STUDENTS' STRATEGIES, EPISODES AND METACOGNITIONS IN THE CONTEXT OF PISA 2003 MATHEMATICAL LITERACY ITEMS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF SOCIAL SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

SERKAN OKUR

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
SECONDARY SCIENCE AND MATHEMATICS EDUCATION

JANUARY 2008

Approval of the thesis:

STUDENTS' STRATEGIES, EPISODES AND METACOGNITIONS IN THE CONTEXT OF PISA 2003 MATHEMATICAL LITERACY ITEMS

submitted by **SERKAN OKUR** in partial fulfillment of the requirements for the degree of **Master of Science in Secondary Science And Mathematics Education Department, Middle East Technical University** by,

Prof. Dr. Canan Özgen ____________________
Dean, Graduate School of **Natural and Applied Sciences**

Prof. Dr. Ömer Geban ____________________
Head of Department, **Secondary Science and Mathematics Education**

Assist. Prof. Dr. Ayhan Kürşat Erbaş ____________________
Supervisor, **Secondary Science and Mathematics Education Dept., METU**

Examining Committee Members:
Prof. Dr. Giray Berberoğlu ____________________
Secondary Science and Mathematics Education Dept., METU

Assist. Prof. Dr. Ayhan Kürşat Erbaş ____________________
Secondary Science and Mathematics Education Dept., METU

Assoc. Prof. Dr. Safure Bulut ____________________
Secondary Science and Mathematics Education Dept., METU

Assist. Prof. Dr. Erdinç Çakıroğlu ____________________
Elementary Education Dept., METU

Dr. Yusuf Koç ____________________
Elementary Education Dept., METU

Date: 21.01.2008

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name: Serkan Okur

Signature :

ABSTRACT

STUDENTS' STRATEGIES, EPISODES AND METACOGNITIONS
IN THE CONTEXT OF
PISA 2003 MATHEMATICAL LITERACY ITEMS

Okur, Serkan
M.S., Department of Secondary Science and Mathematics Education
Supervisor: Assist. Prof. Dr. Ayhan Kürşat Erbaş

January 2008, 154 pages

The purpose of this study was to investigate the problem solving strategies, problem solving episodes, and metacognition of five Turkish students just graduated from elementary school and explore the interplay of these factors on their problem solving success in mathematics. The research data had been collected by clinical interviews and a self monitoring questionnaire followed by the interviews. Ten mathematical problems that participant students had worked on were selected among the released mathematical literacy items used in Programme for International Student Assessment (PISA) 2003.

The problem solving strategies used by participants were coded according to the descriptions given by Posamentier & Krulik (1999). The cognitive-metacognitive problem solving framework developed by Artzt and Armour-Thomas (1992) has been used to observe the problem solving episodes of the participants. The coding system developed by Pappas et al. (2003) has been utilized to examine the major components of metacognition (mistake recognition, adaptability, awareness and

expression of thought) of the participants. The self-monitoring questionnaire responses were analyzed to crosscheck the results obtained from the clinical interviews.

The problem solving behaviors of the participants observed in the study confirmed their academic success levels. The study confirmed that the problem solving success is too complex to be clarified by a unique property or a behavior of the problem solver. The problem solving requires overcoming various obstacles to reach a successful result. Hence, not only the students should have the required mathematical knowledge and a good repertoire of different problem solving strategies, but also they should know when and how to use those strategies, and also they could monitor and regulate their problem solving processes using their metacognitive skills. So mathematics teachers should provide problems that require different problem solving strategies and encourage the students to explore new strategies, to take risks in trying and to discuss failures and successes with peers and teacher.

Keywords: Mathematics education, Programme for International Student Assessment (PISA), Clinical interview, Problem solving strategies, metacognition

ÖZ

PISA 2003 MATEMATİK OKUR YAZARLIĞI SORULARI BAĞLAMINDA ÖĞRENCİ STRATEJİLERİ, ADIMLARI VE ÜSTBİLİŞLERİ

Okur, Serkan
Y. Lisans, Ortaöğretim Fen ve Matematik Alanları Eğitimi Bölümü
Tez Yöneticisi: Yrd. Doç. Dr. Ayhan Kürşat Erbaş

Ocak 2008, 154 sayfa

Bu çalışmanın amacı, İlköğretim okullarından yeni mezun olmuş beş Türk öğrencisinin problem çözme stratejilerinin, problem çözme adımlarının ve üstbilişlerinin incelenmesi, bu faktörlerin problem çözme başarıları üzerindeki etkileşimini araştırmaktır. Araştırma verileri, klinik mülakatlar ve mülakatlar sonrası uygulanan anket ile toplanmıştır. Katılımcı öğrencilerin üzerinde çalıştığı on matematik problemi, Uluslararası Öğrenci Değerlendirme Programı (PISA) 2003'te kullanılan yayımlanmış matematik okur yazarlığı sorularından seçilmiştir.

Katılımcıların kullandığı problem çözme stratejileri Posamentier & Krulik (1999) tarafından verilen tanımlara göre kodlanmıştır. Artzt ve Armour-Thomas (1992) tarafından geliştirilen bilişsel-üstbilişsel problem çözme çerçevesi, katılımcıların problem çözme adımlarını gözlemlemede kullanılmıştır. Katılımcının üstbilişlerinin ana bileşenlerini (hata farkedimi, uyum yeteneği ve düşünceyi ifade) incelemek için Pappas ve diğer. (2003) tarafından geliştirilen kodlama sistemi

kullanılmıştır. Anket yanıtları klinik mülakatlardan elde edilen sonuçları kontrol etmek için analiz edilmiştir.

Katılımcıların çalışmada gösterdikleri problem çözme davranışlarının akademik başarılarıyla paralel olduğu görülmüştür. Çalışma bulguları problem çözme başarısının tek bir değişken ile yada öğrencinin bir davranışı ile açıklamak için fazla kompleks olduğunu göstermiştir. Problem çözme doğru sonuca ulaşmak için birçok engeli aşmayı gerektirmektedir. Bu nedenle öğrencilerin ilgili matematiksel ön gerekliliklere ve değişik problem çözme becerisinden oluşan bir dağarcığa sahip olmanın yanı sıra bunları ne zaman ve nasıl kullanacaklarını bilmeleri ve ayrıca özbilişsel yeteneklerini kullanarak problem çözme süreçlerini gözlemlemeleri ve düzenlemeleri de gerekmektedir. Bu yüzden matematik öğretmenleri öğrencilerine çeşitli problem çözme stratejileri gerektiren problemler sağlamalı ve öğrencilerini yeni stratejiler denemekte, riskler almakta ve başarısızlık ve başarıları arkadaşları ve öğretmenleri tartışabilme konusunda cesaretlendirmelidirler.

Anahtar Kelimeler: Uluslararası Öğrenci Değerlendirme Programı (PISA), Matematik Eğitimi, Klinik Mülakat, Problem Çözme Stratejileri, Üstbiliş

To my family and my dearest wife Yeliz

ACKNOWLEDGEMENTS

I wish to express my gratitude to my supervisor Assist. Prof. Dr. Ayhan Kürşat Erbaş for his valuable guidance, patience and encouragements.

I would also like to thank to Melek Çatak Işık and Şemsettin Beşer for their assistance throughout my research.

I want to thank to Zehra Ekşi and Erdem Dönmezçelik for his valuable support and encouragement.

In addition, I am grateful to Nuray Çavuşoğlu helping me to decide on the problems and the participants of the study.

I also thank the students that have participated willingly to the clinical interviews and gave me opportunity to collect data.

I want to thank my father Eşref Okur, my mother Şennur Okur and my brother Erkan Okur for their constant support and trust in my whole life.

And finally, I want to express my thanks to my dearest wife for her endless love, patience, encouragement and morale support in everyday of our friendship.

TABLE OF CONTENTS

ABSTRACT iv

ÖZ...vi

ACKNOWLEDGEMENTS ix

TABLE OF CONTENTS x

LIST OF TABLES xiii

ABBREVIATIONS xv

CHAPTER

1. INTRODUCTION..1

1.1 Purpose of the study 4

1.2 Research Questions 5

1.3 Significance of the Research 5

1.4 Definition of Terms 6

2. LITERATURE REVIEW..8

2.1 Mathematical Problem 8

2.3 Problem Solving Strategies 9

2.3 Problem Difficulty and Good Problem Solvers 12

2.4 Cognition, Metacognition and Problem Solving 14

2.5 Cognitive and Metacognitive Problem Solving Frameworks 17

2.6 Summary 20

3. METHODOLOGY 22

3.1 Research Design 22
3.1.1 Participant Selection 23
3.1.2 Selection of the mathematical problems for the interviews 25
3.1.3 Data Collection 27
3.1.3.1 Clinical Interviews 27
3.1.3.2 Self Monitoring Questionnaire 28
3.2 Data Analysis 29
3.2.1 Analysis of Problem Solving Strategies of the Participants 29
3.2.2 Analysis of Problem Solving Episodes of the Participants 31
3.2.3 Analysis of the Metacognitions of the Participants 32
3.2.3.1 Mistake Recognition...33
3.2.3.2 Adaptability 33
3.2.3.3 Awareness and Expression of Thinking 33
3.2.4 Analysis for the Difficulties, Failures and Results 34
3.2.5 Analysis of the Responses of the Participants to the Self Monitoring Questionnaire...34
3.3 Validity and Reliability of the study 34
3.4 Assumptions and Limitations 36
3.4.1 Assumptions 36
3.4.2 Limitations...36
4. RESULTS AND CONCLUSIONS 37
4.1 Participants' Results 37
4.2 Participants' Problem Solving Strategies 38
4.3 Participants' Problem Solving Episodes 43
4.4 Participants' Mistake Recognition, Adaptability and Expression of Thinking 50

4.5 Participants' Difficulties, Failures and Results 55

4.6 The Results from the Questionnaire 60

4.7 Conclusions 62

5. DISCUSSIONS, IMPLICATIONS AND RECOMMENDATIONS 65

5.1 Discussion 65

5.2 Implications 67

5.3 Recommendations to Further Research 69

REFERENCES 70

APPENDICES

A. PISA 2003 LEVELS OF PROFIENCY IN MATHEMATICS 76

B. COGNITIVE-METACOGNITIVE FRAMEWORK FOR PROTOCOL ANALYSIS OF PROBLEM SOLVING IN MATHEMATICS 79

C. SELF-MONITORING QUESTIONNAIRE - ENGLISH 84

D. SELF-MONITORING QUESTIONNAIRE - TURKISH 86

E. PROBLEMS USED IN THIS STUDY TRANSLATED INTO TURKISH 88

F. FOLLOW-UP QUESTIONS USED BY EXPERIENCED INTERVIEWERS 97

G. PROBLEMS AND CLINICAL INTERVIEW TRANSCRIPTIONS IN ENGLISH 100

H. STRATEGY USAGES OF THE PARTICIPANTS 143

I. PROBLEM SOLVING EPISODES OF PARTICIPANTS 148

J. MISTAKE RECOGNITION, ADAPTABILITY AND EXPRESSION OF THINKING OF PARTICIPANTS 151

LIST OF TABLES

Table 1. Participants of this Case Study 24
Table 2. Levels and Percentages of Correct Answers of the Chosen Problems in PISA 2003 and Possible Problem Solving Strategies for these Problems 26
Table 3. Scores of the Participants in each Problem and their Total Scores 38
Table 4. Strategies Used by the Participants in Each Problem 39
Table 5. Efficiency Rates of Participants in Different Problem Solving Strategies and Their Total Scores 40
Table 6. Cases Where the Participants Were Successful in Their Second Strategy Attempts 43
Table 7. Cognitive and Metacognitive Episodes of the Participants 44
Table 8. Problem Solving Episodes Which are the Primary Reasons for the Failures of Finding a Correct Answer for Each Participant 48
Table 9. Mistake Recognition and Adaptability of the Participants 50
Table 10. Awareness and Expression of Thinking of the Participants 54
Table 11. Results and Correctness of the Answers When the Participants had Independent Adaptability 56
Table 12. Results and Correctness of the Answers When the Participants had Lack of Mistake Awareness 58
Table 13. Results and Correctness of the Answers When the Participants had Uncertainty 59
Table 14. The Questionnaire Items that have been Responded as Yes by Nearly All Participants 61
Table 15. Descriptions for the Six Levels of Profiency in Mathematics in PISA 2003 76

Table 16. Level and Percentages of Correct Answers in PISA 2003 for Problem 1 100
Table 17. Scores of participants in Problem 1 101
Table 18. Level and Percentages of Correct Answers in PISA 2003 for Problem 2 104
Table 19. Scores of participants in Problem 2 105
Table 20. Level and Percentages of Correct Answers in PISA 2003 for Problem 3 107
Table 21. Scores of participants in Problem 3 108
Table 22. Level and Percentages of Correct Answers in PISA 2003 for Problem 4 113
Table 23. Scores of participants in Problem 4 114
Table 24. Level and Percentages of Correct Answers in PISA 2003 for Problem 5 118
Table 25. Scores of participants in Problem 5 119
Table 26. Level and Percentages of Correct Answers in PISA 2003 for Problem 6 122
Table 27. Scores of participants in Problem 6 123
Table 28. Level and Percentages of Correct Answers in PISA 2003 for Problem 7 127
Table 29. Scores of participants in Problem 7 128
Table 30. Level and Percentages of Correct Answers in PISA 2003 for Problem 8 132
Table 31. Scores of participants in Problem 8 133
Table 32. Level and Percentages of Correct Answers in PISA 2003 for Problem 9 136
Table 33. Scores of participants in Problem 9 136
Table 34. Level and Percentages of Correct Answers in PISA 2003 for Problem 10 140
Table 35. Scores of participants in Problem 10 140
Table 36. Strategies used by the Participants of the Study 143
Table 37. Problem Solving Episodes of the Participants of the Study 148
Table 38. Mistake Recognition, Adaptability and Expression of Thinking of the Participants in the Study 151

ABBREVIATIONS

NCTM	National Council of Mathematics Teachers
OECD	Organization for Economic Co-operation and Development
OKS	Student Selection and Placement Exam for Secondary Education Institutions
ÖSS	University Entrance Examination
PIRLS	Progress in International Reading Literacy Study
PISA	International Student Assessment Program
TIMSS	Trends in International Mathematics and Science Study

CHAPTER 1

INTRODUCTION

Turkey have participated four international comparative studies since 1997: 3rd Trends in International Mathematics and Science Study (TIMSS 1999), Progress in International Reading Literacy Study (PIRLS 2001), and Programme for International Student Assessment (PISA 2003 & PISA 2006). As Berberoğlu (2006) explained "these studies were designed to help researchers, preparers of curricula and educational policy-makers to provide a basis for a better understanding of the functioning of their own educational systems in assessing and restructuring the educational policies of their countries" (p. 4).

TIMSS-R 1999 and PISA 2003 results have shown that the achievement levels of Turkish students are lower than the international average (Mullis et al., 2000; OECD, 2004; Berberoğlu, 2006; Babadoğan & Olkun, 2006; Bulut, 2007). While three-quarters of Organization for Economic Co-operation and Development (OECD) countries' students can perform at least mathematical tasks at Level 2, over half of the Turkish students couldn't manage Level 1 tasks (OECD, 2004) (see Appendix A for level details).

Low performance of Turkish students in mathematics is not unique to international studies. Results of national standardized exams also point out the same direction. For example, Turkish students enter to Student Selection and Placement Exam for Secondary Education Institutions (OKS) at the end of elementary school (i.e., 8th grade). Even though entering is not mandatory, students are placed in various types of secondary schools (e.g., standard public high school, Anatolian high school, vocational technical high school, etc.) according to their results of this exam.

In OKS, students are asked to solve 100 multiple choice questions 25 of which are mathematics items. The mean scores of Turkish students in mathematics questions were 3.35 in 2007 (http://oks2007.meb.gov.tr/oks_ista.htm).

The low performance of Turkish students in International studies (PIRLS, TIMMS, and PISA) and other internal indicators such as some national exams (OKS & University Entrance Examination [ÖSS]) yield to reform efforts in education (Babadoğan & Olkun, 2006). In 1997, compulsory education was increased from 5 to 8 years; in 2002, preschool curriculum for 36-72 months-old children was developed; in 2003, the Board of Education decided to modernize the basic education curriculum, consulting the teachers, faculties of education, and civil society and the new curricula for primary and secondary schools have been implemented in schools "with ongoing changes" since 2004. (Bulut, 2007, pp. 203-204; Worldbank, 2006, p.3). The reforms in Turkish education aimed to change the curriculum from a subject centered to a learner centered one and to shift the education system from a behaviorist approach to more of a constructivist one (Babadoğan & Olkun, 2006 ; Bulut, 2007).

Since the score of the students in OKS exam often determines the type of the secondary school they will attend, the students prepare this exam consisting of multiple choice questions mostly in *dersane* which are private education institutes offering specialized courses for national standardized exams such as OKS and ÖSS rather than their schools. These private courses are usually claimed to focus the students to solve a test item in a shorter time period as possible. Most of the students usually try to memorize how to solve routine problems in a mathematical subject rather than learning the subject in depth. In October 2007, Turkish Ministry of National Education has announced that OKS will be replaced by three Level Determination Exams (SBS) which will be applied at the end of 6th, 7th and 8th grades. One of the reasons for the change is explained as a must to upgrade the assessment according to the new perspective of the new curricula that has been changed since 2004 (MEB, 2007).

The real problem may be the behaviorist point of view which has been very widespread in teaching and learning mathematics in Turkey until recently renewed elementary and secondary grades mathematics curricula (World Bank, 2006). As

Posamentier & Krulik (1998) stated “From the earliest days in school, students are typically taught to solve problems in the most straightforward way possible. This is the way typical mathematics textbook problems are intended to be solved” (p. 15). Having the required knowledge for the mathematical problems is essential for solving the problem (Charles, Lester & O’Daffer, 1987; Schoenfeld, 1992). Also experience has an important role in problem solving so novice problem solvers need to solve many problems to gain that experience (Lesh, 2007; Rudder, 2006; Schoenfeld, 1992). But choosing good problems from textbooks and assigning these problems as homework can not be enough. Most of the time, the homework are done to pass a course or as Posamentier and Krulik (1998) stated, “…a substantial portion of problem solving is done by rote. Students struggle through one problem in the section, the teacher reveals a model solution and the remainder of the problems in the section are solved in the same manner” (p. 15). Imitating the teacher’s solutions in exercises may have the students successful in standard exercises, but these students have great difficulty in solving non-standard problems (Harskamp & Suhre, 2007). They apply the strategy they have memorized and don’t verify their solutions and answers, generally the students misunderstood the problem and/or their strategy doesn’t work or they can’t find an answer (Goldberg, 2003).

Problem solving is not a just method in mathematics, but a major part of learning mathematics where the students deepen their understanding of mathematical concepts by analyzing and synthesizing their knowledge (BoE, 2007; Krulik & Rudnick, 2003, NCTM, 2000). Different problem solving strategies are necessary as students experience new mathematical problems. The teacher’s mission is to create a classroom environment that students are encouraged to explore new strategies, to take risks in trying and to discuss failures and successes with peers and teacher. In such supportive environments, students understand that their solutions are appreciated and develop confidence in their mathematical abilities; hence they develop a willingness to engage in and explore new problems (BoE, 2007; NCTM , 2000).

Metacognitive processes have a positive effect on problem solving performance (Schoenfeld, 1992; Lester, 1994, Yimer & Ellerton, 2006). Students need to learn to monitor and adjust their processes and strategies in solving mathematical problems, so that they can realize how much they know and how good they can apply problem solving strategies (BoE, 2007; NCTM, 2000). Many studies showed that metacognition helps students to overcome difficulties during the mathematical problem solving (Artzt & Armour-Thomas, 1992; Goos & Galbraith, 1996; Goos, Galbraith, & Renshaw, 2000; Pugalee, 2001; Rysz, 2004; Yimer & Ellerton, 2006).

Although international studies and national exams showed that Turkish students have difficulties in mathematics and mathematical problem solving, these studies also showed there are also Turkish students who received high scores. As cited in Education Sector Study (World Bank, 2006):

> Yet it is clear from the above figure that Turkey knows how to produce elite quality education: the share of Turkish students that performed at the highest level on the PISA mathematics exam was 2.4 percent, compared to 2.2 percent for students in the United States. (p. 6)

The literature and curricula recommend some ways to be successful problem solver and highlight the cases why a student can have a failure in solving problems. In this study, I will search how some Turkish students can be successful in solving mathematical literacy items and how and why other Turkish students fail in solving the same problems.

1.1 Purpose of the study

Reviewing the previous literature, I understood that strategies and strategy using abilities are important factors in success of the students in problem solving tasks. Also investigating the cognitive and metacognitive problem solving episodes of the students can give important data about how and why the students can reach a correct answer or not. Mistake recognition, adaptability and expression of thinking are three main components of metacognition; investigating those components may give more data about the failures and success in problem solving process. The purpose of this study was to investigate the problem solving strategies, problem solving episodes, and metacognition of five Turkish students just graduated from elementary school while they are working on mathematical literacy items and explore the interplay of these factors on their success in these items.

1.2 Research Questions

The following research questions will be addressed in this study:

Which problem solving strategies do elementary school graduate Turkish students employ while solving selected PISA 2003 mathematical literacy items?

Which problem solving episodes are observed when elementary school graduate Turkish students while solving selected PISA 2003 mathematical literacy items?

Which mistake recognition, adaptability and expression of thinking behaviors do elementary school graduate Turkish students display while solving selected PISA 2003 mathematical literacy items?

How problem solving strategies, cognitive and metacognitive behaviors of elementary school graduate Turkish students effect their success in PISA 2003 mathematical literacy items?

1.3 Significance of the Research

Mathematical problem solving literature highlights the effects of the problem solving strategies, problem solving episodes, and metacognition of the students on the problem solving, the findings of the present study will add new data about the effects and the interplay of these factors on the problem solving success in mathematics. Previous studies about mathematical problem solving of elementary school Turkish students provided data about the problem solving strategies and methods of 7th and 8th graders (Arslan & Altun, 2007; Karataş & Güven, 2004) and 4th and 5th graders (Yazgan & Bintaş, 2005). Studies also showed that the effects of different problem solving training methods on students' mathematics success (Arslan & Altun, 2007; Koç & Bulut, 2002).In this thesis study, I intended to add more information about the problem solving performances and problem solving strategies of elementary school graduate Turkish students. Yılmaz (2003) showed that metacognitive training helped students in understanding and representing the problem. This study will also give a closer investigation about the cognitive and metacognitive behaviors of these students while solving mathematical problems and the effects of these behaviors on problem solving success.

“Every child may learn mathematics” is the main principle of the new Turkish mathematics curriculum (BoE, 2007). I think that every child may be a good mathematical problem solver should be our further aim. Determining the problem solving abilities of Turkish students can provide us clues about not only their mathematical knowledge but also about the improvements and reforms in educational programs (Karataş & Güven, 2004). In the new mathematics curriculum, expression of thinking, questioning, problem solving and evaluation are listed as some of the roles of the students in learning mathematics (BoE, 2007). The findings of the study will give information about the levels of Turkish students just graduated from elementary school in these roles. The results of the study can also highlight the problem solving habits of the elementary school graduate Turkish students with differing academic success levels. These findings can help the Turkish mathematics teachers and educators when they design mathematical problem solving activities for secondary school students.

Yayan (2003) investigated how out-of-school activities, socioeconomic status, importance given to math, math classroom climate, perception of failure, teacher-centered and student-centered activities explain Turkish, Dutch and Italian students’ mathematics achievement in TIMSS-R (1999). İş Güzel (2006) investigated students’ mathematical literacy skills across Turkey, member and candidate countries of European Union through the PISA 2003. In this study, the Turkish students dealt with problems that were chosen from PISA 2003 mathematical literacy items, the results can give an idea about the effects of problem solving strategies, episodes and metacognition on the success and failures of Turkish students in PISA 2003 mathematical literacy items.

1.4 Definition of Terms

Problem: “A problem exists when you have a goal but do not immediately know how to reach the goal. Thus, a problem consists of three elements: (i) a given state (i.e., current situation), (ii) a goal state (i.e., the desired state of situation) and (iii) obstacles that block you from moving directly from the given state to goal state” (Mayer, 2003, p. 70)

Problem Solving: "Solving a problem means finding a way out of a difficulty, a way out of an obstacle, attaining an aim which was not immediately attainable" (Polya, 1973, p. ix)

Problem solving strategy: a collection of mathematical processes placed in the order in which they may be used (Frobisher, 1994).

Problem Solving Episodes: Schoenfeld (as cited in Artzt & Thomas, 1992) defines an episode as "a period of time during which an individual or a problem-solving group is engaged in one large task". In this study, problem solving episodes refer to the first seven problem solving episodes introduced in cognitive-metacognitive framework of Artzt & Armour-Thomas (1992): Read, Understand, Analyze, Explore, Plan, Implement and Verify.

Metacognition: "Metacognition refers to one's knowledge concerning one's own cognitive processes and products or anything related to them" (Flavell, 1976, p. 232). In this study "three major components, namely recognition of mistakes, adaptability, and awareness and expression of thought" have taken into consideration (Pappas et al., 2003, p. 432).

Recognition of mistakes: "children's ability to monitor their work in order to produce accurate results" (Pappas et al., 2003, p. 432).

Adaptability: "The selection of strategies appropriate for solving specific problems" (Pappas et al., 2003, p. 432).

Awareness and expression of thought: Children's awareness of the solution and ability in expressing the thinking (Pappas et al., 2003).

Case: A problem activity for each participant has been accepted as a case. In this study, 10 problems have been applied to 5 participants. So the study investigated total of 50 cases.

CHAPTER 2

LITERATURE REVIEW

This chapter will give an overview of relevant literature pertinent to the research study investigating the effects of strategies, problem solving episodes, and metacognition of Turkish secondary students on their problem solving success.

2.1 Mathematical Problem

Problem is used in daily life referring a "trouble". For most of the people, a problem reminds an uncontrollable situation or a situation is very difficult to handle. Dictionary definitions of the problem are (1) a (serious) difficulty that needs attention and thought; (2) a question for consideration or for which an answer is needed (Longman Active Dictionary of English, 1989). Mayer (2003) gives the several definitions that we need in the study:

> A problem exists when you have a goal but do not immediately know how to reach the goal. Thus, a problem consists of three elements: (i) a given state (i.e., current situation), (ii) a goal state (i.e., the desired state of situation) and (iii) obstacles that block you from moving directly from the given state to goal state (p.70).
>
> A routine problem is a problem for which the problem solver immediately knows the solution procedure. In contrast, a non-routine problem is a problem for which the solver does not know a solution procedure (p. 71).
>
> A well-defined problem has a clearly specified initial state, goal state and set of operators. For example, "$3X + 2 = 8$, Solve for X". In an ill-defined problem, initial state, goal state and/or set of operators is not clearly

specified. For example, “How can a newly married couple get enough money to buy a new house?” (p. 72).

In mathematics education, “word problems” have been commonly accepted as problems for many years (Frobisher, 1994, p. 152). These problems are also called story problems. These problems can involve one or more calculations. They are routine problems that aim to have the students gain basic problem solving skills. (Altun, 2000). Now, word problems are accepted as exercises or questions. Krulik and Rudnick (2003) explain the distinctions between questions, exercise and problem as (p. 3):

(a)Question: a situation that can be resolved by mere recall and memory

(b)Exercise: a situation that involves drill and practice to reinforce a previously learned skill or algorithm

(c)Problem: A situation that requires analysis and synthesis of previously learned knowledge to resolve.

Polya (1973) states that solving a problem means “finding a way out of a difficulty, a way out of an obstacle, attaining an aim which was not immediately attainable ” (p. ix). NCTM (2000) defines problem solving as “engaging in a task for which the solution method is not known in advance” (p. 52). The new Turkish mathematics curriculum underlines that exercises solution of which are already known cannot be accepted as problems. Also it is remarked that when problems can make the student interested and engaged, and wishful to obtain a solution, the gained mathematical knowledge and skills will be more meaningful (BoE, 2007). Problem solving is not a just method in mathematics, but a major part of learning mathematics. (BoE, 2007; NCTM 2000). Since it requires analysis and synthesis of the previous skills and knowledge, as Krulik and Rudnick (2003) remarks “to succeed in problem solving is to learn how to learn” (p. 7).

