TRACING THE FOOTSTEPS OF THE YOUNG LEIBNIZ IN THE LABYRINTH OF THE CONTINUUM

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ABSTRACT

TRACING THE FOOTSTEPS
OF THE YOUNG LEIBNIZ IN
THE LABYRINTH OF THE CONTINUUM

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This study is an attempt to explicate Gottfried Wilhelm von Leibniz's search for a way out of the labyrinth of the continuum in his early years of philosophizing. The main motive of the study is the belief that it would be worthwhile to see how Leibniz initially goes into the labyrinth and comes across with the riddles contained in it. Accordingly, this thesis is intended to discuss what the problem of the composition of the continuum is for the young Leibniz, which concepts and metaphysical problems are associated with the labyrinth, and what particular difficulties challenge Leibniz in his struggle. More importantly, the study seeks to delineate how Leibniz responds to these difficulties, what kinds of solutions he suggests, and how and why he changes his mind and offers different accounts concerning the composition of the continuum in his early writings. In this search for a way out of the labyrinth, some of the early writings of Leibniz written between 1666 and 1675 were studied with a particular emphasis on those directly related with the labyrinth of the continuum. During the study, the differences and transitions between geometrical, physical, and metaphysical accounts concerning the problem of the composition of the continuum were examined with a special focus on the bridging role of ‘motion’ and the notion of ‘*conatus*.’

Keywords: Leibniz, continuum, continuous, composition of the continuum, cohesion, motion, *conatus*, division, divisibility, indivisible, infinitesimal, substance, infinite, geometrical, physical, metaphysical.

ÖZ

GENÇ LEIBNIZ'İN
SÜREKLİLİK LABİRENTİNDEKİ
ADIMLARININ İZİNDE

Ebetürk, Emre

Yüksek Lisans, Felsefe Bölümü

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Bu çalışma, Gottfried Wilhelm von Leibniz'in erken dönem felsefesinde süreklilik labirentinden çıkış çabalarını anlatmayı amaçlamaktadır. Çalışmanın temel güdüsü, Leibniz'in labirente adım atışının ve bununla birlikte labirentin sorunlarıyla yüzleşmesinin incelenmeye değer olduğuna olan inançtır. Temel alınan bu görüş ışığında, sürekliliğin oluşumu probleminin genç Leibniz için nasıl bir anlam taşıdığı, labirentin hangi kavramlar ve metafizik problemlerle ilişkili olduğu ve Leibniz'i labirentten çıkış çabasında zorlayan güçlüklerin neler olduğu tartışılmıştır. Daha da önemlisi, Leibniz'in ilk çalışmalarında bu güçlüklere ne tür yanıtlar verdiği, ne gibi çözümler ürettiği ve neden ve nasıl fikir değişikliklerine gidip sürekliliğin oluşumuna dair farklı kavrayışlar geliştirdiği aktarılmaya ve açıklanmaya çalışılmıştır. Leibniz'in labirentten çıkış yolu bulma çabasını incelerken, 1666-1675 yılları arasında yazmış olduğu kimi yazılar incelenmiş, süreklilik labirentiyle doğrudan ilişkili olan yazılar üzerinde özellikle durulmuştur. Çalışma boyunca, Leibniz'in sürekliliğin oluşumu problemine yönelik vermiş olduğu geometrik, fiziksel ve metafizik açıklamalar arasındaki farklar ve geçişler irdelenmiş ve bununla ilgili olarak hareketin ve '*conatus*' kavramının birleştirici rolüne özellikle odaklanılmıştır.

Anahtar Sözcükler: Leibniz, süreklilik, sürekli, sürekliliğin oluşumu, kohezyon, hareket, *conatus*, bölünme, bölünebilirlik, bölünemez, sonsuz küçük, töz, sonsuz, geometrik, fiziksel, metafizik.

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There are many other names among my instructors, colleagues, and friends, whose support and patience I will always remember. I know that I cannot adequately express my appreciation to all my family including İrem for always standing by me. Finally, I wish hereby to thank those who somehow contributed to the Department of Philosophy in METU, the magnificent society that provided me with the opportunity to write this thesis.

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ABBREVIATIONS

(FOR THE EDITIONS OF LEIBNIZ'S WRITINGS)

L Loemker, Leroy E., trans. and ed. 1969. *Gottfried Wilhelm Leibniz: Philosophical Papers and Letters*. 2nd ed. Boston: Reidel.

LLC Arthur, Richard T. W., trans. and ed. 2001. *The Labyrinth of the Continuum: Writings on the Continuum Problem, 1672-1686*. New Haven: Yale University Press.

DSR Parkinson, George H. R., trans. and ed. 1992. *De Summa Rerum: Metaphysical Papers, 1675-1676*. New Haven: Yale University Press.

P Parkinson, George H.R., trans. and ed. Morris, Mary trans. 1973. *Gottfried Wilhelm Leibniz: Philosophical Writings*. London: J. M. Dent & Sons.

CHAPTER 1

INTRODUCTION:
THE LABYRINTH OF THE CONTINUUM

It is customary among Leibniz scholars who are deeply concerned with his struggle with the problem of the continuum to mention Leibniz's reference to the two labyrinths. Being a postulant who dares to examine how Leibniz tries to deal with the labyrinth of the continuum in the early years of his philosophy, I will remain loyal to the tradition in this respect, and begin with a quote from Leibniz's "On Freedom." "There are two labyrinths of the human mind," Leibniz states, "one concerns the composition of the continuum, and the other the nature of freedom" (P, 107). Leibniz's deliberations on the nature of freedom are relatively more recognized given many of his famous works on the subject such as the *Theodicy*. However, the other labyrinth, namely the labyrinth of the continuum, is no less essential to Leibniz's metaphysics. Indeed, what awaits the one who is able to make his way out of this labyrinth, is the door opening to a sound philosophy: "… no one will arrive at a truly solid metaphysics who has not passed through that labyrinth [of the continuum]" (Leibniz quoted in Arthur 2001, xxii).

Beginning with the very first years of his interest in philosophy, Leibniz became gradually aware of the dilemmas that come up with any deliberation on continuity. The more Leibniz encountered the riddles and the intricate constitution of the labyrinth, the more explicitly he acknowledged the importance of the problem. Hence Leibniz thought that the problem of the labyrinth is of profound importance which he should work out before establishing a substantial philosophy.

The philosophy of Leibniz is mostly coupled with his brief work, the *Monadology*. This work is considered to be one of the most substantial exhibitions of Leibniz's mature system. However, the *Monadology*, and its fundamental constituents, i.e., monads, are fairly an outcome of years devoted to the problem of the composition of the continuum. This claim is also articulated by Bertrand Russell, who indicated that whatever monads are can only be exposed after giving an account of the continuum, "since Leibniz professes to deduce the existence and nature of monads largely from the need of explaining the continuum (Russell, 108).

In this study, I will try to explicate Leibniz's search for a way out of the labyrinth of the continuum in his early years of philosophizing. Although in the course of this thesis, some ideas concerning the mature philosophy of Leibniz will be hinted at, my primary purpose is not to follow the path that leads to monads. In other words, the reason why this topic is undertaken in this thesis can be explained neither with Leibniz's initial purposes, nor with his outcomes, his purported failure or success on the issue. I consider Leibniz's early efforts in the labyrinth to be worthy of examination on their own. I believe that it would be worthwhile to see how Leibniz initially goes into the labyrinth and comes across with the riddles contained in it. Thereby, I intend to see what indeed the problem of the composition of the continuum is for the young Leibniz; which concepts and metaphysical problems are associated with the labyrinth; and what particular difficulties challenge Leibniz in his struggle. More importantly, I will try to delineate how Leibniz responds to these difficulties; what kind of solutions he suggests; and how and why Leibniz changes his mind and offers different accounts concerning the composition of the continuum within the relatively narrow span of time. On the other hand, in general, I also aim to draw attention to the fact that prospective studies on the concept of continuity, particularly with a Leibnizian support, can provide various openings for the fundamental problems of philosophy.

Before seeking Leibniz's answers to the questions yielded by the labyrinth, it is necessary to discuss why the problem of the continuum is so important a philosophical topic. In order to facilitate this discussion, it would be better to begin with what terms such as 'continuity' or the 'continuum' come to mean, and with which philosophical concepts and problems 'continuity' is related with. I believe that this preliminary discussion by itself will provide a basic idea of why the labyrinth is a perfect and irreplaceable analogy for the problem of the continuum.

1.1 A Bird's-Eye View of the Labyrinth

1.1.1 The Continuum and its Associates

Continuity, in general, is one of the most significant and intricate concepts of philosophy, with subtle yet deep connections to numerous other philosophical concepts and problems. The term 'continuity' derives from a Latin word '*continere*' meaning "to hold together." In its simplest sense, "continuous" means uninterrupted. A continuous entity is called a *continuum*. What is not continuous is usually called *discrete* or discontinuous.

If the continuum is not interrupted, then on the face of it, it can be argued that a continuum contains no gaps; accordingly, in its most general form, one philosophical opposition connected with the problem of the continuum is "being vs. nothing." This opposition can be put into words with the question whether a gap means "nothing", and whether there are gaps (or there is "nothing") in nature, or more particularly in the nature of continuous things.

Again usually, 'continuum' denotes unity (or plenum), while 'discreteness' denotes plurality, which implies a connection with another fundamental opposition: "one vs. many." Following these aspects of continuity, a possible deliberation concerns the question of whether a continuum has to be *undivided*, or of whether it is possible for a continuum to have parts. On the other hand, it is usually held that any continuum can be *divisible* to *infinity*, which means at first sight that it cannot be composed of *indivisible* units. And in fact, this is one of the main reasons behind the argument which excludes the possibility of *atom*s in their usual sense. Hence a further opposition emerges, i.e., *infinite divisibility* vs. *indivisible units*. Another crucial debate that arises here is whether the continuum is *actually infinitely divided* or it is only *potentially infinitely divisible*, which means that the problem embraces the "actual vs. potential" dispute, as well.

The question of whether there are gaps in the universe is mostly associated with continuity in space. However, a similar question may be raised with respect to continuity of time. These are essentially the questions concerning the nature of the spatial and the temporal continua, respectively. Are space and time composed of atomic units or are they continuous wholes? More specifically, is space composed of geometrical points, and is time composed of moments (or instants)? If they are, how does this composition happen? If they are not, what are the spatial and the temporal continua composed of? Are they merely geometrical or ideal continua, or are they real continua composed of actual entities?

Closely related with these questions is one of the most discussed problems of physics and philosophy from the Ancients to the most contemporary space-time debates: the continuity of motion. There are likewise several questions to be raised with regard to the continuity of motion. If there are no gaps in space and time, does it come to mean that a particular motion is continuous (or smooth)? Conversely, if space and time are discrete, or atomic, does it imply that motion is also discrete or cinematographic? Are there real rests in motion? Does the possibility of rests contradict the continuity of motion? What is a continuous motion composed of?

Even if one accepts that the material or physical universe is continuous, there remain various questions to be asked with respect to thoughts and perceptions.

Minds seem to be discontinuous in the sense that the content of one mind is not perfectly communicable to others. Are thoughts and perceptions in a mind continuous with one another? If they are, how does this continuity take place? What are the relationships between the material universe and minds (or perceptions)? Are they continuous or discontinuous? Actually these questions are the upshots of another opposition which can be summarized as the "physical vs. mental."

All these questions concerning the problem of the continuum are still formulated at a level that is simple and even crude. While following Leibniz's path in the labyrinth, more specific and elegant questions will be dealt with by means of Leibniz's fine analysis of the problem. Nevertheless, even at this rudimentary point, I hope I could have been able to hint at the labyrinthine nature of the problem, through the various dilemmas and concepts which are to comprise the network of the paths and channels inside the labyrinth.

On the other hand, for now, it is important to acknowledge that despite their apparent diversity, all these concepts and problems are enclosed within the same labyrinth that accommodates them. Therefore, a plausible solution to the problem of the composition of the continuum may be helpful in understanding the nature of all these concepts and dilemmas. And if one can talk about the nature of the continuum through a basic theory of the continuity, it would be possible to derive outcomes concerning the constituents of the continuum, irrespective of its kind.

1.1.2 The Backyard of the Leibnizian Labyrinth

Given that the problem of the continuum abuts or is interlaced with many other philosophical riddles, it is not surprising that this broad problem is not invented by Leibniz. There are in fact various thinkers who mulled over the riddles of the labyrinth since Ancient philosophy to the present day. There are various figures in the philosophy of history who influenced Leibniz with their search for a way out of the continuum. Some of these figures are the Ancient atomists, Zeno, and

Aristotle from the Ancient times, and Galileo, Descartes, Gassendi, Hobbes, and Cavalieri from early modern philosophy.

It is well known that Ancient philosophy was particularly involved in subjects connected with the "one and the many" problem, and especially with Zeno's paradoxes. But the moderns, as well as Scholastic philosophy, inherited these problems from the Ancients and continued to discuss them seriously. Zeno's paradoxes were closely related with the composition of the continuous space, time, and motion. While these paradoxes laid down the difficulties of considering space and time to be composed of points and instants, respectively, they questioned the reality of motion and the phenomenal world.

Aristotle, on the other hand, was perhaps the most important Ancient figure, who dealt meticulously with Zeno's paradoxes, and systematically examined some key concepts such as 'continuous,' 'contiguous,' and 'discrete;' 'divisible,' and 'divided;' 'actual,' and 'potential;' 'finite,' and 'infinite.' Aristotle set forth definitions of the terms such as 'continuous' and 'contiguous' which were going to be very decisive for the ongoing debate over the problem of the continuum, not to mention their influence on Leibniz. Furthermore, he accounted for the division and the composition of the continuum along his definitions. As is well-known, Scholastic philosophy, which sustained the discussion on continuity, derived many of its features from its interpretation of Aristotle and Plato.

In early 17th century as well, questions created by the labyrinth of the continuum were feverishly under debate. Thus it is not unusual that Leibniz was not the only one of his time who revived the problem once again. Composition of the continuum, infinite divisibility and the possibility of indivisibles began to be popular issues of philosophical and scientific dispute before him. Among many modern thinkers, Pierre Gassendi, Galileo Galilei, Bonaventura Francesco Cavalieri, René Descartes and Thomas Hobbes were the leading names that inspired Leibniz's thoughts to a substantial extent. These men were all deeply involved in and proposed solutions for the problem of the composition of the continuum.

Gassendi was one of the modern atomists, who thought that world is composed of atoms and interspersed void. As is to be seen in the Chapter 2, his account of the composition of motion had considerable influence on Leibniz (L, 362, Leclerc, 123-124, and also see Fisher). Galileo, on the other hand, gave an account for the composition of the continuum of infinitely many unquantifiable indivisibles, which was later differently interpreted by Leibniz, as is to be shown in Chapter 3. Galileo's conception of indivisibles was further developed by the mathematician Cavalieri, from whom Leibniz derived his idea of the formation of figures through the motion of ultimate units. (Arthur 2001, xxviii, xxxv, see also Bell).

