# THE EFFECTS OF A MATHEMATICS TEACHING METHODS COURSE ON PRE-SERVICE ELEMENTARY MATHEMATICS TEACHERS' CONTENT KNOWLEDGE FOR TEACHING MATHEMATICS

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Approval of the Graduate School of Social Sciences

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# ABSTRACT

THE EFFECTS OF A MATHEMATICS TEACHING METHODS COURSE ON PRE-SERVICE ELEMENTARY MATHEMATICS TEACHERS' CONTENT KNOWLEDGE FOR TEACHING MATHEMATICS

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The purpose of this study is to examine the effects of a mathematics teaching methods course on pre-service elementary mathematics teachers' content knowledge for teaching mathematics (CKTM). In order to accomplish this purpose, pre-service mathematics teachers' understanding of basic concepts and procedures in school mathematics, use of mathematical definitions, presentation of mathematical content to students, identification of common errors, misconceptions and solution strategies and evaluation of unusual solution methods were examined with the help of a multiple choice test.

The data were collected from 43 senior pre-service mathematics teachers from a teacher education program at a large public university in Ankara. The participants were given an 83-item test to measure their content knowledge for mathematics teaching at the beginning and after the methods course. The purpose of the pre- and post-test assessment was to measure the amount of change in the participants' knowledge for mathematics teaching. The test was developed and piloted at the University of Michigan in the USA for Learning Mathematics for Teaching (LMT) Project. Quantitative data analysis techniques were used to answer the research questions.

The results indicated that there was a significant effect of the mathematics teaching methods course on pre-service teachers' content knowledge for teaching mathematics. Moreover, the findings showed that there is no significant mean difference between male and female pre-service teachers, and between the pre-service teachers who have taken at least one mathematics teaching elective course and the ones who have not taken any elective course related to mathematics teaching in terms of their CKTM. Also, the study showed that there is a significant positive relationship between pre-service teachers' CKTM and their academic achievement on undergraduate mathematics content courses.

The study is expected to make important contributions to the literature by providing information about whether the methods courses significantly contribute to pre-service teachers' understanding of knowledge for mathematics teaching. Moreover, the findings of the study is hoped to inform teacher educators and policy makers about the needs and improvements in teacher preparation programs.

Keywords: Teacher knowledge, pre-service elementary mathematics teachers, mathematics teaching methods course, content knowledge for teaching mathematics

# ÖZ


MATEMATİK ÖĞRETİMİ YÖNTEMLERİ DERSİNİN İLKÖĞRETİM MATEMATİK ÖĞRETMEN ADAYLARININ MATEMATİK ÖĞRETİMİNE YÖNELİK ALAN BİLGİLERİ ÜZERİNDEKİ ETKİLERİ



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Bu çalışmanın amacı, matematik öğretimi yöntemleri dersinin ilköğretim matematik öğretmen adaylarının matematik öğretimine yönelik alan bilgileri üzerindeki etkilerini incelemektir. Bu amaç doğrultusunda, ilköğretim matematik öğretmen adaylarının temel kavram ve işlemlere yönelik anlayışları, matematiksel tanımlarını kullanışları ve bunları öğrencilere sunuşları, öğrencilerin yaygın hatalarını, kavram yanılgılarını ve çözüm yöntemlerini belirleyişleri ve değişik çözüm yöntemlerini değerlendirme şekilleri çoktan seçmeli bir test yardımıyla incelenmiştir.

Çalışmanın verileri Ankara'daki bir devlet üniversitesinde öğretmen yetiştirme programına devam eden 43 son sınıf öğrencisinden toplanmıştır. Katılımcılara matematik öğretimi yöntemleri dersinin başında ve sonrasında matematik öğretimine yönelik alan bilgilerini ölçen 83 soruluk bir test uygulanmıştır. Ön ve son test ile katılımcıların matematik öğretimine yönelik bilgilerindeki değişimi ölçmek amaçlanmıştır. Bu çalışmada kullanılan test, Michigan Üniversitesi'nde yürütülen Learning Mathematics for Teaching (LMT) projesi kapsamında geliştirilmiş ve pilot çalışmaları yapılmıştır.

Çalışma sonuçları, matematik öğretimi yöntemleri dersinin ilköğretim matematik öğretmen adaylarının matematik öğretimine yönelik bilgileri üzerinde anlamlı bir etkiye sahip olduğunu göstermiştir. Bunun yanı sıra, bulgular kız ve erkek öğretmen adayları arasında, ve matematik öğretimine yönelik seçmeli ders alan ve almayan öğretmen adayları arasında anlamlı bir fark olmadığını ortaya koymuştur. Ayrıca, çalışma öğretmen adaylarının matematik öğretimine yönelik bilgileri ile lisans eğitimi matematik derslerindeki akademik başarıları arasında anlamlı ve pozitif yönlü bir ilişki olduğunu göstermiştir.

Çalışmanın, matematik öğretimi yöntemleri dersinin ilköğretim matematik öğretmen adaylarının matematik öğretimine yönelik alan bilgileri üzerine anlamlı bir katkı sağlayıp sağlamadığına dair bilgi sunması açısından literatüre önemli katkılar sağlayacağı düşünülmektedir. Bunun yanı sıra, çalışmanın öğretmen eğitimcilerini ve eğitim politikacılarını öğretmen yetiştirme programlarındaki ihtiyaçlar ve gelişmeler hakkında bilgilendirilmesi umulmaktadır.

Anahtar Kelimeler: Öğretmen bilgisi, ilköğretim matematik öğretmen adayları, matematik öğretimi yöntemleri dersi, matematik öğretimine yönelik bilgi

To my grandfather,
I wish I had spent more time with you

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Finally, I would like to dedicate this study to my grandfather who died in 1993. I wish I could spend more time with you. I think I had a lot of things that I could learn from you. I miss you a lot, DEDE.

# TABLE OF CONTENTS

## LIST OF TABLES

## LIST OF FIGURES

## LIST OF FORMULAS

## LIST OF ABBREVIATIONS

CK: Curricular Knowledge

CCK : Common Content Knowledge

CKC : Common Knowledge of Content

CKTM : Content Knowledge for Teaching Mathematics

EME: Elementary Mathematics Teacher Education

GEO : Geometry

GPK : General Pedagogical Knowledge

IACTS : Identify, Assess, Challenge, Transform, and Sustain

KCS : Knowledge of Content and Students

KCT : Knowledge of Content and Teaching

LMT : Learning Mathematics for Teaching

MDFQ : Multiplication and Division of Fractions Questionnaire

METU: Middle East Technical University

NCOP : Number Concepts and Operation

PCK : Pedagogical Content Knowledge

PMT: Pre-service Mathematics Teachers

PFA : Patterns, Functions and Algebra

SCK : Specialized Content Knowledge

SETM : Survey of Elementary Teachers of Mathematics

SII : Study of Instructional Improvement

SKC : Specialized Knowledge of Content

SMCK: Subject Matter Content Knowledge

SMK : Subject Matter Knowledge

SUM : Specialized Understanding of Mathematics

# CHAPTER I

# INTRODUCTION

As stated by Horowitz, et al. (2005) "Teaching is not just talking, and learning is not just listening" (p.88). Effective teaching requires knowing the ways of making the subject matter understandable for the students (Ball, Thames, & Phelps, 2007). Thus, teachers play an important role in students' understanding of mathematics effectively (Işıksal, 2006). Many researchers have pointed out that effective teachers understand the need and importance of the knowledge of the subject area, other disciplines, learners, and educational, social and cultural context of the country (Ball, 1991a; Ball & McDiarmid, 1990; Nakiboğlu & Karakoç, 2005).

Moreover, most of the previous studies have emphasized the importance of mathematical content knowledge for teaching among the knowledge bases which should be possessed by teachers (Ball, 1990a; Ball, 1990b; Ball, 1991a; Ball & McDiarmid, 1990; Shulman, 1987). Researches have discussed that teachers should know not only the mathematics deeply and conceptually, but also the ways of making mathematics meaningful for the students. So, teachers should have content knowledge for teaching mathematics which means knowing what and how will be taught (Ball 1991b; Ball, Thames, & Phelps, 2007; Borko, 2004; Ma, 1999; Wilson & Berne, 1999).

Regarding the 'content knowledge for teaching mathematics', some researchers have concentrated on explaining the components of this type of knowledge (Hill & Ball, 2004; Shulman, 1986; 1987). Shulman (1986) divided teacher knowledge into three main categories; namely, subject matter content knowledge (SMCK), pedagogical content knowledge (PCK), and curricular knowledge (CK). SMCK refers to the knowledge about the mathematical content that will be taught, whereas PCK means the knowledge about the ways of teaching the mathematics concerning multiple representations, materials, appropriate and alternative solutions, and students' preconceptions and misconceptions. The curricular knowledge addresses to the characteristics of the curriculum of the subject matter. Even though Shulman (1986) proposed his categorization of content knowledge for any subject area of education, there are many studies in mathematics which have utilized his categorization as a framework (Berenson, et al., 1997; Even, 1993; Even & Tirosh, 1995; Leinhardt & Smith, 1985; Ma, 1990; Zhou, et al., 2006). On the other hand, Hill and Ball (2004) proposed mathematics specific categorization of content knowledge for teaching. Hill and Ball (2004) described the content knowledge as it is composed of specialized knowledge of content (SKC) and common knowledge of content (CKC). While common knowledge of content is the deep knowledge of mathematics itself, specialized knowledge of content is related to the knowledge of how to teach mathematics for understanding. Hill and Ball (2004) concentrated on SKC since they declared that SKC is the knowledge which identifies teaching professions.

The literature includes several studies investigating either pre-service teachers' or in-service teachers' mathematical content knowledge for teaching (Even & Tirosh, 1995; Hill, 2007; Moss, 2006; Quinn, 1997). These studies have showed that pre-service teachers have limited understanding about content knowledge for teaching mathematics. Especially, pre-service teachers and in-service teachers do not have sufficient specialized knowledge of content for teaching mathematics. Thus, researchers have suggested that teacher educators should take the mathematical content knowledge for teaching into consideration seriously while designing their courses. Also, those studies have highlighted the role of methods courses in improving pre-service teachers' content knowledge for teaching mathematics.

The situation of in-service and pre-service teachers' mathematical content knowledge in Turkey is almost the same. More specifically, Turkish in-service and Turkish pre-service teachers are not competent enough in terms of the content knowledge for teaching mathematics, the knowledge about explaining meanings of the concepts and procedures, using multiple representations, using materials, and evaluating students' solutions (Acar, 2005; Ay, 2004; Bütün, 2005; Işıksal, 2006; Sıvacı, 2003; Türnüklü, 2005; Yıldız & Ilgar, 1999). Bütün (2005) have pointed out that the deficiencies of Turkish in-service and pre-service teachers in mathematical content knowledge have influenced their teachings. Based on this existing situation in Turkey, the researchers suggested that teacher education programs should give sufficient importance to content and mathematics methods courses. Furthermore, the studies showed that the mathematics teaching methods course plays an important role in improving pre-service teachers' mathematical content knowledge for teaching (Kinach, 2002; Moss, 2006; Quinn, 1997). Those studies have implied that it is important to conduct studies which examine whether mathematics teaching methods course influences pre-service mathematics teachers' content knowledge improvement. Hence, this study aims to investigate the effect of mathematics teaching methods course on pre-service mathematics teachers' content knowledge for teaching mathematics. More specifically, this study intends to answer the following questions:

1. Is there a significant change in the pre-service elementary mathematics teachers' SETM total scores following their participation in the mathematics teaching methods course?

    1.1. Is there a significant change in the pre-service elementary mathematics teachers' SETM scores in the Number Concepts and Operation Subscale following their participation in the mathematics teaching methods course?

    1.2. Is there a significant change in the pre-service elementary mathematics teachers' SETM scores in the Geometry Subscale following their participation in the mathematics teaching methods course?

    1.3. Is there a significant change in the pre-service elementary mathematics teachers' SETM scores in the Algebra Subscale following their participation in the mathematics teaching methods course?

2. Is there a significant mean difference in the gain scores for male and female pre-service elementary mathematics teachers?

2.1. Is there a significant mean difference in the gain scores of the Number Concepts and Operation Subscale for male and female pre-service elementary mathematics teachers?

2.2. Is there a significant mean difference in the gain scores of the Geometry Subscale for male and female pre-service elementary mathematics teachers?

2.3. Is there a significant mean difference in the gain scores of the Algebra Subscale for male and female pre-service elementary mathematics teachers?

3. Is there a significant mean difference in the posttest scores for male and female pre-service elementary mathematics teachers?

3.1. Is there a significant mean difference in the posttest scores of the Number Concepts and Operation Subscale for male and female pre-service elementary mathematics teachers?

3.2. Is there a significant mean difference in the posttest scores of the Geometry Subscale for male and female pre-service elementary mathematics teachers?

3.3. Is there a significant mean difference in the posttest scores of the Algebra Subscale for male and female pre-service elementary mathematics teachers?

4. Is there a significant mean difference in the posttest scores of pre-service elementary mathematics teachers with respect to their elective course preferences?

5. Is there a relationship between pre-service mathematics teachers' posttest scores and their academic achievement on mathematics content courses?

## 1.1. Motivations for the Study

In Turkey, elementary school students do not perform well in the mathematics as evidenced by the results of TIMSS (Third International Mathematics and Science Study), PISA (The Program for International Student Assessment), OKS (Secondary School Student Selection Exam), and OSS (University Entrance Exam). To state more specifically, the 8$^{th}$ grade students participated to TIMSS in 1999, and the results showed that Turkey was in the 31$^{st}$ place among 38 countries in terms of the mathematics performance (Eğitimi Araştırma ve Geliştirme Dairesi Başkanlığı [EARGED], 2003). Similarly, Turkey was in the 28$^{th}$ place among 40 countries in terms of mathematics achievement in PISA in 2003 (EARGED, 2005). In addition to these international exams, the mathematics performance of Turkish students is not good enough in OKS and OSS. For instance, the mean of the mathematics scores in OKS was found as 1.15 out of 23 in 2004, and 2.35 out of 25 in 2005 (Milli Eğitim Bakanlığı [MEB], 2004, 2005). These results of the international and national exams have indicated that there is a need for the reform movements in mathematics education.

Based on this need, the new curriculum for elementary mathematics education was recently developed. One of the goals of the new curriculum is to teach mathematics for understanding and to make students learn meaningfully. To achieve this, the new curriculum has emphasized using concrete materials and technology in teaching, providing students the opportunity for connecting mathematical concepts or procedures with others, and working cooperatively. Moreover the new elementary school mathematics curriculum has aimed to develop the following five skills in students: (1) Problem Solving, (2) Communication, (3) Reasoning, (4) Estimation Strategies, and (5) Connection (Ministry of National Education, 2005, 2007). However, to implement the new curriculum, the role of teacher is so important that they should possess the necessary characteristics, skills and knowledge (Işıksal, Koç, Bulut, & Atay-Turhan, 2007). Thus, there have been changes in the teacher education curricula (Higher Education Council, 2006). The mathematics teaching methods and pedagogy courses constitutes the biggest part of the new teacher education curricula. This emphasis on those courses is expected to provide more opportunity for pre-service mathematics teachers to enhance their mathematical content and pedagogical content knowledge (Işıksal, Koç, Bulut, & Atay-Turhan, 2007). In fact, having subject matter knowledge and pedagogical content knowledge, and skills of implementing the curriculum based on these knowledge is included in the General Efficiency Criteria for Teaching Profession (TEDP, 2006).

In addition to the role of teachers' implementing the elementary school mathematics curriculum, teachers' mathematical content knowledge affects the achievements of their students (Hill, Rowan, & Ball, 2005). More specifically, the researchers investigated whether and how teachers' mathematical knowledge for teaching helps to increase students' mathematics achievement. Hill and her colleagues (2005) have found that teachers' mathematical knowledge for teaching affects first and third grade students' achievement. Furthermore, Hill, Rowan and Ball (2005) have suggested that improving pre-service programs in a way that they provide pre-service teachers opportunities to gain and improve mathematical content knowledge. Since teachers' knowledge for teaching affects student achievement, it is important to explore how teachers' mathematical content knowledge for teaching can be improved in teacher education programs. More specifically, it is worthwhile to investigate effects of mathematics teaching methods courses on teachers' mathematical knowledge for teaching. To sum up, all these considerations constitute the motivation for this study.

## 1.2. Definitions of Important Terms

The following terms need to be defined for the purpose of this study:

*Content Knowledge for Teaching Mathematics*: The content knowledge for mathematics teaching refers to the knowledge needed for teaching which requires explaining terms and concepts to students, interpreting students' solutions, using representations correctly in the classroom, selecting mathematically correct and appropriate tasks for particular grade level, and providing students with examples of mathematical concepts, algorithms or proofs (Ball & Bass, 2003; Ball, Bass & Hill, 2004; Hill, Rowan, & Ball, 2005). Furthermore, the characteristics of the content knowledge for teaching mathematics are (1) using mathematically correct algorithm in calculations, (2) choosing useful models or examples, (3) identifying students' errors and analyze the source of these errors, (4) choosing and using most useful representations, and (5) using mathematical language correctly (Ball, Hill, & Bass, 2005).

*Specialized Knowledge of Content*: It is one of the components of the content knowledge for teaching mathematics. Specialized knowledge of content is characterized by the following teacher behaviors: (1) Explaining why any specific procedure works and what it means, (2) appraising student methods for solving computational problems, (3) appraising student using novel methods, and (4) being able to determine whether student methods generalizable to other problems (Hill & Ball, 2004).

*Common Knowledge of Content*: It is one of the components of the content knowledge for teaching mathematics and refers to the general mathematical knowledge needed to solve any mathematical task.

*Subject Matter Content Knowledge*: Subject matter content knowledge refers to having the knowledge of facts, concepts or accepted truths, explaining why these concepts are worth learning, and relating the concepts within and without discipline (Shulman, 1986).

*Pedagogical Content Knowledge*: Shulman (1986) defined the pedagogical content knowledge as the knowledge of the most useful examples, the knowledge of alternative and various representations, the knowledge about how the subject matter can be made comprehensible and meaningful to students, the knowledge of the topics that students have difficulty to learn and how to make it easy for the students, the knowledge of preconceptions and misconceptions of the students in different grade levels, and the knowledge of the strategies for overcoming the misconceptions of the students.

*Mathematics Teaching Methods Course*: It is one of the courses offered by elementary mathematics teacher education program to senior pre-service mathematics teachers. The general goal of this course is to make pre-service teachers understand elementary and middle school mathematics concepts deeply, and get insight about how to teach particular mathematical topic.

# CHAPTER II

# REVIEW OF THE LITERATURE

The purpose of this study is to examine effects of a mathematics teaching methods course on pre-service elementary mathematics teachers' knowledge for mathematics teaching. This study is mainly related to the teacher knowledge for teaching mathematics. More specifically, the theoretical framework of this study is centered around two fundamental approaches to teachers' mathematical content knowledge; namely, Shulman's (1986) categorization of content knowledge, and Ball's and her colleagues' (2004) categorization of knowledge of content. In this review of the literature, theoretical background of teacher knowledge, mathematical content knowledge for teaching, and the role of teaching methods courses on teachers' content knowledge are presented.

## 2.1. Theoretical Background of the Teacher Knowledge

One of the important considerations needed to describe an effective teacher is the knowledge base that is possessed by the individual who will teach (Ball, 1991b). An effective teacher should be aware of the importance of the knowledge of the subject area, other disciplines, learners, and educational, social and cultural context of the country (Ball, 1991a; Ball & McDiarmid, 1990, Nakiboğlu & Karakoç, 2005). Shulman (1987) mentioned that the knowledge base of teachers includes the content knowledge about the subject area of the teacher, pedagogical knowledge, curriculum knowledge about his/her subject area and other subject areas, knowledge about learners and their characteristics, knowledge about school districts and school culture, and knowledge about educational values, purposes and philosophies determined by the government.

Many researchers have emphasized on the importance of content knowledge in professional development of teachers (Ball, 1990a; Ball, 1990b; Ball, Thames, &

Phelps, 2007; Borko, 2004; Chinnapan & Lawson, 2005; Even & Tirosh, 1995; Goulding & Suggate, 2001; Grossman, Wilson, & Shulman, 1989; Ma, 1999; McDiarmid, Ball & Anderson, 1989; Shulman, 1987; Wilson & Berne, 1999). Like Shulman (1987), Wilson and Berne (1999) mentioned about the opportunities to talk about subject matter, about students and learning, and about teaching while proposing alternative approaches to teacher learning of professional knowledge. This implies that knowledge about subject matter, knowledge about students and learning, and knowledge about teaching constituted major determinants of professional knowledge. These three knowledge bases having an important role in professional development of teachers have been studied by many researchers since mid-80s (Ball, Thames, & Phelps, 2007; Cochran-Smith, & Lytle, 1999).

In a similar line, Da Ponte and Chapman (2006) recently reviewed the studies about teachers' mathematics knowledge, knowledge of mathematics teaching, beliefs and conceptions, and practice. Based on this review, they found that most teachers lacked the knowledge of mathematics and the knowledge of mathematics teaching. More specifically, the results revealed that most teachers did not have a deep understanding about the content that they taught. The result is so prominent that it revealed the need of more research investigating what pre-service teachers know about the mathematics that they will teach, how their mathematical content knowledge for teaching can be improved during their experiences in teacher education programs, and which courses can help them improve their mathematical content knowledge (Ball, 1991a).

After mid 1980s, the role of teacher needed to promote learning showed a shift from an implementer of the curriculum to an organizer of the learning environment and a scaffolder of student learning (Even & Tirosh, 1995). Even and Tirosh (1995) also stated that mathematical content knowledge became a significant issue for this new role of teacher in promoting student learning. There are many studies in which Shulman's (1986) categorization of content knowledge was utilized as a conceptual framework for their research on teacher knowledge (Ball, 1990b; Ball & McDiarmid, 1990; Borko et al., 1992; Goulding, Rowland & Barber, 2002). The categorization of content knowledge proposed by Shulman (1986) is examined in detail in the following pages. There are also some researchers who formed new frameworks about teacher knowledge built upon Shulman's categorization of content knowledge (Gess-Newsome, 1999; Hill & Ball, 2004; Kolis & Dunlap, 2004; Ma, 1999). Kolis and Dunlap's (2004) theory was *K3P3 Model of Teacher Knowledge*. They used three knowledge bases; content, student, and learning knowledge (Shulman, 1987) as the core knowledge basis. According to Kolis and Dunlap, these three knowledge bases are not isolated from each other; but, intersect with each other. In fact, they defined three knowledge processes each of which lied in the intersection of three knowledge bases. Moreover, Gess-Newsome (1999) proposed two models for constructing the knowledge needed for classroom teaching; Integrative Model and Transformative Model. In the integrative model, the knowledge needed for classroom teaching, that is; PCK, can be constructed by integrating the three categories of teacher knowledge which are subject matter, pedagogy and context. On the other hand, transformative model requires transformation of these three constructs and forming a unique form of knowledge, PCK. In addition to these, Ball and her colleagues (2004) proposed another categorization of content knowledge for mathematics teaching. Their conceptual framework and each type of knowledge they proposed are explained sufficiently in detail in the following pages.

These frameworks mentioned briefly above have an important contribution to the literature about teacher knowledge and teacher education since they provide a theoretical basis to teacher educators for designing their subject matter courses or methods courses. Teacher educators should know what knowledge pre-service teachers possess in order to design the courses in teacher education programs effectively (Işıksal, 2006). So, these frameworks may explicitly affect teachers' professional development if they are used effectively in the courses in teacher education programs. Moreover, these frameworks may implicitly affect student learning because the competency of teachers in their subject area is not only related with their professional works but also student learning (Hill, Rowan, & Ball, 2005). In order to foster student learning, teachers should have a great deal of knowledge they teach. Therefore, teacher education programs should both emphasize content knowledge and provide pre-service teachers with opportunities to develop their understanding of subject matter (Borko, 2004). Leinhardt and Smith (1985) explained the relationship between teacher knowledge and student learning by stating that the more content knowledge teachers have and the more they connect their knowledge into their practice, the more competent the students should become in terms of mathematics. Similarly, Çakıroğlu and Çakıroğlu (2003) pointed out that the efficiency and success of the school curriculum is directly related to teachers who are the most critical people in the implementation of the curriculum effectively.

In this literature review, Shulman's, and Ball's and her colleagues' categorizations of content knowledge are analyzed in detail separately and compared in the context of mathematical content knowledge.

## 2.2. Teachers' Mathematical Content Knowledge for Teaching

There is a common agreement on that the content knowledge of the mathematics teachers is an important component of teacher knowledge (Ball, 1990a; Ball, 1990b; Ball, 1991a; Ball, & McDiarmid, 1990; Ball, Thames, & Phelps, 2007; Borko, 2004; Even & Tirosh, 1995; Goulding & Suggate, 2001; Ma, 1999; Nakiboğlu, & Karakoç, 2005; Shulman, 1987; Wilson & Berne, 1999). In this sense, Ball and her colleagues (2007) explained the importance of mathematical content knowledge for teaching by stating:

> Teachers must know the subject they teach. Indeed, there may be nothing more foundational to teacher competency. The reason is simple. Teachers who do not themselves know a subject well are not likely to have the knowledge they need to help students learn this content. At the same time, however, just knowing a subject well may not be sufficient for teaching. One need only sit in a classroom for a few minutes to notice that the mathematics that teachers work with in instruction is not the same mathematics taught and learned in college classes. In addition, teachers need to know mathematics in ways useful for, among other things, making mathematical sense of student work and choosing powerful ways of representing the subject so that it is understandable to students. It seems unlikely that just knowing more advanced math will satisfy all of the content demands of teaching. What seems most important is knowing the mathematics actually used in teaching (p.45).

She pointed out that knowing mathematics for teaching is not only knowing all the concepts and procedures that they will teach, applying them to the problems and solving the problems correctly, but also knowing the ways of making the subject meaningful for students. In fact, teachers' mathematical content knowledge should include what is to be taught and how to teach it (Grossman, 1990; Ma, 1999). In other words, to be a good teacher, making the mathematical knowledge understandable for the students is as important as knowing mathematics well enough (Ball, Thames, & Phelps, 2007).

In addition, Ball (1990a) explained what the subject matter knowledge for teaching requires by dividing it into two dimensions; *substantive knowledge of mathematics* and *knowledge about mathematics*. The former represents the knowledge of concepts and procedures. It was characterized by the following three criteria: (1) having correct knowledge of concepts and procedures, (2) knowing meaning and reasoning of the concepts and procedures, and (3) appreciating and understanding relationships of the mathematical concepts and procedures. On the other hand, the latter is related to the nature of the mathematical knowledge. For instance, identifying the logical answer, determining the validity of an answer, origin of the mathematics and changes in the mathematics are included in this dimension of subject matter knowledge for teaching. According to Ball (1990a), the integration of these two types of knowledge entails the subject matter knowledge for teaching.

Based on the distinction of substantive knowledge of mathematics and knowledge about mathematics, Ball (1990a) investigated 252 pre-service teacher candidates' subject matter knowledge for teaching division with fractions and feelings about mathematics. The results of the study showed that the majority of the pre-service teachers had difficulty with generating appropriate representation for division of fractions. This was interpreted by the researcher as that they had a narrow understanding of substantive knowledge of division with fractions. At this point, Ball (1990a, 1991a) asserted that teachers should know more than the standard algorithm. In fact, they should have the knowledge of relationship between concepts and procedures, and why procedures work. Ball's interviews with the pre-service teachers indicated that they were viewing knowing about mathematics as remembering rules and applying standard algorithms. Therefore, Ball emphasized that teachers must deeply understand the mathematics that they will teach so as to answer the questions of students like why inverting the second fraction and multiplying them works for division of fractions. This lack of subject matter knowledge for teaching might be overcome through carefully and effectively designed content courses. To state differently, the content courses like Geometry, Number and Numeration, and Function and Proportionality in which concrete models, realistic and contextual problems, group work, and writing journal about the growth of oneself about mathematical content knowledge used effectively had a great influence on pre-service and in-service teachers' content knowledge (Cramer, 2004).

Beside comprising subject matter knowledge by substantive knowledge of mathematics and knowledge about mathematics, Ball's and her colleagues (2004) have recently formed a new categorization of mathematical content knowledge based on Shulman' s (1986) categorization of content knowledge. In that study, it was aimed to study pre-service mathematics teachers' content knowledge for teaching in three domains; Number Concepts and Operations, Geometry, and Algebra regarding the conceptual framework of mathematical content knowledge. Underlying theories in both Shulman's, and Ball's and her colleagues' categorization of content knowledge are presented separately and the distinction between the two is made clearly in below.

