EXPORT DYNAMICS, SIZE AND PRODUCTIVITY OF FIRMS

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ABSTRACT

EXPORT DYNAMICS, SIZE AND PRODUCTIVITY OF FIRMS

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In this thesis we examined the export dynamics at the firm level. A two period model is proposed for the life of firms. The firms may have three different behaviors: staying out of markets, producing for the domestic market, and producing for both the domestic and the export markets. During two periods, firms may enter or exit the markets according to their (expected) profits. All firms are profit maximizing such that they compare the maximum (expected) profits in the domestic and export markets. Firms are also heterogenous so that they have different levels of productivity. We examined changes in investment, market share and profits with respect to changes in the market and firm parameters. The profits and investments of the exporting and non-exporting firms are compared by both analytical and numerical methods.

Keywords: Export dynamics, Firm Productivity, Firm Size, Self-Selection

ÖZ

İHRACAT DİNAMİKLERİ, FİRMA BÜYÜKLÜĞÜ VE ÜRETKENLİK

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Bu çalışmada, ihracat dinamikleri firma düzeyinde incelenmiştir. Firmaların ömürleri için iki dönemlik bir model sunulmuştur. Firmalar üç farklı davranış sergileyebilirler (i) herhangi bir piyasaya girmemek, (ii) sadece iç piyasada üretim yapmak (iii) hem iç piyasaya üretim yapmak hem de ihracat yapmak. Firmalar iki dönemlik süreç boyunca kararlarını sadece kârlarını maksimize eden duruma göre vereceklerdir. Modeldeki firmaların farklı üretkenlik seviyelerinde oldukları varsayılmış ve buna göre bazı karşılaştırmalar yapılmıştır. Firmaların yatırım seviyelerinde, üretimdeki ihracat paylarında ve kâr düzeylerinde, firma ve piyasa parametrelerine göre nasıl değişiklikler olduğu incelenmiştir. İhracatcı olan ve olmayan firmaların kâr ve yatırım düzeylerinin kıyaslaması analitik ve sayısal olarak yapılmıştır. Son olarak değişik üretkenlikteki firmaların iki dönemde ortaya koydukları davranışlar bulunmuş ve bu davranışlar üzerine literatürdeki ampirik sonuçları destekleyen yönde bazı çıkarımlar yapılmıştır.

Anahtar Kelimeler: İhracat dinamikleri, Firma Üretkenliği, Firma Büyüklüğü, Kendiliğinden-Seçim

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To My Parents

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LIST OF SYMBOLS

D	Domestic Market
E	Both Domestic and Export Markets
S	Stay out of both Markets
r	Interest Rate
z	Capital Stock
Q_i	Total Production of Firm i
Q_i^e	Export Production of Firm i
Q_i^d	Domestic Production of Firm i
α	Return to Scale Parameter
A_i	Real Productivity
k_i	Investment Amount of Firm i
k_m	Threshold Investment Amount
x_i	Export Market Share of Firm i
P_i^e	Export Market Price of Firm i
P_i^d	Domestic Market Price of Firm i
B_i^e	Export Market Demand Parameter of Firm i
B_i^d	Doemstic Market Demand Parameter of Firm i
P^e	Export Market Average Price
P^d	Domestic Market Average Price
ϵ_e	Export Market Price Elasticity Parameter
ϵ_d	Domestic Market Price Elasticity Parameter
F^e	Export Market sunk cost
F^d	Domestic Market sunk cost
π^e	Profit in Export Market
π^d	Profit in Domestic Market

CHAPTER 1

INTRODUCTION

The differences in the characteristics of exporting and non-exporting firms have been investigated in the recent literature. Many of these studies find that the exporters are larger, more productive, more capital-intensive, more technology-intensive, and pay higher wages. There exists two complementary explanations for these advantages of exporting firms: (i) the market selection hypothesis, and (ii) the learning hypothesis. The first hypothesis suggests that the firms already more productive have more tendency to enter the export market and the second one proposes that the productivity of the firms which have already entered to the export market increases due to the learning effect.

The very first studies on the idea that export activity and economic growth are related are Beckerman (1962), Kaldor (1970), Balassa (1988). There are two patterns of causation between the growth and exporting. One of them is that exports induce an increase in country's output and productivity. On the other hand, some scholars claim that the direction of causality runs from economic growth to export. There are four main arguments about the export-led growth: (i) exporting activity is an important component of autonomous demand and determines a multiplier effect on investment and output (see Beckerman 1962, Kaldor 1970) both in the exporting and in related sectors in the home economy (see Khan & Khanum 1997). (ii) Second, the growth of the exporting sector promotes a reallocation of resources from the non-trade sector to the export sector itself which, being relatively more productive, raises the overall productivity of the country (Bernard & Jensen 1997, Giles & Williams 2000). (iii) Export is a means to generate foreign currency inflows, required to finance imports (Thrilwall 1980). (iv) Outward orientation may result in efficiency gains for firms, due to the exploitation of economies of scale and learning associated with knowledge pullovers from international contacts (Clerides et al. 1998).

The researchers suggesting that the direction of causality is from productivity growth to exports claim that economic growth produces an enhancement of skills and technologies, which creates the basis for the international competitive advantage and in turn determines exports. Due to the establishment of an international distribution channel and/or to the adaptation of products to foreign standards, export markets includes sunk costs. These sunk costs determine that only the larger and more productive firms will start exporting (Roberts & Tybout 1997, Bernard et al. 2000). Over many empirical studies, Giles & Williams (2000) find that evidence is not conclusive. They review more than one hundred and fifty empirical papers, using either cross-section or time-series data, and do not reach any conclusion about the direction of causality. A positive association between export and economic growth is supported in most cross-country studies. In earlier works this was interpreted as evidence of export-led growth. However, the empirical approach does not allow excluding growth-led export as well.

A number of recent works have tried to analyze the issue at the micro-level, looking at the direction of causality between exporting activity and productivity growth at the level of the firm. The key questions in this stream of literature are "Do more productive firms become exporters?" and "Do exporters become more productive firms?". In fact, the correlation between exporting and productivity can be the resulting forces. More productive firms become exporters, because exporting requires some additional costs, such as expenses related to establishing a distribution channel, transportation costs, or production costs to modify products for international markets. This in turn implies that only the outperforming firms expect to be able to cover these additional costs and will rationally choose to enter the export market. Hence, correlation between productivity and export may arise as a result of the self-selection of better firms to go into the export market. On the other hand, exporters might learn from their presence in international markets for two main reasons: (i) the larger international market allows the exploitation of economies of scale and, (ii) international contacts with buyers and customers are likely to foster knowledge and technology spillovers, such as access to technical expertise, including new product designs and new production methods.

There are many empirical studies (see Bernard & Jensen 1997, Bernard & Wagner 1997, Delgado et al. 2002, Girma et al. 2004, Castellani 2002) suggesting that firms which entered export market had some prior advantage. Bernard & Jensen (1997), consider the sources of the substantial performance advantages at exporting plants and firms. They perform econometric analysis on the U.S. plants and suggest that the advantages of exporting firms are substantial: at any point in time exporters produce more than twice as much output and are more productive. They find that exporting has a positive effect on the probability of plant survival, growth in size and employment but they do not find any evidence of more productivity growth of the exporter firms. In a different paper, Bernard & Jensen (1999) also show that self-selection causes an aggregate growth in productivity, due to a composition effect: exporters grow in size, their share in aggregate productivity rise, and since they have better performances prior to enter the export market, aggregate productivity increases. A similar study on the German manufacturing industry is performed by Bernard & Wagner (1997). The authors find that the exporters are larger, more capital-intensive and employing more white-collar workers than the non-exporting firms. Additionally, they suggest that the higher survival rates of exporters can be easily explained by the superior performance characteristics of plants before exporting not by exporting enhancing performance. The study of Wagner (2002) on the German industry shows that starting to exporting have positive effects on growth of employment but have weaker evidence for a positive effect on labor productivity. Delgado et al. (2002) investigate the total factor productivity differences between exporting and non-exporting firms in Spanish manufacturing sector. Results of this article indicate clearly higher levels of productivity for exporting firms than for non-exporting firms and find evidence supporting the self-selection of more productive firms in the export market while the evidence in favor of learning-by-exporting is rather weak, and limited to younger exporters. The learning-by-exporting hypothesis is supported besides the self-selection hypothesis in Girma et al. (2004). The authors investigates the UK manufacturing firms and state that exporters are more productive and further exporting increases the firm productivity. However, a similar study Greenaway et al. (2005) on the Swedish economy which is extremely open shows that there is no evidence of pre- and post-entry differences in firm level productivity.

Clerides et al. (1998), try to find whether the firms become more efficient after becoming exporters and exporters generate positive externality for domestically oriented producers. They provide probably the more careful and comprehensive attempt to sort out the direction of causality between export and productivity. They estimate a system of two equations, one for the choice to enter the export market, the other for the process which governs unit costs. Using data on plants from Colombia, Mexico and Morocco, they find strong evidence of self-selection and no evidence of learning. Their results are consistent with Bernard & Jensen (1997), Bernard & Wagner (1997), Delgado et al. (2002). Some recent studies Melitz (2003), Jean (2002), Medin (2003), Helpman et al. (2004) have key theoretical contributions. Melitz (2003) investigates the effect of export market entry costs on the distribution of impact of trade across different types of firms. The existence of such costs does not affect the welfare-enhancing properties of trade, but alter the distribution of gains from trade across firms. In other words the more productive ones reap benefits in the form of market share and profit while the less productive firms lose both. And additionally the least productive ones are forced to stay out of trade. Authors think that these trade-induced reallocations towards more efficient firms explain why trade may generate aggregate productivity gains without necessarily improving the productive efficiency of individual firms. Similar results are found in Helpman et al. (2004) which extends the work of Melitz (2003) by giving firms the opportunity of setting up an overseas affiliate. Jean (2002) show that the interactions between international trade and firms' heterogeneity is investigated in a context of monopolistic competition. The results of this study suggest that exporters are larger and more productive than non-exporters and exporters are more atypical in this last respect when the country suffers from a comparative disadvantage in the sector, and that export-driven cross-firm reallocations ("share effects“) significantly increase average productive efficiency. Additionally, authors think that trade is not only a threat that eliminates the least efficient firms but the trade-induced increase in competitive pressure may also be the result of the entry of new producers, attracted by the new profit opportunities carried out by exports. Medin (2003) also has similar conclusions. Furthermore he suggests that small countries have a higher share of exporting firms when there are increasing returns to scale, a reversal of the standard home market effect common to increasing returns/monopolistic competition trade models.

On the other hand, Kraay (1999) investigates the medium and large size Chinese firms and finds some positive learning effects of exporting as found in Castellani (2002) which investigates the firms of Italian manufacturing sector. Aw et al. (1998) compare cross-sectional average productivity of group of firms which have undergone different patterns of transition in and out of the export market, in order to identify the relative importance of self-selection and learning-by-exporting. They identify four different status for the firms: stay out (firms do not export neither in period t, nor in period t + 1), entry (firms do not export in period t and export in period t + 1), exit (firms export in time t and do not export in time t + 1), stay in (firms export both in t and t + 1). They find differences between Taiwanese and South Korean industries in self-selection and learning. For Taiwanese, they find strong evidence of self-selection and, in some sectors, evidence of the learning-by-exporting. For South Korean industries, evidence of self-selection is weaker and no evidence of learning is found.

Comprehensive literature surveys on the export productivity can be found in Tybout (1997) and Greenaway & Kneller (2007) where in both studies the self-selection hypothesis is shown to have strong evidences while the learning by exporting has less conclusive evidences.

In this study we will examine the dynamics of firms in domestic and export market. We look for the evidences of self selection hypothesis and investigate the firm behaviors under different firm and market properties. To simulate the dynamics of firms we set a two period dynamic model. The firms in the model are heterogenous in productivity and face different demands for their products. The behaviors of firms will be examined under different market structures based on the analytical models we derived for the firm profit maximization processes. Additionally some numerical methods are utilized to understand the behaviors. The results are compared with some empirical data and with the empirical studies performed in the literature.

The following chapter (Chapter 2) explains the two period dynamic model, the market properties (especially demand functions), and the production functions of firms. The profit functions and the maximization of these profit functions are investigated in detail.

In Chapter 3, some properties of the maximized profits in the domestic and combined markets are investigated. Some conclusions are derived that help us in understanding the behavior of firms under the changes in the market and firm properties.

The next chapter (Chapter 4) shows the systematically solution of the firm behaviors in each period and has very important conclusions supporting findings in the literature, especially the self-selection hypothesis.

We perform some concluding remarks in the last chapter and show the resemblance of the results of this study with the empirical results in the literature.

CHAPTER 2

PROBLEM FORMULATION

We use a simple two period model to analyze the behavior of firms. At the beginning of the first period, the entrepreneur has three choices: (i) stay out of markets, (ii) enter domestic market, (iii) enter both domestic and export markets. The entrepreneur makes the entry and size decisions simultaneously. She calculates the expected profits in each case and she chooses the optimum firm size (measured by the value of capital stock) that maximizes the profit, and enters the market which has the maximum expected profit or stay outside. If she enters any market than she will learn her productivity and so the profit. The same procedure is valid for the second period. The entrepreneurs calculate their expected or exact profits and decide whether to exit, stay or enter any market. At the beginning of second period entrepreneur knows her productivity in the market which she entered in the previous period. In other words, if she entered the domestic market then she would know the productivity for the domestic market but not the productivity of exporting market. Therefore, she has to use the expected values of export market productivity. If she entered both domestic and exporting markets than she knows both domestic and export market productivity. So she can calculate the expected or exact profits in the second period and decide what to do. The entrepreneur's decision problem is summarized in Figure 2.1.

The entrepreneur is expected to earn nothing if she opts for staying out. This assumption is set to simplify the problem. In fact some entrepreneurs

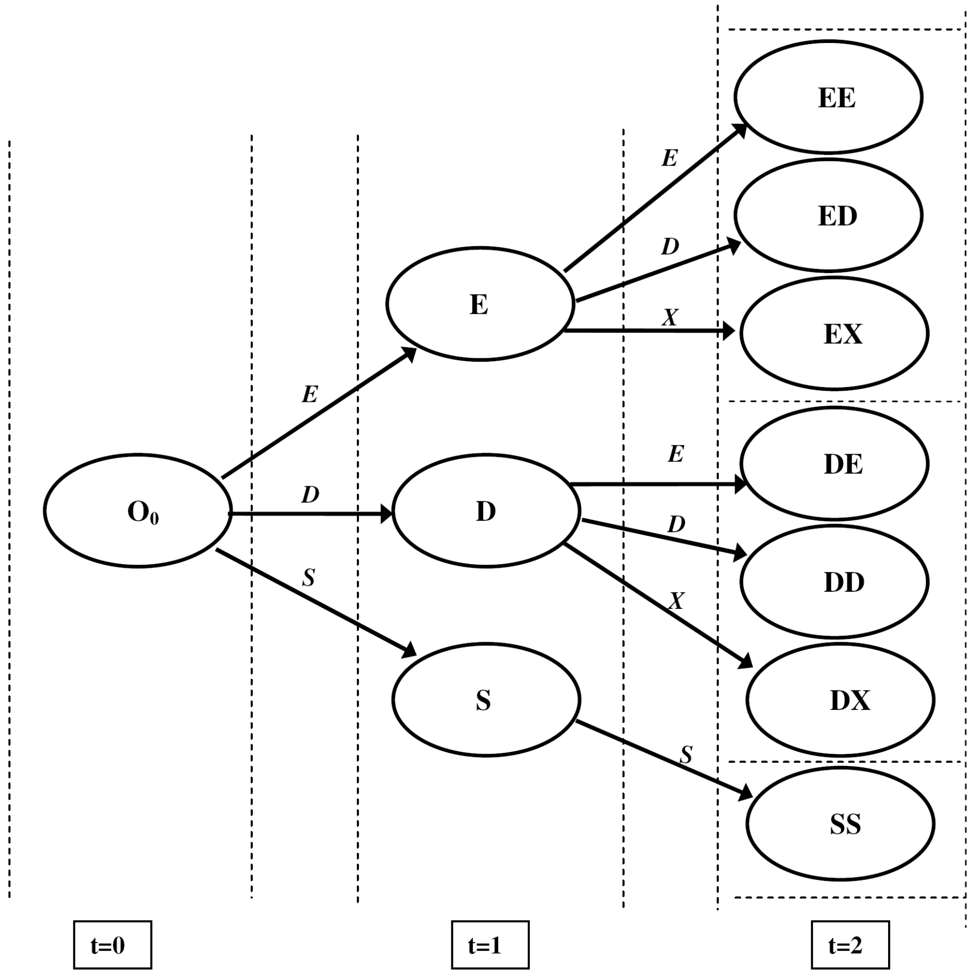


Figure 2.1: Entrepreneur's entry and exit decisions.

may own some amount of capital at the beginning and may gain rz interest payment (r is the interest rate and z is the initial capital amount). However, the inclusion of this gain in the problem will change only the thresholds that the profits should exceed. One would simply apply the threshold rz instead of zero for the necessary cases. In this study the profits should exceed zero which is the gain of staying out of markets and the firms are assumed to have no capital at the beginning and borrow all of the capital with the borrowing interest rate r.

If the entrepreneur i stays in the market for one period after investment, she will earn revenue from her output Q_i which is found according to the following production function with one input:

$$Q_i = A_i(k_i - k_m)^\alpha \qquad (2.1)$$

where A_i is the productivity, k_i is the amount of investment of firm i and α is a returns to scale parameter for the market ($0 \le \alpha \le 1$), and k_m is the minimum amount of capital to start production. If $k_i < k_m$ then there is no production ($Q_i = 0$). k_m is introduced to have variable returns to scale. Note that the returns to scale is the ratio of percentage change in investment to percentage change in production. In other words $RTS = \frac{\Delta Q/Q}{\Delta k/k}$.

$$\begin{aligned}
RTS &= \frac{\partial Q}{\partial k_i}\frac{k}{Q} && (2.2)\\
&= A_i\alpha(k_i - k_m)^{\alpha-1}\frac{k_i}{A_i(k_i - k_m)^\alpha} && (2.3)\\
&= \frac{k_i\alpha}{k_i - k_m} && (2.4)\\
& && (2.5)
\end{aligned}$$

If $k_m < k_i < \frac{k_m}{1-\alpha}$ then $RTS > 1$ (increasing returns to scale). If $k_i = \frac{k_m}{1-\alpha}$ then $RTS = 1$ (constant returns to scale). And if $k_i > \frac{k_m}{1-\alpha}$ then the production function has decreasing returns to scale ($RTS < 1$). If k_m is equal to zero, the production function exhibits decreasing returns to scale for all output levels. In Figure 2.2 two production functions are plotted. In the upper plot the red one is production function with $\alpha = 0.7$ and the blue one is with $\alpha = 0.3$ ($k_m = 10$, $A_i = 10$). As seen as alpha gets higher, the production function gets steeper and production amount increases faster as investment increases. The below plot shows the returns to scale property of the production functions. With higher α value the zone that production has increasing returns to scale extends.

The amount of productions in domestic market Q_i^d and export market Q_i^e will sum to total production Q_i. Therefore, we will relate these production amounts as

$$Q_i^e = x_i Q_i \qquad (2.6)$$

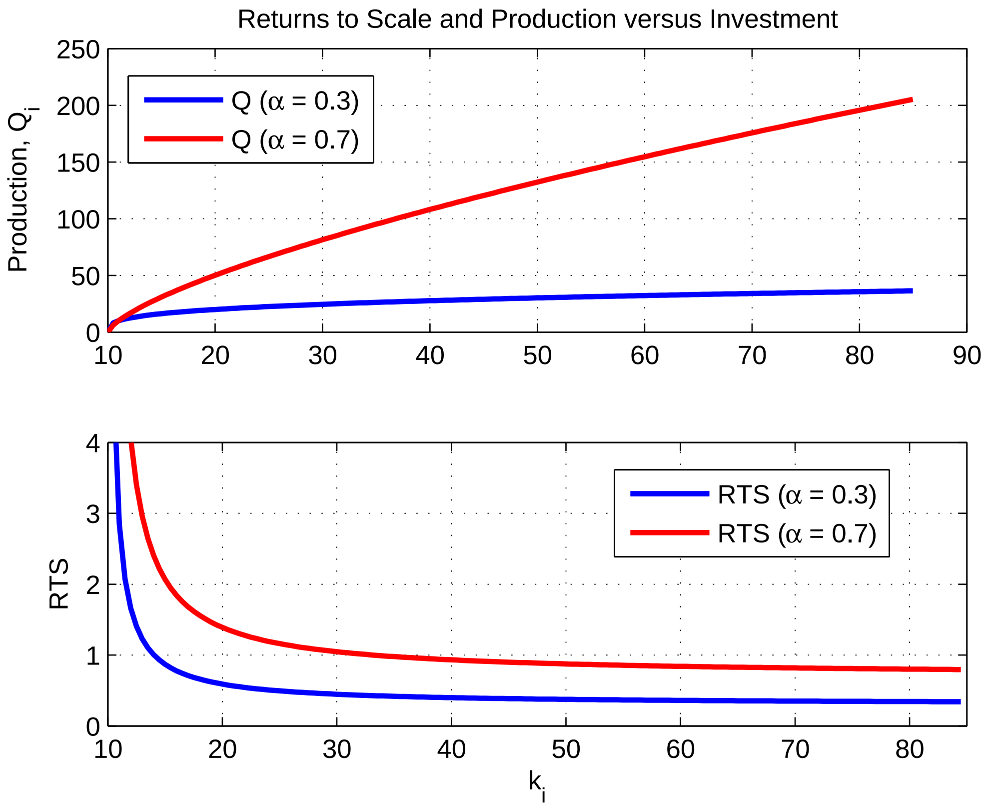


Figure 2.2: Production functions.

$$Q_i^d = (1 - x_i)Q_i \tag{2.7}$$

where x_i is the proportion of production of firm i in export market.

The demand functions for any firm, i in the domestic and export markets are the following.

$$P_i^d = B_i^d P^d (Q_i^d)^{-\epsilon_d} = B_i^d P^d [(1 - x_i)Q_i]^{-\epsilon_d} \tag{2.8}$$

$$P_i^e = B_i^e P^e (Q_i^e)^{-\epsilon_e} = B_i^e P^e (x_i Q_i)^{-\epsilon_e} \tag{2.9}$$

where P_i^d and P_i^e are the prices in domestic and export markets of firm i, respectively and P^d and P^e are the mean prices of domestic and export markets, respectively. B_i^d and B_i^e are the parameters that reflects the market size, number of firms, and product quality and differentiation in the domestic and export market, respectively. The parameter B_i^e also includes the exchange rate. The firms will utilize the expected values of these parameters when they do not know the exact values. We assume that there is no correlation between these parameters. In Figure 2.3, the demand functions for domestic and export markets are shown. The demand in export market is higher than the domestic market since the parameter B_i^e is higher in export market. The parameters ϵ_d and ϵ_e are related with the price elasticity of demand functions. In fact price elasticity of demand functions are $1/\epsilon_d$ and $1/\epsilon_e$ in domestic and export markets, respectively. Therefore, if $0 < \epsilon < 1$, then the demand function is price elastic and if $1 < \epsilon < \infty$ then the demand function has an inelastic price elasticity. In this study we will assume both markets are price elastic, i.e. $0 < \epsilon_d < 1$ and $0 < \epsilon_e < 1$.

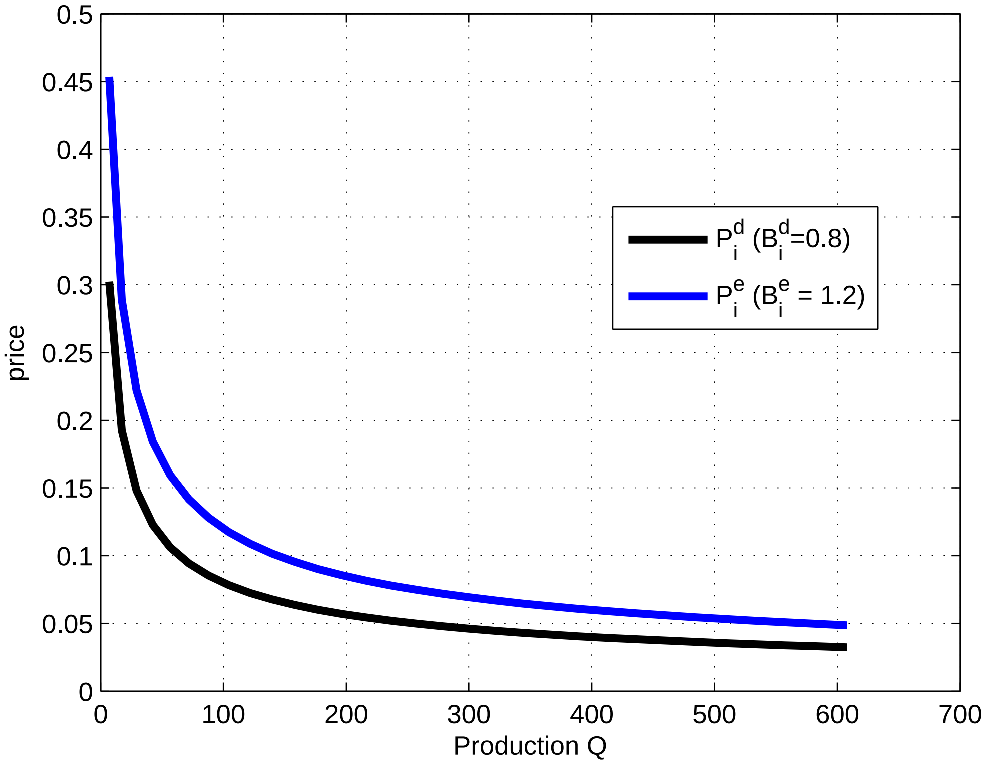


Figure 2.3: Demand functions.

The firms should meet the constant sunk costs of entering the domestic and export markets. F^d and F^e will be called as the sunk costs for the markets domestic and export, respectively. Usually, sunk cost to enter export market is higher than the sunk cost of domestic market ($F^e > F^d$) because establishment of an international distribution channel and/or to the adaptation of products to foreign standards result in additional sunk costs. A firm entering only domestic market for the first time will pay $F^d + k_i$ and a firm entering both domestic and export markets for the first time will pay $F^d + F^e + k_i$.

The firms exiting any market cannot recover the sunk costs. The investments k_i are consumed totally in a period. Therefore, any firm had the investment k_i cannot recover this investment at the end of the periods. Any firm starting a new period in a market should meet the new investment amount.

2.1 Profits of firms

In this section we will define the profits of firms for the cases given in Figure 4.1. At $t = 0$ the firms do not know their productivity and parameters B_i^d and B_i^e in the market. Therefore, each firm will utilize the expected values for these parameters and calculate their expected profits at each level.

Let's describe the profits at first state -from $t = 0$ to $t = 1$. There are three choices for the firms: S, D and E.

1. Stay out of markets (S):

   A firm staying out of the markets will not obtain any profit.

   $$\pi_S = 0 \tag{2.10}$$

2. Enter domestic market (D):

   If firm i enters only the domestic market the firm should guess the price at which its products will be demanded. Therefore, the firm will use the expected value of the price in domestic market, i.e. $E[P_i^d]$. In addition, the firm should pay for the sunk cost F^d since it is entering the domestic market for the first time. The firm also should pay profit maximizing investment k_i. Each firm will find the level of investment by solving the profit maximization problem. The expected profit of firm will become

$$E[\pi_D] = E[P_i^d]Q_i^d - (1+r)(k_i + F^d) \tag{2.11}$$

where expected price is a function of expected product quality, $E[B_i^d]$, the market price P^d, amount of production Q_i^d (will be calculated due to profit maximization) and price elasticity ϵ.

$$E[P_i^d] = E[B_i^d]P^d Q_i^{d-\epsilon_d} \tag{2.12}$$

3. Enter both domestic and export market (E):
   The firm i enters both the domestic and export markets. The firm does not know the productivity and product quality in both markets. Therefore, she will utilize the expected values in the profit calculations, i.e. $E[P_i^d]$ and $E[P_i^e]$. The firm should pay both sunk costs F^d and F^e to enter the markets and additionally the profit maximizing investment level. Therefore, the expected profit of firm will be the expected revenues from each market -$E[P_i^d]Q_i^d$ and $E[P_i^e]Q_i^e$- minus the investment and sunk costs.

$$E[\pi_E] = E[P_i^d]Q_i^d + E[P_i^e]Q_i^e - (1+r)(k_i + F^d + F^e) \tag{2.13}$$

where both expected prices are functions like

$$E[P_i^d] = E[B_i^d]P^d Q_i^{d-\epsilon_d} \tag{2.14}$$

$$E[P_i^e] = E[B_i^e]P^e Q_i^{e-\epsilon_e} \tag{2.15}$$

and the amount of productions in each market is determined by maximizing profit for x_i and k_i. Note that the production in domestic and export markets are related as defined in equation 2.6.

The firms at step $t = 0$ should also estimate their profits for the second period (from $t = 1$ to $t = 2$) to decide whether to enter any market. Therefore, they will consider the following cases.

1. Firm decided to stay out of markets (S) at $t = 0$: Then the only choice of the firms for the second period is again staying out of the markets. Thus, the profit of the firm for the second period is zero.

$$\pi_{SS} = 0 \tag{2.16}$$

2. Firm decided to enter domestic market at $t = 0$: The firm planing to enter domestic market will have the choices of (i) exiting domestic market, (ii) continue in domestic market, and (iii) entering export markets. The profits in these cases will be calculated with the estimations of productivity and product quality in the domestic market.

   (a) Firm exits the market (DX):
   Firm planing to enter the domestic market in the first period will exit the market in the second period. If the firm exits from domestic market, then the profit $\pi_{D,X}$ will become zero.

   $$\pi_{D,X}|_{t=0} = 0 \tag{2.17}$$

   (b) Firm continues in domestic market (DD):
   Firm planing to enter the domestic market in the first period will continue in the domestic market in the second period. If the firm continue with the domestic market then she will pay for the investment k_i and the expected profit at the end of the period will be

   $$E[\pi_{D,D}]|_{t=0} = E[P_i^d]Q_i^d - (1+r)k_i \tag{2.18}$$

   where the expected price is determined by the expected value of productivity, i.e. $E[B_i^d]$. The firm should determine the investment amount k_i by solving the profit maximization problem.

   (c) Firm enters export market (DE):
   Firm planing to enter the domestic market in the first period will continue in the domestic market and additionally enter the export

market in the second period. The firm should pay for the sunk cost F_e to enter export market. Then the expected profit will become

$$E[\pi_{D,E}]|_{t=0} = E[P_i^d]Q_i^d + E[P_i^e]Q_i^e - (1+r)(k_i + F_e) \tag{2.19}$$

3. Firm decided to enter both domestic and export markets at $t = 0$:
The firm planing to enter both domestic and export markets will have the choices of (i) exiting both markets, (ii) exiting export market and continue in domestic market, and (iii) continue in both markets. The profits in these cases will be calculated with the estimations of productivity and product quality in both markets.

    (a) Firm exits both markets (EX):
    Firm planing to enter both the domestic and export markets in the first period will exit both markets in the second period. Since firm exits both markets, it wont get any profit.

$$\pi_{E,X}|_{t=0} = 0 \tag{2.20}$$

    (b) Firm exits export market (stay in domestic market) (ED):
    Firm planing to enter both the domestic and export markets in the first period will exit export market and continue in the domestic market in the second period. The firm should pay only for the investment for second period. The profit of the firm staying only in domestic market will depend on its sales in domestic market.

$$E[\pi_{E,D}]|_{t=0} = E[P_i^d]Q_i^d - (1+r)k_i \tag{2.21}$$

    (c) Firm continues in both markets (EE):
    Firm planing to enter both the domestic and export markets in the first period will continue in both markets in the second period. The firm should pay only for the investment for second period. If the firm

decide to stay in both markets then the profit of it will depend on the sales in both markets.

$$E[\pi_{E,E}]|_{t=0} = E[P_i^d]Q_i^d + E[P_i^e]Q_i^e - (1+r)k_i \tag{2.22}$$

After the firm decided to enter any of the markets at $t = 0$, it will face with the actual values of productivity and B_i^e and B_i^d in the markets it entered. Therefore, the firm learning these parameters will recalculate its expected profits for the second period with these parameters. The expected profits in the second period for the firms survived for the first period will be as the following.

1. Firm entered domestic market at $t = 0$:
   The firm entered domestic market knows its productivity and product quality in domestic market B_i^d. The firm will have the choices of (i) exiting domestic market, (ii) continuing in domestic market, and (iii) entering export markets. The profits in these cases will be calculated with the estimations of productivity and product quality in the export market and the exact values of these parameters in the domestic market.

   (a) Firm exit the market (DX):
   Firm entered the domestic market in the first period will exit the market in the second period. If the firm exits from domestic market, then the profit $\pi_{D,X}$ will become zero.

   $$\pi_{D,X}|_{t=1} = 0 \tag{2.23}$$

   (b) Firm continues in domestic market (DD):
   Firm entered the domestic market in the first period will continue in the domestic market in the second period. The firm should pay only for the investment for second period. If the firm continue with the domestic market then she will pay for the investment k_i. The profit at the end of the second period will be

   $$\pi_{D,D}|_{t=1} = P_i^d Q_i^d - (1+r)k_i \tag{2.24}$$

where price P_i^d is known exactly.

(c) Firm enters export market (DE):

Firm entered the domestic market in the first period will continue in the domestic market and additionally enter the export market in the second period. The firm should pay for the sunk cost F_e to enter export market and the investment amount k_i. The firm knows the price of its products in domestic market but not in export market. Then the expected profit will become

$$E[\pi_{D,E}]|_{t=1} = P_i^d Q_i^d + E[P_i^e] Q_i^e - (1+r)(k_i + F_e) \tag{2.25}$$

2. Firm entered both domestic and export markets at $t = 0$:

The firm entered both domestic and export markets knows both the productivity and product quality in both markets. The firms will have the choices of (i) exiting both markets, (ii) exiting export market and continue in domestic market, and (iii) continuing in both markets. The profits in these cases will be calculated with the exact values of productivity and product quality in both markets.

(a) Firm exits both markets (EX):

Firm entered both the domestic and export markets in the first period will exit both markets in the second period. Since firm exits both markets, it won't get any profit.

$$\pi_{E,X}|_{t=1} = 0 \tag{2.26}$$

(b) Firm exits export market (stays in domestic market) (ED):

Firm entered both the domestic and export markets in the first period will exit export market and continue in the domestic market in the second period. The firm should pay only for the investment for second period. The profit of the firm staying only in domestic market will depend on its sales in domestic market.

$$\pi_{E,D}|_{t=1} = P_i^d Q_i^d - (1+r)k_i \tag{2.27}$$

(c) Firm continues in both markets (EE):

Firm entered both the domestic and export markets in the first period will continue in both markets in the second period. The firm should pay only for the investment for second period. If the firm decide to stay in both markets then the profit of it will depend on the sales in both markets where the firm knows both productivity and product qualities in both market.

$$\pi_{E,E}|_{t=1} = P_i^d Q_i^d + P_i^e Q_i^e - (1+r)k_i \tag{2.28}$$

2.2 The decision process of firms

The firms in this model are as usual profit maximizing firms. Therefore, the firms will calculate their expected profits at the beginning of each period and decide to enter, stay or exit the markets according to the profit maximizing results. There are two decision points. One is at the beginning of first period (at $t = 0$) and second is at the beginning of second period (at $t = 1$). The firm decide its actions according to the profits calculated at these points.

2.2.1 Decision at $t = 0$

Each firm estimates its profits at both periods and take the action of maximizing profit. There are seven possible actions for the firm at this point:

- SS: S at $t = 0$ and S at $t = 1$

$$E[\pi_{S,S}] = (1+r)E[\pi_S]_{t=0} + E[\pi_{S,S}]_{t=0} \tag{2.29}$$

- DX: D at $t = 0$ and X at $t = 1$

$$E[\pi_{D,X}] = (1+r)E[\pi_D]_{t=0} + E[\pi_{D,X}]_{t=0} \tag{2.30}$$

- DD: D at $t = 0$ and D at $t = 1$

$$E[\pi_{D,D}] = (1+r)E[\pi_D]_{t=0} + E[\pi_{D,D}]_{t=0} \tag{2.31}$$

- DE: D at $t = 0$ and E at $t = 1$

$$E[\pi_{D,E}] = (1+r)E[\pi_D]_{t=0} + E[\pi_{D,E}]_{t=0} \tag{2.32}$$

- EX: E at $t = 0$ and X at $t = 1$

$$E[\pi_{E,X}] = (1+r)E[\pi_E]_{t=0} + E[\pi_{E,X}]_{t=0} \tag{2.33}$$

- ED: E at $t = 0$ and D at $t = 1$

$$E[\pi_{E,D}] = (1+r)E[\pi_E]_{t=0} + E[\pi_{E,D}]_{t=0} \tag{2.34}$$

- EE: E at $t = 0$ and E at $t = 1$

$$E[\pi_{E,E}] = (1+r)E[\pi_E]_{t=0} + E[\pi_{E,E}]_{t=0} \tag{2.35}$$

The firm will calculate these expected profits and select the maximum of them. Therefore, the decision at $t = 0$ S, D or E will be determined with respect to the maximum of the seven profits above.

If maximum profit is $E[\pi_{S,S}]$, then action is S.

If maximum profit is any of $\{E[\pi_{D,X}], E[\pi_{D,D}], E[\pi_{D,E}]\}$, then action is D.

If maximum profit is any of $\{E[\pi_{E,X}], E[\pi_{E,D}], E[\pi_{E,E}]\}$, then action is E.

2.2.2 Decision at $t = 1$

The firms are at $t = 1$ they will decide what to do in the second period. They will estimate their profits for the second period and chose the maximum one. Note that some firms will know some of the parameters (productivity and product quality) in the markets they entered in the first period.

For the firm stayed out of markets in the first period will have only one choice which is again staying out of market.

- if S at $t = 0$ then
  - S at $t = 1$

$$E[\pi_{S,S}] = E[\pi_{S,S}]_{t=1} \tag{2.36}$$

If any firm has entered the domestic market then it will have the choices: (i) to exit the market, (ii) to stay in the market, and (iii) to enter the export market while staying in domestic market.

- if D at $t = 0$ then
  - X at $t = 1$

$$E[\pi_{D,X}] = E[\pi_{D,X}]_{t=1} \tag{2.37}$$

  - D at $t = 1$

$$E[\pi_{D,D}] = E[\pi_{D,D}]_{t=1} \tag{2.38}$$

  - E at $t = 1$

$$E[\pi_{D,E}] = E[\pi_{D,E}]_{t=1} \tag{2.39}$$

If the firm has entered both domestic and export markets then it will have the choices: (i) to exit both markets, (ii) to stay in the domestic market and exit export market, and (iii) to continue in both export and domestic markets.

