EIGHTH GRADE STUDENTS' SKILLS IN TRANSLATING AMONG DIFFERENT REPRESENTATIONS OF ALGEBRAIC CONCEPTS

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ABSTRACT

EIGHTH GRADE STUDENTS' SKILLS IN TRANSLATING AMONG DIFFERENT REPRESENTATIONS OF ALGEBRAIC CONCEPTS

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The purpose of this study was to determine eighth grade students' skills of translating among different representations; graphic, table, equation, and verbal sentence; of algebraic concepts. Moreover, it was also aimed to investigate if there is any gender difference regarding the translation skills of students translating multiple representations, and their most common errors in making these translations.

For data collection, 18 schools were selected randomly from 103 elementary schools in Çankaya district of Ankara. Then all of the eighth grade students in each school were selected as sample. In total 705 eighth grade students were participated in the study.

To assess students' translation skills "Translation among different representations of algebraic concepts test" (TADRACT) was developed by researcher. Descriptive statistics were obtained to understand students' achievement in translation process. To compare mean scores of female and male students, the statistical analysis of Independent Samples t-test was used. Every question were examined in detail to determine any misconceptions, and most frequent errors students made in translating among different algebraic representations.

The results of test indicated that 8th grade students had poor skill in translations of four different representations; verbal statement, equation, table, graphic; in algebraic concepts. There was no significant difference between mean scores of girls and mean scores of boys. The most problematic translations were from other representations; equation, table, graphic; to verbal statement, and translations from other three representations; verbal statement, equation, graphic; to table were the easiest translations.

Keywords: Mathematics education, multiple representations, algebra, gender difference

ÖZ

SEKİZİNCİ SINIF ÖĞRENCİLERİNİN CEBİR KAVRAMLARININ FARKLI TEMSİL BİÇİMLERİ ARASINDA DÖNÜŞÜM YAPMA BECERİLERİ

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Bu araştırmanın amacı sekizinci sınıf öğrencilerinin cebir kavramlarının çoklu temsil biçimleri (grafik, tablo, denklem, sözlü anlatım) arasında dönüşüm yapma becerilerini belirlemekti. Ayrıca, kız ve erkek öğrencilerin dönüşüm yapma becerilerinde farklılık olup olmadığı ve öğrencilerin en kolay, en zor bulduğu dönüşümler ve en çok yapılan hataların araştırılması hedeflenmiştir.

Ankara 'nın Çankaya ilçesindeki 103 ilköğretim okulu arasından 18 okul rasgele seçilmiştir ve bu okullardaki tüm sekizinci sınıf öğrencileri örnek olarak alınmıştır. Toplam olarak 705 sekizinci sınıf öğrencisi bu araştırmaya katılmıştır.

Öğrencilerin dönüşüm becerilerini ölçmek için " Cebirsel Kavramların Farklı Temsil Biçimleri Arasında Dönüşüm Yapma" testi araştırmacı tarafından hazırlanmıştır.

Betimleyici istatistikler öğrencilerin dönüşüm işlemindeki başarılarını belirlemek için kullanılmıştır. Bağımsız gruplar t-testi kız ve erkek öğrencilerin test ortalamalarını karşılaştırmak için kullanılmıştır. Testin içindeki her soru öğrencilerin cebirsel temsil biçimleri arasında dönüşüm yapmaları esnasında anlam yanılgılarını ve en sık yapılan hataları belirlemek için incelenmiştir.

Testin sonuçları 8. sınıf öğrencilerin cebir kavramlarının dört temsil biçimi; sözel anlatım, denklem, tablo, grafik; arasında dönüşüm yapmada düşük beceriye sahip olduklarını göstermiştir. Kız öğrencilerin test ortalamaları ile erkek öğrencilerin test ortalamaları arasında anlamlı bir fark bulunmamıştır. En problemli dönüşümler diğer temsil biçimlerinden; denklem, tablo, grafik; sözlü anlatıma yapılan dönüşümler, en kolay dönüşümler ise diğer temsil biçimlerinden; sözlü anlatım, denklem, grafik; tabloya yapılan dönüşümlerdir.

Anahtar Kelimeler: Matematik eğitimi, çoklu temsil biçimleri, cebir, cinsiyet farklılıkları

Dedicated to my father and family of Sert

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CHAPTER I

INTRODUCTION

Mathematics is one of the most difficult school subjects for students in all grades. In Turkey, in the national high school entrance examination, OKS, administered to eighth grade students, the subject that students succeed least is usually mathematics. For example, in 2007 OKS exam, mean of mathematics questions that were answered correctly was 3.3 out of 25. However, mean of correct questions in science was 5.7, in social sciences 8.6, and in Turkish 13.8 (MEB, 2007). During the last decades, researchers have been emphasizing new teaching methods and materials that can be used in mathematics lessons. Many new curricula have been prepared to make reform in mathematics learning and teaching (Erbilgin, 2003). In Turkey, a new mathematics curriculum in reformist nature was initiated in 2004. The aim of these studies is to develop new ways that help students to understand mathematics lessons easily and successfully (Thomasenia, 2000). Development of well-organized representations in mathematics instruction is seen one of the ways that help students' mathematics understanding (De Windth-King & Goldin, 2003).

1.1 Representations in Mathematics Education

According to Lesh, Post and Behr (1987) representations are "external (and therefore observable) embodiments of students' internal conceptualizations" (p.34).

Goldin (2003) defined representation as "configuration of signs, characters, icons or objects that can somehow stand for, or represent something else". According to Kaput (1989; as cited in Cramer, 2003) representations can be something on paper, in physical form or in mind used to understand mathematical situations. Actually, the word representation is not new in mathematics education (Zaskis, & Liljedahl, 2004). Dienes presented the positive effect of representations on students' mathematical learning in 1960 (Lesh, Post, & Behr, 1987). In USA, the mathematics education reform in last two decades has been emphasizing mathematical representations. However, its necessity in mathematics education is recently taken attention (Cuoco, 2002; Goldin & Janvier, 1998).

Representations are seen as aid to both understanding and solving mathematical problems (Greeno & Hall, 1997). They have significant contribution on developing concepts, and problem-solving skill (Schultz & Waters, 2000). According to Greeno & Hall (1997) when students work on a problem; they make drawings, write notes, construct tables, and equations that becomes easier for students to follow the solution and to control the conclusion. Also, representations help students during problem solving process with providing facilities like seeing patterns, and providing different inferences and calculations. Some children develop accurate representations and solve difficult mathematics problems without learning about representations. However, the number of such children is very few and there is a belief that understanding mathematics is innate skill (Goldin, 2003). However, according to representational perspective if representations are used at teaching conceptual subjects and problem solving, students can learn difficult mathematical ideas (Goldin, 1987).

Representations are studied in two parts: Internal and External Representations. Internal representations are representation of interpretations of ideas in the mind (Erbilgin, 2003). Schema, cognitive structures of a person are examples of internal representations (Janvier, Girardon, & Morand, 1993; as cited in Pape & Tchoshanov, 2001). External representations are physical shapes like words, graphics, pictures, equations, tables, etc. According to Sherin (2000), when external representation is used, "representing" and "represented" world are connected to each other. In other words when students work with representations, they are actually attempting to link the abstract mathematical ideas to the real world objects. Zhang (1997) described the relationship between internal and external representations with following statement. When a student drew a diagram or wrote a formula of his/her idea about given problem, he/she represented his/her internal representations as external representations. That is to say, students formed external representations from their internal representations. Generally, researchers mean external representation when they use the term representation. In this study, the term representation will be used as external representation, too.

In schools, representation is taught end in itself that is only one of the representations is used at teaching a mathematical concept. In this way, students cannot learn how to use representations and advantages and disadvantages of them (Greeno & Hall, 1997). However, if students learn mathematics content by using various representations of concepts, they can experience different representations and choose most appropriate one when they are needed (Özgün-Koca, 1998).

1.2 Multiple Representations

Multiple representations are external representations that are used to express same concept in different forms (Özgün-Koca, 1998). When students connect graphical, tabular, symbolic, and verbal descriptions of given situation, or some of these descriptions, they help students understanding mathematical concepts and relationships (Kaput, 1989, Porzio, 1999; as cited in Kwaku,2003). The importance of multiple representations in mathematics education has been researched since early twentieth century. At early 1920's, the National Committee on Mathematical Requirements of the Mathematics Association of America (NCMR) explained the place of different representations on the algebraic and geometric problem solving procedure. (Bidwell & Clason, 1970). In 1960, Dienes suggested in his "Multiple Embodiment Principle" that to provide that students obtain mathematical abstraction mathematical content must be taught in different representational forms (Dienes, 1960; as cited in Lesh, Post, & Behr, 1987). New Turkish mathematics curriculum of middle grade mentions that the relationships between mathematical concepts must be searched and discussed during mathematics lessons. Also, students must learn to associate abstract and concrete objects. Therefore, associating different representations of mathematical concepts, conditions, and operations, and making translations between different representations are some aims of mathematics education. The benefits of multiple representations in solution process of algebraic expressions provide meaningful and qualitative learning (MEB, 2006).

In school mathematics; graphics, symbols, equations are used separately that limits power of representations (NCTM, 2001). For example, when a graphical representation is used during a problem solving process, students cannot understand that symbolic representation of the same problem represents the same information. Most of them think that symbolic representation of the problem represents a different problem (Janvier, 1987). However, multiple representations enable students to see different aspects of the same problem (Dufour-Janvier, Bednarz, Belanger, 1987). Moreover, according to Brenner et al. (1997) combination of concrete, visual, and abstract representations is the most appropriate system for human brain. Furthermore, multiple representations help students in reasoning, problem solving, and learning processes (Özgün-Koca, 1998). Ainsworth (1999) sums up the benefits of multiple representations in three topics: multiple representations (1) support different ideas and processes, (2) constrain interpretations, (3) promote a deeper understanding of the domain.

When students begin studying mathematics by using multiple representations, they have some obstacles, like thinking different representational form of the same problem being a different problem as mentioned above. This obstacle is related to understanding "translations" within and between multiple representations (Yerushalmy, 1997). Translation means changing one representational form to another. Kerslake

(1981; as cited in Pitts, 2003) gave middle school students "$W=2H+3$" equation and asked them to find W for $H=1$ and $H=2$ and draw the graph of this information. 60% of students found W, only 35% of them found the points that would be used at graph, and 19.9% of them drew the graph. This result showed students' difficulty in translating equation to graphic. This is only one of the examples showing translation problems between multiple representations. However, translations among multiple representations are as important as understanding and using multiple representations (Lesh, Post & Behr, 1987). Hitt (1999) argues that teaching translation between representations is a central goal of mathematics education. "If somebody wants to be proficient, he need to be able to use and apply learned concept in one setting to another" (NCTM, 2000, p.20). Many researchers share the idea that students need to learn translation process to understand mathematics topics (Yamada, 2000), because translation process help students solve problems correctly (Lesh et. al., 1987) by making mathematical ideas meaningful for them (Post et al., 1993). Harvey (1991) explains the function of translation as following:

> One must be able to translate mathematical concepts among representations and more importantly reconcile the different information provided by the different representations so as to understand the common abstraction underlying all of them (p.3).

1.3 Multiple Representations in Algebra

Algebra is one of the problematic parts of mathematics lesson. For example, according to TIMSS's results, 1994-1995 study on over 40 countries, students have problems with algebra (Hail, 2000). Also, Turkey was one of the 38 countries that attended TIMSS's 1999 study. The results of the study showed that 8th grade Turkish students had poor performances in algebra items like other mathematical concepts such as measurement and arithmetic (TIMSS, 1999). According to Schwieger (1999), students have negative feelings for algebra because of finding algebra difficult. However, algebra is an important mathematical tool that helps students in problem solving and learning different mathematical concepts. Algebra is a language that helps students in (1) making generalization, (2) answering more than one question about a problem at one time, (3) making connection between quantities, and (4) solving numerical problems (Moses, 1999). According to Thorpe (1999; as cited in Moses, 1999) algebra helps children in equations, symbols, formulas, and following derivations of other subjects. Algebraic concepts are necessary not only in mathematics but also in many other fields which makes it very important (Mourad, 2005).

Although algebra is very important part of mathematics education, students' problems and negative feelings about algebra have not been resolved by educators, yet. In middle grades, students have difficulties in passing from arithmetic to algebraic thinking (Kieran, 1992; as cited in Bednarz, Kierani & Lee,1996). In problem solving, they prefer using arithmetic instead of algebra even if algebra makes the solution procedure easier. Using representational techniques like tables, graphics, words, equations help students to make connection between arithmetic and algebra (Dufour-Janvier et.al., 1987). When representations are used, Rogers and Jones (1997) argue that algebraic thinking has begun.