### 2.2.1. Shulman's Categorization of Content Knowledge

The widely known approach to teacher knowledge is the content knowledge classification proposed by Shulman (1986). Shulman's notion of "pedagogical content knowledge" which is one of the parts of content knowledge of teachers was an important contribution to the literature in mid-1980s (Ball, Thames, & Phelps, 2007). Shulman (1986, 1987) regarded knowledge of subject matter or knowledge of content as a prerequisite for someone who will teach. For him, a teacher's knowledge of the content does not necessarily result in student learning. The point that distinguishes mathematics teachers from other adults like engineers, doctors or physicians who know mathematics well is their knowledge of how to teach mathematical content meaningfully (Shulman, 1986, 1987). This implies that teachers' content knowledge for teaching is one of the major determinants of effective teachers which also differentiate them from other professions who are capable of doing mathematics well.

Teachers should have in depth understanding of subject matter, organization of the subject matter, the ways of identifying students' past learning and deficiencies in their past learning, and the ways of facilitating new understanding by the help of teacher education program they followed. In general, this source of knowledge base of teachers is the content knowledge (Shulman, 1987). Similarly, Grossman (1990) pointed out that the knowledge of subject matter and the knowledge of students are two basic knowledge components that make a person a teacher. Shulman (1986) divided the content knowledge into three main categories; namely, subject matter content knowledge (SMCK), pedagogical content knowledge (PCK), and curricular knowledge. Subject matter content knowledge refers not only to have the knowledge of facts, concepts or accepted truths but also to explain why these concepts are worth learning and to relate the concepts within and without discipline. Shulman's pedagogical content knowledge generally reflects the subject matter knowledge for teaching, and bridges the content knowledge and the teaching as a practice. Pedagogical content knowledge includes the following characteristics: (1) the knowledge of the most useful examples, (2) the knowledge of alternative and various representations, (3) the knowledge about how the subject matter can be made comprehensible and meaningful to students, (4) the knowledge of the topics that students have difficulty to learn and how to make it easy for students, (5) the knowledge of preconceptions and misconceptions of students in different grade levels, and (6) the knowledge of the strategies for overcoming the misconceptions of students (Shulman, 1986). For him, all these characteristics of the pedagogical content knowledge imply that the pedagogical content knowledge is the knowledge of how to teach the subject matter. The third category of the content knowledge is the curricular knowledge (CK). This knowledge includes mainly two dimensions which are vertical curriculum knowledge and lateral curriculum knowledge. The former one represents the knowledge of the sequence of other topics in the curriculum of the subject area (that is, mathematics) in the same year and preceding and later years in order to make connection within topics of the subject. The latter dimension includes the knowledge about the curriculum of the other subject areas that students are studying at the same time in order to relate mathematics with other subject areas (other courses like science). The curricular knowledge also refers to the knowledge about the philosophy of the curriculum, organization of the topics in the curriculum, knowledge of variety of the instructional materials for different grade levels and knowledge of alternative programs, textbook and materials (Shulman, 1986).

Based on Shulman's identification, these three categories of content knowledge are made up of the knowledge base of teachers which is the main source of their teaching. These three categories of content knowledge also reflect that the content knowledge of teachers includes not only the knowledge of what they teach but also the knowledge of how they teach and the knowledge of the curriculum of the subject and other related subjects. After the brief overview of the Shulman's categorization of content knowledge, it is time to focus on Ball's categorization of knowledge of content.

### 2.2.2. Ball's and Her Colleagues' Categorization of Knowledge of Content

The second approach is Ball's and her colleagues' approach of "content knowledge for teaching mathematics", built mainly upon Shulman's notion of PCK (Ball, Thames, & Phelps, 2007). The basic idea which lies on their framework is that being able to use mathematics content knowledge in teaching is more crucial than just knowing mathematics (Ball, Bass, & Hill, 2004). Ball and her colleagues first defined 'mathematical knowledge for teaching' as the mathematical knowledge used to carry out 'work of teaching' (Ball, & Bass, 2003; Ball, Bass, & Hill, 2004; Hill, Rowan, & Ball, 2005). They refer to all the things that teachers do in teaching mathematics by saying 'work of teaching'. Ball and her colleagues (2007) stated that teaching includes knowing and understanding the content of the curriculum, knowing students' preconceptions and misconceptions, knowing how to evaluate and respond to students' errors, and selecting appropriate representations. In order to be more specific, they illustrated that the knowledge needed for explaining terms and concepts to students, interpreting students' solutions, using representations correctly in the classroom, selecting mathematically correct and appropriate tasks for particular grade level, and providing students with examples of mathematical concepts, algorithms or proof are included in the knowledge needed for 'work of teaching' (Ball, & Bass, 2003; Ball, Bass, & Hill, 2004; Hill, Rowan, & Ball, 2005). Furthermore, Ball, Hill and Bass (2005) mentioned some aspects of mathematical knowledge for teaching. These are, (1) using mathematically correct algorithm in calculations, (2) choosing useful models or examples, (3) identifying students' errors and analyzing the source of these errors, (4) choosing and using most useful representations, and (5) using mathematical language correctly. These aspects reflect both the knowledge of mathematics and the knowledge of using this mathematical knowledge in the classroom.

More specifically, Hill and Ball (2004) described mathematical content knowledge for teaching as it is composed of two types of knowledge of content: *specialized knowledge of content* (SKC) and *common knowledge of content (CKC)*. In fact, Ball and her colleagues (2007) stated that they subdivided Shulman's subject matter content knowledge into specialized content knowledge and common content knowledge, and his pedagogical content knowledge into knowledge of content and students and knowledge of content and teaching. They characterized specialized knowledge of content as the knowledge that is unique to teacher who will engage in teaching children mathematics. Hill and Ball (2004) also illustrated SKC by the following teacher behaviors: (1) explaining why any specific procedure works and what it means, (2) appraising student methods for solving computational problems, (3) appraising student using novel methods, and (4) being able to determine whether student methods are generalizable to other problems. For instance, knowing how to represent $\frac{2}{5}$ or 0.30 by using diagrams, how to explain distributive property meaningfully and mathematically correctly or how to judge the alternative solution methods of students for a problem such as 50x47 are included in specialized content knowledge. On the contrary, Hill and Ball (2004) asserted that CKC is not unique to the individual who is teaching mathematics. In fact, a person does not necessarily to be a teacher to have common knowledge of content. The common knowledge of content can be exemplified as (1) being able to compute 50x47, (2) identifying what the 0 power of 2 equals to, and (3) solving word problems satisfactorily. Hill and Ball (2004) declared that mathematical content knowledge for teaching is comprised of both specialized knowledge of content and common knowledge of content. For Hill and Ball, teachers' teaching subject matter competently depends on the combination of these two types of content knowledge.

Even the understanding of content knowledge is a critical issue in teacher education; beginning teachers also need to learn about students and their learning (McDiarmid, Ball, & Anderson, 1989). Ball and her colleagues (2007) defined knowledge of content and students (KCS) as "the knowledge that combines knowing about students and knowing about mathematics" (p. 36). KCS includes knowing the students' conceptions, preconceptions, and misconceptions; knowing what students will find easy, difficult or confusing; and knowing how to respond students' misconceptions or wrong solutions. Another subcategory of the mathematical knowledge for teaching which is drawn in the light of Shulman's PCK is the "knowledge of content and teaching" (KCT). KCT is defined as the "knowledge that combines knowing about teaching and knowing about mathematics" (Ball, Thames, & Phelps, 2007, p.38). Ball illustrated KCT such that deciding the examples which ease teaching of the topic, deciding the most effective method of teaching and the most useful representation, and identifying probing questions to further students' learning. Ball and her colleagues (2007) stated that the tasks included in the work of teaching are related to both specific mathematical understanding and an understanding of pedagogical issues affecting students' learning and their interaction with each other. Ball and her colleagues (2007, p. 40) also related KCS and KCT with PCK by stating:

> …the last two domains-knowledge of content and students and knowledge of content and teaching-coincide with the two central dimensions of pedagogical content knowledge identified by Shulman (1986):
>
> - "the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons" (p. 9), and;

- "the ways of representing and formulating the subject that make it comprehensible to others" (p. 9).

On the other hand, Hill (2007) explained that SKC is different from PCK since it deals with not only teaching task but also pure mathematical content which will be taught. Moreover, a basic difference can be drawn Shulman's view and Ball's view. That is, Ball mentioned about the knowledge needed for *teaching* while Shulman emphasized on the knowledge needed for *teachers*. In spite of this difference, there are some concurrencies between these two views. The following diagram proposed by Ball and her colleagues (2007) presents her categories of mathematical knowledge for teaching and the concurrency with Shulman's (1986) categories.

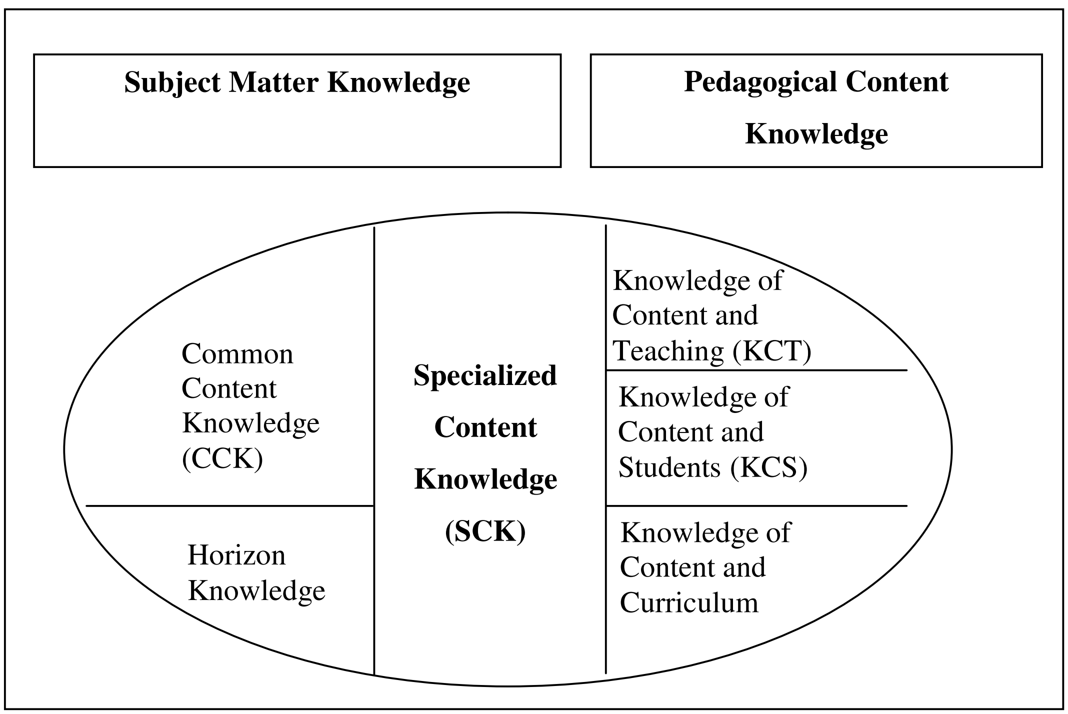


Figure 2.1 Comparison of Shulman's and Ball and her Colleagues' Approach of Mathematical Content Knowledge (Adapted from Ball and her colleagues, 2007, p. 42)

In this diagram, there is a new category of knowledge, “Horizon Knowledge”. Although Ball and her colleagues (2007) hesitated whether horizon knowledge can be seen as a part of subject matter knowledge, she defined this category of knowledge as “Horizon knowledge is an awareness of how mathematical topics are related over the span of mathematics included in the curriculum” (2007, p.42). This diagram briefly pictures out that SKC, CKC and horizon knowledge constitute Shulman’s SMK; and KCT, KCS, and knowledge of content and curriculum compose Shulman’s PCK. This can be interpreted as that Shulman’s categorization is more general than Ball’s one. In this sense, Ball provided more detailed and mathematics-specific categorization of content knowledge. This study will mainly be developed on Ball’s categorization of content knowledge for teaching mathematics.

In addition, Ball and her colleagues (2007) criticized some deficiencies of her own categorizations of mathematical content knowledge for teaching. Firstly, she pointed out that the same knowledge needed in teaching mathematics can be thought as both SKC and KCS. For instance, while analyzing student error or selecting examples, illustrations and representations that deepen student understanding, a teacher may either use SCK or, KCS or both. Another deficiency is that for some tasks the categories of knowledge in teaching mathematics may overlap each other and this makes difficult to measure each one (Ball, Thames, & Phelps, 2007). For example, selecting examples, illustrations and representations that deepen student understanding in the topic of adding fractions can be seen as requiring KCS, on the other hand, this task requires the algorithm of adding two fractions (CCK), explaining students mathematical meaning of adding fractions and representing meaningfully (SCK), and deciding appropriate ways of explaining the points which students have difficulty (KCT). Lastly, for some tasks it is hard to distinguish CCK from SCK (Ball, Thames, & Phelps, 2007). To illustrate this, we can consider the problem of finding a division operation represented in the Figure 2. Whether the knowledge of $11 \div 4 = 2\frac{3}{4}$ is either CCK or SCK can not be easily determined.

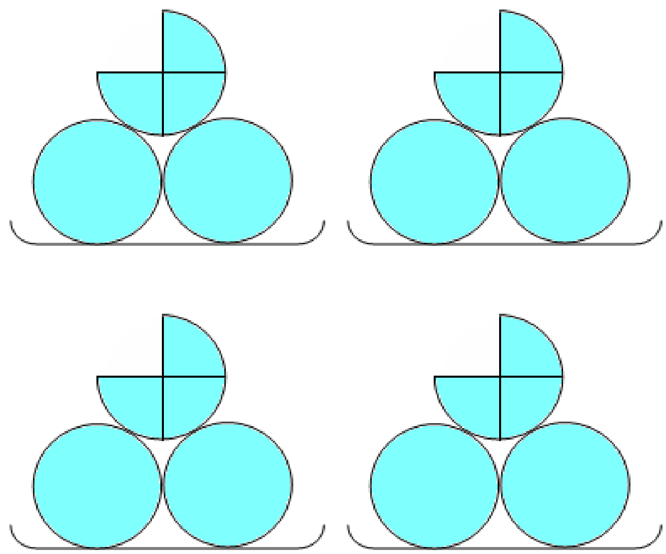

Figure 2.2 Representation of the division operation of 11 $\div$ 4 = $2\frac{3}{4}$ (Van de Walle, 2004, p. 147)

Based on Ball's self-critics, this categorization can be seen as open to improvement. However, her categorization and descriptions of the knowledge for teaching mathematics are clear enough to guide teacher educators by showing what teachers need to know for effective teaching. What is important is to measure teachers' content knowledge by reliable and valid measures, and determine how this knowledge can be developed. Ball, Lubienski, and Mewborn (2001) viewed that understanding the nature and role of mathematical knowledge for teaching and developing it is an unsolved problem. Ball and her colleagues (Ball et al., 2001; Ball et al., 2005) mentioned about two research approaches for this unsolved problem. The first approach deals with the characteristics of the teachers. This approach uses the courses taken, degrees earned, results of basic skills test, or certification received as the indicators of teacher characteristics (Ball, Lubienski, & Mewborn, 2001; Hill, Rowan, & Ball, 2005). The second approach focuses on the teachers' knowledge. This approach is related with pedagogical content knowledge proposed by Shulman (1986, 1987) and mathematical content knowledge proposed by Ball and her colleagues (Ball, Thames, & Phelps, 2007; Hill, & Ball, 2004), partly. In addition to these two approaches, Ball et al. (2001) suggested an alternative approach, which is built on both 'characteristics of teachers' and 'knowledge of teachers'. Since they considered that the courses taken, the credits earned and the degrees attained are also a representation of teachers' mathematical knowledge, they put more emphasis on teaching and teachers' use of mathematical knowledge. In line with the same emphasis, Ball (1991a) and Ma (1999) asserted that teacher's mathematical knowledge is needed to improve the quality of the instruction. Their views can be interpreted as an indicator of a close relationship between the teachers' mathematical knowledge and their teaching. Regarding these, policymakers have provided great support on this issue. However, there has been little improvement on determining whether and when teachers develop their mathematical knowledge (Chinnapan & Lawson, 2005; Grossman, 1990; Hill & Ball, 2004). Serving this purpose, Hill, Schilling and Ball (2004) developed multiple choice tests as Content Knowledge for Teaching Mathematics (CKTM) measures to measure teachers' mathematical content knowledge for teaching. Since 2001, they developed tests and equated form of these tests for the topics of Number Concepts and Operations (NCOP), Patterns, Functions and Algebra (PFA) and Geometry (GEO) for elementary and middle schools curriculum separately (Hill, 2004). All the forms of CKTM measures were piloted and factor analysis were carried out. The tests included the items related with both SKC and CKC. For instance, some items asked to represent a procedure or evaluate students' solution methods (SKC) and some of them asked to solve a word problem or compute an algorithm (CKC). The results of the factor analysis also supported the theoretical distinction between CKC and SKC since the items in these types were loaded in different factors. Hence, Ball and her colleagues developed a valid and reliable measure of teachers' mathematics knowledge for teaching (Hill, Schilling, & Ball, 2004). In this study, one of the CKTM measures will be used as an assessment tool to get information on pre-service teachers' mathematical content knowledge for teaching.

The literature about teacher's mathematical content knowledge also include many other studies using either Shulman's categorization or Ball and her colleagues' categorization or both. Some of these studies are briefly discussed below.

### 2.2.3. Studies Related to Teachers' Mathematical Content Knowledge

There are several studies investigating teachers' or pre-service teachers' mathematical content knowledge in the subject area of mathematics (Chinnapan & Lawson, 2005; Even, 1993; Even, & Tirosh, 1995; Goulding, Rowland, & Barber, 2002; Hill, Rowan, & Ball, 2005; Kinach, 2002; Leinhardt & Smith, 1985; Chinnapan & Lawson, 2005; Van Der Valk & Broekman, 1999). In this part of the literature review, the studies on pre-service teachers' and in-service teachers' mathematical content knowledge are reviewed.

To start, Leinhardt and Smith (1985) examined and compared 4 expert and 4 novice fourth grade mathematics teachers' SMK by using semantic nets, planning nets and flow charts reflecting their knowledge of fractions, interviews, card-sorting task and transcription of their videotaped lessons. The study was based on two core areas of knowledge which are thought important for becoming expert teachers. One is lesson structure knowledge and the other one is SMK. Leinhardt and Smith (1985) briefly described that lesson structure knowledge is the knowledge about planning and performing lesson in a coherent and fluent way and SMK is the knowledge about concepts and procedures and their connections within each and between them as Shulman (1986) defined. For Leinhardt and Smith, SMK is an important supporter for lesson structure. The results of the interviews about fraction knowledge and card sort task requiring sorting 40 math problems and giving rationale for their sorting showed that expert teachers had more knowledge about fraction and more hierarchical structure in their knowledge than novice teachers even though there are varieties in expert teachers' subject matter knowledge levels about fraction (1985). The result of this study pointed out that novice or pre-service teachers should be developed in terms of their content knowledge during teacher preparation programs. To achieve this, the courses in teacher preparation program should address to the conceptual development of pre-service teachers in the topics of elementary mathematics school curriculum.

Another study conducted by Even and Tirosh (1995) investigated teachers' SMK, knowledge about students and presentations of the subject matter. The researchers concentrated on mainly two aspects of PCK proposed by Shulman (1986). These aspects are the knowledge about the subject matter, chosen as functions and undefined mathematical operations in this study, and the knowledge about students. The participants of the study were 162 senior students in the teacher preparation programs of secondary school mathematics teaching. 152 of the participants were required to complete an open-ended questionnaire and 10 of them were interviewed after having administered questionnaires. The results of the study can be summarized such that teachers' SMK about undefined mathematical operations was limited and teachers did not tend to understand students' reasoning and just considered whether the answer of the student was right or wrong. Therefore teachers' presentations of subject matter were no more than telling the rules and procedures owing to their inadequate SMK. Even and Tirosh (1995) also inferred from the results of their study that the two aspects of PCK which are knowledge of subject matter and knowledge about students should be taken into consideration seriously by teacher educators to improve teacher preparation programs. Likewise, Even (1993) investigated pre-service secondary teachers' SMK and its relations with PCK regarding the function concept. 152 pre-service teachers were responded to open-ended questionnaire and 10 additional ones were interviewed. The results of the study showed that pre-service teachers lacked of an understanding the importance and origins of the arbitrariness and univalence nature of the functions. To be more specific, they could distinguish the relations representing a function and not representing a function; but they could not provide reasons for the need and importance of the univalence concept of functions. The most striking conclusion the researcher stated was that understanding mathematical need and importance of concepts can help teachers to make appropriate decisions in their instruction; that is, it can support their PCK (Even, 1993). So, this study implied that pre-service teachers should be given sufficient opportunity to develop an understanding and appreciation of the need and importance of the mathematical topics. This is an important step in improving their teachings which is actually one of the aims of teacher preparation programs. These studies depicted that pre-service teachers know and perform mathematical procedures and algorithms well; but they are not sufficient in explaining the rationale behind these procedures and algorithms, and so the need and importance of them. Regarding Ball and her colleagues' framework (2004), the results of these studies can be interpreted that pre-service teachers just knew the mathematics which is known by any educated people dealing with mathematics. They did not have the mathematical knowledge which a mathematics teacher should know to teach. That is, their common knowledge of content for mathematics teaching was high enough, but their specialized knowledge of content for mathematics teaching is not.

Moreover, Verschaffel, et al. (2005) and Tobias and Itter (2007) investigated pre-service teachers' mathematical content knowledge regarding the gender difference. More specifically, Tobias and Itter (2007) explored the pre-service teachers' mathematical understandings. The participants of their study included 318 female and 79 male first year pre-service teachers from two rural Australian universities. The researchers measured pre-service teachers' mathematical understandings by the competency test which is composed of items assessing mathematical knowledge and skills that an $8^{th}$ grade student should have. The findings of the study showed that most of the pre-service teachers' answers were so similar that the answers of the pre-service teachers who were from different universities did not vary much. Tobias and Itter (2007) explored the difference between male and female pre-service teachers' mathematical competencies for teaching. The results indicated that male pre-service teachers performed better than females. Also, Tobias and Itter (2007) highlighted the most striking point by stating that only half of the pre-service teachers could solve the questions correctly although they were designed for the level of an $8^{th}$ grade student. Similarly, Verschaffel et al. (2005) examined 1475 pre-service teachers' mathematical knowledge and skills considering gender and institute differences. The researchers found significant differences between male and female pre-service teachers' mathematical knowledge in favor of males. Also, the results of this study indicated significant difference between institutes. Thus, Verschaffel et al. (2005) claimed that different institutes do not provide equal opportunities to enhance pre-service teachers' competencies. Furthermore, the results indicated that pre-service teachers lacked of mathematical knowledge and skills when they entered teacher training programs, and even they were not really ready to teach mathematics when they completed the teacher training program. Since the literature do not include many researches investigating pre-service teachers' mathematical knowledge considering gender difference, these studies contributed a lot to the literature review of the present study.

Also, there are some studies investigating teachers' content knowledge in a cross-cultural context (An, Kulm, & Wu, 2004; Berenson, et al., 1997; Ma, 1990; Zhou, et al., 2006). Berenson and her colleagues (1997) investigated pre-service teachers' content knowledge of the area concept in an international study. They asked 25 pre-service teachers from four different countries to write a lesson plan about area concept without using any textbooks. Eight of 25 lesson plans were presented according to the analysis based on Shulman's (1986) content knowledge categorization. More specifically, they analyzed mathematical content knowledge in two dimensions; concept-centered and procedure-centered. It was found that four lesson plans addressed the concept-centered content knowledge while other four addressed the procedure-centered content knowledge. Based on the results of this study, Berenson and her colleagues categorized pre-service teachers into three groups. Accordingly, the first group of the pre-service teachers possesses a combination of concept-centered and procedure-centered content knowledge. The second group of teachers constructs their teaching into procedure-centered content knowledge. This group tried to use concept-centered approach but they could not achieve this. The third group has little conceptual knowledge. They mostly used procedural formulas in teaching area concept. Beside this categorization of teachers regarding their mathematical content knowledge in the topic of area, Berenson et al. (1997) suggested that teacher education programs should be designed to meet the needs of all three groups of teachers. In other words, experiences that make pre-service teachers acquire procedural and conceptual knowledge of topics and identify students' conceptual and procedural explanations should be considered in deciding the content of the courses in teacher education program. Another international study was carried out by Zhou and colleagues (2006). Zhou and colleagues compared SMK, PCK and general pedagogical knowledge (GPK) of 162 U.S. and Chinese 3$^{rd}$

grade teachers guided by Shulman's (1986) teacher knowledge categorization. The results of the study showed that Chinese teachers have more in-depth understanding related to concepts, computation, and word problems of fractions than American teachers. Chinese teachers were also better than American teachers on identifying important points of teaching the fraction concepts and ensuring students' understanding related to PCK; however Chinese teachers performed poorly about psychological and educational theories related with GPK. The result of this study was so similar with Ma's (1999) investigation which compared American and Chinese elementary school teachers' SMK. Ma (1999) had found that Chinese teachers had significantly deeper understanding of subtraction with grouping, multi-digit multiplication, division by fractions, and relationship between perimeter and area than American teachers. These studies showed that teachers' and pre-service teachers' content knowledge for mathematics teaching varies in different countries. This variety might be resulted from the difference between teacher preparation programs in different countries. So, identifying Turkish pre-service mathematics teachers' mathematical content knowledge is a significant issue to both picturize the contributions of teacher preparation programs on pre-service teachers' content knowledge for teaching mathematics and improve the quality of teacher preparation programs directly and elementary mathematics education in Turkey indirectly.

In the literature review of this study, not many studies utilizing Ball's and her colleagues' (2004) categorization of content knowledge as a conceptual framework were come across. This might be because of that Ball's and her colleagues' approach of knowledge of content is new and still on improvement. However, Ball and her colleagues carried out some studies the conceptual frameworks of which were based on their own categorization of content knowledge (Hill, 2007; Hill & Ball, 2004). Both studies carried out by Hill (2007), and Ball and Hill (2004) dealt with teachers' specialized and common content knowledge for mathematics teaching by using the same measurement tool; Content Knowledge for Teaching Mathematics (CKTM) measures. CKTM measures which were designed for the Study of Instructional Improvement (SII) in the context of Learning Mathematics for Teaching (LMT) Project at the University of Michigan in the United States were used in these studies.

These measures were developed to elicit both specialized knowledge of content and common knowledge of content. More detailed information about CKTM measures were provided above. In fact, Survey of Elementary Teachers of Mathematics (SETM) which is one of the measures of CKTM used in the present study was developed in the context of the same project. More specifically, Hill (2007) recently has investigated middle school teachers' mathematical knowledge for teaching, and relationships between teachers' mathematical knowledge for teaching and their subject matter preparation. More than 80% of teachers who participated in this study had taken three or more mathematics courses and most of them had taken mathematics methods courses in their education program. In her study, Hill concentrated on two content areas; namely, number and operations and algebra. The results of the study showed that common content knowledge (CCK) items were found significantly easier than specialized content knowledge (SCK) items since more CCK items in the test were answered correctly. In fact, the item related to evaluating unusual solution methods of students, one of the components of SCK, found by teachers the most difficult. Based on this result, Hill asserted that middle school teachers are not proficient enough to explain and represent the mathematical ideas in unusual solution methods. She also hypothesized that the reason of teachers' finding SCK items more difficult is that most teachers have either no SCK or limited amount of SCK even though they know rules, procedures and algorithms well. Also, Hill tried to answer the question of whether middle school teachers' training influences their content knowledge for teaching. In this context, the results showed that middle school teachers who took mathematics courses and mathematics methods courses performed better on the test. Based on these results, Hill argued that the number of methods and mathematics course work taken can be used to predict levels of mathematical content knowledge for teaching (2007). This study is so important that it examines the effect of teacher training period on teachers' content knowledge development. Especially, the findings about methods courses and mathematics content courses indicated that the mathematics teaching methods courses might be the place where pre-service teachers' both specialized and common content knowledge for teaching mathematics is improved. Thus, the conclusions of this study present the rationale of the setting of the present study; that is implementing the methods course to investigate and improve the pre-service mathematics teachers' mathematical content knowledge for teaching.

