AN APPRAISAL OF CURVILINEAR FORMS IN ARCHITECTURE WITH AN EMPHASIS ON STRUCTURAL BEHAVIOUR: A CASE STUDY ON CHANNEL TUNNEL RAILWAY TERMINAL AT WATERLOO

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Prof. Dr. Canan Özgen
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

Assoc. Prof. Dr. Selahattin Önür
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Assoc. Prof. Dr. Ali İhsan Ünay
Supervisor

Examining Committee Members

Dr. Erhan Karaesmen (METU, ARCH)

Assoc. Prof. Dr. Ali İhsan Ünay (METU, ARCH)

Assist. Prof. Dr. Lale ÖZGENEL (METU, ARCH)

Assist. Prof. Dr. Ali Murat TANYER (METU, ARCH)

Assist. Prof. Dr. Saadet Toker (SDU, ARCH)

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Tuba Çıngı

ABSTRACT

AN APPRAISAL OF CURVILINEAR FORMS IN ARCHITECTURE WITH AN EMPHASIS ON STRUCTURAL BEHAVIOUR: A CASE STUDY ON CHANNEL TUNNEL RAILWAY TERMINAL AT WATERLOO

Çıngı, Tuba
M.S., Department of Architecture
Supervisor: Assoc. Prof. Dr. Ali İhsan Ünay

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Architectural curvilinear form has been on the scene since the time of the first building shelters. *Curve* is the most common form in nature. This phenomenon inspired human beings while they are building structures. Curvilinear form has developed over centuries, via structural enhancements and aesthetic tenets. A symbolic meaning is tailored to curvilinear structures such as use of *domes* in religious buildings. However, the difficulties in the construction process of these forms have been a challenge for people throughout the history. Today, introduction of computer aided design and manufacturing technologies into building industry encourages the use of curvilinear forms in architecture.

This study intends to explore the relationship between structure and architectural curvilinear form. The curvilinear form will be examined basically according to its structural potentials through its geometrical configuration. A computer model of the roof of Channel Tunnel Railway Terminal at Waterloo is generated and with some geometrical modifications for the configuration of the roof, new schemes of structures are obtained. An analytical comparison of structural behavior and efficiency is made via the computer model of the roof and these modified configurations.

Keywords: Curve, curvilinear form, structural behavior, structural modeling, Waterloo Train Terminal.

ÖZ

MİMARİDEKİ EĞRİSEL FORMLARIN YAPISAL DAVRANIŞ ÜZERİNDEN BİR DEĞERLENDİRMESİ: WATERLOO TREN TERMİNALİ ÜZERİNE BİR ÇALIŞMA

Çıngı, Tuba
Yüksek Lisans, Mimarlık Bölümü
Tez Yöneticisi: Doç. Dr. Ali İhsan Ünay

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Eğrisel mimari formlar, ilk barınaklar zamanından bu yana görülmektedir. *Eğri*, doğadaki en yaygın formdur. Bu olgu, insanoğlunu yapı inşa ederken etkilemiştir. Eğrisel formlar yüzyıllar boyunca yapısal ilerlemeler ve estetik ilkeler doğrultusunda gelişmiştir. Eğrisel yapılara, dini yapılarda *kubbe* kullanımında olduğu gibi sembolik anlamlar yüklenmiştir. Fakat bu yapıların yapım sürecindeki zorluklar, tarih boyunca insanlar için bir meydan okuma olmuştur. Günümüzde, bilgisayar destekli tasarım ve üretim teknolojilerinin yapı endüstrisine girmesi, eğrisel formların mimaride kullanımını teşvik etmektedir.

Bu çalışma, yapı ile mimari eğrisel formlar arasındaki ilişkiyi irdelemeyi amaçlamaktadır. Eğrisel form, temel olarak, geometrik kurgusu aracılığıyla yapısal potansiyellerine göre incelenecektir. Waterloo Channel Tunnel Railway Terminal yapısı çatısının sayısal modeli oluşturulacak ve çatı kurgusunda bazı geometrik değişiklikler yoluyla yeni yapısal şemalar elde edilecektir. Yapısal davranış ve verimliliğin analitik bir karşılaştırması, çatının ve bu yeni şemaların sayısal modeli yoluyla yapılacaktır.

Anahtar Kelimeler: Eğri, eğrisel form, yapısal davranış, yapısal modelleme, Waterloo Tren İstasyonu.

To,

Güney Çıngı

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INTRODUCTION

Sheltering, protection, feeding and perpetuation are considered as the most instinctive and vital components of the human life. Sheltering and protection are the two, recognizable as defining the character of the space in which the daily life would mostly take place.[1] Man has to shape certain materials and use them in certain quantities to make his building stand up against the pull of the earth, wind, earthquakes, fires etc, while he is building simple shelter for himself or enclosing large spaces for hundreds of people. Moreover, from earliest times the man has had a natural sense of beauty, so the construction was also conceived according to aesthetic theories, besides the requirements of strength and economy.

Architectural curvilinear form has been on the scene since the time of the first building shelters. “Curve” is the most common form in nature, and this inspired human beings while they are building structures. In “On Growth and Form”, D’Arcy Thompson suggests that the shapes of living things are largely the result of adaptation to physical forces, not behavior and diet.[2] Forms found in nature are shaped for maximum efficiency, transferring the required force with the optimum amount of material and human beings are often inspired by these fundamental principles of nature. They imitate the aesthetical and sometimes structural, even functional qualities that already exist in nature, to realize structures, pieces of art. In some cases wings of a bird, form of a wave or a drop of water can impress a designer. However, not only the aesthetic qualities of nature, but also structural, even functional qualities affect the designer. There are also examples of curvilinear forms created by the nature itself. The arches in nature that are formed by erosion, wind and the like are clear ex-

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[1] Karaesmen, E. Architecture and Engineering, in: *Civil News, Boğaziçi University, Department of Civil Engineering.* (pp. 1-3). 2005, May.

[2] Thompson, D’Arcy. *On growth and form.* The University Press, Cambridge. 1942.

amples that impressed human beings. Thus, the first shelters of human beings had their analogies from the nature.

Vernacular architecture, which is generally constructed of earth, is curvilinear in form by the nature of the materials and the construction techniques. The structural curved arch was discovered by early builders; there are 3200 years old mud brick storage rooms built with arches that still survive in Egypt. As civilizations developed, preference was given to linear forms, yet curved form continued to develop over the centuries, initially as structural enhancements, later as aesthetic tenets. In the East, simplicity and form were combined to create functional curvilinear structures. A symbolic meaning is proposed to curvilinear structures such as use of *domes* in religious buildings besides structural and aesthetic qualities. With the introduction of need for uninterrupted larger spaces in churches, mosques, halls, auditoriums and like, designers became more interested in curvilinear forms as a tool for long spanning. In the 19th century, new metallurgical materials led to new forms of curvilinear architecture. The later part of the century brought the Art Nouveau movement, with its fluid, curvilinear lines. Antonio Gaudi stated that “The straight line belongs to the man, the curve to God.”[3] In the 20th and 21st century, with the introduction of new materials and structural developments, outstanding, mathematically precise optimal load bearing curved forms emerged. The structural economy, achieved by the use of curvature, in fact, has been appreciated since the days of Ancient Rome. It still applies when modern materials are used, because it is ultimately necessary to offer, within the structure, a resistance moment equal to the moment of the applied loads. This requires either curvature or thickness in the horizontal structural members, and over large spans, the latter is necessarily associated with heavy weights.[4] Briefly, the curvilinear forms have been a challenge for builders throughout the history.

This study intends to explore the relationship between structure and architectural curvilinear form. It is obvious that there is no unique relationship between structure and form if we mean simply the geometrical configuration of the structure by form.

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[3] Middendorf, J. Computer aided curvilinear architectural design, in: http://www.johnmiddendorf.com /johnmfiles/UNSW/CADarch/index.htm. Last accessed in December 2006.

[4] Cowan, H. J. *An historical outline of architectural science*. Elsevier Pub. Co., New York. 1966. p. 82.

Different structural actions can be observed for the same loading. Different strengths and stiffnesses may be associated with a single external geometry according to the materials used, the internal detailing, the construction, and the scale. The geometrical configuration is, therefore, only one aspect of structural form, besides the wide range of choice of materials and constructional techniques.

In this study, the curvilinear form will be examined essentially through its structural potentials according to its geometrical configuration. The structural advantages of the curvilinear form will be explored via the computer-based structural analysis of Waterloo Station as a case study. In the thesis, the affect of the material properties and the construction techniques on the structural behavior will also be evaluated briefly with some examples though they will not be the fundamental consideration of the study.

Curvilinear forms are responsive to the leading ideals of structural art, efficiency, economy and elegance, described by Billington in “The Tower and the Bridge”, especially when there is a need of long spanning. These forms have been preferred by well-known designers throughout architectural history, who were looking for not only statical necessities, economic solutions, the ease of construction, but also were in pursuit of satisfying appearance, the “beauty” in their designs. All structural designs, both for long or short spans, are based on the same general concepts. Yet, in long-span structures, the relationship between dead weight, strength and proportions becomes critical. The design criteria have to be far more rational in order to arrive at safe, economical, and aesthetic solutions. Curvilinear forms are utilized in order to optimize these criteria.

The concept of structure is defined in chapter 2, followed by structural ideals, which are **efficiency**, **economy** and **elegance**, stated by Onouye and Billington. The relationship between structure and form is examined and simple elemental forms are introduced with an emphasis on curvilinear forms.

In chapter 3, the concept of curvature in structuring is presented. Curvatures and their mathematical background are represented as a prelude to surface creation methods.

The surface types are characterized according to their creation methods. The mathematics of basic curved forms will be analyzed to be able to understand the nature of their structural behavior.

In chapter 4, the development and utilization of curvilinear structural forms will be discussed within an historical overview. The emphasis will be mostly on structures and structural elements that have marked significant steps forward in widening the range of possible future choice, in history. A group of examples relevant to the theme of the thesis is also introduced.

Chapter 5 focuses on the comparison of structural capacity of linear and curvilinear structures by analyzing the Channel Tunnel Railway Terminal at Waterloo, as a case study. Background of the design survey of the terminal building both for architecture and for structure will be explored in accordance with the realization process. The computer model of the roof of the terminal building is generated by using Structural Analysis Program (SAP) 2000 Nonlinear 7.1 in order to understand the behavior of the structure. With some geometrical modifications for the configuration of the roof, new schemes of structures are also generated in order to make comparison to the curved structure of the roof.

The computer model of the roof in this study is inspired by the roof of Channel Tunnel Railway Terminal at Waterloo but some modifications are made during the modeling process in order to obtain a simplified model that would be appropriate solely for this study. The aim of the study is to observe the outcomes of these analytical models and make comparisons, especially from the point of their geometry, load capacity and efficiency.

CHAPTER 2

STRUCTURE AND FORM IN ARCHITECTURE

Many architects, structural engineers and designers define structure in their own words in many ways; most of these definitions however stem from the same principles. Onouye defines structure as, "something made up of interdependent parts in a definite pattern of organization (Figure 1) –an interrelation of parts as determined by the general character of the whole." He also states that "Structure, particularly in the natural world, is a way of achieving the most strength from the least material through the most appropriate arrangement of elements within a form suitable for its intended use."[1]

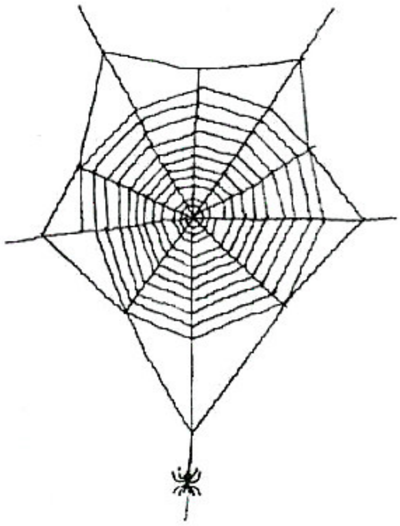

Figure 1. Radial, spiral pattern of the spider web

(Onouye, B. *Statics and strength of materials for architecture and building construction*. Prentice Hall. Upper Saddle River, NJ. 2002.)

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[1] Onouye, B. *Statics and strength of materials for architecture and building construction*. Prentice Hall. Upper Saddle River, NJ. 2002. p. 1.

The primary function of a building structure is to support and redirect loads and forces to the ground in a safe manner. Building structures are facing the forces of wind, the effects of gravity, vibrations, and earthquakes and like. The subject of the structure is all encompassing; everything has its own unique form. A cloud, a sea-shell, a tree, a grain of sand, the human body-each is a miracle of structural design. Buildings, as any other physical entity, require structural frameworks to sustain their existence in a physical form.[2]

To *structure* also means to *build* –to make use of materials in a way as to assemble an interconnected whole that creates space suitable to a particular function or functions and besides, to protect the internal space from undesirable external elements.[3]

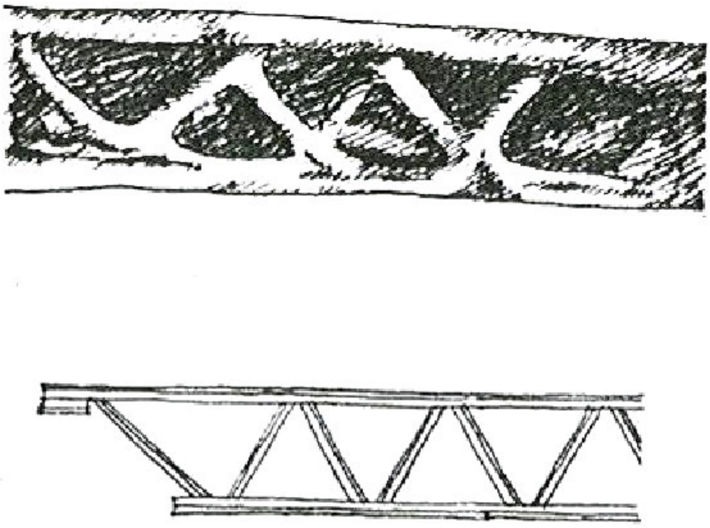

Figure 2. Metacarpal bone from a vulture wing and an open-web steel truss with web members

(Onouye, B. *Statics and strength of materials for architecture and building construction*. Prentice Hall. Upper Saddle River, NJ. 2002.)

A structure, whether large or small, must be stable and durable, must satisfy the intended functions for which it was built, and must achieve economy or efficiency (Figure 2). Structure is all about doing more with using less material to support the

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[2] Ibid. p. 2.

[3] Ibid.

given load or enclose a given volume. It is a matter of balancing structural performance with the cost of achieving it.[4] As Addis quoted from Isaac Newton's *Principia*: "Nature does nothing in vain, and more is in vain when less will serve; for Nature is pleased with simplicity, and affects not the pomp of superfluous causes."

2.1 Structural Design

Structural design is basically a process with the applied forces and the materials that resist these forces. Structurally, a building must never collapse under the action of assumed loads. Moreover, tolerable deformation of the structure should not cause material distress. Onouye briefs the general procedure of designing a structural system with the following phases:

- Conceiving of the basic structural form
- Devising the gravity and lateral force resisting strategy
- Roughly proportioning the component parts
- Developing a foundation scheme
- Determining the structural materials to be used
- Detailed proportioning of the component parts
- Devising a construction methodology

After all these phases have been examined and modified in an iterative manner, the structural elements within the system are then checked mathematically. There are no sets of rules to achieve "good design". The iterative approach is most often employed to arrive at a design solution. The structural principles influence the form of the building, and a logical solution (often an economical one) is always based on a correct interpretation of these principles.[5]

> It can be said that in these works (Gothic Cathedrals, Eiffel Tower, Firth of Froth Bridge), forerunners of great architecture of tomorrow, the relationship between technology and aesthetics that we found in the great buildings of the past has remained intact. It seems to me that this relationship can be defined

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[4] Addis, B. *The art of the structural engineer.* Artemis, London. 1994. p. 1.

[5] Onouye, B. *Statics and strength of materials for architecture and building construction*. p. 3.

> in the following manner: the objective data of the problem, technology and statics (empirical or scientific), suggest the solutions and forms; the aesthetic sensitivity of the designer, who understands the intrinsic beauty and validity, welcomes the suggestion and models it, emphasizes it, proportions it, in a personal manner which constitutes the artistic element in architecture.[6]

2.2 The Ideals of Structural Design

Karaesmen states that every building should fulfill these following requirements: **functionality**, **safety** and **economy**. The functionality is the result of an architectural engineering process in order to reach a higher level of satisfaction proceeds to the building. The safety is another basic requirement since the first and the most essential duty of a building consists to standing up properly without allowing any structurally hazarding fear and bad behavior. The economy is another concept establishing a discipline of efficiency in construction process aiming a lower cost without concession from the safety and the standard of functionality.[7]

Similarly, Onouye states that the main purpose of structural design is to make the building "stand up". In making a building stand up, the principles governing the stability and equilibrium of buildings form the basis of structural thinking.[8] According to Onouye, the principal functional requirements of a building structure are:

1. Stability and equilibrium
2. Strength and stiffness
3. Economy
4. Functionality
5. Aesthetics

The fundamental concept of **equilibrium** is concerned with balancing of forces to ensure that a building and its components will not move. In fact, all structures endure some movement under load, but stable structures have relatively small deformations.

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[6] Nervi, P. L. Aesthetics and technology in building. Harvard University Press, Cambridge. 1965.

[7] Karaesmen, E. *CE 480 Introduction to architectural engineering, Lecture notes.* Boğaziçi University, Civil Engineering Department. Istanbul. 2006.

[8] Ibid. p.10.

A “good structure” is one that achieves a condition of equilibrium with a minimum effort.[9] *Strength and stiffness* of materials are concerned with the stability of a building’s component parts, whereas *static* deals with the theory of general stability.

Economy is another fundamental in the structurally correct buildings. Two kinds of “economy” are present in buildings. One is based on expediency, availability of materials, cost, and constructability. The other “inherent” economy is dictated by the laws of nature.[10]

In *On Growth and Form*, D’Arcy Thompson described how nature creates a great diversity of forms from an inventory of basic principles, as a response to the action of forces. Thompson points out that; “...in short, the form of an object is a diagram of forces; in this sense, at least, that from it we can judge of or deduce the forces that are acting or have acted upon it; in this strict and particular sense, it is a diagram.”[11]

To form as a diagram is an important idea in the application of the principle of *optimization* (maximum output for minimum energy). A clear example of optimization is seen in the Nature; the honeycomb of the bee. These honeycombs are an arrangement of hexagonal cells containing the greatest amount of honey with the least amount of bee wax and are the structure that requires the least energy for the bees to construct.

Functionality and aesthetics are two other principal functional requirements of a building structure, which cannot be discussed separately, yet in different circumstances, one of the concepts may have dominance on the other. For instance, while designing a car park, the functionality will be the first consideration since the structural design should obey some standards of parking. There will be many criteria as for which types of vehicles will the car park will be used, the dimensions of the vehicles, the optimum dimensions for a car to turn comfortably etc. while deciding the appropriate structural system and scheme for the identified function. The main purpose of the structure in this case is to response to the needs of the function. Aesthetic

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[9] Ibid.

[10] Ibid. p. 3.

[11] Ibid. p. 4.

considerations will contribute to the design process in very later phases, in some cases it will not be a consideration for that design at all. In the design process of a monument, a symbolic structure, aesthetics will become the basic consideration and the intentions of the designer will orient the structural design. Generally the designer have the virtual image of the monument, the form, in his/her mind, then he/she will seek for the appropriate structural system to be able to make his/her design "stand up". Sometimes the materials are chosen for the monument and then the "right" structure that matches with those materials is searched for. These are generally extreme cases for the contribution of function and aesthetics in the design processes. Thus, to achieve the "good structure", the principle of *optimization* will come to the scene again.

Billington categorizes the leading ideals of structural art into three categories, which is similar to Onouye's principles: **efficiency, economy** and **elegance**. He also calls these principles as the measurements of three dimensions of structure. The first dimension of a structure is a scientific one, which is measured by **efficiency**. The second dimension is the social dimension of the structure. **Economy** measures this dimension of structure. The third dimension is symbolic, and it is this dimension that opens up the possibility for the new engineering to become structural art. Although there can be no measure for a symbolic dimension, we recognize a symbol by its **elegance and its expressive power**.[12]

Works of structural art flourish when the goals of freedom and discipline are held in balance. The disciplines of structural art are efficiency and economy, and its freedom lies in the potential it offers the designer for the expression of a personal style motivated by the conscious aesthetic search for engineering elegance. The engineers of the nineteenth century had to find ways to use it as efficiently as possible because of the great cost of new industrialized iron. They were employed to build larger and larger structures –longer-span bridges, higher towers, and wider-spanning roofs- all with less material. They tried to find the limits of structure, to make forms that would be light. They began to stretch iron, steel, and reinforced concrete, just as medieval

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[12] Billington, D. P. *The tower and the bridge: the new art of structural engineering.* Princeton University, Princeton. 1983. pp. 16-17.

designers had stretched stone into the skeletal Gothic cathedral. In Britain, the center for early structural art, public works were under the control of shareholders and industrialists. Thus, the engineer had always to work under the discipline of economy consistent with usefulness.[13] Economy has always been a prerequisite to creativity in structural art.

Minimal materials and costs may be necessary, but they are not, of course, sufficient. A third ideal must control the final design: the conscious aesthetic motivation of the designer. The elements of the new art form were, then, efficiency (minimum materials), economy (minimum cost), and elegance (maximum aesthetic expression).[14]

2.3 Structure and Form

The shape of a body is defined as the set of points that represent its observable spatial limits. These points form a line or a surface. We talk about the "form" of a body when, in addition to its geometric properties, its other aspects and properties are taken into consideration. Thus, the form of the body is the characteristic image typically associated with it.[15]

In many cases, it is acceptable to use the terms *form* and *structure* interchangeably. In some cases, however, a distinction between the two terms must be clarified. The term *form* is more suitable for an entity taken as a whole, whereas the term *structure* should be used when the whole is analyzed by its components. As far as works of architecture are concerned, a clarification of the two terms is needed. "Architectural form" can refer either to an entire building or to one of its components, as long as the part displays a certain degree of autonomy. In this manner, the ancient Greek temple, the column, and the capital are all architectural forms. The term "structure", applied to a work of architecture, refers to those elements that contribute to the architecture's strength – load-bearing walls, beams, columns and so on.[16]

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[13] Ibid. p. 5.

[14] Ibid. p. 6.

[15] Zannos, A. *Form and structure in architecture.* Van Nostrand Reinhold, New York. 1987. p. 9.

[16] Ibid.

It is obvious that there is no unique relationship between structure and form if we mean simply the geometrical configuration of the structure as the definition of the form. Different structural actions, different strengths, and different stiffnesses may be associated with a single geometry. They will vary according to the materials used, the cross-sections, the manner and sequence of construction, and the scale.[17]
The geometrical configuration is, therefore, only one aspect of structural form; yet it may still be regarded as the primary determinant. It determines whether equilibrium is possible and, if possible, it determines which types of structural action are possible and rules out others. The choices of materials, internal details, and construction techniques develop some possibilities.[18]

2.3.1 Simple Elemental Forms

The basic classification is again primarily geometrical, although it is also necessary to take into account the support provided and the manner of loading since the structural potential is being considered.

There are two main geometrical distinctions apart from the obvious one between the essentially linear forms and the rest. The first relates to curvature. Of the curved forms in Figure 3, (f) to (j) are singly curved; (k) to (m) are doubly curved with both curvatures of the same sense (known as synclastic); and (n) to (p) are doubly curved with the two curvatures of opposite senses (known as anticlastic). The second relates to cross-sectional dimensions. The forms to the left of the vertical line are of potential structural value with almost negligible thickness; and those to the right of the vertical line, though not incapable of acting in the manner assumed with an almost negligible thickness, do call, in practice for a significant thickness to guard against buckling. It will also be seen, from the point of view of the uses that might be made of them, that some forms are capable only of transmitting loads, either directly (a) and (e) or indirectly (f) and (j), but that the remainder are capable also of enclosing space.[19]

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[17] Mainstone, R. J. *Developments in structural form.* M.I.T. Press, Cambridge. 1975. p. 83.

[18] Ibid. pp. 83-84.

[19] Ibid. p. 85.

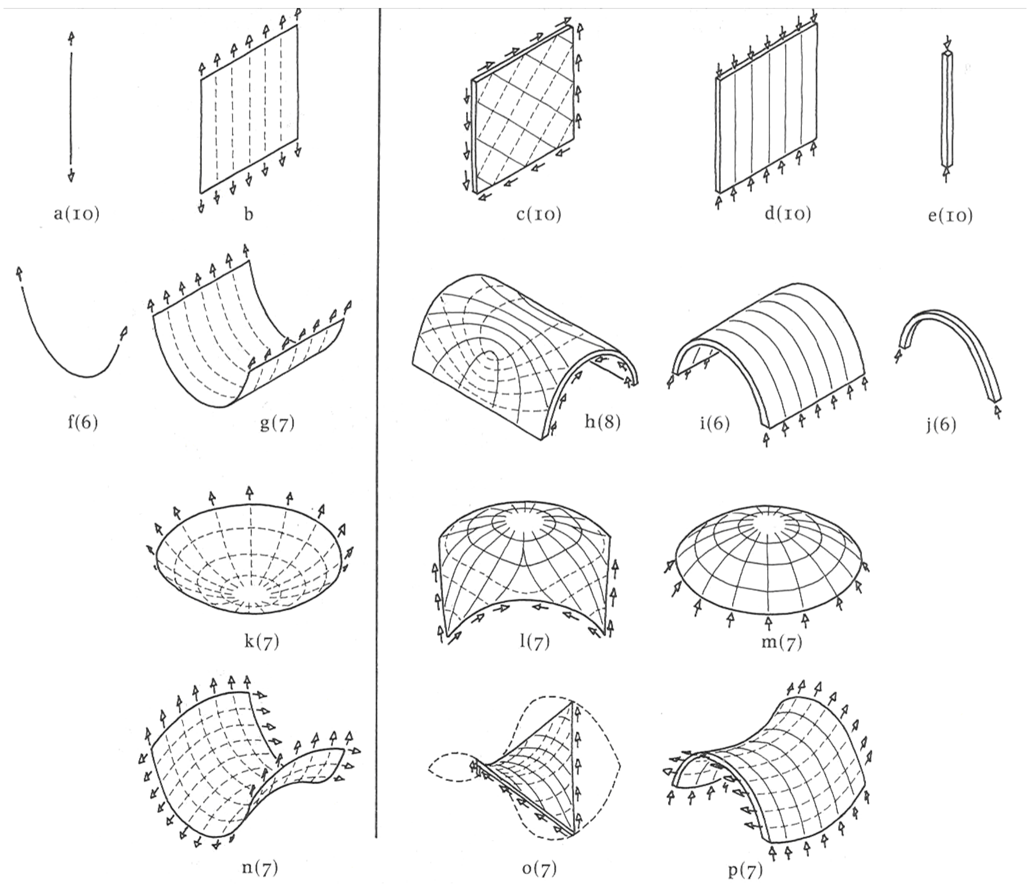

Figure 3. Elemental structural forms

(Mainstone, R. J. *Developments in structural form.* M.I.T. Press, Cambridge. 1975.)

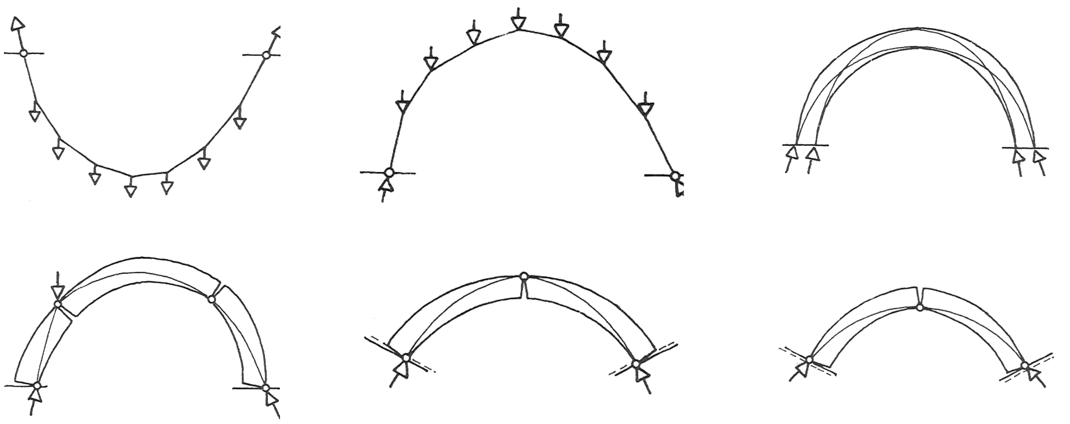

Figure 4. The primary internal actions for the loading indicated by full lines to denote compressions and broken lines to denote tensions

(Mainstone, R. J. *Developments in structural form.* M.I.T. Press, Cambridge. 1975.)

In terms of structural actions, the main distinction between those forms is their primary action whether it is purely tensile, purely compressive, or a combination of tension and compression. As in Figure 4, the primary internal actions for the loading assumed have been indicated by sketching the directions of the principal stresses, using full lines to denote compressions and broken lines to denote tensions.[20]

A further distinction relates to the way in which the tension and compression, where the primary action calls for both, are distributed over the depth or thickness of the cross-section. For all the forms shown in the figure, the primary action calls only for a uniform tension or compression throughout the thickness at any point in any given surface direction. For the forms above the horizontal line, however, the primary action, being a flexural one, calls for both tension and compression within the depth.[21]

2.3.2 Curvilinear Forms

"Curve" is the most common form in nature, and it inspired human beings since the time they were building the first structures for sheltering at the outset. As civilizations developed, besides the linear forms, curved form continues to develop over the centuries, as structural enhancements, as an aesthetic, as a symbolic or religious meaning. The need for larger spaces uninterrupted with the elements of structural system as churches, mosques, halls, auditoriums and like lead designers to be interested in curvilinear forms as a tool for long spanning.

Curvilinear forms are responsive to the leading ideals of structural art, efficiency, economy and elegance, described by Billington. As such, these forms have been preferred by well-known designers who were looking for not only statical necessities, economic solutions, the ease of construction etc but were also in search of an "aesthetic" quality for their designs, throughout the history.

Pier Luigi Nervi was one of those designers who used the yields of curvilinear forms in his buildings. The most spectacular of Nervi's buildings are certainly the domes

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[20] Ibid.

[21] Ibid. p. 86.

and barrel shells he built between 1935 and 1959. These are also the works that best illustrate Nervi's preoccupation with very simple overall shapes made up on interplay of individual elements.

As Nervi stated "My early experiences had formed in me a habit of searching for solutions that were intrinsically and constructionally the most economic, a habit which the many succeeding competitions tenders (almost the totality of my projects) have only succeeded in strengthening."[22] At the same time, the economy was, for Nervi, intimately connected with finding "the method of bringing dead and live loads down to the foundations...with the minimum use of materials." Economy of cost and efficiency of materials were, however, never enough, for as he continued, "I still remember the long and patient work to find an agreement between the static necessities... and the desire to obtain something which for me would have a satisfying appearance."[23] His expectations were materialized in the form of curve.

Figure 5. Sports Hall, Nervi

(www.architecture.uwaterloo.ca/faculty_project. Last accessed in December 2006.)

Gaudi is another well-known designer for his unique curvilinear architecture. He integrated the parabolic arch, nature's organic shapes, and underwater fluidity into his architecture. While designing buildings, he observed the forces of gravity and related

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[22] Nervi, P. L. *Aesthetics and technology in building.* Harvard University Press, Cambridge. 1965. p. 24.

[23] Ibid. pp. 23-24.

catenary principles. The saddle forms of Gaudi had some advantages over more traditional shapes. A saddle shape has opposite curvatures, a downward arch in one direction and an upward arch in the other. Hence, a saddle-shaped roof surface is much stiffer. Moreover, these saddle shapes contain straight lines within their surfaces, something impossible with a spherical surface. Certain imaginary planes, passed through this surface, will intersect them in straight lines. This property of saddle shells makes them easier to construct. For Gaudi, this geometric property had a deep religious meaning, which confirmed his sense of the naturalness of the form. He took this form to be “a miracle of mathematics” and attributed holy properties to the trinity of straight lines which determine any surface.”[24]

Figure 6. Sagrada Familia, Barcelona

(http://www.greatbuildings.com/buildings/Sagrada_Familia.html. Last accessed in December 2006.)

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[24] Collins, G. *Antonio Gaudi.* George Braziller, New York. 1960. p. 23.

All structural designs, whether for long or short spans, are based on the same general concepts. However, in practice, the proportions of short-span structures can be often determined by nonstructural requirements. On the other hand, in long-span structures, the relationship between dead weight, strength and proportions becomes critical, and the design criteria have to be far more rational in order to arrive at safe, economical, and aesthetic solutions.[25]

In order to optimize dead-load efficiency, long-span structures should have their shapes approximate a natural line of pressure, such as parabolic arch, or a catenary. When this is done, the shear-and moment- resisting force always acts at the center of gravity. Hence, the use of curved forms is often efficient as well as appealing. Since there is theoretically only direct axial force in the system and the material requirements are minimized, such curved systems are efficient because they yield overall structural depth for long spans without increasing sectional depth.[26]

In contrast, long-span design of a beam would require much more sectional depth along its entire length, or at least much of it, because the overall moment increases with L^2, deflection with L^4. Therefore, ordinary long-span beams need a lot of additional material to maintain a flat surface. With curved structures, the required moment-resisting forces can be controlled by increasing the rise or the sag of the system as a whole rather than by increasing the cross-sectional depth. As the overall structural depth is maximized, the thrust of an arch or the tension in a cable, as well as local bending requirements, will be minimized.[27]

Long-span structures can be built using flat construction such as heavy girders, trusses, space-frames. However, for spans approximately 30 m, it is often more economical to build a system made up of curved members.[28]

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[25] Lin, T. Y. *Structural concepts and systems for architects and engineers.* Van Nostrand Reinhold Co., New York. 1988. p. 377.

[26] Ibid.

[27] Ibid.

[28] Ibid.

However, curved systems have some disadvantages as well. The main objection to curved systems is the complexities and difficulties faced during the construction processes in comparison to flat surfaces. In addition, the visualization of these forms in three-dimensional renders has been a serious problem. Until now, the limitations of hand drawn architectural drawings sometimes led to linear buildings that are easily rendered in 2D form, whereas three-dimensional curvilinear form is facilitated by three-dimensional rendering tools, not available until recently.