Although students learn solving equations, they can not use representational skills in solution processes (Moseley, & Brenner, 1997). When, algebra-teaching techniques are examined, it is understood that multiple representations are not taught to students in algebra lessons, like other mathematical concepts. In Turkey, students learn only to use equations as a part of solution process to achieve the aim of finding solution instead of understanding algebraic concept. There are research about the effects of multiple representations and translations among them in algebra in Turkey like studies all over the world. Also, new mathematics curriculum has been designed with giving attention to using representations in all subjects of mathematics lessons. On the other hand, there is no research about the middle school students' knowledge about multiple representations and skill of translating among different algebraic representations. However, if developers and operators of new curriculum are aware of students' level, errors, misconceptions about multiple representations, mathematics curriculum and lessons can be designed more beneficial for students.

1.4 Purpose of the Study:

The primary purpose of this study was to determine eighth grade students' skill of translating among different representations of algebraic concepts. The representations focused in this study were graphic, table, equation, and verbal sentence. The second purpose was to investigate the effect of students' gender on their skill of translating multiple representations. It was also aimed to investigate students' most common errors in making these translations.

1.5 Research Questions:

- What are the skills of 8^{th} grade students in making translations among different algebraic representations?
- Is there a significant effect of students' gender on their skills of translating among different algebraic representations?
- What are the most frequent errors students made in translating among different algebraic representations?

1.6 Significance of the Study:

Starting from 2004-2005 school year, a new era began for the elementary school curricula of Turkish education system with implementation of a new curriculum. Mathematics curriculum is one part of this curriculum innovation. New mathematics curriculum's practices will be different in terms of teaching philosophy, content, teacher's role, student's role and materials. Some skills such as connection, reasoning, problem solving, and communication employ a great importance in the new mathematics curriculum. In the new mathematics curriculum representations are shown as central to study of mathematics. According to many research results done about students' use of multiple representations in mathematics education, using multiple representations in mathematics education improves understanding of concepts, and problem solving skill (Brenner, Mayer, & Duran,1997; Özgun-Koca, 1998). In this sense, it is critical to understand students' skills in using representations, especially at the end of elementary education. Better understanding students' flexibilities in working with multiple representations may provide information for the improvement of current practices in school mathematics.

Researchers continue to study about the most appropriate teaching methods for algebraic understanding. According to Janvier (1987) if teachers use multiple representations in algebra lessons, students are able to understand algebraic thinking that is necessary not only in mathematics lesson but also in a lot of other scientific concepts. Moreover, the skill of translation between multiple representations is as important as using representations. Students who can make translation between multiple representations have many tools at solving algebraic problems, and they can appreciate the harmony, consistency, and beauty of mathematics (NCTM, 1989). In this sense, this study will help to explore students' current skills in making translations among different algebraic representations.

1.7 *Definition of Important Terms*

Representation: Configuration of signs, characters, icons or objects that can somehow stand for, or represent something else (Goldin, 2003, p.276).
External (and therefore observable) embodiments of students' conceptualizations for internal representations (Lesh, Post, & Behr, 1987).

Multiple representations: different representations that are used (e.g. graphs, tables, equations, diagrams) at the same time.

Translation among representations: "Translation" is a term that derives from the idea of multiple representations. Translation refers to the psychological processes involved in going from one form of representation to another, for example in going from an equation to a graph and vice versa (Janvier, 1987). A translation always involves two forms of representations (e.g. graphs and tables or equations and tables).

1.8 Assumptions:

The basic assumptions of this study were:

- Respondents answered truthfully.
- Respondents performed to the best of their skill on test.
- Respondents answered with sincerity.
- Respondents understood the terms of the questions as defined by the researcher.

1.9 Limitations:

This study is applied to eighth grade students in Çankaya district of Ankara. The results of this study may not be generalized to all elementary level students in Turkey because of their varying facilities, socio-economic status, and interest in the study.

In addition, this study focused on translations between multiple representations of algebraic concepts that hinder making generalization of the results to all subjects of elementary mathematics.

CHAPTER II

LITERATURE REVIEW

This study focused on understanding skills, misconceptions of 8^{th} grade students in making translations among different algebraic representations and the relationship between gender difference and translation process. In this chapter, a review of recent literature is presented concerning algebra, multiple representations in mathematics education, and multiple representations in algebra.

2.1 Multiple Representations

The place of multiple representations in mathematics education has been proposed in number of theories. According to Lesh, Post and Behr (1987) multiple representations gained importance with Dienes' "Multiple Embodiment Principle". According to this principle, physical representations helped students' mathematical understanding. In addition, Dienes explained multiple representations as an aid in students understanding of mathematical concepts. (Dienes, 1960, 1977; as cited in Lesh, Post, & Behr, 1987). Bruner (1966) classified representations in three categories as enactive, iconic, and symbolic. Enactive representations were through action. In iconic level, students learned concepts by images. Students used symbols in symbolic level. Bruner stated that providing an environment that helped translating experience into powerful representational systems was heart of education. According to Dufour-

Janvier, Bednarz, and Belanger (1987) students had to decide appropriate representations during mathematics lessons. In addition, students had to learn similarities and differences of representations and pass from one representation to other representation.

Constructivist theory examined that students constructed their own knowledge themselves. According to constructivism, students have conceptually understood mathematical concept, then they learned abstract representations to transform mathematical knowledge. Teachers' role in this process were representing concepts in multiple ways in mathematics lessons and ensuring that students use these ways in integrating new concepts. In addition, everyone might understand the same concept from different representations during building his/her knowledge that showed the importance of using multiple representations in mathematics education according to the constructivist theory (Goldin, 1990; Kaput, 1991). Lesh was one of the important researchers who talked about translations between representations and transformations within them. In his multiple representations translation model, five representations that were real-world situations, manipulative, pictures and diagrams, spoken languages, and written symbols were described. According to Lesh, distinct types of representations were not sufficient in mathematics learning. Translations among multiple representations, and transformations between them were as important as representations (Lesh, Post, & Behr, 1987).

The theoretical framework of this study was drawn from Janvier's Representational Translations Model (1987) (Table 2.1). Janvier explained representation as “may be a combination of something written on paper, something existing in the form of physical objects and carefully constructed arrangement of idea in one’s mind” (1987, p. 68). According to Janvier, there were three stages of mathematical understanding. First stage was using different representations to identify mathematical concepts. Second stage was artful management within representations. The last stage was translation from one representation to another. Verbal descriptions or pictures, tables, graphics, and formulas are the four representations of Janvier’s model. Translations among two representations such as from tables to graphics are direct translations according to Janvier. On the other hand, translations among two representations with the help of other representations are indirect representations. When making translations from equation to graphic, making translation from equation to table and then from table to graphic is an example of indirect representation. In this study, it was expected from students to make direct translations of verbal statements, equations, tables, and graphics. According to Dufour-Janvier, et al. (1987) students had to know using representations, rejecting one representation to another in a given mathematical situation, making translations from one representation to another representation. Moreover, students had to have the skill of choosing appropriate representation during mathematical process. Therefore, they concluded that instructional strategies should be improved in a way that they include variety of representations and are flexible to use translation processes in representations. (1987).

Table 2.1 Janvier's Model of Translation Process Between Different Representations

From / To	Situations, Verbal Description	Tables	Graphs	Formulas
Situations, Verbal Description		Measuring	Sketching	Modeling
Tables	Reading		Plotting	Fitting
Graphs	Interpretation	Reading Off		Curve Fitting
Formulas	Parameter Recognition	Computing	Sketching	

2.2 *Research on Multiple Representations*

Many researchers have investigated the effects of multiple representations on mathematics achievement of students. There are many studies regarding multiple representations in various grade levels starting from kindergarten to university level. For example, Smith (1999) examined four third grade children about how they create and use multiple representations in response to problems to make inferences about their capability and view of role of representation. The children created idiosyncratic representations. All children created similar representations, but there were differences in creation and use of representations. Also all children used pictures to solve problems.

Brenner et al. (1997) compared 6th grade American students' problem-representation skills to 6th grade Chinese, Taiwanese, and Japanese students. The reason of this research was researcher's belief that Asian countries' students had high level of achievement at TIMSS because of teaching mathematics different in these countries. A test for mathematical achievement included two parts. In the first part, students solved problems. In the second part, students chose correct representational forms of questions from given answers. Participants were asked two forms of achievement test that included both solution and representation items. After the research, Asian students were twice more correctly answered solution items than American students were. On representational item, Japanese students three times, Chinese and Taiwan students five times as likely as American students to answer correctly. Chinese students had highest scores on representation tasks and visual representation of fractions. Asian students had both stronger basic skills and conceptual skills that gave them advantage in solving more difficult problems. According to Brenner et all., one reason of this success was that in Asian mathematics textbooks multiple representations were used more than in American mathematics textbooks.

Enyedy (1997) used computers to investigate animated representations in probability. Thirteen pair of 7th and 8th grade students was chosen. Pairs studied "coin game", static representation, on the computer. Five pairs were given rules of a game and event tree, animated representation. Remaining eight pairs were given only rules of game. Enyedy concluded that event tree helped students visualize abstract events. Therefore, when probability lessons are designed animated representations can be valuable at students' understanding the concept.

There are also studies about translation among multiple representations. Michaelidou and Gagatsis (2004) examined students' skills in recognizing decimal numbers in different representation systems, translating decimal numbers from one representation to another and identifying relationships between representations by administrating tests to 12-year-old students. Number line was discussed as a geometrical model. Research showed that representations of decimal numbers were not sufficiently developed. In addition, students had difficulties in translation from number line to symbolic expression and vice versa.

The study of Gagatsis, Christou, and Elia (2004) focused on the representations and translations of mathematical relationships. 79 students of 6th grade were asked to represent the idea of the given questions in another representational form. Two theoretical models were evaluated by researchers. In the first one, problem was given in graphical form and students were asked to translate it into verbal, tabular, and symbolic forms. In second, verbal form of the problem was given and students were asked to translate it into other three forms. After the study, researchers concluded that multiple representations did not themselves help students develop mathematical understanding; there had been hierarchy among representations. Some representations were called prototypes that serve as the basis for understanding and connecting representations of same content. For example, graphical representations acted as a prototype for understanding verbal and tabular representations. In addition, tabular representations acted as a prototype to make translations from symbolic representations to others. Gagatsis et al. suggested that translations among representations may be a block to learning mathematics if patterns that using prototypes to adjust the hierarchy were not followed.

In another study, Taber (2001) applied 13 days of instruction to 22 fifth grade students to investigate the role of multiple representations on multiplication of fractions. The instruction was designed according to Lesh's five representational system that includes physical representations, pictures, verbal language, written symbol, and "real-life" problem. The results showed that students were more successful at post-test than pre-test because during the instruction they were pushed to use different representations although some of them did not understand the connections between five representations totally. Taber concluded that students gained skill to create and transform different representations after instruction that could be helpful to students develop their solution strategies to fraction problems.

In Turkey, Kurt (2006) examined middle grade students' skills in translating among different representations regarding the fraction concept and the effect of grade level and gender on these skills. In this study, she used Lesh transition model which included symbolic number line, region, real-life situations, and discrete object models for assessing students' skills in translating representations. 1456 students in 6th, 7th, and 8th grades in Ankara were used as participants. Kurt found out that students' skills to translate representations among fractions were poor. Also, both gender and grade level had significant effects on students' skills. Students had problems especially in number line representations of fractions. Kurt concluded that teachers had to use multiple representations during teaching fraction concept and used more examples to overcome the problems about part-whole conception.

Researchers were also interested in the role of technology at using multiple representations in the classroom. Özgün-Koca (1998) investigated the use of educational technology for multiple representations; equations, graphs, tables; in problem solving. At the end of multiple representation-based instruction, students stated that they were aware of different representations in problem solving, but they preferred to focus on one representation. In addition, students' answers to questionnaire showed that most of them preferred to use equations and graphs in problem solving because to study these representations were easier in computers than other representations. According to Özgün-Koca the major reason for choosing any representation was students' previous knowledge and experience with that representation and personal preferences. Therefore, Özgün-Koca concluded that educators had to provide multiple representation environments in classrooms. Therefore, students had a chance of experiencing all representations.

