SINGLE SHOT HIT PROBABILITY COMPUTATION FOR AIR DEFENSE BASED ON ERROR ANALYSIS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

İNCİ YÜKSEL

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
INDUSTRIAL ENGINEERING

JUNE 2007

Approval of the Graduate School of Natural and Applied Sciences

Prof. Dr. Canan ÖZGEN
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science

Prof. Dr. Çağlar GÜVEN
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Prof. Dr. Levent KANDİLLER
Co-Supervisor

Prof. Dr. Nur Evin ÖZDEMİREL
Supervisor

Examining Committee Members

Prof. Dr. Ömer KIRCA (METU, IE)

Prof. Dr. Nur Evin ÖZDEMİREL (METU, IE)

Prof. Dr. Levent KANDİLLER (Çankaya Unv., IE)

Asst. Prof. Sedef MERAL (METU, IE)

Dr. Hüseyin YAVUZ (ASELSAN)

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name:
İnci YÜKSEL

Signature:

ABSTRACT

SINGLE SHOT HIT PROBABILITY COMPUTATION FOR AIR DEFENSE BASED ON ERROR ANALYSIS

YÜKSEL, İnci

M.S., Department of Industrial Engineering

Supervisor: Prof. Dr. Nur Evin ÖZDEMİREL

Co-Supervisor: Prof. Dr. Levent KANDİLLER

June 2007, 138 pages

In this thesis, an error analysis based method is proposed to calculate single shot hit probability (P_{SSH}) values of a fire control system. The proposed method considers that a weapon and a threat are located in three dimensional space. They may or may not have relative motion in three dimensions with respect to each other. The method accounts for the changes in environmental conditions. It is applicable in modeling and simulation as well as in top down design of a fire control system to reduce the design cost. The proposed method is applied to a specific fire control system and it is observed that P_{SSH} values highly depend on the distance between the weapon and the threat, hence they are time varying. Monte Carlo simulation is used to model various defense scenarios in order to evaluate a heuristic developed by Gülez (2007) for weapon-threat assignment and scheduling of weapons' shots. The heuristic uses the proposed method for P_{SSH} and time of flight computation. It is observed that the difference between the results of simulation and heuristic depends on the scenario used.

Keywords: Air Defense, Error Analysis, Fire Control System, Monte Carlo Simulation, Single Shot Hit Probability

ÖZ

HAVA SAVUNMASI İÇİN HATA ANALİZİNE DAYALI TEK ATIŞTA VURUŞ OLASILIĞI HESABI

YÜKSEL, İnci

M.S., Endüstri Mühendisliği Bölümü

Tez Yöneticisi: Prof. Dr. Nur Evin ÖZDEMİREL

Ortak Tez Yöneticisi: Prof. Dr. Levent KANDİLLER

Haziran 2007, 138 Sayfa

Tezde, bir atış kontrol sisteminin tek atışta vuruş ihtimalinin (TAVİ) hesaplanması için hata analizi temelli bir yöntem önerilmiştir. Önerilen yöntemde, üç boyutlu uzayda konumlanmış bir silah ve bir tehdit dikkate alınmaktadır. Bu silah ve tehdit birbirlerine göre hareketli veya sabit olabilir. Yöntem çevresel şartların değişimini de hesaba katar. Yöntemin, modellemede ve benzetimde kullanıma uygun olması yanında atış kontrol sistemlerinin yukarıdan aşağı tasarımlarını daha az maliyetle gerçekleştirmek amacı ile de kullanılabilir. Önerilen bu yöntem özel bir atış kontrol sisteminde uygulanmıştır ve TAVİ değerlerinin silah ve tehdit arasındaki uzaklığa bağlı olduğu, dolayısıyla zaman ile değiştiği gözlemlenmiştir. Gülez (2007) tarafından geliştirilen silah-tehdit eşlemesi ve atış çizelgelemesi yapan sezgisel yöntemin sonuçlarını değerlendirmek üzere bir Monte Carlo benzetimi geliştirilmiş ve çeşitli savunma senaryoları üzerinde denenmiştir. Bu sezgisel yöntem, TAVİ ve uçuş süresi hesaplamalarında, önerilen yöntemi kullanmaktadır. Sezgisel yöntem ve benzetim sonuçları arasındaki farkın kullanılan senaryoya bağlı olduğu gözlemlenmiştir.

Anahtar Kelimeler: Hava Savunma, Tek Atışta Vuruş İhtimali, Hata Analizi, Monte Carlo Benzetimi, Atış Kontrol Sistemi

ACKNOWLEDGMENT

In the first place, I would like to express my gratitude to Prof. Dr. Nur Evin Özdemirel and Prof. Dr. Levent Kandiller for their precious supervision, advice and guidance throughout the study. I am deeply indebted to them for their patience, encouragement, and support.

I would like to thank to the committee members Prof. Dr. Ömer Kırca and Asst. Prof. Sedef Meral for their participation and valuable comments and suggestions about the thesis.

I am grateful to my manager Dr. Hüseyin Yavuz for his constant help and support. I also thank him for his participation in the committee, his valuable comments and suggestions about the thesis and his support during this study.

I am thankful to ASELSAN Inc. for giving me permission to use the project specific data and project specific studies that I performed, especially for allowing the application of the proposed method to a real system in order to verify the method. I would like to thank to Faruk Mengüç and Bülent Mete for their valuable comments and suggestions about the study. Moreover, I would like to thank to Emre Ayazoğlu, Ramazan Arık and Alper Erdener for their help for determining the error values and consultation during development of the error analysis model.

I would like to express my special thanks to my manager İsmet Atalar and my colleagues Taner Gülez and Aybeniz Yiğit for their motivation and support during the thesis.

I would like to mention my eternal indebtedness to my parents for their love, support and especially their persistent confidence in me. I am also grateful to my sister Sezin. My grand parents deserve special mention for their support and love.

Finally, I would like to express my appreciation to Ekim Ergün, for his friendship, support and patience.

TABLE OF CONTENTS

ABSTRACT iv
ÖZ vi
ACKNOWLEDGMENT viii
TABLE OF CONTENTS x
LIST OF TABLES xii
LIST OF FIGURES xiii

CHAPTERS

1. INTRODUCTION 1
2. LITERATURE SURVEY 5
    2.1. Error Analysis in Fire Control Systems 5
        2.1.1. Error Sources 5
        2.1.2. Error Classification 9
        2.1.3. Error Quantification 10
    2.2. Calculating the Hit Probability 11
        2.2.1. Parameters of Error Distributions 12
        2.2.2. Target Model 13
        2.2.3. Assumptions about Error Distributions and Hit Probability 13
2.3. Monte Carlo Simulation for Surface-to-Air Defense 15
3. SINGLE SHOT HIT PROBABILITY COMPUTATION 22
    3.1. Fire Control Problem 22
    3.2. Error Analysis 22
        3.2.1. Possible Errors of a Fire Control System 23
        3.2.2. Categorization of Errors 27

3.2.3. Error Sources and Their Categorization 29
3.2.4. Quantification of Errors 31
3.3. Computation of Single Shot Hit Probability.................................... 53
3.4. Model Verification.. 56
4. SCENARIO GENERATION and SIMULATION 60
4.1. Scenario Generation.. 60
4.2. Monte Carlo Simulation 65
5. EXPERIMENTATION.. 68
5.1. Experimental Settings 68
5.2. Experimental Results 70
6. CONCLUSION.. 82
REFERENCES 86

APPENDICES

A. SUMMARY OF LITERATURE REVIEW 89
B. NOTATION USED IN ERROR ANALYSIS 101
C. BALLISTIC EQUATIONS 108
D. SIMULATION RESULTS-SUMMARY REPORT 116
E. SIMULATION RESULTS-DETAILED REPORTS of SCENARIO 11122 134
F. SIMULATION RESULTS – SIMULATION OUTPUT of SCENARIO 11122 135
G. SIMULATION RESULTS-AMMUNITION USAGE vs. BLUE SUCCESS 138

LIST OF TABLES

TABLES

Table 1. Error Sources 7

Table 2. Classification of Errors 10

Table 3. Quantification of Errors 11

Table 4. Error Parameters 12

Table 5. Assumptions about Error Distributions and Single Shot Hit Probability Calculations 14

Table 6. Monte Carlo Based Surface to Air Defense Simulations 17

Table 7. Summary of Literature Survey 18

Table 8. Error Sources 30

Table 9. Scenario Type Related Scenario Parameters 61

Table 10. Error Model Related Scenario Parameters 62

Table 11. Scenarios with Heuristic Objective Higher Than Simulation Objective 76

Table 12. Scenarios with Heuristic Objective Lower Than Simulation Objective 77

Table 13. Summary of Literature Survey 90

Table 14. Ballistic Equations 108

Table 15. Summary Results of Simulation Runs 118

Table 16. Report of Ammunition Usage 134

Table 17. Report of Blue Success 134

Table 18. Simulation Output of Scenario 11122 135

LIST OF FIGURES

FIGURES

Figure 1. Geometry of Fire Control Problem 2

Figure 2. Effect of Error in Target Position..23

Figure 3. Effect of Error in Weapon Location...24

Figure 4. Effect of Error in Prediction Angle or Error in Weapon Orientation.....25

Figure 5. Effect of Error in Target's Relative Azimuth Velocity.........................26

Figure 6. Effect of Error in TOF or Error in Range Of Target.............................27

Figure 7. General Geometry of Target Position Error..34

Figure 8. Computation of Target Position Error..34

Figure 9. General Geometry of Error in Weapon Location..................................37

Figure 10. Computation of Weapon Location Error..37

Figure 11. General Geometry of Error in Weapon Orientation............................39

Figure 12. Computation of Weapon Orientation Error Caused by Error in True North...40

Figure 13. Computation of Weapon Stabilization Error and Line of Sight Stabilization Error...40

Figure 14. Flow Chart of Ballistic Equations...44

Figure 15. General Geometry of Target Range Error...46

Figure 16. Computation of True Target Range 47

Figure 17. General Geometry of Error in TOF...48

Figure 18. Computation of TOF Error..49

Figure 19. Flowchart of Output Error Calculation 52

Figure 20. Determination of Single Shot Hit Probability for Bi-Variate Normal Error Distribution...55

Figure 21 Comparison of Empirical P_{SSH} and Theoretical P_{SSH} 58

Figure 22. Comparison of Empirical P_{SSH} and Theoretical P_{SSH} -Revised............59

Figure 23. Simulation Flow Chart 67
Figure 24. Comparison of Heuristic Objective and Simulation Objective for Small Scenarios 72
Figure 25. Comparison of Heuristic Objective and Simulation Objective for Medium Scenarios 73
Figure 26. Comparison of Heuristic Objective and Simulation Objective for Large Scenarios 74
Figure 27. Comparison of construction Objective Falling in 95% C.I. of Simulation Objective 75
Figure 28. Correlation of Ammunition Usage and Blue Success 78
Figure 29. Change of Blue Success/Ammunition Usage Ratio with Experimental Settings 80
Figure 30. Change of Survived Assets/Ammunition Usage Ratio with Experimental Settings 81
Figure 31 Ammunition Usage vs Blue Succes with respect to Scenario Settings 138

CHAPTER 1

INTRODUCTION

Fire control problem is defined as “How can a projectile be fired from a weapon at a target in such a way as to enable the projectile to hit the target?” in Fire Control Series of Department of Defense Handbook (MIL-HDBK-799, pg 2-2). The model associated with fire control systems in this study does not apply to those projectiles known as guided missiles. The term projectile is used in a restricted sense in the thesis, and refers to only bullets, shells and rockets, but not to guided missiles.

General geometry of a fire control problem is illustrated in Figure 1.

According to Figure 1, the weapon is located at point C and the target is detected at point A. Fire control system determines the future position of the target which is presented as B. In fact, it determines the projectile to hit the target, which is initially at point A and which will be at point B according to the target’s predicted position at the time of impact. Thus, fire control system specifies the fire elevation angle (EL) and azimuth lead angle (AZ), both of which are calculated after prediction angle and given initial position of target and target elevation (E).

The fire control problem is the main concern of fire control systems. Fire control systems evaluate various types of data concerning target position, target range, target velocity, environmental conditions and ammunition characteristics in order to calculate the elevation and azimuth angles required for a successful hit of the target.

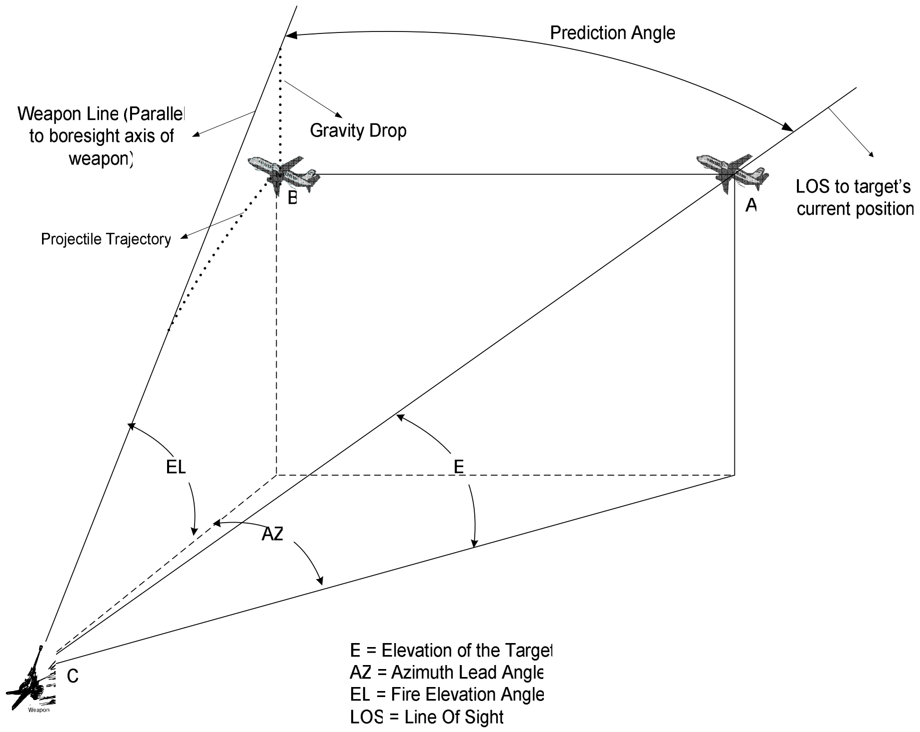

Figure 1. Geometry of Fire Control Problem

The accuracy of a fire control system is measured by its single shot hit probability. Single shot hit probability for a certain fire control system can be estimated from the errors that cause displacement of the actual aiming point from the intended one. There are many error sources that cause a number of errors affecting the single shot hit probability of a fire control system. In addition, single shot hit probabilities are closely related with the distance between the weapon and the target. As the distance between weapon and target changes over time, single shot hit probability values may change drastically. Therefore taking constant values for these probabilities during an engagement is not realistic.

There is a need for single shot hit probability values to be estimated correctly in all types of engagement models such as mathematical programming or simulation. Single shot hit probabilities are the major input parameters for such models. In addition, estimating single shot hit probability accurately is essential for combat modeling and ammunition planning.

On the other hand, single shot hit probability is one of the most important design requirements that a fire control system has to meet. Thus, estimating the single shot hit probability of an alternative design is essential for reducing its cost. A model for estimating single shot hit probability using error analysis is a useful tool for evaluating subsystems that can be used in the system design.

In the thesis, we intend to develop a method of estimating single shot hit probability values taking into consideration a wide variety of error sources, thereby providing valuable input for combat models and fire control system design. In literature, most studies take single shot hit probability values as constant or as user specified input that are derived from operational data. However, the proposed method will compute time (distance) varying single shot hit probability values considering environmental conditions, unlike most of the studies in the literature.

The proposed method for surface to air model in three dimensions is adopted from a surface to surface fire control problem model in two dimensions. The three dimensional movement of threats are also taken into consideration. The model computes single shot hit probability at a stationary or moving weapon firing to a stationary or moving air threat at any position in three dimensional space. The calculated probabilities are used as input for assignment of weapons to threats and scheduling of shots by means of a construction heuristic developed by Gülez (2007). Results of this heuristic are also simulated for a variety of engagement scenarios.

The thesis is organized as follows. Chapter 2 provides a review of relevant literature. Error analysis of a fire control system and single shot hit probability computation are presented in Chapter 3. Chapter 4 describes scenario generation and the Monte Carlo simulation model. Experimental settings and results are discussed in Chapter 5, and the thesis concludes with Chapter 6.

CHAPTER 2

LITERATURE SURVEY

In this chapter, we first review error analysis for fire control systems, then discuss single shot hit probability calculation based on error analysis, and finally review Monte Carlo based engagement simulations for surface-to-air defense using single shot hit probabilities. A summary of the papers reviewed are presented in Appendix A.

2.1. Error Analysis in Fire Control Systems

Error analysis of a fire control system involves detecting error sources, classifying the errors caused by each error source and quantification of errors.

2.1.1. Error Sources

Error sources of a fire control system are system specific though a general list of error sources can be given. In the literature, error sources are discussed at different levels of detail. Some sources only categorize them according to the error they cause and simply mention a generic name such as aiming errors, while some other sources provide detailed list of errors. The US Department of Defense Handbook-Fire Control Systems gives a list of possible error sources for a fire control system mounted on a tank (MILHNDBK-799, 1996, pg 6-32). Macfadzean (1992, pg 111) gives a list of error sources which contribute to error in an anti-aircraft artillery system. Wahlde and Metz (1999) examine the error sources of sniper weapon fire control system. Webb and Rand (2000), Helgert (1971) and Walsh (1955) enumerate some error sources of fire control systems as examples. Cothier (1984) lists error sources caused by command, control and communication system, based on a fire control system without a video tracker. Ender (2006) lists the error sources contributing to the randomness in miss distance calculation. A list of error sources that are used in the papers reviewed is given inTable 1. In the table, we provide a column for our study for comparison purposes.

Table 1. Error Sources

	Error Sources		**Our Study**	**Ender, 2006**	**Webb and Held, 2000**	**Wahlde and Metz, 1999**	**MHNDBK-799, 1996**	**Macfadzean, 1992**	**Cothier, 1985**	**Helgert, 1971**
Weapon and projectile related	Target relative azimuth rate error		+				+			
	Target range error		+			+	+		+	
	Ballistic computation error		+				+	+		
	Weapon control error		1				+			
	Line of sight stabilization error		+				+	+		+
	Weapon stabilization error		+				+	+		+
	Projectile weight		2	+				+		+
	Elevation aiming Azimuth aiming		3	+				+		
	Manual tracking error Weapon pointing error		+			+	+			
	Ammunition temperature level		+		+					
	Ship flexure for ship based guns		4							+
	Gun jump		1			+		+		
	Round to round ammunition dispersion		+			+				+
	Weapon - target altitude		5			+				
	angle of site	U[6]	7				+			
		E/M[6]	+				+			
	coriolis acceleration	U	8				+			
		E/M	8				+			
	Crosswind	U	9	+		+	+	+		
		E/M	+	+		+	+	+		
	muzzle deflection	U	9				+			+
		E/M	+				+			+
	nonstandard air density	U	9	+		+	+	+		+
		E/M	+	+		+	+	+		+
	nonstandard air temperature	U	9	+		+	+	+		+
		E/M	+	+		+	+	+		+
	nonstandard muzzle velocity	U	9	+		+	+	+		
		E/M	+	+		+	+	+		

---

[1] It is accounted for implicitly by other error sources.

[2] It is implicitly accounted for by the governing equations in our system.

[3] This error is calculated by other error sources in our study.

[4] In our study, we are concerned with a tank.

[5] Weapon target altitude is not used explicitly in our system.

[6] U: Uncompensated bias factor, E/M: Error in measuring or estimating the bias factor

[7] It is compensated in our system by ballistic equations.

[8] This is ignored as our system's maximum range is not long.

[9] It is compensated in our system

Table 1 (continued)

Error Sources			Our Study	Ender, 2006	Webb and Held, 2000	Wahlde and Metz, 1999	MHNDBK-799, 1996	Macfadzean, 1992	Cothier, 1985	Helgert, 1971
Weapon and projectile related	projectile jump	U	[9]		+		+			
		E/M	+		+		+			
	range wind	U	[10]	+		+	+	+		+
		E/M	+	+		+	+	+		+
	sight/weapon parallax	U	[10]				+	+		
		E/M	+				+	+		
	trunnion cant	U	[10]			+	+			
	weapon cant	E/M	+			+	+			
	vehicle gun line velocity component	U	[10]				+			
		E/M	+				+			
	vehicle transverse velocity component	U	[10]				+			
		E/M	+				+			
Error sources of sub-error budgets	Sensor alignment		[11]					+		
	Structural flexibility		[11]					+		
	Disturbance torques		[11]					+		
	Instrument granularity		[11]					+		
	Thermal noise		[11]					+		
	Gear backlash		[11]					+		
	Glint		[11]					+		
	Scintillation		[11]					+		
	Propagation conditions		[11]					+		
	Filter dynamics		[11]					+		
	Bearing friction/stiction		[11]					+		
	Servo velocity constant		[11]					+		
	Target motion derivation from the assumed target model		[11]					+		
	Servo noise, jitter		[11]					+		
Command and control system related	Navigational errors		[12]							+
	Incorrect radar tracking data		[12]							+
	Target location error in indirect fire Target prediction error		+				+		+	+

---

[10] It is not compensated in our system.

[11] In our study these are accounted for by related error sources, error sub-budgets are of no concern

[12] This error is implicitly taken into account in target prediction error.

Error Sources		**Our Study**	**Ender, 2006**	**Webb and Held, 2000**	**Wahlde and Metz, 1999**	**MHNDBK-799, 1996**	**Macfadzean, 1992**	**Cothier, 1985**	**Helgert, 1971**
Sight system related	Sight resolution	[13]			+				+
	Optical path bending	[13]			+				

2.1.2. Error Classification

Errors are classified at different levels in the literature. A fire control system specific error classification is presented by some of the sources while others use a general classification of errors i.e. systematic or random error. The most detailed classification is presented in The US Department of Defense Handbook-Fire Control Systems (MILHNDBK-799, 1996, pg 4-37), which is presented in Section 3.2.2 as it is used in our study. Webb and Held's (2000) and Macfadzean's (1992, pg 126) classifications are similar. Groves and Smith (1957) and Wahlde and Metz (1999) present how the classification of errors affects the probability distribution of the impact point. A less detailed classification which is not fire control system specific is proposed by Lee (2006) and Helgert (1971). In addition, Klimack (2005) classifies the errors similarly. Table 2 presents the error classification schemes that are reviewed

---

[13] This error is related with Sniper Weapon's sight system, not relevant in our study.

Table 2. Classification of Errors

Sources	**Classification**	
Yang Weon Lee, 2006 Helgert, 1971	C1	Systematic errors
		Time varying errors
Klimack, 2005	C2	Systematic errors
		Random errors
Webb and Held, 2000	C3	Round-to-round errors
		Occasion-to-occasion errors
		Tank-to-tank errors
Wahlde and Metz, 1999 Groves and Smith, 1957	C4	Variable bias errors
		Random errors
Military Handbook-799, 1996	C5	Fixed biases
		Occasion-to-occasion errors
		Burst-to-burst errors
		Round-to-round errors
Macfadzean, 1992	C6	Round-to-round errors
		Occasion-to-occasion errors

2.1.3. Error Quantification

For each error source, the error quantity that this source causes in the system output has to be determined. In literature, there are several approaches to this quantification process. The US Department of Defense Handbook-Fire Control Systems (MILHNDBK-799, 1996, pg 4-63) presents two methods for quantification of errors. The first one is deriving error propagation functions of the system when the errors are small enough to neglect their nonlinear effects. The second method is solving the governing equations twice, once without any error and once adding the error to parameters. The difference between the two outputs is then computed as a measure of error. Macfadzean (1992, pg 111) quantifies miss distance caused by each error source by differential effects and uses one-sigma error values of the error sources. Wahlde and Metz (1999) also use one-sigma error values of the error sources. Cothier (1984) calculates the command, control and communication errors from the topology of the air defense situation. In literature, errors are not always physically derived from the system. Lee (2006), Helgert (1971) and Edmundson (1961) formulate the errors as Gaussian random processes. Ender (2006) uses Monte Carlo simulation to quantify contribution of each error source to the miss distance. Sources and the methods that they use for error quantification are listed in Table 3.

Table 3. Quantification of Errors

Sources	**Quantification Method**
Lee, 2006 Helgert, 1971	Formulating errors as Gaussian Random Process
Ender ,2006	Monte Carlo simulation
Klimack, 2005	Using the past data of firings
Military Handbook-799, 1996	Error propagation functions
	Solving governing equations once without error and then with appropriate error
Metz, 1999 Macfadzean, 1992	Finding differential effects of one sigma error/finding unit effect of each error source
Cothier, 1985	Using topology of situation

2.2. Calculating the Hit Probability

Hit probability calculation involves determination of error distribution parameters, target model and assumptions about error distribution and hit probability.

2.2.1.Parameters of Error Distributions

Parameters of the error distributions (typically the mean and the variance of error) are either found from the error analysis or derived from historical firing data. Lee (2006), The US Department of Defense Handbook-Fire Control Systems (MILHNDBK-799, 1996, pg 4-63), Macfadzean (1992, pg 115), Wahlde and Metz (1999), Helgert (1971), Grubbs (1964), Groves and Smith (1957) and Walsh (1955) use root sum of squares (RSS) of individual error sources to find the parameters of the error distributions. In order to find single shot hit probability, Groves and Smith (1957), Macfadzean (1992, pg 115) and The US Department of Defense Handbook-Fire Control Systems (MILHNDBK-799, 1996, pg 4-63) threat fixed system biases as the mean and all other errors as the standard deviation of the error distribution. On the other hand, Klimack (2005) and Laurent (1962, 1952) estimate error distribution parameters empirically from historical data of firings. The methods used for estimating error parameters are presented in Table 4.

Table 4. Error Parameters

Sources	Error Parameters
Yang Weon Lee, 2006 Wahlde and Metz,1999 MLHNDBK-799, 1996 Macfadzean, 1992 Helgert, 1971 Grubbs, 1964 Groves and Smith, 1957 Walsh, 1955	RSS of errors of individual error sources
Klimack, 2005 Laurent, 1962 Laurent, 1952	Error parameters are empirically found from the past data of firings
Webb and Held, 2000	Error parameters are not numerically calculated, but dispersion is assumed to have a value proportional to the target size
Edmundson, 1961	Error parameters are calculated from circular error probable (CEP) of the weapon by random sampling

2.2.2.Target Model

Macfadzean (1992, pg 126) models the target by the shape primitives such as circle, rectangle or square. These are used to approximately project target's volume onto the plane which is orthogonal to the projectile's velocity vector at the time of impact. Klimack (2005) and Walsh (1955) also use shapes primitives to model the target.

2.2.3.Assumptions about Error Distributions and Hit Probability

In the literature, assumptions about error distributions and single shot hit probability calculation varies according to the purpose of the studies. A list of these assumptions is provided in Table 5. Although there are commonly used assumptions such as normality and independency of error distributions, some assumptions are specific to the study conducted.

Table 5. Assumptions about Error Distributions and Single Shot Hit Probability Calculations

ASSUMPTIONS	**Our study**	**Yang Weon Lee, 2006**	**Klimack, 2005**	**Webb and Held, 2000**	**Metz, 1999**	**Military Handbook-799, 1996**	**Macfadzean, 1992**	**Helgert, 1971**	**Jaiswal and Sangal, 1969**	**Grubbs, 1963**	**Edmundson, 1959**	**Laurent 1958**	**Laurent 1957**	**Groves and Smith, 1957**	**VanBrocklin and Murray, 1955**	**Walsh, 1955**
Error distributions in elevation and azimuth are normally distributed.	+		+			+	+	+	+	+				+	+	+
Error distributions in elevation and azimuth are independent from each other.	+	+	+	+	+	+	+			+				+	+	+
No systematic errors exist (because system is well designed).		+		+	+										+	
Dispersion in azimuth and elevation are equal to each other.			+												+	+
Targets are aimed at their center of gravity.	+			+												+
Actual aim point is displaced from the intended aim point by an amount that systematic errors determine.	+					+	+	+								
Target is hit when the round intercepts targets vulnerable area.	+		+	+		+	+									+
Target is rectangular.	+			+	+	+	+		+							
Target is circular.						+	+							+		
Individual error sources are independent from each other.	+			+	+	+	+	+								
All error sources are assumed to have normal distributions and are given as one sigma standard deviation values.				+	+	+	+									
Aim point (x_0, y_0) is the center of gravity of the target where x_0 is bias in azimuth error distribution and y_0 is bias of elevation error distribution.			+													
n-rectangular coordinates of the impact point are mutually independent.											+					
Impact point has a normal distribution with zero mean in n-rectangular coordinates.											+					
The aiming error (x, y) has a normal distribution with expected value (0, 0) and the same dispersion in both coordinates.																+
The only effect of the aiming error on the probability distributions is to make the expected value of impact point for each round be equal to (x, y)																+

Table 5 (Continued)

ASSUMPTIONS	Our study	Yang Weon Lee, 2006	Klimack, 2005	Webb and Held, 2000	Metz, 1999	Military Handbook-799, 1996	Macfadzean, 1992	Helgert, 1971	Jaiswal and Sangal, 1969	Grubbs, 1963	Edmundson, 1959	Laurent 1958	Laurent 1957	Groves and Smith, 1957	VanBrocklin and Murray, 1955	Walsh, 1955
Target is punctual.												+	+			
Errors in range and deflection cause the coordinates of the impact points to obey a bivariate normal distribution whose center coincides with the target's center.												+	+			
The target has a uniform vulnerability throughout.									+							

2.3.Monte Carlo Simulation for Surface-to-Air Defense

Ender (2006, pg 268) states that Monte Carlo models are often used when the process has too many phases that account for randomness or too many conditional probabilities. Ender uses Monte Carlo based methods to determine the uncertainty in top down design of an air defense method. Gogolak (1973) compares three Monte Carlo based air defense engagement analysis simulations. These simulations are called DLMNTY, MONTYX and TOOTH. MONTYX makes random weapon-threat assignments and assumes that the interceptors have the same probability of kill. DLMNTY extends MONTYX's capability to the case where two bomber types are contained in a bomber cell. TOOTH models four types of penetrating bombers and two types of interceptors which defense in two waves. MONTYX, DLMNTY and TOOTH take probability of kill as input from the user. Beare (1987) introduces a stochastic Monte Carlo simulation model called PARADE, which is used as main analytical tool for air defense analysis in British Army. He emphasizes the need for integrating this simulation model with a linear programming model in order to reduce the computation time. PARADE is used to measure the performance of the mathematical model which is called potential kill. Potential kill parameters indicate the number of kills achieved by a weapon on a site against a specific track, which targets may follow in specific environmental conditions.

Taylor (1959) used Monte Carlo simulation in NORTAM to estimate the outcome of terminal engagement between interceptor and target. In order to simulate a single terminal engagement, the tracks of bomber and fighter are modeled. The engagement between bomber and fighter is done if some rules are satisfied such as the bomber is in armament range at the time of impact and launch errors are within desired ranges. These ranges and limits are taken as input, probability of target kill is of no concern.

Cline (1961) conducted a survey about use of mathematical modeling and simulation as a technique for weapon system evaluation where the models surveyed are classified. According to this classification, ten out of fifty six models are air defense models which use modeling and simulation as a technique for weapon system evaluation. Fossett et al. (1990) describes COMO III, which is a Monte Carlo based simulation model, as a standard army model for tactical air defense artillery effectiveness studies. They also introduce another Monte Carlo based simulation called ADAGE, which predicts relative effectiveness of combinations of air defense weapons in a division. COMO III and ADAGE use U.S. Department of Defense data sources especially for lethality and terrain data.

In our study, weapon-threat assignment and scheduling is done by a heuristic method and time varying single shot hit probability values calculated for each weapon-threat pair under specific environmental conditions are used for hit assessment. Monte Carlo based surface to air defense simulations used for engagement analysis are listed in Table 6.

Table 6. Monte Carlo Based Surface to Air Defense Simulations

Study	**Simulation Method**	**Scenario Generation**	**Hit assessment**	**Engagement**
Yang Weon Lee, 2006	Event sequenced Monte Carlo Simulation	Targets: Constant speed/diving. Projectiles modeled by: Constant speed/ exponential fit to a range-table data.	If projectile intercepts with target model a hit is obtained.	Specified by input of the maximum and minimum intercept ranges and a parameter that designates either the first intercept at the maximum range or the last intercept at minimum range.
Fossett et al., 1990	Monte Carlo Simulation	N/A[14]	N/A	N/A
Beare, 1987	Monte Carlo Simulation	150 predefined cases are used where a case is defined by given numbers of unit and weapons deployed on particular sites in a particular scenario.	N/A	Mathematical model is used to maximize the kill potential achieved against that track for which it is lowest.
Gogolak, 1973	Monte Carlo Simulation	Bombers and interceptors are entered by the user as inputs.	Equal probability of kill, probability of detection and conversion for interceptors taken as input from the user, are assumed.	Random assignment is made.
Taylor, 1959	Monte Carlo Simulation	N/A	Does not make hit assessment, but considers whether a weapon can engage the target or not.	Engagement is made once it is determined that the target will be in maximum range of the weapon after flight time and fire-control predicted angular launch errors are within armament corrective maneuver limits.

Table 7 summarizes the availability of the topics with which we are concerned

---

[14] N/A: Any information about the topic is not available in the paper.

Table 7. Summary of Literature Survey

STUDY	**Do all possible error sources listed?** [15]	**Is propagation of errors in the system analyzed?** [16]	**Classification Method** [17]	**Quantification of Errors** [18]	**Is contribution of each error source analyzed?** [19]	**Error Parameters** [20]	**Target Model** [21]	**Error Distribution** [21]	**Single Shot Hit Probability Calculation** [21]	**Is model proposed for single shot hit probability calculations verified by firings' data?** [21]	**Scenario Generation** [21]	**Engagement Simulation** [21]
Yang Weon Lee, 2006	No	No	C5	Random Process	N/A	N/A	Shape primitives	N/A	N/A	N/A	Yes	Event-Sequenced Monte Carlo
Ender, 2006	No	Yes	N/A	N/A	N/A	N/A	Circular	Empirical CDF found by Monte Carlo runs	Monte Carlo Simulation	N/A	N/A	N/A
Klimack, 2005	N/A	N/A	C6	N/A	N/A	Historical data	Rectangular	Bivariate Normal Distribution	Integration	N/A	N/A	N/A

---

[15] **Yes**: All error sources are listed, **No:** Some error sources are mentioned, **N/A**: Error sources are not mentioned.

[16] **Yes**: Propagation of all errors mentioned are analyzed, **No:** Propagation of some errors analyzed, **N/A**: Propagation of errors is not mentioned.

[17] **N/A**: No classification, **C1...C6**: Refer to the classification types listed inTable 2.

[18] **N/A**: Any of the errors are quantified.

[19] **Yes**: Contribution of each error mentioned is analyzed, **No**: Contribution of some errors mentioned are analyzed, **N/A:** Contribution of errors is not mentioned.

[20] **RSS**: Root-sum-square of individual errors, **N/A**: Error parameters are not used.

[21] **N/A:** Concept that is in question is not mentioned.

Table 7 (Continued)

STUDY	**Do all possible error sources listed?**[15]	**Is propagation of errors in the system analyzed?**[16]	**Classification Method** [17]	**Quantification of Errors** [18]	**Is contribution of each error source analyzed?** [19]	**Error Parameters** [20]	**Target Model** [21]	**Error Distribution** [21]	**Single Shot Hit Probability Calculation**[21]	**Is model proposed for single shot hit probability calculations verified by firings' data?** [21]	**Scenario Generation** [21]	**Engagement Simulation** [21]
Webb and Held, 2000	No	No	C4	N/A	No	Given	Rectangular	Bivariate Normal Distribution	Integration	N/A	N/A	N/A
Metz, 1999	Yes	No	C3	Differential Effects	Yes	RSS	N/A	Bivariate Normal Distribution	N/A	N/A	N/A	N/A
Military Handbook-799, 1996	Yes	Yes	C1	Propagation Functions/ Governing Equations	No	RSS	Shape primitives	Bivariate Normal Distribution	Integration	N/A	N/A	N/A
Macfadzean, 1992	Yes	Yes	C2	Differential Effects	No	RSS	Shape primitives	Bivariate Normal Distribution	Integration	N/A	N/A	N/A
1Fossett et. al., 1990	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	Monte Carlo
Beare, 1987	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	Defined Scenari os	Monte Carlo
Cothier, 1985	Yes	N/A	N/A	Topology of situation	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A

Table 7 (Continued)

STUDY	Do all possible error sources listed?[15]	Is propagation of errors in the system analyzed?[16]	Classification Method[17]	Quantification of Errors[18]	Is contribution of each error source analyzed?[19]	Error Parameters[20]	Target Model[21]	Error Distribution[21]	Single Shot Hit Probability Calculation[21]	Is model proposed for single shot hit probability calculations verified by firings' data?[21]	Scenario Generation[21]	Engagement Simulation[21]
Dyer, 1974	N/A	N/A	N/A	N/A	N/A	Historical data	Circular	Bivariate Normal Distribution	Maximum Likelihood Estimation	N/A	N/A	N/A
Gogolak, 1973	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	Taken as input	Monte Carlo
Helgert, 1971	No	No	C5	Random Process	No	RSS	N/A	Bivariate Normal Distribution	N/A	N/A	N/A	N/A
Jaiswal and Sangal, 1969	No	No	N/A	N/A	N/A	N/A	Rectangular	Bivariate Normal Distribution	N/A	N/A	N/A	N/A
Grubbs, 1964	No	No	N/A	N/A	N/A	RSS	Circular/ Spherical	Multivariate Normal Distribution	Integration	N/A	N/A	N/A
Laurent, 1962	No	No	N/A	N/A	N/A	Given/ Former data	Punctual	Bivariate Normal Distribution	Integration	N/A	N/A	N/A
Taylor, 1959	No	No	No	No	No	N/A	N/A	N/A	N/A	N/A	N/A	Monte Carlo

Table 7 (Continued)

STUDY	**Do all possible error sources listed?**[15]	**Is propagation of errors in the system analyzed?**[16]	**Classification Method** [17]	**Quantification of Errors** [18]	**Is contribution of each error source analyzed?** [19]	**Error Parameters** [20]	**Target Model** [21]	**Error Distibution** [21]	**Single Shot Hit Probability Calculation**[21]	**Is model proposed for single shot hit probability calculations verified by firings' data?** [21]	**Scenario Generation** [21]	**Engagement Simulation** [21]
Groves and Smith, 1957	No	N/A	C3	N/A	N/A	RSS	Circular	Circular Normal	Integration	N/A	N/A	N/A
Laurent, 1957	No	No	N/A	N/A	N/A	Given/ Former data	Punctual	Bivariate Normal Distribution	Integration	N/A	N/A	N/A
Walsh, 1955	No	No	N/A	N/A	N/A	RSS	Area given	Bivariate Normal Distribution	Integration	N/A	N/A	N/A
VanBrocklin and Murray, 1955	N/A	N/A	N/A	N/A	N/A	Given	Area given	Bivariate Normal Distribution	Polar Planimeter Method	N/A	N/A	N/A
Our Study	Yes	Yes	C1	Governing Equations	Yes	RSS	Rectangular	Bivariate Normal Distribution	Integration	Yes	Yes	Monte Carlo

CHAPTER 3

SINGLE SHOT HIT PROBABILITY COMPUTATION

3.1. Fire Control Problem

Fire control problem is defined as “How can a projectile be fired from a weapon at a target in such a way as to enable the projectile to hit the target?” in Fire Control Series of Department of Defense Handbook (MIL-HDBK-799, pg 2-2). As it was stated before, the model associated with the fire control does not apply to those projectiles known as guided missiles in this study.

The main purpose of the fire control problem is to find the firing elevation and azimuth angles in order to orient the gun barrel (see Figure 1). These angles are calculated under consideration of the target’s initial position, the target’s relative motion with respect to the weapon, projectile characteristics, and effect of environmental conditions on the projectile trajectory.

