AN INVERSE FREE ELECTRON LASER ACCELERATOR AS AN INJECTOR FOR A SYNCHROTRON RADIATION SOURCE

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ABSTRACT

AN INVERSE FREE ELECTRON LASER ACCELARATOR AS AN INJECTOR FOR A SYNCHROTRON RADIATION SOURCE

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An inverse process to the free electron laser operating in the low density Compton regime as a particle accelerator is treated classically. The similarities of A tapered wiggler structure to that of a linear accelerator is shown in the sense of particle bunching and synchrotron oscillations. Trapping efficiency of particles and estimates of gain and loss for various energy ranges are investigated. It is shown that this system is effective for short range, high gradient acceleration.

Keywords: Lasers, Free Electron Laser, Accelerator, Synchrotron Radiation .

ÖZ

BİR SİNKROTRON RADYASYONU KAYNAĞI PÜSKÜRTÜCÜSÜ OLARAK BİR TERS SERBEST ELEKTRON LAZER HIZLANDIRICISI

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Düşük yoğunluk Compton rejiminde, ters serbest elektron lazeri sürecinde bir parçacık hızlandırıcısı klasik olarak çözümlendi. Parçacık kümelenmesi ve sinkrotron salınımları açısından artırımlı dalgalandırıcı yapısının doğrusal hızlandırıcılara benzer sonuçları gösterildi. Parçacıkların tuzaklanma verimliliği ve çeşitli enerji düzeylerindeki kazanç ve kayıpları araştırıldı. Bu sistemin kısa mesafe, yüksek fark hızlandırmasında etkinliği gösterildi.

Anahtar Kelimeler : Lazerler, Serbest Elektron Lazeri, Hızlandırıcı, Sinkrotron Radyasyonu.

Dedicated to my mother, Necla Akın, for her never ending love and meticulous care on her children.

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To Murat Gülver, it's my pleasure to thank for his help in typing the manuscript in word 97. I offer sincere thanks to my parents for their superior assistance by all means,their ever lasting help,love and support.And,this is nothing but a confession, that this work would never end -even would not start at all-, without the encouragement of Hatice, the prettiest jevel of my life forever.

TABLE OF CONTENTS

LIST OF SYMBOLS

I_{tot}: total power emitted by one electron due to Synchrotron Radiation.

e: electron charge = $4.80653199 \times 10^{-10}$ [statcoulomb]

c: speed of light in vacuum = 2.9979×10^{10} [cm/sec].

$m_{\circ}$: electron mass = $9.1093897 \times 10^{-28}$ [gr].

λ_w : undulator period [cm].

B_w: undulator magnetic field [Gauss].

K : undulator parameter [dimensionless].

γ : electron energy normalized to electron rest mass energy [dimesionless].

E_w: undulator Electric field [statvolts/cm].

β : electron velocity normalized to the speed of light [dimesionless].

λ : laser wavelength [cm].

β_z : longitudinal electron velocity normalized to the speed of light.

$\beta_{\perp}$: transverse electron velocity normalized to the speed of light.

$r_{\circ}$: classical electron radius = $2.81794092 \times 10^{-13}$ [cm].

ρ : radius of the electron beam trajectory [cm].

Φ : relative phase difference between the laser field and the electron oscillation [radian].

Δ : the phase shift.

Φ_r : resonant phase of the wiggler structure [radian].

γ_r : resonant energy parameter of the undulator [dimesionless].

η : electron energy difference with the resonant one normalized to the energy of the resonant electron [dimesionless].

ϕ_s : synchronous phase in a synchrotron.

$E_\circ$: Electric field at the focus of a laser beam [statvolts/cm].

$w_\circ$: The beam waist at the focus of a laser [cm].

R : Rayleigh Range [cm].

P : Laser Power [ergs/sec].

$Z_\circ$: Vacuum impedance $= 4.196 \times 10^{-10}$ [statohm].

$m_\circ c^2$: electron rest mass energy $= 0.510999$ [MeV].

n_e: number of electrons per cubic centimeter.

PREFACE

This thesis started with the hope of designing an innovative e^-e^+ collider of energies over few hundred GeV using the inverse free electron laser accelerator (IFELA) principle since the electron accelerators are limited to 300 GeV. However, calculations have shown that high gradient acceleration is not possible for energies over 50 GeV.

In chapter 1, novel methods of particle acceleration and their advantages and disadvantages are discussed. Free Electron Lasers(FEL) and other lasers are compared in the sense of their power and operating mechanisms. Synchrotron Radiation (SR) facilities and their structures are outlined briefly, and SR as a power loss mechanism in accelerators is pointed out. Insertion devices to improve the brightness of SR sources and limits of magnet technology are summarized.

Chapter 2 uses the single particle model of the FEL developed by Colson W.B. and Pellegrini C. The first section describes quantum mechanical single particle stimulated Compton scattering picture, which reveals the upconversion of electron energy at the expense of the optical power or vice versa. The rest of the theory uses classical electrodynamics, and it suffices for a full description of the FEL. Particle bunching at the optical wavelength scale both for constant parameter wiggler and tapered wiggler and the oscillations of nearby particles about the resonant electron are shown theoretically.

In Chapter 3, principles of the inverse process to the FEL for acceleration are discussed. Section 2 and 3 reveal, the contradicting nature of high gradient acceleration and high brightness particle acceleration, and the similarity of Linacs and Synchrotrons with the IFELA in the sense of longitudinal oscillations which is likely to be called synchrotron oscillations. Section 5,6 and 7 investigate the loss and gain properties of an IFELA and end up with the conclusion that the most effective range of acceleration using this process is the interval of few MeV to few tens of GeV. In section 8 and 9 the structure of magnetic field profiles for two tapering options is discussed and the current limitations up to various criteria are determined. Section 10 and 11 clarify the problems with the practical realization of an IFELA and exemplify the experimental verifications, which prove that this method really works. In section 12, the possibility of transverse focusing property of an IFELA according to two different theories is discussed. In this final section, it is recommended as a compact and practical industrial accelerator. The possibility of obtaining laser-like X-ray radiation pulsed in the femtosecond time scale is discussed if IFELA is used as an injector in an SR facility. The investigation of transverse focusing, and emittance properties are still incomplete, but they need a different treatment with a different formulation.

As to the presentation of the material, I tried to keep the thesis as clear as possible, preventing it from esoteric complications, but still keeping track of scientific rituals to protect it from naive simplifications. With the critics and guidance of my supervisor, I hope this point is accomplished.

CHAPTER 1

A GENERAL VIEW

1.1.Introduction

With the advent of the laser, a new era begun not only in optical research fields such as holography and spectroscopy, but also it found some applications in other fields of physics as well. Since that time a great many kind of lasers using gases, solids, liquids, various molecular mixtures or even free electrons as the active medium with different coherence, frequency, power, directionality, and size options have been built even pushing the laser applications to broader fields. Within these large application fields, one of them is the particle acceleration using lasers as the power source.

The study of advanced methods of acceleration for research has been largely concentrated on two quite different areas of applicability. First, the acceleration of electrons and positrons to ultrarelativistic energies (>1TeV) and second, the acceleration of electrons, protons and ions from rest or very low energies to a speed that is a significant fraction of the speed of light. On the other hand various accelerators found some applications in industry. To note a few, linear accelerators and cyclotrons are used in isotope production, in radiation therapy in cancer management, ion implantation polymer modifications, food preservations, disinfection treatment of water, electron beam treatment of metals (0,1-10 MeV), welding, melting, drilling, hardening, film deposition, materials inspection, sterilization, "fine-tuning" of semiconductors, pulse radiolysis, laser pumping (FEL), fusion research, criminology, archeology, art and history, in research with neutron scattering and synchrotron radiation. In the future, commercially available accelerators will require energies in the GeV range and megawatts of beam power (Turner S.,1996)[1]. However, regardless of the application, and range of energy, all accelerator dependent facilities suffer from size and expense. The basic impetus for the development of alternative acceleration methods to conventional synchrotrons, cyclotrons and linacs are the need for increasing the acceleration gradient and decreasing the expenses. Conventional acceleration structures, for instance, klystron driven linear particle accelerators are limited to a maximum acceleration gradient of 50 MeV/m. This is determined by breakdown of the disc loaded waveguide acceleration structure and the maximum power that can be obtained from klystrons(Piestrup M.A.,1985)[2]. Other less advanced conventional structures are limited to 10 MeV/m. To build a 1TeV Linac, then, one needs approximately 100 km of accelerating structure, not an attractive picture since one will run out of space or money (usually money before space). So the prospects of future accelerators are high gradient accelerating structures, high beam quality and cheapness.

1.2.New Methods of Acceleration

The chart in which the increase of energy is plotted as a function of years in the accelerator technology is shown in Figure 1(Sessler M.A.,1985)[3]. This increase of energy is the result of the fact that diverse technologies are applied to the conventional accelerating structures. But to keep increasing, new methods should have been proposed.

The new methods proposed are three fold: collective accelerators (having particles accelerated interacting with the others), laser accelerators, and multibeam accelerators. The first class is out of the scope of this thesis. Laser accelerators include, Inverse Free Electron Laser Accelerator (IFEL), The Plasma Laser accelerator or the Beat Wave Accelerator (BWA), the Inverse Cerenkov Accelerator or Stimulated Cerenkov Interaction Device (SCI). Multibeam Accelerators include the TwoBeam Accelerator (TBA) which utilizes Free Electron Masers instead of a klystron in a different linac structure, and the Wake Field Accelerator, which is capable of achieving gradients of 500 MeV/m if some practical and theoretical problems are solved. Each method has its own advantages and disadvantages.

The reason for the suggestion of various accelerating structures using lasers is the incredibly large optical component in the focus of a laser beam. As a comparison, the electric field at the focus of a powerful laser beam can reach to about 10^4 - 10^6 MV/m whereas that of the Stanford Linear Collider (SLC) reaches to 17 MV/m. Nevertheless, the direction of the electric field to transfer energy to the charged particle is in the wrong direction; transverse to the direction of propagation. Moreover, the field has a finite depth, more clearly the electric field at the focus of the laser falls down to a half fraction in a Rayleigh range and decreases with the distance from the focus.

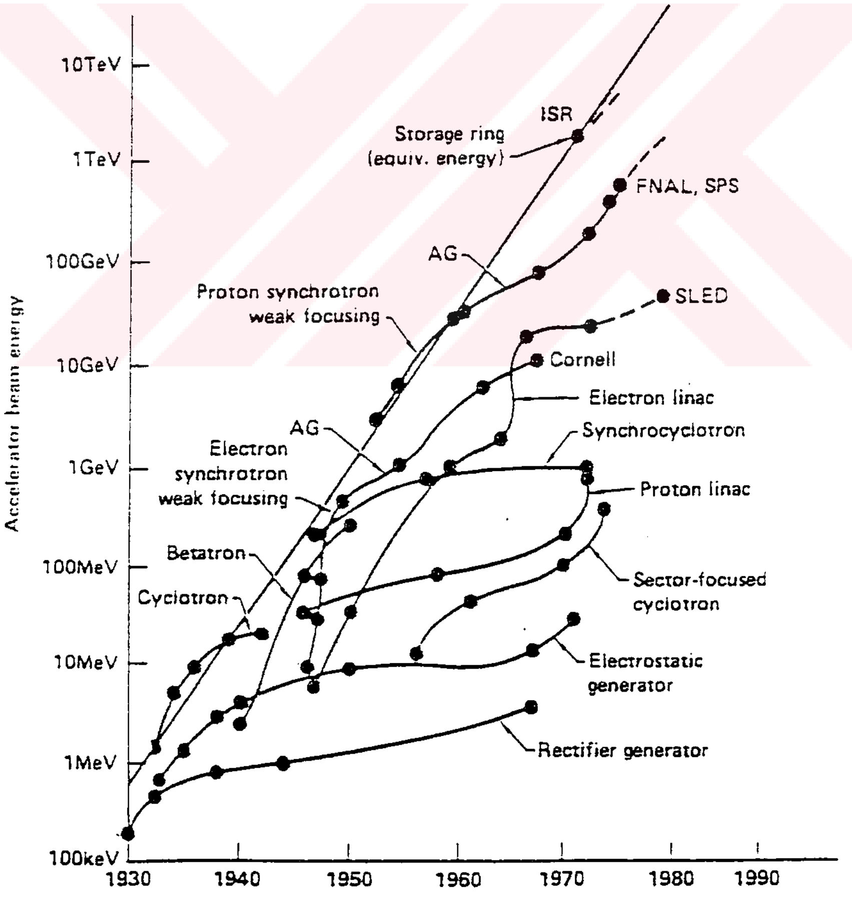

Fig.1. "The Livingston Chart", which shows energy of particle accelerators as a function of years.

To add to the picture, a lightwave travels at the speed of light, which is not possible for a material entity, hence there is a problem of synchronism to couple the light and particle beams unless the wave is slowed down as that in a microwave travelling wave tube slow wave structure, or, another witty solution is found.

Furthermore, the motion of a free electron in a plane electromagnetic wave, being determined by the oscillating electric and magnetic fields of the lightwave, in the overall picture does not accelerate the electron, the fact that there is no continuous acceleration is shown in Figure 2

Consequently, to achieve acceleration using a laser, either the wave must be slowed down being in a medium (e.g. a dielectric gas), or near a surface (by means of evanescent waves etc.),or, the particle must be bent in a periodic manner by some external means to couple with the electric field of the laser field resulting in energy exchange as in a FEL, or, the direction of propagation of the particle beam and the laser beam must be different as in an SCI device to achieve synchronism. Two Inverse Cerenkov Accelerator Structures to achieve synchronism is shown in Figure 3 [2].

Dielectric Gas loading to increase the interaction depth leads to an acceleration gradient of 200 MeV/m. Using this mechanism to accelerate electrons to high energies requires an index of refraction corresponding to about 1 atm of H_2 gas, leading to multiple scattering as a side effect.

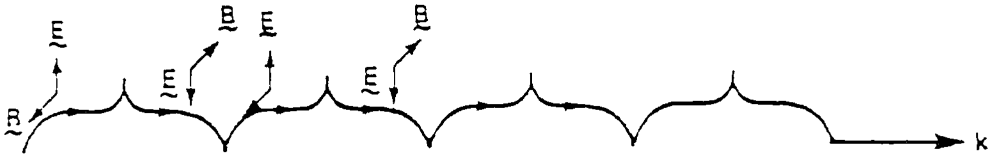

Fig.2. Motion of an electron in a plane electromagnetic wave. The particle motion is determined by a combination of (reversing) transverse electric field which accelerates the particle and reversing transverse magnetic field which bends the particle. The net effect is that a particle is moved along in the direction of the wave, but is not accelerated (from Sessler M.A., 1985)[3].

The mechanism of The Plasma Beat Wave Accelerator (PBWA) proposed by Tajima T. & Dawson J. In 1981[4], where two intense lasers or masers are shone on a plasma of plasma frequency equal to the difference of the frequencies of the lasers, is capable of accelerating plasma electrons to high energies in large flux.

The photon beat excites through the Forward Raman scattering large amplitude plasmons whose phase velocity is close to c in an underdense plasma. The Plasmon electrostatic fields trap electrons and carry them to high energies. This is to utilize free electrons (plasma electrons) non-linearly trapped by the created electrostatic wave and in this sense it is an inverse process to the Free electron Laser [4]. The PBWA could be considered as an IFEL operating in the Raman regime. The plasma frequency is a function of charge density and therefore, the difficult task of Plasma Control makes it unpractical.