In another research study, Hill and Ball (2004) investigated whether summer workshop component of a professional development institute can help elementary school teachers improve their knowledge of mathematics for teaching. The researchers used one of the CKTM measures which include equated forms as pre- and post-test respectively. Teachers participated in summer institutes for 40 to 120 hours in the context of the study mentioned. In these institutes, mathematicians covered elementary mathematics topics like long division and order of operations whereas mathematics educators provide more practical knowledge by presenting activities which are connected with the mathematics learned from mathematicians (Hill & Ball, 2004). First result of the study was that mathematics knowledge of teachers among institutes significantly varied in the pretest. Second, institutes showed difference in increasing teachers' mathematical content knowledge effectively. These two results suggested that the growth in the institutes was the result of the effective program, not of testing threat. The results of the Hill and Ball's study also showed that a single professional development program can be effective in teachers' learning mathematics for teaching (2004). These results are important since they indicated that teacher educators or policymakers can design new programs or improve the old ones so that pre-service teachers can learn mathematical knowledge for teaching in their teacher education programs. These contributions to literature are significant since teachers' knowledge for mathematics teaching influences their teaching practices (Cohen & Hill, 2001).

Also, a researcher used Ball and her colleagues' framework on her dissertation (Moss, 2006). Furthermore, this study investigates strengths and weaknesses of pre-service teachers' specialized understanding of mathematics (SUM) and the contribution of methods course on their improvement of SUM (Moss, 2006). More specifically, Moss (2006) examined 244 pre-service mathematics teachers enrolled in mathematics methods course based on the conceptual framework proposed by Ball and her colleagues (2004). The Content Knowledge for Teaching

Mathematics (CKTM) Instrument which is developed in the context of LMT project was used to measure pre-service teachers' both specialized and common content knowledge related to Number Concepts and Operations, and Geometry. Pre-service teachers were observed to be good at computations and interpretations of geometric definitions while they performed poorly on multiple representations. In general, it was found that pre-service teachers' SUM in geometry was better than those in number and operations. Also, paired sample t-test results showed that pre-service teachers exhibited statistically significant improvement in their content knowledge to teach mathematics effectively even if their improvement was varied in item based analysis. Moreover, Moss (2006) interviewed with instructors of methods courses to identify the learning opportunities which have influenced pre-service teachers SUM. Instructors highlighted the role of class discussions, readings, classroom activities and problem solving, video clips, manipulatives and field experiences. Based on these findings, Moss emphasized the need for more opportunities either in methods courses or content courses so as to train teachers who were competent in terms of specialized understanding of mathematics. Moss also stressed the essentiality of specialized understanding of pre-service teachers by stating: "…In order to encourage their students' mathematical thinking, teachers must be able to appreciate and evaluate the reasonableness of their thinking. However to be able to do this, they must have for themselves a deeper understanding of mathematics." (2006, p.97). Moss's study contributes to the literature review of the present study since it directly investigates the effect of the methods course on pre-service teachers' content knowledge, in particular on SUM, similar to the present study. Although, the present study is not the replication of this study in a different context, the main goals coincide. Therefore, Moss's study gives a rationale for the significance of the present study. In this context, the function of this study is so important for the present study that it revealed the significant effects of the methods course on pre-service teachers' specialized understanding of mathematics.

In addition, there are some research and thesis studies about teachers' mathematical content knowledge in Turkey (Acar, 2005; Ay, 2004; Bütün, 2005; Işıksal, 2006; Sıvacı, 2003; Türnüklü, 2005; Yıldız & Ilgar, 1999). These studies are important since they presented the existing situation of the in-service and pre-service teachers in Turkey in terms of their competency of mathematical content knowledge for teaching.

As for pre-service teachers' own thoughts about their content knowledge for mathematics teaching, Yıldız and Ilgar (1999) investigated proficiencies of the knowledge of mathematics teaching of the last year students enrolled in elementary mathematics teacher education ($4^{th}$ through $8^{th}$ grades) and elementary teacher education ($1^{st}$ through $5^{th}$ grades) programs in a university in Turkey. The pre-services graduated from both programs would teach mathematics in elementary schools. 43 pre-service classroom teachers and 51 pre-service elementary mathematics teachers responded to a 32-item questionnaire which measures their opinions about their adequacy on mathematics teaching. The instrument is a 4 point likert type scale, ranging from "I am fully incompetent" to "I am fully competent". The results of the study showed that most of the pre-service teachers evaluated themselves as competent in preparing appropriate materials for teaching mathematics, and using those materials in an effective way. Pre-service teachers also believed that their proficiency level of teaching natural numbers by using the concept of set and teaching fractions and decimals effectively was high. On the other hand, their proficiency levels in planning classroom activities, selecting appropriate teaching methods and forming problems for mathematical topics were not high enough. Moreover, there was no significant mean difference between pre-service mathematics teachers and pre-service classroom teachers in terms of their proficiencies of the knowledge of mathematics teaching. This study puts forward that pre-service teachers who will teach mathematics in elementary schools did not possess enough mathematical content knowledge to teach mathematics effectively even though they believed that they are proficient in this area. This result is both interesting and significant because of the fact that if teachers and pre-service teachers do not believe that they are proficient in content knowledge, they will not need to learn about mathematics they will teach or to improve their knowledge. This situation requires us to consider the question of "why do most of the pre-service teachers in Turkey believe that they are competent enough in terms of mathematical content knowledge for teaching while they are not actually?" The answer of this question might be that the courses offered by teacher education programs in Turkey mostly focus on pure mathematics courses related to advance mathematics and pedagogy courses. These courses do not provide information about mathematics that they will teach. The last time that pre-service teachers experienced elementary school mathematics is their school years before entering universities. By considering their own proficiencies in doing elementary school mathematics, they might think themselves as proficient in mathematical content knowledge for teaching mathematics. However, computing operations and solving problems well do not guarantee teaching mathematics well. Therefore, teachers should possess mathematical content knowledge for teaching mathematics; that is, they should have both specialized and common content knowledge (Ball, Thames, & Phelps, 2007). Thus, the present study aimed to identify pre-service mathematics teachers' content knowledge for teaching mathematics and whether the mathematics teaching methods course can affect this knowledge development. This study is expected to contribute both theoretically and practically to teacher education programs in Turkey.

In a similar line, Ay (2004) investigated the effect of content and profession lessons on elementary mathematics teachers' professional development and their opinion about the content of the courses they took. He found out that pre-service teachers did not relate the content of the courses with real life applications in the content courses. Pre-service teachers stated that mathematical content courses did not provide a connection between mathematical concepts. As a result, Ay (2004) recommended that either content or methods courses should provide experiences to elementary teachers that they could relate the content of the course with 6.-8. Mathematics Curriculum. This reveals that any courses (content or methods course) may not enhance pre-service teachers' content knowledge development related to elementary school mathematics tasks even though the aim of these courses should be that. Furthermore, Ay (2004) suggested that the courses given in regular teacher preparation should cover all the mathematical topics in elementary mathematics curriculum. Similarly, Işıksal (2006) emphasized that mathematics teaching methods courses should incorporate the mathematics curriculum and the methods of teaching.

Both Ay's (2004) and Işıksal's (2006) point of view about methods courses provided an awareness of the function of the methods courses on Turkish mathematics teacher education. Thus, mathematics teaching methods course was determined as a course in the mathematics teacher education program in Turkey which provides experiences about mathematics that pre-service mathematics teachers will teach.

In addition to teachers' own perception and evaluation of their mathematical content knowledge and the effect of the courses in the teacher preparation program on their content knowledge, Sıvacı (2003) investigated the pre-service classroom teachers' subject matter knowledge of elementary 5$^{th}$ grade mathematics and competencies of professional knowledge by asking them to answer a 30-item test about fractions, sets and operations. 450 last grade students in Classroom Teaching Program in nine education faculties in Turkey participated in Sıvacı's study. The results revealed in general that the pre-service teachers' proficiency level of subject matter knowledge about elementary 5$^{th}$ grade mathematics was 59,86 %. To be more specific, there was a significant mean difference in subject matter knowledge about mathematics in terms of the type of high school from which pre-service teachers graduated, the faculty enrolled, and the level of faculty. This indicated that pre-service teachers' mathematical knowledge was influenced by their experiences in high schools and in universities. The difference between the faculties also showed that teacher education programs in some universities were more successful in developing pre-service teachers' mathematical content knowledge in Turkey. Based on this result, Sıvacı suggested that faculties should design supplementary programs related to the deficiencies of pre-service teachers' mathematical content knowledge and ensure that mathematical content knowledge about each topic should be possessed by pre-service teachers. It is apparent that considering this recommendation seriously will also affect pre-service teachers' future teaching and their students' learning. Faculties taking some actions about improving the quality of the courses by concentrating on the content of the courses and learning opportunities provided in those courses and coming to a consensus about the course criteria might be more reasonable solution to reduce the difference between faculties and increase the pre-service teachers' subject matter professions. Similar to the conclusion of previous study, Bütün (2005) explored the extent and organization of mathematics teachers' pedagogical content knowledge about school mathematics curriculum and how this knowledge influences their teaching methods. For this study, Bütün interviewed three elementary mathematics teachers in Trabzon. In the first part of the data collection procedure, he used semi-structured interviews to determine teachers' beliefs about nature of mathematics, and learning and teaching mathematics. In the second part of the data collection procedure, he used seven scenario type questions to determine their mathematical knowledge. The results of the study showed that mathematics teachers' mathematical content knowledge and their beliefs about the nature of mathematics, and teaching and learning mathematics influenced their teaching methods. More specifically, teachers preferred direct instruction and demonstration methods which rely on mainly rules and procedures owing to their limited understanding of mathematics. Moreover, it was observed that their knowledge was not sufficient to interpret students' explanations. Therefore they tried to design lessons based on the explanation of the topic that was presented in scenarios, but not on the solutions and explanations of the students about those particular topics. Regarding Ball's and her colleagues' categorization of content knowledge, this indicated that the teachers do not have enough specialized content knowledge (SKC) to teach the topic meaningfully and enough knowledge about students and their thinking and learning ways (KCS). Bütün (2005) proposed similar suggestions with other researchers by stating that teacher preparation program should give sufficient importance to content and mathematics teaching methods courses. These courses should provide pre-service teachers' with opportunities to develop conceptual understanding about mathematical topics.

In another study, Türnüklü (2005) investigated the relationship between pedagogical and mathematical content knowledge of pre-service teachers. To determine pedagogical content knowledge, 45 pre-service teachers (last grade) in a university in Turkey were asked to answer four problems. The mean of the grades in mathematical content courses taken throughout their university education was used as an indicator of pre-service teachers' mathematical content knowledge. The results showed that there is a relation between pre-service teachers' pedagogical content knowledge and their mathematical content knowledge. To be more specific, pre-service teachers who are successful in mathematics content courses were found good at using their pedagogical content knowledge. However, it was found that mathematical content knowledge is necessary but not sufficient to develop pedagogical content knowledge. Türnüklü stated that it would be better to emphasize the relationship between pre-service teachers' pedagogical content knowledge and mathematical content knowledge. In this line, Işıksal (2006) investigated pre-service teachers' subject matter knowledge and their pedagogical content knowledge about the topic of multiplication and division of fractions and the relationship between these two types of knowledge. 28 senior students enrolled in a teacher education program at a public university responded to Multiplication and Division of Fractions Questionnaire (MDFQ). 17 volunteered pre-service teachers were also interviewed. Results of this study showed that pre-service teachers correctly answered the questions requiring computing on the multiplication and division operations with fractions. That is, their procedural knowledge was good enough to answer the questions related with operations with fractions. On the other hand, they did not have enough subject matter knowledge to explain the meaning of multiplication and division of fractions and to reason the operations conceptually. This implied that their procedural knowledge was not sufficient to construct conceptual understanding about multiplication and division of fractions. Moreover, this limited subject matter knowledge affected their pedagogical content knowledge; especially knowledge about students' common (mis)conceptions. It was also seen that many pre-service teachers thought that as they had enough knowledge about multiplication and division with fractions. The questions in MDFQ helped them recognize their own deficiencies. Based on these results of this study, Işıksal suggested that there should be some courses in which pre-service teachers could both discuss the meaning of concepts and relationships between concepts, and between concepts and procedures, and experience the conceptions and misconceptions of the students about the topics in elementary school mathematics curriculum. These "content-pedagogy rich" (Işıksal, 2006, p. 206) courses provided pre-service teachers with the opportunity to deepen their subject matter and pedagogical content knowledge, and relate and practice these two components of content knowledge for mathematics teaching. Relating to this troublesome exhibition of pre-service teachers' knowledge and pre-service mathematics teacher education in Turkey, Işıksal (2006) advised to teacher educators to take the suggestion of "content-pedagogy rich" courses into consideration seriously to enhance pre-service teachers' content knowledge and improve their future teaching. These results highlighted once more that pre-service mathematics teachers in Turkey do not have sufficient specialized content knowledge for teaching mathematics. This constitutes an important reason for the need of well-organized courses in teacher preparation programs. This might be achieved by either opening new and qualified courses or improving the quality of the existing courses. Before this, it is important to investigate the effect of these courses, as well. In fact, this is one of the significance of the present study.

In this part, the studies related with mathematics teachers' or pre-service mathematics teachers' content knowledge for teaching mathematics both in Turkey and abroad were summarized briefly. All these studies underlined the importance of content knowledge for teaching mathematics in their professional development and how this knowledge can be developed during teacher education. Especially, the studies carried out in Turkey showed that Turkish pre-service mathematics teachers do not have sufficient content knowledge. Moreover, those studies provided an overview of what has been done to identify and develop pre-service teachers' or in-service teachers' content knowledge for teaching mathematics. The literature also includes some studies related with the role of methods courses on teacher's content knowledge. These studies are presented in the next part.

### 2.3. Role of Methods Courses on Teachers' Content Knowledge

One of the courses offered by mathematics teacher education programs is the mathematics teaching methods course. In fact, the mathematics teaching methods course plays an important role in teacher preparations since it can influence pre-service teachers' content knowledge (Ball, 1989). The literature reviewed in this line brought out that there are some studies investigating the effects of mathematics methods courses on teachers' mathematical content knowledge improvement

(Graeber, 1999; Harder & Talbot, 1997; Kinach, 2002; McDiarmid, Ball, & Anderson, 1989; Quinn, 1997; Simon, 1993).

Firstly, Quinn (1997) investigated the effects of methods courses on mathematical content knowledge of elementary and secondary mathematics teachers. 47 pre-service teachers, 28 of whom were pre-service elementary mathematics teachers, participated in this study. The test which was used to measure meaningful knowledge of mathematical content in this study was composed of 25 multiple choice questions covering most of the topics in elementary mathematics curriculum. The results of the study showed that at the end of the methods course which promoted students' questions, hands-on and cooperative group activities and technology use, pre-service elementary teachers' knowledge of content increased significantly. In spite of this statistically significant increase, pre-service elementary mathematics teachers performed poorly in the questions related to long division, fraction, geometry, and probability and statistics. This result highlighted the need and importance of the methods courses which was focused on a single unit like teaching geometry or teaching number and operations. This indicates that the methods courses and content courses in teacher education programs should emphasize the inadequacies in mathematical content knowledge of pre-service elementary teachers. Quinn also suggested that more time should be spared for the methods courses which combine mathematical content knowledge with pedagogical strategies. In this respect, McDiarmid et al. (1989) emphasized that methods instructors should first help pre-service teachers' developing their own understanding of the subject matter. This might be achieved by concentrating on a few topics during the methods course and providing an environment for pre-services to deepen their understanding of subject matter. Secondly, they proposed that methods instructors should make pre-service teachers learn about students' preconceptions and misconceptions on a particular topic and the way of their learning. Lastly, pre-service teachers should experience using multiple representations in teaching, evaluating students' representations and answers, and evaluating textbooks, activities and instructional tools. These suggestions both provides a method to make methods course more effective and highlights the importance of methods courses in developing and improving pre-service teachers' content knowledge.

In a similar sense, Graeber (1999) explored what important knowledge about mathematics should be included in mathematics methods courses for pre-service teachers. She proposed that during methods courses, pre-service teachers should be given enough opportunity to learn some characteristics of Shulman's PCK which are the knowledge of students' understanding, difficulties, preconceptions and misconceptions and how to make the subject comprehensible to students. Harder and Talbot (1997) also investigated how mathematics methods courses are taught by examining course syllabi from 45 public and private universities. It was found that class discussions, lab experiences, student presentations, and journals and lesson plans were the most frequently used tasks in the methods courses. Whereas there was not too much emphasis on pre-service teachers' mathematical knowledge about the topics in the mathematics curriculum, such as geometry, algebra, numbers and fractions, some tasks like lab experiences including using manipulatives, and writing journals about their own growth of pedagogical content addressed to the mathematical knowledge. Thus, Harder and Talbot's study pointed out that methods courses were prepared to improve pre-service teachers' mathematical knowledge by either explicit or implicit tasks.

Regarding this role of methods courses in pre-service teachers' mathematical content knowledge, Kinach (2002) proposed a cognitive strategy for pre-service mathematics teachers PCK development in methods courses. The cognitive strategy (IACTS) proposed by Kinach was composed of five elements: Identify, Assess, Challenge, Transform, and Sustain. The basic idea of IACTS was eliciting and assessing pre-service teachers PCK within a self-chosen context, then if necessary, challenging their instructional explanations within a new context (context A) and developing them within another context (context B). Kinach designed the mathematics methods courses based on this cognitive strategy on the topic of addition and subtraction with integers. At the end of the study, the cognitive strategy proposed to develop pre-service teachers' PCK was found to have given the pre-service teachers the opportunity of explaining and representing concepts and procedures within different manipulative context. Hence, IACTS could reveal different aspects of pre-service teachers' understanding of a particular topic. Also most of the pre-service teachers preferred teaching for understanding at the end of the methods course. Actually, such a result is desired to reach by the methods courses in teacher preparation programs since teachers' SMK shape their PCK and so their teaching and their students' learning (Kinach, 2002).

In another study, Ball (1989) explained the function of methods course on pre-service teachers' learning to teach. She viewed the methods course as a course addressing not only pedagogical way of acting as a teacher but also the subject matter that will be taught. More specifically, Ball mentioned about the role of methods courses on reflecting fundamental nature of teaching; subject matter knowledge, knowledge about students and how they learn, knowledge about teachers' role and how classroom environment can be designed to foster students' learning mathematics. In addition, Ball put forward that the methods courses give pre-service teachers opportunities to revisit and reconstruct their own past learning and experiences and to learn about appropriate ways of using manipulatives to teach mathematics effectively. Briefly, Ball emphasized the essential and effective role of methods courses in regular teacher education program. Ball's point of view in this study can be seen as an indication of which should be considered by teacher educators. Teacher educators should be aware of the function of methods courses on developing pre-service teachers' knowledge about mathematics.

To sum up, the results of the studies mentioned above revealed the role of methods courses in developing pre-service teachers' mathematical content knowledge. These studies also implied that teacher educators should not overlook the value of subject matter understanding of pre-service teachers in preparing them to teach for understanding (McDiarmid, Ball, & Anderson, 1989). Teacher educators should show sufficient attention to deciding the content and tasks of methods courses so as to promote pre-service teachers' content knowledge and their teaching mathematics. Moreover, the role of subject-specific methods courses in shaping pre-service teachers' understanding of content knowledge should not be overlooked (Grossman, 1990). That is, there might be separate methods courses for each topic in elementary school mathematics; methods course for teaching numbers and operations, for teaching geometry and measurement, for teaching algebra and functions, and for teaching probability. If these courses are designed well enough to make experience pre-service teachers with mathematical concepts, underlying meaning of the algorithms, students' common conceptions and misconceptions related to particular topic, then pre-service teachers might gain more content knowledge for teaching mathematics. Thus, an attempt of examining the effects of a mathematics teaching methods course on pre-service teachers' content knowledge for teaching mathematics is worthwhile since it is believed to provide significant information to both policy makers and teacher educators.

This chapter presented the related literature about in-service and pre-service teachers' mathematical content knowledge and the role of methods course on teacher education. In the next chapter, the methodology of the study is briefly described.

# CHAPTER III

# METHODOLOGY

This study explores the effect of a mathematics teaching methods course on pre-service teachers' content knowledge for teaching mathematics. In previous chapters, research questions, related literature review and the motivations for the study were presented. The goal of this chapter is to describe the method of inquiry. More specifically, the population and sample, variables and the instrument used in data collection are explained in detail. These are followed by the procedure of data collection, the context of the methods courses and data analysis. Also, internal validity threats, and assumptions and limitations of the study are addressed in this chapter.

## 3.1. Population and Sample

In this study, all senior elementary pre-service mathematics teachers enrolled mathematics teaching methods courses in Turkish universities were identified as the target population. The accessible population is all senior pre-service teachers enrolled in mathematics teaching methods courses in the universities in Ankara. The accessible population is the population to which the results will be generalized.

Purposive sampling method was used to obtain the representative sample of this study. Purposive sampling is appropriate in occasions where investigators need to select participants who have particular characteristics needed for the study. Therefore, by purposive sampling method, investigators select people who are believed to provide the data needed (Fraenkel & Wallen, 2006). Two criteria were taken into consideration to select the representative sample. The first one is the language of the instruction of the universities in which the pre-service teachers were enrolled. The researcher needed to study pre-service teachers who were in regular teacher education program in a university where the medium of instruction is English since the measurement tool (SETM) is in English. Thus, the researcher purposively chose pre-service mathematics teachers in a four-year teacher education program, Elementary Mathematics Education at METU because the medium of instruction is English in METU. The second criterion was related to the mathematics teaching methods course. Since the present study investigates the effect of the mathematics teaching methods course on pre-service mathematics teachers' content knowledge for teaching, the participants were selected based on their enrollment in a mathematics teaching methods course during the fall semester of 2007-2008. Since the methods course is given to the pre-service teachers in the last year of elementary mathematics teacher education program, 43 senior pre-service teachers taking the methods course constitutes the sample of the present study. These two criteria provided a sound representative and convenient sample from which the researcher will have a deep insight about the effect of mathematics teaching methods course on pre-service mathematics teachers' content knowledge for teaching mathematics. Figure 3.1. summarizes the sampling procedure visually.

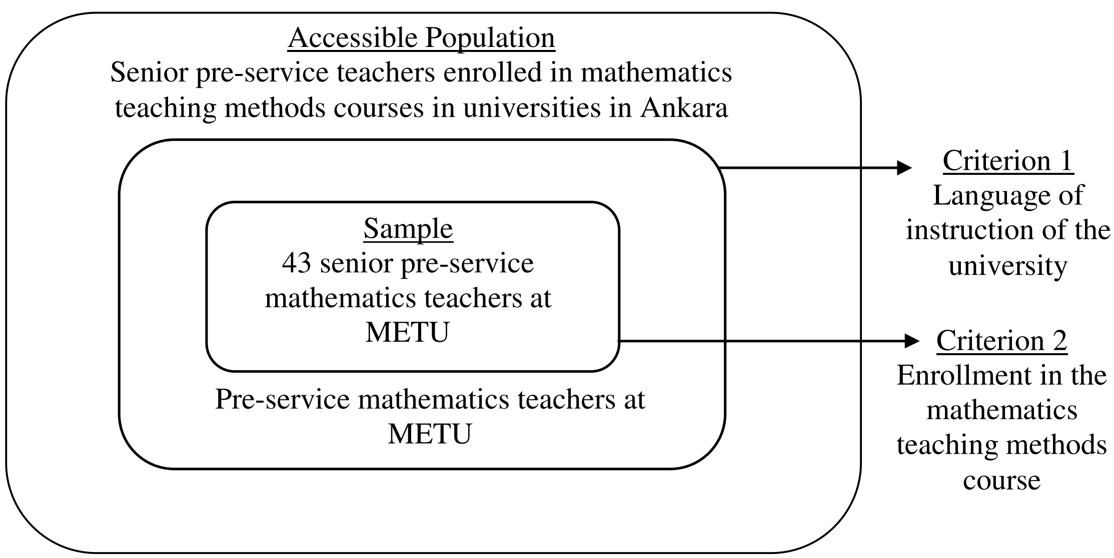


Figure 3.1 Sampling procedure of the study

Moreover, the gender distribution of the sample is given in Figure 3.2. It shows that the sample includes 18 male (41.9%) and 25 female (58.1%) senior pre-service teachers.

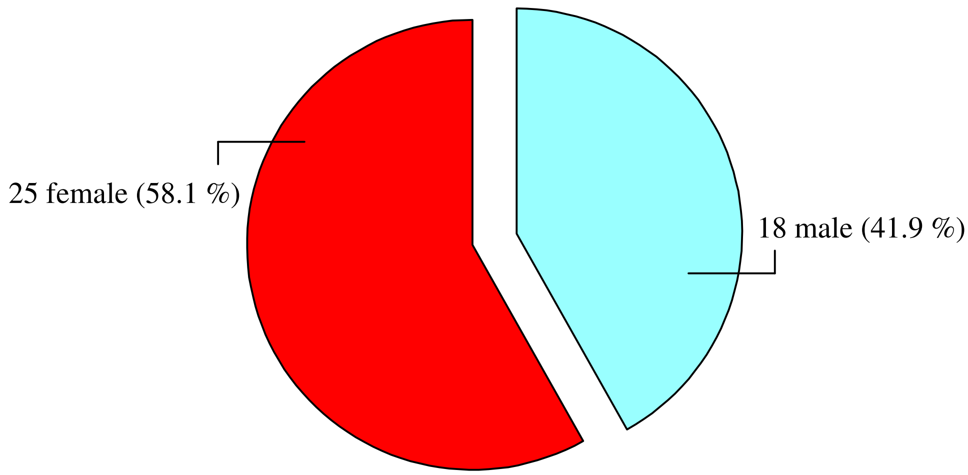


Figure 3.2 Distribution of Participants by Gender

### 3.1.1. The Elementary Mathematics Teacher Education (EME) Program

The Elementary Mathematics Education Teacher Program is an undergraduate program under the Department of Elementary Education. It is a four-year regular teacher education program that aims to educate socially, personally and professionally competent mathematics teachers (Middle East Technical University, General Catalog, 2005).

The students are required to take pure mathematics and science courses, and a few introduction to education courses in their first two years. In the following years, the number of general pedagogical courses and mathematics teaching courses increase while the content courses decrease. More specifically, the program offers students nine mathematics courses from the Department of Mathematics (MATH), 4 pedagogy courses from the Department of Educational Sciences (EDS) and twelve courses related to elementary mathematics and science teaching from the Department of Elementary Education (ELE). The program also includes 2 Physics, 1 Chemistry and 1 Biology content courses from related departments. Moreover, 3 of the courses offered by ELE are field experience courses; School Experience I, School Experience II, and Practice Teaching in Elementary Education. The first course offered in the second semester of the first year is mostly based on observation of the classroom environment and the learning and teaching procedure. Other two courses are offered in their final year and make pre-service teachers actively engaged in teaching practice. All the courses offered in the program are given in Appendix A.

## 3.2. Variables

### 3.2.1. Dependent Variables

The dependent variables of this study are (a) the pre-service teachers' content knowledge for teaching mathematics (CKTM) and (b) CKTM improvement which was measured by the gain scores, the difference between pre- and posttest scores. Both dependent variables are continuous and measured by Survey of Elementary Teachers of Mathematics (SETM).

### 3.2.2. Independent Variables

The major independent variable of this study is the participation in the mathematics teaching methods course. The content of the methods course was determined by the instructor of the course at the beginning of the semester. The pre-service teachers' gender, elective course preferences, and academic achievements on mathematics content courses are other independent variables. To get information about participants' academic achievement on mathematics content courses, their transcripts were examined. The academic achievements on mathematics content courses were found by computing the mean of the grades in related courses. More specifically, the pre-service teachers' academic achievements on mathematics content courses were determined by computing the mean of the grades taken from nine mathematics content courses offered by the teacher education program.

### 3.3. Instrument

The Survey of Elementary Teachers of Mathematics (SETM) was used to measure pre-service teachers' content knowledge for teaching mathematics (CKTM) in the areas of Number Concepts and Operations, Geometry and Algebra. The SETM is one of the CKTM measures developed as part of the Learning Mathematics for Teaching (LMT) project at the University of Michigan in the USA to measure mathematical knowledge for teaching. The most common ways to measure mathematical content knowledge of teachers have been conducting interviews and tests including short-answer questions (Hill & Ball, 2004). Utilizing these measurement methods was one of the hindrances of working with large number of teachers or pre-service teachers (Moss, 2006). To examine teachers' mathematical content knowledge, CKTM measures were developed in response to the need of large scale, reliable and valid multiple choice assessment (Ball, Hill, & Bass, 2005).

