AN INVESTIGATION OF 10TH GRADE STUDENTS' PROOF SCHEMES IN GEOMETRY WITH RESPECT TO THEIR COGNITIVE STYLES AND GENDER

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ABSTRACT

AN INVESTIGATIGATION OF 10TH GRADE STUDENTS' PROOF SCHEMES IN GEOMETRY WITH RESPECT TO THEIR COGNITIVE STYLES AND GENDER

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The purpose of the present study is to identify 10th grade students' use of proof schemes in geometry questions and to investigate the differences in the use of proof schemes with respect to their cognitive style and gender. The sample of the study was 224 tenth grade students from four secondary schools. Of those, 126 participants were female and 98 participants were males.

Data was collected at the end of the academic year 2005-2006 through uses of two data collection instruments: Geometry Proof Test (GPT) and Group Embedded Figure Test (GEFT). GPT, included eleven open-ended questions on triangle concept, was developed by researcher to investigate students' use of proof schemes. The proof schemes reported by Harel and Sowder (1998) were used as a framework while categorizing the students' responses. GEFT developed by Witkin, Oltman, Raskin and Karp (1971) was used to determine cognitive styles of the students as field dependent (FD), field independent (FI) and field mix (FM).

To analyze data, descriptive analyses, repeated measure ANOVA with three proof schemes use scores as the dependent variables and a 2 (gender) x 3 (cognitive styles: FD, FM, FI) multivariate analysis of variance (MANOVA) with three proof schemes use scores as the dependent variables was employed. The results revealed that students used externally based proof schemes and empirical proof schemes significantly more than analytical proof schemes. Furthermore, females used empirical proof schemes significantly more than the males. Moreover, field dependent students used externally based proof schemes in GPT significantly more than field independent students. Also, field independent students use analytical proof schemes significantly more than field dependent mix students. There was no significant interaction between gender and cognitive style in the use of proof schemes.

The significant differences in students' use of proof schemes with respect to their gender and FDI cognitive style connote that gender and FDI cognitive styles are important individual differences and should be taken into consideration as instructional variables, while teaching and engaging in proof in geometry and in mathematics.

Keywords: Proof Schemes, Cognitive Styles, Field Dependence-Independence, Gender, Geometry, Triangles

ÖZ

ONUNCU SINIF ÖĞRENCİLERİNİN GEOMETRİDEKİ İSPAT ŞEMALARININ BİLİŞSEL STİLLERİ VE CİNSİYETLERİNE GÖRE İNCELENMESİNE YÖNELİK BİR ÇALIŞMA

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Bu çalışmanın amacı 10.sınıf öğrencilerinin geometri sorularında kullandıkları ispat şemaları belirlemek ve öğrencilerin bilişsel stilleri ve cinsiyetlerine göre ispat şemaları kullanımlarındaki farklılıkları araştırmaktır. Çalışmanın örneklemi dört ortaöğretim okulundan 224 onuncu sınıf ögrencisidir. Bu öğrencilerden 126'sı kız, 98'i erkektir.

Çalışmanın verileri 2005-2006 akademik dönemi sonunda, iki veri toplama aracı kullanılarak elde edilmiştir. Bu araçlar Geometri İspat Testi (GİT) ve Gizlenmiş Şekiller Grup Testi (GŞGT)'dir. GİT, üçgenlerle ilgili onbir açık uçlu soru içermekte olup, araştırmacı tarafından öğrencilerin ispat şemaları kullanımlarını araştırmak amacıyla geliştirilmiştir. Öğrencilerin cevapları Harel ve Sowder (1998) tarafından geliştirilen ispat şemaları çerçevesinde kategorize edilmiştir. Öğrencilerin bilişsel stillerini alan bağımlı, alan bağımsız ve alan karışık olarak belirlemek için Witkin, Oltman, Raskin ve Karp (1971) tarafından geliştirilen GŞGT kullanılmıştır.

Verileri analiz etmek için, betimsel analiz, yinelenmiş ölçütler ile varyans analizi ve çoklu varyans analizi kullanılmıştır. Ortaya çıkan sonuçlar şöyledir:

öğrenciler dışsal dayanaklı ve deneysel ispat şemalarını analitik ispat şemalarına göre önemli ölçüde daha fazla kullanmaktadır. Kız ögrenciler deneysel ispat şemalarını erkek öğrencilere göre önemli ölçüda daha fazla kullanmaktadır. Bunun yanında, alan bağımlı öğrenciler dışsal dayanaklı ispat şemalarını alan bağımsız öğrencilere göre önemli ölçüde daha fazla kullanmaktadır. Ayrıca, alan bağımsız öğrenciler analitik ispat şemalarını alan bağımlı öğrencilere göre önemli ölçüde daha fazla kullanmaktadır. İspat semalarını kullanımda bilişsel stil ile cinsiyet arasında anlamlı bir ilişki yoktur.

İspat şemaları kullanımında cinsiyet ve bilişsel stile göre ortaya çıkan önemli farklılıklar, cinsiyet ve alan bağımlılık-bağımsızlık bilişsel stilinin önemli kişisel farklılıklar olduğunu, bu farklılıkların eğitim faliyetlerinde, geometride ve matematikte, ispat kavramını öğretirken, birer degişken olarak ele alınması gerektigini ortaya koymaktadır.

Anahtar Kelimeler: İspat Şemaları, Bilişsel Stil, Alan Bağımlılık-Bağımsızlık, Cinsiyet, Geometri, Üçgenler

To My Parents

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TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF ABBREVIATIONS

GPT	: Geometry Proof Test
GEFT	: Group Embedded Figure Test
Cogn. Style	: Cognitive Style
P	: Probability
SD	: Standard Deviation
Kurts	: Kurtosis
Skew	: Skewness
Sig.	: Significance
df	: Degrees of Freedom
η^2	: Effect Size (Eta Square)
F	: F-Statistics
ANOVA	: Analysis of Variance
MANOVA	: Multivariate Analysis of Variance

CHAPTER 1

INTRODUCTION

A proof is an argument needed to validate a statement, an argument that may assume several different forms as long as it is convincing (Hanna, 1989). Sometimes, the two terms, proof and justification, have been interpreted in literature to refer in one way: “what establishes truth for a person or a community” (Harel & Sowder, 2005, p.3). The two terms, however, sometimes have been interpreted differently; while, the term justification refers to “any reason given to convince people (e.g., teachers or other students) of the truth of a statement”, the term (formal mathematical) proof refers to “any justification which satisfies the requirements of rigor, language, etc., demanded by professional mathematicians to accept a mathematical statement as valid within axiomatic system” (Marrades & Gutierrez, 2000, p.89). In the former interpretation, proof connotes an activity from kindergarten on as well as throughout the historical development of the mathematics, whereas, in the latter interpretation the term “proof” often connotes the relatively precise argumentation given by mathematicians (Harel & Sowder, 2005, p.3). Since the school students are exposed to proof not just as the formal process of constructing logically consistent arguments based on axioms, definitions, and theorems but also in a broader set of processes that include argumentation, justification and validation, proof in schools is not interpreted narrowly in terms of formal mathematical proof, but in a broader sense, it is seen as a justification.

There are several roles of proof in mathematics: to verify the correctness of a statement, to explain why a statement is true, to systemize results obtained in a deductive system (a system of axioms, definitions, excepted theorems, etc.), to discover new theorems, to communicate or transmit mathematical knowledge, to construct an empirical theory, to explore the meaning of a definition (or consequences of an assumption, and to incorporate a well-known fact into a new framework (Bell, 1976; de Villiers, 1990, 1999; Hanna, 2000). Though it has several roles, the most important contribution of proof in mathematics education is the promotion of mathematical understanding (Hanna, 2000; Hanna & Jahnke, 1996). Therefore, the concept of proof is one of the key elements in school mathematics (Hanna & Jahnke, 1996; Yackel & Hanna, 2003).

Mathematics educators believe that the place of proof in school mathematics should be enhanced. NCTM Principles and Standards (2000, p.56) recommend that reasoning and proof should be the part of mathematics curriculum at all levels from pre-kindergarten through grade 12. Reasoning and proof standard of NCTM clearly states that students should be able to

- recognize reasoning and proofs as fundamental aspects of mathematics;
- makes and investigate mathematical conjectures;
- develop and evaluates mathematical arguments and proofs;
- select and use various types of reasoning and methods of proofs (p.56)

Although the proof and proving are expected to play much more prominent role in school mathematics, a great number of research studies conducted in the last two decades have given evidence that the students have serious difficulties with proof (e.g., Balacheff, 1991; Bell, 1976; Chazan, 1993; Coe & Ruthven, 1994; Harel & Sowder, 1998; Healy & Hoyles, 2000; Mariotti, 200; Porteous, 1990; Senk, 1985; Solomon, 2006; Weber, 2001). There is a considerable body of research to suggest that school pupils tend to argue at an empirical level rather than on the basis of mathematical structure (Balacheff, 1988; Bell, 1976; Coe & Ruthven, 1994; Stylianides, 2007). Many researchers have generally characterized high school students as empiricists, that is, relying mainly on the demonstration of specific examples to establish the validity of a mathematical conjecture. This finding has been evidenced in several studies where students were asked to explain, justify, or prove a mathematical conjecture (Balacheff, 1988; Bell, 1976; Galbraith, 1981; Healy & Hoyles, 2000; Schoenfeld, 1983,) and to convince others about the truth of a mathematical conjecture (Williams, 1979).

From different points of view, many authors have observed students as they attempt to solve proof problems, and identified students' conception of proof as an important determinant of students' proof practices (Healy & Hoyles, 2000; Harel & Sowder, 1998; Marradez & Guiterrez, 2000; Stylianides, 2006). It was underlined that students' perception of proof plays an important role not only in their thinking and reasoning, but also in the types of argument they produce (Moore, 1994). A significant portion of research on student's conception of proof seeks to categorize the arguments produced by students as proof. An argument that convince students or which a students would use to convince someone is classified as a type of proof scheme (or justification scheme). There has been significant research on proof to classify and characterize students' proof schemes (Balacheff, 1988; Bell, 1976; Chazan, 1993; Fichbein, 1982; Harel & Sowder, 1998; Healy & Hoyles, 2000; Porteous, 1990; Recio & Goldino, 2001; Schoenfeld, 1989; van Dormolen, 1977). Each of these frameworks classifies students' proof from different dimensions such as "empirical" vs "deductive" (Bell, 1976), based on "specific example", "common properties", and "reason about reasoning" (van Dormolen, 1977); based on "naïve empiricism", "crucial example", "generic example", and "thought experiment" (Balacheff, 1988); by "appeal to authority", "example", and "generalizable arguments" (Carpenter, Franke & Levi, 2003), "externally-based", "empirical", "analytic" proof schemes (Harel & Sowder, 1998). Although all these frameworks are remarkable for proof literature, Harel and Sowder's proof schemes is most comprehensive one, because, each of three schemes (externally-based, empirical, analytic) that Harel and Sowder identified is made up of several sub-schemes and these sub-schemes posses some commonalities with other frameworks.

Researcher put forward different factors that may have role in students' conception of proof such as, students' cognitive development (see, for example, Senk, 1989; Tall, 1991), notational difficulties (Selden & Selden, 1995), socio-mathematical norms (Dreyfus, 1999, Yackel & Cobb, 1996), poor conceptual understanding and ineffective proof strategies (Moore, 1994; Weber, 2003). However, little attention has been paid to documenting variables related to individual differences. One of the most important individual differences is the cognitive style. The construct of cognitive style is important factor in education due to its influence on students' learning and learning outcomes (Kogan, 1976; Liu & Reed, 1984; Riding, 2000; Sadler-Smith & Riding, 1999; Saracho, 1997; Smith, 2000; Witkin, Moore & Goodenough, 1977).

Cognitive styles considered one of the important individual differences have been studied extensively since the 1970s and described as the consistent and enduring differences in individual cognitive organization and functioning (Ausubel, Novak & Hanesian, 1978). Studies showed that cognitive style is an important factor in education due to its influence on the student performance (Saracho, 1997), students' learning and learning outcomes (Liu & Reed, 1984; Kogan, 1976, Witkin et al., 1977). One of the most extensively studied cognitive styles is the cognitive style of field dependence-independence (FDI). FDI dimension is defined as 'the extent to which a person perceives part of a field as discrete from the surrounding field as a whole, rather than embedded in the field; the extent to which a person perceives analytically' (Witkin et al., 1977, p. 7).

The relation of cognitive style and academic performance in various fields of science and mathematics has been well documented (Witkin et al., 1977), such as the relation of cognitive styles and concept attainment, concept learning (Morgan, 1997), learning approaches; (Diaz, 1999), learning strategies (Witkin et al. 1977; Tinajero & Paramo, 1998), problem solving ability (Maher, 1982; Reiff ,1992; Squire, 1977) spatial visualization (Arrington, 1987), amount of guidance and the level of abstraction (McLeod, Carpenter, McCornack & Skvarcius, 1978), and general reasoning ability (McLeod & Briggs, 1980). However, FDI cognitive styles influence on students' conception of proof is still unclear.

Another important individual difference is gender. Gender difference in mathematics performance is one of the subjects that have been studied intensively. It was found that the gender differences in mathematics performance are particularly apparent on high cognitive level questions and for high-achieving students (Benbow, 1988; Benbow & Stanley, 1980, 1983; Edwards, 1985; Fennema & Carpenter, 1981;

Fox & Cohn, 1980; Hall & Hoff, 1988; Kissane, 1986; Peterson & Fennema, 1985; Wagner & Zimmerman, 1986). On the other hand, some researchers suggested that gender differences in mathematical performance are declining. They showed that females tend to score as well as males on college mathematics placement tests (Bridgeman & Wendler, 1991) and average female grades in college mathematics courses tend to be as good as or better than those of males (Linn & Kessel, 1994). Others added that boys tend to outperform girls in measurement, proportionality, geometry, spatial geometry, analytic geometry, trigonometry and application of mathematics, whereas, girls have performed better than boys in computing, set operation, and symbolic relation (Fennema, 1974; Jarvis, 1964; Johnson, 1987; Pattison & Grieve, 1984; Robitaille, 1989; Wood, 1976). Linn and Kessel (1996) found that females do better than males on routine math problems, whereas males excel on non-textbook-like problems.

Gallagher and De Lisi (1994) pointed out that gender differences favoring high-ability male students over high-ability female students may be due in part to differences in solution strategies, since males and females use different strategies in response to mathematical problems. For example, Fennema, Carpenter, Jacobs, Franke, and Levi (1998) examined girls' and boys' solution strategies in solving number facts, addition/subtraction, and non-routine problems throughout 3 years and found that girls tended to use more concrete strategies and boys tended to use more abstract strategies. Gallagher and De Lisi (1994) examined the gender differences between male and female high-ability students on a set of difficult conventional and unconventional SAT-M items and found a gender differences in success patterns and in strategy use on conventional and unconventional problems. There were no studies which have investigated gender differences in strategy use in geometry questions. Students' proof schemes' can be viewed as their solution strategies in proof questions. However, we still do not know which strategies, proof schemes, males and females use in proof questions.

