CONTROLLING HIGH QUALITY MANUFACTURING PROCESSES:

A ROBUSTNESS STUDY OF THE LOWER-SIDED TBE EWMA PROCEDURE

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INTRODUCTION

1.1 Controlling High Quality Manufacturing Processes

Many Statistical Process Control (SPC) techniques have been proposed since the 1920's when Walter Shewhart first introduced the concept of control charts (Shewhart, 1925). Since 1940's, SPC charts have found widespread applications in improving the quality of manufacturing processes by monitoring the unusual variability in processes. Shewhart, Cumulative Sum (CUSUM) and Exponentially Weighted Moving Average (EWMA) control charts are usually known as the traditional charts and have been the most commonly used SPC charts for a long time. However, as the emphasis on the quality of products and customer satisfaction increased, high quality processes producing fewer defects arouse. In such industrial environments, the traditional control charts may not perform as well as they used to. The disadvantage of traditional control charts has led to the development of modified versions of the charts for dealing with high quality processes. Therefore, time between events (TBE) control charts, which monitor the time between defects or the good units between consecutive defects rather than to monitor the number of defects in a time interval, were introduced.

Among those charts TBE Exponentially Weighted Moving Average (EWMA) control chart has been widely preferred in many applications in many manufacturing and service industries since they are better at detecting small shifts and easier to implement. In this research, the lower-sided exponential EWMA charts, which are the most widely used type of TBE EWMA, are considered.

1.2 Problem Statement

A widely accepted model for time between events data is the exponential distribution. Hence, TBE EWMA charts for a process modeled with exponential distribution are considered. However, practical applications do not conform to the theory all the time and observations do not always follow the assumed distribution so that the exponential assumption may not be satisfied in some applications. In such situations, it is expected from a control chart to detect the process mean shifts with the same performance whatever the underlying distribution is, such that it should be insensitive to the assumed distribution or act as distribution-free. In other words, a control chart is expected to be *robust* to the assumed distribution. However, since the TBE EWMA chart is designed under the assumption of exponential distribution and the true underlying distribution may be different, it is highly possible that the chart may not perform as designed. Hence, when there may be deviations from the assumed distribution, it is essential to properly design a TBE EWMA that is robust.

1.3 Motivation and Importance of the Study

The exponential EWMA control chart is a widely used statistical process control tool in TBE monitoring. Although the robustness studies of several widely used control charts have been performed in the literature, the robustness of the exponential (TBE) EWMA chart has not been investigated yet. Since the performance of the traditional defect monitoring charts such as Shewhart, CUSUM, or EWMA may be poor in the high quality manufacturing environments, the TBE monitoring charts have been introduced and have become widely popular in industry. Recently, the robustness of the TBE CUSUM has been studied by Borror, Keats, and Montgomery (2003). However, to the best of our knowledge, a study for the TBE EWMA has not been performed.

1.4 Research Objectives

In this research, the sensitivity of the TBE EWMA control chart to the assumption of exponentially distributed observations has been studied. The study aims to show whether the exponential EWMA chart is robust or sensitive to the deviations from the assumed exponential distribution. A detailed analysis is required to evaluate the robustness and to define parameter settings, which would provide a robust TBE EWMA if there exist any.

Weibull and lognormal distributions may be considered as the models representing the departures from exponential distribution. Hence, robustness can be analyzed by changing the parameters of these distributions (see for example Borror, Keats, Montgomery, 2003). Furthermore, by using a Markov Chain approach (Brook and Evans, 1972), the performance of the TBE EWMA chart may be evaluated for various design parameters and under various distribution models.

To evaluate the robustness, performance of the TBE EWMA chart under the exponential distribution may be compared with the performance under the deviated models. Consequently, an analysis may be performed to find out on which parameter combinations the TBE EWMA control chart works more effectively and is robust to the deviations from the exponential distribution.

1.5 Organization of the Research

The thesis is organized as follows. In the next chapter, a review of the literature is provided, where the traditional and the TBE control charts are explained. Robustness studies in the literature are also discussed in this chapter. Chapter 3 explains and provides the details of the methodology used in the robustness study and the design procedure for a lower-sided TBE EWMA chart is explained in Chapter 4. Accordingly, in Chapter 5, performance results for the lower-sided TBE EWMA charts are presented and discussed in order to investigate the robustness. Finally, in Chapter 6 we conclude and provide some future research directions.

CHAPTER 2

LITERATURE REVIEW and BACKGROUND

2.1 Background on Statistical Process Control

Statistical Process Control (SPC) is a collection of statistical and graphical tools used for monitoring, controlling and improving process/product quality. SPC is one of the significant features of Statistical Quality Control (SQC) and introduced in the 1920's when Walter A. Shewhart developed the control chart concept (Shewhart, 1925). The introduction of SPC is also considered as the formal beginning of statistical quality control (Montgomery, 2006).

At first, the importance of the SQC concept was not comprehended clearly and applications were limited to manufacturing processes. Awareness of quality and emphasis on customer satisfaction began to increase in the course of time. Towards the 1980's, as the global market became more competitive, the ability to meet the customer requirements became vital. With the increased emphasis on quality, it became obvious that statistical techniques were necessary to control/improve product quality and reduce costs. Since 1980's, statistical quality control has found a great acceptance and widespread applications in not only the manufacturing processes, but also in service industries (Montgomery, 2006).

Essential objective of SPC is to reduce variation, improve the process to a higher quality level and maintain process stability, (Smith, 2004). This is achieved by detecting unusual sources of variability caused by shifts in the process and taking corrective actions immediately. For such continuous improvement strategies, control charts are the primary SPC tools.

2.1.1 Control Chart Methodology

Control charts deal with two types of variation. The first type, namely "common-cause variation" or "chance causes of variation", is inherent in the process and economically or technologically irremovable. This type of variation is the result of the cumulative effect of random and unavoidable causes. A certain amount of natural variability, caused by the "common-causes", always exists in any process even if it is well designed or maintained carefully. Generally, a probability distribution such as a normal, Poisson, or exponential, is used to model this natural variability. When only common causes of variations are present in a process, the process is considered to be "in-control".

The other type of variation is relatively large in magnitude and is defined as "assignable-cause variation" or "special cause of variation". Assignable causes generally occur due to improper uses of machines, operator errors, or defective raw materials. The variability arising from such sources is larger than the "chance causes of variation" and represents an out-of-control situation. This variability shifts the process to an "out-of-control" state, where a larger proportion of the process output is in the range of unacceptable quality level (Montgomery, 2006). Detecting this unusual variability, occurring as a result of assignable causes, is the primary objective of the control charts.

Control charts can be constructed from either the individual observations or sample averages of the observations. First a control statistic is computed either from these individual or sample observations and then a control chart is formed by plotting this control statistic for the monitored quality characteristic over time.

A graphical display of a control chart is given in Figure 1. Often there is a center line (CL) and two control limits, upper and lower (respectively, UCL and LCL) on the chart. If there is no unusual source of variability (that is the process is in-control), almost all of the observations will be plotted between the two control limits. If one point exceeds the UCL or the LCL, it is taken as an indication of the occurrence of an assignable cause and a shift of the process to an out-of-control state. In order to remove the assignable cause and bring the process into the in-control state, an investigation should be performed to find the source of the variation and corrective actions should be taken.

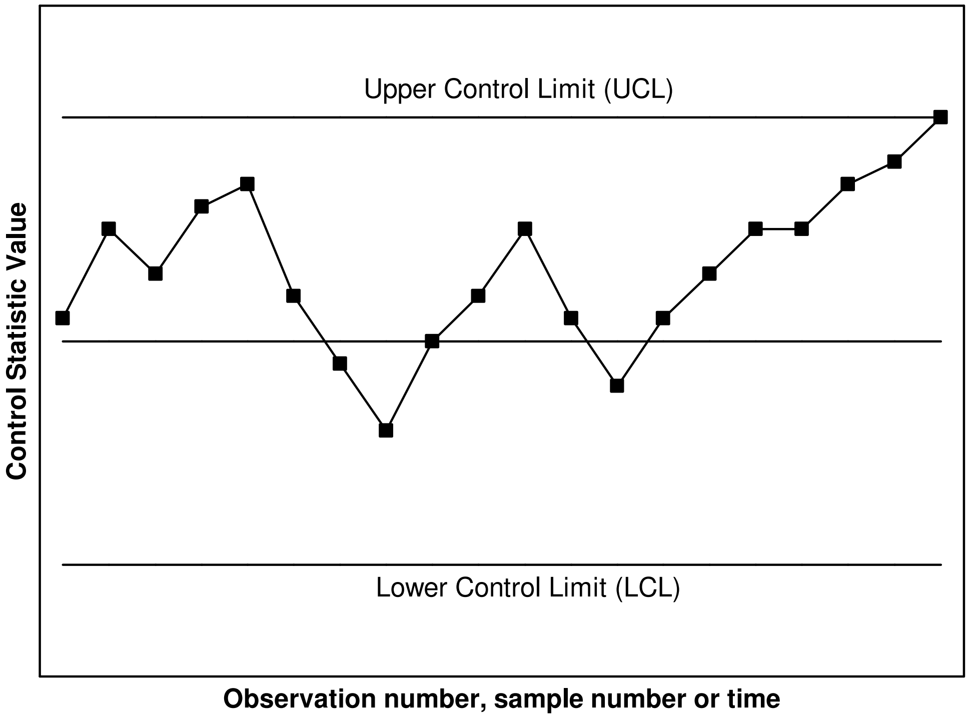

Figure 1: Graphical display of a typical control chart

Two types of data exist for process monitoring; variables and attributes. Variable data refer to the continuous measurements, such as diameter length, weight, viscosity, strength etc. Attribute data are composed of discrete data which are generally in the form of counts such as the number of nonconformities (defects) on a unit of product.

Both variable and attribute data can be monitored with the traditional control charts and their variations. In the following, these charts, namely, Shewhart, Cumulative Sum (CUSUM), and Exponentially Weighted Moving Average (EWMA) will be reviewed.

2.1.2 Shewhart Control Chart

Shewhart control chart was first introduced by Walter Shewhart in 1924 and has been considered as the most widely used control chart for a long time. Shewhart control charts can be constructed for both continuous and discrete data. $\bar{X}$ and R, $\bar{X}$ and S, X and MR charts are the examples for Shewhart control charts constructed for variables. Besides, Shewhart has four types of attributes control charts, which can be listed as p chart, np chart, c chart and u chart. These charts can be constructed both for individual observations and for the average of rational subgroups (sample averages). Detailed information about basic principles of Shewhart control charts can be found in Woodall and Montgomery (1999) or Montgomery (2006).

Shewhart control charts have been the most popular control chart for a long time since they are easy to implement and understand and the required calculations can be manually handled. However, the growing emphasis on the variability reduction and quality/process improvement gave rise to high quality processes in which the variability is reduced and even small process shifts are considered as unaccepted. In such processes, the early detection of not only large shifts but also the small shifts is essential for decreasing the variability.

Since Shewhart charts use only the current observation and ignore the past observations, they are not as much sensitive to small shifts as they are to large shifts. Therefore, Shewhart charts may not be effective in meeting the expectations of such high quality processes. This disadvantage of the Shewhart control charts encouraged the researchers to propose new techniques and gave rise to better charts for statistical process monitoring. These new charts use not only the current but also the past observations. Therefore, more information would be available for detecting small shifts quickly. The two popular control charts utilizing the past data are the cumulative sum (CUSUM) and the exponentially weighted moving average (EWMA) control charts.

2.1.3 Cumulative Sum (CUSUM) Control Chart

The CUSUM chart to monitor a normal mean was first proposed in Page (1954). Most of its popularization may be attributable to Lucas (1976). Unlike the Shewhart chart, CUSUM chart utilizes the past information available from previously plotted points, so has a greater sensitivity for detecting small shifts. The variables CUSUM charts with a normal distribution model of observations have been popular since the 1980's, mostly with the availability of personal computers on the shop floor. On the other hand, CUSUM charts for attributes were also proposed with a Poisson model for the counts of defects (Brook and Evans, 1972; Lucas, 1985).

Interested readers may be referred to Woodall (1986) for a review of the CUSUM chart design, Gan (1991) for optimal design of CUSUM charts, and Gan and Choi (1994) for a computer program for measuring the performance of CUSUM charts.

As can be comprehended from its name, CUSUM accumulates deviations of the past and current observations from the target value and uses the cumulative sum of these deviations in its control statistic. The control statistic of a standardized CUSUM can be written as:

$$C_t^+ = \max\left\{0,\ y_t - k + C_{t-1}^+\right\}$$

$$C_t^- = \max\left\{0,\ -k - y_t + C_{t-1}^-\right\}$$

Here, y_t is the observation at time t, k is the reference value, C_t^+ and C_t^- are the control statistics plotted on separate control charts, called the upper and lower CUSUMs, respectively. Note that C_t^+ accumulates the information of the observations measured above the target value, whereas C_t^- accumulates the information of the observations measured below the target value. Initial value of upper and lower control statistics are generally taken as zero ($C_0^+ = 0$ and $C_0^- = 0$). If either one of the control statistics plotted on the charts exceeds the decision interval, H, the process is deemed to be out-of-control. Detailed information about basic principles of CUSUM control charts can be found in Montgomery (2006), Hawkins and Olwell (1998).

2.1.4 Exponentially Weighted Moving Average (EWMA) Control Charts

In the quality control literature, EWMA charts were first proposed by Roberts (1959). In his research, Roberts (1966) indicated that EWMA and CUSUM charts are competing charts to detect small shifts in the mean of a normal distributed observations. The choice among the two is a largely a matter of personal preference. Similar to the CUSUM charts, EWMA control charts also accumulate current and past information from the observations and are proven to be more effective than Shewhart control charts in detecting small shifts in the process mean. Besides, EWMA charts are considered as being easier to design, implement and understand compared to the CUSUM charts.

Properties of the EWMA charts are evaluated analytically by Box, Jenkins and MacGregor (1974), Robinson and Ho (1978), Hunter (1986), Crowder (1987), and Waldmann (1986). A significant contribution about the design of an optimal EWMA chart was provided by Crowder (1989) and Lucas and Saccucci (1990).

General information on the design of EWMA control charts for monitoring a normal process mean is provided in the following:

Define the EWMA control statistic as:

$$Z_t = (1-\lambda) Z_{t-1} + \lambda X_t \qquad t = 1,2,3\ldots$$

where,

t denotes time or the interval between consecutive defects,

λ is the smoothing constant and assumes a value in the interval $0< \lambda \leq 1$,

Z_t is the EWMA statistic at time t,

X_t represents the individual observation,

$X_1, X_2, X_3, \ldots$ are *i.i.d.* random variables with mean μ and constant variance σ^2, i.e.

$$E(X_t)=\mu$$

$$Var(X_t)=\sigma^2$$

Hence, the EWMA chart can be seen as a weighted average of all past and current observations.

Assume that when the process is in-control, the in-control mean is μ_0. When $\mu \neq \mu_0$, the process is said to be out-of-control.

The initial value of the EWMA statistic, Z_0, is generally set to in-control mean value $Z_0=\mu_0$ since,

$$E(Z_t)=\mu_0$$

The variance of the EWMA statistic is:

$$\sigma^2_{Z_t}=Var(Z_t)=\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2t}\right]\sigma^2$$

Note that as t increases, variance reaches its asymptotic value:

$$\mathrm{Var}(Z_{\infty})=\frac{\lambda}{(2-\lambda)}\sigma^2$$

and hence one may use the asymptotic variance to simplify computations. By using the exact variance, control limits are not constant but parabolic over time and this feature is especially useful in detecting process shifts at startups. In fact, this serves as a natural Fast Initial Response (FIR) feature. However, in practice asymptotic variance is often used since variance quickly approaches its steady-state value. Therefore, in this research asymptotic control limits are considered.

In the design of EWMA control charts upper and lower control limits are set as:

$$h_L=\mu_0-L_1\sigma_{Z_t}$$

$$CL=\mu_0$$

$$h_U=\mu_0+L_2\sigma_{Z_t}$$

where, h_U is the upper control limit, h_L is the lower control limit, and L_1 and L_2 are the width of the control limits in terms of the σ_{Z_t}. L_1 and L_2 are generally taken to be equal and the control limits become symmetric ($L_1 = L_2 = L$). A point exceeding either of the control limits is an indication of the occurrence of an assignable cause. For normally distributed processes, λ and L pairs, which are the parameters of EWMA control charts, are provided in Crowder (1989) and Lucas and Saccucci (1990).

Originally, the EWMA chart was introduced to monitor a mean of a continuous quality characteristic modeled with a normal distribution. However, since attribute data are often encountered in all types of business and industry, there has been a need for an EWMA control chart to monitor the counts of the occurrence of nonconforming items, which is usually modeled by a Poisson distribution. Therefore, an EWMA control chart, which monitors the mean of a Poisson distributed process was also developed (Gan, 1989). Since Gan (1989) used rounded values of the EWMA statistic, a further analysis on the design of a Poisson EWMA was provided by Borror, Champ, Rigdon (1998). In their research the exact EWMA statistic is used in the method so that no information is lost (Testik, McCullough, Borror, 2006). For a detailed review on the attributes control charts see Woodall (1997).

2.2 Time-Between-Events (TBE) Control Charts

The progress of the quality control techniques and new technological developments has led to high quality processes in which small amount of defects occur. In such systems, the number of defects occurring in a given time interval may be so small that often no defect occurs in intervals. In such type of count data environments, control charts based on Poisson assumption may not be effective since the number of counts in a given interval has a high probability of being zero. For such high quality processes, monitoring of time-between-events (TBE) data is suggested since monitoring the time or the good units between consecutive defects is more effective rather than monitoring the counts of defects themselves (Liu et al., 2004).

Since the counts of defects in a process are usually Poisson distributed, the time or the good units between these defects follow an i.i.d. exponential distribution. Hence, TBE processes are usually modeled with an exponential distribution.

The TBE monitoring control charts in the literature are Cumulative Quantity Control (CQC), CQC-r, exponential CUSUM, and exponential EWMA Control Charts. Vardeman and Ray (1985) were the first researchers studying TBE charts. Gan (1994) studied the design of TBE CUSUM and introduced the exponential CUSUM. CQC charts are proposed by Chan, Xie, and Goh (2000). A comprehensive discussion about TBE monitoring charts can be found in Liu et al. (2004).

On the other hand, Gan (1998) explained the design and use of exponential EWMA control charts for TBE monitoring. The design of one-sided (upper and lower) and two-sided exponential EWMA control charts were considered in his research. In a TBE monitoring chart, a decrease in the mean of TBE indicates that more defects are produced whereas, an increase in the mean of TBE indicates that fewer defects are produced. A decrease in the mean of TBE can be monitored with a lower-sided TBE EWMA chart while an increase is monitored with an upper-sided TBE EWMA chart. Since our priority is to detect increases in the defect rate, this research considers lower-sided TBE EWMA control charts. On the other hand, upper-sided TBE

EWMA charts can also be used to monitor process improvements. Below, design information is given about the lower-sided exponential (TBE) EWMA chart. A detailed discussion can be found in Gan (1998):

Lower-Sided TBE EWMA Charts

A lower-sided TBE EWMA chart is designed by plotting Z_t against t, where:

$$Z_t = \min\{B, (1-\lambda) Z_{t-1} + \lambda X_t\} \qquad t=1,2,3\ldots$$

Here, observations X_1, X_2, X_3, ... are time between nonconforming items (interarrival times of defects) distributed as exponentially with the probability density function:

$$f(x) = \frac{1}{\theta} e^{-x/\theta} \qquad x \geq 0$$

Mean (μ), and the standard deviation (σ) of the exponential distribution is θ:

$$\mu = \theta$$
$$\sigma = \theta$$

$B > 0$ is the upper boundary of the control chart. Since the lower-sided TBE EWMA chart monitors the decrease in the TBE mean, it becomes sufficient and effective to bind the chart. With the existence of B, the control statistic measured higher than B ($Z_t > B$), is pulled to the value of B. Therefore, the control statistic is prevented from being far apart from h_L, so that the early detection of shifts is ensured.

The initial value of the EWMA statistic (Z_0) is taken between lower control limit (h_L) and B ($h_L < Z_0 < B$). The lower sided TBE EWMA charts give an out of control alarm when the statistic exceeds the lower control limit ($Z_t \leq h_L$).

The effect of the boundary value (B) was investigated by Gan (1998). He examined the performance of one-sided exponential EWMA charts with various λ values in the range [0.1, 1] for values of $B \in \{1, 2, 5\}$. Gan (1998) showed that the chart's sensitivity increases as B increases until B = 2. No appreciable gain was observed beyond B = 2. Thus, the reasonable choice for boundary was suggested to be B = 2.

In our research, further examination will be performed to find out the effect of B and λ on robustness.

2.3 Robustness Studies of Control Charts

In designing a control chart, an appropriate distribution for the observations should be defined. For the traditional charts like Shewhart, it is usually assumed that the observations follow a normal distribution. For the attributes control charts and count type data, Poisson distribution is usually considered. For the case of TBE charts, a convenient distribution is the exponential as mentioned before. However, it is known that in some cases the observations do not follow the assumed distribution. Especially in practice, deviations from the assumed distribution is common and when there have been deviations from the assumed distribution, it is essential to know that the chart would still show the same or close performance as is designed.

Many researches were conducted in the literature about the sensitivity of the charts to the assumed distribution, that is, robustness. The individual Shewhart control chart has been shown to be sensitive to normality assumption while the $\overline{X}$ control chart is quite robust to the normality assumption (Schilling and Nelson, 1976). The effect of sample size was also considered in Yourstone and Zimmer (1992) and Amin, Reynolds, and Bakir (1995). They investigated the situations where normality assumption causes problems for n>1. On the other hand, the robustness of EWMA control chart to normality was also studied (Borror, Montgomery, and Runger, 1999). In this robustness study, it was assumed that the observations actually follow a t-distribution or a gamma distribution instead of normal distribution. As a result, it was shown that EWMA control chart, designed with the normality assumption, gave almost the same performance when distributions are t and gamma for small λ values. It was concluded that the EWMA control chart is very insensitive and quite robust to the normality assumption.

In another study, Borror, Keats and Montgomery (2003) focused on the robustness of TBE CUSUM to assumption of exponential distribution. In this research, Weibull and lognormal distributions were considered to represent the true underlying distribution. Their results prove that the ability of exponential CUSUM control chart to detect shifts was not drastically affected by the distribution changes. It was concluded that the TBE CUSUM is fairly robust to the assumption of exponential distribution.

CHAPTER 3

THE METHODOLOGY

In this research, we investigate the performance of a lower-sided exponential EWMA chart when the true underlying distribution is different from exponential.

In this chapter, parameter settings to design a robust lower-sided TBE EWMA chart for the assumption of exponential distribution are analyzed. In order to evaluate the performance and robustness of the chart, some performance metrics and a method to compute these metrics are required. In the first section of this chapter, the performance metrics chosen are explained. In the second section, first some background information is provided for modeling a TBE EWMA chart with the Markov Chain Approach. Then, this method is modified and used for calculating the performance metrics under various distributions.

3.1 Control Chart Performance Metrics

Performance of a control chart is usually evaluated by analyzing the properties of its run length (RL) distribution. RL can be defined as the number of points (control chart statistics) plotted on the chart until the first out-of-control signal is generated.

Average Run Length (ARL) is the most common metric used for evaluating the performance of a control chart. ARL can be defined as the expected number of points plotted on a control chart until a point indicates an out-of-control situation. In other words, it is the mean of RL distribution. The average run length of the chart when the process is in-control (i.e. a false alarm performance) is denoted by ARL_0. Since in-control state of the process is desired to be long, it is advantageous for a process to have large ARL_0 values. On the other hand, when the process shifts to an out-of-control state, the early detection of the shift is essential. The time or number of points plotted needed to detect a shift is denoted by ARL_1, and a chart with a small ARL_1 is preferred. Moreover, a control chart with a smaller ARL_1 value for a particular shift magnitude, is generally preferred to the ones with larger ARL_1 values for the same shift (Testik, McCullough, Borror, 2006).

Although ARL is the most widely used performance metric, the performance information obtained by interpreting the ARL values can be misleading in some situations. The in-control RL distribution of an EWMA control chart is known to be quite right skewed. In addition, the skewness of the RL distribution fluctuates as the shift in the process varies. Therefore, ARL would not necessarily be a very "typical" value of the run length (Montgomery, 2006) and important information regarding the performance of a control chart may be missed by interpreting only the ARL values.

At this point, other performance metrics in addition to ARL are required for a better understanding of the RL distribution and the chart performance. These performance metrics, which can be obtained by utilizing higher order moments of the RL, are the percentiles of the run length distribution and standard deviation of the run lengths (SDRL). These two metrics in addition to ARL suffice us to describe the entire profile of RL of the chart.

In this research, performance metrics that will be considered for describing the performance of a lower-sided TBE EWMA control chart are ARLs, SDRLs and 10^{th}, 50^{th}, 90^{th} percentiles of run length distribution.

3.2 Markov Chain Approach for Calculating the Performance Metrics

Brook and Evans (1972) proposed the Markov Chain approach for computing the probability distribution of run lengths and for determining the ARLs of control charts. In the study of Brook and Evans, both continuous and discrete control statistics were considered. For continuous control statistics, discretization is considered and exact probabilities for the approximate model are calculated. They also suggested the use of Markov Chain approach for continuous distributions as an accurate approximation for ARLs. Lucas and Saccucci (1990) used a similar procedure to Brook and Evans (1972) and evaluated the properties of a continuous state Markov Chain by discretizing the infinite state transition probability matrix of an EWMA control chart. The interval between upper and lower control limits were divided into approximately infinite states.

In this research, run length properties of a lower-sided TBE EWMA control chart for monitoring changes in the mean of an exponentially distributed process, is evaluated by using a finite state Markov Chain.

In order to model a control chart using Markov Chain Approach, the in-control region of the chart (the interval between upper and lower control limits (B, h_L)) is divided into N intervals, as shown in Figure 2. The midpoints (m_i) and lower control limit (h_L) are the states of the Markov Chain. The midpoint (m_i) of each interval corresponds to a transient state of the Markov Chain. The lower control limit (h_L) is defined as the $(N+1)^{th}$ state and represents the absorbing state of the Markov Chain. Note that, the continuous values of the EWMA statistic, i.e. Z_t, may fall anywhere between h_L and B. Using a discrete Markov Chain approximation, it is assumed that Z_t will be equal to the midpoint of the corresponding interval. So, when the process is in-control, the control statistic Z_t, is equal to m_i whenever and wherever it is in interval i. If the control statistic Z_t equals to or falls below the lower control limit ($Z_t \leq h_L$), i.e. Z_t passes to the absorbing state, the process is assumed to be out-of-control.