The philosophy of the instrument (SETM) is based on the idea that teachers should have deeper mathematical knowledge than other professions (Hill, Schilling, & Ball, 2004). The instrument includes items measuring specialized knowledge of content (SKC), common knowledge of content (CKC), and knowledge of students and content (KSC). As it was mentioned before, SKC is the knowledge related to the ways of teaching mathematics while CKC was the knowledge related to mathematical content that will be taught (Hill & Ball, 2004). Hence, the items are related to not only how a mathematical problem can be solved or an algorithm can be carried out but also how students' solutions can be evaluated, how a mathematical expression can be represented in multiple ways, how physical materials can be used in mathematical explanations appropriately and accurately, and how the meaning of mathematical concepts, procedures, and algorithms can be explained (Hill, Schilling, & Ball, 2004). At the beginning of the LMT project at the University of Michigan, the test developers chose two mathematical content areas, Number Concepts and Operations to write items because Number Concepts and Operations constitute a broad part of the K-6 curriculum (Ball, Bass, & Hill, 2004; Hill, Schilling, & Ball, 2004). Then they continued to work on Patterns, Functions and Algebra. The initial item development structure is shown in Table 3.1.

Table 3.1 Initial Item Development Structure (Ball, Bass, & Hill, 2004)

| | Domains | |
|---|---|---|
| | Knowledge of mathematics | Knowledge of students and mathematics |
| Number Concepts | | |
| Operations | | |
| Patterns, functions, algebra | | |

This table summarizes that test developers wrote items in three content areas with two domains of teacher knowledge. The shaded part showed that there were no items in that area until 2004. In the following years, other content domains (e.g. geometry) and the middle school forms were added to the measures. All the measurement items were reviewed, revised, rewritten and piloted (Ball, Bass, & Hill, 2004; Hill, Schilling, & Ball, 2004). More specifically, elementary school forms were piloted with 600 elementary teachers and difficulties of each item were computed (Hill, Schilling, & Ball, 2004). At the end, there were mathematical content knowledge for teaching items in the following domains (corresponding grades): (1) Number and operations (K-6, 6-8), (2) Patterns, function, and algebra (K-6, 6-8), and (3) Geometry (3-8). Moreover, rational number, proportional reasoning, geometry, and data, probability and statistics items have been piloted in 2007-2008 (Hill, 2004).

The SETM used in the present study is formed by combining three forms: (1) Number Concepts and Operation 2007A, (2) Geometry 2005A, (3) Algebra 2006B. These forms were chosen based on the consultation with one of the researchers at University of Michigan. Thus, SETM includes three subscales which are Content Knowledge about Number Concepts and Operation (NCOP), Content Knowledge about Geometry (GEO), and Content Knowledge about Algebra (ALG) and the number of items in each subscale is 29, 19, and 35, respectively. The scores which can be taken range from 0 to 29 in the Number Concepts and Operation subscale, from 0 to 19 in the Geometry subscale, and from 0 to 35 in the Algebra subscale.

Since the items were not published owing to the cost of the development, only the released items presented in the web site of LMT project (http://sitemaker.umich.edu/lmt/home) are attached in Appendix B. The actual forms of the measures are obtained by Dr. Yusuf Koç upon his participation into one of the workshop sessions of the LMT project at University of Michigan in August, 2007.

### 3.3.1. Reliability and Validity

Schilling, Blunk and Hill (2007) developed an interpretive validity approach during the validation of the measures. This interpretive approach includes three assumptions and inferences; an elemental assumption, a structural assumption, and an ecological assumption (Schilling, Blunk, & Hill, 2007). More specifically, the elemental assumption is related to that items correspond to the teachers' reasoning about mathematical knowledge for teaching (Hill, Dean, & Goffney, 2007). The structural assumption is about the domain of the mathematical knowledge for teaching and types of knowledge. That is, the items should reflect subject matter area (e.g., Number Concepts and Operations) and content knowledge (i.e., CCK, SCK, and KCS) (Hill, Dean, & Goffney, 2007; Schilling, Blunk, & Hill, 2007). On the other hand, the ecological assumption is that the items reflect the mathematical content knowledge needed to effective teaching and students' meaningful learning (Hill, et al., 2007). For the elemental assumption, the researchers found sufficient evidence to support that the items reflect teachers' mathematical knowledge for teaching (Hill, Dean, & Goffney, 2007). However, there have been some problems with the structural assumption since CCK and SCK items were not loaded in different factors in the factor analysis. KCS items also were appeared as they needed some revision (Schilling, Blunk, & Hill, 2007). In terms of ecological assumption, there were some evidences which supported the relationship between teachers' responses to the items and their teaching effectiveness (Hill, et al., 2007). As these validity studies reflected, the researchers might need to revise the measures and even the theory of teacher knowledge (Schilling, Blunk, & Hill, 2007).

The test developers still work on the reliability and validity studies of the SETM used in this study. However, piloting the test in Turkey is not in the scope of this study. Even the reliability of the test is found high by the test developers, it might be different in Turkey context. This constitutes one of the limitations of this study.

On the other hand, the language of the test, English, was considered as a potential threat to the construct validity. However, the pre-service teachers have been enrolled in the teacher education program where the medium of instruction is English. In spite of the fact that, the correlation between average grades on English courses taken in the teacher education program, and pretest and posttest scores were explored separately.

#### 3.3.1.1. Relationship between Academic Achievement on English Courses and Pretest and Posttest Scores

It was found worthwhile to analyze the relationship between pre-service teachers' academic achievement on English courses and their pretest and posttest scores since the language of the test was English. This analysis would provide information about construct validity of the test scores. Participants' average grades on English courses were determined by computing the mean of the grades on four English courses; namely, Development of Reading And Writing Skills I, Development of Reading and Writing Skills II, Academic Oral Presentation and Advanced Communication Skills. To investigate whether there is a relationship between pre-service mathematics teachers' content knowledge for teaching mathematics and their average grades on English courses, Pearson product-moment correlations were conducted for both pretest and posttest scores.

Before calculating Pearson product-moment correlation, the assumptions of normality, linearity, and homoscedasticity were checked. As it was mentioned in Descriptive Statistics section, the normality of the distribution of pretest and posttest scores was met regarding the corresponding skewness and kurtosis values. To check the normality of the distribution of pre-service teachers' average grades on English courses, basic descriptive statistics were explored. Table 3.2 shows the descriptive statistics of pre-service teachers' average grades on English courses.

Table 3.2 Basic Descriptive Statistics of Average Grades on English Courses

| | Average Grades on English Courses |
|---|---|
| Mean | 2.78 |
| Std. Deviation | 0.65 |
| Minimum | 1.50 |
| Maximum | 3.75 |
| Skewness | -0.31 |
| Kurtosis | -0.97 |

The skewness and kurtosis values were tolerable to interpret that pre-service mathematics teachers' average grades on English courses were normally distributed. In order to calculate correlation coefficient, the relationship between the two variables is required to be linear (Pallant, 2001). The scatterplot of the pre-service teachers' average grades on English courses and the pretest scores, and the scatterplot of the average grades on English courses and posttest scores are presented in Figure 3.3 and Figure 3.4, respectively.

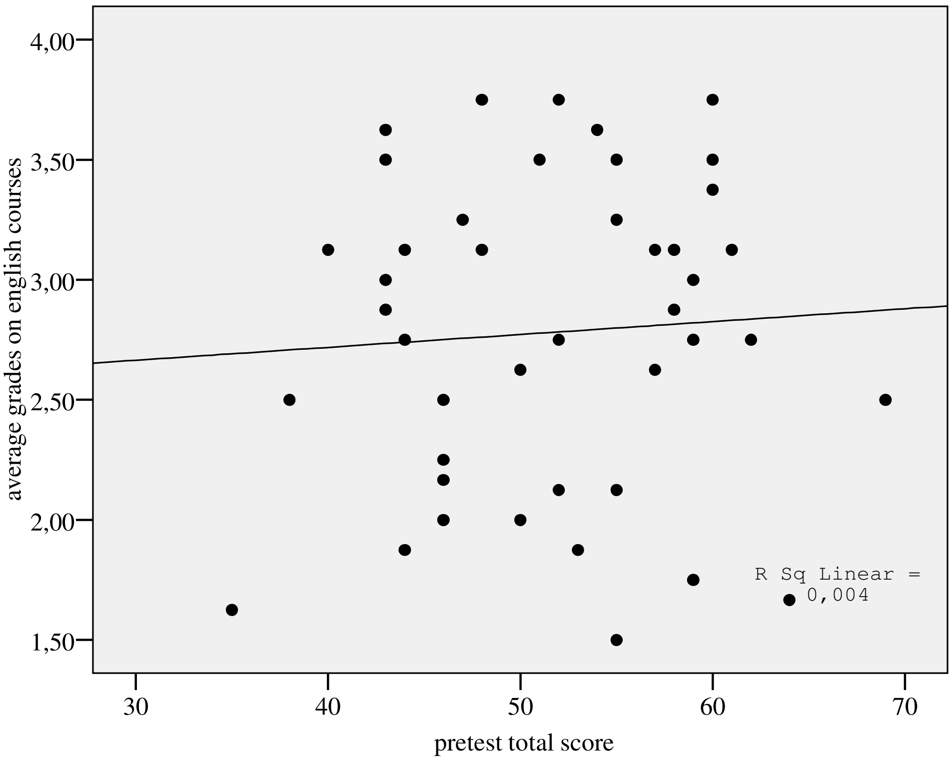


Figure 3.3 Scatterplot of pretest scores and average grades on English courses

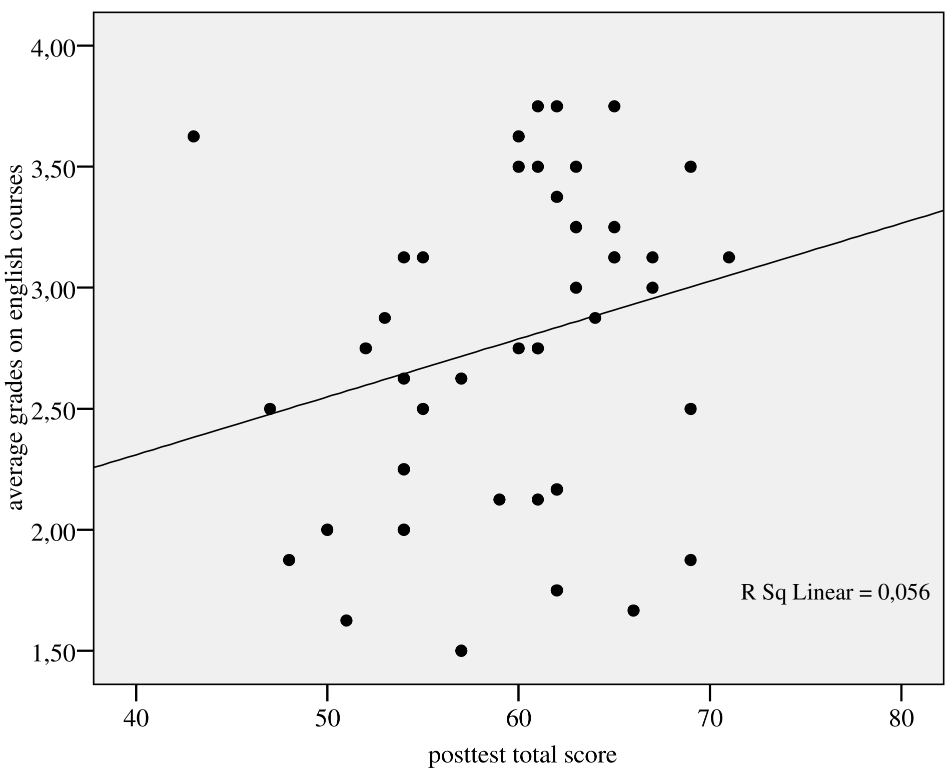


Figure 3.4 Scatterplot of posttest scores and average grades on English courses

The scatterplots indicated that there were no violations in linearity and homoscedaticity assumptions. After checking the assumptions, the bivariate correlation was conducted. The results are presented in Table 3.3.

The findings indicated that there was no significant correlation between pre-service mathematics teachers' total pretest scores and their average grades on English courses [$r = .06$, $n = 43$, $p > .05$]. Similarly, there was no significant correlation between pre-service mathematics teachers' total posttest scores and their average grades on English courses [$r = .24$, $n = 43$, $p > .05$].

Table 3.3 Results of the Bivariate Correlations of Pretest and Posttest Scores and Average Grades on English Courses

| | | Average Grades on English Courses | Pretest | Posttest |
|---|---|---|---|---|
| Average Grades on English Courses | Pearson Correlation | 1 | 0.06 | 0.24 |
| | Sig. (2-tailed) | | 0.69 | 0.12 |
| | N | 43 | 43 | 43 |
| Pretest | Pearson Correlation | 0.06 | 1 | 0.69 |
| | Sig. (2-tailed) | 0.69 | | 0.00 |
| | N | 43 | 43 | 43 |
| Posttest | Pearson Correlation | 0.24 | 0.69 | 1 |
| | Sig. (2-tailed) | 0.12 | 0.00 | |
| | N | 43 | 43 | 43 |

The results indicated that pre-service teachers' academic achievement in English was not related to their performance in the pretest and posttest. This provides information about construct validity of the instrument as it measured the CKTM of pre-service teachers. On the other hand, this information is not strong enough since the English courses in the graduate program either do not include mathematical terms or include a few mathematical terms. Therefore, the researcher prepared an unknown word list as seen in Appendix C and then gave it to the pre-service teachers with the test. Hence, pre-service teachers' proficiencies in English still constitute a limitation for this study.

### 3.4. Data Collection Procedure

The data is collected from senior pre-service mathematics teachers enrolled in Elementary Mathematics Teacher Education program during the fall semester of 2007. The SETM was administered two times as pre- and posttest to compare pre-service teachers' mathematical content knowledge for teaching, and to identify the effect of mathematics teaching methods course. It was used as pretest at the beginning of the mathematics teaching methods course in the fall semester (September, 2007) and as the posttest in the middle of the spring semester (March, 2008). Both the pretest and posttest data were collected by the researcher and the course instructor. A time schedule indicating the data collection procedure chronologically is given in Table 3.4.

Table 3.4 Time schedule for data collection

| Date | Data Collection |
|---|---|
| September 2007<br>(Fall semester - at the beginning of the methods course) | Pretest administration |
| March 2008<br>(Spring semester - after the methods course) | Posttest administration |

The primary method for obtaining data is quantitative because the primary purpose of this study is to investigate the effects of mathematics teaching methods course on pre-service teachers' content knowledge for teaching mathematics (CKTM). Although conducting this study in a quantitative way is not common in the issue of teachers content knowledge (Ball, Bass, & Hill, 2004; Hill, Schilling, & Ball, 2004), it was possible to explore the effects of the methods course on the participants' knowledge of teaching quantitatively.

Since most of the data collection procedure took place in the mathematics teaching methods course, and the effect of methods course on pre-service teachers' content knowledge for teaching was the main issue of this study, the content of the course is described below. The content and the learning opportunities provided in mathematics teaching methods course give more information about the setting of this study.

### 3.4.1. Mathematics Teaching Methods Course

The Methods of Mathematics Teaching course is a last year course in the mathematics teacher education program. The general goal of the course was to make pre-service teachers understand elementary and middle school mathematics concepts and the ideas behind the standard algorithms, rules and formulas and get insight about how to teach a particular mathematical topic.

The students were required to read the textbook of *Elementary and Middle School Mathematics* by Van de Walle (2004). As Van de Walle (2004) mentioned that the book is commonly used by classroom teachers as a resource book. The philosophy of the book is "..., every child can come to believe that he or she is capable of making sense of mathematics" (Van de Walle, 2004). The book consists of two main sections, Teaching Mathematics: Foundations and Perspectives, and Development of Mathematical Concepts and Procedures. The first section is related to reform movements in mathematics teaching, problem-based instruction, and using technology in mathematics teaching. On the other hand, the second section focuses on the K-8 mathematics topics, Number Concepts, Operations, Place Value Development, Whole-Number Computation, Estimation, Fraction Concepts and Computation with Fractions, Decimal and Percent Concepts, Ratio and Proportion Concepts, Measurement and Geometric Concept, Data Analysis and Probability, Algebraic Reasoning, Functions, and Exponents, Integers and Real Numbers (Van de Walle, 2004). In mathematics teaching methods course only concepts of Data Analysis and Probability was not studied by pre-service teachers owing to the limited time. Moreover, the book includes some parts referring to the NCTM Standards. The textbook's reflecting reforms in mathematics education is important since the new elementary mathematics curriculum in Turkey is also designed to the reform movements in mathematics teaching (MEB, 2005, 2007). In addition to the elementary mathematics curriculum, the content of the textbook is consistent with the test used to measure pre-service teachers' content knowledge for teaching mathematics. More specifically, the book includes meaning of the mathematics concepts, and procedures and multiple representations of those concepts, and procedures. For instance, how counters can be used to represent computations with integers is one of the tasks in the textbook. The SETM also included an item related to representation of a subtraction with integers by using counters. Thus, the textbook plays an important role in the mathematics teaching methods course to improve pre-service teachers' mathematical content knowledge.

In this course, the participants were assessed by one midterm, four unannounced quizzes, and one final exam. In addition to these, the pre-service teachers' were expected to complete three assignments; namely, Active Reflection Assignment, Article Critique, and Activity Design Project. Active Reflection Assignments were in-class assignments and were given at the beginning and at the end of the semester. Students were expected to write a reflection paper related to their opinions about the characteristics of effective mathematics teaching and effective mathematics teacher. The second assignment, Article Critique, required students to go to the ULAKBIM library and review an article selected from the Journal of Teaching Mathematics in Middle School. The article review included briefly summarizing the message that is given in the article, interpreting the strengths and the weaknesses of the article, and reasoning about the effect of the article on their teaching. On the other hand, the last assignment required to design three different forms of an activity on the same topic for 6-8$^{th}$ grade mathematics; (1) for regular elementary school students, (2) for students who have learning difficulties or who are less successful, and (3) for gifted students. So, participants were required to gear down and gear up the cognitive demand based on which the activity for regular elementary school students have been constructed. The Activity Design Project also included a reflection paper explaining how the activity was geared up and down and what characteristics of the activities appealed to gifted and less successful students. Hence, these three assignments were targeted to lead pre-service teachers to make self-critics about their beliefs on mathematics teaching, to search for new studies and utilize them for their teaching practices, and to design instructional activities as suitable for students' cognitive abilities. The course syllabus, assignment sheets, and midterm and quiz questions are given in Appendix D.

Moreover, several handouts and worksheets were prepared and distributed to students as in-class activities. In those activities, the pre-service mathematics teachers had the opportunity to experience working on a mathematical task either individually or cooperatively, and understanding elementary school students' solution methods. They dealt with underlying meaning of algorithms, rules and procedures, representing a mathematical expression or operation in multiple ways, using physical materials while working on a mathematical task, discussing the different solution methods of a mathematical problem, and evaluating sample student solutions and understanding students' way of thinking about a specific mathematical topic like integers. To illustrate, pre-service teachers worked on operations with fractions. They discussed what $\frac{4}{5} \times \frac{1}{2}$ means, by how many different ways it can be represented, and how a problem can be constructed representing this operation. Also, they showed operations with natural numbers, and operations with fractions by using both area model and set model. Furthermore, they used algebra tiles to represent polynomials like $2x^2+4xy+y^2$. The mathematics teaching methods course did not only deal with teaching but also the mathematics itself. For instance, the pre-service teachers were given worksheets describing the characteristics of the geometric shapes. The sample worksheets given as in-class activities are given in Appendix E. Moreover, each pre-service teacher was expected to participate in class discussions related to in-class activities and the related readings in the textbook.

Thus, in the mathematics teaching methods course the pre-service elementary mathematics teachers had many learning opportunities to see elementary and middle school students' common (mis)conceptions and to experience classroom activities designed to teach particular mathematical concepts and procedures.

### 3.5. Data Analysis

The participants' responses to the SETM questions were entered on the SPSS 15.0 program. These responses were coded as incorrect (0), or correct (1). Then the data were analyzed by utilizing both descriptive and inferential statistics tools available in SPSS.

### 3.5.1. Descriptive Statistics

The mean, standard deviation, minimum and maximum scores, skewness and kurtosis values of the pretest, posttest, and gain scores, and the pretest, posttest, and gain scores in each subscale; Number Concepts and Operations, Geometry, and Algebra were computed. Also those basic descriptive statistics were presented by gender and by pre-service teachers' elective course preferences. These descriptive statistics indicated a general picture about the pre-service teachers' CKTM improvement.

### 3.5.2. Inferential Statistics

The main goal of this study is to explore the effect of mathematics teaching methods course on pre-service teachers' content knowledge for teaching mathematics. To investigate this, paired-samples t-test was conducted for pretest and posttest scores. Also, paired samples t-tests were employed to examine the mean difference between pretest and posttest of each subscale; Number Concepts and Operations, Geometry, and Algebra.

Moreover, the differences between pre-service teachers' CKTM were examined. Therefore, independent samples t-tests were conducted to identify whether pre-service teachers' CKTM differ by their gender and their elective course preferences. Also, bivariate correlation was employed to investigate the relationship between the pre-service teacher' academic achievement on mathematics content courses and their CKTM. These inferential statistics provided an in-depth analysis of pre-service teacher' content knowledge for teaching mathematics.

Following the significant results, power analysis was conducted to determine whether the conclusion reached by the statistical test was correct (Fraenkel & Wallen, 2006). One of the using power analyses is determining the level of power in a study which has been conducted by using the known effect size and sample size (Murphy & Myors, 2004). To calculate power of the test, G*Power program was used. The G*Power program was designed to calculate power of statistical tests used in social and behavioral research (Faul, Erdfelder, Lang, & Buchner, 2007). The program firstly determines the effect size by Cohen's measures and then calculates the power of any statistical test. Effect sizes were also computed to gain information about the magnitude difference between the two sets of scores or the scores of two groups. Both effect size and power were so important that they presented information about practical significance

In addition, Type 1 and Type 2 errors have possibility to occur on t-tests. Type 1 error occurs when the t-test results show that there is a difference between groups even though there is not really. Type 2 error occurs when the researcher fails to reject null hypothesis, but actually the decision is wrong (Pallant, 2001). In spite of the fact that these two errors inversely related, some precautions were tried to be taken to control these errors. One of the ways of reducing the Type 1 and Type 2 errors is using MANOVA or ANOVA in the statistical analysis. However, using MANOVA or ANOVA is not appropriate in this study because of the small sample size. Since the sample size is not large enough, the assumptions of the MANOVA and ANOVA has violated. Therefore, only Bonferroni approach was used in interpreting the t-test results to control the Type 1 error (Green, Salkind, & Akey, 2000).

### 3.6. Assumptions and Limitations

In this part, the main assumptions and limitations of this study are explained. Firstly, it was assumed that the pre-service teachers responded to the items in the test seriously. Pre-service teachers answered the items of the test 90 minutes in average. It was observed that the pre-service teacher who finished the test at first completed the test about 45 minutes. This indicated that most of the pre-service teachers dealt with the questions seriously.

Moreover, pre-service teachers were assumed to be good enough to read and understand the items of the instrument which is in English since the medium of the instruction of the teacher education program which the participants enrolled in is English. In their courses, they were assessed by the questions which are written in

English and expected to answer the questions in English. Therefore, their proficiencies of the English were assumed to be sufficient to understand and answer the items. Also, the correlation between pre-service teachers' academic achievement on English courses and their test scores showed that there were no significant relation between pre-service teachers' academic achievement on English courses and their pretest and posttest scores. As mentioned before, this provides valuable, but not sufficient information about pre-service elementary mathematics teachers' competencies in English, especially for the mathematical terms. Thus, the researcher considers the English competencies of the pre-service teachers as one of the limitations. In other words, if the test measuring CKTM were in Turkish, the results might change.

Since this study investigates the effect of teaching methods course and the data were gathered in the context of this course, the results of the study was limited to the same courses in which the similar textbook is used and the similar learning opportunities are provided.

Another limitation of the study is using one group. Since there is no control group, comparing the groups in terms of their content knowledge for mathematics teaching was not possible. Lastly, pre-service teachers' characteristics (demographic variables, SES, educational background, interests, etc.) and other learning opportunities in their teacher education program (like field experience and other courses) were not considered in the analysis of this study. These variables might have an influence on pre-service teachers' content knowledge for teaching mathematics.

### 3.7. Validity of the Study

Validity of the study is based on both internal and external validity threats. Validity threats which may influence validity of the results of the study are explained separately as internal validity threats and external validity threats in this section.

### 3.7.1. Internal Validity of the Study

Internal validity refers to the degree to which observed differences on dependent variable is resulted from the independent variable. Internal validity threats occur when the observed results are not related to dependent variable itself, but related to some unintended variables. An experimental study may be subjected to 10 internal validity threats; (1) Subject characteristics, (2) Attitude of subjects, (3) Testing, (4) Instrumentation, (5) Implementation, (6) Mortality, (7) Location, (8) History, (9) Maturation, and (10) Regression (Fraenkel & Wallen, 2006). Paying attention to these possible threats and trying to control them are essential steps in carrying out a study (Fraenkel & Wallen, 2006).

The present study is based on the data obtained from pre-service mathematics teachers enrolled in mathematics teaching methods course. The sample of this study was chosen purposively based on the two criteria as mentioned before. On the other hand, other characteristics of the participants might influence the internal validity of this study. These characteristics are type of high school (either Anatolian Teacher High School or other high school), the learning opportunities provided by other courses, the experience as a student teacher in the field experience, and work experiences. It was assumed that the learning opportunities provided by other courses and the field and work experiences have little or no effect on the results of the study. It is because that most of the participants took only School Experience II related to mathematics teaching while taking methods course and they did not actively participate in teaching in this course. In the second semester they took Practice Teaching in Elementary Education course, but this course did not started when they responded to posttest. Thus, the field experiences were regarded as they did not influence participants' content knowledge for teaching mathematics a lot. However, they took elective courses. These elective courses were added to the analysis of the study and thus they were controlled. When types of their high schools are examined, it was found that most of them (95%) graduated from Anatolian Teacher High School. So, it was assumed that they did not influence the results of the study.

The attitude of the subject is also considered as a possible internal validity threat of this study. To control it, a presentation about the study was made for pre-service teachers. The purpose of the study, the importance of their serious participations and the implicit and explicit benefits of the results of the study were explained. The most important internal validity threat is testing threat since the design of the study is one-group pretest-posttest experimental study. To control testing effect, the time interval between pre- and post-test was lengthened as much as possible. In fact, there is a 6-month gap between pre- and post-test.

Also, instrument decay in scoring procedure did not occur since the test was a multiple-choice test. Using standard conditions in data gathering period helped control both instrumentation, location, and subject attitude threats. Besides, threats about implementation were tried to control by using one group and one instructor. As it was mentioned before, 43 pre-service teachers' were responded to pre-, and post-tests, and so, the mortality threat was not encountered. Regression also is not a threat to the internal validity of this study as well owing to the fact that the selection of the participants was not based on their performance. Moreover, the data were obtained in six moths in standard conditions. For this reason, history threat could not be the internal validity threats of this study. On the other hand, there was a semester holiday in the time interval between pretest and posttest. This might give rise to maturation threat.

To sum up, the most significant internal validity threat seems to be testing threat. However, it is considered seriously throughout the study and possible actions were performed to control it. In addition to this, the researcher role and bias is considered as another significant internal validity threat of instrumentation and implementation. Therefore, the role of the researcher and the course instructor during data collection and implementation is briefly described below.

#### 3.7.1.1. The Researcher Role and Bias

In a quantitative study, the data collectors, scorers or implementers might unconsciously distort the data in favor of the intended outcomes (Fraenkel & Wallen, 2006).

Both tests were administrated in the classroom by the researcher and the instructor. Since the researcher and the instructor of the methods course know the purpose of the study, the researcher and instructor bias might have occurred. To control over this bias, the course instructor did not communicate with pre-service teachers during data collection period. In fact, he was just an outside observer. On the other hand, researcher prepared a list including unknown words and distributed them to the pre-service teachers by attaching to the test in order to reduce the possible questions. In spite of this, some pre-service teachers asked some questions about the items in the test. In such a case, the researcher paid attention to answer the questions in order not to direct the pre-service teacher about the answer of the question. Also, the researcher preferred making a brief explanation to the whole class about the most common questions asked by the pre-service teachers. In spite of this serious effort of the researcher, her attitude, behaviors or actions might affect pre-service teachers while responding the tests.

