

MULTI-ITEM
INVENTORY-ROUTING PROBLEM
FOR AN FMCG COMPANY

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FOR AN FMCG COMPANY**

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ABSTRACT

MULTI-ITEM INVENTORY-ROUTING PROBLEM FOR AN FMCG COMPANY

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In this study, inventory–routing system of a company operating in Fast Moving Consumer Goods (FMCG) industry is analyzed. The company has decided to redesign distribution system by locating regional warehouses between production plants and customers. The warehouses in the system are all allowed to hold stock without any capacity restriction. The customers are replenished by the warehouse to which they have been assigned. Customer stocks are continuously monitored by the warehouse and deliveries are to be scheduled. In this multi–item, two-echelon inventory–distribution system, main problem is synchronizing inventory and distribution decisions. An integrated Mixed Integer Programming optimization model for inventory and distribution planning is proposed with the aim of optimally coordinating inventory management and vehicle routing. The model determines the replenishment periods of items and amount of delivery to each customer; and constructs the delivery routes with the objective of cost minimization. The integrated model is coded in GAMS and solved by CPLEX. The integrated inventory-routing model is simulated with retrospective data of the company. Computational results on test problems are provided to show the effectiveness of the model developed in terms

of the performance measures defined. Moreover, the feasible solution obtained for a period is compared to the realized inventory levels and distribution schedules. Computational results seem to indicate a substantial advantage of the integrated inventory-routing system over the existing distribution system.

Keywords: Inventory-Distribution System, Inventory-Routing Problem (IRP)

ÖZ

HIZLI HAREKET EDEN TÜKETİM MALLARI ŞİRKETİ İÇİN ÇOK ÜRÜNLÜ ENVANTER ROTALAMA PROBLEMİ

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Bu çalışmada, hızlı hareket eden tüketim malları sektöründe faaliyet gösteren bir şirketin envanter-rotalama sistemi analiz edilmiştir. Şirket üretim tesisleri ile müşteriler arasına bölgesel depolar yerleştirerek dağıtım sistemini yeniden tasarlama kararını almıştır. Sistemdeki tüm depoların herhangi bir kapasite kısıtı olmadan stok taşımalarına izin verilmiştir. Müşteriler atanmış oldukları depolar tarafından tedarik edilmektedir. Müşteri stokları depo tarafından sürekli izlenmekte ve sevkiyatlar planlanmaktadır. Bu çok ürünlü, iki katmanlı envanter-dağıtım sisteminde ana sorun envanter ve dağıtım kararlarını senkronize edebilmektir. Envanter yönetimini ve araç rotalamayı en uygun olarak koordine etmek amacıyla bir MIP optimizasyon modeli önerilmiştir. Bu model ürünlerin sevkiyat aralıklarını ve her müşteriye yapılacak sevkiyat miktarını belirlemekte ve maliyet en küçükleme amacıyla sevkiyat rotalarını oluşturmaktadır. Entegre model GAMS'te kodlanmış ve CPLEX ile çözülmüştür. Entegre envanter-rotalama modeli şirketin gerçek verileri ile test edilmiştir. Geliştirilen modelin performans ölçütleri açısından etkililiğini gösterebilmek için test problemlerinden elde edilen sonuçlar sergilenmiştir. Buna ilaveten, bir zaman aralığı

için elde edilen mümkün çözüm, gerçekleşmiş envanter seviyeleri ve dağıtım planları ile karşılaştırılmıştır. Hesaplanmış sonuçların, entegre envanter rotalama probleminin mevcut dağıtım sistemine nazaran gözle görülür bir avantaja işaret ettiği görülmektedir.

Anahtar Kelimeler: Envanter-Dağıtım Sistemi, Envanter-Rotalama Problemi (IRP)

To my parents and my sister Seza

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CHAPTER 1

INTRODUCTION

In this thesis, the inventory-routing problem for a fast moving consumer goods (FMCG) company is studied. The company produces biscuits, cakes, crackers, chocolates and bars. Products are sold at 200,000 sales points, dispersed all around Turkey. The supply chain of the company consists of 182 suppliers, 5 manufacturing plants, 8 regional directorates, 148 distributors, approximately 182,000 retailers and 60 chain markets having 1200 sales points. The distributors and chain markets are immediate customers of the company. Effective management of such a huge supply chain is a key success factor for the company to strengthen its position in the industry and increase its market share.

In the beginning of the year 2004, the company decided to redesign its distribution network in order to reduce high logistics costs and increase the service level to its customers. The distribution network design studies resulted with six regional warehouses, located between the production plants and customers of the company. It is expected that the new network design will reduce the logistics costs to a considerable amount, as a result of the consolidation of demand at the regional warehouses and making palletized high volume shipments (with high capacity trucks) possible. This is because in the new distribution network the company now replenishes the inventories of regional warehouses, instead of distributors or chain markets. Note that we refer to this new system as the proposed system hereafter in the study.

Distribution network design is a strategic level decision and affects the tactical and operational level decisions and systems as well, which should be redesigned on the backbone determined by the network. One such system design problem in our context is to locate regional warehouses between the customers and the plants of the company. This design transforms the single-echelon nature of the distribution network to a two-echelon, in terms of finished-product flow between the company and its immediate customers. Therefore, there is a need to adapt the inventory control system to the new two-echelon structure and redesign the distribution system considering the replenishment of the stocks at the regional warehouses.

The system design problem stated above is considered by Bulur (2005) for the same company as a thesis study. In his study, an inventory-distribution system is designed for the management of the above-mentioned two-echelon structure of the distribution network. The proposed system includes a virtual central warehouse and six regional warehouses. Bulur (2005) investigates the inventory policies to be used in the regional warehouses. However, the decision problems regarding the downstream part of the supply chain was out of the scope of his study. Opening six regional warehouses changes the structure of the distribution between the company and its customers, especially distributors. In the new system, the distributors are replenished from the regional warehouses, not from the plant warehouses.

Another system design problem is to set inventory replenishment policies between the regional warehouses and the customers assigned to them. In such a system design problem, one should focus on the systemwide cost of the distribution system. Because minimizing individual customers' costs seems reasonable for the customers but it does not guarantee the global optimality for the system. Therefore, it should be assumed that the members of the supply chain are managed by a centralized control. Some questions arise as a result of this arrangement, such as

- When to serve a customer?
- How much to deliver a customer when it is served?
- Which delivery routes to use?

In this thesis, we consider inventory replenishment problem of the company from regional warehouses to customer depots together with the transportation costs between them. The decision problems that include inventory and transportation costs and the tradeoff between them are called inventory-routing problems in the literature. In this study, the inventory-routing problem that arises in the two-echelon network design is considered. It is assumed throughout the study that the system with its inventory decisions and shipment methods are managed by the company in order to achieve a systemwide optimization.

The described system resembles a popular approach called ‘vendor managed inventory’ in the literature. The vendor manages the inventory levels of its customers by considering on-hand inventory at their depots and the geographical locations of them. The aim is to minimize the systemwide cost by simultaneously considering inventory and transportation components. The vendor is analogous to the company and the customer is analogous to the distributors in our case.

In the transportation side, there exist different approaches to be utilized. One of them is the direct delivery approach. In the direct delivery case, each customer has a dedicated truck. An amount ordered by a customer is directly delivered by its dedicated truck and then the truck comes back to the regional warehouse. Another approach is to deliver the orders placed by different customers by the same truck called milk run. Meanwhile the truck carries different customers’ loads and delivers them in a sequence. This approach requires determining a route for each truck. In this case, the problem is composed of both inventory decisions and routing decisions.

The proposed system is simulated with the retrospective data of the company in order to see its effects on the system and compare the results with the actual results of the current system, in terms of the performance measures defined for the inventory-routing system.

The thesis consists of five chapters, which are organized as follows:

In Chapter 2, the problem definition and the scope of the study are given. The existing inventory-distribution system and the related problems are stated. The proposed system is explained; the problem environment and the scope are defined. Chapter 3 includes a review of the literature on inventory-routing problems and vendor managed inventory systems. Chapter 4 the inventory routing system is explained. System design motivations are explained briefly and then integrated inventory-routing model is introduced. Model parameters, variables and formulations are discussed in detail. Different extensions of the integrated inventory-routing model are also discussed in this chapter. Finally, in Chapter 5 inventory routing system is evaluated using retrospective data. Performance measures utilized in the evaluation are explained first. Then retrospective data are analyzed to find out the current system performance. Then the results of the inventory routing model are compared with the current system in terms of transportation and inventory performances.

CHAPTER 2

PROBLEM DEFINITION AND SCOPE OF THE STUDY

In this chapter, the current inventory-distribution system of the company and the problems inherent in the system are explained. Then, a logistics network structure for the company is proposed and its expected benefits for the company are stated. Finally, the scope of the study is discussed in detail.

2.1. Current System and the Problems

The current inventory-distribution system from the plants to the distributors and chain markets can be described as a multi-item, single echelon system. The customers of the company are distributors and chain markets that are directly served from the plants. Inventory that belongs to company is held at the plant warehouses. There are also small sized depots at the sites of the seven of eight regional directorates of the company, which are geographically dispersed in Turkey. However, these depots are not used as distribution centers. They are used only to fulfill urgent demands of the distributors, which are very low in amount, and the demands of the chain markets that are usually less than a full-truckload.

Customers place their orders daily to the regional directorates of the company. Regional directorates transfer these orders to the 'shipment department' after the necessary adjustments are made. Orders are processed; routes and truck loads are formed by the shipment department.

Shipment from the plant warehouses to the customers is made by a third-party carrier company that operates a dedicated fleet of trucks for the company, each

having a net loading capacity of 36 m³. Payments to the carrier company are made on the basis of total distance traveled (kms), regardless of the amount of load carried. Because of this payment scheme, shipments are made in full truckload. Distance is calculated as the total length of the route from the warehouses to the customer and from the customer back to the warehouses. This means that the company pays for the empty trucks coming back from the customers. The kind of trucks used is an expensive mode of shipment, in terms of the loading capacity and fuel consumption. Cost per unit volume carried per unit distance traveled is higher than using higher capacity trucks.