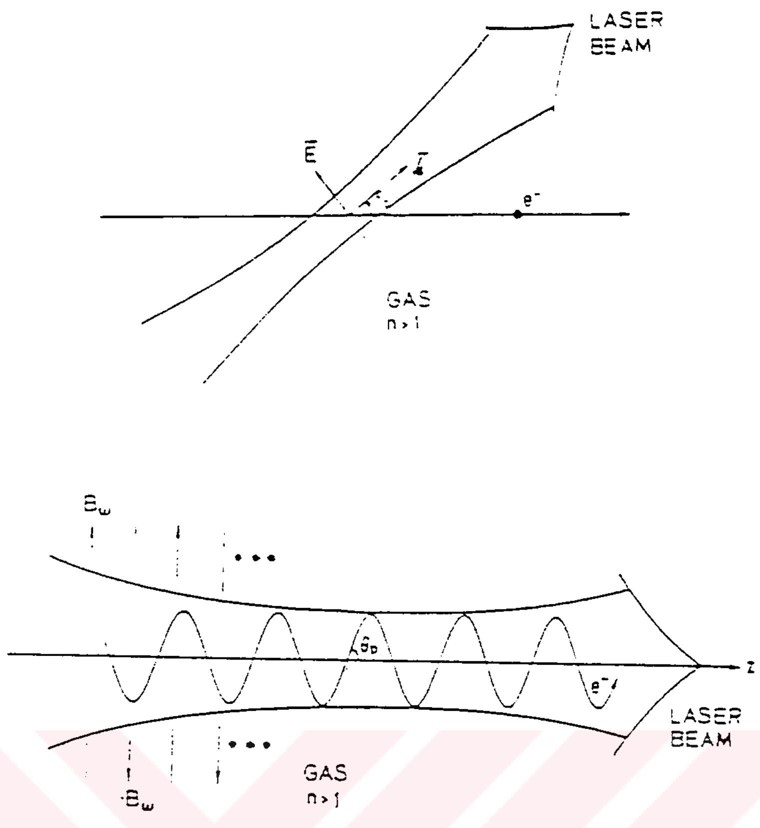

Fig. 3. (a) The stimulated Cerenkov interaction. (b) The gas loaded inverse Cerenkov accelerator (from Piestrup M.A.,1985)

More of a practical concern is the Two Beam Accelerator (TBA) originally suggested by Sessler (1982)[5], since it combines FEL (or maser) with several well known technologies -high current induction linacs microwave waveguides, and travelling wave linac structures (Selph F.B., 1983)[6].

There are two reasons for such a structure to prefer, in the first place, it operates at higher frequencies than present day linacs (3GHz at SLAC, Stanford Linear Accelerator and a potentially 35 GHz FEL is underway at Livermore),

and secondly, FEL is an efficient means of delivering high power microwave energy to a high gradient structure. It is observed that, to get to ultra-high energies in a conventional linac structure keeping the power requirements within bounds, one must go to higher frequency accelerating fields. The stored energy is proportional to the transverse area of the linac, but decreases in inverse proportion of the frequency as the transverse size increases, and the transverse size is proportional to the RF wavelength. To break through this conflicting situation both the frequency and power must be increased, however, at 30 GHz, ten times the frequency of SLAC, none of the available power sources as multibeam klystrons, photocathode klystrons, gyrotrons is sufficiently developed. The FEL is suggested as the Power Source (Keefe D., 1988)[7]. The Two-Beam Accelerator scheme is shown in Figure.4. [8]

An induction accelerator (Low energy beam in Figure 4) produces an intense electron beam of a few Mevs.

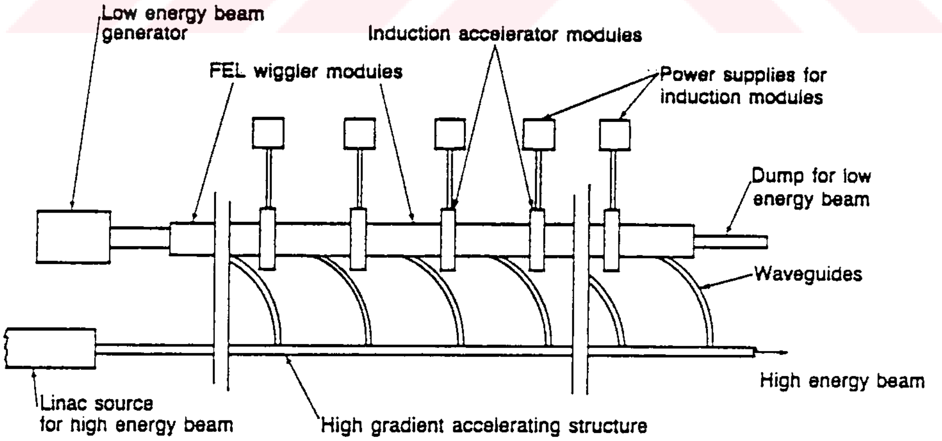

Fig. 4. The Two-Beam Accelerator Scheme (from Hopkins D.B., 1988)[8].

The beam passes through wiggler modules to produce microwave energy. This microwave energy is transferred through a waveguide to the high-energy beam in such a way that longitudinal electric field gradient is produced for acceleration of the high-energy beam. The energy lost from the low energy beam as radiation is restored by induction accelerator modules. This scheme promises acceleration gradients of up to 250 MeV/m and the promising application is for a linear collider of very high energy say 375 on 375 GeV. The Problems are technical such as making a compromise between efficient power transfer and overmoding in waveguides, to adjust lengths of waveguides for proper phase matching etc.

The Inverse Free Electron Laser Accelerator (IFELA) stands as a compact means of particle acceleration. It has the same disadvantage as the TBA; the particle beam to be accelerated has to be already relativistic for the interaction to take place. In the IFELA no medium is present, the electron beam is in vacuum unlike gas loaded inverse Cerenkov case, so that higher field strengths can be obtained. Vacuum also means collision free motion. Large transverse area of the structure, that is, the electron beam being far from any material boundary discards the option of breakdown. There is no problem of phase synchronism, however, there are some certain disadvantages too; the component of the motion parallel to the accelerating field component of the laser field is small, giving rise to a relatively smaller energy exchange, the design of the field profile is not easy since the trajectory of the electron is not a straight line, there is need for B-fields, and there is need for focusing laser to increase the acceleration component and it has its own disadvantages as mentioned before.

1.3. Lasers

The basics of a laser are essentially quantum mechanical. The active medium of a laser consist of either organic dye molecules, or atoms, or semiconductors is given energy through some external means, called the pumping mechanism by means of electrical, chemical or optical processes. When the population inversion is reached, the upper state(s) being more populated than the lower one(s), and once some electrons fall to lower energy states, within their lifetimes, emitting radiation to stimulate the others, the radiation field grows rapidly with a phase correlation giving rise to coherence. The output is built after the radiation bounces back and forth in a cavity standing wave structure gaining energy from the medium and losing some of the energy to reach saturation. This general procedure is treated quantum mechanically. The wavelength of an emitted photon from a transition is definite and there is one and only one kind of transition to set up the output, though there is broadening due to some natural processes such as Doppler broadening, collisions or temperature fluctuations. Basically, the fundamental frequency is certainly determined by the very nature of the quantum mechanical states of the active medium. To change the energy levels of an atom it has to be put in extremely large fields that one can only reach on the surfaces of neutron stars (Merzbacher E.)[9], far from being a possibility on the earth. That's why The U.S. National Bureau of standards (NBS) supports atomic laser development research.

However, an FEL utilizes free electrons as the active medium. Although quantum mechanical treatment is possible, classical treatment suffices. An electron can emit more than one photon, and it usually does, though the frequency of the emitted photon is not necessarily a means of a standard due to magnetic field imperfections in the wiggler and due to energy and momentum fluctuations of the incident electron beam. The high energies and a wide range of continuous tunability with a high degree of coherence and optical properties makes the FEL attractive as a research tool in spectroscopy, as a direct energy weapon in military and a useful source for industrial applications. That's why the U.S. Department of Energy instead of The NBS supports the FEL research.

1.4. Synchrotron Radiation

It is a well known fact that charged particles if accelerated radiate and radiate in a narrow cone if bent at relativistic velocities tangential to the trajectory of its motion with an opening angle of $1 / \gamma$. This fact is first predicted by Ivarenko and Pomeranchouk in 1944, and since visible radiation of this kind was first observed for the first time in 1947 from a 70 MeV synchrotron at General Electric, it is called the synchrotron radiation. Nonetheless this radiation was not welcome at times of its first observation since it was considered as a side effect in the early synchrotrons as a power loss mechanism. But the properties of the radiation making it valuable for the following reasons were later realized. In the first place, a synchrotron source produces naturally polarized radiation; the radiation emitted by an electron in a bending magnet is totally polarized in the plane of the electron orbit. Secondly, the available spectrum ranges from the Far Infrared up to hard X-rays depending on the energy of the electron. Thirdly, The properties of the SR, once parameters of the particle beam are given, are absolutely calculable (Kunz C., 1990)[10]. And Furthermore VUV, and Soft X-ray light, which could be obtained using synchrotron radiation (SR), have the right wavelength [$\lambda \approx 10^{-7}$ -10^{-10} m] for exploring the atomic structure of solids, molecules, and important biological structures. The sizes of atoms, molecules, and proteins as well as the length of chemical bonds, the minimum distances between atomic planes in crystals are in this range (Schlachter A.S., 1994) [11]. Today's SR sources have some advantages over their ancestors. They use not only bending magnets to produce SR, but also same insertion devices to improve their optical properties such as brightness, coherence and flux. These insertion devices consisting of wigglers and undulators; as periodic magnetic structures are shown in Figure 5, where the major differences in between being the degree of transverse excursion of the electron and the forthcoming changes in optical properties. In the wiggler regime, the transverse oscillations of the electrons are so large that the angular deviations of the emitted radiation are wider than the opening angle of the synchrotron radiation. The radiation field is an incoherent sum of the fields by each magnet section. If there are N magnet sections, the gain in intensity relative to a bending magnet of the same strength is 2N. In the case of the undulator, however, the amplitudes of the field radiated at each period of the electron trajectory may interfere, giving rise to a line spectrum. The gain in brilliance can reach to N^2 times that of a period. The machines under construction, as the third generation, have wigglers and undulators, having very small a product of the source size by the divergence of the electron beam, (i.e. low emittance). ALS (Advanced Light Source) facility in this third generation proves to be a good example.

A linac accelerates electrons to some 50 MeV, which then injects them into a booster synchrotron for acceleration to 1.5 GeV. At only 3 MeV electrons are already accelerated to 99% of the speed of light and at 1.5 GeV they are at 99.999996% of the speed of light. They then enter a storage ring of a circumference of 200 meters, to emit third-generation SR. Their schemes are given in Fig 6 & Fig 7(from Schlachter A.S., 1994) [11].

These third generation sources of SR, utilizing electrons or positrons in the energy range of 1-10 GeV are tabulated in Table 1[12] .The reason for using electrons or positrons instead of protons is that the total power emitted by one single particle is given as (Kunz C., 1979)[13]

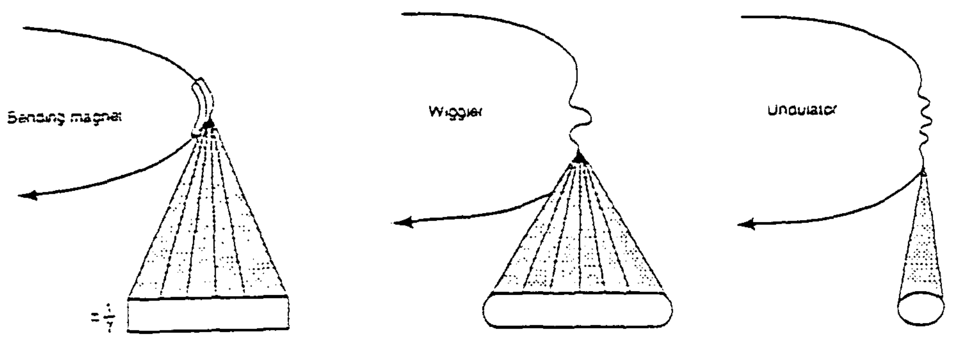

Fig. 5. The radiation beams emitted by a bending magnet, a wiggler, and an undulator (from Petroff Y., 1994)[12].

$$I_{total} = \frac{2}{3}\frac{e^2 c}{r^2}\gamma^4 = \frac{2e^2 c}{3r^2}\frac{E^4}{(mc^2)^4} \qquad (1.1).$$

Where $E = \gamma m_o c^2$ is the particle energy and r is the radius of curvature of the trajectory, and this expression yields much larger power radiated for lighter particles.

Table 1. Third-generation machines under construction (from Petroff Y., 1994)[12].

Name	Place	Maximum Electron Energy (Gev)
In the hard X-rays range		
APS	Argonne/USA	7
SPring-8	Japan	8
In the UV and soft X-rays range		
ALS	Berkeley/USA	1.5
ELETTRA	Trieste/Italy	2.1
BESSY II	Berlin/Germany	1.7
MAX II	Lund/Sweden	1.5
SRRC	Taiwan, Republic of China	1.5

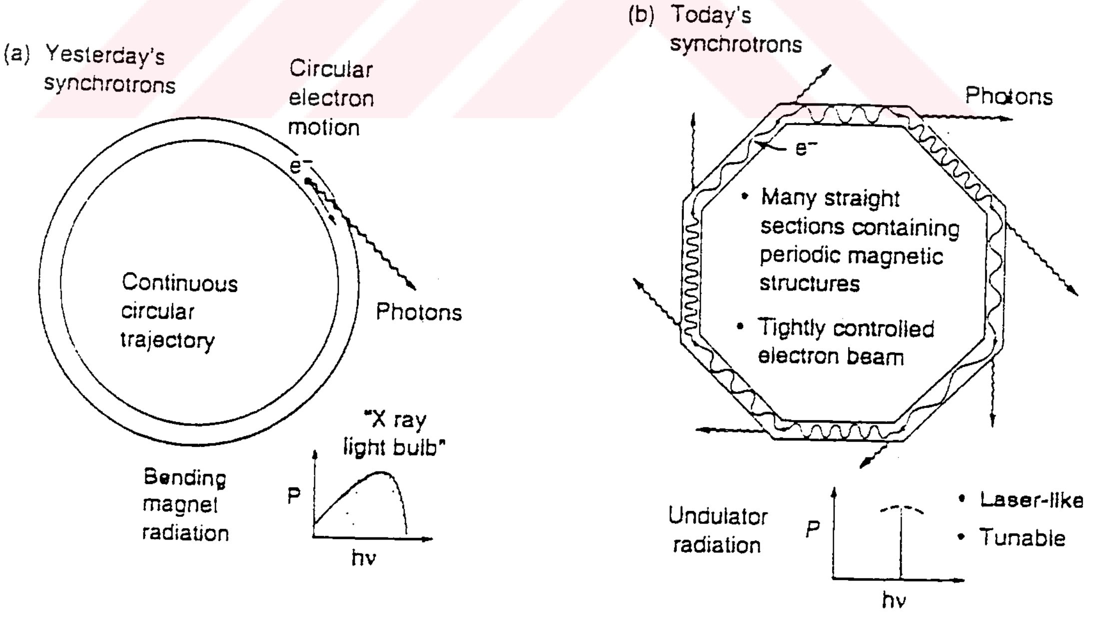

Fig. 6 Characteristics of, a) early synchrotrons, b) Third generation facilities. Note that, wiggler radiation is few orders of magnitude brighter than the bending magnet radiation

Using proton instead of electron yields twelve orders of magnitude smaller amount of energy radiated by S.R. To avoid SR. losses in synchrotrons and accelerators based on particle trajectory bending, their radius of curvature must be as large as possible.