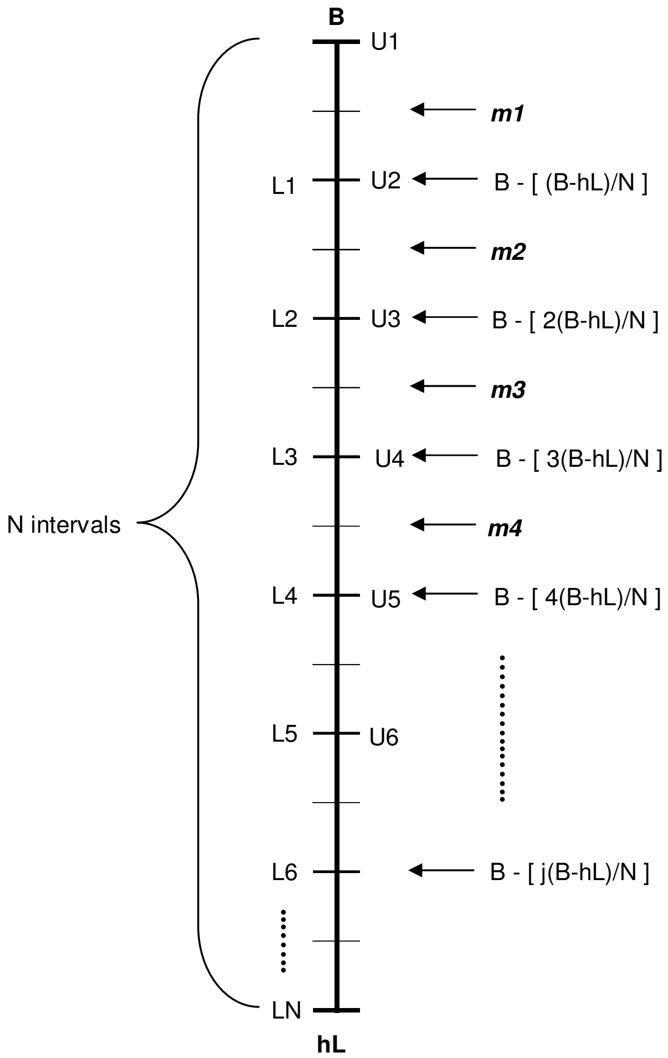

Figure 2: In-control Region Divided into N Subintervals

The number of intervals is determined as 301 to provide enough precision for an accurate approximation. In literature, this number has been taken commonly between 100 and 150 for computational simplicity. It is known that although accuracy can be improved by increasing the number of states, this increment also extends the computation time. However, here N=301 is selected after a trial and error process to provide better accuracy, and since computation times were acceptable.

Here, L_j and U_j be the boundaries of the j^{th} subinterval where,

$$L_j = B\text{-}\left[j \times \frac{(B\text{-}h_L)}{N} \right]$$

$$U_j = B\text{-}\left[(j\text{-}1) \times \frac{(B\text{-}h_L)}{N} \right]$$

and m_i be the midpoint of the i^{th} subinterval such that:

$$m_i = B\text{-}\left[(2i\text{-}1) \times \frac{(B\text{-}h_L)}{2N} \right]$$

As mentioned before, run length (RL) is the number of steps until Z_t reaches the absorbing state from its initial state. The RL distribution of an EWMA is determined by transition probability matrix.

Let P_{ij} be the transition probability of passing from state i to state j. In other words, it is the probability of Z_{t-1} (which is in state i at time t-1) passing to state j at the next time period t. Note that again, the continuous values of the EWMA statistic at time t-1, i.e. Z_{t-1}, may fall anywhere between the upper and lower boundaries of the interval. Using a discrete Markov Chain approximation, it is assumed that Z_{t-1} will be equal to the midpoint of the corresponding interval. So the control statistic Z_t, is equal to m_i whenever and wherever it is in state i. The matrix created from the transition probabilities P_{ij} (i = 1,2,…,N+1, and j = 1,2,…,N+1) is called the transition matrix. The row sum of the transition matrix is 1 and it can be shown as:

$$\begin{array}{r c} & \overbrace{\quad}^{\text{transient states}} \quad \overbrace{\quad}^{\text{absorbing state}(h_L)} \\ \text{transient states} & \left[\begin{array}{c|c} Q & W \\ \hline 0 & 1 \end{array}\right] \\ \text{absorbing state } (h_L) & \end{array}$$

$$P_{ij} = \left[\begin{array}{cccc|c} p_{11} & p_{12} & \cdots & p_{1N} & p_{1(N+1)} \\ p_{21} & p_{22} & \cdots & p_{2N} & p_{2(N+1)} \\ \vdots & \vdots & & \vdots & \vdots \\ p_{N1} & p_{N2} & \cdots & p_{NN} & p_{N(N+1)} \\ \hline 0 & 0 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right\} \text{transient states} \\ \left.\begin{array}{l} \\ \end{array}\right\} \text{absorbing state}(h_L) \end{array}$$

(Column groups: N transient states; absorbing state(h_L))

for $\forall\, i = 1,2,\ldots,N+1$;

$$\sum_{j=1}^{N+1} P_{ij} = 1$$

Transition probabilities:

The transition probabilities can be computed as defined in the following:

First, consider P_{i1}, the probability of moving from state i to the 1st state. Since any value of Z_t larger than B is set equal to B in a lower-sided TBE EWMA chart, the largest statistic always falls between the boundaries of 1st state ($B \geq Z_t \geq L_1$). So, in a lower-sided TBE EWMA control chart, P_{i1} can be written as:

$$P_{i1} = \Pr\left(Z_t \geq L_1 \middle| Z_{t-1} = m_i\right) \qquad i=1,2,3,...,301$$

From state i to state 1 transition probabilities can be written as:

$$P_{i1} = \Pr\left(Z_t \geq L_1 \mid Z_{t-1} = m_i\right)$$

$$= \Pr\left((1-\lambda) Z_{t-1} + \lambda X_t \geq L_1 \mid Z_{t-1} = m_i\right)$$

$$= \Pr\left((1\text{-}\lambda) m_i + \lambda X_t \geq B\text{-}\frac{B\text{-}h_L}{N} \right)$$

$$= \Pr\left((1\text{-}\lambda) B\text{-}\frac{(1\text{-}\lambda)(2i\text{-}1)(B\text{-}h_L)}{2N} + \lambda X_t \geq B\text{-}\frac{B\text{-}h_L}{N} \right)$$

$$= \Pr\left(\lambda X_t \geq B\text{-}\frac{(B\text{-}h_L)}{N}\text{-}(1\text{-}\lambda) B + \frac{(1\text{-}\lambda)(2i\text{-}1)(B\text{-}h_L)}{2N} \right)$$

$$= \Pr\left(X_t \geq B\text{-}\frac{(B\text{-}h_L)}{N\lambda} + \frac{(1\text{-}\lambda)(2i\text{-}1)(B\text{-}h_L)}{2N\lambda} \right)$$

Secondly, transition from state i=1,2,…,301 to state j=2,3,…,301 except the 1st state, transition probabilities can be written as:

$$P_{ij} = \Pr\left(U_j > Z_t > L_j \middle| Z_{t-1} = m_i\right)$$

$$= \Pr\left(B\text{-}\frac{(j\text{-}1)*(B\text{-}h_L)}{N} > (1\text{-}\lambda) Z_{t-1} + \lambda X_t > B\text{-}\frac{j*(B\text{-}h_L)}{N} \mid Z_{t-1} = m_i \right)$$

$$= \Pr\left(B\text{-}\frac{(j\text{-}1)*(B\text{-}h_L)}{N} > (1\text{-}\lambda) m_i + \lambda X_t > B\text{-}\frac{j*(B\text{-}h_L)}{N} \right)$$

$$= \Pr\left(B-\frac{(j-1)*(B-h_L)}{N} > (1-\lambda)\left(B-\frac{(2i-1)*(B-h_L)}{2N} \right) +\lambda X_t > B-\frac{j*(B-h_L)}{N} \right)$$

$$= \Pr\left(B-\frac{(j-1)*(B-h_L)}{N} > (1-\lambda) B - \frac{(1-\lambda)(2i-1)(B-h_L)}{2N} +\lambda X_t > B-\frac{j*(B-h_L)}{N} \right)$$

$$= \Pr\left(\frac{B-\frac{(j-1)*(B-h_L)}{N}-(1-\lambda)B+\frac{(1-\lambda)(2i-1)(B-h_L)}{2N}}{\lambda} > X_t > \frac{B-\frac{j*(B-h_L)}{N}-(1-\lambda)B+\frac{(1-\lambda)(2i-1)(B-h_L)}{2N}}{\lambda} \right)$$

$$= \Pr\left(\begin{array}{l} B-\frac{(j-1)*(B-h_L)}{N\lambda}+\frac{(1-\lambda)(2i-1)(B-h_L)}{2N\lambda} > X_t > \\ B-\frac{j*(B-h_L)}{N\lambda}+\frac{(1-\lambda)(2i-1)(B-h_L)}{2N\lambda} \end{array} \right)$$

Finally, consider $P_{i(N+1)}$, the probability of moving from state i to the absorbing state.

$$P_{i(N+1)} = \Pr\left(Z_t \le h_L \mid Z_{t-1} = m_i \right)$$

$$= \Pr\left((1-\lambda) Z_{t-1} +\lambda X_t \le h_L \mid Z_{t-1} = m_i \right)$$

$$= \Pr\left((1-\lambda) B-\frac{(1-\lambda)(2i-1)(B-h_L)}{2N} +\lambda X_t \le h_L \right)$$

$$= \Pr\left(\lambda X_t \le h_L -(1-\lambda) B+\frac{(1-\lambda)(2i-1)(B-h_L)}{2N} \right)$$

$$= \Pr\left(X_t \le \frac{h_L}{\lambda} - \frac{(1-\lambda)B}{\lambda} + \frac{(1-\lambda)(2i-1)(B-h_L)}{2N\lambda} \right)$$

In the equations above, the transition probabilities are defined in terms of observations X_t, which are assumed to be exponentially distributed with mean μ for an in-control process.

Calculating the ARL:

Consider an $(N+1)\times(N+1)$ matrix **P** of the transition probabilities including the absorbing state. Let $N\times N$ matrix **Q** be the matrix obtained by removing the $(N+1)^{st}$ row and column of P.

$$\mathbf{Q = [P_{ij}] \qquad i , j = 1,2,\ldots,N}$$

Suppose R is an (Nx1) vector such as:

$$\mathbf{R=\left[R_1,R_2,...,R_N\right]'}$$

Given that the initial value of the control statistic equals to the midpoint of state i ($Z_0 = m_i$), let the i^{th} element of the vector **R** be the ARL of this process (R_i = ARL). The vector **R** can be computed by as follows (see for example Brook and Evans, 1972):

$$\mathbf{(I-Q)R=1}$$

$$\mathbf{R= (I-Q)^{-1}1}$$

Here, **I** is an NxN identity matrix and **1** is an Nx1 column vector of ones. Hence,

$$\mathrm{R} = \begin{bmatrix} R_1 \\ \vdots \\ R_N \end{bmatrix} = \left[\begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix}_{NxN} - \begin{bmatrix} p_{11} & \cdots & p_{1N} \\ \vdots & \ddots & \vdots \\ p_{N1} & \cdots & p_{NN} \end{bmatrix}_{NxN} \right]^{-1} \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}_{Nx1}$$

In order to obtain ARL values of the process, the R vector can be calculated from the equation above. The elements of the vector R provides the ARL for corresponding initial states.

Calculating Percentiles of RL:

To calculate the percentiles of run length distribution, the cumulative probabilities of RL are used. Let $\mathbf{F_r}$ be an Nx1 vector of the cumulative probabilities, where:

$$\mathbf{F_r = (I - Q^r)1}$$

Each of the N entries of $\mathbf{F_r}$ is again for one initial value of Z_0. For example, if $Z_0 = m_i$ (initial value is at state i), the i^{th} entry of $\mathbf{F_r}$ gives us the cumulative probability of the run length (White and Keats, 1996). In this study we search for the 10^{th}, 50^{th}, and 90^{th} percentiles of the run length distribution.

To find the 10^{th} percentile, the index r is increased until the targeted entry (i) of $\mathbf{F_r}$ reaches the value of 0.1. When the entry is equal to or greater than 0.1, the corresponding value of r is the 10^{th} percentile of the RL. Similarly, calculations can be done to obtain the 50^{th} and 90^{th} percentiles.

Calculating the Standard Deviation of Run Length (SDRL):

The SDRL is important for assessing the spread of the RL distribution. Let **D** be the Nx1 SDRL vector such that:

$$\mathbf{D} = \left\{\mathbf{R}+2\left[\left(\mathbf{I}-\mathbf{Q}\right)^{-1}-\mathbf{I}\right]\mathbf{R}-\mathbf{R}^{2}\right\}^{0.5}$$

Just like **R** and $\mathbf{F_r}$, i^th entry of **D** corresponds to the initial value $Z_0 = m_i$. It gives us the SDRL (White and Keats, 1996).

Solving equations above, one can obtain the ARLs, 10^th, 50^th, 90^th percentiles and SDRLs for the lower-sided TBE EWMA chart. In this study, the Markov Chain calculations are programmed in Matlab and the code is validated using simulations.

CHAPTER 4

THE DESIGN OF THE ROBUSTNESS EXPERIMENTS

In order to execute the Markov Chain code and obtain the performance metrics, the design parameters for a lower-sided TBE EWMA control chart are required. As mentioned before, the assumed distribution for a TBE EWMA chart is generally defined as exponential in the literature and also in this research. Therefore, in this chapter, for obtaining an exponentially designed lower-sided TBE EWMA chart, the design parameters such as the value of the in-control mean (μ_0), desired in-control Average Run Length (ARL), mean shifts, smoothing constant (λ) values, and boundary (B) values are specified. Then the values of the last unknown parameter, lower control limit (h_L), are determined by using the values of the known parameters. Next, a brief introduction to the Weibull and lognormal distributions is provided. After describing why these distributions are selected for the robustness study, selected parameters for these distributions are discussed.

4.1 Design Parameters for an Exponential Lower-sided TBE EWMA

In designing a lower-sided exponential EWMA control chart, standardized values of the mean, mean shifts, control limit (h_L), the boundary (B) and initial control statistic (Z_0) may be used as proposed in Gan (1998). Standardized values are obtained by dividing the corresponding parameters (Z_0, h_L, B, and mean) by the mean of the observations. A control chart for exponential observations with an in-control mean $\mu_0 = \theta$ and design parameters Z_0, h_L, B, and λ would have the same performance with the standardized control chart with an in control mean $\mu_0 = \theta/\theta = 1$, and design parameters Z_0/θ, h_L/θ, B/θ, and λ. Because, if the observations are exponentially distributed with a mean=θ $\left(X_t \sim \exp(\theta)\right)$, standardized observations are exponentially distributed with mean=1 $\left(\frac{X_t}{\theta} \sim \exp(1)\right)$. That is, the transition probabilities of the actual and the standardized observations are the same:

For example if λ is taken as 1, the control statistic ($Z_t = (1-\lambda)Z_{t-1} + \lambda X_t$) turns into $Z_t = X_t$ and the transition probabilities:

For $X_t \sim \exp(\theta)$,

$$P(X \le t) = 1 - e^{-t}$$

For $\frac{X_t}{\theta} \sim \exp(1)$,

$$P\left(\frac{X}{\theta} \le t\right) = P(X \le \theta t)$$

$$= 1 - e^{-\theta t/\theta} = 1 - e^{-t}$$

When the standardized parameters are multiplied with a constant, the performance of this design is also equivalent to the performance of the standardized design. For example, a lower-sided TBE EWMA chart with parameters λ, B, h_L, Z_0, and μ_0=1 has the same ARL performance with the chart with parameters λ, $5\times B$, $5\times h_L$, $5Z_0$, and μ_0=5 (Gan, 1998). For example again λ is taken as 1, the control statistic ($Z_t = (1-\lambda)Z_{t-1} + \lambda X_t$) turns into $Z_t = X_t$ and the transition probabilities:

For $X_t \sim \exp(1)$, and the design parameters B, h_L, and Z_0;

$$P(X_t \le h_L) = 1 - e^{-h_L}$$

For $X_t \sim \exp(5)$ and the design parameters 5B, 5 h_L and $5Z_0$;

$$P(X_t \le 5h_L) = 1 - e^{-5h_L/5}$$

$$= 1 - e^{-h_L}$$

Hence, standardized parameters can be used even if the mean is not 1. Borror, Keats, Montgomery (2003) also prefer to use standardized parameters in their study. Without loss of generality, in this research we use standardized parameter values for an in-control mean which is equal to 1($\mu_0 = 1$).

To evaluate the performance of the charts, out-of-control process scenarios are needed. Here, out-of-control process is modeled in terms of shifts in the mean of exponentially distributed time-between-events observations. Since standardized parameters are considered in the design of the charts, mean shifts should also be defined in terms of the standardized in-control mean which is 1. Standardized shifted means considered in the study are 0.95, 0.9, 0.8, 0.7, 0.6, 0.5 and 0.2. These shifts also correspond to 1.05, 1.11, 1.25, 1.43, 1.67, 2 and 5, respectively, in terms of standardized defect rates. For example, if the defects are Poisson distributed with mean rate 1, a shift in the defect rate to 5 corresponds to $1/5 = 0.2$ in terms of the standardized exponential mean shift (TBE mean shift). As a reference, a cross tabulation is provided in Table 1. Since we are studying an exponentially distributed EWMA chart which is a TBE control chart, TBE mean shifts are utilized in this research. These mean shifts are quite close to, but more detailed than the values commonly preferred by many researchers in the literature (see for example, Borror, Keats, Montgomery, 2003).

Table 1: Cross Tabulation of Mean Shifts

TBE mean shift	Defect mean shift
0,95	1,05
0,9	1,11
0,8	1,25
0,7	1,43
0,6	1,67
0,5	2
0,2	5

Smoothing constant λ is also a very important design parameter that can be used to tune the chart for specified shift magnitudes. Therefore, a broad range of λ values is considered for investigating the effect of this parameter on the performance of the charts. More specifically, considered λ values are 0.01, 0.05, 0.1, 0.2, 0.4, 0.6, 0.8, and 1. Note that, a λ value of 1 for the EWMA corresponds to a Shewhart control chart. The smallest λ value used in the literature is usually 0.1, however we prefer to use smaller values in order to perform more detailed investigation on robustness.

Although Gan (1998) has suggested a value of 2 for the parameter B in the design of the charts, we have considered several B values; B = 1, B = 2, B = 5 for observing their effect on robustness.

Determining Lower Control Limit (h_L)

The lower-sided exponential EWMA control charts studied in this research are designed so that they all have the same ARL_0 (in-control ARL) performance. Hence, ARL_1 (out-of-control ARL) performances may be fairly compared. For a lower-sided TBE EWMA control chart, lower control limit, h_L, can be determined in combination with the selected design parameters to obtain approximately the same target in-control ARL. A practical value for the target in-control ARL is defined to be 500 (ARL_0=500) for each parameter combination. Note that these designs will also be used under Weibull and lognormal distributions.

To obtain the control limit, Markov Chain calculations were coded in Matlab and a search is performed for the control limit values to yield the desired ARL_0 value for each parameter combination. In Tables 2-4, h_L values for each of the λ and B combinations are provided.

Table 2: λ and h_L combinations yielding the same ARL_0=500, when B=1

λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
h_L	0,871	0,6561	0,5176	0,3577	0,1921	0,1026	0,0471	0,002
B	1	1	1	1	1	1	1	1
Zo	1	1	1	1	1	1	1	1

Table 3: λ and h_L combinations yielding the same ARL_0=500, when B=2

λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
h_L	0,901446	0,68607	0,545071	0,37932	0,204487	0,109064	0,049218	0,002002
B	2	2	2	2	2	2	2	2
Zo	1	1	1	1	1	1	1	1

Table 4: λ and h_L combinations, yield the same ARL_0=500, when B=5

λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
h_L	0,876086	0,685314	0,544453	0,379277	0,205282	0,1098005	0,049686	0,002002
B	5	5	5	5	5	5	5	5
Zo	1	1	1	1	1	1	1	1

Gan (1998) also provided an h_L versus λ plot for determining the lower control limits for some ARL_0 values. Some values obtained from this plot were used to verify the computer program for the Markov Chain computations. Small differences between Gan's and our results are likely and certainly expected since different approaches have been used for calculating the ARL values in two studies. While Gan used differential equations, we consider Markov chain approach. But those differences are approximately 0.04% and quite insignificant.

After all the parameters of the chart have been determined and the chart has been designed, the next step is to calculate the performance metrics (ARLs, SDRLs, percentiles) of the lower sided TBE EWMA chart for the two cases. First case occurs when the chart is designed under the assumption of exponential distribution, and the true underlying distribution is also exponential. The other case occurs when the chart is designed under the assumption of exponential distribution, but in fact the observations follow a distribution different from exponential. For this robustness study, we decided to use several Weibull and lognormal distributions since these distributions' parameters can be determined such that moments and shape of the distributions are similar to exponential distribution. Same distributions were also considered in Borror, Keats, Montgomery (2003). The next two sections provide background information about Weibull and lognormal distributions and describe how we use them in our robustness study.

4.2 Distributions Considered in Robustness Study

4.2.1 Weibull Distribution

The Weibull Distribution is one of the most widely used statistical models for the analysis of reliability problems. Because of its flexibility, it may assume shapes similar to some other distributions, such as exponential and normal. Weibull distribution is a continuous probability distribution and its density function can be written as:

$$f(x) = \begin{cases} \frac{\beta}{\alpha} x^{\beta-1} e^{-x^{\beta}/\alpha} & x>0 \\ 0 & \text{elsewhere} \end{cases}$$

Here, $\beta > 0$ is the shape parameter and $\alpha > 0$ is the scale parameter of the distribution. It is such a flexible distribution that the characteristics of some other distributions can be obtained by revaluating its shape parameter β. From Figure 3, one can observe how the value of the shape parameter β affects the shape of the Weibull distribution. Exponential is a special case of Weibull and can be obtained when $\beta = 1$. As β increases, distribution becomes similar to the normal distribution, and for $\beta = 3.4$ Weibull turns into a normal distribution.

As mentioned before, time between events (TBE) distribution is assumed exponential. In the following sections, we investigate how the performance of lower-sided TBE EWMA control chart is affected when the chart is designed under the assumption of exponential distribution, but in fact the observations follow a Weibull distribution. For this robustness study, Weibull distributions with various β values are considered. The values of β are determined as 1.2, 1.5, 2, 2.5, and 4. The shapes of the Weibull distribution designed with these β values and departures from the assumed exponential distribution are shown in Figure 3.

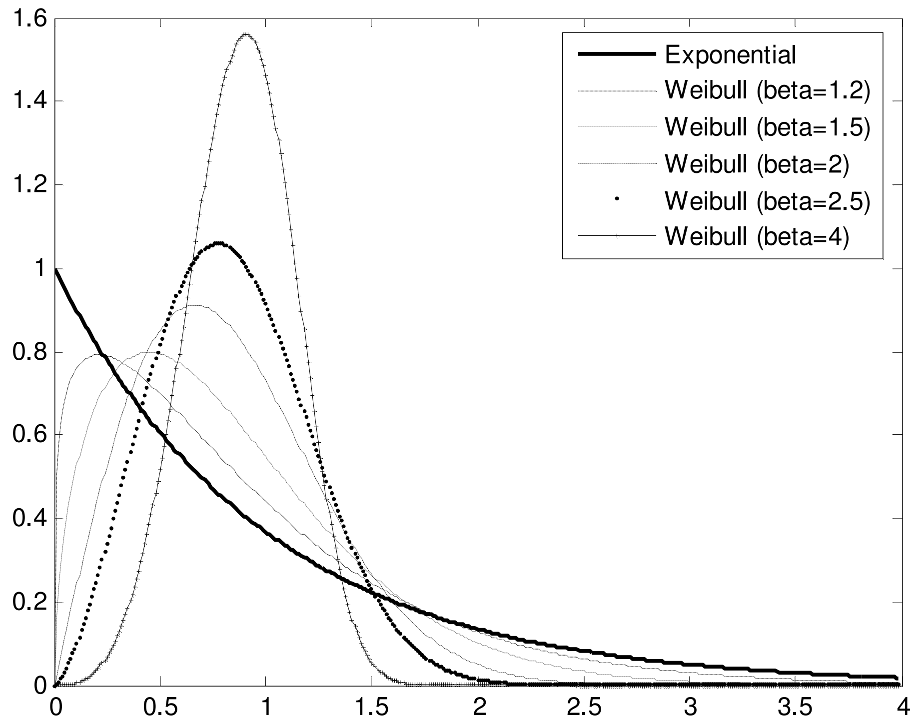

Figure 3: Plot of exponential and considered Weibull distributions with mean=1

After determining the values of the shape parameter β, the values of the scale parameter, α, is selected such that the mean of the Weibull distribution is equal to $1(\mu_0 = 1)$ when the process is in-control and is equal to the considering shift magnitude when the process is out-of-control. By standardizing and equalizing the mean of the distributions, the variations occurring in the performance metrics are assured to be sourced from only the distributions themselves. The determined values of β and α combinations can be observed in Tables 5-10 below:

Table 5: α values when β = 1(exponential case)

β	α	μ	σ
1	1	1	1
1	0,95	0,95	0,9025
1	0,9	0,9	0,81
1	0,8	0,8	0,64
1	0,7	0,7	0,49
1	0,6	0,6	0,36
1	0,5	0,5	0,25
1	0,2	0,2	0,04

Table 6: α values when β = 1.2

β	α	μ	σ
1,2	1,0631	1	0,7004
1,2	1,0099	0,95	0,6321
1,2	0,9568	0,9	0,5674
1,2	0,8505	0,8	0,4483
1,2	0,7442	0,7	0,3432
1,2	0,6379	0,6	0,2522
1,2	0,5315	0,5	0,1751
1,2	0,2126	0,2	0,028

Table 7: α values when β = 1.5

β	α	μ	σ
1,5	1,1077	1	0,461
1,5	1,0524	0,95	0,4161
1,5	0,997	0,9	0,3734
1,5	0,8862	0,8	0,295
1,5	0,7754	0,7	0,2259
1,5	0,6646	0,6	0,1659
1,5	0,5539	0,5	0,1153
1,5	0,2216	0,2	0,0184

Table 8: α values when β = 2

β	α	μ	σ
2	1,1284	1	0,2732
2	1,072	0,95	0,2466
2	1,0155	0,9	0,2213
2	0,9027	0,8	0,1749
2	0,7899	0,7	0,1339
2	0,677	0,6	0,0984
2	0,5642	0,5	0,0683
2	0,2257	0,2	0,0109

Table 9: α values when β = 2.5

β	α	μ	σ
2,5	1,1271	1	0,1831
2,5	1,0707	0,95	0,1652
2,5	1,0144	0,9	0,1483
2,5	0,9017	0,8	0,1172
2,5	0,7889	0,7	0,0897
2,5	0,6762	0,6	0,0659
2,5	0,5635	0,5	0,0458
2,5	0,2254	0,2	0,0073

Table 10: α values when β = 4

β	α	μ	σ
4	1,1033	1	0,0787
4	1,0481	0,95	0,071
4	0,9929	0,9	0,0637
4	0,8826	0,8	0,0504
4	0,7723	0,7	0,0386
4	0,662	0,6	0,0283
4	0,5516	0,5	0,0197
4	0,2206	0,2	0,0031

4.2.2 Lognormal Distribution

The lognormal distribution is also a widely used model for continuous random quantities and used especially in reliability applications to model failure times.