Descartes' main influence on Leibniz's tackling of the problem of the continuum was his conception of actual division of the subtle matter to indefinite particles by means of the motion of parts, and his overall ideas concerning matter and motion (Descartes, 232-240, Arthur 1998, 110, Garber 1982). On the other hand, it was Hobbes whose ideas played a great role in shaping of Leibniz's distinctive accounts of the composition of the continuum on the basis of a conception of animated points, i.e., *conatuses*. That is to say, Hobbes's theory of endeavours and his alternative definition to Euclidean point, and his revival of an Aristotelian conception of continuity proved to be very influential on Leibniz's theories of motion and the continuum and his objective of reconciling modern thought with Aristotelian philosophy (LLC, 359-360, Arthur 1998, 114).

1.1.3 The Scope of the Labyrinth

As is evident from the accounts concerning the composition of the continuum from points, the labyrinth of the continuum has important connections with mathematics, particularly with geometry. A very typical question concerning the mathematical aspect of the problem is of how an extended line can be composed of unextended points. Furthermore, the studies of Leibniz and the above-mentioned thinkers on the problem played the principal role in the foundations of infinitesimal calculus. The units that are assigned to be the ultimate constituents of the continuum were considered to be infinitely small or smaller than any given. This idea was very central to Leibniz's invention of his version of the infinitesimal calculus. Given this connection with mathematics and Leibniz's revolutionary studies on calculus, it is not surprising that the labyrinth of the continuum is usually examined from a mathematical perspective with a special concentration on his relatively later writings, where Leibniz makes clearer distinctions between mathematical, physical and metaphysical domains.

However, this particular interest in mathematics should not eclipse the problem's significance for other concerns. The scope of the concept of continuity is definitely not limited to, or determined by its mathematical implications. There are vital physical considerations, for instance, which are, as Richard Arthur points out, relatively neglected (Arthur 1998, 110). Of course, perhaps in many cases it is rather a matter of choice than mere ignorance, for it is not difficult to see how a manifold web of relations is embedded in this labyrinth of the continuum, as is tried to be portrayed above.

Perhaps a more reasonable approach would be to consider the problem as a whole and through these relations, just as Leibniz himself tried to do. One of the distinctive features of Leibniz's early effort at tackling the problem was his insistence on reconciling the realms of physics and mathematics by means of commensurable accounts. And I maintain that, in the early writings pertaining to the labyrinth of the continuum, this commensurability is rendered possible through the agency of Leibniz's idiosyncratic metaphysics.

Indeed, a slightly deeper interest in the problem demonstrates how intertwined all these domains of mathematics, physics and metaphysics are as regards the labyrinth of the continuum. For instance, as Richard Arthur holds, throughout the development of Leibniz's ideas concerning the problem of the continuum, "brilliant mathematical innovations are in constant interplay with his thoughts on natural philosophy and its metaphysical foundations" (Arthur 2007a, 32).

Therefore, it is not necessary in all accounts of Leibniz to dissociate the different considerations of the problem. On the contrary, perceiving the relations, transitions, and also the divergences between these realms or between Leibniz's different accounts concerning these realms, is considerably important in understanding the essence of the labyrinth as a whole.

The fact that the labyrinth of the continuum is not solely a mathematical problem is in fact evident from Leibniz's writings. In these writings, Leibniz does not solely consider the composition of the continuum from mathematical entities. As Samuel Levey indicates, "the labyrinth of the continuum he frequently mentions is conceived to embrace a host of physical as well as mathematical concerns" (Levey 1999, 95). The physical atoms, *conatuses*, indivisibles with parts and actual magnitudes, bodies, motions, and even minds are considered to be possible constituents of continua in Leibniz's various early writings. Furthermore, this is not solely inferred from his discussions about the constituents. As well as referring to numbers, geometrical points, lines, figures and surfaces, throughout these writings, Leibniz talks about numerous notions with physical and metaphysical connotations such as the *actual* division of the continuum, continuity of motion, time and space, continuity of things in nature, the plenum, vacuum, causes, resistance, impenetrability, cohesion, sensations and so on. Accordingly, it is important to at least be aware of these considerations rather than ignoring them in favor of any particular facet of the labyrinth.

1.2 The Appraisal of the Labyrinth

1.2.1 The Exterior Difficulties and the Secondary Works

Given that the labyrinth of the continuum incorporates various intertwined topics, it would be admitted that establishing a consistent account that effectively meets all the difficulties associated with the labyrinth was not an easy task for Leibniz. Accordingly, as articulated independently by the leading scholars of the subject, there is no single work like the *Monadology* or *Theodicy* that presents the "Leibnizian solution" to the problem of the continuum. Bradley Bassler, who submitted a dissertation on the subject in 1995, points out with reference to Leibniz's expression of "the two labyrinths of philosophy," that although Leibniz had devoted a whole book to the labyrinth of freedom, a systematic or a perfectly "to the point" writing on the second one is missing (Bassler 1995, 2). Similarly, Richard Arthur states, referring to a draft of Leibniz's Paris years, in which Leibniz outlined his prospective treatment of the labyrinth of the continuum as consisting of the topics "on the *composition* of the continuum, time, place, motion of the atom, on the indivisible and the infinite" (DSR, 91), Leibniz could not comply with his wish to write a comprehensive treatise on the subject (Arthur 2001, xxiii).

The lack of such a comprehensive account is definitely an obstacle that makes a study on Leibniz's struggle with the labyrinth difficult. However, this does not mean that there are no writings on this subject. The early writings of Leibniz, particularly for this study, are important in this respect, as well. As is to be elucidated, although Leibniz did not write systematic treatises on the labyrinth, especially one of the early writings, namely the *Theoria Motus Abstracti*, together with other texts Leibniz penned within the same interval of time—and which can be interpreted to rotate around the axis of the *Theoria Motus Abstracti*—can be seen as a candidate account to provide a thoroughgoing way out of the labyrinth. In this and many other writings of the early period, various riddles of the labyrinth are expounded. Furthermore, there is enough evidence for deducing different solutions to the problem of the composition of the continuum even in the short span of time that is to be dealt in this thesis.

A similar difficulty facing an inquiry of Leibniz's thoughts on the problem of the continuum is related with the treatment of the topic in the secondary literature. It appears to be that a few decades ago, there were not many works directly related with the labyrinth of the continuum. In 1986, Richard Arthur stages the problem in his "Leibniz on Continuity" perhaps for the first time in such a serious manner. In 1995, However, Bassler still thinks that even the mere formulation of the continuum problem, though it is the central problem in Leibniz's metaphysics, is so difficult that it is absent in the secondary works on the problem. He encapsulates this oddness in a question: “How can a problem be at once so central and so invisible?” (Bassler 1995, 2).

Four years later, Samuel Levey makes a similar comment: He points out that, while there are improvements regarding the comprehension of Leibniz’s later philosophy, understanding of his thoughts on continuity “remains highly imperfect” (Levey 1999, 81). However, as he himself indicates, there have been improvements, and these have been quite fast, though still limited to the hands of the same few scholars, who are above all, I believe, Richard Arthur and Otto Bradley Bassler.

Richard Arthur, who is the editor and the translator of the wonderful selection of Leibniz’s writings on the problem of the continuum, is therefore one of the main figures which I have frequently appealed to while writing this thesis. His “Introduction” to the relevant selection, which makes a nice story of the development of Leibniz’s ideas somewhat concerning the labyrinth, is a momentous work of analysis on its own. Furthermore, he has written several other articles on the problem which I have made use of in my study. These writings pertain to the diverse aspects of the problem such as the physical and the mathematical ones, though without isolating one from the other.

Likewise, many of Bradley Bassler’s articles related with the subject, together with his dissertation, provided me with great insight. In his dissertation, Bassler tries to consider the problem with a developmental account in a historical scheme, which is similar to the methodology that is to be adopted in this thesis. I would like to express that his analysis of the early writings of Leibniz is so thorough that for several times, I changed my mind after I have read him, or I had to give credit to him for things I had written before accessing and reading some other writings of his.

There are of course other important scholars whose works guided the current study. Samuel Levey’s comprehensive writings delineating the connections between the mathematical and the metaphysical aspects of Leibniz's early accounts were of considerable use especially for the first two chapters of my study. Christia Mercer's meticulous work on the origins of Leibniz's metaphysics was an important source concerning the development of Leibniz's ideas. Daniel Garber's nice summary of Leibniz's early notions of metaphysics and motion, and Stuart Brown's edition concerning the philosophy of young Leibniz were my other main references with respect to Leibniz's early philosophizing. Finally, translations and notes of George Henry Parkinson and Leroy Loemker were the other primary sources that I benefited from.

1.2.2 The Interior Difficulties: Leibniz's Treatment of the Problem

> Now what are the difficulties of the continuum, and how they are evaded? I cannot hope to succeed in making the subject plain, both because it is nearly the most difficult subject in philosophy, and because Leibniz's treatment offers special difficulties to the commentator. (Russell, 109)

Russell is as frank as he is right about the peculiar difficulties of the subject. Besides the complexity of the labyrinth itself, it is not very easy to follow the path of Leibniz's advancement in the labyrinth. Especially with respect to his early years, in which Leibniz was under the influence of diverse schools and philosophers, it demands a careful and thorough inspection of the reasons behind the frequent amendments in his accounts. There are various different accounts and arguments somehow related with the problem, and they are far from being consistent as a whole. Given the situation, one can easily suspect whether the problem or the attempts to solve it is a labyrinth. In this respect, Bassler states that as Leibniz' thought develops, the view of the labyrinth does as well." (Bassler 1995, x). Seeing eye to eye with him, Arthur writes, with reference to Leibniz, that "his own thought on the subject is something of a labyrinth itself, and from a modern point of view many of his pronouncements are apt to seem blatantly contradictory." (Arthur 1986, 107). Arthur later repeats this idea and says with respect to the changing arguments of Leibniz that they are "of such bewildering intricacy that it is hard not to regard them as constituting a labyrinth in themselves" (Arthur 2001, xxv). He emphasizes that it is not easy to track these changes together with their reasons, since “there is no sustained line of argument, no easily discernible explanations for his changes of position” (ibid.).

1.2.3 An Optimistic Inquiry: Replacing “Despite” with “Through”

Although I admit all the above-mentioned difficulties in advance, I will pursue an optimistic objective, and attempt to trace and find out reasons for Leibniz’s amendments to his early theses concerning the solution to the problem of the continuum. Leibniz tried to find a “thread” for the labyrinth of the continuum in various ways. He dedicated himself to produce solutions, which, at the same time, do not contradict other premises or principles of his philosophy. However, each time he proposed a new solution, sooner or later he noticed the new difficulties that solution had brought about. Hence, his ideas continuously changed and gave place to new ones. In this sense, Leibniz was a distinctive philosopher who displayed an example of negating and then surpassing one’s own ideas and philosophical position.

To sum up, the development of the philosophy of Leibniz does not follow a straight line, but rather involves a process that involved ebbs and flows. However, the scholars who delve into this course of development of Leibniz’s philosophy arrive at the seeds of the spectacular system, put into words in the *Monadology*, in his early writings. From this point of view, that is, when looked at the whole picture, it can be argued that there is still a progress and continuity in Leibniz’s system *despite* all the contradictions and inconsistencies.

In this study, I desire to look to see if it is possible to push this argument further, and although implicitly, my move will be to replace the “despite” in the argument with “through.” That is to say, I will try to give an account that suggests the idea that Leibniz’s theses on the problem of the continuum develops (not despite but) through the contradictions he deals with. I believe that these inconsistencies are the locomotives of the progression of his thoughts.

1.3 The Procession in the Labyrinth

My thesis will pursue a developmental account of Leibniz's search for a way out of the labyrinth in his early writings. This account will, as much as is possible, stick to the alleged chronological order of Leibniz's writings. As indicated in the previous section, I will try to elucidate the main ideas contained in these writings which are somehow related with the problem of the continuum, and my aim will be to seek and point out the reasons behind Leibniz's changes of mind.

Needless to say, I will also lay down Leibniz's alternative solutions to the problem of the composition of the continuum. I will discuss these possible solutions which can be derived from Leibniz's writings. These solutions will be presented in harmony with the first three formulations Richard Arthur presents in his "From Actuals To Fictions: Four Phases in Leibniz's Early Thought On Infinitesimals" as the three theories related with the continuum and the infinitesimals. Thus, in each chapter, I will expound only one of the possible solutions which I will delineate in conjunction with Arthur's formulations. I will thoroughly examine these solutions, and discuss their applicability with respect to the geometrical, physical and the metaphysical realms. For each solution, the continuity of motion, and its composition will be of primary importance, since it will prove to be the particular continuum that bridges the geometrical and the physical accounts under the body of metaphysics.

Chapter 2 is a kind of introduction to the important concepts and the riddles of the labyrinth in connection with how they take place in Leibniz's first philosophical writings. These introductory discussions will provide the rudiments of Leibniz's conception of the problem of the continuum.

Among the concepts and riddles to be discussed within Chapter 2, one can speak of divisibility, and the following questions brought about by this concept: *Is every continuum divisible? If a continuum is divisible, does it have to follow that it is infinitely divisible? Does being divisible always imply being divided as well?* In this respect, I will discuss the differences between the actual division and the potential divisibility of the continuum.

Afterwards, the main concern of the labyrinth of the continuum will come to the fore, that is, the composition of the continuum. The questions attached to the problem of the composition of the continuum will address how and of what a continuum is composed, and ask if the answer will imply an actual composition. Furthermore, the question will be elaborated in greater length to inquire whether a continuum can be composed of ultimate units which are qualitatively different than the continuum they are supposed to compose.

In conjunction with the problem of the composition of the continuum, the cohesion of the parts that compose the continuum will turn out to be the second main difficulty to be overcome. This difficulty, which will be named the "problem of cohesion," will require an explanation for how the parts of a continuum cohere so that they can be the parts of a certain body. Henceforth, this question will be raised as regards to all purported solutions to the composition of the continuum.

Within Chapter 2, Leibniz's desire to establish a reconciliatory metaphysics based on a combination of Aristotelian individual substances with simple mechanistic explanations will be discussed. In this respect, the significant role this reconciliation project plays in relation to the composition of the continuum will be acknowledged.

Finally, in Chapter 2, Leibniz's first systematic account of motion, together with his first conception of the continuity of motion will be explicated. This theory of motion will also yield what will be called "the first formulation" suggested for the problem of the composition of the continuum. In this account of motion and the theory of continuum, the key move will be the use of the expression "*smaller than any given*," which is the original form of the idea of infinitesimals in Leibniz's philosophy. By means of this expression, Leibniz will differentiate his theory from many others.