- if E at $t = 0$ then
  - X at $t = 1$

$$E[\pi_{E,X}] = E[\pi_{E,X}]_{t=1} \tag{2.40}$$

- D at $t = 1$

$$E[\pi_{E,D}] = E[\pi_{E,D}]_{t=1} \tag{2.41}$$

- E at $t = 1$

$$E[\pi_{E,E}] = E[\pi_{E,E}]_{t=1} \tag{2.42}$$

The firms will calculate the expected profits according to the actions they took in the previous period and will decide on the action that will maximize their profit.

- For the firm had action S in the first period: action is SS.
- For the firm had action D in the first period:
  - If maximum profit is $E[\pi_{D,X}]$, then action is DX.
  - If maximum profit is $E[\pi_{D,D}]$, then action is DD.
  - If maximum profit is $E[\pi_{D,E}]$, then action is DE.
- For the firm had action E in the first period:
  - If maximum profit is $E[\pi_{E,X}]$, then action is EX.
  - If maximum profit is $E[\pi_{E,D}]$, then action is ED.
  - If maximum profit is $E[\pi_{E,E}]$, then action is EE.

2.3 Profit Maximization

The firms calculating their profits will always try to find the maximum possible profit. So they have to optimize their investment level and share of domestic and export markets to maximize the profit. The firms are uncertain about A_i

and B_i^d and B_i^e before entering the markets. As soon as they enter one of the markets they face with the exact values of these parameters. So before entering the market the firms will utilize the expected values of the unknown parameters and will utilize the exact values as they enter the markets. In both cases the firm should maximize the profit with the taken values of these parameters. Therefore, in this section we will take the values of these parameters as known but one may use the expected values instead of exact values of these parameters. There will be no change in the profit maximization procedure for both cases.

2.3.1 Profit maximization in domestic market

If firm i enters only the domestic market, the expected profit of firm will become

$$E[\pi_D] = P_i^d Q_i^d - (1+r)(k_i + F^d) \tag{2.43}$$

where the investment amount k_i is found by solving the problem of maximization of π_D^0. Note that we assume the value of P_i^d to be known exactly ($E[\mathbf{P}_i^d] = P_i^d$).

Substituting equation 2.8 into 2.43

$$\begin{aligned} \pi_D &= \left(B_i^d P^d Q_i^{d-\epsilon_d}\right) Q_i^d - (1+r)(k_i + F^d) \\ &= \left(B_i^d P^d Q_i^{d1-\epsilon_d}\right) - (1+r)(k_i + F^d) \end{aligned} \tag{2.44}$$

Substituting equation 2.1 into 2.44

$$\begin{aligned} \pi_D &= \left(B_i^d P^d (A_i (k_i - k_m)^\alpha)^{1-\epsilon_d}\right) - (1+r)(k_i + F^d) \\ &= B_i^d P^d A_i^{1-\epsilon_d} (k_i - k_m)^{\alpha(1-\epsilon_d)} - (1+r)(k_i + F^d) \end{aligned} \tag{2.45}$$

Now π_D can be maximized with respect to k_i. The first derivative of π_D with respect to k_i is:

$$\frac{d\pi_D}{dk_i} = (\alpha(1-\epsilon_d)) B_i^d P^d A_i^{1-\epsilon_d} (k_i - k_m)^{\alpha(1-\epsilon_d)-1} - (1+r) \tag{2.46}$$

The solution of $\frac{d\pi_D}{dk_i} = 0$ will give the k_i^{Dmax} value maximizing domestic profit. Then k_i^{Dmax} is found as:

$$k_i^{Dmax} = \left[\frac{1+r}{(\alpha(1-\epsilon_d)) B_i^d P^d A_i^{1-\epsilon_d}}\right]^{\frac{1}{\alpha - \alpha\epsilon_d - 1}} + k_m \tag{2.47}$$

To make sure the first order condition gives the maximizing investment amount one should apply the second derivative test.

$$\frac{d^2\pi_D}{dk_i^2} = [\alpha(1-\epsilon_d))] [\alpha(1-\epsilon_d)-1] B_i^d P^d A_i^{1-\epsilon_d} (k_i^{Dmax} - k_m)^{\alpha(1-\epsilon_d)-2} \quad (2.48)$$

To have a maximum the second partial derivative should be negative. The sufficient condition for the negative second partial derivative is $(1-\epsilon_d)[\alpha(1-\epsilon_d)-1] < 0$. If $1-\epsilon_d < 0$ or $\epsilon_d > 1$ (inelastic domestic demand) then $\alpha(1-\epsilon_d)-1 < 0$ since $\alpha > 0$ and therefore, $(1-\epsilon_d)[\alpha(1-\epsilon_d)-1] > 0$ which means the domestic profit π_D^0 does not have any maximum. One may observe that $\frac{\partial \pi_D^0}{\partial k_i}$ is always negative for this case. Therefore the profit maximizing investment will be zero. In other words firm will not enter the domestic market. On the other hand, if $\epsilon_d < 1$ (price elastic demand) and α satisfies $\alpha(1-\epsilon_d)-1 < 0$ then $\frac{\partial^2 \pi_D^0}{\partial k_i^2}$ becomes negative and a profit maximizing k_i value may exist. The firm would find this point and stay (or enter) the domestic market. And the corresponding maximum profit is (detailed derivation is in Appendix A.1)

$$\begin{aligned} \pi_D^{max} &= \left[\frac{B_i^d P^d A_i^{1-\epsilon_d}}{(1+r)^{\alpha(1-\epsilon_d)}}\right]^{\frac{1}{1-\alpha(1-\epsilon_d)}} \left\{[\alpha(1-\epsilon_d)]^{\frac{\alpha(1-\epsilon_d)}{1-\alpha(1-\epsilon_d)}} - [\alpha(1-\epsilon_d)]^{\frac{1}{1-\alpha(1-\epsilon_d)}}\right\} \\ &\quad -(1+r)k_m - (1+r)F^d \end{aligned}$$

Note that in this maximum profit equation the positiveness of the term

$$\left\{[\alpha(1-\epsilon_d)]^{\frac{\alpha(1-\epsilon_d)}{1-\alpha(1-\epsilon_d)}} - [\alpha(1-\epsilon_d)]^{\frac{1}{1-\alpha(1-\epsilon_d)}}\right\}$$

is a necessary condition to have a positive maximum profit. If this term is negative then the profit is definitely negative. One may show that the conditions $\epsilon_d < 1$ and $\alpha(1-\epsilon_d) < 1$ satisfy this term to be positive. Therefore, necessary but not sufficient conditions for the maximum profit in domestic market being positive are $\epsilon_d < 1$ and $\alpha(1-\epsilon_d) < 1$. If the level of threshold investment k_m and entry sunk cost F^d are sufficiently small to have a positive profit, then the firm will opt to produce in domestic market.

If the P_i^d function is not known exactly and the expected value of it is used in maximizing the profit ($\mathbf{P}_i^d = E[P_i^d] = E[B_i^d] P^d Q_i^{d-\epsilon_d}$), then the equation 2.47

becomes

$$E[k_i^{Dmax}] = \left[\frac{1+r}{(\alpha(1-\epsilon_d))E[B_i^d]P^d A_i^{1-\epsilon_d}}\right]^{\frac{1}{\alpha-\alpha\epsilon_d-1}} + k_m \tag{2.49}$$

Three typical profit functions are shown in Figure 2.4. Profit is plotted versus investment k_i for three different productivity levels: A_1, A_2, and A_3 where $A_1 < A_2 < A_3$ ($\alpha_1 < \frac{1}{(1-\epsilon_d)}$). The firms should find the profit maximizing k_i^{Dmax} value by utilizing equation 2.47. Note that as the productivity of the firm increases the profit maximizing k_i^{Dmax} also increases as seen in Figure 2.5.

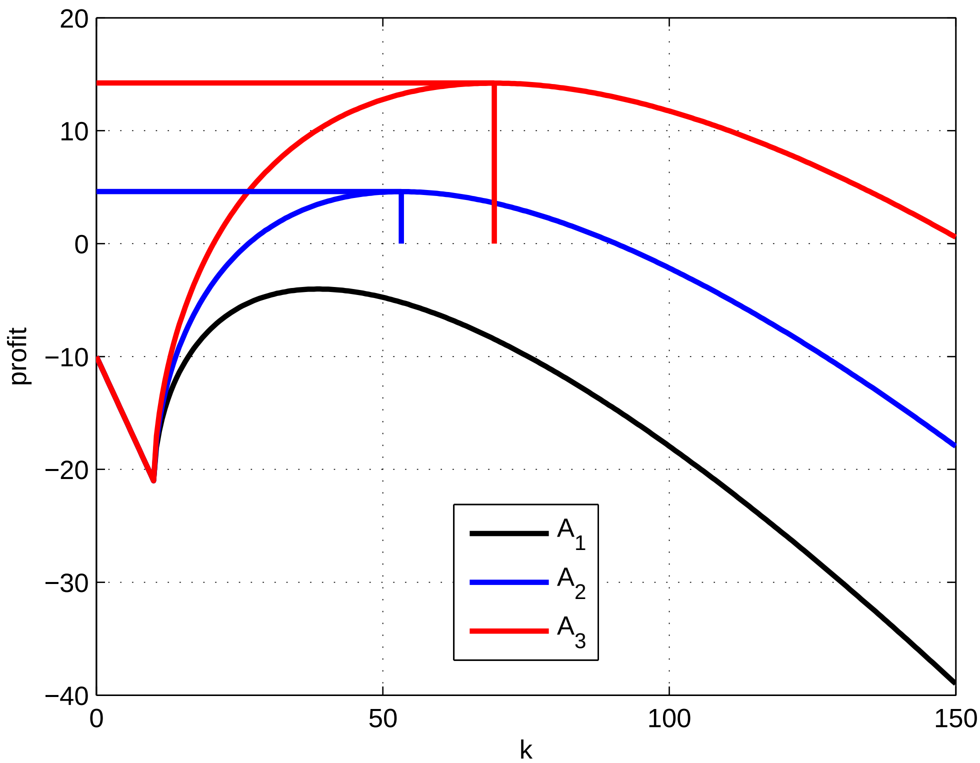


Figure 2.4: Profit versus investment for different levels of productivity.

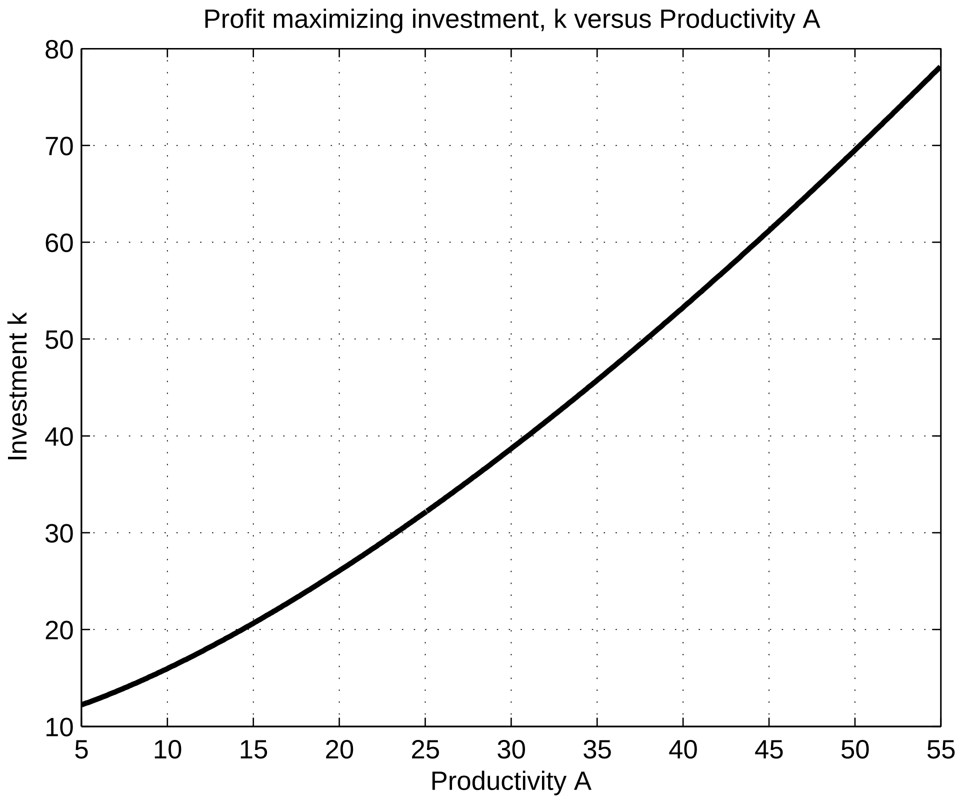


Figure 2.5: Profit maximizing investment k_i^{max} versus productivity A_i in domestic market.

2.3.2 Profit maximization in both domestic and export market

If firm i enters both the domestic and export markets, the profit of firm will become

$$\pi_E = P_i^d Q_i^d + P_i^e Q_i^e - (1+r)(k_i + F^d + F^e) \tag{2.50}$$

where the profit maximizing investment amount k_i^{Emax} is found by solving the problem of maximization of π_E. Recall that, again we assumed that the values of P_i^d and P_i^e are known exactly ($E[\mathbf{P}_i^d] = P_i^d$ and $E[\mathbf{P}_i^e] = P_i^e$).

Substituting equations 2.6, 2.7, 2.8, and 2.9 into 2.50

$$\begin{aligned}\pi_E &= B_i^d P^d((1-x_i)Q_i)^{-\epsilon_d}(1-x_i)Q_i + B_i^e P^e(x_i Q_i)^{-\epsilon_e} x_i Q_i \\ &\quad -(1+r)(k_i + F^d + F^e)\end{aligned}$$

$$= B_i^d P^d((1-x_i)Q_i)^{1-\epsilon_d} + B_i^e P^e(x_i Q_i)^{1-\epsilon_e} - (1+r)(k_i + F^d + F^e) \tag{2.51}$$

Substituting equation 2.1 into 2.51

$$\begin{aligned} \pi_E &= B_i^d P^d((1-x_i)A_i(k_i-k_m)^{\alpha})^{1-\epsilon_d} \\ &+ B_i^e P^e(x_i A_i(k_i-k_m)^{\alpha})^{1-\epsilon_e} \\ &-(1+r)(k_i+F^d+F^e) \end{aligned} \tag{2.52}$$

Before finding the maximum profit, we should show the concavity of π_E. The proof of the concavity of π_E is shown in Appendix A.2. We concluded that if $\alpha < \frac{\epsilon_d}{1-\epsilon_d}$ and $\alpha < \frac{\epsilon_e}{1-\epsilon_e}$ then the profit function is concave (and has a global maximum). If any of the conditions, $\alpha < \frac{\epsilon_d}{1-\epsilon_d}$ and $\alpha < \frac{\epsilon_e}{1-\epsilon_e}$, is not valid then the profit function π_E is indefinite. The examples for the concave and not-concave profit functions are shown in Figure A.1

Now we can continue with the maximization of π_E function. Note that we will assume $\alpha < \frac{\epsilon_d}{1-\epsilon_d}$ and $\alpha < \frac{\epsilon_e}{1-\epsilon_e}$ so that the profit function has a maximum. In the equation 2.51 to maximize profit one should find the optimum x_i and k_i values under the constraints $0 \leq x_i \leq 1$ and $k_i \geq k_m$. Therefore, the problem may be summarized as an optimization problem

$$\begin{aligned} max_{x_i,k_i} \quad & B_i^d P^d((1-x_i)A_i(k_i-k_m)^{\alpha})^{1-\epsilon_d} \\ & + B_i^e P^e(x_i A_i(k_i-k_m)^{\alpha})^{1-\epsilon_e} - (1+r)(k_i+F^d+F^e) \\ subject\ \ to \quad & x_i - 1 \leq 0 \ and\ -x_i \leq 0 \ and\ -k_i + k_m \leq 0 \end{aligned} \tag{2.53}$$

Using Lagrangian methods Sydsaeter & Hammond (1994), we can define the Lagrangian function as

$$\begin{aligned} \mathcal{L}(x_i,k_i) &= B_i^d P^d((1-x_i)A_i(k_i-k_m)^{\alpha})^{1-\epsilon_d} \\ &+ B_i^e P^e(x_i A_i(k_i-k_m)^{\alpha})^{1-\epsilon_e} - (1+r)k_i - F^d - F^e \\ &- \lambda_1 (x_i - 1) - \lambda_2 (-x_i) - \lambda_3 (-k_i + k_m) \end{aligned} \tag{2.54}$$

Equating the partial derivatives of $\mathcal{L}(x_i, k_i)$ to zero

$$\begin{aligned}\frac{\partial \mathcal{L}(x_i,k_i)}{\partial x_i} &= -B_i^d P^d A_i^{1-\epsilon_d}(k_i-k_m)^{\alpha(1-\epsilon_d)}(1-\epsilon_d)(1-x_i)^{-\epsilon_d} \\ &\quad +B_i^e P^e A_i^{1-\epsilon_e}(k_i-k_m)^{\alpha(1-\epsilon_e)}(1-\epsilon_e)x_i^{-\epsilon_e} \\ &\quad -\lambda_1+\lambda_2 \qquad (2.55)\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathcal{L}(x_i,k_i)}{\partial k_i} &= B_i^d P^d A_i^{1-\epsilon_d}(1-x_i)^{1-\epsilon_d}\alpha(1-\epsilon_d)(k_i-k_m)^{\alpha(1-\epsilon_d)-1} \\ &\quad +B_i^e P^e A_i^{1-\epsilon_e}x_i^{1-\epsilon_e}\alpha(1-\epsilon_e)(k_i-k_m)^{\alpha(1-\epsilon_e)-1} \\ &\quad -(1+r)+\lambda_3 \qquad (2.56)\end{aligned}$$

The Kuhn-Tucker conditions are

$$\frac{\partial \mathcal{L}(x_i,k_i)}{\partial x_i} = 0 \qquad (2.57)$$

$$\frac{\partial \mathcal{L}(x_i,k_i)}{\partial k_i} = 0 \qquad (2.58)$$

$$\lambda_1 \geq 0, \quad \lambda_1(1-x_i)=0 \qquad (2.59)$$

$$\lambda_2 \geq 0, \quad \lambda_2 x_i=0 \qquad (2.60)$$

$$\lambda_3 \geq 0, \quad \lambda_3(-k_i+k_m)=0 \qquad (2.61)$$

The solution of equations 2.57, 2.58, 2.59, 2.60, 2.61 gives the optimal x_i and k_i values maximizing π_E^0.

To simplify the lagrangian equations let us call the constants

$$\begin{aligned}\Theta^d &= B_i^d P^d A_i^{1-\epsilon_d} \\ \Theta^e &= B_i^e P^e A_i^{1-\epsilon_e}\end{aligned}$$

Then, the lagrangian functions 2.55 and 2.56 becomes

$$\begin{aligned}\frac{\partial \mathcal{L}(x_i,k_i)}{\partial x_i} &= -\Theta^d(k_i-k_m)^{\alpha(1-\epsilon_d)}(1-\epsilon_d)(1-x_i)^{-\epsilon_d} \\ &\quad +\Theta^e(k_i-k_m)^{\alpha(1-\epsilon_e)}(1-\epsilon_e)x_i^{-\epsilon_e} \\ &\quad -\lambda_1+\lambda_2 \qquad (2.62)\end{aligned}$$

$$\frac{\partial \mathcal{L}(x_i,k_i)}{\partial k_i} = \Theta^d(1-x_i)^{1-\epsilon_d}\alpha(1-\epsilon_d)(k_i-k_m)^{\alpha(1-\epsilon_d)-1}$$

$$+\Theta^e x_i^{1-\epsilon_e}\alpha(1-\epsilon_e)(k_i-k_m)^{\alpha(1-\epsilon_e)-1}$$
$$-(1+r)+\lambda_3 \tag{2.63}$$

This equation set will be solved for 4 different cases.

Case 1: $0 < x_i < 1$ **and** $k_i > k_m$

The constraints are satisfied therefore from equations 2.59, 2.60, 2.61 the lagrangian constants are found as $\lambda_1 = 0$, $\lambda_2 = 0$, and $\lambda_3 = 0$. Therefore, equations 2.62 and 2.63 becomes

$$\begin{aligned}\frac{\partial\mathcal{L}(x_i,k_i)}{\partial x_i} &= -\Theta^d(k_i-k_m)^{\alpha(1-\epsilon_d)}(1-\epsilon_d)(1-x_i)^{-\epsilon_d}\\ &\quad +\Theta^e(k_i-k_m)^{\alpha(1-\epsilon_e)}(1-\epsilon_e)x_i^{-\epsilon_e}\\ &= 0\end{aligned} \tag{2.64}$$

$$\begin{aligned}\frac{\partial\mathcal{L}(x_i,k_i)}{\partial k_i} &= \Theta^d(1-x_i)^{1-\epsilon_d}\alpha(1-\epsilon_d)(k_i-k_m)^{\alpha(1-\epsilon_d)-1}\\ &\quad +\Theta^e x_i^{1-\epsilon_e}\alpha(1-\epsilon_e)(k_i-k_m)^{\alpha(1-\epsilon_e)-1}\\ &\quad -(1+r)\\ &= 0\end{aligned} \tag{2.65}$$

Rearranging 2.64

$$\Theta^d(k_i-k_m)^{\alpha(1-\epsilon_d)}(1-\epsilon_d)(1-x_i)^{-\epsilon_d} = \Theta^e(k_i-k_m)^{\alpha(1-\epsilon_e)}(1-\epsilon_e)x_i^{-\epsilon_e} \tag{2.66}$$

Rearranging 2.65

$$\begin{aligned}1+r &= \Theta^d(1-x_i)^{1-\epsilon_d}\alpha(1-\epsilon_d)(k_i-k_m)^{\alpha(1-\epsilon_d)-1}\\ &\quad +\Theta^e x_i^{1-\epsilon_e}\alpha(1-\epsilon_e)(k_i-k_m)^{\alpha(1-\epsilon_e)-1}\\ &= \Theta^d(k_i-k_m)^{\alpha(1-\epsilon_d)}(1-\epsilon_d)(1-x_i)^{-\epsilon_d}\alpha(1-x_i)(k_i-k_m)^{-1}\\ &\quad +\Theta^e(k_i-k_m)^{\alpha(1-\epsilon_e)}(1-\epsilon_e)x_i^{-\epsilon_e}\alpha x_i(k_i-k_m)^{-1}\end{aligned} \tag{2.67}$$

Substituting equation 2.66 in 2.67

$$\begin{aligned} 1+r &= \Theta^e (k_i - k_m)^{\alpha(1-\epsilon_e)} (1-\epsilon_e) x_i^{-\epsilon_e} \alpha (k_i - k_m)^{-1} \left[(1-x_i) + x_i\right] \\ &= \Theta^e (k_i - k_m)^{\alpha(1-\epsilon_e)-1} (1-\epsilon_e) x_i^{-\epsilon_e} \alpha \end{aligned} \quad (2.68)$$

or

$$\begin{aligned} 1+r &= \Theta^d (k_i - k_m)^{\alpha(1-\epsilon_d)} (1-\epsilon_d)(1-x_i)^{-\epsilon_d} \alpha (k_i - k_m)^{-1} \left[(1-x_i) + x_i\right] \\ &= \Theta^d (k_i - k_m)^{\alpha(1-\epsilon_d)-1} (1-\epsilon_d)(1-x_i)^{-\epsilon_d} \alpha \end{aligned} \quad (2.69)$$

Solving equations 2.68 and 2.69 for x_i in terms of k_i and constants will yield

$$x_i = \left[\frac{\Theta^e (k_i - k_m)^{\alpha(1-\epsilon_e)-1} (1-\epsilon_e)\alpha}{1+r}\right]^{\frac{1}{\epsilon_e}} \quad (2.70)$$

or

$$x_i = 1 - \left[\frac{\Theta^d (k_i - k_m)^{\alpha(1-\epsilon_d)-1} (1-\epsilon_d)\alpha}{1+r}\right]^{\frac{1}{\epsilon_d}} \quad (2.71)$$

Again simplifying the constants as

$$\begin{aligned} \Omega^e &= \frac{\Theta^e (1-\epsilon_e)\alpha}{1+r} = \frac{B_i^e P^e A_i^{1-\epsilon_e} (1-\epsilon_e)\alpha}{1+r} \\ \Omega^d &= \frac{\Theta^d (1-\epsilon_d)\alpha}{1+r} = \frac{B_i^d P^d A_i^{1-\epsilon_d} (1-\epsilon_d)\alpha}{1+r} \end{aligned}$$

And using equations 2.70 and 2.71 we can write the implicit function of profit maximizing k_i^{Emax} value.

$$\left[\Omega^e\right]^{\frac{1}{\epsilon_e}} \left(k_i^{Emax} - k_m\right)^{\frac{\alpha(1-\epsilon_e)-1}{\epsilon_e}} + \left[\Omega^d\right]^{\frac{1}{\epsilon_d}} \left(k_i^{Emax} - k_m\right)^{\frac{\alpha(1-\epsilon_d)-1}{\epsilon_d}} = 1 \quad (2.72)$$

Equation 2.72 seems to be the simplest form of profit maximizing k_i^{Emax} value. However, further analytical solution of k_i^{Emax} is not possible. Therefore, one may solve for the k_i^{Emax} by numerical methods.

To find the profit maximizing export market share x_i^{Emax}, lets again utilize the equations 2.68 and 2.69 to eliminate k_i in the equations.

$$k_i = \left[\frac{\Theta^e (1-\epsilon_e)\alpha}{1+r}\right]^{\frac{1}{1-\alpha(1-\epsilon_e)}} x_i^{\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1}} + k_m \quad (2.73)$$

$$k_i = \left[\frac{\Theta^d (1-\epsilon_d)\alpha}{1+r}\right]^{\frac{1}{1-\alpha(1-\epsilon_d)}} (1-x_i)^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}} + k_m \quad (2.74)$$

or

$$k_i = \left[\Omega^e\right]^{\frac{1}{1-\alpha(1-\epsilon_e)}} x_i^{\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1}} + k_m \quad (2.75)$$

$$k_i = \left[\Omega^d\right]^{\frac{1}{1-\alpha(1-\epsilon_d)}} (1-x_i)^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}} + k_m \quad (2.76)$$

Combining equations 2.75 and 2.76

$$\left[\Omega^e\right]^{\frac{1}{1-\alpha(1-\epsilon_e)}} (x_i^{Emax})^{\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1}} = \left[\Omega^d\right]^{\frac{1}{1-\alpha(1-\epsilon_d)}} (1-x_i^{Emax})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}} \quad (2.77)$$

which is again a very hard to solve problem. We will utilize numerical methods to solve this problem.

The maximum profit in terms of the optimal values becomes

$$\begin{aligned}\pi_E^{max} &= B_i^d P^d \left[(1-x_i^{Emax})A_i(k_i^{Emax}-k_m)^\alpha\right]^{1-\epsilon_d} \\ &+B_i^e P^e \left[x_i^{Emax}A_i(k_i^{Emax}-k_m)^\alpha\right]^{1-\epsilon_e} \\ &-(1+r)(k_i^{Emax}+F^d+F^e) \quad (2.78)\end{aligned}$$

After finding the optimal values of k_i and x_i one can evaluate the maximum profit π_E^{max}. Furthermore, we can write π_E^{max} in terms of k_i^{Emax} by eliminating x_i^{Emax} in the equation 2.78. Rearranging equations 2.70 and 2.71

$$x_i^{Emax} = \left[\frac{B_i^e P^e A_i^{1-\epsilon_e}(1-\epsilon_e)\alpha}{1+r}\right]^{\frac{1}{\epsilon_e}} (k_i^{Emax}-k_m)^{\frac{\alpha(1-\epsilon_e)-1}{\epsilon_e}} \quad (2.79)$$

$$1-x_i^{Emax} = \left[\frac{B_i^d P^d A_i^{1-\epsilon_d}(1-\epsilon_d)\alpha}{1+r}\right]^{\frac{1}{\epsilon_d}} (k_i^{Emax}-k_m)^{\frac{\alpha(1-\epsilon_d)-1}{\epsilon_d}} \quad (2.80)$$

Substituting 2.79 and 2.80 into 2.78

$$\begin{aligned}\pi_E^{max} &= B_i^d P^d \left\{\left[\frac{B_i^d P^d A_i^{1-\epsilon_d}(1-\epsilon_d)\alpha}{1+r}\right]^{\frac{1}{\epsilon_d}} (k_i^{Emax}-k_m)^{\frac{\alpha(1-\epsilon_d)-1}{\epsilon_d}}\right\}^{1-\epsilon_d} \\ &\quad A_i^{1-\epsilon_d}(k_i^{Emax}-k_m)^{\alpha(1-\epsilon_d)} \\ &\quad +B_i^e P^e \left\{\left[\frac{B_i^e P^e A_i^{1-\epsilon_e}(1-\epsilon_e)\alpha}{1+r}\right]^{\frac{1}{\epsilon_e}} (k_i^{Emax}-k_m)^{\frac{\alpha(1-\epsilon_e)-1}{\epsilon_e}}\right\}^{1-\epsilon_e} \\ &\quad A_i^{1-\epsilon_e}(k_i^{Emax}-k_m)^{\alpha(1-\epsilon_e)} \\ &\quad -(1+r)(k_i^{Emax}+F^d+F^e)\end{aligned}$$

$$
\begin{aligned}
= \quad & \left[B_i^d P^d A_i^{1-\epsilon_d} \left[\frac{\alpha(1-\epsilon_d)}{1+r} \right]^{1-\epsilon_d} \right]^{\frac{1}{\epsilon_d}} (k_i^{Emax} - k_m)^{\frac{(1-\epsilon_d)(\alpha-1)}{\epsilon_d}} \\
& + \left[B_i^e P^e A_i^{1-\epsilon_e} \left[\frac{\alpha(1-\epsilon_e)}{1+r} \right]^{1-\epsilon_e} \right]^{\frac{1}{\epsilon_e}} (k_i^{Emax} - k_m)^{\frac{(1-\epsilon_e)(\alpha-1)}{\epsilon_e}} \\
& -(1+r)(k_i^{Emax} + F^d + F^e) \qquad (2.81)
\end{aligned}
$$

We will utilize numerical methods to find the optimal values. A typical profit function with the variables x_i and k_i is shown in Figure 2.6.

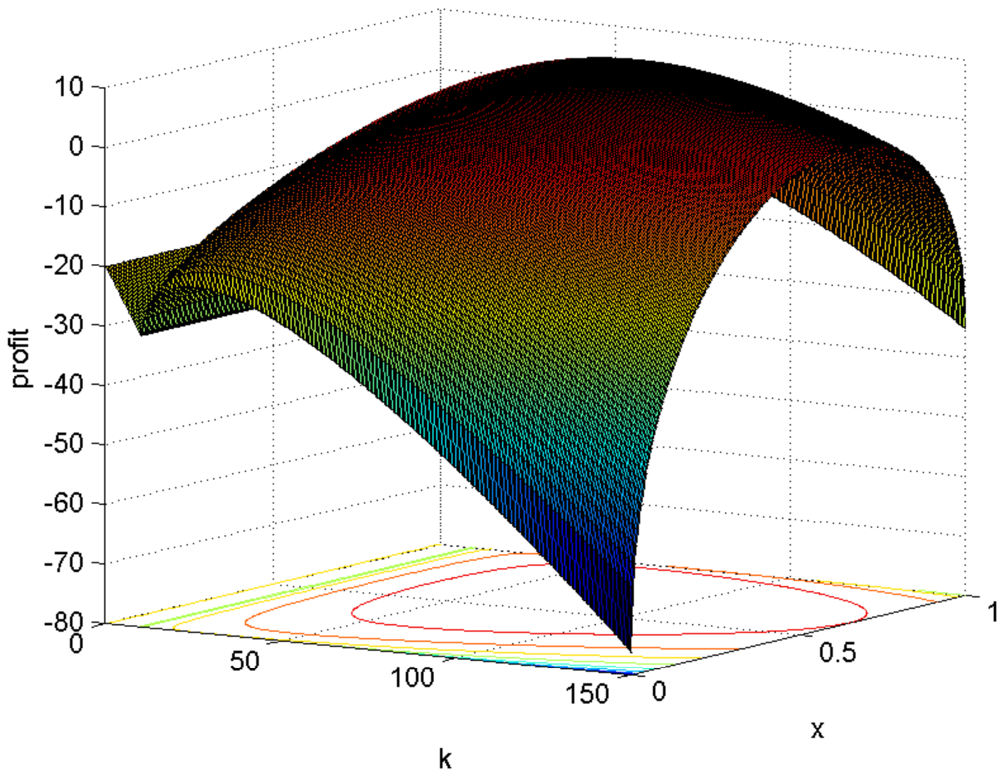


Figure 2.6: Profit function of both markets versus x_i and k_i.

The firm should find the optimal x_i and k_i values leading to maximum profit. One example of profit maximizing x_i and k_i values is shown in Figure 2.7. For this example every parameter of the markets are same except the product qualities such that $B_i^d < B_i^e$. Therefore, the share of the export market is higher than the share of the domestic market ($x_i > 0.5$).

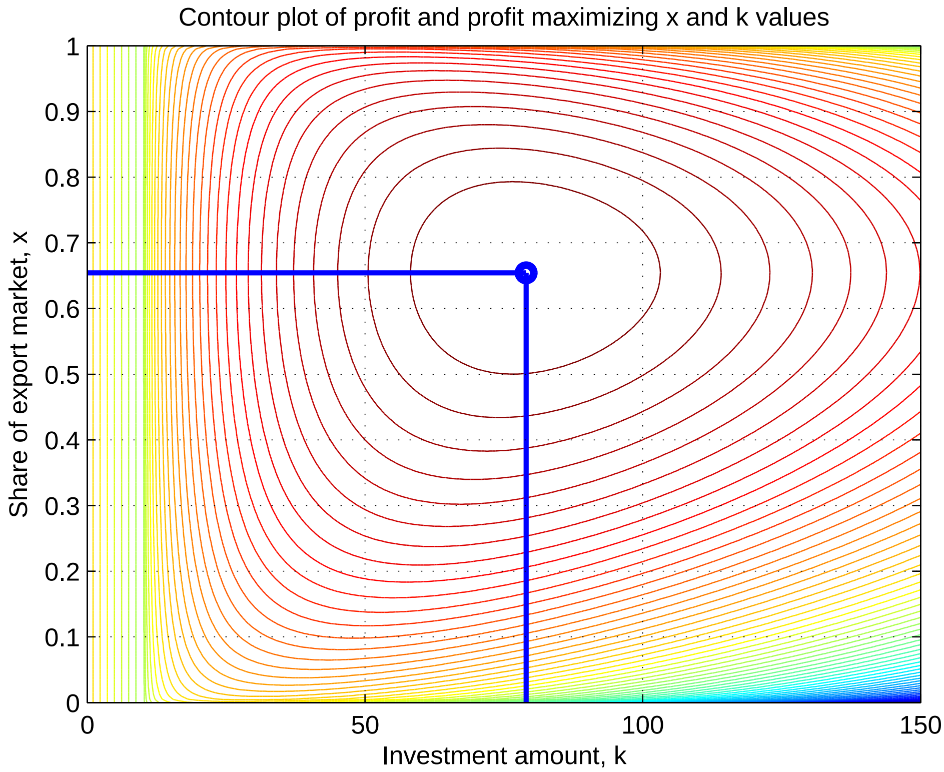
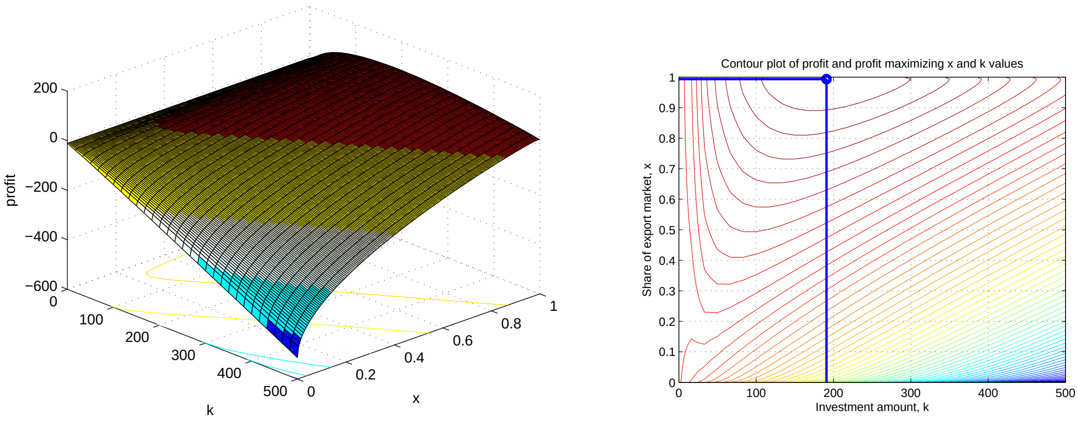


Figure 2.7: Profit Maximizing x_i^{Emax} and k_i^{Emax} values in both domestic and export markets.

In Figure 2.8, the product quality parameters of the markets is set such that $B_i^e >> B_i^d$. Therefore the production in domestic market will be close to zero ($1 - x_i \approx 0$) as seen in Figure 2.8(b). The profit maximizing x_i value is close to 1 but since profit function is concave for this case, maximizing x_i^{Emax} would never become zero.



3-D Plot Contour Plot

Figure 2.8: 3-D and contour plots of the profit function. x_i^{Emax} is close to 1.