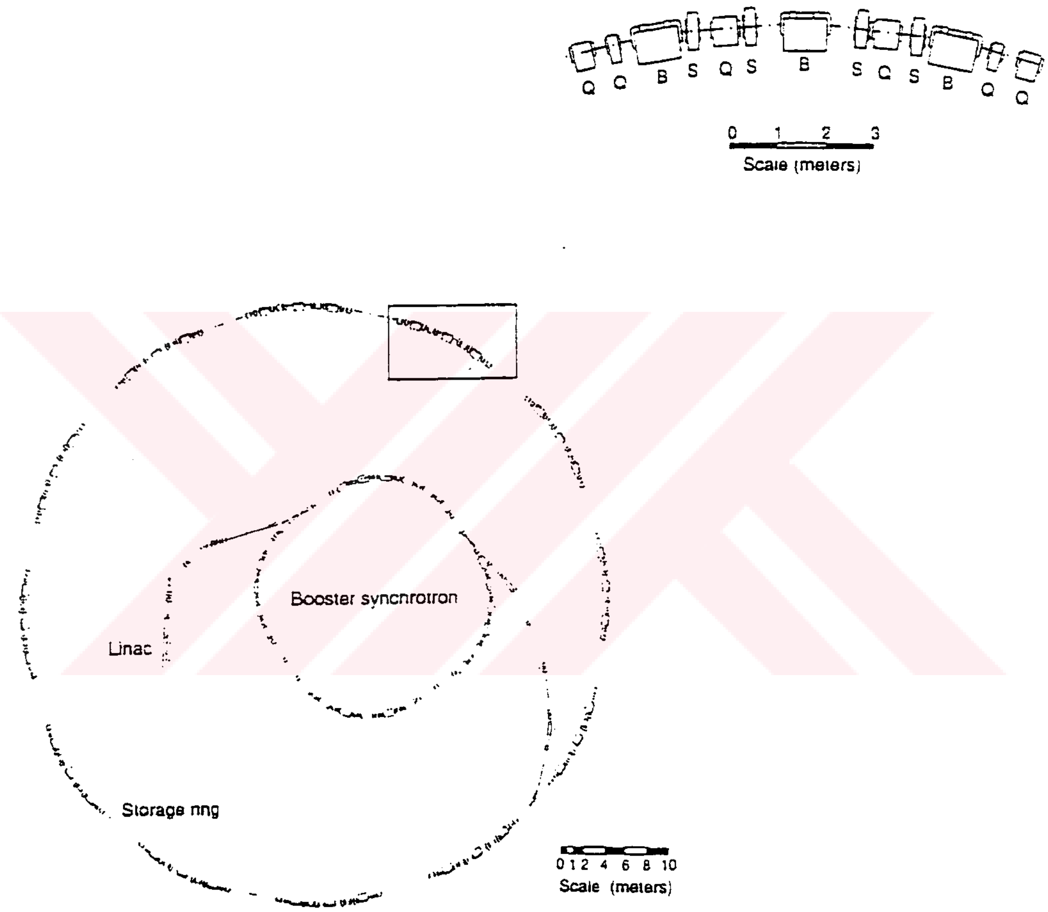

Fig.7. The ALS storage ring has 12 arc-shaped and 12 straight sections. The arc sections are imbedded in a magnetic lattice, which consist of 12 magnet sequences, one for each. (B), (Q), (S) denote bending magnets, quadrupole magnets, and sextupole magnets respectively. Relativistic electrons travel through the storage ring generating SR.

1.5. Insertion Devices

These are the magnetic structures, which causes the electrons to wiggle in the interaction region transverse to the direction of propagation of the electron beam. They can be either a periodic array of alternatively polarized magnets to produce " ideally " sinusoidal magnetic field or an ideally helical magnetic field structure as in figure 8[10]. Principally, one can generate a periodic magnetic field by a helical winding of 2ℓ wires ($\ell = 1,2....$). The case $\ell = 2$ is referred to as quadrupole magnetic field structure, and builds up of two pairs of helical windings with currents in opposite directions (Bilikmen S., 1994).[14] For these two cases we can write the idealized magnetic fields as

$$\overrightarrow{B_w} = \hat{x}\, B_w \cos \frac{2\pi z}{\lambda_w} \qquad (1.2)$$

And

$$\overrightarrow{B_w} = B_w \left(\hat{x} \cos \frac{2\pi z}{\lambda_w} + \hat{y} \sin \frac{2\pi z}{\lambda_w} \right) \qquad (1.3)$$

Where, B_w is the peak magnetic field strength, λ_w is the period length, and z stands for the propagation direction. For the first case we can assume an exact sinusoidal trajectory,

$$y = -a \cos(k_w z) \qquad \text{with } k_w = \frac{2\pi}{\lambda_w} \qquad (1.4).$$

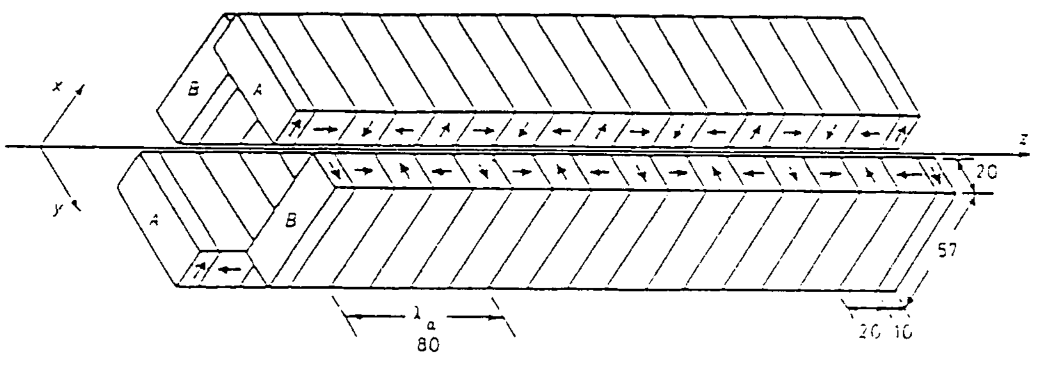

Fig. 8 The magnet design proposed for a helical undulator (from Kunz C.,1990)[10].

$$\frac{dy}{dz} = k_w \mathrm{a} \ \sin(\ k_w\ z) \qquad (1.5).$$

$$\left[\frac{d^2y}{dz^2}\right]_{\max} = k_w{}^2 \mathrm{a} \qquad (1.6).$$

$$\left[\frac{d^2y}{dz^2}\right]_{\max} = \frac{1}{\rho} \qquad , \ \rho \text{ : maximum radius of curvature} \qquad (1.7).$$

This circle and centripetal acceleration picture reads

$$e\vec{\vartheta}\times\vec{\mathrm{B}}_w = m_e\frac{\vartheta^2}{\rho}, \qquad \text{or,} \qquad \rho \cong \frac{\gamma mc}{e\mathrm{B}_w} \qquad (1.8).$$

And the angular deflection given in terms of opening angle $\frac{1}{\gamma}$ units leads to the definition of unitless deflection parameter K.

$$\left[\frac{dx}{dz}\right]_{max} = k_w a = \frac{K}{\gamma} \qquad (1.9).$$

If, $K \leq 1$, then, undulator regime holds.

Formula 7,8 & 9 gives

$$K = \frac{eB_w}{k_w m_o c} = \frac{eB_w \lambda_w}{2\pi m_o c} \qquad (1.10).$$

The deflection parameter, -since it contains only undulator magnetic field and the periodicity as the variable parameters-, is sometimes referred to as undulator parameter. For an IFEL, synchrotron radiation is a nuisance and no matter which regime holds; the interaction radiation field is external and S.R. is a loss mechanism.

The wavelength of the radiated spectrum in a sinusoidal wiggler is given by Kincaid and Howels as, [11]

$$\lambda_i = \frac{\lambda_w}{2i\gamma^2}\left\{1 + \frac{K^2}{2} + \gamma^2\theta^2\right\} \qquad (1.11)$$

Here, i=1 gives the fundamental harmonic and θ stands for the observation angle.

1.6. Magnets

The maximum value of B_w is determined by the choice of technology (electromagnet, permanent magnet or superconducting magnet). A wiggler is supposed to have a small period, (cm), a sufficiently high field amplitude on axis (not less than a few kGauss) , and since the electron beam will wiggle in the structure , it has to provide a sufficiently large aperture (few cm). For a room temperature electromagnet, the above mentioned parameters require current densities well above kA/cm^2 range, the limit imposed by cooling requirements (Gaupp A., 1995).[15] That's why the first FEL wiggler was superconducting.

Examples of the projects and designs are: ACO at Orsay, Brookhaven, LELA at Frascati and Felix at Daresbury. ACO storage ring has wiggler period λ_w= 4cm , K ~ 1.5 with a current density 200 A/mm^2 and an on axis field B_w=0.4 T. Brookhaven designs are supported by a water cooled copper coil with a maximum current density of 150 A/mm^2, giving similar parameters as Orsay. In Frascati case $\lambda_w = 12cm$, $I_{max} = 30\ A/mm^2$, $B_w = 0.5T$, $K \leq 5$. Felix magnet is another similar case with, λ_w= 20 cm, I ~ 10 A/ mm^2 in water-cooled coils (Poole M.W, 1983)[16].

Rare Earth Cobalt (REC) magnets have changed the picture quite impressively. REC alloys are highly anisotropic, having an easy axis of magnetization. The permeability being close to one, blocks of material appear only as surface currents, with current densities 1 kA / mm^2 so these current sheet equivalent materials allow linear superposition exactly as if they were coils greatly simplifying design procedures. Magnetic Fields Larger than 1.25 T are attainable using REC alloys.

The most famous alloys consist of $SmCo_5$, Sm_2Co_{17}, $Nd_2Fe_{14}B$ being mixed in proper ratios. Their sintered blocks of magnetic material are extremely brittle and spontaneous oxidation of dust exposed to air may cause explosions.

The alloy $Nd_2Fe_{14}B$ instead of $SmCo_5$ provides 30% more a B_w than the latter. However, temperature sensitivity and worse radiation resistance of $Nd_2Fe_{14}B$ affects the long term stability.

As to helical magnets, the wiggler for the Stanford FEL employs niobium-titanium conductor to achieve an on axis fields of up to 1.3 T. Permanent magnets can build up the helical system as well. However, varying the magnetic field comes as a major difficulty. If the magnet period is not too short, water-cooled electromagnets will be used for their ease of control and cheapness.

Design considerations for tapered undulators, -the magnetic fields and periods varying on the axis-, requires special attention. In this thesis magnetic field strengths not more than 2 T will be assumed and design engineering for tapered wigglers will be discarded.

CHAPTER 2

FREE ELECTRON LASERS

2.1. Stimulated Scattering Picture

There are two primary components of the FEL, the first is an accelerating mechanism to provide energetic particles, and the second is an insertion device for converting particle kinetic energy to radiation. An optional third component could be a mirror pair as a cavity if the device is to operate in oscillator mode instead of amplifier. The primary difference of the FEL from a synchrotron radiation source is that the radiation in an SR device is still a spontaneous radiation whereas the FEL uses stimulated scattering to obtain laser light.

In a frame moving with the electron in a periodic transverse magnetic and/or electric field(s) with relativistic velocities the fields will transform as (Jackson J.D., 1975)[17]

$$\vec{E}' = \gamma(\vec{E}_w + \vec{\beta}\times\vec{\beta}_w) - \frac{\gamma^2}{\gamma+1}\vec{\beta}\left(\vec{\beta}\cdot\vec{E}_w\right) \qquad (2.1.a)$$

$$\vec{B}' = \gamma(\vec{B}_w - \vec{\beta}\times\vec{E}_w) - \frac{\gamma^2}{\gamma+1}\vec{\beta}(\vec{\beta}\bullet\vec{B}_w) \qquad (2.1.b)$$

where the variables are wiggler electric and magnetic fields and β is the velocity of the electron normalized to the speed of light.

Since the velocity of the electron is primarily on axis and has negligible component transverse, and since the magnetic field is transverse $\vec{\beta}\cdot\vec{B}_w$ term drops. If there is no $\vec{E}_w$ field, then we are left with

$$\vec{E}' = \gamma\vec{\beta}\times\vec{B}_w \qquad (2.2.a)$$

and

$$\vec{B}' = \gamma\vec{B}_w \qquad (2.2.b)$$

Relativistic velocities implies $\beta\sim 1$, so, $|\vec{B}'| \approx |\vec{E}'|$, and taking $\vec{\beta}\approx\vec{\beta}_z$, which really is the case where β_z is the parallel component on axis, the $\vec{E}'$ and $\vec{B}'$ fields are nearly perpendicular. Note that, symmetry of the eqn.s 2.1 implies that a zero transverse magnetic field and an applied periodic transverse electric field, with a different undulator mechanism, leads to very similar results. In fact, there are proposals for electrostatic wigglers, or electromagnetic waves of high intensity or, electrostatic waves that could be obtained by injecting the electron beam into an appropriate plasma, or crystal. Most of them are less convenient, for instance, periodic electrostatic fields due to their extremely high voltage requirements over the surface breakdown limits, hence the Magneto-Bremsstrahlung devices win the elections.

To the electron, the undulator appears as an electromagnetic wave. If say a photon of the same frequency, is fired on the electron in the opposite direction with the virtual photon of the undulator field what is expected? In figure 9 it is pictorially illustrated that the electron will either forward scatter the undulator photon, or back scatter the laser photon.