3.2. Error Analysis

Aim point on the target is assumed to be the center of the target. If there is not any error in the system, the impact point and the aim point coincides after a duration called Time of Flight (TOF). TOF is the amount of time that it takes for the ammunition fired to reach the target. However, any error in the fire control system causes displacement of the impact point. In order to find the amount of displacement, an error analysis has to be carried out.

3.2.1. Possible Errors of a Fire Control System

Possible errors that are seen from the geometry of the fire control problem, as we have identified from various sources, can be stated as follows (MILHNDBK-799, 1996, PG 6.32), (Macfadzean, 1992, pg 111).

1. *Error in Target's Position:* Error in target's initial detection position directly affects the prediction for target's future position as specified in Figure 2.

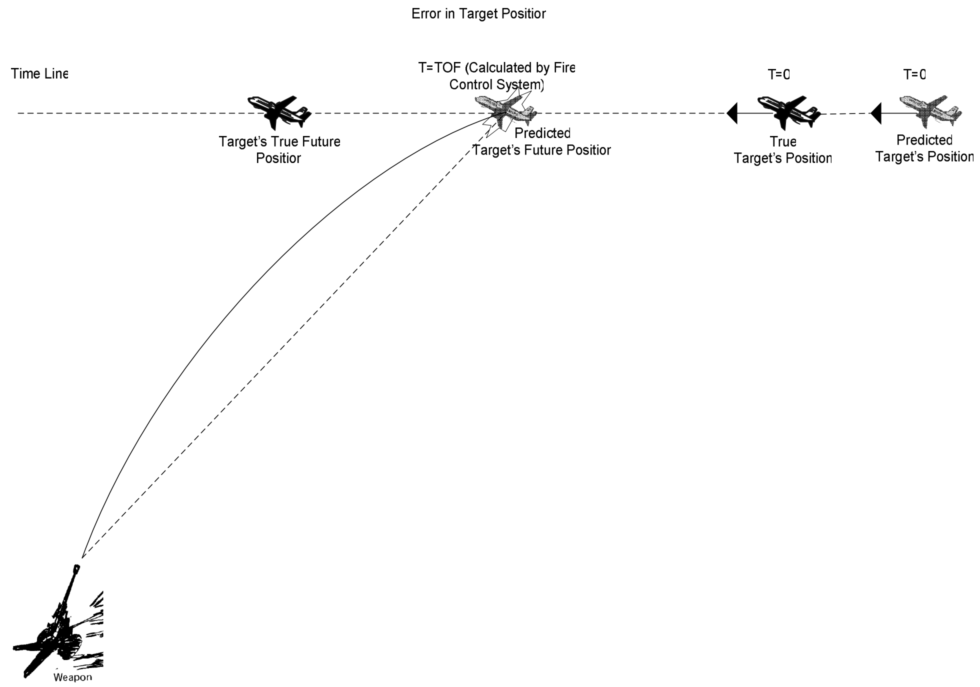

Figure 2. Effect of Error in Target Position

2. *Error in Weapon Location:* Error in weapon location affects range and weapon line orientation. As it affects range, TOF is affected by this error. Figure 3 shows how the general state is affected from this error.

3. *Error in Weapon Orientation:* Weapon is oriented toward the true north coordinate which is detected by a sensor system. The error in this system causes

weapon to orient to a different point. In addition, we have weapon stabilization error causing weapon orientation. This error affects the orientation of weapon line, which is also seen in Figure 4.

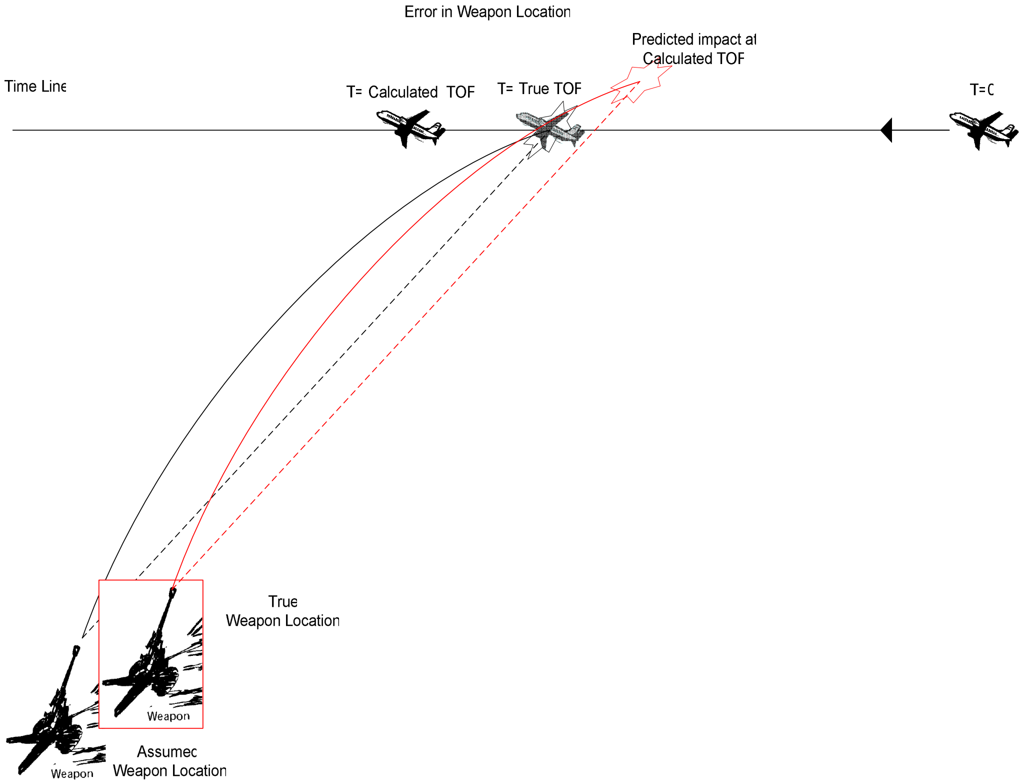

Figure 3. Effect of Error in Weapon Location

4. *Error in Prediction Angle:* Prediction angle is calculated with various inputs taken from the sensors at the fire control system. Input errors in these sensors cause an overall error in the prediction angle. The associated effect is presented in Figure 4 and input errors are explained below.

4.1. *Error in Target's Relative Rate:* Target's relative rate refers to the magnitude of target's relative velocity in three dimensions with respect to the weapon location. We term the velocity in azimuth as the azimuth rate, the velocity in elevation as the elevation rate, and velocity in the direction of weapon line (boresight axis of weapon) the as approach rate. This error directly affects the prediction for target's future position if weapon or target is not stationary. The impact of error in the target's relative azimuth rate when target is not stationary is depicted in Figure 5.

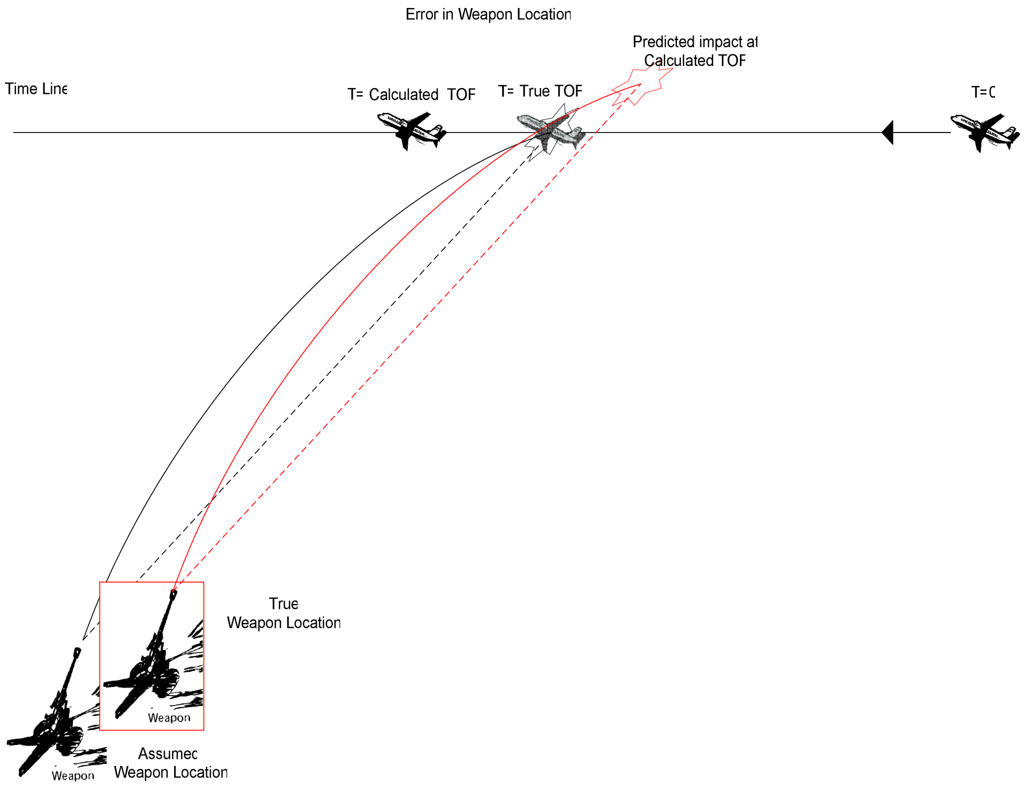

Figure 4. Effect of Error in Prediction Angle or Error in Weapon Orientation

4.2. *Error in Target Range:* Target range is the distance between the weapon and the target. Therefore, related to target's position and weapon location errors. This affects the TOF of the projectile as well as the prediction angle. Target range error has the same effect as TOF error as shown in Figure 6.

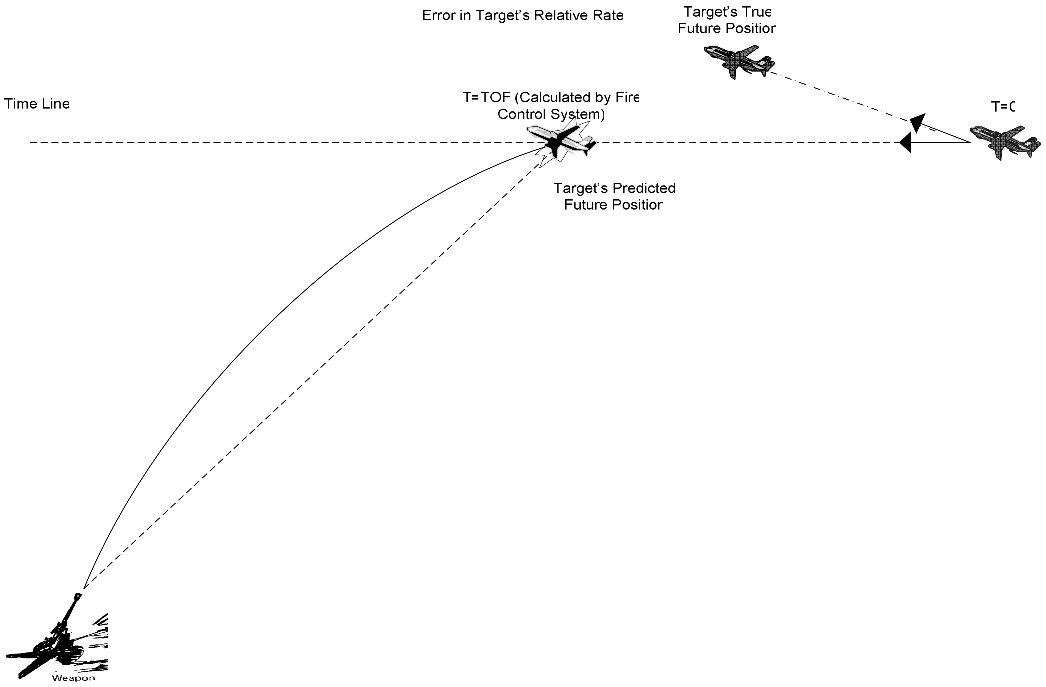

Figure 5. Effect of Error in Target's Relative Azimuth Velocity

5. *Error in Time of Flight (TOF) Calculation:* Error in the calculation of TOF affects the impact point of the projectile as target's future position is estimated wrongly. Figure 6 presents how this error influences the general state.

These errors will be analyzed further and quantified in Section 3.2.4.

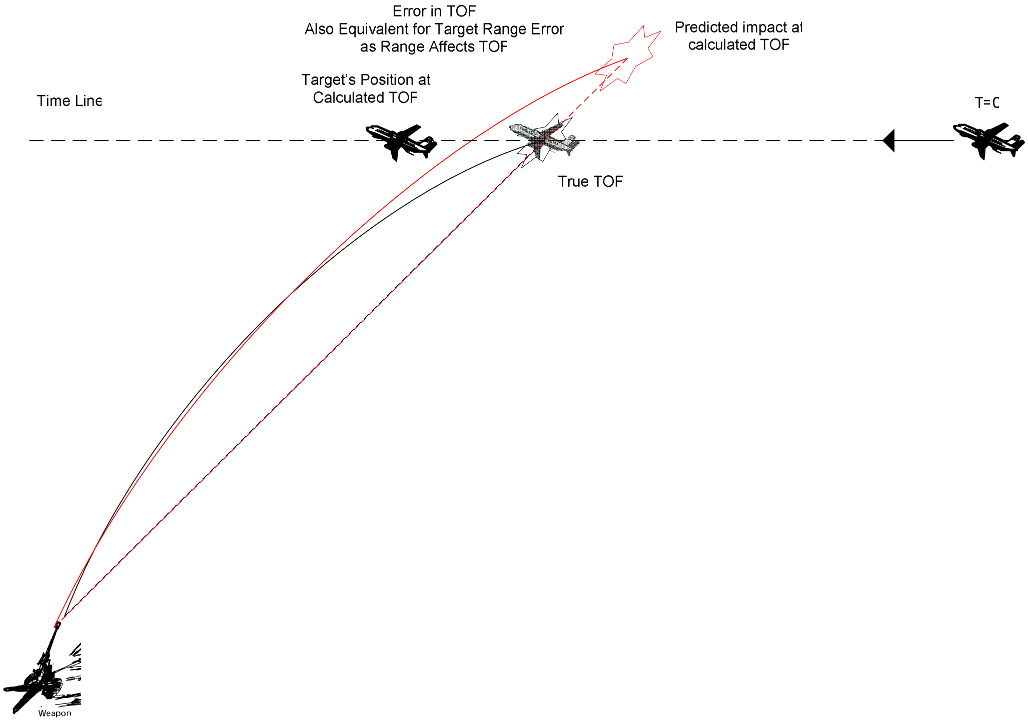

Figure 6. Effect of Error in TOF or Error in Range Of Target

3.2.2. Categorization of Errors

All error sources causing the above errors are categorized as the first step of our error analysis.

We focus on The US Department of Defense Handbook-Fire Control Systems as the main reference (U.S. Department of Defense, 4-37). According to this book, physical errors of a fire control system can be either systematic error or random error. Systematic errors are bias type errors. If systematic errors are present, the center of the impact point distribution is displaced from the true target aim point by an amount specified by the bias. Random errors are dispersion or noise type errors. It is assumed that in the presence of an error of this type, impact points have an elliptical random (typically normal) distribution centered at the true aim point.

In general, target engagement is not limited with a single shot. Multiple rounds can be fired in a single engagement and/or there can be a series of engagements. In such cases, some errors are shared by all rounds during one engagement, some are shared by rounds within a burst, or some others change from round-to-round. For this reason, fire control errors are divided into four categories when we consider the current practice in weapon design. These categories are:

(a) Fixed Biases (μ_1): This error displaces the aiming point from intended aiming point in a specific direction by an amount of μ_1. Examples are gravity drop-off, drift on spin stabilized projectiles, sight/weapon parallax, any other error sources that arise from damaged or out of adjustment fire control equipment. In a well maintained fire control system, correction is made for all fixed biases.

(b) Occasion-to-occasion biases (μ_2): The displacement caused by this error (μ_2) is derived from the distribution formed by the bias μ_1 from (a) and variance of (b). These are errors that vary quite slowly that can be treated as constant in an engagement. They vary randomly from engagement to engagement. Examples are errors due to vehicle cant, changes in air density, and changes in air temperature.

(c) Burst-to-burst biases (μ_3): The displacement caused by this error (μ_3) is derived from the distribution formed by the bias μ_2 from (b) and variance of (c). They have different values for each burst fired during an engagement. Laying error in the case that the reticle is laid onto the target before each burst of fire from automatic cannon by the gunner are some examples

(d) Round-to-round errors (μ_4): This is mainly due to ammunition dispersion. The displacement caused by this error (μ_4) is derived from the distribution formed by the bias μ_3 from (c) and variance of (d).

In error analysis, all the error sources should be examined in order to find out to which error category that they belong. Our categorization identifies whether the error source causes bias, dispersion or both in the weapon system under study. Contribution of each source to the displacement of the impact point from the target aim is calculated. For single shot hit probability calculations, fixed system biases are associated with location parameter (mean) of the impact point distribution, whereas round-to-round, burst-to-burst and occasion-to-occasion biases are associated with the variance. Hence, fixed system biases will be treated as systematic errors, and latter group of biases will be treated as random errors for the sake of simplicity in the rest of the analysis as in (MILHDBK-799, pg 4-44).

3.2.3. Error Sources and Their Categorization

In this part of error analysis error sources that cause errors given in Section 3.2.1 are listed and then categorized according to the scheme given in Section 3.2.2. The errors can be associated with the inputs and outputs of the system:

Input Errors: Errors due to the sensors and components involved in the process of determining the state, and errors of target acquisition and tracking constitute errors in input parameters.

Output Errors: Errors due to the weapon, the gun pointing mechanisms and projectile constitute errors in the output of the system.

The error sources of a fire control system are listed in Table 8. In the table systematic error sources are limited by two items only, because the weapon system of concern compensates for other systematic errors. In the table, the errors sources are classified as input or output error. The “resulting error” numbers given in the last column correspond to the error numbers in Sections 3.2.1 and 3.2.4.

Table 8. Error Sources

Error Sources		Systematic Error	Random Error	Input or Output	Resulting Error
a. Weapon location error			√	I	2
b. Weapon orientation error			√	O	3
c. Manual tracking error			√	O	4
d. Line of sight stabilization error			√	O	3
e. Target relative rate error		√	√	I	4
f. Target range error			√	I	4, 5
g. Target prediction error			√	I	4
h. Ballistic computation error			√	I	4, 5
i. Weapon stabilization error			√	O	3
j. Round-to-round ammunition dispersion			√	O	4
k. Time difference between range measurement and firing		√	√	O	4
l. Bias factors and errors in measurement or estimation of bias parameters	Trunnion cant		√	I	4
	Muzzle deflection		√	I	4
	Crosswind		√	I	4
	Range wind		√	I	4, 5
	Nonstandard muzzle velocity		√	I	4, 5
	Nonstandard air temperature		√	I	4, 5
	Nonstandard air density		√	I	4, 5
	Sight/weapon parallax		√	I	4
	Projectile jump		√	O	4

3.2.4. Quantification of Errors

The next step is to quantify the output error of the fire control system caused by the error sources listed above. That is, propagation of error caused by each source has to be identified through the equations used in the fire control system. Therefore, we present first the flow of errors through the system along with their associated signals. Error in the system output is the final result of all errors associated with the weapon system. The overall system error is expressed in terms of mean bias error and the variance of dispersion for the fire elevation angle (EL) and the azimuth lead angle (AZ). Therefore, a block diagram of system that presents input-output relationships has to be drawn and the equations have to be identified through this diagram in order to illustrate the contribution of each error source to the overall output error. In order to identify input-output relationships in the system, each possible error has to be examined.

The notation used in these calculations is given in Appendix B.

1. *Error in Target's Position:* This error occurs when initial detection of the target and estimation of its coordinates are done by a target acquisition system. If a video tracker is used in the system (which is optional in our system) and target is being seen at firing time, the effect of this error is zero. The contribution of this error source to the total system error is divided in two parts, constant range effect and range effect.

   Error in target's position directly affects target's predicted future position. Therefore, this error affects the fire elevation and azimuth angles in baseline trajectory where no compensation or correction is made. Target position error occurs in three dimensions, and target position error in one direction is independent of target position errors in other two directions. In the *x*-*z* plane, target can be at any location within an ellipse whose dimensions are target position error parameters in *x* and *z* directions, σ_{TPX} and σ_{TPZ} meters. Similarly, target can be at any location within an ellipse in the *y*-*z* plane, whose dimensions are target position error parameters in *y* and *z* directions, σ_{TPY} and

σ_{TPZ} meters. Errors in target position in *x*, *y* and *z* direction cause α_{TPX}, α_{TPY} and α_{TPZ} radian errors in weapon line considering the point on the circle where the greatest distortion occurs. Error in *x* direction only affects the azimuth angle, and errors in *y* and *z* directions affect the elevation angle. Therefore, target position error in azimuth, α_{TP_AZ}, is equal to the target position error in *x* direction and target position error in elevation, α_{TP_EL}, is equal to the root sum squares of target position errors in *y* and *z* directions. This constitutes the constant range effect of the target position error. The geometry of this error is given in Figure 7.

Constant range effect is calculated through the geometry presented in Figure 8 (which applies to both of the error sources) such that:

$$\alpha_{TPX} = \arctan\left(\frac{\sigma_{TPX}}{\sqrt{R^2 - H^2}.Cos(AZ)}\right)$$

$$\alpha_{TPY} = \arctan\left(\frac{\sigma_{TPY}\sqrt{R^2 - H^2}CosAZ}{Cos^2AZ(R^2 - H^2) + H(H - \sigma_{TPY})}\right)$$

$$\alpha_{TPZ} = \arctan\left(\frac{\sigma_{TPZ}H}{H^2 + CosAZ\sqrt{R^2 - H^2}(CosAZ\sqrt{R^2 - H^2} - \sigma_{TPZ})}\right)$$

$$\alpha_{TP\_AZ} = \alpha_{TPX} \quad \text{and} \quad \alpha_{TP\_EL} = \sqrt{\alpha_{TPY}^2 + \alpha_{TPZ}^2}$$

where

σ_{TPX} : Error in target position in *x* direction (meters)
α_{TPX} : Error caused by target position error in *x* direction (radians)
σ_{TPY} : Error in target position in *y* direction (meters)
α_{TPY} : Error caused by target position error in *y* direction (radians)
σ_{TPZ} : Error in target position in *z* direction (meters)

α_{TPZ} : Error caused by target position error in z direction (radians)

α_{TP_EL} : Target position error in elevation (radians)

α_{TP_AZ} : Target position error in azimuth (radians)

AZ : Azimuth angle (radians)

H : Height of target (meters)

R : Target range calculated according to the target position (meters)

When target's position is read from the target acquisition system, laser range finder is not used for measuring the target's range. Instead, target's position is used for ranging. Thus, when a target acquisition system is in use, error in target range, σ_{TPR}, is found from error in target's position instead of using the measurement precision of laser range finder. This constitutes the range effect of target position error. Calculation of the contribution of range effect of target position error, σ_{TPR}, is presented below.

$$\sigma_{TPR} = \sqrt{\left(\sigma_{TPX}{}^{2} + \sigma_{TPY}{}^{2} + \sigma_{TPZ}{}^{2}\right)}$$

where

σ_{TPR} : Range effect of target position (meters)

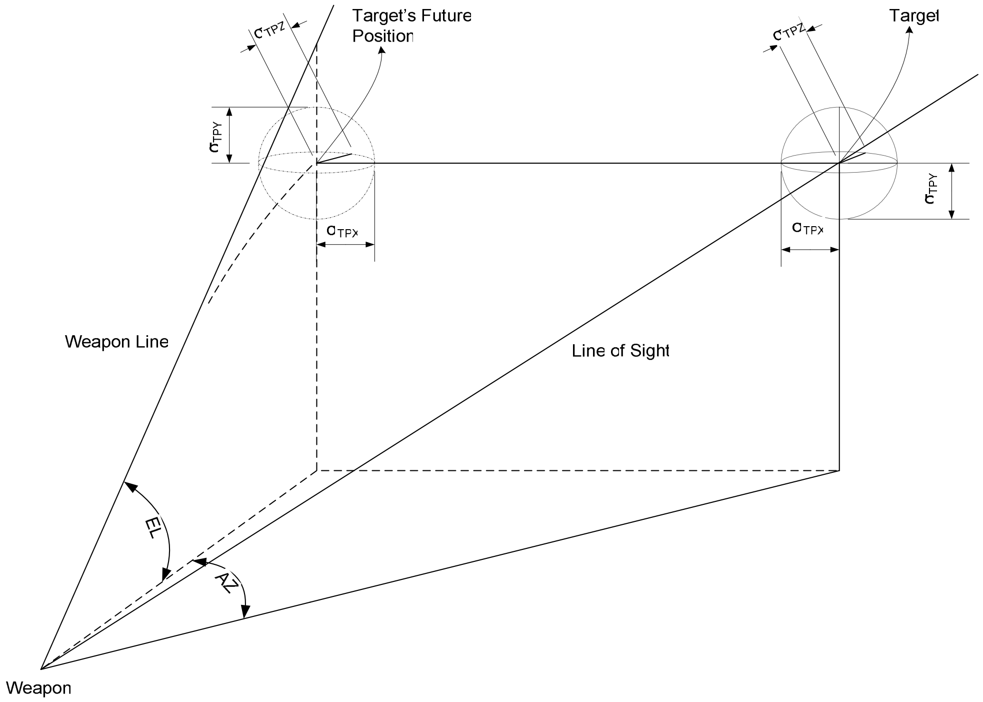

Figure 7. General Geometry of Target Position Error

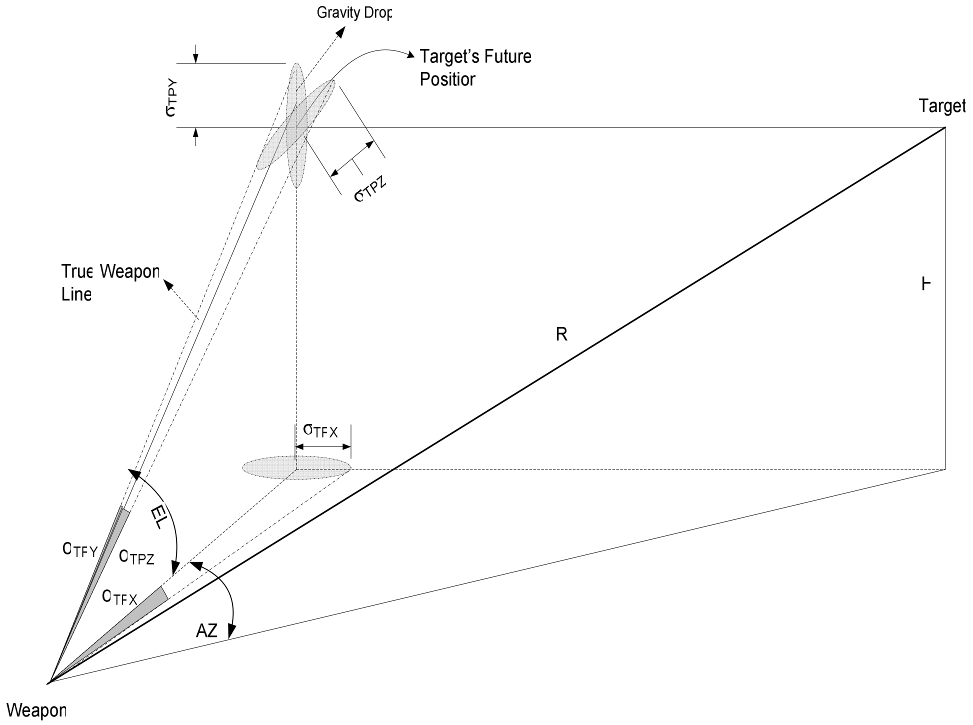

Figure 8. Computation of Target Position Error

2. *Error in Weapon Location:* This error is valid when detection of the target and approximation of the position coordinates of this target is done by target acquisition system. If video tracker is used instead of a target acquisition system, weapon location is not used as an input for fire control solution. Location of the weapon is determined by a sensor system called positioning and direction finding system. This system determines the location of weapon with an error of σ_{WP_X}, σ_{WP_Y} and σ_{WP_Z} meters in three dimensions. Therefore, weapon can be located within an ellipse having dimensions σ_{WP_X}, and σ_{WP_Z} meters in the x-z plane causing a deviation of α_{WP_AZ} radians between the true weapon line and the weapon line in azimuth. Similarly, weapon can be located within an ellipse having dimensions σ_{WP_Y} and σ_{WP_Z} meters in the y-z plane causing a deviation of α_{WP_EL} radians between the true weapon line and the weapon line in elevation. Weapon location error in z direction affects error in elevation but does not affect error in azimuth. Therefore, error in elevation is defined by weapon location error in y and z directions. Since weapon location errors in three dimensions are independent of each other, we can use the root sum of squares of errors caused by weapon location error in y direction and z direction to find the weapon location error in elevation. Figure 9 shows the geometry of weapon location error. Error in weapon location is calculated through the geometry presented in Figure 10 such that:

$$\alpha_{WPX} = \arctan\left(\frac{\sigma_{WPX}}{\sqrt{R^2 - H^2}CosAz}\right),$$

$$\alpha_{WPY} = \arctan\left(\frac{\sigma_{WPY}\sqrt{R^2 - H^2}CosAZ}{Cos^2AZ(R^2 - H^2) + H(H - \sigma_{WPY})}\right),$$

$$\alpha_{WPZ} = \arctan\left(\frac{\sigma_{WPZ}H}{H^2 + CosAZ\sqrt{R^2 - H^2}(CosAZ\sqrt{R^2 - H^2} - \sigma_{WPZ})}\right) ,$$

$\alpha_{WP\_AZ} = \alpha_{WPX}$ and $\alpha_{WP\_EL} = \sqrt{\alpha_{WPY}^{2} + \alpha_{WPZ}^{2}}$

where

σ_{WPX} : Error in weapon location in x direction (meters)

α_{WPX} : Error caused by weapon location error in x direction (radians)

σ_{WPY} : Error in weapon location in y direction (meters)

α_{WPY} : Error caused by weapon location error in y direction (radians)

σ_{WPZ} : Error in weapon location in z direction (meters)

α_{WPZ} : Error caused by weapon location error in z direction (radians)

α_{WP_EL} : Weapon location error in elevation (radians)

α_{WP_AZ} : Weapon location error in azimuth (radians)

AZ : Azimuth angle (radians)

H : Height of target (meters)

R: Target range (meters)

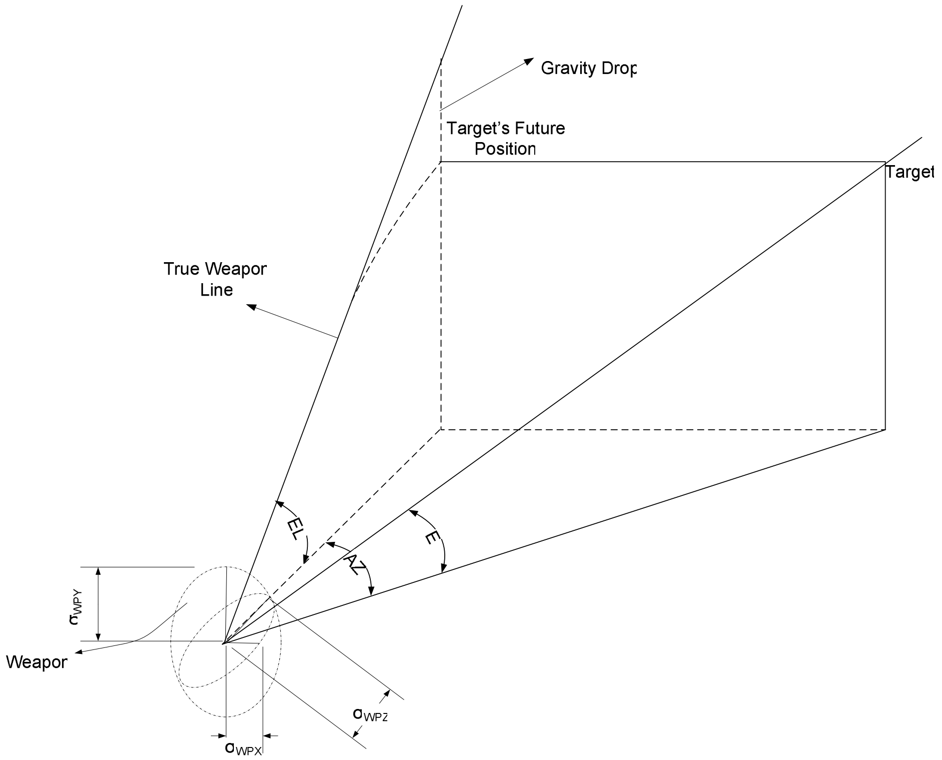

Figure 9. General Geometry of Error in Weapon Location

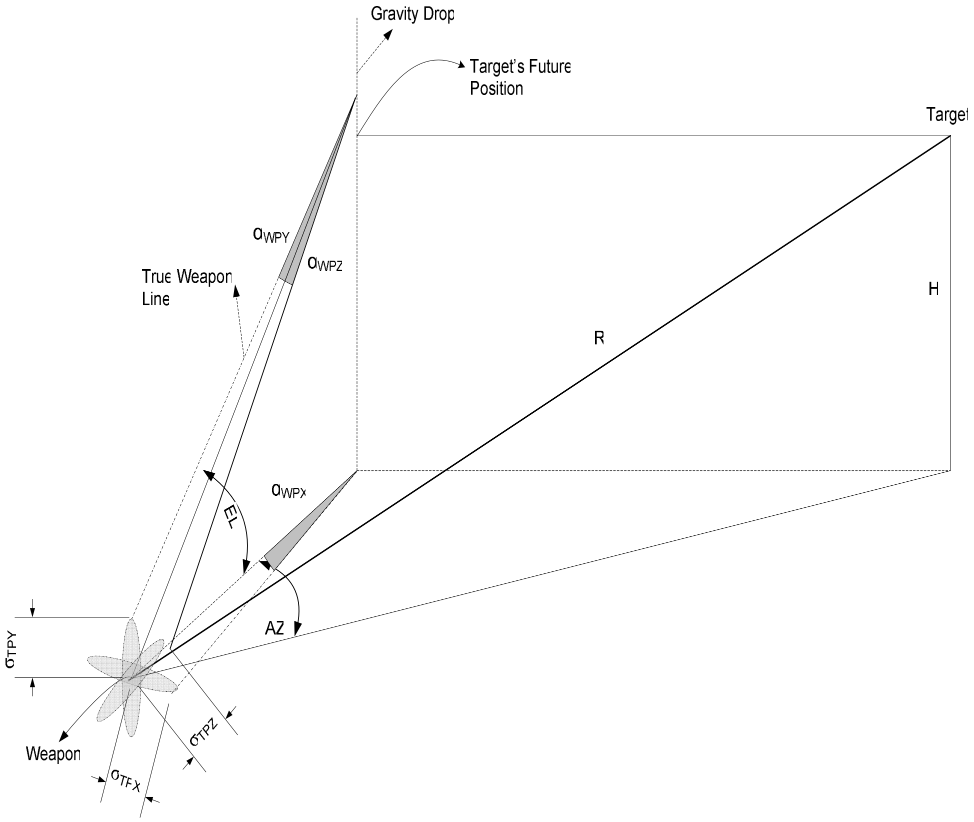

Figure 10. Computation of Weapon Location Error

3. *Error in Weapon Orientation:* The error sources that cause weapon orientation error are measurement error of weapon's positioning and direction finding system, weapon stabilization error and line of sight stabilization error.

   Measurement error of positioning and direction finding system involves error in finding the true north orientation. This system finds true north direction with an error of σ_{WON} radians which causes an angle of distortion of σ_{WON} radians in weapon line direction. The distortion is only in the azimuth angle as weapon is oriented to the north in azimuth.

   Weapon stabilization error involves the deviation of gun barrel from the target point as a result of weapon motion. Gun barrel points the true target point with an error of σ_{WSX} (σ_{WSY}) radians in azimuth (elevation), which causes an angle of distortion of σ_{WSX} (σ_{WSY}) radians in weapon line direction in azimuth (elevation).

   One other error source that causes this error is line of stabilization error. Line of stabilization error is resulted by weapon motion, and weapon is oriented to the true target point error of σ_{LSX} radians in azimuth and σ_{LSY} radians in elevation. Figure 11 shows the geometry of weapon orientation error.

