TRAJECTORY ESTIMATION IN DIRECTIONAL DRILLING USING BOTTOM HOLE ASSEMBLY (BHA) ANALYSIS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

SERKAN DOĞAY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
PETROLEUM & NATURAL GAS ENGINEERING

DECEMBER 2007

Approval of the thesis:

TRAJECTORY ESTIMATION IN DIRECTIONAL DRILLING USING BOTTOM HOLE ASSEMBLY (BHA) ANALYSIS

submitted by **SERKAN DOĞAY** in partial fulfillment of the requirements for the degree of **Master of Science in Petroleum and Natural Gas Engineering Department, Middle East Technical University** by,

Prof. Dr. Canan Özgen __________
Dean, Graduate School of **Natural and Applied Sciences**

Prof. Dr. Mahmut Parlaktuna __________
Head of Department, **Petroleum and Natural Gas Eng.**

Prof. Dr. Mustafa Verşan Kök __________
Supervisor, **Petroleum and Natural Gas Eng. Dept., METU**

Asst. Prof. Dr. Evren Özbayoğlu __________
Co-Supervisor, **Petroleum and Natural Gas Eng. Dept., METU**

Examining Committee Members:

Prof. Dr. Mahmut Parlaktuna ______________________
Petroleum and Natural Gas Eng. Dept., METU

Prof. Dr. Mustafa Verşan KöK ______________________
Petroleum and Natural Gas Eng. Dept., METU

Assoc. Prof. Dr. Serhat Akın ______________________
Petroleum and Natural Gas Eng. Dept., METU

Asst. Prof. Dr. Evren Özbayoğlu ______________________
Petroleum and Natural Gas Eng. Dept., METU

Hüseyin Ali Doğan, M.Sc ______________________
Petroleum and Natural Gas Engineer, TPAO

Date: 05/12/2007

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name : Serkan DOĞAY

Signature :

ABSTRACT

TRAJECTORY ESTIMATION IN DIRECTIONAL DRILLING USING BOTTOM HOLE ASSEMBLY (BHA) ANALYSIS

DOĞAY, Serkan
M.SC., Department of Petroleum&Natural Gas Engineering
Supervisor: Prof. Dr. Mustafa Verşan KÖK
Co-Supervisor: Assist. Prof. Dr. Evren ÖZBAYOĞLU

December 2007, 85 pages

The aim of this study is to combine the basic concepts of mechanics on drill string which are related to directional drilling, thus finding a less complicated and more economical way for drilling directional wells. Slick BHA, which has no stabilizers attached and single stabilizer BHA are analyzed through previously derived formulas gathered from the literature that are rearranged for this study. An actual directional well is redrilled theoretically with a slick BHA and a computer program is assembled for calculating the side force and direction of the well for single stabilizer BHA. Influence of controllable variables on drilling tendency is investigated and reported. The study will be useful for well trajectory and drill string design in accordance with the drilling phase. Also, by using available data from offset wells, drilling engineer can back-calculate the formation anisotropy index (FAI) that is often used for optimizing well trajectories and predicting drilling tendency on new wells in similar drilling conditions. After analysing the directional well data used in this study, it has been concluded that the well could be drilled without a steerable tool if the kick of point (KOP) is not a shallower depth. If the KOP is kept similar, the same curvature could not be achieved without a steerable tool.

Keywords: Directional drilling, FAI, KOP

ÖZ

YÖNLÜ SONDAJ DİZİSİ KUVVET ANALİZİ İLE YÖN TAYİNİ

DOĞAY, Serkan
Yüksek Lisans, Petrol ve Doğalgaz Mühendisliği
Tez Yöneticisi: Prof. Dr. Mustafa Verşan KÖK
Ortak Tez Yöneticisi: Y.Doç.Dr. Evren ÖZBAYOĞLU

Aralık 2007 85 sayfa

Bu çalışmanın amacı mekaniğin yönlü sondaja ait temel kavramlarını bir araya getirip kuvvet ve baskı analizleri ile yönlü kuyuların sondajı için daha kolay uygulanabilir ve ekonomik bir yol bulmaktır. Merkezleyicisi olmayan çıplak dizi ve tek merkezleyicili dizi literatürde var olan formüllerin ışığında incelenmiş, formüller yön analizi için düzenlenmiştir. Bu çalışmanın sonucunda yönlü olarak açılmış bir kuyu parametrelerde değişiklik yapılarak çıplak dizi ile de teorik olarak açılmış ve alternatif bir yol olarak sunulmuştur. Tek merkezleyicili dizi yön analizi için ise bir bilgisayar programı yazılmıştır. Parametrelerin sondaj üzerine etkileri araştırılmış ve değerlendirilmiştir. Sonuçlar yönlü kuyuların planlama ve sondaj aşamalarında yararlı olacak niteliktedir. Çevre kuyuların verileri kullanılarak hesaplanabilen formasyon verilerinin (FAI) diğer kuyuların planlamasında kullanılabileceği gösterilmiştir. Çalışmada kullanılan yönlü sondaj verileri analiz edildiğinde bahsi geçen kuyunun yön değişikliğine başlanılan noktanın (KOP) daha sığ bir derinliğe çekilmesi durumunda yönlü sondaja ihtiyaç duyulmadan açılabileceği gözlenmiştir. Ancak, KOP 'in ayni noktada kalması durumunda, yönlü sondaj gereçleri kullanılarak elde edilen dönüşün sağlanamadığı görülmüştür.

Anahtar kelimeler: Yönlü sondaj, FAI, KOP

To My Lovely Wife And My Parents

ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to my supervisor Prof. Dr. Mustafa V. Kök and my co-supervisor Assist. Prof. Dr. Evren Özbayoğlu for their guidance, advice, criticism, encouragements and insight throughout the research.

I would like to express my gratitude to my family and my friends Emre Yusuf Yazıcıoğlu, Yılmaz Yılmaz, Yusuf Ziya Pamukçu and Mehmet Ali Kaya for their generous attitude and support.

I would like to express my gratitude to Faculty of Engineering for enrolling me as a graduate student at Middle East Technical University (METU).

TABLE OF CONTENTS

LIST OF FIGURES

NOMENCLATURE

BSI = Bit Steerability Index

B_b = Bit Steerability in 'B' axis, ft/lbf

B_s = Bit Steerability in 'S' axis, ft/lbf

C = Drilling parameter, (ft/hr)/lbf

EI = Bending stiffness, lbf/in^2

ΔF = Footage Drilled, ft

h = Formation Anisotropy Index

H_o = Side force at bit, lbf

h_o = Dimensionless Side force at bit, lbf

H_{ST} = Stabilizer force, lbf

h_{ST} = Dimensionless stabilizer force, lbf

KD = Formation Drillability in 'D' axis, ft/lb

KE = Formation Drillability in 'E' axis, ft/lb

l = Dimensionless tangency length

L = Tangency Length, ft

p = Unit weight of Drill String, lbf.ft

R = Resultant Force, lbf

Rcl = Radial clearance, in.

r = Dimensionless radial clearance

WOB = Weight on bit, lbf

KOP = Kick off point

BHA= Bottom hole assembly

MWD= Measurement while drilling

DC= Drill collar

DP= Drill pipe

Greek Letters

α = Initial Hole Inclination, deg

α_n = True drilling direction in anisotropic formation and anisotropic bit, deg

β = Bit tilt angle, deg

δ = Drilling direction in isotropic formation and anisotropic bit, deg

Φ = Drilling direction in isotropic condition, deg

γ = Formation Dip Angle, deg

ψ = Drilling direction in Anisotropic Formation, deg

ρ_m= Drilling fluid density, ppg

ρ_{st}= Steel density, ppg

CHAPTER 1

INTRODUCTION

Drill string is the major component of a rotary drilling system which generally consists of Kelly (in Kelly drive systems), drill pipes, drill collars and stabilizers. The bit is made up to the drill collars by means of a bit sub. The rotation produced by the rotary table is transmitted to the Kelly, which directs the rotary motion to the drill string down to the bit. In order to achieve penetration part of the weight of drill collars is transferred to the bit so called Weight on Bit (WOB).

If the WOB is increased above a certain value called the critical weight on bit, buckling of drill collars may occur. To ensure mechanical integrity of drillstring, it is necessary to predict the expected loading on each part and then show that these loads do not lead to failure of the drillstring. It is clear that loads should be predicted as accurately as possible to allow safe, economical drillstring designs.[1]

Advanced control mechanisms for controlling wellbore trajectory during drilling are complex and costly while the industry is being forced for using these systems due to the increase in number of directional well projects all over the world. In this manner, several studies have been carried out for controlling wellbore deviation with simple analytical equations that would replace the simulators and to have further insight in controlling mechanisms.[2]

As will be discussed in the next sections, hole inclination and trajectory are affected by several factors such as angle (vertical deviation), direction (azimuth or horizontal deviation), formation characteristics, drilling parameters, and BHA. Effects of some factors are easily quantified, while others are not that easy.

Early studies of drilling mechanics were based upon the perfect verticality of the holes, later; from field experiences it has been shown that drilling such vertical holes is impossible, all wells have an inclination even if the formations are homogenous and isotropic. In following study the hole verticality assumption have been removed and hole size, drill collar size, placement of stabilizers in drill collar string was studied. The experience with Seminole fields in Oklahoma during the late twenties, made the industry realize that drilling does not necessarily follow the intended trajectory[3]. Something happens down hole which makes the drillstring deviate from its course. The efforts to understand the cause for deviation of drillstrings led to their mechanical analysis using the concept of structural mechanics for the drilling operations.

In the past 40 years, significant progress in the theoretical analysis of hole deviation problems has been made. The pioneering work has been primarily a result of the efforts by Lubinski and Woods[4]. In 1950, Lubinski considered the buckling of a drill string in a straight vertical hole, a problem also considered by Willers[5] in 1941. It was concluded that very low bit weights must be used to prevent hole deviation resulting from drill collar buckling. The use of conventional stabilizers was proposed in 1951 by MacDonald and Lubinski[6] as a method for permitting greater bit weights to be carried without drill collar buckling, these authors pointed out that a 2^{o} nearly vertical spiral hole can cause severe key seating and drill pipe wear, whereas a 3^{o} straight inclined hole with deviation all in one direction, while not vertical, will not result in serious drilling or producing problems. Studies were continued with an investigation of straight inclined holes by Lubinski and Woods[7] in 1953. They concluded that perfectly vertical holes cannot be drilled even in isotropic formations unless extremely low bit weights are used. They postulated that constant drilling conditions produce holes of constant inclination angle and varying conditions cause the hole to drill at a new equilibrium angle. This analysis was not concerned with drill string buckling since it was based on an equilibrium solution in which the drill string was presumed to lie along the lower side of the hole above the point of tangency. Weight of the drill collars below the point of tangency tends to force the hole toward the vertical, whereas the weight on bit tend to force hole away from vertical.

The concept of an anisotropic formation was introduced as an empirical method for explaining actual drilling data and as a means for extrapolating known deviation data to other conditions of bit weight, drill collar size and clearance. This analysis permits computation of the change of equilibrium hole angle when conditions are varied. In 1954, practical charts[8] were made available for solving equilibrium hole angle problems for straight inclined holes and the analysis was extended to apply large angles.

Use of stabilizers in straight inclined holes was consided by Woods and Lubinski[9]. They computed the additional weight which can be carried without an increase of hole angle as a result of the use of a stabilizer, and determined the optimum location for the stabilizer.

Lubinski[10] computed the influence of doglegs on fatigue failures of drillpipe and presented a method for measuring dog-leg severity. He pointed out that very large collar-to-hole clearances can lead to fatigue failure of drill collar connections, and that rotating with the bit off bottom can be worse than drilling with the full weight of the drill collars on the bit in highly inclined holes when inclination decreases with depth in the dog-leg.

Equilibrium solutions for straight inclined holes given in the references cited above are not applicable when buckling occurs or when the holes are curved. However, the problem of the instability of a drill collar in an inclined hole has been considered by Bogy and Paslay [11,12] (1964) as well as the problem of helical post-buckling equilibrium.

Since the pioneering work by Lubinski[4], the drilling industry has come to accept and appreciate the importance of analysis of BHA, which is now regarded as important in controlling the deviation tendencies of well trajectory, especially in directional, horizontal and extend reach wells.

As a matter of fact, in the drilling industry, one of the most critical steps about the well design and also about drilling operations is controlling the hole deviation. As the surface coordinates and the main targets are declared, the well designer has to complete his designs through some assumptions and some experiences about the formations and their behavior. Accordingly low formation data lead to some deviation problems while drilling, which can cause in missing the targets or spending extra time and money for catching the targets. Once a well is drilled and the data is obtained, a comprehensive study must be held in order to design new wells which will be discussed throughout the study.

CHAPTER 2

LITERATURE REVIEW

2.1 Directional Drilling

In order to perform a mechanical investigation to the drill string, it is beneficial to examine the uses of non-straight holes, which has many applications all over the world.

Directional drilling is the science of drilling non-vertical holes, accordingly, controlled directional drilling is the science of deviating a well bore along a planned course to a subsurface target whose location is a given lateral distance and direction from the vertical. At a specified vertical depth, this definition is the fundamental concept of controlled directional drilling even in a well bore which is held as close to vertical as possible as well as a deliberately planned deviation from the vertical.

In earlier times, directional drilling was used primarily as a remedial operation, either to sidetrack around stuck tools, bring the well bore back to vertical, or in drilling relief wells to kill blowouts. Interests in controlled directional drilling began about 1929 after new and rather accurate means of measuring hole angle was introduced during the development of Seminole, Oklahoma field.

The first application of oil well surveying occurred in the Seminole field of Oklahoma during the late 1920's. Subsurface geologists found it extremely difficult to develop logical contour maps on the oil sands or other deep key beds. The acid bottle inclinometer was introduced into the area and disclosed the reason for the problem; almost all the holes were crooked, having as much as 50 degrees inclination at some check points.

In the spring of 1929, a directional inclinometer with a magnetic needle was brought into the field. Holes that indicated an inclination of 45 degrees with the acid bottle were actually 10 or 11 degrees less in deviation. The reason was that the acid bottle reading chart had not been corrected for the meniscus distortion caused by capillary pull. Thus better and more accurate survey instruments were developed over the following years. The use of these inclination instruments and the results obtained showed that in most of the wells surveyed, drill stem measurements had very little relation to the true vertical depth reached, and that the majority of the wells were "crooked". Some of the wells were inclined as much as 38 degrees off vertical. Directional drilling was employed to straighten crooked holes.

In the early 1930’s the first controlled directional well was drilled in Huntington Beach, California. The well was drilled from an onshore location into offshore oil sands using whipstocks, knuckle joints and spudding bits. An early version of the single shot instrument was used to orient the whipstock. Controlled directional drilling was initially used in California for unethical purposes, that is, to intentionally cross property lines. In the development of Huntington Beach Field, two mystery wells completed in 1930 were considerably deeper and yielded more oil than other producers in the field which by that time had to be pumped. The obvious conclusion was that these wells had been deviated and bottomed under the ocean. This was acknowledged in 1932, when drilling was done on town lots for the asserted purpose of extending the producing area of the field by tapping oil reserves beneath the ocean along the beach front.

Controlled directional drilling had received rather unfavorable publicity until it was used in 1934 to kill a wild well near Conroe, Texas. The Madeley No.1 had been spudded a few weeks earlier and, for a while, everything had been going normally. But after a while the well developed a high pressure leak in its casing, and before long, the escaping pressure created a monstrous crater that swallowed up the drilling rig. The crater, approximately 170 feet in diameter and of unknown depth, filled with oil mixed with sand in which oil boiled up constantly at the rate of 6000 barrels per day. As if that were not enough, the pressure began to channel through upper formations and started coming to the surface around neighboring wells, creating a very bad situation indeed. It was decided that there was nothing to do except let the well blow and hope that it would eventually bridge itself over[13].

In the meantime, however, it was suggested that an offset well to be drilled and deviated so that it would bottom out near the borehole of the cratered well. Then mud under high pressure could be pumped down this offset well so that it would channel through the formation to the cratered well and thus control the blow out. The suggestion was approved and the project was completed successfully, to the gratification of all concerned. As a result, directional drilling became established as one way to overcome wild wells, and it subsequently gained favorable recognition from both companies and contractors. With typical oilfield ingenuity, drilling engineers and contractors began applying the principles of controlled directional drilling whenever such techniques appeared to be the best solution to a particular problem.

Current expenditures for hydrocarbon production have dictated the necessity of controlled directional drilling, and today it is no longer the dreaded operation that it once was. Probably the most important aspect of controlled directional drilling is that it enables producers all over the world to develop subsurface deposits that could never be reached economically in any other manner[13].

2.1.1 Applications of Directional Drilling

a) Side Tracking

Side tracking was the original directional drilling technique. Initially, sidetracks were “blind", in other words, azimuth of the well was not known. The objective is simply to deviate the well in a short distance. The technique may also be used for unexpected geological changes or shifting the well to another position as illustrated in Fig.1[13].

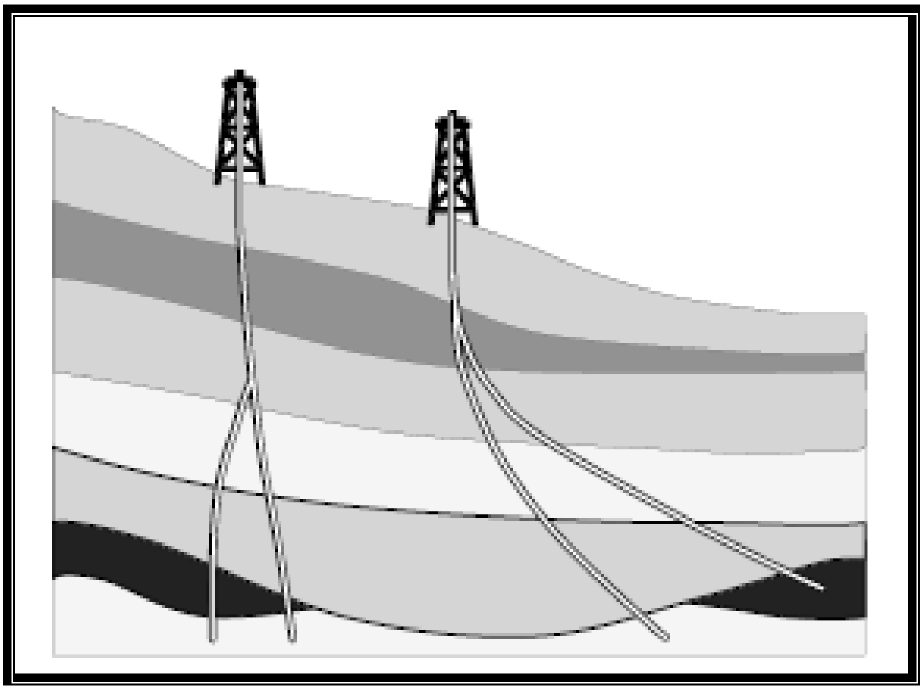

Fig.1 Side Tracking[13]

b) Inaccessible Locations

Targets located which are impossible to reach such as cities, rivers or in environmentally sensitive areas make it necessary to locate the drilling rig some distance away. A directional well is drilled to reach the target in such locations. Fig.2[13] illustrates a reservoir below a city where the rig is located away from the location.

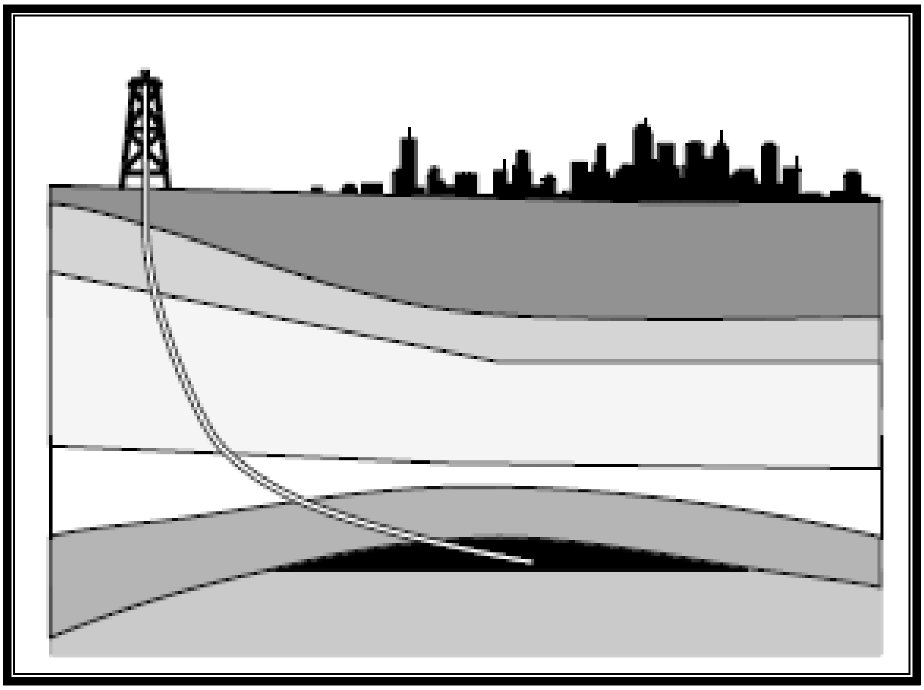

Fig.2 Inaccessible locations[13]

c) Salt Dome Drilling

Salt domes have been found to be natural traps of oil accumulating in strata beneath the overhanging hard cap. There are severe drilling problems associated with drilling a well through salt formations. A widely used solution is to drill a directional well to reach the reservoir as illustrated in Fig.3[13], thus avoiding the problem of drilling through the salt.

