IDENTIFICATION OF HANDLING MODELS FOR ROAD VEHICLES

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KUTLUK BİLGE ARIKAN

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IDENTIFICATION OF HANDLING MODELS FOR ROAD VEHICLES

submitted by **KUTLUK BİLGE ARIKAN** in partial fulfillment of the requirements for the degree of **Doctor of Philosophy in Mechanical Engineering Department, Middle East Technical University** by,

Prof. Dr. Canan Özgen ____________________
Dean, Graduate School of Natural and Applied Sciences

Prof. Dr. Kemal İder ____________________
Head of Department, Mechanical Engineering

Prof. Dr. Y. Samim Ünlüsoy ____________________
Supervisor, Mechanical Engineering Dept., METU

Examining Committee Members:

Prof. Dr. M. Kemal Özgören ____________________
Mechanical Engineering Dept., METU

Prof. Dr. Y. Samim Ünlüsoy ____________________
Mechanical Engineering Dept., METU

Prof. Dr. Tuna Balkan ____________________
Mechanical Engineering Dept., METU

Prof. Dr. Kemal Leblebicioğlu ____________________
Electrical and Electronics Engineering Dept., METU

Asst. Prof. Bülent İrfanoğlu ____________________
Mechatronics Engineering Dept., Atılım University

Date: 07.03.2008

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name : Kutluk Bilge ARIKAN

Signature :

ABSTRACT

IDENTIFICATION OF HANDLING MODELS FOR ROAD VEHICLES

ARIKAN, KUTLUK BİLGE
Ph.D. Mechanical Engineering Department
Supervisor: Prof. Dr. Y. Samim ÜNLÜSOY

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This thesis reports the identification of linear and nonlinear handling models for road vehicles starting from structural identifiability analysis, continuing with the experiments to acquire data on a vehicle equipped with a sensor set and data acquisition system and ending with the estimation of parameters using the collected data. The 2 degrees of freedom (dof) linear model structure originates from the well known linear bicycle model that is frequently used in handling analysis of road vehicles. Physical parameters of the bicycle model structure are selected as the unknown parameter set that is to be identified. Global identifiability of the model structure is analysed, in detail, and concluded according to various available sensor sets. Physical parameters of the bicycle model structure are estimated using prediction error estimation method. Genetic algorithms are used in the optimization phase of the identification algorithm to overcome the difficulty in the selection of initial values for parameter estimates. Validation analysis of the identified model is also presented. Identified model is shown to track the system response successfully. Following the linear model identification, identification of 3 dof nonlinear models are studied. Local identifiability analysis is done and optimal input is designed using the same procedure for linear model structure identification. Practical identifiability analysis is performed using Fisher Information Matrix. Physical parameters are estimated using the data from simulated experiments. High accuracy estimates are obtained. Methodology for nonlinear handling model identification is presented.

Keywords: Bicycle Model, Structural Identifiability, Practical Identifiability, Parameter Estimation, Prediction Error, Genetic Algorithm, Input Design

ÖZ

YOL TAŞITLARI İÇİN DÖNÜŞ MODELLERİ TANILANMASI

ARIKAN, KUTLUK BİLGE
Doktora, Makina Mühendisliği Bölümü
Tez Yöneticisi: Prof. Dr. Y. Samim ÜNLÜSOY

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Bu tezde, yol taşıtları için doğrusal ve doğrusal olmayan dönüş modellerinin tanılanmaları, yapısal tanılanabilirlik analizinden başlamış, algılayıcı ve veri toplama sistemi ile donanmış fiziksel araç üzerinde yapılan deneyler ve bu deneylerde elde edilen veri kullanılarak gerçekleştirilen parameter kestirimleri ile sonuçlandırılmıştır. Tanılanan 2 serbestlik dereceli doğrusal model, taşıtların dönüş dinamikleri ile ilgili çalışmalarda yaygın olarak kullanılan tek izli araç modelinden faydalanarak kurulmuştur. Tek izli araç modelinde yer alan fiziksel parametreler, kestirilecek parameter kümesi içinde yer almaktadır. Bu modelin genel tanılanabilirliği, algılayıcı kümeleri seçeneklerine gore detaylıca incelenmiştir. Fiziksel parametreler, öngörü hatası kestirimi yöntemi ile elde edilmişlerdir. Eniyileme yöntemi olarak genetic algoritma kullanılarak, kestirimlerin doğru değerlere yakınsamasında engel olabilecek ilk değer sorununun giderilmesi sağlanmıştır. Tanılanan modelin geçerliliği sınanmış, modelin gerçek sistemin tepkisini başarılı bir şekilde takip ettiği görülmüştür. Doğrusal model yapılarının tanılanmasının ardından, 3 serbestlik dereceli doğrusal olmayan model yapılarının tanılanmaları üzerinde çalışılmıştır. Yerel tanılanabilirlik analizleri doğrusal olmayan model yapıları üzerinde yapılmıştır. Eniyi girdi tasarımı da doğrusal model yapılarının tanılanmaları için faydalanılan eniyi girdi tasarımı yaklaşımı kullanılarak gerçekleştirilmiştir. Uygulamada tanılanabilirlik analizleri de doğrusal olmayan model yapısı üzerinde, Fisher Bilgi Matrisi marifetiyle tatbik edilmiştir. Fiziksel parametreler, deney benzetimlerinden elde edilen veri kullanılarak kestirilmiş, başarılı sonuçlar elde edilmiştir.

Anahtar Kelimeler: Tek İzli Araç Modeli, Yapısal Tanılanabilirlik, Uygulamada Tanılanabilirlik, Parametre Kestirimi, Öngörü Hatası, Genetik Algoritma, Girdi Tasarımı

To my parents

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LIST OF SYMBOLS

a	: Distance from front axle to the center of gravity of the vehicle
A_a	: Augmented system matrix
A_y^P	: Lateral acceleration at point P
A_i, B_i	: Simplified nonlinear tire model parameters
b	: Distance from rear axle to the center of gravity of the vehicle
B_a	: Augmented input matrix
c	: Distance from rear axle to velocity sensor position
C_a	: Augmented output matrix
$C_{a1,2}$	: Front and rear axle linear cornering stiffness
C_f	: Front axle cornering stiffness
C_r	: Rear axle cornering stiffness
c_ϕ	: Total roll damping of the vehicle about roll axis
$c_{\phi1,2}$	: Resulting roll damping coefficients of the front and rear suspensions about roll axis
$c_{\theta1,2}$	: Front and rear suspensions resulting linear damping coefficients
d	: Distance from the front axle to accelerometer position
D_a	: Augmented direct coupling matrix
D_M	: Set of values that θ may take in the model structure
F	: Rayleigh's dissipation function
F_{ae}	: Aerodynamic force resisting to motion in $-x_G$ axis
F_{cf}	: Cornering force of front axle
F_{cr}	: Cornering force of rear axle
h	: Height of center of gravity from roll axis

J : Yaw moment of inertia

$k_{\phi 1,2}$: Resulting roll stiffness coefficients of the front and rear suspensions about roll axis

$k_{\theta 1,2}$: Front and rear suspensions resulting linear stiffness coefficients

k_{ϕ} : Total roll stiffness of the vehicle about roll axis

L : Wheel base

m : Vehicle mass

M : Fisher Information Matrix

N : Number of measurements used in identification algorithm

r : Yaw velocity

R_v : Covariance matrix of measurement noise

R : Yaw velocity

S : Surface area of vehicle normal to air flow in $-x_G$ axis.

$S(i)$: Matrix of output sensitivities to parameters

T : Kinetic energy of the system

U : Forward velocity (constant)

U_e : Potential energy of the system

v : Vehicle Side-slip velocity

v_Q : Side slip velocity at point Q

V_{Gx_G} : Component of absolute velocity of mass center G in body reference frame (G, x_G, y_G, z_G).

V_{Gy_G} : Component of absolute velocity of mass center G in body reference frame (G, x_G, y_G, z_G).

V_{Gz_G} : Component of absolute velocity of mass center G in body reference frame (G, x_G, y_G, z_G).

$V_N(\theta)$: Cost function to be minimized

w_a : Accelerometer measurement noise

w_g : Gyroscope measurement noise

α_f : Slip angle of front wheels

α_r : Slip angle of rear wheels

α_t : Transient slip angle

α_{ss} : Steady-state value of the slip angle

$\alpha_{ss1,2}$: steady-state slip angles for front and rear wheels.

$\varepsilon_{1,2}$: Front and rear axle roll-steer coefficient

ρ : Air density

δ : Steering angle of front wheels

σ : Relaxation length

$\hat{\theta}_N$: Parameter estimates

θ_r : Inclination angle of roll axis

ω_{x_G} : Components of absolute angular velocity of vehicle in body reference frame (G, x_G, y_G, z_G).

ω_{y_G} : Components of absolute angular velocity of vehicle in body reference frame (G, x_G, y_G, z_G).

ω_{z_G} : Components of absolute angular velocity of vehicle in body reference frame (G, x_G, y_G, z_G).

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CHAPTER I

INTRODUCTION

In recent years, identification and estimation studies on vehicle dynamics research appear in literature more frequently. Identified mathematical models of road vehicles provide for comprehensive analysis of the handling characteristics and assists ongoing design and development studies. Linear bicycle model is a well known and frequently used model for studying vehicle handling dynamics. Estimating the physical parameters of model structures based on linear bicycle model enables the engineers to directly use the model in real vehicle simulations. Some typical applications published in literature are presented in references [1-5]. Various models and identification techniques are utilized in these studies.

Identification process can be carried out using either parametric or nonparametric models [6]. Parametric models may have black-box or grey-box structures. Black box structures utilize appropriate mathematical functions to describe the input and output relationship. On the other hand, grey box models employ the physical relationships describing system dynamics [7-9]. Use of grey box models may reduce the number of parameters to be estimated. However, in this case, reaching optimum parameter estimates becomes more complex due to the increased number of local minima.

In order to end up with reliable models, identification experiments should be designed prior to the collection of input/output data during tests. Experiment design can be regarded in two stages, namely, the qualitative design and the quantitative design [10]. Qualitative experiment design includes the selection of input ports and output ports from which the data will be collected. This selection should be made based on the structural identifiability of the parameters to be estimated. In other words, inputs and outputs should be selected in such a way that the desired parameters become identifiable. If the identifiability of the structure is ensured, estimation of a unique set of parameters can then be expected. This will also improve the performance of the optimization algorithm [11]. Quantitative part of the experiment design focuses on reaching parameter estimates with low uncertainty [10]. It is desired to get the maximum information from the collected data. So as to extract maximum information, optimal inputs should be designed employing a suitable criterion. Input design concerns the shape, amplitude, duration, and frequency content of the input signal, and sampling time implemented during the identification experiments. Use of optimal inputs increases the accuracy of the parameter estimation.

In this thesis, it is aimed to identify 2 degrees of freedom (dof) and 3 dof handling models for road vehicles. Former ones have linear mathematical model structures, whereas latter ones have both linear and nonlinear structures. 2 dof models are mainly based on the linear bicycle model composed of physical parameters. On the other hand, both black-box and grey-box structures are desired to be identified for 3 dof dynamics identification.

In this study, an experimental work using a vehicle equipped with a sensor set and data acquisition system is also described. Collected data are then imported in the identification algorithm and the physical parameters of the bicycle model structure are estimated using prediction error estimation method. In the case of physical parameter estimation, initial values of the estimates become more critical due to the local minima problem. To overcome this difficulty, genetic algorithms are utilized in the optimization process. Validation analysis of the identified model is also presented.

The experimental part is carried out with support from TOFAŞ. Acquired data during these experiments is utilized for the purpose of 2 dof model identification.

CHAPTER 2

ESTIMATION AND IDENTIFICATION IN VEHICLE DYNAMICS RESEARCH

Developments in sensor technology and digital technology accelerate the improvements in automotive industry. By the use of new materials, design of higher performance engines, tires result in extreme vehicles in today's world. However, these high power and high performance components require complex control systems, complex mathematical models and advanced identification routines. That is why, in the first sentence the importance of the improvements in sensor and digital technologies are emphasized.

Considering the trend of development above, one can find many research studies about estimation and identification practices about vehicle dynamics. Before classification of the studies in literature, it would be better to give some basics about parameter and state estimation and system identification.

2.1 State Estimation

In 1963, David G. Luenberger initiated the theory of observers for state reconstruction of linear dynamical systems. Since then, owing to its utility and its intimate connection with fundamental system concepts, observer theory continues to be a fruitful area of research and has been substantially developed in many different directions. In view of this, the observer has come to take its pride of place in linear multivariable control alongside the optimal linear regulator and the Kalman filter [12].

The major thrust in the development of observers for multivariable linear causal systems came from the introduction of state-space methods in the time domain by Kalman in 1960. The immediate impact of state-space methods was the strikingly direct solution of many longstanding of control in a new multivariable system context. The designed control systems are normally of the linear state feedback type and, if they are to be implemented, call for the complete availability of the state vector of the system. It is frequently the case, however, that even in low-order systems it is either impossible or in appropriate, from practical considerations, to measure all the elements of the system state vector. This problem is to be overcome to retain many useful properties of linear feedback control. The observer provides an elegant and practical solution to this problem.

The need for state reconstruction or observation is not limited to deterministic systems only. In many engineering applications, it is necessary to estimate a single or multi-dimensional signal, embedded in a stochastic noise process, either continuously or at distinct time points over an observation interval [13]. Generally, at any given time point, an estimate can be formed from measurements by solution of a known mathematical relationship describing the behavior of the system. Often in practical estimation problems, a reliance on either measurements or a mathematical description of the system behavior alone provides state estimates with insufficient accuracy to satisfy the application's requirements. Therefore, it is of considerable practical interest to study optimal strategies that can be used to combine signal measurements and mathematical descriptions of the system behavior in a wide class of state estimation and signal estimation problems. It is of particular interest to determine an optimal estimation strategy satisfying a minimum variance of error cost function for a wide class of state estimation problems in which the signals or states to be estimated are embedded in a stochastic noise process. Estimation problems of this type were first evaluated by N. Wiener in 1942. The development of data processing methods for dealing with random variables can be traced to Gauss, who invented the technique of deterministic least-squares and employed it in a relatively simple orbit measurement problem [15]. The next significant contribution to the broad subject of estimation theory occurred more than 100 years later when Fisher, working with probability density functions, introduced the approach of maximum likelihood estimation. Utilizing random process theory, Wiener set forth a procedure for the frequency domain design of statistically optimal filters.

Wiener's study led to a great amount of activity in the field of signal estimation and it has importance in bringing the statistical point of view into communication and control theory. Under very general conditions, for a wide class of problems with signals embedded in stochastic noise systems, Wiener presented an optimal linear minimum variance of error (least-squares) estimation strategy to combine measurements with a mathematical description of signal behavior. For estimation problems of this type, Wiener determined the form of an optimal linear minimum variance of error estimation filter in terms of frequency-domain concepts such as transfer functions and spectral densities. This optimal estimation filter takes the form of a continuous-time linear filter, called Wiener filter. This filter is not easily modified for discrete-time systems, and when modified, it does not provide a recursive, computationally efficient algorithm. Further, the Wiener filter can be complicated when the signal is multi-dimensional, and it requires that the noise and signal processes be statistically stationary.

In 1960, R. E. Kalman reformulated the problem in terms of state-space concepts. Under a general set of conditions, for systems described by a state-space model, Kalman described the form of an optimal linear minimum variance estimation filter for the optimal estimation of the current system state from a blend of system dynamics and measurements on the system. This estimation filter takes the form of a computationally efficient sequential and recursive discrete-time algorithm called the discrete-time Kalman filter. In 1961, Kalman and Bucy presented a similar solution for the continuous-time state estimation problem, called the continuous-time Kalman filter. The earliest applications of the Kalman filter dealt with satellite orbit determination. Today, Kalman filters are commonly found in navigation, tracking, and industrial control problems as well [13].

For linear systems, observing the state of a dynamical system has been extensively studied. For nonlinear systems, the theory of observers is not nearly as complete or successful as it is for linear systems. Applying linear observer theory to nonlinear problems has had success, but has by no means closed the book on nonlinear observer design. Instead, attempts continue to be made to construct nonlinear observers using tools from nonlinear systems theory [16].

Kalman filtering methods are also applied to nonlinear systems. When the Kalman filter is applied to the linear representation of a nonlinear system it is called a linearized Kalman filter. In some Kalman filtering applications, where divergence occurs due to significant nonlinearity in the system dynamics, satisfactory results can be obtained by using a near-optimal state estimation filter called the extended Kalman filter. The extended Kalman filter is obtained applying the generalized Kalman filter algorithm to a linearized system model, where in this case the nonlinear system is linearized about the Kalman filter's estimated trajectory rather than a recomputed nominal trajectory.

In cases where system models are uncertain or ill-defined, fuzzy and neural network estimators can be used instead of the conventional estimation techniques. They can be used within a hybrid solution for estimation problem together with a Kalman filter [14].

2.1.1 Estimation for Stochastic Systems

Estimation is the process of extracting information from data. Data can be used to infer the desired information and may contain errors. Modern estimation methods use known relationships to compute the desired information from the measurements, taking account of measurement errors, the effects of disturbances and control actions on the system, and prior knowledge of the information. These methods are based on the statistical estimation theory [13].

The principles of statistical estimation theory are employed, generally, to determine structures and methods that allow for parameter estimation to be performed, in some sense, optimally. The Kalman filter is an application of statistical estimation theory, or estimation theory, which has been applied to numerous practical estimation problems.

Estimation theory is a field of statistics concerned with the technique for estimating the value of a set of unknown parameters, x, from a set of measurements Y containing information about x. In estimation theory, estimates for x, denoted by $\hat{x}$ are formed from Y by using an estimation rule called an estimator. An optimal estimator, in some sense, is tried to be found for x. The essential estimation problem, then, is to find an estimator for x that acts upon a set of observations Y, containing information about x, to provide in some sense good or optimal estimates for x. To select an optimal estimator, the relative performance evaluation of different estimators has led to statistical notion of loss or cost.

The most popular optimality criterion for estimation is the Bayes criteria. The Bayes criteria, for any given estimation problem, selects the optimal estimator to be the estimator that minimizes expectations of the loss function. An estimator that satisfies the Bayes criteria for a given estimation problem is said to be a Bayes estimator for the problem. Bayes estimators with squared-error loss functions are called minimum variance of error estimators. When the loss function for the Bayes criteria is the squared-error loss function, the optimal linear estimator is called a linear minimum variance (LMV) of error estimator [13].

2.1.2. Estimation Filters

In practical engineering problems, estimation is typically performed as an ongoing process in which measurements are continuously updated in time and input to an estimation filter where parameter estimation is performed repeatedly in time. An estimation filter provides at time t an estimate, $\hat{x}(t+\alpha \mid t)$, for the value of signal x at some time t+α, which is an estimate for x(t+α). When α is greater than zero, the estimation filter is called a predictive filter. When α is less than zero, the estimation filter is called a smoothing filter. When α is equal to zero, the estimation filter is called as a filter.

A linear minimum variance of error estimation filter is an estimation filter that at any given time is a linear minimum variance of error estimator. A linear minimum variance of error estimation filter is a minimum variance of error estimation filter over the class of all linear filters [13].

There are assumptions in the derivation of the Kalman filter. The Kalman filter should be used under the consideration of these assumptions. The process and measurement noise random processes, are uncorrelated zero mean white noise random processes with known auto covariance functions. The initial system state is a random vector that is uncorrelated to both the process and measurement noise random processes. The initial system state has a known mean and known covariance matrix. Under the Gaussian restrictions, the Kalman filter is a linear estimation filter that also minimizes the variance of error from among all estimation filters, linear or nonlinear, that is the Kalman filter is not simply an LMV estimation filter for the estimation of the system state, but it is a minimum variance of error estimation filter as well.

Consider the stochastic model of a discrete time system

$$x_k = Ax_{k-1} + Bu_k + w_{k-1}$$
$$z_k = Hx_k + v_k$$

where

x_k	state vector
z_k	measurement vector
u_k	deterministic system input
w_k	process noise (plant disturbance)
v_k	measurement noise
Q	process noise covariance
R	measurement noise covariance
P_k^-	priori estimate error covariance
P_k	posteriori estimate error covariance

The discrete Kalman filter equations are as follows [13]

Time update equations (predict):

$$\hat{x}_k^- = A\,\hat{x}_{k-1} + B\,u_k$$
$$P_k^- = A\,P_{k-1}\,A^T + Q$$

Measurement update equations (correct):

$$K_k = P_k^-\,H^T\left(H\,P_k^-\,H^T + R\right)^{-1}$$
$$\hat{x}_k = \hat{x}_k^- + K_k\left(z_k - H\,\hat{x}_k^-\right)$$
$$P_k = \left(I - K_k\,H\right)P_k^-$$

The discrete Kalman filter equations can be derived from different ways. Derivation from the orthogonality principle and derivation from the innovations approach are two of them.

2.1.2.1 Alternate forms and extensions of the Kalman filter algorithm

- The discrete Kalman filter was developed for systems with a known linear state-space model. However, in practical applications there often exists great uncertainty about some of the system model parameters. In some applications an incorrect system model may lead to divergence. Several extensions of the conventional Kalman filter algorithm that use the incoming measurements to either identify the unknown system model parameters or to otherwise minimize the effect of an incorrect system model on the Kalman filter algorithm. These types of filters are called adaptive (or self learning) Kalman filters. Generally, when an adaptive Kalman filter is applied to an incorrect system model it provides an estimate that is more optimal (that is, that has a smaller variance of error) than the estimate provided by the conventional Kalman filter algorithm under the incorrect system model assumption. And in many cases when divergence is due to system modeling errors, the application of an adaptive Kalman filter can provide satisfactory results from an otherwise divergent filter [13], [17].

- In some situations it may be necessary to apply Kalman filtering methods to system models with system disturbance and measurement noise processes with known, but not necessarily zero, means. Kalman filtering algorithm can be modified to provide an optimal LMV estimation filter for such situations. This modified recursive algorithm is called the generalized Kalman filter. In addition to this generalized assumption, the case where the plant disturbance and measurement noise random processes may be correlated at the same time point can be also considered. Generalized Kalman filter algorithm is modified to handle the described case. This sequential and recursive algorithm for the optimal LMV estimate provides the most general form of discrete Kalman filter and is called the general discrete Kalman filter. Continuous-time Kalman filter can be extended in a similar manner and the general continuous-time Kalman filter can be obtained [13].

- Kalman filter has found many applications with nonlinear systems. In many Kalman filtering applications, an approximate linear system model is assumed for a nonlinear system, and this approximate linear system model then forms the basis for the Kalman filter utilization. When the Kalman filter algorithm is applied to such a linear representation of a nonlinear system, it is called a linearized Kalman filter. However, sometimes the linear approximation leads to divergence. In these situations, where divergence occurs due to significant nonlinearity in the system dynamics, satisfactory results can sometimes be obtained by using a Kalman filter algorithm modification called the extended Kalman filter [12], [13], [17]. In this case, the nonlinear system is linearized about the Kalman filter's estimated trajectory.

- The unscented Kalman filter is proposed as a derivative free alternative to the extended Kalman filter in the frame work of state estimation. The unscented Kalman filter consistently outperforms extended Kalman filter in terms of prediction and estimation errors [18].

Besides the conventional observers and filters fuzzy and neuro-fuzzy estimators are also used. Such an estimator is useful in cases where system model is either uncertain or ill-defined. An example in the literature uses multiple fuzzy models to track the motion of a target, which performs a priori unknown maneuver, based upon uncertain measurements [14].

2.2 System Identification

As a definition, system identification deals with the problem of building mathematical models of dynamical systems based on the observed data from the systems [11]. The construction of a model from data involves the following basics

- The data: The input-output data are sometimes recorded during a specifically designed identification experiment, where the user may determine which signals to measure and when to measure them and may also choose the input signals. The object with experiment design is thus to make these choices so that the data become maximally informative, subject to constraints. In other cases user may not have the possibility to affect the experiment, but must use data from the normal operation of the system.

- The set of models: A set of candidate models is obtained within a collection of models. This is the most important and, at the same time, the most difficult choice of the system identification procedure. Sometimes the model set is obtained after careful modeling. Then a model with some unknown physical parameters is constructed from basic physical laws and other well-established relationships. In other cases standard linear models may be employed, without reference to the physical background. Such a model set, whose parameters are basically viewed as vehicles for adjusting the fit to the data and do not reflect physical considerations in the system, is called a black box. Model sets with adjustable parameters with physical interpretation may, accordingly, be called gray boxes.

- Determining the best model in the set: This is the identification method. The assessment of model quality is typically based on how the models perform when they attempt to reproduce the measured data.

After having settled the on the preceding three choices, at least implicitly, a particular model has been arrived. That is the one in the set that best describes the data according to the chosen criterion. It then remains to test whether this model is good enough, that is, whether it is valid for its purpose. Such tests are known as model validation. They involve various procedures to assess how the model relates to observed data, to prior knowledge, and to its intended use. Deficient model behavior in these respects makes the model to be rejected, while good performance will develop a certain confidence in the model. A model can never be accepted as a final and true description of the system. Rather, it can at best be regarded as a good enough description of certain aspects that are interested in.

The identification procedure has a logical flow of collecting data firstly, then choosing a model set, then picking the best model in this set. If the model first obtained will not pass the model validation tests, then steps of the procedure are revised.

The model may be deficient for a variety of reasons:

- The numerical procedure failed to find the best model according to the criterion.

- The criterion was not well chosen.

- The model set was not appropriate, in that it did not contain any “good enough” description of the system.

- The data set was not informative enough to provide guidance in selecting good models.

The major part of an identification application in fact consists of addressing these problems.

2.2.1 Parameter Estimation Methods

A set of candidate models has been selected, and it is parametrized as a model structure using a parameter vector. The search for the best model within the set then becomes a problem of determining or estimating that parameter vector. There are many different ways of organizing such a search and also different views on what one should search for.

The ability of the models to describe the observed data should be evaluated. Thus the prediction error is formulated. A good model is one that is good at predicting, that is, one that produces small prediction errors when applied to the observed data. Note that there is considerable flexibility in selecting various predictor functions, and this gives a corresponding freedom in defining good models in terms of prediction performance.

The error in the estimate of unknown parameters $\tilde{\theta} = \theta - \hat{\theta}$ provides the quantity having the greatest interest in an estimation problem. Since it is a random vector, its value is generally unknown and must be described probabilistically. Certainly, the most desirable estimate is one for which the estimation error equals zero. In general, an estimator cannot be expected to be the best in this sense because Cramer-Rao inequality provides a lower bound on the covariance of the estimation error. Since the bound is not necessarily equal to the zero matrix, any estimator, based on a finite amount of data, must be expected to contain some error. Then, it is reasonable to attempt to define estimators which minimize the estimation error in some prescribed sense.

2.2.1.1 Least Squares Estimation

Least squares can be regarded as a deterministic approach to the estimation problem. Suppose n parameters, denoted as the n dimensional vector x, are to be estimated from M measurements, denoted as the M dimensional vector z. The parameters x and measurements z are assumed to be related according to

$$z = H\,x + v.$$

The (Mxn) matrix H is assumed to be known and the number of measurements is at least as large as the number of unknown parameters. In addition, H has maximal rank n. The vector v represents unknown errors that occur in the measurement of x.

For least squares estimation the estimator is chosen to minimize the sum of the square of the errors. More precisely, $\hat{x}_{LS}$ is defined as the least squares estimator of x given the data z if it minimizes

$$L = \frac{1}{2}\sum_{i=1}^{M} v_i^2 = \frac{1}{2} v^T v$$

$$= \frac{1}{2}(z - H\,x)^T (z - H\,x)$$

Since H has been assumed to have maximal rank, the inverse of (H^T H) exists and the least squares estimator is found to be

$$\hat{x}_{LS} = (H^T H)^{-1} H^T z$$

The error in the estimator is a linear function of the measurement errors v and it has mean value of zero. Therefore, $\hat{x}_{LS}$ is called as unbiased estimator. This seems to be a desirable property. However, biased estimators also arise and can provide advantages relative to unbiased estimators.

The least squares cost function can be generalized by introducing a symmetric, positive definite weighting matrix W.

$$L = \frac{1}{2}(z - H\,x)^T W\,(z - H\,x)$$

The elements of W can be chosen to emphasize (or deemphasize) the influence of specific measurements upon the estimate $\hat{x}_{LS}$.

The error covariance matrix provides a measure of the behavior of the estimator. It is natural to attempt to determine the estimator that will minimize the error variances. An important performance index, which can be regarded as the probabilistic version of least squares, is provided by the mean square error

$$L = E\left[(x - \hat{x})^T (x - \hat{x})\right]$$

For nonlinear systems, this performance index can yield results which are substantially different than the least squares index. Unbiased estimators which are linear and minimize the mean square error are referred to as best, linear, unbiased estimators and denoted BLUE [19].

Many other topics could be discussed in conjunction with least squares. Also, other forms and extensions of least squares estimation exists.

Considering the estimators such as least squares estimators, the properties that characterize the class of good estimators are as follows

- The estimator should be based upon all available data.

- The estimate should be acceptable (i.e., when evaluated with any data set, it should always yield an estimate that represents a possible value of the parameter).

- The estimator should be consistent so that the estimate converges in probability to the parameter.

- The estimator commonly should be unbiased (although biased estimators may sometimes be useful).

- The estimator should be efficient in the sense that its error covariance is as small as possible.

The latter two properties are often required only in asymptotic sense as the number of data samples becomes large. In addition, it is desirable that the probability distribution of the estimator error be asymptotically Gaussian. Then, the analysis of the performance of the estimator is made simpler.

2.2.1.2 Maximum a posteriori and maximum likelihood estimators

An estimator proposed by Pearson around 1900 is obtained by the method of moments. The idea behind the method is simple and intuitively appealing. It is practical in the sense that it frequently leads to a computationally realizable estimator. However, it has little theoretical justification with the result that its appeal is primarily intuitive.

The method of moments is based on the preceding observation regarding the consistency of the sample moments. Thus, for large samples, it is reasonable to expect that the sample moment provides a good approximation of the moments of the distribution. In fact, the method of moments is generally inefficient and so is not utilized except in the absence of better estimators [11], [19].

Consider the density function $f(z|\theta)$ where z is the measurement vector and θ is the vector of parameters to be estimated. Regarding it as a function of unknown parameter θ, it is reasonable to choose as an estimator of θ that maximizes $f(z|\theta)$. This estimator can be interpreted as providing the value of θ that makes the measurement most likely.