   Error in weapon location is calculated through the geometry presented in Figure 12 and Figure 13 such that

$$\alpha_{WOX} = \sqrt{\sigma_{WON}^{2} + \sigma_{WSX}^{2} + \sigma_{LSX}^{2}}\ ,\quad \alpha_{WOY} = \sqrt{\sigma_{WSY}^{2} + \sigma_{LSY}^{2}}\ ,$$

   where

   α_{WOX} : Weapon orientation error in azimuth, (radians)
   α_{WOY} : Weapon orientation error in elevation, (radians)

σ_{WON} : Error of true north direction found by weapon positioning and direction finding system (radians)

σ_{WSX} : Weapon stabilization error in azimuth, (radians)

σ_{WSY} : Weapon stabilization error in elevation, (radians)

σ_{LSX} : Line of sight stabilization error radius in azimuth (radians)

σ_{LSY} : Line of sight stabilization error radius in elevation (radians)

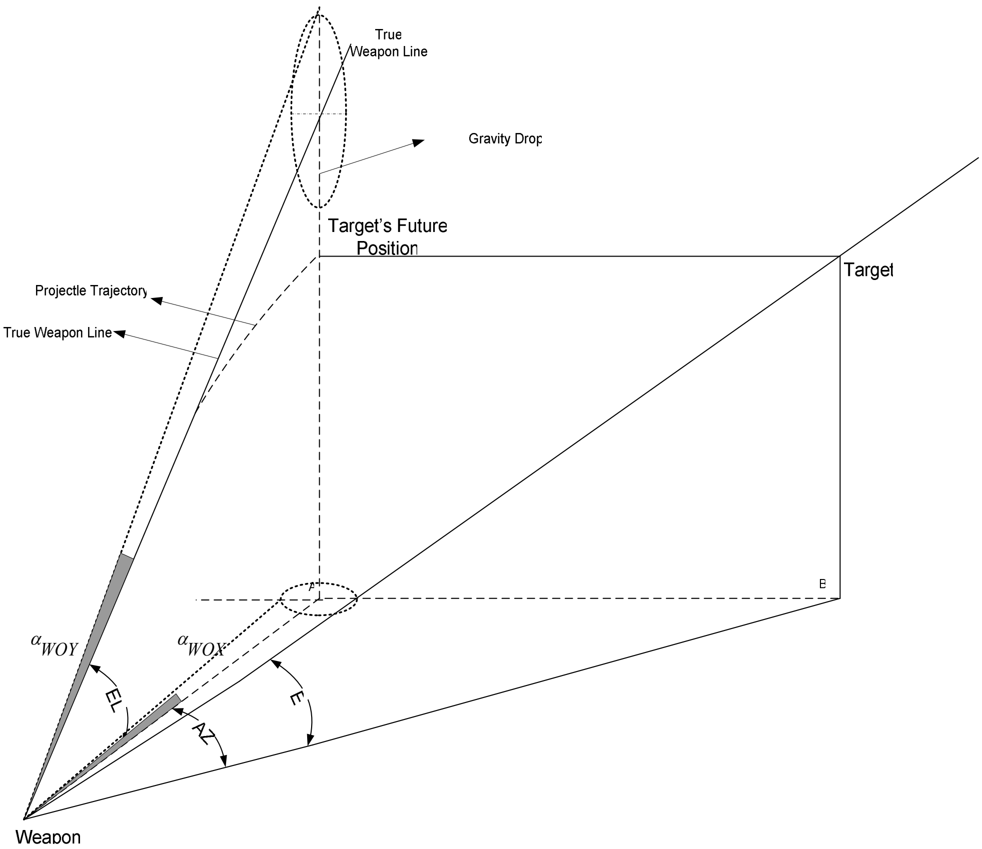

Figure 11. General Geometry of Error in Weapon Orientation

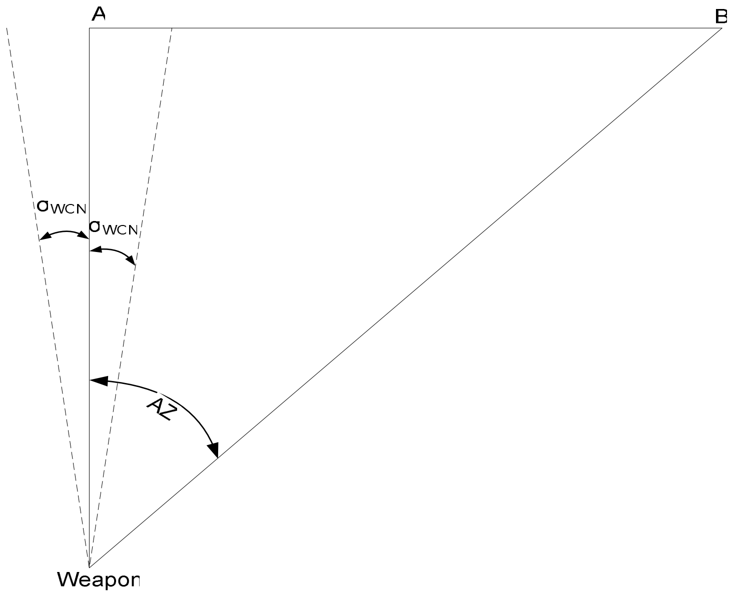

Figure 12. Computation of Weapon Orientation Error Caused by Error in True North

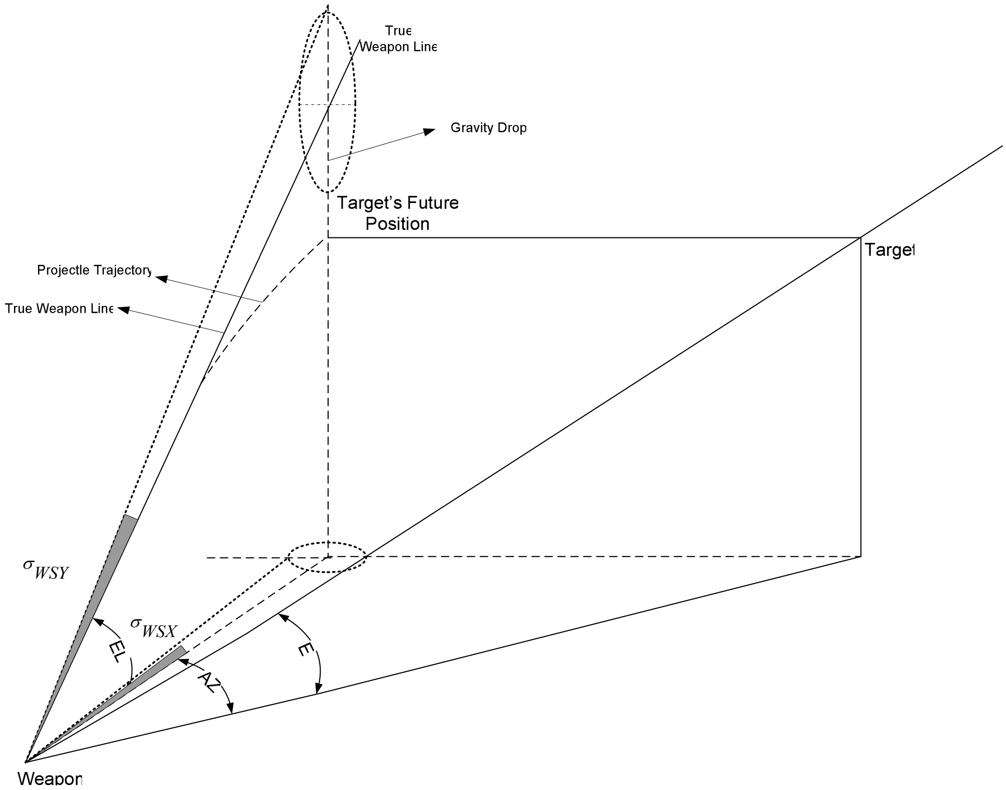

Figure 13. Computation of Weapon Stabilization Error and Line of Sight Stabilization Error

4. *Error in prediction angle:* Error in prediction angle corresponds to the errors of elevation of target (EL) and azimuth lead angle (AZ). Fire control problem is solved for elevation and azimuth, EL and AZ are found through ballistic computations, and some errors are tried to be compensated. Uncompensated biases cause systematic error and compensated biases cause random error due to measurement errors. In addition, the fire control system that we concern is a man in the loop system. That means human sense and reaction time affects the solution of fire control problem as well as appliying this solution by the system. Manual tracking error and time between target range measurement and firing the ammunition are caused by human sense and reaction time.

   Quantification of these errors can be done through error propagation functions which are found via derivation of ballistic equations (MIL-HDBK-799, Section 4-4). This derivation is based on the assumption that the errors are small enough to neglect the nonlinear effects. However, they are not neglected in our analysis. For this purpose, the equations are first solved for the case with no error, and solved again for the case in which appropriate errors are added. Then, the output errors are obtained by comparing these two results (MIL-HDBK-799, Section 4-63).

   Root mean square error (RMSE) is used for calculating the contribution of each of the error sources to the output error. However, RMSE is not available for all error sources. Instead, the accuracy of the sensors of measurement is known for some of the error sources in the form of an interval ±Δ. For these error sources, the error is assumed to be uniformly distributed in the given interval and the RMSE is found as the standard deviation of uniform distribution, i.e.

$$RMSE = \sqrt{\int_{-\Delta}^{\Delta} \frac{x^2}{2\Delta} dx} = \frac{2\Delta}{\sqrt{12}}$$

The output error of each source is obtained from the ballistic equations used for fire control solution through the process given below:

For each of the error sources:

S1. Find fire control solution through the fire control equations assuming no error exists.

$$f(y, x_1,x_2,\ldots x_n) = O_0,$$

where

$f(y, x_1,x_2,\ldots,x_n)$: Ballistic equation system that is used to find fire control solution,

y : Parameter associated with the error source which will be specified in "random prediction angle errors" and "systematic prediction angle errors" explained below,

$x_1,x_2,\ldots x_n$: Other parameters needed to obtain fire control solution,

O_0 : Output with no error.

S2. Find fire control solution through the fire control equations by perturbing the original parameter by the error associated with the error source considered:

S2.1. If the error source causes systematic error:

$$\alpha_y = \left| f(\sigma_y + y, x_1,x_2,\ldots x_n) - O_0 \right|,$$

where

α_y : Output error of error source,

σ_y : RMSE associated with the error source.

S2.2. If the error source causes random error:

$$\alpha_y = 0.5\left(\left| f(y+\sigma_y, x_1, x_2,\ldots x_n) - O_0 \right| + \left| f(y-\sigma_y, x_1, x_2,\ldots x_n) - O_0 \right| \right)$$

Error in prediction angle can be examined in two parts:

4.1. Random prediction angle errors: This error is associated with accuracy of inputs and precision of ballistic computations. In order to find out the propagation of these errors in the system, the input flow has to be examined.

Figure 14 presents the flowchart which is used to calculate ballistic angles. The formulas for nine types of ammunition used in the fire control system under study are given in Appendix C. The inputs of ballistic equations are taken via sensor systems located on the fire control system as presented in the flowchart. Inputs correspond to the compensated bias factors. However, even if the bias factor is compensated through ballistic equations, measurement errors in sensor systems cause error in the orientation of the weapon line or in the coordinate of the aiming point.

Random error sources causing error sources in the prediction angle are listed below:

a) Measurement in target range (caused by either precision of laser range finder or target prediction error) (TR),
b) Ballistic computation error (BC),
c) Manual tracking error (MT),
d) Round-to-round ammunition dispersion (AD),
e) Errors associated with bias factors and errors in measurement or estimation of bias parameters
    a. Trunnion cant (θ),
    b. Muzzle deflection (MD),
    c. Wind Velocity (Crosswind-Range wind) (W),
    d. Wind Direction (Crosswind-Range wind) (WD),
    e. Propellant Temperature (Nonstandard muzzle velocity) (Tg),
    f. Nonstandard air temperature (T),
    g. Air Pressure (Nonstandard air density) (P),
    h. Distance between mirror and gun barrel (Sight/weapon parallax) (XR, YR),
    i. Boresight Distance (Sight/weapon parallax) (R_{Bor}),
    j. Zeroing Angles (Projectile Jump) (A_{Z0}, E_{Z0}),
    k. Target relative azimuth rate (TRR_AZ),
    l. Target relative elevation rate (TRR_EZ).

Each of these error sources is annotated as *y* in the quantification process given above.

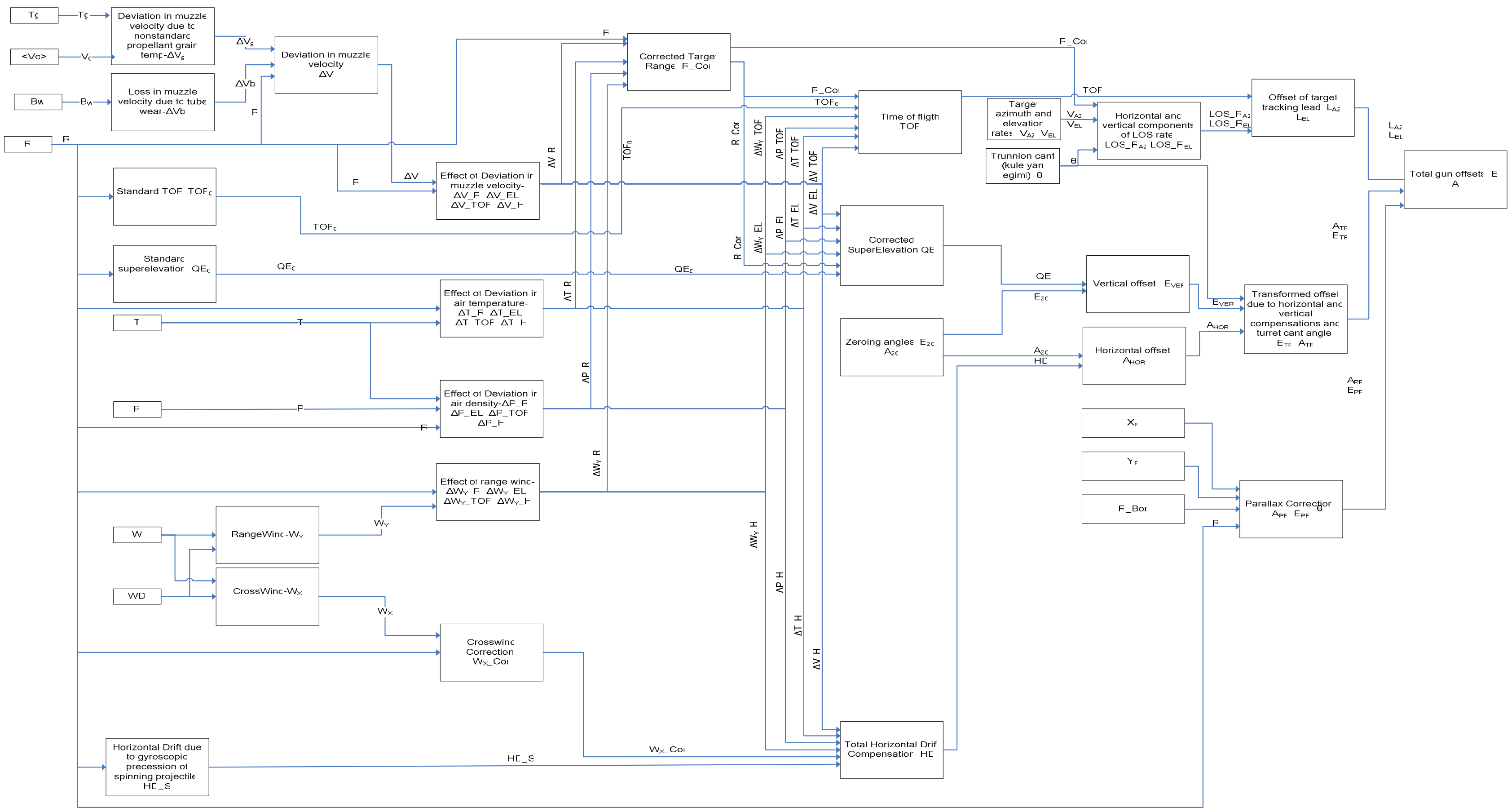

Figure 14. Flow Chart of Ballistic Equations

4.2. Systematic prediction angle errors: There are two error sources that cause systematic errors in the system under study. These are:

a) time period between range measurement and firing ammunition, and
b) target's relative rate in the direction of weapon line.

During the time period between range measurement and firing the ammunition, target's movement is ignored by the system of concern. In addition, target's relative rate (velocity) in the direction of the weapon line cannot be compensated by the current system, causing ignorance of target's movement during TOF. Because of these two error sources, the current system makes the calculation of ballistic angles in elevation and azimuth by assuming an incorrect target range, thus the impact point is displaced from the center of gravity of the hit zone. General geometry of these two errors caused by target's relative motion is presented in Figure 15. In order to handle this error properly, the true range is calculated through the geometry given in Figure 16. Then, ballistic azimuth and elevation angles are calculated through the ballistic formulas with the knowledge of the true range. The difference of ballistic angles found by using the true range and the measured range reflects the displacement between the intended impact point and the actual impact point in both azimuth and elevation, which are prediction angle biases in azimuth (μ_{AZ}) and elevation (μ_{EL}).

$$\mathrm{R}_{\mathrm{true}} = \sqrt{(\Delta t.V_{AZ})^2 + (\sqrt{\mathrm{R}^2_{measured} - \mathrm{H}^2} - (TOF + \Delta t).Vz)^2 + (\mathrm{H} - \Delta t.V_{EL})^2},$$

where

Δt: Time period between target range measurement and firing (seconds)

$\mathrm{R}_{\mathrm{true}}$: Target range at the time of firing (meters)

$\mathrm{R}_{measured}$: Measured target range (meters)

V_{AZ}: Target's relative azimuth rate (m/s)

V_{EL}: Target's relative elevation rate (m/s)
V_Z: Target's relative rate in the direction of weapon line (m/s)
H: Altitude of the target (m)

R_{true} is one of the error sources annotated by y in the error quantification process given above.

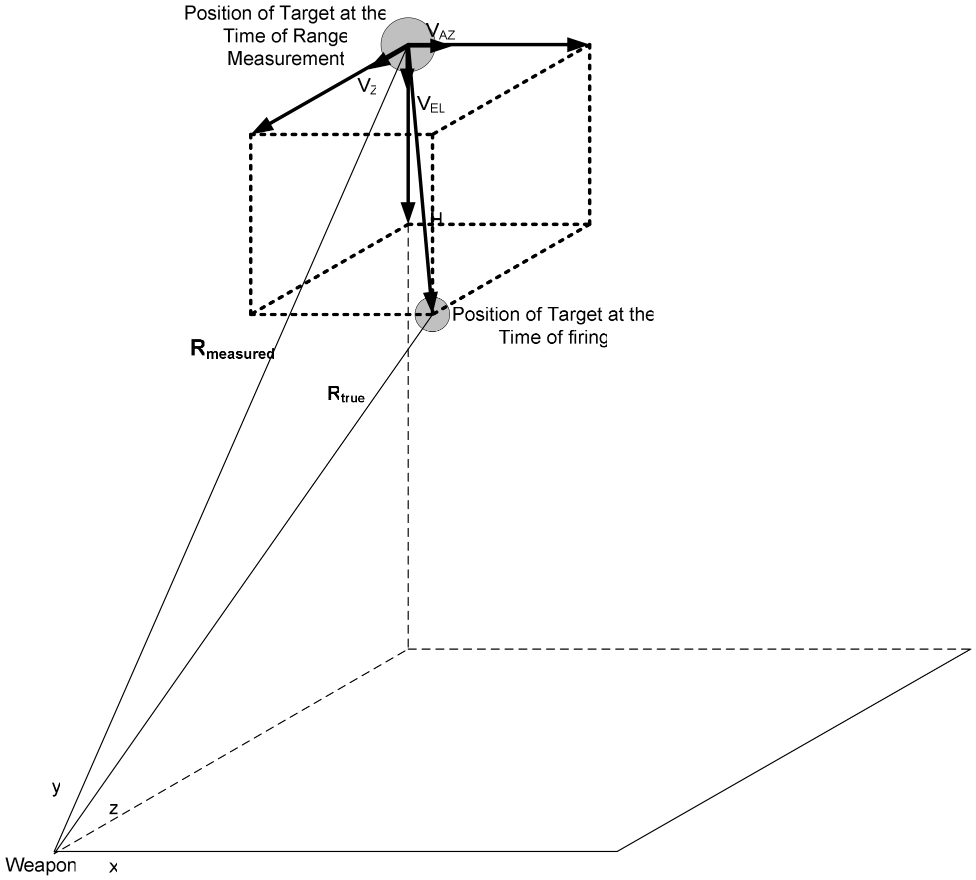

Figure 15. General Geometry of Target Range Error

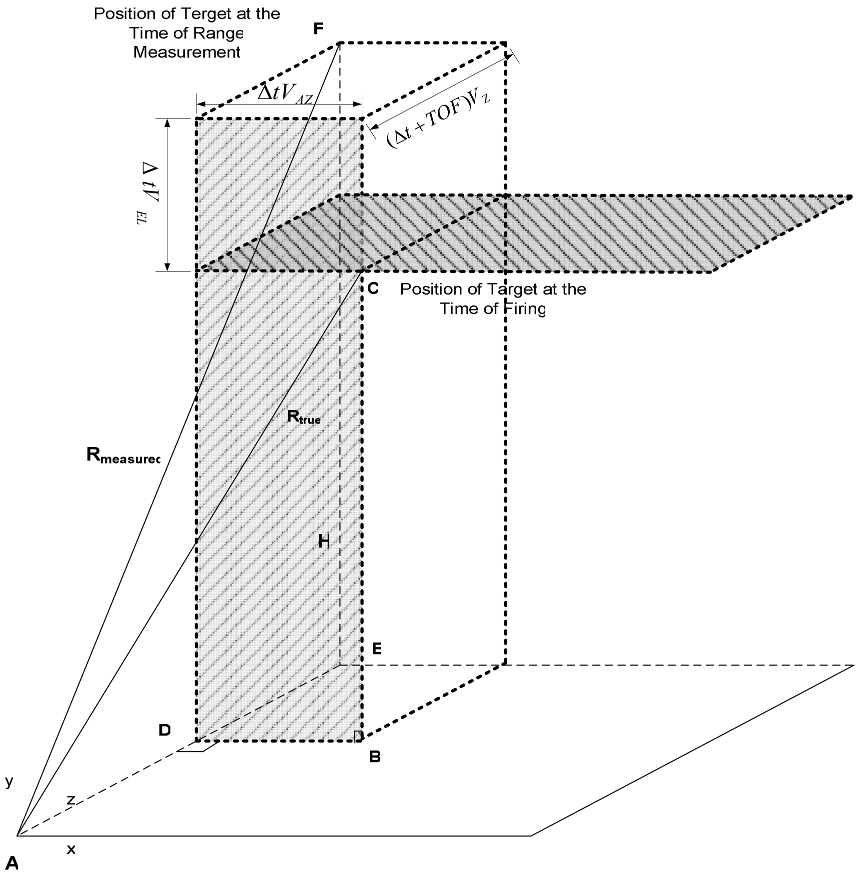

Figure 16. Computation of True Target Range

5. *Error in Time of Flight:* This error directly affects the prediction of future target position. Error in TOF is caused by many error sources, which also leads into an error in prediction angle (or miss-distances as called by Macfadzean, 1992, pg 115). The errors associated with these common sources have to be treated differently as TOF error and the prediction angle error are not independent. The geometry of error in TOF is given in Figure 17.

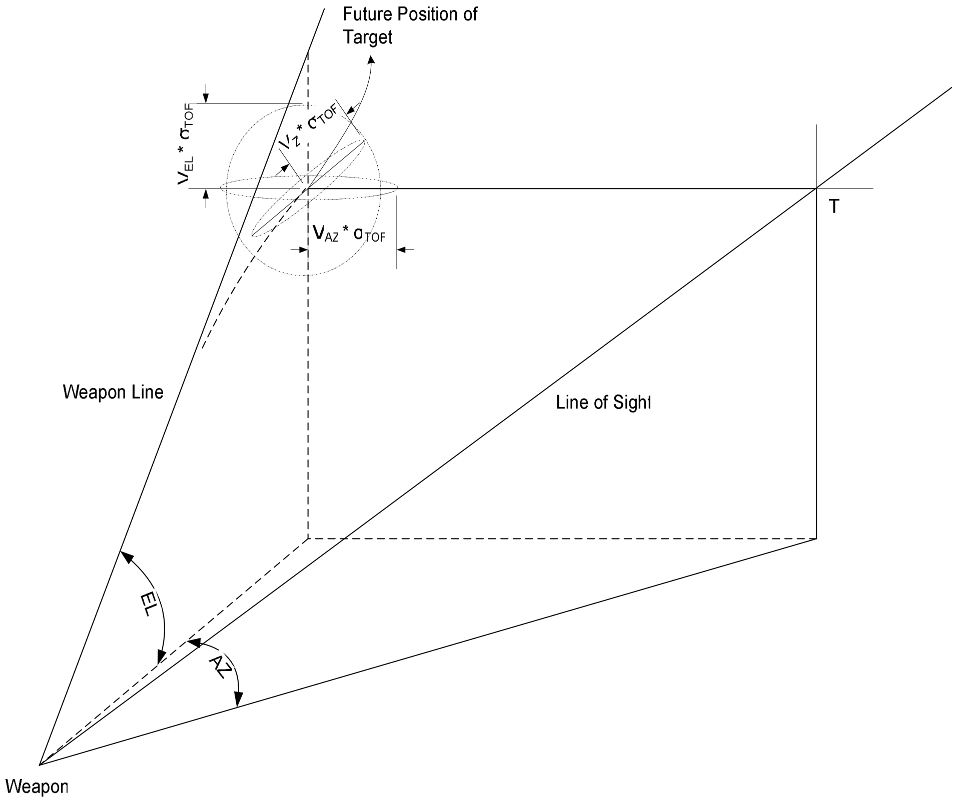

Figure 17. General Geometry of Error in TOF

Error in TOF caused by error source *X* is calculated through the geometry presented in Figure 18 such that:

$$\alpha_{X\_TOF\_AZ} = \arctan\left(\frac{\sigma_{X\_TOF}V_{AZ}}{\sqrt{R^2-H^2}-\sigma_{X\_TOF}V_z}\right),$$

$$\alpha_{X\_TOF\_EL} = \arctan\left(\frac{H-\sigma_{x\_TOF}V_{EL}}{\sqrt{R^2-H^2}-\sigma_{X\_TOF}V_z}\right) - \arctan\left(\frac{H}{\sqrt{R^2-H^2}}\right),$$

where

σ_{X_TOF}: Error in TOF caused by error source *X* (seconds)

$\alpha_{X\_TOF\_AZ}$: TOF error in azimuth caused by error source X (radians)

$\alpha_{X\_TOF\_EL}$: TOF error in elevation caused by error source X (radians)

R: Target range (meters)

H: Target's altitude (meters)

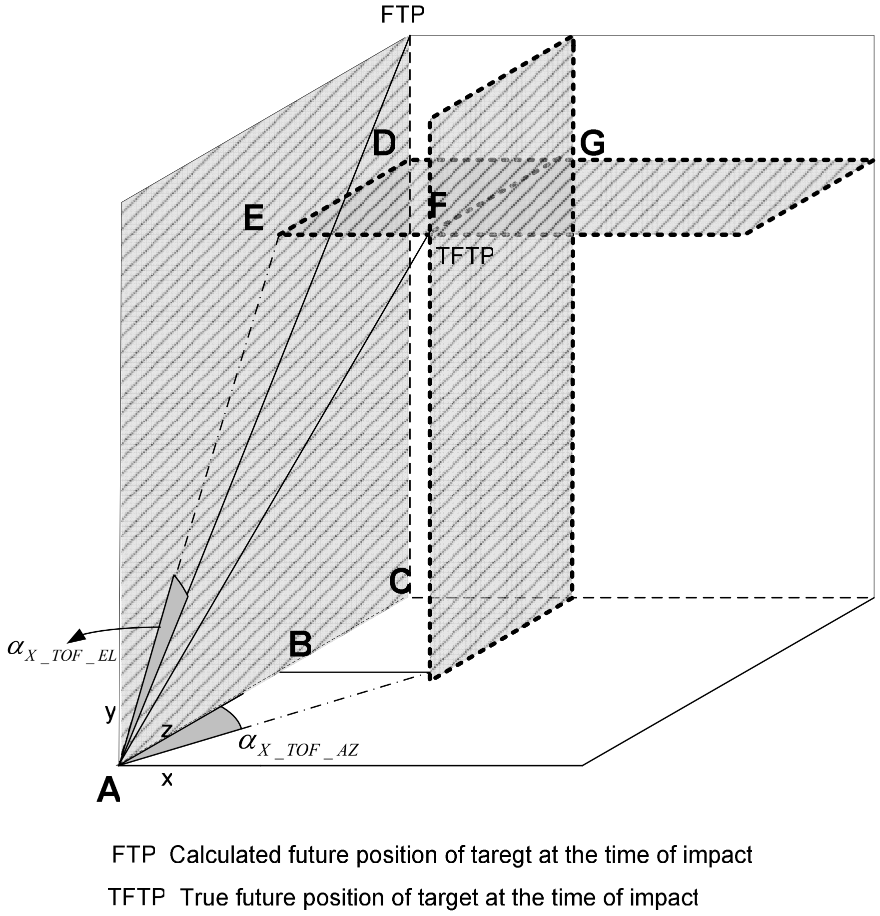

Figure 18. Computation of TOF Error

Error in TOF is computed through ballistic equations as the effect of measurement errors are considered through the TOF calculations in these equations.

Because TOF and prediction angle errors are not independent, they have to be unified for each and every error source listed below in order to calculate the total contribution of each error source to the output error.

Error sources that both cause TOF error and prediction angle error are:

a) Measurement in target range (caused by either precision of laser range finder or target prediction error);
b) Ballistic computation error;
c) Errors associated with bias factors and errors in measurement or estimation of bias parameters
    i. Wind velocity (range wind),
    ii. Wind direction (range wind),
    iii. Propellant temperature (nonstandard muzzle velocity),
    iv. Nonstandard air temperature,
    v. Air pressure (nonstandard air density).

Assume that σ_X is the RMSE of one of the error sources above. In general, this source causes error in prediction angle in azimuth and elevation, $\alpha_{X\_A\_AZ}$ radians and $\alpha_{X\_A\_EL}$ radians, respectively. In addition, σ_X RMSE causes σ_{X_TOF} seconds TOF error. These three values are found by the procedure given under "Error in Prediction Angle", which involves solving the ballistic equations without and with error and taking the difference in the output. σ_{X_TOF} seconds of TOF error results in displacement of moving target in three directions, azimuth, elevation and direction of weapon line, $V_{AZ}.\sigma_{X\_TOF}$, $V_{EL}.\sigma_{X\_TOF}$, $V_Z.\sigma_{X\_TOF}$ meters, respectively; given that V_{AZ}, V_{EL}, V_Z are the relative rates of target with respect to the weapon in these three directions. These displacements in three dimensions introduce displacement of weapon line in azimuth $\alpha_{X\_TOF\_AZ}$ radians and in elevation $\alpha_{X\_TOF\_EL}$ radians. The total contribution of each error source above to the total system error is root sum of squares of the prediction angle error and TOF error caused by the error source (Macfadzean, 1992, pg 122). Therefore, total error of error source *X* is

$\alpha_{X\_AZ} = \sqrt{\alpha^2_{X\_A\_AZ} + \alpha^2_{X\_TOF\_AZ}}$ radians in azimuth and

$\alpha_{X\_EL} = \sqrt{\alpha^2_{X\_A\_EL} + \alpha^2_{X\_TOF\_EL}}$ radians in elevation.

The overall process of calculating contributions of error sources to the output error is presented by the flowchart given in Figure 19. At the end of the error analysis, overall systematic error and random error of the system is obtained. These are used to obtain error distributions which are essential for the computation of single shot hit probability. Overall systematic error component of the system is the root sum of squares (RSS) of individual systematic errors caused by all error sources. They are assumed to be independent. Random errors are also assumed to be independent for each of the error sources and overall random error component of the system is found from the RSS of individual random errors caused by each error source.

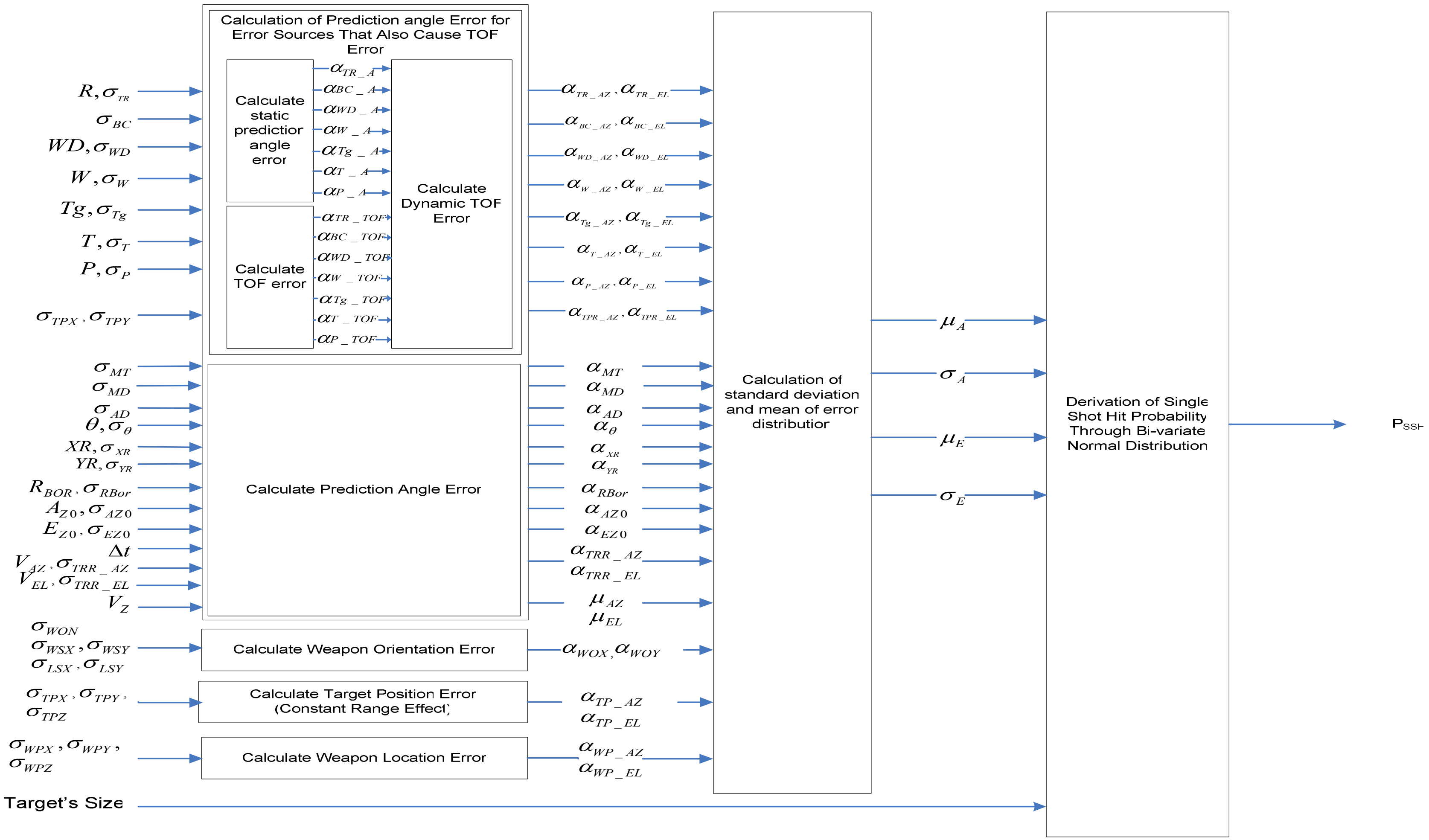

Figure 19. Flowchart of Output Error Calculation

3.3. Computation of Single Shot Hit Probability

Single shot hit probability (P_{SSH}) is the probability that a single round hits the target. P_{SSH} can be computed by using the error distributions of the fire control system, which are drawn from the random and systematic errors obtained as a result of the error analysis given as in Section 3.2. In addition, a target model is needed to obtain the single shot hit probability.

Error analysis reduces deviation of weapon line in a two dimensional plane in terms of aimuth and elevation angles although the target has a three dimensional volume. In order to compute P_{SSH}, this three dimensional target volume is projected onto the two dimensional plane which is orthogonal to the projectile (Macfadzean, 1992, pg 126). When real targets are considered, this projection is a complex process. Hence, target modeling is needed. Macfadzean states that circle, square, ellipse and rectangle are most frequently used “shape primitives” for modeling either targets or vulnerable target components.

Error analysis is done in two dimensions based on two-dimensional target projection. Two probability distributions are found for errors in azimuth and in elevation. These distributions have the following parameters:

μ_A = Root sum of squares (RSS) of systematic error components in azimuth,
μ_E = RSS of systematic error components in elevation,
σ_A = RSS of random errors caused by each error source in azimuth,
σ_E = RSS of random errors caused by each error source in elevation.

Before passing to single shot hit probability calculation, it is essential to state the assumptions.

A1. Error distributions in elevation and azimuth are normal and independent of each other. Therefore, the probability density function used for two-dimensional fire control problem and formulated in rectangular coordinates is bi-variate normal distribution.

A2. All targets aiming at our weapons or assets are assumed to be cylindrical in shape, so that the projection of a target to the plane, which is orthogonal to the projectile trajectory, is always rectangular with its edges aligned to the two coordinate axes. We refer to this projection as the “hit zone”.

A3. The fire control system is well-designed and most of the systematic errors are accounted for in the ballistic equations and compensated by the system. However, since target’s relative rate (velocity) in the direction of the weapon and time period between range measurement and firing the ammunition cannot be compensated in the present system, mean of error distribution in azimuth and mean of error distribution in elevation are different from zero, i.e. $\mu_A \neq 0$ and $\mu_E \neq 0$. We are also concerned with random errors σ_A and σ_E.

A4. Intended impact point is displaced from the center of gravity of the hit zone by an amount determined by μ_A and μ_E.

A5. If the actual impact point (after errors) falls into the hit zone, the threat is assumed to be neutralized. This assumption implies that the hit zone is the vulnerable zone for the target. It may be the silhouette of the target, a component of the target, or it may include a region surrounding the target as long as a hit in this zone results in neutralization.

Determination of single shot hit probability (P_{SSH}) is illustrated in Figure 20 for bi-variate normal error distribution and the assumptions above. There are two error distributions whose parameters are found from error analysis: azimuth error distribution and elevation error distribution. These error distributions introduce a plane on which the actual impact point falls, for the target at range R. This plane is orthogonal to the trajectory of the projectile at the time of impact. Therefore, hit zone is on this plane. The assumption about target model guarantees that hit zone is a rectangle with its edges aligned to the coordinate directions *a* and *e.* In order to have a successful hit, error in azimuth should be between a_1 and a_2, and error in elevation should be between e_1 and e_2 given that the intended impact point is center of gravity of the hit zone. Thus, given that azimuth error distribution and elevation error distribution are independent from each other, the product of the probability that azimuth error is within a_1 and a_2 and the probability that elevation error is within e_1 and e_2 gives the probability that the impact point is in the hit zone. Thus, single shot hit probability is calculated through bivariate normal distribution.

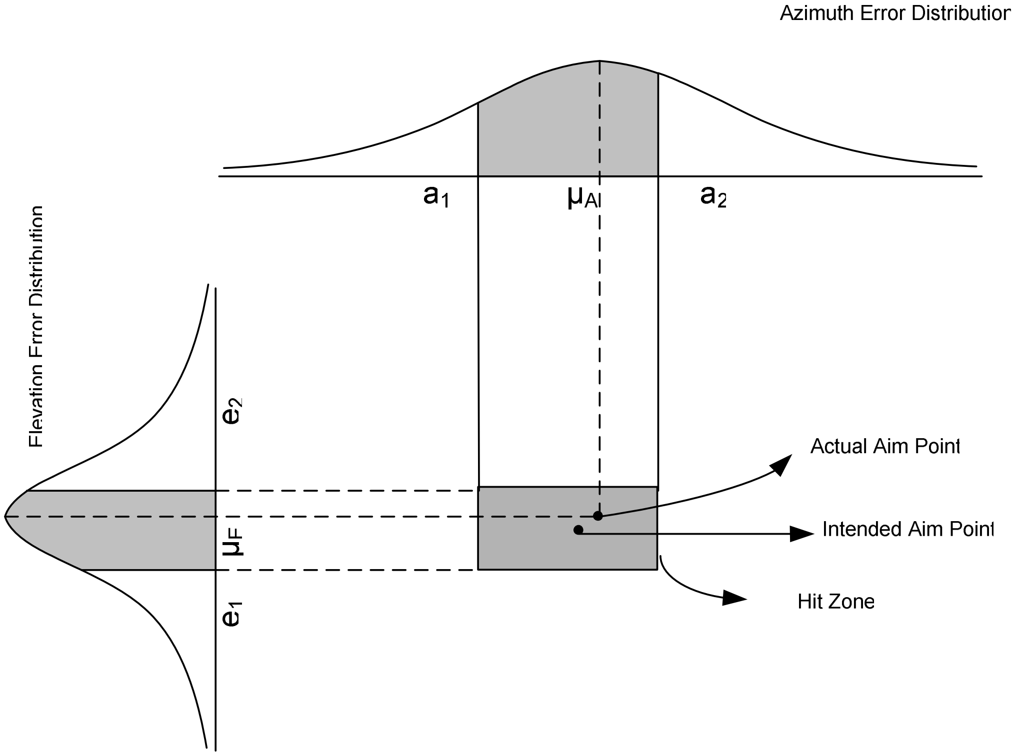

Figure 20. Determination of Single Shot Hit Probability for Bi-Variate Normal Error Distribution

Recall that, the probability density function of bi-variate normal distribution of random variables *X* and *Y* is as follows:

$$f(x,y)=\frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}}\exp\left\{\frac{-1}{2(1-\rho^2)}\left[\frac{(x-\mu_X)}{\sigma_X{}^2}-\frac{2\rho(x-\mu_X)(y-\mu_Y)}{\sigma_X\sigma_Y}-\frac{(y-\mu_Y)}{\sigma_Y{}^2}\right]\right\}$$

(3.1)

where ρ is the correlation coefficient.

The cumulative distribution function of bi-variate normal distribution evaluated over a rectangular region is:

$$F(x,y)=\iint\limits_{A_R}\frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}}\exp\left\{\frac{-1}{2(1-\rho^2)}\left[\frac{(x-\mu_X)}{\sigma_X{}^2}-\frac{2\rho(x-\mu_X)(y-\mu_Y)}{\sigma_X\sigma_Y}-\frac{(y-\mu_Y)}{\sigma_Y{}^2}\right]\right\}dxdy$$

(3.2)

In P$_{\text{SSH}}$ computation, azimuth error and elevation error represented by random variables *A* and *E* are assumed to be uncorrelated; therefore the probability density function given by equation 3.1 reduces to 3.3.

$$f(a,e)=\frac{1}{2\pi\sigma_A\sigma_E}\exp\left\{\frac{-1}{2}\left(\frac{(a-\mu_A)}{\sigma_A{}^2}-\frac{(e-\mu_E)}{\sigma_E{}^2}\right)\right\} \qquad (3.3)$$

The respective cumulative distribution function is given by equation 3.4.

$$F(a,e)=\iint\limits_{A_R}\frac{1}{2\pi\sigma_A\sigma_E}\exp\left\{\frac{-1}{2}\left[\frac{(a-\mu_A)}{\sigma_A{}^2}-\frac{(e-\mu_E)}{\sigma_E{}^2}\right]\right\}da\,de \qquad (3.4)$$

where A_R is the area of the projection of target or target's vulnerable region on the plane which is orthogonal to projectile velocity.