In this chapter, the concept of *form* and *structure* are explained from architectural and engineering points of views. The relationship between structure and form is examined, since the image of the building and the system that makes the building stand up are complementary concepts.

Moreover, simple elemental forms are introduced with an emphasis on curvilinear forms, as understanding these forms is fundamental to grasp the overall concept of curvature.

The structural ideals, efficiency, economy and elegance are represented referring to some authors. These concepts will be discussed further on the analysis of Waterloo Station throughout its structure.

CHAPTER 3

THE CONCEPT OF CURVILINEARITY IN STRUCTURING

A sheet of paper held in the hand, bends and cannot support its own weight. If the same sheet of paper is given a slight upward curvature, it is capable of supporting its own weight and some additional load (Figure 7). The upward curvature increases the stiffness and the load-carrying capacity of the paper because it locates some material away from the "neutral axis," so that the bending rigidity of the sheet is considerably increased. As a result, the new carrying capacity of the paper is obtained not by increasing the amount of material used, but by giving it proper form.[1]

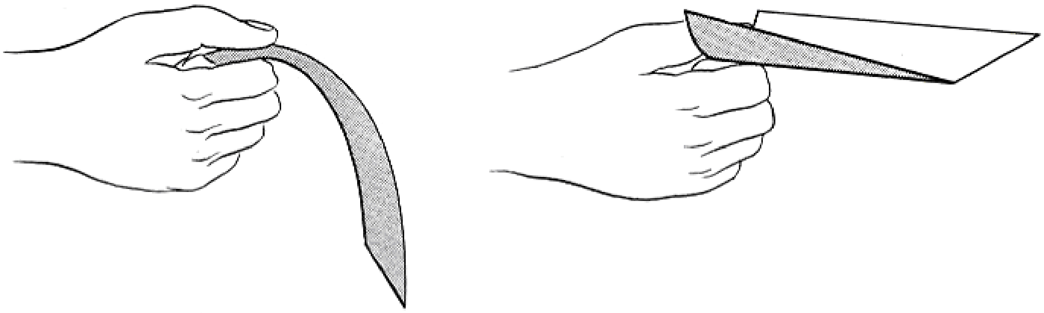

Figure 7. Form-resistant structure

(Salvadori, M. G. *Structure in architecture.* Prentice-Hall Inc., New Jersey. 1963.)

Structures in which strength is obtained by shaping the material according to the loads they must carry are called **form-resistant structures.** For instance, membranes are purely tensile form resistant structures as they depend on curvature and twist to carry loads. If the same membrane were turned upside down, it would be a form-resistant structure developing only compression, that is, the two-dimensional antifu-

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[1] Salvadori, M. G. *Structure in architecture.* Prentice-Hall Inc., New Jersey. 1963. p. 322.

nicular for those loads. Thin shells are another example for form-resistant structures since they are thin enough not to develop considerable bending stresses, but thick enough to carry loads by compression, tension, and shear. Thin shells allow the economical construction of domes and other curved roofs of varied form and exceptional strength. They are among the most sophisticated expressions of modern structural design.[2]

The form resistant structures mentioned above owe their efficiency to curvatures and twist. In order to understand their structural action, a general overview over their purely geometrical characteristics should be done.

3.1 Curvatures

The curvature of a surface at a point is displayed by cutting it with a plane rotated around the normal to the surface at that point. The curvature varies as the plane rotates. It may be up or down in all directions, or up in some and down in others.[3]

The intersection of a dome with a plane passing through its normal has a **downward** curvature. For spherical domes, the curvatures are all identical. For the other types of dome, they change from a maximum to a minimum as the plane rotates (Figure 8). Conversely, the curvatures of a dish are all up. Surfaces like domes or dishes, in which the curvature changes value around a point but it always up or down, are called **synclastic.** If we call downward curvatures positive and upward curvatures negative, domes have positive curvatures and dishes negative curvatures in all directions. Surfaces with positive or negative curvature at all points are **nondevelopable**, since they cannot be flattened without stretching them (Figure 9). Their stiffness and strength stems, in large part, from their resistance to those deformations which tends to flatten them.[4]

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[2] Ibid.

[3] Ibid. p. 324.

[4] Ibid.

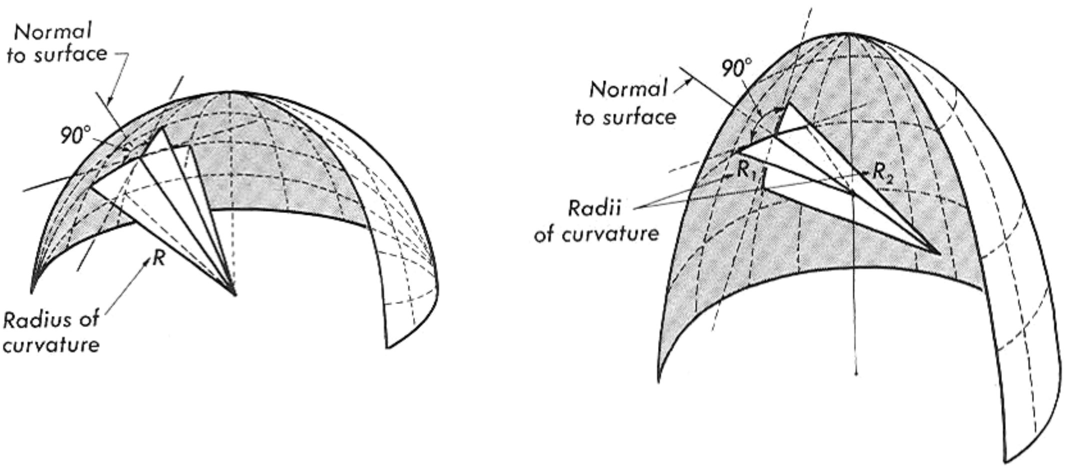
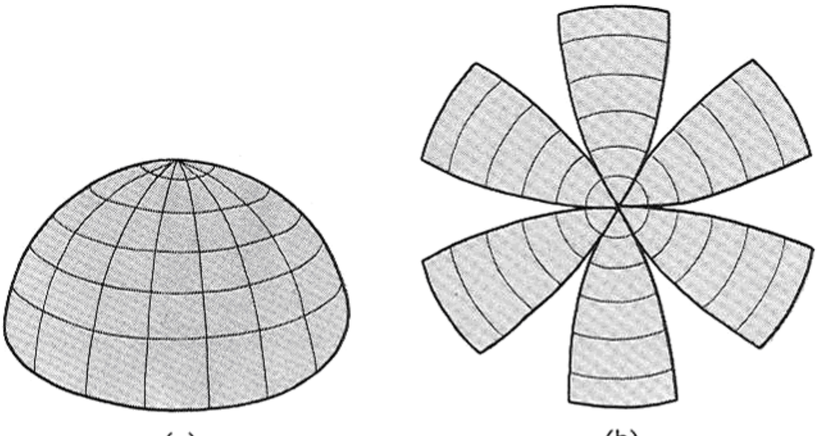


Figure 8. Curvatures of a spherical dome

(Salvadori, M. G. *Structure in architecture.* Prentice-Hall Inc., New Jersey. 1963.)



(a) (b)

Figure 9. Development of a synclastic surface

(Salvadori, M. G. *Structure in architecture.* Prentice-Hall Inc., New Jersey. 1963.)

The forms with the greatest natural stiffness are, in general, the synclastic ones. In the domical vault or shell continuously supported around its base (Figure 3m), the load may be carried entirely by compression (Figure 10b) or by a combination of compression and tension (Figure 10a). A similarly shaped membrane (Figure 3k) must, however, act wholly in tension, because its negligible thickness will cause buckling under compression. It is difficult to ensure this purely tensile action if it hangs from the supports as shown and the only loading is that of self-weight plus varying and uncontrolled air pressures on both sides. Thus, unless it is used for the storage of liquids or weighed down by a heavy cladding, it is best used inverted and given a tensile prestress throughout by maintaining an internal air pressure greater than any likely external pressure.[5]

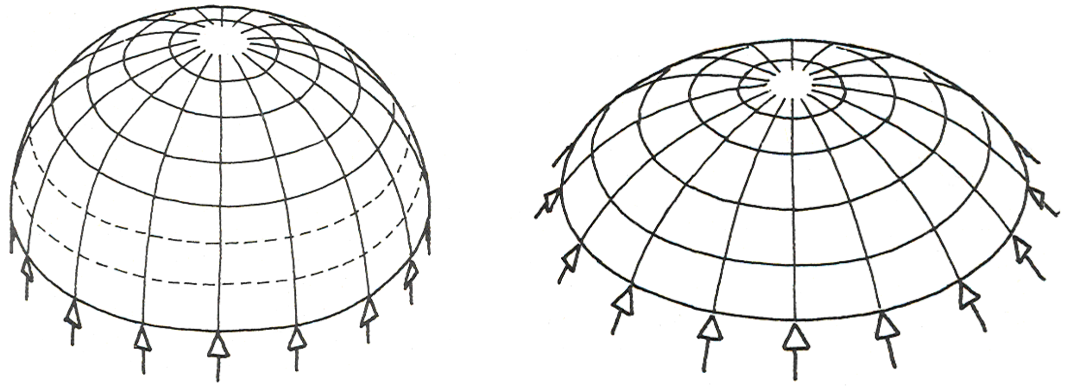


Figure 10. Internal static equilibrium of domical shells.
Principal compressions are shown in the upper sketches by full lines and principal tensions are shown by broken lines.

(Mainstone, R. J. *Developments in structural form.* M.I.T. Press, Cambridge. 1975.)

Surfaces, like those of the barrel roofs or gutter channels, with curvature always positive or always negative but vanishing in one direction, are called **developable** surfaces. They can be flattened without stretching them. Developable surfaces are noticeably less stiff and strong than synclastic surfaces.[6]

If a horse saddle is cut with a vertical rotating plane its curvature changes not only in value, but also in sign. As the cutting plane rotates around its axis, the curvature of the saddle changes gradually from positive to negative values, and vice versa; thus, the curvatures must vanish in two directions. Saddle or **anticlastic** surfaces have, in general, two directions of zero curvature which means that two directions along which straight lines lie on the surface (Figure 11). They are also nondevelopable.[7]

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[5] Mainstone, R. J. *Developments in structural form.* M.I.T. Press, Cambridge. 1975. pp. 88-89.

[6] Salvadori, M. G. *Structure in architecture.* p. 326.

[7] Ibid.

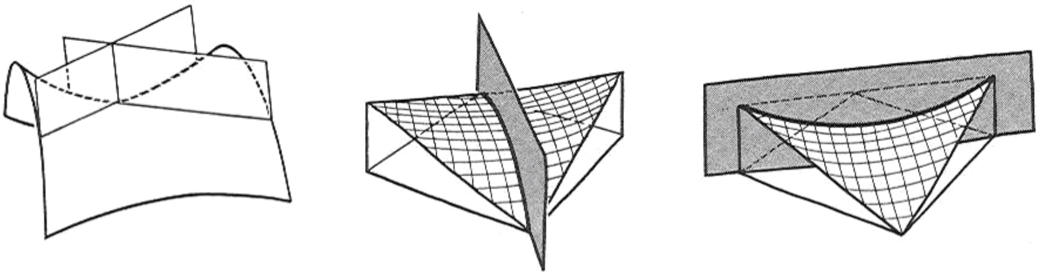

Figure 11. Saddle surface and principal curvature lines of saddle surface

(Salvadori, M. G. *Structure in architecture.* Prentice-Hall Inc., New Jersey. 1963.)

Anticlastic forms, unless prestressed, will tend to act in compression in the directions of maximum arch-like curvature and in tension in directions of maximum catenary-like curvature (Figure 3o and p). Ideally, if there is enough thickness to resist buckling under compression and if the surface geometry is so chosen as to call for principal tensions and compressions of equal and uniform magnitudes throughout, the saddle form (Figure 3p) may be cut along lines midway between the directions of these principal tensions and compressions and will then transmit only continuous and uniform shears to the edge members (Figure 3o). Again the membrane form must act wholly in tension (Figure 3n), and this calls for tensile prestressing in the direction of the arch-like curvature. Such prestressing may be applied through continuous rigid edge members giving, if the curvatures are sufficient and all compression is eliminated, the stiffest form of membrane. Alternatively, it can be applied by means of guy ropes as in a tent.[8]

To be able to envision the curvatures of a surface, the surface must cut with a plane perpendicular to it. In Figure 12, it is shown that, a perpendicular plane parallel to the axis cuts the cylinder along a straight line in a cylinder, representing a lack of curvature in this direction. A cut at right angles to the axis has a large curvature, while cuts in any other direction show smaller curvatures. Thus, the sections acquire curvatures that vary from zero in one direction to a maximum value at right angles to it as the cutting plane rotates around the normal to the cylinder. The two perpendicular directions in which the curvatures become respectively maximum and minimum are called the **principal directions of curvature** of the membrane surface. If we take an ele-

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[8] Mainstone, R. J. *Developments in structural form.* pp. 88-89.

ment of the cylinder with sides parallel to these principal directions, it is seen that the element has no twist. Nevertheless, an element with sides not parallel to the principal directions has twist (Figure 13a and b). This property is not peculiar to cylinder. It is convenient to mark principal curvature directions by the lack of twist -that is, of slope change-in these directions.[9]

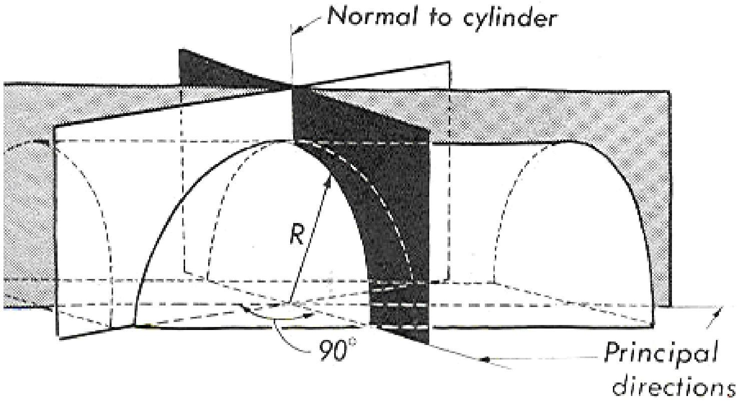

Figure 12. Cylinder curvatures

(Salvadori, M. G. *Structure in architecture.* Prentice-Hall Inc., New Jersey. 1963.)

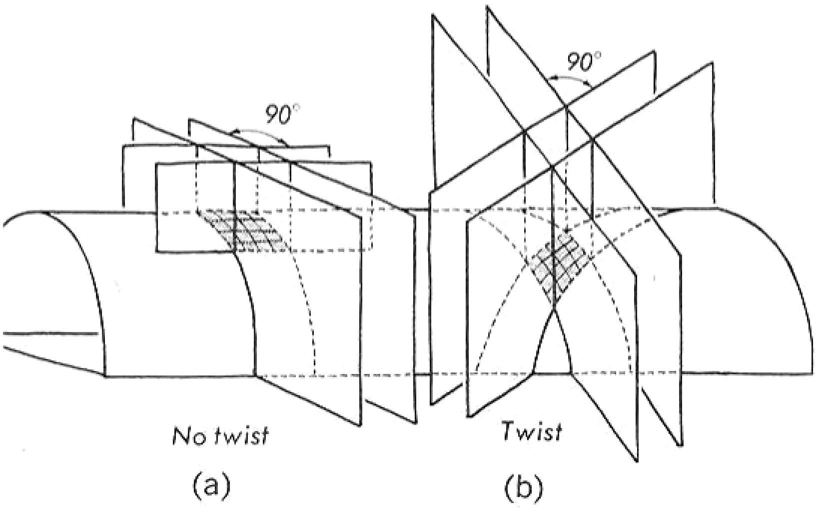

Figure 13. Cylinder twist

(Salvadori, M. G. *Structure in architecture.* Prentice-Hall Inc., New Jersey. 1963.)

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[9] Salvadori, M. G. *Structure in architecture.* pp. 300-302.

The directions of principal curvature on a surface can be marked by the means of small crosses and the pattern of **lines of principal curvature** can be obtained**.** Figure 14 indicates the principal curvature lines for a general surface.[10]

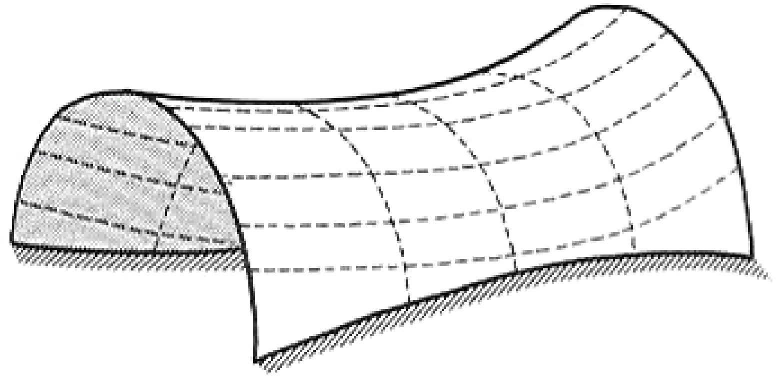

Figure 14. Principal curvature lines

(Salvadori, M. G. *Structure in architecture.* Prentice-Hall Inc., New Jersey. 1963.)

3.1.1 Rotational Surfaces

Rotational surfaces are described by the rotation of a plane curve around a vertical axis (Figure 15). The plane may have a variety of shapes, so it provides a variety of domes. The most commonly used dome is spherical; its surface is obtained by rotating an arc of circle around a vertical axis.[11]

Elliptical domes are described by half an ellipse rotating around its vertical axis (Figure 16a); their action is not as efficient as the action of a spherical dome because the top of the shell is flatter, and the reduction in curvature introduces a tendency to buckle. In contrast, the parabolic dome (Figure 16b) has a sharper curvature at its top and offers structural advantages even in comparison with the sphere.[12]

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[10] Ibid. p. 302.

[11] Ibid. p. 328.

[12] Ibid. p. 330.

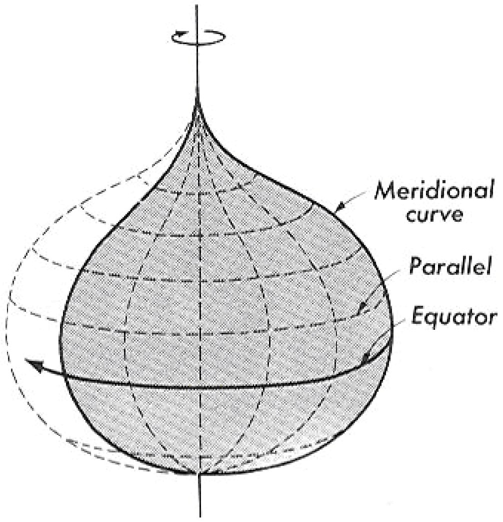

Figure 15. Rotational surface

(Salvadori, M. G. *Structure in architecture.* Prentice-Hall Inc., New Jersey. 1963.)

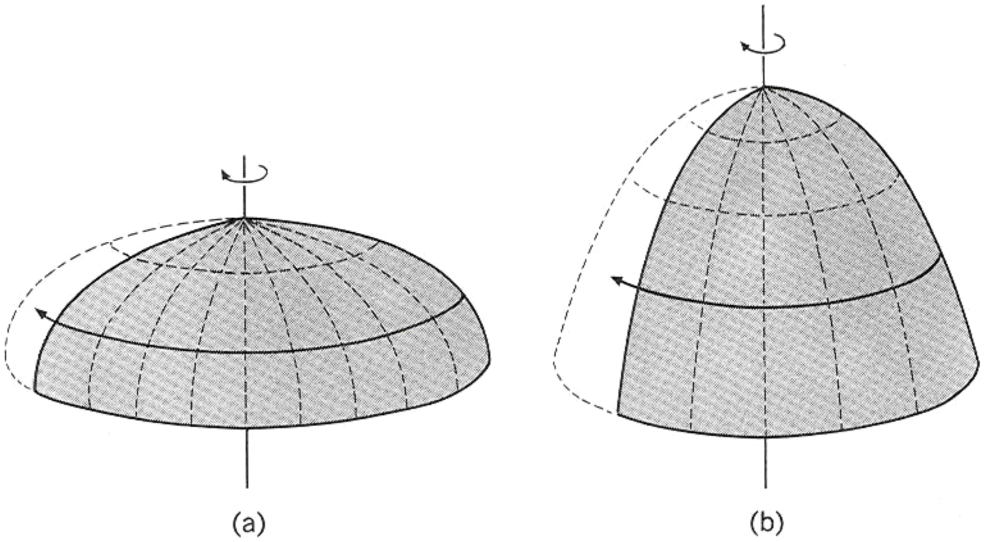

Figure 16. Elliptical and parabolic surfaces of revolution

(Salvadori, M. G. *Structure in architecture.* Prentice-Hall Inc., New Jersey. 1963.)

A cone is a surface described by rotating a straight line around a vertical axis. The lack of curvature in the radial direction of the cone makes the forming of cones in reinforced concrete to some extent simpler than the forming of regular domes.[13]

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[13] Ibid.

Circular curved barrels are obtained by rotating around a vertical axis the upper half of a circle or any other curve not intersecting the axis. Such surfaces are called torus (Figure 17).[14]

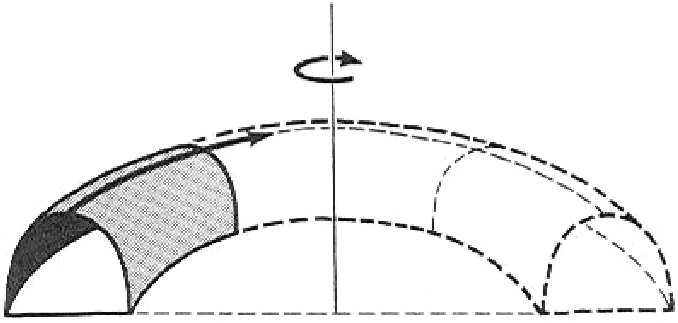

Figure 17. Torus surface

(Salvadori, M. G. *Structure in architecture.* Prentice-Hall Inc., New Jersey. 1963.)

3.1.2 Translational Surfaces

Translational surfaces are obtained by sliding a plane curve, usually at right angles to the first, while maintaining it vertical. A cylinder is obtained by translating a horizontal straight line along a vertical curve or vice versa. Depending on the curve, the cylinder may be circular, parabolic, or elliptic (Figure 18).[15]

The translation of a vertical parabola with downward curvature on a perpendicular parabola with downward curvature generates a surface called an **elliptic paraboloid**, which covers a rectangular area. Its horizontal sections are ellipses; its vertical sections are parabolas (Figure 19). If the two parabolas are identical, the paraboloid covers a square area, and its horizontal sections become circles. The elliptic paraboloid was the first shape used to build a thin concrete shell in the beginning of 1900's.[16]

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[14] Ibid.

[15] Ibid. p. 332.

[16] Ibid.

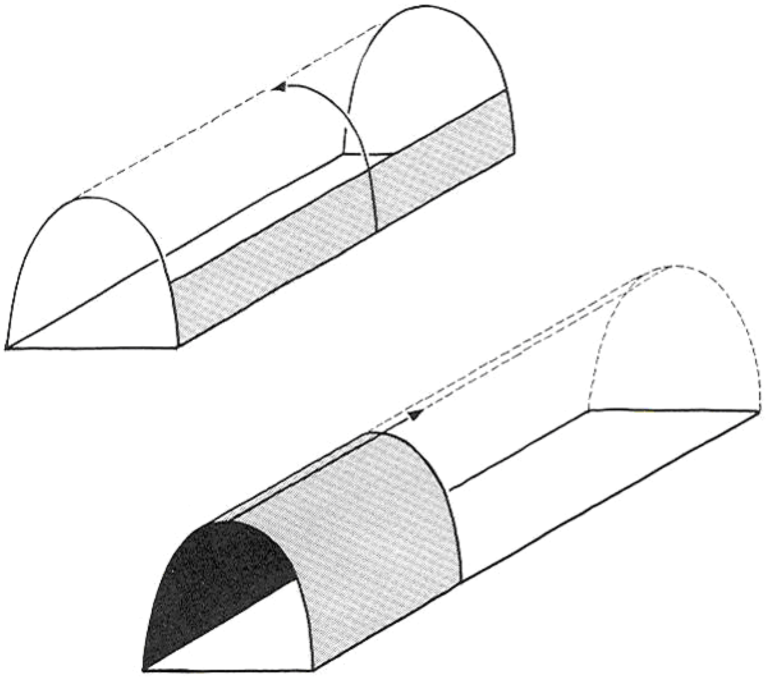

Figure 18. Cylindrical surfaces

(Salvadori, M. G. *Structure in architecture.* Prentice-Hall Inc., New Jersey. 1963.)

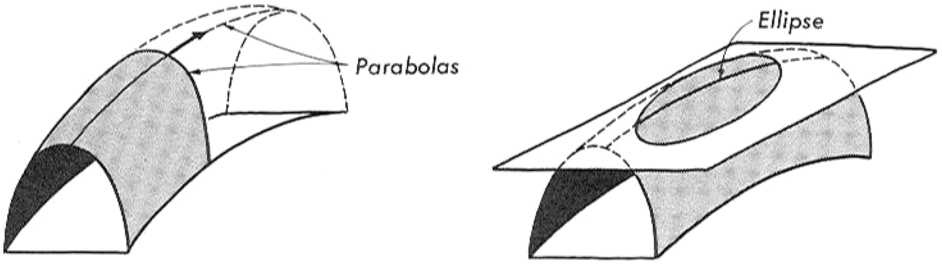

Figure 19. Elliptic paraboloid

(Salvadori, M. G. *Structure in architecture.* Prentice-Hall Inc., New Jersey. 1963.)

A **hyperbolic paraboloid** is obtained by translating a parabola with downward curvature on a parabola with upward curvature (Figure 20a). Its horizontal sections are two separate branches of a curve called hyperbola, while its vertical principal sections are parabolas (Figure 20b). As for the saddle surfaces, the curvature of the hyperbolic paraboloid vanishes in two directions, but for a hyperbolic paraboloid these directions are the same at all points. The construction of such surfaces in reinforced concrete is simple because of the use of the straight planks in the direction of the lines of zero curvature.[17]

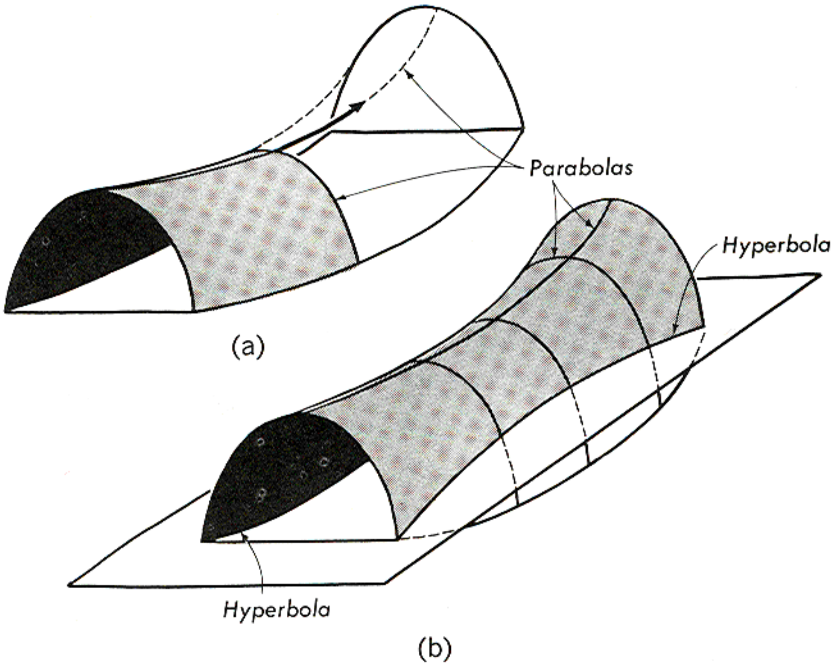

Figure 20. Hyperbolic paraboloid

(Salvadori, M. G. *Structure in architecture.* Prentice-Hall Inc., New Jersey. 1963.)

The principle of hyperbolic paraboloid is illustrated in Figure 21. ABCD is a square on plan; F is vertically above C; and at the other diagonally opposite corner, E is vertically above A; so that the original square has been “warped” into the doubly curved surface EBFD. If we divide all the sides into an equal number of parts and join by straight lines from BF to ED, and from EB to DF, the surface formed is curved in two directions contrary to the straight lines. The curved line BD illustrates that the surface curves up from the low corner B and then down to the other low corner D, forming a parabola. The cross-section cutting the two corners B and D is shown in Figure 19b. On the other hand, the surface curves down from the high corner E and then up to the other high corner F, forming a parabola, the cross-section being shown at (c). All cross-sections parallel to the diagonals BD and EF are parabolic. Although

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[17] Ibid.

this doubly curved surface appears to require complicated formwork, the formwork can actually consist of straight timber joists spanning along the straight lines shown in Figure 21.[18]

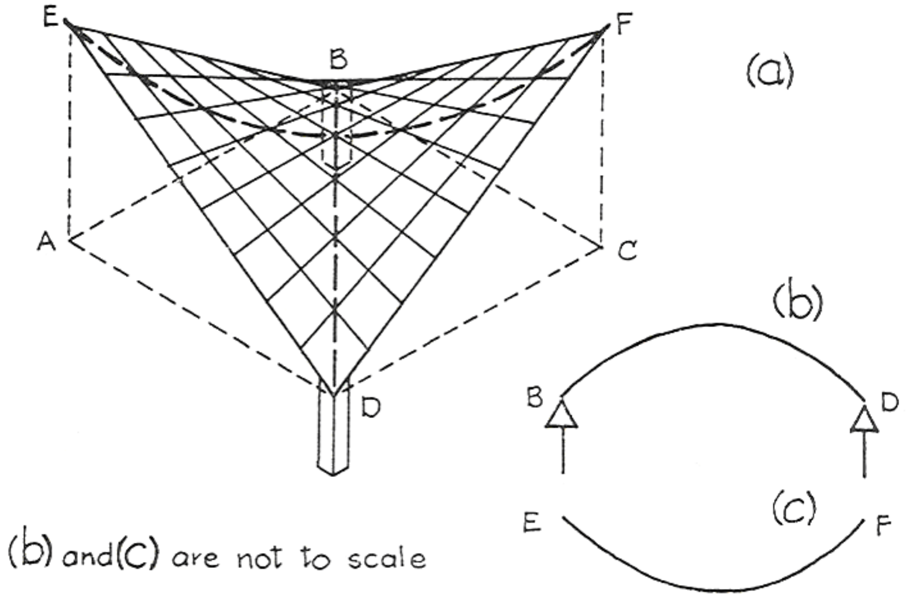

Figure 21. The principle of hyperbolic paraboloid

(Morgan, W. *Elements of structure; an introduction to the principles of building and structural engineering.* Pitman, London. 1964.)

3.1.3 Ruled Surfaces

A **ruled surface** is the surface generated by sliding the two ends of a straight-line segment on two separate curves. The cylinder is a ruled surface generated by sliding a horizontal line segment on two identical vertical curves .When the two curves are two straight lines askew in space, the ruled surface is a hyperbolic paraboloid (Figure 22).[19]

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[18] Morgan, W. *Elements of structure; an introduction to the principles of building and structural engineering.* Pitman, London. 1964. pp. 179-180.

[19] Salvadori, M. G. *Structure in architecture.* p. 336.

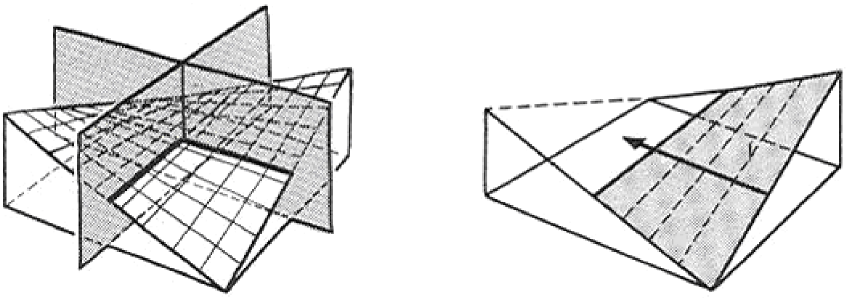

Figure 22. Straight-line generatrices of hyperbolic paraboloid

(Salvadori, M. G. *Structure in architecture.* Prentice-Hall Inc., New Jersey. 1963.)

Conoidal surfaces are a different kind of ruled surfaces generated by sliding a straight-line segment on two different curves lying in parallel planes. The conoid is called circular, parabolic, or elliptic depending on whether its end curve is an arc of a circle, a parabola, or an ellipse.[20]

3.1.4 Surfaces of Special Type

The elementary, mathematically defined surfaces may be combined in different ways to obtain surfaces that are more complex. Two cylindrical shells may be intersected at right angles to cover a rectangular area with a “groined vault” (Figure 23). Parallel cylinders with curvatures generate an undulated roof similar to a folded plate (Figure 23).[21]

Scalloped ringed roofs may be obtained by joining sectors of cones with curvatures alternately upward and downward. A parabolic cylinder may be undulated to transform it into a surface with curvatures in two directions rather than in one, thus stiffening it. Spherical domes may be undulated for the same purpose (Figure 24).[22]

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[20] Ibid.

[21] Salvadori, M. G. *Structure in architecture.* p. 338.

[22] Ibid. p. 340.

There is no reason to limit the curvilinear forms to forms easily definable by geometrical formulas. Structurally sound, "free" shapes may be invented. If the designer is familiar with the structural behavior of the basic geometric forms, then the boundaries of his/her imagination will be widened.