2.3 Research on Multiple Representations in Algebra

2.3.1 Algebra

According to Lesh, Post, and Behr (1987), students learned limited perspective of algebra in schools. Students learned writing and doing operations on equations instead of "thinking" about algebra. Clement, Lochhead, and Monk (1981) developed simple questions about translations into and of algebraic notation. Only, fewer than 50% university students solved the questions. The most common error was "reversing error”. That is for example students wrote 4C=5S instead of 5C=4S. In addition, translations from tables and pictures were also problematic. According to Clement, Lochhead, and Monk, these errors were not trivial and the reason of these errors was that students did not learn to construct formula in mathematics lessons. Usually, students were given formula and asked to manipulate it. Therefore, students had to learn translations between words into algebraic notation and vice versa that could help them to solve their problems at translation process between practical situation and mathematical notation. After one year, Clement (1982) showed that college students could not solve simple algebraic word problems. The error was reversals again. In addition, there were reversal errors not only in translations from equation to words but also in translations from pictures to equations and table to equations. Stacey and MacGregor (1993) investigated errors in formulating algebraic equations and reached a new explanation of these errors. According to him, students from verbal statement a cognitive model of compared unequal quantities. Stacey and MacGregor (1997) gave a table to 14-year-old students. They asked a question about the variables of table and then asked students to verbalize the given information in table. Most students gave correct answer to question, but they could not describe the relationships between variables in words. In addition, researchers asked to write algebraic symbol of given information in table format and only half of students who gave correct answer to question were able to write algebraic symbol of information. Stacey and McGregor (1997) concluded that students saw patterns, but they could not express them algebraically. Moseley and Brenner (1997) concluded that beginning algebra students had difficulty at acting on a variable. Because curriculum presented algebra as a computational tool that produced mistaken generalizations. According to Roger and Jones (1997), students had to work on symbolic relationships for beginning of "algebraic thinking" in their mind. Thornton (2001) explained three approaches of algebra instruction. In symbolic approach letters and variables were began to use by students. However, this approach was not sufficient for algebraic thinking, because generally students memorized rules without much understanding during algebraic problem solution. At this time, patterns approach used to ask students to generalize patterns. At this approach, students could see patterns, but they had problems at writing information given in patterns as an equation or verbalizing. In function approach, students represented concepts in words, symbols, tables, and graphs. Therefore, Thornton reached the need to use multiple representations during algebra instruction.

2.3.2 Multiple Representations in Algebra

There are studies about students' performance in using multiple representations in algebra, the effects of multiple representations on both teaching and learning of algebra, and new curriculum designs that include multiple representations in algebra concepts.

One of the studies about the effects of multiple representations on algebra is Stacey and MacGregor's study (2000). They dealt with five aspects of arithmetic, which were the essential foundations for learning algebra. These were seeing the operation, not just the answer; understanding the equal signs; understanding the properties of numbers, being able to use all numbers; and working without a practical context. For investigating first aspect a table that explained the relation between x and y unknowns was shown to 14 years old students and asked them to explain this relationship. Many of the students gave correct answer when giving a value of *x* and asking the value of *y*. However, only three-quarter of these students could explain the relationship between x and y. Only half of the students who gave the correct answer wrote the equation of this relationship. Researchers concluded that students could use the relationships among representations correctly, but many could not explain them with words or symbols. Students often saw patterns in the numbers, but they could not express them algebraically.

In their study, Brenner et al. (1995) aimed to redesign a pre-algebra unit to emphasize problem representation skills. This 20-day unit about functions was applied to middle school students. Seven pre-algebra classes from 7th and 8th grades were selected. Four classes participated in representation–based unit that focus on problem representation skills by having students use algebra to represent mathematical relationships with other representations including tables, graphs, pictures, diagrams as a treatment group. Three classes participated in textbook lessons which were traditional instructions focusing on symbol manipulation as a comparison group. Prior to instructional unit pretests and following the unit posttests were given to students. Results showed that representational skills were learnable in classrooms with an appropriate instruction. Comparison group learned to use different methods to solve mathematics problems, but the students at this group failed to use relationships among representations. On the other hand, the students participated in representation-based unit was more likely to use appropriate tables, diagrams, equations. They used more representations after treatment. Treatment group produced greatest pretest-to-posttest gain than control group. Therefore, researchers concluded that a unit could be designed to emphasize representational skills and these skills helped students to solve problems.

Mourad (2005) conducted a study to compare the effects of two teaching methods for an 8th grade algebra unit. A classroom was divided into experiment and control groups. 17-day unit on linear functions was applied to students. The experiment group used a method that involved activities based on inductive reasoning, multiple representations, and guided discovery. Combination of patterns and functions were used to teach algebra and students were guided to translate functions from one representation to another. The control group applied traditional teaching. The main point of the study was graphing and writing linear equations. The unit that experiment group used completed inductive activities that involved presentation of linear situations as graphs, tables, words, symbols and translation of these representations. Also, activities were designed to help students to select preferred representation. During the unit, quizzes were administered and at the end of the unit students answered exam questions. The experimental group students made translation to graphs much better than control group students did. Because of learning different representations, experimental group students learned different choices to solve problems. Researcher concluded that by making some differences on activities this unit could increase the success on algebra functions.

In an algebra project that was prepared by Silva, Moses, Rivers, and Johnson (1990) a five step process was used to overcome students' barriers in making transition to thinking algebraically. The aim of this project was to show the relationships between algebra and the physical world. During this project, students engaged in some physical events by going to trip to make link between physical world and mathematics. After the trips that represented the physical events, students made pictures of the places they visited, or draw graphs. In addition, they talked and wrote about events. Finally, students developed their own representations for various operations about the physical events. After this project, student succeeded in algebra that was understood from their success in entering college preparatory mathematics.

Cai (2004) reported two studies about U.S. and Chinese students' thinking about mathematics problem solving. First study examined the relationship between U.S. and Chinese students' selection of solution strategies and representations about algebraic problems and their learning opportunities to algebra. U.S. sample consisted of 6th, 7th, 8th grade students and Chinese sample consisted of 4th, 5th, 6th grade students. Chinese students used "national unified textbooks". Teachers encouraged students to solve problems in different ways. On the other hand, U.S. teachers did not follow textbooks exactly and they used patterns. After the algebra unit, students were asked four problems and wanted to explain their solutions. The investigations of answers showed that Chinese students rarely used visual representations; however, U.S. students' degree of using visual representations increased after learning algebraic concepts. Second study examined the impact of teachers' beliefs. Chinese and U.S. teachers evaluated their students responses and researchers interviewed with them. This study showed that U.S. and Chinese teachers both had different learning goals and emphasis on teaching of problem solving. For example, U.S. teachers gave more importance to visual representations and concrete materials, on the other hand Chinese teachers did not give importance to visual strategy. Also according to Chinese teachers, students should learn general strategies. However, U.S. teachers thought that it did not matter what strategies students used as long as they could solve problems. These different beliefs were the effect of Chinese and U.S. students' thinking and preferences about algebra. When Chinese and U.S. students' achievements were compared, Chinese students had early successful introduction of algebraic concepts. In conclusion, Chinese students' early algebra learning could not be explained by the differences between U.S. and Chinese students' thinking about mathematical problem solving (study 1). On the other hand, teachers' different beliefs in two nations were the reason of these differences. Moreover, researchers suggested that the effects of other factors' such as culture, history must have been investigated.

Students' difficulties in solving algebraic word problems were explored by Lochhead (1988). Seven questions were asked to 150 students. The results showed that many students had difficulties in solving algebraic word problems, mostly in translating written language to mathematical language. Students had problems (1) when they were asked to write an equation to represent the relationship between two variables given in a tabular form, (2) when they were asked to write sentence to represent to given two-variable linear equation, and (3) when they were asked to write an equation to represent the relationship between two variables given in pictorial form. The researcher suggested that providing students practice at the translating process might be one of the solutions to overcome these problems.

Students' skills to work with algebraic variables were researched by Moseley and Brenner (1997). Fifteen students from pre-algebra classroom were taught algebraic variables with an experimental curriculum emphasizing multiple representation skills. These students used tables, graphs, variables, and functions and translated representations to each other. 12 students were taught with traditional instruction as a comparable group. Pretest and posttest interviews were done with students before and after five weeks of instruction. Researchers compared pretest and posttest interviews results about two word problems and two graphical problems. This comparison showed that students receiving multiple representation curriculum were more able to act on variables than other students receiving traditional curriculum. In addition, they were more successful at integrating variables into equations and representing them as graphical representation. Researchers concluded that representational skills helped students to make link between knowledge of algebra and real world knowledge of problem solving. Multiple representation curriculum both influenced type and frequency of representations used and students' performance in solving word problems. Finally, researchers suggested that long term benefits of multiple representation curriculum needs to be researched.

Swafford and Langrall (2000) investigated 6th grade students' use of equations to describe contextual problem situations before teaching students formal algebra. Their aim was to determine whether students could solve special case problems, generalize the relationships, and use symbolic representations to solve problems. For this study, ten students were chosen from a 6th grade classroom. Transcribed interviews, students' written work, interviewers' notes, and data summaries were used as data sources. Six verbal problems including direct variation, linear relationships, arithmetic sequence, exponential relationships, and inverse variation were asked students to write general equations that showed the relationship between dependent and independent variables. If students had problems in writing equations, they were asked to construct tables and then asked about the relationship. The results showed that 6th grade students successfully generalized problems involving specific cases and wrote equations. There were more students who could describe the relationships than who could show them symbolically. The students who had difficulty in symbolic representation also had problems in describing general relationships verbally. In addition, not all of the students who described relationships wrote appropriate equations. Moreover, many of the students who could write correct equations did not use them in solving problems. Tables diverted students' attention. Researchers concluded that examining a problem through different representations was beneficial for students if also the links between representations were taught.

Hail (2000) conducted a study to research the role of multiple representations on learning basic algebraic concepts. A four-week experiment was applied twenty nine seventh grade students in pre-algebra classroom. During experiment, multiple representations were used to teach variables, equations, and equation solving process. Participants were given written documents; assignments, worksheets, and prompts. In addition, Hail made interview with students, and examined videotapes of lessons as a data resource. Study results showed that manipulative helped students on variable operation, solving equations, and avoiding errors. Also, students understood that they did not have to use only abbreviation for variable with the help of graphs. According to Hail the reason of the positive effects of graphs and manipulative was that mathematical actions became less abstract when they were taught with these representations. Participants preferred mostly graphs for solving equation even they did not understand it totally. They were not successful in solving equations because they could not understand connections between representations. Moreover, they were not flexible at transition among representations that affect their achievements.

Using concrete representations in algebra were also investigated. Sharp (1995) used algebra tiles in high school algebra to construct translations between algebra and physical systems. Five high school algebra classrooms joined the study. Treatment group used algebra tiles to study adding, subtracting, multiplying, and factoring equations. Control group did not use any manipulative. Two experiments were conducted during the study. At first experiment, only during the factorization unit algebra tiles were used, at second one at all operations of algebraic expressions algebra tiles used by the students in the treatment group. After the same quizzes and examples, there were no differences between test scores obtained form treatment and control group students. However, some data showed representations helped students to solve problems. For example, some students from treatment group drew pictures of algebra tiles that helped them to solve problems. Also, some low achieving students from treatment group began to try to solve problems after learning with algebra tiles. Sharp concluded that although algebra tiles did not increase test sores, students learned to visualize problems with the help of them.

According to Cunningham (2005) to understand function concept better, students should be given opportunities to transfer algebraic, numeric, and graphic representations in classroom. At this point the time that teachers spend on transfer problems and the frequency of they use these type of problems on their assessment came into prominence. For these reasons, he researched 28 algebra teachers who taught algebra in grades 8 through10. A survey contained six transfer problems about linear functions was applied to teachers as a data source. At this survey teachers were required to circle both the number of class periods they taught the transfer problems and number of times they used this problems at their assessment. After analyzing results of surveys, Cunningham concluded that graphic to numeric transfer problems were used less frequently than other type of transfer problems by teachers during the lessons. Also teachers used these problem less frequently on assessment. Moreover, according to many researchers (Knuth, 2000; Schornfeld 1993) translations from graphics to equation were the most problematic translations for students. Researcher concluded that although students had problems at translating graphical representations to numerical representations, teacher did not give opportunity to students to master these transfer problems. He suggested teachers should be aware of students' difficulties in transfer problems and spend more time on problematic translation problems during the lessons.