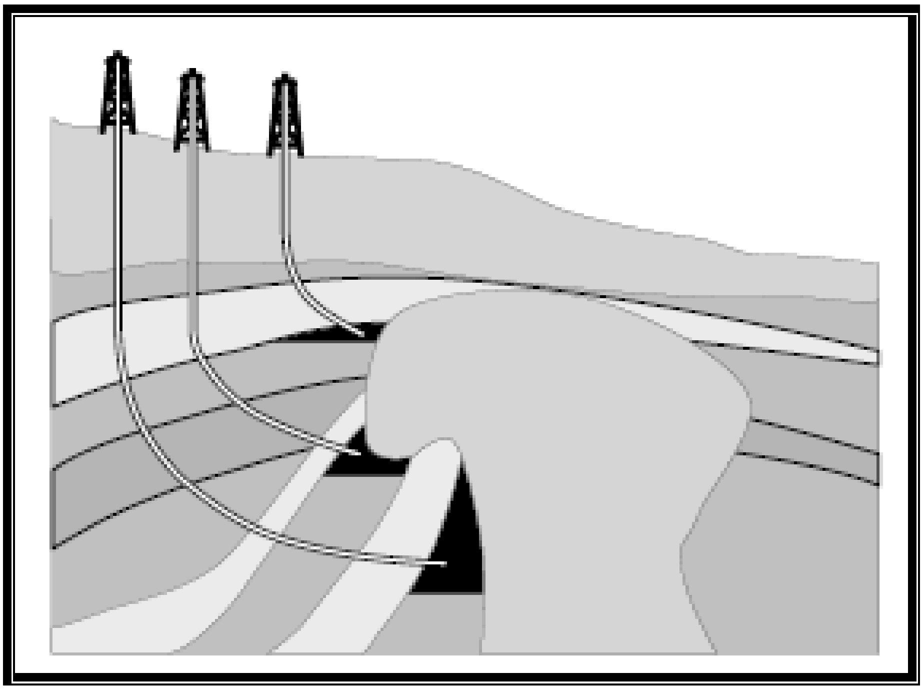

Fig.3 Salt dome drilling[13]

d) Fault Controlling

Crooked holes are common when drilling nominally vertical. This is often due to faulty sub-surface formations. It is often easier to drill a directional well into such formations without crossing the fault lines as illustrated in Fig.4[13]. The well illustrated on the left side does not touch the fault line thus drilling is safer and easier compared to the well on the right side

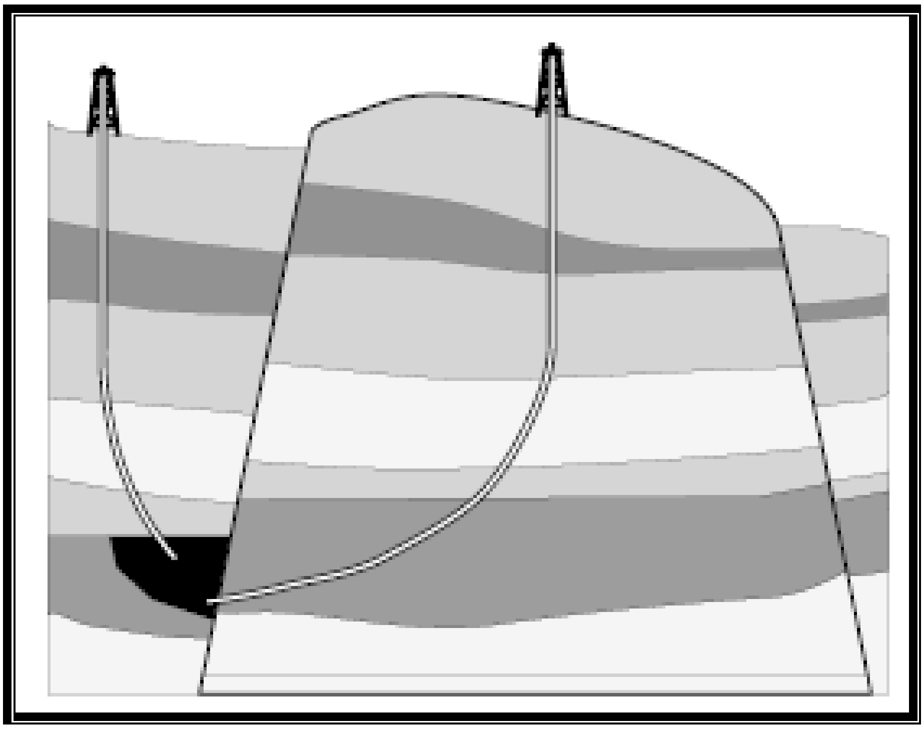

Fig.4 Fault Controlling[13]

e) Multiple-Exploration Wells from a Single Wellbore

A single well bore can be plugged back at a certain depth and deviated to drill a new well. A single well bore is sometimes used as a point of departure to drill others. It allows exploration of structural locations without drilling other complete wells.Enhanced reach drilling is one axample for that as illustrated in Fig.5[13].

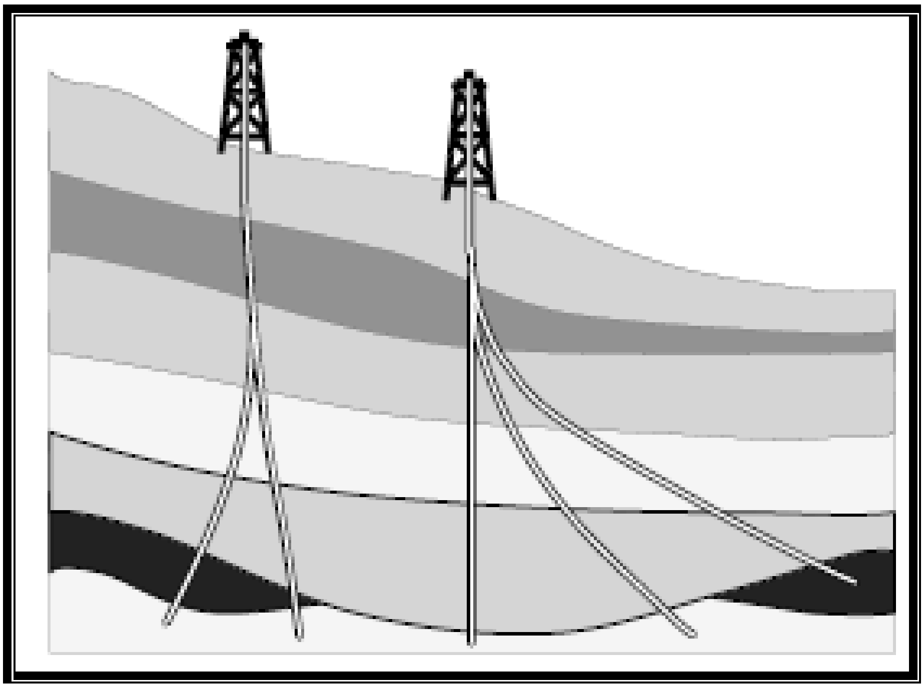

Fig.5 Multiple-Exploration Wells from a Single Wellbore[13]

f) Onshore Drilling

Reservoirs located below large bodies of water which are within drilling reach of land are being tapped by locating the wellheads on land and drilling directionally underneath the water. This saves money as the land rigs are much cheaper. A directional well drilled from land, reaching below sea is illustrated in Fig.6[13].

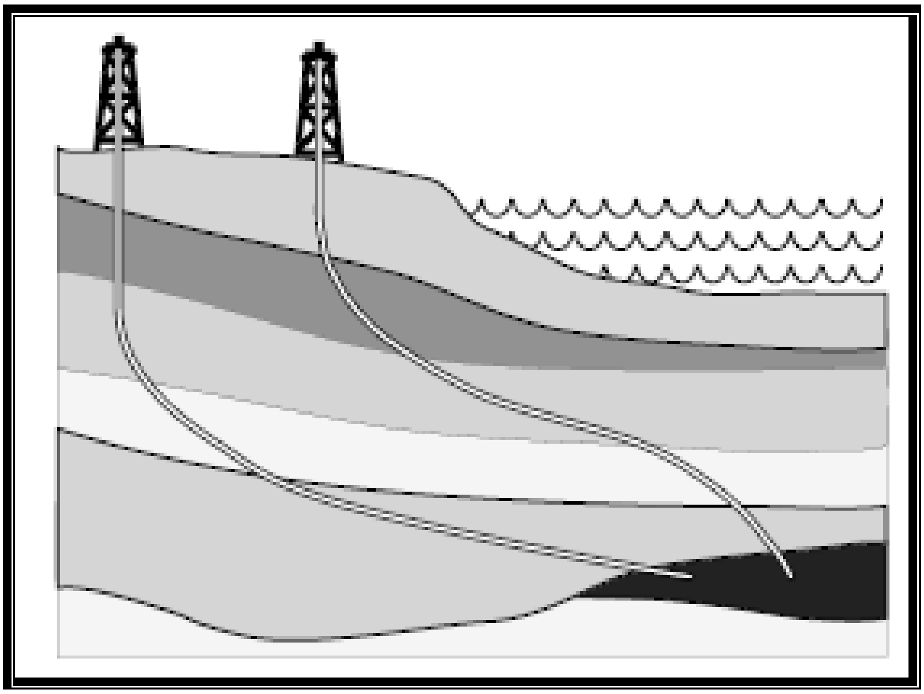

Fig.6 Onshore Drilling[13]

g) Relief Well

The objective of a directional relief well is to intercept the bore hole of a well which is blowing and allow it to be "killed" as shown in Fig.7[13]. The bore hole causing the problem is the size of the target. To locate and intercept the blowing well at a certain depth, a carefully planned directional well must be drilled with great precision.

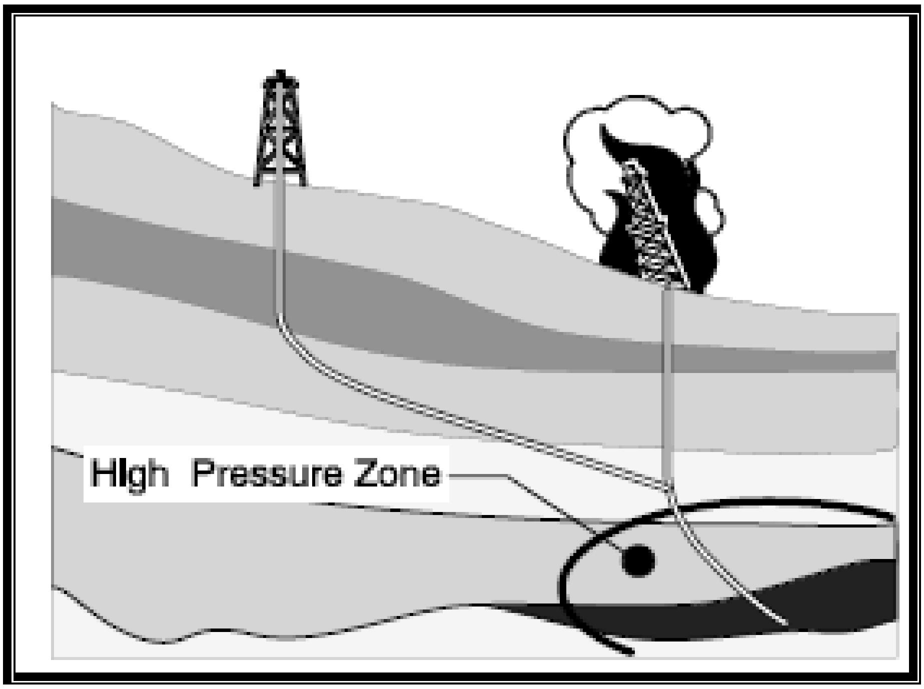

Fig.7 Relief Well[13]

h) Horizontal Wells

Reduced production in a field may be due to many factors, including gas and water coning or formations with good but vertical permeability. Engineers can then plan and drill a horizontal drainhole as illustrated in Fig.8[13]. Horizontal wells are divided into long, medium and short-radius designs, based on the buildup rates used.

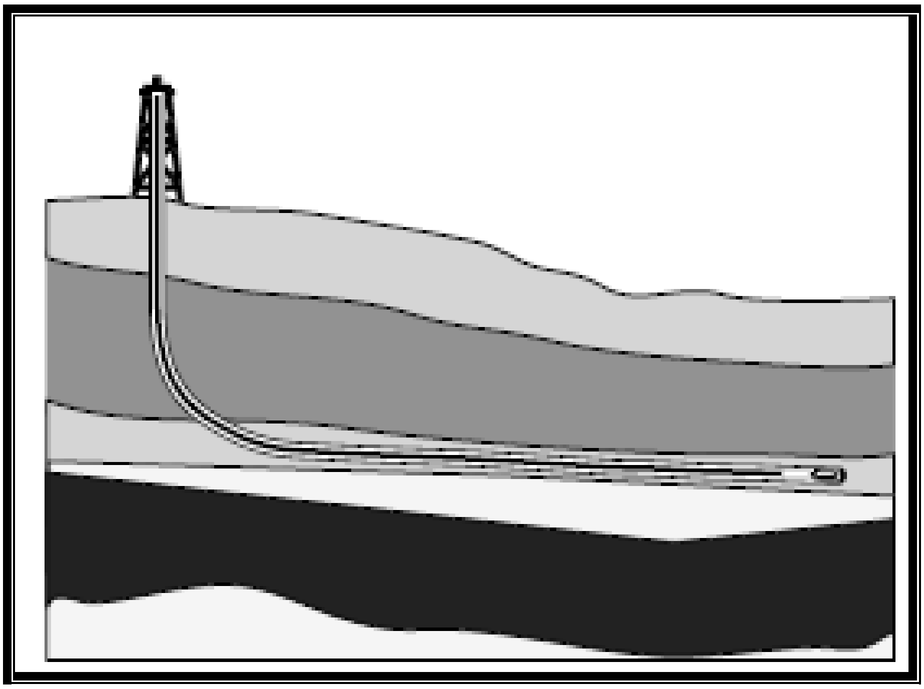

Fig.8 Horizontal Well[13]

i) Offshore Multiwell Drilling

Directional drilling from a multiwell offshore platform is the most economic way to develop offshore oil fields as illustrated in Fig.9[13]. In such cases, wells should have trajectories which will prevent anticollision. Accordingly wells must be designed and drilled directional. By this method, several wells can be drilled using one platform which reduces the costs.

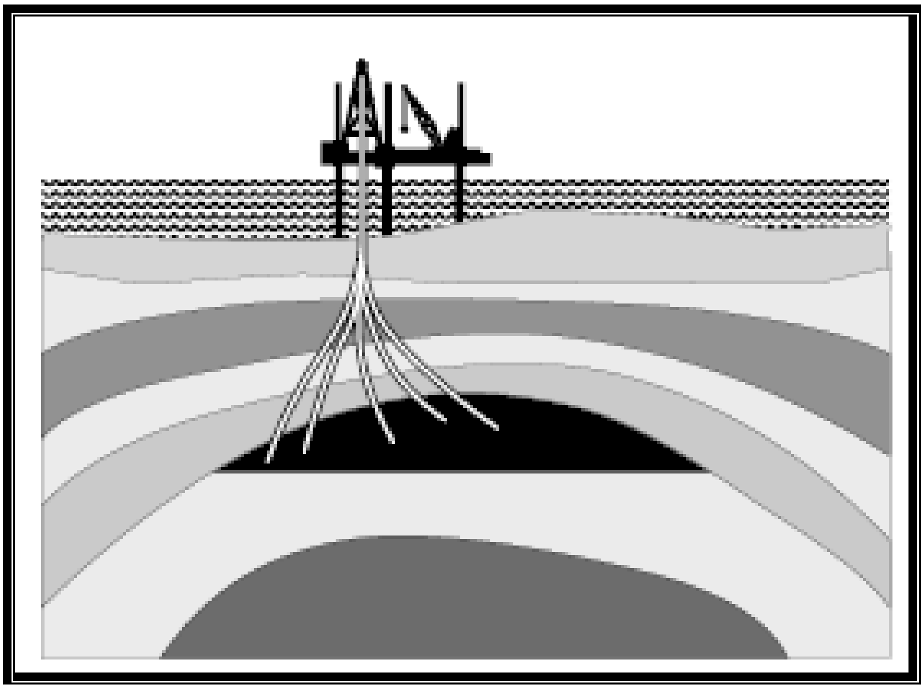

Fig.9 Offshore Multiwell Drilling[13]

j) Multiple Sands From a Single Wellbore

In this application, a well is drilled directionally to intersect several inclined oil reservoirs as illustrated in Fig.10[13]. This allows completion of the well using a multiple completion system. The well may have to enter the targets at a specific angle to ensure maximum penetration of the reservoirs due to the directional plan.

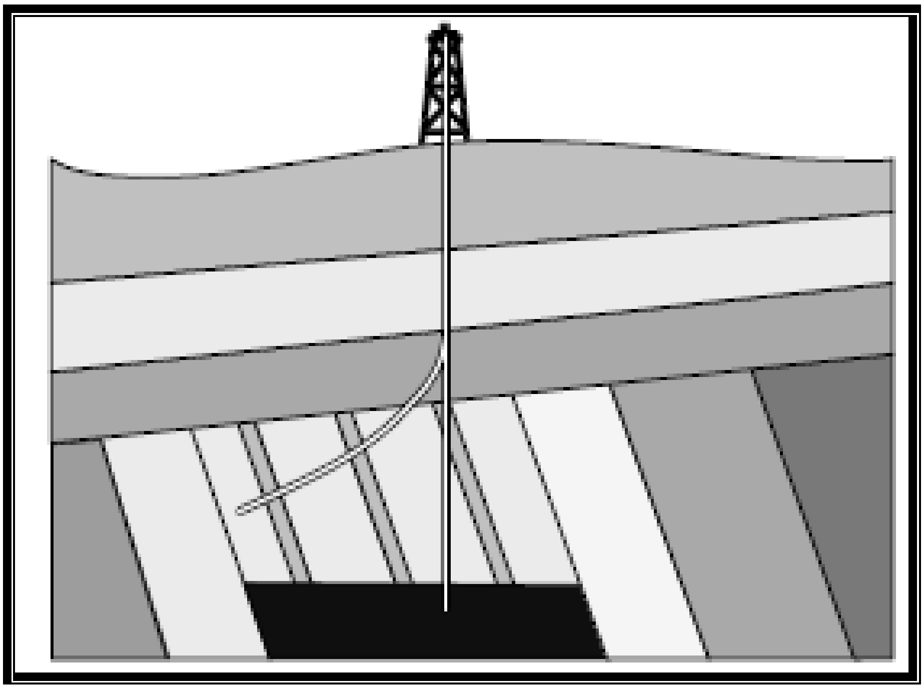

Fig.10 Multiple Sands from a Single Wellbore[13]

k) Controlling Vertical Wells

Directional techniques are also used to “straighten crooked holes”. When unplanned deviation occurs in a well which is supposed to be vertical, various directional techniques can be used to bring the well back to vertical as planned. This is one of the earliest applications of directional drilling as illustrated in Fig.11[13].

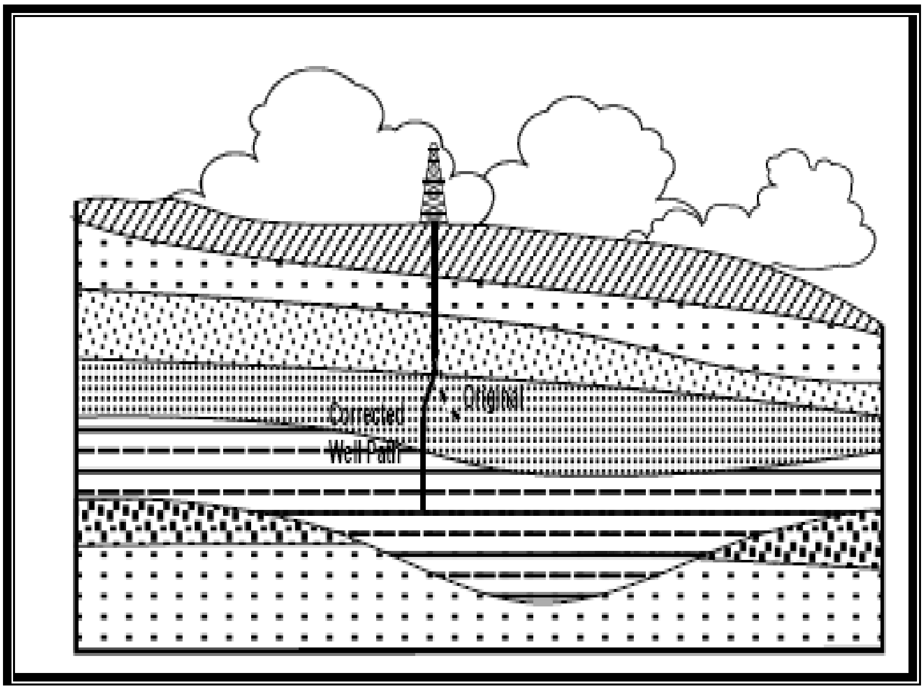

Fig.11 Controlling Vertical Wells[13]

2.2 Factors Affecting Hole Inclination

2.2.1 Anisotropic Formations

Formation is considered as one of the main factors affecting hole deviation. This can be traced to the way the rock fails under the action of the bit. Homogenity of the rocks help the bit drill more vertical while nonhomogeneous formations such as faults, dipping plates cause the bit to tilt one way and start to deviate the hole.