The density function $f(z|\theta)$ is called the likelihood function. Because of the exponential character of many density functions, it is convenient to deal with the log likelihood function $\ln[f(z|\theta)]$. Certainly, $f(z|\theta)$ is maximized by choosing θ to maximize $\ln[f(z|\theta)]$.

The maximum likelihood estimator is intuitively appealing. Unlike the intuitively appealing method of moments, the maximum likelihood approach can be shown to exhibit many theoretical properties that confirm that it is a good estimator. Maximum likelihood estimators can be shown, under reasonable conditions, to be consistent and asymptotically efficient. Also, the probability distribution of the estimator is asymptotically normal.

Bayes' rule states that the a posteriori density function $f(\theta|z)$ is related to the a priori density $f(\theta)$ according to

$$f(\theta|z) = f(z|\theta)\ [f(\theta)/f(z)]$$

Following the discussion of maximum likelihood estimation, it is reasonable to consider choosing an estimator that maximizes the a posteriori density given the measurements z.

The maximum a posteriori and the maximum likelihood estimators are seen to have an obvious relationship. Suppose that the a priori density $f(\theta)$ is uniform over the range of values of θ for which $f(z|\theta)$ is significantly greater than zero. Then, $f(\theta|z)$ is maximized by the same value of θ which maximizes the likelihood function $f(z|\theta)$. This case arises when there is no a priori information about θ available. Thus, the maximum a posteriori estimator differs from the maximum likelihood estimator when a priori density $f(\theta)$ is available and causes the maximum of the likelihood function in $f(z|\theta)$ to shift.

There are other parameter estimation techniques available in literature. In the previous chapters only the popular ones are explained briefly.

2.2.2 Nonlinear System Identification

When changing from a linear to nonlinear model, it may occur that the nonlinear model, if it is not chosen flexible enough, performs worse than the linear one. A good strategy for avoiding this undesirable effect is to use a nonlinear model architecture that contains a linear model as a special case. Overall model output is the sum of the linear and the nonlinear model parts. This strategy is very appealing because it ensures that the overall nonlinear model performs better than the linear model [20].

Modeling and identification of nonlinear dynamic systems is a challenging task because nonlinear processes are unique in the sense that they do not share many properties. A major goal for any nonlinear system modeling and identification scheme is universalness: that is, the capability of describing a wide class of structurally different systems.

External dynamics strategy is by far the most frequently applied nonlinear dynamic system modeling and identification approach [20]. It is based on the nonlinear input/output model. The name "external dynamics" stems from the fact that the nonlinear dynamic model can be clearly separated into two parts: a nonlinear static approximator and an external dynamic filter bank. In principle, any model architecture can be chosen for the approximator. However, from the large number of approximator inputs it is obvious that the approximator should be able to cope with relatively high dimensional mappings, at least for high order systems. Typically, the filters are chosen as simple time delays. Then they are referred to as tapped-delay lines, and if the approximator is chosen as a neural network the whole model is usually called a time delay neural network. Many properties of the external dynamics approach are independent of the specific choice of the approximator.

When widening the focus to also include black box identification of nonlinear dynamic systems, the problem of selecting model structures becomes increasingly difficult. Neural networks are used for constructing such black box models [3], [19]. Multilayer perceptron (MLP) network is good at learning nonlinear relationships from a set of data. Thus, in the pursuit for a family of model structures suitable for identification of nonlinear dynamic systems, it is natural to bring up MLP networks. By making this choice, the model structure selection is basically reduced to dealing with the following issues; selecting the inputs to the network and selecting an internal network architecture. An often used approach is to reuse the input structures from the linear models while letting the internal architecture be feedforward MLP network. This approach has several following attractive advantages

- It is a natural extension of the well known linear model structures

- The internal architecture can be expanded gradually as a higher flexibility is needed to model more complex nonlinear relationships.

- The structural decisions required by the user are reduced to a level that is reasonable to handle.

- Suitable for design of control systems.

Recurrent or real time identification is also available in neural network dynamic system models. Real time recurrent learning algorithm, also known as simultaneous backpropagation, has been proposed in literature [18].

Fuzzy system models and neuro-fuzzy architectures are also available in literature [14], [20].

Least squares estimation is also applied for estimation of parameters of nonlinear system models [21]. Its solution is seen to require the solution of a system of nonlinear algebraic equations which call for orthogonality between the residual and the matrix of first partial derivatives of the observation equation relative to the unknown parameters. Linearization of the observation equation led to the definition of an iterative search procedure called the Gauss method. Alternatively, approximating the cost function by a quadratic leads to another iterative procedure called the Newton method. Extended Kalman filter is a well known method for estimating the parameters from nonlinear measurements that are generated in real time [22]. In some cases, extended Kalman filter may have some convergence problems and least squares estimation of different form may give better results compared to the extended Kalman filter estimates [22].

2.3 Estimation and Identification Studies about Vehicles

There are many applications available including estimation and system identification in vehicle research studies. In this thesis, identification of vehicle handling dynamics and estimation of related physical parameters of road vehicles are focused. Before discussing the studies on this subject, a rough classification is given below.

- Estimation of vehicle states: Estimated states depend on the form of the mathematical model and the available measurements. Forward velocity, yaw velocity, side slip angle, roll velocity are some of the estimated states, [1, 2, 23, 24, 25, 26, 27]. Estimated states are used to monitor some variables or to use as feedback signals in control systems.

- Estimation of tire/road surface friction coefficient: This parameter is essential to optimize the control systems such as ABS and TCS, [25, 28, 29, 30].

- Estimation of tire forces: It is used to generate mathematical models for tires and it can also be used for control purposes, [25, 28, 29].

- Moment of inertia estimation**:** Inertia tensor may be identified by using special test setups or by utilizing the collected data from vehicle during the special maneuvers, [31, 32]. Inertial parameters are basic for simulations and control system design.

- Estimation of cornering stiffness: It is a key parameter for both simulation and controller design purposes. It may be estimated to generate mathematical models for tires, [1, 27, 33].

- Estimation of suspension parameters: In the mathematical modeling of coupled handling and roll dynamics, it is essential to use roll damping and stiffness of suspension system within the parameters. Estimation of them may be required for controller design purposes also. Ride comfort models also utilize suspension parameters, [2, 3, 34, 35].

Some other estimation practices include the estimation of terrain parameters, road grade, aerodynamic drag, lift coefficient, road bank angle, wind gust, rolling resistance, driver model parameters, etc.

Considering the identification of handling dynamics, most of the studies employ extended Kalman filter to estimate parameters together with the states.

This type of parameter estimation requires defining the parameters as additional states. Linear bicycle model is the commonly utilized model structure in most of the applications. Optimal input design is not considered in general. It is also seen that validation tests are generally limited in these studies.

In addition, identifiability and observability analysis are generally missing in these applications. On the other hand some studies carry sensitivity analysis to check the identifiability [36, 37]. Besides, genetic algorithm based optimization technique is not so usual in the identification practices of vehicle dynamics applications. These items may be considered as some of the motivating points in this thesis.

2.4 Utilized Hardware in the Identification of Vehicle Dynamics

Since the identification requires experimental input-output data, some tests should be conducted on the vehicle equipped with some special kind of hardware. Sensors, data acquisition system, some actuators, indicators, etc. are employed hardware components during the experiments.

- Data Acquisition System: In order to acquire the data during the experiment data acquisition system is required, figure 1. For identification purposes, applied inputs and sensor outputs should be acquired during the tests with the appropriate resolution. These systems need some types of software to process the data, make settings, etc. In some experiments data acquisition system should also be able to generate some actuation signals.

Figure 1 Data acquisition system [38]

- Steering Wheel angle and Torque Measurement System: In order to measure the steering input and applied steering wheel torque, a sensor system is required. Such systems are mounted in the steering system by replacing the existing steering wheel. In basic handling models steering input is the single input to the system. Besides, for constant forward velocity models of coupled roll and handling, it is also the only input to the system.

Figure 2 Steering wheel measurement system

- Wheel Speed Measurement System: Measurement of rotational wheel speeds may be used to detect the wheel slip that is used in tire models. Wheel speed measurement is required if longitudinal and lateral force interactions are included in tire force modeling part.

- Wheel Torque and Force Measurement System: Applied torque to the wheels is an input to the system. In addition, the forces on wheels are important variables of vehicle dynamics. Such sensor systems measure the vertical load on wheels and forces and moments generated by tires. These may be used for the identification of tire models of the vehicle by the experimental data acquired during road tests.

Figure 3 Wheel torque and tire force measurement system [39]

- Velocity Sensors: Measuring forward and lateral velocities is critical in experiments, fig. 4. Forward velocity may be kept constant in experiments such that it becomes a parameter of the system. Side slip velocity has small amplitude compared to the forward velocity. Output of the lateral velocity sensor might be conditioned by amplification to match the sensor output to A/D converter's input range.

- Inertial Measurement Units: Accelerations, rotational velocities, and orientations of the vehicle are required variables in most of the experiments.

Inertial measurement units are employed to measure these quantities, fig. 5. These units include 3 axes accelerometers, 3 axes gyroscopes, mostly magnetometers, and software to estimate some variables and filter out some undesired noise components. These units build up the basic sensor sets utilized in physical tests to acquire data for identification purposes.

- Steering Robot: Designed steering inputs can be applied to the vehicle by using steering robots. Figure 6 shows the one of the most popular steering robots used in physical experiments [42]. Closed loop experiments can also be conducted using inertial measurement systems together with this robot. Experiments may be designed to excite the vehicle with desired amplitude of roll or yaw motion using the feedback from inertial measurement unit.

In addition some inertial measurement units may include global positioning sensors (GPS). Closed loop experiments may be conducted using position feedback and vehicle can follow a predefined or designed path during the tests by the actuation of steering robots.

Figure 4 Optical velocity sensor [40]

Figure 5 Inertial measurement unit [41]

Figure 6 Steering robot [42]

CHAPTER 3

SCOPE OF THE THESIS

The study is focused on the identification of the handling and cornering dynamics of an automobile. The complicated mathematical model of an automobile will be calculated using experimental data of driving tests.

The need for an identified mathematical model is stated as follows.

3.1 Analysis of the handling dynamics

Straight road stability and performance of the vehicles were analyzed and improved in the previous years. Firstly braking scenarios were studied and researchers focused on the design of control systems to improve the stability and performance of the vehicles during braking. Anti blockage system (ABS) was the control system designed for this purpose. After the performance of such systems had reached a certain level, researchers began to deal with the case of accelerating on a straight road. Traction control systems (TCS) were designed to improve the vehicle performance

Next, researchers have been interested in cornering performance of vehicles. Control systems such as vehicle dynamics control (VDC), electronic stability control (ESP) have been designed to improve the cornering and handling stability and performances. The vehicle has the most complex dynamics when both steering and driving/braking inputs are applied. The analysis of such a motion requires complicated mathematical models, simulations and some road tests. Complex control systems for this dynamics can be designed and implemented using mathematical models describing the coupled dynamics of the vehicle. In the design stage of the vehicle mathematical models and computer simulations are used. However, all the parameters cannot be known in many cases. Therefore, identification of the mathematical models after manufacturing might be required and useful for analysis and design stages. This will enable to get higher performance and reach the available potential of the vehicle.

3.2 Replacement of impossible and/or dangerous road tests with simulations

The performance of a vehicle can be tested on special test areas. Some special inputs are applied to the vehicle by test pilots. The limits of the vehicle might be observed during these tests. However, such experiments might be dangerous for the pilot in the test vehicle.

Even in standard tests, the pilot cannot apply the same inputs on the same road for each trail throughout the experiment. This might affect the results and performance of the experiments. For example, testing the performance of a new part or modifications may not be achieved in the desired manner. Comparisons may not be healthy in such cases. Mathematical models of the vehicle can be used in the simulations of the road tests that are dangerous or difficult to repeat for each time.

In addition to this, test areas for all kinds of driving experiments are not available in our country. Vehicles have to be sent abroad for these tests to be carried out or they might be altogether ignored in some cases. Therefore, it becomes a necessity to carry these tests in computer simulations to minimize the numbers of actual road tests. The mathematical model for these simulations can be identified using available and relatively simple road experiments but these models should reflect the limiting behavior and characteristics of the vehicle.

3.3 Design and modification of systems

Road conditions and constraints in Turkey are quite different from those of other countries. Vehicles designed for European conditions are manufactured and sold for use in our country. In such cases, some systems need to be modified. In particular, suspension systems require some modifications to suit the roads and improve the ride comfort of the passengers. It should be mentioned that the suspension system also has some affects on the tractive and handling performances. Modifications in suspension parameters to improve the ride comfort may disturb the handling performance. Therefore, vehicle parameters should be optimized considering both ride and handling dynamics. This brings a need to use a mathematical model reflecting the effects of suspension parameters on ride and handling responses.

Considering these needs and applications, it is desired to identify and calculate mathematical model(s) of a designed and manufactured vehicle

- to be used in handling dynamics and cornering performance analysis,
- to be able to simulate road tests realistically,
- to be used in suspension optimization.

It is also desired to perform qualitative and quantitative design of the experiments considering the identifiability analysis and optimal input design.

The procedure is summarized in the following figure [6].

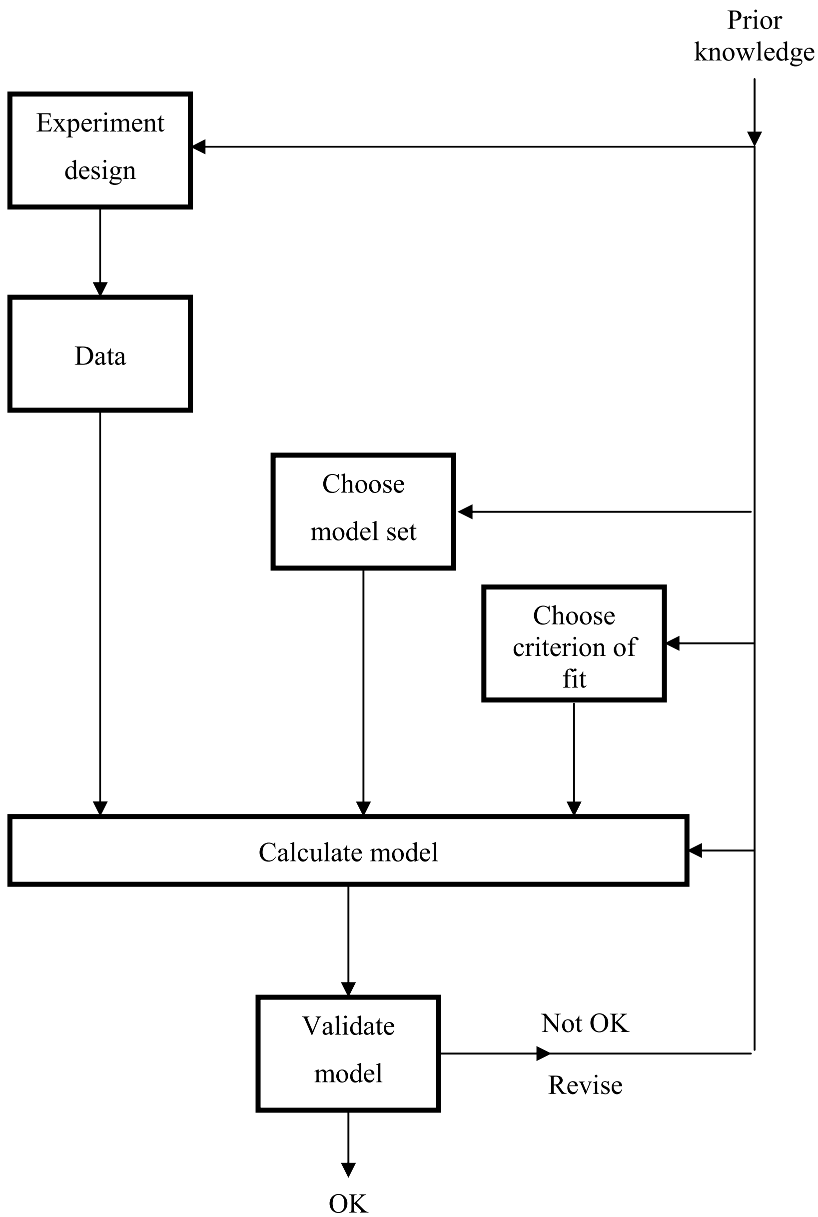


Figure 7 Steps of identication

CHAPTER 4

IDENTIFICATION OF 2 DEGREES OF FREEDOM HANDLING MODELS

The presentation of this chapter is organized in two parts. Firstly, structural global identifiability of a mathematical model, the classical linear bicycle model, with physical parameters is analyzed. In particular, identifiability of the physical parameter sets using individual sensors and different sets of sensors is examined in detail. In the second part, experiments carried on a vehicle equipped with a sensor set and data acquisition system and the use of measured data within the identification algorithm are described. Physical parameters of the bicycle model structure are estimated using prediction error estimation method. Genetic algorithms are used in the optimization phase of the identification algorithm to overcome the difficulty in the selection of initial values for parameter estimates. Validation analysis of the identified model is also presented.

A linear model of a vehicle has certain limitations, such as low lateral accelerations, say below 0.3g's. However, it still gives valuable information about the basic handling behavior of a vehicle without extensive measurements on the vehicle, and once identified can be used in simulations without need to carry out further tests on the road. To be able to simulate the motion of the vehicle at higher lateral accelerations, however, a somewhat more complicated nonlinear model will be necessary.

Linear bicycle model represents the motion of the vehicle under the following conditions; constant forward speed, low lateral acceleration, say below 0.3g's, and rigid suspensions [32]. These conditions result in the generation of linear tire cornering forces with the following relations.

$$\begin{aligned} F_{cf} &= C_f \, \alpha_f \\ F_{cr} &= C_r \, \alpha_r \end{aligned} \tag{1}$$

Figure 8 shows the schematic representation of the bicycle model. A set of moving axes is attached to the center of gravity of the vehicle. The plane motions of the vehicle are represented in terms of the forward, side slip, and yaw velocities.

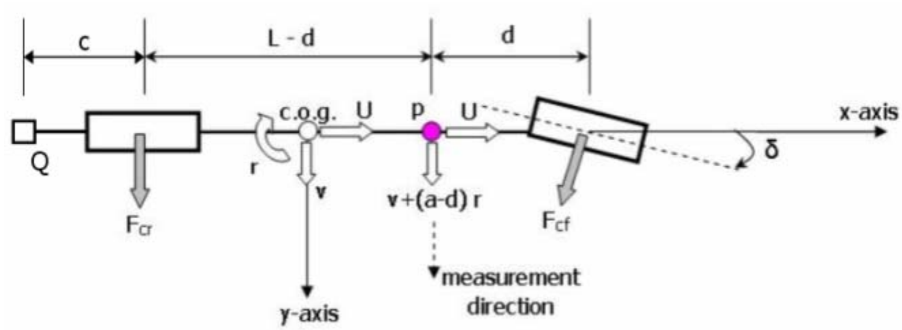


Figure 8 Schematic representation of bicycle model

By applying the Newton's laws of motion, continuous-time state equations for the bicycle model are derived and given below.

$$\begin{bmatrix} \dot{v} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \dfrac{C_f + C_r}{m\,U} & \dfrac{a\,C_f - b\,C_r}{m\,U} - U \\ \dfrac{a\,C_f - b\,C_r}{J\,U} & \dfrac{a^2\,C_f + b^2\,C_r}{J\,U} \end{bmatrix} \begin{bmatrix} v \\ r \end{bmatrix} + \begin{bmatrix} -\dfrac{C_f}{m} \\ -\dfrac{a\,C_f}{J} \end{bmatrix} \delta \tag{2}$$

4.1 Output equations

In order to complete state-space model of the vehicle, output equations should be given. Three sensors are assumed to be placed on the vehicle to represent the output of the system; an accelerometer to measure the lateral acceleration, a gyroscope to measure the yaw rate, and a velocity sensor to measure the side slip velocity. Locations of the sensors on the vehicle determine the format of the output equations. It is assumed that the position of the vehicle center of gravity is unknown. Therefore the accelerometer for lateral acceleration is assumed to be placed at some point P on the body fixed x-axis. Its position with respect to the front axle is given by parameter d in figure 1. The output equation for the accelerometer is given below.

$$A_y^P = C_{acc}\begin{bmatrix} v \\ r \end{bmatrix} + D_{acc}\delta \tag{3}$$

In this equation, the coefficients of the state vector and the steering input are given by :

$$\begin{aligned} C_{acc} &= \left[\frac{C_f + C_r}{m\,U} + (a-d)\frac{(a\,C_f - b\,C_r)}{J\,U} \quad \frac{a\,C_f - b\,C_r}{m\,U} + (a-d)\frac{(a^2\,C_f + b^2\,C_r)}{J\,U} \right] \\ D_{acc} &= -\frac{C_f}{m} - (a-d)\frac{a\,C_f}{J} \end{aligned} \tag{4}$$

The gyroscope can be placed anywhere on the vehicle under the constraint that its measurement axis is perpendicular to the body x-y plane. Output equation for the gyroscope can be written as follows:

$$R = C_{gyro}\begin{bmatrix} v \\ r \end{bmatrix} + D_{gyro}\,\delta \tag{5}$$

where

$$\begin{aligned} C_{gyro} &= \begin{bmatrix} 0 & 1 \end{bmatrix} \\ D_{gyro} &= 0 \end{aligned} \tag{6}$$

Velocity sensor can be placed at the rear end of the vehicle that is placed a distance *c* away from the rear axle, at point Q, as illustrated in figure 1. Output equation is given as below:

$$v_Q = C_v\begin{bmatrix} v \\ r \end{bmatrix} + D_v\,\delta \tag{7}$$

where

$$\begin{aligned} C_v &= \begin{bmatrix} 1 & -(L + c + a) \end{bmatrix} \\ D_v &= 0 \end{aligned} \tag{8}$$

State-space model of the vehicle is completed by the output equations of the corresponding sensors or sensor sets. Unknown parameters of the model to be estimated are {a, J, C_f, C_r}. Other parameters {m, U, d, c} are either easy to measure or known. Once the parameter set to be identified is selected, the answers must be found to the questions:

-Which sensors should be used in the experiments to reach this set correctly?

-Is the available set of sensors sufficient to estimate the selected parameters uniquely?

Answers to these questions require a detailed identifiability analysis of the model structure and they constitute the frame of the qualitative experiment design for identification.

4.2 Structural identifiability analysis

There are various techniques to analyse the identifiability of the model structures [10, 11, 43, 44]. For linear model structures (i.e. linear in input) there are two main techniques to check the structural identifiability. The first one is the transfer function method and the second one is based on the similarity transformation approach [10]. In this study the former approach is employed. However, the latter should also give same results due to the linearity of the model. The basic definitions for structural identifiability are provided below [43].

<u>Definition 1:</u> The parameter θ_i is structurally globally identifiable for the input class $\mathcal{U}$ if, and only if, for almost any $\theta \in D_M$ one has

$$\left.\begin{array}{l} \theta^* \in D_M \\ y^*(\theta^*, t) \equiv y(\theta, t), \forall t \in IR^+, \forall u \in \mathcal{U} \end{array}\right\} \Rightarrow \theta_i^* = \theta_i$$

where u is the input to the model and y is the output of the model.

<u>Definition 2:</u> The model $M(\theta)$ is structurally globally identifiable if, and only if, all its parameters θ_i are structurally globally identifiable.

Since the model is linear, definitions above can be regarded as follows.

Model structure $G(s,\theta)$ is globally identifiable at θ^* if

$$G(s,\theta) = G(s,\theta^*), \quad \theta \in D_M \Rightarrow \theta = \theta^* \tag{10}$$

where G(s) is the transfer function between the input and the output.

Identifiability analysis is first carried for each individual sensor. Parameters that are identifiable by the use of each sensor are determined. $G_1(s)$, $G_2(s)$ and $G_3(s)$ are the transfer functions of the system between the steering input and lateral acceleration at point P, yaw rate, and side slip velocity at point Q, respectively. All relevant nonlinear equation sets in identifiability analysis are analysed by linearising them about point θ^*. Jacobian matrix of the equation set with respect to θ is formed. Uniqueness of the solution is concluded by the determinant of the Jacobian. A detailed analysis is given in following sections. Also, each nonlinear equation system is solved by MatLab and/or Maple symbolic solvers. According to the uniqueness of solutions, global identifiabilty of the structures are concluded.

4.2.1 Accelerometer at point P

For the accelerometer, the analysis of structural identifiability starts as given below using the definitions:

$$G_1(s,\theta) = G_1(s,\theta^*), \quad \theta \in D_M \Rightarrow \theta = \theta^* \tag{11}$$

where $\theta = \{a, J, C_f, C_r, U\}$

$$\frac{c_1 s^2 + c_2 s + c_3}{s^2 + c_4 s + c_5} = \frac{c_1^* s^2 + c_2^* s + c_3^*}{s^2 + c_4^* s + c_5^*} \tag{12}$$

Coefficients c_i, i=1,..,5 are nonlinear functions of the physical variables.

$$c_1 = C_f \, \frac{m\, a\, (d-a) - J}{m\, J} \tag{13}$$

$$c_2 = \frac{C_f \, C_r \, L \, (L-d)}{m\, U\, J} \tag{14}$$

$$c_3 = \frac{C_f\ C_r\ L}{m\ J} \tag{15}$$

$$c_4 = -\frac{a^2\ C_f + b^2\ C_r}{U\ J} - \frac{C_f + C_r}{U\ m} \tag{16}$$

$$c_5 = \frac{C_f\ C_r\ L^2}{m\ J\ U^2} - \frac{b\ C_r - a\ C_f}{J} \tag{17}$$

Equation (12) gives 5 nonlinear algebraic equations in terms of the physical parameters of the system. These nonlinear equations are generated as follows.

$$c_i = c_i^*,\ i = 1\text{-}5 \tag{18}$$

Equations (18) can be written in terms of the physical parameters.

$$C_f\ \frac{m\ a\ (d - a) - J}{m\ J} = C_f{}^*\ \frac{m\ a^*\ (d - a^*) - J^*}{m\ J^*} \tag{19}$$

$$\frac{C_f\ C_r\ L\ (L - d)}{m\ U\ J} = \frac{C_f^*\ C_r^*\ L\ (L - d)}{m\ U\ J^*} \tag{20}$$

$$\frac{C_f\ C_r\ L}{m\ J} = \frac{C_f^*\ C_r^*\ L}{m\ J^*} \tag{21}$$

$$\frac{a^2\ C_f + (L - a)^2\ C_r}{U\ J} + \frac{C_f + C_r}{U\ m} = \frac{a^{*2}\ C_f{}^* + (L - a^*)^2\ C_r{}^*}{U\ J^*} + \frac{C_f{}^* + C_r{}^*}{U\ m} \tag{22}$$

$$\frac{C_f\ C_r\ L^2}{m\ J\ U^2} - \frac{(L - a)\ C_r - a\ C_f}{J} = \frac{C_f^*\ C_r^*\ L^2}{m\ J^*\ U^2} - \frac{(L - a^*)\ C_r{}^* - a\ C_f{}^*}{J^*} \tag{23}$$

If there exists a unique solution for the parameter set such that $\theta = \theta^*$, then model structure in Equation (12) is structurally globally identifiable. In fact, there are 5 equations and 4 unknowns considering the equation system above. It means one additional parameter can be included in the parameter set θ. Let's assume that the forward velocity of the vehicle, U, is unknown. Determinant of the Jacobian of the system formed by Equations (19) to (23) is nonzero. Therefore a unique solution should be expected for the system. These equations are solved and a unique solution in the following form resulting in an identifiable model structure is obtained.

$$a = a^*,\ J = J^*,\ C_f = C_f^*,\ C_r = C_r^*,\ U = U^*$$

Similarly, instead of forward velocity, parameter d that specifies the location of the accelerometer can be set as an unknown parameter. Parameter set to be identified becomes $\theta = \{a, J, C_f, C_r, d\}$. Nonzero determinant of the Jacobian indicates the existence of a unique solution and the nonlinear set of equations gives a unique solution in the following form resulting in a globally identifiable model structure.

$$a = a^*,\ J = J^*,\ C_f = C_f^*,\ C_r = C_r^*,\ d = d^*$$

4.2.2 A Gyro

Identifiability analysis for a single gyro employs the following equations.

$$G_2(s,\theta) = G_2(s,\theta^*),\quad \theta \in D_M \Rightarrow \theta = \theta^* \tag{24}$$

where $\theta = \{a, J, C_f, C_r\}$.

$$\frac{c_6 s + c_7}{s^2 + c_4 s + c_5} = \frac{c_6^* s + c_7^*}{s^2 + c_4^* s + c_5^*} \tag{25}$$

Coefficients c_i, i=4,..,7 are nonlinear functions of the physical variables. c_4 and c_5 are given in Equations (16) and (17), and c_6 and c_7 are given below.

$$c_6 = \frac{-a\ C_f}{m\ J} \tag{26}$$

$$c_7 = \frac{C_f\ Cr\ L}{m\ J\ U} \tag{27}$$

Equation (25) gives 4 nonlinear algebraic equations in terms of the physical parameters of the system. These nonlinear equations are generated as follows.

$$c_j = c_j^*,\ j{=}4\text{-}7 \tag{28}$$

In terms of the physical parameters, Equations (28) can be written in the form of nonlinear algebraic equations as follows.

$$\frac{a^2\ C_f + (L-a)^2\ C_r}{U\ J} + \frac{C_f + C_r}{U\ m} = \frac{a^{*2}\ C_f^{\ *} + (L-a^*)^2\ C_r^{\ *}}{U\ J^*} + \frac{C_f^{\ *} + C_r^{\ *}}{U\ m} \tag{22}$$

$$\frac{C_f\ C_r\ L^2}{m\ J\ U^2} - \frac{(L-a)\ C_r - a\ C_f}{J} = \frac{C_f^{\ *}\ C_r^{\ *}\ L^2}{m\ J^*\ U^2} - \frac{(L-a^*)\ C_r^{\ *} - a\ C_f^{\ *}}{J^*} \tag{23}$$

$$\frac{a\ C_f}{m\ J} = \frac{a^*\ C_f^{\ *}}{m\ J^*} \tag{29}$$

$$\frac{C_f\ Cr\ L}{m\ J\ U} = \frac{C_f^{\ *}\ Cr^*\ L}{m\ J^*\ U} \tag{30}$$

There are 4 equations and 4 unknowns in the expressions. Determinant of the Jacobian of the system formed by related equations is nonzero. Therefore a unique solution should be expected for the system. These equations are solved and they give unique solution in such a form that $\theta = \theta^*$ resulting in identifiable model structure. Therefore the model structure in Equation (25) is globally identifiable. Similar to the single accelerometer case, theoretically a single gyro may be enough to reach a unique set of the physical parameter set.