Chapter 3 focuses on a very important piece of writing from Leibniz's early philosophy, the *Theoria Motus Abstracti*, which is directly connected with the problem of the continuum. This theory marks a crucial milestone in Leibniz's philosophy, since it presages, particularly by means of the notion of '*conatus*,' Leibniz's later conception of simple substances. Moreover, this theory distinguishes itself from the previous accounts by means of several novel ideas Leibniz brings to the fore. Here Leibniz does not only give a geometrical account of motion and the continuum, he also establishes the bonds of metaphysics with geometry and physics.

The initial distinctive feature of this new theory is Leibniz's consideration of the continuum as consisting of infinitely many actual parts, which sets him apart from Aristotle and Descartes in this respect. The second crucial move of Leibniz in the *Theoria Motus Abstracti* is the denial of minimal (or Euclidean) points that are claimed to be composing the continuum. Leibniz bases his denial of minima on a "part-whole argument" which he frequently resorts to and which will therefore be discussed, particularly in Chapter 3. In place of minima, he introduces a fairly new conception: the 'indivisibles'. In Chapter 3, this idiosyncratic conception of indivisibles is to be expounded in detail, and the differences of indivisibles from Euclidean points will be clarified. Basically, the indivisibles, as opposed to minima, have magnitudes, even if they are smaller than any given.

These arguments eventually lead to the second formulation of Leibniz's solution to the problem of the continuum: "The continuum is composed of indivisibles, defined as parts smaller than any assignable, with no gaps between them. Indivisibles have indistant parts, but no extension …" (Arthur 2007a, 8). In the same way as in the previous chapter, the applicability of this solution to phenomenal world will be questioned, again with a special attention to the relations between the ideal, physical, and the metaphysical domains. In the final section, the chapter will include a succinct account of the brief period in which Leibniz abruptly adopts ideas that are quite contrary to those of the *Theoria Motus Abstracti*.

The theory of motion constructed upon the central notion of the *Theoria Motus Abstracti*, i.e., '*conatus*' will also be discussed in this chapter. This discussion consists of the meaning of the term '*conatus*', its association with motion, and its role in the composition of continuous motion. Furthermore, it will be seen that in the *Theoria Motus Abstracti*, '*conatus*' is not only indispensable for an account of motion, but it also has its place in the definitions of 'body' and 'mind'. As a final point, the use Leibniz makes of *conatuses* in the explanation of cohesion of bodies will be examined.

In Chapter 4, I will be discussing the first few manuscripts Leibniz wrote during the years he stayed in Paris. The main text that will be referred to will be "De Minimo et Maximo," in which, I believe, Leibniz furthers the theory of motion expounded in the *Theoria Motus Abstracti* to its limits. However, before discussing this theory, I will discuss Leibniz's explanation of cohesion through *conatuses*, which is still maintained in the early Paris years, though with slight changes, which are to be detailed.

While the distinctive feature of the account that will be depicted in Chapter 3 is a new conception of indivisibles, the demarcation of "De Minimo et Maximo" from the *Theoria Motus Abstracti* comes about by Leibniz's renunciation of indivisibles. So in this chapter, the reasons behind this renunciation will be thoroughly discussed.

Instead of the indivisibles of the *Theoria Motus Abstracti*, in "De Minimo et Maximo" Leibniz stages infinitesimal quantities, which are extended but still smaller than any given. Accordingly, some of the main focus of concern will be to explain what these infinitesimal quantities suggest and how the continuum is composed of them. During the clarification of these infinitesimal things, the infinite will be brought under the spotlight as arguably the most puzzling riddle of the labyrinth.

In Chapter 4, it will be proclaimed that the status of motion within the solution to the composition of the continuum, and within Leibniz's metaphysics, gains more prominence than ever before, to the extent where the account of motion and the solution to the composition of the continuum begin to overlap. As in the previous chapters, I will examine Arthur's formulation of this solution which is a rather geometrical one. Thus the tension between the geometrical, metaphysical, and physical accounts will continue to be a notable point of contention. I will try to explore what Leibniz has in mind with respect to the correspondence between these domains and applicability of his accounts. At this point, I will again take into consideration the unifying role of the metaphysics of motion and *conatuses* with respect to the ideal-geometrical and real-physical realms.

Finally, Leibniz's transition to a metaphysics based on sensations of minds from a metaphysics grounded upon a system of continuous motions and *conatuses* will be examined. In what follows this examination, I will reflect on the possible reasons behind the gradual disappearing of the theory of *conatuses* from the philosophy of Leibniz, with a particular attention to Leibniz's changing ideas about the actuality of infinitesimals.

CHAPTER 2

THE EARLIEST WRITINGS:
THE RUDIMENTS OF
THE LABYRINTH OF THE CONTINUUM

The period between the time when Leibniz was seriously involved in philosophy and the advent of his journey to Paris may be singled out on account of its certain characteristics. Writings pertaining to this period manifest the enthusiasm, ambition and also the confusion of a bright young man, who has such sublime aspirations that no one of his age would easily imagine. As is to be expected of a student however ingenious, his first thoughts and writings bear many references to his instructors and influences.[1] Nevertheless, the richness of these references renders his first works "original" in the sense that his ideas have a variety of origins. Furthermore, given his explicit intention to reconcile different thoughts and beliefs, his eclecticism would be acknowledged.

This juvenile period, for which Leibniz sometimes prides and sometimes derides himself, can indeed be said to have culminated with the *Theoria Motus Abstracti* (Theory of Abstract Motion), written in the winter of 1670-71. According to Richard Arthur, Leibniz's first proposed solution to the problem of the composition of the continuum is articulated in this work (Arthur 2001, xxxii). He

[1] See Mercer 1999 for a nice portrayal of Leibniz's early teachers.

supports this interpretation by pointing out Leibniz's early letters in which the philosopher maintains that the theory he adopted in *Theoria Motus Abstracti* is seemingly the only possible solution to the problem (Arthur 2007a, 10, fn.13). Otto Bradley Bassler, on the other hand, writes in his dissertation that Leibniz's April letter to Thomasius, which is dated 1669, "may be seen as Leibniz' first attempt to provide a systematic approach for resolving the continuum problem" (Bassler 1995, 136). But one certain thing is that they agree on the significance of the *Theoria Motus Abstracti*. For Bassler, it is the summit of Leibniz's early metaphysical position, in which the centrality of the continuum problem for a sound metaphysical basis for physics is apparent. He thinks that the *Theoria Motus Abstracti* includes new and comparatively more distinct thoughts, which will be a basis for Leibniz's future ideas concerning the problem (ibid., xi, 210).

Owing to its important place in the development of Leibniz's theses on the continuum problem, in Chapter 3 of this study, the *Theoria Motus Abstracti* will be explored in detail. However, even this early work has a quite extensive background and this chapter will mainly concentrate on other important works which prepare this background of the *Theoria Motus Abstracti*. In order to reflect similarities and divergences between these works, I classified them under three main subtitles, which examine the published version of Leibniz's doctoral dissertation, two of his important early theological writings, and his April letter to Thomasius, respectively. These works are not only important because they are essential for an understanding of the metaphysical foundations of the later works; they also present the bases and the main concerns of the problem of the continuum in their simplest forms. In these writings, one witnesses the appearance of many notions and questions in close association with the problem of the continuum, such as the 'composition' and the 'division' of the continuum, 'infinite divisibility', 'cohesion', 'impenetrability', the 'geometrical' and the 'physical'. Besides all these, Leibniz's letter to Thomasius includes a basic theory of motion, from which a general formulation of a way out of the labyrinth of the continuum is derived.

Thus, in this chapter, while examining these early writings, I will try to give an idea of the fundamental concepts mentioned above and clarify their importance with regard to the problem of the continuum. In this respect, I also aim to call attention to the relation between Leibniz's efforts to establish a metaphysical framework and the difficulties that the labyrinth of the continuum brings about. Similarly, it will be seen how different motives of Leibniz, such as the theological, or Aristotelian ones, affect the route he chooses in the labyrinth. Finally, I aspire to depict the first systematic conception of continuity that Leibniz conveys in his April 1669 letter to Thomasius.

Given that Leibniz's early influences are various, these early works reflect quite an extensive background which began to be formed even before Leibniz was seriously engaged in the philosophy of the moderns. Although this claim is a little controversial, it is often stated that Leibniz was still under the influence of Scholastic philosophy when he was 15-16.[2] Nevertheless, in his university years, Leibniz was drawn towards the ideas of the moderns, particularly by the revival of ancient thoughts and the atomist philosophy and hence, by the problem of the continuum.

As is often articulated, Leibniz was a reconciler throughout his life. Besides his early works trying to reunite Catholicism with Protestantism, he developed a passion to reconcile Aristotle and in part some elements of Scholastic philosophy and theology with the moderns.[3] On the one hand, he was critical of solely mechanistic explanations which disregard Aristotle or which were in plain contradiction with some theological doctrines. On the other hand, he sided with the simple explanatory methods of the very same moderns against the Scholastics, who obscured Aristotelian thought with superfluous additions. His objective of reconciling Aristotle with modern thought is manifest in his pieces of writings

[2] See Mercer 2001, 64-65, Arthur 2001, xxviii.

[3] See Mercer 2001 chs. 1 and 2, Levey 1999, 82, Bassler 1995, 59, 81, Arthur 2001, xxix.

pertaining to that period. These writings also contain many ideas about his first thoughts about continuity.[4]

Although the motives and the influences are similar, there are striking contrasts between the *Theoria Motus Abstracti* and its antecedents. These differences are so crucial that it is possible to use them so as to classify diverse approaches Leibniz adopts throughout his struggle with the problem of the continuum. More specifically, it is important to observe that, Leibniz's diverging conceptions of (the) continuity (of motion, matter, things etc.), or some of the metaphysical assumptions involved in these conceptions, particularly those displayed in his April letter to Thomasius, in the *Theoria Motus Abstracti* and in "De Minimo et Maximo" will recur in similar forms in different writings later in his career. Nevertheless, in order to perceive the similar motives and influences behind these differing accounts, now it is more appropriate to begin briefly with the earliest writings, particularly keeping an eye on how Leibniz modifies his attempts to revive an Aristotelian conception of substance in accord with a modern perspective.

2.1 *Dissertatio De Arte Combinatoria* (1666): Mathematical Rudiments of the Labyrinth of the Continuum

Leibniz is known to have published his *Dissertatio De Arte Combinatoria* (*Dissertation on the Art of Combinations*; hereafter *Dissertatio*) when he was twenty. Later, in his *On the General Characteristic*, he undervalues this work, regretting that he did not have the opportunity to have been grown up in Paris and got acquainted with mathematics earlier, as Pascal did (L, 222-223). However, as he himself acknowledges, some of the main ideas of the dissertation, particularly his combinatorial method and the ideas related with logic, are considered to be

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[4] There are many names, such as Samuel Levey, Otto Bassler and Richard Arthur who lay emphasis on this reconciliation project, particularly by comparing Leibniz's reading of Aristotle with modern philosophy. Arthur indicates that Leibniz attempts this not only because he believes that these two are compatible, but also because he thinks that the new physics of the moderns requires such a sound metaphysical foundation. (cf. Arthur 2001, xxix, Bassler 1995, 45-46, 54-55, Levey 1999, 82)

very important for his later thought, especially for a grounding for his ideal of a universal method and for his calculus (ibid.).[5] Moreover, this work, which is an extension of Leibniz's studies in his university years, is also a nice indication of his continuing departure from the Scholastics, as he explicitly reproves them, though he yet does not essentially pursue the moderns either.

Bradley Bassler sees the *Dissertatio* as a sign of Leibniz's thought that a solution to the problem of the continuum is only possible "through a coordination of mathematical considerations with fundamentally physical issues" (Bassler 1995, 129-130).[6] In this dissertation, Leibniz initiates his life-long project of a universal language, which is to be constituted from the most fundamental concepts and in turn from the combinations and propositions derived from these concepts. Here, starting from allegedly basic theorems and with the help of arithmetical combination and permutation, Leibniz tries to shed light on logic, metaphysics, physics and as a student of law, even on some practical subjects such as ethics and justice. Thus, it would be unfair to read this text as a merely mathematical one. Leibniz here systematically deals with fundamental philosophical issues[7], which I will now try to make clear.

2.1.1 God, Nature and the Infinite

As is to be admitted, the very fundamental concepts to found this universal language are also the most crucial ones. And the difficulties of elucidating them often makes philosophers postpone their grand projects. The *infinite* is without doubt one of those fundamental philosophical concepts, which troubled Leibniz to a great deal. Furthermore, as is going to be clear, the conception Leibniz has of the infinite will always prove to be decisive in his struggle with the problem of the continuum.

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[5] See Bassler 1995, 108, Loemker in L, 73, Rutherford 1995, 227-228.

[6] For a detailed discussion of the role of the *Dissertatio de Arte Combinatoria* for the problem of continuum in Leibniz, see Bassler 1995, 108-130

[7] In this sense, Bassler (1995) suggests that this dissertation may be interpreted as first draft of Leibniz's system (54).

Being equally fundamental, two instantiations of the infinite, namely *God* or *Mind* and *nature* or *phenomena* were waiting to be treated on the common plane of a universal language by the prospective doctor of law, while a new world of thought was flourishing. God was indestructible, therefore eternal, and also had infinite power and wisdom. Minds were also considered to be eternal, as long as they were taken to be analogous to God, given that both God and minds were *incorporeal* "substances." On the other hand, *corporeal* "substances" making up nature or the phenomenal world, were infinitely many, and had an infinite variety of various qualities. They might be, for instance, of infinitely small magnitudes, in infinitely many different shapes or colors. But somehow there was a relation between these two infinities, the mystery of which has kept many philosophers busy for years. And the question of whether a continuum in any form exists may arguably take this relation as its initial concern because of its decisive aspects. Since to show how the world of minds and phenomena are bridgeable is to show the possibility of the most comprehensive continuum. In a sense, this explains how Leibniz begins his *Dissertatio*. I think that Leibniz's universal method or language required at least a conceptual continuity between these two infinities. And in the first part of his *Dissertatio*, Leibniz tried to bridge them with another fundamental concept, 'motion'. From then on, 'motion' frequently undertook that bridging function between the phenomena and the minds.

The first part of the *Dissertatio* begins with God, i.e., the incorporeal infinite. Leibniz here seeks to demonstrate the existence of God, by positing God as an incorporeal *substance* having infinite power. In his second definition, he defines substance as "whatever moves or is moved" (L, 73). According to his axioms, there has to be a mover for anything moving, and if all the parts of a thing are moved, it means that the *whole* is moved. In the end, what Leibniz suggests is that God is the mover of the entire corporeal world, which is a whole. This relation of the mover and the moved, therefore, is the first connection Leibniz provides by means of motion, though apparently this connection at the same time marks the edge where these two worlds fall apart.