3.4. Model Verification

Our model for calculating single shot hit probability for fire control systems based on error analysis is applied to a fire control system in order to find its single shot hit probability theoretically. In order to verify the model, various single shot hit probabilities are calculated with our model using the inputs taken from the actual firing tests of the system, which were conducted as system operational tests as the last phase of system design process. These theoretical results are compared with the empirical results of actual firing tests. Figure 21 presents the results of this comparison. As emphasized in the error analysis, the single shot hit probabilities are drawn from the error distributions using the RMSE values, so these distributions represent an expected error variation of one-sigma. This means that about two thirds of the time the actual error will be less than the theoretical error and P_{SSH} value will be higher than the theoretical one (Metz, 1999). If the results in Figure 21 are examined, tests with id numbers 15, 23, 27 and 34 result in single shot hit probabilities lower than the theoretical ones in contrast with the expectation. When the test reports belonging to the tests with given id numbers are investigated, it is observed that several problems with the weapon were recorded at the time of testing. Figure 22 shows empirical versus theoretical P_{SSH} values without these tests.

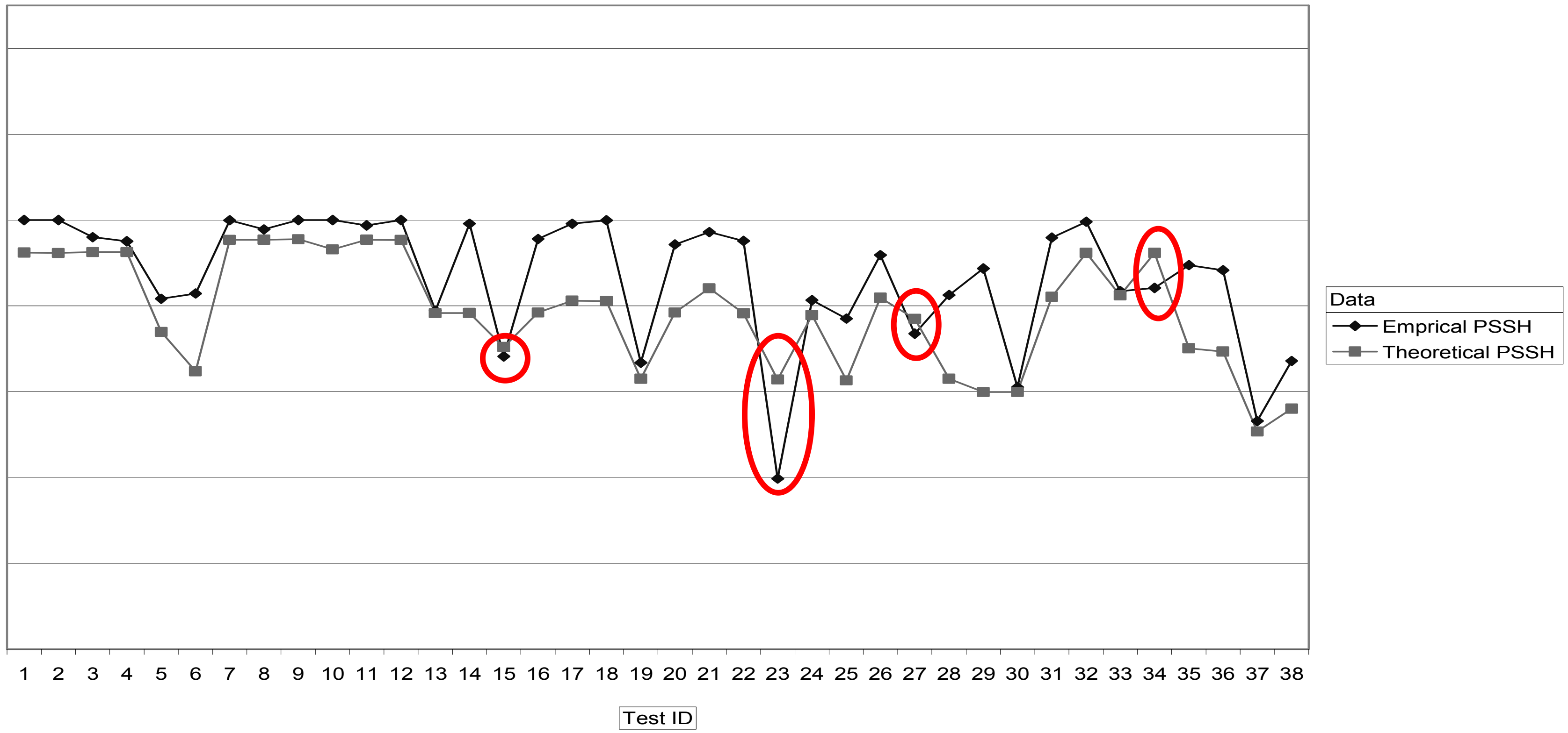

Figure 21 Comparison of Empirical P_{SSH} and Theoretical P_{SSH}

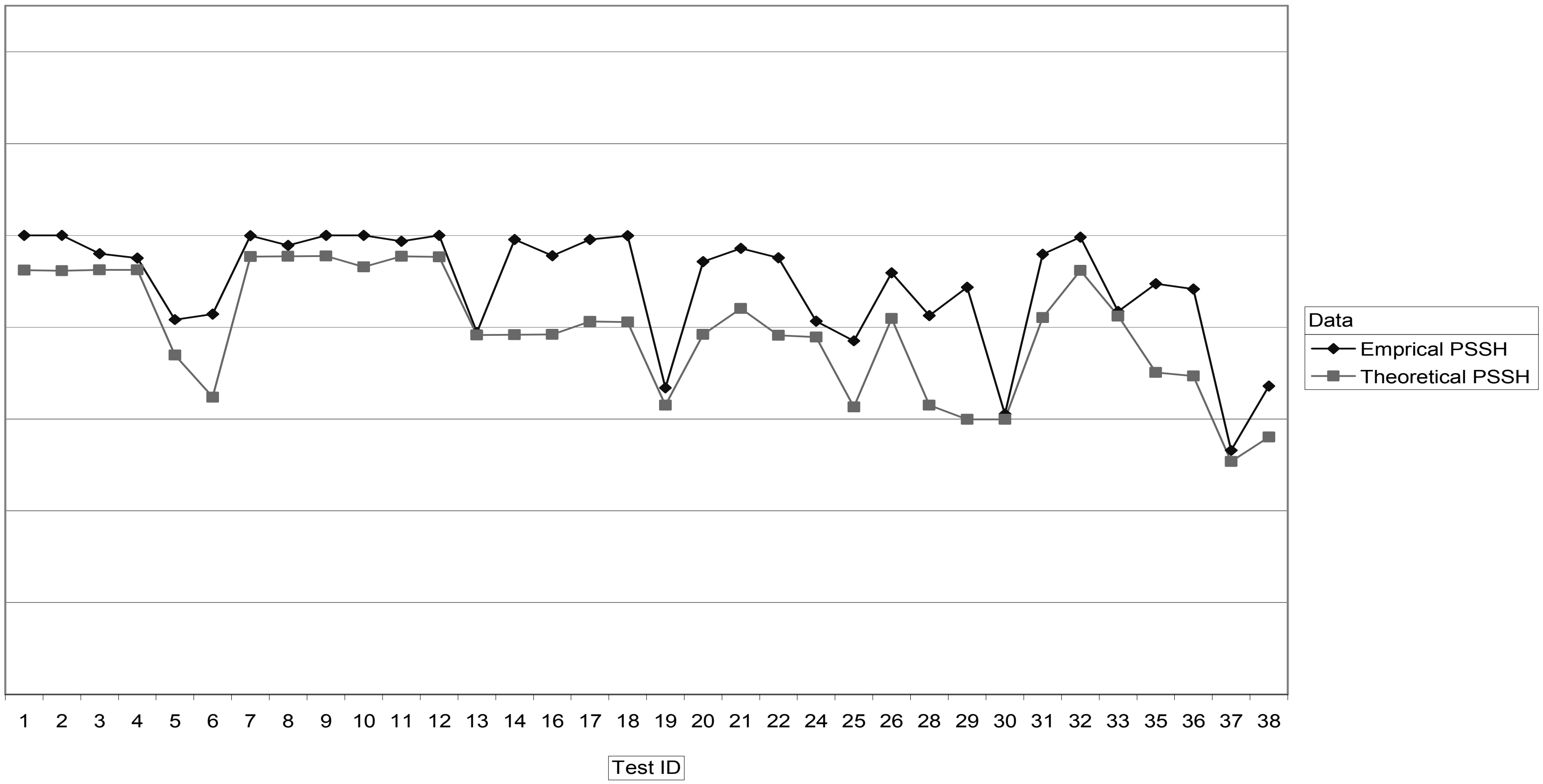

Figure 22. Comparison of Empirical P_{SSH} and Theoretical P_{SSH} -Revised

CHAPTER 4

SCENARIO GENERATION and SIMULATION

4.1. Scenario Generation

In a typical scenario, there are weapons which protect the assets against the attack of threats in a particular tactical air defense environment. Weapons and assets are the targets of the threats. Thus, in order to generate a scenario, weapon, asset and threat characteristics have to be determined and the environmental conditions have to be specified. In addition, the matching between threats and their targets has to be made. Therefore, scenario parameters that will be generated can be classified as weapon related, threat related, asset related and environmental. Scenario size is determined by the weapons, threats and assets.

Scenario parameters also include the parameters for the model that is used for P_{SSH} computation, i.e. the error parameters. All weapons are assumed to have video trackers and all shots are fired at the threats such that the weapon operators can see with the optical equipment of the fire control system. Thus, it is assumed that laser range finder is used to measure the target range, and target position and weapon location have no effect on total system error. Target position error and weapon location error are taken as zero.

Parameters for scenario identification are listed in Table 9. The number and location of weapons, assets and threats are related with the scenario size, and the threat level in the scenario. These scenario dependent decisions are made during experimental settings which are explained in Section 5.1. Each weapon can be a target of threats with the probability of p_{dt} which is determined by the experimental settings. Assets are not identical and this is reflected by different weights. When a threat is generated, it is assigned to a target which can be either a weapon or an asset. A threat is assigned to only one target during a scenario. Threat type is randomly generated. A threat can be either single ammunition which directly aims at its target or a weapon such as an airplane or a helicopter. If the threat is a weapon, it flies over its target at a constant altitude and comes to the closest point to its target before it fires. A threat's type and the target's location that the threat is aiming at are used to determine its direction. A threat can hit its assigned target with a probability of *q*. This probability is assumed to be the overall probability of hit during threat's attack and it is determined from different distributions according to the target's type.

Table 9. Scenario Type Related Scenario Parameters

Weapon	Number of weapons	$DU(W_L, W_U)$
	Weapon location on x-y plane	$(U(X_L, X_U), U(Y_L, Y_U))$
	Weapon is also a target or not	is also a target with probability p_{dt}
	Ammunition inventory	$DU(M_L, M_U)$
Asset	Number of assets	$DU(A_L, A_U)$
	Asset location	$(U(X_L, X_U), U(Y_L, Y_U))$
	Asset weight (value)	DU(1, 10)
Threat	Number of threats	$DU(T_L, T_U)$
	Threat location (x, y, z)	$(U(X_L, X_U), U(Y_L, Y_U), U(500,2000))$
	Target to attack	Can be an asset or a weapon depending on the type of defense
	Threat type	A single ammunition
		An aircraft or helicopter
	Threat's probability of hitting its target	U(0.7, 0.9) if threat is a single ammunition U(0.3, 0.7) if threat is an aircraft

Parameters for P_{SSH} the model are weapon and threat related parameters and environmental parameters that are the input of P_{SSH} calculation. These are listed in Table 10. Threat related parameters are generated in accordance with its type. Threat type affects the threat's speed and magnitude i.e. its length and width.

Table 10. Error Model Related Scenario Parameters

Weapon	Sight system	Direct optical
		TV camera
		Thermal camera
	Target tracking type	Manual
		Automatic
	Ammunition type	DU(1, 9)
	Trunnion cant angle	U(0, 20) degrees
	Boresight range	U(1000, 2000) meters
	Zeroing angles	U(0,1) mil
Threat	Length	Depending on the type of threat
	Width	Depending on the type of threat
	Speed of threat	Depending on the type of threat
Environmental conditions	Air temperature	U(19,44) ^{o}C
	Air pressure	U(904,916) mbar
	Wind direction	U(0,360) degrees
	Wind speed	U(0,30) m/sec
	Grain temperature	U(12,39) ^{o}C

Parameters related with the environmental conditions are generated from base probability distributions specific to parameter type at the beginning of the scenario. However, they are not constant for each shot in the scenario. For each shot these parameters vary from the base scenario values by an amount determined for the parameter specific deviation distributions, while all other parameters are generated once and remain constant during the scenario.

The procedure used for scenario generation is as follows:

S1. Determine scenario size and type:
  1.1. Set lower and upper limits on number of weapons (W_L, W_U), assets (A_L, A_U) and threats (T_L, TU) according to experimental settings.
  1.2. Set lower and upper limits of coordinates of the battle area as $X_L = Y_L = -1000$ m and $X_U = Y_U = 1000$ m.
  1.3. Determine type of defense parameter p_{dt} according to experimental settings.
S2. Generate weapon characteristics:
  2.1. Determine the number of weapons from DU(W_L, W_U).
  2.2. For each weapon,
    2.2.1. Determine location (x, y) from U(X_L, X_U) and U(Y_L, Y_U).
    2.2.2. Decide if it is also a target according to p_{dt}.
    2.2.3. Determine sight system from DU(1, 3).
    2.2.4. Determine tracking type from DU(1, 2).
    2.2.5. Determine ammunition type from DU(1, 9).
    2.2.6. Determine cant angle from U(0, 20).
    2.2.7. Determine boresight range from U(1000, 2000).
    2.2.8. Determine zeroing angles in elevation and azimuth from U(0, 1).
S3. Generate asset characteristics:
  3.1. Determine the number of assets from DU(A_L, A_U).
  3.2. For each asset
    3.2.1. Determine location (x, y) from U(X_L, X_U) and U(Y_L, Y_U).
    3.2.2. Determine weight (value) of the asset from DU(1, 10).
S4. Generate threat characteristics:

4.1. Determine the number of threats from DU(T_L, T_U)

4.2. For each threat,

4.2.1. Determine detection location (x, y, z) from U(X_L, X_U), U(Y_L, Y_U) and U(500, 2000).

4.2.2. Determine the target that the threat will attack.

4.2.3. Determine the threat type from DU(1, 2).

4.2.4. Determine probability of threat hitting its target:

If threat is single ammunition, determine probability from U(0.7, 0.9).

If threat is a weapon, determine probability from U(0.3, 0.7).

4.2.5. Determine direction:

If threat is single ammunition, find direction of the threat such that it directly arms at the target.

If threat is a weapon, find direction of the threat such that it flies over the target.

4.2.6. Determine speed:

If threat is single ammunition, speed ~ U(200, 300).

If threat is a weapon, speed ~ U(100, 250).

4.2.7. Determine length:

If threat is single ammunition, length ~ U(3, 8).

If threat is a weapon, length ~ U(5, 10).

4.2.8. Determine width:

If threat is single ammunition, width ~ U(1, 3).

If threat is a weapon, width ~ U(5, 10).

S5. Generate weapons' ammunition quantity:

5.1. Set lower and upper limits on quantity of ammunition (M_L, M_U) according to the experimental settings.

5.2. For each weapon generated, determine ammunition quantity from DU (M_L, M_U).

S6. Generate base values of environmental conditions:

6.1. Determine air temperature from U(19, 44).

6.2. Determine air pressure from U(904, 916).

6.3. Determine wind direction from U(0, 360).

    - 6.4. Determine wind speed from U(0, 30).
    - 6.5. Determine grain temperature from U(12, 39).
- S7. For each shot in the scenario, determine the deviation of
    - 7.1. air temperature around its base value from U(-0.5, 0.5),
    - 7.2. air pressure around its base value from U(-1, 1),
    - 7.3. wind direction around its base value from U(-2, 2),
    - 7.4. wind speed around its base value from U(-0.5, 0.5),
    - 7.5. grain temperature around its base value from U(-0.5, 0.5).

4.2. Monte Carlo Simulation

Error analysis based calculation of P_{SSH} for fire control systems is used in a construction heuristic that is developed by Gülez (2007). This heuristic assigns the weapons to the threats and schedules the shots of the weapons so as to maximize the sum of survival probabilities weighted by the values of assets. Time varying P_{SSH} values, which are found by the method proposed in this study, are used in the construction heuristic. The heuristic also uses the proposed model to calculate the time of flight of the projectiles.

In order to evaluate the outcome of the assignments and the schedule formed by the heuristic, a Monte Carlo simulation is performed in this study. The process of scenario simulation is described by a flow chart in Figure 23. There are two types of events in the event list of the simulation, a projectile fired by a weapon reaches the threat that it is assigned to, and a threat reaches the asset that it is attacking. The first type of events is scheduled by the construction heuristic with the objective of maximizing survival probability of the assets. The second type of events are determined before the simulation and scheduled according to the type of threats and their speed. Ammunition type threats reach their assets at the end of their time of flight. Weapons type threats are assumed to fire when they reach the closest point to the asset that they are assigned to.

Monte Carlo simulation is used to make hit assessment of projectiles fired by the weapons at the threats, and hit assessment of the assets that are attacked by the threats. Threats are assumed to be neutralized when they have a successful hit from any weapon, as it is also stated in hit zone definition under assumptions in Section 3.3. Therefore, when a threat is hit, then the future shots scheduled at that threat from the weapons are cancelled. In the same way, when a weapon is hit by any threat, its future shots are also cancelled.

Sum of survival probabilities weighted by values of assets is taken as the primary performance measure of the simulation. Number of replications is increased until the relative precision of the 95% confidence interval for this performance measure falls below a certain fraction (taken as 0.1 in this study). The performance measure of the simulation is also compatible with the objective function of the heuristic; hence a comparison can be made between the results of the heuristic and the simulation.

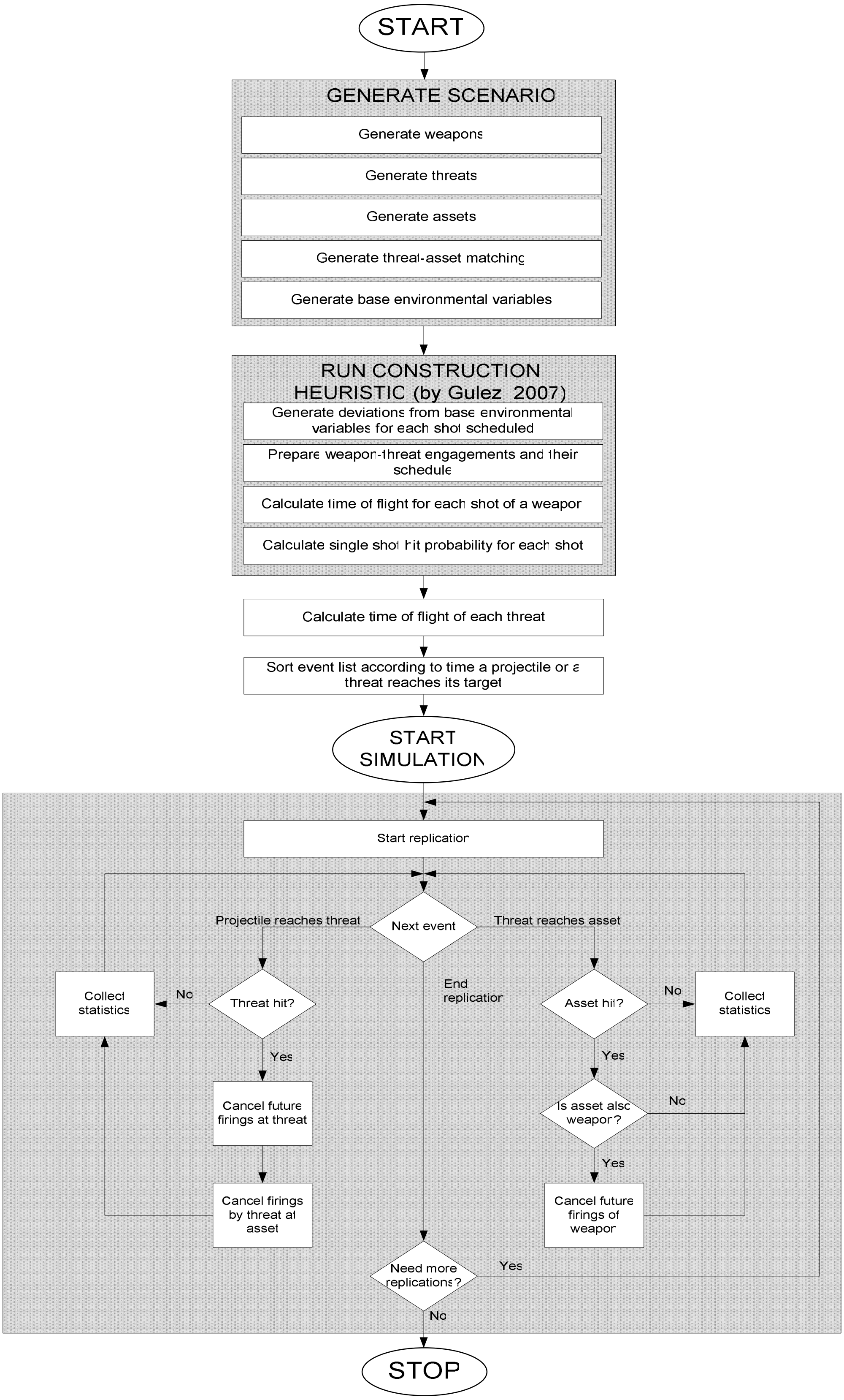

Figure 23. Simulation Flow Chart

CHAPTER 5

EXPERIMENTATION

5.1. Experimental Settings

We conducted simulation experiments on a variety of scenarios. Our experimental factors in a scenario are defense type, scenario size, threat level and ammunition level.

Defense Type (D): Defense type determines whether weapons are also targets or not. We have three levels of defense type:

- Area Defense (DA): Only the assets are the targets of the threats. None of the weapons are targets and they defend assets against threats.

- Point Defense (DP): There are no passive assets. All weapons are also assets and targets of the threats.

- Mixture of point and area defense (DM): There are assets that are targets of the threats, but a weapon can also be target with probability 0.5.

We determine defense type by setting $p_{dt} = 0$ for DA, $p_{dt} = 1$ for DP and $p_{dt} = 0.5$ for DM.

Scenario size (S): Scenario size is related with the number of assets and number of weapons. Scenario size determines the upper and lower limits of the number of weapons and number of assets. There are three levels. These are:

- .

- Small scenario (SS): Number of weapons is between $W_L = 2$ and $W_U = 10$. Number of assets is between $A_L = 1$ and $A_U = 5$

- Medium scenario (SM): Number of weapons is between $W_L = 11$ and $W_U = 20$. Number of assets is between $A_L = 6$ and $A_U = 10$.

- Large scenario (SL): Number of weapons is between $W_L = 21$ and $W_U = 40$. Number of assets is between $A_L = 11$ and $A_U = 20$.

Threat level (T): Threat level determines the relationship between the number of weapons and the number of threats. There are three types of threat level:

- Low Threat Level (TL): The number of threats is lower than the number of weapons. If the number of weapons generated for a particular scenario is w, than the number of threats is generated from DU $(1, w-1)$.

- Equal Threat Level (TE): The number of threats is equal to the number of weapons generated for a particular scenario.

- High Threat Level (TH): The number of threats is higher than the number of weapons. If the number of weapons generated for a particular scenario is w, than the number of threats is generated from w+DU $(1, w)$.

Ammunition Level (A): Ammunition level is related with the ammunition inventory. There are two ammunition levels:

- Unlimited Ammunition (AU): Maximum number of ammunition that a weapon can fire during a scenario, which is denoted as d_j^{max} for weapon j, is equal to the total number of ammunitions that this weapon can fire at each of the threats, given the set up time of the weapon. If each weapon has more than this maximum number of ammunitions, each weapon's ammunition inventory becomes unlimited for a specific scenario. Thus ammunition inventory for weapon j, d_j, is taken as

$$d_j = d_j^{\max}, \quad \text{for all } j$$

where

$$d_j^{\max} = \left\lceil \frac{total\ duration}{set\ up\ time} \right\rceil$$

- Limited Ammunition (AL): For each of the weapons, number of ammunition is limited depending on maximum number of ammunition that a weapon can fire during a scenario such that:

$$d_j \sim DU(1, \frac{d_j^{\max}}{2}), \text{for all } j$$

Scenarios are prepared according to the experimental settings. For each combination of four experimental settings, five independent scenarios are generated. Therefore, we experimented with a total of $3^3 \times 2 \times 5 = 270$ scenarios.

5.2. Experimental Results

Detailed results for 270 scenarios including the 95% confidence intervals for the weighted sum of survival probabilities, ammunition usage, blue success and number of assets hit are given in Appendix D. A sample of simulation output report is provided in Appendix E for a scenario and the simulation output of that scenario is given in Appendix F.

For each scenario construction heuristic and simulation are compared by using the weighted sum of survival probabilities, which will be called the objective function here after. We investigated whether the objective function value of the construction heuristic falls into the 95% confidence interval of the simulation objective function value at the end of the simulation runs for each scenario. Figure 24, Figure 25 and Figure present the comparison for small, medium and large scenarios, respectively. These figures show that the construction objective function tends to fall in the 95% confidence interval of the simulation objective more often as the scenario size increases. In 61 of the 90 small scenarios, the construction objective is in the confidence interval of the simulation objective. The same numbers are 70 and 75 for medium and large scenarios. The main reason for this is the number of threats attacking a particular asset and the number of weapons assigned to a particular threat increases as the scenario size increases. The increase in these numbers decreases the probability of an asset or a threat not to be hit at the end of many simulation runs.

When the direction of deviation of construction heuristic's objective function values from the 95% confidence interval of simulation objective function values is examined Figure 27 is obtained.

When Figure 27 is examined, it is seen that there are scenarios whose construction objective is lower than the lower limit of 95% confidence interval of simulation objective. There are also scenarios whose construction objective is higher than the upper limit of 95% confidence interval of simulation objective. These scenarios are listed in Table 11 and Table 12.

Construction objective value of a scenario may be higher than the upper limit of the 95% confidence interval of simulation objective in the case when an asset is reached with a high probability. This is possible if all weapons have low PSSH values against a threat attacking that asset, so it can not be hit in almost all of the simulation replications and destroys the asset. This case can occur when at least one of the circumstances listed below happens.

- Only one weapon is assigned to the threat (SS, TH, TE)
- Asset that is attacked by the threat is not defended by a weapon that it is close, therefore P_{SSH} against the threat is low (DA, DM)

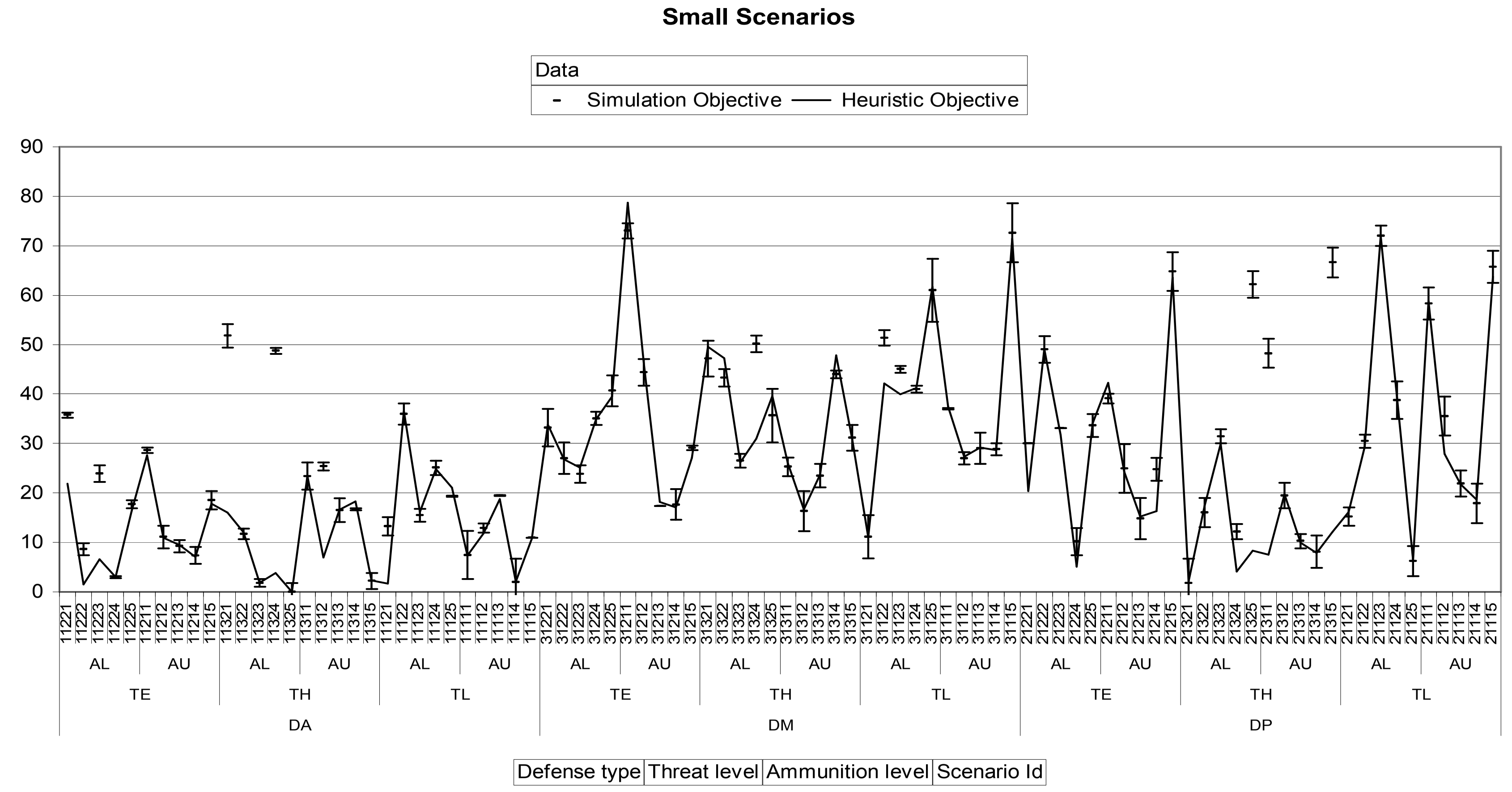

Figure 24. Comparison of Heuristic Objective and Simulation Objective for Small Scenarios

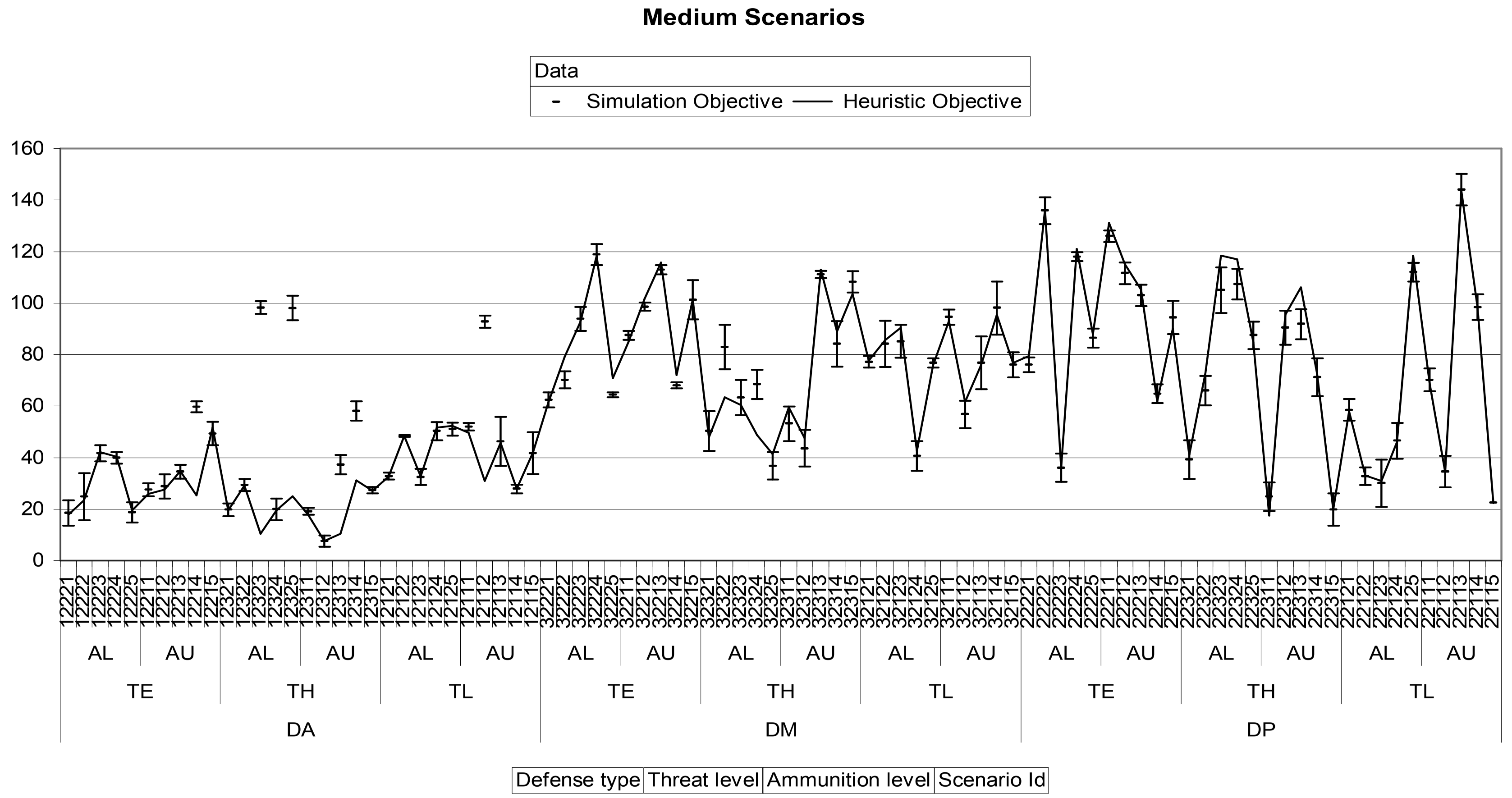

Figure 25. Comparison of Heuristic Objective and Simulation Objective for Medium Scenarios

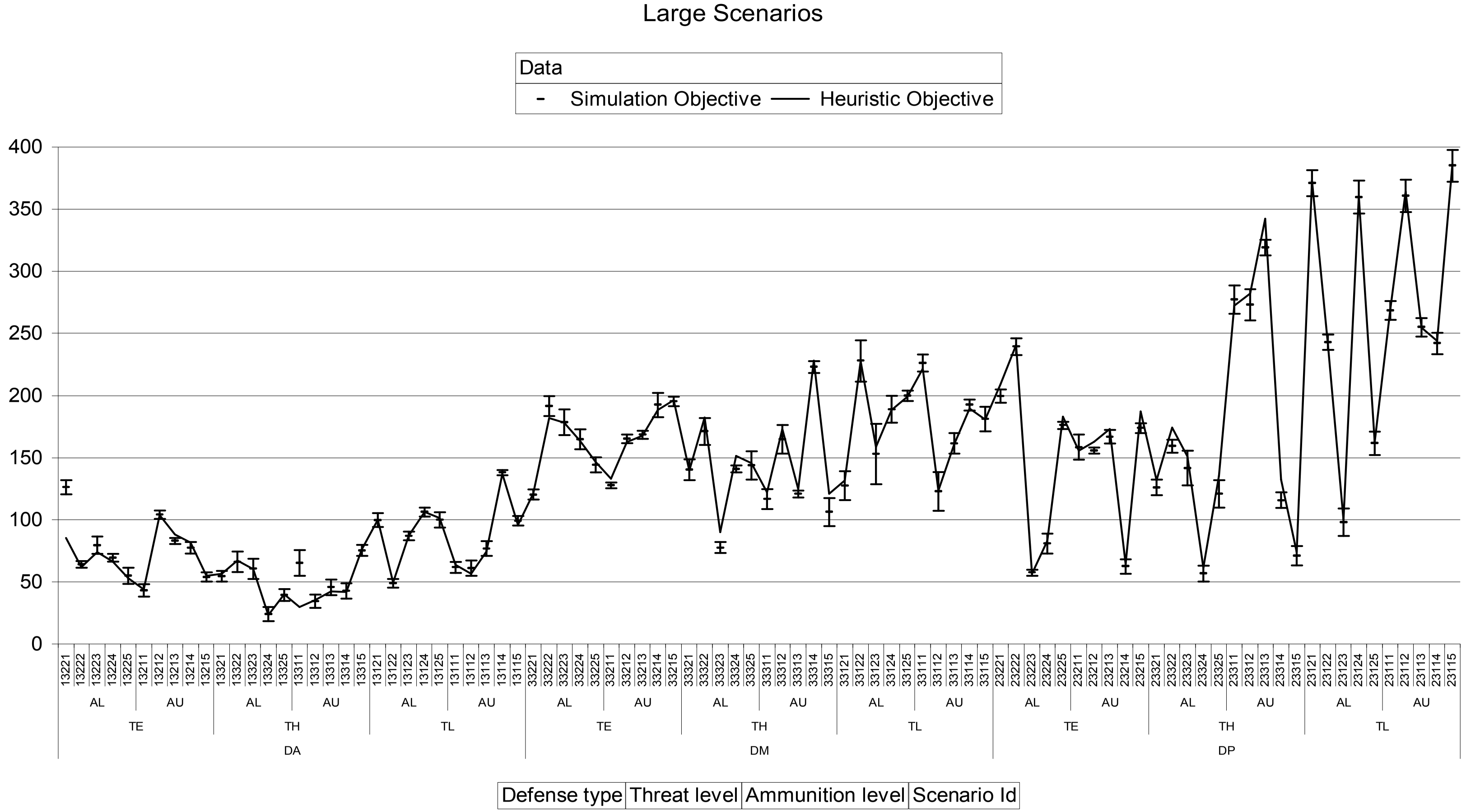

Figure 26. Comparison of Heuristic Objective and Simulation Objective for Large Scenarios

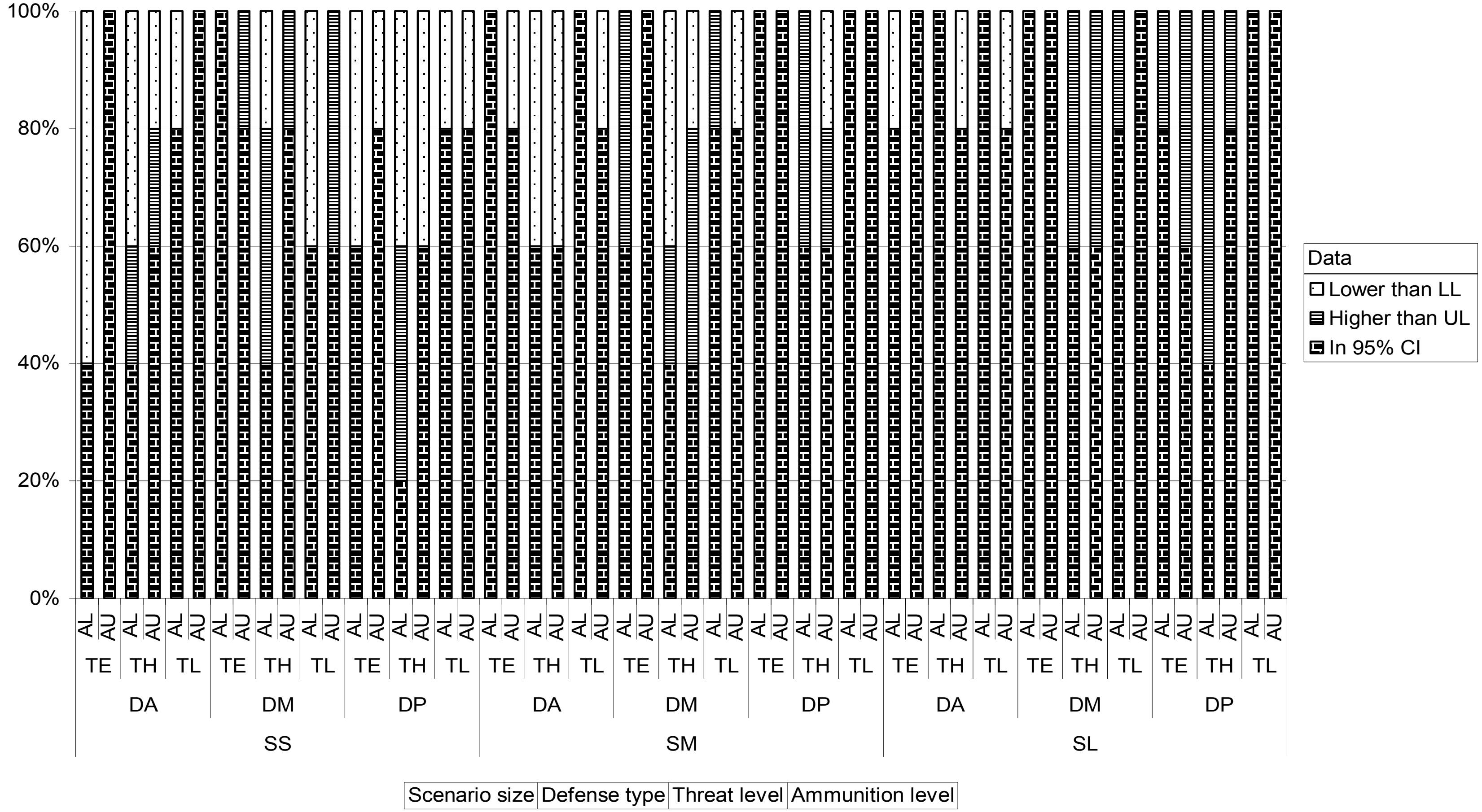

Figure 27. Comparison of construction Objective Falling in 95% C.I. of Simulation Objective

