b*x^3+c/x^2	0.058385	0.000000	0.037497	0.965275
151	a+b*log(x)^2+c/x^1.5	0.059883	0.000000	0.039446	0.963471
152	a+b*log(x)^2+c*x^.5	0.062043	0.000000	0.042343	0.960788
153	a+b*log(x)^2+c/x^2	0.062548	0.000000	0.043035	0.960147
154	a+b*x^.5+c/x^2	0.063052	0.000000	0.043732	0.959502
155	a+b*log(x)+c/x^.5	0.063889	0.000000	0.044900	0.958420
156	a+b*x^.5*log(x)+c*log(x)/x^2	0.067090	0.000000	0.049512	0.954148
157	a+b*x^.5*log(x)	0.064465	0.000000	0.049869	0.953818
158	a+b*log(x)^2+c*log(x)/x	0.067751	0.000000	0.050493	0.953240
159	a+b*exp(x)+c*x^.5	0.067856	0.000000	0.050648	0.953096
160	a*x^b*exp(-c*x)	0.069820	-0.072601	0.053623	0.950342
161	x^a*exp(b-c*x)	0.069820	-0.072601	0.053623	0.950342
162	a*b^x*x^c	0.069820	-0.072601	0.053623	0.950342
163	1/(a+b*x+c*x^2)	0.070580	-0.106775	0.054797	0.949254
164	a/(1+b*x+c*x^2)	0.070580	-0.106775	0.054797	0.949254
165	a+b*x^3+c*log(x)/x^2	0.070785	0.000000	0.055115	0.948960
166	a+b*x^.5+c*log(x)/x	0.071485	0.000000	0.056210	0.947945
167	a+b*x^3+c*log(x)/x	0.074475	0.000000	0.061011	0.943500
168	a+b*log(x)+c/x	0.076101	0.000000	0.063704	0.941005
169	a+b*x^3	0.073382	0.000000	0.064620	0.940158

Table 26(continued) – Results of Curve Fitting Software for (1-SSHP) Values

Rank	Model	Standard Error	Residual Sum	RSS	R2
170	a+b*log(x)^2+c*log(x)/x^2	0.076675	0.000000	0.064670	0.940111
171	a+b*log(x)^2	0.073415	0.000000	0.064677	0.940105
172	a+b*log(x)+c*exp(-x)	0.077237	0.000000	0.065621	0.939231
173	a*x^(b*x)	0.076511	-0.087293	0.070247	0.934946
174	Third order inverse polynomial	0.084813	0.000000	0.071932	0.933386
175	a+b*log(x)+c/x^1.5	0.085806	0.000000	0.080988	0.924999
176	a+b/x^.5+c*exp(-x)	0.091821	0.000000	0.092742	0.914114
177	a+b*x^.5+c*log(x)/x^2	0.091953	0.000000	0.093008	0.913868
178	a+b/x+c*exp(-x)	0.092122	0.000000	0.093351	0.913550
179	a+b*log(x)+c/x^2	0.093085	0.000000	0.095313	0.911734
180	1-exp(-a*x^b)	0.089505	-0.412248	0.096134	0.910973
181	a+b*x^.5	0.090462	0.000000	0.098200	0.909060
182	a+b*log(x)+c*log(x)/x	0.097314	0.000000	0.104171	0.903531
183	a+b/x^.5+c/x	0.098744	0.000000	0.107255	0.900675
184	a+b*exp(x)+c*log(x)	0.102801	0.000000	0.116248	0.892347
185	a+b*log(x)/x+c/x^2	0.105522	0.000000	0.122483	0.886572
186	exp(a+b*x)	0.101528	-0.101774	0.123696	0.885449
187	a*b^x	0.101528	-0.101774	0.123696	0.885449
188	a*exp(b*x)	0.101528	-0.101774	0.123696	0.885449
189	a+b/x^.5+c/x^1.5	0.111729	0.000000	0.137318	0.872834
190	a+b/x^.5+c*log(x)/x	0.113369	0.000000	0.141379	0.869073
191	a+b*log(x)/x+c/x^1.5	0.114486	0.000000	0.144178	0.866481
192	a+b*log(x)/x+c/x	0.118425	0.000000	0.154271	0.857135
193	a+b/x^.5+c/x^2	0.121587	0.000000	0.162616	0.849406
194	a*b^(1/x)*x^c	0.123597	-0.095347	0.168037	0.844386
195	exp(a+b/x+c*log(x))	0.123597	-0.095347	0.168037	0.844386
196	a+b*log(x)+c*log(x)/x^2	0.132099	0.000000	0.191950	0.822241
197	a+b*log(x)/x+c*exp(-x)	0.133203	0.000000	0.195174	0.819255
198	a+b/x+c/x^1.5	0.134333	0.000000	0.198499	0.816176
199	a+b*exp(x)+c/x^.5	0.135382	0.000000	0.201612	0.813294
200	First order logarithm	0.139932	0.000000	0.234970	0.782402
201	a+b*log(x)	0.139932	0.000000	0.234970	0.782402
202	Second order inverse polynomial	0.146475	0.000000	0.236005	0.781443
203	a+b/x^.5+c*log(x)/x^2	0.159666	0.000000	0.280426	0.740307
204	a+b*exp(x)+c/x	0.160909	0.000000	0.284810	0.736246
205	1/(1+a*x)	0.151096	0.084230	0.296791	0.725151
206	a+b/x^1.5+c/x^2	0.166601	0.000000	0.305315	0.717258
207	a*(1+x)^b	0.163323	-0.106100	0.320095	0.703570
208	x/(a+b*x+c*sqr(x))	0.171944	-0.152787	0.325213	0.698830
209	a+b/x+c*log(x)/x^2	0.174439	0.000000	0.334720	0.690026
210	a+b*exp(x)+c/x^1.5	0.178143	0.000000	0.349086	0.676723
211	a+b/x^1.5+c*log(x)/x^2	0.180537	0.000000	0.358531	0.667976
212	a+b*log(x)/x^2+c/x^2	0.181922	0.000000	0.364053	0.662862
213	a+b*exp(x)+c*log(x)/x^2	0.183218	0.000000	0.369257	0.658042

Table 26(continued) – Results of Curve Fitting Software for (1-SSHP) Values

Rank	Model	Standard Error	Residual Sum	RSS	R2
214	a+b*exp(x)+c*exp(-x)	0.183794	0.000000	0.371583	0.655889
215	a*x^b	0.179617	-0.103556	0.387147	0.641475
216	a+b*exp(x)+c/x^2	0.188802	0.000000	0.392110	0.636880
217	a+b*log(x)/x^2+c*exp(-x)	0.192071	0.000000	0.405803	0.624198
218	a+b/x^.5	0.186191	0.000000	0.416007	0.614749
219	x^a	0.179187	0.135431	0.417402	0.613457
220	a+b/x^1.5+c*exp(-x)	0.197350	0.000000	0.428418	0.603256
221	a+b*exp(x)+c*log(x)/x	0.221904	0.000000	0.541657	0.498388
222	a+b*exp(x)	0.214829	0.000000	0.553817	0.487127
223	First order inverse polynomial	0.221794	0.000000	0.590311	0.453332
224	a+b/x	0.221794	0.000000	0.590311	0.453332
225	a+b*log(x)/x+c*log(x)/x^2	0.234884	0.000000	0.606876	0.437991
226	a+b*log(x)/x^2	0.237421	0.000000	0.676422	0.373587
227	a*exp(b/x)	0.238967	-0.054737	0.685264	0.365399
228	a+b/x^1.5	0.244973	0.000000	0.720139	0.333102
229	a+b/x^2+c*exp(-x)	0.260231	0.000000	0.744920	0.310153
230	a+b*exp(-x)	0.254104	0.000000	0.774829	0.282455
231	a+b/x^2	0.258712	0.000000	0.803185	0.256196
232	1-exp(-x^a)	0.251530	1.010693	0.822473	0.238334
233	a*x^(b/x)	0.264177	0.254584	0.837472	0.224443
234	a+b*log(x)/x	0.287169	0.000000	0.989595	0.083567
235	a+b*cos(x)+c*sin(x)	0.301904	0.000000	1.002608	0.071516
236	exp(-x^a)	0.342936	1.582301	1.528866	0.000000
237	1-1/(x^a)	0.456757	1.520412	2.712145	0.000000
238	a*x/(b+x)	0.483908	3.421335	2.810002	0.000000
239	a*b*x/(1+b*x)	0.483908	3.421335	2.810002	0.000000
240	x/(a*x+b)	0.483908	3.421336	2.810002	0.000000
241	a^(1/x)	0.488250	-2.930974	3.099045	0.000000
242	1/(a+b*log(x))	0.568298	5.453261	3.875558	0.000000
243	1-exp(-a*x^2)	0.588347	4.235718	4.499972	0.000000
244	a/(1+b*x)	0.620147	6.739981	4.614987	0.000000
245	x/(a+b*x-c*x^2)	0.650296	5.999204	4.651733	0.000000
246	1/(a+b*x)	0.635130	7.009408	4.840682	0.000000
247	a*cos(x)+b*sin(x)	0.677290	7.868593	5.504668	0.000000
248	exp(x-a)	0.656401	7.729239	5.601205	0.000000
249	cos(x+a)	0.896775	7.530687	10.454663	0.000000
250	sin(x+a)	0.896775	7.530687	10.454663	0.000000
251	1/(x+a)	1.066573	6.343206	14.788516	0.000000

APPENDIX C

Solutions of Forty Test Scenarios

The solutions of forty test scenarios are not given here due to their length. Instead, they are supplied in the given CD-ROM.

APPENDIX C

Solutions of Scenarios With Five Methods

Table 27 – Baron Global Solution for Scenario 101

Baron Global - 101					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	5.3050	8.7806	0.314686
1	1	2	8.7806	11.6429	0.542260
1	1	3	11.6429	14.0000	0.812415
2	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542260
2	2	3	11.6429	14.0000	0.812415
Computation Time (sec):		**21600**	**Objective Function Value:**		**0.942021**

Table 28 – Baron Global Solution for Scenario 102

Baron Global - 102					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	5.3050	8.7806	0.406880
1	1	2	8.7806	11.6429	0.658362
1	1	3	11.6429	14.0000	0.903081
2	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542260
2	2	3	11.6429	14.0000	0.812415
Computation Time (sec):		**21600**	**Objective Function Value:**		**0.961047**

Table 29 – Baron Global Solution for Scenario 103

Baron Global - 103					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	8.7806	11.6429	0.658362
1	1	3	11.6429	14.0000	0.903081
1	2	1	5.3050	8.7806	0.361039
2	1	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542260
2	2	3	11.6429	14.0000	0.812415
Computation Time (sec):		**21600**	**Objective Function Value:**		**0.961533**

Table 30 – Baron Global Solution for Scenario 104

Baron Global - 104					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	3	11.6429	14.0000	0.903081
1	2	2	8.7806	11.6429	0.571298
1	2	3	11.6429	14.0000	0.824273
2	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542260
2	2	1	5.3050	8.7806	0.247605
Computation Time (sec):		**21600**	**Objective Function Value:**		**0.956889**

Table 31 – Baron Global Solution for Scenario 105

Baron Global - 105					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	8.7806	11.6429	0.542260
1	1	3	11.6429	14.0000	0.812415
1	2	3	11.6429	14.0000	0.812415
2	1	1	5.3050	8.7806	0.314686
2	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542260
Computation Time (sec):		**21600**	**Objective Function Value:**		**0.924887**

Table 32 – Baron Global Solution for Scenario 106

Baron Global - 106					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	5.3050	8.7806	0.406880
1	1	2	8.7806	11.6429	0.658362
1	1	3	11.6429	14.0000	0.903081
2	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542260
2	2	3	11.6429	14.0000	0.812415
Computation Time (sec):		**21600**	**Objective Function Value:**		**0.955329**

Table 33 – Baron Global Solution for Scenario 107

Baron Global - 107					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	5.3050	8.7806	0.406880
1	1	2	8.7806	11.6429	0.658362
1	1	3	11.6429	14.0000	0.903081
2	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542260
2	2	3	11.6429	14.0000	0.812415
Computation Time (sec):		**21600**	**Objective Function Value:**		**0.955329**

Table 34 – Baron Global Solution for Scenario 108

Baron Global - 108					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	3	11.6429	14.0000	0.903081
1	2	2	8.7806	11.6429	0.571298
1	2	3	11.6429	14.0000	0.824273
2	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542260
2	2	1	5.3050	8.7806	0.247605
Computation Time (sec):		**21600**	**Objective Function Value:**		**0.948026**

Table 35 – Baron Local Solution for Scenario 101

Baron Local - 101					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	1.0847	5.3050	0.314686
1	1	3	8.7806	11.6429	0.542260
1	1	4	11.6429	14.0000	0.812415
2	1	2	5.3050	8.7806	0.314686
2	2	1	8.7806	11.6429	0.542260
2	2	2	11.6429	14.0000	0.812415
Computation Time (sec):		**1.516**	**Objective Function Value:**		**0.933371**

Table 36 – Baron Local Solution for Scenario 102

Baron Local - 102					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	5.3050	8.7806	0.903081
1	1	2	8.7806	11.6429	0.571298
1	1	3	11.6429	14.0000	0.824273
2	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542260
2	2	3	11.6429	14.0000	0.247605
Computation Time (sec):		**1.453**	**Objective Function Value:**		**0.961047**

Table 37 – Baron Local Solution for Scenario 103

Baron Local - 103					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	5.3050	8.7806	0.406880
1	1	2	8.7806	11.6429	0.658362
1	1	3	11.6429	14.0000	0.903081
2	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542260
2	2	3	11.6429	14.0000	0.812415
Computation Time (sec):		**1.532**	**Objective Function Value:**		**0.961047**

Table 38 – Baron Local Solution for Scenario 104

Baron Local - 104					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	5.3050	8.7806	0.542260
1	2	1	5.3050	8.7806	0.812415
1	2	3	11.6429	14.0000	0.812415
2	1	2	8.7806	11.6429	0.314686
2	1	3	11.6429	14.0000	0.314686
2	2	2	8.7806	11.6429	0.542260
Computation Time (sec):		**1.500**	**Objective Function Value:**		**0.945583**

Table 39 – Baron Local Solution for Scenario 105

Baron Local - 105					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	5.3050	8.7806	0.903081
1	1	3	8.7806	11.6429	0.571298
1	1	4	11.6429	14.0000	0.824273
2	1	1	1.0847	5.3050	0.314686
2	2	1	8.7806	11.6429	0.542260
2	2	2	11.6429	14.0000	0.247605
Computation Time (sec):		**1.515**	**Objective Function Value:**		**0.919152**

Table 40 – Baron Local Solution for Scenario 106

Baron Local - 106					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	5.3050	8.7806	0.658362
1	1	2	8.7806	11.6429	0.903081
1	1	3	11.6429	14.0000	0.361039
2	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542260
2	2	3	11.6429	14.0000	0.812415
Computation Time (sec):		**1.421**	**Objective Function Value:**		**0.955329**

Table 41 – Baron Local Solution for Scenario 107

Baron Local - 107					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	8.7806	11.6429	0.406880
1	1	2	11.6429	14.0000	0.658362
1	2	2	5.3050	8.7806	0.903081
2	2	1	1.0847	5.3050	0.314686
2	2	3	8.7806	11.6429	0.542260
2	2	4	11.6429	14.0000	0.812415
Computation Time (sec):		**1.435**	**Objective Function Value:**		**0.951037**

Table 42 – Baron Local Solution for Scenario 108

Baron Local - 108					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	3	8.7806	11.6429	0.658362
1	2	1	8.7806	11.6429	0.571298
1	2	2	11.6429	14.0000	0.824273
2	1	1	1.0847	5.3050	0.158464
2	1	2	5.3050	8.7806	0.314686
2	1	4	11.6429	14.0000	0.812415
Computation Time (sec):		**1.427**	**Objective Function Value:**		**0.933879**

Table 43 – Linear Model Solution for Scenario 101

Linear Model - 101					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	2	1	5.3050	8.7806	0.314686
1	2	2	8.7806	11.6429	0.542260
1	2	3	11.6429	14.0000	0.812415
2	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542260
2	1	3	11.6429	14.0000	0.812415
Computation Time (sec):		**0.266**	**Objective Function Value:**		**0.942021**

Table 44 – Linear Model Solution for Scenario 102

Linear Model - 102					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	8.7806	11.6429	0.658362
1	2	3	8.7806	11.6429	0.542260
1	2	4	11.6429	14.0000	0.812415
2	1	2	11.6429	14.0000	0.812415
2	2	1	1.0847	5.3050	0.158464
2	2	2	5.3050	8.7806	0.314686
Computation Time (sec):		**0.204**	**Objective Function Value:**		**0.943990**

Table 45 – Linear Model Solution for Scenario 103

Linear Model - 103					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	11.6429	14.0000	0.903081
1	2	3	8.7806	11.6429	0.571298
1	2	4	11.6429	14.0000	0.824273
2	1	1	8.7806	11.6429	0.542260
2	2	1	1.0847	5.3050	0.158464
2	2	2	5.3050	8.7806	0.314686
Computation Time (sec):		**0.251**	**Objective Function Value:**		**0.956576**

Table 46 – Linear Model Solution for Scenario 104

Linear Model - 104					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	2	1	5.3050	8.7806	0.361039
1	2	2	8.7806	11.6429	0.571298
1	2	3	11.6429	14.0000	0.824273
2	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542260
2	1	3	11.6429	14.0000	0.812415
Computation Time (sec):		**0.157**	**Objective Function Value:**		**0.947217**

Table 47 – Linear Model Solution for Scenario 105

Linear Model - 105					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	3	11.6429	14.0000	0.812415
1	2	1	5.3050	8.7806	0.314686
1	2	3	11.6429	14.0000	0.812415
2	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542260
2	2	2	8.7806	11.6429	0.542260
Computation Time (sec):		**0.141**	**Objective Function Value:**		**0.924887**

Table 48 – Linear Model Solution for Scenario 106

Linear Model - 105					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	8.7806	11.6429	0.658362
1	2	1	1.0847	5.3050	0.158464
1	2	2	5.3050	8.7806	0.314686
2	1	2	11.6429	14.0000	0.812415
2	2	3	8.7806	11.6429	0.542260
2	2	4	11.6429	14.0000	0.812415
Computation Time (sec):		**0.235**	**Objective Function Value:**		**0.929610**

Table 49 – Linear Model Solution for Scenario 107

Linear Model - 107					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	8.7806	11.6429	0.658362
1	2	3	8.7806	11.6429	0.571298
1	2	4	11.6429	14.0000	0.824273
2	1	2	11.6429	14.0000	0.812415
2	2	1	1.0847	5.3050	0.158464
2	2	2	5.3050	8.7806	0.314686
Computation Time (sec):		**0.204**	**Objective Function Value:**		**0.934313**

Table 50 – Linear Model Solution for Scenario 108

Linear Model - 108					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	2	1	5.3050	8.7806	0.361039
1	2	2	8.7806	11.6429	0.571298
1	2	3	11.6429	14.0000	0.824273
2	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542260
2	1	3	11.6429	14.0000	0.812415
Computation Time (sec):		**0.188**	**Objective Function Value:**		**0.933202**

Table 51 – Construction Heuristic Solution for Scenario 101

Construction Heuristic - 101					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
2	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
2	2	1	5.3050	8.7806	0.314686
1	2	2	8.7806	11.6429	0.542261
1	2	3	11.6429	14.0000	0.812418
Computation Time (sec):		**0.027**	**Objective Function Value:**		**0.942022**