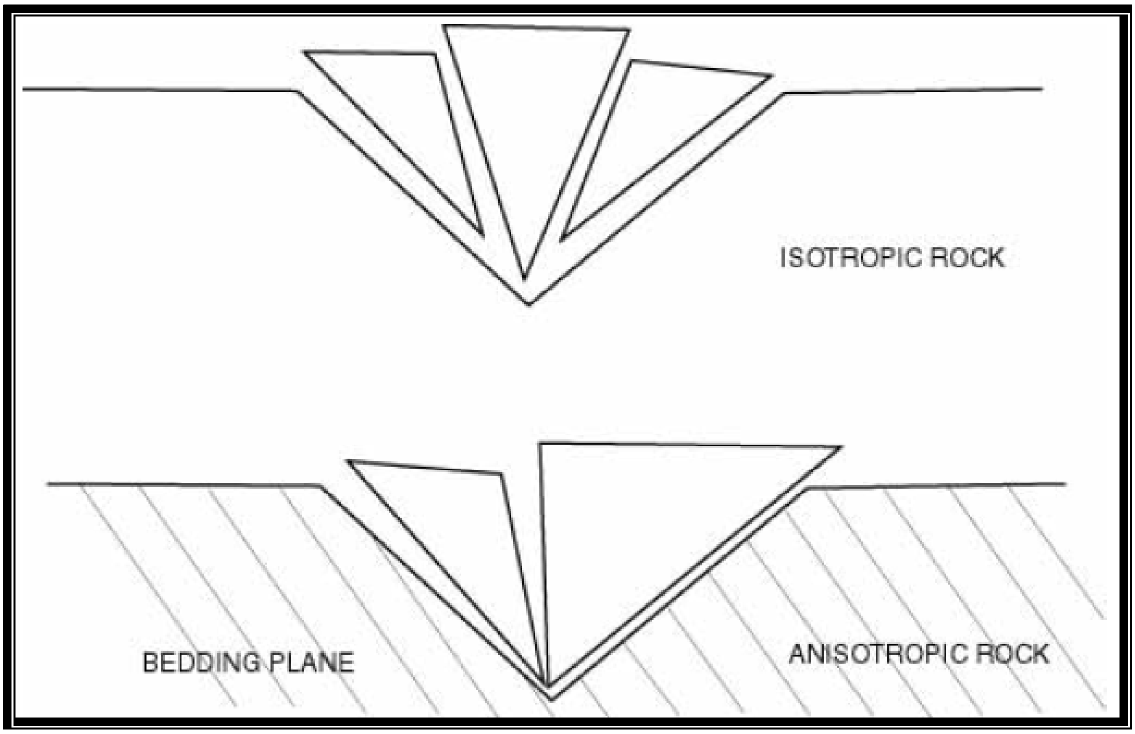

Fig.12 Isotropic vs. Anisotropic Formation

Anisotropic formation may be defined as; A formation with directionally dependent variables. The changes in homogenity or differences in formation hardness are primary causes of bit deviation. Formation hardness will affect the penetration rate, and this will determine the amount of time the bit or the stabilizer will be abrading the hole wall and enlarging the wellbore or wearing out itself. Field experience shows that bits generally tend to drill up dip when the bedding planes have dips of less than 45^o and to drill down dip when bedding dips are grater than 60^o (bedding planes dip is referenced to vertical). The quantification of anisotropic failure effect of rock on wellbore deviation was originally introduced from the works of Lubinski and Woods. It can be shown that the formation dip angle, γ, the hole inclination angle, α, resultant force angle, Ø, and formation anisotropy index, h, are related by the following equation.

$$h = 1 - \tan(\gamma - \alpha) \div \tan(\gamma - \theta) \quad (2\text{-}1)$$

2.2.2 Formation Drillability Theory

Investigation of the cause of hole deviation led to a theory proposed by Sultanov and Shandalov[14] which seeks to explain hole angle change in terms of the difference in drilling rates in hard and soft dipping formations. Presumably angle in the hole changes bacause the bit drills slower in that portion of the hole in the hard formation.

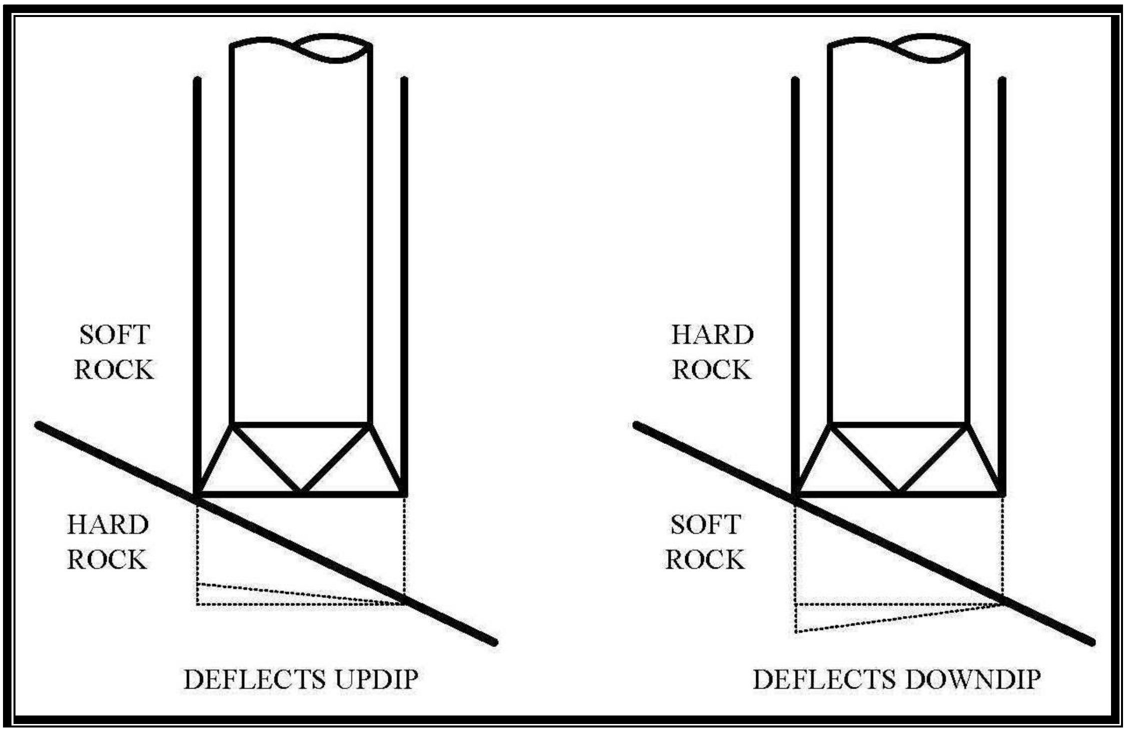

Fig.13 Formation Drillability Theory of Hole Deviation

Inherent in this theory is the underlying assumption that the bit weight is distributed uniformly over the bottom of the hole. It predicts updip deviation when drilling into harder rock and downdip in softer rock.

2.2.3 Miniature Whipstock Theory

Drilling experiments have been made by Hughes Tool Co. in which an artificial formation composed of glass plates has been drilled with the hole inclined to the laminations. In these tests the plates fractured perpendicular to the bedding plane, creating miniature whipstocks. If such whipstocks are created when laminated rock fractures perpendicular to bedding planes, they could cause updip drilling. This theory offers a possible qualitative explanation to hole deviation in slightly dipping formations; however, it does not explain the downdip drilling which occurs in steeply dipping formations[14].

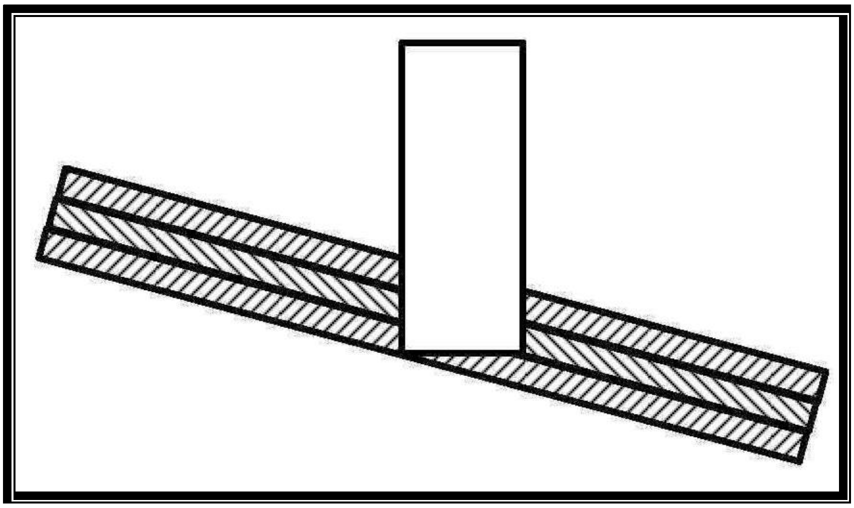

Fig.14 Miniature Whipstock Theory

2.2.4 Drill Collar Moment Theory

When a bit drills from a soft to a hard formation the weight on bit is not distributed evenly along the bottom of the hole. Since more of the weight on bit is taken by the hard formation, a moment is generated at the bit. Such a moment changes the pendulum length to the point of tangency as well as the side force at the bit. The variation of side force is not the same when drilling from soft to hard formations as when drilling from hard to soft and therefore, can affect a change of hole inclination. It is believed that the drill string moment theory offers a possible quantitative alternative to the anisotropic formation theory.

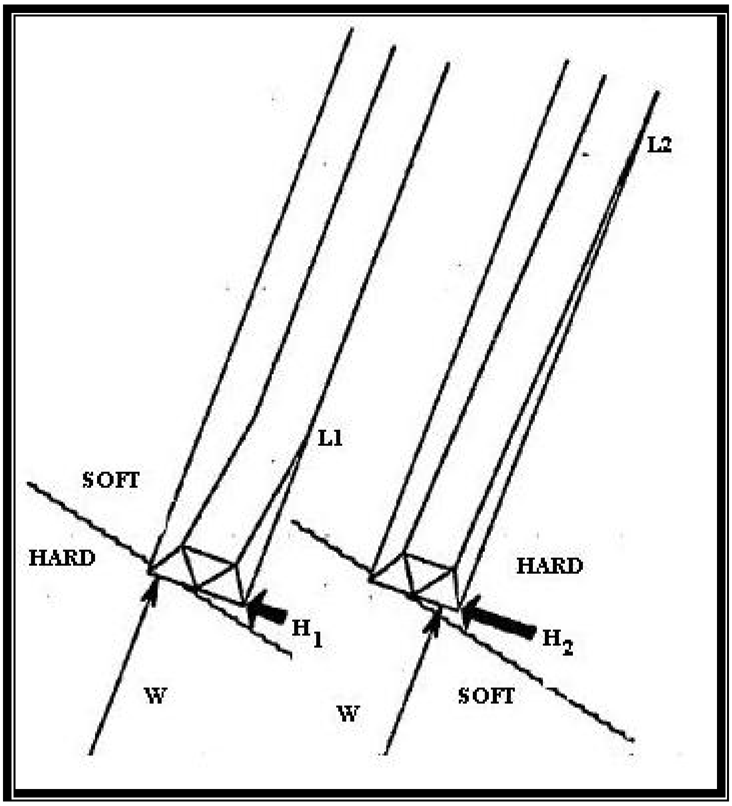

Fig.15 Drill Collar Moment in Drilling of Dipping Formations

2.2.5 Bit Steerability Index

Mechanics of BHA has received much more serious attention compared to formation and drill bit interaction in their relation to the effect on the direction of drill bit penetration so far.

The drilling ratio R was proposed by Bradley[15] (1975) to describe a bit's drilling ability with respect to angular direction, where R is defined as;

$$R = \frac{R_{\eta}}{R_{\eta=0}} \qquad (2\text{-}2)$$

where Rη is the drilling rate of the bit at an angle η in an isotropic rock and Rη=0 is the drilling rate of the same bit at η=0 at the same force in the same rock.

Bradley showed that mill tooth or insert bits have much stronger preference for drilling forward while the diamond bits are designed with more of a cutting structure along the lateral face of the bit. He also showed that for normal drilling situation (high WOB), the assumption that the bit drills in the direction of the resultant force in 'isotropic' rock appears reasonable, although his works were based on conceptual bits.

The study of bit behavior in anisotropic formation was performed by Cheatham[16]. In his work, it is assumed that rock drillability can be described by three constants representing the drilling rate in three orthogonal directions and the bit itself can be represented by two constants representing the drilling rate for the bit along the axis (face) and in the radial direction (side).

However, Cheatham's work requires support of experimental data to determine the bit constants (side and face) and the three orthogonal rock constants. Millhiem and Warren[17] conducted a full-scale experiment to measure the side cutting characteristics of a bit or stabilizer. They concluded that bits or stabilizers will cut laterally. The side-cutting rate is a function of penetration rate, contact force, component design, and rock type.

The concept of Bit Anisotropy Index (BAI) has been discussed in the literature so far. However, a slightly different term is used for Bit Anisotropy. The Bit Steerability Index (BSI) is defined as the relative difference of rock bit ability to drill in an axial and the lateral direction.

CHAPTER 3

STATEMENT OF THE PROBLEM

Recently, number of directional wells drilled with steerable bottom hole assemblies is increasing rapidly. Steering tools for these wells rises up the drilling costs. The aim of this study is to investigate possibilities for replacing steering tools with simpler BHA's to drill cheaper where offset well data is available. In order to understand the mechanical behavior of the drillstring, BHA has to be analyzed through theoretical force analysis. In this study the BHA is categorized in two groups. The first one is the slick assembly which has usages in some specific well conditions while the other is the single stabilizer assembly which is more commonly used in the industry than the slick assembly.

In the first phase, free body diagrams of both BHA types are analyzed through Miska[18] and Lubinski[4] 's derivations. Their studies are addressing the side forces playing on the bit which is the main parameter in hole deviation. Using the derived formulae, a directional well that is drillied with steerable BHA will be analyzed and, drilling the same well with a slick assembly will be discussed. A computer program will be built and discussed for the slick assembly model in order to determine the side forces and the drilling direction.

Simulation of different drilling parameters and conditions for both slick and single stabilizer assemblies will be used for preparing figures. The graphics will be analyzed and discussed, in order to reach general conclusions for directional drilling.

CHAPTER 4

MATHEMATICAL MODELING

The derivations of Miska[18] and Lubinski[4] will be used for modeling 2 basic BHA types. The first type is the slick assembly which has no stabilizers attached, where the second type has 1 stabilizer attached. Fig.16 shows the both BHA types.

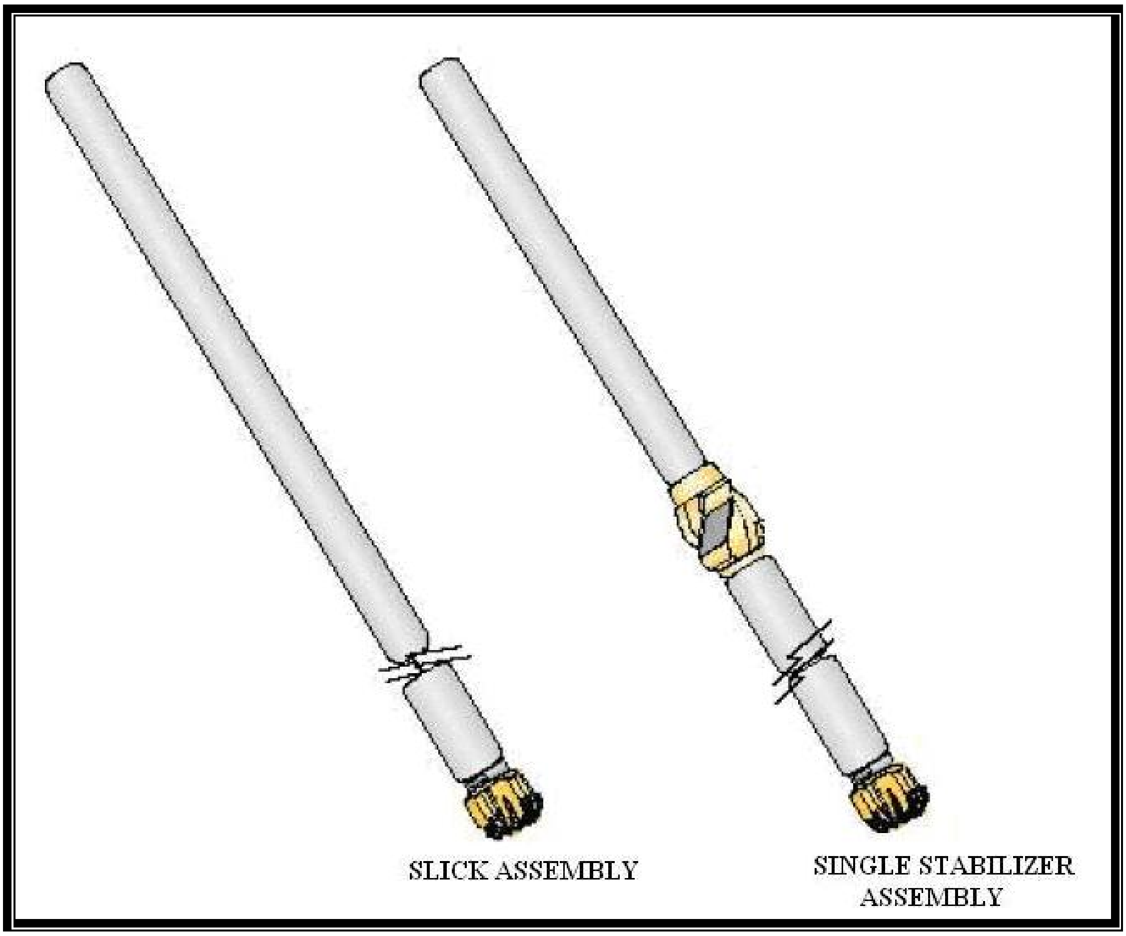

Fig.16 BHA Illustrations Used in Simulations

Slick and Single stabilizer BHA can be modeled starting with the following assumptions;

1. All BHA members are uniform all through the string
2. The wellbore walls are rigid and the bore is in gage
3. The X-coordinate is always tangent to the center line of the wellbore at the initial
4. Above the bit, drill collars contact the wall of the hole at “Point of tangency”. Above that point, the drill string lies on the lower side of the wellbore
5. The drill string behaves elastically
6. Moment at bit equals to zero
7. Dynamic effect of the drillstring and the drilling fluid are ignored
8. WOB is much bigger than the axial component of the weight of the drill collars

4.1 Slick Assembly Modeling

Fig.17 illustrates the free body diagram of a slick assembly in a straight inclined wellbore. Let’s consider the cross section MM’ at point P(X,Y). From the elementary beam equation,

$$S = EI\frac{d^3Y}{dX^3} \tag{4-1}$$

$$S = pX\sin(\alpha+\beta) - WOB\sin\beta + H_o\cos\beta \tag{4-2}$$

where S: Shear force at point P

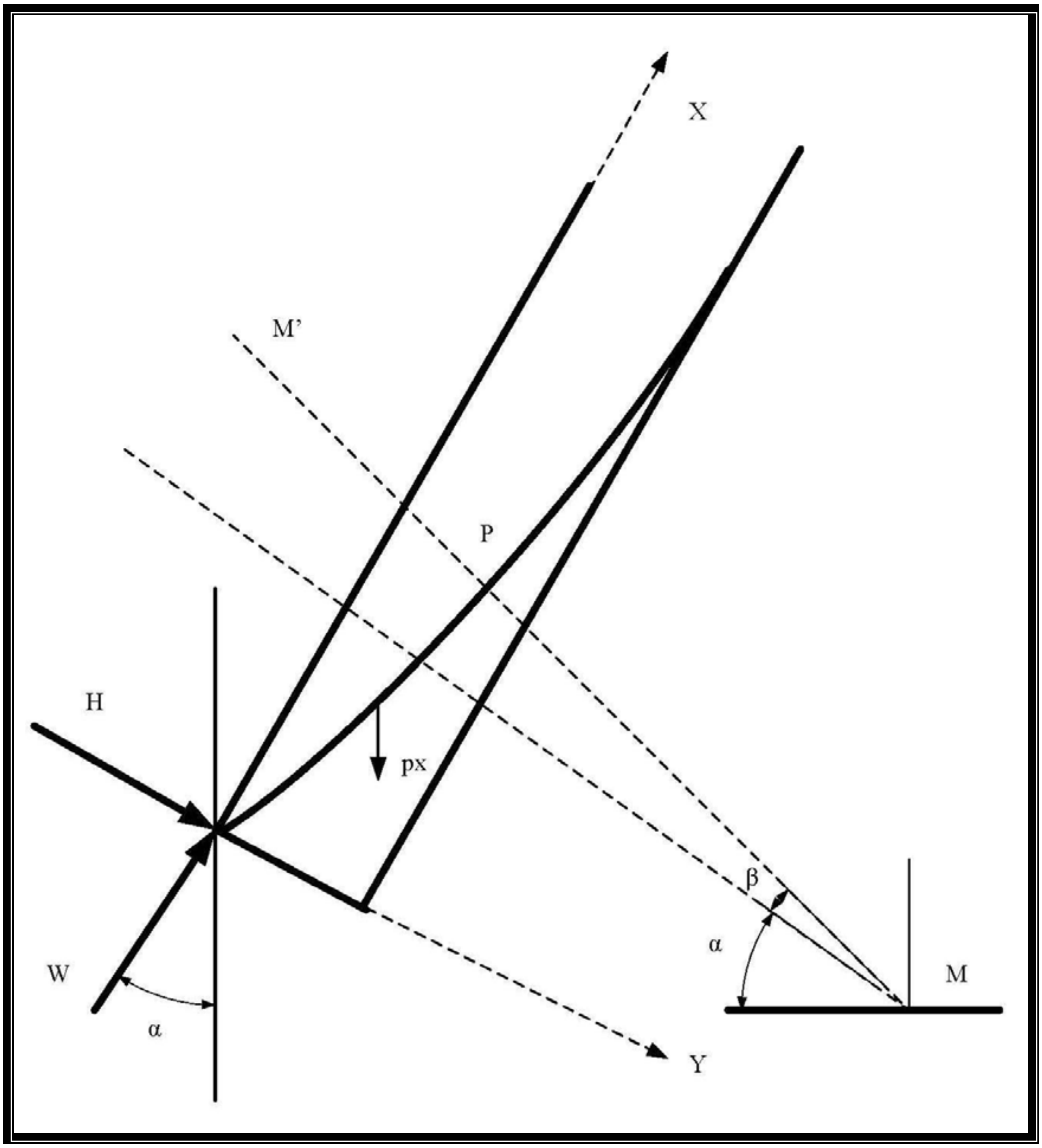

Fig.17 Free Body Diagram of a Slick Assembly

By taking into account all forces acting on the M-M' cross-section;

$$\frac{EI}{\cos\beta}\frac{d^3Y}{dX^3} = pX\sin\alpha - (WOB - pX\cos\alpha)\tan\beta + H_o \qquad (4\text{-}3)$$

For large WOB's (larger than the axial component of the pendulum weight of drill collar) Equation (4-3) is transformed to;

$$EI\frac{d^3Y}{dX^3} = pX\sin\alpha - WOB\frac{dY}{dX} + H_o \qquad (4\text{-}4)$$

In order to simplify the solution of equation (4-4), it can be written in dimensionless form as;

$$\frac{d^3y}{dx^3} = x\sin\alpha - \frac{dy}{dx} + h_o \quad . \qquad (4\text{-}5)$$

$$y = c_1 + c_2\sin x + c_3\cos x + \frac{x^2}{2}\sin\alpha + h_o x \qquad (4\text{-}6)$$

c_1, c_2, c_3, h_o, are unknowns of equation (4-6). In order to solve the equation, boundary conditions must be defined.

