DOWNLINK TRANSMISSION TECHNIQUES FOR MULTI USER MULTI INPUT MULTI OUTPUT WIRELESS COMMUNICATIONS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

ADEM COŞKUN

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
ELECTRICAL AND ELECTRONICS ENGINEERING

AUGUST 2007

Approval of the thesis:

DOWNLINK TRANSMISSION TECHNIQUES FOR MULTI USER MULTI INPUT MULTI OUTPUT WIRELESS COMMUNICATIONS

submitted by **ADEM COŞKUN** in partial fulfillment of the requirements for the degree of **Master of Science** in **Electrical and Electronics Engineering Department, Middle East Technical University** by

Prof. Dr. Canan Özgen __________________

Dean, Graduate School of **Natural and Applied Sciences**

Prof. Dr. İsmet Erkmen __________________

Head of Department, **Electrical and Electronics Engineering**

Assist. Prof. Dr. Çağatay Candan __________________

Supervisor, **Electrical and Electronics Engineering Dept., METU**

Examining Committee Members:

Prof. Dr. Yalçın Tanık __________________

Electrical and Electronics Engineering Dept., METU

Assist. Prof. Dr. Çağatay Candan __________________

Electrical and Electronics Engineering Dept., METU

Assist. Prof. Dr. Ali Özgür Yılmaz __________________

Electrical and Electronics Engineering Dept., METU

Assist. Prof. Dr. İlkay Ulusoy __________________

Electrical and Electronics Engineering Dept., METU

Ahmet Doğrusöz, PhD __________________

ASELSAN

Date:

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name:

Signature

ABSTRACT

DOWNLINK TRANSMISSION TECHNIQUES FOR MULTI USER MULTI INPUT MULTI OUTPUT WIRELESS COMMUNICATIONS

COŞKUN, Adem

M.S., Department of Electrical and Electronics Engineering
Supervisor: Assist. Prof. Dr. Çağatay CANDAN

August 2007, 81 pages

Multi-user MIMO (MIMO-MU) communication techniques make use of available channel state information at the transmitter to mitigate the inter-user interference. The goal of these techniques is to provide the least interference at the mobile stations by applying a precoding operation.

In this thesis a comparison of available techniques in the literature such as Channel Decomposition, SINR Balancing, Joint-MMSE optimization is presented. Novel techniques for the MIMO multi-user downlink communication systems, where a single stream is transmitted to each user are proposed. The proposed methods, different from the other methods in the literature, use a simple receiver to combat the interference. It has been shown that MRC based receivers are as good as more complicated joint MMSE receivers.

Keywords: Maximal Ratio Combining (MRC), Signal to interference-plus-noise ratio (SINR) Balancing, Multi-Input Multi Output (MIMO) antennas, Inter user Interference, Multiple Access, Downlink

ÖZ

ÇOK GİRDİLİ ÇOK ÇIKTILI VE ÇOK KULLANICILI KABLOSUZ HABERLEŞME İÇİN İNİŞ BAĞI İLETİM TEKNİKLERİ

COŞKUN, Adem

Yüksek Lisans, Elektrik ve Elektronik Mühendisliği Bölümü
Tez Yöneticisi: Y.Doç.Dr. Çağatay CANDAN

Ağustos 2007, 81 sayfa

Çok girdili çok çıktılı (MIMO) ve çok kullanıcılı haberleşme teknikleri, kanal durum bilgisinden faydalanarak kullanıcılar arasındaki girişimi azaltırlar. Bu teknikler ön-kodlama işlemleri uygulayarak mobil istasyonlardaki girişimi en aza indirmeyi amaçlar.

Bu tezde literatüde yeralan Kanalın Ayrıştırılması, SINR (İşaretin girişim-artı-gürültüye oranı) Dengelenmesi, Ortak MMSE optimizasyonu teknikleri karşılaştırılmakta ve her bir kullanıcısı tekil veri akışı alan çok kullanıcılı MIMO iniş bağı haberleşme sistemleri için yeni teknikler önerilmektedir. Önerilen metotlar girişimle mücadele ederken, literatürdeki diğer metotlardan farklı olarak basit yapılı alıcılar kullanmaktadır. Tezde MRC tabanlı alıcıların daha karmaşık Ortak MMSE alıcıları kadar iyi çalıştıkları gösterilmiştir.

Anahtar Kelimeler: Azami oran birleştiricisi (MRC), İşaretin girişim-artı-gürültüye oranının (SINR) dengelenmesi, Çok girdili çok çıktılı (MIMO) antenler, Kullanıcılar arası girişim, Çoklu erişim, İniş Bağı

ACKNOWLEDGEMENTS

Before starting, I should express my sincere respects and thanks from the bottom of my hearth for my thesis supervisor Cagatay Candan. Without his kind efforts and intelligence I don't think that I would have lead to that stage of my master of science study in METU. During that thesis study I have always found his guidance beside me, which was the one and only support of mine through that work. With his contributions to my academic knowledge, I really feel myself more in Telecommunications now.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ABBREVATIONS

BER	: Bit Error Rate
BS	: Base Station
BC	: Broadcast Channel
CDMA	: Code Division Multiple Access
CSI	: Channel State Information
CSIT	: Channel State Information at the Transmitter
DFE	: Decision Feedback Equalizer
DPC	: Dirty Paper Coding
FCR	: Fixed Combining Rule
FDMA	: Frequency Division Multiple Access
IID	: Independently Identically Distributed
JMMSE	: Joint Minimum Mean Squared Error
MAC	: Multiple Access Channel
MAI	: Multiple Access Interference
MIMO	: Multi Input Multi Output
MIMO-MU	: Multi User MIMO
MIMO-SU	: Single User MIMO
MISO	: Multi Input Single Output
ML	: Maximum Likelihood
MMSE	: Minimum Mean Squared Error
MRC	: Maximum Ratio Combining
OFDM	: Orthogonal Frequency-Division Multiplexing
QAM	: Quadrature Amplitude Modulation
QPSK	: Quadrature Phase Shift Keying
SDMA	: Space Division Multiple Access
SINR	: Signal to Interference plus Noise Ratio
SISO	: Single Input Single Output

SNR	: Signal to Noise Ratio
ST	: Space Time
STBC	: Space Time Block Coding
STC	: Space-Time Coding
STTC	: Space-time Trellis Coding
SUC	: Successive-Interference-Cancellation
SVD	: Singular Value Decomposition
TDMA	: Time Division Multiple Access
T-MMSE	: Total Minimum Mean Square Error
TrMMSE	: Transmit Minimum Total Mean Square Error
ZF	: Zero Forcing

CHAPTER 1

INTRODUCTION

Wireless signal transmission technology provides its users a convenient, low-cost and an easy way of communication. However the efficiency and reliability concerns of wireless systems bring many challenges. Different from a wired system, the most distinct difficulty, associated with wireless system design, originates from the nature of the *wireless channels*. Wireless channels are *time-varying* and with a *fading* nature, including variations in the received signal power over the distance due to *path loss* and *shadowing*. The propagation environment of a wireless signal is the *space* with many *reflective/refractive* surfaces on the way of signal from its origin to its destination point. The time varying and *multipath* fading nature of the wireless channel occurs because of the lack of precise control of the medium. *Multipath effect* exists as the receipt of multiple versions of the transmitted signal with different delays at the receiver. These different signal versions, reached to the receivers, have constructive or destructive effects onto each other resulting with *fading* [1].

Diversity, which is the effect that causes the generation of multiple independently faded replicas of the same signal, might be introduced to the wireless systems to combat with fading. By sending signals, that carry the same information through different independent paths, multiple independently faded replicas of data symbols are obtained at the receiver ends [2]. Appropriate analytical manipulations on the received replicas of the transmitted signal may offer a noteworthy success in reshaping the transmitted signal and more reliable detection can be achieved. Diversity can be realized over different forms such as; time, frequency, space etc. providing that the slots, used for replica generation, be independent from each other (*coherency*). *Spatial diversity* (which may also be called *antenna diversity*) can be obtained by placing multiple antennas at the transmitter and/or the receiver, which would generate multiple *diversity branches* in between the transmitter and the receiver. Therefore, independent signal paths can be created in between individual transmit and receive antennas, providing that the multiple antennas are sufficiently separated. *MIMO (Multi-input Multi-output)* is the configuration for this multi antenna solution. Error probability as a function of the SNR, for different numbers of diversity branches *L,* is plotted in Figure 1.1. It can be understood from Figure 1.1 that, increasing *L* dramatically decreases the error probability [2]. The slopes of the curves are getting higher proportional to the total number of diversity branches, promising a more reliable communication of less error.

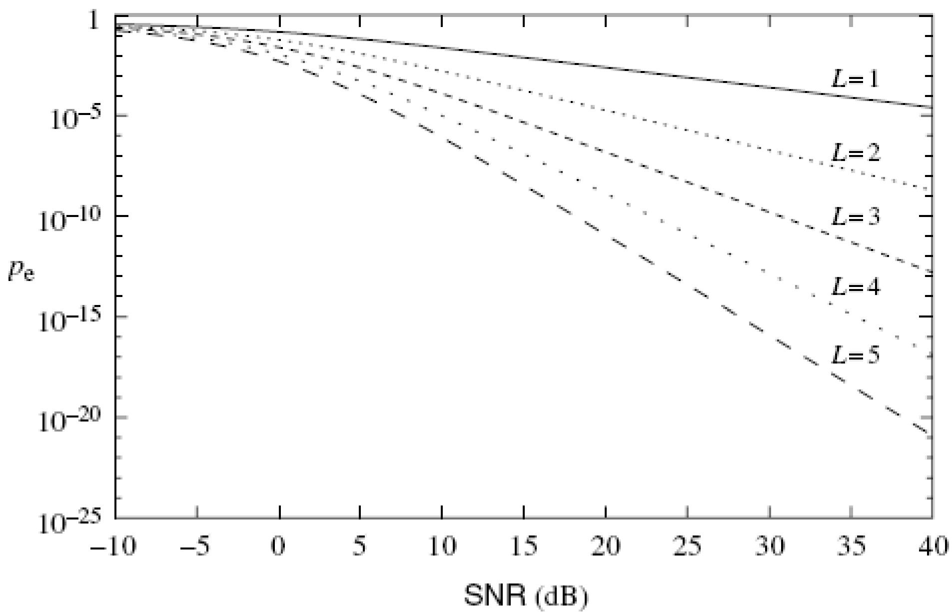

Figure 1.1 Average error probability plot of a MIMO system with different numbers of diversity branches

Similarly, a wireless link, where there is only one single antenna at the transmitter and at the receiver, can be named *SISO* (*Single input Single Output*). To overcome multipath fading in a *SISO* system, *equalization*, *coding* or *interleaving* techniques can be used [3].

Multiplexing independent data streams in space is called *Spatial Multiplexing (SM)* [2]. Since we have multiple input and output points in our communication link, multiple independent streams can also be loaded simultaneously to the system with the use of MIMO. In [4] it has been shown the *spatial multiplexing* may result in a theoretical *capacity* gain of *min(M,N),* where *N* is the number of receiving and *M* is the number of transmitting antennas. Thus, in addition to providing *diversity*, the use of *MIMO* antennas may also increase the communication capacity between a point to another point. However the ability of the multiple antenna channels to provide combinations of *diversity* and *spatial multiplexing* has to be determined and the trade-off between how much each type of gain one can achieve has to be explored before using *MIMO* wireless link.

Moreover, with a proper beamforming scheme, the array of antennas at the receiver and/or transmitter may let the desired signal to add coherently at the receiver [5]. Thus, multiple antennas may also provide an increase in *SNR*.

MIMO and the use of space have also extensions on *coding*. *Space Time Codes (STC*), [6, 7], which combines the space and time slots for generating reliable coding schemes, provide *coding gain* in addition to providing diversity. There are several approaches to STC, which are different in coding structures (e.g. *Space Time Block Coding (STBC), Space-time trellis Coding (STTC)*). STC may also be shown as a generalization of *delay diversity* [8].

CONTENT OF THE THESIS RESEARCH

With the next chapter, we start by providing a review of the MIMO receiver architectures and some important subjects that will frequently be used throughout the thesis, such as the *SINR* (Signal to Interference plus Noise Ratio) and *the Duality Principle*. Therewith, we will make the problem description of a wireless multiple access (multi user) MIMO channel, where several mobile users are coordinated by a single base station. Such a channel is called *MIMO-MU (Multi-user MIMO).* The effect of SINR in *downlink* signal transmission side of the multi user MIMO communication will be mainly concentrated in that chapter.

In the literature there are several approaches, proposed for the mitigation of the negative effects of SINR evident in multiple access downlink signal transmission. A number of previously proposed approaches will be explained in Chapter 3. A detailed comparison of those techniques will be given and the advantages and disadvantages of each technique will be explained, supported with computer simulations.

In Chapter 4, we will focus on a multi user MIMO downlink communication scenario, in which each mobile user receives a *single stream* of data, where the users are utilized with multiple antennas. Also for this special case, the performance of the techniques, given in Chapter 3, will be simulated. Besides, three novel techniques will be proposed, each having different functional and/or performance benefits.

Although the mobile users might contain many antennas for signal reception, the scenario in Chapter 4 puts forward a very simple situation, where they only receive single streams of data. Therefore one of the design criterions of the new proposals will be simplifying the receiver structures of mobile users after the antenna reception, regardless of the complex antenna arrays of the users. That is a plausible criterion, which also lets the users make us of their multiple antennas from diversity point of view without adding extra implementations after the signal reception. For the proposed techniques, we will examine the use of *MRC (Maximal Ratio Combining)* at the receivers and the transmit precoding will be optimized for such receivers.

CHAPTER 2

PROBLEM DESCRIPTION

In this chapter, we present the description of *MIMO-MU* downlink communication problem. We will first examine the receivers for single user MIMO systems and then the details of multi user receivers with SINR relations and quality results will be given. Throughout this thesis all the *channels*, which are denoted by the transfer functions of the communication system, are assumed to be *flat fading* (frequency flat). The elements of the channels are independently identically distributed *(i.i.d.)* zero mean complex Gaussian random variables with unit variance. The elements of the additive noise vector at each receiving antenna are also independently identically distributed *(i.i.d.)* zero mean complex Gaussian random variables with variance σ^2.

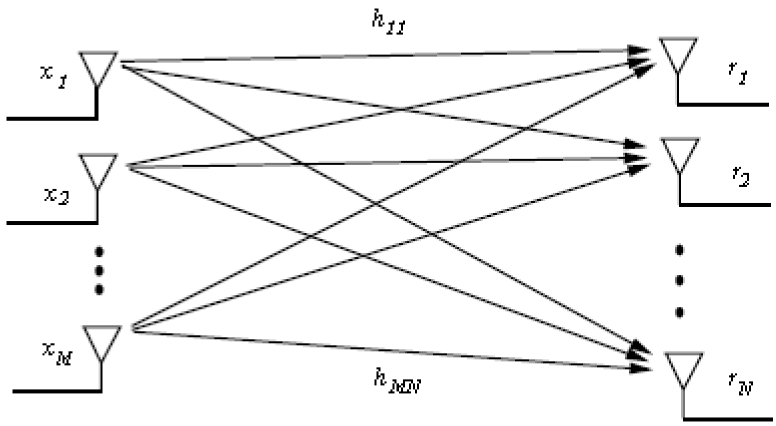

Figure 2.1 Single User MIMO system configuration

2.1 SINGLE USER MIMO RECEIVERS

Consider a narrowband (flat-fading) point-to-point wireless link employing *N* receive antennas and *M* transmitting antennas, which is shown in Figure 2.1. The object of the transmission is to transmit *M* data symbols, indicated with vector *x*. Thus the channel can be modeled by a matrix *H* of dimension *NxM*. If each transmitting antenna is fed by one data symbol, x_i , then the the signal model can be constructed as;

$$\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1M} \\ h_{21} & \ddots & & h_{2M} \\ \vdots & & \ddots & \vdots \\ h_{N1} & h_{N2} & \cdots & h_{NM} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_N \end{bmatrix} \tag{2.1}$$

$$r = Hx + n \tag{2.2}$$

where *r* is the received vector with dimension *Nx1* and *x* is the transmitted signal vector of dimension *Mx1*. *n* is the additive white Gaussian noise vector, with variance σ^2 for its every vector component. The complex channel coefficients h_{ij} represents the channel gains from transmit antenna *j* to receive antenna *i*. The transmit power constraint at the transmitter is;

$$E\{xx^H\} = I, \quad or\ similarly \quad E\{trace[xx^H]\} = M \tag{2.3}$$

which means for every data symbol, a transmit power value of *1* is assigned. Notation $E\{\ \}$ represents expectation operation.

In order to remove the effects of interfering streams and the noise and to make use of the multiple antennas, occupied both at the transmitter and at the receiver, a filter *V* should be placed after the signal reception in finding the linear estimate of *x*. This filter may also be called "*decoder*". In such a single user MIMO antenna system (*MIMO-SU*) the data symbols can be collected as shown in Figure 2.2.

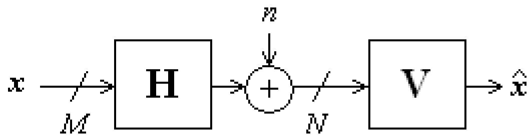

Figure 2.2 MIMO-SU communication channel

The linear estimate of *x,* found manupilating the received vector *r,* can be formulated as follows;

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \vdots \\ \hat{x}_M \end{bmatrix} = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1N} \\ v_{21} & \ddots & & v_{2N} \\ \vdots & & \ddots & \vdots \\ v_{M1} & v_{M2} & \cdots & v_{MN} \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{bmatrix} \tag{2.4}$$

$$\hat{x} = Vr = V\left(Hx + n\right) \tag{2.5}$$

where V is an MxN matrix for processing the received signal. For MIMO-SU, the most common receiver structures, used for vector decoding, are *ML* (Maximum Likelihood), *MMSE* (Minimum Mean Squared Error), *MF* (Matched Filter) and *ZF* (Zero Forcing) receivers. During receiver considerations, it will be assumed that the realization of the channel H is estimated and known to the receiver. In addition to these receiver structures, SVD (Singular Value Decomposition) method will also be explained in this chapter. This technique requires transmitter side precoding, using the channel state information as will be explained later.

2.1.1 ML Receiver

The ML method forms a set of vectors, which are possible to be received at one channel use, by using the apriori channel knowledge and the data symbols to be used. Afterwards, it chooses the element of the set that minimizes the average probability of error and assigns the data symbol vector that the chosen vector corresponds to. Therefore, assuming the noise is additive white Gaussian, ML chooses the vector x that solves;

$$\hat{x} = \arg\min_{x} \|r - Hx\|_F^2 \tag{2.6}$$

where the optimization can be performed through an exhaustive search over all candidate vector symbols x, as explained. Where A is the size of the data symbol set (e.g. $A=4$ for *QPSK*) and M is the number of symbols, transmitted per channel usage, an ML receiver implements a search over A^M possibilities for every received vector. There are several fast algorithms called *sphere decoding* to reduce the complexity [9].

ML technique gives the minimum probability of decoding error solution for a MIMO-SU communication system. Thus, ML is the most appropriate technique if hardware cost of its implementation (complexity) can be afforded. The Viterbi algorithm can be used to implement ML receivers.