Case 2: $x_i = 0$ **and** $k_i > k_m$

For this case the export market share is constant at $x_i = 0$. The lagrangian constants becomes: $\lambda_1 = 0$, $\lambda_2 > 0$, and $\lambda_3 = 0$. And the first order conditions (equations 2.55 and 2.56) becomes

$$\frac{\partial \mathcal{L}(x_i, k_i)}{\partial x_i} = -B_i^d P^d A_i^{1-\epsilon_d}(k_i - k_m)^{\alpha(1-\epsilon_d)}(1-\epsilon_d) + \lambda_2 = 0 \tag{2.82}$$

$$\frac{\partial \mathcal{L}(x_i, k_i)}{\partial k_i} = B_i^d P^d A_i^{1-\epsilon_d}\alpha(1-\epsilon_d)(k_i - k_m)^{\alpha(1-\epsilon_d)-1} - (1+r) = 0 \tag{2.83}$$

Solving for k_i in equation 2.83

$$k_i^{Emax} = \left[\frac{1+r}{B_i^d P^d A_i^{1-\epsilon_d}\alpha(1-\epsilon_d)}\right]^{\frac{1}{\alpha(1-\epsilon_d)-1}} + k_m \tag{2.84}$$

which is the same solution with profit maximization in domestic market (Section 2.3.1, Equation 2.47). Therefore, the profit maximization in domestic market is a corner solution of profit maximization in both markets. The corresponding lagrangian constant becomes

$$\lambda_2 = B_i^d P^d A_i^{1-\epsilon_d}(k_i^{Emax} - k_m)^{\alpha(1-\epsilon_d)}(1-\epsilon_d) \tag{2.85}$$

if the term on the right side is greater than zero ($\lambda_2 > 0$) then this case (case 2) is valid and profit maximizing export market share becomes $x_i = 0$. The corresponding maximum profit becomes (substituting k_i and $x_i = 0$ values into 2.52),

$$\begin{aligned}
\pi_E^{max} &= B_i^d P^d \left((1 - x_i) A_i (k_i^{Emax} - k_m)^{\alpha}\right)^{1-\epsilon_d} \\
&\quad + B_i^e P^e \left(x_i A_i (k_i^{Emax} - k_m)^{\alpha}\right)^{1-\epsilon_e} - (1+r)(k_i^{Emax} + F^d + F^e) \\
&= \left[\frac{B_i^d P^d A_i^{1-\epsilon_d}}{(1+r)^{\alpha(1-\epsilon_d)}}\right]^{\frac{1}{1-\alpha(1-\epsilon_d)}} \left\{[\alpha(1-\epsilon_d)]^{\frac{\alpha(1-\epsilon_d)}{1-\alpha(1-\epsilon_d)}} - [\alpha(1-\epsilon_d)]^{\frac{1}{1-\alpha(1-\epsilon_d)}}\right\} \\
&\quad - (1+r)(k_m + F^d + F^e) \qquad (2.86)
\end{aligned}$$

Case 3: $x_i = 1$ **and** $k_i > k_m$

For this case the export market share is constant at $x_i = 1$. The lagrangian constants becomes: $\lambda_1 > 0$, $\lambda_2 = 0$, and $\lambda_3 = 0$. And the corresponding first order conditions (equations 2.55 and 2.56) are

$$\frac{\partial \mathcal{L}(x_i, k_i)}{\partial x_i} = B_i^e P^e A_i^{1-\epsilon_e} (k_i - k_m)^{\alpha(1-\epsilon_e)} (1-\epsilon_e) - \lambda_1 \qquad (2.87)$$

$$\frac{\partial \mathcal{L}(x_i, k_i)}{\partial k_i} = B_i^e P^e A_i^{1-\epsilon_e} \alpha (1-\epsilon_e)(k_i - k_m)^{\alpha(1-\epsilon_e)-1} - (1+r) \qquad (2.88)$$

Solving for k_i in equation 2.88

$$k_i^{Emax} = \left[\frac{1+r}{B_i^e P^e A_i^{1-\epsilon_e} \alpha(1-\epsilon_e)}\right]^{\frac{1}{\alpha(1-\epsilon_e)-1}} + k_m \qquad (2.89)$$

And the corresponding lagrangian constant λ_1 becomes

$$\lambda_1 = B_i^e P^e A_i^{1-\epsilon_e} (k_i^{Emax} - k_m)^{\alpha(1-\epsilon_e)} (1-\epsilon_e) \qquad (2.90)$$

If the term on the right side is greater than zero ($\lambda_1 > 0$) then this case (case 3) is valid and profit maximizing export market share becomes $x_i = 1$. Substituting k_i and $x_i = 1$ values into 2.52 we can find the maximum profit as

$$\begin{aligned}
\pi_E^{max} &= B_i^d P^d \left((1 - x_i^{Emax}) A_i (k_i^{Emax} - k_m)^{\alpha}\right)^{1-\epsilon_d} \\
&\quad + B_i^e P^e \left(x_i^{Emax} A_i (k_i^{Emax} - k_m)^{\alpha}\right)^{1-\epsilon_e} - (1+r)(k_i^{Emax} + F^d + F^e) \\
&= \left[\frac{B_i^e P^e A_i^{1-\epsilon_e}}{(1+r)^{\alpha(1-\epsilon_e)}}\right]^{\frac{1}{1-\alpha(1-\epsilon_e)}} \left\{[\alpha(1-\epsilon_e)]^{\frac{\alpha(1-\epsilon_e)}{1-\alpha(1-\epsilon_e)}} - [\alpha(1-\epsilon_e)]^{\frac{1}{1-\alpha(1-\epsilon_e)}}\right\} \\
&\quad - (1+r)(k_m + F^d + F^e) \qquad (2.91)
\end{aligned}$$

Case 4: $0 < x_i < 1$ **and** $k_i = k_m$

The maximizing investment amount is constant at $k_i = k_m$. The lagrangian constants becomes: $\lambda_1 = 0$, $\lambda_2 = 0$, and $\lambda_3 > 0$. And the corresponding first order conditions (equations 2.55 and 2.56) are

$$\begin{aligned}\frac{\partial \mathcal{L}(x_i, k_i)}{\partial x_i} &= 0 \\ \frac{\partial \mathcal{L}(x_i, k_i)}{\partial k_i} &= -(1+r) + \lambda_3 = 0\end{aligned}$$

From equation 2.92, $\lambda_3 = 1+r > 0$. And the profit maximizing export market share can be any value satisfying $0 < x_i < 1$. The corresponding Maximum profit becomes

$$\begin{aligned}\pi_E^{max} &= B_i^d P^d ((1-x_i)A_i(k_i - k_m)^\alpha)^{1-\epsilon_d} \\ &\quad + B_i^e P^e (x_i A_i (k_i - k_m)^\alpha)^{1-\epsilon_e} - (1+r)(k_i + F^d + F^e) \\ &= -(1+r)(k_m + F^d + F^e)\end{aligned}$$

which is non-positive since $k_m \geq 0$, $F^d \geq 0$, and $F^e \geq 0$.

2.4 Comparison of Maximum Profits π_D^{max} and π_E^{max}

We will state some conclusions about the maximum profits in this section. Recall the profit maximizing investment k_i^{Dmax} for domestic market (Equation 2.47).

$$k_i^{Dmax} = \left[\frac{1+r}{(\alpha(1-\epsilon_d))\Theta^d}\right]^{\frac{1}{\alpha(1-\epsilon_d)-1}} + k_m \tag{2.92}$$

and the profit maximizing investment k_i^{Emax} in export market in equation 2.74

$$k_i^{Emax} = \left[\frac{\Theta^d(1-\epsilon_d)\alpha}{1+r}\right]^{\frac{1}{1-\alpha(1-\epsilon_d)}} (1 - x_i^{Emax})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}} + k_m \tag{2.93}$$

where

$$\Theta^d = B_i^d P^d A_i^{1-\epsilon_d}$$

Combining these equations, we can write k_i^{Emax} in terms of k_i^{Dmax} as

$$k_i^{Emax} - k_m = \left[k_i^{Dmax} - k_m\right](1 - x_i^{Emax})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}} \qquad (2.94)$$

Note that $(1 - x_i^{Emax})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}} > 1$ since $\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1} < 0$ for $0 < x_i^{Emax} < 1$. Therefore, we can simply state that k_i^{Emax} is bigger than k_i^{Dmax} for $0 < x_i < 1$. Also, recall that these maximizing investment levels are found for concave profit functions. If these functions are not concave then we will have the corner solutions as explained before.

If the sunk cost paid for entering the export market is zero, then the solution of π_E at $x_i = 0$ becomes domestic profit function π_D. Therefore, considering the concavity of π_E (say all of the conditions stated above are satisfied) there will be higher values of π_E for every $0 < x_i < 1$ with respect to π_D. Note that π_D is the solution of π_E at $x_i = 0$. This results in that the profit function π_E will be greater even at $k_i = k_i^{Dmax}$. Let's call the maximum of π_E at $k_i = k_i^{Dmax}$ as $\pi_E^{max}|_{k_i^{Dmax}}$ satisfying $\pi_E^{max}|_{k_i^{Dmax}} > \pi_D^{max}$. In fact, since $k_i^{Emax} > k_i^{Dmax}$ and profit function is concave then the global maximum of π_E will be greater then the one at $k_i = k_i^{Dmax}$. Therefore, we can conclude that π_E^{max} will be always higher then the π_D^{max} under the condition both profit functions are concave. In Figure 2.9 3D plots of the profit function of firm only in domestic market π_D and the profit function of firm in both markets π_E are shown. Note that blue surface is the extension of π_D along $0 \leq x_i \leq 1$ for better comparison. For this figure $F^e = 0$. As seen both of the profit functions are concave. As in the previous results the profit maximizing k_i^{Dmax} value is lower than the ones for both domestic and export markets k_i^{Emax}. And the maximum profit π_E^{max} is higher then the π_D^{max}. Also note that since π_E is concave it has higher values at $k_i = k_i^{Dmax}$ along $0 < x_i < x_i^{ub}$ where x_i^{ub} is the upper x_i value that π_E exceeds π_D.

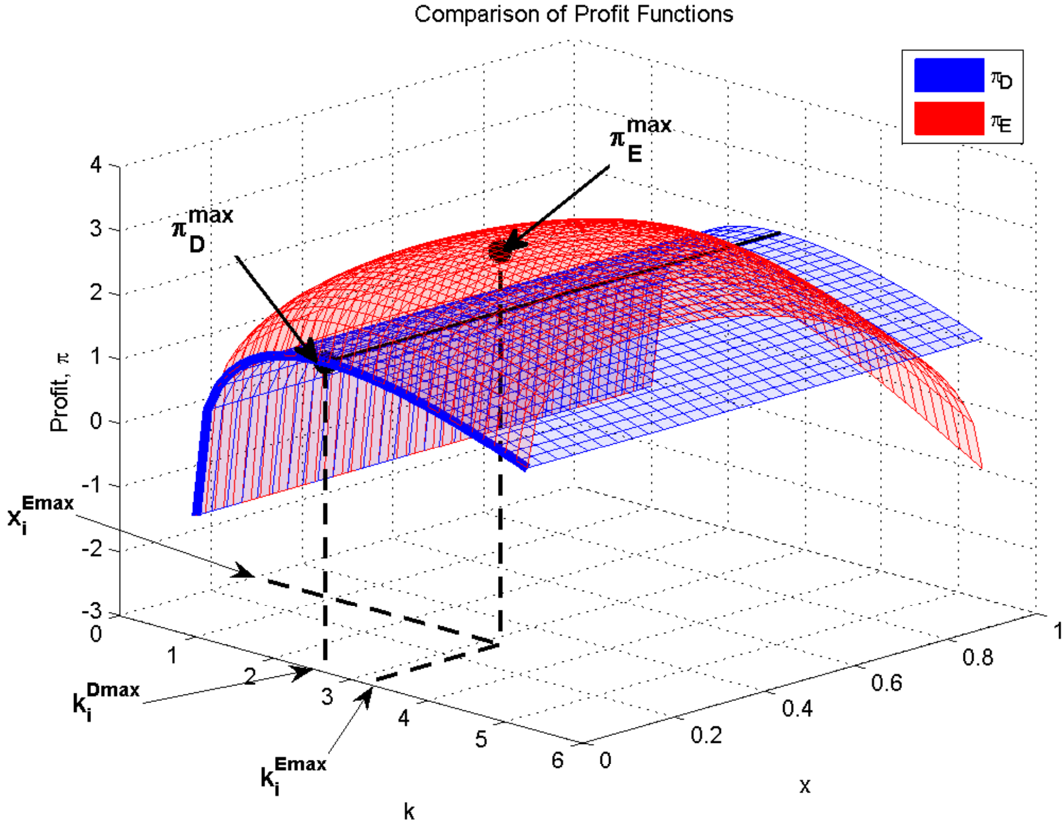


Figure 2.9: Comparison of π_E and π_D.

CHAPTER 3

COMPARATIVE STATICS ANALYSIS OF PROFITS AND INVESTMENT

In this chapter we will analyze the behavior of profit functions, maximum profits and profit maximizing investment levels under the variations of production and demand functions. In the following sections the effects of market parameters on domestic and export market maximum profits are analyzed. Some numerical examples are also provided. For the numerical examples the parameters are taken as $B_i^d = 1$, $B_i^e = 1$, $F^d = 2$, $F^e = 2$, $r = 0.1$, $P^d = 1$, $P^e = 1$, $\epsilon_e = 0.5$, $\epsilon_d = 0.5$, $k_m = 2$, $\alpha = 0.5$ unless some other value is declared.

3.1 Effects of Parameters on Maximum Profit in Domestic Market

The profit maximizing investment amount were found in section 2.3.1 as

$$k_i^{max} = \left[\frac{\theta B_i^d P^d A_i^{1-\epsilon_d}}{1+r}\right]^{\frac{1}{1-\theta}} + k_m \tag{3.1}$$

and the corresponding maximum profit is

$$\pi_D^{max} = \left[\frac{B_i^d P^d A_i^{1-\epsilon_d}}{(1+r)^\theta}\right]^{\frac{1}{1-\theta}} \left[\theta^{\frac{\theta}{1-\theta}} - \theta^{\frac{1}{1-\theta}}\right] - (1+r)(k_m + F^d) \tag{3.2}$$

where

$$0 < \theta = \alpha(1-\epsilon_d) < 1 \tag{3.3}$$

Let us investigate the behavior of maximum profit and maximizing investment amount with respect to changes in market parameters.

3.1.1 Effects of Productivity, A_i

Change of k_i^{max} with respect to change in A_i:

The change of maximizing investment level with respect to change in productivity A_i.

$$\frac{dk_i^{max}}{dA_i} = \left[\frac{\theta B_i^d P^d}{1+r}\right]^{\frac{1}{1-\theta}} \left[\frac{1-\epsilon_d}{1-\theta}\right] A_i^{\frac{\theta-\epsilon_d}{1-\theta}} \tag{3.4}$$

where $\frac{1-\epsilon_d}{1-\theta} > 0$. The profit maximizing investment level increases as productivity A_i increases. The second deriative

$$\frac{d^2k_i^{max}}{dA_i^2} = \left[\frac{\theta B_i^d P^d}{1+r}\right]^{\frac{1}{1-\theta}} \left[\frac{1-\epsilon_d}{1-\theta}\right] \left[\frac{\theta-\epsilon_d}{1-\theta}\right] A_i^{\frac{2\theta-\epsilon_d-1}{1-\theta}} \tag{3.5}$$

If $\theta - \epsilon_d = \alpha(1-\epsilon_d) - \epsilon_d > 0$ or $\alpha > \frac{\epsilon_d}{1-\epsilon_d}$ then $\frac{dk_i^{max}}{dA_i}$ increases as A_i increases. This means the change in maximizing investment amount per productivity change will rise as productivity increases. If the reverse is valid ($\alpha < \frac{\epsilon_d}{1-\epsilon_d}$) then $\frac{dk_i^{max}}{dA_i}$ will decrease as productivity increases.

Change of π_D^{max} with respect to change in A_i:

The change in maximum domestic profit, π_D^{max} with respect to productivity A_i is

$$\frac{d\pi_D^{max}}{dA_i} = \left[\frac{B_i^d P^d}{(1+r)^\theta}\right]^{\frac{1}{1-\theta}} \left[\theta^{\frac{\theta}{1-\theta}} - \theta^{\frac{1}{1-\theta}}\right] \frac{1-\epsilon_d}{1-\theta} A_i^{\frac{1-\epsilon_d}{1-\theta}-1} \tag{3.6}$$

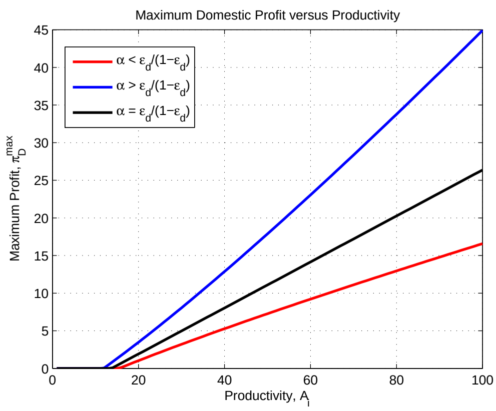
Since $\frac{1-\epsilon_d}{1-\theta} > 0$, the maximum domestic profit increases as productivity of firm increases. The second derivative is

$$\frac{d^2\pi_D^{max}}{dA_i^2} = \left[\frac{B_i^d P^d}{(1+r)^\theta}\right]^{\frac{1}{1-\theta}} \left[\theta^{\frac{\theta}{1-\theta}} - \theta^{\frac{1}{1-\theta}}\right] \left[\frac{1-\epsilon_d}{1-\theta}\right] \left[\frac{\theta-\epsilon_d}{1-\theta}\right] A_i^{\frac{2\theta-\epsilon_d-1}{1-\theta}} \tag{3.7}$$

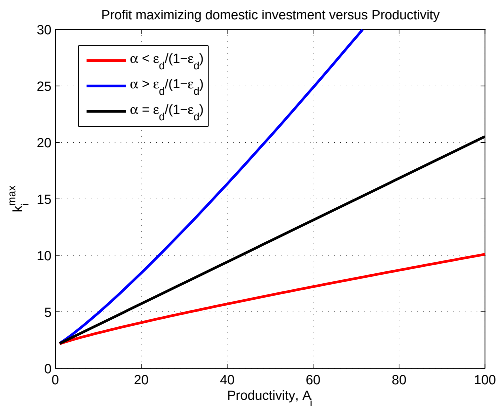
Again if $\alpha > \frac{\epsilon_d}{1-\epsilon_d}$ then $\frac{\partial\pi_D^{max}}{\partial A_i}$ increases as A_i increases. In other words the gain by each productivity increase will rise as productivity gets higher. If $\alpha = \frac{\epsilon_d}{1-\epsilon_d}$

then the change of change in maximum profit is constant with respect to change in A_i. And if $\alpha < \frac{\epsilon_d}{1-\epsilon_d}$ then the increase in maximum profit per productivity gets smaller as productivity increases.

In Figure 3.1, the change in profit maximizing investment k_i^{max} and maximum domestic profit π_D^{max} are plotted under three conditions: (i) $\frac{d^2\pi_D^{max}}{dA_i^2} > 0$ or $\alpha < \frac{\epsilon_d}{1-\epsilon_d}$ $(\alpha = 0.5)$, (ii) $\frac{d^2\pi_D^{max}}{dA_i^2} < 0$ or $\alpha > \frac{\epsilon_d}{1-\epsilon_d}$ $(\alpha = 0.8)$, and (iii) $\frac{d^2\pi_D^{max}}{dA_i^2}$ or $\alpha = \frac{\epsilon_d}{1-\epsilon_d}$ $(\alpha = 0.667)$. ϵ is taken 0.4 for each case.



(a) π_D^{max} vs A_i



(b) k_i^{max} vs A_i

Figure 3.1: Effects of price elasticity of demands on the maximum profit and profit maximizing investment in domestic market

Considering the above results, we can state that (i) as the productivity of firms increases the investment amounts also increase, (ii) as productivity of firm increases the profit of the firm increases, or the firms with higher productivity will invest more and earn more.

3.1.2 Effects of B_i^d

Change of k_i^{max} with respect to change in B_i^d:

The change in profit maximizing investment, k_i^{max} with respect to parameter B_i^d is

$$\frac{dk_i^{max}}{dB_i^d} = \left[\frac{\theta P^d A_i^{1-\epsilon_d}}{1+r}\right]^{\frac{1}{1-\theta}} \left[\frac{1}{1-\theta}\right] (B_i^d)^{\frac{\theta}{1-\theta}} \tag{3.8}$$

The profit maximizing investment amount increases as B_i^d parameter increases. The second derivative

$$\frac{d^2k_i^{max}}{d(B_i^d)^2} = \left[\frac{\theta P^d A_i^{1-\epsilon_d}}{1+r}\right]^{\frac{1}{1-\theta}} \left[\frac{1}{1-\theta}\right] \left[\frac{\theta}{1-\theta}\right] (B_i^d)^{\frac{2\theta-1}{1-\theta}} \tag{3.9}$$

shows that the first derivative $\frac{dk_i^{max}}{dB_i^d}$ is increasing as B_i^d increases since $\frac{d^2k_i^{max}}{d(B_i^d)^2} > 0$.

Change of π_D^{max} with respect to change in B_i^d:

The change in maximum domestic profit, π_D^{max} with respect to parameter B_i^d is

$$\frac{d\pi_D^{max}}{d(B_i^d)} = \left[\frac{P^d A_i^{1-\epsilon_d}}{(1+r)^\theta}\right]^{\frac{1}{1-\theta}} \left[\theta^{\frac{\theta}{1-\theta}} - \theta^{\frac{1}{1-\theta}}\right] \left[\frac{1}{1-\theta}\right] \left(B_i^d\right)^{\frac{\theta}{1-\theta}} \tag{3.10}$$

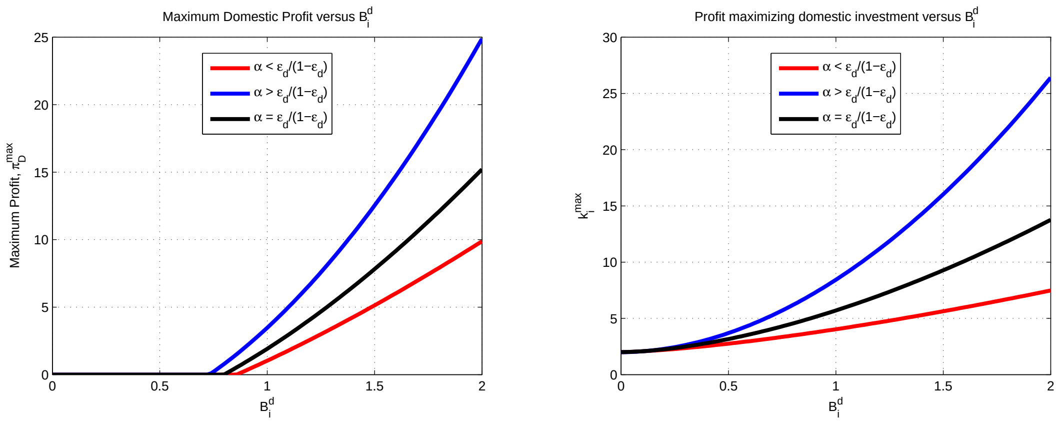
As the B_i^d parameter increases the maximum profit also increases since $\frac{d\pi_D^{max}}{dB_i}$ is always positive. For the behavior of the change in the maximum profit corresponding to the changes in B_i^d, let us investigate the second derivative

$$\frac{d^2\pi_D^{max}}{d(B_i^d)^2} = \left[\frac{P^d A_i^{1-\epsilon_d}}{(1+r)^\theta}\right]^{\frac{1}{1-\theta}} \left[\theta^{\frac{\theta}{1-\theta}} - \theta^{\frac{1}{1-\theta}}\right] \left[\frac{1}{1-\theta}\right] \left[\frac{\theta}{1-\theta}\right] \left(B_i^d\right)^{\frac{2\theta-1}{1-\theta}} \tag{3.11}$$

As it is seen above $\frac{d^2\pi_D^{max}}{d(B_i^d)^2}$ is always positive. Therefore, the increase in the maximum profit per B_i^d increase gets higher as B_i^d increases.

In Figure 3.2, the maximum domestic product and profit maximizing domestic investment are plotted versus parameter B_i^d for three different cases

($\alpha < \frac{\epsilon_d}{1-\epsilon_d}$ ($\alpha = 0.5$), $\alpha > \frac{\epsilon_d}{1-\epsilon_d}$ ($\alpha = 0.8$), and $\alpha = \frac{\epsilon_d}{1-\epsilon_d}$) ($\alpha = 0.667$). $\epsilon = 0.4$ for each case. As it is seen in each case the profit and investment grows with an increasing velocity.



(a) π_D^{max} vs B_i^d (b) k_i^{max} vs B_i^d

Figure 3.2: Effects of B_i^d on the maximum profit and profit maximizing investment in the domestic market.

These results show that as the demand in domestic market increases the firms will invest more and gain more profit. Or say if a firm faces with a higher demand for its own product then it will invest more and have a higher profit than the firm with a less demand parameter B_i^d.

3.1.3 Effects of sunk cost F^d

Change of k_i^{max} with respect to change in F^d:

The change in maximizing investment, k_i^{max} with respect to the sunk cost F^d is

$$\frac{dk_i^{max}}{dF^d} = 0 \tag{3.12}$$

Therefore, the amount of sunk cost does not have any effect on the profit maximizing investment amount. Whether the sunk cost increases or decreases the investment will not change. If maximized profit is negative because the sunk cost is very high then the profit maximizing investment and the maximum profit is zero since the firm will not produce (enter the market).

Change of π_D^{max} with respect to change in F^d:

The change in maximum domestic profit, π_D^{max} with respect to the sunk cost F^d is

$$\frac{d\pi_D^{max}}{dF^d} = -(1+r) \tag{3.13}$$

This means the maximum profit shifts in the reverse direction with the proportion $(1+r)$ of change in the sunk cost.

In Figure 3.3 the effect of domestic market sunk cost on the maximum profit and optimal k_i amount are observed. As it is seen the change in sunk cost does not effect the optimal investment level if the profit is positive. As F^d increases the maximum profit decreases with the same amount up to the point where profit becomes negative. The values of parameters for this figure: $\epsilon_d = 0.4$, $\epsilon_e = 0.5$, and $\alpha = 0.5$.

To summarize, the sunk cost is a means that does not affect the investment amount but it is the constant amount that is paid by earned profit. If the profit is not enough to overcome this cost then the firm will not enter the market and produce anything.

3.1.4 Effects of threshold investment amount k_m

Change of k_i^{max} with respect to change in k_m:

The change in maximizing investment, k_i^{max} with respect to threshold investment amount k_m is

$$\frac{dk_i^{max}}{dk_m} = 1 \tag{3.14}$$

The profit maximizing investment shifts with the same amount of change in minimum required investment.

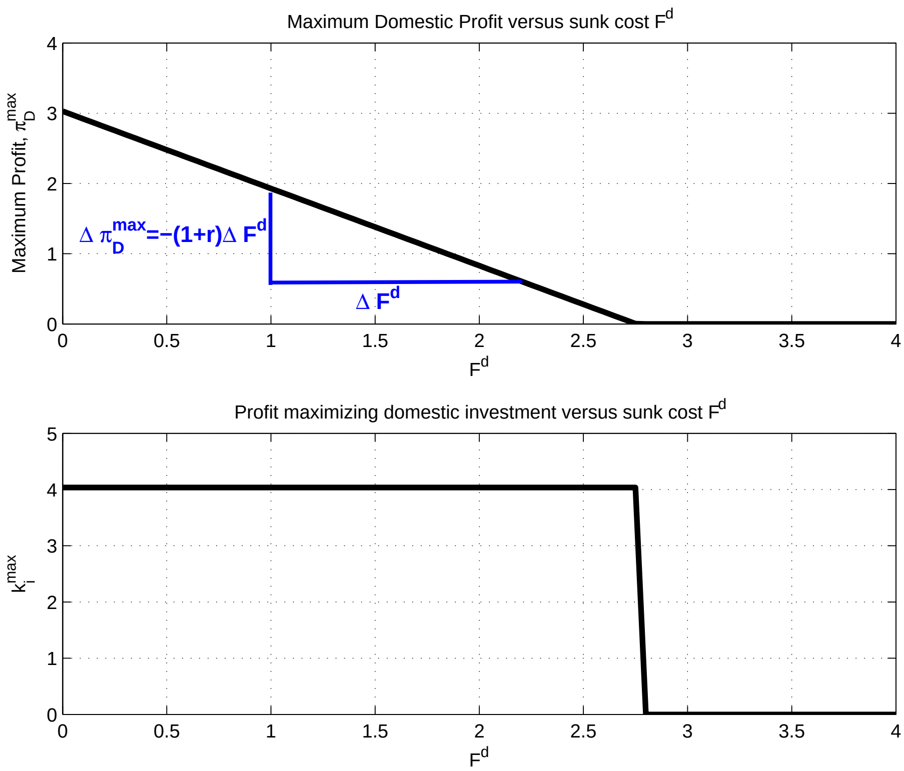


Figure 3.3: Effects of F^d on the maximum profit and profit maximizing investment in the domestic market.

Change of π_D^{max} with respect to change in k_m:

The change in maximum domestic profit, π_D^{max} with respect to threshold investment amount k_m is

$$\frac{d\pi_D^{max}}{dk_m} = -(1+r) \tag{3.15}$$

As the threshold investment amount increases the maximum profit decreases with the amount $-(1+r)\Delta k_m$.

In Figure 3.4 the change in maximum profit and optimal investment are plotted with respect to change in threshold investment level k_m. As seen the profit decreases with the constant proportion of $-(1+r)$ $(r = 0.1)$. The optimal investment level k_i^{max} increases with the same amount of increase in k_m. In both plots, the values are set to zero when the profit becomes negative.

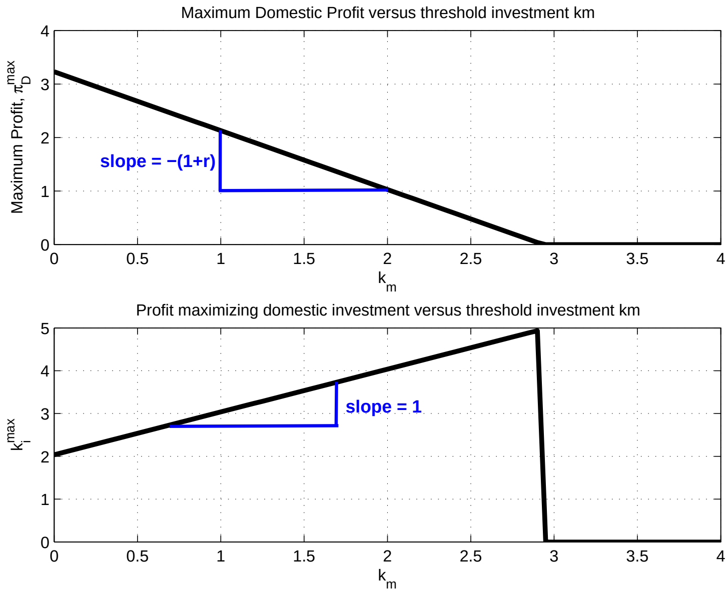


Figure 3.4: Effects of k_m on the maximum profit and profit maximizing investment in the domestic market.

The constant threshold investment, k_m is the amount that the firm should exceed to start production. In fact the difference $k_i - k_m$ is the amount that results in profit. Therefore, this difference does not change if the threshold changes. This means, as the threshold increases the investment should increase with the same amount to cover the profit maximizing difference $k_i - k_m$. However since the investment is paid by borrowing the profit will decrease with the ratio $1 + r$ as threshold increases.

3.1.5 Effects of interest rate r

Change of k_i^{max} with respect to change in r

The change in maximizing investment, k_i^{max} with respect to interest rate r is

$$\frac{dk_i^{max}}{dr} = \left[\theta B_i^d P^d A_i^{1-\epsilon_d}\right]^{\frac{1}{1-\theta}} \left[-\frac{1}{1-\theta}\right] (1+r)^{\frac{\theta}{1-\theta}} \tag{3.16}$$

Thus, the profit maximizing investment and interest rate are inverse proportional. As interest rate increases k_i^{max} decreases.

Change of π_D^{max} with respect to change in r

The change in maximum domestic profit, π_D^{max} with respect to interest rate r is

$$\frac{d\pi_D^{max}}{dr} = \left[B_i^d P^d A_i^{1-\epsilon_d}\right]^{\frac{1}{1-\theta}} \left[\theta^{\frac{\theta}{1-\theta}} - \theta^{\frac{1}{1-\theta}}\right] \left[-\frac{\theta}{1-\theta}\right] (1+r)^{\frac{2\theta-1}{1-\theta}} \tag{3.17}$$

which is always negative. Therefore, the maximum profit in the domestic market is decreasing with the rise in interest rate.

In Figure 3.5 the change in maximum profit and optimal investment are plotted with respect to change in interest rate r. As seen both the profit and optimal investment decreases as k_m increases. In both plots, the values are set to zero when the profit becomes negative.

Not surprisingly, the interest rate has negative effects on both the investment and profit amounts. As the borrowing interest rate of the firms increases the firms opt to utilize less investment and therefore gain less profit.

3.2 Effects of Parameters on Maximum Profit of the Firm Producing in Both Markets

The profit maximization problem for the firm producing in both markets is defined in section 2.3.2. We can find the response (increasing or decreasing) of profit maximizing investment amount k_i^{max}, profit maximizing export market share x_i^{max}, and maximum profit π_E^{max} with respect to changes of some critical

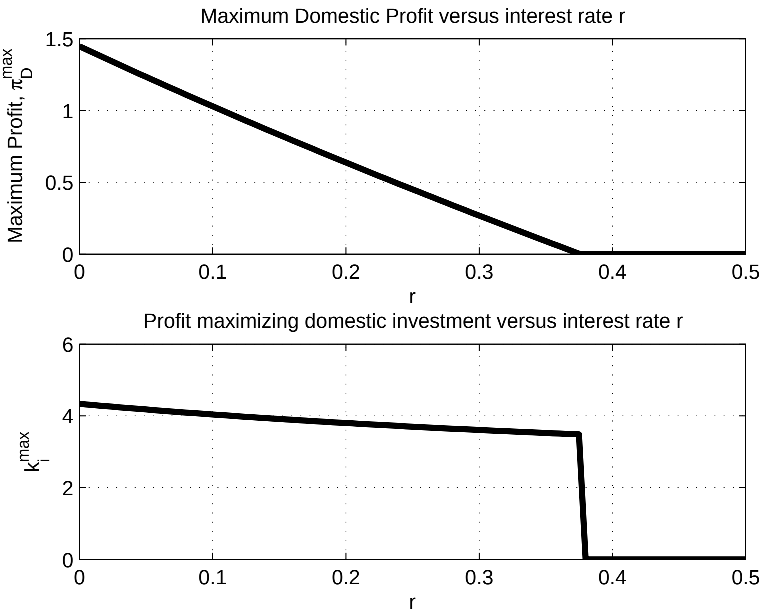


Figure 3.5: Effects of r on the maximum profit and profit maximizing investment in the domestic market.

variables. The exact solutions of these responses in terms of market parameters can not be found, but we can at least find the directions (positiveness or negativeness). Recall the equations for these profit maximizing parameters:

$$[\Omega^e]^{\frac{1}{\epsilon_e}} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_e)-1}{\epsilon_e}} + \left[\Omega^d\right]^{\frac{1}{\epsilon_d}} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_d)-1}{\epsilon_d}} = 1 \quad (3.18)$$

$$[\Omega^e]^{\frac{1}{1-\alpha(1-\epsilon_e)}} (x_i^{max})^{\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1}} = \left[\Omega^d\right]^{\frac{1}{1-\alpha(1-\epsilon_d)}} (1 - x_i^{max})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}} \quad (3.19)$$

where

$$\Omega^e = \frac{\Theta^e(1-\epsilon_e)\alpha}{1+r} = \frac{B_i^e P^e A_i^{1-\epsilon_e}(1-\epsilon_e)\alpha}{1+r}$$
$$\Omega^d = \frac{\Theta^d(1-\epsilon_d)\alpha}{1+r} = \frac{B_i^d P^d A_i^{1-\epsilon_d}(1-\epsilon_d)\alpha}{1+r}$$

Substituting the constant terms

$$\left[\frac{B_i^e P^e A_i^{1-\epsilon_e}(1-\epsilon_e)\alpha}{1+r}\right]^{\frac{1}{\epsilon_e}} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_e)-1}{\epsilon_e}}$$

$$+\left[\frac{B_i^d P^d A_i^{1-\epsilon_d}(1-\epsilon_d)\alpha}{1+r}\right]^{\frac{1}{\epsilon_d}} (k_i^{max}-k_m)^{\frac{\alpha(1-\epsilon_d)-1}{\epsilon_d}} = 1 \tag{3.20}$$

and

$$\left[\frac{B_i^e P^e A_i^{1-\epsilon_e}(1-\epsilon_e)\alpha}{1+r}\right]^{\frac{1}{1-\alpha(1-\epsilon_e)}} (x_i^{max})^{\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1}} =$$

$$\left[\frac{B_i^d P^d A_i^{1-\epsilon_d}(1-\epsilon_d)\alpha}{1+r}\right]^{\frac{1}{1-\alpha(1-\epsilon_d)}} (1-x_i^{max})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}} \tag{3.21}$$

The maximum profit in terms of optimal values is

$$\begin{aligned} \pi_E^{max} &= B_i^d P^d \left[(1-x_i^{max})A_i(k_i^{max}-k_m)^{\alpha}\right]^{1-\epsilon_d} && (3.22)\\ &+B_i^e P^e \left[x_i^{max} A_i(k_i^{max}-k_m)^{\alpha}\right]^{1-\epsilon_e} \\ &-(1+r)(k_i^{max}+F^d+F^e) && (3.23) \end{aligned}$$

3.2.1 Effects of Productivity, A_i

Change of k_i^{max} with respect to change in A_i:

To find the response of profit maximizing investment to changes in productivity we use implicit differentiation method (detailed derivation is in Appendix A.3). The resulting derivative is

$$\frac{dk_i^{max}}{dA_i} = -\frac{\Phi_1^e + \Phi_1^d}{\Phi_2^e + \Phi_2^d} > 0 \tag{3.24}$$

where the variables Φ_1^e and Φ_1^d are always positive and Φ_2^e and Φ_2^d are always negative. Then the productivity growth leads to increase in maximizing investment or the firm with higher productivity (ceteris paribus) will pay for higher investment to maximize the profit.

Change of x_i^{max} with respect to change in A_i:

To find the response of profit maximizing x_i^{max} to changes in productivity we again use the implicit differentiation method. The detailed derivation is in

Appendix A.4. The result is as follows

$$\begin{array}{l} \frac{dx_i^{max}}{dA_i} < 0 \quad if \;\; \epsilon_d < \epsilon_e \\ \frac{dx_i^{max}}{dA_i} = 0 \quad if \;\; \epsilon_d = \epsilon_e \\ \frac{dx_i^{max}}{dA_i} > 0 \quad if \;\; \epsilon_d > \epsilon_e \end{array} \tag{3.25}$$

meaning that if the elasticity of domestic market is higher than the export market ($\epsilon_d < \epsilon_e$), the firm will decrease its export market share x_i as its productivity increases. In other words the firms with higher productivity will have a lower share of export market if the domestic market is more elastic. The vice versa is also valid. If export market is more elastic then the share of export market will be higher or the firm with higher productivity will produce more for export market to have its profit maximum.