Table 52 – Construction Heuristic Solution for Scenario 102

Construction Heuristic - 102					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
2	1	1	8.7806	11.6429	0.542261
1	1	2	11.6429	14.0000	0.903086
2	2	1	1.0847	5.3050	0.158464
2	2	2	5.3050	8.7806	0.314686
1	2	3	8.7806	11.6429	0.542261
1	2	4	11.6429	14.0000	0.812418
Computation Time (sec):		**0.028**	**Objective Function Value:**		**0.953609**

Table 53 – Construction Heuristic Solution for Scenario 103

Construction Heuristic - 103					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
2	1	1	8.7806	11.6429	0.542261
1	1	2	11.6429	14.0000	0.903086
2	2	1	1.0847	5.3050	0.158464
2	2	2	5.3050	8.7806	0.314686
1	2	3	8.7806	11.6429	0.571296
1	2	4	11.6429	14.0000	0.824266
Computation Time (sec):		**0.026**	**Objective Function Value:**		**0.956577**

Table 54 – Construction Heuristic Solution for Scenario 104

Construction Heuristic - 104					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
2	1	1	8.7806	11.6429	0.542261
1	1	2	11.6429	14.0000	0.903086
2	2	1	1.0847	5.3050	0.066208
2	2	2	5.3050	8.7806	0.247605
1	2	3	8.7806	11.6429	0.571296
1	2	4	11.6429	14.0000	0.824266
Computation Time (sec):		**0.026**	**Objective Function Value:**		**0.951941**

Table 55 – Construction Heuristic Solution for Scenario 105

Construction Heuristic - 105					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
2	1	1	1.0847	5.3050	0.158464
2	1	2	5.3050	8.7806	0.314686
1	1	3	8.7806	11.6429	0.542261
1	1	4	11.6429	14.0000	0.812418
2	2	1	8.7806	11.6429	0.542261
1	2	2	11.6429	14.0000	0.812418
Computation Time (sec):		**0.027**	**Objective Function Value:**		**0.919154**

Table 56 – Construction Heuristic Solution for Scenario 106

Construction Heuristic - 106					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
2	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.903086
2	2	1	5.3050	8.7806	0.314686
1	2	2	8.7806	11.6429	0.542261
1	2	3	11.6429	14.0000	0.812418
Computation Time (sec):		**0.028**	**Objective Function Value:**		**0.946973**

Table 57 – Construction Heuristic Solution for Scenario 107

Construction Heuristic - 107					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
2	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.903086
2	2	1	5.3050	8.7806	0.314686
1	2	2	8.7806	11.6429	0.571296
1	2	3	11.6429	14.0000	0.824266
Computation Time (sec):		**0.027**	**Objective Function Value:**		**0.950492**

Table 58 – Construction Heuristic Solution for Scenario 108

Construction Heuristic - 108					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
2	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.903086
2	2	1	5.3050	8.7806	0.247605
1	2	2	8.7806	11.6429	0.571296
1	2	3	11.6429	14.0000	0.824266
Computation Time (sec):		**0.028**	**Objective Function Value:**		**0.948026**

Table 59 – Improvement Heuristic Solution for Scenario 101

Improvement Heuristic - 101					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
2	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
2	2	1	5.3050	8.7806	0.314686
1	2	2	8.7806	11.6429	0.542261
1	2	3	11.6429	14.0000	0.812418
Computation Time (sec):		**0.047**	**Objective Function Value:**		**0.942022**

Table 60 – Improvement Heuristic Solution for Scenario 102

Improvement Heuristic - 102					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
2	1	1	8.7806	11.6429	0.542261
1	1	2	11.6429	14.0000	0.903086
2	2	1	1.0847	5.3050	0.158464
2	2	2	5.3050	8.7806	0.314686
1	2	3	8.7806	11.6429	0.542261
1	2	4	11.6429	14.0000	0.812418
Computation Time (sec):		**1.073**	**Objective Function Value:**		**0.953609**

Table 61 – Improvement Heuristic Solution for Scenario 103

Improvement Heuristic - 103					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
2	1	1	8.7806	11.6429	0.542261
1	1	2	11.6429	14.0000	0.903086
2	2	1	1.0847	5.3050	0.158464
2	2	2	5.3050	8.7806	0.314686
1	2	3	8.7806	11.6429	0.571296
1	2	4	11.6429	14.0000	0.824266
Computation Time (sec):		**0.186**	**Objective Function Value:**		**0.956577**

Table 62 – Improvement Heuristic Solution for Scenario 104

Improvement Heuristic - 104					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
2	1	1	8.7806	11.6429	0.542261
1	1	2	11.6429	14.0000	0.903086
2	2	1	1.0847	5.3050	0.066208
2	2	2	5.3050	8.7806	0.247605
1	2	3	8.7806	11.6429	0.571296
1	2	4	11.6429	14.0000	0.824266
Computation Time (sec):		**1.377**	**Objective Function Value:**		**0.951941**

Table 63 – Improvement Heuristic Solution for Scenario 105

Improvement Heuristic - 105					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
2	1	1	1.0847	5.3050	0.158464
2	1	2	5.3050	8.7806	0.314686
1	1	3	8.7806	11.6429	0.542261
1	1	4	11.6429	14.0000	0.812418
2	2	1	8.7806	11.6429	0.542261
1	2	2	11.6429	14.0000	0.812418
Computation Time (sec):		**0.543**	**Objective Function Value:**		**0.919154**

Table 64 – Improvement Heuristic Solution for Scenario 106

Improvement Heuristic - 106					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
2	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.903086
2	2	1	5.3050	8.7806	0.314686
1	2	2	8.7806	11.6429	0.542261
1	2	3	11.6429	14.0000	0.812418
Computation Time (sec):		**1.327**	**Objective Function Value:**		**0.946973**

Table 65 – Improvement Heuristic Solution for Scenario 107

Improvement Heuristic - 107					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
2	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.903086
2	2	1	5.3050	8.7806	0.314686
1	2	2	8.7806	11.6429	0.571296
1	2	3	11.6429	14.0000	0.824266
Computation Time (sec):		**0.220**	**Objective Function Value:**		**0.950492**

Table 66 – Improvement Heuristic Solution for Scenario 108

Improvement Heuristic - 108					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
2	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.903086
2	2	1	5.3050	8.7806	0.247605
1	2	2	8.7806	11.6429	0.571296
1	2	3	11.6429	14.0000	0.824266
Computation Time (sec):		**0.967**	**Objective Function Value:**		**0.948026**

Table 67 – Baron Global Solution for Scenario 201

Baron Global - 201					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	2	1	11.6429	14.0000	0.314686
1	2	3	5.3050	8.7806	0.812415
1	3	1	11.6429	14.0000	0.314686
2	1	1	5.3050	8.7806	0.314686
2	1	3	5.3050	8.7806	0.812415
2	2	2	11.6429	14.0000	0.542260
3	1	2	8.7806	11.6429	0.542260
3	3	2	8.7806	11.6429	0.542260
3	3	3	8.7806	11.6429	0.812415
Computation Time (sec):		**21600**	**Objective Function Value:**		**1.912599**

Table 68 – Baron Global Solution for Scenario 202

Baron Global - 202					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	11.6429	14.0000	0.542260
1	1	3	8.7806	11.6429	0.812415
1	3	2	11.6429	14.0000	0.542260
2	2	3	8.7806	11.6429	0.812415
2	3	1	11.6429	14.0000	0.314686
2	3	3	5.3050	8.7806	0.812415
3	1	1	11.6429	14.0000	0.314686
3	2	1	5.3050	8.7806	0.314686
3	2	2	5.3050	8.7806	0.542260
Computation Time (sec):		**21600**	**Objective Function Value:**		**2.883177**

Table 69 – Baron Global Solution for Scenario 203

Baron Global - 203					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	3	5.3050	8.7806	0.903081
1	3	1	11.6429	14.0000	0.361039
1	3	2	5.3050	8.7806	0.571298
2	2	1	8.7806	11.6429	0.406880
2	2	2	5.3050	8.7806	0.658362
2	2	3	8.7806	11.6429	0.903081
3	1	1	11.6429	14.0000	0.361039
3	1	2	5.3050	8.7806	0.571298
3	3	3	8.7806	11.6429	0.812415
Computation Time (sec):		**21600**	**Objective Function Value:**		**1.951345**

Table 70 – Baron Global Solution for Scenario 204

Baron Global - 204					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	11.6429	14.0000	0.658362
1	1	2	8.7806	11.6429	0.903081
1	3	2	11.6429	14.0000	0.361039
2	2	1	5.3050	8.7806	0.406880
2	2	2	5.3050	8.7806	0.658362
2	2	3	8.7806	11.6429	0.903081
3	3	1	11.6429	14.0000	0.158464
3	3	3	1.0847	5.3050	0.542260
3	3	4	8.7806	11.6429	0.812415
Computation Time (sec):		**21600**	**Objective Function Value:**		**2.927617**

Table 71 – Baron Global Solution for Scenario 205

Baron Global - 205					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	3	11.6429	14.0000	0.903081
1	2	3	11.6429	14.0000	0.903081
1	3	3	11.6429	14.0000	0.903081
2	1	2	8.7806	11.6429	0.542260
2	2	1	5.3050	8.7806	0.314686
2	3	1	5.3050	8.7806	0.314686
3	1	1	5.3050	8.7806	0.314686
3	2	2	8.7806	11.6429	0.542260
3	3	2	8.7806	11.6429	0.542260
Computation Time (sec):		**21600**	**Objective Function Value:**		**1.954626**

Table 72 – Baron Global Solution for Scenario 206

Baron Global - 206					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	3	11.6429	14.0000	0.903081
1	2	2	11.6429	14.0000	0.903081
1	3	4	11.6429	14.0000	0.903081
2	1	1	11.6429	14.0000	0.314686
2	1	2	5.3050	8.7806	0.542260
2	3	3	8.7806	11.6429	0.542260
3	2	1	8.7806	11.6429	0.542260
3	3	1	8.7806	11.6429	0.158464
3	3	2	1.0847	5.3050	0.314686
Computation Time (sec):		**21600**	**Objective Function Value:**		**2.937368**

Table 73 – Baron Global Solution for Scenario 207

Baron Global - 207					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	3	11.6429	14.0000	0.903081
1	2	3	11.6429	14.0000	0.903081
1	3	3	11.6429	14.0000	0.903081
2	1	1	11.6429	14.0000	0.361039
2	1	2	5.3050	8.7806	0.571298
2	2	2	8.7806	11.6429	0.571298
3	2	1	8.7806	11.6429	0.314686
3	3	1	5.3050	8.7806	0.314686
3	3	2	5.3050	8.7806	0.542260
Computation Time (sec):		**21600**	**Objective Function Value:**		**1.957476**

Table 74 – Baron Global Solution for Scenario 208

Baron Global - 208					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	3	11.6429	14.0000	0.903081
1	2	3	11.6429	14.0000	0.903081
1	3	3	11.6429	14.0000	0.903081
2	1	1	11.6429	14.0000	0.361039
2	3	1	5.3050	8.7806	0.361039
2	3	2	5.3050	8.7806	0.571298
3	1	2	8.7806	11.6429	0.542260
3	2	1	8.7806	11.6429	0.314686
3	2	2	5.3050	8.7806	0.542260
Computation Time (sec):		**21600**	**Objective Function Value:**		**2.944292**

Table 75 – Baron Global Solution for Scenario 209

Baron Global - 209					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	11.6429	14.0000	0.314686
1	2	3	5.3050	8.7806	0.812415
1	3	1	11.6429	14.0000	0.314686
2	1	3	5.3050	8.7806	0.812415
2	2	1	11.6429	14.0000	0.314686
2	3	3	5.3050	8.7806	0.812415
3	1	2	11.6429	14.0000	0.542260
3	2	2	8.7806	11.6429	0.542260
3	3	2	8.7806	11.6429	0.542260
Computation Time (sec):		**21600**	**Objective Function Value:**		**1.894946**

Table 76 – Baron Global Solution for Scenario 210

Baron Global - 210					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	2	1	11.6429	14.0000	0.314686
1	2	3	5.3050	8.7806	0.812415
1	3	3	11.6429	14.0000	0.542260
2	1	1	8.7806	11.6429	0.542260
2	3	1	8.7806	11.6429	0.158464
2	3	4	1.0847	5.3050	0.812415
3	1	2	11.6429	14.0000	0.812415
3	2	2	11.6429	14.0000	0.542260
3	3	2	8.7806	11.6429	0.314686
Computation Time (sec):		**21600**	**Objective Function Value:**		**2.849677**

Table 77 – Baron Global Solution for Scenario 211

Baron Global - 211					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	11.6429	14.0000	0.658362
1	1	2	8.7806	11.6429	0.903081
1	3	1	11.6429	14.0000	0.222265
2	2	1	1.0847	5.3050	0.406880
2	2	2	5.3050	8.7806	0.658362
2	2	3	8.7806	11.6429	0.903081
3	3	2	11.6429	14.0000	0.314686
3	3	3	5.3050	8.7806	0.542260
3	3	4	8.7806	11.6429	0.812415
Computation Time (sec):		**21600**	**Objective Function Value:**		**1.937175**

Table 78 – Baron Global Solution for Scenario 212

Baron Global - 212					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	1.0847	5.3050	0.903081
1	3	1	11.6429	14.0000	0.222265
1	3	2	1.0847	5.3050	0.361039
2	2	1	5.3050	8.7806	0.406880
2	2	2	5.3050	8.7806	0.658362
2	2	3	8.7806	11.6429	0.903081
3	1	1	11.6429	14.0000	0.571298
3	3	3	8.7806	11.6429	0.542260
3	3	4	8.7806	11.6429	0.812415
Computation Time (sec):		**21600**	**Objective Function Value:**		**2.901338**

Table 79 – Baron Global Solution for Scenario 213

Baron Global - 213					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	3	11.6429	14.0000	0.903081
1	2	3	11.6429	14.0000	0.903081
1	3	3	11.6429	14.0000	0.903081
2	2	1	11.6429	14.0000	0.314686
2	2	2	5.3050	8.7806	0.542260
2	3	1	8.7806	11.6429	0.314686
3	1	1	5.3050	8.7806	0.314686
3	1	2	5.3050	8.7806	0.542260
3	3	2	8.7806	11.6429	0.542260
Computation Time (sec):		**21600**	**Objective Function Value:**		**1.945505**

Table 80 – Baron Global Solution for Scenario 214

Baron Global - 214					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	3	8.7806	11.6429	0.903081
1	3	2	11.6429	14.0000	0.658362
1	3	3	8.7806	11.6429	0.903081
2	1	1	11.6429	14.0000	0.314686
2	2	1	5.3050	8.7806	0.314686
2	2	2	5.3050	8.7806	0.542260
3	1	2	8.7806	11.6429	0.542260
3	2	3	8.7806	11.6429	0.812415
3	3	1	11.6429	14.0000	0.314686
Computation Time (sec):		**21600**	**Objective Function Value:**		**2.919517**

Table 81 – Baron Global Solution for Scenario 215

Baron Global - 215					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	3	11.6429	14.0000	0.903081
1	2	3	11.6429	14.0000	0.903081
1	3	3	11.6429	14.0000	0.903081
2	1	1	11.6429	14.0000	0.361039
2	2	1	5.3050	8.7806	0.361039
2	3	1	5.3050	8.7806	0.361039
3	1	2	5.3050	8.7806	0.542260
3	2	2	8.7806	11.6429	0.542260
3	3	2	8.7806	11.6429	0.542260
Computation Time (sec):		**21600**	**Objective Function Value:**		**1.949177**

Table 82 – Baron Global Solution for Scenario 216

Baron Global - 216					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	11.6429	14.0000	0.903081
1	2	3	11.6429	14.0000	0.903081
1	3	4	11.6429	14.0000	0.903081
2	1	1	11.6429	14.0000	0.571298
2	2	1	8.7806	11.6429	0.361039
2	3	2	5.3050	8.7806	0.361039
3	2	2	5.3050	8.7806	0.542260
3	3	1	8.7806	11.6429	0.158464
3	3	3	1.0847	5.3050	0.542260
Computation Time (sec):		**21600**	**Objective Function Value:**		**2.927179**

Table 83 – Baron Local Solution for Scenario 201

Baron Local - 201					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	8.7806	11.6429	0.542260
1	2	1	8.7806	11.6429	0.542260
1	3	2	5.3050	8.7806	0.314686
2	2	2	11.6429	14.0000	0.812415
2	3	3	8.7806	11.6429	0.542260
2	3	4	11.6429	14.0000	0.812415
3	1	1	5.3050	8.7806	0.314686
3	1	3	11.6429	14.0000	0.812415
3	3	1	1.0847	5.3050	0.158464
Computation Time (sec):		**8.125**	**Objective Function Value:**		**1.904149**

Table 84 – Baron Local Solution for Scenario 202

Baron Local - 202					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	8.7806	11.6429	0.542260
1	2	1	8.7806	11.6429	0.542260
1	3	2	5.3050	8.7806	0.314686
2	2	2	11.6429	14.0000	0.812415
2	3	3	8.7806	11.6429	0.542260
2	3	4	11.6429	14.0000	0.812415
3	1	1	5.3050	8.7806	0.314686
3	1	3	11.6429	14.0000	0.812415
3	3	1	1.0847	5.3050	0.158464
Computation Time (sec):		**8.469**	**Objective Function Value:**		**2.879389**

Table 85 – Baron Local Solution for Scenario 203

Baron Local - 203					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	8.7806	11.6429	0.658362
1	1	2	11.6429	14.0000	0.903081
1	3	1	5.3050	8.7806	0.361039
2	2	1	1.0847	5.3050	0.251587
2	2	3	8.7806	11.6429	0.658362
2	2	4	11.6429	14.0000	0.903081
3	2	2	5.3050	8.7806	0.314686
3	3	2	8.7806	11.6429	0.542260
3	3	3	11.6429	14.0000	0.812415
Computation Time (sec):		**7.375**	**Objective Function Value:**		**1.947661**

Table 86 – Baron Local Solution for Scenario 204

Baron Local - 204					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	8.7806	11.6429	0.658362
1	3	1	1.0847	5.3050	0.222265
1	3	4	11.6429	14.0000	0.824273
2	2	1	8.7806	11.6429	0.658362
2	2	2	11.6429	14.0000	0.903081
2	3	3	8.7806	11.6429	0.470810
3	1	1	5.3050	8.7806	0.361039
3	1	3	11.6429	14.0000	0.824273
3	3	2	5.3050	8.7806	0.314686
Computation Time (sec):		**6.734**	**Objective Function Value:**		**2.915017**

Table 87 – Baron Local Solution for Scenario 205

Baron Local - 205					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	8.7806	11.6429	0.658362
1	2	2	8.7806	11.6429	0.658362
1	3	1	1.0847	5.3050	0.251587
2	1	2	11.6429	14.0000	0.812415
2	2	1	5.3050	8.7806	0.314686
2	3	3	8.7806	11.6429	0.542260
3	2	3	11.6429	14.0000	0.812415
3	3	2	5.3050	8.7806	0.314686
3	3	4	11.6429	14.0000	0.812415
Computation Time (sec):		**4.906**	**Objective Function Value:**		**1.924681**