4.1.1 Boundary Conditions for Slick BHA

The boundary conditions are chosen at the bit and at the point of tangency.

At bit;

$$x = 0$$
$$y = 0 \Rightarrow y' = 0$$
$$\Rightarrow y'' = 0$$

At point of tangency;

$$x = l$$
$$y = \frac{r}{m_1} \Rightarrow y' = 0$$
$$\Rightarrow y'' = 0$$

In order to determine the side force at the bit, which is crucious in calculating the directional behavior of the well, and the length of the point of tangency, namely H_o and l, equation (4-6) must be solved with the boundary conditions.

Detailed derivations and solutions can be found in Appendix-C and Appendix-D.

4.2 Single Stabilizer Model in Straight Inclined Wellbore

In both vertical and directional directional drilling, the usefulness of slick BHA is limited, due to the restrictions on WOB and the geometrical properties of the drill collars. In order to perform a better degree of control in directional drilling, stabilizers are used at different positions from the bit. Primary purpose of a stabilizer is to control the drilling deviation while a near bit stabilizer is sometimes recommended for keeping the bit on its axis and thus supplying more life to the bit by protecting the bearings and the cutting structure. Fig. 18 illustrates the free body diagram of an assembly with one stabilizer lying in an inclined wellbore. The placement of stabilizer plays an important part in controlling the side force at the bit.

Selection of the proper type of stabilizer and placing it is usually based upon the analysis of the drilling data from offset wells. Welded blade stabilizer can be used in soft formations while integral blade stabilizers are recommended in hard formations is one example for the usage.

If the stabilizer is placed far enough from the bit, then the unsupported length of drill collar will increase. This tends to increase the side force at the bit and will cause the BHA to drill toward vertical. Such practice is used if the drilling engineer intends to reduce the inclination angle of a wellbore. This is known as the ‘pendulum effect’.

On the other hand, if the stabilizer is placed closer to the bit, side force will decrease and it is also possible that the side force will change in sign (direction). The bit will be pushed toward the high side of the hole and creating the ‘fulcrum effect’. This will cause the BHA to increase the wellbore inclination angle or building hole. Zero in side force is also possible by arranging the stabilizer position. In this case, the BHA will keep drilling straight ahead or no change in the wellbore inclination. Other advantage of BHA with stabilizer is to reduce the chance for differential sticking of the drill collars. Supporting the drill collar will keep the drill collar off the wellbore wall thereby eliminating contact with the mud cake and causing differential sticking.

However, stabilizers also create problems during drilling. Disadvantage of the stabilizer is the requirement for drilling trips to change the stabilizer placement to accommodate changes in drilling direction.

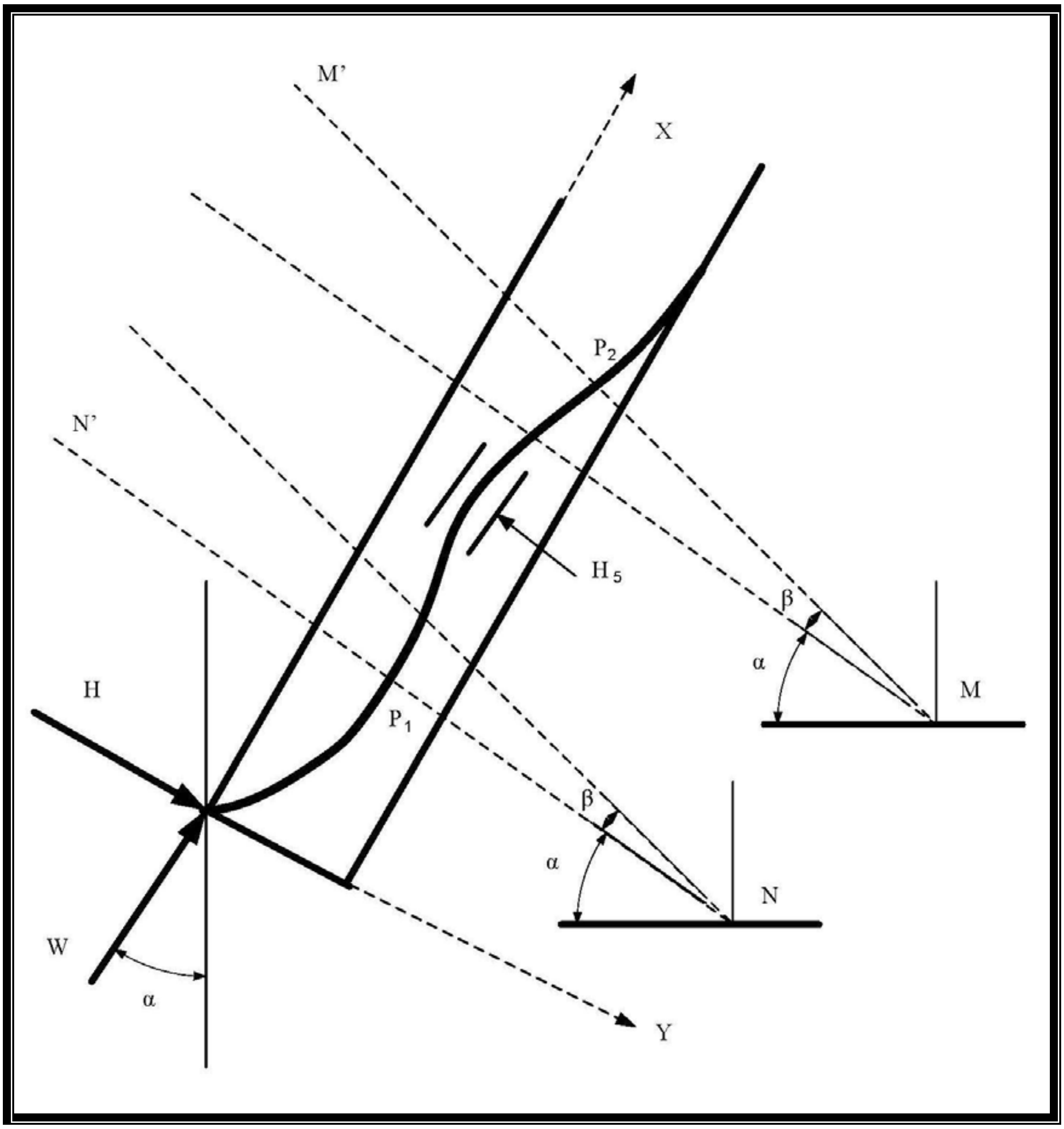

Fig.18 Free Body Diagram of a Single Stabilizer Assembly

In modeling single stabilizer assembly, the BHA has to be divided into two segments and analyzed separately. The first segment of the BHA in Fig.17 is the part of the drill string between the bit and the stabilizer ($0 < X < X_1$). X_1 is the distance of the stabilizer from the bit. The second segment is the part of drill string located above the stabilizer to the drill collar contact point (point of tangency, l).

The stabilizer is modeled as a point type stabilizer, and in this model, it is forced to contact the wellbore. Consequence of this is a reaction force will be presented at the stabilizer, the stabilizer force, H_{ST}. The differential equation for the first segment can be written as follows;

$$\mathrm{E}l\,\frac{d^3Y_{A,N}}{dX^3_{A,N}} + W\,\frac{dY_{A,N}}{dX_{A,N}} = H_o + pX_{A,N}\sin\alpha_N \tag{4-7}$$

Equation (4-7) is valid only for $0 < X_{A,N} < X_1$. While for the second segment, it is desired to take into account the presence of stabilizer force, H_{st}, and hence;

$$\mathrm{E}l\,\frac{d^3Y_{B,N}}{dX^3_{B,N}} + W\,\frac{dY_{B,N}}{dX_{B,N}} = H_o - H_{ST} + pX_{B,N}\sin\alpha_N \tag{4-8}$$

Where the equation is valid only for $X_1 < X_{B,N} < l$

It is noticeable that a positive side force at the bit H_o occurs if the bit is pushed toward the high side of the hole. The positive side force at the stabilizer, however, is for the stabilizer pushing on the lower part of the hole. It is also important that the sign of the side force and the sign of the deflection at the stabilizer must be consistent, e.g. both positive or both negative. For certain equilibrium configurations, the stabilizer does not contact either the upper or the lower side of the hole. In such a case, we say that the stabilizer is floating and the side force at the stabilizer is nill[18].

4.2.1 Boundary Conditions for Single Stabilizer BHA

Sets of boundary conditions for equation 4-7 and 4-8 can be listed as follows;

At Bit, X = 0;

$$Y_{A,N}(0) = 0$$

$$Y''_{A,N}(0) = 0$$

At Stabilizer, X = X_1;

$$Y_{A,N}(X_1) = Y_{B,N}(X_1)$$

$$Y''_{A,N}(X_1) = Y''_{B,N}(X_1)$$

$$Y'_{A,N}(X_1) = Y'_{B,N}(X_1)$$

At Tangency Point, X = L;

$$Y_{B,N}(LN) = R_N = Rcl + (L_N - F)\tan(\alpha_{N-1} - \alpha_N)$$

$$Y''_{B,N}(L) = 0$$

$$Y'_{B,N}(L) = 0$$

Equation (4-8) can be written in dimensionless form as;

$$y''' = \frac{\mathrm{d}^3\mathrm{y}}{\mathrm{dx}^3} = d\frac{dy^2}{dX^2} \div dX\frac{dX}{dx} = \frac{m_2^2}{m_1}y'''m_2 = \frac{m_2^3}{m_1}Y''' \qquad (4\text{-}9)$$

Rewriting;

$$\frac{m_1Wy'''}{m_2^2 p\sin\alpha} + \frac{m_1W^2y'}{\mathrm{E}lp\sin\alpha} = \frac{m_3m_2Wh}{p\sin\alpha} + \frac{m_2^2Wx}{\mathrm{E}l} \qquad (4\text{-}10)$$

Introducing the scaling parameters m_1, m_2, and m_3 the dimensionless forms of the differential equations are written in the form;

$$Y'''_a + Y'_a = h_o + x_a \quad .. \quad (4\text{-}11)$$

$$Y'''_b + Y'_b = h_o - h_{stb} + x_b \quad . \quad (4\text{-}12)$$

Solving the equations 4-11 and 4-12 along with the corresponding boundary conditions we obtain;

$$\sin(x_1 - l) + (1 + h_1)\cos(x_1 - l) - \sin x_1 + (\cot x_1)(1 + c_{st} - \cos x_1 - 0.5x_1^2 - h_o x_1) + h_{st}$$

(4-13)

$$h_0 = \frac{1 + c_{st} - 0.5x_1^2 - \cos(x_1 - l) + (l + h_1)\sin(x_1 - l)}{x_1} \quad (4\text{-}14)$$

$$h_1 = h_0 - h_{st} = \frac{c_{st} - c - \cos(x_1 - l) + l\sin(x_1 - l) - 0.5(x_1^2 - l^2) + 1}{x_1 - l + \sin(l - x_1)} \quad (4\text{-}15)$$

Analysis of equations 4-13, 4-14, 4-15 leads to a conclusion that once the dimensionless distance to the stabilizer, x_1 is known, then equation 4-13 can be solved for the dimensionless distance to the point of tangency, l. As the l value is obtained, the dimensionless side forces at the bit and the stabilizer can be found from equations 4-14 and 4-15 respectively.

To calculate the drilling direction in isotropic and anisotropic conditions equations 4-5 and 4-6 can be applied.

Solutions to Equation 4-9 are shown in Appendix E.

CHAPTER 5

TRANSIENT TRAJECTORY ANALYSIS SIMULATIONS

5.1 Direction of Drilling

5.1.1 Slick Bottom Hole Assembly

In order to figure out a drill ahead model, the first step is to determine pre-known parameters those to be used with the assigned formulas, which are listed as follows:

Hole diameter: 12.25”
Outside diameter of drill collars: 8.0”
Inside diameter of drill collars: 3.0”
Hole inclination angle: 3^0
Mud weight: 11 ppg
Weight on bit: 20000 lb
Formation dip: 15^0
FAI: 0.045

$$K_b = 1 - \frac{\rho_m}{\rho_{st}} \Rightarrow K_b = 1 - \frac{11}{65.5} = 0.83$$

$$p = W_{DC} \times K_b \Rightarrow p = 147 \times 0.83 = 122 lb / ft$$

Moment of inertia;

$$I = \frac{\pi}{64}(OD^4 - ID^4) \Rightarrow I = \frac{\pi}{64}[(\frac{8}{12})^4 - (\frac{3}{12})^4] = 9.5\times10^{-3}\ ft^4$$

$$m_1 = \frac{EIp\sin\alpha}{W^2} \Rightarrow m_1 = \frac{4320\times10^6\times9.5\times10^{-3}\times122\times\sin 3}{20000^2} = 0.655\,ft$$

$$m_2 = \sqrt{\frac{\mathrm{E}l}{W}} \Rightarrow m_2 = \sqrt{\frac{4320\times10^6\times9.5\times10^{-3}}{20000}} = 45.3\,ft$$

$$m_3 = EI\frac{m_1}{m_2^3} \Rightarrow 4320\times10^6\times9.5\times10^{-3}\times\frac{0.655}{45.3^3} = 289.2lb$$

The apparent wellbore radius is;

$$r = \frac{0.5}{12}(12.25 - 8) = 0.177\,ft$$

$$\frac{r}{m_1} = l\tan\frac{l}{2} - \frac{l^2}{2} \Rightarrow 0.270$$

Solving for l;

$l = 1.49$

$$h_o = \tan\frac{l}{2} - l \Rightarrow h_o = -0.567$$

$$H_o = m_3\times h_o \Rightarrow 289.2\times(-0.567) = -163.98lb$$

Which is side force at the bit. If the formation was isotropic, the BHA would have dropping tendency as the sign of the side force is negative. To determine the directional tendency in anisotropic and dipping formation;

$$\phi = \arctan(\frac{H_o}{WOB}) + \alpha$$

$$\phi = 2.53^0$$

$$\psi = \gamma - \arctan[(1-h)\tan(\gamma - \phi)]$$

$$\psi = 3.1^0$$

Since the instantaneous rock bit displacement "ψ" is greater than the initial hole inclination "α" the assembly will have building tendencies. It is therefore apparent that whether BHA will have building or dropping tendency depends not only upon the BHA composition and WOB, but also the formation-bit interaction.

5.1.2 Single Stabilizer Bottom Hole Assembly

The example hole parameters are as follows;

Hole diameter: 12.25"
Outside diameter of drill collars: 9.0"
Inside diameter of drill collars: 3.0"
Stabilizer location: 15 ft above the bit
Hole inclination angle: 25^0
Mud weight: 10.5 ppg
Weight on bit: 55000 lb
Formation dip: 0, isotropic

$$K_b = 1 - \frac{\rho_m}{\rho_{st}} \Rightarrow K_b = 1 - \frac{10.5}{65.5} = 0.84$$

$$p = W_{DC} \times K_b \Rightarrow p = 192 \times 0.84 = 161.2 lb/ft$$

Moment of inertia;

$$I = \frac{\pi}{64}(OD^4 - ID^4) \Rightarrow I = \frac{\pi}{64}[(\frac{9}{12})^4 - (\frac{3}{12})^4] = 1.53 \times 10^{-2} ft^4$$

$$m_1 = \frac{EIp \sin\alpha}{W^2} \Rightarrow m_1 = \frac{4320 \times 10^6 \times 1.53 \times 10^{-2} \times 161.2 \times \sin 25}{55000^2} = 1.492 ft$$

$$m_2 = \sqrt{\frac{\mathrm{E}I}{W}} \Rightarrow m_2 = \sqrt{\frac{4320 \times 10^6 \times 1.53 \times 10^{-2}}{55000}} = 34.701 ft$$

$$m_3 = EI\frac{m_1}{m_2^3} \Rightarrow 4320 \times 10^6 \times 1.53 \times 10^{-2} \times \frac{1.492}{34.701^3} = 2360 lb$$

Dimensionless distance to the stabilizer;

$$x_1 = \frac{15}{34.701} = 0.432$$

Dimensionless clearance at the stabilizer and at point of tangency;

$$c_{st} = \frac{0.5 \times 0.25}{12 \times 1.492} = 0.007$$

$$c = \frac{0.5x(12.25 - 9)}{12x1.492}$$

Solving numerically equations 4-13, 4-14, 4-15 ;

$$l = 1.835$$

From equation 4-13;

$$h_1 = 1.2664$$

Consequently the dimensionless side forces;

$h_o = 0.426$ hence; $H_o = 1007lb$

From equation D-2, resultant force angle is;

$$\phi = \arctan(\frac{1007}{55000}) + 25 = 26.05^0$$

Which means that hole is in building tendency with these parameters, however, if the stabilizer is placed 60 ft above the bit instead of 15 ft;

$$K_b = 1 - \frac{\rho_m}{\rho_{st}} \Rightarrow K_b = 1 - \frac{10.5}{65.5} = 0.84$$

$$p = W_{DC} \times K_b \Rightarrow p = 192 \times 0.84 = 161.2lb/ft$$

Moment of inertia;

$$I = \frac{\pi}{64}(OD^4 - ID^4) \Rightarrow I = \frac{\pi}{64}[(\frac{9}{12})^4 - (\frac{3}{12})^4] = 1.53 \times 10^{-2} ft^4$$

$$m_1 = \frac{EIp\sin\alpha}{W^2} \Rightarrow m_1 = \frac{4320\times10^6\times1.53\times10^{-2}\times161.2\times\sin 25}{55000^2} = 1.492\,ft$$

$$m_2 = \sqrt{\frac{EI}{W}} \Rightarrow m_2 = \sqrt{\frac{4320\times10^6\times1.53\times10^{-2}}{55000}} = 34.701\,ft$$

$$m_3 = EI\frac{m_1}{m_2^3} \Rightarrow 4320\times10^6\times1.53\times10^{-2}\times\frac{1.492}{34.701^3} = 2360lb$$

Dimensionless distance to the stabilizer;

$$x_1 = \frac{60}{34.701} = 1.729$$

Dimensionless clearance at the stabilizer;

$$c_{st} = \frac{0.5\times0.25}{12\times1.492} = 0.007$$

Dimensionless clearance at point of tangency;

$$c = \frac{0.5\times(12.25-9)}{12\times1.492} = 0.09$$

Solving numerically equations 4-13, 4-14, 4-15;

$$l = 3.267$$

From equation 4-13;

$$h_1 = -2.71$$

Consequently the dimensionless side forces;

$h_o = 0.63$ hence; $H_o = -1482lb$

From equation D-2, resultant force angle is;

$$\phi = \arctan(\frac{-1482}{55000}) + 25 = 23.5^0$$

For the first assembly (stabilizer 15 ft above bit) the drillstring will possess building tendencies while for the second assembly (stabilizer 60 ft above the bit) the string will possess dropping tendency which shows the effect of stabilizer location.

In both cases, the formation is assumed to be isotropic, for an anisotropic case where the formation dip is 15^0 and the formation anisotropy index is found to be 0.045, following results are obtained:

For the first case (Stabilizer 15 ft above bit);

$$\psi = 15 - \arctan[(1 - 0.045)]\tan[(15 - 26.05)] = 25.6^0$$

For the second case (Stabilizer 60 ft above bit);

$$\psi = 15 - \arctan[(1 - 0.045)]\tan[(15 - 23.5)] = 23.1^0$$

CHAPTER 6

RESULTS AND DISCUSSION

Calculating the drilling direction of a given assembly is the first step in a transient trajectory prediction. To perform this calculation, the BHA and wellbore properties such as the diameter of the hole, the type of BHA used, if present, stabilizer diameter are required, as well as the operating parameters and the formation parameters.

Scenarios for different wells are used to simulate actual wells, followed by preparation of the graphics and commenting on them. The types of BHA's used in the simulations are both single stabilizer and slick assemblies. In the single stabilizer BHA, the place of stabilizer varies from 45 to 60 ft from the bit. The formation varies from anistropic to isotropic to cover different geologies and comment on the factors independent of the formation. The hole size is 12.25" and the mud used in the drilling is 11 to 12.5 ppg.