In his proof, Leibniz uses what he thinks is a kind of Aristotelian prime mover argument, claiming that a body is either moved by another body, or by an incorporeal.[8] If the mover is not an incorporeal, lest the argument falls into an infinite regression, bodies should move each other continuously. But as mentioned, when all the parts are moving, the whole, which is, according to Leibniz, of infinitely many parts, is also moving. Therefore, another mover is required for the origin of the motion of this whole. And if infinite regression is not desired, then the cause of motion should be an incorporeal. God, who by definition has infinite power and hence capacity "to move the infinite" is the only incorporeal substance to move this whole (ibid., 73-74).

2.1.2 Division and Parts of the Continuum

Leibniz holds in his dissertation that every body, being a whole, is of infinitely many parts, or as is written in one of the axioms, has an infinite number of parts. Leibniz thinks that this is equivalent to the commonly used expression that "the continuum is infinitely divisible" (ibid., 74), which finally leads us to the core of the subject matter of this thesis.

As will be frequently indicated, *divisibility* of the continuum together with the conditions of this divisibility is one of the most crucial concerns of the problem of the continuum. This concern brings several questions to the forefront: *Is every continuum divisible? If a continuum is divisible, does it have to follow that it is infinitely divisible? Does being divisible always imply being divided as well?* These questions are key to the many subtitles that are to be discussed under the problem of the continuum.

Hence, Leibniz's acknowledgment that the continuum is infinitely divisible signifies perhaps for the first time that the problem of the continuum is one of the tasks to confront in establishing a sound metaphysics. Furthermore, although Leibniz does not deliberate on it, his consideration of a body, which has an

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[8] Later, his reading of Aristotle in this respect will be objected to by his professor Jacob Thomasius.

infinite number of parts, as an infinitely divisible continuum is very important in this respect. Since here Leibniz seems to think that being infinitely divisible is equivalent to having infinitely many parts. And the fact that Leibniz does not deliberate on this particular thought indicates that in his early years, he does not have in his head a distinction of actual-potential division, or ideal-real entities yet. For in time Leibniz prefers to distinguish being actually divided, which is a characteristic of the discontinuous things in nature, from being infinitely divisible, which is a characteristic of ideal or geometrical continua. Nevertheless, it would be better not to go into the deeper details here before Leibniz makes these distinctions himself.

Yet there *are* distinctions that Leibniz makes in this dissertation as well. For instance, Leibniz distinguishes 'being' both from 'quality' and 'quantity', which are modes (or affections) of substance, and maintains that 'quality' is "something absolute," while 'quantity' is something relative to the parts of a thing. Since their quantities say something about the relation between two different parts, or a part with its whole, quantity denotes a relation (ibid.). The parts which are related with each other represent a union. And together with their union, Leibniz describes, they are a *whole*. According to his reasoning, a unity is an abstraction from one being, whereas each whole, when abstracted from a unity, is called *number*.[9] After that, Leibniz argues that 'quantity' is the number of parts. Since number always denotes a unity, number of parts turns out to be determinate. But when the number of parts is not *known*, 'quantity' can be used to signify a relation to another 'quantity'.

Of course there are mathematical and even some theological motives behind Leibniz's reasoning, and all of its possible metaphysical implications will not be our concern. But at this point, it is important to draw attention to the *epistemological aspect* of the problem of the continuum. Number of parts of a

[9] Leibniz's argument that the 'whole' and 'number' are abstractions of the intellect is related with one of his criticisms of Scholastics for their belief that "number arises only from the division of the continuum and cannot be applied to incorporeal beings" (L, 76-77). In contrast, he thinks that since number is an abstraction from any being, it can be applied to any possible thought of a unity.

continuum is directly related with the divisibility of the continuum. If a continuum is said to be divisible or divided into an infinite number of parts, two epistemological claims can be derived from this argument. One of them concerns the knowledge of the number of the parts, while the other has to do with the knowledge of *the infinite*. It is seen in Leibniz's dissertation that as far as number denotes a unity, the number of a thing's parts is determinate, and the indeterminateness of the number of parts appears to be linked with the capacity of our intellect, because there is an inexplicit point beyond which the number of parts is not known to us. However, in Axiom 4, bodies are said to have an infinite *number* of parts (which means that, at that time, the 'infinite' was not taken to be indefinite by Leibniz). To say that the number of parts is infinite may imply that the 'infinite' is a number, given that the number of parts is a unity. On the other hand, in the *Dissertatio*, Leibniz also states that one infinite can be greater than another (ibid., 75). In other words, he believes that two infinities are comparable. However, Leibniz does not sustain these ideas for a long time and I think particularly his deeper interest in Galileo will play a big role in the change of his ideas on the infinite, which will be discussed in the following chapters. However, as for the *Dissertatio*, it can surely be claimed that Leibniz's conception of 'infinite' is yet far from being sophisticated as it will be later in his career.

As is declared in Chapter 1, the problem of the continuum is first of all the problem of the *composition* of the continuum. However, with respect to *Dissertatio*, there is not much to find in the name of answers to the questions how and of what a continuum is actually composed. Leibniz does not specify what kind of things the infinitely many parts of the physical or metaphysical continuum are. Nevertheless, reasoning primarily on arithmetical grounds, Leibniz indicates that a whole is composed of minima, that is, "parts now not to be considered as further divisible" (ibid., 78), which are all equal to one another.

Accordingly, it would be claimed that in *Dissertatio*, Leibniz considers the continuum to be composed of atomic, indivisible units, which are perfectly homogeneous, or more truly, identical, with each other. However, it can be inferred that these units are not homogeneous with the continuum they compose, since for Leibniz, the continuum is infinitely divisible, whereas these minima are not further divisible. Thus Leibniz in fact encounters the utmost riddle of the labyrinth of the continuum: *How can a continuum be composed of the constituents which are heterogeneous with it?*

Although Leibniz does not explicitly refer to the corporeal or physical continuum, a similar conception of continuity is also perceptible as regards bodies. Leibniz thinks that mathematics in fact deals with 'beings,' and number "rightly belongs to metaphysics" (ibid., 77). Owing to his mathematical outlook, here Leibniz treats bodies as analogous to numbers, which are wholes. Although these wholes "can be broken up into parts … there are common parts in the different smaller wholes themselves" (ibid.). Thus, these smallest unities appear to be analogous to minima. Therefore, with respect to bodies, it would not be wrong to say that Leibniz adopts here a kind of atomist standpoint, where the world, which is a whole, is composed of atomic wholes or units.

In any case, Leibniz does not ask the question of how a body, or any part, can be composed of atomic units yet continuous and of infinitely many parts at the same time, despite the fact that the idea of body having an infinite number of parts, and being an infinitely divisible continuum, clashes with the traditional sense of atoms. Furthermore, juxtaposing minimum parts does not necessarily produce a continuum. In *Dissertatio*, there is no explanation for the question how these parts compose a continuum. And this latter question is above all related with the problem of accounting for *cohesion*. The problem of cohesion, which primarily asks how different parts of a continuum cohere so that they are the same parts of a continuous whole, is another inevitable riddle of the labyrinth of the continuum, which we will come across throughout the early writings of Leibniz.

In brief, the reason why the *Dissertatio* is included in the current study is that it provides an opening into many such fundamental aspects of the problem of the continuum in their simplest forms, together with an understanding of Leibniz's early motives and standpoint. Although they are fundamental, the notions he uses and the distinctions he makes are not yet clear or sophisticated. Nevertheless, this work gives ideas about Leibniz's future discussions revolving around these notions and distinctions. Leibniz's analysis of part and whole is plainly connected to the problem of the composition of the continuum. On the other hand, divisibility of the continuum into the parts that compose it will always remain a task to account for, owing to the epistemological questions about the number of parts and infinite divisibility that arise with it. Furthermore, these epistemological questions and Leibniz's consideration of wholes as abstractions imply his much later distinction of real and ideal, and of physical with the mathematical or geometrical. Finally, in this dissertation, perhaps where this distinction is really vague, Leibniz bridges corporeals with incorporeals, i.e. God or minds, through motion. And it will soon be noticed that motion not only has a bridging function between these two, but also between the ultimate corporeals themselves. Each of the concerns above will be discussed within the following writings of Leibniz, which will be dealt with in a chronological order.

2.2 Early Theological Writings: Metaphysical Rudiments of the Labyrinth of the Continuum

After becoming a doctor of law, Leibniz had a broad project on his mind based on a new outlook on theological issues with a different philosophical approach, which would be a blend of Aristotelian metaphysics, modern physics and mathematics with some Scholastic and religious concerns. In this way, he was hoping to shed light on Christian doctrines, which were left in the shadows thanks to the obscurity of the Scholastics and disparaging critiques of the moderns. He planned this work, called *Catholic Demonstrations*, as consisting of four main parts and a prolegomena. While the prolegomena was intended to expound main notions and principles pertaining to physics, metaphysics, mathematics and logic, the remaining parts were projected to demonstrate the existence of God, the immortality of the soul, some Christian doctrines, and the divine authority of the church (Mercer 2001, 68). Here two important essays written in support of this project, namely the *Confessio Naturae contra Atheistas* and *De Transsubstantione* will be considered. The fact that these texts are theologically oriented does not eclipse their being metaphysically substantial, particularly in the sense that several fundamental notions such as magnitude, figure, form, matter, motion, substance, space, extension and impenetrability, are examined in a way that would give a general opinion about Leibniz's early attitude towards metaphysics.

Besides giving an understanding of Leibniz's early metaphysical perspective, going over the *Confessio Naturae contra Atheistas* and *De Transsubstantione* provides the path to Leibniz's first solutions to the problem of the continuum. Although Leibniz mainly has a theological motivation, in these papers he shows us his awareness of the centrality of notions such as cohesion and divisibility for the problem. Furthermore, it would be observed that the boundaries of the problem are extensive as it comes out that some fundamental metaphysical questions and the answers given to them are decisive for the possible solutions of the problem of the continuum. With respect to these theological papers, these questions mainly concern the essential qualities of bodies, their principle of motion and autonomy within a purportedly Aristotelian framework. However, it will take a little while to get to the point where these questions are connected to the problem of the continuum.

2.2.1 *Confessio Naturae contra Atheistas* (1668): God against the Atomists

In the *Confessio Naturae contra Atheistas* (Confession of Nature Against the Atheists; hereafter *Confessio*), one sees a confrontation of Leibniz's religious concerns with his methodical inclinations. Here he articulates his doubts concerning the spreading influence of the atomist and mechanical accounts of the modern philosophers, with respect to the relation of metaphysics and science with religion. In fact, he appreciates some of those modern or mechanical accounts regarding natural phenomena, particularly because, unlike the Scholastics, they avoid appealing unnecessarily to incorporeals or God.[10] Nevertheless, those who further these ideas to impiety and fostering atheism seem to worry him to a greater extent. He exposes his assent clearly by quoting Horace in Latin: "Nec Deus intersit, nisi dignus vindice nodus inciderit"[11] (L, 110). However, his main concern in this work is indeed to point out the existence of such a knot, where God should intervene. And here Leibniz tries to solve this knot via an Aristotelian approach.

Leibniz agrees with the moderns on their tenet that as far as possible, natural phenomena can and should be explained by the primary qualities of body, which are *magnitude*, *figure*, and *motion*. But on the other hand, he claims that there is no way to derive these primary qualities themselves *from* the nature or essence of the bodies. A body is essentially defined as "that which exists in space" and apparently these primary qualities of magnitude, figure and motion are not included in this definition. Accordingly, Leibniz argues that the origin of these qualities stem from an incorporeal principle. For him, without resting upon an incorporeal, a body is not self-subsistent, and an account of it would not be self-sufficient (ibid.). Throughout the first part of the *Confessio*, he sets about to prove this claim.[12]

First, Leibniz questions why a body has a certain magnitude and figure but not another. He claims that this cannot be answered by an appeal to the nature of bodies, because their main constituent, which is matter, is "indeterminate as to any definite figure" (ibid., 111). If one does not prefer to give an explanation through an incorporeal, then for Leibniz, there remain only two alternatives. The

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[10] He mentions Galileo, Bacon, Gassendi, Descartes, Hobbes and Digby. (L, 110)

[11] "And let no god intervene, unless a knot come worthy of such a deliverer." (Loemker's translation, L, 119)

[12] Christia Mercer, who examines these earliest writings in detail, notes that Leibniz oversimplifies the ideas of the moderns. The fact that some of Leibniz's conclusions are not in contradiction with some of the moderns such as Gassendi and Descartes, particularly on the issue of the cause of motion, is a confirmation of her claim: "For instance, Descartes maintains that motion is a mode *of* extension, even though it has to be added *to* extension by God" (Mercer and Sleigh, 74, also see Mercer 2001, 70-81, and also see Garber 1982, 166).

first of them is to suggest that bodies have their definite figures from eternity. But Leibniz maintains that "[e]ternity cannot be considered the cause of anything" (ibid.), and it cannot be a real reason for a certain figure of the body. But if one tries to explain it with the motion of another body, which is the way the moderns choose, then this argument goes to infinity. For one has to explain why that body had its certain previous form before the action of the other body brought about the new figure. And if the same reason is given, then the argument will fall into an infinite regress.

Thereby, it is seen that, though Leibniz's approval of modern ideas seems to have increased, he still uses an argument similar to the one he used in his *Dissertatio de Arte Combinatoria*. This similarity is more evident in the account for motion. Much the same way as in the argument about figure, Leibniz talks about the two possible ways of giving an account of motion of a body which is "left to itself" or at rest without appealing to an incorporeal (ibid.). One gives an explanation either by assuming motion as eternal, or by suggesting that contiguous bodies—bodies that are touching each other—move each other by their motion. However, for Leibniz, just for the same reasons given in the case of figure, these ways of explanation are "futile" (ibid.). Because still one has to explain why a body is not at rest from the eternity, or in the second case, how and why the mover itself is moved, without falling into an infinite regress.

Here it is worth noting that Leibniz makes a distinction between motion and the mobility of a body. This distinction can be seen as the origin of the dispute about whether Leibniz considered motion as unreal in his early writings. Leibniz defines mobility as being able to change place, and motion as change of place (ibid.). In a sense, this implies an actual-potential distinction, where mobility denotes a potential for the actual state of motion. Nevertheless, Leibniz thinks that this distinction leads to the conclusion that "mobility arises from the nature of a body, but that motion itself does not" (ibid.), which suggests that the potential for motion is essential to the body though the actual state of its motion is not.

In the *Part II* of the *Confessio*, Leibniz makes use of the concept of motion for a proof of the immortality of the human mind, as well. According to this argument, action means a variation of the essence, and "the essence of a body is being in space" (ibid., 113). So, when a body moves, it changes its position (or existence) in space, and therefore, it has an essential variation. Since this variation is a result of motion, Leibniz concludes that "every action of a body is motion" (ibid.). If something is not a body, then by definition it is not in space, and therefore cannot move. And if something cannot move, it is indestructible and immortal, since dissolution and corruption occurs only through motions of the parts of a body. For Leibniz, human mind is immortal, because it is distinguished from body by one of its actions, which is 'thinking'. And since "all motion has parts, by Aristotle's demonstration and common agreement," thinking is not a motion since it is an action without parts (ibid.). Years later, in 1675, Leibniz maintains the same idea that "whatever has separable parts cannot think" with the same motive to demonstrate that minds must be indestructible (DSR, 9).