Change of π_E^{max} with respect to change in A_i:

Finding the derivative of π_E^{max} with respect to A_i is a very complicated computation. However, we can simply show that the maximum profit increases as productivity increases. Let's first consider the total profit function of both markets for $0 \leq x_i \leq 1$ and $k_i > k_m$

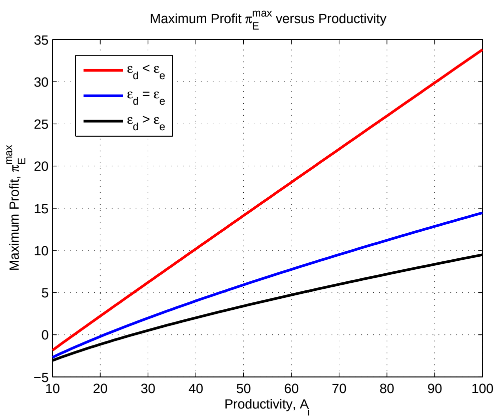
$$\begin{aligned} \pi_E &= B_i^d P^d \left[(1-x_i)A_i(k_i-k_m)^{\alpha}\right]^{1-\epsilon_d} \\ &\quad + B_i^e P^e \left[x_i A_i (k_i-k_m)^{\alpha}\right]^{1-\epsilon_e} \\ &\quad -(1+r)(k_i+F^d+F^e) \end{aligned} \tag{3.26}$$

The derivative of this function with respect to A_i is

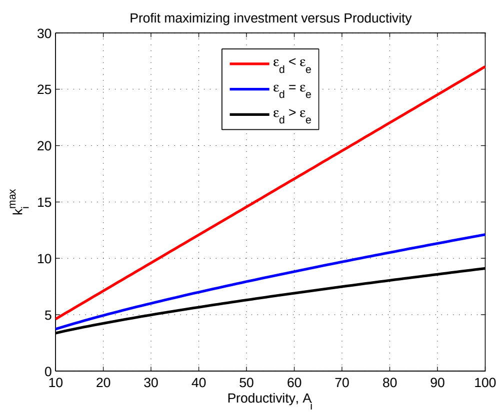
$$\begin{aligned} \frac{d\pi_E}{dA_i} &= B_i^d P^d (1-\epsilon_d) A_i^{-\epsilon_d} \left[(1-x_i)(k_i-k_m)^{\alpha}\right]^{1-\epsilon_d} \\ &\quad + B_i^e P^e (1-\epsilon_e) A_i^{-\epsilon_e} \left[x_i (k_i-k_m)^{\alpha}\right]^{1-\epsilon_e} \end{aligned} \tag{3.27}$$

Equation 3.27 shows that $\frac{d\pi_E}{dA_i}$ is always positive for every $0 \leq x_i \leq 1$ and $k_i > k_m$. Therefore, the profit increases for every $0 \leq x_i \leq 1$ and $k_i > k_m$. We can conclude that if profit increases for every value of x_i and k_i, then the maximum profit will be higher when the productivity increases.

In Figure 3.6 and 3.7, the maximum profit of both markets π_E^{max}, the optimal investment amounts k_i^{max}, and the optimal market shares x_i^{max} versus productivity are plotted under three conditions: $\epsilon_d < \epsilon_e$ ($\epsilon_d = 0.4$), $\epsilon_d = \epsilon_e$ ($\epsilon_d = 0.5$), $\epsilon_d > \epsilon_e$ ($\epsilon_d = 0.6$). $\epsilon_e = 0.5$ and $\alpha = 0.7$ for each case. As derived above the maximum profit and maximizing investment increases as productivity increases in each case: $\epsilon_d < \epsilon_e$, $\epsilon_d = \epsilon_e$, and $\epsilon_d > \epsilon_e$. In Figure 3.7 we observe that the optimal export market share x_i^{max} shows different behavior under these three conditions. If the elasticity of both markets are same, then there will be no change in the export market share. If the domestic market is more elastic $\epsilon_d < \epsilon_e$ then the share of domestic market $1 - x_i$ will increase and if reverse is valid $\epsilon_d > \epsilon_e$ then the export share will increase.



(a) π_E^{max} vs A_i



(b) k_i^{max} vs A_i

Figure 3.6: Effects of productivity A_i on the maximum profit and profit maximizing investment for the exporter firm.

The above analytical results show that the productivity increase will increase the investment and the profit of firm as we found for the domestic market and the change in the market shares will depend on the price elasticities of markets. The firms will prefer to produce more for the market which is more price elastic

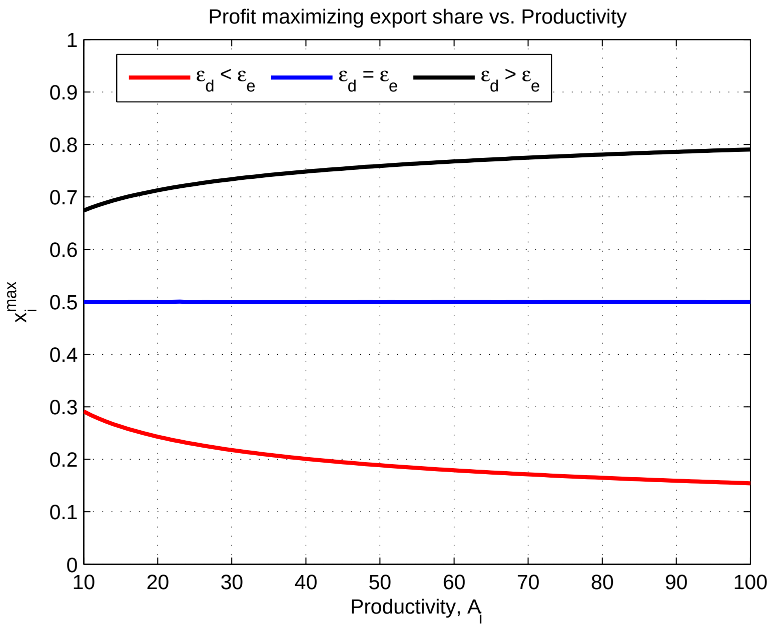


Figure 3.7: Effects of productivity A_i on the profit maximizing market share x_i^{max} for the exporter firm.

and as productivity increases the share of the market which is more elastic will rise. In other words the firms with higher productivity will produce more for the elastic market with respect to the less productive firms. Usually the export markets are more price elastic (there are more firms in export market, etc.); hence the more productive firms usually work more for selling in export market than selling in domestic market with respect to less productive firms.

3.2.2 Effects of B_i^d and B_i^e

Change of k_i^{max} with respect to change in B_i^d and B_i^e:

We again utilize the differentiation of k_i^{max} with respect to B_i^d or B_i^e to observe the effects. According to the derivations in Appendix A.5

$$\frac{dk_i^{max}}{dB_i^d} = -\frac{R_{Bd}}{R_{k1} + R_{k2}} \tag{3.28}$$

$$\frac{dk_i^{max}}{dB_i^e} = -\frac{R_{Be}}{R_{k1} + R_{k2}} \tag{3.29}$$

where variables R_{k1} and R_{k2} are always negative and R_{Bd} and R_{Be} are always positive. Therefore, $\frac{dk_i^{max}}{dB_i^d}$ and $\frac{dk_i^{max}}{dB_i^e}$ are positive, which means the profit maximizing investment amount is increasing with rise in the market demands.

Change of x_i^{max} with respect to change in B_i^d and B_i^e:

The effects of shifts in market demands on the market shares are derived in Appendix A.6.

$$\frac{dx_i^{max}}{dB_i^d} = \frac{C_{Bd}}{C_{x1} + C_{x2}} \tag{3.30}$$

$$\frac{dx_i^{max}}{dB_i^e} = -\frac{C_{Be}}{C_{x1} + C_{x2}} \tag{3.31}$$

where $C_{Bd} > 0$, $C_{Be} > 0$ and $C_{x1} < 0$ and $C_{x1} < 0$. Therefore, $\frac{dx_i^{max}}{dB_i^d}$ is always negative while $\frac{dx_i^{max}}{dB_i^e}$ is always positive. Therefore, the increase in the domestic market parameter B_i^d will result in a decrease in the export market share and vice versa. In addition, the increase in export market parameter B_i^e results in increase in export market share and a decrease in this parameter leads to a decrease in export market share x_i.

Change of π_E^{max} with respect to change in B_i^d and B_i^e:

Finding the derivative of π_E^{max} with respect to B_i^d or B_i^e is very difficult since the relations are in closed form. However we can simply show that the maximum profit increases as B_i parameters increase. Lets first consider the total profit function of both markets for $0 \leq x_i \leq 1$ and $k_i > k_m$

$$\begin{aligned}\pi_E &= B_i^d P^d \left[(1-x_i)A_i(k_i-k_m)^\alpha\right]^{1-\epsilon_d} \\ &\quad + B_i^e P^e \left[x_i A_i(k_i-k_m)^\alpha\right]^{1-\epsilon_e} \\ &\quad -(1+r)(k_i+F^d+F^e)\end{aligned} \tag{3.32}$$

The derivative of this function with respect to B_i^d is

$$\frac{d\pi_E}{dB_i^d} = P^d \left[(1-x_i)A_i(k_i-k_m)^\alpha\right]^{1-\epsilon_d} \tag{3.33}$$

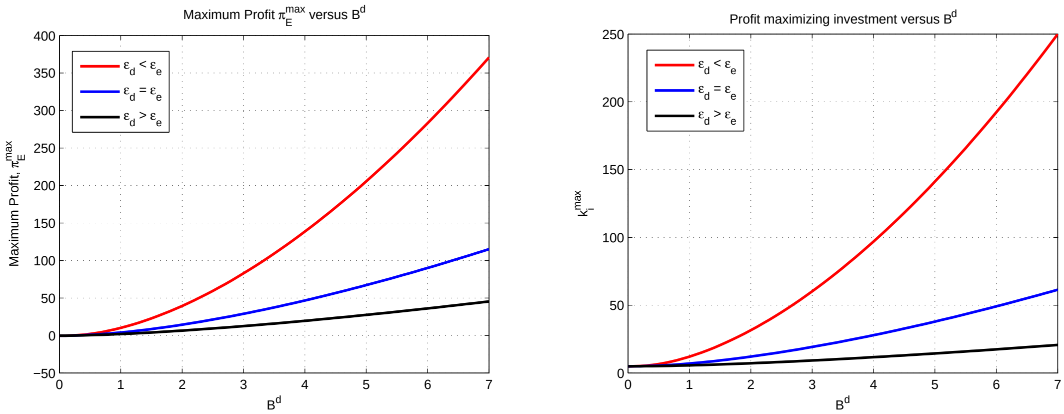
and for B_i^e is

$$\frac{d\pi_E}{dB_i^e} = P^e \left[x_i A_i (k_i - k_m)^\alpha\right]^{1-\epsilon_e} \tag{3.34}$$

Equations 3.33 and 3.34 show that $\frac{d\pi_E}{dB_i^d}$ and $\frac{d\pi_E}{dB_i^e}$ are always positive for every $0 \leq x_i \leq 1$ and $k_i > k_m$. Therefore the profit increases for every $0 \leq x_i \leq 1$ and $k_i > k_m$. We can conclude that if profit increases for every value of x_i and k_i then the maximum profit will also be higher when the B_i parameters increases.

In Figures 3.8 and 3.9 the maximum profit of both markets π_E^{max}, the optimal investment amounts k_i^{max}, and the optimal market shares x_i^{max} versus B_i^d are plotted under three conditions: $\epsilon_d < \epsilon_e$ ($\epsilon_d = 0.4$), $\epsilon_d = \epsilon_e$ ($\epsilon_d = 0.5$), $\epsilon_d > \epsilon_e$ ($\epsilon_d = 0.6$). $\epsilon_e = 0.5$, $\alpha = 0.7$, $B_i^e = 1.0$ for each case. As derived above the maximum profit and maximizing investment increases as B_i^d increases in each case: $\epsilon_d < \epsilon_e$, $\epsilon_d = \epsilon_e$, and $\epsilon_d > \epsilon_e$. The increase of the case $\epsilon_d < \epsilon_e$ is faster than the others. One may conclude that as the domestic market demand gets more price elastic the gain from the increase in B_i^d gets greater. In Figure 3.9 we observe that the optimal export market share x_i^{max} decreases for each three conditions. However, the decrease in the case the domestic market is more elastic $\epsilon_d < \epsilon_e$ is faster than the other two conditions. In fact as the demand gets more price elastic in domestic market, the speed of decrease in export market share increases. Note that the export market share at $B_i^d = 0$ is 100% ($x_i = 1$). As the B_i^d parameter grows x_i gets smaller but never become zero (theoretically) because $B_i^e = 1 > 0$ and therefore the profit maximization always result in a combination of two markets. When $B_i^e = 0$ then the export market share is 0%. Similar numerical results (not shown here) can be imagined for the B_i^e parameter.

One may simply conclude that as the demands in any of the markets expands the firms increase their investments and by the way the profits. If the increase is in domestic market demand then the firm choose to produce more for the domestic market and vice versa. If the increase is in export market demand then the share of export market will be more. In other words if the demand in



(a) π_E^{max} vs B_i^d (b) k_i^{max} vs B_i^d

Figure 3.8: Effects of B_i^d on the maximum profit and profit maximizing investment for the exporter firm.

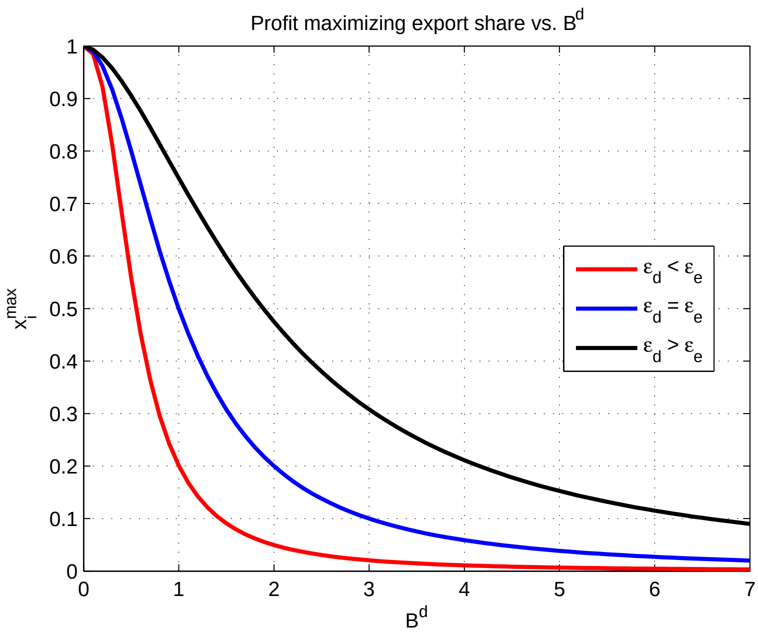


Figure 3.9: Effects of B_i^d on the profit maximizing market share x_i^{max} for the exporter firm.

domestic market is higher for any firm, then it produces more for the domestic market with respect to the one having less demand. The same is valid for the export market demand. In each case the firm has higher profit.

3.2.3 Effects of F^d and F^e

Change of k_i^{max} with respect to change in F^d and F^e:

If we investigate the profit maximizing investment equation in 3.20, we observe that neither F^d nor F^e has any effect on this solution. Therefore, we can conclude that any change in market entry sunk costs would not change the profit maximizing investment amount. In mathematical words

$$\frac{dk_i^{max}}{dF^d} = \frac{dk_i^{max}}{dF^e} = 0 \tag{3.35}$$

Change of x_i^{max} with respect to change in F^d and F^e:

Investigating the profit maximizing export market share equation in 3.21, we again observe that neither F^d nor F^e has any effect on this solution. Therefore, we can conclude that any change in market entry sunk costs would not change the profit maximizing export market share, x_i^{max}. In mathematical words

$$\frac{dx_i^{max}}{dF^d} = \frac{dx_i^{max}}{dF^e} = 0 \tag{3.36}$$

Change of π_E^{max} with respect to change in F^d and F^e:

To observe the effects of F^d and F^e on the maximum profit π_E^{max} lets investigate the equation 3.22

$$\begin{aligned} \pi_E^{max} &= B_i^d P^d \left[(1 - x_i^{max}) A_i (k_i^{max} - k_m)^\alpha\right]^{1-\epsilon_d} & (3.37) \\ &\quad + B_i^e P^e \left[x_i^{max} A_i (k_i^{max} - k_m)^\alpha\right]^{1-\epsilon_e} \\ &\quad - (1 + r)(k_i^{max} + F^d + F^e) & (3.38) \end{aligned}$$

The derivatives of this maximum profit with respect to entry costs are

$$\frac{d\pi_E^{max}}{dF^d} = \frac{\pi_E^{max}}{dF^e} = -(1 + r) \tag{3.39}$$

which means the maximum profit changes in the opposite direction with the same amount of change in entry costs. If one of the entry costs increases, say ΔF amount, then the maximum profit will decrease with the same amount.

In fact the profit function shifts upwards and downwards with the decrease or increase in the entry cost, respectively.

A typical example of changes in π_E^{max}, k_i^{max}, and x_i^{max} with respect to change in F^d are plotted in Figure 3.10. As seen there is no change in profit maximizing investment and export market shares while the maximum profit decreases with the slope $-(1+r)$. The market parameters for this figure are: $F^e = 2$, $\epsilon_d = 0.4$, $\epsilon_e = 0.5$, $\alpha = 0.7$, $A_i = 40$.

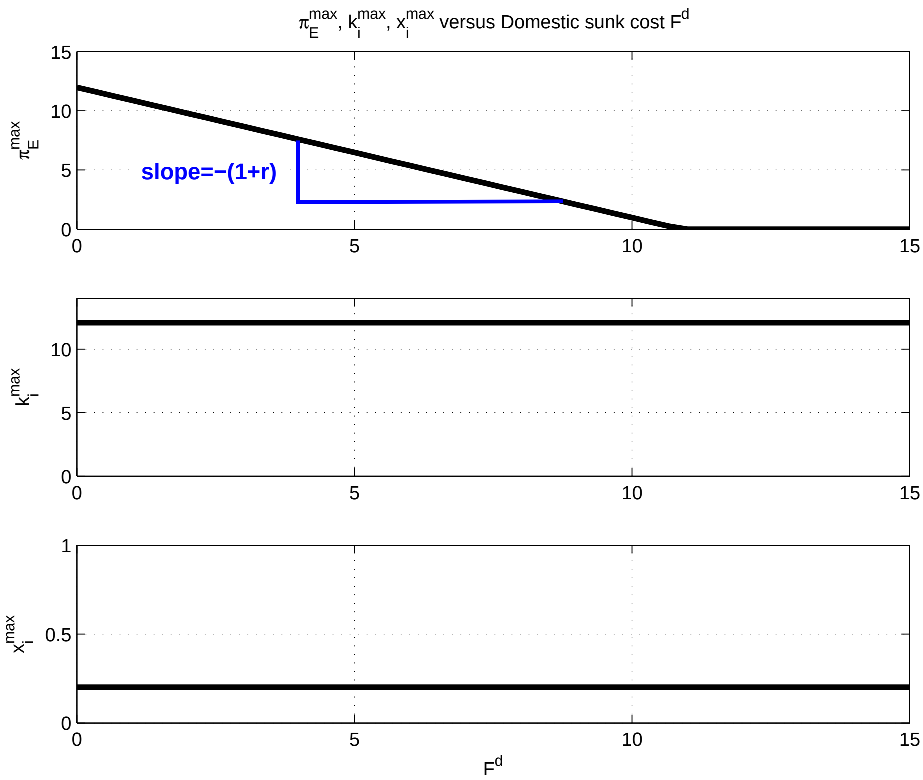


Figure 3.10: Effects of F^d on the maximum profit, profit maximizing investment and market share for the exporter firm.

As we found for the domestic market profit maximization, again the entry costs do not effect the optimal investment and market share values but instead sunk costs are the constant means that should be paid by earning revenue. If the revenue is not high enough to pay for these sunk costs then the firm will not produce for both markets (It may produce only in domestic market if domestic profit is positive).

3.2.4 Effects of k_m

Change of k_i^{max} with respect to change in k_m:

One can simply show that $dk_i^{max}/dk_m = 1$ by utilizing implicit differentiation methods in equation 3.20. Therefore, any increase in k_m would result in the same amount of increase of maximizing investment amount.

Change of x_i^{max} with respect to change in k_m:

Investigating equation 3.21, one may observe that there is no relation between x_i and k_m. In other words

$$\frac{dx_i^{max}}{dk_m} = 0 \tag{3.40}$$

Therefore, the change in minimum required investment level does not change the profit maximizing export market share.

Change of π_E^{max} with respect to change in k_m:

The derivative of maximum profit in equation 3.22 with respect to k_m is

$$\begin{aligned} \frac{d\pi_E^{max}}{dk_m} &= B_i^d P^d \left[(1-x_i^{max})A_i\right]^{1-\epsilon_d} \alpha(1-\epsilon_d)(k_i^{max}-k_m)^{\alpha(1-\epsilon_d)-1}\left(\frac{dk_i^{max}}{dk_m}-1\right) \\ &\quad + B_i^e P^e \left[x_i^{max}A_i\right]^{1-\epsilon_e} \alpha(1-\epsilon_e)(k_i^{max}-k_m)^{\alpha(1-\epsilon_e)-1}\left(\frac{dk_i^{max}}{dk_m}-1\right) \\ &\quad -(1+r)\frac{dk_i^{max}}{dk_m} \end{aligned} \tag{3.41}$$

Note that the derivative of x_i^{max} is not included this differentiation since $\frac{dx_i^{max}}{dk_m} = 0$. And recall that $\frac{dk_i^{max}}{dk_m} = 1$, then equation 3.41 becomes

$$\frac{\pi_E^{max}}{dk_m} = -(1+r)$$

Which means an increase in minimum investment level (say Δk_m) will result in $(1+r)\Delta k_m$ amount of decrease in the maximum profit.

The changes in π_E^{max}, k_i^{max}, and x_i^{max} with respect to change in k_m are plotted in Figure 3.11. As seen there is no change in the export market share while the maximum profit and optimal investment amounts changes with the slopes $-(1+r)$ and 1, respectively. The market parameters for this figure are: $F^e = 2$, $F^d = 2$ $\epsilon_d = 0.4$, $\epsilon_e = 0.5$, $\alpha = 0.7$, $A_i = 20$.

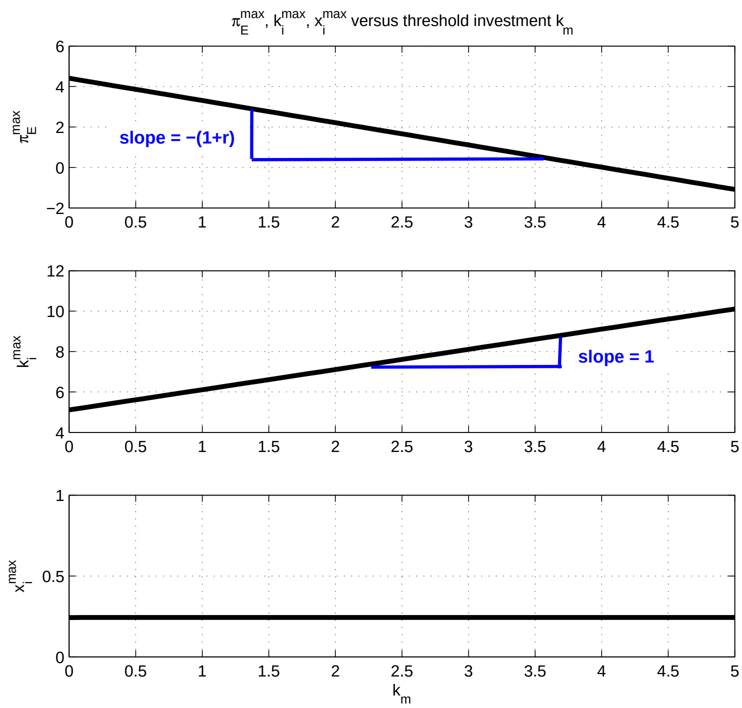


Figure 3.11: Effects of k_m on the maximum profit, profit maximizing investment and market share for the exporter firm.

The threshold investment amount does not change the market shares because this is only the least amount that the firm should afford to start production. As in domestic market profit maximization, again the necessary part of investment to maximize the profit is $k_i - k_m$; thus the firm will not change the difference $k_i - k_m$ to to maximize the profit and increase (or decrease) the same amount of increase (or decrease) of threshold investment. However, the maximum profit decreases with the ratio $1+r$ as this amount increases since this is the borrowed capital.

3.2.5 Effects of r

Change of k_i^{max} with respect to change in r:

The effect of changes in the interest rate on the investment level is derived in Appendix A.7. The implicit differentiation results in

$$\frac{dk_i^{max}}{dr} = \frac{\eta_{r1} + \eta_{r2}}{\eta_{k1} + \eta_{k2}} \tag{3.42}$$

where constants η_{r1}, η_{r2} are positive and η_{k1}, η_{k2} are all negative. Therefore, the derivative $\frac{dk_i^{max}}{dr}$ is negative. In other words the increase in interest rate would result in decrease in the profit maximizing investment amount and viceversa.

Change of x_i^{max} with respect to change in r:

The derivation of $\frac{dx_i^{max}}{dr}$ is in Appendix A.8.

$$\begin{array}{lll} \frac{dx_i^{max}}{dr} < 0 & if & \epsilon_d > \epsilon_e \\ \frac{dx_i^{max}}{dr} = 0 & if & \epsilon_d = \epsilon_e \\ \frac{dx_i^{max}}{dr} > 0 & if & \epsilon_d < \epsilon_e \end{array} \tag{3.43}$$

If domestic market is more elastic than any increase in interest rate will increase the export market share and if export market is more elastic then any increase in interest will decrease the export market share.

Change of π_E^{max} with respect to change in r:

Finding the derivative of π_E^{max} with respect to r is very difficult since the relations are in closed form. However we can simply show that the maximum profit decreases as r increases. Lets first consider the total profit function of both markets for $0 \leq x_i \leq 1$ and $k_i > k_m$

$$\begin{aligned} \pi_E &= B_i^d P^d \left[(1-x_i)A_i(k_i-k_m)^\alpha\right]^{1-\epsilon_d} \\ &\quad + B_i^e P^e \left[x_i A_i(k_i-k_m)^\alpha\right]^{1-\epsilon_e} \\ &\quad -(1+r)(k_i+F^d+F^e) \end{aligned} \tag{3.44}$$

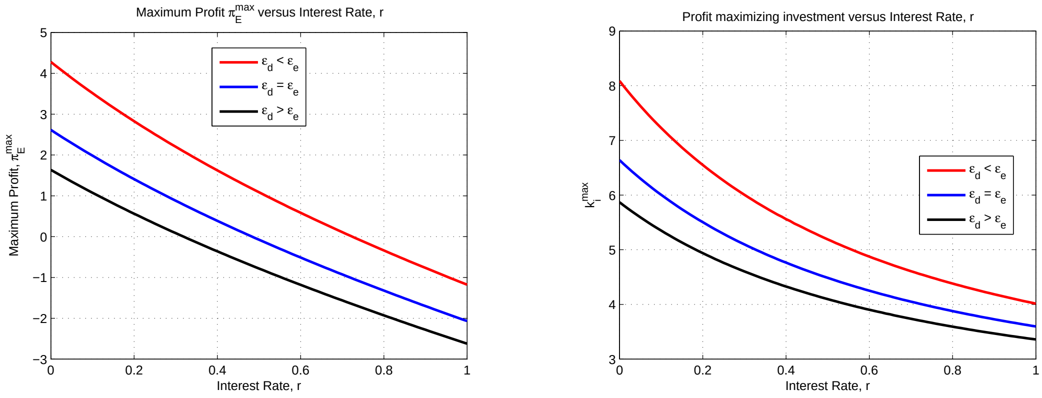
The derivative of this function with respect to r is

$$\frac{d\pi_E}{dr} = -k_i$$

Since $k_i > k_m$, $\frac{d\pi_E}{dr}$ is always negative for every $0 \leq x_i \leq 1$ and $k_i > k_m$. Therefore the profit decreases for every $0 \leq x_i \leq 1$ and $k_i > k_m$ in the case of an increase in investment. We can conclude that if profit decreases for every value of x_i and k_i then the maximum profit will also be lower when the interest increases.

In Figures 3.12 and 3.13 the maximum profit of both markets π_E^{max} and the optimal investment amounts k_i^{max}, and the optimal market shares x_i^{max} versus interest rate r are plotted under three conditions: $\epsilon_d < \epsilon_e$ ($\epsilon_d = 0.45$), $\epsilon_d = \epsilon_e$ ($\epsilon_d = 0.5$), $\epsilon_d > \epsilon_e$ ($\epsilon_d = 0.55$). $\epsilon_e = 0.5$, $\alpha = 0.7$, $A = 30$ for each case. As shown above maximum profit and optimal investment lowers as the interest rate in the market increases. The change of the export market differs according to the price elasticity of markets as seen in Figure 3.13. If domestic market is more elastic than the export market share increases with the increase in interest rate and vice versa. If the elasticity of markets are same then the share of markets do not change.

To summarize, the borrowing interest rate has negative effect on the investment and profit as we found in domestic market. On the other hand the change in the market shares depends on the price elasticities of demand functions. If the they are same the market shares does not change. If they are different then the share of the more elastic market decreases as the interest rate increases.



(a) π_E^{max} vs r (b) k_i^{max} vs r

Figure 3.12: Effects of interest rate r on the maximum profit and profit maximizing investment for the exporter firm.

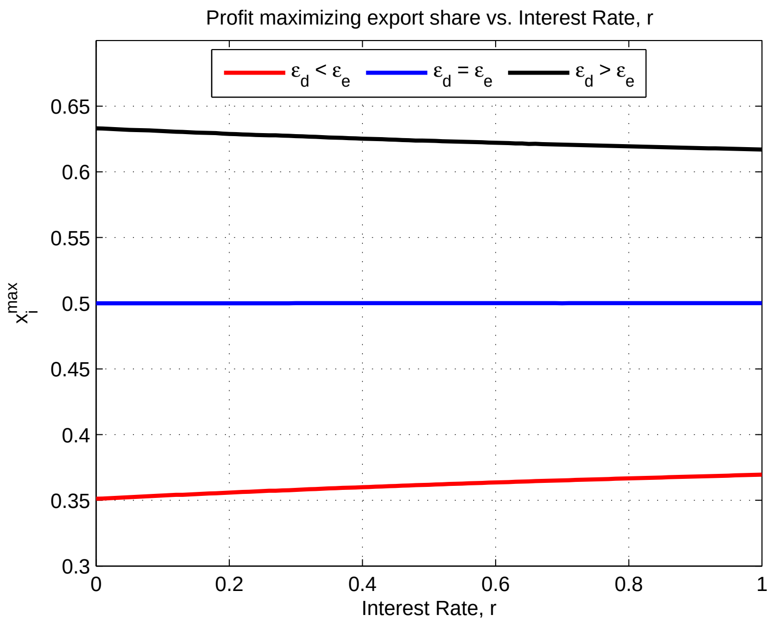


Figure 3.13: Effects of interest rate r on the profit maximizing market share x_i^{max} for the exporter firm.

3.3 Comparison of behaviors of Domestic and Export Markets

By using the above derivations we can compare the changes of maximum profits in domestic and both domestic and export markets. Recall the equation 2.94

$$k_i^{Emax} - k_m = \left[k_i^{Dmax} - k_m\right](1 - x_i)^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}} \tag{3.45}$$

If we differentiate this equation with respect to A_i we can find the difference between the changes in profit maximizing optimal investment amounts of domestic and domestic & export markets.

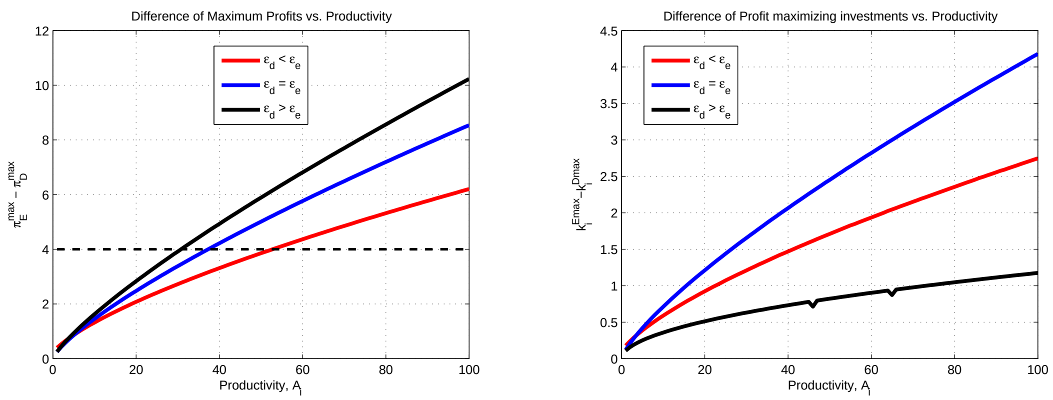
$$\begin{aligned}\frac{\partial(k_i^{Emax}-k_m)}{\partial A_i} &= \frac{\partial(k_i^{Dmax}-k_m)}{\partial A_i}(1-x_i)^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}} \\ &- \left[k_i^{Dmax}-k_m\right]\left[\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}\right](1-x_i)^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}-1} \\ &\frac{\partial x_i^{Emax}}{\partial A_i} \qquad (3.46)\end{aligned}$$

$$\begin{aligned}\frac{\partial k_i^{Emax}}{\partial A_i}-\frac{\partial k_i^{Dmax}}{\partial A_i} &= \frac{\partial k_i^{Dmax}}{\partial A_i}\left[(1-x_i)^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}}-1\right] \\ &- \left[k_i^{Dmax}-k_m\right]\left[\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}\right](1-x_i)^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}-1} \\ &\frac{\partial x_i^{Emax}}{\partial A_i} \qquad (3.47)\end{aligned}$$

In the equation 3.47, the term $(1-x_i)^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}}$ is always greater than 1 for $0 < x_i < 1$. And since $\alpha(1-\epsilon_d)-1$ is always negative the difference $\frac{\partial k_i^{Emax}}{\partial A_i}-\frac{\partial k_i^{Dmax}}{\partial A_i}$ becomes always positive for $\frac{\partial x_i^{Emax}}{\partial A_i} > 0$ which is satisfied when $\epsilon_d > \epsilon_e$ (domestic market is less price elastic than the export market) (Equation 3.25). Therefore, if the export market is more elastic then the increase in productivity will result in the increase of the gap between the optimal investment amounts. For the other cases the derivations of the behaviors of optimal investment and maximum profits are very hard to solve problems with analytical methods. Instead, we will utilize the numerical methods. For the following numerical solutions the parameters of markets and firms are $r = 0.1$, $P^d = 1$, $P^e = 1$, $F^d = 0$, $F^e = 0$ $B_i^d = 1.0$, $B_i^e = 1.0$, $A_i = 40$ unless something different is declared. Note that domestic and export market sunk costs are taken as zero to prevent negative profits. In this section we only deal with the change of the difference between maximized profit levels with respect to productivity and parameters B_i^e and B_i^d to conclude some dynamics of the firms during decision process. If these profit levels are not greater than the sunk costs then the profit will be negative and the firms will not enter any of the markets. Hence, here we assume that the sunk costs are zero. On the other hand, exporting sunk cost is taken as zero because the only additional payment in the combined market is the export market sunk cost. Therefore, we will show the gaps for the case where there is no export sunk cost to see clearly the difference between the maximum profits and optimal values. We will show F^e as a threshold that the gap between the maximum profits should exceed for the firm to enter the combined market.

In Figure 3.14 the difference of the maximum profits and the difference of the optimal investment amounts are plotted under three different conditions: $\epsilon_d < \epsilon_e$ ($\epsilon_d = 0.40$), $\epsilon_d = \epsilon_e$ ($\epsilon_d = 0.5$), $\epsilon_d > \epsilon_e$ ($\epsilon_d = 0.60$). $\epsilon_e = 0.5$, $\alpha = 0.7$ for each case. In Figure 3.14(a) the change of the difference of maximum profits ($\pi_E^{max} - \pi_D^{max}$) with respect to increasing productivity A_i is shown. As seen from the figure for each case $\epsilon_d < \epsilon_e$, $\epsilon_d = \epsilon_e$, and $\epsilon_d > \epsilon_e$ the gap between the profits grows. The gap for the case $\epsilon_d > \epsilon_e$ grows faster than the other cases. And the gap for $\epsilon_d < \epsilon_e$ is the slowest one. The maximum profit for the combined market is always higher than the maximum profit of domestic market. But for the lower productivity levels this gap can not exceed the sunk cost. As the productivity gets higher the combined market maximum profit grows faster than the maximum profit of domestic market so that exceeds this additional payment. Therefore, the firms with lower productivity opts to enter only domestic market while the ones with higher productivity opt to enter both markets. If the export market is more elastic then the gap exceeds F^e at a lower productivity level. Note that, if the firm is already in the export market then she would not pay for the export market sunk cost F^e and therefore the maximum profit of combined market always becomes higher than the domestic market maximum profit and so she definitely opt to stay in the export market whether the productivity level is low or high.

In Figure 3.14 (b) the gap between the optimal investment amounts $k_i^{Emax} - k_i^{Dmax}$ is plotted versus productivity. Again the gap between optimal investments gets greater as the productivity increases. The optimal value of investment for the combined market is always higher than the optimal value in domestic market for higher productivity levels. The firms (if able to) should choose to make higher investments for profit maximization in combined market. On the other hand, opposite to the gap of maximum profits, the gap between optimal investment grows fastest for equal price elasticities and lower for the other cases. We cannot compare the speeds of cases $\epsilon_d > \epsilon_e$ and $\epsilon_d < \epsilon_e$, since the highest one will depend on the numerical values.



(a) $\pi_E^{max} - \pi_D^{max}$ vs A_i (b) $k_i^{Emax} - k_i^{Dmax}$ vs A_i

Figure 3.14: Effects of productivity A_i on the gap between maximum profits and profit maximizing investments.

In Figure 3.15 the change in the export market share with respect to increase in productivity is plotted for the same three cases. As seen for the case $\epsilon_d < \epsilon_e$ the export market share decreases. For $\epsilon_d = \epsilon_e$ this share does not change and for $\epsilon_d > \epsilon_e$ it increases. However, as stated above the profit gap grows in every case (whether the export share decreases or increases).