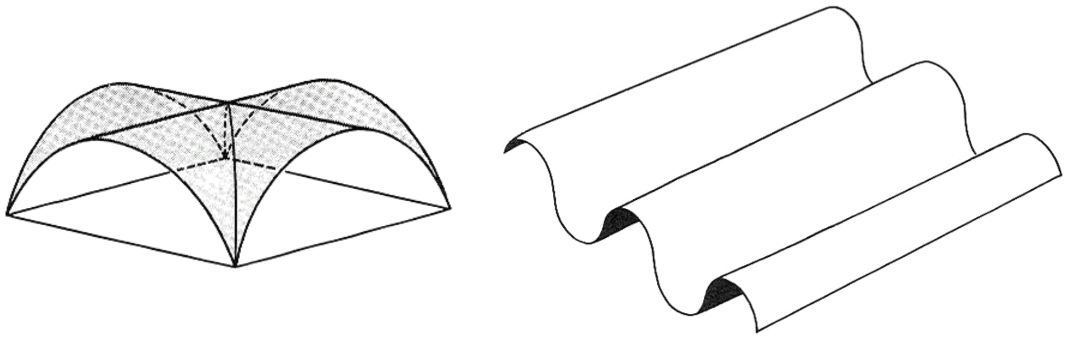

Figure 23. Intersecting cylinders and undulated cylindrical surface

(Salvadori, M. G. *Structure in architecture.* Prentice-Hall Inc., New Jersey. 1963.)

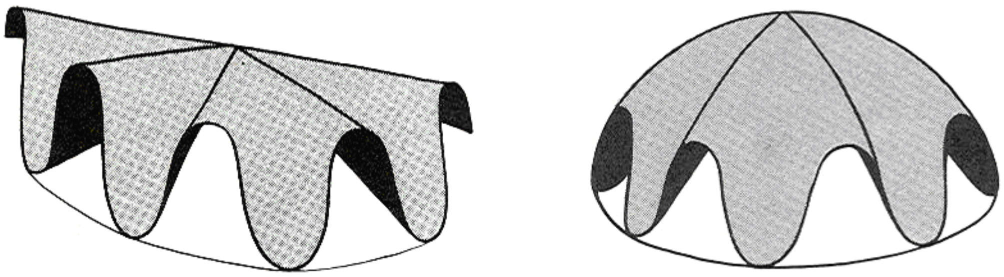

Figure 24. Undulated conical surface and undulated spherical sector

(Salvadori, M. G. *Structure in architecture.* Prentice-Hall Inc., New Jersey. 1963.)

3.2 Discussion of Structural Advantages for Arches, Vaults and Domes

Since the beginning of history, human beings have tried to span distances using arch construction. Essentially, arch required materials resisting only compression, and large quantities of materials like stone or mud bricks were easily available. Later, fired brick, concrete, and steel were produced and used for arch construction.[23]

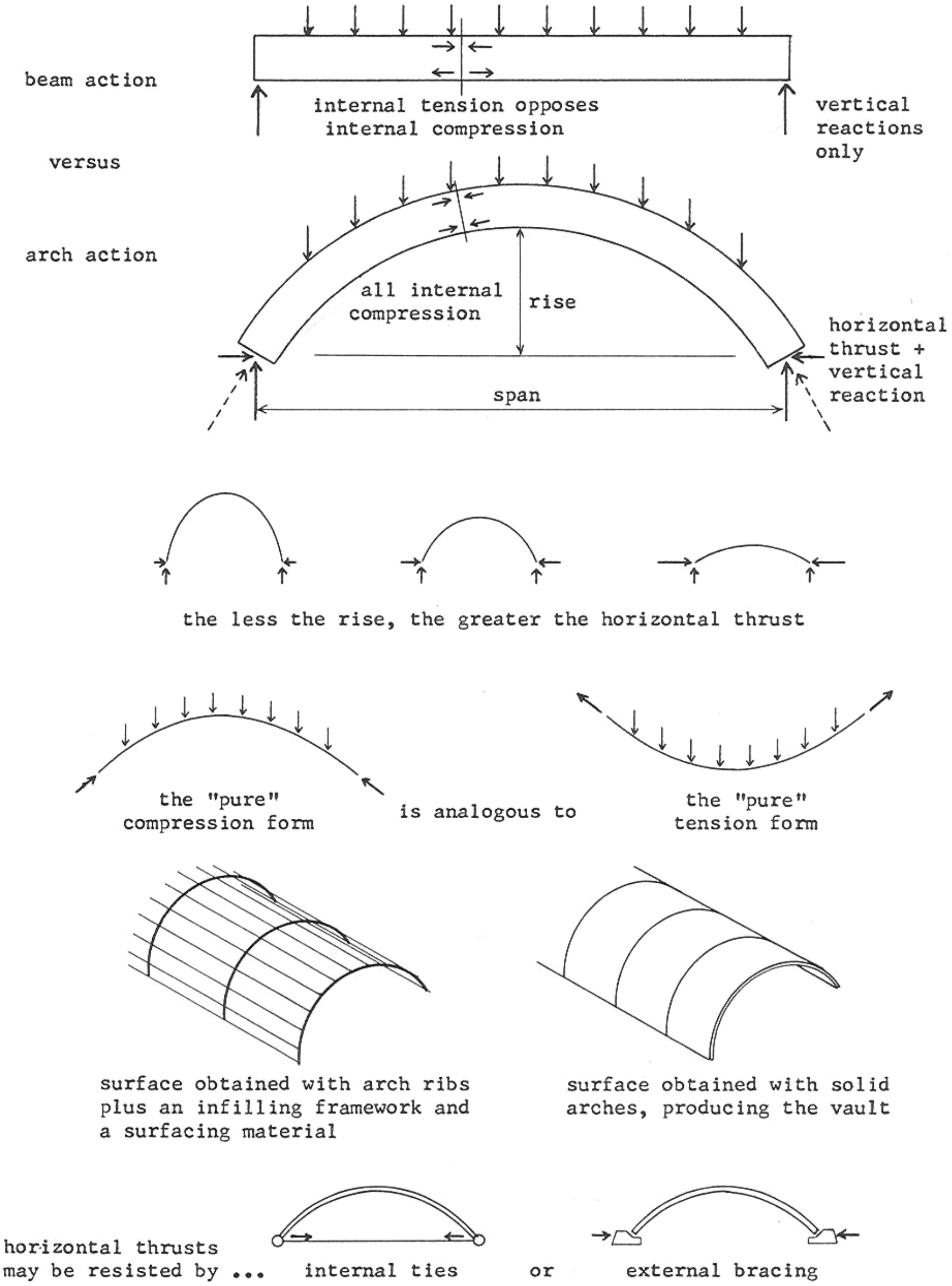

Figure 25. Basic aspects of arches

(Ambrose, J. E. *Building structures.* Wiley, New York. c1993.)

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[23] Lin, T. Y. *Structural concepts and systems for architects and engineers.* Van Nostrand Reinhold Co., New York. 1988. p. 378.

The arched forms have been recognized from early times as forming well resistant structural components. Although, the concept of flexural or axial compressive resistance was not known by the early contractors, they realized the structural advantages of the arched forms empirically. They preferred arched forms also for a visual effect.[24]

The basic concept in an arch is to develop a spanning structure by the use of only internal compression (Figure 25). The profile of the "pure" arch is geometrically resulted from the loading and support conditions. For a single-span arch with no fixity at the base in the form of moment resistance, with supports at the same level, and with a uniformly distributed load on the entire span, the resulting form is that of a second-degree curve, or a parabola. Various other curves –circular or elliptical- can be used, but the basic form is the parabola if the load is primarily gravity.[25]

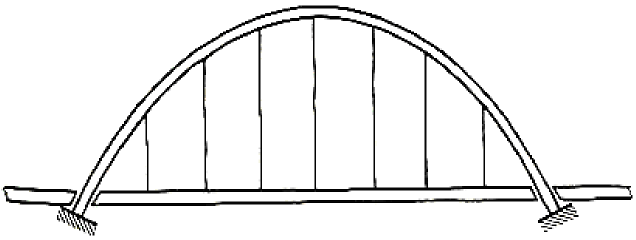

Figure 26. Funicular arch

(Salvadori, M. G. *Structure in architecture.* Prentice-Hall Inc., New Jersey. 1963.)

The parabolic shape assumed by a cable carrying loads uniformly distributed horizontally may be inverted to give the ideal shape of an arch developing only compression under this type of load (Figure 26). The Spanish architect Gaudi determined the form of the arches in the church of the Sacred Family in Barcelona by this method.[26]

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[24] Karaesmen, E. *CE 480 Introduction to architectural engineering, Lecture notes.* Boğaziçi University, Civil Engineering Department. Istanbul. 2006.

[25] Ambrose, J. E. *Building structures.* Wiley, New York. c1993. p. 37.

[26] Salvadori, M. G. *Structure in architecture.* p. 130.

The basic issues of statics in arch design are illustrated in Figure 27. A uniform load, (w) units per linear foot, is supplied along the projected horizontal length of the arch. Due to the symmetry, the vertical component of the end reactions is V= wL/2= W/2. This load reaction is the same as for a simple beam and, similarly, there is no shear across the midspan of the arch, as illustrated in the one-half arch freebody. Taking the moment about the crown, we obtain

$$M_{resist} = Hh = wL/2\ (L/2 - L/4) \text{ and } C = H = wL^2/8h = WL/8h$$

Where C, H is analogous to the C, T forces in a beam and (h) is the overall height of the arch. Obviously, (h) is much larger than (d) for the beam design and C, H << C,T for a beam and (H) can be tension or compression.[27]

Since equilibrium requires that H be constant across the arch, a parabolic curve would theoretically result in no moment on the arch section. The resultant of V and H follows the natural line of pressure, and the reaction at the supports is $R = \sqrt{H^2 + V^2}$.[28]

In practice, it should be noted that an arch is not generally subjected to a uniform load. First, there is usually a difference in the depth and the weight of the arch per meter of span. Then the weight per meter varies again because the inclination of the arch rib itself varies. Furthermore, live load may act as another consideration. Therefore, in final design, it is always necessary to design the arch to carry a certain amount of bending moment.[29] At any case, flexural moment is smaller than at the case of straight lining beam because of the lateral components of the reaction force as seen in Figure 27.

In modern arch construction, materials such as reinforced concrete or steel are used. Since these materials can resist tension as well as compression, they can carry local

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[27] Lin, T. Y. *Structural concepts and systems for architects and engineers.* p. 379.

[28] Ibid.

[29] Ibid. p. 381.

moments and arch ribs can be quite slender compared with the span. However, it is often necessary to stiffen the ribs against buckling.[30]

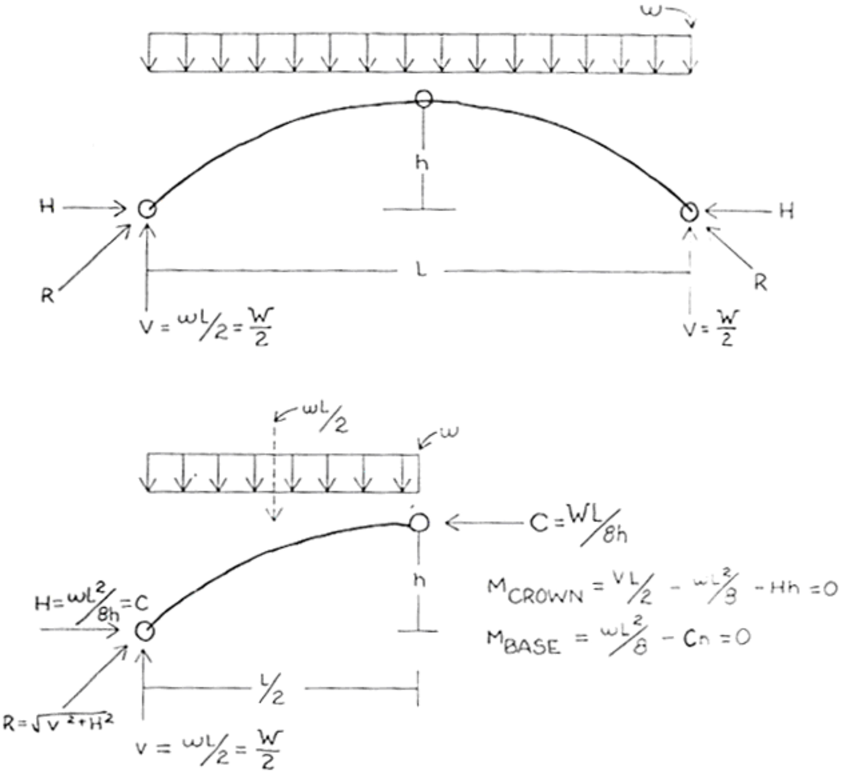

Figure 27. The statics of a three-hinged arch

(Lin, T. Y. *Structural concepts and systems for architects and engineers.* Van Nostrand Reinhold Co., New York. 1988.)

To understand the structural behavior of an arch, it is necessary to understand the nature of its basic configuration. The three most common cases are those shown in Figure 28, which are the fixed arch (a), the two-hinged arch (b) and the three-hinged arch (c).[31]

The fixed arch is used most commonly in reinforced concrete bridge and tunnel construction. It is not feasible for very long span arches to maintain the fixed condition at the base. So this form is generally used in short to medium spans. The fixed arch is

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[30] Lin, T. Y. *Structural concepts and systems for architects and engineers.* p. 382.

[31] Ambrose, J. E. *Building structures.* p. 38.

highly indeterminate in its action and is subject to internal stress and abutment forces as a result of the thermal expansion and contraction. The two-hinged arch is commonly used for long spans. The pinned base is more feasible for a large arch and is not subjected to forces as a result of thermal change to the degree that the fixed support is. The three-hinged arch is generally preferred for medium-span building roof structures. The pinned bases are more easily developed than fixed ones, making shallow bearing-type foundations rational for the medium –span structure. Moreover, the thermal expansion and contraction of the arch segments will cause vertical movement at the peak pin, but have no considerable effect on the bases. This simplifies the foundation design. Construction can often be prefabricated.[32]

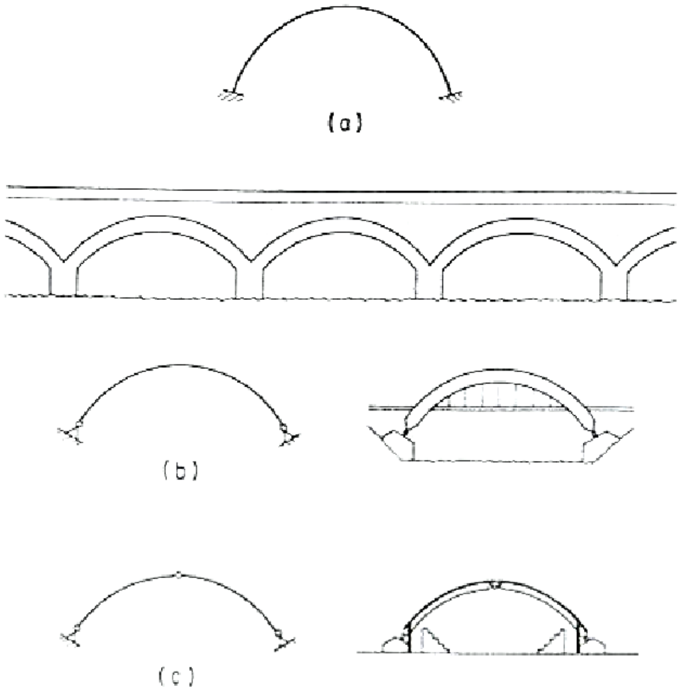

Figure 28. Types of arches

(Ambrose, J. E. *Building structures.* Wiley, New York. c1993.)

In architecture, a vault means a roof or floor constructed in arch form. The simplest type of vault, shown in Figure 29, is known as a *barrel vault.* Barrel vault is a long arch thrusting against the supporting walls along their whole length. These walls

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[32] Ibid.

have to be thick in order to resist the thrusts and the openings must be small not to reduce the efficiency of the abutments.[33]

The introduction of the *intersecting vault or groined vault* is a great structural advance (Figure 29). Referring to Figure 30, where the two vaults meet, curved intersecting lines called *groins* are formed. These lines give an arch form as shown by CDB. Assuming square bay and semicircular transverse and longitudinal arches, the span of each arch is equal to the length of side of a square (such as AC or AB). The span of the "arch" CB, however, is equal to the diagonal of the square, and if the rise of the diagonal "arch" formed by the groin is equal to the rise of the transverse and longitudinal arches, then arch CDB is a semi-ellipse. It is therefore flatter than arch AB.[34]

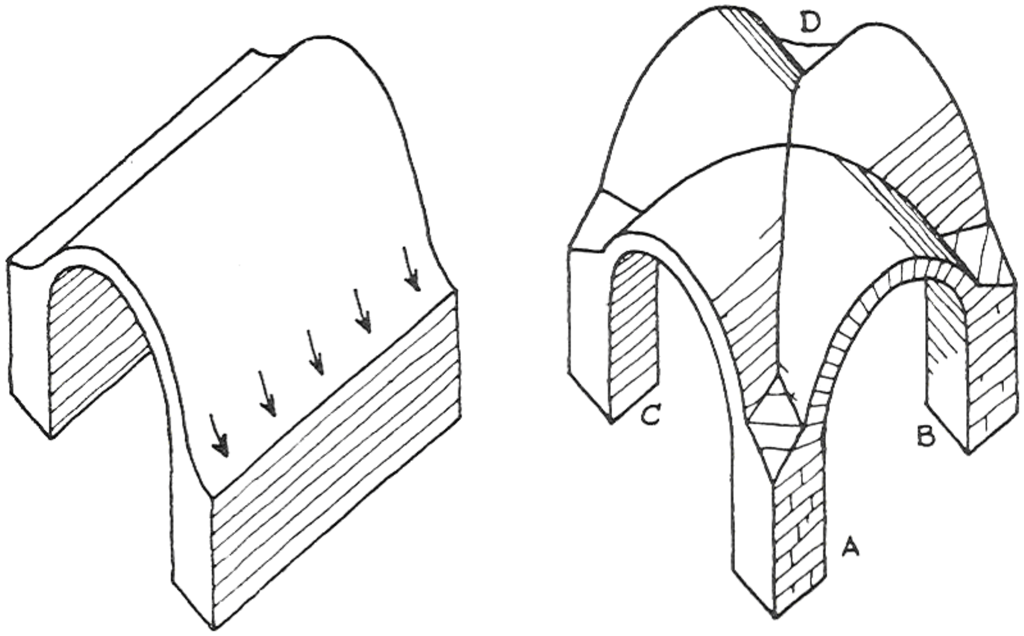

Figure 29. Barrel and groined vault

(Morgan, W. *Elements of structure; an introduction to the principles of building and structural engineering.* Pitman, London. 1964.)

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[33] Morgan, W. *Elements of structure; an introduction to the principles of building and structural engineering.* pp. 51-52.

[34] Ibid. p. 53.

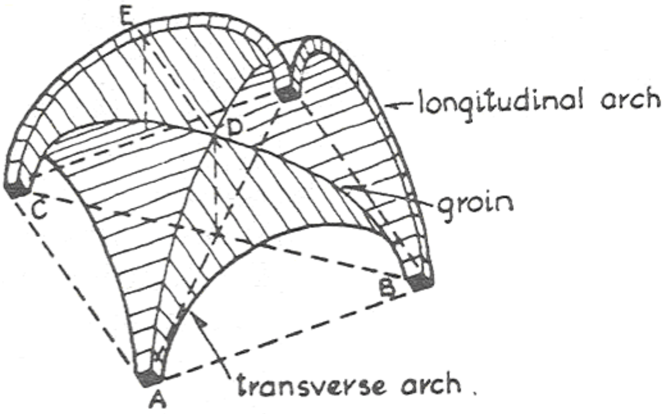

Figure 30. The groins and the arches forming the vault

(Morgan, W. *Elements of structure; an introduction to the principles of building and structural engineering.* Pitman, London. 1964.)

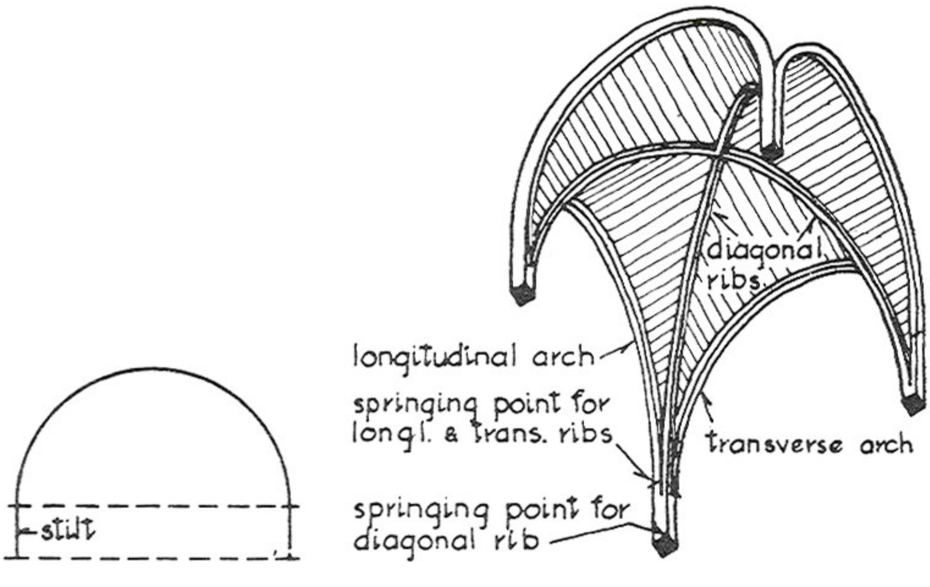

Figure 31. Stilted ribbed vault

(Morgan, W. *Elements of structure; an introduction to the principles of building and structural engineering.* Pitman, London. 1964.)

A new development in vaulting technique made the groins very important structurally. The development consisted in constructing ribs along the groin lines so that these ribs formed load-bearing arches spanning diagonally (Figure 31). It was an advantage to make these diagonal ribs or arches semicircular so as to reduce the thrust in these arches. The transverse and longitudinal arches had to be constructed to give a satisfactory structural and aesthetic roof. One expedient was to stilt these arches, i.e. to make them semicircular by having their springing points higher than those of the diagonal ribs or arches. In this type of ribbed vault all the loads are taken by the constructional framework or skeleton formed by the ribs or arches, which transfer the loads and thrusts to the supporting piers. The vault was completed by filling in between the ribs, and since this infilling had only its weight to carry; it could be made only nearly 10 cm thick.[35]

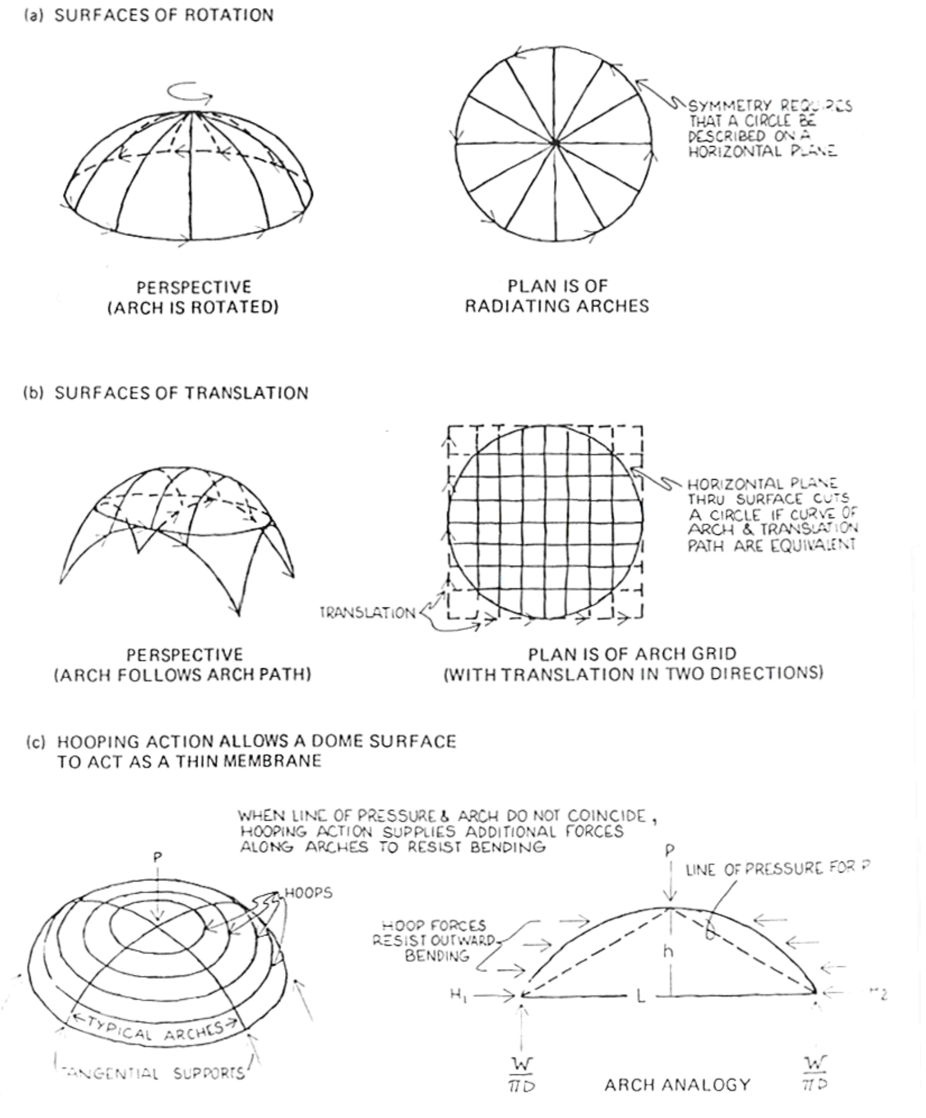

Figure 32. Domes generated by rotation or translation

(Lin, T. Y. *Structural concepts and systems for architects and engineers.* Van Nostrand Reinhold Co., New York. 1988.)

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[35] Ibid. p. 54.

If the single arch is rotated about its vertical axis, the form generated is a dome (Figure 32). A dome can also be visualized as an arch translated over another arch (Figure 32b). This structural form is a circle in plan, in contrast to the vault, which relates to a rectangle, or cross form.[36] The primary action in a dome is similar to that in an arch. Ideally, the main compressive force starts from the top of the arches and goes down along the radial or orthogonal ribs to the support system. However, dome arches do not always overlap with the ideal line of pressure determined by the load distribution on each arch unit. Therefore, the arches will bend unless the dome surface is capable of providing circumferential hooping action to resist local bending (Figure 32c). When this horizontal hooping action is present, the surface behaves as a membrane and domes can be much thinner than simple arches.[37]

Figure 33 shows a radial ribbed spherical dome that carries a vertical load. The line of pressure must be redirected by horizontal forces to make it follow the radial ribs of the dome. In this example, the necessary horizontal forces are provided by hoop compression. Consequently, the edge forces are transmitted into a support subsystem along the bottom edge of the dome.

Dome surfaces behave this way because they are not “developable”; in other words, they cannot be flattened out under load (Figure 33b). Hence, they are naturally efficient and can reduce dead load to a minimum. A dome has the least weight if it carries only the membrane compression along the arches. Shape is the important factor and for a given dead load condition, it is possible to have the radial system of ribs of a shape such that natural pressure line follows the surface curvature without producing any bending moment. If this is done, no circumferential hooping force is required around the dome to carry the dead load. When live-load conditions are small compared to dead load, only small hooping resistance is required and the design is made simpler.[38]

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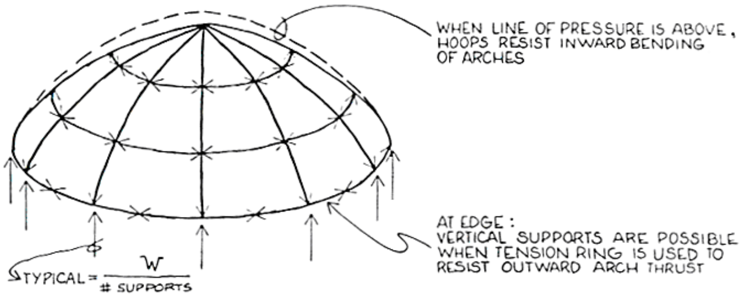
[36] Ambrose, J. E. *Building structures.* p. 39.

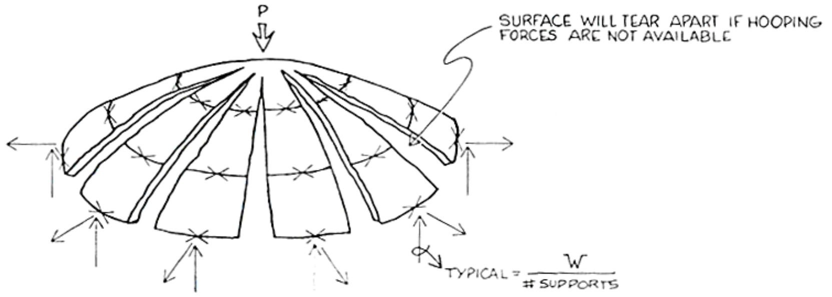
[37] Lin, T. Y. *Structural concepts and systems for architects and engineers.* p. 397.

[38] Ibid. pp. 397-398.

(a) HOOPING ACTION COMPRESSIVE WHEN LINE OF PRESSURE ABOVE SURFACE



(b) HOOPING FORCES RESULT BECAUSE DOMES ARE NOT DEVELOPABLE



(c) SHAPE MAY BE FAR AWAY FROM NATURAL LINE OF PRESSURE IF HOOP FORCES ARE ADEQUATE

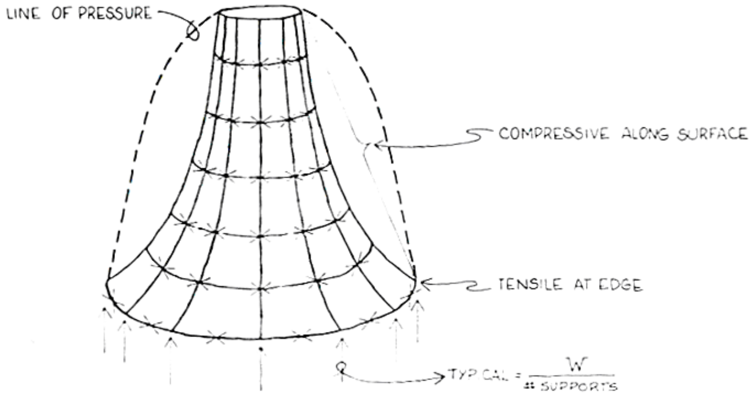


Figure 33. Hooping action for domes

(Lin, T. Y. *Structural concepts and systems for architects and engineers.* Van Nostrand Reinhold Co., New York. 1988.)

3.3 Surface Structures

Surface structures are the structures consisting of thin, extensive surfaces which function structurally primarily by resolving only internal forces within their surfaces (Figure 34). Figure 35 shows the difference between in-plane and out of plane force resolution. The wall in resisting compression, in stabilizing the building by resisting in-plane shear, and in spanning like a beam acts as a surface structure. The vault and the dome are real surface structures.[39]

The purest surface structures are the tension surfaces since they are often made of materials capable of any significant out-of-plane resistance. The canvas tent and the rubber balloon are all limited in capability to tension resistance within the planes of their surfaces. The forms they assume, then, must all be completely “pure”. In fact, the “pure” compression surface is sometimes derived by simulating it in reverse with a tension surface. Compression surfaces must necessarily be more rigid than tension ones, due to the possibility of buckling. Because of this increased stiffness, they are difficult to use in a way, which avoids developing out-of-plane bending and shear.[40]

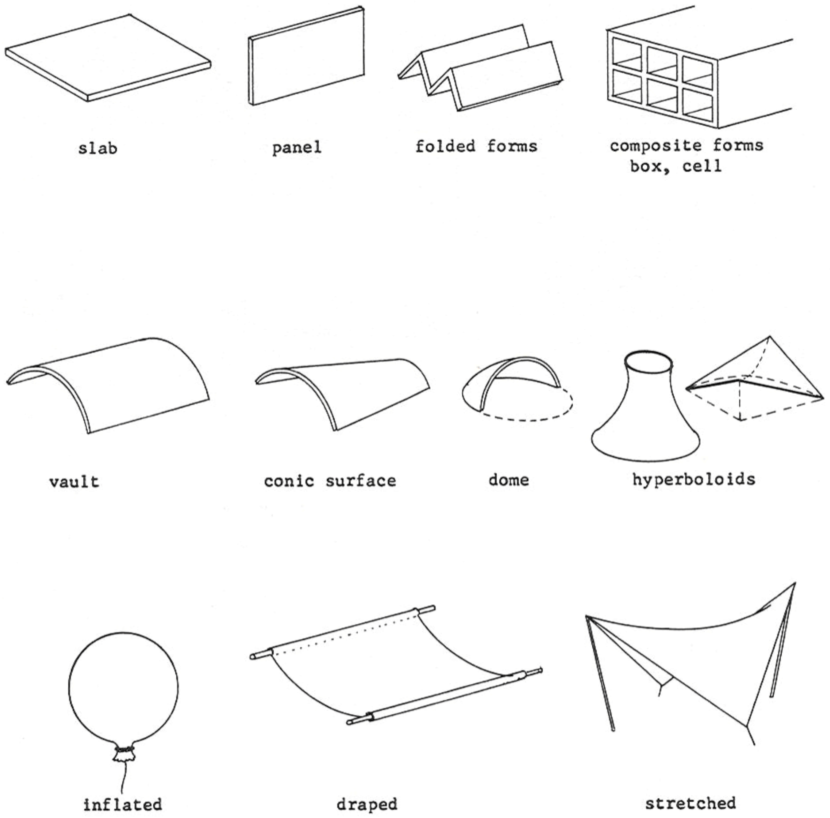


Figure 34. Surface structure –basic forms
(a) Flat surfaces (b) Curved surfaces (c) Tension surfaces

(Ambrose, J. E. *Building structures primer.* Wiley, New York. 1967.)

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[39] Ambrose, J. E. *Building structures primer.* Wiley, New York. 1967. pp. 93-96.

[40] Ibid.