It will soon be acknowledged that the ideas about the nature of the body and the origin and the makeup of its primary qualities (particularly of motion) are significant for an understanding of the problems of continuity that Leibniz deals with. On the other hand, in the *Confessio*, the notion of *cohesion* appears to be the one that is most directly related with the problem. By itself, cohesion of a body has been an important problem haunting in philosophy and physics. Simply put, this problem concerns the question of how a body persists without its parts being disbanded. Thus, its connection with the problem of continuity is obvious in the sense that the cohering parts of a body actually form a continuum. In other words, as is already indicated in the previous section, the problem of cohesion mainly asks how parts of a whole cohere so that they form a continuum. Thus, in its broad sense, a sound explanation would be required for the cohesion of the constituents of any continuum. On the other hand, as will be illustrated below, it is Leibniz who suggests the need for a further explanation concerning the cohesion *within* the constituents themselves.

As is to be expected, some of the early moderns try to provide a solution to the problem of cohesion without resorting to God or any other incorporeal entity. While most of them draw inspiration from the ancient atomists in their accounts—which are about to be discussed below—Galileo makes use of a different but again originally ancient idea claiming that cohesion of parts is a consequence of "nature's repugnance to vacuum."[13]

Galileo's account of the continuum suggests that a finite continuum can be composed of infinitely many unquantifiable indivisible parts (*parti non quante*)[14] and unquantifiable voids (Galileo, 33). Even though Galileo's demonstration is a geometrical one, he thinks that the same idea can be considered concerning bodies. He believes that between the ultimate particles of a solid body, which are taken analogous to unquantifiable indivisible points that compose simple lines, there are miniscule voids. In line with his supposition of nature's repugnance to vacuum, Galileo argues that these unassignable voids provide the coherence of the parts. He mentions this view in a dialogue of his *Two New Sciences* through the interlocutor named Salviati:

> … Seeing clearly that a repugnance to the void is undoubtedly what prevents the separation of two slabs except by great force, and that still more force is required to separate the two parts of the marble or bronze column, I cannot see why this [repugnance to the void] must not likewise exist and be the cause of coherence between smaller parts, right on down to the minimum ultimate [particles] of the same material. (ibid., 26, brackets belong to Stillman Drake)

After a while, Christiaan Huygens replaces the idea of nature's repugnance to vacuum with the pressure of a kind of matter subtler than air (Arthur 1998, 115). However, Leibniz notices that in each of these accounts there was a common problem which rendered them insufficient: None of them accounts for the

[13] cf. Arthur 1998, 113-115, Arthur 2001, xxxvii-xxxviii. In his translation of Galileo's *Two New Sciences* Stillman Drake prefers to use 'void' instead of 'vacuum'.

[14] Unquantifiable simply means that which has no quantity, or magnitude. Drake writes in the Glossary section of his Galileo translation that the "essential characteristic of 'unquantifiable parts' is that they are uncountable and are devoid of size" (Galileo, xxxvi). See also Smith's "Galileo's Theory of Indivisibles: Revolution or Compromise?"

cohesion within the ultimate constituents. In what follows, I will try to explain how he articulates this objection in the *Confessio*.

After discussing the primary qualities of bodies, Leibniz looks into the origins of the firmness of bodies which he characterizes by resistance, cohesion, and reflection, and expresses that no philosopher can explain these three properties through the primary qualities of bodies (L, 112). Precisely here, he refers to the above mentioned atomist account of cohesion, which was upheld by ancient atomists such as Leucippus, Democritus and Epicurus, and by Pierre Gassendi and John Chrysostum Magnenus, whom Leibniz calls "their modern followers" (ibid). This account is based on an idea of interlocking bodies through some sort of hooks and rings. Leibniz's objection is that they still have to explain how these "interlocking instruments themselves" cohere. He rightfully asks "[m]ust we assume hooks on hooks to infinity?" (ibid.). Afterwards, he talks about a modification of this account which he thinks is possible but still insufficient, this time based on the diversity of atoms themselves:

> There remains only one answer which these most subtle philosophers can make to such objections; they may assume certain indivisible corpuscles, which they call atoms, as the ultimate elements of bodies, which, by their varied shapes, variously combined, bring about the various qualities of sensible bodies. But no reason for cohesion and indivisibility appears with these ultimate corpuscles. (ibid.)

Thus, for Leibniz what is erroneous about these suggested explanations is that all of them *assume* the cohesion within the ultimate constituents in advance. And for him, at least for the time being, the modern standpoint cannot provide a solution to the cohesion of bodies without falling into contradiction or infinite regress.

Before he suggests his own solution to the cohesion problem, together with the questions regarding the origin of the primary qualities of bodies, Leibniz also criticizes another solution to the problem of cohesion, which was suggested only by ancients and briefly proposed that atoms, or parts of atoms, cohere because they have no vacuum between one another. Leibniz says that, this argument comes to mean that each and every body in contact with another coheres with the other body, which is absurd. Thus, for Leibniz, contiguity, that is, being merely in contact, is not a sufficient condition of cohesion, and together with this idea, one can see that Leibniz began to tell continuity and contiguity apart in his *Confessio* (ibid.).

As a result, Leibniz thinks he has demonstrated that qualities of bodies can ultimately be explained only by means of an incorporeal being, which is God. He suggests that,

> [i]n explaining the atoms, we may therefore rightly resort to God, who endows with firmness these ultimate elements of things. I marvel that neither Gassendi, nor any other of these most acute philosophers of our century has noticed this excellent opportunity to demonstrate the divine existence. For through the ultimate analysis of bodies, it becomes clear that nature cannot dispense with the help of God. (ibid.)

From these sentences, it can be inferred that Leibniz is nevertheless inclined to a kind of atomism with a divine support, though he does not specify the characteristics of these 'atoms' yet. Furthermore, as mentioned before, even if the idea of atoms still seems in contradiction with his claim in the *Dissertatio* that every body has an infinite number of parts and continuum is infinitely divisible, it can be said that Leibniz's arguments on divisibility, atoms and continuum are still not so well defined in these early writings. For instance, although Leibniz raises an objection to Galileo's account of cohesion, he does not argue against his unquantifiable points and voids. This silence concerning these matters may imply that Leibniz does not yet have a clear grasp of the significance of the problem of the composition of the continuum in constituting a sound metaphysics.

However, concerning his objections to atomist accounts of cohesion, Leibniz's purposes are evident. While he posits God as the incorporeal principle, he adds that God does not move bodies one by one, but instead he provides for their moving each other. Thus in this way, he reserves a space for mechanical explanations retaining a God, which is responsible for the harmony among the bodies by means of determining their magnitude, figure and motion through his omnipotence and omniscience. And this conception of harmony granted in advance to the world of phenomena obviously implies his future notion of the *pre-established harmony*, which is grounded on his Principle of Harmony, or Principle of the Best, which Leibniz gradually constitutes in the following years.

However, in some respects Leibniz seems dissatisfied with his account and conclusions, which ground every important property of bodies in God, leaving no space for the autonomy of minds. His dissatisfaction can indeed be inferred from his ensuing amendments.

First of all, the causal connection between God and bodies remains as blurred as the relation between the primary qualities of bodies themselves.[15] Seemingly, Leibniz tries to establish a metaphysics of substance with an Aristotelian outlook, but by this quite unfounded connection between the corporeal and mechanical world and the sole incorporeal principle, that is, God, his standpoint becomes unpersuasive, both for the moderns and their religion oriented opponents. Second, the role of minds as substantial forms is ignored and reduced only to divine mind, and that is one of the major adjustments to be found in *De Transsubstantione*.

2.2.2 *De Transsubstantione* (c1668): An Aristotelian Reconciliation

The significance of *De Transsubstantione* (On Transubstantiation) for the current study is to point out the compelling role that Leibniz's attempt to reconcile an Aristotelian conception of individual substance with the mechanical explanations of body and motion will play in his future engagements with the problem of continuity. This essay is an even more religiously oriented piece of writing, which takes the possibility of transubstantiation as its subject, and tries to face the difficulties regarding the doctrine of the Eucharist. Nevertheless, again it has very important implications which are rather metaphysical, for Leibniz tries to show the possibility of transubstantiation by means of a metaphysical proof. Hence he indicates that his proof "depends on interpretation of the terms 'substance",

[15] As is indicated in Mercer and Sleigh, "[b]ecause God causes the shape by moving matter … it is not clear whether the shape of the shoe is strictly in God or in the object." (Mercer and Sleigh, 77)

‘appearances’ or ‘accidents’, and ‘numerical identity’” which he develops “on the basis of their meanings as accepted by the Scholastics” (ibid., 115).

In *De Transsubstantione*, Leibniz firstly repeats the line of reasoning he used in the *Confessio* to show that every action of a body is motion, and then his argument that bodies do not have their principle of motion within themselves. But this time, as is just implied, the latter one is revised so as to replace the incorporeal principle, which is previously equated with God, with “concurrent mind” (ibid., 116). Furthermore, the conclusion in *De Transsubstantione* is different as well. Leibniz argues that body is not a substance by itself, since here, he defines substance as self-subsistent and that which has its principle of action within itself: “If that which has a principle of action within itself is a body, it has a principle of motion within itself,” however, “[n]o body has a principle of motion within itself apart from a concurrent mind” (ibid., 115-116).[16]

Nevertheless, this means that the same body, when taken *together with* a concurrent mind, is considered to be a substance, for only minds have their principle of action: “Thus the substance of body is union with a sustaining mind” (ibid, 116). On the other hand, a body taken apart from a concurrent mind is an accident or only an appearance (ibid.).

Thus, in *De Transsubstantione*, Leibniz replaces God as the incorporeal principle, with the concurrent mind. Against a possible objection that, in his account there is only one mind for all the substances, Leibniz emphasizes that concurrent mind is not what divine mind is; it is specific to each substance (ibid., 118). If the human body is in question, substance is its union with the human mind, whereas the substance of a body without “reason is union with the universal mind, or God.” (ibid.). Although all these arguments ultimately lead to the justification of transubstantiation, which will not be dealt with here, it can be interpreted that Leibniz restores a new version of Aristotelian primary substance by bringing mind

[16] Mercer (2001) points that this is the first time Leibniz puts forward that principle of self-activity, which will always remain in his philosophy (85).

and body together and by claiming that the body is not self-subsistent without its corresponding mind. So, on the one hand, he tries to argue against atomists and tries to provide a metaphysical ground for theology, and on the other hand, he seeks to conserve a kind of Aristotelian conception of individual substance, in a fashion that does not dispense with the autonomy of bodies. And in this way, Leibniz opens space for mechanical explanations, as well.[17]

Now, perhaps at this point, the crucial position this conception of substance occupies with respect to the problem of the continuum is still not very explicit. However, these substances are in fact complex versions of what Leibniz will later assume to be the simple constituents of the continuum in his abstract or geometrical accounts. This fact that Leibniz's conception of substance is a decisive factor for the composition of the continuum, is also articulated by Arthur:

> … the idea that mechanism requires a foundation in individual substances whose principal activity is mind forms the starting point for all his further attempts on the continuum problem. (Arthur 2001, xxix)

However, all these connections will be much clearer when we meet those accounts which try to suggest solution to the question how a continuum is composed of simple units.

The ambiguity is much bigger when it comes to 'motion' in *De Transsubstantione*. It is already indicated that here Leibniz argues that motion does not follow from the nature of a body. In other words, a body does not have its principle of motion within itself. But whether this comes to mean that 'motion' is an unreal property is not very certain. In any case, 'motion' is still taken as one of the three primary qualities of body, together with 'magnitude and 'figure'. If the term 'real' denotes an "essential" property, then the claim that motion is unreal would be right, in the sense that motion is not an essential property, for it is

[17] Bradley Bassler sees an original philosophical style in this attempt, which, he thinks, "would seek an autonomy of the bodily for the purposes of physical analysis but would nonetheless retain the absolute metaphysical dependence of the bodily on the mental conceived of as substantial. (Bassler 1995, 146-147)

caused from without. In other words, it is not contained in the nature of bodies, as opposed to mobility. On the other hand, if a real property is a substantial property, then one can derive from *De Transsubstantione* that motion is a substantial property of body in union with its mind, insofar as a body in its union with a mind is a substance, and mind, which is a part of this substance, is the principle of the motion that its corresponding body has.

Furthermore, in the same paper, there is an interesting footnote written against the Cartesians arguing that space and extension "really differ from body, because otherwise motion would not be a real thing, and a vacuum would be necessary (L, 120). There are several points to infer from this note. First of all, Leibniz distinguishes body from mere space and extension. This is a very crucial step in the way to his accounts of the continuum. For by distinguishing the realm of corporeals, or determinate matter, from mere space and extension, Leibniz separates two main domains of continuity. The first domain, which is the spatial, extensional or geometrical continuum, is the ideal and the potential one. And the second domain, material, or corporeal continuum, is the physical and the actual one. However, to repeat again, these distinctions inferred from what has been examined so far, are still not clearly made or elucidated by Leibniz himself.

As a second point to infer from this small footnote of Leibniz, it is possible to argue that Leibniz seems not to be in favor of the idea of a vacuum, since he seems to avoid concluding that the vacuum is necessary. Although Leibniz is still not explicit in this respect, this is a critical matter of choice in relation to the problem of the composition of the continuum. On the face of it, the possibility of vacua would be considered to be in contradiction with the idea of the continuum. However, as is indicated, some of Leibniz's influences, such as Galileo, or modern atomists, speak of the "existence of vacuum," and furthermore, Galileo incorporates voids in his account of the composition of the continuum. In this respect, it is worth noting in advance that the choice with regard to vacuum is decisive for any account of the continuum Leibniz would present.

And finally, it can be inferred that Leibniz does not consider motion as *unreal* in *De Transsubstantione*. While separating space and extension from bodies, Leibniz seems to attribute motion to bodies as a real thing, not to space or extension, which appears to be ideal. However, still one should first settle on the meaning of 'real' and 'unreal' for Leibniz, which is definitely hard to decide by looking at his earliest writings. Nevertheless, the status of the motion with respect to the nature of bodies continue to remain as the weakest as well as the most critical link of Leibniz's reconciliation project, because his ideas concerning motion proves to be the locomotive of his differing hypothesis, which is to be confirmed in the somewhat radical shift of his letter to Thomasius.