The behaviors of the gaps between the maximum profits and the optimal investment amounts under the change of B^d parameter are plotted in Figures 3.16(a) and (b) under three different conditions: $\epsilon_d < \epsilon_e$ ($\epsilon_d = 0.40$), $\epsilon_d = \epsilon_e$ ($\epsilon_d = 0.5$), $\epsilon_d > \epsilon_e$ ($\epsilon_d = 0.60$). $\epsilon_e = 0.5$, $\alpha = 0.7$, and $B^e = 1$ for each

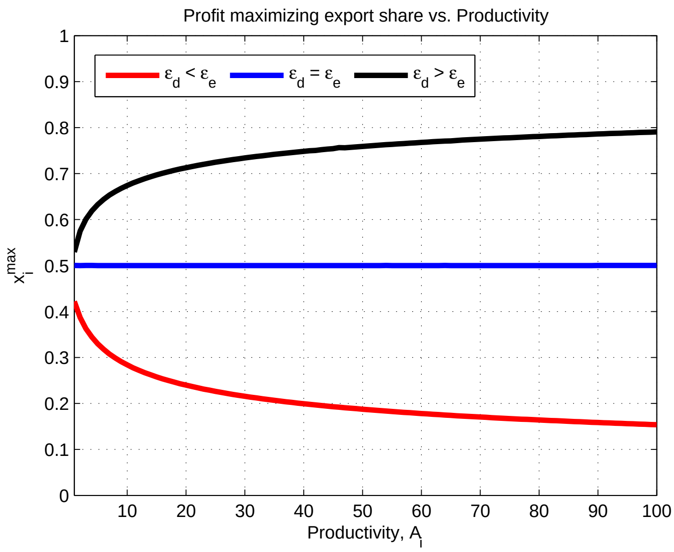
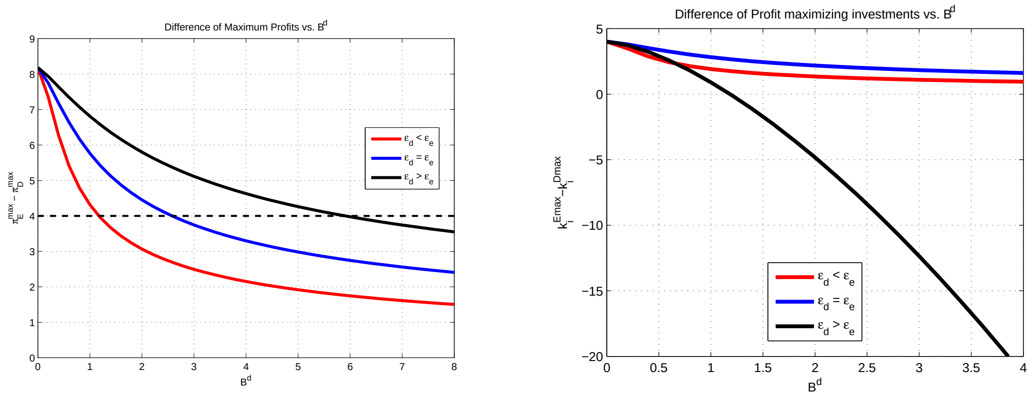


Figure 3.15: Effects of productivity A_i on the profit maximizing market share.

case. As seen form the Figure 3.16(a), the maximum profit of combined market π_E^{max} is always higher than the maximum profit of domestic market. Although it gets lower, the difference never gets zero or negative (or probably gets zero at infinity). The threshold at F^e is exceeded at the beginning. Therefore, for the firms with lower B^d parameter the preferred market is the combination of domestic and export markets. As the B^d parameter gets higher the gap between the maximum profits will not exceed the export market sunk cost and so the firm will prefer to enter only domestic market. On the other hand if the domestic market is more price elastic then the gap will decrease faster. Therefore, for the case domestic market is more price elastic the firms will choose to enter only the domestic market at lower B_i^d values. Note again that if the firm is already in the export market then she will not pay for the sunk cost F^e and so the profit in combined market will be higher whether the B^d parameter is lower or higher.

Investigating the Figure 3.16(b) we observe that the optimal investment gap decreases for each case but never becomes zero ($k_i^{Emax} > k_i^{Dmax}$) for $\epsilon_d \leq \epsilon_e$ (domestic market is more/equal price elastic). For the case where export market is more price elastic the optimal investment in domestic market exceeds the one in combined market but the recall that the maximum profit of domestic market is still lower than the maximum profit of combined market.



(a) $\pi_E^{max} - \pi_D^{max}$ vs B^d

(b) $k_i^{Emax} - k_i^{Dmax}$ vs B^d

Figure 3.16: Effects of B^d on the gap between maximum profits and profit maximizing investments.

In Figure 3.17 the share of export market versus B_i^d is plotted. For every case the export market share decreases as B_i^d increases. Note that the speed of decline in x_i is faster for the case domestic market is more price elastic.

In Figure 3.18(a), the gap between the maximum profits $\pi_E^{max} - \pi_D^{max}$ versus the export market parameter B_i^e is plotted for again under three different conditions: $\epsilon_d < \epsilon_e$ ($\epsilon_d = 0.40$), $\epsilon_d = \epsilon_e$ ($\epsilon_d = 0.5$), $\epsilon_d > \epsilon_e$ ($\epsilon_d = 0.60$). $\epsilon_e = 0.5$, $\alpha = 0.7$, and $B^d = 1$ for each case. This gap is positive for every $B_i^e > 0$. Therefore, the combined market maximum profit is greater than the domestic market maximum profit. The firms will opt to enter the combined market when this gap exceeds the threshold F^e. When the export market is more price elastic the speed of grow of the gap is higher then the other cases. Therefore, the firms

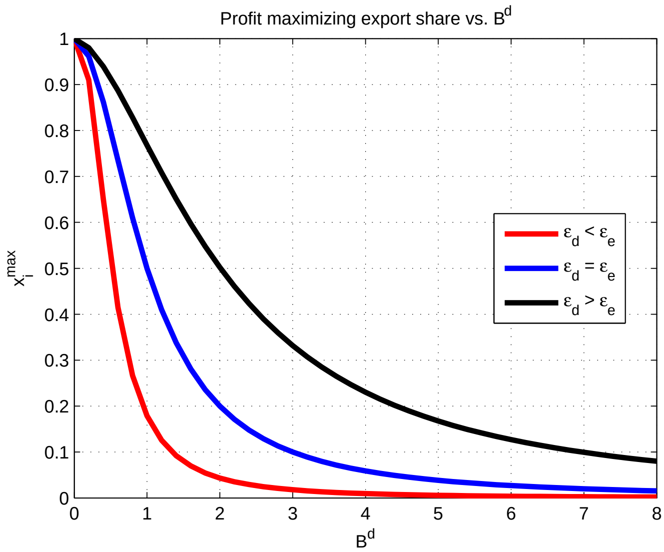
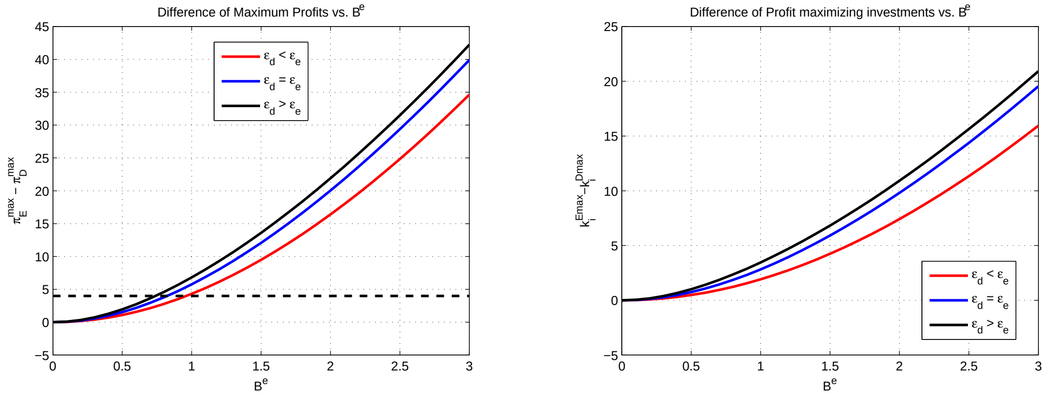


Figure 3.17: Effects of B^d on the profit maximizing market share.

enter the combined market at lower B_i^e values for this case. Again note that if the firm is already in the combined market then she will not pay the exporting sunk cost F^e. The firms already in export market will definitely choose to stay in the export market since the gap is always positive.

In the Figure 3.18(b), the profit maximizing investment differences versus parameter B_i^e is plotted. As seen this gap is negative for the case domestic market is less elastic because the optimal value of the profit maximization in domestic market is higher than the one in combined market. Whether the domestic or export market is more price elastic the gap continues to increase as the B_i^e parameter increases.

The change of export market share with respect to the changes in B_i^e is plotted in Figure 3.19. In each case the export market share increases. This increase is faster when the export market price elasticity is higher.



(a) $\pi_E^{max} - \pi_D^{max}$ vs B^e (b) $k_i^{Emax} - k_i^{Dmax}$ vs B^e

Figure 3.18: Effects of B^e on the gap between maximum profits and profit maximizing investments.

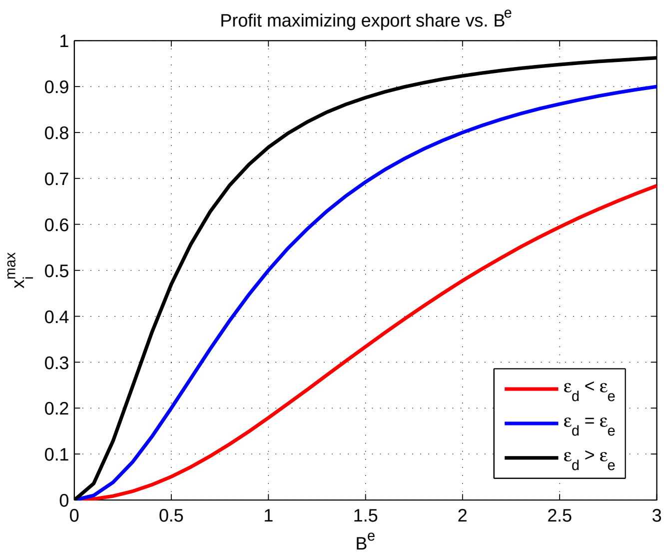


Figure 3.19: Effects of B^e on the profit maximizing market share.

3.4 Performance Measures

To compare the performances or efficiencies of the exporters and non-exporters some measures like profit per investment (k_i), profit per total investment ($k_i + F^d + F^e$), total production per investment may be utilized. We derive some analytical relationships between these measures of the exporters and non-exporters.

3.4.1 Profit/Investment Ratio

The profit per investment can be used as a measure to compare the performances of exporters and non-exporters. The relation between the profit/invetment ratios of the exporters and non-exporters are derived in Appendix A.9 as

$$\begin{aligned}\frac{\pi_E^{max}}{k_i^{Emax} - k_m} - \frac{\pi_D^{max}}{k_i^{Dmax} - k_m} &= x_i^{Emax} \frac{1+r}{\alpha(1-\epsilon_e)(1-\epsilon_d)}(\epsilon_e - \epsilon_d) \\ &+ \frac{(1+r)k_m + F^d}{(k_i^{Emax} - k_m)} \left((1 - x_i^{Emax})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}} - 1 \right) \\ &- \frac{F^e}{k_i^{Emax} - k_m} \end{aligned} \tag{3.48}$$

Using this relation we can consider the following cases.

1. No threshold investment and no sunk costs:
   For the simplest case, let's assume $k_m = F^d = F^e = 0$. Then equation 3.48 becomes

   $$\frac{\pi_E^{max}}{k_i^{Emax}} - \frac{\pi_D^{max}}{k_i^{Dmax}} = x_i^{Emax} \frac{1+r}{\alpha(1-\epsilon_e)(1-\epsilon_d)}(\epsilon_e - \epsilon_d)$$

   The term on the right side of the equation is positive if $\epsilon_d < \epsilon_e$ and negative if $\epsilon_e < \epsilon_d$. This means that the profit/efficient investment ratio will be higher in the market which is less price elastic if there is no threshold investment and no entry costs. In Figure 3.20 the differences of profit/investment ratios of export and domestic market vs productivity is plotted under three different conditions: $\epsilon_d < \epsilon_e$ ($\epsilon_d = 0.45$, $\epsilon_e = 0.5$), $\epsilon_d = \epsilon_e$ ($\epsilon_d = 0.5$, $\epsilon_e = 0.5$), $\epsilon_d > \epsilon_e$ ($\epsilon_d = 0.55$, $\epsilon_e = 0.45$). $\alpha = 0.7$, and $B^e = B^d = 1$ for each case. As seen from the figure the profit/investment ratio is always positive but decreasing for case $\epsilon_d < \epsilon_e$. As the productivity of firms increase the gain per investment difference decreases for the exporter firms. For the case, price elasticity of markets are equal $\epsilon_d = \epsilon_e$

the profit/investment ratios of the markets are equal, therefore the difference is always zero on the figure. If the export market is more price elastic then the gain per investment will be higher for the non-exporters. This difference is greater for the more productive firms.

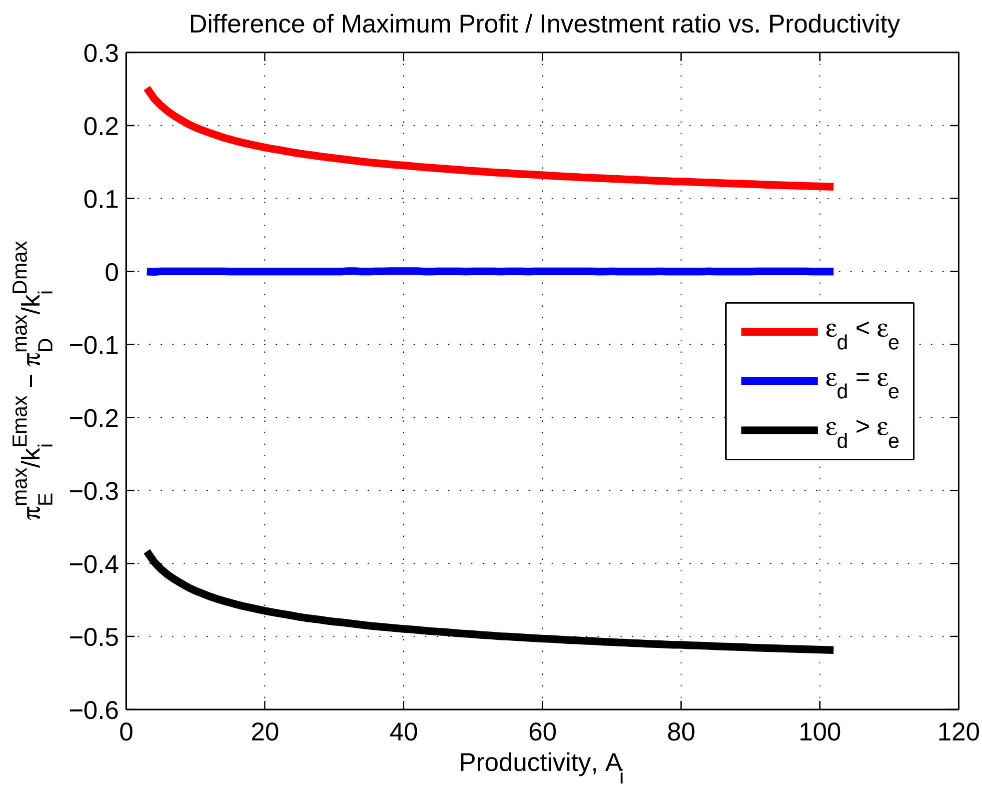
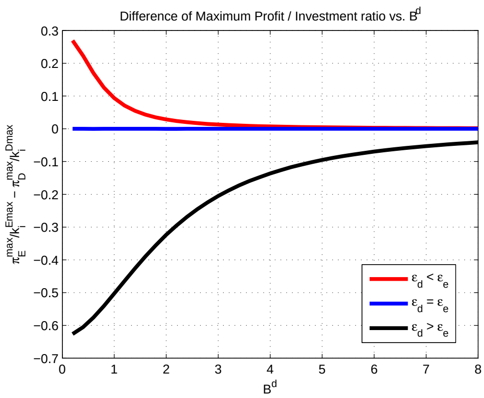


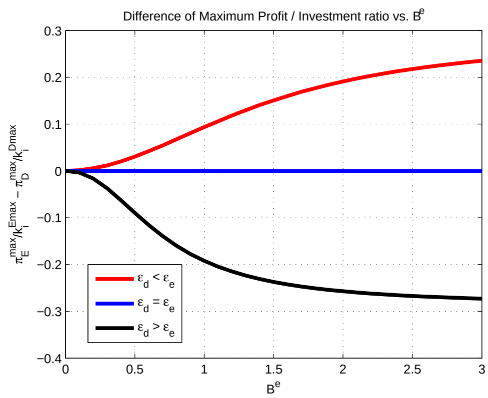
Figure 3.20: Effects of A_i on the profit/investment difference of domestic and export markets.

In Figures 3.21(a) and (b) the differences of profit/investment ratios of export and domestic market vs B^d and B_i^e are plotted under three different conditions: $\epsilon_d < \epsilon_e$ ($\epsilon_d = 0.45$, $\epsilon_e = 0.55$), $\epsilon_d = \epsilon_e$ ($\epsilon_d = 0.5$, $\epsilon_e = 0.5$), $\epsilon_d > \epsilon_e$ ($\epsilon_d = 0.55$, $\epsilon_e = 0.45$). $A_i = 60$, $\alpha = 0.7$, and $B^e = 1$ and $B^d = 1$

in figures (a) and (b) respectively for each case. As seen from the figures (a) and (b) the difference of the profit/investment ratios $\frac{\pi_E^{max}}{k_i^{Emax}} - \frac{\pi_D^{max}}{k_i^{Dmax}}$ is positive when domestic market is more price elastic, zero when the price elasticity of markets are same, and negative if export market is more price elastic. Note that the differences converges to zero as the demand in export market increases and demand in export market contracts.



(a) $\frac{\pi_E^{max}}{k_i^{Emax}} - \frac{\pi_D^{max}}{k_i^{Dmax}}$ vs B^d



(b) $\frac{\pi_E^{max}}{k_i^{Emax}} - \frac{\pi_D^{max}}{k_i^{Dmax}}$ vs B^e

Figure 3.21: Effects of B_i^d and B_i^e on the profit/investment ratio difference of domestic and export markets. No threshold Investment and no sunk costs.

2. Threshold investment is positive and no sunk costs:

   For this case, $k_m > 0$ and $F^d = F^e = 0$. Then equation 3.48 becomes

$$\frac{\pi_E^{max}}{k_i^{Emax} - k_m} - \frac{\pi_D^{max}}{k_i^{Dmax} - k_m} = x_i^{Emax} \frac{1+r}{\alpha(1-\epsilon_e)(1-\epsilon_d)}(\epsilon_e - \epsilon_d) + \frac{(1+r)k_m}{(k_i^{Emax} - k_m)} \left((1 - x_i^{Emax})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}} - 1 \right) \tag{3.49}$$

   The second term on the right side is always positive. Therefore, if domestic market is equal or more elastic than the export market $\epsilon_d \leq \epsilon_e$, then the difference is always positive and so the profit/investment ratio is always higher in the export market. If the export market price elasticity is higher then the difference is positive when

$$x_i^{Emax}\frac{1+r}{\alpha(1-\epsilon_e)(1-\epsilon_d)}(\epsilon_d-\epsilon_e) < \frac{(1+r)k_m}{(k_i^{Emax}-k_m)}\left((1-x_i^{Emax})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}}-1\right)$$

and negative if

$$x_i^{Emax}\frac{1+r}{\alpha(1-\epsilon_e)(1-\epsilon_d)}(\epsilon_d-\epsilon_e) > \frac{(1+r)k_m}{(k_i^{Emax}-k_m)}\left((1-x_i^{Emax})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}}-1\right)$$

Note that for the case exporter gain per efficient investment is higher we can conclude that the gain per investment is also higher (see Appendix A.9). However if non-exporter's gain from efficient investment is higher then the direction of gain per investment will depend on the parameter values.

In Figure 3.22 the differences of profit/investment ratios of export and domestic market vs productivity is plotted under three different conditions: $\epsilon_d < \epsilon_e$ ($\epsilon_d = 0.45$, $\epsilon_e = 0.55$), $\epsilon_d = \epsilon_e$ ($\epsilon_d = 0.5$, $\epsilon_e = 0.5$), $\epsilon_d > \epsilon_e$ ($\epsilon_d = 0.55$, $\epsilon_e = 0.45$). $\alpha = 0.8$, and $B^e = B^d = 1$, $k_m = 1$ for each case. As seen from the figure, if the domestic market is equal or more price elastic then the gain per ivestment is always positive. For the higher productive firms this gain is less (converging to zero). For the case where the export market is more elastic the difference of profit/investment ratios is positive for the less productive firms and negative for the higher productive firms.

In Figures 3.23(a) and (b) the differences of profit/investment ratios of export and domestic market vs B_i^d and B_i^e are plotted under three different conditions: $\epsilon_d < \epsilon_e$ ($\epsilon_d = 0.45$, $\epsilon_e = 0.55$), $\epsilon_d = \epsilon_e$ ($\epsilon_d = 0.5$, $\epsilon_e = 0.5$), $\epsilon_d > \epsilon_e$ ($\epsilon_d = 0.55$, $\epsilon_e = 0.45$). $A_i = 100$, $\alpha = 0.8$, and $B^e = 1$ and $B^d = 1$ in figures (a) and (b) respectively for each case. As seen from the figures (a) and (b) the difference of the profit/investment ratios $\frac{\pi_E^{max}}{k_i^{Emax}} - \frac{\pi_D^{max}}{k_i^{Dmax}}$ is positive and decreasing when domestic market is equal

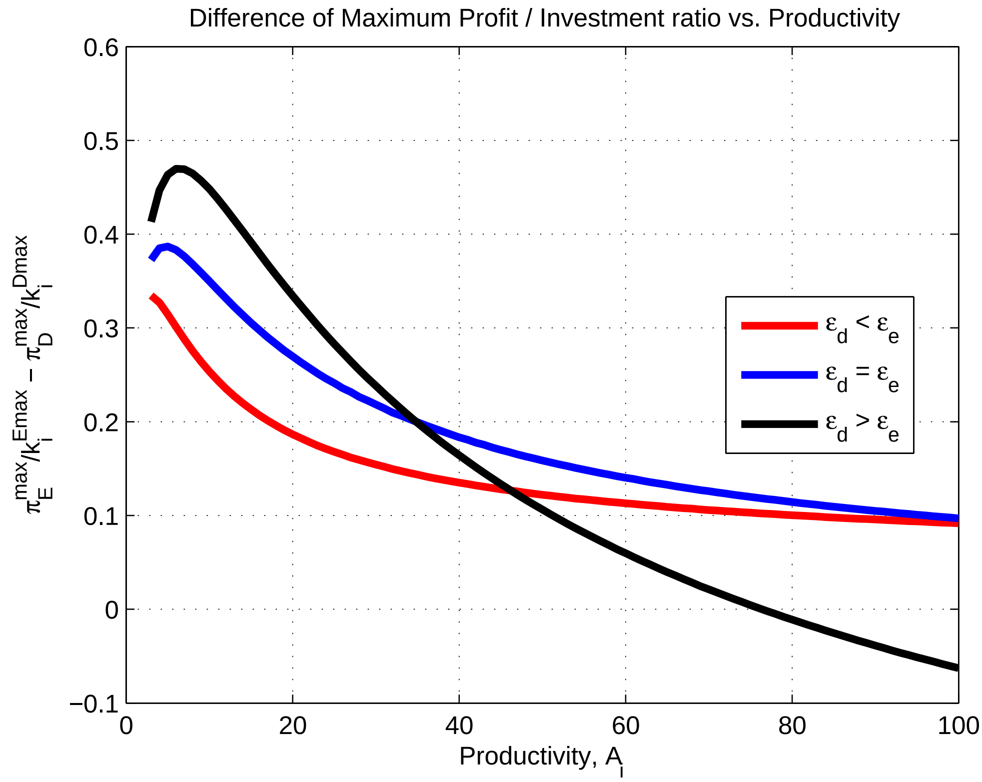
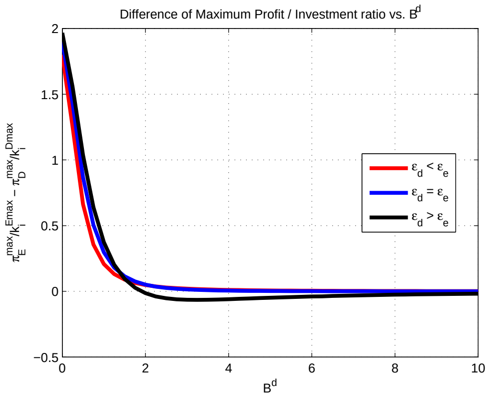


Figure 3.22: Effects of A_i on the profit/investment difference of domestic and export markets. Positive threshold investment and no sunk costs

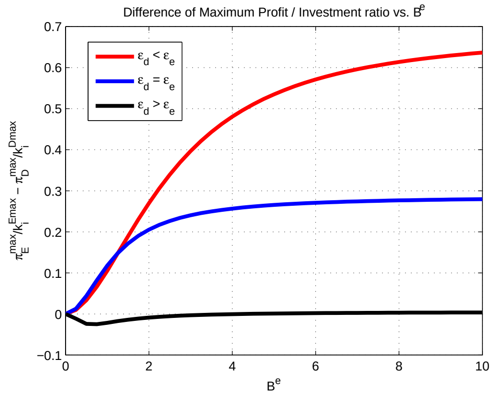
or more price elastic (converges to zero). If the export market is more price elastic then this difference is positive for very low domestic demand and becomes negative for the higher domestic demands. Note that the differences converges to zero as the demand in export market increases and demand in export market contracts.

Additionally in figure 3.24 the results derived by the same parameters except $A_i = 20$ is shown. As seen from the figure the gap between the gain per investment never becomes zero. The less productive firms may always find the gain per investment in export market more.

3. Threshold investment and domestic market sunk cost are positive:



(a) $\frac{\pi_E^{max}}{k_i^{Emax}} - \frac{\pi_D^{max}}{k_i^{Dmax}}$ vs B^d



(b) $\frac{\pi_E^{max}}{k_i^{Emax}} - \frac{\pi_D^{max}}{k_i^{Dmax}}$ vs B^e

Figure 3.23: Effects of B_i^d and B_i^e on the profit/investment ratio difference of domestic and export markets. Positive threshold investment and no sunk costs.

For this case, $k_m > 0$ and $F^d > 0$ and $F^e = 0$. Then equation 3.48 becomes

$$\begin{aligned}\frac{\pi_E^{max}}{k_i^{Emax} - k_m} - \frac{\pi_D^{max}}{k_i^{Dmax} - k_m} &= x_i^{Emax} \frac{1+r}{\alpha(1-\epsilon_e)(1-\epsilon_d)}(\epsilon_e - \epsilon_d) \\ &+ \frac{(1+r)k_m}{(k_i^{Emax} - k_m)} \left((1 - x_i^{Emax})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}} - 1\right) \\ &+ \frac{(1+r)F^d}{(k_i^{Emax} - k_m)} \left((1 - x_i^{Emax})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}} - 1\right)\end{aligned} \tag{3.50}$$

The only difference between the equations 3.49 and 3.50 is the positive term

$$\frac{(1+r)F^d}{(k_i^{Emax} - k_m)} \left((1 - x_i^{Emax})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}} - 1\right)$$

Domestic market sunk cost has negative effect on gain per investment amounts of both export and domestic markets but affects domestic market ratio more than the export market's. The numerical results will be similar with the previous case but this time the gain per investment in export market is more higher than the one in domestic market.

In Figure 3.25 the differences of profit/investment ratios of export and

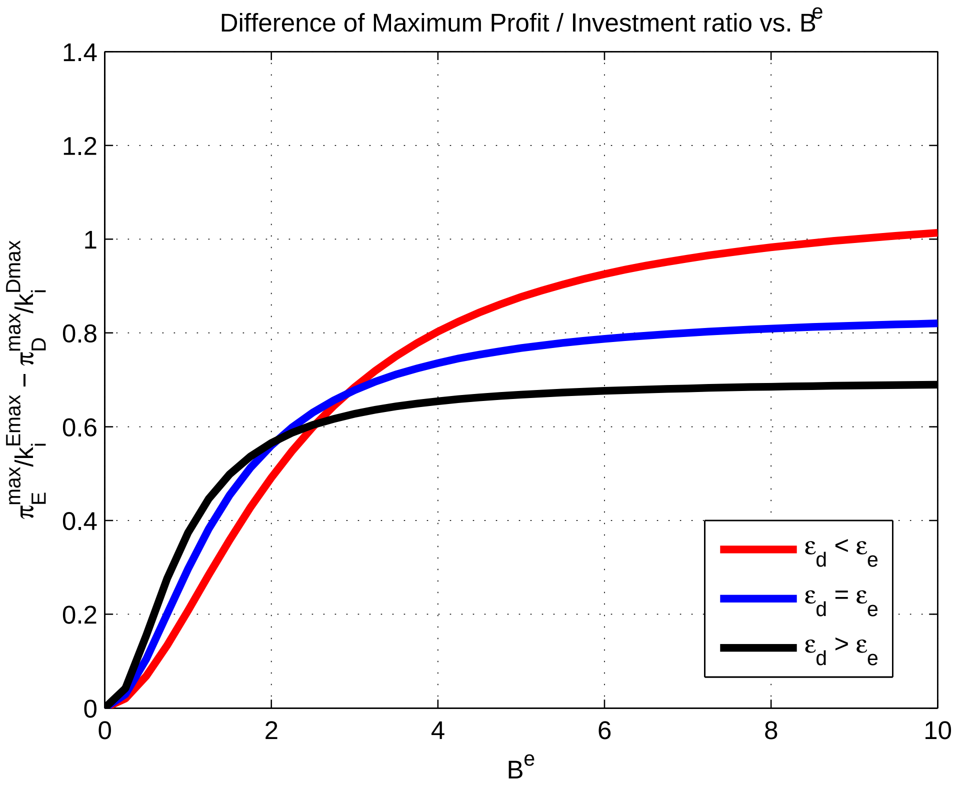


Figure 3.24: Effects of A_i on the profit/investment difference of domestic and export markets. Positive threshold investment and no sunk costs

domestic market vs productivity is plotted under three different conditions: $\epsilon_d < \epsilon_e$ ($\epsilon_d = 0.45$, $\epsilon_e = 0.55$), $\epsilon_d = \epsilon_e$ ($\epsilon_d = 0.5$, $\epsilon_e = 0.5$), $\epsilon_d > \epsilon_e$ ($\epsilon_d = 0.55$, $\epsilon_e = 0.45$). $\alpha = 0.8$, $B^e = B^d = 1$, $k_m = 1$ and $F^d = 1$ for each case. As seen from the figure the exporter's gain per investment is higher than the non-exporter's gain the productivity of firm is higher or lower for $\epsilon_d \leq \epsilon_e$. On the other hand the firms with higher productivity will gain more per investment when they are non-exporters. The results are similar with the previous case except for some productiveness levels the gain per investment for the same productive exporters is higher then the gain of non-exporters in this case while the reverse is valid for the previous case.

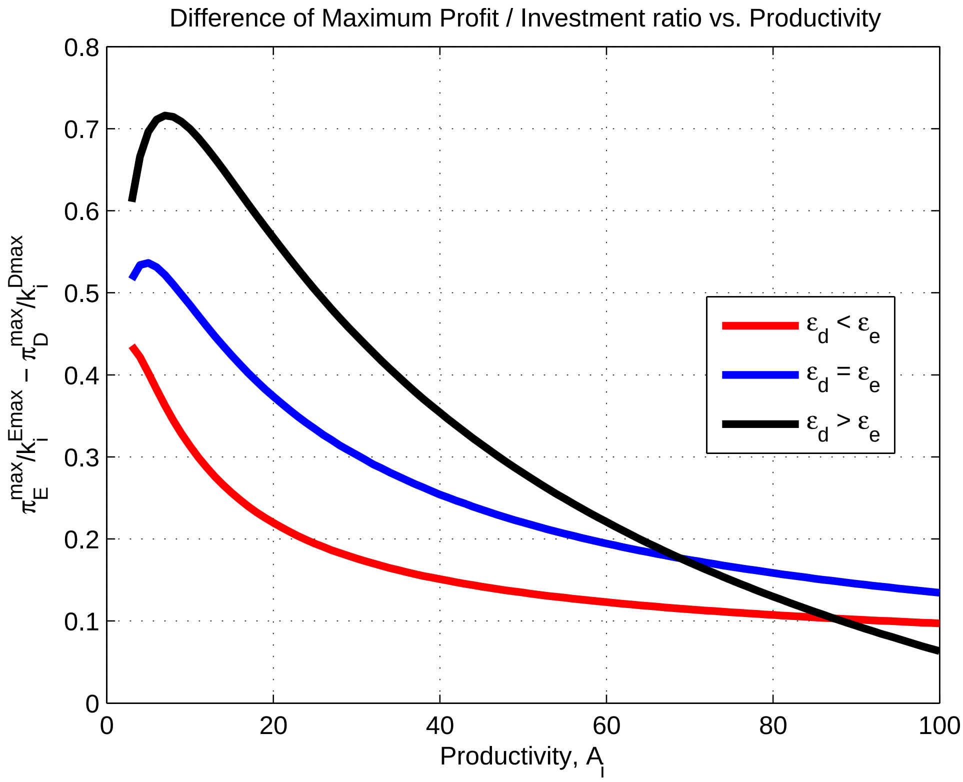
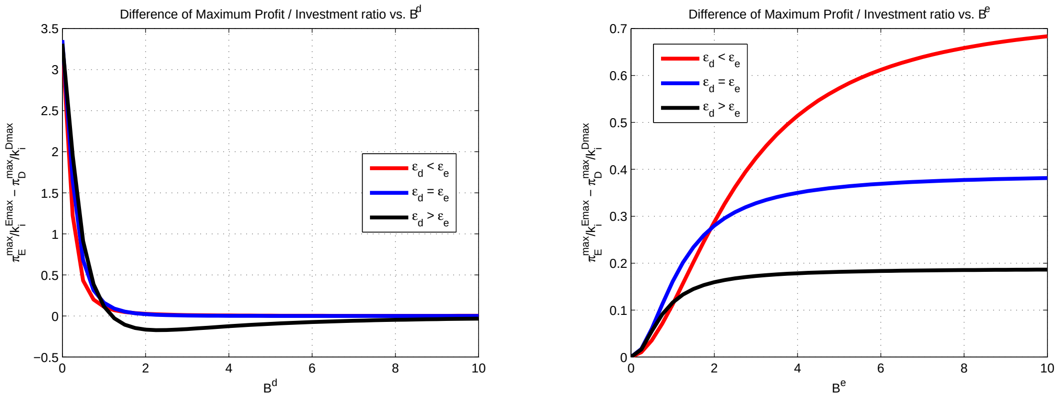


Figure 3.25: Effects of A_i on the profit/investment difference of domestic and export markets. Positive threshold investment and positive domestic sunk cost.

In Figures 3.26(a) and (b) the differences of profit/investment ratios of export and domestic market vs B_i^d and B_i^e are plotted under three different conditions: $\epsilon_d < \epsilon_e$ ($\epsilon_d = 0.45$, $\epsilon_e = 0.55$), $\epsilon_d = \epsilon_e$ ($\epsilon_d = 0.5$, $\epsilon_e = 0.5$), $\epsilon_d > \epsilon_e$ ($\epsilon_d = 0.55$, $\epsilon_e = 0.45$). $A_i = 100$, $\alpha = 0.8$, $F^d = 1$. $B^e = 1$, and $B^d = 1$ in figures (a) and (b) respectively for each case. The demand expansion in domestic market results in increase in the profit/investment ratio for both exporter and non-exporters. However, the increase of the non-exporter is higher than the exporter. The gap converges to zero as the domestic demand expands and becomes negative for higher productive firms when the export market is more price elastic.



(a) $\frac{\pi_E^{max}}{k_i^{Emax}} - \frac{\pi_D^{max}}{k_i^{Dmax}}$ vs B^d (b) $\frac{\pi_E^{max}}{k_i^{Emax}} - \frac{\pi_D^{max}}{k_i^{Dmax}}$ vs B^e

Figure 3.26: Effects of B_i^d and B_i^e on the profit/investment ratio difference of domestic and export markets. Positive threshold investment and positive domestic sunk cost.

4. Threshold investment, domestic and export market sunk costs are positive:

   For this case, $k_m > 0$ and $F^d > 0$ and $F^e > 0$. Then equation 3.48 is the case to be considered. The difference from the previous case is the negative term:

$$-\frac{(1+r)F^e}{k_i^{Emax} - k_m}$$

   Export market sunk cost decreases only export market gain per investment therefore the results in the previous case will shift downwards. The amount of shift will be different at each productivity level since k_i^{Emax} is different for each productivity; in fact, the more productive firms' profit/investment ratio will be less affected since $-\frac{(1+r)F^e}{k_i^{Emax}-k_m}$ is less for higher k_i^{Emax} values. Recall that we showed k_i^{Emax} values are higher for more productive firms. Therefore, the more productive firms are less affected due to the export market sunk costs. In Figure 3.27 the change of the gap with respect to productivity increases is shown under three different conditions: $\epsilon_d < \epsilon_e$ ($\epsilon_d = 0.45$, $\epsilon_e = 0.55$), $\epsilon_d = \epsilon_e$ ($\epsilon_d = 0.5$, $\epsilon_e = 0.5$),

$\epsilon_d > \epsilon_e$ ($\epsilon_d = 0.55$, $\epsilon_e = 0.45$). $A_i = 100$, $\alpha = 0.8$, $F^d = 1$, $F^e = 2$, $B^e = 1$, $B^d = 1$. Comparing with the results in Figure 3.25 we see that the less productive firms are more affected due to the export market sunk cost.

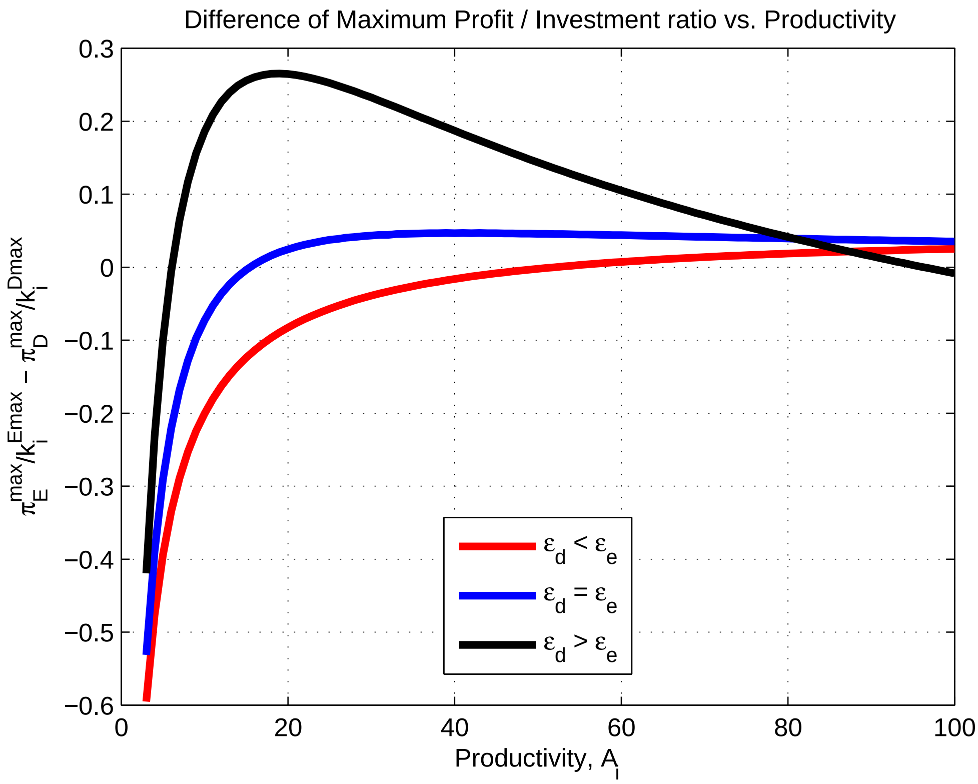
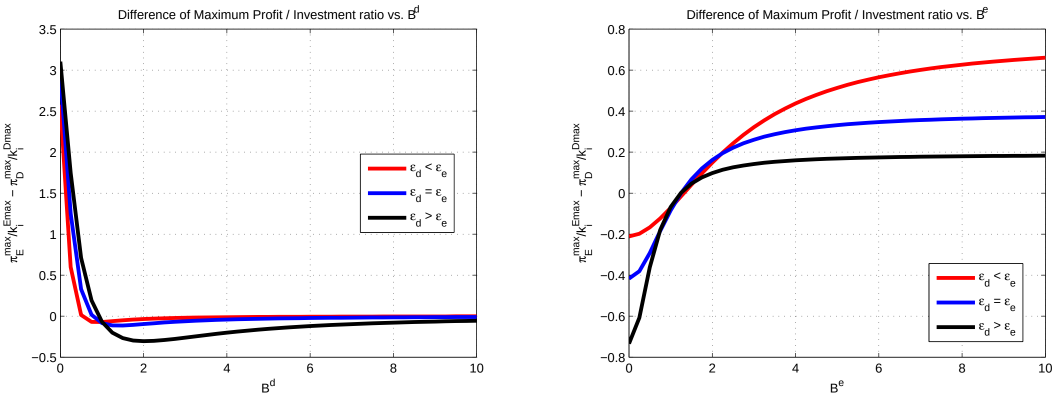


Figure 3.27: Effects of A_i on the profit/investment difference of domestic and export markets. Positive threshold investment and positive domestic and export market sunk cost.