A random variable X is said to be lognormally distributed with mean μ and standard deviation σ if the natural logarithm of this variable (ln(X)) is normally distributed with mean *m* and standard deviation *s*. Lognormal probability density function can be written as:

$$f(x) = \begin{cases} \dfrac{\exp\left(-\dfrac{1}{2}\left(\dfrac{\log(x) - m}{s} \right)^2 \right)}{xs\sqrt{2\pi}} & x > 0 \\ 0 & otherwise \end{cases}$$

Here, the parameter *m* is the scale parameter and can be any real number; the parameter *s* is the shape parameter and must be a positive real number. The mean and standard deviation of the lognormal distribution are defined as:

$$\mu = \exp\left(\left[2m + s^2\right]/2\right)$$

$$\sigma = \sqrt{\exp\left[2m + 2s^2\right] - \exp\left[2m + s^2\right]}$$

Again the values of *m* and *s* are selected such that this combination makes the mean of the lognormal distribution μ equal to the mean of the exponential distribution (which is 1 when the process is in-control and is equal to the considering mean shift when the process is out-of-control) The determined values of *m* and *s* combinations are provided in Tables 11-14 below:

Table 11: *m* values when *s* = 0.94

m	*s*	μ	σ
-0,4418	0,94	1	1,4196
-0,4931	0,94	0,95	1,2812
-0,5472	0,94	0,9	1,1498
-0,665	0,94	0,8	0,9084
-0,7985	0,94	0,7	0,6956
-0,9527	0,94	0,6	0,5110
-1,135	0,94	0,5	0,3549
-2,051	0,94	0,2	0,0568

Table 12: *m* values when *s* = 0.974

m	*s*	μ	σ
-0,47438	0,974	1	1,5822
-0,5256	0,974	0,95	1,4281
-0,5797	0,974	0,9	1,2816
-0,6975	0,974	0,8	1,0126
-0,831	0,974	0,7	0,7753
-0,9851	0,974	0,6	0,5697
-1,1675	0,974	0,5	0,3956
-2,084	0,974	0,2	0,0633

Table 13: *m* values when *s* = 1.092

m	*s*	μ	σ
-0,5962	1,092	1	2,2953
-0,6475	1,092	0,95	2,0715
-0,7016	1,092	0,9	1,8591
-0,8194	1,092	0,8	1,4689
-0,9529	1,092	0,7	1,1247
-1,107	1,092	0,6	0,8264
-1,2894	1,092	0,5	0,5738
-2,206	1,092	0,2	0,0917

Table 14: *m* values when *s* = 1.182

m	*s*	μ	σ
-0,6986	1,182	1	3,0433
-0,7499	1,182	0,95	2,7466
-0,8039	1,182	0,9	2,4654
-0,9217	1,182	0,8	1,9479
-1,0552	1,182	0,7	1,4915
-1,2094	1,182	0,6	1,0957
-1,3917	1,182	0,5	0,7609
-2,308	1,182	0,2	0,1217

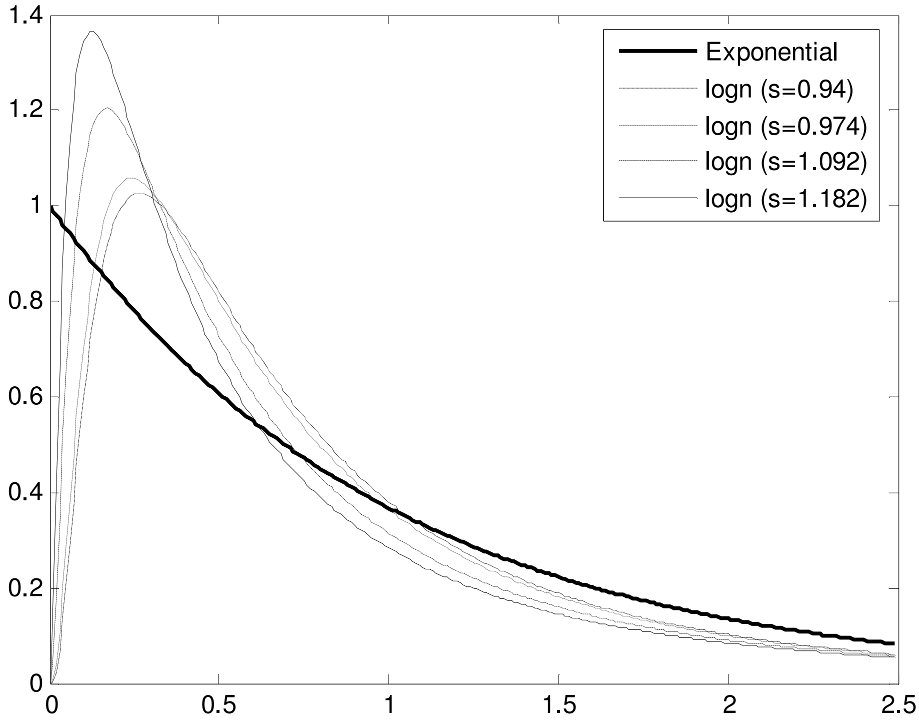

Figure 4: Plot of exponential and considered lognormal distributions with mean=1

In Figure 4, exponential and lognormal distributions with mean = 1, constructed with determined *m* and *s* values, are plotted. It can be seen that the lognormal distribution is positively skewed and the skewness increases as *s* increases.

CHAPTER 5

RESULTS AND DISCUSSION

In the first section of this chapter, performance results obtained using the Markov Chain approach are displayed. Computed in-control and out-of-control performance metrics (ARLs, SDRLs and percentiles) for exponential, Weibull and lognormal distributions are presented in several tables. In the second section, these computed performance metrics are analyzed and compared in order to investigate the robustness of the lower-sided TBE EWMA control chart to non-exponential observations.

5.1 Performance Results

As noted previously, TBE EWMA charts are designed under the assumption of exponentially distributed observations. Hence, the performance metrics for the lower-sided TBE EWMA chart are first provided for the exponential case (control case). Next, performance results for the test cases with Weibull and lognormal distributed observations are presented.

5.1.1 Performance Results for the Exponential Case

Assume that the true underlying distribution of the observations is exponential with an in-control mean 1 ($\mu_0 = 1$) and standard deviation 1 ($\sigma = 1$). Consider the parameter combinations (λ, B, h_L) provided in Tables 2-4 for the lower-sided TBE EWMA chart for the exponential observations. These designs are used as the parameters of the Markov Chain approach in computing the performance metrics, where the in-control case is $\mu_0 = 1$ and the out-of-control cases are $\mu_1 = 0.2, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95$. Calculated in-control and out-of-control performance metrics are classified according to the considered boundary (B) values. The results for the B = 2 case are provided in Table 15. The other cases for B = 1 and B = 5 are presented in Appendix A. Note that here the B = 2 case is specifically selected and will be discussed in the following section.

Table 15: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B = 2) when true underlying distribution is exponential

mean	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,901446	0,68607	0,545071	0,37932	0,204487	0,109064	0,049218	0,002002
	B	2	2	2	2	2	2	2	2
	Z_0	1	1	1	1	1	1	1	1
1.00									
ARL_0		500,00	500,04	500,00	500,00	500,00	500,01	500,00	500,00
SDRL		497,62	486,69	489,74	493,08	495,62	496,89	497,63	499,50
10%		68	65	62	59	57	55	55	53
50%		340	351	350	349	348	348	347	347
90%		1149	1134	1138	1142	1146	1147	1148	1151
0.95									
ARL_1		233,90	282,28	313,42	349,20	388,14	412,28	431,82	475,03
SDRL		210,10	265,93	301,98	341,86	383,63	409,10	429,42	474,53
10%		50	45	43	43	45	46	48	50
50%		167	201	221	244	270	287	300	329
90%		507	629	707	794	888	945	991	1093
0.90									
ARL_1		134,61	170,05	201,46	244,62	299,54	337,60	370,58	450,05
SDRL		105,07	152,22	189,27	236,93	294,91	334,37	368,17	449,55
10%		40	34	32	33	36	38	41	48
50%		103	124	143	172	209	235	258	312
90%		271	368	448	553	684	773	850	1036
0.80									
ARL_1		65,15	74,87	91,08	122,14	175,43	221,37	267,21	400,10
SDRL		37,10	56,90	78,40	114,05	170,62	218,05	264,75	399,60
10%		29	23	21	20	23	26	30	43
50%		56	58	67	87	123	154	186	277
90%		114	149	193	271	398	505	612	921
0.70									
ARL_1		41,25	41,20	47,15	63,46	100,76	140,54	186,48	350,15
SDRL		17,29	24,80	34,92	55,30	95,82	137,13	183,99	349,65
10%		23	18	15	14	15	18	22	37
50%		37	35	37	47	71	98	130	243
90%		64	73	93	135	226	319	426	806
0.60									
ARL_1		29,86	26,67	27,94	35,03	57,09	86,10	125,13	300,20
SDRL		9,34	12,19	16,66	27,09	52,12	82,64	122,61	299,70
10%		20	15	12	11	10	12	15	32
50%		28	24	23	27	41	61	88	208
90%		42	43	50	70	125	194	285	691
0.50									
ARL_1		23,34	19,24	18,59	20,91	32,26	50,78	80,00	250,25
SDRL		5,45	6,51	8,42	13,40	27,34	47,29	77,45	249,75
10%		17	12	10	9	8	8	11	27
50%		22	18	16	17	24	36	56	174
90%		31	28	30	38	68	112	181	576
0.20									
ARL_1		14,14	10,22	8,55	7,24	6,94	8,48	13,40	100,40
SDRL		1,16	1,08	1,18	1,50	2,73	5,21	10,88	99,90
10%		13	9	7	6	4	4	4	11
50%		14	10	8	7	6	7	10	70
90%		16	12	10	9	10	15	28	231

5.1.2 Performance Results for the Weibull Case

As a second case, assume that the true underlying distribution of the observations is not exponential but Weibull for the lower-sided TBE EWMA chart designed under the assumption of exponential distribution. Several Weibull distributions the shape and scale parameter combinations (β,α) of which are presented in Tables 6-10 are considered as test cases. As the shape parameters, β = 1.2, 1.5, 2, 2.5, and 4 are selected to represent the departures from exponential with increasing β values.

In order to obtain performance metrics of the chart for the Weibull case, the transition probabilities of the Markov Chain code is modified to account for the Weibull distribution. However, the design parameter combinations (λ, h_L, B), which are determined for exponential distributed observations, are unchanged. In other words, the same design parameter combinations are used in the code in order to experience the performance of a chart designed under the assumption of exponential distribution but when the true underlying distribution is Weibull. In-control and out-of-control performance metrics are obtained by executing the modified Markov Chain code for the considered cases (parameter combinations of β and α, and for each of the design parameter combinations λ, h_L, B). In order to obtain the in-control performance metrics, (β,α) combinations that yield a mean of 1 for the Weibull distribution are selected as (β = 1.2, α = 1.0631), (β = 1.5, α = 1.1077), (β = 2, α = 1.1284), (β = 2.5, α = 1.1271) (β = 4, α = 1.1033). Furthermore, out-of-control performance metrics are evaluated for the means shifted to 0.95, 0.9, 0.8, 0.7, 0.6, 0.5, and 0.2. Note that, to obtain the out-of-control performance metrics for the mean shifts, (β,α) combinations yielding the desired shifted mean values for the Weibull distribution are selected. For example, for a shifted mean value of 0.5, the following parameter combinations are used (β = 1.2, α = 0.5315), (β = 1.5, α = 0.5539), (β = 2, α = 0.5642), (β = 2.5, α = 0.5635), (β = 4, α = 0.5516). Table 16 presents the in-control and out-of-control ARLs, SDRLs, and percentiles computed for the TBE EWMA design with B = 2 when the observations follow a Weibull distribution with β = 1.5. For ease of readability, the remaining 14 tables are provided in Appendix B.

Table 16: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B=2) when true underlying distribution is Weibull (β=1.5)

	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,901446	0,68607	0,545071	0,37932	0,204487	0,109064	0,049218	0,002002
	B	2	2	2	2	2	2	2	2
mean	Z_0	1	1	1	1	1	1	1	1
1.00									
ARL_0		1536,2	6195,2	10.615	15.647	19.233	19.483	18.207	13.015
SDRL		1499	6171,8	10.600	15.638	19.228	19.480	18.204	13.015
10%		197	674	1132	1656	***	***	***	1372
50%		1075	4301	7362	***	***	***	***	***
90%		3489	***	***	***	***	***	***	***
0.95									
ARL_1		381,74	1831,4	4076,9	7840,5	12.187	13.971	14.277	12.053
SDRL		321,62	1802,5	4059,8	7830,9	12.182	13.968	14.275	12.052
10%		91	219	445	835	1289	***	***	1270
50%		286	1278	2831	5438	***	***	***	***
90%		800	4179	9365	***	***	***	***	***
0.90									
ARL_1		165,56	629,62	1628,6	3918,5	7621,1	9888,8	11.072	11.114
SDRL		110,77	598,58	1610,1	3908,2	7615,6	9885,2	11.069	11.113
10%		60	94	188	422	808	1045	***	***
50%		136	446	1135	2719	5284	***	***	***
90%		310	1409	3726	9009	***	***	***	***
0.80									
ARL_1		67,73	128,40	313,46	989,97	2869,5	4756,4	6424,9	9313,8
SDRL		28,99	99,89	294,12	978,87	2863,6	4752,6	6422,3	9313,3
10%		37	38	50	114	308	505	679	982
50%		62	99	223	690	1991	3298	***	***
90%		106	258	697	2265	6600	***	***	***
0.70									
ARL_1		41,41	50,00	86,60	264,99	1027,4	2152,9	3528,2	7622,9
SDRL		12,39	27,09	68,81	253,68	1021,3	2149	3525,5	7622,4
10%		28	24	25	38	114	230	374	804
50%		39	43	66	187	714	1494	2446	***
90%		58	85	176	595	2358	4952	8121	***
0.60									
ARL_1		29,71	28,52	35,96	81,42	351,77	908,32	1811,4	6049
SDRL		6,51	10,41	20,83	70,59	345,52	904,24	1808,6	6048,5
10%		22	18	16	18	43	99	193	638
50%		29	26	30	60	246	631	1256	***
90%		38	42	63	173	802	2086	4167	***
0.50									
ARL_1		23,17	19,67	20,40	31,53	117,62	353,56	854,99	4602,5
SDRL		3,76	4,94	7,90	21,75	111,42	349,42	852,16	4602
10%		19	14	12	12	18	41	93	485
50%		23	19	19	25	83	246	594	***
90%		28	26	31	60	263	809	1965	***
0.20									
ARL_1		14,25	10,20	8,55	7,28	7,68	13,17	38,27	1165,1
SDRL		0,81	0,77	0,84	1,14	2,90	9,37	35,45	1164,6
10%		13	9	8	6	5	5	7	123
50%		14	10	8	7	7	10	27	808
90%		15	11	10	9	11	25	84	2682

*** indicates very large values for the percentiles where the computations were truncated

5.1.3 Performance Results for the Lognormal Case

In the third and the last case, assume that the true underlying distribution of the observations is again not exponential but lognormal for the lower-sided TBE EWMA chart designed under the assumption of exponential distribution. Several lognormal distributions are considered with the shape and scale parameter combinations (*s, m*) presented in Tables 11-14. The shape parameters are determined as *s* = 1.182, *s* = 1.092, *s* = 0.974, and *s* = 0.94.

In order to obtain performance metrics for the lognormal case, similar to the Weibull case, the transition probabilities of the Markov Chain code is modified to account for the lognormal distribution. However, the design parameter combinations (λ, h_L, B) determined for the exponential distributed observations still remain unchanged.

In-control and out-of-control performance metrics are obtained by executing the modified Markov Chain code for the considered cases (for the determined parameter combinations of (*s, m*), and for each of the design parameter combinations (λ, hl, B). In order to obtain in-control performance metrics, (*s, m*) combinations that yield a mean of 1 for the lognormal distribution are selected as (*s* = 0.94, *m* = -0.4418), (*s* = 0.974, *m* = -0.47438), (*s* = 1.092, *m* = -0.5962), (*s* = 1.182, *m* = -0.6986). The Markov Chain code, executed with those parameters, gives the in-control ARLs, SDRLs, and percentiles of the chart. Moreover, out-of-control performance metrics are evaluated for the means shifted to 0.95, 0.9, 0.8, 0.7, 0.6, 0.5, and 0.2. Note that, to obtain the out-of-control performance metrics for the mean shifts, (*s,m*) combinations yielding the desired shifted mean values for the lognormal distribution are selected. For example, in order to obtain out-of-control performance metrics for a shifted mean value 0.7, the following parameter combinations are used (*s* = 0.94, *m* = -0.7985), (*s* = 0.974, *m* = -0.831), (*s* = 1.092, *m* = -0.9529), (*s* = 1.182, *m* = -1.0552). Table 17 presents the in-control and out-of-control performance metrics (ARLs, SDRLs, and 10^{th}, 50^{th}, 90^{th} percentiles) computed for the lower-sided TBE EWMA design of B = 2 under lognormal distribution (*s* = 0.94). For the ease of readability, the rest of the tables, which present the results for the design of B = 1, and B = 5 under various lognormal distributions, are provided in Appendix C.

Table 17: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B = 2) when true underlying distribution is Lognormal (s = 0.94)

mean	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,901446	0,68607	0,545071	0,37932	0,204487	0,109064	0,049218	0,002002
	B	2	2	2	2	2	2	2	2
	Z_0	1	1	1	1	1	1	1	1
1.00									
ARL_0		399,50	430,09	572,91	1.093,5	4.300,8	19.702	141.310	2.427.000.000
SDRL		405,82	414,99	560,27	1.084,4	4.295,0	19.698	141.310	2.427.000.000
10%		56	59	72	123	***	***	***	***
50%		264	303	401	761	***	***	***	***
90%		929	971	1303	2506	***	***	***	***
0.95									
ARL_1		203,94	242,26	331,78	651,57	2.658,3	12.544	92.665	1.723.800.000
SDRL		189,37	225,29	318,50	642,29	2.652,4	12.540	92.663	1.723.800.000
10%		44	41	47	77	***	***	***	***
50%		141	173	234	454	***	***	***	***
90%		450	536	747	1488	***	***	***	***
0.90									
ARL_1		123,86	147,69	201,19	395,34	1.640,7	7.918,2	60.031	1.205.500.000
SDRL		101,96	130,06	187,65	385,92	1.634,8	7.914,0	60.028	1.205.500.000
10%		36	31	33	50	178	***	***	***
50%		91	108	144	277	1139	***	***	***
90%		256	317	446	898	3770	***	***	***
0.80									
ARL_1		63,12	67,86	85,64	156,02	626,34	3.076,8	24.236	559.540.000
SDRL		39,48	50,90	72,46	146,62	620,30	3.072,5	24.233	559.540.000
10%		28	22	21	25	71	***	***	***
50%		52	53	64	111	436	***	***	***
90%		114	134	180	347	1434	***	***	***
0.70									
ARL_1		40,80	38,85	44,00	69,07	242,75	1.156,6	9.232,5	238.840.000
SDRL		19,31	23,58	31,84	60,05	236,73	1.152,3	9.229,5	238.840.000
10%		23	17	15	15	31	126	***	***
50%		36	32	35	51	170	803	***	***
90%		66	70	85	147	551	2658	***	***
0.60									
ARL_1		29,86	25,81	26,48	34,91	97,60	421,87	3.285,7	91.552.000
SDRL		10,72	12,28	15,54	26,53	91,73	417,58	3.282,7	91.552.000
10%		19	14	12	11	16	48	***	***
50%		27	22	22	27	69	294	***	***
90%		44	42	47	69	217	966	***	***
0.50									
ARL_1		23,51	18,90	17,96	20,19	42,05	151,04	1.082	30.480.000
SDRL		6,35	6,88	8,20	12,56	36,46	146,81	1.079	30.480.000
10%		17	12	10	9	9	20	117	***
50%		22	17	16	16	31	106	751	***
90%		32	28	29	37	90	342	2488	***
0.20									
ARL_1		14,47	10,20	8,52	7,17	6,78	8,87	22,87	210.580
SDRL		1,32	1,23	1,32	1,55	2,56	5,39	20,06	210.580
10%		13	9	7	6	4	4	5	***
50%		14	10	8	7	6	7	17	***
90%		16	12	10	9	10	16	49	***

*** indicates very large values for the percentiles where the computations were truncated

5.2 DISCUSSION

Robustness of the lower-sided TBE EWMA chart can be evaluated by comparing the in-control and out-of-control performance metrics computed for both the control and the test cases.

First, in-control and out-of-control performances are compared, respectively, in the following two sections. Next, by considering both the in-control and out-of control performance results, robustness of the lower-sided TBE EWMA chart is evaluated. Finally, design parameters to provide an almost distribution-free TBE EWMA will be discussed.

5.2.1 In-control Performance Comparisons

It has been noted in the literature that the in-control performance of the control charts may be affected significantly when the underlying distribution deviates from the assumed distribution of the observations. In this section, the effect of misspecified distributions on the in-control performance of the TBE EWMA chart, specifically on the ARL_0 performance is shown.

For comparison purposes, the ARL_0 values for the considered exponential, Weibull and lognormal distributions are summarized in Tables 18-20 for the cases of B = 1, B = 2, and B = 5, respectively. For the detailed results with SDRLs and percentiles, interested readers are referred to Appendix A, B, C.

Note that control charts are designed to yield an in-control ARL value of 500 (approximately) under the exponential distribution. Here, the target ARL_0 value of 500 for the exponential distribution is compared with the ARL_0 values computed for the Weibull and lognormal distribution cases.

Table 18: In-control ARLs for exponential, various Weibull and various lognormal distributions when the chart is designed with B = 1

Design Parameters	EWMA							Shewhart
λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
h_L	0,871	0,6561	0,5176	0,3577	0,1921	0,1026	0,0471	0,002
B	1	1	1	1	1	1	1	1
Z_o	1	1	1	1	1	1	1	1
Exponential	500	500	500	500	500	500	500	500
Weibull								
β=1,2	1.029,9	1.617,9	1.911,2	2.142,3	2.253,6	2.237,5	2.148,1	1.865,4
β=1,5	3.358,5	12.020	18.103	22.962	24.097	22.454	19.429	13.035
β=2	35.457	588.870	1.220.200	1.648.200	1.463.500	1.120.800	767.190	318.320
β=2,5	617.790	48.263.000	121.640.000	150.130.000	98.065.000	57.870.000	30.248.000	7.539.300
β=4	3,93E+10	1,98E+14	4,77E+14	2,33E+14	3,84E+13	8,72E+12	1,88E+12	9,26E+10
lognormal								
s=1,182	153,71	116,91	116,13	139,71	255,34	571,15	1.690,2	653.350
s=1,092	201,67	174,29	192,00	270,82	637,42	1.788,9	6.680,8	7.478.500
s=0,974	311,64	352,60	477,96	909,19	3.324,3	13.596	74.766	528.830.000
s=0,94	361,65	456,49	670,72	1.425,1	6.086,9	28.354	178.480	2.443.300.000

Table 19: In-control ARLs for exponential, various Weibull and various lognormal distributions when the chart is designed with B = 2

Design Parameters	EWMA							Shewhart
λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
h_L	0,901446	0,68607	0,545071	0,37932	0,204487	0,109064	0,049218	0,002002
B	2	2	2	2	2	2	2	2
Z_o	1	1	1	1	1	1	1	1
Exponential	500	500	500	500	500	500	500	500
Weibull								
β=1,2	792,15	1263,8	1558,2	1848,8	2069,4	2122,1	2095,3	1863,2
β=1,5	1536,2	6195,2	10.615	15.647	19.233	19.483	18.207	13.015
β=2	4.981,6	141.310	401.100	750.030	915.020	828.490	668.560	317.690
β=2,5	19.775	5.081.400	21.947.000	45.460.000	47.965.000	36.303.000	24.406.000	7.520.500
β=4	3.890.300	1,40E+12	1,30E+13	2,05E+13	8,99E+12	3,35E+12	1,20E+12	9,22E+10
Lognormal								
s=1,182	224,99	143,77	129,93	139,77	229,44	488,52	1.506,8	650.670
s=1,092	274,33	199,99	200,37	252,43	535,29	1.441,0	5.763,9	7.442.200
s=0,974	364,89	356,22	437,03	736,18	2.461,2	9.829,5	60.616	525.550.000
s=0,94	399,50	430,09	572,91	1.093,5	4.300,8	19.702	141.310	2.427.000.000

Table 20: In-control ARLs for exponential, various Weibull and various lognormal distributions when the chart is designed with B = 5

Design Parameters	EWMA							Shewhart
λ	**0,01**	**0,05**	**0,1**	**0,2**	**0,4**	**0,6**	**0,8**	**1**
h_L	**0,876086**	**0,685314**	**0,544453**	**0,379277**	**0,205282**	**0,1098005**	**0,0496857**	**0,002002**
B	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**
Z_o	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
Exponential	500	500	500	500	500	500	500	500
Weibull								
β=1,2	1.648,1	1.257,5	1.550	1.833,9	2.031,5	2.079,1	2.059,8	1.863,2
β=1,5	16.391	5.972,9	10.406	15.371	18.503	18.646	17.514	13.015
β=2	1.137.200	119.650	375.830	721.110	856.290	770.800	625.590	317.690
β=2,5	29.571.000	3.429.100	19.111.000	42.385.000	43.615.000	32.938.000	22.323.000	7.520.500
β=4	7,43E+07	2,73E+11	7,62E+12	1,65E+13	7,38E+12	2,79E+12	1,02E+12	9,22E+10
Lognormal								
s=1,182	217,24	146,77	132,73	144,82	235,27	492,79	1.490,1	650.670
s=1,092	371,68	203,46	203,41	259,26	542,33	1.434,7	5.619,9	7.442.200
s=0,974	1.178,7	354,44	435,30	746,17	2.441,5	9.558,8	57.634	525.550.000
s=0,94	1.930,2	432,46	576,17	1.103,6	4.234,4	18.997	133.150	2.427.000.000

5.2.1.1 In-control Performance Comparison for Weibull Cases

Tables 18-20 indicate that all the in-control ARL values computed for the Weibull distributions are higher than 500 for each design parameter combination of λ, h_L, B. In some cases, where the shape parameter β and the smoothing constant λ get larger, ARL_0 values may take extremely high values such as billions. Note that similar results were also observed by Borror, Keats, and Montgomery (2003) in the study of robustness for the TBE CUSUM chart.

As β increases, the variance of the considered Weibull distribution decreases. Since the in-control observations fluctuate with a smaller variance in the case of larger β values, the possibility of an in-control observation to gets close to or exceed the control limit decrease. Therefore, false alarm rates decrease and ARL_0 values increase. For example, in Table 18, for $\lambda = 0.01$ and for the $\beta = 1.2$ case, the ARL_0 value is computed as 1029. However, as β increases to $\beta = 4$, the ARL_0 value increases extremely and takes the value of $39{,}33\text{x}10^9$.

In-control ARLs also increase as the smoothing constant λ increases. However, the huge increase of ARL_0's are interrupted as the value of λ reaches a value between 0.2 and 0.6. After λ values reach 0.6, the ARL_0 also reaches its peak value and starts to decrease. However, for any parameter combination of λ, ARL_0s are still measured extremely larger than 500.

Although the ARL_0 results under all of the considered Weibull distributions are larger compared to the exponential case of 500, further analyses on SDRL and percentiles should be performed for understanding the behavior of run length better. The Tables in Appendix B show that the in-control SDRL values under the considered Weibull distributions are similar to the corresponding ARL_0 values. Note also that the 10^{th} and 50^{th} (median) percentiles of the run lengths are smaller than the ARL_0 values, which indicate that shorter run lengths than the ARL_0 value are more likely. Besides, 90^{th} percentiles are quite larger than the corresponding ARL_0 values, which indicate that significantly larger ARL_0 values are also possible due to the skewness of run length distribution.

As mentioned earlier, higher values of ARL_0 are desirable since the expected time to signal a false alarm gets larger. Therefore, in-control ARLs for the considered Weibull distributions (see Tables 18-20) indicate that the lower-sided TBE EWMA chart will rarely signal a false alarm regardless of the severe departure of the Weibull distribution from exponential. However, there is a tradeoff between ARL_0 and ARL_1 values. Although higher ARL_0 values are desirable, the out-of-control ARLs would also be higher. Consequently, in order to announce the extremely high ARL_0 values as good, the ARL_1 values for the corresponding cases should also be checked.

5.2.1.2 In-control Performance Comparison for Lognormal Cases

Unlike the case of the Weibull observations, the expected in-control ARL values for the lower-sided TBE EWMA charts are generally smaller than the designed ARL_0 value of 500 when the underlying distribution is lognormal. The results in Tables 18-20 indicate that smaller ARL_0 values are obtained generally when λ is small and the shape parameter of the lognormal distribution s gets larger, especially when s equals to 1.182 (s = 1.182).