2.3 From a letter to Thomasius (1669): Development of the First Account of the Continuum

With all its ambiguities and eager claims, Leibniz's April 30th letter to Thomasius is an indicator of a philosophical personality at the start of his career, whose mind is to some extent confused, partly because of his aim to reconcile different schools and thoughts. For such an objective bears the difficulty of blending and comparing different but analogous concepts of different philosophies. At first glance, this letter is Leibniz's manifestation of this attempt of reconciliation to his professor. It is essential for a study surveying the origins of the continuity problem, which would be admitted even on account of the fact that Leibniz makes clear what he understands from 'continuum' and 'discontinuum'. But prior to this, this letter is a valuable piece in that it manifests Leibniz's first serious metaphysical ideas that are to be published.[18] In fact, his metaphysical discussions are not separate from the matters concerning the continuum; rather they often get intertwined with them. This is obvious with respect to the discussion that turns around the concept of motion. In his April 1669 letter to Thomasius, Leibniz articulates for the first time a theory of motion which he improves in another work

[18] Mercer (2001) has a stronger claim that "in earlier works like *Dissertatio De Arte Combinatoria*, Leibniz had discussed metaphysical topics and offered some suggestions about the proper elements of metaphysics … but the published letter to Thomasius constitutes his first attempt to offer a fully articulated and original theory of substance." (99).

called *De Rationibus motus*, a series of drafts written right after this letter. And this theory of motion has much to say about the continuity of motion. But what is more important is that the framework that is derived from this theory of motion can be translated as the first formulation suggested for the problem of the continuum, which describes to a certain extent what Richard Arthur does in one of his essays that is to be discussed here.

In his letter written to his professor, Leibniz openly sets forth his aim to reconcile the moderns with Aristotle, although his highly esteemed teacher thinks the other way.[19] And this time Leibniz is straighter than before, when he says "I maintain the rule which is common to all these renovators of philosophy, that only magnitude, figure, and motion are to be used in explaining corporeal properties" (L, 94). But he also believes that the philosophy of the moderns, which is based on these fundamental properties, is reconcilable with Aristotelian philosophy to such an extent that "the whole of Aristotle's eight books can be accepted without injury to the reformed philosophy" (ibid.).

However, what he means by Aristotelian ideas is certainly not the Scholastic interpretations of them. Leibniz claims that the Scholastics misinterpreted Aristotle, as he has previously pointed out in *Dissertatio De Arte Combinatoria* particularly for the issues of metaphysics, logic, and law. For him, the same objection can be raised against matters concerning phenomena. Indeed, the possibility of reconciliation lies in his belief that the primary physical properties of bodies which are determined to be magnitude, figure, and motion by new philosophers can take the place of Aristotle's fundamental notions matter, form, and change without bringing about contradictions. Furthermore, since Leibniz believes that those ideas of modern philosophers were indeed derived from Aristotelian philosophy, he holds this reconciliation to be not only possible, but also necessary, in the sense that they should already be explainable through each other (ibid., 94-95).

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[19] "[T]hose people who repeat the same old song that the ancient Aristotle can be reconciled with sacred scripture … should be met with derision" (Thomasius quoted in Mercer 2001 34).

As is indicated before, another criterion of Leibniz's preference of the moderns is that their ideas are simpler when compared to the superfluous notions and entities of the Scholastics. Even though theirs are hypotheses just as the hypotheses of the Scholastics, they are "clearer and more intelligible" since they do not unnecessarily assume incorporeals in bodies, and instead try to explain phenomena only by means of "magnitude, figure, and motion" (ibid., 95, 101).

2.3.1 Primary Matter and the Formation of Bodies through Motion

In this respect, Leibniz carries out a blend of a geometrical and physical account in conformity with fundamental Aristotelian notions. First of all, by introducing *primary matter*, Leibniz separates the ideal, geometrical realm, from the corporeal one. Here, he defines primary matter as a continuous mass, nature of which is extension and impenetrability (or antitypy). Since it is taken to be a continuous mass, it may be argued that it leaves out the possibility of vacuum in its usual sense as well. In this sense, primary matter differs from space. Because in this writing Leibniz defines space as "primary extended being or a mathematical body" (ibid., 100). On the other hand, primary matter is a "secondary extended being," and aside from the mathematical one, it has a "physical body" (ibid).

The nature of a physical body consists in its being impenetrable or resistant in addition to its being extended. While to be extended for a body is identified with being in space, impenetrability is defined as a body's "being constrained either to give way to another being of this kind which strikes it or to stop it" (ibid.) and elsewhere as the "impossibility of being in the same space with another thing" (ibid., 101).

"Being impenetrable and able to give way to others" comes to mean that bodies are able to move. That is why Leibniz thinks that mobility follows from matter's being impenetrable. However, physical bodies differ from mere primary matter by their motion, because, although mobility follows from its being impenetrable, the primary matter is in itself at rest. In other words, it just has a *potential* for motion. Primary matter is prior to all forms, for it has existence on its own. Through motion, its homogeneity is replaced by diversity. That is to say, although primary matter is a *continuum* composed of *parts* at rest, these parts obtain forms through motion. Borrowing the terms of Averroists, Leibniz describes matter as having *interminate* quantity which, as he claims, is equivalent to "indefinite" in Descartes.[20] Given that primary matter is a continuum, Leibniz says that "it is not cut into parts" (ibid.).

Here seems to be a conflict concerning the continuity and the "parts" of matter. In the very same paragraph, Leibniz says on the one hand that "[n]ow this *continuous* mass which fills the world is, as long as all its *parts* are at rest, primary matter …," and on the other hand that "[f]or so long as matter is *continuous*, it is *not* cut into parts …" (LLC, 337; italics are mine). I am not sure if this uncertainty can be explained by the difference implied by the expression "cut into," but Leibniz seems to separate actual division from the potential one. In this manner, it can be assumed that those parts of this continuous mass are only potentially parts. Thus, just as the primary matter has a potential to move, it also has a potential to take determinate forms. As Samuel Levey indicates, the primary matter is indifferently divisible into parts in any number of ways, but it is not actually divided" (Levey 1999, 83).

Bassler inquires, on the other hand, whether one can handle this contradiction by assuming that the limits of the parts of primary matter are themselves not "parts," so they do not disturb the continuity, or that these limits are "one" with respect to Aristotle's definition of 'continuity' (Bassler 1995, 66).[21] But nevertheless, as he points out, one cannot say that this continuum is homogeneous, since that matter can still be heterogeneous with those boundaries (ibid., 66, fn. 21).

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[20] Samuel Levey prefers to use "unbounded" instead of "interminate in Levey 1999, 82. For his justification, see Levey 1998, 73, fn. 27.

[21] Leibniz occasionally uses Aristotle's definition of continuous and contiguous in *Physics*. Aristotle suggests that things are contiguous when they are in contact and consecutive, which is not different from today's dictionary definition of the word. On the other hand, by continuous things, he means those whose limits form a unity (Aristotle, 138).

As is emphasized several times before, another important question related with this part-whole relation in a continuum is the question how those parts cohere so that they form that particular whole. Even if they are not actually parts, primary matter's coherence as a whole requires an explanation. It will be recalled that earlier in his *Confessio*, Leibniz argues against atomists for not accounting for the cohesion of interlocking instruments and atoms themselves, and he exercises this objection as an opportunity to prove the existence of God. However, heretofore in his letter to Thomasius, Leibniz does not give an explanation for the cohesion of the parts of the primary matter, either for the whole, that is, for the continuous primary matter itself, or for the independent unities, i.e. for the cohesion within the individual parts. One possible way for Leibniz to explain cohesion here would be matter's being impenetrable, because being impenetrable necessitates maintaining a certain figure without being disintegrated. But in any case, Leibniz does not give a detailed explanation about *how* this cohesion takes place.

Being loyal to the modern approach, and in the direction of his Aristotelian project, in his letter, Leibniz equates form with figure, which is defined as the boundary of the body. Having boundaries takes for granted a discontinuity among parts, denoting a division of matter. If matter is created as a discontinuous entity, or if it entails vacua, according to Leibniz, it means that there were determinate forms from the beginning. However, if primary matter is continuous from the beginning, then this division, and in turn figures and boundaries originate from motion (ibid., 96). And this latter option is what Leibniz seems to adopt in his letter.

Here for the first time, Leibniz describes Aristotelian conception of continuum as composed of bodies, whose extremities are one (ibid.). Accordingly, he considers discontinuity as the direct opposite of continuity, while taking contiguity as a subset of discontinuity. In other words, contiguous bodies are not continuous, but discontinuous. In this sense, Leibniz thinks that discontinuous parts, and in turn forms, may come about in two different ways: either by a separation by means of force, in which case there is a vacuum between, or by the motion of parts "in different directions," but being still contiguous, that is, in contact (ibid.). He gives an example of two spheres one within the other, and say that they are contiguous yet discontinuous, since they move in different directions. Together with this idea on the generation of form and boundaries, Leibniz ends up making his first *clear* definitions of 'continuous' and 'discontinuous' parallel to those of Aristotle.[22] On the other hand, the possibility he left for the vacuum by mentioning it brings him a little away from Aristotle and nearer to atomists, although he is still in favor of the second explanation.[23]

There is a difficulty about the generation of forms from the primary matter that will challenge Leibniz within the time subsequent to the letter to Thomasius. Here Leibniz prefers the idea that figures of physical bodies are formed through motion from the continuous primary matter at rest, rather than being eternally determinate, as he discussed in the *Confessio*. But we know that Leibniz previously addressed the moderns the question why a physical body has its certain figure but not another, and his own explanation consisted of appealing to an incorporeal principle. However, in some respects, this suggestion was not satisfying for him, because one still had to explain the division of or separation of parts from the primary matter. First of all, if forms are generated through division of primary matter, there is no sufficient reason why this should not be a division to infinity. And if this division or separation yields infinite number of parts, and if the figure of these parts can further change—and as will be clear soon, they can—bodies' being impenetrable becomes open to discussion. Because there would be no parts which cannot be penetrated by any motion, if there is an infinite separation.

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[22] cf. Crockett, 127.

[23] In accordance with the second explanation, one may argue that Leibniz explains cohesion with a common motion, since parts are separated from each other by their motions. As far as I understood, Samuel Levey defends this view in Levey 1999, 101. If this is what Leibniz has in mind, it would be argued that Leibniz will develop this thought in his future writings, beginning from the *Theoria Motus Abstracti*.

As a matter of fact, Leibniz soon notices that his former claim of infinite divisibility of matter and the continuum is in contradiction with its impenetrability. On the other hand, Leibniz does not easily abandon 'impenetrability', because he thinks that it would mean an utter dissolution of matter itself, unless another account for the resistance of bodies is found. (Bassler 1995, 168-169).

The annoyance caused by this problem becomes perceptible in a series of drafts penned between Leibniz's April 1669 letter to Thomasius and the *Theoria Motus Abstracti*. These drafts together are called *De Rationibus motus*, and this work will be discussed in more details below. In one of these drafts, as a possible way out of the problem, Leibniz designates motion as the reason for resistance of bodies. He argues that only bodies in motion can resist other bodies (ibid.). The reasoning behind this argument relies on a former idea that is articulated both in the *Confessio* and in *De Transsubstantione*: Every action of a body is motion. And since 'resistance' is a kind of action, then it should be a motion, as well. Hence, bodies can resist only if they have motion. (ibid., 176-177).

However, one may think that there is a quite odd situation here. In his letter to Thomasius, Leibniz derives impenetrability, and therefore the resistance of matter from the impossibility of two bodies to share the same space. On the other hand, he derives the possibility of motion, or mobility, from matter's being impenetrable, or resistant. But in *De Rationibus motus*, Leibniz explains resistance by motion, the possibility of which has previously been explained by resistance. Thus in this respect, I find it difficult to decide whether to resist means to move, or to move means to resist, and I think the only possible answer seems to be "both."

There is another crucial consequence of explaining resistance in terms of motion. If only bodies in motion are capable of resistance, then a body at rest becomes thoroughly penetrable, that is, indistinguishable from empty space. In the letter to Thomasius, Leibniz argues that there is a "sensible quality, which occurs in all bodies and only in bodies and by which man may distinguish body from nobody, as if by a criterion and [b]eyond doubt this is mass or antitypy, together with extension" (L, 101). Therefore, after Leibniz begins to explain resistance by motion, primary matter which is at rest becomes equivalent to space or "nobody", because without impenetrability, it has nothing in its nature aside from extension. Hence, for a while, motion remains the criterion for distinguishing bodies from space and primary matter.

Heretofore, we have talked about matter and form of the Aristotelian terminology. Now, the last main notion to square with modern terminology is '*change*'. Leibniz here shares the ideas of the moderns who suggest that all sorts of change can be explained solely by motion. In this fashion, generation and corruption are indeed consequences of the motion of parts. For Leibniz, not only the qualities of a body, but also the substance itself may change by motion. Nevertheless, he remarks that generation and corruption are not motions themselves; rather they are the "end of motion" where motion instantaneously halts (ibid., 97). Here Leibniz also depicts figure as "the last instant of motion" (ibid.), and according to him, this explains why forms or figures "consists in something *indivisible* and cannot be increased or decreased" (ibid., italics are mine). And hereby the idea of 'indivisibles', symbols of indestructibility, becomes an essential element of the discussions on the problem of the continuum. Since an indivisible implies being not subject to change or annihilation, it would not be surprising that Leibniz considers the idea of it preferable, given that he believes minds to be eternal. Hence, with their advantages and disadvantages, which will be made clear in the course of this study, indivisibles become a part of Leibniz's structurally varying systems.

Regarding the derivation of figures from motion, Bassler reminds us of Thomasius' objections against Leibniz's reconciliation project that Leibniz reduces "physical analysis to geometrical considerations" (Bassler 1995, 81). Parenthetically, it is worth noting that this is a very important discussion, since throughout Leibniz's struggle with the problem of the continuum, it sometimes becomes really difficult to distinguish whether Leibniz reasons on a geometrical (or mathematical) or a physical plane. In addition to these, there seems to be a metaphysical plane, which I think bridges the other two. Now, with respect to geometry, Thomasius claims that Aristotle denies that it is a science, whereas Leibniz sees it as a true science, which he claims to show with reference to Aristotle's texts (L, 98). For Leibniz, a true science deals with substance, and so does geometry, in which case the substance is space. Figure, existence of which depends on space, "is a substance, or … something *substantive* (ibid.). Thus, attributing figure, or what is merely spatial, a "substantial" characteristic, Leibniz establishes a geometrical ground for his physical and metaphysical analyses. Similarly, unlike the Scholastics, who think that mathematics is not a perfect science since "it does not always demonstrate from causes" (ibid., 98), Leibniz believes that it does:

> For it demonstrates figures from motion; from the motion of a point a line arises, from the motion of a line a surface, from the motion of a surface a body. The rectangle is generated by the motion of one straight line along another, the circle by the motion of a straight line around an unmoved point. Thus the constructions of figures are motions, and the properties of figures, being demonstrated from their constructions, therefore come from motion, and hence a priori, from a cause. (ibid.)

