LIGHT CONE QCD SUM RULES AND MESON PHYSICS

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I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all materials and results that are not original to this work.

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ABSTRACT

Light Cone QCD Sum Rules and Meson Physics

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In this thesis, we applied Light Cone QCD sum rules (LCSR) to several problems in meson physics. Semileptonic $B \to \eta \ell \nu$ decays are important to get information on Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. We calculated form factors of this decay in LCSR frame. Our results are confirmed by later studies and these similar studies had been used for enhancing result on b to u quark transition matrix element of CKM matrix by BaBar collaboration. We used LCSR method also for calculating coupling constant of radiative $\rho \to \eta\gamma$ decay since the analysis of the vector particle to pseudoscalar particle radiative decay with η and η' mesons in final state can provide insights to the long standing issue of the η and η' mixing. Our result $g_{\rho\eta\gamma} = (1.4 \pm 0.2)$ is very close to experimental value $g_{\rho\eta\gamma} = (1.42 \pm 0.12)$. We also calculated magnetic moment of the ρ meson in LCSR frame which is an important parameter since it is strongly related to internal structure of hadron. Our result $\mu = 2.3 \pm 0.5$ in units of $(e/2m_\rho)$, is in better agreement with lattice QCD results than traditional QCD sum rules. Quark contents of light scalar mesons are still under debate and we analyzed $\phi \to KK$ decays which is important for understanding the quark content of the f_0 meson. Our final result is $g_{\phi KK} = 4.9 \pm 0.8$ which is in well agreement with existing experimental result $g_{\phi KK} = 4.8$.

Keywords: Standard Model, QCD Sum Rules, LCQCD Sum Rules, B meson, Rare Decays.

ÖZ

Işık Konisi Kuantum Renk Dinamiği Toplam Kuralları ve Mezon Fiziği

Kanık, İnanç
Doktora, Fizik Bölümü
Tez Yöneticisi: Prof. Dr. Mustafa Savcı

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Bu tezde, ışık konisi kuantum renk dinamiği toplam kurallarını (LCSR) mezon fiziğindeki bir çok probleme uyguladık. Yarıleptonik $B \to \eta \ell \nu$ bozunumları Cabibbo-Kobayashi-Maskawa (CKM) matris elemanları hakkında bilgi edinmek için önemlidir. Bu bozunumun yapı çarpanlarını LCSR çerçevesinde hesapladık. Sonuçlarımız daha sonra yapılmış benzer çalışmalarla desteklenmiş ve bu benzer çalışmalar BaBar deneysel grubu tarafından CKM matrisinin b kuarktan u kuarka geçiş elemanının daha hassas hesaplanması için kullanılmıştır. Vektör bir parçacığın sözdeskaler bir parçacığa ışınımsal bozunumu eğer son ürün olarak η ve η' parçacıklarını içerebiliyorsa, η ve η' parçacıklarının karışması olayına dair önemli bilgiler sağlayabilir. Bu nedenle LCSR metodunu ışınımsal $\rho \to \eta\gamma$ bozunumunun etkileşim sabitini hesaplamak için kullandık. Sonucumuz $g_{\rho\eta\gamma} = (1.4 \pm 0.2)$ deneysel sonuç olan $g_{\rho\eta\gamma} = (1.42 \pm 0.12)$ ile uyum içerisindedir. Ayrıca hadronun iç yapısına çok bağlı olduğu için önemli bir parametre olan manyetik momenti ρ mezonu için LCSR çerçevesinde hesapladık. Sonuç olarak $(e/2m_\rho)$ cinsinden $\mu = 2.3 \pm 0.5$ şeklinde bulundu ki bu sonuç geleneksel QCD toplam kurallarından elde edilen sonuca göre diğer metotlardan elde edilen sonuçlarla daha uyumludur. Hafif skaler mezonların kuark içerikleri hala tartışmalıdır ve $\phi \to$

KK bozunumu f_0 mezonunun kuark içeriğini anlamak için önemli olduğundan bu bozunumu inceledik. Sonuç olarak bulduğumuz $g_{\phi KK} = 4.9 \pm 0.8$ deneysel değer olan $g_{\phi KK} = 4.8$ ile uyum içerisindedir.

Anahtar Kelimeler: Standart Model, QCD toplam kuralları, Işık konisi toplam kuralları, B mezonu, Nadir bozunmalar.

to my friends and family..

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CHAPTER 1

INTRODUCTION

In nature there are four fundamental forces known which are electromagnetic, weak, strong and gravitational forces. The Standard Model (SM) is a quantum field theory which describes three of these forces; electromagnetic, weak and strong interactions. With SM, physicists try not only to describe nature but also to understand it by unifying concepts and basing theories on simpler ideas. Last century shows symmetry can be a good foundation for a theory. With the idea of symmetry in mind physicists came up with QCD and electroweak theory. QCD is the theory describing strong interaction and quantum electroweak theory is the theory which describes and unifies electromagnetic and weak interaction. They both are based on internal symmetries of fields; QCD is invariant under SU(3) gauge transformations and electroweak theory is invariant under SU(2)$\otimes$U(1) gauge transformations. In QCD, there are three kinds of charge which are called color charges, named as red, green, blue. Gluons are the particles that is mediating strong interaction between color charged particles and there are eight of them. Particles carrying electroweak forces are photons, W^+, W^- and Z^0 bosons. All these forces carrying particles are obeying boson statistics so they have integer spins. Because of the gauge invariance they have to obey, fields in these theories have been expected to be massless so their particle like excitations that is mediating the forces. But this not the case for W^+, W^- and Z^0 bosons which have all masses around 90 GeV. This puzzle has been solved by Sheldon Glashow, Abdus Salam and Steven Weinberg with Glashow-Weinberg-Salam(GWS) model by 1968. They were awarded the 1979 Nobel Prize in Physics for their work. They basically used Higgs mechanism to give masses to bosons at low energies. Higgs mechanism depends on the idea that if we had a field whose is not zero on the ground state and if we couple this field to other fields suitably they gain mass when the system is in the ground state. This non-zero field called as Higgs

field. Despite the whole lagrangian is carrying SU(2)⊗U(1) gauge symmetry due to interaction with the field whose ground state amplitude is not zero electroweak mediating bosons gain mass except photon. Within their work they also predicted two particles Z^0 and Higgs boson. W^+, W^- bosons were postulated previously to explain charge carrying characteristic of weak interaction. But a neutral weak interaction idea was new to physics world by that time. Z^0 has been observed indirectly in CERN by 1973. Electrons seemingly move by themselves interpreted as interaction of a neutrino with electron through Z^0 boson. In 1983, all W^+, W^- and Z^0 bosons are produced and observed directly in Super Proton Synchrotron at CERN. Unlike other bosons proposed by GWS model higgs boson is the only particle that has not been observed directly or indirectly, which is also only accepted but unobserved particle by SM.

Matter in SM is formed by fermionic spin 1/2 elementary particles, leptons and quarks. Each group consists of six particles, which are related in pairs. The six quarks are paired in the three generations; up and down quarks form the first generation, charm and strange quarks form the second generation, top and bottom quarks form the third. The six leptons are similarly arranged in three generations electron and electron-neutrino, muon and muon-neutrino, tau and tau-neutrino. In each generation, there are one unit charge difference between particles and they can change into each other through charged weak interactions. Also it should be noted that the up (u), down (d) and strange (s) quarks are called light quarks and the charm (c), bottom (b), and top (t) are called as heavy quarks.

Most famous of leptons is electron. That is because muon and tau are much heavier than electron that they do not live long and decay into electron through weak interactions so they are not abundant in universe as much as electron. On the other hand electron is everywhere in universe and formed atoms with protons and neutrons with the help of electromagnetic interaction. Neutrinos are abundant too but since they can just interact through weak interaction they are hard to observe.

Main difference between leptons and quarks are, leptons do not participate in strong interactions. Quarks can interact strongly since they have color charges.

Table 1.1: Lepton and quark properties [11]. Electric charge is given in terms of electron charge, e=1.602176487(40)10^{-19} Coulombs

Name	*ElectricCharge*	*Mass(MeV)*
Leptons		
Electron (e)	e	0.51099892 ± 0.00000004
Muon (μ)	e	105.658369 ± 0.000009
Tau (τ)	e	$1776.99^{+0.29}_{-0.26}$
Electron Neutrino(v_e)	0	< 0.000002
Muon Neutrino(v_μ)	0	< 0.19
Tau Neutrino(v_τ)	0	< 18.2
Quarks		
Up (u)	2/3 e	1.5 to 3.3
Down (d)	-1/3 e	3.5 to 6.0
Strange(s)	-1/3 e	104^{+26}_{-34}
Charm (c)	2/3 e	1270^{+70}_{-110}
Bottom (b)	-1/3 e	4200^{+170}_{-70}
Top (t)	2/3 e	171200± 2100

Quarks form composite particles by interacting strongly which are called hadrons. Quarks are not observed isolated, outside hadrons yet. When a quark is forced out of a hadron, energy given to quark for forcing out should be so high that it will be enough to create new quark, antiquark pairs. So it will create new quark pairs until it loses most of its energy. Forced quark and emerging quarks will form new hadrons and will stay bounded. These phenomenon is called color confinement and it is not explained fully yet. Nature does not allow particles with net color charge to live by themselves. So all hadrons have net zero color charge.

1.1 Hadrons

Mesons and baryons are hadron groups, grouped according to the number of valance quarks, two and three valance quarks respectively. There are also claims on the existence of pentaquarks which have five valance quarks, four quark states which have four valance quarks, glueballs which only consist of gluons and hadronic molecules but they are not confirmed experimentally yet. On the

other hand many mesons and baryons observed experimentally. To identify a hadron we have to define its quantum numbers corresponding to the representations of the Poincaré group and its valance quark content. Quantum numbers corresponding to the representations of the Poincaré group are given in the form J^{PC}(m), where J is the spin, P is the parity, C is charge conjugation parity, and m, the mass.

Parity of a particle is determined from behavior of particle wave function under sign change of one spatial coordinate. If particle's wave function remains same it is parity even or +1 state, if it change sign it is odd or -1 state. All four forces except weak forces are parity invariant, it means product of parity of incoming particles and product of parity of outgoing particles will be same for them. In weak interactions only left handed neutrinos and right handed anti-neutrinos participate in interactions but parity transformation takes left handed particles to right handed ones so weak interaction is not invariant under parity transformation and it will not conserve product of parity of particles in interactions. One last note for parity is the parity of an anti-fermion is opposite the parity of the fermion, while the parity of an anti-boson is the same as the parity of the boson.

Charge conjugation exchanges particle and anti-particle. For charged particles, this is the same thing as changing the sign. The difference occurs only for neutral particles which are not their own anti-particle, like K^0, D^0, B^0, neutrinos etc.. For photon since both electric and magnetic field will change sign, photon is an odd charge-parity particle. Electromagnetism, gravity and the strong interaction are all invariant under charge conjugation transformation. Universe will not change for these forces if all particles changed with anti-particles and anti-particles with particles. But again weak forces are violating charge conjugation symmetry. Under charge conjugation transformation a left handed neutrino will transform into left handed anti-neutrino which is not participating weak interactions.

Valance quark content defined with quantum numbers like isospin, strangeness, charmness, bottomness and topness. Strangeness, charmness, bottomness and topness quantum numbers give respective valance quark number. Isospin concept was introduced by Werner Heisenberg in 1932 to explain symmetries of the

then newly discovered neutron. Since neutron and proton are similar to each other in mass, physicists want to think them as different states of same particle. Since there are two states, namely proton and neutron, it is thought in similar way to spin and they are called states of isospin 1/2 particle. $I_z = 1/2$ corresponds to proton and $I_z = -1/2$ corresponds to neutron. When new mesons and baryons are observed, they are grouped under isospin groups in the same manner. For example delta baryons are 3/2 spin baryons with four members who has similar mass though they have different charges like in proton-neutron couple. Since there are four of them, they form an isospin 3/2 group with each $I_z = 3/2, 1/2, -1/2, -3/2$ correspond to different delta baryon. But with increasing number of baryons and mesons, physicists realized that isospin symmetry group can be subgroup of even larger symmetry group. The larger symmetry was first realized and named the Eight-fold Way by Murray Gell-Mann. Gell-Mann proposed the existence of up, down and strange quarks which would belong to the fundamental representation of the SU(3) flavor symmetry. SU(3) flavor symmetry was a badly broken symmetry due to high mass of strange quark with respect to up and down quark mass. But SU(3) flavor symmetry idea gave birth to quark model. After the quark model was established, it was realized that the isospin was related to the up and down quark content of particles. The relation is $I_z = 1/2[(N_u - N_{\bar{u}}) - (N_d - N_{\bar{d}})]$ where N_u, $N_{\bar{u}}$, N_d and $N_{\bar{d}}$ are the number of up quark, up antiquark, down quark and down antiquark, respectively.

1.1.1 Mesons

Existence of mesons had been predicted by Hideki Yukawa in 1935. After the observation of pion, which is a meson, experimentally by Cecil Powell in 1947, Hideki Yukawa took nobel price for this prediction in 1949. By the time Hideki Yukawa predicted mesons he did not know about QCD and quarks. He predicted pions and their approximate mass while trying to explain nuclear forces between proton and neutron, and finite range of nuclear forces. After quarks and QCD had established, it is understood that mesons have internal structure and formed by one quark and an antiquark pair which are called valance quarks. Valance quarks are inside a sea of virtual quark-antiquark pairs and virtual gluons binding them

together. Spin of a meson will be total angular momentum of the system which is the total spin of quarks (S) plus orbital angular momentum(L) carried by quarks. Parity(P) and charge conjugation parity(C) quantum numbers of a meson will be:

$$C = (-1)^{L+S}, \tag{1.1}$$

$$P = (-1)^{L+1}. \tag{1.2}$$

Mesons whose quarks have non-zero orbital momentum are accepted as the excited states of zero orbital momentum ones, so mesons under interest have quarks with zero orbital momentum. Hence, since mesons formed by two spin 1/2 particles they can have spin 0, pseudoscalar mesons, and spin 1 , vector mesons. Even though they are created and annihilated inside the nucleus constantly, they are not observed easily outside the nucleus since they are all unstable and have short lifetimes.

1.1.2 Baryons

Like mesons, baryons are also composite particles formed by quarks. But unlike mesons, baryons have three valance quarks or three antiquarks which are inside a sea of virtual quark-antiquark pairs and gluons. Spin of a baryon will be total angular momentum of the system which is total spin of quarks (S) plus orbital angular momentum(L) carried by quarks. Particle physicists are mostly interested in baryons whose quarks has zero orbital angular momentum since non-zero ones accepted as excited states of zero orbital angular momentum baryons. Hence, since baryons are formed by three spin 1/2 particles they can have spin 1/2 and spin 3/2. Their parity can be calculated as:

$$P = (-1)^{L}. \tag{1.3}$$

So baryons with zero orbital angular momentum have even parity. In any interaction, the same number of baryons must appear in the initial and final states, this is called baryon number conservation. The conservation of baryon number

makes the parity of baryons irrelevant.

Most famous of baryons are proton and neutron, they are called nucleons since nucleus of atoms are formed by them. They are plentiful in universe since they have long life time. Most baryons are not stable and they decay into nucleons as the final product.

Classification rules for baryons had been determined by the particle data group [11]. These rules are;

- Baryons with three u and/or d quarks are Nucleons(N) (I = 1/2) or Δ's (I = 3/2).
- Baryons with two u and/or d quarks are Λ's (I = 0) or Σ's (I = 1). If the third quark is heavy, its identity is given by a subscript.
- Baryons with one u or d quark are Ξ's (I = 1/2). One or two subscripts are used if one or both of the remaining quarks are heavy.
- Baryons with no u or d quarks are Ω's (I = 0), and subscripts indicate any heavy quark content.
- Baryons that decay strongly have their masses as part of their names. For example, Σ^0 does not decay strongly, but $\Delta^{++}(1232)$ does.
- Baryons in total angular momentum J = 3/2 configuration which have the same symbols as their J = 1/2 counterparts are denoted by an asterisk (*).

Physicists measure lots of parameters of baryons and mesons experimentally. QCD is believed to be true theory of strong interactions which should allow physicists to be able to calculate measured parameters of baryons and mesons theoretically. Unfortunately there is no methods available at the moment to calculate all properties of hadrons to the desired accuracy, starting from QCD lagrangian. To understand QCD better physicists tried to follow the way they followed during progressing victorious QED and they tried to apply perturbation theory to QCD unfortunately its success was partial for QCD. Reliability of perturbation theory depends on the smallness of coupling constant of theory.

For QED coupling constant is around 1/137 for low energy scales and increasing slowly with energy scale of interactions we are taking into account. Unlikely coupling constant of QCD decreases with energy, this behavior is called asymptotic freedom [1]. Above energy scale $\Lambda_{QCD} \sim 0.50GeV$ coupling constant is less than 1 and we can assume perturbative approach holds. But below energy scale Λ_{QCD} perturbative QCD will not be applicable. Unfortunately for hadrons, interactions below energies Λ_{QCD} are important and we cannot do perturbative calculations.

Non-perturbative methods are needed for making calculations with QCD and predicting properties of hadrons. Still no method could be established that is valid for all hadrons and that can calculate properties of hadrons to any desired accuracy. But physicists came up with lots of methods to approach problem with partial successes. Chiral perturbation theory, heavy quark effective theory, QCD sum rules (QCDSR), lattice calculations are some of the main non-perturbative methods developed to handle calculations. Every one of them has their own virtues and weaknesses. One of the most powerful method is QCDSR ([2]) that has been proposed by M. Shifman, A. Vainshtein and V. Zakharov. QCDSR has been derived from QCD lagrangian and it can be used in most physical processes involving QCD and analytical results can be acquired. There are methods like lattice QCD that can calculate physical parameters more precise with respect to QCDSR but it can calculate less number of parameter. Lattice QCD can only acquire numerical values and it does not give any qualitative picture. On the other hand QCD sum rules give analytical answers so physical picture behind the scene can be understood better. Despite this wide application range, there are cases where QCDSR have trouble. When meson-to-meson transition form factors are considered, there are cases where QCDSR may not work well. For example when B meson to M transitions considered, where M is a light meson, higher order matrix elements grow with mass of beauty quark making the operator product expansion used in QCDSR non-convergent. To overcome shortcomings of QCDSR, QCD light cone sum rules(LCSR) had been developed in late 80s [3], [4], [5]. LCSR are a variation QCDSR which is very effective to apply problems involving meson to meson transition. It has been tested on several problems up to date, in [6] it is compared and retested with recent experimental results on $D \to (\pi, K) l\nu$ decays and LCSR had produced results in very good agreement

with experimental results.

In this thesis, LCSR will be applied to several problems in meson physics. But first QCD sum rules and Light Cone QCD sum rules (LCSR) will be introduced in chapter 2. In chapter 3, LCSR will be used to analyze B to η meson decay. To illustrate how to calculate properties of hadrons with LCSR, in chapter 4 we will analyze radiative decay of ρ meson into η. In chapter 5 we will calculate magnetic moment of ρ meson and at last chapter we will apply LCSR to $\phi \to KK$. In chapters 3, 4, 5, 6 the articles [7], [8], [9], [10] will be followed respectively.

Table 1.2: Pseudoscalar and Vector Mesons.

Particle Name	**Particle Symbol**	**Antiparticle Symbol**	**Quark Content**	**Rest Mass** (MeV/c^2)	J^{PC}
$PseudoscalarMesons$					
Pion	π^+	π^-	$u\overline{d}$	139.57018(35)	0^-
Pion	π^0	Self	$\frac{u\overline{u}-d\overline{d}}{\sqrt{2}}$	134.977 ± 0.001	0^{-+}
Eta meson	η	Self	$\frac{u\overline{u}+d\overline{d}-2s\overline{s}}{\sqrt{6}}$	547.51 ± 0.18	0^{-+}
Eta prime	$\eta'(958)$	Self	$\frac{u\overline{u}+d\overline{d}+s\overline{s}}{\sqrt{3}}$	957.78 ± 0.14	0^{-+}
Kaon	K^0	$\overline{K}^0$	$d\overline{s}$	497.468 ± 0.022	0^-
Kaon	K^+	K^-	$u\overline{s}$	493.677 ± 0.016	0^-
K-Short	K_S	Self	$\frac{d\overline{s}-s\overline{d}}{\sqrt{2}}$	497.7	0^-
K-Long	K_L	Self	$\frac{d\overline{s}+s\overline{d}}{\sqrt{2}}$	497.7	0^-
D meson	D^+	D^-	$c\overline{d}$	$1,869.62 \pm 0.20$	0^-
D meson	D^0	$\overline{D}^0$	$c\overline{u}$	$1,864.84 \pm 0.17$	0^-
strange D	D_s	D_s	$c\overline{s}$	$1,968.49 \pm 34$	0^-
B meson	B^+	B^-	$u\overline{b}$	$5,279.13 \pm 31$	0^-
B meson	B^0	$\overline{B}^0$	$d\overline{b}$	$5,279.50 \pm 33$	0^-
strange B	B_s	$\overline{B}_s$	$s\overline{b}$	$5,366.1 \pm 6$	0^-
charmed B	B_c	B_c	$c\overline{b}$	$6,368.49 \pm 5$	0^-
$VectorMesons$					
Charged rho	$\rho^+(770)$	$\rho^-(770)$	$u\overline{d}$	775.4(4)	1^{--}
Neutral rho	$\rho^0(770)$	Self	$\frac{u\overline{u}-d\overline{d}}{\sqrt{2}}$	775.49(34)	1^{--}
Phi meson	$\varphi(1020)$	Self	$s\overline{s}$	$1,019.445(20)$	1^{--}
J/Psi	J/ψ	Self	$c\overline{c}$	$3,096.916(11)$	1^{--}
Upsilon meson	Υ	Self	$b\overline{b}$	$9,460.3(26)$	1^{--}

CHAPTER 2

QCD AND LIGHT CONE QCD SUM RULES

In QCD, the conventional perturbative approaches often fail because of the confining and the strong coupling nature of the theory. In 1979, M. Shifman, A. Vainshtein and V. Zakharov proposed the method of QCD sum rules to solve this problem which then has become a widely used tool in the problems associated with the hadronic phenomenology [2]. After they proposed their sum rules, other sum rules are also developed for QCD. To identify sum rules in [2], they will be called as SVZ sum rules. In this chapter mostly [12] and chapter 2 of [13] will be followed.