2.3 Problem Solving Strategies

A strategy is defined as “a plan of action resulting from strategy or intended to accomplish a specific goal” in the dictionary (The American Heritage Dictionary of the English Language, 2000). In a sense of mathematical problem solving, a strategy is a collection of mathematical processes placed in the order in which they may be used (Frobisher, 1994).

Polya's book *How to Solve It* also started the discussion of problem solving strategies or heuristics (Schoenfeld, 1992). The word "heuristic" comes from the same Greek root (`ευρισκω) as "eureka" meaning "to find". Heuristic means "serving to discover". Polya (1973) explained the aim of heuristics as "to study the methods and rules to discovery and invention" (p. 12). Polya (1973) introduced the heuristics (exploiting) analogy, auxiliary elements, decomposing and recombining, induction, specialization, variation and working. These strategies in Polya's book have been appreciated, but they have also been criticized for being "descriptive rather than prescriptive" (Schoenfeld, 1992, p. 353).

Different problem solving strategies are needed in solving various mathematical problems (BoE, 2007; Charles, Lester & O'Daffer, 1987; Schoenfeld, 1992). Solving a problem a strategy can be used, but in some problems several strategies may be used together to find a solution (BoE, 2007; NCTM, 2000). Schoenfeld (1992) emphasizes that if the strategies are tried to be taught as algorithms, the students cannot develop the desired skills. In NCTM 2000 and in the new Turkish mathematics curriculum, it is underlined that teacher's must create a classroom environment that students are encouraged to explore new strategies, so that students can understand that their solutions are appreciated and develop confidence in their mathematical abilities; hence they develop a willingness to engage in and explore new problems (BoE, 2007; NCTM, 2000).

Charles, Lester & O'Daffer (1987) listed the strategies that might be introduced in a problem solving program (p.10):

1. Guess, check, revise,
2. Draw a picture,
3. Act out the problem,
4. Use of objects,
5. Look for a pattern,
6. Choose an operation,
7. Solve a simpler problem,
8. Make a table,

9. Write an equation,
10. Use logical reasoning,
11. Work backward.

Although different names have been given to the strategies, Posamentier and Krulik (1998) list the major problem solving strategies that can be used in solving mathematical problems as (pp.4-5) :

1. Working backwards
2. Finding a pattern
3. Adopting a different point of view
4. Solving a simpler, analogous problem (specification without loss of generality)
5. Considering extreme cases
6. Making a drawing (visual representation)
7. Intelligent guessing and testing
8. Accounting for all possibilities
9. Organizing data
10. Logical reasoning

Posamentier and Krulik (1998) also explained how these strategies are used in daily life and introduced some problems that can be solved using these strategies.

Özkaya (2000) investigated the students' uses and preferences in problem solving strategies in geometry problems. She has shown that students prefer "unconventional" problem solving strategies (consisting of adapting a known procedure, visual abstraction from given procedure, deriving an equation, estimation) to "conventional" problem solving strategies (consisting of trial and error, counting, direct calculation, using a simpler form of the problem, finding a pattern) in solving a geometry test.

Karataş and Güven (2004) examined and discussed 8th grade Turkish students' successes and failures in word problem solving process. Four word problems have been implemented to five students by using clinical interviews. The data showed that students usually explain problems by using variable and have difficulties in writing equations if they misunderstand the problem.

Yazgan and Bintaş (2005) examined the 4th and 5th grade students' learning and using of problem solving strategies. Strategies were determined as guess and check, look for a pattern, make a drawing, work backward, simplify the problem and make a systematic list. It was observed that 4th and 5th grade students can informally use problem solving strategies without any training; strategies can be learned by 4th and 5th grade students and training had a positive effect on students' problem solving success.

Arslan and Altun (2007) designed a trial study in order to teach problem solving strategies and to find out at which level 7th and 8th class students have learnt them and discussed the use of problem solving strategies. The strategies consist of six heuristic strategies known as Simplify the Problem, Guess and Check, Look for a Pattern, Make a Drawing, Make a Systematic List and Work Backward. The results showed that low ability pupils significantly benefited from the learning environment, especially in the use of simplify the problem and work backward strategies.

The findings of researchers on problem solving of Turkish elementary school students showed that Turkish students can use some problem solving strategies without any training and the training had a positive effect on the usage of problem solving strategies and students' problem solving success (Arslan & Altun, 2007; Karataş & Güven, 2004; Yazgan & Bintaş, 2005).

2.3 Problem Difficulty and Good Problem Solvers

The definition of problem and problem solving bring the discussion of difficulty of a problem. In 1970s and early 1980s, researchers tried to determine the factors of problem difficulty (Lester, 1994). Schoenfeld (as cited in Mayer, 2003) stated that "it is the particular relationship between the individual and the task that makes the task a problem for that person". Lester (1994) emphasizes that

> Problem difficulty is not so much a function of various task variables as it is of characteristics of the problem solver, such as traits (e.g., spatial visualization ability, ability to attend to structural features of problems), dispositions (i.e., beliefs and attitudes), and experiential background (e.g., instructional history, familiarity with types of problems). (pp. 664-665)

So the same problem can be difficult or challenging for some people, and easy for some others. This idea yields to the research of *special* problem solvers. Many researchers (e.g., Schoenfeld, 1985; Silver, 1985) studied the distinctions between "good" and "poor" problem solvers (alternatively, "successful" and "unsuccessful" problem solvers, or "expert" and "novice" problem solvers). (Lesh & Zawojewski, 2007; Lester, 1994)

Lester (1994) summarizes the Schoenfeld's studies about the differences of good problem solvers from poor problem solvers in at least five important respects:

1. Good problem solvers know more than poor problem solvers and what they know, they know differently-their knowledge is well connected and composed of rich schemata.
2. Good problem solvers tend to focus their attention on structural features of problems, poor problem solvers on surface features.
3. Good problem solvers are more aware than poor problem solvers of their strengths and weaknesses as problem solvers.
4. Good problem solvers are better than poor problem solvers at monitoring and regulating their problem-solving efforts.
5. Good problem solvers tend to be more concerned than poor problem solvers about obtaining "elegant" solutions to problems. (p. 665)

Using the framework of Polya (1973) and the characteristics of good problem solvers, Rudder (2006) investigated the strategies and thought processes of American and Singaporean students while performing mathematics problem solving tasks, to measure the differences in the students' performance. During the task-based interviews students completed twelve problem solving tasks while thinking aloud. The American and Singaporean students both demonstrated two similar factors not gaining completely correct results when completing the problems solving tasks:

- Lack of checking leads to errors
- Not deciding on a method leads to errors (p.191).

The American students had three other factors:

- Feelings about a problem affects students performance
- Not understanding the question leads to errors

- Difficulty with foundational concepts leads to errors (p. 191).

Observing differences of good problem solvers from poor problem solvers, we can observe whether a student tend to be a good or poor problem solver. As Lester (1994) points out that “although, we can learn a lot by comparing the behaviors of experts and novices” (p. 665), characteristics of the expert problem solvers “cannot be directly taught to non experts” (Lesh, 2007, p. 767).

2.4 Cognition, Metacognition and Problem Solving

Gagne and Medsker (1996) defines problem solving as “higher level cognitive activity, either novel or routine, that requires previous learning of various types and that may result in new learning” (p. 124). Mayer (2003) states that problem solving includes three features that make it a “directed cognitive processing” (p. 71). He explains the features as “(a) Cognitive – problem solving occurs internally in one’s cognitive system, (b) Process – problem solving involves mental computation in which a mental operation is applied to a mental representation, and (c) Directed – problem solving is based on one’s goal and results in an activity intended to solve a problem.”(p. 71).

Similar to Mayer, Gagne and Medsker (1996) lists the key features of problem solving as (1) the task requires a solution or sets a performance goal, but the solution process may not be a defined procedure, and there may be a variety of correct solutions; (2) some degree of search takes place in the performers thinking process; (3) the performer uses previously learned rules, verbal information, and cognitive strategies to reach a solution or achieve the goal; and (4) in the process of solving the problem, the performer may learn a higher order rule or cognitive strategy that will help solve similar problems in the future (pp. 124-125).

We can see not only cognitive but also metacognitive perspectives of problem solving in these features. Although distinguishing what is cognitive from what is metacognitive is not easy (Garofalo & Lester, 1985; Goos & Galbraith, 1996), when metacognition is studied, the literature most of the time gave the following description of Flavell (as cited in Garofalo & Lester, 1985) as a guideline:

> Metacognition refers to one's knowledge concerning one's own cognitive processes and products or anything related to them, e.g., the

> learning-relevant properties of information or data. . . . Metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects on which they bear, usually in the service of some concrete goal or objective.

Using this explanation, Garofalo and Lester (1985) gives the relation between cognition and metacognition as "cognition involved in doing, whereas metacognition is involved in choosing and planning what to do and monitoring what is being done" (p. 164).

Garofalo and Lester (1985) explains that metacognitive knowledge involves "the interaction of person, task, and strategy" (p.168). Person knowledge includes "one's assessment of one's own capabilities and limitations with respect to mathematics in general and also with respect to particular mathematical topics or tasks" (p. 167).Mathematical task knowledge includes "one's beliefs about the subject of mathematics as well as beliefs about the nature of mathematical tasks" (p. 167). Mathematical strategy knowledge includes "knowledge of algorithms and heuristics, but it also includes a person's awareness of strategies to aid comprehending problem statements, organizing information or data, planning solution attempts, executing plans, and checking results" (p. 168). Schoenfeld (as cited in Yimer & Ellerton, 2006) listed the metacognitive processes as assessing one's own knowledge, formulating a plan of attack, selecting strategies, and monitoring and evaluating progress. Researchers remark that metacognitive processes have a positive effect on problem solving performance (Schoenfeld, 1992; Lester, 1994, Yimer & Ellerton, 2006). In mathematics curricula, it is remarked that students need to learn to monitor and adjust the strategies (BoE, 2007; NCTM, 2000).

Lester (1994) emphasizes "the degree to which metacognition influences problem-solving activity has not been resolved-indeed", and lists the three results have come to be generally accepted as (p. 666):

1. Effective metacognitive activity during problem solving requires knowing not only what and when to monitor, but also how to monitor. Moreover, teaching students how to monitor their behavior is a difficult task.

2. Teaching students to be more aware of their cognitions and better monitors of their problem-solving actions should take place in the context of learning specific mathematics concepts and techniques (general metacognition instruction is likely to be less effective).
3. The development of healthy metacognitive skills is difficult and often requires "unlearning" inappropriate metacognitive behaviors developed through previous experience.

Yılmaz (2003) investigated the effect of metacognitive training on seventh grade students' problem solving performance. The results showed that the training didn't make a difference in students' problem solving performances but it helped students in understanding and representing the problem. Many studies have confirmed that metacognition helps students to overcome difficulties during the problem solving process (Artzt & Armour-Thomas, 1992; Goos & Galbraith, 1996; Goos, Galbraith, & Renshaw, 2000; Pugalee, 2001; Rysz, 2004; Yimer & Ellerton, 2006).

Pappas et al. (2003) conceptualize metacognition as involving three major components: recognition of mistakes, adaptability, and awareness and expression of thought (p. 432):

> Recognition of mistakes (Garafalo & Lester, 1985) refers to children's ability to monitor their work in order to produce accurate results. For example, children learn to see when numbers have been added incorrectly.
>
> Adaptability (Shrager & Siegler,1998) refers to the selection of strategies appropriate for solving specific problems. For example, children learn that counting individual objects is an effective yet cumbersome strategy when solving a problem like 25+12, and adapt by selecting an alternative method like counting from the larger number.
>
> A third component of metacognition is awareness and expression of thought (Carr & Jessup, 1995). For instance, children gradually become aware of the fact that the answer to 5 + 2 was determined by counting on their fingers, and learn to describe the process in a reasonably coherent fashion.

2.5 Cognitive and Metacognitive Problem Solving Frameworks

Polya (1973) emphasizes that effective problem-solving consists of four main phases: understanding the problem, devising a plan, carrying out the plan, and looking back. Understanding the problem includes labeling and identifying unknowns, condition(s), and data, and determining the solubility of the problem.

Devising a plan includes deciding an appropriate technique or problem solving strategy, restating the problem if necessary. Carrying out the plan, the chosen strategy is used to solve the problem step by step. The strategy can be changed if necessary. In the looking back phase, the solution is checked if the answer is reasonable according to the given problem. Other problems can be considered with the same strategy, and other strategies can be checked to solve the same problem.

This four phase model yield to a great amount of research on problem solving. Garofalo and Lester (1985), says that “the four phases (understanding, planning, carrying out the plan, and looking back) serve as a framework for identifying a multitude of heuristic processes that may foster successful problem solving” (p. 169). But they claim that “Polya's conceptualization considers metacognitive processes only implicitly” (p.169).

Schoenfeld (as cited in Goos et al., 2000) developed a procedure involving five episodes: Reading, Analysis, Exploration, Planning /Implementation, and Verification. Goos et al. (2000) emphasize that “this framework specifies the ideal cognitive and metacognitive characteristics of each episode, which can be compared with students’ observed problem solving behaviors” (p. 3). Harskamp & Suhre (2007) evaluated the effectiveness of a student controlled computer program for high school mathematics based on instruction principles derived from Schoenfeld’s theory of problem solving. The computer program allowed students to choose problems and to make use of hints during different episodes of solving problems (analyzing the problem, selecting appropriate mathematical knowledge, making a plan, carrying it out, and checking the answer against the question asked). The analysis showed that “hints during different episodes differ in their effectiveness to improve problem solving” (p. 837). Use of hints that are connected to the solution approach (Plan) and feedback on the correct solution approach (Verify) were most effective for both weak and strong problem solvers. This result supports the comment of Garofalo & Lester

(1985) who remarked that "metacognitive decisions, especially managerial ones, between these episodes can have powerful effects on solution attempts" (p. 169).

Garofalo and Lester (1985) developed a cognitive-metacognitive framework that comprises four categories of activities involved in performing a mathematical task: orientation, organization, execution, and verification. They claimed that the four categories are "related to, but are more broadly defined than, Polya's four phases" (p. 171).

Stillman and Galbraith (1998) focused both on the mathematical processing and the underlying cognitive and metacognitive activities that led to that processing of female students at the senior secondary level using the cognitive metacognitive framework of Garofalo and Lester (1985). Their finding showed that "on average more time was spent on orientation and execution activities with little time being spent on organization and verification activities, however, the successful groups spent less time on orientation than the other groups" (p. 183). Rysz (2004) also used the cognitive metacognitive framework of Garofalo and Lester (1985) by adding a fifth category labeled as lack of metacognition to identify metacognitive thoughts adult students had while learning elementary probability and statistics concepts and while problem solving, alone and with other students. The results showed that "students can earn above-average grades using limited or no metacognition, but those who provided evidence of cognitive awareness and self-monitoring were better able to report an understanding of probability and statistics concepts" (p. ii).

Artzt and Armour-Thomas (1992) introduced another cognitive-metacognitive framework that "attempts to show a synthesis of the problem-solving steps identified in mathematical research by Garofalo and Lester, Polya, and Schoenfeld, and of cognitive and metacognitive levels of problem solving behaviors studied within cognitive psychology, in particular, by Flavell" (p. 140). They separated Schoenfeld's Plan/Implement episode into two distinct categories, and expand the episodic categories for the coding of student behaviors in groups to include understanding the problem and watching and listening. The eight episodes are: (i) reading, (ii) understanding, (iii) analysis, (iv) exploring, (v) planning, (vi) implementing, (vii) verifying, (viii) watching and listening. Each of the eight problem-solving episodes was categorized as cognitive or metacognitive.

Characteristics of each episode, as defined by Artzt and Armour-Thomas, are given in Appendix B.

Goos *et.al* (2000) investigated the metacognitive self monitoring strategies used by senior secondary school students while working individually on a mathematics problem using the framework of Artzt and Armour-Thomas (1992). Although there were instances of successful self-monitoring in the participants, it was found that

> Even if students do review their progress towards the goal, check their calculations while they work, and attempt to verify the accuracy and sense of their answer, their worthy metacognitive intentions will be foiled if they are unable to recognize when they are stuck, have no alternative strategy available, cannot find their error (or cannot fix it if they do find it), or fail to recognize nonsensical answers (p. 16).

Teong (2003) modified Artzt and Armour-Thomas' framework and used this metacognitive framework CRIME - Careful reading, Recall possible strategies, Implement possible strategies, Monitor, and Evaluation- to observe the effects of metacognitive training on the problem solving abilities of 11-12 year old Singaporean students. The participants used a computer-based program WordMath in training. The findings of the study "revealed that the ability to know when and how to use metacognitive behaviors when they are needed are important determinants to successful word-problem solving" (p. 53). But the study also showed that "the occurrence of metacognitive behaviors on its own does little to ensure successful word-problem solving" (p. 53). Jones (2006) examined the cognitive processes used during problem-solving tasks of two middle school students with different levels of mathematics anxiety and self-esteem in a qualitative case study. The study showed that a "contributing factor to having success with cognitive processes is combining them with other mathematical procedures, concepts, and problem solving heuristics" (p. 104). The findings of Goos et al. (2000), Teong (2003) and Jones (2006) confirmed the conclusions of Artzt and Armour-Thomas (1992) who have remarked that "the interrelationship between metacognitive and cognitive processes is complex, and an appropriate interplay between the two is necessary for successful problem solving to occur" (p. 161).

2.6 Summary

Although word problems have been accepted as problems in mathematics education for many years, the latest literature agrees that the problem must require analysis and the synthesis of the previous knowledge, and a challenge where attaining the aim which is not immediately attainable (Altun, 2000; BoE, 2007; Frobisher, 1994, Krulik & Rudnick, 2003; NCTM, 2000, Polya, 1973). Problem solving is not a just method in mathematics, but a major part of learning mathematics (BoE, 2007; NCTM 2000). It is a process where the students deepen their understanding of mathematical concepts by analyzing and synthesizing their knowledge and learn how to learn (BoE, 2007; Krulik & Rudnick, 2003, NCTM, 2000).

Difficulty of a problem depends on the characteristics, abilities and experience of the problem solver more than the properties of the task. (Lester, 1994, Mayer, 2003). Many researchers (e.g., Schoenfeld, 1985; Silver, 1985) studied the distinctions between "good" and "poor" problem solvers, but it is understood that the properties of experts cannot be directly taught to nonexperts (Lesh & Zawojewski, 2007; Lester, 1994).

Polya (1973) presented a four phase model for effective problem-solving (understanding the problem, devising a plan, carrying out the plan, and looking back) and introduced some heuristics which leaded to a great amount of research on problem solving and problem solving strategies. Different problem solving strategies are needed in solving various mathematical problems. Solving a problem a strategy can be used, but in some problems several strategies may be used together to find a solution (BoE, 2007; NCTM, 2000; Polya, 1973; Schoenfeld, 1992). The findings of researchers on problem solving of Turkish elementary school students showed that Turkish students can use some problem solving strategies without any training and the training had a positive effect on the usage of problem solving strategies and students' problem solving success (Arslan & Altun, 2007; Koç & Bulut, 2003; Karataş & Güven, 2004; Yazgan & Bintaş, 2005).

Schoenfeld (as cited in Goos et al. ,2000) developed a procedure involving five episodes: (Reading, Analysis, Exploration, Planning /Implementation, and Verification); Garofalo and Lester (1985) developed a cognitive-metacognitive framework that comprises four categories of activities involved in performing a mathematical task: orientation, organization, execution, and verification. These two frameworks added cognitive and metacognitive perspectives to the problem model of Polya (1973). Then Artzt & Armour Thomas (1992) introduced another cognitive-metacognitive framework that "attempts to show a synthesis of the problem-solving steps identified in mathematical research by Garofalo and Lester, Polya, and Schoenfeld, and of cognitive and metacognitive levels of problem solving behaviors studied within cognitive psychology, in particular, by Flavell" (p. 140). The eight episodes are: (i) reading, (ii) understanding, (iii) analysis, (iv) exploring, (v) planning, (vi) implementing, (vii) verifying, (viii) watching and listening. These frameworks are useful in observing the cognitive and metacognitive behaviors in problem solving episodes.

Flavell (1979) defines metacognition as "one's knowledge concerning one's own cognitive processes and products or anything related to them" (p. 232). Garofalo & Lester (1985) gives the relation between cognition and metacognition as "cognition involved in doing, whereas metacognition is involved in choosing and planning what to do and monitoring what is being done" (p. 165). Researchers have showed that problem solving requires efficient use of cognitive and metacognitive behaviors (Artzt & Armour Thomas, 1992; Garofalo & Lester, 1985; Gagne & Medsker, 1996; Lester, 1994, Mayer, 2003). Jones (2006) showed that a "contributing factor to having success with cognitive processes is combining them with other mathematical procedures, concepts, and problem solving heuristics" (p. 104). Other studies showed that metacognitive processes are important factors on problem solving performance (Schoenfeld, 1992; Lester, 1994, Yimer & Ellerton, 2006) and metacognition helps students to overcome difficulties during the problem solving process (Artzt & Armour-Thomas, 1992; Goos & Galbraith, 1996; Goos et al., 2000; Pugalee, 2001; Rysz, 2004; Yılmaz, 2003, Yimer & Ellerton, 2006).

In summary, the literature showed me that to investigate the reasons for the problem solving successes and failures of the students; problem solving strategies, problem solving episodes (cognitive and metacognitive behaviors during these episodes) and metacognitions of the students should be taken into consideration.

CHAPTER 3

METHODOLOGY

This chapter outlines the methodology for the research project investigating the problem solving strategies, problem solving episodes, and metacognition of Turkish secondary students while solving the PISA 2003 mathematical literacy items. The details of the methodology of the research are explained in detailed sections below.

3.1 Research Design

Merriam (1998) suggests that qualitative research "help us understand and explain the meaning of social phenomena with as little disruption of the natural setting as possible" (p.5). In particular, Merriam (1998) emphasizes that

> "A case study design is employed to gain an in-depth understanding of the situation and meaning for those involved. The interest is in process rather than outcomes, in context rather than a specific variable, in discovery rather than confirmation" (p.19).

In this study, qualitative research strategies are utilized for collecting and analyzing data in order to gain understanding about the problem solving strategies, problem solving episodes, and metacognition of five Turkish students just graduated from elementary school and explore the interplay of these factors on their problem solving success in mathematical literacy items selected from PISA 2003.

The following research questions will be addressed in this study:

Which problem solving strategies do elementary school graduate Turkish students employ while solving selected PISA 2003 mathematical literacy items?

Which problem solving episodes are observed when elementary school graduate Turkish students while solving selected PISA 2003 mathematical literacy items?

Which mistake recognition, adaptability and expression of thinking behaviors do elementary school graduate Turkish students display while solving selected PISA 2003 mathematical literacy items?

How problem solving strategies, cognitive and metacognitive behaviors of elementary school graduate Turkish students effect their success in PISA 2003 mathematical literacy items?

3.1.1 Participant Selection

As Merriam (1998) stated "sample selection in qualitative research is usually (but not always) non-random, purposeful and small, as opposed to the larger, more random sampling of quantitative research"(p. 8). So, I decided to use a purposeful sampling in the research. Patton (1990) recommends specifying a minimum sample "based on expected reasonable coverage of the phenomenon given the purpose of the study" (p. 186). Merriam (1998) recommends choosing "the factors relevant to the study's purpose" to determine the size of the sample (p. 66). The four criteria for choosing the participants used in this research study are given as follows:

The first criterion was having students of the same grade level. The students in the study were elementary school graduate students who were beginning to ninth grade. The second criterion was the academic success of the participants. The participants had graduated from different elementary schools so I used the OKS success levels as a guide to determine participants' success levels. The third criterion the researcher looked for was the students' willingness to participate in the research study. The students and their parents had been informed about the study. All the participants were willing to attend the study. The last criterion was teacher recommendations. Since I haven't taught the students, the candidate list had been formed by their teachers according to the first three criteria. Using the above mentioned criteria, I selected 4 females and one male student attending to a dersane in Ankara with the assistance of their mathematics teachers in there. Aylin and Bora were two participants who had been recommended as successful problem solvers.

Deniz and Emel had been chosen as novice problem solvers. Ceylan had been chosen as the poor problem solver. The OKS success percentages and school types are introduced in Table 1. Here is a summary of the teachers ‘comments about the participants:

Aylin doesn’t hesitate to ask when she has difficulties. She doesn’t directly accept what she is taught, she examines and asks questions. She likes solving many mathematical problems. She likes discussing solutions with her peers. Bora has a high self confidence, he likes presenting his ideas. But he can sometimes react learning new things and force his own ideas. In general, he likes discovering things by himself. Deniz has the attitude to learn as much as she need, she doesn’t ask for more in learning. If she has difficulties, she asks questions; but she can quit when she feels too much challenge. She likes learning from others. Emel likes learning with her peers. She tries to learn the algorithms; she doesn’t examine the solution methods. She can quit by accepting that she doesn’t know how to solve that type of problems. Ceylan doesn’t have confidence in mathematics; she tries to memorize algorithms and formulas. To do that she likes solving the same problem with the same method to be sure that she has learned.

Table 1. Participants of this Case Study

Student Name (Gender)	OKS Success (over 818359 students)	Secondary School
Aylin (F)	0.005%	Science High School
Bora (M)	0.160%	Private High School
Deniz (F)	6.891%	Anatolian High School
Emel (F)	8.652%	Anatolian High School
Ceylan (F)	73.247%	General High School

Note. Student names have been changed with a pseudonym that preserving the gender.

3.1.2 Selection of the mathematical problems for the interviews

Hunting (1997) says that "time needed to be set aside for an interview depends on the age of the student…with 10-12 year old children the time might range from 35 to 50 minutes" (p. 151). The suggested time for the whole interview was approximately 40-45 minutes. So, ten problems were decided to be used in the pilot study. Selection of problems to be used in the study was guided by three main criteria recommended by Goos & Galbraith (1996):

1. The questions had to be relevant to the students' classroom experience if the protocols were to provide insights into thinking processes typical of the subject area;
2. The questions had to be challenging enough to require, and elicit, metacognitive behavior, while simultaneously being within the capacity of the subjects to solve with existing knowledge;
3. The questions needed to contain a blend of genuine 'problems' and routine exercises, so that initial success on the latter would help put the students at ease at the start of each think-aloud session. (p. 238)

The problems have been chosen from PISA 2003 mathematical literacy items that have been published in OECD website. These items are in the level of 15-year-olds, and they concern with how students analyze, reason and communicate (OECD, 2003). Difficulty levels of the items and the percentages for the correct answers for the items have been used in choice of problems with different difficulty. Possible problem solving strategies have also been considered.

The selected PISA 2003 problems were translated into Turkish by the researcher according to the translation notes provided with the problems. The language of the problems has been checked by two mathematics teachers. The selected problems for the research are provided in Appendix G. The scoring criteria and scores of participants are also provided for each problem in Appendix G. The translated problems are provided in appendix D. Table 2 summarizes the levels of the problems and the percentages of the correct answers of Turkish students and all students participated in PISA 2003.

Table 2. Levels and Percentages of Correct Answers of the Chosen Problems in PISA 2003 and Possible Problem Solving Strategies for these Problems

Problem No	Level	Correct PISA 2003 Total	Correct PISA 2003 TR	Possible Problem Solving Strategies
1*	Level 2	64.58%	46.96%	Finding a pattern, Visual representation
2*	Level 2	72.73%	61,47%	Logical Reasoning
3*	Level 6	24.90%	20,21%	Intelligent guessing & Testing, Logical Reasoning
4	Not Available	Not Available	Not Available	Accounting for all possibilities, Logical reasoning
5	Level 3	60.46%	40.98%	Accounting for all possibilities, Logical reasoning
6*	Level 3	58.67%	36.56%	Organizing Data, Logical reasoning, Visual Representation
7	Level 6	19.38%	11.64%	Finding a pattern, Logical Reasoning
8	Level 3	60.41%	38.75%	Logical reasoning
9	Level 4	46.90%	27.98%	Finding a pattern, Organizing Data
10	Level 4	47.48%	25.43%	Working backwards, Accounting for all possibilities, Intelligent guessing & Testing

* Levels are determined by the researcher according to the percentages of correct answers in total PISA 2003 as no information has been published about the levels of these items in the OECD reports.