The computer program developed to solve the single stabilizer case, a bisection method is employed to solve the equations 4-13, 4-14, 4-15. As the wellbore radius, outer diameter of DC'S, place of stabilizer, stabilizer clearance, initial hole angle, mud weight and weight on bit is given as input, the side force and the final hole inclination are the outputs. To note, computer program codes which are given in Appendix A, are for 15^0 dipping formation and FAI is 0.045. Parameters can be changed from the formula

Fig.19 and Fig.20 are drawn for an initially inclined 12 ¼” hole drilled with a slick assembly. 9”x3” 147 lb/ft drill collars are used for this sample well. MW is 11 ppg. Analysis of Fig.19 leads to a conclusion that as the WOB value is increased, the side force increases. It is desirable to note side forces are negative for this situation which means in isotropic and non-dipping conditions a dropping tendency is expected that is clearly seen in Fig.20.

Fig.20 is drawn for 3 different geological conditions, as for isotropic condition, 15 degree dipping formation and 20 degree dipping formation. FAI is 0.045 for dipping formations As the initial inclination of the well is 3^0, it is obvious that for the isotropic conditions the well has a decreasing trend of dropping tendency as the weight on bit is increased.

For the 15 degree dipping formation the well is expected to build angle in an increasing trend as the weight on bit value is increased. In a 20 degree dipping formation, the assembly is in a bigger value of building angle. In three conditions inclination seems to be linearly increasing with the increasing WOB up to a certain value, but above that the linearity of the slope is not carried.

Fig.19 WOB vs Side Force for Slick Assembly

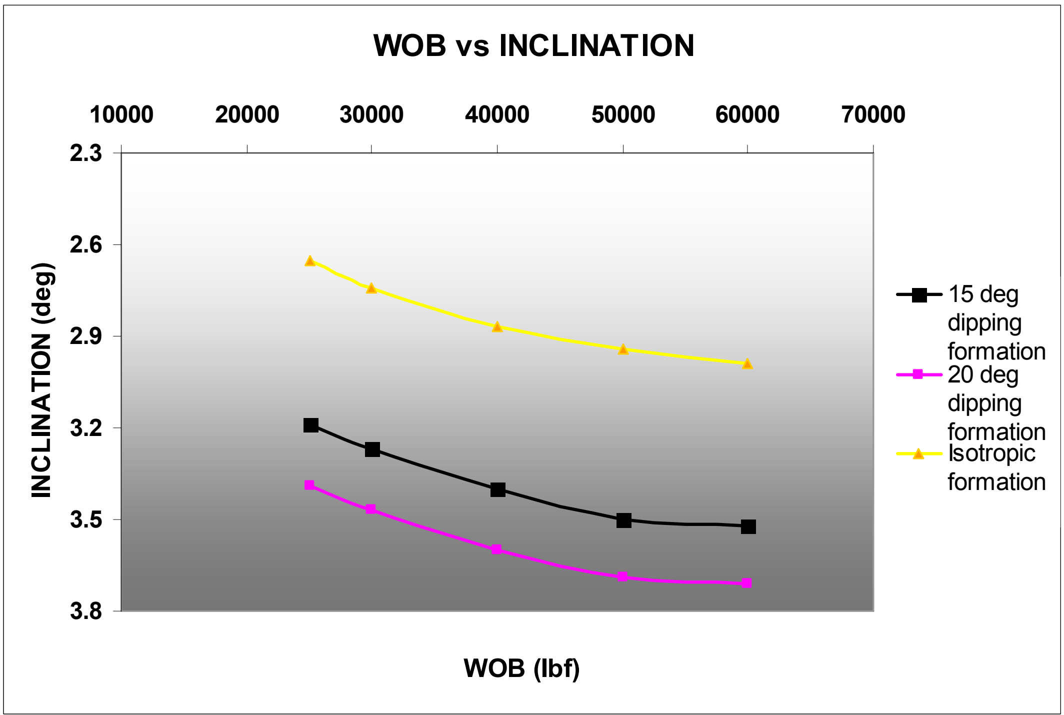

Fig.20 WOB vs Inclination for Slick Assembly

Next sample is a 12 ¼" initially 3^0 inclined hole drilled with two different BHA members and at different formation dips. The BHA is a single stabilizer assembly. Stabilizer diameter is 12" and placed 60 ft above the bit. FAI is 0.045 for dipping formations. WOB is 80000 lbs Fig.21 shows the relationship between the initial hole inclination and side force for two different bottom hole assemblies. The bottom hole assembly consisting of 8" drillcollars tend to have bigger side forces than the 9" one in all initial inclination scenarios, which means that bottom hole assembly composed of 8" drillcollars will have more building tendency than the 9" one for given anisotropic conditions.

Fig.22 shows the trend of a BHA composed of 8"x3" 147 lb/ft DC's at 3 different geological formations. As clearly seen from the graph, 15 degree dipping formation is in building trend while 10 degree dipping formation is in dropping trend for bigger initial inclination values. For smaller initial inclinations both BHA's are building. For three of them it can be declared that as the initial inclination increases, the assembly is whether decreasing the build rate or increasing the drop rate.

Fig.23 shows the behavior of two different strings in 15 degree dipping formation. It is obvious that smaller diameter drill collars have more building, less dropping tendency than the larger diameter drill collars.

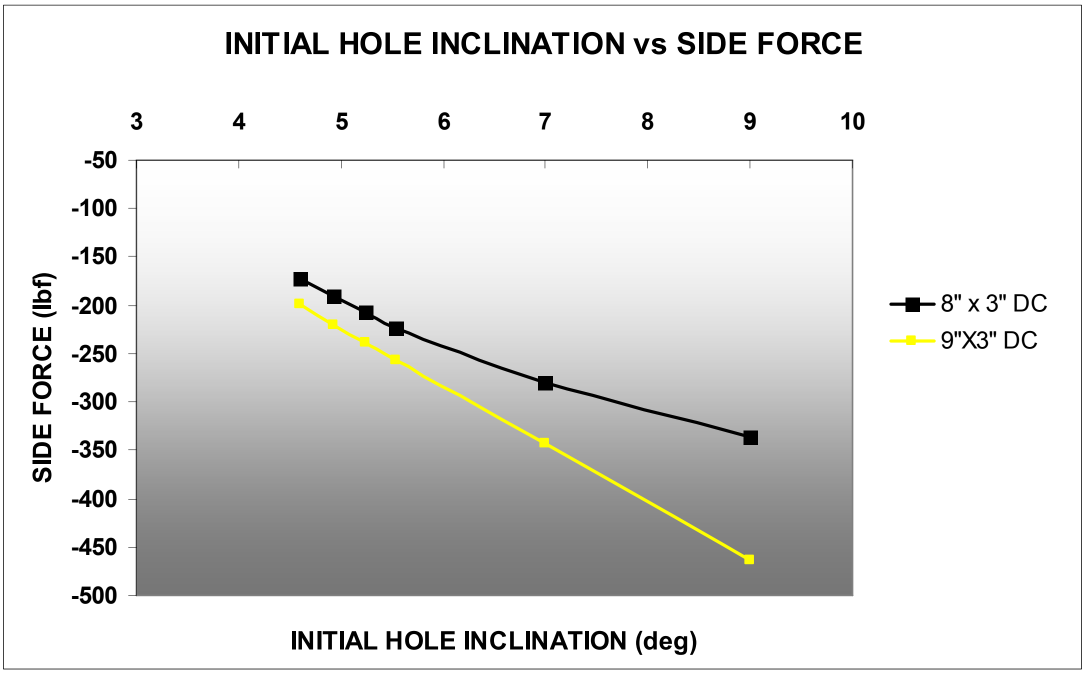

Fig.21 Initial Hole Inclination vs Side Force for Single Stabilizer Assembly

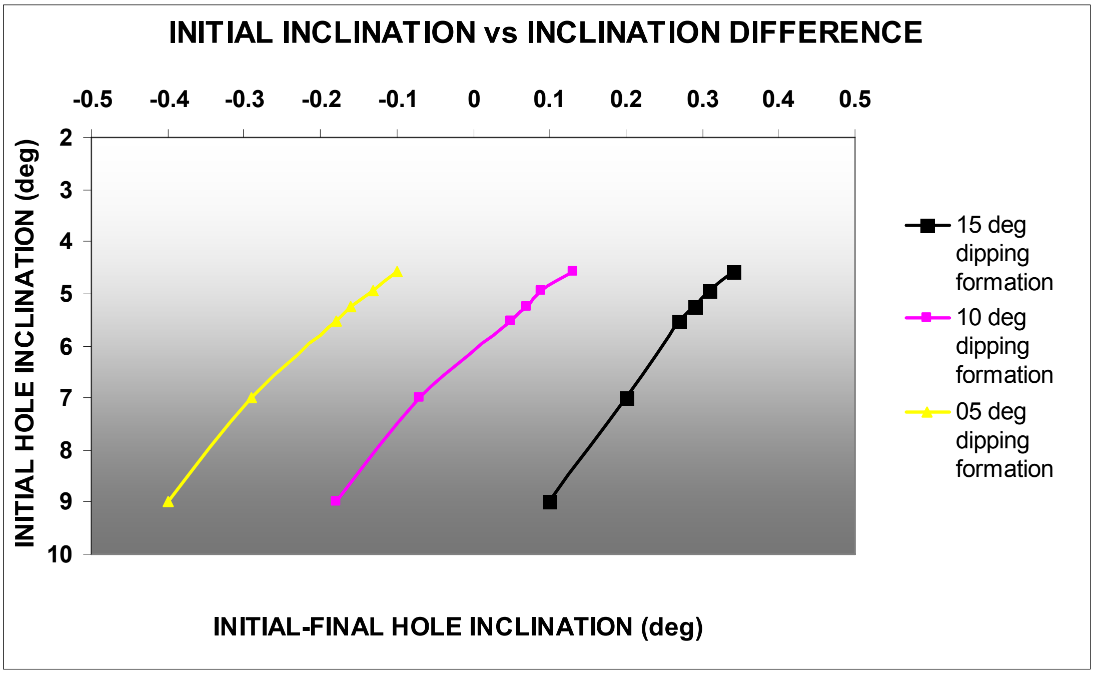

Fig.22 Initial Inclination vs Δ Inclination

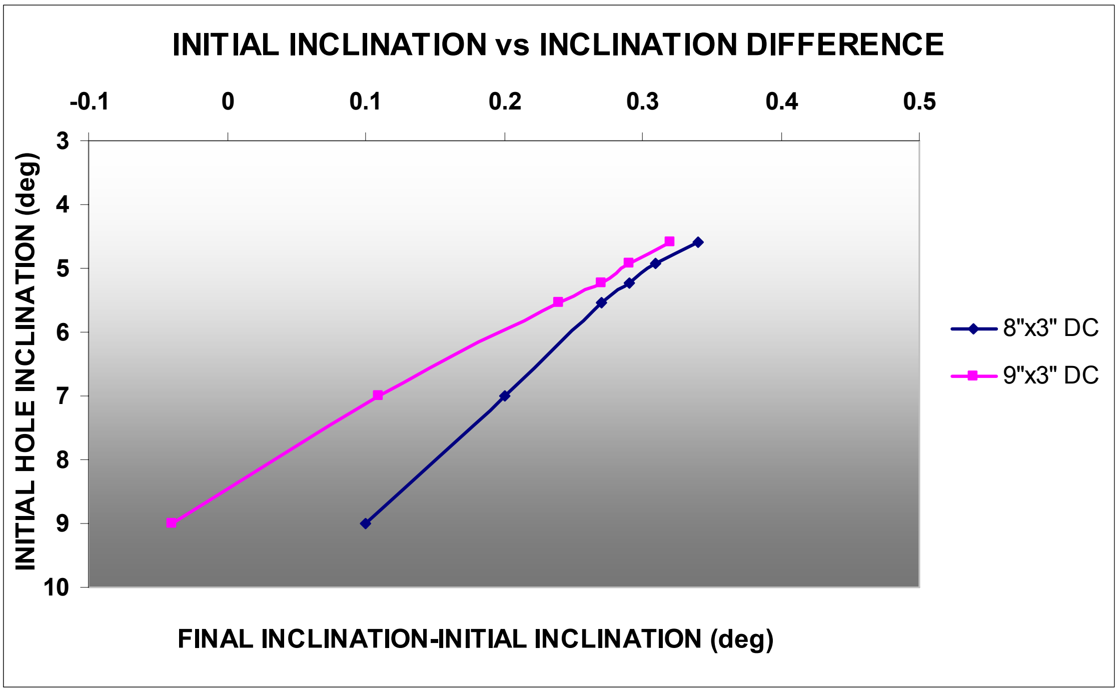

Fig.23 Initial Inclination vs Δ Inclination

Following sample is a 12 ¼" initially 3^0 inclined hole drilled with two different BHA members and at different formation dips. The BHA is a single stabilizer assembly. Stabilizer diameter varies from 12" to 12.25" and placed 60 ft above the bit. FAI is 0.045 for dipping formations. WOB is 65000 lbs. Fig.24 indicates that for a 12 ¼" well, as the stabilizer clearance decreases, side force also decrases which limits the building tendency of the string.

The same conclusion is gathered also from Fig.25 in three different geological conditions where FAI is 0.045 for dipping formations. As the dipping increases, the building tendency of the string increases which is also obvious in previous graphs.

Fig.26 examines the relationship between the FAI and the final inclination in two different geologies. As the FAI increases the final inclination also increases for both geologies which shows that the directional tendency is higher for anisotropic formations than the isotropic ones.

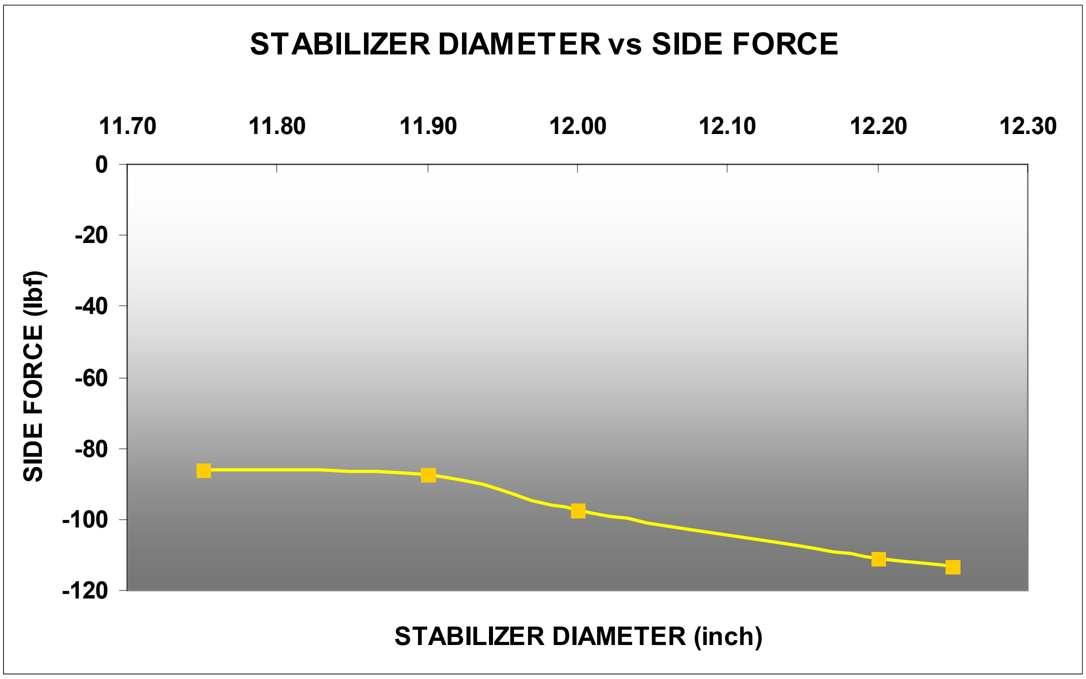

Fig.24 Stabilizer Diameter vs Side Force for Single Stabilizer Assembly

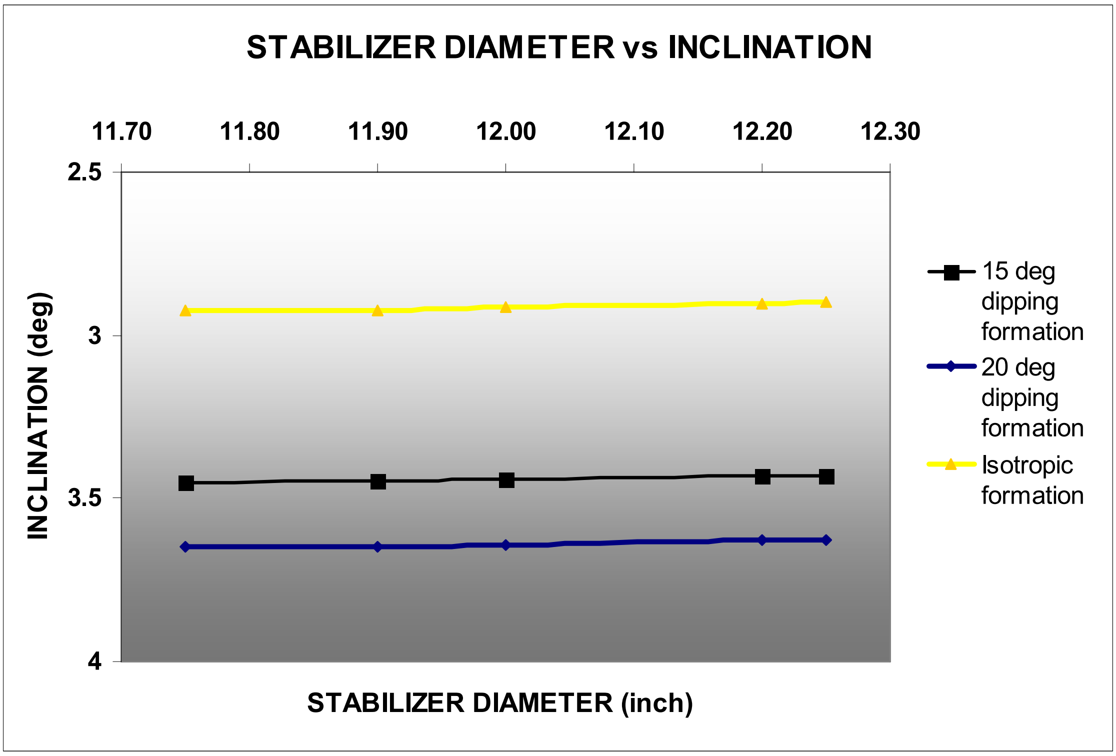

Fig.25 Stabilizer Diameter vs Inclination for Single Stabilizer Assembly

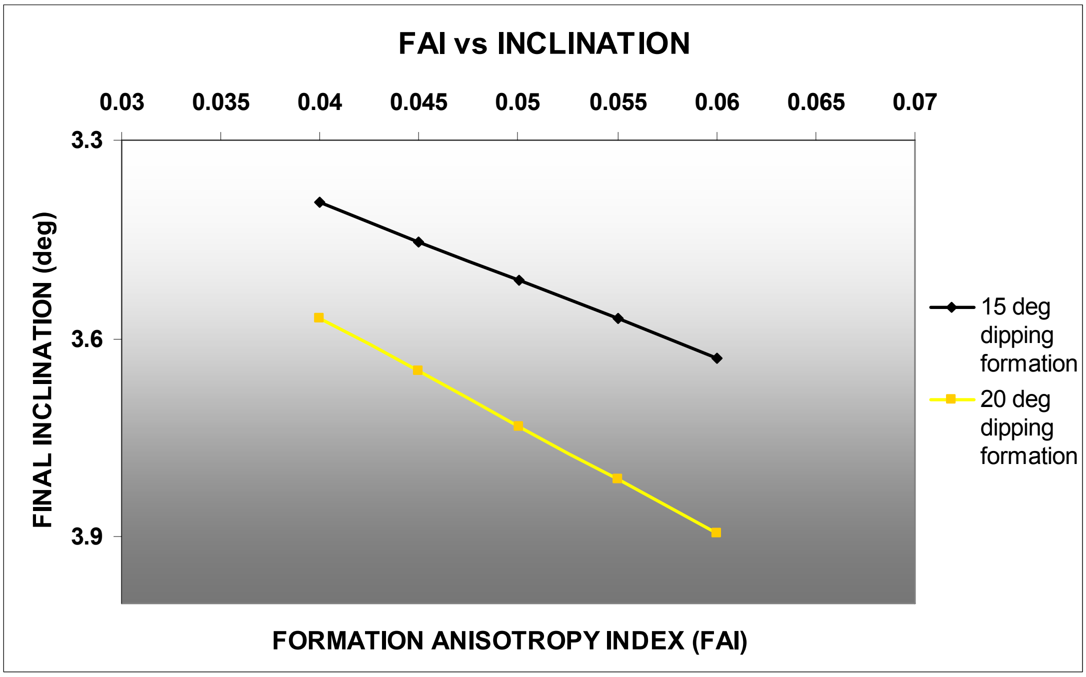

Fig.26 FAI vs Inclination for Single Stabilizer Assembly

6.1 Field Practices for Slick Assembly:

The X-1 well is drilled with a 12 ¼" insert bit. BHA is composed of 8"x3" 147 lb/ft slick Drill Collars and a bent sub made up to 1.4^0. MW is 11 ppg, WOB is 30000 lbs all through the curved section. KOP is 1992 m. with 3^0 inclination. Lithology is a mixture of marl and shale, consistent all through the wellFinal angle is 38.97^0. According to the logs, formation dip is 25^0. The azimuth of the last inclination just before the kick-off point is 320^0 while the final target azimuth is 306.3^0 which are very similar. Based upon the rig-site data, studying the behavior of a slick assembly without any direction tool at the same physical conditions is economically beneficial for further wells in the same field. Thus a theoretical analysis is conducted for the X-1 well. For the calculations, all drilling parameters are known, as mentioned above, formation dip angle is known from the previous logs, in order to obtain the "h" values of the field, the data above the Kick off Point is used as that interval was drilled with a slick assembly due to loss circulation problems.