Consequently, since motion is portrayed as the generation of figures, which are geometrical in essence, it falls within the field of geometry. Here Bassler asserts that this means "it cannot simultaneously belong to the realm of the physical" (Bassler 1995, 84). However, this argument should not leave out a crucial idea which Leibniz will later restage: Notwithstanding their being solely geometrical figures, here Leibniz derives *things* from motion, rather than explaining motion by means of things. Besides, the moment Leibniz incorporates 'motion' and causality into geometry, borders of metaphysics inevitably verge on those of geometry.

Notwithstanding this approach between geometry and metaphysics through motion, so far the ontological status of 'motion' still remains vague. But before the status of motion with regard to the nature of bodies and to "real things" is discussed, there is a question remaining to be addressed: Although motion is the cause of figures, the origin of motion itself is not yet clear, and it seems that it cannot be accounted for by means of impenetrability alone. And therefore, this one is maybe the most complicated and controversial matter of this letter.

2.3.2 Problem of the Reality of Motion and the Continuous Creation

Leibniz asserts that “matter in itself is devoid of motion” (L, 99), which means that the cause or the origin of motion of bodies is something other than bodies themselves. In this respect, he compares Aristotle with the Scholastics, and claims that the Scholastics misunderstood the role of forms in accounting for the motion in Aristotle’s philosophy by considering form as “immaterial being, though insensible in bodies, which spontaneously imparts motion to a body” (ibid). In Leibniz’s interpretation form is the cause of motion, but not “the primary one” (ibid.). In association with the modern perspective, the idea is that figure does indeed have a kind of physical effect on the motion of the body it belongs to, just as a body can physically determine another’s motion. However, according to Leibniz “primary form,” is “really abstracted from matter, namely mind, which is at the same time the efficient cause” (ibid.). This primary form, Leibniz maintains, is the first and the main principle of motion. And among the substantial forms, only mind is the primary form, which is “really abstracted from matter,” and which can freely and spontaneously generate motion. Furthermore, Leibniz claims to have this reasoning derived from Aristotle’s prime mover argument, in which, for Leibniz, the unmoved mover was proposed as a necessary upshot of the fact that bodies do not have their principle of motion within themselves (ibid.).[24]

Now, it would be helpful to sum up the critical but complicated status of motion, before passing to the somewhat radical conclusion of the letter. First, motion is signified to be the cause of figure, figure being identified with form. Hence, motion appears to be the *formal* cause of the body, since body is *formed* through motion. On the other hand, form or figure is still “the principle of motion *within* its own body” (ibid., italics are mine). Furthermore mind, which is the only

[24] For an objection to Leibniz’s claims on the prime mover argument and for a discussion of Thomasius’s response, see Bassler, 86-7.

substantial *form* for Leibniz, is said to be the primary cause of motion. Thus there is a kind of peculiar reciprocal causal relation between motion and forms. Motion, is the cause of the figure of a "body" and hence the destroyer of the continuity of primary matter, whereas mind as the primary form causes motion.[25]

However, just as he did in his *Confessio*, Leibniz distinguishes motion from mobility, defining mobility as the tendency to move, and as mentioned above, mobility follows from the nature of matter, which is being extended and impenetrable. According to Leibniz, one cannot assume anything which does not follow from these two primary properties owing to his assertion that "nothing ought to be supposed in bodies whose cause cannot be discovered in their first or constitutive principles" (ibid., 101). And accordingly, he argues that motion is not derived from those two primary properties of the body, and therefore, "strictly speaking," it is *not* "a real entity in bodies" (ibid., 102). Thus, although the cause of motion is mind, which is the primary substantial form, it seems that mind is not considered as one of the "first or constitutive principles" of body. From this point of view, boundaries and figure, as well as change which is instantiated by generation, corruption etc., are claimed to be brought about by an unreal entity, namely motion.

On the other hand, it is observed that 'motion' is what distinguishes bodies from mere space and matter. From this perspective, it can be interpreted as an essential property of bodies, although Leibniz thinks that it is not a real entity. Furthermore, the idea in *De Rationibus motus* that motion is the cause of resistance, and therefore it is what distinguishes bodies from "nothing" supports the thought that motion is essential to bodies. However, even by considering this later work, it is

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[25] Mercer (2001) also points out to this difficulty concluding that Leibniz is "silent about exactly how form is supposed to be both an arrangement of matter and a principle of motion" (122). Then she suggests that this obstacle can be surmounted by considering the difference between *body qua matter* and *body qua form*. After being generated by a concurring mind, body qua form becomes the cause of the arrangement of parts of a matter, i.e. its figure. Therefore, being impenetrable, body does "*cause* motion in the sense that, when it is struck it mill move, and when it strikes another body, it will move the latter" (ibid.). Similarly, Bassler writes that "[a]ccording to the Leibniz of the letter to Thomasius, body consists of a figure in space insofar as its form is concerned, and of antitypy or resistance insofar as its matter is concerned" (178).

still too early to reach this conclusion. First, the possibility of bodies being at rest is not completely denied, and rather taken as a "limiting case" (cf. Bassler 1995, 181-184). Second, Leibniz is consistent in his view that real properties of a body should be derivable from its nature. Therefore, provided that motion is not derivable from the definition of body itself, Leibniz still would not hold motion as a real property of bodies. And accordingly, as long as resistance is explained by motion, and motion is not a real property, it becomes difficult to attribute resistance to bodies as a real property as well.

The most interesting part of the April letter to Thomasius, on the other hand, is Leibniz's radical conclusion including his continuous creation thesis. In fact, this is a kind of uncomplicated theory of motion, and it answers the question of whether motion is continuous or not. Here Leibniz claims to have demonstrated that

> … whatever moves is continuously created, and bodies are something at any instant in assignable motion, but that they are nothing at any time midway between the instants in motion—a view that has never been heard of until now, but which is clearly necessary, and will silence the atheists. (L, 102)

It is odd that Leibniz states that this is an original thought while Descartes, before him, put forward a version of the continuous creation thesis. Richard Arthur thinks that the original part of his account may be his idea that the creation is *continuous*, in the sense that the times at which the body does not exist are, by Arthur's expression, *unassignable*—that is, no quantity can be assigned to these times—but the motion is "metaphysically" discontinuous since it only consists in these instants (Arthur 2007a, 6).

So, at the end of the letter, it is inferred that motion is not "real" in the sense that there is nothing actually *in* motion, for bodies do not exist between the instants. But this by itself does not make motion completely unreal, if it does not make bodies so. Bodies are said to exist only at the assignable instants of their motion in which they are created over and over again. Nevertheless, whether motion does *really exist* in those assignable instants, or whether reality excludes motion is still not very clear. And it seems to remain vague unless the constitution of those instants is clearly defined.

The whole picture is also interesting with respect to the reality of bodies and their so-called primary qualities. Primary matter is composed of parts, which are not bodies, which are not substances, but which are still somehow “parts” of an unreal—or as one might say ‘ideal’— continuum or “parts” in a substantial space. And this continuum is broken by motion, which is a seemingly geometrical, i.e. an unreal property, and hence bodies are *formed*. But these bodies exist only at the instants of their “unreal” motion. But whether they are “real,” when they are “something” is also not elucidated.

Furthermore, by connecting motion to the continuous creation, ‘motion’ becomes completely dependent on the creator. Accordingly, since motion is the property that distinguishes bodies from mere space and provides them with their forms, autonomy of bodies, or of individual substances comprising of body and mind, becomes less meaningful. And this is something really unpleasant for Leibniz when his initial motives of establishing a metaphysics that is grounded on an Aristotelian conception of individual substances and compatible with the moderns is considered.

In other words, while still playing its role as a bridge between mind and body in a substance, motion remains *unessential* to both of them, and although its source is the primary substantial form, it turns out to be unreal: “Thus motion is no more essential to body, or deducible from its nature, than it is in the Gassendist and Cartesian mechanical philosophies he is opposing.” (Arthur 2001, xxxi)[26] Its being dependent, unessential and unreal, on the other hand, brings out the inevitable conclusion that the substantial connection Leibniz tries to construct between body and mind becomes unreal, too.

[26] Also see Mercer and Sleigh 76-78.

2.3.3 Subsequent Reconsiderations: God is Discharged from the Duty

As a result of these unfavorable consequences, in the time following Leibniz's April letter to Thomasius, considerable changes occur in Leibniz's thinking. Unsurprisingly, these changes disclose Leibniz's disturbance with respect to the "dependent" status of 'motion'. For 'motion's being originally dependent on God's intervention rendered the possibility of an autonomous individual substance doubtful. This adverse outcome would more likely support the position of the moderns rather than that of Leibniz, since it was Leibniz's claim that an Aristotelian conception of substance would be the best way through which phenomena can be accounted for. And God's intervention at each instant was totally undesirable for such a conception.

One of the clearest indications of this disturbance before *De Rationibus motus* is written is Leibniz's revisions to his April letter to Thomasius.[27] Leibniz attaches a revised version of his letter to a preface that was going to be published within a book of Marius Nizolius. Although the main body of the letter is left as it is, there are some critical changes somehow connected to the above-mentioned problem. Here Leibniz tries to feature the idea that 'thinking' and individual minds should have a more direct role in motion of bodies. Furthermore, he emphasizes that a body cannot have any motion unless the principle of it is provided by an incorporeal.

However, the most critical revision is the one that seems to lay aside the greatest incorporeal principle of all, i.e., God. In this published version, Leibniz omits the paragraph where he advances his continuous creation thesis. This choice is very important in the sense that it makes Leibniz ask several questions all over again. These questions concern the reality of motion and bodies, their causes, and their connection with the relevant incorporeal principle; the continuity of motion and

[27] The grounds for his revision are neatly elucidated in Mercer 2001, 137-144, to which I am indebted for my comments. But see also Garber 1982.

the structure of time instants. And what Leibniz searches for in the ensuing span of time is nothing but the answers to them.

Indeed, *De Rationibus motus* is a product that reflects this search for answers. This series of drafts written in the years 1669 and 1670 were never published, and it is hard to say that there is a consistency between the drafts comprising *De Rationibus motus*. This is mostly because of the gradual transformation of Leibniz's opinions during this period. Furthermore, although this work is projected to be a commentary on Wren and Huygen's laws of motion, in the meantime, in his drafts Leibniz settles accounts with his past opinions.

A significant part of Leibniz's reconsideration of his views has already been told during the discussion of 'impenetrability' within the letter to Thomasius. Together with *De Rationibus motus*, Leibniz makes motion essential for resistance, and therefore, essential for bodies as well.[28] However, together with the revision of his letter to Thomasius, Leibniz discharges God from the duty of continuous creation of motion and bodies. And having cut the link between the motion and its primary reason, i.e. God, Leibniz needs to find a sufficient explanation for mobility and motion, so that he can reformulate his theory of motion.

In fact, Leibniz pens his more systematic and relatively more sufficient explanation of motion that is in harmony with his initial aims in his *Theoria Motus Abstracti*. But before it, in *De Rationibus motus* he goes on to improve his theory of motion outlined in the unrevised version of his April letter to Thomasius. However, its new version illustrated in *De Rationibus motus* is in many respects, at odds with Leibniz's forthcoming hypotheses in the *Theoria Motus Abstracti* and the *Theoria Motus Concreti*, and it is still a draft which is not published by Leibniz.

[28] And in place of 'impenetrability', he speaks of elasticity and inelasticity, degrees of which depend on the cohesion between parts of a body. (Bassler 1995, 181).

2.4 The First Formulation: Points and Gaps

Although Leibniz abandons the theory of motion he outlines in his letter to Thomasius and in the drafts of *De Rationibus motus*, I believe that this simple theory of motion is still important, because it is possible to extract a formulation from it concerning the composition of the continuum. If this formulation can be put in a generalized form, it can be considered as the origin of what I will call the synopsis of one of the main possible solutions that Leibniz proposes to the problem of the composition of the continuum. Furthermore, the ideas that are to be contained in this formulation would also recur in more or less different forms in some of the future writings of Leibniz.

At this point, I would like to say that in search for deriving formulations from the different accounts of Leibniz, I draw inspiration from Richard Arthur who suggests a formulation from the theory of motion that is offered in Leibniz's letter to Thomasius and *De Rationibus motus* in his "From Actuals To Fictions: Four Phases in Leibniz's Early Thought on Infinitesimals." In fact, in his essay, he nicely analyses Leibniz's early efforts to find a solution to the problem of the continuum, and classifies these efforts into four phases, each of which represents a kind of theory of the continuum in association with Leibniz's conception of "infinitely small." This theory of motion, together with its counterpart in *De Rationibus motus*, stands for one of these phases, although in this form it apparently demonstrates the *discontinuity* of motion. I will be discussing Arthur's first formulation with regard to the composition of the continuum, as soon as the additions Leibniz makes to this theory of motion in *De Rationibus motus* are covered in what follows.

2.4.1 "Unassignable" Gaps within Motion: Rudiments of Infinitesimals

The addition Leibniz makes in *De Rationibus motus* can be traced back to Pierre Gassendi. Gassendi is one of the modern atomists of the 17th century, who presents a conception of world as consisting of very hard atoms and void interspersed between these atoms. Accordingly, he thinks that there is a discontinuity in the physical realm as opposed to the infinitely divisible mathematical one, although the discontinuity of parts is not something we can distinguish by senses (LLC, 362).[29] Similarly Gassendi thinks that motion is not continuous, but contains several rests in it. Furthermore, he explains the varieties in the speed of motion on account of the interspersed rests:

> [J]ust as we conceive the light of the midday sun to be the brightest, and then the various degrees of light down to pure darkness to be created by the intermixture of more or less darkness, so may we conceive motion by which we infer Atoms are carried through the void … to be the swiftest; then all the degrees there are from this to a pure rest, are made up of more or fewer particles of rest intermixed. (ibid., 363)

Now the addition that Leibniz makes to his theory of motion is clearly based upon this Gassendian idea of states of rest interfused within the motion of a body, where the speed of motion is dependent on the number of those rests. The relevant passage in *De Rationibus motus* is indeed a short one and goes as follows:

> Whatever moves more slowly does so because of several little intervals of rest (*quietulas*) interspersed. What moves more quickly does so because of fewer. A little interval of rest is an existence in the same place for a time smaller than any given. (Leibniz quoted in Arthur 2007a, 5)[30]

Richard Arthur points out that the expression "a time smaller than any given" is different from "the smallest" unit of time, and rather implies an infinitely small quantity, which is the main difference of Leibniz's idea of rest from the finite but insensible rests of Gassendi. Arthur inspects this idea in connection with what he calls the "unassignable" times in the April letter to Thomasius.[31] He suggests that these intervals here denote a time longer than a *moment*. In other words, Leibniz does not compose time out of minima or likewise space from points. Accordingly, Arthur thinks that Leibniz is in *De Rationibus motus* in favor of a continuous motion interrupted by "unassignable" intervals of time. He thinks this is another

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[29] See also Fisher, and Leclerc, 123-124 .