3.1.3 Data Collection

In a case study research, data can be collected by interviews, observations and various documents. (Merriam, 1998). I collected the data by

- Clinical Interviews
- Solution sheets
- Self-Monitoring Questionnaire

3.1.3.1 Clinical Interviews

Merriam (1998) emphasize that "in qualitative research, interviewing is often the major source of the qualitative data needed for understanding the phenomenon under study" (p. 91). He also remarks that "most common is the semi-structured interview that is guided by a set of questions and issues to be explored, but neither the exact wording nor the order of the questions is predetermined" (p. 93).When observing the mathematical skills of the students, the interaction of the students and the tasks can be observed by clinical interviews (Baki, Karataş & Güven, 2002). I conducted five clinical interviews to collect data using ten predefined mathematical problems. The interviews have conducted on 15th of September in 2007 in one of the available classroom in the dersane where the participants were attending.

I informed the students that the research was not investigating on their correct answers, but more importantly how they thought while doing problem solving. The participants were also informed that there was no time limit for any problems as long as they have eager to solve that problem. The clinical interviews ranged from 35 to 45 minutes when these ten problems were solved and the questionnaire was filled by each participant.

Hunting emphasizes the advantage of clinical interviews as "the data source (the student) and the data analyzer and interpreter (the teacher-clinician) can engage directly in interactive communications. The teacher-clinician "reads the play" as the play proceeds" (p. 145). In a clinical interview, the students should be carefully observed but their explanations should not be influenced by the researcher (Baki et al., 2002). Since I had no clinical interview experience myself, I read the recommendations and transcriptions in previous studies (Artzt& Armour-Thomas, 1992; Baki et al., 2002; Hunting,1997). I observed the students in the interviews and asked questions only when participants needed scaffolding or I needed the participants to express their mental processes, responses and implementations. I tried to use the follow-up question suggested by Hunting (1997, pp. 153-154) such as:

- Can you tell me what you are thinking?
- Can you say out loud what you are doing?
- Can you tell me how you worked that out? How did you know? How did you decide?
- Was that just a lucky guess?
- The other day another student told me.. .
- Do you know what _ means?
- Do you know a way to check whether you are right?
- Why?
- Pretend you are the teacher. Could you explain what you think to a younger child? How would you explain?

Detailed descriptions of the follow up questions as described by Hunting (1997) are provided in Appendix F.

The students did the computations and wrote the solutions on the solution sheets. They were also given instructions to complete each task while thinking out loud by verbally expressing their thoughts as they work on the problems. Merriam (1998) emphasizes the importance of completeness of the recordings and recommends video recording to analyze the observations after the interview, when possible. Artz and Armour Thomas (1992), Stillman and Galbrath (1998) and Teong (2003) also used video recording in this manner. The video tape recordings helped me to analyze participants' nonverbal behaviors in different tasks.

3.1.3.2 Self Monitoring Questionnaire

Following completion of all problems, the participants completed a self monitoring questionnaire. The questionnaire was adapted by Goos et al. (2000) from Fortunato, Hecht, Tittle, and Alvearez (as cited in Goos et al., 2000). This questionnaire was designed to "allow the students to evaluate themselves, regarding what they thought they did during the problem solving activity" (Goos et al., 2000, p. 5). The questionnaire consisted of total of twenty-one statements to which students

responded by ticking boxes marked Yes, No, or Unsure in three sections. The questionnaire has been translated by the researcher and the language of the questionnaire has been checked by English speaker mathematics teacher. The original Questionnaire and Turkish translation is provided in Appendices D and E.
The Questionnaire data provided me with the opportunity to compare the findings obtained from the interviews and the observations.

3.2 Data Analysis

Merriam (1998) emphasizes the importance of using multiple data in case studies as "On-site investigation of the case involves observing what is going on, talking informally and formally with people, and examining documents and materials that are part of the context" (p. 137). As Patton (1990) points out, "By using a combination of observations, interviewing, and document analysis, the fieldworker is able to use different data sources to validate and cross-check findings" (p. 244). To find answers to the research questions, I used a combination of observations, interviewing, and document analysis. I transcribed the video recordings of the interviews to clinical interview transcriptions that are presented in Appendix G in English. The transcriptions involve my observations, students' responses and answers. The research data were analyzed according to "Problem Solving Strategies", "Problem Solving Episodes", and "Metacognitions". Questionnaire responses of the participants were also analyzed to crosscheck the results. I took the assistance of a mathematics education doctoral student in checking the codings I have made. After I had finished the codings for the strategies, episodes and metacognitions of the participants, the second rater checked the correctness of the codings for the first four questions of all participants. A full agreement between the codings I and the second rater had made was reached at the end. I have made the necessary changes in coding and analyses accordingly.

3.2.1 Analysis of Problem Solving Strategies of the Participants

The problem solving strategies that are used by participants were coded according to the definitions given by Posamentier & Krulik (1999). For each problem, the strategies used by the participants were listed. "Strategies yield to

Incorrect or No Answer" and "Strategies yield to Correct Answer" were coded for each participant in each problem by the researcher. The numbers of attempts of using different problem solving strategies of each participant when they find correct or partially correct answers in each problem were also coded (see Appendix H for full analysis). The definitions of the strategies based on the descriptions of Posamentier & Krulik (1999) are:

Working backwards

Students reverse the steps that produced an end result which can lead to the starting value which is required to find.

Finding a pattern

Student try to find a rule or pattern to explain the situation and solve the problem according to the pattern.

Adopting a different point of view

Student adopt a different point of view than that to which he or she was led initially by the problem

Solving a simpler, analogous problem (specification without loss of generality)

Student tries to solve a simpler problem to see the solution of the original problem.

Considering extreme cases

The student considers the extreme cases of the variables that do not change the nature of the problem.

Making a drawing (visual representation)

The student draws a figure or diagram etc. to represent the given data in the problem visually.

Intelligent guessing and testing

Student make a guess and test it against the conditions of problem, the next guess is based upon the information obtained from the previous guess.

Accounting for all possibilities

Student tries to list all the possible conditions in the problem and evaluate or check each condition to find the one that suits to the aim of the problem. The listing should be organized to account all of the possibilities.

Organizing data

Student organizes the given data in a table or a systematical listing.

Logical reasoning

The student uses logical reasoning (deduction or induction or abduction)

The "Strategies yield to Incorrect or No Answer" and "Strategies yield to Correct Answer" were coded for each problem (see Table 4). The problem solving strategies and number of uses of these strategies have been analyzed for each participant (see Table 5). The cases, which the participants have found a correct answer after their second attempt, were analyzed (see Table 6).

3.2.2 Analysis of Problem Solving Episodes of the Participants

I used the cognitive-metacognitive framework of Artzt and Armour-Thomas (1992) to observe the problem solving episodes. The last episode "Watching and listening" episode was designed to observe watching and listening behaviors in a group study which was not investigated in the study; so this episode type was not included in the discussions. Characteristics of each episode, summarized from the descriptions of Artzt and Armour-Thomas (1992, pp. 173-175) are as follows:

- **Episode 1: Reading the problem (cognitive)**

  The student reads the problem.

- **Episode 2: Understanding the problem (metacognitive)**

  The student considers domain-specific knowledge that is relevant to the problem. Domain-specific knowledge includes recognition of the linguistic, semantic, and schematic attributes of the problem in his or her own words and represents the problem in a different form.

- **Episode 3: Analyzing the problem (metacognitive)**

  The student decomposes the problem into its basic elements and examines the implicit or explicit relations between the givens and goals of the problem.

- **Episode 4: Planning (metacognitive)**

  The student selects steps for solving the problem and a strategy for combining them that might potentially lead to problem solution if implemented.

- **Episode 5a: Exploring (cognitive)**

  The student executes a trial-and-error strategy in an attempt to reduce the discrepancy between the givens and the goals.

- **Episode 5b: Exploring (metacognitive)**

The student monitors the progress of his or her or others' attempted actions thus far and decides whether to terminate or continue working through the operations.

- **Episode 6a: Implementing (cognitive)**

The student executes a strategy that grows out of his or her understanding, analysis, and/or planning decisions and judgments.

- **Episode 6b: Implementing (metacognitive)**

The student engages in the same kind of metacognitive process as in the exploring (metacognitive) phase of problem solving, monitoring the progress of his or her attempted actions.

- **Episode 7a: Verifying (cognitive)**

The student evaluates the outcome of the work by checking computational operations.

- **Episode 7b: Verifying (metacognitive)**

The student evaluates the solution of the problem by judging whether the outcome reflected adequate problem understanding, analysis, planning, and/or implementation.

More detailed descriptions are given Appendix B. For each problem, the problem solving episodes of the participants were coded (see Appendix I for full analysis). Cognitive and metacognitive episodes of each participant have been analyzed (see Table 7). The episodes which were the reasons of failures of each participant have been analyzed (see Table 8).

3.2.3 Analysis of the Metacognitions of the Participants

To observe the main components of metacognition, I also used the coding system developed by Pappas et al. (2003) and examined children's behaviors and verbalizations on each of ten problems. For each problem, I have analyzed the mistake recognition; adaptability and expression of thinking of each participant (see Appendix J for full analysis). Mistake recognition, adaptability and expression of thinking of each participant in total ten problems have been analyzed (see Table 9 and 10). The details and descriptions of levels of each component have been introduced in subsections.

3.2.3.1 Mistake Recognition

"Mistake recognition" of the participants in each problem has been coded according to following definitions as described by Pappas et al. (2003, p. 436):

- **Lack of Awareness:** The child is unaware of a mistake and can solve the problem only with external guidance.
- **Uncertainty:** The child's questioning tone of voice, facial expression (e.g., a guilty smile), or attempt to distract the interviewer in order to avoid responding indicates a lack of certainty concerning the correctness of the answer.
- **Awareness and Spontaneous Correction:** Without prompting by the interviewer, the child recognizes a mistake and changes an incorrect answer.

3.2.3.2 Adaptability

"Adaptability" of the participants in each problem has been coded according to following definitions as described by Pappas et al. (2003, pp. 436-437). Adaptability with Adult Assistance which exists on the original coding has not been used since the participants haven't been assisted by the researcher.

- **Lack of Adaptability**: The child is unaware that an approach is unsuccessful or cumbersome and does not employ a different strategy even when the interviewer suggests an alternative.
- **Awareness of the Need for Change**: The child is aware that a strategy is unsuccessful or cumbersome, wants to abandon it, but cannot implement another one.
- **Independent Adaptability**: The child spontaneously changes to another strategy if the present one is not effective. The child may verbalize this change in strategies or simply implement an alternative strategy.

3.2.3.3 Awareness and Expression of Thinking

"Awareness and Expression of Thinking" of the participants in each problem has been coded according to following definitions as described by S. Pappas et al. (2003, pp. 436-437):

- **No Response or Ambiguous Description**: The child does not respond or provides an ambiguous response.
- **Inaccurate Description:** A description is provided, but it clearly differs from the actual strategy used.
- **Partial Description:** The child verbalizes some of the processes used to solve a given task, often the last strategy employed, but not all.
- **Full Description:** The child accurately describes the strategies used to solve a given task.

3.2.4 Analysis for the Difficulties, Failures and Results

First, I analyzed the cases which the participants had a difficulty or failure in solving the problems but they could reach a correct answer showing independent adaptability (see Table 11). Then I analyzed how lack of mistake awareness affected the problem solving episodes and the results (see Table 12). Finally, I analyzed how uncertainty of mistake awareness affected the problem solving episodes and the results (see Table 13).

3.2.5 Analysis of the Responses of the Participants to the Self Monitoring Questionnaire

I analyzed the items that participant commonly responded as "Yes" according to this mapping (see Table 14). Since the questionnaire have been applied only once to each participant, the other data sources have been used to validate and crosscheck the results.

3.3 Validity and Reliability of the study

Merriam (1998, p. 199) summarizes the reliability and validity of a qualitative case study as:

> Assessing the validity and reliability of a qualitative study involves examining its components parts, as you might in other types of research. As Guba and Lincoln (1981, p. 378) point out, in an experimental study you can "talk about the validity and reliability of the instrumentation, the appropriateness of the data analysis techniques, the degree of relationship between the conclusions drawn and the data upon which they presumably

> rest, and so on." It is "not whit different" in a qualitative case study – "were the interviews reliably and validly constructed; was the content of the documents properly analyzed; do the conclusions of the case study rest upon the data?"

Keeping those in mind, I chose the data collection tools and data analysis methods which have been introduced earlier considering the trustworthiness of the study –or the research validity- during research design and the data collection.

My first concern was trustworthiness or the internal validity of the study which "deals with how research findings match with reality" (p. 201). Triangulation - "using multiple data sources or methods to confirm researcher findings"- is one of the ways that Merriam (1998) suggested for enhancing the internal validity on a qualitative research (p. 204). The triangulation of the interviews, observations and questionnaire data helped further this process and enhanced the internal validity of this research study. Merriam (1998) also suggests "conducting repeated or long term observations to demonstrate internal validity of the study" (p. 201). I applied ten problems which allowed observing the participants in similar cases and seeing the patterns in participants' activities. Finally, I enhanced the trustworthiness by "peer examination" (Merriam, 1998, p. 201) as a mathematics education doctoral student has checked the codings I have made for the strategies, episodes and metacognitions of the participants.

My second concern was consistency or the reliability of the qualitative study which "refers to the extent to which research findings can be replicated" (Merriam, 1998, p. 205). Yin (2003) suggests "to make as many steps as operational as possible… so that an auditor could repeat the procedures and arrive at the same results" (pp. 37-38). The criteria for participant and problem selections, the coding systems to analyze the interview data and the questionnaire applied, the methods that were used in this study are introduced in details to lead further studies that search the similar questions in the same settings. Triangulation of the data also "strengthened the reliability of the study as well as the internal validity" (Merriam, 1998, p. 207).

My final concern was transferability or external validity which "is concerned with the extent to which the findings of the study can be applied to other situations" (Merriam, 1998, p. 207). Merriam (1998) suggested three strategies to enhance the external validity of the research study results. The first strategy is for the researcher to give rich, thick description. I tried to provide enough description about the participants, the problems, the data collection methods and the methods for analysis, so that "readers will be able to decide how well the research study matches the research situation" (Merriam, 1998, p. 211). The second strategy is the typicality or modal category of the research. I described how typical the participants of the study are with the other students in same categories in the country to "give the readers opportunity of comparisons with their own situations" (Merriam, 1998, p. 211). The final strategy is using a multisite design. I "maximized diversity in the phenomenon of interest" using purposeful sampling and purposeful tasks (Merriam, 1998, p. 212).

3.4 Assumptions and Limitations

3.4.1 Assumptions

The main assumptions of the present study are the following:

1. All participants answered the measurement instruments accurately and sincerely.
2. Participants did not have linguistic problems in the problem solving process.

3.4.2 Limitations

The limitations of the present study are as listed below:

1. This study was limited to the five students graduated from elementary school in Ankara at the beginning of autumn semester of 2007-2008 academic years.
2. A self monitoring questionnaire, which require the participant to respond truthfully and willingly, were applied.

CHAPTER 4

RESULTS AND CONCLUSIONS

In this chapter, I will explain the results and the conclusions of this study researching the following questions:

Which problem solving strategies do elementary school graduate Turkish students employ while solving selected PISA 2003 mathematical literacy items?

Which problem solving episodes are observed when elementary school graduate Turkish students while solving selected PISA 2003 mathematical literacy items?

Which mistake recognition, adaptability and expression of thinking behaviors do elementary school graduate Turkish students display while solving selected PISA 2003 mathematical literacy items?

How problem solving strategies, cognitive and metacognitive behaviors of elementary school graduate Turkish students effect their success in PISA 2003 mathematical literacy items?

Ten problems that have been used in this study are introduced in Appendix G. Moreover transcriptions of the clinical interviews with each participant are presented in Participants' Work subsections of each problem in Appendix G.

4.1 Participants' Results

The total scores of the participants in the study confirmed their OKS success levels of the participants (see Table 3). Successful students of the study were the participants are the students who were most successful in OKS exam: Aylin and Bora. Aylin and Bora managed to find the answers of all the problems in all levels.

The participant with the worst score of the study was the one who had the worst OKS score: Ceylan. Ceylan could only solve two problems correctly and found a partial answer in a problem. The levels of the problems were Level 2 and Level 3. Deniz and Emel were expected to be novice problem solvers according to their OKS scores and teacher comments. Their scores in the case study confirmed that. They have solved more than the half of the problems. They could solve one of the Level 4 problems and they had partially correct answer in one of the Level 6 problems.

Table 3. Scores of the Participants in each Problem and their Total Scores

Participant	Proficiency Levels of the Items										Total Score
	Level 2		NA*	Level 3			Level 4		Level 6		
	Pr 1**	Pr 2**	Pr 4	Pr 5	Pr 6**	Pr 8	Pr 9	Pr 10	Pr 3**	Pr 7	
Aylin	1	1	1	1	1	1	1	1	1	1	10
Bora	1	1	1	1	1	1	1	1	1	1	10
Ceylan	1	-	-	1	-	0.5	-	-	-	-	2.5
Deniz	1	1	1	0.75	1	-	1	-	-	0.75	6.5
Emel	-	1	1	1	-	1	1	-	-	0.75	5.75

* PISA 2003 proficiency level for this problem cannot be determined as no related information can be accessed (including the OECD mean for the item)

** Levels are determined by the researcher according to the percentages of correct answers in total PISA 2003 as no information has been published about the levels of these items in the OECD reports.

4.2 Participants' Problem Solving Strategies

In this section, the findings related to the following research questions will be presented:

Which problem solving strategies do elementary school graduate Turkish students employ while solving selected PISA 2003 mathematical literacy items?

To answer the first question, the strategies used by the five participants in each problem are listed. This data have been represented fully in Appendix H. It is observed that in this case study, the participants used 7 different strategies of the 10 strategies presented by Posamentier & Krulik (1999). These strategies are:

1. Working backwards (in Problem 10)
2. Finding a pattern (in Problems 1, 7 and 9)
3. Making a drawing (visual representation) (in Problems 1 and 7)
4. Intelligent guessing and testing (in Problem 3)
5. Accounting for all possibilities (in Problem 4,5 and 10)
6. Organizing data (in Problems 6 and 9)
7. Logical reasoning (in Problems 2,3,4,5,6,7,8 and 9)

Table 4. Strategies Used by the Participants in Each Problem

Problem No	**Strategies yield to Incorrect or No Answer**	**Strategies yield to Correct Answer**
1	Visual Representation , Finding a pattern	Visual Representation , Finding a pattern
2	-	Logical Reasoning
3	Intelligent guessing and testing , Logical Reasoning	Intelligent guessing and testing
4	Logical Reasoning	Accounting for all possibilities, Logical Reasoning
5	-	Accounting for all possibilities, Logical Reasoning
6	Organizing Data,	Organizing Data , Logical Reasoning
7	Finding a pattern , Logical Reasoning	Visual Representation, Logical Reasoning
8	Logical Reasoning	Logical Reasoning
9	Organizing Data, Logical Reasoning	Finding a pattern, Organizing Data, Logical Reasoning
10	Working Backwards , Accounting for all possibilities	Working Backwards , Accounting for all possibilities

While investigating the strategies that yielded the participants to a correct answer for each problem, I observed that the same strategies yielded other participants to an incorrect or no answer. In Table 4, strategies that yield to an incorrect or no answer and strategies that yield to correct answer are listed for each problem. The data confirmed that choosing a correct strategy for a problem doesn't guarantee a correct answer.

Table 5. Efficiency Rates of Participants in Different Problem Solving Strategies and Their Total Scores

Participants	Problem Solving Strategies							Total Score
	Working Backwards	Finding a Pattern	Visual Representation	Intelligent Guessing and testing	Accounting for all possibilities	Organizing Data	Logical Reasoning	
Bora	1/1	2/2	2/3	1/1	2/2	2/2	6/6	10
Aylin	-	2/2	1/1	1/2	3/3	2/2	6/7	10
Deniz	-	2/3	1/1	0/1	2/3	2/3	5/6	6.5
Emel	0/1	0/2	1/1	-	2/3	1/2	5/6	5.75
Ceylan	-	1/1	0/1	-	1/2	0/1	2/5	2.5

Note. a/b means that the participant used the related strategy *b* times and *a* of the attempts resulted in a correct answer

I also analyzed the strategies used by each participant. Table 5 shows the strategies that have been used by each participant with efficiency and total points of each participant achieved in the case study. The numbers of different problem strategies tried by participants with different success levels were nearly the same. For instance, Bora tried 7; Aylin, Deniz and Emel tried 6 and Ceylan tried 5 different strategies throughout all of the problems.

Bora tried 7 different strategies and managed to use all the strategies efficiently. He managed to use Working backwards in Problem 10; finding a pattern in Problems 1 and 9, Intelligent guessing and testing in problem 3, Accounting for all possibilities in Problems 5 and 10, Organizing data in Problems 6 and 9 and Logical reasoning in problems 2, 3, 4, 5, 6, 7 and 8 efficiently and found the correct answer in his first attempt. He failed to find the correct answer in the first problem with visual representation in his first attempt, but he changed his strategy and found the answer using finding a pattern strategy (see Appendix G and H for details).

Aylin tried 6 different strategies and managed to use all the strategies efficiently. In Problem 3, she couldn't find the answer using intelligent guessing and testing strategy in her first attempt but she retried the strategy and found the correct answer in her second attempt. In Problem 4, she misunderstood the problem and tried logical reasoning strategy in solving the problem, but then she understood her mistake and found the correct answer using accounting for all possibilities and logical reasoning strategies. In the other seven problems, she found the correct answer in her first attempt: she used finding a pattern strategy in problems 1 and 9; logical reasoning strategy in problems 2, 5, 6, 7 and 8, accounting for all possibilities strategy in problem 5, organizing data strategy in problem 6 and 9; and visual representation strategy in problem 7 (see Appendix G and H for details).

Deniz tried 6 different strategies and used 5 of them efficiently: she efficiently used logical reasoning in problems 2; accounting for all possibilities and logical reasoning in problems 4 and 5; organizing data strategy in problem 6 and visual representation in problem 7 in her first attempt. In problem 1, she failed to find the answer with finding a pattern strategy in her first attempt but she retried the strategy and found the correct answer in her second attempt. In problem 9, she retried organizing data strategy although she failed in first attempt; she managed to find the answer in her second attempt. She used intelligent guessing and testing in problem 3, and accounting for all possibilities in problem 10; but she failed to find a correct answer in those two problems (see Appendix G and H for details).

Emel tried 6 different strategies and used 4 of them efficiently: she efficiently used logical reasoning in problems 2 and 8; accounting for all possibilities and logical reasoning in problems 4 and 5; visual representation and logical reasoning in problem 7. She couldn't use finding a pattern strategy efficiently in Problem 1 and 7; she used accounting for all possibilities and working backwards in Problem 10 but she couldn't find the correct answer (see Appendix G for details).

Ceylan tried 5 different strategies through the problems but she could only use 3 of these strategies efficiently to find a correct or partially correct answer. She used finding a pattern strategy in Problem 1, accounting for all possibilities and logical reasoning strategies in Problem 5; and logical reasoning strategy in Problem 8 efficiently. She couldn't use visual representation in Problem 1, and organizing data in Problem 6 efficiently to find a correct answer. Ceylan who has tried the least number of strategies also used least number of strategies efficiently among five participants (see Appendix G for details).

As seen in Table 5, the number of the different problem solving strategy usage and efficiency can explain the ranks in the total scores. The data showed how efficiently students use different problem solving strategies seems to create difference in the results.

As seen in Appendix H, it is represented that the participants found the correct answer in their first attempt in 27 cases. Table 6 summarizes the other 8 cases, where participants kept on trying the same strategy or tried a new strategy and managed to find a correct answer. Aylin and Bora, who usually found appropriate strategies in their first attempts, could find the appropriate usage of a strategy in their second attempts when they have troubles in their first attempts. Deniz and Emel got one third of their points in their second attempts. Deniz took 2 points of her total of 6.5 points and Emel took 1.75 points of her total of 5.75 points when they keep on trying a new strategy. Ceylan had difficulty in trying a different strategy in the problems she had troubles. She managed to find a correct answer changing her strategy in only 1 of possible 8 problems. This fact caused her unsuccessful result. In summary, the results show that when the students have enough problem solving strategy repertoires and keep on trying these strategies, it can make a difference in the total results.

Table 6. Cases Where the Participants Were Successful in Their Second Strategy Attempts

Problem No	Participant	Strategies yield to Incorrect or No Answer	Strategies yield to Correct Answer	Points	Successful at Attempt
1	Bora	Visual Representation	Visual Representation , Finding a pattern	1	2nd
1	Ceylan	Visual Representation	Finding a pattern	1	2nd
1	Deniz	Finding a pattern	Finding a pattern	1	2nd
3	Aylin	Intelligent guessing and testing	Intelligent guessing and testing	1	2nd
4	Aylin	Logical Reasoning	Accounting for all possibilities & Logical Reasoning	1	2nd
7	Emel	Finding a pattern	Visual Representation& Logical Reasoning	0.75	2nd
9	Deniz	Organizing Data	Organizing Data & Finding a pattern	1	2nd
9	Emel	Organizing Data	Organizing Data	1	2nd

4.3 Participants' Problem Solving Episodes

In this section, the findings related to the following research questions will be presented:

Which problem solving episodes are observed when elementary school graduate Turkish students while solving selected PISA 2003 mathematical literacy items?

Although the participants were encouraged to talk aloud, when they had silence for a while, I asked the participants to explain when their latter actions didn't give clue about their behaviors. To answer the first question, I analyzed the video recordings to investigate the problem solving episodes of each participant for the ten problems according to the cognitive-metacognitive problem solving framework. The number of episodes and the order of these episodes can be observed in that table in Table 27 in Appendix I.

The first findings of the analysis were as expected from the design of the framework, in a problem solving activity (see Appendix I for further details):

- All episodes don't need to occur to find a correct answer. For example, Aylin and Bora never used exploration but found the correct answer in all problems.
- The problem solving episodes can occur in different orders. For example, in most problems implementation is observed after analysis, in problem 1, Emel analyzed the problem after implementation.
- Some episodes can be observed several times in a problem. For example, in Problem 4, Emel read the problem twice, analyzed the problem for three times.

Table 7. Cognitive and Metacognitive Episodes of the Participants

Participant	Problem Solving Episodes									
	Read	Understand	Analyze	Plan	Explore		Implement		Verify	
	C	M	M	M	C	M	C	M	C	M
Aylin	11 (10)	11 (10)	12 (10)	12 (10)	-	-	14 (10)	3 (3)	-	2 (2)
Bora	10 (10)	10 (10)	10 (10)	10 (10)	-	-	11 (10)	1 (1)	1 (1)	3 (3)
Ceylan	11 (10)	10 (10)	8 (8)	7 (7)	4 (4)	2 (2)	9 (9)	-	-	-
Deniz	10 (10)	10 (10)	11 (10)	11 (10)	4 (4)	4 (4)	12 (10)	1 (1)	-	-
Emel	10 (10)	10 (10)	10 (9)	9 (9)	4 (2)	3 (2)	10 (9)	2 (2)	-	1 (1)
Total	52 (50)	51 (50)	51 (47)	49 (46)	12 (10)	9 (8)	57 (48)	7 (7)	1 (1)	6 (6)

Note. C: Cognitive ; M: Metacognitive ;
xx(xx): Number of times the episode observed(Number of different problems). e.g., 11(10) means that the episode is observed 11 times in total of 10 different problems

The analysis showed that the participants displayed the Reading and Understanding episodes in all cases. Analysis, Plan, and Implementation (as Cognitive behavior) episodes were observed nearly in all cases. (see Table 7 and Appendix G and I for full details).