Furthermore, the role of the researcher and instructor during the mathematics teaching methods course was also important. The course instructor had determined the textbook of the course, and assignments before he saw the test items. In fact Van de Walle's (2004) book has been using in the mathematics teaching methods course for three years in Elementary Mathematics Teacher Education Program at METU. On the other hand, the instructor determined the in-class activities before the classes by considering the appropriateness of the readings in the book. He did not solve any of the questions in the test during the methods course. In fact, he focused on the examples in the textbook and worksheet. He performed the great effort to make pre-service teachers about the mathematical content knowledge that they will teach, and the effective ways of teaching mathematics for meaningful learning rather than training them for the test.

The researcher participated approximately 25% of the methods lessons during the whole semester. Even though she was an observer in most of the courses, she participated in some of the class discussions. During these class discussions, the researcher did not refer to the items in the test. Also, she did not take place in the process of preparing midterm, final and quiz questions. Thus, the researcher tried to reduce the effect of being a research assistant in the mathematics teaching methods course.

Even though the researcher and the course instructor followed several strategies which are mentioned above, their roles in data collection and implementation processes could not be underestimated. The researcher and course instructor bias might threat the study internally. Hence, this constitutes an another limitation for the study.

### 3.7.2. External Validity of the Study

External validity is defined as "the extent that the results of a study can be generalized from a sample to a population" (Fraenkel & Wallen, 2006, p.108). It requires both population generalizability and ecological generalizability.

Population generalizability is related to generalizability of the results of the study to the intended population (Fraenkel & Wallen, 2006). 43 senior pre-service teachers enrolled in teaching methods course at METU constituted the sample of this study. The intended population was determined as senior pre-service teachers enrolled in mathematics teaching methods courses in universities in Ankara. There are 4 universities in Ankara which include Elementary Mathematics Teacher Education Program. Since there are approximately 300 senior pre-service mathematics teachers in elementary mathematics education program in the universities in Ankara, the selected sample size is more than 10% of the intended population. This is an indication of population generalizability of the study. Nevertheless, there was a specific characteristic of the sample which is that they should have enrolled in a teacher education program where the medium of language is English. This characteristic of the sample reduces the population generalizability of the study.

The ecological generalizability is related to the generalizability of the study to other conditions and settings (Fraenkel & Wallen, 2006). This is one of the limitations of this study. The ecological generalizability of this study is limited to the mathematics teaching methods courses in which the similar textbook is used and the similar learning opportunities are provided.

# CHAPTER IV

# RESULTS AND CONCLUSIONS

This chapter aims to present the results of the study in two main sections. The first section includes descriptive statistics of pretest and posttest scores, and gain scores, the difference between posttest and pretest scores. The second section deals with the inferential statistics obtained by the statistical analysis. Also this chapter presents to the conclusions regarding the results of the study.

## 4.1. Descriptive Statistics

Descriptive statistics concerning the participants' pretest scores, posttest scores, and gain scores are presented in this section. More specifically, this section summarizes the descriptive statistics of the scores for the whole group, by gender, and by elective course preferences.

### 4.1.1. Descriptive Statistics for Whole Group

43 participants responded to the Survey of Elementary Teachers of Mathematics (SETM) both in pretest and posttest. Table 4.1 summarizes the pretest and posttest, and the gain scores, the difference between pretest scores and posttest scores.

Table 4.1 Pretest, Posttest, and Gain Scores for the Whole Group

| | Pretest (out of 83) | Posttest (out of 83) | Gain (Posttest-pretest) |
|---|---|---|---|
| N | 43 | 43 | 43 |
| Mean | 51.65 | 59.70 | 8.05 |
| Std. Deviation | 7.66 | 6.49 | 5.67 |
| Minimum | 35.00 | 43.00 | -3.00 |
| Maximum | 69.00 | 71.00 | 21.00 |
| Skewness | -0.04 | -0.54 | 0.34 |
| Kurtosis | -0.60 | -0.16 | -0.14 |

As given in Table 4.1, while pre-service elementary mathematics teachers have a mean score of 51.65 (*SD* = 7.66) on the pretest, their mean score in the posttest is 59.70 (*SD* = 6.49) out of 83. Thus, the average gain scores of pre-service mathematics teachers is found as 8.05 (*SD* = 5.67). This gain score constitutes 9.70% of 83, the highest possible score in the test. Moreover, the minimum score in the pretest (35) went up to 43 in the posttest. However, the increase in the maximum score from pretest to posttest was not so large that it only moved from 69 to 71. Table 4.1 also presents the skewness and kurtosis values for the whole group. Those values indicated that the distributions of the pretest and posttest scores, and gain scores for the whole group are approximately normally distributed.

The pretest, posttest and gain scores in each subscale of the SETM are shown in Table 4.2. This table indicated that the mean score of the Number Concepts and Operation (NCOP) Subscale in the pretest, 16.88 (*SD* = 3.20), increased to 19.72 (*SD* = 2.79) which makes the average gain score 2.84 (*SD* = -3.00). Considering the change between the pre- and posttest, while the minimum score in the NCOP Subscale increased from 10 to 14, the maximum score did not change (*max* = 24.00). The mean scores in the Geometry (GEO) Subscale showed an increase from 13.42 (*SD* = 2.57) in the pretest to 15.58 (*SD* = 2.30) in the posttest. Both minimum and maximum scores increased from pretest (*min* = 7, *max* = 18) to posttest (*min* = 11, *max* = 19) in the GEO Subscale. Also, the average the gain scores in the GEO subscale was found as 2.16 (*SD* = -2.00). Also, there is a similar increase in the mean scores from the pretest (*M* = 21.35, *SD* = 4.13) to posttest (*M* = 24.40, *SD* = 3.27) in the Algebra (ALG) Subscale. The average the gain score was 3.05 (*SD* = -4.00). When minimum and maximum scores were examined, it was observed that both of them increased from pretest (*min* = 13, *max* = 28) to posttest (*min* = 17, *max* = 32).

Table 4.2 Pretest, Posttest, and Gain Scores in the Subscales for the Whole Group

| | | Pretest | Posttest | Gain |
|---|---|---|---|---|
| Number Concepts and Operation Subscale (number of items=29) | N | 43 | 43 | 43 |
| | Mean | 16.88 | 19.72 | 2.84 |
| | Std. Deviation | 3.20 | 2.79 | -3.00 |
| | Minimum | 10.00 | 14.00 | 9.00 |
| | Maximum | 24.00 | 24.00 | 2.88 |
| | Skewness | -0.39 | -0.68 | -0.01 |
| | Kurtosis | -0.24 | -0.58 | -0.74 |
| Geometry Subscale (number of items=19) | Mean | 13.42 | 15.58 | 2.16 |
| | Std. Deviation | 2.57 | 2.30 | -2.00 |
| | Minimum | 7.00 | 11.00 | 9.00 |
| | Maximum | 18.00 | 19.00 | 2.51 |
| | Skewness | -0.34 | -0.72 | 0.82 |
| | Kurtosis | -0.33 | -0.60 | 0.54 |
| Algebra Subscale (number of items=35) | Mean | 21.35 | 24.40 | 3.05 |
| | Std. Deviation | 4.13 | 3.27 | -4.00 |
| | Minimum | 13.00 | 17.00 | 12.00 |
| | Maximum | 28.00 | 32.00 | 3.87 |
| | Skewness | -0.13 | -0.10 | 0.52 |
| | Kurtosis | -1.21 | 0.12 | -0.12 |

Moreover, the maximum scores that could be taken in each subscale are different since each subscale of SETM consists of different amount of the items. When the average gain scores in each subscale are considered, it was observed that mean of the test scores increased 9.79% ($M$ = 2.84) in the NCOP Subscale, 11.37% ($M$ = 2.16) in the GEO Subscale, and 8.71% ($M$ = 3.05) in the ALG Subscale. This indicated that pre-service elementary mathematics teachers improved their performance relatively more in the Geometry Subscale.

Table 4.2 also shows the skewness and kurtosis values of pretest, posttest, and gain scores for each subscale. These values provide information about the nature of the distribution of the scores in each section. Based on these values, distribution of pretest, posttest, and gain scores were regarded as normally distributed.

### 4.1.2. Descriptive Statistics Considering Gender

25 female and 18 male pre-service teachers participated in this study. In order to gain descriptive information about whether gender differences exist with respect to the pretest, posttest, and gain scores, Table 4.3 presents the mean scores by gender.

Table 4.3 Pretest, Posttest, and Gain Scores by Gender

| | Pretest | | Posttest | | Gain | |
|---|---|---|---|---|---|---|
| | Male | Female | Male | Female | Male | Female |
| N | 18 | 25 | 18 | 25 | 18 | 25 |
| Mean | 53.39 | 50.40 | 59.50 | 59.84 | 6.11 | 9.44 |
| Std. Deviation | 8.12 | 7.22 | 6.57 | 6.55 | 5.84 | 5.24 |
| Minimum | 35.00 | 38.00 | 48.00 | 43.00 | -3.00 | 0.00 |
| Maximum | 69.00 | 64.00 | 69.00 | 71.00 | 16.00 | 21.00 |
| Skewness | -0.35 | 0.12 | -0.09 | -0.89 | 0.41 | 0.60 |
| Kurtosis | 0.30 | -1.08 | -0.88 | 0.63 | -0.60 | 0.35 |

The above table shows that while male pre-service teachers have a mean score of 53.39 (*SD* = 8.12), female pre-service teachers have a mean of 50.40 (*SD* = 7.22) in the pretest. This means that male pre-service elementary mathematics teachers performed slightly better than the females in the pretest. On the other hand, the mean score of the female pre-service teachers (*M* = 59.84, *SD* = 6.55) is found very close to the males' (*M* = 59.50, *SD* = 6.57) in the posttest. This indicated that the females showed about 3-point more improvement than male participants in terms of mathematical content knowledge. The means of males and females on gain scores confirms this finding as they indicated that female pre-service teachers' test scores increased 11.37% (*M* = 9.44, *SD* = 5.24) while male pre-service teachers showed a 7.36% (*M* = 6.11¸ *SD* = 5.84) improvement on the test. When the minimum and maximum scores examined in Table 4.3, it was seen that in the pretest, the minimum score of females (*min* = 38) was higher than the minimum of males (*min* = 35) whereas the maximum score of the females (*max* = 64) was smaller than the maximum of males (*max* = 69). However, in the posttest both minimum score (48) and maximum score (69) for male and female pre-service teachers were the same. In addition, the skewness and kurtosis values in the Table 4.3 indicated that the distribution of both males' and females' pretest, posttest, and gain scores were normal.

Furthermore, the descriptive statistics of the scores of each subscale of the test were explored regarding gender, and shown in Table 4.4.

Table 4.4 Pretest, Posttest, and Gain Scores in the Subscales by Gender

| | | Pretest | | Posttest | | Gain | |
|---|---|---|---|---|---|---|---|
| | | Male | Female | Male | Female | Male | Female |
| | N | 18 | 25 | 18 | 25 | 18 | 25 |
| Number Concepts and Operation Subscale | Mean | 16.83 | 16.92 | 19.39 | 19.96 | 2.56 | 3.04 |
| | Std. Deviation | 4.00 | 2.56 | 3.07 | 2.61 | 3.40 | 2.49 |
| | Minimum | 10.00 | 12.00 | 14.00 | 14.00 | -3.00 | -2.00 |
| | Maximum | 24.00 | 21.00 | 23.00 | 24.00 | 9.00 | 7.00 |
| | Skewness | -0.37 | -0.35 | -0.54 | -0.80 | 0.35 | -0.40 |
| | Kurtosis | -0.59 | -0.84 | -1.18 | 0.16 | -0.72 | -0.88 |
| Geometry Subscale | Mean | 13.89 | 13.08 | 15.17 | 15.88 | 1.28 | 2.80 |
| | Std. Deviation | 2.68 | 2.50 | 2.50 | 2.15 | 2.76 | 2.14 |
| | Minimum | 7.00 | 9.00 | 11.00 | 11.00 | -2.00 | 0.00 |
| | Maximum | 18.00 | 17.00 | 18.00 | 19.00 | 9.00 | 8.00 |
| | Skewness | -0.74 | -0.12 | -0.57 | -0.85 | 1.39 | 1.00 |
| | Kurtosis | 1.28 | -1.04 | -1.08 | -0.09 | 2.27 | 0.30 |
| Algebra Subscale | Mean | 22.67 | 20.40 | 24.94 | 24.00 | 2.28 | 3.60 |
| | Std. Deviation | 3.94 | 4.07 | 2.90 | 3.51 | 3.86 | 3.86 |
| | Minimum | 13.00 | 15.00 | 20.00 | 17.00 | -4.00 | -3.00 |
| | Maximum | 28.00 | 27.00 | 32.00 | 30.00 | 12.00 | 12.00 |
| | Skewness | -1.04 | 0.45 | 0.81 | -0.34 | 1.00 | 0.26 |
| | Kurtosis | 0.78 | -1.21 | 0.76 | -0.43 | 1.40 | -0.41 |

In the Number Concepts and Operation Subscale, the mean score of female pre-service teachers ($M$ = 16.92, $SD$ = 2.56) was higher than male pre-services' ($M$ = 16.83, $SD$ = 4.00) in the pretest, while their mean scores were very close to each other in the posttest ($M_{female}$ = 19.96, $SD_{female}$ = 2.61; $M_{male}$=19.39, $SD_{male}$ = 3.07). The average gain scores indicated that the percentages of the increase in the scores from the pretest to posttest was higher in females which is 10.48% ($M$ = 3.04, $SD$ = 2.49) than in males which was 8.83% ($M$ = 2.56¸ $SD$ = 3.40).

Similar to the total pretest scores, the minimum score of females ($min$ = 12) was higher than the minimum score of males ($min$ = 10) in the NCOP Subscale in the pretest. On the contrary, the maximum score of females ($max$ = 21) was smaller than the maximum of males (*max* = 24) in the pretest. The posttest scores of the males and females indicated that their minimum score in NCOP Subscale were 14. On the other hand, female pre-service teachers (*max* = 24) have a slightly higher maximum score than males (*max* = 23) in the posttest.

In the Geometry Subscale, it was observed that the mean scores of males and females were very close to each other both in the pretest ($M_{\text{male}}$ = 13.89, $SD_{\text{male}}$ = 2.68; $M_{\text{female}}$ = 13.08, $SD_{\text{female}}$ = 2.50) and posttest ($M_{\text{male}}$ = 15.17, $SD_{\text{male}}$ = 2.50; $M_{\text{female}}$=15.88, $SD_{\text{female}}$ = 2.15). However, while the mean score of males was slightly higher in the pretest, the mean score of females was higher in the posttest. Both minimum and maximum scores of males and females also showed a slight increase. In more detail, males' minimum score went up from 7 to 11, and the maximum score stayed same (*max* = 18). On the other hand, female pre-service teachers rose their minimum score from 9 to 11, and their maximum score from 17 to 19. All these results indicated that the improvement of female pre-service teachers in the Geometry Subscale was larger than that of males. The gain scores showed similar results such that males increased their scores 6.74% (*M* = 1.28, *SD* = 2.76) from pretest to posttest while females increased 14.74% (*M* = 2.80, *SD* = 2.14).

In Algebra Subscale, the female pre-service teachers (*M* = 20.40, *SD* = 4.07) performed lower than males (*M* = 22.67, *SD* = 3.94) in the pretest. This situation was preserved in the posttest with a slight difference between the mean score of males (*M* = 24.94, *SD* = 2.90) and females (*M* = 24.00, *SD* = 3.51). Both males and females increased their minimum and maximum scores from pretest ($min_{\text{male}}$ = 13, $max_{\text{male}}$ = 28; $min_{\text{female}}$ = 15, $max_{\text{female}}$ = 27) to posttest ($min_{\text{male}}$ = 20, $max_{\text{male}}$ = 32; $min_{\text{female}}$ = 17, $max_{\text{female}}$ = 30) as well. This improvement can also be seen in the gain scores in Table 4.4. More specifically, the average gain score of male pre-service teachers were 2.28 (*SD* = 3.86), reflecting 6.51% improvement, whereas the females have a mean score of 3.60 (*SD* = 3.86) in the gain scores, reflecting a 10.29% improvement. This statistics indicated that female pre-service teachers showed more improvement than males in the Algebra Subscale, like in other subscales. However, this does not mean that males performed lower than females in the Algebra Subscale. In fact, male pre-service teachers had higher mean scores both in pretest and posttest. The result indicating that female pre-service teachers showed more improvement was due to the fact that they performed lower than males in the pretest.

Table 4.4 also provided the information about the distribution of the scores of males and females. When the skewness and kurtosis values are examined, there was only one violation which was on the male pre-service teachers' gain scores in the Geometry Subscale. Even though the skewness values, 1.39, could be acceptable for normal distribution, the kurtosis value, 2.27, exceeded the interval between -2 and 2. Yet, all skewness and kurtosis values seen in Table 4.4 can be accepted as tolerable for the normal distribution.

### 4.1.3. Descriptive Statistics Considering Participant's Elective Course Preferences

Pre-service teachers enrolled in the teacher education program at METU take six elective courses. These elective courses could be related to teaching mathematics, teaching other subject areas, or any interest areas like history of jazz or tennis. When the elective courses taken by participants were examined, it was seen that some of the participants took elective courses related to mathematics teaching while some others did not. As Figure 4.1 indicates, 29 (67%) pre-service teachers took one or more elective courses related to mathematics teaching like problem solving in mathematics, whereas 14 (33%) of them preferred elective courses related to educational areas or other subject areas like environmental education or history of music.

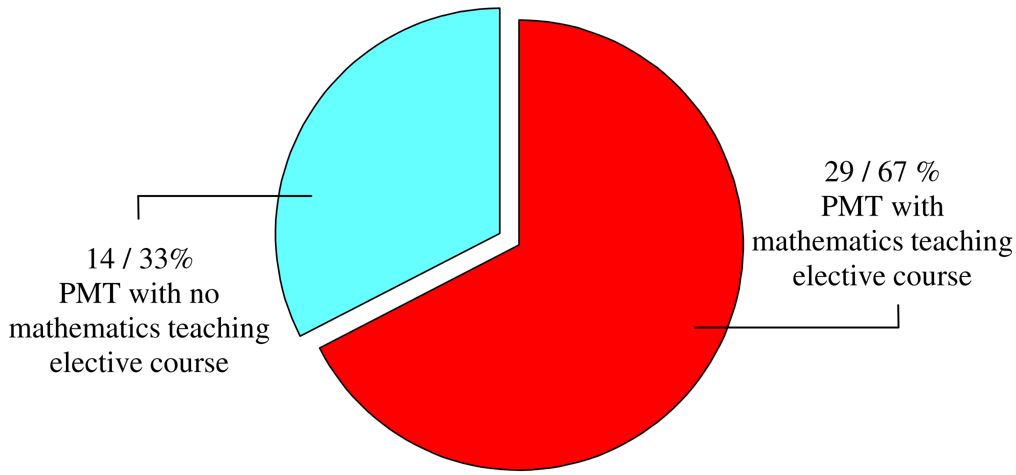


Figure 4.1 Distribution of the participants by their preferences of elective courses

When the transcripts of the pre-service teachers participated in this study are examined, it was found that mainly the following seven courses related to mathematics teaching had been chosen: (1) SSME 440-Teaching of Geometry Concepts, (2) SSME 486-Problem Solving in Mathematics, (3) SSME 550-Technology in Mathematics Education, (4) SSME 456-Laboratory Applications in Mathematics Teaching, (5) ELE 430-Exploring Geometry with Dynamic Geometry Applications, (6) ELE 467-Creative Drama in Elementary Mathematics and Science Education, and (7) ELE 482-Projects in Elementary Science and Mathematics Education. The reason why these students take these courses might be that they support their knowledge about teaching mathematics. Table 4.5 showed the frequencies of pre-service teachers taking the elective courses related to mathematics teaching.

Table 4.5 Frequencies of Pre-service Teachers and Elective Courses Related to Mathematics Teaching

| Elective Courses Related with Mathematics Teaching | Number of Pre-service Teachers |
|---|---|
| Teaching of Geometry Concepts | 14 |
| Laboratory Applications in Mathematics Teaching | 9 |
| Problem Solving in Mathematics | 24 |
| Projects in Elementary Science and Mathematics Education | 4 |
| Technology in Mathematics Education | 1 |

Pre-service mathematics teachers were put into two groups considering their elective course preferences. The first group consisted of the pre-service mathematics teachers who took at least one mathematics teaching course as elective and the second group consisted of pre-service teachers who did not take any mathematics teaching course as elective. The descriptive statistics of the scores of the two groups were given in Table 4.6.

Table 4.6 Pretest, Posttest, and Gain Scores by Elective Course Preferences of Pre-service Teachers

| | Pretest | | Posttest | | Gain | |
|---|---|---|---|---|---|---|
| | Math Teaching Elective Course | No Math Teaching Elective Course | Math Teaching Elective Course | No Math Teaching Elective Course | Math Teaching Elective Course | No Math Teaching Elective Course |
| N | 29 | 14 | 29 | 14 | 29 | 14 |
| Mean | 51.59 | 51.79 | 59.83 | 59.43 | 8.24 | 7.64 |
| Std. Deviation | 8.30 | 6.41 | 6.62 | 6.44 | 5.63 | 5.97 |
| Minimum | 35.00 | 43.00 | 43.00 | 48.00 | -3.00 | -1.00 |
| Maximum | 69.00 | 62.00 | 71.00 | 69.00 | 21.00 | 18.00 |
| Skewness | -0.08 | 0.24 | -0.73 | -0.13 | 0.37 | 0.33 |
| Kurtosis | -0.61 | -1.33 | 0.19 | -0.54 | 0.34 | -0.71 |

The descriptive statistics seen in Table 4.6 showed that the mean scores of pre-service teachers who took at least one mathematics teaching course as an elective ($M_{\text{pre}} = 51.59$, $SD_{\text{pre}} = 8.30$; $M_{\text{post}} = 59.83$, $SD_{\text{post}} = 6.62$) and pre-service teachers who did not take any mathematics teaching course ($M_{\text{pre}} = 51.79$, $SD_{\text{pre}} = 6.41$; $M_{\text{post}} = 59.43$, $SD_{\text{post}} = 6.44$) were nearly same both in the pretest and posttest. Both groups of pre-service mathematics teachers performed better in the posttest. When the minimum and maximum scores of groups were examined, it was observed that pre-service teachers who did not took any mathematics teaching course as elective (*min* = 43, *max* = 62) had higher minimum score and lower maximum score than those who took at least one mathematics teaching elective course (*min* = 35, *max* = 69) in the pretest. In the posttest, both groups increased their minimum and maximum scores. Still, pre-service teachers who did not take any mathematics teaching course as an elective (*min* = 48, *max* = 69) had higher minimum score and lower maximum score than those who took at least one mathematics teaching elective course (*min* = 43, *max* = 71) in the posttest. The average gain scores also indicated that both pre-service teachers taking at least one teaching mathematics elective course (*M* = 8.24, 9.93%, *SD* = 5.63) and pre-service teachers not taking any mathematics teaching elective course (*M* = 7.64, 9.20%, *SD* = 5.97) showed nearly equal improvement.

Furthermore, the skewness and kurtosis values were used to interpret the distribution of pretest, posttest, and gain scores for both groups of pre-service teachers. The values in Table 4.6 indicated that the pretest, posttest, and gain scores were approximately normally distributed.

Beside the descriptive statistics about the scores on the total test, the scores in each subscale of the test were examined. Table 4.7 summarizes the basic descriptive statistics of pretest, posttest, and gain scores in the subscales of the test considering the elective courses taken by the pre-service mathematics teachers.

Table 4.7 Pretest, Posttest, and Gain Scores in the Subscales by Elective Course Preferences of Pre-service Teachers

| | | Pretest | | Posttest | | Gain | |
|---|---|---|---|---|---|---|---|
| | | Math Teaching Elective Course | No Math Teaching Elective Course | Math Teaching Elective Course | No Math Teaching Elective Course | Math Teaching Elective Course | No Math Teaching Elective Course |
| | N | 29 | 14 | 29 | 14 | 29 | 14 |
| Number Concepts and Operation Subscale | Mean | 17.10 | 16.43 | 19.69 | 19.79 | 2.59 | 3.36 |
| | Std. Deviation | 3.12 | 3.43 | 2.84 | 2.22 | 2.57 | 3.48 |
| | Minimum | 11.00 | 10.00 | 14.00 | 11.00 | -2.00 | -3.00 |
| | Maximum | 24.00 | 21.00 | 24.00 | 19.00 | 7.00 | 9.00 |
| | Skewness | -0.09 | -0.92 | -0.54 | -0.94 | -0.07 | -0.17 |
| | Kurtosis | -0.44 | -0.14 | -0.64 | 0.01 | -1.22 | -0.56 |
| Geometry Subscale | Mean | 13.45 | 13.36 | 15.83 | 15.07 | 2.38 | 1.71 |
| | Std. Deviation | 2.51 | 2.79 | 2.22 | 2.46 | 2.21 | 3.07 |
| | Minimum | 9.00 | 7.00 | 11.00 | 11.00 | -1.00 | -2.00 |
| | Maximum | 17.00 | 18.00 | 19.00 | 18.00 | 8.00 | 9.00 |
| | Skewness | -0.27 | -0.48 | -0.94 | -0.38 | 0.97 | 1.00 |
| | Kurtosis | -0.89 | 1.00 | 0.01 | -1.21 | 0.73 | 0.78 |
| Algebra Subscale | Mean | 21.03 | 22.00 | 24.31 | 24.57 | 3.28 | 2.57 |
| | Std. Deviation | 4.41 | 3.53 | 3.42 | 3.03 | 4.11 | 3.41 |
| | Minimum | 13.00 | 15.00 | 17.00 | 20.00 | -4.00 | -2.00 |
| | Maximum | 28.00 | 26.00 | 30.00 | 32.00 | 12.00 | 9.00 |
| | Skewness | 0.11 | -0.84 | -0.48 | 1.23 | 0.40 | 0.87 |
| | Kurtosis | -1.31 | -0.24 | -0.31 | 1.91 | -0.12 | 0.00 |

In the Number Concepts and Operation Subscale, pre-service teachers who took at least one mathematics teaching course as an elective ($M$ = 17.10, $SD$ = 3.12) performed slightly better than the ones who did not take any elective course related to mathematics teaching ($M$ = 16.43, $SD$ = 3.43) in the pretest. On the other hand, the mean scores of the two groups were very close to each other in the posttest. The means of the gain scores showed that the improvement of pre-service teachers who did not take any elective course related with mathematics teaching ($M$ = 3.36, 11.59%, $SD$ = 3.48) was higher than the ones who took at least one elective course related with mathematics teaching ($M$ = 2.59, 8.93%, $SD$ = 2.57). On the other hand, pre-service teachers who took teaching mathematics courses ($M_{pre}$ = 13.45, $SD_{pre}$ = 2.51; $M_{post}$ = 15.83, $SD_{post}$ = 2.22) and who did not take any of those courses ($M_{pre}$ = 13.36, $SD_{pre}$ = 2.79; $M_{post}$ = 15.07, $SD_{post}$ = 2.46) performed nearly same both in pretest and posttest in the Geometry Subscale. Also the gain scores showed that the pre-service teachers who took at least one mathematics teaching course as an elective ($M$ = 2.38, 12.5%, $SD$ = 2.21) showed more improvement than those who did not take any mathematics teaching elective course ($M$ = 1.71, 9%, $SD$ = 3.07).

On the contrary to other subscales, in the Algebra Subscale the mean score of pre-service teachers who did not take elective course related with mathematics teaching ($M_{pre}$ = 22.00, $SD_{pre}$ = 3.53; $M_{post}$ = 24.57, $SD_{post}$ = 3.03) were slightly higher than pre-service teachers who took at least one mathematics teaching course ($M_{pre}$ = 21.03, $SD_{pre}$ = 4.41; $M_{post}$ = 24.31, $SD_{post}$ = 3.42) both in the pretest and posttest. Also the mean of the gain scores of pre-service teachers who took at least one mathematics teaching course as an elective ($M$ = 3.28, 9.37%, $SD$ = 4.11) was higher than the mean score of those who did not take any elective course related to mathematics ($M$ = 2.57, 7.34%, $SD$ = 3.41). This result indicated that pre-service teachers who took at least one elective course related with mathematics teaching improved their content knowledge for teaching algebra more than others.