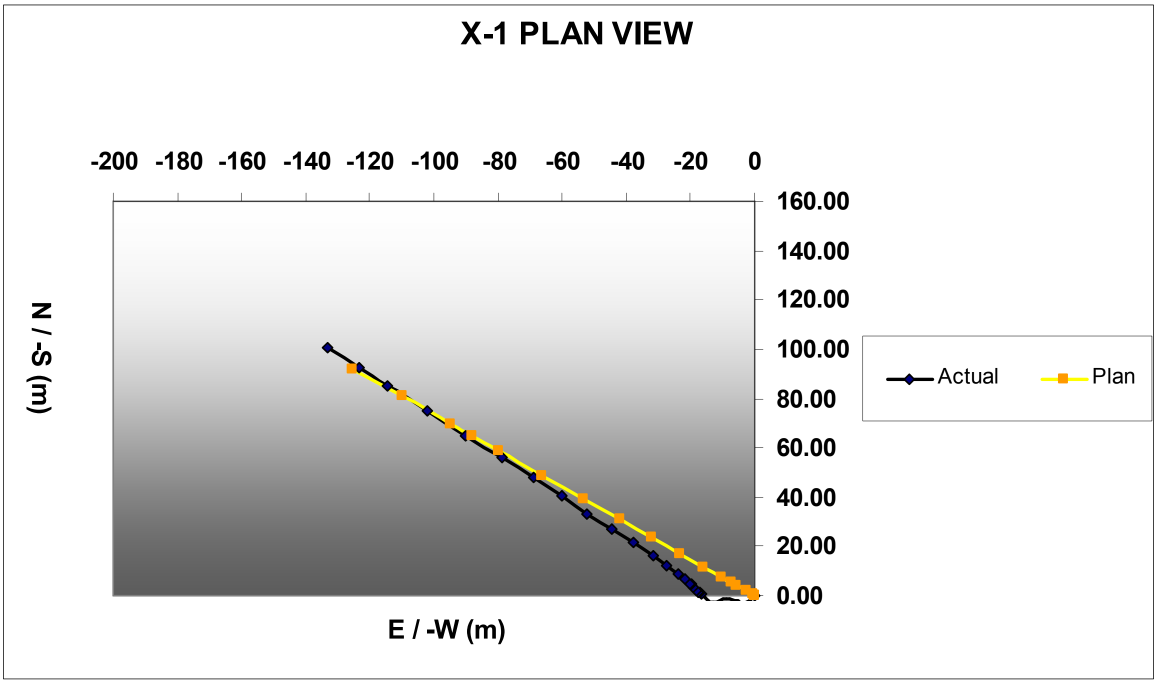

Fig.27 Plan View of X-1 Drilled with Steerable Assembly

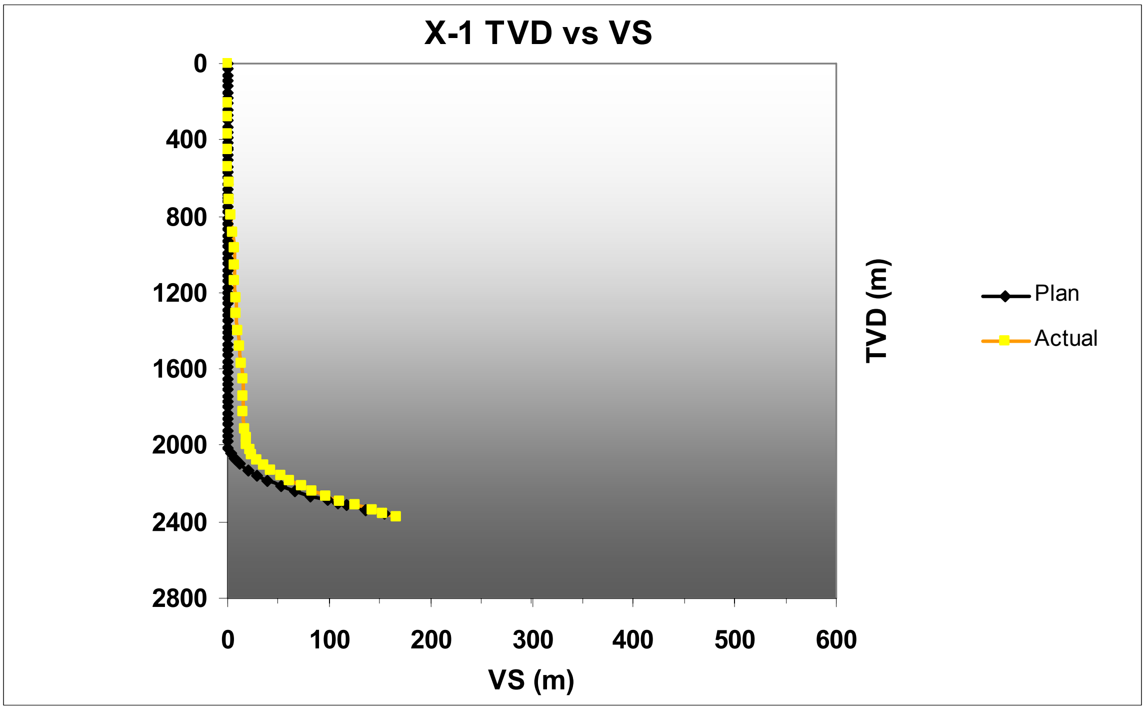

Fig.28 TVD vs VS of X-1 Drilled with Steerable Assembly

Lengths DLS =	30,0 m					Target Az. =	306,3 deg		
						Target TVD =	2370 m		
Stn No	**MD (m)**	**Inc.**	**Azim.**	**TVD (m)**	**N/-S (m)**	**E/-W (m)**	**VS (m)**	**DLS**	**Tool**
1	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	Whead
2	204,00	0,54	223,45	204,00	-0,70	-0,66	0,12	0,08	Actual
3	280,00	0,58	213,75	279,99	-1,28	-1,12	0,15	0,04	Actual
4	366,00	0,86	215,07	365,99	-2,17	-1,73	0,11	0,10	Actual
5	452,00	0,75	238,79	451,98	-2,99	-2,59	0,32	0,12	Actual
6	538,00	0,32	270,00	537,97	-3,28	-3,31	0,72	0,18	Actual
7	623,00	0,55	295,61	622,97	-3,10	-3,91	1,32	0,10	Actual
8	710,00	0,93	304,10	709,96	-2,53	-4,87	2,43	0,14	Actual
9	796,00	1,07	289,81	795,95	-1,86	-6,21	3,90	0,10	Actual
10	882,00	0,99	282,10	881,94	-1,44	-7,69	5,35	0,06	Actual
11	968,00	0,64	264,58	967,93	-1,33	-8,89	6,38	0,15	Actual
12	1054,00	0,71	240,10	1053,92	-1,64	-9,83	6,96	0,10	Actual
13	1140,00	0,81	242,04	1139,92	-2,19	-10,83	7,44	0,04	Actual
14	1225,00	0,90	250,37	1224,91	-2,69	-11,99	8,07	0,05	Actual
15	1311,00	0,67	282,65	1310,90	-2,81	-13,12	8,91	0,17	Actual
16	1397,00	1,32	319,08	1396,89	-1,95	-14,26	10,34	0,31	Actual
17	1483,00	1,50	317,19	1482,86	-0,38	-15,67	12,41	0,06	Actual
18	1569,00	0,70	317,92	1568,84	0,84	-16,79	14,03	0,28	Actual
19	1655,00	0,21	330,83	1654,84	1,37	-17,22	14,69	0,17	Actual
20	1741,00	0,24	320,00	1740,84	1,64	-17,41	15,00	0,02	Actual
21	1827,00	0,80	320,00	1826,84	2,24	-17,91	15,76	0,20	Actual
22	1913,00	1,75	320,00	1912,81	3,71	-19,14	17,62	0,33	Actual
23	1963,00	0,50	320,00	1962,80	4,46	-19,77	18,58	0,75	Actual
KOP	1992,00	3,00	311,20	1991,79	5,05	-20,43	19,45	2,59	Actual
24	2021,00	4,98	316,38	2020,72	6,47	-21,87	21,45	2,08	Actual
25	2050,00	7,50	316,95	2049,54	8,76	-24,03	24,55	2,61	Actual
26	2079,00	10,74	314,98	2078,17	12,05	-27,23	29,08	3,37	Actual
27	2108,00	14,19	310,96	2106,48	16,30	-31,83	35,30	3,68	Actual
28	2137,00	17,40	308,40	2134,39	21,32	-37,91	43,18	3,40	Actual
29	2165,00	19,41	309,87	2160,95	26,90	-44,76	52,00	2,21	Actual
30	2194,00	20,46	310,90	2188,21	33,31	-52,29	61,87	1,15	Actual
31	2223,00	22,43	310,58	2215,20	40,23	-60,33	72,44	2,04	Actual
32	2252,00	25,13	311,52	2241,74	47,91	-69,14	84,09	2,82	Actual
33	2280,00	28,20	309,17	2266,76	56,03	-78,72	96,62	3,48	Actual
34	2309,00	31,00	308,14	2291,97	64,98	-89,91	110,93	2,94	Actual
35	2338,00	33,38	309,48	2316,51	74,66	-101,95	126,36	2,57	Actual
36	2367,00	35,00	310,16	2340,50	85,10	-114,46	142,63	1,72	Actual
37	2386,00	37,20	309,14	2355,85	92,24	-123,08	153,80	3,60	Actual
38	2407,00	38,97	309,14	2372,38	100,42	-133,13	166,74	2,53	Actual

Fig.29 Surveys of X-1 Drilled with Steerable Assembly

Drilling Anisotropy index, h values from depth 1600 to 1992 are obtained by back calculation form the final angles. An example for the 1741-1828 m. interval is as follows;

$$K_b = 1 - \frac{\rho_m}{\rho_{st}} \Rightarrow K_b = 1 - \frac{11}{65.5} = 0.83$$

$$p = W_{DC} \times K_b \Rightarrow p = 147 \times 0.83 = 122 lb / ft$$

Moment of inertia;

$$I = \frac{\pi}{64}(OD^4 - ID^4) \Rightarrow I = \frac{\pi}{64}[(\frac{8}{12})^4 - (\frac{3}{12})^4] = 9.5\times10^{-3}\ ft^4$$

$$m_1 = \frac{EIp\sin\alpha}{W^2} \Rightarrow m_1 = \frac{4320\times10^6\times9.5\times10^{-3}\times122\times\sin 0.24}{30000^2} = 0.023\,ft$$

$$m_2 = \sqrt{\frac{\mathrm{E}l}{W}} \Rightarrow m_2 = \sqrt{\frac{4320\times10^6\times9.5\times10^{-3}}{30000}} = 37\,ft$$

$$m_3 = EI\,\frac{m_1}{m_2^3} \Rightarrow 4320\times10^6\times9.5\times10^{-3}\times\frac{0.023}{37^3} = 18.6lb$$

The apparent wellbore radius is;

$$r = \frac{0.5}{12}(12.25 - 8) = 0.177\,ft$$

$$\frac{r}{m_1} = l\tan\frac{l}{2} - \frac{l^2}{2} \Rightarrow 7.7$$

Solving for l;

l = 2.68

$$h_o = \tan\frac{l}{2} - l \Rightarrow h_o = 1.58$$

$$H_o = m_3\times h_o \Rightarrow 18.6\times1.58 = 29.3lb$$

$$\phi = \arctan(\frac{H_o}{WOB}) + \alpha$$

$$\phi = 0.3^0$$

$$\psi = \gamma - \arctan[(1-h)\tan(\gamma - \phi)]$$

$$0.8 = 25 - \arctan[(1-h)\tan(25-0.3)]$$

Solving the equation for h;

h= 0.045

From the back calculations from 1600 to 1992 meters, the average drilling anisotropy index, h is 0.045 which will be used in following calculations.

With the 12 ¼" slick assembly at 40000 lb Weight on Bit, 1992-2021 m. interval, where the angle is built from 3^0 to 4.98^0 with the steerable bottom hole assembly, is redrilled theoretically as follows;

$$K_b = 1 - \frac{\rho_m}{\rho_{st}} \Rightarrow K_b = 1 - \frac{11}{65.5} = 0.83$$

$$p = W_{DC} \times K_b \Rightarrow p = 147 \times 0.83 = 122 lb/ft$$

Moment of inertia;

$$I = \frac{\pi}{64}(OD^4 - ID^4) \Rightarrow I = \frac{\pi}{64}[(\frac{8}{12})^4 - (\frac{3}{12})^4] = 9.5 \times 10^{-3} ft^4$$

$$m_1 = \frac{EIp\sin\alpha}{W^2} \Rightarrow m_1 = \frac{4320 \times 10^6 \times 9.5 \times 10^{-3} \times 122 \times \sin 3}{40000^2} = 0.17 ft$$

$$m_2 = \sqrt{\frac{\mathrm{E}l}{W}} \Rightarrow m_2 = \sqrt{\frac{4320 \times 10^6 \times 9.5 \times 10^{-3}}{40000}} = 32\,ft$$

$$m_3 = EI\,\frac{m_1}{m_2^3} \Rightarrow 4320 \times 10^6 \times 9.5 \times 10^{-3} \times \frac{0.17}{32^3} = 213lb$$

The apparent wellbore radius is;

$$r = \frac{0.5}{12}(12.25 - 8) = 0.177\,ft$$

$$\frac{r}{m_1} = l\tan\frac{l}{2} - \frac{l^2}{2} \Rightarrow 1.04$$

Solving for l;

$l = 1.97$

$$h_o = \tan\frac{l}{2} - l \Rightarrow h_o = -0.46$$

Side force at the bit is;

$$H_o = m_3 \times h_o \Rightarrow 213 \times (-0.46) = -98lb$$

The resultant force angle is;

$$\phi = \arctan(\frac{H_o}{WOB}) + \alpha$$

$\phi = 2.86^0$

For the 25^0 dipping formation with 0.045 FAI

$$\psi = \gamma - \arctan[(1-h)\tan(\gamma - \phi)] = 3.77^0$$

As shown in Figure 29, the steerable assembly drilled a 29 m. interval with a 1.9^0 inclination difference. For a slick assembly to drill same section, 40000 lb Weight on bit is desired, the inclinations from 1992–2022 m. are shown below in the Figure 30.

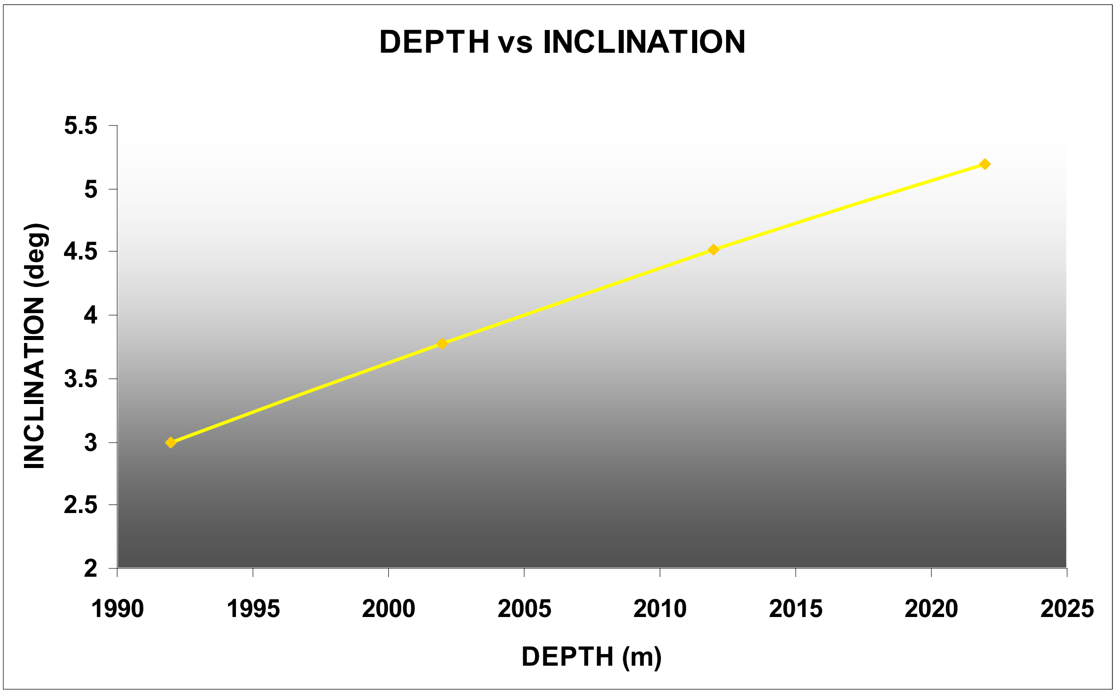

Fig.30 Depth vs Inclination of X-1 for Slick Assembly (1992m-2022)

In Fig 30, the depth intervals are 10 m with about 0.77^0 for each. The Dogleg Severity for this interval is;

$5.2 - 3 = 2.2^0 / 30m$ which is an applicable value for actual drilling conditions.

The maximum DLS is 3.68^0 in 2079-2108 m. interval for the steerable assembly. In order to drill that section with a slick assembly, 60000 lbs WOB is suitable, proven as follows;

$$K_b = 1 - \frac{\rho_m}{\rho_{st}} \Rightarrow K_b = 1 - \frac{11}{65.5} = 0.83$$

$$p = W_{DC} \times K_b \Rightarrow p = 147 \times 0.83 = 122 lb / ft$$

Moment of inertia;

$$I = \frac{\pi}{64}(OD^4 - ID^4) \Rightarrow I = \frac{\pi}{64}[(\frac{8}{12})^4 - (\frac{3}{12})^4] = 9.5 \times 10^{-3} ft^4$$

$$m_1 = \frac{EIp \sin\alpha}{W^2} \Rightarrow m_1 = \frac{4320 \times 10^6 \times 9.5 \times 10^{-3} \times 122 \times \sin 10.74}{60000^2} = 0.26 ft$$

$$m_2 = \sqrt{\frac{EI}{W}} \Rightarrow m_2 = \sqrt{\frac{4320 \times 10^6 \times 9.5 \times 10^{-3}}{60000}} = 26.2 ft$$

$$m_3 = EI \frac{m_1}{m_2^3} \Rightarrow 4320 \times 10^6 \times 9.5 \times 10^{-3} \times \frac{0.26}{26.2^3} = 593 lb$$

The apparent wellbore radius is;

$$r = \frac{0.5}{12}(12.25 - 8) = 0.177 ft$$

$$\frac{r}{m_1} = l \tan\frac{l}{2} - \frac{l^2}{2} \Rightarrow 0.68$$

Solving for l;

$l = 1.83$

$$h_o = \tan\frac{l}{2} - l \Rightarrow h_o = -0.53$$

Side force at the bit is;

$$H_o = m_3 \times h_o \Rightarrow 593 \times (-0.53) = -314lb$$

The resultant force angle is;

$$\phi = \arctan(\frac{H_o}{WOB}) + \alpha$$

$$\phi = 10.44^0$$

For the 25^0 dipping formation with 0.045 FAI

$$\psi = \gamma - \arctan[(1-h)\tan(\gamma - \phi)]$$

$$\psi = 11^0$$

Using the data, Fig.31 is obtained;

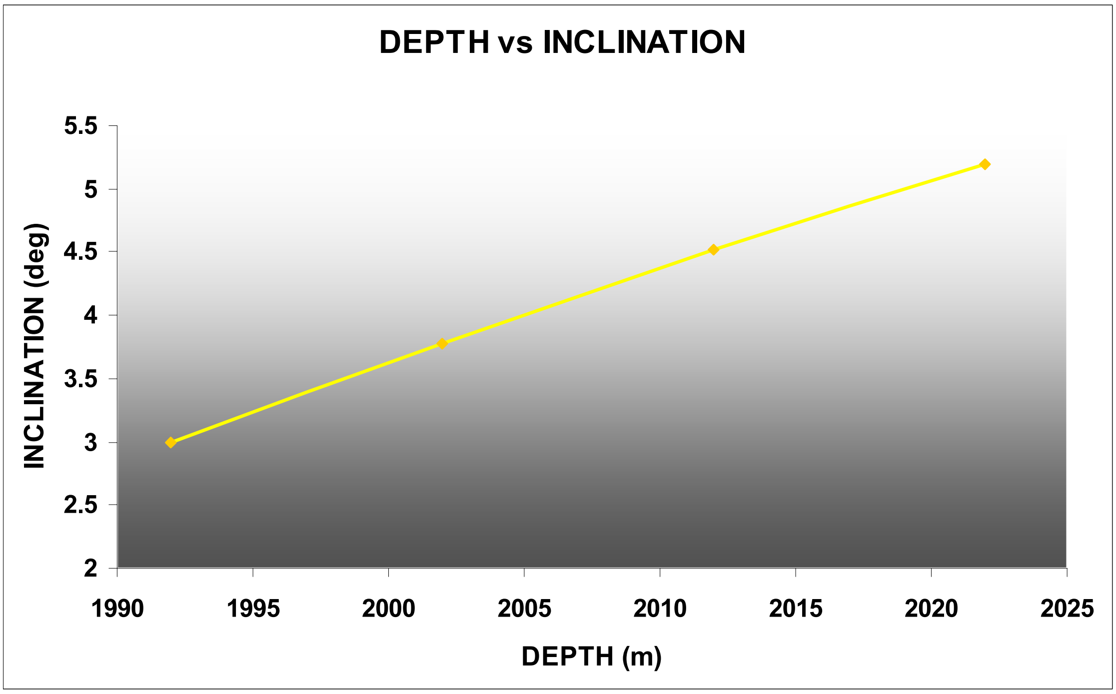

Fig.31 Depth vs Inclination of X-1 for Slick Assembly (2079m-2105m)

In Fig 31, the depth intervals are 2m with about 0.26^0 for each. Accordingly, Dogleg Severity is ; 3.9^0/30 m. which is again an applicable value for drilling conditions.