In SVZ sum rules, hadrons are represented by their interpolating quark currents taken at large virtualities. The main idea of SVZ sum rules is injecting these quark currents into QCD vacuum and then following their evolution in order to get information on the hadron they represent. First a quark current correlator will be introduced and calculated in terms of phenomenological parameters. Then it will be calculated in deep Euclidean region with both perturbative and non-perturbative contributions included by using operator product expansion (OPE) [14]. Physical quantities of interest are determined by matching these two representations of the correlator. So quark currents which will be injected into QCD vacuum should be selected such that their correlation's phenomenological part will contain desired physical parameters. For example, if decay constant or mass for a vector particle will be calculated, one could use the following correlator function :

$$\Pi_{\mu\nu} = i \int d^4x e^{iqx} \left\langle 0 \left| \mathrm{T}\{J_\nu(x) J_\mu^\dagger(0)\} \right| 0 \right\rangle . \tag{2.1}$$

Here J is the interpolating quark current that will be used to represent hadron

under investigation. J should have same quantum numbers and quark contents with hadron it represents. For example for ρ meson which is isotopic spin 1 vector meson, quark current should be selected as:

$$J_\mu = \frac{\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d}{\sqrt{2}}. \tag{2.2}$$

When unit operator formed by complete set of hadronic states inserted between these currents in eq.2.1 an equation in terms of decay constant and mass of ρ meson will be acquired. It will be illustrated in the following section. When the current given in eq.2.2 used, due to conservation of electromagnetic current and transversality of $\Pi_{\mu\nu}$ correlator function can be written as:

$$\Pi_{\mu\nu} = (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi(q^2). \tag{2.3}$$

$\Pi(q^2)$ is called as polarization operator.

2.1 Spectral Density

Imaginary part of polarization operator at positive values of q^2 is called spectral density.

$$\rho(s) = \frac{1}{\pi} Im\Pi(s) \ , \quad s = q^2 > 0. \tag{2.4}$$

In fact spectral density represents hadronized QCD. It has peaks in amplitude at energies where a hadron creation with quantum numbers of chosen current will be observed. These peaks has a width that is called width of the hadron. At energies less than two times hadron creation energy a continuous spectra aside from peaks will emerge. That part is the two or more particle creation part and it starts from the energies below two particle energy because they may form bound states. It will look like:
for one hadron. From spectral density one can get all the information on the spectrum and the width of the hadrons. So exact calculation of spectral density is what physicists dream of but it is not possible with current methods.

One can write an idealized analytical spectral density function which has zero

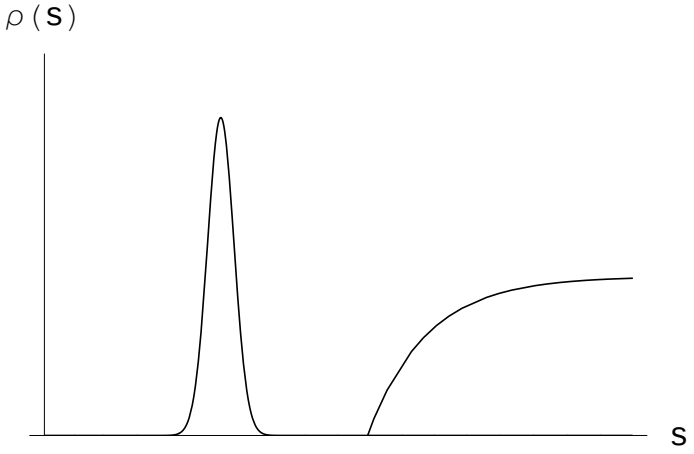

Figure 2.1: Form of spectral density when there is one hadron

hadron widths. In this case peaks in spectral density will be represented by dirac delta functions. If complete set of hadronic states inserted between currents in eq.2.1:

$$2Im\Pi_{\mu\nu}(q) = \sum_n \langle 0 \left| j_\nu \right| n \rangle \left\langle n \left| j_\mu^\dagger \right| 0 \right\rangle d\tau_n (2\pi)^4 \delta^{(4)}(q - p_n), \tag{2.5}$$

here n is running over all possible hadronic states that can be created by chosen current, $d\tau_n$ denotes the integration over the phase space volume of these states. Let H be the lowest lying hadron in the channel defined by chosen current and let's assume current chosen is such that H a vector meson than following definition can be done:

$$\left\langle H(q) \left| j_\mu^\dagger \right| 0 \right\rangle = f_H m_H \epsilon_\mu^{(H)*}, \tag{2.6}$$

where m_H is mass of hadron, $\epsilon_\mu^{(H)}$ is polarization of hadron which is perpendicular to momentum q_μ due to current conservation . f_H is called decay constant of H and it is a parameter related to nonperturbative QCD. If eqs.2.3, 2.5, 2.6 put together,

$$Im\Pi(q^2) = \pi f_H^2 \delta(q^2 - m_H^2) + \pi \rho^c(q^2) \theta(q^2 - s_0) \tag{2.7}$$

will be acquired. s_0 is the threshold where continuum and higher resonance states starts, and ρ^c is part of the spectral density representing continuum states, excited states and higher resonance states. As it has been already mentioned before that

will be below two particle creation, may even be less than m_H because of bound states.

$\Pi(q^2)$ had represented in terms of phenomenological parameters in eq.2.7, which holds at positive values of q^2 where hadron picture is dominant. Next, as stated earlier $\Pi(q^2)$ should be calculated by using OPE in deep Euclidean region with both perturbative and nonperturbative contributions included.

2.2 Operator Product Expansion

For large negative $q^2 << -\Lambda_{QCD}$ main contribution to polarization function $\Pi(q^2)$ come from short spatial distances and times[12]. In this regime polarization function could be calculated by using OPE. OPE says that correlation function of two currents in short distances can be expanded as the sum of local operators with space time dependent coefficients, like:

$$\mathrm{T}\{j_2(x)j_1(0)^\dagger\} = \sum_d C_d(x)O_d, \tag{2.8}$$

here $C_d(x)$ are called Wilson coefficients, and O_d is set of local operators who are lorentz covariant and gauge invariant. They are ordered according to their dimensionality, higher dimensional operators has less contribution to expansion. When QCD is considered, some examples to possible operators are $G^2_{\mu\nu}$ which has dimension 4 and $\bar{q}q\bar{q}q$ which has dimension 6.

If OPE applied to polarization operator it becomes:

$$\Pi(q^2) = \sum_d C_d(q^2) \langle O_d \rangle , \tag{2.9}$$

$\langle O_d \rangle$ has used for $\langle 0 | O_d | 0 \rangle$. Please note above equation valid for short spatial distances and times. So polarization operator can be calculated for large negative $q^2 << \Lambda_{QCD}$ using 2.9. Of course one needs to know Wilson coefficients for calculating it.

Let us consider following correlation function for two vector current :

$$\Pi_{\mu\nu}(q) = i \int d^4x e^{iq.x} \left\langle 0 \left| \mathrm{T}\{\overline{\Psi}(x)\gamma_\mu \Psi(x), \overline{\Psi}(0)\gamma_\nu \Psi(0)\} \right| 0 \right\rangle , \tag{2.10}$$

where Ψ represents a light quark field with mass m. If it is expanded using OPE:

$$\Pi_{\mu\nu}(q) = (q_\mu q_\nu - q^2 g_{\mu\nu}) \sum_d C_d(q^2) \left\langle O_d \right\rangle , \tag{2.11}$$

the lowest order operator in the expansion will be unit operator and corresponding Wilson coefficient is the perturbative part of polarization operator, $C_0(q^2) = \Pi^{pert}(q^2)$. To acquire $\Pi(q^2)^{pert}$ up to order $\alpha_s = \frac{g_s^2}{4\pi}$ where g_s is quark-gluon coupling constant, one needs to calculate diagrams in fig.2.2. In [12] result is given as:

$$C_0(q^2) = \Pi^{pert}(q^2) = -\frac{1}{4\pi^2}\left(1 + \frac{\alpha_s}{\pi} ln \frac{-q^2}{4m^2}\right). \tag{2.12}$$

If one wants to calculate higher orders in OPE, theory at hand should be an-

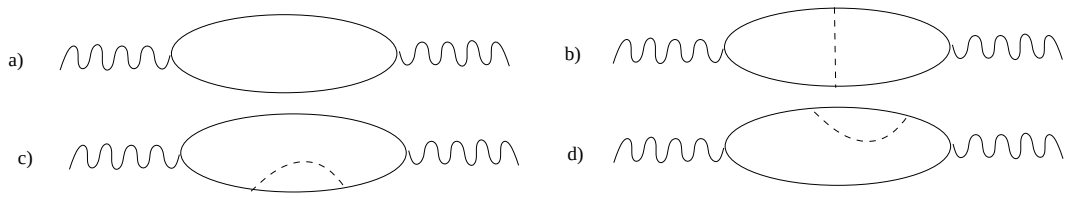

Figure 2.2: In these diagrams dashed lines represents gluon, normal lines quarks. (a) quarks freely propagates (b,c,d) quarks emit and absorb back one gluon

alyzed at short distances. Thanks to asymptotic freedom, for short distances which means large energy and momentum, correlation function of hadronic currents can be written perturbatively. But we will decompose fields into two parts; quantum part and background one. We will contract the quantum part of the fields considering they are in an external field. Here external fields will be QCD vacuum fields. New diagrams will emerge because of the existence of vacuum fields. Both for light and heavy quarks there will be interaction of quarks with background gluons (fig.2.3a,b,c). For light quarks, quarks and antiquarks may

interchange with their vacuum counterpart (2.3d), and combined interactions of quarks and gluons are possible. These two events only available for light quarks because heavy quark condensation in QCD vacuum is really small. Next order

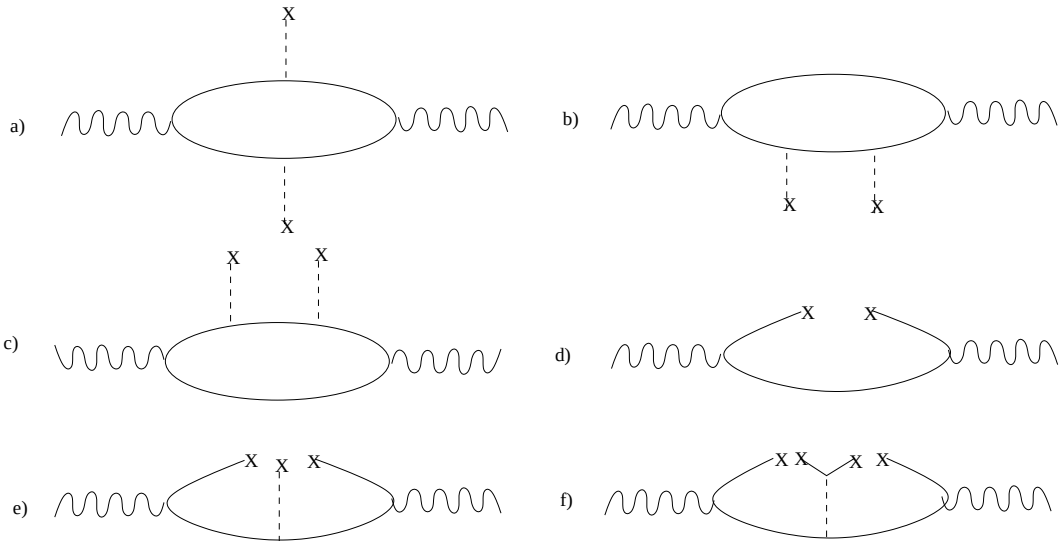

Figure 2.3: (a,b,c) gluon , (d)quark,(e)quark gluon and (f)four quark condensate contribution diagrams

operator after unit operator in the expansion is $\overline{\Psi}\Psi$. For this order the diagram in fig.2.3(d) should be calculated, keeping two background quark field and contracting the rest will give:

$$\Pi_{\mu\nu}^{(\overline{\Psi}\Psi)}(q) = i\int d^4x e^{iq.x} \quad \left\langle 0 \left| \overline{\Psi}^i_{vac}(x)\gamma_\mu S^{ij}(x,0)\gamma_\nu \Psi^j_{vac}(0) \right.\right. \\ \left.\left. + \ \overline{\Psi}^j_{vac}(0)\gamma_\nu S^{ji}(0,x)\gamma_\mu \Psi^i_{vac}(x) \right| 0 \right\rangle . \qquad (2.13)$$

Here Ψ_{vac} is background part of the quark field and S is free quark propagator. $\Psi(x)$ will be expanded as:

$$\Psi(x) = \Psi(0) + x_\mu \overrightarrow{D_\mu}\Psi(0) + ... \qquad (2.14)$$

When eq.2.14 used in eq.2.13 there will be terms including brakets like $\left\langle 0 \left| \overline{\Psi}\Psi \right| 0 \right\rangle$ and $\left\langle 0 \left| \overline{\Psi}\overrightarrow{D}\Psi \right| 0 \right\rangle$. These can be related by using equation of motion $\not{D}\Psi =$

$-im\Psi$, m is the mass of the propagating quark. At last:

$$\Pi_{\mu\nu}(q)^{(\overline{\Psi}\Psi)} = \frac{2m}{q^4}\left\langle 0\left|\overline{\Psi}\Psi\right|0\right\rangle \tag{2.15}$$

will be acquired.

Wilson coefficient for $\left\langle\overline{\Psi}\Psi\right\rangle$ has been calculated:

$$C(q^2) = \frac{2m}{q^4}. \tag{2.16}$$

To go further in the expansion other diagrams of fig.2.3 should be calculated. When all Wilson coefficients up to dimensionality 6 are taken into account, the result will be:

$$\begin{aligned}\Pi(q^2) = & - \frac{1}{4\pi^2}(1+\frac{\alpha_s}{\pi})ln\frac{-q^2}{4m^2} + \frac{2m\left\langle\overline{\Psi}\Psi\right\rangle}{q^4} + \frac{\alpha_s\left\langle G^{(a)}_{\mu\nu}G^{(a)\mu\nu}\right\rangle}{12\pi q^4} \\ & + \frac{m^3}{3q^8}\left\langle g_s\overline{\Psi}\sigma_{\mu\nu}\frac{\lambda^a}{2}G^{(a)\mu\nu}\Psi\right\rangle + \frac{2\pi\alpha_s}{q^6}\left[\left\langle(\overline{\Psi}\gamma_\mu\gamma_5\frac{\lambda^a}{2}\Psi)(\overline{\Psi}\gamma^\mu\gamma_5\frac{\lambda^a}{2}\Psi)\right\rangle\right. \\ & + \left.\frac{2}{9}\left\langle(\overline{\Psi}\gamma_\mu\frac{\lambda^a}{2}\Psi)(\overline{\Psi}\gamma^\mu\frac{\lambda^a}{2}\Psi)\right\rangle\right],\end{aligned} \tag{2.17}$$

which had been given in [12]. λ^a are generators of SU(3) which is the gauge group of QCD, check appendix B for details.

Numerical values of condensates are mostly obtained by fitting data. But long time before sum rules physicists know quark condensate value of QCD vacuum. This parameter is order parameter for chiral symmetry breaking. Some of the vacuum condensate values are:

$$\begin{aligned}\left\langle\overline{\Psi}\Psi\right\rangle &= -(240\pm 10MeV)^3, \\ \frac{\alpha_s}{\pi}\left\langle G^{(a)}_{\mu\nu}G^{(a)\mu\nu}\right\rangle &= (0.012GeV^4)\pm 30\%.\end{aligned} \tag{2.18}$$

So polarization function had calculated in terms of vacuum condensate values for large negative q^2, where quark picture is dominant. Next, it had to be matched with phenomenological part.

2.3 Bridging quarks and hadron

$\Pi(q^2)$ had given in terms of vacuum condensations for large negative q^2, and for positive q^2 in terms of hadronic parameters. If one can relate two parts, hadronic parameters and vacuum condensations can be related. Since there are only five or six parameters important in vacuum condensates part, explaining huge hadronic data with them will be a great achievement.

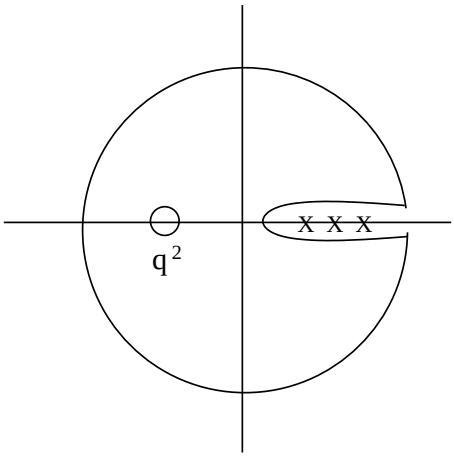

Figure 2.4: Contour to calculate function at the point shown by small circle. Crosses show hadronic thresholds.

To relate them cauchy formula can be used for $\Pi(q^2)$, along the contour in fig.2.5:

$$\begin{aligned}\Pi(q^2) &= \frac{1}{2\pi i}\oint_C dz\frac{\Pi(z)}{z-q^2} = \frac{1}{2\pi i}\oint_{|z|=R} dz\frac{\Pi(z)}{z-q^2} \\ &+ \frac{1}{2\pi i}\int_0^R dz\frac{\Pi(z+i\epsilon)-\Pi(z-i\epsilon)}{z-q^2}. \end{aligned} \tag{2.19}$$

To get rid of the integral over circle, R will be taken to infinity. But integrand may not vanish or vanish fast enough as R goes to infinity. In most cases that can be easily cured by subtracting first few terms of taylor expansion from $\Pi(q^2)$. For example if $\Pi(0)$ subtracted from integral over circle, it will be:

$$\frac{1}{2\pi i}\oint_{|z|=R} dz\frac{\Pi(z)}{z-q^2} - \frac{1}{2\pi i}\oint_{|z|=R} dz\frac{\Pi(z)}{z} = \frac{q^2}{2\pi i}\oint_{|z|=R} dz\frac{\Pi(z)}{z(z-q^2)}. \tag{2.20}$$

That integrand clearly vanishs faster. One should take enough subtraction terms to make the integral over circle zero, $\Pi(q^2) - \Pi(0) - \frac{d\Pi(0)}{dq^2}q^2 - ..$, usually more than a few terms are not needed. These terms will be called as subtractions and note that they are constants or polynomial in q^2. For the second integral one can use:$\Pi(z + i\epsilon) - \Pi(z - i\epsilon) = 2iIm\Pi(q^2)$, which is acquired from Schwartz reflection principal. From spectral density polarization operator relation eq.2.7, it is expected that $Im\Pi(q^2)$ will be zero before our hadronic states start to form at energies $E_0 = minimum(m_H^2, s_0)$. And eq.2.19 will become:

$$\Pi(q^2) = \frac{1}{\pi}\int_{E_0}^{\infty} ds \frac{Im\Pi(s)}{(s - q^2)} + subtractions. \tag{2.21}$$

If also eq.2.7 has been considered above equation becomes:

$$\Pi(q^2) = \frac{f_H^2}{(m_H^2 - q^2)} + \int_{E_0}^{\infty} ds \frac{\rho^c(s)}{(s - q^2)} + subtractions. \tag{2.22}$$

A link had been established but eq.2.22 is still not good enough. ρ^c is not well known and one needs to supress contributions coming from it. Borel transformation can be applied in order to improve situation. Borel transformation will be defined as:

$$\hat{\mathfrak{B}}_M^2(q^2) = lim \frac{1}{(n-1)!}(q^2)^n(-\frac{d}{dq^2})^{n-1}, \quad q^2 \to \infty, n \to \infty, \frac{Q^2}{n} \equiv M^2 fixed \tag{2.23}$$

M^2 is called Borel parameter. For example:

$$\begin{aligned}
\hat{\mathfrak{B}}_M^2(q^2)\left(\frac{1}{(s-q^2)^n}\right) &= \frac{1}{(k-1)!}\frac{e^{-s/M^2}}{(M^2)^{k-1}}, \\
\hat{\mathfrak{B}}_M^2(q^2)e^{\alpha q^2} &= \delta\left(\frac{1}{M^2} - \alpha\right), \\
\hat{\mathfrak{B}}_M^2(q^2)lnq^2 &= -1, \\
\hat{\mathfrak{B}}_M^2(q^2)q^{2n} &= 0, \qquad n\epsilon N^+.
\end{aligned} \tag{2.24}$$

If Borel transformation is applied to eq.2.22:

$$\Pi(M^2) = f_H^2 e^{-m_H^2/M^2} + \int_{E_0}^{\infty} ds \rho^c(s) e^{-s/M^2}. \tag{2.25}$$

As illustrated, new equation is free from unknown constant $\Pi(0)$ and contributions coming from states heavier than m_H had been suppressed. It is possible to estimate integral over higher resonance and continuum states. When q^2 goes to minus infinity all condensate contributions can be neglected and it can be assumed that $\Pi(q^2) = \Pi^{(pert.)}(q^2)$, where $\Pi^{(pert.)}$ is the perturbative QCD part of the polarization operator. This limit points out as $q^2 \to -\infty$:

$$q^2 \int_{E_0}^{\infty} ds \frac{Im\Pi(s)}{s(s-q^2)} \simeq q^2 \int_{4m^2}^{\infty} ds \frac{Im\Pi^{(pert.)}(s)}{s(s-q^2)}. \tag{2.26}$$

This is known as the global quark hadron duality and asymptotic form of the both integrands should be same. So it can be assumed as $Im\Pi(s) = Im\Pi^{(pert.)}(s)$ at large s values, which is known as local quark hadron duality. Integral over continuum can be approximated as:

$$\int_{E_0}^{\infty} ds \rho^c(s) e^{-s/M^2} \simeq \frac{1}{\pi} \int_{s_0}^{\infty} ds Im\Pi^{(pert.)}(s) e^{-s/M^2}, \tag{2.27}$$

here s_0 does not have to be same as E_0, it should be fitted and expected to be around the energies where excited states begin.