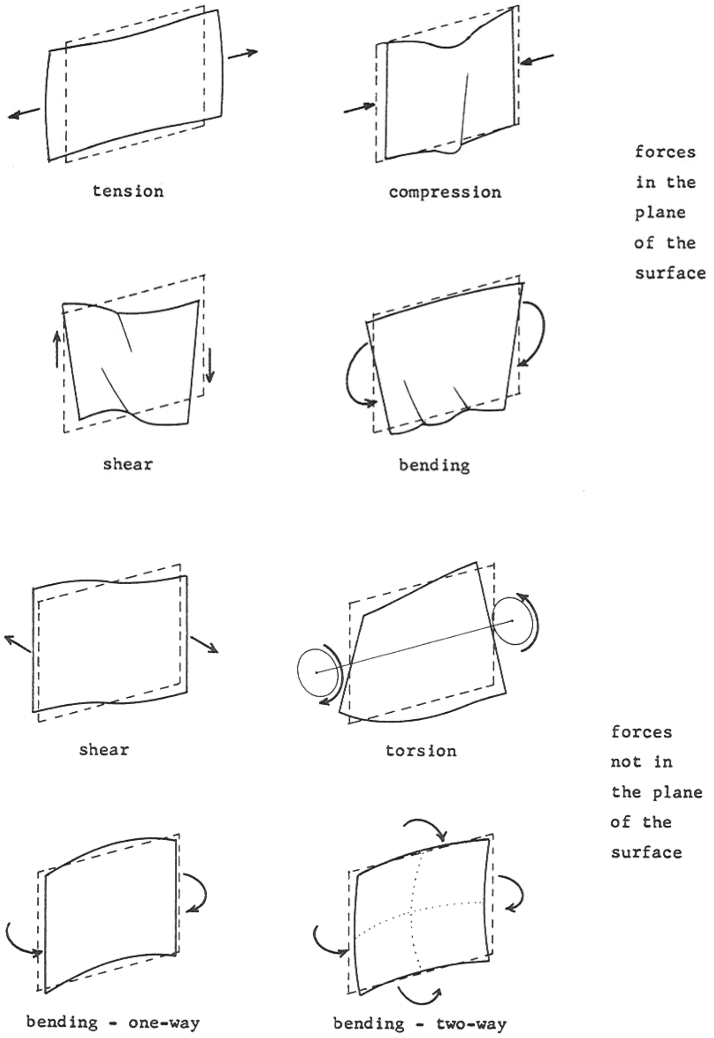

Figure 35. Surface structures – force resolution

(Ambrose, J. E. *Building structures primer.* Wiley, New York. 1967.)

Compression resistive surface structures having curved forms are called shells. The egg, the light bulb, the plastic bubble are all examples of shells. Reinforced concrete is the most appropriate material for these surfaces in the building scale. Shells are suitable for both simple and complex geometries. Edges, corners, openings and point supports are potential locations of high stress and out-of-plane bending; and as a consequence, reinforcing is often necessary –usually consisting of monolithically cast ribs in concrete. The existence of these stiffening ribs modifies the pure surface structure character of the shells and results in complex behaviors.[41]

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[41] Ibid.

3.3.1 Development of Membrane Theory

A membrane resisting forces only within its surface (Figure 36) can form a structural member. A membrane does not require thickness to resist bending or twisting moments or transverse shear forces, so the membrane can be made very thin within the constructional limitations.[42]

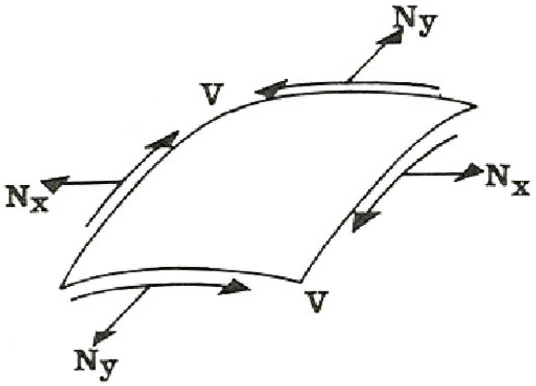

Figure 36. The forces is a membrane surface
A membrane can resist only those forces that are within its surface. The only possible membrane forces are two tensile or compressive forces N_x and N_y at right angles and shear forces V, as shown.

(Cowan, H. J. *Science and building: Structural and environmental design in the nineteenth and twentieth centuries*. John Wiley & Sons, New York. 1978.)

A really thin shell cannot be produced if there are substantial flexural stresses (Figure 37). The internal resistance moment of a section is formed by the flexural tension and compression, and the lever arm between their lines of action. The shell has to be thick enough to accommodate the lever arm. Corrugations may also be useful in some cases.[43]

A thin shell can exist without bending. This is easily demonstrated with a soap film, which is quite stable under tensile, compressive and shear forces acting in the surface of the membrane, but breaks immediately if subjected to bending. Most shells are statically determinate if there is no bending moments. In practice, a shell cannot be

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[42] Cowan, H. J. *Science and building: Structural and environmental design in the nineteenth and twentieth centuries*. John Wiley & Sons, New York. 1978. p. 151.

[43] Ibid.

supported to give statically determinate edge conditions; but for some geometric proportions, the bending stresses can be restricted to thicker edges, where the shell is supported.[44]

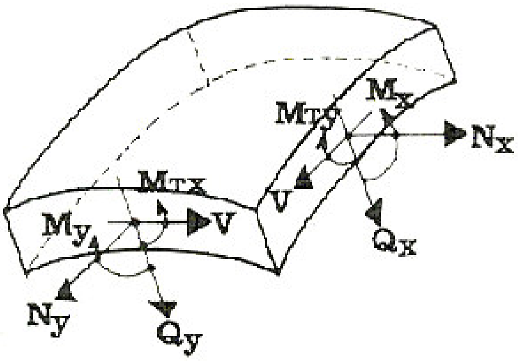

Figure 37. A thick shell
A "thick" shell can also resist bending moments at right angles to one another, M_x and M_y, twisting moments at right angles to one another, M_{Tx} and M_{Ty}, and transverse shear forces, Q_x and Q_y.

(Cowan, H. J. *Science and building: Structural and environmental design in the nineteenth and twentieth centuries*. John Wiley & Sons, New York. 1978.)

The dome over the Planetarium in Jena was probably the first reinforced concrete structure deliberately designed as a membrane by Dischinger and Bauersfeld in 1923. It is only 3 cm thick over a span of 14 m. Since it is a hemispherical dome, its horizontal reaction is fully absorbed by the hoop tension, which is resisted by the reinforcement within the surface of the shell, which is different from the heavy weight of the traditional masonry dome.[45]

While the membrane theory is stated by G. Lame and E. Clapeyron, the first application to the architectural structures was made by F. Dischinger in 1928. He derived the membrane stresses from the conditions of the equilibrium of the forces acting on an element of the shell, and obtained simple solutions for domes and cylindrical shells base don the circle, parabola, ellipse, catenary and cycloid. F. Aimond added the membrane solution for the hyperbolic paraboloid in 1936.[46]

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[44] Ibid.

[45] Ibid.

[46] Ibid.

The membrane theory applies only if the forces within the shell are compatible with the reactions at its boundaries, which is a rare condition. In the hemispherical shell of the Jena Planetarium, the hoop forces are balanced within the shell, and the meridianal forces (at right angles to hoop forces) require only a continuous vertical reaction around the circumference of the dome. If the dome is supported on a wall, the boundary conditions are satisfied. Yet, a hemispherical dome finds few opportunities in the modern architectural applications. In a shallow dome, the hoop force is still absorbed within the shell; the meridianal membrane force, on the contrary, requires an inclined reaction, which is not supplied by the supports in most buildings. It is therefore normal practice to insert a ring tie. The ring tie, however, creates bending stresses within the shell. In a shallow shell, the hoop stresses at the edge of the dome are compressive so that the shell tends to contract on loading; while the tie expands when the dome is loaded. Since the tie is joined to the shell, the shell is subjected to bending, and its thickness must be increased near the edge of the dome to accommodate the flexural stresses. Since the bending stresses depend on the relative elastic deformation of the shell and the tie, they are statically indeterminate.[47]

The same considerations are applicable for most shell forms. The cylindrical barrel vault, probably the most widely used architectural shell, requires transverse ties for stability in order to absorb the horizontal reaction of the arches at the ends of the shell. In shallow hyperbolic paraboloids, a load uniformly distributed in plan gives rise to membrane shear stresses, which are uniform over the entire shell.[48]

The bending stresses are generally important near the boundaries and gradually disappear towards the inner portion of the shell. In that case, it is possible to design the shell by the membrane theory and increase the thickness of concrete and the quantity of reinforcement empirically in the boundary region.[49]

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[47] Cowan, H. J. *An historical outline of architectural science*. Elsevier Pub. Co., New York. 1966. p. 87.

[48] Ibid.

[49] Ibid.

3.4 Suspension Structures

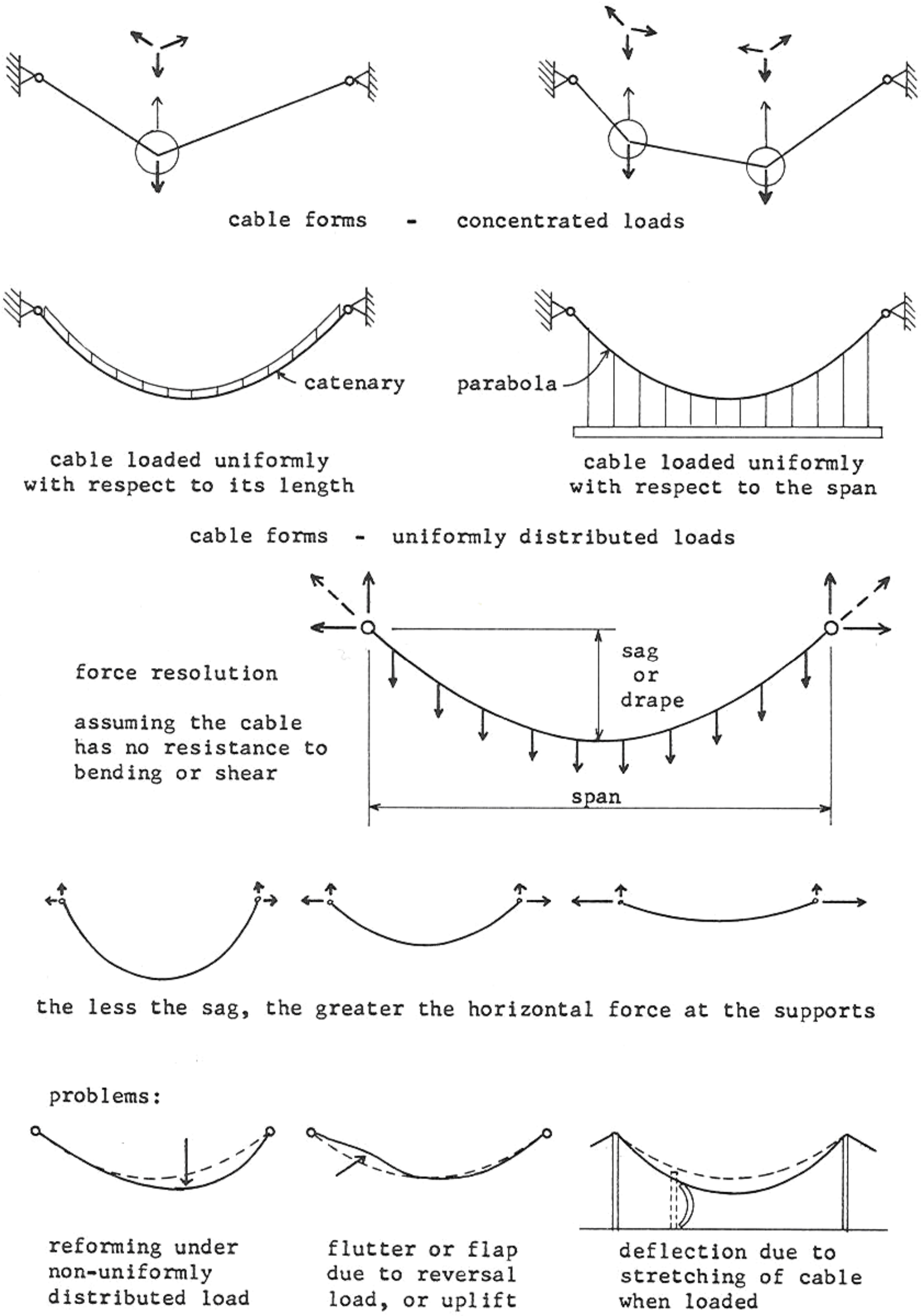

Figure 38. Basic aspects of cable systems

(Ambrose, J. E. *Building structures primer.* Wiley, New York. 1967.)

Curved forms can also be created by the suspension structures. The suspension structure was firstly used by primitive societies, using vines or strands woven from grass or shredded bamboo as the material. Human beings achieved impressive spans by these structures at that time; footbridges of 35 m span have been recorded. Yet, this system reached its peak for great span capability with the development of steel. This system developed with the development of steel as chain and link, and later the cable woven of drawn wire (Figure 38).[50]

Structurally, the single draped cable is simply the inverse of the arch in both geometry and internal force. The compression-arch parabola is turned down to produce the tension cable. Span-to-sag ratio and horizontal inward thrust at the supports have their parallels in the arch behavior.[51]

Flexible suspension structures are statically determinate (Figure 39). In a roof structure, additional cables are required at right angles to support the roof sheeting, and for stability, these cables must be interconnected as a net. The result is statically indeterminate. A high-tensile steel cable spanning 50 m, which is a relatively modest span, tensioned to the stress of 500MPa, extends 125 mm, which is not a small deformation. Moreover, it is necessary to prestress the cable at right angles due to their sensitivity to thermal movement.[52]

As a result, it is necessary to determine the geometry of a network of interconnected cables under the combined action of loads and prestressing forces.

In this chapter, the concept of curvilinearity in structuring is presented. Curvatures and their mathematical background are explained briefly. The mathematics and structural advantages of arch, vault, dome, surface and suspension structures are analyzed to be able to understand the nature of their structural behavior. All these mathematical background will be a prelude for the next chapter about the development and utilization of the curvilinear structural forms to make easier to understand that structural evolution throughout the history.

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[50] Ambrose, J. E. *Building structures primer.* Wiley, New York. 1967. p. 91.

[51] Ibid.

[52] Cowan, H. J. *Science and building: Structural and environmental design in the nineteenth and twentieth centuries.* pp. 178-179.

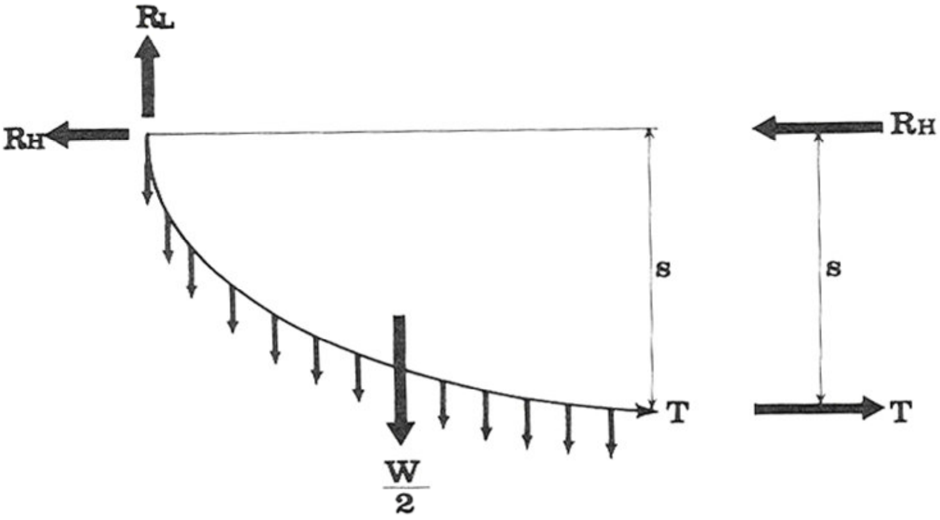

Figure 39. The diagram of a cable
Cable provides a moment resisting the vertical forces because it sags. The resistance moment is formed by the horizontal reaction R_H, the cable tension T, and the sag s.

(Cowan, H. J. *Science and building: Structural and environmental design in the nineteenth and twentieth centuries.* John Wiley & Sons, New York. 1978.)

CHAPTER 4

DEVELOPMENT AND UTILIZATION OF CURVILINEAR STRUCTURAL FORMS: A HISTORICAL OVERVIEW

In outlining the development and utilization of curvilinear structural forms throughout history, the emphasis will be mostly on structures and structural elements that have marked significant steps forward in widening the range of possible future choice. They illustrate in various ways the kinds of problems to be overcame in structural design and the solutions for overcoming them. Moreover, they mark significant steps in facing these problems and illustrate the growing freedom of choice of form that has been acquired.

It is impossible to include every example of curvilinear structures here but a number of themes relevant to the object of the thesis are selected. Issues related to layout and structural forms are fundamental to the architect and engineer when designing a building of curvilinear structure, thus referring to examples during the design process may be helpful. When referring to examples, it is important to understand the basis of the design and its relationship to other developments. It is for this reason that emphasis has been placed on history and the evolution of technological development of curvilinear structures. Examples hare selected to help to understand the overall picture of the evolution of the curvilinear forms as well as to demonstrate what is possible and what kinds of new developments may take place.

4.1 Arches

Arch, a fundamental construction system in architecture, used to span the space between walls, piers, or other supports and to create a roof or a ceiling. An arch is a rigid span curving upward between two points of support. It appears in a variety of structures, such as an arcade, formed by a row of arches, supported by load-bearing arches, a roof, a bridge, or as a single, freestanding triumphal or memorial arch.

This chapter concentrates on developments that were structurally significant rather than aesthetic or similar attempts, except where it is relevant to the developments of complete structural forms. Such developments, in the case of the arch, were ones that allowed spanning wider gaps that permit to improve strength and stability and to reduce dependency on heavy abutments that took advantage of the potentials of new materials that facilitates construction.[1]

Arches have been built since prehistoric times. The earliest attempts were probably simple adaptations of naturally occurring forms like a fallen log or a boulder wedged between two others.

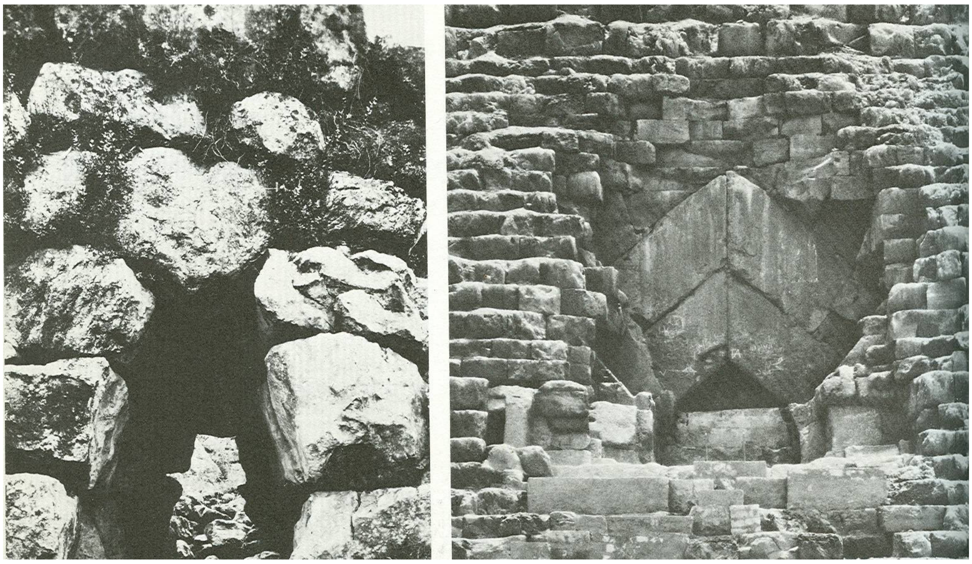

Figure 40. Bridge between Tiryns and Epidaurus (Left) and Nurth Entrance, Pyramid of Cheops (Mainstone, R. J. *Developments in structural form.* M.I.T. Press, Cambridge. 1975.)

The brick and stone arch may be taken as the first examples for this form. Among the early manmade forms that may have been contributed to this development are (1) an entirely straight copy of the wedged boulder (Figure 40a); (2) the alternative simplest

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[1] Mainstone, R. J. *Developments in structural form.* M.I.T. Press, Cambridge. 1975. p. 97.

form consisting of two long blocks of stone inclined inwards to meet as an inverted V (Figure 40b); (3) the equivalent of the latter consisting of two logs similarly inclined towards each other, or bundles of reeds set more vertically in the ground and then bent over until their free ends met and could be tied together, as in the hut (Figure 41).[2]

Figure 41. Domical huts

(Karaesmen, E. Architecture and Engineering, in: *Civil News, Boğaziçi University, Department of Civil Engineering.* (pp. 1-3). 2005, May.)

In the marshes of southern Iraq, roughly parabolic “arches” up to 6 m in span have been made by setting bundles of giant reeds in the ground in two rows spaced that far apart, then bending their heads to meet one another and tying them together. [3] Their action is not the purely compressive characteristic of pure arch action but it was probably a hint for using bricks instead of saplings or reeds.

Alongside these forms, it should be also noted the early and widespread construction of “false” arches (Figure 42) because it seems likely that the arch was developed from the use of corbels. In these arches, blocks or bricks are cantilevered horizontally on one another so that the succeeding layers reduced the span. At the end, the two

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[2] Ibid. p. 98.

[3] Maxwell, G. *People of the reeds.* Harper, New York. 1957. pp. 162-163.

sides meet at the centre. The corbels are subject to tension on the upper face. This form of construction was used more than three thousand years ago, e.g. in the subterranean Treasury of Atreus at Mycenae, ca. 1185 B.C.[4]

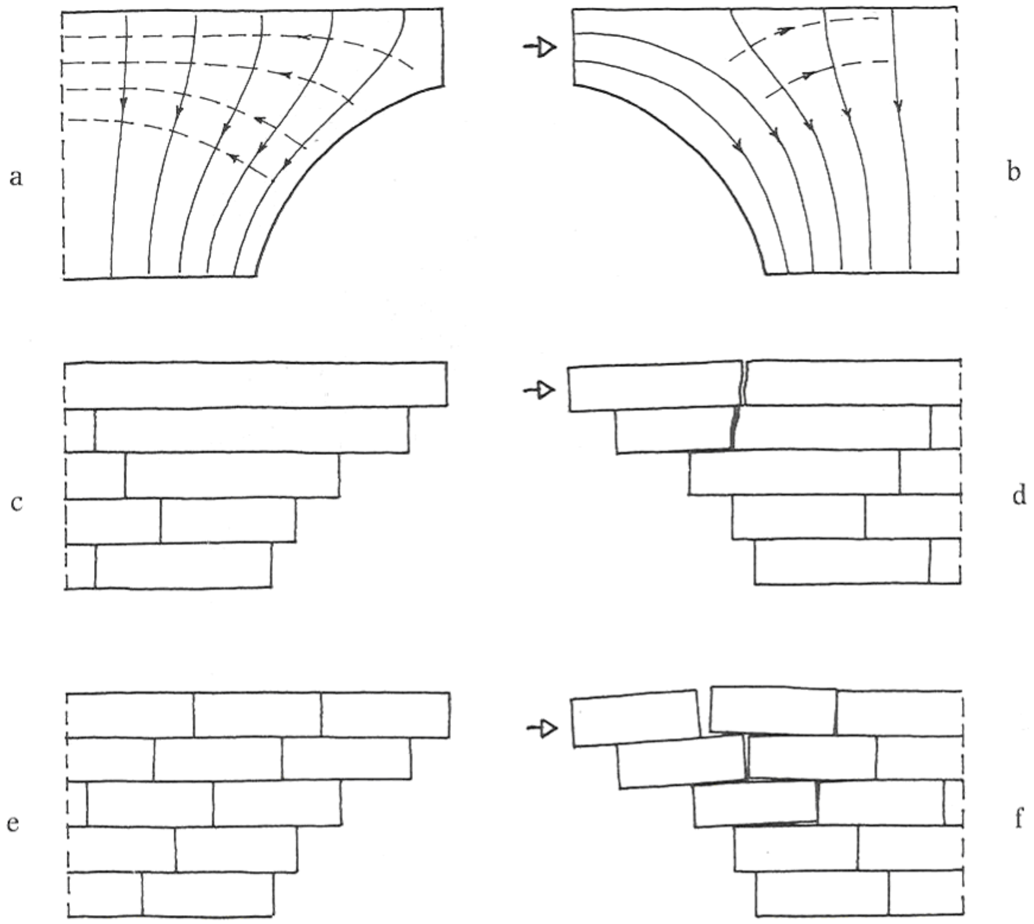

Figure 42. Internal static equilibrium of a false arch
(a, b); alternative details of construction (c, e); possible modes of collapse (d, f). Principal compressions are shown in a and b by full lines, and principal tensions by broken lines

(Mainstone, R. J. *Developments in structural form.* M.I.T. Press, Cambridge. 1975.)

It was in Greece, Rome and Etruria that true voussoir arches began to appear in stone in about the fourth century B.C. There, they not only replaced the massive lintel but also the false arch. Instead of a fairly direct copying of the brick arches of Egypt or farther east, a hint from the arch-like forms that tended to appear at intervals in the

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[4] Cowan, H. J. *An historical outline of architectural science*. Elsevier Pub. Co., New York. 1966. p. 4.

polygonal masonry often used for walls at the time, was taken. There was more experience in the use of timber.[5]

Monumental works of the classical era were not constructed with arch forms acting in pure compression. Although techniques of construction were known in classical times, the statical function and the counterbalancing of the voussoirs' thrusts did not inspire the architects of the classical Greece to use them. Mycenaean gates and tombs had triangular pseudoarches or corbelled arches, which do not act in compression. Wall gates of the classical era were constructed with arches, whose members acted as real voussoirs, and with corbelled arches. Nevertheless, geometric design intentions were still more important rather than statical function and its expression.[6]

West gate in the city wall at Falerii Novi, Italy (Figure 43) shows that the early arches were semicircular and had voussoirs of considerable depth even for very modest spans. The depth of voussoirs is close to two-thirds of the radius.[7]

Figure 43. Gateway (Porta di Giove), Falerii Novi

(http://spazioinwind.libero.it/popoli_antichi/Italici/Falerii%20Novi.html. Last accessed in December 2006.)

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[5] Boyd, T. D. The Arch and the Vault in Greek Architecture, in: American Journal of Archaeology. (pp. 83-100). 1978, Vol. 82, No. 1.

[6] Zannos, A. *Form and structure in architecture*. Van Nostrand Reinhold, New York. 1987. p. 21.

[7] Mainstone, R. J. *Developments in structural form.* p. 102.

As experience grew in the next two centuries and the great cautiousness of these very deep arches was recognized, designers became more daring. The profiles of these are still circular arcs, but the depths of the voussoirs are not much greater than one-tenth of the radius for spans up to 25 m.[8]

Apart from the type of arch seen at Falerii Novi, the Romans introduced a number of other innovations. They were; (1) the arch of pentagonal voussoirs bonded into the spandrel masonry alongside and above, or of flat-topped voussoirs constituting both arch and spandrel; (2) the flat or lintel arch; and (3) the arch of joggled voussoirs.[9]

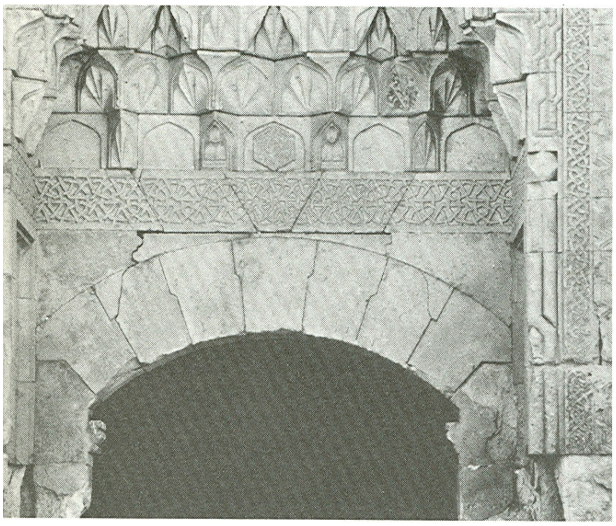

Figure 44. Doorway, Sultan Han, between Kayseri and Sivas

(Mainstone, R. J. *Developments in structural form.* M.I.T. Press, Cambridge. 1975.)

In the first, the arch could not accommodate itself to a spreading of its supports brought out by its outward thrusts simply by hinging rotations of the voussoirs. It was more likely to do so relative slipping of the voussoirs. The second was a variant of the first, with a horizontal instead of a curved soffit. Its depth had to be sufficient to contain a thrust line in equilibrium with the loads. Moreover, it should not have been so flat to cause the supports to be pushed apart far enough to permit a collapse.

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[8] Ibid.

[9] Clarke, S. *Ancient Egyptian construction and architecture.* Dover Publications, New York. 1990. p. 187.

This meant that accommodation to actual spreading was again to occur largely by relative slipping of the voussoir. The use of joggled voussoirs was more convenient to reduce the possibility of slipping. An example is illustrated in Figure 44. Joggling would also have had the advantage of making construction easier when the centring was not completely rigid. Its value in earthquakes may have been an important reason for its widespread adoption in Turkish masonry arches of the Seljuk and Ottoman periods. The extreme elaboration of the interlocking of the voussoirs in many of the Ottoman examples must, however, be considered as purely decorative.[10]

In the first century A.D., even in buildings in which concrete was extensively used for vaults, supporting arches were constructed of stone or brick over their full widths. Besides their proven strengths, such arches had overcome the need for soffit formwork and been ready sooner to stand on their own and carry other loads. These advantages probably were the reasons for their choice of use in structures like the Roman Colosseum (Figure 45) and the Pantheon.[11]

Figure 45. Roman Colosseum

(http://www.greatbuildings.com/cgi-bin/gbi.cgi/Roman_Colosseum.html/cid_aj1299_b.html. Last accessed in December 2006.)

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[10] Mainstone, R. J. *Developments in structural form.* pp. 102-103.

[11] Ibid. p. 105.

In most of the important Roman monumental vaulted buildings, columns and architraves coexisted with arches. As in all morphological constructions, the column and the architraves were three-dimensional decorative elements of wall constructions whose openings were bridged with arches or vaults. In this construction, a dissonance between form and statical function can be observed. This conflict ends when the horizontal architraves are taken away and the arches are designed as circular architraves that transfer the loads directly to columns. This solution later became a primary structural or morphological characteristic of Romanesque and Byzantine architecture.[12]

Besides the development of structural and architectural form, there were various departures from the simple circular-arc profile –usually a full semi-circle– preferred by the Romans. Structurally, the ideal profile will always be that conforms exactly to a thrust line in equilibrium with all the loads (Figure 46). If it is preferred to minimize the horizontal thrust, the rise should also be greater than it is in the semicircular arch. If it is preferred to minimize the horizontal thrust, the rise should also be greater than it is in the semicircular arch. Some of the profiles that have been actually adopted as the pointed profiles of many Gothic arches and arched ribs nearly answered these structurally desirable requirements. It seems more likely that the profiles were chosen primarily for the advantages they offered in ease of construction and in solving some of the aesthetic problems associated with groined and ribbed vaults having semicircular transverse profiles.[13] Roman arches and domes were almost invariably circular. This may have been probably because of their structural advantages for the construction; but it is more likely that the circle was used because it was regarded as the most perfect curve.[14]

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[12] Zannos, A. *Form and structure in architecture*. p. 29.

[13] Mainstone, R. J. *Developments in structural form.* p. 106.

[14] Cowan stated in his *An historical outline of architectural science that* " Even though structural evidence was to the contrary, this view was reiterated, on geometric or religious grounds, up to the late nineteenth century." And quoted from John Ruskin, Stones of Venice:

"Many architects, especially the worst, have been curious in designing out-of-the-way arches-elliptical arches, so called, and other singularities. The good architects have been generally content, and we for the present will be so, with God's arch, the arch of the rainbow and of the apparent Heaven, and which the sun shapes for us as it sets and rises."

The idea to obtain the best profile by inverting a similarly loaded hanging chain appears to have been achieved at in the latter part of the seventeenth century. In the eighteenth and nineteenth centuries, more scientific choice of form and the possibilities of masonry construction got closer to their limits. In 1903, Sejourne completed a bridge in Luxembourg with twin arch ribs of 85 m span. By then, the masonry arch lost its dominance. Iron, steel and reinforced concrete had all come into use.[15]

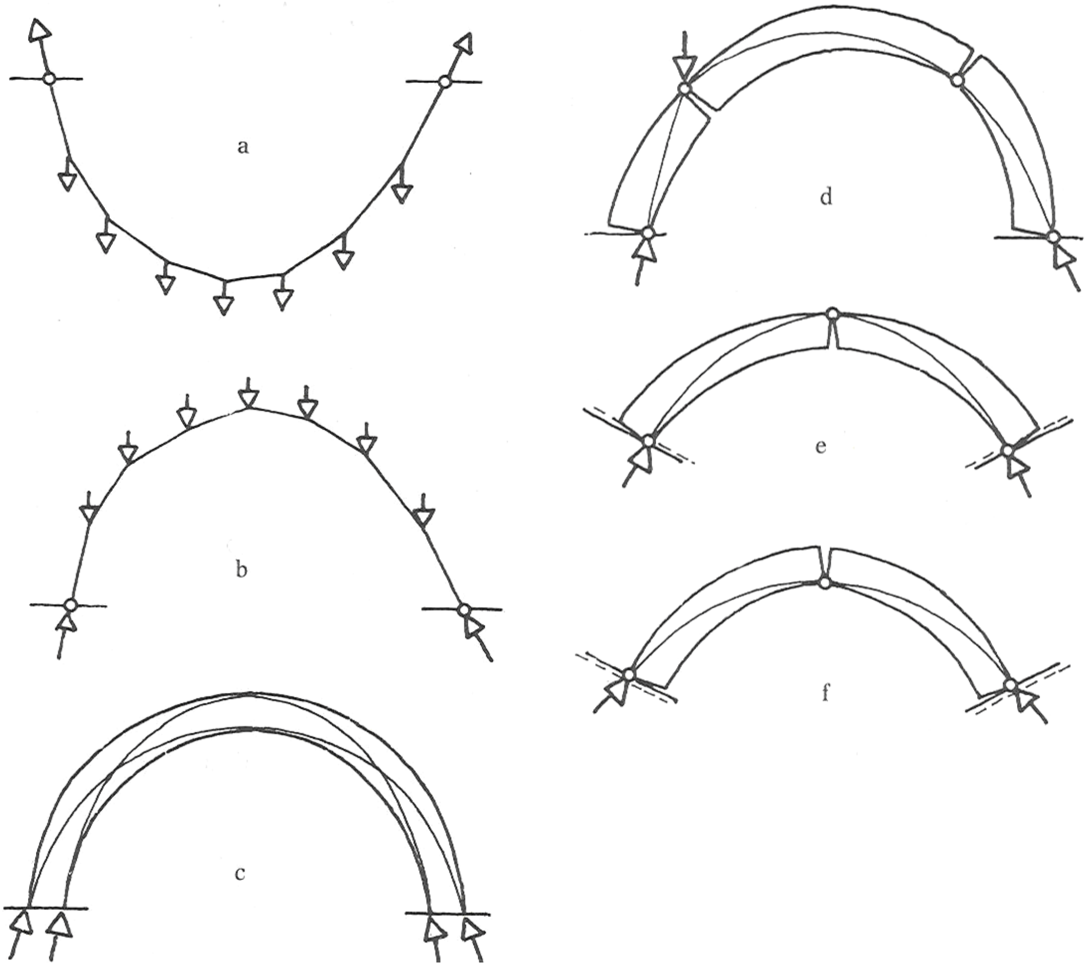

Figure 46. Static equilibrium of the catenary and arch
Continuous lines drawn within the arches c to f are possible thrust-lines.