Plants are dedicated to different product families. Each plant holds inventory of its finished-goods only in its own warehouse that is located nearby. On the other hand, distributors and chain markets hold inventories in their own warehouses. They are free in determining their inventory levels and ordering decisions. Transportation cost related to the transfer between the plants' warehouses and the customers' depots belongs to the company. The company utilizes 36 m³ (6 tons) trucks for the transportation. Since there is no transportation cost incurred by the customers, they place the orders in an irregular manner. The main driving force in the ordering decision is to gain incentives related to the ordering quotas. This practice results in high fluctuations in distributors' orders and very high inventories in their warehouses according to their sales.

The schematic representation of the current system can be seen in Figure 2.1.

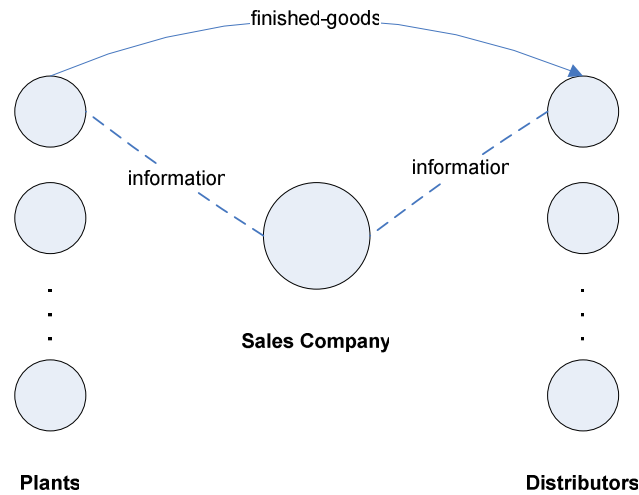


Figure 2.1. Schematic representation of the current system

The main problem in the current inventory-distribution system of the company arises from the forecasting and incentive systems being used. Forecasts, which are determined at the end of a year by the top managers of the company, are based on the total sales target for the next year. Then this total amount is divided into months and distributed among the sales regions according to the sales potentials of the regions. These targets are not in terms of stock keeping units (SKU), but in terms of the total weight. Monthly total sales targets for the following three months are sent to the regional directorates every month. Regional directorates partition the total monthly amounts among the distributors and chain markets. Finally, distributors and chain market managers split these total amounts into 30 product groups that constitute all the SKUs of the company according to past sales data. These sales targets of the product groups are consolidated at the regional directorates and finalized after necessary adjustments. Those approved amounts become the monthly sales targets of the distributors. The distributors are then asked to split the product group targets among the SKUs of each group as forecasts. Those SKU-based forecasts are consolidated at the eight regional directorates and sent to the marketing department. Marketing department aggregates the monthly forecasts that come from the sales regions and splits them

into weeks using some multipliers determined according to the customer order patterns in a month. While splitting these amounts, promotional and advertisement activities are also considered, hence some small amounts are added to the calculated forecasts. This means that the total of weekly forecasts for a month for an SKU is always greater than the total sales target of that SKU in that month.

Apparently, the system operates as a “push” (forecast-driven) system. The sales company places the orders of rolling 12 weeks on every Monday. According to these orders, the production company plans the next week’s production. The responsibility of production department is to make the production according to weekly plan and make the products ready to be shipped for the following Monday. There are mainly three problems here. Firstly, periods of the planning horizon do not coincide with periods of the forecast horizon. Production plans are made weekly, however the sales targets are monthly. Monthly production is almost uniformly distributed among the weeks of a month, but the majority of the orders from the distributors is placed during the last two weeks (mainly the last week) of the month. Secondly, monthly total of the production orders is always greater than the monthly sales targets because of rationing game among customers in case of scarce inventory at the plant warehouse. Customers intend to place orders higher than the required amount especially for fast moving products. Thirdly, although the production orders are fulfilled in SKU basis, distributors get incentives when they realize their product group sales targets. Group-target realization based incentive system causes imbalances in terms of sales among the SKUs that constitute the product group.

At first sight, high inventories seem to be as a problem only for the distributor not for the company because inventory-carrying cost is incurred by the distributor. However, high inventory levels have impacts on different aspects of the system. Since the company is a fast moving consumer goods manufacturer, the products should be presented to the markets as soon as possible. The product life-cycle is short, ranging from 9 to 12 months. In the current system, there are three different stockpiles of the inventory throughout the supply chain. The production plant is

the first stage that the product is stocked. The second is the customer of the company, namely the distributors. The last stage is the sales points that the product meets the end-consumer. At the first stage, the waiting time is short because the production is done to make the product available at the beginning of the next week. Therefore, inventory turnover is one week, or two weeks at most in case of late withdrawals at this stage. Likewise, the waiting time is short at the last stage, because of the fast moving characteristic of the product. However, the waiting time at the second stage, in the distributors' warehouses, is longer compared with the other stages. Long waiting times make the products perish in the inventory. The unsuitable stocking conditions fasten this process. The perished products are sent back to the company as 'returns'. The distributors do not charge any cost for the returns. Returns are changed with the new products at the same price. Furthermore, there is no limit on returned volumes. This is another factor affecting the distributors ordering behaviors.

2.2. The Proposed Logistics System

By the year 2004, with the changes that occurred in the market and in the economy, the company decided to develop strategic plans for the years ahead, when the rules of the game are expected to be much more different. It was realized that to be successful in the competition cost minimization, service level maximization and product and process differentiation would become more important. Especially service level improvement in every echelon of the supply chain in the FMCG industry is more crucial than in any other industry. Production costs could no longer be reduced, as every effort to reduce production costs has already been made by all of the companies in the industry. Logistics, then, seems to be the most promising area for cost reduction and process improvement. What's more, the customers become more demanding day by day. They want to carry less inventory than before, as holding inventory is no longer profitable as it used to be; instead the customers prefer to be closer to the production plants so that their requirements can be met within a shorter lead-time. As a result of all these, the company has started a project in the field of logistics.

Bulur and Çölova (2004) have shown that when the logistics network is redesigned by the opening of six regional warehouses (distribution centers) between the company and its customers, logistics costs can be reduced by a considerable amount, while the response time to the customers is shortened. They have extended the location-allocation model developed by Güler et.al. (2004) and found out that logistics costs can be significantly reduced when six regional warehouses are opened.

Six regional warehouse locations and the provinces assigned to them are colored on a map. The map representation of the service areas of the regional warehouses can be seen in Figure 2.2. Each color on the map represents the service area of each individual regional warehouse.

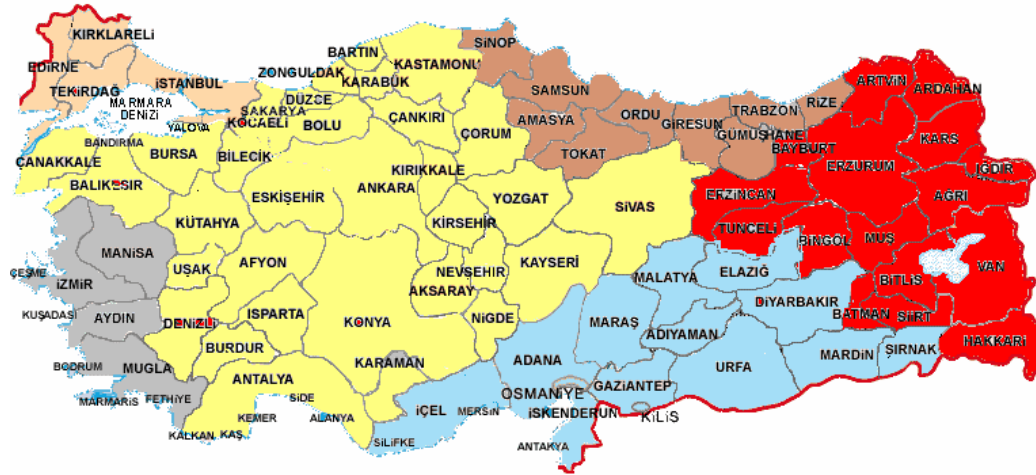


Figure 2.2. Service regions of the warehouses

In the Bulur and Çölova model, direct shipments from the plant warehouses to the regional warehouses are made by high-capacity trucks with a palletized-load. On the other hand, outbound shipment from the regional warehouses to the customers is made by small trucks and no palletizing is used. Cost reduction in the model relative to current practice comes about mainly from using high-capacity trucks

that carry full truckloads between the plants and the regional warehouses, enabling the shipment of the products over a considerable length of the route by a cheaper transportation mode. Although warehouse-operating costs increase with the addition of six regional warehouses, the reduction in the transportation costs outweighs the increase in the warehouse operating costs.

The proposed logistics network enables the company to become closer to its customers and to deliver their orders with shorter lead-times. The new network design is expected to increase customer satisfaction and to decrease their stock levels.

The schematic representation of the proposed system can be seen in Figure 2.3.

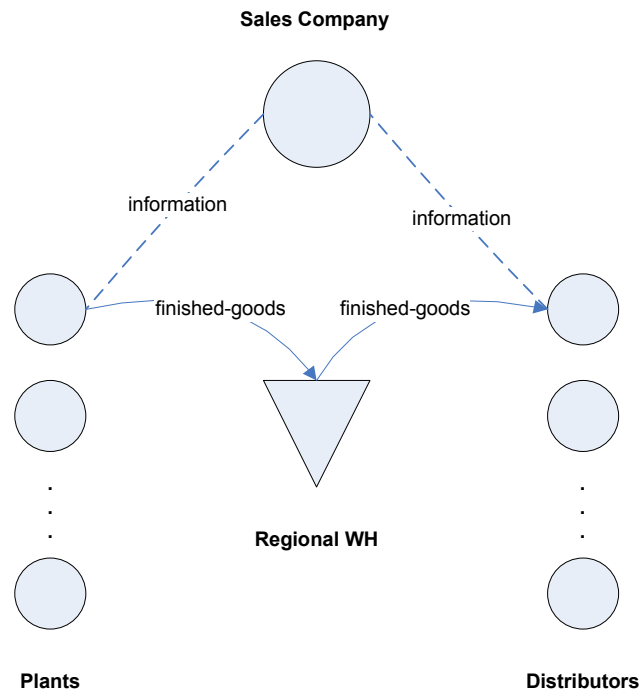


Figure 2.3. Schematic representation of the proposed system by Bulur and Çölova

In the proposed logistic network design, how to replenish customers' stock is one of the main problems. Moreover, the replenishment problem has to be analyzed together with the modes of transportation between the regional warehouse and the customers. Possible questions in the proposed design are:

When to replenish customers stock?