For illustration let us take $H = \rho$ and use current given in eq.2.2. Now if we take Borel transformation of eq.2.17 and equate eq.2.27 and with use of eq.2.27 an equation for f_ρ will be acquired:

$$f_\rho^2 = e^{m_\rho^2/M^2} \left(\frac{M^2}{4\pi^2}(1+\frac{\alpha_s}{\pi})(1-e^{-\frac{s_0}{M^2}}) + \frac{2(m_u+m_d)\left\langle \overline{\Psi}\Psi \right\rangle}{M^2} + \frac{\left\langle \alpha_s G^{(a)}_{\mu\nu} G^{(a)\mu\nu} \right\rangle}{12\pi M^2} \right), \tag{2.28}$$

result has given up to dimensionality 4. Now values of s_0 and M^2 had to be determined. The approach developed until now does not hold for small M^2 where the contribution of higher dimensional condensates are stronger so M^2 should be kept large. But for very large values of M^2 quark hadron duality cannot be trusted enough hence M^2 should not be too large. So we will start from energies that higher vacuum condensates are suppressed enough but continuum part does

not contribute more than 30 percent. Such working window for eq.2.28 is given as $0.5 < M^2 < 1.2GeV^2$ in [12].

In general when QCD sum rule procedure applied, one needs draw result versus M^2, where such a picture will be acquired: The region shown will be SVZ rule

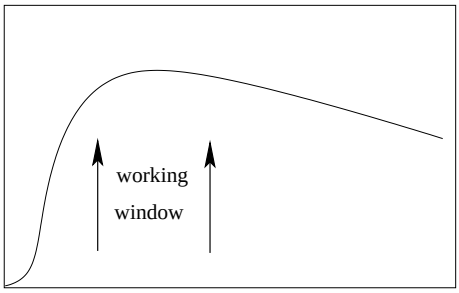

Figure 2.5: General picture calculated value vs. M^2 when sum rule applied

working window. There is no guarantee at every problem such a window can be defined.

s_0 will be fixed with the demand of f_ρ to be maximally stable in working window. It is known it should be around the start of excited states. If graphs of calculated value plotted for different values of s_0 around the start of excited states, one can determine most stable case. For the channel considered now, it starts around $1.5 \sim 2.0GeV^2$. Maximum stability can be achieved by fixing $f_\rho = 1.7Gev^2$ for eq.2.28, it is illustrated in [12].

Now f_ρ can be calculated from eq.2.28. For α_s, its value at $1GeV$ will be taken which is approximately 0.5. Within working window f_ρ will change between 210 - 217 MeV. Uncertainty will get larger when uncertainties of vacuum condensates had been considered too, that will add uncertainty around %1. During calculations higher order terms of OPE expansion had been neglected that will create uncertainty around %5, and limited accuracy of perturbative contributions will give extra %3 uncertainty. When summed up total error will be around %10. Result will be:

$$f_\rho = 213 \pm 20MeV. \tag{2.29}$$

For comparison experimental value is :

$$f_\rho^{exp} = 216 \pm 5MeV. \tag{2.30}$$

2.4 Light Cone QCD Sum Rules

SVZ sum rules can be generalized to $A \to B+C$ kind of decays. Either three currents correlation can be written or instead of vacuum bra $< B(q)|$ or $< C(q)|$ can be used, which will represent B or C hadron on mass shell state with momentum q in the final state. Our correlation function will look like:

$$\Pi(p,q) = i \int d^4x e^{ipx} \langle B(q) \left| \mathrm{T}\{j_1(x) j_2(0)\} \right| 0 \rangle . \tag{2.31}$$

In [15], to calculate coupling constant for $D^* \to D + \pi$ decay $\overline{d}\gamma_\mu c$ and $\overline{c} i\gamma_5 u$ has been used. Correlator function sandwiched between π meson state and vacuum state. After contracting free propagating part, correlator function will be:

$$\begin{aligned} \Pi_\mu(p,q) &= i \int d^4x e^{ipx} \left\langle \pi(q) \left| \mathrm{T}\{\overline{d}(x)\gamma_\mu c(x), \overline{c}(0) i\gamma_5 u(0)\} \right| 0 \right\rangle , \\ &= i \int \frac{d^4x d^4k}{(2\pi)^4 (m_c^2 - k^2)} e^{i(p-k)x} \left[m_c \left\langle \pi(q) \left| \overline{d}(x)\gamma_\mu\gamma_5 u(0) \right| 0 \right\rangle \right. \\ &+ \left. k^\nu \left\langle \pi(q) \left| \overline{d}(x)\gamma_\mu\gamma_\nu\gamma_5 u(0) \right| 0 \right\rangle \right] , \end{aligned} \tag{2.32}$$

where for free quark propagator:

$$iS^0(x,0) = \langle 0 \left| \mathrm{T}\{c(x), \overline{c}(0)\} \right| 0 \rangle = \int \frac{d^4k}{(2\pi)^4 i} e^{-ikx} \frac{\not{k} + m_c}{m_c^2 - k^2} \tag{2.33}$$

has been used. m_c is the mass of charm quark. If fields expanded like in eq.2.14, after integration:

$$\Pi_\mu(p,q) = i \frac{m_c}{m_c^2 - p^2} \sum_{n=0}^{\infty} \frac{(2p.q)^n}{(m_c^2 - p^2)^n} M_n q_\mu \tag{2.34}$$

will be acquired where

$$\left\langle \pi(q) \left| \overline{d}(0) \overleftarrow{D}_{\alpha 1} \overleftarrow{D}_{\alpha 2} ... \overleftarrow{D}_{\alpha n} \gamma_\mu \gamma_5 u(0) \right| 0 \right\rangle = i^n q_\mu q_{\alpha 1} q_{\alpha 2} ... q_{\alpha n} M_n + ... \tag{2.35}$$

There is a problem here. $\xi = (2p.q)/(m_c^2 - p^2) = [(p+q)^2 - p^2]/(m_c^2 - p^2)$ is a dimensionless parameter and it is small only if $(p+q)^2 \simeq p^2$. If it is not small one have to keep infinite number of matrix elements M_n in the expansion.

Light Cone QCD Sum Rules is variant of QCD sum rules which is designed to overcome above mentioned difficulty. One will take Wilson coefficients as a function of x^2 and make the expansion for small values of x^2, that is why it is called light cone QCD sum rules. Vacuum condensates will take weight according to their twist, which is defined as dimension minus spin of the operator. For example:

$$\begin{aligned} \left\langle \pi(q) \left| \overline{d}(x) \gamma_\mu \gamma_5 u(0) \right| 0 \right\rangle &= i q_\mu f_\pi \int_0^1 du e^{iuqx} [\phi_\pi(u) + x^2 g_1(u) + O(x^4)] \\ &+ f_\pi \left(x_\mu - \frac{x^2 q_\mu}{qx} \right) \int_0^1 du e^{iuqx} g_2(u), \end{aligned} \tag{2.36}$$

here ϕ_π has associated with twist two operators so it is the most dominant term. It represents the distribution in the fraction of the light-cone momentum of the pion carried by a constituent quark. g_1 and g_2 are related to twist four operators. These functions appear instead of vacuum condensations in the expansion. The price for using LCSR is we will have functions instead of numbers.

After this expansion procedure of LCSR is not very different from SVZ sum rules. In next chapters, we will give more details with examples. As first application of LCSR, it will be applied to semileptonic decay of B meson to η meson. LCSR is well suited to this subject since it is a heavy meson to light meson transition and in this kind of problems traditional QCD sum rules have problems.

CHAPTER 3

SEMILEPTONIC $B \to \eta\ell\nu$ DECAY IN LIGHT CONE QCD

The paradigm of CP violation related to the structure of CKM matrix in the Standard Model (and its beyond) is fueling an impressive experimental programme for the study of both exclusive and inclusive B-decays. The data from BaBar and Belle open new era of B meson physics, and open a real possibility for detecting semileptonic decay modes of B mesons. The semileptonic decay modes of B mesons are much cleaner samples then the nonleptonic decay modes since in these modes, there does not exist any problems connected with the presence of a third strongly interacting particle. For this reason, the study of semileptonic decays is one of the efficient ways for the determination of the CKM matrix elements. For example V_{cb} has been determined from semileptonic meson decays [16]. CLEO collaboration [17] have measured the branching ratios of $B^0 \to \pi^-\ell^+\nu$ and $B^0 \to \rho^-\ell^+\nu$ which leads to $|V_{ub}| = (3.25 \pm 0.14^{+0.21}_{-0.29} \pm 0.55)\, 10^{-3}$. It is well known that for accurate determination of CKM matrix elements we need more reliable determination of transition form factors. Another point that makes $B \to \eta\ell\nu$ semileptonic decays interesting is due to the fact that they also give information on the $\eta - \eta'$ mixing (see for example [21, 22]). As pointed out in [23] too, LCSR are important to calculate heavy meson to light meson transition form factors. So we will calculate the transition form factors for $B \to \eta\ell\nu$ decay in LCSR method. In section 3.1, we derive the sum rules for the transition form factors of $B \to \eta\ell\nu$ decay. Section 3.2 is devoted to the numerical analysis and contain a summary of the results and our conclusions.

3.1 Light Cone QCD Sum Rules for the $B \to \eta$ transition form factors

The $B \to \eta$ weak transition form factors $f_+(q^2)$ and $f_-(q^2)$ are defined as:

$$\langle \eta(p)|\bar{u}\gamma_\mu(1+\gamma_5)b|B(p+q)\rangle = 2f_+p_\mu + (f_+ + f_-)q_\mu \,. \tag{3.1}$$

In this section, we calculate these form factors using light cone QCD sum rule method. We adopt the usual strategy of light cone sum rules method by considering the following correlator function:

$$\Pi_\mu(p,q) = i\int d^4x e^{iqx}\ \langle \eta(p)|T\left\{\bar{u}(x)\gamma_\mu(1+\gamma_5)b(x)\bar{b}(0)i(1+\gamma_5)u(0)\right\}|0\rangle. \tag{3.2}$$

The main reason for choosing the chiral $\bar{b}i(1+\gamma_5)u$ current instead of the $\bar{b}i\gamma_5 u$ current, which have been used in the calculation of the $B \to \pi$ weak form factor [15], is that in this case, twist-3 wave functions which are the main inputs in LCSR method and which bring the main uncertainty to the prediction, do not contribute (see below, see also [24]).

First let us consider the hadronic representation of the correlator. By inserting a complete set of states with the same quantum numbers as the operator $\bar{b}i(1+\gamma_5)u$ between the currents and isolating the pole term of the lowest pseudo scalar B meson, we get the following hadronic representation of the correlator:

$$\begin{aligned}\Pi_\mu(p,q) &= \frac{\langle \eta|\bar{u}\gamma_\mu b|B\rangle\langle B|\bar{b}i\gamma_5 u|0\rangle}{m_B^2-(p+q)^2} + \sum_h \frac{\langle \eta|\bar{u}\gamma_\mu b|h\rangle\langle h|\bar{b}i(1+\gamma_5)u|0\rangle}{m_h^2-(p+q)^2} \\ &= \Pi_1(q^2,(p+q)^2)p_\mu + \Pi_2(q^2,(p+q)^2)q_\mu. \end{aligned}\tag{3.3}$$

The sum in eq.3.3 takes into account the contributions of the higher states and continuum. Note that the intermediate states h contain not only pseudo scalar resonances of masses greater the m_B, but also scalar resonances with $J^P = 0^+$ corresponding to the operator $\bar{b}u$.

For the invariant amplitudes Π_i, one can write a general dispersion relation in

the B meson momentum squared, $(p+q)^2$, as:

$$\Pi_i(q^2,(p+q)^2) = \int ds \frac{\rho_i(s)}{s-(p+q)^2} + \ subtractions, \tag{3.4}$$

where *subtractions* are polynomials in $(p+q)^2$ and the spectral densities corresponding to eq.3.3 are given as:

$$\rho_1(s) = 2f_+(q^2)\frac{m_B^2 f_B}{m_b}\delta(s-m_B^2) + \rho_1^h(s), \tag{3.5}$$

$$\rho_2(s) = (f_+ + f_-)\frac{m_B^2 f_B}{m_b}\delta(s-m_B^2) + \rho_2^h(s). \tag{3.6}$$

The first terms in eqs.3.5 and 3.6 represent the ground state B-meson contribution and are easily obtained from eq.3.3 using eq.3.1 and the definition

$$\langle B|\bar{b}i\gamma_5 d|0\rangle = \frac{m_B^2 f_B}{m_b},$$

whereas ρ_i^h represent the spectral densities of the higher resonances and the continuum. The spectral densities ρ_i^h can be approximated by invoking the quark-hadron duality ansatz:

$$\rho_i^h(s) = \rho_i^{QCD}\theta(s-s_0). \tag{3.7}$$

So for the hadronic representation of the invariant amplitudes Π_i we get:

$$\begin{aligned}
\Pi_1 &= \frac{2f_+(q^2)m_B^2 f_B}{m_b(m_B^2-(p+q)^2)} + \int_{s_0}^{\infty} ds \frac{\rho_1^{QCD}(s)}{s-(p+q)^2} + \ subtractions, \\
\Pi_2 &= \frac{(f_+ + f_-)m_B^2 f_B}{m_b(m_B^2-(p+q)^2)} + \int_{s_0}^{\infty} ds \frac{\rho_2^{QCD}(s)}{s-(p+q)^2} + \ subtractions.
\end{aligned} \tag{3.8}$$

For obtaining sum rules for the form factors f_+ and f_- we must calculate the correlator function in QCD. It can be done by using the light cone OPE method; i.e. by expanding the T product of currents near the light cone $x^2 \simeq 0$. After

contracting the b-quark fields we get

$$\Pi_\mu(p,q) = i\int d^4x e^{iqx}\langle\eta(p)|\bar{u}(x)\gamma_\mu(1+\gamma_5)S_b(x)(1+\gamma_5)u(0)|0\rangle, \tag{3.9}$$

where S_b is the full propagator of the b quark. Its explicit expression is given by the following expression

$$\begin{aligned} iS_b(x) &= iS_b^0(x) - igs\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\int_0^2 dv\left\{\frac{1}{2}\frac{\not{k}+m_b}{(m_b^2-k^2)^2}G^{\mu\nu}(vx)\sigma_{\mu\nu}\right. \\ &\left. + \frac{1}{m_b^2-k^2}vx_\mu G^{\mu\nu}(vx)\gamma_\nu\right\}. \end{aligned} \tag{3.10}$$

Here $S_b^0(x)$ is the free quark propagator of the massive b quark:

$$S_b^0(x) = \frac{m_b^2}{4\pi^2}\frac{K_1(m_b\sqrt{-x^2})}{\sqrt{-x^2}} - i\frac{m_b^2}{4\pi^2}\frac{\not{x}}{x^2}K_2(m_b\sqrt{-x^2}), \tag{3.11}$$

where K_i are the Bessel functions.

From eqs.3.9-3.11 it follows that, in order to calculate the theoretical part of the correlator function, the matrix elements of the nonlocal operators between η meson and vacuum states are needed.

Here we would like to do following remark: in all the following calculations we neglect the $\eta-\eta'$ mixing since it is very small [16], and therefore we choose as the interpolating current for the η meson, the SU(3) octet axial vector current:

$$J_\mu = \frac{1}{\sqrt{6}}(\overline{u}\gamma_\mu\gamma_5 u + \overline{d}\gamma_\mu\gamma_5 d - 2\overline{s}\gamma_\mu\gamma_5 s). \tag{3.12}$$

In order to simplify the notation we use $\overline{q}\Gamma_\mu q$ to denote $\frac{1}{\sqrt{6}}(\overline{u}\gamma_\mu\gamma_5 u + \overline{d}\gamma_\mu\gamma_5 d - 2\overline{s}\gamma_\mu\gamma_5 s)$ and introduce notation $F_\eta = \frac{f_\eta}{\sqrt{6}}$, where f_η determined from:

$$\langle 0|\overline{q}(0)\Gamma_\mu q(0)|\eta(q)\rangle = -if_\eta q_\mu. \tag{3.13}$$

An important observation is that the terms in $S_b(x)$ containing odd number of gamma matrices do not contribute, i.e. leading twist-3 terms do not give

any contribution. Up to twist-4, the η meson wave functions are defined in the following way [26] :

$$\begin{aligned}
\langle\eta(p)|\bar{q}\gamma_\mu\gamma_5 q|0\rangle &= -if_\eta p_\mu \int_0^1 du e^{-iupx}\left[\varphi_\eta(u)+\frac{1}{16}m_\eta^2 x^2 A(u)\right] \\
&\quad -\frac{i}{2}f_\eta m_\eta^2 \frac{x_\mu}{px}\int_0^1 du e^{-iupx}B(u), \qquad (3.14)\\
\langle\eta(p)|\bar{q}\gamma_\mu\gamma_5 g_s G_{\alpha\beta} q|0\rangle &= f_\eta m_\eta^2\left[p_\beta\left(g_{\alpha\mu}-\frac{x_\alpha p_\mu}{px}\right)-p_\alpha\left(g_{\beta\mu}-\frac{x_\beta p_\mu}{px}\right)\right] \\
&\quad \times\int \mathcal{D}\alpha_i\varphi_\perp(\alpha_i)e^{-ipx(\alpha_1+u\alpha_3)} \\
&\quad +f_\eta m_\eta^2\frac{p_\mu}{px}(p_\alpha x_\beta - p_\beta x_\alpha)\int \mathcal{D}\alpha_i\varphi_\parallel(\alpha_i)e^{-ipx(\alpha_1+u\alpha_3)}, \\
&\qquad (3.15)\\
\langle\eta(p)|\bar{q}g_s\tilde{G}_{\alpha\beta}\gamma_\mu|0\rangle &= if_\eta m_\eta^2\left[p_\beta\left(g_{\alpha\mu}-\frac{x_\alpha p_\mu}{px}\right)-p_\alpha\left(g_{\beta\mu}-\frac{x_\beta p_\mu}{px}\right)\right] \\
&\quad \times\int \mathcal{D}\alpha_i\tilde{\varphi}_\perp(\alpha_i)e^{-ipx(\alpha_1+u\alpha_3)} \\
&\quad +if_\eta m_\eta^2\frac{p_\mu}{px}(p_\alpha x_\beta - p_\beta x_\alpha)\int \mathcal{D}\alpha_i\tilde{\varphi}_\parallel(\alpha_i)e^{-ipx(\alpha_1+u\alpha_3)}, \\
&\qquad (3.16)
\end{aligned}$$

where $\tilde{G}_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\sigma\lambda}G^{\sigma\lambda}$, $\mathcal{D}\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3\delta(1-\alpha_1-\alpha_2-\alpha_3)$. $\bar{q}(x)$, $q(0)$, $G_{\mu\nu}(ux)$, $\tilde{G}_{\mu\nu}(ux)$ are used in shorter form as $\bar{q}$, q, $G_{\mu\nu}$, $\tilde{G}_{\mu\nu}$ respectively. In eqs.3.14-3.16, the $\varphi_\eta(u)$ is the leading twist-2, $A(u)$ and part of $B(u)$ are two particle twist-4, $\varphi_\parallel(\alpha_i)$, $\varphi_\perp(\alpha_i)$, $\tilde{\varphi}_\parallel(\alpha_i)$ and $\tilde{\varphi}_\perp(\alpha_i)$ are three particle twist-4 wave functions.