Exploration (Cognitive) wasn't observed in the problem solving activities of two successful participants Aylin and Bora (see Table 7 and Appendix I for further details). Ceylan, Deniz and Emel tried exploration in several problems. Exploration (Cognitive) episode was observed in 10 cases before Analysis and Plan episodes. When the Exploration (as Metacognitive behavior) leaded the participants to appropriate Analysis episode (see Appendix G and I for further details):

- In problem 1, Ceylan understood that drawing will not lead her to an answer, she analyzed that using the square numbers will help in finding the pattern and she found the correct answer.
- In problem 1, 6 and 9, Deniz started exploring (cognitive) the problem, then metacognitive exploration leaded to analysis and she found the correct answer in those problems. In problem 10, Deniz started exploring (cognitive) the problem, then metacognitive exploration leaded to analysis -but she found an incorrect answer after an error in implementation-.
- In problem 9, Emel also started exploring the problem, after the analysis, she found the correct answer by accounting for all possibilities.

When the Exploration (as Metacognitive behavior) leaded the participant to inappropriate Analysis episode, the student ended with an incorrect answer (see Appendix G and I for further details):

- In problem 10, Ceylan explored the problem by "accounting the possibilities". After trying the first possibility, she decided that the strategy is incorrect (Exploration(Metacognitive)), the she analyzed the problem incorrectly and planned to add the biggest prices which leaded her to an incorrect answer.

When the students stuck in Exploration (cognitive), they quit or had an incorrect answer (see Appendix G and I for further details):

- In problem 6 and 9, Ceylan stuck in exploration (cognitive), she quit in Problem 6, and she guessed an answer which was incorrect.
- Similarly, in problem 6, Emel tried exploring an answer, using metacognitive exploration she restarted exploring but she quit at her third attempt.

In 36 cases, Analysis has been displayed before exploration (cognitive) (see Appendix G and I for further details):

- Aylin and Bora; they forced Analysis in all problems.
- Ceylan displayed analysis before exploration in problems 2,4,5,7 and 8; but only in problems 5 and 8, she could analyze the problem appropriately.
- Deniz displayed analysis before exploration in problems 2,3,5,7 and 8. In problems 2, 5 and 7, she could reach a correct answer.
- Emel displayed analysis before exploration in problems 2, 3, 5, 7, 8 and 10. She managed to find correct answer In problems 2, 5, 7 and 8.

In 48 cases, Implementation (cognitive) was observed; in 7 cases, it was followed by Implementation (Metacognitive) episode (See Table 7). When the students judged their implementations or discovered mistakes in their implementations, they could reach a correct answer in a correct Implementation (Cognitive) (see Appendix G and I for further details):

- In problem 3 and 10, Aylin judged her implementation and found the correct answer in the second implementation(cognitive) attempt.
- In problem 1, Bora discovered his mistake in his drawing and found the correct answer by redrawing the figure in the second implementation(cognitive) attempt.
- In problem 4, Deniz discovered that she need the sum of the faces to finish the implementation and found the correct answer.

When students judged their implementations, in 2 cases, they yield to appropriate Analysis (see Appendix G and I for further details):

- In Problem 4, Aylin judged her implementation, she reanalized the problem and found the correct answer with a correct plan and implementation(cognitive).

- In Problem 3, Deniz judged her implementation, she reanalized the problem and had a correct plan, but she made a calculation error in implementation(cognitive) and found an incorrect answer.

Although a student judged her implementations in one case, she yielded to an inadequate Analysis (see Appendix G and I for further details):

- In Problem 1, Emel judged her implementation, she decided to reanalize the problem but stuck at the Analysis episode.

Verification (Cognitive and Metacognitive) was observed in 7 cases. Verification (Metacognitive) episode was observed in 6 of these cases :

- In problem 3 and 4, Aylin verified whether the answer satisfy what is asked in the problem. In Problem 4, she found her reading mistake in verification which yielded her to reading episode and she found a correct answer in her second attempt.
- In Problem 1, Bora verified the answer of the problem with an alternative strategy. In Problems 3 and 10, he verified whether the answer satisfy what is asked in the problem.
- In Problems 10, Emel verified whether the answer satisfy what is asked in the problem.

Verification (Cognitive) was observed in one case:

- In Problem 10, Bora verified his calculations to be sure for the answer.

The participants may have had the Verification (Cognitive and Metacognitive) mentally without talking aloud, but I have to note that participants had the tendency to pass to the next problem whenever they found an answer.

Table 8. Problem Solving Episodes Which are the Primary Reasons for the Failures of Finding a Correct Answer for Each Participant

Participant	Problem Solving Episodes									
	Read	Understand	Analyze	Plan	Explore		Implement		Verify	
	C	M	M	M	C	M	C	M	C	M
Aylin	-	-	-	-	-	-	-	-	-	-
Bora	-	-	-	-	-	-	-	-	-	-
Ceylan	-	<3>	<2>	-	<2>	-	<1>	-	-	-
Deniz	-	-	<3>	-	-	-	<2>	-	-	-
Emel	<1>	<1>	<2>	-	<1>	-	-	-	-	-
Total	<1>	<4>	<7>	-	<3>	-	<3>	-	-	-

Note. C: Cognitive ; M: Metacognitive; <x>: The number of cases where the episode is the primary reason for the failure of finding a correct solution

I also determined the episodes that were the primary reasons for the failures of finding a correct answer for each participant. The failures and mistakes leaded to 14 incorrect and 4 partially correct answers. In one case, Reading episode was the primary reason for the failure of finding a correct solution (see Table 8 and Appendix G and I for further details):

- In Problem 10, Emel misread the price of the set of wheels, so she found an incorrect answer, although there was no error in the latter episodes.

In 4 cases, Understanding episode was the primary reason for the failure of finding a correct solution (see Table 8 and Appendix G and I for further details):

- In problems 2,3 and 4, Ceylan couldn't understand the problem which made appropriate Analysis or Exploration (Cognitive) impossible , she found incorrect answers in all these 3 cases.
- In problem 3, Emel misunderstood what was asked in the problem, she suggested an incorrect rule for the answer.

In 7 cases, the problems in Analysis episode leaded to inappropriateness of Plan or Implementation (Cognitive) episodes (see Table 8 and Appendix G and I for further details):

- In problems 7, Ceylan analyzed the given conditions incorrectly: she thought that it will be enough if the side lengths are 6 and 10 which was incorrect and she chose the incorrect options. In problem 10, Ceylan had an incorrect analysis after exploration, she decided the chose the product with highest prices, which was incorrect. In both problems, she found incorrect answers.
- In problems 5, Deniz analyzed the problem requirement incorrectly; she checked only one pairs of faces which leaded to partially correct answer. In Problem 7, Deniz analyzed the side lengths of the figure; but she could get a partially correct answer by deciding 3 of 4 figures. In Problem 8, Deniz analyzed the conditions incorrectly; she added the price of the complete skateboard which was incorrect and found incorrect answer.
- In problem 1, Emel understood that her pattern suggestion was not correct but she stuck in analysis episode and couldn't change the incorrect result. In Problem 7, Emel analyzed the side lengths of the figure; but she could get a partially correct answer by deciding 3 of 4 figures.

In 3 cases, the participant failed at Exploration (Cognitive) (see Table 8 and Appendix G and I for further details):

- In problems 6 and 9, Ceylan explored the problems using her previous experience, but she couldn't display metacognitive exploration behavior which can lead her to analysis. In problem 6, she quit the problem with no answer, in problem 9 she guessed an incorrect option beyond the given multiple choice options.
- In problem 6, Emel also tried to memorize similar problem solutions and explored a solution, she displayed metacognitive exploration twice but she quit at her third trial.

And finally in 3 cases, the mistakes in the Implementation (Cognitive) led to partially correct or incorrect result (see Table 8 and Appendix G and I for further details):

- In problem 8, Ceylan had a mistake in calculating the sum of the correct prices in finding the highest price.
- In problems 3 and In Problem 10, Deniz started accounting for all possibilities for a skateboard with the closest price less than 120, she suggested prices "60,14,16,20" for the answer, but prices "65,14,16,20" were the correct answer.

4.4 Participants' Mistake Recognition, Adaptability and Expression of Thinking

In this section, the findings related to the following research question will be presented:

Which mistake recognition, adaptability and expression of thinking behaviors do elementary school graduate Turkish students display while solving selected PISA 2003 mathematical literacy items?

Table 9. Mistake Recognition and Adaptability of the Participants

		Mistake Recognition			Adaptability		
Participant	Correct Answer at First Attempt	Lack of Awareness	Uncertainty	Awareness and Correction	Lack of Adaptability	Awareness for the need of change	Independent Adaptability
Aylin	8	-	-	2	-	-	2
Bora	9	-	-	1	-	-	1
Ceylan	1	3	5	1	7	1	1
Deniz	3	4	1	2	5		2
Emel	4	2	3	1	3	2	1
Total	25	9	9	7	15	3	7

I investigated the 50 cases to analyze the mistake recognition, adaptability and expression of thinking of the participants. In 25 cases the participants found the correct answers (full 1 point) in their first attempts (See Appendix H). It is remarkable that 17 of these cases belong to the most successful two participants (See Table 9).

I investigated the mistake awareness and adaptability in the other 25 cases where the participants had mistakes and difficulties. In 9 cases, the participants showed lack of mistake awareness:

- In Problem 3, Ceylan didn't understand her mistake because of the lack of required knowledge "variables and coefficients", she assigned values of the variables as coefficients (see Appendix G for details). In problem 8, Ceylan miscalculated the sum and didn't realize that (see Appendix G for details). And in problem 10, Ceylan couldn't realize that the price of the skateboard shouldn't be more than 120 and suggested an answer where the price was 137 (see Appendix G for details).
- In Problem3, Deniz miscalculated the total scores for cars Sp, N1 and KK so she couldn't see that the rule she suggested didn't make the car Ca a winner (see Appendix G for details). In Problem 5, Deniz checked the rule of being number cube incorrectly, so she couldn't see her mistake (see Appendix G for details). In Problem 8, Deniz didn't realize that the first product in the list given in problem was a complete skateboard price; so added this value to the sum and found incorrect answer (see Appendix G for details). In problem 10, Deniz suggested prices "60,14, 16, 20" for the answer, where prices "65, 14,16, 20" were the correct answer since the sum is more closer to 120, but she didn't realize that possibility.
- In Problem 3, Emel didn't understand her mistake because she misunderstood what is asked (see Appendix G for details). In Problem 10, Emel misread a value in the problem and although she verified her

calculations and reread the problem for the aim, she didn't check the values, so she couldn't see her mistake (see Appendix G for details).

In 9 cases they showed uncertainty:

- In Problem 2, Ceylan said she was not sure of the answer because she couldn't understand what "3xS" meant, because of the lack of required knowledge "variables and coefficients" (see Appendix G for details). In Problem 4, Ceylan guessed the faces of the dice, she was so unsure that she assigned 5 for both top and bottom of dice 3; then she changed the top value to 4 but her behaviors showed taht she was not sure (see Appendix G for details). In Problem 6, Ceylan was not sure what she was doing: instead of dividing 200 by 12, she tried adding so many 12s to get 200, she got confused. When I asked her what she would do next after finding the multiples, she couldn't say what she would do next and she quit the problem (see Appendix G for details). In Poblem 7, she tried to find the similarities in the shapes to guess an answer but she wasn't sure what she was doing (see Appendix G for details). In Problem 9, she couldn't explain why she has chosen 6 as the answer (see Appendix G for details).
- In Problem 7, Deniz said that she wasn't sure about her answer (see Appendix G for details).
- In Problem 1, Emel said that there may be a pattern different than her answer to satisfy the problem (see Appendix G for details). In Problem 6, Emel couldn't decide what to do, she quit all attempts since she was uncertain (see Appendix G for details). In Problem 7, Emel said that she guessed the side lengths of the parallelogram; she wasn't sure(see Appendix G for details).

In 3 of the cases that participants were uncertain, the participants showed awareness for need of change in their methods but they couldn't go further:

- In Problem 1, Emel said that there may be a pattern different than her answer to satisfy the problem but she couldn't change her answer (see Appendix G for details). In Problem 6, she tried to apply different cases from her past experience, but she quit at the end(see Appendix G for details).

- In Problem 2, Ceylan said she couldn't change her answer because of the lack of required knowledge "variables and coefficients" (see Appendix G for details).

In 7 cases, the participants started with lack of awareness or uncertainty about their mistakes but they had independent adaptability to modify or change their strategies to find the correct method and the correct answer (see Appendix G and J and Table 9 for details):

- In Problem 3, Aylin was uncertain about her plan, but then she correctly applied intelligent guessing and testing strategy in her second attempt (see Appendix G for details). In Problem 4, Aylin wrote 42 as the answer then she realized that she misunderstood the problem; she reread the problem and found the correct answer in her second attempt (see Appendix G for details).
- In Problem 1, Bora wrote 9 as the answer but then he realized that he drew the figure incorrectly and solved the problem correctly in his second attempt (see Appendix G for details).
- In Problem 1, Ceylan realized that she couldn't draw the figure correctly and solved the problem correctly in his second attempt by finding the pattern by the number of squares (see Appendix G for details).
- In Problem 1, Deniz wrote 9 as the answer but then she realized that the pattern she has thought was not correct; she decided the correct pattern and found the correct answer in her second attempt (see Appendix G for details). In Problem 9, Deniz was uncertain about what to do, but she found the answer after organizing the given data (see Appendix G for details).
- In Problem 9, Emel guessed 9 as the answer but then she realized that it was not correct; she found the answer after organizing the given data (see Appendix G for details).

In the other 15 cases where participants have lack of mistake awareness (in 9 cases) or uncertainty (6 cases), the participants the participants showed lack of adaptability to change their solutions (see Appendix G and J and Table 9 for details). In summary, mistake awareness and adaptability of the participants have a remarkable positive effect the problem solving success of the students.

Table 10. Awareness and Expression of Thinking of the Participants

Participant	Awareness and Expression of Thinking			
	No Response or Ambiguous Description	Inaccurate Description	Partial Description	Full Description
Aylin	-	-	-	10
Bora	-	-	-	10
Ceylan	4	-	-	6
Deniz	-	-	1	9
Emel	-	-	-	10
Total	4	-	1	45

I investigated the awareness and expression of thinking of the participants to see its effect in problem solving success (see Appendix H). The participants could generate a full description of their process and calculations in 45 cases (see Table 10 for details). In 36 of these cases, the participants found a correct or partially correct answer. The data shows that generating full description of the thinking doesn't guarantee that the participant will have a correct answer.

In 4 cases, Ceylan displayed no response or ambiguous description of thinking: in problems 4, 7, 9 and10, she couldn't describe what she had tried. She couldn't find a correct answer in those problems. In 3 cases, Deniz could show a partial description of thinking in problems 3, 5 and 10. In these cases, she mainly fully described her plan but it wasn't fully what she had applied in her problem solving activity. What is remarkable that in 6 of these cases, when the participants cannot generate full description of their problem solving steps, they couldn't find a correct answer (see Appendix G and J and Table 10 for details). In summary, difficulty in awareness and expression of thinking of the participant seem to be a sign of a possible incorrect or no answer.

4.5 Participants' Difficulties, Failures and Results

In previous sections, I have analyzed what the problem solving strategies, problem solving episodes and mistake recognition, adaptability and expression of thinking of students can tell about the results of the students. A main purpose of this study was to investigate deeply why and how the participants ended up with a correct or an incorrect answer in the problem solving activities. In this section, the previous findings will be combined and related to the following research question will be presented:

How problem solving strategies, cognitive and metacognitive behaviors of elementary school graduate Turkish students effect their mathematical problem solving success while solving selected problems from PISA 2003?

As it was remarked in section 4.4, in 25 cases (of total 50 cases), the participants found the correct answer in their first attempts. It is remarkable to note that in all these 25 cases, Reading, Understanding, Analysis and Implementation (Cognitive) episodes were observed in a row (see Appendix I for details).

In 25 cases the participants had a difficulty or failure in solving the problems (see section 4.4 for details). In 7 of these 25 cases, the participants could reach a correct answer where they have showed independent adaptability (see section 4.4 for details). Table 11 summarizes how independent adaptability effected in problem solving episodes in these 7 cases. As seen in Table 11, when the participants could display independent adaptability, they could overcome the difficulties in Exploration (Cognitive) and Implementation (Cognitive). Independent adaptability was observed in Exploration (Metacognitive) and Implementation (Metacognitive) episodes, respectively:

- In 4 cases, where Exploration (Metacognitive) yield to appropriate Analysis, Plan and Implementation (Cognitive) episodes in a row, participants reached to the correct answer (see Table 11).
- In 3 cases, Implementation (Metacognitive) yield to appropriate Implementation (Cognitive) and in one case it yield to appropriate Analysis Plan and Implementation (Cognitive) episodes in a row (see Table 11).

Table 11. Results and Correctness of the Answers When the Participants had Independent Adaptability

Problem No.	Participant	Difficulty in	Result of "Independent Adaptability"	Correctness of the Answer
1	Bora	Implementation(C)	Implementation(M) leaded to appropriate Implementation(C)	Correct
1	Ceylan	Exploration(C)	Exploration(M) leaded to Analysis. Appropriate Analysis, Plan and Implementation(C)	Correct
1	Deniz	Exploration(C)	Exploration(M) leaded to Analysis. Appropriate Analysis, Plan and Implementation(C)	Correct
3	Aylin	Implementation(C)	Implementation(M) leaded to appropriate Implementation(C)	Correct
4	Aylin	Implementation(C)	2 Explorations(M) leaded to appropriate Analysis, Plan and Implementation(C)	Correct
9	Deniz	Exploration(C)	Exploration(M) leaded to Analysis. Appropriate Analysis, Plan and Implementation(C)	Correct
9	Emel	Exploration(C)	Exploration(M) leaded to Analysis. Appropriate Analysis, Plan and Implementation(C)	Correct

Note. (C): Cognitive. (M): Metacognitive

I will also present the 18 of other cases where participants showed Awareness and Correctness displaying Independent Adaptability. The problem solving episodes where the participants failed had already been introduced in section 4.3 (see Table 8). In these cases:

- The participant had partially correct answers in 4 cases: Ceylan got 0.5 points in Problem 8; Deniz got 0.75 points in problems 5 and 7; Emel got 0.75 points in Problem 7).
- The participants had incorrect answers in 12 cases: Ceylan found incorrect answers in Problems 2, 3, 4, 7, 9 and 10; Deniz found incorrect answers in Problems 3, 8 and 10; and Emel found incorrect answers in Problems 1,3 and 10.
- The participants quit with missing answers in 2 cases: Ceylan and Emel quit Problem 6.

Table 12 summarizes how the lack of mistake awareness affected the problem solving episodes and results of the participants. As seen in Table 12, in all 9 cases, the participants displayed lack of mistake awareness combined with lack of Adaptability:

- In Problem 10, Emel read a value in the problem incorrectly which resulted in an incorrect answer although she had Verification (Metacognitive) episode. She had verified whether her answer makes sense with what the problem asked, but she couldn't recognize her mistake in reading (see Appendix G for details).
- In Problem 3, Ceylan and Emel had difficulty in Understanding episode, lack of awareness leaded to inadequate Analysis, Plan and Implementation (Cognitive) episodes in a row which resulted in an incorrect answer (see Appendix G for details).
- When Deniz (in problems 5 and 8) and Ceylan (in Problem 10) had difficulty in Analysis episode, lack of mistake awareness leaded to inadequate Plan and Implementation (Cognitive) episodes in a row, which also had resulted in incorrect answer (see Appendix G for details).

- When Deniz (in problems 3 and 10) and Ceylan (in Problem 3) had a mistake in Implementation (Cognitive) episode, lack of verification leaded in incorrect or partially correct answers (see Appendix G for details).

Table 12. Results and Correctness of the Answers When the Participants had Lack of Mistake Awareness

Problem No.	Participant	Mistake in	Adaptability of the participant	Result of "Lack of Mistake Awareness"	Correctness of the Answer
3	Ceylan	Understanding	Lack of Adaptability	Inadequate Analysis, Plan and Implementation(C)	Incorrect
3	Deniz	Implementation(C)	Lack of Adaptability	Lack of Verification(C)	Incorrect
3	Emel	Understanding	Lack of Adaptability	Inadequate Analysis, Plan and Implementation(C)	Incorrect
5	Deniz	Analysis	Lack of Adaptability	Inadequate Plan and Implement(C)	Partially Correct
8	Ceylan	Implementation(C)	Lack of Adaptability	Lack of Verification(C)	Partially Correct
8	Deniz	Analysis	Lack of Adaptability	Inadequate Plan and Implementation(C)	Incorrect
10	Ceylan	Analysis	Lack of Adaptability	Inadequate Plan and Implementation(C)	Incorrect
10	Deniz	Implementation(C)	Lack of Adaptability	Lack of Verification(C)	Incorrect
10	Emel	Reading	Lack of Adaptability	Verification(M) couldn't help	Incorrect

Note. (C): Cognitive. (M): Metacognitive

Table 13. Results and Correctness of the Answers When the Participants had Uncertainty

Prob. No.	Participant	Difficulty in	Adaptability of the participant	Result of "Uncertainty"	Correctness of the Answer
1	Emel	Analysis	Awareness for the need of change	Implementation(M) leaded to Inadequate Analysis, Plan and Implementation(C)	Incorrect
2	Ceylan	Understanding	Awareness for the need of change	Inadequate Analysis, Plan and Implementation(C)	Incorrect
4	Ceylan	Understanding	Lack of Adaptability	Inadequate Analysis, Plan and	Incorrect
6	Ceylan	Exploration(C)	Lack of Adaptability	Inadequate Exploration(C)	No Answer
6	Emel	Exploration(C)	Lack of Adaptability	Exploration(M) leaded to Inadequate Exploration(C)	No Answer
7	Ceylan	Analysis	Lack of Adaptability	Inadequate Plan and Implementation(C)	Incorrect
7	Deniz	Analysis	Lack of Adaptability	Inadequate Plan and Implementation(C)	Partially Correct
7	Emel	Analysis	Lack of Adaptability	Inadequate Plan and Implementation(C)	Partially Correct
9	Ceylan	Exploration(C)	Lack of Adaptability	Kept on Explore(C) and guessed an option	Incorrect

Note. (C): Cognitive. (M): Metacognitive

Table 13 summarizes how the uncertainty of the participants affected the problem solving episodes and the correctness of the answers:

- In Problem 1, Emel and in Problem 3, Ceylan showed awareness for need of change in their processes; in those cases the participants had failed in Analysis episode(see Appendix G for details).
- In Problem 6, Ceylan and Emel couldn't display adaptability and stuck in Exploration (Cognitive) and finally quit the problem with no answer (see Appendix G for details).
- In Problem 9, Ceylan couldn't display adaptability and guessed an incorrect answer (see Appendix G for details).
- In Problem 7, Ceylan, Deniz and Emel had difficulty in analysis, since they couldn't display adaptability and they ended with inadequate plans and implementations (Cognitive). Ceylan had incorrect answer and Deniz and Emel had partially correct answer in Problem 7 (see Appendix G for details).

4.6 The Results from the Questionnaire

The questionnaire has been applied to the participants at the end of clinical interview sessions. I wanted the participants to mark "Yes" if they thought that the questionnaire item was valid in most of the ten problems. As Goos et.al (2000) emphasized, since it is unwise to accept participants' self-reports directly, "the students' questionnaire responses must be interpreted in the light of their actual problem solving behavior" (p. 10). Table 14 displays the questionnaire items that have been responded as *Yes* by the most of the participants. In this section, I will comment on what the responses can tell us and how the interviews and observations confirm those.

The responses to items 2 and 5 may show us that four participants were aware of the importance of understanding (see Table 14). I observed that the participants tended not to go on unless they understood the problem. I want to note that Ceylan was the only participant didn't mark "Yes" to these questions and as seen in problem solving episodes section, she failed in understanding in 3 problems and failed in analysis in 2 problems. These data may show that she underestimate the importance of understanding.

Table 14. The Questionnaire Items that have been Responded as Yes by Nearly All Participants

Self-Monitoring Questionnaire Item	Number of Responses		
	Yes	No	Not Sure
2. I made sure that I understood what the problem was asking me.	4	1	0
5. I identified the information that was given in the problem.	4	0	1
6. I thought about different approaches I could try for solving the problem.	5	0	0
8. I made a mistake and had to redo some working.	5	0	0
12. I checked my calculations to make sure they were correct.	5	0	0
15. I thought about different ways I could have solved the problem.	4	1	0

The responses to items 12 and 8 may show us that all participants are aware of importance checking the calculations and corrections (see Table 14). Researcher observed that most of the time the participants didn't check their calculations when they are sure of themselves or when they were bored of long calculations. As it is introduced in problem solving episodes subsection, participants verified their results in 7 cases of 50, and only one of them was the check of calculations. I also have to note that 4 of these cases belong to Bora which is most successful in this case study in all perspectives.

The responses to items 6 and 15 may show us that all participants were aware of the importance of trying different strategies (see Table 14). As introduced in strategies subsection, seven problems were solved by trying second strategies. In eight more problems the effort of the participants trying a second strategy was not enough to find a correct answer. The findings of the study show that the participant didn't have the negative belief that every problem has only a way of solving.

In the end the questionnaire, only two participants wrote their comments. Bora commented that he liked to try to solve a problem in many ways although he achieved an answer. Using my observations and comments of Bora's teachers, I can say that Bora tends to be "more concerned than poor problem solvers about obtaining "elegant" solutions to problems" which has been described as a property that distinguish good problem solvers from the poor ones (Lester, 1994, p. 665). Emel commented that the problems were different from the problems she had studied in text books, so she had tried to use her logical reasoning as much as she can. Emel used logical reasoning strategy in six problems and she was successful in four of these problems.

4.7 Conclusions

The results and the problem solving behaviors of the participants observed in the study confirmed their academic success levels. Successful students of the study were the participants are the students who were most successful in OKS exam and the participant with the worst score of the study was the one who had the worst OKS score (see section 4.1 for more details).

The results of the study showed that choosing a correct strategy doesn't guarantee a correct answer. The students who can use different types of strategies efficiently are more successful in mathematical problem solving (See Table 4 and section 4.2 for further details). Two successful participants tend to use different strategies efficiently and change the strategy when they need. When students try to remember the problem solving strategies in similar problems and try these methods even though they are not appropriate for the current problem, which can be seen in the low efficiency rates of different problem solving strategies (See Table 5 and section 4.2 for further details).

Mistake recognition, adaptability and expression of thinking are the main components of metacognition of the students in the context of mathematical problem solving. It is confirmed in this study that, when the students exhibit awareness and correction, they reach a correct answer (see section 4.4 for details). The results of the analysis also showed that when the students have the independent adaptability, they can change their lack of mistake awareness or uncertainty of mistake awareness to awareness and correction (see section 4.4 for details).Lack of mistake awareness or uncertainty leads to either a partially correct answer, an incorrect answer or no answer at all. In other cases, when the students display awareness for a need of change, they cannot decide a new strategy and usually quit the problem with no answer. When the students show lack of adaptability, the failure is inevitable and it ends with an incorrect or partially correct answer (see section 4.4 for details). Finally, this study confirmed that when the students can generate full description of their processes, they mostly reach a correct or partially correct answer and difficulty in awareness and expression of thinking of the participant seem to be a sign of a possible incorrect or no answer (see section 4.4 for details).

The findings of this study confirmed that all problem solving episodes described in cognitive-metacognitive framework of Artzt and Armour-Thomas (1992) do not need to occur in a successful problem solving; the students may bypass some episodes and be successful in solving the problem. Moreover, the problem solving episodes can be observed in the different orders and some problem solving episodes can be observed more than once (see section 4.3 for details).

Reading and Understanding episodes are observed in nearly every problem solving activity (see section 4.3 and Table 7 for details). Having the required knowledge does not guarantee success in any problem solving episodes or a correct answer, but lack of required knowledge for the problem results in failure in Understanding and/or Analysis episodes (see section 4.5 for details).

The students can reach a correct answer in their first attempt when they can have appropriate Analysis, Plan and Implementation (Cognitive) episodes in a row. Expert problem solvers tend to force this path. When students bypass Analysis and Plan episodes, Exploration (Cognitive) leads to a correct answer only when Exploration (Metacognitive) leads to Analysis episode. In that case, the students should have appropriate Analysis, Plan and Implementation (Cognitive) episodes in a row. Students can manage this when they have the independent adaptability. Having appropriate Exploration (Metacognitive) and Implementation (Metacognitive) episodes, the students can change the inefficient strategy with an appropriate strategy or they can find a way to use their present strategy efficiently (see Table 11 for details). On the other hand, when students cannot display Exploration (Metacognitive) and Implementation (Metacognitive) behaviors, they can stuck in Exploration (Cognitive) episode (see Tables 12 and 13 for details).