2.1.2 MF Receiver

To maximize the output SNR, *Match Filtering* concept [3] might also be implemented for the multiple antenna receivers. *Spatial match filtering* requires the received signals by each antenna to be co-phased and weighted before combining, according to their signal strengths. Referring back to Equation 2.1, if h_{ij} are the complex channel gains, the weights of match filtering are h_{ij}^*, where superscript * denotes complex conjugation [5]. Equation (2.1) can also be written as follows;

$$\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{bmatrix} = \begin{bmatrix} \tilde{h}_1^H \\ \tilde{h}_2^H \\ \vdots \\ \tilde{h}_N^H \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_N \end{bmatrix} \tag{2.7}$$

where $\widetilde{h}_k^{\,H}$ represents a row of channel coefficients seen by the k^{th} antenna of the receiver. Then the estimate is calculated as;

$$\hat{x} = \sum_{i=1}^{N} \widetilde{h}_i^{\,H} r_i \tag{2.8}$$

Hermitian operation, $(\ )^{H}$, is equivalent to the transpose conjugate operation. The operation in (2.8) is equivalent to *maximal ratio combining (MRC)* for a single transmitting antenna and *N* receiving antenna system. We may conclude with the Equation (2.9) that, the decoder at the receiver might be chosen as;

$$V = H^{H} \tag{2.9}$$

However it should be kept in mind that, this solution would only be optimum in the absence of the other *interfering streams,* only when the desired signal and noise are present at the receiver (i.e. no spatial multiplexing). That is to say when $x_1 = x_2 = \cdots = x_M$.

2.1.3 Zero Forcing (ZF) Receiver

The transmitter might be multiplexing multiple data streams to the channel simultaneously. In that case the streams would obviously interfere with each other. When the interfering streams are present, the received signals can be combined to maximize the signal power over the interference, rather than just SNR, as it was in match filtering. The *ZF filter* separates the received signal into non-interfering components. Its definition is;

$$V = (H^{H} H)^{-1} H^{H} \tag{2.10}$$

which is the *Moore-Penrose pseudoinverse* of *H*. In case of having a square channel matrix, the equation simplifies into the inverse of the channel, that is $V = H^{-1}$.

ZF receiver decomposes the matrix channel into *M* parallel scalar channels and forces to cancel out the intersymbol interference completely, ignoring the noise amplification problem after zero-forcing. Despite its simplicity, there are some important drawbacks of using a ZF filter. The noise amplification problem causes ZF receiver to give low performance at low SNR.

Using an algebraic analogy; the input signals are the unknowns in a linear system of equations with the observations given by the measurements made at the receiver array. Provided that, enough observations (receiving antennas) are available, the input signals can be identified uniquely [5]. Therefore the number of receiving antennas, via which the observations are collected, shall be larger than or equal to the number of transmitting antennas (i.e $N \geq M$), assuming that M different data are sent from each transmitting antenna in order to make use of *spatial multiplexing*. In short, in a MIMO-SU system, where ZF is used to collect the sent streams, N should be kept larger than or equal to M, which can obviously be expanded for the other types of receivers, explained in this chapter.

2.1.4 MMSE Receiver

Where the ZF receiver forces for zero interference, the MMSE receiver allows some of signals in the interfering subspace by minimizing MSE (Mean Squared Error).

$$MSE = E\left(\|Vr - x\|^2\right) \tag{2.11}$$

The following manipulation might be conducted on (2.11)

$$\begin{aligned} MSE &= E\left((Vr - x)(Vr - x)^H\right) \\ &= VR_r V^H + I - H^H V^H - VH \\ &= (V - H^H R_r^{-1}) R_r (V - H^H R_r^{-1})^H + I - H^H R_r^{-1} H \\ &= (V - H^H R_r^{-1}) R_r (V - H^H R_r^{-1})^H + N_0 (H^H H + N_0 I)^{-1} \end{aligned} \tag{2.12}$$

where $R_r = E\left(rr^H\right)$, received autocorrelation matrix. In (2.12) only the first term depends of the selection of V [9]. In order to make this term zero, V should be chosen as;

$$V = H^H R_r^{-1} = H^H (HH^H + N_0 I)^{-1} = (HH^H + N_0 I)^{-1} H^H \qquad (2.13)$$

As can be seen from the above-derived formulation of MMSE receiver, it differes from the ZF receiver as it sets a balance in between the condition of the channel and the noise variance. As $N_0 \to 0$, MMSE approaches ZF receiver and similarly as $N_0 \to \infty$, MMSE approaches MF receiver.

2.1.5 SVD Solution

We have reviewed some methods, which may be used at the receivers of a single user *MIMO* link. These methods design the decoders at the receivers, according to the condition of the channel, the knowledge of which is always available at the receiver. However, if the transmitter is somehow informed about the channel knowledge, then a *pre-processing* may also be conducted at the transmitter prior to the sent of the signal. A basic linear algebra knowledge, *SVD (Singular Value Decomposition)*, is used for that purpose. Figure 2.2 is adapted with the installment of pre-processing (precoding) matrix at the transmitter as follows;

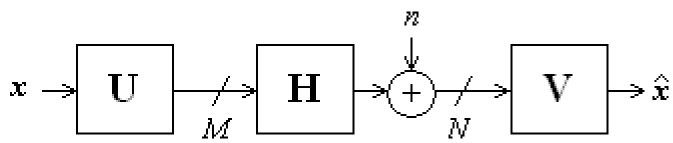

Figure 2.3 MIMO-SU communication channel with preprocessing

$$\hat{x} = V\left(HUx + n\right) = VHUx + Vn \qquad (2.14)$$

$E\{trace[UU^H]\} = P_T$, where P_T is the total transmit power. Therefore *precoding matrix U* also carries the total transmit power information. The channel matrix H has a singular value decomposition (SVD) of;

$$H = S\Lambda D^H \qquad (2.15)$$

where S and D are (rotation) unitary matrices and Λ is a rectangular matrix, whose diagonal elements are non-negative real numbers (i.e. singular values of H, which are called "λ") and off-diagonal elements are zero.

With that knowledge on linear algebra, a decoder *V* and precoder *U* can be occupied at the receiver and transmitter respectively, having the following equalities;

$$U = D \quad \text{and} \quad V = S^H \tag{2.16}$$

As long as the unitary matrices result with identity, when they are multiplied with their hermitian, the precoding and decoding operation would remove the rotation matrices from the equation of the received signal. Then we can rewrite the equation (2.14) as follows;

$$\hat{x} = VS\Lambda D^H Ux + Vn = S^H S\Lambda D^H Dx + S^H n = \Lambda x + S^H n \tag{2.17}$$

It can easily be understood from (2.17) that the SVD solution generates parallel channels in between the transmitter and receiver, where the number of parallel channels will be equal to the number of non-zero singular values of the channel *H*, being equal to $\min(N, M)$, because of the rectangular shape of diagonal matrix Λ. Note that, the result in (2.17) corresponds to the *spatial multiplexing* considerations of the previous chapter, where variety of data symbols can be fed to the wireless channel through these parallel links simultaneously.

Some other popular non-linear receiver structures can also be used to receive MIMO data such as *MMSE-Successive-Interference-Cancellation (SUC) Receiver* also known as *Decision feedback equalizer (DFE).*

2.2 MULTI USER MIMO

In case of a network of multiple users or a medium shared by multiple mobile users, being coordinated by a base station (BS), the communication of each user should be conducted through an assigned share of the common medium (e.g.

time slots, frequency, spreading code etc.), which leads to another problem named *Multiple access* problem. Conventional multiple access methods like TDMA (Time Division Multiple Access), CDMA (Code Division Multiple Access) and FDMA (Frequency Division Multiple Access) make use of with different schemes of *frequency*, *code* or *time sharing*. That may also be extended to a scheme, based on space sharing, for servicing multiple users at the same time simultaneously.

MIMO-MU, which stands for Multi-User MIMO, is a sub topic of MIMO. The idea of supporting multi-users (MU) simultaneously, by employing multiple antennas at base station, is called MIMO-MU. For multi user case, the base station communicates with the multiple users simultaneously in the same frequency channel by exploiting differences in spatial signatures at the base-antenna array induced by spatially dispersed users. This technique is also known as *SDMA* (Space Division Multiple Access). SDMA allows channel reuse within a cell to increase spectral efficiency. [10]

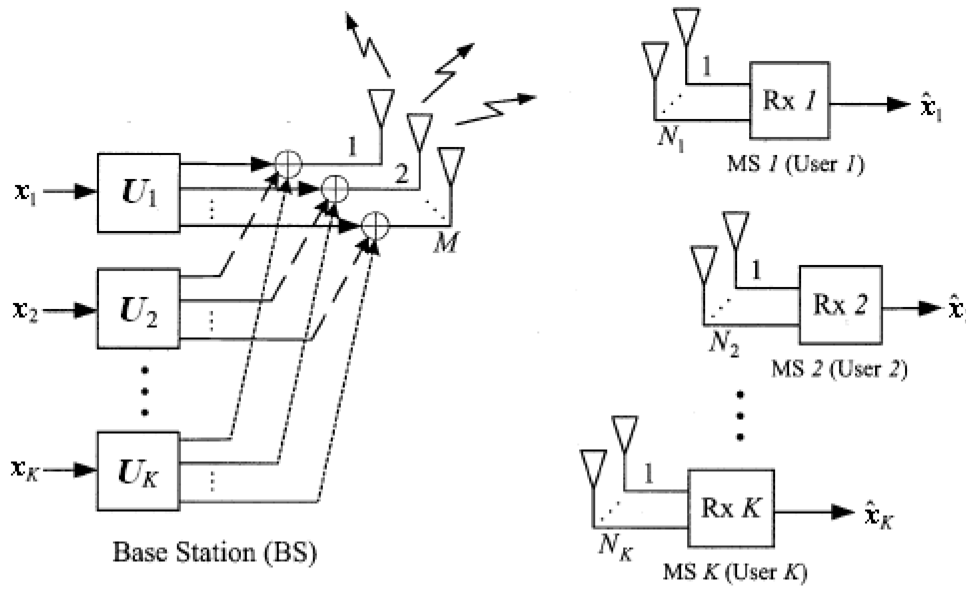

Figure 2.4 Multi User MIMO system configuration

As normal, the communication in between the users and the base station is to occur in two directions. These are the *uplink*, where a group of users transmit data symbols to the same base station, and the *downlink*, where the base station attempts to transmit signals to multiple users.

In Figure 2.4 [11] a MIMO-MU scenario is realized, where the base station coordinates the communication of K users via its M antennas. For the rest of the thesis the transmission is assumed to take place in a single cell communication system, where single base station serves K number of users, as in Figure 2.4.

2.2.1 MIMO-MU Uplink Channel

The uplink MIMO-MU channel can be thought as an extension of single user MIMO channel (MIMO-SU). Therefore array processing, detection or other receiver design techniques suitable for single user MIMO application can also be applied for uplink channel, with slight adaptations. All receiver architectures such as ZF, MMSE and ML are applicable for uplink channel with the same tradeoffs between complexity and diversity/SNR performance. The performance may be increased if some feedback is possible from the base station to each user upon the channel conditions of other users, which is an exhaustive feedback scenario [12].

In order to extend our discussions on the uplink channel, assume a wireless communication scenario, where single antenna users communicate to the base station (See the Figure 2.5). After the reception of multiple signals, the base station estimates the k^{th} user's bits by applying a decoding operation (u_k) on to the received signal.

Therefore the following equation can be put forward by taking Figure 2.5 into consideration;

$$\hat{x}_k = u_k \sum_{m=1}^{K} H_m \sqrt{q_m} x_m + n \tag{2.18}$$

where q is the power value assigned to each information symbol transmitted to the base station and u_k is the decoding vector used at the BS in order to find out the data symbols sent by user k.

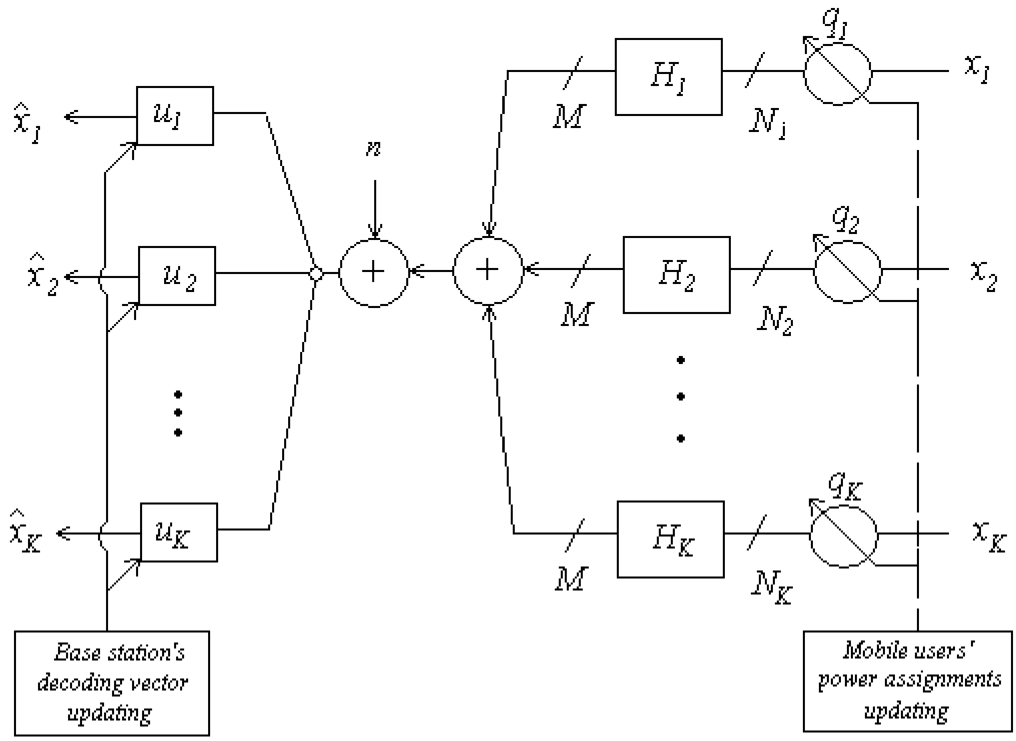

Figure 2.5 Functional block diagram for uplink channel for single antenna mobile users

2.2.2 MIMO-MU Downlink Channel Problem

In multi-user downlink communication, the main problem is due to the fact that, each receiver sees its own channel but unaware of other users' channels. In the downlink, as a result of the lack of coordination between the users, the data symbols sent to the other users cause *interference* at each user, which is called *inter-user interference (*or *Multiple Access Interference (MAI))*. In other words, if each user is equipped with a single antenna and receives a single stream and if there are 10 users, the transmitted bits to other 9 users interfere with the bit transmitted to the intended user. This interference can not be mitigated solely at the receiver. A partial or full cooperation among the overall system might be allowed, requiring communication also between the users. However, instead of increasing the complexity burden of the mobile users, a channel preconditioning called *channel precoding* at the transmitter (i.e. *transmit precoding*) is the ideal solution for mitigating the interference at the users. Therefore the base station shall be authorized to take this duty upon itself, by collecting *channel state information (CSI)* from all of the users and shaping the signal suitable to each user.

It is obvious that the channel information can only be informed to the transmitter by means of feeding back the *CSI* to the transmitter *(CSIT),* which is a challenging problem that needs an extra transmission cost for the system (e.g., a *frequency-division duplex transmission* might be introduced for feedback data [13]). That is the drawback of setting a precoding scheme at transmitter side. Rather than the feedback, training or pilot symbols broadcasting at high signal power in the uplink might also be used to explore the channel knowledge. For instance that can be realized in a *time-division duplex system,* without exceeding the coherent time. Also detailed work on *Limited capacity feedback,* with several discussions on its value for different schemes, were presented in [14], [15]. In these works, how to make use of the partial or imperfect CSI, when the instantaneous perfect knowledge is not available and its costs, are analyzed. The availability of instantaneous perfect channel state information at the transmitter is one of the most critical assumptions of this thesis.

The main question in the downlink side is how to implement the *transmit precoding* at the base station in order to let the users to receive their own data, while eliminating the unwanted interference of other users' data. For downlink signaling, there are several techniques introduced in order to obtain an interference free communication. Such techniques will be discussed later in the next chapter.

Similar to the uplink channel, Figure 2.6 can be drawn for downlink channel, where p_k is the power value assigned to data symbols of user *k,* where each mobile user is equipped with single antenna. Therefore the following equation can be put forward, by taking Figure 2.6 into consideration, for the estimate of the signal of user *k*;

$$\hat{x}_k = H_k^H \sum_{m=1}^{K} u_m \sqrt{p_m} x_m + n \tag{2.19}$$

u_m is the *transmit precoding* vectors, derived for the mitigation of interference. On (2.19) and that of functional diagram in Figure 2.6, multi-user transmission approach is based on linear processing, which assumes that the transmitted signal is generated by a linear combination of data symbols, *x*, contained in u_m vectors [13].

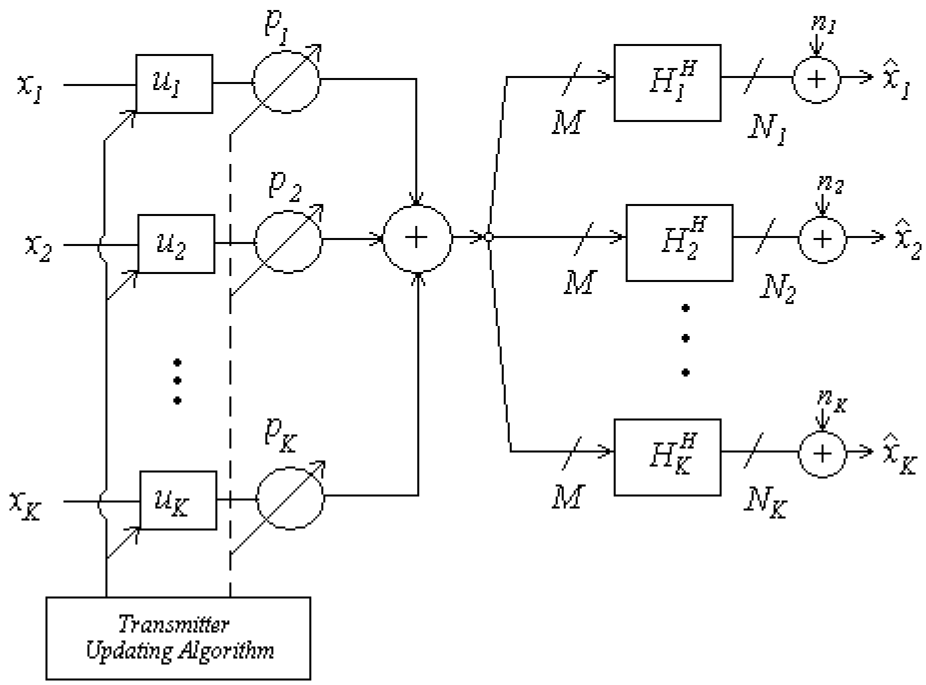

Figure 2.6 Functional block diagram for downlink channel for single antenna mobile users

Without precoding and power allocation (u_m and p_m) for a downlink communication scenario with *K* users, the received signal at user *k* will be;

$$r_k = \underbrace{H_k x_k}_{signal\ for\ user\ k} + \underbrace{\sum_{\substack{l=1 \\ l \neq k}}^{K} H_l x_l}_{interference} + n_k \tag{2.20}$$

A simple mathematical representation of the interference concept is given in Equation (2.20). The first argument in the equation is the signal value intended to be received by user *k,* however the second term, which is the superposition of the other users' signaling information, is the interference. The noise, n_k, received by user *k,* is assumed to be additive white Gaussian in time and space. On the other hand the interfering term, caused from other users, will have an effect like a colored noise in the equation. The interference term should be eliminated as much as possible.

2.2.3 SINR (Signal to Interference plus noise ratio)

SINR is the performance parameter of the MIMO-MU communication system. Broadly in (2.20) a mathematical notation is given for interference. Now we would like to demonstrate the strengths of signals and the interfering terms in a communication scenario in more detail.