A dissertation about the effects of spatial skill and achievement on usage of multiple representations was explored by Erbilgin (2003). To provide data, case study method was used. Four students from 8th grade were selected representing one high achieving-high spatial skill, one high achieving-low spatial skill, one low achieving-high spatial skill, one low achieving-low spatial skill. Research results showed that both spatial skill and achievement affect students' use of multiple representations and preferences of them. Also, translation from different representations to tables was easy for students, and all of them could classify tables, and verbal representations. Erbilgin suggested that multiple representations positively influenced understanding of mathematics concepts.

In Turkey, Erbaş (1999) conducted a study to investigate students' difficulties and misconceptions in elementary algebra. He selected 217 ninth grade students and three algebra test developed by researcher were applied to these students. After study, his results showed that one of the errors of students in algebra was in formulation of simpler algebraic equations. When students translated a verbal statement into an equation, they made reversal error that was explained before. He concluded that students had difficulties and errors in both forming and solving simple algebraic equations.

Akkuş Çıkla (2004) investigated the effects of multiple representations-based instructions on seventh grade Turkish students' algebra performance, attitudes toward mathematics, and representation preference compared to the conventional teaching in her dissertation. Algebra achievement test, translations among representations skill test, and Chelsea diagnostic algebra test to assess algebra performance; mathematics attitude scale to assess students' attitudes toward mathematics; representation preference inventory to determine students' preferences, and interviews were used during the study. Her results showed that multiple representation based instruction made significant effect on students' algebra performance, because it provided students visualization in algebraic objects, connections between algebraic ideas, and translational skills in algebra problem solving. On the other hand, during the traditional teaching only drawing graph was taught as a translation among representations. Experimental group students were better problem solvers and experimental group students had more tendencies to use different representational modes rather than the symbolic mode for solving algebra problems after the treatment. Experimental group students could use different multiple representations for algebra problems and found appropriate representations for the given algebra problem.

2.4 Gender in Mathematics

The role of gender in mathematics education has been researched since beginning of the 1900s. In1974, Fennema published her first article about gender and mathematics that supported the ideas that there were differences in girls' and boys' learning mathematics. In addition, in the same year Maccoby and Jacklin (1974) concluded after their study that there were gender differences in verbal , visual-space and general mathematics skills.

In 1980, Sherman conducted Macoby and Jacklin's study again and found that there was a small difference between females' and males' mathematics achievement in high school level.

In 1990, Fennema and Leder concluded that there was an effect of teachers on gender difference in mathematics, but they were not sure that whether teachers' different interaction with females and males was major reason of gender differences. Cai (1995) conducted a study to investigate gender differences in mathematics problems. He found that males were more successful than females in mathematics problems in spite of computation problems. However, Gardner and Engelhard (1999) found that males were more successful in computation problems. This difference showed that there was inconsistency among research findings about the relationship between content domain and gender differences. Also, they concluded that girls performed better than boys in elementary and middle school level, but boys performed better than girls in high school. Moreover, females performed better on algebra than males, but males performed better than females on geometry and problem solving (Carlton & Harris, 1989).

In addition, studies showed that females performed better than males on open-constructed items in test; on the other hand, males were more successful on multiple-choice items. The reason of females' high performance on open-ended items was documented as females' better language skills that help them expressing their ideas (Oui Liu, 2006).

On the other hand, many studies found no difference between females' and males' mathematics achievement. Mai (1995) examined British, Columbian, Ontario, Hong Kong, and Japanese students to investigate gender differences in mathematics achievement. 13-year-olds students did not show any significant difference between females' and males' mathematics performance. Comple, Hombo, and Mazzeo (2000) analyzed mathematics achievement of students aged 9, 13, and 17 from 1970 to 1999. They found that males had better performance than females in 1970. However, there was no significant difference between genders in mathematics achievement in 1999.

In international assessments, gender difference in mathematics education was researched, too. Boys outperformed girls in 2000 and 2003 PISA (Oui Liu, 2006). In addition, TIMSS data showed that in general, eight grade boys performed better than girls in mathematics assessment from 1995 to 2003 (TIMSS, 2000a, 2000b, 2004).

In Turkey, Dursun and Dede (2004) determined the factors of students' low performance in mathematics by asking questions to eight elementary school mathematics teachers in Sivas. After research, they concluded that gender seemed the least important factor of this low performance. Dinç-Artut and Tarım (2006) interviewed with 728 primary school students to identify knowledge about place value concept. They concluded that difficulties of students were similar from the point of view of gender. Furthermore, Akkuş- Çıkla (2004) did not see the effect of gender on seventh grade students' performance in algebra lesson in her study. On the other hand, Kurt (2006) concluded that girls were more successful than boys in translations among different representations of fractions in elementary level.

As it seen, there is a lack of consistency in research findings about gender and mathematics achievement. Therefore, there is need for more research about the relationship between gender and mathematics both in Turkey and other countries.

2.5 Summary

The importance of multiple representations in mathematics education was researched by many scholars and the models that could be used at application of multiple representations to mathematical concept were designed (Dienes, 1960; Bruner, 1966; Lesh, Post, & Behr, 1987, Janvier, 1987). According to these theories, many researches were conducted all over the world about the importance of multiple representations in different concepts of mathematics. Probability (Enyedy, 1997), decimal numbers (Michaelidou and Gagatsis, 2004), fraction (Kurt, 2006; Taber, 2001), problem solving (Brenner et. al., 1997) were some of most investigated concepts. In addition, because of being one of the problematic concepts in elementary mathematics, the effects of multiple representations and translations between them on students' difficulties and errors about algebra were investigated (Clement, Lochhead, & Monk, 1981; Stacey & MacGregor, 1997, Thornton, 2001). According to these researchers, there were positive effects of using multiple representations and translations among them in algebra learning. Therefore, some researchers suggested to redesign algebra unit to emphasize representational skills (Mourad, 2005; Brenner et al., 1995). Cunnigham (2005) explained the teachers' role that they should be aware of students' problems in adapting mathematical concepts to multiple representations to prepare more efficient mathematical units. In addition, the relationship between gender and mathematics was researched since 1970s. However, there is no consistency in research findings that show the need to make studies about the role of gender on different concepts of mathematics and multiple representation usage during mathematics lessons.

CHAPTER III

METHOD

In this chapter, procedures of the study are presented. It includes details on design, population and sample, measuring instrument, variables, procedure, pilot study, and data analyses used in this study.

3.1 Design

In this study, descriptive statistics were used to summarize collection of data in a clear and understandable way. In educational studies, much of the information is collected through numbers of test scores, percentages, frequencies and the like. Descriptive Statistics are used to present numerical data to help researcher to simply large amounts of data in a sensible way (Fraenkel & Wallen, 2006). Measures of central tendency; the mode, the median, the mean; measure of variable; standard deviation; and histogram were used as techniques for summarizing quantitative data. Frequency table was used as technique for summarizing categorical data. In addition, Independent Samples t-test was used to compare mean scores of female and male students.

3.2 Population and Sample

All 8th grade public school students in Ankara were target population of this study. All 8th grade public school students in Çankaya district of Ankara was selected as accessible population. There were 103 elementary schools in Çankaya district. Among them,18 schools were chosen as the sample that the results of this sample would be generalized to accessible population. These 18 schools were selected randomly from 103 elementary schools and then all of the eighth grade students in each school were selected as sample. 705 eighth grade students were participated in the study. This selection of schools rather than students is known as cluster random sampling. Cluster random sampling is more effective with larger numbers of clusters (Fraenkel, & Wallen, 2006). After evaluation of students' answers, researcher decided to accept students who did not respond to eight and more questions as not participated to study. When these students were eliminated, the answer sheets of 684 8th grade public elementary school students were used as data of this study. As given in Table 3.1, 49.6 % of the sample were female, 50.4 % were male.

Table 3.1 Distribution of Gender in the Sample

	N	Percent
Females	333	49.6
Males	341	50.4
Total	674	100

3.3 Measuring Instrument

3.3.1 Translation among Different Representations of Algebraic Concepts Test

The aim of this study was to explore 8th grade students' skills of translating among different representations of algebraic concepts, students' errors in making translations among different algebraic representations, and the effects of gender to the translation skills. To assess students' translation skills "Translation among different representations of algebraic concepts test" (TADRACT) was developed by researcher and used in the study (see Appendix A). In developing instrument, several other instruments that were used in Turkey and other countries with the similar aims were reviewed. Interview questions of Erbilgin's study (2003), the word problem representation test designed by Brenner et. al. (1995), representations preference inventory and translations among representations skill test designed by Akkuş-Çıkla (2004) were some of the instruments that the researcher benefited from. Based on Janvier's Representational Translations Model (1987), translations among four representations that were graphic, equation, table, and verbal statement were used in the items of TADRACT. In every item, algebraic information was given in one of the four representation modes and students were asked to translate the given representation to one of the other three representations. As given in Table 3.2, in items 1, 6, and 9 students were asked to make translations from graphical representation to equation, table, and verbal statement; in items 2,10, and 11 from table to verbal statement, equation, and graphic; in items 3,4, and 7 from verbal statement to table, graphic, and equation; in items 5, 8, and 12 from equation to table, graphic, and verbal statement respectively.

Table 3.2 The required Representational Translations within Questions

From / To	***Equation***	***Verbal statement***	***Table***	***Graphic***
Equation		***12***	***5***	***8***
Verbal statement	***7***		***3***	***4***
Table	***10***	***2***		***11***
Graphic	***1***	***9***	***6***	

Note. The numbers in the table indicate the question numbers, not the number of questions.

Every question was given in a table and students were expected to write the answer of the question to the empty place on the right hand side of the question. In addition, the given representation and desired representation were written on top of every question to make it easier for students to understand what is expected from them. Students were also asked to show all their work on the paper.

A scoring guide to evaluate TADRACT was prepared by researcher. "1" point was given to correct answers and "0" point was given to both incorrect answers and missing answers. In addition, partial scoring was done to partially correct answers. Because of unit problems in items contained translation to verbal statement, labeling problems in items contained translation to equation and table, and both labeling and drawing problems in items contained translation to graphic, some answers were scored partial.(see Appendix B).

3.3.2 Pilot Study

Pilot study including12 questioned test was conducted with 47 eighth grade students chosen from two public schools to evaluate content, level and appropriateness of questions, design of the test, efficiency of time that students had. The results of the test were evaluated with Statistical Package for the Social Sciences (SPSS). It was seen that there were some problematic parts at content, and design of the questions. The problematic parts of the questions were re-arranged and corrected by the researcher. During the pilot study, it was understood that one lesson (40 minutes) was appropriate to answer questions. The corrected version of the data collection tool was studied by one faculty member, one research assistant from the Department of Elementary Mathematics Education at Middle East Technical University (METU), and two elementary school teachers working in the schools where the pilot study was conducted. The instrument reviewed based on the appropriateness of its aim, and students' knowledge level, language, writing style, and expression style. Based on their feedback, last version of the test was formed.

3.3.3 Validity and Reliability of Measuring Instrument

For validity of the TADRACT, it was reviewed by one faculty member, one research assistant from the Department of Elementary Mathematics Education of Middle East Technical University (METU), and two elementary school teachers to assure the content and face validity. They checked the instrument according to its aim, intelligibility of questions, students' grade level, language, writing style, and formation of questions. According to their suggestions, changes in the instrument were made. To check internal consistency of instrument, split-half reliability estimate was calculated with the help of SPSS. Gutman Split half was found to be .79 that indicated satisfactory reliability.

3.4 Variables

In this study, the statistical analyses were mainly descriptive. For the inferential statistics, there were one dependent and one independent variable. Students' raw scores on translations among algebraic representations skills was dependent variable of this study. Gender was the independent variable of this study.

3.5 Procedure

Before application of the study on selected schools, researcher asked permission to administer the instrument on selected schools from Ministry of Education. Test was administered to 705 eighth grade elementary school students in one lesson time, 40 minutes, during 2006-2007 spring semester. First, the aim and importance of this study were explained to students briefly by researcher and students were requested to answer questions earnestly. Then, test papers included 4 pages were given to students and researcher controlled that whether students wrote intended information such as gender and classroom. At that time students were informed that the results of this study would not effect their mathematics achievement, and their personal information would not be used during evaluation process. In addition, students were explained that errors and false answers were as important as correct answers. Therefore, it was important to answer all questions even if students were not sure of their response.