Failure in Analysis episode generally leads to incorrect answer. When the students display independent adaptability, they can discover their mistakes in one of the Implementation (Metacognitive) or Verification (Metacognitive) episodes and return to Analysis episode. Similarly failure in Understanding episode leads to incorrect answer; unless the student displays independent adaptability and the Verification (Metacognitive) occurs. A mistake in Reading episode can also lead to incorrect answer. The students can overcome the mistake displaying independent adaptability and discovering the mistake in Implementation (Metacognitive) or Verification (Metacognitive) episodes, which leads them back to Reading episode (See Table 12 and 13 and section 4.5 for details).

After Exploration (Metacognitive) and Implementation (Metacognitive) episodes, the students usually find their mistakes or change their strategies (see section 4.5 and Table 11 for details). Students rarely display Verification (Cognitive and Metacognitive); they tend to pass to the next problem immediately when they find an answer. On the other hand, successful students tend to verify whether their answer make sense with the aim of the problem (see section 4.5 and Table 11 for details).

CHAPTER 5

DISCUSSIONS, IMPLICATIONS AND RECOMMENDATIONS

This chapter restates the purpose and interprets the results of the present study in discussion. Then, implications and recommendations are mentioned.

5.1 Discussion

The purpose of this study was to investigate the effects of problem solving strategies, problem solving episodes, and metacognition of five Turkish students graduated from elementary school on their problem solving success. The answer the following research questions will be discussed in this subsection:

Which problem solving strategies do elementary school graduate Turkish students employ while solving selected PISA 2003 mathematical literacy items?

Which problem solving episodes are observed when elementary school graduate Turkish students while solving selected PISA 2003 mathematical literacy items?

Which mistake recognition, adaptability and expression of thinking behaviors do elementary school graduate Turkish students display while solving selected PISA 2003 mathematical literacy items?

How problem solving strategies, cognitive and metacognitive behaviors of elementary school graduate Turkish students effect their success in PISA 2003 mathematical literacy items?

For the present study, the research data had been collected by the clinical interviews done with five student participants individually. The interviews conducted by the researcher consisted of the students completing ten mathematical problems.

The participants were informed that the aim of the study was not investigating their correct answers, but to observe how they thought during the problem solving process. So, the participants were given instructions to think aloud and verbally express their thoughts as they worked on the problems. The participants were also informed that there was no time limit for any problems as long as they have eager to solve the problem. The clinical interviews for each participant were video recorded. At the end of the clinical interviews, a self monitoring questionnaire was applied to the participants.

After the analysis of the data, it was found that although the problem solving strategies are very important in solving the problem, but finding a correct strategy does not guarantee a successful answer. This result confirms Rudder (2006) who had observed the same findings on American and Singaporean students.

In general, the problem solving behaviors of the participants observed in the study confirmed their academic success levels. Successful students of the study were the participants are the students who were most successful in OKS exam and the participant with the worst score of the study was the one who had the worst OKS score. Successful students tend to analyze the problem and choose a strategy and implement their plan. They can use different problem solving strategies effectively. Successful students in the study seem to confirm observations of the researchers about the good problem solvers (Lesh, 2007; Lester, 1994; Schoenfeld 1985, 1992).

Both cognitive and metacognitive verification are rare in the study. Participants tend to pass to the next problem immediately when they find an answer. Successful students used metacognitive verification to be sure that they found what the problem asked. Lack of verification was one of common factors that Rudder (2006) had observed in American and Singaporean students which yield to incorrect answers. Although, every problem solving framework or model emphasizes the importance or verification since Polya (1973)'s four phase model, lack of verification seems to be a universal property of students.

Various studies have shown that metacognition helps students to deal with the difficulties during the problem solving process (Goos & Galbraith, 1996; Goos, Galbraith, & Renshaw, 2000; Pugalee, 2001; Yimer & Ellerton, 2006). Although students have difficulties in any episode of the problem solving, they can use their metacognitive skills to detect the mistake or missing parts of the process and adapt themselves independently to make the required changes. Students also can try exploration skipping analysis and planning; their exploration is successful when it yields the student to analysis. Lack of awareness and/or adaptability of the students in the problem seem to explain the success or the failures in those cases. Lack of required knowledge seems to affect the understanding and analysis of the students. In fact, students can have problems in understanding and analysis even if they have the required knowledge. The students can have failures in cognitive explorations in lack of awareness and adaptability; and uncertainty rarely results in independent adaptability which leads to a missing or incorrect answer.

Pappas et al. (2003) concluded that "metacognitive and verbal skills… may also lead teachers to conclude that children who can describe thinking and explain ideas are more mathematically competent than children who are less articulate" (p. 448). Results of the present study confirmed that conclusion. Turkish students could generate full description of their works in 44 cases, 36 of these cases resulted in correct or partially correct answer. When Turkish students displayed ambiguous or partial descriptions of their work, they resulted in no answer or incorrect answer.

To sum up, the study confirmed that the problem solving success is too complex to be clarified by a unique property or a behavior of the problem solver. The problem solving requires overcoming various obstacles to reach a successful result. As Lesh (2007) stated "no strategy, process, behavior or characteristic should be expected to always be productive for every problem" (p. 778). Hence, not only the students should have the required mathematical knowledge and a good repertoire of different problem solving strategies, but also they should know when and how to use those strategies, and also they could monitor and regulate their problem solving processes using their metacognitive skills.

5.2 Implications

In this study, I observed that the participants didn't have the negative belief that every problem has a unique way of solving. But when poor problems solvers are uncertain about their solutions, they may not try (or think) another way and stuck in exploring the same ineffective method. So, as Goos et al. (2000) remarked, we should not only "encourage students to monitor and regulate" their process but also ensure that they are "well equipped to respond" (p. 14). As suggested in mathematics curricula, the teachers should create a classroom environment that students are encouraged to explore new strategies, to take risks in trying and to discuss failures and successes with peers and teacher (BoE, 2007; NCTM , 2000). Hence students can understand that their solutions are appreciated and develop confidence in their mathematical abilities; hence they develop a willingness to engage in and explore new problems (BoE, 2007; NCTM , 2000).

Although being an expert problem solver cannot be taught just being told the properties of an expert problem solver (Lesh, 2007; Lester, 1994), at least keeping these cases in mind may help the novice problem solvers gain experience in solving problems in a more conscious way. As Garofalo & Lester (1985) remarked metacognitive decisions between episodes are crucial, and as Harskamp and Suhre (2007) showed in their studies, hints and feedbacks during Analysis, Plan and Verify episodes can improve effectiveness of problem solving of students. The teachers should apply activities where behaviors and difficulties in different problem solving episodes can be observed, and the teachers should improve their observation and scaffolding abilities to help the students giving the efficient hints or feedbacks.

The follow-up questions suggested by Hunting (1997) (see Appendix F) helped me when I needed the participants to express their mental processes responses and implementations. The teachers can use these questions to encourage students to express their thoughts. These questions also can trigger students' thinking and metacognitive processes without giving direct hints about the solution.

Various studies showed that Turkish students can use some problem solving strategies without any training (Arslan & Altun, 2007; Yazgan & Bintaş, 2005). In the present study, the participants haven't given any training about problem solving strategies and 7 different problem solving strategies have been observed in their solutions. It is not possible to search back which of these strategies have been gained by the participants without special training. But this study showed which of these strategies the students have difficulties. We know that the students can learn new strategies and training had a positive effect on students' problem solving success (Arslan & Altun, 2007; Koç & Bulut, 2003; Yazgan & Bintaş, 2005). As it is advised in mathematics curricula, the teachers should provide problems that require different problem solving strategies and encourage the students to try new strategies solving the same problem even if they already found an answer (BoE, 2007; NCTM , 2000).

5.3 Recommendations to Further Research

Following are some recommendations for further research on the effects of problem solving strategies, problem solving episodes, and metacognition of Turkish students on their problem solving success.

In this study, the problem solving strategies, problem solving episodes, and metacognition of five Turkish students just graduated from elementary school has been investigated. More research can be done to investigate Turkish students in different grade levels.

As Artzt & Armour-Thomas (1992) remarks "the interrelationship between metacognitive and cognitive processes is complex, and an appropriate interplay between the two is necessary for successful problem solving to occur" (p. 161). Using the cognitive-metacognitive framework of Artzt and Armour-Thomas (1992) was helpful to observe not only the problem solving episodes but also cognitive and metacognitive processes of the participants. Also the coding system of Pappas et al. (2003) helped me to deeply analyze the metacognitive behaviors of the participants.

In this study, a self monitoring questionnaire has been applied to the participants to observe their monitoring and regulation about their problem solving behaviors. The responses have also been used to observe possible problem solving habits and believes of the participants but the findings were limited. The researchers who will deeply investigate the habits and beliefs can consider using another questionnaire.

In this study, according to research settings, the researcher hadn't give hints or clues when the participants have difficulties. So, the adaptability of the participants when they have assistance hasn't been observed. Researchers who need to observe this behavior should consider the required updates in the research settings.

REFERENCES

Altun, M. (2000). İlköğretimde Problem Çözme Öğretimi. *Millî Eğitim Dergisi* (147). Retrieved May 10, 2007 from http://yayim.meb.gov.tr/dergiler/147/altun.htm

Arslan, Ç., & Altun, M. (2007). Learning To Solve Non-routine Mathematical Problems. *İlköğretim Online* 6(1), 50-61.

Artzt, A. , & Armour-Thomas, E. (1992). Development of a cognitive-metacognitive framework for protocol analysis of mathematical problem solving in small groups. *Cognition and Instruction* (9), 137-175.

Babadoğan, C., & Olkun, S. (2006). Program Development Models and Reform in Turkish Primary School Mathematics Curriculum. *International Journal for Mathematics Teaching and Learning* .

Baki, A., Karataş, İ., & Güven, B. (2002). Klinik Mülakat Yöntemi ile Problem Çözme Becerilerinin Değerlendirilmesi. *Ulusal 5. Fen Bilimleri ve Matematik Eğitimi Kongresi.* Ankara: UFMEK.

Berberoğlu, G. (2006). *Results of International Student Assessment Studies Which Turkey Must Consider*. Retrieved November 12, 2007 from http://siteresources.worldbank.org/INTTURKEY/Resources/361616-1142415001082/Giray_Berberoglu_ENG.pdf

BoE. (2007). *Talim ve Terbiye Kurumu Baskanligi İlkögretim Matematik Dersi 6–8 Sınıflar Öğretim Programı ve Kılavuzu.* Ankara: MEB.

Bulut, M. (2007). Curriculum Reform in Turkey: A Case of Primary School Mathematics Curriculum. *Eurasia Journal of Mathematics, Science & Technology Education , 3* (3), 203-212.

Charles, R., Lester, F., & O'Daffer, P. (1987). *How to Evaluate Progress in Problem Solving.* Reston: National Council of Teachers of Mathematics.

Frobisher, L. (1994). Problems, Investigations and an Investigative Approach. In A. Orton, & G. Wain, *Issues in Mathematics* (pp. 150-173). London: Cassell.

Gagne, R. M., & Medsker, K. L. (1996). *The conditions of learning. Training applications.* Fort Worth, TX: Harcourt Brace College Publishers.

Garofalo, J., & Lester, F. (1985). Metacognition, Cognitive Monitoring, and Mathematical Performance. *Journal for Research in Mathematics Education , 16* (3), 163-176.

Geiger, V., & Galbraith, P. (1998). Developing a diagnostic framework for evaluating student approaches to applied mathematics. *International Journal of Mathematics, Education, Science, and Technology* (29), 533-559.

Goldberg, P. D., & Bush, W. S. (2003). Using metacognitive skills to improve 3rd graders' math problem solving. *Focus on Learning Problems in Mathematics.* Retrieved November 15, 2007 from http://findarticles.com/p/articles/mi_m0NVC/is_4_25/ai_n6126743

Goos, M., & Galbraith, P. (1996). Do It This Way! Metacognitive Strategies in Collaborative Mathematical Problem. *Educational Studies in Mathematics , 30* (3), 229-260.

Goos, M., Galbraith, P., & Renshaw, P. (2000). *A Money Problem:A Source of Insight into Problem Solving Action.* Retrieved June 16, 2007 from http://www.cimt.plymouth.ac.uk/journal/pgmoney.pdf

Harskamp, E., & Suhre, C. (2007). Schoenfeld’s problem solving theory in a student controlled learning environment. *Computers & Education* (49), 822–839.

Hunting, R. P. (1997). Clinical interview methods in mathematics education research and practice. *The Journal of Mathematical Behavior , 16*, 145-165.

Jones, V. O. (2006). *Cognitive Processes during Problem Solving of Middle School Students with Different Levels of Mathematics Anxiety And Self Esteem: Case Studies.* Unpublished doctoral dissertation, Florida State University, Florida. Retrieved November 15, 2007 from http://etd.lib.fsu.edu/theses/available/etd-07072006-195753/ unrestricted/voj_dissertation.pdf

İş Güzel, Ç.(2006). *A cross-cultural comparison of the impact of human and physical resource allocations on students' mathematical literacy skills in the programme for international student assessment (PISA) 2003.* Unpublished doctoral dissertation, Middle East Technical University, Ankara, Turkey.

Karataş, İ., & Güven, B. (2004). 8. Sınıf öğrencilerinin problem çözme becerilerinin belirlenmesi: Bir özel durum çalışması. *Milli Eğitim Dergisi,* 163. Retrieved November 10, 2007 from http://yayim.meb.gov.tr/dergiler/163/karatas.htm

Koç, Y., & Bulut, S. (2002). Effects of Cooperative and Individualistic Problem Solving Methods on Mathematical Problem Solving Performance. *Hacettepe Üniversitesi Egitim Fakültesi Dergisi* (22), pp. 82-90.

Krulik, S. & Rudnick, J.A. (2003). *Problem Solving: A Handbook for Senior High School teachers*. Eastbourne: Antony Rowe Ltd.

Lesh, R., & Zawojewski, J. S. (2007). Problem solving and modeling. In F. Lester (Ed.), *Second Handbook of research on mathematics teaching and learning*. Greenwich, CT: Information Age Publishing.

Lester, F. K. (1994). Musing about mathematical problem solving research: 1970-1994. *Journal for Research in Mathematics Education,* 25(6), 660-675.

Longman active study dictionary of English (18th ed). (1989). Harlow, Essex: Longman

Mayer, R. E. (2003). Mathematical Problem solving. In J. M. Royer, *Mathematical Cognition* (pp. 69-92). Connecticut: Information Age Publishing.

MEB. (2007). *64 Soruda Ortaöğretime Geçiş Sistemi.* Retrieved November 15, 2007 from http://www.meb.gov.tr/duyurular/duyurular2007/64sorudaoges/oges64soru.pdf

Merriam, S. B. (1998). *Qualitative Research and Case Study Applications in Education* (Rev. ed.). San Francisco: Jossey-Bass Publishers.

MONE. (2007). *National Education Statistics Formal Education 2006-2007.* Retrieved November 11, 2007 from http://sgb.meb.gov.tr/istatistik/meb_istatistikleri_orgun_egitim_2006_2007.pdf

Mullis, I.V.S., Martin, M.O., Gonzales, E.J., Gregory, K.D., Garden, R.A., O'Connor, K.M., Chrostowski, S.J., & Smith, T.A. (2000). *TIMSS 1999 International Mathematics Report: Findings from IEA's Repeat of the Third International Mathematics and Science Study at the Eighth Grade.* Chestnut Hill, MA: Boston College. Retrieved November 13, 2007 from http://timss.bc.edu/timss1999i/pdf/T99i_Math_All.pdf

NCTM. (2000). *Principles and Standards for School Mathematics.* Reston: VA: Author.

OECD. (2003). *The PISA 2003 Assessment framework: Mathematics, reading, science and problem solving knowledge and skills.* Paris: OECD.

OECD. (2004). *Learning for tomorrow's world -First Results From PISA 2003: Executive Summary.* Paris: OECD.

OECD. (2005). *PISA 2003 Technical Report.* Paris: OECD.

Pappas, S., Ginsburg, H. P., & Jia, M. (2003). SES differences in young children's metacognition in the context of mathematical problem solving. *Cognitive Development , 18* (3), 431-450.

Patton, M. Q. (1990). *Qualitative Evaluation Methods (2nd Edition).* Thousand Oaks, CA: Sage.

Polya, G. (1973). *How to solve it: A new aspect of mathematical method* (2nd ed.). Princeton, NJ: Princeton.

Posamentier, A. S., & Krulik, S. (1998). *Problem-Solving Strategies for Efficient and Elegant Solutions.* California: Corwin Press,Inc.

Pugalee, D. K. (2001). Writing, mathematics, and metacognition: Looking for connections through students' work in mathematical problem solving. *School Science and Mathematics , 101* (5), 236-245.

Rudder, C. A. (2006). *Problem Solving: Case Studies Investigating the Strategies Used By Secondary American and Singaporean Students.* Unpublished doctoral dissertation, Florida State University, Florida. Retrieved September 17, 2007 from http://etd.lib.fsu.edu/theses/available/etd-07102006-135937/unrestricted/ car_dissertation.pdf

Rysz, T. (2004). *Metacognition in Learning Elementary Probability and Statistics.* Unpublished doctoral dissertation, University of Cincinnati, Ohio. Retrieved June 15, 2007 from http://www.stat.auckland.ac.nz/~iase/publications/dissertations/04.Rysz.Dissertation.pdf

Schoenfeld, A. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grows, *Handbook of research on mathematics teaching and learning* (pp. 334-370). New York: Macmillan.

Stillman, G. A., & Galbraith, P. L. (1998). Applying mathematics with real world connections: Metacognitive characteristics of secondary students. *Educational Studies in Mathematics, 36*(2), 157-195.

Teong, S. K. (2003). The effect of metacognitive training on mathematical word-problem solving. *Journal of Computer Assisted Learning, 19*(1), 46 – 55.

The American Heritage Dictionary of the English Language (4th ed). (2000). Boston: Houghton Mifflin

World Bank. (2006). *Turkey Education Sector Study-Report No:32450-TU.* Washington DC: World Bank.

Yazgan, Y., & Bintaş, J. (2005). İlköğretim Dördüncü ve Beşinci Sınıf Öğrencilerinin Problem Çözme Stratejilerini Kullanabilme Düzeyleri: Bir Öğretim Deneyi. *Hacettepe Üniversitesi Egitim Fakültesi Dergisi* (28), pp. 210-218.

Yılmaz, B. (2003). *Effects of metacognitive training on seventh grade students' problem solving performance.* Unpublished master's thesis, Boğaziçi Universitesi, Istanbul, Turkey.

Yimer, A., & Ellerton, N. F. (2006). Cognitive and Metacognitive Aspects of Mathematical Problem Solving: An Emerging Model. *MERGA 2006.* Wahroonga: MERGA.

Yin, R. K. (2003) *Case Study Research: Design and Methods*. (3rd ed.) Thousand Oaks, CA: Sage.

APPENDIX A

PISA 2003 LEVELS OF PROFIENCY IN MATHEMATICS

Taken from First Results from PISA 2003: Executive Summary (OECD, 2004, p. 5)

Table 15. Descriptions for the Six Levels of Profiency in Mathematics in PISA 2003

Score	Level	What students can typically do?
668	Level 6	Students can conceptualize, generalize, and utilize information based on their investigations and modeling of complex problem situations. They can link different information sources and representations and flexibly translate among them. Students at this level are capable of advanced mathematical thinking and reasoning. These students can apply insight and understanding along with a mastery of symbolic and formal mathematical operations and relationships to develop new approaches and strategies for dealing with novel situations. Students at this level can formulate and precisely communicate their actions and reflections regarding their findings, interpretations, arguments and the appropriateness of these to the original situations.

Table 15. Descriptions for the Six Levels of Profiency in Mathematics in PISA 2003 (Continued)

Score	Level	What students can typically do?
606	Level 5	Students can develop and work with models for complex situations, identifying constraints and specifying assumptions. They can select, compare, and evaluate appropriate problem-solving strategies for dealing with complex problems related to these models. Students at this level can work strategically using broad, well-developed thinking and reasoning skills, appropriately linked representations, symbolic and formal characterizations, and insight pertaining to these situations. They can reflect on their actions and formulate and communicate their interpretations and reasoning.
544	Level 4	Students can work effectively with explicit models for complex concrete situations that may involve constraints or call for making assumptions. They can select and integrate different representations, including symbolic ones, linking them directly to aspects of real-world situations. Students at this level can utilize well-developed skills and reason flexibly, with some insight, in these contexts. They can construct and communicate explanations and arguments based on their interpretations, arguments and actions.
482	Level 3	Students can execute clearly described procedures, including those that require sequential decisions. They can select and apply simple problem-solving strategies. Students at this level can interpret and use representations based on different information sources and reason directly from them. They can develop short communications reporting their interpretations, results and reasoning.

Table 15. Descriptions for the Six Levels of Profiency in Mathematics in PISA 2003 (Continued)

420	Level 2	Students can interpret and recognize situations in contexts that require no more than direct inference. They can extract relevant information from a single source and make use of a single representational mode. Students at this level can employ basic algorithms, formulae, procedures, or conventions. They are capable of direct reasoning and making literal interpretations of the results.
358	Level 1	Students can answer questions involving familiar contexts where all relevant information is present and the questions are clearly defined. They are able to identify information and to carry out routine procedures according to direct instructions in explicit situations. They can perform actions that are obvious and follow immediately from the given stimuli.

APPENDIX B

COGNITIVE-METACOGNITIVE FRAMEWORK FOR PROTOCOL ANALYSIS OF PROBLEM SOLVING IN MATHEMATICS

Taken from Artzt & Armour-Thomas (1992, pp.172-175)

The following framework outlines the interactive relationship between metacognitive and cognitive processes in mathematical problem solving. The episodic categories are described theoretically and empirically. The level or levels of cognition associated with each category are indicated as well. Note that during the course of problem solving these episodes need not occur in the order listed, may occur several times, and may indeed be bypassed completely.

Episode 1: ***Reading the problem (cognitive)***

Description: The student reads the problem.

Indicators: The student is observed as reading the problem or listening to someone else read the problem. The student may be reading the problem silently or aloud to the group.

Episode 2: Understanding the problem (metacognitive)

Description: The student considers domain-specific knowledge that is relevant to the problem. Domain-specific knowledge includes recognition of the linguistic, semantic, and schematic attributes of the problem in his or her own words and represents the problem in a different form.

Indicators: The student may be exhibiting any of the following behaviors:

(a) restating the problem in his or her own words; (b) asking for clarification of the meaning of the problem; (c) representing the problem by writing the key facts or by making a diagram or list; (d) reminding himself or herself or others of the requirements of the problem, for example, "Remember, we must use the exact number that is asked for in the problem"; (e) stating or asking himself or herself whether he or she has done a similar problem in the past; and (f) discussing the presence or absence of important pieces of information.

Episode 3: Analyzing the problem (metacognitive)

Description: The student decomposes the problem into its basic elements and examines the implicit or explicit relations between the givens and goals of the problem.

Indicators: The student is engaging in an attempt to simplify or reformulate the problem. An attempt is made to select an appropriate perspective of the problem and to reformulate it in those terms. Examples of statements reflecting that such analysis is occurring are: "After you use all the given information, it becomes an easy problem of addition," and "Because the total is a multiple of five, I think the answer must be divisible by five."

Episode 4: Planning (metacognitive)

Description: The student selects steps for solving the problem and a strategy for combining them that might potentially lead to problem solution if implemented. The student may also select a representation for the information in the problem. In addition, the student may assess the status of the problem solution and make decisions for change if necessary.

Indicators: The student describes an approach that he or she intends to use to solve the problem. This may be in the form of steps to be taken or strategies to be used. Examples of statements that reflect planning include the following: "Let's use the given information first and see what the problem looks like after that; "Let's work backwards by estimating an answer and see how it must be adjusted to fit the problem"; "Let's draw a chart and fill in the numbers; "Let's think of a different way to go about this; and

"Let's check back to see where we went wrong."

Episode 5a: Exploring (cognitive)

Description: The student executes a trial-and-error strategy in an attempt to reduce the discrepancy between the givens and the goals.

Indicators: The student engages in a variety of calculations without any apparent structure to the work. There is no visible sequence to the operations performed by the student.

Episode 5b: Exploring (metacognitive)

Description: The student monitors the progress of his or her or others' attempted actions thus far and decides whether to terminate or continue working through the operations. This differs from analysis in that it is less well structured, and it is further removed from the original problem. If one comes across new information during exploration, he or she may return to analysis in the hope of using that information to better understand the problem.

Indicators: (a) The student draws away from the problem to ask himself or herself or someone else what has been done during the exploration. Examples of such statements are: "What are you doing?" and "What am I doing?" (b) The student gives suggestions to other students about what to try next in the exploration. An example of such a comment is: "It's getting too big; try it with one less." (c) The student evaluates the status of the exploration. Examples of such statements are: "This isn't getting us anywhere," and
"I think that's the answer!"

Episode 6a: Implementing (cognitive)

Description: The student executes a strategy that grows out of his or her understanding, analysis, and/or planning decisions and judgments. Unlike exploration, the student's actions are characterized by a quality of systematicity and deliberateness in transforming the givens into the goals of the problem.

Indicators: The student appears to be engaging in a coherent and well structured series of calculations. There is evidence of an orderly procedure.

Episode 6b: Implementing (metacognitive)

Description: The student engages in the same kind of metacognitive process as in the exploring (metacognitive) phase of problem solving, monitoring the progress of his or her attempted actions. Unlike the exploratory phase, however, the metacognitive decisions build on, check, or revise those previously considered decisions. Furthermore, the student may consider a reallocation of his or her problem-solving resources, given the time constraint within which the problem must be solved.

Indicators: During the implementation phase, the student draws away from the work to see what has been done or where it is leading. The following examples of statements reflect this: "Okay, I used all the given conditions, and now I will start adding what is left"; "Wait. You forgot to use the second point"; and "This is taking too long. Try skipping the odd numbers."

Episode 7a: Verifying (cognitive)

Description: The student evaluates the outcome of the work by checking computational operations.

Indicators: The student redoes the computational operations he or she did before to check that it was done correctly.

Episode 7b: Verifying (metacognitive)

Description: The student evaluates the solution of the problem by judging whether the outcome reflected adequate problem understanding, analysis, planning, and/or implementation. Should the student discover a discrepancy in this comparison search, he or she engages in new decision making for correcting the faulty metacognitive and/or cognitive processing that led to the incorrect solution. The ability to adjust one's thinking on the basis of evaluative information is another indication of self-regulatory competence. Should the evaluation of problem solution indicate an adequacy of or congruence with metacognitive and cognitive processing, the mental reiteration ends.

Indicators: After the student has decided that the solution or part of the solution has been obtained, he or she may review the work in several ways:

(a) The student checks the solution process to see whether it makes sense. For example, "When we simplified the problem, did we use all of the given information?" (b)The student checks to see if the solution satisfies the conditions of the problem. For example, "Does our answer satisfy both of the properties that were asked for?" (c) The student explains to a groupmate how the solution was obtained. For example, "I knew it had to be a big number, so I started with the largest numbers first."

Episode 8: Watching and listening (uncategorized)

Description: This category only pertains to students who are working with other people. The student is attending to the ideas and work of others.

Indicators: The student appears to be listening to a group member who is talking or watching a group member who is writing.

APPENDIX C

SELF-MONITORING QUESTIONNAIRE - ENGLISH

Place a tick in the appropriate column to show how you were thinking before, during, and after working on the problem.

BEFORE you started to solve the problem—what did you do?

	YES	NO	UNSURE
1. I read the problem more than once.	—	—	—
2. I made sure that I understood what the problem was asking me.	—	—	—
3. I tried to put the problem into my own words.	—	—	—
4. I tried to remember whether I had worked on a problem like this before.	—	—	—
5. I identified the information that was given in the problem.	—	—	—
6. I thought about different approaches I could try for solving the problem.	—	—	—

AS YOU WORKED on the problem—what did you do?