Table 88 – Baron Local Solution for Scenario 206

Baron Local - 206					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	1.0847	5.3050	0.251587
1	2	2	11.6429	14.0000	0.903081
1	3	1	5.3050	8.7806	0.406880
2	1	2	5.3050	8.7806	0.314686
2	2	1	8.7806	11.6429	0.542260
2	3	3	11.6429	14.0000	0.812415
3	1	3	8.7806	11.6429	0.542260
3	1	4	11.6429	14.0000	0.812415
3	3	2	8.7806	11.6429	0.542260
Computation Time (sec):		**5.141**	**Objective Function Value:**		**2.905358**

Table 89 – Baron Local Solution for Scenario 207

Baron Local - 207					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	8.7806	11.6429	0.658362
1	1	2	11.6429	14.0000	0.903081
1	2	4	11.6429	14.0000	0.903081
2	3	1	5.3050	8.7806	0.361039
2	3	2	8.7806	11.6429	0.571298
2	3	3	11.6429	14.0000	0.824273
3	2	1	1.0847	5.3050	0.158464
3	2	2	5.3050	8.7806	0.314686
3	2	3	8.7806	11.6429	0.542260
Computation Time (sec):		**7.453**	**Objective Function Value:**		**1.946796**

Table 90 – Baron Local Solution for Scenario 208

Baron Local - 208					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	8.7806	11.6429	0.658362
1	3	1	5.3050	8.7806	0.406880
1	3	3	11.6429	14.0000	0.903081
2	1	2	11.6429	14.0000	0.824273
2	2	2	5.3050	8.7806	0.361039
2	3	2	8.7806	11.6429	0.571298
3	2	1	1.0847	5.3050	0.158464
3	2	3	8.7806	11.6429	0.542260
3	2	4	11.6429	14.0000	0.812415
Computation Time (sec):		**7.484**	**Objective Function Value:**		**2.922946**

Table 91 – Baron Local Solution for Scenario 209

Baron Local - 209					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	8.7806	11.6429	0.542260
1	2	1	8.7806	11.6429	0.542260
1	3	2	5.3050	8.7806	0.314686
2	2	2	11.6429	14.0000	0.812415
2	3	3	8.7806	11.6429	0.542260
2	3	4	11.6429	14.0000	0.812415
3	1	1	5.3050	8.7806	0.314686
3	1	3	11.6429	14.0000	0.812415
3	3	1	1.0847	5.3050	0.158464
Computation Time (sec):		**8.515**	**Objective Function Value:**		**1.889293**

Table 92 – Baron Local Solution for Scenario 210

Baron Local - 210					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	2	1	8.7806	11.6429	0.542260
1	3	1	5.3050	8.7806	0.314686
1	3	2	8.7806	11.6429	0.542260
2	1	3	8.7806	11.6429	0.542260
2	2	2	11.6429	14.0000	0.812415
2	3	3	11.6429	14.0000	0.812415
3	1	1	1.0847	5.3050	0.158464
3	1	2	5.3050	8.7806	0.314686
3	1	4	11.6429	14.0000	0.812415
Computation Time (sec):		**8.266**	**Objective Function Value:**		**2.839219**

Table 93 – Baron Local Solution for Scenario 211

Baron Local - 211					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	8.7806	11.6429	0.658362
1	1	3	11.6429	14.0000	0.903081
1	3	1	5.3050	8.7806	0.361039
2	1	1	5.3050	8.7806	0.314686
2	2	1	5.3050	8.7806	0.406880
2	2	2	8.7806	11.6429	0.658362
3	2	3	11.6429	14.0000	0.812415
3	3	2	8.7806	11.6429	0.542260
3	3	3	11.6429	14.0000	0.812415
Computation Time (sec):		**6.828**	**Objective Function Value:**		**1.925973**

Table 94 – Baron Local Solution for Scenario 212

Baron Local - 212					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	3	1	1.0847	5.3050	0.222265
1	3	2	5.3050	8.7806	0.361039
1	3	4	11.6429	14.0000	0.824273
2	2	1	5.3050	8.7806	0.406880
2	2	2	8.7806	11.6429	0.658362
2	2	3	11.6429	14.0000	0.903081
3	1	1	8.7806	11.6429	0.571298
3	1	2	11.6429	14.0000	0.824273
3	3	3	8.7806	11.6429	0.542260
Computation Time (sec):		**7.377**	**Objective Function Value:**		**2.888927**

Table 95 – Baron Local Solution for Scenario 213

Baron Local - 213					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	5.3050	8.7806	0.406880
1	2	1	8.7806	11.6429	0.658362
1	3	1	1.0847	5.3050	0.251587
2	1	2	8.7806	11.6429	0.542260
2	1	3	11.6429	14.0000	0.812415
2	2	2	11.6429	14.0000	0.812415
3	3	2	5.3050	8.7806	0.314686
3	3	3	8.7806	11.6429	0.542260
3	3	4	11.6429	14.0000	0.812415
Computation Time (sec):		**4.828**	**Objective Function Value:**		**1.908077**

Table 96 – Baron Local Solution for Scenario 214

Baron Local - 214					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	5.3050	8.7806	0.406880
1	2	1	8.7806	11.6429	0.658362
1	3	1	1.0847	5.3050	0.251587
2	1	2	8.7806	11.6429	0.542260
2	2	2	11.6429	14.0000	0.812415
2	3	3	8.7806	11.6429	0.542260
3	1	3	11.6429	14.0000	0.812415
3	3	2	5.3050	8.7806	0.314686
3	3	4	11.6429	14.0000	0.812415
Computation Time (sec):		**4.750**	**Objective Function Value:**		**2.872845**

Table 97 – Baron Local Solution for Scenario 215

Baron Local - 215					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	8.7806	11.6429	0.658362
1	1	2	11.6429	14.0000	0.903081
1	3	3	11.6429	14.0000	0.903081
2	2	2	5.3050	8.7806	0.361039
2	3	1	5.3050	8.7806	0.361039
2	3	2	8.7806	11.6429	0.571298
3	2	1	1.0847	5.3050	0.158464
3	2	3	8.7806	11.6429	0.542260
3	2	4	11.6429	14.0000	0.812415
Computation Time (sec):		**7.500**	**Objective Function Value:**		**1.939503**

Table 98 – Baron Local Solution for Scenario 216

Baron Local - 216					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	2	1	5.3050	8.7806	0.406880
1	3	1	1.0847	5.3050	0.251587
1	3	4	11.6429	14.0000	0.903081
2	1	1	8.7806	11.6429	0.571298
2	1	2	11.6429	14.0000	0.824273
2	3	2	5.3050	8.7806	0.361039
3	2	2	8.7806	11.6429	0.542260
3	2	3	11.6429	14.0000	0.812415
3	3	3	8.7806	11.6429	0.542260
Computation Time (sec):		**8.904**	**Objective Function Value:**		**2.903884**

Table 99 – Linear Model Solution for Scenario 201

Linear Model - 201					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	2	2	8.7806	11.6429	0.542260
1	3	2	8.7806	11.6429	0.542260
1	3	3	11.6429	14.0000	0.812415
2	1	1	5.3050	8.7806	0.314686
2	1	3	11.6429	14.0000	0.812415
2	2	3	11.6429	14.0000	0.812415
3	1	2	8.7806	11.6429	0.542260
3	2	1	5.3050	8.7806	0.314686
3	3	1	5.3050	8.7806	0.314686
Computation Time (sec):		**0.282**	**Objective Function Value:**		**1.912598**

Table 100 – Linear Model Solution for Scenario 202

Linear Model - 202					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	2	2	8.7806	11.6429	0.542260
1	3	2	8.7806	11.6429	0.542260
1	3	3	11.6429	14.0000	0.812415
2	1	1	5.3050	8.7806	0.314686
2	1	3	11.6429	14.0000	0.812415
2	2	3	11.6429	14.0000	0.812415
3	1	2	8.7806	11.6429	0.542260
3	2	1	5.3050	8.7806	0.314686
3	3	1	5.3050	8.7806	0.314686
Computation Time (sec):		**0.235**	**Objective Function Value:**		**2.883176**

Table 101 – Linear Model Solution for Scenario 203

Linear Model - 203					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	8.7806	11.6429	0.658362
1	1	3	11.6429	14.0000	0.903081
1	3	3	11.6429	14.0000	0.824273
2	1	1	5.3050	8.7806	0.314686
2	2	1	5.3050	8.7806	0.406880
2	2	3	11.6429	14.0000	0.903081
3	2	2	8.7806	11.6429	0.542260
3	3	1	5.3050	8.7806	0.314686
3	3	2	8.7806	11.6429	0.542260
Computation Time (sec):		**0.156**	**Objective Function Value:**		**1.948084**

Table 102 – Linear Model Solution for Scenario 204

Linear Model - 204					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	8.7806	11.6429	0.658362
1	1	3	11.6429	14.0000	0.903081
1	3	3	11.6429	14.0000	0.824273
2	1	1	5.3050	8.7806	0.314686
2	2	1	5.3050	8.7806	0.406880
2	2	3	11.6429	14.0000	0.903081
3	2	2	8.7806	11.6429	0.542260
3	3	1	5.3050	8.7806	0.314686
3	3	2	8.7806	11.6429	0.542260
Computation Time (sec):		**0.188**	**Objective Function Value:**		**2.920522**

Table 103 – Linear Model Solution for Scenario 205

Linear Model - 205					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	8.7806	11.6429	0.658362
1	1	3	11.6429	14.0000	0.903081
1	3	3	11.6429	14.0000	0.903081
2	2	1	5.3050	8.7806	0.314686
2	2	3	11.6429	14.0000	0.812415
2	3	1	5.3050	8.7806	0.314686
3	1	1	5.3050	8.7806	0.314686
3	2	2	8.7806	11.6429	0.542260
3	3	2	8.7806	11.6429	0.542260
Computation Time (sec):		**0.172**	**Objective Function Value:**		**1.944364**

Table 104 – Linear Model Solution for Scenario 206

Linear Model - 206					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	8.7806	11.6429	0.658362
1	1	3	11.6429	14.0000	0.903081
1	3	3	11.6429	14.0000	0.903081
2	2	1	5.3050	8.7806	0.314686
2	2	3	11.6429	14.0000	0.812415
2	3	1	5.3050	8.7806	0.314686
3	1	1	5.3050	8.7806	0.314686
3	2	2	8.7806	11.6429	0.542260
3	3	2	8.7806	11.6429	0.542260
Computation Time (sec):		**0.188**	**Objective Function Value:**		**2.929162**

Table 105 – Linear Model Solution for Scenario 207

Linear Model - 207					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	8.7806	11.6429	0.658362
1	1	3	11.6429	14.0000	0.903081
1	3	3	11.6429	14.0000	0.903081
2	1	1	5.3050	8.7806	0.361039
2	2	1	5.3050	8.7806	0.361039
2	2	3	11.6429	14.0000	0.824273
3	2	2	8.7806	11.6429	0.542260
3	3	1	5.3050	8.7806	0.314686
3	3	2	8.7806	11.6429	0.542260
Computation Time (sec):		**0.172**	**Objective Function Value:**		**1.948793**

Table 106 – Linear Model Solution for Scenario 208

Linear Model - 208					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	8.7806	11.6429	0.658362
1	1	3	11.6429	14.0000	0.903081
1	3	3	11.6429	14.0000	0.903081
2	1	1	5.3050	8.7806	0.361039
2	2	1	5.3050	8.7806	0.361039
2	2	3	11.6429	14.0000	0.824273
3	2	2	8.7806	11.6429	0.542260
3	3	1	5.3050	8.7806	0.314686
3	3	2	8.7806	11.6429	0.542260
Computation Time (sec):		**0.172**	**Objective Function Value:**		**2.933592**

Table 107 – Linear Model Solution for Scenario 209

Linear Model - 209					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	2	2	8.7806	11.6429	0.542260
1	3	2	8.7806	11.6429	0.542260
1	3	3	11.6429	14.0000	0.812415
2	1	2	8.7806	11.6429	0.542260
2	1	3	11.6429	14.0000	0.812415
2	2	3	11.6429	14.0000	0.812415
3	1	1	5.3050	8.7806	0.314686
3	2	1	5.3050	8.7806	0.314686
3	3	1	5.3050	8.7806	0.314686
Computation Time (sec):		**0.188**	**Objective Function Value:**		**1.894945**

Table 108 – Linear Model Solution for Scenario 210

Linear Model - 210					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	2	2	8.7806	11.6429	0.542260
1	3	2	8.7806	11.6429	0.542260
1	3	3	11.6429	14.0000	0.812415
2	1	2	8.7806	11.6429	0.542260
2	1	3	11.6429	14.0000	0.812415
2	2	3	11.6429	14.0000	0.812415
3	1	1	5.3050	8.7806	0.314686
3	2	1	5.3050	8.7806	0.314686
3	3	1	5.3050	8.7806	0.314686
Computation Time (sec):		**0.141**	**Objective Function Value:**		**2.847869**

Table 109 – Linear Model Solution for Scenario 211

Linear Model - 211					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	8.7806	11.6429	0.658362
1	1	3	11.6429	14.0000	0.903081
1	3	3	11.6429	14.0000	0.824273
2	1	1	5.3050	8.7806	0.314686
2	2	1	5.3050	8.7806	0.406880
2	2	3	11.6429	14.0000	0.903081
3	2	2	8.7806	11.6429	0.542260
3	3	1	5.3050	8.7806	0.314686
3	3	2	8.7806	11.6429	0.542260
Computation Time (sec):		**0.172**	**Objective Function Value:**		**1.931547**

Table 110 – Linear Model Solution for Scenario 212

Linear Model - 212					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	8.7806	11.6429	0.658362
1	1	3	11.6429	14.0000	0.903081
1	3	3	11.6429	14.0000	0.824273
2	1	1	5.3050	8.7806	0.314686
2	2	1	5.3050	8.7806	0.406880
2	2	3	11.6429	14.0000	0.903081
3	2	2	8.7806	11.6429	0.542260
3	3	1	5.3050	8.7806	0.314686
3	3	2	8.7806	11.6429	0.542260
Computation Time (sec):		**0.109**	**Objective Function Value:**		**2.887447**

Table 111 – Linear Model Solution for Scenario 213

Linear Model - 213					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	8.7806	11.6429	0.658362
1	1	3	11.6429	14.0000	0.903081
1	3	3	11.6429	14.0000	0.903081
2	2	1	5.3050	8.7806	0.314686
2	2	3	11.6429	14.0000	0.812415
2	3	1	5.3050	8.7806	0.314686
3	1	1	5.3050	8.7806	0.314686
3	2	2	8.7806	11.6429	0.542260
3	3	2	8.7806	11.6429	0.542260
Computation Time (sec):		**0.110**	**Objective Function Value:**		**1.935243**

Table 112 – Linear Model Solution for Scenario 214

Linear Model - 214					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	8.7806	11.6429	0.658362
1	1	3	11.6429	14.0000	0.903081
1	3	3	11.6429	14.0000	0.903081
2	2	1	5.3050	8.7806	0.314686
2	2	3	11.6429	14.0000	0.812415
2	3	1	5.3050	8.7806	0.314686
3	1	1	5.3050	8.7806	0.314686
3	2	2	8.7806	11.6429	0.542260
3	3	2	8.7806	11.6429	0.542260
Computation Time (sec):		**0.172**	**Objective Function Value:**		**2.910920**

Table 113 – Linear Model Solution for Scenario 215

Linear Model - 215					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	8.7806	11.6429	0.658362
1	1	3	11.6429	14.0000	0.903081
1	3	3	11.6429	14.0000	0.903081
2	1	1	5.3050	8.7806	0.361039
2	2	1	5.3050	8.7806	0.361039
2	2	3	11.6429	14.0000	0.824273
3	2	2	8.7806	11.6429	0.542260
3	3	1	5.3050	8.7806	0.314686
3	3	2	8.7806	11.6429	0.542260
Computation Time (sec):		**0.188**	**Objective Function Value:**		**1.939672**

Table 114 – Linear Model Solution for Scenario 216

Linear Model - 216					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	8.7806	11.6429	0.658362
1	1	3	11.6429	14.0000	0.903081
1	3	3	11.6429	14.0000	0.903081
2	1	1	5.3050	8.7806	0.361039
2	2	1	5.3050	8.7806	0.361039
2	2	3	11.6429	14.0000	0.824273
3	2	2	8.7806	11.6429	0.542260
3	3	1	5.3050	8.7806	0.314686
3	3	2	8.7806	11.6429	0.542260
Computation Time (sec):		**0.298**	**Objective Function Value:**		**2.915350**

Table 115 – Construction Heuristic Solution for Scenario 201

Construction Heuristic - 201					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
3	2	1	1.0847	5.3050	0.158464
3	2	2	5.3050	8.7806	0.314686
2	2	3	8.7806	11.6429	0.542261
1	2	4	11.6429	14.0000	0.812418
2	3	1	8.7806	11.6429	0.542261
1	3	2	11.6429	14.0000	0.812418
Computation Time (sec):		**0.036**	**Objective Function Value:**		**1.903615**

Table 116 – Construction Heuristic Solution for Scenario 202

Construction Heuristic - 202					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
3	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542261
1	2	3	11.6429	14.0000	0.812418
3	3	1	5.3050	8.7806	0.314686
2	3	2	8.7806	11.6429	0.542261
1	3	3	11.6429	14.0000	0.812418
Computation Time (sec):		**0.054**	**Objective Function Value:**		**2.883178**

Table 117 – Construction Heuristic Solution for Scenario 203

Construction Heuristic - 203					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.361039
3	1	2	8.7806	11.6429	0.571296
1	1	3	11.6429	14.0000	0.903086
3	2	1	5.3050	8.7806	0.247605
2	2	2	8.7806	11.6429	0.658363
2	2	3	11.6429	14.0000	0.903086
2	3	1	5.3050	8.7806	0.314686
1	3	2	8.7806	11.6429	0.571296
1	3	3	11.6429	14.0000	0.824266
Computation Time (sec):		**0.083**	**Objective Function Value:**		**1.948621**

Table 118 – Construction Heuristic Solution for Scenario 204

Construction Heuristic - 204					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.361039
3	1	2	8.7806	11.6429	0.571296
1	1	3	11.6429	14.0000	0.903086
2	2	1	8.7806	11.6429	0.658363
2	2	2	11.6429	14.0000	0.903086
3	3	1	1.0847	5.3050	0.158464
2	3	2	5.3050	8.7806	0.314686
1	3	3	8.7806	11.6429	0.571296
1	3	4	11.6429	14.0000	0.824266
Computation Time (sec):		**0.084**	**Objective Function Value:**		**2.926943**

Table 119 – Construction Heuristic Solution for Scenario 205

Construction Heuristic - 205					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.903086
3	2	1	1.0847	5.3050	0.158464
3	2	2	5.3050	8.7806	0.314686
2	2	3	8.7806	11.6429	0.542261
1	2	4	11.6429	14.0000	0.903086
2	3	1	8.7806	11.6429	0.542261
1	3	2	11.6429	14.0000	0.903086
Computation Time (sec):		**0.096**	**Objective Function Value:**		**1.950021**

Table 120 – Construction Heuristic Solution for Scenario 206

Construction Heuristic - 206					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.903086
3	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542261
1	2	3	11.6429	14.0000	0.903086
3	3	1	5.3050	8.7806	0.314686
2	3	2	8.7806	11.6429	0.542261
1	3	3	11.6429	14.0000	0.903086
Computation Time (sec):		**0.106**	**Objective Function Value:**		**2.939428**