For a downlink scenario of a base station, having *6* antennas, and *3* users, where each user is equipped with two antennas, the channel matrix H can be written as;

$$H = \begin{bmatrix} \overbrace{h_{11} \quad h_{12}}^{User1} & \overbrace{h_{13} \quad h_{14}}^{User2} & \overbrace{h_{15} \quad h_{16}}^{User3} \\ h_{21} \quad h_{22} & h_{23} \quad h_{24} & h_{25} \quad h_{26} \\ h_{31} \quad h_{32} & h_{33} \quad h_{34} & h_{35} \quad h_{36} \\ h_{41} \quad h_{42} & h_{43} \quad h_{44} & h_{45} \quad h_{46} \\ h_{51} \quad h_{52} & h_{53} \quad h_{54} & h_{55} \quad h_{56} \\ h_{61} \quad h_{62} & h_{63} \quad h_{64} & h_{65} \quad h_{66} \end{bmatrix} \tag{2.21}$$

H is a 6x6 matrix, composed of 6x1 channel vectors of each user's antennas. h_{ij} denotes the channel coefficient between of the i^{th} tranmitting antenna j^{th} receiving antenna. The first two columns of the H matrix are the channel coefficients seen by the antennas of first user. Other users' coefficients are also listed two by two columns. Instead of coefficient representation, the channel vectors may also be denoted as $\widetilde{h}_k$, which is a column vector of channel coefficients seen by the k^{th} receiving antenna. Note that the channel matrix model in (2.21) is different from its single user correspondent presented in (2.1).

$$H = \begin{bmatrix} \overbrace{\widetilde{h}_1 \quad \widetilde{h}_2}^{User1} & \overbrace{\widetilde{h}_3 \quad \widetilde{h}_4}^{User2} & \overbrace{\widetilde{h}_5 \quad \widetilde{h}_6}^{User3} \end{bmatrix} = \begin{bmatrix} H_1 & H_2 & H_3 \end{bmatrix} \qquad (2.22)$$

If each user receives a single stream, then each user has one precoding vector shown by u_1, u_2 *and* u_3. With the precoding vectors, the signal part of the received vector at all users become;

$$r = H^H U p x \Rightarrow \begin{matrix} User1\left\{\begin{matrix} \\ \\ \end{matrix}\right. \\ User2\left\{\begin{matrix} \\ \\ \end{matrix}\right. \\ User3\left\{\begin{matrix} \\ \\ \end{matrix}\right. \end{matrix} \begin{bmatrix} \widetilde{h}_1^H \\ \widetilde{h}_2^H \\ \hline \widetilde{h}_3^H \\ \widetilde{h}_4^H \\ \hline \widetilde{h}_5^H \\ \widetilde{h}_6^H \end{bmatrix} \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix} \begin{bmatrix} \sqrt{p_1} x_1 \\ \sqrt{p_2} x_2 \\ \sqrt{p_3} x_3 \end{bmatrix} \qquad (2.23)$$

which may also be written as

$$r \Rightarrow \begin{matrix} User1\left\{\begin{matrix} \\ \\ \end{matrix}\right. \\ User2\left\{\begin{matrix} \\ \\ \end{matrix}\right. \\ User3\left\{\begin{matrix} \\ \\ \end{matrix}\right. \end{matrix} \begin{bmatrix} \mathbf{\widetilde{h}_1^H u_1 \sqrt{p_1}} & \widetilde{h}_1^H u_2 \sqrt{p_2} & \widetilde{h}_1^H u_3 \sqrt{p_3} \\ \mathbf{\widetilde{h}_2^H u_1 \sqrt{p_1}} & \widetilde{h}_2^H u_2 \sqrt{p_2} & \widetilde{h}_2^H u_3 \sqrt{p_3} \\ \hline \widetilde{h}_3^H u_1 \sqrt{p_1} & \mathbf{\widetilde{h}_3^H u_2 \sqrt{p_2}} & \widetilde{h}_3^H u_3 \sqrt{p_3} \\ \widetilde{h}_4^H u_1 \sqrt{p_1} & \mathbf{\widetilde{h}_4^H u_2 \sqrt{p_2}} & \widetilde{h}_4^H u_3 \sqrt{p_3} \\ \hline \widetilde{h}_5^H u_1 \sqrt{p_1} & \widetilde{h}_5^H u_2 \sqrt{p_2} & \mathbf{\widetilde{h}_5^H u_3 \sqrt{p_3}} \\ \widetilde{h}_6^H u_1 \sqrt{p_1} & \widetilde{h}_6^H u_2 \sqrt{p_2} & \mathbf{\widetilde{h}_6^H u_3 \sqrt{p_3}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad (2.24)$$

As with the H matrix, r matrix also has separated rows for each user. The bold entries in r matrix of (2.24) represent the signal values intended to each user. However, the rest of the values are the interferences to the simultaneous transmission. The signal value, to be transmitted to one of the users, effects the rest of the users as an interference. The ideal case is the one where the non-bold values on r matrix should become zero.

Let us generate a new matrix similar to r but composing of the power values rather than the signal values, in other words taking magnitude square of every entry in (2.24) we get;

$$\begin{matrix} User1\left\{ \begin{matrix} \\ \\ \end{matrix} \right. \\ User2\left\{ \begin{matrix} \\ \\ \end{matrix} \right. \\ User3\left\{ \begin{matrix} \\ \\ \end{matrix} \right. \end{matrix} \begin{bmatrix} \mathbf{p_1 u_1{}^H R_1 u_1} & p_2 u_2{}^H R_1 u_2 & p_3 u_3{}^H R_1 u_3 \\ \mathbf{p_1 u_1{}^H R_2 u_1} & p_2 u_2{}^H R_2 u_2 & p_3 u_3{}^H R_2 u_3 \\ \hline p_1 u_1{}^H R_3 u_1 & \mathbf{p_2 u_2{}^H R_3 u_2} & p_3 u_3{}^H R_3 u_3 \\ p_1 u_1{}^H R_4 u_1 & \mathbf{p_2 u_2{}^H R_4 u_2} & p_3 u_3{}^H R_4 u_3 \\ \hline p_1 u_1{}^H R_5 u_1 & p_2 u_2{}^H R_5 u_2 & \mathbf{p_3 u_3{}^H R_5 u_3} \\ p_1 u_1{}^H R_6 u_1 & p_2 u_2{}^H R_6 u_2 & \mathbf{p_3 u_3{}^H R_6 u_3} \end{bmatrix} \tag{2.25}$$

where $R_k = \widetilde{h}_k \widetilde{h}_k^H$, which is a realization of spatial correlation matrix of channel gains. In the following two sections, mathematical formulations of downlink and uplink SINR will be derived for both single antenna and multiple antenna mobile users.

2.2.3.1 SINR for single antenna at the Mobile Users

Based on the idea given with Figure 2.5, Figure 2.6 and with the power matrix in (2.25), the uplink SINR of the k^{th} user, realized at the BS, can be expressed as;

$$SINR_k^{UL} = \frac{q_k \left| u_k{}^H H_k \right|^2}{N_0 + \sum_{j \neq k} q_j \left| u_k{}^H H_j \right|^2} \tag{2.26}$$

and the downlink SINR, realized at the k^{th} user, can be expressed as;

$$SINR_k^{DL} = \frac{p_k \left| u_k{}^H H_k \right|^2}{N_0 + \sum_{j \neq k} p_j \left| u_j{}^H H_k \right|^2} \tag{2.27}$$

2.2.3.2 SINR for multiple antenna at the Mobile Users

The mobile users might also be equipped with multiple antennas. Figure 2.7 and Figure 2.8 might be accepted as extensive versions of Figure 2.5 and Figure 2.6.

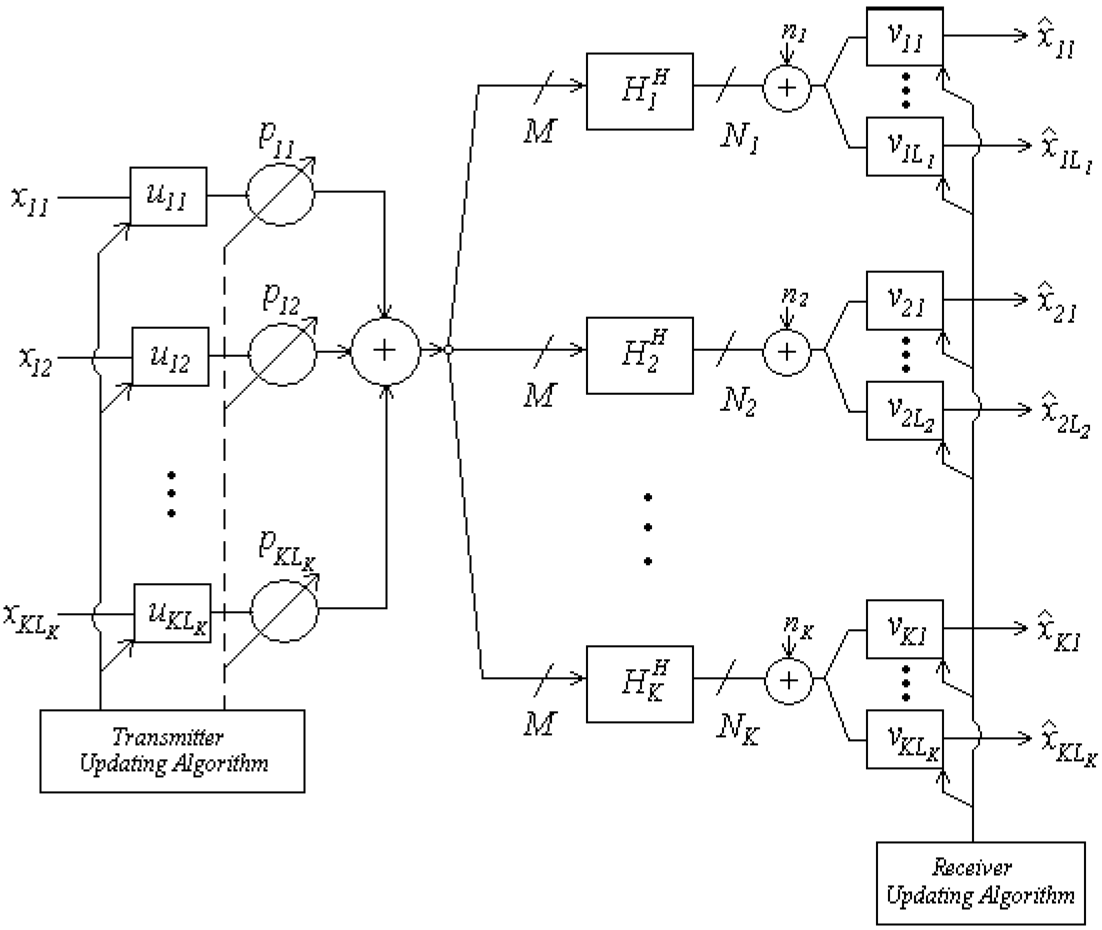

Figure 2.7 Functional block diagram for downlink channel for multiple antenna mobile users

In the figures, subscripts are used in a way that for instance; p_{ij} represents the power scaling value for j^{th} data stream of user *i*. Variables *u* and *v* are precoding and decoding vectors as common. In total $\sum_{i=1}^{K} L_i$ data streams are transmitted simultaneously. Transmitter and Receiver updating algorithms set the power values and the precoding & decoding vectors. Right hand side of the communication represents the users, while the left hand side is the base station.

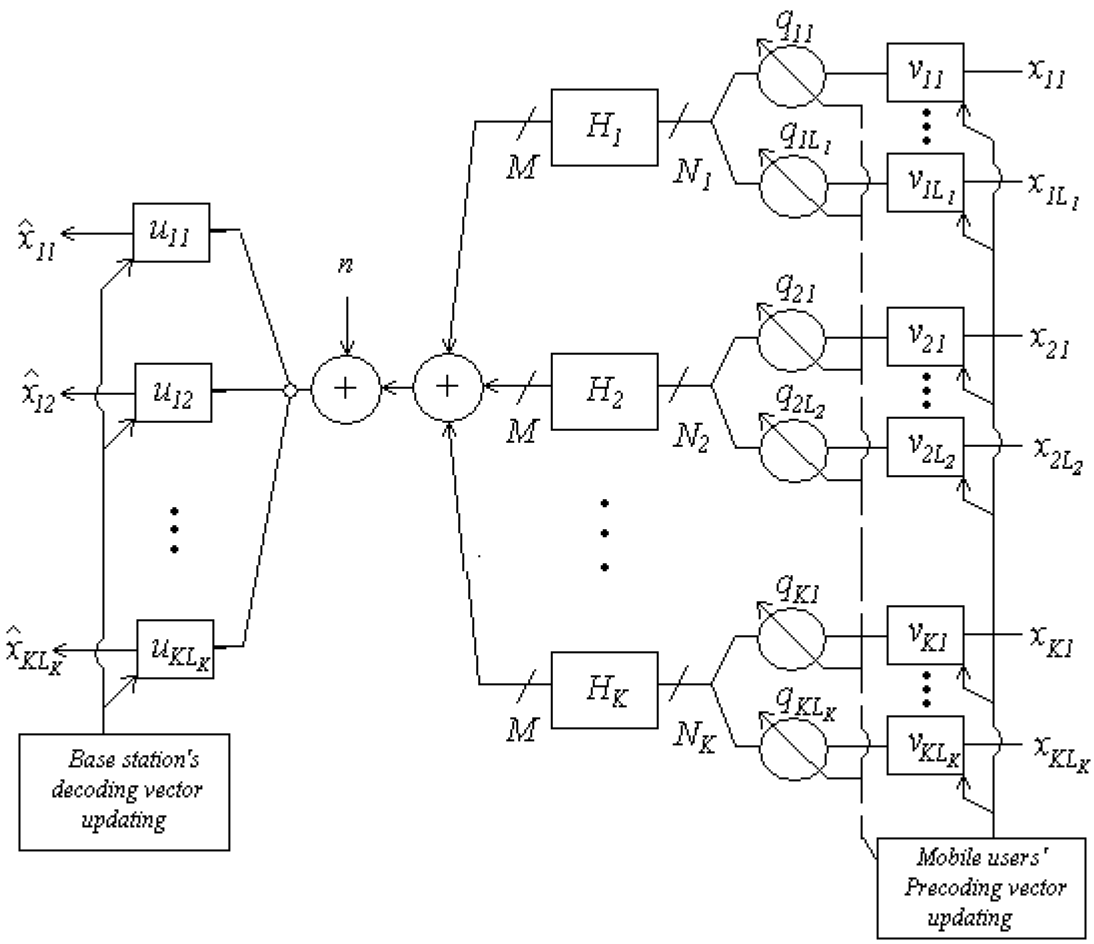

Figure 2.8 Functional block diagram for uplink channel for multiple antenna mobile users

When mobile users are equipped with multiple antennas, processing at transmitter and receivers are used to eliminate intra-user and inter-user interference.

For the downlink transmission, the SINR realized at the j^{th} data stream received by user k is (Superscripts DL and UL refer to downlink and uplink quantities, respectively) [16];

$$SINR_{kj}^{DL} = p_{kj} \frac{v_{kj}^{H} S_{kj}^{DL} v_{kj}}{v_{kj}^{H} T_{kj}^{DL} v_{kj}} \tag{2.28}$$

where,

$$S_{kj}^{DL} = H_{k}^{H} u_{kj} u_{kj}^{H} H_{k} \tag{2.29}$$

$$T_{kj}^{DL} = \underbrace{\sum_{l=1, l\neq j}^{L_k} p_{kl} H_{k}^{H} u_{kl} u_{kl}^{H} H_{k}}_{\text{intra-user-interference}} + \underbrace{\sum_{i=1, i\neq k}^{K} \sum_{m=1}^{L_i} p_{im} H_{k}^{H} u_{im} u_{im}^{H} H_{k}}_{\text{inter-user-interference}} + \sigma^2 I \tag{2.30}$$

Intra-user-interference, which may also be called *inter-stream-interference*, is the interference that is caused in between the multiple data streams of a user. Thus, if a user receives only one data stream, the intra-user-interference term in the equation becomes “zero”. Ideal condition is to set the total interference to zero. It should be kept in mind that the transmit precoding matrix U should satisfy the power constraint $E\left(\|pUx\|^2\right) = p^2.trace\left(U^H U\right) = P_T$, where $\|.\|$ and *tr(.)* denote the vector 2-norm and trace operation of a matrix, respectively, and P_T is the total transmission power [17].

With a similar fashion, for the uplink transmission channel of the multiple antenna mobile users, the SINR realized at the j^{th} data stream sent by user k to the Base station is;

$$SINR_{kj}^{UL} = q_{kj} \frac{u_{kj}^{H} S_{kj}^{UL} u_{kj}}{u_{kj}^{H} T_{kj}^{UL} u_{kj}} \tag{2.31}$$

Given that,

$$S_{kj}^{UL} = H_{k} v_{kj} v_{kj}^{H} H_{k}^{H} \tag{2.32}$$

$$T_{kj}^{UL} = \sum_{l=1, l\neq j}^{L_k} q_{kl} H_k v_{kl} v_{kl}^H H_k^H + \sum_{i=1, i\neq k}^{K} \sum_{m=1}^{L_i} q_{im} H_i^H v_{im} v_{im}^H H_i^H + \sigma^2 I \qquad (2.33)$$

As long as the communication is assumed to take place in a single cell communication systems, no consideration on *inter-cell interference* was made.

2.2.4 Uplink-Downlink Duality

Uplink – downlink duality is an important result, which is used in the optimization of SNR values for the downlink channel. Figure 2.5 and Figure 2.6 shows the uplink and downlink channels for single antenna mobile users.

Uplink-downlink duality states that if the set of SINR values for the uplink (2.26) and the downlink channel (2.27) are the same, that is

$$SINR_k^{DL} = SINR_k^{UL} = SINR_k \qquad (2.34)$$

then both systems, uplink and downlink, can operate at the same total power. We can show this as follows [2]:

Let us define;

- $a = (a_1, \cdots, a_K)^t$, $b = (b_1, \cdots, b_K)^t$
- $a_k = \dfrac{SINR_k^{DL}}{(1 + SINR_k^{DL})\left|h_k^* u_k\right|^2}$, $b_k = \dfrac{SINR_k^{UL}}{(1 + SINR_k^{UL})\left|u_k^* h_k\right|^2}$
- A is a *KxK* matrix to have component *(k,j)* equal to $\left|u_j^* h_k\right|^2$
- $q=[q_1 \ \ldots \ q_N]$, $p=[p_1 \ \ldots \ p_N]$,

In the uplink case with a given set of u_k vectors that can reach a given set of SNR's; the power at each user is uniquely determined as the solution of the following linear equation system:

$$(I_k - diag\{b_1, \cdots, b_K\} A^t) q = N_0 b \qquad (2.35)$$

In the downlink case with a given set of u_k vectors that can reach a given set of SNR's; the power assigned to each at each user is uniquely determined as the solution of the following linear equation system:

$$(I_K - diag\{a_1,\cdots,a_K\}A)p = N_0 a \qquad (2.36)$$

The total power used at the downlink and uplink channels are given as $\sum_{k=1}^{N} p_k$ and $\sum_{k=1}^{N} q_k$. The claim is that the total power in both channels are the same. To see this we first solve (2.35) and (2.36) for the transmit powers and we get.

$$p = N_0(I_K - diag\{a_1,\cdots,a_K\}A)^{-1}a = N_0(D_a - A)^{-1}1 \qquad (2.37)$$

$$q = N_0(I_K - diag\{b_1,\cdots,b_K\}A^t)^{-1}b = N_0(D_b - A^t)^{-1}1 \qquad (2.38)$$

where $D_a = diag\left(1/a_1,\cdots, 1/a_K \right)$, $D_a = diag\left(1/b_1,\cdots, 1/b_K \right)$ and 1 is the vector of all 1's. To achieve the same SINR in the downlink and its dual uplink a=b and we conclude

$$\sum_{k=1}^{K} P_k = N_0 1^t (D_a - A)^{-1} 1 = N_0 1^t \left[(D_a - A)^{-1} \right]^t 1 = N_0 1^t \left(D_a - A^t \right)^{-1} 1 = \sum_{k=1}^{K} Q_k \qquad (2.39)$$

The uplink – duality has been shown to be valid when users have multiple antennas as well. [18]

CHAPTER 3

MIMO-MU DOWNLINK PROCESSING

In the previous chapter, a detailed discussion on the *interference* problem in a wireless multiple access environment was made and its effects on the downlink side were emphasized. In this part we will study on multi-user MIMO (*MIMO-MU*) downlink processing methods to mitigate both inter-user and intra user interferences. Therefore the ultimate goal for these methods would be set as to establish high *SINR* channels between transmitter and each user. Linear processing techniques are of interest because of their non-complicated nature and simplicity [13].