Motivated by a lack of research about individual differences in students' proof conceptions, it was aimed to investigate students' proof conception, specifically proof schemes, and the effect of cognitive style and gender on the students' proof schemes using Harel and Sowder's proof schemes framework.

1.1 Purpose of the Study

This study aimed to investigate the tenth grade students' conceptions of proof, specifically their proof schemes in geometry. The study was guided by two main research questions: (a) Are there any significant differences between 10^{th} grade students' proof schemes use in geometry? (b) Are there any significant differences between students' proof schemes use in geometry with respect to their cognitive styles and gender?

1.2 Significance of the Study

The significance of this study lies in documenting the role of cognitive style and gender on students' conception of proof, particularly on proof schemes that students use in geometry. The results of this study will shed light on students' proof performance. Mathematics educators can better understand the students' strength and difficulties in proof; identify difficulties that may stem from differences in gender or cognitive style and design their instruction in a way that meet the needs of students with different cognitive style.

1.3 Definition of the Terms

In this section some important terms that are used in the present study are defined as followings.

1. **Proof Schemes:** Proof schemes can be defined as individual schemes of making conjectures, truths, and convictions when doing a proof.
2. **Externally-Based Proof Schemes:** Externally-based proof schemes refer to one of the categories of the proof schemes in which students read the problem

once and begin comprising arguments based on some outside sources like teacher, textbooks, etc., or they manipulate symbolic expression in the problem with little or no understanding.

3. **Empirical Proof Schemes:** Empirical proof schemes refer to one of the categories of the proof schemes in which students evaluate their conjectures by using their experience, past works, intuition, or reflections.

4. **Analytic Proof Schemes:** Analytic proof schemes refer to one of the categories of the proof schemes in which students evaluate their conjectures by using logical deduction in such a way that they generate ideas from either key issue of the problem or undefined terms and axioms.

5. **Proof Schemes Use Score:** Proof schemes use score refers to how many times a students use the schema from the categories of all proof schemes in Geometry Proof Test.

6. **Cognitive Style:** Cognitive style refers "consistent individual differences in these ways of organizing and processing information and experience (Messick, 1976, p.4). Cognitive styles of participants cognitive style are determined with respect to their scores on Group Embedded Figure Test. Cognitive Style is classified in to three group such as field independent, field dependent, and field neutral. The classification procedure is explained in Chapter 3.

CHAPTER 2

LITERATURE REVIEW

In this chapter, the review of the literature on proof will be presented under five sections: functions of proof, students' view on the functions of proof, students' ability to construct proof, students understanding of proof and categorizations of students' proof. In subsequent sections, the review of literature on cognitive styles, and gender will be presented.

2.1 Functions of Proof

In the research literature on mathematical proof, there are several aspects of proof that have been referred to as the functions of proof (Bell, 1976; de Villiers, 1999; Hanna & Jehnke, 1993; Schoenfeld, 1992). Traditionally, proof has been seen almost exclusively in terms of the verification of the correctness of mathematical statements. According to this aspect, proof is used mainly to remove either personal doubt and/or those of skeptics. This idea has gained supports of several researchers (Bell, 1976; Hanna, 1989; Rav, 1999; Volmink, 1990). Hanna (1989) stated that

> A proof is an argument needed to validate a statement, an argument that may assume several different forms as long as it is convincing. (p.20)

In a similar vein, Bell (1976) declared that

> The mathematical meaning of proof carries three senses. The first is verification or justification, concerned with the truth of a proposition. (p.24)

Beside the verification aspect, some researchers emphasized other functions of proof. For example, Bell (1978) points out that mathematical proof is concerned “not simply with the formal presentation of arguments, but with the student’s own activity of arriving at conviction, of making verification, and of communicating convictions about results to others” (p.48). Hersh (1993) has claimed that proof has one purpose in the world of mathematical research: that of providing conviction. He explicitly restates this belief as followings:

> In mathematical research, the purpose of proof is to convince. The test of whether something is a proof is whether it convinces qualified judges…..a proof is just a convincing argument, as judged by competent judges….In mathematical practice, in the real life of living mathematicians, proof is convincing argument, as judged by qualified judges. (p.389)

Hersh (1993) strongly believes that whereas the primary role of proof in the mathematics community is to convince, in schools and at undergraduate level its role is to explain. Hanna (1995) elaborates this idea as followings;

> While in mathematical practice the main function of proof is justification and verification, its main function in mathematics education is surely that of explanation. (p. 47)

Hanna (1998) has called for using proofs to create a meaningful experience; that is, as a means to help students understand why results are true. Hersh (1993) and Hanna (1995) assert that the main function of proof in the classroom should be to promote understanding by explaining.

de Villiers (1990, 1999) points out that proofs have multiple functions that go beyond mere verification and that can also be developed in computer environments: such as explanation (providing insight into why it is true), discovery (the discovery or invention of new results), communication (the negotiation of meaning), intellectual challenge (the self-realization/fulfillment derived from constructing a proof), systematization (the organization of various results into a deductive system of axioms, concepts and theorems). Recently, Hanna (2000) has provided a comprehensive list of the various purposes of mathematical proof:

1. verification (concerned with the truth of a statement)
2. explanation (providing insight into why it is true);
3. systematization (the organization of various results into a deductive system of axioms, major concepts and theorems);
4. discovery (the discovery or invention of new results);
5. communication (the transmission of mathematical knowledge);
6. construction of an empirical theory;
7. exploration of the meaning of a definition or the consequences of an assumption;
8. incorporation of a well-known fact into a new framework and thus viewing it from a fresh perspective. (p. 9)

2.2 Students' View on the Functions of Proof

Research showed that proof in school mathematics traditionally has been perceived by students as a formal and, often meaningless, exercise to be done for the teacher (Alibert, 1988). Research also showed that students are unable to distinguish between different forms of mathematical reasoning such as explanation, argumentation, verification and proof (Dreyfus, 1999; Hanna & Jahnke, 1996).

Researcher put forward some reasons for the students' difficulties. For example, Hanna (2000) believed that the idea of verification of correctness of mathematical statements has one-sidedly dominated teaching practice and research on the teaching of proof. Similarly, De Villies (1998) claimed that teaching approaches often tend to concentrate on verification and devalue or omit exploration and explanation. Harel and Sowder (1998) suggested that "we impose on students proof methods and implication rules that in many cases are utterly extraneous to what convince them" (p. 237). Hence, students' experiences with proof often are limited to verifying the truth of statements that they know have been proven before and, in many cases, are intuitively obvious to them (Knuth, 2002, p64). Such experiences often lead students to view proof as a procedure for confirming what is already known to be true (Schoenfeld, 1994); as a game that you already know what the result of it (Wheeler, 1990, p. 3). Thus, as Schoenfeld (1994) declare that, "in most instructional contexts proof has no personal meaning or explanatory power for students" (p. 75). Balacheff (1991) claims that if students do not engage in proving processes it is not because they are not able to do so, but rather because they do not see any reason or feel any need for it. Research studies have invariably shown that students fail to see a need for proof because all too often they are asked to prove things that are obvious to them.

2.3 Students' Ability to Construct Proof

Research showed that many students have found the study of proof difficult (e.g., Balacheff, 1991; Bell, 1976; Chazan, 1993; Coe & Ruthven, 1994; Healy & Hoyles, 2000; Porteous, 1990; Senk, 1985). Clements and Battista (1992) point out that teachers' attempts to teach formal mathematical proof to secondary school students (frequently during short periods of time) were not successful. Williams (1980) declared that despite the efforts made by mathematics educators to promote students' conception of proof, only the ablest students have understanding of it (Williams, 1980).

In literature, several empirical studies reported that students have difficulties with proof. For example; Usiskin (1987) studied 99 high school geometry classes in five states in U.S. and found that at the end of their geometry course, 28% of the students couldn't do a simple triangle concurrence proof, and only 31% of the students were judged to be competent in constructing proof. Senk (1985) reported that only 30% of the students in full-year geometry courses that teach proof reach 75% mastery level in proof writing. Beside, even those students that succeeded to function in the proving ritual were not always aware of its meaning. They rarely saw the point of proving, and/or the need to prove, especially when the statement to be proved was given as a ready-made fact without any discovery by the learners. Selden and Selden (1995) investigated students' abilities to construct or validate a proof structure of a mathematical statement. They analyzed data from tests and examinations of 61 students, which attended introductory mathematics courses at the university level. There were 8.5% correct answers, and these answers were given by only 13.5% of the students. Selden and Selden (1995) conclude that deficits in identifying the logical structure of a statement will entail deficits in constructing a proof structure for these statements.

2.4 Students' Understanding of Proof

Researchers point out that students also fail to distinguish between different forms of mathematical reasoning such as explanation, argumentation, verification and proof (Dreyfus, 1999; Hanna & Jahnke, 1996). Schoenfeld (1985) emphasized that students' perspective on the role of proof is either to confirm something intuitively obvious or to verify something already known to be true. Beside, Vinner (1983) pointed out that high school students view the general proof as a method to examine and to verify a particular case or as a process of finding some more evidence.

Research showed that many adolescents and adults do not genuinely believe general mathematical statements (Chazan, 1993; Fischbein, 1982; Martin & Harel, 1989; Porteous, 1990; Schoenfeld, 1989; Vinner, 1983). Even when students seem to understand the function of proof in the mathematics classroom (Hanna, 1989; De Villiers, 1990; Godino & Recio, 1997) and to recognize that proofs must be general, they still frequently fail to employ proof to secure beliefs in the truth of their warrants, preferring instead to rely on more data (Coe & Ruthven, 1994; Fischbein, 1982; Healy & Hoyles, 2000; Rodd, 2000; Simon, 2000; Vinner, 1983). A large percentage of students believes that, even after they have proved, checking more examples is desirable (Fischbein & Kedem, 1982; Vinner, 1983). Porteous (1990) contended that checking a particular case implies that the participant did not genuinely believe "the general."

Fishbein and Kedem (1982), and Martin and Harel (1989) noticed that students do not consider formal proofs as always guaranteeing that the conditional statements are valid for all cases. Researchers noted that, after offering a formal proof, the students still gave examples in order to check whether the statement was empirically true or not.

Martin & Harel (1989) showed that college students believe that a proof of a general statement concerning a geometric object does not guarantee that the statement is true for all instances of that object; a proof only guarantees that the statement is true for those instances where objects are spatially similar to the figure referred to the proof. Fischbein & Kadem (1982) presented deductive proofs of statements to 15- to 17-year-olds. One statement and its proof involved geometry (ABCD is a quadrilateral and P, Q, R, S are the midpoints of its sides; one must prove that PQRS is a parallelogram) and the other involved an algebraic statement similar to Problem 1 (prove the expression E = n3 – n is divisible by 6 for every n, n being any positive integer). The participants were asked if they accepted the general validity of the proofs. A number of questions were then posed, such as "V is a doubter. He thinks that we have to check at least a hundred quadrilaterals in order to be sure that PQRS is a parallelogram. What is your opinion?" and if you consider that… [a correct proof has been given] for the theorem 'n3 – n is divisible by 6 for every n' then answer the following question: Do you consider that further checks (by using other numbers) are necessary to increase your confidence in the validity of the theorem? (Fischbein & Kedem, 1982, p. 129). Fischbein and Kadem reported that only a minority of students judged that empirical checks would not increase their confidence in a proof that had already been accepted as valid and general. Fischbein argued that this apparently contradictory behavior was due to the fact that while students were being asked about a mathematical proof, their experience was mostly with empirical proof:

Vinner (1983) described an investigation with 365 students from grades 10 and 11. In a regular mathematics lesson they proved the statement, that each number of the form n3 - n is divisible by 6 (n an integer). The next day the students were presented three solutions of the problem "Prove that (59)3 - 59 is divisible by 6." The first solution was a simple calculation, the second was the proof of the general statement with n = 59, and the third solution was a reference to the general statement and its proof. About one third of the students preferred the second solution. Moreover, 23% of the students were not able to accept the universal validity of a proven mathematical statement.

Many researchers point out that school students have a tendency to use inductive reasoning to validate conjectures in mathematics (Balacheff, 1988; Bell, 1976; Van Dormolen, 1977) rather than to prove them deductively. For example, Porteous (1986) pointed out that a very high percentage of 11-to-16 years-old students do not appreciate the significance of deductive proof in geometry, algebra, and general mathematical reasoning. Martin and Harel (1989) provided preservice elementary teachers with correct deductive, incorrect deductive and inductive arguments for the same statements. Every inductive argument they presented was accepted as a valid mathematical proof by more than half of their students; the acceptance rates for the deductive arguments were not much higher than those for the inductive ones; and the false deductive proofs were accepted by close to half of the students.

Schoenfeld (1986) claimed that after a full year geometry course in college, most students do not see the point of using deductive reasoning in geometric constructions, and that they are still naive empiricists whose approach to constructions is an empirical 'guess and test' loop. They produce proofs because the teacher demands them, not because they recognize that they are necessary in their practice (Balacheff, 1988).

Schoenfeld (1985) describes, in a study of geometrical problem solving, students' focus on methods that he labels naive empiricism: To test ideas by constructing figures, and then determine the correctness of the ideas by the shapes of the figures. This approach often caused different types of failure. Often the students did not attempt to use the mathematical properties of the objects to construct some kind of deductive reasoning, even though their resources were sufficient and proper reasoning could have helped them to make considerable progress.

Coe and Ruthven (1994) found that when proof contexts are data-driven, and students are expected to form conjectures by generalization or counter example, then students' proof strategies are primarily empirical. It seems that in such a context students are willing to replace deductive argument by a sufficiently diverse set of instances.

Hoyles et al. (1998) made a significant contribution to the field with their recent systematic investigation into junior high school students' reasoning on geometric number patterns, understanding of logical implication, proof constructions, and views of proof. The results showed that even high-attainers had great difficulties in generating proofs and are more likely to rely on empirical proofs. However, the majority of students valued general and explanatory arguments. Although around one third of English high-attainers had no idea of the validity of these empirical arguments, more than half gave completely correct evaluations.

The situation in students' understandings of disproving is similar to proving. Many students are not convinced by a counterexample and view it as an exception that does not contradict the statement in question (Galbraith, 1981; Harel & Sowder, 1998). Galbraith (1995), for example, found that many secondary-school students did not accept that a single counterexample suffices to disprove a claim in mathematics. Even if students seem to understand the special role and status of counterexamples, they are unable to generate a correct counterexample. In an attempt to generate a counterexample, they either give an example that does not satisfy the necessary conditions or an example that is impossible (Zaslavsky & Ron, 1998).This research showed that students tend to argue that an invalid statement is right, view a counterexample as a non-example or exception, and are impeded by the ability to generate counterexamples.