	YES	NO	UNSURE
7. I checked my work step by step as I went through the problem.	—	—	—
8. I made a mistake and had to redo some working.	—	—	—
9. I reread the problem to check that I was still on track.	—	—	—
10. I asked myself whether I was getting any closer to a solution.	—	—	—
11. I had to rethink my solution method and try a different approach.	—	—	—

AFTER YOU FINISHED working on the problem—what did you do?

	YES	NO	UNSURE
12. I checked my calculations to make sure they were correct.	—	—	—
13. I looked back over my solution method to check that I had done what the problem asked.	—	—	—
14. I asked myself whether my answer made sense.	—	—	—
15. I thought about different ways I could have solved the problem.	—	—	—

Did you use any of these ways of working?

	YES	NO	UNSURE
16. I "guessed and checked".	—	—	—
17. I used algebra to set up some equations to solve.	—	—	—
18. I drew a diagram or picture.	—	—	—
19. I wrote down important information.	—	—	—
20. I felt confused and couldn't decide what to do.	—	—	—
21. I used other ways to work on the problem.	—	—	—

(If you have ticked YES for Question 21, please write a sentence or two in the space below to explain what you did)

APPENDIX D

SELF-MONITORING QUESTIONNAIRE - TURKISH

Problemden önce, problem sırasında ve problemi çözdükten sonra ilgili kısımlardaki sorular için EVET, HAYIR, EMİN DEĞİLİM cevaplarından birini seçin.

Problemi çözmeden ÖNCE – neler yaptın?

	Evet	Hayır	Emin Değilim
1. Problemi birden fazla okudum.	__	__	__
2. Problemin ne sorduğunu anladığımdan emin oldum.	__	__	__
3.Problemi kendi sözcüklerimle yeniden yazdım.	__	__	__
4. “Daha önce böyle bir problem ile karşılaşmış mıydım?” diye düşündüm.	__	__	__
5. Önce problemde verilen bilgiyi saptadım.	__	__	__
6. Problemi çözebilmek için değişik yollar düşündüm.	__	__	__

Problem üzerinde UĞRAŞIRKEN – neler yaptın?

	Evet	Hayır	Emin Değilim
7. Problemi çözerken her adımda yaptıklarımı kontrol ettim.	__	__	__
8. Hata yaptığımı gördüm ve bazı işlemleri baştan yapmak zorunda kaldım.	__	__	__
9. “Doğru yolda mıyım?” diye problemi tekrar okudum.	__	__	__
10. Kendime “Çözüme yaklaştım mı?” diye sordum.	__	__	__
11. Çözüm yolumu tekrar düşündüm ve yeni bir yol denedim.	__	__	__

Problem çözümü BİTTİKTEN SONRA – neler yaptın?

	Evet	Hayır	Emin Değilim
12. "İşlemlerim doğru mu?" diye bir daha kontrol ettim.	__	__	__
13. "Problemde istenileni buldum mu?" diye çözüm yolumu tekrar kontrol ettim.	__	__	__
14. Kendime "Sonuç mantıklı mı?" diye sordum.	__	__	__
15. "Problemi başka hangi yollarla çözebilirdim?" diye düşündüm.	__	__	__

Çözüm sırasında bunlardan birini/birkaçını kullandın mı?

	Evet	Hayır	Emin Değilim
16. Tahmin ettim ve kontrol ettim.	__	__	__
17. Çözümümde denklemler kullandım.	__	__	__
18. Grafik veya resim çizdim.	__	__	__
19. Önemli bilgileri not aldım.	__	__	__
20. Kafam karıştı ve ne yapacağımı bilemedim.	__	__	__
21. Başka bir yol denedim.	__	__	__

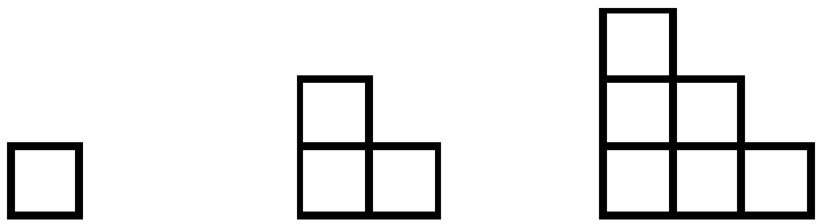
21. Soruya EVET dediyseniz, lütfen bir sonraki sayfada açıklayınız.

APPENDIX E

PROBLEMS USED IN THIS STUDY TRANSLATED INTO TURKISH

Problem 1

Rasim birim karelerden oluşan bir örüntü oluşturuyor. İşte adımlar:



Adım 1 Adım 2 Adım 3

Gördüğün gibi, Adım 1'de bir kare, Adım 2'de 3 kare ve Adım 3'te altı kare kullanıyor.

Adım 4'te kaç kare kullanmalıdır?

Cevap: .... kare

Problem 2

Bir araba dergisi yeni çıkan arabaları değerlendirirken bir değerlendirme sistemi kullanmakta ve "Yılın Arabası" ödülünü en yüksek puanı alan arabaya vermektedir. Beş yeni araba bu sistemde değerlendirilmiştir. Değerlendirme sonuçları tabloda verilmiştir.

Araba	Güvenlik (G)	Yakıt Kullanımı (Y)	Dış Görünüş (D)	İç Görünüş (I)
Ca	3	1	2	3
M2	2	2	2	2
Sp	3	1	3	2
N1	1	3	3	3
KK	3	2	3	2

Puanlama şu şekildedir:

3 puan = Mükemmel

2 puan = İyi

1 puan = Vasat

Araba dergisi arabanın toplam puanını hesaplarken bireysel puanlarının katsayılı toplamını kullanıyor. Bu kural:

Toplam Puan = (3 x G) + Y + D + I

"Ca" marka arabanın toplam puanını hesaplayınız.

"Ca" marka arabanın Toplam Puanı: ....

Problem 3

Bir araba dergisi yeni çıkan arabaları değerlendirirken bir değerlendirme sistemi kullanmakta ve "Yılın Arabası" ödülünü en yüksek puanı alan arabaya vermektedir. Beş yeni araba bu sistemde değerlendirilmiştir. Değerlendirme sonuçları tabloda verilmiştir.

Araba	Güvenlik (G)	Yakıt Kullanımı (Y)	Dış Görünüş (D)	İç Görünüş (I)
Ca	3	1	2	3
M2	2	2	2	2
Sp	3	1	3	2
N1	1	3	3	3
KK	3	2	3	2

Puanlama şu şekildedir:

3 puan = Mükemmel

2 puan = İyi

1 puan = Vasat

"Ca" marka arabanın üreticisi Toplam Puanı veren kuralın adil olmadığını öne sürüyor.

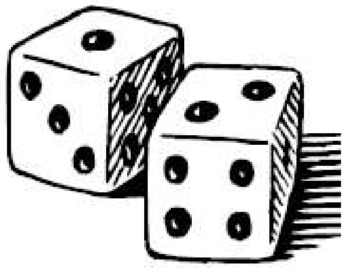
Sadece "Ca" marka arabanın kazanan olmasını sağlayacak bir kural yazınız. Yazdığınız kural dört değişkeni de içermelidir. Değişken katsayıları pozitif tam sayı olmalıdır.

Toplam Puan = ………× G + ………× Y + ………× D + ………× I.

Problem 4

Sağda, iki zarın resmi verilmiştir. Zarlara ait kural şu şekilde verilmiştir:

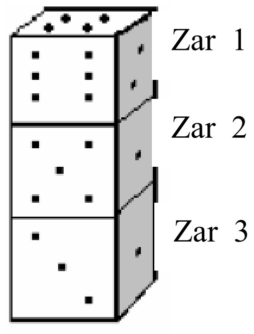
Zarın zıt yüzlerindeki toplam nokta sayısı 7'dir.



Sağda üst üste konmuş üç adet zar görülmektedir.

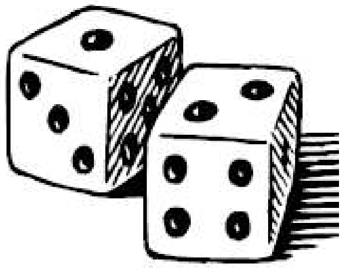
Zar 1'in üstünde dört nokta vardır.

Göremediğimiz 5 dikey yüzdeki toplam nokta sayısı nedir (Zar 1'in altı, Zar 2'nin üstü ve altı, Zar 3'ün üstü ve altı)?



Toplam :............................

Problem 5

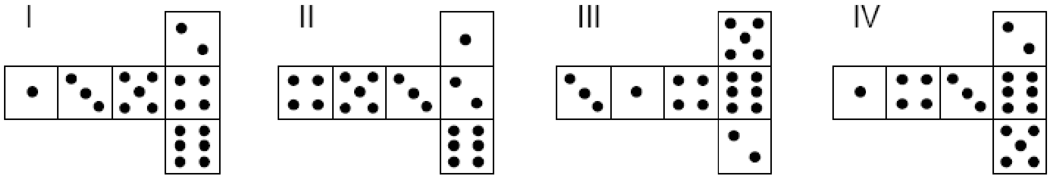


Sağda, iki zarın resmi verilmiştir. Zarlara ait kural şu şekilde verilmiştir:

Zarın zıt yüzlerindeki toplam nokta sayısı her zaman 7'dir.

Siz de kartonu kesip, katlayıp, yapıştırarak sayı küpü hazırlayabilirsiniz.

Bu birçok yolla yapılabilir. Aşağıda dört sayı küpünün açık hallerini ve yüzlerindeki noktaları görebilirsiniz.



Hangi şekiller katlanınca zıt yüzlerindeki noktaların toplam sayısı 7 olma kuralını sağlar? Her şekil için tabloda "Evet" ya da "Hayır"'ı işaretleyin.

Şekil	Zıt yüzlerindeki noktaların toplam sayısı 7 olma kuralını sağlar?
I	Evet / Hayır
II	Evet / Hayır
III	Evet / Hayır
IV	Evet / Hayır

Problem 6

Bir kitap rafı yapabilmek için marangoz şu malzemeleri kullanmaktadır:

4 uzun tahta panel,
6 kısa tahta panel,
12 küçük uç,
2 büyük uç ve
14 vida

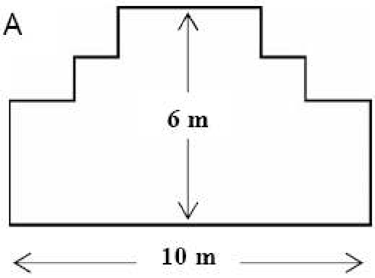
Marangozun elinde 26 uzun tahta panel, 33 kısa tahta panel, 200 küçük uç, 20 büyük uç ve 510 vida vardır.
Bu marangoz elindeki malzeme ile kaç tane kitap rafı yapabilir?

Cevap: ....

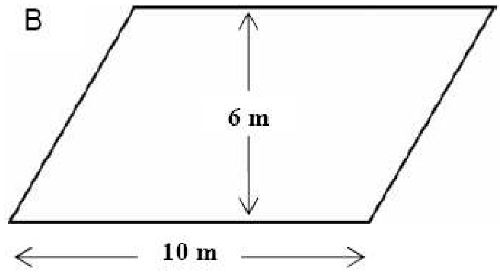
Problem 7

Bir marangoz 32 metre kereste kullanarak bir bahçe yatağının etrafını çevreleyecektir. Düşündüğü tasarımlar aşağıdaki gibidir:

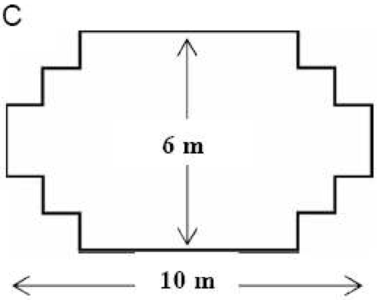
A



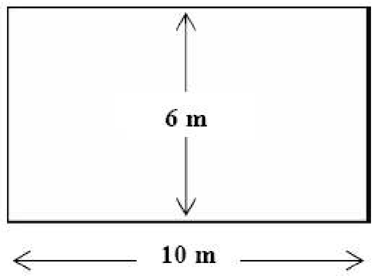
B



C



D

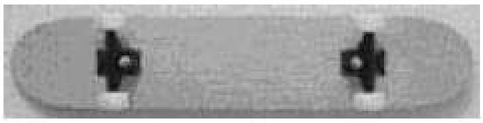
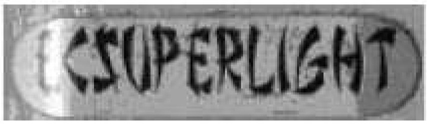
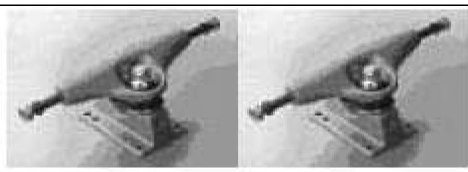
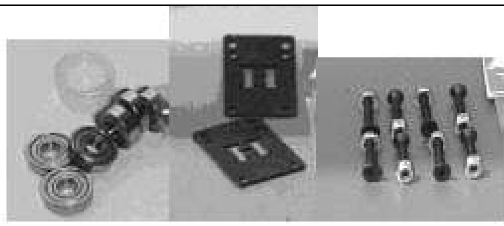


Her bir tasarım için "Evet" veya "Hayır"ı daire içine alın.

Bahçe Yatağı Tasarımı	Bu tasarım ile tam 32 metre tahta kullanılarak bahçe yatağı yapılabilir mi?
Tasarım A	Evet / Hayır
Tasarım B	Evet / Hayır
Tasarım C	Evet / Hayır
Tasarım D	Evet / Hayır

Problem 8 - 9 - 10

Erdem kaykay meraklısıdır. KAYKAYCILAR mağazasını ziyaret ederek fiyatları kontrol ediyor. Bu mağazada hazır kaykay alınabiliyor. Yada isterseniz bir gövde, dört tekerlek, iki dingil ve bir takım hırdavat paketi kullanarak kendi kaykayınızı oluşturabilirsiniz. Ürün fiyatları şöyle:

Ürün	Fiyatlar (PAR cinsinden)	
Hazır kaykay	82 veya 84	
Gövde	40, 60 veya 65	
4 adet tekerlek	14 veya 36	
2 dingil	16	
Gerekli hırdavat paketi (Somun, vida ve rulmanlar)	10 veya 20	

PAR hayali bir para birimidir.

Problem 8

Erdem kendi kaykayını oluşturmak istiyor. En düşük ve en yüksek maliyet ne olur?

(a) En düşük maliyet: .... PAR.

(b) En yüksek maliyet: .... PAR.

Problem 9

Mağaza üç farklı gövde, iki farklı tip tekerlek seti ve iki farklı hırdavat seti öneiyor. Dingil için tek bir öneri var. Erdem kaç farklı kaykay oluşturabilir?

A) 6

B) 8

C)10

D)12

Problem 10

Erdem'in 120 PAR parası var ve toplayabileceği en pahalı kaykayı oluşturmak istiyor. Her bir parça için ne kadar PAR harcayabilir. Cevaplarınızı tabloya yazın.

Parça	Fiyatı (PAR cinsinden)
Gövde	
Tekerlekler	
Dingiller	
Hırdavat	

APPENDIX F

FOLLOW-UP QUESTIONS USED BY EXPERIENCED INTERVIEWERS

Hunting, (1997, pp. 5-6) suggests some general forms of follow-up question used by experienced interviewers:

- ***Can you tell me what you are thinking?***

This question is useful after about 10 seconds of silence where it is not certain that productive mental activity is taking place.

- ***Can you say out loud what you are doing?***

When a student seems to be engaged in thought, after giving a short time, the interviewer may interrupt. Indicators of activity include inaudible utterances, scratch work on paper, motor activity such as tapping, eye and other body movements.

- ***Can you tell me how you worked that out? How did you know? How did you decide?***

A student may respond with an answer to a problem without any apparent clue as to the way the answer was obtained. These questions are intended to convey to the student that you are interested in how the result was determined. As such it is designed to encourage a verbal explanation.

- ***Was that just a lucky guess?***

If the student makes a response but does not give an explanation, then this question often has the effect of putting the student at ease and relieving tension. Sometimes in an effort to obtain information students will respond with the first thing that comes into their head. Students are generally happy to admit guessing.

- ***The other day another student told me…***

If there are grounds for supposing that the student isn't confident about the solution offered, or the interviewer wants to test the strength of a conviction, an alternative solution from a neutral and anonymous third party may be proposed for consideration. The advantage of attributing the alternative solution to a third party is that the student could feel it in his or her best interests to agree with a view emanating from the interviewer, just because the source of that view has power and status in the situation.

- ***Do you know what _ means?***

Success on a task may depend on knowledge of a particular term used in presentation of the problem. Potentially problematic vocabulary can be nullified by clarifying the meaning of the term. Teachers being teachers have an uncontrollable urge to teach. Should a teacher explain a point during an interview? The answer to this question rests on whether the teacher primarily intends to assess the status of the student's mathematical knowledge. It is not wrong to provide a student information. In fact, there are benefits in seeing how far the student is able to progress on the basis of some assistance. It may be that the information provided allows the student to incorporate other knowledge previously untapped. It is worth bearing in mind that the interview itself is a learning experience for the student. The extent to which the teacher digresses into a didactic frame during an interview will dictate how much progress will be made through the interview given the time available. We generally discourage teachers from digressing during formal training.

- ***Do you know a way to check whether you are right?***

Problem solutions, particularly those involving basic arithmetic operations, can be checked by means of estimation, rounding, or the appropriate inverse operation. Encouraging checking provides another window into a student's depth of understanding.

- ***Why?***

In response to an explanation a student may make an assertion. Asking why is a sensible way of encouraging further explanation.

- ***Pretend you are the teacher. Could you explain what you think to a younger child? How would you explain?***

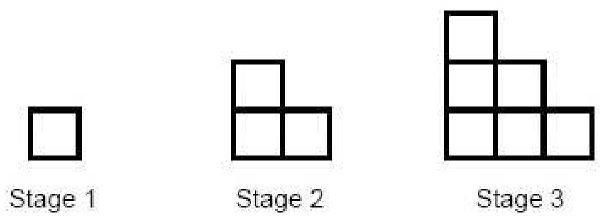
Encouraging children to formulate viewpoints or design settings for younger children provides an opportunity to capture their understanding of a situation or problem.

APPENDIX G

PROBLEMS AND CLINICAL INTERVIEW TRANSCRIPTIONS IN ENGLISH

Problem 1: Step Pattern

Robert builds a step pattern using squares. Here are the stages he follows.



As you can see, he uses one square for Stage 1, three squares for Stage 2 and six for Stage 3.

How many squares should he use for the fourth stage?

Table 16. Level and Percentages of Correct Answers in PISA 2003 for Problem 1

Problem No	PISA 2003 Level*	Correct PISA 2003 Total	Correct PISA 2003 Turkey
1	Level 2	64.58%	46.96%

*Level is determined by the researcher according to the percentages of correct answers in total PISA 2003 as no information has been published about the levels of these items in the OECD reports.

This problem is chosen as first to be a warm up for the children; and a warm up for the researcher to start to know the participants. The students don't need any further mathematics knowledge. The pattern can be found using the shapes or using the number of the squares in each shape. The question asks the fourth stage, so they don't need to generalize the pattern.

STEP PATTERN SCORING

Full Credit

1 point : 10.

No Credit

0 point : Other responses or Missing answer.

Table 17. Scores of participants in Problem 1

Participant	Aylin	Bora	Ceylan	Deniz	Emel
Score	1	1	1	1	0

Participant's Work

Aylin

[She read the question and thought about it.]

A: The pattern goes like 1, 2, 3... The answer should be 10. ***[The answer is correct]***

R: Did you see any similar questions before?

A: Yes, I have seen some.

R: Can you explain your solution, please.

[I asked this to see if she is sure about her answer.]

A: The squares that are added are increasing each time... In Shape 3, three squares are added. In shape 4, if we add 4, the result will be 10. ***[The explanation is correct.]***

R: Ok. Can you note it down? ***[She wrote the explanation on paper.]***

R: Did you think an alternative way?

A: I think there was a formula... n-1 something. I forgot it now.

[She saw that the pattern is summation]

Bora

[He read the question.]

B: Hmm, I can draw this. It may be easier.

[He tried to draw quickly, and counted the squares.]

B: 9. I can also do it with an alternative way.

R: Can you explain how you found it.

B: It starts from this corner. Then it is designed with a space on top. Then if we continue similarly, we add 4, and it is.

[He understood his mistake. He was surprised.]

R: You have said 9 before, Think again...

B: Hmm, just a minute... ***[He examined his drawing and he drew again.]***

R: It is OK, now. ***[He changed his answer as 10.]***

B: The alternative can be like this: 1, 3, 6... 2 added, 3 added, 4 added.

R: Have you seen this problem before.

B: No I haven't

Ceylan

[She examined the problem for a while. Then she wrote 7.]

[Then she drew a figure. The figure was incorrect. She counted the squares. Then she looked at the figures given in the problem. She crossed out 7 and wrote 10.]

R: Can you explain your solution?

C: for example, they have added 2 here. And they have added 3. So I put 4 here and the result is 10. ***[The answer and explanation is correct.]***

Deniz

[She read the problem and examined the figures.]

D: 1, 3, 6... 3, 3… Did they add 3? It is 9.

R: Can you explain a little bit more?

D: No it's not 9. 3, 6... 1 is added, 2 is added... 10 is the answer.

[That is correct.]

R: Can you explain why?

D: Here 2 is added, here 3is added, here 4 will be added and 10 is the result.

[The explanation is correct.]

Emel

[She examined the question and wrote the solution.]

E: I think there are different patterns for steps. Of course there is a pattern but... In first step there is 1; in third step it is twice, that is 6. Then there should have been 4 squares in second step.

E: Maybe it is funny but I think for this step, it is 1 more than the twice. Then it is twice in third step. Then it will be 1 more than the twice in fourth step. So I found 9.

[Her pattern suggestion doesn't satisfy the shapes and numbers given in the problem.]

R: Can you find another pattern?

E: Of course there is one...

R: A pattern for both odd and even numbers...

E: I thought that the rule is increasing…

E: 1 3 6... I think there is another pattern but I can't find... ***[She tried a new pattern but it is also not correct. She quit.]***

Problem 2: The Best Car 1

A car magazine uses a rating system to evaluate new cars, and gives the award of "The Car of the Year" to the car with the highest total score. Five new cars are being evaluated, and their ratings are shown in the table.

Car	Safety Features (S)	Fuel Efficiency (F)	External Appearance (E)	Internal Fittings (T)
Ca	3	1	2	3
M2	2	2	2	2
Sp	3	1	3	2
N1	1	3	3	3
KK	3	2	3	2

The ratings are interpreted as follows:

3 points = Excellent
2 points = Good
1 point = Fair

To calculate the total score for a car, the car magazine uses the following rule, which is a weighted sum of the individual score points:

Total Score = (3 x S) + F + E + T

Calculate the total score for Car "Ca". Write your answer in the space below.

Total score for "Ca": ....

Table 18. Level and Percentages of Correct Answers in PISA 2003 for Problem 2

Problem No	PISA 2003 Level*	Correct PISA 2003 Total	Correct PISA 2003 Turkey
2	Level 2	72.73%	61,47%

*Level is determined by the researcher according to the percentages of correct answers in total PISA 2003 as no information has been published about the levels of these items in the OECD reports.

Problem 2 is a warm-up for Problem 3. The problem requires the knowledge of coefficients and variables which is a prerequisite to solve the problem 3. The correct answer is 15.

THE BEST CAR SCORING
Full Credit
1 point: 15.
No Credit
0 point: Other responses or missing answer

Table 19. Scores of participants in Problem 2

Participant	Aylin	Bora	Ceylan	Deniz	Emel
Score	1	1	0	1	1

Participant's Work

Note: The variables used by the participant in the problem are translated to English ones.

Aylin

[She examined the problem.]

A: All right. Shall I solve it?

R: How did you solve it? What did you think?

A: I will put the values in the formula.

R: Ok. ***[She wrote the solution. That's correct.]***

A: It's 15.

Bora

[He examined the problem. He wrote the answer. The answer is correct.]

R: Can you explain what you did?

B: In the computation of the total score, it says we have to multiply "Safety" by 3, and the sum of the others... So I took their sum... And this multiplied by 3 is 9. The total is 15.

R: Can you please write it down? ***[He wrote down the solution. It's correct.]***

Ceylan

[She examined the problem. She wrote "3S + 1 +2 +3 =9"]

C: Is it 9?

R: Can you show your solution? ***[She wrote "(3xS) + 1 +2 +3 =9"]***

C: That's it.

R: Can you explain it? What did you think?

C: Fuel; Internal, External...When you add them, it is 6. Since this is 3, total is 9.

[The answer is wrong. She started to read the next problem. After she solved it, I asked her if she was sure for both questions.]

S: Do you want to check both questions?

C: I didn't understand (3xS)... But I am sure for the third question.

Deniz

[She examined the problem. She wrote "3.3 = 9 + 1 +2 +3 =15". She wrote "15" as answer. The answer is correct.]

R: Can you explain this?

D: I put these in the formula...

Emel

[She examined the problem. She wrote "3x3 + 1 +2 + 3 = 15" She circled "15"]

R: Can you explain it?

E: At first I didn't understand but... the individual score points... here it says S, it means security... It's the abbreviation. Her it says S. 3 times 3, then fuel is 1, external appearance is 2, internal appearance is 3. I found 15 using the sum.

Problem 3: The Best Car 2

A car magazine uses a rating system to evaluate new cars, and gives the award of "The Car of the Year" to the car with the highest total score. Five new cars are being evaluated, and their ratings are shown in the table.

Car	Safety Features (S)	Fuel Efficiency (F)	External Appearance (E)	Internal Fittings (T)
Ca	3	1	2	3
M2	2	2	2	2
Sp	3	1	3	2
N1	1	3	3	3
KK	3	2	3	2

The ratings are interpreted as follows:

3 points = Excellent
2 points = Good
1 point = Fair

The manufacturer of car "Ca" thought the rule for the total score was unfair.

Write down a rule for calculating the total score so that Car "Ca" will be the winner.

Your rule should include all four of the variables, and you should write down your rule by filling in positive numbers in the four spaces in the equation below.

Total score = ...× S + ...× F + ...× E + ...× T.

Table 20. Level and Percentages of Correct Answers in PISA 2003 for Problem 3

Problem No	PISA 2003 Level*	Correct PISA 2003 Total	Correct PISA 2003 Turkey
3	Level 6	24.90%	20,21%

*Level is determined by the researcher according to the percentages of correct answers in total PISA 2003 as no information has been published about the levels of these items in the OECD reports.

This problem not only requires the knowledge of coefficients and variables as in Problem 2 but also “Logical Reasoning” and/or “Intelligent Guessing & Testing” strategies to be solved. The correct rule that will make “Ca” the winner is counted as correct answer.

THE BEST CAR 2 SCORING
Full Credit
1 point: Correct rule that will make “Ca” the winner.
No Credit
0 point: Other responses or missing answer

Table 21. Scores of participants in Problem 3

Participant	Aylin	Bora	Ceylan	Deniz	Emel
Score	1	1	0	0	0

Participant’s Work

Note: The variables used by the participant in the problem are translated to English ones.

Aylin

[She examined the problem. She wrote 2 for S and T. She wrote 4 for F and 6 for E]

A: This is it. I think so.

R: Can you explain? ***[The answer is not correct.]***

A: First I looked at S values. When we multiply S values by 2, none of them is bigger than Ca’s. Ca’s is bigger or equal to. 2 is enough for here. Here (E), I can’t write 2 or 3, 5 is enough but I wrote 6. Here I can’t write 3, when we multiply 3 by 3, the others are bigger. So I wrote F.

[Silence for few seconds.]

R: Ok...Please, continue...

A: But the others are increasing too... ***[She understood her mistake.]***

R: Ok. If you restart the problem, what will you do? ***[She thought a little.]***

R: You can write what you are thinking...

[She started to write the previous explanation. Then she stopped and reread the problem.]

A: F and E will always be smaller than others'.