(Mainstone, R. J. *Developments in structural form.* M.I.T. Press, Cambridge. 1975.)

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[15] Mainstone, R. J. *Developments in structural form.* p. 106.

While the masonry arch remained dominant, the only alternatives were timber, and from the latter part of the eighteenth century, cast iron. In relation to stone, brick and concrete, they have a better performance in tension. They could span wider gaps and carry higher imposed loads as simple beams than a stone lintel was able to do.[16]

The first major timber arches were built by Apollodorus of Damascus in A.D. 106. They are represented in one of the reliefs on Trajan's Column in Rome and, according to Dio Cassius, there were twenty-one spans that were more than 30 m.[17]

Some of the finest surviving medieval timber arches are in Westminster Hall, London (Figure 47). In this hall, each arch was built up from three sets of timbers side by side, originally held together by oak pins. The roof was completed in 1402 when the large timbers necessary for its span of 20.5m were still easily obtainable.[18]

Figure 47. Westminster Hall, London

(http://moment.mit.edu/imageLibrary/images/large_images/Fig18.JPG. Last accessed in December 2006.)

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[16] Ibid.

[17] Ibid. pp. 106-107.

[18] Ibid. p. 104.

The first cast-iron arches either followed almost equally closely the existing timber forms or masonry prototypes as the bridge over the Wear at Sunderland built between 1793 and 1796. In comparison with most masonry arches, a noticeable difference was that, much shallower profiles could nearly be always adopted.[19]

Wrought iron and steel, possessed higher tensile strengths and be formed into long members of uniform cross-section which could be joined together by rivets, bolds, welds to develop fully these tensile strengths, although it was weaker in compression than cast iron. As a result of their introduction, it became possible to reduce the amount of material by the advantage of reduced liability to buckling that could be achieved through a greater strength and stiffness in bending, where it was needed to span larger distances by the arched form.[20] The Gateway Arch at St Louis (Figure 48) has been an outstanding example of these type of arched forms. It has a span of 192m and is 192 m high. The profile was chosen to give fairly uniform compressive stress under dead load.

Figure 48. Gateway Arch, St Louis, Saarinen

(http://www.arrakeen.ch/usaaug98/098%20%20St.Louis%20Gateway%20Arch.JPG. Last accessed in December 2006.)

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[19] Ibid. p. 108.

[20] Ibid. pp. 108-109.

New strong concretes made from artificial cements became available in the nineteenth century. By the time that concrete arches of significant span were again constructed, embedded steel bars or rolled sections was used as reinforcement. Where bars were used as reinforcement, the forms usually closely resembled masonry forms. Where heavier rolled sections were used, the resultant form was resembled a steel arch, simply covered with concrete for protection and to give added lateral stiffness.[21]

The development of new forms, based essentially on the use of relatively thin slabs, as the most efficient elementary form for concrete reinforced with bars, was largely the work of three men towards the end of the nineteenth and in the early decades of the twentieth century. They were Hennebique, Maillart, and Freyssinet.[22]

Today, arch-like forms are still preferred by designers especially in bridge design. Lusitania Bridge, Merida designed by Santiago Calatrava, constructed between 1988 and 1991, and has an expressive structural element, a huge steel arch of 34 m high, that spans 189 m. Despite to the huge span, the structure still has a simple, elegant fluidity.[23]

4.2 Vaults

Longer spans were needed extensively through medieval periods with structural components covering voluminous spaces. The **vault** representing a more advanced structural from conceptually and technologically was developed to satisfy the needs of those facilities. [24]

It is not easy to specify the date of the invention of the vault although many ancient vaults still exist. In Thebes, vaults can be found that date back to 2200 B.C. In

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[21] Ibid. p. 110.

[22] Ibid.

[23] Sharp, D, ed. *Santiago Calatrava.* E & FN Spon, London. 1994. p. 37.

[24] Karaesmen, E. *CE 480 Introduction to architectural engineering, Lecture notes.* Boğaziçi University, Civil Engineering Department. Istanbul. 2006.

Europe, besides the well-known vault forms of Orchomenos and Mycenae, which date from 3000-1500 B.C., there exist even earlier Neolithic remains (5600 B.C.) in the village of Khirokitia in Cyprus. In the classical Greek era, vaults were used very seldom, and mostly in technical rather than in architectural works. It is not easy to state exactly how the vault was invented. However, judging by the time of its use as well as by its function, the "false" vault may be regarded as the forerunner of the vault. Figure 49 shows the structures of vaults built in Mesopotamia during the Parthian and the Sassanid eras. These examples show that ancient builders invented the vault intuitively by way of the corbelled vault.[25]

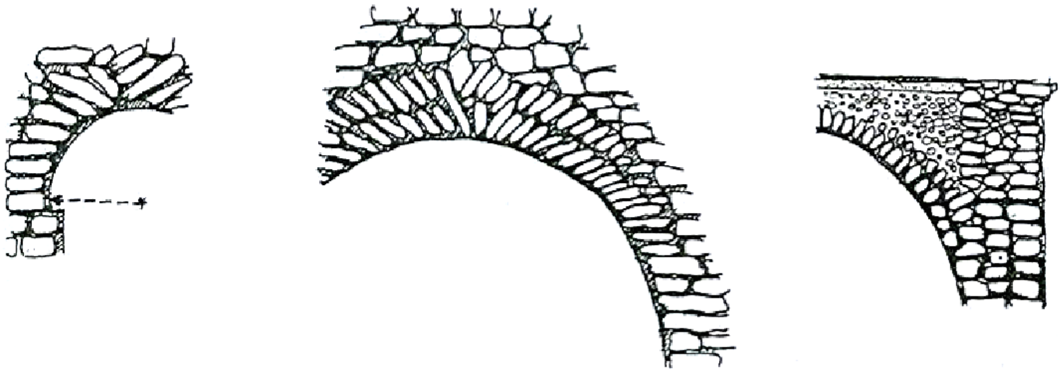

Figure 49. Vault construction systems in Mesopotamia
The stones at the base are positioned as in a corbelled vault system of construction, while near the top they are placed almost like voussoirs.

(Zannos, A. *Form and structure in architecture.* Van Nostrand Reinhold, New York. 1987.)

Vault constructions had been used, mainly by the Sumerians, the Persians, and the Assyrians. The origins of Roman and Byzantine vault construction must be sought in the East. The Roman contribution to vault construction came later. The Romans established the vault not only as a system of construction but also as a morphological element of monumental architecture.[26]

Seljukian works should be referred as the most striking examples of this evolution. Seljukian vault should be considered as a major step in the history of construction.

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[25] Zannos, A. *Form and structure in architecture*. pp. 56-57.

[26] Ibid. p. 31.

On Anatolia, few of them are still standing. There are also remains with partially injured components through wars and disasters especially earthquakes. Karatay Medresesi in Konya is of the outstanding examples at Anatolia.[27]

While Roman architecture used the arch and the vault in non-existence of or within a post-and-lintel system, the vault became the dominant feature in following eras. The statical function of the vault became the central theme of expression in Byzantine, Romanesque, and Gothic architecture.[28]

The early developments of groined vaults were closely parallel to the dome. To construct them in cut stone introduced on the complex cutting of the blocks forming the groins. This is well illustrated in the Tomb of Theodoric at Ravenna. Casting throughout in concrete prevented this difficulty. Groined vaults became commonplace in and around Rome during the first century A.D. Later, embedded brick ribs were introduced at the groins as they had been introduced into the continuous surfaces of the concrete domes and barrel vaults. Examples can be seen in Severan substructures in Palantine and in the Baths of Diocletian. Spans exceeding 20 m were achieved.[29]

While it was difficult to cut the stones for the groins of the vault, it was more difficult to form adequate groins in a brick vault of the same surface geometry as the Roman concrete vaults. Many Romanesque groined vaults were constructed in a manner not very different from early Roman concrete construction. The blocks were laid as voussoirs, but were only roughly cut to shape before being bedded in large quantities of mortar.[30]

The vault-construction system, established during the Roman period, reached its most important expression in Byzantine and Gothic architecture. When buildings of

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[27] Karaesmen, E. *CE 480 Introduction to architectural engineering, Lecture notes.* Boğaziçi University, Civil Engineering Department. Istanbul. 2006.

[28] Ibid. pp. 31-32.

[29] Mainstone, R. J. *Developments in structural form.* p. 130.

[30] Ibid.

the Byzantine, Romanesque, and Gothic styles are studied, it is realized that vault-construction systems can express the action of forces. The forms of these works tell how forces balance, how they change direction etc. In these works, aesthetic appreciation derives from the expression of statical function. Statical function in Byzantine architecture is, of course, expressed in a very different way from that of classical Greek architecture, because the statical function and geometry of these two systems are so different. In classical Greek architecture, the articulated members of the building bring out the fact that each one functioned in a different way. In Byzantine architecture, the entire building had a uniform statical function visually accessible to the observer. In Gothic architecture, creative expression was inspired by the art of transferring forces, but differently from Byzantine architecture. In the West, the dominant character was analytic and rational, seeking for the technical perfection. Thus, the Romanesque style developed independently in the West and was transformed into linear forms that became continuously thinner and taller. Vaults and groin vaults were articulated into arches and ribbed vaults, walls separated into a skeleton of columns. During the Renaissance, there was an increasing tendency to express forces by means of building forms. This tendency developed gradually until it finally dominated and characterized Baroque architecture. The curve, in both the sections and plans of the façade became an undulated curved surface in Baroque architecture. The most important construction system of the Renaissance was the vault construction system. It has been widely used since the ancient times and in monumental structures, such as the domes of the Florence Cathedral and St. Peter's in Rome.[31]

In the simple groined vault, the support roles of the missing triangular sections of the two intersecting barrels are taken over by diagonal arches created at the groins within the thickness of the vault. These arches carry a much more concentrated load than the rest of the vault. The groins are naturally stiff like any other crease in a surface, so that there is a little risk of their buckling.[32]

The alternative method was to construct diagonal ribs at the groins before filling in the intervening webs or severies. This method was adapted by Gothic builders and

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[31] Ibid. pp. 37-50.

[32] Mainstone, R. J. *Developments in structural form.* p. 130.

had far-reaching consequences. Ribs constructed in this way, had been used in the Islamic world since the tenth century, only in connection with domes and domical vaults (Figure 50). They were used in places where there was no need for them structurally.[33]

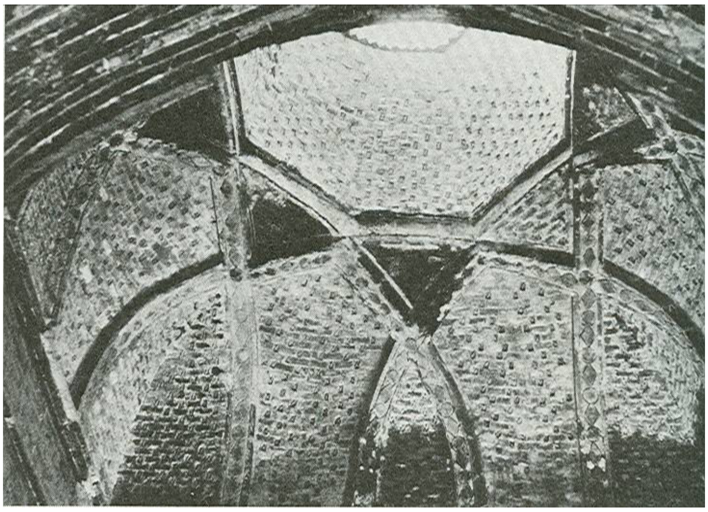

Figure 50. Small vault of the library, Friday Mosque, Isfahan

(Mainstone, R. J. *Developments in structural form.* M.I.T. Press, Cambridge. 1975.)

When applied to a groined vault, such ribs had a much important role. Initially, they were used primarily as cover strips for irregularities in the groins, and secondarily as aids to construction and as potential stiffeners in the vault. As fully developed in the latter part of the twelfth and early thirteenth centuries, they became an essential constructional aid having importance both structurally and aesthetically.[34]

The later elaboration of the basic diagonal rib system by adding more and more ribs would similarly have helped during construction, as it reduced the sizes of the compartment between the ribs and as a result simplified the construction of the webs. It would also have stiffened the webs without the need to be arched up in the direction of spanning. In the fourteenth, fifteenth and sixteenth centuries, in central Europe,

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[33] Godard, A. *The art of Iran.* Praeger, New York. 1965. pp. 259-325.

[34] Mainstone, R. J. *Developments in structural form.* p. 131.

the continuity of the whole vaulting-system was emphasized by the ribs spreading over its surface as a continuous net.[35]

Figure 51. Bibliotheque Nationale de Paris

(http://www.edithcaldwell.com/gallery.php?category=4&id=73. Last accessed in December 2006.)

Some nineteenth-century works, such as the Bibliotheque Sainte-Genevieve (1843) and the Bibliotheque Nationale de Paris (1861) By Pierre Francois Henri Labrouste are among the first buildings that made use of new materials such as iron. In these buildings, iron replaced stone, but this change did not change the system of construction and the form of the building: the bearing system is still of the vault construction type (Figure 51).[36]

Viollet-le-Duc was a French architect and theorist, famous for his restorations of medieval buildings. He had an important role in the Gothic Revival in France as he was in the public discourse on “honesty” in architecture, which eventually transcended all revival styles, to inform the moving spirit of Modernism. Viollet-le-Duc applied the lessons he had derived from Gothic architecture, seeing beneath the atmospheric appeal that drew his British contemporaries to especially what he conceived of its ra-

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[35] Ibid.

[36] Zannos, A. *Form and structure in architecture.* pp. 52-53.

tional structural systems, to modern building materials such as cast iron in vaults. His approach to both medieval and modern architecture was severely rational. Basic intervention theories of historic preservation are framed in the dualism of the retention of the status quo versus a "restoration" that creates something that never actually existed in the past. Viollet-le-Duc wrote that restoration is a "means to reestablish (a building) to a finished state, which may in fact never have actually existed at any given time."[37]

4.3 Domes

Dome is the last step of a long walk starting from "arch" and making an intermediate step at "vault". As sheltering is one of the basic requirements for human beings to survive, very simple dome-shaped huts constructed with reeds and timbers covered with turfs etc. were probably the earliest man-made forms.[38] The materials were probably transformed to solid rammed earth or mud, then, into mud-brick or stone by the time. The first conical domes of mud or mud-brick, have survived from as early as the sixth or fifth century B.C. Stone is a more durable material, and was being used for the dome by about the middle of the second century B.C. By 1330 B.C., monumental proportions for this time were achieved in "Treasury of Atreus" or "Tomb of Agemennon" at Mycenae, with a diameter at the base of 14.5 m. At this time, it was getting widespread in southern and Western Europe, too.[39]

The early stone domes, in particular, were false domes, constructed like the false arches. They were always capped by a single larger stone.[40] Some early brick domes were constructed in a slightly different way. The horizontal brickwork was arching up a little at two opposite points on the circumference of the circular base wall and canting it forwards a little. Successive courses made two conical fans that met each other above the transverse diameter.[41]

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[37] Viollet-le-Duc, E. *The foundations of architecture*. George Braziller, New York. 1854. p. 195.

[38] Rapoport, A. *House form and culture.* Prentice-Hall, New Jersey. 1969. pp. 19-20.

[39] Mainstone, R. J. *Developments in structural form.* p. 116.

[40] Ibid.

[41] Frankfort, H. *Tell Asmar and Khafaje; the first season's work in Eshnunna.* Univ. of Chicago Press, Chicago. 1932.

An advantage of these freehand methods of constructions was that they were suitable for to variations on the basic circular plan. It was not essential, for the horizontal projections of the individual blocks of a false dome to be the same at all the points in each horizontal course right from the start. When covering a room that was square in plan, it was merely necessary to start the forward projection at a lower level in the corners than elsewhere and then to extent it progressively towards the centers of the sides until a near-circular base was achieved and finally the dome is completed. Dwellings with corbelled domes in Harran, Turkey are the later examples for this technique (Figure 52).[42]

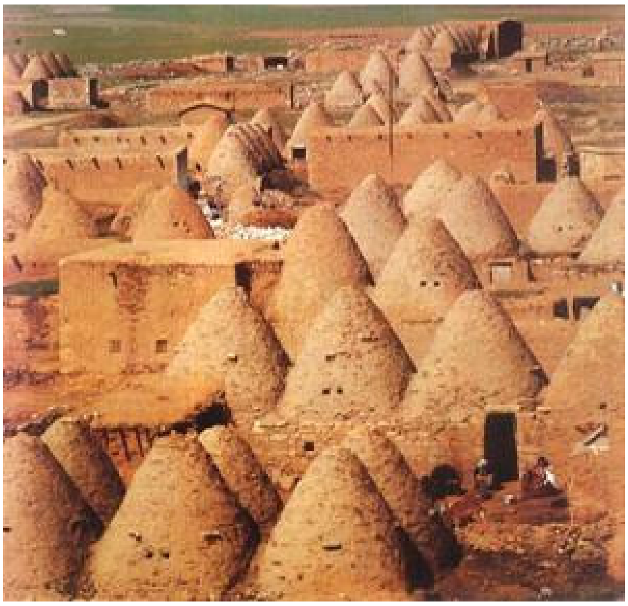

Figure 52. Dwellings with corbelled domes in Harran, Turkey

(http://lostingrovont.typepad.com/photos/uncategorized/harran.jpg. Last accessed in December 2006.)

The true dome of cut stone –with each stone bedded more or less at right angles to the profile of the inner surface- come later than the voussoir arch, and did not develop directly from it. There were probably several reasons for it. One was the greater difficulty of cutting the blocks to fit closely each other. Another was the fact that individual blocks in the upper part of the dome would tend to slide forwards until it was complete. These reasons were probably responsible for the long-continued

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[42] Mainstone, R. J. *Developments in structural form.* p. 117.

use of the false form in places where its construction was further simplified by the ready availability of a stone easily cut into long squared blocks. In addition, there were not a strong demand for the improvement of this form in the third, second and first centuries B.C. Despite the wide distribution of simple circular hut forms, large circular buildings or spaces within buildings were rare, at that time.[43]

The early development of the concrete dome is a little obscure. The earliest surviving of concrete domes are those of the *frigidaria* or cooling-rooms of the Stabian and Forum Baths at Pompeii, constructed in the late second or early first century B.C. They have a conical form with the open eye at the top that was also typical of later Roman concrete domes. They are about 6 m in diameter.[44]

A hundred years later, this conical form had transformed into an internally hemi-spherical one, as in the Temple of Mercury at Baia. Being also a bath, its dome again rose to an open central eye from a circular base, but its span of 21.5 m greatly exceeded the earlier domes at Pompeii.[45]

To realize the architectural and structural potential of the basic domical form, it is necessary to look at to the period between the reconstruction under Nero that followed the great fire in Rome in A.D. 64 and the completion of the Pantheon as it now stands under Hadrian in about A.D. 128.[46]

The dome of the Pantheon constructed between A.D. 118 and 126 had an internal diameter of 43.3 m remained unequalled until the Renaissance. Internally, it rose from its circular base as a hemisphere, though the mass was lightened in the lower parts and its surface was broken up by five rows of coffers diminishing in height towards the top. As seen in the drawing (Figure 53), neither the cylindrical drum that supports the dome nor the dome itself is a simple solid mass of concrete. The drum

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[43] Ibid.

[44] Vitruvius, P. *On architecture.* Dover Publications, Inc., New York. 1960. pp. 157-159.

[45] Maiuri, A. *Pompeii.* La Libreria Dello Stato, Rome. 1949. pp. 241-253.

[46] MacDonald, W. L. The architecture of the Roman Empire. Yale University Press, New Haven. 1986. p. 3.

consists of eight bays, each with a large opening to the interior and one of them open also to the exterior for access, alternating with eight others with closed internal voids. All these openings and voids are bridged by deep arches. The lower third of the dome, which is coffered internally but appears externally to be a simple continuation of the drum, has corresponding internal voids, similarly arched over. Above these voids, where it has a stepped profile externally, it is constructed in layers with progressively lighter *caementa* as it rises towards an open eye at the top.[47] This dome, served as the model not only for most subsequent Roman concrete domes but also, less directly, for many constructed of other materials.

The earlier architecture was firstly concerned with mass and sculptural form rather than the internal space. There were, of course, some tendencies to emphasize the interior as in assembly halls, basilicas etc but these tendencies were limited by the spanning limitations of simple beam and truss types of roof. The Roman concrete dome tackled with these limitations and permitted an architecture of large unencumbered interiors.[48]

Structurally the new forms were almost as revolutionary. Yet, they were still not benefiting from the increased possibilities of structural action that were conferred on them by their double curvature. Their thickness at the crown or eye was usually between one-tenth and one-fifteenth of the radius. Full exploitation of the double curvature might have permitted a reduction of these thicknesses to about 1/200th of the radius provided that the other conditions were appropriate.[49]

Transition from non-circular plans to domical forms had been always a problem. At the small spans of the four supporting arches at Jerash, the problem of the transition could be avoided, by making the radius of the dome itself considerably greater than that of its circular springing-line at the level of the arch crowns and simply carrying it down with the same radius to the points where the arches met. When this proce-

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[47] Mainstone, R. J. *Structure in architecture: history, design, and innovation.* Aldershot, Hampshire; Brookfield. 1999. p. 150.

[48] Mainstone, R. J. *Developments in structural form.* pp. 119-120.

[49] Ibid. p. 120.

dure is followed, the parts below the circular springing-line of the dome proper are referred to as merging pendentives. Its drawbacks were the increase in radius and more difficult construction because of the flatter profile of the dome.[50]

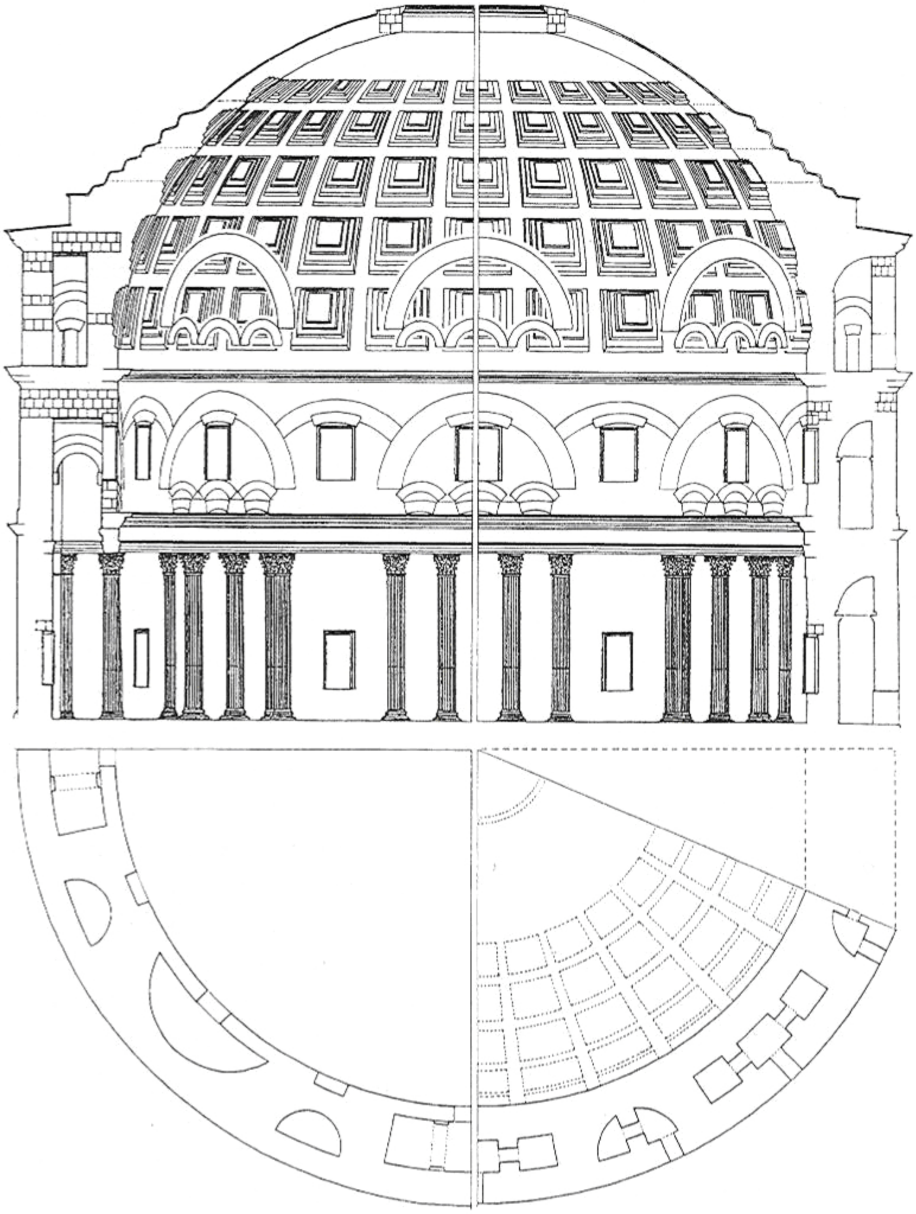

Figure 53. The Pantheon's internal structure seen in elevation and vertical section and in plan at the intermediate level and the springing level of the dome

(Mainstone, R. J. *Structure in architecture: history, design, and innovation.* Aldershot, Hampshire; Brookfield. 1999.)

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[50] Ibid. p. 123.

One way of avoiding these drawbacks was to build up similar part-spherical triangular pendentives, but afterwards to continue construction with more vertical springings and a reduced radius of curvature. This was the method adopted for the sixth-century early Byzantine dome of Hagia Sophia in Istanbul (Figure 54). The other way, usually favored by Islamic architects was to first span the corners of the ground plan by means of secondary arches named as squinches as in the fine eleventh-century Seljuk dome of the Gunbad-i-Kharka in the Friday Mosque in Isfahan.[51]

Figure 54. Dome of Hagia Sophia, Istanbul

(http://www.columbia.edu/cu/gsapp/BT/EEI/MASONRY/masonry1.html. Last accessed in December 2006.)

In overall design, there were two important innovations. The first was the incorporation of circumferential ties around the base. The second was the dome formed of two distinct shells separated by a void. It had the advantages of divorcing weathering surface from the inner shell, and of reducing the weight. It also permitted an increase in the external size and height of the dome to make it more imposing without necessarily increasing the internal height. The earliest known masonry double domes were Iranian tomb towers from the eleventh century.[52]

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[51] Ibid.

[52] Creswell, K. A. *Early muslim architecture: Umayyad, early Abbasids and Tulunids.* Clarendon Pr., Oxford. 1932. pp. 65-131.

Hagia Sophia, Istanbul, by Isidoros and Anthemios was constructed between A.D. 532 and 537 but the dome was replaced in 563 after an earthquake. The great dome of the Hagia Sophia is 31 m in diameter. The church is almost square in plan to the level of the gallery roofs, if the later peripheral buttressing and of four ramps which projected from the four corners to give access to galleries are divested. Above a lesser square in the centre, with sides averaging 30.975 m. measured between the skirtings, the dome is carried on the arches and pendentives spanning between four main piers. At the east and west its outward thrusts are counterbalanced and carried downward and outward by the inward thrusts of the two great semidomes which abut the main eastern and western arches and are themselves carried on secondary arches spanning between the main piers and secondary piers lying further back on the extremities of the main square. The construction material of the building, with only minor exceptions is brick, with courses of stone inserted at the springings of arches and vaults.[53]

The dome of the Roman Pantheon marked the main peak of a period of intensive development of wide-spanning concrete vaults. This dome was competed but never challenged in the subsequent years of the Western Empire. In the east, Justinian's church of St. Sophia in Istanbul, where the central dome in which there were crowned a complex system of billowing semidomes had a diameter nearly the two thirds of that of the Pantheon.

The outstanding double dome in the West was Brunelleschi's octagonal domical vault over the crossing of Florence Cathedral, Santa Maria del Fiore constructed between 1420 and 1436 (Figure 55). Brunelleschi created the dome 84 m above the ground with a diameter of 42 m and a rise of 32 m. the dome was octagonal on the outside but covered up its true construction on the inside. He developed a framework of nine circles within the external octagonal envelope of the dome. The dome consisted of two shells, which contained inner horizontal rings or circular arches. The components of the arch were held together by forces induced by their own weight and when the arch was finished, it became stable.[54] With the completion of the dome

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[53] Mainstone, R. J. *Structure in architecture: history, design, and innovation.* pp. 23-28.

[54] Margolius, I. *Architects + engineers = structures.* Wiley-Academy, Great Britain. 2002. p. 22.

of Santa Maria del Fiore in Florence was the pre-eminence of the Pantheon's dome lost.

Figure 55. The dome of Santa Maria del Fiore

(http://cv.uoc.es/~991_04_005_01_web/fitxer/cupula1.gif. Last accessed in December 2006.)

The *dome* was an engineering achievement with its dimensions and stiffness besides its symbolic and religious meaning in the Ottoman Empire; thus, it is unavoidable that throughout the history of Ottoman's, Mimar Koca Sinan, who is a master builder with his outstanding arched and domed structures, worked as the chief architect for sultans Selim I, Süleyman I, Selim II and Murad III.

Sinan was interested in the concept of unity within his structures, as a consequence, he burdened an active structural task for the members of the secondary structural system as semidomes etc, and he integrated the secondary structural system with the primary structure, the dome. The Şehzade Mosque, constructed between 1543 and 1548, is an important example in the evolution of this principle in a rationalist manner. In Süleymaniye Mosque, which has similarities with Hagia Sophia in the structural manner, Sinan had improved the idea of unity in structure and this interference resulted in a more dynamic relationship with the primary and secondary structural system. Edirnekapı Mihrimah Sultan Mosque (Figure 56), which is one of the most impressive buildings of Sinan, derives attention with its purity in its morphology and structure. Selimiye Mosque is the most nominative and hence the most mature one according to the structural system. Although the dimensions of Sinan's domes were not as large as those in Pantheon or Florence Cathedral, devising his architecture on dome structures and his consistency provides him a privileged position.[55]

Figure 56. Mihrimah Sultan Mosque, Istanbul

(http://cmes.hmdc.harvard.edu/ecmes/photo/cultural_exchange. Last accessed in December 2006.)

The Saint Peter's Basilica was completed in 1590 with the contribution of many designers during the design process. The dome, out of masonry with iron tension rings, has a height of 137 m and diameter of 42 m. The original iron tension rings placed in

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[55] Erzen, J. N. *Mimar Sinan cami ve külliyeleri: tasarim süreci üzerine bir inceleme.* ODTU Mimarlik Fakultesi, Ankara. 1991. pp. 69-78.

the masonry dome by Michelangelo were apparently insufficient so that serious cracking appeared in the dome by the 18th century. When cracks occurred on St. Peter's dome, Giovanni Poleni tried to check the shape of the dome by using the sketch shown in Figure 57 and by experimenting on its mirror image. He constructed a scale model of the antifunicular (inverted funicular) curve of the dome by suspending balls from a string; the weight of each ball was proportionate to the weight of the corresponding voussoir. The publication of his findings in 1748 represented a first application of a funicular curve to the design of the thrust line of a dome. This method is commonly used today to determine the shape of arch, shell or membrane structures.[56]

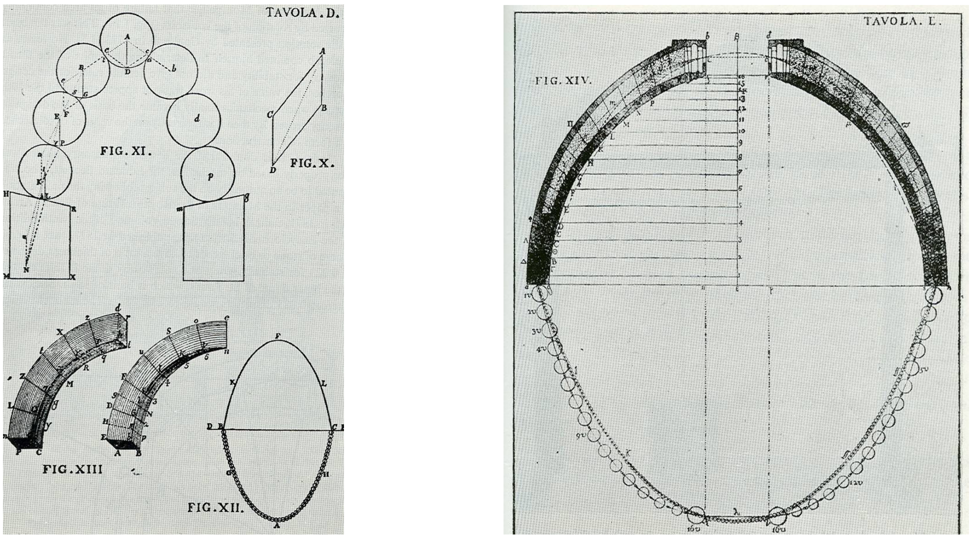

Figure 57. Poleni's use of the catenary for the solution of the masonry arch and his solution to the correct line of thrust for the Dome of St. Peter's

(Cowan, H. J. *An historical outline of architectural science.* Elsevier Pub. Co., New York. 1966.)