Which customers to be replenished on a given day?

How to deliver customers, in which order?

2.3. Scope of the Study

Not the chain-market channel, only the distributor channel is considered in this study. In the current sales mix, the portion of distributor channel constitutes 80% of total sales.

Prior to this study, the decisions already made in the context of the logistics project in the company are the followings:

- the locations of the regional warehouses
- the inventory policy to be used in the new two-echelon logistics network (from plants to the regional warehouses)
- the method to compute the inventory control parameters for the regional warehouses
- the type of vehicles to be used for the shipments from the plant warehouses to the regional warehouses
- the number of trucks needed and the method for the replenishment of the inventories at the regional warehouses
- the assignment of the customers to the regional warehouses
- the type of vehicle to be used in customer order shipments from the regional warehouses to the customers

The remaining decision problems to be solved before implementing the proposed system are the followings:

- the inventory policy to be used at the customers' side in the new logistics network
- the methods of inventory replenishments at the customers' warehouses
- the number of trucks needed for the proposed inventory routing system

This work aims to complete these missing parts in the proposed logistics network and to verify the expected benefits in terms of inventory holding cost and transportation cost.

The inventory-distribution system studied has the following characteristics:

- multi-item system with almost 200 SKUs produced and distributed
- each regional warehouse acting like a plant warehouse for all SKUs
- There are six regional warehouses and 180 distributors each assigned to one regional warehouse
- demand at the customers consisting of the daily orders received from the sales points in their service area

CHAPTER 3

LITERATURE SURVEY

Supply chain management has gained an ever growing importance. Over the past years, design and management strategies of supply chains are widely studied in the literature. The inventory decisions of upstream and downstream members of the supply chain and the transportation techniques among them constitute the focus of these studies. The main objective of the studies is to minimize systemwide costs and increase efficiency of the operations.

Early studies on the issues of today's supply chains started in the 1950's following the improvements in the linear programming formulations. Dantzig and Ramser (1959) studied the truck-dispatching problem as a generalization of traveling salesman problem. There is a fleet of gasoline delivery trucks operating between a bulk terminal and a large number of service stations. The objective is to satisfy the demand of the stations while minimizing the total distance traveled by the fleet. A linear programming formulation is given to obtain optimum routing solution to the problem.

Clarke and Wright (1964) extended the problem by adding variable truck capacities. The fleet of varying capacity trucks travels from a central depot to a number of delivery points. The study develops an iterative procedure for rapid selection of an optimum or near-optimum solution among possible routes.

The majority of such earlier works focus on designing minimum path network algorithms. This literature explicitly considers the costs related to travel distance.

However, it typically neglects inventory costs related to shipment size. On the other hand, substantial literature is also available on inventory control especially on variants of the “economic order quantity” (EOQ). EOQ models focus on trading off fixed reorder costs against inventory costs that increase with order size. Thus, this literature explicitly models inventory costs related to shipment size. However, it typically represents transportation costs simply as part of a constant that measures reorder costs, and generally neglects the dependence of transportation costs on distribution network design.

The interrelationship between the inventory decisions and transportation planning has been addressed in further studies. Federgruen and Zipkin (1984) studied a combined problem of allocating scarce resource available at some central depot among several stochastic demand locations. This study is an extension of the classical vehicle routing problem and known as the first optimization model that combines inventory and routing costs. Such models that integrate vehicle routing with inventory control problems are called inventory-routing models. In their study a set of vehicle routes were designed to minimize total cost while satisfying capacity constraints and meeting customer requirements. The vehicles leave a depot, deliver customers’ demand and return to the depot. The study suggests a nonlinear integer programming formulation for the combined problem of inventory and routing. Their approach decomposes the main problem into one nonlinear inventory allocation subproblem and a number of traveling salesman subproblems. A similar problem but without restricting on the number of customers that can be served on a given vehicle route was studied by Chien et al. (1989).

Another class of inventory-routing problem is the infinite-horizon models with deterministic demand. The product is pulled by each retailer at a given constant rate and the problem is to determine an infinite-horizon transportation policy that minimizes the sum of inventory and vehicle routing costs. Anily and Federgruen (1990) studied such a problem by considering a distribution system with a depot and many geographically dispersed retailers. Retailers face external constant

deterministic demand. The stock is distributed from the depot by a fleet of capacitated vehicles through efficient routes. Inventories are kept at the retailers but not at the depot itself. The objective of the study is to determine feasible replenishment strategies minimizing long-run average transportation and inventory costs.

In a further research Anily and Federgruen (1993) extended their earlier analysis to the case where central inventories may be kept at the warehouse. The former problem becomes a two-echelon inventory decision problem combined with vehicle routing strategies. Viswanathan and Mathur (1997) studied a multi product generalization of the distribution system considered by Anily and Federgruen (1990) which restricts the number of products to one. They presented a new heuristic that generates a stationary nested joint replenishment policy (SNJRP) for the problem.

Bertazzi et al. (2002) studied a multi period model with deterministic demand in which a set of products is shipped from a common supplier to several retailers. Each retailer determines a minimum and a maximum level of inventory of each product. Each retailer must be visited by a vehicle of given capacity before its inventory reaches the minimum level. The products are delivered by the supplier in such quantities that the maximum level of inventory is reached at the retailer. The problem is to determine the retailers to be visited and the route of the vehicle for each discrete time instant. A heuristic algorithm is presented in the study and the solutions obtained with different objective functions are compared. To have a better understanding of the model and the proposed heuristic, a case of a single product and a single vehicle is investigated. The study can be extended to the case of multiple products and vehicles.

Besides these studies on inventory routing, in late 1980's a new trend based on this problem in logistics has emerged; vendor managed resupply or vendor managed inventory. It refers to a situation in which a supplier manages the inventory replenishment of its customers. The approach offers a framework for

synchronizing inventory and transportation decisions. It is popularized by the implementations of Wal-Mart and Procter & Gamble in the grocery industry. Successful VMI systems are then implemented by other companies, including Campbell Soup and Johnson & Johnson, and Barilla.

Campbell Soup's Continuous Replenishment Program is known as one of the first implementations of vendor-managed inventory (VMI) in industry. The program was designed to improve the efficiency of inventory management throughout the supply chain. The supplier (Campbell Soup) assumed responsibility for managing retailer inventories. It ships products from several plant warehouses to retailer distribution centers (DCs), from which the stores operated by the retailer are resupplied. Cachon and Fisher (1997) developed a forecasting and inventory management system for the company. The study proposes simple inventory management rules to operate the program and it tests these rules with a simulation using actual demand data provided by the company.

Following the successful implementations of the new VMI approach, several studies have been carried out in this area. Waller et al. (1999) provided insight into the approach. Their study describes VMI benefits and explains why these benefits have resulted from VMI. The study also describes some underlying technologies required to make the system work. A simulation model that examines VMI quantitatively is introduced in order to understand the effect of key variables.

The inventory-routing problem is closely related with the vendor managed inventory systems. This relationship was stated in a study by Campbell et al. (1998). The study discusses the inventory routing problem. The problem captures the basic characteristics of the vendor managed resupply situation. In the VMI context, inventory-routing problem differs from traditional vehicle routing problems because it is based on customers' usage rather than customers' orders. They consider the repeated distribution of a single product, from a single facility, to a set of n customers having deterministic demands and local inventory

capacities. A fleet of m homogenous vehicles, each with capacity Q , is available for the distribution of the product. The objective is to minimize the average distribution costs without causing stockouts at any of the customers.

Çetinkaya and Lee (2000) presented an analytical model for coordinating inventory and transportation in VMI systems. They consider a vendor realizing a sequence of random demands from a group of retailers located in a given geographical region. the vendor uses a certain kind of (s, S) policy for replenishing its inventory, and a time-based, shipment-consolidation policy for delivering customer demands. Their study computes the optimum replenishment quantity and dispatch frequency for the case of random demands by developing a renewal-theoretic model.

The importance of inventory routing problem (IRP) in vendor managed inventory replenishment systems has been discussed by many researchers in recent years. In their study, Kleywegt et al. (2002) have stated the IRP as one of the core problems that has to be solved when implementing VMI replenishment systems. They shortly discussed the potential benefits and requirements to obtain the benefits of the VMI programs. Their study is on the problem of determining optimal policies for the distribution of single product from a single supplier to multiple customers. For this purpose, a special case of the IRP in which only one customer is visited on each vehicle route is considered. This special case of the IRP is called the IRP with Direct Deliveries (IRPDD). The study formulates the inventory routing problem with direct deliveries as a Markov decision process, and proposes methods to find good solutions. In another study, Kleywegt et al. (2004) extended both their former formulation and approach to handle multiple deliveries per trip.

Motivated by the need to solve the inventory routing problem in VMI replenishment Campbell and Savelsbergh (2004a) studied the delivery volume optimization for a given sequence of customer visits. They developed a linear time algorithm for determining a delivery schedule for a route to maximize total delivery volume.

Campbell and Savelsbergh (2004b) studied a solution approach for the inventory-routing problem. They develop a two-phase approach based on decomposing the set of decisions. Phase I (planning) considers a coarse approximation of the problem, with daily decisions, over a k-day planning horizon, and a delivery schedule is created. Based on the results of Phase I, Phase II (scheduling) considers a model with decision accuracy in terms of minutes rather than days for the first j days, and constructs a set of delivery routes. The first phase utilizes integer programming, whereas the second phase employs routing and scheduling heuristics.