2.5 Categorization of Students Proof

2.5.1 Van Hiele Levels and Proof

The concept of Van Hiele levels originally proposed as a model of learning geometry. Most work concerning van Hiele levels has mostly centered on geometrical understanding and reasoning (Burger & Shaughnessy, 1986, Lunkenbein, 1983). However, Van Hiele levels have been applied to proof, in particular to geometric proof. Bell et al. (1983) provide the following summary of the five van Hiele levels:

Level I "is characterized by the perception of geometric figures in their totality as entities ... judged according to their appearance. The pupils do not see the parts of the figure, nor ... relationships among components ...and among the figures themselves. The child can memorize the names of these figures relatively quickly, recognizing the figures by their shapes alone."

In level II, the pupil "begins to discern the components of the figures; he also establishes relationships among these ... and between individual figures. The properties of the figures are established experimentally: are described, but not yet formally defined."

Pupils who have reached level III **"**establish relations among the properties of a figure and among the figures themselves. The pupils are now able to discern the possibility of one property following from another and the role of definition is clarified ... The order of logical conclusion is developed with the help of the textbook or the teacher."

At level IV "the significance of deduction as a means of constructing and developing all geometric theory' is recognized. The role of axioms becomes clear, and 'the students can now see the various possibilities for developing a theory proceeding from various premises."

Level V "corresponds to the modern (Hilbertian) standard of rigor. A person at this level develops a theory without making any concrete interpretation." (P.222-223)

Hoffer (1981) summarized these levels under the titles of recognition, analysis, ordering, deduction and rigor. Levels 4 and 5 are clearly closely related to the construction of proofs at a formal level whereas level 3, and to a certain extent level 2, can be seen as relating to proof at an informal level. Van Hiele, quoted in Bell et al (1983), illustrates the transition from one level to the next in the following statement:

> "At each level, appears in an extrinsic manner what was intrinsic on the previous level. At the first level, the figures were in fact just determined by their properties, but one who is thinking at this level is not conscious of these properties". (Bell, 1983; p.201)

2.5.2 Bell's Categories of Justifications

Bell (1976) identified two categories of students' justifications used in proof problems:

1. Empirical justification
2. Deductive justification

Empirical justification is characterized by the use of examples as element of conviction, while deductive justification is characterized by the use of deduction to connect data with conclusions. Within each category, Bell identified a variety of types: The types of empirical answers correspond to different degrees of completeness of checking the statement in the whole (finite) set of possible examples. The types of deductive answers correspond to different degrees of completeness of constructing deductive arguments

2.5.3 Carpenters et al.'s Categories of Justifications

Based on their research into primary classroom practices Carpenter and colleagues (2003) categorize students' attempts to justify mathematical statements, into three;

1. Appeal to authority
2. Justification by example
3. Generalizable arguments.

By appealing to authority, what convinces the student and what the student does to persuade others arises from some external sources, such as, textbooks or authority figures, a parent or a teacher, or in some cases a peer. Children who appeal to authority most often know that an idea is true because someone they trust has told them. However, Carpenter and colleagues contend that students justify by example more commonly than they appeal to authority. Furthermore, they believe that children do not routinely make generalizable arguments but that as they proceed into the middle grades they begin to see that arguing by examples is limiting and can be encouraged to try to employ more general forms of argumentation.

2.5.4 Van Dormolen's Categories of Proof

Instead of focusing on students' justification, van Dormolen (1977) had characterized levels of functioning for three categories of proof. He produced his taxonomy of proof as follows:

1. Focus on a particular example
2. Use of an example as a generic embodiment of a concept
3. Use of general and deductive argument.

2.5.5 Balacheff's Categories of Justifications

Balacheff (1988, 1991) extended van Dormolen's taxonomy of proof, distinguished between which he called pragmatic justifications and conceptual justifications and outlined four types of justification under them.

1. Naïve empiricism

2. Crucial experiment
3. Generic example
4. Thought experiment

The category of pragmatic justification based on the use of examples includes the first three types, whereas, the category of conceptual justification based on abstract formulations of properties and of relationships among properties included the last type.

> "... pragmatic proofs are those having recourse to actual action or showings, and by contrast, conceptual proofs are those which do not involve action and rest on formulations of the properties in question and relations between them" (Balacheff, 1988:p.217).

At the first level (Naïve empiricism), a statement to be proved is checked in a few (somewhat randomly chosen) cases. After verifying some cases, the student concludes that an assertion is valid from a small number of cases, and then the conjecture is true.

At the second level (Crucial experiment), a statement is checked in a careful selected example. The student deals more explicitly with the question of generalization by examining a case that is not very particular. If the assertion holds in that case, it is validated. The difference from Naive empiricism is mainly that the students are aware of the problem of generality. For example; Balacheff provided an example of two students who have a conjecture about the number of diagonals of polygons. To verify their conjecture, they chose a polygon with a large number of sides, believing that if it works for this case, it will always work.

At the third level (Generic example), justification is based on operations or transformations on an example which is selected as a characteristic representative of a class. The students develop arguments based on a 'generic example'. Although a particular case is the focus, it is used not as a particular case, but rather as an example of a class of objects.

At the fourth level (Thought experiment) students begin to detach their explanations from particular examples, and begin to move from practical to intellectual proofs. An abstract member of a class is discussed. The proof is indicated by looking at the properties of the objects, not at the effects of operations on the object. For example, a student described the reason why each vertex had (n-3) diagonals and the reason why n(n-3) was divided by 2.

2.5.6 Harel and Sowder's Categories of Proof Schemes

Tracking the levels of justification in a similar mode to Carpenter, Franke, and Levi (2003), Sowder and Harel (1998) identified the kinds of proof schemes used by students:

1. Externally based proof schemes
2. Empirical proof schemes
3. Analytic proof schemes.

Harel and Sowder (1998) define a proof scheme (or justification scheme) to be the arguments that a person uses to convince herself and others of the truth or falseness of a mathematical statement.

> "A person's proof scheme consists of what constitutes ascertaining and persuading for that person […] As defined, ascertaining and persuading are entirely subjective and can vary from person to person, civilisation to civilisation, and generation to generation within the same civilisation" (Harel & Sowder 1998, p.242).

They characterize seven major types of proof schemes, grouped into the three categories of proof schemes (Harel and Sowder, 1998; Sowder and Harel, 1998; Harel, 2002).

Externally based proof scheme includes justification based on the authority of a source external to students, like teacher, textbook, etc. Students appeal to an external authority or to the form of an argument (ritualistic or symbolic) to determine mathematical validity (Harel and Sowder, 1998). Harel and Sowder (1998) believed that instruction emphasizing the structure and symbols of proofs might reinforce students' desires to memorize proofs, which may, in turn, lead to ritualism. Students who engage in ritualism may determine that a proof is valid based solely on its appearance rather than for its content.

Empirical proof scheme includes justifications based solely on examples (inductive type) or, more specifically, drawings (perceptual type); the way things look. Students appeal to specific examples or perceived patterns for validation.

Analytical proof scheme includes justification based on generic arguments or mental operations that result in, or may result in, formal mathematical proofs. The arguments given are general and involve mathematical reasoning. Such arguments or operations can be based on general aspects of a problem (transformational type) or contain different related situations, resulting in deductive chains based on elements of an axiomatic system (structural or axiomatic type). Students who hold analytic proof schemes use logical deductions to validate conjectures. Transformational proof **sc**heme is based on "operations on [mental] objects and anticipations of the operations' results" (1998, p. 258) through deductive reasoning. Such transformations may be restricted by the student's sense of the mathematical context, generality requirements, or the mode of justification. Thus, a transformational proof scheme is seen as a restrictive analytic scheme. The axiomatic proof scheme is based on a person's understanding that a proof must derive from undefined terms and axioms. This, too, can be restricted by the student's intuitive sense of mathematical structure.

2.6 Individual Differences in Mathematics Achievement

Researchers have examined a number of possible underlying factors and potential predictors of mathematics achievement, including effective variables such as attitude, motivation, and self-concept and non-intellectual variables such as gender, socioeconomic status, and ethnicity; and cognitive factors such as reasoning skills, spatial visualization and so on (Dillon & Schemek, 1983). After the style construct become apparent in educational psychology literature, thousand of articles have been written about differentiated patterns of perceiving, thinking, and problem solving in various situations under different conditions (Morgan, 1997). The term cognitive style was developed by cognitive psychologists conducting research into problem solving and sensory and perceptual abilities (Stenberg & Grigorenko, 2001). Then, cognitive style studies have entered scholarly domains of education, psychology, anthropology, social work, counseling and many others and it has been considered as one of the individual differences in cognition.

"Cognitive" refers to the process involved in the overall act of processing information in becoming knowledgeable (Morgan, 1997). It includes perception, judgment values, and memory. Style is defined as "habitual pattern or preferred ways of doing something (e.g., thinking, learning, and teaching) that are consisted over long periods of time and across many areas of activity and that remains virtually the same (Stenberg & Grigorenko, 2001). As a part of the term cognitive style, style implies that as individual, we employ personal characteristics in the acquisition of knowledge, and more often than not, approach a learning experience in ways that differ from other individuals (Morgan, 1997). Simply, cognitive style refers to an individual's way of processing information. Cognitive style theory suggests that individuals utilize different patterns in acquiring knowledge (Morgan, 1997).

2.6.1 Cognitive Style

Cognitive style theorist and researcher Messick (1976) emphasized that individual differences makes a difference;

> Each individual has a preferred ways of organizing what he sees and remembers and thinks about. Consistent individual differences in these ways of organizing and processing information and experience have come to be called cognitive styles….They are conceptualized as stable attitudes, preferences, or habitual strategies determining a persons'

typical modes of perceiving, remembering, thinking and problem solving (Messick, 1976, p. 4-5)

Cognitive styles have been studied extensively since the 1970s and described as the consistent and enduring differences in individual cognitive organization and functioning (Ausubel et.al, 1978). Morgan (1997) describes the cognitive style as the psychological dimensions that indicate the individual differences in preferred ways of organizing and processing information. Keefe (1982) stated that cognitive styles are the "cognitive, affective, and physiological traits that serve as relatively stable indicators of how learners perceive, interact with, and respond to the learning environment". Saracho (1997) stated that cognitive styles include stable attitudes, preferences, or habitual strategies that distinguish the individual styles of perceiving, remembering, thinking, and solving problems.

2.6.2 Characteristics of Cognitive Style

It has been proposed that cognitive style is stable over time and likely to show little change (Riding & Rayner, 1998). There are several important characteristics of style. The first is differentiation between the concept of style and strategies. According to Stenberg and Grigorenko (2001), at the basic level, style and strategies can be distinguished by "degree of consciousness" involved. Style operates without individual awareness, whereas strategies involve a conscious choice of alternatives. Two terms are used interchangeably by some authors (Cronbach & Snow, 1977), but, in general, strategy is used for task-or-context dependent situation, whereas style implies a higher degree of stability falling midway between ability and strategy (Stenberg & Grigorenko, 2001).

The second important characteristic is related to nature of cognitive style itself. Many studies showed that cognitive style is independent of other construct (individual difference dimensions) such as intelligence, personality, and gender (Riding & Rayner, 1998). Also, cognitive style is distinguished from ability.

Considering its nature as distinct form ability, McKenna (1984) highlighted four distinguishing characteristics of cognitive style;

- Ability is more concerned with the level of performance, while style focus on the manner of performance
- Ability is unipolar while style is bipolar.
- Ability has values attached to it such that one end of ability dimension is valued and the other is not, while for a style dimension, neither end is considered better overall.
- Ability has a narrower range of application than style.

Beside, Morgan (1997) point out that cognitive style is not an indication of one's level of intelligence, but a description of the unique ways employed by learners in acquiring new information. (p.6)

2.6.3 Cognitive Style and Education

Students learning and their subsequent academic performance critically depend on the way they manipulate and process information contained in taught material (Druyan & Levin, 1996). Morgan (1997) emphasizes the emergence of several theories in education and psychology about various strategies in processing information during classroom experiences. Since cognitive style is related to a person's psychological and educational preferences in ways of organizing and processing information and is a part of the individual's personality, it is considered as an important factor in education due to its influence on the student performance (Saracho, 1997).

Studies show that cognitive style influence the way students learn and how they learn (Liu & Reed, 1984). Some researcher pointed out that cognitive style is an important variable in students' learning and learning outcomes (Kogan, 1976, Witkin et al., 1977). Witkin pointed out that in the study that cognitive style interact with instructional factors and influences educational practices, such as " how students learn, how teacher teach, how teacher and students interact" (Witkin et al., 1977).

2.6.4 Cognitive Style and Learning Style

Learning style is defined as an individual's preferred method for assimilating information, in an active learning cycle (Kolb, 1984). In educational and psychological domain, some researchers substitute the term learning style for the term cognitive style. In some situations, the terms cognitive style and learning style are used interchangeably. However, a distinction is made by some researchers. For example, Riding and Sadler-Smith (1997) made a distinction between the cognitive style and learning style. They defined the cognitive style as a core characteristic of the individual, while learning styles are seen as strategies and ways of adapting the material to use it as effectively as possible. Cognitive style is frequently included under the broader term of learning style.

The significance of an individual's cognitive style and learning style to the performance in various learning situations has been explored by many authors over the years (Kolb, 1985; Riding & Sadler-Smith, 1997; Laurillard, 1993; Ford, 2000).

2.6.5 Categorization of Cognitive Style

Since the mid-1940s, there have been many works which have contributed emergence of several models and labels of cognitive style and learning style. Curry's (1983) clarified the current cognitive and learning style model with by dividing models into three levels like the layers of an "onion", but this onion was divided into four levels recently:

- Instructional & Environmental Preferences are describing the outermost layers of the onion, which are usually observable traits. Models of Dunn and Dunn, and Reichmann and Grasha are based on this preference.

- Social Interaction Models consider ways in which the interaction between the individual and social context will result in certain strategies. William Perry, Mary Belenky, and Marcia Baxter Magolda developed learning style models as based on this model.

- Information Processing Models describe the middle layer in the onion, and try to understand the processes by which information is obtained, sorted, stored, and used. Kolb's, Howard Gardner's, and Gregoric's studies are based on this model.

- Personality Models describe the innermost layer of the onion, the level at which our personality traits shape the orientations. Myers-Briggs' and Witkin's studies are based on this model.

According to this categorization, cognitive style is the innermost layer of the onion, and considered as the most stable one for people. While passing outwards from the center, the constructs (cognitive style, learning style, and learning preferences) become open to introspection, more context-dependent and less-fixed (Sadler-Smith, 2001). Cunningham-Atkins et al. (2004) defines the cognitive style as the underlying aspect of an individual's style and as being most likely to influence their approach to learning. Cognitive style is considered as the most stable preference related to information processing and is regarded as a narrower concept than the concept of learning style.