The majority of later double or multiple masonry domes were constructed with more widely spaced shells that were, structurally, independent of one another. This gave much greater freedom for the Baroque desire for impressive height externally. St Mark's in Venice is an example for this.[57]

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[56] Zannos, A. *Form and structure in architecture.* pp. 50-51.

[57] Mainstone, R. J. *Developments in structural form.* p. 129.

While masonry domes and vaults of the traditional type were still built in the early 19th and 20th century, a number of remarkable curved iron structures, such as Joseph Paxton's Crystal Palace in London 1851 and Henri Labrouste's Bibliotheque Nationale in Paris in 1868 were produced. Iron and steel construction was considered as an assembly of linear elements formed into curves as required, and this emphasis continued at the time when the first curve reinforced concrete structures were built. The dome of the Melbourne Public Library, at the time of its construction in 1911 the largest reinforced concrete dome in the world, was designed as a ribbed structure.[58]

In the following year, the completion of the Centenary Hall in Breslau marked a turning point. It was the first concrete building which surpassed the Pantheon in size (in steel this had been achieved half a century earlier), and the for the next forty years there was a marked tendency towards the reduction in weight and greater elegance, rather than an increase in span.[59]

4.4 Shells

Although there were a few earlier experiments, the real further development of the thin shell began with the construction of a planetarium dome and then of a larger and flatter dome at Jena in the early 1920's. These were constructed by erecting first triangulated nets of light steel bars, then suspending interior formwork from these and spraying on a thin layer of concrete. The concrete was only 30 mm thick in the first dome and 60 mm in the second for spans of 16 m and 40 m, respectively.[60] Being a hemispherical dome, its horizontal reaction is fully absorbed by the hoop tension, which is resisted by the reinforcement within the surface of the shell. This may be contrasted with heavy weight of the traditional masonry dome.[61]

Afterwards, the theoretical and practical advantages of the hyperbolic paraboloid having been realized (ideally a uniform state of stress throughout the shell, coupled

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[58] Cowan, H. J. *An historical outline of architectural science*. p. 82.

[59] Ibid.

[60] Mainstone, R. J. *Developments in structural form.* p. 134.

[61] Cowan, H. J. *An historical outline of architectural science*. p. 85.

with the possibility of generating the surface by straight lines only); thin shells of this form were also constructed. Accordingly, by the mid 1930s, most of the basic possibilities of using reinforced concrete to construct true shell forms that benefited from the inherent stiffness of double curvature had been explored, including the use of prestressed ties where necessary. However, these developments had been limited in the use of structures like factories and market halls, for a time. [62]

The main post-war development for the shells is the introduction of prestressing, which enables it to be lifted off the formwork; the creation of free shell forms (Figure 58) and a great increase in the maximum span. The most promising of the new shell forms are the hyperbolic paraboloid and the conoid. Both can be formed by straight lines, which greatly reduce the cost of the formwork, different from the dome.[63]

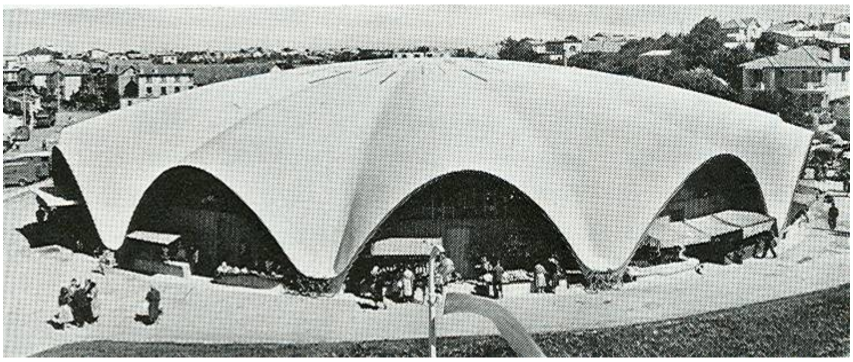

Figure 58. Market Hall at Royan, France
A free shaped dome, 60 m in diameter.

(Cowan, H. J. *An historical outline of architectural science.* Elsevier Pub. Co., New York. 1966.)

Architectural interest was really awakened about twenty years later. (Gaudi's much earlier proposals to use forms very similar to the hyperbolic paraboloid for the vaults remained unexecuted). This awakening started with the Saarinen's Kresge Audotorium, Yamasaki's St Louis Air Terminal, and Candela's Church of the Miraculous Virgin, all constructed in 1954-5.[64]

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[62] Mainstone, R. J. *Developments in structural form.* p. 134.

[63] Cowan, H. J. *An historical outline of architectural science.* p. 90.

[64] Mainstone, R. J. *Developments in structural form.* p. 134.

Candela broke the monopoly of academic science on thin shells, and showed how beauty and utility could combine and open up limitless new possibilities of form. After some early explorations, he settled on one type of form, the hyperbolic paraboloid. Like all structural artists, Candela had some difficulties with some of his works from which he learned and improved. His success came primarily from his central aesthetic motive and his recognition that thin shell behavior could only come from observations of full-scale structures in service. For Candela, thin shell design was not stimulated by thin-shell theory. He used only the simplest type of mathematical theory, called the membrane theory.[65]

The St Louis Air Terminal was roofed by three structural independent thin-groined vaults set side by side and separated by the triangular panels of glazing. The free edges and the groins were stiffened and strengthened by external ribs. Since the diagonal ribs collected the most of the loads, the four-point support of each shell was quite adequate.[66]

Two years later, Nervi's smaller Sports Palace and Esquillan's CNIT Hall (Figure 59) broke new ground structurally in two ways. In the former, Nervi adapted to the requirements of large flat domes to eliminate the need for soffit formwork. The entire soffits were formed from precast units, which also served as stiffening ribs for the thin in-situ shells subsequently cast on top.[67]

In the CNIT Hall, a complete double shell was constructed to obtain the necessary additional stiffness required for the great span of 206 m, the relatively low rise, and the basic choice of form. The form approximated to a triangular groined vault with three horizontal ridges and very slight circumferential curvatures below the ridges so that the loads would be transmitted as directly as possible to the three supports. To

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[65] Billington, D. P. *The tower and the bridge: the new art of structural engineering.* Princeton University, Princeton. 1983. pp. 190-192.

[66] Mainstone, R. J. *Developments in structural form.* pp. 135-136.

[67] Billington, D. P. *The tower and the bridge: the new art of structural engineering.* pp. 176-181.

make the two shells, only 60 mm thick, act in harmony, they were connected at intervals by precast concrete webs and transverse diaphragms of the same thickness.[68]

Figure 59. CNIT Hall, France

(http://www.viaggiaresempre.it/Defense6.JPG. Last accessed in December 2006.)

Saarinen’s TWA Terminal at Kennedy Airport illustrates the extent to which, a free sculptural modeling of the form is possible without any need for the massiveness of Roman concrete vaults. On the other hand, Utzon’s original proposals for the Sydney Opera House serve as a reminder that there are limits, set by the inevitable laws of static equilibrium.[69]

Timber was another material used for the shell-like forms. After Otto had built some relatively small domes in this way in the 1960s in Esses, Berkeley and Montreal, the real demonstration of what could be done came in 1973-5 with the construction of large pavilion in Mannheim. This pavilion consists of two large halls and connecting

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[68] Mainstone, R. J. *Developments in structural form.* pp. 137-138.

[69] Ibid. p. 138.

walkways on an irregular curving plan, all covered by a single continuously curved roof. A hanging-chain model gave the essential configuration.[70]

Figure 60. TWA Terminal, New York

(http://www.greatbuildings.com/buildings/TWA_at_New_York.html. Last accessed in December 2006.)

Although the use of shell roofs has spread in the last two decades, the high cost of the formwork and design difficulties always become important objections. The creation of curved shapes composed of straight or linear elements may therefore be regarded as a constructional advance, rather than as a return to older methods.[71]

4.5 Catenaries, Slung Membranes and Cable Nets

Catenary can be described as a curved structural element, which spans in the same way as a hanging chain or *catena* rather than the particular curve assumed by a freely hanging uniform chain loaded only by its own weight. It is structurally, the simplest possible form after the straight tie.[72]

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[70] Ibid. p. 139.

[71] Cowan, H. J. *An historical outline of architectural science.* p. 90.

[72] Mainstone, R. J. *Developments in structural form.* p. 113.

Slung ropes made from natural fibers serving as very simple temporary bridges or as supports for tent-like shelters were probably the first man-made catenaries. The materials used in Mesopotamia such as mud-brick were not of high quality, and this probably caused the discovery of the catenary arch, which alone is subject to pure compression under its own weight. For example, the Great Arch of the Palace of Ctesiphon built in 550 A.D. with mud bricks, lightly burnt, and set in clay mortar, has a span of 27m and a height of 33.6m. Its survival for 14 centuries is remarkable.[73] In Roman times, catenaries were developed on quite a large scale for the temporary shade canopies of theatres and amphitheatres.[74]

The slung membrane is the first tensile counterpart of the dome or vault, and was not seen until some time after the first simple domical huts. In spite of widespread experience with sails, it had hardly developed beyond the large circus-tent until the early 1950s.[75]

This delay is related with some obstacles to be overcome. The first was the relatively low tensile strength of most fabrics and the difficulty of making effective joints to transmit tension than of making joints subject primarily to compression. The second was the natural flexibility of thin, singly curved tensile forms and the risk of flutter and billow in the wind. The third was the difficulty of overcoming the flexibility by adopting forms with marked double curvature without, thus, introducing excessive tensions at some points.[76]

The much greater importance of fairly uniform stresses to utilize fully the available tensile strengths of the thin tensile membrane is its main difference from the compression structure typified by the Gothic cathedral or Gaudi's design for the church of the Sagrada Familia. In these structures, the thicknesses are sufficient to accommodate a satisfactory system of internal thrusts even if there are considerable depar-

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[73] Cowan, H. J. *An historical outline of architectural science*. p. 5-6.

[74] King, R. *Brunelleschi's dome: how a Renaissance genius reinvented architecture.* Walker & Co., New York. 2000.

[75] Mainstone, R. J. *Developments in structural form.* p. 140.

[76] Ibid.

tures from an ideal geometry. Stresses are also low enough for it to matter little if they are far from uniform. For the membrane, they do matter and choice of surface geometry is therefore more restricted. Moreover, the only doubly curved membrane that can match the ability of the singly curved membrane to assume automatically an ideal configuration is the soap bubble or soap film, ,which stretches or contracts until the surface tension is uniform.[77]

The obstacle of limited fabric strengths has been overcome since 1970s. In earlier large slung roofs, a continuous fabric had to be replaced as the principle load-bearing medium by an equivalent network of wire cables, by single parallel strands in singly curved roofs, or by double-layer arrangements of cables held apart by vertical struts and tensioned against them.[78]

More frequently, cable networks have been used and their stiffness has been provided by double curvature and prestressing. One set of cables has been slung between peripheral arches, cables or trusses to carry the whole weight. The other set, intersecting the first one at right angles, has been tensioned against it to hold it steady (Figure 3n). Constant curvatures are preferred for the avoidance of the excessive local tensions.[79]

Before computers came to scene in the design process, it was possible to achieve a moderate satisfactory geometry by adjustments of cable lengths during erection. Otto, the pioneer design of such roofs, used preliminary models such as soap-films stretched directly over wire frames or between flexible threads, but it was too difficult to achieve this for large span roofs. So, computers became essential for the more accurate theoretical analyses.[80]

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[77] Ibid.

[78] Ibid.

[79] Ibid.

[80] Ibid. pp. 140-141.

4.6 Air Supported and Pneumatic Structures

The stretched membrane supported only by internal air pressure for purposes of space enclosure came to scene later than the slung membranes and cable nets. After early successful experience with Radomes in the 1950s, it became an inexpensive choice for small temporary structures and for covering swimming baths and recreation halls. For this type of membranes, two possible forms exist. First one is the single-skin air-supported structure held up by a pressure difference between the enclosed space and the exterior. The second is the double-skin pneumatic structure held up by the stiffening effect of a higher pressure in the space between the two skins. [81]

The ideal form is that of a floating soap-bubble, continuously tied down at the base. Other synclastic forms are also possible but will lead to varying tensions in the membrane as the curvature varies. The anticlastic curvatures are possible only as transitions between zones of synclastic curvature just as waists can occur in a child's balloon only between bulbous zones.[82]

The most successful design up till now has probably been the American Pavilion for Expo 70 in Osaka (Figure 61). It was constructed over a partly excavated elliptical area, 140 m long and 80 m wide. The U.S. Pavilion has five primary components. The roof is made of a translucent fiberglass fabric. The walls are formed by an earth berm, which supports a concrete ring. The ring balances the lateral loads of the cables that span the roof and is superelliptical in shape. A system of blowers maintains internal air pressure and so provides the air columns that support the roof. The vinyl-coated fiberglass fabric was stiffened by a diagonal cable net made from bridge strand and anchored in a concrete compression ring at the top of the berm. The superelliptical shape of the pavilion represents a relatively new structural form. The super ellipse was formulated only 12 years ago and published for the first time in 1959. The Osaka pavilion is a super ellipse with an exponent of 2.5. If the exponent

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[81] Otto, F. *Tensile structures; design, structure, and calculation of buildings of cables, nets, and membranes.* The MIT Press, Cambridge. 1973.

[82] Mainstone, R. J. *Developments in structural form.* pp. 143-144.

were 2, the shape would be an ellipse. If it were to approach infinity, the shape would become rectangular.[83]

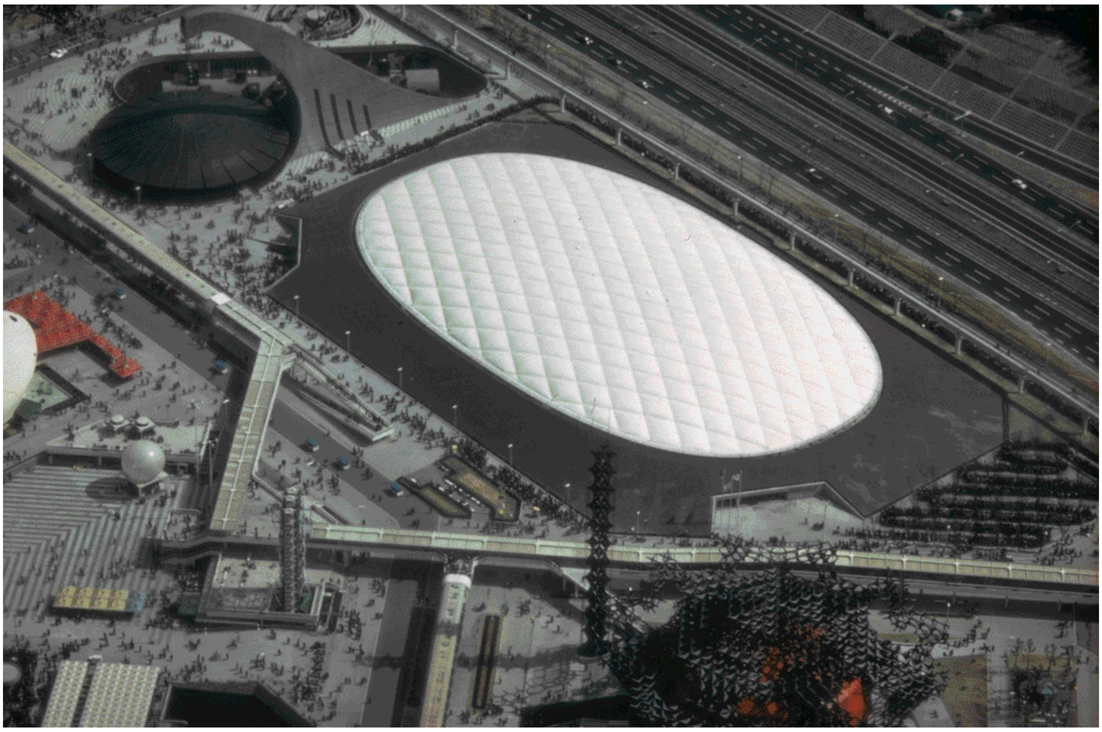

Figure 61. American Pavilion for Expo 70 in Osaka

(Villecco, M. The Infinitely Expandable Future of Air Structures, in. *Architectural Forum.* (pp. 40-43). 1970, September, vol. 133.)

4.7 Contemporary Curvilinear Structural Forms

The 19th and 20th century buildings, structurally speaking, have widened further the range of choice for wide-span enclosures. Nevertheless, the types of roof have mostly paralleled fairly closely the earlier forms. Various multiple-arch forms have replaced the barrel vault; doubly curved shells, cable nets membranes, and space frames have replaced the dome and groined vault.[84]

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[83] Villecco, M. The Infinitely Expandable Future of Air Structures, in. *Architectural Forum.* (pp. 40-43). 1970, September, vol. 133.

[84] Mainstone, R. J. *Developments in structural form.* p. 237.

The integration of new materials and new construction techniques to architecture as post-tensioning, pre-tensioning the concrete, using timber, iron ribs and steel in advanced techniques offered different choices for wide-spanning. Architecturally, this new materiality brought new vision and expression to traditional curvilinear form. Structurally, these materials and techniques provided architects and engineers to enclosure wider spans. Using steel trusses was one of the basic systems for wide spanning. Early trusses were out of timber especially utilized by ancient Greeks. From 19th century, architects and engineers utilize trusses out of steel in various configurations. Besides the steel trusses had linear schemes, designers used curved trusses so as to utilize the structural potentials and aesthetic impression of *curve*. The curved trusses were seen especially at glasshouses and train sheds.

The adoption of a simpler total form was first seen in large glasshouses or conservatories in which the glazing was entirely carried by timber or iron ribs. The early examples being the Great Conservatory at Chatsworth built between 1837 and 1840 and the Palm House at Kew built ten years later. [85]

The chief nineteenth-century development of the simple domed form may be represented by dome of the Halle au Ble in Paris having a circle of an internal diameter of 39 m and the Albert Hall in London having an elliptical plan with a diameter varying from 57 m to 67 m. Like the dome of the Roman Pantheon, both of these stood directly on drum-like walls.[86]

Not long after the completion of the Palm House in 1848, the eventual simplification of the arched form came with the great train sheds of railway stations, chiefly with at St Pancras, built between 1868 and 1869. At St Pancras, the entire 73 m width of the terminal was spanned at intervals of 9 m by great-trussed arches rising directly from platform level (Figure 62).[87]

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[85] Ibid.

[86] Ibid. p. 241.

[87] Meeks, C. L. V. *The railroad station; an architectural history.* Yale University Press, New Haven. 1956. pp. 197-222.

In 1968, the great arched train shed of St Pancras Station was completed. Designed by William Henry Barlow, it was the longest spanning roof up to that time, and remained so until the Galerie des Machines was constructed in 1889. The shed has a span over 73 m with height of 30 m above the rails and a length of 209 m.[88]

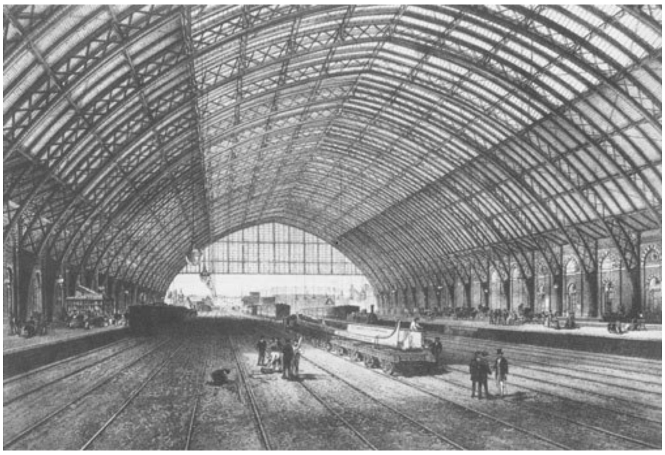

Figure 62. St Pancras Station

(http://www.usc.edu/dept/architecture/slide/ghirardo/CD3/StPancrasStation.jpg. Last accessed in December 2006.)

The continuous arch tied down at its feet, as at St Pancras, and without significant freedom of rotation there, was, however, a structure with a fairly high degree of statical indeterminacy. An arch with a central pin and pinned supports was statically determinate. In the latter part of the nineteenth century, this had some advantages that the stresses in it and the reactions at the support could be easily calculated and was not highly affected by the little movements of supports or the thermal expansions and contractions of the arch.[89]

The first large-scale example of the use of the three-pinned arch was in the Galerie des Machines built for the 1889 Paris Exhibition (Figure 63). It innovated the structural principle of the three-pinned arch, pioneered the use of structural steel. The 3 m deep trussed-arches spanned 114 m. Each trussed-arch was made up of two sections

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[88] Wilkinson, C. *Supersheds: the architecture of long-span, large volume buildings.* Butterworth Architecture, Oxford. 1996. p. 8.

[89] Mainstone, R. J. *Developments in structural form.* p. 239.

joined at the top with a pin. At the base, the trusses were tapered to a hinged joint, ensuring an exact distribution of the stresses and the materials used.[90]

Figure 63. Galerie des Machines, Paris

(http://e3.uci.edu/clients/bjbecker/SpinningWeb/adams1889galeriemachines7a.jpg. Last accessed in December 2006.)

Examples of the further development of the arched form and the corrugated barrel form are the aircraft hangars built by Nervi between 1935 and 1941 and a large exhibition and two sports halls that were built between 1948 and 1960. The first of the three structures was exhibition hall in Turin. Both this and the larger sports hall employed the type of the precast “voussoir”.[91]

The TWA Terminal, constructed between 1956 and 1962, was conceived as a large-scale piece of sculpture. The roof was designed as four shells, each trapezoidal in plan, symmetrical about a central ridge, and cantilevered out from just two points symmetrically placed on each side of the ridge. As a result, there is a continuous succession of smoothly-flowing curves and a spatial experience that remains unique, inside.[92]

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[90] Wilkinson, C. *Supersheds: the architecture of long-span, large volume buildings.* p. 6.

[91] Nervi, P. L. *Structures*. F.W. Dodge Corp., New York. 1956. pp. 32-39 and 66-81.

[92] Mainstone, R. J. *Developments in structural form.* p. 245.

The Waterloo Terminal, finished in 1993, built on a restricted site alongside an existing terminal has an axis of a continuous curve with a width 48.5 m at the terminal and 32.7m at the open departure end. The internal profile was highly unsymmetrical –much steeper on the outside of the curve in order to give adequate clearance to the trains with no platform on this side. For this reason, the arches were again three-pinned as in the Galerie des Machines.[93]

At the Kansai International Airport Terminal building (Figure 64) finished in 1994, the main structure consists of trapezoidal trusses of tubular steel members formed in a gently rolling curve that arched main span of 83 m. their depth varies across the section, extending to 4 m depth at the maximum span.[94]

Figure 64. Kansai International Airport Terminal building

(http://www.aij.or.jp/eng/prizes/design/photo24.gif. Last accessed in December 2006.)

In the Leipzig Neue Messe Glass Halls, designed by Marg of Von Gerkan Marg, completed in 1996 with a span of 80 m, more conventional vaulted steel structure was used to enclose a huge wintergarden measuring 237.5 x 79 x 28 m high. The structure is composed of an external orthogonal single-layer grid shell of uniform diameter steel tubes stiffened by primary arches at 25 m centers.[95]

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[93] Davey, P. Waterloo International, in: *Architectural Review.* (pp. 26-44). 1993, September.

[94] Wilkinson, C. *Supersheds: the architecture of long-span, large volume buildings.* p. 128

[95] Ibid. p. 140.

Centre des nouvelles industries et technologies (CNIT) Building constructed between 1956 and 1958 is a double thin shell made of reinforced concrete. The shell covers an area of 22500 m^2 and distance between supports is 218 m. The building is the largest concrete shell in the world in terms of square meter of area covered per support.[96]

The Eden project, designed by Nicholas Grimshaw & Partners, housed in a former china quarry in Cornwall, is a giant botanical garden opened to the public in March 2001. Stretching over 26,000m sq, the project comprises of eight geodesic domes forming two biomes for trees and plants. Inspired by the laws of nature, the structures are made up of connecting hexagons, pentagons and triangles, forming a sphere. The domes of the Eden Project are the largest geodesic domes built yet, reaching 60m in height in places and spanning enough land to house 29 football pitches. They are made of lightweight galvanized steel tubing of varying sizes, created and cut by a computer.[97]

Figure 65. Eden project and section through the dome

(http://www.mckerracher.org/images/edenproject.jpg. Last accessed in December 2006.)

Yet the predominant architectural form in the 20th century remained linear since the limitations of hand drawn architectural drawings leads to linear buildings that are easily rendered in 2D form, whereas three dimensional curvilinear form is facilitated

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[96] Structurae. *Centre des nouvelles industries et technologies.*http://en.structurae.de/structures/data/index.cfm?ID=s0000019. Last accessed in December 2006.

[97] Arup. *The Eden Project.* http://www.arup.com/europe/feature.cfm?pageid=307. Last accessed in November 2006.

by three dimensional rendering tools, which are not available until recently. Now, in 21st century, architectural curvilinear form rises again with the integration of technology and computer to the design and manufacturing process in architecture. Any form that can be computed by means of digital mediums can now be constructed.

Figure 66. The Bubble, BMW's exhibition pavilion by Bernard Franken and ABB Architekten

(Kolarevic, B. Digital Morphogenesis, in: *Architecture in the digital age: Design and manufacturing*. (pp. 11-28). Branko Kolarevic, ed. Spon Press, New York. 2003.)

CHAPTER 5

ANALYSIS OF CURVILINEAR FORM THROUGH A CONTEMPORARY STRUCTURE: A CASE STUDY ON TERMINAL BUILDING, WATERLOO

International Terminal at Waterloo Station, London, designed by the architecture firm Nicholas Grimshaw & Partners, is a railway station built on a complex, constrained site. The International Terminal is an addition to Waterloo Station, a London train station built in 1922. The terminal was completed in May 1993. The station is a symbolic and actual threshold between Britain and the Continent, and the first monument of a new railway age, when high-speed trains will compete with aeroplanes as the most effective form of travel within Europe. Nicholas Grimshaw views the Channel Tunnel Railway Terminal at Waterloo as a "heroic railway station with the same function as a 21st century airport."[1]

The terminal is a multifaceted transport interchange: a railway station, which, basically, functions like an airport. Located in central London, it is situated in a constrained urban setting and has an important mission to answer the demands of 15 million international rail passengers per year.

Channel Tunnel Railway Terminal at Waterloo is chosen for this study intentionally since the building has an outstanding attitude beyond the contemporary curvilinear structures. The structure owes this status to the achievement of structural design that is clearly expressed with the curvilinear long span arches, the harmony between the functionality and structural approach of the building, and the aesthetic qualities it articulates.

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[1] Powell, K. *Structure, space and skin: the work of Nicholas Grimshaw & partners*. Phaidon, London. 1995. p. 26.

In this chapter, the architectural and structural progress of the design process and evolution of the construction period of the building will be explained. With the purpose of understanding the behavior of the structure, a computer model inspired by the roof of Channel Tunnel Railway Terminal at Waterloo will be generated by using Structural Analysis Program (SAP) 2000 Nonlinear 7.1. With some geometrical modifications for the configuration of the roof, new schemes of structures are obtained in order to make comparison to the curved structure of the roof. The outcomes of these comparisons will be analyzed from the point of their geometry, load capacity and efficiency searching for the correspondence with the structural ideals defined deeply in Chapter 2, which are *efficiency*, *economy* and *elegance*.

5.1 Architecture of the building

The tight urban situation next to the existing Waterloo station, the neighboring buildings and live electric rails, the London Underground tunnels were the main constraints at the outset of the design process. The severe constraints of the site helped to generate novel architecture and structure, which continues the tradition of innovative British railway sheds that began in the nineteenth century in London with King's Cross (Lewis Cubitt, 1851-1852), Paddington (I.K. Brunel, 1852-1854) and St Pancras (W.H. Barlow, 1863-1868) Stations.[2]

The intention while designing the project was to build a 'streamlined terminal' through which passengers could pass with the minimum fuss at maximum speed. The site, adjacent to the existing national rail station, was only just wide enough to accommodate the necessary five tracks. Limited by live electric rails and shallow London Underground tunnels beneath, the terminal needed to be 'streamlined' structurally, also to be 'streamlined' in terms of its internal organization.[3] After extensive analysis of the traffic and passenger flows, and detailed study of the site geometry, the initial design for the station was presented in mid-1988.[4]

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[2] Margolius, I. *Architects + engineers = structures.* Wiley-Academy, Great Britain. 2002. p. 81.

[3] Grimshaw Architects. *Waterloo International Terminal.* http://www.grimshaw-architects.com/grimshaw/print/projectdata.php?id=36. Last accessed in November 2006.

[4] McGuckin, S. *Project profile: Waterloo International.* British Cement Association Publication, Berkshire. 1994. p. 6.

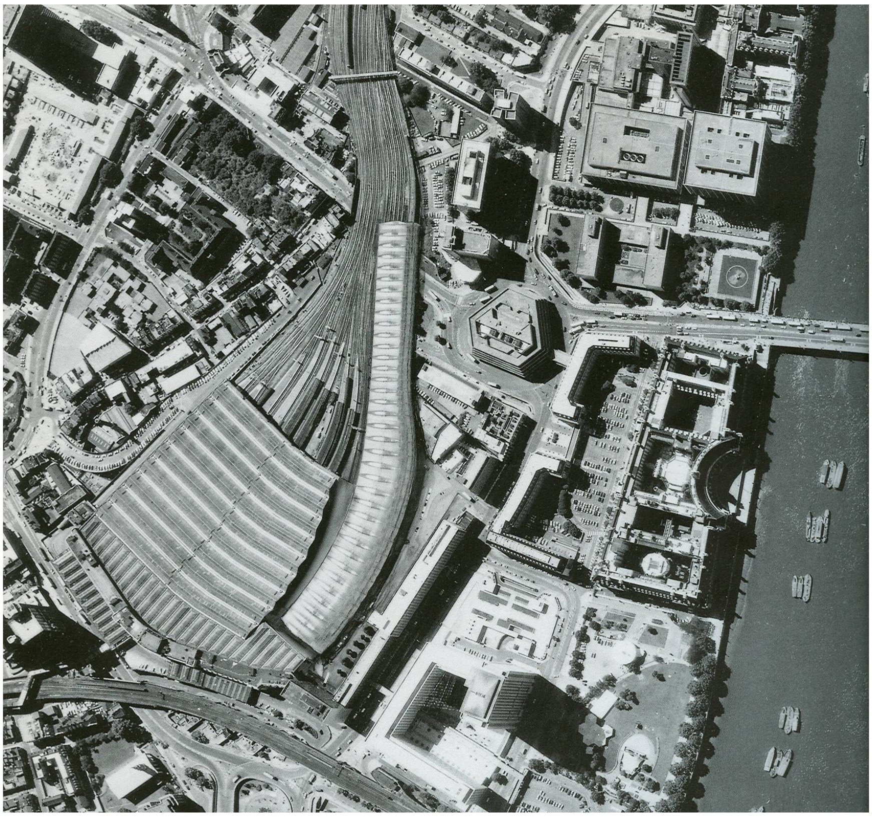

Figure 67. Aerial view of the 400 m long terminal roof in relation to the surrounding cityscape

(Powell, K. *Structure, space and skin: the work of Nicholas Grimshaw & partners*. Phaidon, London. 1995.)

In its design, the station reflects the tradition of the railway halls of the 19th century, but at the same time, it is a symbol of a new age of rail travel. The 400-m-long structure composed of 36 arched trusses follow the twisting line of the railway tracks. The structure tapers in width from 48.5 m at the northern end to 32.7 m at the southern end.[5]

This funnel shape was a consequence of the turning radius of the trains, the confines of the busy city-center site and the limited width of the site and is carried through the station's four levels. [6] One level is a basement that provides short-term car parking; levels two and three house arrivals and departures respectively; and level four; the upper floor is devoted to the platforms.[7] The platforms are located at the southern

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[5] Bahnhof in London, in: *Detail*. (p. 674). 1995, vol. 4.

[6] McGuckin, S. *Project profile: Waterloo International.* p. 6.

[7] Ibid. p. 3.

end, at the upper level beneath the arched roof. Waiting areas, shops and offices are situated at lower levels, which are naturally lighted through the glazed west facade.[8]

Although the project is reminisced with the outstanding roof, almost 90% of the project is concerned with work carried out underground. This comprises the reused brick vaults underneath the mainline station, some of which house the catering and support services, and others through which large crowds are guided, efficiently but pleasurably, through the spaces beneath the tracks.[9] The internal organization of the floors has been arranged to provide easy orientation to passengers. Departures and Arrivals are assigned a level each, to encourage a single direction of passenger movement on each floor. For all customers, there is a clear, linear progression in the terminal.[10]

5.2 Structure of the building

A 1.8 m-deep basement floor slab, spanning the shallow London Underground Bakerloo and Northern lines, forms a raft foundation to the structure. There lies arrivals floor slab above, and the two slabs together form a heavily reinforced concrete box, which carries the track support and platform structures. The space formed by these structures contains the arrival and departure areas.[11]

Over 200 m long, this building extends from the concourse at Waterloo Station to a series of road bridges over Leake Street. Its roof is formed by the platform structure, which extends a further 200 m on existing brick arches and is mounted on sliding bearings to accommodate thermal expansion. Concrete double-height shear walls are located below the platforms, providing both longitudinal and transverse fixity. The shear walls are exposed and compromised twin diagonal strut/tie members enclosing feature voids.[12]

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[8] Bahnhof in London, in: *Detail.* (p. 674). 1995, vol. 4.

[9] Powell, K. *Structure, space and skin: the work of Nicholas Grimshaw & partners.* p. 35.

[10] Grimshaw Architects. *Waterloo International Terminal.* http://www.grimshaw-architects.com/grimshaw/print/projectdata.php?id=36. Last accessed in November 2006.

[11] McGuckin, S. *Project profile: Waterloo International.* p. 6.

[12] Ibid. p. 7.