In addition to these descriptive statistics about the scores on each subscale, Table 4.7 provided skewness and kurtosis values of distributions of scores. Based on these values, it could be said that there is no violation about normal distribution of the scores on each subscale of the test.

### 4.2. Inferential Statistics

The results of the paired sample t-tests, independent sample t-tests regarding gender difference and elective course differences of the pre-service mathematics teachers and bivariate correlations are presented with the corresponding research questions below. Because of the small sample size, t- tests are conducted, rather than MANOVA and ANOVA. Also, adjusted alpha levels are used in appropriate conditions to reduce the possibility of Type I error.

#### 4.2.1. The Effect of the Mathematics Teaching Methods Course on Content Knowledge for Teaching Mathematics

To investigate whether there is a significant effect of mathematics teaching methods course on the pre-service mathematics teachers' (PMTs') content knowledge for teaching mathematics (CKTM), paired samples t-tests were utilized. The results of these tests are presented with research questions in the following paragraphs.

##### 4.2.1.1. Research Question 1

*Is there a significant change in the pre-service elementary mathematics teachers' SETM total scores following their participation in the mathematics teaching methods course?*

A paired samples t-test was conducted to evaluate the impact of the mathematics teaching methods course on pre-service teachers' content knowledge for teaching mathematics. Before conducting paired samples t-test, the normality assumption was verified such that the skewness and kurtosis values of the gain scores indicated normal distribution as stated in descriptive statistics. Table 4.8 shows the results of the paired samples t-test.

Table 4.8 Paired Samples t-test Results for the Difference between Pretest and Posttest

| | | Mean difference | Std. Deviation | t | df | Sig. (2-tailed) |
|---|---|---|---|---|---|---|
| Pair 1 | Total Pretest Score - Total Posttest Score | -8.05 | 5.68 | -9.29 | 42 | .000 |

The findings showed that there was a statistically significant increase in SETM total scores from pretest (*M* = 51.65, *SD* = 7.66) to posttest (*M* = 59.70, *SD* = 6.49), $t(42) = -9.29$, $p < .01$. The average gain score was found as 8.05.

The effect size interpreted by eta squared statistic which was computed by Formula 4.1 (Pallant, 2001).

$$\text{Eta squared} = \frac{t^2}{t^2 + N - 1} \tag{4.1}$$

The eta squared statistic was found as 0.67 which indicates a large effect size according to Cohen's (1977) guidelines for paired samples t-test. Moreover, the power of the t-test was calculated by G*Power, power analysis program, which was designed to calculate power of statistical tests used in social and behavioral research (Faul, Erdfelder, Lang, & Buchner, 2007). The program initially calculated the effect size according to Cohen's measures as .87 which indicates large effect size. Then, the power was computed as .99. The large effect size and high power indicated that there is a practical difference between the pretest and posttest scores of the pre-service teachers in addition to statistical significance.

In addition, paired samples t-tests were conducted to evaluate the impact of the mathematics teaching methods course on pre-service teachers' content knowledge for teaching mathematics related to each subscale of the SETM; Number Concepts and Operation Subscale, Geometry Subscale, and Algebra Subscale. To check the normality assumption, the skewness and kurtosis values of the gain scores on each subscale were examined. As stated in descriptive statistics, the difference scores on each subscale were accepted as normally distributed. Table 4.9 presented the results of the paired samples t-tests for each subscale.

Table 4.9 Paired Samples t-test for Each Subscale of the SETM

| | | Mean difference | Std. Deviation | t | df | Sig. (2-tailed) |
|---|---|---|---|---|---|---|
| Pair 1 | Pretest NCOP Score - Posttest NCOP Score | -2.84 | 2.88 | -6.46 | 42 | .000 |
| Pair 2 | Pretest GEO Score - Posttest GEO Score | -2.16 | 2.51 | -5.66 | 42 | .000 |
| Pair 3 | Pretest ALG Score – Posttest ALG Score | -3.05 | 3.87 | -5.16 | 42 | .000 |

Bonferroni approach to control for Type 1 error across the three pairs, a *p* value of less than .003 ( .01/ 3 = .003). Therefore, the results of the paired samples t-tests were analyzed according to the adjusted alpha level. According to this adjusted alpha level, there was a significant increase in the Number Concepts and Operation Subscale from the pretest ($M$ = 16.88, $SD$ = 3.20) to posttest ($M$ = 19.72, $SD$ = 2.79), $t(42) = -6.46$, $p < .003$. The mean difference (posttest score on NCOP – pretest score on NCOP) was 2.84. Also, the effect size computed by eta squared statistic (.50) was found as a large effect according to Cohen's (1977) guidelines. Similarly, there was a significant increase in the subscale of Geometry from pretest ($M$ = 13.42, $SD$ = 2.57) to posttest ($M$ = 15.58, $SD$ = 2.30), $t(42) = -5.66$, $p < .003$. The mean difference (posttest score on GEO – pretest score on GEO) was found as 2.16. The effect size computed by eta squared statistic (.43) was a large effect size according to Cohen's (1977) guidelines for paired samples t-test. In the Algebra Subscale, there was a significant increase from the pretest ($M$ = 21.35, $SD$ = 4.13) to posttest ($M$ = 24.40, $SD$ = 3.27), $t(42) = -5.16$, $p < .003$. The mean difference (posttest score on ALG – pretest score on ALG) was found as 3.05. The eta squared statistic (.39) indicated a large effect size. Even though the mean differences of each pairs were seemed close to each other, the increase in the mean scores from the pretest to posttest varied due to the difference of the number of the questions in each subscale. So, the mean differences of each subscale were converted into percentages to compare them accurately. The percentages of the average scores were computed as multiplying the mean difference of pairs by 100, and then dividing by the number of items in the corresponding subscale. Thus, the average gain scores was 9.79% in the NCOP Subscale, 11.37% in the GEO Subscale, and 8.71% in the ALG Subscale. This indicated that pre-service teachers showed the most improvement in the Geometry Subscale. Furthermore, the power of the test was found nearly .90 in each section. These results revealed that there was both statistically and practically significant difference between the pretest and posttest scores in each section due to the large effect size and power. Hence, it can be asserted that the mathematics teaching methods course contributed to pre-service mathematics teacher's content knowledge for teaching mathematics.

### 4.2.2. Pre-service Mathematics Teachers' Content Knowledge for Teaching Mathematics Considering Gender Difference

The descriptive statistics showed some slight differences between male and female pre-service teachers' scores on the SETM in Table 4.3. To explore the mean difference of pre-service teachers' CKTM improvement in the mathematics teaching methods course, gain scores were chosen as the dependent variable and independent samples t-tests were conducted for gain scores of male and female pre-service teachers. Moreover, independent-samples t-tests were employed for posttest scores to investigate whether there is a significant mean difference between male and female pre-service mathematics teachers' CKTM. The posttest scores were also chosen as the dependent variable for independent samples t-test since the posttest scores show the mathematical content knowledge of pre-service teachers who have taken mathematics teaching methods course. The results of the tests were presented with corresponding research questions below.

#### 4.2.2.1. Research Question 2

*Is there a significant mean difference in the total gain scores for male and female pre-service elementary mathematics teachers?*

An independent samples t-test was conducted to compare gain scores for males and females. Before conducting the independent samples t-test, the normality assumption was verified. As stated previously, the distributions of total gain scores of males and females were approximately normal. Table 4.10 presents the results of the independent samples t-test.

Table 4.10 Independent Samples t-test Results for Gain Scores with respect to Gender

| | | Levene's Test for Equality of Variances | | | | | |
|---|---|---|---|---|---|---|---|
| | | f | Sig. | Mean Difference | t | df | Sig.(2-tailed) |
| Total Gain Score | Equal variances assumed | 0.65 | 0.42 | -3.33 | -1.96 | 41 | 0.06 |
| | Equal variances not assumed | | | -3.33 | -1.92 | 34.25 | 0.06 |

The results indicated that the homogeneity of variance assumption was met at the 0.01 alpha level. Therefore, the corresponding values were considered to interpret the results. According to this, there was no significant difference in the gain scores for male pre-service mathematics teachers (*M* = 6.11, *SD* = 5.84) and female pre-service mathematics teachers [*M* = 9.44, *SD* = 5.24; $t(41) = -1.96$, $p = 0.06$]. The mean difference between males and females was found as 3.33. The magnitude of the differences in the means was determined by eta squared statistics (.07) computed by the Formula 4.2 (Pallant, 2001).

$$\text{Eta squared} = \frac{t^2}{t^2 + (N1 + N2 - 2)} \qquad (4.2)$$

According to Cohen's (1977) guidelines, the magnitude of the mean differences in males and females was moderate. This moderate effect size means that there is a slight difference between mean scores of males and females in terms of gain scores in practice, but this difference was not statistically significant.

Moreover, gender difference was examined for the gain scores of each subscale. Therefore, independent samples t-tests were conducted for each subscale and the results were analyzed based on the adjusted alpha level to control Type 1 error. Table 4.11 shows the results of the independent sample t-tests.

Table 4.11 Independent Samples t-test Results for Gain Scores of Each Subscale with respect to Gender

| | | Levene's Test for Equality of Variances | | | | | |
|---|---|---|---|---|---|---|---|
| | | f | Sig. | Mean Difference | t | df | Sig.(2-tailed) |
| Gain Score of NCOP | Equal variances assumed | 0.84 | 0.09 | -0.48 | -0.54 | 41 | 0.59 |
| | Equal variances not assumed | | | -0.48 | -0.51 | 29.56 | 0.61 |
| Gain Score of GEO | Equal variances assumed | 0.93 | 0.34 | -1.52 | -2.04 | 41 | 0.05 |
| | Equal variances not assumed | | | -1.52 | -1.95 | 30.81 | 0.06 |
| Gain Score of ALG | Equal variances assumed | 0.14 | 0.70 | -1.32 | -1.11 | 41 | 0.27 |
| | Equal variances not assumed | | | -1.32 | -1.11 | 36.78 | 0.27 |

As table 4.11 showed that the homogeneity of variance assumptions of the independent sample t-tests for each subscale were met. Since there are three subscales in the test, the adjusted alpha level was found as 0.03 by using Bonferroni approach ( .01\ 3 = .003). Considering the adjusted alpha level, there was no significant difference in the gain scores of Number Concepts and Operation Subscale for male pre-service mathematics teachers ($M$ = 2.56, $SD$ = 3.40) and female pre-service mathematics teachers [$M$ = 3.04, $SD$ = 2.49; $t(41)$ = -0.54, $p$ = 0.59]. Similarly, in the Geometry Subscale there was no significant difference in the gain scores for male pre-service mathematics teachers ($M$ = 1.28, $SD$ = 2.76) and female pre-service mathematics teachers [$M$ = 2.80, $SD$ = 2.14; $t(41)$ = -2.04, $p$ > 0.03]. Also, there was no statistically significant mean difference in the gain scores of

Algebra Subscale for male pre-service mathematics teachers (*M* = 2.28, *SD* = 3.86) and female pre-service mathematics teachers [*M* = 3.60, *SD* = 3.86; $t(41) = -1.11$, $p = 0.27$]. These results indicated that both males and females showed statistically same amount of improvement in three subscales of the SETM, Number Concepts and Operation, Geometry, and Algebra.

#### 4.2.2.2. Research Question 3

*Is there a significant mean difference in the posttest scores for male and female pre-service elementary mathematics teachers?*

An independent samples t-test was conducted to compare total posttest scores for male and female pre-service mathematics teachers. The normality of the distribution of posttest scores for males and females were met as stated in descriptive statistics. The results of independent samples t-test for posttest scores of males and females are shown in Table 4.12.

Table 4.12 Independent Samples t-test Results for Posttest Scores with respect to Gender

| | | Levene's Test for Equality of Variances | | | | | |
|---|---|---|---|---|---|---|---|
| | | f | Sig. | Mean Difference | t | df | Sig.(2-tailed) |
| Total Posttest Score | Equal variances assumed | 0.04 | 0.84 | -0.34 | -0.17 | 41 | 0.87 |
| | Equal variances not assumed | | | -0.34 | -0.17 | 36.73 | 0.87 |

Table 4.12 showed that equal variances are assumed in the independent samples t-test. According to the results of the independent samples t test, there was no significant mean difference in the posttest scores for male pre-service mathematics teachers (*M* = 59.50, *SD* = 6.57) and female pre-service mathematics teachers [*M* = 59.84, *SD* = 6.55; $t(41) = -1.17$, $p = .87$]. The mean difference between males and females was found as .34. The magnitude of the differences in the means was determined by eta squared statistics (.03) by the Formula 4.2. The eta squared

statistics indicated that the magnitude of the difference between males' and females' mean scores on the posttest was very small (Cohen, 1977). This means that there was no obvious difference between male and female pre-service teachers' CKTM in practice.

In addition, the difference between male and female pre-service teachers' CKTM in each subscale was explored. To achieve this purpose the independent samples t-tests were conducted for the posttest scores of the participants in each subscale. The results are presented in Table 4.13.

Table 4.13 Independent Samples t-test Results for the Posttest Scores of Each Subscale with respect to Gender

| | | Levene's Test for Equality of Variances | | | | | |
|---|---|---|---|---|---|---|---|
| | | f | Sig. | Mean Difference | t | df | Sig.(2-tailed) |
| Posttest Score of NCOP | Equal variances assumed | 1.69 | 0.20 | -0.57 | -0.66 | 41 | 0.51 |
| | Equal variances not assumed | | | -0.57 | -0.64 | 32.93 | 0.53 |
| Posttest Score of GEO | Equal variances assumed | 0.75 | 0.39 | -0.71 | -1.00 | 41 | 0.32 |
| | Equal variances not assumed | | | -0.71 | -0.99 | 33.19 | 0.33 |
| Posttest Score of ALG | Equal variances assumed | 1.09 | 0.30 | 0.94 | -0.93 | 41 | 0.36 |
| | Equal variances not assumed | | | 0.94 | -0.96 | 40.15 | 0.34 |

As seen in Table 4.13, the Levene's statistics for each subscale were higher than the alpha level. This indicated that the homogeneity of variance assumptions of the independent sample t-tests for each subscale was met. Bonferroni approach was used to control for Type 1 error across three subscales. With the Bonferroni method, the alpha level used in the independent sample t-tests divided by the number of the three since there was three subscales. Then the adjusted alpha level was found as .003. Thus, the results of the independent samples t-tests for the posttest scores of each subscale showed that there was no statistically significant difference in the posttest scores of Number Concepts and Operation Subscale for male pre-service mathematics teachers ($M$ = 19.39, $SD$ = 3.07) and female pre-service mathematics teachers [$M$ = 19.96, $SD$ = 2.61; $t$(41) = -0.66, $p$ = 0.51]. In Geometry Subscale there was no significant mean difference in the posttest scores for males ($M$ = 15.17, $SD$ = 2.50) and females [$M$ = 15.88, $SD$ = 2.15; $t$(41) = -1.00, $p$ = 0.32]. Similar with the other subscales, there was no statistically significant difference in the posttest scores of Algebra Subscale for male ($M$ = 2.94, $SD$ = 2.90) and female pre-service mathematics teachers [$M$ = 24.00, $SD$ = 3.51; $t$(41) = -0.93, $p$ = 0.36]. Thus, at the end of the mathematics teaching methods course, the amount of CKTM in Number Concepts and Operation, Geometry, and Algebra were statistically same for both male and female pre-service mathematics teachers.

#### 4.2.3. Pre-service Mathematics Teachers' Content Knowledge for Teaching Mathematics Considering Elective Course Preferences

As summarized in the descriptive statistics section, 29 (67%) pre-service teachers took at least one elective course related to mathematics teaching whereas 14 (33%) of them preferred taking elective courses related to other educational areas or other subject areas. An independent-samples t-test was conducted to investigate whether there is a significant mean difference between pre-service mathematics teachers' CKTM with respect to their elective course preferences. Pre-service teachers' posttest scores were chosen as the dependent variable in the t-test because posttest scores were reflecting pre-service teachers' CKTM at the end of the methods course. The research question and results of the test are presented below.

##### 4.2.3.1 Research Question 4

*Is there a significant mean difference in the posttest scores of pre-service elementary mathematics teachers with respect to their elective course preferences?*

An independent samples t-test was conducted to compare posttest scores of pre-service mathematics teachers regarding their elective course preferences. Prior to conducting independent samples t-test, the normality assumption was verified. As stated in descriptive statistics, the posttest scores of pre-service teachers according to their elective course preferences were approximately normally distributed. Table 4.14 summarizes the results of the independent samples t-test.

Table 4.14 Independent Samples t-test Results with respect to Elective Course Preferences

| | | Levene's Test for Equality of Variances | | | | | |
|---|---|---|---|---|---|---|---|
| | | f | Sig. | Mean Difference | t | df | Sig. (2-tailed) |
| Total Posttest Score | Equal variances assumed | 0.02 | 0.89 | 0.40 | 0.19 | 41 | 0.85 |
| | Equal variances not assumed | | | 0.40 | 0.19 | 26.43 | 0.85 |

The findings indicated that the homogeneity of variance assumption was met at 0.01 alpha level. Therefore, the corresponding values were considered to interpret the results. As it is seen in the table, there was no significant difference in the posttest scores of pre-service teachers who took at least one mathematics teaching elective course ($M$ = 59.83, $SD$ = 6.62) and who did not take any mathematics teaching course as elective [$M$ = 59.43, $SD$ = 6.44; $t(41)$ = .19, $p$ = .85]. The mean difference between the two groups of pre-service teachers was found as 0.40. The magnitude of the differences in the means was very small. This small difference indicated that both pre-service teachers who took at least one elective course related with mathematics and who did not take any elective course related with mathematics teaching were nearly at the same level in terms of content knowledge for teaching mathematics.

### 4.2.4. Relationship between Academic Achievement on Mathematics Content Courses and Content Knowledge for Teaching Mathematics

It was found reasonable to investigate the relationship between the pre-service teachers' academic achievement on mathematics content courses and their CKTM since the test measures pre-service teachers' mathematical content knowledge for teaching. Pre-service teachers' academic achievements on mathematics content courses were determined by computing the mean of their grades on nine mathematics content courses; namely, Fundamentals of Mathematics, Calculus I, Calculus II, Introductory Discrete Mathematics, Analytical Geometry, Elementary Geometry, Basic Algebraic Structures, Introduction to Differential Equations, and Linear Algebra. To compute the average grades on mathematics content courses, pre-service teachers' grades were converted into numerical form such that AA, BA, BB, CB, CC, DC, DD, FD, and FF correspond to 4.00, 3.50, 3.00, 2.50, 2.00, 1.50, 1.00, 0.50, and 0.00, respectively. Then the mean of these grades of mathematics courses was calculated. To investigate whether there was a relationship between pre-service mathematics teachers' content knowledge for teaching mathematics and their average grade of mathematics content courses, Pearson product-moment correlations were conducted for the posttest scores and the average grade on the mathematics content courses.

#### 4.2.4.1. Research Question 5

*Is there a relationship between pre-service mathematics teachers' posttest scores and their academic achievement on mathematics content courses?*

The relationship between pre-service teachers' posttest scores on SETM and their academic achievement on mathematics content courses was investigated by utilizing Pearson product-moment correlation coefficient. Before calculating Pearson product-moment correlation, preliminary analyses were performed to ensure that there was no violation of the assumptions of normality, linearity and homoscedasticity. As it was mentioned in the Descriptive Statistics section in Table 4.1, the distribution of total posttest scores was approximately normal based on the skewness and kurtosis values. To check the normality of the distribution of pre-service teachers' average grades on mathematics content courses, basic descriptive statistics were explored. Table 4.15 presents the descriptive statistics of pre-service teachers' average grades on mathematics content courses.

Table 4.15 Pre-service Mathematics Teachers' Average Grades on Mathematics Content Courses

| | Average Grades on Math Courses |
|---|---|
| Mean | 2.01 |
| Std. Deviation | 0.47 |
| Minimum | 1.13 |
| Maximum | 3.50 |
| Skewness | 0.35 |
| Kurtosis | 1.25 |

The skewness and kurtosis values indicated that pre-service mathematics teachers' average grades on mathematics content courses were approximately normally distributed.

Another assumption which should be checked was linearity. To be more specific, in order to calculate correlation coefficients accurately, the relationship between the two variables is required to be linear (Pallant, 2001). Figure 4.2 shows the scatterplot generated to investigate the relationship between the posttest scores and average grades on mathematics content courses.

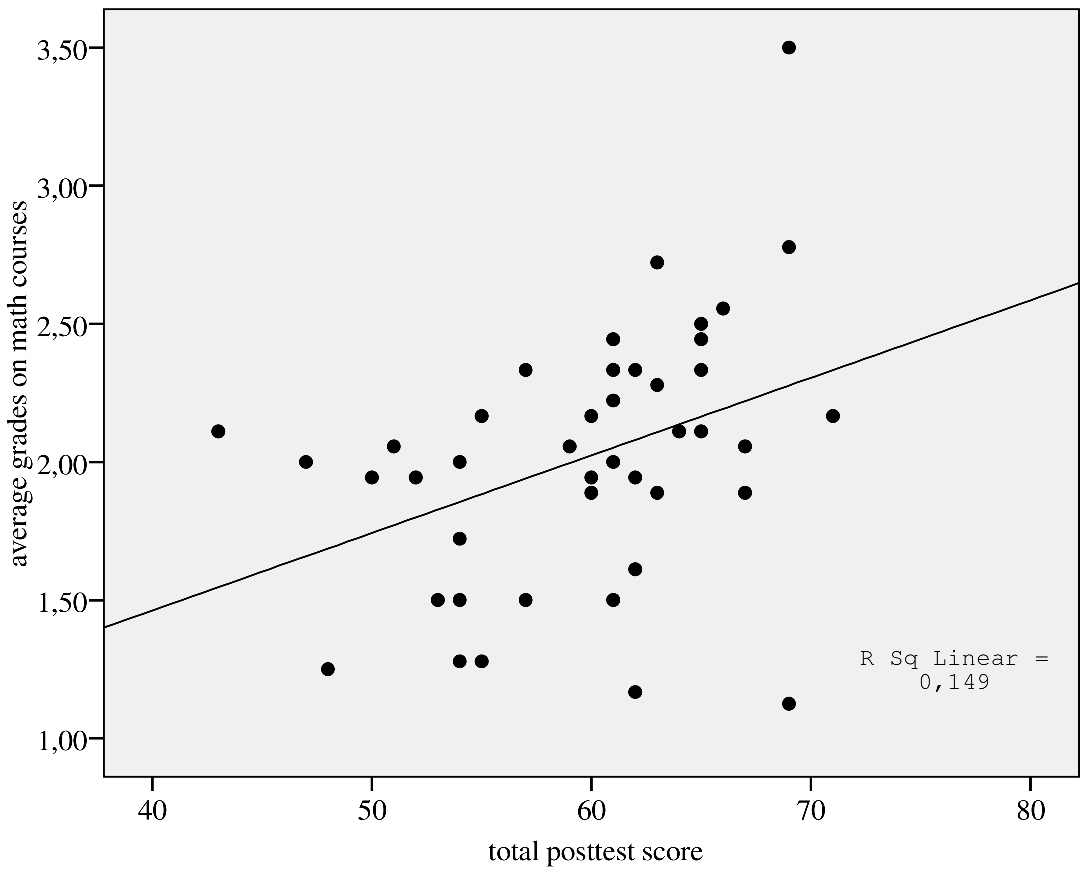


Figure 4.2 Scatterplot of posttest scores and Average Grades on Mathematics Content Courses

The distribution of the scores on the scatterplot showed that the relationship between the variables was linear. In addition, it was shown in the scatterplot that the scores are almost evenly spread in a cigar shape. This indicated that the homoscedasticity assumption was also met (Pallant, 2001). The results of Pearson product-moment correlation are presented in Table 4.16.

Table 4.16 Results of the Bivariate Correlations of Posttest Scores and Average Grades on Mathematics Content Courses

| | | Average Grades on Math Courses | Total Posttest Score |
|---|---|---|---|
| Average Grades on Math Courses | Pearson Correlation | 1 | 0.39(*) |
| | Sig. (2-tailed) | | 0.01 |
| | N | 43 | 43 |
| Total Posttest Score | Pearson Correlation | 0.39(*) | 1 |
| | Sig. (2-tailed) | 0.01 | |
| | N | 43 | 43 |

The results revealed that there was a significant medium positive correlation between pre-service mathematics teachers' total posttest scores and average grades on mathematics content courses [$r$ = .39, $n$ = 43, $p$ < .05], with high academic achievement on mathematics content courses with high total posttest score. The pre-service teachers' average grades on mathematics content courses explain nearly 15 percent of the variance in participants' total posttest scores on the SETM ($r^2 = 14.89$).

Hence, the results indicated that pre-service teachers' academic achievement on mathematics content courses are positively correlated with content knowledge for teaching mathematics. More specifically, pre-service teacher having high grades on mathematics content courses has more content knowledge for teaching mathematics.

## 4.3. Summary

The results of the statistical analyses explored the impact of the mathematics teaching methods course on pre-service mathematics teachers' content knowledge improvement for teaching mathematics. The findings of the study showed that a well-organized mathematics teaching methods course could improve pre-service mathematics teachers' CKTM. In the present study, improvements of pre-service teachers' content knowledge was observed in each subscale; Number Concepts and Operation, Geometry, and Algebra. More specifically, pre-service mathematics teachers improved their content knowledge for teaching mathematics by nearly 10% at the end of the mathematics teaching methods course. Even though the rate of this improvement can be seen as small, the difference between pretest and posttest scores was found as statistically and practically significant. Also, the increase in the mean scores was 9.79% in the NCOP Subscale, 11.37% in the GEO Subscale, and 8.71% in the ALG Subscale. This indicated that pre-service teachers showed the most improvement in Geometry Subscale.

Furthermore, the mathematical content knowledge improvement of female pre-service teachers was not different from the male pre-service teachers. This shows that the mathematics teaching methods course contributed to females' and males' CKTM nearly equally. When the CKTM of pre-service teachers who have taken the mathematics teaching methods course are considered almost ready to teach compared in terms of gender, it was found that both male and female pre-service teachers were at the same level of CKTM. Similarly, the mathematical content knowledge of pre-service teachers did not differ in terms of their preferences of elective courses related with mathematics teaching. That is, pre-service teachers who took elective courses related to mathematics teaching and who did not take one of those elective courses had approximately similar amount of CKTM at the end of the methods course.

In addition, the findings of the study showed that pre-service mathematics teachers' level of CKTM is significantly related with their academic achievement on mathematics content courses. This result indicated that pre-service teachers who were successful in mathematics content courses would have more mathematical content knowledge for teaching.

Another conclusion of this study was about the relationship between academic achievement on English courses and pretest and posttest scores of pre-service teachers. Since there was no relation between pre-service teachers' CKTM and their average grades on English courses, it can be inferred that their scores on SETM measuring mathematical content knowledge was not related to pre-service teachers' competencies in English. This is a finding which strengthens the construct validity of the test.

In conclusion, the results of the study evinced the effect of mathematics teaching methods course on pre-service mathematics teacher' CKTM, the differences between pre-service teachers in terms of CKTM, and the variables which are related with pre-service teachers' CKTM improvement.

# CHAPTER V

# DISCUSSION, IMPLICATIONS AND RECOMMENDATIONS

The main purpose of this study is to explore the effect of a mathematics teaching methods course on pre-service teachers' content knowledge for teaching mathematics. The research questions have been investigated by utilizing the statistical tests in the previous chapter. This chapter deals with reasoning about the results of the study and comparing those of the studies in the literature, implications of the study, and recommendations for practice and further studies.

## 5.1. Discussion

The results of the study indicated that the mathematics teaching methods course has potential to influence pre-service teachers' content knowledge for teaching mathematics positively. More specifically, pre-service teachers' pretest scores on SETM measuring content knowledge for teaching mathematics significantly increased at the end of the method course as seen in the posttest scores and gain scores. Furthermore, the positive change was not only observed in the total scores of SETM, but also similar improvement of mathematical content knowledge for teaching occurred in each subscale of SETM; namely, Number Concepts and Operation, Geometry, and Algebra.