Gaur and Fisher (2004) describe the development and implementation of a system to solve a periodic inventory routing problem at Albert Heijn, a supermarket chain in the Netherlands. A vehicle routing and delivery-scheduling problem occurs once every three to six months. Given hourly demand forecasts for each store, travel times and distances, cost parameters, and various transportation constraints, the firm seeks to determine a weekly delivery schedule specifying the times when each store should be replenished from a central distribution center, and to determine the vehicle routes that service these requirements at minimum cost. The system has three modules. The inventory routing module jointly determines the delivery times of stores and vehicle routes to service them, ignoring the variability of demand and fleet-size constraints. The objective of the module is to achieve minimum cost configuration of inventories and routes. This problem is called periodic inventory routing problem (IRP) throughout their study. Using the periodic IRP as a subproblem, the truck assignment module incorporates constraints on truck fleet-size and various time restrictions such that the fixed and variable costs of transportation are minimized. Finally, the workload-balancing module readjusts the departure times of routes within feasibility constraints and given time windows to match the target workload profile of the DC as closely as possible.

CHAPTER 4

INVENTORY-ROUTING SYSTEM DESIGN

In this chapter, the approach developed for the management of inventory-routing system for the proposed logistics network is discussed. In literature, two cases of inventory replenishment problem are widely discussed. In both cases, inventory levels of customers are continuously monitored. By simultaneously considering inventory holding costs and transportation costs, one decides when and how much to deliver to the customers. The two cases differ in the modes of transportation. One of them is the direct-delivery case. In this case, the customers are directly replenished from a central stock facility. A vehicle travels to a customer and returns back to the depot after delivery. On the other hand, in routed-delivery case, a vehicle leaves the depot, follows a routing sequence in delivering customers and returns to the depot. Such a delivery method is called “milk-run”. In this manner, our study is of routed-delivery case. The underlying assumptions in the design and the mathematical models representing the system are presented explicitly throughout the chapter.

The proposed system by Bulur and Çölova (2004) is a typical vendor managed inventory system with regional warehouses. In the context of the problem, the vendor is the production company. The company is assumed to locate six regional warehouses, and keep inventories in these warehouses. The customers are assigned to these six warehouses and their inventories are managed by the company. Prior to our study, the locations of the regional warehouses and the customers assigned to each warehouse are determined by the company. The

remaining design problem is how to manage customer inventories in the proposed system.

The design for the management of the distribution system of the company is based on the routed-delivery case of the inventory replenishment problems. In the routed-delivery case, each distributor is delivered from the regional warehouse to which it was assigned. The inventory of a distributor is continuously monitored by electronic data interchange (EDI). A web-based distributor information system is designed and implemented, in order to improve information sharing between the company and its distributors. According to their inventory positions, the company decides the day and the sequence that the distributors are visited. Routes are formed such that they start from a regional warehouse, pass through the distributors that should be delivered and terminate at the regional warehouse. The company simultaneously decides the amounts of delivery of each product to each distributor and determines the routes in order to minimize the total cost, which consists of inventory holding and transportation costs. Each distributor is assigned to one regional warehouse and cannot be served by another warehouse.

4.1. Integrated Inventory-Routing Model

Inventory management models generally have objectives of minimizing total costs (inventory holding and fixed ordering costs) while achieving a desired customer service level. When minimizing the costs, service level is taken into account by incurring a penalty cost for the shortages, but it is not the primary objective of the system. However, service level is very important for the FMCG industry, in which the company operates. The main objective is to satisfy all demands of distributors on time. Because of the presence of many substitutable SKUs and brands in the market, lack of brand loyalty and high percentage of impulse buys, penetration (availability at the retailer) of a product becomes the ultimate goal in the retail industry. High penetration rates can be achieved by effective distribution of products by distributors. The availability of products in distributor depots is a prerequisite for an effective distribution. On the other hand, routing models

generally have objectives of minimizing total costs having different components (fixed operating cost of a vehicle, fixed and variable shipping costs along arcs), or minimizing total distance traveled on the routes. Combining these two different objectives in a single model is not easy. The inventory routing model in this study is an attempt to combine these two conflicting decision problems into a single model and provide an integrated solution approach to the company's inventory and distribution management problem.

The logistic system considered in this study consists of a manufacturing unit located in Eskisehir producing different items. The manufactured products are delivered to six regional warehouses. Inbound delivery (from manufacturing unit to regional warehouse) takes place by means of a fleet of homogenous high-volume trucks with limited capacity, which are available at the plant. Demand of a distributor, which is considered deterministically known, is fulfilled by the regional warehouse that the distributor was assigned in advance. Outbound delivery (from regional warehouse to distributors) takes place by means of a fleet of homogenous low-volume vehicles with limited capacity, which are available at the regional warehouse. The movement of a vehicle implies a fixed usage cost, which is related to vehicle insurance and depreciation or drivers' rewards, and a variable shipping cost depending on both transported quantities and traveled distance. We assume that each vehicle utilized in both inbound and outbound delivery can make at most one trip during each day. For the outbound delivery, a vehicle cannot deliver to the same distributor more than once for each trip, and it has to get back to the regional warehouse at the end of its trip. On the other hand, different vehicles cannot deliver to the same customer in the same period, and therefore partial servicing is not allowed. Each distributor may keep inventory as well as regional warehouses, so that demand of distributors may be satisfied in advance. Therefore, our aim is the definition of a joint inventory and routing plan, minimizing inventory and transportation costs, while fulfilling demand requirements.

The system described may be well depicted as a network, in which each node represents the regional warehouse or a distributor and arcs express the existence of a route connection between a pair of nodes. Consequently, we decided to develop a mixed-integer optimization model to describe the problem.

Indices

For each variable or parameter of the model, the following have been defined:

$t = 1, \dots, T$ denoting the time period in days;

$j = 1, \dots, J$ denoting the product;

$k \in W$ denoting the distributor (regional warehouse is associated to the value $k = 0$ and the whole set of nodes is indicated as $N = W \cup \{0\}$);

$v = 1, \dots, V$ denoting the low-volume vehicle.

Variables

Variables of the model are defined as follows:

P_{jt} = quantity of product j transported from plant to regional warehouse in period t ;

I_{jkt} = inventory level for product j at the end of period t at node k ;

Y_t = binary variable assuming value 1 if a vehicle travels from plant to regional warehouse in period t ;

q_{jkt}^v = quantity of product j delivered to distributor k with vehicle v in period t ;

x_{ilt}^v = aggregated quantity transported on arc (i, l) by vehicle v in period t ;

z_{ilt}^v = binary variable assuming value 1 if vehicle v travels along arc (i, l) in period t .

Parameters

Parameters of the model are defined as follows:

h_{jkt} = inventory holding cost for a unit of product j at node k in period t ;

s_t = fixed cost of traveling from plant to regional warehouse in period t ;

f_t = fixed usage cost for a low-volume vehicle in period t ;

r_{il} = fixed cost of traveling from node i to node l ;

c_{jil} = variable shipping cost for each unit of product j along arc (i, l) ;

d_{jkt} = demand for product j of distributor k in period t ;

C = available vehicle capacity (homogenous low-volume vehicles);

K = available truck capacity (homogenous high-volume trucks);

M = a sufficiently big number for modeling reasons;

The Model

The integrated inventory-routing model is formulated as

$$\mathbf{Min} \sum_{t=1}^T \left\{ \sum_{j=1}^J \sum_{k \in N} h_{jkt} * I_{jkt} + s_t * Y_t + \sum_{v=1}^V \left[\sum_{l \in W} f_t * z_{0lt}^v + \sum_{\substack{i, l \in N \\ i \neq l}} [r_{il} * z_{ilt}^v + c_{il} * x_{ilt}^v] \right] \right\} \quad (1)$$

subject to

$$P_{jt} + I_{j0t-1} - I_{j0t} = \sum_{k \in W} \sum_{v=1}^V q_{jkt}^v \quad t = 1, \dots, T; \quad j = 1, \dots, J \quad (2)$$

$$\sum_{v=1}^V q_{jkt}^v + I_{jkt-1} - I_{jkt} = d_{jkt} \quad t = 1, \dots, T; \quad j = 1, \dots, J; \quad k \in W \quad (3)$$

$$\sum_{j=1}^J P_{jt} \leq K * Y_t \quad t = 1, \dots, T \quad (4)$$

$$\sum_{\substack{i \in N \\ i \neq k}} x_{ikt}^v - \sum_{\substack{m \in N \\ m \neq k}} x_{kmt}^v = \sum_{j=1}^J q_{jkt}^v \quad t = 1, \dots, T; \quad v = 1, \dots, V; \quad k \in W \quad (5)$$

$$\sum_{i \in W} \sum_{v=1}^V x_{i0t}^v - \sum_{m \in W} \sum_{v=1}^V x_{0mt}^v = - \sum_{k \in W} \sum_{v=1}^V \sum_{j=1}^J q_{jkt}^v \quad t = 1, \dots, T \quad (6)$$

$$x_{ilt}^v \leq C * z_{ilt}^v \quad i \in N; \quad l \in N; \quad i \neq l; \quad t = 1, \dots, T; \quad v = 1, \dots, V \quad (7)$$

$$\sum_{j=1}^J q_{jkt}^v \leq C * \sum_{\substack{i \in N \\ i \neq k}} z_{ikt}^v \quad t = 1, \dots, T; \quad v = 1, \dots, V; \quad k \in W \quad (8)$$

$$\sum_{l \in W} z_{0lt}^v \leq 1 \quad t = 1, \dots, T; \quad v = 1, \dots, V \quad (9)$$

$$\sum_{\substack{i \in N \\ i \neq l}} z_{ilt}^v - \sum_{\substack{m \in N \\ m \neq l}} z_{lmt}^v = 0 \quad t = 1, \dots, T; \quad v = 1, \dots, V; \quad l \in N \quad (10)$$