In Figures 3.28 (a) and (b) the differences of profit/investment ratios of export and domestic market vs B_i^d and B_i^e are plotted under three different conditions: $\epsilon_d < \epsilon_e$ ($\epsilon_d = 0.45$, $\epsilon_e = 0.55$), $\epsilon_d = \epsilon_e$ ($\epsilon_d = 0.5$, $\epsilon_e = 0.5$), $\epsilon_d > \epsilon_e$ ($\epsilon_d = 0.55$, $\epsilon_e = 0.45$). $A_i = 100$, $\alpha = 0.8$, $F^d = 1$, $F^e = 4$ for each case. $B^e = 1$, and $B^d = 1$ in figures (a) and (b) respectively. As seen from the figure (a) the exporters gain per investment shifts downwards with respect to the result in 3.26(a). Therefore, as the domestic market expands the gain per investment gets higher in the non-exporting case whether the domestic or export market is more price elastic. As seen in figure (b), the gain per investment for non-exporting firms is higher for lower B_i^e values. As the export market continues to expand the gain in exporting exceeds the gain the non-exporting.



(a) $\frac{\pi_E^{max}}{k_i^{Emax}} - \frac{\pi_D^{max}}{k_i^{Dmax}}$ vs B^d (b) $\frac{\pi_E^{max}}{k_i^{Emax}} - \frac{\pi_D^{max}}{k_i^{Dmax}}$ vs B^e

Figure 3.28: Effects of B_i^d and B_i^e on the profit/investment ratio difference of domestic and export markets. Positive threshold investment and positive domestic and export market sunk cost.

The results show that the price elasticity of the market is an important parameter affecting the gain per investment. If the export market is less price elastic then the gain of the exporting firm from each investment is higher for any productiveness. However, if the export market is more price elastic then the ratio profit/investment may be higher for the exporting firms if they are less productive. For the higher productive firms the gain due to production only in domestic market is higher then the exporting case. The higher productive firms invests more and the returns from investments gets lower as the investment increases. Therefore the ratios gets closer to each other (or the difference converges to zero). This is mainly due to the decreasing returns to scale properties of the markets. In addition, the previous results show that the lower productive firms stay in domestic market and the higher productive ones enter the export market. These two results show that the self-selection of firms results in the less efficient usage of the sources in a country.

3.4.2 Profit/Total Investment Ratio

In some industries the entry costs of markets are observable. In other words the costs that firms pay for entering the markets, can be measured. For these firms the ratio $\frac{\pi}{k_i+F^d+F^e}$ can be utilized as a measure of the efficiency. In the previous section we found the relation between the ratios $\frac{\pi_E^{max}}{k_i^{Emax}-k_m}$ and $\frac{\pi_D^{max}}{k_i^{Dmax}-k_m}$. For the efficiency criterion profit/total investment (where total investment is $k_i + F^d + F^e$) one should compare the ratios $\frac{\pi_E^{max}}{k_i^{Emax}+F^d+F^e}$ and $\frac{\pi_D^{max}}{k_i^{Dmax}+F^d}$. We can rearrange these ratios as:

$$
\begin{aligned}
\frac{\pi_E^{max}}{k_i^{Emax}+F^d+F^e} &= \frac{\pi_E^{max}}{k_i^{Emax}-k_m}\left(\frac{k_i^{Emax}-k_m}{k_i^{Emax}+F^d+F^e}\right) \\
\frac{\pi_D^{max}}{k_i^{Dmax}+F^d} &= \frac{\pi_D^{max}}{k_i^{Dmax}-k_m}\left(\frac{k_i^{Dmax}-k_m}{k_i^{Dmax}+F^d}\right)
\end{aligned} \tag{3.51}
$$

If $\frac{\pi_E^{max}}{k_i^{Emax}-k_m} > \frac{\pi_D^{max}}{k_i^{Dmax}-k_m}$ and $\frac{k_i^{Emax}-k_m}{k_i^{Emax}+F^d+F^e} > \frac{k_i^{Dmax}-k_m}{k_i^{Dmax}+F^d}$ then $\frac{\pi_E^{max}}{k_i^{Emax}+F^d+F^e} > \frac{\pi_D^{max}}{k_i^{Dmax}+F^d}$ and similarly if $\frac{\pi_E^{max}}{k_i^{Emax}-k_m} < \frac{\pi_D^{max}}{k_i^{Dmax}-k_m}$ and $\frac{k_i^{Emax}-k_m}{k_i^{Emax}+F^d+F^e} < \frac{k_i^{Dmax}-k_m}{k_i^{Dmax}+F^d}$ then $\frac{\pi_E^{max}}{k_i^{Emax}+F^d+F^e} < \frac{\pi_D^{max}}{k_i^{Dmax}+F^d}$. For the other cases the result will depend on the numerical values. Furthermore, we may state that the export market sunk cost has a negative effect on this efficiency measure of the exporter and does not affect the non-exporter's efficiency. Therefore, for higher F^e values the efficiency of the exporter will decrease drastically and become worse than the non-exporter's efficiency.

3.4.3 Production/Investment Ratio

The ratio, production per investment may also be utilized as a measure for the efficiencies of the firms. We can simply show the relation between the ratios $\frac{Q_i}{k_i^{max}-k_m}$ of exporters and non-exporters

$$\frac{Q_i^{exporter}}{k_i^{Emax}-k_m} = \frac{A_i(k_i^{Emax}-k_m)^\alpha}{k_i^{Emax}-k_m} = \frac{A_i}{(k_i^{Emax}-k_m)^{1-\alpha}} \tag{3.52}$$

$$\frac{Q_i^{non-exporter}}{k_i^{Dmax}-k_m} = \frac{A_i(k_i^{Dmax}-k_m)^\alpha}{k_i^{Dmax}-k_m} = \frac{A_i}{(k_i^{Dmax}-k_m)^{1-\alpha}} \tag{3.53}$$

It is obvious that the production (Q_i) per efficient investment ($k_i^{max}-k_m$) is higher for the non-exporter. Furthermore, if there is no threshold investment then production per investment ratio of non-exporter is definitely higher. The relation between the ratios Q_i/k_i^{max} may be derived from the equations

$$\frac{Q_i^{exporter}}{k_i^{Emax}} = \frac{A_i}{(k_i^{Emax}-k_m)^{1-\alpha}}\left(1-\frac{k_m}{k_i^{Emax}}\right) \tag{3.54}$$

$$\frac{Q_i^{non-exporter}}{k_i^{Dmax}} = \frac{A_i}{(k_i^{Dmax}-k_m)^{1-\alpha}}\left(1-\frac{k_m}{k_i^{Dmax}}\right) \tag{3.55}$$

where $1-\frac{k_m}{k_i^{Emax}} > 1-\frac{k_m}{k_i^{Dmax}}$. Therefore, the production per investment comparison of exporter and non-exporter will depend on the threshold investment ratio. For lower thresholds (or $k_i^{Emax} >> k_m$ and $k_i^{Dmax} >> k_m$) the effect of threshold investment decreases and the non-exporter firm has a better efficiency of production per investment. This is mainly due to the decreasing returns to scale property of the market. However, if the threshold investment is close to the investment amounts then the market will have increasing returns to scale and the exporters efficiency will be better.

3.4.4 Revenue Normalized with Domestic Price/Total Investment Ratio

The revenue per investment may also be a efficiency measure for the comparison of the exporters and non-exporters. The revenue and the profit has the relation

$$TR^E = P_i^d Q_i^d + P_i^e Q_i^e = \pi_E^{max} + (1+r)(k_i^{Emax} + F^d + F^e) \quad (3.56)$$

$$TR^D = P_i^d Q_i^d = \pi_D^{max} + (1+r)(k_i^{Dmax} + F^d) \quad (3.57)$$

The ratio $TR/P^d/(k_i + F^d + F^e)$ in both markets will be

$$\frac{TR^E}{k_i^{Emax} + F^d + F^e}\frac{1}{P^d} = \frac{1}{P^d}\frac{\pi_E^{max}}{k_i^{Emax} + F^d + F^e} + \frac{1}{P^d}(1+r) \quad (3.58)$$

$$\frac{TR^D}{k_i^{Dmax} + F^d}\frac{1}{P^d} = \frac{1}{P^d}\frac{\pi_D^{max}}{k_i^{Dmax} + F^d} + \frac{1}{P^d}(1+r) \quad (3.59)$$

One may simply observe that the relation between the revenue (normalized with domestic prices) per total investment has the same relation with the profit per total investment ratios which were mentioned in the previous sections.

CHAPTER 4

ANALYSIS OF EXPORT MARKET DYNAMICS

The firm decision process is explained in the chapter Problem Formulation. In this chapter we will analyze this process in detail considering the levels and changes of productivity A_i and market parameters B_i^d and B_i^e using the findings in the chapter Comparative Statics Analysis of Profits and Investment. Remember the diagram of decision process of firms as summarized in Figure 4.1. In the following sections we will analyze the firm decision process at the beginning ($t = 0$), at $t = 1$ and lastly discuss the results.

4.1 Decision at $t = 0$

There are three possible paths for the firm at this point. For both periods firms do not know the B_i^d and B_i^e parameters and instead use the same expected values of them in both periods. Therefore, the profit maximization for each period will result in the same optimal investment and export market share. The only difference between the maximum profits will be the market entry costs F^d and/or F^e. For simplicity lets call the maximum profits for $F^d = 0$ and $F^e = 0$ as:

$$max\left\{E[P_i^d]Q_i^d - (1+r)k_i\right\} = E[\pi_D^{maxP}] \qquad (4.1)$$

$$max\left\{E[P_i^d]Q_i^d + E[P_i^e]Q_i^e - (1+r)k_i\right\} = E[\pi_E^{maxP}] \qquad (4.2)$$

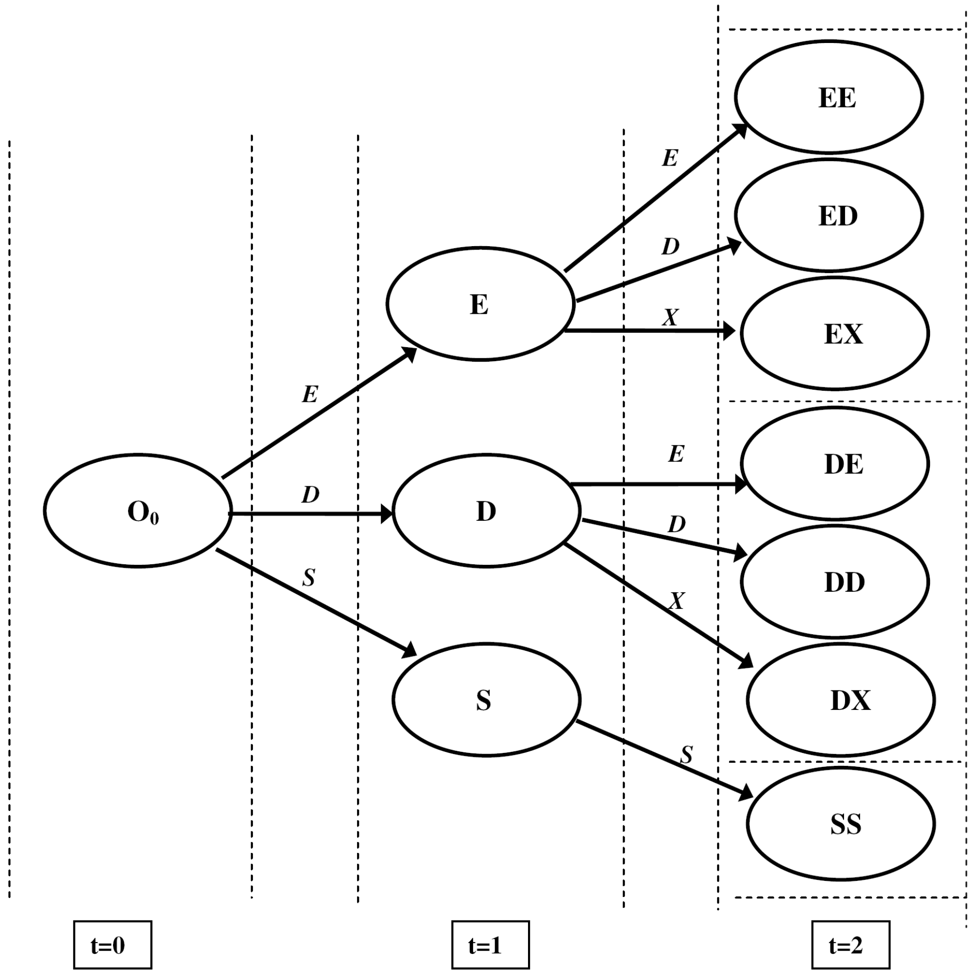


Figure 4.1: Entrepreneur's entry and exit decisions.

(4.3)

Recall that in the profit maximization F^e and F^d values does not change the optimal k_i and x_i values.

- SS: S at $t = 0$ and S at $t = 1$

  Recall that the profit of the firm not entering any market is zero.

$$E[\pi_{S,S}] = (1 + r)E[\pi_S]_{t=0} + E[\pi_{S,S}]_{t=0} = 0; \tag{4.4}$$

Therefore, if the profits in the markets are not positive then the firms will opt to stay outside the market.

- DX: D at $t = 0$ and X at $t = 1$

  If the firm plans to enter the domestic market at $t = 0$ and exit the market (exit profit is zero.) in the second period the expected total profit of the firm will become

$$\begin{aligned} E[\pi_{D,X}] &= (1+r)E[\pi_D]_{t=0} + E[\pi_{D,X}]_{t=0} \\ &= (1+r)\left(E[P_i^d]Q_i^d - (1+r)(k_i + F^d)\right) \\ &= (1+r)\left(E[P_i^d]Q_i^d - (1+r)k_i\right) - (1+r)^2F^d \\ &= (1+r)E[\pi_D^{maxP}] - (1+r)^2F^d \end{aligned} \tag{4.5}$$

  The firm does not have any definite knowledge about the demand function it will face when it enters the domestic market. Therefore, the expected value of B_i^d parameter is utilized.

- DD: D at $t = 0$ and D at $t = 1$

  The firm is planning to continue in domestic market in both periods for this case. Again the B_i^d parameter is unknown for both periods. Firm will gain $E[\pi_D]_{t=0}$ in the first period and $E[\pi_{D,D}]_{t=0}$ in the second period. The only difference between these profits is that in the first period firm will pay for the domestic sunk cost but in the second will not. Recall that the existence of F^d does not effect the profit maximizing investment amount k_i (equation 3.12) and therefore the solution of maximization problem in both periods will give the same result. Therefore, we can write $E[\pi_D]_{t=0}$ in terms of $E[\pi_{D,D}]_{t=0}$ as $E[\pi_D]_{t=0} = E[\pi_{D,D}]_{t=0} - (1+r)F^d$. The resulting profit becomes

$$\begin{aligned} E[\pi_{D,D}] &= (1+r)E[\pi_D]_{t=0} + E[\pi_{D,D}]_{t=0} \\ &= (2+r)E[\pi_{D,D}]_{t=0} - (1+r)^2F^d \\ &= (2+r)\left(E[B_i^d]P^dQ_i^{d^{1-\epsilon_d}} - (1+r)k_i\right) - (1+r)^2F^d \end{aligned}$$

$$= \quad (2+r)E[\pi_D^{maxP}] - (1+r)^2 F^d \tag{4.6}$$

- DE: D at $t = 0$ and E at $t = 1$

  If the firm find it more profitable then may enter domestic market in the first period and export market in the second period. Note that the profit maximizing investment level may differ between the periods. The corresponding total profit becomes

$$\begin{aligned}
E[\pi_{D,E}] &= (1+r)E[\pi_D]_{t=0} + E[\pi_{D,E}]_{t=0} \\
&= (1+r)\left(E[P_i^d]Q_i^d - (1+r)(k_i + F^d)\right) \\
&\quad + \left(E[P_i^d]Q_i^d + E[P_i^e]Q_i^e - (1+r)(k_i + F^e)\right) \\
&= (1+r)\left(E[P_i^d]Q_i^d - (1+r)k_i\right) \\
&\quad + \left(E[P_i^d]Q_i^d + E[P_i^e]Q_i^e - (1+r)k_i\right) \\
&\quad -(1+r)^2 F^d - (1+r)F^e \\
&= (1+r)E[\pi_D^{maxP}] + E[\pi_E^{maxP}] \\
&\quad -(1+r)^2 F^d - (1+r)F^e
\end{aligned} \tag{4.7}$$

- EX: E at $t = 0$ and X at $t = 1$

  If the EX case is more profitable then the firm will enter both domestic and export market in the first period and exit both markets in the second period (exit profit is zero). The firms planning to perform EX will evaluate the expected profits as

$$\begin{aligned}
E[\pi_{E,X}] &= (1+r)E[\pi_E]_{t=0} + E[\pi_{E,X}]_{t=0} \\
&= (1+r)\left(E[P_i^d]Q_i^d + E[P_i^e]Q_i^e - (1+r)(k_i + F^d + F^e)\right) \\
&= (1+r)\left(E[P_i^d]Q_i^d + E[P_i^e]Q_i^e - (1+r)k_i\right) \\
&\quad -(1+r)^2 F^d - (1+r)^2 F^e \\
&= (1+r)E[\pi_E^{maxP}] - (1+r)^2 F^d - (1+r)^2 F^e
\end{aligned} \tag{4.8}$$

- ED: E at $t = 0$ and D at $t = 1$

  For this case the firm profit maximization result in the choice of entering both export and domestic market in the first period and exiting export market in the second period. The resulting profit becomes

$$\begin{aligned}
E[\pi_{E,D}] &= (1+r)E[\pi_E]_{t=0} + E[\pi_{E,D}]_{t=0} \\
&= (1+r)\left(E[P_i^d]Q_i^d + E[P_i^e]Q_i^e - (1+r)(k_i + F^d + F^e)\right) \\
&\quad + E[P_i^d]Q_i^d - (1+r)k_i \\
&= (1+r)\left(E[P_i^d]Q_i^d + E[P_i^e]Q_i^e - (1+r)k_i\right) \\
&\quad + \left(E[P_i^d]Q_i^d - (1+r)k_i\right) - (1+r)^2F^d - (1+r)^2F^e \\
&= (1+r)E[\pi_E^{maxP}] + E[\pi_D^{maxP}] - (1+r)^2F^d - (1+r)^2F^e
\end{aligned} \tag{4.9}$$

- EE: E at $t = 0$ and E at $t = 1$

  Another option for the firm is to enter both markets in the first period and stay in both in the second period. The corresponding expected profit of this strategy is

$$\begin{aligned}
E[\pi_{E,E}] &= (1+r)E[\pi_E]_{t=0} + E[\pi_{E,E}]_{t=0} \\
&= (1+r)\left(E[P_i^d]Q_i^d + E[P_i^e]Q_i^e - (1+r)(k_i + F^d + F^e)\right) \\
&\quad + \left(E[P_i^d]Q_i^d + E[P_i^e]Q_i^e - (1+r)k_i\right) \\
&= (2+r)\left(E[P_i^d]Q_i^d + E[P_i^e]Q_i^e - (1+r)k_i\right) \\
&\quad -(1+r)^2F^d - (1+r)^2F^e \\
&= (2+r)E[\pi_E^{maxP}] - (1+r)^2F^d - (1+r)^2F^e
\end{aligned} \tag{4.10}$$

The firm will calculate these expected profits and select the maximum of them. Therefore, the decision at $t = 0$ S, D or E will be determined with respect to the maximum of the seven profits above.

- If maximum profit is $E[\pi_{S,S}]$, then action is S.

  If the profits found for other cases is less than zero then the best choice for the firm is to stay out of both markets.

- If maximum profit is any of $\{E[\pi_{D,X}], E[\pi_{D,D}], E[\pi_{D,E}]\}$, then action is D. Lets compare the profits in the cases : DX, DD, and DX. Investigating the above results we observe that the difference between profits of case DX and DD is

$$\begin{aligned} E[\pi_{D,D}] - E[\pi_{D,X}] &= (2+r)E[\pi_D^{maxP}] - (1+r)^2 F^d \\ &\quad - \left[(1+r)E[\pi_D^{maxP}] - (1+r)^2 F^d\right] \\ &= E[\pi_D^{maxP}] \end{aligned} \tag{4.11}$$

If the maximum profit in the domestic market (without sunk cost) is positive then the firm will definitely prefer case DD over DX. And if $E[\pi_D^{maxP}]$ is negative then $E[\pi_{D,X}]$ will also be negative which means staying out of markets is more profitable. Therefore, no firm will opt to enter the domestic market while planing to exit the market in the second period. The future plan DX is definitely unfeasible.

Comparing the profits of cases DD and DE

$$\begin{aligned} E[\pi_{D,E}] - E[\pi_{D,D}] &= (1+r)E[\pi_D^{maxP}] + E[\pi_E^{maxP}] \\ &\quad -(1+r)^2 F^d - (1+r)F^e \\ &\quad - \left\{(2+r)E[\pi_D^{maxP}] - (1+r)^2 F^d\right\} \\ &= E[\pi_E^{maxP}] - E[\pi_D^{maxP}] - (1+r)F^e \end{aligned} \tag{4.12}$$

Recall that in chapter 2 (section 2.4) we showed that $E[\pi_E^{maxP}] \geq E[\pi_D^{maxP}]$. Therefore, the difference $E[\pi_E^{maxP}] - E[\pi_D^{maxP}]$ is nonnegative. If this difference is sufficiently high such that $E[\pi_E^{maxP}] - E[\pi_D^{maxP}] > (1+r)F^e$ then the entrepreneur will prefer to plan the future of the firm as entering domestic market in the first period and additionally entering export market in the second period. If the export market sunk cost is sufficiently high so that the difference of profits in the case DE and in the case DD is lower then F^e then the firm will opt to plan the future as entering in the domestic market for the first period and stay in this market for the second period.

- If maximum profit is any of $\{E[\pi_{E,X}], E[\pi_{E,D}], E[\pi_{E,E}]\}$, then action is E. The firm will enter the export market for the first period if any of the profits in the cases: EX, ED, and EE exceeds other profits. Lets again first compare these three profits

$$\begin{aligned}
E[\pi_{E,D}] - E[\pi_{E,X}] &= (1+r)E[\pi_E^{maxP}] + E[\pi_D^{maxP}] \\
&\quad -(1+r)^2F^d - (1+r)^2F^e \\
&\quad -\left\{(1+r)E[\pi_E^{maxP}] - (1+r)^2F^d - (1+r)^2F^e\right\} \\
&= E[\pi_D^{maxP}] \qquad (4.13)
\end{aligned}$$

$$\begin{aligned}
E[\pi_{E,E}] - E[\pi_{E,X}] &= (2+r)E[\pi_E^{maxP}] - (1+r)^2F^d - (1+r)^2F^e \\
&\quad -\left\{(1+r)E[\pi_E^{maxP}] - (1+r)^2F^d - (1+r)^2F^e\right\} \\
&= E[\pi_E^{maxP}] \qquad (4.14)
\end{aligned}$$

$$\begin{aligned}
E[\pi_{E,E}] - E[\pi_{E,D}] &= (2+r)E[\pi_E^{maxP}] - (1+r)^2F^d - (1+r)^2F^e \\
&\quad -\left\{(1+r)E[\pi_E^{maxP}] + E[\pi_D^{maxP}]\right. \\
&\quad \left.-(1+r)^2F^d - (1+r)^2F^e\right\} \\
&= E[\pi_E^{maxP}] - E[\pi_D^{maxP}] \qquad (4.15)
\end{aligned}$$

The above equations can be investigated under three conditions. (i) $E[\pi_E^{maxP}] > E[\pi_D^{maxP}] > 0$: for this condition the most profitable case is EE since $E[\pi_{E,E}]$ is positive and greater then other profits. (ii) $E[\pi_E^{maxP}] > 0 > E[\pi_D^{maxP}]$: the most profitable case is again EE since $E[\pi_{E,E}]$ is positive and greater then other profits. (iii) $0 > E[\pi_E^{maxP}] > E[\pi_D^{maxP}]$: in this condition since the profit of the firms is negative, neither of the EX, ED, and EE is profitable and the firm would prefer to stay out of markets at the beginning.

Therefore if the firm plans to enter both markets at the beginning then his future plan is neither of exiting export market or exiting both markets. The firm definitely plans to perform EE if he finds it more profitable.

Additionally we can compare the profits of firms performing EE and DE as

$$\begin{aligned} E[\pi_{E,E}] - E[\pi_{D,E}] &= (2+r)E[\pi_E^{maxP}] - (1+r)^2F^d - (1+r)^2F^e \\ &\quad - \left\{(1+r)E[\pi_D^{maxP}] + E[\pi_E^{maxP}] - (1+r)^2F^d - (1+r)F^e\right\} \\ &= (1+r)\left(E[\pi_E^{maxP}] - E[\pi_D^{maxP}]\right) - r(1+r)F^e \end{aligned} \quad (4.16)$$

Recall that if the firm plans to perform DE then the condition was found as $E[\pi_E^{maxP}] - E[\pi_D^{maxP}] > (1+r)F^e$. One can simply show that if $E[\pi_E^{maxP}] - E[\pi_D^{maxP}] > (1+r)F^e$ then $(1+r)\left(E[\pi_E^{maxP}] - E[\pi_D^{maxP}]\right) - r(1+r)F^e > 0$. Therefore, the firm will definitely choose to perform EE instead of DE.

The difference between the expected profits of EE and DD are:

$$\begin{aligned} E[\pi_{E,E}] - E[\pi_{D,D}] &= (2+r)E[\pi_E^{maxP}] - (1+r)^2F^d - (1+r)^2F^e \\ &\quad - \left\{(2+r)E[\pi_D^{maxP}] - (1+r)^2F^d\right\} \\ &= (2+r)\left(E[\pi_E^{maxP}] - E[\pi_D^{maxP}]\right) - (1+r)^2F^e \end{aligned} \quad (4.17)$$

If this difference is positive or $E[\pi_E^{maxP}] - E[\pi_D^{maxP}] > \frac{(1+r)^2}{2+r}F^e$, then the firm will opt to plan performing EE. If the reverse is valid ($E[\pi_E^{maxP}] - E[\pi_D^{maxP}] < \frac{(1+r)^2}{2+r}F^e$) then it will opt to plan performing DD.

According to the above conclusions, in fact there are three choices of firms: SS, DD, EE. At the beginning ($t = 0$) all firms are uncertain about the parameters B_i^d and B_i^e. Therefore, each firm will use the expected values of these parameters to calculate their expected maximum profits, expected maximizing investment amounts, and expected maximizing export market shares. Lets assume that every firm will expect the same demand function so the same level of B_i^d and B_i^e parameters. In other words $E[B_i^d] = E[B_i^e] = 1$ for $i = 1..N$ where N is the number of firms. We will observe the case where there are firms with various amounts of productiveness. Therefore lets say the productiveness of all of the firms ranges from 0 to 145. We will utilize the interest rate r as 10%. The threshold investment k_m is 2 for the following numerical examples.

In the following four figures 4.2, 4.3, 4.4, and 4.5 the plots of the maximized amounts of $E[\pi_{E,E}]$ and $E[\pi_{D,D}]$ are shown on the same axes with varying A_i. These four figures are the four possible events that may occur during the firm decision process and they differ only in the amounts of market entry sunk costs. Investigating each case:

1. $F^d = 2$ and $F^e = 2$:

   In this case $E[\pi_{E,E}]$ exceeds the $E[\pi_{D,D}]$ before they get positive values (Figure 4.2). Therefore the firms with lower productivity stay out of the markets and the firms with higher productivity enters both markets.

2. $F^d = 2$ and $F^e = 8$:

   The difference of this case with respect to first one is that only F^e increases. Therefore, the profit function $E[\pi_{E,E}]$ shifts down such that for some region of A_i the positive $E[\pi_{D,D}]$ exceeds $E[\pi_{E,E}]$ (Figure 4.3). For this case we observe that for the lowest portion of A_i firms prefer staying out of the markets. For some middle portion, they prefer to perform DD and so enter to the domestic market. For the remaining upper portion, the firms with higher productivity opt to enter both markets and planing to stay there in the second period (EE).

3. $F^d = 2$ and $F^e = 14$:

   Another possibility is that F^e is so high that $E[\pi_{E,E}]$ can not exceed $E[\pi_{D,D}]$ within the region of productivity (Figure 4.4). Note that $E[\pi_{E,E}]$ would exceed $E[\pi_{D,D}]$ for higher values of productivity but we assumed that the maximum available A_i value is 145. The lower productive firms opt to stay out of market (S) and plans to stay out markets in the second period too. The firms with higher productivity opt to enter only domestic market (D).

4. $F^d = 10$ and $F^e = 14$:
   In Figure 4.5, the entry cost of domestic market is higher than the previous cases. Therefore both profit curves shift down with the same amount (with respect to third condition). Both profits are negative and the firm plans to stay out of the markets for both periods (SS). Thus, firms whether their productivity is high or low will opt to stay out of the markets. Again note that the profit curves get positive values for higher A_i values but not in the defined maximum available productivity.

Firms may choose to enter any of the markets or stay out of the market due to the market and firm parameters. However, in total the above conclusions state that firms with higher productivity will have the chance of entering markets. If the export market entry cost is low enough then firms with sufficient amount of productivity will prefer producing in both markets (E). If the market entry cost of export market is sufficiently high then the firms with middle productivity will opt to enter only domestic market (D) while the highest productive ones still opt to enter both markets (E).

4.2 Decision at $t = 1$

After the decision at $t = 0$, the firms will have the following options:

- S in the first period:
  For the firm stayed out of markets in the first period will have only one choice which is again staying out of market. The corresponding profit is zero.

$$E[\pi_{SS}] = 0 \tag{4.18}$$

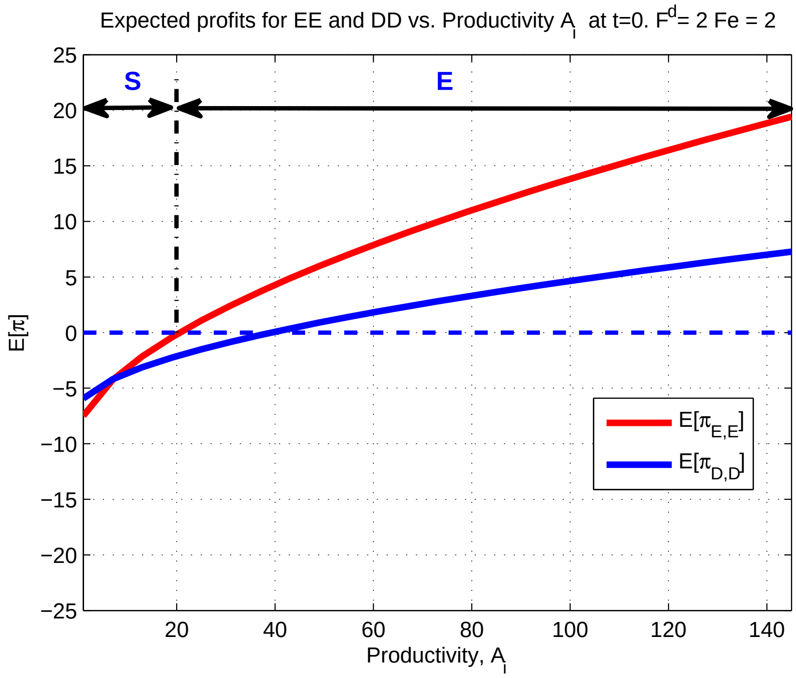


Figure 4.2: Profits $E[\pi_{E,E}]$ and $E[\pi_{D,D}]$ versus A_i. $F^d = 2$ and $F^e = 2$.

- D in the first period:

  The firm found the domestic profit positive and the difference in equation 4.17 as negative at $t = 0$ and entered the domestic market with the future plan of staying in domestic market in the second period. The firm entered domestic market in the first will have the choices: (i) exiting the market, (ii) staying in the market, and (iii) entering the export market while staying in domestic market. The firm learned about its domestic market parameter B_i^d but still do not know the export market parameter B_i^e and utilizing $E[B_i^e]$ instead. The corresponding profits will be

$$\pi_{DX} = 0 \tag{4.19}$$

$$\pi_{DD} = P_i^d Q_i^d - (1+r)k_i \tag{4.20}$$

$$E[\pi_{DE}] = P_i^d Q_i^d + E[P_i^e] Q_i^e - (1+r)(k_i + F_e) \tag{4.21}$$

(4.22)

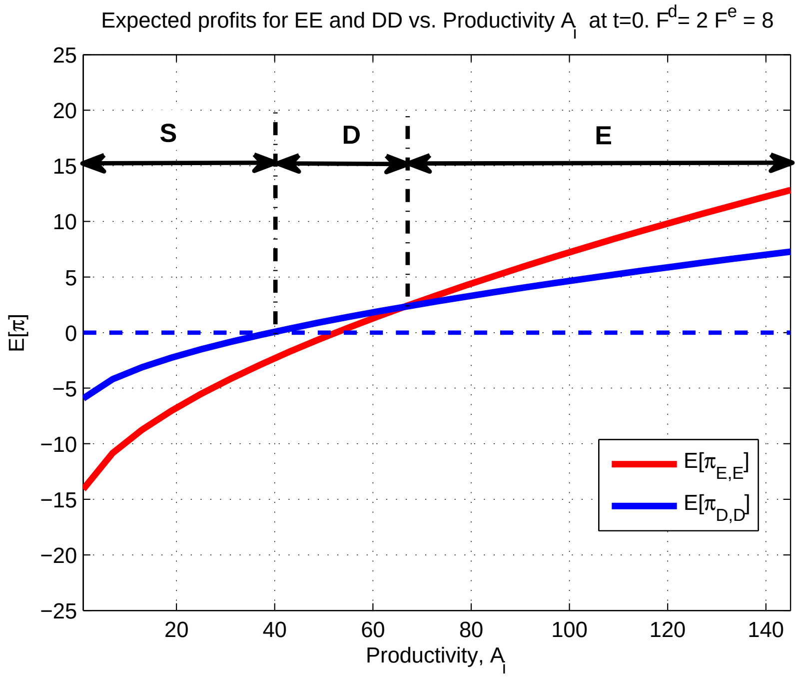


Figure 4.3: Profits $E[\pi_{E,E}]$ and $E[\pi_{D,D}]$ versus A_i. $F^d = 2$ and $F^e = 8$.

The exact value of B_i^d would be higher or lower than or equal to the expected one. We will examine these cases.

- $\mathbf{B}_i^d = E[B_i^d]$:

  The exact value of the parameter, $\mathbf{B}_i^d$ is same with the expected one. therefore, $\pi_{D,D} = E[\pi_{D,D}]$ and one can simply show that if $(2+r)\left(E[\pi_E^{maxP}] - E[\pi_D^{maxP}]\right) - (1+r)^2 F^e < 0$ then $E[\pi_E^{maxP}] - E[\pi_D^{maxP}] - (1+r)F^e < 0$. Therefore, the firm will continue in the domestic market if there is no difference in the expected and exact values of parameter B_i^d.

- $\mathbf{B}_i^d > E[B_i^d]$:

  If the firm underestimated the value of B_i^d in the first period then the exact value of B_i^d will be higher in the second period. We derived that $\frac{\partial(\pi_E^{max} - \pi_D^{max})}{\partial B_i^d} < 0$ in the previous chapter. Therefore, as B_i^d

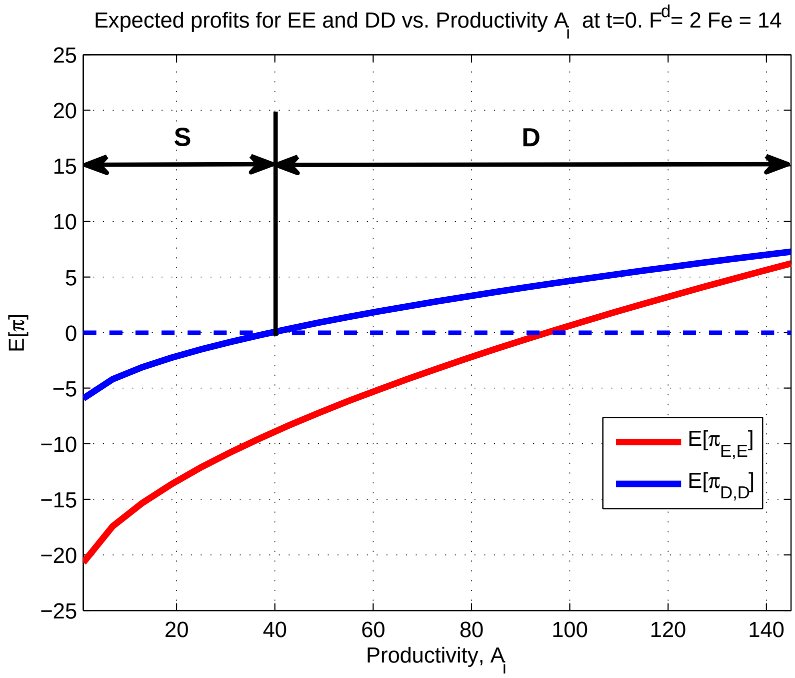


Figure 4.4: Profits $E[\pi_{E,E}]$ and $E[\pi_{D,D}]$ versus A_i. $F^d = 2$ and $F^e = 14$.

increases, the gap between the maximum profits decreases but never gets zero. If the gap $E[\pi_E^{maxP}] - E[\pi_D^{maxP}] < (1+r)F^e$ and B_i^d increases, then this gap gets smaller so that it can not overcome the sunk cost $(1+r)F^e$. Hence, the firm will continue in the domestic market if B_i^d increases. In Figure 4.6 profits $E[\pi_E^{maxP}]$, $E[\pi_E^{maxP}]$ versus productivity A_i are plotted for $\mathbf{B}_i^d = E[B_i^d]$ ($B_i^d = 1.0$) and $\mathbf{B}_i^d > E[B_i^d]$ ($B_i^d = 1.3$). The sunk costs are $F^d = 2$ and $F^e = 8$. Figure 4.6 is a good example to illustrate the effect of increase in B_i^d. Both profit curves shift up and the gap $\pi_D^{maxP} - E[\pi_E^{maxP}]$ increases so that firm continues in the domestic market.