In this scheme, one can adjust the energy of the electron such that the wavelength of the spontaneous radiation corresponds to the wavelength of the laser radiation. There are two possibilities: (1) the electron scatters the undulator virtual photons ahead and loses momentum.(2) The electron scatters the laser photon backward and gains momentum. Using the Compton diagram with $\phi_c' =$ 180° and $\theta_c' = 0^\circ$, with the conservation of energy and momentum, eliminating the electron momentum then there is a frequency shift $\Delta\omega' = \omega_i' - \omega_s'$ due to electron recoil

$$\frac{\Delta\omega'}{\omega_i'} = \frac{mc^2}{2\hbar\omega_i'} \qquad (2.3)$$

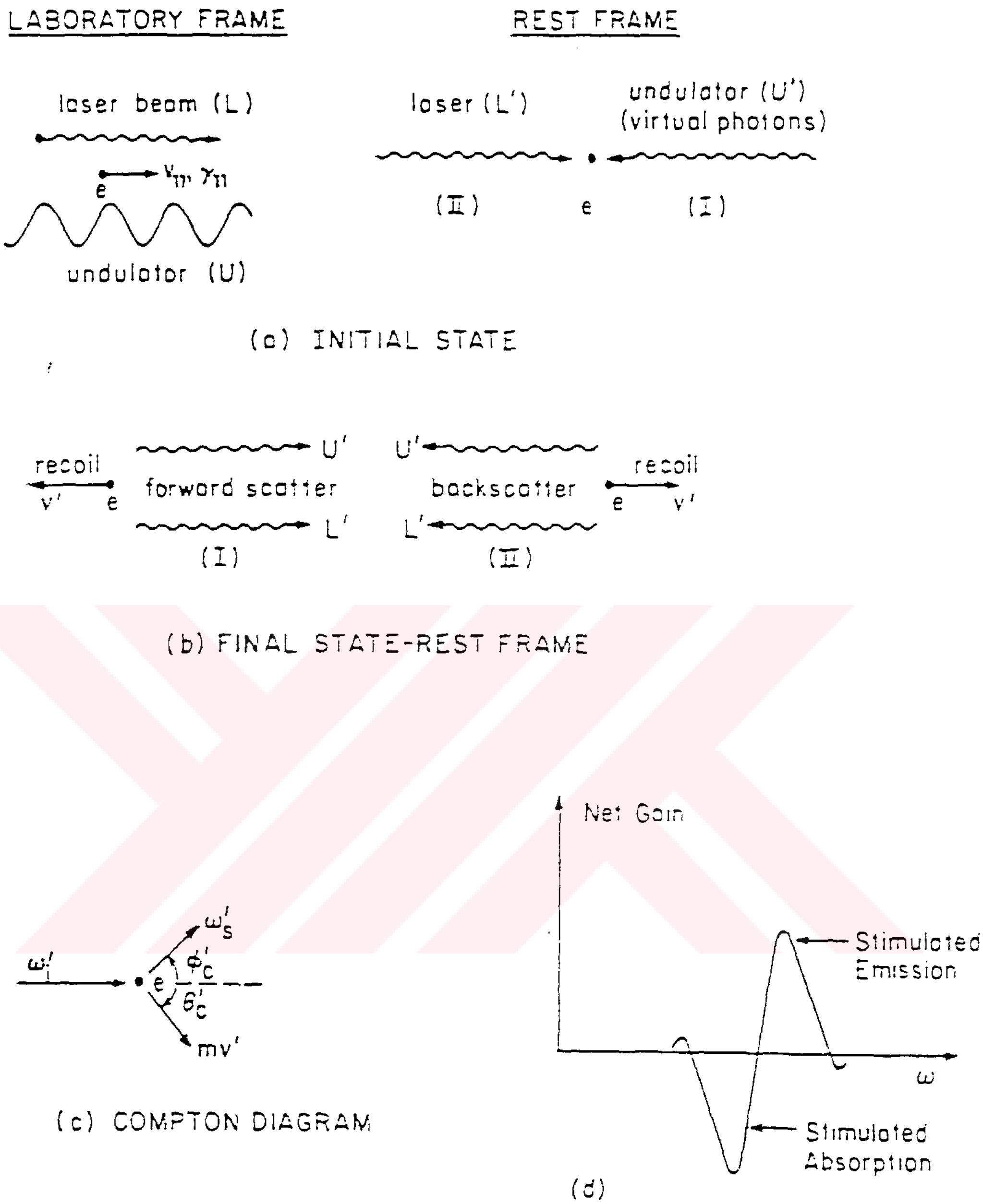

Fig. 9 The Compton scattering interaction diagram for a single electron for the accelerator case (from Marshall T.C., 1985)[18].

The processes in 1 and 2 are opposite so they occur at different wavelengths, and this is mentioned on figure 9.d.

The Weiszacker-Williams approximation that the wiggler field is replaced by a free optical wave in the pseudo-rest frame, gives the idea of electron acceleration or field amplification. Here the spontaneous radiation is described as a scattering of a wiggler photon off an electron at rest on resonance, and stimulated by laser photon of the same polarization.

The scattering of an external photon back into the wiggler field where the process is stimulated by the virtual photon is equally probable. For electrons exactly on resonance, the two rates are equal and the net effect is zero.At resonance, a FEL provides no gain, and this is shown in figure 11.b& 9.d. In the system running in Figure 10 the FEL operates in the amplifier mode and the CO_2 laser light is amplified 24 MeV being the resonance energy of the electrons. Note the similarity of the gain curve of fig 9.d. with figure 11.

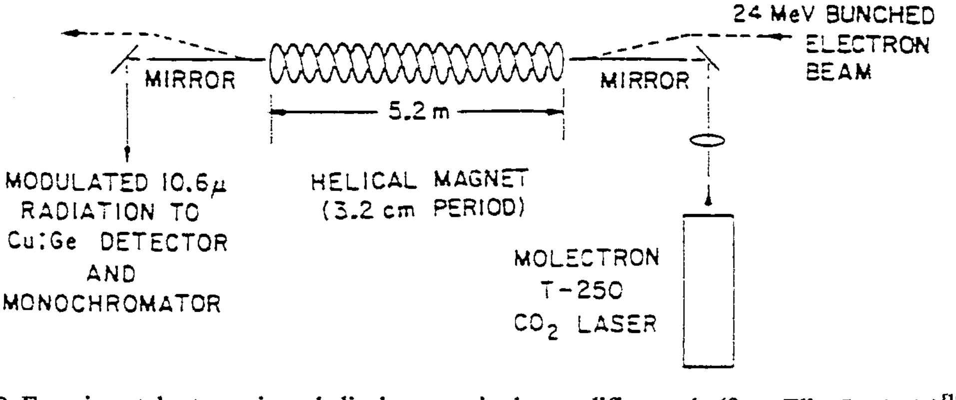

Fig. 10. Experimental setup using a helical magnet in the amplifier mode (from Elias L., 1976)[19].

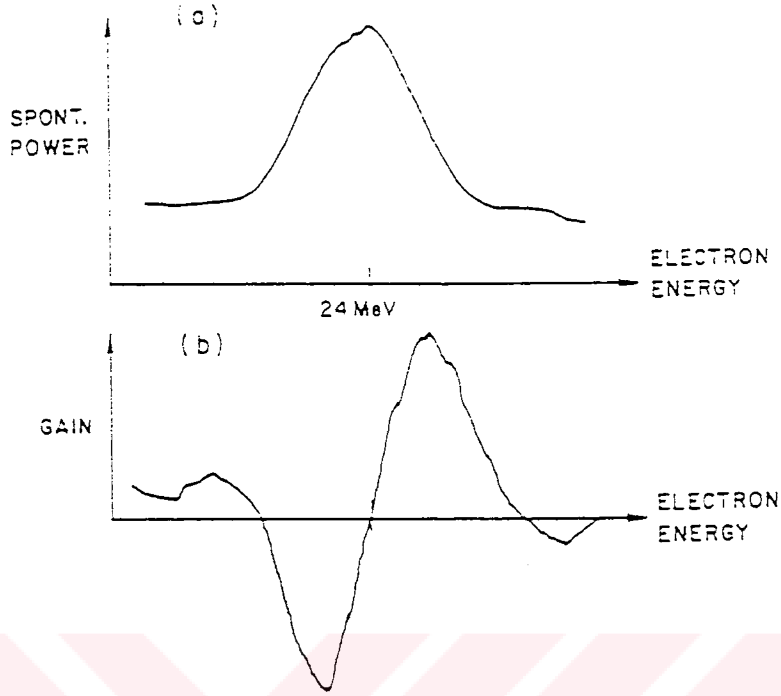

Fig. 11. a) The spontaneous power at 10.6 μm as a function of electron energy. b) The gain curve as a function of energy for the setup illustrated in figure 10.(from Elias L.,1976)[19].

The gain curve being asymmetric faces us to think that the mean kinetic energy of the electron beam E_{kin} changes. By energy conservation the optical energy E_{opt} then changes accordingly

$$\Delta E_{opt} = -\Delta E_{kin} \qquad (2.4.).$$

To gain more insight into the mechanism alternative classical description has to be presented.

2.2. Classical Description

Free electron laser wiggler schemes using sinusoidal, helical and uniform magnetic field structures have been proposed. The reason for using a periodic structure is to obtain a periodic motion sinusoidally or helically. One can obtain a helical trajectory using a uniform magnetic field as well, which is a situation likely to be met in magnetosphere, astrophysical magnetic fields and laboratory devices; Colson and Ride in 1979[20] have analized a FEL process such as the one in figure 12. They also analyzed the situation where the radiation from spiraling electrons enhance the energy of leading electrons, which can also be considered as a laser accelerator[21].

The situations to be discussed is (1) the electron on a helical trajectory is shone by a circularly polarized laser, if the electric field component of the laser is parallel to the direction of the motion of the electron the field loses energy to the electron and vice versa (2). The electron moves on a sinusoidal trajectory and shone by a linearly polarized laser, the polarization direction matches the plane of the electron motion and, the above mentioned situation applies. The growing radiation field means a FEL and the growing electron energy the IFEL accelerator. The analysis of a helical trajectory is simpler since the parallel component of β is not a function of z unlike a sinusoidal wiggler.

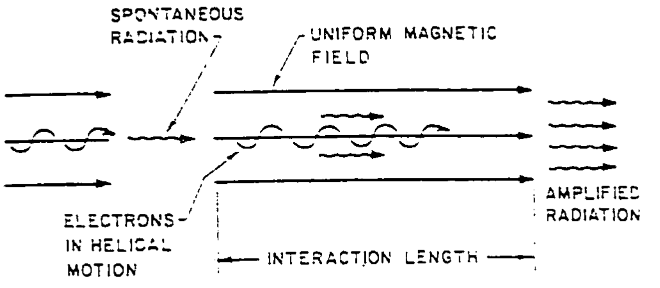

Fig. 12. Relativistic electrons spiral along the field lines of a uniform magnetic field, and emit circularly-polarized, monoenergetic radiation, in the forward direction. Depending on the initial conditions this radiation may further induce the emission of laser light or acceleration of the forward electrons at the expense of the radiation field (from Colson W.B., 1979)[20].

To make an analysis of one electron one has to get rid of the Raman Regime conditions,[18] where the optical field couples with the collective plasma oscillations. In the Raman Regime, current density is high, electron beam energies are low, so that space charge forces affect the interaction unlike the Compton regime, the regime discussed in figure 9. The condition is that the beam plasma wavelength being much larger than the interaction length. That is the frequency of the collective oscillations will be much smaller than the beat wave frequency of the laser and the wiggler frequencies. Imposing an upper limit on the electron density to lower the frequency of plasma oscillations (Liu C.S., 1994)[22], The Compton Regime condition is satisfied, hence we can analyze what is going on, on an electron.

To achieve a significant energy transfer between a particle and a wave over an extended distance requires a synchronism condition between the electron velocity and the phase velocity of the wave. This sets a problem since velocity of electron is less than c.

As a synchronism condition, in the electron rest frame (pseudo-rest if sinusoidal wiggler applies), in the same interval as a wiggler period flows on the electron, a light wavelength advances on the electron too, so that, ignoring a significant energy exchange as a first order approximation, the initial conditions are met at the end of this time interval. This implies

$$\frac{\lambda_w + \lambda}{c} = \frac{\lambda_w}{< \vartheta_z >} \qquad (2.4).$$

where $< \vartheta_z >$ is the average velocity in the z- direction, λ_w and λ are wiggler and optical wavelengths. To describe this condition in other words, the electron slips half an optical wavelength in traversing half the wiggler period, so as to keep the dot product, $\vec{\vartheta}.\vec{E}$, always of the same sign, to increase or to decrease the laser field strength corresponding to FEL and IFEL conditions. The condition to be kept as it is for the time being for synchronism is

$$\frac{\lambda}{\lambda_w} = \frac{c}{< \vartheta_z >} - 1 = \frac{1 - < \beta_z >}{< \beta_z >} \qquad (2.5).$$

where β_z is the electron velocity component parallel to the wiggler axis normalized to the speed of light.

2.3. Helical Wiggler in Classical Description

On a helical trajectory no point has any symmetry imperfections or rather "privilage". On the other hand, on a sinusoidal trajectory if an electron has more or less a constant velocity, the forward component changes as a function of space coordinates. This is the case even if the energy exchange is not taken into account.

The equation of motion for an electron in a helical wiggler with an optical pump and under the influence of interactions with other particles (Pellegrini C.,1983)[23], is given by

$$\frac{d\vec{p}}{dt} = e\vec{E} + e\vec{\beta} \times (\vec{B} + \vec{B}_w) + \text{space charge force} + \text{radiation reaction force.} \quad (2.6)$$

If the force due to wiggler magnetic field is larger than the synchrotron losses, the condition for radiation reaction, to be negligible, applies

$$\frac{2}{3}(r_{\circ}\gamma B_w)^2 < eB_w \quad (2.7).$$

with

$$r_{\circ} = \frac{e^2}{4\pi\varepsilon_{\circ} mc^2}$$ $r_{\circ}$: classical electron radius

and

$$\gamma^2 < \frac{m_\circ c^2}{e\mathrm{B}_w r_\circ} \tag{2.8}$$

For $\mathrm{B}_w = 1\mathrm{T}$, $\gamma < 10^6$, that is ***one has to take into account the radiation reaction in the TeV range, but for GeV energies it is safe to neglect.***

The condition for neglecting the space charge forces is [23] expressed as

$$\mathrm{E}_{sc} < \frac{\mathrm{E}}{\gamma} \tag{2.9}$$

Here E_{sc} is the longitudinal space charge electric field. By substitution we obtain

$$\frac{n_e m_\circ c^2 \lambda r_\circ}{e} < \frac{\mathrm{E}}{\gamma} \Rightarrow n_e < \frac{e\mathrm{E}}{m_\circ c^2 \gamma \lambda r_\circ} \tag{2.10}$$

For $\gamma = 1000$ $\lambda = 10^{-6} m$ $\mathrm{E} = 10^6 V/m$ the maximum electron density allowed using equation 2.10 is 10^{13}/cm. Electric fields could be three orders of magnitude larger allowing much denser objects. And finally, for large enough wiggler magnetic fields

$$\frac{d\vec{p}}{dt} = e\vec{\mathrm{E}} + e\vec{\beta} \times \vec{\mathrm{B}}_w \tag{2.11}$$

Since the energy exchange is due to the optical component

$$\frac{d\varepsilon}{dt} = m_\circ c^2 \frac{d\gamma}{dt} = e\vec{\vartheta} . \vec{\mathrm{E}} \tag{2.12a}$$

With

$$\vec{p} = \vec{\beta}\gamma m_\circ c \quad ,$$

$$\frac{d\vec{p}}{dt} = m_\circ c(\gamma \frac{d\vec{\beta}}{dt} + \vec{\beta}\frac{d\gamma}{dt}).$$

Picking up the magnetic field terms one obtains

$$\frac{d\vec{\beta}}{dt} = \frac{e}{m_\circ c\gamma}(\vec{\beta} \times \vec{\mathrm{B}}_w). \tag{2.13a}$$

In equation pairs 12 &13 the energy exchange due to optical field and the trajectory improvement due to wiggler magnetic field are obvious:

$$\dot{\vec{\beta}} = \frac{e}{m_\circ c\gamma}\vec{\beta} \times \vec{\mathrm{B}}_w \tag{2.13.b}$$

$$\dot{\gamma} = \frac{e}{m_\circ c}\vec{\mathrm{E}} \cdot \vec{\beta} \tag{2.12.b}$$

In a wiggler period, the projection of the helical trajectory on the x-y plane, z being the propagation direction, completes a circle of radius ρ. Writing $\vec{\beta}$ in terms of its parallel and perpendicular components to $\hat{z}$

$$\vec{\beta} = \beta_z \hat{z} + \vec{\beta}_\perp, \tag{2.14}$$

the circumference of the circle, being $2\pi\rho$ is traced with velocity $\beta_\perp c$, in the meantime, the electron advances a distance λ_w with velocity c $\beta_z \approx c$.Then the radius is

$$\rho = \frac{\beta_{\perp}\lambda_w}{2\pi} \tag{2.15}$$

Since there is no point of privilage on the helical trajectory, using symmetry

$$\overrightarrow{\beta_{\perp}} = U\left\{\hat{x}\cos(\frac{2\pi z}{\lambda_w}) + \hat{y}\sin(\frac{2\pi z}{\lambda_w})\right\} \tag{2.16}$$

With

$$\overrightarrow{\mathrm{B}_w} = \left(\mathrm{B}_w \cos(\frac{2\pi z}{\lambda_w}), \mathrm{B}_w \sin(\frac{2\pi z}{\lambda_w}), 0 \right)$$

Substituting in equation (2.13.b.) gives for U the equality

$$U = \frac{e\lambda_w \mathrm{B}_w}{2\pi mc\gamma} = \frac{K}{\gamma} \quad .$$

K, in this equation is the same as the formerly determined deflection (or undulator) parameter for the sinusoidal case.