It can be inferred from the results of the study that pre-service teachers' knowledge about solving a mathematical problem in different ways, carrying out the algorithms correctly, evaluating students' solutions appropriately, representing mathematical expression in multiple ways, using physical materials in mathematical expressions or operations appropriately and accurately, and explaining the meaning of the mathematical concepts, procedures and algorithms, which were measured by SETM showed significant improvement at the end of the methods course (Hill, Schilling, & Ball, 2004). More briefly, the mathematics teaching methods course influenced pre-service mathematics teachers' knowledge related to the mathematics that they will teach at elementary schools. It might be because that the methods course dealt with most of the mathematical topics that pre-service teachers will teach in elementary schools, like Numbers and Operations, Algebra, Geometry, and Measurement. Moreover, pre-service teachers' have experienced discussing the mathematical tasks conceptually, representing mathematical operations or expressions in multiple ways, evaluating students' solution strategies, and handling with students' misconceptions in each mathematical topic in the methods course. Therefore, pre-service teachers could have gained both common knowledge of content and specialized knowledge of content for teaching mathematics in this course.

Furthermore, the literature includes many studies the results of which support the findings of the present study (Ball, 1989; Hill, 2007; Kinach, 2002; Moss, 2006; Quinn, 1997). These studies emphasized the role of methods courses in developing pre-service teachers' mathematical content knowledge. To be more specific, Moss (2006) reported that pre-service teachers' content knowledge for teaching mathematics related to Number and Operation, and Algebra showed significant improvement at the end of the methods course. Also, the results of Moss's study revealed that pre-service teachers' specialized understanding of mathematics in Geometry was better than Number and Operation. The findings of the present study are in agreement with Moss' study such that Moss's study has shown the significant difference between total pretest and posttest scores, like the present study. Similarly, the improvement of pre-service teachers' mathematical content knowledge for teaching in the Geometry Subscale (11.37 %) was found higher than the improvement of in the Number Concepts and Operation Subscale (9.79%). In another study, Hill (2007) put forward that the middle school teachers who took mathematics courses and mathematics methods courses performed better on the test, one of the CKTM measures. By considering this argument as an indicator of role of methods course on content knowledge development, this result can be viewed as another agreement with the results of the present study.

Besides, the literature does not include enough quantitative studies investigating teacher knowledge. One of these quantitative studies was Moss's

(2006) study. Moss has found a statistically significant improvement on PMTs' CKTM following the methods course. However, the statistical analysis has revealed small effect size in Moss's study. In this aspect, the present study has an important contribution to the literature by presenting the effect of mathematics teaching methods course on CKTM with a large effect size and large power even the sample size was small. Thus, a positive effect of methods courses on pre-service teachers' CKTM exists and the magnitude of this effect is so large that it has importance for the practice of teacher education.

Even the change of pre-service teachers' CKTM was found statistically significant, there might be some other factors enhancing pre-service teachers' CKTM, rather than the mathematics teaching methods course itself. For instance, the field experience of pre-service teachers might influence their mathematical content knowledge for teaching. However, pre-service teachers did not have considerable teaching opportunities in the field experiences. They mostly observed the teachers and students in the course of School Experience II. Moreover, the pre-service elementary mathematics teachers took the course of ELE 420 (Practice Teaching in Elementary Education) in the second semester. Since the posttest was administered on March in the second semester, it would be better to consider whether this course influenced pre-service teachers' CKTM. Yet, in the first two weeks of the course the required permissions were obtained from the Ministry of National Education and the schools which the pre-service teachers will go for student teaching. Thus, pre-service teachers started their field experience after about a month, but attended a two-hour course in the Department of Elementary Mathematics Education during this period. This implies that pre-service teachers did not experience with student teaching until March. Therefore, the effect of field experience courses might not be considered as important.

Another factor having the potential to influence pre-service teachers' CKTM was the elective courses related to mathematics teaching. There were some suggestions made by other researchers about content courses. Işıksal (2006) suggested "content-pedagogy rich" courses to enhance pre-service teachers' content knowledge. Therefore, in the present study pre-service teachers were put into two groups according to their elective courses; first group consisted of the participants who took at least one elective course related to mathematics teaching and the second group included the ones who did not take any mathematics teaching course as an elective. However, the results of the present study did not verify the researchers' prediction since it was found that there was no statistically significant mean difference between their posttest scores. This result might have occurred due to several reasons. The first reason might be related to the sample size. Since the sample is small (N=43), the difference between means of the groups might not be large enough to be detected in the statistical analysis. Also, the pre-service teachers in the first group, who took at least one elective course related to mathematics teaching, did not take the same courses. For example, one pre-service teacher has taken only Teaching of Geometry Concepts course whereas another one has taken Problem Solving in Mathematics course. The number of mathematics teaching elective courses which was taken was not equal, either. For instance, while one pre-service teacher has taken only one mathematics teaching elective course, another one has taken three mathematics teaching elective courses. Moreover, pre-service teachers did not take these courses in the same semester or from the same instructors. Their grades taken from these courses were not the same. All these factors might probably influence the results. Another reason might be that the courses related to teaching mathematics might not really provide the opportunities to improve mathematical content knowledge for teaching. Lastly, pre-service teachers might not have got enough benefit about how to teach the subject matter enough in these courses. That is, they could not relate the pedagogical knowledge with mathematics which is taught at elementary schools. In spite of the fact that there were too many variables influencing the results of the statistical analysis, it was found worthwhile to investigate the differences between pre-service teachers regarding their elective courses as most of the past studies suggested. The result pointed out that pre-service teachers' elective courses related with mathematics teaching were not a statistically influential factor for CKTM improvement.

Another finding of the present study was that there was no significant difference between the male and female pre-service teachers in terms of their mathematical content knowledge improvement during the methods course. Even though most of the previous studies have not investigated gender difference in teacher knowledge, a few studies put forward that male pre-service teachers' mathematical knowledge were better than females' (Tobias & Itter, 2007; Verschaffel, Janssens, & Janssen, 2005). In the present study, it was expected that there might be mean differences in favor of females since female pre-service teachers participated into the class activities considerably higher than the male participants. However, no difference was found on pre-service teachers' mathematical content knowledge for teaching in posttest and gain scores, and in the Number Concepts and Operation, Geometry, and Algebra Subscale in terms of gender. This indicated that both male and female pre-service teachers have similar opportunities to improve their CKTM in mathematics teaching methods course. This conclusion can be interpreted that the activities, tasks and assignments used in the methods course did not favor neither males nor females. In other words, the instructor of the mathematics teaching methods course might have considered the equity principle in terms of gender in his instruction, and this helps all pre-service teachers have enough experiences to improve their mathematical content knowledge for teaching. As a result, male and female pre-service teachers completed the mathematics teaching methods course possessed nearly same amount of CKTM.

In addition to these, there are some studies in the literature which have emphasized the effect of mathematics content courses on pre-service teachers' content knowledge (Ay, 2004; Hill, 2007; Türnüklü, 2005). Ay (2004) has stated that mathematics content courses did not provide a connection within mathematical concepts, or between mathematical concepts and their real life applications. On the other hand, Hill (2007) found that pre-service teachers who took mathematics content courses and methods course have more mathematical content knowledge for teaching. In Türnüklü's (2005) study, the mean grades of mathematics content courses were used as an indicator of pre-service teachers' mathematical content knowledge. All these studies informed the researcher of the present study to investigate the relationship between pre-service teachers' academic achievement of the mathematics content courses and their content knowledge for teaching mathematics. The results of the present study are consistent with Hill’s and Türnüklü’s arguments that the academic achievement on mathematics content courses is significantly correlated with pre-service teachers’ posttest scores on the SETM. In contrast to Ay’s (2004) study, the results of the present study put forward that the mathematics content courses might have provided a deep insight into mathematics for pre-service teachers, and the pre-service teachers could have related this insight with pedagogical knowledge.

The mathematics content courses offered in the elementary mathematics education program include topics of advanced mathematics, but not the mathematics that pre-service teachers will teach. More specifically, the mathematics content courses which pre-service elementary mathematics teachers took are related to set theory, functions and composition of functions, basic counting, discrete probability, binary operations, limits and integrals, linear differential equations, Euclidean and non-Euclidean Geometry, fundamental principals of analytic geometry, and matrices, determinants and systems of linear equations, in general (Middle East Technical University, General Catalog, 2005). Even though these mathematical issues are more advanced than the mathematics which pre-service mathematics teachers will teach in elementary schools, they might have helped pre-service teachers think about the reasons and meanings of mathematical theorems and their way of proofing. This might have helped pre-service teachers understand the meaning of rules or algorithms and the reasons of why a particular procedure works. Thus, pre-service teachers might benefit from these courses to improve their mathematical content knowledge for teaching.

Furthermore, the results of the study pointed out that pre-service mathematics teachers’ content knowledge about Geometry improved more than Number Concepts and Operation, and Algebra. This improvement might be resulted from both the courses of Analytical Geometry and Elementary Geometry offered by the Department of Mathematics, and the elective course of Teaching of Geometry Concepts offered by the Department of Secondary Science and Mathematics Education. It might be because of the fact that the content of the Analytic Geometry and Elementary Geometry courses are related to elementary school mathematics than other mathematics content courses. To state more specifically, Analytic Geometry course includes Cartesian coordinates in plane and space, and translation and rotation in the plane, while the content of Elementary Geometry contains polygons, geometric solids, properties of circles and triangles, angle measurement in radians and degrees, trigonometric functions, and Phytagorean theorem (Middle East Technical University, General Catalog, 2005). Despite these mathematical issues are dealt with in an advanced way, all of them are taught in elementary schools. So, these courses might also affect the pre-service teachers' improvement of mathematical content knowledge for teaching Geometry. Also the course of Teaching of Geometry Concepts was taken some of the pre-service teachers as an elective course. Since development of geometric concepts and methods of teaching geometry emphasizing on active learning are interested in this course, it might also have influenced pre-service teachers' content knowledge improvement about teaching Geometry.

To sum up, the mathematics teaching methods course plays an important role in improving pre-service teachers' content knowledge for teaching mathematics when it is designed to help pre-service teachers understand the elementary school mathematics conceptually, and how to teach it. The methods courses allowing pre-service teachers to experience constructing discussion about mathematical concepts, algorithms and procedures, reasoning about underlying meaning of those, using materials appropriately to teach a mathematical topic, using multiple representations for a particular mathematical task, understanding elementary school students' (mis)conceptions in a particular mathematical topic, and evaluating students' solution strategies might improve pre-service mathematics teachers' content knowledge for teaching. Hence, mathematics teaching methods courses have an important role in elementary mathematics teacher education programs.

## 5.2. Recommendations and Implications

This study offers significant information to teacher educators, program developers, and policy makers on how pre-service mathematics teachers' content knowledge for teaching mathematics can be improved. Even though the study underlines that the mathematics teaching methods courses have potential to influence pre-service teachers' content knowledge for teaching mathematics from a statistical perspective, pre-service teachers need more opportunities to understand mathematics conceptually. The pre-service teachers' improvement of mathematical content knowledge is an important issue, because teachers who have more and deep mathematical content knowledge will teach mathematics more meaningfully (Moss, 2006).

To improve pre-service teachers' CKTM, there should be more emphasis on the quality of both content courses and mathematics teaching courses. The content of the mathematics teaching methods course have implied that pre-service teachers should have the opportunity to work in groups on a mathematical task, to discuss the reasons why a particular algorithm or rule works and meaning of the concepts and procedures, to experience different solution ways of elementary school students, to evaluate different solution methods, and to use physical materials and multiple representation while teaching a mathematical topic in order to enhance their content knowledge for teaching mathematics (Ball, 1991a; Işıksal, 2006). Besides, the mathematics teaching methods course might be divided into several courses regarding the subject areas of mathematics. For instance, there might be a geometry teaching methods course, algebra teaching methods course, and a probability and statistics teaching methods course. In this way, more time could be devoted for each subject area of school mathematics. Thus, pre-service teachers might have the opportunity of deeply studying each subject area. Also, these courses should be designed to make pre-service teachers aware of their deficiencies about mathematical content knowledge in mathematics content courses (Ball, 1989, 1991a; Ball & McDiarmid, 1990; Nakiboğlu & Karakoç, 2005).

Furthermore, the Department of Mathematics Education and the Department of Mathematics should collaborate with each other. This may provide teacher educators to know more about pre-service mathematics teachers' mathematical knowledge and use this information while designing their courses.

In addition to these, replications of the present study in other education faculties in Turkey or in other countries are recommended to determine whether the results will be similar. Thus, the students in the elementary mathematics teacher education programs in different faculties can be compared in terms of content knowledge for teaching mathematics. This might be important to make mathematics teacher education programs possess the same standards in Turkey. In replication studies, using both the experimental and control groups is highly suggested. Also, there is a need of longitudinal studies to investigate whether pre-service teachers could reflect their CKTM on their teaching, or whether the effect of mathematics teaching methods course on pre-service teachers' CKTM would be permanent or temporary.

Also, researchers can design experimental studies to compare the effect of several learning opportunities on pre-service teachers' mathematical content knowledge. To illustrate, the major activity might be watching videos in the mathematics teaching methods course for one group while the major activity of the methods course might be using physical materials for the second group. The researcher can compare the effect of these two learning opportunities on pre-service teachers' CKTM.

Finally, the Elementary Teacher Education Program has recently changed. The new program includes the courses related to content knowledge (50%), teaching profession knowledge and skills (30%), and the general cultural knowledge such as Principles of Kemal Atatürk I and II, Oral Communication, Written Communication, and Community Service (20%) (Higher Education Council, 2006). This indicates that the new program has remarkable emphasis on content and teaching knowledge. Since the sample of the present study includes the pre-service elementary mathematics teachers who follow the old program of Elementary Mathematics Teacher Education, conducting a similar study with the pre-service teachers who enroll in the new Elementary Teacher Education program might provide worthwhile information about the functions of the courses in the new Elementary Teacher Education Program.

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# APPENDICES

## APPENDIX A

### Elementary Mathematics Education Program

**FIRST YEAR**

| **First Semester** | | **Second Semester** | |
|---|---|---|---|
| MATH111 | Fundamentals of Mathematics | MATH 112 | Introductory Discrete |
| MATH119 | Calculus with Analytic Geometry | MATH 120 | Mathematics Calculus for Functions of Several Variables |
| PHYS 181 | Basic Physics I | PHYS 182 | Basic Physics II |
| ENG 101 | Development of Reading and Writing Skills I | ENG 101 | Development of Reading and Writing Skills II |
| EDS 119 | Introduction to Teaching Profession | ELE 132 | School Experience I |
| IS 100 | Introduction to Information Technologies and Applications | | |

**SECOND YEAR**

| **Third Semester** | | **Fourth Semester** | |
|---|---|---|---|
| MATH115 | Analytical Geometry | MATH 116 | Basic Algebraic Structures |
| MATH201 | Elementary Geometry | MATH 219 | Introduction to Differential Equations |
| CHEM283 | Introductory General Chemistry | BIO 106 | General Biology |
| EDS 221 | Development and Learning | ELE 224 | Instructional Planning and Evaluation |
| ENG 211 | Academic Oral Presentation Skills | ELE 300 | Computer Applications in Education |
| HIST 2201 | Principles of Kemal Atatürk I | HIST 2202 | Principles of Kemal Atatürk II |

**THIRD YEAR**

| **Fifth Semester** | | **Sixth Semester** | |
|---|---|---|---|
| MATH260 | Linear Algebra | ELE 240 | Probability and Statistics |
| ELE 317 | Instructional Development and Media in Mathematics | ELE 332 | Laboratory Applications in Science II |
| ELE 331 | Laboratory Applications in Science I | ELE 336 | Methods of Science and Mathematic Teaching |
| TURK 305 | Oral Communication | EDS 304 | Classroom Management |
| | Elective I | TURK 306 | Written Communication |
| | Elective II | | Elective III |

**FOURTH YEAR**

| **Seventh Semester** | | **Eighth Semester** | |
|---|---|---|---|
| ELE 437 | School Experience II | ELE 420 | Practice Teaching in Elementary Education |
| ELE 443 | Methods of Mathematics Teaching | ELE 448 | Textbook Analysis in Mathematics Education |
| ENG 311 | Advanced Communication Skills | EDS 424 | Guidance |
| | Elective IV | | Elective VI |
| | Elective V | | |

**APPENDIX B**

**The Released Items of the LMT Project**

**STUDY OF INSTRUCTIONAL IMPROVEMENT/
LEARNING MATHEMATICS FOR TEACHING**

# *MATHEMATICAL KNOWLEDGE FOR TEACHING (MKT) MEASURES*

## MATHEMATICS RELEASED ITEMS
## 2005

University of Michigan, Ann Arbor
610 E. University #1600 Ann
Arbor, MI 48109-1259 (734) 647-
5233
www.sitemaker.umich.edu/lmt

 Measures development supported by NSF grants REC-9979873, REC- 0207649, EHR-0233456 & EHR 0335411, and by a subcontract to CPRE on Department of Education (DOE), Office of Educational Research and Improvement (OERI) award #R308A960003.

April 12, 2007

Dear Colleague:

Thank you for your interest in our survey items measuring mathematical knowledge for teaching. To orient you to the items and their potential use, we explain their development, intent, and design in this letter.

The effort to design survey items measuring teachers' knowledge for teaching mathematics grew out of the unique needs of the *Study of Instructional Improvement* (SII). SII is investigating the design and enactment of three leading whole school reforms and these reforms' effects on students' academic and social performance. As part of this research, lead investigators realized a need not only for measures which represent school and classroom processes (e.g., school norms, resources, teachers' instructional methods) but also teachers' facility in using disciplinary knowledge in the context of classroom teaching. Having such measures will allow SII to investigate the effects of teachers' knowledge on student achievement, and understand how such knowledge affects program implementation. While many potential methods for exploring and measuring teachers' content knowledge exist (i.e., interviews, observations, structured tasks), we elected to focus our efforts on developing survey measures because of the large number of teachers (over 5000) participating in SII.

Beginning in 1999, we undertook the development of such survey measures. Using theory, research, the study of curriculum materials and student work, and our experience, we wrote items we believe represent some of the competencies teachers use in teaching elementary mathematics – representing numbers, interpreting unusual student answers or algorithms, anticipating student difficulties with material. With the assistance of the University of California Office of the President[1], we piloted these items with K-6 teachers engaged in mathematics professional development. This work developed into a sister project to SII, *Learning Mathematics for Teaching* (LMT). With funding from the National Science Foundation, LMT has taken over instrument development from SII, developing and piloting geometry and middle school items.

We have publicly released a small set of items from our projects' efforts to write and pilot survey measures. We believe these items can be useful in many different contexts: as open-ended prompts which allow for the exploration of teachers' reasoning about mathematics and student thinking; as materials for professional development or teacher education; as exemplars of the kinds of mathematics teachers must know to teach. We encourage their use in such contexts. <u>However, this particular set of items is, as a group, NOT appropriate for use as an overall measure, or scale, representing teacher knowledge.</u> In other words, one <u>cannot</u> calculate a teacher score that reliably indicates either level of content knowledge or growth over time.

We ask users to keep in mind that these items represent steps in the process of developing measures. In many cases, we released items that failed, statistically speaking, in our piloting; in these cases, items may contain small mathematical ambiguities or other imperfections. If you have comments or ideas about these items, please feel free to contact one of us by email at the addresses below.

These items are the result of years of thought and development, including both qualitative investigations of the content teachers use to teach elementary mathematics, and quantitative field trials with large numbers of survey items and participating teachers. <u>Because of the intellectual effort put into these items by SII investigators, we ask that *all* users of these items satisfy the following requirements:</u>

[1] Elizabeth Stage, Patrick Callahan, Rena Dorph, principals.Please request permission from SII for any use of these items. To do so, contact Geoffrey Phelps at gphelps@umich.edu. Include a brief description of how you plan to use the items, and if applicable, what written products might result.

1) In any publications, grant proposals, or other written work which results from use of these items, please cite the development efforts which took place at SII by referencing this document:

Hill, H.C., Schilling, S.G., & Ball, D.L. (2004) Developing measures of teachers' mathematics knowledge for teaching. Elementary School Journal 105, 11-30.

3) Refrain from using these items in multiple choice format to evaluate teacher content knowledge in any way (e.g., by calculating number correct for any individual teacher, or gauging growth over time). Use in professional development, as open-ended prompts, or as examples of the kinds of knowledge teachers might need to know is permissible.

You can also check the SII website (http://www.sii.soe.umich.edu/) or LMT website (http://www.sitemaker.umich.edu/lmt) for more information about this effort.

Below, we present three types of released item – elementary content knowledge, elementary knowledge of students and content, and middle school content knowledge. Again, thank you for your interest in these items.

Sincerely,

| | |
|---|---|
| Deborah Loewenberg Ball | Heather Hill |
| Dean, School of Education | Associate Professor |
| William H. Payne Collegiate Professor | Harvard Graduate School of Education |
| University of Michigan | |

**Study of Instructional Improvement/Learning Mathematics for Teaching**

Mathematical Knowledge for Teaching Measures
(MKT measures) Released Items, 2005
ELEMENTARY CONTENT KNOWLEDGE ITEMS

1. Ms. Dominguez was working with a new textbook and she noticed that it gave more attention to the number 0 than her old book. She came across a page that asked students to determine if a few statements about 0 were true or false. Intrigued, she showed them to her sister who is also a teacher, and asked her what she thought.

Which statement(s) should the sisters select as being true?
(Mark YES, NO, or I'M NOT SURE for each item below.)

| | Yes | No | I'm not sure |
|---|---|---|---|
| a) 0 is an even number. | 1 | 2 | 3 |
| b) 0 is not really a number. It is a placeholder in writing big numbers. | 1 | 2 | 3 |
| c) The number 8 can be written as 008. | 1 | 2 | 3 |

2. Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

| Student A | Student B | Student C |
|---|---|---|
| 35<br>x 25<br>1 25<br>+7 55<br>8 75 | 35<br>x25<br>175<br>+700<br>875 | 35<br>x 25<br>25<br>150<br>100<br>+600<br>875 |

Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

| | Method would work for all whole numbers | Method would NOT work for all whole numbers | I'm not sure |
|---|---|---|---|
| a) Method A | 1 | 2 | 3 |
| b) Method B | 1 | 2 | 3 |
| c) Method C | 1 | 2 | 3 |

3. Ms. Harris was working with her class on divisibility rules. She told her class that a number is divisible by 4 if and only if the last two digits of the number are divisible by 4. One of her students asked her why the rule for 4 worked. She asked the other students if they could come up with a reason, and several possible reasons were proposed. Which of the following statements comes closest to explaining the reason for the divisibility rule for 4? (Mark ONE answer.)

a) Four is an even number, and odd numbers are not divisible by even numbers.

b) The number 100 is divisible by 4 (and also 1000, 10,000, etc.).

c) Every other even number is divisible by 4, for example, 24 and 28 but not 26.

d) It only works when the sum of the last two digits is an even number.

4. Ms. Chambreaux's students are working on the following problem:

Is 371 a prime number?

As she walks around the room looking at their papers, she sees many different ways to solve this problem. Which solution method is correct? (Mark ONE answer.)

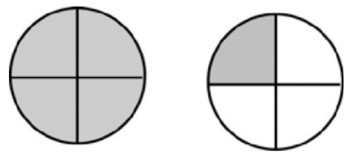
a) Check to see whether 371 is divisible by 2, 3, 4, 5, 6, 7, 8, or 9.

b) Break 371 into 3 and 71; they are both prime, so 371 must also be prime.

c) Check to see whether 371 is divisible by any prime number less than 20.

d) Break 371 into 37 and 1; they are both prime, so 371 must also be prime.

5. Mrs. Johnson thinks it is important to vary the whole when she teaches fractions. For example, she might use five dollars to be the whole, or ten students, or a single rectangle. On one particular day, she uses as the whole a picture of two pizzas. What fraction of the two pizzas is she illustrating below? (Mark ONE answer.)



a) 5/4

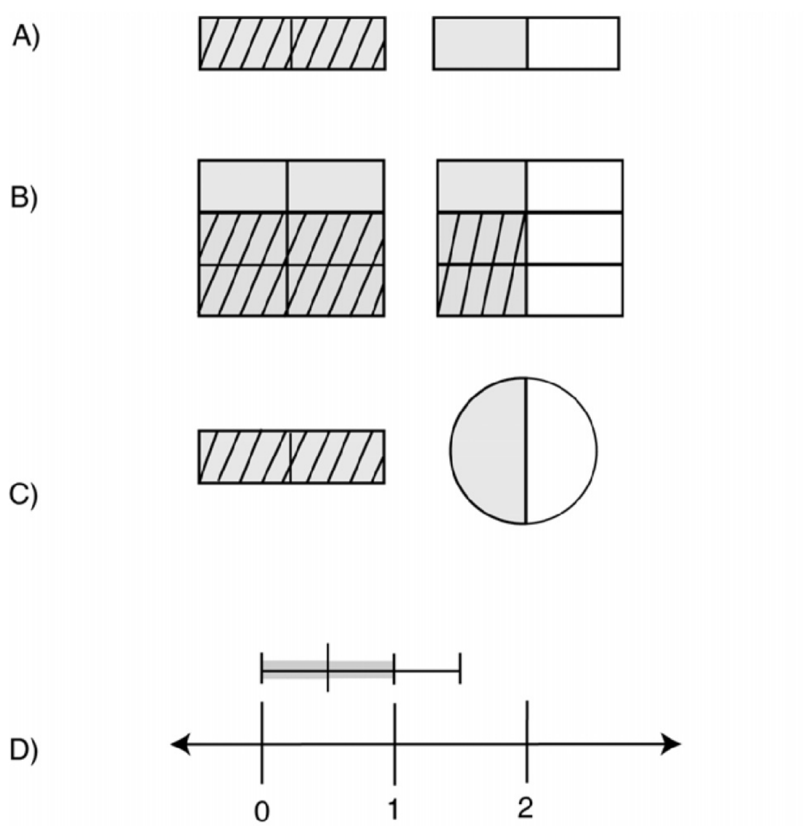
b) 5/3

c) 5/8

d) 1/4

6. At a professional development workshop, teachers were learning about different ways to represent multiplication of fractions problems. The leader also helped them to become aware of examples that do not represent multiplication of fractions appropriately.

Which model below cannot be used to show that $1\frac{1}{2} \times \frac{2}{3} = 1$? (Mark ONE answer.)



7. Which of the following story problems could be used to illustrate $1\frac{1}{4}$ divided by $\frac{1}{2}$ (Mark YES, NO, or I'M NOT SURE for each possibility.)

| | Yes | No | I'm not sure |
|---|---|---|---|
| a) You want to split $1\frac{1}{4}$ pies evenly between two families. How much should each family get? | 1 | 2 | 3 |
| b) You have $1.25 and may soon double your money. How much money would you end up with? | 1 | 2 | 3 |
| c) You are making some homemade taffy and the recipe calls for $1\frac{1}{4}$ cups of butter. How many sticks of butter (each stick = $\frac{1}{2}$ cup) will you need? | 1 | 2 | 3 |

8. As Mr. Callahan was reviewing his students' work from the day's lesson on multiplication, he noticed that Todd had invented an algorithm that was different from the one taught in class. Todd's work looked like this:

$$
\begin{array}{r}
983 \\
\underline{\times\ \ 6} \\
488 \\
\underline{+5410} \\
5898
\end{array}
$$

What is Todd doing here? (Mark ONE answer.)

a) Todd is regrouping ("carrying") tens and ones, but his work does not record the regrouping.

b) Todd is using the traditional multiplication algorithm but working from left to right.

c) Todd has developed a method for keeping track of place value in the answer that is different from the conventional algorithm.

d) Todd is not doing anything systematic. He just got lucky – what he has done here will not work in most cases.

9. Mr. Garrett's students were working on strategies for finding the answers to multiplication problems. Which of the following strategies would you expect to see some elementary school students using to find the answer to 8 x 8? (Mark YES, NO, or I'M NOT SURE for each strategy.)

| | Yes | No | I'm not sure |
|---|---|---|---|
| a) They might multiply 8 x 4 = 32 and then double that by doing 32 x 2 = 64. | 1 | 2 | 3 |
| b) They might multiply 10 x 10 = 100 and then subtract 36 to get 64. | 1 | 2 | 3 |
| c) They might multiply 8 x 10 = 80 and then subtract 8 x 2 from 80: 80 – 16 = 64. | 1 | 2 | 3 |
| d) They might multiply 8 x 5 = 40 and then count up by 8's: 48, 56, 64. | 1 | 2 | 3 |

10. Students in Mr. Hayes' class have been working on putting decimals in order. Three students — Andy, Clara, and Keisha — presented 1.1, 12, 48, 102, 31.3, .676 as decimals ordered from least to greatest. What error are these students making? (Mark ONE answer.)

a) They are ignoring place value.

b) They are ignoring the decimal point.

c) They are guessing.

d) They have forgotten their numbers between 0 and 1.

e) They are making all of the above errors.