There are more than 30 different cognitive style labels used by many researchers. These labels were categorized by Riding and Cheema (1991) into two dimensions: the wholistic-analytic dimension, and the verbal-imagery dimension. Wholistic-analytic dimension dealt with the structure and organization of the content (Riding & Sadler-Smith, 1992), while the verbal-imagery dimension dealt with the mode of presentation (Riding & Douglas, 1993). The most prominent style label for wholistic-analytic dimension of cognitive styles is field-dependence/field-independence (FD/FI). FD/FI originated in Witkin's work (Witkin, 1974; Witkin et al., 1977; Witkin & Goodenough, 1981). It is extensively studied by several researches and has had wide application to educational problems (Saracho, 1997; Tang, 2003).

2.6.5.1 Field Dependency

As stated before, there are many different cognitive style definitions and labels determined by many researchers. Witkin and his associates (Witkin & Goodenough, 1979; Witkin et al., 1977; Witkin et.al., 1971) developed the concept of field dependence/ independence (FDI) to differentiate two distinct cognitive styles. The cognitive style of FDI is based on individual's tendency of perception the surroundings. The FDI dimensions are defined as 'the extent to which a person perceives part of a field as discrete from the surrounding field as a whole, rather than embedded in the field; the extent to which a person perceives analytically' (Witkin et al., 1977, p. 7). Field dependency describes the specific ways individual process information. Witkin (1977) believed that these differences influence how students interact with the learning environment and how students approach a learning task and learning materials.

2.6.5.2 History and development

Witkin and his colleagues developed the concept of field dependency in the 1940s with research on human perception of upright direction (Witkin et al., 1977). These studies showed a consistent pattern in strategies used by people to perform a task in which their physical orientation was varied in relation to a task they had to perform (Witkin & Goodenough, 1981). Some subject tended to use the cues of the visual field (i.e., the surrounding visual elements) while the others relied on internal gravitational referents. These suggest that people have preferred ways of integrating the diverse source of information available for locating the upright direction, and this became the basis for the concept of field dependency.

The first two methods of exploring how people located the upright direction were the rod-and-frame test (RFT) and body adjustment test (BAT) (Witkin et al., 1971). The objective of these tests is for the subjects to perform the task without influence from the surroundings. Witkin and Goodenough conducted the study based on the assumption that there might be a relationship between how rapidly and accurately individuals locate the upright direction by using physical senses and how well they could separate a part form a larger whole in which the part was embedded. Separating a part from its larger whole was seen as a feature of many problem solving tasks (Witkin & Goodenough, 1981). In the body adjustment task, participants were seated in a small room that was tilted to the right or left. Participants' chairs were tilted and they were asked to correct the tilt of their chairs to the upright vertical position. It was found that some individuals oriented themselves to the tilt of the room while others correctly oriented themselves to the true vertical direction. The road-and-frame-test eliminated the gravitational cue inherent in BAT but still required participants to view a tilted illuminated square frame. A luminous rod was suspended within the frame, pivoting from the same center point as frame. The participants were instructed to move the tilted rod to the upright position within the tilted frame. Consistent from the findings form BAT, some people used frame to define the upright direction while the others used their bodies' internal references.

Individuals who are able to orient themselves along the true vertical in a room despite confusing physical and visual cues generated by a tilted floor and movable chair were described as field- independent. Their sense of vertical originated from internal body awareness of gravity and was nor affected by misleading visual clues created by the environment. Individuals who aligned themselves along a vertical axis relative to misleading environment were labeled as field –dependent. They relied on visual cues from external environment to determine placement and did not rely on an internal, bodily awareness of gravity. Thus, the concept of field dependency describes how people perceive, acquire, and act on knowledge of their surrounding (Witkin et al., 1977).

Researcher later broadened the field dependency concept from perception of the upright direction to include perceptual and intellectual problem solving (Witkin et al., 1977). The field dependency concept reflects how individuals function within their environments. Field-independent individuals, characterized by reliance on an internal reference, were discovered to be more capable of cognitive restructuring and disembedding skill than field-dependent individuals, who are characterized by reliance on external references. Field-independent individuals tend to have a more articulated self concept with clear boundaries between internal attributes, feeling and needs, and the external social environment (Witkin & Goodenough, 1981). Field-dependents are more global or undifferentiated while field-independent individuals tend to be more articulated or differentiated (Saracho, 1989).

2.6.5.3 Characteristics

According to Witkin and Goodenough (1981), cognitive style is the mode of self consistency of cognitive restructuring competence, and is bipolar in nature and stable over time. The bipolar nature of field dependency is a continuum in which people are at different points of two extremes (Morgan, 1997). Witkin et al. (1977) found that field-independence is distinct from field-dependence in perceptual, intellectual and psychological characteristics, and these characteristics are highly consistent and stable over time. Early studies on field dependency focused on perceptual differences. Witkin (1977) found that there were individual differences in perception and the extent of differentiation was reflected in the degree of field dependency. Later, Witkin and his colleagues broadened their studies from perception differences into intellectual functioning that specifically related to analytical ability and individual differences in the modes of thinking.

In psychological dimension of cognitive style, Saracho (1984) summarized that field-dependent individuals usually rely on surrounding field and authority to make their decisions. They observe the faces of those around them for information, prefer to be with people, and experience their environment in a relatively global fashion as they conform to the prevailing field or context. Saracho also state that “field-dependent person tend to be able to abstract elements from surrounding field and to solve problems identifying critical elements out of context, remaining socially detached, independent of authority, and analytic” (p.44). He concluded that field-independent individuals have more advantages than field dependent individuals in logical thinking and problem solving skills. Field-dependent individuals are more sensitive to social cues, and field-independent individuals are more independent in decision making and more competitive in social interaction.

Since the field dependency describes the specific ways individual process information, students with different types of field dependency use different strategies in responding to complicated learning situation. Research findings have reported that due to their different ways of processing information, field-dependent and field-independent students approach learning task in qualitatively different ways, make use of different strategies, and require different aids in classroom learning and instruction.

Yea-Ru Chuang (1999) contended that FI learners tend to solve problems through intuition and use of trial-and-error strategies, as opposed to FD learners, who perceive objects as a whole and look more for more uni-dimensional relationships. According to Miller (1997) FD learners 'prefer externally defined goals and organization' while FI learners 'can provide their own structure for learning activities'

Field-independent students prefer to use internal referents to define information, whereas field-dependent students use external referents (Witkin,1977). For example, in problem solving activities, field independent students would like to define the problem based on their own understanding of situation and find a solution, more self-motivated. Field-dependent students would be more dependent on clues provided by teacher to define the problem (they are responsive to external reinforcement,) and work with other people to solve problem, they are better at learning social material and learning it in a social way (Mcleod et.al., 1978).

Beside, field-dependent students find it more difficult than do field-independent students to recognize information, attend salient cues, and impose structure on information when there is none present (Witkin & Goodenough, 1981). That is; it is more challenging for field dependent students to deal with a more complex situation than field-independent students, and that field-dependent students often need more help with separating critical element form its background.

From Witkin and his collegues' studies it is cleared that field dependent people use external referents to guide them in processing information, while the field independent people use internal referents. Saracho's (1997) summary of the characteristics of the field-dependent and independent people is presented in Table 2.1.

Table 2.1 Characteristics of the field-dependent and field-independent people (Saracho, 1997)

Field Dependent People	**Field Independent People**
tend to be global	tend to be more analytical
take longer to solve the same kinds of problems	can solve problems whose materials require structuring
are guided by the organization of the field as a whole	can abstract an item from the surrounding field
use global defenses, such as repression and denial	employ specialized defenses such as intellectualization and isolation
influenced by authority figures or by peers	are independent of authority
use external sources of information for self-definition	dependent on their on values and standards
have a strong interest in people, respond to people's emotional expressions, and like to have people around them	impersonal and socially detached
prefer occupations which require involvement with others, such as elementary school teaching, selling, or rehabilitation counseling	favor occupation in which working with others is not essential, such as astronomy or physics
oriented to subject areas related most directly to people, such as socialsciences	favor impersonal abstract subjects, e.g. mathematics, physicalsciences

As clear now, a field-dependent person is global, holistic, uncertain, and dependent upon others, while a field-independent person is analytic, confident, and self-reliant. The final work of Witkin suggests that FDI covers three major constructs:

1. reliance on internal vs. external referents;
2. cognitive restructuring skills; and
3. interpersonal competencies (Witkin & Goodenough, 1981).

2.6.5.4 Measurement

Following Witkin et al. research the Embedded Figures Test (EFT) and the Group Embedded Figure Test (GEFT) were developed (Witkin et al., 1977). In these tests, individual is shown a simple geometric figure and then shown a more complex geometric figure that has within it the simple form. The individual is required to find the simple geometric figure within the more complex geometric figure within a limited amount of time. There are total of three sections for the test. The first section is a practice section where the score is not considered in the total score. The individual's ability to locate the simple figure without being distracted by the complex figure indicates the degree to which the individual is field-dependent, field-neutral, or field-independent. Test questions have score. Respondents scoring within one standard deviation above the mean are considered to be field-independent compared to their field-dependent counterparts, whose scores are located one standard deviation below the mean. Respondents around the mean are considered to be field-neutral or field-mixed. The GEFT instrument has a reliability of .82 (Witkin et al., 1971). Researchers think that how people respond to GEFT indicates their general tendencies in learning, perceiving, and understanding the world (Witkin et al., 1971). These cognitive learning style differences are important for educators to understand and act upon in order for each student to benefit fully from the educational process (Witkin et al., 1977). FI and FD scores measured by the GEFT are supposedly not correlated with intelligence or ability (Witkin & Goodenough, 1979; Witkin et.al., 1977; Witkin et.al., 1971).

2.6.6 FDI and Academic Performance

The practical implication of FDI cognitive styles for education has indicated that the individuals' different cognitive styles bear direct impact upon their academic performance. Several studies have shown that FI subjects tend to outperform FI subjects in various developmental-cognitive tasks (Case, 1974, 1975; Case & Globerson, 1974; Ehri & Muzio, 1974; Globerson, 1977; Linn, 1978; Ronning et al., 1984). For example; to investigate overall academic achievement, Tinajero and Paramo (1997) analyzed various subjects of the school curriculum in a single sample of 408 students (215 boys and 193 girls) aged between 13 and 16. They found that FI boys and girls performed better than FD ones in all of the subjects considered. This result supported previous findings that FDI is related to overall academic achievement (Saracho, 1997; Witkin et al., 1977). Niaz (1989) studied proportional reasoning competence among science majors to investigate difference in the performance between FD and FI students. From a group of over three hundred university students, Niaz reported that during certain experiments FD students demonstrate less proportional reasoning skill than their FI counterparts. Marinas (1999) investigated effect of spatial and FDI abilities on students problem solving strategies in the mathematical tasks in Logo environment. Qualitative data was collected from four elementary students interacting with the computer on problem solving-mapping tasks on ratio/proportion, coordinate system, and area. Marinas found that FDI seemed to affect the choice of problem solving strategies. While the two FI students were quicker to find patterns and use this strategy in solving the problems, FD students spend more time trying to understand and use trial-and-error methods to solve the task.

2.6.7 Proof schemes and Cognitive Styles

There was little research that investigated students' proof schemes in relation with cognitive styles or other individual differences domains. Only Housman and Porter (2003) have used Harel and Sowder's framework to investigate above average students' proof schemes along with the learning strategies that students use to learn a mathematical concept from its definition. To assess students' proof schemes, the students were provided seven conjectures on algebra and geometry, and asked to state whether each was true or false, and asked provide written proofs. Researchers found that the students who wrote and were convinced by deductive arguments were successful in reformulating concepts and using examples. However, the students who were most convinced by external factors were unsuccessful in generating examples, using examples, and reformulating concepts.

2.6.8 Gender

2.6.8.1 Gender Differences in Mathematics Performance

As an individual difference, gender differences in mathematics performance are one of the subjects that have been studied intensively within the fields of psychology and education. Many empirical studies in these fields have shown that gender differences in mathematics learning are not evident during early school years, but boys begin to outperform girls in the mathematics learning during the intermediate school years, and go further in many mathematical areas during the high school years (Armstrong, 1981; Block, 1976; Burton et al., 1986; Crosswhite et al., 1985; Fennema, 1974, 1980, 1984; Fox, 1980; Leder, 1985; Mccoby & Jacklin, 1974,; Petterson & Fennema, 1985; Hyde, Fennema, & Lamon, 1990).

Fennema says that "although there appears to be a general consensus about male advantage in math, the timing of emergence of gender differences is not clear enough". For example, during the elementary and middle school years, few overall gender differences in mathematics are found when total scores are examined, though differences are found on some subtests as early as first grade (Lummis & Stevenson, 1990). Geary (1994, p228) concluded that gender differences in mathematical problem solving is found as early as the first grade. Benbow (1988) reported a strong evidence for the existence of gender differences by age 12; some other researchers (Hyde et al., 1990; Marshall, 1984) have argued that gender differences in mathematical problem solving do not emerge until adolescence. In her review of published studies Fennema (1974) concluded that

> No significant differences between boys' and girls' mathematics achievement were found before boys and girls entered elementary school or during early elementary years. In upper elementary and early high school years significant differences were not always apparent. However, when significant differences did appear they were more apt to be in the boys' favor when higher-level cognitive tasks were being measured and in the girls' favor when lower-level cognitive tasks were being measured. (pp. 136-137)

Leahey and Guo (2001) conduct a research to document the magnitude of the gender differences across age, among different subsamples (e.g., high scorers and for different subtest (e.g., reasoning). Researcher used large national data set and curvilinear growth models to examine gender differences in mathematical trajectories from elementary school through high school. Leahey and Gou concluded that "despite relatively equal starting points in elementary school, and relatively equal slopes, boys have a faster rate of acceleration which leads a slight gender differences in geometry by the 12th grade.

On the other hand, a substantial body of research reported that whether or not gender differences are found in performance seems to depend on factors, such as, content and format of the test administered (Armstrong, 1985; Hanna, 1986; Hanna et al., 1988; Kimball,1989; Marshall, 1983; Pattison & Grieve,1984; Senk & Usiskin, 1985; Silver et al., 1988; Smith & Walker, 1988), the age level of the participants (Carpenter et al., 1989; Dossey, 1985; Fennema & Carpenter, 1981; Hilton & Berglund, 1974; Joffe & Foxman, 1988; Leder, 1988b), and whether classroom grades and standardized test of achievement are considered (Kimball, 1989).

Many researcher declared that the differences are particularly apparent on high cognitive level questions and for high-achieving students (Benbow, 1988; Benbow & Stanley, 1980, 1983; Edwards, 1985; Fennema & Carpenter, 1981; Fox &

Cohn, 1980; Hall & Hoff ,1988; Kissane, 1986; Peterson & Fennema, 1985; Wagner & Zimmerman, 1986). Benbow (1992) reports that males outnumber females in both high ability and low ability groups in mathematics from adolescence through adulthood, although the differences are more pronounced in the higher achievement group. On the other hand, in their meta-analysis of literature on gender difference, Hyde et al (1990) cautioned against the tendency to over generalize the occurrence of gender differences from highly selected group to the broader student body. They suggested that gender differences in mathematical performance are declining. For example; some studies showed that females tend to score as well as males on collage math placement test (Bridgeman & Wendler, 1991) and average female grades in collage math course tend to be as good as or better than those of males (Linn & Kessel, 1994).