As s increases, variance of the considered lognormal distribution also increases (see Tables 11-14). Since the in-control observations fluctuate with a larger variance, possibility of an in-control observation to exceed the control limit increases. Although the mean is unchanged, the chart signals false alarms more frequently and hence, smaller ARL_0 values are expected to be observed. As can be observed in Tables 18-20, smaller (worse) ARL_0 values are generally expected when the parameter s of the lognormal distribution is greater than 1. On the other hand, the lower-sided TBE EWMA chart is not affected much by the increase in the variability due to lognormal distribution for the cases when s is less than 1. For example, in Table 20, for the design parameters λ=0.01, h_L=0.876086, and B=5; the ARL_0 values are computed as 1930, 1178, 372, and 217 for the s values 0.94, 0.974, 1.092, and 1.182, respectively. Based on these results, it can be concluded that when the true underlying distribution is lognormal with s larger than 1, in-control performance of the exponentially designed lower-sided TBE EWMA chart gets worse.

Although the ARL_0 results point out that the expected false alarm rate is generally higher for the lognormal case in contrast to the exponential case, further analyses relying on SDRL and percentile values should again be considered. It can be observed from SDRLs and percentile values in Tables in Appendix C that the RL distribution is right skewed and has a standard deviation similar to the ARL_0 value when the process is in-control. By considering 10^{th} and 50^{th} (median) percentiles, one can interpret that shorter, i.e. worse, run lengths are more likely. However, since the run length distribution is skewed, and the 90^{th} percentiles are measured quite larger than ARL_0 values, there is always a possibility of the chart to perform better and experience desirable in-control run lengths.

5.2.2 Out-of-Control Performance Comparisons

A robust control chart would perform similar to the designed performance regardless of the underlying distribution. Up to this point, in-control performance analysis has been done. As mentioned earlier, interpreting only the in-control performance metrics is not sufficient to make general conclusions about the sensitivity of a lower-sided TBE EWMA chart to non-exponential distributions. In order to make more meaningful and correct interpretations about the robustness of the chart, out-of-control performance metrics should also be analyzed in addition to in-control ones. Therefore, out-of-control performance comparisons of the TBE EWMA chart under the Weibull and lognormal distributions are considered in the following two sections.

To compare the out-of-control performances of the considered distributions, computed results are summarized in the Tables 22-27. Only the ARL_1 values are displayed in these tables. For ease of readability, out-of-control SDRLs and percentiles are provided in detailed tables in Appendix A, B, C.

5.2.2.1 Out-of-Control Performance Comparison for Weibull Cases

Out-of-control ARL results under various Weibull distributions and for a range of mean shifts are presented in Tables 22-24. In the Tables, ARL_1 values that lie within ±10 % interval of the corresponding ARL_1 values of the exponential case are highlighted. These highlighted cases imply that the lower-sided TBE EWMA chart is not very sensitive to the assumed exponential distribution for the corresponding design parameters and the mean shift. For example, it can be observed from Table 23 that the chart is generally not very sensitive to the assumption of exponential distribution for the mean shifts 0.2, 0.5, 0.6, 0.7, and 0.8 when the smoothing constant values are $\lambda \leq 0.4$, $\lambda \leq 0.1$, $\lambda \leq 0.05$, $\lambda \leq 0.05$ and $\lambda = 0.01$, respectively.

Note that $\lambda = 0.01$ is the smallest smoothing constant considered in this research and it can detect a wide range of mean shifts quickly, even smaller shifts such as 0.7 or 0.8. For example, consider a small mean shift from 1 to 0.7. In this case, the ARL_1 performance under the exponential distribution is 41.25. The ARL_1 performances for all the considered Weibull distributions are also so close to 41 even for the case of $\beta = 4$, which signifies a severe departure of the Weibull distribution from the exponential distribution. From Table 23, consider another small λ, say 0.05. Such a lower-sided TBE EWMA chart under various Weibull cases is observed to have similar ARL_1 performances with the exponential cases for all the mean shifts in the range 0.2-0.7. Note also that, variation of the run length distribution decreases as the shift size increases and λ value decreases. It can also be observed from tables in Appendix B that the run length distributions are slightly skewed for the highlighted cases, which can be seen from the comparison of the medians (50^{th} percentiles) with the ARL_1 values. For detailed analyses of the run length distributions, all the out-of-control performance metrics including ARL_1s, SDRLs and the percentiles experienced under the control and the test cases should be compared.

Table 21: Out-of-control Performance Metrics
under exponential and considered Weibull distributions
for the case of $\lambda = 0.01$ and the mean shift = 0.7

	Control Case	Test Cases				
Performance Metrics	Exponential	Weibull (β=1.2)	Weibull (β=1.5)	Weibull (β=2)	Weibull (β=2.5)	Weibull (β=4)
ARL_1	41,25	41,49	41,41	41,19	41,09	41,14
SDRL	17,29	14,95	12,39	9,79	8,23	5,88
10%	23	25	28	30	31	34
50%	37	39	39	40	40	41
90%	64	61	58	54	52	49

In Table 21, performance metrics obtained for the case of $\lambda = 0.01$ and the mean shift 0.7 are presented as a typical behavior for the highlighted cases. It can be seen from this table that as β increases, the run length distributions become more condensed due to the small SDRL values. Moreover, medians get closer to the corresponding ARL_1 values, which lead to more symmetric run length distributions.

Considering also the in-control performances, the lower-sided TBE EWMA with a small smoothing constant such as $\lambda = 0.01$ and 0.05 is observed to be robust to Weibull distributed observations for a wide range of mean shifts.

It is known for an EWMA chart that as λ increases, the performance in detecting small shifts decreases. This is due to the longer process history kept in the control statistic. Besides, Tables in Appendix B point out that by increasing the λ values, SDRLs of the run length distributions also increase, especially for small mean shifts in the range of 0.7-0.95. As the run length variation increases, ARL_1s increase and medians decrease. Medians are generally observed to be smaller than ARL_1 values, which indicate that the run length distribution becomes positively skewed. That is, even extremely high run lengths are possible. These cases are generally encountered for the λ values and shifts magnitudes taking place out of the highlighted area.

Consequently, an increase of the λ value also increases the sensitivity of the chart to the assumed exponential distribution and as a result, robustness of the chart decreases. For example, consider again Table 23, the case of $\lambda = 0.2$. The lower-sided TBE EWMA chart with $\lambda = 0.2$ is insensitive to the assumption of the exponential distribution only when the mean shifts from 1 to 0.2. For smaller shifts in the range 0.5 to 0.95, the chart is quite sensitive to the assumption of exponential distribution and performs worse under the Weibull cases even for the $\beta = 1.2$ case. Moreover, for the λ values larger than 0.2, insensitivity to the assumed distribution is not observed in any of the tables.

As a result, considering both in-control and out-of-control performances, it can be concluded that for smaller values of λ, say less than 0.2, the lower-sided TBE EWMA charts are observed to be insensitive and robust to the assumption of exponential distribution for a wide range of mean shifts.

Table 22: Out-of-control ARLs under exponential and considered Weibull distributions for several mean shifts when B = 1

		TBE Mean Shifts (Standardized)						
		0,2	0,5	0,6	0,7	0,8	0,9	0,95
lower-sided EWMA								
	Exponential	18	30,05	38,47	53,1	83,7	168,9	275,6
λ = 0,01	Weibull (β=1,2)	18	30,18	38,89	54,4	89,3	211,0	418,8
h_L= 0,871	Weibull (β=1,5)	17,98	30,26	39,16	55,4	94,2	269,8	751,9
B = 1	Weibull (β=2)	17,98	30,28	39,33	56,1	98,4	360,4	2.001
Z_0=1	Weibull (β=2,5)	17,98	30,26	39,36	56,4	100,5	455,1	6.007
	Weibull (β=4)	17,98	30,20	39,32	56,6	102,8	858,6	507.680
lower-sided EWMA								
	Exponential	11,48	22,11	31,12	49,3	91,5	201,1	313,6
λ = 0,05	Weibull (β=1,2)	11,48	22,56	32,84	56,6	128,9	408,7	799,1
h_L= 0,6561	Weibull (β=1,5)	11,47	22,94	34,50	66,3	210,2	1.315	3.905
B = 1	Weibull (β=2)	11,46	23,20	36,15	81,3	497,5	12.765	86.186
Z_0=1	Weibull (β=2,5)	11,45	23,29	37,10	96,8	1.369	179.560	2,97E+06
	Weibull (β=4)	11,43	23,32	38,38	157,7	75.462	2,26E+09	7,16E+11
lower-sided EWMA								
	Exponential	9,31	21,16	32,8	57,1	110,9	232,1	340,3
λ = 0,1	Weibull (β=1,2)	9,31	22,53	38,3	80,3	208,5	620,1	1089
h_L= 0,5176	Weibull (β=1,5)	9,30	24,06	46,6	133,3	583,4	3.227	7.707
B = 1	Weibull (β=2)	9,29	25,84	62,6	338,6	4.187	73.773	307.970
Z_0=1	Weibull (β=2,5)	9,28	27,08	83,7	1.012	39.835	2,38E+06	1,78E+07
	Weibull (β=4)	9,24	29,33	230,6	64.702	1,01E+08	2,58E+11	1,23E+13
lower-sided EWMA								
	Exponential	7,77	24,4	42,1	77,2	145,1	272,1	370,0
λ = 0,2	Weibull (β=1,2)	7,82	30,2	63,7	150,6	372,5	912,0	1.406
h_L= 0,3577	Weibull (β=1,5)	7,85	41,3	122,3	448,3	1.759	6.645	12.513
B = 1	Weibull (β=2)	7,85	71,1	419,5	3.465	30.607	245.350	6,52E+05
Z_0=1	Weibull (β=2,5)	7,84	130,9	1.724	32.950	6,60E+05	1,14E+07	4,29E+07
	Weibull (β=4)	7,80	1.294	238.320	5,71E+07	1,27E+10	2,23E+12	2,47E+13
lower-sided EWMA								
	Exponential	7,48	38,1	67,8	117,6	197,5	319,9	401,7
λ = 0,4	Weibull (β=1,2)	7,91	67,1	148,9	318,5	646,3	1.240	1.682
h_L= 0,1921	Weibull (β=1,5)	8,45	167,4	527,7	1.565	4.256	10.571	16.128
B = 1	Weibull (β=2)	9,18	891,2	5.079	25.931	114.990	439.820	816.220
Z_0=1	Weibull (β=2,5)	9,80	5.383	54.679	4,73E+05	3,42E+06	2,02E+07	4,56E+07
	Weibull (β=4)	11,29	1,68E+06	9,28E+07	3,74E+09	1,13E+11	2,51E+12	1,03E+13
lower-sided EWMA								
	Exponential	9,3	58,6	98,4	157,2	239,9	352,3	421,4
λ = 0,6	Weibull (β=1,2)	11,3	132,5	264,9	492,6	859,7	1.420	1.792
h_L= 0,1026	Weibull (β=1,5)	15,2	477,8	1240,6	2.904	6.197	12.208	16.691
B = 1	Weibull (β=2)	25,7	4.429	17.591	60.071	178.690	471.190	735.740
Z_0=1	Weibull (β=2,5)	45,5	43.315	2,60E+05	1,28E+06	5.328.600	18.839.000	3,35E+07
	Weibull (β=4)	312,4	4,60E+07	9,22E+08	1,35E+10	1,51E+11	1,30E+12	3,48E+12
lower-sided EWMA								
	Exponential	14,4	86,9	134,6	197,6	278,2	378,2	436,2
λ = 0,8	Weibull (β=1,2)	22,0	229,5	403,0	659,5	1020,4	1.509	1.808
h_L= 0,0471	Weibull (β=1,5)	43,2	1013,5	2.139	4.107	7.309	12.227	15.505
B = 1	Weibull (β=2)	141,0	12.333	35.142	87.755	196.530	402.730	560.680
Z_0=1	Weibull (β=2,5)	478,5	150.820	577.080	1,87E+06	5,28E+06	1,33E+07	2,03E+07
	Weibull (β=4)	20.211	2,78E+08	2,55E+09	1,80E+10	1,03E+11	4,80E+11	9,70E+11
lower-sided EWMA								
	Exponential	100,5	250,5	300,5	350,5	400,5	450,5	475,5
λ = 1	Weibull (β=1,2)	270,8	812,2	1.011	1.216	1.427	1.644	1.754
h_L= 0,002	Weibull (β=1,5)	1.167	4.610	6.058	7.634	9.328	11.131	12.071
B = 1	Weibull (β=2)	12.736	79.581	1,15E+05	1,56E+05	203.720	257.810	287.300
Z_0=1	Weibull (β=2,5)	134.840	1,33E+06	2,10E+06	3,09E+06	4,32E+06	5,79E+06	6,63E+06
	Weibull (β=4)	1,48E+08	5,79E+09	1,20E+10	2,22E+10	3,79E+10	6,07E+10	7,54E+10

Table 23: Out-of-control ARLs under exponential and considered Weibull distributions for several mean shifts when B = 2

		TBE Mean Shifts (Standardized)						
		0,2	0,5	0,6	0,7	0,8	0,9	0,95
lower-sided EWMA								
	Exponential	14,14	23,34	29,86	41,25	65,15	134,6	233,9
λ = 0,01	**Weibull (β=1,2)**	14,21	23,31	29,87	41,49	66,84	149,8	292,5
h_L= 0,901446	**Weibull (β=1,5)**	14,25	23,17	29,71	41,41	67,73	165,6	381,7
B = 2	**Weibull (β=2)**	14,20	22,95	29,46	41,19	68,20	183,5	552,0
Z_0=1	**Weibull (β=2,5)**	14,11	22,83	29,34	41,09	68,61	198,8	793,2
	Weibull (β=4)	14,00	22,92	29,15	41,14	69,35	233,5	2.772
lower-sided EWMA								
	Exponential	10,22	19,24	26,67	41,2	74,9	170,0	282,3
λ = 0,05	**Weibull (β=1,2)**	10,21	19,47	27,61	45,1	93,6	278,9	564,0
h_L= 0,68607	**Weibull (β=1,5)**	10,20	19,67	28,52	50,0	128,4	629,6	1.831
B = 2	**Weibull (β=2)**	10,19	19,81	29,41	56,8	219,6	3.145	19.004
Z_0=1	**Weibull (β=2,5)**	10,18	19,85	29,89	62,6	399,2	21.594	2,92E+05
	Weibull (β=4)	10,13	19,83	30,45	76,9	4.187	2,66E+07	5,17E+09
lower-sided EWMA								
	Exponential	8,55	18,59	27,9	47,1	91,1	201,5	313,4
λ = 0,1	**Weibull (β=1,2)**	8,55	19,45	31,3	60,2	145,4	437,3	811,0
h_L= 0,545071	**Weibull (β=1,5)**	8,55	20,40	36,0	86,6	313,5	1.629	4.077
B = 2	**Weibull (β=2)**	8,53	21,43	43,8	166,0	1.410	20.824	89.630
Z_0=1	**Weibull (β=2,5)**	8,52	22,08	52,1	349,8	8.231	372.440	2,80E+06
	Weibull (β=4)	8,51	23,04	86,3	6.352	4,54E+06	6,84E+09	2,95E+11
lower-sided EWMA								
	Exponential	7,24	20,9	35,0	63,5	122,1	244,6	349,2
λ = 0,2	**Weibull (β=1,2)**	7,27	24,8	48,6	108,7	267,6	695,7	1.132
h_L= 0,37932	**Weibull (β=1,5)**	7,28	31,5	81,4	265,0	990,0	3.919	7.841
B = 2	**Weibull (β=2)**	7,28	47,1	215,8	1.461	11.553	94.143	2,67E+05
Z_0=1	**Weibull (β=2,5)**	7,27	72,5	664,8	9.937	1,70E+05	2,86E+06	1,15E+07
	Weibull (β=4)	7,23	353,9	37.238	6,31E+06	1,08E+09	1,62E+11	1,87E+12
lower-sided EWMA								
	Exponential	6,94	32,3	57,1	100,8	175,4	299,5	388,1
λ = 0,4	**Weibull (β=1,2)**	7,27	52,8	114,1	245,1	514,7	1.049	1.479
h_L= 0,204487	**Weibull (β=1,5)**	7,68	117,6	351,8	1027,4	2.870	7.621	12.187
B = 2	**Weibull (β=2)**	8,19	518,0	2.726	13.313	59.346	242.450	475.830
Z_0=1	**Weibull (β=2,5)**	8,57	2.591	23.911	1,93E+05	1.370.400	8.568.300	2,05E+07
	Weibull (β=4)	9,34	462.830	2,23E+07	8,02E+08	2,23E+10	4,94E+11	2,16E+12
lower-sided EWMA								
	Exponential	8,5	50,8	86,1	140,5	221,4	337,6	412,3
λ = 0,6	**Weibull (β=1,2)**	10,1	107,5	215,3	408,5	738,0	1.276	1.653
h_L= 0,109064	**Weibull (β=1,5)**	13,2	353,6	908,3	2.153	4.756	9.889	13.971
B = 2	**Weibull (β=2)**	20,7	2.849	11.000	37.396	114.210	319.250	519.010
Z_0=1	**Weibull (β=2,5)**	33,8	24.404	1,40E+05	6,79E+05	2.859.100	10.697.000	1,99E+07
	Weibull (β=4)	176,3	1,75E+07	3,29E+08	4,55E+09	4,95E+10	4,42E+11	1,24E+12
lower-sided EWMA								
	Exponential	13,4	80,0	125,1	186,5	267,2	370,6	431,8
λ = 0,8	**Weibull (β=1,2)**	20,1	203,6	360,7	599,3	946,6	1.434	1.740
h_L= 0,049218	**Weibull (β=1,5)**	38,3	855,0	1.811	3.528	6.425	11.072	14.277
B = 2	**Weibull (β=2)**	118,0	9.650	27.355	68.861	157.740	335.040	476.870
Z_0=1	**Weibull (β=2,5)**	379,1	110.050	415.620	1,35E+06	3,88E+06	1,01E+07	1,59E+07
	Weibull (β=4)	13.611	1,65E+08	1,47E+09	1,02E+10	5,82E+10	2,82E+11	5,90E+11
lower-sided EWMA								
	Exponential	100,4	250,3	300,2	350,2	400,1	450,1	475,0
λ = 1	**Weibull (β=1,2)**	270,5	811,2	1.010	1.215	1.426	1.642	1.752
h_L= 0,002	**Weibull (β=1,5)**	1.165	4.603	6.049	7.623	9.314	11.114	12.053
B = 2	**Weibull (β=2)**	12.710	79.422	1,14E+05	1,56E+05	203.310	257.300	286.720
Z_0=1	**Weibull (β=2,5)**	134.500	1,33E+06	2,10E+06	3,08E+06	4,31E+06	5,78E+06	6,61E+06
	Weibull (β=4)	1,47E+08	5,76E+09	1,20E+10	2,21E+10	3,78E+10	6,05E+10	7,51E+10

Table 24: Out-of-control ARLs under exponential and considered Weibull distributions for several mean shifts when B=5

		TBE Mean Shifts (Standardized)						
		0,2	**0,5**	**0,6**	**0,7**	**0,8**	**0,9**	**0,95**
lower-sided EWMA								
	Exponential	13,95	28,0	36,7	51,3	80,9	160,9	263,3
λ = 0,01	**Weibull (β=1,2)**	14,15	31,8	42,8	62,7	109,3	280,8	592,7
h_L= 0,876086	**Weibull (β=1,5)**	14,28	38,9	53,8	82,7	167,4	724,0	2.692
B = 5	**Weibull (β=2)**	14,21	56,0	80,4	125,1	289,8	3.429	41.910
Z_0=1	**Weibull (β=2,5)**	13,98	82,2	127,0	194,1	409,0	7.888	2,65E+05
	Weibull (β=4)	13,21	263,5	541,2	998,1	1,71E+03	4,67E+03	83.940
lower-sided EWMA								
	Exponential	10,18	19,22	26,7	41,3	75,1	170,5	282,8
λ = 0,05	**Weibull (β=1,2)**	10,17	19,47	27,7	45,3	94,2	280,1	564,3
h_L= 0,685314	**Weibull (β=1,5)**	10,15	19,66	28,6	50,3	129,4	625,8	1.796
B = 5	**Weibull (β=2)**	10,13	19,77	29,4	57,0	217,6	2.928	16.900
Z_0=1	**Weibull (β=2,5)**	10,10	19,80	29,9	62,6	380,4	17.605	2,17E+05
	Weibull (β=4)	10,05	19,79	30,6	76,6	2.989	9,83E+06	1,38E+09
lower-sided EWMA								
	Exponential	8,50	18,54	27,9	47,2	91,2	201,6	313,6
λ = 0,1	**Weibull (β=1,2)**	8,50	19,41	31,2	60,3	145,5	436,6	808,3
h_L= 0,544453	**Weibull (β=1,5)**	8,49	20,38	36,0	86,7	312,7	1.612	4.017
B = 5	**Weibull (β=2)**	8,48	21,43	43,9	165,6	1.384	19.984	84.992
Z_0=1	**Weibull (β=2,5)**	8,47	22,10	52,3	345,3	7.815	338.480	2,49E+06
	Weibull (β=4)	8,45	23,12	86,0	5.619	3,46E+06	4,54E+09	1,84E+11
lower-sided EWMA								
	Exponential	7,23	20,9	35,0	63,4	121,9	244,1	348,7
λ = 0,2	**Weibull (β=1,2)**	7,27	24,8	48,5	108,4	266,4	691,0	1.124
h_L= 0,379277	**Weibull (β=1,5)**	7,29	31,5	81,2	263,4	980,0	3.864	7.716
B = 5	**Weibull (β=2)**	7,28	47,1	214,1	1.438	11.275	91.169	2,58E+05
Z_0=1	**Weibull (β=2,5)**	7,27	72,2	653,0	9.616	1,62E+05	2,70E+06	1,08E+07
	Weibull (β=4)	7,24	343,0	34.327	5,58E+06	9,21E+08	1,34E+11	1,52E+12
lower-sided EWMA								
	Exponential	6,90	31,9	56,4	99,4	173,4	297,2	386,4
λ = 0,4	**Weibull (β=1,2)**	7,22	52,0	112,0	240,0	503,4	1.027	1.449
h_L= 0,205282	**Weibull (β=1,5)**	7,62	115,0	342,1	995,2	2.770	7.339	11.727
B = 5	**Weibull (β=2)**	8,12	498,4	2.602	12.630	56.008	227.780	446.120
Z_0=1	**Weibull (β=2,5)**	8,50	2.448	22.357	1,79E+05	1.261.200	7.835.400	1,87E+07
	Weibull (β=4)	9,25	409.900	1,93E+07	6,84E+08	1,87E+10	4,10E+11	1,78E+12
lower-sided EWMA								
	Exponential	8,4	49,8	84,4	137,9	218,0	334,5	410,2
λ = 0,6	**Weibull (β=1,2)**	10,0	104,8	209,3	396,5	716,6	1.243	1.614
h_L= 0,1098	**Weibull (β=1,5)**	13,0	341,1	872,9	2.063	4.547	9.448	13.355
B = 5	**Weibull (β=2)**	20,3	2.700	10.372	35.108	106.830	297.690	483.340
Z_0=1	**Weibull (β=2,5)**	32,7	22.697	1,30E+05	6,24E+05	2.614.600	9.741.400	1,81E+07
	Weibull (β=4)	164,6	1,53E+07	2,84E+08	3,89E+09	4,19E+10	3,71E+11	1,04E+12
lower-sided EWMA								
	Exponential	13,2	78,2	122,4	182,9	263,2	367,5	429,9
λ = 0,8	**Weibull (β=1,2)**	19,8	197,7	349,7	581,3	920,2	1.400	1.704
h_L= 0,0496857	**Weibull (β=1,5)**	37,3	821,8	1.737	3.378	6.148	10.610	13.703
B = 5	**Weibull (β=2)**	113,5	9.121	25.771	64.694	147.870	313.580	446.180
Z_0=1	**Weibull (β=2,5)**	359,8	102.250	384.480	1,24E+06	3,56E+06	9,28E+06	1,45E+07
	Weibull (β=4)	12.389	1,45E+08	1,28E+09	8,84E+09	5,01E+10	2,42E+11	5,05E+11
lower-sided EWMA								
	Exponential	100,4	250,3	300,2	350,2	400,1	450,1	475,0
λ = 1	**Weibull (β=1,2)**	270,5	811,2	1.010	1.215	1.426	1.642	1.752
h_L= 0,002	**Weibull (β=1,5)**	1.165	4.603	6.049	7.623	9.314	11.114	12.053
B = 5	**Weibull (β=2)**	12.710	79.422	1,14E+05	1,56E+05	203.310	257.300	286.720
Z_0=1	**Weibull (β=2,5)**	134.500	1,33E+06	2,10E+06	3,08E+06	4,31E+06	5,78E+06	6,61E+06
	Weibull (β=4)	1,47E+08	5,76E+09	1,20E+10	2,21E+10	3,78E+10	6,05E+10	7,51E+10

5.2.2.2 Out-of-Control Performance Comparison for Lognormal Cases

ARL results for each considered boundary value, i.e. B = 1, B = 2, B = 5, are presented respectively in Tables 25-27 under the exponential and the lognormal distributions when the mean shifts are 0.2, 0.5, 0.6, 0.7, 0.8, 0.9, and 0.95. Note again that, ARL_1 values that lie within ±10 % interval of the corresponding ARL_1 values of the exponential case are highlighted in the tables.

The ARL_1s provided in these tables indicate that when λ is smaller than 0.2, the ability of the TBE EWMA to detect shifts in the range 0.2-0.6 is not affected much from the underlying distribution of the observations. On the other hand, for smaller mean shifts in the range 0.7-0.95 and for smaller λ values, unlike to the Weibull case, the charts under the lognormal distributions generally detect the considered shifts faster than the charts under the exponential case. Moreover, as the shape parameter s of the lognormal increases, ARL_1 and SDRL values decrease. The smallest ARL_1 values are measured when the true underlying distribution is lognormal with s = 1.182. However, as the shape parameter decrease from s = 1.182 to s = 0.94, the ARL_1 and the SDRL values increase and approach to the values computed for the exponential case. For example, in Table 25, when $\lambda = 0.1$ and the process mean shifts from 1 to 0.7, ARL_1 for the exponential distribution is computed as 57, whereas ARL_1 values under lognormal distributions with s = 1.182, s = 1.092, s = 0.974, and s = 0.94 are computed as 32, 37, 48, and 52, respectively. On the other hand, the Tables in Appendix C illustrate that run length distribution for lognormal cases are highly skewed such that the median of the RL are generally computed quite smaller than the ARL_1 values. Hence, quite larger run lengths are possible to occur due to the skewness.

It is known that small ARL_1 values are desirable over the larger ones for a fixed in-control performance. Unfortunately, the in-control performances computed for some of the lognormal distributions, under which very small ARL_1s are achieved, are not fixed or not above the targeted value. Thus, a one to one comparison is difficult, the patterns in both the in-control and out-of-control performances should be considered together to draw a conclusion.

As mentioned earlier, the computed ARL_0 values for almost all λ values (except for the 0.01 and 0.05 values) are larger than the target ARL_0 of 500 when the true underlying distribution is lognormal with $s = 0.94$ and $s = 0.974$. In contrast, for the remaining two lognormal distributions with $s = 1.182$ and $s = 1.094$, ARL_0 values are generally smaller than the target ARL_0 value of 500 for most of the λ values.