5.3 Construction period

During the construction process, many obstacles had to be faced and overcome. The narrowness of the site was one of the main obstacles. Construction depth was limited by the London Underground tunnels and height by the desire to keep the height of the new platform as close as possible to the level of the domestic station. Other obstacles included a new Underground station concourse, a British rail tunnel, four banks of escalators, and neighboring buildings. In addition, there was also the hazard of working close to the live electric rails.[13]

To ensure that none of the structures either beneath or bordering the site was damaged by demolition, excavation or construction, an extensive geotechnical survey was undertaken. This included drilling 40 boreholes to depths between 15 m and 70 m; high quality sampling using thin wall tubes and rotary coring; in-situ testing; 20 trial pits next to the existing foundations; and 15 rotary cores of the existing foundations. Before the basement could be built, 55000 m^3 of excavations were required.[14]

Foundations had to be completed in three stages as a consequence of the restricted site. To begin, the first portion of the raft foundation was cast in the middle of the site, from which the perimeter diaphragm wall and sheet-piled walls were propped during the second stage. Finally, when the perimeter works were finished, the props were removed and concrete poured to complete the raft.[15]

Construction of the track support structure began over the arches at the north end of the terminal. Columns in this area were precast to shorten the construction time. Once the shear walls had been completed and tied into the slab, the temporary restraining false work was taken away and moved forward for the next section. The program required the track and platform structure to be finished first to allow the erection of the roof to begin on time. The departure floor slab, constructed at a later

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[13] Ibid. p. 8.

[14] Ibid.

[15] Ibid. p. 9.

stage, used traditional false work supported from the arrivals slab below and back-propped to the basement slab.[16]

After assembly of the truss sections on the concrete deck, the 36 arches were positioned by mobile cranes. A new bay was started every two-three weeks, and was clad immediately after completion.[17]

5.4 The roof structure

The focus of both technical skill and architectural spectacle is the roof. The roof can be lower and flatter since it does not have to compete with steam and smoke, but the result is not like that since it is a successor of Victorian train sheds. Yet it is not a copy but a response to its own circumstance: its tapering span, and its narrow, sinuous plan, is determined by site and the track layout.[18] The asymmetric form of the trusses responding to the dictates of the site layout derives from the position of a single track tight onto the western edge of the site where the roof must rise more steeply in order to accommodate the height of the trains.[19] This western side is clad entirely in glass with the structure of the roof clearly expressed. The western side becomes a public showcase for the trains, and allows arriving passengers to glimpse Westminster and the River Thames.[20]

The Terminal's main structure is a 400-m-long steel and glass tube that tapers from a width of 48.5 m. to 32.7 m. The Terminal tube consists of 36 asymmetrical arches. While they decrease in size as the structure tapers, the arches are identical in design. The roof has an remarkable attitude since it interprets an image of a distorted barrel vault or a curved surface architecturally, yet, structurally speaking, it is a system of 36 arches.

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[16] Ibid. p. 10.

[17] Ibid.

[18] Powell, K. *Structure, space and skin: the work of Nicholas Grimshaw & partners*. pp. 28-32.

[19] Grimshaw Architects. *Waterloo International Terminal.* http://www.grimshaw-architects.com/grimshaw/print/projectdata.php?id=36. Last accessed in November 2006.

[20] Powell, K. *Structure, space and skin: the work of Nicholas Grimshaw & partners*. pp. 28-32.

The filigree roof structure consists of pairs of bowstring trusses joined off-centre to form flattened three-pin arches. The design is a complex variation on these bowstring arches. A typical bowstring arch functions like a bow: A thick member is held in curved compression by a tension cable.[21]

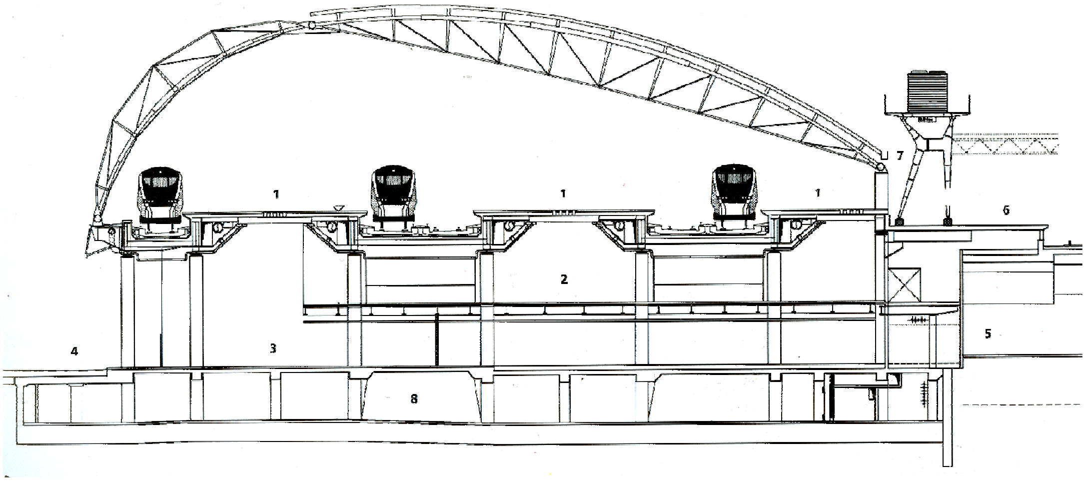

Figure 68. Cross section of the terminal

(Powell, K. *Structure, space and skin: the work of Nicholas Grimshaw & partners*. Phaidon, London. 1995.)

Each arch was made from two prismatic bowstring trusses connected at a central knuckle joint. The larger truss (major truss), located at the eastern side, had two telescoping, hollow compression booms on its upper face and a single smaller solid tension bar beneath to stop the truss spreading under load. Due to asymmetric arch profile, the small truss (minor truss) was reversed, with two tension rods forming the outer chord with a single, internal compression strut. The tops of larger trusses were sheathed in glass and stainless steel cladding while the smaller trusses were fully glazed on the inside of the chords showing the exposed outer steelwork.[22]

To provide adequate space for train access at this point, the minor roof truss is set outside the glass envelope. The major truss consists of two tubular outer compression

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[21] Bahnhof in London, in: *Detail.* (p. 674). 1995, vol. 4.

[22] Margolius, I. *Architects + engineers = structures.* pp. 80-81.

booms and a solid lower boom. As a result of the asymmetric geometry, the stresses in the minor truss are reversed: the inner boom is in compression, and the two outer booms are in tension.[23]

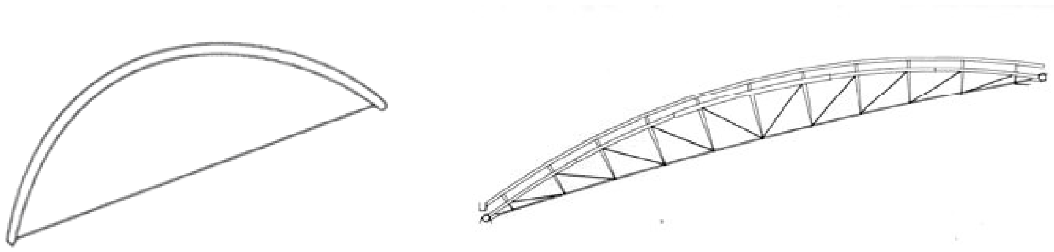

Figure 69. Bow and the bow-string arch

(http://www.owlnet.rice.edu/~arch214/waterloo.pdf. Last accessed in November 2006.)

The trusses derive their elegance partly from this inversion of structure and skin (and of tension and compression members) as they pass from east to west, but also from the tapering of their members, created out of telescoping circular sections. This allows the members to respond, with economy of material, to the distribution of forces in each truss.[24]

A cable-strung bowstring arch operates like a typical beam: The upper member is in compression, while the cable is in tension. Yet in the roof of Waterloo Terminal, the trusses do not employ cables, which can resist tension. Instead, the Waterloo bowstring trusses utilize metal rods as the rods can resist to compressive force, the compression and tension forces can switch. This switch occurs due to uplift. Normally, dead loads press down on the arch, causing the upper members to be in compression and the lower ones in tension. Nevertheless, when wind loads push up against the structure, the lower member goes into compression and the upper member is in tension.[25]

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[23] Bahnhof in London, in: *Detail*. (p. 674). 1995, vol. 4.

[24] Powell, K. *Structure, space and skin: the work of Nicholas Grimshaw & partners*. p. 32.

[25] Rice University Information Technology. *Waterloo Station International Terminal.* http://www.owlnet.rice.edu/~arch214/waterloo.pdf. Last accessed in November 2006.

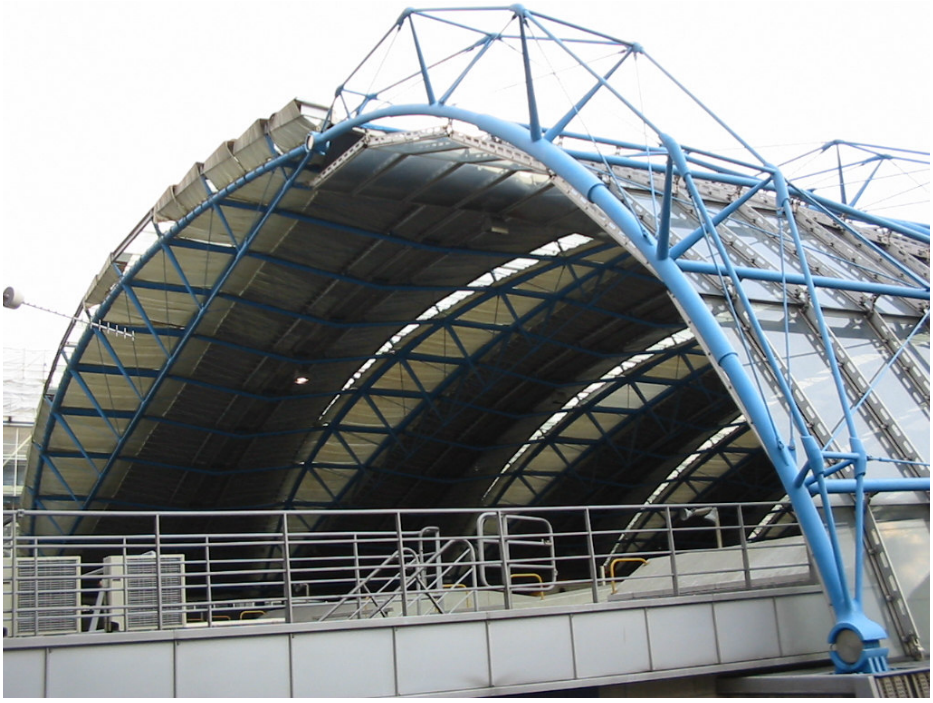

Figure 70. The asymmetric arch

(Ünay, A. İ. 2000)

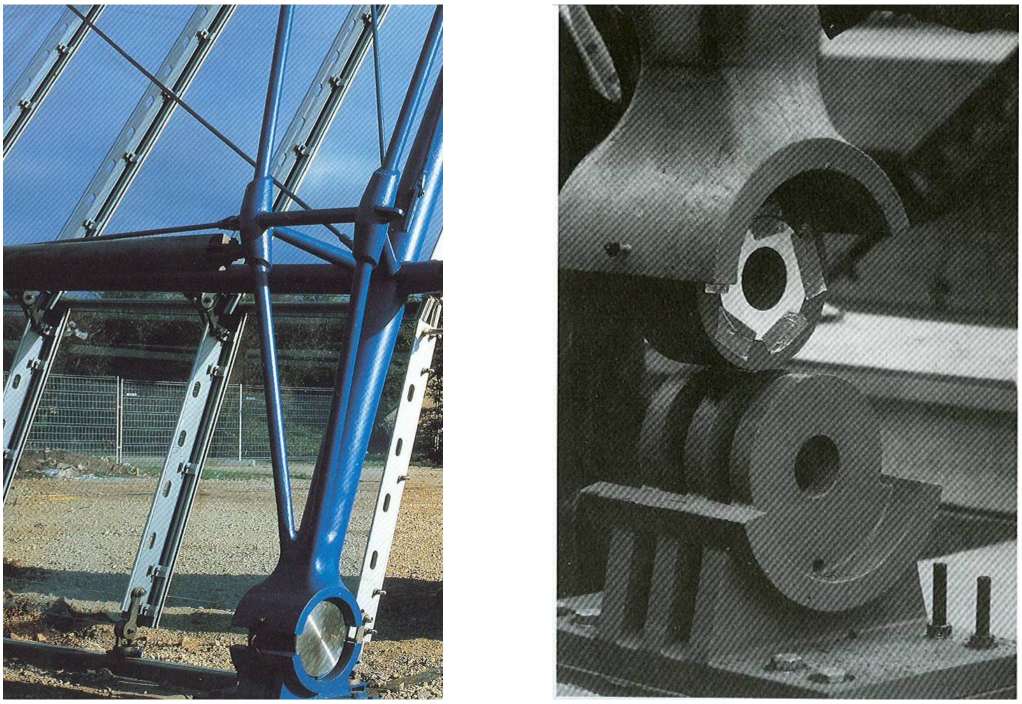

Figure 71. Pin joint detail

(Powell, K. *Structure, space and skin: the work of Nicholas Grimshaw & partners*. Phaidon, London. 1995.)

The two bowstring trusses are connected by a cast steel hinge with stainless steel bolts at the point where the roof glazing and cladding meet. Zero moment is created where bowstring sections meet other, because they allow the segments to rotate freely connection points. Each pair of arches is structurally independent, insuring that if any structural unit is damaged the others will still stand.[26] The structure is strengthened by diagonal tension rods in the plan of the lower boom between the trusses on the western side. The whole structure is designed to be light in weight and to use a minimum amount of steel.[27]

Figure 72. The glazing of the curvilinear roof

(http://www.owlnet.rice.edu/~arch214/waterloo.pdf. Last accessed in November 2006.)

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[26] Ibid.

[27] Bahnhof in London, in: *Detail*. (p. 674). 1995, vol. 4.

The glazing of this curvilinear roof was accomplished using standard-size, rectangular elements throughout. The glazing system is fixed to the secondary structure with hinged stainless steel castings.[28] The minor part of the arch is clad on its underside with overlapping panels of glass. Because each 3-pin assembly is slightly smaller than the one before, the glass gradually tapers. The cladding on the major side of the arch sits above the structure. In between, metal decking tilts inward, providing easy water drainage, and creating an undulating appearance.[29]

Figure 73. An inner view of the terminal expressing the glazing and metal deck

(Ünay, A. İ. 2000)

The arched roof of the train shed follows the curve of the railway, and increases in span down the length of the station to accommodate the increase in the width of the platforms. The roof is supported by a series of three-pin arches. Each arch and the related cladding are different as the roof changes width along the curved tracks. The variability of the arches is visible in both plan and isometric views.[30]

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[28] Bahnhof in London, in: *Detail*. (p. 674). 1995, vol. 4.

[29] Rice University Information Technology. *Waterloo Station International Terminal.* http://www.owlnet.rice.edu/~arch214/waterloo.pdf. Last accessed in November 2006.

[30] Szalapaj, P. Parametric Propagation of Form, in: *Architecture Week,* http://www.architectureweek.com /2001/0919/tools_1-2.html. Last accessed in December 2006.

The modeling of the tapering arched roof was an important phase within the whole design process. Using conventional computer aided design (CAD) modeling techniques; a single arch form could be modeled and then duplicated 35 times, with adjustments for the curvature of the track in plan. A difficult process of resizing individual trusses and arches would then need to be carried out. The complexity and variation in the size and shape of the structural elements involved in the train shed were possible because of the application of structural analysis CAD techniques. Instead of modeling each arch separately, a single parametric model of an arch was modeled, so that it encoded the underlying design rules for the whole family of arches. Afterwards, the complete roof model became a series of *instances* of this parametric arch, each with a different value for the span parameter.[31]

International Terminal Waterloo was completed in May 1993, within budget (£130m). Since its completion, it has won a number of architectural awards, including the Mies van der Rohe Pavilion award for European Architecture (1994) and the RIBA President's Building of the Year Award (1994).[32]

5.5 The computer-based structural analysis of the roof structure

A digital model of the roof arch is reproduced using Autodesk AutoCAD 2000, in drawing interchange format (dxf). With this representation, the geometric layout of the arch is aimed to be realized only with line segments. These line segments would represent the arch bow, compression and tension rods of the structure. The connection points of these members are abstracted as single nodes as this study deals with the overall behavior of the arch, and do not involve a finite element analysis (FEA).

Next step was to import the geometric data to the SAP (Structural Analysis Program) 2000 7.1. All line segments were converted to a frame/cable element in the new medium and was automatically assigned a constant frame section predefined in the software. At this stage, new frame/cable sections are defined considering thicknesses

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[31] Ibid.

[32] Grimshaw Architects. *Waterloo International Terminal.* http://www.grimshaw-architects.com/grimshaw/print/projectdata.php?id=36. Last accessed in November 2006.

of different parts of the arch, which are assigned to the structure to reflect the actual member configuration. For the sake of simplifying the problem without comprising the essence of the behavior, seven different pipe sections are defined to include both the thickest part of the major truss, which is defined as 600 mm in diameter, and the rods, 50 mm in diameter, used in the structure.[33]

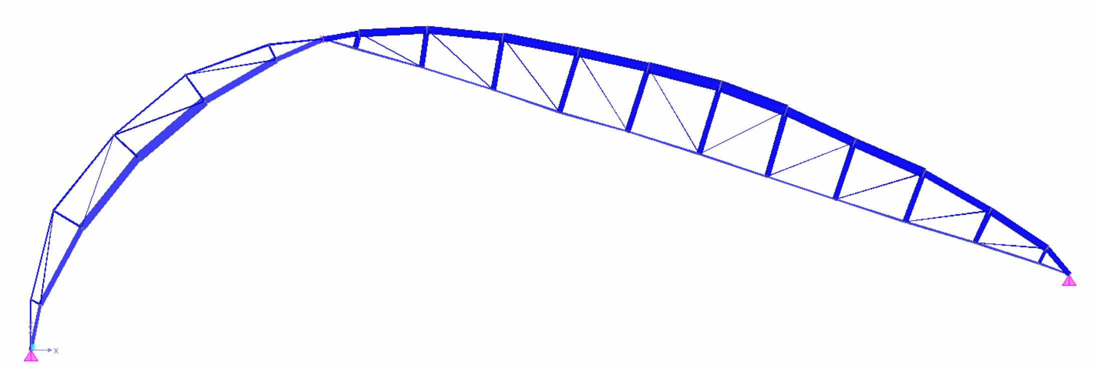

Figure 74. The analytical model of the arch of Waterloo terminal

Support conditions of the two ends of the arch are set to reflect the existing hinge restraints. The hinge is restrained for translation in all three axes and only allowed rotational movement in the longitudinal axis, which accurately reflects the actual behavior. Third hinge at the intersection of the two arch segments is also restrained to allow one rotational movement and translational movement in three axes. Moreover, two ends of the tension/compression rods in the structure are released for moment forces to represent the connection type used in the actual structure.

Forces, on the other hand, are restricted as dead -including glazing- and wind loads for simplicity. Accumulating additional loads such as snow would not considerably change the behavior, but affect the displacement values and member force values. Dead load is calculated by the software itself as an extension of material/section assignments to the frame/cable members. An additional glazing load of 0.5kN/m^2 is added to the dead load case. Wind load is calculated using the Turkish Standards

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[33] The model is based on the data that Assoc. Prof. Ali İhsan Ünay prepared for a previous study as SAP90 text file.

TS498. The structure is assumed to have a wind load (q) of 0.8 kN/m^2 that corresponds to the height of 9 to 20 meters. Considering the 11-meter longitudinal spans between the arches, the resulting force is derived from the formula 0.8q as 7.04kN/m. This value is influenced on the main members of the minor arch perpendicularly. Accordingly, major arch is loaded with 3.52kN/m as an uplift force, which corresponds to the 0.4q.

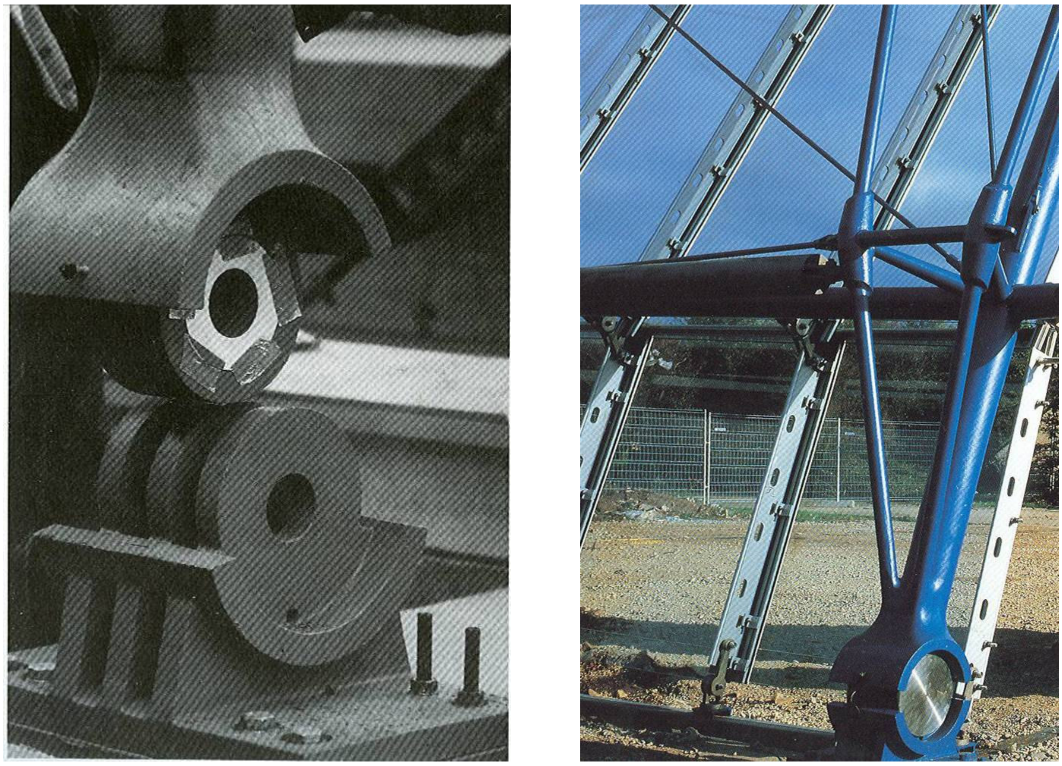


Figure 75. The detail of the hinges at the supports

(Powell, K. *Structure, space and skin: the work of Nicholas Grimshaw & partners.* Phaidon, London. 1995.)

Based on this accurate model of the roof arch, member definitions are modified for the analytical comparisons without changing the geometry. The thickness of the arch bows are unified through all sections, and they are relatively refined in order to meet the assumed loading conditions and to prevent an over design condition. Since the study will only involve dead load and wind load conditions, keeping the actual section thicknesses that are obviously designed to withstand more loading conditions would render a result that the section has excess dead weight and much more structural capacity than the given loadings. This model will be labeled as M1 in this study.

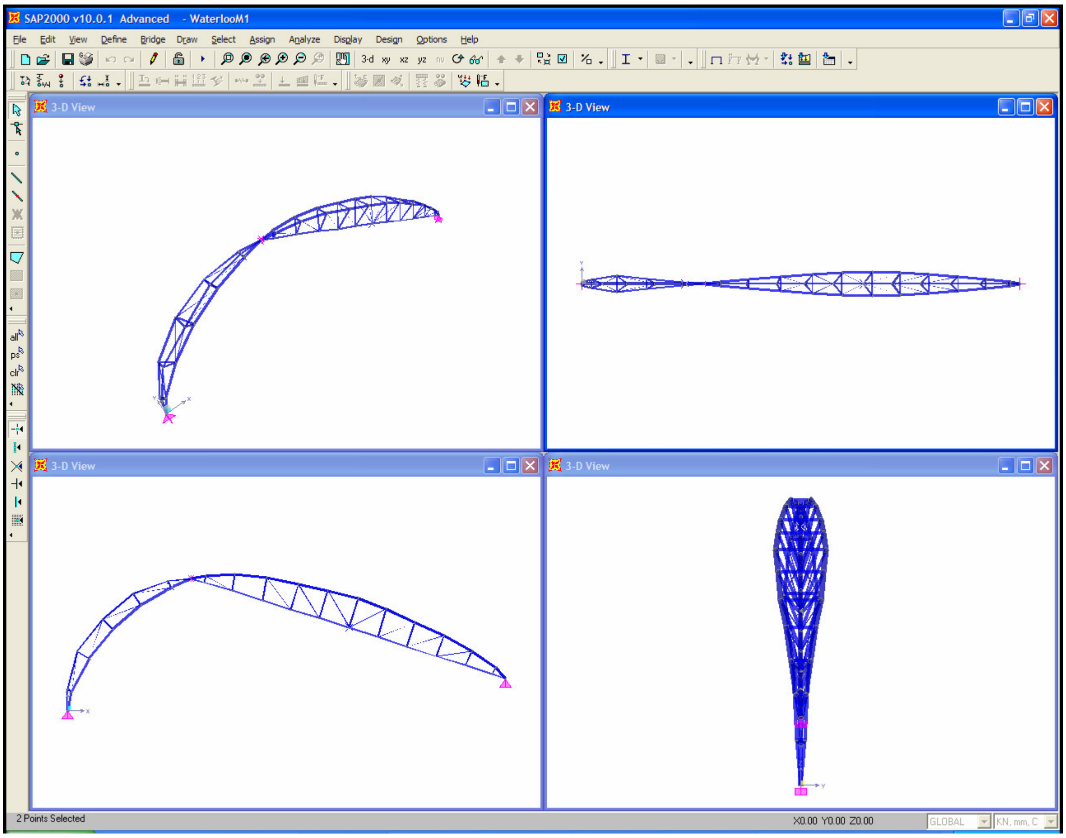


Figure 76. The analytical model of M1

To compare with M1, two models having the same span were generated. First model is a straight truss member that keeps the fundamental configuration and mean beam depth constant throughout the section. The truss member sections are modified to equalize the overall weight of the beam with the base model. Thus, a comparison of displacements between two schemes is made in terms of the concept of efficiency. The analysis involved the dead load condition, but not the wind load case considering the horizontal position of the check model. This model will be labeled as M2.

Second model is developed to represent a closer scheme to the base model. The scheme is shaped with linear truss elements of the mean depth that base model utilize. Member sections are modified as in the first case in order to keep the overall weight equal to the base model. This case involved dead and wind loading cases together. This model will be labeled as M3 in this study.

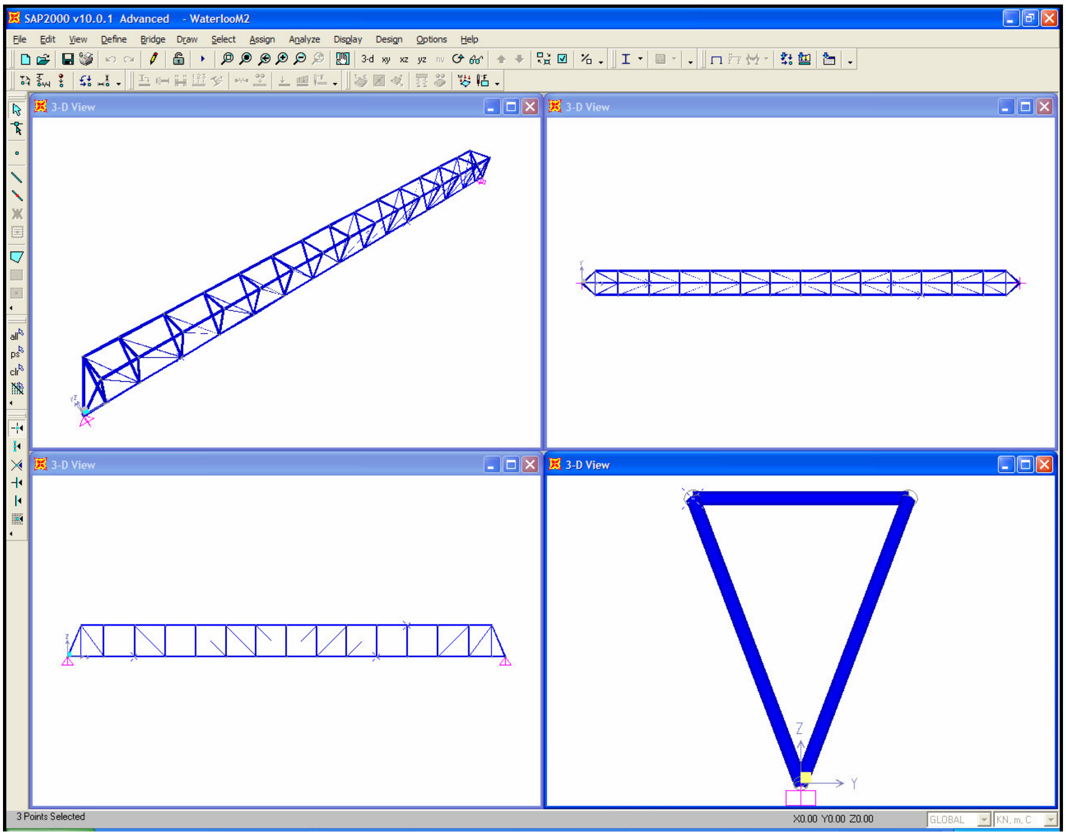


Figure 77. The analytical model of M2

There are some parameters to understand the behavior of the structure. One of these parameters is the displacement of nodes, which is a significant value for the analysis of a structure to display how the geometry of the elements and the original location of joints had changed under that specific loading. There are some limits for the values of displacements, which are defined not only according to structural concerns but also according to serviceability conditions. When the values obtained for the displacement exceed a certain value, a failure is expected not necessarily because of the structural insufficiencies but also functional inadequacies. Displacement is a parameter to understand the physical problems that are encountered. During these observations, the most critical points are considered because a structure should be designed according to the weakest point of the whole structural system.

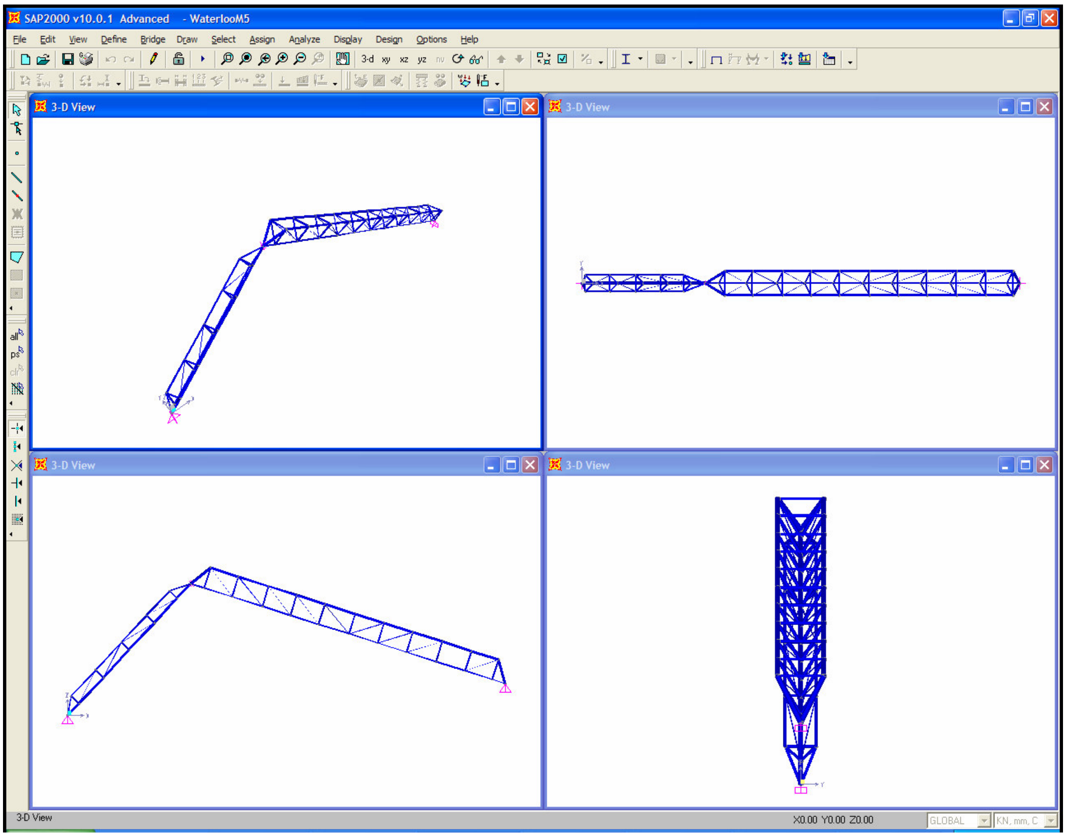
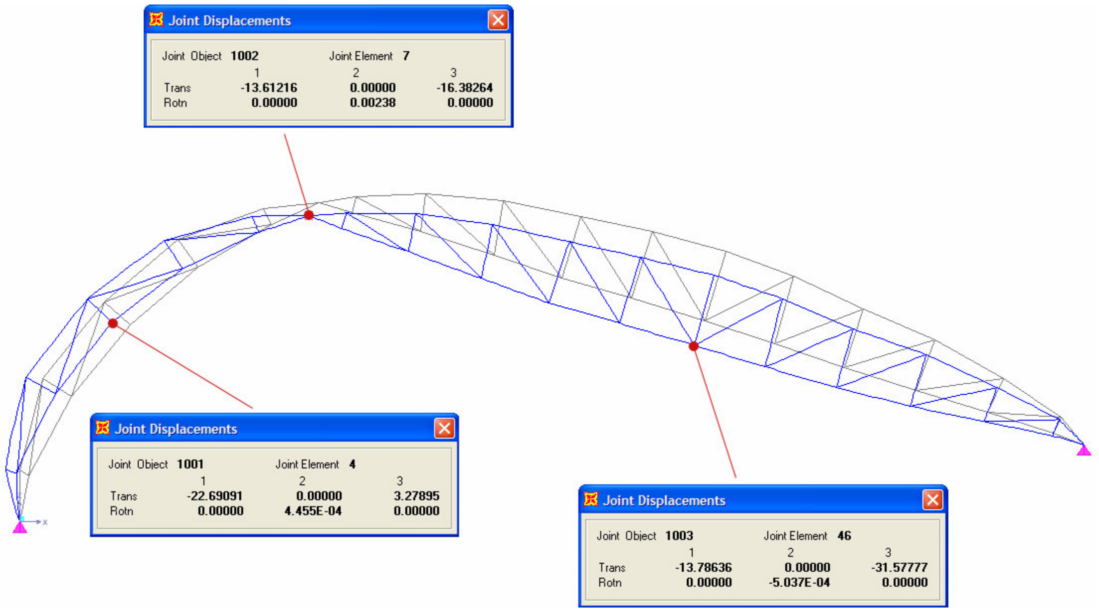


Figure 78. The analytical model of M3

In this study, first case involves the comparison of the displacements of M1 and M2 under dead load. The displacement results are illustrated in Figure 79 and 80 for the critical points of the trusses.