R: Ok...

A: Then I need to raise the Security S and the Internal I.

[She checked her first answer. She understood that it was not correct.]

R: Leave the first answer. You can restart by drawing a line.

[She wrote _S _F_E _I.]

A: If I call the coefficient of F as x, I will have a loss of xF.

[She wrote x under F and –xF.]

A: Internal, External... Here (E) also has the loss. ***[She wrote -2xy.]***

A: I can do it, but it's hard to explain.

R: Don't think about explaining to me. You can try anything. You have enough space on paper.

A: Yes I need to do some trials...

R: What do you have in mind?

A: These are lowering, these are increasing the total. So I must find the suitable coefficients...

R: Can you write one?

A: For example, I write 1 for these (F &E), so they are less.

R: What about the other coefficients? ***[She did some trials on the table.]***

A: I say 2 for this. ***[She tried 3S+1F+1E+2I for each car.]***

A: Yes ***[The answer is correct.]***

Bora

[He read the problem.]

B: I had calculated the total score of Ca in the first problem. Now I have to calculate the others to see if it is best or worst.

[He calculated the total score for all of them.]

B: So KK is the best. Then I need a new way. To make Ca the best, I have to multiply its best scores by bigger numbers.

[He showed the first column.]

B: KK is the best. From here, it can be this or Sp. I have to multiply S and I by the bigger coefficients. And multiply these ones by smallest coefficients. Especially E.

B: I can multiply the first one by 4 but I don't need to, because the others are the same. I can multiply this one (I) by 3. I will multiply this (F) by one, and this (E) by one. I can say 2 for the other one (S)...

[He checked the total scores of Ca, Sp and KK in mind.]

B: Yes, I think it's correct. Maybe I can check the other cars but this is true.

R: what is your final answer?

[He checked the others.]

B: This satisfies. Yeah, this is the answer. ***[The answer is correct.]***

Ceylan

[She read the problem. She wrote 3 for S, 1 for F, 2 for E and 3 for I.]

R: Can you explain why you decided on these numbers?

C: Because G is 3, Y is 1, D is 2, I is 2, No it is 3... ***[She paused.]***

R: So you say if we put the values in the formula, it will be done.

C: Yes.

[The answer is not correct. She hasn't checked the answer. Since she couldn't understand the problem, the researcher asked her whether she was sure for both questions.]

R: Do you want to check both questions?

C: I didn't understand (3xS)... But I am sure for the second question.

Deniz

[She read the problem.]

D: I will write the suitable numbers so that this will be the best.

D: Then I have to calculate all of them.

[She is not so happy about it. She calculated all quickly in the first paper.]

D: Will this be the best?

R: Yes.

D: I take this (S) 3, this (I) is 3, this (F) is 1, and this (E) is 1.

R: Why are these numbers?

D: To make it bigger than 16.

R: Is it your final answer?

D: It is 17 now. Wait I need to check the others.

D: I have to redo it then… ***[She checked the table.]***

D: If first one is 4. ***[She calculated 4S for each car.]***

D: 1 F… ***[She calculated 1F for each car.]***

D: And this (I) is 2.

[She calculated 2I for each car. She had miscalculations.]

[She checked the table. She calculated the sums without D, then she calculated them by adding 1D, but she forgot to add D in last 3 cars.]

D: 4 for S, 1 for F, 1 for E, 2 for I. ***[The answer is not correct.]***

R: Can you explain this?

D: This numbers satisfy the rule.

R: Can you explain your solution?

D: I tried values that make Ca to be bigger.

Emel

[She read the problem.]

E: Ca brand thinks that the rule is not fair. Find a rule such that only Ca is the best.

E: Then we have to calculate all of them. ***[She calculated them all, correctly.]***

E: We will something and Ca will win. The total score is the sum. Am I right?

[Silence for a few seconds]

R: A rule that makes Ca the only best car in total score.

E: I see. I make all these 3 then.

[She calculated the Ca total if all values are 3.]

E: Then this is 18, and all of them are less.

R: How?

E: If we change these all to 3 here (table given in the problem), it is ok. Right? Or can't we?

[She tried to change values in the table.]

R: You don't have a right to change the table. You can change the coefficients.

E: I see then I give the coefficients the bigger values. Is there any obligation for this?

R: Any rule that makes Ca the only winner is Ok.

E: I can give 5 for this and 10 for this one.

[She wrote 10G + 5Y + 7D + 6I. She calculated Ca as 67.]

E: I found this.

R: Can you explain it?

E: We want to make Ca the only winner but we can't change the table. So I changed the coefficients.

R: Ok

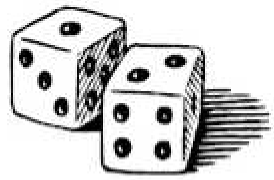
E: I think this is true.

Problem 4: Number Cubes 1

On the right, there is a picture of two dice.

Dice are special number cubes for which the following rule applies:

> The total number of dots on two opposite faces is always seven.



On the right you see three dice stacked on top of each other. Die 1 has four dots on top.

How many dots are there **in total** on the five horizontal sides that you cannot see (bottom of die 1, top and bottom of die 2 and die 3)?

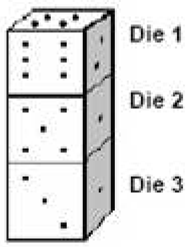


Table 22. Level and Percentages of Correct Answers in PISA 2003 for Problem 4

Problem No	PISA 2003 Level	Correct PISA 2003 Total	Correct PISA 2003 Turkey
4	Not Available	Not Available	Not Available

This problem has been chosen for comparing the strategies for Problem 5. The problem requires spatial visualization. “Logical reasoning” and/or “accounting for all possibilities” strategies can be used to solve the problem. The correct answer is 17.

NUMBER CUBES 1 SCORING
Full Credit
1 point: 17.
No Credit
0 point: Other responses or missing answer

Table 23. Scores of participants in Problem 4

Participant	Aylin	Bora	Ceylan	Deniz	Emel
Score	1	1	0	1	1

Participant's Work

Aylin

[She read the question.]

A: We will count these and we will not count these, that is what I understood. So ...

[She wrote 14 x 3 = 42. She tried to guess the missing points on each side of the dice. She reread the problem. She overlined the previous solution.]

R: Can you explain your thoughts?

A: I have read the problem incorrectly. I tried to find the sum of the faces that are seen. I found the sum of the faces except the ones in the middle. I understood the problem in my second reading...

[She tried to find the horizontal faces by eliminating the front faces. She correctly found them for the second dice. Then suddenly she wrote 3+7+7 = 17]

A: 17.

R: Can you explain?

A: It is already given in the problem. Sum of two opposite faces is 7. Then sum of ups and down is 7. I will add two opposite faces twice. It is 14. Opposite of 4 is 3. 14 plus 3 is 17.

[Her answer and solution are correct.]

Bora

[He read the problem. He reread it loudly.]

B: The sum of opposite faces is 7. For example opposite of 1 is 6. Hmm... It is asked the sum of the 5 horizontal faces. 7 plus 7… the opposite of 4 is 3. The sum is 17.

[He wrote 17 as answer. It is correct.]

R: Can you write down the solution.

B: Ok.

Ceylan

[She read the problem .She draw the figure. But she didn't put all the spots on the dice.]

R: Can you explain you thinking?

C: I try to find it in my mind...

R: What is your strategy? How do you plan to solve this problem?

R: Let me ask this: Can you say what is asked in the problem?

C: Bottom of Die 1, top and bottom of Die 2, top and bottom of Die 3...

R: What do you know?

C: Bottom of Die 3 is 5. ***[It can be 2 or 5.]***

S: Please, note down your decisions.

[She wrote "Bottom of Die 3 is 5".] [She wrote top of Die 3 is 5. Then she changed it to 4. It is incorrect.] [She wrote bottom of Die2 is 4. It can be 4 or 3.] [She wrote top of Die 2 is 6. It is not correct.] [She wrote bottom of Die 1 is 5. It is not correct.]

R: What is your answer?

C: 24. ***[She wrote 24 as answer. The answer is incorrect.]***

Deniz

[She examined the problem.]

D: Opposite faces?

R: When you hold a die like this, you touch the opposite faces.

[She examined the dice. She wrote 1-3-5 near the first die.]

D: Which sum is 7?

R: Each time you hold a die from the opposite faces, the sum of the points on them is 7.

D: Then if here is 6 the next face is 1, right?

R: Not the next, the opposite face...

D: How can it be?

R: Think of the poles of the earth as opposite...

D: I got it now. Then it is 3 ...

R: Can you tell me what is given and what is asked in the problem?

D: The sum of the five horizontal faces...

D: Here is 3. Here is 4...

[She wrote 3 for bottom of Die 1.] [She wrote 3 for Top of Die 2, and 4 for bottom of Die 2. In fact they may be converse. But she chose the correct pair] [She wrote 4 for Top of Die 3, and 3 for bottom of Die 3. This is impossible because the front face is 3.She chose an incorrect pair. But their sum is 7 at least.]

D: That is it. No I need the sum, the answer is 17.

Emel

[She read the problem.]

E: Which sum is 7?

R: Each time you hold a die from the opposite faces, the sum of the points on them is 7.

E: Then this is 3. Since the top is 4 and the sum is 7, bottom is 3.

[She wrote 3 for bottom of Die 1.]

E: I will ask something. Are the faces of Die 1 and Die 2 related?

R: The rule is given for each die.

E: Then I can’t say that top of Die 2 is 4.

R: What is asked in the problem?

E: The sum of the points in the 5 horizontal faces...

R: You found the bottom of Die 1.

E: Yes.

R: What about Die 2?

E: The front is 5, then the back is 2. Here is 1 then the back is 6. Then 3 and 4 are left for the top and bottom. Let me write 4 for top and 3 for bottom.

[She wrote 4 for Top of Die 2, and 3 for bottom of Die 2. In fact, we can’t know which is top or bottom but she chose the correct pair.]

E: In Die 3, here is 1 then the opposite is 6. Here is 3 then the opposite is 4. 2 and 5 are left.

R: Yes.

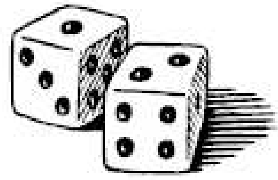
E: Which one is 2 and which one is 5? ***[Silence for a few seconds.]***

E: It doesn’t matter. The sum is 7 anyway.

E: 3, 4 : 7. 7, 3 : 10. 10, 7 :17. The sum is 17.

Problem 5: Number Cubes 2

On the right, there is a picture of two dice.

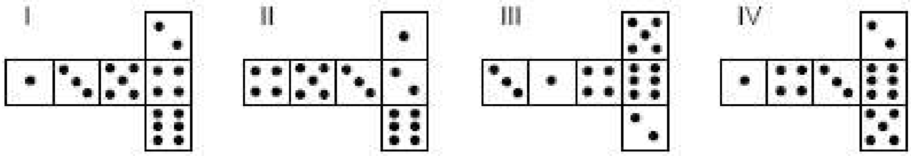


Dice are special number cubes for which the following rule applies:

> The total number of dots on two opposite faces is always seven.

You can make a simple number cube by cutting, folding and gluing cardboard. This can be done in many ways. In the figure below you can see four cuttings that can be used to make cubes, with dots on the sides.

Which of the following shapes can be folded together to form a cube that obeys the rule that the sum of opposite faces is 7? For each shape, circle either "Yes" or "No" in the table below.



Shape	Obeys the rule that the sum of opposite faces is 7?
I	Yes / No
II	Yes / No
III	Yes / No
IV	Yes / No

Table 24. Level and Percentages of Correct Answers in PISA 2003 for Problem 5

Problem No	PISA 2003 Level	Correct PISA 2003 Total	Correct PISA 2003 Turkey
5	Level 3	60.46%	40.98%

The problem requires spatial visualization. “Logical reasoning” and “Accounting for all possibilities” strategies can be used to solve the problem. The correct answer is “No, Yes, Yes, No”.

NUMBER CUBES 2 SCORING
Full Credit
1 point: No, Yes, Yes, No, in that order.
Partial Credit:
0.75 points: Exactly three correct.
No Credit
0 point: Other responses or missing answer

Table 25. Scores of participants in Problem 5

Participant	Aylin	Bora	Ceylan	Deniz	Emel
Score	1	1	1	0.75	1

Participant’s Work

Aylin

[She read the question.] [She circled II and III.]

A: I think these are the true answers, but is there a trick in the question?

R: Can you explain what you think?

A: If you fold this, 4 and 3, 5 and 2, 6 and 1 will be one on the opposite of each other. And this one, too.

A: These ones don’t obey the rule, but I thought whether there was any other way to fold them.

R: Is there any other way?

A: I don’t think so. ***[She marked for I and IV, YES for II and III.]***

B: Can you note it down your reasons? ***[The explanation is correct.]***

Bora

[He read the question.]

B: This one doesn't obey. ***[He scratched out I.]***

B: This one obeys. ***[He circled II.]***

B: This one obeys, too. ***[He circled III.]***

B: This one doesn't obey. ***[He scratched out IV.]***

[He marked the answers. All of them are correct.]

R: How did you do? Can you explain little?

B: Well, if you fold it, there must be a blank.

B: When these ones (6, 2) opposed, it makes 8. This one (I) doesn't obey.

B: Here (II) 7, 7, 7. It obeys.

B: Here (III) 7, 7, 7, too. It obeys.

B: Here (IV), there is one 7 but others don't obey.

Ceylan

[She read the question.] [She wrote I II III IV.]

C: I can't understand what the opposite sides mean.

R: If you hold the dice, you hold it from the opposite sides.

C: I see...

[She marked NO for I, YES for II and III, and NO for IV. All are correct.]

R: Can you explain shortly?

C: If you fold this (III), that one and that one became opposite. In II, it obeys, too. In I, it doesn't, in IV it doesn't either.

Deniz

[She read the question.]

[She marked YES for I, II, III and NO for IV. Three of them are correct, first one is incorrect.]

R: Can you explain shortly?

D: These are opposites. 4 and 3 makes 7.

[She only checked the rule for a pair in Dice I. So her answer is incorrect.]

D: Here, it makes 7, too and here too, but here not.

[She only checked the rule for two pairs in Dice IV. So her answer is correct.]

Emel

E: The same rule again. ***[She remembered the previous problem]***

[She read the problem.]

E: This one obeys, but this one does not.

[In Dice I, she showed that 4 and 3 give 7, but 5 and 1 do not.]

E: Must all opposite faces obey the rule?

R: The rule is written in the problem..

E: This one (II) obeys. This one (III) doesn't. Sorry, it obeys; I had seen 5 as 6.

[She corrected it.]

E: This one doesn't obey.

R: Can you mark them in the table?

[She marked NO for I, YES for II and III, NO for IV. All are correct.]

Problem 6: Bookshelves

To complete one set of bookshelves a carpenter needs the following components:

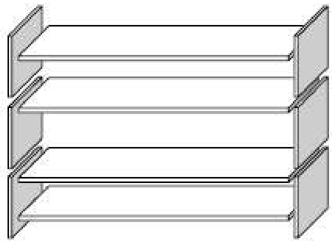
4 long wooden panels,

6 short wooden panels,

12 small clips,

2 large clips and

14 screws.



The carpenter has in stock 26 long wooden panels, 33 short wooden panels, 200 small clips, 20 large clips and 510 screws.

How many sets of bookshelves can the carpenter make?

Table 26. Level and Percentages of Correct Answers in PISA 2003 for Problem 6

Problem No	PISA 2003 Level*	Correct PISA 2003 Total	Correct PISA 2003 Turkey
6	Level 3	58.67%	36.56%

*Level is determined by the researcher according to the percentages of correct answers in total PISA 2003 as no information has been published about the levels of these items in the OECD reports.

The problem requires spatial visualization. “Organizing data” and/or “Logical Reasoning” strategies can be used to solve the problem. The correct answer is 5.

BOOKSHELVES SCORING
Full Credit
1 point: 5.
No Credit
0 point: Other responses or missing answer

Table 27. Scores of participants in Problem 6

Participant	Aylin	Bora	Ceylan	Deniz	Emel
Score	1	1	0	1	0

Participant's Work

Aylin

[She read the question. She was silent for a while.]

R: Can you explain what you think?

A: First, I look the multiples. I try to find which one that has the smallest multiple.

R: Usually, you like to solve the problem in your mind, don't you?

[She smiled. She wrote 6x and 5x. She divided 200 by 12 and wrote 16x. She divided 510 by 14 and wrote 36x.]

A: Only 5 can be done. ***[She wrote 5 as an answer.]***

A: Because, this one has the smallest multiple. If we want to make more than 5, we don't have enough short wooden panels, for example.

R: Can you write down your solution shortly. ***[Her explanation was correct.]***

Bora

[He read the question. He read it loudly. He tried to understand the problem. He was little confused because of the figure. But he read it again and understood.]

B: Now, I have 26, 33, 200, 20, 510 ... For one set, we need 4 long wooden panel ... I have 26 long panel ... Of course It can't be divided exactly. When I divided it by 4, 6 sets can be done.

B: When I divided 33 by 6, 5 sets can be done.

B: When I divided 200 by 12… ***[He couldn't do it in his mind, he calculated on paper.]***

B: 16.

B: 20 divided by 2, gives 10

B: 500 divided 14... ***[He couldn't do it in his mind, he calculated on paper.]***

B: 36.

B: This one restricts us. ***[He circled the 5.]***

B: One of them would restrict us. Here it is this one. I can make 5 sets with them. It doesn't matter that I have more from the rest. ***[He wrote 5 as an answer. Answer was correct.]***

Ceylan

[She read the question. She was silent for a while.]

R: Can you explain me the problem?

C: Here it is written what we need; here it is written what we have... It asks us, how many we can make.

R: How can you solve this problem?

C: I think like this: What is multiplied by 4 makes 26 gives us how many we can make that.

R: If you want, do it by writing.

[She wrote 4.6=24; 6.5 = 30; 12. ; 2.10=20; and 14.]

[She multiplied 24 by 12, and then scratched out it.]

[She tried to find 200 by adding 12s. After adding so many, she found 208. But she made mistakes in the calculations. She counted the 12s. When she found, she tried to multiply 12 and 17. Then she counted then again. She multiplied 12 and 19. She found 128. She subtracted 12 2 times from that. She found 104.]

R: Can you explain what you are doing?

C: Here, I added the 12s to find 200, but it couldn't. I can't do.

R: If you want, try to explain what you want to do next.

C: I want to find this (the multiple of 12). Then I will find this (the multiple of 14). I'll do this way.

R: If you find those numbers, can you tell the answer?

C: ... ***[She was silent for a while.]***

R: Let's say those numbers are 12 and 45. What would your answer be?

C: ... ***[She was silent for a while.]***

R: You're are little confused, aren't you ?

C: I can make 24 items.

Deniz

[She read the question.]

D: We have 26 items, so we can do it 6 sets, 4 items in each. For 33, we can do 5 sets. ***[She divided 200 by 12.]***

D: 12 sets can be done. 10 sets can be done.

[She divided 510 by 14 and found 36.] [She thought a little bit.]

R: You found these values. What is the value asked in the problem?

D: How many book sets can carpenter make with the material he has... 6 sets form here...

D: And then... ***[Silence for a while.]***

R: In your opinion, what is the answer? Can these values help?

D: ... ***[Silence for a while.]***

R: Well, let's talk about this. Can you tell me why you found these values ?

D: I found, how many he can make using each of them.

R: And you are stuck, there…

D: Do we need the sum? ***[Silence for a while.]***

R: If you want, you can quit, but we have enough time.

[She thought for a while.]

D: When he makes one set, here 5 remain, and here 4...

[She wrote 4, and then stopped.]

R: Can you explain what you tried?

D: Here, we have 6 sets. When he makes one, 5 remain. ***[Silence for a while.]***

R: So, what is next? ***[She thought for a while.]***

D: I see now, there are only 5 here. He can't make more than that.

R: Can you find the answer now?

D: It is 5. ***[The answer was correct.]***

Emel

[She read the question. She was silent for a while.]

R: Well, what does the problem say? What is given, what is asked?

E: Bookshelf materials are given. We have a lot of material. It asks us, how many sets can be done?

E: Well, let's sum all of the given. If we can make 1 console from that item, we can find how many sets we can make with items we have.

[She tried to use her ratio knowledge.]

E: if we make 1 set with 38 items… let's add of them, it makes 789. If 1 for 38, what is for 789?

[She divided 789 by 38, since it was not divisible, she thought for a while.]

E: It doesn't. I remember some kind of questions, when you divide, you can find the answer.

R: Well, what did you try? You divide it, since it is not divisible, do you say this is not the solution?

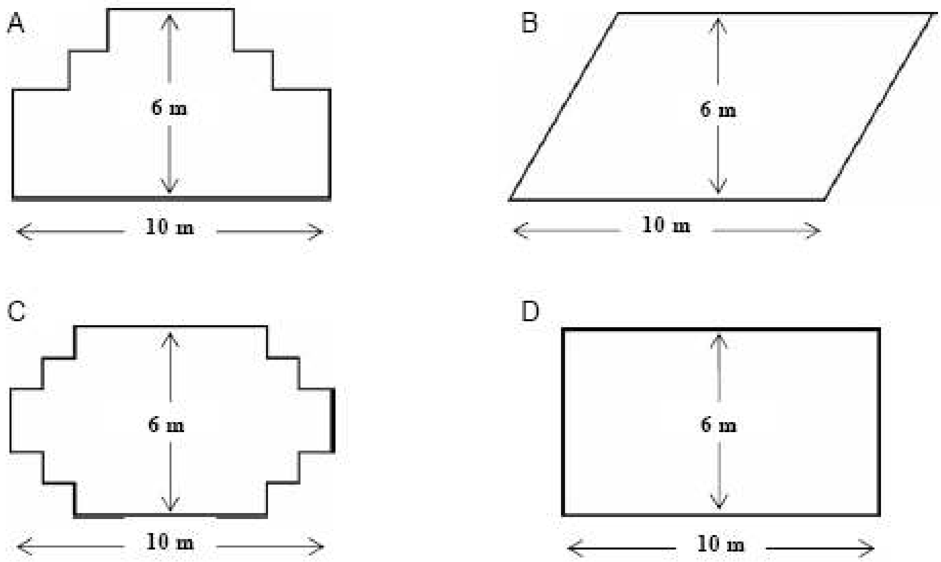
E: Yes. I don't know. This question is too hard for me...

R: You can think for a while, if you want.

E: I can try something like that. I can sum the panels and the others, separately. For example we need 4 long 6 short 10 panels. Here we need, 33 and 26, total 49 panel ... No, 49 is not divisible by 10... ***[She quit the problem.]***

Problem 7: Carpenter

A carpenter has 32 metres of timber and wants to make a border around a garden bed. He is considering the following designs for the garden bed.



Circle either "Yes" or "No" for each design to indicate whether the garden bed can be made with 32 metres of timber.

Garden bed design	**Using this design, can the garden bed be made with 32 metres of timber?**
Design A	Yes / No
Design B	Yes / No
Design C	Yes / No
Design D	Yes / No

Table 28. Level and Percentages of Correct Answers in PISA 2003 for Problem 7

Problem No	PISA 2003 Level	Correct PISA 2003 Total	Correct PISA 2003 Turkey
7	Level 6	19.38%	11.64%

The problem requires spatial visualization. “Logical reasoning” and/or and/or “visual representation strategies” can be used. The correct answers are “Yes, No, Yes, Yes”, respectively. Finding all is given full credit; any three of four is given partial credit. All the others are given no credit for this problem.

CARPENTER SCORING 1
Full Credit
1 point: Exactly four correct
Design A Yes
Design B No
Design C Yes
Design D Yes
Partial Credit
0.75 point: Exactly three correct.
No Credit
0 point: Two or fewer correct or missing answer.

Table 29. Scores of participants in Problem 7

Participant	Aylin	Bora	Ceylan	Deniz	Emel
Score	1	1	0	0.75	0.75

Participant’s Work

Aylin

[She read the question and thought about it.]

[She wrote 32 behind the figure A. She circled the A.]

A: This is an oblique rectangle. That’s why, I guess we can set 6 this one.

[She thought for a while. She circled the C and D.]

A: These (C, D) are already true.

[She turned back to the figure B. She marked the edge which completes the shape to the rectangle. She thought.]

R: Can you explain what do you think?

A: It is possible but I can't prove it to myself.

R: Can you explain why it is possible?

A: We can calculate the area of a parallelogram by height. Here is 10 again... I wonder, is it possible to think it as a rectangle?

R: Can you tell me what is asked in the problem; mathematically?

A: The circumference…

[She thought for a while. She scratched out B.]

R: Can you explain why?

A: I see now. Here is the height; it is the hypotenuse so it can't be definitely same. The value must be more than 32.

R: Can you note it down to the solution part?

[Her explanation was correct. All answers were true.]

Bora

[He read the question and thought about it. He read it loudly.]

B: 32 meters to border around. Hmm, then it is the circumference.

[He completed the Figure A to a rectangle.]

B: Figure A is ok.

R: Can you explain why?

B: I can do it in two ways. Let me tell my way. I don't need to know these values. Their sum must be equal 10 like the opposite side. 10 plus 10 makes 20. We can say that the sum of these little edges is equal to 6. The area … sorry the circumference is same as the rectangle…. 2 times sum of the two edges, gives 32. Yeah, that is it.

B: But here (B), I can't do same. Since it is a parallelogram, so surely this edge is more than 6... 32 metres is not enough. So Figure B is not Ok. Figure C is the same as Figure A, it can be completed to a rectangle. This one (D) is already a rectangle. So it is OK, too.

[He marked the answers. All was true.]

R: Can you note down why you can't use Figure C?

[He wrote "The edge of the parallelogram must be bigger than 6 by Pythagoras' Theorem or the rule of being a triangle…".]

B: Let me show on the figure. This is the biggest angle so this edge will be bigger than 6.

Ceylan

[She read the problem and thought about it.]

[She marked YES for A, YES for B, YES for C. Then she changed YES to NO for C. She marked YES for D.]

R: Can you explain shortly?

C: Here (C) is not 10 meter. I think so.

R: Can you explain for all?

C: Here (A) and here (B) it is 10 meter to 6 meter, but here (C) it is not.

[Her explanation was wrong. Two of her answers were wrong.]

Deniz

[She read the question and thought about it.]

[She wrote the dimensions on the figures. On parallelogram, she wrote 10 to all edges. She marked NO for B and YES for D.]

D: Is it 10 meters here (C)?

R: The answer is in the problem...

[She guessed some values to short edges of Figure C. She wrote NO for C.]

[She guessed some values long edges of Figure A. She got bored. She marked YES for A...]

R: Can you explain it very shortly?

D: Here, I guessed some values according the givens. For all the figures, I calculated the sum.

R: The sum?

D: The circumferences. I'm sure for D, but I'm little confused about A.

[Some of the answers are not correct. She is not sure of her solution.]

Emel

[She read the question and thought about it. She wrote the dimensions on the figures. On parallelogram, she wrote 10 to other edge. She marked NO for B and YES for D.]

E: We can choose B.

R: Are you sure?

E: This edge is 10, height is 6, so the area is the same.

R: Can you explain me what the question asks?

E: We try to find that, whether we can make a garden bed with 32 meters of timber.

R: Well, for each, what do you need to check?

E: I check whether the circumference is 32. I see now, their sum is 32. We can make a rectangle with these. This (A) can be chosen. ***[She marked YES for A.]***

B: It is parallelogram. It can be chosen, too. ***[She marked YES for B.]***

R: Can you explain it?

E: I think these (sides) can be 6, too. ***[Her explanation and answer were wrong.]***

E: This can be completed to the rectangle.

[She guessed some values for the edges. It didn't fit. She gave new values. At last, she became satisfied.]

E: Or, let's cut these here ... This (D) is already rectangle … This must be done too.

[She wrote YES for D.]

R: What's your last decision for C?

E: I guess, this can be completed to a rectangle, too.

R: What's your answer then?

E: If we give values these, it can be completed to 6.

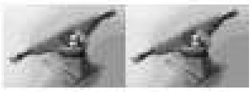
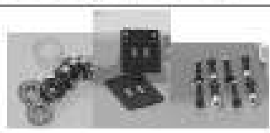
[She added the values. She became sure. She marked YES for C too.]