Table 11. Scenarios with Heuristic Objective Higher Than Simulation Objective

Scenario Size	Defense Type	Threat level	Ammunition level	Scenario Id	Heuristic Objective Function	Simulation Objective Function	Lower Limit	Upper Limit
SS	DA	TH	AL	11325	0.00	0.00	0.00	0.00
			AU	11314	18.27	16.67	15.20	18.13
	DM	TE	AU	31211	78.65	73.00	68.07	77.93
		TH	AL	31322	47.27	43.27	39.32	47.23
				31325	39.58	35.65	32.43	38.86
			AU	31314	47.87	44.00	40.17	47.83
		TL	AU	31111	37.44	37.00	37.00	37.00
				31113	29.19	29.00	29.00	29.00
	DP	TH	AL	21321	2.17	1.74	1.60	1.89
				21322	17.40	16.04	14.76	17.32
SM	DM	TE	AL	32222	79.06	70.18	63.51	76.86
				32225	70.75	64.38	58.06	70.69
		TH	AL	32325	41.42	36.72	33.76	39.68
			AU	32311	59.47	53.16	48.87	57.46
				32312	47.38	43.54	40.09	46.98
		TL	AL	32123	90.43	85.10	81.03	89.17
	DP	TH	AL	22322	72.85	66.00	60.65	71.35
				22323	118.54	105.00	94.74	115.26
			AU	22313	106.15	91.80	85.45	98.15
SL	DM	TH	AL	33323	90.00	77.53	70.15	84.92
				33324	151.36	140.80	132.14	149.46
			AU	33312	172.95	164.60	158.50	170.70
				33315	120.79	106.33	96.97	115.69
		TL	AL	33123	158.69	152.80	147.14	158.46
	DP	TE	AL	23221	209.13	199.50	191.58	207.42
			AU	23213	172.98	166.80	162.37	171.23
				23215	187.20	173.70	162.32	185.08
		TH	AL	23323	150.85	141.60	133.48	149.72
				23324	61.35	56.70	52.47	60.93
				23325	135.43	120.80	110.84	130.76
			AU	23314	132.25	115.56	104.79	126.32

Table 12. Scenarios with Heuristic Objective Lower Than Simulation Objective

Scenario Size	Defense Type	Threat level	Ammunition level	Scenario Id	Heuristic Objective Function	Simulation Objective Function	Lower Limit	Upper Limit
SS	DA	TE	AL	11221	21.91	35.75	33.05	38.45
				11222	1.46	8.65	7.89	9.41
				11223	6.59	23.92	21.55	26.29
		TH	AL	11321	16.01	51.78	46.88	56.67
				11324	3.84	48.71	44.00	53.42
			AU	11312	6.91	25.36	23.23	27.50
		TL	AL	11121	1.63	13.24	12.66	13.82
	DM	TH	AL	31324	30.99	50.13	46.15	54.12
		TL	AL	31122	42.18	51.37	46.45	56.28
				31123	39.97	45.00	40.83	49.17
	DP	TE	AL	21221	20.42	30.07	28.18	31.97
				21224	5.14	10.16	9.35	10.96
			AU	21214	16.32	24.74	23.09	26.39
		TH	AL	21324	4.07	12.18	10.99	13.37
				21325	8.34	62.23	56.25	68.21
			AU	21311	7.52	48.24	43.86	52.61
				21315	12.15	66.62	60.26	72.98
		TL	AL	21123	71.99	72.00	72.00	72.00
			AU	21112	27.87	35.53	32.34	38.72
SM	DA	TE	AU	12214	25.32	59.70	55.48	63.92
		TH	AL	12323	10.39	98.24	88.78	107.69
				12325	24.90	98.04	90.00	106.08
			AU	12313	10.37	37.24	34.17	40.32
				12314	31.10	58.06	54.53	61.58
		TL	AU	12112	30.90	92.76	83.60	101.92
	DM	TH	AL	32322	63.38	82.90	76.78	89.03
				32324	48.73	68.45	63.51	73.39
			AU	32315	103.62	108.20	104.56	111.84
		TL	AU	32114	95.42	98.10	96.43	99.77
	DP	TH	AU	22311	17.36	24.83	22.61	27.04
SL	DA	TE	AL	13221	85.45	126.09	115.72	136.45
		TH	AU	13311	29.92	65.32	59.68	70.96
		TL	AU	13112	56.37	61.00	58.45	63.55

Construction objective of a scenario may be lower than the lower limit of the 95% confidence interval of simulation objective in the case when an asset survives with a high probability. This is possible if any threat considered dangerous in the heuristic is almost always hit during the simulation runs and cannot reach the asset that it is attacking. This case can be observed at least one of the following happens.

- More than one weapon may be assigned to the threat (TL)
- A weapon is assigned to the threat more than once (AU)
- The asset is defended by more than one weapon (DA, DM)
- There is a single threat attacking the asset (SS)

It is also essential to find out if there is a correlation between the ammunition usage and blue success (defender's success rate). It is important to observe if the success level can be increased by increasing the ammunition usage. However, the values should be scaled with respect to the scenario size (by number of threats) for observing the correlation. Figure 28 presents the relationship between the ammunition usage and the blue success.

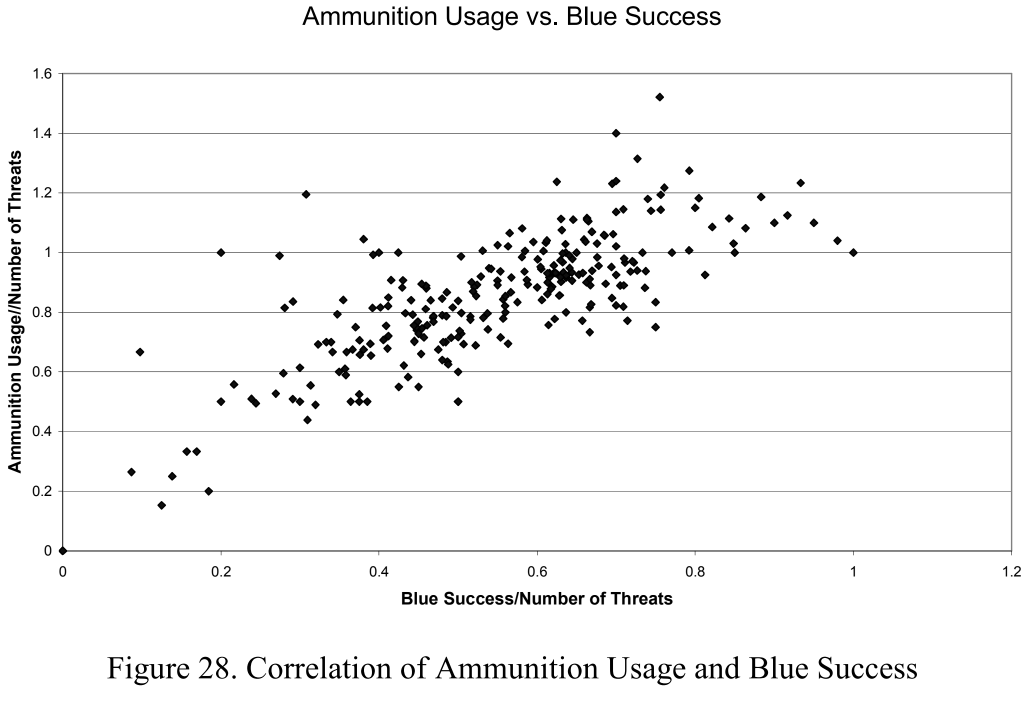

Figure 28. Correlation of Ammunition Usage and Blue Success

As it is seen from Figure 28, the blue success in general increases as the ammunition usage increases. The change of blue success value with the change in ammunition usage based on the experimental settings is presented in Figure 29. According to the figure, the ratio is less sensitive to the change of ammunition level setting when threat level setting is TE (equal threat level). The deviation increases when the threat level increases. The correlation of ammunition usage and blue success based on scenario types is presented in Appendix G.

Another key performance factor is the ratio of survived assets to the ammunition used. The change of this ratio with respect to experimental settings is presented in Figure 30. The pattern of change in the ratio with the experimental settings is similar to the pattern observed in Figure 29.

Change in the ratio of neutralized threats to number of threats with experimental settings is also investigated. However, consistent pattern is not observed. In addition, the ratio of blue success to the number of threats and total ammunition used for scenario types is investigated, but it is seen that the change in ratio with the experimental settings has no observable pattern.

Computation times for simulations are 30.28–2224.23 seconds for small scenarios, the same times are 54.46-7377.31 and 275.32-34150.68 seconds for medium and large scenarios. For a particular scenario size times increase when threat level is high (TH). Major part of the computation is spent on PSHH and TOF calculation as a specific PSSH value is calculated in 19.23 seconds and a specific TOF value is calculated in 4.96 seconds on the average

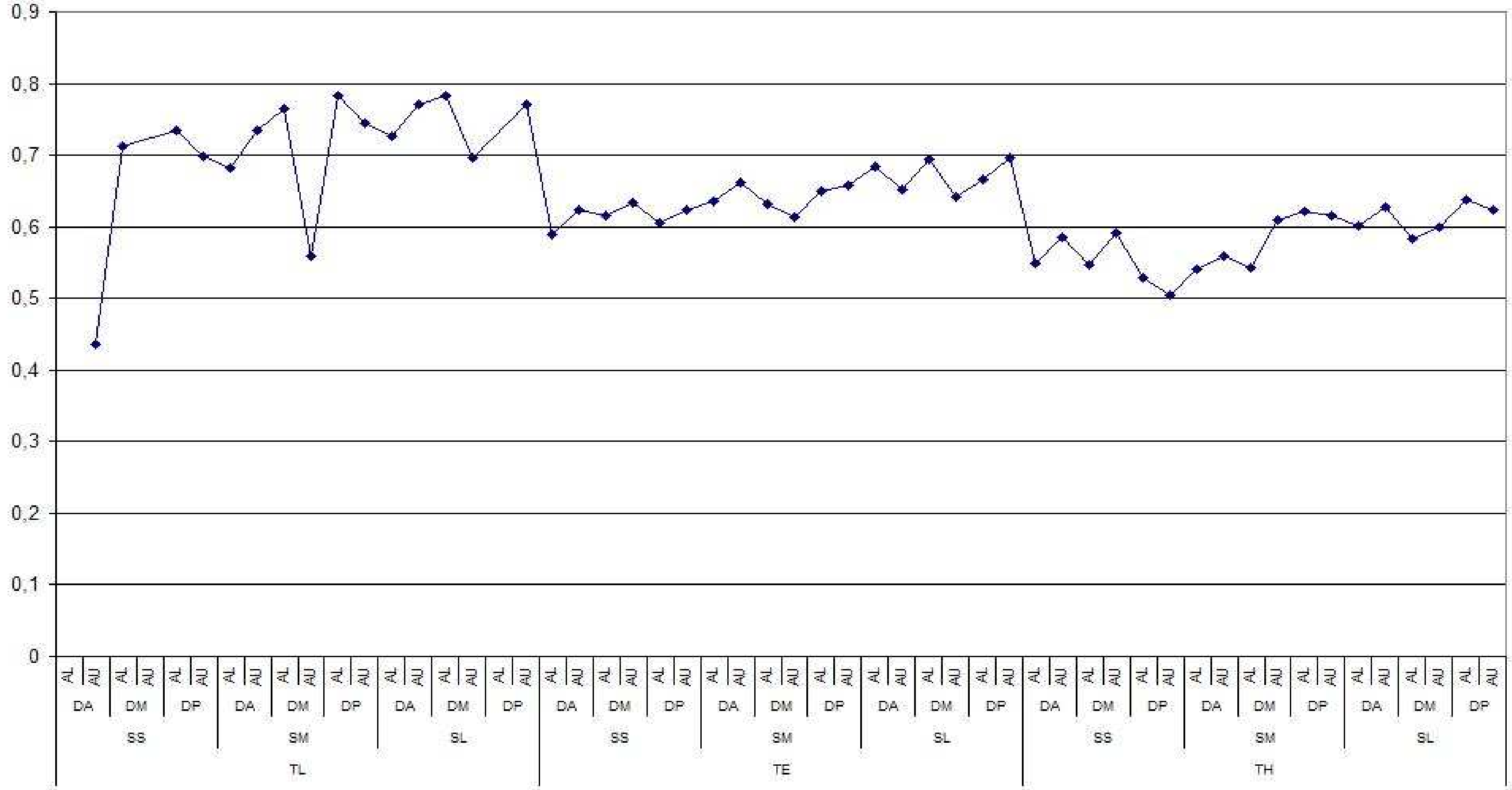

Figure 29. Change of Blue Success/Ammunition Usage Ratio with Experimental Settings

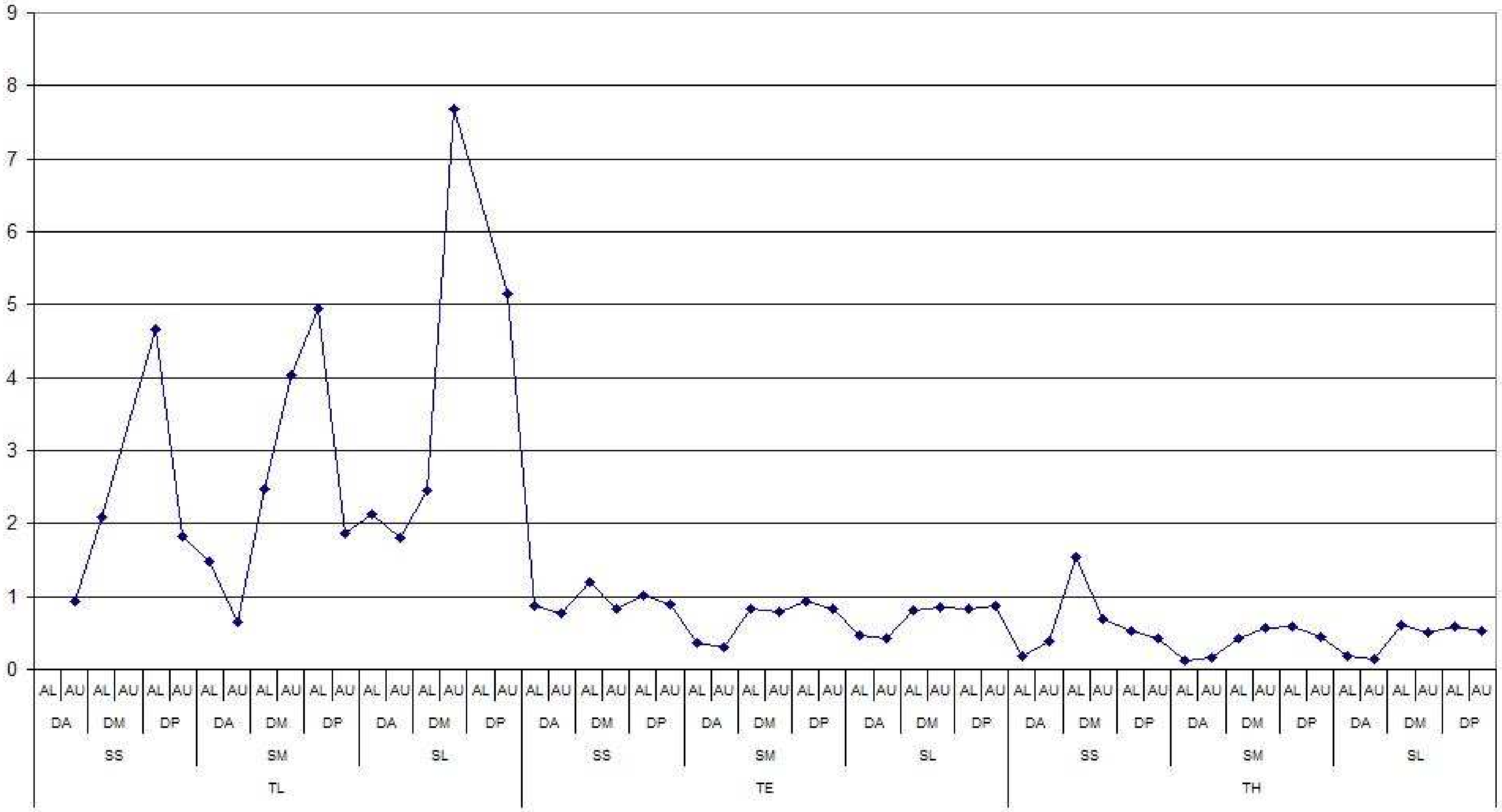

Figure 30. Change of Survived Assets/Ammunition Usage Ratio with Experimental Settings

.CHAPTER 6

CONCLUSION

In this thesis, we proposed a method for calculation of single shot hit probability (P_{SSH}) values of fire control systems by performing an error analysis. In addition, we developed a Monte Carlo simulation model for defense scenarios to evaluate the weapon-threat assignments and the schedule of weapons' shots, which are done by a construction heuristic developed by Gülez (2007).

Our P_{SSH} calculation method involves examination of various error sources contributing to displacement of the actual aim point from the intended point. The amount of displacement is evaluated as the total system error. P_{SSH} value for a weapon is derived from the error distributions of weapon's fire control system in three dimensions. The weapon and the threat are located at two different points in three dimensional space. They may have three dimensional relative velocities. Environmental conditions at the time of firing are also taken into account by the proposed method. Therefore, the proposed method considers P_{SSH} values are fire control system depended as calculated by the error analysis we propose. The error sources as well the associated magnitudes may vary from system to system.

The P_{SSH} values calculated by means of the proposed method indicate that they change considerably as the distance between the weapon and the threat changes. Hence, P_{SSH} values are subject to a drastic change by the time of firing the projectile when either the threat or weapon is not stationary. It is observed that this change in P_{SSH} values also depend on the relative velocity of the threat with respect to the weapon. In addition, P_{SSH} values are affected by the environmental conditions.

The method proposed can be used in the top-down design of a fire control system where the main concern is to satisfy the primal system requirement, the single shot hit probability. The method can be used to evaluate design alternatives as well as to choose among the available subsystems that will be used in the fire control system. Therefore, the method proposed in this work help to reduce the cost of fire control system design.

The application of the proposed method indicated that taking P_{SSH} values as constant throughout the engagement is not correct. The main contribution of the thesis is that P_{SSH} values are time varying for a particular weapon and threat pair having relative motion with respect to each other. It is not realistic to assume constant values in mathematical programming models for engagement and combat simulation.

A Monte Carlo simulation is performed in order to evaluate the outcome of the weapon-target assignments and scheduling of shots by a construction heuristic developed by Gülez (2007). Scenarios are generated with different experimental settings for performing the simulation and evaluating the construction heuristic. Construction heuristic and simulation results are compared. It is observed that construction heuristic’s objective tends to fall in the 95% confidence interval of the simulation objective, especially in large scenarios. It is also observed that ammunition usage and blue success rates are positively correlated. In addition, the variation in the number of threats neutralized per unit of ammunition used with respect to the change in threat level is similar to the variation in the number of assets survived per unit of ammunition used with respect to the change in threat level. For scenarios having the same number of threats and weapons and for scenarios having more threats than weapons, these two ratios are robust to the changes in other experimental settings. However, for scenarios having fewer threats than weapons, there can be large fluctuations in these two ratios when the experimental settings change.

The P_{SSH} computation method proposed together with the engagement simulation tool can be used for several purposes at strategic level. They can be used for ammunition planning in a given a specific tactical environment. The effects of environmental conditions on the outcome of engagements can be evaluated through the proposed method that makes use of the heuristic and simulation. In addition, this model can be used to evaluate various air defense strategies.

A potential future research issue is to integrate the proposed P_{SSH} computation method in simulation models and simulators. Air defense strategies can be evaluated more realistically when this method is integrated with flight simulation models of the threats, so that the response of the fire control system in terms of hit probability against maneuver of the threats can be observed.

The computation of P_{SSH} and TOF values are time consuming for real-time use of the model. In order to use this method together with the construction heuristic (Gülez, 2007) and simulation model as a decision support tool at the operational level, the computation time of P_{SSH} and TOF has to be reduced. This can be done by tabulating the P_{SSH} and TOF values for certain parameters and using a table look up strategy instead of calculating these values each time in the construction heuristic, or simulation. In addition, to maximize the P_{SSH}, a feed back for the best aim point can be provided by fire control system by real-time use of the method for P_{SSH} computation. Thus, P_{SSH} can be optimized during the shot.

In order to calculate the engagement hit probability or kill probability of a fire control system, single shot hit probability values are must. Hence, the proposed method can be extended to calculate kill probability of ammunition whose damage function is known or to calculate engagement probability of a fire control system which fires a known number of projectiles during a given time window. Moreover, the proposed method with the damage function of ammunition can be used for estimating the area that one can defend. Thus, the proposed method can be used in planning sector allocation of weapons.

Our assumption can be relaxed to allow irregular target silhouette (hit zone) shapes other than the rectangle. Also, normality assumption of error distributions may be relaxed to consider other probability distributions.

The current method considers only the artillery type of ammunition. Similar methods need to be developed to estimate P_{SSH} values for guided missiles in order to complete air defense models.

Another potential future work is developing a similar method for ahead ammunitions which are programmed before firingin order to explode at a given point. They have a lethal volume. The target is neutralized if its vulnerable region intersects with the lethal volume of the ammunition, so target model will be three dimensional.

The proposed method with the engagement simulation can be used to identify the major error sources that are more effective in fire control system’s performance in terms of single shot hit probability. After these error sources are identified, improvements can be made in fire control system to reduce the effect of these error sources in order to increase the firing performance of the fire control system.

REFERENCES

Beare, G. C. (1987), "Linear Programming in Air Defense Modeling", *The Journal of the Operational Research Society*, Vol.38, No.10, pp. 889-905.

Beaumont, P. (2004), Multi-Platform Coordination and Resource Management in Command and Control, M.Sc. Thesis, l'Universite Laval.

Cline, R. E. (1961), "A survey and Summary of Mathematical Models as Applied to Weapon System Evaluation", The University of Michigan, Technical Report No: 61-276, Ohio.

Cothier, P. E. (1982), "Assessment of Timeliness in Command and Control", M.Sc. Thesis, Massachusetts Institute of Technology, Cambridge.

David, I., Amil Alalouf (1998), "On Optimal Aiming To Hit a Linear Target", Department of Industrial Engineering and Management, Ben-Gurion University of the Negev, Beer-Sheva.

DiDonato, A. R., M. P. Jarnagin (1961), "Integration of General Bivariate Gaussian Distribution over an Offset Circle", *Mathematics of Computation*, Vol.15, No.76, pp. 375-382.

Duncan, R. L. (1964), "Hit Probabilities for Multiple Weapon Systems", *SIAM Review*, Vol. 6, No. 2, pp. 111-114.

Dyer, D. D. (1974), "Estimation in a Truncated Circular Normal Distribution with Ballistic Applications", *Operations Research*, Vol. 22, No. 1, pp. 197-201.

Edmundson, H. P. (1961), "The Distribution of Radial Error and Its Statistical Application in War Gaming", *Operations Research*, Vol.9, No.1, pp. 8-21.

Ender, T. R. (2006), "A Top-Down, Hierarchical, System-of-Systems Approach to the Design of an Air Defense Weapon", PhD. Thesis, School of Eurospace Engineering, Georgia Institute of Technology.

Fossett, C. A., Dale Harrison; Harry Weintrob; Saul I. Gass (1991), "An Assessment Procedure for Simulation Models: A Case Study", *Operations Research*, Vol. 39, No. 5, pp. 710-723.

Genz, A. (1992), “Numerical Computation of Multivariate Normal Probabilities”, Washington State University, Washington.

Gülez, T. (2007), “Weapon-Target Allocation And Scheduling For Air Defense With Time Varying Hit Probabilities”, M.Sc. Thesis, Middle East Technical University, Ankara.

Gogolak, V. V. (1973) “Quick Evaluation of Computer Models for Air Defense”, Analytic Services Inc., Virginia.

Groves, A. R., Ed S. Smith (1957), “Salvo Hit Probabilities for Offset Circular Targets”, *Operations Research*, Vol. 5, No. 2, pp. 222-228.

Grubbs, F. E. (1964), “Approximate Circular and Noncircular Offset Probabilities of Hitting”, *Operations Research*, Vol. 12, No. 1, pp. 51-62.

Helgert, H. J. (1971), On the Computation of Hit Probability, *Operations Research*, Vol. 19, No.3, pp. 668-684.

Jaiswal, N. K., P. P. Sangal (1972), “Expected Damage Area for Stick and Triangular Pattern Bombing”, *Operations Research*, Vol. 20, No. 2, pp. 344-349.

Klimack, W. K. (2005), “Simple Probability of Hit Corrections of Adjusted Target Exposure”, Operations Research Center of Excellence, Technical Report, New York.

Laurent, Andre G. (1957), “Bombing Problems A Statistical Approach”, *Operations Research*, Vol.5, No.1, pp. 75-89.

Laurent, Andre G. (1962), “Bombing Problems A Statistical Approach II”, *Operations Research*, Vol.10, No.3, pp. 380-387.

Lee, Y. W. (2007), “Neural Solution To The Target Intercept Problems In A Gun Fire Control System”, *Neurocomputing*, Issue: 70, pp. 689-696.

Lilliefors, H. W. (1957), “A Hand Computation Determination of Kill Probability for Weapons Having Spherical Lethal Volume”, *Operations Research*, Vol. 5, No. 3, pp. 416-421.

Macfadzean, R. (1992), “Surface Based Air Defense System Analysis”, Artech House, Boston, London, pp.87-133.

McNolty, F. (1967), “Kill Probability for Multiple Shots”, *Operations Research*, Vol.15, No.5, pp. 165-169.

McNolty, F. (1965), “Kill Probability When Lethal Effect is Variable”, *Operations Research*, Vol.15, No.3, pp. 478-482.

McNolty, F. (1962), “Kill Probability When The Weapon Bias Is Variable”, *Operations Research*, Vol.10, No.5 pp. 693-701.

Metz, D., Raymond Von Wahle (1999), “Sniper Weapon Fire Control Error Budget Analysis”, Army Research Laboratories, Aberdeen.

Taylor, J. L. (1959), “Development and Application of a Terminal-Air-Battle Model”, *Operations Research*, Vol. 7, No. 6. , pp. 783-796.

U.S. Department of Defense Handbook: Fire Control Systems – General, 1996, MIL-HDBK-799 AR.

Vanbrocklin, G. R. Jr., P. G. Murray (1956), “A Polar- Planimeter Method for Determining the Probability of Hitting a Target”, *Operations Research*, Vol.4, No.1, pp. 87-91.

Walsh, J. E.(1955), “Approximate Salvo Kill Probabilities for Small and Medium Sized Targets when Cumulative Damage is Unimportant”, *Journal of the Operations Research Society of America*, Vol. 3, No. 1, pp. 69-76.

Washburn, A. (2002), “Notes on Firing Theory”, Naval Postgraduate School, Monterey, California.

Webb, D. W., Bruce J. Held (2000), “Modeling of Tank Gun Accuracy under Two Different Zeroing Methods”, Rice University, Houston.

Yüksel, İ., Nur Evin Özdemirel, Levent Kandiller (2007), “Atış Kontrol Sistemi Hata Analizi ile Tek Atışta Vuruş İhtimalinin Hesaplanması”, Ulusal Savunma Uygulamaları Modelleme ve Simülayon Konferansı, Ankara.

APPENDIX A

SUMMARY OF LITERATURE REVIEW

The papers that are read throughout the study are given in table Table 13. These papers are studied according to the problem that is presented, the method proposed, the application if available and relevance to our study.

Table 13. Summary of Literature Survey

Author	Title	Problem Definition	Method	Application	Relevance
Beare, 1987	Linear programming in air defense modeling	To determine the most effective mix and deployment of air defense weapons to defend a given set of assets against a range of air threats	An optimizing model, together with a simulation model is used	Optimizing model and simulation called PARADE is compared	Stochastic Monte Carlo simulation is used as a tool for air defense analysis.
Beaumont, 2004	Multi-platform coordination and resource management in command and control	In this thesis, it is addressed that how a multi agent system that focuses specifically on some particular aspects of the C2, in order to reduce the complexity of the domain, is developed. The primary focus is on the battle planning, resource assignment and engagement control processes.		Design of a decision-support for anti-air warfare on Canadian frigates.	Probability of kill is used to find the objective function of the mathematical model however, for each of the weapon type this probability is taken as a constant.
Brocklin and Murray,1956	A polar planimeter method for determining the probability of hitting a target	- Determining probability of falling a plane area when the distribution of hits in the plane of this area is a normal bivariate distribution centered at some fixed point in the plane	A polar planimeter method is developed	Numerical examples are given	The purpose of the study is to determine the hit probability without using any computer easily. It is relevant as it uses the same assumptions, but has no contribution to our study.

Table 13 (Continued)

Author	Title	Problem Definition	Method	Application	Relevance
Cline, 1961	A survey and summary of mathematical and simulation models as applied to weapon system evaluation	To conduct a survey in order to obtain information concerning the utilization of mathematical modeling and simulation as technique for weapon system evaluation.	-	-	The projects are summarized which use mathematical modeling and simulation techniques.
Cothier,1984	Assessment of timelines in command and control	-Determining single shot hit probability for a fire control system which uses a command and control system to forecast target location -Relationship between single shot hit probability and parameters used to forecast target location such as tracking time	Assessing the effectiveness of command control and communications systems.	A methodology for assessing the effectiveness of C^3 systems is extended to include timeliness.	The methodology for evaluating measures of effectiveness is illustrated through application to an idealized weapon system.
DiDonato and Jarnagin, 1961	Integration of the general bivariate Gaussian distribution over an offset circle	How to calculate bivariate Gaussian cumulative probability function over an offset circle			It can be used in calculation of multivariate normal cdf.

Table 13 (Continued)

Author	Title	Problem Definition	Method	Application	Relevance
Duncan,1964	Hit probabilities for multiple weapon systems	Determining the probability of hitting a target with at least one missile from a random circular salvo	Maximizes the probability of hitting the target with at least one round using number of rounds, area of target and standard error. First calculates the probability that the target lies within the area covered by the salvo and calculates the probability that the target is hit if it lies in this area. Assumes standard error which is the error caused by misdetection of the target is given.	Proves that increasing the salvo density towards the center of the also pattern does not lead to a significant increase in the hit probability.	-
Dyer, 1974	Estimation in a truncated circular normal distribution with ballistic applications	Maximum likelihood estimates of the parameters of a circular normal distribution truncated outside a circular region is found in this paper	Gives mathematical formulation of maximum likelihood estimates of the parameters of a circular normal distribution truncated outside a circular region	A numerical example is given to illustrate the method.	This paper gives the bivariate normal cumulative function to calculate the hit probability. It is relevant for our model's verification methods as it proposes a method to find the maximum likelihood estimates of mean and standard deviation parameters (which we found after error analysis) after N rounds are fired to a specific circular target.

Table 13 (Continued)

Author	Title	Problem Definition	Method	Application	Relevance
Ender, 2006	A top-down, hierarchical, system-of-systems approach to the design of an air defense weapon	How to design a weapon system	A design method is proposed after evaluating the alternatives available	Results of analysis are available.	The error sources related with fire control system is given. Their 1 sigma errors are used in the error analysis, the error analysis is paraphrased from Macfadzean's book. Monte Carlo simulation is used in order to fit error distributions and single shot hit probability is calculated via these distributions.
Fossett et. Al. 1991	An assessment procedure for simulation models: A case study	The objective is to develop and test a method for evaluating simulation models and to illustrate how it can provide insights into a simulation's strengths and weaknesses, especially in terms of identifying areas for improvements	Evaluative methodology is proposed	Methodology was systematically applied to three Army simulation models that were used in the acquisition of air defense systems.	The Army simulation models are evaluated, one of them COMOIII is a standard Army model for tactical air defense artillery effectiveness studies which is based on Monte Carlo simulation.
Genz, 1992	Numerical computation of multivariate normal probabilities	To calculate multivariate normal cumulative probability function	MATLAB's mvncdf.m is used to calculate bivariate normal probability cdf : Implements the unnumbered equation between (3) and (4) in Section 2.2 of Genz (2004), integrating in terms of theta between asin(rho) and +/- pi/2, using adaptive quadrature.		Calculation of bivariate normal probabilities

Table 13 (Continued)

Author	Title	Problem Definition	Method	Application	Relevance
Groves and Smith, 1957	Salvo hit probabilities for offset circular targets	Determine hit probability by a single integration for the firing of one salvo around at a circular target of a specified radius, the probability that at least one of the n missiles will hit the target.	The method is numerical other than analytic. The target is partitioned into K regions, and probability that at least one missile hit the target is calculated through the sum of product of probability that at least one missile hits the target and probability that center of impact is in the specified region. The proposed method applies strictly to offset circles.	A sample problem is given and the special cases are told. For irregular targets, the target can be modeled by several circles in order to use this method. For highly irregular targets it is suggested in this paper to use the method proposed by G. R. Van Brocklin Jr. and P. G. Murray's paper "A Polar Planimeter Method for Determining the Probability of Hitting a Target" in 1956.	It is relevant because of its assumptions about the problem. In addition to that first special case given in sample problem is calculation of single shot hit probability of a fire control system. It is presented here that if a single shot is done, the total error is calculated by root sum of squares of burst to burst biases and round to round biases. (Supports our assumption told in classification of errors part.)
Grubbs, 1963	Approximate circular and noncircular offset probabilities of hitting	Probability of hitting a circular target (two dimensional case) or a spherical target (three dimensional case) whether delivery errors are equal or unequal and also for point of aim or center of impact of the rounds either coinciding with the target centroid or offset from it.	The geometry of the problem is given and mathematical formulation that is used to find probability of hitting a circular or spherical target given the geometry that uses this geometry is presented.	The problem of hitting a circular target is examined for two dimensional case and the problem of hitting a spherical target is examined for three dimensional case. A literature review about the computation of probability of hit is given and the examples given in literature are presented as well.	—The part that presents the geometry of the problem is relevant for our problem as this part presents how probability of hit is found from the error distributions geometrically. —Additional Points of Interest part is also important as it is presented how errors from error sources are summed up to have an error distribution parameter. This part is also important as it tells the relationship of the problem with coverage problems and probability of kill calculations.

Table 13 (Continued)

Author	Title	Problem Definition	Method	Application	Relevance
Jaiswal and Sangal, 1972	Expected damage area for stick and triangular pattern bombing	Problem is to determine expected damaged area of a rectangular target in two cases where bombing patterns are different e.g. Stick and triangular pattern assuming bombing and aiming errors are normally distributed and the damage function of each bomb follows a noncircular exponential square fall-off law.	Mathematical formulation is given to evaluate two bombing patterns given the assumptions according to the area damaged which is used as measure of effectiveness	Two bombing patterns are evaluated numerically according to the mathematical formulation proposed.	The assumptions about the aiming errors: they are taken as normally distributed-(assumptions and notation, 3–4–6)
Klimack,2005	Simple probability of hit corrections for adjusted target exposures	When obtaining data for analysis of military ground combat systems, often for use in high resolution simulations, the data may not be available as required. One case is considered here, where probability of hit, hp, is available for a particular exposure of the vehicle as a target, but another exposure is required. But the case is restricted with only target geometry.	A methodology for recalculating probability of hit for a different target from a given probability of hit and given data is proposed. Measure of spread is treated as standard deviation of normal probability distribution. Bivariate normal distribution is assumed with zero mean and equal standard deviations.	The methodology is applied to the BMP-3 and Bradley Fighting Vehicle	Treat Hit Assessment Scheme: More detailed treatment is common, where damage is assessed as none, a mobility kill (M kill), a firepower kill (F kill), a combination MF kill, and a catastrophic kill (K kill). These results to the target are, respectively, no effect, loss mobility, loss of target ability to fire, loss of mobility and firing capability, and loss of the entire target vehicle. Each has an associated probability.

Table 13 (Continued)

Author	Title	Problem Definition	Method	Application	Relevance
Laurent, 1957	Bombing problems-A statistical approach	The problem is obtaining the probability of a hit in a given area from bomber's viewpoint by using error distributions and from opponent's viewpoint using past impact points' distribution.	The proposed method obtains the best estimate of the probability of a hit in a given area, with a known or unknown target, on the basis of the information given by the points of impact during a flight.	Estimation of probability of hit with circular and non circular error distributions is given. In addition to that probability of hit is estimated for areas with different geometries.	Assumptions used are relevant.
Laurent, 1962	Bombing problems-A statistical approach II	The problem is the same as the one in "Bombing Problems-A Statistical Approach " however in this paper the target is moving according to a law depending on a set of variables Z and linear in its coefficients B.			
Lee, 2006	Neural solution to the target intercept problems in a gun fire control system	The online derivation of gunfire control adjustments to minimize the miss distance between a target and the projectiles	A time delay neural network is implemented in order to develop the miss distance correction filter using neural networks	An event sequenced Monte Carlo simulation model is developed with subsections of input, event generation, gun performance evaluation and output.	Gun process can be used in modeling the error distributions

Table 13 (Continued)

Author	Title	Problem Definition	Method	Application	Relevance
Lilliefors, 1957	A hand computation determination of kill probability for weapons having lethal volume	A method for calculating the probability of killing a point target is proposed for these situations: the lethal volume is a sphere centered on the burst point of the weapon the distribution of the weapon's burst points is described by a three dimensional multivariate normal distribution about the target	Problem is evaluated with three dimensional multivariate normal probability pdf assuming deviations in three dimensions are equal to each other in order to keep the problem as simple as to solve by hand.	A numerical example is given to illustrate the method.	The computation of deviations is not given. The computation is simplified to use for hand computations and the target is a point target. It is relevant for multivariate normal probability assumption for calculating probability of hit.
McNolty, 1962	Kill probability when the weapon bias is randomly distributed	Presents derivations of formulas for probability of killing a point target which is distributed about origin of the xy plane according to circular normal density when the bias of the weapon system is randomly distributed in accordance with a prescribed density function	-Derivation of SSKP when weapon bias is not constant -Numerical evaluation of formulas	-Derivation of SSKP in two dimensional case when bias is distributed according to Gamma/ Maxwell's/ Beta distribution. -Derivation of SSKP in three dimensional case when bias is distributed according to Gamma/ Maxwell's/ Rayleigh distribution.	Not relevant since it assumes a point target which is distributed about origin of the xy plane according to circular normal density and lethal circle/volume of the weapon is given with radius R and center point that is offset from the origin a distance given by weapon bias.