Consequently, by considering both the ARL_0 and the ARL_1 performances, we conclude that when the true underlying distribution is lognormal with $s < 1$, the lower-sided TBE EWMA chart with $\lambda = 0.1$ and $\lambda = 0.2$ is quite robust for the wide range of shifts considered. Note that although the ARL_1 performances of the charts with $\lambda = 0.01$ and $\lambda = 0.05$ are insensitive to the assumption of exponential distribution, ARL_0 performances of these charts deviate significantly from the designed performance. On the other hand, despite superior ARL_1 performances for the cases with $s > 1$, robustness is a concern due to worse ARL_0 performances.

Table 25: Out-of-control ARLs for exponential and various lognormal distributions for several mean shifts when B = 1

		TBE Mean Shifts (Standardized)						
		0,2	0,5	0,6	0,7	0,8	0,9	0,95
lower-sided EWMA								
λ = 0,01	Exponential	18	30,05	38,5	53,1	83,7	168,9	275,6
h_L = 0,871	logn (s=1,182)	18	28,84	35,6	45,9	62,7	93,0	117,8
B = 1	logn (s=1,092)	18	29,23	36,5	48,0	68,1	108,1	144,3
Z_0=1	logn (s=0,974)	18	29,63	37,5	50,5	75,3	133,5	195,8
	logn (s=0,94)	18	29,72	37,7	51,1	77,4	142,2	215,9
lower-sided EWMA								
λ = 0,05	Exponential	11,48	22,1	31,1	49,3	91,5	201,1	313,6
h_L = 0,6561	logn (s=1,182)	11,41	19,9	25,4	34,0	48,1	72,5	91,2
B = 1	logn (s=1,092)	11,43	20,5	26,9	37,6	56,7	94,3	126,6
Z_0=1	logn (s=0,974)	11,45	21,3	28,9	43,0	72,9	146,2	222,1
	logn (s=0,94)	11,46	21,5	29,5	44,8	79,0	170,3	271,6
lower-sided EWMA								
λ = 0,1	Exponential	9,31	21,2	32,8	57,1	110,9	232,1	340,3
h_L = 0,5176	logn (s=1,182)	9,18	17,4	23,1	32,1	46,8	71,9	90,9
B = 1	logn (s=1,092)	9,22	18,4	25,4	37,5	59,9	103,3	139,7
Z_0=1	logn (s=0,974)	9,26	19,7	29,1	47,9	90,3	195,7	302,0
	logn (s=0,94)	9,27	20,1	30,3	52,0	104,4	246,9	401,0
lower-sided EWMA								
λ = 0,2	Exponential	7,77	24,4	42,1	77,2	145,1	272,1	370,0
h_L = 0,3577	logn (s=1,182)	7,51	16,8	23,8	35,1	53,9	85,7	109,1
B = 1	logn (s=1,092)	7,58	18,7	28,5	46,2	79,6	144,2	196,9
Z_0=1	logn (s=0,974)	7,66	22,1	38,2	74,3	159,9	372,2	579,4
	logn (s=0,94)	7,68	23,3	42,3	88,1	206,6	528,6	864,0
lower-sided EWMA								
λ = 0,4	Exponential	7,48	38,1	67,8	117,6	197,5	319,9	401,7
h_L = 0,1921	logn (s=1,182)	6,61	21,7	34,6	56,4	93,1	154,3	198,6
B = 1	logn (s=1,092)	6,85	28,3	50,9	94,4	178,2	337,8	464,5
Z_0=1	logn (s=0,974)	7,19	45,0	101,5	240,2	579,4	1.397	2.160
	logn (s=0,94)	7,30	53,2	130,7	338,7	893,6	2.351	3.792
lower-sided EWMA								
λ = 0,6	Exponential	9,26	58,6	98,4	157,2	239,9	352,3	421,4
h_L = 0,1026	logn (s=1,182)	7,27	38,2	67,4	118,0	203,3	344,1	444,3
B = 1	logn (s=1,092)	8,01	61,8	126,3	253,8	499,0	957,0	1.313
Z_0=1	logn (s=0,974)	9,34	147,5	390,0	1.002	2.484	5.924	9.020
	logn (s=0,94)	9,85	202,8	588,3	1.652	4.448	11.469	18.128
lower-sided EWMA								
λ = 0,8	Exponential	14,37	86,9	134,6	197,6	278,2	378,2	436,2
h_L = 0,0471	logn (s=1,182)	11,18	100,8	191,4	348,1	609,2	1.030	1.324
B = 1	logn (s=1,092)	14,27	216,0	473,8	982,4	1.939	3.668	4.973
Z_0=1	logn (s=0,974)	22,04	835,7	2.353	6.124	14.907	34.247	50.922
	logn (s=0,94)	25,86	1.363	4.191	11.841	31.068	76.538	117.640
lower-sided EWMA								
λ = 1	Exponential	100,5	250,5	300,5	350,5	400,5	450,5	475,5
h_L = 0,002	logn (s=1,182)	2.106	44.466	87.323	157.310	265.350	425.200	529.470
B = 1	logn (s=1,092)	8.274	308.890	688.040	1.382.100	2.568.800	4,49E+06	5,83E+06
Z_0=1	logn (s=0,974)	89.825	9,10E+06	2,53E+07	6,15E+07	1,36E+08	2,76E+08	3,85E+08
	logn (s=0,94)	211.620	3,07E+07	9,21E+07	2,40E+08	1,21E+09	1,18E+09	1,74E+09

Table 26: Out-of-control ARLs for exponential and various lognormal distributions for several mean shifts when B = 2

		TBE Mean Shifts (Standardized)						
		0,2	0,5	0,6	0,7	0,8	0,9	0,95
lower-sided EWMA								
λ = 0,01	Exponential	14,14	23,34	29,86	41,2	65,1	134,6	233,9
h_L= 0,901446	logn (s=1,182)	14,27	23,58	29,81	40,1	59,2	101,7	145,3
B = 2	logn (s=1,092)	14,35	23,63	29,95	40,6	61,0	110,1	164,9
Z_0=1	logn (s=0,974)	14,45	23,56	29,92	40,8	62,8	120,9	194,5
	logn (s=0,94)	14,47	23,51	29,86	40,8	63,1	123,9	203,9
lower-sided EWMA								
λ = 0,05	Exponential	10,22	19,24	26,67	41,20	74,87	170,0	282,3
h_L= 0,68607	logn (s=1,182)	10,20	18,12	23,65	32,64	48,47	79,0	104,9
B = 2	logn (s=1,092)	10,20	18,44	24,47	34,79	54,36	96,4	135,9
Z_0=1	logn (s=0,974)	10,32	19,02	25,80	38,30	65,05	134,1	211,3
	logn (s=0,94)	10,20	18,90	25,81	38,85	67,86	147,7	242,3
lower-sided EWMA								
λ = 0,1	Exponential	8,55	18,59	27,94	47,15	91,08	201,5	313,4
h_L= 0,545071	logn (s=1,182)	8,48	16,27	21,84	30,92	46,53	75,0	97,8
B = 2	logn (s=1,092)	8,50	16,89	23,39	34,77	56,36	100,9	140,4
Z_0=1	logn (s=0,974)	8,53	17,77	25,82	41,73	77,45	169,2	266,6
	logn (s=0,94)	8,52	17,96	26,48	44,00	85,64	201,2	331,8
lower-sided EWMA								
λ = 0,2	Exponential	7,24	20,91	35,03	63,46	122,1	244,6	349,2
h_L= 0,37932	logn (s=1,182)	7,03	15,63	22,13	32,84	51,0	82,9	107,1
B = 2	logn (s=1,092)	7,09	17,04	25,59	41,15	70,9	130,2	180,2
Z_0=1	logn (s=0,974)	7,15	19,37	32,27	60,30	126,4	293,3	461,0
	logn (s=0,94)	7,17	20,19	34,91	69,07	156,0	395,3	651,6
lower-sided EWMA								
λ = 0,4	Exponential	6,94	32,26	57,1	100,8	175,4	299,5	388,1
h_L= 0,204487	logn (s=1,182)	6,24	19,56	30,8	49,9	82,2	137,0	177,2
B = 2	logn (s=1,092)	6,43	24,57	43,1	78,7	147,4	280,2	387,2
Z_0=1	logn (s=0,974)	6,70	36,47	78,4	179,3	424,7	1.022	1.587
	logn (s=0,94)	6,78	42,05	97,6	242,8	626,3	1.641	2.658
lower-sided EWMA								
λ = 0,6	Exponential	8,48	50,8	86,1	140,5	221,4	337,6	412,3
h_L= 0,109064	logn (s=1,182)	6,79	33,3	57,9	100,5	172,6	292,6	378,8
B = 2	logn (s=1,092)	7,40	51,5	103,0	204,2	399,3	766,2	1.053,6
Z_0=1	logn (s=0,974)	8,47	113,2	289,4	728,1	1.786	4.255	6.495
	logn (s=0,94)	8,87	151,0	421,9	1.157	3.077	7.918	12.544
lower-sided EWMA								
λ = 0,8	Exponential	13,40	80,0	125,1	186,5	267,2	370,6	431,8
h_L= 0,049218	logn (s=1,182)	10,45	89,5	169,0	306,9	537,8	913,1	1.177
B = 2	logn (s=1,092)	13,14	185,6	404,2	8,36E+02	1.653	3.142	4.273
Z_0=1	logn (s=0,974)	19,71	678	1,89E+03	4,89E+03	1,19E+04	2,75E+04	4,11E+04
	logn (s=0,94)	22,87	1.082	3,29E+03	9,23E+03	2,42E+04	6,00E+04	9,27E+04
lower-sided EWMA								
λ = 1	Exponential	100,4	250,3	300,2	350,2	400,1	450,1	475,0
h_L= 0,002	logn (s=1,182)	2.100	44.304	86.995	1,57E+05	2,64E+05	4,23E+05	5,27E+05
B = 2	logn (s=1,092)	8.245	3,08E+05	684.980	1,38E+06	2,56E+06	4,47E+06	5,80E+06
Z_0=1	logn (s=0,974)	89.416	9,05E+06	2,51E+07	6,11E+07	1,35E+08	2,75E+08	3,83E+08
	logn (s=0,94)	210.580	3,05E+07	9,16E+07	2,39E+08	5,60E+08	1,21E+09	1,72E+09

Table 27: Out-of-control ARLs for exponential and various lognormal distributions for several mean shifts when B = 5

		TBE Mean Shifts (Standardized)						
		0,2	0,5	0,6	0,7	0,8	0,9	0,95
lower-sided EWMA								
$\lambda = 0{,}01$	Exponential	13,95	28,01	36,7	51,3	80,9	160,9	263,3
$h_L = 0{,}876086$	logn (s=1,182)	12,94	23,92	31,0	42,3	62,6	105,2	146,4
B = 5	logn (s=1,092)	13,03	25,44	33,9	48,0	75,8	143,6	220,0
$Z_0 = 1$	logn (s=0,974)	13,15	28,16	39,2	59,7	107,2	265,7	511,1
	logn (s=0,94)	13,19	29,16	41,3	64,5	122,0	340,7	727,1
lower-sided EWMA								
$\lambda = 0{,}05$	Exponential	10,18	19,22	26,69	41,30	75,1	170,5	282,8
$h_L = 0{,}685314$	logn (s=1,182)	10,19	18,16	23,74	32,86	49,0	80,1	106,8
B = 5	logn (s=1,092)	10,19	18,47	24,56	35,01	54,9	97,7	138,0
$Z_0 = 1$	logn (s=0,974)	10,19	18,83	25,60	38,11	64,9	133,9	210,7
	logn (s=0,94)	10,20	18,92	25,88	39,06	68,4	149,0	244,2
lower-sided EWMA								
$\lambda = 0{,}1$	Exponential	8,50	18,54	27,91	47,16	91,2	201,6	313,6
$h_L = 0{,}544453$	logn (s=1,182)	8,43	16,23	21,84	31,00	46,9	76,0	99,5
B = 5	logn (s=1,092)	8,44	16,84	23,37	34,83	56,6	101,8	142,0
$Z_0 = 1$	logn (s=0,974)	8,46	17,67	25,71	41,59	77,2	168,6	265,6
	logn (s=0,94)	8,47	17,92	26,47	44,07	85,9	202,2	333,5
lower-sided EWMA								
$\lambda = 0{,}2$	Exponential	7,23	20,90	35,01	63,4	121,9	244,1	348,7
$h_L = 0{,}379277$	logn (s=1,182)	7,03	15,69	22,28	33,2	51,9	84,9	110,4
B = 5	logn (s=1,092)	7,09	17,08	25,70	41,4	71,7	132,5	184,2
$Z_0 = 1$	logn (s=0,974)	7,15	19,40	32,34	60,5	127,0	295,6	465,8
	logn (s=0,94)	7,17	20,21	34,97	69,2	156,5	397,4	656,0
lower-sided EWMA								
$\lambda = 0{,}4$	Exponential	6,90	31,9	56,4	99,4	173,4	297,2	386,4
$h_L = 0{,}205282$	logn (s=1,182)	6,21	19,5	30,8	50,0	82,9	139,2	180,8
B = 5	logn (s=1,092)	6,39	24,4	42,9	78,4	147,4	281,6	390,7
$Z_0 = 1$	logn (s=0,974)	6,65	36,0	77,3	176,4	417,9	1.008	1.570
	logn (s=0,94)	6,73	41,4	95,8	237,8	613,0	1.608	2.611
lower-sided EWMA								
$\lambda = 0{,}6$	Exponential	8,40	49,8	84,4	137,9	218,0	334,5	410,2
$h_L = 0{,}1098$	logn (s=1,182)	6,74	32,9	57,4	99,8	172,1	293,3	380,8
B = 5	logn (s=1,092)	7,33	50,6	101,1	200,8	393,5	758,4	1.046
$Z_0 = 1$	logn (s=0,974)	8,38	110,0	280,0	703,3	1.726	4.120	6.301
	logn (s=0,94)	8,78	146,1	406,1	1.110	2.952	7.607	12.070
lower-sided EWMA								
$\lambda = 0{,}8$	Exponential	13,21	78,2	122,4	182,9	263,2	367,5	429,9
$h_L = 0{,}0496857$	logn (s=1,182)	10,32	87,5	165,2	300,4	527,8	899,2	1.161
B = 5	logn (s=1,092)	12,92	179,8	390,9	808,8	1.602	3.052	4.158
$Z_0 = 1$	logn (s=0,974)	19,27	645,8	1.791	4.633	11.285	26.097	39.006
	logn (s=0,94)	22,31	1.025	3.099	8.685	22.777	56.449	87.209
lower-sided EWMA								
$\lambda = 1$	Exponential	100,4	250,3	300,2	350,2	400,1	450,1	475,0
$h_L = 0{,}002$	logn (s=1,182)	2.100	44.304	86.995	1,57E+05	264.300	423.480	527.320
B = 5	logn (s=1,092)	8.245	3,08E+05	6,85E+05	1,38E+06	2,56E+06	4,47E+06	5,80E+06
$Z_0 = 1$	logn (s=0,974)	89.416	9,05E+06	2,51E+07	6,11E+07	1,35E+08	2,75E+08	3,83E+08
	logn (s=0,94)	210.580	3,05E+07	9,16E+07	2,39E+08	5,60E+08	1,21E+09	1,72E+09

5.2.3 The Effect of the B Parameter on Robustness

In the previous sections, we analyzed the in-control and out-of-control performance metrics without discussing the effects of the charts' B parameters. In the following, we evaluate the performance metrics computed with the three particular B values, i.e. B = 1, B = 2, B = 5, and explore their effects on the charts' robustness.

Initially, consider the B = 1 and B = 2 designs. It has been observed from Tables 18 and 19 that the selection of these B values does not affect the in-control performances of the charts significantly. However, from the Tables 22-23 (Weibull) and the Tables 25-26 (lognormal), it can be observed that B = 2 design has smaller out-of-control ARLs than the B = 1 design. As λ increases, this distinction begins to decrease and when $\lambda = 1$ (i.e. a Shewhart chart), B = 1 and B = 2 designs have almost the same performances. Table 28 combines the results from the in-control and out-of control performance analyses and presents λ and B combinations for which the lower-sided TBA EWMA chart is robust. It can be seen from this table that the chart with B = 2 design is generally robust to the assumption of exponential distribution for more cases. The smallest λ value considered in this research is 0.01. If smaller λ values ($\lambda < 0.01$) were considered, B = 1 designs may also be expected to be comparably robust as the B = 2 designs. However, such small λ values may create computational difficulties in the studies. Hence, λ values smaller than 0.01 are generally not appreciated in the applications.

Considering the B = 2 and the B = 5 designs, although both designs generally perform similarly for $\lambda > 0.01$ cases, at the smallest $\lambda = 0.01$, B = 5 design can not perform well resulting in relatively large ARL_1 values. Hence, in Table 28, no robustness is observed for B = 5 designs at small λ values and at small mean shifts.

Therefore, the charts with B = 2 design may be suggested for robustness and the performance comparisons in this study are generally based on this result.

Table 28: Resulting Robustness Table

	TBE Mean Shifts						
	0.2	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**0.95**
Robustness for B=1	Robust for $\lambda = 0.01\text{-}0.2$	Robust for $\lambda=0.01\text{-}0.1$	Robust for $\lambda=0.01\text{-}0.05$	Robust for $\lambda=0.01$	No robustness observed	No robustness observed	No robustness observed
Robustness for B=2	Robust for $\lambda = 0.01\text{-}0.4$	Robust for $\lambda=0.01\text{-}0.1$	Robust for $\lambda=0.01\text{-}0.05$	Robust for $\lambda=0.01$	Robust for $\lambda=0.01$	No robustness observed	No robustness observed
Robustness for B=5	Robust for $\lambda = 0.01\text{-}0.4$	Robust for $\lambda=0.05\text{-}0.1$	Robust for $\lambda=0.05$	No robustness observed	No robustness observed	No robustness observed	No robustness observed

5.2.4 Design Recommendations for a Robust Lower-Sided TBE EWMA Chart

In designing a robust TBE EWMA chart, the design parameters should be selected with respect to the intended shift magnitude for detection. For example, in order to detect a large mean shift in the range of 0.2-0.5, relatively large λ values such as $\lambda = 0.1$, or $\lambda = 0.2$ in a B=2 design may be preferred. On the other hand, in order to detect smaller mean shifts in the range of 0.5-0.8, smaller λ values such as $\lambda = 0.01$ or $\lambda = 0.05$ may be suggested.

In conclusion, a TBE EWMA chart may be designed to be robust to departures from the assumed exponential distribution of the observations. However, in the extreme cases, when the true underlying distribution deviates significantly from the assumed distribution, robustness may be a serious issue and care should be taken.

CHAPTER 6

CONCLUSION and FUTURE WORK

6.1 Conclusion

Common use and wide applications of statistical quality control techniques and technological developments gave rise to high quality processes producing fewer defects. In such industrial environments, the traditional control charts may not perform effectively. In order to deal with the high quality processes and improve the drawbacks of traditional control charts, time-between-events (TBE) control charts were developed. Among those charts, TBE Exponentially Weighted Moving Average (EWMA) control chart has been a widely preferred one in many applications and a lower-sided TBE EWMA chart is considered in this research.

TBE EWMA control charts monitor the time between defects or the good units between consecutive defects. Since a widely accepted model for the time-between-events data is the exponential distribution, TBE processes are generally modeled with exponential distribution. However, sometimes observations may follow a different distribution. In such situations where the true underlying distribution of observations is different from the assumed exponential distribution, it is highly possible that the chart may not perform as designed. Hence, in this study, we investigate if the ability of a lower-sided TBE EWMA chart to detect shifts is affected by the deviations from the assumed distribution of the observations or if the chart can act distribution free, that is, robust.

In this study, exponential distributed observations are substituted with Weibull and lognormal distributed observations to represent the deviations from the assumed distribution. As the test cases, shape parameters are determined as $\beta=1.2$, $\beta=1.5$, $\beta=2$, β=2.5, β=4 for the Weibull distributions and *s* = 1.182, *s* = 1.092, *s* = 0.974, and *s* = 0.94 for the lognormal distributions. Several lower-sided TBE EWMA charts are designed with the assumption of exponentially distributed observations. As the design parameters for the TBE EWMA chart, considered smoothing constant values are λ=0.01, λ=0.05, λ=0.1, λ=0.2, λ=0.4, λ=0.6, λ=0.8, and λ=1 and the boundary values are B=1, B=2, B=5. Furthermore, standardized observations are utilized in the research. Hence, the ARL_1 in-control mean value is 1 and the investigated mean shifts are 0.95, 0.9, 0.8, 0.7, 0.6, 0.5 and 0.2.

In order to evaluate the performance and robustness of the chart, some performance metrics and a method to compute these metrics were required. Since run length distribution of an EWMA control chart is known to be quite skewed, by interpreting only the ARL values, important information regarding the performance of a control chart might be missed. Hence, in addition to the widely used performance metric ARL, standard deviation of the run lengths (SDRL) and 10^{th}, 50^{th}, 90^{th} percentiles of the run length distribution have also been considered. A Markov Chain Approach of Brook and Evans (1972) has been modified and used for evaluating the performance of a TBE EWMA control chart. Thus, the performance metrics for the assumed distribution (exponential) of observations and for the test cases (various Weibull and lognormal distributions) are computed by using the Markov Chain code generated from this approach.

The results of the Markov Chain code demonstrate the robust nature of the lower-sided TBE EWMA to departures from the assumed underlying distribution (exponential). It has been shown that the charts designed with B = 2 are generally more robust to the departures compared to the B = 1 or the B = 5 designs. Hence, we focused on and considered B = 2 designs for the robustness analysis in this research. It has been observed that the robustness of the charts is also affected from the changes in the λ values.

The lower-sided TBE EWMA charts under considered Weibull distributions generally perform close to the control case (exponential) at small values of λ for a wide range of mean shifts. However, as λ values increase, the robustness tends not to hold for small mean shifts. In addition to that, the charts become also less robust to small shifts as the shape parameter β increases, that is, deviation from the assumed distribution increases. By considering both in-control and out-of-control performance comparisons, one can conclude that for smaller values of λ, say less than 0.2, the lower-sided TBE EWMA charts are observed to be robust to exponential distribution for a wide range of mean shifts.

Under the lognormal distribution, the charts generally seem to be robust to both small and large mean shifts for a wide range of λ values, say 0.01-0.4. However, although out-of-control performances are better compared to the exponential case, in-control performances of the charts under some lognormal distributions, say lognormal distributions with s > 1, may be below the targeted values and are not very desirable. Thus, the robustness of the lower-sided TBE EWMA charts under lognormal cases where s > 1 may be a concern. On the other hand, when s < 1, the lower-sided TBE EWMA charts have better in-control and out-of-control performances compared to the exponential case. Thus, one can conclude that if the true underlying distribution is lognormal with $s < 1$, the lower-sided TBE EWMA charts with λ = 0.1 and λ = 0.2 are quite robust for a wide range of shifts.

Although in some extreme cases, say lognormal with parameter s > 1, the robustness of the lower-sided TBE EWMA charts may be a concern, it has been shown for smaller λ values that these charts are generally robust to departures from the assumed exponential distribution. In other words, the ability of the lower-sided TBE EWMA charts to detect shifts in the range of 0.2-0.8 is not significantly affected by the distribution of the observations.

The obvious inference from this study is that users of lower-sided TBE EWMA procedures need not be concerned about certain small departures from the exponential time-between-events distribution generally when the chart is designed with $\lambda < 0.2$ and B = 2. However, when the true underlying distribution deviates significantly from the assumed distribution, robustness may be a serious issue and should be handled carefully.

6.2 Suggestions for Future Work

In this thesis, a robustness study for the lower-sided time-between-events EWMA control charts, which are used monitoring decreases in the mean of a TBE process, are performed. For the future research, robustness studies are recommended to be performed for the upper-sided TBE EWMA charts, which monitor increases in the mean of a TBE process and are generally used for monitoring process improvements. Also future researchers might extend such studies and perform a robustness study for the two-sided TBE EWMA charts, which monitor both the decreases and increases in the mean of a TBE process.

Although the EWMA control charts are generally used to monitor the mean of a quality characteristic, there are few studies concentrating on variability monitoring. A robustness study of the EWMA charts to the shifts in the process variability can also be a future research topic.

This research considers Weibull and lognormal distributed observations for representing the deviations from the assumed exponential distribution. In addition to these test cases, other distributions may also be utilized; however, we expect the results not to differ significantly from the ones of this research.

Moreover in this research, individual observations are employed in order to compute the control statistic of the lower-sided TBE EWMA charts. Instead of individual observations, sample averages of observations may be investigated and the effect of sample size on robustness can be analyzed.