$$\sum_{\substack{i \in N \\ i \neq l}} z_{ilt}^v \leq 1 \quad t = 1, \dots, T; \quad v = 1, \dots, V; \quad l \in N \quad (11)$$

$$P \geq 0, I \geq 0, q \geq 0, x \geq 0; \quad (12)$$

$$Y, z \in \{0,1\}. \quad (13)$$

The objective function (1) of the model includes both inventory costs (central and local inventories) and logistics costs (fixed and variable transportation). Constraints 2 and 3, respectively, assure the balance at the regional warehouse among inventory and deliveries and the demand fulfillment for each distributor, preserving the possibility of building up local inventories. Constraints 4 limit truck capacity usage between plant and regional warehouse and force the binary truck utilization variables. The analogous vehicle capacity restriction is imposed by constraints 7 for the transportation between regional warehouse and distributors. Although only constraints with $i = 0$ are strictly necessary for the capacity check, also the other constraints are considered to enforce logical conditions on the binary variables. Constraints 5 and 6 are the flow conservation equations, assuring, respectively, flow balancing at each node and the collecting at the regional warehouse. Although constraints 6 are aggregated with respect to the distribution fleet, it was necessary to set constraints separately for each vehicle, to avoid transshipment at intermediate nodes. Constraint 8 ensures that if any product is delivered to a distributor, then there must be at least one vehicle enters to that distributor, with the vehicle capacity restriction preserved. Constraints 9 account for the fact that, in each period, at most one trip can be performed, by each vehicle. Equalities 10 express the trip integrity constraints, ensuring that each vehicle gets back to the regional warehouse at the end of the delivery route. Finally, constraints 11 restrict deliveries of different vehicles to the same distributor in the same period, and therefore partial servicing is not allowed. Because equations 5 and 6 for demand fulfillment have been introduced, typical subtour breaking constraints are unnecessary in this case.

4.2. Extensions of the Integrated Inventory-Routing Model

There are some assumptions in the above formulation. We will discuss these assumptions and the corresponding extensions of the Integrated Inventory-Routing Model here in this part of the thesis.

4.2.1. A fleet of Nonhomogenous Vehicles

In the problem context, delivery takes place by means of a fleet of homogenous vehicles with limited capacity between regional warehouse and distributors. Extensions to account for a nonhomogenous fleet can be easily accomplished by introducing in constraints 7 and 8 and in the objective function suitable parameters of C^v , f_t^v , r_{il}^v , c_{il}^v . By introducing these parameters, constraints 7 and 8 become;

$$x_{ilt}^v \leq C^v * z_{ilt}^v \quad t = 1, \dots, T; \quad v = 1, \dots, V \quad i \in N; l \in N; i \neq l; \quad (14)$$

$$\sum_{j=1}^J q_{jkt}^v \leq C^v * \sum_{\substack{i \in N \\ i \neq k}} z_{ikt}^v \quad t = 1, \dots, T; \quad v = 1, \dots, V; \quad k \in W \quad (15)$$

Routing portion of the objective function is as such:

$$\sum_{v=1}^V \left[\sum_{l \in W} f_t^v * z_{0lt}^v + \sum_{\substack{i, l \in N \\ i \neq l}} [r_{il}^v * z_{ilt}^v + c_{il}^v * x_{ilt}^v] \right]$$

4.2.2. Stockout Costs

In our formulation, it is assumed that products have to be delivered to distributors, to ensure fulfillment of the demand, which is considered deterministically known. Extensions may be devised to take into account stockout costs, so that the issue of distributor portfolio selection may be considered. In this manner, inventory level

variable I_{jkt} is split into two components and a new parameter representing backorder cost is defined.

I_{jkt}^+ = positive inventory level for product j at the end of period t at node k ;

I_{jkt}^- = negative inventory level for product j at the end of period t at node k ;

b_{jkt} = backorder cost for a unit of product j at node k in period t ;

Then, constraints 3 will include both components of inventory variables:

$$\sum_{v=1}^V q_{jkt}^v + (I_{jkt-1}^+ - I_{jkt-1}^-) - (I_{jkt}^+ - I_{jkt}^-) = d_{jkt} \quad t = 1, \dots, T; j = 1, \dots, J; k \in W \quad (16)$$

Inventory portion of the objective function is as such:

$$\sum_{j=1}^J \sum_{k \in N} (h_{jkt} * I_{jkt}^+ + b_{jkt} * I_{jkt}^-)$$

4.2.3. Warehouse Capacity

In the model, each distributor is allowed to keep inventory, so that demand may be satisfied in advance. However, there is no limit on the amount of inventory, which can be held by distributors. By introducing the capacity parameter and new constraints, the model will restrict the inventory levels at the distributor.

H_k = warehouse capacity at node k ;

$$\sum_{j=1}^J I_{jkt} \leq H_k \quad t = 1, \dots, T; \quad j = 1, \dots, J; \quad k \in W \quad (17)$$

Similarly, capacity restriction for the regional warehouse can be imposed by employing the same constraints with $k \in N$.

4.2.4. Multiple Truck between the Plant and Regional WH

By defining the Y_t as a binary variable, the model implicitly assumes that there is one truck utilized between the plant and a warehouse. Y_t variables may have a value of 1 at most on each day. However, some regional warehouses facing high demands may need to have multiple deliveries on a given day. To handle such a

situation, the Y_t variables defined as integer rather than binary in the modified formulation. Furthermore, following constraints are added to the model in order to maintain its solvability.

$$\sum_{t=1}^T Y_t \leq \left\lfloor \frac{\sum_j \sum_{k \in W} \sum_{t=1}^T (d_{jkt} - I_{jk0})}{K} \right\rfloor \quad (18)$$

$$Y_t \leq \left\lfloor \frac{\sum_{k \in W} \sum_{j=1}^J (d_{jkt} - I_{jkt-1})}{K} \right\rfloor \quad t = 1, \dots, T \quad (19)$$

4.2.5. Production Capacity

The logistic system considered in this study assumed a manufacturing unit producing different items in unlimited quantities. In other words, there is no capacity constraint on production. Although such an assumption seems unrealistic, it separates production and logistics (inventory and distribution) decisions. It is reasonable to an extent because the model studied in this thesis is an attempt to optimize the logistics decisions of the company. However, production capacity constraints can be incorporated in the model by adding warehouse index to the P_{jt} variables. By defining,

$u \in U$ denoting the regional warehouse;

$B_t =$ production capacity of manufacturing unit in period t ;

$P_{jut} =$ quantity of product j transported from plant to regional warehouse u in period t ;

Then the capacity restriction of manufacturing facility is imposed by constraints,

$$\sum_{u \in U} \sum_{j=1}^J P_{jut} \leq B_t \quad t = 1, \dots, T \quad (20)$$

CHAPTER 5

EVALUATION OF THE INVENTORY-ROUTING SYSTEM

In this chapter, experimentations with the inventory-routing systems developed are explained. In order to evaluate and investigate the performance of the proposed systems, the models are simulated with retrospective data.

In this study, the focus is to design an inventory-routing system for the company and verify the benefits of restructuring the current inventory replenishment methods of the customers by using vendor managed inventory approach. Comparison of the performances of the current and the proposed system is very essential in this work. Therefore, instead of using generated data or constructing some alternative scenarios, retrospective data is used to simulate the approach developed in the previous chapter. By this way, the existing system and the proposed system can be compared in terms of performance measures of the inventory-routing system.

The model developed has been tested on several random problems. Random instances generated to test the model ranged from problems with ten different products and four customers (of İzmir region) to be served, to problems with 12 customers (of Adana region) with 10 products distributed. In evaluation, problem instances have been obtained by using real cost coefficients and capacities.

The computational tests have been executed on an Intel(R) Pentium(R) M processor of 1.80GHz, 1GB RAM. The algorithm was implemented using GAMS IDE 2.0.20.0 and the CPLEX Library is used to solve the MIP problem.

Computational times ranged from a few minutes to one hour, for the largest problems.

5.1. Performance Measures

The main issue in any inventory-routing system is the trade-off between the inventory and transportation costs. Total cost for such a system is composed of inventory holding cost, fixed and variable transportation costs. The cost comparison between the current replenishment system and the inventory-routing system will give be an indicator about the performance of the new replenishment system. The comparison is based on the past data (2005) provided by the company. Provided data include orders of distributors, shipment day to distributors and amount of product shipped to them.

The performance measures to be evaluated in this work can be summarized as follows:

- Average amount of inventory held in the system (regional warehouse and distributors); comparison of the inventory levels of the existing system and the inventory-routing system
- Transportation cost in terms of number of deliveries and total distance traveled; existing system vs. inventory-routing system

There are also some performance measures specific to solvability of the model Solution time and quality of a solution obtained in each test are the performance measures specific to the model in this study.

5.2. Data and Parameters

As mentioned before, experimentation is made by the retrospective data. For the evaluation of the inventory routing system, initially the regional warehouses and the assigned provinces are considered. By a former study, Bulur and Çölova (2005) have studied a proposed system of six regional warehouses located between the plants and the distributors of the company. The distributors are also assigned to these six regional warehouses. The warehouses are located in Adana,

Erzurum, Eskisehir, Istanbul, Izmir and Samsun. Among these six regional warehouses, Izmir, Samsun and Adana are selected as pilot regions to test and validate the models and discuss the results. The cities and the warehouses that are assigned to are presented in Table 5.1 below:

Table 5.1 Cities that are assigned to warehouse in Izmir, with distances

Cities	Assigned to WH	Distances to WH (kms)
AYDIN	İZMİR	140
İZMİR		10
MANİSA		46
MUĞLA		239

High-capacity trucks are used in inbound transportation (between plant warehouses and regional warehouses) each having a volume of 90 m³ corresponds to 10 tons of capacity. Regular trucks are used for outbound transportation (shipment from regional warehouses to distributors) each of which has a net capacity of 40 m³, total weight of 6 tons. Trucks are paid based on the distance travelled. High-capacity trucks are paid 1.10 YTL/km and regular trucks are paid 0.80 YTL/km. These costs correspond to variable transportation cost term in our formulation.