4.2.3 Velocity sensor at point Q

Identifiability analysis for a single velocity sensor results in the following equations.

$$G_3(s,\theta) = G_3(s,\theta^*), \quad \theta \in D_M \Rightarrow \theta = \theta^* \tag{31}$$

where $\theta = \{a, J, C_f, C_r\}$.

$$\frac{c_8 s + c_9}{s^2 + c_4 s + c_5} = \frac{c_8^* s + c_9^*}{s^2 + c_4^* s + c_5^*} \tag{32}$$

Coefficients c_i, i=4, 5, 8, 9 are nonlinear functions of the physical variables. c_4 and c_5 are given by Equations (16) and (17), and the expressions for c_8 and c_9 are given below.

$$c_8 = C_f \frac{m\, a\, (a + c + L) - J}{m\, J} \tag{33}$$

$$c_9 = C_f \frac{(m\, a\, U^2) - C_r\, L\, (2\, a + c)}{m\, J\, U} \tag{34}$$

Equation (32) gives 4 nonlinear algebraic equations in terms of the physical parameters of the system. These nonlinear equations are generated as follows.

$$c_k - c_k^* = 0, \text{ k=4,5,8,9} \tag{35}$$

Equation set is modified in such a way that numerators of the nonlinear equations above form the new nonlinear algebraic equation system. Each denominator of the original system contains parameter J, resulting in a condition that J should not be equal to zero. Nonlinear equations are solved by

Maple and there exists 4 solutions for the system. Each solution has J = 0 term and none of them gives $\theta = \theta^*$. J = 0 makes the determinant of the Jacobian of the new equation system zero. Therefore this model structure consisting of the given output equation is not globally identifiable with the parameter set θ.

Parameter set is reduced to have 3 physical parameters. All alternatives with 3 equations and 3 unknowns are analysed and no unique solution of nonlinear equation set is reached. It is of interest to note that at most two parameters can be identifiable. Thus, in order to have globally identifiable parameters using a single velocity sensor at point Q, following combinations are possible.

$\{C_r, a\}$ are assumed to be known (in addition to m, U, and c): Then $\{C_f, J\}$ are identifiable.

$\{C_f, a\}$ are assumed to be known (in addition to m, U, and c): Then $\{C_r, J\}$ are identifiable.

$\{a, J\}$ are assumed to be known (in addition to m, U, and c): Then $\{C_f, C_r\}$ are identifiable.

$\{C_f, C_r\}$ are assumed to be known (in addition to m, U, and c): Then $\{a, J\}$ are identifiable.

4.2.4 Accelerometer at point P and a gyro

It is seen that the use of a single accelerometer or a single gyro is sufficient to have an identifiable model structure when the mass and the forward speed of the vehicle are known, and d is measured. If both an accelerometer and a gyro are used together in identification experiments, there exist 7 nonlinear algebraic equations that come from the transfer functions in the identifiability analysis. In this case, one may question if it is possible to estimate some other parameters, in addition to the 4 physical parameters previously selected. In order to answer this question and to discover the potential of such a sensor set, it is assumed that the mass of the vehicle, forward speed, and the distance of the point P to the front axle are also unknown. Using an accelerometer and a gyro forms the most probable sensor set that can be used in the practical applications and experiments due to the in market sensor sets combining accelerometers and gyros.

In such a case, equations (19) to (23), (29), and (30) form the set of nonlinear algebraic equations that are used for the identifiability analysis of the modified parameter set:

$\theta = \{m, a, J, C_f, C_r, U, d\}$

To analyse the set of equations they are linearised about θ^*.

$$J_M = \begin{bmatrix} \frac{\partial c_1}{\partial \theta_1} & \frac{\partial c_1}{\partial \theta_2} & \cdots & \frac{\partial c_1}{\partial \theta_7} \\ \frac{\partial c_2}{\partial \theta_1} & \ddots & & \frac{\partial c_2}{\partial \theta_7} \\ \vdots & & \ddots & \vdots \\ \frac{\partial c_7}{\partial \theta_1} & \cdots & \cdots & \frac{\partial c_7}{\partial \theta_7} \end{bmatrix}_{\theta^*} \tag{36}$$

where J_M is the Jacobian matrix and θ_j is the j^{th} element of θ.

The linearized system can be written as

$$J_M \theta^T = c^*$$

where $c^* = \{c_1^*, \cdots, c_7^*\}^T$.

For the given θ, it can be shown that determinant of the Jacobian matrix, J_M, is zero. This may result in inconsistency or infinitely many solutions for the linearized system. It is thus not suprising to see that the nonlinear symbolic solver in MatLab™/Maple™ gives infinitely many solutions for the 7 equations-7 parameters system above.

Considering the linearized system a 6x6 (6 equations, 6 parameters) subsystem is searched within the equation system. All of the 6x6 submatrices are formed and determinants of each of 49 submatrices are summarized in the following matrix.

$$S_{det} = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{17} \\ d_{21} & \ddots & & d_{27} \\ \vdots & & \ddots & \vdots \\ d_{71} & \cdots & \cdots & d_{77} \end{bmatrix}$$

where d_{ij} represents the determinant of the submatrix formed by removing the i^{th} row and j^{th} column of J. It is indicated below whether the determinant of each 6x6 subsystem is zero (Z) or nonzero (NZ).

$$S_{det} = \begin{bmatrix} NZ & Z & NZ & NZ & NZ & Z & Z \\ NZ & Z & NZ & NZ & NZ & Z & Z \\ NZ & Z & NZ & NZ & NZ & Z & Z \\ NZ & Z & NZ & NZ & NZ & Z & Z \\ NZ & Z & NZ & NZ & NZ & Z & Z \\ NZ & Z & NZ & NZ & NZ & Z & Z \\ NZ & Z & NZ & NZ & NZ & Z & Z \end{bmatrix} \tag{37}$$

For example if the first row (first equation) is rejected and the first parameter, mass, is assumed to be known; the determinant of the related submatrix is nonzero, in equation (38). Thus there exists a possibility of a unique solution for the 6x6 system. This nonlinear system has a unique solution when the mass is known and $m = m^*$.

$$a = a^*,\ J = J^* \frac{m}{m^*},\ C_f = C_f^* \frac{m}{m^*},\ C_r = C_r^* \frac{m}{m^*},\ U = U^*,\ d = d^*$$

It is concluded that when mass is known and $m = m^*$, then the rest of the parameters are identifiable. The model structure is structurally identifiable for all θ^*. Therefore it is globaly identifiable.

S_{det} provides some interesting results. For example, if the third element in the parameter set is known, and any one of the equations is eliminated; then the model structure is globally identifiable. The same conclusion is reached if either C_f or C_r is known. If only U or d is known, however, then the remaining parameter set is not identifiable.

4.2.5 Velocity sensor at Q and a gyro

A combination of a velocity sensor at point Q and a gyro is now considered. Transfer functions related to these sensors present six nonlinear algebraic equations, equations (22), (23), (29), (30), $c_8 = c_8^*$ and $c_9 = c_9^*$. Therefore the parameter set is enlarged to include 6 parameters,

$\theta = \{m, a, J, C_f, C_r, U\}$.

A similar analysis as in the previous part shows that if the mass of the vehicle is known, the remaining parameters can be determined uniquely and the model structure is globally identifiable.

4.3 Steering input

Steering angle of the front wheels is the only input to the bicycle model structure. It has a major impact on the quality of the data to be collected. The input should be rich enough to excite all the necessary modes of the system [45-47]. While considering the richness of the input, output and input amplitudes should be limited so as to assure the validity of the proposed linear model.

During the experiments, an experienced test pilot applied the steering inputs with different shapes and amplitudes. No attempt has been made to optimize the applied steering inputs used in the experimental runs. However, the richness of the inputs are checked according to [6] and [48].

It is known that the applied input is persistently exciting of order n, if the power spectrum is different from zero at, at least, n points in the interval $-\pi < \omega \leq \pi$. Assume that the model is given in transfer function form and the numerator and the denominator of the model structure have the same degree n; then the input should be persistently exciting of order 2n+1. This requires that the power spectrum should be nonzero at 2n+1 points in the interval $-\pi < \omega \leq \pi$ [6, 18]. However, rich inputs are not necessarily optimum.

Richness of the applied input that is used in the identification algorithm, shown in Figure 9a, is checked using its power spectrum in Figure 9b. It is seen that the power spectrum satisfies the condition of persistent excitation.

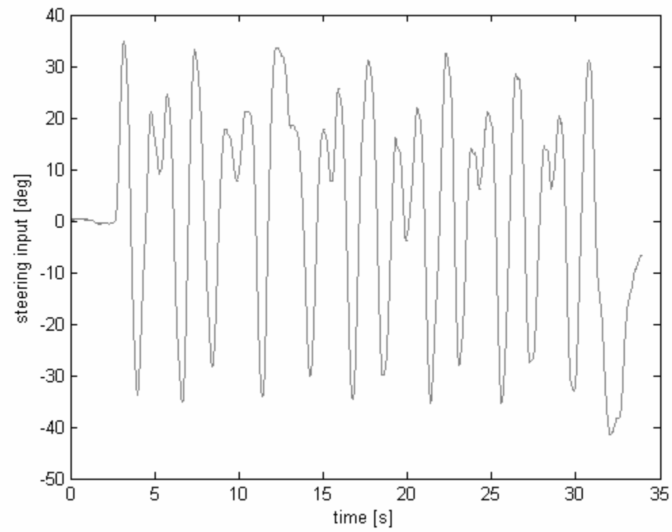


Figure 9.a Sample rich steering input

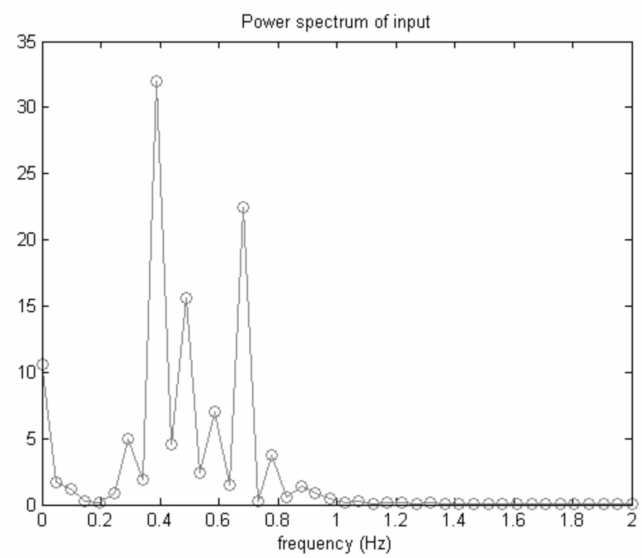


Figure 9.b Power spectrum of the sample steering input

4.4 Experiments

Test vehicle is instrumented with the following sensors:

i) Inertial platform that contains three accelerometers and three gyroscopes, figure 3. Only one of the accelerometers and one of the gyroscopes are employed for the identification purposes. Remaining sensors are utilized to check test conditions.

ii) Forward velocity sensor.

iii) Steering wheel sensor to measure the input.

Figure 10 Inertial platform at point P on the experimental vehicle

Steering wheel provides the input to the system and it should be measured. Forward velocity is kept constant during experiments and becomes a parameter of the model. Therefore, in addition to the accelerometer and the gyroscope, a steering wheel angle sensor and a forward velocity sensor are mounted on the vehicle as shown in Figures 11 and 12. The data acquisition system is shown in Figure 13.

Figure 11 Forward and lateral velocity sensors, at point Q, on the experimental vehicle

Figure 12 Steering wheel system on the experimental vehicle

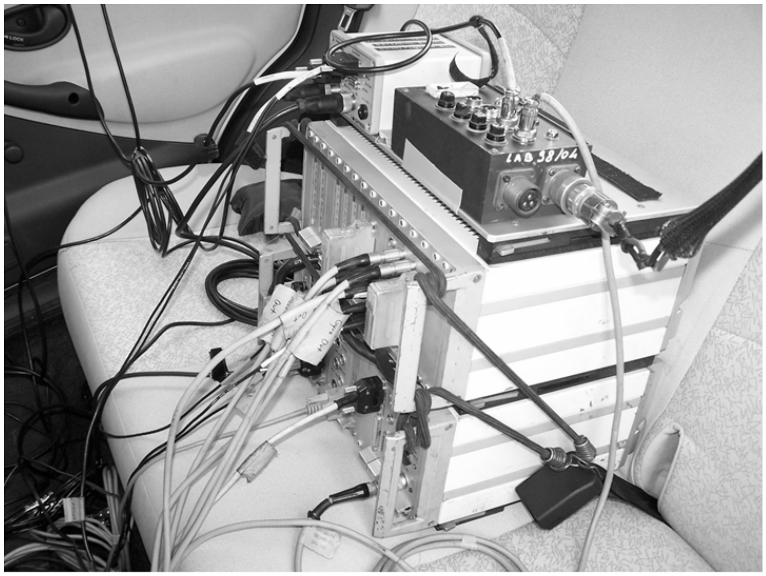


Figure 13 Data acquisition system

Limitations of the road restrict the input shape to be applied during the tests. Tests require a certain straight region of the test circuit to collect data for identification. Length of the straight region determines the duration of the test.

Data is collected with a sampling frequency of 100 Hz. This is far away from the expected bandwidth of the yaw and lateral dynamics of the vehicle that is less than 4 Hz. Experiment durations are between 30 to 50 seconds.

During the experiments various constant forward velocities are set. As the selected constant forward velocity is increased, the amplitude of the steering input is reduced to stay within the conditions for which the linear bicycle model is valid.

Designed steering inputs have not been used in the experiments. Because there is not an actuation system available that tracks the designed inputs via computer. It has been aimed to generate persistent inputs to the vehicle by the test pilot and use of some indications on the steering wheel, fig. 12. Pilot has tried to apply periodic inputs to the system by using 4 or 5 indicated steering angles (to track a combination of 2 or 3 sinusoids as much as possible!). The persistence of excitation can be checked by the power spectrum of the input signal [6], [48].

A sample set of the input and the corresponding measurements are given in Figures 14 to 21.

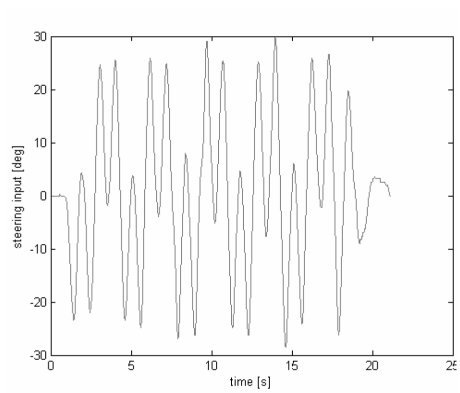


Figure 14 Steering input

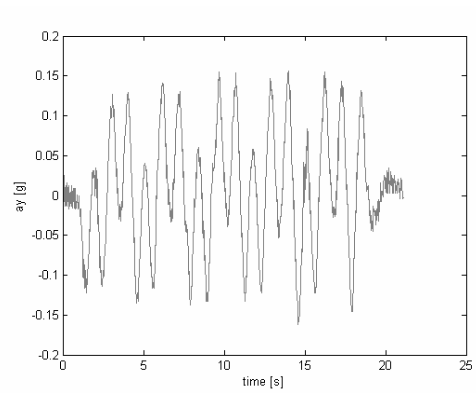


Figure 15 Lateral acceleration

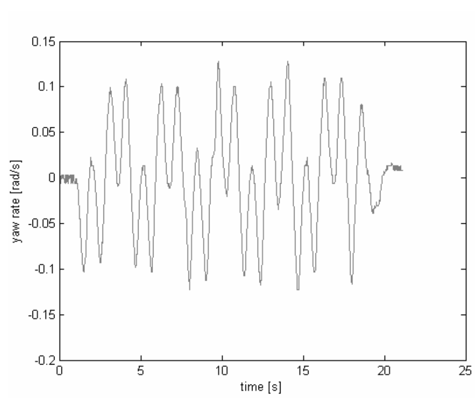


Figure 16 Yaw rate

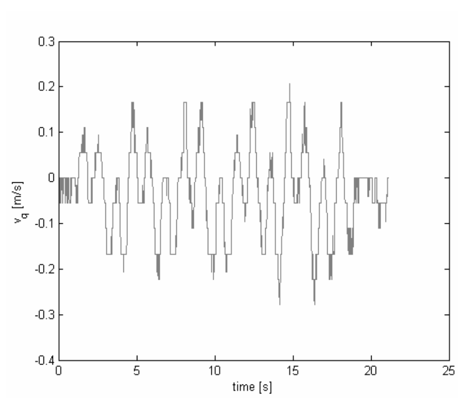


Figure 17 Lateral velocity at point Q

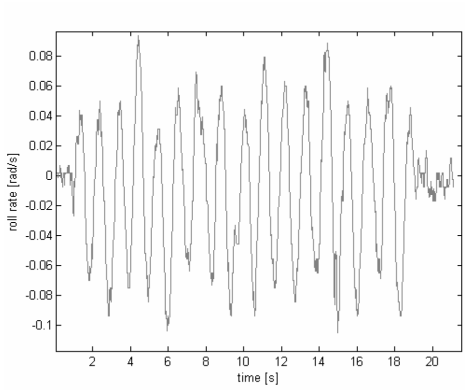


Figure 18 Roll rate

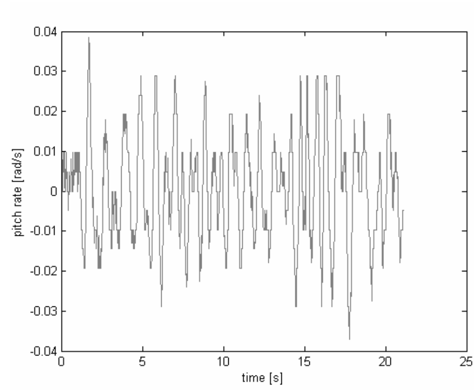


Figure 19 Pitch rate

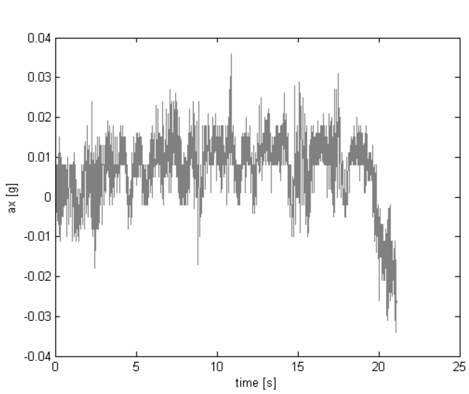


Figure 20 Forward acceleration

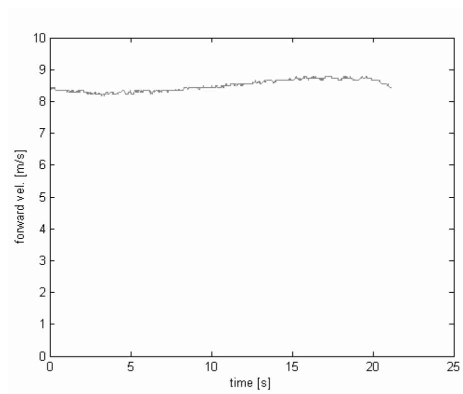


Figure 21 Forward velocity

4.5 Identification algorithm

Outputs of the accelerometer to measure the lateral acceleration and gyro placed at point P are processed in the identification algorithm to estimate parameters. Three cases, namely; use of a single accelerometer, a single gyro and a sensor set composed of an accelerometer and a gyro are considered separately in experimental measurements for the identification algorithm. Output equation for each case is given below.

$$y_{acc} = C_{acc}\begin{bmatrix} v \\ r \end{bmatrix} + D_{acc}\delta + w_a$$: lateral acceleration measurement at point P.

$$y_{gyro} = C_{gyro}\begin{bmatrix} v \\ r \end{bmatrix} + D_{gyro}\delta + w_g$$: yaw velocity measurement.

$$y_{set} = \begin{bmatrix} y_{acc} \\ y_{gyro} \end{bmatrix} = C_{set}\begin{bmatrix} v \\ r \end{bmatrix} + D_{set}\delta + \begin{bmatrix} w_a \\ w_g \end{bmatrix}$$: lateral acceleration and yaw velocity measurements where,

$$C_{set} = \begin{bmatrix} \frac{C_f + C_r}{m\,U} + (a-d)\frac{(a\,C_f - b\,C_r)}{J\,U} & \frac{a\,C_f - b\,C_r}{m\,U} + (a-d)\frac{(a^2\,C_f + b^2\,C_r)}{J\,U} \\ 0 & 1 \end{bmatrix}$$
$$D_{set} = \begin{bmatrix} -\frac{C_f}{m} - (a-d)\frac{a\,C_f}{J} \\ 0 \end{bmatrix} \tag{38}$$

Prediction error method is used to identify the models. This method requires the representation of a predictor model in the form [6]:

$$\hat{y}_m(t \mid t-1) = f\left(Z^{t-1}\right) \tag{39}$$

with the input-output data $Z^N = \{ u(1), y(1), u(2), y(2), ..., u(N), y(N) \}$.

The predictor is parameterized in the following form:

$$\hat{y}_m(t \mid \theta) = f\left(Z^{t-1}, \theta\right) \tag{40}$$

Here, θ is the set of the parameters to be estimated. The estimation problem is transformed into a minimization problem of the following type :

$$\hat{\theta}_N = \arg\min_{\Theta} V_N(\hat{\theta})$$
$$V_N(\hat{\theta}) = \sum_{t=1}^{N} \left\| \left(y(t) - f\left(Z^{t-1}, \hat{\theta} \right) \right) \right\|^2 \tag{41}$$

The utilized predictor in this study is a discrete time state space model in directly parameterized innovations form.

$$\hat{x}(t+1,\theta) = A(\theta)\,\hat{x}(t) + B(\theta)\,u(t) + K(\theta)\left[y(t) - C(\theta)\,\hat{x}(t,\theta) - D(\theta)\,u(t)\right]$$
$$\hat{y}(t|\theta) = C(\theta)\,\hat{x}(t,\theta) + D(\theta)\,u(t) \tag{42}$$

$$\text{prediction error}: e(t) = y(t) - C(\theta)\,\hat{x}(t,\theta) - D(\theta)\,u(t) \tag{43}$$

In the foregoing predictor, matrices A, B, C, and D are discrete time approximations of the matrices in continuous time bicycle model equations.

K matrix is the Kalman gain which depends on the matrices A and C together with the covariances of process and measurement noise. Instead of using those parameters, K can be parameterized directly in terms of θ. Predictor then becomes a much simpler function of θ. In the use of a single accelerometer Kalman gain matrix is given as $K = [k_{11}\ k_{21}]^T$. Similarly, in identification using a single gyro, Kalman gain becomes $K = [k_{11}\ k_{21}]^T$. Finally, using accelerometer and gyro together, K takes the following form.

$$K = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

Parameter sets to be estimated in the predictors are given as:

- using a single accelerometer: $\theta = \{a, J, C_f, C_r, k_{11}, k_{21}\}$,
- using a single gyro: $\theta = \{a, J, C_f, C_r, k_{11}, k_{21}\}$,
- using accelerometer and gyro together: $\theta = \{a, J, C_f, C_r, k_{11}, k_{12}, k_{21}, k_{22}\}$.

The cost function to be minimized is given in equation (3):

$$V_N(\hat{\theta}) = \frac{1}{N}\sum_{i=1}^{N}\left[e_1^{\ 2}(i) + e_2^{\ 2}(i)\right] \tag{44}$$

where $e_1(.)$ and $e_2(.)$ are the elements of prediction error vector.

Determination of the initial values for the numerical optimization algorithms presents an important problem in such cases. In addition, estimation of the physical parameters increases the local minima and the optimization surface becomes more complex [8, 9]. Strong dependence of the estimate values on the initial values and the presence of local minima problem have necessitated the use of the genetic algorithm (GA) for optimization purposes. GA requires many trials for the selection of size and range of initial population, cross over fraction, type of mutation, etc., so as to converge to some solution. However, even when the identifiability of the parameter set is guaranteed and GA is utilized, local minima still generate problems in the optimization process dealing with noisy data. Constraints and limits should be specified for the values of the parameters to be identified to ease the solution of convergence problems. This brings the need for prior information on the parameter set.

4.6 Estimated parameters

Values of parameter estimates using accelerometer and gyro data together are given in Table 1.

Table 1 Estimated parameters using both accelerometer and gyro data

C_f [N/rad]	C_r [N/rad]	a [m]	J [kg.m^2]	k_{11} [s]	k_{12} [m]	k_{21} [rad.s/m]	k_{22}
-39170	-24380	0.82	1900	-0.28	-0.44	-0.06	1.05

Data used in the identification algorithm is compared with the output of the identified predictor and continuous time bicycle model in Figures 22 and 23.

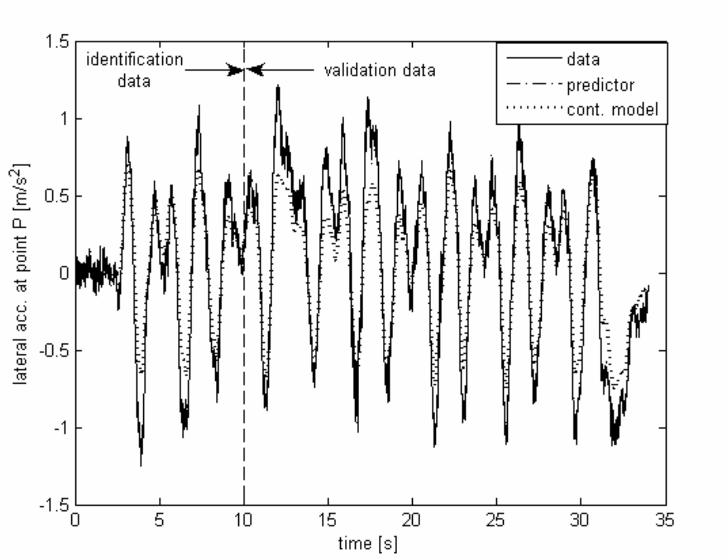


Figure 22 Comparison of predictor response, continuous model response and accelerometer output - both accelerometer and gyro

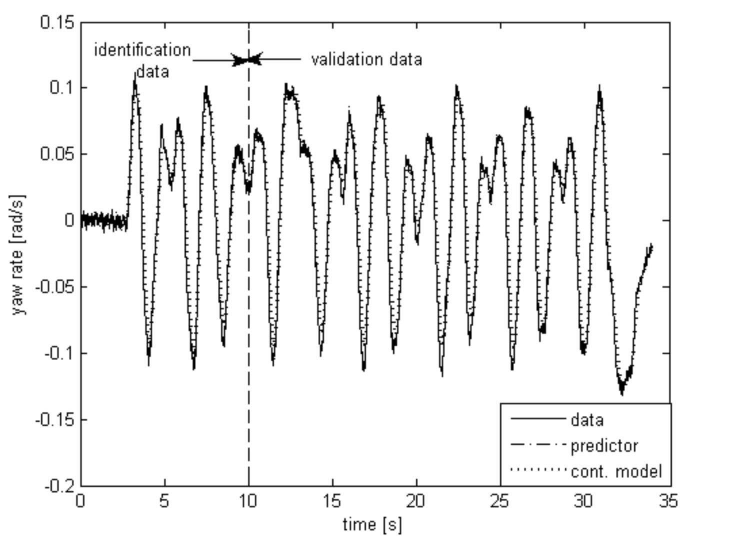


Figure 23 Comparison of predictor response, continuous model response and gyroscope output - both accelerometer and gyro

Data in the first 10 seconds is used in identification algorithm. Remaining data in the figures are used in the verification of the identified predictor. It is seen that the predictor response tracks the actual measurements very well throughout the identification duration.

Table 2 lists the estimated parameters using a single accelerometer. Some differences between the parameter estimates in Tables 1 and 2 are observed. It is noted that the cost functions obtained in 2 cases are also different. Cost function in case of using accelerometer and gyro together is about 9.6×10^{-4} whereas it is approximately 1×10^{-3} for the use of single accelerometer case. This situation is attributed to the presence of local minima which is more difficult to avoid with one sensor even though GA is used. Using accelerometer and gyro measurements together in identification algorithm improves the accuracy of the estimates.

Table 2 Estimated parameters using a single accelerometer

C_f	C_r	a	J	k_{11}	k_{21}
-37600 N/rad	-35970N/rad	0.98 m	1500 kg m^2	-0.30	0.17

Data used in the identification algorithm is compared with the output of the identified predictor and continuous time bicycle model in Figure 24.

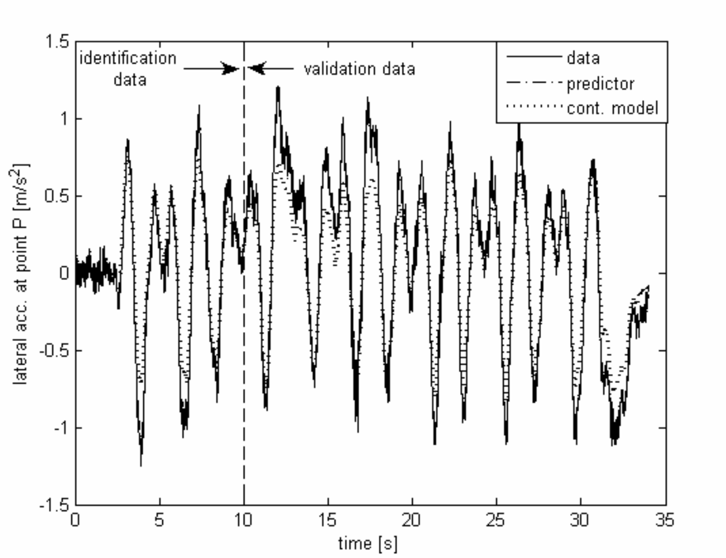


Figure 24 Comparison of predictor response, continuous model response and accelerometer output - single accelerometer

Finally estimated parameters of the vehicle model using a single gyro are given in Table 3.