The answers were coded as "1" for correct answers, and "0" for incorrect answers. The questions that were not answered evaluated as wrong answers. In addition, partial scoring was applied to partially correct answers. A scoring guide was used to help scoring the students' responses (Appendix B). In order to decide how to interpret the missing values first, two mean scores were calculated. One of the mean scores considered missing values as o, and the other considered them just as missing and used rest of the responses to calculate the mean scores. The total means of these two variables were found to be 30 and 34 out of 100. Although when the missing values were taken as zero in scoring the overall mean decreased, the difference were considered to be negligible. In addition, if a student is not responding to a question, it may also mean she or he does not has a sufficient answer for that question. For these reasons, missing values were interpreted same as incorrect responses, in other words they were scored as 0.However, to minimize the negative effects of missing responses on the interpretation of the results, students who did not answer more than eight questions were considered not to participate to study.

3.6 Data Analysis

To investigate the research questions quantitative analysis of data and analysis of students' responses to questions were done.

3.6.1 Quantitative Analysis

Both descriptive and inferential statistics were carried out by SPSS. Descriptive statistics of frequencies, mean, median, minimum, and maximum values, standard deviation, skewness, and kurtosis were obtained to understand students' achievement in translation process. To compare mean scores of female and male students, the statistical analysis of Independent Samples t-test was used.

3.6.2 Analysis of Students' Errors

Every question were examined in detail to understand misconceptions, and most frequent errors students made in translating among different algebraic representations. While scoring students' responses, common errors were coded and their frequencies were determined.

CHAPTER IV

RESULTS

The purpose of this study was to examine 8th grade students' achievement in translations among algebraic representations. It was also aimed to investigate the most difficult, and easiest translations, as well as, the most common errors of students in translation process. Finally, this study intended to investigate the relationships between gender and students' achievement in translation process

In this chapter, the analysis of the data that were collected from 705 8th grade students in 18 elementary schools of Çankaya districts in Ankara was presented.

First, the descriptive statistics of scores that students had after solving 12-questioned test to understand their achievement in this study are described. Second, the descriptive statistics of each question to investigate the difficult and easy questions are explained. Third, the descriptive statistics of questions grouped triple according to the information, given in questions were expected to translate which representation to examine translations to verbal statement, equation, table, and graphic are reported. Fourth, the most common errors that students made in translation among representations are described. Finally, the reason of question that whether there is relationship between students achievement on this study and gender or not is reported.

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4.1 Descriptive Statistics of SCORE

The first step of data analysis involved the descriptive statistics about the mean scores of students on the "Translation among different representations of algebraic concepts test" (TADRACT). Table 4.1 represents mean, median, mode, standard deviation, skewness, kurtosis, minimum and maximum values of the students' scores in the TADRACT.

Table 4.1 Descriptive Statistics of Students' TADRACT Scores

	N	Min.	Max.	Mean	Median	Mode	Std	Skewness	Kurtosis
TADRACT Scores	674	0	12	3.8	3.5	2	2.69	.63	-.17

Students' responses to each question of the test were marked between 0 and1 that showed students had maximum 12 points, and minimum 0 point from the test. As seen from Table 4.1, there were students who received scores on the both ends of the scale, 0 and 12. The number of students who had 12 out of 12 was only 3, on the other hand the number of students who had 0 out of 12 was 68. The mean score of students was stated as 3.8 that showed poor skill in translating among algebraic representations. Similarly, the median score was 3.5 and the mode was 2 that showed also poor skill of students.

Standard deviation of the score was 2.69. This low standard deviation showed that the observations were less spread out. The value of skewness was .63. This positive value of skewness indicated that data were skewed right because of most scores tended to be low. Also, the kurtosis value was -.17 that showed the flat distribution of scores. As Figure 4.1 indicated that the data was mostly located on the left side of the histogram.

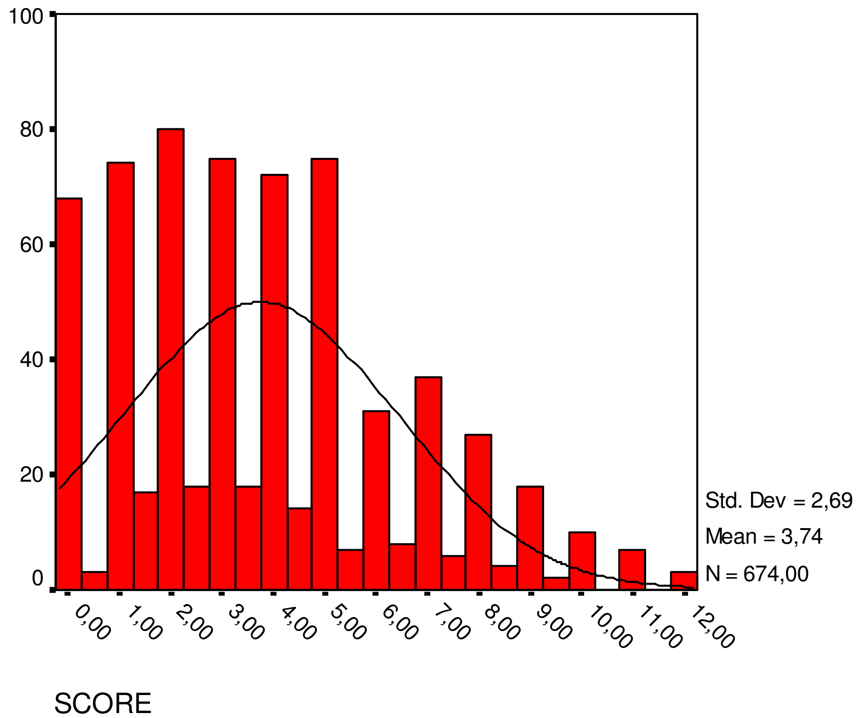


Figure 4.1 Histogram of students' score

In Table 4.2, the number of students, mean and standard deviations of their scores based on 18 elementary schools in which this study was conducted was given. The highest mean score was 4.8 out of 12 in schools 2 and 14. The lowest mean score was 1.8 in schools 9 and 13. Standard deviations were between 1.6 and 3.1. This low standard deviation showed that the observations were less spread out in each school.

Table 4.2 Descriptive Statistics of Each School

	N	Mean	Std
School 1	35	4.4	2.4
School 2	39	4.8	2.9
School 3	56	4.2	2.3
School 4	70	4.3	2.5
School 5	50	4.4	2.6
School 6	68	4.1	3.0
School 7	68	4.5	2.9
School 8	12	3.7	3.1
School 9	41	1.8	1.6
School 10	18	2.7	2.3
School 11	17	1.6	1.6
School 12	24	1.9	1.6
School 13	24	1.8	2.3
School 14	37	4.8	2.7
School 15	30	4.0	2.5
School 16	28	3.3	2.0
School 17	14	3.3	2.5
School 18	43	2.6	2.6

4.2 Descriptive Statistics of Each Question

In this section, frequencies and percentages of correct and incorrect responses computed were presented to investigate the most difficult and easiest questions.

Table 4.3 Distribution of Questions According to Answers

	Completely Correct		Incorrect or Missing		Partially Correct	
	f	%	f	%	f	%
Q1 (G – E)	131	19.4	543	80.6	-	-
Q2 (T – V)	332	49.3	342	50.7	-	-
Q3 (V– T)	249	36.9	406	60.2	19	
Q4 (V – G)	270	40.1	290	43.0	114	12.0
Q5 (E – T)	242	35.9	432	64.1	-	-
Q6 (G– T)	235	34.9	425	63.1	14	2.1
Q7 (V - E)	81	12.0	593	88.0	-	-
Q8 (E – G)	95	14.1	558	82.8	21	3.1
Q9 (G – V)	188	27.9	451	66.9	35	5.2
Q10 (T - E)	276	40.9	397	58.9	1	0.1
Q11 (T – G)	277	41.4	395	58.6	2	0.3
Q12 (E – V)	27	4.0	647	96.0	-	-

Note. T: Table, V: Verbal Statement, E: Equation. G: Graphic

As it can be seen in Table 4.3, the most difficult item was 12. In this question, students were asked to translate an equation to verbal statement. Only 4% (27 students out of 674) of participants gave correct answer to this question. The easiest item was 2. It was about translation from table to verbal statement. 49.3% (332 students out of 674) of participants gave correct answer to this question. In addition, the least incorrect (or missing) answers were given to question 4 which was about translation from verbal statement to graphic.

4.3 Descriptive Statistics of Translations among Representations

In this section, questions involving translations to verbal statement, equation, table, or graphic were presented by giving frequencies and percentages of correct, incorrect, and missing answers to investigate each translation skill individually

4.3.1. Translations to Verbal Statement

Table 4.4 Descriptive Statistics of Three Questions involving Translations to Verbal Statement.

	All questions answered correctly		All questions answered incorrectly.		All questions missing	
	f	%	f	%	f	%
To Verbal	14	2.1	168	24.9	222	32.9

Note. Partially correct responses were not included in the table.

In the TADRACT, there were questions about translation from table to verbal sentence (Q2), translation from graphics to verbal sentence (Q9), and translation from equation to verbal sentence. Only 14 participants gave correct answers to all of these three questions about translations to verbal sentence as seen in Table 4.4. As seen from Table 4.5, the easiest one was translation from table to verbal sentence, 49.3 % of students gave correct answer. The most difficult one is translation from equation to verbal sentence, only 4 % of students gave correct answer.

Table 4.5 Descriptive Statistics of Each Question Involved Translation to Verbal Statement

	Correct		Incorrect		Missing	
	f	%	f	%	f	%
Q2 (T-V)	332	49.2	322	47.8	20	3.0
Q9 (G-V)	188	27.9	358	53.1	93	13.8
Q12(E-V)	27	4.0	455	67.5	192	28.5

4.3.2 Translations to Equation

Table 4.6 Descriptive Statistics of Three Questions involving Translations to Equation

	All questions answered correctly		All questions answered incorrectly		All questions missing	
	f	%	f	%	f	%
To Equation	47	7.0	200	29.7	202	30.0

Note. Partially correct responses were not included in the table.

Another group of questions involved translations to equation. These questions were translation from graphics to equation (Q1), translation from verbal statement to equation (Q7), and translation from table to equation (Q10). 47 participants gave correct answers all of three questions about translations to equation as shown in Table 4.6. The least number of correct responses in this group were for the question involving a translation from verbal statement to equation (Q7). The highest percent of correct responses, on the other hand were for the question involving a translation from table to equation (Table 4.7).

Table 4.7 Descriptive Statistics of Each Question Involved Translation to Equation

	Correct		Incorrect		Missing	
	f	%	f	%	f	%
Q1(G-E)	131	19.4	475	70.5	68	10.1
Q7(V-E)	81	12.0	509	75.5	84	12.5
Q10(T-E)	276	40.9	261	38.7	136	20.2

4.3.3 Translations to Table

As it can be seen in Table 4.8, 74 students gave correct answers to all of three questions about translations to table. The questions that were about translations to tabular representation were involved translation from verbal sentence to table (Q3), translation from equation to table (Q5), and translation from graphic to table (Q6).

Table 4.8 Descriptive Statistics of Three Questions involving Translations to Table

	All questions answered correctly		All questions answered incorrectly		All questions missing	
	f	%	f	%	f	%
To Table	74	11.0	156	23.1	131	19.4

Note. Partially correct responses were not included in the table.

According the results displayed in Table 4.9, questions involving the translations to table had relatively higher percentage of correct responses. However, the percent of correct responses were still about 35%.When the answers of these three questions were compared, it was seen that the number of students who gave correct and incorrect answers are too close to each other.

Table 4.9 Descriptive Statistics of Each Question involved Translation to Table

	Correct		Incorrect		Missing	
	f	%	f	%	f	%
Q3(V-T)	249	36.9	389	57.7	17	2.5
Q5(E-T)	242	35.9	357	53.0	75	11.1
Q6(G-T)	235	34.9	353	52.4	72	10.7

4.3.4 Translations to Graphic

51 students out of 674 gave correct answers to all three questions about

translation to graphic. These questions were translation from verbal sentence to graphic (Q4), translation from equation to graphic (Q8), and translation from table to graphic (Q11). As it was shown in Table 4.11, translation from verbal sentence to graphic (Q4) was the easiest one among the three. The most difficult one was translation from equation to graphic (Q8).

Table 4.10 Descriptive Statistics of Three Questions involving Translation to Graphic

	All questions answered correctly		All questions answered incorrectly		All questions missing	
	f	%	f	%	f	%
To Graphic	51	7.6	130	19.3	159	23.6

Note. Partially correct responses were not included in the table.