Daily sales and inventory levels of distributors for each stock keeping unit (SKU) throughout the year 2005 are used as the demand data for the evaluation of the inventory-routing system. These sales and stocks are in units of kilograms. As mentioned previously the company manufactures about 200 SKUs. Instead of embedding all SKUs into the proposed models, the analysis is carried out on the representative SKUs. 10 representative SKUs that have similar characteristics (seasonality, share in total sales) are chosen among the products of the firm.

5.3. Experimentation of the Inventory-Routing System

Since the orders of the distributors accumulate at the end of a month because of the incentive system currently in use, their orders are used as the demand data in the experimentation. Experimentation is carried out on four cities in İzmir region. In order to compare the results with the current system, the cities are thought as if they are replenished from a central depot in Eskisehir. In other words, inventory-routing model is utilized in current configuration of distribution network. Ten representative products are selected among the products of the firm so that at least one of them exists in the past replenishments from the plant to these four cities. In order to compare our inventory routing model with the current distribution system, the planning horizon is set as 30 days (a month). The realized replenishments of the current distribution system for 30 days are presented in the following table:

Table 5.2 Total distance traveled in the current distribution system

FROM	TO	DISTANCE (kms)	# TRAVELED	TOTAL DISTANCE TRAVELED (kms)
ESKİSEHİR	AYDIN	483	15	14490 x 2
	İZMİR	412	25	20600 x 2
	MANİSA	394	8	6304 x 2
	MUĞLA	502	14	14056 x 2
TOTAL			62	27725 x 2 = 55450

For the 30 days of test period, 62 replenishments have been done from plant to the selected cities. The total distance traveled is calculated by multiplying distance by 2 for accounting the return after delivery. Therefore, total distance traveled is 55450 kms in the current system.

In order to analyze the effectiveness of inventory-routing system on distribution strategies, initial inventories of the distributors are set to the level that was realized in the current system. Furthermore, final inventories in the model are forced to be same as the inventory levels that are realized at the end of the test period. Distributor orders data is taken as demand in the model. Within the same

test period of 30 days, the inventory routing model results in a different distribution strategy having less distance traveled according to the current system. Results of our model are presented in the following table:

Table 5.3 Total distance traveled in the inventory-routing model

FROM	TO	DISTANCE (kms)	# TRAVELED	TOTAL DISTANCE TRAVELED (kms)
DEPOT	AYDIN	483	6	2898
	İZMİR	412	27	11124
	MANİSA	394	4	1576
	MUĞLA	502	7	3514
AYDIN	DEPOT	483	6	2898
	İZMİR	130	0	0
	MANİSA	156	0	0
	MUĞLA	99	1	99
İZMİR	AYDIN	130	1	130
	DEPOT	412	26	10712
	MANİSA	36	0	0
	MUĞLA	229	0	0
MANİSA	AYDIN	156	0	0
	DEPOT	394	4	1576
	İZMİR	36	0	0
	MUĞLA	255	0	0
MUĞLA	AYDIN	99	0	0
	DEPOT	502	8	4016
	İZMİR	229	0	0
	MANİSA	255	0	0
TOTAL			90	38543

The model results in 44 replenishment routes starting from the depot and all routes end at the depot. In the table, numbers of times arcs are traveled in the network and corresponding distances are presented. Totally, 38543 kms is traveled in the inventory-routing model with the same initial inventory, orders and ending inventory data. This is 30% lower than the realized distribution schedule of the current system.

Another performance criterion in our experimentation is inventory levels. Daily inventory levels of distributors for 30-days test period are used in order to

compare the results of current distribution strategy and the inventory routing model. For the comparison, realized inventory levels of representative products at the beginning of the test period are taken as the initial inventories for each distributor respectively. Likewise, realized inventory levels at the end of the test period are defined as the ending inventories in the model. As such, the effect of inventory routing on inventory levels will be tested by having same starting and ending points with the current distribution system. Realized daily inventory levels of the current system and the daily inventories of the inventory routing model are plotted on the graph below.

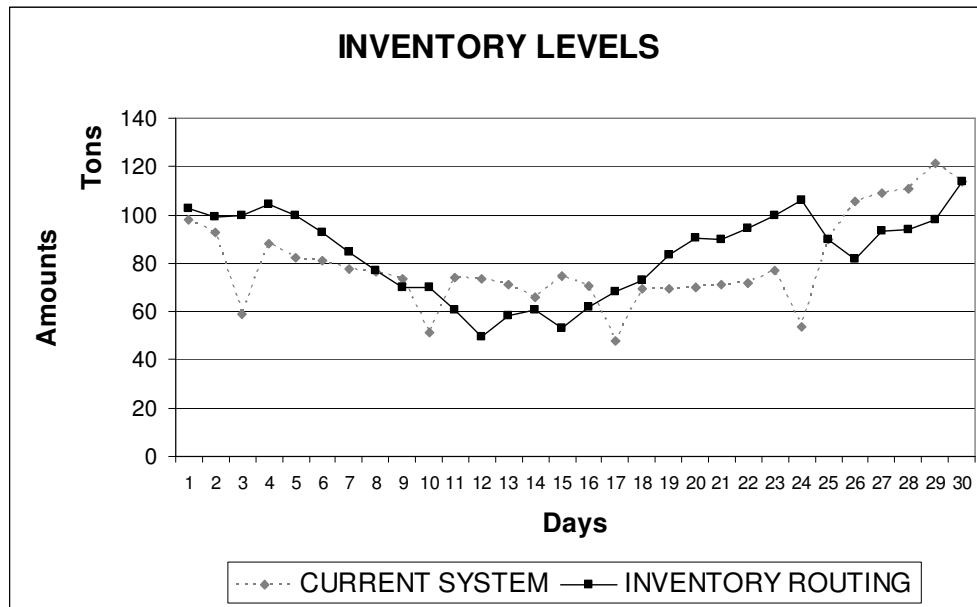


Figure 5.1. Total inventory held in system with ending inventory requirements.

Gray dashed line in the graph represents total daily inventories of the distributors in the Izmir region for the 30 days of test period. These inventory levels are based on the distributors' stock data for the selected test period. Black line in the graph represents stock data of inventory routing model. Starting with the same inventory with the current system, the stocks are lowered by the inventory routing model in the first 15 days of test period. However, since the ending inventories are forced

to be same with the current system, stock levels are continuously increases in the remaining days. On the average 80 tons of inventory was held daily in the current system. However, average daily inventory increases by 5% and is 84 tons in inventory routing model. The main reason of this increase in inventory levels is the ending inventories that the model is forced to yields. Individual graphs of distributors' inventory levels are plotted and presented as Appendix A.

The assumption of equal beginning and ending inventories is required in order to compare the behavior of the model during the test period. However, this assumption may force inventory routing model to yield more stock than actually needed. Originated from this idea, ending inventory requirement is dropped from the model in order to observe its behavior. Such a configuration results in continuously decrease in inventory levels. Since there is no ending inventory requirements system inventory decreases drastically and diminishes at the end of test period. Individual graphs of distributors' inventory levels are plotted and presented as Appendix B.

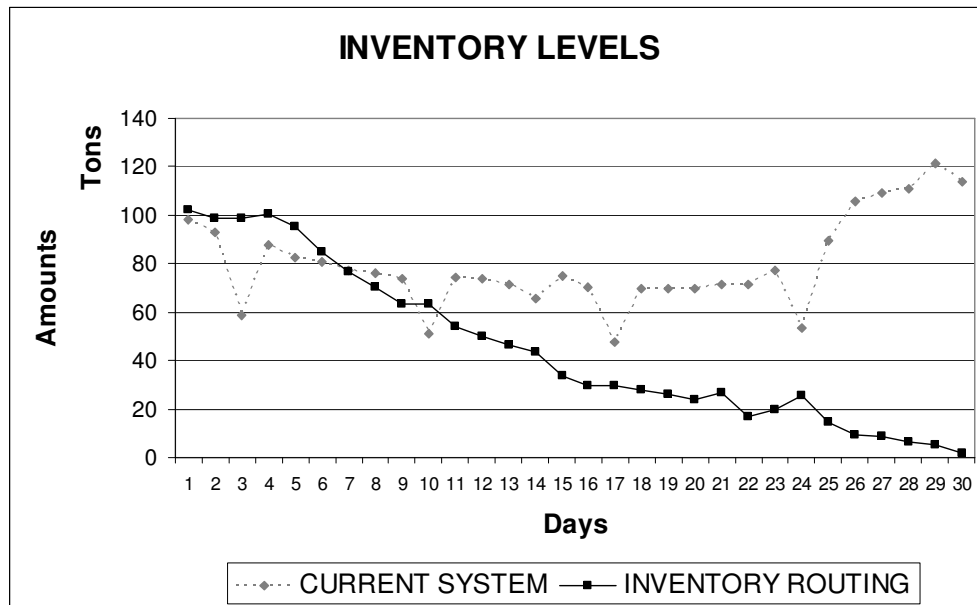


Figure 5.2. Total inventory held in system without ending inventory requirements.

CHAPTER 6

CONCLUSION AND DIRECTIONS FOR FUTURE RESEARCH

In this research, inventory-distribution system of a company operating in the Fast Moving Consumer Goods (FMCG) industry is analyzed. After previous studies performed in the company, it is decided to change the distribution network, by opening six regional warehouses that will operate as distribution centers between the company and its customers. Within this new logistics network structure, the distributors are replenished from the regional warehouse rather than plants. Therefore, the distribution system between a regional warehouse and the distributors assigned to that warehouse becomes an inventory-routing system and a model to optimize inventory costs and transportation costs of the system is required.