Table 3 Estimated parameters using a single gyro

C_f	C_r	a	J	k_{11}	k_{21}
-56350 N/rad	-48480 N/rad	0.80 m	1320 kg m^2	1.03	1.72

Genetic algorithm allows the use of a population as initial guesses for the parameter estimates. It has the ability to evaluate many points in the parameter space. However, it carries a drawback in the convergence side of the optimization problem. The difference in values of estimated parameters in Tables 1-3 is mainly due to this property of genetic algorithm. The accuracy of the estimated parameters for each sensor set can be improved by combining genetic algorithm based optimization with gradient based techniques. Starting to search with genetic algorithm and then switching to a gradient based search algorithm is accessible to overcome both the initial value problem and subsequent local minima problems and the convergence problem. The characteristic of input may also outcome the difference in the values of parameters in Tables 1-3. The input is not designed to be optimal for estimation of parameters of concern.

Data used in the identification algorithm is compared with the output of the identified predictor and continuous time bicycle model in Figure 25.

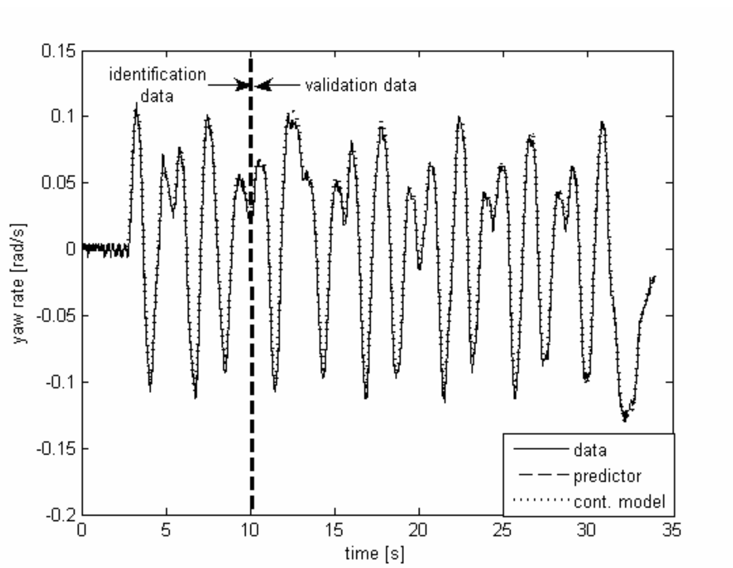


Figure 25 Comparison of predictor response, continuous model response and gyroscope output- single gyro

4.7 Model validation

Identified model is verified comparing its response with a measurement that has not been used within the identification algorithm. The response of the continuous time linear bicycle model with estimated parameters is compared with the measured data and predictor response in Figures 25 and 26 for the case of using accelerometer and gyro together. Validation data set belongs to a test with constant forward speed of 7 m/s which is different from the identification set which has been obtained at a constant forward speed of 5.6 m/s.

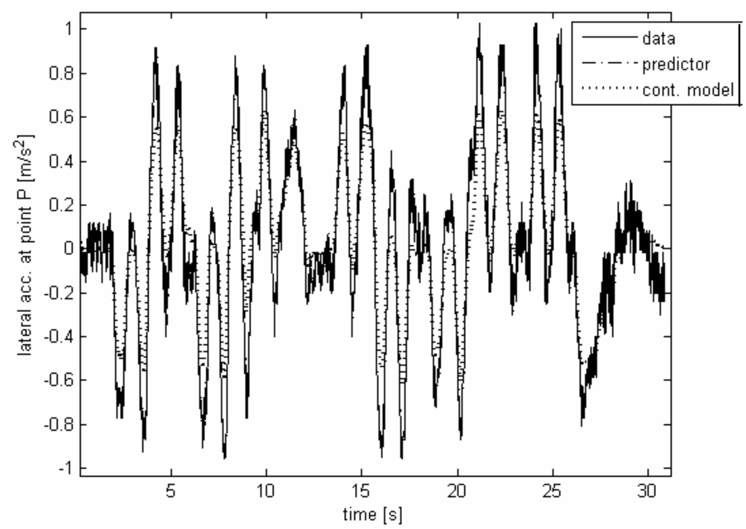


Figure 25 Comparison of predictor response, continuous time model response, and accelerometer output - both accelerometer and gyro

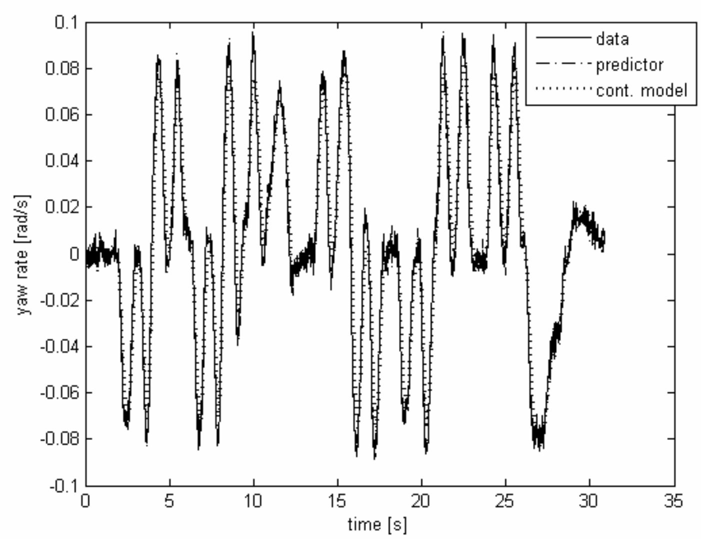


Figure 26 Comparison of predictor response, continuous time model response, and gyro output - both accelerometer and gyro

Identified predictor and corresponding continuous time model responses in case of using a single accelerometer are also compared with the validation data set in Figure 27.

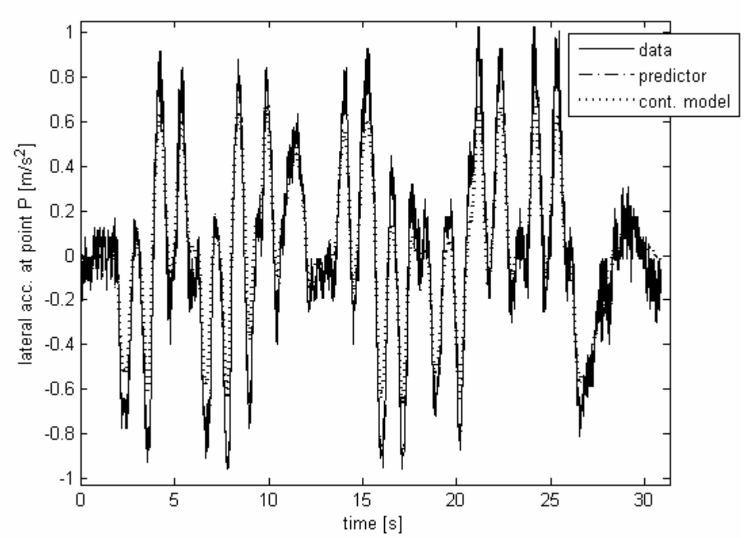


Figure 27 Comparison of predictor response, continuous time model response, and accelerometer output –single accelerometer

Identified predictor and corresponding continuous time model responses in case of using a single gyroscope are also compared with the validation data set in Figure 28.

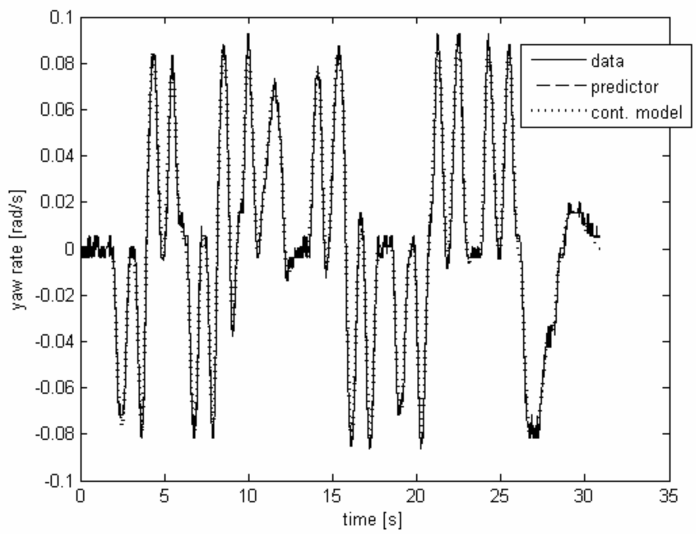


Figure 28 Comparison of predictor response, continuous time model response, and gyroscope output – single gyro

4.8 Optimal Input Desıgn

Qualitative part of the experiment design deals with the inputs and outputs regarding the identifiability property. On the other hand design of the inputs is the subject of quantitative design. Shape and duration of the inputs, sampling time, limitations and constraints on the amplitudes are all determined in this stage. In order to design an optimal input generally nominal values for the parameters –apriori information about the system- are utilized.

Steering input to the wheels is the only input to the model. It has a major impact on the quality of the data. The input should be rich enough to excite all necessary modes of the system [45], [69]. While considering the richness of the input, some constraints should be kept in mind also. Output and input amplitudes should be limited so as to assure the validity of the proposed model.

In literature there are some alternative methods to optimize inputs for identification [6], [45], [48], [50]. Information content of the data is expressed in some kind of matrices such as Fisher's information matrix. Errors in parameter estimates are related with these matrices. The cost function to be minimized is expressed in terms of the elements of the information matrix.

In this report the methodology given is based on the study of Morelli that is achieved at NASA for the identification of high performance aircrafts [46, 49]. In that study the input is designed so that sensitivities of the model outputs to parameters are high and correlations among the sensitivities are low. Constant experiment duration is taken. Limitations on amplitudes are imposed. Square wave inputs are generated. As an example, in this part sum of sinusoids type of input is optimized. The technique is explained below.

The system model is given as

$$\begin{aligned} \dot{x} &= f(x,\theta,u) \\ y &= h(x,\theta,u) \\ z(i) &= y(i) + v(i),\ i = 1,..,N \end{aligned} \tag{45}$$

$$\begin{aligned} &E\{v(i)\} = 0 \\ &E\{v(i)v(j)^T\} = R_v\delta_{ij} \text{ assumed Gaussian.} \end{aligned} \tag{46}$$

Constraints on input and output amplitudes are given as

$$\begin{aligned} &\left|u_j(t)\right| \le \mu_j \quad \forall t \quad j = 1,\ldots,n_i \\ &\left|y_k(t)\right| \le \varsigma_k \quad \forall t \quad k = 1,\ldots,n_o \end{aligned} \tag{47}$$

Matrix of output sensitivities to the parameters is S(i)

$$S(i) = \left. \frac{\partial\, y(i)}{\partial\, \theta} \right|_{\theta=\hat{\theta}} \tag{48}$$

Information matrix M defined as

$$M = \sum S(i)^T R_v^{-1} S(i) \tag{49}$$

expresses the information content of an experiment. Dispersion matrix, D is equal to the inverse of information matrix M. Cramer-Rao lower bounds for the parameter standard errors are computed as the square root of the diagonal elements of D. They are the minimum achievable standard errors of the parameters using an asymptotically unbiased and efficient estimator. Cost function to be minimized for constant experiment duration is the sum of the squares of the Cramer-Rao bounds for parameter standard errors.

Output sensitivity for the j^{th} parameter appears as the j^{th} column of the sensitivity matrix. They are computed as

$$\frac{d}{dt}\left(\frac{\partial x}{\partial \theta_i}\right) = \frac{\partial f(x,\theta,u)}{\partial x}\frac{\partial x}{\partial \theta_i} + \frac{\partial f(x,\theta,u)}{\partial \theta_i} \tag{50}$$

$$\frac{\partial y}{\partial \theta_i} = \frac{\partial h(x,\theta,u)}{\partial x}\frac{\partial x}{\partial \theta_i} + \frac{\partial h(x,\theta,u)}{\partial \theta_i} \tag{51}$$

$$\frac{\partial x}{\partial \theta_i}(0) = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \quad i = 1,..,q \tag{52}$$

In order to find optimum inputs, nominal values for the parameters should be used to solve the foregoing equations with the state equations. To achieve this, estimated values of the parameters from experiments without optimal inputs may be used within the model.

Following additional states, outputs and matrices are defined to solve the system.

$$\frac{\partial x}{\partial \theta_1} = \begin{bmatrix} \dfrac{\partial x_1}{\partial \theta_1} \\ \\ \dfrac{\partial x_2}{\partial \theta_1} \end{bmatrix} = \begin{bmatrix} x_3 \\ \\ x_4 \end{bmatrix} \tag{53}$$

$$\frac{\partial x}{\partial \theta_2} = \begin{bmatrix} \dfrac{\partial x_1}{\partial \theta_2} \\ \\ \dfrac{\partial x_2}{\partial \theta_2} \end{bmatrix} = \begin{bmatrix} x_5 \\ \\ x_6 \end{bmatrix} \tag{54}$$

$$\frac{\partial x}{\partial \theta_3} = \begin{bmatrix} \dfrac{\partial x_1}{\partial \theta_3} \\ \\ \dfrac{\partial x_2}{\partial \theta_3} \end{bmatrix} = \begin{bmatrix} x_7 \\ \\ x_8 \end{bmatrix} \tag{55}$$

$$\frac{\partial x}{\partial \theta_4} = \begin{bmatrix} \dfrac{\partial x_1}{\partial \theta_4} \\ \\ \dfrac{\partial x_2}{\partial \theta_4} \end{bmatrix} = \begin{bmatrix} x_9 \\ \\ x_{10} \end{bmatrix} \tag{56}$$

$$\frac{\partial y}{\partial \theta_1} = \begin{bmatrix} \dfrac{\partial y_1}{\partial \theta_1} \\ \\ \dfrac{\partial y_2}{\partial \theta_1} \end{bmatrix} = \begin{bmatrix} y_3 \\ \\ y_4 \end{bmatrix} \tag{57}$$

$$\frac{\partial y}{\partial \theta_2} = \begin{bmatrix} \dfrac{\partial y_1}{\partial \theta_2} \\ \\ \dfrac{\partial y_2}{\partial \theta_2} \end{bmatrix} = \begin{bmatrix} y_5 \\ \\ y_6 \end{bmatrix} \tag{58}$$

$$\frac{\partial y}{\partial \theta_3} = \begin{bmatrix} \dfrac{\partial y_1}{\partial \theta_3} \\ \\ \dfrac{\partial y_2}{\partial \theta_3} \end{bmatrix} = \begin{bmatrix} y_7 \\ \\ y_8 \end{bmatrix} \tag{59}$$

$$\frac{\partial y}{\partial \theta_4} = \begin{bmatrix} \dfrac{\partial y_1}{\partial \theta_4} \\ \\ \dfrac{\partial y_2}{\partial \theta_4} \end{bmatrix} = \begin{bmatrix} y_9 \\ \\ y_{10} \end{bmatrix} \tag{60}$$

$$\frac{\partial A}{\partial \theta_1} = A_{\theta 1}, \; \frac{\partial A}{\partial \theta_2} = A_{\theta 2}, \; \frac{\partial A}{\partial \theta_3} = A_{\theta 3}, \; \frac{\partial A}{\partial \theta_4} = A_{\theta 4} \tag{61}$$

$$\frac{\partial B}{\partial \theta_1} = B_{\theta 1}, \; \frac{\partial B}{\partial \theta_2} = B_{\theta 2}, \; \frac{\partial B}{\partial \theta_3} = B_{\theta 3}, \; \frac{\partial B}{\partial \theta_4} = B_{\theta 4} \tag{62}$$

$$\frac{\partial C}{\partial \theta_1} = C_{\theta 1}, \; \frac{\partial C}{\partial \theta_2} = C_{\theta 2}, \; \frac{\partial C}{\partial \theta_3} = C_{\theta 3}, \; \frac{\partial C}{\partial \theta_4} = C_{\theta 4} \tag{63}$$

$$\frac{\partial D}{\partial \theta_1} = D_{\theta 1}, \ \frac{\partial D}{\partial \theta_2} = D_{\theta 2}, \ \frac{\partial D}{\partial \theta_3} = D_{\theta 3}, \ \frac{\partial D}{\partial \theta_4} = D_{\theta 4} \tag{64}$$

Augmented state space matrices become as follows

$$A_a = \begin{bmatrix} A & Z & Z & Z & Z \\ A_{\theta 1} & A & Z & Z & Z \\ A_{\theta 2} & Z & A & Z & Z \\ A_{\theta 3} & Z & Z & A & Z \\ A_{\theta 4} & Z & Z & Z & A \end{bmatrix} \tag{65}$$

$$B_a = \begin{bmatrix} B \\ B_{\theta 1} \\ B_{\theta 2} \\ B_{\theta 3} \\ B_{\theta 4} \end{bmatrix} \tag{66}$$

$$C_a = \begin{bmatrix} C & Z & Z & Z & Z \\ C_{\theta 1} & C & Z & Z & Z \\ C_{\theta 2} & Z & C & Z & Z \\ C_{\theta 3} & Z & Z & C & Z \\ C_{\theta 4} & Z & Z & Z & C \end{bmatrix} \tag{67}$$

$$D_a = \begin{bmatrix} D \\ D_{\theta 1} \\ D_{\theta 2} \\ D_{\theta 3} \\ D_{\theta 4} \end{bmatrix} \tag{68}$$

$$Z = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{69}$$

The new state vector and output vector are as follows

$$x_a = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 & x_9 & x_{10} \end{bmatrix}^T \tag{70}$$

$$y_a = \begin{bmatrix} y_1 & y_2 & y_3 & y_4 & y_5 & y_6 & y_7 & y_8 & y_9 & y_{10} \end{bmatrix}^T \tag{71}$$

State equations become

$$\begin{aligned} \dot{x}_a &= A_a x_a + B_a u \\ y_a &= C_a x_a + D_a u \end{aligned} \tag{72}$$

with

$x_a(0)=0$.

Sensitivity matrix is given as

$$S(i) = \begin{bmatrix} y_3(i) & y_5(i) & y_7(i) & y_9(i) \\ y_4(i) & y_6(i) & y_8(i) & y_{10}(i) \end{bmatrix} \tag{73}$$

Optimization is carried with genetic algorithm implemented in MatLab. Below is the form of the proposed steering wheel input.

$$\delta_{SW}(t) = a_1 \sin(\omega_1 t) + a_2 \sin(\omega_2 t) + a_3 \sin(\omega_3 t) \tag{74}$$

Optimization algorithm will give the optimum amplitude and frequency values of the sinusoids. Amplitude constraints are imposed on them not to violate the linear model assumption. Frequencies are limited within such a region that the interested modes are included. In literature, it is said that full amplitude square type inputs will give better results [46, 49].

Figures 29 to 32 give the examples for the test inputs applied to the system during the experiments.

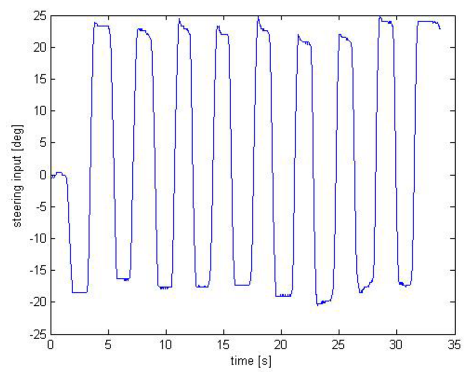


Figure 29 Sample steering input

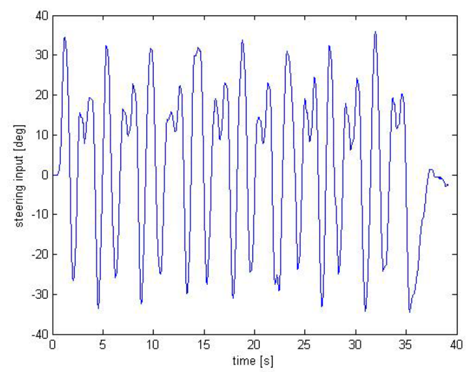


Figure 30 Sample steering input

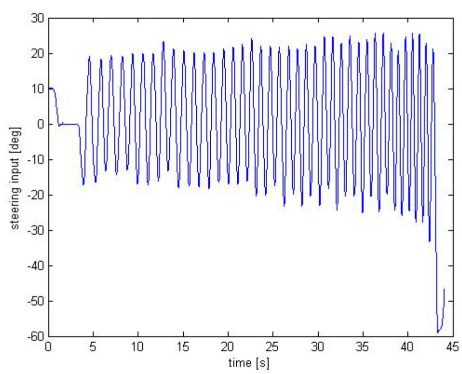


Figure 31 Sample steering input

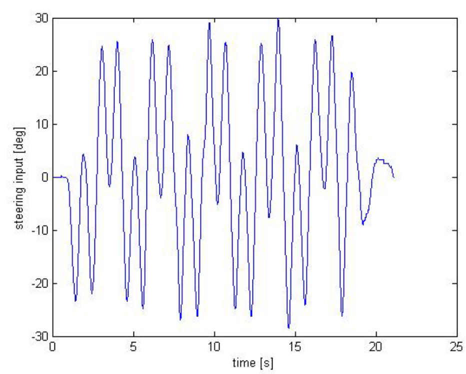


Figure 32 sample steering input

The input design technique described above can also be utilized to design optimal inputs for the experiments of 3 dof model structure identification.

A sample optimal input is given in Figure 33.

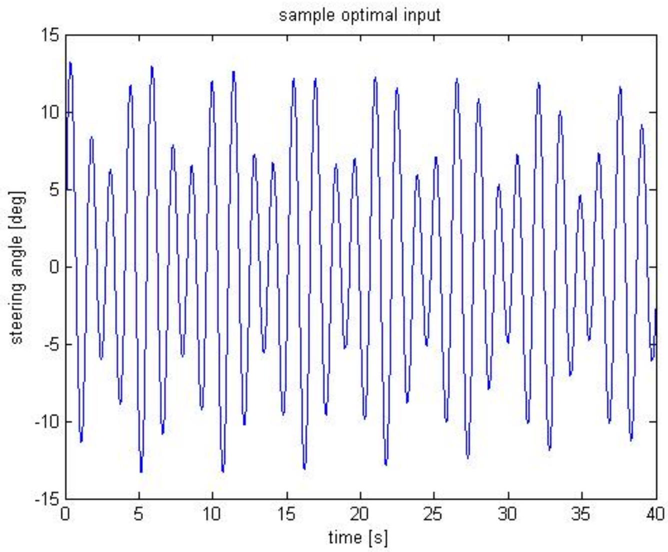


Figure 33 An optimal steering input

CHAPTER 5

IDENTIFICATION OF 3 DOF HANDLING MODELS

In addition to lateral dynamics and yaw dynamics, rolling motion of the vehicle has sensible affect on the handling characteristics. In addition, the limited range of the linear bicycle model makes researchers use more complicated models. The coupled dynamics of lateral, yaw and the roll motions can thus be modeled using a 3 dof linear and nonlinear model structures.

Various types of mathematical models are utilized for the purpose of the thesis. 5 dof and 4 dof nonlinear models are used to design and simulate the experiments. These models are employed to present *simulated acquired data* prior to the tests performed on the real vehicle.

5 dof simulation model is composed of longitudinal, lateral, yaw, roll, and the pitch dynamics. Eliminating the pitch motion gives the 4 dof nonlinear simulation model. Since that, handling dynamics is of the primary concern; forward acceleration/deceleration may be disregarded. Constant forward velocity assumption can be valid. Elimination of longitudinal dynamics by constant forward velocity assumption reveals 3 dof model structures. Grey-box model structures in this chapter are based on these 3 dof model structures. Both linear and nonlinear models are constructed.

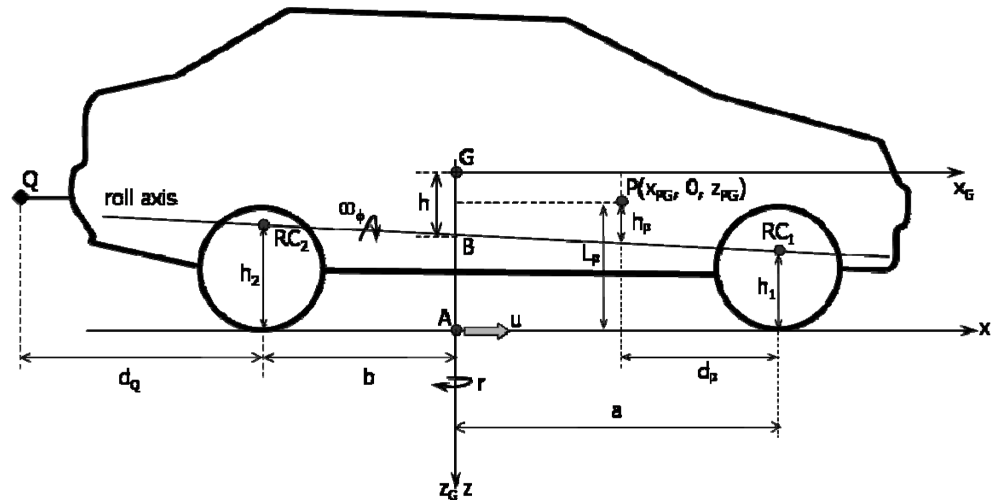
Coupled longitudinal and handling dynamics has a really complex nature due to the characteristics of tires [51]. Force generation nature of pneumatic tires depends mainly on the generated slip angle and longitudinal slip values. In case of pure longitudinal motion or pure cornering motion with constant speed, one can eliminate the affect of slip angle on the longitudinal dynamics or longitudinal slip on the handling dynamics [51]. In order to be able to use pure side slip conditions to model the force generation of tires, forward velocity during cornering is assumed to be constant.

In both 4 dof and 5 dof simulation models, Magic Formula tire model is used to model the cornering force generation characteristics of pneumatic tires [51, 52, 53]. The original Magic Formula tire model outputs steady-state lateral tire force to the normal force and side slip angle inputs. However, due to the characteristics of the maneuvers in experiments, transient forces are needed. Therefore, the modified form of the Magic Formula tire model is used to reflect the generated transient forces in simulation models [51, 54].

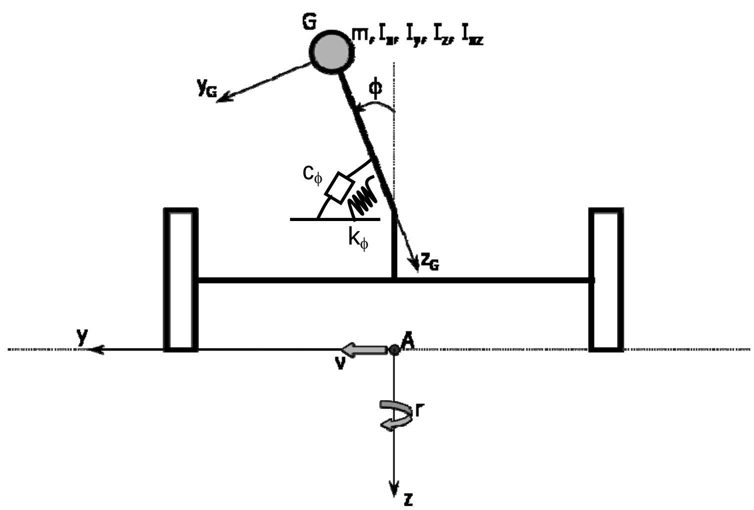
5.1 Nonlinear Simulation Models

Lagrange equations are used to derive the equations of motion of the system [51, 55]. Figures 34.a, 34.b, and 34.c show the simplified physical model and schematic representation of the vehicle below.

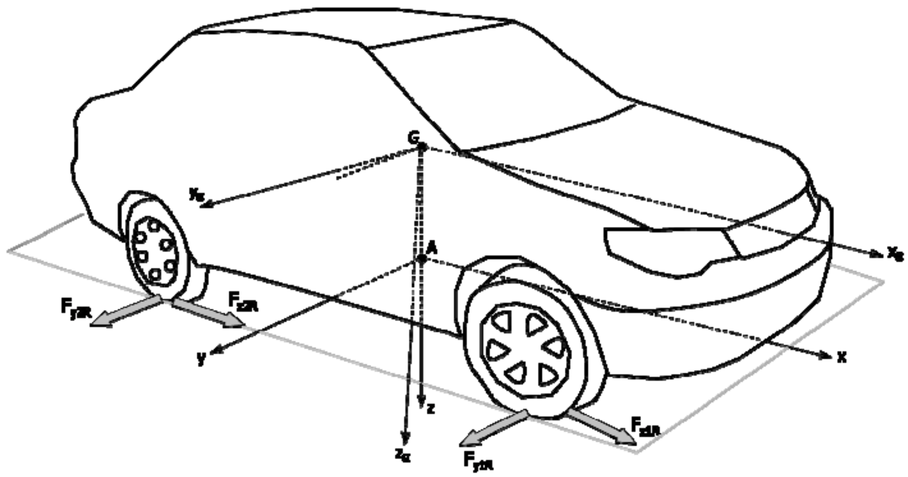
Initial generalized coordinates are; X, Y, Ψ, ϕ, θ. However, to get a better format in wiriting the output equations based on employed sensor set, generalized coordinates shall be modified. Modified generalized coordinates are; u, v, r, ϕ, θ. To write the Lagrange equations in terms of modified coordinates, Equations 48-52 are utilized. X and Y are the coordinates of point A with respect to inertial reference frame, Ψ is the yaw angle, and θ is the pitch angle.