Problem 8: Skateboard 1

Eric is a great skateboard fan. He visits a shop named SKATERS to check some prices.

At this shop you can buy a complete board. Or you can buy a deck, a set of 4 wheels, a set of 2 trucks and a set of hardware, and assemble your own board.

The prices for the shop's products are:

Product	Price in zeds	
Complete skateboard	82 or 84	
Deck	40, 60 or 65	
One set of 4 Wheels	14 or 36	
One set of 2 Trucks	16	
One set of hardware (bearings, rubber pads, bolts and nuts)	10 or 20	

Eric wants to assemble his own skateboard. What is the minimum price and the maximum price in this shop for self-assembled skateboards?

(a) Minimum price: .... zeds.

(b) Maximum price: .... zeds.

Table 30. Level and Percentages of Correct Answers in PISA 2003 for Problem 8

Problem No	PISA 2003 Level	Correct PISA 2003 Total	Correct PISA 2003 Turkey
8	Level 3	60.41%	38.75%

"Logical Reasoning" strategy can be used to solve the problem. The correct answers are Minimum price: 80 and maximum price: 137.

SKATEBOARD 1 SCORING

Full Credit

1 point: Both the minimum (80) and the maximum (137) correct.

Partial Credit

0.5 point: Only the minimum (80) correct.

0.5 point: Only the maximum (137) correct.

No Credit

0 point: Other responses or missing answer.

Table 31. Scores of participants in Problem 8

Participant	Aylin	Bora	Ceylan	Deniz	Emel
Score	1	1	0.5	0	1

Participant's Work

Aylin

[She read the question.]

[She wrote 40 +14 +16 +10]

A: We choose the ones with the lowest prices.

[She wrote the answer for first part as 80. It is correct.]

[She wrote 65 + 36 + 16 + 20 = 52 + 85 = 137. She wrote 137 as an answer. She wrote the explanation.]

[Explanation is correct.]

Bora

[He read the question.]

B: Yes. The lowest and highest costs... He wants to create his own skate. He doesn't buy it. The body ... for minimum cost 40, 14, 16, 10... Total 80 is the minimum.

[He wrote down the answer. His answer is correct.]

B: Maximum 65, 36, 16, 20.

[He calculated the sum and wrote 137 as the answer.]

R: Can you note down your thoughts? ***[Explanation was correct.]***

Ceylan

[She read the problem. She was silent for a while.]

R: Did you understand what is given?

C: Why are there 3 prices here?

R: There are 3 different options for the deck.

[She wrote 40 +14 +16 +10 and wrote 80 as an answer for the lowest price.]

[She wrote 65 + 36 + 16 + 20 = 127, and wrote 127 for the highest price. She made a miscalculation.]

R: Can you explain shortly?

C: Here, I add the lowest ones, and here I add the highest ones. ***[Explanation was correct.]***

Deniz

[She read the question.]

[She added the lowest value in every row. She found 162 which was incorrect answer. Because she added the ready skateboard price. She made same mistake in the maximum cost and found 221.]

R: Can you explain? How did you do?

D: By summing the lowest and highest ones here.

[Her explanation was correct but, she found two incorrect answers.]

Emel

[She read the question and thought about it.]

E: For the minimum cost, we choose the ones with the lowest prices. For this one I choose 40. And this one 16, this one 10, minimum total cost is 80.

[She wrote 40+14+16+10=80. Her answer was correct.]

E: For the maximum cost, we must choose the ones with the lowest prices.

[She wrote 65+36+16+20 =137. Her answer was correct.]

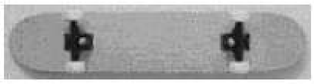
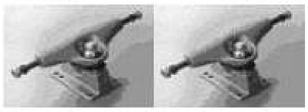
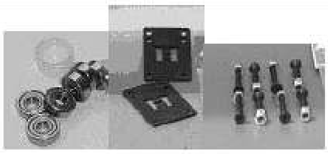
E: My explanation was weird but…

Problem 9: Skateboard 2

Eric is a great skateboard fan. He visits a shop named SKATERS to check some prices.

At this shop you can buy a complete board. Or you can buy a deck, a set of 4 wheels, a set of 2 trucks and a set of hardware, and assemble your own board.

The prices for the shop's products are:

Product	**Price in zeds**	
Complete skateboard	82 or 84	
Deck	40, 60 or 65	
One set of 4 Wheels	14 or 36	
One set of 2 Trucks	16	
One set of hardware (bearings, rubber pads, bolts and nuts)	10 or 20	

The shop offers three different decks, two different sets of wheels and two different sets of hardware. There is only one choice for a set of trucks.

How many different skateboards can Eric construct?

A 6
B 8
C 10
D 12

Table 32. Level and Percentages of Correct Answers in PISA 2003 for Problem 9

Problem No	PISA 2003 Level	Correct PISA 2003 Total	Correct PISA 2003 Turkey
9	Level 4	46.90%	27.98%

"Organizing data" and/or "Finding a pattern" and/or "logical reasoning" strategies can be used. The correct answer is 12.

SKATEBOARD 2 SCORING
Full Credit
1 point: D. 12.
No Credit
0 point: Other responses or missing answer.

Table 33. Scores of participants in Problem 9

Participant	Aylin	Bora	Ceylan	Deniz	Emel
Score	1	1	0	1	1

Participant's Work

Aylin

[She read the question.]

A: Answer is 12.

R: Can you explain why?

A: We have 3 here. We can choose among this 3. We create a table like this.

[She drew a table.]

A: If we have 2 types of wheels, we can among this 2. We have only one type of trucks, and 2 types of hardware. So when we multiply these, it gives 12.

R: Can you explain why you multiply those?

A: Because we have 3 body and 2 that. Each of these can be combined each o f those. When we multiply the numbers, the result gives the answer.

[Her explanation was correct.]

Bora

[He read the problem.][He wrote 3.2.3.1 = 12.]

B: 12...

R: Can you explain why?

B: We have 3 types of deck. I can choose one of the decks and choose the other parts. There are 2 types of wheels and 2 types of trucks… We can multiply all. It is permutation multiplication.

Ceylan

[She read the question.]

[She circled the option for 6.]

C: 6

R: Can you explain why?

C: Well... We have 3 different decks and 2 different wheels. As you see...

[Her answer was wrong. Her explanation was partially correct.]

Deniz

[She read the question, she wrote 3 decks, 2 wheels and 2 trucks below the first page.]

[She wrote 1 1 1 1... She tried to find the combinations. She circled the 12.]

R: Can you explain why?

D: In math, it is called something...

R: Name is not important.

D: Maybe, there is no name for it but ... I multiply all of these.

R: Why?

D: Because, the skateboard is made by using each of them.

R: So?

D: I solve it with that multiplication rule.

R: Can you explain the first thing (1 1 1 1) ?

D: It was nonsense. I tried to see what happens if there were one of each…

[She said that trial was nonsense but she remembered the rule by trying that.]

Emel

[She read the problem and thought about it. Below the first page, she wrote 3 decks, 2 wheels and 2 hardware.]

E: Let's try that: We have 3 types of decks. We can choose one of them…

[She wrote 40 60 65.]

E: Here let's choose 14. ***[She wrote 14 below each of 40 60 65.]***

E: Now, we have to do a triple thing, but I haven't understood yet.

E: Let's write 14 16 20 there. ***[She wrote 16 and 20 each of all.]***

E: Now, let's ... Each of these... ***[She circled 14, 16 and 20.]***

E: 9, the answer is 9. ***[She noticed that, 9 is not an option.]***

E: Well ... In fact, it is very simple question...

R: No problem, you have time... ***[If she was alone, she may have quit.]***

E: It is a very simple question but ... I can't do that.

R: Can you explain what you tried to do?

E: Now. There is only one axe ... and 2 this, 2 that and 3 body... Then, for each of these ... ***[She counted the possibilities...]***

E: Then... we can try that... These are 4. 4 times 3... I say 12.

R: Well, how did you find that?

E: I start with just one deck. ***[She counted the 4 combination for choosing the rest in loud.]***

E: For each, there is 4 different way. We have already 3 different decks. Total is 12.

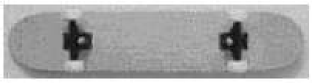
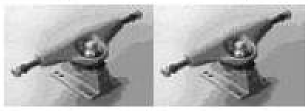
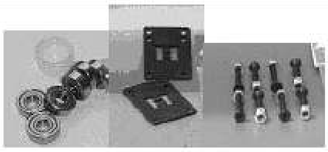
[Her answer was correct.]

Problem 10: Skateboard 3

Eric is a great skateboard fan. He visits a shop named SKATERS to check some prices.

At this shop you can buy a complete board. Or you can buy a deck, a set of 4 wheels, a set of 2 trucks and a set of hardware, and assemble your own board.

The prices for the shop's products are:

Product	Price in zeds	
Complete skateboard	82 or 84	
Deck	40, 60 or 65	
One set of 4 Wheels	14 or 36	
One set of 2 Trucks	16	
One set of hardware (bearings, rubber pads, bolts and nuts)	10 or 20	

Eric has 120 zeds to spend and wants to buy the most expensive skateboard he can afford.

How much money can Eric afford to spend on each of the 4 parts? Put your answer in the table below.

Part	Amount (zeds)
Deck	
Wheels	
Trucks	
Hardware	

Table 34. Level and Percentages of Correct Answers in PISA 2003 for Problem 10

Problem No	PISA 2003 Level	Correct PISA 2003 Total	Correct PISA 2003 Turkey
10	Level 4	47.48%	25.43%

"Working backwards" and/or "Accounting for all possibilities" strategies can be used to solve the problem. The correct answer is "65 on Deck, 14 on Wheels, 16 on trucks, 20 on hardware".

SKATEBOARD 3 SCORING

Full Credit

1 point: 65 zeds on a deck, 14 on wheels, 16 on trucks and 20 on hardware.

No Credit

0 point: Other responses or missing answer.

Table 35. Scores of participants in Problem 10

Participant	Aylin	Bora	Ceylan	Deniz	Emel
Score	1	1	0	0	0

Participant's Work

Aylin

[She read the question and thought about it.]

[She tried 60, 36, 16 and 10. Total was 122. She scratched out it. She made some testing. She circled the 115. She wrote 65, 14, 16 and 20.]

R: Can you explain?

A: I found the sum of the prices in different combinations. My first attempt was over 120. Then I found 115.

R: Can you note down shortly?

[She wrote solution. Her answer is correct.]

Bora

[He read the problem.]

B: 120… The maximum cost was 137.

[He examined the numbers. He tried something. He wrote 60, 14, 16 and 20.]

R: Can you explain how you do? ***[He changed 60 to 65.]***

B: I can try a different way too. Maximum is 137. I have to decrease it 17, in some way. I can't do it by changing the trucks. When I decrease 10 in hardware and decrease 5 in the first one, it isn't enough. The best is decreasing the cost of the wheels; it lowers the cost by 22... When we select the body as 40, it makes a bigger change as 25. When we decrease the cost of the wheels, total cost became 117.

B: I can try this; 40,36,16,20... That may give a better solution.

R: What's your final decision?

B: Something's wrong there. In the last one, total cost is 112. But, the first answer was 117; yes it is the correct answer.

[He wrote solution. He thought that the sum was 117 where it was 115 but his answer was correct.]

Ceylan

[She read the question.]

[She wrote 40, 14, 16 and 10.]

R: How did you find this? Can you explain?

C: Well, I think this is not correct.

R: You can do it again.

[She scratched out the old solution. She added 65, 36, 16 and 20. She found 137.]

C: The answer is this. ***[She wrote her answer. Her answer was wrong.]***

[She was bored to try more.]

Deniz

[She read the question.]

[She added the values near the table in the first page. She reread the problem.]

D: Why did I add this?

[She saw that the ready skateboard price is not asked in problem 10. She excluded that price. But she didn't remember the first question. She made some trials in the first page.] [She wrote 60, 14, 16 and 20 as an answer. She was bored. She started problem 9, she did not check the result.]

Emel

[She read the question and thought about it.]

E: Hmm, the most expensive... Then we need to choose most expensive body... In the previous problem, we chose the most expensive ones ... yeah, it was 137... it's too much. ***[She remembered Problem 8.]***

E: Let's say 65 for the deck…

E: For wheels... Let's say 16...

[The options for the wheels were 14 or 36, but she wrote 16.She added 65 and 16. She subtracted 81 from 120 and found 39.]

E: We have left 39. If we choose most expensive ones for the others, it makes 36. It is enough for us. ***[She added the numbers and found 137.]***

E: But, the sum of these is 137. Something's wrong …

R: Do you want to check?

E: No, it is not 137, it is 117. This is OK.

[She did not check the values. 16 is not an option for the wheels. So the answer is incorrect.]

APPENDIX H

STRATEGY USAGES OF THE PARTICIPANTS

Table 36. Strategies used by the Participants of the Study

Prob. No	Participant	Used Knowledge	Strategies yield to Incorrect or No Answer	Strategies yield to Correct Answer	Score	Successful at Attempt
1	Aylin	Numbers	-	Finding a pattern	1	1st
1	Bora	Numbers	Visual Representation	Visual Representation , Finding a pattern	1	2nd
1	Ceylan	Numbers	Visual Representation	Finding a pattern	1	2nd
1	Deniz	Numbers	Finding a pattern	Finding a pattern	1	2nd
1	Emel	Numbers	Finding a pattern	-	0	-
2	Aylin	Variables, Coefficients	-	Logical Reasoning	1	1st
2	Bora	Variables, Coefficients	-	Logical Reasoning	1	1st
2	Ceylan	Addition	-	-	0	-
2	Deniz	Variables, Coefficients	-	Logical Reasoning	1	1st
2	Emel	Variables, Coefficients	-	Logical Reasoning	1	1st

Table 36. Strategies used by the Participants of the Study (Continued)

Prob. No	Participant	Used Knowledge	Strategies yield to Incorrect or No Answer	Strategies yield to Correct Answer	Score	Successful at Attempt
3	Aylin	Variables, Coefficients	Intelligent guessing and testing	Intelligent guessing and testing	1	2nd
3	Bora	Variables, Coefficients	-	Intelligent guessing and testing	1	1st
3	Ceylan	-	Logical Reasoning	-	0	-
3	Deniz	Variables, Coefficients	Intelligent guessing and testing	-	0	-
3	Emel	Variables, Coefficients	Logical Reasoning	-	0	-
4	Aylin	Spatial visualization, Addition	Logical Reasoning	Accounting for all possibilities & Logical Reasoning	1	2nd
4	Bora	Spatial visualization, Addition	-	Logical Reasoning	1	1st
4	Ceylan	Spatial visualization, Addition	-	-	0	-
4	Deniz	Spatial visualization, Addition	-	Accounting for all possibilities & Logical Reasoning	1	1st
4	Emel	Spatial visualization, Addition	-	Accounting for all possibilities & Logical Reasoning	1	1st
5	Aylin	Spatial visualization, Addition	-	Accounting for all possibilities, Logical Reasoning	1	1st

Table 36. Strategies used by the Participants of the Study (Continued)

Prob. No	Participant	Used Knowledge	Strategies yield to Incorrect or No Answer	Strategies yield to Correct Answer	Score	Successful at Attempt
5	Bora	Spatial visualization, Addition	-	Accounting for all possibilities, Logical Reasoning	1	1st
5	Ceylan	Spatial visualization, Addition	-	Accounting for all possibilities, Logical Reasoning	1	1st
5	Deniz	Spatial visualization, Addition	-	Accounting for all possibilities, Logical Reasoning	0,75	1st
5	Emel	Spatial visualization, Addition	-	Accounting for all possibilities, Logical Reasoning	1	1st
6	Aylin	Numbers, Division	-	Organizing Data , Logical Reasoning	1	1st
6	Bora	Numbers, Division	-	Organizing Data , Logical Reasoning	1	1st
6	Ceylan	Numbers, Division, Addition	Organizing Data	-	0	-
6	Deniz	Numbers, Division	-	Organizing Data , Logical Reasoning	1	1st
6	Emel	Numbers, Division	-	-	0	-
7	Aylin	Circumference of a rectangle, Circumference of a polygon	-	Visual Representation& Logical Reasoning	1	1st
7	Bora	Circumference of a rectangle, Circumference of a polygon	-	Visual Representation& Logical Reasoning	1	1st
7	Ceylan	Circumference of a rectangle,	Logical Reasoning	-	0	-
7	Deniz	Circumference of a rectangle, Circumference of a polygon	-	Visual Representation& Logical Reasoning	0.75	1st

Table 36. Strategies used by the Participants of the Study (Continued)

Prob. No	Participant	Used Knowledge	Strategies yield to Incorrect or No Answer	Strategies yield to Correct Answer	Score	Successful at
7	Emel	Circumference of a rectangle, Circumference of a polygon	Finding a pattern	Visual Representation& Logical Reasoning	0.75	2nd
8	Aylin	Minimum value, Maximum value, Addition	-	Logical Reasoning	1	1st
8	Bora	Minimum value, Maximum value, Addition	-	Logical Reasoning	1	1st
8	Ceylan	Minimum value, Maximum value, Addition	-	Logical Reasoning	0,5	1st
8	Deniz	Minimum value, Maximum value, Addition	Logical Reasoning	-	0	-
8	Emel	Minimum value, Maximum value, Addition	-	Logical Reasoning	1	1st
9	Aylin	Counting Principle	-	Organizing Data, Finding a pattern	1	1st
9	Bora	Counting Principle	-	Organizing Data, Finding a pattern	1	1st
9	Ceylan	-	Logical Reasoning	-	0	-
9	Deniz	Counting Principle	Organizing Data	Organizing Data, Finding a pattern	1	2nd
9	Emel	Counting Principle	Organizing Data	Organizing Data	1	2nd
10	Aylin	Addition	-	Accounting for all possibilities	1	1st

Table 36. Strategies used by the Participants of the Study (Continued)

Prob. No	Participant	Used Knowledge	Strategies yield to Incorrect or No Answer	Strategies yield to Correct Answer	Score	Successful at
10	Bora	Addition	-	Working Backwards , Accounting for all possibilities	1	1st
10	Ceylan	Addition	Accounting for all possibilities	-	0	-
10	Deniz	Addition	Accounting for all possibilities	-	0	-
10	Emel	Addition	Working Backwards , Accounting for all possibilities	-	0	-

APPENDIX I

PROBLEM SOLVING EPISODES OF PARTICIPANTS

Table 37. Problem Solving Episodes of the Participants of the Study

Prob. No	Participant	Read	Understand	Analysis	Plan	Explore		Implement		Verify		Score
		C	M	M	M	C	M	C	M	C	M	
1	Aylin	1	2	3	4	-	-	5	-	-	-	1
1	Bora	1	2	3	4	-	-	5 7	6	-	8	1
1	Ceylan	1	2	5	6	3	4	7	-	-	-	1
1	Deniz	1	2	5	6	3	4	7	-	-	-	1
1	Emel	1	2	3 7	4	-	-	5	6	-	-	0
2	Aylin	1	2	3	4	-	-	5	-	-	-	1
2	Bora	1	2	3	4	-	-	5	-	-	-	1
2	Ceylan	1	2	3	4	-	-	5	-	-	-	0
2	Deniz	1	2	3	4	-	-	5	-	-	-	1
2	Emel	1	2	3	4	-	-	5	-	-	-	1
3	Aylin	1	2	3	4	-	-	5 7	6	-	8	1
3	Bora	1	2	3	4	-	-	5	-	-	6	1
3	Ceylan	1	2	-	3	-	-	4	-	-	-	0

Note. Numbers in the episodes refer to the order of actions of the participants

Table 37. Problem Solving Episodes of the Participants of the Study (Continued)

Prob. No	Participant	Read	Understand	Analysis	Plan	Explore		Implement		Verify		Score
		C	M	M	M	C	M	C	M	C	M	
3	Deniz	1	2	3 7	4 8	-	-	5 9	6	-	-	0
3	Emel	1	2	3	4	-	-	5	-	-	-	0
4	Aylin	1 7	2 8	3 9 13	4 10 14	-	-	5 11 15	6 12	-	-	1
4	Bora	1	2	3	4	-	-	5	-	-	-	1
4	Ceylan	1	2	3	4		-	5	-	-	-	0
4	Deniz	1	2	3	4	-	-	5 7	6	-	-	1
4	Emel	1	2	3	4	-	-	5 7	6	-	8	1
5	Aylin	1	2	3	4	-	-	5	-	-	6	1
5	Bora	1	2	3	4	-	-	5	-	-	-	1
5	Ceylan	1	2	3	4	-	-	5	-	-	-	1
5	Deniz	1	2	3	4	-	-	5	-	-	-	0.75
5	Emel	1	2	3	4	-	-	5	-	-	-	1
6	Aylin	1	2	3	4	-	-	5	-	-	6	1
6	Bora	1	2	3	4	-	-	5	-	-	6	1
6	Ceylan	1	2	-	-	3	-	-	-	-	-	0
6	Deniz	1	2	5	6	3	4	7	-	-	-	1
6	Emel	1	2	-	-	3 5 7	4 6	-	-	-	-	0
7	Aylin	1	2	3	4	-	-	5	-	-	6	1
7	Bora	1	2	3	4	-	-	5	-	-	6	1

Note. Numbers in the episodes refer to the order of actions of the participants

Table 37. Problem Solving Episodes of the Participants of the Study (Continued)

Prob. No	Participant	Read	Understand	Analysis	Plan	Explore		Implement		Verify		Score
		C	M	M	M	C	M	C	M	C	M	
7	Ceylan	1	2	3	4	-	-	5	-	-	-	0
7	Deniz	1	2	3	4	-	-	5	-	-	-	0.75
7	Emel	1	2	3	4	-	-	5	-	-	-	0.75
8	Aylin	1	2	3	4	-	-	5	-	-	-	1
8	Bora	1	2	3	4	-	-	5	-	-	-	1
8	Ceylan	1	2	3	4	-	-	5	-	-	-	0.5
8	Deniz	1	2	3	4	-	-	5	-	-	-	0
8	Emel	1	2	3	4	-	-	5	-	-	-	1
9	Aylin	1	2	3	4	-	-	5	-	-	-	1
9	Bora	1	2	3	4	-	-	5	-	-	-	1
9	Ceylan	1	2	-	-	3	-	-	-	-	-	0
9	Deniz	1	2	5	6	3	4	7	-	-	-	1
9	Emel	1	2	5	6	3	4	7	-	-	-	1
10	Aylin	1	2	3	4	-	-	5 7	6	-	-	1
10	Bora	1	2	3	4	-	-	5	-	6	7	1
10	Ceylan	1	2	5	6	3	4	7	-	-	-	0
10	Deniz	1	2	5	6	3	4	7	-	-	-	0
10	Emel	1	2	3	4	-	-	5	-	-	6	0

Note. Numbers in the episodes refer to the order of actions of the participants

APPENDIX J

MISTAKE RECOGNITION, ADAPTABILITY AND EXPRESSION OF THINKING OF PARTICIPANTS

Table 38. Mistake Recognition, Adaptability and Expression of Thinking of the Participants in the Study

		Mistake Recognition			Adaptability			Awareness and Expression of Thinking				
Prob. No	Participant	Lack of Awareness	Uncertainty	Awareness and Correction	Lack of Adaptability	Awareness for the need of change	Independent Adaptability	No Response or Ambiguous Description	Inaccurate Description	Partial Description	Full Description	Score
1	Aylin	-	-	-	-	-	-	-	-	-	1	1
1	Bora	-	-	1	-	-	1	-	-	-	1	1
1	Ceylan	-	-	1	-	-	1	-	-	-	1	1
1	Deniz	-	-	1	-	-	1	-	-	-	1	1
1	Emel	-	1	-	-	1	-	-	-	-	1	0
2	Aylin	-	-	-	-	-	-	-	-	-	1	1
2	Bora	-	-	-	-	-	-	-	-	-	1	1
2	Ceylan	-	1	-	-	1	-	-	-	-	1	0
2	Deniz	-	-	-	-	-	-	-	-	-	1	1
2	Emel	-	-	-	-	-	-	-	-	-	1	1

Table 38. Mistake Recognition, Adaptability and Expression of Thinking of the Participants in the Study (Continued)

Prob. No	Participant	Mistake Recognition			Adaptability			Awareness and Expression of Thinking				Score
		Lack of Awareness	Uncertainty	Awareness and Correction	Lack of Adaptability	Awareness for the need of change	Independent Adaptability	No Response or Ambiguous Description	Inaccurate Description	Partial Description	Full Description	
3	Aylin	-	-	1	-	-	1	-	-	-	1	1
3	Bora	-	-	-	-	-	-	-	-	-	1	1
3	Ceylan	1	-	-	1	-	-	-	-	-	1	0
3	Deniz	1	-	-	1	-	-	-	-	-	1	0
3	Emel	1	-	-	1	-	-	-	-	-	1	0
4	Aylin	-	-	1	-	-	1	-	-	-	1	1
4	Bora	-	-	-	-	-	-	-	-	-	1	1
4	Ceylan	-	1	-	1	-	-	-	-	-	1	0
4	Deniz	-	-	-	-	-	-	-	-	1	-	1
4	Emel	-	-	-	-	-	-	-	-	-	1	1
5	Aylin	-	-	-	-	-	-	-	-	-	1	1
5	Bora	-	-	-	-	-	-	-	-	-	1	1
5	Ceylan	-	-	-	-	-	-	-	-	-	1	1
5	Deniz	1	-	-	1	-	-	-	-	-	1	0.75
5	Emel	-	-	-	-	-	-	-	-	-	1	1
6	Aylin	-	-	-	-	-	-	-	-	-	1	1

Table 38. Mistake Recognition, Adaptability and Expression of Thinking of the Participants in the Study (Continued)

Prob. No	Participant	Mistake Recognition			Adaptability			Awareness and Expression of Thinking				Score
		Lack of Awareness	Uncertainty	Awareness and Correction	Lack of Adaptability	Awareness for the need of change	Independent Adaptability	No Response or Ambiguous Description	Inaccurate Description	Partial Description	Full Description	
6	Bora	-	-	-	-	-	-	-	-	-	1	1
6	Ceylan	-	1	-	1	-	-	-	-	-	1	0
6	Deniz	-	-	1	-	-	1	-	-	-	1	1
6	Emel	-	1	-	-	1	-	-	-	-	1	0
7	Aylin	-	-	-	-	-	-	-	-	-	1	1
7	Bora	-	-	-	-	-	-	-	-	-	1	1
7	Ceylan	-	1	-	1	-	-	1	-	-	-	0
7	Deniz	-	1	-	1	-	-	-	-	-	1	0.75
7	Emel	-	1	-	1	-	-	-	-	-	1	0.75
8	Aylin	-	-	-	-	-	-	-	-	-	1	1
8	Bora	-	-	-	-	-	-	-	-	-	1	1
8	Ceylan	1	-	-	1	-	-	-	-	-	1	0.5
8	Deniz	1	-	-	1	-	-	-	-	-	1	0
8	Emel	-	-	-	-	-	-	-	-	-	1	1
9	Aylin	-	-	-	-	-	-	-	-	-	1	1
9	Bora	-	-	-	-	-	-	-	-	-	1	1

Table 38. Mistake Recognition, Adaptability and Expression of Thinking of the Participants in the Study (Continued)

Prob. No	Participant	Mistake Recognition			Adaptability			Awareness and Expression of Thinking				Score
		Lack of Awareness	Uncertainty	Awareness and Correction	Lack of Adaptability	Awareness for the need of change	Independent Adaptability	No Response or Ambiguous Description	Inaccurate Description	Partial Description	Full Description	
9	Ceylan	-	1	-	1	-	-	1	-	-	-	0
9	Deniz	-	-	1	-	-	1	-	-	-	1	1
9	Emel	-	-	1	-	-	1	-	-	-	1	1
10	Aylin	-	-	-	-	-	-	-	-	-	1	1
10	Bora	-	-	-	-	-	-	-	-	-	1	1
10	Ceylan	1	-	-	1	-	-	1	-	-	-	0
10	Deniz	1	-	-	1	-	-	-	-	1	-	0
10	Emel	1	-	-	1	-	-	-	-	-	1	0