A number of empirical studies have shown that boys tend to outperform girls in measurement, proportionality, geometry, spatial geometry, analytic geometry, trigonometry and application of mathematics (Battista, 1990; Fennema, 1980; Fennema & Carpenter,1981; Garden, 1989; Hanna, 1986; Linn & Pulos, 1983; Marshall, 1983; Martin & Hoover; 1987; Pattison & Grieve, 1984; Robitaille, 1989; Sabers et al., 1987; Shuard, 1986; Wood, 1976), whereas, girls have performed better than boys in computing, set operation, and symbolic relation (Brandon et al., 1987; Fennema, 1974; Jarvis, 1964; Johnson, 1987; Meece et al, 1982; Pattison & Grieve, 1984; Robitaille, 19896; Wood, 1976). Linn & Kessel (1996) found that females do better than males on routine math problems, whereas males excel on nontexbook-like problems.

In the past, researchers have used, almost exclusively, multiple choice tasks to examine the gender differences (Marshall, 1983). How male and female students differ in solving more complex problems, such as proof problems, is less investigated. Due to the use of multiple choice items the gender differences in most of the previous studies were examined and reported only in terms of mean scores or percent correct and incorrect rather than providing an analysis of the differences in solution processes. A few studies that examined gender differences in students' mathematical thinking (e.g., Fennema et al., 1998; Gallagher & De Lisi, 1994).

2.6.8.2 Gender Differences in Strategy use

Gallagher and De Lisi (1994) pointed out that gender differences favoring high-ability male students over high-ability female students may be due in part to differences in solution strategies, since males and females use different strategies in response to mathematical problems.

In a longitudinal study of gender differences in mathematics, Fennema et al. (1998) examined 38 girls' and 44 boys' solution strategies in solving number facts, addition/subtraction, and nonroutine problems throughout 3 year. They found that girls tended to use more concrete strategies and boys tended to use more abstract strategies. At the end of the third year, girls used more standard algorithms than boys and boys were more successful than girls on problems that required flexibility.

Gallagher and De Lisi (1994) examined the gender differences between 25 male and 22 female high-ability students on a set of difficult conventional and unconventional SAT-M items. Although researchers found no difference in overall performance of males and females, they found gender differences in success patterns and in strategy use on conventional and unconventional problems. Specifically, female students were more likely than male students to correctly solve conventional problems using algorithmic strategies; male students were more likely than female students to correctly solve unconventional problems using logical estimation or insight.

2.6.8.3 Gender Differences in Geometry and Proof

Students' proof schemes can be considered as the solution strategies that students used to response questions required proving and justification. However, there is no study which has investigated gender differences in students' solution strategies or proof schemes in geometry proof questions. Beside, there was little attempt in proof literature in investigating gender differences in students proof performances. Senk and Usiskin (1983) investigated gender differences in the ability to write proof in geometry, with a large-scale national (U.S.) study. Senk and Usiskin (1983) reported that after taking a standard high-school geometry course, even though adolescent males typically perform better than their female peers on geometric ability tests, no gender difference found in high-school students' ability to write geometric proofs. However, Senk and Usiskin's study focused on how well students write geometric proof instead of the strategies or proof schemes that students might used to answer proof questions.

CHAPTER 3

METHODOLOGY

The following sections include the descriptions of the research problem and hypotheses, research design, participants, definitions of terms, variables, and development of measuring tools, procedures, methods used to analyze data, assumptions and limitations.

3.1 Research Problem and Hypothesis of the Study

In this section, the research problem, related sub-problems and associated hypothesis of the present study will be stated briefly. The main research questions that guided the present study were as follows;

RQ1: Are there any significant differences among 10^{th} grade students' proof schemes use in geometry?

RQ2: Are there any significant differences between students' proof schemes use in geometry with respect to their cognitive styles and gender?

In order to examine the sub-problems following null hypothesis are defined.

$H_0 1$: There is no significant mean difference between 10^{th} grade students' externally based, empirical and analytical proof schemes use scores in GPT.

$H_0 2$: There are no statistically significant differences between 10^{th} grade field dependent and field independent students' externally based, empirical and analytical proof schemes use scores in GPT.

$H_0 3$: There are no statistically significant differences between 10^{th} grade male and female students' externally based, empirical and analytical proof schemes use scores in GPT.

$H_0 4$: There is no statistically significant interaction between cognitive styles and gender of students with respect to students' externally based, empirical and analytical proof schemes use scores.

The null hypotheses stated above will be tested at significance level of 0.05.

3.2 Research Design and Research Variables

The purpose of the present study was to investigate 10^{th} grade students' proof schemes and difference in students' proof schemes with respect to their cognitive styles and gender. Therefore the present study was stand on both descriptive and causal-comparative design (Frankel & Wallen, 1996).

The independent and dependent variables of the present study are presented in Table 3.1 below.

Table 3.1 Research Variables of the Study

Independent variables		Dependent Variables
Cognitive Style	Gender	
Field Independent	Male	Externaly-Based Proof Schemes Use
Field Dependent	Female	Emprical Proof Schemes Use
Field Mix		Axiomatic Proof Schemes Use

3.3 Participants of the Study

The accessible population of this study was all tenth grade students in Anatolian and Private High Schools in Çankaya district in 2005-2006 spring semesters. Total number of students in these types of schools in Çankaya district of Ankara is almost 2600. The participants of the study were 224 tenth grade students from four secondary schools.

At the beginning, approximately 10% of the accessible population was planed to be included as the sample of the study. Prior to the study necessary permissions was taken from the Turkish Ministry of Education for 17 schools in Çankaya district. Although all these schools were contacted, only five schools (three Anatolian high schools, two private high schools) were willing to participate in the study. Because of time restriction, and absences of students in schools, it could not be possible to reach all the students from five high schools. Therefore, only 224 students from three schools (three Anatolian high schools and one private high school) were participated in the study. The number of students participated in the study from two types of school was presented in Table 3.2.

Table 3.2 The number of students participated in the study from three Anatolian High Schools and one Private High School

	Anatolian High Schools	Private High School
Females	89	37
Males	71	27
Total	160	64

Of 224 participants, 126 participants were female and 98 participants were males. Participants ranged in age from 16 to 18 years (Mean=16.69, SD=.43, Med=17). All students participated in the study has a grade of 4 or 5 (over a scale of 5) on geometry course that they took in previous semester.

The Anatolian high schools are the schools that accept students who take the exam called as Ortaöğretim Kurumları Sınavı (OKS) and conducted by Turkish Ministry of Education. This exam includes 100 multiple choices questions and measures the students' knowledge and skills in four domains: Turkish Literature, Mathematics, Science, and Social Sciences. The minimum score that have to be taken from the exam is 160. Regarding to their total scores on the exam and the quota of the schools, students are placed to the Anatolian high schools they preferred.

The private high schools in Ankara are the schools that accept students according to a private school entrance exam conducted by the school. Similar to OKS, this exam includes 100 multiple choices questions and measures the students' knowledge and skills in four domains: Turkish literature, mathematics, science, and social sciences. Regarding to their total scores on this test, students are ranked. The number of students as many as the quota of the school is accepted to the school.

As indicated, students must fulfill certain requirement to attend in Anatolian high schools and private high schools. Thus the level of the students in those schools is high relative to their counterparts in most regular state high schools. Literature put forward that the largest gender difference have found among high ability student population (Hyde et al, 1990; Gallagler & DeLisi, 1994). Therefore it was decided to conduct the study with a sample of students enrolled in these schools so as to expose gender differences in the use of proof schemes.

3.4 Measuring Instruments

In the present study following instruments were used as measuring instruments.

1. Group Embedded Figure Test (GEFT)
2. Geometry Proof Test (GPT)

Below, development process of the each measuring instruments are explained.

3.4.1 Group Embedded Figure Test (GEFT)

Group Embedded Figure Test developed by Witkin, Oltman, Raskin and Karp (1971) was used to determine cognitive styles of the participants in the study. GEFT is widely used in cognitive style studies as a tool for determining individuals' cognitive styles in a manner of field dependence / field independence. In this present study GEFT's Turkish version was used (Cebeciler, 1988).

The test includes three parts: the first part including seven items was preparatory and not included in the assessment. The assessment was made on the second and the third parts. The second and the third parts each including nine items required students to find the embedded figures given in the complex figures. Sample items in GEFT were presented in Appendix D.

One point was given for each true response to the items and zero point was given for each incorrect response. The total score ranges from 0 to 18. It was important that each part of GEFT was administered in a limited time. The time given for the first part was two minutes and for the second and the third parts each was five minutes. For the main study, the KR-20 internal consistency reliability obtained was .86.

The classification of students into the categories of cognitive style was determined with regard to the mean and standard deviation of the group scores as proposed in the previous studies (Cebeciler, 1988; Kahtz & Kling, 1999; Liu & Reed, 1994). Students who scored one-standard deviation above the mean were grouped as field independent (FI) and students who scored one-half standard deviation below the mean were grouped as field dependent (FD). Students who take scores one standard deviation around the mean were categorized as field mix (FM).

3.4.2 Geometry Proof Test (GPT)

The Geometry proof test was designed by the researcher to identify students' proof schemes use in a geometric context. At the beginning, a pool of 22 open-ended geometry questions on triangle concept requiring proving and justifying was developed by conferring with the textbooks and several geometry books. The questions in the pool were accord with 10^{th} grade geometry goals and objectives of National Mathematics Curriculum (Genel Ortaöğretim Geometri Dersi Programi, 1992), and covered very fundamental notions in triangle concept: (a) the types and properties of triangles, (b) angle-side relations in triangles, (c) Elements of a triangle (d) similarity in triangles and (e) theorems in triangles. Considering the literature on students' conception proof and students' proof schemes (Goldino and Recio, 1999; Heally & Hoyles, 2000; Harel & Sowder, 1998, Sowder & Harel, 1998), the format of questions was determined as open-ended to gain insight deeper understanding of students' proof schemes. Questions were composed of conjectures required justification as true or false, theorems required proof. There are six questions on the angle types and properties of triangles, four questions on angle-side relations in triangles, four questions on basic elements of a triangle, four questions on similarity in triangles and, four questions on theorems in triangles. Half of the questions in each category were selected to form GPT. The table of specifications for the question in GPT is presented in Table 3.3, below.

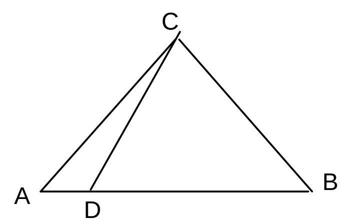
Table 3.3 Table of Specifications of the Questions in GPT

Question	Types and properties of triangles	Angle-side relation in triangles	Elements of a triangle	Similarity in triangles	Theorems in triangles
1	✓				
2		✓			
3			✓	✓	
4				✓	
5					✓
6	✓				
7	✓				
8	✓				
9			✓		
10		✓			
11					✓

A graduate student from a Mathematics Department and a mathematics educator from Secondary Science and Mathematics Education Department of Middle East Technical University were asked to check questions in GPT with respect to the content (representativeness of the concept of triangle, adequacy of number of questions, relevancy to the proof or justification tasks, appropriateness to the tenth grade geometry curriculum and the level of the students), the format (clarity and languages). The graduate students and the mathematics educator both confirmed that questions were suitable according to content (representativeness of the concept of triangle, adequacy of number of questions, relevancy to the proof or justification tasks, appropriateness to the tenth grade geometry curriculum and the level of the students). However, they declared that wording of some questions were difficult to understand. Regarding their recommendations on the format of the questions, the problematic questions were revised in order to make wording clear.

In order to check the clarity and difficulty of questions, and determine the test duration, the GPT comprising eleven questions was piloted with two 10th grade students attending to an Anatolian high school in the second semester of 2005-2006 academic years. One of the students was asked to complete the test in ninety minutes to determine the time required to complete the test. This student completed her solutions on eleven questions in a given time and confirmed that the items were clear to understand and the difficulties of them were appropriate for their level. The GPT was administered to the second student by a one-to-one interview for the aim of checking whether the items were designed or pitched at an appropriate level for the students and meet the desired properties. Similar to the first student, this student stated no difficulty in understanding the questions in GPT. The interview data were audiotaped. As a result of the pilot testing, all questions were decided to be used in the main study. Hence, the final version of GPT comprised eleven questions. Sample questions of GPT were given in Table 3.4, and GPT was given in Appendix C.

Table 3.4 Sample questions in GPT

Question 1 Is it possible to draw a triangle in which the sum of all possible pair of interior angles is less than 120 degree? Justify your answer.
Question 11 Prove that, in triangle ABC if $\lvert AB \rvert = \lvert AC \rvert$, then $\lvert AC \rvert > \lvert DC \rvert$ 
Question 12 Prove that in any triangle ABC the line segment DE which connects the midpoints of the side AB and AC is parallel to BC and has half of the length of BC.

Students' responses to eleven questions were investigated so as to expose the proof schemes they use in the triangle concept. The proof schemes reported by Harel and Sowder (1998) were used as a framework while categorizing the students' responses. Based on Harel & Sowder's works (1998, 2005), the key characteristics of each main schemes and their sub-categories were summarized. The key characteristics of proof schemes were controlled by an associate professor, and a research assistant in mathematics education from Secondary Science and Mathematics Education Department at Middle East Technical University. A full agreement was satisfied with discussion. The key characteristics of Harel and Sowder's proof schemes were presented in Appendix A and the examples from students' proof schemes use were presented in Appendix B.

In order to establish the extent of consensus on use of the categorization for responses given to the proof questions, inter-rater reliability coefficient was computed. Randomly selected 40 tests were scored by the first researcher and a research assistant in Secondary Science and Mathematics Education Department of Middle East Technical University. Inter-coder agreement was calculated by the ratio of the total number of agreements to the total number of coding and found as .92. The differences were discussed until resolved.

Students' responses on eleven questions in GPT were coded as authoritarian, ritual; symbolic, perceptual, example based transformational or axiomatic proof schemes. Frequency scores for GPT were created for the use of externally based proof schemes (by adding the number of responses coded as authoritarian, ritual, or symbolic), empirical proof schemes (by adding the number of responses coded as perceptual or example based), and analytical proof schemes (by adding the number of responses coded as transformational or axiomatic). Therefore, a student has three scores for GPT. Because a student could choose to use any proof scheme to answer a question, the possible range of the scores for each proof scheme category was between 0 and 11. Thus, a score of 0, for example, for externally based proof schemes meant that the student never chose to use externally based proof schemes to answer the geometry questions presented. Whereas, a score of 11 for externally based proof schemes meant the students always chose to use externally based proof schemes to answer the questions.