Table 4.11 Descriptive Statistics of Each Question involved Translation to Graphic

	Correct		Incorrect		Missing	
	f	%	f	%	f	%
Q4(V-G)	270	40.1	270	40.1	20	3.0
Q8(E-G)	95	14.1	439	65.1	119	17.7
Q11(T-G)	277	41.4	323	47.9	72	10.7

According to results displayed in Table 4.4, Table 4.6, Table 48, and Table 4.10; the most problematic translation was from other representations (table, equation, graphic) to verbal statement. In addition, the most correct answers were given to questions about translation from other three representations to table. On the other hand, the number of students who was successful at translations to table was only 74 out of 674 which was very low number.

4.4 Most Frequent Errors

In this part, students' most frequent errors were reported. These errors were given based on the representation types.

4.4.1 Translation to Verbal Sentence

Most of the participants tried to translate given information to verbal statement. However, some of them wrote inadequate information to explain the relationships in algebraic problems. In addition, some others wrote completely wrong information about relationships.

Question 2:

*T*he relationship of time of bicycle rent and the amount of money that a bicycle rent company earned was given in following table. Translate the given information to verbal statement.

Renting time (hour)	1	2	3	4	5
Earning money (YTL)	25	50	75	100	125

Figure 4.2 Question involving translation form table to verbal statement

In question 2 (Figure 4.2), it was expected from students to translate the given information to verbal statement. One of the most frequent errors about this question was that the answer was true but inadequate. For example, they wrote:

"There is right proportion between time and amount of money."

"The amount of money increases as time passed."

"Company earned 25 YTL in an hour, 50 YTL in two hours, 75 YTL in three hours an etc."

"The amount of earning money increases 2 times as time passed."

"The amount of earning money increased 25 times as time passed."

Some students form a question from the given information instead of explaining it with their own words.

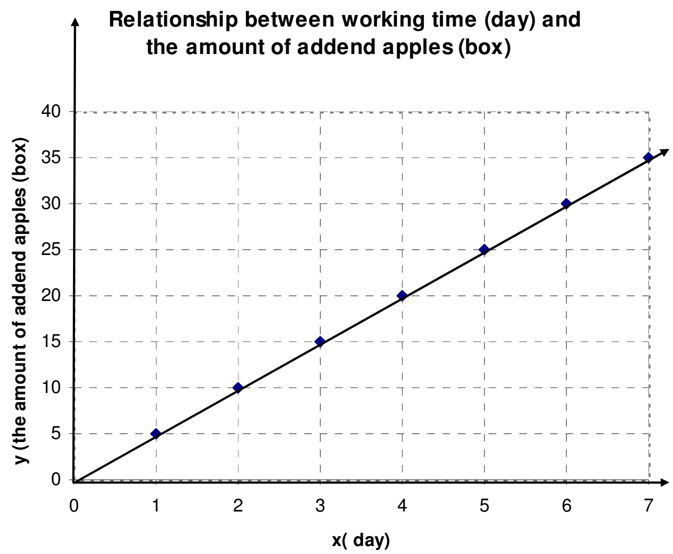
"How much money the company earns after 5 hours if they earn 25 YTL in an hour?"

In question 9 (Figure 4.3) it was asking to translate graphical representation to verbal statement. The answers of this question, like the previous one, there were verbal sentences, but they were wrong. Some of the most common inadequate answers are given below (numbers in parentheses represent the frequency of similar responses):

"There is direct proportion between working time and the amount of apple boxes that workers picked." (f=52)

"Workers pick 5 boxes in one day, 10 boxes in two days etc." (f=47)

Question 9:



In an orange garden, workers pick oranges in boxes. Translate the relationship between the working time (day) and the amount of reaped apples (box) that was given in graphical form to verbal sentence.

Figure 4.3 Question involving translation from graphic to verbal statement

Some students could form verbal statement from given information. However, they had the problem with the relationship between time and amount of boxes:

> "Every day apple boxes are picked 5 more than the apple boxes have picked the day before." (f=37)

<u>Question 12</u>

y= 300-10x

The equation given above shows the relationship between the number of questions that Zeynep has answered for homework by now and the number of questions she have to answer to finish her homework. *x* represents the number of questions that Zeynep has solved, and *y* represents the number of remaining questions. Explain this relationship in your own words.

Figure 4.4 Question involving translation form equation to verbal statement

Question 12 (Figure 4.4) was asking to translate the given information in equation form to verbal statement.13.9 % of answers were true but inadequate about the relationships between the number of questions that Zeynep has answered for homework by now and the number of questions she has to answer to finish her homework.

"There is inverse proportion between *x* and y."

"When x increases, *y* decreases."

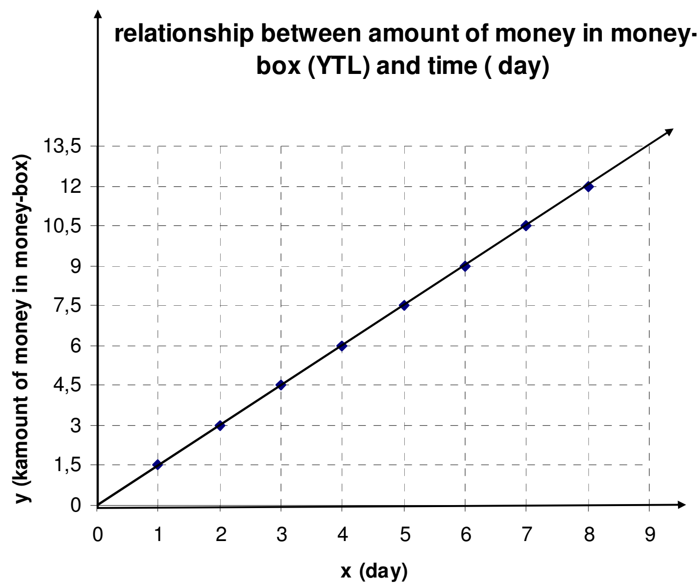
"The number of remaining questions that Zeynep has to answer is found when 10 times the time passed by that time is subtracted from 300."

10.6 % of participants made wrong translation to verbal statement

4.4.2 Translation to Equation

The common error of this kind of translation was about using unknowns in equation.

Question 1:



A child takes money from his father to put in his moneybox. In the graphic, the relationship between the amount of money that this child has in his money-box and the amount of time passed is showed. Translate this relationship to equation.

Figure 4.5 Question involving translation from graphic to equation

Question 1 (Figure 4.5) was asking to translate given information in graphical form to equation. Because of the fact that unknowns x and y that would be used in equation were given in the graphic in this example, there was not problem at explanation of unknowns. On the other hand, 11.5 % of students used x and y in the place of each other.

"$1.5y = x$"

<u>Question 7:</u>

A worker earns money as the time he works. Every hour that he works, he earns 10 YTL. Show the relationship between earning money and time passed in an equation.

Figure 4.6 Question involving translation from verbal statement to equation

Question 7 (Figure 4.6) was asking to translate given information in verbal statement to equation. In this question the unknowns that would be used as variables in the equation were not given. Students had to create unknowns themselves. At that time, 6.8 % of them had inadequate responses in which the unknowns were not explained by students, which made it impossible to assess the response. For example, they wrote:

"y= 10x"

without explaining what y and x stands for.

In addition, 23% of participants explained the unknowns briefly. However, they used the unknowns in wrong places that made the answer completely incorrect.

x= working time (hour) y = the amount of money that worker earned

"$x=10y$"

Moreover, some students drew table instead of writing an equation.

Question 10:

A	50	80	110	140
B	25	40	55	70

The table shows the relationship between height that a ball reach when it is threw upward (A) and height that this ball jumps after it hits the ground first time (B). Write the equation that shows this relationship.

Figure 4.7 Question involving translation from table to equation

In question 10 (Figure 4.7), it was asking to translate given information in table to equation. Of all students 19% of them (f=128) did not write anything to answer this question. In addition, 13.3% of the participants specified that they did not know to write equation. 64 students (9.5 %) tried to write equation but they made errors. Most frequent error was writing inverse proportion of A to B.

"2A=B"

4.4.3 Translations to Table

Question 3:
A motorbike moves at a rate of 80km per hour. Verbal statement explains that a motorbike travels at a rate of 80km per hour. Draw a table that shows the relationship between time passed (hour), and distance (km).

Figure 4.8 Question involving translation from verbal statement to table

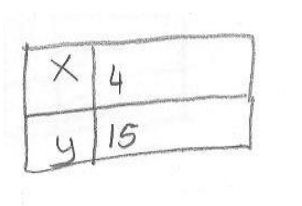
In Question 3 (Figure 4.8), it was asking to translate information given in verbal statement to table. Among all participants 13% (f=87) tried to draw a graphic instead of table. Most problematic part of the tables was labeling. Some of students drew velocity - timetable instead of distance - timetable.

Question 5:
y = 15+4x
The relationship between y and x variables is given in the equation. Draw a graphic that shows this relationship.

Figure 4.9 Question that involves translation from equation to table

Question 5 (Figure 4.9) involved a translation from equation to verbal statement. Among all students 26.7% (f=180) did not write anything to answer this question.15.2 % of participants (f= 103) prepared wrong tables (Figure 4.10). In addition, among all students 3.4% drew graphic instead of table.

Student 1:



Student 2:

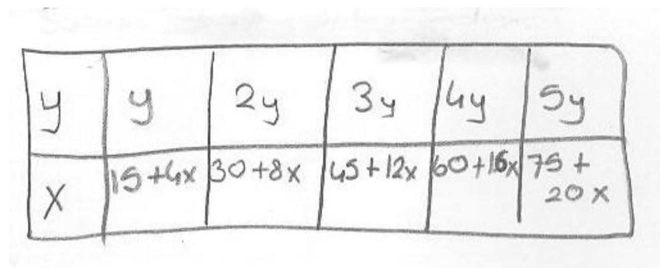
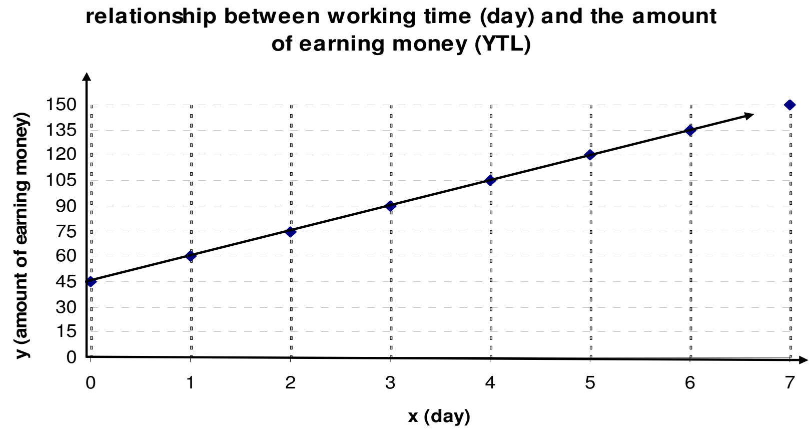


Figure 4.10 Examples of incorrect responses involving tables drawn for question 5

Question 6:

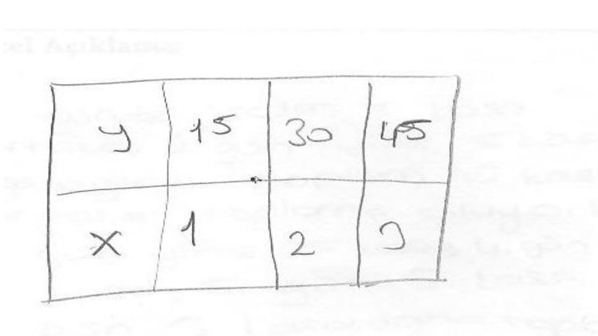


A boy works in a bookstore to save money for a new computer. Graphic shows the relationship between working time (day) and the amount of saving money (YTL). This boy has had money before starting his job in bookstore. Draw a table that shows relationship between amounts of money that boy have saved by now and working time.

Figure 4.11 Question involving translation from graphic to table

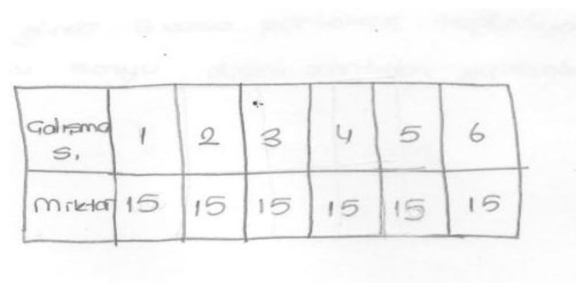
In question 6 (Figure 4.11) it was asking to translate graphical representation to tabular representation. The difference of the question 6 from other graphic questions was that the initial point was not "0". The boy had 45 YTL before starting to gain money.145 students, 21.5 % of participants, did not give any answer to this question. 161 students, 23.8 % of participants, drew wrong table (Figure 4.12).