Table 121 – Construction Heuristic Solution for Scenario 207

Construction Heuristic - 207					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.571296
1	1	3	11.6429	14.0000	0.903086
3	2	1	1.0847	5.3050	0.158464
3	2	2	5.3050	8.7806	0.314686
2	2	3	8.7806	11.6429	0.571296
1	2	4	11.6429	14.0000	0.903086
2	3	1	8.7806	11.6429	0.571296
1	3	2	11.6429	14.0000	0.903086
Computation Time (sec):		**0.068**	**Objective Function Value:**		**1.953180**

Table 122 – Construction Heuristic Solution for Scenario 208

Construction Heuristic - 208					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.571296
1	1	3	11.6429	14.0000	0.903086
3	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.571296
1	2	3	11.6429	14.0000	0.903086
3	3	1	5.3050	8.7806	0.314686
2	3	2	8.7806	11.6429	0.571296
1	3	3	11.6429	14.0000	0.903086
Computation Time (sec):		**0.107**	**Objective Function Value:**		**2.943256**

Table 123 – Construction Heuristic Solution for Scenario 209

Construction Heuristic - 209					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
3	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542261
1	2	3	11.6429	14.0000	0.812418
3	3	1	5.3050	8.7806	0.314686
2	3	2	8.7806	11.6429	0.542261
1	3	3	11.6429	14.0000	0.812418
Computation Time (sec):		**0.094**	**Objective Function Value:**		**1.894947**

Table 124 – Construction Heuristic Solution for Scenario 210

Construction Heuristic - 210					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	8.7806	11.6429	0.542261
1	1	2	11.6429	14.0000	0.812418
3	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542261
1	2	3	11.6429	14.0000	0.812418
3	3	1	1.0847	5.3050	0.158464
2	3	2	5.3050	8.7806	0.314686
2	3	3	8.7806	11.6429	0.542261
1	3	4	11.6429	14.0000	0.812418
Computation Time (sec):		**0.127**	**Objective Function Value:**		**2.849679**

Table 125 – Construction Heuristic Solution for Scenario 211

Construction Heuristic - 211					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.361039
3	1	2	8.7806	11.6429	0.571296
1	1	3	11.6429	14.0000	0.903086
2	2	1	8.7806	11.6429	0.658363
2	2	2	11.6429	14.0000	0.903086
3	3	1	1.0847	5.3050	0.158464
2	3	2	5.3050	8.7806	0.314686
1	3	3	8.7806	11.6429	0.571296
1	3	4	11.6429	14.0000	0.824266
Computation Time (sec):		**0.138**	**Objective Function Value:**		**1.935632**

Table 126 – Construction Heuristic Solution for Scenario 212

Construction Heuristic - 212					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.361039
3	1	2	8.7806	11.6429	0.571296
1	1	3	11.6429	14.0000	0.903086
2	2	1	8.7806	11.6429	0.658363
2	2	2	11.6429	14.0000	0.903086
3	3	1	1.0847	5.3050	0.158464
2	3	2	5.3050	8.7806	0.314686
1	3	3	8.7806	11.6429	0.571296
1	3	4	11.6429	14.0000	0.824266
Computation Time (sec):		**0.116**	**Objective Function Value:**		**2.900874**

Table 127 – Construction Heuristic Solution for Scenario 213

Construction Heuristic - 213					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.903086
3	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542261
1	2	3	11.6429	14.0000	0.903086
3	3	1	5.3050	8.7806	0.314686
2	3	2	8.7806	11.6429	0.542261
1	3	3	11.6429	14.0000	0.903086
Computation Time (sec):		**0.077**	**Objective Function Value:**		**1.945508**

Table 128 – Construction Heuristic Solution for Scenario 214

Construction Heuristic - 214					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	8.7806	11.6429	0.542261
1	1	2	11.6429	14.0000	0.903086
3	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542261
1	2	3	11.6429	14.0000	0.903086
3	3	1	1.0847	5.3050	0.158464
2	3	2	5.3050	8.7806	0.314686
2	3	3	8.7806	11.6429	0.542261
1	3	4	11.6429	14.0000	0.903086
Computation Time (sec):		**0.056**	**Objective Function Value:**		**2.922021**

Table 129 – Construction Heuristic Solution for Scenario 215

Construction Heuristic - 215					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.571296
1	1	3	11.6429	14.0000	0.903086
3	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.571296
1	2	3	11.6429	14.0000	0.903086
3	3	1	5.3050	8.7806	0.314686
2	3	2	8.7806	11.6429	0.571296
1	3	3	11.6429	14.0000	0.903086
Computation Time (sec):		**0.053**	**Objective Function Value:**		**1.948951**

Table 130 – Construction Heuristic Solution for Scenario 216

Construction Heuristic - 216					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
2	1	1	8.7806	11.6429	0.571296
1	1	2	11.6429	14.0000	0.903086
3	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.571296
1	2	3	11.6429	14.0000	0.903086
3	3	1	1.0847	5.3050	0.158464
3	3	2	5.3050	8.7806	0.314686
2	3	3	8.7806	11.6429	0.571296
1	3	4	11.6429	14.0000	0.903086
Computation Time (sec):		**0.072**	**Objective Function Value:**		**2.926947**

Table 131 – Improvement Heuristic Solution for Scenario 201

Improvement Heuristic - 201					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
3	2	1	1.0847	5.3050	0.158464
3	2	2	5.3050	8.7806	0.314686
2	2	3	8.7806	11.6429	0.542261
1	2	4	11.6429	14.0000	0.812418
2	3	1	8.7806	11.6429	0.542261
1	3	2	11.6429	14.0000	0.812418
Computation Time (sec):		**0.580**	**Objective Function Value:**		**1.903615**

Table 132 – Improvement Heuristic Solution for Scenario 202

Improvement Heuristic - 202					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
3	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542261
1	2	3	11.6429	14.0000	0.812418
3	3	1	5.3050	8.7806	0.314686
2	3	2	8.7806	11.6429	0.542261
1	3	3	11.6429	14.0000	0.812418
Computation Time (sec):		**0.887**	**Objective Function Value:**		**2.883178**

Table 133 – Improvement Heuristic Solution for Scenario 203

Improvement Heuristic - 203					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.361039
3	1	2	8.7806	11.6429	0.571296
1	1	3	11.6429	14.0000	0.903086
2	2	1	8.7806	11.6429	0.658363
2	2	2	11.6429	14.0000	0.903086
2	3	1	1.0847	5.3050	0.158464
1	3	2	5.3050	8.7806	0.361039
1	3	3	8.7806	11.6429	0.571296
3	3	4	11.6429	14.0000	0.812418
Computation Time (sec):		**1.456**	**Objective Function Value:**		**1.948771**

Table 134 – Improvement Heuristic Solution for Scenario 204

Improvement Heuristic - 204					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.361039
3	1	2	8.7806	11.6429	0.571296
1	1	3	11.6429	14.0000	0.903086
2	2	1	8.7806	11.6429	0.658363
2	2	2	11.6429	14.0000	0.903086
3	3	1	1.0847	5.3050	0.158464
2	3	2	5.3050	8.7806	0.314686
1	3	3	8.7806	11.6429	0.571296
1	3	4	11.6429	14.0000	0.824266
Computation Time (sec):		**1.163**	**Objective Function Value:**		**2.926943**

Table 135 – Improvement Heuristic Solution for Scenario 205

Improvement Heuristic - 205					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.903086
3	2	1	1.0847	5.3050	0.158464
3	2	2	5.3050	8.7806	0.314686
2	2	3	8.7806	11.6429	0.542261
1	2	4	11.6429	14.0000	0.903086
2	3	1	8.7806	11.6429	0.542261
1	3	2	11.6429	14.0000	0.903086
Computation Time (sec):		**1.292**	**Objective Function Value:**		**1.950021**

Table 136 – Improvement Heuristic Solution for Scenario 206

Improvement Heuristic - 206					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.903086
3	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542261
1	2	3	11.6429	14.0000	0.903086
3	3	1	5.3050	8.7806	0.314686
2	3	2	8.7806	11.6429	0.542261
1	3	3	11.6429	14.0000	0.903086
Computation Time (sec):		**0.654**	**Objective Function Value:**		**2.939428**

Table 137 – Improvement Heuristic Solution for Scenario 207

Improvement Heuristic - 207					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.571296
1	1	3	11.6429	14.0000	0.903086
3	2	1	1.0847	5.3050	0.158464
3	2	2	5.3050	8.7806	0.314686
2	2	3	8.7806	11.6429	0.571296
1	2	4	11.6429	14.0000	0.903086
2	3	1	8.7806	11.6429	0.571296
1	3	2	11.6429	14.0000	0.903086
Computation Time (sec):		**0.915**	**Objective Function Value:**		**1.953180**

Table 138 – Improvement Heuristic Solution for Scenario 208

Improvement Heuristic - 208					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.571296
1	1	3	11.6429	14.0000	0.903086
3	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.571296
1	2	3	11.6429	14.0000	0.903086
3	3	1	5.3050	8.7806	0.314686
2	3	2	8.7806	11.6429	0.571296
1	3	3	11.6429	14.0000	0.903086
Computation Time (sec):		**1.045**	**Objective Function Value:**		**2.943256**

Table 139 – Improvement Heuristic Solution for Scenario 209

Improvement Heuristic - 209					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
3	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542261
1	2	3	11.6429	14.0000	0.812418
3	3	1	5.3050	8.7806	0.314686
2	3	2	8.7806	11.6429	0.542261
1	3	3	11.6429	14.0000	0.812418
Computation Time (sec):		**0.076**	**Objective Function Value:**		**1.894947**

Table 140 – Improvement Heuristic Solution for Scenario 210

Improvement Heuristic - 210					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	8.7806	11.6429	0.542261
1	1	2	11.6429	14.0000	0.812418
3	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542261
1	2	3	11.6429	14.0000	0.812418
3	3	1	1.0847	5.3050	0.158464
2	3	2	5.3050	8.7806	0.314686
2	3	3	8.7806	11.6429	0.542261
1	3	4	11.6429	14.0000	0.812418
Computation Time (sec):		**0.461**	**Objective Function Value:**		**2.849679**

Table 141 – Improvement Heuristic Solution for Scenario 211

Improvement Heuristic - 211					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.361039
3	1	2	8.7806	11.6429	0.571296
1	1	3	11.6429	14.0000	0.903086
2	2	1	8.7806	11.6429	0.658363
2	2	2	11.6429	14.0000	0.903086
3	3	1	1.0847	5.3050	0.158464
2	3	2	5.3050	8.7806	0.314686
1	3	3	8.7806	11.6429	0.571296
1	3	4	11.6429	14.0000	0.824266
Computation Time (sec):		**1.100**	**Objective Function Value:**		**1.935632**

Table 142 – Improvement Heuristic Solution for Scenario 212

Improvement Heuristic - 212					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.361039
3	1	2	8.7806	11.6429	0.571296
1	1	3	11.6429	14.0000	0.903086
2	2	1	8.7806	11.6429	0.658363
2	2	2	11.6429	14.0000	0.903086
3	3	1	1.0847	5.3050	0.158464
2	3	2	5.3050	8.7806	0.314686
1	3	3	8.7806	11.6429	0.571296
1	3	4	11.6429	14.0000	0.824266
Computation Time (sec):		**1.170**	**Objective Function Value:**		**2.900874**

Table 143 – Improvement Heuristic Solution for Scenario 213

Improvement Heuristic - 213					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.903086
3	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542261
1	2	3	11.6429	14.0000	0.903086
3	3	1	5.3050	8.7806	0.314686
2	3	2	8.7806	11.6429	0.542261
1	3	3	11.6429	14.0000	0.903086
Computation Time (sec):		**1.447**	**Objective Function Value:**		**1.945508**

Table 144 – Improvement Heuristic Solution for Scenario 214

Improvement Heuristic - 214					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	8.7806	11.6429	0.542261
1	1	2	11.6429	14.0000	0.903086
3	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542261
1	2	3	11.6429	14.0000	0.903086
3	3	1	1.0847	5.3050	0.158464
2	3	2	5.3050	8.7806	0.314686
2	3	3	8.7806	11.6429	0.542261
1	3	4	11.6429	14.0000	0.903086
Computation Time (sec):		**0.375**	**Objective Function Value:**		**2.922021**

Table 145 – Improvement Heuristic Solution for Scenario 215

Improvement Heuristic - 215					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.571296
1	1	3	11.6429	14.0000	0.903086
3	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.571296
1	2	3	11.6429	14.0000	0.903086
3	3	1	5.3050	8.7806	0.314686
2	3	2	8.7806	11.6429	0.571296
1	3	3	11.6429	14.0000	0.903086
Computation Time (sec):		**1.327**	**Objective Function Value:**		**1.948951**

Table 146 – Improvement Heuristic Solution for Scenario 216

Improvement Heuristic - 216					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
2	1	1	8.7806	11.6429	0.571296
1	1	2	11.6429	14.0000	0.903086
3	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.571296
1	2	3	11.6429	14.0000	0.903086
3	3	1	1.0847	5.3050	0.158464
3	3	2	5.3050	8.7806	0.314686
2	3	3	8.7806	11.6429	0.571296
1	3	4	11.6429	14.0000	0.903086
Computation Time (sec):		**0.015**	**Objective Function Value:**		**2.926947**

Table 147 – Baron Global Solution for Scenario 301

Baron Global - 301					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	5.3050	8.7806	0.314686
1	3	2	8.7806	11.6429	0.542260
1	5	2	8.7806	11.6429	0.542260
2	1	2	8.7806	11.6429	0.542260
2	2	1	5.3050	8.7806	0.314686
2	3	1	5.3050	8.7806	0.314686
3	2	2	8.7806	11.6429	0.542260
3	2	3	11.6428	14.0000	0.812415
3	5	3	11.6428	14.0000	0.812415
4	4	2	8.7806	11.6429	0.542260
4	4	3	11.6428	14.0000	0.812415
4	6	1	5.3050	8.7806	0.314686
5	3	3	11.6428	14.0000	0.812415
5	4	1	5.3050	8.7806	0.314686
5	6	2	8.7806	11.6429	0.542260
6	1	3	11.6428	14.0000	0.812415
6	5	1	5.3050	8.7806	0.314686
6	6	3	11.6428	14.0000	0.812415
Computation Time (sec):		**21600**	**Objective Function Value:**		**3.825197**

Table 148 – Baron Global Solution for Scenario 302

Baron Global - 302					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	2	3	11.6428	14.0000	0.812415
1	3	2	8.7806	11.6429	0.542260
1	4	2	8.7806	11.6429	0.542260
2	1	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542260
2	5	3	11.6428	14.0000	0.812415
3	4	3	11.6428	14.0000	0.812415
3	5	2	8.7806	11.6429	0.542260
3	6	2	8.7806	11.6429	0.542260
4	1	2	8.7806	11.6429	0.542260
4	2	1	5.3050	8.7806	0.314686
4	3	3	11.6428	14.0000	0.812415
5	3	1	5.3050	8.7806	0.314686
5	5	1	5.3050	8.7806	0.314686
5	6	3	11.6428	14.0000	0.812415
6	1	3	11.6428	14.0000	0.812415
6	4	1	5.3050	8.7806	0.314686
6	6	1	5.3050	8.7806	0.314686
Computation Time (sec):		**21600**	**Objective Function Value:**		**5.766353**

Table 149 – Baron Global Solution for Scenario 303

Baron Global - 303					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	5.3050	8.7806	0.406880
1	1	3	11.6428	14.0000	0.903080
1	6	3	11.6428	14.0000	0.903080
2	2	2	8.7806	11.6429	0.658362
2	2	3	11.6428	14.0000	0.903080
2	6	1	5.3050	8.7806	0.361038
3	1	2	8.7806	11.6429	0.571297
3	4	1	5.3050	8.7806	0.406880
3	4	3	11.6428	14.0000	0.903080
4	2	1	5.3050	8.7806	0.406880
4	4	2	8.7806	11.6429	0.658362
4	6	2	8.7806	11.6429	0.571297
5	5	1	5.3050	8.7806	0.406880
5	5	2	8.7806	11.6429	0.658362
5	5	3	11.6428	14.0000	0.903080
6	3	1	5.3050	8.7806	0.406880
6	3	2	8.7806	11.6429	0.658362
6	3	3	11.6428	14.0000	0.903080
Computation Time (sec):		**21600**	**Objective Function Value:**		**3.935343**

Table 150 – Baron Global Solution for Scenario 304

Baron Global - 304					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	5.3050	8.7806	0.406880
1	6	2	8.7806	11.6429	0.658362
1	6	3	11.6428	14.0000	0.903080
2	2	1	8.7806	11.6429	0.658362
2	2	2	11.6428	14.0000	0.903080
2	6	1	5.3050	8.7806	0.361038
3	1	1	1.0846	5.3050	0.222265
3	1	3	8.7806	11.6429	0.571297
3	1	4	11.6428	14.0000	0.824273
4	4	1	5.3050	8.7806	0.406880
4	4	2	8.7806	11.6429	0.658362
4	4	3	11.6428	14.0000	0.903080
5	5	1	5.3050	8.7806	0.406880
5	5	2	8.7806	11.6429	0.658362
5	5	3	11.6428	14.0000	0.903080
6	3	1	5.3050	8.7806	0.406880
6	3	2	8.7806	11.6429	0.658362
6	3	3	11.6428	14.0000	0.903080
Computation Time (sec):		**21600**	**Objective Function Value:**		**5.906018**

Table 151 – Baron Global Solution for Scenario 305

Baron Global - 305					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	3	1	8.7806	11.6429	0.658362
1	5	2	8.7806	11.6429	0.658362
1	6	3	11.6428	14.0000	0.903080
2	1	1	5.3050	8.7806	0.314686
2	2	1	5.3050	8.7806	0.314686
2	6	2	8.7806	11.6429	0.542260
3	1	3	11.6428	14.0000	0.812415
3	4	1	1.0846	5.3050	0.158463
3	6	1	5.3050	8.7806	0.314686
4	2	3	11.6428	14.0000	0.903080
4	3	2	11.6428	14.0000	0.903080
4	5	3	11.6428	14.0000	0.903080
5	4	2	5.3050	8.7806	0.314686
5	4	3	8.7806	11.6429	0.542260
5	4	4	11.6428	14.0000	0.812415
6	1	2	8.7806	11.6429	0.542260
6	2	2	8.7806	11.6429	0.542260
6	5	1	5.3050	8.7806	0.314686
Computation Time (sec):		**21600**	**Objective Function Value:**		**3.888241**

Table 152 – Baron Global Solution for Scenario 306

Baron Global - 306					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	3	2	8.7806	11.6429	0.658362
1	3	3	11.6428	14.0000	0.903080
1	6	3	11.6428	14.0000	0.903080
2	1	3	11.6428	14.0000	0.812415
2	2	2	8.7806	11.6429	0.542260
2	6	2	8.7806	11.6429	0.542260
3	4	2	8.7806	11.6429	0.542260
3	5	3	11.6428	14.0000	0.812415
3	6	1	5.3050	8.7806	0.314686
4	1	2	8.7806	11.6429	0.658362
4	2	3	11.6428	14.0000	0.903080
4	5	2	8.7806	11.6429	0.658362
5	1	1	5.3050	8.7806	0.314686
5	4	1	5.3050	8.7806	0.314686
5	5	1	5.3050	8.7806	0.314686
6	2	1	5.3050	8.7806	0.314686
6	3	1	5.3050	8.7806	0.314686
6	4	3	11.6428	14.0000	0.812415
Computation Time (sec):		**21600**	**Objective Function Value:**		**5.859342**