3.5 Procedure

The present study started with a review of literature about the variables in the research question. After a review of related literature, data collection instruments were developed. Geometry proof test was developed and then piloted with tenth grade students in April 2006. The schools that would be included in the study were determined. In order to administer the tests to the schools, necessary permission was obtained form Ankara Directorate of Education through the Presidency of Middle

East Technical University. The researcher participated administration process of the tests in schools those were willing to participate to the study.

GEFT and GPT were administered in June 2006 in the classroom period in the given order by the first researcher, giving two or three days between two administrations. When administering GEFT, the students were given simple figures on a separate sheet and were reminded to find the same simple figures embedded in the complex figures in GEFT, without changing directions and size of the figures. When administering GPT, students were reminded to justify their answers as clear as possible. Ninety minutes were given to students to finish the test.

3.6 Data Collection

The present study was executed at the end of the academic year 2005-2006. First of all, GEFT was administered to 300 students participated in the study in June 2006. When administering GEFT, restricted time limits were satisfied by the researcher. After two or three days, GPT was administered to the 230 students. 224 students who were attended in both tests were participated in the study. Data were analyzed using Statistical Package for Social Science 11.5 for Windows (SPSS 11.5).

3.7 Data Analyses

Data obtained through GEFT and GPT were coded and transferred to the computer environment using SPSS 11.5 package program. Descriptive statistics such as mean, median, frequency, standard deviation, skewness, and kurtosis of the dependent variables were calculated by levels of independent variables. Repeated measure ANOVA was used to test the difference between students externally based proof schemes use, empirical proof schemes use and axiomatic proof schemes use in GPT. A 2 (gender) x 3 (cognitive styles: FD, FM, FI) multivariate analysis of variance (MANOVA) with three proof schemes use scores as the dependent variables was employed to test the effect of gender and cognitive style on students' proof schemes scores in GPT. For each significant MANOVA, a separate univariate tests for the dependent variables was carried out as a follow-up analysis. Significant differences were assessed at the .05 level for all comparisons (α=.05). Due to a inflated risk of Type 1 errors, Bonferonni correction was applied to the subsequent univariate tests.

3.8 Assumptions of the Study

For the purpose of this study, it was assumed that the administration of GEFT and GPT were completed under standard conditions, Another assumption was that all participants of the present study answered the measuring instruments accurately and sincerely.

3.9 Limitation of the Study

One of the main limitations of this study was the restricted sample for the study. This study was limited with 224 tenth grade students from four high schools (three Anatolian high schools, one private high school) at the second semester in 2005-2006academic year. Although 17 high schools were contacted initially, only 5 schools voluntarily participated in the study. A threat to the external validity due to selection of sample groups could not be completely eliminated. A more generalized study would be including other types of schools such as science high schools and public schools.

Participated students' geometry course grades were 4 or 5 over 5 point scale. This also restricts the generalizability of the study to the low-achiever students in geometry. In order to increase generalizability of the study, further studies can include students from different achievement levels.

Nevertheless, replication of the study with students of different grade levels, and in different school districts and at different times could increase the generalization of the findings.

CHAPTER 4

RESULTS

The sections that follow include results of analysis that are conducted to obtain statistical evidence for the claims of the present study. The results of the descriptive statistics tests are presented in the first section. The results of the inferential statistics tests for research questions and hypotheses associated to them are presented in the remaining sections of the present chapter.

4.1 Results of Descriptive Statistics

The analyses of the present study based on the data obtained from two tests; Group Embedded Figure Test (GEFT) and Geometry proof test (GPT). As stated in the previous chapter the GEFT was used to investigate the cognitive style of the participants, while GPT was used to investigate students' proof schemes use in geometry.

The descriptive statistics for two tests used in the study are presented in Table 4.1, Table 4.2, Table 4.3, Table 4.4 and Table 4.5. Mean, median, standard deviation, skewness, and kurtosis values for each dependent variable are given with respect to the levels of independent variables.

Table 4.1 Descriptive Statistics for GEFT Scores of Males and Females

	N	Mean	Med	SD	Skewness	Kurtosis
Females	126	10.17	10.00	4.124	-.064	-1.027
Males	98	10.08	10.00	4.837	-.127	-1.212
Total	224	10.13	10.00	4.440	-.104	-1.087

Table 4.2 The Number and Percents of Males and Females Distributed Over Cognitive Styles

	Field Dependent	Field Independent	Field Mix
Females	28 (12.5%)	25 (11.2%)	73 (32.5%)
Males	29 (12.9%)	28 (12.5%)	41 (18.3%)
Total	57 (25.4%)	53 (23.6%)	114(50.8%)

As seen in Table 4.1, mean of the GEFT scores of females and males were close to each other. The skewness and kurtosis values of the GEFT scores were between ±2.0, so they were in an acceptable range for a normal distribution (George & Mallery, 2001). In Table 4.2, the cognitive styles of students determined by mean and standard deviations of the groups are presented. As seen, the number of field mix (FM) students was nearly twice of the number of field dependent (FD) and field independent (FI) students. The number of FD females, the number of FD males, the number of FI females and the number of FI males were nearly equal to each other. However the number of FM females was twice of the number of FD males.

Tables 4.3 Descriptive Statistics for Externally Based Proof Schemes Use in terms of Gender and Cognitive Styles in GPT

	Females					Males				
	N	Mean	SD	Skew	Kurts	N	Mean	SD	Skew	Kurts
FD	28	3.68	1.44	-.054	-1.12	29	4.03	2.04	.435	-.350
FI	25	2.64	1.55	.662	.117	28	2.82	2.01	1.076	.666
FM	73	3.38	1.68	.397	.664	41	3.63	1.97	.726	.169
Total	126	3.30	1.63	.344	.143	98	3.52	2.04	.652	-.188

Table 4.4 Descriptive Statistics for Empirical Proof Schemes Use in terms of Gender and Cognitive Styles in GPT

	Females					Males				
	N	M	SD	Skew	Kurts	N	M	SD	Skew	Kurts
FD	28	4.25	1,81	-.401	-.020	29	3.17	1.64	-.244	-.626
FI	25	4.12	1.13	.312	.906	28	3.64	1.19	-.083	-.463
FM	73	3.84	1.64	.232	.205	41	3.10	1.53	.665	1.563
Total	126	3.98	1.59	.038	.157	98	3.28	1.48	.092	.262

Table 4.5 Descriptive Statistics for Analytic Proof Scheme Use in terms of Gender and Cognitive Styles in GPT

	Females					Males				
	N	M	SD	Skew	Kurts	N	M	SD	Skew	Kurts
FD	28	.64	.58	.285	-1.527	29	.76	.689	1.042	.555
FI	25	2.12	.711	-.530	-.360	28	1.39	.685	-.427	-1.242
FM	73	1.11	.71	.131	-1.369	41	1.63	.788	.117	-.865
Total	126	1.21	.723	-1.22	1.143	98	1.31	.753	.236	-.954

The results of descriptive statistics for externally-based, empirical and analytic proof schemes use scores in terms of gender and cognitive styles are presented in Table 4.3, Table, 4.4, and Table 4.5, respectively. As seen in Table 4.3, Table 4.4 and Table 4.5, the skewness and kurtosis values of the students externally-based, empirical and analytic proof schemes use scores were between ±2.0, so they were in an acceptable range for a normal distribution (George & Mallery, 2001).

As seen in Table 4.3, the means of FD females and males' externally based proof schemes use scores on GPT were higher than the means of FI and FM females and males scores on GPT. Table 4.4 shows that the means of female FD, FI and FM students' empirical proof schemes use scores were higher than the means of male FD, FI and FM students' empirical proof schemes use score. Table 4.5 shows that the means of FD and FM males' analytical proof schemes use scores were higher than the means o FD and FM females' analytical proof schemes use score. However, the mean of FI females' analytical proof schemes use scores was higher than the means of FI males' analytical proof schemes use scores.

4.2 Results of Inferential Statistics

In this section results of the testing research sub-problems of the present study are presented. The inferential statistics were used to test research questions through their associated hypothesis.

4.2.1 The First Sub-problem and Its Hypothesis

Before testing this hypothesis it would be better to give the frequencies and percents of each proof scheme used in GPT in Table 4.6 below.

As seen in Table 4.6, the frequencies of three proof schemes use vary from question to question. However, totally, the frequency of empirical proof schemes use (817) and the frequency of externally based proof schemes (769) were more than the frequency of analytical proof schemes uses (268). This implies that students used empirical proof schemes more frequently and they used analytical proof schemes less frequently. In order to see whether the differences between the frequencies of students proof schemes use were significant or not, the first research question was tested by means of its' associated null hypothesis.

As seen in Table 4.6, the frequencies of three proof schemes use vary from question to question. However, totally, the frequency of empirical proof schemes use (817) and the frequency of externally based proof schemes (769) were more than the frequency of analytical proof schemes uses (268). This implies that students used

empirical proof schemes more frequently and they used analytical proof schemes less frequently. In order to see whether the differences between the frequencies of students proof schemes use were significant or not, the first research question was tested by means of its' associated null hypothesis.

Table 4.6 Frequencies and Percents of Proof Schemes Use in GPT

	Externally Based		Empirical		Analytic	
Questions	Frequency	Percent	Frequency	Percent	Frequency	Percent
1	16	8.8	119	65.7	46	25.4
2	55	26.4	143	68.8	10	4.8
3	30	15.2	163	82.3	5	2.5
4	103	75.2	25	18.2	9	6.6
5	146	70.9	10	4.9	50	24.4
6	99	51.0	77	39.7	18	9.3
7	50	27.6	87	48.1	44	24.3
8	67	43.2	73	47.1	15	9.7
9	25	27.2	47	51.1	20	21.7
10	50	35.7	67	47.9	23	16.4
11	128	74.4	6	3.5	38	22.1
Total	769	41.5	817	44.06	268	14.4

To test the $H_0 1$, a one-way repeated measure ANOVA was used. For the present study, proof schemes used was within-subject factor with three levels (externally based, empirical, analytic) and proof schemes use scores are dependent variables. It was recommended that when there are more than two levels of within-subject factor it is better to use multivariate tests instead of standard univariate F test (Green, Salkind & Akey, 2000). In the present case, there are three levels. Therefore, multivariate tests were used.

There are two assumptions of multivariate tests. These are normality and independency of difference scores between the levels of factor. Since the sample of the present study was random sample and the tests were conducted under the control of researcher, the assumption of independency of cases was satisfied. The normality assumption was controlled by the skewness and kurtosis values of proof schemes use scores as shown in Table 4.7 and by the skewness and kurtosis values of difference scores as shown in Table4.8. George and Mallery (2001) stated that kurtosis value between ±1.0 is considered as excellent; however, a kurtosis value between ±2.0 is also acceptable for many cases. Therefore, it was decided that skewness and kurtosis values of proof schemes use scores as shown in Table 4.7 and skewness and kurtosis values of difference scores as shown in Table4.8 were in approximately acceptable range for a normal distribution.

Table 4.7 Descriptive Statistics for Students Proof Schemes Use scores

	N	Mean	SD	Skewness	Kurtosis
Externally Based	224	3.39	1.82	0.573	0.126
Empirical	224	3.67	1.58	0.097	0.141
Analytical	224	1.25	1.42	1.420	1.830

Table 4.8 Descriptive Statistics for Paired Differences

Difference Scores	N	Mean	SD.	Skewness	Kurtosis
Exter.Based - Empirical	224	0.27	2.73	-0.501	0.74
Analytical - Empirical	224	-2.42	2.34	0.435	0.671

The results of the multivariate tests indicated that a significant proof schemes use effect with Wilk's Λ=.45, F (2, 222) = 135.80, p=.000 η^2=.55. In other words, there is a significant mean difference among the proof schemes use scores of students in GPT ($p<.05$). According to Cohen's classification for effect sizes, .01 is small, .09 is medium, and .25 or grater is large (Cohen, 1988). It can be seen that the value of the .55 for effect size is a large effect size.

To test which means of proof scheme use differ from each other, a pair-wise comparison was conducted. Paired samples t-test was used for this purpose. As seen in Table 4.7, the results of paired sample t-test indicated that there are significant mean differences between empirical and analytical proof schemes use scores in GPT ($p<.016$) with a large effect size (η^2=.39) and also there are significant mean difference between the externally-based and analytical proof schemes use score (($p<.025$) with a large effect size (η^2=.52), however, there is no difference between the externally based and empirical proof schemes use scores. The means and standard deviation scores for proof schemes use scores are given in Table 4.8.

Table 4.7 Results of Paired Samples T-Test for Types of Proof Schemes Use Scores

Paired Differences	Mean	SD	t	df	p	η^2
Pair1 Extern-Empirical	-.28	2.73	-1.513	223	.132	.01
Pair2 Extern-Analytical	2.15	2.72	11.807	223	.000	.39
Pair3 Empirical-Analytical	2.42	2.35	15.469	223	.000	.52

*p<.05

Table 4.8 Means and Standard Deviations of Types of Proof Scheme Use Scores in GPT

	Mean	SD
Externally Based	3.40	1.821
Empirical	3.67	1.584
Analytic	1.25	1.427

As the results of the paired-samples t-test in Table 4.7 and mean and standard deviation of three types of proof schemes in Table 4.8 indicate, students use externally based proof schemes and empirical proof schemes significantly more than analytical proof schemes ($Mean_{extern}$ =3.40, SD_{extern} =1.821; $Mean_{empir}$ =3.67, $SD_{empiric}$ =1.584; $Mean_{analy}$ =1.25, SD_{analy} =1.427). However, there is no significant difference between students' use of externally based and empirical proof schemes. As seen in Table 4.6, 41.5 % of the students' responses were externally based, 44.06 % of the students responses were empirical and only 14.4 % of the students responses were analytical.

4.2.2 The Second Sub-problem and Its Hypotheses

Second sub-problem of the study is "What is the effect of gender and cognitive style on students proof schemes use?" This sub-problem was tested on GPT by means of following three null hypotheses:

$H_0 2$: There are no statistically significant differences between 10^{th} grade male and female students' externally based, empirical and analytical proof schemes use scores in GPT.

$H_0 3$: There are no statistically significant differences between 10^{th} grade field dependent and field independent and field mix students' externally based, empirical and analytical proof schemes use scores in GPT.

$H_0 4$: There is no statistically significant interaction between cognitive styles and gender of students with respect to students' externally based, empirical and analytical proof schemes use scores.

In order to test these hypotheses, two-way MANOVA was used. Gender and cognitive styles were independent variables and types of proof schemes used in GPT were dependent variables for the analyses. $H_0 2$ and $H_0 3$ represent main effects of gender and cognitive styles and $H_0 4$ represent interaction effect between gender and cognitive style. There are three assumption of MANOVA. One of the assumptions is randomness and independence of participants. As stated before, sample of the present study was random sample and the tests were conducted under the control of researcher. Second assumption is normality of dependent variables across each population at all levels of factors. In the previous section on the results of descriptive statistics, the skewness and kurtosis values for all levels of gender and cognitive styles for each dependent variable were given in Table 4.3, Table 4.4, and Table 4.5. In some of the population, these values are between ±2.0. However, they were still in acceptable range for a normal distribution. The last assumption is homogeneity of covariance matrices. The results of the analysis to evaluate the assumption of homogeneity of variance-covariance matrices were satisfactory, with Box's M test producing $M = 37.874$, $F(30, 56723) = 1.211$, $p > .05$.