We denote *K,* as the number of users, and *M,* as the number of transmitting antennas placed at the transmitter side. N_k is the number of antennas at user *k*, which means in total there are $\sum_{k=1}^{K} N_k$ receiving antennas. The notation $(M,(N_1 x N_{2 \ldots} N_K))$ represents the numbers of antennas at the base and at each mobile station. The techniques, to be explained in this chapter, can be used together with other access techniques such as TDMA, CDMA etc. The instantaneous channel state information (*CSI*) is assumed to be always available at the transmitter, which means the transmitter is fully aware of the coefficients of the communication channel.

It should be kept in mind that, not only the number of antennas but also the number of data streams that will simultaneously be transmitted to the users has an important role on designing the algorithm to be used. We first introduce a chart on Figure 3.1, drawn according to the requirements of the mobile system, based both on the number of the data-streams and the number of receiving antennas.

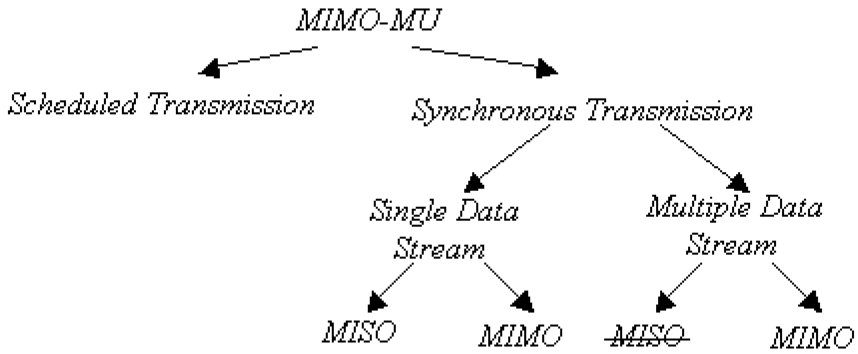

Figure 3.1 MIMO-MU antenna and stream comparison chart

On Figure 3.1 *synchronous transmission* stands for simultaneous data transmission in the space domain. A scenario of interest for next-generation systems, where the users might also be equipped with multiple antennas, is on the bottom level of the chart, which is *MIMO*. *MIMO,* which requires the use of antenna arrays both at the base station and at every user, is an optional feature for a single stream receiving system; but is desirable due to increased diversity. Else, multiple data streams may also be transmitted to each multiple antenna user, as long as it is known from *spatial multiplexing* considerations in Chapter 1 and 2. A heterogeneous communication system might also be considered, where some of the users are with single and some with multiple antennas, even where the number of antennas at each user might differ from each other.

For a *MISO* system, where the receiving user has only one single antenna, it is mathematically impossible to receive multiple data streams simultaneously. Therefore MISO condition was deleted in Figure 3.1 for multiple data stream transmission. Using multiple antennas at the receiver is a necessity if multiple streams are to be delivered to the users.

3.1 MIMO-MU DOWNLINK TECHNIQUES

There are some conventional techniques, which can be used to solve the downlink MIMO-MU problem, such as applying *ZF* or *MMSE* before transmission. Later, more recent techniques, having better performance, will be discussed.

3.1.1 Zero Forcing at transmitter

The simplest way of avoiding the inter-user interference is forcing all interference terms be zero. Different from the single user ZF technique, for multi user case the processing is made at the transmitter, where receiver receives its data without any decoding technique. Similar to the single user case, the preprocessing matrix is chosen as the pseudoinverse of the channel matrix, which can be expressed as; $U = H^t = H^H (HH^H)^{-1} = (HH^H)^{-1} H^H$. Technique may also be referred to as *channel inversion*, for the case where H is square. The number of transmitting antennas shall be larger than or equal to the total number of receiving antennas of all users, which is vice versa in single user ZF receiver case.

There is another drawback in ZF transmitter technique. Let h_i be the 1st column of H and if two or more user channels h_i are close to each other, then some users will receive very little power (power reduction) and we get the transmitter dual of the noise enhancement problem observed in single user ZF receivers [10]. In other words, at this scenario the technique will force to separate the two identical channels and spend most of the transmitter power for this and therefore the available power for other users will be decreased. We may call ZF solution as a power inefficient way of solving downlink MIMO problem for this reason. ZF at transmitter is advantageous when SNR is high.

The main reason, why ZF type of receiver has several drawbacks, is due to its algorithm, which goes for having zero interference at each user. However a limited amount of interference at each receiver allows one to consider a larger set of potential solutions that can provide higher capacity for a given transmit condition [13].

3.1.2 Transmit-MMSE

With a similar idea, instead of ZF, the transmitter can make precoding with MMSE criterion, which gives a better performance as expected. This technique in the literature is called "*Transmit MMSE*" (TrMMSE).

We assume a simple receiver structure, where each user is equipped with a single antenna. Based on the MMSE criterion, given in (2.11), the problem statement can be expressed for multi user case as;

$$U = \arg\min_{U} E\left(\|\hat{x} - x\|^2\right) \quad \text{s.t. } trace\left(UU^H\right) = P_T \tag{3.1}$$

where the power constraint states that, the total average transmit energy after transformation by *U* is equal to P_T. *U* is a *MxK* matrix that is composed of the precoding vector of each user. *x* is *Kx1* data symbol vector, composing of data symbols of every single antenna user at each of its rows. Since we assume *x* contains i.i.d. zero mean complex random variables with unit variance and uncorrelated with the noise, we can rewrite the equation as follows [19];

$$U = \arg\min_{U}\left\{E\left(\left\|H^H Ux + n - x\right\|^2\right)\right\} = \arg\min_{U}\left\{E\left(\left\|H^H Ux - x\right\|^2\right) + E\left(\|n\|^2\right)\right\} \tag{3.2}$$

where *H* is the channel matrix, generated by concatenating the channels of every user that is $H = [H_1, H_2 \cdots H_K]$. From the power constraint $trace\left(UU^H\right) = P_T$ we can obtain $\frac{trace\left(UU^H\right)}{P_T} = 1$ and therefore

$$U = \arg\min_{U}\left\{E\left(\left\|H^H Ux - x\right\|^2\right) + E\left(\|n\|^2\right)\frac{trace\left(UU^H\right)}{P_T}\right\} \tag{3.3}$$

By setting

$$\beta = \frac{E\left(\|n\|^2\right)}{P_T} = \frac{K\sigma^2}{P_T} \tag{3.4}$$

and using the equality

$$E\left(\left\|H^H Ux - x\right\|^2\right) = trace\left(\left(H^H U - I\right)\left(H^H U - I\right)^H\right) \tag{3.5}$$

where $E\left(xx^H\right) = I$, the MSE equation is equivalent to

$$U = \arg\min_U \left\{trace\left(\left(H^H U - I\right)\left(H^H U - I\right)^H\right) + \beta.trace\left(UU^H\right)\right\} \tag{3.6}$$

By denoting the cost function in the minimization argument of (3.6) as

$$g(U) = trace\left(\left(H^H U - I\right)\left(H^H U - I\right)^H\right) + \beta.trace\left(UU^H\right) \tag{3.7}$$

and taking $\nabla_U g(U) = 0$, the solution to the problem, formulated in (3.1), can be given by;

$$U = \left(HH^H + \beta I\right)^{-1} H = H(H^H H + \beta I)^{-1} \tag{3.8}$$

Note that the multi-user precoding vectors, *U*, are calculated similar to the single user MMSE receivers in (2.13). β is called the *loading factor*, maximizing the SNR at the receiver when the scheme is used. In Figure 3.2 the block diagram for TrMMSE is given. Because for TrMMSE the total power is equally distributed among all users' precoding vectors, no power consideration is realized on Figure 3.2. Like using the ZF at the transmitter for *MIMO-MU downlink*, this approach also requires that the number of transmit antennas be larger than the total number of receive antennas.

TrMMSE or ZF at the transmitter could still be employed in a wireless network of *multiple antenna users*, but is not a particularly efficient solution, since forcing two closely spaced antennas, belonging to a single user to receive different signals, would require extra power, when the channels for these antennas are highly correlated. It also aims to mitigate the interference totally at the transmitter precoding operation, ignoring the possibility of receiver employed beamforming.

One solution to this problem is to use block channel inversion or block diagonalization [20]. This approach is essentially a generalization of channel inversion that optimizes the power transfer to a group of antennas rather than a single antenna [13]. In [21] a low complexity user selection algorithm was also added to that of system for more improved performance.

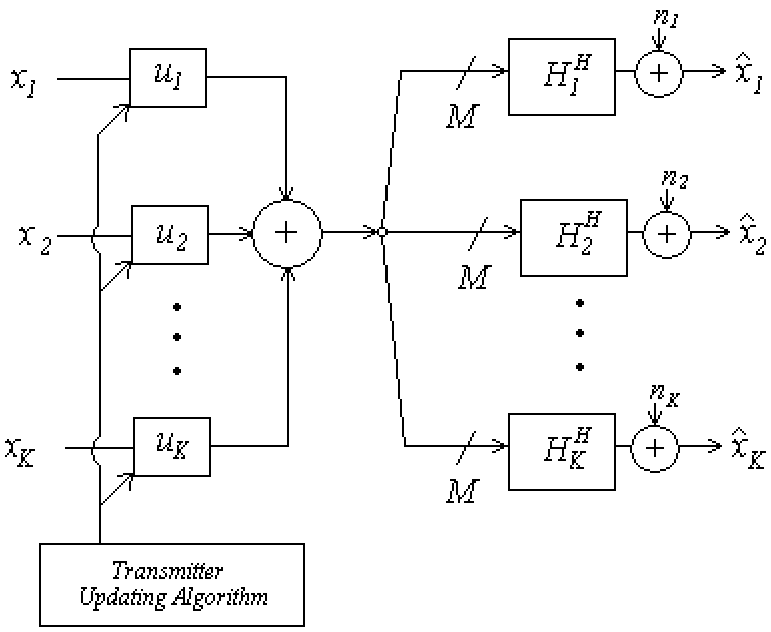

Figure 3.2 Functional block diagram for TrMMSE algorithm for single antenna mobile users

3.1.3 Channel Decomposition

One way of avoiding the interference is to isolate each user's data from each other and to designate several parallel interference-free communication lines in between the users and the transmitter. In order to do so, a transmit preprocessing technique at the transmitter may service the system, using the idea of *Channel Decomposition* [11]. The precoding vectors, generated by the Channel decomposition method, turn the MIMO-MU downlink channel into multiple parallel single-user MIMO channels.

For instance, a system of 6x(2,2,2), where the channel decomposition method is utilized, each user will experience the same performance as they were in a 2x2 MIMO-SU system. Once channels are decomposed among individual users, any single user MIMO receiver technique can be applied to collect the sent data. Figure 3.3 represents a comparison of downlink MIMO-SU system with MIMO-MU system, where Channel Decomposition method is used. As can be seen from Figure 3.3; after the decomposition each user operates as if it was a single user system with 2 transmitting and 2 receiving antennas. Note that, performance plots of corresponding cases of multi user and single user overlaps onto each other in Figure 3.3.

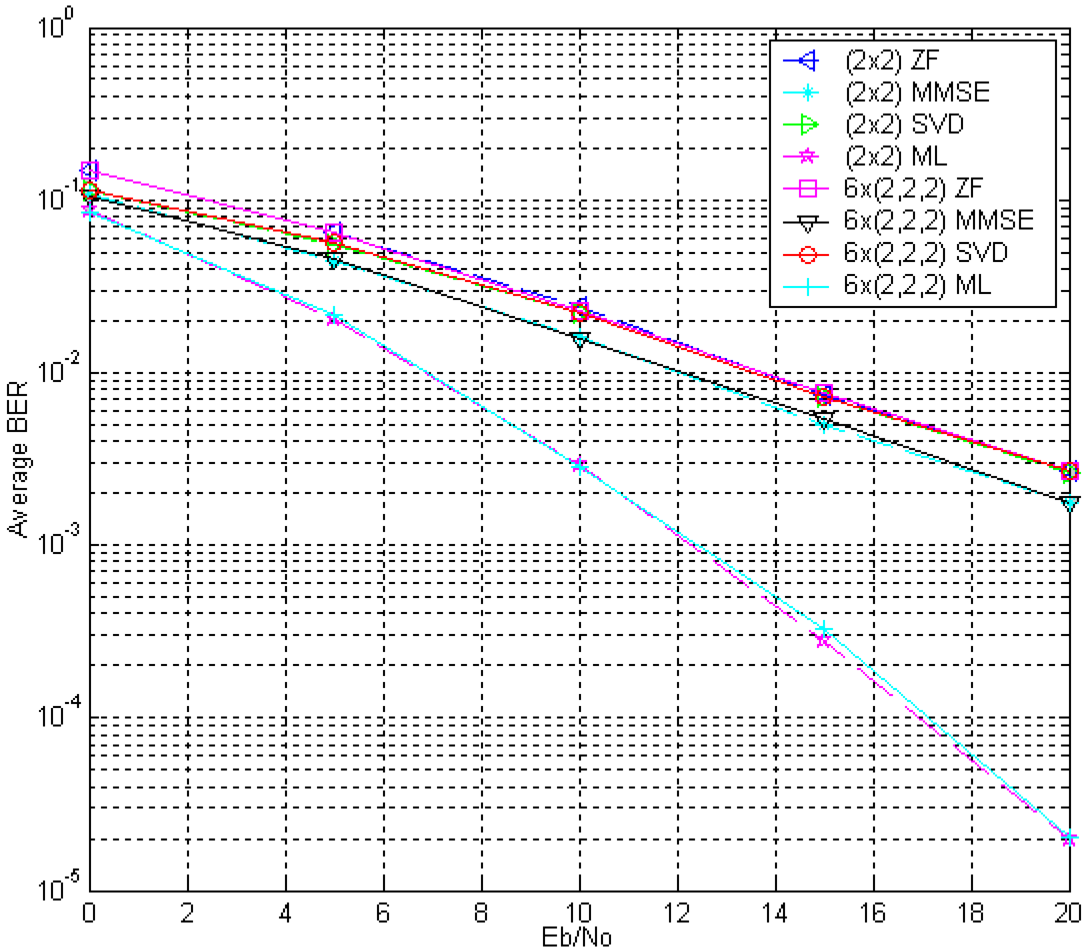

Figure 3.3 Comparison of downlink MIMO-SU system with MIMO-MU system where Channel Decomposition method is used

The configuration of a downlink communication scenario was given in Figure 2.4, which can also be assumed for this technique too. In Figure 2.4 assume that k^{th} user receives a L_k *x1* vector of x_k, where L_k is the number of data symbols, sent simultaneously to user *k*. U_k is a MxL_k precoding matrix assigned to user *k,* the procedure of finding which will later be explained. Each column of U_k is the preprocessing vector of an individual data symbol, aimed to simultaneously be transmitter to user *k*.

If we assume that, the power assigned for each vector of U_k matrices are same and there is no decoding algorithm at the users, then the received signals can be expressed by a vector of length N_k, which is given by

$$\hat{x}_k = H_k^H \sum_{m=1}^{K} U_m x_m + n \tag{3.9}$$

In such a system where channel decomposition is the concern, we will use the idea of "Null Space" from linear algebra. *U* vectors should be selected such that *inter-user interference* at any user is zero. In order to reach that result, U_k should be calculated for all users using the following equation,

$$U_k = \underset{trace(U_k U_k^{\ H}) \leq P_k}{\arg} \begin{pmatrix} H_1^H U_k = 0 \\ \ldots \\ H_{k-1}^H U_k = 0 \\ H_{k+1}^H U_k = 0 \\ \ldots \\ H_K^H U_K = 0 \end{pmatrix} \tag{3.10}$$

which means, U_k has to be in the null space (or "kernel") of all the channels in over all system, except the channel of user *k*. Each precoding vector should be calculated one by one taking the channels of other users into account, where the power of each vector should not exceed the power value set to that vector.

There is also an important constraint for the number of transmitting antennas, which is a constraint depending on the size of the calculated null spaces.

Channel decomposition method states that, the number of transmitting antennas has to be larger than the sum of the number of receive antennas of any *K-1* users, which can be written as;

$$M > \max\left(\sum_{i=1,i\neq k}^{K} N_i, k = 1,2,...,K\right) \tag{3.11}$$

The constraint mandates quite high number of antennas at the base station. In case of a mobile communication scenario of many mobile users, equipped with several antennas, this constraint imposes the transmitter to occupy high number of antennas. Moreover, If (3.11) is violated, the communication link between the base station and some of the users might be corrupted.

The constraint on (3.11) guarantees at least one independent channel for every user. Under that condition, more than one data stream can be transferred simultaneously to a user. If we indicate the number of data streams for each user as D_k, then;

$$D_k = M - \sum_{i=1,i\neq k}^{K} N_i, k = 1,2,..., K \tag{3.12}$$

$D_k > 1$ means there are more than one precoding vectors for user *k*, which are orthogonal to each other. (3.12) also indicates that an increase at the number of transmitting antennas also increases the number of precoding vectors of each user. For instance adding one more antenna at the base station will add an independent channel for each user. Consider a scenario, where a user in the system has a large number of receiving antennas in comparison to other users. In that case, the users with many antennas impose many nulling conditions, making the overall design more difficult. Furthermore the algorithm has another drawback. It attempts thoroughly to eliminate the interference and give no consideration of the noise [22].

The decomposed vectors of *U* can also be manipulated by some criteria such as BLAST or SVD techniques. Once the signal is received by a mobile user, any receiving techniques in Chapter 2 can be applied to the received data. Before applying one of the single user receiver techniques, instead of the channel in between the base station and the user, the *"effective channel",* realized by the user, (channel of precoding between users) should be used in the equations. In Figure 3.4 a block diagram of the technique can be found, where each user recovered their data streams by SVD solution. *S* and *V* are found by the use of rotation matrices of SVD solution. Also note that, the performance comparisons in Figure 3.3 were evaluated with the use of *SVD*.