Table 13 (Continued)

Author	Title	Problem Definition	Method	Application	Relevance
McNolty, 1965	Kill probability when lethal effect is variable	Presents derivations of expressions for the basic single-shot kill probability when kill is dependent on target prediction and intercept errors, and the conditional probability that a kill will occur given that a point of impact is a specified distance r from the target when this probability is variable. (in kill-no-kill approach this probability is 1 in lethal circle or sphere; 0 outside)	-Derivation of SSKP when conditional kill probability is variable -Combination of kill-no-kill approach and drop off of lethal flux.	-Calculation of two dimensional SSKP with circular normal and elliptical normal cases -Calculation of three dimensional SSKP with spherical normal and ellipsoidal normal cases	Not relevant since it assumes a point target which is distributed about origin of the xy plane according to circular normal density and lethal circle/volume of the weapon is given with radius R and center point that is offset from the origin a distance given by weapon bias.
McNolty, 1967	Kill probability for multiple shots	Probability of 1)killing a target at least one in N tosses of the lethal circle 2)- killing the target n times in N tosses 3)- requiring less than or equal to m shots to kill l the target exactly once 4)killing the target at least once when bias is randomly distributed. 5)assuming target is randomly located according to an offset circular normal distribution and remains in its unknown position throughout N circle		Probability of kill is derived for single shot case, multiple shot case, and multiple shot case with random bias.	Not relevant since it assumes a point target which is distributed about origin of the xy plane according to circular normal density and lethal circle/volume of the weapon is given with radius R and center point that is offset from the origin a distance given by weapon bias.

Table 13 (Continued)

Author	Title	Problem Definition	Method	Application	Relevance
McNolty, 1967	Kill probability for multiple shots	-Probability of killing a target at least one in N tosses of the lethal circle -Probability of killing the target n times in N tosses -Probability of requiring less than or equal to m shots to kill l the target exactly once -Probability of killing the target at least once when bias is randomly distributed. assuming target is randomly located according to an offset circular normal distribution and remains in its unknown position throughout N independent tosses of a lethal circle		Probability of kill is derived for single shot case, multiple shot case, and multiple shot case with random bias.	Not relevant since it assumes a point target which is distributed about origin of the xy plane according to circular normal density and lethal circle/volume of the weapon is given with radius R and center point that is offset from the origin a distance given by weapon bias.
Taylor, 1959	Development and application of a terminal-air-battle model	The development and application of a terminal engagement model of a Monte Carlo type fighter-bomber battle is presented	A Monte Carlo simulation model called NORTAM is developed in order to estimate the outcome of terminal engagement between interceptor and target.	A list of general applications of NORTAM is given.	Engagement analysis is done such that determining whether an engagement is done or not by using maneuver limits of the target, target range and fire control system errors.
Wahlde and Metz, 1999	Sniper weapon fire control error budget analysis	Error budget analysis of sniper weapon	Finding error budget through error analysis and investigate the effects of error sources to the total system error and probability of hit	Error budget of Sniper Weapon (with fire control) is calculated and compared with the error of baseline system (without fire control)	—Classification of errors -Error analysis

Table 13 (Continued)

Author	Title	Problem Definition	Method	Application	Relevance
Walsh, 1955	Approximate salvo kill probabilities for small and medium sized targets when cumulative damage is unimportant	Calculating the kill and hit probabilities of salvo-firing situations where the cumulative damage contribution is unimportant	Presents some approximate probability expressions of an analytical nature	--	This paper is relevant as it presents how to calculate hit probability given some error sources, the general types of errors, target's vulnerable region and target modeling, and the assumptions made.
WashBurn, 2002	Notes on firing theory	—Computations related to firing theory	-	-	Calculation of the probability that a weapon kills a target or a sensor detects a target
Webb and Held, 2000	Modeling of tank gun accuracy under two different zeroing methods	Models for first shot accuracy under two common zeroing processes, comparison of these two	Uses a linear model to estimate azimuth or elevation jump, drawbacks of using this model included.	-Probability of hit is calculated under fleet zero and individual zero method and effect of temperature to probability of hit is examined under two different methods.	-Derivation of single shot hit probability -Determination of bias errors - Terminology used in paper and the methods told are important for understanding errors in a fire control system and determining gun accuracy from these errors.

APPENDIX B

NOTATION USED IN ERROR ANALYSIS

(A mil is a unit of angular measurement equal to 1/6400 of 360 deg. It is used in gunnery applications, and it is convenient because one mil subtends approximately 1 m at 1000 m.)

In notation, σ denotes the root mean square error in measurement of a specific error source and α denotes this error source's quantity of contribution to total system error.

EL: Elevation angle, mil

AZ: Azimuth angle, mil

TR: Target range measurement

BC: Ballistic computation

MT: Manual Tracking

AD: Ammunition dispersion

MD: Muzzle deflection

TOF: Time of Flight (corrected), seconds

ΔV: Total deviation in muzzle velocity, meters per second

ΔV_g: Deviation in muzzle velocity due to nonstandard propellant grain temperature, meters per second

ΔV_b: Loss in muzzle velocity due to tube wear, meters per second

T_g: Propellant grain temperature, ^{o}C

B_W: measured tube wear, meters

V_0: standard muzzle velocity specific to each round type, meters per second

W: measured wind velocity, meters per second

WD: direction of wind, radians

W_X: Crosswind velocity, meters per second

W_Y: Range wind velocity, meters per second

ΔW_Y_R: Deviation in range due to range wind, meters

ΔW_Y_EL: Deviation in elevation angle of target due to range wind, mil

ΔW_Y_TOF: Deviation in TOF due to range wind, second

ΔW_Y_H: Deviation in drift due range wind, mil

R: measured target range, meters

ΔV_R: Deviation in range due to deviation in muzzle velocity, meters

ΔV_EL: Deviation in elevation angle of target due to deviation in muzzle velocity, mil

ΔV_TOF: Deviation in TOF due to deviation in muzzle velocity, seconds

ΔV_H: Deviation in drift due to deviation in muzzle velocity, mil

T: Nonstandard air temperature, ^{o}C

ΔT_R: Deviation in range due to deviation in air temperature, meters

ΔT_EL: Deviation in elevation angle of target due to deviation in air temperature, mil

ΔT_TOF: Deviation in TOF due to deviation in air temperature, seconds

ΔT_H: Deviation in drift due to deviation in air temperature, mil

P: Nonstandard air pressure, mbar

ΔP_R: Deviation in range due to deviation in air density, meters

ΔP_EL: Deviation in elevation angle of target due to deviation in air density, mil

ΔP_TOF: Deviation in TOF due to deviation in air density, seconds

ΔP_H: Deviation in drift due to deviation in air density, mil

R_Cor: Corrected target range, meters

QE_0: Standard superelevation, mil

TOF_S: TOF for standard conditions, seconds

QE: Corrected superelevation, mil

W_X_Cor: crosswind correction, mil

HD_S: horizontal drift compensation for the gyroscopic precision of the spinning projectile for spin stabilized projectiles, mil

HD: total horizontal drift compensation, mil

θ: trunnion cant, radians

E_{Z0}: vertical zeroing angle pertinent to round type, mil

A_{Z0}: horizontal zeroing angle pertinent to round type, mil

L_{AZ}: horizontal offset of target tracking lead, mil

L_{EL}: vertical offset of target tracking lead, mil

E_{HOR}: vertical offset, mil

A_{HOR}: horizontal offset, mil

E_{TR}: transformed elevation offset due to trunnion cant, mil

A_{TR}: transformed azimuth offset due to trunnion cant, mil

V_{AZ}: Target relative rate in azimuth, meters per second

V_{Z}: Target relative rate in the direction of weapon line, meters per second

V_{EL}: Target relative rate in elevation, meters per second

LOS_R_{AZ}: horizontal component of line of sight rate, mil/s

LOS_R_{EL}: vertical component of line of sight rate, mil/s

E_{PR}: parallax angle in elevation, mil

A_{PR}: parallax angle in azimuth, mil

E: total offset in elevation, mil

A: total offset in azimuth, mil

$R_B$or: target range measured at boresight correction

X_{R}: Horizontal component of distance between mirror and gun barrel, meters

Y_{R} : Vertical component of distance between mirror and gun barrel, meters

EL : Fire elevation Angle, radians

AZ : Azimuth Angle, radians

H : Height of Target, meters

α_{WOX} : Weapon Orientation Error in azimuth, radians

α_{WOY} : Weapon Orientation Error in elevation, radians

σ_{WON} : Error of true north direction found by weapon positioning and direction finding system, radians

σ_{WSX} : Weapon Stabilization error in azimuth, radians

σ_{WSY} : Weapon Stabilization error in elevation, radians

σ_{LSX} : Line of stabilization Error in Azimuth, meters

σ_{LSY} : Line of stabilization Error in Elevation, meters

σ_{WPX} : Error in Weapon Location in x Direction, meters

α_{WPX} : Error Caused by Weapon Location Error in x Direction, radians

σ_{WPY} : Error in Weapon Location in y Direction, meters

α_{WPY} : Error Caused by Weapon Location Error in y Direction, radians

σ_{WPZ} : Error in Weapon Location in z Direction, meters

α_{WPZ} : Error Caused by Weapon Location Error in z Direction, radians

α_{WP_EL} : Weapon Location Error in Elevation, radians

α_{WP_AZ} : Weapon Location Error in Azimuth, radians

σ_{TPX} : Error in Target Position in x Direction, meters)

α_{TPX} : Error Caused by Target Position Error in x Direction, radians

σ_{TPY} : Error in Target Position in y Direction, meters

α_{TPY} : Error Caused by Target Position Error in y Direction, radians

σ_{TPZ} : Error in Target Position in z Direction, meters

α_{TPZ} : Error Caused by Target Position Error in z Direction, radians

α_{TP_EL} : Target Position Error in Elevation, radians

α_{TP_AZ} : Target Position Error in Azimuth, radians

σ_{TPR} : Range Effect of Target Position, meters

α_{TRR_AZ} : Prediction angle error caused by measurement error of target's relative rate in azimuth, radians

α_{TRR_EL} : Prediction angle error caused by measurement error of target's relative rate in elevation, radians

σ_{TRR_AZ} : Measurement error of target's relative rate in azimuth, meters per second

σ_{TRR_EL} : Measurement error of target's relative rate in elevation, meters per second

σ_{MT} : Manual tracking error, radians

α_{MT} : Prediction angle error caused by manual tracking error, radians

σ_{TR} : Target range measurement error, meters

α_{TR_A} : Prediction angle error caused by target range measurement error, radians

α_{TR_TOF}: TOF error caused by target range measurement error, seconds

α_{TR_AZ}: Total prediction angle error caused by target range measurement in azimuth, radians

α_{TR_EL}: Total prediction angle error caused by error in target range measurement in elevation, radians

σ_{BC}: Error in ballistic computation, radians

α_{BC_A}: Prediction angle error caused by error in ballistic computation, radians

α_{BC_TOF}: TOF error caused by error in ballistic computation, seconds

α_{BC_AZ}: Total prediction angle error caused error in ballistic computation in azimuth, error

α_{BC_EL}: Total prediction angle error caused error in ballistic computation in azimuth, error

σ_{AD}: Round-to-round ammunition dispersion, radians

α_{AD}: Prediction angle error caused by round-to-round ammunition dispersion error, radians

σ_{θ}: Error in trunnion cant measurement, radians

α_{θ}: Prediction angle error caused by error in trunnion cant measurement, radians

σ_{MD}: Error in Muzzle deflection measurement

α_{MD}: Prediction angle error caused by error in Muzzle deflection measurement, radians

σ_{W}: Error in wind speed measurement, meters per second

α_{W_A}: Prediction angle error caused by error in wind speed measurement, radians

α_{W_TOF}: TOF angle error caused by error in wind speed measurement, seconds

α_{W_AZ}: Total prediction angle error caused by error in wind speed measurement in azimuth, radians

α_{W_EL}: Total prediction angle error caused by error in wind speed measurement in elevation, radians

σ_{WD} : Error in wind direction measurement, degrees

α_{WD_A} : Prediction angle error caused by error in wind direction measurement, radians

α_{WD_TOF} : TOF angle error caused by error in wind direction measurement, seconds

α_{WD_AZ} : Total prediction angle error caused by error in wind direction measurement in azimuth, radians

α_{WD_EL} : Total prediction angle error caused by error in wind direction measurement in elevation, radians

σ_{Tg} : Error in propellant temperature, ^{o}C

α_{Tg_A} : Prediction angle error caused by error in propellant temperature measurement, radians

α_{Tg_TOF} : TOF angle error caused by error in propellant temperature measurement, seconds

α_{Tg_AZ} : Total prediction angle error caused by error in propellant temperature measurement in azimuth, radians

α_{Tg_EL} : Total prediction angle error caused by error in propellant temperature measurement in elevation, radians

σ_{T} : Error in air temperature, ^{o}C

α_{T_A} : Prediction angle error caused by error in air temperature measurement, radians

α_{T_TOF} : TOF angle error caused by error in air temperature measurement, seconds

α_{T_AZ} : Total prediction angle error caused by error in air temperature measurement in azimuth, radians

α_{T_EL} : Total prediction angle error caused by error in air temperature measurement in elevation, radians

σ_{P} : Error in air pressure, mbar

α_{P_A} : Prediction angle error caused by error in air pressure measurement, radians

α_{P_TOF} : TOF angle error caused by error in air pressure measurement, seconds

α_{P_AZ} : Total prediction angle error caused by error in air pressure measurement in azimuth, radians

α_{P_EL} : Total prediction angle error caused by error in air pressure measurement in elevation, radians

σ_{YR} : Error in vertical component of distance between mirror and gun barrel measurement, meters

α_{YR} : Prediction angle error caused by error in vertical component of distance between mirror and gun barrel measurement, radians

σ_{XR} : Error in horizontal component of distance between mirror and gun barrel measurement, meters

α_{XR} : Prediction angle error caused by error in horizontal component of distance between mirror and gun barrel measurement, radians

σ_{RBor} : Error in target range measured at boresight correction measurement, meters

α_{RBor} : Prediction angle error caused by error in target range measured at boresight correction measurement, radians

σ_{EZ0} : Error in zeroing angle measurement in elevation, radians

α_{EZ0} : Prediction angle error caused by error in zeroing angle measurement in elevation, radians

σ_{AZ0} : Error in zeroing angle measurement in azimuth, radians

α_{AZ0} : Prediction angle error caused by error in zeroing angle measurement in azimuth, radians

μ_{AZ} : Prediction angle bias in azimuth caused from time period between ranging and firing, and target’s relative rate in the direction of weapon line, radians

μ_{EL} : Prediction angle bias in elevation caused from time period between ranging and firing, and target’s relative rate in the direction of weapon line, radians

APPENDIX C

BALLISTIC EQUATIONS

Table 14. Ballistic Equations

Projectile Type	Equation	Result is	Unit
Type 1	$\Delta V_g = V_0 * (C_0 + C_1 * T_g + C_2 * T_g{}^2)$	Deviation in muzzle velocity due to grain temperature	m/s
	$\Delta V_b = c * B_W$	Deviation in muzzle velocity due to tube wear	m/s
	$\Delta V = \Delta V_g - \Delta V_b$	Deviation in muzzle velocity	m/s
	$W_X = -W * \sin(WD)$	Crosswind	m/s
	$W_Y = -W * \cos(WD)$	Range Wind	m/s
	$\Delta V\_R = -\Delta V * (C_0 + C_1 * R + C_2 * R^2 + C_3 * R^3)$	Deviation in target range due to change in muzzle velocity	m
	$\Delta T\_R = -100 * ((T - 15)/288.15) * (C_0 + C_1 * R + C_2 * R^2 + C_3 * R^3)$	Deviation in target range due to change in air temperature	m
	$\Delta P\_R = -((P * 100/(285.9 * (273.15 + T))) - 1.2299)/1.2299 * 100 * (C_0 + C_1 * R + C_2 * R^2 + C_3 * R^3)$	Deviation in target range due to change in air density	m
	$\Delta \mathrm{W}_{\mathrm{Y}}\_R = 1.94384/3.6 * \mathrm{W}_{\mathrm{Y}} * (C_0 + C_1 * R + C_2 * R^2 + C_3 * R^3)$	Deviation in target range due to range wind	m
	$R\_Cor = R + \Delta \mathrm{V\_R} + \Delta \mathrm{T\_R} + \Delta \mathrm{P\_R} + \Delta \mathrm{W_X\_R}$	Corrected target range	m
	$TOF = C_0 + C_1 * R\_Cor + C_2 * R\_Cor^2 + C_3 * R\_Cor^3$	Projectile's time of flight	s
	$QE = C_0 + C_1 * R\_Cor + C_2 * R\_Cor^2 + C_3 * R\_Cor^3$	Corrected superelevation	mil
	$W_X\_Cor = W_X/10 * (C_0 + C_1 * R + C_2 * R^2 + C_3 * R^3)$	Crosswind correction	mil

Table 14 (Continued)

Projectile Type	Equation	Result is	Unit
	$HD\_S = C_0 + C_1 * R + C_2 * R^2 + C_3 * R^3$	Horizontal drift compensation for the gyroscopic precision of the spinning projectile for spin stabilized projectiles	mil
	$HD = W_X\_Cor - HD\_S$	Total horizontal drift compensation	mil
	$A_{HOR} = HD - A_{Z0}$	Horizontal Offset	mil
	$E_{VER} = QE - E_{Z0}$	Vertical Offset	mil
	$A_{TR} = A_{HOR} * \cos(\theta) + E_{VER} * \sin(\theta)$	Transformed azimuth offset due to trunnion cant	mil
	$E_{TR} = E_{VER} * \cos(\theta) - A_{HOR} * \sin(\theta)$	Transformed elevation offset due to trunnion cant	mil
	$LOS\_R_{AZ} = V_{AZ} / R\_Cor * Cos(\theta)$	Horizontal component of line of sight rate	mil/s
	$LOS\_R_{EL} = -1 * V_{AZ} / R\_Cor * Sin(\theta)$	Vertical component of line of sight rate	mil/s
	$L_{AZ} = LOS\_R_{AZ} * TOF$	Horizontal offset of target tracking lead	mil
	$L_{EL} = LOS\_R_{EL} * TOF$	Vertical offset of target tracking lead	mil
	$A_{PR} = (X_R / R - X_R / R\_Bor) * 1018.6$	Parallax azimuth angle	mil
	$E_{PR} = (Y_R / R - Y_R / R\_Bor) * 1018.6$	Parallax elevation angle	mil
	$A = A_{TR} + L_{AZ} + A_{PR}$	Total offset in azimuth	mil
	$E = E_{TR} + L_{EL} + E_{PR}$	Total offset in elevation	mil
Type 4	$\Delta V_g = C_0 + C_1 * T_g + C_2 * T_g^2 + C_3 * T_g^3$	Deviation in muzzle velocity due to grain temperature	m/s
	$\Delta V_b = c * B_W$	Deviation in muzzle velocity due to tube wear	m/s
	$\Delta V = \Delta V_g - \Delta V_b$	Deviation in muzzle velocity	m/s
	$W_X = -W * \sin(WD)$	Crosswind	m/s
	$W_Y = -W * \cos(WD)$	Range Wind	m/s
	$\Delta V\_EL = -\Delta V * (C_0 + C_1 * R + C_2 * R^2 + C_3 * R^3)$	Deviation in elevation angle of target due to deviation in muzzle velocity	mil

Table 14 (Continued)

Projectile Type	**Equation**	**Result is**	**Unit**
Type 4	$\Delta V\_EL = -\Delta V * (C_0 + C_1 * R + C_2 * R^2 + C_3 * R^3)$	Deviation in elevation angle of target due to deviation in muzzle velocity	mil
	$\Delta V\_TOF = \Delta V * (C_4 + C_5 * R + C_6 * R^2 + C_7 * R^3)$	Deviation in TOF due to deviation in muzzle velocity	s
	$\Delta T\_EL = -100 * ((T - 15)/288.15) * (C_0 + C_1 * R + C_2 * R^2 + C_3 * R^3)$	Deviation in elevation angle of target due to deviation in air temperature	Mil
	$\Delta T\_TOF = 100 * ((T - 15)/288.15) * (C_4 + C_5 * R + C_6 * R^2 + C_7 * R^3)$	Deviation in TOF due to deviation in air temperature	s
	$\Delta P\_EL = -((P * 100/(285.9 * (273.15 + T))) - 1.2299)/1.2299 * 100 * (C_0 + C_1 * R + C_2 * R^2 + C_3 * R^3)$	Deviation in elevation angle of target due to deviation in air density	Mil
	$\Delta P\_TOF = ((P * 100/(285.9 * (273.15 + T))) - 1.2299)/1.2299 * 100 * (C_4 + C_5 * R + C_6 * R^2 + C_7 * R^3)$	Deviation in TOF due to deviation in air density	s
	$\Delta \mathrm{W_Y}\_EL = \mathrm{W_Y}/3.6 * (C_0 + C_1 * R + C_2 * R^2 + C_3 * R^3)$	Deviation in elevation angle of target due to range wind	mil
	$\Delta \mathrm{W_Y}\_TOF = -\mathrm{W_Y}/3.6 * (C_4 + C_5 * R + C_6 * R^2 + C_7 * R^3)$	Deviation in TOF due to range wind	s
	$QE_0 = C_0 + C_1 * R + C_2 * R^2 + C_3 * R^3$	Standard superelevation	mil
	$TOF_S = C_4 + C_5 * R + C_6 * R^2 + C_7 * R^3$	TOF for standard conditions	S
	$QE = QE_0 + \Delta V\_EL + \Delta T\_EL + \Delta P\_EL + \Delta W_X\_EL$	Corrected superelevation	mil
	$TOF = TOF_S + \Delta V\_TOF + \Delta T\_TOF + \Delta P\_TOF + \Delta W_X\_TOF$	Time of Flight (corrected),	s
	$W_X\_Cor = W_X/10 * (C_0 + C_1 * R + C_2 * R^2 + C_3 * R^3)$	Crosswind correction	mil
	$HD\_S = 0$	Horizontal drift compensation for the gyroscopic precision of the spinning projectile for spin stabilized projectiles	mil
	$HD = W_X\_Cor - HD\_S$	Total horizontal drift compensation	mil

Table 14 (Continued)

Projectile Type	Equation	Result is	Unit
Type 4	$A_{HOR} = HD - A_{Z0}$	Horizontal Offset	mil
	$E_{VER} = QE - E_{Z0}$	Vertical Offset	mil
	$A_{TR} = A_{HOR} * \cos(\theta) + E_{VER} * \sin(\theta)$	Transformed azimuth offset due to trunnion cant	mil
	$E_{TR} = E_{VER} * \cos(\theta) - A_{HOR} * \sin(\theta)$	Transformed elevation offset due to trunnion cant	mil
	$LOS\_R_{AZ} = V_{AZ} / R\_Cor * Cos(\theta)$	Horizontal component of line of sight rate	mil/s
	$LOS\_R_{EL} = -1 * V_{AZ} / R\_Cor * Sin(\theta)$	Vertical component of line of sight rate	mil/s
	$L_{AZ} = LOS\_R_{AZ} * TOF$	Horizontal offset of target tracking lead	mil
	$L_{EL} = LOS\_R_{EL} * TOF$	Vertical offset of target tracking lead	mil
	$A_{PR} = (X_R / R - X_R / R\_Bor) * 1018.6$	Parallax azimuth angle	mil
	$E_{PR} = (Y_R / R - Y_R / R\_Bor) * 1018.6$	Parallax elevation angle	mil
	$A = A_{TR} + L_{AZ} + A_{PR}$	Total offset in azimuth	mil
	$E = E_{TR} + L_{EL} + E_{PR}$	Total offset in elevation	mil
Type 5	$\Delta V_g = C_0 + C_1 * T_g + C_2 * T_g{}^2 + C_3 * T_g{}^3$	Deviation in muzzle velocity due to grain temperature	m/s
	$\Delta V_b = c * B_W$	Deviation in muzzle velocity due to tube wear	m/s
	$\Delta V = \Delta V_g - \Delta V_b$	Deviation in muzzle velocity	m/s
	$W_X = -W * \sin(WD)$	Crosswind	m/s
	$W_Y = -W * \cos(WD)$	Range Wind	m/s
	$\Delta V\_EL = -\Delta V * (C_0 + C_1 * R + C_2 * R^2 + C_3 * R^3)$	Deviation in elevation angle of target due to deviation in muzzle velocity	mil
	$\Delta V\_TOF = \Delta V * (C_4 + C_5 * R + C_6 * R^2 + C_7 * R^3)$	Deviation in TOF due to deviation in muzzle velocity	s

Table 14 (Continued)

Projectile Type	Equation	Result is	Unit
Type 5	$\Delta T\_EL = -100*((T-15)/288.15)*(C_0+C_1*R+C_2*R^2+C_3*R^3)$	Deviation in elevation angle of target due to deviation in air temperature	Mil
	$\Delta T\_TOF = 100*((T-15)/288.15)*(C_4+C_5*R+C_6*R^2+C_7*R^3)$	Deviation in TOF due to deviation in air temperature	s
	$\Delta P\_EL = -((P*100/(285.9*(273.15+T)))-1.2299)/1.2299*100*(C_0+C_1*R+C_2*R^2+C_3*R^3)$	Deviation in elevation angle of target due to deviation in air density	Mil
	$\Delta P\_TOF = ((P*100/(285.9*(273.15+T)))-1.2299)/1.2299*100*(C_4+C_5*R+C_6*R^2+C_7*R^3)$	Deviation in TOF due to deviation in air density	s
	$\Delta \mathrm{W}_Y\_EL = \mathrm{W}_Y/3.6*(C_0+C_1*R+C_2*R^2+C_3*R^3)$	Deviation in elevation angle of target due to range wind	mil
	$\Delta \mathrm{W}_Y\_TOF = -\mathrm{W}_Y/3.6*(C_4+C_5*R+C_6*R^2+C_7*R^3)$	Deviation in TOF due to range wind	s
	$QE_0 = C_0+C_1*R+C_2*R^2+C_3*R^3$	Standard superelevation	mil
	$TOF_S = C_4+C_5*R+C_6*R^2+C_7*R^3$	TOF for standard conditions	S
	$QE = QE_0+\Delta V\_EL+\Delta T\_EL+\Delta P\_EL+\Delta W_X\_EL$	Corrected superelevation	mil
	$TOF = TOF_S+\Delta V\_TOF+\Delta T\_TOF+\Delta P\_TOF+\Delta W_X\_TOF$	Time of Flight (corrected),	s
	$W_X\_Cor = W_X/10*(C_0+C_1*R+C_2*R^2+C_3*R^3)$	Crosswind correction	mil
	$HD\_S = 0$	Horizontal drift compensation for the gyroscopic precision of the spinning projectile for spin stabilized projectiles	mil
	$HD = W_X\_Cor - HD\_S$	Total horizontal drift compensation	mil
	$A_{HOR} = HD - A_{Z0}$	Horizontal Offset	mil
	$E_{VER} = QE - E_{Z0}$	Vertical Offset	mil
	$A_{TR} = A_{HOR}*\cos(\theta)+E_{VER}*\sin(\theta)$	Transformed azimuth offset due to trunnion cant	mil
	$E_{TR} = E_{VER}*\cos(\theta)-A_{HOR}*\sin(\theta)$	Transformed elevation offset due to trunnion cant	mil

Table 14 (Continued)

Projectile Type	Equation	Result is	Unit
Type 5	$LOS\_R_{AZ} = V_{AZ} / R\_Cor * Cos(\theta)$	horizontal component of line of sight rate	mil/s
	$LOS\_R_{EL} = -1 * V_{AZ} / R\_Cor * Sin(\theta)$	vertical component of line of sight rate	mil/s
	$L_{AZ} = LOS\_R_{AZ} * TOF$	horizontal offset of target tracking lead	mil
	$L_{EL} = LOS\_R_{EL} * TOF$	vertical offset of target tracking lead	mil
	$A_{PR} = (X_R / R - X_R / R\_Bor) * 1018.6$	Parallax azimuth angle	mil
	$E_{PR} = (Y_R / R - Y_R / R\_Bor) * 1018.6$	Parallax elevation angle	mil
	$A = A_{TR} + L_{AZ} + A_{PR}$	Total offset in azimuth	mil
	$E = E_{TR} + L_{EL} + E_{PR}$	Total offset in elevation	mil
Type 9	$\Delta V_g = C_0 + C_1 * T_g + C_2 * T_g{}^2$	Deviation in muzzle velocity due to grain temperature	m/s
	$\Delta V_b = c * B_W$	Deviation in muzzle velocity due to tube wear	m/s
	$\Delta V = \Delta V_g - \Delta V_b$	Deviation in muzzle velocity	m/s
	$W_X = -W * \sin(WD)$	Crosswind	m/s
	$W_Y = -W * \cos(WD)$	Range Wind	m/s
	$\Delta V\_EL = -\Delta V * (C_0 + C_1 * R + C_2 * R^2 + C_3 * R^3 + C_4 * R^4)$	Deviation in elevation angle of target due to deviation in muzzle velocity	mil
	$\Delta V\_TOF = \Delta V * (C_5 + C_6 * R + C_7 * R^2 + C_8 * R^3)$	Deviation in TOF due to deviation in muzzle velocity	s
	$\Delta V\_H = \Delta V * (C_9 + C_{10} * R + C_{11} * R^2 + C_{12} * R^3 + C_{13} * R^4)$	Deviation in drift due to deviation in muzzle velocity	mil
	$\Delta T\_EL = -100 * ((T - 15) / 288.15) * (C_0 + C_1 * R + C_2 * R^2 + C_3 * R^3 + C_4 * R^4)$	Deviation in elevation angle of target due to deviation in air temperature	mil

Table 14 (Continued)

Projectile Type	Equation	Result is	Unit
Type 9	$\Delta T\_TOF = 100*((T-15)/288.15)*(C_5+C_6*R+C_7*R^2+C_8*R^3)$	Deviation in TOF due to deviation in air temperature	s
	$\Delta T\_H = 100*((T-15)/288.15)*(C_9+C_{10}*R+C_{11}*R^2+C_{12}*R^3+C_{13}*R^4+C_{14}*R^5)$	Deviation in drift due to deviation in air temperature	mil
	$\Delta P\_EL = -((P*100/(285.9*(273.15+T)))-1.2299)/1.2299*100*(C_0+C_1*R+C_2*R^2+C_3*R^3+C_4*R^4+C_5*R^5)$	Deviation in elevation angle of target due to deviation in air density	mil
	$\Delta P\_TOF = ((P*100/(285.9*(273.15+T)))-1.2299)/1.2299*100*(C_6+C_7*R+C_8*R^2+C_9*R^3)$	Deviation in TOF due to deviation in air density	s
	$\Delta P\_H = ((P*100/(285.9*(273.15+T)))-1.2299)/1.2299*100*(C_{10}+C_{11}*R+C_{12}*R^2+C_{13}*R^3+C_{14}*R^4)$	Deviation in drift due to deviation in air density	mil
	$\Delta \mathrm{W}_Y\_EL = \mathrm{W}_Y/3.6*(C_0+C_1*R+C_2*R^2+C_3*R^3+C_4*R^4)$	Deviation in elevation angle of target due to range wind	mil
	$\Delta \mathrm{W}_Y\_TOF = -\mathrm{W}_Y/3.6*(C_5+C_6*R+C_7*R^2+C_8*R^3)$	Deviation in TOF due to range wind	s
	$\Delta \mathrm{W}_Y\_H = -\mathrm{W}_Y/3.6*(C_9+C_{10}*R+C_{11}*R^2+C_{12}*R^3+C_{13}*R^4)$	Deviation in drift due range wind	mil
	$QE_0 = C_0+C_1*R+C_2*R^2+C_3*R^3+C_4*R$	Standard superelevation	mil
	$TOF_S = C_5+C_6*R+C_7*R^2+C_8*R^3$	TOF for standard conditions	s
	$QE = QE_0+\Delta V\_EL+\Delta T\_EL+\Delta P\_EL+\Delta W_X\_EL$	Corrected superelevation	mil
	$TOF = TOF_S+\Delta V\_TOF+\Delta T\_TOF+\Delta P\_TOF+\Delta W_X\_TOF$	Time of Flight (corrected),	s
	$W_X\_Cor = W_X/10*(C_0+C_1*R+C_2*R^2+C_3*R^3+C_4*$ $+C_5*R^5)$	Crosswind correction	mil
	$HD\_S = C_0+C_1*R+C_2*R^2+C_3*R^3+C_4*R^4+C_5*R^5$	horizontal drift compensation for the gyroscopic precision of the spinning projectile for spin stabilized projectiles	mil
	$HD = W_X\_Cor-HD\_S-\Delta V\_H-\Delta T\_H-\Delta P\_H-\Delta W_Y\_H$	Total horizontal drift compensation	mil
	$A_{HOR} = HD - A_{Z0}$	Horizontal Offset	mil

Table 14 (Continued)

Projectile Type	Equation	Result is	Unit
Type 9	$E_{VER} = QE - E_{Z0}$	Vertical Offset	mil
	$A_{TR} = A_{HOR} * \cos(\theta) + E_{VER} * \sin(\theta)$	Transformed azimuth offset due to trunnion cant	mil
	$E_{TR} = E_{VER} * \cos(\theta) - A_{HOR} * \sin(\theta)$	transformed elevation offset due to trunnion cant	mil
	$LOS\_R_{AZ} = V_{AZ} / R\_Cor * Cos(\theta)$	horizontal component of line of sight rate	mil/s
	$LOS\_R_{EL} = -1 * V_{AZ} / R\_Cor * Sin(\theta)$	vertical component of line of sight rate	mil/s
	$L_{AZ} = LOS\_R_{AZ} * TOF$	horizontal offset of target tracking lead	mil
	$L_{EL} = LOS\_R_{EL} * TOF$	vertical offset of target tracking lead	mil
	$A_{PR} = (X_{R} / R - X_{R} / R\_Bor) * 1018.6$	Parallax azimuth angle	mil
	$E_{PR} = (Y_{R} / R - Y_{R} / R\_Bor) * 1018.6$	Parallax elevation angle	mil
	$A = A_{TR} + L_{AZ} + A_{PR}$	Total offset in azimuth	mil
	$E = E_{TR} + L_{EL} + E_{PR}$	Total offset in elevation	mil

APPENDIX D

SIMULATION RESULTS-SUMMARY REPORT

A summary report is generated at the end of simulation runs. For each scenario, reported statistics and data in summary report are listed below.

- Scenario information
    - Scenario id number
    - Defense type
    - Scenario size
    - Threat level
    - Ammunition level
    - Number of weapons
    - Number of assets
    - Number of threats
    - Number of targets
- Objective function value
    - Construction heuristic objective
    - Simulation objective
    - Standard deviation of simulation objective
    - 95 % confidence level half length of simulation objective
    - Lower limit of 95% confidence interval of simulation objective
    - Upper limit of 95% confidence interval of simulation objective
- Ammunition usage statistics
    - Average number of ammunition fired
    - Standard deviation of number of ammunition fired
    - 95 % confidence level half length of number of ammunition fired

    - Lower limit of 95% confidence interval of number of ammunition fired
    - Upper limit of 95% confidence interval of number of ammunition fired
- Blue success statistics
    - Average number of threats neutralized
    - Standard deviation of number of threats neutralized
    - 95 % confidence level half length of number of threats neutralized
    - Lower limit of 95% confidence interval of number of threats neutralized
    - Upper limit of 95% confidence interval of number of threats neutralized
- Hit asset statistics
    - Average number of threats neutralized
    - Standard deviation of number of assets hit
    - 95 % confidence level half length of number of assets hit
    - Lower limit of 95% confidence interval of number of assets hit
    - Upper limit of 95% confidence interval of number of assets hit

270 scenarios are generated for experimentation and their simulation summary report is presented in Table 15.