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APPENDIX A

PERFORMANCE RESULTS FOR THE LOWER-SIDED TBE EWMA CHARTS UNDER EXPONENTIAL DISTRIBUTION

Table A-1: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B = 1) when true underlying distribution is exponential:

mean	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,871	0,6561	0,5176	0,3577	0,1921	0,1026	0,0471	0,002
	B	1	1	1	1	1	1	1	1
	Z_o	1	1	1	1	1	1	1	1
1.00									
ARL_0		499,46	500,24	500,31	499,97	500,24	500,11	499,83	500,50
SDRL		449,1311	479,2	486,82	491,56	495,24	496,65	497,29	500,00
10%		98	72	65	60	57	56	55	53
50%		362	353	351	349	348	348	347	347
90%		1084	1124	1134	1140	1145	1147	1148	1152
0.95									
ARL_1		275,61	313,64	340,34	370	401,73	421,39	436,21	475,50
SDRL		226,365	292,45	326,72	361,51	396,69	417,9	433,67	475,00
10%		72	52	48	47	47	48	48	51
50%		207	224	240	259	280	293	303	330
90%		570	695	766	841	918	966	1001	1094
0.90									
ARL_1		168,85	201,08	232,08	272,05	319,88	352,29	378,23	450,50
SDRL		121,98	179,91	218,38	263,5	314,81	348,79	375,67	450,00
10%		57	40	37	36	38	40	42	48
50%		134	146	165	191	223	245	263	312
90%		327	435	517	615	730	807	868	1037
0.80									
ARL_1		83,68	91,48	110,91	145,09	197,5	239,94	278,15	400,50
SDRL		43,74	71,07	97,27	136,47	192,38	236,41	275,58	400,00
10%		41	27	24	23	25	28	32	43
50%		73	70	81	103	138	167	194	278
90%		141	184	238	323	448	548	637	922
0.70									
ARL_1		53,1	49,27	57,07	77,19	117,63	157,17	197,62	350,50
SDRL		19,96	30,48	43,88	68,6	112,47	153,6	195,04	350,00
10%		32	21	18	16	17	20	23	37
50%		49	41	44	56	83	110	138	243
90%		79	89	114	167	264	357	452	806
0.60									
ARL_1		38,47	31,12	32,77	42,11	67,76	98,43	134,59	300,50
SDRL		10,64	14,42	20,44	33,71	62,59	94,84	132	300,00
10%		27	17	14	12	12	14	17	32
50%		37	28	27	32	49	69	94	208
90%		53	50	59	86	149	222	307	691
0.50									
ARL_1		30,05	22,11	21,16	24,35	38,09	58,62	86,94	250,50
SDRL		6,16	7,44	9,95	16,34	32,98	55,02	84,34	250,00
10%		23	14	12	9	9	9	11	27
50%		29	21	19	20	28	42	61	174
90%		38	32	34	46	81	130	197	576
0.20									
ARL_1		18	11,48	9,31	7,77	7,48	9,26	14,37	100,50
SDRL		1,19	1,17	1,28	1,66	3,07	5,88	11,8	100,00
10%		17	10	8	6	5	4	4	11
50%		18	11	9	7	7	7	11	70
90%		20	13	11	10	11	17	30	231

Table A-2: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B = 5) when true underlying distribution is exponential:

mean									
	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,876086	0,685314	0,544453	0,379277	0,205282	0,1098005	0,0496857	0,002002
	B	5	5	5	5	5	5	5	5
	Z_o	1	1	1	1	1	1	1	1
1.00									
ARL_0		500,00	500,00	500,00	500,00	500,00	500,00	500,00	500,00
SDRL		468,55	487,39	490,12	493,15	495,76	496,97	497,69	499,50
10%		83	64	61	59	56	55	55	53
50%		356	350	350	349	348	348	347	347
90%		1110	1135	1138	1142	1146	1147	1148	1151
0.95									
ARL_1		263,26	282,77	313,57	348,73	386,45	410,22	429,90	475,03
SDRL		224,67	267,00	302,46	341,43	382,05	407,12	427,56	474,53
10%		61	44	43	43	45	46	47	50
50%		196	201	221	244	269	285	299	329
90%		556	631	708	793	884	941	987	1093
0.90									
ARL_1		160,85	170,53	201,63	244,12	297,23	334,48	367,47	450,05
SDRL		121,81	153,15	189,72	236,46	292,70	331,32	365,10	449,55
10%		48	33	32	33	35	38	41	48
50%		126	124	143	172	207	233	255	312
90%		319	370	449	552	678	766	843	1036
0.80									
ARL_1		80,94	75,11	91,17	121,88	173,36	217,96	263,25	400,10
SDRL		47,05	57,43	78,69	113,81	168,61	214,68	260,83	399,60
10%		35	23	21	20	23	26	30	43
50%		69	58	67	87	122	152	183	277
90%		142	150	194	270	393	498	603	921
0.70									
ARL_1		51,30	41,30	47,16	63,37	99,43	137,89	182,91	350,15
SDRL		23,22	25,10	35,09	55,22	94,55	134,52	180,45	349,65
10%		27	17	15	14	15	18	21	37
50%		46	35	37	47	70	97	128	243
90%		82	74	93	135	223	313	418	806
0.60									
ARL_1		36,67	26,69	27,91	35,01	56,38	84,40	122,44	300,20
SDRL		13,33	12,37	16,75	27,08	51,45	80,96	119,94	299,70
10%		22	14	12	11	10	12	15	32
50%		34	24	23	27	41	60	86	208
90%		54	43	50	70	123	190	279	691
0.50									
ARL_1		28,01	19,22	18,54	20,90	31,91	49,82	78,24	250,25
SDRL		8,41	6,62	8,47	13,41	27,02	46,35	75,71	249,75
10%		18	12	10	9	8	8	11	27
50%		27	18	16	17	24	36	55	174
90%		39	28	30	38	67	110	177	576
0.20									
ARL_1		13,95	10,18	8,50	7,23	6,90	8,40	13,21	100,40
SDRL		2,37	1,14	1,20	1,51	2,71	5,15	10,69	99,90
10%		11	9	7	6	4	4	4	11
50%		14	10	8	7	6	7	10	70
90%		17	12	10	9	10	15	27	231

APPENDIX B

PERFORMANCE RESULTS FOR THE LOWER-SIDED TBE EWMA CHARTS UNDER VARIOUS WEIBULL DISTRIBUTIONS

Table B-1: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B = 1) when true underlying distribution is Weibull (β = 1.2)

	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,871	0,6561	0,5176	0,3577	0,1921	0,1026	0,0471	0,002
	B	1	1	1	1	1	1	1	1
mean	Z_0	1	1	1	1	1	1	1	1
1.00									
ARL_0		1029,9	1617,9	1911,2	2142,3	2253,6	2237,5	2148,1	1865,4
SDRL		964,962	1593	1895,9	2133	2248,3	2233,8	2145,5	1864,9
10%		167	193	215	234	242	239	229	197
50%		734	1129	1329	1488	1564	1552	1490	1293
90%		2287	3693	4381	4921	5182	5147	4943	4295
0.95									
ARL_1		418,83	799,14	1088,7	1406	1681,8	1791,9	1807,5	1754
SDRL		354,68	773,55	1073	1396,7	1676,5	1788,2	1804,9	1753,5
10%		100	107	129	157	182	192	193	185
50%		312	562	759	977	1167	1243	1254	1216
90%		881	1807	2486	3225	3866	4121	4159	4038
0.90									
ARL_1		211	408,69	620,1	912,04	1239,7	1420,1	1508,5	1643,9
SDRL		151,14	382,73	604,08	902,51	1234,3	1416,4	1505,9	1643,4
10%		72	66	80	105	135	153	161	174
50%		168	291	435	635	861	985	1046	1140
90%		407	907	1407	2088	2848	3265	3470	3785
0.80									
ARL_1		89,3	128,94	208,54	372,51	646,28	859,73	1020,4	1427,3
SDRL		42,21	103,83	192,25	362,72	640,74	855,99	1017,7	1426,8
10%		47	35	37	48	73	94	110	151
50%		80	98	150	261	450	597	708	990
90%		144	264	459	845	1481	1975	2346	3286
0.70									
ARL_1		54,42	56,62	80,31	150,58	318,5	492,61	659,48	1216,1
SDRL		17,75	34,35	64,58	140,68	312,85	488,81	656,79	1215,6
10%		35	24	22	25	39	55	72	129
50%		51	47	61	107	223	343	458	843
90%		78	101	164	334	726	1129	1515	2800
0.60									
ARL_1		38,89	32,84	38,31	63,72	148,94	264,85	403,04	1010,8
SDRL		9,196	14,02	24,01	54,02	143,22	260,99	400,33	1010,3
10%		28	19	16	15	21	31	45	107
50%		37	30	32	47	105	185	280	701
90%		51	51	70	134	335	605	924	2327
0.50									
ARL_1		30,18	22,56	22,53	30,21	67,13	132,45	229,52	812,15
SDRL		5,25	6,71	10,05	21,11	61,44	128,56	226,79	811,65
10%		24	15	13	11	12	17	27	86
50%		30	21	20	24	48	93	160	563
90%		37	31	36	58	147	300	525	1869
0.20									
ARL_1		18	11,48	9,31	7,82	7,91	11,28	22,03	270,797
SDRL		1,01	1	1,1	1,49	3,23	7,66	19,33	270,296
10%		17	10	8	6	5	4	5	29
50%		18	11	9	8	7	9	16	188
90%		19	13	11	10	12	21	47	623

Table B-2: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B = 1) when true underlying distribution is Weibull (β = 1.5)

	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,871	0,6561	0,5176	0,3577	0,1921	0,1026	0,0471	0,002
	B	1	1	1	1	1	1	1	1
mean	Z_0	1	1	1	1	1	1	1	1
1.00									
ARL_0		3358,5	12020	18103	22962	24097	22454	19429	13035
SDRL		3271,1	11990	18085	22952	24092	22450	19426	13034
10%		493	1293	1923	***	2544	2369	2049	1374
50%		2355	8340	12553	***	16705	15565	13468	***
90%		7619	27638	41660	***	55479	51696	44733	***
0.95									
ARL_1		751,93	3904,7	7707,3	12513	16128	16691	15505	12071
SDRL		662,6	3873,3	7689,2	12503	16123	16688	15503	12071
10%		158	440	828	***	***	***	1636	1272
50%		550	2716	5348	***	***	***	***	***
90%		1615	***	***	***	***	***	***	***
0.90									
ARL_1		269,81	1314,9	3227,3	6644,7	10571	12208	12227	11131
SDRL		189,33	1282	3208,3	6634	10565	12205	12224	11130
10%		94	168	357	710	1119	***	1291	1173
50%		216	992	2243	***	***	***	***	***
90%		516	2985	7406	***	***	***	***	***
0.80									
ARL_1		94,2	210,2	583,35	1759,2	4255,9	6197,3	7309	9327,7
SDRL		38,54	177,83	563,26	1747,9	4249,9	6193,3	7306,3	9327,2
10%		54	51	80	195	454	***	773	983
50%		86	156	411	1223	2952	***	***	***
90%		145	442	1317	4036	9792	***	***	***
0.70									
ARL_1		55,42	66,27	133,35	448,33	1564,8	2904,2	4107,3	7634,4
SDRL		15,08	39,25	113,90	436,66	1558,6	2900,1	4104,5	7633,9
10%		38	29	31	58	170	310	435	805
50%		53	56	99	314	1087	2014	***	***
90%		75	117	282	1017	3595	***	***	***
0.60									
ARL_1		39,16	34,50	46,63	122,31	527,69	1240,6	2138,9	6058
SDRL		7,63	13,19	29,66	110,80	521,29	1236,4	2136,1	6057,5
10%		30	21	19	23	61	134	228	639
50%		38	32	38	88	368	861	1483	***
90%		49	52	85	267	1207	2851	4921	***
0.50									
ARL_1		30,26	22,94	24,06	41,28	167,44	477,83	1013,5	4609,5
SDRL		4,32	5,82	10,03	30,69	161,01	473,59	1010,7	4609
10%		25	17	14	14	23	54	109	486
50%		30	22	22	32	118	333	703	***
90%		36	31	37	81	377	1095	2330	***
0.20									
ARL_1		17,98	11,47	9,30	7,85	8,45	15,22	43,23	1166,8
SDRL		0,83	0,83	0,92	1,28	3,42	11,29	40,38	1166,3
10%		17	11	8	6	5	5	7	123
50%		18	11	9	8	8	12	31	809
90%		19	12	10	9	13	30	96	2686

Table B-3: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B = 1) when true underlying distribution is Weibull (β = 2)

mean									
	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,871	0,6561	0,5176	0,3577	0,1921	0,1026	0,0471	0,002
	B	1	1	1	1	1	1	1	1
	Z_o	1	1	1	1	1	1	1	1
1.00									
ARL_0		35.457	588.870	1.220.200	1.648.200	1.463.500	1.120.800	767.190	318.320
SDRL		35.338	588.840	1.220.200	1.648.200	1.463.500	1.120.800	767.190	318.320
10%		***	***	***	***	***	***	***	***
50%		***	***	***	***	***	***	***	***
90%		***	***	***	***	***	***	***	***
0.95									
ARL_1		2.000,90	86.186	307.970	652.210	816.220	735.740	560.680	287.300
SDRL		1.865,50	86.148	307.950	652.200	816.210	735.740	560.680	287.300
10%		333	***	***	***	***	***	***	***
50%		***	***	***	***	***	***	***	***
90%		***	***	***	***	***	***	***	***
0.90									
ARL_1		360,44	12.765	73.773	245.350	439.820	471.190	402.730	257.810
SDRL		245,93	12.723	73.750	245.340	439.810	471.180	402.730	257.810
10%		131	1383	***	***	***	***	***	***
50%		292	***	***	***	***	***	***	***
90%		680	***	***	***	***	***	***	***
0.80									
ARL_1		98,35	497,52	4.186,5	30.607	114.990	178.690	196.530	203.720
SDRL		32,85	453,85	4.161,2	30.594	114.990	178.690	196.530	203.720
10%		62	92	464	***	***	***	***	***
50%		93	359	***	***	***	***	***	***
90%		142	1089	***	***	***	***	***	***
0.70									
ARL_1		56,15	81,34	338,65	3.465,3	25.931	60.071	87.755	155.990
SDRL		12,03	47,65	313,77	3.451,4	25.924	60.067	87.752	155.990
10%		42	36	58	378	***	***	***	***
50%		55	68	243	***	***	***	***	***
90%		72	143	747	***	***	***	***	***
0.60									
ARL_1		39,33	36,15	62,59	419,52	5.078,6	17.591	35.142	114.580
SDRL		5,98	11,88	41,82	405,58	5.071,4	17.587	35.139	114.580
10%		32	24	24	57	542	1857	***	***
50%		39	34	50	295	***	***	***	***
90%		47	52	117	948	***	***	***	***
0.50									
ARL_1		30,28	23,20	25,84	71,09	891,17	4.428,5	12.333	79.581
SDRL		3,36	4,74	9,85	58,39	883,84	4.423,9	12.330	79.580
10%		26	18	16	19	100	471	1302	***
50%		30	23	24	53	620	3071	***	***
90%		35	29	39	147	2042	***	***	***
0.20									
ARL_1		17,98	11,46	9,29	7,85	9,18	25,75	140,98	12.736
SDRL		0,67	0,66	0,73	1,04	3,70	21,40	137,93	12.735
10%		17	11	8	7	6	7	18	1342
50%		18	11	9	8	8	19	99	***
90%		19	12	10	9	14	54	321	***

Table B-4: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B = 1) when true underlying distribution is Weibull (β = 2.5)

mean									
	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,871	0,6561	0,5176	0,3577	0,1921	0,1026	0,0471	0,002
	B	1	1	1	1	1	1	1	1
mean	Z_0	1	1	1	1	1	1	1	1
1.00									
ARL_0		617.790	48.263.000	121.640.000	150.130.000	98.065.000	57.870.000	30.248.000	7.539.300
SDRL		617.650	48.263.000	121.640.000	150.130.000	98.065.000	57.870.000	30.248.000	7.539.300
10%		***	***	***	***	***	***	***	***
50%		***	***	***	***	***	***	***	***
90%		***	***	***	***	***	***	***	***
0.95									
ARL_1		6.007	2.965.000	17.758.000	42.920.000	45.569.000	33.544.000	20.252.000	6.631.200
SDRL		5.827	2.964.900	17.758.000	42.920.000	45.569.000	33.544.000	20.252.000	6.631.200
10%		796	***	***	***	***	***	***	***
50%		***	***	***	***	***	***	***	***
90%		***	***	***	***	***	***	***	***
0.90									
ARL_1		455,12	179.560	2.376.200	11.401.000	20.223.000	18.839.000	13.272.000	5.793.600
SDRL		309,38	179.510	2.376.100	11.401.000	20.223.000	18.839.000	13.272.000	5.793.600
10%		168	***	***	***	***	***	***	***
50%		368	***	***	***	***	***	***	***
90%		857	***	***	***	***	***	***	***
0.80									
ARL_1		100,48	1.369,2	39.835	659.660	3.421.800	5.328.600	5.283.400	4.316.000
SDRL		28,42	1.316,2	39.806	659.650	3.421.800	5.328.600	5.283.400	4.316.000
10%		69	192	4223	***	***	***	***	***
50%		96	966	***	***	***	***	***	***
90%		138	3084	***	***	***	***	***	***
0.70									
ARL_1		56,41	96,75	1.012	32.950	473.140	1.282.900	1.870.400	3.090.200
SDRL		10,00	57,48	982,41	32.934	473.130	1.282.800	1.870.400	3.090.200
10%		44	43	133	***	***	***	***	***
50%		55	81	710	***	***	***	***	***
90%		70	171	2291	***	***	***	***	***
0.60									
ARL_1		39,36	37,10	83,67	1.724,20	54.679	259.580	577.080	2.101.900
SDRL		4,93	10,74	59,63	1.708,50	54.671	259.580	577.080	2.101.900
10%		33	26	30	196	***	***	***	***
50%		39	35	66	1200	***	***	***	***
90%		46	51	161	3950	***	***	***	***
0.50									
ARL_1		30,26	23,29	27,08	130,86	5.382,6	43.315	150.820	1.332.500
SDRL		2,76	3,99	9,60	116,40	5.374,7	43.310	150.810	1.332.500
10%		27	19	17	27	574	***	***	***
50%		30	23	25	95	3733	***	***	***
90%		34	29	40	282	12384	***	***	***
0.20									
ARL_1		17,98	11,45	9,28	7,84	9,80	45,52	478,49	134.840
SDRL		0,57	0,57	0,61	0,87	3,95	40,83	475,29	134.840
10%		17	11	9	7	6	9	53	***
50%		18	11	9	8	9	33	333	***
90%		19	12	10	9	15	99	1098	***

Table B-5: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B = 1) when true underlying distribution is Weibull (β = 4)

mean									
	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,871	0,6561	0,5176	0,3577	0,1921	0,1026	0,0471	0,002
	B	1	1	1	1	1	1	1	1
	Z_0	1	1	1	1	1	1	1	1
1.00									
ARL_0		3,93E+10	1,98E+14	4,77E+14	2,33E+14	3,84E+13	8,72E+12	1,88E+12	9,26E+10
SDRL		3,93E+10	1,98E+14	4,77E+14	2,33E+14	3,84E+13	8,72E+12	1,88E+12	9,26E+10
10%		***	***	***	***	***	***	***	***
50%		***	***	***	***	***	***	***	***
90%		***	***	***	***	***	***	***	***
0.95									
ARL_1		507.680	7,16E+11	1,23E+13	2,47E+13	1,03E+13	3,48E+12	9,70E+11	7,54E+10
SDRL		507.400	7,16E+11	1,23E+13	2,47E+13	1,03E+13	3,48E+12	9,70E+11	7,54E+10
10%		***	***	***	***	***	***	***	***
50%		***	***	***	***	***	***	***	***
90%		***	***	***	***	***	***	***	***
0.90									
ARL_1		858,62	2,26E+09	2,58E+11	2,23E+12	2,51E+12	1,30E+12	4,80E+11	6,07E+10
SDRL		635,31	2,26E+09	2,58E+11	2,23E+12	2,51E+12	1,30E+12	4,80E+11	6,07E+10
10%		283	***	***	***	***	***	***	***
50%		671	***	***	***	***	***	***	***
90%		1685	***	***	***	***	***	***	***
0.80									
ARL_1		102,80	75.462	1,01E+08	1,27E+10	1,13E+11	1,51E+11	1,03E+11	3,79E+10
SDRL		19,99	75.391	1,01E+08	1,27E+10	1,13E+11	1,51E+11	1,03E+11	3,79E+10
10%		79	***	***	***	***	***	***	***
50%		101	***	***	***	***	***	***	***
90%		129	***	***	***	***	***	***	***
0.70									
ARL_1		56,59	157,68	64.702	5,71E+07	3,74E+09	1,35E+10	1,80E+10	2,22E+10
SDRL		6,67	104,84	64.665	5,71E+07	3,74E+09	1,35E+10	1,80E+10	2,22E+10
10%		48	62	***	***	***	***	***	***
50%		56	127	***	***	***	***	***	***
90%		65	294	***	***	***	***	***	***
0.60									
ARL_1		39,32	38,38	230,63	238.320	9,28E+07	9,22E+08	2,55E+09	1,20E+10
SDRL		3,25	8,22	198,66	238.300	9,28E+07	9,22E+08	2,55E+09	1,20E+10
10%		35	29	53	***	***	***	***	***
50%		39	37	170	***	***	***	***	***
90%		44	49	489	***	***	***	***	***
0.50									
ARL_1		30,20	23,32	29,33	1.294,40	1.675.200	4,60E+07	2,78E+08	5,79E+09
SDRL		1,82	2,69	8,78	1.276,20	1.675.200	4,60E+07	2,78E+08	5,79E+09
10%		28	20	20	153	***	***	***	***
50%		30	23	28	903	***	***	***	***
90%		33	27	41	2957	***	***	***	***
0.20									
ARL_1		17,98	11,43	9,24	7,80	11,29	312,43	20.211	1,48E+08
SDRL		0,36	0,50	0,46	0,61	4,60	307,04	20.207	1,48E+08
10%		18	11	9	7	7	38	2133	***
50%		18	11	9	8	10	218	***	***
90%		18	12	10	8	17	712	***	***

Table B-6: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B = 2) when true underlying distribution is Weibull (β = 1.2)

	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,901446	0,68607	0,545071	0,37932	0,204487	0,109064	0,049218	0,002002
	B	2	2	2	2	2	2	2	2
mean	Z_0	1	1	1	1	1	1	1	1
1.00									
ARL_0		792,15	1263,8	1558,2	1848,8	2069,4	2122,1	2095,3	1863,2
SDRL		776,34	1245,8	1545,8	1841	2064,7	2118,8	2092,8	1862,7
10%		105	149	175	202	222	226	223	197
50%		550	881	1084	1284	1436	1472	1453	1292
90%		1804	2886	3572	4247	4759	4882	4821	4289
0.95									
ARL_1		292,55	564,02	810,98	1132,2	1479,3	1652,9	1739,9	1751,9
SDRL		254,06	542,19	796,92	1123,8	1474,4	1649,5	1737,4	1751,4
10%		66	79	98	127	160	177	186	185
50%		215	398	566	787	1027	1147	1207	1214
90%		623	1270	1849	2596	3400	3802	4003	4033
0.90									
ARL_1		149,82	278,93	437,32	695,69	1049,3	1276,2	1433,5	1642
SDRL		109,28	255,54	422,27	686,80	1044,2	1272,8	1431	1641,5
10%		49	50	60	81	115	138	153	173
50%		119	201	308	485	729	886	994	1138
90%		292	612	987	1590	2409	2934	3297	3780
0.80									
ARL_1		66,84	93,56	145,36	267,65	514,74	738,04	946,60	1425,6
SDRL		33,60	71,18	129,84	258,19	509,44	734,49	944,05	1425,1
10%		33	29	29	37	59	81	102	151
50%		59	73	106	188	358	513	657	988
90%		111	186	314	604	1178	1695	2176	3282
0.70									
ARL_1		41,49	45,08	60,22	108,72	245,07	408,53	599,34	1214,6
SDRL		14,95	25,82	45,65	99,18	239,59	404,86	596,74	1214,1
10%		25	20	19	20	31	46	65	128
50%		39	38	47	78	172	284	416	842
90%		61	79	120	238	557	936	1377	2796
0.60									
ARL_1		29,87	27,61	31,25	48,58	114,11	215,34	360,65	1009,6
SDRL		7,94	11,43	18,31	39,40	108,55	211,58	358,00	1009,1
10%		21	16	14	13	17	26	40	107
50%		29	25	26	37	81	150	251	700
90%		40	43	55	100	256	491	827	2324
0.50									
ARL_1		23,31	19,47	19,45	24,76	52,77	107,48	203,64	811,18
SDRL		4,60	5,77	8,22	16,29	47,28	103,68	200,96	810,68
10%		18	13	11	10	10	15	24	86
50%		23	18	17	20	38	76	142	562
90%		29	27	30	46	114	243	465	1867
0.20									
ARL_1		14,21	10,21	8,55	7,27	7,27	10,12	20,14	270,47
SDRL		0,99	0,92	1,01	1,33	2,81	6,62	17,48	269,97
10%		13	9	7	6	5	4	5	29
50%		14	10	8	7	7	8	15	188
90%		15	11	10	9	11	19	43	622

Table B-7: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B = 2) when true underlying distribution is Weibull (β = 2)

	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,901446	0,68607	0,545071	0,37932	0,204487	0,109064	0,049218	0,002002
	B	2	2	2	2	2	2	2	2
mean	Z_0	1	1	1	1	1	1	1	1
1.00									
ARL_0		4.981,6	141.310	401.100	750.030	915.020	828.490	668.560	317.690
SDRL		4.909,4	141.280	401.080	750.020	915.010	828.480	668.560	317.690
10%		590	***	***	***	***	***	***	***
50%		3.475	***	***	***	***	***	***	***
90%		11.376	***	***	***	***	***	***	***
0.95									
ARL_1		551,96	19.004	89.630	267.290	475.830	519.010	476.870	286.720
SDRL		457,66	18.966	89.609	267.280	475.820	519.000	476.870	286.720
10%		137	***	***	***	***	***	***	***
50%		416	***	***	***	***	***	***	***
90%		1.147	***	***	***	***	***	***	***
0.90									
ARL_1		183,48	3.144,6	20.824	94.143	242.450	319.250	335.040	257.300
SDRL		109,74	3.103,2	20.801	94.131	242.440	319.240	335.040	257.300
10%		76	368	***	***	***	***	***	***
50%		156	2192	***	***	***	***	***	***
90%		326	7187	***	***	***	***	***	***
0.80									
ARL_1		68,20	219,56	1.409,6	11.553	59.346	114.210	157.740	203.310
SDRL		23,66	181,86	1.385,2	11.540	59.340	114.200	157.740	203.310
10%		42	56	170	1229	***	***	***	***
50%		64	165	985	***	***	***	***	***
90%		99	456	3214	***	***	***	***	***
0.70									
ARL_1		41,19	56,83	165,97	1.461,1	13.313	37.396	68.861	155.670
SDRL		9,79	28,98	143,45	1.447,6	13.306	37.392	68.858	155.670
10%		30	29	38	166	***	***	***	***
50%		40	50	122	1017	***	***	***	***
90%		54	94	353	3347	***	***	***	***
0.60									
ARL_1		29,46	29,41	43,77	215,79	2.725,8	11.000	27.355	114.350
SDRL		5,12	9,02	25,57	202,67	2.718,8	10.996	27.352	114.350
10%		23	20	20	35	294	***	***	***
50%		29	28	37	154	1892	***	***	***
90%		36	41	77	480	6267	***	***	***
0.50									
ARL_1		22,95	19,81	21,43	47,12	517,96	2.848,6	9.649,7	79.422
SDRL		2,98	3,98	7,35	35,45	510,86	2.844,0	9.646,7	79.421
10%		19	15	14	15	61	304	***	***
50%		23	19	20	36	361	1976	***	***
90%		27	25	31	93	1183	6553	***	***
0.20									
ARL_1		14,20	10,19	8,53	7,28	8,19	20,74	118,03	12.710
SDRL		0,59	0,62	0,67	0,93	2,99	16,53	115,01	12.710
10%		14	10	8	6	5	6	15	***
50%		14	10	8	7	7	16	83	***
90%		15	11	9	8	12	42	268	***

Table B-8: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B = 2) when true underlying distribution is Weibull (β = 2.5)

	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,901446	0,68607	0,545071	0,37932	0,204487	0,109064	0,049218	0,002002
	B	2	2	2	2	2	2	2	2
mean	Z_0	1	1	1	1	1	1	1	1
1.00									
ARL_0		19.775	5.081.400	21.947.000	45.460.000	47.965.000	36.303.000	24.406.000	7.520.500
SDRL		19.674	5.081.300	21.947.000	45.460.000	47.965.000	36.303.000	24.406.000	7.520.500
10%		***	***	***	***	***	***	***	***
50%		***	***	***	***	***	***	***	***
90%		***	***	***	***	***	***	***	***
0.95									
ARL_1		793,23	292.070	2.799.300	11.503.000	20.523.000	19.918.000	15.856.000	6.614.700
SDRL		667,35	292.020	2.799.300	11.503.000	20.523.000	19.918.000	15.856.000	6.614.700
10%		191	***	***	***	***	***	***	***
50%		594	***	***	***	***	***	***	***
90%		1662	***	***	***	***	***	***	***
0.90									
ARL_1		198,79	21.594	372.440	2.863.500	8.568.300	10.697.000	10.123.000	5.779.100
SDRL		109,84	21.545	372.410	2.863.400	8.568.300	10.697.000	10.123.000	5.779.100
10%		90	***	***	***	***	***	***	***
50%		172	***	***	***	***	***	***	***
90%		342	***	***	***	***	***	***	***
0.80									
ARL_1		68,61	399,20	8.231,2	169.630	1.370.400	2.859.100	3.877.000	4.305.200
SDRL		20,26	353,37	8.203,1	169.620	1.370.400	2.859.100	3.877.000	4.305.200
10%		46	83	***	***	***	***	***	***
50%		66	291	***	***	***	***	***	***
90%		95	859	***	***	***	***	***	***
0.70									
ARL_1		41,09	62,55	349,82	9.936,6	193.400	679.450	1.347.000	3.082.400
SDRL		8,23	30,68	323,31	9.921,6	193.400	679.450	1.347.000	3.082.400
10%		31	33	61	***	***	***	***	***
50%		40	55	251	***	***	***	***	***
90%		52	102	771	***	***	***	***	***
0.60									
ARL_1		29,34	29,89	52,12	664,84	23.911	140.430	415.620	2.096.700
SDRL		4,31	7,93	31,29	649,95	23.903	140.430	415.620	2.096.700
10%		24	21	23	83	***	***	***	***
50%		29	29	43	465	***	***	***	***
90%		35	40	93	1511	***	***	***	***
0.50									
ARL_1		22,83	19,85	22,08	72,46	2.590,5	24.404	110.050	1.329.200
SDRL		2,53	3,33	6,83	59,17	2.582,7	24.399	110.050	1.329.200
10%		20	16	15	19	280	***	***	***
50%		23	19	21	54	1798	***	***	***
90%		26	24	31	149	5955	***	***	***
0.20									
ARL_1		14,11	10,18	8,52	7,27	8,57	33,76	379,06	134.500
SDRL		0,41	0,52	0,58	0,78	3,05	29,21	375,89	134.500
10%		14	10	8	6	6	8	43	***
50%		14	10	8	7	8	25	264	***
90%		15	11	9	8	13	72	869	***