Let the circularly polarized electric field component of the incoming laser field be given by

$$\vec{\mathrm{E}} = \mathrm{E}_{\circ}\left\{\hat{x}\sin\left(\frac{2\pi z}{\lambda} - \omega t + \phi_{\circ}\right) + \hat{y}\cos\left(\frac{2\pi z}{\lambda} - \omega t + \phi_{\circ}\right)\right\}$$

or

$$\vec{E} = E_{\circ}\left\{\hat{x}\sin(\frac{2\pi}{\lambda}(z - ct) + \phi_{\circ}) + \hat{y}\cos(\frac{2\pi}{\lambda}(z - ct) + \phi_{\circ})\right\} \tag{2.17}$$

where, $\phi_{\circ}$ is the epoch angle or initial phase of the field. Substituting in equation (2.12.b) gives

$$\dot{\gamma} = -\frac{eE_\circ K}{m_\circ c\gamma}\sin\Phi \tag{2.18}$$

where

$$\Phi = \frac{2\pi}{\lambda_w}z + \frac{2\pi}{\lambda}(z - ct) + \phi_\circ \tag{2.19}$$

being the relative phase difference between the laser field and the electron oscillation.

For a net energy charge the relative phase must be constant. Let the limits of time be 0 and $\frac{\lambda_w}{c\beta_z}$, and that of z be 0 and λ_ω ,in particular, the initial and final values of the variables throughout a wiggler period set as default.

Then the phase shift is to be given by

$$\Delta = \Phi \Bigg]_{\substack{t = \lambda_w / \beta_z \bullet c \\ z = \lambda_w}} - \Phi \Bigg]_{\substack{t = 0 \\ z = 0}}$$

namely

$$\Delta = 2\pi + \frac{2\pi\lambda_w}{\lambda} - \frac{2\pi}{\lambda}\frac{\lambda_w}{\beta_z}.$$

To be in synchronism $\Delta = 0$, and this implies

$$\frac{\lambda}{\lambda_w} = \frac{1-\beta_z}{\beta_z} \quad . \tag{2.20}$$

Note the similarity of this with equation 2.5, that they suggest physically the same thing.

Since $\gamma^2 = \dfrac{1}{1-\beta^2}$

and

$$\beta^2 = \beta_z^{\ 2} + \beta_\perp^{\ 2} = 1 - \frac{1}{\gamma^2} \Rightarrow \beta_z = \sqrt{1-\left(\frac{1}{\gamma^2}+\frac{K^2}{\gamma^2}\right)} \qquad (2.21).$$

If K is not too large

$$\beta_z \cong 1 - \frac{1+K^2}{2\gamma^2} \qquad (2.22).$$

Substituting in eqn(2.20), with $\beta_z \approx 1$, that is, setting the denominator as one, the numerator will determine the result as

$$\frac{\lambda}{\lambda_w} = \frac{1+K^2}{2\gamma^2}$$

This defines a resonant energy given by the following expression

$$\gamma_r^{\ 2} = \frac{\lambda_w}{2\lambda}(1+K^2) \qquad (2.23)$$

Writing equation 2.19 as

$$\Phi = \left(\frac{2\pi}{\lambda_w} + \frac{2}{\lambda}\right)z - \frac{2\pi c}{\lambda}t + \phi_\circ$$

$$\Phi = \left(\frac{2\pi}{\lambda_w} + \frac{2\pi}{\lambda}\right)z - \omega t + \phi_\circ \qquad (2.24).$$

taking the time derivative (Milloni P.W.,1988)[24] and using $\dot{z} = \beta_z c$.

$$\dot{\Phi} = \left(\frac{2\pi}{\lambda_w} + \frac{2\pi}{\lambda}\right)\beta_z c - \frac{2\pi c}{\lambda}$$

$$\dot{\Phi} = 2\pi c\left(\frac{1}{\lambda} + \frac{1}{\lambda_w}\right)\left(1 - \frac{1+K^2}{2\gamma^2}\right) - \frac{2\pi c}{\lambda}$$

$$\dot{\Phi} = \frac{2\pi c}{\lambda_w}\left[1 - \left(1 + \frac{\lambda_w}{\lambda}\right)\frac{1+K^2}{2\gamma^2}\right] \tag{2.25}$$

since $\frac{\lambda_w}{\lambda}$ is a large amount in compared to 1, approximately

$$\dot{\Phi} = \frac{2\pi c}{\lambda_w}\left[1 - \frac{\lambda_w}{\lambda}\frac{1+K^2}{2\gamma^2}\right] \tag{2.26}$$

If one sets $\dot{\Phi} = 0$, the condition for the relative phase to keep constant, inside the parenthesis equal to zero gives the same condition as (eqn 2.23); it is of crucial importance to note that as particle exchanges energy with the radiation field, it steps out of resonance so either λ_w and B_w or K(both λ_w and B_w) must be adjusted to keep satisfying the resonance condition. Changing the parameters are referred to as tapered wiggler.

Equations (2.26),(2.18) and (2.23) give

$$\dot{\gamma} = -\frac{eE_\circ K}{m_\circ c\gamma}\sin\Phi$$

$$\dot{\Phi} = \frac{2\pi c}{\lambda_w}\left(1 - \frac{\gamma_r^2}{\gamma^2}\right)$$

(2.27a&b)

Equation (2.18), as the energy exchange term, for a synchronous particle for which $\dot{\Phi} = 0$ throughout the wiggler, claims that depending on the value of Φ, the electron can gain energy from the radiation field ($\dot{\gamma} > 0$), or vice versa. But it is not very easy to insert electron bunches in a fraction of an optical wavelength, so we can anticipate a uniform distribution of initial phases, and inevitably there will be a small distribution of electron energies. At a first glance one is not to expect a net absorption or stimulated emission from such an electron beam.

Using $\vartheta_z = c\beta_z = \left(1 - \frac{1+K^2}{2\gamma^2}\right)c$

and

$$\gamma_r^2 = \frac{\lambda_w}{2\lambda}(1+K^2).$$

$$\vartheta_z \approx c\left(1 - \frac{\lambda \gamma_r^2}{\lambda_w \gamma^2}\right) \tag{2.26}$$

and assuming a constant parameter wiggler

$$\dot{\vartheta}_z \cong \frac{2c\lambda\gamma_r^2}{\lambda_w \gamma^3}\dot{\gamma} \tag{2.27}$$

Then for non-resonant electrons, the energy exchange means a modulation of longitudinal velocities, faster electrons catching up to the slower ones. The electrons are bunched on the laser wavelength scale, since an electron injected half an optical period later or earlier experiences an energy exchange of equal magnitude, but opposite sign

So FEL interaction "heats" the electron beam, increasing their energy spread. On resonance the bunching occurs at a phase of the optical field where there is no net exchange of energy. The vital question here is; do particles initially nearby in energy and phase to the synchronous particle remain nearby in this phase space throughout the interaction process?

2.4. The Constant Parameter Wiggler

Very close energies are chosen to the resonant particle, so that, for the energy difference normalized to synchronous energy

$$\eta = \frac{\gamma - \gamma_r}{\gamma_r}$$

satisfying

$$\eta << 1 \tag{2.28}$$

then, for the equation pair

$$\dot{\gamma} = -\frac{e\mathrm{E}_{\circ}K}{\gamma m_{\circ}c}\sin\Phi$$

and

$$\dot{\Phi} = \frac{2\pi c}{\lambda_w}\left(1 - \frac{\gamma_r^{\,2}}{\gamma^2}\right)$$

Mathematical digressions yield, using

$$\eta = \frac{\gamma}{\gamma_r} - 1 \quad ,$$

$$\dot{\eta} = \frac{\dot{\gamma}\cdot\gamma_r - \dot{\gamma}_r\gamma}{\gamma_r^{\ 2}}$$

Note that, for a constant parameter wiggler $\dot{\gamma}_r = 0$.

$$\dot{\eta} = -\frac{eE_\circ K}{\gamma_r m_\circ c\gamma}\sin\Phi = -\frac{eE_\circ \beta_\perp}{m_\circ c\gamma_r}\sin\Phi \qquad (2.29)$$

$$\dot{\Phi} = \frac{2\pi c}{\lambda_w}\left(\frac{\gamma^2 - \gamma^2{}_r}{\gamma_r^{\ 2}}\right) = \frac{2\pi c}{\lambda_w}\left(\frac{\gamma+\gamma_r}{\gamma_r}\right)\left(\frac{\gamma-\gamma_r}{\gamma_r}\right) = \frac{4\pi c}{\lambda_w}\eta \qquad (2.30)$$

The coupled equations

$$\dot{\eta} = -\frac{eE_\circ \beta_\perp}{m_\circ c\gamma_r}\sin\Phi$$

and ,

$$\dot{\Phi} = \frac{4\pi c}{\lambda_w}\eta$$

Remind us of a canonical conjugate pairs where the canonical variables are η and Φ.

Using the Hamilton's equations

$$\dot{q}_i = \frac{\partial H}{\partial p_i} \qquad \text{and} \qquad \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

The Hamiltonian of the motion of an electron in a FEL structure is

$$H = \frac{2\pi c}{\lambda_w}\eta^2 + \frac{eE_\circ \beta_\perp}{m_\circ c\gamma_r}\cos\Phi \qquad (2.31).$$

Corresponding trajectories in phase space are shown in Figure 13. The Hamiltonian is similar to the one describing the motion of a pendulum, with a region of bounded motion and that of an unbounded motion divided by a separatrix.[15][23]

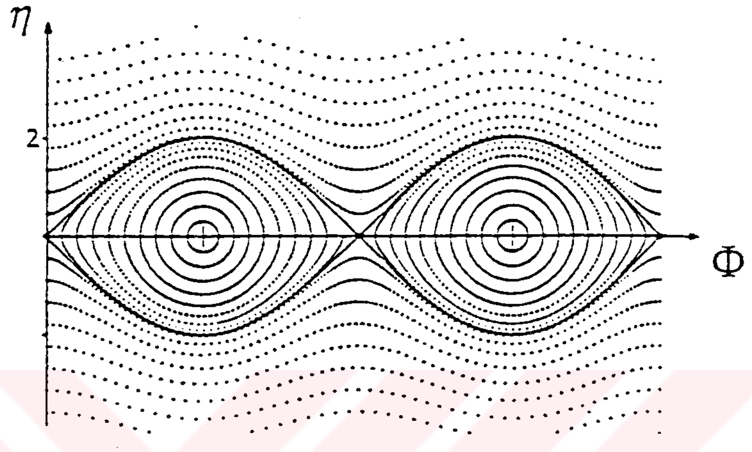

Fig. 13. Description of trajectories in phase space for a constant parameter wiggler. The horizontal coordinate is the phase difference Φ between the electric field vector and the transverse motion of the electron a range of 4π corresponds to two optical wavelengths.The vertical coordinate η is the normalized energy deviation being the canonical conjugated variable to Φ (from Gaupp A.,1995).[15]

2.5. Tapered Wiggler

As the electron gains or loses energy, it will step out of resonance if it is a synchronous particle. That's why the parameter of the wiggler has to be changed so as to keep satisfying the resonance condition.

Assuming again electrons of only slightly off-resonance, the time rate of change of normalized energy difference reads

$$\dot{\eta} = \frac{d}{dt}\left(\frac{\gamma - \gamma_r}{\gamma_r}\right) = \frac{\dot{\gamma}\gamma_r - \dot{\gamma}_r\gamma}{\gamma_r^{\ 2}} \cong \frac{\dot{\gamma}}{\gamma_r} - \frac{\dot{\gamma}_r}{\gamma_r}$$

using

$$\frac{d}{dt}\left(\gamma^2 - \gamma_r^{\ 2}\right) \cong 2\gamma_r\left(\dot{\gamma} - \dot{\gamma}_r\right)$$

substituting expressions for $\dot{\gamma}$ and $\dot{\gamma}_r$ and considering the equality

$$\gamma_r\dot{\eta} = \dot{\gamma} - \dot{\gamma}_r$$

We get the coupled equations

$$\dot{\eta} = -\frac{e\mathrm{E}_{\circ}\mathrm{K}}{m_{\circ}c\gamma_r^{\ 2}}\left(\sin\Phi - \sin\Phi_r\right) \qquad (2.32.).$$

$$\dot{\Phi} = \frac{4\pi c}{\lambda_w}\eta. \qquad (2.33).$$

Defining $\omega_{\circ} = \dfrac{2\pi c}{\lambda_w}$,then gives (2.34).

$$\dot{\Phi} = 2\omega_{\circ}\eta \qquad (2.35).$$

$$\dot{\eta} = -\frac{\omega_{\circ}}{2}\left(\frac{\Omega}{\omega_{\circ}}\right)^2\left(\sin\Phi - \sin\Phi_r\right) \qquad (2.36).$$

where

$$\Omega^2 = \frac{4\pi e\mathrm{E}_{\circ}K}{\lambda_w m_{\circ}c\gamma_r^{\ 2}} \qquad (2.37).$$

Changing the time variable as $\tau = \omega_\circ t$ and substituting in (eqns 2.35 & 2.36) results in the equation pair

$$\frac{d\Phi}{d\tau} = 2\eta \qquad (2.38).$$

$$\frac{d\eta}{d\tau} = -\frac{1}{2}\left(\frac{\Omega}{\omega_\circ}\right)^2 \left(\sin\Phi - \sin\Phi_r\right). \qquad (2.39).$$

This gives a Hamiltonian of the form[23]

$$\mathrm{H} = \eta^2 + V(\Phi)$$

where

$$V(\Phi) = -\frac{1}{2}\left(\frac{\Omega}{\omega_\circ}\right)^2 (\cos\Phi + \Phi\sin\Phi_r) \qquad (2.40).$$

is the ponderomotive potential attached to describe the motion of the electrons

In Figure 14, this potential is plotted as a function of phase.