G
Q
roll axis
P(x_PG, 0, z_PG)
x_G
h
h_P
RC_2
RC_1
B
L_P
h_2
h_1
A
u
x
d_Q
b
r
d_P
a
z_G z

(34.a)


G
m, I_x, I_y, I_z, I_xz
y_G
ϕ
c_ϕ
k_ϕ
z_G
y
A
v
r
z

(34.b)



(34.c)

Figure 34 Axes and velocities on physical model

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i}-\frac{\partial T}{\partial q_i}+\frac{\partial U_e}{\partial q_i}+\frac{\partial F}{\partial \dot{q}_i}=Q_i \tag{75}$$

$$u=\dot{X}\cos(\Psi)+\dot{Y}\sin(\Psi) \tag{76}$$

$$v=-\dot{X}\sin(\Psi)+\dot{Y}\cos(\Psi) \tag{77}$$

$$r=\dot{\Psi} \tag{78}$$

$$\frac{\partial T}{\partial \dot{X}}=\frac{\partial T}{\partial u}\frac{\partial u}{\partial \dot{X}}+\frac{\partial T}{\partial v}\frac{\partial v}{\partial \dot{X}}=\frac{\partial T}{\partial u}\cos(\Psi)-\frac{\partial T}{\partial v}\sin(\Psi) \tag{79}$$

$$\frac{\partial T}{\partial \dot{Y}}=\frac{\partial T}{\partial u}\frac{\partial u}{\partial \dot{Y}}+\frac{\partial T}{\partial v}\frac{\partial v}{\partial \dot{Y}}=\frac{\partial T}{\partial u}\sin(\Psi)+\frac{\partial T}{\partial v}\cos(\Psi) \tag{80}$$

$$\frac{\partial T}{\partial \dot{\Psi}}=\frac{\partial T}{\partial r} \tag{81}$$

$$\frac{\partial T}{\partial \Psi}=\frac{\partial T}{\partial u}v-\frac{\partial T}{\partial v}u \tag{82}$$

Equations of motion of the system are derived using the equations below.

$$\frac{d}{dt}\frac{\partial T}{\partial u}-r\frac{\partial T}{\partial v}=Q_u \tag{83}$$

$$\frac{d}{dt}\frac{\partial T}{\partial v}+r\frac{\partial T}{\partial u}=Q_v \tag{84}$$

$$\frac{d}{dt}\frac{\partial T}{\partial r}-v\frac{\partial T}{\partial u}+u\frac{\partial T}{\partial v}=Q_r \tag{55}$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\phi}}-\frac{\partial T}{\partial \phi}+\frac{\partial U}{\partial \phi}+\frac{\partial F}{\partial \dot{\phi}}=Q_\phi \tag{86}$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\theta}}-\frac{\partial T}{\partial \theta}+\frac{\partial U}{\partial \theta}+\frac{\partial F}{\partial \dot{\theta}}=Q_\phi \tag{87}$$

Generalized forces, Q_i, are composed of only tire forces and aerodynamic forces.

$Q_i=\sum_j \vec{F}_j\cdot\frac{\partial \vec{r}_j}{\partial q_i}$, q_i : i^{th} generalized coordinate, $\vec{r}_j$: position vector to j^{th} force in inertial reference frame.

$$Q_u=(F_{x1R}+F_{x1L})\cos(\delta)-(F_{y1R}+F_{y1L})\sin(\delta)+(F_{x2R}+F_{x2L}) \tag{88}$$

$$Q_v=(F_{x1R}+F_{x1L})\sin(\delta)+(F_{y1R}+F_{y1L})\cos(\delta)+(F_{y2R}+F_{y2L}) \tag{89}$$

$$Q_r = a(F_{x1R} + F_{x1L})\sin(\delta) + a(F_{y1R} + F_{y1L})\cos(\delta) - b(F_{y2R} + F_{y2L}) \tag{90}$$

$$Q_\theta = 0 \tag{91}$$

$$\begin{aligned} Q_\phi &= (F_{x2R} + F_{x2L})(b\sin(\theta) + h_a\cos(\theta)) + \\ &(F_{y1R} + F_{y1L})(a\sin(\phi)\cos(\theta) + h_a\sin(\phi)\sin(\theta)) + \\ &(F_{y2R} + F_{y2L})(h_a\sin(\phi)\sin(\theta) - b\sin(\phi)\cos(\theta)) + F_{ae} \end{aligned} \tag{92}$$

Note that, Q_θ above is written for $F_{x1R,L}$=0.

F_{ae}: aerodynamic force resisting to motion in -x_G axis = $F_{ae} = \frac{1}{2}C_f\,\rho\,S\,u^2$

where

C_f: Aerodynamic coefficient,

ρ: Air density

S: Surface area of vehicle normal to air flow in -x_G axis.

It is assumed that the complete mass of the vehicle rolls about the roll axis given in Figure 34. This assumption reduces the number of parameters necessary for both simulation and identification.

T: Kinetic energy of the system

U_e: Potential energy of the system

F: Rayleigh's dissipation function

$$\begin{aligned} T = \frac{1}{2}m\left[V_{Gx_G}^{\;2} + V_{Gy_G}^{\;2} + V_{Gz_G}^{\;2}\right] + \frac{1}{2}I_x\omega_{x_G}^{\;2} + \frac{1}{2}I_y\omega_{y_G}^{\;2} + \\ \frac{1}{2}I_z\omega_{z_G}^{\;2} - I_{xz}\omega_{x_G}\omega_{z_G} \end{aligned} \tag{93}$$

Due to the symmetry of the vehicle, product of inertia terms, I_{xy} and I_{yz}, are assumed to be zero.

$V_{Gx_G}, V_{Gy_G}, V_{Gz_G}$: components of absolute velocity of mass center G in body reference frame (G, x_G, y_G, z_G).

ω_{x_G} , ω_{y_G} , ω_{z_G} : components of absolute angular velocity of vehicle in body reference frame (G, x_G, y_G, z_G).

$$V_{Gx_G} = u\cos(\theta) + v\sin(\theta)\sin(\phi) - h(\sin(\phi) + \cos(\phi)\dot{\theta}) \tag{94}$$

$$V_{Gy_G} = v\cos(\phi) - r\ h\sin(\theta)\cos(\phi) + h\cos(\theta_r)\dot{\phi} \tag{95}$$

$$V_{Gz_G} = u\sin(\theta) - v\cos(\theta)\sin(\phi) \tag{96}$$

θ_r: inclination angle of roll axis $\cong (h_2 - h_1)/L$

$$\omega_{x_G} = -\sin(\theta)\cos(\phi)r + \dot{\phi}\cos(\theta_r) \cong -\theta\ r + \dot{\phi} \tag{97}$$

$$\omega_{y_G} = \sin(\phi)\ r + \cos(\phi)\dot{\theta} \cong \phi\ r + \dot{\theta} \tag{98}$$

$$\omega_{z_G} = \cos(\theta)\cos(\phi)\ r - \dot{\phi}\sin(\theta_r) - \sin(\phi)\dot{\theta} \cong r - \dot{\phi}\ \theta_r - \phi\ \dot{\theta} \tag{99}$$

Rayleigh's dissipation function is given below.

$$F = \frac{1}{2}\left(c_{\phi 1} + c_{\phi 2}\right)\dot{\phi}^2 + \frac{1}{2}\left(a^2\ c_{\theta 1} + b^2\ c_{\theta 2}\right)\dot{\theta}^2 \tag{100}$$

$c_{\phi 1,2}$: Resulting roll damping coefficients about roll axis

$c_{\theta1,2}$: Front and rear suspensions resulting linear damping coefficients

Potential energy function is given below.

$$U_e = \frac{1}{2}\left(k_{\phi1} + k_{\phi2}\right)\phi^2 + \frac{1}{2}\left(a^2\ k_{\theta1} + b^2\ k_{\theta2}\right)\theta^2 - \frac{1}{2} m\ g\ h\ \phi^2 \qquad (101)$$

$k_{\phi1,2}$: Resulting roll stiffness coefficients about roll axis

$k_{\theta1,2}$: Front and rear suspensions resulting linear stiffness coefficients

5.1.1 Magic Formula Tire Model

In nonlinear simulation models, Magic Formula tire model with transient slip angles are utilized [53]. Models are derived considering 4 wheels unlike the bicycle model. Below are the equations for slip angles and tire's lateral force.

$$\frac{\sigma}{u}\dot{\alpha}_t + \alpha_t = \alpha_{ss} \qquad (102)$$

where

α_t : transient slip angle

α_{ss} : steady-state value of the slip angle

$$\alpha_{ss1} = \delta + \varepsilon_1\phi - \frac{v + a\ r}{U} \qquad (103)$$

$$\alpha_{ss2} = \varepsilon_2\phi - \frac{v - b\ r}{U} \qquad (104)$$

where

$\alpha_{ss1,2}$: steady-state slip angles for front and rear wheels. They are used in the tire model where necessary.

$$\alpha_y = \tan(\alpha_t) \tag{105}$$

$$df_z = \frac{F_z - F_{z0}}{F_{z0}} \tag{106}$$

$$C_y = p_{cy1} \tag{107}$$

$$D_y = \mu_y F_z \tag{108}$$

$$\mu_y = p_{Dy1} + p_{Dy2}\ df_z \tag{109}$$

$$E_y = (p_{Ey1} + p_{Ey2}\ df_z)(1 - p_{Ey3}\ \mathrm{sgn}(\alpha_y)) \tag{110}$$

$$K_y = p_{Ky1} F_{z0} \sin\left[2 \arctan\left(\frac{F_z}{p_{Ky2} F_{z0}}\right)\right] = B_y C_y D_y \tag{111}$$

F_y is the generated lateral force of tire.

$$F_y = D_y \sin\left[C_y \arctan\left(B_y \alpha_y - E_y \left(B_y \alpha_y - \arctan\left(B_y \alpha_y\right)\right)\right)\right] \tag{112}$$

$B_y C_y D_y$ is the cornering stiffness term;

$$B_y C_y D_y = \left.\frac{\partial F_y}{\partial \alpha_y}\right|_{\alpha_y = 0} \tag{113}$$

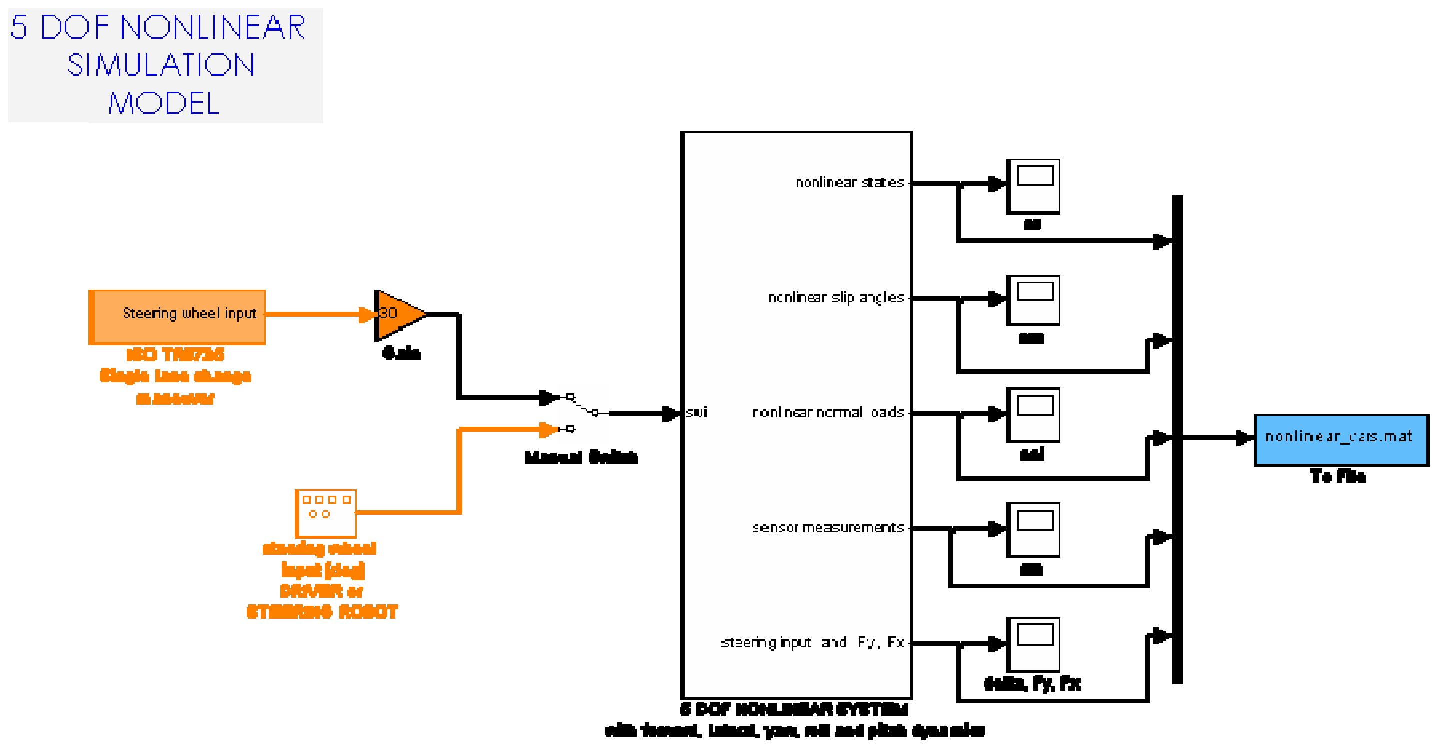
5.1.2 4-Dof and 5-Dof Simulation Models in MatLab/Simulink

4 and 5 dof simulation models are utilized for analyzing the vehicle dynamics, couplings and nonlinearities present in vehicle dynamics. Simulated experiments are achieved on these models. These models are constructed in MatLab/Simulink, fig. 35 and fig. 36.

Nonlinear dynamics of the vehicle is presented in models by using Embedded MatLab Functions and m-file S-functions mainly. In fact, it is known that there exist some alternatives about construction of models in Simulink. It is desired to run the models in real-time. However, due to the usage of m-file S-functions it is not possible. It is desired to change the m-file S-functions with other alternatives such as C S-functions.

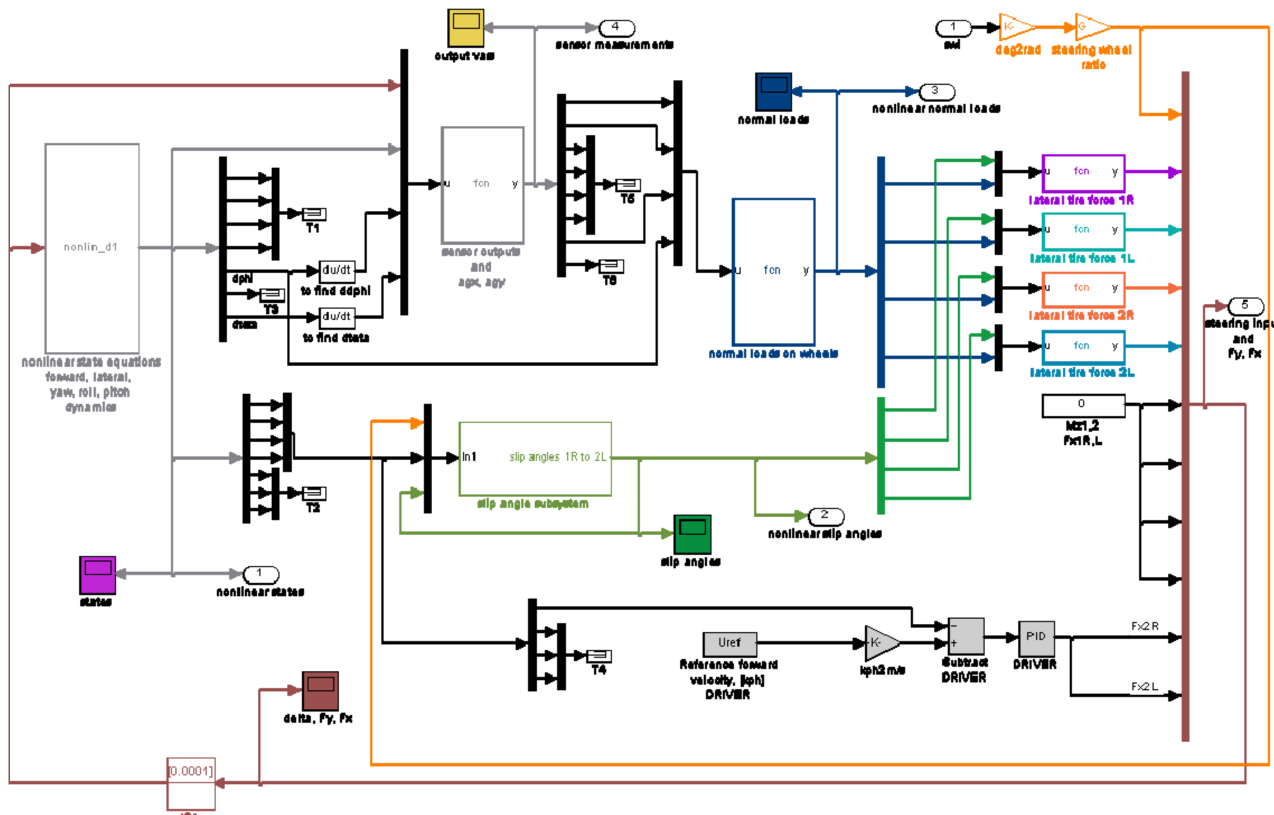
Magic Formula tire model is utilized in these models as nonlinear tire models. Dynamic tire model is employed due to the concerned maneuvers for system identification. Pure slip condition is assumed. It means during maneuvers, tire generates cornering forces and negligible amount of traction/braking force. This simplifies the structure of the Magic Formula model, reduces the number of parameters necessary for simulations, and the interaction due to the longitudinal and lateral force generation can be neglected. However, for the future studies it is also desired to add the interaction of concern and generate a more general model.

A simple driver model is also placed in the 4 and 5 dof models. It is desired to represent the constant velocity condition of experiments. In real experiments, it is not so easy to keep the forward velocity constant during maneuvers. Therefore there might be some unavoidable variations in the magnitude of velocity. This is simulated by use of a PID controller to keep the velocity constant as a driver.



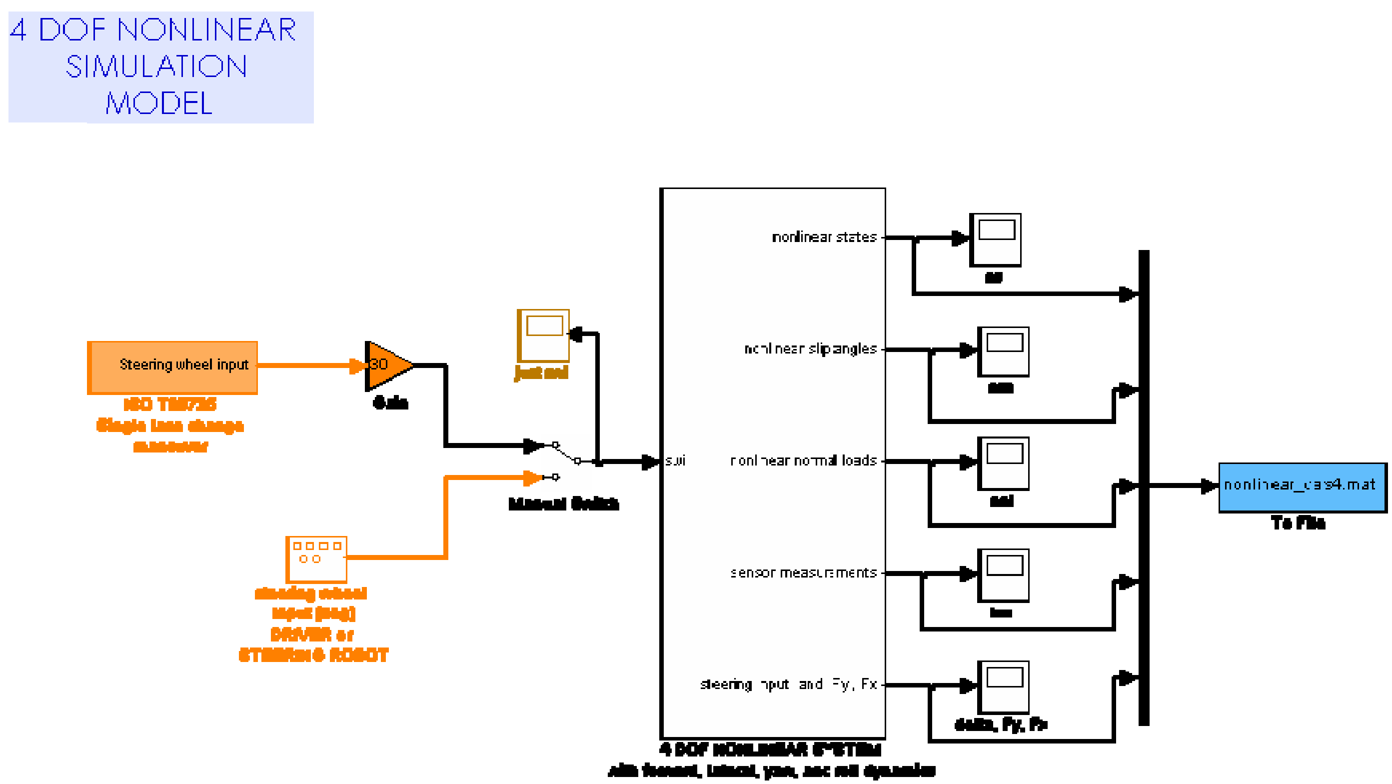
5 DOF NONLINEAR SIMULATION MODEL
Steering wheel input
30
Manual Switch
swi
nonlinear states
nonlinear slip angles
nonlinear normal loads
sensor measurements
steering input and Fy, Fx
delta, Fy, Fx
nonlinear_vars.mat
To File

(35.a)

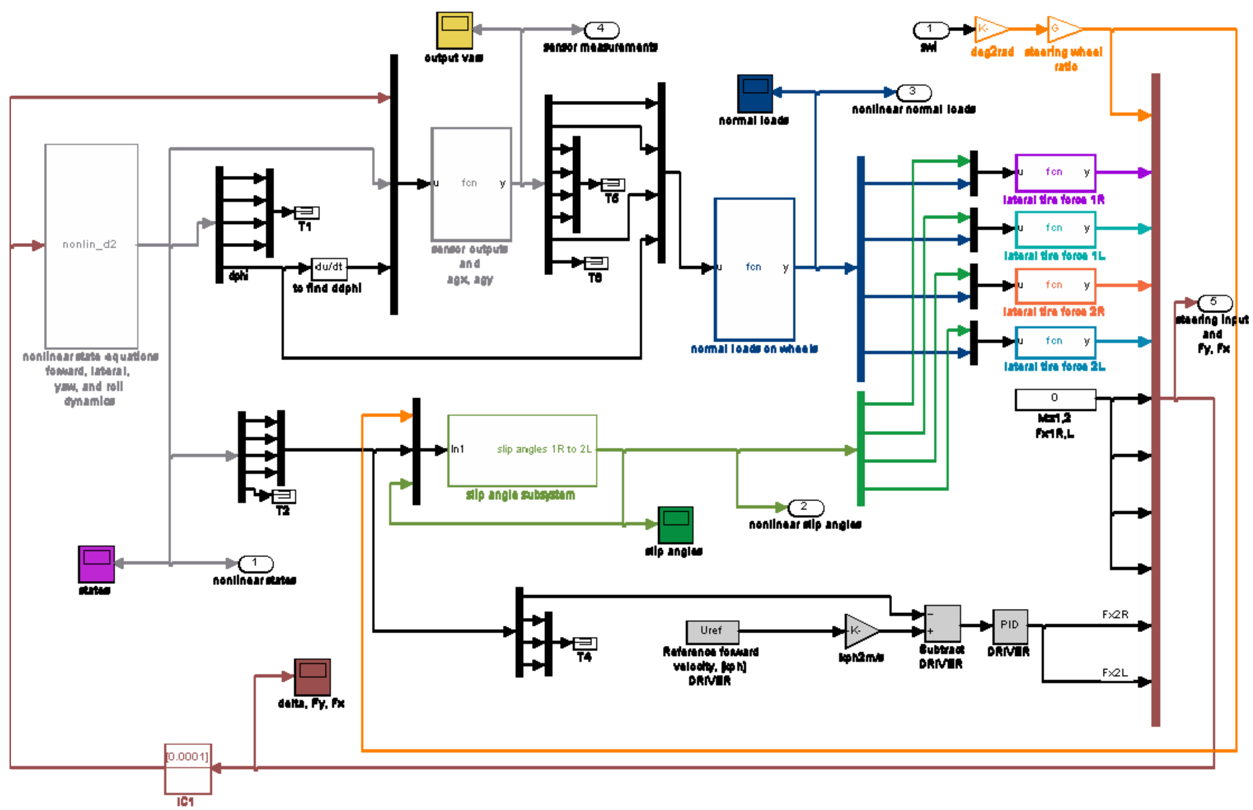


(35.b)

Figure 35 5-dof nonlinear model in Simulink


4 DOF NONLINEAR
SIMULATION
MODEL
Steering wheel input
30
Gain
Manual Switch
nonlinear states
nonlinear slipangles
nonlinear normal loads
sensor measurements
steering input and Fy, Fx
To File

(36.a)



(36.b)

Figure 36 4-dof nonlinear model in Simulink

5.2 3-Dof Simulation and Identification Model Structures

In addition to the 4 dof and 5 dof nonlinear simulation models, 3 dof linear and nonlinear model structures are derived based on the similar Lagrange equations given above. These models are used for simulations and they also form basic structures in identification practices.

5.2.1 3-Dof Linear Simulation and Identification Model Structures

Using the Lagrange equations above, one can find the resulting linear model with states $\{v, r, \phi, d\phi/dt\}^T$. It is constructed in MatLab/Simulink, fig. 37. Linearized slip angles and tire forces are used.

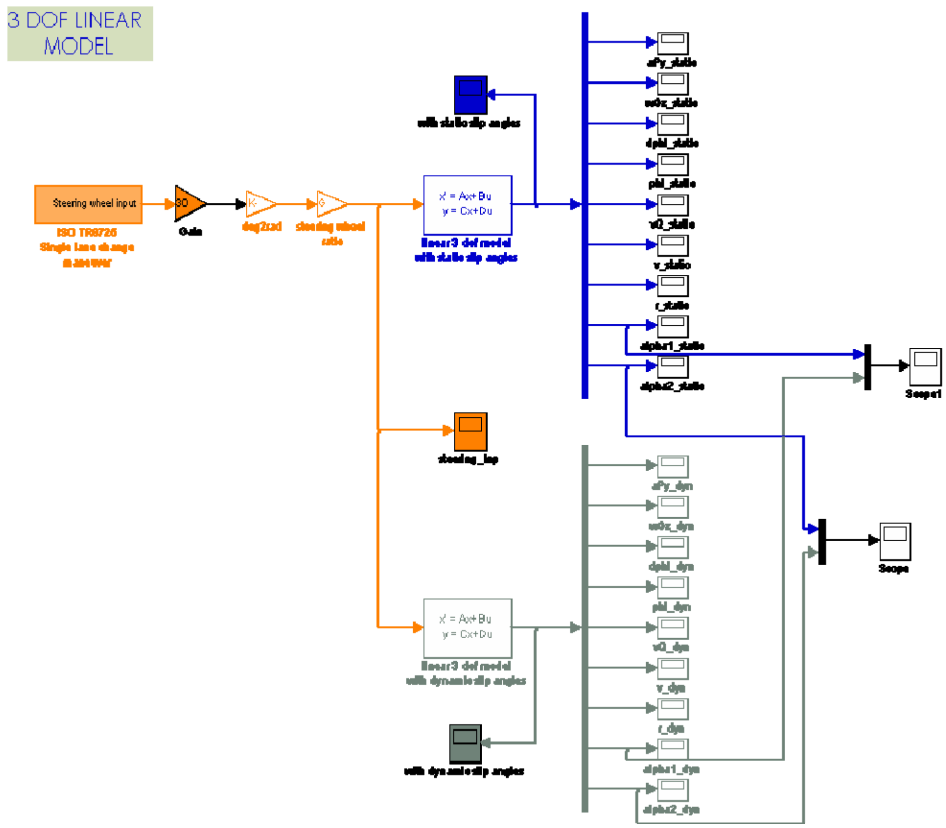


Figure 37 3-dof linear model in Simulink

Parameter set of the model structure is given below.

$$\theta_{lin} = \{a, I_x, I_z, I_{xz}, h_1, h_2, h, k_{\phi 1}, k_{\phi 2}, c_{\phi 1}, c_{\phi 2}, C_{a1}, C_{a2}, \varepsilon_1, \varepsilon_2\} \tag{114}$$

$C_{a1,2}$: Front and rear axle linear cornering stiffness

$\varepsilon_{1,2}$: Front and rear axle roll-steer coefficient

Parameters that are outside the set above are easy to measure. This can be used as the largest set to be estimated in identification algorithm. Details of the experiment design and identification practices for linear 3 dof model structures are not discussed in this thesis. They are left for future reports and papers.

5.2.2 3-Dof Nonlinear Simulation and Identification Model Structures

2 dof linear bicycle model has some limitations as discussed in the previous chapters. In literature it is also stated that estimation of physical parameters based on this model may result velocity dependent parameter estimates. In other words, values of estimated parameters may change with the forward velocity [56].

In addition, it is experienced that even in the experimental conditions of 2 dof model identification, the roll motion is apparent also. However, its affect is ignored by the use of 2 dof linear model structure.

Considering the scope of the thesis and points above, nonlinear model structures including roll dynamics that is coupled with side slip and yaw dynamics are constructed. Nonlinear 3 dof model structures may be also employed to estimate the parameters of nonlinear tire models, and analyze their affect on handling response.

All purposes and needs that are discussed above are also shaped with the available identifiability analysis techniques and identification algorithms that are to be employed. The nonlinear model structure should carry details enough for the desired purposes and it should be as simple as possible to be processed by a structural identifiability algorithm. To meet the requirements, the model has 8 states with nonlinear dynamic tire models. States are as follows;

$$x = \{v, r, \phi, \dot{\phi}, F_{y1R}, F_{y1L}, F_{y2R}, F_{y2L}\}^T. \tag{115}$$

Due to the large amount of equations that arise in the identifiability analysis, the nonlinear tire model is reduced into following form.

Cornering stiffness term:

$$K_y = p_{Ky1} F_{z0} \sin\left[2 \arctan\left(\frac{F_z}{p_{Ky2} F_{z0}}\right)\right] = B_y C_y D_y \tag{116}$$

$$F_y = K_y \alpha = p_{Ky1} F_{z0} \sin\left[2 \arctan\left(\frac{F_z}{p_{Ky2} F_{z0}}\right)\right] \alpha \tag{117}$$

This makes the system give response to the load transfers in cornering. Load transfers change the lateral force produced by the tire. Therefore, nonlinear model with 4 wheels is constructed.