Table 153 – Baron Global Solution for Scenario 307

Baron Global - 307					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	4	11.6428	14.0000	0.903080
1	3	3	11.6428	14.0000	0.903080
1	6	2	11.6428	14.0000	0.903080
2	1	2	5.3050	8.7806	0.361038
2	2	2	11.6428	14.0000	0.824273
2	4	1	5.3050	8.7806	0.361038
3	1	1	1.0846	5.3050	0.158463
3	3	2	8.7806	11.6429	0.542260
3	5	2	5.3050	8.7806	0.314686
4	2	1	8.7806	11.6429	0.658362
4	4	3	11.6428	14.0000	0.903080
4	5	4	11.6428	14.0000	0.903080
5	3	1	5.3050	8.7806	0.361038
5	4	2	8.7806	11.6429	0.571297
5	5	3	8.7806	11.6429	0.571297
6	1	3	8.7806	11.6429	0.542260
6	5	1	1.0846	5.3050	0.158463
6	6	1	8.7806	11.6429	0.542260
Computation Time (sec):		**21600**	**Objective Function Value:**		**3.896961**

Table 154 – Baron Global Solution for Scenario 308

Baron Global - 308					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	5	3	11.6428	14.0000	0.903080
1	6	2	8.7806	11.6429	0.658362
1	6	3	11.6428	14.0000	0.903080
2	1	2	11.6428	14.0000	0.824273
2	2	1	1.0846	5.3050	0.222265
2	5	2	8.7806	11.6429	0.571297
3	3	1	5.3050	8.7806	0.314686
3	4	1	5.3050	8.7806	0.314686
3	6	1	5.3050	8.7806	0.314686
4	1	1	8.7806	11.6429	0.658362
4	3	2	8.7806	11.6429	0.658362
4	3	3	11.6428	14.0000	0.903080
5	2	2	5.3050	8.7806	0.361038
5	4	3	11.6428	14.0000	0.824273
5	5	1	5.3050	8.7806	0.361038
6	2	3	8.7806	11.6429	0.542260
6	2	4	11.6428	14.0000	0.812415
6	4	2	8.7806	11.6429	0.542260
Computation Time (sec):		**21600**	**Objective Function Value:**		**5.863434**

Table 155 – Baron Global Solution for Scenario 309

Baron Global - 309					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	3	1	5.3050	8.7806	0.314686
1	3	3	11.6428	14.0000	0.812415
1	6	3	11.6428	14.0000	0.812415
2	2	1	5.3050	8.7806	0.314686
2	2	3	11.6428	14.0000	0.812415
2	4	2	8.7806	11.6429	0.542260
3	3	2	8.7806	11.6429	0.542260
3	4	1	5.3050	8.7806	0.314686
3	6	2	8.7806	11.6429	0.542260
4	1	1	5.3050	8.7806	0.314686
4	5	1	5.3050	8.7806	0.314686
4	5	3	11.6428	14.0000	0.812415
5	1	2	8.7806	11.6429	0.542260
5	4	3	11.6428	14.0000	0.812415
5	5	2	8.7806	11.6429	0.542260
6	1	3	11.6428	14.0000	0.812415
6	2	2	8.7806	11.6429	0.542260
6	6	1	5.3050	8.7806	0.314686
Computation Time (sec):		**21600**	**Objective Function Value:**		**3.789891**

Table 156 – Baron Global Solution for Scenario 310

Baron Global - 310					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	3	2	5.3050	8.7806	0.314686
1	3	3	8.7806	11.6429	0.542260
1	6	2	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542260
2	5	2	8.7806	11.6429	0.542260
2	5	3	11.6428	14.0000	0.812415
3	3	1	1.0846	5.3050	0.158463
3	6	1	1.0846	5.3050	0.158463
3	6	4	11.6428	14.0000	0.812415
4	1	1	8.7806	11.6429	0.542260
4	2	1	5.3050	8.7806	0.314686
4	3	4	11.6428	14.0000	0.812415
5	2	3	11.6428	14.0000	0.812415
5	4	1	8.7806	11.6429	0.542260
5	5	1	5.3050	8.7806	0.314686
6	1	2	11.6428	14.0000	0.812415
6	4	2	11.6428	14.0000	0.812415
6	6	3	8.7806	11.6429	0.542260
Computation Time (sec):		**21600**	**Objective Function Value:**		**5.699353**

Table 157 – Baron Global Solution for Scenario 311

Baron Global - 311					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	3	11.6428	14.0000	0.903080
1	6	2	8.7806	11.6429	0.658362
1	6	3	11.6428	14.0000	0.903080
2	2	3	11.6428	14.0000	0.903080
2	3	2	5.3050	8.7806	0.314686
2	6	1	5.3050	8.7806	0.361038
3	1	1	5.3050	8.7806	0.361038
3	4	1	5.3050	8.7806	0.406880
3	4	3	11.6428	14.0000	0.903080
4	2	1	5.3050	8.7806	0.406880
4	2	2	8.7806	11.6429	0.658362
4	4	2	8.7806	11.6429	0.658362
5	1	2	8.7806	11.6429	0.571297
5	5	1	8.7806	11.6429	0.658362
5	5	2	11.6428	14.0000	0.903080
6	3	1	1.0846	5.3050	0.251586
6	3	3	8.7806	11.6429	0.658362
6	3	4	11.6428	14.0000	0.903080
Computation Time (sec):		**21600**	**Objective Function Value:**		**3.920312**

Table 158 – Baron Global Solution for Scenario 312

Baron Global - 312					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	4	11.6428	14.0000	0.903080
1	6	2	8.7806	11.6429	0.658362
1	6	3	11.6428	14.0000	0.903080
2	2	1	5.3050	8.7806	0.406880
2	2	2	8.7806	11.6429	0.658362
2	2	3	11.6428	14.0000	0.903080
3	1	2	5.3050	8.7806	0.361038
3	1	3	8.7806	11.6429	0.571297
3	4	1	8.7806	11.6429	0.658362
4	1	1	1.0846	5.3050	0.158463
4	4	2	11.6428	14.0000	0.903080
4	6	1	5.3050	8.7806	0.361038
5	5	1	5.3050	8.7806	0.406880
5	5	2	8.7806	11.6429	0.658362
5	5	3	11.6428	14.0000	0.903080
6	3	1	5.3050	8.7806	0.406880
6	3	2	8.7806	11.6429	0.658362
6	3	3	11.6428	14.0000	0.903080
Computation Time (sec):		**21600**	**Objective Function Value:**		**5.887633**

Table 159 – Baron Global Solution for Scenario 313

Baron Global - 313					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	3	11.6428	14.0000	0.903080
1	3	3	11.6428	14.0000	0.903080
1	6	4	11.6428	14.0000	0.903080
2	2	1	5.3050	8.7806	0.314686
2	4	1	5.3050	8.7806	0.314686
2	4	2	8.7806	11.6429	0.542260
3	1	1	5.3050	8.7806	0.314686
3	2	2	8.7806	11.6429	0.542260
3	5	1	8.7806	11.6429	0.542260
4	3	2	8.7806	11.6429	0.658362
4	4	3	11.6428	14.0000	0.903080
4	5	2	11.6428	14.0000	0.903080
5	1	2	8.7806	11.6429	0.542260
5	2	3	11.6428	14.0000	0.812415
5	6	1	1.0846	5.3050	0.158463
6	3	1	5.3050	8.7806	0.314686
6	6	2	5.3050	8.7806	0.314686
6	6	3	8.7806	11.6429	0.542260
Computation Time (sec):		**21600**	**Objective Function Value:**		**3.880155**

Table 160 – Baron Global Solution for Scenario 314

Baron Global - 314					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	3	2	8.7806	11.6429	0.658362
1	6	2	8.7806	11.6429	0.658362
1	6	3	11.6428	14.0000	0.903080
2	1	2	8.7806	11.6429	0.542260
2	5	3	11.6428	14.0000	0.812415
2	6	1	5.3050	8.7806	0.314686
3	2	1	5.3050	8.7806	0.314686
3	3	1	5.3050	8.7806	0.314686
3	4	1	5.3050	8.7806	0.314686
4	1	3	11.6428	14.0000	0.903080
4	3	3	11.6428	14.0000	0.903080
4	5	1	5.3050	8.7806	0.406880
5	1	1	5.3050	8.7806	0.314686
5	2	3	11.6428	14.0000	0.812415
5	5	2	8.7806	11.6429	0.542260
6	2	2	8.7806	11.6429	0.542260
6	4	2	8.7806	11.6429	0.542260
6	4	3	11.6428	14.0000	0.812415
Computation Time (sec):		**21600**	**Objective Function Value:**		**5.829073**

Table 161 – Baron Global Solution for Scenario 315

Baron Global - 315					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	3	3	11.6428	14.0000	0.903080
1	5	2	11.6428	14.0000	0.903080
1	6	3	11.6428	14.0000	0.903080
2	2	4	11.6428	14.0000	0.824273
2	4	1	5.3050	8.7806	0.361038
2	4	2	8.7806	11.6429	0.571297
3	1	1	5.3050	8.7806	0.314686
3	2	1	1.0846	5.3050	0.158463
3	3	2	8.7806	11.6429	0.542260
4	1	2	8.7806	11.6429	0.658362
4	1	3	11.6428	14.0000	0.903080
4	5	1	8.7806	11.6429	0.658362
5	2	2	5.3050	8.7806	0.361038
5	2	3	8.7806	11.6429	0.571297
5	4	3	11.6428	14.0000	0.824273
6	3	1	5.3050	8.7806	0.314686
6	6	1	5.3050	8.7806	0.314686
6	6	2	8.7806	11.6429	0.542260
Computation Time (sec):		**21600**	**Objective Function Value:**		**3.879759**

Table 162 – Baron Global Solution for Scenario 316

Baron Global - 316					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	3	1	5.3050	8.7806	0.406880
1	6	1	5.3050	8.7806	0.406880
1	6	3	11.6428	14.0000	0.903080
2	2	1	1.0846	5.3050	0.222265
2	2	3	8.7806	11.6429	0.571297
2	5	3	11.6428	14.0000	0.824273
3	1	1	5.3050	8.7806	0.314686
3	5	1	5.3050	8.7806	0.314686
3	6	2	8.7806	11.6429	0.542260
4	1	3	11.6428	14.0000	0.903080
4	3	3	11.6428	14.0000	0.903080
4	4	2	11.6428	14.0000	0.903080
5	2	2	5.3050	8.7806	0.361038
5	3	2	8.7806	11.6429	0.571297
5	5	2	8.7806	11.6429	0.571297
6	1	2	8.7806	11.6429	0.542260
6	2	4	11.6428	14.0000	0.812415
6	4	1	8.7806	11.6429	0.542260
Computation Time (sec):		**21600**	**Objective Function Value:**		**5.836166**

Table 163 – Baron Local Solution for Scenario 301

Baron Local - 301					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	8.7806	11.6429	0.542260
1	2	2	5.3050	8.7806	0.314686
1	5	1	8.7806	11.6429	0.542260
2	3	2	5.3050	8.7806	0.314686
2	3	4	11.6429	14.0000	0.812415
2	6	1	5.3050	8.7806	0.314686
3	2	1	1.0847	5.3050	0.158464
3	2	3	8.7806	11.6429	0.542260
3	4	3	11.6429	14.0000	0.812415
4	3	1	1.0847	5.3050	0.158464
4	5	2	11.6429	14.0000	0.812415
4	6	2	8.7806	11.6429	0.542260
5	1	2	11.6429	14.0000	0.812415
5	3	3	8.7806	11.6429	0.542260
5	6	3	11.6429	14.0000	0.812415
6	2	4	11.6429	14.0000	0.812415
6	4	1	5.3050	8.7806	0.314686
6	4	2	8.7806	11.6429	0.542260
Computation Time (sec):		**207.179**	**Objective Function Value:**		**3.808097**

Table 164 – Baron Local Solution for Scenario 302

Baron Local - 302					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	2	3	8.7806	11.6429	0.542260
1	2	4	11.6429	14.0000	0.812415
1	3	1	5.3050	8.7806	0.314686
2	1	3	11.6429	14.0000	0.812415
2	5	1	8.7806	11.6429	0.542260
2	6	1	5.3050	8.7806	0.314686
3	1	2	8.7806	11.6429	0.542260
3	2	2	5.3050	8.7806	0.314686
3	6	2	8.7806	11.6429	0.542260
4	2	1	1.0847	5.3050	0.158464
4	5	2	11.6429	14.0000	0.812415
4	6	3	11.6429	14.0000	0.812415
5	3	2	8.7806	11.6429	0.542260
5	3	3	11.6429	14.0000	0.812415
5	4	3	11.6429	14.0000	0.812415
6	1	1	5.3050	8.7806	0.314686
6	4	1	5.3050	8.7806	0.314686
6	4	2	8.7806	11.6429	0.542260
Computation Time (sec):		**204.785**	**Objective Function Value:**		**5.757766**

Table 165 – Baron Local Solution for Scenario 303

Baron Local - 303					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	8.7806	11.6429	0.658362
1	1	3	11.6429	14.0000	0.903081
1	6	3	8.7806	11.6429	0.658362
2	2	1	5.3050	8.7806	0.406880
2	2	2	8.7806	11.6429	0.658362
2	6	4	11.6429	14.0000	0.824273
3	4	1	5.3050	8.7806	0.406880
3	4	2	8.7806	11.6429	0.658362
3	6	1	1.0847	5.3050	0.222265
4	2	3	11.6429	14.0000	0.903081
4	4	3	11.6429	14.0000	0.903081
4	6	2	5.3050	8.7806	0.361039
5	1	1	5.3050	8.7806	0.361039
5	5	1	8.7806	11.6429	0.658362
5	5	2	11.6429	14.0000	0.903081
6	3	1	5.3050	8.7806	0.406880
6	3	2	8.7806	11.6429	0.658362
6	3	3	11.6429	14.0000	0.903081
Computation Time (sec):		**186.819**	**Objective Function Value:**		**3.928757**

Table 166 – Baron Local Solution for Scenario 304

Baron Local - 304					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	5	2	8.7806	11.6429	0.571298
1	6	1	5.3050	8.7806	0.406880
1	6	2	8.7806	11.6429	0.658362
2	2	1	8.7806	11.6429	0.658362
2	2	2	11.6429	14.0000	0.903081
2	6	3	11.6429	14.0000	0.824273
3	4	1	5.3050	8.7806	0.406880
3	4	2	8.7806	11.6429	0.658362
3	4	3	11.6429	14.0000	0.903081
4	1	3	11.6429	14.0000	0.812415
4	3	3	8.7806	11.6429	0.470810
4	5	3	11.6429	14.0000	0.812415
5	1	1	5.3050	8.7806	0.361039
5	1	2	8.7806	11.6429	0.571298
5	5	1	5.3050	8.7806	0.406880
6	3	1	1.0847	5.3050	0.251587
6	3	2	5.3050	8.7806	0.406880
6	3	4	11.6429	14.0000	0.903081
Computation Time (sec):		**208.874**	**Objective Function Value:**		**5.866369**

Table 167 – Baron Local Solution for Scenario 305

Baron Local - 305					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	2	2	8.7806	11.6429	0.658362
1	3	1	8.7806	11.6429	0.658362
1	6	4	11.6429	14.0000	0.903081
2	1	2	5.3050	8.7806	0.314686
2	5	3	11.6429	14.0000	0.812415
2	6	1	1.0847	5.3050	0.158464
3	4	1	8.7806	11.6429	0.542260
3	6	2	5.3050	8.7806	0.314686
3	6	3	8.7806	11.6429	0.542260
4	2	3	11.6429	14.0000	0.903081
4	3	2	11.6429	14.0000	0.903081
4	4	2	11.6429	14.0000	0.903081
5	1	1	1.0847	5.3050	0.158464
5	2	1	5.3050	8.7806	0.314686
5	5	2	8.7806	11.6429	0.542260
6	1	3	8.7806	11.6429	0.542260
6	1	4	11.6429	14.0000	0.812415
6	5	1	5.3050	8.7806	0.314686
Computation Time (sec):		**157.370**	**Objective Function Value:**		**3.883875**

Table 168 – Baron Local Solution for Scenario 306

Baron Local - 306					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	3	4	11.6429	14.0000	0.903081
1	4	3	11.6429	14.0000	0.903081
1	6	4	11.6429	14.0000	0.903081
2	3	3	8.7806	11.6429	0.542260
2	5	2	11.6429	14.0000	0.812415
2	6	1	1.0847	5.3050	0.158464
3	2	3	11.6429	14.0000	0.812415
3	4	1	5.3050	8.7806	0.314686
3	4	2	8.7806	11.6429	0.542260
4	1	2	11.6429	14.0000	0.903081
4	3	2	5.3050	8.7806	0.406880
4	5	1	8.7806	11.6429	0.658362
5	2	1	5.3050	8.7806	0.314686
5	3	1	1.0847	5.3050	0.158464
5	6	2	5.3050	8.7806	0.314686
6	1	1	8.7806	11.6429	0.542260
6	2	2	8.7806	11.6429	0.542260
6	6	3	8.7806	11.6429	0.542260
Computation Time (sec):		**155.995**	**Objective Function Value:**		**5.854562**

Table 169 – Baron Local Solution for Scenario 307

Baron Local - 307					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	8.7806	11.6429	0.658362
1	3	1	8.7806	11.6429	0.658362
1	4	1	8.7806	11.6429	0.658362
2	1	1	5.3050	8.7806	0.361039
2	2	2	8.7806	11.6429	0.571298
2	4	2	11.6429	14.0000	0.824273
3	2	3	11.6429	14.0000	0.812415
3	5	1	1.0847	5.3050	0.158464
3	6	3	8.7806	11.6429	0.542260
4	1	3	11.6429	14.0000	0.903081
4	3	2	11.6429	14.0000	0.903081
4	6	4	11.6429	14.0000	0.903081
5	2	1	5.3050	8.7806	0.361039
5	5	4	11.6429	14.0000	0.824273
5	6	1	1.0847	5.3050	0.222265
6	5	2	5.3050	8.7806	0.314686
6	5	3	8.7806	11.6429	0.542260
6	6	2	5.3050	8.7806	0.314686
Computation Time (sec):		**149.887**	**Objective Function Value:**		**3.883107**

Table 170 – Baron Local Solution for Scenario 308

Baron Local - 308					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	3	11.6429	14.0000	0.903081
1	3	4	11.6429	14.0000	0.903081
1	6	4	11.6429	14.0000	0.903081
2	2	1	8.7806	11.6429	0.571298
2	2	2	11.6429	14.0000	0.824273
2	5	1	8.7806	11.6429	0.571298
3	1	1	5.3050	8.7806	0.314686
3	4	3	11.6429	14.0000	0.812415
3	6	2	5.3050	8.7806	0.314686
4	1	2	8.7806	11.6429	0.658362
4	4	1	5.3050	8.7806	0.406880
4	6	3	8.7806	11.6429	0.658362
5	3	2	5.3050	8.7806	0.361039
5	3	3	8.7806	11.6429	0.571298
5	5	2	11.6429	14.0000	0.824273
6	3	1	1.0847	5.3050	0.158464
6	4	2	8.7806	11.6429	0.542260
6	6	1	1.0847	5.3050	0.158464
Computation Time (sec):		**152.146**	**Objective Function Value:**		**5.847805**