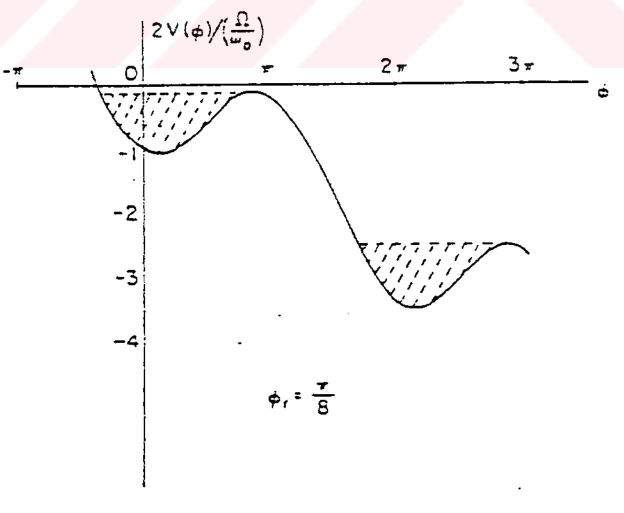

Fig. 14. Ponderomotive Potential well in an axially variable wiggler for $\Phi = \frac{\pi}{8}$ (from Pellegrini C.,1983)[23]

2.6 Synchrotron Oscillations

The coupling of the electron transverse motion to the transverse optical field causing an energy exchange gives rise to a longitudinal slow motion superimposed on the rapid motion throughout the wiggler. Consider the case of a constant parameter wiggler illustrated in Fig.13. The physics of the electron dynamics can be described as if the electron sees a sinusoidal potential of period of a wavelength, co-propagating along the axis at a speed of the resonant particle. There is no physical meaning associated with this ponderomotive potential except that it describes the longitudinal effects due to the transverse coupling. The motion is slow and the motion of the trapped particles is described by a pendulum equation. The particles nearby the synchronous particle wander around and with an analogy to synchrotrons this motion is likely to be called synchrotron oscillation; since a ´trapped` particle traces out a closed trajectory in the phase $(\eta - \Phi)$ space with a well defined synchrotron frequency.

The ponderomotive potential corresponding to a tapered wiggler for which the resonant phase is chosen to be $\Phi_r = \frac{\pi}{8}$ is shown in Figure 14. The shaded area represents a potential well where electrons are trapped and oscillate around the synchronous phase. The parameters of the wiggler are so adjusted that the synchronous phase will be kept constant throughout the interaction and the synchronous energy for the resonant particle will change correspondingly.

Constant parameter wiggler FEL offers an efficiency of 1 % FEL interaction, this could be improved regaining the energy of the used electrons in a storage ring or collecting the energy in an electrostatic structure. The efficiency than will yield 20 %, if 95 % of the electrons are captured, quite a good efficiency for a laser. However, this additional structure adds to the investment costs. Tapered wigglers are suggested for FEL efficiency enhancement, and they offer 25 % efficiency without using storage rings and extra inventory. Nevertheless, they have their own problems such as trapping efficiency, and wiggler design (Boehmer H., 1981)[25].

CHAPTER 3

DESIGN CONSIDERATIONS FOR AN IFELA

3.1. Principles

Although the interaction process is similar, there are essential differences between the FEL and the IFELA. The objective in the FEL is to obtain lasing using synchrotron radiation, and in most analysis, Born approximation, that the trajectory of the electron beam is not changed due to radiative losses, is set as a first principle. For the IFELA, on the other hand, since the objective is to increase the energy of the particle, giving rise to an inevitable trajectory improvement, Born approximation fails.

Bunching of the electrons on the optical wavelength scale can be considered as a mechanism making the system similar to phase correlated dipole radiators each λ apart, and so giving rise to lasing. For the case of the IFELA, the emitted radiation is an energy loss process,-the problem of lasing or incoherent emission of radiation is not a question of concern-,but the interaction process leading to bunching is an automatic means of bunching the particles on the optical wavelength scale which is a desirable case for some accelerator applications. In the FEL, tapered wiggler is optional for magnet design, due to the difficulty of building the magnetic field profile and is preferable for efficiency enhancement. For the IFELA, the constant parameter wiggler is limited to a narrow accelerating energy difference, tapering is almost inevitable. Only for slight energy changes constant parameter wiggler, with a well-determined initial phase injection and wiggler length gives an observable result.

To keep satisfying the resonance condition (eqn.2.23) either B_w or λ_w will be adjusted, in the design increasing the latter is no problem, but magnet technology limits the maximum value of B_w. On the other hand, the expression for the radius of the trajectory (equation 2.15) with the equation for resonance condition (equation 2.23) states the following: i) for the case of magnetic field tapering the magnetic field increases as γ. This in turn when substituted in the expression for the radius, gives a constant ρ. ii) For the case of magnet period tapering, for large deflection parameter neglecting one in (equation 2.23), the energy is proportional to the power three over two of the periodicity.[This in turn will fail to satisfy equation. 2.22., so to improve the resonance condition, but it is not a crucial problem] Substituting in equation 2.15, the radius is proportional to one and a half power of the periodicity. This has a drawback, however, increasing radius in the focus of a laser means even smaller accelerating component since laser to be focussed will operate in TEM_{00} mode Gaussian Intensity Distribution. Furthermore, increasing ρ will force the magnet gap to larger values making the on axis magnetic field smaller, which, then exhibits another disadvantage.

As the laser source one needs high power ones such as continuous wave CO_2 lasers, or Pulsed Nd: glass lasers or FELs. Pulsed lasers may offer higher energies at a first glance, but for long range acceleration, for the accelerated electrons to keep going on with the nextcoming pulse there is a problem of phase matching. Cw CO_2 lasers offer high powers and they operate in high efficiencies (~30%). Mode locking may result in longer coherence lengths in gas lasers, so the length of the accelerating region limited by the coherence length for the initial particles to keep trapped in the appropriate phase will be larger. Smaller coherence lengths will yield unpredictable results.

3.2. Trapping Potential

To find the maximum and minimum of the ponderomotive well potential expression for tapered wiggler (equation 2.40) using the derivative with respect to Φ gives:

$$-\sin\Phi + \sin\Phi_r = 0 \tag{3.1}$$

And this is satisfied for $\Phi = \Phi_r$ and, $\Phi = \pi - \Phi_r$. The former gives the minimum and the latter the maximum in equation 2.40. Using the Hamiltonian expression for the potential well depth, one obtains

$$\eta^2{}_{\max} = V(\pi - \Phi_r) - V(\Phi_r)$$

or

$$\eta_{\max} = \frac{\Omega}{\omega_0}\left\{\cos\Phi_r - \left(\frac{\pi}{2} - \Phi_r\right)\sin\Phi_r\right\}^{\frac{1}{2}}. \tag{3.2}$$

The particles in the phase depth between $\Phi = \pi - \Phi_r$ and $\Phi_{\min}$, where $V(\Phi_{\min}) = V(\pi - \Phi_r)$, and the potential depth $\eta < \eta_{\max}$, are trapped, in the ponderomotive potential. The particles exceeding these phase and energy limitations will simply spill out of the potential well, they are no longer trapped, and hence they are not accelerated.

The energy exchange term, equation 2.27, for an FEL or an IFELA, may make someone to think of choosing a larger synchronous phase Φ_r, -(the phase finally reaching to $\Phi_r = \frac{\pi}{2}$)-, but this in equation 3.2 gives a zero potential depth, i.e., no particle is trapped in the well, none is being accelerated but the resonant one.

The maximum value of trapping potential depth $\eta_{\max}$ and the phase potential depth will come up with choosing $\Phi_r = 0$, but this maximum rate of trapping means no acceleration at all. ***Then a compromise has to be accomplished between high gradient acceleration and charged particle beam brightness for an optimum value of*** Φ_r.

3.3 The Analogy Between Linacs, Synchrotrons and IFELAs

For both the case of the linac and the synchrotron, an accelerating structure, possibly a resonant cavity with an electric field component oscillating parallel to the direction of motion of the electron, transfers energy from the particle, but it can extract energy as well to act as a decelerator if the appropriate phase conditions are satisfied. By intuition, one can imply a resonant particle for which energy and phase conditions to keep accelerating will be satisfied throughout. The linac and synchrotron schemes are shown in figure 15.

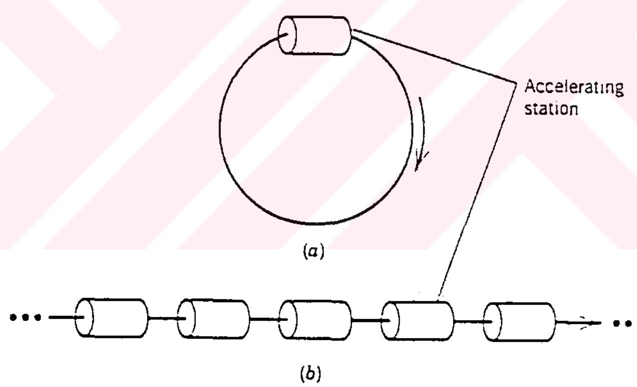

Fig.15. A sequence of accelerating stations as in a synchrotron (a) or in a linac(b) (from Edwards D.A.,1983)[26].

In the neighborhood of this phase, there is a region of phase stability, due to the fact that a particle ahead of this ϕ_s, gains energy more slowly, becoming slower than the wave, so that, it will drift back towards ϕ_s, whereas a particle little behind receives some extra acceleration, and will catch up with and lead ϕ_s. At the next station the electrons will exchange their situation for the previous station, so will oscillate about the synchronous phase ϕ_s gaining energy all on the average at the correct rate (Hereward H.G., 1970)[27].

Acceleration or deceleration of the electron depends on the phase angle between the velocity of the electron and the component of the electric field parallel to the motion. The energy exchange term is then written to be

$$m_\circ c^2 \frac{d\gamma}{dt} = \vec{J}.\vec{\mathrm{E}} \tag{3.3}$$

where J is the current through the station.

This implies

$$\frac{d\gamma}{dz} = \frac{e\mathrm{E}_z}{m_\circ c^2} \sin\phi_s, \tag{3.4}$$

where, E_z denotes the peak electric field component parallel to the beam propagation direction, and ϕ_s, is the synchronous phase in a synchrotron or linac. Writing the energy exchange term for the FEL, equation 2.18 as a function of the longitudinal distance through the wiggler gives, in analogy with the previous equation

$$\frac{d\gamma}{dz} = \frac{e\mathrm{E}_\circ K}{m_\circ c^2 \gamma} \sin\Phi_r \tag{3.5}$$

For the case of synchrotron, since the electrons are loosing energy through the device as SR, correct amount of energy must be transferred to keep synchronism. If phase, and/or energy is not matched, the electron will gain or lose energy due to

$$\frac{d}{dz}(\gamma - \gamma_r) = \frac{eE_z}{m_\circ c^2}[\sin\phi - \sin\phi_s] . \qquad (3.6)$$

The similarity of equations (2.32) and (3.6), and , that of (3.3) with (3.4) leads to a similarity between Linacs and Synchrotrons with IFELA; they have a definite synchrotron oscillation frequency and resonant phase, where, in the IFELA, coupling with the electric field is transverse instead of longitudinal, unlike the former constituents.

3.4 Optimization of Parameter

As in the case of brightness and high gradient acceleration there are some other conflicting concepts as well, so they desire optimization.

The electric field profile of a focused TEM_{00} mode laser beam is given as a function of longitudinal component z, radial component ρ, beam waist $W_\circ$, maximum electric field just at the center of the focus $E_\circ$ as

$$E(z,\rho) = E_\circ \frac{w_\circ}{w(z)} \exp\left(\frac{-\rho}{w(z)}\right)^2 . \qquad (3.7)$$

Where the beam size w(z) is given in terms of beam waist and Rayleigh Range as

$$w(z) = w_\circ \left\{1 + \left(\frac{z}{R}\right)^2\right\}^{\frac{1}{2}} \tag{3.8}$$

where,

$$R = \frac{\pi w_\circ^{\ 2}}{\lambda} \tag{3.9}$$

and,

$$E_\circ = \left(\frac{4PZ_\circ}{\pi w_\circ^{\ 2}}\right)^{\frac{1}{2}} \tag{3.10}$$

Here, P stands for the laser power, and $Z_\circ$ for the vacuum impedance.

Equation 3.7 implies decreasing accelerating field linearly with the distance from the focus and exponentially with the distance from the axis. Electron will be kept close to the axis, but this in turn means a large SR loss at higher energies due to equation 1.1.

Using Equations 3.9 and 3.8 , one can infer that for a smaller beam waist and hence a larger maximum electric field attainable, the Rayleigh Range must be kept small. But this in turn means that the electric field dies out much faster, and the range of acceleration is much more limited in the focus of a laser. Without decreasing the Rayleigh Range, to have larger accelerating electric fields one can use a smaller wavelength laser, so this is the reason for choosing Nd: glass or Nd: YAG lasers operating at 1.06 μm instead of an 10.6 μm CO_2 Laser. But this conclusion does not simply mean that, for a "Gedanken Experiment" with very small wavelength high power lasers (none of which exists yet), one is allowed to go to indefinitely small beam waist. In other words, although equation 3.7 suggests an infinitely extending electric field in the space, at $\rho = w(z)$, the field becomes zero, viz., the minimum value of the beam waist is limited by the radius of the electron beam trajectory. If the radius of the trajectory becomes larger than the laser beam width, no matter how powerful the laser is, it would not work at all.

3.5 Power Loss Due to Synchrotron Radiation

An electron moving on an exactly linear trajectory will not interact with a co-propagating electromagnetic wave, and hence the IFELA is based on the principle that, to couple the electron beam and the electromagnetic beam, particle trajectory will be bent. Particle trajectory bending is naturally a reason for sychrotron radiation emission and the due loss of the electron energy. If this system is used to accelerate protons or ions, namely, heavier particles, which is possible for applications except for the particle acceleration to obtain SR in short wavelength range, this power loss due to SR is not a problem according to equation 1.1. This loss mechanism is a problem for electron or positron acceleration, i.e., for light particles. In figure 16,The Livingston Chart in figure 1 is shown for electron and proton accelerators separately. The maximum attainable energy in electron acceleration appears to be few hundred GeV. For the case of The Inverse Free Electron Laser Accelerator mechanism, Synchrotron Radiation is supposed to be the essential limitation factor for the case of electron acceleration, although it is negligibly small for heavier particles.

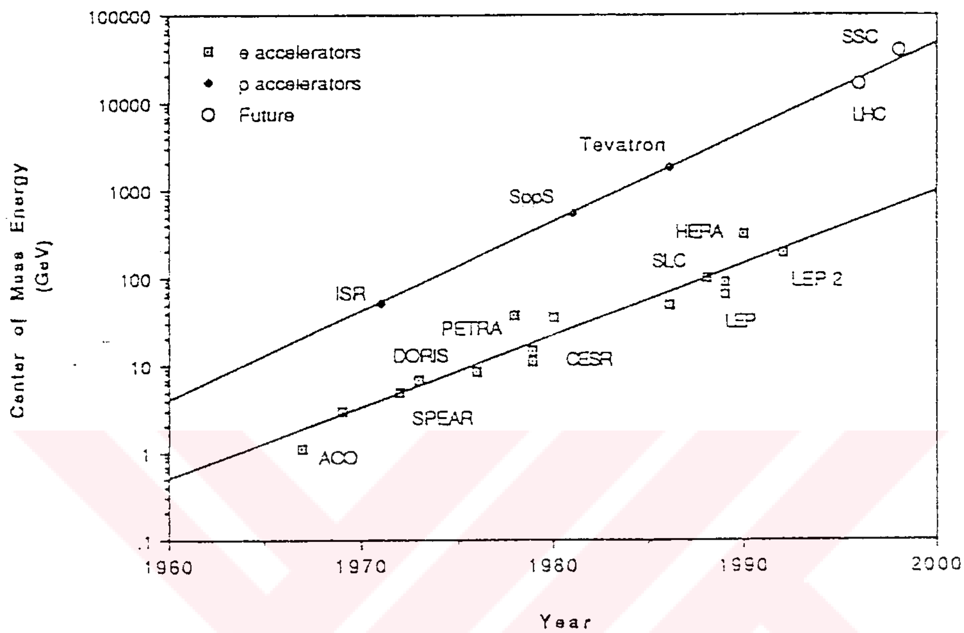

Fig. 16 The Livingstone Chart for various electron and proton accelerators (from Edwards D.A. 1983)[26].