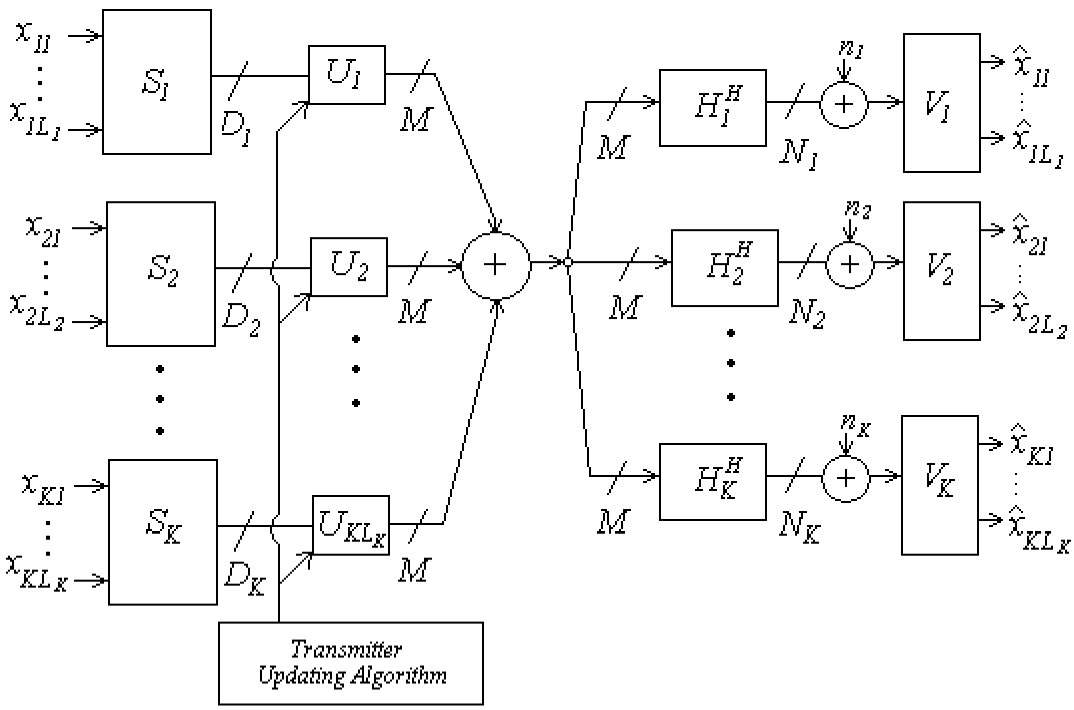

Figure 3.4 Functional block diagram for downlink transmission of Channel Decomposition method for multiple antenna mobile users

Channel Decomposition is similar to making ZF at the transmitter. The critical difference is that, ZF at the transmitter treat each receiving antenna individually and makes both intra-user and inter-user interference zero at any receive antenna, where channel decomposition nulls only the inter user interference and applies single user receiving algorithms for the interference free channels of users. ZF at transmitter would obviously have the same performance with channel decomposition algorithm, when ZF receiver is utilized at every mobile user.

In the literature various similar techniques or that of extensions can also be found such as [23] and [24].

3.1.3.1 Single Stream transmission for MIMO-MU with Channel Decomposition

The transmit antenna constraint of channel decomposition method, given in (3.11), might be avoided with further calculations. *Iterative Nu-SVD* algorithm, given in [23], improves the constraint as;

$$M > \max\left(\sum_{i=1, i \neq k}^{K} D_i , k = 1,2,...,K \right) \tag{3.13}$$

where D_i is the number of data streams transmitted to user *i*. Equation (3.13) allows the base station to determine the amount of its antennas according to the total number of data streams transmitted to mobile users. In case, where users aim to receive single data stream, (3.13) becomes;

$$M \geq K \tag{3.14}$$

which reduces the hardware complexity at the base station. According to *Iterative Nu-SVD algorithm* [23] the received vector at the BS is defined as follows;

$$\hat{x}_k = V_k \left(H_k^H \sum_{m=1}^{K} Q_m B_m x_m + n \right) \tag{3.15}$$

which means every data symbol at the BS is precoded with *Q and B* matrices consequently and afterwards given to the channel. Thus, the precoding matrix for user *k* is U_k=Q_k B_k. When the data stream is received by a mobile user, it is manipulated with V_k *matrix.*

The *Nu-SVD* combines the SVD solution with the Null Space usage in the channel decomposition method. Therefore, two different precoding operations are required prior to the transmission. In *Nu-SVD,* precoder Q_k contains the null space vectors found by channel decomposition method and precoder B_k is designed by applying the SVD solution to $H_k Q_k$.

Similar to problem formulation in (3.10), the precoding matrix Q should be selected so that, at the users all the interference terms be zero. Therefore the following problem might be defined in (3.16), which means Q_k has to be in the null space (or "kernel") of the *effective channels* of all users, except the effective channel of user *k*. Different from [11] effective channel of user k is $V_k H_k^H$, because of the installment of an extra decoding vector at the mobile users.

$$Q_k = \underset{0 \, ztrace(U_k U_k^H) \leq P_k}{\arg} \begin{pmatrix} V_1 H_1^H U_k = 0 \\ \dots \\ V_{k-1} H_{k-1}^H U_k = 0 \\ V_{k+1} H_{k+1}^H U_k = 0 \\ \dots \\ V_K H_K^H U_K = 0 \end{pmatrix} \tag{3.16}$$

Q_k represents the basis of evaluated null space and B_k denotes the coordinate transformation under that basis. Similar to [11] the aim in [23] is to generate a precoding and decoding scheme so that

$$V_k H_k U = \begin{bmatrix} 0_{1 \cdots k-1} & \underbrace{\Lambda_k}_{kth \;\; subblock \;\; matrix} & 0_{k+1, \cdots K} \end{bmatrix} \quad \text{or} \quad V_k H_k Q_k B_k = \Lambda_k \tag{3.17}$$

where $U = [U_1, \cdots, U_K]$. (3.17) is an solution, expected, as all the columns of U lays in the null space of H, except the k^{th} column. In that case the optimization rule in [23] is that;

$$(V_k, U_k)_{opt} = \arg \max_{V_k, U_k} \|\Lambda_k\|^2 \tag{3.18}$$

or as Q is found, we may also write down (3.18) as

$$(V_k, B_k)_{opt} = \arg\max_{V_k, B_k} \|\Lambda_k\|^2 \tag{3.19}$$

In [25] it has been shown that the SVD can be taken as the optimal solution in achieving the capacity for a single user MIMO-link. Therefore V_k, B_k can be updated by;

$$V_k = S_k \quad \text{and} \quad B_k = D_k \tag{3.20}$$

where $SVD(H_k Q_k) = S_k \Lambda_k D_k{}^H$. Using this solution, an iterative algorithm is formulated as follows

TABLE I Algorithmic solution of the Iterative Nu-SVD problem for single stream transmission

---

1: Initialize: n⇐0, $V_i^{(0)} = I_{D_j x Nr_i}, \forall i$

2: **repeat**

3: Compute Q_i, I=1, …, K using (3.16)

4: Compute B_i and V_i, I=1, …, K using (3.20)

5: **until** convergence

Nu-SVD method iteratively finds the precoding and decoding matrices, used at the downlink communication. This method will also be used in the next chapter, when the case of single stream MIMO-MU scenario is concentrated.

3.1.4 SINR Balancing

The techniques, we have described so far, conduct algorithms only to find proper precoding vectors. Where the SINR is the most critical constraint to achieve more with the wireless system, the amount of power, assigned to each user, should also be adjusted. The reason is that; if the levels of SINRs at the users are needed to be modified, changing the power, transmitted to an individual user, would obviously change the interference for all other users. Therefore a better performance would be leaded by introducing a dynamic transmit power assignment among users according to their channel condition. This necessitates a precoding solution where the precoding vectors and power weights are needed to be jointly optimized.

While this class of solutions achieve their desired objectives optimally, they are iterative in nature, and therefore have a substantially higher computational cost compared to channel decomposition and transmit MMSE/ZF schemes [13].

The SINR balancing technique [26] introduces a downlink beamforming algorithm using the *duality principle,* details of which was given in the previous chapter. Therefore the objective is to find optimal downlink power assignments, *p, q* and spatial signatures, *u*, for obtaining an SINR levels same for all users;

$$SINR = SINR_1^{DL} = SINR_2^{DL} = \cdots = SINR_K^{DL} \tag{3.21}$$

which are also equal to the uplink SINR levels as mandated by the duality principle. Rather than using SINR levels, a cost function may also be introduced for assigning different levels of SINRs for individual users such that;

$$C_i^{DL} = \frac{SINR_i^{DL}}{\gamma_i} \tag{3.22}$$

where $C_1^{DL} = C_2^{DL} = \cdots = C_K^{DL}$. SINR Balancing technique intakes target threshold values ($\gamma_1,....,\gamma_K$), which are different from each other and being set according to the SINR need of each signal destination point. In the next generation systems there will be an increasing need to support heterogeneous wireless services, which means the needs of the users would be different. The algorithm balances the SINR qualities proportional to the SINR needs. One may choose the threshold levels same for all user, in order to provide same servicing to all. In the simulations of this technique, the thresholds will be selected as equal for all of the participated users.

In literature, SINR balancing was studied for two different scenarios. First is the case where all the users receive one data stream and having a single antenna [26]. The second is the case where users may receive more than one data stream simultaneously, which mandates the use of decoding vectors at the receivers [16]. In this section we will mainly evaluate the results of [26] for discussions on SINR balancing, however the results of [16] will shortly be explained later in this section.

Duality principle sets same vectors for precoding (at downlink) and decoding (at uplink) at the base station. Therefore Figure 2.5 and Figure 2.6 can be accepted as the functional block diagrams for single stream downlink and uplink transmissions of SINR Balancing method. With slight changes the SINR equations given in (2.26) and (2.27) are updated respectively;

$$SINR_i^{DL} = p_i \frac{u_i^H R_i u_i}{Q_i} \quad , \quad Q_i = \sum_{\substack{k=1 \\ k \neq i}}^{K} p_k u_k^H R_i u_k + \sigma_i^2 \tag{3.23}$$

$$SINR_i^{UL} = q_i \frac{u_i^H R_i u_i}{u_i^H T_i u_i} \quad , \quad T_i = \sum_{\substack{k=1 \\ k \neq i}}^{K} q_k R_k + \sigma_i^2 I \tag{3.24}$$

While using SINR Balancing algorithm for the solution of downlink transmission, u_i vectors for precoding at the BS and p_i values for power assignment should be found. As long as the solution of uplink dual is easier to evaluate, algorithm first aims to solve the uplink dual for finding u vectors.

Considering the uplink SINR equation (3.24) if we aim to maximize the SINR level, u vectors should be chosen as;

$$\hat{u}_i = \arg\max_{u_i} q_i \frac{u_i^H R_i u_i}{u_i^H T_i u_i} \tag{3.25}$$

Thus the optimum precoding vector for each user can be evaluated by finding the dominant generalized eigenvectors of the matrix pairs (R_i, T_i) for $1 \leq i \leq K$.

For the solution of the equation in (3.25) q_i values (the uplink power assignments) has to be priorly found. In [26] it has been shown that q_i values may be evaluated by ;

$$\Lambda(\widetilde{U}, P_{\max}) q_{ext} = \lambda_{\max}\left(\Lambda(\widetilde{U}, P_{\max})\right) q_{ext} \quad \text{with } \left[q_{ext}\right]_{K+1} = 1 \qquad (3.26)$$

where $q_{ext} = [q_1, q_2 \ldots q_K, 1]$ and P_{max} is the total transmit power. The optimal power vector q is obtained as the first K components of the dominant eigenvector of $\Lambda(\widetilde{U}, P_{\max})$, which can be scaled so that its last component equals one.

An iterative algorithm for solving of uplink dual should be used for balancing the SINR levels of the users. After the iterations, the optimal u vectors will be found and then equation (3.27) can be used for finding the downlink power assignments.

$$\Upsilon(\widetilde{U}, P_{\max}) \widetilde{p}_{ext} = \lambda_{\max}\left(\Upsilon(\widetilde{U}, P_{\max})\right) \widetilde{p}_{ext} \quad \text{with } \left[p_{ext}\right]_{K+1} = 1 \qquad (3.27)$$

where $p_{ext} = [p_1, p_2 \ldots p_K, 1]$. According to the solution, p_{ext} is the dominant eigenvector of $\Upsilon(\widetilde{U}, P_{\max})$ with its last component scaled to one.

Υ and Λ are called coupling matrices, the representations of which are given in Appendix D.

It has also been shown in [26] that, at each step of the SINR balancing procedure, the algorithm maximizes the *worst SINR* among all of the users. Therefore the optimization problem can also be formulated as maximizing of the worst case SINR, that is to say;

$$\max\left(\min_{1\le i\le K}\left[\frac{SINR_i^{DL}}{\gamma_i}\right]\right) \text{ under a total power constraint} \tag{3.28}$$

Algorithmic Solution of the SINR Balancing Problem:

An algorithm for finding the precoding vectors, u_k, and power assignments, p_k and q_k, is proposed as in Table II.

TABLE II Algorithmic solution of the SINR Balancing problem for single antenna mobile users

---

1: Initialize: $n \Leftarrow 0$, $q^{(0)} = [0,.....,0]^T$

2: $\widetilde{R}_i \Leftarrow R_i / \sigma_i^2$, $1 \le i \le K$

3: $\sigma_i^2 \Leftarrow 1$, $1 \le i \le K$

4: ***repeat***

5: $n \Leftarrow n+1$

6: $u_i^{(n)} \Leftarrow e_{\max}(\widetilde{R}_i, T_i(q^{(n-1)}))$, $1 \le i \le K$

7: $u_i^{(n)} \Leftarrow u_i^{(n)} / \|u_i^{(n)}\|_2$, $1 \le i \le K$

8: *solve* $\Lambda(U^{(n)}, P_{\max})\begin{bmatrix} q^{(n)} \\ 1 \end{bmatrix} = \lambda_{\max}(n)\begin{bmatrix} q^{(n)} \\ 1 \end{bmatrix}$

9: ***until*** $\lambda_{\max}(n-1) - \lambda_{\max}(n) < \varepsilon$

10: compute the optimum downlink power allocation p by solving

$$\Upsilon(U^{(n)}, P_{\max})\begin{bmatrix} p \\ 1 \end{bmatrix} = \lambda_{\max}(n)\begin{bmatrix} p \\ 1 \end{bmatrix}$$

In the algorithm the iteration number is shown by notation n. At each iteration, algorithm first keeps the transmitted power assignments fixed and chooses the precoding vectors, later in the same iteration it does fix the vectors and finds out the power levels.

A natural extension of this problem is to consider a case where the users are also equipped with multiple antennas, where one may transmit multiple streams for each user. However for this case the users also need decoders for decoding multiple data streams. Therefore the aim is again same; jointly optimizing the power allocation and transmit-receive filters for all users by taking the advantage of the duality between the uplink and downlink [16]. The *S* and *T* matrices, calculated in (2.29), (2.30), (2.32) and (2.33) previously, are valid for multiple data stream transmission case. The downlink block diagram in Figure 2.7 and uplink block diagram in Figure 2.8 are used for multiple antenna case.

Similar to the idea given with the equation (3.25), the precoder and decoders can be found by finding the dominant generalized eigenvectors of the matrix pairs of *S and T* matrices;

$$v_{kj}^{(n)} = \hat{e}_{\max}(S_{kj}^{DL}, T_{kj}^{DL}) \ , \ \ 1 \le k \le K \ while \ 1 \le l \le L_k \tag{3.29}$$

$$u_{kj}^{(n)} = \hat{e}_{\max}(S_{kj}^{UL}, T_{kj}^{UL}) \ , \ \ 1 \le k \le K \ while \ 1 \le l \le L_k \tag{3.30}$$

An algorithmic solution of the SINR Balancing problem for multiple antenna mobile users is given in [16] with some slight adaptations on the coupling matrices Υ and Λ.

Rather than SINR, in the literature there are also several works, which focuses on different constraint in order to generate duality in between the uplink and downlink such as [27], where the problem is minimizing the maximal normalized MSE of all users for fairness and the sum of all normalized MSE for overall efficiency. In [28] SINR Balancing solution was combined with the SVD solution.

3.1.5 Joint MMSE Transmitter and Receiver Design

We have already reviewed some downlink techniques, designed for multiple stream transmission. These algorithms require multiple antennas, employed at the users, as been reasoned with the discussions on Figure 3.1. That means the users also have arrays of antennas and it is possible to make *beamforming* also at the receivers to further improve the system BER performance. The Joint-MMSE (*JMMSE)* technique designs the transmitter and receiver for the downlink of MIMO-MU jointly, where the interference and noise for each of the users is balanced by allocating the transmit power effectively among them. The algorithms of [16] and JMMSE are the most computationally expensive ones over all the schemes, we have discussed so far, but they also offer the best performance as we will see later in the numerical results section.

JMMSE technique introduces an iterative method to solve the multiuser downlink problem like the SINR Balancing method. As an optimization criterion, it uses *minimum total mean square error* (T-MMSE), which aims to design a set of transmitter and receiver precoding and decoding matrices so that the summation of *MSE* of every user could be minimized under the constraint of BS power.

$$\min_{U_1,U_2,\ldots U_k,V_1,V_2,\ldots,V_k} \sum_{i=1}^{K} MSE_i \,, \qquad \text{s.t.} \;\; tr\left(\sum_{i=1}^{K} U_i^H U_i\right) = P \tag{3.31}$$

where P is the total transmit power for the base station and U is the precoding matrix for the transmitting side. That means the optimization is made not only being subject to TMMSE but also for a given total transmit power. In [29], the minimization of MSE criterion is previously studied for single user MIMO systems.

Again taking the Figure 2.7 into consideration, the estimate of sent data symbols could be expressed as;

$$\hat{x}_k = V_k \; (H_k^H \sum_{m=1}^{K} U_m x_m + n) \tag{3.32}$$

The downlink and uplink block diagram of JMMSE solution is similar to the ones, given in Figure 2.7 and Figure 2.8. The optimization problem is defined as follows [22], which is adapted from the uplink solution of [30]. For the k^{th} user, the MSE could be calculated as,

$$MSE_k = E\|\hat{x} - x\|^2 = tr\left(E(\hat{x} - x)(\hat{x} - x)^H\right)$$

$$= tr\left(V_k H_k \left(\sum_{i=1}^{K} U_i U_i^H \right) H_k^H V_k^H + N_0 V_k V_k^H - U_k^H H_k^H V_k^H - V_k H_k U_k + I \right) \quad (3.33)$$

We will use an algebraic knowledge "*Lagrange multiplier*" to solve the MSE equation. The Lagrange dual objective function could be constructed as follows

$$L(V_1,...,V_k;U_1,...,U_k;\mu) = \mu\left(tr\left(\sum_{i=1}^{K} U_i^H U_i \right) - P \right)$$

$$+ \sum_{i=1}^{K} tr\left(V_i H_i \left(\sum_{i=1}^{K} U_i U_i^H \right) H_i^H V_i^H + N_0 V_i V_i^H - U_i^H H_i^H V_i^H - V_i H_i U_i + I \right) \quad (3.34)$$

where μ is the Lagrange multiplier. A detailed description about Lagrange Multipliers can be found in Appendix B. Taking derivative of the equation (3.34) with respect to $(V_1,...,V_k;U_1,...,U_k)$ and equating them to 'zero', we could solve out the optimum precoding and decoding matrices for this optimization problem.

$$\frac{\partial L(V_1,...,V_k;U_1,...,U_k;\mu)}{\partial U_i^*} = \left(\sum_{i=1}^{K} H_i^H V_i^H V_i H_i + \mu I \right) U_i - H_i^H V_i^H = 0 \quad (3.35)$$

$$U_i = \left(\sum_{i=1}^{K} H_i^H V_i^H V_i H_i + \mu I \right)^{-1} H_i^H V_i^H \quad (3.36)$$

And the decoding matrix is calculated as;

$$\frac{\partial L(V_1,...,V_k;U_1,...,U_k;\mu)}{\partial V_i^*} = V_i H_i \left(\sum_{i=1}^{K} U_i U_i^H \right) H_i^H + N_0 V_i - U_i^H H_i^H = 0 \quad (3.37)$$

$$V_i = U_i^H H_i^H \left(H_i \left(\sum_{i=1}^{K} U_i U_i^H \right) H_i^H + N_0 I \right)^{-1} \quad (3.38)$$

Note that, (3.38) also states that the decoder of each user depends on only its own channel, when the precoders are known. Those results are similar to optimum joint linear transmitter and receiver designs of a single user MIMO system, where design is to minimize MSE of the received signal [29]. The optimum transmitter and receiver could be realized through a joint iterative algorithm given in Table III. The optimal transmitter and receiver beamformers are dependent on each other. Thus, for the algorithm in Table III, typically some arbitrary initial values are chosen, and the transmitter and receiver side beamformers are iteratively recalculated until the convergence criterion is met. [13]

TABLE III Algorithmic solution of the JMMSE problem

1: Initialize: n⇐0, $V_i^{(0)} = I_{D_I x N r_i}$, $\forall i$

2: **repeat**

3: Compute U_i, I=1, …, K using (3.34) and $V_i^{(0)}$

4: Compute V_i, I=1, …, K using (3.36) and U_i

5: **until** $\sum_{i=1}^{K} \|V_i(n+1) - V(n)\|_F^2 < \varepsilon$

where $I_{D_I x N r_i}$ is a $D_i x N r_i$ matrix with its *(m,n)*th entry is "zero" for *m≠n* and "1" for *m=n*. Notation *n* denotes the index of iteration and $\| \ \|_F^2$ is the squared Frobenius norm. Furthermore the calculation of the Lagrange multiplier, μ, should also be found throughout the algorithm (see Appendix C)

In each iterative step of Joint MMSE (JMMSE) method the total mean square error is monotonously diminishing, where it is lower bounded to 0 [22]. Therefore the method guarantees the convergence. It should also be realized that

JMMSE algorithm has consideration of noise and a dynamic transmit power assignment among individual users.