Thus, after the KOP, the well can be drilled with a slick BHA with the positive effect of WOB alternating from 40000 lbs to 60000 lbs without any DLS problem up to 2108 m.

After this depth, as the inclination increases, larger weight on bit values are required to follow the path. For the practical conditions it is not possible to increase weight on bit more than 60000 lbs for a 12 ¼" bit at a slick assembly, this handicap can be solved by kicking off from a shallower depth in order to reach the target.

6.2 Single Stabilizer Assembly Simulation

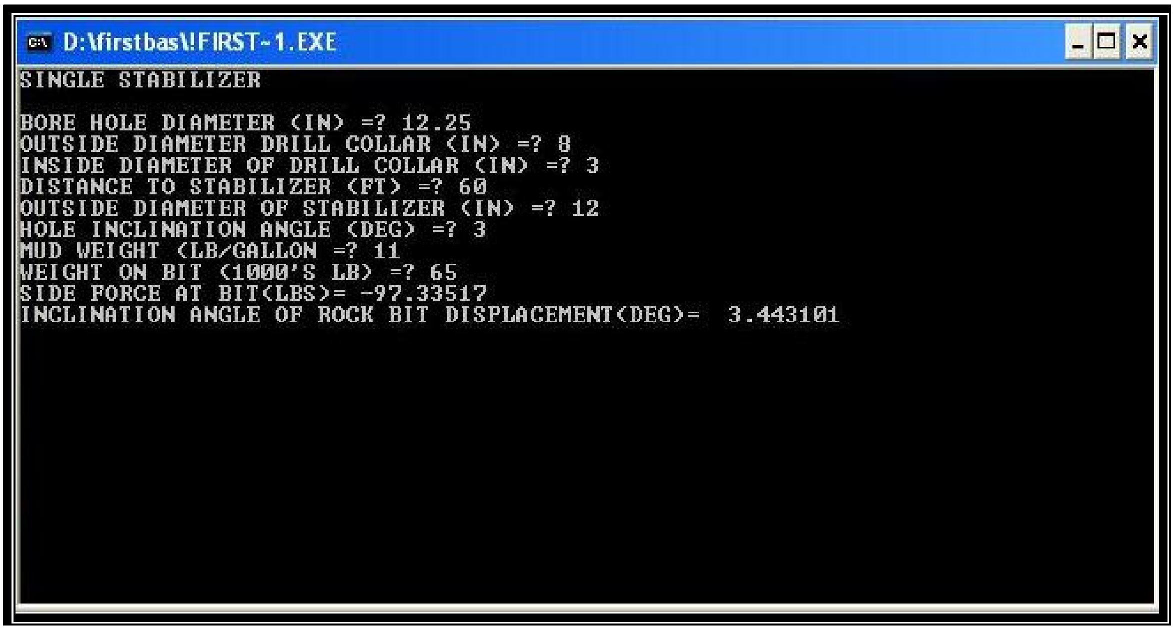

Fig.32 Computer Program Screenshot-1

The computer program outputs' aim is to investigate the instantaneous rock bit displacements in a continuous way where the well starts from 3^0 (with approval by both gyroscopic survey and MWD prior to the casing run, and with a pre-known azimuth) inclination and carries up to 6.9^0 in actual well parameters. As the side force is always negative, it is predicted to have dropping tendency, but with the effect of formation anisotropy, well possessed building tendency up to the target. The remaining screenshots up to 6.9^0 can be found in Appendix B.

CHAPTER VII

CONCLUSION

The force analysis of slick and single stabilizer BHA's and the examinations through the results and graphs obtained, enabled this study to reach to some general conclusions which are beneficial and cost reducing for the directional drilling operations, or in some cases the results may be used as an alternative way for conventional directional drilling operations. Following is the list of conclusions gathered throughout the study;

- Higher WOB values tend to deviate well from vertical both for slick and single stabilizer assemblies.

- Position of stabilizers, stabilizer-hole wall clearances governs the BHA behavior and therefore the drilling tendencies. Bigger stabilizer clearances give the drill string more building tendency.

- Diameter of drill collars used in the BHA affects the directional tendency. Bigger diameter drill collars have less directional tendency than the smaller ones.

- Directional tendency may alter from build to drop at different formation dips and formation anisotropy indexes. Thus, same parameters may not result in the same ways in different formations

- The drilling tendency for anisotropic formation and anisotropic bit is a function of formation properties, bit properties, initial inclination angle and the resultant force at the bit.

- Formation dip strongly affects the final hole angle and directional tendency. As the dipping increases, directional tendency also increases.

- Formation anisotropy index affects the final hole angle and directional tendency. Anisotropic formations have more building tendencies.

- Initial hole inclination affects the final hole angle and directional tendency in a negative way. More vertical wells have more building tendencies. As the slope increases, weight on bit must be gradually increased in order to continue building.

- Type of BHA used affects the final hole angle.

- Theoretically, in isotropic formation conditions hole can be drilled vertical when the side force at the bit becomes zero. While in anisotropic conditions, vertical hole can be drilled when the side force at the bit and the inclination angles become constant.

REFERENCES

1. P.R. Paslay., “Stress Analysis of Drillstrings”,SPE 27976,Aug.1994, 187-190.

2. Bernt S.Aadnoy. ,“Analytical Models for design of Wellpath and BHA”SPE 77220,Sep.2002,2-4.

3. Wilson, G. E., “How to Drill a Usable Hole, Part I“, World Oil ,Aug.1960,6-8.

4. Lubinski, A.,“A Study of the Buckling of Rotary Drilling Strings,” Drilling and Production Practice, 1950,178 -214.

5. Willers, F.A.,”Das Knicken schwerer Geatsnge” Z. Zngew. Math. u Mech.,1941,Vol. 21,No. 1, 43.

6. MacDonald, G. C, and Lubinski, A., “Straight-Hole Drilling in Crooked-Hole Country”, Drill. and Prod. Prac. , API,1951,80.

7. Lubinski, A. and Woods, H. B., “Factors Affecting the Angle of Inclination and Dog-Legging in Rotary Borehole”, Drill. and Prod. Prac.,API,1953,222.

8. Woods, H.B. and Lubinski, A., “Practical Charts for Solving Problems on Hole Deviation”, Drill and Prod. Prac., API,1954,56.

9. Woods and Lubinski, A., “Use Of Stabilizers in Controlling Hole Deviation”, Drill. and Prod. Prac., API,1955,165.

10. Lubinski, A.,“Maximum Permissible Dog Legs in Rotary Borehole”, JPT,1961,175.

11. Bogy, D. B. and Paslay, P. R.,“Buckling of Drill Pipe in an Inclined Hole”, Trans., ASME, Series B,May, 1964,214-220.

12. Paslay, P. R. and Bogy, D. B.,“The Stability of a Circular Rod Laterally Constrained to be in Contact with an Inclined Circular Cylinder”, Jour of Applied Mech.,Dec.1964,608-610.

13. Directional Drililing Training manual, Schlumberger, Houston,Texas, Jan.1997,1-20.

14. Sultanov, B. and Shandalov, G.,“Effects of Geological Conditions on Well Deviation“,Geol i Raw,1961, No. 3, 107.

15. Bradley, W.B., “Factors Affecting the Control of Borehole angle in Straight and Directional Wells”, JPT, June 1975, 679-688.

16. Cheatham, J.B., “A Theoretical Model for Directional Drilling Tendency of a Drill Bit in Anisotropic Rock”, SPE 10642,1-5.

17. Millheim, K., “Side Cutting Characteristics of Rock Bits and Stabilizers While Drilling”, SPE 7518,Oct.1978,10

18. Miska, S., “Drill String Composition, Properties, and Design” Socorro, New Mexico, June 1991,10-30.

19. Murphey, et. al., “Hole Deviation and Drill String Behavior”, SPE 1259, 1966,1-10.

20. Chan, H., “Mathematical Modelling of Transient Wellbore Trajectory For Simple BHA”, Master’s Thesis, University of Tulsa, 2001.

21. Suchato, P., “Description of the Initial Part of Transient Borehole Trajectory Behavior in Two-Dimensions”, Master’s Thesis, University of New Mexico,1985.

22. Dawson, R., Paslay, P.R., “Drillpipe Buckling in Inclined Holes”, Journal of Petroleum Technology, October, 1984.

APPENDIX A

COMPUTER PROGRAM

```
5 CLS
10 PRINT  “SINGLE STABILIZER MODELLING”
15 PRINT
20 INPUT  “BOREHOLE DIAMETER (IN)  = “;DH:DH=DH/12
25 INPUT  “OUTSIDE DIAMETER OF DRILL COLLAR (IN)
=”;OD:OD=OD/12
30 INPUT ” INSIDE DIAMETER OF DRILL COLLAR (IN) =”;ID:ID=ID/12
35 INPUT ”DISTANCE TO STABILIZER (FT) =”;X1
40 INPUT “OUTSIDE DIAMETER OF STABILIZER (IN) =”;DST:DST=DST/12
45 INPUT “HOLE INCLINATION ANGLE (DEG)=”;A:A=A/57.29
50 INPUT “MUD WEIGHT (LB/GALLON) =”;DE
55 INPUT”WEIGHT ON BIT (1000’S LB) =”;W:W=W*1000
60 E=4.176E+09
65 I=3.1416*(OD^4-ID^4)/64
70 EI=E*I
75 WD=3.1416*(OD^2-ID^2)*489/4
80 S=(DH-DST)/2
85 R=(DH-OD)/2
90 P=WD*(1-DE/65.469)
```

```
95 GOSUB 135

100 CDC=R/M1

105 L1=X1/M2

110 CST=S/M1

115 GOSUB 155

120 H=HO/M3

125 Teta=ATN(h/W)*180/3.1416+HA*57.29

130 Phi=15-ATN((1-0.045)*tan((15-Teta)*3.1416/180))*180/3.1416

135 PRINT"SIDE FORCE AT BIT F(LBS)=";H

140 PRINT"INCLINATION ANGLE OF ROCK BIT

DISPLACEMENT(DEG)=";Phi

145 END

150 M1=EI*P*SIN(A)/W^2

155 M2=SQR(EI/W)

160 M3=P*SIN(A)*SQR(EI/W)

165 RETURN

170 LL=0.1 : LH=10.0 : L=(LL+LH)/2

175 A=CST-CDC+1+0.5*(L^2-L1^2)-COS(L-L1)-L*SIN(L-L1)

180 B=L1-L+SIN(L-L1)

185 HB=A/B

190 C1B=CDC-1-0.5*L^2-HB*L

195 C2B=COS(L)+(L+HB)*SIN(L)
```

```
200 C3B=SIN(L)-(L+HB)*COS(L)205 HO=(1+CST-0.5*L1^2-COS(L-L1)-
(L+HB)*SIN(L-L1))/L1
210 C3A=(1+CST-0.5*L1^2-COS(L1)-HO*L1)/SIN(L1)
215 F=(1-C2B)*SIN(L1)+(C3B-C3A)*COS(L1)+HB-HO
220 IF ABS(F)<0.0001 THEN GOTO 225
225 IF F>0 THEN GOTO 220
230 LH=L:L=(LH+LL)/2 :GOTO 160
235  LL=L:L=(LL+LH)/2 :GOTO 160
240 LL=0.1 : LH=L1 : X=(LL+LH)/2
245 FP=-SIN(X)+C3A*COS(X)+X+HO
250  IF ABS(FP)<0.00001 THEN GOTO 255
255 IF FP>0 THEN GOTO 250
260 LH=X:X=(LH+LL)/2 :GOTO 230
265 LL=X:X=(LL+LH)/2 :GOTO 230
270 YM=-1+COS(X)+C3A*SIN(X)+0.5*X^2+HO*X
275 IF YM=CDC THEN GOTO 275
280 IF YM>CDC THEN GOTO 275
285 RETURN
290 PRINT"CONTACT POINT BETWEEN BIT AND STABILIZER"
295 END
```