Table 171 – Baron Local Solution for Scenario 309

Baron Local - 309					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	3	2	5.3050	8.7806	0.314686
1	3	3	8.7806	11.6429	0.542260
1	6	1	1.0847	5.3050	0.158464
2	2	2	8.7806	11.6429	0.542260
2	6	3	8.7806	11.6429	0.542260
2	6	4	11.6429	14.0000	0.812415
3	2	1	5.3050	8.7806	0.314686
3	2	3	11.6429	14.0000	0.812415
3	6	2	5.3050	8.7806	0.314686
4	1	1	5.3050	8.7806	0.314686
4	5	1	8.7806	11.6429	0.542260
4	5	2	11.6429	14.0000	0.812415
5	1	2	8.7806	11.6429	0.542260
5	1	3	11.6429	14.0000	0.812415
5	3	1	1.0847	5.3050	0.158464
6	3	4	11.6429	14.0000	0.812415
6	4	1	8.7806	11.6429	0.542260
6	4	2	11.6429	14.0000	0.812415
Computation Time (sec):		**199.636**	**Objective Function Value:**		**3.778768**

Table 172 – Baron Local Solution for Scenario 310

Baron Local - 310					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	3	1	5.3050	8.7806	0.314686
1	5	1	8.7806	11.6429	0.542260
1	6	1	1.0847	5.3050	0.158464
2	1	2	8.7806	11.6429	0.542260
2	3	2	8.7806	11.6429	0.542260
2	6	3	8.7806	11.6429	0.542260
3	2	3	8.7806	11.6429	0.542260
3	6	2	5.3050	8.7806	0.314686
3	6	4	11.6429	14.0000	0.812415
4	2	2	5.3050	8.7806	0.314686
4	2	4	11.6429	14.0000	0.812415
4	5	2	11.6429	14.0000	0.812415
5	1	1	5.3050	8.7806	0.314686
5	2	1	1.0847	5.3050	0.158464
5	3	3	11.6429	14.0000	0.812415
6	1	3	11.6429	14.0000	0.812415
6	4	1	8.7806	11.6429	0.542260
6	4	2	11.6429	14.0000	0.812415
Computation Time (sec):		**199.105**	**Objective Function Value:**		**5.689142**

Table 173 – Baron Local Solution for Scenario 311

Baron Local - 311					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	1	5.3050	8.7806	0.406880
1	3	3	11.6429	14.0000	0.824273
1	6	2	8.7806	11.6429	0.658362
2	2	1	5.3050	8.7806	0.406880
2	6	1	5.3050	8.7806	0.361039
2	6	3	11.6429	14.0000	0.824273
3	1	3	11.6429	14.0000	0.824273
3	4	3	8.7806	11.6429	0.658362
3	4	4	11.6429	14.0000	0.903081
4	2	2	8.7806	11.6429	0.658362
4	2	3	11.6429	14.0000	0.903081
4	4	2	5.3050	8.7806	0.406880
5	1	2	8.7806	11.6429	0.571298
5	5	1	8.7806	11.6429	0.658362
5	5	2	11.6429	14.0000	0.903081
6	3	1	5.3050	8.7806	0.406880
6	3	2	8.7806	11.6429	0.658362
6	4	1	1.0847	5.3050	0.222265
Computation Time (sec):		**212.813**	**Objective Function Value:**		**3.884818**

Table 174 – Baron Local Solution for Scenario 312

Baron Local - 312					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	3	11.6429	14.0000	0.903081
1	6	2	8.7806	11.6429	0.658362
1	6	3	11.6429	14.0000	0.903081
2	2	1	5.3050	8.7806	0.406880
2	2	3	11.6429	14.0000	0.903081
2	6	1	5.3050	8.7806	0.361039
3	1	1	5.3050	8.7806	0.361039
3	1	2	8.7806	11.6429	0.571298
3	4	1	5.3050	8.7806	0.406880
4	2	2	8.7806	11.6429	0.658362
4	4	2	8.7806	11.6429	0.658362
4	4	3	11.6429	14.0000	0.903081
5	3	3	8.7806	11.6429	0.542260
5	5	1	8.7806	11.6429	0.658362
5	5	2	11.6429	14.0000	0.903081
6	3	1	1.0847	5.3050	0.251587
6	3	2	5.3050	8.7806	0.406880
6	3	4	11.6429	14.0000	0.903081
Computation Time (sec):		**180.723**	**Objective Function Value:**		**5.885464**

Table 175 – Baron Local Solution for Scenario 313

Baron Local - 313					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	3	3	11.6429	14.0000	0.903081
1	4	3	11.6429	14.0000	0.903081
1	6	4	11.6429	14.0000	0.903081
2	2	2	11.6429	14.0000	0.812415
2	4	2	8.7806	11.6429	0.542260
2	6	3	8.7806	11.6429	0.542260
3	1	4	11.6429	14.0000	0.812415
3	4	1	5.3050	8.7806	0.314686
3	6	2	5.3050	8.7806	0.314686
4	2	1	8.7806	11.6429	0.658362
4	5	1	8.7806	11.6429	0.658362
4	5	2	11.6429	14.0000	0.903081
5	1	1	1.0847	5.3050	0.158464
5	3	1	5.3050	8.7806	0.314686
5	6	1	1.0847	5.3050	0.158464
6	1	2	5.3050	8.7806	0.314686
6	1	3	8.7806	11.6429	0.542260
6	3	2	8.7806	11.6429	0.542260
Computation Time (sec):		**121.260**	**Objective Function Value:**		**3.867694**

Table 176 – Baron Local Solution for Scenario 314

Baron Local - 314					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	4	3	11.6429	14.0000	0.903081
1	6	2	8.7806	11.6429	0.658362
1	6	3	11.6429	14.0000	0.903081
2	4	2	8.7806	11.6429	0.542260
2	5	3	8.7806	11.6429	0.542260
2	5	4	11.6429	14.0000	0.812415
3	2	3	11.6429	14.0000	0.812415
3	4	1	5.3050	8.7806	0.314686
3	5	2	5.3050	8.7806	0.314686
4	1	1	8.7806	11.6429	0.658362
4	3	2	8.7806	11.6429	0.658362
4	3	3	11.6429	14.0000	0.903081
5	2	1	5.3050	8.7806	0.314686
5	3	1	5.3050	8.7806	0.314686
5	6	1	5.3050	8.7806	0.314686
6	1	2	11.6429	14.0000	0.812415
6	2	2	8.7806	11.6429	0.542260
6	5	1	1.0847	5.3050	0.158464
Computation Time (sec):		**144.295**	**Objective Function Value:**		**5.827279**

Table 177 – Baron Local Solution for Scenario 315

Baron Local - 315					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	2	1	8.7806	11.6429	0.658362
1	4	1	8.7806	11.6429	0.658362
1	5	2	5.3050	8.7806	0.406880
2	4	2	11.6429	14.0000	0.824273
2	6	1	1.0847	5.3050	0.222265
2	6	2	5.3050	8.7806	0.361039
3	2	2	11.6429	14.0000	0.812415
3	3	3	8.7806	11.6429	0.542260
3	6	3	8.7806	11.6429	0.542260
4	1	2	11.6429	14.0000	0.903081
4	3	4	11.6429	14.0000	0.903081
4	6	4	11.6429	14.0000	0.903081
5	1	1	8.7806	11.6429	0.571298
5	3	2	5.3050	8.7806	0.361039
5	5	3	8.7806	11.6429	0.571298
6	3	1	1.0847	5.3050	0.158464
6	5	1	1.0847	5.3050	0.158464
6	5	4	11.6429	14.0000	0.812415
Computation Time (sec):		**178.285**	**Objective Function Value:**		**3.861642**

Table 178 – Baron Local Solution for Scenario 316

Baron Local - 316					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	3	2	8.7806	11.6429	0.658362
1	3	3	11.6429	14.0000	0.903081
1	6	3	11.6429	14.0000	0.903081
2	2	2	8.7806	11.6429	0.571298
2	5	1	5.3050	8.7806	0.361039
2	5	2	8.7806	11.6429	0.571298
3	1	4	11.6429	14.0000	0.812415
3	2	1	5.3050	8.7806	0.314686
3	5	3	11.6429	14.0000	0.812415
4	2	3	11.6429	14.0000	0.903081
4	3	1	5.3050	8.7806	0.406880
4	6	2	8.7806	11.6429	0.658362
5	1	1	1.0847	5.3050	0.222265
5	4	1	8.7806	11.6429	0.571298
5	4	2	11.6429	14.0000	0.824273
6	1	2	5.3050	8.7806	0.314686
6	1	3	8.7806	11.6429	0.542260
6	6	1	5.3050	8.7806	0.314686
Computation Time (sec):		**148.523**	**Objective Function Value:**		**5.833085**

Table 179 – Linear Model Solution for Scenario 301

Linear Model - 301					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	3	1	5.3050	8.7806	0.314686
1	4	1	8.7806	11.6429	0.542260
1	5	3	8.7806	11.6429	0.542260
2	2	2	8.7806	11.6429	0.542260
2	5	1	1.0847	5.3050	0.158464
2	5	2	5.3050	8.7806	0.314686
3	1	1	5.3050	8.7806	0.314686
3	1	3	11.6429	14.0000	0.812415
3	3	3	11.6429	14.0000	0.812415
4	1	2	8.7806	11.6429	0.542260
4	3	2	8.7806	11.6429	0.542260
4	6	1	5.3050	8.7806	0.314686
5	2	3	11.6429	14.0000	0.812415
5	4	2	11.6429	14.0000	0.812415
5	6	2	8.7806	11.6429	0.542260
6	2	1	5.3050	8.7806	0.314686
6	5	4	11.6429	14.0000	0.812415
6	6	3	11.6429	14.0000	0.812415
Computation Time (sec):		**0.529**	**Objective Function Value:**		**3.816547**

Table 180 – Linear Model Solution for Scenario 302

Linear Model - 302					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	3	1	5.3050	8.7806	0.314686
1	4	1	8.7806	11.6429	0.542260
1	5	3	8.7806	11.6429	0.542260
2	2	2	8.7806	11.6429	0.542260
2	5	1	1.0847	5.3050	0.158464
2	5	2	5.3050	8.7806	0.314686
3	1	1	5.3050	8.7806	0.314686
3	1	3	11.6429	14.0000	0.812415
3	3	3	11.6429	14.0000	0.812415
4	1	2	8.7806	11.6429	0.542260
4	3	2	8.7806	11.6429	0.542260
4	6	1	5.3050	8.7806	0.314686
5	2	3	11.6429	14.0000	0.812415
5	4	2	11.6429	14.0000	0.812415
5	6	2	8.7806	11.6429	0.542260
6	2	1	5.3050	8.7806	0.314686
6	5	4	11.6429	14.0000	0.812415
6	6	3	11.6429	14.0000	0.812415
Computation Time (sec):		**0.764**	**Objective Function Value:**		**5.757702**

Table 181 – Linear Model Solution for Scenario 303

Linear Model - 303					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	3	1	8.7806	11.6429	0.571298
1	3	2	11.6429	14.0000	0.824273
1	5	3	8.7806	11.6429	0.571298
2	4	1	5.3050	8.7806	0.314686
2	5	1	1.0847	5.3050	0.066208
2	5	2	5.3050	8.7806	0.247605
3	1	1	8.7806	11.6429	0.571298
3	4	2	8.7806	11.6429	0.658362
3	6	3	8.7806	11.6429	0.571298
4	1	2	11.6429	14.0000	0.812415
4	2	1	5.3050	8.7806	0.406880
4	6	1	1.0847	5.3050	0.222265
5	2	2	8.7806	11.6429	0.542260
5	2	3	11.6429	14.0000	0.812415
5	5	4	11.6429	14.0000	0.903081
6	4	3	11.6429	14.0000	0.824273
6	6	2	5.3050	8.7806	0.314686
6	6	4	11.6429	14.0000	0.812415
Computation Time (sec):		**0.264**	**Objective Function Value:**		**3.841385**

Table 182 – Linear Model Solution for Scenario 304

Linear Model - 304					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	3	1	8.7806	11.6429	0.571298
1	3	2	11.6429	14.0000	0.824273
1	4	2	5.3050	8.7806	0.247605
2	2	2	8.7806	11.6429	0.658362
2	5	1	5.3050	8.7806	0.247605
2	5	2	8.7806	11.6429	0.470810
3	1	1	5.3050	8.7806	0.361039
3	4	4	11.6429	14.0000	0.903081
3	6	3	11.6429	14.0000	0.824273
4	1	2	8.7806	11.6429	0.542260
4	5	3	11.6429	14.0000	0.812415
4	6	1	5.3050	8.7806	0.361039
5	1	3	11.6429	14.0000	0.824273
5	2	3	11.6429	14.0000	0.812415
5	4	1	1.0847	5.3050	0.066208
6	2	1	5.3050	8.7806	0.247605
6	4	3	8.7806	11.6429	0.571298
6	6	2	8.7806	11.6429	0.542260
Computation Time (sec):		**0.265**	**Objective Function Value:**		**5.772686**

Table 183 – Linear Model Solution for Scenario 305

Linear Model - 305					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	2	3	11.6429	14.0000	0.903081
1	5	2	8.7806	11.6429	0.658362
1	6	3	11.6429	14.0000	0.903081
2	1	1	8.7806	11.6429	0.542260
2	5	1	5.3050	8.7806	0.314686
2	6	1	5.3050	8.7806	0.314686
3	1	2	11.6429	14.0000	0.812415
3	3	2	8.7806	11.6429	0.542260
3	4	3	8.7806	11.6429	0.542260
4	2	2	8.7806	11.6429	0.658362
4	4	1	1.0847	5.3050	0.251587
4	4	2	5.3050	8.7806	0.406880
5	3	1	5.3050	8.7806	0.314686
5	5	3	11.6429	14.0000	0.812415
5	6	2	8.7806	11.6429	0.542260
6	2	1	5.3050	8.7806	0.314686
6	3	3	11.6429	14.0000	0.812415
6	4	4	11.6429	14.0000	0.812415
Computation Time (sec):		**0.359**	**Objective Function Value:**		**3.860986**

Table 184 – Linear Model Solution for Scenario 306

Linear Model - 306					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	3	1	5.3050	8.7806	0.406880
1	4	1	5.3050	8.7806	0.406880
1	6	2	11.6429	14.0000	0.903081
2	3	3	11.6429	14.0000	0.812415
2	5	1	1.0847	5.3050	0.158464
2	5	2	5.3050	8.7806	0.314686
3	1	1	8.7806	11.6429	0.542260
3	4	2	8.7806	11.6429	0.542260
3	5	3	8.7806	11.6429	0.542260
4	1	2	11.6429	14.0000	0.903081
4	2	2	4.5585	5.3050	0.369290
4	6	1	8.7806	11.6429	0.658362
5	2	3	8.7806	11.6429	0.542260
5	3	2	8.7806	11.6429	0.542260
5	5	4	11.6429	14.0000	0.812415
6	2	1	0.1782	5.3050	0.136453
6	2	4	11.6429	14.0000	0.812415
6	4	3	11.6429	14.0000	0.812415
Computation Time (sec):		**0.452**	**Objective Function Value:**		**5.821320**

Table 185 – Linear Model Solution for Scenario 307

Linear Model - 307					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	3	1	8.7806	11.6429	0.658362
1	4	2	8.7806	11.6429	0.658362
1	6	2	5.3050	8.7806	0.406880
2	4	1	5.3050	8.7806	0.361039
2	5	1	0.0901	5.3050	0.201808
2	5	2	4.4859	5.3050	0.327429
3	1	1	5.3050	8.7806	0.314686
3	3	2	11.6429	14.0000	0.812415
3	4	3	11.6429	14.0000	0.812415
4	1	2	8.7806	11.6429	0.658362
4	5	3	8.7806	11.6429	0.658362
4	6	4	11.6429	14.0000	0.903081
5	1	3	11.6429	14.0000	0.824273
5	2	2	11.6429	14.0000	0.824273
5	6	1	1.0847	5.3050	0.222265
6	2	1	8.7806	11.6429	0.542260
6	5	4	11.6429	14.0000	0.812415
6	6	3	8.7806	11.6429	0.542260
Computation Time (sec):		**0.311**	**Objective Function Value:**		**3.860437**

Table 186 – Linear Model Solution for Scenario 308

Linear Model - 308					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	3	1	8.7806	11.6429	0.658362
1	4	2	8.7806	11.6429	0.658362
1	6	2	5.3050	8.7806	0.406880
2	4	1	5.3050	8.7806	0.361039
2	5	1	0.0901	5.3050	0.201808
2	5	2	4.4859	5.3050	0.327429
3	1	1	5.3050	8.7806	0.314686
3	3	2	11.6429	14.0000	0.812415
3	4	3	11.6429	14.0000	0.812415
4	1	2	8.7806	11.6429	0.658362
4	5	3	8.7806	11.6429	0.658362
4	6	4	11.6429	14.0000	0.903081
5	1	3	11.6429	14.0000	0.824273
5	2	2	11.6429	14.0000	0.824273
5	6	1	1.0847	5.3050	0.222265
6	2	1	8.7806	11.6429	0.542260
6	5	4	11.6429	14.0000	0.812415
6	6	3	8.7806	11.6429	0.542260
Computation Time (sec):		**0.359**	**Objective Function Value:**		**5.818162**

Table 187 – Linear Model Solution for Scenario 309

Linear Model - 309					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	3	1	5.3050	8.7806	0.314686
1	4	2	11.6429	14.0000	0.812415
1	5	3	11.6429	14.0000	0.812415
2	1	3	8.7806	11.6429	0.542260
2	3	2	8.7806	11.6429	0.542260
2	5	1	5.3050	8.7806	0.314686
3	2	4	11.6429	14.0000	0.812415
3	3	3	11.6429	14.0000	0.812415
3	6	1	8.7806	11.6429	0.542260
4	1	1	1.0847	5.3050	0.158464
4	1	2	5.3050	8.7806	0.314686
4	4	1	8.7806	11.6429	0.542260
5	1	4	11.6429	14.0000	0.812415
5	2	2	5.3050	8.7806	0.314686
5	2	3	8.7806	11.6429	0.542260
6	2	1	1.0847	5.3050	0.158464
6	5	2	8.7806	11.6429	0.542260
6	6	2	11.6429	14.0000	0.812415
Computation Time (sec):		**0.436**	**Objective Function Value:**		**3.764234**

Table 188 – Linear Model Solution for Scenario 310

Linear Model - 310					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	3	1	5.3050	8.7806	0.314686
1	4	2	11.6429	14.0000	0.812415
1	5	3	11.6429	14.0000	0.812415
2	1	3	8.7806	11.6429	0.542260
2	3	2	8.7806	11.6429	0.542260
2	5	1	5.3050	8.7806	0.314686
3	2	4	11.6429	14.0000	0.812415
3	3	3	11.6429	14.0000	0.812415
3	6	1	8.7806	11.6429	0.542260
4	1	1	1.0847	5.3050	0.158464
4	1	2	5.3050	8.7806	0.314686
4	4	1	8.7806	11.6429	0.542260
5	1	4	11.6429	14.0000	0.812415
5	2	2	5.3050	8.7806	0.314686
5	2	3	8.7806	11.6429	0.542260
6	2	1	1.0847	5.3050	0.158464
6	5	2	8.7806	11.6429	0.542260
6	6	2	11.6429	14.0000	0.812415
Computation Time (sec):		**0.436**	**Objective Function Value:**		**5.648466**