– $\mathbf{B}_i^d < E[B_i^d]$:

In this case the firm overestimates the parameter B_i^d in the first period. So we have to examine what happens when B_i^d decreases. As in the previous case $\frac{\partial(\pi_E^{max} - \pi_D^{max})}{\partial B_i^d} < 0$. So, as B_i^d decreases the gap

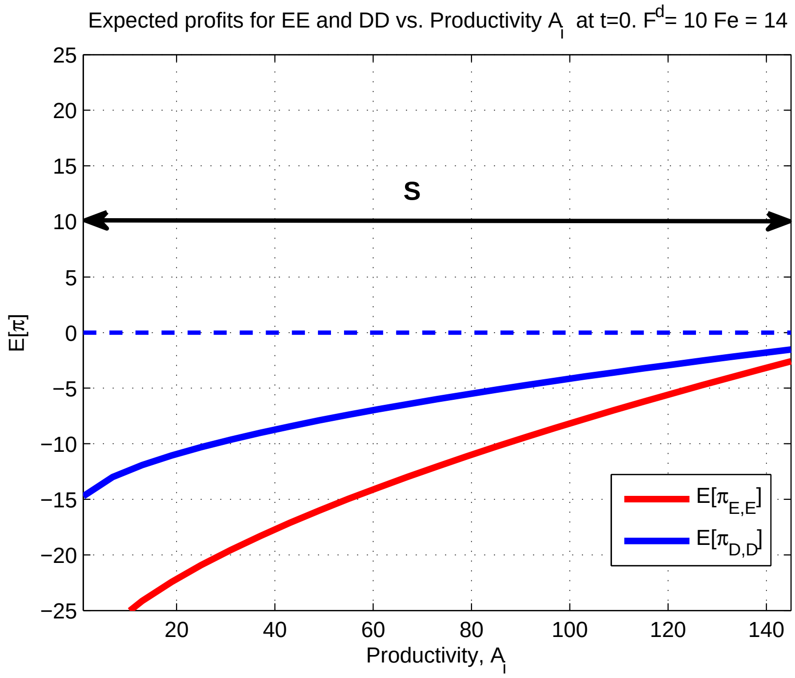


Figure 4.5: Profits $E[\pi_{E,E}]$ and $E[\pi_{D,D}]$ versus A_i. $F^d = 10$ and $F^e = 14$.

between the profits increases, but at the same time both of the profit levels also decreases. Therefore, in some situations the decrease in B_i^d would result in $E[\pi_E^{maxP}] - E[\pi_D^{maxP}] > (1+r)F^e$ while $E[\pi_E^{maxP}] > 0$ so that the firm opts to enter the export market. And for some cases both $E[\pi_E^{maxP}] < 0$ and $E[\pi_D^{maxP}] < 0$ so that the firm opts to exit the domestic market. Or there may be no change in the conditions $E[\pi_E^{maxP}] - E[\pi_D^{maxP}] < (1+r)F^e$ and $E[\pi_D^{maxP}] > 0$, so that the firm will stay in the domestic market. In Figure 4.7, profits $E[\pi_E^{maxP}]$, $E[\pi_E^{maxP}]$ versus productivity A_i are plotted for $B_i^d = E[B_i^d]$ ($B_i^d = 1.0$) and $B_i^d < E[B_i^d]$ ($B_i^d = 0.6$). The sunk costs are $F^d = 2$ and $F^e = 8$. This figure shows the effect of decrease in B_i^d. As seen from the figure the firms with lower productivity exits the domestic market. The highest productive ones opt to enter the export market and the ones in the middle stay only in the domestic market. In

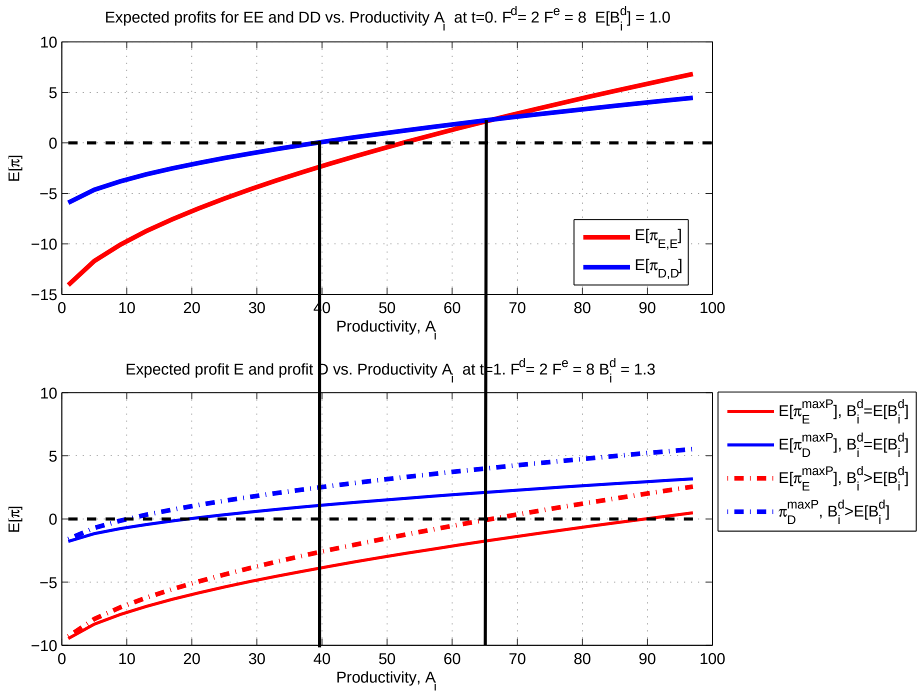


Figure 4.6: Effect of underestimation of B_i^d

each case the profits of firms decrease. Note that, although there are exiting non-exporters for this level of overestimation there is no exiting exporter firm. The exporters still have positive profits and will continue in the export market. Therefore, we can conclude that the exporters have less tendency to exit the market with respect to non-exporters.

- E in the first period:

  The firms opted to enter the export market in the first period will have the knowledge of all parameters. Therefore, they will be certain on their decisions. These firms will have the choices: (i) exit both markets, (ii) stay in the domestic market and exit export market, and (iii) continue in

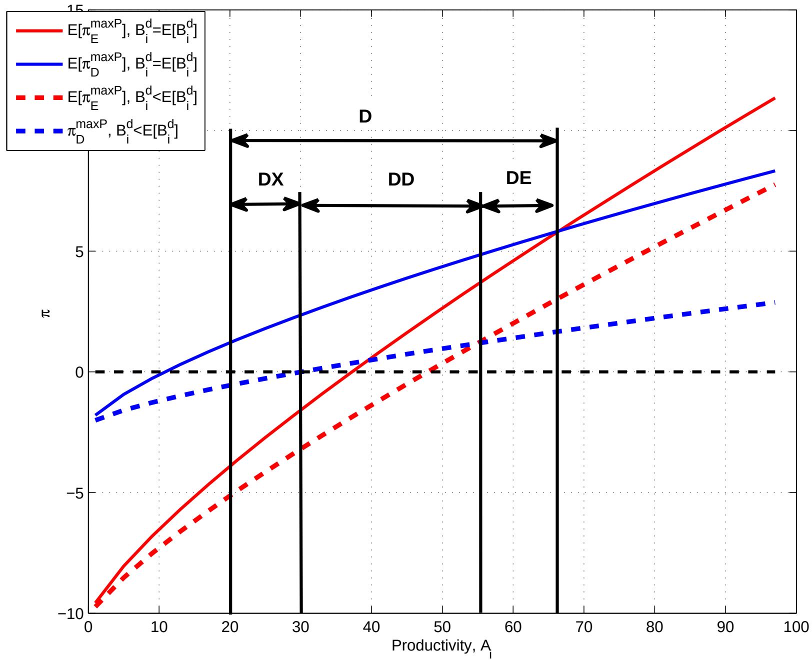


Figure 4.7: Effect of overestimation of B_i^d

both export and domestic markets. The profits will be

$$\pi_{EX} = 0 \tag{4.23}$$

$$\pi_{ED} = P_i^d Q_i^d - (1+r)k_i \tag{4.24}$$

$$\pi_{EE} = P_i^d Q_i^d + P_i^e Q_i^e - (1+r)k_i \tag{4.25}$$

(4.26)

The comparison of these profits will depend on the estimation errors. The firm may underestimate or overestimate the parameters B_i^d and B_i^e. However, whether the exact value is lower or higher the profit π_E^{maxP} will be higher then the π_D^{maxP} since there is no sunk cost in this period. Therefore, the firm will never opt to exit the export market while continuing in the domestic market (ED). On the other hand the differences between the estimated and exact values of these parameters would lead to negative profit and the firm may leave both of the markets since $0 > \pi_E^{maxP} > \pi_D^{maxP}$. In Figure 4.8 the changes in π_E^{maxP} with respect to decrease of B_i^d and B_i^e are shown. Profits π_E^{maxP} for $B_i^d = 1$ & $B_i^e = 1$, $B_i^d = 0.8$ & $B_i^e = 0.8$, $B_i^d = 0.5$ & $B_i^e = 0.5$ are plotted. The sunk costs are $F^d = 2$ and $F^e = 8$. If the firm over estimated the parameters such that the exact values are $B_i^d = 0.8$ & $B_i^e = 0.8$, then the firm will not change its decision since π_E^{maxP} is still positive. however if the firm with lower productivity made a drastic error in the estimations such that $B_i^d = 0.5$ & $B_i^e = 0.5$, then it will exit both of the markets. The more productive ones will still have the chance to make positive profit. Note that π_D^{maxP} is not shown on these case but one should be sure of that $\pi_E^{maxP} > \pi_D^{maxP}$ for every A_i.

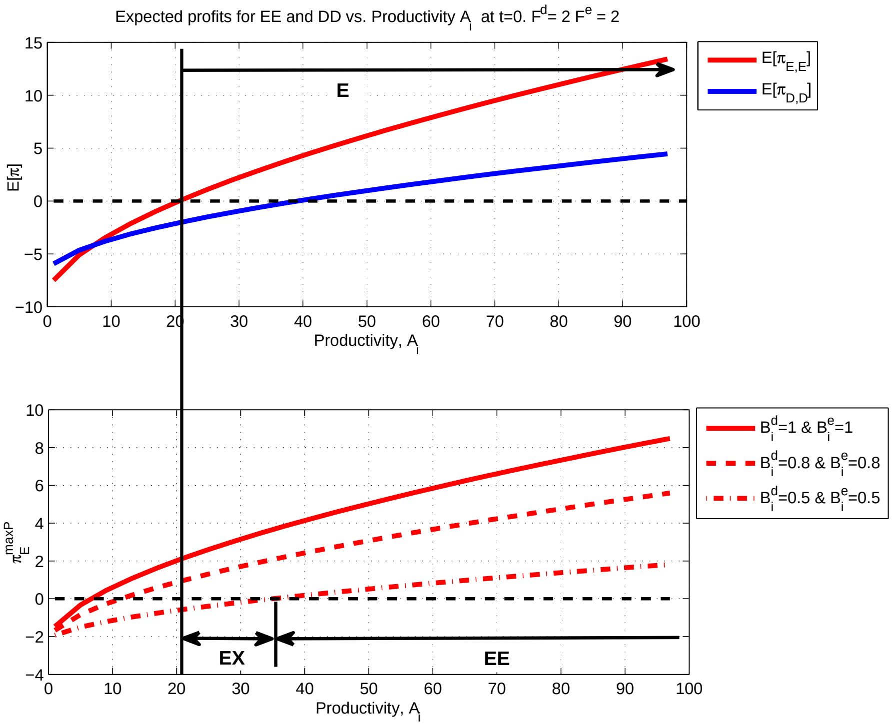


Figure 4.8: Effects of overestimation in both B_i^d and B_i^e.

4.3 Discussions

According to the findings, we can summarize some main conclusions

- Exporters are more productive:

  The analytical and numerical derivations show that the more productive firms have more tendency to be exporters. This supports the self-selection hypothesis in the literature (Greenaway & Kneller (2007), Bernard & Jensen (1997), Bernard & Wagner (1997), Delgado et al. (2002), Girma et al. (2004), Castellani (2002)). More productive firms become exporters, because they can afford the additional costs of exporting such as expenses related to establishing a distribution channel, transportation costs, or production costs to modify products for international markets. Therefore, the correlation between productivity and export status may arise as a result of the self-selection of better firms to go into the export market. Furthermore, the self-selection of exporting firms leads to reallocation of resources from less efficient to more efficient firms which may be the cause of export-led growth. On the other hand, (not included in our model) exporters might learn from their presence in international markets. This may be caused by two main reasons: (i) the larger international market allows the exploitation of economies of scale and, (ii) international contacts with buyers and customers are likely to foster knowledge and technology pullovers, such as access to technical expertise, including new product designs and new production methods. The learning-by-exporting phenomena is not included in our model but one may simply observe the effects of productivity rises considering the analytical and numerical derivations we performed.

- Exporters have no tendency to become non-exporter and less tendency to exit both markets:

  As we stated above once a firm becomes an exporter (pay for the high sunk costs and had a great investment) then the firm will always find more profitable to be an exporter. Recall that if there is no sunk cost then exporter's maximum profit (whether it is negative or positive) is always higher than the non-exporter's maximum profit. Therefore, there are two options for an exporter: (i) to stay as an exporter if there is no drastic expectation error in the profit or (ii) exit the market if the firm overestimated the demand parameters with a big error. However, in some models the exporting may require paying for some additional amount of sunk cost (say f^e) at each period. This may result in the domestic profit to be higher than the exporting profit for some lower productivity levels (as we showed for F^e). For these models the exporters with the lowest productivity may leave the export market and produce only in domestic market. On the other hand, since the exporters are more productive and so have larger sizes they have less tendency to leave the markets with respect to the non-exporters which are less productive and smaller in size.

- Exporters are larger:
  We analytically derived that for the same productivity level the exporters have more profit maximizing investment amounts. Furthermore, we showed that if the productivity of a firm is higher then the profit maximizing investment amount is also higher. These two statements proves that the exporters (which are more productive) invests much more than the non-exporters. Additionally they pay for the export market entry costs. If we define the firm size as the sum of investment amount and entry costs (or entry investment) then the exporter will larger. A good example is in Figure 4.9. Profits $E[\pi_{E,E}]$ and $E[\pi_{D,D}]$ versus productivity A_i (left) and versus firm size (right) are plotted. The entry costs are $F^d = 2$ and $F^e = 8$. The expected profits $E[\pi_{E,E}]$ and $E[\pi_{D,D}]$ versus the productivity is plotted on the left side (same with Figure 4.3). The profit of domestic market (blue curve) is higher than the profit of export market in the productivity interval $40 - 65$. For the higher productivity values

the export profit exceeds non-exporting profit. The figure on the right is the plot of again profits with respect to the firm size. Note that firm size is $k_i^{Dmax} + F^d$ for the non-exporting firm and $k_i^{Emax} + F^d + F^e$ for the exporting firm. As seen from the figure the greatest non-exporter firm size is less than 6 while the size of smallest firm in export market is greater than 15. Although there is a little productivity difference between the largest non-exporter and the smallest exporter, they have very different firm size. Because if any firm decides to enter the export market it should invest more and pay for the additional entry investment.

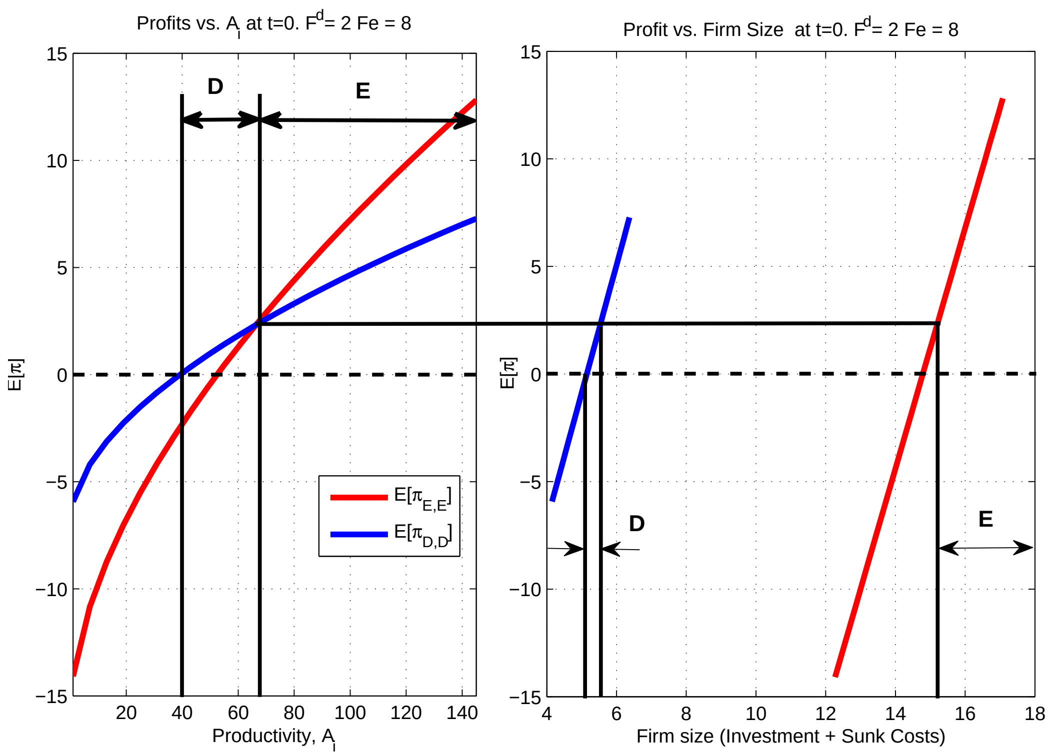


Figure 4.9: Profit and the firm size comparison for exporter and non-exporter firms.

- Contraction in Domestic demand may force firms to become exporters: As we found in the numerical example in Figure 4.7 the overestimation of B_i^d parameter before entering the market resulted in the decrease in both the domestic and exporting profits. And the least productive firms had to exit the domestic market while the intermediate productive firms continue in the domestic market. The interesting result is the behavior of the most productive non-exporter firms. They opt to enter the export market since the exporting profit is higher than the domestic profit for their productivity levels. However, note that the profit in export market is still less then the previous period's profit of domestic market. Additionally, the exporters increase the export market share due to the contraction in the domestic demand (section 3.2.2). The 2001 economic crisis in Turkey is a good example for this finding. In Figure 4.10 the ratio of volume of exporting (general) to the GDP in Turkey is plotted for the years 1990-2006 (TUIK 2007). As seen in 2001 there is a drastic increase in export/GDP ratio. Since there has been a contraction in the domestic demand the non-exporting firms opted to enter the export market to maximize their profit. The depreciation in the exchange rate also has a positive effect in the rise of export volume.

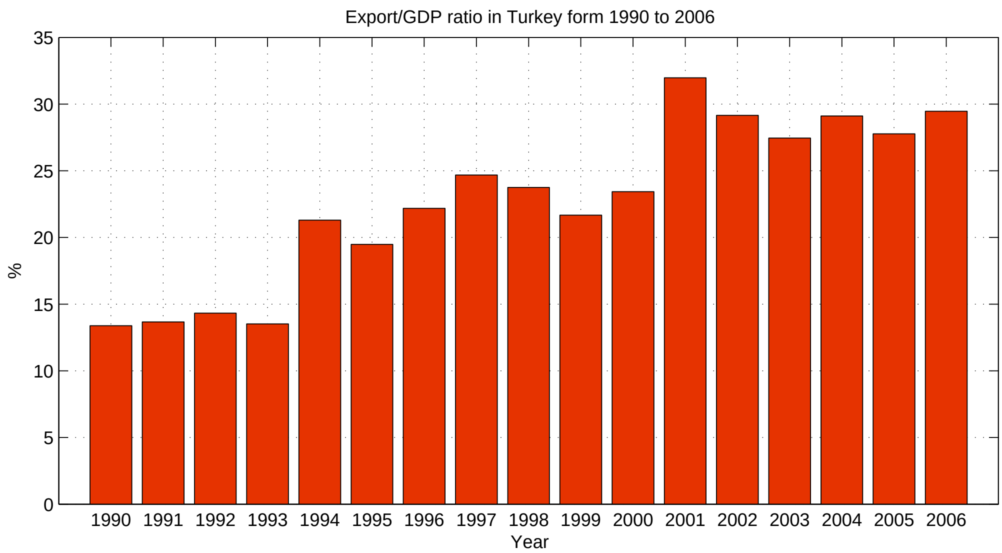


Figure 4.10: Export/GDP ratio in Turkey from 1990 to 2006.

CHAPTER 5

CONCLUSION

In this thesis we examined the export dynamics at the firm level. A two period model is proposed for the life of firms. The firms may have three different behaviors: staying out of markets, producing for the domestic market, and producing for both the domestic and the export markets. During the two periods, firms may enter or exit the markets according to the profit maximization preferences. All firms are profit maximizing firms such that they compare the maximum available profits in the domestic and export markets. The firms are also heterogenous so that they have different levels of productivity.

We examined the changes in investment, market share and profits of firms with respect to the changes in the market and firm parameters. The profits and investments of the exporting and non-exporting firms are compared by both analytical and numerical methods.

Our main findings can be summarized as follows. First, exporters are more productive since they can afford the additional sunk costs of exporting such as expenses related to establishing a distribution channel, transportation costs, or production costs to modify products for international markets. This supports the self-selection hypothesis suggested by Greenaway & Kneller (2007), Bernard & Jensen (1997), Bernard & Wagner (1997), Delgado et al. (2002), Girma et al. (2004), Castellani (2002)). Second, the size of exporters are greater than that of non-exporters because they have to invest more and additionally afford for the export entry costs. Third, exporters have less tendency to become non-exporters and less tendency to exit both markets because they are more productive and larger in size. Lastly, contraction in domestic demand may force firms to become exporters. The most productive non-exporter firms may opt to enter the export market since it becomes more profitable after a domestic demand contraction.

The second hypothesis on the export-led growth is learning-by-exporting. In our model we showed the supportive evidences for the self-selection. For the future work, our model may be expanded to include learning by exporting.

Appendix A

NECESSARY DERIVATIONS

A.1 Domestic Market Maximum Profit

We can write maximum domestic profit π_D^{max} in terms of firm and market parameter. Recall the domestic profit equation

$$\pi_D = B_i^d P^d A_i^{1-\epsilon_d}(k_i - k_m)^{\alpha(1-\epsilon_d)} - (1+r)(k_i + F^d) \tag{A.1}$$

and the profit maximizing investment amount

$$k_i^{Dmax} = \left[\frac{1+r}{(\alpha(1-\epsilon_d))B_i^d P^d A_i^{1-\epsilon_d}}\right]^{\frac{1}{\alpha-\alpha\epsilon_d-1}} + k_m \tag{A.2}$$

substituting A.2 into A.1 one would get

$$\begin{aligned}
\pi_D^{max} &= B_i^d P^d A_i^{1-\epsilon_d}\left\{\left[\frac{\alpha(1-\epsilon_d)B_i^d P^d A_i^{1-\epsilon_d}}{1+r}\right]^{\frac{1}{1-\alpha(1-\epsilon_d)}} + k_m - k_m\right\}^{\alpha(1-\epsilon_d)} \\
&\quad -(1+r)\left\{\left[\frac{\alpha(1-\epsilon_d)B_i^d P^d A_i^{1-\epsilon_d}}{1+r}\right]^{\frac{1}{1-\alpha(1-\epsilon_d)}} + k_m\right\} - (1+r)F^d \\
&= B_i^d P^d A_i^{1-\epsilon_d}\left[\frac{\alpha(1-\epsilon_d)B_i^d P^d A_i^{1-\epsilon_d}}{1+r}\right]^{\frac{\alpha(1-\epsilon_d)}{1-\alpha(1-\epsilon_d)}} \\
&\quad -(1+r)\left[\frac{\alpha(1-\epsilon_d)B_i^d P^d A_i^{1-\epsilon_d}}{1+r}\right]^{\frac{1}{1-\alpha(1-\epsilon_d)}} - (1+r)k_m - (1+r)F^d \\
&= \left[B_i^d P^d A_i^{1-\epsilon_d}\right]^{\frac{1}{1-\alpha(1-\epsilon_d)}}(1+r)^{\frac{-\alpha(1-\epsilon_d)}{1-\alpha(1-\epsilon_d)}}\left[\alpha(1-\epsilon_d)\right]^{\frac{\alpha(1-\epsilon_d)}{1-\alpha(1-\epsilon_d)}} \\
&\quad - \left[B_i^d P^d A_i^{1-\epsilon_d}\right]^{\frac{1}{1-\alpha(1-\epsilon_d)}}(1+r)^{\frac{-\alpha(1-\epsilon_d)}{1-\alpha(1-\epsilon_d)}}\left[\alpha(1-\epsilon_d)\right]^{\frac{1}{1-\alpha(1-\epsilon_d)}} \\
&\quad -(1+r)k_m - (1+r)F^d
\end{aligned}$$

$$= \left[\frac{B_i^d P^d A_i^{1-\epsilon_d}}{(1+r)^{\alpha(1-\epsilon_d)}}\right]^{\frac{1}{1-\alpha(1-\epsilon_d)}} \left\{[\alpha(1-\epsilon_d)]^{\frac{\alpha(1-\epsilon_d)}{1-\alpha(1-\epsilon_d)}} - [\alpha(1-\epsilon_d)]^{\frac{1}{1-\alpha(1-\epsilon_d)}}\right\}$$
$$-(1+r)k_m - (1+r)F^d \tag{A.3}$$

A.2 The proof of concavity of π_E

The sufficient conditions for the concavity are (i) $\frac{\partial^2 \pi_E}{\partial x_i^2} \leq 0$ (ii) $\frac{\partial^2 \pi_E}{\partial k_i^2} \leq 0$, and (iii) $\frac{\partial^2 \pi_E}{\partial x_i^2}\frac{\partial^2 \pi_E}{\partial k_i^2} - \left(\frac{\partial^2 \pi_E}{\partial k_i \partial x_i}\right)^2 \geq 0$.

Before continuing with differentiation lets simplify 2.52 as

$$\begin{aligned}\pi_E &= \gamma_d(1-x_i)^{1-\epsilon_d}(k_i-k_m)^{\alpha(1-\epsilon_d)} \\ &\quad +\gamma_e x_i^{1-\epsilon_e}(k_i-k_m)^{\alpha(1-\epsilon_e)} - (1+r)(k_i+F^d+F^e)\end{aligned} \tag{A.4}$$

where

$$\begin{aligned}\gamma_d &= B_i^d P^d A_i^{1-\epsilon_d} \\ \gamma_e &= B_i^e P^e A_i^{1-\epsilon_e}\end{aligned}$$

The first and second order derivatives with respect to x_i is

$$\begin{aligned}\frac{\partial \pi_E}{\partial x_i} &= -\gamma_d(1-\epsilon_d)(1-x_i)^{-\epsilon_d}(k_i-k_m)^{\alpha(1-\epsilon_d)} \\ &\quad +\gamma_e(1-\epsilon_e)x_i^{-\epsilon_e}(k_i-k_m)^{\alpha(1-\epsilon_e)}\end{aligned} \tag{A.5}$$

$$\begin{aligned}\frac{\partial^2 \pi_E}{\partial x_i^2} &= -\gamma_d(1-\epsilon_d)\epsilon_d(1-x_i)^{-1-\epsilon_d}(k_i-k_m)^{\alpha(1-\epsilon_d)} \\ &\quad -\gamma_e(1-\epsilon_e)\epsilon_e x_i^{-1-\epsilon_e}(k_i-k_m)^{\alpha(1-\epsilon_e)}\end{aligned} \tag{A.6}$$

For every $0 \leq x_i \leq 1$ and $k_i > k_m$, $\frac{\partial^2 \pi_E}{\partial x_i^2}$ is negative.

The first and second order derivatives with respect to k_i is

$$\begin{aligned}\frac{\partial \pi_E}{\partial k_i} &= \gamma_d(1-x_i)^{1-\epsilon_d}\alpha(1-\epsilon_d)(k_i-k_m)^{\alpha(1-\epsilon_d)-1} \\ &\quad +\gamma_e x_i^{1-\epsilon_e}\alpha(1-\epsilon_e)(k_i-k_m)^{\alpha(1-\epsilon_e)-1} - (1+r)\end{aligned} \tag{A.7}$$

$$\begin{aligned}\frac{\partial^2 \pi_E}{\partial k_i^2} &= \gamma_d(1-x_i)^{1-\epsilon_d}\alpha(1-\epsilon_d)\left[\alpha(1-\epsilon_d)-1\right](k_i-k_m)^{\alpha(1-\epsilon_d)-2} \\ &\quad +\gamma_e x_i^{1-\epsilon_e}\alpha(1-\epsilon_e)\left[\alpha(1-\epsilon_e)-1\right](k_i-k_m)^{\alpha(1-\epsilon_e)-2}\end{aligned} \tag{A.8}$$

For every $0 \leq x_i \leq 1$ and $k_i > k_m$, $\frac{\partial^2 \pi_E}{\partial k_i^2}$ is negative since $\alpha(1-\epsilon_d)-1<0$ and $\alpha(1-\epsilon_e)-1<0$.

The third condition for concavity is $\frac{\partial^2 \pi_E}{\partial x_i^2}\frac{\partial^2 \pi_E}{\partial k_i^2} - \left(\frac{\partial^2 \pi_E}{\partial k_i \partial x_i}\right)^2 \geq 0$. Since the equations get so complicated, lets derive each term separately

$$\begin{aligned}\frac{\partial^2 \pi_E}{\partial k_i \partial x_i} &= -\gamma_d(1-x_i)^{-\epsilon_d}\alpha(1-\epsilon_d)^2(k_i-k_m)^{\alpha(1-\epsilon_d)-1} \\ &\quad +\gamma_e x_i^{-\epsilon_e}\alpha(1-\epsilon_e)^2(k_i-k_m)^{\alpha(1-\epsilon_e)-1} \end{aligned} \tag{A.9}$$

The square of this equation is

$$\begin{aligned}\left[\frac{\partial^2 \pi_E}{\partial k_i \partial x_i}\right]^2 &= \left[\gamma_d(1-x_i)^{-\epsilon_d}\alpha(1-\epsilon_d)^2(k_i-k_m)^{\alpha(1-\epsilon_d)-1}\right]^2 \\ &\quad + \left[\gamma_e x_i^{-\epsilon_e}\alpha(1-\epsilon_e)^2(k_i-k_m)^{\alpha(1-\epsilon_e)-1}\right]^2 \\ &\quad -2\left[\gamma_d(1-x_i)^{-\epsilon_d}\alpha(1-\epsilon_d)^2(k_i-k_m)^{\alpha(1-\epsilon_d)-1}\right] \\ &\quad\quad \left[\gamma_e x_i^{-\epsilon_e}\alpha(1-\epsilon_e)^2(k_i-k_m)^{\alpha(1-\epsilon_e)-1}\right] \\ &= \gamma_d^2(1-x_i)^{-2\epsilon_d}\alpha^2(1-\epsilon_d)^4(k_i-k_m)^{2[\alpha(1-\epsilon_d)-1]} \\ &\quad +\gamma_e^2 x_i^{-2\epsilon_e}\alpha^2(1-\epsilon_e)^4(k_i-k_m)^{2[\alpha(1-\epsilon_e)-1]} \\ &\quad -2\gamma_e\gamma_d x_i^{-\epsilon_e}(1-x_i)^{-\epsilon_d}\alpha^2(1-\epsilon_e)^2(1-\epsilon_d)^2 \\ &\quad\quad (k_i-k_m)^{\alpha(1-\epsilon_e)+\alpha(1-\epsilon_d)-2} \end{aligned} \tag{A.10}$$

And the multiplication $\frac{\partial^2 \pi_E}{\partial x_i^2}\frac{\partial^2 \pi_E}{\partial k_i^2}$ is

$$\begin{aligned}\frac{\partial^2 \pi_E}{\partial x_i^2}\frac{\partial^2 \pi_E}{\partial k_i^2} &= \left[-\gamma_d(1-\epsilon_d)\epsilon_d(1-x_i)^{-1-\epsilon_d}(k_i-k_m)^{\alpha(1-\epsilon_d)}\right. \\ &\quad \left.-\gamma_e(1-\epsilon_e)\epsilon_e x_i^{-1-\epsilon_e}(k_i-k_m)^{\alpha(1-\epsilon_e)}\right] \\ &\quad \left[\gamma_d(1-x_i)^{1-\epsilon_d}\alpha(1-\epsilon_d)\left[\alpha(1-\epsilon_d)-1\right](k_i-k_m)^{\alpha(1-\epsilon_d)-2}\right. \\ &\quad \left.+\gamma_e x_i^{1-\epsilon_e}\alpha(1-\epsilon_e)\left[\alpha(1-\epsilon_e)-1\right](k_i-k_m)^{\alpha(1-\epsilon_e)-2}\right] \\ &= -\gamma_d^2(1-x_i)^{-2\epsilon_d}\alpha\epsilon_d(1-\epsilon_d)^2\left[\alpha(1-\epsilon_d)-1\right](k_i-k_m)^{2([\alpha(1-\epsilon_d)-1]} \\ &\quad -\gamma_e^2 x_i^{-2\epsilon_e}\alpha\epsilon_e(1-\epsilon_e)^2\left[\alpha(1-\epsilon_e)-1\right](k_i-k_m)^{2[\alpha(1-\epsilon_e)-1]} \\ &\quad -\gamma_d\gamma_e(1-x_i)^{-1-\epsilon_d}x_i^{1-\epsilon_e}\alpha\epsilon_d(1-\epsilon_e)(1-\epsilon_d)\left[\alpha(1-\epsilon_e)-1\right] \\ &\quad\quad (k_i-k_m)^{\alpha(1-\epsilon_d)+\alpha(1-\epsilon_e)-2} \\ &\quad -\gamma_d\gamma_e(1-x_i)^{1-\epsilon_d}x_i^{-1-\epsilon_e}\alpha\epsilon_e(1-\epsilon_e)(1-\epsilon_d)\left[\alpha(1-\epsilon_d)-1\right] \end{aligned}$$

$$
\begin{aligned}
& (k_i - k_m)^{\alpha(1-\epsilon_d)+\alpha(1-\epsilon_e)-2} \\
= \quad & -\gamma_d^2(1-x_i)^{-2\epsilon_d}\alpha\epsilon_d(1-\epsilon_d)^2\left[\alpha(1-\epsilon_d)-1\right](k_i-k_m)^{2([\alpha(1-\epsilon_d)-1]} \\
& -\gamma_e^2 x_i^{-2\epsilon_e}\alpha\epsilon_e(1-\epsilon_e)^2\left[\alpha(1-\epsilon_e)-1\right](k_i-k_m)^{2[\alpha(1-\epsilon_e)-1]} \\
& -\gamma_d\gamma_e x_i^{-\epsilon_e}(1-x_i)^{-\epsilon_d}\alpha(1-\epsilon_e)(1-\epsilon_d)(k_i-k_m)^{\alpha(1-\epsilon_d)+\alpha(1-\epsilon_e)-2} \\
& \left\{\frac{x_i}{1-x_i}\epsilon_d\left[\alpha(1-\epsilon_e)-1\right]+\frac{1-x_i}{x_i}\epsilon_e\left[\alpha(1-\epsilon_d)-1\right]\right\}
\end{aligned}
$$

Now finding the difference between the above terms

$$
\begin{aligned}
& \frac{\partial^2\pi_E}{\partial x_i^2}\frac{\partial^2\pi_E}{\partial k_i^2}-\left(\frac{\partial^2\pi_E}{\partial k_i\partial x_i}\right)^2 = \\
& \gamma_d^2(1-x_i)^{-2\epsilon_d}\alpha(1-\epsilon_d)^2(k_i-k_m)^{2[\alpha(1-\epsilon_d)-1]} \\
& \left\{-\epsilon_d\left[\alpha(1-\epsilon_d)-1\right]-\alpha(1-\epsilon_d)^2\right\} \\
& +\gamma_e^2 x_i^{-2\epsilon_e}\alpha(1-\epsilon_e)^2(k_i-k_m)^{2[\alpha(1-\epsilon_e)-1]} \\
& \left\{-\epsilon_e\left[\alpha(1-\epsilon_e)-1\right]-\alpha(1-\epsilon_e)^2\right\} \\
& +2\gamma_e\gamma_d x_i^{-\epsilon_e}(1-x_i)^{-\epsilon_d}\alpha(1-\epsilon_e)(1-\epsilon_d)(k_i-k_m)^{\alpha(1-\epsilon_e)+\alpha(1-\epsilon_d)-2} \\
& \quad\left\{\frac{1}{2}\frac{x_i}{1-x_i}\epsilon_d\left[1-\alpha(1-\epsilon_e)\right]+\frac{1}{2}\frac{1-x_i}{x_i}\epsilon_e\left[1-\alpha(1-\epsilon_d)\right]+\alpha(1-\epsilon_e)(1-\epsilon_d)\right\}
\end{aligned} \tag{A.11}
$$

Calling the positive variables X, and Y as

$$
\begin{aligned}
X &= \gamma_d(1-x_i)^{-\epsilon_d}\alpha^{0.5}(1-\epsilon_d)(k_i-k_m)^{\alpha(1-\epsilon_d)-1} \\
Y &= \gamma_e x_i^{-\epsilon_e}\alpha^{0.5}(1-\epsilon_e)(k_i-k_m)^{\alpha(1-\epsilon_e)-1}
\end{aligned}
$$

And performing the simplification

$$
a = -\epsilon_d\left[\alpha(1-\epsilon_d)-1\right]-\alpha(1-\epsilon_d)^2 = \epsilon_d-\alpha(1-\epsilon_d) \tag{A.12}
$$

$$
c = -\epsilon_e\left[\alpha(1-\epsilon_e)-1\right]-\alpha(1-\epsilon_e)^2 = \epsilon_e-\alpha(1-\epsilon_e) \tag{A.13}
$$

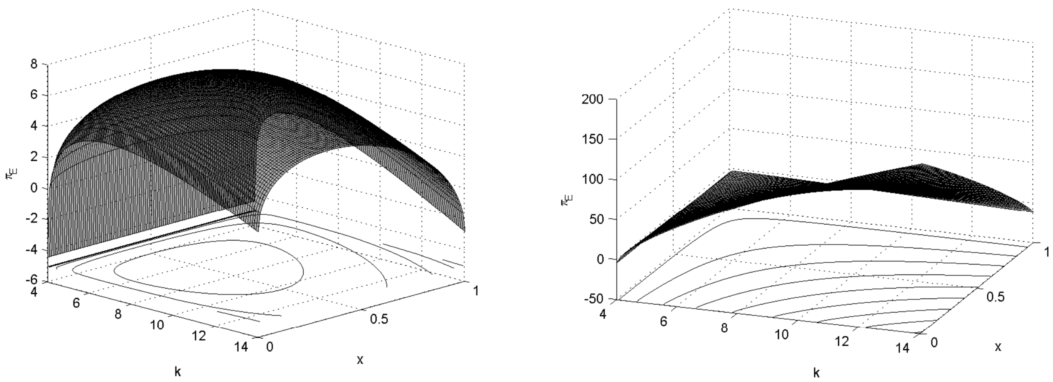
And defining one more variable as

$$
b = \frac{1}{2}\frac{x_i}{1-x_i}\epsilon_d\left[1-\alpha(1-\epsilon_e)\right]+\frac{1}{2}\frac{1-x_i}{x_i}\epsilon_e\left[1-\alpha(1-\epsilon_d)\right]+\alpha(1-\epsilon_e)(1-\epsilon_d)
$$

then equation A.11 becomes

$$
\frac{\partial^2\pi_E}{\partial x_i^2}\frac{\partial^2\pi_E}{\partial k_i^2}-\left(\frac{\partial^2\pi_E}{\partial k_i\partial x_i}\right)^2 = aX^2+2bXY+cY^2
$$

which is in quadratic form. One may observe that a is positive if $\epsilon_d - \alpha(1-\epsilon_d) > 0$ or $\alpha < \frac{\epsilon_d}{1-\epsilon_d}$ and c is positive if $\alpha < \frac{\epsilon_e}{1-\epsilon_e}$. The term b is always positive. And we know that the variables X and Y in quadratic equation are always positive. Therefore, we can conclude that if $\alpha < \frac{\epsilon_d}{1-\epsilon_d}$ and $\alpha < \frac{\epsilon_e}{1-\epsilon_e}$ then $\frac{\partial^2 \pi_E}{\partial x_i^2} \frac{\partial^2 \pi_E}{\partial k_i^2} - \left(\frac{\partial^2 \pi_E}{\partial k_i \partial x_i} \right)^2$ is positive definite and so the profit function is concave (and has a global maximum), if any of the conditions, $\alpha < \frac{\epsilon_d}{1-\epsilon_d}$ and $\alpha < \frac{\epsilon_e}{1-\epsilon_e}$, is not valid then the profit function π_E is indefinite. In Figure A.1 there are two samples of profit function. The function on the left side is concave satisfying the conditions $\alpha < \frac{\epsilon_d}{1-\epsilon_d}$ and $\alpha < \frac{\epsilon_e}{1-\epsilon_e}$, while the one on the right side does not satisfy these conditions.