Here we should note that that matrix element $\langle\eta(p)|\bar{u}\gamma_\mu G^{\alpha\beta}(ux)\sigma_{\alpha\beta}u|0\rangle = 0$ due to the parity invariance of strong interactions and using the identity

$$\gamma_\mu\sigma_{\alpha\beta} = i(g_{\mu\alpha}\gamma_\beta - g_{\mu\beta}\gamma_\alpha)+\epsilon_{\mu\alpha\beta\rho}\gamma^\rho\gamma_5.$$

Inserting eqs.3.10 and 3.11 into eq.3.9 and using definitions of η-meson wave functions (eqs.(3.14)-(3.16)) for the theoretical part of the correlation function

we get:

$$\begin{aligned}
\Pi_\mu^{th} &= i\sqrt{2}\int d^4x e^{iqx}\left[\int_0^1 du e^{iupx}\left\{-\frac{F_\eta m_b^2}{32\pi^2}(16\varphi_\eta + Am_\eta^2 x^2)\frac{K_1(m_b\sqrt{-x^2})}{\sqrt{-x^2}}p_\mu\right.\right. \\
&- \left.\frac{1}{4\pi^2}F_\eta m_b^2 m_\eta^2 B\frac{K_1(m_b\sqrt{-x^2})}{\sqrt{-x^2}}\frac{x_\mu}{px}\right\} \\
&+ \int\mathcal{D}\alpha\int_0^1 du e^{i(\alpha_2+u\alpha_3)px}\left\{\frac{F_\eta m_\eta^2 m_b}{12\pi^2}(\varphi_\parallel+\tilde{\varphi}_\parallel)K_0(m_b\sqrt{-x^2})\frac{x_\mu p^2 - p_\mu(px)}{px}\right. \\
&+ \left.\left.\frac{1}{12\pi^2}F_\eta m_\eta^2 m_b(\varphi_\perp+\tilde{\varphi}_\perp)K_0(m_b\sqrt{-x^2})\frac{p^2x_\mu + 2(px)p_\mu}{px}\right\}\right]. \qquad (3.17)
\end{aligned}$$

The next task is to carry out the fourier transformation and then to take the Borel transform with respect to the variable $(p+q)^2$ in order to suppress the contributions of the higher states and the continuum and also in order to eliminate the subtraction terms. Finally we identify the same structures both at the hadronic and quark gluon levels. Subtraction of the continuum contribution is done by employing the quark-hadron duality as mentioned before. This amounts to restricting the spectral integral to $s \leq s_0$. (for more details about this procedure see the appendix A and also [15]). For the theoretical part of the correlator (eq.3.2) function we get:

$$\begin{aligned}
\Pi_\mu^{th} &= \frac{\sqrt{2}F_\eta m_\eta^2}{3}I_1^1(\varphi_\parallel+\tilde{\varphi}_\parallel-2\varphi_\perp-2\tilde{\varphi}_\perp)p_\mu \\
&+\frac{\sqrt{2}F_\eta m_\eta^4}{3m_b}\tilde{I}_2^2(\varphi_\parallel+\tilde{\varphi}_\parallel+\varphi_\perp+\tilde{\varphi}_\perp)p_\mu \\
&+2\sqrt{2}F_\eta m_b J_1^0(\varphi_\eta)p_\mu - \sqrt{2}\frac{F_\eta}{16}m_b m_\eta^2 J_1^2(A)p_\mu - \sqrt{2}F_\eta m_\eta^2\tilde{J}_2^1(B)p_\mu \\
&-\sqrt{2}F_\eta m_\eta^2 J_2^1(B)q_\mu + \frac{\sqrt{2}}{3m_b}F_\eta m_\eta^4 I_2^2(\varphi_\parallel+\tilde{\varphi}_\parallel+\varphi_\perp+\tilde{\varphi}_\perp)q_\mu, \qquad (3.18)
\end{aligned}$$

where the functions $I_n^m(\varphi)$, $J_n^m(\varphi)$, $\tilde{I}_n^m(\varphi)$, and $\tilde{J}_n^m(\varphi)$ are defined in the appendix A.

Equating the coefficients of the corresponding p_μ and q_μ structures in eqs.3.3-3.6 and 3.18 we get the following sum rules for the form factors f_+ and $f_+ + f_-$

respectively:

$$f_+ = \frac{m_b F_\eta e^{\frac{m_B^2}{M^2}}}{\sqrt{2} m_B^2 f_B}\left[\frac{m_\eta^2}{3} I_1^1(\varphi_\parallel + \tilde{\varphi}_\parallel - 2\varphi_\perp - 2\tilde{\varphi}_\perp) + \frac{m_\eta^4}{3m_b}\tilde{I}_2^2(\varphi_\parallel + \tilde{\varphi}_\parallel + \varphi_\perp + \tilde{\varphi}_\perp)\right.$$
$$\left. + 2m_b J_1^0(\varphi_\eta) - \frac{m_b m_\eta^2}{16} J_1^2(A) - m_\eta^2 \tilde{J}_2^1(B)\right], \quad (3.19)$$

$$f_+ + f_- = \frac{\sqrt{2} F_\eta m_b}{m_B^2 f_B} e^{\frac{m_B^2}{M^2}}\left[-m_\eta^2 J_2^1(B) + \frac{m_\eta^4}{3m_b} I_2^2(\varphi_\parallel + \tilde{\varphi}_\parallel + \varphi_\perp + \tilde{\varphi}_\perp)\right]. \quad (3.20)$$

eqs.3.19 and 3.20 are our final results for the $B \to \eta$ transition form factors.

3.2 Numerical Analysis

From eqs.3.19 and 3.20 it follows that the main input parameters of these sum rules are the η meson wave functions. The explicit expressions of the wave functions $\varphi_\eta(u)$, $A(u)$, $B(u)$ and $\varphi_\parallel(\alpha_i)$, $\varphi_\perp(\alpha_i)$, $\tilde{\varphi}_\parallel(\alpha_i)$, and $\tilde{\varphi}_\perp(\alpha_i)$ are given in [26].

The next input parameter of the sum rule is leptonic decay constant F_η. It should be noted that for η meson case, the situation is more complicated compared to the π and K meson cases, due to the mixing with η'. As we already noted that we neglect mixing between η and η'. More recent analysis show that the coupling to the octet current η is $f_\eta = 159\ MeV$ [33]which we use in our numerical calculations.

In our calculations we neglect QCD radiative corrections. Therefore for consistency we neglect QCD corrections to the leptonic decay $f_B = 160\ MeV$ [15] (see also [29]).

Having these input parameters, we can carry out our numerical calculation. According to QCD sum rules philosophy, first of all, a suitable range for the auxiliary Borel parameter M^2 should be found such that the numerical results are stable. The lower limit of M^2 is determined by the requirement that the terms $\sim M^{-2n}$, $(n > 1)$ remain sub dominant. The upper bound of M^2 is determined by requiring that the contributions of the higher resonances and the continuum

are less then 30% of the total result. Our numerical calculations leads that both requirements are satisfied in the region $10\,GeV^2 < M^2 < 16\,GeV^2$. It should be noted that the light cone QCD sum rules predictions for the form factors are reliable at the region

$$q^2 \lesssim m_b^2 - 2m_b\chi, \tag{3.21}$$

where χ is a typical hadronic scale of roughly $0.5\,GeV$. Then $q^2 \lesssim 18\,GeV^2$.

In figs.3.1 and 3.2, the M^2 dependencies of the form factors $f_+(q^2)$ and $f_-(q^2)$ is depicted for three different values of $q^2 = 0\,,\ 5\,,\ 10\ GeV^2$ for two different values of the continuum threshold $s_0 = 35\,,\ 40\ GeV^2$. From these figures, we see that $f_+(q^2)$ and $f_-(q^2)$ vary weakly as M^2 varies in the region $10\,GeV^2 < M^2 < 16\,GeV^2$ up to $q^2 = 18\,GeV^2$.

Having the working region of M^2, our next problem is to find the dependence of the form factors $f_+(f_-)(q^2)$ on q^2 at given values of M^2, namely at $M^2 = 12\,GeV^2$ and $M^2 = 16\,GeV^2$, at $s_0 = 35\,GeV^2$ and $s_0 = 40\,GeV^2$. We find that $f_+^\eta(0) = 0.15 \pm 0.3$ and $f_-^\eta(0) = -0.16 \pm 0.2$. As we already noted, in the region $q^2 \geq 18\,GeV^2$, applicability of the light cone QCD sum rules is questionable. In order to extend our results to the full physical region, we look some parameterization of the form factors in such a way that in the region, $0 \leq q^2 \leq 18\,GeV^2$, this parameterization coincide with light cone QCD sum rules predictions. A good parameterization of the q^2 dependence can be given in terms of three parameters as:

$$f_i(q^2) = \frac{f_i(0)}{1 - a_F\left(\frac{q^2}{m_B^2}\right) + b_F\left(\frac{q^2}{m_B^2}\right)^2}. \tag{3.22}$$

The values of these parameters for the $B \to \eta$ transition form factors $f_+(f_-)$ are obtained as $a_F = 1.07 \pm 0.08$ ($a_F = 1.1 \pm 0.1$) and $b_F = 0.19 \pm 0.16$ ($b_F = 0.28 \pm 0.19$) where the quoted errors are only due to the variations of the continuum threshold, s_0, and the Borel mass, M^2.

It should be noted that the form factors of $B \to \eta$ transition can be related from $B \to \pi$ transition form factors using $SU(3)_F$ symmetry. For example, the

value of $f_+(q^2=0)=0.15$ obtained via $SU(3)_F$ symmetry is consistent with our prediction of $f_+(q^2=0)$.

Semileptonic decays of B meson investigated by later works too. In [30] B to η transition form factors calculated with different current correlator and it reproduced very similar result. Also B to $\eta\prime$ transition form factors calculated [31]. Not only semileptonic decays of B but semileptonic decays of B_s and B_c had been investigated in [32].

Finally, we would like to note that the background for the $B \to \eta\ell\nu$ decay would be much smaller than that for the $B \to \pi\ell\nu$ decay, due to the much lower multiplicity, since the background caused by $B \to \eta X$ is one order of magnitude smaller than that of $B \to \pi X$.

In conclusion, we calculate the transition form factors for $B \to \eta\ell\nu$ decay using the light cone QCD sum rules. These form factors can be used for enhancing statistical analyzes on determining bottom to up quark transition matrix element, V_{ub}, of CKM matrix. In a recent experimental result of BaBar collaboration [33], form factors of B to η transition has been used to enhance information on V_{ub}. V_{ub} has been estimated $3.6 \pm 0.2 \pm 0.1^{+0.6}_{-0.4}$ from form factors calculated with LCSR.

In following chapter, LCSR will be applied to radiative ρ to η decay. In this decay since photon is real, we had to do calculations where photon's momentum square is zero and this is out of region where traditional QCD sum rules works.

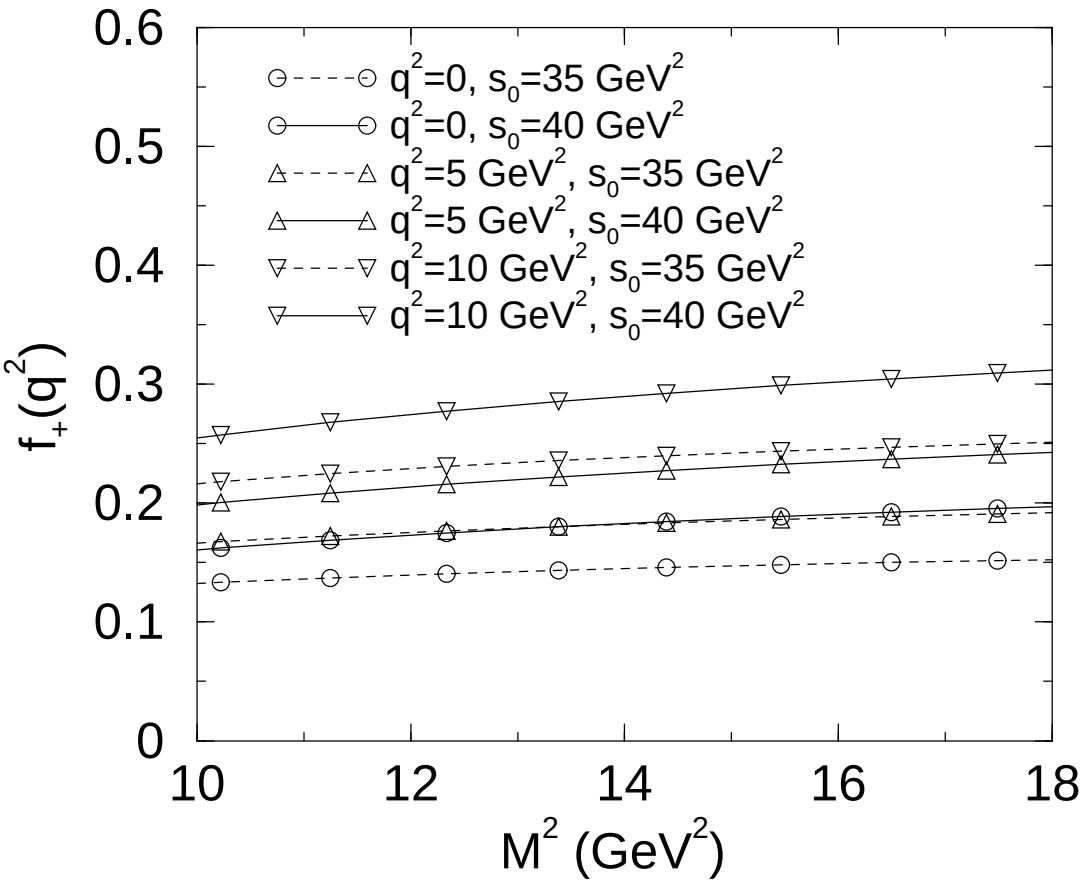

Figure 3.1: The dependence of the form factor $f_+(q^2)$ on the Borel parameter M^2 at $q^2 = 0$, 5, and 10 GeV^2 for $s_0 = 35$ GeV^2 and $s_0 = 40$ GeV^2.

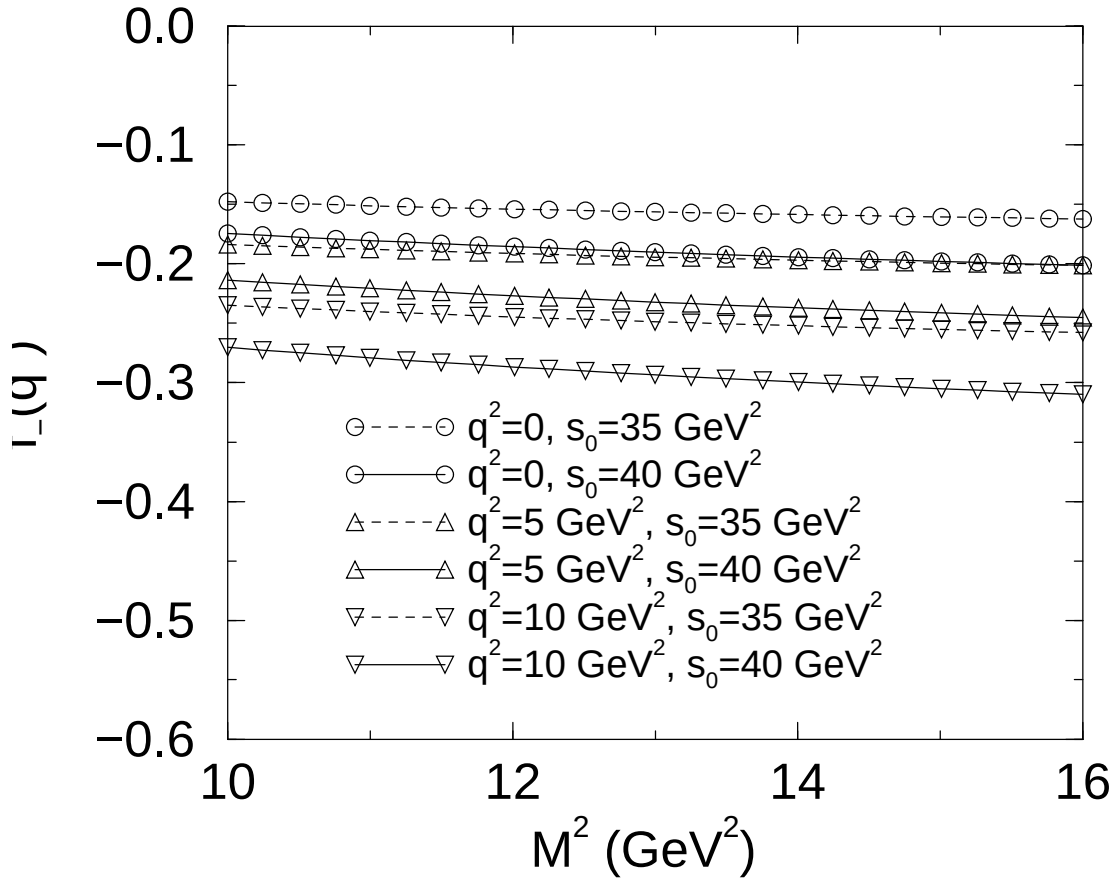

Figure 3.2: The same as fig.3.1 but for the form factor $f_-(q^2)$.

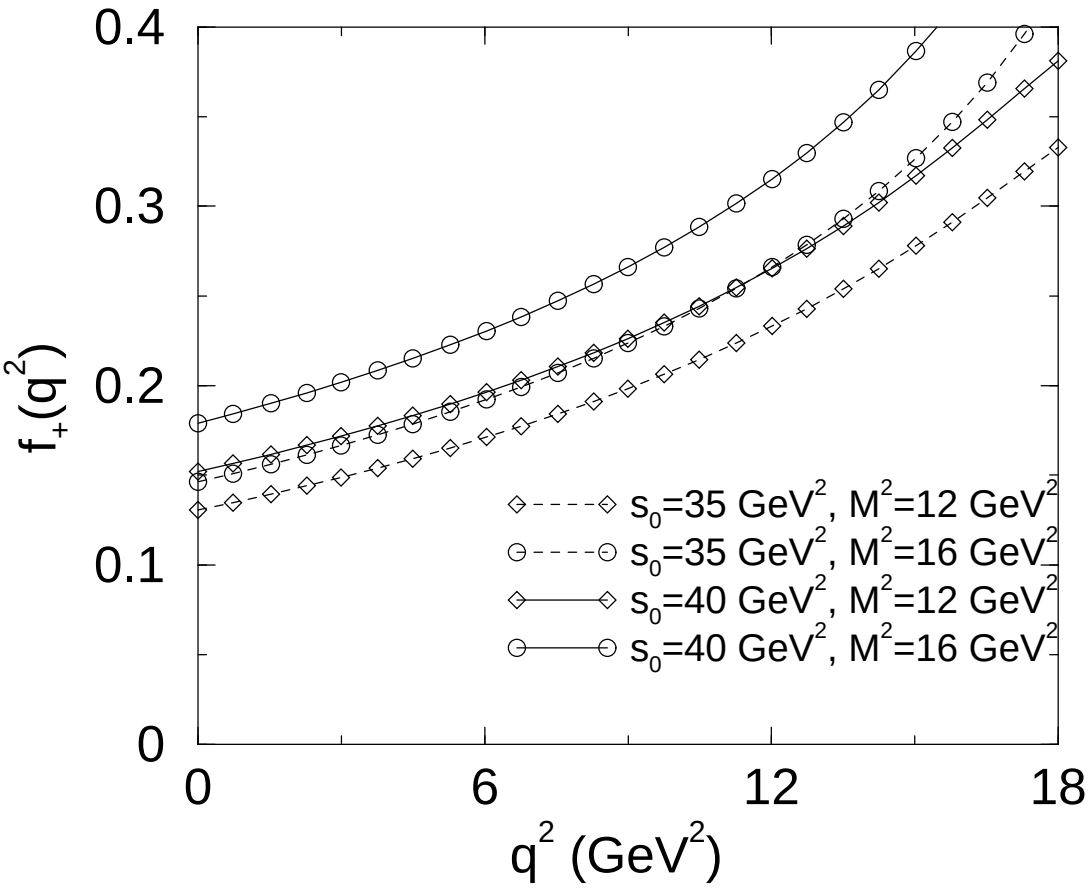

Figure 3.3: The dependence of the form factor $f_+(q^2)$ on q^2 at the continuum threshold $s_0 = 35\ GeV^2$ and $s_0 = 40\ GeV^2$ and at the Borel mass $M^2 = 12\ GeV^2$ and $M^2 = 16\ GeV^2$.

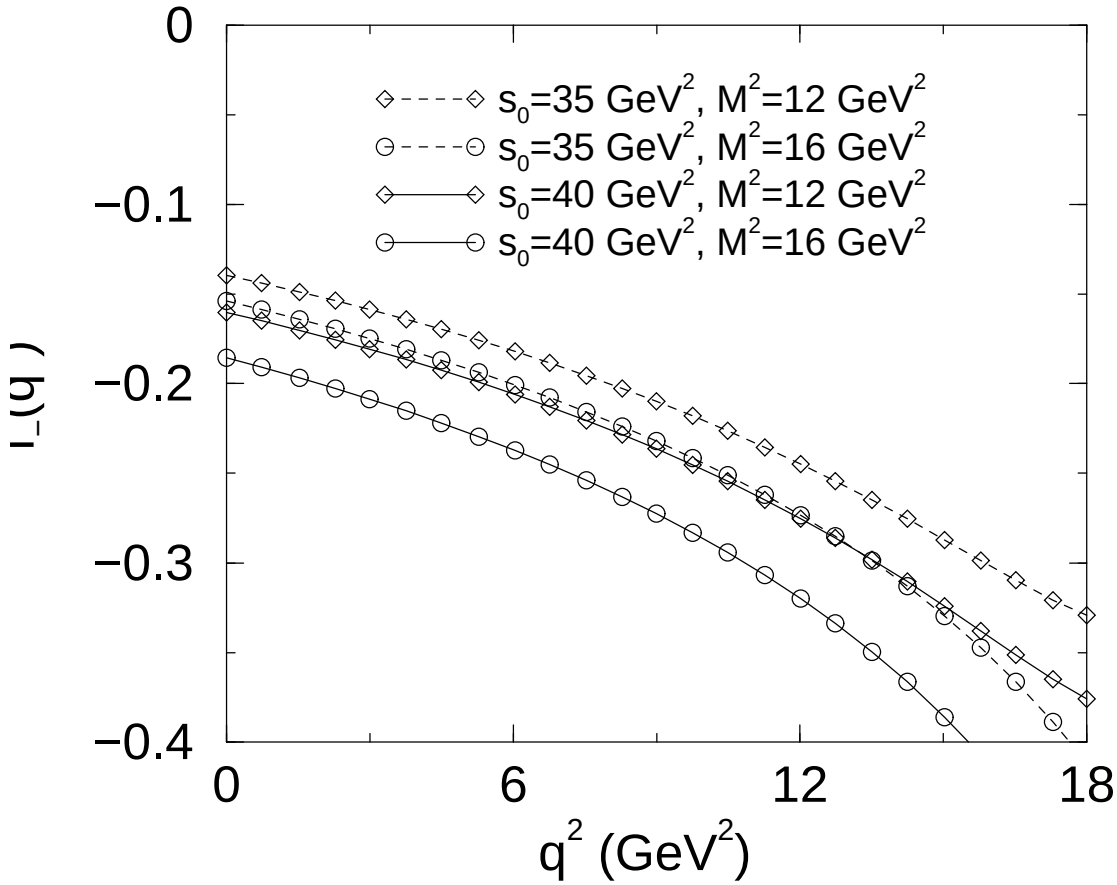

Figure 3.4: The same as fig.3.3 but for the form factor $f_{-}(q^2)$.