Table 189 – Linear Model Solution for Scenario 311

Linear Model - 311					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	3	1	5.3050	8.7806	0.361039
1	4	2	8.7806	11.6429	0.470810
1	5	1	5.3050	8.7806	0.361039
2	3	3	11.6429	14.0000	0.812415
2	4	1	5.3050	8.7806	0.314686
2	6	1	1.0847	5.3050	0.222265
3	1	1	8.7806	11.6429	0.571298
3	3	2	8.7806	11.6429	0.542260
3	6	4	11.6429	14.0000	0.824273
4	2	2	8.7806	11.6429	0.658362
4	4	3	11.6429	14.0000	0.903081
4	5	2	8.7806	11.6429	0.542260
5	2	3	11.6429	14.0000	0.812415
5	5	3	11.6429	14.0000	0.903081
5	6	2	5.3050	8.7806	0.314686
6	1	2	11.6429	14.0000	0.812415
6	2	1	5.3050	8.7806	0.247605
6	6	3	8.7806	11.6429	0.542260
Computation Time (sec):		**0.234**	**Objective Function Value:**		**3.826963**

Table 190 – Linear Model Solution for Scenario 312

Linear Model - 312					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	3	1	5.3050	8.7806	0.361039
1	4	2	8.7806	11.6429	0.470810
1	5	1	5.3050	8.7806	0.361039
2	3	3	11.6429	14.0000	0.812415
2	4	1	5.3050	8.7806	0.314686
2	6	1	1.0847	5.3050	0.222265
3	1	1	8.7806	11.6429	0.571298
3	3	2	8.7806	11.6429	0.542260
3	6	4	11.6429	14.0000	0.824273
4	2	2	8.7806	11.6429	0.658362
4	4	3	11.6429	14.0000	0.903081
4	5	2	8.7806	11.6429	0.542260
5	2	3	11.6429	14.0000	0.812415
5	5	3	11.6429	14.0000	0.903081
5	6	2	5.3050	8.7806	0.314686
6	1	2	11.6429	14.0000	0.812415
6	2	1	5.3050	8.7806	0.247605
6	6	3	8.7806	11.6429	0.542260
Computation Time (sec):		**0.265**	**Objective Function Value:**		**5.748773**

Table 191 – Linear Model Solution for Scenario 313

Linear Model - 313					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	8.7806	11.6429	0.658362
1	3	1	5.3050	8.7806	0.406880
1	6	1	5.3050	8.7806	0.406880
2	3	3	11.6429	14.0000	0.812415
2	5	1	5.3050	8.7806	0.314686
2	6	2	8.7806	11.6429	0.542260
3	1	1	5.3050	8.7806	0.314686
3	3	2	8.7806	11.6429	0.542260
3	4	1	5.3050	8.7806	0.314686
4	1	3	11.6429	14.0000	0.903081
4	2	3	11.6429	14.0000	0.903081
4	5	2	8.7806	11.6429	0.658362
5	2	2	8.7806	11.6429	0.542260
5	5	3	11.6429	14.0000	0.812415
5	6	3	11.6429	14.0000	0.812415
6	2	1	5.3050	8.7806	0.314686
6	4	2	8.7806	11.6429	0.542260
6	4	3	11.6429	14.0000	0.812415
Computation Time (sec):		**0.467**	**Objective Function Value:**		**3.841404**

Table 192 – Linear Model Solution for Scenario 314

Linear Model - 314					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	2	8.7806	11.6429	0.658362
1	3	1	5.3050	8.7806	0.406880
1	6	1	5.3050	8.7806	0.406880
2	3	3	11.6429	14.0000	0.812415
2	5	1	5.3050	8.7806	0.314686
2	6	2	8.7806	11.6429	0.542260
3	1	1	5.3050	8.7806	0.314686
3	3	2	8.7806	11.6429	0.542260
3	4	1	5.3050	8.7806	0.314686
4	1	3	11.6429	14.0000	0.903081
4	2	3	11.6429	14.0000	0.903081
4	5	2	8.7806	11.6429	0.658362
5	2	2	8.7806	11.6429	0.542260
5	5	3	11.6429	14.0000	0.812415
5	6	3	11.6429	14.0000	0.812415
6	2	1	5.3050	8.7806	0.314686
6	4	2	8.7806	11.6429	0.542260
6	4	3	11.6429	14.0000	0.812415
Computation Time (sec):		**0.483**	**Objective Function Value:**		**5.759918**

Table 193 – Linear Model Solution for Scenario 315

Linear Model - 315					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	3	11.6429	14.0000	0.903081
1	3	1	1.0847	5.3050	0.251587
1	6	2	5.3050	8.7806	0.406880
2	1	2	8.7806	11.6429	0.571298
2	4	3	11.6429	14.0000	0.824273
2	5	1	5.3050	8.7806	0.361039
3	1	1	5.3050	8.7806	0.314686
3	3	3	8.7806	11.6429	0.542260
3	5	2	8.7806	11.6429	0.542260
4	2	1	11.6429	14.0000	0.903081
4	6	1	1.0847	5.3050	0.251587
4	6	3	8.7806	11.6429	0.658362
5	3	4	11.6429	14.0000	0.824273
5	4	1	5.3050	8.7806	0.361039
5	5	3	11.6429	14.0000	0.824273
6	3	2	5.3050	8.7806	0.314686
6	4	2	8.7806	11.6429	0.542260
6	6	4	11.6429	14.0000	0.812415
Computation Time (sec):		**0.358**	**Objective Function Value:**		**3.831494**

Table 194 – Linear Model Solution for Scenario 316

Linear Model - 316					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
1	1	3	11.6429	14.0000	0.903081
1	3	1	1.0847	5.3050	0.251587
1	6	2	5.3050	8.7806	0.406880
2	1	2	8.7806	11.6429	0.571298
2	4	3	11.6429	14.0000	0.824273
2	5	1	5.3050	8.7806	0.361039
3	1	1	5.3050	8.7806	0.314686
3	3	3	8.7806	11.6429	0.542260
3	5	2	8.7806	11.6429	0.542260
4	2	1	11.6429	14.0000	0.903081
4	6	1	1.0847	5.3050	0.251587
4	6	3	8.7806	11.6429	0.658362
5	3	4	11.6429	14.0000	0.824273
5	4	1	5.3050	8.7806	0.361039
5	5	3	11.6429	14.0000	0.824273
6	3	2	5.3050	8.7806	0.314686
6	4	2	8.7806	11.6429	0.542260
6	6	4	11.6429	14.0000	0.812415
Computation Time (sec):		**0.421**	**Objective Function Value:**		**5.775731**

Table 195 – Construction Heuristic Solution for Scenario 301

Construction Heuristic - 301					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	5.3050	8.7806	0.314686
3	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
6	2	1	1.0847	5.3050	0.158464
5	2	2	5.3050	8.7806	0.314686
3	2	3	8.7806	11.6429	0.542261
1	2	4	11.6429	14.0000	0.812418
4	3	1	8.7806	11.6429	0.542261
2	3	2	11.6429	14.0000	0.812418
6	4	1	5.3050	8.7806	0.314686
4	4	2	8.7806	11.6429	0.542261
2	4	3	11.6429	14.0000	0.812418
6	5	1	1.0847	5.3050	0.158464
5	5	2	5.3050	8.7806	0.314686
3	5	3	8.7806	11.6429	0.542261
1	5	4	11.6429	14.0000	0.812418
4	6	1	8.7806	11.6429	0.542261
2	6	2	11.6429	14.0000	0.812418
Computation Time (sec):		**0.073**	**Objective Function Value:**		**3.807231**

Table 196 – Construction Heuristic Solution for Scenario 302

Construction Heuristic - 302					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	5.3050	8.7806	0.314686
3	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
6	2	1	1.0847	5.3050	0.158464
5	2	2	5.3050	8.7806	0.314686
3	2	3	8.7806	11.6429	0.542261
1	2	4	11.6429	14.0000	0.812418
4	3	1	8.7806	11.6429	0.542261
2	3	2	11.6429	14.0000	0.812418
6	4	1	5.3050	8.7806	0.314686
4	4	2	8.7806	11.6429	0.542261
2	4	3	11.6429	14.0000	0.812418
6	5	1	1.0847	5.3050	0.158464
5	5	2	5.3050	8.7806	0.314686
3	5	3	8.7806	11.6429	0.542261
1	5	4	11.6429	14.0000	0.812418
4	6	1	8.7806	11.6429	0.542261
2	6	2	11.6429	14.0000	0.812418
Computation Time (sec):		**0.043**	**Objective Function Value:**		**5.766357**

Table 197 – Construction Heuristic Solution for Scenario 303

Construction Heuristic - 303					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	5.3050	8.7806	0.314686
3	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
6	2	1	1.0847	5.3050	0.158464
5	2	2	5.3050	8.7806	0.314686
3	2	3	8.7806	11.6429	0.542261
1	2	4	11.6429	14.0000	0.812418
4	3	1	8.7806	11.6429	0.542261
2	3	2	11.6429	14.0000	0.812418
6	4	1	5.3050	8.7806	0.314686
4	4	2	8.7806	11.6429	0.542261
2	4	3	11.6429	14.0000	0.812418
6	5	1	1.0847	5.3050	0.158464
5	5	2	5.3050	8.7806	0.314686
3	5	3	8.7806	11.6429	0.542261
1	5	4	11.6429	14.0000	0.812418
4	6	1	8.7806	11.6429	0.542261
2	6	2	11.6429	14.0000	0.812418
Computation Time (sec):		**0.052**	**Objective Function Value:**		**3.931997**

Table 198 – Construction Heuristic Solution for Scenario 304

Construction Heuristic - 304					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	5.3050	8.7806	0.314686
3	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
6	2	1	1.0847	5.3050	0.158464
5	2	2	5.3050	8.7806	0.314686
3	2	3	8.7806	11.6429	0.542261
1	2	4	11.6429	14.0000	0.812418
4	3	1	8.7806	11.6429	0.542261
2	3	2	11.6429	14.0000	0.812418
6	4	1	5.3050	8.7806	0.314686
4	4	2	8.7806	11.6429	0.542261
2	4	3	11.6429	14.0000	0.812418
6	5	1	1.0847	5.3050	0.158464
5	5	2	5.3050	8.7806	0.314686
3	5	3	8.7806	11.6429	0.542261
1	5	4	11.6429	14.0000	0.812418
4	6	1	8.7806	11.6429	0.542261
2	6	2	11.6429	14.0000	0.812418
Computation Time (sec):		**0.053**	**Objective Function Value:**		**5.915191**

Table 199 – Construction Heuristic Solution for Scenario 305

Construction Heuristic - 305					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	5.3050	8.7806	0.314686
3	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
6	2	1	1.0847	5.3050	0.158464
5	2	2	5.3050	8.7806	0.314686
3	2	3	8.7806	11.6429	0.542261
1	2	4	11.6429	14.0000	0.812418
4	3	1	8.7806	11.6429	0.542261
2	3	2	11.6429	14.0000	0.812418
6	4	1	5.3050	8.7806	0.314686
4	4	2	8.7806	11.6429	0.542261
2	4	3	11.6429	14.0000	0.812418
6	5	1	1.0847	5.3050	0.158464
5	5	2	5.3050	8.7806	0.314686
3	5	3	8.7806	11.6429	0.542261
1	5	4	11.6429	14.0000	0.812418
4	6	1	8.7806	11.6429	0.542261
2	6	2	11.6429	14.0000	0.812418
Computation Time (sec):		**0.048**	**Objective Function Value:**		**3.900042**

Table 200 – Construction Heuristic Solution for Scenario 306

Construction Heuristic - 306					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	5.3050	8.7806	0.314686
3	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
6	2	1	1.0847	5.3050	0.158464
5	2	2	5.3050	8.7806	0.314686
3	2	3	8.7806	11.6429	0.542261
1	2	4	11.6429	14.0000	0.812418
4	3	1	8.7806	11.6429	0.542261
2	3	2	11.6429	14.0000	0.812418
6	4	1	5.3050	8.7806	0.314686
4	4	2	8.7806	11.6429	0.542261
2	4	3	11.6429	14.0000	0.812418
6	5	1	1.0847	5.3050	0.158464
5	5	2	5.3050	8.7806	0.314686
3	5	3	8.7806	11.6429	0.542261
1	5	4	11.6429	14.0000	0.812418
4	6	1	8.7806	11.6429	0.542261
2	6	2	11.6429	14.0000	0.812418
Computation Time (sec):		**0.047**	**Objective Function Value:**		**5.878856**

Table 201 – Construction Heuristic Solution for Scenario 307

Construction Heuristic - 307					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	5.3050	8.7806	0.314686
3	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
6	2	1	1.0847	5.3050	0.158464
5	2	2	5.3050	8.7806	0.314686
3	2	3	8.7806	11.6429	0.542261
1	2	4	11.6429	14.0000	0.812418
4	3	1	8.7806	11.6429	0.542261
2	3	2	11.6429	14.0000	0.812418
6	4	1	5.3050	8.7806	0.314686
4	4	2	8.7806	11.6429	0.542261
2	4	3	11.6429	14.0000	0.812418
6	5	1	1.0847	5.3050	0.158464
5	5	2	5.3050	8.7806	0.314686
3	5	3	8.7806	11.6429	0.542261
1	5	4	11.6429	14.0000	0.812418
4	6	1	8.7806	11.6429	0.542261
2	6	2	11.6429	14.0000	0.812418
Computation Time (sec):		**0.052**	**Objective Function Value:**		**3.906359**

Table 202 – Construction Heuristic Solution for Scenario 308

Construction Heuristic - 308					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	5.3050	8.7806	0.314686
3	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
6	2	1	1.0847	5.3050	0.158464
5	2	2	5.3050	8.7806	0.314686
3	2	3	8.7806	11.6429	0.542261
1	2	4	11.6429	14.0000	0.812418
4	3	1	8.7806	11.6429	0.542261
2	3	2	11.6429	14.0000	0.812418
6	4	1	5.3050	8.7806	0.314686
4	4	2	8.7806	11.6429	0.542261
2	4	3	11.6429	14.0000	0.812418
6	5	1	1.0847	5.3050	0.158464
5	5	2	5.3050	8.7806	0.314686
3	5	3	8.7806	11.6429	0.542261
1	5	4	11.6429	14.0000	0.812418
4	6	1	8.7806	11.6429	0.542261
2	6	2	11.6429	14.0000	0.812418
Computation Time (sec):		**0.048**	**Objective Function Value:**		**5.886513**

Table 203 – Construction Heuristic Solution for Scenario 309

Construction Heuristic - 309					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	5.3050	8.7806	0.314686
3	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
6	2	1	1.0847	5.3050	0.158464
5	2	2	5.3050	8.7806	0.314686
3	2	3	8.7806	11.6429	0.542261
1	2	4	11.6429	14.0000	0.812418
4	3	1	8.7806	11.6429	0.542261
2	3	2	11.6429	14.0000	0.812418
6	4	1	5.3050	8.7806	0.314686
4	4	2	8.7806	11.6429	0.542261
2	4	3	11.6429	14.0000	0.812418
6	5	1	1.0847	5.3050	0.158464
5	5	2	5.3050	8.7806	0.314686
3	5	3	8.7806	11.6429	0.542261
1	5	4	11.6429	14.0000	0.812418
4	6	1	8.7806	11.6429	0.542261
2	6	2	11.6429	14.0000	0.812418
Computation Time (sec):		**0.049**	**Objective Function Value:**		**3.789894**

Table 204 – Construction Heuristic Solution for Scenario 310

Construction Heuristic - 310					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	5.3050	8.7806	0.314686
3	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
6	2	1	1.0847	5.3050	0.158464
5	2	2	5.3050	8.7806	0.314686
3	2	3	8.7806	11.6429	0.542261
1	2	4	11.6429	14.0000	0.812418
4	3	1	8.7806	11.6429	0.542261
2	3	2	11.6429	14.0000	0.812418
6	4	1	5.3050	8.7806	0.314686
4	4	2	8.7806	11.6429	0.542261
2	4	3	11.6429	14.0000	0.812418
6	5	1	1.0847	5.3050	0.158464
5	5	2	5.3050	8.7806	0.314686
3	5	3	8.7806	11.6429	0.542261
1	5	4	11.6429	14.0000	0.812418
4	6	1	8.7806	11.6429	0.542261
2	6	2	11.6429	14.0000	0.812418
Computation Time (sec):		**0.076**	**Objective Function Value:**		**5.699358**

Table 205 – Construction Heuristic Solution for Scenario 311

Construction Heuristic - 311					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	5.3050	8.7806	0.314686
3	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
6	2	1	1.0847	5.3050	0.158464
5	2	2	5.3050	8.7806	0.314686
3	2	3	8.7806	11.6429	0.542261
1	2	4	11.6429	14.0000	0.812418
4	3	1	8.7806	11.6429	0.542261
2	3	2	11.6429	14.0000	0.812418
6	4	1	5.3050	8.7806	0.314686
4	4	2	8.7806	11.6429	0.542261
2	4	3	11.6429	14.0000	0.812418
6	5	1	1.0847	5.3050	0.158464
5	5	2	5.3050	8.7806	0.314686
3	5	3	8.7806	11.6429	0.542261
1	5	4	11.6429	14.0000	0.812418
4	6	1	8.7806	11.6429	0.542261
2	6	2	11.6429	14.0000	0.812418
Computation Time (sec):		**0.072**	**Objective Function Value:**		**3.923350**

Table 206 – Construction Heuristic Solution for Scenario 312

Construction Heuristic - 312					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	5.3050	8.7806	0.314686
3	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
6	2	1	1.0847	5.3050	0.158464
5	2	2	5.3050	8.7806	0.314686
3	2	3	8.7806	11.6429	0.542261
1	2	4	11.6429	14.0000	0.812418
4	3	1	8.7806	11.6429	0.542261
2	3	2	11.6429	14.0000	0.812418
6	4	1	5.3050	8.7806	0.314686
4	4	2	8.7806	11.6429	0.542261
2	4	3	11.6429	14.0000	0.812418
6	5	1	1.0847	5.3050	0.158464
5	5	2	5.3050	8.7806	0.314686
3	5	3	8.7806	11.6429	0.542261
1	5	4	11.6429	14.0000	0.812418
4	6	1	8.7806	11.6429	0.542261
2	6	2	11.6429	14.0000	0.812418
Computation Time (sec):		**0.063**	**Objective Function Value:**		**5.887013**

Table 207 – Construction Heuristic Solution for Scenario 313

Construction Heuristic - 313					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	5.3050	8.7806	0.314686
3	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
6	2	1	1.0847	5.3050	0.158464
5	2	2	5.3050	8.7806	0.314686
3	2	3	8.7806	11.6429	0.542261
1	2	4	11.6429	14.0000	0.812418
4	3	1	8.7806	11.6429	0.542261
2	3	2	11.6429	14.0000	0.812418
6	4	1	5.3050	8.7806	0.314686
4	4	2	8.7806	11.6429	0.542261
2	4	3	11.6429	14.0000	0.812418
6	5	1	1.0847	5.3050	0.158464
5	5	2	5.3050	8.7806	0.314686
3	5	3	8.7806	11.6429	0.542261
1	5	4	11.6429	14.0000	0.812418
4	6	1	8.7806	11.6429	0.542261
2	6	2	11.6429	14.0000	0.812418
Computation Time (sec):		**0.061**	**Objective Function Value:**		**3.891016**