Table 15. Summary Results of Simulation Runs

Scenario									Objective Function						Ammo usage					Blue success					Hit asset				
Scenario Id	**Scenario Size**	**Scenario Size**	**Threat Level**	**Ammo Level**	**Number of Weapons**	**Number of Assets**	**Number of Threats**	**Number of Targets**	**Heuristic Objective**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**
11111	DA	SS	TL	AU	4	3	4	3	7.3	7.4	5.3	0.5	6.9	8	4.8	0.4	0	4.7	4.8	1.2	0.7	0.1	1.2	1.3	1.9	0.8	0.1	1.8	2
11112	DA	SS	TL	AU	5	2	1	2	11.8	12.8	2.9	1.2	11.6	14.1	1	0	0	1	1	0.4	0.5	0.2	0.2	0.6	0.4	0.5	0.2	0.2	0.6
11113	DA	SS	TL	AU	6	5	6	5	18.7	19.5	7.7	1.7	17.8	21.1	4	0	0	4	4	2	0.9	0.2	1.9	2.2	1.6	0.9	0.2	1.4	1.8
11114	DA	SS	TL	AU	4	2	4	2	2.1	2	3.6	0.2	1.8	2.2	4.2	0.6	0	4.1	4.2	1.5	0.6	0	1.5	1.6	1.5	0.6	0	1.5	1.5
11115	DA	SS	TL	AU	4	2	1	2	10.7	11	2.5	0.8	10.1	11.8	1	0	0	1	1	0.7	0.5	0.2	0.5	0.8	0.2	0.4	0.1	0	0.3
11121	DA	SS	TL	AL	10	2	9	2	1.6	13.2	12.1	0.6	12.7	13.8	6.6	6	0.3	6.3	6.8	4.5	4.2	0.2	4.3	4.7	1.8	1.7	0.1	1.7	1.9
11122	DA	SS	TL	AL	9	5	3	5	36.5	36	3.2	2.3	33.7	38.3	3.7	0.8	0.6	3.1	4.3	2.8	0.4	0.3	2.5	3	0.1	0.3	0.2	0	0.3
11123	DA	SS	TL	AL	7	2	1	2	16.1	15.5	4.5	1.3	14.2	16.8	0	0	0	0	0	0	0	0	0	0	0.5	0.5	0.1	0.4	0.6
11124	DA	SS	TL	AL	6	5	4	5	24.8	25.1	4	1.7	23.4	26.8	4.9	0.7	0.3	4.6	5.2	3	0.8	0.4	2.7	3.4	0.6	0.6	0.3	0.3	0.8
11125	DA	SS	TL	AL	3	4	3	4	21.1	19.3	3.7	1.9	17.4	21.2	1	0	0	1	1	0.5	0.5	0.3	0.2	0.7	1.9	0.3	0.2	1.7	2.1
11211	DA	SS	TE	AU	8	5	8	5	27.6	28.7	5.4	2.3	26.3	31	8.5	0.8	0.3	8.2	8.9	4.5	1.1	0.5	4.1	5	1.6	0.7	0.3	1.3	1.9
11212	DA	SS	TE	AU	5	3	5	3	10.9	11.1	3.8	1.1	10	12.2	1	0	0	1	1	0.9	0.3	0.1	0.8	1	1.5	0.6	0.2	1.3	1.7
11213	DA	SS	TE	AU	4	3	4	3	9.4	9.2	3.9	0.8	8.5	10	2.8	0.7	0.1	2.7	3	1.6	0.5	0.1	1.5	1.7	0.9	0.8	0.2	0.7	1
11214	DA	SS	TE	AU	3	3	3	3	7	7.3	2.9	0.6	6.8	7.9	2.4	0.5	0.1	2.3	2.5	0.8	0.6	0.1	0.7	1	1.2	0.7	0.1	1.1	1.3
11215	DA	SS	TE	AU	7	4	7	4	17.8	18.5	6.2	1.8	16.8	20.3	4.1	0.3	0.1	4	4.2	3.1	0.7	0.2	2.9	3.3	1.9	0.7	0.2	1.7	2.1
11221	DA	SS	TE	AL	3	5	3	5	21.9	35.7	15.3	2.7	33	38.4	1	0.6	0.1	0.9	1.1	0.5	0.6	0.1	0.4	0.6	1.9	1.4	0.3	1.7	2.2
11222	DA	SS	TE	AL	9	2	9	2	1.5	8.7	8.6	0.8	7.9	9.4	8.8	10.7	0.9	7.9	9.7	5.4	6.6	0.6	4.8	6	1.3	1.7	0.1	1.2	1.5

Table 15 (Continued)

Scenario									Objective Function						Ammo usage					Blue success					Hit asset				
Scenario Id	**Scenario Size**	**Scenario Size**	**Threat Level**	**Ammo Level**	**Number of Weapons**	**Number of Assets**	**Number of Threats**	**Number of Targets**	**Heuristic Objective**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**
11223	DA	SS	TE	AL	10	4	10	4	6.6	23.9	17.8	2.4	21.5	26.3	7.9	6.9	0.9	7	8.8	4.4	4	0.5	3.9	5	2.4	2.2	0.3	2.1	2.7
11224	DA	SS	TE	AL	9	3	9	3	3	2.9	4.5	0.2	2.7	3.1	6.1	1.2	0	6	6.1	3.4	0.9	0	3.4	3.5	2.6	0.6	0	2.6	2.6
11225	DA	SS	TE	AL	4	4	4	4	16.3	17.7	3.7	1.6	16.1	19.3	3.6	1.2	0.5	3.1	4.1	2.5	1	0.4	2.1	3	0.7	0.7	0.3	0.4	0.9
11311	DA	SS	TH	AU	4	5	5	5	23.4	23.4	2.6	1.9	21.5	25.3	4.4	0.7	0.5	3.9	4.9	2.3	0.7	0.5	1.8	2.8	1.3	0.5	0.3	1	1.6
11312	DA	SS	TH	AU	9	3	12	3	6.9	25.4	19.6	2.1	23.2	27.5	7.6	6.7	0.7	6.9	8.3	5.8	5.2	0.6	5.3	6.4	2.1	2	0.2	1.9	2.3
11313	DA	SS	TH	AU	5	4	6	4	16.6	16.5	2.5	1.3	15.2	17.8	4.4	0.6	0.3	4.1	4.8	2.7	0.7	0.4	2.3	3.1	1.5	0.5	0.3	1.2	1.8
11314	DA	SS	TH	AU	5	5	10	5	18.3	16.7	2.3	1.5	15.2	18.1	5.6	0.5	0.3	5.3	5.9	2.2	0.6	0.4	1.8	2.5	2.9	0.3	0.2	2.7	3.1
11315	DA	SS	TH	AU	9	2	12	2	2.3	2.2	3.9	0.1	2	2.3	11.3	0.9	0	11.3	11.4	7.3	1.2	0	7.2	7.3	1.7	0.4	0	1.7	1.8
11321	DA	SS	TH	AL	6	5	11	5	16	51.8	41.3	4.9	46.9	56.7	7.7	8.1	1	6.8	8.7	4.9	5.2	0.6	4.3	5.5	2.8	3	0.4	2.4	3.2
11322	DA	SS	TH	AL	5	4	10	4	12	11.7	5.7	0.9	10.8	12.6	5.3	0.7	0.1	5.2	5.4	2.7	0.9	0.1	2.6	2.8	2.2	0.7	0.1	2	2.3
11323	DA	SS	TH	AL	4	2	7	2	1.8	1.8	3.7	0.1	1.7	1.9	4.3	0.4	0	4.3	4.3	2.5	0.8	0	2.5	2.5	1.8	0.4	0	1.8	1.8
11324	DA	SS	TH	AL	8	4	9	4	3.8	48.7	88.4	4.7	44	53.4	7.6	15.3	0.8	6.7	8.4	4	8.1	0.4	3.5	4.4	2.4	4.9	0.3	2.2	2.7
11325	DA	SS	TH	AL	9	2	18	2	0	0	0	0	0	0	8.9	0.9	0.6	8.3	9.5	4.4	1.4	1	3.4	5.4	2	0	0	2	2
12111	DA	SM	TL	AU	12	10	12	10	49.7	51.9	7.3	4.9	47	56.8	8.9	1.3	0.9	8	9.8	6.5	0.8	0.6	5.9	7	2.7	1.2	0.8	1.9	3.5
12112	DA	SM	TL	AU	12	7	12	7	30.9	92.8	74.2	9.2	83.6	101.9	8.9	9.5	1.2	7.8	10.1	5.5	5.9	0.7	4.7	6.2	2.6	3	0.4	2.3	3
12113	DA	SM	TL	AU	19	10	18	10	45.7	46.2	8.4	3.1	43.1	49.3	22.9	2.1	0.8	22.2	23.7	14.3	1.4	0.5	13.7	14.8	2.5	1.3	0.5	2	3
12114	DA	SM	TL	AU	13	7	6	7	27.9	27.8	3.2	2.3	25.5	30.1	5	0	0	5	5	4.5	0.7	0.5	4	5	1.2	0.4	0.3	0.9	1.5

Table 15 (Continued)

Scenario									Objective Function						Ammo usage					Blue success					Hit asset				
Scenario Id	**Scenario Size**	**Scenario Size**	**Threat Level**	**Ammo Level**	**Number of Weapons**	**Number of Assets**	**Number of Threats**	**Number of Targets**	**Heuristic Objective**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**
12115	DA	SM	TL	AU	17	7	12	7	41.8	41.7	5.5	3.9	37.8	45.6	13.5	1.3	0.9	12.6	14.4	11	0.8	0.6	10.4	11.6	0.8	0.8	0.6	0.2	1.4
12121	DA	SM	TL	AL	14	7	5	7	32.6	32.8	6	2.5	30.3	35.2	3.2	0.4	0.2	3	3.4	2.4	0.6	0.3	2.1	2.7	1.3	0.9	0.4	0.9	1.7
12122	DA	SM	TL	AL	18	7	3	7	48.4	48.4	6.6	4.7	43.7	53.1	3	0	0	3	3	2.2	0.8	0.6	1.6	2.8	0.7	0.8	0.6	0.1	1.3
12123	DA	SM	TL	AL	18	7	4	7	32.8	32.4	6.9	2.7	29.7	35.2	3	0	0	3	3	1.5	1.1	0.4	1.1	1.9	1.3	1	0.4	0.9	1.7
12124	DA	SM	TL	AL	20	10	9	10	51.6	50.3	3	2.1	48.2	52.4	8.7	1.1	0.8	7.9	9.5	6.5	1	0.7	5.8	7.2	1.9	0.6	0.4	1.5	2.3
12125	DA	SM	TL	AL	15	10	12	10	52.3	51.1	6.4	4.6	46.5	55.7	10	0.7	0.5	9.5	10.5	6.9	1	0.7	6.2	7.6	3	0.7	0.5	2.5	3.5
12211	DA	SM	TE	AU	16	8	16	8	25.8	27.6	5.1	2.4	25.1	30	11.2	0.7	0.3	10.9	11.5	7.8	1.1	0.5	7.2	8.3	3.7	0.9	0.4	3.2	4.1
12212	DA	SM	TE	AU	17	7	17	7	27.5	28.8	8.8	2.4	26.4	31.2	15.2	0.8	0.2	15	15.4	10	1.3	0.4	9.6	10.4	3	1.1	0.3	2.6	3.3
12213	DA	SM	TE	AU	20	9	20	9	34.7	34.5	10.7	2.4	32.1	36.8	18.7	1.1	0.3	18.4	18.9	12.9	1.5	0.3	12.5	13.2	3.8	1.4	0.3	3.5	4.1
12214	DA	SM	TE	AU	20	8	20	8	25.3	59.7	37.6	4.2	55.5	63.9	17.2	15.2	1.7	15.5	18.9	12.3	10.8	1.2	11	13.5	3.7	3.4	0.4	3.3	4.1
12215	DA	SM	TE	AU	16	9	16	9	51.8	49.3	12	4.8	44.5	54.1	21	1.9	0.7	20.3	21.8	11.6	1.5	0.6	11	12.2	2.5	1.4	0.6	1.9	3
12221	DA	SM	TE	AL	18	7	18	7	17.9	18.5	6.1	1.4	17.1	19.8	15.4	0.9	0.2	15.2	15.6	9.4	1.4	0.3	9.1	9.7	3.6	0.9	0.2	3.4	3.8
12222	DA	SM	TE	AL	17	9	17	9	23.4	24.8	8.2	2.1	22.7	26.9	10.6	0.6	0.1	10.4	10.7	7.3	1.1	0.3	7	7.6	4.2	1.1	0.3	3.9	4.5
12223	DA	SM	TE	AL	16	9	16	9	42.1	41.7	5.3	3.8	37.9	45.5	18.3	1.3	0.9	17.4	19.2	12.1	1.1	0.8	11.3	12.9	2.3	1.2	0.8	1.5	3.1
12224	DA	SM	TE	AL	12	8	12	8	40.2	39.9	5.3	3.8	36.1	43.7	8.1	0.7	0.5	7.6	8.6	4.4	1	0.7	3.7	5.1	3.1	0.7	0.5	2.6	3.6
12225	DA	SM	TE	AL	19	7	19	7	19.4	18.8	7.1	1.2	17.5	20	17.4	1.4	0.2	17.2	17.7	11.7	1.2	0.2	11.5	11.9	4.2	1	0.2	4.1	4.4
12311	DA	SM	TH	AU	20	9	35	9	17.8	19.1	6.9	1.3	17.8	20.4	24.5	1.4	0.3	24.3	24.8	15.6	1.6	0.3	15.3	15.9	5.6	1.1	0.2	5.4	5.8

Table 15 (Continued)

Scenario									Objective Function						Ammo usage					Blue success					Hit asset				
Scenario Id	**Scenario Size**	**Scenario Size**	**Threat Level**	**Ammo Level**	**Number of Weapons**	**Number of Assets**	**Number of Threats**	**Number of Targets**	**Heuristic Objective**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**
12312	DA	SM	TH	AU	19	7	37	7	7.5	7.5	6.3	0.4	7.2	7.9	27.8	1.1	0.1	27.7	27.8	16.6	1.8	0.1	16.4	16.7	6	0.8	0.1	6	6.1
12313	DA	SM	TH	AU	15	8	30	8	10.4	37.2	30.2	3.1	34.2	40.3	20.8	15	1.5	19.3	22.3	11.7	8.5	0.9	10.8	12.5	6.6	4.8	0.5	6.1	7.1
12314	DA	SM	TH	AU	18	8	25	8	31.1	58.1	27	3.5	54.5	61.6	22.6	15.8	2.1	20.6	24.7	13.7	9.7	1.3	12.5	15	3.5	2.6	0.3	3.1	3.8
12315	DA	SM	TH	AU	12	9	14	9	27.1	27.3	3.5	2.5	24.8	29.8	13.9	0.7	0.5	13.4	14.4	5.5	1.2	0.8	4.7	6.3	3.9	0.6	0.4	3.5	4.3
12321	DA	SM	TH	AL	16	8	23	8	19.4	19.7	7.6	1.5	18.3	21.2	18.1	1.2	0.2	17.8	18.3	11.9	1.4	0.3	11.6	12.1	4.4	1.2	0.2	4.2	4.7
12322	DA	SM	TH	AL	19	10	36	10	29.1	29.3	11	2.3	27	31.6	32	1.5	0.3	31.7	32.3	16.5	1.8	0.4	16.2	16.9	7.2	1.1	0.2	6.9	7.4
12323	DA	SM	TH	AL	20	7	35	7	10.4	98.2	102.8	9.5	88.8	107.7	28.7	35.9	3.3	25.4	32	14.4	18.1	1.7	12.7	16.1	6	7.5	0.7	5.3	6.7
12324	DA	SM	TH	AL	18	8	26	8	19.4	19.9	8.1	1.7	18.2	21.7	32	2.2	0.5	31.5	32.5	18.1	2	0.4	17.6	18.5	4.4	1.2	0.3	4.1	4.6
12325	DA	SM	TH	AL	15	9	29	9	24.9	98	86.2	8	90	106.1	20.1	22.6	2.1	18	22.2	9.4	10.7	1	8.4	10.4	6	6.8	0.6	5.4	6.6
13111	DA	SL	TL	AU	38	12	11	12	63.6	61.6	9.1	5.8	55.8	67.3	10.3	0.6	0.4	9.9	10.6	6.8	1.4	0.9	5.9	7.7	2.2	1.4	0.9	1.3	3.1
13112	DA	SL	TL	AU	37	13	17	13	56.4	61	3.6	2.5	58.5	63.5	17	0.7	0.5	16.5	17.5	13.1	1	0.7	12.4	13.8	1.9	0.6	0.4	1.5	2.3
13113	DA	SL	TL	AU	36	13	7	13	74.2	76.8	9.8	7	69.8	83.8	5.3	0.5	0.3	5	5.6	4.3	0.7	0.5	3.8	4.8	1.4	1.2	0.8	0.6	2.2
13114	DA	SL	TL	AU	24	18	4	18	137.1	138	4.2	3	135	141	4.4	0.7	0.5	3.9	4.9	3.6	0.5	0.4	3.2	4	0.2	0.4	0.3	0	0.5
13115	DA	SL	TL	AU	36	16	13	16	95.8	99.1	9	6.5	92.6	105.6	13.1	1.2	0.9	12.2	14	10.3	1.3	0.9	9.4	11.2	1.2	1	0.7	0.5	1.9
13121	DA	SL	TL	AL	27	16	8	16	100.1	99.7	6.9	4.9	94.8	104.6	7.5	0.5	0.4	7.1	7.9	5.9	0.6	0.4	5.5	6.3	1.8	0.8	0.6	1.2	2.4
13122	DA	SL	TL	AL	33	15	32	15	49.6	48.7	4.8	3.4	45.3	52.1	33.4	1.6	1.1	32.3	34.5	21.1	1.3	0.9	20.2	22	6.1	0.9	0.6	5.5	6.7
13123	DA	SL	TL	AL	37	12	2	12	87.3	86.9	3.5	2.5	84.4	89.4	2.2	0.4	0.3	1.9	2.5	1.9	0.3	0.2	1.7	2	0.1	0.3	0.2	0	0.3

Table 15 (Continued)

Scenario									Objective Function						Ammo usage					Blue success					Hit asset				
Scenario Id	Scenario Size	Scenario Size	Threat Level	Ammo Level	Number of Weapons	Number of Assets	Number of Threats	Number of Targets	Heuristic Objective	avg	std	hl	ll	ul	avg	std	hl	ll	ul	avg	std	hl	ll	ul	avg	std	hl	ll	ul
13124	DA	SL	TL	AL	38	20	11	20	106.5	106.2	6.4	4.6	101.6	110.8	12.6	1	0.7	11.9	13.3	7.8	1	0.7	7.1	8.5	2.1	1.4	1	1.1	3.1
13125	DA	SL	TL	AL	37	16	14	16	101.5	99.9	5.2	3.7	96.2	103.6	7	0	0	7	7	5.1	0.6	0.4	4.7	5.5	4.1	0.6	0.4	3.7	4.5
13211	DA	SL	TE	AU	26	12	26	12	43.7	43.2	7.5	4.2	39	47.4	24.7	1.2	0.7	24	25.3	16.7	1.7	0.9	15.7	17.6	4.3	1.4	0.8	3.5	5
13212	DA	SL	TE	AU	23	20	23	20	103.8	104.2	11.4	8.1	96.1	112.3	21.9	1	0.7	21.2	22.6	13.9	1.6	1.1	12.8	15	6	1.6	1.2	4.8	7.2
13213	DA	SL	TE	AU	33	20	33	20	88	83	11.4	8.1	74.9	91.1	31	1.1	0.8	30.2	31.8	22.1	1.5	1.1	21	23.2	6.3	1.8	1.3	5	7.6
13214	DA	SL	TE	AU	25	16	25	16	81.8	77.4	8	5.7	71.7	83.1	28.5	1.7	1.2	27.3	29.7	18.6	1.3	0.9	17.7	19.5	4.5	1.2	0.8	3.7	5.3
13215	DA	SL	TE	AU	24	14	24	14	54.8	54	10.1	4.7	49.2	58.7	26.7	1.2	0.6	26.1	27.2	15.5	2	0.9	14.6	16.4	5.9	1.5	0.7	5.1	6.6
13221	DA	SL	TE	AL	26	18	26	18	85.5	126.1	43.5	10.4	115.7	136.5	28.7	17	4.1	24.7	32.8	17.3	10.4	2.5	14.8	19.8	4.9	3.3	0.8	4.1	5.7
13222	DA	SL	TE	AL	27	15	27	15	61.8	64.1	7.4	5.3	58.8	69.4	26.1	1.6	1.1	25	27.2	19.2	1.4	1	18.2	20.2	4.5	1	0.7	3.8	5.2
13223	DA	SL	TE	AL	24	16	24	16	73.9	79.4	8.9	6.4	73	85.8	24.5	1.3	0.9	23.6	25.4	16.8	2.1	1.5	15.3	18.3	3.2	1.5	1.1	2.1	4.3
13224	DA	SL	TE	AL	22	15	22	15	66.5	69.4	8.5	6.1	63.3	75.5	17.2	0.8	0.6	16.6	17.8	10.3	1.6	1.2	9.1	11.5	5.2	0.9	0.7	4.5	5.9
13225	DA	SL	TE	AL	38	16	38	16	53	55	9.4	4.3	50.7	59.3	30.4	2.1	0.9	29.4	31.3	24.2	1.2	0.6	23.6	24.7	5.9	1.5	0.7	5.2	6.5
13311	DA	SL	TH	AU	36	12	55	12	29.9	65.3	36.7	5.6	59.7	71	51	33.1	5.1	45.9	56	35.9	23.4	3.6	32.3	39.5	7.2	4.9	0.8	6.4	7.9
13312	DA	SL	TH	AU	25	13	35	13	35.3	34.3	10.6	3.4	30.9	37.6	34.6	1.5	0.5	34.1	35	17.6	2.3	0.7	16.9	18.4	7.6	1.5	0.5	7.1	8
13313	DA	SL	TH	AU	30	14	49	14	42.2	45.6	5	3.6	42	49.2	52.4	2.4	1.7	50.7	54.1	32.8	2.2	1.6	31.2	34.4	7.5	1.3	0.9	6.6	8.4
13314	DA	SL	TH	AU	34	16	50	16	41.8	42.8	7.9	3.7	39.1	46.5	45.4	1.8	0.8	44.6	46.2	29.4	1.4	0.7	28.7	30	9	1.5	0.7	8.3	9.6
13315	DA	SL	TH	AU	39	20	50	20	76.2	75.3	12	6.2	69.1	81.5	47.8	2.4	1.2	46.6	49	31.1	1.7	0.9	30.2	32	10.1	1.4	0.7	9.3	10.8

Table 15 (Continued)

Scenario									Objective Function						Ammo usage					Blue success					Hit asset				
Scenario Id	**Scenario Size**	**Scenario Size**	**Threat Level**	**Ammo Level**	**Number of Weapons**	**Number of Assets**	**Number of Threats**	**Number of Targets**	**Heuristic Objective**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**
13321	DA	SL	TH	AL	22	14	32	14	56.3	54.4	10.6	4.5	49.9	58.9	29.3	1.5	0.6	28.7	30	20.4	2.3	1	19.4	21.3	6	1.7	0.7	5.3	6.8
13322	DA	SL	TH	AL	24	18	30	18	67.1	66.1	10.8	6	60.1	72.1	27.6	2	1.1	26.5	28.7	15.9	1.9	1	14.8	16.9	7.2	1.7	0.9	6.3	8.1
13323	DA	SL	TH	AL	39	16	63	16	60.2	60.5	11.4	5.8	54.6	66.3	58.6	1.9	1	57.6	59.6	39.4	1.9	1	38.4	40.3	9.2	1.3	0.7	8.6	9.9
13324	DA	SL	TH	AL	32	15	56	15	23.1	24.1	9.7	2.1	22	26.2	50.8	1.5	0.3	50.5	51.1	24.1	2.6	0.6	23.5	24.7	11.6	1.2	0.3	11.3	11.8
13325	DA	SL	TH	AL	31	15	56	15	40.1	39.4	8.2	3.7	35.6	43.1	52.5	2.4	1.1	51.4	53.6	31	3	1.4	29.6	32.4	10.3	1	0.4	9.9	10.8
21111	DP	SS	TL	AU	6	0	4	6	58.7	58.3	5.3	3.8	54.5	62.1	3	0	0	3	3	3	0	0	3	3	0.7	0.5	0.3	0.4	1
21112	DP	SS	TL	AU	3	0	2	3	27.9	35.5	10.9	3.2	32.3	38.7	3	1.7	0.5	2.5	3.6	1.5	1	0.3	1.2	1.8	0.3	0.6	0.2	0.2	0.5
21113	DP	SS	TL	AU	9	0	4	9	21.8	21.9	2.5	1.8	20.1	23.7	2.7	0.7	0.5	2.2	3.2	1.9	0.3	0.2	1.7	2.1	1.7	0.8	0.6	1.1	2.3
21114	DP	SS	TL	AU	6	0	5	6	18.6	17.9	3.2	1.4	16.5	19.3	3.8	0.9	0.4	3.4	4.2	2.3	0.7	0.3	2	2.6	1.5	0.8	0.3	1.2	1.9
21115	DP	SS	TL	AU	8	0	3	8	63.8	65.7	4.3	3.1	62.6	68.8	2.8	1.5	1.1	1.7	3.9	1.9	0.3	0.2	1.7	2.1	0.7	0.5	0.3	0.4	1
21121	DP	SS	TL	AL	5	0	2	5	16.1	15.2	2.7	1.5	13.7	16.7	1	0	0	1	1	0.4	0.5	0.3	0.1	0.7	1.2	0.7	0.4	0.8	1.6
21122	DP	SS	TL	AL	5	0	5	5	29.3	30.5	7.2	2.7	27.8	33.1	2.2	0.4	0.1	2	2.3	1.5	0.7	0.2	1.3	1.8	1.6	0.8	0.3	1.3	1.9
21123	DP	SS	TL	AL	8	0	1	8	72	72	0	0	72	72	1	0	0	1	1	1	0	0	1	1	0	0	0	0	0
21124	DP	SS	TL	AL	5	0	3	5	39.3	38.8	7.4	3.1	35.6	41.9	0.5	0.7	0.3	0.2	0.8	0.4	0.5	0.2	0.2	0.6	1.1	0.7	0.3	0.8	1.4
21125	DP	SS	TL	AL	5	0	4	5	6.5	6.2	0.6	0.5	5.7	6.7	2	0	0	2	2	1.5	0.5	0.4	1.1	1.9	1.9	0.3	0.2	1.7	2.1
21211	DP	SS	TE	AU	6	0	6	6	42.3	39.1	5.4	3.6	35.5	42.7	6	1.2	0.8	5.2	6.8	2.5	0.8	0.6	2	3.1	2.1	0.5	0.4	1.7	2.5
21212	DP	SS	TE	AU	10	0	10	10	24.3	24.9	2.5	1.8	23.1	26.7	9.8	0.8	0.6	9.2	10.4	7.1	0.7	0.5	6.6	7.6	1.7	0.8	0.6	1.1	2.3

Table 15 (Continued)

Scenario									Objective Function						Ammo usage					Blue success					Hit asset				
Scenario Id	**Scenario Size**	**Scenario Size**	**Threat Level**	**Ammo Level**	**Number of Weapons**	**Number of Assets**	**Number of Threats**	**Number of Targets**	**Heuristic Objective**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**
21213	DP	SS	TE	AU	4	0	4	4	15.2	14.8	1.9	1.4	13.4	16.2	4.6	1	0.7	3.9	5.3	3.2	0.6	0.5	2.7	3.7	0.3	0.5	0.3	0	0.6
21214	DP	SS	TE	AU	5	0	5	5	16.3	24.7	9.7	1.6	23.1	26.4	3	1.7	0.3	2.7	3.3	1.7	1.2	0.2	1.5	2	1.7	1.4	0.2	1.5	2
21215	DP	SS	TE	AU	10	0	10	10	63.6	64.8	9.7	5.4	59.4	70.2	6.6	0.5	0.3	6.3	6.9	4.5	0.8	0.5	4.1	5	2.8	1.1	0.6	2.2	3.4
21221	DP	SS	TE	AL	5	0	5	5	20.4	30.1	11.3	1.9	28.2	32	4.7	3	0.5	4.2	5.2	2.9	2	0.3	2.6	3.3	0.9	1.1	0.2	0.8	1.1
21222	DP	SS	TE	AL	8	0	8	8	49.1	49	5.7	4.1	44.9	53.1	7.4	0.7	0.5	6.9	7.9	6.5	0.5	0.4	6.1	6.9	1	0.8	0.6	0.4	1.6
21223	DP	SS	TE	AL	6	0	6	6	31.7	33.1	5.4	2.4	30.7	35.5	5	1	0.4	4.6	5.5	3.6	0.5	0.2	3.4	3.9	1.3	0.8	0.3	0.9	1.6
21224	DP	SS	TE	AL	3	0	3	3	5.1	10.2	5.4	0.8	9.4	11	2	1.5	0.2	1.8	2.2	0.3	0.5	0.1	0.2	0.4	1.3	1.1	0.2	1.1	1.4
21225	DP	SS	TE	AL	7	0	7	7	34.1	33.7	5.7	2.6	31.1	36.3	3.4	0.5	0.2	3.2	3.7	2.2	0.6	0.3	2	2.5	2.2	0.8	0.4	1.8	2.6
21311	DP	SS	TH	AU	4	0	8	4	7.5	48.2	48.1	4.4	43.9	52.6	4.4	5.4	0.5	3.9	4.9	2.5	3.2	0.3	2.2	2.8	3	3.7	0.3	2.6	3.3
21312	DP	SS	TH	AU	9	0	11	9	19.7	19.4	4.1	1.6	17.9	21	6.6	0.9	0.4	6.2	6.9	3.1	0.8	0.3	2.8	3.4	4.1	1	0.4	3.7	4.5
21313	DP	SS	TH	AU	5	0	7	5	10	10.3	3.1	0.7	9.5	11	4.1	0.3	0.1	4	4.2	2.5	1	0.2	2.3	2.7	2.4	0.8	0.2	2.3	2.6
21314	DP	SS	TH	AU	8	0	13	8	7.9	8.1	2.2	0.7	7.4	8.8	10.9	0.9	0.3	10.6	11.2	3.8	1.3	0.4	3.4	4.2	3.9	1.1	0.4	3.6	4.3
21315	DP	SS	TH	AU	5	0	10	5	12.2	66.6	63.5	6.4	60.3	73	7.1	8.4	0.8	6.2	7.9	3.8	4.5	0.5	3.3	4.2	3.5	4.2	0.4	3.1	4
21321	DP	SS	TH	AL	3	0	5	3	2.2	1.7	1.3	0.1	1.6	1.9	1.3	0.5	0.1	1.3	1.4	0.4	0.5	0.1	0.4	0.5	2.1	0.7	0.1	2.1	2.2
21322	DP	SS	TH	AL	5	0	6	5	17.4	16	4.6	1.3	14.8	17.3	5.1	0.4	0.1	5	5.2	2.5	0.9	0.2	2.2	2.7	1.8	0.9	0.3	1.5	2
21323	DP	SS	TH	AL	5	0	6	5	30	31.5	5.6	3.1	28.3	34.6	6.2	0.8	0.4	5.8	6.6	3.7	1	0.5	3.1	4.2	1.1	0.7	0.4	0.7	1.5
21324	DP	SS	TH	AL	4	0	8	4	4.1	12.2	8.2	1.2	11	13.4	4	3.3	0.5	3.5	4.5	3.1	2.6	0.4	2.7	3.5	2.6	2.2	0.3	2.3	2.9

Table 15 (Continued)

Scenario									Objective Function						Ammo usage					Blue success					Hit asset				
Scenario Id	Scenario Size	Scenario Size	Threat Level	Ammo Level	Number of Weapons	Number of Assets	Number of Threats	Number of Targets	Heuristic Objective	avg	std	hl	ll	ul	avg	std	hl	ll	ul	avg	std	hl	ll	ul	avg	std	hl	ll	ul
21325	DP	SS	TH	AL	4	0	6	4	8.3	62.2	72.2	6	56.2	68.2	3.1	4.3	0.4	2.7	3.4	1.4	2.1	0.2	1.3	1.6	2.8	4	0.3	2.5	3.1
22111	DP	SM	TL	AU	13	0	7	13	68.9	70.2	4	2.9	67.3	73.1	5.4	0.7	0.5	4.9	5.9	5	0	0	5	5	1.3	0.7	0.5	0.8	1.8
22112	DP	SM	TL	AU	13	0	11	13	34.3	34.5	4.7	3.4	31.1	37.9	9	0.9	0.7	8.3	9.7	7.8	1.3	0.9	6.9	8.7	1.5	1.6	1.1	0.4	2.6
22113	DP	SM	TL	AU	19	0	7	19	143.6	144	6.5	4.7	139.3	148.7	7.8	0.8	0.6	7.2	8.4	5.9	0.9	0.6	5.3	6.5	1	0.8	0.6	0.4	1.6
22114	DP	SM	TL	AU	18	0	16	18	98.5	98.4	5.8	4.1	94.3	102.6	19.1	1.1	0.8	18.3	19.9	12.1	1.3	0.9	11.2	13	1.6	1	0.7	0.9	2.3
22115	DP	SM	TL	AU	12	0	4	12	22.2	22.4	1.3	0.9	21.5	23.3	4.1	0.6	0.4	3.7	4.5	2.2	0.6	0.5	1.7	2.7	0.8	0.6	0.5	0.3	1.3
22121	DP	SM	TL	AL	12	0	2	12	57.8	58.5	2.4	1.7	56.8	60.2	2	0	0	2	2	1.7	0.5	0.3	1.4	2	0.3	0.5	0.3	0	0.6
22122	DP	SM	TL	AL	12	0	9	12	33	32.7	2.2	1.6	31.1	34.3	8.6	1.1	0.8	7.8	9.4	6.1	1	0.7	5.4	6.8	1.1	0.7	0.5	0.6	1.6
22123	DP	SM	TL	AL	13	0	13	13	30.8	30	2.4	1.8	28.2	31.8	12.1	1.1	0.8	11.3	12.9	8.1	0.7	0.5	7.6	8.6	3	0.8	0.6	2.4	3.6
22124	DP	SM	TL	AL	16	0	2	16	45.9	46.5	1.6	1.1	45.4	47.6	1	0	0	1	1	1	0	0	1	1	0.5	0.5	0.4	0.1	0.9
22125	DP	SM	TL	AL	19	0	16	19	118.5	112	10.7	7.6	104.4	119.6	11.2	1.1	0.8	10.4	12	7.7	2.4	1.7	6	9.4	5	1.3	1	4	6
22211	DP	SM	TE	AU	16	0	16	16	131.1	126	10.8	7.7	118.3	133.7	17.2	1.6	1.2	16	18.4	10.1	0.9	0.6	9.5	10.7	3.4	1.1	0.8	2.6	4.2
22212	DP	SM	TE	AU	17	0	17	17	115	111.6	12.1	8.7	102.9	120.3	15.6	1.2	0.8	14.8	16.4	10.7	1.4	1	9.7	11.7	4.6	1.3	1	3.6	5.6
22213	DP	SM	TE	AU	13	0	13	13	105.1	103	9.5	6.8	96.2	109.8	13.9	0.6	0.4	13.5	14.3	8.7	0.8	0.6	8.1	9.3	2.7	0.9	0.7	2	3.4
22214	DP	SM	TE	AU	13	0	13	13	61.9	64.8	7.9	5.7	59.1	70.5	9.3	1.2	0.8	8.5	10.1	7.2	1	0.7	6.5	7.9	2.2	1.3	0.9	1.3	3.1
22215	DP	SM	TE	AU	14	0	14	14	90.1	94.4	7.4	5.3	89.1	99.7	15.9	0.9	0.6	15.3	16.5	9.8	0.9	0.7	9.1	10.5	2.2	0.9	0.7	1.5	2.9
22221	DP	SM	TE	AL	13	0	13	13	79.5	76	9.4	6.7	69.3	82.7	10.9	0.6	0.4	10.5	11.3	6.5	1.6	1.1	5.4	7.6	3.5	1.2	0.8	2.7	4.3

Table 15 (Continued)

Scenario									Objective Function						Ammo usage					Blue success					Hit asset				
Scenario Id	**Scenario Size**	**Scenario Size**	**Threat Level**	**Ammo Level**	**Number of Weapons**	**Number of Assets**	**Number of Threats**	**Number of Targets**	**Heuristic Objective**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**
22222	DP	SM	TE	AL	19	0	19	19	136.2	135.9	9.9	7.1	128.8	143	13.2	0.8	0.6	12.6	13.8	10.7	0.9	0.7	10	11.4	3.9	1.1	0.8	3.1	4.7
22223	DP	SM	TE	AL	14	0	14	14	36.1	36	2	1.4	34.6	37.4	15.2	1.2	0.9	14.3	16.1	11.5	1.2	0.8	10.7	12.3	2	0.7	0.5	1.5	2.5
22224	DP	SM	TE	AL	17	0	17	17	121	118	12.3	8.8	109.2	126.8	15	0.8	0.6	14.4	15.6	7.3	1.3	1	6.3	8.3	5.2	1.2	0.9	4.3	6.1
22225	DP	SM	TE	AL	20	0	20	20	88	86.4	5.8	4.1	82.3	90.5	13.1	2	1.4	11.7	14.5	7.8	0.9	0.7	7.1	8.5	5.6	1	0.7	4.9	6.3
22311	DP	SM	TH	AU	17	0	32	17	17.4	24.8	11.7	2.2	22.6	27	24.6	13.5	2.6	22	27.2	14.4	8.2	1.5	12.8	15.9	9.6	5.5	1	8.5	10.6
22312	DP	SM	TH	AU	16	0	21	16	95.5	90.4	12.5	9	81.4	99.4	22.7	1.2	0.8	21.9	23.5	12.2	1.1	0.8	11.4	13	4.7	1.6	1.1	3.6	5.8
22313	DP	SM	TH	AU	16	0	22	16	106.2	91.8	13.6	6.4	85.4	98.2	19.6	1.1	0.5	19.1	20.1	12.1	1.6	0.7	11.4	12.8	5.8	1.5	0.7	5.1	6.5
22314	DP	SM	TH	AU	18	0	33	18	72.6	71.2	8	5.7	65.5	76.9	26.3	1.3	0.9	25.4	27.2	14.3	2.4	1.7	12.6	16	9.1	1	0.7	8.4	9.8
22315	DP	SM	TH	AU	13	0	19	13	19.8	19.8	2.6	1.8	18	21.6	16.9	0.7	0.5	16.4	17.4	13.4	1.6	1.1	12.3	14.5	3.1	1.3	0.9	2.2	4
22321	DP	SM	TH	AL	13	0	15	13	41	39.2	4.1	3	36.2	42.2	13.5	1.4	1	12.5	14.5	9.2	1.1	0.8	8.4	10	3.2	1	0.7	2.5	3.9
22322	DP	SM	TH	AL	17	0	20	17	72.8	66	7.5	5.4	60.6	71.4	15.8	1.5	1.1	14.7	16.9	9.6	1.5	1.1	8.5	10.7	6	1.2	0.9	5.1	6.9
22323	DP	SM	TH	AL	18	0	23	18	118.5	105	14.3	10.3	94.7	115.3	21.8	1.4	1	20.8	22.8	12.4	1.9	1.4	11	13.8	7.5	1.4	1	6.5	8.5
22324	DP	SM	TH	AL	18	0	32	18	117	107.3	25.5	10.3	97	117.6	24.8	3.3	1.3	23.5	26.1	16.5	2.9	1.2	15.3	17.7	7.3	2.6	1	6.2	8.3
22325	DP	SM	TH	AL	18	0	25	18	84.8	87.5	6.8	4.9	82.6	92.4	22.3	1.3	1	21.3	23.3	13.1	2.8	2	11.1	15.1	5.5	1	0.7	4.8	6.2
23111	DP	SL	TL	AU	25	0	3	25	269.1	268.4	5.7	4.1	264.3	272.5	2.2	0.4	0.3	1.9	2.5	2	0	0	2	2	0.6	0.5	0.4	0.2	1
23112	DP	SL	TL	AU	35	0	11	35	364.2	360.8	11.4	8.1	352.7	368.9	10.3	0.5	0.3	10	10.6	7.9	1.3	0.9	7	8.8	2.2	1	0.7	1.5	2.9
23113	DP	SL	TL	AU	29	0	19	29	254.7	255	14.3	10.3	244.7	265.3	16.1	1	0.7	15.4	16.8	13.2	1.3	0.9	12.3	14.1	3.5	1.4	1	2.5	4.5

Table 15 (Continued)

Scenario									Objective Function						Ammo usage					Blue success					Hit asset				
Scenario Id	**Scenario Size**	**Scenario Size**	**Threat Level**	**Ammo Level**	**Number of Weapons**	**Number of Assets**	**Number of Threats**	**Number of Targets**	**Heuristic Objective**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**
23114	DP	SL	TL	AU	26	0	8	26	244.1	242	11.4	8.1	233.9	250.1	7.2	0.4	0.3	6.9	7.5	5.3	0.9	0.7	4.6	6	1.8	1.1	0.8	1	2.6
23115	DP	SL	TL	AU	39	0	5	39	386.1	385	8.5	6.1	378.9	391.1	5.9	0.7	0.5	5.4	6.4	3.7	0.5	0.3	3.4	4	0.5	0.8	0.6	0	1.1
23121	DP	SL	TL	AL	38	0	1	38	372.9	371	3.2	2.3	368.7	373.3	0	0	0	0	0	0	0	0	0	0	0.9	0.3	0.2	0.7	1.1
23122	DP	SL	TL	AL	36	0	4	36	241.5	242.9	4.7	3.4	239.5	246.3	2.2	0.4	0.3	1.9	2.5	1.7	0.5	0.3	1.4	2	1.3	0.7	0.5	0.8	1.8
23123	DP	SL	TL	AL	27	0	12	27	99.9	98	4.7	3.4	94.6	101.4	11.1	1	0.7	10.4	11.8	8.4	1	0.7	7.7	9.1	2.5	1.2	0.8	1.7	3.3
23124	DP	SL	TL	AL	35	0	13	35	360.5	359.7	13.8	9.8	349.9	369.6	10.7	0.9	0.7	10	11.4	9.1	1.2	0.9	8.2	10	2.3	1.3	0.9	1.4	3.2
23125	DP	SL	TL	AL	34	0	4	34	162.6	161.5	5.3	3.8	157.7	165.3	2.2	0.4	0.3	1.9	2.5	1.8	0.6	0.5	1.3	2.3	1.7	1.1	0.8	0.9	2.5
23211	DP	SL	TE	AU	22	0	22	22	155.4	158.4	11.8	8.4	150	166.9	26	1.2	0.9	25.1	26.9	17.7	1.4	1	16.7	18.7	2.2	1.5	1.1	1.1	3.3
23212	DP	SL	TE	AU	24	0	24	24	163	155.7	15.3	11	144.7	166.7	17.2	1.2	0.9	16.3	18.1	12	1.2	0.8	11.2	12.8	6.7	1.7	1.2	5.5	7.9
23213	DP	SL	TE	AU	35	0	35	35	173	166.8	6.2	4.4	162.4	171.2	29.5	1.6	1.2	28.3	30.7	19.5	1	0.7	18.8	20.2	7.2	1	0.7	6.5	7.9
23214	DP	SL	TE	AU	40	0	40	40	64.3	62.4	4.1	2.9	59.5	65.3	35.6	1.3	1	34.6	36.6	26.7	1.3	1	25.7	27.7	8.8	2	1.5	7.3	10.3
23215	DP	SL	TE	AU	25	0	25	25	187.2	173.7	15.9	11.4	162.3	185.1	24.6	2	1.4	23.2	26	16.9	2.3	1.7	15.2	18.6	5.7	1.8	1.3	4.4	7
23221	DP	SL	TE	AL	35	0	35	35	209.1	199.5	11.1	7.9	191.6	207.4	37	1.9	1.4	35.6	38.4	24	1.4	1	23	25	6.5	1.6	1.1	5.4	7.6
23222	DP	SL	TE	AL	31	0	31	31	240.6	239.4	16	11.4	228	250.8	27.6	1.6	1.1	26.5	28.7	22	2	1.4	20.6	23.4	4.4	1.8	1.3	3.1	5.7
23223	DP	SL	TE	AL	37	0	37	37	56.5	57.4	4	2.9	54.5	60.3	37.2	1.7	1.2	36	38.4	22.5	2	1.4	21.1	23.9	8.3	2	1.4	6.9	9.7
23224	DP	SL	TE	AL	28	0	28	28	83.2	80.8	6.5	4.6	76.2	85.4	21.8	1	0.7	21.1	22.5	15.6	1.6	1.1	14.5	16.7	7.8	1.6	1.2	6.6	9
23225	DP	SL	TE	AL	23	0	23	23	183.1	176	15.8	11.3	164.7	187.3	25.6	1.5	1.1	24.5	26.7	14.5	1.2	0.8	13.7	15.3	5.4	1.6	1.1	4.3	6.5