CHAPTER 4

RADIATIVE $\rho \to \eta\gamma$ DECAY IN LIGHT CONE QCD

Radiative transition between vector (V) and pseudoscalar (P) mesons represent an important source of information on low energy hadron physics. These transitions are governed by magnetic dipole (M1) radiation of the photon and had played one of the central roles for checking the predictions of quark model and SU(3) symmetry, as well as it was very useful in the determination of the magnetic dipole moment of $N^*(1535)$ in $\gamma N \to \eta N$ process [35]. Recently the theoretical activity on the VPγ magnetic dipole transition have increased (see [34] and references therein), in particular, due to the fact that the analysis of the radiative $V \to P\gamma$ decay with η and η' mesons in final state can provide insights to the long standing issue of the η and η' mixing (for review see [33] and references therein). The usual parametrization of $\eta - \eta'$ mixing in the octet–singlet basis, which will be used in this work, is as follows: the current-particle matrix elements are defined as

$$\langle 0|J^{\alpha}_{5\mu}|P(p)\rangle = -if^{\alpha}_{P}p_{\mu}(\alpha = 8; 0; P = \eta, \eta'), \tag{4.1}$$

where $J^{8}_{5\mu}$, the $SU(3)_F$ octet axial vector current, is given by

$$J^{8}_{5\mu} = \frac{1}{\sqrt{6}}(\overline{u}\gamma_{\mu}\gamma_{5}u + \overline{d}\gamma_{\mu}\gamma_{5}d - 2\overline{s}\gamma_{\mu}\gamma_{5}s), \tag{4.2}$$

and $J^{0}_{5\mu}$, the $SU(3)_F$ singlet current is given by:

$$J^{0}_{5\mu} = \frac{1}{\sqrt{3}}(\overline{u}\gamma_{\mu}\gamma_{5}u + \overline{d}\gamma_{\mu}\gamma_{5}d + \overline{s}\gamma_{\mu}\gamma_{5}s). \tag{4.3}$$

Two mixing angles θ_8 and θ_0 are required in order to consistently describe mixing [36]. Accordingly the couplings in eq.4.1 can be defined as follows

$$f_\eta^8 = f_8 cos\theta_8 \quad , \quad f_\eta^0 = -f_0 sin\theta_0, \tag{4.4}$$
$$f_{\eta'}^8 = f_8 sin\theta_8 \quad , \quad f_{\eta'}^0 = f_0 cos\theta_0. \tag{4.5}$$

Alternatively, two independent axial vector currents with distinct flavors can be considered

$$\begin{aligned} J_{5\mu}^q &= \frac{1}{\sqrt{2}}(\overline{u}\gamma_\mu\gamma_5 u + \overline{d}\gamma_\mu\gamma_5 d), \\ J_{5\mu}^s &= \overline{s}\gamma_\mu\gamma_5 s, \end{aligned} \tag{4.6}$$

and the couplings of these currents with η and η' mesons are defined similarly to eq.4.1 :

$$f_\eta^q = f_q cos\varphi_q \quad , \quad f_{\eta'}^q = f_q sin\varphi_q, \tag{4.7}$$
$$f_\eta^s = -f_s sin\varphi_s \quad , \quad f_{\eta'}^s = f_s cos\varphi_s. \tag{4.8}$$

As we see that in each basis there are two angles and in [33] it was shown that, to a very good accuracy, the mixing can described in terms of single angle φ since $|\varphi_s - \varphi_q|/|\varphi_s + \varphi_q| << 1$ which is also confirmed by a QCD sum rules calculation [25].

We will follow the first approach, neglecting the mixing angles θ_0 and θ_8 due to their smallness and calculate the coupling constant of $\rho \to \eta\gamma$ decay in framework of light cone QCD sum rules (more about of light cone QCD sum rules and its applications can be found in [19] and [18]).

In section 4.1 we derive light cone QCD sum rules for $\rho \to \eta\gamma$ decay constant, section 4.2 is devoted to the numerical analysis of the sum rules and contain our conclusions.

4.1 Light Cone QCD Sum Rules for $\rho \to \eta\gamma$ Coupling Constant

In this section we calculate the coupling constant of $\rho \to \eta\gamma$ decay using light cone QCD sum rules method. In order to calculate this coupling constant we consider the following two point correlation function

$$\Pi_{\mu\nu} = i \int d^4x e^{ip_2 x} \langle 0|T\left\{J^{\eta}_{\nu}(x) J^{\rho}_{\mu}(0)\right\}|0\rangle_{\gamma}, \tag{4.9}$$

where γ denotes the external electromagnetic field, and J^{η}_{ν} and J^{ρ}_{μ} are the interpolating currents with η and ρ meson quantum numbers. Here we would make the following remark. As we already noted that both of the mixing angles in eq.4.5) are small, in the following discussion we will neglect the mixing, i.e we take $J^{\eta}_{\mu} \equiv J^{8}_{5\mu}$.

At the phenomenological level the eq.4.9 can be expressed as:

$$\Pi_{\mu\nu} = \sum \frac{\langle 0|J^{\eta}_{\nu}|\eta(p_2)\rangle \langle \eta(p_2)|\rho(p_1)\rangle_{\gamma} \langle \rho(p_1)|J^{\rho}_{\mu}|0\rangle}{(p_2^2 - m_{\eta}^2)(p_1^2 - m_{\rho}^2)}, \tag{4.10}$$

where $p_1 = p_2 + q$ and q is the photon momentum. The matrix elements entering eq.4.10 are defined as

$$\langle 0|J^{\rho}_{\mu}|\rho\rangle = m_{\rho} f_{\rho} \varepsilon^{\rho}_{\mu}, \tag{4.11}$$

$$\langle 0|J^{8}_{5\nu}|\eta(p_2)\rangle = -i f^{8}_{\eta} p_{2\nu}. \tag{4.12}$$

The remaining matrix element $\langle \eta(p_2)|\rho(p_1)\rangle_{\gamma}$ which describes the M1 transition,can be parameterized as,

$$\langle \eta(p_2)|\rho(p_1)\rangle_{\gamma} = e\epsilon_{\mu\nu\alpha\beta} \varepsilon^{\rho}_{\mu} p_{1\nu} \varepsilon^{\gamma}_{\alpha} q_{\beta} F(q^2), \tag{4.13}$$

where ε^{γ} is the photon polarization vector. Since the photon is real, we need the value of $F(q^2)$ only at the point $q^2 = 0$. We can use an alternative parametrization

for the $\rho\eta\gamma$ vertex:

$$L_{int} = -\frac{e}{m_\rho} g_{\rho\eta\gamma} \epsilon_{\mu\nu\alpha\beta} (\partial_\mu \rho_\nu - \partial_\nu \rho_\mu)(\partial_\alpha A_\beta - \partial_\beta A_\alpha). \tag{4.14}$$

Comparing eqs. (4.13) and (4.14) we see that

$$F(q^2 = 0) \equiv \frac{g_{\rho\eta\gamma}}{m_\rho}. \tag{4.15}$$

Using eqs. (4.10) – (4.14), for the physical part of the sum rules we get

$$\Pi^{ph}_{\mu\nu} = \frac{g_{\rho\eta\gamma} f_\rho f^8_\eta p_{2\nu} \epsilon_{\mu\sigma\alpha\beta} p_{1\sigma} \varepsilon^\gamma_\alpha q_\beta}{(p_2^2 - m_\eta^2)(p_1^2 - m_\rho^2)}. \tag{4.16}$$

Our next task is the calculation of correlator eq.4.1 from the QCD side. The correlator receives both perturbative and non-perturbative contributions. In calculation of the non-perturbative contributions by the OPE on the light cone one needs to know the matrix elements of nonlocal operators between vacuum and the photon states;i.e. $\langle \gamma(q)|\bar{q}\Gamma_i q|0\rangle$ where Γ_i is an arbitrary Dirac matrix. These matrix elements can be expressed in terms of photon wave functions with definite twist. In calculations we neglect twist 3 three particle photon wave functions since their contributions are small. Twist two and twist four photon wave functions are defined as [3, 37, 38]:

$$\begin{aligned}
\langle \gamma(q)|\bar{q}\sigma_{\alpha\beta} q|0\rangle &= ie_q \langle \bar{q}q\rangle \int_0^1 e^{iuqx} \{(\varepsilon_\alpha q_\beta - \varepsilon_\beta q_\alpha)[\chi\varphi(u) + x^2 [g_1(u) - g_2(u)]] \\
&\quad + [qx(\varepsilon_\alpha x_\beta - \varepsilon_\beta x_\alpha) + \varepsilon x(x_\alpha q_\beta - q_\alpha x_\beta)] g_2(u)\} du, \qquad (4.17) \\
\langle \gamma(q)|\bar{q}\gamma_\alpha\gamma_5 q|0\rangle &= \frac{f}{4} e_q \epsilon_{\mu\alpha\beta\rho} \varepsilon^\alpha q^\beta x^\rho \int_0^1 du e^{iuqx} \psi(u), \qquad (4.18)
\end{aligned}$$

where for simplicity we use $\overline{q}\Gamma q$ to denote $J^8_{5\mu} = \frac{1}{\sqrt{6}}(\overline{u}\Gamma u + \overline{d}\Gamma d - 2\overline{s}\Gamma s)$.

In eq.4.17 and 4.18 e_q is the corresponding quark charge, χ is the magnetic susceptibility, $\varphi(u)$, and $\psi(u)$ are the leading twist two and $g_1(u)$, and $g_2(u)$ are the twist four photon wave functions.

After standard calculations we get the following expression for correlator from

QCD side in the coordinate representation:

$$\Pi_{\mu\nu} = \frac{1}{\sqrt{12}}\frac{i}{16}(e_u - e_d)\int \frac{d^4x e^{ip_2x}}{\pi^4 x^6}\{[6(\varepsilon x)\epsilon_{\alpha\beta\nu\mu}q_\alpha x_\beta - 6(qx)\epsilon_{\alpha\beta\nu\mu}\varepsilon_\alpha x_\beta + 6x^2\epsilon_{\alpha\beta\nu\mu}\varepsilon_\alpha q_\beta - 12x_\nu\epsilon_{\alpha\beta\rho\mu}\varepsilon_\alpha q_\beta x_\rho] + f\pi^2x^2[x_\nu\epsilon_{\alpha\beta\rho\mu}\varepsilon_\alpha q_\beta x_\rho + x_\mu\epsilon_{\alpha\beta\rho\nu}\varepsilon_\alpha q_\beta x_\rho]\int_0^1 due^{iuqx}\psi(u)\}. \quad (4.19)$$

The sum rules for $g_{\rho\eta\gamma}$ are obtained by equating the phenomenological and theoretical parts (in eq.4.19) it is necessary to perform Fourier transformation first) of the correlator.

Performing double Borel transformation on variables $p_2^2 = p^2$ and $p_1^2 = (p+q)^2$ on both sides of the correlation function in order to suppress the contributions of the continuum and higher states (procedure of subtraction in light cone sum rules one can find in chapter 2), and also to remove the subtraction terms in the dispersion relation, we obtain the following sum rules for $g_{\rho\eta\gamma}$ coupling constant

$$g_{\rho\eta\gamma} = \frac{e^{[m_\rho^2/M_1^2 + m_\eta^2/M_2^2]}(e_u - e_d)}{2\sqrt{3}f_\eta^u f_\rho}(1-u_0)\{\frac{3}{2\pi^2}M^2E_0(s_0/M^2) + f\psi(u_0)\}, \quad (4.20)$$

where $E_0(s_0/M^2) = 1 - e^{-s_0/M^2}$ is the function used to subtract continuum, s_0 is the continuum and

$$u_0 = \frac{M_1^2}{M_1^2 + M_2^2}, M^2 = \frac{M_1^2M_2^2}{M_1^2 + M_2^2}, \quad (4.21)$$

where M_1^2 and M_2^2 are the Borel mass parameters in ρ and η channels. Note that in eq.4.21 we take into account $F_\eta^u = F_\eta^d$. The masses of η meson is close to the ρ meson mass. For this reason, it is natural to set $M_1^2 = M_2^2 = 2M^2$ from which it follows that $u_0 = 1/2$.

4.2 Numerical Analysis

In this section we present our numerical result on $g_{\rho\eta\gamma}$ coupling constant. From sum rules eq.4.20 we see that for estimating $g_{\rho\eta\gamma}$ coupling constant first of all

one needs to know the photon wave function $\psi(u)$. It was shown in [3, 38] that the photon wave function do not deviate remarkably from its asymptotic form which is given by $\psi(u) = 1$ [38, 37]. The values of the other constants appearing in the sum rules are: $m_\rho = 0.77\ GeV$, $m_\eta = 0.55\ GeV$, $f = 0.028 GeV^2$. Leptonic decay constant of ρ meson $f_\rho = 0.15\ GeV$ follows from experimental result of the $\rho \to e^+e^-$ decay, $\Gamma(\rho \to e^+e^-) = (6,85 \pm 0,11) KeV$[39]. More recent analysis shows that the coupling of η meson with the octet axial vector current is $f_\eta^8 = 0.159\ GeV$ [33] and this result we will use in our analysis.

In fig.4.1 we present the dependence of the coupling constant $g_{\rho\eta\gamma}$ on the Borel parameter M^2 at three different values of the continuum threshold: $s_0 = 1.4\ GeV^2$, $1.6\ GeV^2$, $1.8\ GeV^2$. Since the Borel mass M^2 is an auxiliary parameter and the physical quantities should not depend on it, we must look for the region where $g_{\rho\eta\gamma}$ is practically independent of M^2. We obtain that this condition is satisfied when $1\ GeV^2 \leq M^2 \leq 1.4\ GeV^2$. From this figure we also obtained that the variation of s_0 from $s_0 = 1.4\ GeV^2$ to $s_0 = 1.8\ GeV^2$ causes a change on the result on $g_{\rho\eta\gamma}$ of about 10%. Therefore one can say that the result $g_{\rho\eta\gamma}$ is insensitive to s_0 and M^2. Our final prediction on the coupling constant is

$$g_{\rho\eta\gamma} = (1.4 \pm 0.2), \tag{4.22}$$

where the error is attributed to the variation of s_0, M^2 and neglected twist three photon wave functions.

At the end we would like to compare our prediction on $g_{\rho\eta\gamma}$ with experimental result. The decay width of the $\rho \to \eta\gamma$ decay is given by

$$\Gamma(\rho \to \eta\gamma) = \frac{\alpha g_{\rho\eta\gamma}^2}{24} m_\rho (1 - m_\eta^2/m_\rho^2)^3. \tag{4.23}$$

The experimental value is $\Gamma(\rho^0 \to \eta\gamma) = (57 \pm 10)\ KeV$[39]. Using this value of $\Gamma(\rho^0 \to \eta\gamma)$, the $g_{\rho\eta\gamma}$ coupling constant is obtained from eq.4.23 as:

$$g_{\rho\eta\gamma} = (1.42 \pm 0.12), \tag{4.24}$$

which is very close to the sum rule prediction.

Finally we note that the coupling constant $g_{\omega\eta\gamma}$ can be obtained from $g_{\rho\eta\gamma}$ with the help of the relation $g_{\rho\eta\gamma} = 3g_{\omega\eta\gamma}$

In the following chapter, an important hadronic parameter of ρ meson, magnetic moment, will be calculated from $\rho(p, \varepsilon^r)$ meson to $\rho(p', \varepsilon^{r'})$ transition form factors under external magnetic field. LCSR has advantage over SVZ sum rules again because photon's momentum square will be zero.

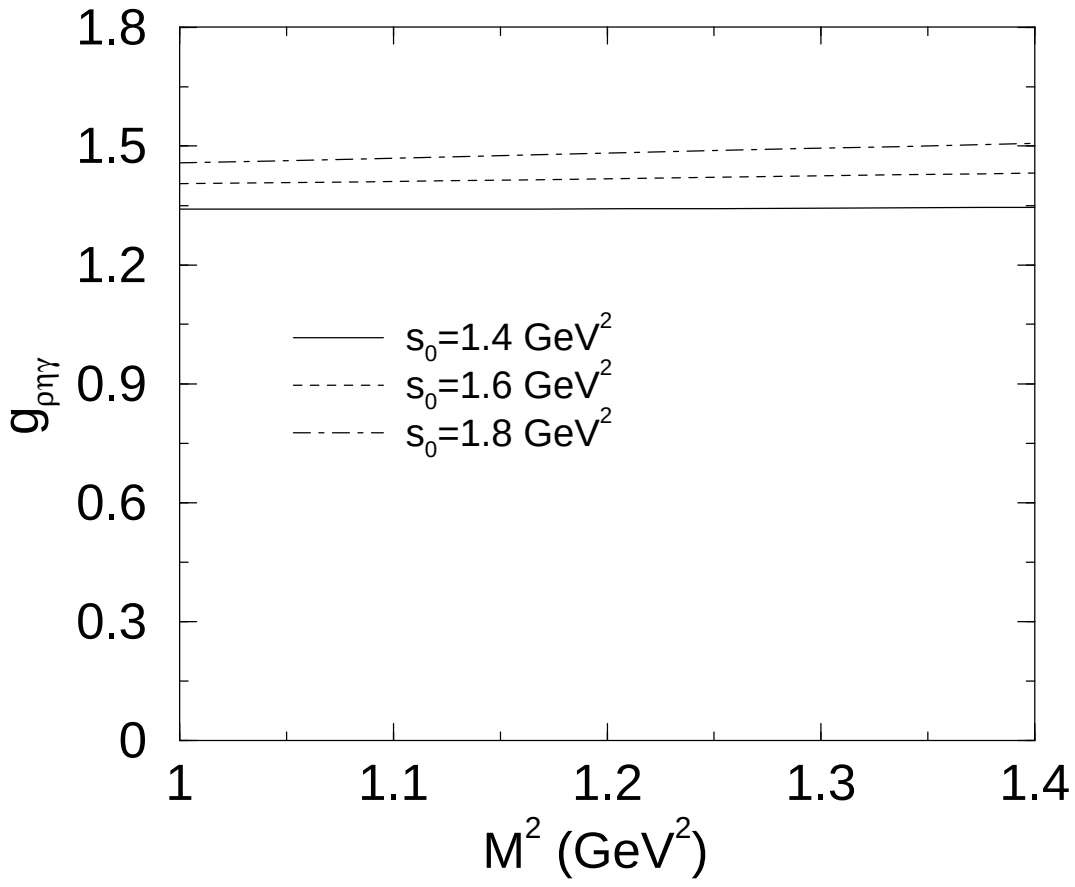

Figure 4.1: The dependence of the $g_{\rho\eta\gamma}$ coupling constant on the Borel parameter M^2 at three different values of the continuum threshold $s_0 = 1.4\ GeV^2$, $s_0 = 1.6\ GeV^2$ and $s_0 = 1.8\ GeV^2$.

CHAPTER 5

MAGNETIC MOMENT OF THE ρ MESON IN QCD LIGHT CONE SUM RULES

One of the important static characteristic of hadrons is their magnetic moment. Magnetic moments of nucleons are calculated in the framework of the QCD sum rule method in [40, 41], using the external field technique, and using the same approach magnetic moment of the ρ meson is calculated in [42].

Furthermore, it should be mentioned here that, in [43] form factors of the ρ meson are calculated at intermediate momentum transfer by using the three–point QCD sum rules method, and then extrapolating these form factors to $Q^2 = 0$ (this point lies outside the applicability region of the method).

In this chapter, we present an independent calculation of the magnetic moment of the ρ meson in QCD light cone sum rules method .

The LCSR is successfully applied to a variety of problems in hadron physics. Magnetic moment of the nucleon is first obtained in LCSR in [4]. Magnetic moments of the octet and decuplet baryons are calculated in [44] and [27], respectively, and [45] and [46] are recent articles where LCSR applied to calculation of baryonic magnetic moments.

In section 5.1, LCSR for the ρ meson magnetic moment is obtained. In section 5.2, our numerical results and a comparison with the results of the other approaches is presented.

5.1 LCSR for the ρ meson magnetic moment

In this section we calculate the ρ meson magnetic moment in LCSR. We consider the following correlator of two vector currents in the external electromagnetic field

$$\Pi_{\mu\nu}(p,q) = i \int d^4x e^{ipx} \left\langle 0 \left| \mathrm{T}\{j_\nu(x) j_\mu^\dagger(0)\} \right| 0 \right\rangle_\gamma , \tag{5.1}$$

where the subscript γ denotes the external electromagnetic field, $j_\nu(x) = \bar{u}\gamma_\nu d(x)$ is the vector current with the ρ meson quantum number.