Transient tire forces are employed by using the following state equations.

$$\frac{\sigma}{u} \dot{F}_y + F_y = p_{Ky1} F_{z0} \sin\left[2 \arctan\left(\frac{F_z}{p_{Ky2} F_{z0}}\right)\right] \alpha \tag{118}$$

The parameter set of this model structure is given below.

$$\theta_{nonlin} = \{a, I_x, I_z, I_{xz}, h_1, h_2, h, k_{\phi 1}, k_{\phi 2}, c_{\phi 1}, c_{\phi 2}, \varepsilon_1, \varepsilon_2, p_{Ky1}, p_{Ky2}\} \tag{119}$$

Parameters that are outside the set above are easy to measure. This is can be used as the largest set to be estimated in identification algorithm. However, it will be reduced according to the identifiability analysis method.

5.3 Simulation Results of the Models

Model structures are simulated in single lane change maneuvers. First one is a more severe maneuver with higher lateral acceleration. Second one is a relatively moderate maneuver. Following figures show the assumed measurements during the experiments. It is assumed that the vehicle is equipped with a steering wheel sensor to measure the input to the system, an accelerometer placed at point P to measure the lateral acceleration (see Figure 28.a), an inertial platform to measure the angular velocity ω_{Gz}, angular velocity ω_{Gx}, roll angle ϕ, and a velocity sensor to measure the side slip velocity at point Q, (see Figure 34.a).

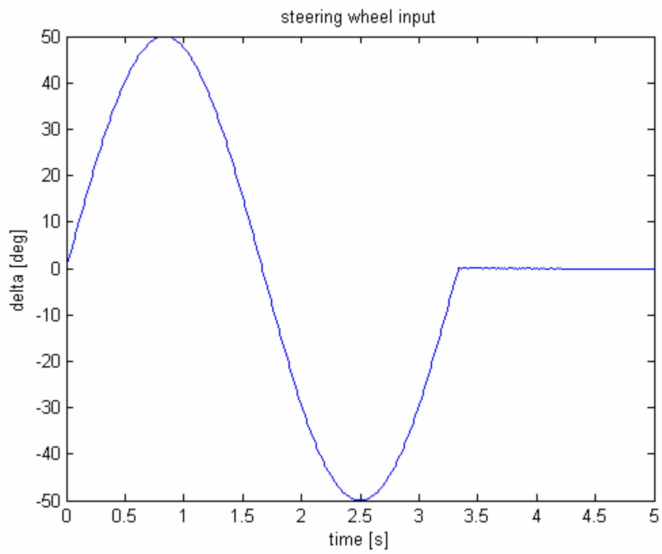


Figure 38 Steering input for maneuver 1

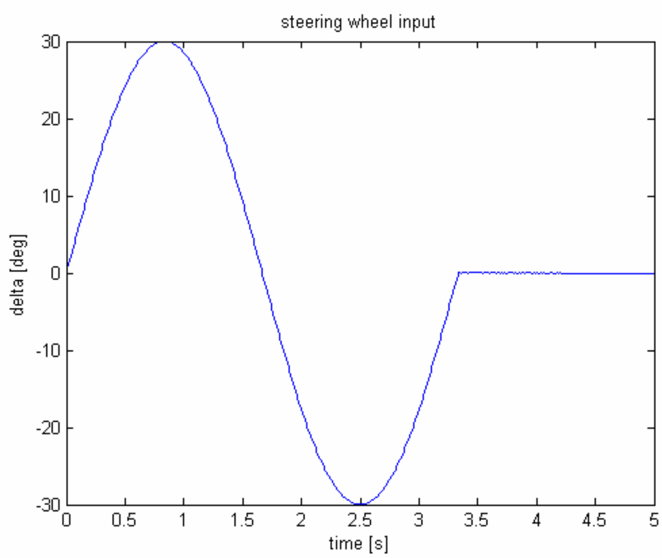


Figure 39 Stering input for maneuver 2

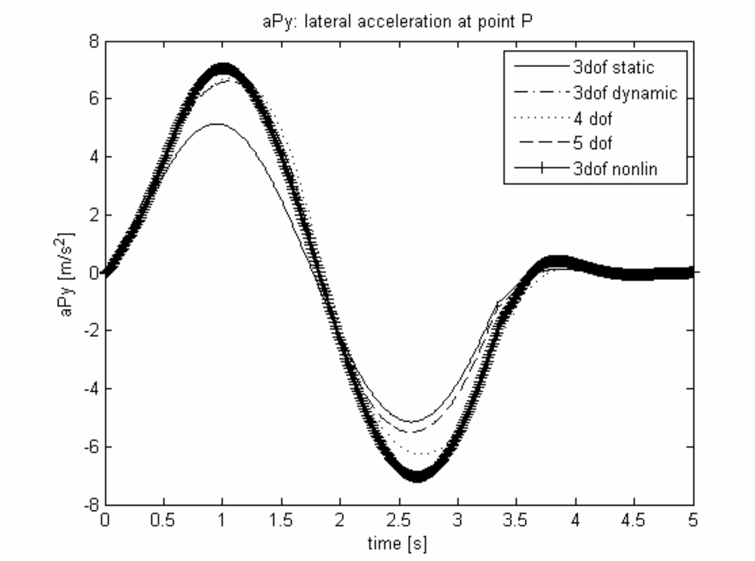


Figure 40 Lateral acc. at P for maneuver 1

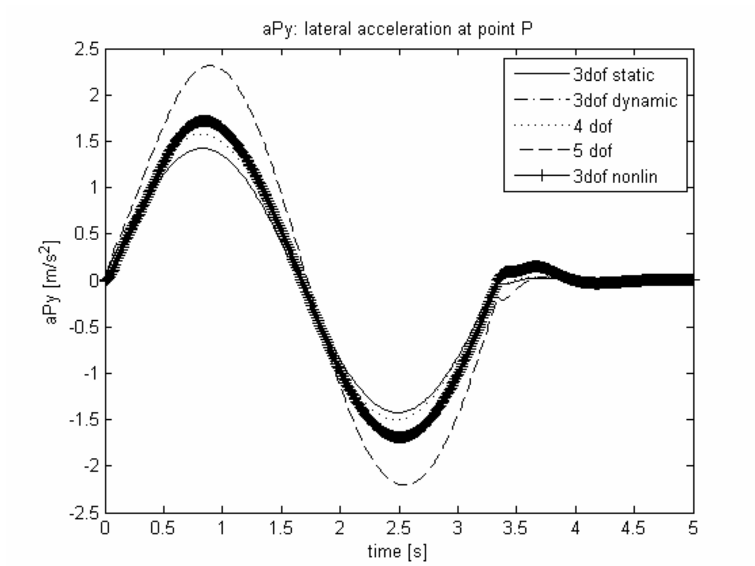


Figure 41 Lateral acc. at P for maneuver 2

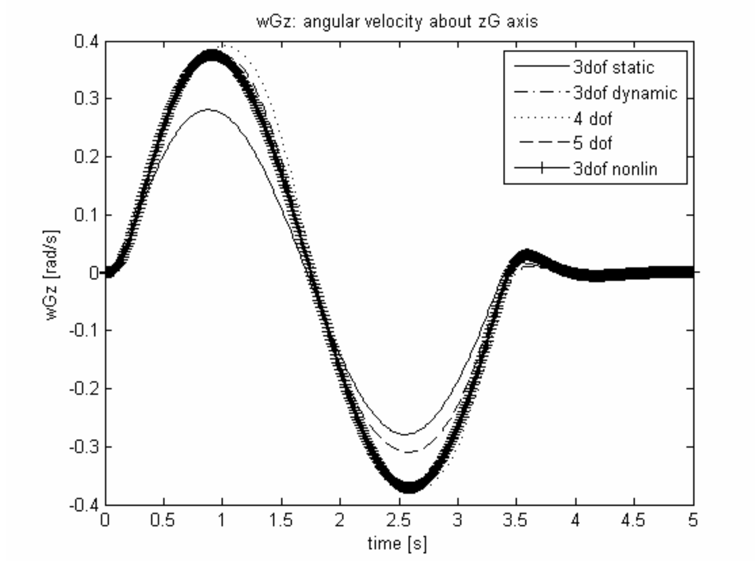


Figure 42 Angular vel. about z_G axis for maneuver 1

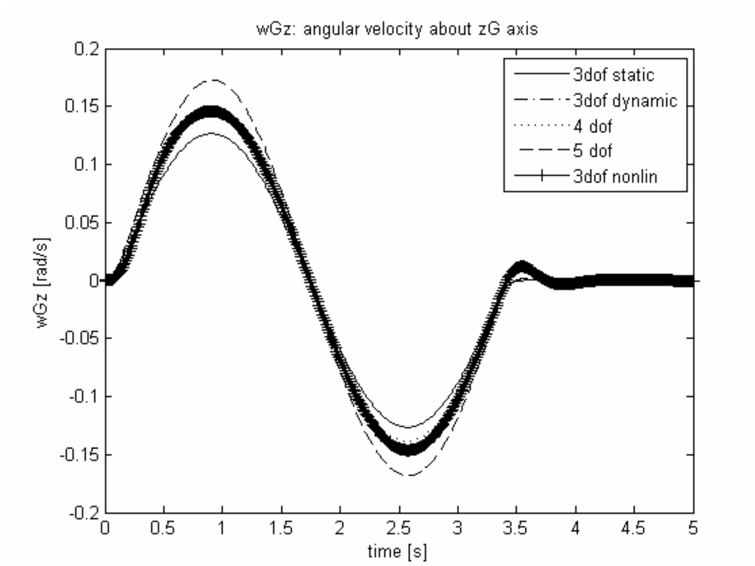


Figure 43 Angular ve. about z_G axis for maneuver 2

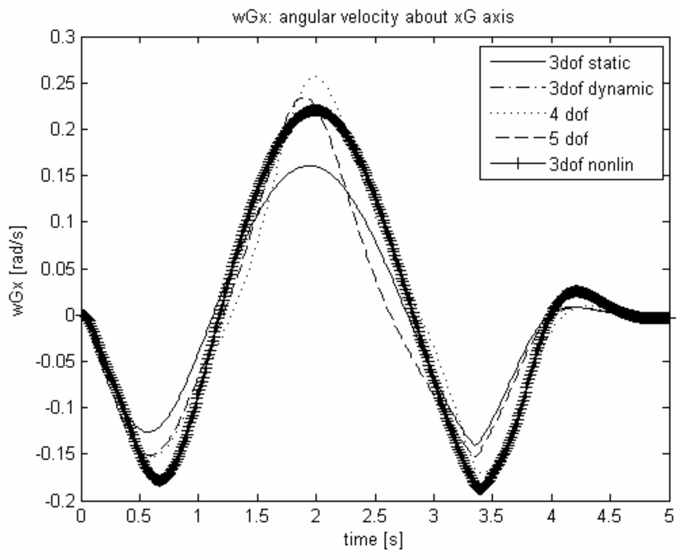


Figure 44 Angular vel. about x_G axis for maneuver 1

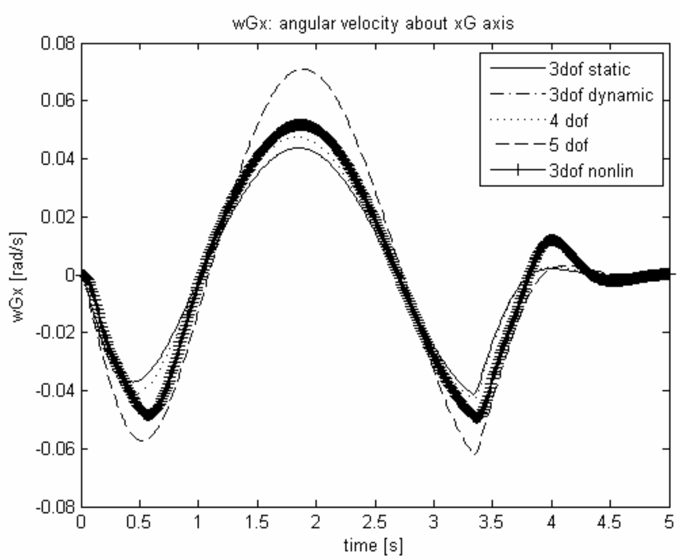


Figure 45 Angular vel. about x_G axis for maneuver 2

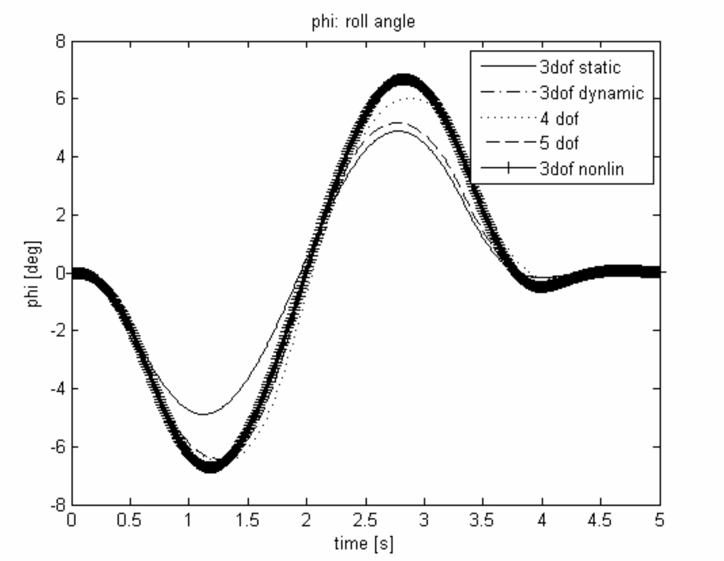


Figure 46 Roll angle for maneuver 1

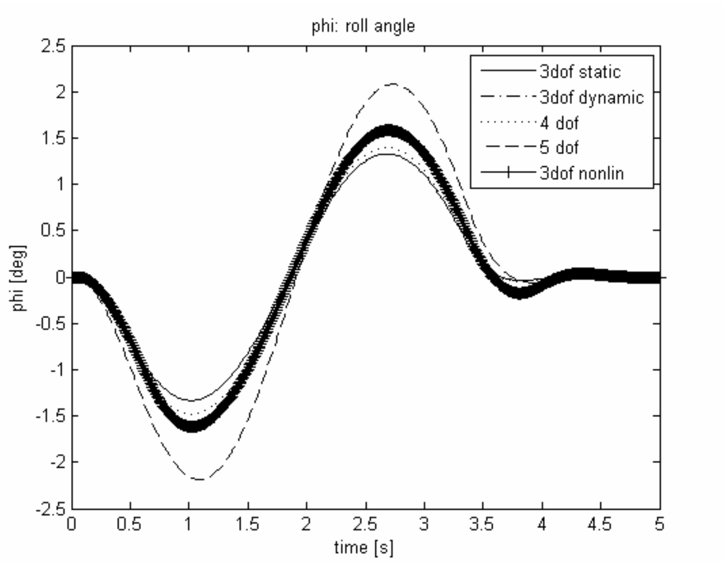


Figure 47 Roll angle for maneuver 2

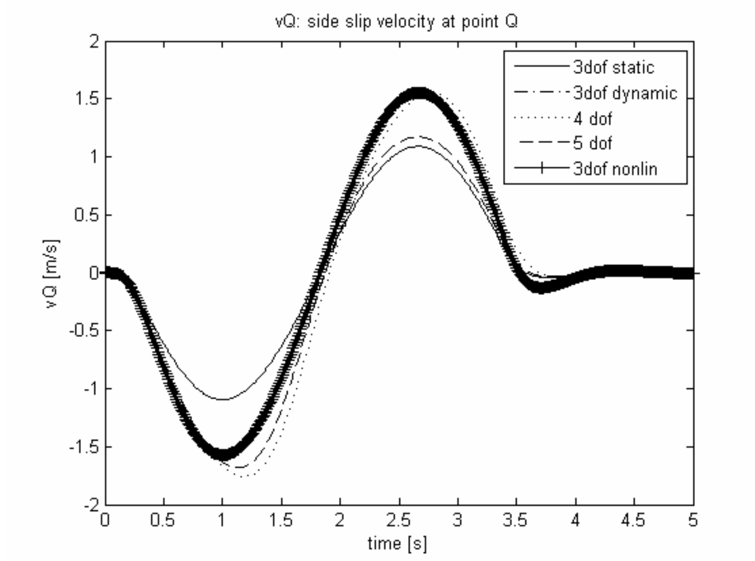


Figure 48 Slip vel. at point Q for maneuver 1

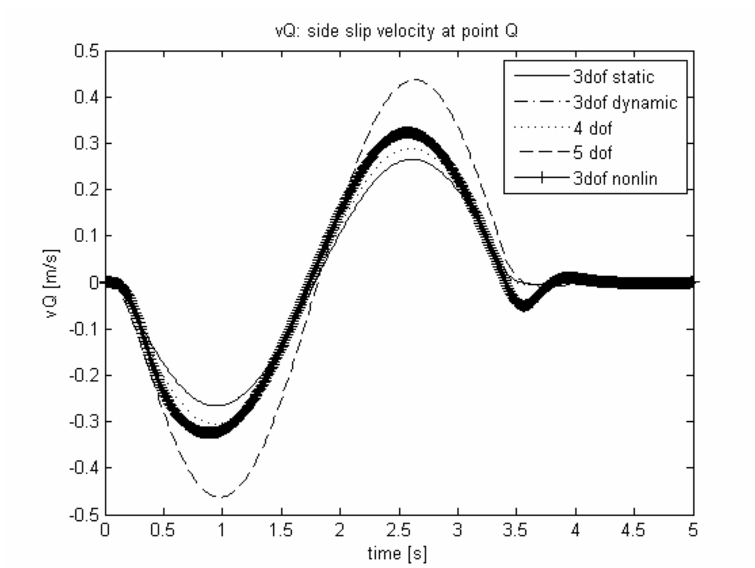


Figure 49 Slip vel. at point Q for maneuver 2

These plots are used to analyze nonlinearites and affects of pitch and forward dynamics on simulated measurements during experiments. 3 dof nonlinear model structure with the assumptions above, give compatible responses according to the 4 dof and 5 dof models.

5.4 Structural Identifiability Analysis

Structural identifiability analysis given in linear model identification part is a straight forward technique for model structures linear in input. However, identifiability analysis techniques for nonlinear (in input) model structures are not so clear. Implementation of each technique is case dependent, i.e. each case brings its own problems to be solved. Generally large numbers of nonlinear equations are reached as a result and even with today's computation power it may not be possible to solve them and comment about the identifiability of the model structure. Therefore, concerned model structure is processed with alternative forms of tire force generation model to simplify the resultant equations.

Selected tire model is generated using the cornering stiffness term of Magic Formula model. This stiffness term also depends on normal load. Therefore, cornering force is a function of slip angle and normal force.

$$F_{yss} = K_y \alpha = p_{Ky1} F_{z0} \sin\left[2 \arctan\left(\frac{F_z}{p_{Ky2} F_{z0}}\right)\right]\alpha \tag{120}$$

$$\frac{\sigma}{u}\dot{F}_y + F_y = F_{yss} \tag{121}$$

To process the model in identifiability analysis algorithm, F_{yss} is replaced by its second order Taylor Series approximation. It has such a mathematical form:

$$F_{yss} = -(A\,F_z + B)\,\alpha \tag{120}$$

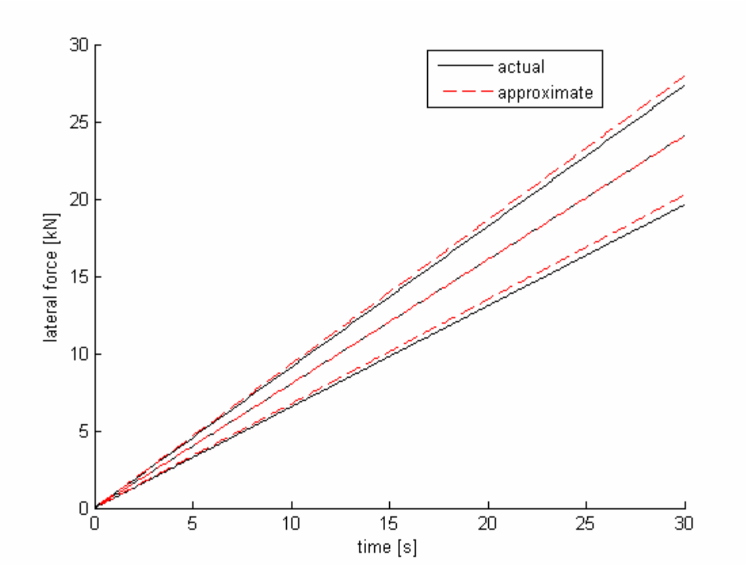


Figure 50 Tire model and its approximation for different normal loads on tire

This approximate form is especially suitable for identifiability analysis with differential algebra techniques. Structural identifiability of nonlinear model structures can be analyzed by the following techniques [11, 43, 44, 56, 57].

1 - Linearization of the model structure and the use of transfer function and similarity transformation techniques. By linearizing the model, identifiability of some parameters may be lost. It is known that nonlinear model structures are generally more identifiable compared to linear model structures [57].

2 - Taylor Series expansion method. It is generated by using the Taylor series expansion of measurement equations. Since measurement vector is unique, all its derivatives are unique. The coefficients of the expansion term are used to check the identifiability. If the solutions for the parameter set are uncountable then the system is unidentifiable. It is locally identifiable if the solutions are countable and it is globally identifiable if there is a unique solution. This technique gives sufficient condition of the identifiability analysis. In addition, the coefficients are very large to solve for the parameters [56, 60].

3 - Vajda's approach based on the local state isomorphism. It is generally used to check the structural identifiability of control systems.

4 - Differential algebraic methods. It is based on the elimination of state variables in measurement equations [11]. Finding characteristic set of ideal defined by the state-space model of the system is required. Ljung's approach tries to rearrange the model structure as a linear regression. Today, many alternative forms and techniques of differential algebraic type of analysis are generated to check structural identifiability. In this study, one of the differential algebraic techniques developed by Xia and Moog is employed for the identifiability analysis [58, 59, 60].

5.4.1 Outline of the Identifiability Analysis Method

The nonlinear model structure is given as follows [58].

$$\begin{aligned} \dot{x} &= f(x, \theta, u) \\ y &= h(x, \theta, u) \\ x(0, \theta) &= x_0 \end{aligned} \tag{123}$$

where $x \in \Re^n$, $u \in \Re^m$, $y \in \Re^p$, and $\theta \in \Re^q$.

It is assumed that,

$$\mathrm{rank} \frac{\partial\, h(x, \theta, u)}{\partial\, x} = p$$

This assumption presents that measurements are linearly independent. Therefore, observable representations could be assumed to be generated.

The system is said to be algebraically identifiable if there exist a $T > 0$, a function

$\Phi: R^q \times R^{(k+1)m} \times R^{(k+1)p} \rightarrow R^q$ such that

$$\det \frac{\partial \Phi}{\partial \theta} \neq 0$$

and

$$\Phi(\theta, u, \dot{u}, \ldots, u^{(k)}, y, \dot{y}, \ldots, y^{(k)}) = 0 \tag{124}$$

hold on [0,T], for all $(\theta, u, \dot{u}, \ldots, u^{(k)}, y, \dot{y}, \ldots, y^{(k)})$.

$u, \dot{u}, \ldots, u^{(k)}$, and $y, \dot{y}, \ldots, y^{(k)}$ are the derivatives of input and output. The input is continuous and k-times differentiable on [0, T].

The proposed methodology checks for the algebraic identifiability, i.e. the parameters can be represented as functions of inputs, outputs and their successive derivatives. Xia and Moog prove that algebraic identifiability implies structural identifiability. It means if a system is algebraically identifiable, then it is structurally identifiable [58].

Practical implementation of their method requires successive generation of time derivatives of measurement equations till there exist enough equations to eliminate state variables. This technique is mainly heuristic in nature. There exist more systematic elimination algorithms such as Rosenfeld-Groebner algorithm to reach observable representation as characteristic set of the system equations. In such techniques, ranking of the variables is critical. The complexity of the coefficients of the characteristic set or computation time depends on the type of ranking. This is one of the most critical points in differential algebraic methods. The same results may be reached but the resultant equations may be more and more complex.

5.4.2 Implementation of the Identifiability Analysis Method

In order to reduce the difficulty of dealing with algebraic equations of very complex nature, the number of output equations is increased in this study. It is assumed that there are 9 sensors available to measure the following variables (see Figure 28):

y_1 : lateral acceleration at point P,
y_2 : yaw velocity,
y_3 : roll angle,
y_4 : roll velocity,
y_5 : lateral velocity at point Q,
y_6 : F_{y1R}, cornering force at front right tire,
y_7 : F_{y1L}, cornering force at front left tire,
y_8 : F_{y2R}, cornering force at rear right tire,
y_9 : F_{y2L}, cornering force at rear left tire.

Remember that state variables are (see Figure 28);

x_1 : side slip velocity at point A,
x_2 : yaw velocity,
x_3 : roll angle,
x_4 : roll velocity,
x_5 : F_{y1R}, cornering force at front right tire,

x_6 : F_{y1L}, cornering force at front left tire,

x_7 : F_{y2R}, cornering force at rear right tire,

x_8 : F_{y2L}, cornering force at rear left tire.

The sensor set is possible by considering the available hardware components in previous chapters. It is also a feasible question to ask what if there is a limited sensor set. Reaching an observable representation by using the successive derivatives may not be possible due to the very large and complex amount of coefficients. Pages of terms for a single coefficient may cause an *out of memory* problem during the process. Similar computational problem is faced in the use of Groebner algorithm. Therefore, such a sensor set and simplified model structure are generated to have reduced number of output derivatives to eliminate the state variables. One example of the observable map is given below.

$$\Phi_1 : y_1 - \dot{y}_5 - (L_P + D_Q - D_P)\dot{y}_2 - U\, y_2 - (L_P - h_a)\dot{y}_4 \tag{125}$$

Other observable representations are derived from the other measurement equations. The general structure of the observable functions is given below.

$$\Phi_j : \beta_{j1} y_1 + \beta_{j2}\dot{y}_1 + \beta_{j3} y_2 + \beta_{j4}\dot{y}_2 + \cdots + \beta_{js}\dot{y}_5 + \cdots + \beta_{jb} y_i^{\,2} + \cdots + \beta_{jz} u\, y_i + \beta_{jz} y_i y_n y_f + \cdots \tag{126}$$

β_{ji} 's are coefficients of the observable functions and they are nonlinear functions of physical parameters.

Let $B = \begin{bmatrix}\beta_{11} & \ldots & \beta_{ij}\end{bmatrix}^T$ is the vector of all coefficients obtained in the complete set of observable representations of 9 output equations. The number of coefficients is around 100 for the case in this analysis.

According to the implicit function theorem, if

$$\det \frac{\partial B}{\partial \theta} \neq 0 \tag{127}$$

one can locally solve B with respect to θ.

The parameter set that is required to be estimated by the identification of 3 dof nonlinear model structure is given below.

$$\theta = \{a, I_z, I_x, I_{xz}, h_a, c_\phi, k_\phi, h, A_1, B_1, A_2, B_2, \sigma, \varepsilon_1, \varepsilon_2\} \tag{128}$$

So as to reduce the number of parameters, some assumptions are utilized. For example, A_i and B_i are assumed to be same for the tires on the same axle, σ is same for all tires, roll axis is parallel to the road surface, etc.

Implementation of the theory states that parameter set,θ, with 15 parameters is identifiable with the given sensor set. In fact, at the beginning, the parameter set, with 17 elements, contains roll damping and roll stiffness terms separate for front and rear axles, i.e.,

$$\theta = \{a, I_z, I_x, I_{xz}, h_a, c_{\phi 2}, c_{\phi 2}, k_{\phi 1}, k_{\phi 2}, h, A_1, B_1, A_2, B_2, \sigma, \varepsilon_1, \varepsilon_2\} \tag{129}$$

where

$c_\phi = c_{\phi 1} + c_{\phi 2}$ and $k_\phi = k_{\phi 1} + k_{\phi 2}$.

By the idetifiability analysis,

$$\mathrm{rank} \frac{\partial B}{\partial \theta} = 15 . \tag{130}$$

It means 15 of 17 parameters are identifiable. When the state and output equations are analyzed, it is seen that roll stiffness and roll damping terms always exist in $c_{\phi 1} + c_{\phi 2}$ and $k_{\phi 1} + k_{\phi 2}$ forms. Rearranging the parameter set by using total roll stiffness and total roll damping of the vehicle 15 of 15 parameters are identifiable. This may be improved by using complete normal load equations which are trimmed to simplify the state equations.

In fact some other parameters that are not included in the parameter set, θ, can also be identifiable using the available coefficients. For example, forward velocity, U, which is constant during the experiments, is also identifiable with the given sensor set.

5.5 Optimal Input Design

As discussed in the previous parts, experiment design for system identification also requires design of optimal inputs to excite the relevant modes of the system [47, 49]. The aim of designed inputs is to maximize the information content of the acquired data. In order to achieve this, the similar procedure is employed that was experienced in the 2 dof model structure identification practice. It is based on the generation of Fisher Information Matrix. Inverse of this matrix is known as the dispersion matrix of whose diagonal elements contain the Cramer-Rao lower bounds for the parameter standard errors. Employed cost function to be minimized is the sum of the squares of the Cramer-Rao bounds for parameter standard errors.