The displacement of the mid point of minor truss that is labeled as Joint 1001 in model M1 has a translation of -22.7mm in X direction, and 3.3mm in Z direction under dead load condition. Similarly, Joint 1002 and 1003 have a translation of -13.7mm, -13.8mm in X, and -16.4mm, -31.6mm in Z direction, respectively. However, Joint 1002 on M2 has a translational value of -72.0mm in Z direction. Thus, the highest displacement value on M1, which belongs to Joint 1003, is less than the half of the value of the displacement belonging to the most critical point on M2. Having the same span and amount of material used, two structures has obviously different displacements under same loading conditions, which can be elucidated by the use of the curvilinear geometrical configuration. In this case, use of curve leads to a structurally efficient form in terms of displacements.

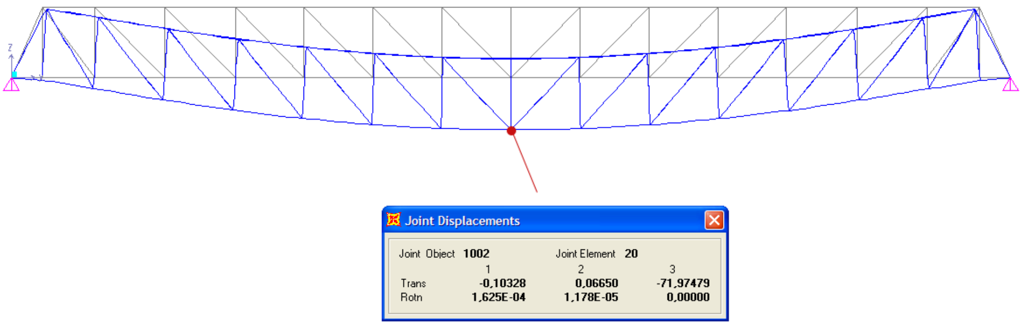


```
Joint Displacements
Joint Object  1002          Joint Element  7
                1             2             3
Trans      -13.61216       0.00000     -16.38264
Rotn         0.00000       0.00238       0.00000
```

```
Joint Displacements
Joint Object  1001          Joint Element  4
                1             2             3
Trans      -22.69091       0.00000       3.27895
Rotn         0.00000     4.455E-04       0.00000
```

```
Joint Displacements
Joint Object  1003          Joint Element  46
                1             2             3
Trans      -13.78636       0.00000     -31.57777
Rotn         0.00000    -5.037E-04       0.00000
```

Figure 79. Deformed shape of M1 under dead load



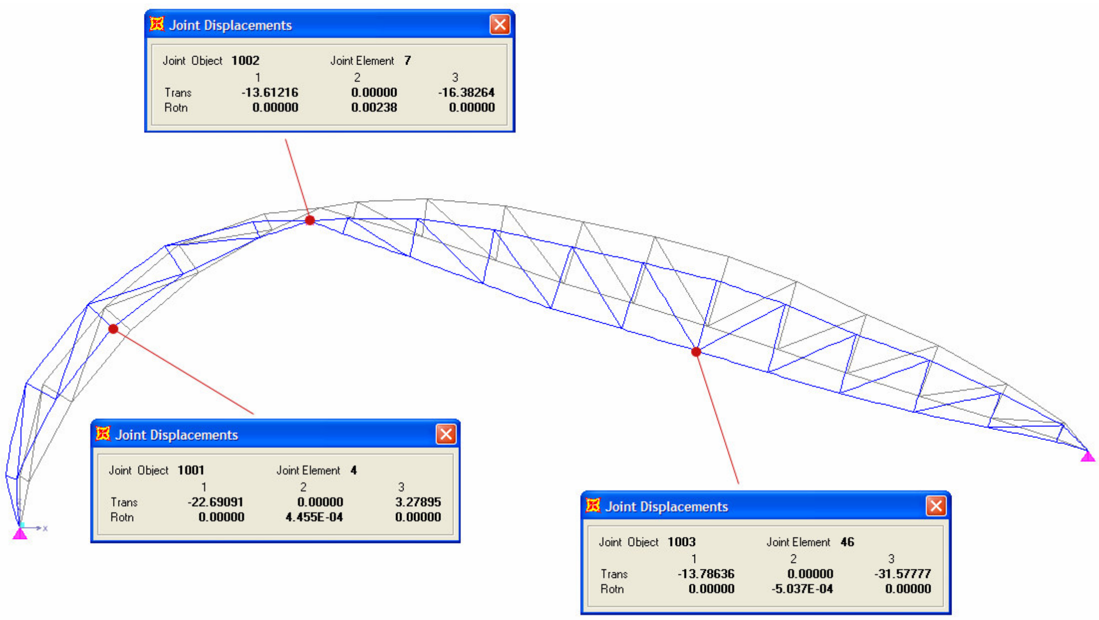
```
Joint Displacements
Joint Object  1002          Joint Element  20
                1             2             3
Trans       -0,10328       0,06650     -71,97479
Rotn       1,625E-04     1,178E-05       0,00000
```

Figure 80. Deformed shape of M2 under dead load

Table 1. Comparison of critical points of M1 and M2 under dead load case

Critical Points	Deflections		
	1 (mm)	2 (mm)	3 (mm)
M1_1001	-22.69	0.00	3.27
M1_1002	-13.61	0.00	-16.38
M1_1003	-13.78	0.00	-31.57
M2_1001	-0.10	0.06	-71.97

In the second case, unlike the first one, a system that responds to the spatial qualities needed for the specific function is proposed. To satisfy this condition, a three-hinged system composed of linear trusses, which principally utilizes an arch behavior as in the case of inverted V discussed in chapter 4, is exposed to dead, wind and uplift loads. Displacement of critical points of M1 and M3 are compared under this load cases and combinations. Figure 81-86 shows the results.

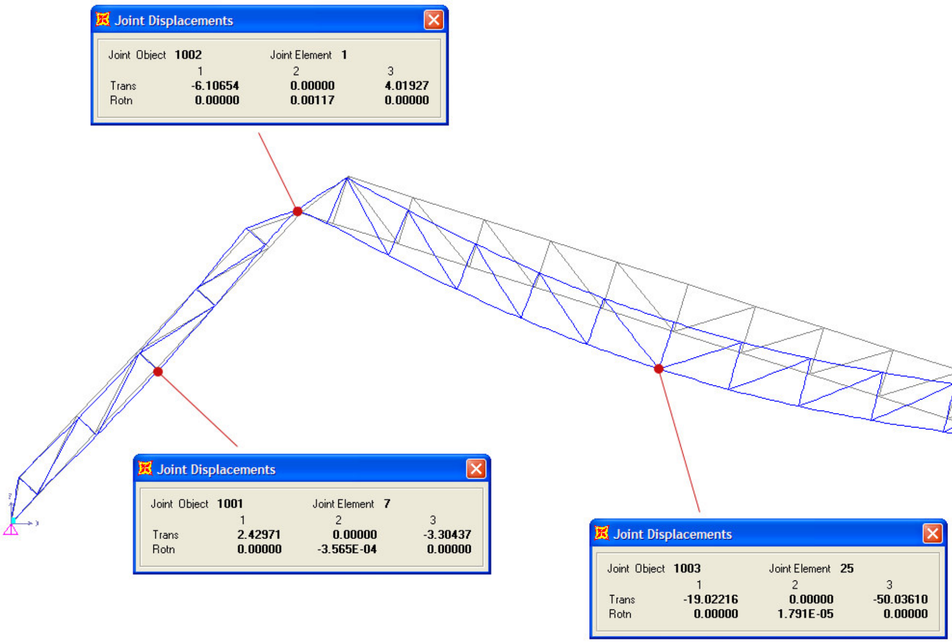


```
Joint Displacements
Joint Object  1002          Joint Element  7
              1             2             3
Trans    -13.61216       0.00000     -16.38264
Rotn       0.00000       0.00238       0.00000
```

```
Joint Displacements
Joint Object  1001          Joint Element  4
              1             2             3
Trans    -22.69091       0.00000       3.27895
Rotn       0.00000     4.455E-04       0.00000
```

```
Joint Displacements
Joint Object  1003          Joint Element  46
              1             2             3
Trans    -13.78636       0.00000     -31.57777
Rotn       0.00000    -5.037E-04       0.00000
```

Figure 81. Deformed shape of M1 under dead load



```
Joint Displacements
Joint Object  1002          Joint Element  1
              1             2             3
Trans     -6.10654       0.00000       4.01927
Rotn       0.00000       0.00117       0.00000
```

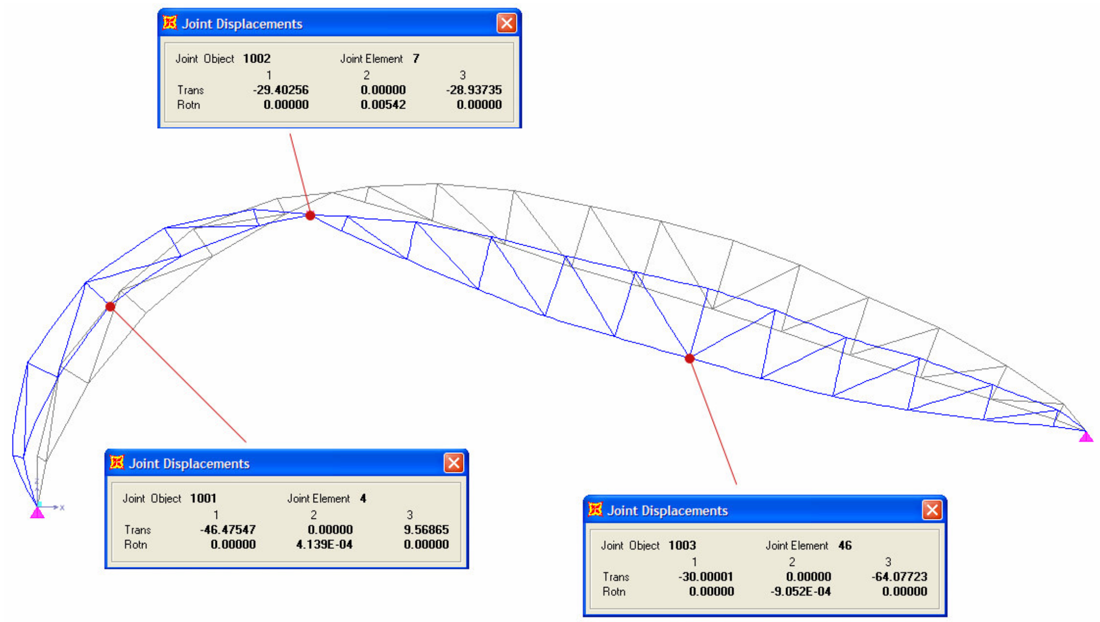
```
Joint Displacements
Joint Object  1001          Joint Element  7
              1             2             3
Trans      2.42971       0.00000      -3.30437
Rotn       0.00000    -3.565E-04       0.00000
```

```
Joint Displacements
Joint Object  1003          Joint Element  25
              1             2             3
Trans    -19.02216       0.00000     -50.03610
Rotn       0.00000     1.791E-05       0.00000
```

Figure 82. Deformed shape of M3 under dead load

Table 2. Comparison of critical points of M1 and M3 under dead load case

Critical Points	Deflections		
	1 (mm)	2 (mm)	3 (mm)
M1_1001	-22.69	0.00	3.27
M1_1002	-13.61	0.00	-16.38
M1_1003	-13.78	0.00	-31.57
M3_1001	2.42	0.00	-3.30
M3_1002	-6.10	0.00	4.01
M3_1003	-19.02	0.00	-50.03

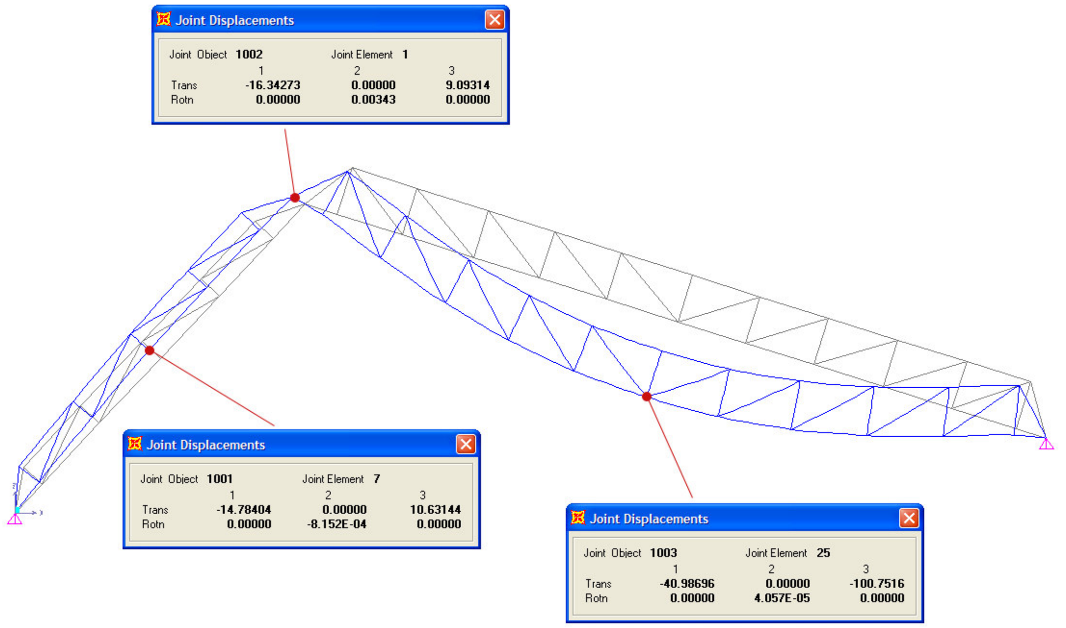


```
Joint Displacements
Joint Object  1002            Joint Element  7
                 1               2               3
Trans     -29.40256         0.00000       -28.93735
Rotn        0.00000         0.00542         0.00000
```

```
Joint Displacements
Joint Object  1001            Joint Element  4
                 1               2               3
Trans     -46.47547         0.00000         9.56865
Rotn        0.00000       4.139E-04         0.00000
```

```
Joint Displacements
Joint Object  1003            Joint Element  46
                 1               2               3
Trans     -30.00001         0.00000       -64.07723
Rotn        0.00000      -9.052E-04         0.00000
```

Figure 83. Deformed shape of M1 under dead load and wind (1) load



```
Joint Displacements
Joint Object  1002            Joint Element  1
                 1               2               3
Trans     -16.34273         0.00000         9.09314
Rotn        0.00000         0.00343         0.00000
```

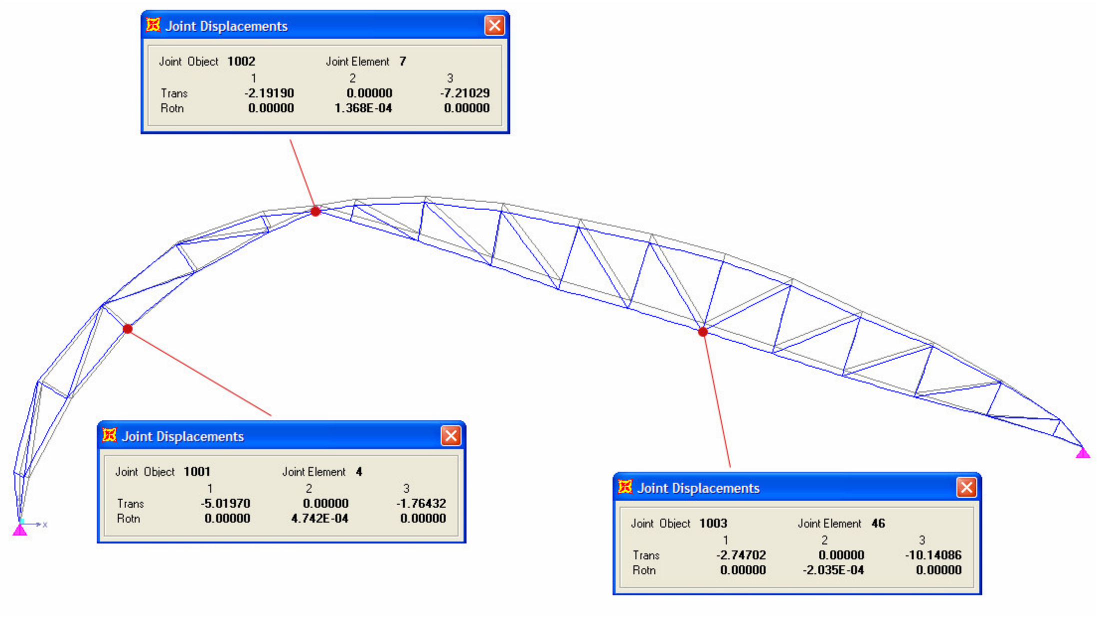
```
Joint Displacements
Joint Object  1001            Joint Element  7
                 1               2               3
Trans     -14.78404         0.00000        10.63144
Rotn        0.00000      -8.152E-04         0.00000
```

```
Joint Displacements
Joint Object  1003            Joint Element  25
                 1               2               3
Trans     -40.98696         0.00000       -100.7516
Rotn        0.00000       4.057E-05         0.00000
```

Figure 84. Deformed shape of M3 under dead load and wind (1) load

Table 3. Comparison of critical points of M1 and M3 under dead and wind (1) load case

Critical Points	Deflections		
	1 (mm)	2 (mm)	3 (mm)
M1_1001	-46.47	0.00	9.56
M1_1002	-29.40	0.00	-28.93
M1_1003	-30.00	0.00	-64.07
M3_1001	-14.78	0.00	10.63
M3_1002	-16.34	0.00	9.09
M3_1003	-40.98	0.00	-100.75

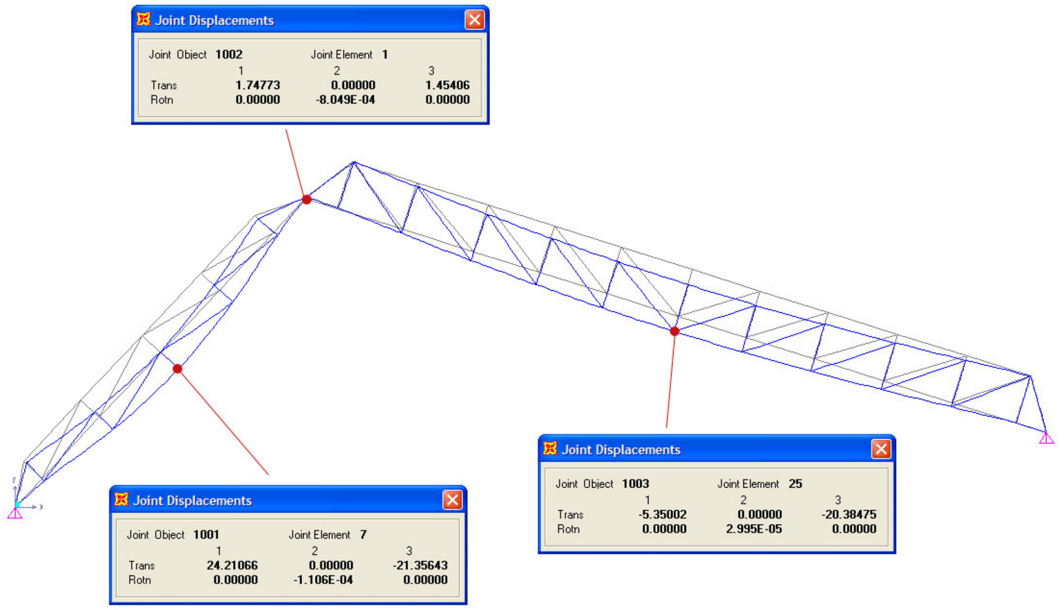


```
Joint Object  1002          Joint Element  7
                 1              2              3
Trans      -2.19190        0.00000       -7.21029
Rotn        0.00000      1.368E-04        0.00000
```

```
Joint Object  1001          Joint Element  4
                 1              2              3
Trans      -5.01970        0.00000       -1.76432
Rotn        0.00000      4.742E-04        0.00000
```

```
Joint Object  1003          Joint Element  46
                 1              2              3
Trans      -2.74702        0.00000      -10.14086
Rotn        0.00000     -2.035E-04        0.00000
```

Figure 85. Deformed shape of M1 under dead load and wind (2) load



```
Joint Object  1002          Joint Element  1
                 1              2              3
Trans       1.74773        0.00000        1.45406
Rotn        0.00000     -8.049E-04        0.00000
```

```
Joint Object  1001          Joint Element  7
                 1              2              3
Trans      24.21066        0.00000      -21.35643
Rotn        0.00000     -1.106E-04        0.00000
```

```
Joint Object  1003          Joint Element  25
                 1              2              3
Trans      -5.35002        0.00000      -20.38475
Rotn        0.00000      2.995E-05        0.00000
```

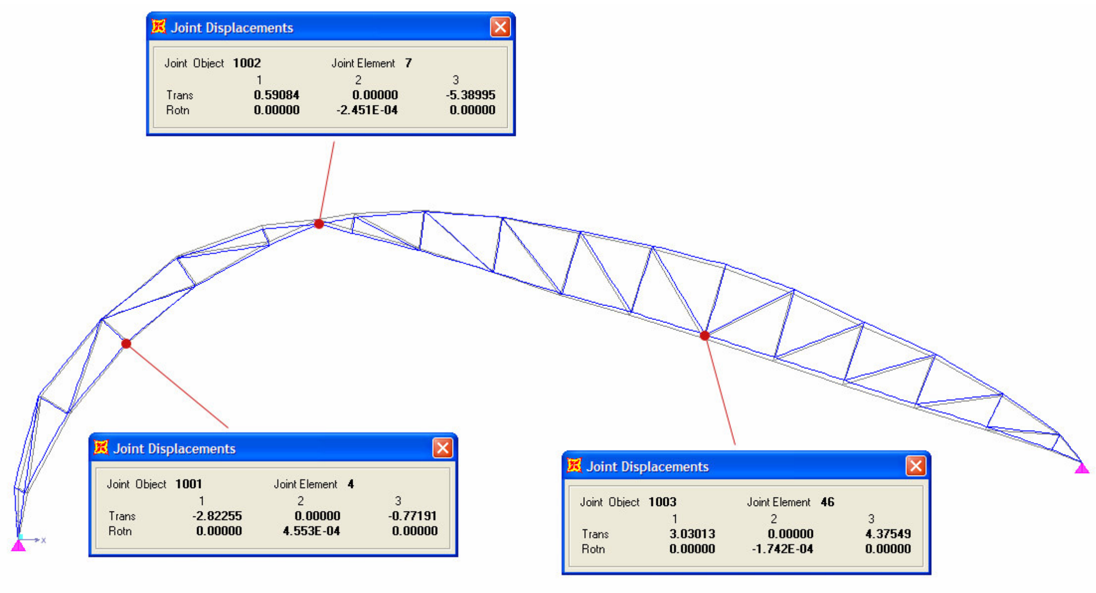
Figure 86. Deformed shape of M3 under dead load and wind (2) load

Table 4. Comparison of critical points of M1 and M3 under dead and wind (2) load case

Critical Points	Deflections		
	1 (mm)	2 (mm)	3 (mm)
M1_1001	-5.01	0.00	-1.76
M1_1002	-2.19	0.00	-7.21
M1_1003	-2.74	0.00	-10.14
M3_1001	24.21	0.00	-21.35
M3_1002	1.74	0.00	1.45
M3_1003	-5.35	0.00	-20.38

Since the structural behavior of the inverted V system is similar to the behavior of the arch, the values of displacements are lower than M2. Yet the displacement values of M1 are lower than M3 except Joint 1002.

There are many considerations while designing a structure as dead load, wind load, snow load, earthquake etc. As the structure studied here is the roof of a terminal building, the uplift force becomes another important consideration in this case. As discussed at the beginning of this chapter, a cable-strung bowstring arch operates like a typical beam: The upper member is in compression, while the cable is in tension. However, at Waterloo Terminal, the bowstring arches utilize metal rods as the rods can resist to compressive force, the compression and tension forces can switch. This switch occurs due to uplift. Normally, dead loads press down on the arch, causing the upper members to be in compression and the lower ones in tension. Nevertheless, when wind loads push up against the structure, the lower member goes into compression and the upper member is in tension. In Figure 87 and 88, the results of displacements of the critical points of M1 and M3 under the combination of dead and uplift loads are illustrated.



```
Joint Displacements

Joint Object  1002            Joint Element  7
                 1               2               3
Trans         0.59084         0.00000        -5.38995
Rotn          0.00000        -2.451E-04       0.00000
```

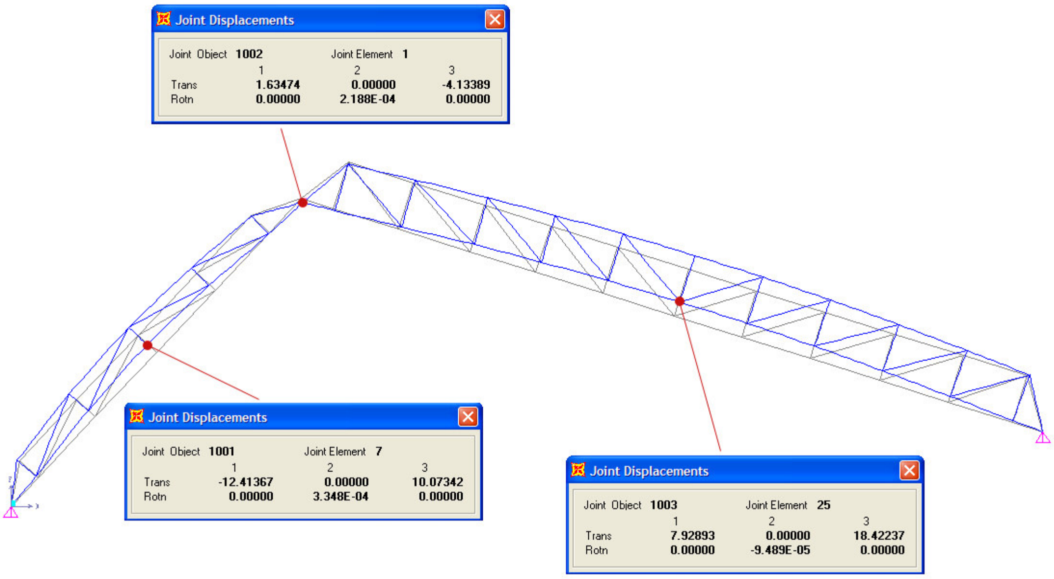
```
Joint Displacements

Joint Object  1001            Joint Element  4
                 1               2               3
Trans        -2.82255         0.00000        -0.77191
Rotn          0.00000         4.553E-04       0.00000
```

```
Joint Displacements

Joint Object  1003            Joint Element  46
                 1               2               3
Trans         3.03013         0.00000         4.37549
Rotn          0.00000        -1.742E-04       0.00000
```

Figure 87. Deformed shape of M1 under dead load and uplift force



```
Joint Object 1002        Joint Element 1
             1            2            3
Trans        1.63474      0.00000      -4.13389
Rotn         0.00000      2.188E-04    0.00000
```

```
Joint Object 1001        Joint Element 7
             1            2            3
Trans        -12.41367    0.00000      10.07342
Rotn         0.00000      3.348E-04    0.00000
```

```
Joint Object 1003        Joint Element 25
             1            2            3
Trans        7.92893      0.00000      18.42237
Rotn         0.00000      -9.489E-05   0.00000
```

Figure 88. Deformed shape of M3 under dead load and uplift force

Table 5. Comparison of critical points of M1 and M3 under dead and uplift force

Critical Points	Deflections		
	1 (mm)	2 (mm)	3 (mm)
M1_1001	-2.82	0.00	-0.77
M1_1002	0.59	0.00	-5.38
M1_1003	3.03	0.00	4.37
M3_1001	-12.41	0.00	10.07
M3_1002	1.63	0.00	-4.13
M3_1003	7.92	0.00	18.42

According to the interpretation of results represented in Tables, the curvilinear form of the roof has lower displacements at the critical points under the defined loading cases in comparison to the other systems generated with linear trusses. As expressed in Table 1, M1 has lower displacements than M2 under dead load case. Table 2 shows the results of the comparison of critical points of M1 and M3 under dead load case. In Table 3, the results of the ccomparison of critical points of M1 and M3 under dead load and wind (1) case and in Table 4, the results of the comparison of critical points of M1 and M3 under dead load and wind (2) case are expressed. From the results it is clear that M1, the curvilinear geometrical shape represents better results than M2 and M3 having linear configurations. Table 5 shows the displacements of the points of M1 and M3 under dead and uplift force, and M1 expresses lower values than M3. Since the analyzed building is a terminal building, the uplift force is a vital criterion for the structural design. The curvilinear configuration provides M1 better stiffness under the combination of dead and uplift loads. The displacement of nodes is a significant value for the analysis of a structure to display how the geometry of the elements and the original location of joints had changed under that specific loading. The displacement values affect not only the structural concerns but also serviceability conditions. Therefore, the lower values obtained by the curvilinear configuration of M1 brings out structural achievement the terminal building. Besides the structural advantages, the curved shape provides the terminal building elegance and expressive power.

CHAPTER 6

CONCLUSION

Curvilinear forms have been recognized as forming well-resistant structural components since the early times. Although the structural advantages stemmed from the concept of flexural or axial compressive resistance was not known by the early builders, they recognized the structural potentials of this form empirically. Moreover, the construction of structural elements in curved forms was easily possible with masonry technique, which was the most widely used material at that time. Curved form developed over centuries as structural enhancements and aesthetic tenets. With the introduction of need for uninterrupted larger spaces, designers became more interested in curvilinear forms. The structural economy achieved by the use of curvature, has always been applied through the history because it is necessary to offer within the structure a resistance moment equal to the moment of the applied loads.

In this study, a survey of curvilinear forms in architecture has been done with an emphasis on structural behavior. The curvilinear form has been examined through its structural potentials according to its geometrical configuration. The concept of structure and the structural ideals were reviewed and the relationship between structure and form was examined in regard to curvilinear forms. The concept of curvature in structuring was also presented since the mathematical background was a prelude to understand the nature of their structural behavior. The development and utilization of curvilinear structural forms has been discussed briefly within an historical overview with emphasis on structures that have marked significant steps terms of widening the range of possible further choices in the history. The study also focused on the comparison of structural capacity of linear and curvilinear structures by analyzing the Channel Tunnel Railway Terminal at Waterloo, as a case study. Background of the design survey of the terminal building was explored through its architectural and structural processes.

An analytical model of the roof of the terminal building was generated, yet some modifications are made during the modeling process in order to obtain a simplified model keeping the geometrical configuration constant that would be appropriate only for this study. The thicknesses of the pipe sections used in the main arch vary in the structure, but in the model proposed, M1, the sections are unified under four basic types. Two models having the same span were generated in order to make comparison to the curved structure of the roof to be able to observe the outcomes considering their geometry, load capacity and efficiency.

When these analytical models are analyzed, the results express that the curvilinear form of the roof has lower displacements at the critical points under the defined loading cases in comparison to the two other systems composed of linear trusses using the same span, same materials, and same dead weight. The displacements are higher in model M2, which is a trussed beam. They get lower when the horizontal beam is modified to a three-hinged system labeled as M3, yet the trussed arch of the terminal, M1, has the smallest displacement values.

The interpretations of the results reveal that the curvilinear structure was inspired by the arch of the terminal are utilized efficiently in terms of displacement under the defined loading cases. Considering one of the essential concepts of ideal structure, *economy,* the curvilinear structure demonstrates accomplished outcomes, since lower displacements are achieved with the same amount of material in M1. Furthermore, in the original structure of the terminal, the thicknesses of the sections of the main arches vary considering the required structural capacity and get thinner where possible.

Efficiency is another quality of a structure, which is a scientific one coexistent with *economy* since the efficient utilization of structure corresponds with the expediency, cost, constructability; in short, the economy of the structure, and the “inherent” economy dictated by the laws of nature. The curvilinear structure is more efficient in comparison to the linear configurations with respect to the results of displacements.

The third quality is symbolic, and it is this concept that opens up the possibility for the new engineering to be structural art. The *elegance* and its expressive power exhibit the symbolic dimension. Although there can not be a single solution for aesthetic consideration, the International Terminal Waterloo is widely accepted as an outstanding elegant example among the contemporary curvilinear structures with its alliance of architecture and structure, form and function and with its expressive power as a building.

It is obvious that the geometrical configuration affects the success of the structure. Concerning these three important qualities, in this study, the curvilinear form displays successful results in terms of these qualities under the cases defined in the study. As examined in the case study, the structural advantages the "curve" proposed, even in the situation that the architectural considerations such as spatial and aesthetics are ignored, are indispensable for the designers. Therefore, the structural stability provided by the geometrical configuration of *curve* is utilized by contemporary designers in not only traditional forms but also free-formed contemporary structures. Besides the structural advantages, the symbolic and aesthetic values that curvilinear form proposed have always appeal the designers' interest. These advantages widen the imagination of the designer and offer possibilities for extreme designs. The undeniable contribution of utilization of *curve* to the field of architecture broadens the structural limits further and new projects are accomplished.

Despite the structural and aesthetic advantages and efficiency that the curved form provides, the limitations of hand drawn architectural drawings and the difficulties faced during construction processes leads the linear forms to be the predominant architectural forms until 20th century. Yet, in the 21st century, curvilinear form arises again with the developments in construction techniques and innovative progress in manufacturing techniques and construction materials. These developments appeal contemporary architects and engineers to curved three-dimensional complexity especially in the case of long-spanning buildings having a symbolic character. Unusual forms are attempted in our days with the return of *curve* to architecture. Some of these modern challenges are questioned from aesthetic and structural point of view but they could generally be qualified as impressive achievements.

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