A recent improvement was made in [17], where power assignments of JMMSE method are further optimized due to stronger subchannels.

3.2 NUMERICAL COMPARISONS

In this part of the chapter, we will compare performance graphics of the techniques discussed for MIMO-MU downlink communications. The results are obtained by simulations, carried out on MATLAB, including the performance comparison given in Figure 3.3. The number of runs at each simulation point is determined by taking the lowest error level into consideration. The total number of runs at each simulated SNR value were chosen as *at least* 10 times greater than the power of lowest BER level of the logarithmic chart. Same considerations were also made for the rest of the simulations in this thesis. The number of runs corresponds to; *the number of channel realizations* and *the number of data symbol vector realizations* together. The number of data symbol vector realizations were chosen as 10 to 50 depending on the run time of the simulation. For instance, where the BER plot of a downlink method reaches the lowest value of 10^{-6}, total number of runs should be selected as (10x1.000.000), that is 10.000.000 times. We may reach this number by simulating *at least* 1.000.000 different channel realizations with 10 different data symbol vector realizations at each channel realization.

Channels are Rayleigh fading with unit variance and zero mean. In the simulation, each of the channel components was generated with Gaussian imaginary and real parts, each having ½ variance and zero mean. In the simulations quadrature-phase-shift keying (QPSK) is utilized.

The estimates are also assumed to pass through a *minimum distance receiver* at the output of the downlink algorithms at the users. Minimum Distance Receiver picks the element from the unit energy QPSK data symbol set, which is closest in Euclidean distance to the estimate of the signal.

The simulations are conducted to illustrate the *average bit error rate* (BER) performances of the downlink methods. The procedure of finding the BER performance is as follows; after each channel use, the estimates of the transmitted symbols are taken from the output of minimum distance receiver at each user. The estimates are compared with the transmitted data symbols and the total number of symbol errors is found at each user. Note that, if single data stream is transmitted to each user, then it corresponds to 1 symbol transmission per channel use. Multiple streams of data transmission correspond to multiple symbols per channel use. After finding the number of symbol errors at each user, they are summed to find the *total number of symbol errors* and then averaged to the total number of users. As long as QPSK is utilized in our simulatons, every 1 symbol error corresponds to 2 bits error. The average BER can be calculated in this fashion. A detailed description on the SNR calculations can be found In Appendix A.

TABLE IV A qualitative comparison of downlink algorithms

	Dynamic Power Assignment	Noise Consideration	No Limitation on # of Transmit Antennas	No Iteration
SINR Balancing [26]	√	√	√	*X*
JMMSE [22]	√	√	√	*X*
Channel Decomposition [11]	*X*	*X*	*X*	√
TrMMSE [19]	*X*	√	√	√

In Table IV a qualitative comparison of four techniques is given according to some important design criteria of a downlink algorithm. Note that the properties of every algorithm are different from each other for solving the downlink problem. Therefore one may choose any of these according to the system needs.

Two different scenarios, one for MISO-MU for single stream transmission and one for MIMO-MU multiple stream transmission, were simulated at the next two sections.

3.2.1 MISO-MU for Single stream transmission

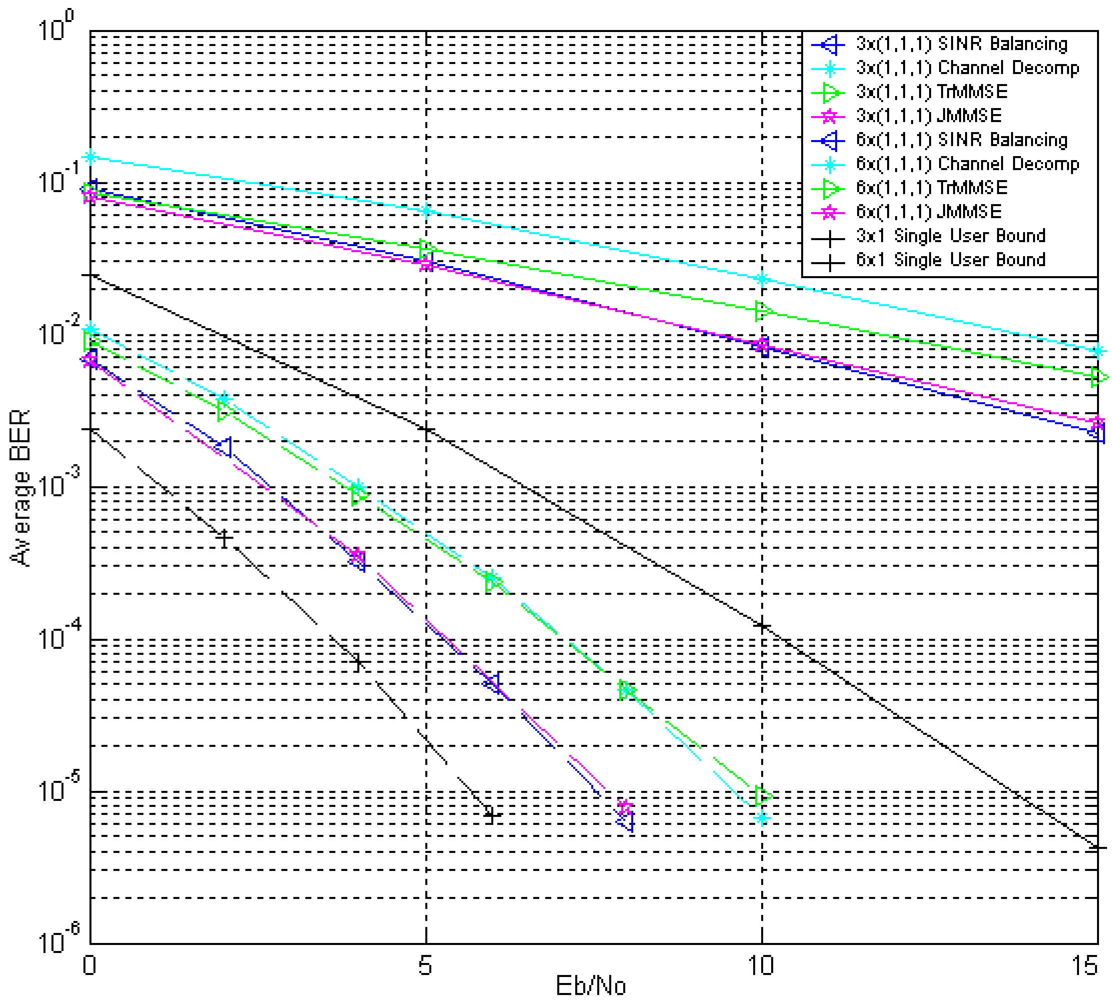

Figure 3.5 Performance comparison between MISO-MU downlink algorithms for single data stream transmission

For MISO-MU single data stream transmission case, we represent our simulation results for a 3x(1,1,1) and a 6x(1,1,1) communication system on Figure 3.5.

As expected, increasing the number of transmitting antennas at the BS improves the average BER performance of the system. The diversity order of the 3x(1,1,1) system is around *one*, which is again expected since 3 transmitting antennas servicing 3 users with a single antenna. The diversity order increases by one, as the number of diversity branches increase by one. Ultimately, the slope of the curve, drawn for 6x(1,1,1) system, reaches four.

In Figure 3.5 it has shown that the SINR Balancing and JointMMSE (JMMSE), which runs more complex algorithms to converge, have better performances among others. It should also be taken into consideration that, when there is only one receiving branch at each of the mobile users, the decoder V of JMMSE algorithm becomes a scalar instead of a vector, where SINR Balancing doesn't need any decoding due to its design for single antenna receivers.

In Figure 3.5 single user bounds are also given. We can see that when the number of transmitting antennas increase, we get more closer curves to these bounds. These bounds were generated by using the optimal beamforming, assuming that the channel is known at the transmitter. It has been shown in [31] that the optimal single user precoder is proportional to the eigenvector of $H^H H$ that corresponds to the largest eigenvalue.

3.2.2 MIMO-MU for Multiple stream transmission

For MIMO-MU multiple data stream transmission case, we represent our simulation results for a *6x(2,2,2)* and a *10x(2,2,2)* communication system in Figure 3.6, where each user receives 2 symbols per channel use.

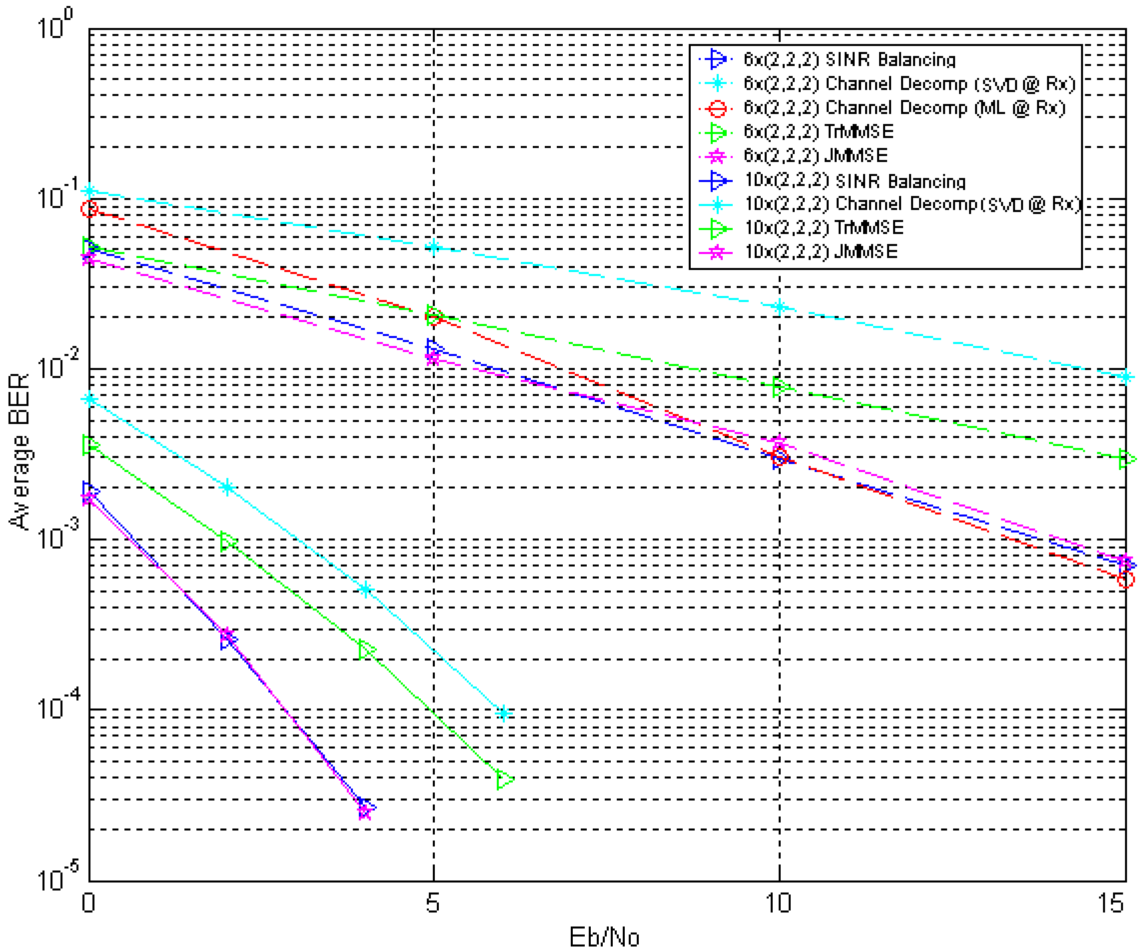

Figure 3.6 Performance comparison between MIMO-MU downlink algorithms for multiple stream transmission

The slope of the curves tends to be *one* for the cases where $\sum_{r=1}^{K} N_r = M$ and increases upto almost *five* with an increase of *four* in the number of transmitting braches. The Channel decomposition (with SVD) technique performs a similar success for *3x(1,1,1)* and *6x(2,2,2).*

The Channel Decomposition with ML receiver case was also tested for the $\sum_{r=1}^{K} N_r = M$ case, in order to show its better performance when SNR is high. Similar with the MISO result in Figure 3.5, JMMSE and SINR Balancing techniques have better performances for multiple data stream transmission with multiple antennas at the mobile users.

CHAPTER 4

SINGLE STREAM MIMO-MU DOWNLINK

In the previous chapter, we analyzed linear algorithms for the solution of downlink problem of multi user MIMO. The focus was, whether the users are occupied with single or multiple antennas for single or multiple simultaneous data stream receipt. In this Chapter, we will examine a special case, where a single stream MISO user upgrades its system to multiple antennas to realize *diversity* gain, rather than spatial multiplexing. In other words we examine the case of upgrading a single stream MISO architecture to a single stream MIMO architecture.

Single stream MIMO-MU means, despite having multiple antennas, the mobile users still receive single streams of data. A decoder, jointly optimized with the transmitter, or the use of a complex receiver structure might be an unnecessary burden for the multiple antenna mobile users, as long as they receive only a single data stream. Therefore, the object of this chapter is to present simplest possible type of receivers, in addition to achieving a better performance.

MRC (Maximal Ratio Combining) is a simple technique, which can be implemented by any coherent receiver, capable of channel estimation and combining. Therefore it does not bring an architectural complexity for the receiver that implements it. Furthermore, MRC may also be used by a receiver, which is able to switch its receiving antennas on and off. In case of a need for more diversity gain, that receiver may activate its multiple antennas, by also applying MRC at the output of its antennas to get the estimate of the sent signal. Note that MRC corresponds to the same solution with *matched filtering*, explained in Chapter 2.

ML and MMSE are more powerful in mitigation of MAI. However with respect to MRC, extra stages are required (e.g. pre-whitening for MMSE) and an increased computational cost is brought to the receivers with the implementation of MMSE and ML methods. Therefore they are not preferable from the simplicity point of view. In the later numerical comparison section, it will also be shown that the BER performance of MRC is almost same when the MMSE method is used for single stream MIMO-MU.

We propose three novel methods for single receiver mobile users, each of which receive single data stream. The first method is based on using Fixed Combining Rule (FCR) at the receivers, which finds a set of precoders optimal in the MSE sense, when decoding vectors are set to a pre-determined vector. The second method optimizes the same SINR among all users at the input of MRC receivers, called PreMRC SINR Balancing. The third method, PostMRC SINR Balancing, optimizes the same SINR at the output of MRC receivers. Second and the third methods are the extensions of the SINR Balancing method [26].

Downlink methods, discussed so far, may also be used for single stream MIMO case. However, the application of Joint MMSE, SINR Balancing (with multiple receiving antenna) methods, which have the better performance among other downlink methods, needs the usage of a secondary channel for the transmission of decoding vectors. When the secondary channel is not available (when system is designed as a MISO system at the beginning), the option is not realizable. For the novel techniques of this chapter, the decoding at the receivers is made on a predetermined basis without any need of coordination in between the receiver and the transmitter for the generation of the decoders, which removes the secondary channel need. However, the use of a simple receiver at the mobile users (MRC or any other predetermined combining scheme) brings an extra algorithmic intensity for the transmitter, while selecting the precoding vectors to compensate the lack of higher complexity receivers.

4.1 PROBLEM DESCRIPTION

For simplicity, consider a two-user communication system, where the transmitter with *M* antennas serves two users, each equipped with a two antennas. Assuming that MRC is set as the receiving architecture at each mobile user, the following figure can be realized;

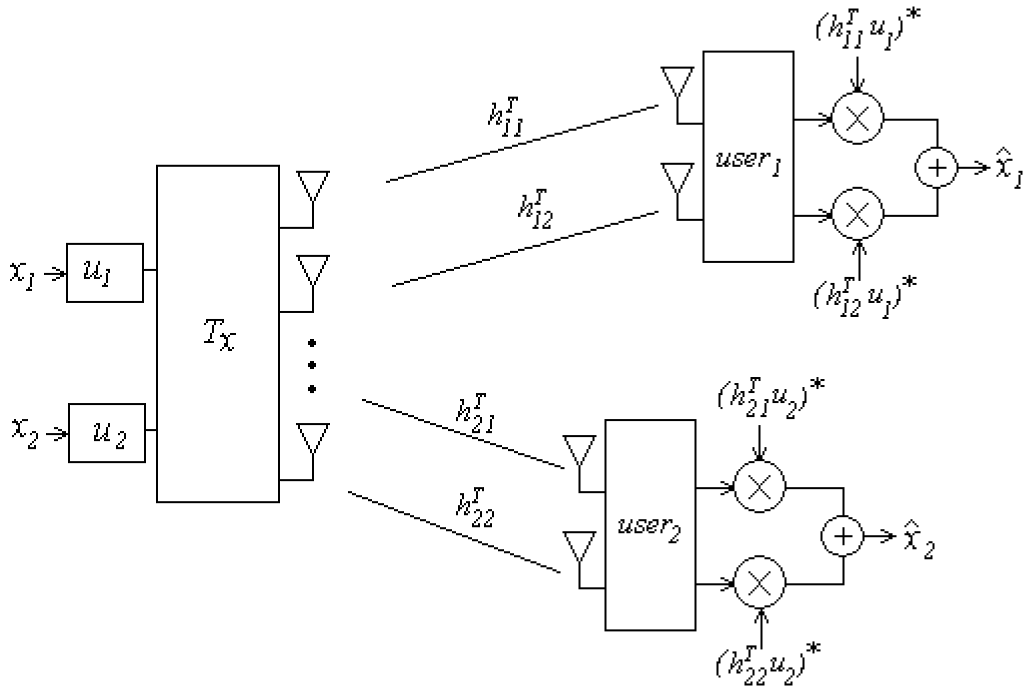

Figure 4.1 MIMO-MU with MRC at the mobile users

We assume that the channel is flat fading zero mean unit variance coefficients independent from each other. Similar to the matrix given in (2.23), for this scenario we have the following matrix for the received signals,

$$
\begin{matrix} User1\left\{ \begin{matrix} \\ \\ \end{matrix} \right. \\ User2\left\{ \begin{matrix} \\ \\ \end{matrix} \right. \end{matrix}
\begin{bmatrix} r_{11} \\ r_{12} \\ r_{21} \\ r_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{h_{11}^T u_1} & h_{11}^T u_2 \\ \mathbf{h_{12}^T u_1} & h_{12}^T u_2 \\ h_{21}^T u_1 & \mathbf{h_{21}^T u_2} \\ h_{22}^T u_1 & \mathbf{h_{22}^T u_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_{11} \\ n_{12} \\ n_{21} \\ n_{22} \end{bmatrix} \tag{4.1}
$$

$$
r_k = H_k \sum_{l=1}^{2} u_l x_l \tag{4.2}
$$

In the equation above, x_k denotes the data symbol transmitted to the k^{th} user and u_k is the precoding vector for the transmission of x_k. The bold entries in r matrix represents the signal values intended to be received by each user, others are the interfering terms. The goal of transmit precoding is the selection of u_1 and u_2 so that the interference terms are eliminated as much as possible. The vector h_{km}^T is a row vector of dimension *1xM* whose complex valued entries are the channel coefficients between the m^{th} antenna and the M antennas at the transmitter, for user k. Additive noise vector n is composed of complex Gaussian i.i.d. components with variance of σ^2. The total available power at the transmitter is P_T, where $\sum_{k=1}^{K} |u_k|^2 = P_T$

4.1.1 Fixed Combining Rule (FCR)

FCR at the receiver is the simplest combination rule to be applied for benefiting from the multiple observations obtained via multiple receiving antennas. For providing simplicity, the FCR approach sets a pre-determined vector at the receivers for combining these multiple observations. The pre-determined combination vector is the same for all users.