Firstly, let us calculate the phenomenological part of the correlator. By inserting a complete set of states between the currents in eq.5.1 with quantum numbers of the ρ meson, we obtain the following representation of the correlator

$$\Pi_{\mu\nu} = \frac{\langle 0|j_\nu|\rho(p)\rangle \langle \rho(p)|\rho(p')\rangle_\gamma \langle \rho(p')|j_\mu^\dagger|0\rangle}{(p^2 - m_\rho^2)(p'^2 - m_\rho^2)} + \cdots \quad , \tag{5.2}$$

where $p' = p + q$, q is the photon momentum and $\cdots$ describe higher states and continuum contributions. The matrix element $\langle 0|j_\nu|\rho(p)\rangle$ is determined as

$$\langle 0|j_\nu|\rho(p)\rangle = \frac{m_\rho^2}{g_\rho} \varepsilon_\nu(p). \tag{5.3}$$

Assuming parity and time–reversal invariance, the electromagnetic vertex of the ρ meson can be written in terms of three form factors [47]

$$\begin{aligned} \langle \rho(p,\varepsilon^r)|\rho(\rho',\varepsilon^{r'})\rangle_\gamma = & -\varepsilon^\rho (\varepsilon^r)^\alpha (\varepsilon^{r'})^\beta \Big\{ G_1(Q^2) g_{\alpha\beta}(p+p')_\rho + G_2(Q^2)(q_\alpha g_{\rho\beta} - q_\beta g_{\rho\alpha}) \\ & - \frac{1}{2m_\rho^2} G_3(Q^2) q_\alpha q_\beta (p+p')_\rho \Big\}, \end{aligned} \tag{5.4}$$

where ε_ρ is the photon and $(\varepsilon^r)^\alpha$, $(\varepsilon^{r'})^\beta$ are the ρ meson vector polarizations. The Lorentz invariant form factors $G_i(Q^2)$ are related to the charge, magnetic

and quadropole form factors through the relations

$$\begin{aligned} F_C &= G_1 + \frac{2}{3}\eta F_{\mathcal{D}}, \\ F_M &= G_2, \\ F_{\mathcal{D}} &= G_1 - G_2 + (1+\eta)G_3, \end{aligned} \tag{5.5}$$

where $\eta = Q^2/4m_\rho^2$ is a kinematical factor. At zero momentum transfer, these form factors are proportional to the usual static quantities of charge, magnetic moment μ and quadropole moment $\mathcal{D}$:

$$\begin{aligned} eF_C(0) &= e, \\ eF_M(0) &= 2m_\rho\mu, \\ eF_{\mathcal{D}}(0) &= m_\rho^2\mathcal{D}. \end{aligned} \tag{5.6}$$

Using eqs.5.2)-5.4 and performing summation over polarizations of the ρ meson, for the phenomenological part of the correlator we get:

$$\begin{aligned} \Pi_{\mu\nu} = {} & \frac{m_\rho^4}{g_\rho^2}\varepsilon^\rho \frac{1}{(m_\rho^2 - p^2)[m_\rho^2 - (p+q)^2]} \\ & \times \left\{ G_1(Q^2)(p+p')_\rho \left[g_{\mu\nu} - \frac{p_\mu p_\nu}{m_\rho^2} - \frac{p'_\mu p'_\nu}{m_\rho^2} + \frac{p'_\mu p_\nu}{2m_\rho^4}(Q^2 + 2m_\rho^2) \right] + G_2(q^2) \left[q_\mu g_{\nu\rho} \right. \right. \\ & \left. - q_\nu g_{\mu\rho} - \frac{p_\nu}{m_\rho^2}\left(q_\mu p_\rho - \frac{1}{2}Q^2 g_{\mu\rho} \right) + \frac{p'_\mu}{m_\rho^2}\left(q_\nu p'_\rho + \frac{1}{2}Q^2 g_{\nu\rho} \right) - \frac{p'_\mu p_\nu p_\rho}{m_\rho^4}Q^2 \right] \\ & \left. - \frac{1}{2m_\rho^2} G_3(Q^2)(p+p')_\rho \left[q_\mu q_\nu - \frac{p_\nu q_\mu}{m_\rho^2}\frac{1}{2}Q^2 + \frac{p'_\mu q_\nu}{m_\rho^2}\frac{1}{2}Q^2 - \frac{p_\nu p'_\mu}{m_\rho^4}\frac{1}{4}(Q^2)^2 \right] \right\}, \end{aligned} \tag{5.7}$$

where $Q^2 = -q^2$. Throughout our analysis, only the values of the form factors at $Q^2 = 0$ are needed. Additionally, using $p' = p + q$ and $q\varepsilon = 0$, eq.5.7 can be

simplified and final answer for the phenomenological part can be written as

$$\Pi_{\mu\nu} = \frac{m_\rho^4}{g_\rho^2}\frac{\varepsilon^\rho}{[m_\rho^2-(p+q)^2]}\Bigg\{2p_\rho F_C(0)\left[g_{\mu\nu}-\frac{p_\mu p_\nu}{m_\rho^2}-\frac{p_\mu q_\nu}{m_\rho^2}\right] + F_M(0)\left[q_\mu g_{\nu\rho}-q_\nu g_{\mu\rho}+\frac{1}{m_\rho^2}p_\rho(p_\mu q_\nu - p_\nu q_\mu)\right]-\Big(F_C(0)+F_{\mathcal{D}}(0)\Big)\frac{p_\rho}{m_\rho^2}q_\nu q_\mu\Bigg\}. \tag{5.8}$$

In order to extract out the magnetic moment of the ρ meson from eq.5.8, we will chose the structure $(p\varepsilon)(p_\mu q_\nu - p_\nu q_\mu)$. Hence, the phenomenological part of the correlator for the above–mentioned structure can be written as

$$\Pi = \frac{m_\rho^2}{g_\rho^2}\frac{1}{(m_\rho^2-p^2)[m_\rho^2-(p+q)^2]}\,\mu, \tag{5.9}$$

where μ is the ρ meson magnetic moment in units of $e/2m_\rho$.

Our next task is calculation of the correlator in eq.5.1 from the QCD side. The correlator receives perturbative and nonperturbative contributions. The perturbative part corresponds to on–shell photon emission from virtual quarks and it is described by the triangle diagram (see fig.5.2). In order to calculate the nonperturbative contributions (see fig.5.2), we need the matrix elements of the nonlocal operators between the vacuum and the photon states, i.e., $\langle\gamma(q)|\bar{q}(x)\Gamma_i(0)|0\rangle$, where Γ is an arbitrary Dirac matrix. In our calculations we take into account twist–2, 3 and 4 photon wave functions (more about the photon wave functions, see [48]). In what follows we present definitions whose wave functions give con-

tribution only to the structure $(p\varepsilon)(p_\mu q_\nu - p_\nu q_\mu)$.

$$\langle\gamma(q)|\bar{q}(x)\gamma_\mu q(0)|0\rangle = ee_q f_{3\gamma}\left(\varepsilon_\mu - q_\mu\frac{\varepsilon x}{qx}\right)\int_0^1 due^{iuqx}\psi^{(v)}(u), \tag{5.10}$$

$$\langle\gamma(q)|\bar{q}(x)\gamma_\mu\gamma_5 q(0)|0\rangle = -\frac{1}{4}ee_q f_{3\gamma}\epsilon_{\mu\alpha\beta\rho}\varepsilon^\alpha q^\beta x^\rho\int_0^1 due^{iuqx}\psi^{(a)}(u), \tag{5.11}$$

$$\begin{aligned}\langle\gamma(q)|\bar{q}(x)\sigma_{\mu\nu}q(0)|0\rangle &= -iee_q\langle\bar{q}q\rangle(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu)\int_0^1 due^{iuqx}\left\{\chi\phi_\gamma(u) + \frac{x^2}{16}A(u)\right\},\\ &-iee_q\langle\bar{q}q\rangle\left[x_\nu\left(\varepsilon_\mu - q_\mu\frac{\varepsilon x}{qx}\right) - x_\mu\left(\varepsilon_\nu - q_\nu\frac{\varepsilon x}{qx}\right)\right]\int_0^1 due^{iuqx}h_\gamma(u),\end{aligned} \tag{5.12}$$

where $\phi_\gamma(u)$ is twist–2, $\psi^{(v)}(u)$ and $\psi^{(a)}(u)$ are twist–3, $A(u)$ and $h_\gamma(u)$ are twist–4 photon wave functions, respectively, and χ is the magnetic susceptibility. It should be noted here that, there are several other functions $T_i(\alpha_i)$ and $\widetilde{S}(\alpha_i)$ (for their definitions, see [48]) that also give contribution to the above–mentioned structure. But their contributions are proportional to the quark mass (in our case u and d quark masses) and therefore irrelevant in the massless quark case.

After some effort, we get the following expression for the correlator from QCD side in the x–representation

$$\begin{aligned}\Pi_{\mu\nu} &= e\int_0^1 du\int dxe^{i(p+uq)x}\varepsilon x(x_\mu q_\nu - x_\nu q_\mu)\\ &\times\left\{(e_d - e_u)\left[\frac{3}{4\pi^4x^6} - f_{3\gamma}\frac{\psi^{(a)}(u)}{8\pi^2x^4} + \frac{i}{2}f_{3\gamma}\frac{\psi^{(v)}(u)}{\pi^2(qx)x^4} - \frac{m_0^2}{384(qx)}\langle\bar{q}q\rangle^2 h_\gamma(u)\right]\right.\\ &\left. - \frac{(e_d + e_u)}{8(qx)}h_\gamma(u)\right\}.\end{aligned} \tag{5.13}$$

Using eq.5.13 and after performing Fourier transformation, the result for the structure $(p\varepsilon)(q_\nu p_\mu - q_\mu q_\nu)$ can be obtained. The sum rules for the ρ meson can be obtained after applying double Borel transformation on the variables p^2 and $(p+q)^2$, which suppresses the continuum and higher states contributions (about this procedure, see) and then matching both representations of the correlators.

Finally, for the above–mentioned structure we get the following sum rule for the

ρ meson magnetic moment

$$\mu = \frac{g_\rho^2}{m_\rho^2} e^{m_\rho^2/M^2}(e_u - e_d)\left\{ \frac{3}{8\pi^2} M^2 f_0(s_0/M^2) + \frac{f_{3\gamma}}{2}\psi^{(a)}(u_0) - 2f_{3\gamma}\Psi^{(v)}(u_0) \right\}, \tag{5.14}$$

where

$$\Psi^{(v)}(u) = \int_0^u \psi^{(v)}(v)dv, \tag{5.15}$$

and, the function

$$f_0(s_0/M^2) = 1 - e^{-s_0/M^2} \tag{5.16}$$

is used to subtract continuum contributions, and naturally, the Borel parameters M_1^2 and M_2^2 are set to be equal to each other, i.e., $M_1^2 = M_2^2 \equiv 2M^2$ since we are dealing with just a single meson, and hence

$$u_0 = \frac{M_1^2}{M_1^2 + M_2^2} = \frac{1}{2}. \tag{5.17}$$

Note that, the last two terms in eq.5.13 disappear after double Borel transformation is performed.

The main reason why we choose the structure $(p\varepsilon)(q_\nu p_\mu - q_\mu q_\nu)$ is that, the term proportional to the magnetic susceptibility χ does not give any contribution, and hence the main uncertainty coming from the definition of χ is absent in the sum rule.

5.2 Numerical analysis

In this section we present our numerical analysis on the ρ meson magnetic moment. It follows from eq.5.14 that, in order to perform further numerical analysis one needs to know the photon wave functions $\psi^{(a)}(u)$ and $\psi^{(v)}(u)$. The explicit

expressions of the functions are

$$\psi^{(v)}(u) = 10u(1-3u+2u^2) - \frac{15}{8}u(w_\gamma^A - 3w_\gamma^V)(1-10u+30u^2-35u^3+14u^4),$$
$$\psi^{(a)}(u) = \frac{5}{2}\left[1+\frac{9}{16}w_\gamma^V - \frac{3}{24}w_\gamma^A\right][1-(2u-1)^2][5(2u-1)^2-1]. \quad (5.18)$$

The values of the input parameters w_γ^V, w_γ^A and $f_{3\gamma}$ are given in [48] to have the values: $w_\gamma^V = (3.8 \pm 1.8)$, $w_\gamma^A = -(2.1 \pm 1.0)$ and $f_{3\gamma} = -(3.9 \pm 2.0) \times 10^{-3}\ GeV^{-2}$. The remaining input parameters are $m_\rho = 0.77\ GeV$ and $g_\rho^2/4\pi = 1.27$.

In fig.5.3 we present the dependence of the magnetic moment on M^2 at three different values of the continuum threshold: $s_0 = 1.5\ GeV^2$, $s_0 = 1.8\ GeV^2$ and $s_0 = 2.0\ GeV^2$. Note that, M^2 in the sum rule is an auxiliary parameter and the physical quantities are expected to be independent of it. Therefore, one must look for a region of M^2 for which the magnetic moment μ be practically independent of it. The lower limit of M^2 is determined by the requirement that terms $\sim M^{-2n}$ $(n > 1)$ remain subdominant. In other words, large power corrections must be absent in the sum rule. The upper bound of M^2 is determined by demanding that the contributions of the higher resonances and continuum are less than, for example, 30% of the total result. Our numerical calculation shows that these requirements are satisfied in the region $1.0\ GeV^2 \leq M^2 \leq 1.4\ GeV^2$ and magnetic moment in this region is practically independent of M^2. We also see from this figure that as s_0 varies from $s_0 = 1.5\ GeV^2$ to $s_0 = 2.0\ GeV^2$, the magnetic moment of the ρ meson changes by an amount of approximately 10%. Therefore we can conclude that the result seems to be almost insensitive to the change in s_0 and M^2 in the above–mentioned region. The final result for the magnetic moment of the ρ meson turns out to be

$$\mu = 2.3 \pm 0.5, \quad (5.19)$$

in units of $(e/2m_\rho)$, where the error can be attributed to the variations in s_0, M^2 and uncertainties in the values of $f_{3\gamma}$, w_γ^V and w_γ^A. At the end, we would like to present a comparison of our result on the ρ meson magnetic moment, with the

ones existing in literature. In the Dyson–Schwinger based models, the ρ meson magnetic moment is estimated to have the value $\mu = 2.69$ [50], $2.5 \leq \mu \leq 3$ [51] in units of $e/2m_\rho$. Covariant and Light front approaches with constituent quark model, both, predict $\mu = 2.23 \pm 0.13$ [52] and in light front formalism it is estimated to be $\mu = 1.83$ [53]. The magnetic moment of ρ meson was calculated long time ago in [54], by considering the low energy limit of the radiative amplitudes in conjunction with the amplitude calculated by the hard–pion technique and found that

$$\frac{16\pi^2\alpha^2 g_\rho^2}{m_\rho^2 \int ds \sigma_{e^+e^-\to n}} < \mu_\rho < 2. \tag{5.20}$$

The ρ meson magnetic moment was also calculated in lattice theory which predicted $\mu_\rho = 2.25(34)$ [55]. As has already been noted, the magnetic moment of the ρ meson in the framework of the traditional QCD sum rule in the presence of external field, is calculated in [42] and it is obtained that $\mu = 1.5 \pm 0.3$. Our result is closer to the predictions of the works [52] and [55]. In a later work [56], this work had been extended and electromagnetic form factors of ρ meson had been calculated.

Finally, we would like to discuss briefly the question how to measure the magnetic moment of ρ meson in experiments. At present, even upper bound for the magnetic and quadropole moments of ρ meson are absent. The very short lifetime does not allow the use of vector–meson–electron scattering or spin procession technique [57] to measure the above–mentioned quantities.

An alternative method for determination of the multipole moments of particles is based on soft photon emission off the hadrons was proposed in [58], since the photon carries information on higher multipoles of the emitting particles. The main idea of this work is that the amplitude for radiative process can be expressed as a power expansion in the photon energy w as follows

$$M = \frac{A}{w} + Bw^0 + Cw + \cdots \tag{5.21}$$

The electric charge contribute to the amplitude at order w^{-1} and the contribution

coming from magnetic moment is proportional to w^0. Therefore, by measuring the cross section or decay width of the radiative process and neglecting terms linear in w, one can determine the magnetic moments of charged particles.

In [58] and [59], the possibility of measuring the magnetic moment of the charged ρ meson in radiative production and decays of such mesons are mentioned and it is claimed that, combined angular and energy distributions of radiated photons is an efficient tool in measuring the magnetic moment of the charged ρ meson.

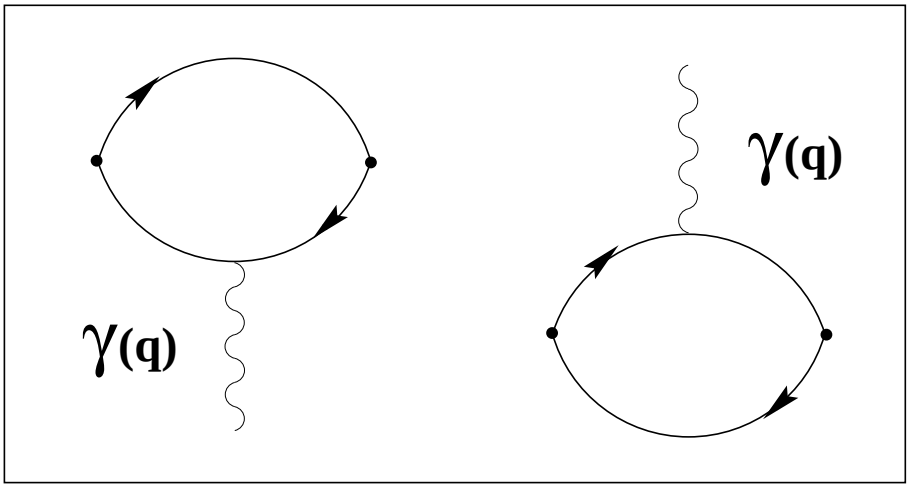

Figure 5.1: Diagrams describing perturbative contribution to the correlator in eq.5.1.

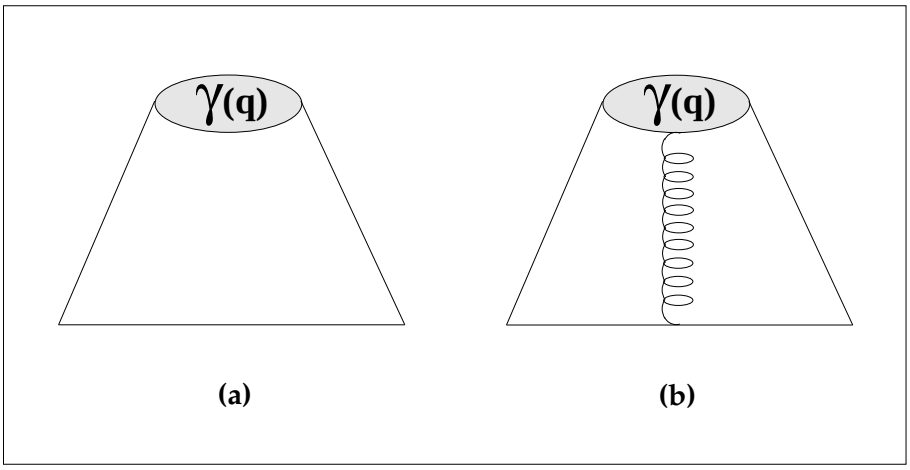

Figure 5.2: Diagrams describing nonperturbative contribution to the correlator in eq.5.1. Here, fig.5.2(a) corresponds to the leading order contribution and fig.5.2(b) corresponds to the gluon correction to the correlator in eq.5.1. In these figures, the wavy line represents gluon, and solid lines represent quark fields, respectively.

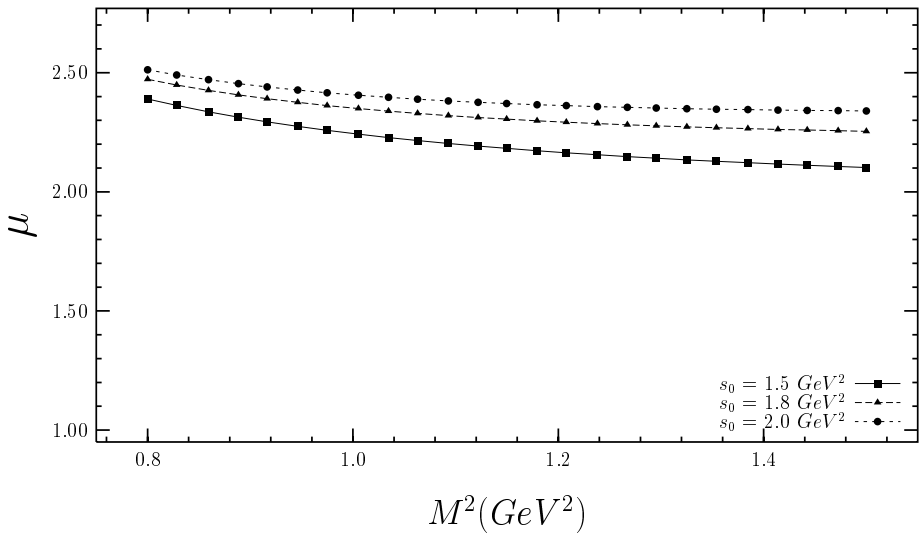

Figure 5.3: The dependence of the magnetic moment of the ρ meson on the Borel parameter M^2, at three different values of the continuum threshold; $s_0 = 1.5\ GeV^2$, $s_0 = 1.8\ GeV^2$ and $s_0 = 2.0\ GeV^2$.