Student 1:



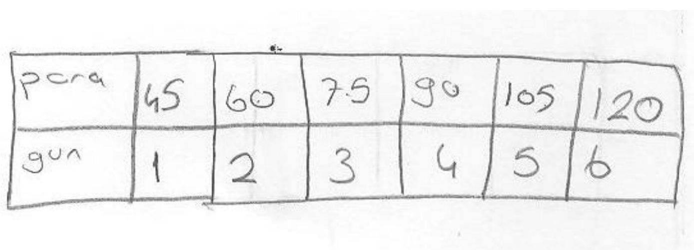
y	15	30	45
x	1	2	3

Student 2:



Gdrama s.	1	2	3	4	5	6
Miktar	15	15	15	15	15	15

Student 3:



para	45	60	75	90	105	120
gun	1	2	3	4	5	6

Figure 4.12 Examples of incorrect responses involving tables drawn for question 6

127 students, 18.2 of participants, did not show that the amount of money was 45 YTL at the beginning.

4.4.4 Translations to Graphic

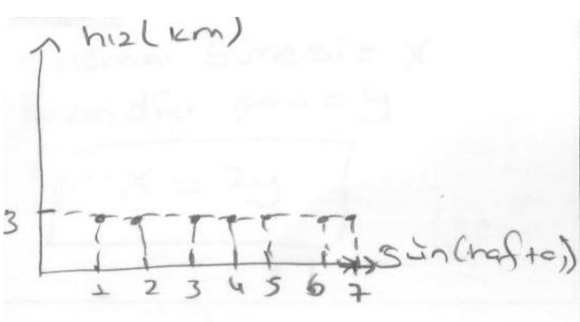
None of the students wrote title of the graphic. There were errors at labeling and drawings. Many participants did not label their graphs or labeling was inverse. In addition, some students did not determine points on the graphic; some others did not connect the points.

Question 4:
Ayşe goes for a three km run everyday. At the end of a week, she have run totally 21 km. Draw a graphic that shows relationship between time passed (day) and the amount of distance that Ayşe have run by that time (km).

Figure 4.13 Question involving translation from verbal statement to graphic

Question 4 (Figure 4.13) was asking to translate verbal representation to graphical representation.102 students, 15.1 % of participants, drew wrong graphics. Most common examples to wrong graphics were given in Figure 4.14.

Student 1:



Student 2:

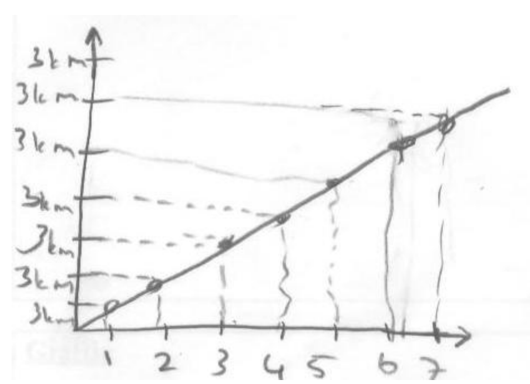


Figure 4.14 Examples of incorrect responses involving graphics drawn for question 4

Question 8:

y =26x

Draw the graphic that shows relationship between x and y variables that is given in equation format.

Figure 4.15 Question involving translation from equation to graphic

Question 8 (Figure 4.15) involved translation from equation to graphical representation. Among all students 15.1% (f=102) did not give answer to this question.122 students, 18.1 % of participants, drew wrong graphics. Most common error was drawing *x* and *y* axis in place of each other. In addition, some students drew table instead of graphic.

Question 11:

x	1	2	3	4	5
y	5	10	15	20	25

Table states the relationship between x and y variables. Draw the graphic of this relationship

Figure 4.16 Question involving translation from table to graphic

In question 11 (Figure 4.16), it was asking to translate tabular representation to graphical representation. Among all participants, 10.2% did not give any answer to this question. In addition, 222 students, 32.9% of participants, drew wrong graphics. 196 of these students drew x and y axis in place of each other.

4.5 Relationship between Gender and Translation Skill

Null Hypothesis

"There is no significant effect of gender on the population mean values of 8th grade students' translation scores."

As indicated in Table 4.11 to evaluate whether the mean value of the test score of females differs from the mean value of the test score of males independent t-test was used. The mean value of scores for females was 3.7 and the mean value of scores for males was 3.78 that showed there was no significant difference between mean values of scores for males and females. In addition, in t-test for equality of means significance was .69. It was bigger than .05 that showed there were no significant difference between mean value of scores for males and females. The negative t value (t= -.399) indicated that the mean value of TADRACT score of males was not significantly greater than the mean value of TADRACT score of females.

Table 4.12 Independent Sample t-test scores

	N	Mean	SD	t	Sig
female	333	3.7000	2.7202	-.399	.69
male	341	3.7827	2.6581		

4.6 Summary of Results

After the study, it was understood that 8th grade students had poor skills in translations of four different representations in algebraic concepts. 12th question was most difficult question that asked translation from equation to verbal statement. 2nd question was the easiest question that asked translation from table to verbal statement. Although the most correct answers were given to question 2, less than half of all students, 49.3 %, gave correct answer.

In addition, it is understood that the most problematic translations were from other representations to verbal statement. Translations from other three representations to table were easiest. However, only 74 students out of 674 gave correct answer to translations to table.

Most frequent error in translations from equation, table, and graphic to verbal statement was that students gave true but inadequate information.

Most frequent error in translations from verbal statement, table, and graphic to equation was labeling. Students had problems at both creating unknowns and using them in equation.

Most frequent error in translations from verbal statement, equation, and graphic to table was labeling.

Most frequent errors in translation from verbal statement, equation, table to graphic was both labeling and drawing. Students did not label or labeled x and y axis in place of each other. Also, some students did not determine points on the graphic or some others did not connect the points to each other.

When the relationship between gender and students' achievement at translation process was investigated, it was understood that gender did not affect 8th students' achievement in this study. That is girls and boys had nearly same mean scores.

CHAPTER V

DISCUSSION

With the new era in education system in Turkey, use of multiple representations in mathematics teaching and learning has gained importance. There has been a common idea that algebra has been one of the difficult subjects of elementary level mathematics and multiple representations might help students to understand better algebraic concepts easily.

Because of the fact that multiple representations are emphasized in the curriculum, and it is important dimension of conceptual understanding, this study aimed to investigate eighth grade students' skills in translating among different representations of algebra concepts. Furthermore, it was also aimed to investigate the common problems in making these translations and to determine relatively easier and difficult translation tasks. Lastly, it was aimed to find out if gender has any effect on students' skills in translation process.

Participants of this study were eighth grade students in public elementary schools. Data collection instrument was a test that included 12 tasks for translations among four representations. In this chapter, the results of this study are discussed.

5.1 Students' Performance in TADRACT

According to descriptive statistics of TADRACT 8th grade students had low skill in translations among verbal, symbolic, graphical, and tabular representations. Considering the Turkish students' insufficient levels of performance in national tests and in international comparison of mathematics performances, these results could be considered 'expected'. The mean score of students was 3.7 out of 12. Although the questions in the test seemed trivial at first that did not involve a problem solving tasks, students still had difficulties. Similar results were obtained by Kurt (2006) on the fraction concepts. One reason of such a result might be that both in the textbooks and during the mathematics instruction, students have very limited chance to work with different representations of mathematics concepts. Especially in algebra unit, very little attention is given to tabular representation. Most of the algebraic representations are handled separately without paying attention to translating them in multiple directions. It may be possible that students solve the questions if they were asking to find numerical answers. Besides, during the administration of the test some students stated that they did not learn these representations and translations among these representations in any mathematical concept.

5.2 Students' Performance in Each Question

The most problematic type of question was the translation from equation to verbal statement. Also, the least correct answers were given in questions that involved the translations from other three representations to verbal statement. These results were not surprising. According to Stacey and McGregor (1997), students had problems in explaining representations in words. Moreover, after his research Lochhead (as cited in Coxford, 1988) concluded that translating linear equations to verbal statement was a big problem for students.

The highest correct response rate was for the item that involved translation from table to verbal statement. This result contradicted with Stacey and McGregor's result (1997) that supported problem in verbal explanation of representations. The reason of this result might be that information was given in tabular representation format. According to Swafford and Langrall (2000), students preferred tables, because they were more understandable than other representations.

In addition, students had problems in questions contained translations from other three representations to equation. The most problematic one was translation from verbal statement to equation. This result was supported by Lochhead (as cited in Coxford, 1988). In his study, he concluded that students were not successful at translating written language (verbal statement) to mathematical language (equation). These results showed that students had problems both translating equation to verbal statement and verbal statement to equation.

Unlike with other translations, the number of correct responses to questions about translations from other three representations- verbal statement, table, and equation- to table was relatively high and very close to each other when compared with other translations. This result was consistent with the result found by Erbilgin (2003).

She concluded that translating different representations to table were easy for 8th grade students. Furthermore as indicated above although translation to verbal statement was most problematic process, translation from table to verbal statement was easiest question for students. In addition, not only translating other representations to tables, but also translating tables to other representations were relatively easy for students as compared to other translations in this study. Number of correct responses to questions about translations from table to equation was high when compared with translations from verbal statement to equation and translations from graphic to equation.

In addition, when students' answers to questions that included translations to tables were investigated, it was understood that most students did not use tables as unique representation. In other words, tables were used as a tool to translate given information to another representation, mostly to graphic. In fact, some students believed that tables and graphics are used together and tables were prepared only before drawing a graph. In fact, this way of using tables is a typical approach in mathematics textbooks and classrooms.

The last group of questions that contained translations from other three representations to graphic were problematic, too. However, in mathematics lessons students usually learn graphing in a title as a separate subject. It was showed that the reason of students' low skill in translation process did not depend on the lack of these representations in curriculum. Although graphic was thought students in mathematics lessons, students did not have enough knowledge about graphical representations.

This result may be due to the fact that students did not have experience in relating different representations to each other.

Most frequent error at translations to verbal statement was giving inadequate information. There may be two reasons of this error. Students' low skill in not only translating, but also understanding representations may be one of these errors. For students, in mathematics lessons finding true answer of questions is more important than understanding and explaining problems in their own words, because they study for OKS that requires solving many questions in a limited time. Students focus on answer rather than question. The second reason may be concerned with literature that shows the importance of relationship of mathematics with other lessons. Even if students understand relationship of variables given one of the representations, they cannot explain this relationship with their own words.

Most frequent error at translations to equation was about unknowns. Some students forgot to explain unknowns and some others used unknowns in place of each other that showed not only students' low skill in writing equations but also their low skill in understanding relationships between variables. In questions involving table and graphing most frequent error was about labeling. Students learned about graphing and table during mathematics and science lessons. However, this knowledge is not enough to prepare graph or table totally correct.

After learning algebra with the help of representations, students are more successful than before (Moseley, Brenner, 1997; Akkuş-Çıkla, 2004). However, in general students do not have adequate knowledge and experience about using representations in mathematic subjects. In the former mathematics curriculum that was in use during the time of data collection, these translations were not emphasized. 8th grade students learned only writing equations from verbal statement that was also found to be a problematic in this study. They learned to use one kind of representation in a question like drawing graphic from given points. However, according to Brenner et al. (1995) representational skills can be learned in classroom. Their study showed that after students learned algebra with redesigned unit, students learned to use different methods, and used more representations in algebra problems. Also Mourad (2005), Moseley and Brenner (1997), and Akkuş- Çıkla (2004) gave point to positive effects of redesigned curriculum on students mathematics achievement. Therefore, one of the solutions of this low skill in translating different representations on algebraic concepts may be designing new mathematics units emphasizing multiple representations skills. In Turkey, the new national curriculum emphasizing multiple representation skills were designed and began to be used in six grade students in 2005-2006 academic year. After two years, when 8th grade students learned algebraic concepts with the help of new curriculum, the effects of this curriculum will be understood. On the other hand, new curriculum is not sufficient by itself to provide students use of multiple representations and translations among them. Teachers' knowledge (Özgün-Koca, 1998; Cunningham, 2005), teachers' beliefs about representations (Cai, 2004) and textbooks (Brenner et. all, 1997; Cai, 2004) are as important as curriculum. According to Cai, students' thinking about role of representations in mathematics lesson is due to beliefs of teachers and their emphases according to their beliefs about representations.