The transmitter of the FCR-based communication system should create precoding vectors so that the pre-determined combination vectors at the receivers achieve the optimal result. The optimization criterion is determined as the minimization of the total mean square error (MSE) of the overall system after the combination operation, which has previously been examined during the discussions on JMMSE method.

For a two user case we assume that, both users implement $[1 \quad 1]^T / \sqrt{2}$ as the combination operation. The goal is to minimize $E\left\{\sum_{k=1}^{2} |x_k - \hat{x}_k|^2\right\}$ where $E\{\ \}$ is the expectation operation and $\hat{x}$ is the estimate of x formed with the stated rule, that is;

$$\hat{x}_k = \frac{1}{\sqrt{2}}(r_{k1} + r_{k2}) \tag{4.3}$$

Then $\hat{x}$ can be written as

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \underbrace{\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}}_{G} \left(\underbrace{\begin{bmatrix} h_{11}^T \\ h_{12}^T \\ h_{21}^T \\ h_{22}^T \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} u_1 & u_2 \end{bmatrix}}_{U} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_{11} \\ n_{12} \\ n_{21} \\ n_{22} \end{bmatrix} \right) \tag{4.4}$$

$$\hat{x} = GHUx + \tilde{n} \tag{4.5}$$

where $x = [x_1 \quad x_2]^T$ and n is a *2x1* vector whose entries are zero mean σ^2 variance additive white Gaussian noise.

Remember that (3.36) and (3.38) were the optimal precoder and decoder for downlink of JMMSE method, which was also using the same optimization criterion with FCR. As long as there would be no need for finding decoders (because they are fixed), the equation of the optimal precoding matrix derived in (3.36), might be adapted for FCR as follows;

$$U^H = (H^H G^H GH + \mu I)^{-1} H^H G^H \tag{4.6}$$

where μ is Lagrange multiplier to satisfy the power constraint $Tr\{U^H U\} = P_T$, which leads to following equation for the unknown variable *v*

$$\sum_{k=1}^{K} \frac{\lambda_k}{(\lambda_k + \mu)} = P_T \tag{4.7}$$

where λ_k is the k^{th} eigenvalue of the $H^H G^H GH$ matrix, similar to the JMMSE method given in Appendix C .

Note that, it is possible to use any other combination matrix G in the calculation above. The success of this operation is studied by numerical studies at a later section.

4.1.2 PreMRC SINR Balancing

Instead of locating fixed decoders at the receivers for combining, now we will let the use of a dynamic combining method, which will be the *MRC*. At the beginning of this chapter various advantages of using MRC at the mobile users was already discussed. *SINR Balancing* method will be utilized with MRC receivers.

The definition of the SINR, objected to be balanced at every user, is the critical concept for adjusting the precoding vectors at the transmitter. *SINR levels at the input of MRC receiver* is taken into consideration by the *PreMRC SINR Balancing.*

If we go back to two user case, the total signal power-over-interference-plus-noise at the input of MRC receivers in two observations (r_{11}, r_{12} in (4.1)) for the first user is;

$$preSINR_1 = \frac{\left|h_{11}^T u_1\right|^2 + \left|h_{12}^T u_1\right|^2}{\left|h_{11}^T u_2\right|^2 + \left|h_{12}^T u_2\right|^2 + 2\sigma^2} \tag{4.8}$$

The tSNR relation can also be written as follows:

$$preSINR_1 = \frac{p_1 \widetilde{u}_1^H \left(h_{11}^* h_{11}^T + h_{12}^* h_{12}^T\right) \widetilde{u}_1}{p_2 \widetilde{u}_2^H \left(h_{11}^* h_{11}^T + h_{12}^* h_{12}^T\right) \widetilde{u}_2 + 2\sigma^2} \quad (4.9)$$

where $\widetilde{u}$ is normalized (unit norm) precoding vectors and $u = \sqrt{p_k}\,\widetilde{u}$. With this notation $\sum_{k=1}^{K} p_k = P_T$ becomes the power constraint.

In the previous chapter the SINR Balancing method was studied. That study in [26] refers to a system, in which every user has a single antenna for single stream transmission. With definition of preSINR in (4.9), it is possible to apply same method in [26] to balance preSINR levels of multi antenna users. The only difference between two optimization goals is that, preSINR involves rank two matrices, $\left(h_{11}^* h_{11}^T + h_{12}^* h_{12}^T\right)$, instead of rank one matrices. But this does not bring any setback for the iterative solution proposed in [26]. Therefore the algorithm in Table II can also be used for this method with slight changes (i.e. Replace $R_i = h_{i1} h_{i1}^H + h_{i2} h_{i2}^H$ and $\gamma_1, \ldots., \gamma_K = 1$ with their correspondents in the Table II).

With this method the ratio of total signal power to the total interference and noise power is maximized.

4.1.3 Post MRC SINR Balancing

Instead of dealing with the SINR value at the input of the MRC receiver, SINR at the output of the combiner may also be used in SINR Balancing method, since we know that the MRC processing at the receivers should further increase SINR at the output (See Appendix E). Therefore we may expect to realize at better performance by considering the SINRs at the outputs of the MRC receivers of each user.

Moreover the precoding vectors, found by SINR Balancing method, may also be adapted based on the receiver architecture prior to the transmission, as long as it is already known by the transmitter. *FCR* technique has benefited from that idea, for the generation of its algorithm. Lets define the received data symbol for user *m,*

$$\hat{x}_k = v_{k1} r_{k1} + v_{k2} r_{k2} \tag{4.10}$$

For the case of two users, the estimates $\hat{x}_1$ and $\hat{x}_2$ can be written as;

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} v_1^T \\ v_2^T \end{bmatrix} \begin{bmatrix} h_{11}^T \\ h_{12}^T \\ h_{21}^T \\ h_{22}^T \end{bmatrix}}_{\widetilde{H}} \begin{bmatrix} u_1 & u_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} v_1^T \\ v_2^T \end{bmatrix} \begin{bmatrix} n_{11} \\ n_{12} \\ n_{21} \\ n_{22} \end{bmatrix}}_{\widetilde{n}} \tag{4.11}$$

In (4.11) v_k is the combination vector for the user k. If we denote the rows of $\widetilde{H}$ with $\widetilde{h}_1$ and $\widetilde{h}_2$ equation (4.11) can be rewritten as follows;

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \begin{bmatrix} \widetilde{h}_1^T u_1 & \widetilde{h}_1^T u_2 \\ \widetilde{h}_2^T u_1 & \widetilde{h}_2^T u_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \widetilde{n}_1 \\ \widetilde{n}_2 \end{bmatrix} \tag{4.12}$$

In (4.12) $\widetilde{n}_1$ and $\widetilde{n}_2$ denote samples of additive white noise with variance $\|v_1\|^2 \sigma^2$ and $\|v_2\|^2 \sigma^2$ respectively. From the equation presented in (4.12) the output SINR for the first user can be written as

$$postSINR_1 = \frac{\left|\widetilde{h}_1^T u_1\right|^2}{\left|\widetilde{h}_1^T u_2\right|^2 + \sigma^2 \|v_1\|^2} \tag{4.13}$$

The SINR relation presented in (4.13) is in the form that can be used by the SINR Balancing algorithm.

MRC requires the addition of every received signal after operating each with the hermitian of the effective channel. Where the effective channel for user k is $H_{eff} = H_k u_k$, the MRC receiver v can be calculated as;

$$v_k = \left(H_k u_k\right)^H \tag{4.14}$$

where H_k is composed of the channel information of user *k,* that is h_{k1} and h_{k2} for two user scenario.

In Table V we present an extension on SINR Balancing algorithm for the worst case optimization of the SINR after MRC.

TABLE V Algorithmic solution of the PostMRC problem

1: Initialize: $n \Leftarrow 0,\ v_1^{(0)} = \cdots = v_K^{(0)} = [1,.....,1]^T / \sqrt{M}$

2: ***repeat***

3: $\quad n \Leftarrow n+1$

4: $\quad \widetilde{h}_i^{\,T} \Leftarrow \left(v_i^{\,(n-1)}\right)^T \begin{bmatrix} h_{i,1}^T \\ \vdots \\ h_{i,M}^T \end{bmatrix},\ 1 \le i \le K$

5: $\quad \sigma_i^2 \Leftarrow \sigma^2 \left\| v_i \right\|_2,\ 1 \le i \le K$

6: $\quad$ *Use SINR Balancing algorithm given in Table I*

$\quad$ *on* $\widetilde{h}_k$ *and* σ_i^2 *to find* $u_i^{(n)}$, $1 \le i \le K$

7: $\quad v_i^{(n)} \Leftarrow conjugate\left(\begin{bmatrix} h_{i,1}^T \\ \vdots \\ h_{i,M}^T \end{bmatrix} u_i^{(n)}\right),\ 1 \le i \le K$

8: ***until*** $\left\| v_i^{(n)} - v_i^{(n-1)} \right\|^2 < \varepsilon,\ ,\ 1 \le i \le K$

9: return $\{u_1, u_1, \cdots u_K\}$

The algorithm is given for a system with K users each equipped with M antennas. Throughout the balancing algorithm, the channel matrix $\widetilde{H}$ has to be updated because of the reaon that made us proceed from (4.11) to (4.12) (that corresponds to the step 4 in the *PostMRC* algorithm on Table V).

Note that, although both PostMRC and PreMRC SINR Balancing methods run the same SINR Balancing algorithm, PostMRC has a second loop in its iterations for updating the $\widetilde{H}$ at each turn, which increases the time of run at the transmitter but gives better performance.

4.2 NUMERICAL COMPARISONS

In this section the three novel algorithms; *FCR, PreMRC SINR Balancing and PostMRC SINR Balancing,* will be compared by computer simulation. The assumptions and simulation criteria are same with the information provided in Section 3.2.

The single user bounds are obtained with a similar fashion as in the previous chapter. Different from the previous user bound discussions, MRC receiver implementation is assumed at the receivers for optimum diversity combining.

In Figure 4.2, we provide sample BER performance comparison between our three proposed algorithms and the previously explained two algorithms (Joint MMSE and the Channel Decomposition solution with SVD). The system is a 6x(2,2,2) system where L=1, the number of data streams transmitted to each of the branches.

The *PostMRC* has the same performance with *JMMSE*. Also it can be realized from the figure that, the algorithms, which has the most computational complexity, have the better performance. In comparison with Figure 3.5, where the performance graphs of an 6x(1,1,1) system is given, channel decomposition gives a worsened performance as the number of receiving branches increases. That is due to the fact that, for Channel Decomposition, when the number of branches increases for a user, the null space of other users decreases. That resuls with a shortening in the space of the communication.

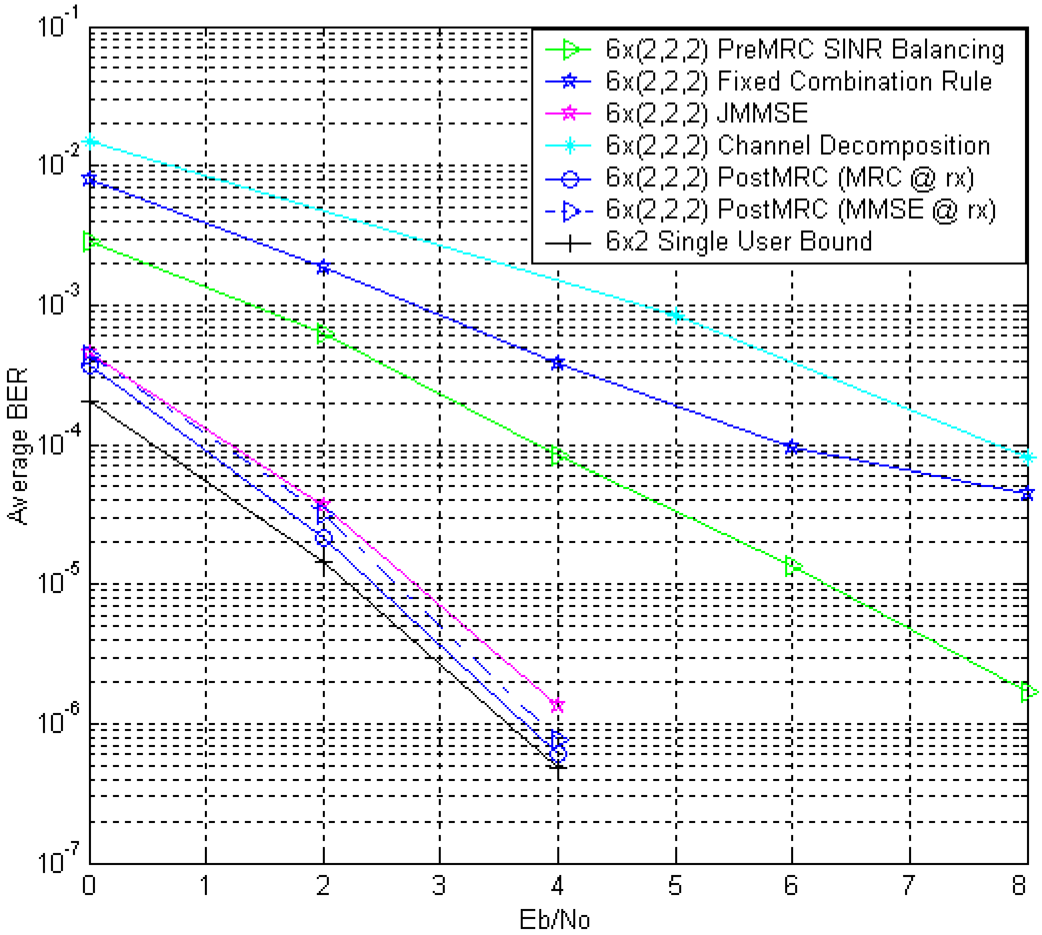

Figure 4.2 Performance comparison between algorithms used for single data stream MIMO-MU of 6x(2,2,2)

PreMRC SINR Balancing and *Fixed Combination rule(FCR)* also has poor performances due to their simple architectures. Since the MRC output, which is the decision variable entering into slicer, is not directly optimized by these methods, both schemes exhibit an error floor at high SNR values. Notice that, if MMSE receiver is replaced with the MRC at the receivers of PostMRC, we get a result more closer to the optimal beamforming result as SNR increases .

For the system in Figure 4.2 there is no limitation on number of antennas for choosing the algorithm to be used, because of the fact that the total number of receiving antennas are equal to the number of transmitting antennas. However when the number of transmit antennas decrease, the channel decomposition method can not be used because of antenna limitation. The next figure, Figure 4.3, gives the simulation results for a 3x(2,2,2) system.

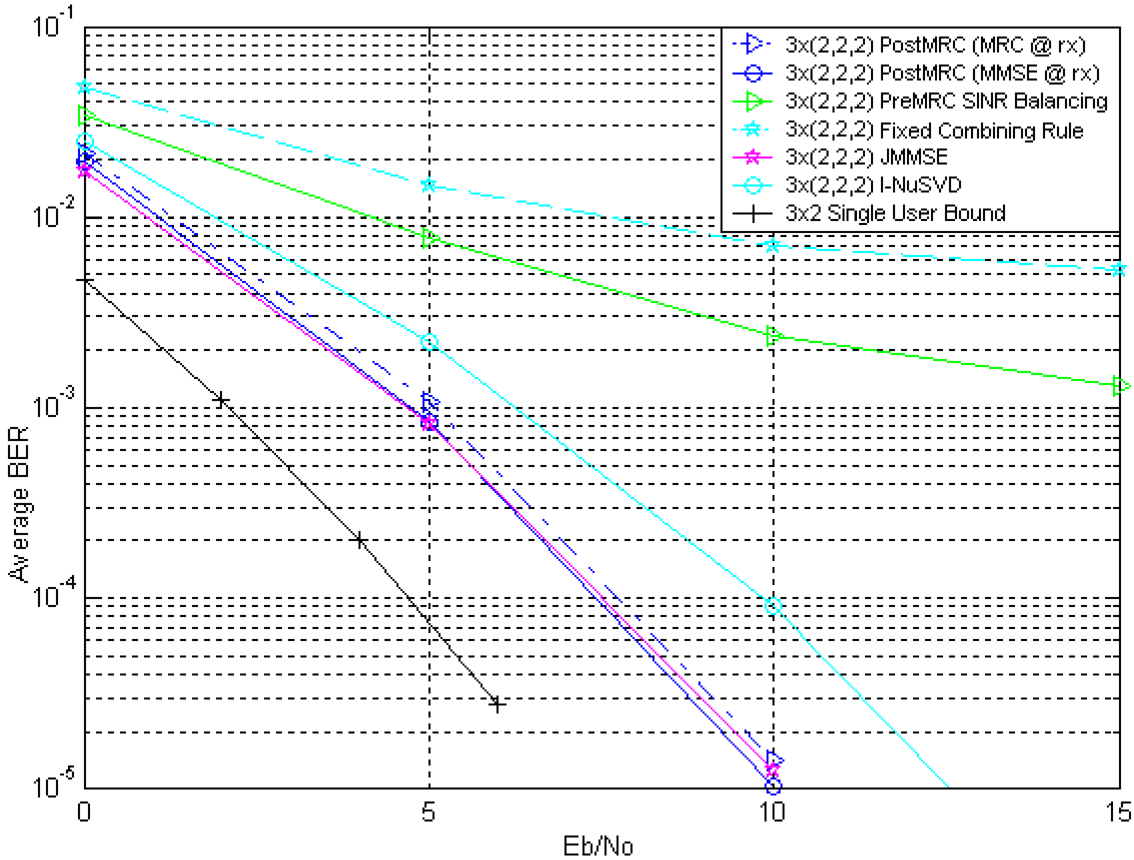

Figure 4.3 Performance comparison between algorithms used for single data stream MIMO-MU of 3x(2,2,2)

Even for this limiting condition the *PostMRC* gives a better performance. Because the total number of receive antennas do not allow generating adequate null space for users, the channel decomposition [11] method for *3x(2,2,2)* system is replaced with *Iterative Nu-SVD* [22]. *FCR* and *PreMRC SINR Balancing has* severe error floors due to multi-user interference.

CHAPTER 5

CONCLUSION

In this thesis, downlink algorithms for MIMO-MU wireless communication systems have been investigated. There are several of these algorithms, claiming different approaches for the problems of multiple access (MA) in space domain. As been mentioned, throughout the thesis, the main problem with the MIMO multiple access environment is the interference in the system.

Firstly, some conventional preprocessing methods, such as Zero Forcing at transmitter and MMSE at the transmitter (TrMMSE, [19]) were discussed. These algorithms are not powerful but easy to implement. Similar to single user *MMSE and ZF* results, when the transmit antenna condition is inadequate, one might face error floors in the implementation of the preprocessing method.