Results of two-way MANOVA with gender and cognitive style as independent variables and three types of proof schemes use score as dependent variables given in Table 4.9.

Table 4.9 Results of two-way MANOVA and Univarite ANOVA for Proof Scheme Use Scores in GPT

	Wilk's Λ	F	Hyp. df	Error df	Sig.	η^2
Gender	.94	4.41	3	216	.005	.058
ExternallyB.		1.051	1	218	.306	.005
Empirical		11.92	1	218	.001	.052
Analytic		.076	1	218	.78	.000
Cognitive Style	.89	4.10	6	432	.001	.054
ExternallyB.		5.727	2	218	.004	.050
Empirical		1.36	2	218	.25	.012
Analytic		9.25	2	218	.000	.078
Gender*Cogn. Style	.96	1.23	6	432	.287	.017

The third null hypothesis of the second research question is "$H_0$4: There is no statistically significant interaction between cognitive styles and gender of students with respect to students' externally-based, empirical and analytical proof schemes use scores in GPT". As seen in Table 4.9, Wilk's Λ indicates that there is not overall significant interaction between gender and cognitive style among the mean scores of three proof schemes use in GPT (Wilk's Λ=.96, F(6,432)=1.23, p=.287). The means and standard deviations of students' proof schemes use scores with respect to their gender and cognitive styles are given in Table 4.10.

Table 4.10 The Means and Standard Deviations of Students' Proof Schemes Use Scores in GPT with respect to Gender and Cognitive Style

	ExternallyB.				Empirical				Analytical			
	Females		Males		Females		Males		Females		Males	
	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD
FD	3.68	1.44	4.03	2.04	4.25	1.81	3.17	1.64	.64	.58	.76	.86
FM	3.38	1.68	3.63	1.97	3.84	1.64	3.10	1.53	1.11	.71	1.63	.68
FI	2.64	1.55	2.82	2.01	4.12	1.13	3.64	1.19	2.12	.71	1.39	.78

The first null hypothesis of the second research question is "$H_0$2: There are no statistically significant differences between 10th grade male and female students' externally-based, empirical and analytical proof schemes use scores in GPT". As seen in Table 4.9, there is a overall significant mean difference among three proof schemes use score in GPT with respect to gender (Wilk's Λ=.95, F (3,216)=3.98, p=.005, η^2=.05) with a small effect size. A univariate ANOVA for each dependent variable with respect to gender and cognitive style was conducted for follow-up tests. The results of univariate ANOVA for proof schemes use scores in GPT are also given in Table 4.9.

Results of univariate analysis indicate that a significant mean difference in empirical gender schemes use score in GPT with respect to gender as shown in Table 4.10 ($F(1,218)=11.92$, $p=.001$, $\eta^2=.052$) with a small to moderate effect size. The means and standard deviations of proof schemes use scores of males and females are given in Table 4.11.

As seen in Table 4.11, females' mean scores of empirical proof schemes use are significantly higher than males' mean scores of empirical proof schemes use ($Mean_{female}=3.98$, $SD_{female}=1.59$; $Mean_{male}=3.28$, $SD_{male}=1.48$). This means that females use empirical proof schemes significantly more than the males. Table 4.10 and Table 4.11 shows that there is no statistically significant mean difference between the externally based proof schemes use scores of males and females with $F(1,218)=1.051$, $p=.306$, $\eta^2=.005$) ($Mean_{female}=3.30$, $SD_{female}=1.63$; $Mean_{male}=3.52$, $SD_{male}=2.04$). Similarly, there is no statistically significant mean difference between the analytical based proof schemes use scores of males and females with $F(1,218)=.076$, $p=.78$, $\eta^2=.000$ ($Mean_{female}=1.21$, $SD_{female}=.723$; $Mean_{male}=1.31$, $SD_{male}=.753$).

Table 4.11 Means and Standard Deviations of Females' and Males' Proof Schemes Use Scores in GPT

	Externally Based		Empirical		Analytical	
	Mean	SD	Mean	SD	Mean	SD
Females	3.30	1.63	3.98	1.59	1.21	.723
Males	3.52	2.04	3.28	1.48	1.31	.753

The second null hypothesis of the second research question is "$H_0 3$: There are no statistically significant differences between 10th grade field dependent, field independent and field mix students proof schemes use scores in GPT".

As MANOVA results seen in Table 4.9 indicate, there is a overall significant mean difference among three proof schemes use score in GPT with respect to cognitive style (Wilk's Λ = .89, F (6,432)= 4.10, p=.001, η^2=.054) with a small to moderate effect size. Results of univariate analysis shown in Table 4.9 indicate that there is a significant mean difference in externally based proof schemes use score in GPT with respect to cognitive style (F(2,218)=5.727, p=.004, η^2=.050) with a small to moderate effect size. Similarly, there is a significant mean difference in analytical proof schemes use score in GPT with respect to cognitive style (F(2,218)= 9.25, p=.00, η^2=.078) with a small effect size. However, there is no significant mean difference in empirical proof schemes use scores in GPT with respect to cognitive style (F (2,218)= 1.36, p=.25). Mean and standard deviations of students' proof schemes use score in GPT are given in Table 4.12. As there are three types of cognitive style, Bonferroni post-hoc analysis was conducted to find out which pairs of cognitive styles produce overall significant difference in externally based and analytical proof schemes use scores. The results of Bonferroni post-hoc analysis are given in Appendix E and means and standard deviations of students' proof schemes use scores with respect to their cognitive styles are given in Table 4.12.

Table 4.12 Means and Standard Deviations of Students' Proof Schemes Use Scores in GPT with respect to Cognitive Style

	Externally-based		Empirical		Analytical	
	Mean	SD	Mean	SD	Mean	SD
FD	3.78	1.76	3.70	1.80	.70	1.08
FM	3.47	1.78	3.57	1.63	1.30	1.47
FI	2.74	1.79	3.87	1.17	1.74	1.47

Bonferroni post-hoc analysis results indicated that there is a significant mean difference between externally based proof schemes use scores of FD and FI students (p=.004). As seen in Table 4.12 mean of the field dependent students' externally based proof schemes use scores are significantly higher than the mean of field independent students' externally-based proof schemes use scores ($Mean_{FD}$ =3.78 SD_{FD} =1.76; $Mean_{FI}$ =2.74, SD_{FI} =1.79). This means that FD students use externally based proof schemes in GPT more than do FI students. On the other hand, there is no significant mean difference between externally based proof schemes use scores of FD and FM students ($Mean_{FD}$ =3.78 SD_{FD} =1.76; $Mean_{FM}$ =3.47, SD_{FM} =1.78). Similarly, there is no significant mean difference between FM students' and FI students' externally based proof schemes use scores ($Mean_{FM}$ =3.47, SD_{FM} =1.78; $Mean_{FI}$ =2.74, SD_{FI} =1.79).

Bonferroni post-hoc analysis results also indicated that there is a significant mean difference between analytical proof schemes use scores of FI and FD students (p=.000). As seen in Table 4.12, FI students' analytical proof schemes use scores is significantly higher than the mean of the FD students' analytical proof schemes use scores ($Mean_{FI}$ =1.74, SD_{FI} =1.47; $Mean_{FD}$ =.70, SD_{FD} =1.08). These mean that; FI students use analytical proof schemes significantly more than FD students. On the other hand, there is no significant mean difference between analytical proof schemes use scores of FI and FM students ($Mean_{FI}$ =1.74, SD_{FI} =1.47; $Mean_{FD}$ =.70, SD_{FD} =1.08) and between analytical proof schemes use scores of FD and FM students ($Mean_{FD}$ =.70, SD_{FD} =1.08, $Mean_{FM}$ =3.47, SD_{FM} =1.78).

Bonferroni post-hoc analysis results indicated no significant mean difference between empirical proof schemes use scores of FD, FI and FM students.

CHAPTER 5

DISCUSSIONS, CONCLUSIONS, AND IMPLICATIONS

This chapter includes discussion, conclusion and interpratation of results, educational implications and recomendation for further research.

5.1 Discussions of the Results

There were two main purposes of this study; (a) to investigate types of proof schemes that students use in geometry and (b) to explore the differences between students' proof schemes use in geometry with respect to their cognitive styles and gender? The study was specifically concerned with the differences between students' proof schemes use in geometry and the effects of gender and cognitive style on students' proof schemes use in geometry. In order to reach these purposes, Group Embedded Figure Test (GEFT) and Geometry proof test (GPT) were administered to 224 tenth grade students. GEFT was used to identify students' cognitive styles and GPT was used to identify types of proof schemes that students used to answer questions in GPT. Frequency scores for GPT were created for the use of each type of proof schemes and identified as proof schemes use score. Proof schemes use scores were analyzed with respect to students' gender and cognitive styles. Hypotheses were tested by one-way repeated-measures ANOVA, and two-way MANOVA. In addition to these, descriptive analyses were utilized. In this section, results of the study will be discussed.

Discussions were based on research questions proposed in the study. The first question was regarding the differences between students' proof schemes use in geometry. The results showed that students used externally based proof schemes and empirical proof schemes significantly more than analytical proof schemes. Specifically, 41.5 % of the students' responses were externally based, 44.06 % of the students' responses were empirical. Only 14.4 % of the students' responses were analytical. One would think that this result was surprising, because the students who participate in this study were enrolled in Anatolian High Schools and Private High School and were at least above average students. However, this result was not surprising since the similar results were founded by several researches, which suggest that there is an absence of deductive proof schemes and pervasiveness of the empirical proof scheme among the students at all grade levels (Fischbein & Kedem, 1982; Harel & Sowder, 1998; Schoenfeld, 1985; Vinner, 1983). Harel and Sowder (2005) declared that prevalence of empirical proof schemes for most students seems to be international. Hoyles et al. (1998) showed that even high-attainers had great difficulties in generating proofs and are more likely to rely on empirical proofs.

The second research question was about the effect of students' gender and cognitive styles, specifically field dependence/field independence cognitive styles, on students' proof schemes use in geometry. Regarding to gender, it was found that there was a significant difference in the empirical proof schemes use. Particularly, females used empirical proof schemes significantly more than the males. This result is consistent with the results of the studies which indicated a gender difference in strategy use in problem solving (Fennema et al., 1998; Gallagher & De Lisi, 1994), though it is inconsistent with the study conducted by Senk and Usiskin (1983) which found no gender differences in the ability to write proof in geometry. The reason for the inconsistency might be that Senk and Usiskin focused on the extent to which secondary school geometry students write proof similar to the theorems or exercises commonly used geometry texts, rather than the strategies or schemes that students used in the proof questions. Although, females' and males' ability to write correct and complete proof is similar, they can use different strategies to write a proof. Thus, though the effects size of gender differences in empirical proof schemes use was small, it was reasonable that females use empirical proof schemes (perceptual and example based proof schemes) significantly more than males. As indicated in literature review, females tended to use concrete strategies rather than abstract ones. It can be considered that females use examples (as in example based proof scheme) and drawings (as in perceptual proof scheme) in proofs to concretize the proof situation. On the other hand, it is equivocal that this finding supports the results of previous studies that indicated males' superiority in geometry during the high school level (Battista, 1990; Fennema, 1980; Fennema & Carpenter, 1981; Hanna, 1986; Leahey & Guo, 2001; Martin & Hoover; 1987), because these studies were interested in students' overall performance on a multiple choice test, rather than analysis of students solutions.

Another component of the second research question was students' field dependent (FD)/field independent (FI) cognitive style. Results regarding to FD/FI cognitive styles were as following; FD students used externally based proof schemes in GPT significantly more than FI students. FI students used analytical proof schemes significantly more than FD and FM students. And, FM students use analytical proof schemes significantly more than FD students. There is no significant difference in the use of empirical proof schemes among FD, FI and FM students. Although the effects size of differences between the uses of proof schemes was small, these results were highly consistent with the characteristics of FD/FI cognitive styles presented in the literature review. Analytical proof schemes include justification based on logical reasoning and formal arguments, whereas, externally-based proof schemes include justifications based on the authority of an external source, such as teacher, textbook (as in authoritarian proof schemes) or the form of an argument (as in ritual and symbolic proof schemes) (Harel & Sowder, 1998). As literature put forward, FI individuals perceive and process information analytically and systematically, base their reasoning on abstract materials and formal discussions, on the other hand, FD individuals process information in an intuitive, global, holistic and passive way and base their reasoning on concrete material and informal discussion (Biggs, Fitzgerald, & Atkinson, 1970; Heath, 1964; Liu & Red, 1994; Saracho, 1997; Witkin et al, 1977). FI individuals apply external referents in processing and refining information whereas, FD individuals are attentive to and make use of prevailing social frame of reference as a primary source of information (Witkin & Goodenough, 1977) FI individuals define the problem based on their own understanding of the situation

(Saracho, 2003), while FD students are more dependent on clues provided by teacher to define the problem and work with other people to solve the problem. It can be seen that the characteristics of FI and FD individuals are corresponding to the characteristics of analytical proof schemes use and externally based proof schemes use, respectively. Consequently, it was reasonable that FI students used analytical proof schemes significantly more than FD students, and FD students used externally based proof schemes significantly more than FI students.

As indicated in literature and method sections, there is a third cognitive style label; field-mix (FM) refers to people who do not have such a clear orientation as FD or FI, but rather fall in the middle of the continuum of FD and FI (Liu & Reed, 1994). The result that FI students used of analytical proof schemes significantly more than FM students, whilst FM students used analytical proof schemes significantly more than FD students is consistent with characteristics of FM students.

Although in the first research question, significant differences were found in students' use of empirical proof schemes, no significant difference was found in the use of empirical proof schemes among FD, FI and FM cognitive styles. These results might indicate that bipolar nature of FDI cognitive style manifest itself either in analytic or in externally based proof schemes use.

5.2 Conclusions

In the light of results of the study, following conclusions can be stated.

1. There is a significant mean difference among the proof schemes use scores of students in GPT. Students use externally based proof schemes and empirical proof schemes significantly more than analytical proof schemes. However, there is no significant difference between students' use of externally based and empirical proof schemes.
2. Females' mean scores of empirical proof schemes use are significantly higher than males' mean scores of empirical proof schemes use. This means that; females use empirical proof schemes significantly more than the males.

However, there is no statistically significant mean difference between the means of externally based proof schemes and analytical proof schemes use scores of males and females.

3. The mean of FD students' externally based proof schemes use scores is significantly higher than the mean of FI students' externally-based proof schemes use scores. This means that FD students use externally based proof schemes in GPT significantly more than FI students. On the other hand, there is no significant mean difference between field dependent students' and field neutral students' externally based proof schemes use scores. Similarly, there is no significant mean difference between FM students' and FI students' externally based proof schemes use scores.
4. The mean of FI students' analytical proof schemes use scores is significantly higher than the mean FD students' analytical proof schemes use scores. This means that; FI students use analytical proof schemes significantly more than FD.
5. There is no significant mean difference in empirical proof schemes use scores of FD, FI and FM students.
6. There is no significant interaction between gender and cognitive style among the mean scores of three proof schemes use in GPT.