11. You are working individually with Bonny, and you ask her to count out 23 checkers, which she does successfully. You then ask her to show you how many checkers are represented by the 3 in 23, and she counts out 3 checkers. Then you ask her to show you how many checkers are represented by the 2 in 23, and she counts out 2 checkers. What problem is Bonny having here? (Mark ONE answer.)

a) Bonny doesn't know how large 23 is.

b) Bonny thinks that 2 and 20 are the same.

c) Bonny doesn't understand the meaning of the places in the numeral 23.

d) All of the above.

12. Mrs. Jackson is getting ready for the state assessment, and is planning mini-lessons for students focused on particular difficulties that they are having with adding columns of numbers. To target her instruction more effectively, she wants to work with groups of students who are making the same kind of error, so she looks at a recent quiz to see what they tend to do. She sees the following three student mistakes:

$$\text{I)}\quad \begin{array}{r} {\scriptstyle 1}\phantom{0} \\ 38 \\ 49 \\ +\,65 \\ \hline 142 \end{array} \qquad\qquad \text{II)}\quad \begin{array}{r} {\scriptstyle 1}\phantom{0} \\ 45 \\ 37 \\ +\,29 \\ \hline 101 \end{array} \qquad\qquad \text{III)}\quad \begin{array}{r} {\scriptstyle 1}\phantom{0} \\ 32 \\ 14 \\ +\,19 \\ \hline 64 \end{array}$$

Which have the same kind of error? (Mark ONE answer.)

a) I and II

b) I and III

c) II and III

d) I, II, and III

13. Ms. Walker’s class was working on finding patterns on the 100’s chart. A student, LaShantee, noticed an interesting pattern. She said that if you draw a plus sign like the one shown below, the sum of the numbers in the vertical line of the plus sign equals the sum of the numbers in the horizontal line of the plus sign (i.e., 22 + 32 + 42 = 31 + 32 + 33). Which of the following student explanations shows sufficient understanding of why this is true for all similar plus signs? (Mark YES, NO or I’M NOT SURE for each one.)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

| | Yes | No | I’m not sure |
|---|---|---|---|
| a) The average of the three vertical numbers equals the average of the three horizontal numbers. | 1 | 2 | 3 |
| b) Both pieces of the plus sign add up to 96. | 1 | 2 | 3 |
| c) No matter where the plus sign is, both pieces of the plus sign add up to three times the middle number. | 1 | 2 | 3 |
| d) The vertical numbers are 10 less and 10 more than the middle number. | 1 | 2 | 3 |

14. Mrs. Jackson is getting ready for the state assessment, and is planning mini-lessons for students around particular difficulties that they are having with subtracting from large whole numbers. To target her instruction more effectively, she wants to work with groups of students who are making the same kind of error, so she looks at a recent quiz to see what they tend to do. She sees the following three student mistakes:

| I | II | III |
|---|---|---|
| $\begin{array}{r} {}^{4}\ {}^{12} \\ \cancel{5}0\cancel{2} \\ -\ 6 \\ \hline 406 \end{array}$ | $\begin{array}{r} {}^{4}\quad {}^{15} \\ 3\cancel{5}00\cancel{5} \\ -\quad 6 \\ \hline 34009 \end{array}$ | $\begin{array}{r} {}^{6\,9\,8\,15} \\ \cancel{7}\cancel{0}\cancel{0}\cancel{5} \\ -\quad 7 \\ \hline 6988 \end{array}$ |

Which have the same kind of error?  (Mark ONE answer.)

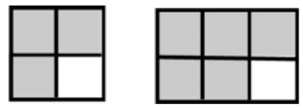
a)  I and II

b)  I and III

c)  II and III

d)  I, II, and III

15. Takeem's teacher asks him to make a drawing to compare $\frac{3}{4}$ and $\frac{5}{6}$. He draws the following:



and claims that $\frac{3}{4}$ and $\frac{5}{6}$ are the same amount. What is the <u>most likely</u> explanation for Takeem's answer (Mark ONE answer.)?

a) Takeem is noticing that each figure leaves one square unshaded.

b) Takeem has not yet learned the procedure for finding common denominators.

c) Takeem is adding 2 to both the numerator and denominator of $\frac{3}{4}$, and he sees that that equals $\frac{5}{6}$

d) All of the above are equally likely.

16. A number is called "abundant" if the sum of its proper factors exceeds the number. For example, 12 is abundant because 1 + 2 + 3 + 4 + 6 > 12. On a homework assignment, a student incorrectly recorded that the numbers 9 and 25 were abundant. What are the most likely reason(s) for this student's confusion? (Mark YES, NO or I'M NOT SURE for each.)

| | Yes | No | I'm not sure |
|---|---|---|---|
| a) The student may be adding incorrectly. | 1 | 2 | 3 |
| b) The student may be reversing the definition, thinking that a number is "abundant" if the number exceeds the sum of its proper factors. | 1 | 2 | 3 |
| c) The student may be including the number itself in the list of factors, confusing proper factors with factors | 1 | 2 | 3 |
| d) The student may think that "abundant" is another name for square numbers. | 1 | 2 | 3 |

17. Students sometimes remember only part of a rule. They might say, for instance, "two negatives make a positive." For each operation listed, decide whether the statement "two negatives make a positive" sometimes works, always works, or never works. (Mark SOMETIMES, ALWAYS, NEVER, or I'M NOT SURE)

| | Sometimes works | Always works | Never works | I'm not sure |
|---|---|---|---|---|
| a) Addition | 1 | 2 | 3 | 4 |
| b) Subtraction | 1 | 2 | 3 | 4 |
| c) Multiplication | 1 | 2 | 3 | 4 |
| d) Division | 1 | 2 | 3 | 4 |

18. Mrs. Smith is looking through her textbook for problems and solution methods that draw on the distributive property as their primary justification. Which of these familiar situations could she use to demonstrate the distributive property of multiplication over addition [i.e., a (b + c) = ab + ac]? (Mark APPLIES, DOES NOT APPLY, or I'M NOT SURE for each.)

| | Applies | Does not apply | I'm not sure |
|---|---|---|---|
| a) Adding $\frac{3}{4}+\frac{5}{4}$ | 1 | 2 | 3 |
| b) Solving $2x - 5 = 8$ for x | 1 | 2 | 3 |
| c) Combining like terms in the expression $3x^2 + 4y + 2x^2 - 6y$ | 1 | 2 | 3 |
| d) Adding 34 + 25 using this method: $\begin{array}{r} 34 \\ \underline{+25} \\ 59 \end{array}$ | 1 | 2 | 3 |

19. Students in Mr. Carson's class were learning to verify the equivalence of expressions. He asked his class to explain why the expressions a – (b + c) and
a – b – c are equivalent. Some of the answers given by students are listed below.

Which of the following statements comes closest to explaining why a – (b + c) and a – b – c are equivalent? (Mark ONE answer.)

a) They're the same because we know that a – (b + c) doesn't equal a – b + c, so it must equal a – b – c.

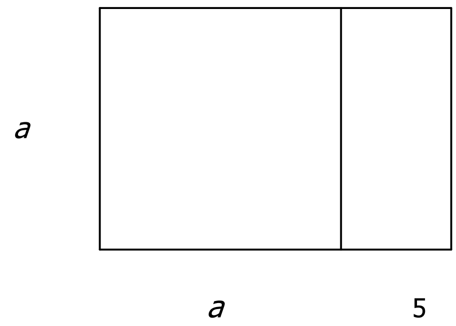
b) They're equivalent because if you substitute in numbers, like a=10, b=2, and c=5, then you get 3 for both expressions.

c) They're equal because of the associative property. We know that a – (b +c) equals (a – b) – c which equals a – b – c.

d) They're equivalent because what you do to one side you must always do to the other.

e) They're the same because of the distributive property. Multiplying (b + c) by –1 produces –b – c.

20. Ms. Whitley was surprised when her students wrote many different expressions to represent the area of the figure below. She wanted to make sure that she did not mark as incorrect any that were actually right. For each of the following expressions, decide whether the expression <u>correctly represents</u> or <u>does not correctly represent</u> the area of the figure. (Mark REPRESENTS, DOES NOT REPRESENT, or I'M NOT SURE for each.)




| | | Correctly represents | Does not correctly represent | I'm not sure |
|---|---|---|---|---|
| a) | $a^2 + 5$ | 1 | 2 | 3 |
| b) | $(a + 5)^2$ | 1 | 2 | 3 |
| c) | $a^2 + 5a$ | 1 | 2 | 3 |
| d) | $(a + 5)a$ | 1 | 2 | 3 |
| e) | $2a + 5$ | 1 | 2 | 3 |
| f) | $4a + 10$ | 1 | 2 | 3 |

21. Ms. Hurlburt was teaching a lesson on solving problems with an inequality in them. She assigned the following problem.

$-x < 9$

Marcie solved this problem by reversing the inequality sign when dividing by $-1$, so that $x > -9$. Another student asked why one reverses the inequality when dividing by a negative number; Ms. Hurlburt asked the other students to explain. Which student gave the best explanation of why this method works? (Mark ONE answer.)

a) Because the opposite of x is less than 9.

b) Because to solve this, you add a positive x to both sides of the inequality.

c) Because $-x < 9$ cannot be graphed on a number line, we divide by the negative sign and reverse the inequality.

d) Because this method is a shortcut for moving both the x and 9 across the inequality. This gives the same answer as Marcie's, but in different form: $-9 < x$.

## APPENDIX C

### Meaning of Some Words

**Gear:** dişli çark

**Dividend:** bölünen

**Divisor:** bölen

**Quotient:** bölüm

**Colored chips:** Renkli şerit

**Cross-multiplication:** İçler dışlar çarpımı

**Messy:** karısık

**To Pause:** Susmak ve beklemek

**Undetermined:** belirsiz

**Raffle tickets:** Piyango bileti

**Mixture:** karışım

**Leftover:** geri kalan

**Plow up:** ekmek, sürmek

**Corn:** mısır

**Acute angle:** dar açı

**Peg:** çivi

**Stump each other:** Birbirine soru sormak

**Compile:** Derlemek, oluşturmak.

**Clue:** ipucu

**Tesselation:** süsleme

**Paint:** boya

**Vertex:** köşe

**Ensue:** Oluşmak, meydana gelmek

**Pose:** Ortaya (bir soru) atmak

**Numerator:** kesrin payı

**Denominator:** kesrin paydası

**Toothpicks:** kürdan

**Linear function:** doğrusal(1.dereceden) fonksiyon

**Quadratic function:** 2. Dereceden fonksiyon

**Exponential Function:** Üssel Fonksiyon (Artan üslü fonksiyon)

**Penny:** para birimi

**Debate:** tartışma

**İnequality:** eşitsizlik

**Distributive property:** dağılma özelliği

**Mathematically Accurate:** matematiksel olarak doğru

**Comprehensible:** anlaşılabilir, açık

**İnspection:** Deneme

**Elimination:** Yok etme

**Substitution:** Yerine Koyma

**Roll up:** Yukarı doğru yuvarlamak

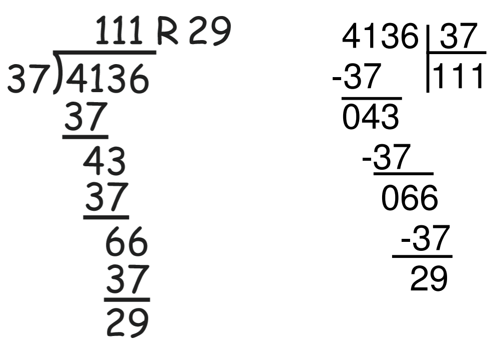
**Roll down:** Aşağı doğru yuvarlanmak

**Drop from top:** Yukardan bırakmak

**Thrown downward:** Aşağıya fırlatmak

**Supplemental:** Tamamlayıcı, bütünleyici

**Pint:** Yarım litrelik sıvı ölçü birimi




The two algorithms in the left are the different representations of the same division.

## APPENDIX D

## Course Syllabus

**Methods of Mathematics Teaching (2-2) 3**
Fall 07

### Readings

Van De Walle, J. A. (2004). Elementary and Middle School Mathematics: Teaching Developmentally, 5th Ed., Allyn & Bacon.
Other readings will be assigned by the instructor during the semester.

### Assignments

**Attendance, Classroom Participation and Readings (100 pts)**

**Active Reflection Assignment (50 pts)**
You will be asked to reflect on characteristics of effective mathematics teaching and effective mathematics teacher. This will be an inclass assignment, and will be given at the beginning and end of the semester; so, please ensure that you do not miss this inclass assignment. I will give you the dates. This assignment consists of two phases; **September 25 (phase 1) & December 26 (phase 2)**

**Article Critique (100 pts)**
You are required to review one article from Teaching Mathematics in the Middle School. The ULAKBIM library next to OSYM holds the recent and previous issues of the journal. So, you need to visit the library to have copies of the article that you want to review for this class. I will give you the assignment guidelines later in the semester. **Due October 17**

**Activity Design Project (200 pts)**
Each student is expected to design 3 instructional activities for 6-8 grade mathematics. I will give you the assignment guidelines later in the semester. **Due December 19**

**Classroom Presentation (50 pts)**
Each of you will make an activity presentation in class. You will be given twenty minutes for your presentation. You're welcome to present one of the activities from your activity design project. The schedule of the presentations will be arranged in the second week of the semester.

**Quizzes (100 pts)**
There will be four announced quizzes.

**Midterm (200 pts)**
There will be one midterm exam covering the course topics. **November 14**

**Final (200 pts)**

The final will cover all course topics. **January 14.**

**Attendance & participation**

Students are expected to attend all class sessions unless they have a documented evidence of medical excuse preventing their attendance. Students are also expected to arrive on time, and stay the entire class session. If you miss six (or more) class sessions without documented excuse, and/or if you establish a pattern of tardiness in class, the highest final grade you can earn in the class will be CC. Two instances of tardiness will be considered as being equivalent to one absence. If you have to miss a class because of an excused reason, it is your responsibility to provide instructor with evidence of doctor's visit no later than the next class session. After an absence, you should obtain class notes, hand-outs, other information from your classmates.

IMPORTANT NOTE: Students will receive FF if they miss 10 or more class sessions with or without excused reasons. Thus, if you are sick for more than 10 class sessions, you may not pass the class regardless of your performance on the assignments and tests because the course requires full and active student participation for a passing grade.

**Academic Misconduct**: I hope there will be no need to worry about academic misconduct (cheating, plagiarism, etc.). Plagiarism will not be tolerated. I will follow the university policies to deal with such cases.

**Policy on Late Assignments**

The expectation is that assignments will be turned in by the announced due dates. I will accept assignments after the due date but your grade will decrease by 10% of the allocated points for each calendar day the assignment is late. For example, a project worth 50 points that is turned in two days late will receive 10 fewer points (50 points x 10% per day x 2 days) than it would have if it had been turned in on time.

For all your assignments, the following general issues will be considered for grading purposes:

- Has the student done what was asked for and specified in the description of the assignment?
- Are the ideas discussed relevant for mathematics teaching and learning?
- Is the work clearly presented and properly written? Are the ideas well developed? Are they coherently woven together and presented in an orderly fashion?
- Does the work demonstrate that the student spent time and thought in completing the assignment? Is the work thoughtful, insightful?
- Has the student made connections to pertinent readings discussed in class and to the literature on the subject under study?

You will be given a tentative schedule next week.

## **Assignment Sheets**

Activity Design Project

Due date December 19

For this assignment, you need to design a mathematics teaching activity for elementary students. The activity can be at any grade level and on any topic.
The activity is designed for regular elementary school students. After designing the activity, you need to design two more activities on the same topic at the same grade level. One of the activities will be more challenging than the first one. The second activity will be designed for a class of gifted students. The third activity will be less challenging than the first one. While the second activity will be for gifted students, the third activity will be for students with learning difficulties or less successful students.
In brief, you need to design three activities altogether; one at a medium level, one at the top, and one at the very low. Yet, all three activities will be designed at same grade level on the same topic. So, you need to **gear down** and **gear up** the cognitive demand required by the first activity.
Besides the activities, you need to write a reflection where you explain how you increased and decreased the level of the first activity. Justify your claims. It is important that you specifically explain what characteristics of the activities are targeted for gifted and less successful students. The paper should be at least two pages.

What to turn in

Activity #1 (at the medium level) 50 POINTS
Activity #2 (for gifted students) 50 POINTS
Activity #3 (for less successful students) 50 POINTS
Reflection paper 50 POINTS

Article Critique

Due October 17

For the critique, you are to choose a "main" article from any issue of Mathematics Teaching in the Middle School (MTMS). That is, choose an article that is at least 3 pages and focuses on an issue. (Editorials, reviews, and other short essays are informative but I want you to look at more extensive articles.) The journal is available at the Tubitak-ULAKBIM Library. Please, visit the following link for Frequently-Asked-Questions about the library:

http://www.ulakbim.gov.tr/sss/

Critiques should be two to three pages in length (word processed, double spaced) and focus on a (or the) major issue raised in the article. Remember to clearly state the name of article you are reviewing and to include the volume and issue number of the journal. In terms of content for your critique, you must include the following three components.

1. a brief summary of the message the author wishes to convey (provide enough detail to allow the reader of your critique to understand your comments — it is better to fully describe key points and omit lesser points than it is to mention lots of points and leave the reader wondering which of those were the most important)

2. your opinions about the strengths and/or weaknesses of the message (be careful to note which of the comments you make are from the author and which are your reactions to the author)

3. how you will think or teach differently after reading the article (or reasons why the article will have no affect on your teaching).

In evaluating your critiques, the following will be considered:

• substance of the critique: Does the critique

  - show evidence that you have understood the central message contained in the article?

  - identify strengths and/or weaknesses in the article, or at least comment on the author's ability to portray her or his message?

  - show in-depth reflection on the author's message and the implications of that message for you?

• composition skills. Does the critique

  - communicate your ideas clearly?

  - contain carefully formed sentences and paragraphs?

  - contain a well-structured flow of ideas?

• preparation of manuscript. Does the critique

  - show evidence that you read and carefully revised your work?

  - show that you took care to eliminate spelling and grammatical errors?

  - indicate care in the general appearance of the paper?

Points will be assigned as follows:

100 points: An unusually insightful critique which includes a very clear summary, easy to follow reaction to the key points, and clear, honest commentary about the implications of the article for you.

85 points: A thoughtful and carefully prepared critique showing some very keen insights and reflections.

75 points: A well prepared paper that captures the essence of the ideas in the manuscript and provides reasonable reaction to them.

60 pts: A critique which is on the right track but misses important details or shows only minimal insight into the implications of the article.

40 pts: A critique which fails on more than one of the above criteria.

## Midterm Questions

November 14, 2007

ELE443 Fall 07
Midterm
Out of 200 points

Please, answer the following questions. Each question is worth 20 points.

1. Describe conceptual knowledge of mathematics and procedural knowledge of mathematics. Provide examples of each.
2. Explain the difference between practice and drill, and what each can provide.
3. Make up four story problems: one join, one part-part-whole, one separate, and one compare problem. For all four problems, use the same number family: 8, 9, 17.
4. Make up two multiplication word problems to illustrate the difference between equal groups and multiplicative comparison.
5. Describe the difference between measurement division and partition division problems. Write one story problem for each.
6. Make up realistic division problems where the remainder is dealt with in each of these three ways: (a) it is discarded (but not left over); (b) it is made into a fraction; (c) it forces the answer to the next whole number?
7. How are traditional algorithms different from invented strategies? Explain the benefits of invented strategies over traditional algorithm.
8. Use a compensation strategy for these: 72 X 34, 35 X 320.
9. Give examples of three categories of fraction models? Why are set models more difficult for younger children?
10. Explain the difference between numerator and denominator for elementary school students (grades 4 thru 8)?

## Quizzes

QUIZ 1

1. Write down a 3-part activity that describes teachers' actions in before, during and after parts of the activity.

QUIZ 2

1. Write three types of estimation. Explain each of them and give an example for each type.

QUIZ 3

1. What are the first three levels of Van Hiele? What do we expect from the students at those levels?

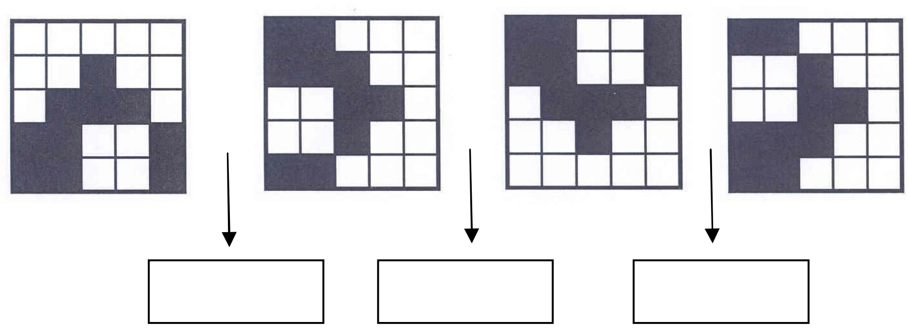
**APPENDIX E**

**Sample Worksheets**

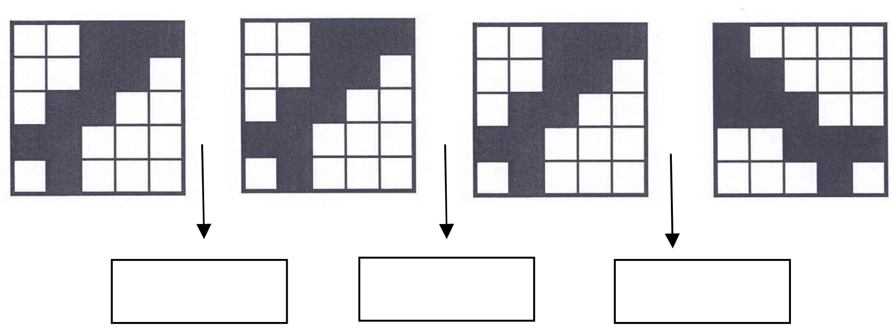
**Worksheet 1**

(Adapted from Kroner, L. R., 1994, p.39)

**1.** Discuss the motion in the grid pattern with the one next to it. Write the type of the motion in the boxes below.



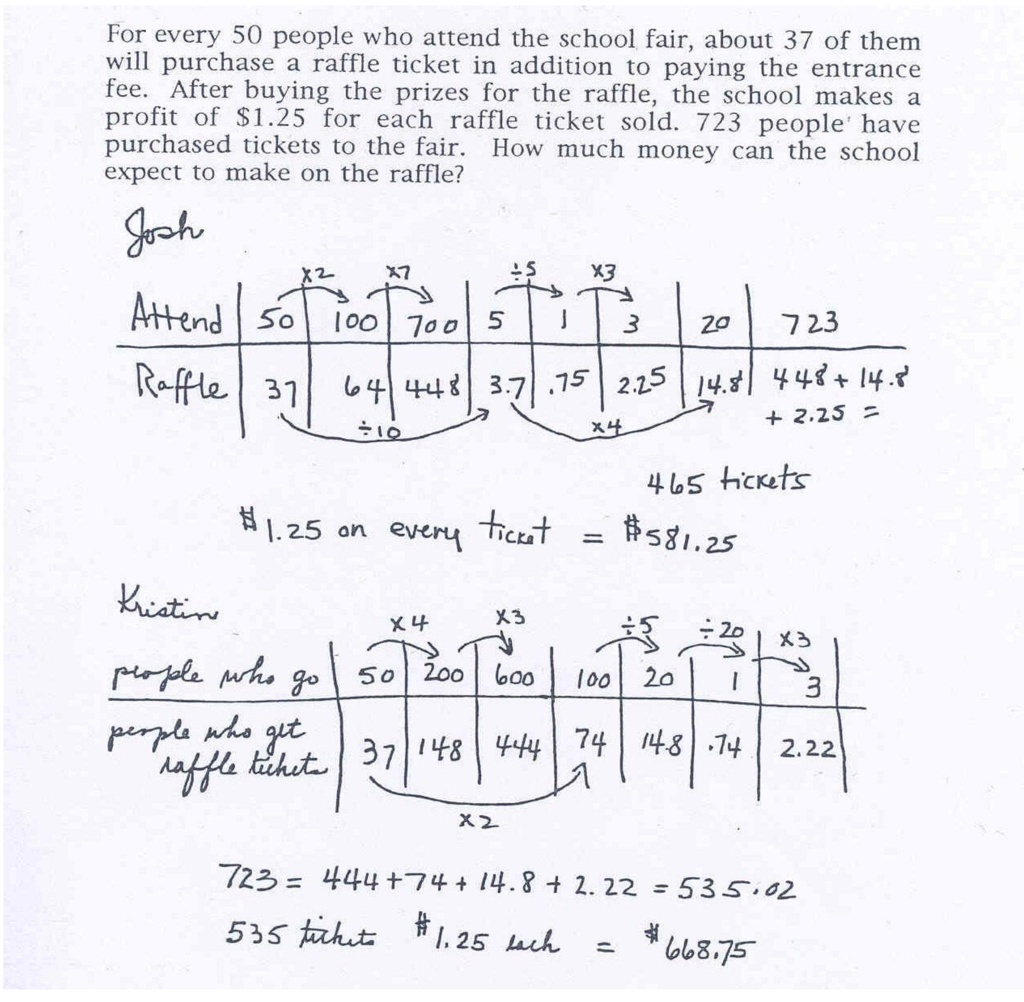
**2.** Discuss the motion in the grid pattern with the one next to it. Write the type of the motion in the boxes below.



# Worksheet 2

(Adapted from Lamon, S., 2005, p. 203)

Analyze the students' solution methods given below. Discuss with your partner why they came up with different responses and possible misconceptions of the students. How could you evaluate these responses?



For every 50 people who attend the school fair, about 37 of them will purchase a raffle ticket in addition to paying the entrance fee. After buying the prizes for the raffle, the school makes a profit of $1.25 for each raffle ticket sold. 723 people have purchased tickets to the fair. How much money can the school expect to make on the raffle?

Josh

×2 ×7 ÷5 ×3

| Attend | 50 | 100 | 700 | 5 | 1 | 3 | 20 | 723 |
|---|---|---|---|---|---|---|---|---|
| Raffle | 37 | 64 | 448 | 3.7 | .75 | 2.25 | 14.8 | 448 + 14.8 + 2.25 = |

÷10 ×4

465 tickets

$1.25 on every ticket = $581.25

Kristin

×4 ×3 ÷5 ÷20 ×3

| people who go | 50 | 200 | 600 | 100 | 20 | 1 | 3 |
|---|---|---|---|---|---|---|---|
| people who get raffle tickets | 37 | 148 | 444 | 74 | 14.8 | .74 | 2.22 |

×2

723 = 444 + 74 + 14.8 + 2.22 = 535.02

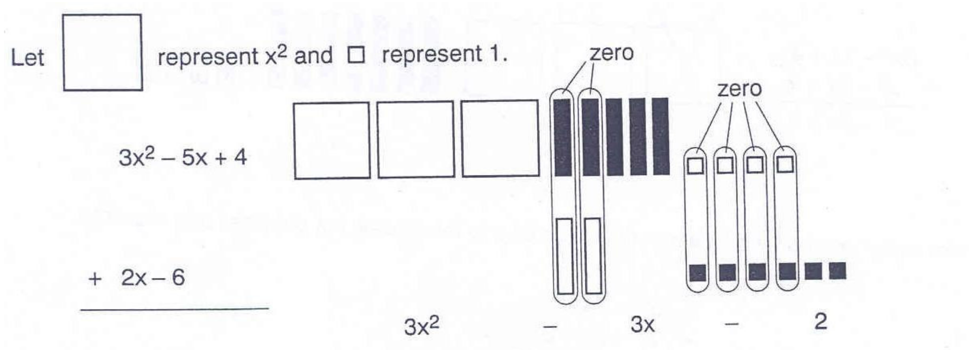
535 ticket $1.25 each = $668.75

**Worksheet 3**

(Adapted from Howden, H., 1994, p.11)

Analyze the example below.
By using algebra tiles, add the following polymials. Show your works by drawing.

Example:




1.

$$
\begin{array}{r}
x^2 + 2xy + 3y^2 \\
+\ \ 2x^2 + xy - y^2 \\
\hline
\end{array}
$$

2.

$$
\begin{array}{r}
4a^2 - 3b^2 \\
2ab + b^2 \\
+\ \ b^2 - ab - 2a^2 \\
\hline
\end{array}
$$