Table 208 – Construction Heuristic Solution for Scenario 314

Construction Heuristic - 314					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	5.3050	8.7806	0.314686
3	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
6	2	1	1.0847	5.3050	0.158464
5	2	2	5.3050	8.7806	0.314686
3	2	3	8.7806	11.6429	0.542261
1	2	4	11.6429	14.0000	0.812418
4	3	1	8.7806	11.6429	0.542261
2	3	2	11.6429	14.0000	0.812418
6	4	1	5.3050	8.7806	0.314686
4	4	2	8.7806	11.6429	0.542261
2	4	3	11.6429	14.0000	0.812418
6	5	1	1.0847	5.3050	0.158464
5	5	2	5.3050	8.7806	0.314686
3	5	3	8.7806	11.6429	0.542261
1	5	4	11.6429	14.0000	0.812418
4	6	1	8.7806	11.6429	0.542261
2	6	2	11.6429	14.0000	0.812418
Computation Time (sec):		**0.057**	**Objective Function Value:**		**5.844042**

Table 209 – Construction Heuristic Solution for Scenario 315

Construction Heuristic - 315					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	5.3050	8.7806	0.314686
3	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
6	2	1	1.0847	5.3050	0.158464
5	2	2	5.3050	8.7806	0.314686
3	2	3	8.7806	11.6429	0.542261
1	2	4	11.6429	14.0000	0.812418
4	3	1	8.7806	11.6429	0.542261
2	3	2	11.6429	14.0000	0.812418
6	4	1	5.3050	8.7806	0.314686
4	4	2	8.7806	11.6429	0.542261
2	4	3	11.6429	14.0000	0.812418
6	5	1	1.0847	5.3050	0.158464
5	5	2	5.3050	8.7806	0.314686
3	5	3	8.7806	11.6429	0.542261
1	5	4	11.6429	14.0000	0.812418
4	6	1	8.7806	11.6429	0.542261
2	6	2	11.6429	14.0000	0.812418
Computation Time (sec):		**0.058**	**Objective Function Value:**		**3.897902**

Table 210 – Construction Heuristic Solution for Scenario 316

Construction Heuristic - 316					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	5.3050	8.7806	0.314686
3	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
6	2	1	1.0847	5.3050	0.158464
5	2	2	5.3050	8.7806	0.314686
3	2	3	8.7806	11.6429	0.542261
1	2	4	11.6429	14.0000	0.812418
4	3	1	8.7806	11.6429	0.542261
2	3	2	11.6429	14.0000	0.812418
6	4	1	5.3050	8.7806	0.314686
4	4	2	8.7806	11.6429	0.542261
2	4	3	11.6429	14.0000	0.812418
6	5	1	1.0847	5.3050	0.158464
5	5	2	5.3050	8.7806	0.314686
3	5	3	8.7806	11.6429	0.542261
1	5	4	11.6429	14.0000	0.812418
4	6	1	8.7806	11.6429	0.542261
2	6	2	11.6429	14.0000	0.812418
Computation Time (sec):		**0.058**	**Objective Function Value:**		**5.853895**

Table 211 – Improvement Heuristic Solution for Scenario 301

Improvement Heuristic - 301					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	5.3050	8.7806	0.314686
3	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.812418
6	2	1	1.0847	5.3050	0.158464
5	2	2	5.3050	8.7806	0.314686
3	2	3	8.7806	11.6429	0.542261
1	2	4	11.6429	14.0000	0.812418
4	3	1	8.7806	11.6429	0.542261
2	3	2	11.6429	14.0000	0.812418
6	4	1	5.3050	8.7806	0.314686
4	4	2	8.7806	11.6429	0.542261
2	4	3	11.6429	14.0000	0.812418
6	5	1	1.0847	5.3050	0.158464
5	5	2	5.3050	8.7806	0.314686
3	5	3	8.7806	11.6429	0.542261
1	5	4	11.6429	14.0000	0.812418
4	6	1	8.7806	11.6429	0.542261
2	6	2	11.6429	14.0000	0.812418
Computation Time (sec):		**0.567**	**Objective Function Value:**		**3.807252**

Table 212 – Improvement Heuristic Solution for Scenario 302

Improvement Heuristic - 302					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
6	1	1	5.3050	8.7806	0.314686
4	1	2	8.7806	11.6429	0.542261
2	1	3	11.6429	14.0000	0.812418
5	2	1	5.3050	8.7806	0.314686
3	2	2	8.7806	11.6429	0.542261
1	2	3	11.6429	14.0000	0.812418
5	3	1	5.3050	8.7806	0.314686
3	3	2	8.7806	11.6429	0.542261
1	3	3	11.6429	14.0000	0.812418
6	4	1	5.3050	8.7806	0.314686
4	4	2	8.7806	11.6429	0.542261
2	4	3	11.6429	14.0000	0.812418
6	5	1	5.3050	8.7806	0.314686
4	5	2	8.7806	11.6429	0.542261
2	5	3	11.6429	14.0000	0.812418
5	6	1	5.3050	8.7806	0.314686
3	6	2	8.7806	11.6429	0.542261
1	6	3	11.6429	14.0000	0.812418
Computation Time (sec):		**1.682**	**Objective Function Value:**		**5.766357**

Table 213 – Improvement Heuristic Solution for Scenario 303

Improvement Heuristic - 303					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.361039
1	1	2	8.7806	11.6429	0.658363
1	1	3	11.6429	14.0000	0.903086
4	2	1	5.3050	8.7806	0.406880
2	2	2	8.7806	11.6429	0.658363
2	2	3	11.6429	14.0000	0.903086
6	3	1	8.7806	11.6429	0.658363
6	3	2	11.6429	14.0000	0.903086
6	4	1	1.0847	5.3050	0.222265
4	4	2	5.3050	8.7806	0.406880
3	4	3	8.7806	11.6429	0.658363
3	4	4	11.6429	14.0000	0.903086
5	5	1	5.3050	8.7806	0.406880
5	5	2	8.7806	11.6429	0.658363
5	5	3	11.6429	14.0000	0.903086
4	6	1	5.3050	8.7806	0.361039
2	6	2	8.7806	11.6429	0.571296
1	6	3	11.6429	14.0000	0.903086
Computation Time (sec):		**2.690**	**Objective Function Value:**		**3.932498**

Table 214 – Improvement Heuristic Solution for Scenario 304

Improvement Heuristic - 304					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	5.3050	8.7806	0.361039
3	1	2	8.7806	11.6429	0.571296
1	1	3	11.6429	14.0000	0.903086
4	2	1	5.3050	8.7806	0.406880
2	2	2	8.7806	11.6429	0.658363
2	2	3	11.6429	14.0000	0.903086
6	3	1	5.3050	8.7806	0.406880
6	3	2	8.7806	11.6429	0.658363
6	3	3	11.6429	14.0000	0.903086
4	4	1	5.3050	8.7806	0.406880
3	4	2	8.7806	11.6429	0.658363
3	4	3	11.6429	14.0000	0.903086
4	5	1	5.3050	8.7806	0.314686
5	5	2	8.7806	11.6429	0.658363
5	5	3	11.6429	14.0000	0.903086
2	6	1	5.3050	8.7806	0.361039
1	6	2	8.7806	11.6429	0.658363
1	6	3	11.6429	14.0000	0.903086
Computation Time (sec):		**1.046**	**Objective Function Value:**		**5.915191**

Table 215 – Improvement Heuristic Solution for Scenario 305

Improvement Heuristic - 305					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.542261
1	1	3	11.6429	14.0000	0.903086
5	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542261
1	2	3	11.6429	14.0000	0.903086
3	3	1	8.7806	11.6429	0.542261
4	3	2	11.6429	14.0000	0.903086
6	4	1	1.0847	5.3050	0.158464
6	4	2	5.3050	8.7806	0.314686
3	4	3	8.7806	11.6429	0.542261
4	4	4	11.6429	14.0000	0.903086
6	5	1	1.0847	5.3050	0.158464
5	5	2	5.3050	8.7806	0.314686
2	5	3	8.7806	11.6429	0.542261
1	5	4	11.6429	14.0000	0.903086
3	6	1	8.7806	11.6429	0.542261
4	6	2	11.6429	14.0000	0.903086
Computation Time (sec):		**2.019**	**Objective Function Value:**		**3.900048**

Table 216 – Improvement Heuristic Solution for Scenario 306

Improvement Heuristic - 306					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
6	1	1	5.3050	8.7806	0.314686
3	1	2	8.7806	11.6429	0.542261
4	1	3	11.6429	14.0000	0.903086
5	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542261
1	2	3	11.6429	14.0000	0.903086
5	3	1	5.3050	8.7806	0.314686
2	3	2	8.7806	11.6429	0.542261
1	3	3	11.6429	14.0000	0.903086
6	4	1	5.3050	8.7806	0.314686
3	4	2	8.7806	11.6429	0.542261
4	4	3	11.6429	14.0000	0.903086
6	5	1	5.3050	8.7806	0.314686
3	5	2	8.7806	11.6429	0.542261
4	5	3	11.6429	14.0000	0.903086
5	6	1	5.3050	8.7806	0.314686
2	6	2	8.7806	11.6429	0.542261
1	6	3	11.6429	14.0000	0.903086
Computation Time (sec):		**2.612**	**Objective Function Value:**		**5.878856**

Table 217 – Improvement Heuristic Solution for Scenario 307

Improvement Heuristic - 307					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.314686
2	1	2	8.7806	11.6429	0.571296
1	1	3	11.6429	14.0000	0.903086
3	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.571296
1	2	3	11.6429	14.0000	0.903086
5	3	1	8.7806	11.6429	0.571296
4	3	2	11.6429	14.0000	0.903086
6	4	1	1.0847	5.3050	0.158464
5	4	2	5.3050	8.7806	0.361039
6	4	3	8.7806	11.6429	0.542261
4	4	4	11.6429	14.0000	0.903086
6	5	1	1.0847	5.3050	0.158464
3	5	2	5.3050	8.7806	0.314686
2	5	3	8.7806	11.6429	0.571296
1	5	4	11.6429	14.0000	0.903086
5	6	1	8.7806	11.6429	0.571296
4	6	2	11.6429	14.0000	0.903086
Computation Time (sec):		**1.554**	**Objective Function Value:**		**3.906417**

Table 218 – Improvement Heuristic Solution for Scenario 308

Improvement Heuristic - 308					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
6	1	1	5.3050	8.7806	0.314686
5	1	2	8.7806	11.6429	0.571296
4	1	3	11.6429	14.0000	0.903086
3	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.571296
1	2	3	11.6429	14.0000	0.903086
3	3	1	5.3050	8.7806	0.314686
2	3	2	8.7806	11.6429	0.571296
1	3	3	11.6429	14.0000	0.903086
6	4	1	5.3050	8.7806	0.314686
5	4	2	8.7806	11.6429	0.571296
4	4	3	11.6429	14.0000	0.903086
6	5	1	5.3050	8.7806	0.314686
5	5	2	8.7806	11.6429	0.571296
4	5	3	11.6429	14.0000	0.903086
3	6	1	5.3050	8.7806	0.314686
2	6	2	8.7806	11.6429	0.571296
1	6	3	11.6429	14.0000	0.903086
Computation Time (sec):		**2.090**	**Objective Function Value:**		**5.886513**

Table 219 – Improvement Heuristic Solution for Scenario 309

Improvement Heuristic - 309					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	5.3050	8.7806	0.314686
4	1	2	8.7806	11.6429	0.542261
2	1	3	11.6429	14.0000	0.812418
5	2	1	5.3050	8.7806	0.314686
3	2	2	8.7806	11.6429	0.542261
1	2	3	11.6429	14.0000	0.812418
6	3	1	5.3050	8.7806	0.314686
3	3	2	8.7806	11.6429	0.542261
1	3	3	11.6429	14.0000	0.812418
6	4	1	5.3050	8.7806	0.314686
4	4	2	8.7806	11.6429	0.542261
2	4	3	11.6429	14.0000	0.812418
5	5	1	5.3050	8.7806	0.314686
3	5	2	8.7806	11.6429	0.542261
2	5	3	11.6429	14.0000	0.812418
6	6	1	5.3050	8.7806	0.314686
4	6	2	8.7806	11.6429	0.542261
1	6	3	11.6429	14.0000	0.812418
Computation Time (sec):		**2.331**	**Objective Function Value:**		**3.789894**

Table 220 – Improvement Heuristic Solution for Scenario 310

Improvement Heuristic - 310					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	8.7806	11.6429	0.542261
2	1	2	11.6429	14.0000	0.812418
3	2	1	8.7806	11.6429	0.542261
1	2	2	11.6429	14.0000	0.812418
5	3	1	1.0847	5.3050	0.158464
4	3	2	5.3050	8.7806	0.314686
3	3	3	8.7806	11.6429	0.542261
1	3	4	11.6429	14.0000	0.812418
5	4	1	5.3050	8.7806	0.314686
2	4	2	8.7806	11.6429	0.542261
6	4	3	11.6429	14.0000	0.812418
6	5	1	5.3050	8.7806	0.314686
4	5	2	8.7806	11.6429	0.542261
2	5	3	11.6429	14.0000	0.812418
6	6	1	1.0847	5.3050	0.158464
4	6	2	5.3050	8.7806	0.314686
3	6	3	8.7806	11.6429	0.542261
1	6	4	11.6429	14.0000	0.812418
Computation Time (sec):		**1.761**	**Objective Function Value:**		**5.699541**

Table 221 – Improvement Heuristic Solution for Scenario 311

Improvement Heuristic - 311					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	5.3050	8.7806	0.361039
3	1	2	8.7806	11.6429	0.571296
1	1	3	11.6429	14.0000	0.903086
2	2	1	5.3050	8.7806	0.406880
2	2	2	8.7806	11.6429	0.658363
2	2	3	11.6429	14.0000	0.903086
6	3	1	5.3050	8.7806	0.406880
6	3	2	8.7806	11.6429	0.658363
6	3	3	11.6429	14.0000	0.903086
4	4	1	5.3050	8.7806	0.406880
3	4	2	8.7806	11.6429	0.658363
3	4	3	11.6429	14.0000	0.903086
4	5	1	5.3050	8.7806	0.314686
5	5	2	8.7806	11.6429	0.658363
5	5	3	11.6429	14.0000	0.903086
4	6	1	5.3050	8.7806	0.361039
1	6	2	8.7806	11.6429	0.658363
1	6	3	11.6429	14.0000	0.903086
Computation Time (sec):		**1.035**	**Objective Function Value:**		**3.923350**

Table 222 – Improvement Heuristic Solution for Scenario 312

Improvement Heuristic - 312					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	5.3050	8.7806	0.361039
3	1	2	8.7806	11.6429	0.571296
1	1	3	11.6429	14.0000	0.903086
4	2	1	5.3050	8.7806	0.406880
2	2	2	8.7806	11.6429	0.658363
2	2	3	11.6429	14.0000	0.903086
6	3	1	5.3050	8.7806	0.406880
6	3	2	8.7806	11.6429	0.658363
6	3	3	11.6429	14.0000	0.903086
3	4	1	8.7806	11.6429	0.658363
3	4	2	11.6429	14.0000	0.903086
4	5	1	5.3050	8.7806	0.314686
5	5	2	8.7806	11.6429	0.658363
5	5	3	11.6429	14.0000	0.903086
4	6	1	1.0847	5.3050	0.222265
2	6	2	5.3050	8.7806	0.361039
1	6	3	8.7806	11.6429	0.658363
1	6	4	11.6429	14.0000	0.903086
Computation Time (sec):		**2.441**	**Objective Function Value:**		**5.891579**

Table 223 – Improvement Heuristic Solution for Scenario 313

Improvement Heuristic - 313					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	5.3050	8.7806	0.314686
3	1	2	8.7806	11.6429	0.542261
4	1	3	11.6429	14.0000	0.903086
5	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.542261
1	2	3	11.6429	14.0000	0.903086
6	3	1	5.3050	8.7806	0.314686
2	3	2	8.7806	11.6429	0.542261
1	3	3	11.6429	14.0000	0.903086
6	4	1	5.3050	8.7806	0.314686
3	4	2	8.7806	11.6429	0.542261
4	4	3	11.6429	14.0000	0.903086
5	5	1	5.3050	8.7806	0.314686
2	5	2	8.7806	11.6429	0.542261
4	5	3	11.6429	14.0000	0.903086
6	6	1	5.3050	8.7806	0.314686
3	6	2	8.7806	11.6429	0.542261
1	6	3	11.6429	14.0000	0.903086
Computation Time (sec):		**0.985**	**Objective Function Value:**		**3.891016**

Table 224 – Improvement Heuristic Solution for Scenario 314

Improvement Heuristic - 314					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	8.7806	11.6429	0.542261
4	1	2	11.6429	14.0000	0.903086
2	2	1	8.7806	11.6429	0.542261
1	2	2	11.6429	14.0000	0.903086
5	3	1	1.0847	5.3050	0.158464
3	3	2	5.3050	8.7806	0.314686
2	3	3	8.7806	11.6429	0.542261
1	3	4	11.6429	14.0000	0.903086
6	4	1	5.3050	8.7806	0.314686
5	4	2	8.7806	11.6429	0.542261
4	4	3	11.6429	14.0000	0.903086
6	5	1	5.3050	8.7806	0.314686
3	5	2	8.7806	11.6429	0.542261
4	5	3	11.6429	14.0000	0.903086
6	6	1	1.0847	5.3050	0.158464
3	6	2	5.3050	8.7806	0.314686
2	6	3	8.7806	11.6429	0.542261
1	6	4	11.6429	14.0000	0.903086
Computation Time (sec):		**2.786**	**Objective Function Value:**		**5.844091**

Table 225 – Improvement Heuristic Solution for Scenario 315

Improvement Heuristic - 315					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
3	1	1	5.3050	8.7806	0.314686
5	1	2	8.7806	11.6429	0.571296
4	1	3	11.6429	14.0000	0.903086
3	2	1	5.3050	8.7806	0.314686
2	2	2	8.7806	11.6429	0.571296
1	2	3	11.6429	14.0000	0.903086
6	3	1	5.3050	8.7806	0.314686
2	3	2	8.7806	11.6429	0.571296
1	3	3	11.6429	14.0000	0.903086
6	4	1	5.3050	8.7806	0.314686
5	4	2	8.7806	11.6429	0.571296
4	4	3	11.6429	14.0000	0.903086
3	5	1	5.3050	8.7806	0.314686
2	5	2	8.7806	11.6429	0.571296
4	5	3	11.6429	14.0000	0.903086
6	6	1	5.3050	8.7806	0.314686
5	6	2	8.7806	11.6429	0.571296
1	6	3	11.6429	14.0000	0.903086
Computation Time (sec):		**1.045**	**Objective Function Value:**		**3.897902**

Table 226 – Improvement Heuristic Solution for Scenario 316

Improvement Heuristic - 316					
Weapon	**Threat**	**Round**	**Eng. Start**	**Eng. End**	**Probability**
5	1	1	8.7806	11.6429	0.571296
4	1	2	11.6429	14.0000	0.903086
2	2	1	8.7806	11.6429	0.571296
1	2	2	11.6429	14.0000	0.903086
3	3	1	1.0847	5.3050	0.158464
3	3	2	5.3050	8.7806	0.314686
2	3	3	8.7806	11.6429	0.571296
1	3	4	11.6429	14.0000	0.903086
5	4	1	5.3050	8.7806	0.361039
6	4	2	8.7806	11.6429	0.542261
4	4	3	11.6429	14.0000	0.903086
6	5	1	5.3050	8.7806	0.314686
5	5	2	8.7806	11.6429	0.571296
4	5	3	11.6429	14.0000	0.903086
6	6	1	1.0847	5.3050	0.158464
3	6	2	5.3050	8.7806	0.314686
2	6	3	8.7806	11.6429	0.571296
1	6	4	11.6429	14.0000	0.903086
Computation Time (sec):		**2.365**	**Objective Function Value:**		**5.854001**