The total amount of energy radiated by an electron per unit length of the trajectory in a helical undulator (Pellegrini C., 1982)[28] is given by the equation

$$S = \frac{8}{3}\pi^2 r_\circ \frac{m_\circ c^2}{\lambda_w} K^2 \gamma^2 \qquad (3.11.)$$

where, $r_\circ = 2.8179 \times 10^{-15}$ meters, denotes the classical electron radius, γ stands for the particle energy, in terms of the rest mass energy of the electron $m_\circ c^2 = 0.511 MeV$, $K = \dfrac{eB_w \lambda_w}{2\pi m_\circ c^2}$ is the dimensionless deflection parameter, or undulator parameter.

Substituting the expression for K results in, the expression for the radiated energy per unit length to become

$$S = \frac{2}{3} \frac{r_\circ e^2}{m_\circ c^2} B_w{}^2 \gamma^2 \tag{3.12}$$

The energy lost is a function of the magnetic field strength B_w and the energy of the particle normalized to electron rest mass units γ, all the others are constants. In this form, the energy loss due to synchrotron radiation being much smaller for a proton than for an electron is obvious, since the charge of an electron equals that of the proton apart from the sign. But their mass is quite different. ***This means that the IFELA could also operate to accelerate protons or heavier charged particles, even more effectively than for electrons.***

In the following calculations, I will confidently assume that the energy loss for unit helical trajectory length approximates that of unit distance in the propagation direction of the beam. This assumption is nothing but the truth as far as the theory outlined in chapter 2 presupposes that the motion of the electron to be almost linear and transverse motion is considered just as slight –but crucial- perturbations. Not to be that descriptive $\beta_z \approx 1$, $\beta_\perp << 1$, which means $c\beta_z \approx c$, in other words, the dominant velocity component is the forward one. Note that, equation 2.15 for the radius of the trajectory sets the same condition as a prerequisite.

The appropriate unit system for microscopic problems, especially the electrodynamics of individual charged particles, is recommended as the Gaussian system of units by J.D.Jackson (page 817 of reference 17). This unit system is used for the below calculations, but SI correspondents are also noted whereever a sensation due to familiarity is necessary. Then equation 3.12. reads for the energy loss per centimeter as a function of magnetic field strength B_w in Gauss and γ (dimensionless) energy parameter.

$$S = 0.528485 \times 10^{-25} B_w{}^2 . \gamma^2 \qquad (3.13)$$

This energy loss, if given in terms of electron rest mass units $m_\circ c^2$, may give a deeper feeling. So the dimensionless energy loss parameter per centimeter, $S_\gamma = \dfrac{S}{m_\circ c^2}$,is given as

$$S_\gamma = 6.46 \times 10^{-20} B_w{}^2 . \gamma^2 \qquad (3.14.)$$

This equation tells us that the rate of energy loss due to synchrotron radiation is a function of energy itself (note that $\gamma = \dfrac{Energy}{m_\circ c^2}$), and a function of the magnetic field strength, but not a function of the wiggler periodicity λ_w.

For an estimate of energy loss, set B_w= 1 Tesla, which is not a science fiction with rare earth cobalt permanent magnet alloys. Then

$$S_\gamma = 6.46\times10^{-12}\gamma^2 / cm = 6.46\times10^{-10}\gamma^2 / meter \quad . \tag{3.15}$$

The energy loss per meter in terms of MeV units is

$$\Delta E = 0.511 MeV \times S_\gamma \tag{3.16}$$

In the Table 2, using these formulas, energy loss per meter due to SR is given for various electron energy ranges with a magnetic field strength $B_w = 1$ T.

The wiggler scheme for constant magnetic field case B_w = constant, as in Table 2, yields the increase of SR losses to increase as γ^2. For this case λ_w increases to keep satisfying synchronism condition.

Equation 2.23 for the synchronism condition in the case of constant period wiggler, magnetic field tapering gives the magnetic field strength as a function of energy as

$$B_w = \frac{2\pi m_o c^2}{e\lambda_w}\left\{\frac{2\lambda\gamma_r^{\ 2}}{\lambda_w} - 1\right\}^{\frac{1}{2}} \quad . \tag{3.17.}$$

This shows that, B_w is proportional to γ_r. As a result, SR loss for a constant period wiggler increases as γ^4.

One may simply imply wiggler period tapering taking also into account that magnetic field strength has an upper limit set by the technology whereas periodicity is unlimited, and the fact that, SR losses increases much slowly.

Table.2 Synchrotron Radiation Loss at various energy ranges for wiggler magnetic field strength of 1 T and an Nd:YAG Laser of power 10TW.

Electron Energy = $\gamma m_{\circ} c^2$ (GeV)	γ [dimensionless]	Energy loss per meter ΔE
~ 0.005	10	0.1264 eV
~ 0.05	10^2	12.64 eV
~ 0.5	10^3	1.264 keV
~ 5	10^4	0.1264 MeV
~ 50	10^5	12.64 MeV
~ 500	10^6	1.264 GeV

But this is not the whole story. Equation 2.15 for the radius of the trajectory gives

$$\rho = \frac{e}{4\pi^2 m_{\circ} c^2} \quad \frac{B_w \lambda_w{}^2}{\gamma} \tag{3.18.}$$

which implies more or less a constant radius for magnetic field tapering $(\lambda_w = constant)$.

However, the synchronism condition

$$\gamma_r{}^2 = \frac{\lambda_w}{2\lambda}\left(1 + K^2\right)$$

implies that, (for the constant magnetic field case)

$\gamma_r^{\,2} \;\alpha\; \lambda_w^{\,3}$.

Substituting $\lambda_w \;\alpha\; \gamma_r^{2/3}$, in the equation 3.18 yields

$\rho \;\alpha\; \gamma_r^{1/3}$.

This simply means that, as one goes to higher enegies, keeping the electron in the proximity of the central axis of the laser beam where the accelerating electric field is maximum due to Gaussian beam profile, is a problem. Briefly, periodicity tapering decreases the SR losses but it decreases the gain as well.

3.6 The Energy Gain

The energy exchange term equation 2.18 for a free electron laser in an IFEL accelerator can be written as

$$m_\circ c^2 \frac{d\gamma}{dt} = \frac{e\mathrm{E}_c K c}{\gamma} \sin\Phi .$$

The rate of change of energy is then

$$\frac{dE}{dt} = \frac{e\mathrm{E}_\circ K c}{\gamma} \sin\Phi$$

Using $\beta_z \sim 1$ and $\dot{z} = \beta_z c$, one gets

$$\frac{d\mathrm{E}}{dz} = \frac{e\mathrm{E}_\circ K}{\gamma} \sin\Phi .$$

By substituting the expression for K given by equation 1.10 the energy gain of the electron per unit undulator length is obtained as

$$\frac{dE}{dz} = \frac{e^2 E_\circ B_w \lambda_w}{2\pi m_\circ c^2 \gamma} \sin\Phi . \qquad (3.19.)$$

One can immediately imply from this equation that ***for a constant period wiggler***, since $B_w \ \alpha \ \gamma_r$, ***the rate of energy gain per unit undulator length is almost constant***, *whereas*, ***for a constant magnetic field wiggler since*** $\lambda_w \ \alpha \ \gamma_r^{2/3}$, ***the energy gain is proportional to*** $\gamma_r^{-1/3}$.

Furthermore, decrease of electric field due to extension of the radius of the trajectory is to be considered.

The condition for synchronism results in

$$\boxed{\gamma_r^{\ 2} = \frac{\lambda_w}{2\lambda}\left[1 + \frac{e^2 B_w^{\ 2} \lambda_w^{\ 2}}{4\pi^2 m_\circ^{\ 2} c^4}\right]} . \qquad (3.20.a)$$

For a Neodymium laser of $\lambda = 1.06\mu m$ and a wiggler peak magnetic field $B_w = 1$ T, the above mentioned synchronism equation in Gaussian units reads

$$\gamma_r^{\ 2} = \frac{\lambda_w}{2.12} \times 10^4 \left[1 + 0.87095 \lambda_w^{\ 2}\right]$$

or

$$4108.25\lambda_w^{\ 3} + 4716.98\lambda_w = \gamma_r^{\ 2} \qquad (3.20.b)$$

This third order equation for the wiggler period will be solved at various electron energies. The expression for the rate of change of energy with distance, equation 3.19, with the following value of parameters, B_w = 1 T and a laser power of 10 TW=10^{13} Watts, Rayleigh Range of R=4 m, and a beam waist $w_\circ = 1.172 \times 10^{-1} cm$, gives the value of maximum attainable electric field at the center of the focus:

$E_\circ = 2.3 \times 10^6$ statvolts/cm.

And equation 3.19 reads, for the particle energy change per centimeter

$$\frac{dE}{dz} = 1.03 \times 10^{-3} \frac{\lambda_w}{\gamma_r} \sin \Phi_r \qquad (3.21.)$$

The rate of change of energy (in units of electron rest mass energy) per unit length (cm)

$$\frac{d\gamma}{dz} = \frac{1.03 \times 10^{-3}}{m_\circ c^2} \frac{\lambda_w}{\gamma_r} \sin \Phi_r \qquad (3.22.)$$

or

$$\frac{d\gamma}{dz} = 1.2576 \times 10^3 \frac{\lambda_w}{\gamma_r} \sin \Phi_r \qquad (3.23.)$$

At this point, the design engineer for the wiggler magnetic field will either prefer large resonant phase for high gradient acceleration, or smaller phase for brighter acceleration. I choose two extreme cases where Φ_r= 70° and Φ_r= 30°. With the assumption of a cold electron beam, that is , energy spread of which is zero, the first one traps the electrons satisfying $70^\circ < \Phi < 110^\circ$, the second one traps $30^\circ < \Phi < 150^\circ$. Presumably, the electrons will be uniformly distributed along the propagation direction, the first one traps roughly 34% of the electrons, and the second 87% of them.

The radius of the electron trajectory in equation 2.15 is given as

$$\rho = \frac{eB_w \lambda_w^2}{4\pi^2 m_\circ c^2 \gamma_r} \quad . \qquad (3.24.)$$

If we again set $B_w = 1$ T, it gives

$$\rho = 0.1486 \frac{\lambda_w}{\gamma_r^2} \quad .$$

To compare the radius of the electron beam trajectory with the beam waist $w_\circ = 1.172 \times 10^{-1} cm$, we solved equation 3.20 for various electron energies. The results are listed in Table 3.

Table 3 clearly shows that, as the energy of the electron increases the energy gain per unit wiggler length decreases. A comparison with SR loss (Table 2) shows that, we have at about 50 GeV the net gain of around 50 MeV/m. At 0.5 TeV loss exceeds gain about two orders of magnitude. As a result we can conclude that, ***the IFELA is a practical device up to 50 GeV, but TeV range is not attainable***, so that other means of acceleration must be considered for this energy range. So we will forget about the design for an electron collider, more adequate proposal would be industrial accelerators of GeV energies assisted by a CO_2 or Nd laser.

Table 3. Results of the energy gain calculations for a Nd laser of power 1 TW, and a fixed B_w =1T

Energy = $\gamma m_o c^2$ (GeV)	γ	λ_w (cm)	ρ (cm)	Energy Gain Per meter $\Phi_r = 30^\circ$	$\Phi_r = 70^\circ$
0.05	10^2	1.34	2.67×10^{-3}	430.5 MeV.	608.8 MeV.
0.5	10^3	6.24	5.94×10^{-3}	200.6 MeV.	283.6 MeV.
5	10^4	28.18	0.0118	90.5 MeV.	127.9 MeV.
50	10^5	134.5	0.0267	43.2 MeV.	61.0 MeV.
500	10^6	624.4	0.06	15.6 MeV.	22.1 MeV.

3.7. The Net Energy Gain

Having outlined the gain and loss behaviors of IFELAs with a common peak magnetic field of 1 Tesla and Nd:YAG laser power of 10TW, one can immediately obtain the net gain per meter of the accelerator. The net gain, and the ratio of the effective accelerating electric field to the electric field on the axis at the focus for various electron energies are listed in table 4.

At about 50 GeV, the accelerating gradient reduces to the value that could be obtained with advanced linear accelerator structures using conventional techniques. Therefore an IFELA is not an effective accelerating ***structure candidate to build e^-e^+ colliders over 300 GeV, which is the value for***

the state-of-the-art technology for the time being. So the motive to build an IFELA is for the industry, reaching energies of GeV, with an acceptable beam brightness and a high gradient of acceleration.

Table.4. Net energy gain for an Electron at various energy ranges of the electron beam.

Electron Energy (GeV)	Ratio of the accelerating field to the field on the axis	Net Gain Per Meter (MeV)	
		$\Phi_r = 30^\circ$	$\Phi_r = 70^\circ$
0.05	0.9995	430.5	608.8
0.500	0.9974	200.6	283.6
5	0.9899	90.4	127.8
50	0.9494	30.5	48.3
500	0.7694	-1248	-1241

Using the table, one can infer that the effect of the Gaussian beam profile prevails as the electron energy increases, and that the TeV range is not attainable. The choice of resonant phase is extremely important in determining the gradient of acceleration. Industrial accelerators of tens of GeV range can be build with a compromise between the beam brightness and the acceleration gradient, up to the application.

An alternative is as a booster accelerator for a Synchrotron Radiation source operating in the X-ray region providing high quality bright and bunched, high energy, charged particles in the tens of GeV range.

3.8. Magnetic Field Profiles

There are two major options for tapering, either B_w or λ_w will be tuned with the energy of the electron to keep synchronous. Intermediate options where both are changed accordingly are possible, but I discard those cases here since they will bring unnecessary complications into the picture.

For the periodicity tapering consider the synchronism equation written in the form

$$\gamma_r^2 = \frac{e^2 B_w^2}{8\pi^2 m_o^2 c^4 \lambda} \lambda_w^3 + \frac{\lambda_w}{2\lambda} . \tag{3.25}$$

For large values of energy (for 1T, 500 MeV say), the last term is negligible and

$$\boxed{\lambda_w \cong 62.438 \times 10^{-3} \gamma_r^{2/3}}.$$

Substituting in the equation for the gain (equation 3.23) ***with a synchronous phase of*** 70° ***gives***

$$\gamma_r^{1/3}\frac{d\gamma_r}{dz} = 73.786571$$

then, intergrating over the accelerator from z = 0 to z = L results in

$$\boxed{\gamma_r^{4/3}(L) = \gamma_r^{4/3}(0) + 98.382094 \times L}$$

where L is in centimeters.