Fisher Information Matrix requires generating the sensitivities of outputs to parameter changes. System model is given below.

$$\begin{aligned} \dot{x} &= f(x, \theta, u) \\ y &= h(x, \theta, u) \\ z(i) &= y(i) + v(i), \; i = 1,.., N \end{aligned} \tag{131}$$

where “v” is the measurement noise.

$$E\{v(i)\} = 0 \tag{132}$$

$$E\{v(i)v(j)^T\} = R_v \delta_{ij} \text{ assumed Gaussian.} \tag{133}$$

Matrix of output sensitivities to parameters is

$$S(i) = \left. \frac{\partial\, y(i)}{\partial\, \theta} \right|_{\theta=\hat{\theta}} \tag{134}$$

Fisher Information Matrix is formed as below.

$$M = \sum S(i)^T R_v^{-1} S(i)\,. \tag{135}$$

In order to find sensitivities, additional states, state equations and outputs are defined.

$$\frac{d}{dt}\left(\frac{\partial x}{\partial \theta_i}\right) = \frac{\partial f(x,\theta,u)}{\partial x}\frac{\partial x}{\partial \theta_i} + \frac{\partial f(x,\theta,u)}{\partial \theta_i} \tag{136}$$

$$\frac{\partial y}{\partial \theta_i} = \frac{\partial h(x,\theta,u)}{\partial x}\frac{\partial x}{\partial \theta_i} + \frac{\partial h(x,\theta,u)}{\partial \theta_i} \tag{137}$$

$$\frac{\partial x}{\partial \theta_i}(0) = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}, i = 1,..,q \tag{138}$$

In order to overcome the computational problems due to the large number of parameters, the parameter set is reduced once more. The final form that is used in input optimization and identification is given below. Other parameters are easy to measure or estimated by some other means.

$$\theta = \{c_\phi, k_\phi, h, A_1, B_1, A_2, B_2, \sigma, \varepsilon_1, \varepsilon_2\} \tag{139}$$

There are 10 parameters, 8 states, and 9 outputs of concern. In order to find the sensitivities of outputs with respect to parameters, an augmented system is constructed with 88 states and 99 outputs! Additional states are defined below.

$$\frac{\partial x}{\partial \theta_1} = \begin{bmatrix} \dfrac{\partial x_1}{\partial \theta_1} \\ \vdots \\ \dfrac{\partial x_8}{\partial \theta_1} \end{bmatrix} = \begin{bmatrix} x_9 \\ \vdots \\ x_{16} \end{bmatrix} \tag{140}$$

$$\frac{\partial x}{\partial \theta_2} = \begin{bmatrix} \dfrac{\partial x_1}{\partial \theta_2} \\ \vdots \\ \dfrac{\partial x_8}{\partial \theta_2} \end{bmatrix} = \begin{bmatrix} x_{17} \\ \vdots \\ x_{24} \end{bmatrix} \tag{141}$$

$$\frac{\partial x}{\partial \theta_3} = \begin{bmatrix} \dfrac{\partial x_1}{\partial \theta_3} \\ \vdots \\ \dfrac{\partial x_8}{\partial \theta_3} \end{bmatrix} = \begin{bmatrix} x_{25} \\ \vdots \\ x_{32} \end{bmatrix} \tag{142}$$

$$\frac{\partial x}{\partial \theta_4} = \begin{bmatrix} \dfrac{\partial x_1}{\partial \theta_4} \\ \vdots \\ \dfrac{\partial x_8}{\partial \theta_4} \end{bmatrix} = \begin{bmatrix} x_{33} \\ \vdots \\ x_{40} \end{bmatrix} \tag{143}$$

$$\frac{\partial x}{\partial \theta_5} = \begin{bmatrix} \frac{\partial x_1}{\partial \theta_5} \\ \vdots \\ \frac{\partial x_8}{\partial \theta_5} \end{bmatrix} = \begin{bmatrix} x_{41} \\ \vdots \\ x_{48} \end{bmatrix} \tag{144}$$

$$\frac{\partial x}{\partial \theta_6} = \begin{bmatrix} \frac{\partial x_1}{\partial \theta_6} \\ \vdots \\ \frac{\partial x_8}{\partial \theta_6} \end{bmatrix} = \begin{bmatrix} x_{49} \\ \vdots \\ x_{56} \end{bmatrix} \tag{145}$$

$$\frac{\partial x}{\partial \theta_7} = \begin{bmatrix} \frac{\partial x_1}{\partial \theta_7} \\ \vdots \\ \frac{\partial x_8}{\partial \theta_7} \end{bmatrix} = \begin{bmatrix} x_{57} \\ \vdots \\ x_{64} \end{bmatrix} \tag{146}$$

$$\frac{\partial x}{\partial \theta_8} = \begin{bmatrix} \frac{\partial x_1}{\partial \theta_8} \\ \vdots \\ \frac{\partial x_8}{\partial \theta_8} \end{bmatrix} = \begin{bmatrix} x_{65} \\ \vdots \\ x_{72} \end{bmatrix} \tag{147}$$

$$\frac{\partial x}{\partial \theta_9} = \begin{bmatrix} \frac{\partial x_1}{\partial \theta_9} \\ \vdots \\ \frac{\partial x_8}{\partial \theta_9} \end{bmatrix} = \begin{bmatrix} x_{73} \\ \vdots \\ x_{80} \end{bmatrix} \tag{148}$$

$$\frac{\partial x}{\partial \theta_{10}} = \begin{bmatrix} \frac{\partial x_1}{\partial \theta_{10}} \\ \vdots \\ \frac{\partial x_8}{\partial \theta_{10}} \end{bmatrix} = \begin{bmatrix} x_{81} \\ \vdots \\ x_{88} \end{bmatrix} \tag{149}$$

The additional outputs in the augmented system are given below.

$$\frac{\partial y}{\partial \theta_1} = \begin{bmatrix} y_{10} \\ \vdots \\ x_{18} \end{bmatrix} \tag{150}$$

$$\frac{\partial y}{\partial \theta_2} = \begin{bmatrix} y_{19} \\ \vdots \\ x_{27} \end{bmatrix} \tag{151}$$

$$\frac{\partial y}{\partial \theta_3} = \begin{bmatrix} y_{28} \\ \vdots \\ x_{36} \end{bmatrix} \tag{152}$$

$$\frac{\partial y}{\partial \theta_4} = \begin{bmatrix} y_{37} \\ \vdots \\ x_{45} \end{bmatrix} \tag{153}$$

$$\frac{\partial y}{\partial \theta_5} = \begin{bmatrix} y_{46} \\ \vdots \\ x_{54} \end{bmatrix} \tag{154}$$

$$\frac{\partial y}{\partial \theta_6} = \begin{bmatrix} y_{55} \\ \vdots \\ x_{63} \end{bmatrix} \tag{155}$$

$$\frac{\partial y}{\partial \theta_7} = \begin{bmatrix} y_{64} \\ \vdots \\ x_{72} \end{bmatrix} \tag{156}$$

$$\frac{\partial y}{\partial \theta_8} = \begin{bmatrix} y_{73} \\ \vdots \\ x_{81} \end{bmatrix} \tag{157}$$

$$\frac{\partial y}{\partial \theta_9} = \begin{bmatrix} y_{82} \\ \vdots \\ x_{90} \end{bmatrix} \tag{158}$$

$$\frac{\partial y}{\partial \theta_{10}} = \begin{bmatrix} y_{91} \\ \vdots \\ x_{99} \end{bmatrix} \tag{159}$$

Sensitivity matrix is formed as below.

$$S(i) = \begin{bmatrix} y_{10}(i) & \cdots & y_{91}(i) \\ \vdots & \cdots & \vdots \\ y_{18}(i) & \cdots & y_{99}(i) \end{bmatrix} \tag{160}$$

It is required to select an input signal form and a single or a couple of parameters of the form that are calculated by the optimization routine. For nonlinear model identification, generally an input type that covers the whole operating range is preferred. In this study chirp type of input signal is selected. That varies within a certain range and it is a typical form of input that is implemented to analyze the real physical vehicle in frequency domain. Its amplitude and final frequency are the arguments of the optimization algorithm to minimize the cost function. There may be some other forms of input structures that can be utilized in optimal input studies. In this study it is just presented to explain the input design for optimal identification experiments.

To solve the augmented system and optimize the input signal, following Simulink model, Figure 51, is employed within an m-file that runs the optimization algorithm. Similar to the previous input design application for 2 dof model identification, genetic algorithm based optimization is preferred.

It is known that A_1, B_1, A_2, and B_2 are nonlinear functions of p_{yk1} and p_{yk2} that are parameters of Magic Formula. It is seen that A's and B's are identifiable with the concerned sensor set. It is also interesting to analyze the identifiability of p_{yk1} and p_{yk2} with the known A and B values. Using the implicit function theorem to check for a unique solution of the parameters, it is found that only one of p_{yk1} and p_{yk2} is identifiable.

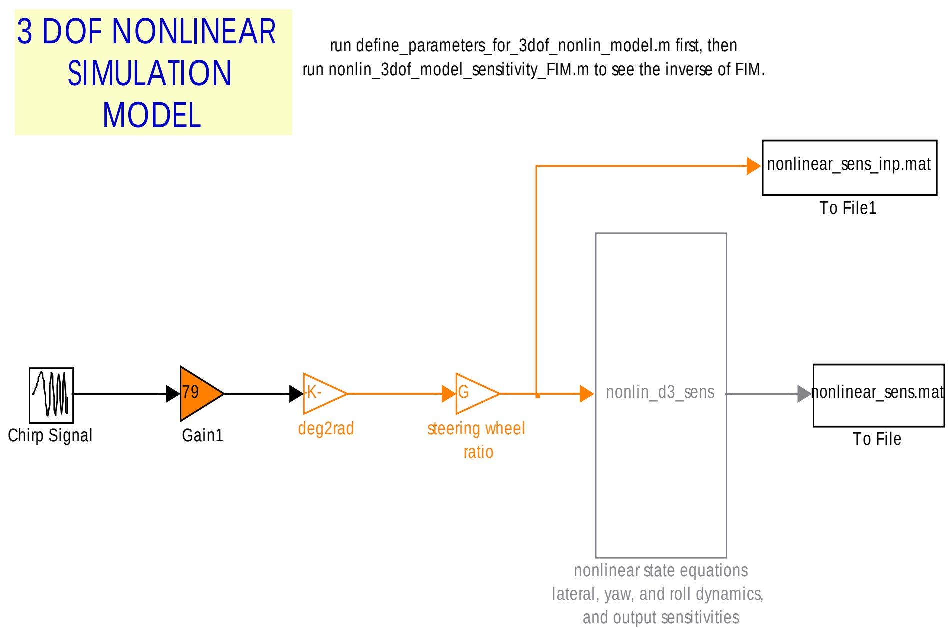


Figure 51 Model used in optimal input design and sensitivity analysis

5.6 Practical Identifiability

Sensitivities of outputs with respect to parameters include a lot of information. Considering nominal values, examples of sensitivity responses are given in the following figures. Figure 52 shows the steering input utilized in the simulation to generate sensitivities.

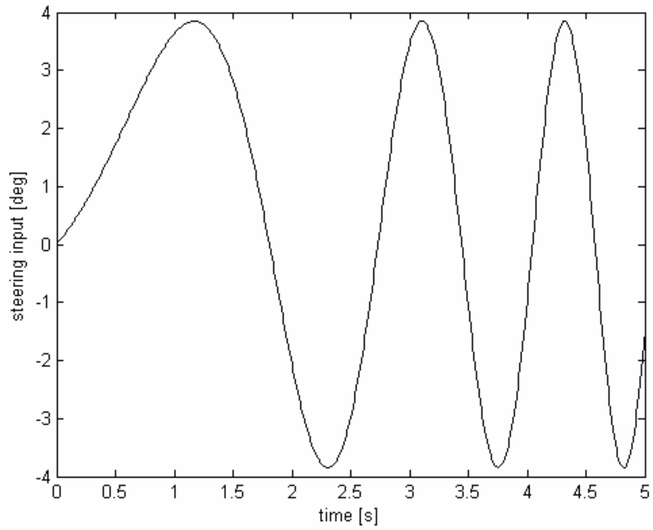


Figure 52 Steering input used in sensitivity analysis

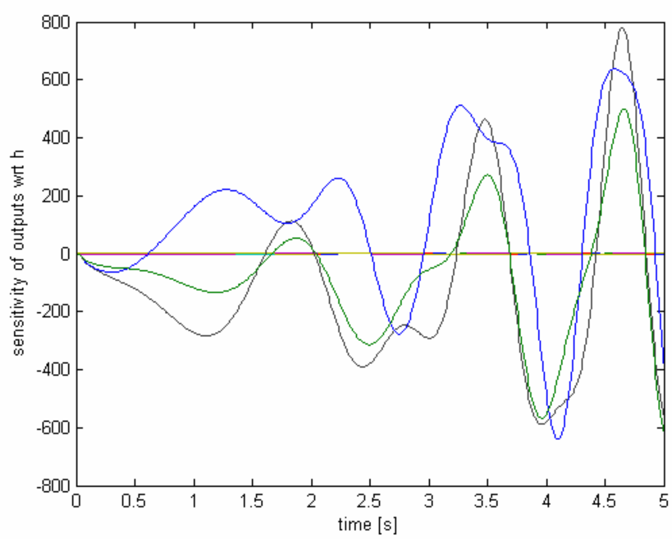


Figure 53 Sensitivity of outputs with respect to h

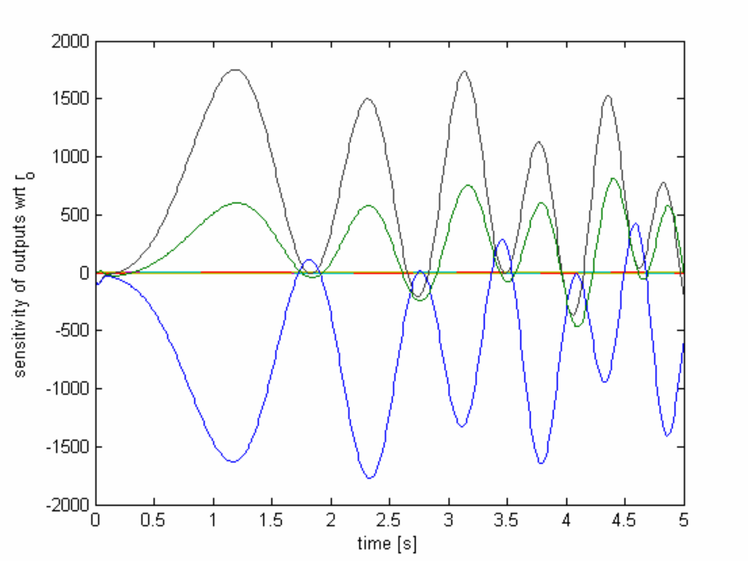


Figure 54 Sensitivity of outputs with respect to relaxation length

Structural or theoretical identifiability searches if it is possible to estimate the values of parameters by using noise-free measurements and an exact model structure. Although necessary, structural identifiability is not sufficient to guarantee successful parameter estimation from noisy data. The concept of practical identifiability covers the identifiability property of the model considering number of acquired data and affect of measurement noise. Therefore, practical identifiability is as important as the structural identifiability of model structure.

In the practical identifiability analysis, sensitivities of outputs with respect to the parameters are employed. If the sensitivity functions are linearly dependent the model is not practically identifiable. This represents a high correlation between the parameter estimates [61, 62, 63]. If two parameters are highly correlated, the change in the model output due to a change in one of the parameters can be compensated by an appropriate change in the other parameter value. This condition prevents determining unique parameter estimates even the model output is highly sensitive to the changes in individual parameters. Fisher Information Matrix can be used to check the practical identifiability of the system [64, 65, 66, 67]. If the determinant of the Fisher Information Matrix is not zero, i.e. not singular, then the model is identifiable. If the inverse of the matrix cannot be obtained, i.e. if it is singular, then the model is not identifiable. This means that sensitivity equations show linear dependence.

Using the output of the model in Figure 51, practical identifiablity test is also generated within this study. Since Fisher Information Matrix is generated by the Simulink model and the related m-file, then its rank can be checked easily. It is seen that Fisher Information Matrix has full rank for the measurement and parameters sets with the employed input discussed above.

5.7 Estimation of Parameters

Identifiability analysis and input design are the qualitative and quantitative parts of experiment design for the system identification. In this study no real experiment on physical vehicle is achieved for the purpose of nonlinear model identification. Simulated experiments are achieved on mathematical models and sensor outputs that are corrupted with noise are stored together with the applied input during the simulated experiment. These data are utilized in identification algorithm to find the parameter estimates.

The cost function minimized during identification algorithm is given below.

$$V_N(\hat{\theta}) = \frac{\sum_{i=1}^{N}\left[e_1^2(i)+e_2^2(i)+e_3^2(i)+e_4^2(i)+e_5^2(i)+e_6^2(i)+e_7^2(i)+e_8^2(i)+e_9^2(i)\right]}{\sum_{i=1}^{N}\left[y_1^2(i)+y_2^2(i)+y_3^2(i)+y_4^2(i)+y_5^2(i)+y_6^2(i)+y_7^2(i)+y_8^2(i)+y_9^2(i)\right]}100\,[\%] \qquad (161)$$

where,

$$e_j(i) = y_j(i) - y_m(i)\,,\ j = 1\ ,..,\ 9 \text{ and } i = 1\ ,..,\ N \qquad (162)$$

$y_j(i)$: measured j^{th} output,

$y_{jm}(i)$: j^{th} output of model (estimated output by model).

The parameter estimates are obtained by

$$\hat{\theta}_N = \arg_\theta \min\left(V_N(\hat{\theta})\right). \tag{163}$$

Parameter estimates take the values minimizing the cost function $V_N(\theta)$.

This cost function is *normed root mean square output error* [68]. It is very suitable for comparing different model structures. Genetic algorithm is used as an optimization tool to find the parameter estimates. A small population with 7 genes is used in the identification example given below. In fact such a small size of population is not common in optimization with genetic algorithm. This is preferred to speed up the algorithm. In identification practices with real experimental data, population size of about 50 will be chosen.

The simulated experiment has duration of 20 seconds. This duration is relatively small compared to the previous physical experiments. Duration is limited to reduce the computational time in computer and force the algorithm to get the estimates from the experiments with designed optimal inputs. The input in the experiment is designed by the procedure explained above. Following figures show the input and a typical set of acquired sensor data during the experiments.

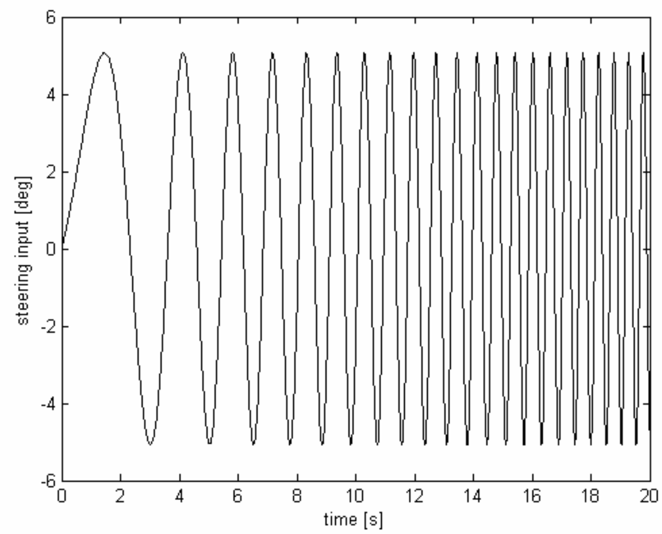


Figure 55 Steering input of the simulated experiment

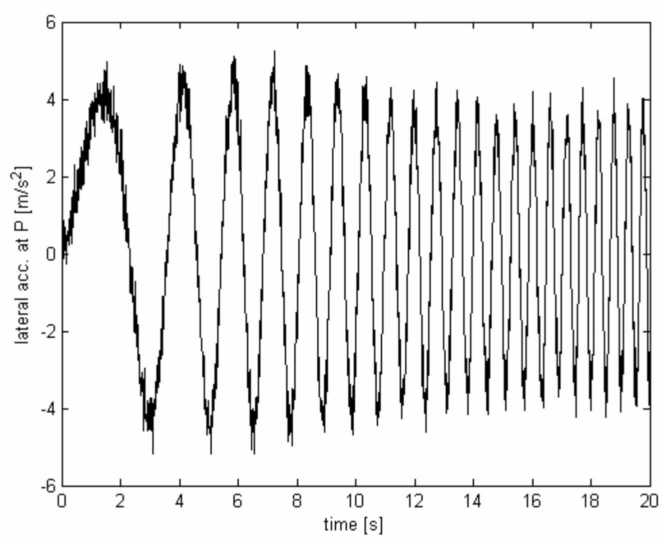


Figure 56 Lateral acceleration at point P

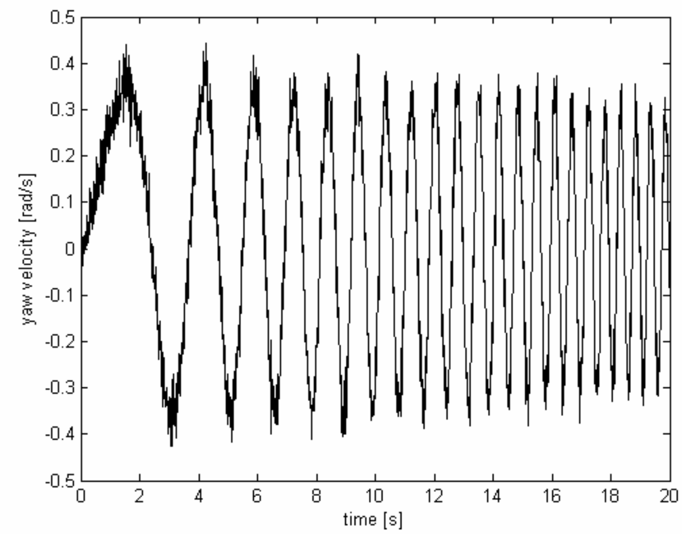


Figure 57 Yaw response

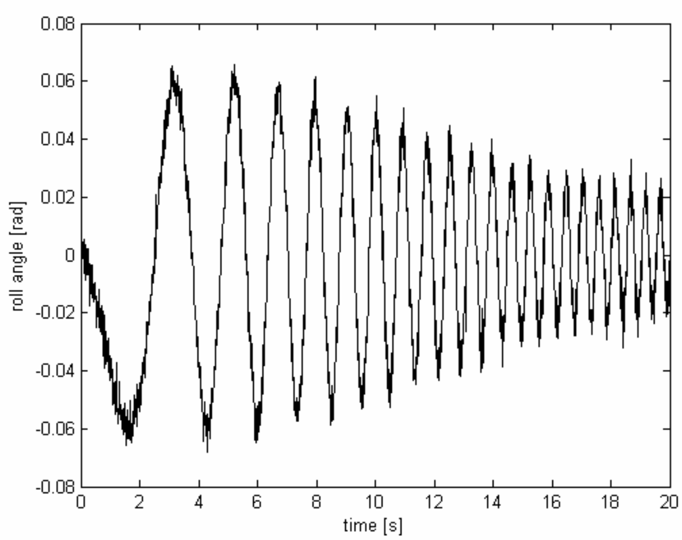


Figure 58 Roll angle

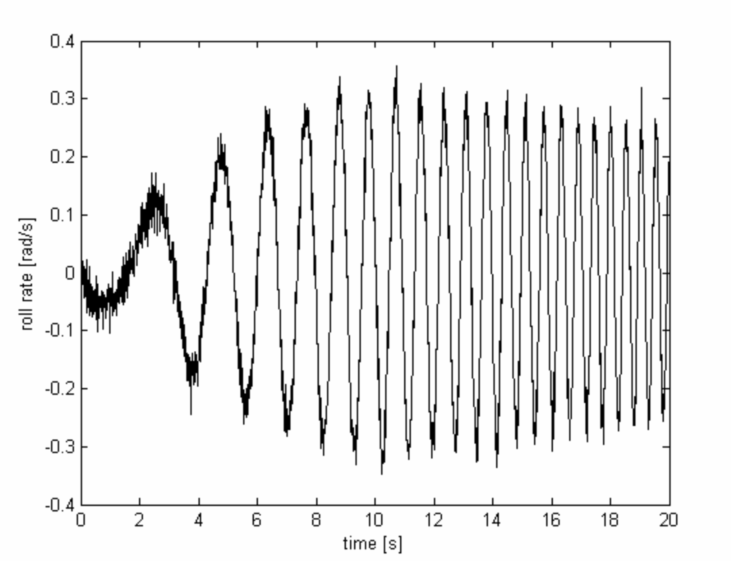


Figure 59 Roll rate

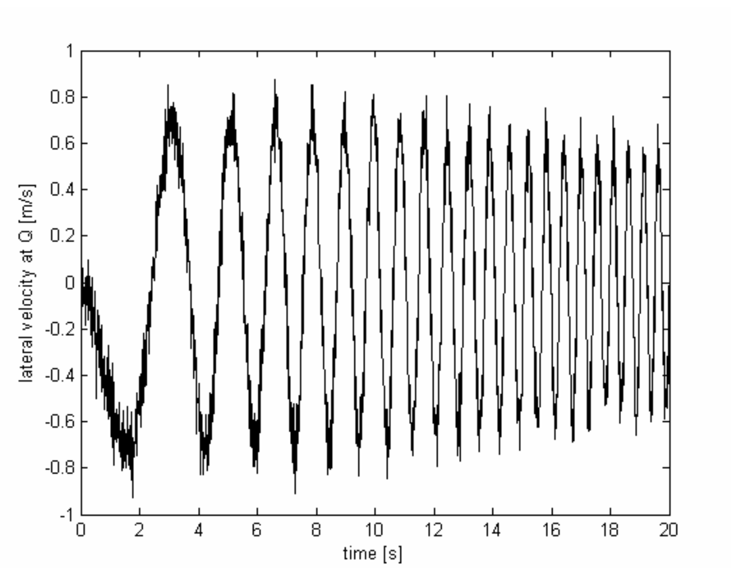


Figure 60 Side slip velocity at point Q

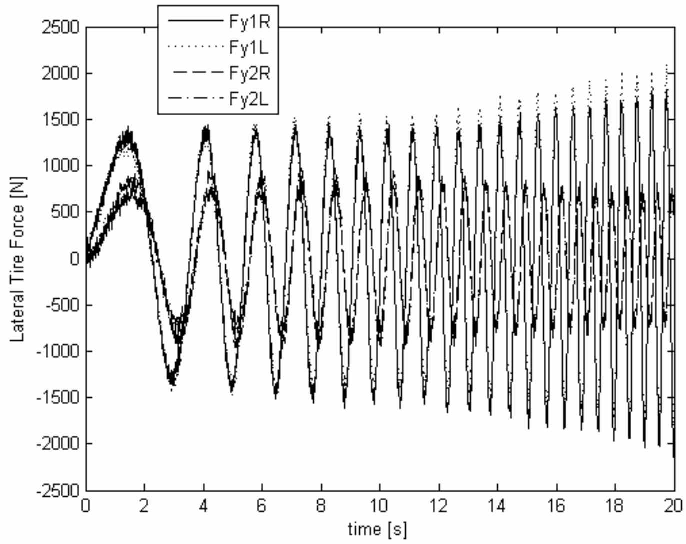


Figure 61 Lateral forces generated at tires

The identification algorithm has revealed a set of parameters with a cost function of approximately 6% normed root mean square error which is acceptable for such applications. Further improvements may be reached if the algorithm processes with a larger data set and larger population in the optimization algorithm.

Estimated parameters are compared with the actual ones used in the simulations of experiments shown in table below.

Table 4 Actual and estimated parameters of nonlinear model identification

	c_ϕ [N.m/(rad/s)]	k_ϕ [N.m/rad]	h [m]	ε_1	ε_2
Actual	5100	41000	0.54	0.074	0.01
Estimated	4533	37700	0.44	0.061	0.03
	A_1 [1/rad]	**B_1 [N/rad]**	**A_2 [1/rad]**	**B_2 [N/rad]**	**σ [m]**
Actual	-7.4	-16620	-9	-13660	0.09
Estimated	-6.3	-16700	-8.7	-15600	0.12

Identified model responses are compared with the measurements used for identification purpose in the following figures.