5.3 Educational Implications

The significant differences in students' use of proof schemes with respect to their gender and FDI cognitive style connote that gender and FDI cognitive styles are important individual differences and should be taken into consideration as instructional variables, while teaching and engaging in proof in geometry or in mathematics. Understanding the differences in proof schemes of males and females and FD, FM and FD students is of the same value with understanding students' strengths and weaknesses in proof. By understanding the relationship between students' gender and cognitive styles and their proof schemes, mathematics educators can design an effective instruction in such a way that with appropriate methods and tools, instruction can meet the needs of students with different gender or cognitive styles. For instance, teaching FD students analytical proofs by encouraging them engage in logical deduction that they are tended to use less or not able to use, can lead to eliminate disadvantages due to the cognitive style.

As proposed in literature and once again this study portrayed, extensive use of empirical proof schemes among all students is another important issue that mathematics educators must be concern. Carefully planned instructional interventions can bring students to see a need to refine and alter their current proof schemes into analytic proof schemes. Teachers' emphasis on logical reasoning, even in the absence of explicit treatment in a curriculum, might influence students' use of logic themselves.

5.4 Recommendations for Further Study

It is possible for this study to illuminate further studies by the following recommendations.

- The sample size should be increased
- Students at different grade levels and from different types of school can be included.
- For a deep investigation qualitative methods can be utilized.
- Students' proof schemes use can be investigated considering different domains in geometry or mathematics, such as, algebra, trigonometry, probability, calculus, etc.
- Students' proof schemes use can be investigated for different types of questions.

- An experimental study can be conducted to examine the effect of teaching analytical proof schemes.

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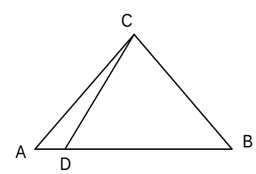
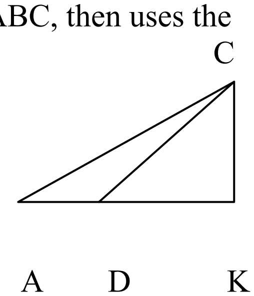
APPENDIX A

THE KEY CHARACTERISTICS OF HAREL and SOWDER'S PROOF SCHEMES

Externally-Based	**Characteristics**
Authoritarian Proof Scheme	-Unable to develop reasons about why a proof is correct -Use teachers or textbooks arguments to determine the correctness of proof -Memorize theorems -Apply formulas
Ritual Proof Scheme	-Provide superficial arguments -Limit the connection between the arguments and proof -Look for similar proof process -Mimic on other proof's process
Symbolic Proof Scheme	-Do not understand the meaning of symbols -Be able to provide arguments, but may not be meaningful -Prove by manipulating mathematical symbols -Write a mathematical statements by using symbols -Use well known symbolic algorithms -Make symbolic manipulation the first and following steps of a proof
Empirical	
Perceptual Prof Scheme	-Connect the hypothesis with proof steps via drawing, but perhaps ignore logical arguments -Determine the correctness of a proof by drawings. -Persuade peers by drawing -Make conclusions based on one or more drawings
Example-Based Proof Scheme	-Lack logical arguments -Determine the correctness of a proof by examples -Convince others by showing examples -Construct a proof by showing examples
Analytical	
Transformational Proof Scheme	-Construct consisted steps -Apply logical rules to the previous statements of a proof -Determine the key issue -Convince others by providing logical reasoning
Axiomatic Proof Scheme	-Set up limited set of undefined terms -Prove using linear methods -Follow traditional proof process (forward method of a proof) -Develop an axiomatic system -Prove how a theorem follows from the axiomatic system

APPENDIX B

THE EXAMPLES FROM STUDENTS PROOF SCHEMES USE

Externally Based	**Specimen for Students' Justification Schemes Use**
Authoritarian Proof Scheme	The line that is drawn into a triangle like $\lvert CD \rvert$ is never longer than the side next to it.
Ritual Proof Scheme	
Symbolic Proof Scheme	
Empirical	
Perceptual Proof Scheme	It looks like that we have shorten the one side of an isosceles triangle and lessen the one angle. 
Example-Based Proof Scheme	
Analytical	
Transformational Proof Scheme	Student draws a perpendicular line CK from C to side AB and forms a right triangle into ABC, then uses the Pythagorean theorem.  $\lvert DK\rvert^2+\lvert CK\rvert^2=\lvert DC\rvert^2$ $\lvert AK\rvert^2+\lvert CK\rvert^2=\lvert AC\rvert^2$ Since $\lvert AK\rvert > \lvert DK\rvert$, then $\lvert AC\rvert > \lvert DC\rvert$
Axiomatic Proof Scheme	

APPENDIX C

GEOMETRİ TESTİ

1. Hangi iki açısı seçilirse seçilsin, bu açıların ölçüsü toplamı 120 dereceden daha küçük olan bir üçgen çizmek mümkün müdür? Cevabınızı matematiksel olarak açıklayınız.

2. Aşağıda bazı elemanları verilmiş olan üçgenlerin, cetvel ve pergel yardımıyla çizilip çizilemeyeceğini belirtiniz. Çizilip çizilememe nedenini matematiksel olarak açıklayınız

   a) Birbirine eşit iki kenar uzunluğu ve bu kenarlar arasındaki açının ölçüsü bilinen üçgen

   b) Üç kenarı birbirinden farklı, bir kenarının uzunluğu ve bu kenara komşu olan iki açısının ölçüsü bilinen üçgen

   c) İki açısının ölçüsü ve hipotenüsün uzunluğu bilinen dik üçgen

   d) Üç açısının ölçüsü bilinen üçgen

3. Aşağıda verilen ifadelerin doğru yada yanlış olduklarını ispatlayınız.

a) Taban uzunlukları ve bu tabana ait yüksekliklerinin uzunlukları eşit iki üçgen eştir.

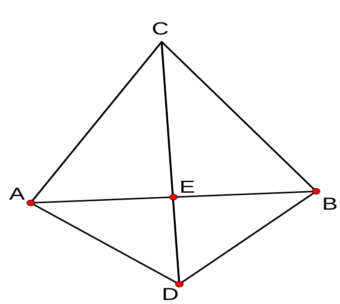
b) Bir üçgenin yüksekliği ait olduğu kenarı ikiye böler.

c) Bir üçgende bir kenara ait kenarortayın uzunluğu herzaman aynı kenara ait yüksekliğin uzunluğundan daha daha fazladır.

d) Bir üçgende bir kenarı ikiye bölen doğru herzaman karşı köşeden geçer.

4. Aşşağıdaki şekilde $|AC| = |BC|$ ve $|AD| = |BD|$ ise

AB $\perp$ CD olduğunu ispatlayınız.



5. Bir açısı 30 derece olan bir dik üçgende, bu açının karşısındaki kenarın hipotenüsün uzunluğunun yarısı olduğunu ispatlayınız.

6. Aşağıda, eşkenar üçgene ait özellikler sıralanmıştır.Bu özellikleri kullanarak kendi eşkenar üçgen tanımınızı yazınız ,daha sonra, tanımınızdan faydalanarak kullanmadığınız özelliklerin doğruluğunu ispatlayınız.

I. İç açılarının toplamı 180 derece
II. Birbirine eşit üç açısı var
III. Birbirine eşit üç kenarı var
IV. Birbirine eşit üç açısı ve üç kenarı var
V. Her üç kenara ait yükseklikler bu kenarları ikiye böler
VI. Açıortay, kenarortay ve yükseklikleri eş.

7. Aşağıda, dik üçgene ait özellikler sıralanmıştır. Bu özellikleri kullanarak kendi dik üçgen tanımınızı yazınız ,daha sonra, tanımınızdan faydalanarak kullanmadığınız özelliklerin doğruluğunu ispatlayınız.

I. İç açılarının toplamının 180 derece
II. Bir açısı 90 derece
III. İki dar açısı var
IV. İki açısının toplamının 90 derece
V. Yüksekliklerden birisi aynı zamanda kenar olan
VI. En uzun kenara ait kenarortayın uzunluğu, bu kenarın uzunluğunun yarısı kadar

8. Aşağıda, ikizkenar üçgene ait özellikler sıralanmıştır. Bu özellikleri kullanarak kendi ikizkenar üçgen tanımınızı yazınız,daha sonra, tanımınızdan faydalanarak kullanmadığınız özelliklerin doğruluğunu ispatlayınız.

I. İç açılarının toplamının 180 derece

II. İki açısının ölçüsü eşit

III. İki kenar uzunluğu ve iki açısının ölçüsü eşit

IV. En az iki kenarının uzunluğu eşit

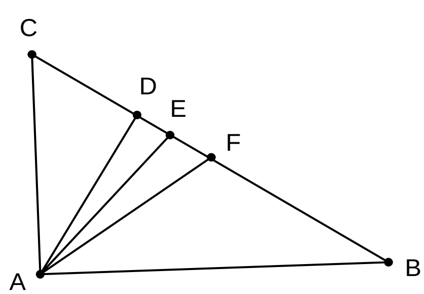
V. Üç kenarından ikisinin uzunluğu eşit

VI. İki kenarortayının uzunluğu eşit

VII. Bir açıortayı aynı zamanda kenarortay ve yükseklik olan

VIII. Eş kenarlara ait kenarortay ve yükseklikleri eş.

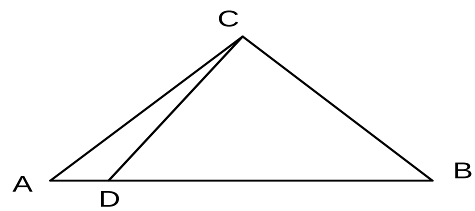
9. Ayşe ile Deniz "Tek Tahminle Üçgenin Yardımcı Elemanlarını Bul" oyunu oynuyorlar. Ayşe, aşşağıda verilen ABC dik üçgeninde CB hipotenüsüne ait açıortay, kenarortay ve yüksekliği Aslı' nın görmeyeceği şekilde çiziyor. $|AB| > |AC|$ ve $|FC| > |FB|$ olduğunu söylüyor. Deniz'in tek seferde AD, DE ve AF doğru parçalarından hangisinin açıortay, kenarortay ve yükseklik olduğunu bilmesinin mümkün olup olmadığını matematiksel olarak açıklayınız.



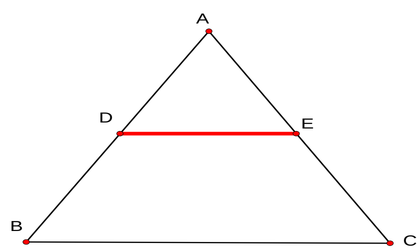
10. Yandaki ABC üçgeninde

$|AB| = |AC|$ ise $|AC| > |DC|$

olduğunu ispatlayınız.

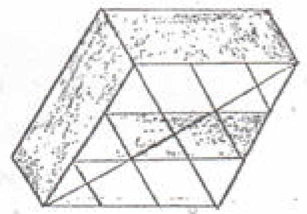


11. Herhangi bir ABC üçgeninde AB ve AC kenarlarının orta noktaları olan D ve E noktalarını birleştiren DE doğru parçasının üçüncü kenar BC ye parallel ve bu kenarın yarısı uzunluğunda oldugunu ispatlayınız.

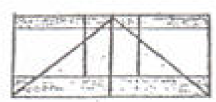


APPENDIX D

SAMPLE ITEMS FROM GEFT



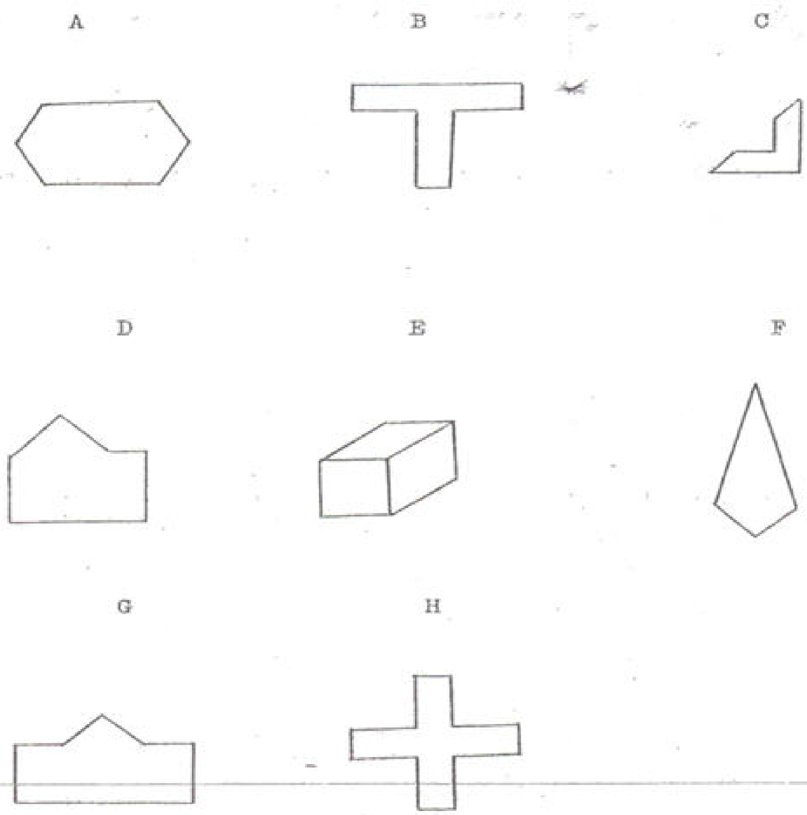
Basit şekil "A" yı bulunuz.



Basit şekil "C" yi bulunuz.

Basit şekil "A" yı bulunuz.

BASİT ŞEKİLLER



APPENDIX E

RESULTS OF BONFERRONI POST-HOC ANALYSIS FOR COGNITIVE STYLES

Dependent Variable	Cog Style (I)	Cog Style(J)	Mean Diff.(I-J)	Std.Error	Sig.
ExternallyB.	FD	F	.39	.29	.557
		FI	1.12*	.34	.004
	FM	FD	-.39	.29	.557
		FI	.74	.29	.042
	FI	FD	-1.12*	.34	.004
		FM	-.74	.29	.042
Empirical	FD	FM	.13	.25	1.00
		FI	-.17	.29	1.00
	FM	FD	-.13	.25	1.00
		FI	-.30	.25	.745
	FI	FD	.17	.29	1.00
		FM	.30	.25	.745
Analytical	FD	FM	-.60*	.22	.024
		FI	-1.03*	.26	.000
	FM	FD	.60*	.22	.024
		FI	-.44	.22	.168
	FI	FD	1.03*	.26	.000
		FM	.44	.22	.168