APPENDIX B

COMPUTER PROGRAM OUTPUTS

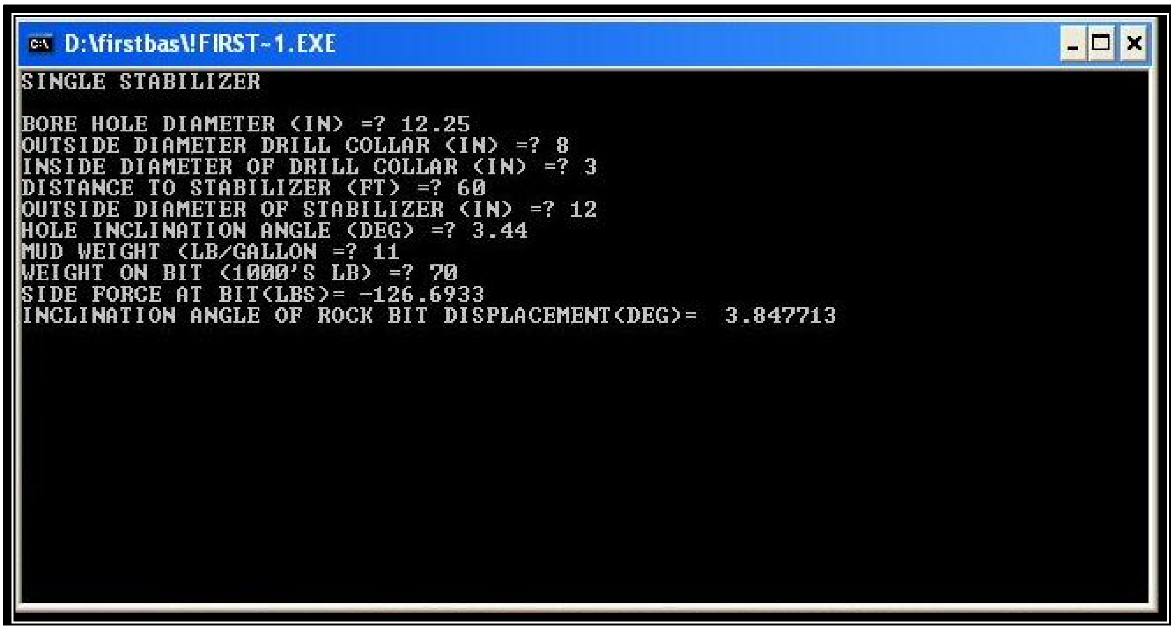

Fig.33 Computer Program Screenshot-2

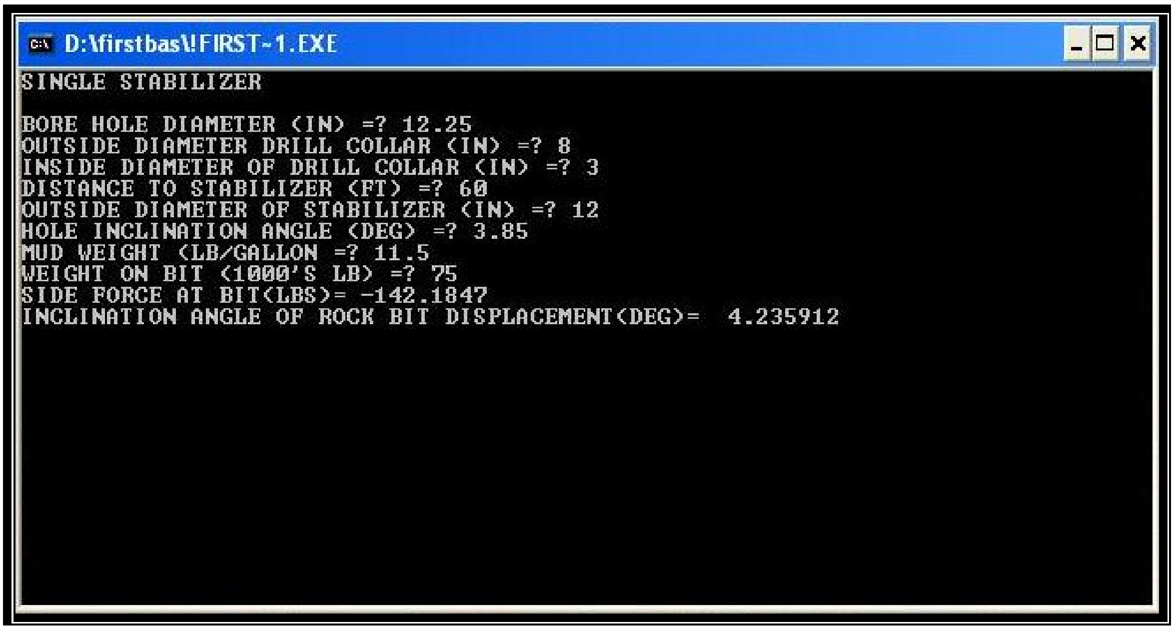

Fig.34 Computer Program Screenshot-3

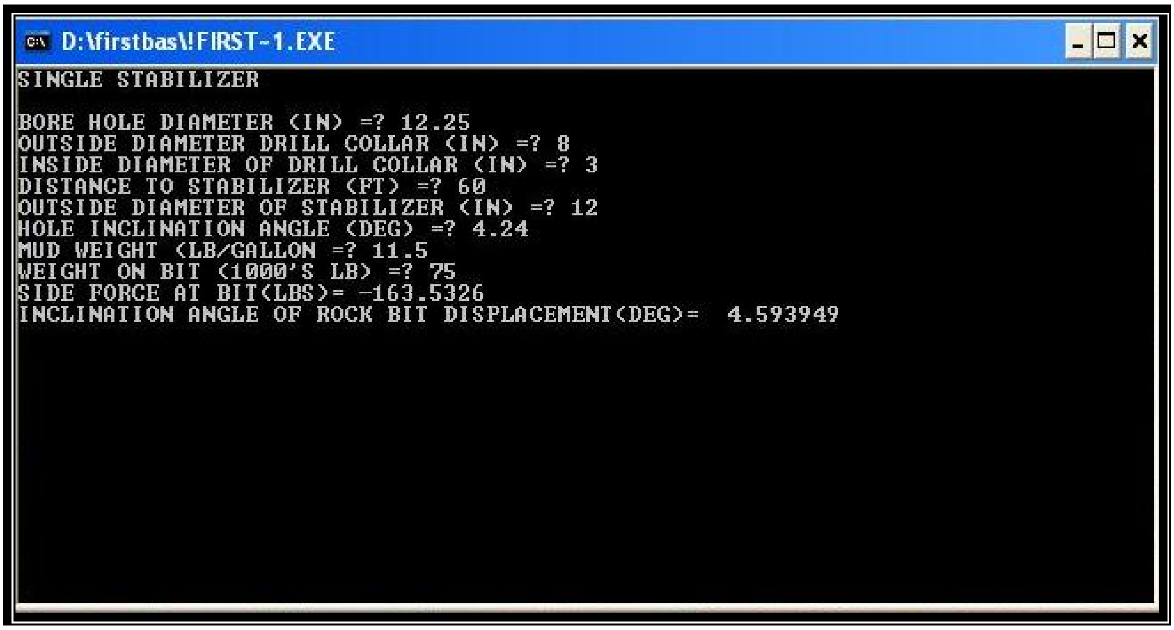

Fig.35 Computer Program Screenshot-4

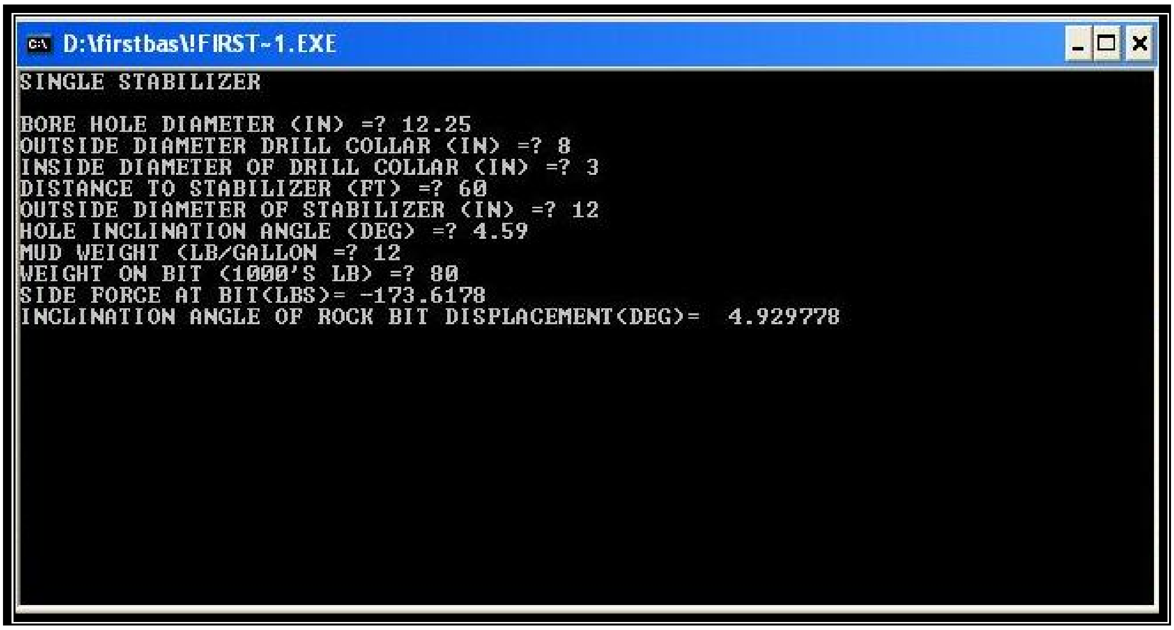

Fig.36 Computer Program Screenshot-5

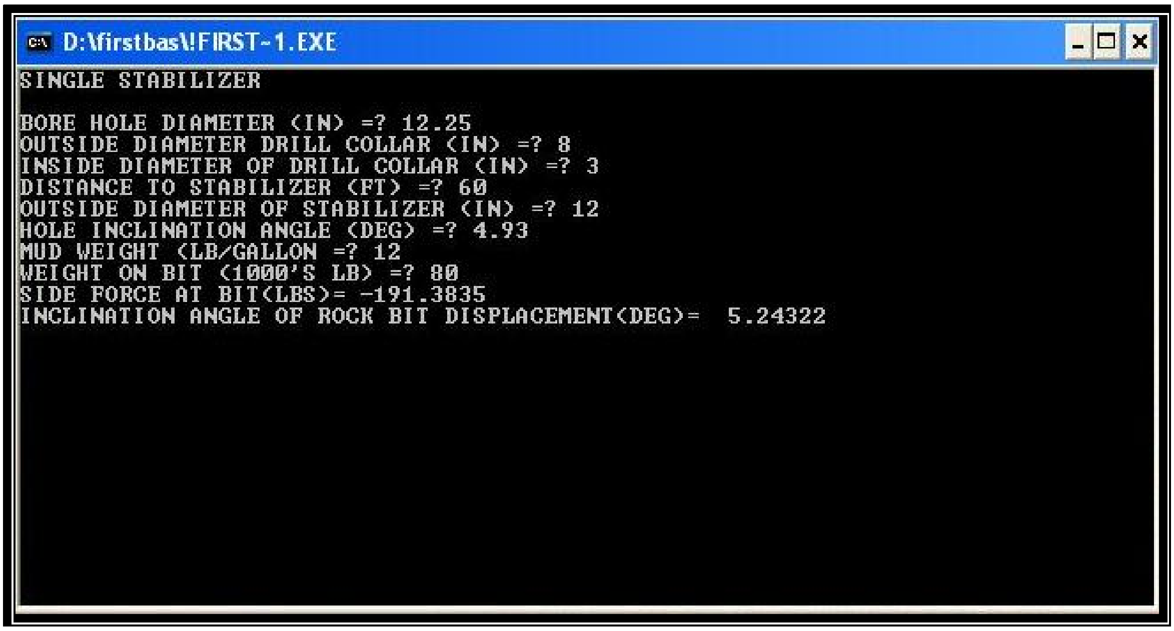

Fig.37 Computer Program Screenshot-6

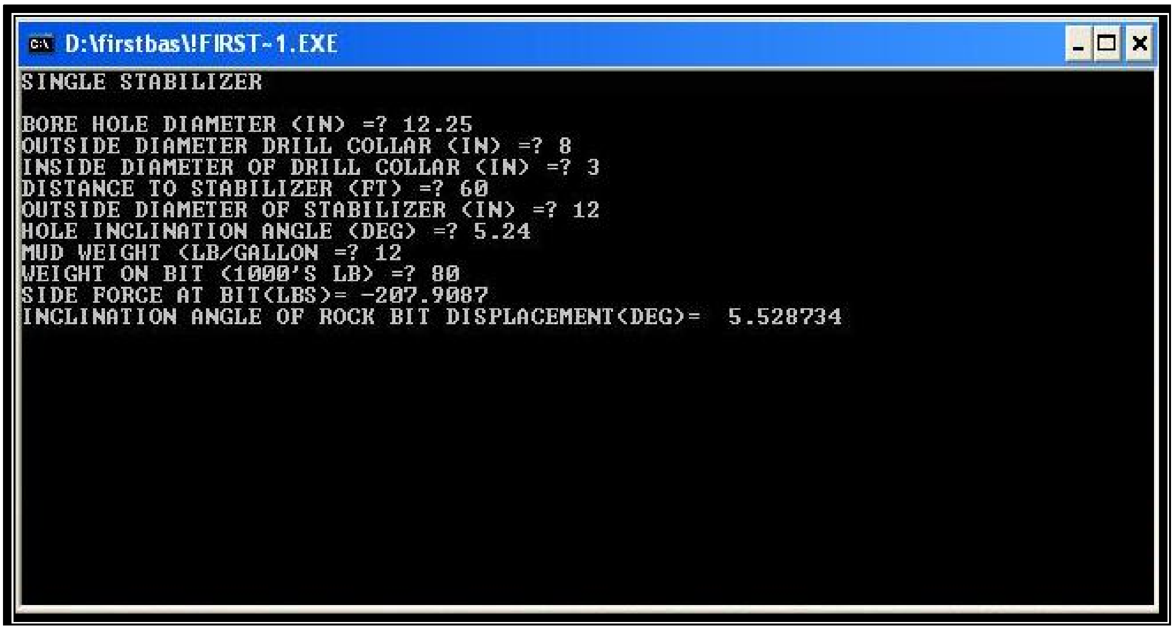

Fig.38 Computer Program Screenshot-7

Fig.39 Computer Program Screenshot-8

Fig.40 Computer Program Screenshot-9

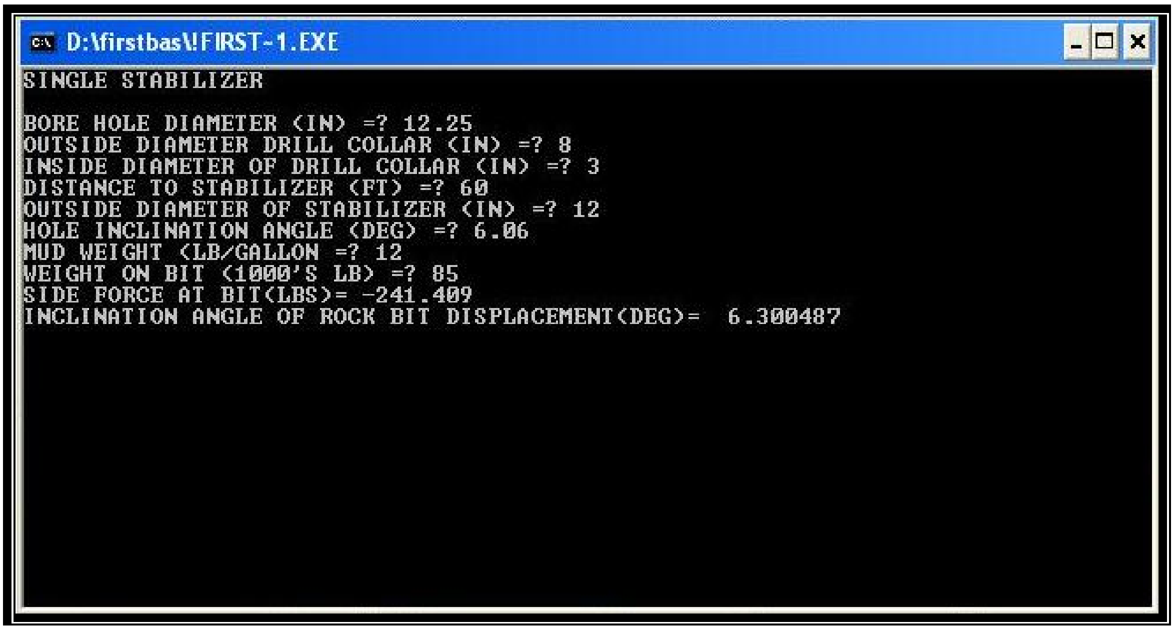

Fig.41 Computer Program Screenshot-10

Fig.42 Computer Program Screenshot-11

Fig.43 Computer Program Screenshot-12

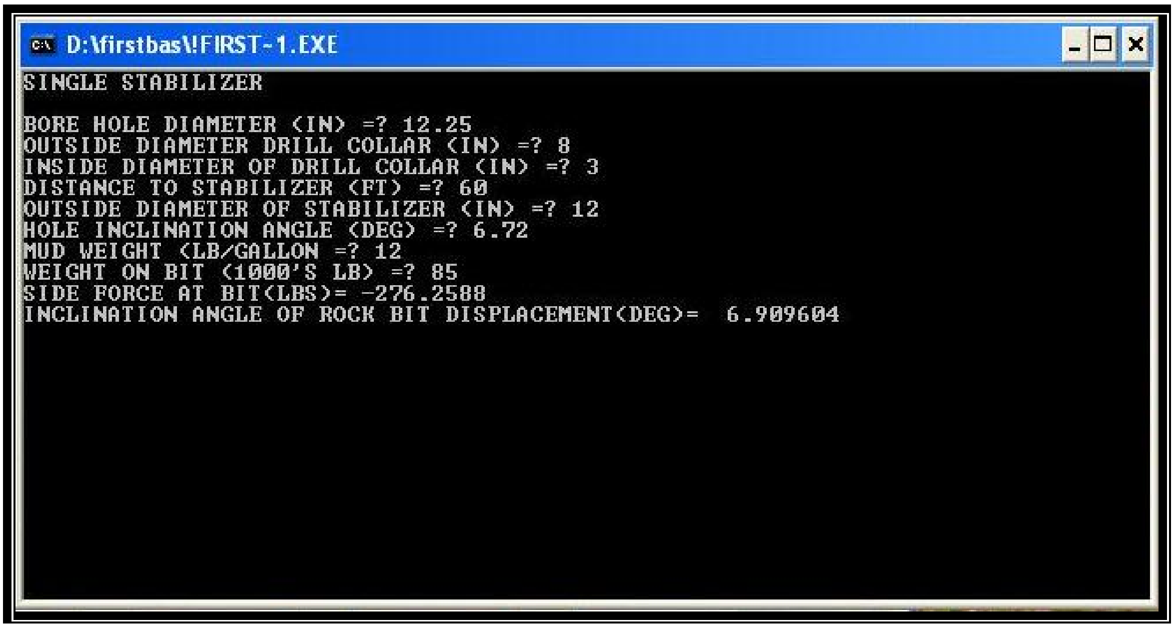

Fig.44 Computer Program Screenshot-13

APPENDIX C

SLICK ASSEMBLY MODELING

$$S = EI \frac{d^3Y}{dX^3} \qquad \text{(C-1)}$$

$$S = pX \sin(\alpha + \beta) - WOB \sin\beta + H_o \cos\beta \qquad \text{(C-2)}$$

Dividing both sides by cosß;

$$\frac{EI}{\cos\beta} \frac{d^3Y}{dX^3} = pX \frac{\sin(\alpha + \beta)}{\cos\beta} + - WOB \tan\beta + H_o$$

Substituting;

$$\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$$

$$\frac{EI}{\cos\beta} \frac{d^3Y}{dX^3} = pX \sin\alpha + pX \cos\alpha \tan\beta - WOB \tan\beta + H_o$$

$$\frac{EI}{\cos\beta} \frac{d^3Y}{dX^3} = pX \sin\alpha - (WOB - pX \cos\alpha) \tan\beta + H_o \qquad \text{(C-3)}$$

As $\tan\beta = \frac{dY}{dX}$

$$\frac{EI}{\cos\beta}\frac{d^3Y}{dX^3} = pX\sin\alpha - (WOB - pX\cos\alpha)\frac{dY}{dX} + H_o$$

For small ß, cos ß approaches 1, hence;

$$EI\frac{d^3Y}{dX^3} = pX\sin\alpha - (WOB - pX\cos\alpha)\frac{dY}{dX} + H_o$$

For large WOB's;

$$EI\frac{d^3Y}{dX^3} = pX\sin\alpha - WOB\frac{dY}{dX} + H_o \qquad \text{(C-4)}$$

Let;

$$Y = m_1 y \Rightarrow dY = m_1 dY$$
$$X = m_2 x \Rightarrow dX = m_2 dX$$
$$H_o = m_3 h_o \Rightarrow dH_o = m_3 dh_o$$

$$\Rightarrow \frac{dY}{dX} = \frac{m_1}{m_2}\frac{dy}{dx}$$

$$\Rightarrow \frac{d^2Y}{dX^2} = \frac{d}{dx}\left(\frac{m_1}{m_2}\frac{dy}{dx}\right) = \frac{m_1}{m_2^2}\frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{d^3Y}{dX^3} = \frac{d}{dx}\left(\frac{m_1}{m_2^2}\frac{d^2y}{dx^2}\right) = \frac{m_1}{m_2^3}\frac{d^3y}{dx^3}$$

$$\Rightarrow EI\frac{m_1}{m_2^3}\frac{d^3y}{dx^3} = pm_2 x\sin\alpha - WOB\frac{m_1}{m_2}\frac{dy}{dx} + m_3 h_o$$

Multiply the equation by $\frac{WOB}{pEI}m_2$;

$$\frac{WOB}{p}\frac{m_1}{m_2^2}\frac{d^3y}{dx^3} = \frac{WOB}{EI}m_2^2 x\sin\alpha - \frac{WOB}{pEI}m_1\frac{dy}{dx} + \frac{WOB}{pEI}m_2 m_3 h_o$$

Let;

$$\frac{WOB^2}{pEI}m_1 = 1 \Rightarrow m_1 = \frac{pEI\sin\alpha}{WOB^2}$$

$$\frac{WOB}{pEI}m_2^2 = 1 \Rightarrow m_2 = \sqrt{\frac{EI}{WOB}}$$

$$\frac{WOB}{pEI}m_2 m_3 = 1 \Rightarrow \frac{WOB}{pEI}\sqrt{\frac{EI}{WOB}}m_3 = 1 \Rightarrow m_3 = p\sqrt{\frac{EI}{WOB}}$$

Hence equation (C-4) can be written in dimensionless form as;

$$\frac{d^3y}{dx^3} = x\sin\alpha - \frac{dy}{dx} + h_o \qquad \text{(C-5)}$$

$$\Rightarrow \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) + y = x\sin\alpha + h_o$$

$$\Rightarrow \frac{d^2y}{dx^2} + y = \frac{x^2}{2}\sin\alpha + h_o x + c_1$$

$$y = c_1 + c_2\sin x + c_3\cos x + \frac{x^2}{2}\sin\alpha + h_o x \qquad \text{(C-6)}$$

Boundary conditions for Slick BHA;

At bit;

$$x = 0$$
$$y = 0 \Rightarrow y' = 0$$
$$\Rightarrow y'' = 0$$

At point of tangency;

$$x = l$$
$$y = \frac{r}{m_1} \Rightarrow y' = 0$$
$$\Rightarrow y'' = 0$$

Solving equation (C-5) with the boundary conditions yield the equation for the centerline of the drill collar in the dimensionless form;

$$y = -1 + \cos x + (\frac{1 - \cos l}{\sin l}) + \sin x + \frac{1}{2}x^2 + h_o x$$

Valid for $0 \le x \le l$ and the following equations;

$$h_o = \tan\frac{l}{2} - l \qquad \text{(C-7)}$$

$$\frac{r}{m_1} = l\tan\frac{l}{2} - \frac{l^2}{2} \qquad \text{(C-8)}$$

APPENDIX D

SOLUTION FOR SLICK ASSEMBLY MODELLING

$$\Rightarrow y = c_1 + c_2 \sin x + c_3 \cos x + \frac{x^2}{2} \sin\alpha + h_o x$$

$$y' = c_2 \cos x - c_3 \sin x + x \sin\alpha + h_o$$

$$y'' = -c_2 \sin x - c_3 \cos x + \sin\alpha$$

$$\Rightarrow 0 = c_1 + c_3$$
$$\Rightarrow 0 = -c_3 + \sin\alpha$$
$$\frac{r}{m_1} = c_1 + c_2 \sin l + c_3 \cos l + \frac{l^2}{2} \sin\alpha + h_o l$$
$$0 = c_2 \cos l - c_3 \sin l + l \sin\alpha + h_o$$
$$0 = -c_2 \sin l - c_3 \cos l + \sin\alpha$$

$$\Rightarrow c_3 = \sin\alpha$$
$$c_1 = -\sin\alpha$$
$$c_2 = \frac{-2r + m_1(-2 - l^2 + 2\cos l + 2l \sin l)\sin\alpha}{2m_1(l \cos l - \sin l)}$$

$$h_o = \frac{-2r \cos l + m_1(-2 - (l^2 - 2)\cos l + 2l \sin l)\sin\alpha}{2m_1(l \cos l - \sin l)} \qquad \text{(D-1)}$$

As the actual dimensional side force can be calculated as;

$$H_o = m_3 \times h_o$$

To calculate the directional tendency of the assembly force angle is calculated as follows;

$$\phi = \arctan(\frac{H_o}{WOB}) + \alpha \qquad \text{(D-2)}$$

Finally, the inclination angle of the rock bit displacement can be found;

$$\psi = \gamma - \arctan[(1-h)]\tan[(\gamma - \phi)] \qquad \text{(D-3)}$$

APPENDIX E

SINGLE STABILIZER MODELLING

$$\mathrm{E}l\,\frac{d^3Y_{A,N}}{dX^3_{A,N}} + W\,\frac{dY_{A,N}}{dX_{A,N}} = H_o + pX_{A,N}\sin\alpha_N \qquad \text{(E-1)}$$

Equation (E-1) is valid only for $0 < X_{A,N} < X_1$. While for the second segment, we have to take into account the presence of stabilizer force, H_{ST}, and hence

$$\mathrm{E}l\,\frac{d^3Y_{B,N}}{dX^3_{B,N}} + W\,\frac{dY_{B,N}}{dX_{B,N}} = H_o - H_{ST} + pX_{B,N}\sin\alpha_N \qquad \text{(E-2)}$$

which this equation is valid only for $X_1 < X_{B,N} < L$

Sets of boundary conditions for equation 3-12 and 3-13 can be listed below.

At Bit, X = 0;

$$Y_{A,N}(0) = 0$$

$$Y''_{A,N}(0) = 0$$

At Stabilizer, $X = X_1$;

$$Y_{A,N}(X_1) = Y_{B,N}(X_1)$$

$$Y''_{A,N}(X_1) = Y''_{B,N}(X_1)$$

$$Y'_{A,N}(X_1) = Y'_{B,N}(X_1)$$

At Tangency Point, X = L;

$$Y_{B,N}(LN) = R_N = Rcl + (L_N - F)\tan(\alpha_{N-1} - \alpha_N)$$

$$Y''_{B,N}(L) = 0$$

$$Y'_{B,N}(L) = 0$$

To make the equation dimensionless, the following parameters are used;

$Y = m_1 y$ *and* $R = m_1 r$

$X = m_2 x$ *and* $L = m_2 l$

$H_o = m_3 h_o$

By applying chain rule;

$$y' = \frac{dy}{dx} = \frac{dy}{dX}\frac{dX}{dx} = \frac{Y'}{m_1}m_2 = \frac{m_2}{m_1}Y'$$

$$y'' = \frac{d^2y}{dx^2} = d\frac{dy}{dX} \div dX\frac{dX}{dx} = \frac{m_2}{m_1}y''m_2 = \frac{m_2^2}{m_1}Y''$$

$$y''' = \frac{d^3y}{dx^3} = d\frac{dy^2}{dX^2} \div dX\frac{dX}{dx} = \frac{m_2^2}{m_1}y'''m_2 = \frac{m_2^3}{m_1}Y''' \qquad \text{(E-3)}$$

Rewrite equation (E-1);

$$ElY''' + WY' = H + pX\sin\alpha$$

$$\frac{Y'''}{p\sin\alpha}+\frac{WY'}{\mathrm{E}lp\sin\alpha}=\frac{H}{p\sin\alpha}+\frac{X}{\mathrm{E}l}$$

$$\frac{m_1y'''}{m_2^3 p\sin\alpha}+\frac{m_1Wy'}{m_2\mathrm{E}lp\sin\alpha}=\frac{m_3h}{p\sin\alpha}+\frac{m_2x}{\mathrm{E}l} \qquad \text{(E-4)}$$

Multiply equation by Wm_2;

$$\frac{m_1Wy'''}{m_2^2 p\sin\alpha}+\frac{m_1W^2y'}{\mathrm{E}lp\sin\alpha}=\frac{m_3m_2Wh}{p\sin\alpha}+\frac{m_2^2Wx}{\mathrm{E}l} \qquad \text{(E-5)}$$

Solving the for m_1, m_2, m_3 we get;

$$m_1=\frac{\mathrm{E}lp\sin\alpha}{W^2}$$

$$m_2=\sqrt{\frac{\mathrm{E}l}{W}}$$

$$m_3=EI\frac{m_1}{m_2^3}$$

Introducing the scaling parameters m_1, m_2, and m_3 the dimensionless forms of the differential equations are written in the form;

$$Y'''_a+Y'_a=h_o+x_a \qquad \text{(E-6)}$$

$$Y'''_b+Y'_b=h_o-h_{stb}+x_b \qquad \text{(E-7)}$$

The corresponding boundary conditions are;

at $x = 0$,

$y_a(0) = 0$ *and* $y''_a(0) = 0$

at $x = x_1$

$y_a(x_1) = y_b(x_1) = \pm c_{st}$

$y'_a(x_1) = y'_b(x_1)$

$y''_a(x_1) = y''_b(x_1)$

at $x = l$

$y_b(l) = r \div m_1 = c$

$y_b'(l) = 0$

$y_b''(l) = 0$

Solving the equations E-6 and E-7 along with the corresponding boundary conditions we obtain;

$$\sin(x_1 - l) + (1 + h_1)\cos(x_1 - l) - \sin x_1 + (\cot x_1)(1 + c_{st} - \cos x_1 - 0.5x_1^2 - h_o x_1) + h_{st} \quad \text{(E-8)}$$

$$h_0 = \frac{1 + c_{st} - 0.5x_1^2 - \cos(x_1 - l) + (l + h_1)\sin(x_1 - l)}{x_1} \quad \text{(E-9)}$$

$$h_1 = h_0 - h_{st} = \frac{c_{st} - c - \cos(x_1 - l) + l\sin(x_1 - l) - 0.5(x_1^2 - l^2) + 1}{x_1 - l + \sin(l - x_1)} \quad \text{(E-10)}$$