In this study, gender did not effect students' achievement. However, Kurt (2006) concluded that gender affected students' achievement on translations among representations about fraction. Girls were more successful than boys. Such varying results may be due to the nature of representations covered in the test. Further studies are needed to understand better the gender differences in representational skills.

5.3 Internal Validity

Internal validity means the degree to which differences in dependent variable are related to only independent variable (Fraenkel & Wallen, 2006). Location threat means that the results of the study are related to location in which the study is applied. Hot, noisy, poorly lighted, unpleasant conditions might affect test scores. This threat reduced by incorporating consistency in subjects' conditions as closely as possible. Instrumentation threat means that the results of the study are due to the instrument or data collector. Researcher reduced this threat by administrating test in classrooms herself. There was not mortality threat, researcher loses some of the participants that affect the results, because of the fact that test was applied each student one time. The elementary schools that test was conducted were randomly selected to reduce subject characteristic threat that means the characteristics of subjects may influence the results of the study.

5.4 External Validity

External validity means the degree to which results are generalizable to groups and environments outside the research. Population generalizability refers to degree to which sample represents population (Fraenkel & Wallen, 2006). In this study to generalize target population was not possible because of time, effort, and money. Therefore, accessible population, Çankaya district of Ankara, was selected by researcher and schools were chosen randomly from accessible population that provided the results presented in the study. However, answer sheets of some students who did not solve more than eight questions were not used during evaluation. This omission might create threat to generalize results to accessible population.

Ecological generalizability refers to the extent that results of the study can be generalized to other conditions (Fraenkel & Wallen, 2006). The conditions of all schools were similar to each other in this study. Test was conducted in similar public schools, similar classroom conditions like lighting, sitting arrangement. In addition, textbooks used in mathematics lessons were same. Therefore, ecological generalizability occurred in this study.

5.5 Implications

The results of this study have implications for elementary grade instruction on the concept of algebra. This study focused on eighth grade students' skill in translations among different representations of algebraic concepts. Based on the findings of this research study, it can be suggested that more attention should be given to not only translations among representations but also usage of each representation in algebraic concepts of elementary mathematics lessons. Teachers should encourage students to use representations and to make translations among representations. In addition, based on the findings of the other studies, multiple representations should be used in different subject areas of elementary mathematics curriculum.

Another suggestion that could be drawn from this study is that students need to focus on a mathematical concept itself rather than true answers of questions with memorizing solution methods. Teachers should give chance to their students in verbal explanations of the concepts that would decrease problems in verbal representations.

5.6 Recommendations for Further Research

Based on the conclusions from this study, it is evident that more studies are needed to research multiple representations in mathematics education. Further research could be about the place of multiple representations in new mathematics curriculum in Turkey, and the effects of new curriculum on students' translation skills about algebraic concepts. Two years later, eight grade students will be learning algebra with new mathematics curriculum that focuses on multiple representations. A study that investigate students' translation skills in the new curriculum and relates the findings to the current study would be very beneficial.

In addition, development of curriculum is not sufficient by itself to make changes in education if teachers are ready to implement new curriculum in mathematics lessons. Researchers suggest that teachers' knowledge and beliefs about multiple representations affect usage of multiple representations in mathematics lessons. Therefore, mathematics teachers' and preservice teachers' knowledge and beliefs about multiple representations and translations among different representations could be the subject of further research.

This study was applied to eighth grade students to evaluate their translational skills towards the end of the elementary school. However, representations in mathematics education can be used in primary school, even in kindergarten. Further research can be conducted by different grade levels to understand better the changes in translation skills of students.

One of the aims of this study was to understand students' errors and problematic translations with the help of their answers to test questions. However, a qualitative case study that comprises individuals contributes much understanding of inside of students' mind. Further research can be conducted to obtain in-depth information about multiple representations and translations in multiple representations in one of the subject of elementary school mathematics.

The gender effect on mathematics education is searched all over the world. There is not a lot of research about the relationship between multiple representations and gender. Further research can look closely to this relationship in both algebra and other subjects of elementary grade mathematics

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APPENDICES

APPENDIX A
Translation Among Different Representations of Algebraic Concepts Test

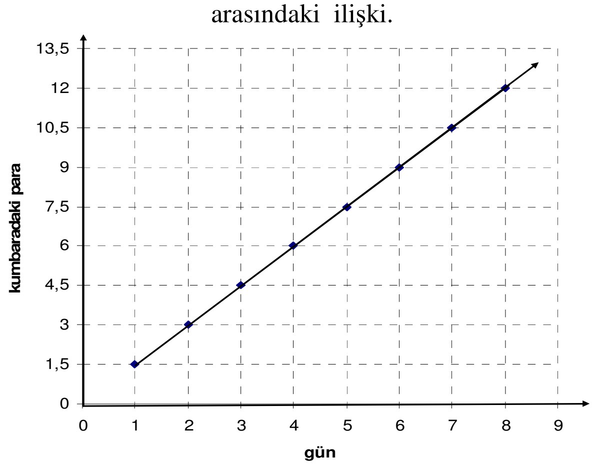
Öğrencinin:

Adı Soyadı: Sınıfı: Cinsiyet:

Grafik:

1)

Kumbaradaki para miktarı(YTL) ve geçen süre (gün) arasındaki ilişki.



Bir çocuk her gün babasından kumbarası için para almaktadır. Yukarıda bu çocuğun kumbarasındaki para miktarı ile geçen süre (gün) arasındaki ilişkiyi gösteren grafik verilmiştir.

Yan tarafa bu ilişkiyi gösteren denklemi yazınız.

Denklem:

Tablo:

2)

Kiralama süresi (saat)	1	2	3	4	5
Kazanılan para (YTL)	25	50	75	100	125

Yukarıdaki tabloda bisiklet kiralama şirketinin kazandığı para ile kiralama süresi (saat) arasındaki ilişkiyi gösteren tablo verilmiştir.

Yan tarafa bu ilişkiyi kendi kelimelerinizle açıklayınız.

Sözel Açıklama:

Sözel Açıklama: **3)** Bir motosiklet saatte ortalama 80 km. hızla gitmektedir. Yukarıda bir motosikletin bir saatte ortalama 80 km yol aldığı sözlü olarak ifade edilmektedir. Yan tarafa yolculuk süresi (saat) ile gidilen yol (km) arasındaki ilişkiyi gösteren tabloyu oluşturunuz.	**Tablo:**

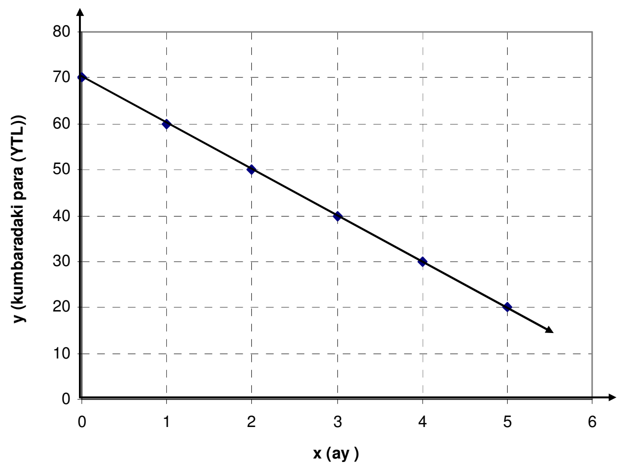
Sözel Açıklama: **4)** Ayşe sabah sporu yaparken her gün 3 km koşuyor. Bir hafta sonunda toplam 21 km koşmuş oluyor. Yukarıda sözlü olarak ifade edilen durumla ilgili olarak geçen süre (gün) ile o güne kadar toplam kaç km koşulduğunu gösteren grafiği yan tarafa çiziniz.	**Grafik:**

Denklem: **5)** $y = 15 + 4x$ Yukarıdaki denklemde x ile y değişkenleri arasındaki ilişkiyi gösteren tabloyu yan tarafa oluşturunuz.	**Tablo:**

Grafik:

6)

Para miktarı (YTL) ile geçen süre (ay) arasındaki ilişki



Bir öğrenci biriktirdiği para ile her ay bir oyun CD si almaktadır. Yukarıdaki grafik öğrencinin para miktarı ile geçen süre (ay) arasındaki ilişkiyi göstermektedir.

Yan tarafa bu ilişkiyi gösteren tabloyu oluşturunuz.

Tablo:

Sözel Açıklama:

7) Bir fabrikada çalışan bir işçi çalıştığı süre kadar para almaktadır. Bu işçi çalıştığı saat başına 10 YTL para kazanmaktadır. 8 saat çalıştığı bir günün sonunda 80 YTL para kazanmıştır.

Yan tarafa.işçinin çalışma süresi (saat) ile kazanılan para arasındaki ilişkiyi gösteren denklemi yazınız .

Denklem:

Denklem:	**Grafik:**
8) $$y = 26x$$ Yukarıdaki denklemde *x* ile *y* değişkenleri arasındaki ilişkiyi gösteren grafiği yan tarafa çiziniz.	
Grafik: **9)** Çalışılan süre (gün) ile toplanan portakal miktarı (kasa) arasındaki ilişki  Bir portakal bahçesinde köylüler portakal toplamaktadır. Çalışılan süre (gün) ile toplanan portakal miktarı (kasa) arasındaki ilişkiyi gösteren grafik yukarıda verilmiştir. Bu ilişkiyi yan tarafa kendi cümlelerinizle açıklayınız.	**Sözel Açıklama:**

Tablo: 10)	Denklem:
 	A | 50 | 80 | 110 | 140 |
B | 25 | 40 | 55 | 70 | Yukarıdaki tablo bir topun yukarı atıldığında ulaştığı yükseklik (*A*) ile yere ilk çarptıktan sonra sıçradığı yüksekliğin (*B*) ilişkisini göstermektedir. Bu ilişkiyi gösteren denklemi yan tarafa yazınız.	

Tablo:
10)

A	50	80	110	140
B	25	40	55	70

Yukarıdaki tablo bir topun yukarı atıldığında ulaştığı yükseklik (*A*) ile yere ilk çarptıktan sonra sıçradığı yüksekliğin (*B*) ilişkisini göstermektedir.

Bu ilişkiyi gösteren denklemi yan tarafa yazınız.

Denklem:

Tablo:
11)

x	1	2	3	4	5
y	5	10	15	20	25

Yukarıda *x* ile *y* değişkenleri arasındaki ilişkiyi gösteren tablo verilmiştir.

Yan tarafa bu ilişkiyi gösteren grafiği çiziniz.

Grafik:

Denklem:
12)

$$y = 300 - 10x$$

Yukarıdaki denklem Zeynep'in dönem ödevi için çözmesi gereken soru sayısını göstermektedir.

x değişkeni Zeynep'in bir saatte ortalama çözdüğü soru sayısını , y değişkeni ise geriye kalan çözmesi gereken soru sayısı gösteriyorsa x ile y değişkenleri arasındaki ilişkinin sözel açıklamasını yan tarafa yazınız.

Sözel Açıklama:

APPENDIX B
Scoring Guide to Evaluate TADRACT

Questions	Score levels
Translation to Verbal Statement (Items 2, 9, and 12)	1 point – Response is completely correct This response is to explaining the relationship of variables. 0.8 point – Response is partially correct This response is explaining relationship with incorrect unit or no unit. 0 point – Response is incorrect, irrelevant, inadequate, or missing.
Translation to Equation (Items 1, 7, and 10)	1 point – Response is correct This response is writing Equation that represents relationship between variables 0.8 point - Response is partially correct This response is writing equation without labeling. 0 point – Response is incorrect, irrelevant, inadequate, or missing.
Translation to Table (Items 3, 5, and 6)	1 point – Response is completely correct This response is drawing table 0.8 point –Response is partially correct This response is drawing table without labeling. 0 point – Response is incorrect, irrelevant, inadequate, or missing.

Translation to Graphic (Items 4, 8, and 11)	1 point – Response is correct This response is drawing graphic. Response is partially correct. 0.6 point - This response is drawing graph without both determining and connecting points 0.5 point - This response is drawing graph without labeling without labeling. 0 point – Response is incorrect, irrelevant, inadequate, or missing.