$\alpha < \frac{\epsilon_d}{1-\epsilon_d}$ and $\alpha < \frac{\epsilon_e}{1-\epsilon_e}$ $\qquad$ $\alpha > \frac{\epsilon_d}{1-\epsilon_d}$ and $\alpha < \frac{\epsilon_e}{1-\epsilon_e}$

Figure A.1: The profit function π_E under two conditions. The left one is a concave function and the one on the right is not concave.

A.3 Derivation of $\frac{dk_i^{max}}{dA_i}$ in Export Market

To find the response of profit maximizing investment to changes in productivity we should use implicit differentiation method. Before taking the derivative lets first simplify equation 3.20 as

$$\Psi^e A_i^{\frac{1-\epsilon_e}{\epsilon_e}} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_e)-1}{\epsilon_e}} + \Psi^d A_i^{\frac{1-\epsilon_d}{\epsilon_d}} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_d)-1}{\epsilon_d}} = 1 \qquad \text{(A.14)}$$

where

$$\begin{aligned} \Psi^e &= \left[\frac{B_i^e P^e (1-\epsilon_e)\alpha}{1+r}\right]^{\frac{1}{\epsilon_e}} \\ \Psi^d &= \left[\frac{B_i^d P^d (1-\epsilon_d)\alpha}{1+r}\right]^{\frac{1}{\epsilon_d}} \end{aligned}$$

Note that Ψ^e and Ψ^d are positive constants. The implicit derivative of equation A.14 is

$$\begin{aligned} &\Psi^e \frac{1-\epsilon_e}{\epsilon_e} A_i^{\frac{1-\epsilon_e}{\epsilon_e}-1} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_e)-1}{\epsilon_e}} dA_i \\ &+\Psi^e A_i^{\frac{1-\epsilon_e}{\epsilon_e}} \frac{\alpha(1-\epsilon_e)-1}{\epsilon_e} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_e)-1}{\epsilon_e}-1} dk_i^{max} \\ &+\Psi^d \frac{1-\epsilon_d}{\epsilon_d} A_i^{\frac{1-\epsilon_d}{\epsilon_d}-1} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_d)-1}{\epsilon_d}} dA_i \\ &+\Psi^d A_i^{\frac{1-\epsilon_d}{\epsilon_d}} \frac{\alpha(1-\epsilon_d)-1}{\epsilon_d} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_d)-1}{\epsilon_d}-1} dk_i^{max} = 0 \qquad \text{(A.15)} \end{aligned}$$

Calling new variables as

$$\begin{aligned} \Phi_1^e &= \Psi^e \frac{1-\epsilon_e}{\epsilon_e} A_i^{\frac{1-\epsilon_e}{\epsilon_e}-1} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_e)-1}{\epsilon_e}} \\ \Phi_2^e &= \Psi^e A_i^{\frac{1-\epsilon_e}{\epsilon_e}} \frac{\alpha(1-\epsilon_e)-1}{\epsilon_e} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_e)-1}{\epsilon_e}-1} \\ \Phi_1^d &= \Psi^d \frac{1-\epsilon_d}{\epsilon_d} A_i^{\frac{1-\epsilon_d}{\epsilon_d}-1} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_d)-1}{\epsilon_d}} \\ \Phi_2^d &= \Psi^d A_i^{\frac{1-\epsilon_d}{\epsilon_d}} \frac{\alpha(1-\epsilon_d)-1}{\epsilon_d} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_d)-1}{\epsilon_d}-1} \end{aligned}$$

The variables Φ_1^e and Φ_1^d are always positive and Φ_2^e and Φ_2^d are always negative $(\alpha(1-\epsilon_d)-1<0)$. Then

$$\frac{dk_i^{max}}{dA_i} = -\frac{\Phi_1^e + \Phi_1^d}{\Phi_2^e + \Phi_2^d} > 0$$

which means the productivity growth leads to increase in maximizing investment or the firm with higher productivity (ceteris paribus) will pay for higher investment to maximize the profit.

A.4 Derivation of $\frac{dx_i^{max}}{dA_i}$ in Export Market

To find the response of profit maximizing x_i^{max} to changes in productivity we should use implicit derivative method. Before taking the derivative lets first simplify equation 3.21 as

$$\begin{aligned} \xi^e A_i^{\frac{1-\epsilon_e}{1-\alpha(1-\epsilon_e)}} (x_i^{max})^{\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1}} &= \xi^d A_i^{\frac{1-\epsilon_d}{1-\alpha(1-\epsilon_d)}} (1-x_i^{max})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}} \\ \frac{\xi^e}{\xi^d} A_i^{\left[\frac{1-\epsilon_e}{1-\alpha(1-\epsilon_e)} - \frac{1-\epsilon_d}{1-\alpha(1-\epsilon_d)}\right]} (x_i^{max})^{\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1}} &= (1-x_i^{max})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}} \end{aligned} \tag{A.16}$$

where

$$\begin{aligned} \xi^e &= \left[\frac{B_i^e P^e (1-\epsilon_e)\alpha}{1+r}\right]^{\frac{1}{1-\alpha(1-\epsilon_e)}} \\ \xi^d &= \left[\frac{B_i^d P^d (1-\epsilon_d)\alpha}{1+r}\right]^{\frac{1}{1-\alpha(1-\epsilon_d)}} \end{aligned}$$

are positive constants. The implicit derivative of equation A.16 is

$$\begin{aligned} & \frac{\xi^e}{\xi^d}\left[\frac{1-\epsilon_e}{1-\alpha(1-\epsilon_e)} - \frac{1-\epsilon_d}{1-\alpha(1-\epsilon_d)}\right] A_i^{\left[\frac{1-\epsilon_e}{1-\alpha(1-\epsilon_e)} - \frac{1-\epsilon_d}{1-\alpha(1-\epsilon_d)}\right]-1} (x_i^{max})^{\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1}} dA_i \\ & + \frac{\xi^e}{\xi^d} A_i^{\left[\frac{1-\epsilon_e}{1-\alpha(1-\epsilon_e)} - \frac{1-\epsilon_d}{1-\alpha(1-\epsilon_d)}\right]} \frac{\epsilon_e}{\alpha(1-\epsilon_e)-1} (x_i^{max})^{\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1}-1} dx_i^{max} \\ = & -\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}(1-x_i^{max})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}-1} dx_i^{max} \end{aligned} \tag{A.17}$$

Calling the variables

$$\begin{aligned} \beta_1 &= \frac{\xi^e}{\xi^d}\left[\frac{1-\epsilon_e}{1-\alpha(1-\epsilon_e)} - \frac{1-\epsilon_d}{1-\alpha(1-\epsilon_d)}\right] A_i^{\left[\frac{1-\epsilon_e}{1-\alpha(1-\epsilon_e)} - \frac{1-\epsilon_d}{1-\alpha(1-\epsilon_d)}\right]-1} (x_i^{max})^{\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1}} \\ \beta_2 &= \frac{\xi^e}{\xi^d} A_i^{\left[\frac{1-\epsilon_e}{1-\alpha(1-\epsilon_e)} - \frac{1-\epsilon_d}{1-\alpha(1-\epsilon_d)}\right]} \frac{\epsilon_e}{\alpha(1-\epsilon_e)-1} (x_i^{max})^{\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1}-1} \\ \beta_3 &= -\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}(1-x_i^{max})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}-1} \end{aligned} \tag{A.18}$$

The derivative becomes

$$\frac{dx_i^{max}}{dA_i} = \frac{\beta_1}{\beta_3 - \beta_2}$$

In this equation the variable β_3 is always positive and β_2 is always negative. Therefore the sign of $\frac{dx_i^{max}}{dA_i}$ depends on β_1. If

$$\frac{1-\epsilon_e}{1-\alpha(1-\epsilon_e)} - \frac{1-\epsilon_d}{1-\alpha(1-\epsilon_d)}$$

is negative then β_1 is negative and so $\frac{dx_i^{max}}{dA_i}$. Vice versa is also valid. Performing some manipulation, one can simply show that

$$\begin{array}{lll} \frac{dx_i^{max}}{dA_i} < 0 & if & \epsilon_d < \epsilon_e \\ \frac{dx_i^{max}}{dA_i} = 0 & if & \epsilon_d = \epsilon_e \\ \frac{dx_i^{max}}{dA_i} > 0 & if & \epsilon_d > \epsilon_e \end{array} \tag{A.19}$$

In economic means, if the elasticity of domestic market is higher than the export market ($\epsilon_d < \epsilon_e$) the firm will decrease its export market share x_i as its productivity increases. In other words the firms with higher productivity will have a lower share of export market if the domestic market is more elastic. The vice versa is also valid. If export market is more elastic then the share of export market it will be higher and the firm with higher productivity will produce more for export market to have its profit maximum.

A.5 Derivation of $\frac{dk_i^{max}}{dB_i}$ in Export Market

The change of profit maximizing k_i^{max} with respect to change in B_i^d or B_i^e can be found by implicit differentiation. Let us first simplify the equation 3.20 as

$$\delta^e (B_i^e)^{\frac{1}{\epsilon_e}} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_e)-1}{\epsilon_e}} + \delta^d (B_i^d)^{\frac{1}{\epsilon_d}} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_d)-1}{\epsilon_d}} = 1$$

where the positive constants δ^e and δ^d are

$$\begin{aligned} \delta^e &= \left[\frac{P^e A_i^{1-\epsilon_e}(1-\epsilon_e)\alpha}{1+r}\right]^{\frac{1}{\epsilon_e}} \\ \delta^d &= \left[\frac{P^d A_i^{1-\epsilon_d}(1-\epsilon_d)\alpha}{1+r}\right]^{\frac{1}{\epsilon_d}} \end{aligned}$$

Again utilizing the implicit derivative

$$\begin{aligned} & \delta^e (B_i^e)^{\frac{1}{\epsilon_e}} \frac{\alpha(1-\epsilon_e)-1}{\epsilon_e} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_e)-1}{\epsilon_e}-1} dk_i^{max} \\ & + \delta^d \frac{1}{\epsilon_d} (B_i^d)^{\frac{1}{\epsilon_d}-1} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_d)-1}{\epsilon_d}} dB_i^d \qquad \text{(A.20)} \\ & + \delta^d (B_i^d)^{\frac{1}{\epsilon_d}} \frac{\alpha(1-\epsilon_d)-1}{\epsilon_d} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_d)-1}{\epsilon_d}-1} dk_i^{max} = 0 \qquad \text{(A.21)} \end{aligned}$$

and rearranging

$$\frac{dk_i^{max}}{dB_i^d} = -\frac{R_{Bd}}{R_{k1} + R_{k2}} \tag{A.22}$$

where

$$\begin{aligned}
R_{Bd} &= \delta^d \frac{1}{\epsilon_d} (B_i^d)^{\frac{1}{\epsilon_d}-1} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_d)-1}{\epsilon_d}} \\
R_{k1} &= \delta^e (B_i^e)^{\frac{1}{\epsilon_e}} \frac{\alpha(1-\epsilon_e)-1}{\epsilon_e} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_e)-1}{\epsilon_e}-1} \\
R_{k2} &= \delta^d (B_i^d)^{\frac{1}{\epsilon_d}} \frac{\alpha(1-\epsilon_d)-1}{\epsilon_d} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_d)-1}{\epsilon_d}-1}
\end{aligned}$$

The variables R_{k1} and R_{k2} are always negative and R_{Bd} is always positive. Therefore, $\frac{dk_i^{max}}{dB_i^d}$ is positive, which means the profit maximizing investment amount is increasing with rise in firm's domestic market parameter B_i^d. The same result can be found in a similar way for parameter B_i^e.

$$\frac{dk_i^{max}}{dB_i^e} = -\frac{R_{Be}}{R_{k1} + R_{k2}} \tag{A.23}$$

where

$$R_{Be} = \delta^e \frac{1}{\epsilon_e} (B_i^e)^{\frac{1}{\epsilon_d}-1} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_e)-1}{\epsilon_e}}$$

Again $\frac{dk_i^{max}}{dB_i^e}$ is always positive.

A.6 Derivation of $\frac{dx_i^{max}}{dB_i}$ in Export Market

Rearranging 3.21 for simplification

$$(B_i^e)^{\frac{1}{1-\alpha(1-\epsilon_e)}} \gamma^e (x_i^{max})^{\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1}} = (B_i^d)^{\frac{1}{1-\alpha(1-\epsilon_d)}} \gamma^d (1 - x_i^{max})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}}$$

where constants γ^e and γ^d are such positive constants

$$\begin{aligned}
\gamma^e &= \left[\frac{P^e A_i^{1-\epsilon_e} (1-\epsilon_e)\alpha}{1+r} \right]^{\frac{1}{1-\alpha(1-\epsilon_e)}} \\
\gamma^d &= \left[\frac{P^d A_i^{1-\epsilon_d} (1-\epsilon_d)\alpha}{1+r} \right]^{\frac{1}{1-\alpha(1-\epsilon_d)}}
\end{aligned}$$

utilizing the implicit derivative for B_i^d

$$(B_i^e)^{\frac{1}{1-\alpha(1-\epsilon_e)}}\gamma^e\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1}(x_i^{max})^{\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1}-1}dx_i^{max} =$$

$$\frac{1}{1-\alpha(1-\epsilon_d)}(B_i^d)^{\frac{1}{1-\alpha(1-\epsilon_d)}-1}\gamma^d(1-x_i^{max})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}}dB_i^d \quad (A.24)$$

$$-(B_i^d)^{\frac{1}{1-\alpha(1-\epsilon_d)}}\gamma^d\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}(1-x_i^{max})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}-1}dx_i^{max} \quad (A.25)$$

Then the derivative is

$$\frac{dx_i^{max}}{dB_i^d} = \frac{C_{Bd}}{C_{x1}+C_{x2}} \quad (A.26)$$

where

$$\begin{aligned}
C_{Bd} &= \frac{1}{1-\alpha(1-\epsilon_d)}(B_i^d)^{\frac{1}{1-\alpha(1-\epsilon_d)}-1}\gamma^d(1-x_i^{max})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}}\\
C_{x1} &= (B_i^e)^{\frac{1}{1-\alpha(1-\epsilon_e)}}\gamma^e\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1}(x_i^{max})^{\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1}-1}\\
C_{x2} &= (B_i^d)^{\frac{1}{1-\alpha(1-\epsilon_d)}}\gamma^d\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}(1-x_i^{max})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}-1}
\end{aligned}$$

One may observe that $C_{Bd} > 0$ and $C_{x1} < 0$ and $C_{x1} < 0$. Therefore, $\frac{dx_i^{max}}{dB_i^d}$ is always negative. With a similar way

$$\frac{dx_i^{max}}{dB_i^e} = -\frac{C_{Be}}{C_{x1}+C_{x2}} \quad (A.27)$$

where

$$C_{Be} = \frac{1}{1-\alpha(1-\epsilon_e)}(B_i^e)^{\frac{1}{1-\alpha(1-\epsilon_e)}-1}\gamma^e(x_i^{max})^{\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1}} \quad (A.28)$$

Since, C_{Be} is always positive, the derivative $\frac{dx_i^{max}}{dB_i^e}$ is always positive, too.

The results show that the increase in the domestic market parameter B_i^d will result in a decrease in the export market share and vice versa. In addition, the increase in export market parameter B_i^e results in increase in export market share and a decrease in this parameter leads to a decrease in export market share x_i.

A.7 Derivation of $\frac{dk_i^{max}}{dr}$ in Export Market

To find the derivative of k_i^{max} with respect r lets rearrange equation 3.20 as

$$\begin{aligned} & [\varphi^e]^{\frac{1}{\epsilon_e}} (1+r)^{-\frac{1}{\epsilon_e}} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_e)-1}{\epsilon_e}} \\ + \quad & \left[\varphi^d\right]^{\frac{1}{\epsilon_d}} (1+r)^{-\frac{1}{\epsilon_d}} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_d)-1}{\epsilon_d}} = 1 \end{aligned} \tag{A.29}$$

where the constants are

$$\begin{aligned} \varphi^e &= B_i^e P^e A_i^{1-\epsilon_e} (1-\epsilon_e)\alpha \\ \varphi^d &= B_i^d P^d A_i^{1-\epsilon_d} (1-\epsilon_d)\alpha \end{aligned}$$

The differentiation of A.29 is

$$\begin{aligned} 0 = \quad & -[\varphi^e]^{\frac{1}{\epsilon_e}} \left(\frac{1}{\epsilon_e}\right) (1+r)^{-\frac{1}{\epsilon_e}-1} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_e)-1}{\epsilon_e}} dr \\ & + [\varphi^e]^{\frac{1}{\epsilon_e}} (1+r)^{-\frac{1}{\epsilon_e}} \left(\frac{\alpha(1-\epsilon_e)-1}{\epsilon_e}\right) (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_e)-1}{\epsilon_e}-1} dk_i^{max} \\ & - \left[\varphi^d\right]^{\frac{1}{\epsilon_d}} \left(-\frac{1}{\epsilon_d}\right) (1+r)^{-\frac{1}{\epsilon_d}-1} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_d)-1}{\epsilon_d}} dr \\ & + \left[\varphi^d\right]^{\frac{1}{\epsilon_d}} (1+r)^{-\frac{1}{\epsilon_d}} \left(\frac{\alpha(1-\epsilon_d)-1}{\epsilon_d}\right) (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_d)-1}{\epsilon_d}-1} dk_i^{max} \end{aligned} \tag{A.30}$$

Rearranging

$$\frac{dk_i^{max}}{dr} = \frac{\eta_{r1} + \eta_{r2}}{\eta_{k1} + \eta_{k2}} \tag{A.31}$$

where constants η_{r1}, η_{r2} are positive and η_{k1}, η_{k2} are all negative.

$$\begin{aligned} \eta_{r1} = \quad & [\varphi^e]^{\frac{1}{\epsilon_e}} \left(\frac{1}{\epsilon_e}\right) (1+r)^{-\frac{1}{\epsilon_e}-1} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_e)-1}{\epsilon_e}} \\ \eta_{k1} = \quad & [\varphi^e]^{\frac{1}{\epsilon_e}} (1+r)^{-\frac{1}{\epsilon_e}} \left(\frac{\alpha(1-\epsilon_e)-1}{\epsilon_e}\right) (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_e)-1}{\epsilon_e}-1} \\ \eta_{r2} = \quad & \left[\varphi^d\right]^{\frac{1}{\epsilon_d}} \left(-\frac{1}{\epsilon_d}\right) (1+r)^{-\frac{1}{\epsilon_d}-1} (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_d)-1}{\epsilon_d}} \\ \eta_{k2} = \quad & \left[\varphi^d\right]^{\frac{1}{\epsilon_d}} (1+r)^{-\frac{1}{\epsilon_d}} \left(\frac{\alpha(1-\epsilon_d)-1}{\epsilon_d}\right) (k_i^{max} - k_m)^{\frac{\alpha(1-\epsilon_d)-1}{\epsilon_d}-1} \end{aligned} \tag{A.32}$$

Therefore, the derivative $\frac{dk_i^{max}}{dr}$ is negative. In other words the increase in interest rate would result in decrease in the profit maximizing investment amount and viceversa.

A.8 Derivation of $\frac{dx_i^{max}}{dr}$ in Export Market

For the differentiation of x_i^{max} with respect to r, simplifying the equation 3.21 as

$$\begin{aligned} &[\varphi^e]^{\frac{1}{1-\alpha(1-\epsilon_e)}} (1+r)^{-\frac{1}{1-\alpha(1-\epsilon_e)}} (x_i^{max})^{\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1}} = \\ &\left[\varphi^d\right]^{\frac{1}{1-\alpha(1-\epsilon_d)}} (1+r)^{-\frac{1}{1-\alpha(1-\epsilon_d)}} (1-x_i^{max})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}} \end{aligned} \tag{A.33}$$

or

$$\begin{aligned} &\frac{[\varphi^e]^{\frac{1}{1-\alpha(1-\epsilon_e)}}}{[\varphi^d]^{\frac{1}{1-\alpha(1-\epsilon_d)}}} (1+r)^{\frac{1}{1-\alpha(1-\epsilon_d)}-\frac{1}{1-\alpha(1-\epsilon_e)}} (x_i^{max})^{\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1}} = \\ &\qquad (1-x_i^{max})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}} \end{aligned} \tag{A.34}$$

Calling

$$\kappa = \frac{[\varphi^e]^{\frac{1}{1-\alpha(1-\epsilon_e)}}}{[\varphi^d]^{\frac{1}{1-\alpha(1-\epsilon_d)}}}$$

and rewriting A.34

$$\begin{aligned} &\kappa(1+r)^{\frac{1}{1-\alpha(1-\epsilon_d)}-\frac{1}{1-\alpha(1-\epsilon_e)}} (x_i^{max})^{\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1}} = \\ &\qquad (1-x_i^{max})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}} \end{aligned} \tag{A.35}$$

The implicit differentiation will be

$$\begin{aligned} &\kappa \left[\frac{1}{1-\alpha(1-\epsilon_d)} - \frac{1}{1-\alpha(1-\epsilon_e)}\right] (1+r)^{\frac{1}{1-\alpha(1-\epsilon_d)}-\frac{1}{1-\alpha(1-\epsilon_e)}-1} (x_i^{max})^{\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1}} dr \\ &+\kappa(1+r)^{\frac{1}{1-\alpha(1-\epsilon_d)}-\frac{1}{1-\alpha(1-\epsilon_e)}} \left[\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1}\right] (x_i^{max})^{\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1}-1} dx_i^{max} \\ &= \\ &-\left[\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}\right] (1-x_i^{max})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}} dx_i^{max} \end{aligned} \tag{A.36}$$

Then the derivative becomes

$$\frac{dx_i^{max}}{dr} = \frac{\phi_{r1}}{\eta_{x2} - \eta_{x1}} \tag{A.37}$$

where

$$\begin{aligned}
\phi_{r1} &= \kappa \left[\frac{1}{1-\alpha(1-\epsilon_d)} - \frac{1}{1-\alpha(1-\epsilon_e)} \right] \\
&\quad (1+r)^{\frac{1}{1-\alpha(1-\epsilon_d)} - \frac{1}{1-\alpha(1-\epsilon_e)} - 1} (x_i^{max})^{\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1}} \\
\eta_{x1} &= \kappa (1+r)^{\frac{1}{1-\alpha(1-\epsilon_d)} - \frac{1}{1-\alpha(1-\epsilon_e)}} \left[\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1} \right] (x_i^{max})^{\frac{\epsilon_e}{\alpha(1-\epsilon_e)-1} - 1} dx_i^{max} \\
\eta_{x2} &= - \left[\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1} \right] (1 - x_i^{max})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}} dx_i^{max}
\end{aligned}$$

Note that $(\eta_{x2} - \eta_{x1})$ is always positive and the sign of ϕ_{r1} depends on

$$\frac{1}{1-\alpha(1-\epsilon_d)} - \frac{1}{1-\alpha(1-\epsilon_e)}$$

If $\epsilon_d < \epsilon_e$ then this term is positive. If $\epsilon_d > \epsilon_e$ then this term is negative and if $\epsilon_d = \epsilon_e$ then the derivative is zero. Therefore

$$\begin{aligned}
\tfrac{dx_i^{max}}{dr} &< 0 \quad if \;\; \epsilon_d > \epsilon_e \\
\tfrac{dx_i^{max}}{dr} &= 0 \quad if \;\; \epsilon_d = \epsilon_e \\
\tfrac{dx_i^{max}}{dr} &> 0 \quad if \;\; \epsilon_d < \epsilon_e
\end{aligned} \tag{A.38}$$

This statement says if domestic market is more elastic than any increase in interest rate will increase the export market share and if export market is more elastic then any increase in interest will decrease the export market share.

A.9 Profit/Invsetment Ratio

The previous analytical solutions are so arranged that we can derive the ratio $\frac{\pi^{max}}{k_i^{max}-k_m}$ simpler than the ratio $\frac{\pi^{max}}{k_i^{max}}$. Therefore, lets first show the relation between these ratios.

To reach the profit/investment ratio from the profit/efficient investment (where efficient investment is $k_i^{max} - k_m$) we can use the simple relations:

$$\frac{\pi_E^{max}}{k_i^{Emax}} = \frac{\pi_E^{max}}{k_i^{Emax} - k_m} \left(1 - \frac{k_m}{k_i^{Emax}} \right)$$

$$\frac{\pi_D^{max}}{k_i^{Dmax}} = \frac{\pi_D^{max}}{k_i^{Dmax} - k_m}\left(1 - \frac{k_m}{k_i^{Dmax}}\right) \tag{A.39}$$

where due to equation 2.94 we know that $k_i^{Emax} > k_i^{Dmax}$. Thus, $1 - \frac{k_m}{k_i^{Emax}} > 1 - \frac{k_m}{k_i^{Dmax}}$. Therefore, if we find that $\frac{\pi_E^{max}}{k_i^{Emax}-k_m} \geq \frac{\pi_D^{max}}{k_i^{Dmax}-k_m}$ then $\frac{\pi_E^{max}}{k_i^{Emax}} > \frac{\pi_D^{max}}{k_i^{Dmax}}$. On the other hand if the finding is $\frac{\pi_E^{max}}{k_i^{Emax}-k_m} < \frac{\pi_D^{max}}{k_i^{Dmax}-k_m}$ then the relation between $\frac{\pi_E^{max}}{k_i^{Emax}}$ and $\frac{\pi_D^{max}}{k_i^{Dmax}}$ will depend on the numerical values. Note that if $k_i^{Emax} >> k_m$ and $k_i^{Dmax} >> k_m$ then the result of comparison of $\frac{\pi_E^{max}}{k_i^{Emax}-k_m}$ and $\frac{\pi_D^{max}}{k_i^{Dmax}-k_m}$ will be same for $\frac{\pi_E^{max}}{k_i^{Emax}}$ and $\frac{\pi_D^{max}}{k_i^{Dmax}}$.

Now we can derive the relation between the ratios $\frac{\pi^{max}}{k_i^{max}-k_m}$ of both markets.

Rearranging the domestic market maximum profit as

$$\begin{aligned}\pi_D^{max} &= B_i^d P^d A_i^{1-\epsilon_d}(k_i^{Dmax} - k_m)^{\alpha(1-\epsilon_d)} - (1+r)(k_i^{Dmax} + F^d) \\ &= B_i^d P^d A_i^{1-\epsilon_d}(k_i^{Dmax} - k_m)^{\alpha(1-\epsilon_d)} - (1+r)(k_i^{Dmax} - k_m) \\ &\quad -(1+r)(k_m + F^d)\end{aligned} \tag{A.40}$$

dividing each side by $k_i^{Dmax} - k_m$

$$\begin{aligned}\frac{\pi_D^{max}}{k_i^{Dmax} - k_m} &= B_i^d P^d A_i^{1-\epsilon_d}(k_i^{Dmax} - k_m)^{\alpha(1-\epsilon_d)-1} - (1+r) \\ &\quad -\frac{(1+r)(k_m + F^d)}{k_i^{Dmax} - k_m}\end{aligned} \tag{A.41}$$

With a similar way rearranging the export market profit

$$\begin{aligned}\pi_E^{max} &= B_i^d P^d \left[(1 - x_i^{Emax})A_i(k_i^{Emax} - k_m)^{\alpha}\right]^{1-\epsilon_d} \\ &\quad +B_i^e P^e \left[x_i^{Emax} A_i(k_i^{Emax} - k_m)^{\alpha}\right]^{1-\epsilon_e} \\ &\quad -(1+r)(k_i^{Emax} - k_m) - (1+r)(k_m + F^d + F^e)\end{aligned} \tag{A.42}$$

dividing each side by $k_i^{Emax} - k_m$

$$\begin{aligned}\frac{\pi_E^{max}}{k_i^{Emax} - k_m} &= B_i^d P^d (1 - x_i^{Emax})^{1-\epsilon_d} A_i^{1-\epsilon_d}(k_i^{Emax} - k_m)^{\alpha(1-\epsilon_d)-1} \\ &\quad +B_i^e P^e (x_i^{Emax})^{1-\epsilon_e} A_i^{1-\epsilon_e}(k_i^{Emax} - k_m)^{\alpha(1-\epsilon_e)-1} \\ &\quad -(1+r) - \frac{(1+r)(k_m + F^d + F^e)}{k_i^{Emax} - k_m}\end{aligned} \tag{A.43}$$

For comparison of the profits per efficient investments $(k_i^{max} - k_m)$ we will examine the differences of the above ratios

$$\begin{aligned}
\frac{\pi_E^{max}}{k_i^{Emax} - k_m} - \frac{\pi_D^{max}}{k_i^{Dmax} - k_m} &= B_i^d P^d (1 - x_i^{Emax})^{1-\epsilon_d} A_i^{1-\epsilon_d} (k_i^{Emax} - k_m)^{\alpha(1-\epsilon_d)-1} \\
&+ B_i^e P^e (x_i^{Emax})^{1-\epsilon_e} A_i^{1-\epsilon_e} (k_i^{Emax} - k_m)^{\alpha(1-\epsilon_e)-1} \\
&-(1+r) - \frac{(1+r)(k_m + F^d + F^e)}{k_i^{Emax} - k_m} \\
&- B_i^d P^d A_i^{1-\epsilon_d} (k_i^{Dmax} - k_m)^{\alpha(1-\epsilon_d)-1} + (1+r) \\
&+ \frac{(1+r)(k_m + F^d)}{k_i^{Dmax} - k_m} \qquad \text{(A.44)}
\end{aligned}$$

rearranging

$$\begin{aligned}
\frac{\pi_E^{max}}{k_i^{Emax} - k_m} - \frac{\pi_D^{max}}{k_i^{Dmax} - k_m} &= B_i^d P^d (1 - x_i^{Emax})^{1-\epsilon_d} A_i^{1-\epsilon_d} (k_i^{Emax} - k_m)^{\alpha(1-\epsilon_d)-1} \\
&- B_i^d P^d A_i^{1-\epsilon_d} (k_i^{Dmax} - k_m)^{\alpha(1-\epsilon_d)-1} \\
&+ B_i^e P^e (x_i^{Emax})^{1-\epsilon_e} A_i^{1-\epsilon_e} (k_i^{Emax} - k_m)^{\alpha(1-\epsilon_e)-1} \\
&+ \frac{(1+r)(k_m + F^d)}{k_i^{Dmax} - k_m} - \frac{(1+r)(k_m + F^d + F^e)}{k_i^{Emax} - k_m} \qquad \text{(A.45)}
\end{aligned}$$

Now, we will utilize the relation between the optimal investment amounts of both markets (equation 2.94)

$$k_i^{Dmax} - k_m = \left[k_i^{Emax} - k_m\right] (1 - x_i^{Emax})^{\frac{\epsilon_d}{1-\alpha(1-\epsilon_d)}} \qquad \text{(A.46)}$$

substituting this equation into A.45

$$\begin{aligned}
\frac{\pi_E^{max}}{k_i^{Emax} - k_m} - \frac{\pi_D^{max}}{k_i^{Dmax} - k_m} &= \\
& B_i^d P^d (1 - x_i^{Emax})^{1-\epsilon_d} A_i^{1-\epsilon_d} (k_i^{Emax} - k_m)^{\alpha(1-\epsilon_d)-1} \\
& - B_i^d P^d A_i^{1-\epsilon_d} \left((k_i^{Emax} - k_m)(1 - x_i^{Emax})^{\frac{\epsilon_d}{1-\alpha(1-\epsilon_d)}} \right)^{\alpha(1-\epsilon_d)-1} \\
& + B_i^e P^e (x_i^{Emax})^{1-\epsilon_e} A_i^{1-\epsilon_e} (k_i^{Emax} - k_m)^{\alpha(1-\epsilon_e)-1} \\
& + \frac{(1+r)(k_m + F^d)}{(k_i^{Emax} - k_m)(1 - x_i^{Emax})^{\frac{\epsilon_d}{1-\alpha(1-\epsilon_d)}}} \\
& - \frac{(1+r)(k_m + F^d + F^e)}{k_i^{Emax} - k_m} \qquad \text{(A.47)}
\end{aligned}$$

rearranging

$$\begin{aligned}\frac{\pi_E^{max}}{k_i^{Emax}-k_m} - \frac{\pi_D^{max}}{k_i^{Dmax}-k_m} = \; & B_i^d P^d A_i^{1-\epsilon_d}(k_i^{Emax}-k_m)^{\alpha(1-\epsilon_d)-1} \\ & \left((1-x_i^{Emax})^{1-\epsilon_d}-(1-x_i^{Emax})^{-\epsilon_d}\right) \\ & +B_i^e P^e (x_i^{Emax})^{1-\epsilon_e} A_i^{1-\epsilon_e}(k_i^{Emax}-k_m)^{\alpha(1-\epsilon_e)-1} \\ & +\frac{(1+r)(k_m+F^d)}{(k_i^{Emax}-k_m)}\left((1-x_i^{Emax})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}}-1\right) \\ & -\frac{(1+r)F^e}{k_i^{Emax}-k_m}\end{aligned} \tag{A.48}$$

note that

$$\begin{aligned}(1-x_i^{Emax})^{1-\epsilon_d}-(1-x_i^{Emax})^{-\epsilon_d} &= (1-x_i^{Emax})^{-\epsilon_d}(1-x_i^{Emax}-1) \\ &= -x_i^{Emax}(1-x_i^{Emax})^{-\epsilon_d}\end{aligned}$$

then the first two terms (say C_1 and C_2) on the right side of equation A.48 becomes

$$\begin{aligned}C_1+C_2 &= B_i^d P^d A_i^{1-\epsilon_d}(k_i^{Emax}-k_m)^{\alpha(1-\epsilon_d)-1}\left(-x_i^{Emax}(1-x_i^{Emax})^{-\epsilon_d}\right) \\ & \quad +B_i^e P^e (x_i^{Emax})^{1-\epsilon_e} A_i^{1-\epsilon_e}(k_i^{Emax}-k_m)^{\alpha(1-\epsilon_e)-1} \\ &= x_i^{Emax}\left[-B_i^d P^d A_i^{1-\epsilon_d}(k_i^{Emax}-k_m)^{\alpha(1-\epsilon_d)-1}(1-x_i^{Emax})^{-\epsilon_d}\right. \\ & \quad \left.+B_i^e P^e (x_i^{Emax})^{-\epsilon_e} A_i^{1-\epsilon_e}(k_i^{Emax}-k_m)^{\alpha(1-\epsilon_e)-1}\right]\end{aligned} \tag{A.49}$$

rearranging equation 2.70

$$(x_i^{Emax})^{-\epsilon_e} = \frac{1+r}{B_i^e P^e A_i^{1-\epsilon_e}(k_i^{Emax}-k_m)^{\alpha(1-\epsilon_e)-1}(1-\epsilon_e)\alpha} \tag{A.50}$$

$$(1-x_i^{Emax})^{-\epsilon_d} = \frac{1+r}{B_i^d P^d A_i^{1-\epsilon_d}(k_i^{Emax}-k_m)^{\alpha(1-\epsilon_d)-1}(1-\epsilon_d)\alpha} \tag{A.51}$$

substituting these terms into equation A.49

$$\begin{aligned}C_1+C_2 &= x_i^{Emax}\left[\frac{1+r}{\alpha(1-\epsilon_e)}-\frac{1+r}{\alpha(1-\epsilon_d)}\right] \\ &= x_i^{Emax}\frac{1+r}{\alpha}\left[\frac{1}{1-\epsilon_e}-\frac{1}{1-\epsilon_d}\right] \\ &= x_i^{Emax}\frac{1+r}{\alpha(1-\epsilon_e)(1-\epsilon_d)}(\epsilon_e-\epsilon_d)\end{aligned}$$

then equation A.48 becomes

$$\frac{\pi_E^{max}}{k_i^{Emax} - k_m} - \frac{\pi_D^{max}}{k_i^{Dmax} - k_m} = x_i^{Emax} \frac{1+r}{\alpha(1-\epsilon_e)(1-\epsilon_d)}(\epsilon_e - \epsilon_d) + \frac{(1+r)(k_m + F^d)}{(k_i^{Emax} - k_m)} \left((1 - x_i^{Emax})^{\frac{\epsilon_d}{\alpha(1-\epsilon_d)-1}} - 1\right) - \frac{(1+r)F^e}{k_i^{Emax} - k_m} \tag{A.52}$$

Equation A.52 shows the relation between the profit per investment amounts of exporters and non-exporters.

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