Evaluation for L = 10 cm gives at $\gamma_r(0) = 10^3$

E = 500 MeV , $\gamma_r(L) \cong 1073$,

the gain is 373.03 MeV/m.

And for the case of magnetic field tapering, again neglecting the last term in equation 3.25, ***with say E = 5 GeV,*** $\gamma_r = 10^4$,

$$B_w \cong \left[\frac{2\pi m_o c^2}{e}\sqrt{\frac{2\lambda}{\lambda^3}}\right]\gamma_r \qquad (3.26.)$$

$$B_w = 0.1560\gamma_r (Gauss)$$

Substituting in the energy gain equation **with** $\Phi_r = 70^\circ$ yields

$$\frac{d\gamma_r}{dz} = 1.8443316$$

and integrating from 0 to L gives

$$\boxed{\gamma_r(L) = \gamma_r(0) + 1.84433116.L(cm).}$$

Notice that ***for constant period wiggler, the electron energy and the magnetic field strength increases linearly with the distance.*** For the accelerator at hand with a Rayleigh Range of 4 m and an accelerator length of 8 m at

$\gamma_r(0) = 10^4$, $\gamma_r(L) = 11475$, a very slight change of magnetic field will be necessary.

3.9 Current Limitations

Courant E.D. and Pellegrini C. [29] have analyzed in 1985, the IFELA taking into account the radiation reaction with a massy theory. ***However, using table 4 and the calculations on page 30, the TeV range, where the radiation reaction becomes effective is over the performance of the IFELA,*** so the theory outlined here suffices, radiation reaction is out of consideration for realistic cases.

Using a Nd:YAG laser power of 10 TW hence a peak electric field on axis of $E_\circ = 2.3\times 10^6$ statvolts/cm, the condition for neglecting the space charge forces (equation 2.10) is

$$n_e < \frac{eE_\circ}{m_\circ c^2 \gamma_r \lambda r_\circ}$$

or

$$n_e < \frac{4.5127\times 10^{19}}{\gamma_r} \text{ electrons per cubic centimeter.}$$

The charge density upper limit is inversely proportional to the electron energy.

Taking the radius of the beam as 0.25 mm to keep in the laser beam profile, but still ignoring the charge scattering -this ignorance is not so kind, due to the fact that the electron beam trajectory radius being much smaller, the trajectories will overlap giving rise to scattering for denser cases-, one can obtain the corresponding current limitation as

$$\boxed{I[statamps] = Area[cm^2]n_e[cm^{-3}]e[statcoul]v_z[cm/\mathrm{sec}]}$$

Figure 17. Pictorial representation of the electron dynamics and particle bunching on the optical wavelength scale, in a helical wiggler (exaggerating the transverse excursions). Current through the cross section of the axes is proportional to the velocity, particle charge, density and area of the particle beam.

$$I < \frac{4.244563 \times 10^8}{\gamma_r}[amperes]$$

where we made use of the fact $v_z \sim c$.

In table 5 the charge density and current limitations due to this criterion are scheduled. ***One may conclude from the data in this table that currents of kA or more is allowed for realistic IFELA.***

As to the current, however, there is another limitation due to the laser power and the accelerator performance as well, and the most stringent one will determine the current limit.

Table 5. Charge density and current limitations for a laser power of 10 TW and a wavelength of 1.06 μm.

Electron Energy	γ_r	**n_e[cm^{-3}]**	**I [kA]**
50 MeV	10^2	4.5127×10^{17}	4.2445×10^3
500 MeV	10^3	4.5127×10^{16}	4.2445×10^2
5 GeV	10^4	4.5127×10^{15}	4.2445×10^1
50 GeV	10^5	4.5127×10^{14}	4.2445

Throughout the length of the accelerator the electrons will be absorbing energy. If the power of the laser is not high enough to cover this range before the radiation field dies out, the electrons will no longer be accelerated. This limitation could be written as

$$\frac{I[statamps]}{e[statcoul]} < \frac{P[ergs/\sec]}{m_\circ c^2(\gamma_r(L)-\gamma_r(0))} ,$$

and converting the units into the Gaussian system, it results in

$$\boxed{I[kA] < \frac{1.9436\times 10^4}{\gamma_r(L)-\gamma_r(0)}} .$$

For an accelerator of length 10m and a gradient of 100 MeV/m, the energy gain 1 GeV leads to a current limit of

I < 10 kA

As the energy gain increases the current limitation decreases accordingly. Generally this criterion dominates as the tighter limitation, especially for larger accelerators of high gradient structure.

3.10 Hot Topics

To reach TeV energies IFELA is not an adequate tool. Tajima T. [30], on the other hand, have proposed the Plasma Beat Wave accelerator (1983) on a plasma of density 10^{18}-10^{20} cm^{-3} with a field gradient of 1-10 GeV/cm, to reach energies in 1-1000 TeV range.

For the GeV range energies IFELA has the problem of keeping the laser beam focused for long enough distances. The transfer of optical power is important for the Two Beam Accelerator too, which is another candidate for novel colliders over 350 GeV. Zakowicz W.[31] , in 1984 proposed dielectric coating the inner walls of a metallic waveguide for reduction of loss for the transmission of optical radiation. If ways to avoid multimoding is achieved, attenuation is not a problem, and the focusing will no longer be a problem. On the other hand, another analysis which sounds very nice comes alternatively from Baryshevsky V.G. and Metelitsa O.N. [32] (1994). They discuss the possibility of mutual focusing of the electron beam and the electromagnetic wave in a FEL, due to the facts that, the averaged force acting upon an electron beam is directed towards the maximum amplitude region of an electromagnetic wave; namely, towards the axis, and that the slight change of refractive index in the beam leads to optical guiding just like that in a fiber. If utilized, this idea reveals that an IFELA will no longer need transverse focusing elements as quadrupole magnets etc., and very low emittance beams will be available.

3.11 Experimental Verification, And Future Prospects

The IFEL process for charged particle acceleration has been proven to work experimentally.

At the University of Columbia a FEL is used to generate 1.5 mm wavelength radiation. This high power radiation is used to accelerate some of the electrons in another accelerator (Wernick I., 1992) [33].

At the Accelerator Test Facility in Brookhaven National Labs, a CO_2 laser of 1-2 GW power interacts with electrons along a wiggler of 0.47 m in a sapphire circular waveguide of inner diameter 2.8 cm. The injection energy is 40.0 MeV and exit energy is 42.3 MeV(Van Steenbergen A.,1996)[34].

The most important problem related to the realization of an IFELA is with keeping the laser beam focused over long distances. Multiple focusing is not an option, since the passage of the electron beam through a medium leads to scattering problems, etc. The use of optical waveguides to convey and to confine the focused laser radiation over a considerable distance is in progress now. The fundamental modes of both circular and rectangular waveguides are appropriate for interaction to take place, but low loss and very small area waveguides have to be developed.

The use of a fictitious future IFELA will be operating probably up to one hundred GeV, and they provide high energy particle bunches for a synchrotron radiation source of X-ray region. Lasing in the X-ray region is now waiting for the development of X-ray mirrors. But SR in the X-ray region has high directionality, high degree of coherence and a very narrow bandwidth, with a properly designed wiggler structure and a very high quality beam, a Synchrotron Radiation source may operate as an X-ray laser.

Various parameters for some of the third generation synchrotron radiation facilities are listed in Table 6. The IFELA we have been working can well be proposed as an injector instead of a linac or a synchrotron. It will provide bunched particles on the optical wavelength scale and the current will be few kA. The emittance properties of an IFELA has to be studied if the objective is to use it as an injector for an SR source, because SR sources and FELs are extremely sensitive to emittance. However if theory of Metelitsa O.N. and Baryshevsky V.G. [32] would work, no one will have to bother about emittance.

3.12 Transverse Behavior

In chapter 2, the longitudinal behavior of the electron beam is studied in a lengthy procedure. Free electron generators of coherent radiation, in the radar and microwave wavelengths as, say, klystrons, travelling wave tubes, and magnetrons, all have the common feature of particle bunching, and hence the coherence properties of the emitted radiation are attributable to this phenomenon in these devices, so is the case for the FEL too, the particles are bunched on the optical wavelength scale in an FEL.

In these bunches, electrons having different energies and phases trace out a closed trajectory in the energy-phase space about that of the synchronous particle.

And these oscillations, in an analogy to accelerators are called synchrotron oscillations. This longitudinal behavior may be utilized to obtain very small period pulses in a Synchrotron Radiation facility. For instance, the electron beam from an IFELA, reaching GeV energies and using and an Nd:YAG laser as the power source, will finally be bunched on a scale about 1 μm. Let these bunches be then sent to a bending magnet, and the output will be a laser-like x-ray, lasting about a fraction of a femtosecond, with a period of femtosecond range. These pulses are extremely important for investigation of some chemical reactions. On the other hand, by using bending magnets instead of undulators to obtain such pulses, one will sacrifice brightness of the optical beam.

Table 6. Salient parameters for four third-generation, low-energy, synchrotron-radiation facilities (from Schlachter A.S., 1994)[11].

	Super ACO	**ALS**	**ELETTRA**	**SOLEIL**
Energy	0.8 GeV	1.5 GeV	1.5 - 2 GeV	2.15 GeV
Circumference	72 m	197 m	259 m	200 m
Emittance	35 nm rad	< 10 nm rad	4 – 7 nm rad	17 – 36 nm rad
Wavelength	18.6 Å	7.9 Å	9.1 Å	2.5 Å
Particle	Positron	electron	electron	positron
Injector	Linac	synchrotron	linac	synchrotron
Straight sections	6	10	11	12
Undulator length	3.2 m	4.5 m	6 m	4 – 5 m
Beam lifetime	6 hours	4 – 6 hours	4 – 10 hours	14 – 24 hours
Begin operation	1988	1993	1995	2000

In designing an accelerator, transverse behavior is as important as the longitudinal one of the particle beam. Pellegrini C.,in 1982[35], has discussed the possibility of transverse focusing for an e^-e^+ collider of 300 GeV IFELA (energy of which we have shown to be inexpedient for the IFEL process). Taking into account the longitudinal magnetic field to satisfy the Maxwell's equations since our ideally sinusoidal and helical magnetic fields fail to satisfy $\vec{\nabla} \times \vec{B} = 0$, then having a parallel component of the form

$$B_z = -\left(\frac{2\pi}{\lambda_w}\right) B_w \left\{ x\cos\left(\frac{2\pi z}{\lambda_w}\right) + y\sin\left(\frac{2\pi z}{\lambda_w}\right) \right\}$$

where, the on axis field is zero but it grows up receding from the axis, as soon as the larger orbit electrons are more deflected onto the axis a possibility of transverse focusing appears. This seems to be a way of naturally focusing the beam without using quadrupole or sextupole magnets, which will add up to the investment costs and physical complexities. However this magnetic field profile still is not the real one since it tends to infinity as one goes to further distances from the axis. But, as far as a natural process for focusing is important, also in the sense of improving the optical quality of a possible SR source, since it is sensitive to beam quality, if an IFELA is used as an injector instead of the synchrotron in Figure 7, it is worth studying.

The theory of Barshevsky V.G.[32], Metelista O.N. and Dubovskaya I.Ya. for the mutual focusing of electron and laser beams sets the condition $a_s \sim K$ or K being larger as a prerequisite where $a_s = \dfrac{eE_o\lambda}{2\pi m_o c^2}$ and $K = \dfrac{eB_w\lambda_w}{2\pi m_o c^2}$, because otherwise defocusing properties would prevail. To guarantee this condition either wiggler periodicity and transverse magnetic field must be increased or laser field and wavelength must be decreased. Increasing wigler period is not a problem, in fact, it is easier to design. Increasing transverse magnetic field however will lead to the electron trajectories to become chaotic in a helical wiggler if a guide magnetic field is taken into account (Bilikmen S. & Abu Safa, 1994)[14]. Decreasing the laser power is undesired, as this will abolish one of the advantages of an IFELA, high gradient acceleration. But substituting $E_{\circ} = 2.3\times10^{6}$ statvolts/cm, $\lambda = 1.06\mu m$, $B_w = 10^4$ Gauss. The condition is satisfied for any period larger than 2.4×10^{-2} cm, which is always the case, because period smaller than cm is not possible. Natural focusing is evidently satisfied for these parameters.

Transverse oscillations of electrons in an accelerator are first observed in a betatron and since that time they are called betatron oscillations. The transverse focusing properties of adding up a guide magnetic field or the effect of mutual focusing of laser and electron beams will probably lead to betatron oscillations. With the actual magnetic field configuration they need a more precise treatment but this is over the scope of this thesis.

CHAPTER 4

CONCLUSION

In this thesis we have discussed the limitations and prospects of the Inverse Free Electron Laser process as a future accelerator. The IFELA is not found to be an adequate device for TeV range electron energies due to SR losses, but is a very effective tool to obtain tens of GeV electron energies bunched on the optical wavelength scale with an acceptable brightness and a high acceleration gradient. An accelerator of such properties will be very suitable as a compact booster for a third generation soft x-ray synchrotron source, tandem to a linac structure and, such a high brightness and high gradient accelerator is the most desired one for the industrial applications. The most essential problem is for low loss single mode optical waveguides, say cylindrical waveguides of a high power fundamental mode, to couple with the electron motion. Once invested and magnet design is accomplished using permanent magnet and adding a guide magnetic field, the power consumption and transverse focusing of the electron beam is not a problem, the IFELA will operate for various applications.

For a precise treatment of an IFELA, the following should be taken into account. Keeping the laser beam focused over large distances in a hollow optical waveguide will lead to the problem of coupling the electron with many waveguides modes. They all will have different phase velocities and attennuation, hence, the electron trajectory may be stochastic. It is a possibility to let a lossy part of the waveguide and since the fundamental mode dies out more slowly, the higher order modes will be eliminated. But this decreases the efficiency of the accelerator. Solution of the waveguide problem is equally usefull for a Two Beam Accelerator, but essential for a practical IFELA.

For a perfect treatment, the field profile in the focus of a laser should not be represented by an infinitely large plane wave, but rather, since it is limited to a region of space, it must be represented by a superposition of plane waves traveling in different directions.

Through the interaction region optical field transfers a considerable amount of its energy to the particle beam, so its damping is to be considered.

As to the magnetic fields we have assumed ideally sinusoidal and ideally helical magnetic fields, which, both fail to satisfy $\vec{\nabla}\times\vec{B}=0$, so adding up a magnetic filed in the direction of propagation is inevitable (nature would not break the rule for the sake of looking simple). This is to add a guiding magnetic field as in the case of microwave TWTs, but, Bilikmen S. & Abu-Safa M. [14] have definitely shown that, for large transverse magnetic fields adding a guide magnetic field in a quadrapole structure leads to electron trajectories to become chaotic.

There are the problems of the effects of transverse field variations, which we ignored and effects of magnetic field fringing on the electron trajectories at the entrance and nozzle, which we did not consider.

With the idealized, above mentioned picture, it is predicted that the IFELA is the appropriate candidate for high quality beams of energies up to a few tens of GeV. For a collider of a few hundred GeV, the Two Beam Accelerator, and for energies of 1-1000 TeV, The Plasma Beat Wave Accelerator are more promising cases. There is still too much to do for these accelerators.

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