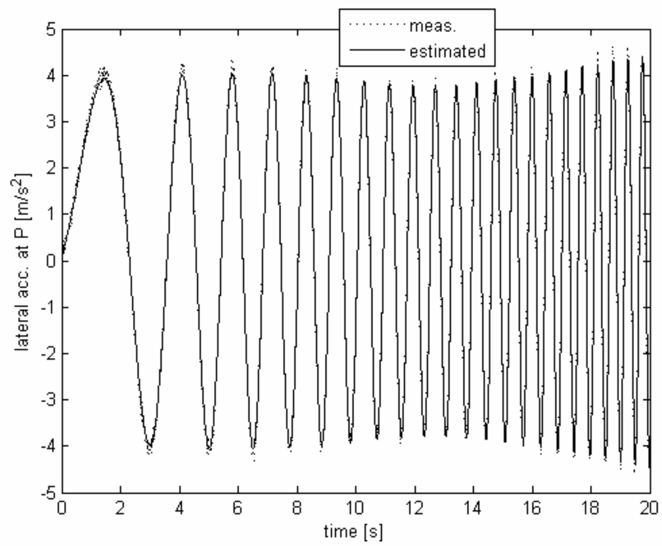


Figure 62 Lateral accelerations at point P

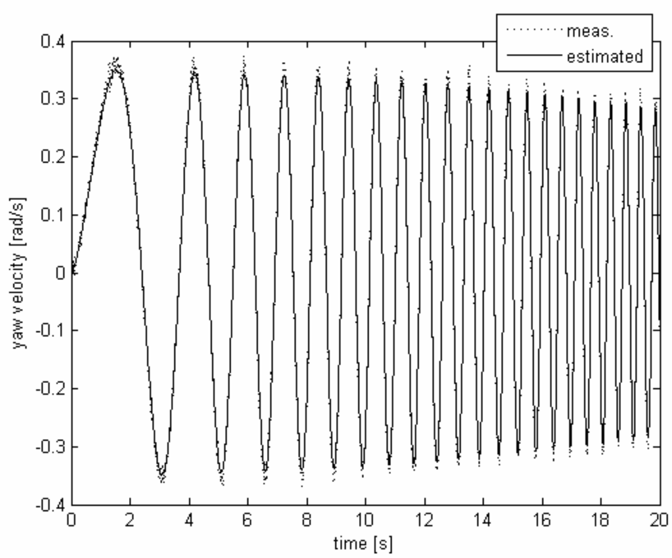


Figure 63 Yaw responses

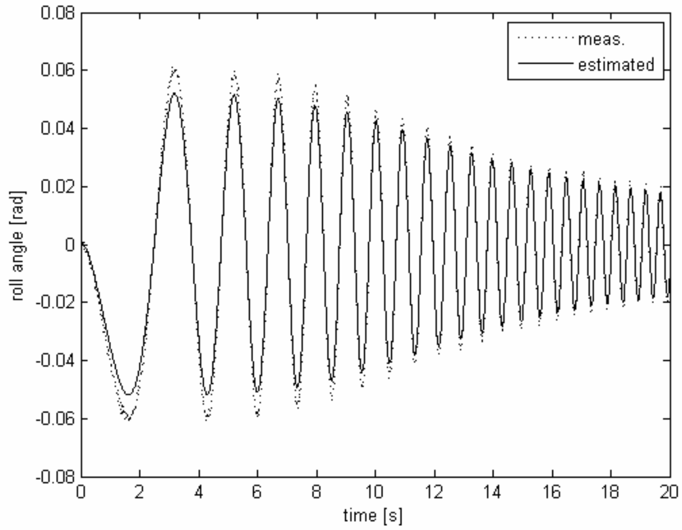


Figure 64 Roll angles

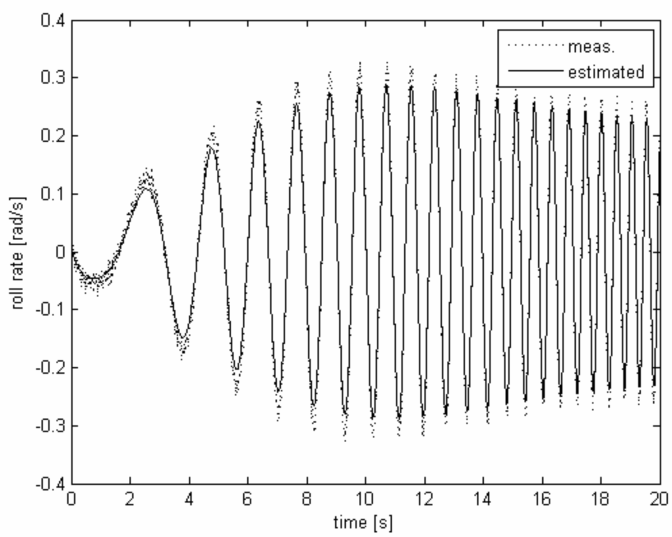


Figure 65 Roll rates

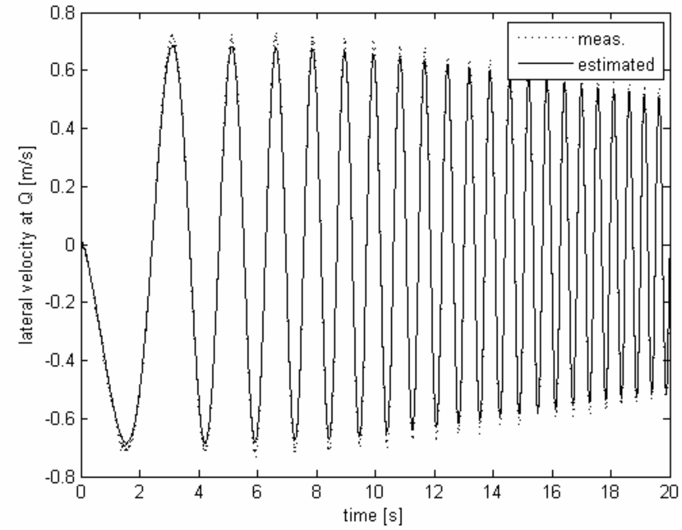


Figure 66 Side slip velocities at point Q

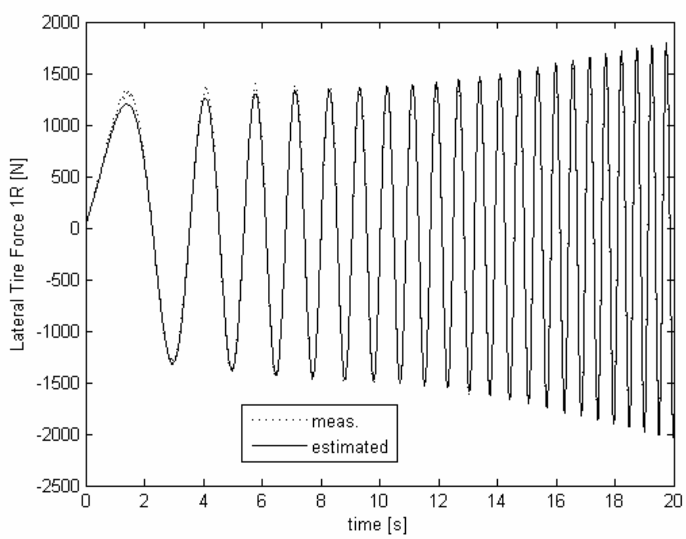


Figure 67 Lateral forces at tire 1R

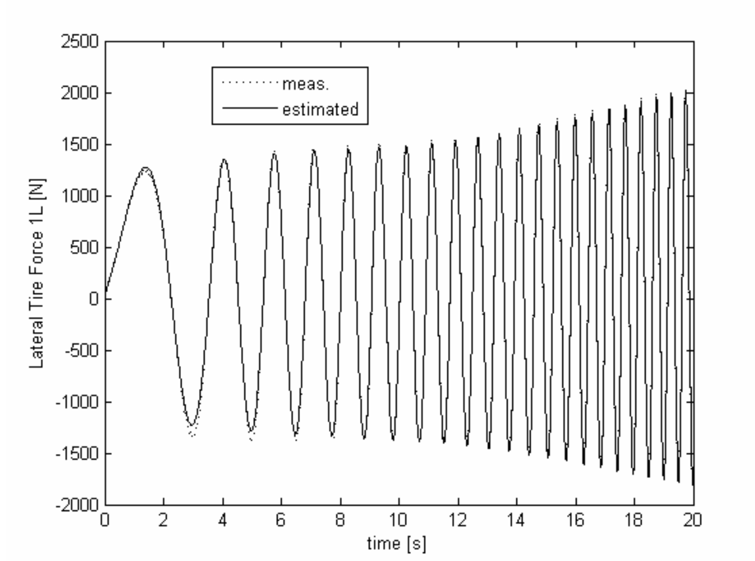


Figure 68 Lateral forces at tire 1L

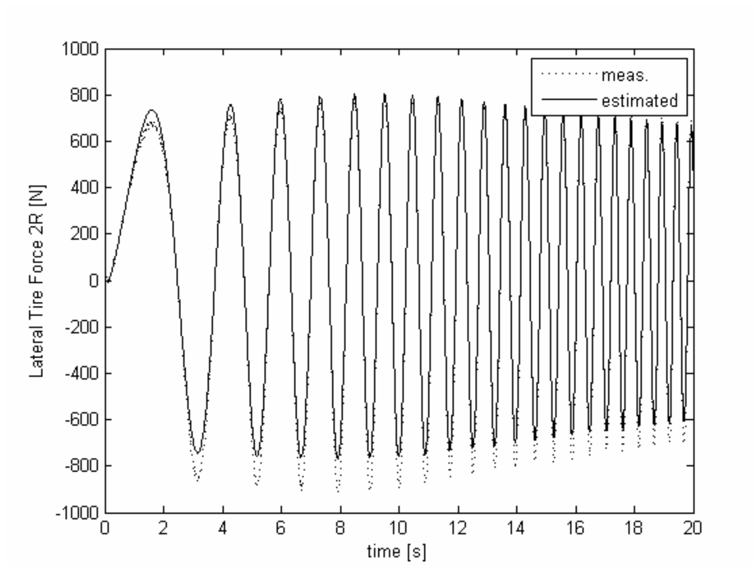


Figure 69 Lateral tire forces at 2R

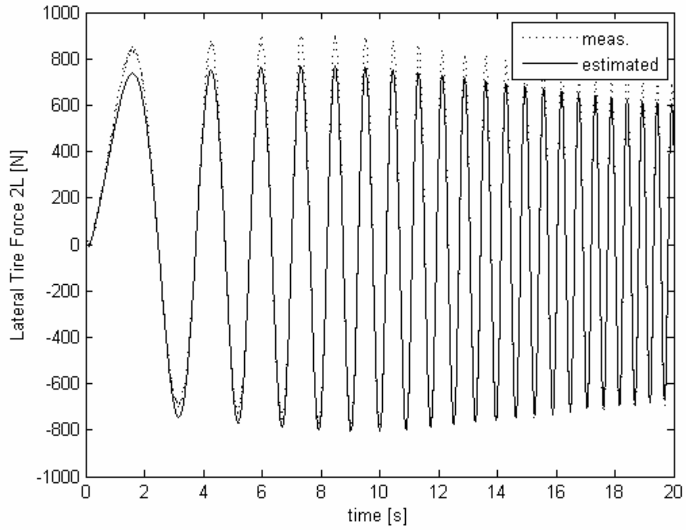


Figure 70 Lateral tire force at 2L

Figures above show that some outputs track the experimental data worse than others. It might be due to the sensitivity of the outputs with respect to parameters. Errors in parameter estimates make such differences on the model outputs.

In addition to the comparisons above, responses of the identified model are also compared with the validation data set. Following figures present the validation of the identified model. In those figures, deterministic model response, of whose outputs are corrupted with noise and assumed as acquired experimental data, are also plotted to compare with the noisy data and estimated outputs.

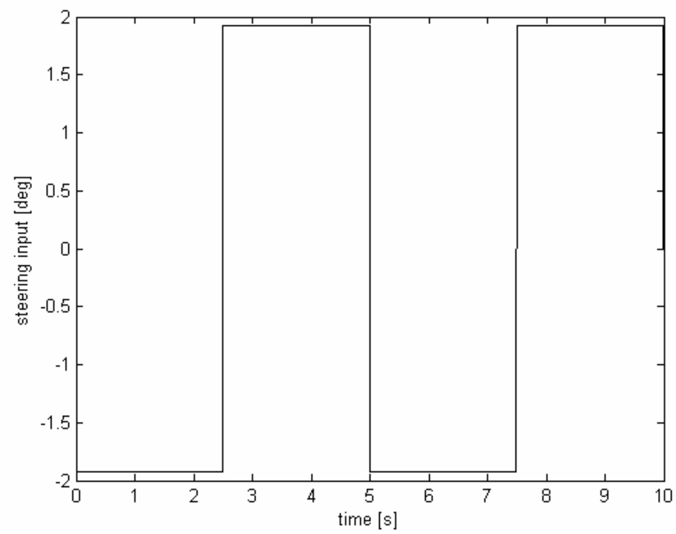


Figure 71 Steering input in validation data set

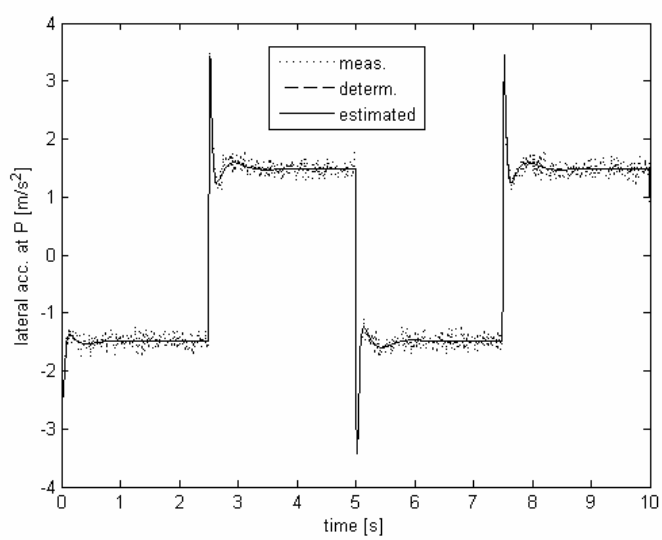


Figure 72 Lateral acceleration at point P

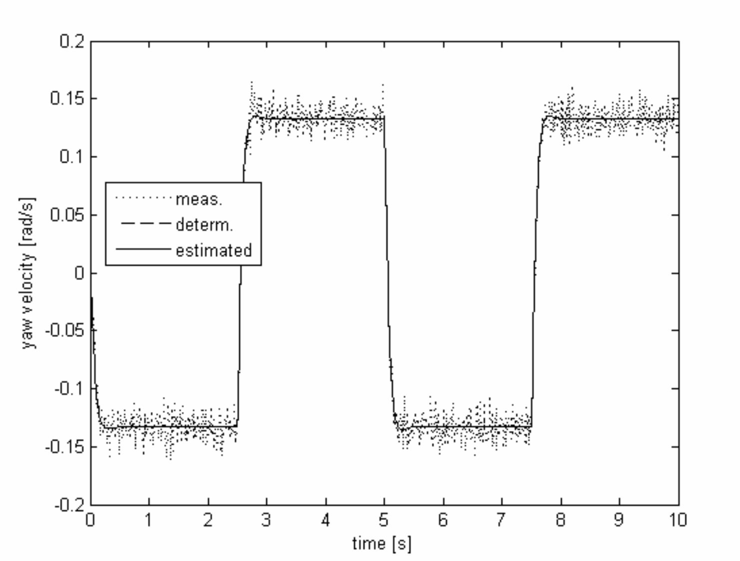


Figure 73 Yaw rates

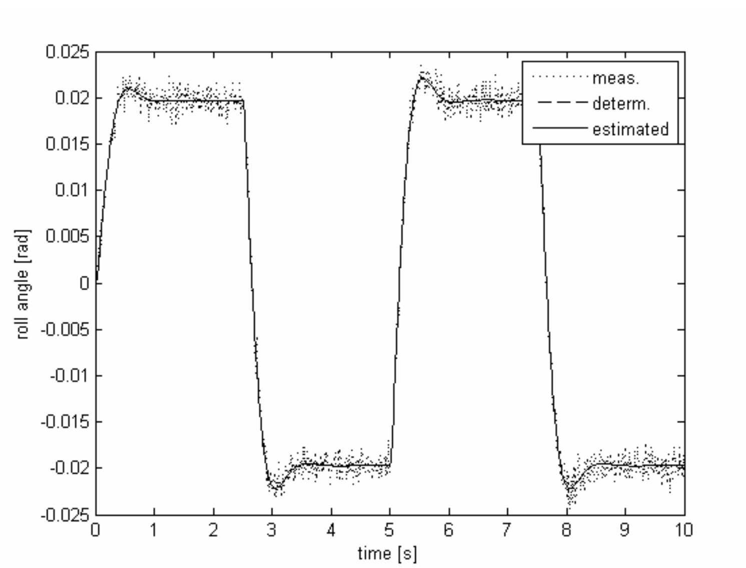


Figure 74 Roll angles

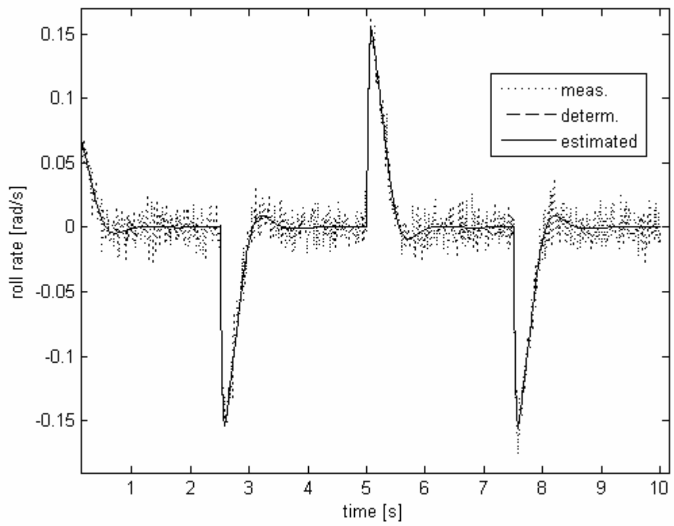


Figure 75 Roll rates

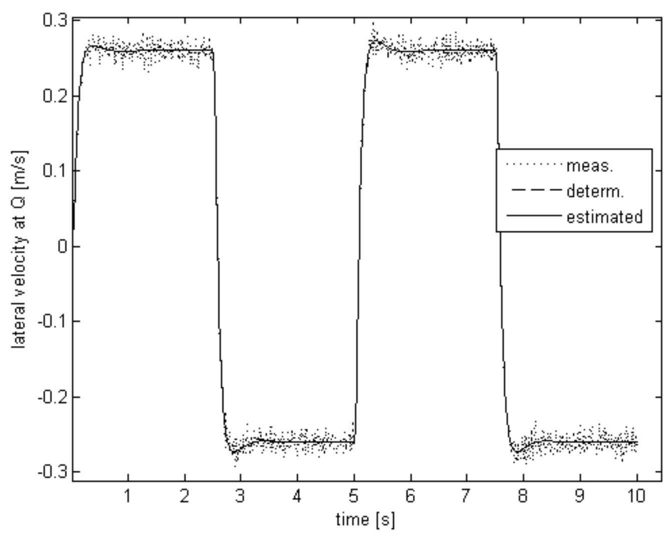


Figure 76 Side slip velocities at point Q

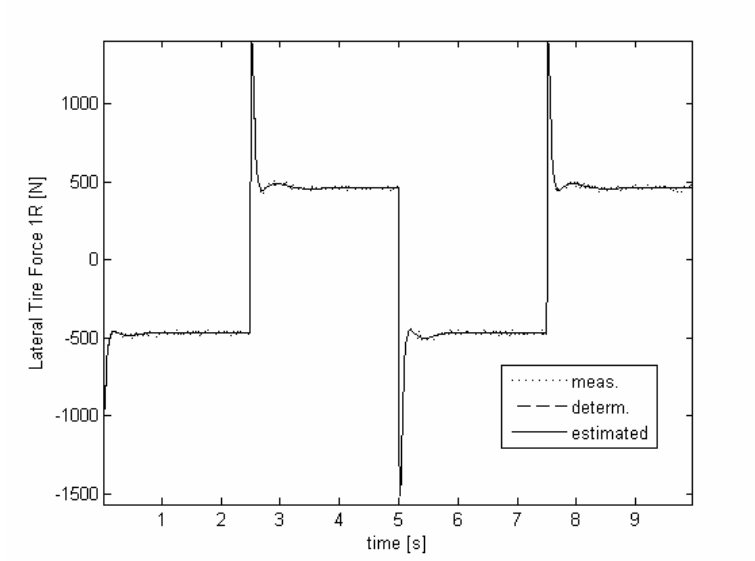


Figure 77 Lateral forces at tires 1R

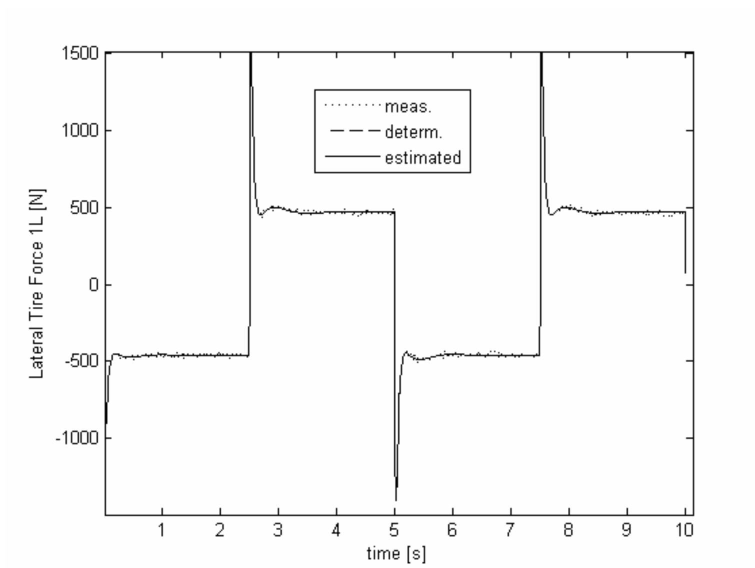


Figure 78 Lateral forces at tires 1L

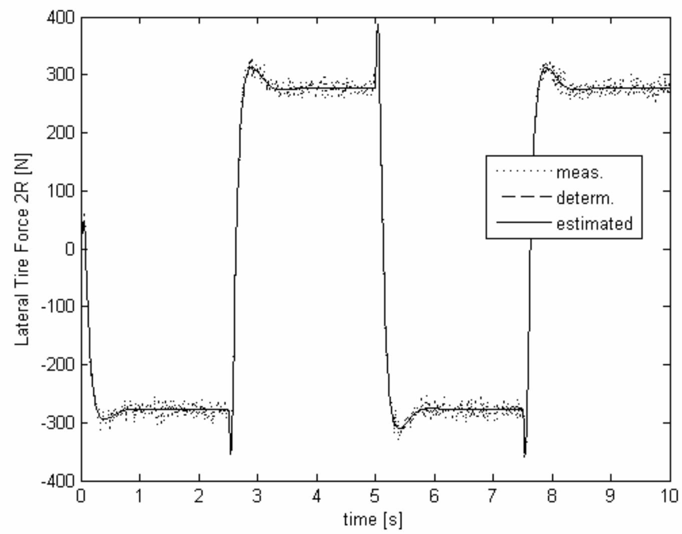


Figure 79 Lateral tire forces at 2R

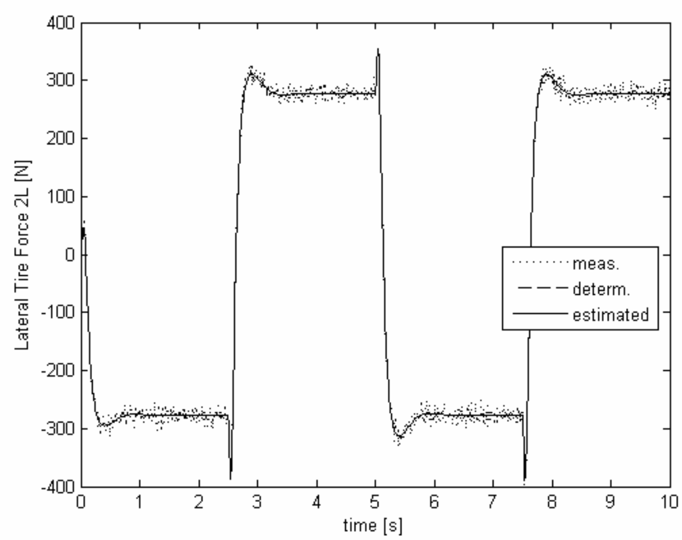


Figure 80 Lateral tire forces at 2L

It seen that validation data set is successfully tracked by the identified model responses. In addition, there are no biases with the deterministic model responses and identified model responses.

CHAPTER 6

CONCLUSION and DISCUSSION

This study is based on the research to identify linear and nonlinear handling models of a physical vehicle that is equipped with sensors and a data acquisition system. It is desired to implement the identification methodology on vehicle dynamics area from experiment design stage to the optimization phase. Starting model is linear bicycle model and the final structure is the nonlinear handling model with coupled roll, yaw and side slip dynamics to expose the affects of roll motion on handling characteristics.

After the linear 2 dof model structure is shaped, identifiability analysis of the handling model is performed for individual sensors and sets of sensors using the transfer function technique. Global identifiability is searched within the analysis. The analysis shows that even a single accelerometer or a gyro is sufficient to make physical parameters of interest identifiable. However, to improve the accuracy of the estimates, a sensor set consisting of an accelerometer and a gyro is utilized as the output in the identification algorithm. Improving the accuracy of the estimates is also critical due to the existence of process and measurement noises. In the model, the position of the accelerometer need not be known as it can also be identified. It can be placed somewhere on the body y axis. This simplifies the design of experiments. Structural identifiability analysis forms the qualitative design stage of the experiment design for identification purpose. Results of this study have been implemented in the identification procedure to estimate the physical parameters of a passenger car model based on linear handling models.

In the application part of the study, a vehicle is instrumented and experiments are carried out such that data can be collected for the identification of linear predictor parameters based on linear bicycle model.

So as to improve the accuracy of the estimates, a sensor set consisting of an accelerometer and a gyro is utilized as the system output and the input for the identification algorithm. Physical parameters of the model are estimated using the prediction error estimation method. In order to overcome the problems arising due to the selection of the initial value for the parameters and local minima, GA is utilized as the optimization technique. Selection of GA parameters is also critical. It affects the values of estimated parameter. Constraint optimization is achieved by using upper and lower bounds for the parameter values. This brings the need of prior information about the values of parameters.

Genetic algorithm makes use of a population as initial guesses for the parameter estimates. This makes the optimization algorithm overcome the problems about initial values of parameter estimates. On the other hand, it has a weakness in the convergence performance. It is desired to start the optimization with genetic algorithm and then to go on with a gradient based search algorithm to overcome both the initial value problem and subsequent local minima problems and the convergence problem in the further identification study of more complicated handling models.

Identified model structure can also be used for state estimation purposes. It can be shown that using a single gyro or an accelerometer guarantees the observability of the system. Therefore, identified Kalman predictor using a single gyro (or accelerometer) enables the estimation of states, namely v and r.

Identified predictors and linear model structures are validated by using data sets that are not utilized for identification purposes. It is seen that the identified linear bicycle model and the predictor simulate the actual vehicle very well. Even though the estimated parameter values differ in using different sensor sets due mostly to the use of pure genetic algorithm, local minima, and use of suboptimal inputs, validation tests show that using a single accelerometer or a single gyro may provide acceptable predictors and models. It is also desired to use optimal inputs that are designed for the experiments by use of a steering robot in the following identification study of complicated handling models. This will also improve the accuracy of estimated parameters.

2 dof linear handling model can be improved by adding some terms reflecting affects of load transfers without the abusing the linearity. Used methodology can also be implemented on such models. It is interesting to analyze the identifiability of the modified model structure with the available sensor set.

Design of the experiments for nonlinear model identification is also studied in the thesis. Structural identifiability analysis for nonlinear model structures is not as clear and systematic as in the case of identifiability analysis of linear model structures. Taylor series expansion and differential algebraic methods are focused. Former gives the sufficient condition for identifiability. This means using very high orders of derivatives of outputs may show that the model structure is identifiable. However, the order is not certain and those derivatives may reveal very complex coefficients to solve. Therefore Taylor Series expansion is not favored and latter one is preferred in the thesis study.

Employed identifiability method is based on differential algebraic point of view. It requires the elimination of state variables from the output equations by using the successive derivatives of the outputs. The common problem of identifiability analysis of nonlinear systems again limits the study. Even with today's computational power such equations with huge number and amount of coefficients cannot be handled and solved by symbolic computation packages. So as to overcome such extreme equations, model structure is simplified by considering some assumptions, and modifications of the terms. For example trigonometric terms are replaced by their series expansion approximates. Sensor set is also enriched to reduce the computational problem. These factors reduced the size of the observable representations that are used to reach the coefficients that are nonlinear functions of physical parameters. This technique shows that 15 parameters of the vehicle model structure are structurally locally identifiable using the discussed sensor set. This means that these parameters can be represented by input, outputs and their derivatives.

In addition the structural identifiability, practical identifiability of the system model is also discussed. Even the structure is theoretically identifiable, the identifiablity of the parameters is not guaranteed due to the existence of noisy measurements and characteristics of the inputs. The sensitivities of outputs to system parameters are derived for the optimal input design purpose. These sensitivities are also important in practical identifiability analysis of the system. High correlation of the estimated parameters is not desired. It degrades the practical identifiability of the system. Rank of Fisher Information Matrix is used to check the the practical identifiability of the system. If it is not full rank, linear dependencies exist within the sensitivity functions. In fact, the practical identification analysis techniques and algorithms are easier to evaluate when compared to structural identiability analysis techniques.

Considering the optimal input design and practical identifiability, necessary algorithms are generated in MatLab Simulink environments. These algorithms generate a systematic way that can be utilized in the identification practices using real experimental data from a physical vehicle and in real experimental conditions with reduced sensor set.

The practical identifiability analysis techniques, sensitivity functions will be studied in the further studies of vehicle dynamics identification and analysis. This may reduce the experiment design phase of the identification procedure. Also, the parameters and settings of the genetic algorithm used in optimization stage may be discussed and studied in the further applications to improve the accuracy of the estimated parameters.

As the identifiability analysis techniques reach computationally simpler forms, nonlinear model structure used in this thesis might be improved and trimmed terms may be replaced to analyze more complex model structures. Reducing the sensor set and analyzing the identifiablity of the system is also an interesting study. Similar to the relevant chapter of 2-dof model structures, identifiability of parameters in case of a use of possible sensor combinations or single sensors may be observed. Additional measurement of normal forces on tires, or direct measurement of slip angles may reveal important results in identifiability analysis and identification applications.

Black-box model structures may also be utilized for the identification of vehicle handling dynamics. In this study, just NNARX type model structures are utilized, however, more suitable black-box structures such as state-space neural networks may be employed in future studies.

Identifed parametric model structures may also be utilized in fault diagnosis practices in handling dynamics research.

Report of 2 dof model identification part, *Identification of linear handling models for road vehicles*, is submitted to *Vehicle System Dynamics* journal. It is accepted and expected to be published in 2008.

Other journal and conference papers belonging to the thesis study are listed below.

- Arıkan, K., B., Ünlüsoy, Y. S., Araç Dinamiğinin Tek İzli Araç Modeli Yardımıyla Tanılanması, *Makina Tasarım ve İmalat Dergisi*, Cilt 6, Sayı 2, pp. 1-10, 2004.

- Arıkan, K., B., Ünlüsoy, Y. S., Tek İzli Araç Modelinde Fiziksel Parametrelerin Kestirimi, Otomatik Kontrol Türk Milli Komitesi Otomatik Kontrol Kongresii, TOK'05, 2005.

- Arıkan, K., B., Ünlüsoy, Y. S., Çelebi, A., O., Korkmaz, İ., Identification of Linear Bicycle Model of an Automobile Using Experimental Data, 3. Otomotiv Teknolojileri Kongresi, OTEKON'06, 2006.

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CURRICULUM VITAE

PERSONAL INFORMATION

Surname, Name: ARIKAN, Kutluk Bilge
Nationality: Turkish (TC)
Date and Place of Birth: 23 August 1974, Ankara
Marital Status: Single
Phone: 0 542 389 0906
E-mail: kutluk.arikan@gmail.com

EDUCATION

Degree	Institution	Year of Graduation
MS	METU Mechanical Engineering	2000
BS	METU Mechanical Engineering	1997
High School	Ankara Özel Yükseliş Koleji	1992

WORK EXPERIENCE

Year	Place	Enrollment
2003-Present	Atılım University, Mechatronics Engineering Department	Instructor
1997-2003	METU Mechanical Engineering Department	Research Assistant

FOREIGN LANGUAGES

Advanced English, Beginner German

PUBLICATIONS

- Arıkan, K., B., Ünlüsoy, Y. S., Korkmaz, İ., Çelebi, A., O., Identification of linear handling models for road vehicles, *Vehicle System Dynamics*, (In Press), 2008.

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RESEARCH INTERESTS

Automotive, Control System Design, Applied Control, Identification, Estimation, Mechatronics, Mobile and Flying Robotics, Sensor Systems, Vibration

HOBBIES

Music, Photography, Compilation Album Design, Reading, Football, Basketball, Handball, Movies