The system considered in this study differs from the other studies in the field of vehicle routing or inventory control. There exist some combination of routing and inventory decisions affecting the systemwide cost of distribution. There are multiple items replenished together by a truck. Therefore, an approach consisting of both routing and inventory costs developed for the multi-item inventory-routing system of the company. Furthermore, some extensions of the approach under specific circumstances are discussed briefly.

The inventory-routing model determines the replenishment amounts of all products from a regional warehouse to distributors assigned to it. A MIP model is constructed to determine the optimal vehicle routes and amount of

replenishments, to minimize the systemwide transportation and inventory cost. In order to overcome the multi-item complexity of the problem, truckload is summed over products. This accumulation overcomes difficulty of multi-item vehicle routing problems makes the inventory-routing model solvable in a reasonable time. Extensions of inventory routing model includes; 1) a fleet of nonhomogenous vehicles, 2) stockout costs, 3) warehouse capacity, 4) multiple truck between plant and regional warehouse and 5) production capacity of plants. 1) By introducing suitable parameters, constraints 7 and 8 are replaced by constraints 14 and 15. 2) By splitting inventory variable into two components, stockout costs are embedded in the model. 3) Warehouse capacity parameter and corresponding constraints stand for the capacity restrictions in distributors. 4) By converting the binary truck variable to an integer variable, the model is allowed to assign multiple trucks between the plant and regional warehouse. 5) Production capacity constraint is utilized by restricting the amount transported from plant to warehouses by the capacity of plants.

The inventory-routing model is experimented with retrospective data of the company, to compare the results with the current system. A test period of 30 days is chosen and 10 representative are selected for the experimentation.

The experimentation of the approach is carried out on Izmir warehouse. There are four cities replenished from Izmir warehouse, these are Aydin, Izmir, Manisa and Mugla. In order to compare with the current distribution performance the model is configured as if these four cities are replenished from Eskisehir. Initial and final inventories are set to actual values. The inventory-routing model resulted with pretty well in terms of transportation costs. Total distance traveled in the model is 30% lower than actual traveled distance of the current system. However, total inventory hold in the system (4 distributors) is 5% higher than the observed values. This is because of the reinforcements on the beginning and ending inventories in the model. In order to overcome the ending inventory effects ending inventory requirements are omitted from the model. Then the results are pretty well in terms of system inventory.

Less distance traveled and lower inventory levels are not the only advantages of the inventory-routing system. Higher utilization of fleet of trucks is also attained by inventory-routing approach. Moreover, the transportation cost decreases, as a result of bulk shipments from regional warehouses to distributors. Although an additional cost emerges by opening regional warehouses, this cost is less than the cost advantage obtained from transportation. As a result it can be said that, with the proposed inventory-routing system, better customer service level can be obtained with less logistics costs.

The proposed inventory-distribution system is very much affected by the inventory levels of distributors. Since the replenishments are scheduled according to inventory levels of the distributors, the accuracy and correctness of inventory data is essential for inventory-routing system. On the other hand incentive system of the company is based on orders of distributors. Since there is no order from distributors a study that aims the incentive system can be the first improvement area after this study.

Other possible future studies may be on the operations between distributors and retailers that they serve. In addition, applying vendor-managed inventory (VMI) at the retailers can bring many more advantages to the distribution system of the distributors and it is worthwhile to study the effects of VMI on the system.

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APPENDIX A

DISTRIBUTORS INVENTORY LEVELS UNDER ENDING INVENTORY REQUIREMENT

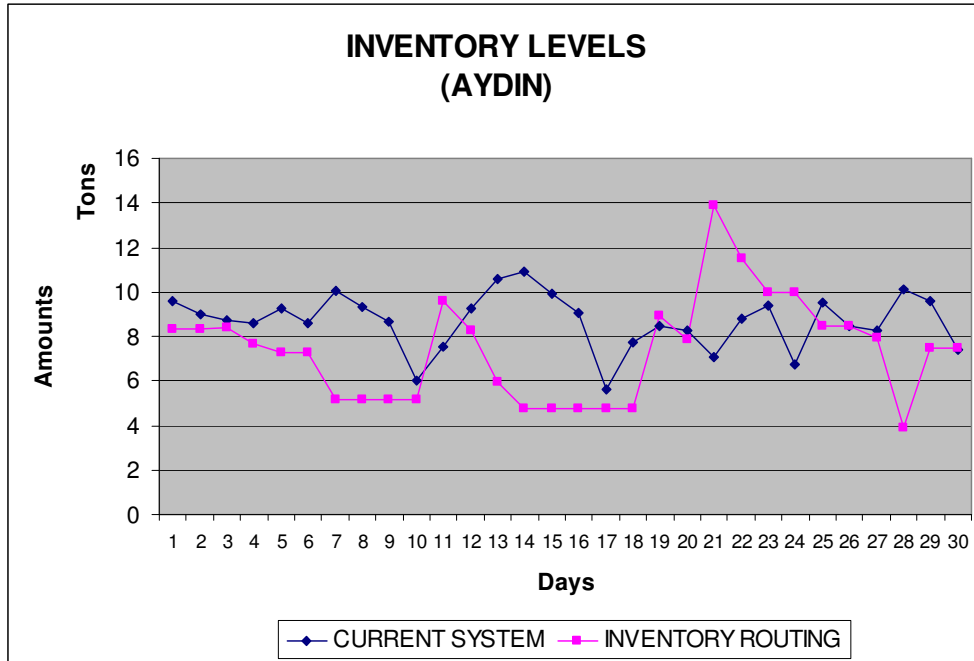


Figure A.1. Total inventory held in AYDIN with ending inventory requirements.

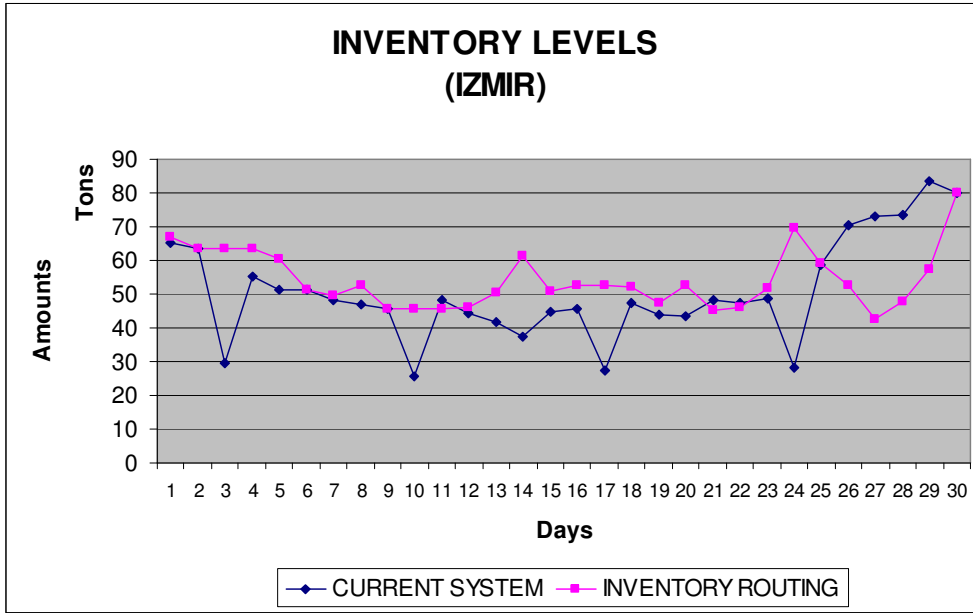


Figure A.2. Total inventory held in IZMIR with ending inventory requirements.

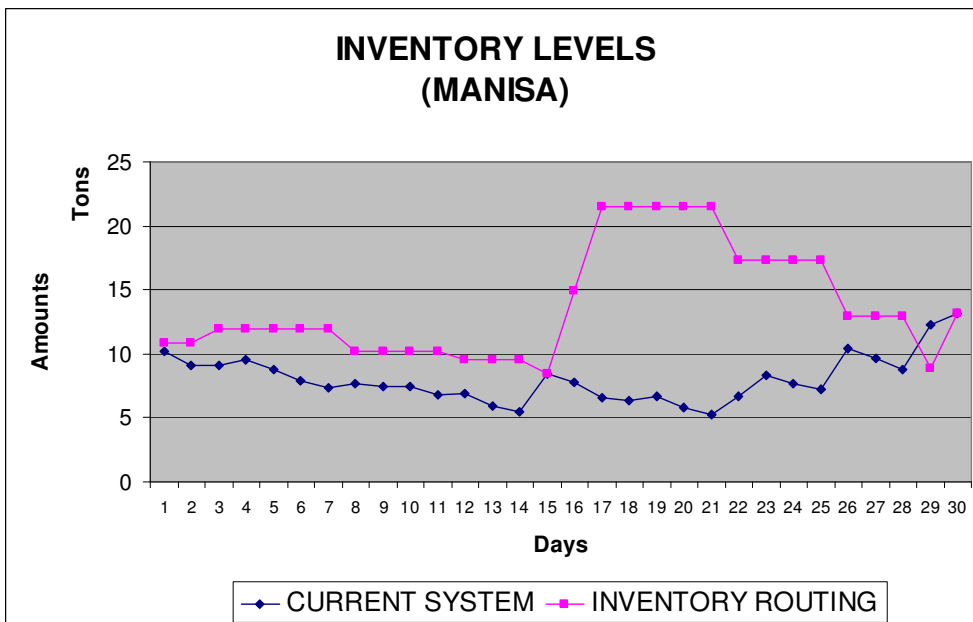


Figure A.3. Total inventory held in MANISA with ending inventory requirements.