Table B-9: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B = 2) when true underlying distribution is Weibull (β = 4)

	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,901446	0,68607	0,545071	0,37932	0,204487	0,109064	0,049218	0,002002
	B	2	2	2	2	2	2	2	2
mean	Z_0	1	1	1	1	1	1	1	1
1.00									
ARL_0		3.890.300	1,40E+12	1,30E+13	2,05E+13	8,99E+12	3,35E+12	1,20E+12	9,22E+10
SDRL		3.890.200	1,40E+12	1,30E+13	2,05E+13	8,99E+12	3,35E+12	1,20E+12	9,22E+10
10%		***	***	***	***	***	***	***	***
50%		***	***	***	***	***	***	***	***
90%		***	***	***	***	***	***	***	***
0.95									
ARL_1		2.771,5	5,17E+09	2,95E+11	1,87E+12	2,16E+12	1,24E+12	5,90E+11	7,51E+10
SDRL		2.564,8	5,17E+09	2,95E+11	1,87E+12	2,16E+12	1,24E+12	5,90E+11	7,51E+10
10%		479	***	***	***	***	***	***	***
50%		1987	***	***	***	***	***	***	***
90%		6112	***	***	***	***	***	***	***
0.90									
ARL_1		233,46	26.615.000	6,84E+09	1,62E+11	4,94E+11	4,42E+11	2,82E+11	6,05E+10
SDRL		111,33	26.615.000	6,84E+09	1,62E+11	4,94E+11	4,42E+11	2,82E+11	6,05E+10
10%		122	***	***	***	***	***	***	***
50%		208	***	***	***	***	***	***	***
90%		378	***	***	***	***	***	***	***
0.80									
ARL_1		69,35	4.186,9	4.541.300	1,08E+09	2,23E+10	4,95E+10	5,82E+10	3,78E+10
SDRL		14,61	4.122,8	4.541.300	1,08E+09	2,23E+10	4,95E+10	5,82E+10	3,78E+10
10%		52	499	***	***	***	***	***	***
50%		68	2922	***	***	***	***	***	***
90%		89	9557	***	***	***	***	***	***
0.70									
ARL_1		41,14	76,95	6.352,4	6.311.600	8,02E+08	4,55E+09	1,02E+10	2,21E+10
SDRL		5,88	35,89	6.317,6	6.311.600	8,02E+08	4,55E+09	1,02E+10	2,21E+10
10%		34	42	***	***	***	***	***	***
50%		41	68	***	***	***	***	***	***
90%		49	123	***	***	***	***	***	***
0.60									
ARL_1		29,15	30,45	86,35	37.238	2,23E+07	3,29E+08	1,47E+09	1,20E+10
SDRL		3,06	5,76	59,05	37.219	2,23E+07	3,29E+08	1,47E+09	1,20E+10
10%		25	24	33	***	***	***	***	***
50%		29	30	69	***	***	***	***	***
90%		33	38	163	***	***	***	***	***
0.50									
ARL_1		22,92	19,83	23,04	353,88	462.830	1,75E+07	1,65E+08	5,76E+09
SDRL		1,83	2,25	5,55	336,87	462.820	1,75E+07	1,65E+08	5,76E+09
10%		21	17	17	53	***	***	***	***
50%		23	20	22	251	***	***	***	***
90%		25	23	30	793	***	***	***	***
0.20									
ARL_1		14,00	10,13	8,51	7,23	9,34	176,30	13.611	1,47E+08
SDRL		0,06	0,38	0,50	0,53	3,11	171,05	13.608	1,47E+08
10%		14	10	8	7	6	23	***	***
50%		14	10	9	7	9	124	***	***
90%		14	11	9	8	13	399	***	***

Table B-10: Performance metrics of exponentially designed Lower-sided TBE EWMA charts (designed with B = 5) when true underlying distribution is Weibull (β = 1.2)

	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,876086	0,685314	0,544453	0,379277	0,205282	0,1098005	0,0496857	0,002002
	B	5	5	5	5	5	5	5	5
mean	Z_0	1	1	1	1	1	1	1	1
1.00									
ARL_0		1.648,1	1.257,5	1.550	1.833,9	2.031,5	2.079,1	2.059,8	1.863,2
SDRL		1.608,5	1.240,5	1.538	1.826,1	2.027,0	2.075,9	2.057,4	1.862,7
10%		210	148	174	200	218	222	219	197
50%		1154	877	1078	1274	1410	1442	1429	1292
90%		3743	2873	3553	4212	4672	4783	4740	4289
0.95									
ARL_1		592,67	564,31	808,35	1.123,7	1.449,2	1.613,8	1.704,1	1751,90
SDRL		537,51	543,27	794,67	1.115,3	1.444,4	1.610,5	1.701,7	1751,40
10%		111	78	97	126	157	173	182	185
50%		429	398	565	781	1006	1120	1182	1214
90%		1293	1272	1843	2576	3331	3712	3921	4033
0.90									
ARL_1		280,79	280,13	436,65	690,97	1.026,6	1242,80	1.399,7	1.642,0
SDRL		225,01	257,35	421,93	682,11	1.021,7	1239,40	1.397,2	1.641,5
10%		75	50	59	81	113	134	150	173
50%		215	202	307	482	713	862	971	1138
90%		573	615	986	1579	2357	2857	3220	3780
0.80									
ARL_1		109,31	94,22	145,46	266,35	503,44	716,58	920,19	1.425,6
SDRL		64,06	72,22	130,17	256,91	498,19	713,06	917,66	1.425,1
10%		46	28	29	37	58	79	99	151
50%		94	73	106	188	351	498	639	988
90%		193	188	315	601	1152	1645	2116	3282
0.70									
ARL_1		62,70	45,33	60,27	108,40	240,05	396,48	581,33	1.214,6
SDRL		27,56	26,32	45,88	988,84	234,62	392,84	578,75	1.214,1
10%		33	20	19	20	30	45	64	128
50%		57	38	47	78	168	276	404	842
90%		99	80	120	237	546	908	1335	2796
0.60									
ARL_1		42,83	27,68	31,24	48,51	112,05	209,30	349,72	1.009,6
SDRL		14,97	11,69	18,43	39,36	106,54	205,57	347,08	1.009,1
10%		26	16	14	13	17	25	39	107
50%		40	25	26	37	79	146	243	700
90%		63	43	55	100	251	477	802	2324
0.50									
ARL_1		31,82	19,47	19,41	24,76	51,98	104,75	197,71	811,18
SDRL		9,31	5,92	8,29	16,30	46,53	100,98	195,04	810,68
10%		21	13	11	10	10	14	23	86
50%		31	18	17	20	38	74	138	562
90%		44	27	30	46	113	236	452	1867
0.20									
ARL_1		14,15	10,17	8,50	7,27	7,22	10,00	19,77	270,47
SDRL		2,46	0,99	1,04	1,34	2,79	6,52	17,12	269,97
10%		11	9	7	6	5	4	4	29
50%		14	10	8	7	6	8	15	188
90%		17	11	10	9	11	18	42	622

Table B-11: Performance metrics of exponentially designed Lower-sided TBE EWMA charts (designed with B = 5) when true underlying distribution is Weibull (β = 1.5)

	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,876086	0,685314	0,544453	0,379277	0,205282	0,1098005	0,0496857	0,002002
	B	5	5	5	5	5	5	5	5
mean	Z_o	1	1	1	1	1	1	1	1
1.00									
ARL_0		16.391	5.972,9	10.406	15.371	18.503	18.646	17.514	13.015
SDRL		16.360	5.951,0	10.392	15.362	18.498	18.643	17.511	13.015
10%		***	***	***	***	***	***	***	***
50%		***	***	***	***	***	***	***	***
90%		***	***	***	***	***	***	***	***
0.95									
ARL_1		2.691,9	1.795,6	4.016,7	7.716,3	11.727	13.355	13.703	12.053
SDRL		2.613,4	1.767,8	4.000,1	7.706,8	11.722	13.351	13.701	12.052
10%		355	214	***	***	***	***	***	***
50%		1891	1253	***	***	***	***	***	***
90%		6096	4098	***	***	***	***	***	***
0.90									
ARL_1		723,96	625,83	1.612,2	3.863,8	7.339,1	9.447,5	10.610	11.114
SDRL		637,80	595,69	1.594,0	3.853,6	7.333,7	9.443,9	10.608	11.113
10%		150	93	186	***	***	***	***	***
50%		531	443	1123	***	***	***	***	***
90%		1554	1402	3689	***	***	***	***	***
0.80									
ARL_1		167,38	129,39	312,68	980,04	2.770,2	4.547,1	6.148,4	9.313,8
SDRL		101,07	101,45	293,62	968,96	2.764,4	4.543,3	6.145,8	9.313,3
10%		69	37	50	113	***	***	***	***
50%		142	99	223	683	***	***	***	***
90%		299	261	695	2242	***	***	***	***
0.70									
ARL_1		82,72	50,34	86,70	263,37	995,22	2.062,6	3.377,7	7.622,9
SDRL		35,08	27,78	69,11	252,09	989,15	2.058,7	3.375,0	7.622,4
10%		45	23	24	38	110	221	***	***
50%		76	43	66	186	692	1431	***	***
90%		129	86	177	592	2284	4744	***	***
0.60									
ARL_1		53,77	28,60	35,98	81,20	342,15	872,94	1.736,9	6.049,0
SDRL		18,00	10,71	21,01	70,40	335,95	868,88	1.734,1	6.048,5
10%		33	17	16	18	42	96	185	***
50%		51	26	30	60	239	606	1205	***
90%		78	43	63	173	780	2005	3996	***
0.50									
ARL_1		38,86	19,66	20,38	31,52	114,98	341,14	821,79	4.602,5
SDRL		11,27	5,10	7,99	21,76	108,82	337,01	818,98	4.602,0
10%		26	14	12	12	18	40	89	***
50%		37	19	19	25	82	238	570	***
90%		54	26	31	60	257	780	1889	***
0.20									
ARL_1		14,28	10,15	8,49	7,29	7,62	12,97	37,29	1.165,1
SDRL		2,53	0,84	0,87	1,15	2,88	9,19	34,48	1.164,6
10%		11	9	9	6	5	5	6	123
50%		14	10	8	7	7	10	27	808
90%		18	11	10	9	11	25	82	2682

Table B-12: Performance metrics of exponentially designed Lower-sided TBE EWMA charts (designed with B = 5) when true underlying distribution is Weibull (β = 2)

	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,876086	0,685314	0,544453	0,379277	0,205282	0,1098005	0,0496857	0,002002
	B	5	5	5	5	5	5	5	5
mean	Z_o	1	1	1	1	1	1	1	1
1.00									
ARL_0		1.137.200	119.650	375.830	721.110	856.290	770.800	625.590	317.690
SDRL		1.137.300	119.620	375.810	721.110	856.280	770.800	625.590	317.690
10%		***	***	***	***	***	***	***	***
50%		***	***	***	***	***	***	***	***
90%		***	***	***	***	***	***	***	***
0.95									
ARL_1		41.910	16.900	84.992	257.900	446.120	483.340	446.180	286.720
SDRL		41.797	16.864	84.972	257.890	446.120	483.340	446.170	286.720
10%		***	***	***	***	***	***	***	***
50%		***	***	***	***	***	***	***	***
90%		***	***	***	***	***	***	***	***
0.90									
ARL_1		3.429,0	2.928,1	19.984	91.169	227.780	297.690	313.580	257.300
SDRL		3.276,6	2.888,1	19.962	91.157	227.770	297.680	313.580	257.300
10%		***	***	***	***	***	***	***	***
50%		***	***	***	***	***	***	***	***
90%		***	***	***	***	***	***	***	***
0.80									
ARL_1		289,78	217,65	1.383,8	11.275	56.008	106.830	147.870	203.310
SDRL		174,52	180,89	1.359,8	11.262	56.001	106.820	147.860	203.310
10%		120	55	167	***	***	***	***	***
50%		246	163	967	***	***	***	***	***
90%		516	453	3155	***	***	***	***	***
0.70									
ARL_1		125,06	57,04	165,64	1.437,7	12.630	35.108	64.694	155.670
SDRL		48,43	29,69	143,39	1.424,3	12.623	35.103	64.692	155.670
10%		72	28	37	164	***	***	***	***
50%		117	50	122	1001	***	***	***	***
90%		189	96	352	3293	***	***	***	***
0.60									
ARL_1		80,43	29,44	43,87	214,11	2.602,4	10.372	25.771	114.350
SDRL		25,85	9,35	25,85	201,03	2.595,4	10.367	25.768	114.350
10%		50	19	20	34	***	***	***	***
50%		77	28	37	152	***	***	***	***
90%		115	42	77	476	***	***	***	***
0.50									
ARL_1		56,02	19,77	21,43	47,06	498,42	2.699,9	9121,20	79.422
SDRL		16,87	4,16	7,47	35,42	491,37	2.695,4	9118,20	79.421
10%		36	15	14	15	59	***	***	***
50%		54	19	20	36	348	***	***	***
90%		78	25	31	93	1138	***	***	***
0.20									
ARL_1		14,21	10,13	8,48	7,28	8,12	20,27	113,55	12.710
SDRL		2,55	0,71	0,70	0,94	2,97	16,09	110,54	12.710
10%		11	9	8	6	5	6	15	***
50%		14	10	8	7	7	15	80	***
90%		18	11	9	8	12	41	258	***

Table B-13: Performance metrics of exponentially designed Lower-sided TBE EWMA charts (designed with B = 5) when true underlying distribution is Weibull (β = 2.5)

mean	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,876086	0,685314	0,544453	0,379277	0,205282	0,1098005	0,0496857	0,002002
	B	5	5	5	5	5	5	5	5
	Z_0	1	1	1	1	1	1	1	1
1.00									
ARL_0		29.571.000	3.429.100	19.111.000	42.385.000	43.615.000	32.938.000	22.323.000	7.520.500
SDRL		29.571.000	3.429.100	19.111.000	42.385.000	43.615.000	32.938.000	22.323.000	7.520.500
10%		***	***	***	***	***	***	***	***
50%		***	***	***	***	***	***	***	***
90%		***	***	***	***	***	***	***	***
0.95									
ARL_1		265.440	216.850	2.489.800	10.785.000	18.713.000	18.104.000	14.516.000	6.614.700
SDRL		265.240	216.800	2.489.700	10.785.000	18.713.000	18.104.000	14.516.000	6.614.700
10%		***	***	***	***	***	***	***	***
50%		***	***	***	***	***	***	***	***
90%		***	***	***	***	***	***	***	***
0.90									
ARL_1		7.887,9	17.605	338.480	2.700.300	7.835.400	9.741.400	9.278.100	5.779.100
SDRL		7.620,6	17.558	338.450	2.700.200	7.835.400	9.741.400	9.278.100	5.779.100
10%		***	***	***	***	***	***	***	***
50%		***	***	***	***	***	***	***	***
90%		***	***	***	***	***	***	***	***
0.80									
ARL_1		409,04	380,39	7.814,8	161.950	1.261.200	2.614.600	3.562.600	4.305.200
SDRL		216,36	335,95	7.787,1	161.940	1.261.200	2.614.600	3.562.600	4.305.200
10%		192	80	***	***	***	***	***	***
50%		359	278	***	***	***	***	***	***
90%		690	818	***	***	***	***	***	***
0.70									
ARL_1		194,09	62,58	345,32	9.616,3	179.290	624.320	1.241.500	3.082.400
SDRL		68,79	31,41	319,14	9.601,3	179.280	624.310	1.241.500	3.082.400
10%		116	32	60	***	***	***	***	***
50%		184	55	248	***	***	***	***	***
90%		285	103	761	***	***	***	***	***
0.60									
ARL_1		126,96	29,91	52,27	653,02	22.357	129.750	384.480	2.096.700
SDRL		41,53	8,32	31,65	638,18	22.349	129.750	384.480	2.096.700
10%		78	21	23	82	***	***	***	***
50%		122	29	43	457	***	***	***	***
90%		182	41	93	1484	***	***	***	***
0.50									
ARL_1		82,20	19,80	22,10	72,15	2.448,1	22.697	102.250	1.329.200
SDRL		26,17	3,56	6,99	58,90	2.440,4	22.693	102.250	1.329.200
10%		52	16	15	19	***	***	***	***
50%		79	19	21	54	***	***	***	***
90%		117	24	31	149	***	***	***	***
0.20									
ARL_1		13,98	10,10	8,47	7,27	8,50	32,69	359,83	134.500
SDRL		2,51	0,62	0,61	0,79	3,02	28,17	356,67	134.500
10%		11	9	8	6	6	8	41	***
50%		14	10	9	7	8	24	250	***
90%		17	11	9	8	12	69	824	***

Table B-14: Performance metrics of exponentially designed Lower-sided TBE EWMA charts (designed with B = 5) when true underlying distribution is Weibull (β = 4)

	λ	**0,01**	**0,05**	**0,1**	**0,2**	**0,4**	**0,6**	**0,8**	**1**
	h_L	**0,876086**	**0,685314**	**0,544453**	**0,379277**	**0,205282**	**0,1098005**	**0,0496857**	**0,002002**
	B	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**
mean	**Z_0**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
1.00									
ARL_0		7,43E+07	2,73E+11	7,62E+12	1,65E+13	7,38E+12	2,79E+12	1,02E+12	9,22E+10
SDRL		7,43E+07	2,73E+11	7,62E+12	1,65E+13	7,38E+12	2,79E+12	1,02E+12	9,22E+10
10%		***	***	***	***	***	***	***	***
50%		***	***	***	***	***	***	***	***
90%		***	***	***	***	***	***	***	***
0.95									
ARL_1		83.940	1,38E+09	1,84E+11	1,52E+12	1,78E+12	1,04E+12	5,05E+11	7,51E+10
SDRL		82.110	1,38E+09	1,84E+11	1,52E+12	1,78E+12	1,04E+12	5,05E+11	7,51E+10
10%		***	***	***	***	***	***	***	***
50%		***	***	***	***	***	***	***	***
90%		***	***	***	***	***	***	***	***
0.90									
ARL_1		4666,10	9.825.500	4,54E+09	1,34E+11	4,10E+11	3,71E+11	2,42E+11	6,05E+10
SDRL		2918,10	9.825.400	4,54E+09	1,34E+11	4,10E+11	3,71E+11	2,42E+11	6,05E+10
10%		***	***	***	***	***	***	***	***
50%		***	***	***	***	***	***	***	***
90%		***	***	***	***	***	***	***	***
0.80									
ARL_1		1708,80	2989,10	3.457.300	9,21E+08	1,87E+10	4,19E+10	5,01E+10	3,78E+10
SDRL		641,10	2927,80	3.457.300	9,21E+08	1,87E+10	4,19E+10	5,01E+10	3,78E+10
10%		***	***	***	***	***	***	***	***
50%		***	***	***	***	***	***	***	***
90%		***	***	***	***	***	***	***	***
0.70									
ARL_1		998,08	76,64	5618,90	5,58E+06	6,84E+08	3,89E+09	8,84E+09	2,21E+10
SDRL		370,33	37,01	5584,70	5,58E+06	6,84E+08	3,89E+09	8,84E+09	2,21E+10
10%		575	41	***	***	***	***	***	***
50%		945	67	***	***	***	***	***	***
90%			125	***	***	***	***	***	***
0.60									
ARL_1		541,15	30,57	85,97	34.327	1,93E+07	2,84E+08	1,28E+09	1,20E+10
SDRL		200,00	6,41	59,09	34.309	1,93E+07	2,84E+08	1,28E+09	1,20E+10
10%		313	23	32	***	***	***	***	***
50%		512	30	69	***	***	***	***	***
90%		806	39	163	***	***	***	***	***
0.50									
ARL_1		263,50	19,79	23,12	342,97	409.900	1,53E+07	1,45E+08	5,76E+09
SDRL		96,37	2,58	5,81	326,05	409.890	1,53E+07	1,45E+08	5,76E+09
10%		153	17	17	51	***	***	***	***
50%		250	20	22	243	***	***	***	***
90%		391	23	31	768	***	***	***	***
0.20									
ARL_1		13,21	10,05	8,45	7,24	9,25	164,64	12.389	1,47E+08
SDRL		2,33	0,47	0,52	0,55	3,08	159,43	12.385	1,47E+08
10%		11	10	8	7	6	22	***	***
50%		13	10	8	7	9	116	***	***
90%		16	11	9	8	13	372	***	***

APPENDIX C

PERFORMANCE RESULTS FOR THE LOWER-SIDED TBE EWMA CHARTS UNDER VARIOUS LOGNORMAL DISTRIBUTIONS

Table C-1: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B=1) when true underlying distribution is Lognormal (s=0.94):

mean									
	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,871	0,6561	0,5176	0,3577	0,1921	0,1026	0,0471	0,002
	B	1	1	1	1	1	1	1	1
	Z_o	1	1	1	1	1	1	1	1
1.00									
ARL_0		361,65	456,49	670,72	1.425,1	6.086,9	28.354	178.480	2.443.300.000
SDRL		312,11	432,74	654,59	1.414,6	6.080,5	28.350	178.480	2.443.300.000
10%		82	69	85	160	***	***	***	***
50%		267	324	470	991	***	***	***	***
90%		768	1020	1523	3268	***	***	***	***
0.95									
ARL_1		215,85	271,56	401,05	864,01	3.792,4	18.128	117.640	1.735.300.000
SDRL		169,17	248,33	385,07	853,52	3.786,0	18.124	117.640	1.735.300.000
10%		63	50	57	100	405	***	***	***
50%		165	196	283	602	2631	***	***	***
90%		436	595	903	1976	8724	***	***	***
0.90									
ARL_1		142,17	170,27	246,92	528,55	2.350,5	11.469	76.538	1.176.400.000
SDRL		98,84	147,79	231,21	518,12	2.344,1	11.464	76.535	1.176.400.000
10%		52	38	40	65	253	***	***	***
50%		113	125	176	370	1631	***	***	***
90%		271	363	548	1203	5404	***	***	***
0.80									
ARL_1		77,35	79,01	104,37	206,58	893,57	4.447,5	31.068	1.213.500.000
SDRL		40,85	58,54	89,52	196,38	887,17	4.443,0	31.064	1.213.500.000
10%		38	26	24	31	100	***	***	***
50%		67	62	77	146	621	***	***	***
90%		130	155	221	462	2049	***	***	***
0.70									
ARL_1		51,11	44,77	51,98	88,08	338,67	1.652,0	11.841	240.350.000
SDRL		20,37	26,63	38,34	78,34	332,34	1.647,6	11.838	240.350.000
10%		31	20	17	18	41	178	***	***
50%		46	37	40	64	237	1146	***	***
90%		78	79	102	190	772	3798	***	***
0.60									
ARL_1		37,70	29,46	30,32	42,32	130,68	588,32	4.191,1	92.115.000
SDRL		11,41	13,54	18,07	33,24	124,53	583,88	4.188,0	92.115.000
10%		26	17	14	13	19	66	***	***
50%		35	26	25	32	92	409	***	***
90%		53	47	54	86	293	1349	***	***
0.50									
ARL_1		29,72	21,46	20,12	23,33	53,19	202,77	1.362,7	30.661.000
SDRL		6,80	7,45	9,22	15,05	47,31	198,40	1.359,6	30.661.000
10%		23	14	11	10	11	25	146	***
50%		28	20	18	19	39	142	946	***
90%		39	31	32	43	115	461	3134	***
0.20									
ARL_1		17,99	11,46	9,27	7,68	7,30	9,85	25,86	211.620
SDRL		1,38	1,32	1,40	1,69	2,86	6,22	23,00	211.620
10%		17	10	8	6	5	4	5	***
50%		18	11	9	7	6	8	19	***
90%		20	13	11	10	11	18	56	***

Table C-2: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B = 1)

when true underlying distribution is Lognormal (s = 0.974):

	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,871	0,6561	0,5176	0,3577	0,1921	0,1026	0,0471	0,002
	B	1	1	1	1	1	1	1	1
mean	Z_o	1	1	1	1	1	1	1	1
1.00									
ARL_0		311,64	352,60	477,96	909,19	3.324,3	13.596	74.766	5,29E+08
SDRL		265,09	329,97	462,45	898,99	3.318,1	13.591	74.763	5,29E+08
10%		74	58	64	105	356	***	***	***
50%		231	251	336	633	2306	***	***	***
90%		657	782	1080	2080	7646	***	***	***
0.95									
ARL_1		195,79	222,05	301,97	579,39	2.160,4	9.019,9	50.922	3,85E+08
SDRL		151,83	199,97	286,65	569,22	2.154,1	9.015,5	50.919	3,85E+08
10%		59	43	46	70	233	***	***	***
50%		150	161	214	405	1499	***	***	***
90%		393	482	675	1321	4966	***	***	***
0.90									
ARL_1		133,47	146,18	195,69	372,19	1.396,9	5.923,6	34.247	2,76E+08
SDRL		92,48	124,81	180,66	362,10	1.390,6	5.919,2	34.244	2,76E+08
10%		49	34	34	48	153	***	***	***
50%		107	108	140	261	970	***	***	***
90%		254	309	431	844	3208	***	***	***
0.80									
ARL_1		75,30	72,86	90,34	159,92	579,37	2.483,6	14.907	1,36E+08
SDRL		40,30	53,34	76,14	150,10	573,12	2.479,1	14.904	1,36E+08
10%		37	25	22	26	67	266	***	***
50%		65	57	67	114	404	1723	***	***
90%		128	142	189	355	1326	5713	***	***
0.70									
ARL_1		50,48	43,01	47,94	74,27	240,18	1.002,2	6.124	6,15E+07
SDRL		20,65	25,56	34,86	64,89	234,03	997,78	6.121	6,15E+07
10%		30	19	17	16	31	110	***	***
50%		45	36	38	54	168	696	***	***
90%		77	76	93	159	545	2302	***	***
0.60									
ARL_1		37,48	28,90	29,10	38,25	101,47	389,99	2.352,8	2,53E+07
SDRL		11,73	13,45	17,26	29,49	95,49	385,63	2.349,8	2,53E+07
10%		26	16	13	12	16	45	251	***
50%		35	25	24	29	72	272	1632	***
90%		53	46	52	77	226	892	5414	***
0.50									
ARL_1		29,63	21,26	19,72	22,11	44,96	147,48	835,68	9,10E+06
SDRL		7,05	7,56	9,10	14,08	39,26	143,21	832,64	9,10E+06
10%		22	14	11	9	10	19	91	***
50%		28	19	17	18	33	104	580	***
90%		39	31	32	40	96	334	1920	***
0.20									
ARL_1		17,99	11,45	9,26	7,66	7,19	9,34	22,04	89.825
SDRL		1,45	1,38	1,44	1,73	2,82	5,78	19,23	89.825
10%		16	10	8	6	5	4	5	***
50%		18	11	9	7	6	8	16	***
90%		20	13	11	10	11	17	47	***