The Channel Decomposition techniques ([11], [23], [17]) aims to generate parallel independent channels among all mobile users. This provides an interference-free communication for all of the users. Mathematically, those algorithms use the null space idea and selects the precoding vectors from the null spaces of the users' channels. We have observed that the drawback for [11] is that, it has a strict antenna constraint at the BS. It is required that the total number of antennas at the base station should be larger than or equal to the size of largest null space generated among the users' channels. In [23] that drawback is somehow avoided by jointly optimizing the receivers and the transmitter and generating the null space, not only according to the receive antennas, but also according to the number of data streams to be transmitted to one user.

The decomposition techniques have no consideration of noise, which also effects their performance. Another disadvantage is that, an increase in the number of receive antennas causes an unfair advantage for the user, which has increased its number of receiving branches. Moreover, if the number of transmitting antennas is at the limit condition, given in (3.11), an extra receive antenna at one of the users would need an increase in the transmit branch quantity.

There are other iterative techniques for better performance. SINR Balancing [26, 16] methods maximize the SINR of the user, which has the worst SINR. They also puts forward a solution considering the noise. JMMSE [22] solution aims to minimize the total MMSE of the overall system. These techniques, including the channel decomposition method in [23], run iterative algorithms and requires the receivers to be informed about the optimal decoding vectors before the start of communication. Due to their computational and hardware cost, these methods may not be preferred. However these are the algorithms, which perform closer to the optimal beamforming solutions.

One concern of this thesis is to investigate the existing methods for a multi user case, where users are equipped with multiple antennas but receiving single data streams. The existing methods might be adapted for that situation. However for better performance, algorithms, requiring special decoding vectors at the receiver, should be selected. Despite each user is receiving only single stream, such a requirement causes an unnecessary load for the mobile users. Design of a method, which does not need an extra decoding algorithm and which applies simple receiver structures, was set as one of the goals for the thesis.

We have proposed three novel techniques for the stated scenario. The critical point on the design of these techniques is that, the users implement the simplest possible receiver structure, which is the MRC. Therefore the goal was not only to succeed in finding a near optimal solution, but also to propose techniques which does not have high complexity at the receiver ends. We also assume a practical use for the multiple antennas of the mobile users. In the examined scenario, users may switch the antennas on and off to realize additional diversity gain as needed.

The First proposed method is *Fixed Combining Rule (FCR)*. It is the simplest method that the receivers have predetermined decoders. Transmitter adapts its precoding vectors according to the predetermined decoding vector. The second is the *PreMRC SINR Balancing*, which is an extension of [26]. In that technique SINR at the input of each MRC receiver is optimized. These two algorithms might be preferred, when the transmitter has many antennas and therefore system is not interference limited. For that case second antenna installment significantly improves system performance.

The third proposed method, *PostMRC SINR Balancing*, sets an iterative algorithm to optimize the worst case SINR after MRC combining. It has been observed that the simple MRC receiver performs near optimal results. We attain the performance of Joint MMSE optimization (which uses more complicated MMSE receivers) after Post MRC SINR Balancing.

MRC is known to be optimal only when the signal is free of interference and noise is additive white Gaussian. The performance of MRC will be sub-optimal and therefore an MMSE receiver should have better BER performance, if the system has colored noise or interference. Despite this, we have chosen MRC as the receiving rule on purpose, in order to simplify the harware of the mobile users' receiver and made use of the precoding operation to compensate the gap of performance between MRC and MMSE. We have observed that with PostMRC SINR Balancing a BER performance is achieved, identical to a system where the receivers are jointly optimized with the transmitter (JMMSE).

FUTURE WORK

A future study might be conducted for improving the performance of those downlink techniques proposed in Chapter 4, (especially for PreMRC Balancing and Fixed Combining Rule). For instance, the structure of combination matrix G in FCR might be changed by defining another optimization criterion on it.

Rather the performance criterion might also be changed in these future works. In our simulations the average BER performance of all users was taken into consideration. For instance the criterion might also be set as maximizing the minimum performance user and that of performance graphics might be drawn.

For the multi-user MIMO channel, given a constraint on the total transmitted power, it is possible to allocate varying fractions of that power to different users in the network [13] due to the relative attenuation levels of the users or dependency between the users channels. That makes the *capacity* considerations of a MIMO-MU system more complex and a critical for choosing the algorithm to be used. The capacity of the MIMO multiple-user channel might also be analyzed using proposed methods, respect to different channel conditions (e.g. different amplitudes of channel components might be defined in the simulations).

Flat-fading and narrowband channels are assumed for simulations, however for many current and next-generation wireless communications applications, this assumption does not hold. Wideband or frequency-selective fading channels suffer from intersymbol interference and a fading characteristic that varies significantly across the frequency band [13]. The proposed downlink techniques might be simulated for a frequency selective environment rather than a flat fading (non-frequency selective) one, which would be a more realistic case where the data rate of the transmission causes a large delay spread. Further, common frequency selective channel techniques like OFDM and/or the use of different modulation schemes such as QAM (for increase of symbol interval [32]) etc. might also be used.

In this thesis no techniques, proposed for multi-antenna receivers, is considered for some common drawbacks associated with wireless systems such as; inter-cell interference of multicell systems, crosstalk, near far effect etc. For a more realistic fashion these very well known problems should also be investigated and/or adaptations on the downlink algorithms may also be made.

It has been assumed throughout the thesis research that, the instantaneous perfect channel knowledge is available at the transmitter, therefore the feedback cost of channel information hasn't been discussed for the novel proposed methods. As one of the aims of this thesis was to simplify the receivers, a further work might be conducted to simplify the feedback burden of the users and both simplifications might associate in a more simplified mobile user architecture. Feedback burden of the channel state information is also an important concept to be analyzed, which should be fed back to the transmitter during the coherence time of the channel. Moreover, the research might be extended with sensitivity analyzes of the numerical methods that finds the eigenvectors.

There are also several sub-topics in MIMO communications, which haven't been examined throughout this work; *coding (e.g. Space-Time coding (STC)), scheduling, nonlinear precoding* (e.g. *Dirty Paper Coding (DPC) in [34]*) are some of these. Discussions of these, for the solution of downlink problem of MIMO-MU, would open new aspects for research, especially for the special case, where single stream transmission is aimed at the MIMO-MU wireless system.

REFERENCES

[1] A. Goldsmith, Wireless Communication, Stanford Univetrsity 2004

[2] D. Tse, P. Viswanath, Fundamentals of Wireless Communications, Cambridge University press, 2005

[3] J.G. Proakis, Digital Communications. 4th Ed. New York: McGraw-Hill, 2001.

[4] I. E. Telatar, Capacity of Multi-antenna Gaussian Channels, European Transactions on Telecommunications, Volume 10, No 6, November 1999, Page(s):585–595

[5] H. Bölcskei, D. Gesbert, C. B. Papadias, A. V. D. Veen, Space-Time Wireless Systems from Array Processing to MIMO Communications, Cambridge University Press, 2006

[6] V. Tarokh, N. Seshadri, A. R. Calderbank, Space-time Codes for High Data Rate Wireless Communication: Performance Criterion and Code Construction, IEEE Transactions on Information Theory, Volume 44, Issue 2, March 1998, Page(s):744 – 765

[7] S. Alamouti, A Simple Transmit Diversity Technique for Wireless Communications, IEEE Journal on Selected Areas Communications, Volume 16, Issue 8, October 1998, Page(s):1451 – 1458

[8] R. T. Derryberry, S. D. Gray, D. M. Ionescu, G. Mandyam, B. Raghothaman, Transmit Diversity in 3G CDMA Systems, IEEE Communications Magazine, April 2002

[9] J. R. Barry, E. A. Lee, D. G. Messerschnitt, Digital Communications, Kluver Academic Publishers, 3rd Edition

[10] A. Paulraj, R. Nabar, D. Gore, Introduction to Space-Time Wireless Communications, 2003, Cambridge University Press

[11] L. U. Choi, R. D. Murch, A Transmit Preprocessing Technique for Multiuser MIMO Systems Using a Decomposition Approach, IEEE Transactions On Wireless Communications, Volume 3, Issue 1, January 2004, Page(s):20 – 24

[12] A. S. Dakdouki, N. K. Mykhaylov, A. A. Skopa, Downlink Processing Algorithms for Multi-Antenna Wireless Communications, IEEE Communications Magazine, Volume 43, Issue 1, January 2005, Page(s):122 – 127

[13] Q. H. Spencer, C. B. Peel, A. Lee Swindlehurst, M. Haart, An Introduction to the Multi-user MIMO Downlink, IEEE Communications Magazine, Volume 42, Issue 10, October 2004, Page(s):60 – 67

[14] J. Love, W. Heath Jr., W. Santipach, M.L. Honig, What is the Value of Limited Feedback for MIMO Channels, IEEE Communication Magazine, Volume 42, Issue 10, October 2004, Page(s):54 – 59

[15] H. Lee, M. Shin, C. Lee, An Eigen-based MIMO Multiuser Scheduler with Partial Feedback Information, IEEE Communications letters, Volume 9, Issue 4, April 2005, Page(s):328 – 330

[16] A. Khachan, A. Tenenbaum, R. S. Adve, Linear Processing for the Downlink in Multiuser MIMO Systems with Multiple Data Streams, IEEE International Conference on Communications, Volume 9, June 2006, Page(s):4113 – 4118

[17] J. Joung, Y. H. Lee, Regularized Channel Diagonalization for Multiuser MIMO Downlink Using a Modified MMSE Criterion, IEEE Transactions on Signal Processing, Volume 55, Issue 4, April 2007, Page(s):1573 – 1579

[18] M. Schubert, S. Shi, E.A. Jorswieck, H. Boche, Downlink Sum-MSE Transceiver Optimization for Linear Multi-User MIMO Systems, IEEE Conference Record of the Thirty-Ninth Asilomar Conference Signals, Systems and Computers, November 2005, Page(s):1424 – 1428

[19] L. U. Choi, Multi-user MISO and MIMO Transmit Signal Processing for Wireless Communication, PhD Thesis submitted to the Hong Kong University of Science and Technology, March 2003

[20] Q. H. Spencer, A. L. Swindlehurst, M. Haardt, Zero-Forcing Methods for Downlink Spatial Multiplexing in Multiuser MIMO Channels, IEEE Transactions on Signal Processing, Volume 52, Issue 2, February 2004, Page(s):461 – 471

[21] Z. Shen, R. Chen, J. G. Andrews, R. W. Heath, Jr., B. L. Evans, Low complexity User Selection Algorithms for Multiuser MIMO Systems with Block Diagonalization, IEEE Transactions on Signal Processing, Volume 54, Issue 9, September 2006, Page(s):3658 – 3663

[22] J. Zhang, Y. Wu, S. Zhou, J. Wang, Joint Linear Transmitter and Receiver Design for the Downlink of Multiuser MIMO Systems, IEEE Communications Letters, Volume 9, Issue 11, November 2005, Page(s):991 - 993

[23] Z. Pan, K. K. Wong, T. S. Ng, Generalized Multiuser Orthogonal Space-Division Multiplexing, IEEE Transactions on Wireless Communications, Volume 3, Issue 6, November 2004, Page(s):1969 – 1973

[24] Q. H. Spencer, A. Lee Swindlehurst, A Hybrid Approach to Spatial Multiplexing in Multiuser MIMO Downlinks, EURASIP Journal on Wireless Communications and Networking, Volume 4, Issue 7, November 2004, Page(s): 739 - 754

[25] G. G. Raleigh, J. M. Cioffi, Spatio-temporal Coding for Wireless Communications, IEEE Transactions on Communications, Volume 46, Issue 3, March 1998, Page(s):357 – 366

[26] M. Schubert, H. Boche, Solution of the Multiuser Downlink Beamforming Problem with Individual SINR constraints, IEEE Transactions on Vehicular Technology, Volume 53, Issue 1, January 2004, Page(s):18 – 28

[27] S. Shi, M. Schubert, MMSE Transmit Optimization for Multi-user Multi-antenna Systems, IEEE international conference on Acoustic Speech and Signal Processing Proceedings, Volume 3, March 2005, Page(s):iii/409 - iii/412 Vol. 3

[28] K. E. Dawui, D. T. M. Slock, Multiuser-MIMO Downlink Tx-Rx Design Based on SVD Channel Diagonalization and Multiuser Diversity, IEEE Conference Record of the Thirty-Ninth Asilomar Conference on Signals, Systems and Computers, November 2005, Page(s):1493 – 1497

[29] H. Sampath, P. Stoica, A. Paulraj, Generalized Linear Precoder and Decoder Design for MIMO Channels Using the Weighted MMSE Criterion, IEEE Transactions on Communications, Volume 49, Issue 12, December 2001, Page(s):2198 – 2206

[30] S. Serbetli, A. Yener, Transceiver Optimization for Multiuser MIMO Systems, IEEE Transactions on Signal Processing, Volume 52, Issue 1, January 2004, Page(s):214 – 226

[31] E. G. Larsson, P. Stoica, Space-Time Block Coding for Wireless Communications, Cambridge University Press, 2003

[32] M. Schwarz, Mobile Wireless Communications, Cambridge University Press, 2005

[34] M. Costa, Writing on Dirty Paper, IEEE Transactions on Information Theory, Volume 29, Issue 3, May 1983, Page(s):439 – 441

[35] S Jensen, An Introduction to Lagrange Multipliers, at "http://www.slimy.com /~steuard/teaching /tutorials /Lagrange.html"

APPENDIX A

In order to compare the performances of different architectures or different algorithms, (*Transmission_power_per_bit* over *Noise_variance)* is accepted as the SNR constraint.

In the computer simulations that can be realized in different ways. For instance; if we fix the total transmission power to a constant value, say "1", the value of noise variance should be changed according to the SNR value to be simulated.

$$SNR = \left(Transmission\_power\_per\_bit \middle/ Noise\_\mathrm{var}iance \right) = \frac{E_b}{\sigma^2} \tag{A.1}$$

then,

$$SNR = \frac{\left(\dfrac{E_t}{total\_number\_of\_bits\_per\_channel\_usage} \right)}{\sigma^2} \tag{A.2}$$

$$\sigma^2 = \frac{1}{total\_number\_of\_bits\_per\_channel\_usage} \tag{A.3}$$

where E_t is the total power at the transmitter and E_b is the transmission power per bit, which is calculated by dividing the total number of bits per one transmission to the total transmit power which is "1". Total number of bits per channel usage is calculated by multiplying "*bits per symbol*" with the "*total number of transmismitted symbols per channel usage*". For instance, for QPSK, bits per symbol is *2,* and if there are *3* users, which are being serviced with single data stream, then *transmismitted symbols per channel usage* is *3.* That means in total 6 bits are transmitted per channel usage.

In another way, the noise variance might be kept constant and the total transmit power might be increased, as the SNR value, to be tested, is intended to be increased.

APPENDIX B

Consider a two dimensional case, where we have a function $f(x,y)$ to maximize subject to $g(x,y)=c$. [35]

The usual way, in which we extremize a function in multivariable calculus, is to set,

$$\nabla\big(f(x,y)\big)=0 \tag{B.1}$$

where ∇ is the gradient operation. One answer for such an optimization problem is to add a new variable μ to the problem and to define a new function to extremize

$$\nabla\big(F(x,y,\mu)\big)=\nabla\big(f(x,y)+\mu.g(x,y)-c\big)=0 \text{ for } \mu\neq 0 \tag{B.2}$$

For JMMSE case the relevant functions are

$$MSE_k = E\|\hat{x}-x\|^2 \Rightarrow f(x,y) \tag{B.3}$$

$$tr\left(\sum_{i=1}^{K} U_i^H U_i\right)=P \Rightarrow g(x,y)=c \tag{B.4}$$

$$L(V_1,...,V_k;U_1,...,U_k;\mu) \Rightarrow F(x,y,\mu) \tag{B.5}$$

APPENDIX C

In this part we will show mathematically how to update the Lagrange Multiplier μ in JMMSE method in Chapter 3 and the proposed FCR algorithm in Chapter 4.

Denote $A=\sum_{i=1}^{K} H_i^H V_i^H V_i H_i$, where the singular value decomposition of matrix A is $S\Lambda S^H$. Then,

$$P = tr\left(\sum_{i=1}^{K} U_i^H U_i\right) = tr\left((A+\mu I)^{-1} A (A+\mu I)^{-1}\right) = tr\left(S(\Lambda+\mu I)^{-1}\Lambda(\Lambda+\mu I)^{-1} S^H\right)$$

$$= \sum_{i=1}^{M} \frac{\lambda_i}{(\lambda_i+\mu)^2} \tag{C.1}$$

λ_i s are the singular values of matrix *A*. The right hand side of the equation takes value from 0 to $+\infty$. Hence the solutions for μ always exist. Solving the final equation at most *2M* roots can be found some of which might be imaginary solutions. By testing the real solutions to find the one, which gives the minimum MSE, we could finally get μ.

APPENDIX D

SINR Balancing algorithm [26] introduces two coupling matrix notations, Υ and Λ as follows;

$$\Upsilon(U, P_{\max}) = \begin{bmatrix} D\Psi(U) & D\sigma \\ \frac{1}{P_{\max}} 1^T D\Psi(U) & \frac{1}{P_{\max}} 1^T D\sigma \end{bmatrix} \quad \text{(D.1)}$$

$$\Lambda(U, P_{\max}) = \begin{bmatrix} D\Psi^T(U) & D\sigma \\ \frac{1}{P_{\max}} 1^T D\Psi^T(U) & \frac{1}{P_{\max}} 1^T D\sigma \end{bmatrix} \quad \text{(D.2)}$$

where,

- $\sigma = \begin{bmatrix} \sigma_1^2 & \ldots & \sigma_K^2 \end{bmatrix}$
- $1 = \begin{bmatrix} 1 & \ldots & 1 \end{bmatrix}^T$
- The spatial covariance matrix; $R_i = \varepsilon\{h_i(t) h_i^H(t)\}, 1 \le i \le K$
- $[\Psi(U)]_{ik} = \begin{cases} u_k^H R_i u_k, k \ne i \\ 0, k = i \end{cases}$
- $D = diag\{(\gamma_1 /(\tilde{u}_1^H R_1 \tilde{u}_1)), \ldots, (\gamma_K /(\tilde{u}_K^H R_K \tilde{u}_K))\}$

APPENDIX E

For the discussions in Chapter 4, it has been stated that; applying Maximal Ratio Combining (MRC) improves the SINR of the received signal (Figure 4.1). Therefore *postMRC* method would give a better performance than the *preMRC* gives. This can be proven as follows: The received signal of the first user can be alternatively expressed as $r_1 = \tilde{h}_1 x_1 + \tilde{h}_2 x_2 + n$. Here $\tilde{h}_1$ is the effective channel that antennas of the first user see when precoding is implemented at the transmitter. In other words $\tilde{h}_1$ and $\tilde{h}_2$ are the first column and second column of the matrix in (4.1) associated with the rows of user 1 (first two rows). When MRC is implemented on r_1, the output SINR becomes the left hand side of

$$\frac{\left|\tilde{h}_1^H \tilde{h}_1\right|^2}{\left|\tilde{h}_1^H \tilde{h}_2\right|^2 + \tilde{h}_1^H \tilde{h}_1 \sigma_n^2} \geq \frac{\tilde{h}_1^H \tilde{h}_1}{\tilde{h}_2^H \tilde{h}_2 + \sigma^2} = preSINR \qquad \text{(E.1)}$$

And the right hand side of inequality is the input SNR. The equality in written Cauchy-Schwarz inequality that is $\left|\tilde{h}_1^H \tilde{h}_2\right| \leq \left(\tilde{h}_1^H \tilde{h}_1\right)\left(\tilde{h}_2^H \tilde{h}_2\right)$. Therefore the SNR at the input optimized by the proposed system is further increased after MRC operation.