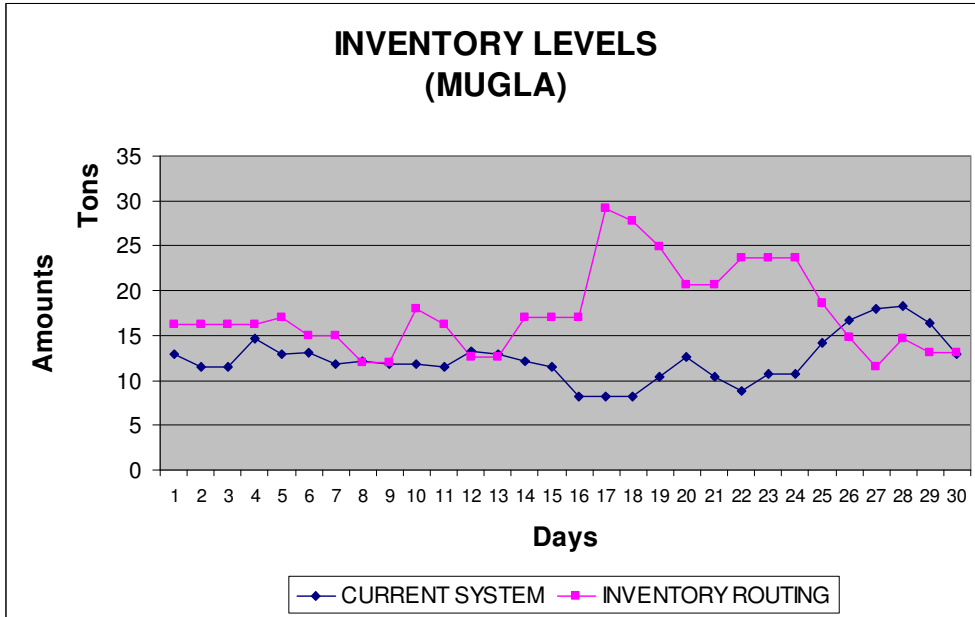


Figure A.4. Total inventory held in MUGLA with ending inventory requirements.

APPENDIX B

**DISTRIBUTORS INVENTORY LEVELS WITHOUT ENDING
INVENTORY REQUIREMENT**

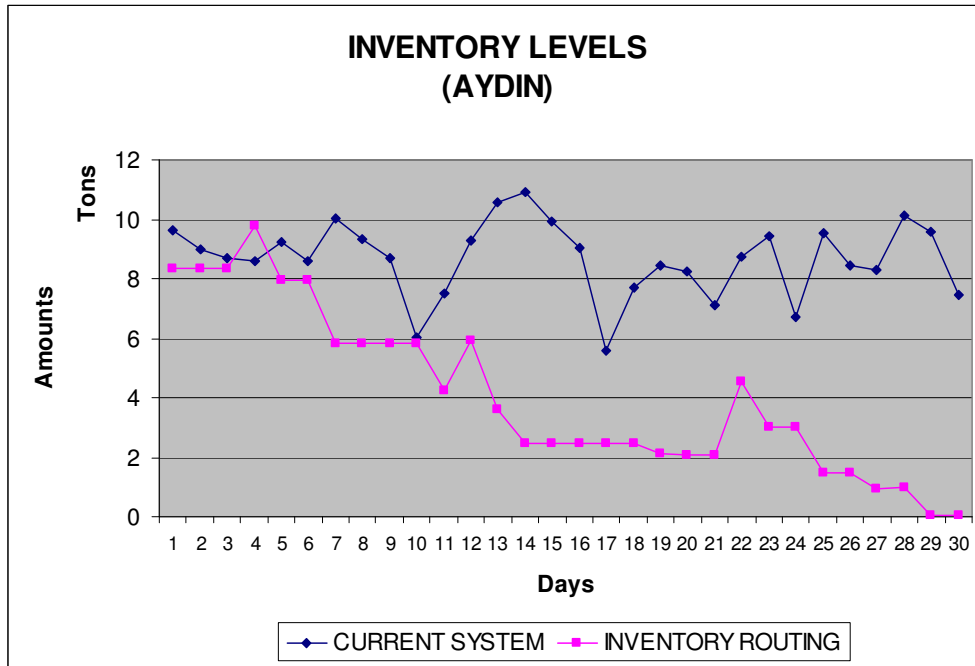


Figure B.1. Total inventory held in AYDIN without ending inventory requirements.

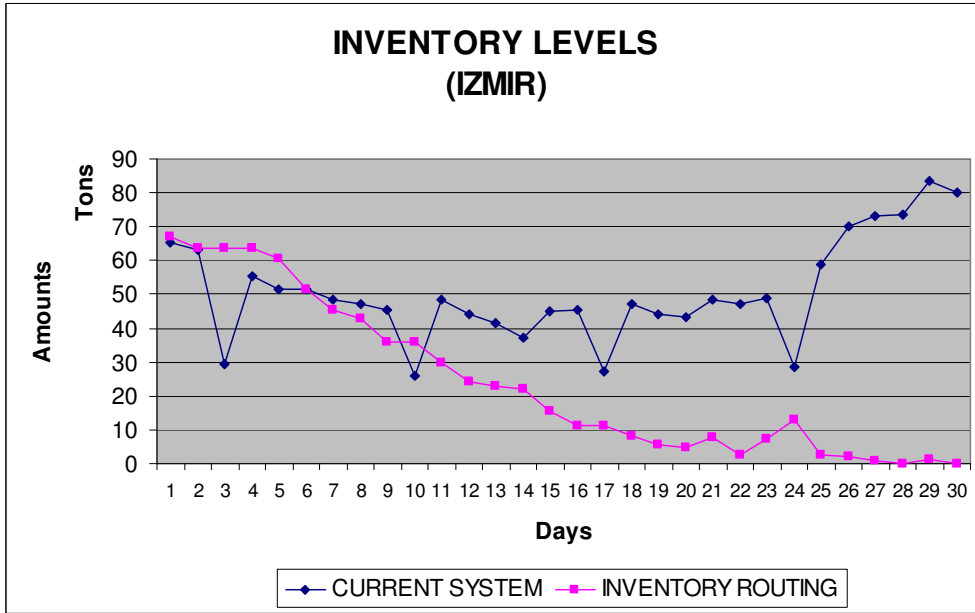


Figure B.2. Total inventory held in IZMIR without ending inventory requirements.

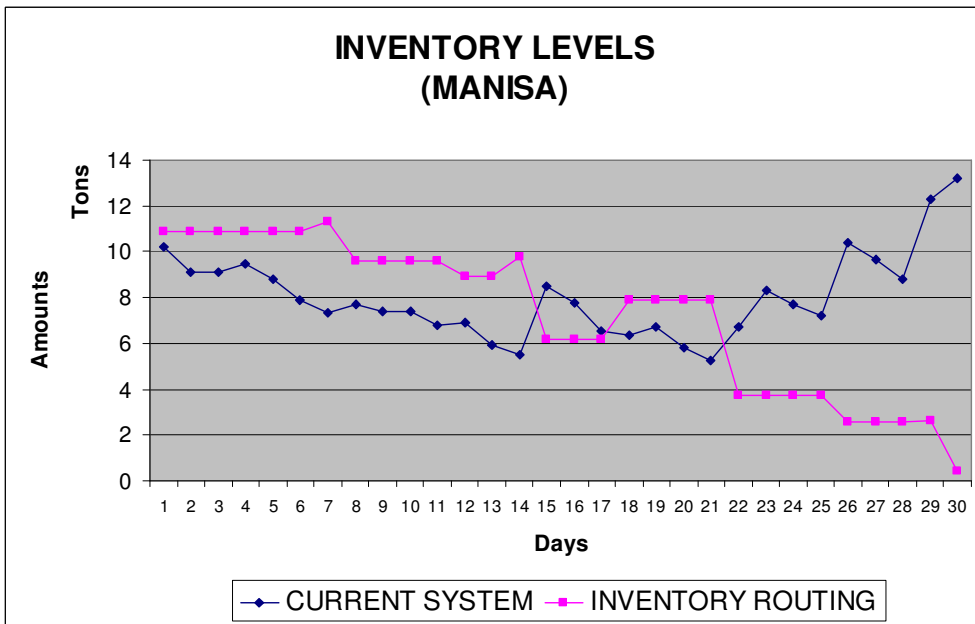


Figure B.3. Total inventory held in MANISA without ending inventory requirements.

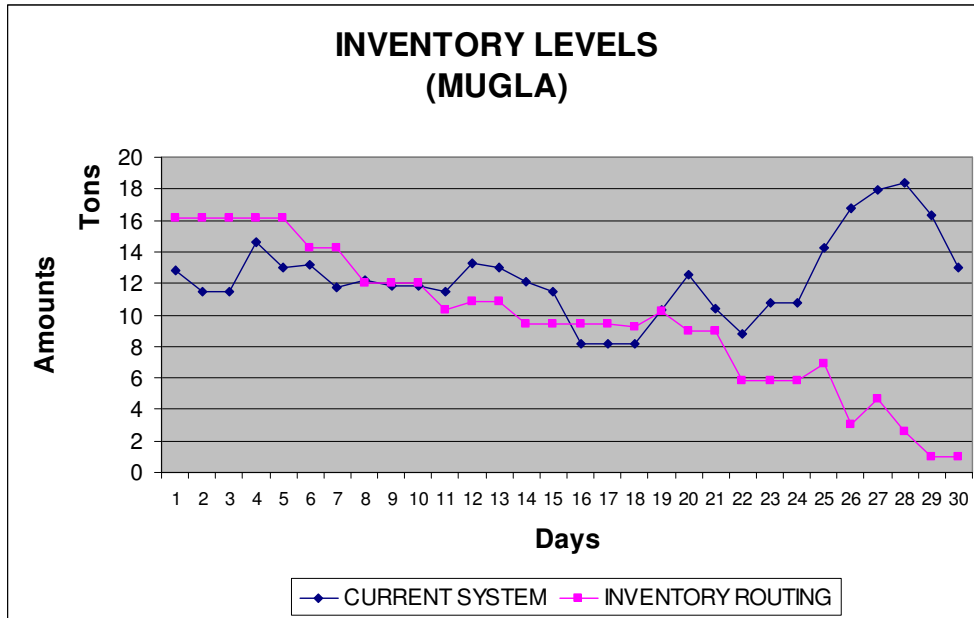


Figure B.4. Total inventory held in MUGLA without ending inventory requirements.