Table 15 (Continued)

Scenario									Objective Function						Ammo usage					Blue success					Hit asset				
Scenario Id	**Scenario Size**	**Scenario Size**	**Threat Level**	**Ammo Level**	**Number of Weapons**	**Number of Assets**	**Number of Threats**	**Number of Targets**	**Heuristic Objective**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**
23311	DP	SL	TH	AU	31	0	38	31	272.1	277.2	16.2	11.6	265.6	288.8	35.4	1.4	1	34.4	36.4	25	1.2	0.8	24.2	25.8	5.8	1.5	1.1	4.7	6.9
23312	DP	SL	TH	AU	36	0	44	36	282.3	273	23.1	16.5	256.5	289.5	45.6	2.2	1.6	44	47.2	29.1	2.2	1.6	27.5	30.7	8.7	2.3	1.7	7	10.4
23313	DP	SL	TH	AU	40	0	58	40	342.4	319	34	24.3	294.7	343.3	58	2.2	1.6	56.4	59.6	36.9	2.8	2	34.9	38.9	11	3.1	2.2	8.8	13.2
23314	DP	SL	TH	AU	30	0	58	30	132.3	115.6	21.6	10.8	104.8	126.3	49.1	2.7	1.4	47.7	50.4	27.8	1.7	0.9	27	28.7	15.6	2.7	1.3	14.2	16.9
23315	DP	SL	TH	AU	33	0	42	33	74	71.1	5.8	4.2	66.9	75.3	43.5	0.8	0.6	42.9	44.1	25	1.6	1.2	23.8	26.2	9.3	1.9	1.4	7.9	10.7
23321	DP	SL	TH	AL	28	0	34	28	132	126	9.4	6.7	119.3	132.7	32.9	1.4	1	31.9	33.9	21.5	0.7	0.5	21	22	7	1.6	1.1	5.9	8.1
23322	DP	SL	TH	AL	31	0	44	31	174.3	159.2	21.8	15.6	143.6	174.8	38.9	2.4	1.7	37.2	40.6	22.9	3.1	2.2	20.7	25.1	11.1	2.7	2	9.1	13.1
23323	DP	SL	TH	AL	25	0	28	25	150.9	141.6	11.3	8.1	133.5	149.7	19.4	1.8	1.3	18.1	20.7	14.2	1.1	0.8	13.4	15	7.3	1.4	1	6.3	8.3
23324	DP	SL	TH	AL	30	0	49	30	61.3	56.7	5.9	4.2	52.5	60.9	44.9	2.4	1.7	43.2	46.6	27.8	2.6	1.9	25.9	29.7	11.1	2	1.4	9.7	12.5
23325	DP	SL	TH	AL	35	0	62	35	135.4	120.8	18	10	110.8	130.8	48.8	2.3	1.3	47.5	50.1	29.1	2.4	1.3	27.8	30.4	14.9	3	1.7	13.2	16.5
31111	DM	SS	TL	AU	4	5	2	8	37.4	37	0	0	37	37	1.2	0.4	0.3	0.9	1.5	1	0	0	1	1	1	0	0	1	1
31112	DM	SS	TL	AU	3	3	2	6	27.4	27	3.8	2.7	24.3	29.7	1	0	0	1	1	0.6	0.5	0.4	0.2	1	0.6	0.7	0.5	0.1	1.1
31113	DM	SS	TL	AU	4	2	2	5	29.2	29	0	0	29	29	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1
31114	DM	SS	TL	AU	6	4	1	6	28.6	28.8	3.8	2.7	26.1	31.5	0	0	0	0	0	0	0	0	0	0	0.8	0.4	0.3	0.5	1.1
31115	DM	SS	TL	AU	10	4	5	12	71.5	72.6	3.2	2.3	70.3	74.9	4	0	0	4	4	2.8	0.6	0.5	2.3	3.3	1.2	0.6	0.5	0.7	1.7
31121	DM	SS	TL	AL	10	3	10	6	11	11.1	5.9	1	10.1	12.1	7	0	0	7	7	3.4	1.3	0.2	3.2	3.6	3.7	0.9	0.2	3.6	3.9
31122	DM	SS	TL	AL	5	2	4	5	42.2	51.4	13.2	4.9	46.5	56.3	3.3	1.7	0.6	2.6	3.9	2.7	1.4	0.5	2.1	3.2	0.9	0.8	0.3	0.6	1.2

Table 15 (Continued)

Scenario									Objective Function						Ammo usage					Blue success					Hit asset				
Scenario Id	Scenario Size	Scenario Size	Threat Level	Ammo Level	Number of Weapons	Number of Assets	Number of Threats	Number of Targets	Heuristic Objective	avg	std	hl	ll	ul	avg	std	hl	ll	ul	avg	std	hl	ll	ul	avg	std	hl	ll	ul
31123	DM	SS	TL	AL	8	4	8	9	40	45	5.8	4.2	40.8	49.2	5	0	0	5	5	3.9	0.9	0.6	3.3	4.5	2.3	0.9	0.7	1.6	3
31124	DM	SS	TL	AL	5	5	4	7	41.3	41	3.3	2.3	38.7	43.3	2	0	0	2	2	2	0	0	2	2	1.2	0.4	0.3	0.9	1.5
31125	DM	SS	TL	AL	9	4	3	11	61.7	61	5.5	3.9	57.1	64.9	2.1	0.3	0.2	1.9	2.3	1	0	0	1	1	1.2	0.6	0.5	0.7	1.7
31211	DM	SS	TE	AU	10	4	10	11	78.7	73	6.9	4.9	68.1	77.9	12.4	0.7	0.5	11.9	12.9	7	0.9	0.7	6.3	7.7	2.4	1.1	0.8	1.6	3.2
31212	DM	SS	TE	AU	8	3	8	9	46.2	44.4	4.2	3	41.4	47.4	9.9	0.9	0.6	9.3	10.5	5	0.8	0.6	4.4	5.6	1.9	0.9	0.6	1.3	2.5
31213	DM	SS	TE	AU	7	5	7	6	18.1	17.3	4.4	1.4	15.9	18.7	5	0	0	5	5	3.4	0.9	0.3	3.1	3.7	2.2	0.8	0.3	2	2.5
31214	DM	SS	TE	AU	8	3	8	5	17	17.6	6	1.5	16.2	19.1	6	0.7	0.2	5.9	6.2	3.6	1	0.2	3.3	3.8	2	0.8	0.2	1.8	2.2
31215	DM	SS	TE	AU	3	4	3	5	27.1	29.1	4	2.7	26.4	31.8	3.1	0.3	0.2	2.9	3.3	2.5	0.7	0.5	2.1	3	0.4	0.7	0.5	0	0.8
31221	DM	SS	TE	AL	7	2	7	6	33.9	33.2	7.8	2.9	30.3	36.1	6.2	0.4	0.2	6	6.4	4.3	1	0.4	4	4.7	1.6	1.1	0.4	1.2	2
31222	DM	SS	TE	AL	3	3	3	4	26.8	27	3.7	2.6	24.4	29.6	2.6	0.5	0.4	2.2	3	1.7	0.5	0.3	1.4	2	0.9	0.7	0.5	0.4	1.4
31223	DM	SS	TE	AL	10	2	10	7	25.1	23.8	6.4	1.5	22.4	25.3	8.9	0.9	0.2	8.7	9.2	4.5	1.5	0.3	4.2	4.9	2.7	1.2	0.3	2.5	3
31224	DM	SS	TE	AL	3	5	3	7	34.8	35.1	5.4	3.3	31.8	38.3	2	0	0	2	2	1.1	0.3	0.2	0.9	1.2	1.2	0.6	0.3	0.8	1.5
31225	DM	SS	TE	AL	9	5	9	7	39.4	40.7	8.9	3	37.6	43.7	7.2	0.9	0.3	6.9	7.5	4.8	0.9	0.3	4.5	5.1	2.2	1	0.4	1.8	2.5
31311	DM	SS	TH	AU	5	5	6	7	26	25.3	5.2	1.9	23.4	27.2	5.9	0.2	0.1	5.8	6	1.6	1	0.4	1.3	2	2.2	0.5	0.2	2	2.4
31312	DM	SS	TH	AU	3	2	4	4	16.7	16.3	2.9	1.3	15	17.6	3.3	0.5	0.2	3.1	3.5	2.2	0.4	0.2	2	2.4	1	0.6	0.3	0.8	1.3
31313	DM	SS	TH	AU	10	5	18	9	23.7	23.5	7.8	2.1	21.4	25.6	13.3	0.7	0.2	13.1	13.5	9	1.1	0.3	8.7	9.3	5.9	1.1	0.3	5.7	6.2
31314	DM	SS	TH	AU	10	3	19	10	47.9	44	9.3	3.8	40.2	47.8	13.6	0.7	0.3	13.3	13.9	8.7	1.6	0.7	8	9.4	4.7	1.1	0.4	4.3	5.2

Table 15 (Continued)

Scenario									Objective Function						Ammo usage					Blue success					Hit asset				
Scenario Id	**Scenario Size**	**Scenario Size**	**Threat Level**	**Ammo Level**	**Number of Weapons**	**Number of Assets**	**Number of Threats**	**Number of Targets**	**Heuristic Objective**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**
31315	DM	SS	TH	AU	3	5	6	7	30.9	31.1	4.3	3.1	28	34.2	5.3	0.7	0.5	4.8	5.8	3.6	0.8	0.6	3	4.2	0.9	0.9	0.6	0.3	1.5
31321	DM	SS	TH	AL	9	5	16	12	49.6	47.2	8.9	3.2	44	50.4	13	1.3	0.5	12.5	13.4	7.3	1.4	0.5	6.9	7.8	4.9	1.2	0.4	4.4	5.3
31322	DM	SS	TH	AL	8	4	10	11	47.3	43.3	5.9	4	39.3	47.2	7.5	0.5	0.4	7.2	7.9	4.1	1.1	0.8	3.3	4.9	3.9	0.9	0.6	3.3	4.5
31323	DM	SS	TH	AL	5	3	8	6	26.2	26.5	7.9	2.6	23.9	29.2	6.7	0.6	0.2	6.5	6.9	2.8	0.8	0.3	2.6	3.1	2.6	0.9	0.3	2.3	2.9
31324	DM	SS	TH	AL	3	4	4	7	31	50.1	21.3	4	46.1	54.1	1	0.7	0.1	0.9	1.1	0.6	0.6	0.1	0.4	0.7	1.8	1.7	0.3	1.5	2.1
31325	DM	SS	TH	AL	10	2	11	8	39.6	35.6	6.3	3.2	32.4	38.9	11.6	1.1	0.5	11.1	12.2	7.5	1.2	0.6	6.9	8.2	2.1	0.9	0.5	1.6	2.5
32111	DM	SM	TL	AU	15	9	7	17	93	94.5	4	2.8	91.7	97.3	4.3	0.7	0.5	3.8	4.8	2.1	0.7	0.5	1.6	2.6	2	0.8	0.6	1.4	2.6
32112	DM	SM	TL	AU	12	8	10	13	61.6	56.7	7.8	5.3	51.5	62	5.1	0.3	0.2	4.9	5.3	2.9	0.8	0.6	2.4	3.5	4.8	1.3	0.8	4	5.7
32113	DM	SM	TL	AU	17	7	14	15	76	76.8	7.6	5.5	71.3	82.3	12	0.7	0.5	11.5	12.5	8.8	1.3	0.9	7.9	9.7	2.1	1.4	1	1.1	3.1
32114	DM	SM	TL	AU	19	10	11	20	95.4	98.1	2.3	1.7	96.4	99.8	11.9	1.4	1	10.9	12.9	9.5	0.7	0.5	9	10	0.7	0.5	0.3	0.4	1
32115	DM	SM	TL	AU	12	7	1	13	76.8	76	5.2	3.7	72.3	79.7	1	0	0	1	1	0.2	0.4	0.3	-0.1	0.5	0.6	0.5	0.4	0.2	1
32121	DM	SM	TL	AL	19	8	7	13	77.7	77.2	3.2	2.3	74.9	79.5	5.4	0.7	0.5	4.9	5.9	4.6	0.5	0.4	4.2	5	1.8	0.4	0.3	1.5	2.1
32122	DM	SM	TL	AL	16	9	4	15	85.6	84.2	5.9	4.2	80	88.4	2.1	0.3	0.2	1.9	2.3	1.5	0.5	0.4	1.1	1.9	1.7	1.1	0.8	0.9	2.5
32123	DM	SM	TL	AL	15	10	9	16	90.4	85.1	5.7	4.1	81	89.2	6.2	0.4	0.3	5.9	6.5	4.7	1.1	0.8	3.9	5.5	2.7	0.8	0.6	2.1	3.3
32124	DM	SM	TL	AL	17	7	14	9	42.6	40.6	6.1	3.7	36.9	44.3	12.5	0.5	0.3	12.2	12.9	9.6	1	0.6	9	10.2	2.9	1.2	0.7	2.2	3.6
32125	DM	SM	TL	AL	12	7	9	13	75.8	76.8	9.1	6.5	70.3	83.3	8.2	0.4	0.3	7.9	8.5	6	1.1	0.8	5.2	6.8	2	0.9	0.7	1.3	2.7
32211	DM	SM	TE	AU	18	7	18	17	84.9	87.5	10.5	7.5	80	95	15.4	0.8	0.6	14.8	16	11.3	1.6	1.1	10.2	12.4	4.2	1.2	0.9	3.3	5.1

Table 15 (Continued)

Scenario									Objective Function						Ammo usage					Blue success					Hit asset				
Scenario Id	**Scenario Size**	**Scenario Size**	**Threat Level**	**Ammo Level**	**Number of Weapons**	**Number of Assets**	**Number of Threats**	**Number of Targets**	**Heuristic Objective**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**	**avg**	**std**	**hl**	**ll**	**ul**
32212	DM	SM	TE	AU	16	10	16	19	101.6	98.6	8.1	5.8	92.8	104.4	16.1	1.2	0.9	15.2	17	8.5	1.4	1	7.5	9.5	4.5	1	0.7	3.8	5.2
32213	DM	SM	TE	AU	20	10	20	22	115.7	112.9	12.4	8.8	104.1	121.8	18.6	1.3	0.9	17.7	19.5	12.3	1.7	1.2	11.1	13.5	4.9	1.3	0.9	4	5.8
32214	DM	SM	TE	AU	19	9	19	17	72.2	68	8.2	5.9	62.1	73.9	19.4	1.2	0.8	18.6	20.2	10.7	1.5	1.1	9.6	11.8	4.7	1.2	0.8	3.9	5.5
32215	DM	SM	TE	AU	19	9	19	19	101.4	101.3	7.4	5.3	96	106.6	21.2	1	0.7	20.5	21.9	12.6	1.6	1.1	11.5	13.7	3.8	1.4	1	2.8	4.8
32221	DM	SM	TE	AL	13	7	13	14	61.6	62.4	10.2	5.6	56.8	68	11.8	1.1	0.6	11.2	12.4	5.4	1.4	0.7	4.7	6.1	3.7	1.1	0.6	3.1	4.3
32222	DM	SM	TE	AL	20	7	20	16	79.1	70.2	9.9	6.7	63.5	76.9	15.6	1.6	1.1	14.5	16.7	10.6	1.1	0.8	9.9	11.4	5.2	1.4	0.9	4.2	6.1
32223	DM	SM	TE	AL	19	9	19	19	92.8	93.9	8.2	5.9	88	99.8	15.7	1.8	1.3	14.4	17	12.7	0.9	0.7	12	13.4	3.2	1.2	0.9	2.3	4.1
32224	DM	SM	TE	AL	19	10	19	22	118.2	118.8	10.4	7.4	111.4	126.2	21.1	1.2	0.9	20.2	22	12.6	1.3	1	11.6	13.6	3.9	1.2	0.9	3	4.8
32225	DM	SM	TE	AL	20	8	20	16	70.8	64.4	11.8	6.3	58.1	70.7	14.6	1.3	0.7	13.9	15.3	9	1.6	0.8	8.2	9.8	6	1.7	0.9	5.1	6.9
32311	DM	SM	TH	AU	18	9	30	15	59.5	53.2	15.9	4.3	48.9	57.5	23.9	1.9	0.5	23.4	24.4	15.1	1.4	0.4	14.7	15.5	7.9	2	0.5	7.4	8.5
32312	DM	SM	TH	AU	12	8	22	12	47.4	43.5	11.2	3.4	40.1	47	18	1.3	0.4	17.6	18.3	8.8	2.2	0.7	8.2	9.5	5.4	1.6	0.5	4.9	5.9
32313	DM	SM	TH	AU	16	8	21	20	112.8	111.1	12.8	9.2	101.9	120.3	18.2	1.5	1.1	17.1	19.3	10.2	1.4	1	9.2	11.2	5.8	1.8	1.3	4.5	7.1
32314	DM	SM	TH	AU	20	8	23	20	89	84.1	9.7	6.9	77.2	91	20.2	1.4	1	19.2	21.2	14.2	1.7	1.2	13	15.4	5.4	2	1.4	4	6.8
32315	DM	SM	TH	AU	19	8	26	21	103.6	108.2	5.1	3.6	104.6	111.8	24.2	1.8	1.3	22.9	25.5	16	1.1	0.8	15.2	16.8	4.8	0.4	0.3	4.5	5.1
32321	DM	SM	TH	AL	14	7	19	14	47.8	50.3	8.7	4.3	46	54.7	19.1	1.2	0.6	18.5	19.7	11.1	1.4	0.7	10.4	11.8	4.2	1.3	0.7	3.6	4.9
32322	DM	SM	TH	AL	19	8	31	18	63.4	82.9	27.9	6.1	76.8	89	23.8	11	2.4	21.4	26.2	14.5	7	1.5	13	16.1	8.4	4.3	0.9	7.5	9.3
32323	DM	SM	TH	AL	13	9	20	14	60.4	63.3	10.7	6.2	57.1	69.5	13.6	0.9	0.5	13	14.1	8.2	1.4	0.8	7.4	9	5.7	1.5	0.9	4.8	6.6

Table 15 (Continued)

Scenario									Objective Function						Ammo usage					Blue success					Hit asset				
Scenario Id	Scenario Size	Scenario Size	Threat Level	Ammo Level	Number of Weapons	Number of Assets	Number of Threats	Number of Targets	Heuristic Objective	avg	std	hl	ll	ul	avg	std	hl	ll	ul	avg	std	hl	ll	ul	avg	std	hl	ll	ul
32324	DM	SM	TH	AL	13	9	23	15	48.7	68.4	22.9	4.9	63.5	73.4	18.7	9	1.9	16.8	20.7	9	4.5	1	8	10	6.9	3.6	0.8	6.1	7.7
32325	DM	SM	TH	AL	16	7	29	12	41.4	36.7	9.6	3	33.8	39.7	23	1.2	0.4	22.6	23.4	10.1	1.7	0.5	9.6	10.6	7	1.5	0.4	6.5	7.4
33111	DM	SL	TL	AU	37	17	30	35	221.6	226.1	7.6	5.4	220.7	231.5	35.6	2.4	1.7	33.9	37.3	26.5	1.4	1	25.5	27.5	2.4	1.5	1.1	1.3	3.5
33112	DM	SL	TL	AU	32	19	32	29	123.4	122.9	9.4	6.7	116.2	129.6	33.3	1.5	1.1	32.2	34.4	19.6	1.5	1.1	18.5	20.7	7.6	1.8	1.3	6.3	8.9
33113	DM	SL	TL	AU	24	13	1	25	161	161.4	3.4	2.4	159	163.8	1.4	0.5	0.4	1	1.8	0.7	0.5	0.3	0.4	1	0.2	0.4	0.3	0	0.5
33114	DM	SL	TL	AU	26	15	9	32	189.5	192.4	11	7.9	184.5	200.3	7	0	0	7	7	5.6	1.2	0.8	4.8	6.4	2	1.2	0.9	1.1	2.9
33115	DM	SL	TL	AU	27	16	2	30	180.8	181	4.2	3	178	184	2	0	0	2	2	1.7	0.5	0.3	1.4	2	0.2	0.4	0.3	0	0.5
33121	DM	SL	TL	AL	36	13	34	27	131.6	127.4	14	10	117.4	137.4	30.8	2.6	1.9	28.9	32.7	21.9	2.8	2	19.9	23.9	6.9	2.1	1.5	5.4	8.4
33122	DM	SL	TL	AL	26	20	5	32	227.6	227.9	3.5	2.5	225.4	230.4	5.2	0.4	0.3	4.9	5.5	4.9	0.3	0.2	4.7	5	0.1	0.3	0.2	0	0.3
33123	DM	SL	TL	AL	39	12	35	32	158.7	152.8	7.9	5.7	147.1	158.5	29.9	1.8	1.3	28.6	31.2	19.6	2.8	2	17.6	21.6	8.6	1.3	0.9	7.7	9.5
33124	DM	SL	TL	AL	23	13	15	27	188.4	188.9	8.2	5.9	183	194.8	14.1	1.3	0.9	13.2	15	10.9	1	0.7	10.2	11.6	1.8	0.9	0.7	1.1	2.5
33125	DM	SL	TL	AL	29	14	11	30	199	199.9	5.4	3.9	196	203.8	9.7	0.8	0.6	9.1	10.3	8.1	1	0.7	7.4	8.8	1.7	0.9	0.7	1	2.4
33211	DM	SL	TE	AU	26	13	26	26	133.1	127.6	9	6.4	121.2	134	27.6	1.6	1.2	26.4	28.8	18.1	1	0.7	17.4	18.8	4.9	1	0.7	4.2	5.6
33212	DM	SL	TE	AU	33	15	33	31	162.4	165	7.3	5.2	159.8	170.3	34	0.8	0.6	33.4	34.6	22.3	0.9	0.7	21.6	23	5.2	1.4	1	4.2	6.2
33213	DM	SL	TE	AU	26	19	26	31	167.7	168.4	19.5	13.9	154.5	182.3	25.6	1.8	1.3	24.3	26.9	15.1	1.9	1.3	13.8	16.4	5	2.3	1.6	3.4	6.6
33214	DM	SL	TE	AU	27	18	27	31	188.4	192.4	8.7	6.2	186.2	198.6	26.3	1.7	1.2	25.1	27.5	17	1.6	1.2	15.8	18.2	4.2	1.1	0.8	3.4	5
33215	DM	SL	TE	AU	38	15	38	32	196.3	195.3	15.3	10.9	184.4	206.3	37.2	1.5	1.1	36.1	38.3	24.5	2.4	1.7	22.8	26.2	5.8	2.3	1.7	4.1	7.5

Table 15 (Continued)

Scenario									Objective Function						Ammo usage					Blue success					Hit asset				
Scenario Id	Scenario Size	Scenario Size	Threat Level	Ammo Level	Number of Weapons	Number of Assets	Number of Threats	Number of Targets	Heuristic Objective	avg	std	hl	ll	ul	avg	std	hl	ll	ul	avg	std	hl	ll	ul	avg	std	hl	ll	ul
33221	DM	SL	TE	AL	38	13	38	29	121.2	120.3	15.8	11.3	109	131.6	35.1	1.2	0.9	34.2	36	24.2	0.9	0.7	23.5	24.9	8.1	2.2	1.6	6.5	9.7
33222	DM	SL	TE	AL	33	17	33	32	182	191.4	17.5	12.6	178.9	204	31.4	1.7	1.2	30.2	32.6	22.9	1.9	1.3	21.6	24.2	5.6	2.3	1.7	3.9	7.3
33223	DM	SL	TE	AL	34	15	34	30	178.1	178.5	8.8	6.3	172.2	184.8	33	1.8	1.3	31.7	34.3	24.5	0.8	0.6	23.9	25.1	4.3	0.9	0.7	3.6	5
33224	DM	SL	TE	AL	32	15	32	30	163.5	164.8	8.8	6.3	158.5	171.1	31.7	0.7	0.5	31.2	32.2	20.5	1.7	1.2	19.3	21.7	6	1.3	1	5	7
33225	DM	SL	TE	AL	23	18	23	28	146.3	144.1	10.7	7.7	136.4	151.8	21.4	1.8	1.3	20.1	22.7	14.1	1.7	1.2	12.9	15.3	4.5	1.4	1	3.5	5.5
33311	DM	SL	TH	AU	34	13	53	32	121.8	116.7	14.7	10.5	106.2	127.2	46.1	2.5	1.8	44.3	47.9	27.5	2.1	1.5	26	29	12.2	2.7	1.9	10.3	14.1
33312	DM	SL	TH	AU	36	20	53	38	173	164.6	8.5	6.1	158.5	170.7	54.5	2.1	1.5	53	56	33.7	1.7	1.2	32.5	34.9	11.2	1	0.7	10.5	11.9
33313	DM	SL	TH	AU	33	15	64	34	124.6	120.7	15.6	11.2	109.5	131.9	57.6	2.8	2	55.6	59.6	33.1	2.8	2	31.1	35.1	15.2	1.5	1.1	14.1	16.3
33314	DM	SL	TH	AU	29	20	34	39	227.9	222.9	18.5	13.2	209.7	236.1	33.9	1.1	0.8	33.1	34.7	21.5	2.1	1.5	20	23	7.1	2.6	1.9	5.2	9
33315	DM	SL	TH	AU	36	14	70	35	120.8	106.3	14.7	9.4	97	115.7	50.4	1.9	1.2	49.2	51.6	28.8	2	1.3	27.5	30.1	18.7	2.4	1.5	17.1	20.2
33321	DM	SL	TH	AL	22	17	24	29	139.4	140.2	10.5	7.5	132.7	147.7	22.7	1.7	1.2	21.5	23.9	13	2.4	1.8	11.2	14.8	5.4	1.3	1	4.4	6.4
33322	DM	SL	TH	AL	40	17	55	40	182.1	171.1	18.1	13	158.1	184.1	46.2	1.9	1.3	44.9	47.5	25.6	2.5	1.8	23.8	27.4	16	1.9	1.3	14.7	17.3
33323	DM	SL	TH	AL	24	12	47	24	90	77.5	13.3	7.4	70.2	84.9	30.9	1.8	1	29.9	31.9	17.7	2.3	1.3	16.4	19	14.6	1.5	0.9	13.7	15.5
33324	DM	SL	TH	AL	27	19	44	34	151.4	140.8	12.1	8.7	132.1	149.5	35.9	2	1.4	34.5	37.3	21.7	2	1.4	20.3	23.1	11.5	2.1	1.5	10	13
33325	DM	SL	TH	AL	30	20	54	37	145.6	143.7	18	12.9	130.9	156.6	42.5	1.3	0.9	41.6	43.4	26.2	2.4	1.7	24.5	27.9	15.4	2.5	1.8	13.6	17.2

APPENDIX E

SIMULATION RESULTS-DETAILED REPORTS of SCENARIO 11122

Table 16. Report of Ammunition Usage

Replication ID	Ammo Type 1	Ammo Type 2	Ammo Type 3	Ammo Type 4	Ammo Type 5	Ammo Type 6	Ammo Type 7	Ammo Type 8	Ammo Type 9	Total Ammo Used	Threat Type 1	Threat Type 2
1	0	0	0	0	1	1	0	2	0	4	0	0
2	0	0	0	0	1	1	0	2	0	4	0	1
3	0	0	0	0	1	2	0	1	0	4	0	0
4	0	0	0	0	1	2	0	2	0	5	0	0
5	0	0	0	0	1	1	0	1	0	3	0	0
6	0	0	0	0	1	1	0	1	0	3	0	0
7	0	0	0	0	1	2	0	2	0	5	0	1
8	0	0	0	0	1	1	0	1	0	3	0	0
9	0	0	0	0	1	1	0	1	0	3	0	0
10	0	0	0	0	1	1	0	1	0	3	0	0

Table 17. Report of Blue Success

Replication ID	Ammo Type 1	Ammo Type 2	Ammo Type 3	Ammo Type 4	Ammo Type 5	Ammo Type 6	Ammo Type 7	Ammo Type 8	Ammo Type 9	Total Success	Assets Hit	Threat Type 1	Threat Type 2
1	0	0	0	0	1	1	0	1	0	3	0	0	0
2	0	0	0	0	1	1	0	0	0	2	1	0	1
3	0	0	0	0	0	2	0	1	0	3	0	0	0
4	0	0	0	0	0	2	0	1	0	3	0	0	0
5	0	0	0	0	1	1	0	1	0	3	0	0	0
6	0	0	0	0	1	1	0	1	0	3	0	0	0
7	0	0	0	0	0	2	0	0	0	2	0	0	0
8	0	0	0	0	1	1	0	1	0	3	0	0	0
9	0	0	0	0	1	1	0	1	0	3	0	0	0
10	0	0	0	0	1	1	0	1	0	3	0	0	0

APPENDIX F

SIMULATION RESULTS – SIMULATION OUTPUT of SCENARIO 11122

Table 18. Simulation Output of Scenario 11122

Replication ID	Time To Fire the Projectile	Time to Impact	Assignment Type	Firer Id	Firer Type	Target Id	Target Type	Order of Shots	Probability of Hit	Success	Usage
1	2.52	4.97	1	5	6	1	2	1	0.99	1	1
1	2.61	5.31	1	8	5	2	1	1	0.82	1	1
1	3.77	6.63	1	2	8	3	2	1	0.71	0	1
1	4.97	7.67	1	6	8	1	2	2	0.96	0	0
1	5.31	8.18	1	5	6	2	1	2	0.70	0	0
1	6.63	9.36	1	1	8	3	2	2	0.59	1	1
1	8.56	8.56	2	2	1	13	0	1	0.88	0	0
1	9.39	13.33	2	3	2	14	0	1	0.34	0	0
1	10.74	12.50	2	1	2	12	0	1	0.58	0	0
2	2.52	4.97	1	5	6	1	2	1	0.99	1	1
2	2.61	5.31	1	8	5	2	1	1	0.82	1	1
2	3.77	6.63	1	2	8	3	2	1	0.71	0	1
2	4.97	7.67	1	6	8	1	2	2	0.96	0	0
2	5.31	8.18	1	5	6	2	1	2	0.70	0	0
2	6.63	9.36	1	1	8	3	2	2	0.59	0	1
2	8.56	8.56	2	2	1	13	0	1	0.88	0	0
2	9.39	13.33	2	3	2	14	0	1	0.34	1	1
2	10.74	12.50	2	1	2	12	0	1	0.58	0	0
3	2.52	4.97	1	5	6	1	2	1	0.99	1	1
3	2.61	5.31	1	8	5	2	1	1	0.82	0	1
3	3.77	6.63	1	2	8	3	2	1	0.71	1	1
3	4.97	7.67	1	6	8	1	2	2	0.96	0	0
3	5.31	8.18	1	5	6	2	1	2	0.70	1	1
3	6.63	9.36	1	1	8	3	2	2	0.59	0	0
3	8.56	8.56	2	2	1	13	0	1	0.88	0	0
3	9.39	13.33	2	3	2	14	0	1	0.34	0	0
3	10.74	12.50	2	1	2	12	0	1	0.58	0	0
4	2.52	4.97	1	5	6	1	2	1	0.99	1	1
4	2.61	5.31	1	8	5	2	1	1	0.82	0	1

Table 18 (Continued)

Replication ID	**Time To Fire the Projectile**	**Time to Impact**	**Assignment Type**	**Firer Id**	**Firer Type**	**Target Id**	**Target Type**	**Order of Shots**	**Probability of Hit**	**Success**	**Usage**
4	3.77	6.63	1	2	8	3	2	1	0.71	0	1
4	4.97	7.67	1	6	8	1	2	2	0.96	0	0
4	5.31	8.18	1	5	6	2	1	2	0.70	1	1
4	6.63	9.36	1	1	8	3	2	2	0.59	1	1
4	8.56	8.56	2	2	1	13	0	1	0.88	0	0
4	9.39	13.33	2	3	2	14	0	1	0.34	0	0
4	10.74	12.50	2	1	2	12	0	1	0.58	0	0
5	2.52	4.97	1	5	6	1	2	1	0.99	1	1
5	2.61	5.31	1	8	5	2	1	1	0.82	1	1
5	3.77	6.63	1	2	8	3	2	1	0.71	1	1
5	4.97	7.67	1	6	8	1	2	2	0.96	0	0
5	5.31	8.18	1	5	6	2	1	2	0.70	0	0
5	6.63	9.36	1	1	8	3	2	2	0.59	0	0
5	8.56	8.56	2	2	1	13	0	1	0.88	0	0
5	9.39	13.33	2	3	2	14	0	1	0.34	0	0
5	10.74	12.50	2	1	2	12	0	1	0.58	0	0
6	2.52	4.97	1	5	6	1	2	1	0.99	1	1
6	2.61	5.31	1	8	5	2	1	1	0.82	1	1
6	3.77	6.63	1	2	8	3	2	1	0.71	1	1
6	4.97	7.67	1	6	8	1	2	2	0.96	0	0
6	5.31	8.18	1	5	6	2	1	2	0.70	0	0
6	6.63	9.36	1	1	8	3	2	2	0.59	0	0
6	8.56	8.56	2	2	1	13	0	1	0.88	0	0
6	9.39	13.33	2	3	2	14	0	1	0.34	0	0
6	10.74	12.50	2	1	2	12	0	1	0.58	0	0
7	2.52	4.97	1	5	6	1	2	1	0.99	1	1
7	2.61	5.31	1	8	5	2	1	1	0.82	0	1
7	3.77	6.63	1	2	8	3	2	1	0.71	0	1
7	4.97	7.67	1	6	8	1	2	2	0.96	0	0
7	5.31	8.18	1	5	6	2	1	2	0.70	1	1
7	6.63	9.36	1	1	8	3	2	2	0.59	0	1
7	8.56	8.56	2	2	1	13	0	1	0.88	0	0
7	9.39	13.33	2	3	2	14	0	1	0.34	0	1
7	10.74	12.50	2	1	2	12	0	1	0.58	0	0
8	2.52	4.97	1	5	6	1	2	1	0.99	1	1
8	2.61	5.31	1	8	5	2	1	1	0.82	1	1
8	3.77	6.63	1	2	8	3	2	1	0.71	1	1
8	4.97	7.67	1	6	8	1	2	2	0.96	0	0
8	5.31	8.18	1	5	6	2	1	2	0.70	0	0
8	6.63	9.36	1	1	8	3	2	2	0.59	0	0
8	8.56	8.56	2	2	1	13	0	1	0.88	0	0
8	9.39	13.33	2	3	2	14	0	1	0.34	0	0

Table 18 (Continued)

Replication ID	Time To Fire the Projectile	Time to Impact	Assignment Type	Firer Id	Firer Type	Target Id	Target Type	Order of Shots	Probability of Hit	Success	Usage
8	10.74	12.50	2	1	2	12	0	1	0.58	0	0
9	2.52	4.97	1	5	6	1	2	1	0.99	1	1
9	2.61	5.31	1	8	5	2	1	1	0.82	1	1
9	3.77	6.63	1	2	8	3	2	1	0.71	1	1
9	4.97	7.67	1	6	8	1	2	2	0.96	0	0
9	5.31	8.18	1	5	6	2	1	2	0.70	0	0
9	6.63	9.36	1	1	8	3	2	2	0.59	0	0
9	8.56	8.56	2	2	1	13	0	1	0.88	0	0
9	9.39	13.33	2	3	2	14	0	1	0.34	0	0
9	10.74	12.50	2	1	2	12	0	1	0.58	0	0
10	2.52	4.97	1	5	6	1	2	1	0.99	1	1
10	2.61	5.31	1	8	5	2	1	1	0.82	1	1
10	3.77	6.63	1	2	8	3	2	1	0.71	1	1
10	4.97	7.67	1	6	8	1	2	2	0.96	0	0
10	5.31	8.18	1	5	6	2	1	2	0.70	0	0
10	6.63	9.36	1	1	8	3	2	2	0.59	0	0
10	8.56	8.56	2	2	1	13	0	1	0.88	0	0
10	9.39	13.33	2	3	2	14	0	1	0.34	0	0
10	10.74	12.50	2	1	2	12	0	1	0.58	0	0

APPENDIX G

SIMULATION RESULTS-AMMUNITION USAGE vs BLUE SUCCESS

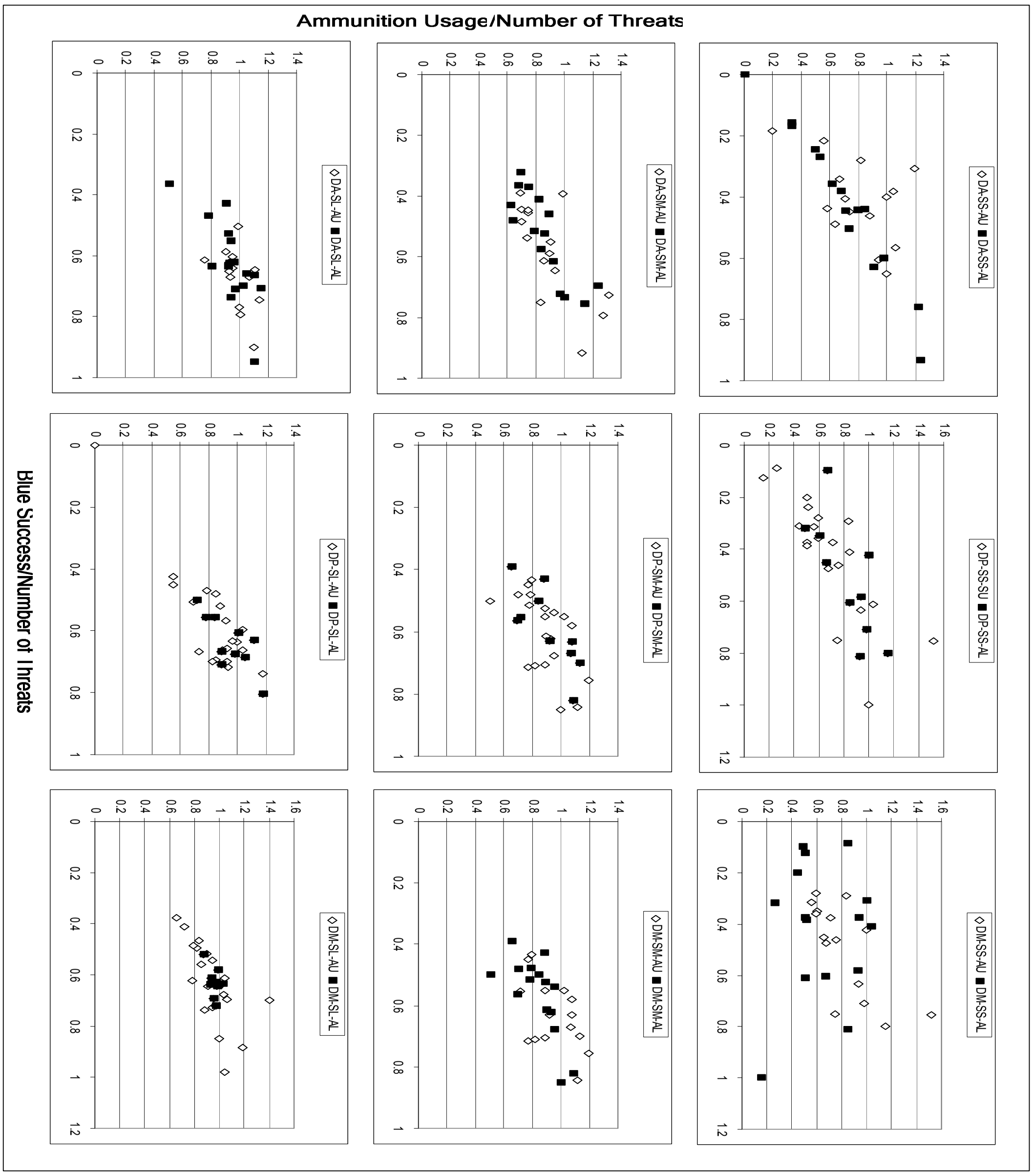

Figure 31 Ammunition Usage vs Blue Succes with respect to Scenario Settings