Table C-3: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B = 1)

when true underlying distribution is Lognormal (s = 1.092):

mean									
	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,871	0,6561	0,5176	0,3577	0,1921	0,1026	0,0471	0,002
	B	1	1	1	1	1	1	1	1
	Z_0	1	1	1	1	1	1	1	1
1.00									
ARL_0		201,67	174,29	192,00	270,82	637,42	1.788,9	6.680,8	7.478.500
SDRL		163,37	155,00	178,45	261,62	631,57	1.784,7	6.677,9	7.478.500
10%		55	36	32	37	72	192	***	***
50%		152	127	137	191	444	1241	***	***
90%		414	376	424	612	1460	4114	***	***
0.95									
ARL_1		144,32	126,57	139,69	196,94	464,50	1.312,7	4.972,5	5.829.100
SDRL		107,86	107,78	126,37	187,83	458,66	1.308,5	4.969,5	5.829.100
10%		47	30	27	29	54	142	***	***
50%		112	94	101	139	324	911	***	***
90%		285	267	304	442	1062	3017	***	***
0.90									
ARL_1		108,12	94,34	103,34	144,24	337,79	956,98	3.667,8	4.492.400
SDRL		73,64	76,15	90,31	135,24	331,97	952,81	3.664,9	4.492.400
10%		41	26	23	23	41	105	389	***
50%		87	71	76	103	236	665	2543	***
90%		204	193	221	320	770	2198	8442	***
0.80									
ARL_1		68,07	56,72	59,86	79,64	178,24	499,02	1.939,2	2.568.800
SDRL		37,54	39,86	47,52	70,94	172,50	494,86	1.936,3	2.568.800
10%		33	21	17	16	24	56	207	***
50%		58	45	45	58	125	347	1345	***
90%		117	109	122	172	403	1144	4461	***
0.70									
ARL_1		47,99	37,55	37,54	46,21	94,45	253,82	982,37	1.382.100
SDRL		21,05	22,13	26,02	37,90	88,84	249,69	979,43	1.382.100
10%		28	17	14	12	15	30	106	***
50%		43	31	30	35	67	177	682	***
90%		75	66	71	96	210	579	2258	***
0.60									
ARL_1		36,51	26,92	25,41	28,48	50,87	126,26	473,81	688.040
SDRL		12,60	12,89	14,78	20,66	45,44	122,20	470,87	688.040
10%		24	15	12	10	10	17	53	***
50%		33	23	21	22	37	89	329	***
90%		53	44	45	55	110	285	1087	***
0.50									
ARL_1		29,23	20,51	18,38	18,74	28,28	61,83	215,98	308.890
SDRL		7,83	7,76	8,62	11,45	23,10	57,86	215,98	308.890
10%		21	13	10	8	8	10	25	***
50%		27	18	16	15	21	44	151	***
90%		39	31	30	34	58	137	494	***
0.20									
ARL_1		17,98	11,43	9,22	7,58	6,85	8,01	14,27	8.274,4
SDRL		1,69	1,57	1,61	1,83	2,69	4,66	11,59	8.273,9
10%		16	10	8	6	4	4	4	***
50%		18	11	9	7	6	7	11	***
90%		20	13	11	10	10	14	29	***

Table C-4: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B = 1) when true underlying distribution is Lognormal (s = 1.182):

mean									
	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,871	0,6561	0,5176	0,3577	0,1921	0,1026	0,0471	0,002
	B	1	1	1	1	1	1	1	1
	Z_0	1	1	1	1	1	1	1	1
1.00									
ARL_0		153,71	116,91	116,13	139,71	255,34	571,15	1.690,2	653.350
SDRL		120,05	99,61	103,82	131,21	249,80	567,13	1.687,3	653.350
10%		46	28	23	22	32	64	181	***
50%		118	87	84	99	179	397	1172	***
90%		310	247	251	311	581	1310	3888	***
0.95									
ARL_1		117,78	91,23	90,87	109,10	198,58	444,28	1.324,1	529.470
SDRL		85,54	74,38	78,78	100,70	193,07	440,27	1.321,2	529.470
10%		40	25	20	19	26	50	142	***
50%		92	69	67	78	139	309	919	***
90%		229	188	193	240	450	1018	3045	***
0.90									
ARL_1		93,04	72,46	71,95	85,70	154,34	344,07	1.030,3	425.200
SDRL		62,29	56,10	60,12	77,42	148,87	340,07	1.027,5	425.200
10%		36	22	18	16	21	40	111	***
50%		75	55	54	62	109	240	715	***
90%		174	145	150	187	348	787	2369	***
0.80									
ARL_1		62,71	48,09	46,80	53,95	93,12	203,32	609,18	265.350
SDRL		34,93	32,78	35,56	45,95	87,75	199,35	606,31	265.350
10%		30	18	15	13	15	25	67	***
50%		53	38	36	40	66	142	423	***
90%		108	91	93	114	207	463	1399	***
0.70									
ARL_1		45,88	34,02	32,06	35,13	56,45	118,03	348,15	157.310
SDRL		20,85	19,81	21,48	27,47	51,20	114,11	345,28	157.310
10%		26	16	13	11	11	16	39	***
50%		40	28	26	27	41	83	242	***
90%		73	60	60	71	123	267	798	***
0.60									
ARL_1		35,61	25,43	23,10	23,78	34,62	67,45	191,38	87.323
SDRL		13,00	12,30	13,20	16,52	29,55	63,60	188,53	87.322
10%		23	14	11	9	8	11	23	***
50%		32	22	19	19	26	48	134	***
90%		52	41	40	45	73	150	437	***
0.50									
ARL_1		28,84	19,88	17,42	16,80	21,71	38,16	100,77	44.466
SDRL		8,30	7,76	8,20	9,96	16,84	34,41	97,95	44.465
10%		21	13	10	8	7	7	13	***
50%		27	18	15	14	17	28	71	***
90%		40	30	28	30	44	83	228	***
0.20									
ARL_1		17,96	11,41	9,18	7,51	6,61	7,27	11,18	2.106,4
SDRL		1,88	1,71	1,73	1,89	2,59	4,06	8,58	2.105,9
10%		16	10	8	6	4	4	4	222
50%		18	11	9	7	6	6	9	1460
90%		20	13	11	10	10	13	22	4850

Table C-5: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B = 2) when true underlying distribution is Lognormal (s = 0.974):

	λ	**0,01**	**0,05**	**0,1**	**0,2**	**0,4**	**0,6**	**0,8**	**1**
	h_L	**0,901446**	**0,68607**	**0,545071**	**0,37932**	**0,204487**	**0,109064**	**0,049218**	**0,002002**
	B	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**
mean	**Z_o**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
1.00									
ARL_0		364,89	356,22	437,03	736,18	2.461,2	9.829,5	60.616	525.550.000
SDRL		373,63	341,73	425,05	727,45	2.455,5	9.825,4	60.613	525.550.000
10%		52	51	57	85	264	***	***	***
50%		238	251	307	513	1708	***	***	***
90%		852	801	991	1684	5660	***	***	***
0.95									
ARL_1		194,52	211,27	266,55	460,95	1.587,0	6.494,6	41.082	382.780.000
SDRL		182,94	195,11	254,00	452,02	1.581,2	6.490,4	41.079	382.780.000
10%		41	37	39	57	172	***	***	***
50%		133	151	189	322	1102	***	***	***
90%		432	465	597	1050	3647	***	***	***
0.90									
ARL_1		120,89	134,14	169,19	293,26	1.021,6	4.255,1	27.516	274.670.000
SDRL		101,51	117,35	156,40	284,22	1.015,7	4.250,9	27.514	274.670.000
10%		35	29	29	39	113	***	***	***
50%		88	98	121	206	710	***	***	***
90%		252	287	373	663	2345	***	***	***
0.80									
ARL_1		62,79	65,05	77,45	126,40	424,69	1.786,0	11.913	134.730.000
SDRL		40,57	48,76	64,95	117,41	418,80	1.781,8	11.910	134.730.000
10%		27	21	19	21	50	192	***	***
50%		51	50	58	90	296	1239	***	***
90%		115	129	162	279	970	4107	***	***
0.70									
ARL_1		40,81	38,30	41,73	60,30	179,34	728,10	4.889,4	61.149.000
SDRL		20,12	23,46	30,10	51,66	173,49	723,88	4.886,4	61.149.000
10%		22	17	15	14	24	80	***	***
50%		35	32	33	45	126	506	***	***
90%		66	69	81	128	405	1671	***	***
0.60									
ARL_1		29,92	25,80	25,82	32,27	78,44	289,37	1.886,8	25.132.000
SDRL		11,24	12,51	15,26	24,21	72,76	285,17	1.883,8	25.132.000
10%		19	14	12	10	13	34	201	***
50%		27	22	21	25	56	202	1309	***
90%		44	42	46	64	173	661	4341	***
0.50									
ARL_1		23,56	19,02	17,77	19,37	36,47	113,20	677,67	9.052.900
SDRL		6,67	7,10	8,25	11,98	31,06	109,07	674,68	9.052.900
10%		17	12	10	8	9	16	74	***
50%		22	17	15	16	27	80	471	***
90%		32	28	28	35	77	255	1557	***
0.20									
ARL_1		14,45	10,32	8,53	7,15	6,70	8,47	19,71	89.416
SDRL		1,39	1,30	1,37	1,59	2,53	5,05	16,94	89.415
10%		13	9	7	6	4	4	5	***
50%		14	10	8	7	6	7	15	***
90%		16	12	10	9	10	15	42	***

Table C-6: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B = 2)

when true underlying distribution is Lognormal (s = 1.092):

	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,901446	0,68607	0,545071	0,37932	0,204487	0,109064	0,049218	0,002002
	B	2	2	2	2	2	2	2	2
mean	Z_o	1	1	1	1	1	1	1	1
1.00									
ARL_0		274,33	199,99	200,37	252,43	535,29	1.441,0	5.763,9	7.442.200
SDRL		289,50	189,86	190,49	244,71	529,98	1.437,1	5.761,1	7.442.200
10%		40	32	30	34	61	155	610	***
50%		171	141	142	177	373	1000	***	***
90%		651	447	449	571	1226	3313	***	***
0.95									
ARL_1		164,90	135,91	140,36	180,22	387,24	1.053,6	4.273,3	5.801.000
SDRL		161,85	124,24	130,08	172,40	381,90	1.049,7	4.270,4	5.801.000
10%		34	26	24	26	46	115	453	***
50%		108	97	100	127	270	731	***	***
90%		375	298	310	405	885	2421	***	***
0.90									
ARL_1		110,10	96,37	100,88	130,21	280,20	766,20	3.141,5	4.471.000
SDRL		98,36	83,86	90,42	122,35	274,84	762,27	3.138,7	4.471.000
10%		30	22	20	21	34	84	334	***
50%		76	70	73	93	196	532	***	***
90%		237	206	219	290	638	1759	***	***
0.80									
ARL_1		61,04	54,36	56,36	70,90	147,45	399,26	1.653,2	2.556.800
SDRL		43,62	41,46	45,97	63,12	142,10	395,31	1.650,3	2.556.800
10%		24	18	15	14	20	46	177	***
50%		47	41	42	52	104	278	1147	***
90%		116	108	116	153	333	914	3803	***
0.70									
ARL_1		40,56	34,79	34,77	41,15	78,72	204,24	836,12	1.375.800
SDRL		22,73	22,38	24,84	33,64	73,45	200,30	833,26	1.375.800
10%		21	15	12	11	13	25	91	***
50%		34	28	27	31	56	143	580	***
90%		69	64	67	85	174	465	1922	***
0.60									
ARL_1		29,95	24,47	23,39	25,59	43,13	102,95	404,22	684.980
SDRL		13,02	12,86	14,09	18,47	38,01	99,06	401,35	684.980
10%		18	13	1	9	9	14	45	***
50%		26	21	19	20	31	73	281	***
90%		46	41	42	50	93	232	927	***
0.50									
ARL_1		23,63	18,44	16,89	17,04	24,57	51,52	185,63	307.560
SDRL		7,84	7,67	8,25	10,37	19,68	47,72	182,77	307.560
10%		16	11	9	8	7	9	22	***
50%		22	16	14	14	19	37	130	***
90%		33	28	28	31	50	114	424	***
0.20									
ARL_1		14,35	10,20	8,50	7,09	6,43	7,40	13,14	8.244,8
SDRL		1,63	1,49	1,55	1,73	2,46	4,18	10,50	8.244,3
10%		13	9	7	6	4	4	4	***
50%		14	10	8	7	6	6	10	***
90%		16	12	10	9	10	13	27	***

Table C-7: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B = 2) when true underlying distribution is Lognormal (s = 1.182):

	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,901446	0,68607	0,545071	0,37932	0,204487	0,109064	0,049218	0,002002
	B	2	2	2	2	2	2	2	2
mean	Z_0	1	1	1	1	1	1	1	1
1.00									
ARL_0		224,99	143,77	129,93	139,77	229,44	488,52	1.506,8	650.670
SDRL		243,60	135,75	121,34	132,73	224,44	484,78	1.504,0	650.670
10%		34	25	22	21	29	55	161	***
50%		134	101	93	99	161	340	1045	***
90%		541	321	288	313	522	1120	3466	***
0.95									
ARL_1		145,34	104,95	97,83	107,13	177,20	378,75	1.176,7	527.320
SDRL		147,38	95,53	88,91	100,04	172,18	375,00	1.173,9	527.320
10%		30	21	19	18	23	43	126	***
50%		91	75	70	76	124	264	816	***
90%		335	229	214	237	401	867	2706	***
0.90									
ARL_1		101,72	78,97	75,02	82,87	136,99	292,63	913,14	423.480
SDRL		94,92	68,67	65,92	75,77	131,98	288,88	910,37	423.480
10%		27	19	16	15	19	34	99	***
50%		68	57	55	60	96	204	634	***
90%		223	168	161	182	309	669	2099	***
0.80									
ARL_1		59,16	48,47	46,53	51,04	82,18	172,65	537,77	264.300
SDRL		45,22	37,44	37,39	44,02	77,21	168,89	534,99	264.300
10%		22	16	13	12	13	22	59	***
50%		44	37	35	38	59	121	374	***
90%		116	97	95	108	183	393	1235	***
0.70									
ARL_1		40,07	32,64	30,92	32,84	49,87	100,54	306,89	156.710
SDRL		24,49	21,66	22,01	26,02	44,98	96,81	304,10	156.710
10%		20	14	11	10	10	14	35	***
50%		32	26	24	25	36	71	214	***
90%		70	61	60	67	108	227	703	***
0.60									
ARL_1		29,81	23,65	21,84	22,13	30,82	57,95	168,97	86.995
SDRL		14,34	13,07	13,34	15,60	26,07	54,27	166,19	86.994
10%		17	12	10	8	8	9	20	***
50%		26	20	18	17	23	41	118	***
90%		47	40	39	42	65	129	385	***
0.50									
ARL_1		23,58	18,12	16,27	15,63	19,56	33,26	89,47	44.304
SDRL		8,75	8,07	8,22	9,43	15,00	29,67	86,71	44.304
10%		16	11	9	7	6	7	12	***
50%		21	16	14	13	15	24	63	***
90%		34	28	27	28	39	72	202	***
0.20									
ARL_1		14,27	10,20	8,48	7,03	6,24	6,79	10,45	2.100,1
SDRL		1,84	1,67	1,68	1,82	2,40	3,71	7,89	2.099,6
10%		13	9	7	5	4	4	3	222
50%		14	10	8	7	6	6	8	1456
90%		16	12	10	9	9	12	21	4835

Table C-8: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B = 5)

when true underlying distribution is Lognormal (s = 0.94):

mean	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,876086	0,685314	0,544453	0,379277	0,205282	0,109801	0,049686	0,002002
	B	5	5	5	5	5	5	5	5
	Z_o	1	1	1	1	1	1	1	1
1.00									
ARL_0		1.930,2	432,46	576,17	1.103,6	4.234,4	18.997	133.150	2.427.000.000
SDRL		1.904,5	418,28	564,04	1.094,7	4.228,8	18.993	133.150	2.427.000.000
10%		226	58	72	124	451	***	***	***
50%		1345	304	403	768	2937	***	***	***
90%		4411	977	1311	2530	9743	***	***	***
0.95									
ARL_1		727,12	244,19	333,54	656,04	2.610,7	12.070	87.209	1.723.800.000
SDRL		686,35	227,93	320,66	646,91	2.604,9	12.066	87.206	1.723.800.000
10%		114	41	47	77	280	***	***	***
50%		517	174	235	458	1811	***	***	***
90%		1621	541	751	1499	6004	***	***	***
0.90									
ARL_1		340,66	149,01	202,16	397,36	1.608,4	7.607,1	56.449	1.205.500.000
SDRL		296,51	131,94	188,95	388,06	1.602,6	7.602,9	56.446	1.205.500.000
10%		74	31	33	50	175	***	***	***
50%		251	109	144	278	1117	***	***	***
90%		727	321	448	903	3696	***	***	***
0.80									
ARL_1		122,05	68,39	85,94	156,49	613,02	2.951,5	22.777	559.540.000
SDRL		83,54	51,79	72,97	147,16	607,08	2.947,3	22.774	559.540.000
10%		44	22	21	25	70	315	***	***
50%		99	53	64	111	427	2047	***	***
90%		231	136	181	348	1404	6791	***	***
0.70									
ARL_1		64,47	39,06	44,07	69,21	237,76	1.110,3	8.685,3	238.840.000
SDRL		33,89	24,01	32,06	60,23	231,82	1.106,1	8.682,3	238.840.000
10%		31	17	15	15	30	121	***	***
50%		56	32	35	51	167	771	***	***
90%		109	70	86	148	540	2551	***	***
0.60									
ARL_1		41,26	25,88	26,47	34,97	95,84	406,14	3.099,3	91.552.000
SDRL		17,02	12,51	15,65	26,61	90,04	401,88	3.096,3	91.552.000
10%		23	14	12	11	15	47	329	***
50%		38	23	22	27	68	283	2149	***
90%		63	42	47	70	213	930	7132	***
0.50									
ARL_1		29,16	18,92	17,92	20,21	41,44	146,13	1.025,3	30.480.000
SDRL		9,68	7,01	8,25	12,60	35,91	141,93	1.022,3	30.480.000
10%		19	12	10	9	9	19	111	***
50%		27	17	16	17	30	103	712	***
90%		42	28	29	37	88	331	2357	***
0.20									
ARL_1		13,19	10,20	8,47	7,17	6,73	8,78	22,31	210.580
SDRL		2,12	1,29	1,33	1,56	2,54	5,31	19,51	210.580
10%		11	9	7	6	4	4	5	***
50%		13	10	8	7	6	7	16	***
90%		16	12	10	9	10	16	48	***

Table C-9: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B = 5)

when true underlying distribution is Lognormal (s = 0.974):

	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,876086	0,685314	0,544453	0,379277	0,205282	0,109801	0,049686	0,002002
	B	5	5	5	5	5	5	5	5
mean	Z_o	1	1	1	1	1	1	1	1
1.00									
ARL_0		1.178,7	354,44	435,30	746,17	2.441,5	9.558,8	57.634	525.550.000
SDRL		1.153,2	341,48	423,86	737,67	2.436,0	9.554,8	57.631	525.550.000
10%		148	49	56	86	262	***	***	***
50%		824	249	305	520	1694	***	***	***
90%		2681	799	987	1707	5615	***	***	***
0.95									
ARL_1		511,06	210,71	265,57	465,79	1.569,5	6.300,6	39.006	382.780.000
SDRL		474,11	195,78	253,45	457,03	1.563,9	6.296,6	39.003	382.780.000
10%		88	36	39	57	170	***	***	***
50%		366	150	188	326	1090	***	***	***
90%		1129	466	596	1061	3607	***	***	***
0.90									
ARL_1		265,65	133,92	168,62	295,64	1.008,0	4.120,1	26.097	274.670.000
SDRL		226,20	118,12	156,19	286,72	1.002,3	4.116,1	26.094	274.670.000
10%		62	29	29	39	111	438	***	***
50%		197	98	121	208	700	2857	***	***
90%		560	288	372	669	2314	9482	***	***
0.80									
ARL_1		107,22	64,89	77,21	127,02	417,90	1.725,5	11.285	134.730.000
SDRL		72,33	49,27	64,96	118,10	412,12	1.721,3	11.282	134.730.000
10%		39	21	19	21	49	185	***	***
50%		87	50	57	91	291	1197	***	***
90%		201	129	162	281	955	3968	***	***
0.70									
ARL_1		59,69	38,11	41,59	60,49	176,39	703,3	4.633,2	61.149.000
SDRL		31,34	23,74	30,14	51,89	170,64	699,1	4.630,3	61.149.000
10%		29	16	14	14	24	78	***	***
50%		52	31	33	45	124	489	***	***
90%		100	69	81	128	399	1614	***	***
0.60									
ARL_1		39,22	25,60	25,71	32,34	77,26	280,03	1.791,3	25.132.000
SDRL		16,29	12,67	15,29	24,30	71,65	275,86	1.788,4	25.132.000
10%		22	14	12	10	13	33	191	***
50%		36	22	21	25	55	195	1243	***
90%		60	42	46	64	171	639	4121	***
0.50									
ARL_1		28,16	18,83	17,67	19,40	36,01	109,97	645,8	9.052.900
SDRL		9,42	7,20	8,27	12,02	30,65	105,88	642,8	9.052.900
10%		18	12	10	8	9	15	71	***
50%		26	17	15	16	27	77	449	***
90%		40	28	28	35	76	248	1483	***
0.20									
ARL_1		13,15	10,19	8,46	7,15	6,65	8,38	19,27	89.416
SDRL		2,12	1,34	1,38	1,60	2,52	4,99	16,51	89.415
10%		11	9	7	6	4	4	5	***
50%		13	10	8	7	6	7	14	***
90%		16	12	10	9	10	15	41	***

Table C-10: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B = 5)

when true underlying distribution is Lognormal (s = 1.092):

	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,876086	0,685314	0,544453	0,379277	0,205282	0,109801	0,049686	0,002002
	B	5	5	5	5	5	5	5	5
mean	Z_0	1	1	1	1	1	1	1	1
1.00									
ARL_0		371,68	203,46	203,41	259,26	542,33	1.434,7	5.619,9	7.442.200
SDRL		352,63	194,13	194,17	251,86	537,26	1.430,9	5.617,1	7.442.200
10%		62	31	30	34	62	155	***	***
50%		261	143	144	182	377	996	***	***
90%		831	456	456	587	1242	3299	***	***
0.95									
ARL_1		219,99	138,01	142,03	184,21	390,67	1.045,7	4.158,2	5.801.000
SDRL		194,89	126,94	132,26	176,64	385,54	1.041,8	4.155,4	5.801.000
10%		48	26	24	26	46	114	441	***
50%		159	98	101	130	272	726	***	***
90%		474	303	314	414	893	2403	***	***
0.90									
ARL_1		143,64	97,68	101,81	132,54	281,64	758,36	3.051,6	4.471.000
SDRL		116,69	85,62	91,74	124,88	276,46	754,52	3.048,8	4.471.000
10%		40	22	20	21	34	83	324	***
50%		108	71	74	94	197	527	2116	***
90%		295	209	221	295	642	1741	7023	***
0.80									
ARL_1		75,81	54,89	56,64	71,71	147,37	393,48	1.601,5	2.556.800
SDRL		50,25	42,27	46,51	64,06	142,15	389,60	1.598,7	2.556.800
10%		29	18	15	14	20	45	171	***
50%		62	42	42	52	104	274	1111	***
90%		141	110	117	155	333	901	3684	***
0.70									
ARL_1		48,03	35,01	34,83	41,44	78,38	200,76	808,80	1.375.800
SDRL		25,66	22,79	25,08	34,01	73,22	196,88	805,97	1.375.800
10%		23	15	12	11	13	25	88	***
50%		41	28	27	31	56	140	561	***
90%		81	65	67	86	174	457	1859	***
0.60									
ARL_1		33,86	24,56	23,37	25,70	42,86	101,11	390,95	684.980
SDRL		14,60	13,08	14,20	18,62	37,82	97,26	388,10	684.980
10%		19	13	11	9	9	14	44	***
50%		31	21	19	20	31	71	272	***
90%		53	41	42	50	92	228	896	***
0.50									
ARL_1		25,44	18,47	16,84	17,08	24,40	50,64	179,78	307.560
SDRL		8,87	7,80	8,31	10,44	19,57	46,87	176,94	307.560
10%		16	11	9	8	7	9	21	***
50%		24	16	14	14	18	36	125	***
90%		37	28	28	31	50	112	410	***
0.20									
ARL_1		13,03	10,19	8,44	7,09	6,39	7,33	12,92	8.244,8
SDRL		2,12	1,54	1,56	1,74	2,45	4,14	10,29	8.244,3
10%		11	9	7	6	4	4	4	***
50%		13	10	8	7	6	6	10	***
90%		16	12	10	9	10	13	26	***

Table C-11: Performance metrics of exponentially designed lower-sided TBE EWMA charts (designed with B = 5) when true underlying distribution is Lognormal (s = 1.182):

	λ	0,01	0,05	0,1	0,2	0,4	0,6	0,8	1
	h_L	0,876086	0,685314	0,544453	0,379277	0,205282	0,109801	0,049686	0,002002
	B	5	5	5	5	5	5	5	5
mean	Z_o	1	1	1	1	1	1	1	1
1.00									
ARL_0		217,24	146,77	132,73	144,82	235,27	492,79	1.490,1	650.670
SDRL		204,03	139,62	124,89	138,18	230,52	489,18	1.487,4	650.670
10%		42	24	21	21	29	55	159	***
50%		151	103	94	102	165	343	1034	***
90%		483	329	295	325	536	1130	3427	***
0.95									
ARL_1		146,40	106,80	99,49	110,37	180,83	380,75	1.161,1	527.320
SDRL		128,27	98,04	91,18	103,61	176,05	377,12	1.158,4	527.320
10%		36	21	18	18	23	43	125	***
50%		105	76	71	79	127	265	806	***
90%		313	235	218	245	410	872	2670	***
0.90									
ARL_1		105,21	80,14	76,01	84,95	139,20	293,26	899,20	423.480
SDRL		84,98	70,35	67,40	78,11	134,38	289,61	896,48	423.480
10%		31	19	16	15	19	34	97	***
50%		78	58	55	61	98	204	624	***
90%		215	172	164	187	314	671	2067	***
0.80									
ARL_1		62,55	48,97	46,86	51,89	82,91	172,11	527,76	264.300
SDRL		42,02	38,24	38,04	45,04	78,09	168,44	525,02	264.300
10%		25	16	13	12	13	21	58	***
50%		50	37	35	38	59	120	367	***
90%		116	99	96	111	185	392	1212	***
0.70									
ARL_1		42,30	32,86	31,00	33,19	50,04	99,84	300,44	156.710
SDRL		23,31	22,07	22,33	26,48	45,27	96,18	297,69	156.710
10%		21	14	11	10	10	14	34	***
50%		36	26	24	25	36	70	209	***
90%		72	61	60	68	109	225	688	***
0.60									
ARL_1		31,00	23,74	21,84	22,28	30,80	57,40	165,21	86.995
SDRL		13,93	13,29	13,49	15,81	26,15	53,78	162,45	86.995
10%		18	12	10	8	7	9	20	***
50%		28	20	18	17	23	41	115	***
90%		49	41	39	43	65	127	377	***
0.50									
ARL_1		23,92	18,16	16,23	15,69	19,50	32,91	87,48	44.304
SDRL		8,72	8,20	8,30	9,53	15,01	29,37	84,74	44.304
10%		15	11	9	7	6	7	12	***
50%		22	16	14	13	15	24	61	***
90%		35	29	27	28	39	71	198	***
0.20									
ARL_1		12,94	10,19	8,43	7,03	6,21	6,74	10,32	2.100,1
SDRL		2,15	1,71	1,69	1,83	2,40	3,68	7,77	2.099,6
10%		11	9	7	5	4	3	3	222
50%		13	10	8	7	5	6	8	1456
90%		16	12	10	9	9	12	20	4835