CHAPTER 6

$\phi \to KK$ DECAY IN LIGHT CONE QCD

Light scalar mesons constitute a remarkable exception of the quark model systematization of mesons and their nature still need to be unambiguously established [60].

Particularly, the nature $f_0(980)$ meson is under debate. According to the naive $\bar{q}q$ picture and strong coupling with kaons, $f_0(980)$ can be interpreted as a pure $\bar{s}s$ state [61]–[63]. However, this interpretation does not explain mass degeneracy between $f_0(980)$ and isovector $a_0(980)$, which is interpreted as a $(\bar{u}u - \bar{d}d)/\sqrt{2}$ state. It is also interpreted as a four quark $\bar{q}q\bar{s}s$ [64] bound state of hadrons [65]–[67] and as a result of a process known as hadronic dressing [61, 68].

For understanding the content of the f_0 meson several alternatives have been suggested: For example, analysis of $\phi \to f_0\gamma$ decay [64]–[69] and investigation of the ratio $\Gamma(a_0 \to f_0\gamma)/\Gamma(\phi \to f_0\gamma)$ [66, 67] are believed to be the most promising ones for this purpose.

The $\phi \to f_0\gamma$ decay is a very efficient tool for this purpose, since the branching ratio is essentially dependent on the content of f_0. For example, if f_0 is a pure $\bar{s}s$ state, the branching ratio is $\sim 10^{-5}$, while if f_0 is composed of four quarks then the branching ratio is expected to be $\sim 10^{-4}$.

The strong coupling constants $g_{\phi K^+K^-}$ and $g_{f_0K^+K^-}$ are among the important hadronic parameters entering to the analysis involving ϕ and $f_0(980)$. Indeed, the kaon loop diagrams contributing $\phi \to f_0\gamma$ are expected to be in terms of $g_{f_0K^+K^-}$, as well as $g_{\phi K^+K^-}$. The coupling constant $g_{f_0K^+K^-}$ is studied in light

cone QCD sum rules [69] (more about light cone QCD sum rules and and its applications can be found in [18, 19]).

We will calculate the strong coupling constant $g_{\phi K^+K^-}$ in light cone QCD sum rules method in this chapter. It should be noted that this constant can be obtained from experimental data on ϕ meson decays. The goal in this chapter is twofold: Firstly, can we get new information about the quark content of ϕ meson comparing experimental data with theoretical results. Secondly, how does light cone QCD work for the asymmetric case, i.e., with different Borel mass parameters corresponding to different mass channels.

In section 6.1, we derive sum rules for the $g_{\phi K^+K^-}$ coupling constant. In section 6.2, we present our numerical results and conclusion.

6.1 Sum rules for $g_{\phi K^+K^-}$ coupling constant

In this section we calculate the strong coupling constant $g_{\phi K^+K^-}$ in light cone QCD sum rules. This coupling constant is defined by the following matrix element:

$$\langle K^-(q)\phi^0(p,\varepsilon)|K^+(p+q)\rangle, \tag{6.1}$$

where the momentum assignment is specified in brackets and ε_μ is the polarization vector of the ϕ meson. In order to calculate the strong coupling constant $g_{\phi K^+K^-}$ we consider the following correlator function

$$\Pi_{\mu\nu}(p,q) = i\int d^4x e^{ipx}\left\langle K(q)\left|\mathrm{T}\left\{J^\phi_\nu(x)\bar{J}^K_\mu(0)\right\}\right|0\right\rangle, \tag{6.2}$$

where the quark current $J^K_\mu = \bar{u}\gamma_\mu\gamma_5 s$ is the axial vector current and $J^\phi_\nu = \bar{s}\gamma_\mu s$ is the interpolating current for the ϕ meson.

The correlator function, in general, can be written in terms of the following five independent invariant functions

$$\Pi_{\mu\nu}(p,q) = \Pi_1 g_{\mu\nu} + \Pi_2 p_\mu p_\nu + \Pi_3 p_\mu q_\nu + \Pi_4 q_\mu p_\nu + \Pi_5 q_\mu q_\nu. \tag{6.3}$$

Therefore, our first problem is to choose the kinematical structure. For this aim, we consider the phenomenological part of the correlator function. This part can be written as

$$\Pi_{\mu\nu} = \sum \frac{\langle K^-(q)\phi^0(p)|K^+(p+q)\rangle\langle K^+(p+q)|J^K_\mu|0\rangle\langle 0|J^\phi_\nu|\phi(p)\rangle}{(p^2-m_\phi^2)[(p+q)^2-m_K^2]}. \tag{6.4}$$

The matrix elements entering eq.6.4 are defined as

$$\begin{aligned}\langle K^+(p+q)|J^K_\mu|0\rangle &= f_K(p+q)_\mu,\\ \langle 0|J^\phi_\nu|\phi(p)\rangle &= m_\phi f_\phi \varepsilon_\nu.\end{aligned} \tag{6.5}$$

Using eqs.6.4 and 6.5, we get for the physical part

$$\Pi_{\mu\nu} = \frac{g_{\phi K^+K^-} f_K m_\phi f_\phi}{(p^2-m_\phi^2)[(p+q)^2-m_K^2]}\,(p_\mu+q_\mu)\left(q_\nu+\frac{1}{2}p_\nu\right). \tag{6.6}$$

It follows from this expression that the only the structures $p_\mu q_\nu$, $q_\mu q_\nu$, $q_\mu p_\nu$ and $p_\mu p_\nu$ give contribution to the correlator function. In further analysis, we will choose the structure $p_\mu q_\nu$ from which the corresponding invariant structure

$$\Pi = \frac{g_{\phi K^+K^-} f_K m_\phi f_\phi}{(p^2-m_\phi^2)[(p+q)^2-m_K^2]}, \tag{6.7}$$

follows.

Our next task is the calculation of the correlator function from QCD side. This calculation can be carried out by using light cone operator product expansion method, in which we work with large momenta, i.e., $-p^2$ and $-(p+q)^2$ are both large. The correlator function, then, can be calculated as an expansion near to the light cone $x^2 \simeq 0$. The expansion involves matrix elements of the nonlocal operators between vacuum and the kaon states, i.e., in terms of kaon wave functions with increasing twist.

After lengthy calculations, we get the following expression for the invariant func-

tion which is proportional to the structure $p_\mu q_\nu$

$$\Pi(p^2,(p+q)^2) = if_K\int_0^1 du\Bigg\{\frac{4ug_2(u)}{\Delta^2}+\frac{\varphi_K(u)}{\Delta}-4\frac{g_1(u)+G_2(u)}{\Delta^2}\left(1+\frac{2m_s^2}{\Delta}\right)$$
$$+\frac{m_K^2}{3}\frac{\varphi_\sigma(u)}{\Delta^2}\Bigg\}+if_K\int_0^1 du\,u\int\mathcal{D}\alpha_i\frac{2}{\Delta_2^2}\Big[2\varphi_\perp(\alpha_i)+\varphi_\parallel+2\tilde{\varphi}_\perp(\alpha_i)\Big]$$
$$+if_K\int_0^1 du\int_0^1 d\alpha_3\int_0^{1-\alpha_3}d\alpha_1\frac{1}{\Delta_2^2}\Big[2\varphi_\perp(\alpha_i)-\varphi_\parallel+2\tilde{\varphi}_\perp(\alpha_i)-\tilde{\varphi}_\parallel(\alpha_i)\Big]$$
$$+2if_K\Bigg\{\int_0^1 du(u-1)\int_0^1 d\alpha_3\frac{4\hat{F}(\alpha_3)\{pq+m_K^2[1+\alpha_3(u-1)]\}}{\Delta_1^3}$$
$$+\int_0^1 du\int_0^1 d\alpha_3\int_0^{1-\alpha_3}d\alpha_1\frac{4F(\alpha_i)[pq+m_K^2(\alpha_1+u\alpha_3)]}{\Delta_2^3}\Bigg\}, \tag{6.8}$$

where

$$\begin{aligned}\Delta &= m_s^2-(p+qu)^2,\\ \Delta_1 &= m_s^2-[p+q(1+(u-1)\alpha_3]^2,\\ \Delta_2 &= m_s^2-[p+q(\alpha_1+u\alpha_3)]^2,\end{aligned} \tag{6.9}$$

and,

$$\hat{F}(\alpha_3) = -\int_0^{\alpha_3}dt\int_0^{1-t}d\alpha_1\Phi(\alpha_1,1-\alpha_1-t,t), \tag{6.10}$$

$$F(\alpha_i) = -\int_0^{\alpha_1}dt\Phi(t,1-\alpha_3-t,\alpha_3), \tag{6.11}$$

$$\Phi(\alpha_i) = \varphi_\parallel(\alpha_i)+\varphi_\perp(\alpha_i)+\tilde{\varphi}_\parallel(\alpha_i)+\tilde{\varphi}_\perp(\alpha_i). \tag{6.12}$$

The functions in eq.6.8 are defined as

$$\langle K(q)|\bar{u}(x)\gamma_\mu\gamma_5 s(0)|0\rangle = -if_K q_\mu \int_0^1 du e^{iuqx}[\varphi_K(u) + x^2 g_1(u)] + f_K\left(x_\mu - \frac{q_\mu x^2}{qx}\right)\int_0^1 du e^{iuqx} g_2(u), \quad (6.13)$$

$$\langle K(q)|\bar{u}(x)\sigma_{\mu\nu}\gamma_5 s(0)|0\rangle = i(q_\mu x_\nu - q_\nu x_\mu)\frac{f_K m_K^2}{6m_s}\int_0^1 du e^{iuqx}\varphi_\sigma(u), \quad (6.14)$$

and

$$G(u) = -\int_0^u g_2(u)dv. \quad (6.15)$$

The matrix elements involving quark–gluon field are determined as

$$\langle K(q)|\bar{u}(x)\gamma_\mu\gamma_5 g_s G_{\alpha\beta}(ux)s(0)|0\rangle = f_K\left[q_\beta\left(g_{\alpha\mu} - \frac{x_\alpha q_\mu}{qx}\right) - q_\beta\left(g_{\beta\mu} - \frac{x_\beta q_\mu}{qx}\right)\right]\int \mathcal{D}\alpha_i \varphi_\perp(\alpha_i) e^{iqx(\alpha_1+u\alpha_3)} + f_K\frac{q_\mu}{qx}(q_\alpha x_\beta - q_\beta x_\alpha)\int \mathcal{D}\alpha_i\varphi_\parallel(\alpha_i)e^{iqx(\alpha_1+u\alpha_3)} \quad (6.16)$$

$$\langle K(q)|\bar{u}(x)\gamma_\mu g_s \tilde{G}_{\alpha\beta}(ux)s(0)|0\rangle = if_K\left[q_\beta\left(g_{\alpha\mu} - \frac{x_\alpha q_\mu}{qx}\right) - q_\beta\left(g_{\beta\mu} - \frac{x_\beta q_\mu}{qx}\right)\right]\int \mathcal{D}\alpha_i \tilde{\varphi}_\perp(\alpha_i) e^{iqx(\alpha_1+u\alpha_3)}, + if_K\frac{q_\mu}{qx}(q_\alpha x_\beta - q_\beta x_\alpha)\int \mathcal{D}\alpha_i\tilde{\varphi}_\parallel(\alpha_i)e^{iqx(\alpha_1+u\alpha_3)}, \quad (6.17)$$

where $\tilde{G}_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\rho\sigma}G^{\rho\sigma}$, $\mathcal{D}\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1-\alpha_1-\alpha_2-\alpha_3)$.

The sum rule for $g_{\phi K^+K^-}$ is obtained by equating the phenomenological, eq.6.7, and theoretical, eq. 6.8, parts.

In order to suppress the contributions of the continuum and higher states, we perform double Borel transformation over the variables $-p^2$ and $-(p+q)^2$ on both sides of eqs.6.7 and 6.8, and obtain the following expression for the correlator

function

$$
\begin{aligned}
f_K m_\phi \quad f_\phi \quad g_{\phi KK} e^{-m_p hi^2/M_1^2} e^{-m_K^2/M_2^2} &= f_K e^{-m_0^2/M^2} \Bigg\{ M^2 \varphi_K(u_0) + 4u_0 g_2(u_0) \\
- \quad 4[g_1(u_0) + G_2(u_0)] + \frac{m_K^2}{3}\varphi_\sigma(u_0) &- 4\frac{m_s^2}{M^2}[g_1(u_0) + G_2(u_0)] \\
+ \quad \left(\int_0^{1-u_0} d\alpha_3 \int_{u_0-\alpha_3}^{u_0} d\alpha_1 + \int_{1-u_0}^{1} d\alpha_3 \int_{u_0-\alpha_3}^{1-\alpha_3} d\alpha_1 - \int_{u_0}^{1} d\alpha_3 \int_{u_0-\alpha_3}^{0} d\alpha_1 \right) & \\
\times \quad \left(2\frac{u_0-\alpha_1}{\alpha_3} \frac{2\varphi_\perp(\alpha_i) + \varphi_\parallel(\alpha_i) + 2\tilde{\varphi}_\perp(\alpha_i)}{\alpha_3} + \frac{\Phi(\alpha_i)}{\alpha_3} - \frac{2}{\alpha_3}\frac{dF(\alpha_i)}{d\alpha_1} \right) & \\
- \quad 2\int_{1-u_0}^{1} \frac{\hat{F}'(\alpha_3)}{\alpha_3} d\alpha_3 - 2\int_{1-u_0}^{1} d\alpha_3 \frac{F(1-\alpha_3, 0, \alpha_3)}{\alpha_3} \Bigg\}, & \qquad (6.18)
\end{aligned}
$$

where

$$
M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2} \ , \quad u_0 = \frac{M_1^2}{M_1^2 + M_2^2} \ , \quad \text{and,} \quad m_0^2 = m_s^2 + m_K^2 u_0(1-u_0). \qquad (6.19)
$$

Subtraction of the continuum and higher states is carried out by employing the quark–hadron duality, i.e., continuum contribution, which is represented in terms of the spectral density obtained from QCD side, by equating it to the one obtained from QCD side, but starting from some given threshold. The prescription for subtraction the contribution of the continuum in light cone version of the sum rule is proposed in [15] (see also [27],[49]). In [15] and in many works, the symmetric point $M_1^2 = M_2^2 = 2M^2$ (i.e., $u_0 = 1/2$) is considered, and then the continuum subtraction is implemented by means of the simple substitution

$$
e^{-m^2/M^2} \to e^{-m^2/M^2} - e^{-s_0/M^2}, \qquad (6.20)
$$

in the leading twist term (in our case leading twist term is the wave function $\varphi_K(u)$). But this prescription is not adequate in our case, where the Borel parameters and masses of different channels are not equal. In the present work we will follow the analysis given in [69], where the prescription for continuum subtraction through use of the Borel parameters with different masses in the respective channels is proposed, and properties of the wave functions are exploited.

Namely, the leading twist–2 wave function can be exploited as a power series

$$\varphi_K(u) = \sum_k b_k (1-u)^k, \tag{6.21}$$

in order to calculate its contribution in the duality region. Here we will neglect the continuum subtraction in the higher twist terms altogether, due to their small contribution to the theoretical part of the sum rules. Here, we will neglect the continuum subtraction in all higher twist terms, due to their small contribution to the theoretical part of the sum rules.

The final result for the $g_{\phi KK}$ coupling is given as

$$\begin{aligned}
g_{\phi KK} = & \frac{1}{m_\phi f_\phi} e^{m_\phi^2/M_1^2} e^{m_K^2/M_2^2} e^{-m_0^2/M^2} \Bigg\{ M^2 \sum_k b_k \left(\frac{M^2}{M_1^2}\right)^k \times \\
& \left[\; 1 - e^{-(s_0-m_s^2)/M^2} \sum_{i=0}^{k} \frac{1}{i!} \left(\frac{s_0-m_s^2}{M^2}\right)^i + \frac{m_K^2 M^2}{M_1^2 M_2^2} \frac{e^{-(s_0-m_s^2)/M^2}}{(k+1)!} \left(\frac{s_0-m_s^2}{M^2}\right)^{k+1} \right] \\
& + \; 4u_0 g_2(u_0) - 4[g_1(u_0) + G_2(u_0)] + \frac{m_K^2}{3}\varphi_\sigma(u_0) - 4\frac{m_s^2}{M^2}[g_1(u_0) + G_2(u_0)] \\
& + \; \left(\int_0^{1-u_0} d\alpha_3 \int_0^{u_0} d\alpha_1 + \int_{1-u_0}^{1} d\alpha_3 \int_{u_0-\alpha_3}^{1-\alpha_3} d\alpha_1 - \int_{u_0}^{1} d\alpha_3 \int_{u_0-\alpha_3}^{0} d\alpha_1 \right) \\
& \left[\; 2\frac{u_0-\alpha_1}{\alpha_3^2} \Big(2\varphi_\perp(\alpha_i) + \varphi_\parallel(\alpha_i) + 2\tilde{\varphi}_\perp(\alpha_i) \Big) + \frac{\Phi(\alpha_i)}{\alpha_3} - \frac{2}{\alpha_3}\frac{dF(\alpha_i)}{d\alpha_1} \right] \\
& - 2\int_{1-u_0}^{1} d\alpha_3 \frac{\hat{F}'(\alpha_3)}{\alpha_3} - 2\int_{1-u_0}^{1} d\alpha_3 \frac{F(1-\alpha_3,0,\alpha_3)}{\alpha_3} \Bigg\},
\end{aligned} \tag{6.22}$$

where s_0 is the smallest continuum contribution.

6.2 Numerical analysis

In this section we present our numerical calculation on $g_{\phi KK}$ coupling constant. It follows from eq.6.22 that the main input parameters are the kaon wave functions. The theoretical framework for their determination is based on an expansion in terms of the matrix elements of conformal operators [4]. In particular, for the

leading twist–2 wave function $\varphi_K(u)$ defined in eq.6.13, the expansion goes into Gegenbauer polynomials:

$$\varphi_K(u,\mu^2) = 6u(1-u)\left[1+\sum_{n=1}^{\infty} a_{2n}(\mu^2)C_{2n}^{3/2}(2u-1)\right], \tag{6.23}$$

where $a_2(1\ GeV) = 0.2$ [70].

Analogously φ_σ is defined as

$$\varphi_\sigma(u) = 6(u-u^2)\left[1+\left(5\eta_3 - \frac{1}{2}\eta_3 w_3 - \frac{7}{20}\rho^2 - \frac{3}{5}\rho^2\tilde{a}_2\right)C_2^{3/2}(2u-1)+..\right], \tag{6.24}$$

where at the $\mu = 1\ GeV$ scale $\eta_3 = 0.015$, $w_3 = -3$, $\tilde{a}_2 = 0.2$. Here, the factor $\rho = m_s^2/m_K^2$ takes into account the boson mass corrections (see [26]). The twist–4 wave functions $\varphi_\parallel(\alpha_i)$, $\varphi_\perp(\alpha_i)$, $\tilde{\varphi}_\parallel(\alpha_i)$ and $\tilde{\varphi}_\perp(\alpha_i)$, including the meson mass corrections are given as (see [4] and [26])

$$\begin{aligned}
\varphi_\perp(\alpha_i) &= 30m_K^2\alpha_3^2(2\alpha_1-1-\alpha_3)\left[h_{00}+h_{01}\alpha_3+\frac{h_{10}}{2}(5\alpha_3-3)+\cdots\right],\\
\varphi_\perp(\alpha_i) &= 120m_K^2\alpha_1(1-\alpha_1-\alpha_3)\alpha_3\left[a_{10}(1-2\alpha_1-\alpha_3)+\cdots\right],\\
\tilde{\varphi}_\perp(\alpha_i) &= -30m_K^2\alpha_3^2\Big\{h_{00}(1-\alpha_3)+h_{01}[\alpha_3(1-\alpha_3)+6\alpha_1(1-\alpha_1-\alpha_3)\\
&\quad + h_{10}[\alpha_3(1-\alpha_3)-\frac{3}{2}[\alpha_1^2+(1-\alpha_1-\alpha_3)^2]+\cdots\Big\},\\
\tilde{\varphi}_\perp(\alpha_i) &= 120m_K^2\alpha_1(1-\alpha_1-\alpha_3)\alpha_3[v_{00}+v_{10}(3\alpha_3-1)+\cdots\Big],
\end{aligned} \tag{6.25}$$

where

$$
\begin{aligned}
h_{00} &= v_{00} = -\frac{1}{3}\eta_4 \ , \\
h_{01} &= \frac{7}{4}\eta_4 w_4 - \frac{3}{20}a_2, \\
h_{10} &= \frac{7}{2}\eta_4 w_4 + \frac{3}{20}a_2, \\
v_{10} &= \frac{21}{8}\eta_4 w_4 \ , \\
a_{10} &= \frac{21}{8}\eta_4 w_4 - \frac{9}{20}a_2,
\end{aligned}
\tag{6.26}
$$

with $\eta_4(\mu = 1\ GeV) = 0.6$ and $w_4(\mu = 1\ GeV) = 0.2$ [4, 26].

The values of other input parameters appearing in eq.6.22 are: $m_s = 0.14\ GeV$ [71], $m_K = 0.4937\ GeV$, $m_\phi = 1.02\ GeV$. Leptonic decay constant of ϕ meson, $f_\phi = 0.234\ GeV$, follows from the experimental result of the $\phi \to \ell^+\ell^-$ decay [72]. The threshold s_0 which is varied around the value $s_0 = 1.1\ GeV^2$, is determined from the analysis of two-point function sum rules for f_K [73].

Having all input parameters, we now proceed by carrying out numerical calculation. The dependence of $g_{\phi KK}$ on Borel masses M_1^2 and M_2^2 at two fixed values of $s_0 = 1.1\ GeV^2$ and $s_0 = 1.2\ GeV^2$ is presented in figs.6.1 and 6.2, respectively. According to the QCD sum rule method ranges of the auxiliary Borel parameters M_i^2 should be found such that the result for $g_{\phi KK}$ be practically independent of them.

From these figures we see that, such regions indeed do exist. When M_1^2 and M_2^2 are varied in the regions $2\ GeV^2 \leq M_1^2 \leq 4\ GeV^2$ and $0.8\ GeV^2 \leq M_2^2 \leq 1.4\ GeV^2$, the result for $g_{\phi KK}$ seems to be independent of the Borel parameters. It should be noted here that, the result changes slightly when the continuum threshold is fixed to the value $s_0 = 1.2\ GeV^2$. The final result for $g_{\phi KK}$ is

$$
g_{\phi KK} = 4.9 \pm 0.8. \tag{6.27}
$$

At this point, let us discuss sources of the uncertainties. $SU(3)$ flavor symmetry breaking effects in kaon distribution amplitudes which we neglected, can play

essential role, since we can explore wide range of u and hence smoothing the effects of the shape of wave function. Additional uncertainty arises from the value of m_s. All these factors can cause an uncertainty about 5–10%. Moreover, the errors coming from the variations in the continuum threshold and Borel masses, change the result about 10%. If all these uncertainties are taken into account, the resulting error is about 20%, which is quoted in eq.6.27.

Finally, we would comment that, existing experimental results on $\phi \to KK$ decay predicts $g_{\phi KK} = 4.8$. So, obviously, we see that our result is quite close to the experimental value. Therefore, we conclude that the quark content of ϕ is $\bar{s}s$, and for channels with different masses and different Borel parameters, light cone QCD sum rules work quite well.

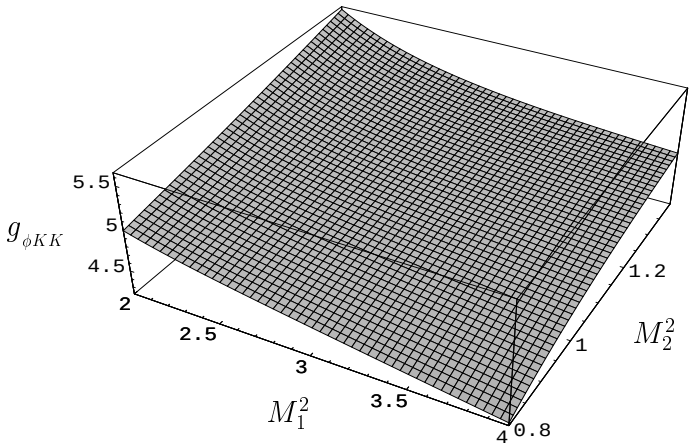

Figure 6.1: The dependence of the coupling constant $g_{\phi KK}$ on the Borel parameters M_1^2 and M_2^2, at the fixed value $s_0 = 1.1\ GeV^2$ of the continuum threshold.

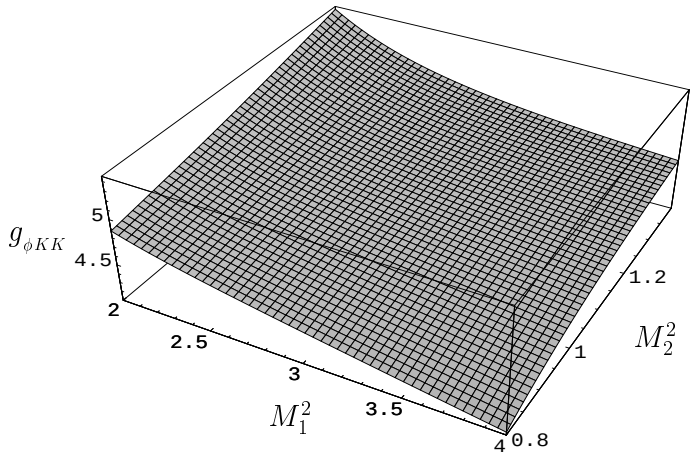

Figure 6.2: The same as figure above, but at the fixed value $s_0 = 1.2\ GeV^2$ of the continuum threshold

CHAPTER 7

Conclusion

Even though QCD is well established theory of strong interactions, physicists are having hard to time to do calculations with QCD . This is mostly because perturbation theory fails in most calculations when applied to QCD due to big coupling constant of the theory. Coupling constant of QCD unlike coupling constant of QED increases when energy of interaction decrease. Above energy scale $\Lambda_{QCD} \sim 0.50GeV$, coupling constant is less than 1 and one can assume perturbative approach holds. Unfortunately for hadrons, interactions below energies Λ_{QCD} are important and properties of hadrons cannot be estimated by using perturbative calculations. Physicists came up with non-perturbative methods to handle QCD calculations. There are lots of effective theories on QCD but most powerful ones are derived from QCD lagrangian. QCDSR is one of the theories that is derived from QCD lagrangian. Though errors in QCDSR will be a little worse with respect to other methods, it has wide application range and you can get analytical results. Even though QCDSR is successful for most problems when heavy meson to light meson transition form factors are considered, QCDSR may not work well. For example when B meson to M transitions considered, where M is a light meson, higher order matrix elements grow with mass of beauty quark making OPE non-convergent. To overcome these difficulties a variation of QCDSR, LCSR had been developed.

Semileptonic decays of B meson are important to get information on CKM matrix elements which are important for explaining CP-violation within framework of SM. For extracting information on b quark to u quark transition matrix element of CKM , form factors of $B \to \eta\ell\nu$ decay will be useful. For this aim, we analyze the correlation of currents $\bar{u}\gamma_\mu(1+\gamma_5)b$ and $\bar{b}(1+\gamma_5)u$ sandwiched between vacuum and η state and calculate the transition form factors for $B \to \eta\ell\nu$ decay using the LCSR. Calculations are done to twist 4 accuracy. Due convenience of selected

currents there is no twist 3 contribution. We extrapolate result and give it in the form:

$$f_i(q^2) = \frac{f_i(0)}{1 - a_F\left(\frac{q^2}{m_B^2}\right) + b_F\left(\frac{q^2}{m_B^2}\right)^2}\,, \tag{7.1}$$

where values of these parameters for the $B \to \eta$ transition form factors $f_+(f_-)$ are obtained as $a_F = 1.07 \pm 0.08$ ($a_F = 1.1 \pm 0.1$) and $b_F = 0.19 \pm 0.16$ ($b_F = 0.28 \pm 0.19$). Semileptonic decays of B meson investigated by later works too. In [30] B to η transition form factors calculated with different current correlator and they reproduced very similar result. This result had been used in a recent experimental result of BaBar collaboration [33] to enhance information on V_{ub}. V_{ub} has been estimated $3.6 \pm 0.2 \pm 0.1^{+0.6}_{-0.4}$ from form factors calculated with LCSR.

Radiative transition between vector (V) and pseudoscalar (P) mesons represent an important source of information on low energy hadron physics. The analysis of the radiative $V \to P\gamma$ decay with η and η' mesons in final state can provide insights to the long standing issue of the η and η' mixing. LCSR is also effective when used in radiative meson to meson transitions. So we consider correlation function of $J^\eta_\nu = \frac{1}{\sqrt{6}}(\overline{u}\gamma_\mu\gamma_5 u + \overline{d}\gamma_\mu\gamma_5 d - 2\overline{s}\gamma_\mu\gamma_5 s)$ and $J^\rho_\mu = \bar{u}\gamma_\mu d$ interpolating currents to acquire information on radiative ρ to η meson decay. We predict the coupling constant $g_{\rho\eta\gamma} = (1.4 \pm 0.2)$ using LCSR. It is very close to experimental value $g_{\rho\eta\gamma} = (1.42 \pm 0.12)$[39].

Magnetic moment of hadrons are important source of information on hadron physics since it is a parameter which is strongly related to internal structure of hadron. If $\rho(p, \varepsilon^r)$ meson to $\rho(p', \varepsilon^{r'})$ transition is considered under external magnetic field it is possible to calculate magnetic moment of ρ meson within LCSR. Form factors of this transition had been studied and from values of form factors when photon's momentum square is zero, magnetic moment of ρ meson can be calculated. QCDSR cannot be applied safely to this problem since OPE will be non-convergent when photon's momentum square approaches zero. The final result for the magnetic moment of the ρ meson turns out to be $\mu = 2.3 \pm 0.5$ in units of $(e/2m_\rho)$. The ρ meson magnetic moment was also calculated in lattice theory which predicted $\mu_\rho = 2.25(34)$ [55]. As has already been noted, the

magnetic moment of the ρ meson in the framework of the traditional QCD sum rule in the presence of external field, is calculated in [42] and it is obtained that $\mu = 1.5 \pm 0.3$. Our result is closer to the predictions of the works [52] and [55]. In a later work [56], this work had been extended and electromagnetic form factors of ρ meson had been calculated.

Quark contents of light scalar mesons are still under debate. For understanding the content of the f_0 meson, analysis of $\phi \to f_0\gamma$ decay [64]–[69] and investigation of the ratio $\Gamma(a_0 \to f_0\gamma)/\Gamma(\phi \to f_0\gamma)$ [66, 67] are believed to be promising since branching ratio of $\phi \to f_0\gamma$ decay is strongly depend on quark content of f_0. For analyzing $\phi \to f_0\gamma$ decay, $g_{\phi KK}$ is an important hadronic parameter because of the kaon loops contributing this decay. For this purpose, we had calculated $g_{\phi KK}$ in light cone QCD, our final result is $g_{\phi KK} = 4.9 \pm 0.8$. We use $\bar{u}\gamma_\mu\gamma_5 s$ for K and $\bar{s}\gamma_\mu s$ for ϕ as interpolating current. Existing experimental results on $\phi \to KK$ decay predicts $g_{\phi KK} = 4.8$. So obviously, we see that our result is quite close to the experimental value. Therefore we conclude that the quark content of ϕ is $\bar{s}s$, and for channels with different masses and different Borel parameters, light cone QCD sum rules work quite well.

To sum up we apply light cone QCD sum rules variety of problems in meson physics successfully. Obtained results in this thesis can shed light into forthcoming experiments.

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Appendix A

A-1

In this appendix, the derivations of the explicit forms of the functions, I_i, and J_i, appearing in Eq. (3.18) are presented and the method to calculate the continuum subtractions is demonstrated. For this purpose, consider a general term of the correlation function :

$$\Pi(p,q) = \int d^4x e^{iqx} \int_0^1 du \int \mathcal{D}\alpha_i e^{i(\alpha_2+u\alpha_3)px} \varphi(\alpha_i) f(u) \frac{K_\nu(m_b\sqrt{-x^2})}{(\sqrt{-x^2})^\nu}, \quad \text{(A-1)}$$

where $\mathcal{D}\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1-\alpha_1-\alpha_2-\alpha_3)$ After switching to Euclidean space and carrying out the x integration, one obtains:

$$\Pi(p,q) = -i\frac{2\pi^2}{m_b^2} \int du \int \mathcal{D}\alpha_i dt t^{1-\nu} \varphi(\alpha_i) f(u) e^{-\frac{t}{2m_b}(Q^2+m_b^2)}, \quad \text{(A-2)}$$

where $Q^2 = (q+kp)^2 = q^2\bar{k} - p^2 k\bar{k} + k(q+p)^2$ and $k = \alpha_2 + u\alpha_3$, $\bar{k} = 1-k$, and all the appearing momenta are Euclidean. In order to obtain Eq. (A-2), the following representation of the Bessel function $K_\nu(x)$ is used:

$$\frac{K_\nu(m_b\sqrt{x_E^2})}{(\sqrt{x_E^2})^\nu} = \frac{1}{2}\int_0^\infty \frac{dt}{t^{\nu+1}} \exp\left[-\frac{m_b}{2}\left(t + \frac{x_E^2}{t}\right)\right], \quad \text{(A-3)}$$

where x_E is the Euclidean position vector. Carrying out the Borel transformation with respect to $(p+q)^2$ using the identity:

$$\mathcal{B}e^{-\alpha(p+q)^2} = \delta\left(\frac{1}{M^2} - \alpha\right), \quad \text{(A-4)}$$

where $\mathcal{B}$ stands for the Borel transformation and M^2 is the Borel parameter and carrying out the t integral, one obtains:

$$\Pi(M^2) = -\frac{4i\pi^2}{m_b}\int du \int \mathcal{D}\alpha_i \left(\frac{2m_b}{kM^2}\right)^{1-\nu} e^{-s(k)}\frac{\varphi(\alpha_i)f(u)}{k}, \tag{A-5}$$

where

$$s(k) = \frac{m_b^2 - q^2\bar{k} + p^2 k\bar{k}}{k}, \tag{A-6}$$

where we have switched back to Minkowskian spacetime. Note that one can write the factors $\frac{2m_b}{kM^2}$ is the integrand as a differential operator acting on the exponential in the integrand and hence:

$$\Pi(M^2) = -\frac{4i\pi^2}{m_b}\left(-\frac{\partial}{\partial m_b}\right)^{1-\nu}\int du\mathcal{D}\alpha_i e^{-s(k)}\frac{\varphi(\alpha_i)f(u)}{k}. \tag{A-7}$$

Note that

$$e^{-\frac{s(k)}{M^2}} = \int_0^\infty ds e^{-\frac{s}{M^2}}\delta(s - s(k)), \tag{A-8}$$

which is nothing but the spectral representation of the exponential. The contributions of the higher states and the continuum are subtracted by replacing a cutoff, s_0, instead of the infinity as the upper limit of the spectral integral. Hence the effect of subtraction of the continuum and the higher states is to restrict the integration region in Eq. (A-7) to the regions of k for which $0 \leq s(k) \leq s_0$ which implies that $\delta \leq k \leq 1$ where

$$\delta = \frac{m_\eta^2 + q^2 - s_0 + \sqrt{(m_\eta^2 + q^2 - s_0)^2 - 4m_\eta^2(q^2 - m_b^2)}}{2m_\eta^2}, \tag{A-9}$$

where $p^2 = m_\eta^2$. Hence the contributions of the term given in Eq. (A-1) to the sum rules after Borel transformation and the continuum is subtracted is given

by:

$$\Pi(M^2) = -\frac{4i\pi^2}{m_b}\left(-\frac{\partial}{\partial m_b}\right)^{1-\nu}\int du\mathcal{D}\alpha_i \frac{\varphi(\alpha_i)f(u)}{k}e^{-\frac{s(k)}{M^2}}, \tag{A-10}$$

where the integration region is determined by the conditions:

$$\begin{aligned} \delta \leq k \leq 1, \\ 0 \leq u\,,\ \alpha_i \leq 1, \end{aligned} \tag{A-11}$$

and the Dirac's delta function in the definition of $\mathcal{D}\alpha_i$ fixes $\sum_i \alpha_i = 1$.

In order to obtain the contributions to the sum rules from terms containing additional powers of x_μ one can use the trick of replacing x_μ by differentiation with respect to q_μ. For terms containing a factor of px in the denominator, one can use the following trick: in order not to have any singularity at $px = 0$, the integral of these wave functions in the absence of the exponential should cancel. Hence, for these terms only, one can write:

$$e^{i\alpha px} \to e^{i\alpha px} - 1 = ipx\int_0^\alpha dk e^{ikpx} \tag{A-12}$$

and the rest of the calculation is similar to the presented one. Note that the subtracted 1 does not contribute.

When the contributions of the continuum is subtracted and the Borel transformation is carried out for the other terms, the following functions which appear

in Eq. (3.18) are encountered:

$$I_1^n(\varphi) = \left(-\frac{\partial}{\partial m_b}\right)^n \left[\int_0^{1-\delta} d\alpha_1 \int_0^{1-\delta-\alpha_1} d\alpha_3 \int_0^1 du + \int_0^{1-\delta} d\alpha_1 \int_{1-\delta-\alpha_1}^{1-\alpha_1} \int_{\frac{\delta-1+\alpha_1+\alpha_3}{\alpha_3}}^1 du\right] \frac{\varphi(\alpha_1, 1-\alpha_1-\alpha_3, \alpha_3)}{1-\alpha_1-\alpha_3+u\alpha_3} e^{-\frac{s(1-\alpha_1-\alpha_3+u\alpha_3)}{M^2}}, \tag{A-13}$$

$$I_2^n(\varphi) = \left(-\frac{\partial}{\partial m_b}\right)^n \left[\int_0^{1-\delta} d\alpha_1 \int_0^{1-\delta-\alpha_1} d\alpha_3 \int_0^1 du \int_\delta^{1-\alpha 1-\alpha_3+u\alpha_3} dk + \int_0^{1-\delta} d\alpha_1 \int_{1-\delta-\alpha_1}^{1-\alpha_1} d\alpha_3 \int_{\frac{\delta-1+\alpha_1+\alpha_3}{\alpha_3}}^1 du \int_\delta^{1-\alpha_1-\alpha_3+u\alpha_3} dk\right] \frac{\varphi(\alpha_1, 1-\alpha_1-\alpha_3, \alpha_3)}{1-\alpha_1-\alpha_3+u\alpha_3} e^{-\frac{s(1-\alpha_1-\alpha_3+u\alpha_3)}{M^2}}, \tag{A-14}$$

$$\tilde{I}_2^n(\varphi) = \left(-\frac{\partial}{\partial m_b}\right)^n \left[\int_0^{1-\delta} d\alpha_1 \int_0^{1-\delta-\alpha_1} d\alpha_3 \int_0^1 du \int_\delta^{1-\alpha 1-\alpha_3+u\alpha_3} dk + \int_0^{1-\delta} d\alpha_1 \int_{1-\delta-\alpha_1}^{1-\alpha_1} d\alpha_3 \int_{\frac{\delta-1+\alpha_1+\alpha_3}{\alpha_3}}^1 du \int_\delta^{1-\alpha_1-\alpha_3+u\alpha_3} dk\right] \varphi(\alpha_1, 1-\alpha_1-\alpha_3, \alpha_3) e^{-\frac{s(1-\alpha_1-\alpha_3+u\alpha_3)}{M^2}}, \tag{A-15}$$

$$J_1^n(\varphi) = \left(-\frac{\partial}{\partial m_b}\right)^n \int_\delta^1 du \frac{\varphi(u)}{u} e^{-\frac{s(u)}{M^2}}, \tag{A-16}$$

$$J_2^n(\varphi) = \left(-\frac{\partial}{\partial m_b}\right)^n \int_\delta^1 du \int_\delta^u dk \frac{\varphi(u)}{k} e^{-\frac{s(k)}{M^2}}, \tag{A-17}$$

$$\tilde{J}_2^n(\varphi) = \left(-\frac{\partial}{\partial m_b}\right)^n \int_\delta^1 du \int_\delta^u dk \varphi(u) e^{-\frac{s(k)}{M^2}}. \tag{A-18}$$

A-2 QCD

In Standart Model (SM) Quantum Chromodynamics (QCD) is the theory of strong interaction. It is the interaction between quarks and gluons and with this interaction quarks and gluons form baryons like proton, neutron etc. and mesons like pion, kaon etc. QCD is SU(3) non-abelian gauge theory and its lagrangian is given as follows:

$$\mathfrak{L} = -\frac{1}{4} G^{(a)}_{\mu\nu} G^{(a)\mu\nu} + \sum_f \overline{q_f}(i \not{D} - m_f) q_f, \tag{A-19}$$

where G is the gauge field strength tensor with color index is given in parenthesis, and q is the fermion fields whose mass is m_f, f is running over quark flavors(u,d,...), $\not{D}$ is the covariant derivative. $G^{(a)}_{\mu\nu}$ and $\not{D}$ is defined in terms of vector potential $A^{(a)}_{\mu}$ as:

$$G^{(a)}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu, \tag{A-20}$$

$$\not{D} = \gamma^i (\partial_i + i g A^a_i \frac{\lambda_a}{2}), \tag{A-21}$$

g is gauge coupling parameter, γ^i are Dirac matrixes, λ_a are generators of SU(3) and f^{abc} is structure constants of SU(3). f^{abc} is totally antisymmetric and related to λ_a through:

$$[\lambda_a, \lambda_b] = 2i f_{abc} \lambda_c. \tag{A-22}$$

For SU(3) infinite number of irreducible representation can be selected. In 3 dimensional fundamental group representation of SU(3), group generators are

given as:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \text{(A-23)}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \text{(A-24)}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} \end{pmatrix}, \quad \text{(A-25)}$$

where in this case structure constants are:

$$\begin{aligned} f_{123} &= 1, \\ f_{147} = f_{246} = f_{257} = f_{345} &= 1/2, \\ f_{156} = f_{367} &= -1/2, \\ f_{458} = f_{678} &= \sqrt{3}/2. \end{aligned} \quad \text{(A-26)}$$

Rest of them can be acquired by using antisymmetry property. Equations of motion for QCD are:

$$(i \not{D} - m_f) q_f = 0, \quad \text{(A-27)}$$

$$D^\mu F^{(a)}_{\mu\nu} = g \sum_f \overline{q_f} \frac{\lambda_a}{2} \gamma_\nu q_f. \quad \text{(A-28)}$$

Despite its simple looking form in closed indices if it is written in open form it can be seen they are quite complex, nonlinear and hard to solve equations.

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