[30] Bassler reminds us that there is no implication that there are different "grades of rest" that has an influence upon the velocity of bodies. The sole factor is the frequency of these rests. (Bassler 1995, 200).

[31] Bradley Bassler, on the other hand, prefers the expression "inaccessibly small" in place of "smaller than any given." See Bassler 1998, 3, fn.7 for his justification. I believe that they are tantamount, in the sense that they both try not to imply Descartes' "indefinite," which Leibniz in the *Theoria Motus Abstracti* claims to be "not in the thing, but in the thinker" (L, 139).

point that distinguishes Leibniz from Gassendi, since Gassendi has in mind a discontinuum, which is "not discernible by the senses" (ibid.). Although Leibniz himself uses the term "unassignable" firstly in the *Theoria Motus Abstracti* and the term "infinitesimal" later in his career, Arthur considers the idea of an interval "smaller than any given" in connection with the discussion of "unassignable" times to constitute the rudiments of the concept of *infinitesimal*s (ibid).

It is nevertheless worth noting that there is a difference between those similar ideas articulated in the April letter to Thomasius and in *De Rationibus motus*. Whereas in the letter, Leibniz asserts that bodies "are nothing at any time midway between the instants in motion," in *De Rationibus motus* they are only "at rest." But as stated in the previous drafts, Leibniz regarded bodies at rest as equivalent to empty space, which is barely different from 'nothing'. I think this difference can be understood given Leibniz's exclusion of the continuous creation thesis, in which the bodies exist only at the instants they are created.

Now, proceeding from this similarity, and also keeping in mind Leibniz's later theories that parallel these two, Arthur identifies the idea of interspersed unassignable intervals of rests as one of the theories of continuum that is to be found in Leibniz's philosophy:

> the continuum is composed of assignable points separated by unassignable gaps; in particular, the motion of a body consists in its creation at assignable instants, its being non-existent in between. (ibid.)

In fact, later in the *Phoranomus*, Leibniz himself declares that in the past he had a conception of the continuum as composed of points and a theory of motion based upon the existence of little rests: "… since I was not yet versed in geometry, I persuaded myself that the continuum consists of points, and that a slower motion is one interrupted by small intervals of rest" (Leibniz quoted in Arthur 1998, 113). Nevertheless, Arthur's formulation may be considered an oversimplification insofar as it takes 'gaps' and 'rests' as commensurable, because 'gaps' seem to imply, whether spatial or temporal, something that in fact is *not*, or that cannot be sensed, whereas 'rest' is a possible state of bodies which can be sensed. Yet there are two possible ways to defend Arthur's formulation. In a sense, each of Leibniz's arguments can still be taken to be based on a kind of mathematical or abstract account. Where 'rests' or 'gaps' correspond to 'zero', the composition of the continuity of motion can be examined in a way that parallels that of space or time, which is indeed a feature that makes this formulation meaningful. In this sense, motion can be seen as one of the continua (or discontinua), the other two being the spatial continuum and the temporal continuum. The second reason is more persuasive and as a matter of fact, more consequential. It can be argued that 'rest' is essentially a kind of 'gap', particularly if one recalls Leibniz's identifying it with 'nothing'. In this sense, what we ordinarily call 'rest' is nothing other than a limiting case, just as in the case of what we ordinarily call 'vacuum'. A body in absolute rest accordingly corresponds to 'nothing', just as an absolute vacuum does. Otherwise, it only denotes a magnitude of motion, smaller than any given.[32] When thought of in this way, 'gap' may have a universal definition, applicable to all of those three kinds: *that in which no motion exists*.

One may reasonably argue that there is however a fundamental problem in calling that formulation—the formulation of the composition of the continuum from assignable points and interspersed unassignable gaps—a theory of *continuum*, while there are gaps within. Bradley Bassler thinks that by those rests, motion is "discretized" and therefore not continuous (Bassler 1995, 199). As a matter of fact Arthur is also aware of the "… tension between the claim that the motion is continuous, in that there is no assignable instant at which it will not exist, and that it is discontinuous, in the sense that there really are gaps in motion, even if unassignable …" (Arthur 2007a, 8). And he adds that Leibniz becomes aware of this tension later in his career, particularly in his dialogue *Pacidius Philalethi*.

[32] There is still a difference between temporal or spatial gaps, and possible gaps within motion. One can think of temporal or spatial gaps without making reference to body. But from the usual standpoint, it seems impossible for a motion to be considered apart from any body or from anything that is in motion (even if it consists in a point), no matter whether it is taken only in a geometrical account. But one of Leibniz's later thoughts, which will be examined in this thesis, and in which motion is deemed as the essence of existence, provides a different outlook that would turn that "usual perspective" over.

Furthermore, this problem is in one sense related with the distinction between geometrical and physical accounts or similarly with the distinction between phenomena and what Leibniz propounds as its mathematical or abstract ground. As is to be seen soon, Leibniz thinks that there is always a deviation between the two, though a negligibly small, and more importantly, an inconsequential one.[33]

As a result, I believe that considering the formulation Richard Arthur puts forward as one of Leibniz's theories of the continuum is an attractive idea, which is useful for categorizing diverging attempts of Leibniz for a way out of the labyrinth. As is indicated, the conclusion that this is an account of a continuous motion is open to debate. Indeed, it does not need to yield a continuum in order to be related with the labyrinth of the continuum. The formulation still concerns continuity, and in the end, it gives a scheme about the composition of the continuum, that will later recur in similar forms with different contents. And this abstract scheme is applicable to geometrical, metaphysical and physical analyses of the continuum, though the constituents of the formulation should be explicated.

In accordance with the framework of the account which demonstrates the continuum as composed of assignable points between which there are unassignable gaps, it would be maintained that the continuum is composed of assignable units, which could be instantiated by very small but "assignable" points, substances, atoms, spaces, durations with respect to geometrical, metaphysical, corporeal, spatial and temporal continua, with the relevant "unassignable" gaps interspersed within each of them. However, what 'assignable' and 'unassignable' suggest for each of these continua should still be explained. For instance, it is implied, particularly with respect to the time units, that these assignable units are not minima, that is, moments which do not imply duration, or an extension of time. Nevertheless, for that time being it is not explicit in the given accounts. Even if it were explicit, with respect to all these

[33] The relation between Leibniz's theories based upon mathematical models and the phenomena is particularly elaborated by Philip Beeley. For a condensed version of his ideas, see his article in Brown 1999.

different kinds of units, one would expect to know how they differ from minima but still can be the ultimate units of the continuum. And this is in fact a question which Leibniz tries to answer beginning with the *Theoria Motus Abstracti*.

2.4.2 Concluding Remarks: From the Rudiments to the Continuum

In fact, it is not only the formulae derived as the possible solutions for the labyrinth of the continuum that is important for this study. As is indicated before, one of the main purposes is to go along with Leibniz in this labyrinth, and see how and why he amends his ideas and theories. And so far we have taken a long way in the labyrinth, beginning from Leibniz's dissertation, through the theological writings and ending with his April 1669 letter to Thomasius and its wake. Now it is possible to recapitulate this thread we have followed so far.

The way inside the labyrinth begins with a conceptual "continuum" founded between two fundamentally different infinities, namely God or incorporeal minds and bodies or the corporeal world, by means of the fragile mediation of motion. Then it is seen that initially Leibniz takes bodies to be analogous to mere matter or even to space, being infinitely divisible and of infinite number of parts. But when it comes to the knowledge of this infinite division, the deviation between geometrical and physical accounts, and therefore between ideal and real, comes gradually into sight.

In Leibniz's theological writings, it is perceived that his being "in-between" among Aristotelian, scholastic/religious and modern ideas shapes his thoughts. There he argues against the moderns claiming that the primary qualities of bodies, which are magnitude, figure, and motion, do not follow from their nature. And these qualities together with the cohesion of parts can ultimately be explained solely by an incorporeal principle. First, he takes this incorporeal principle to be God. But afterwards, he notices that this is something that disregards the autonomy of minds in union with their corresponding bodies, which Leibniz aims to incorporate to his system through an Aristotelian conception of individual substances. Therefore, he gives this role of God to the individual minds, which are somehow connected to their corresponding bodies. But in both cases, the relation between corporeals and incorporeals is left ambiguous. Finally, Leibniz distinguishes bodies from mere space and extension, by their motion, the reality of which is continued to be discussed for a long time.

In his letter to Thomasius, Leibniz continues to follow the same thread, and distinguishes primary matter from bodies. Primary matter, which is devoid of form and at rest, is continuous. But whether it is a *real* entity is dubious. On the one hand, it differs from space, which is mathematical, by its resistance, but on the other hand, it is not a body since it is at rest. In fact, It seems as an ideal continuum composed of "parts" that are not really parts at all, since they have no determinate figures. Perhaps the best word to describe primary matter is potential: It is potentially divisible to bodies with forms, and it has a potential for motion. Together with motion, which is a gift of God to the primary matter at rest, there emerge bodies with determinate boundaries separating each of them from the others. Although there are no vacua in nature, bodies together form a discontinuum, since they are not continuous but contiguous. Thus, it would not be wrong to say that there is a corporeal discontinuum in nature. On the other hand, bodies are recreated in each and every instant of their motion, and they do not exist between the instants. If those gaps are ignored because they are unassignable, motion is continuous, but if gaps are considered, then motion is not continuous and what exists during the time between the instants is *nothing*. Thus although Leibniz avoids spatial voids, he comes to posit temporal vacua within his account of continuous creation. And last, given that 'motion' is unreal, mind and body seem to be discontinuous, in the sense that there is no real plane to bridge these two parts of one individual substance.

As Leibniz gets more familiar with the philosophy of Thomas Hobbes and his favorite notion of *conatus*, he chooses to abandon his conception of little rests and continuous creation. Although the influence of Hobbes manifests itself in the final draft of *De Rationibus motus* as well as in other pieces of writings written immediately after it, and is even more obvious in Leibniz's most systematic writing of the period, i.e. *Theoria Motus Abstracti*, the most explicit confirmation of Hobbes' influence can be found in Leibniz's own letter to Thomas Hobbes. In this letter written in July 1670, Leibniz expresses his admiration for Hobbes' general opinions about motion. Since many comments Leibniz writes in this letter are incorporated into the *Theoria Motus Abstracti*, I find it appropriate to discuss them in the next chapter. Therefore, finally, it is time to deal with this work without lingering further.

CHAPTER 3

THE THEORIA MOTUS ABSTRACTI: THE CONTINUUM OF INDIVISIBLES

This chapter will mainly focus on the most important writing concerning the labyrinth of the continuum among the earliest works of Leibniz, which is called the *Theoria Motus Abstracti* (Theory of Abstract Motion)[34]. This work is known to have been prepared together with the *Theoria Motus Concreti*[35], in the winter of 1670-71. *Theoria Motus Abstracti* can be interpreted as a clear indication of the fundamentality of the problem of the composition of the continuum for Leibniz's metaphysics. Likewise, it is equally fundamental for a philosophical conception of physics. As Bassler writes "[i]n the TMA [*Theoria Motus Abstracti*] Leibniz makes the centrality of the continuum problem for the metaphysical foundations of physics apparent by placing his discussion of it at the very beginning of his work" (Bassler 1995, 227).

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[34] The full title is *Theory of Abstract Motion, or, The Universal Reasons for Motions, Independent of Sense and the Phenomena*. This work is presented to the French Royal Society.

[35] The *Theoria Motus Concreti* is submitted to the London Royal Society. This work is also known as the *Hypothesis Physica Nova* (A New Physical Hypothesis) since the full title of the work on the cover page is *A New Physical Hypothesis; by which the causes of most of the Phenomena of Nature are derived from a certain unique universal motion, supposed in our world, without disdaining either the Tychonians or the Copernicans*, and the text begins with the title *Theory of Concrete Motion; or, A Hypothesis About the Reasons for the Phenomena of Our World*. Thus "Theoria Motus Concreti" and "Hypothesis Physica Nova" are taken as alternative headings. According to Richard Arthur's cataloging, the *Theory of Abstract Motion* is added to the *Hypothesis Physica Nova*, since Leibniz's hypothesis is further included in this writing, which is written successively.

On the other hand, it is the *Theoria Motus Concreti* that is concerned with nature in general, or the world of phenomena, often tied with concepts such as divisibility and elasticity. In the *Theoria Motus Concreti*, Leibniz speaks as if there is continuity in the realm of things, as he also holds in some of his much later writings. In agreement with his *Dissertatio*, he explicitly states that the continuum is divisible to infinity and further infers from this general thought that every sensible quality one perceives in a thing can also be perceived in a much smaller thing, and this goes on to infinity. Likewise, every "atom" is a kind of world, which contains infinite species within itself. In Leibniz's famous words, there are "worlds within worlds to infinity" (LLC, 338). However, for Leibniz, deriving this outcome from our sensibility is not a sufficient account by itself, in the sense that it is still a reasoning from analogy, and demands a causal, rational explanation, as he also defended against Wren and Huygens in *De Rationibus motus*. But giving a rational and systematic account appears to be more of a task to be undertaken in the *Theoria Motus Abstracti*.

It would not be unfair to say that the new hypothesis Leibniz introduces in the *Theoria Motus Abstracti* is in general based on Hobbes' concept of *conatus* (translated as "striving" or "endeavor" in different works). But Leibniz's sources of inspiration are not limited to Hobbes. This early work is again to some extent a blend of Aristotelian, Scholastic, Cartesian, and some other modern ideas, among which there are those concerning not only philosophy, but also physics, chemistry and biology. While some of his thoughts can be seen as a direct appropriation of those ideas, some others seem to emerge as arguments against certain governing thoughts of the period. Thus, although the *Theoria Motus Abstracti* is "without doubt an eclectic account" as Richard Arthur remarks (Arthur 2001, xxxii), the work that emerges is nevertheless a resourceful one.

Certainly, this work bears some critical inconsistencies as well, and they are disdained by some scholars, and seemingly by Leibniz himself as disappointing ideas, which would not have been written by a mathematically equipped Leibniz. However, it is to be admitted that these inconsistencies Leibniz dealt with prove out to be the propelling motives that push Leibniz's arguments further. Moreover, although there are various questionable reasonings, some of them are very important for his future studies, including the revolutionary mathematical ones, such as his invention of calculus. Besides, some other thoughts, particularly those concerning motion, suggest invaluable insights for the moderns and even today for us, as well.

3.1 Infinitely Many Actual Parts

In the section titled *Praedemonstrablia* (Predemonstrable Foundations) of the *Theoria Motus Abstracti*, Leibniz expounds item by item the fundamental principles of his theory. Each principle is called a 'foundation'. The first two foundations together is the statement of the idea which stands for the first step of his account for the composition of the continuum in the *Theoria Motus Abstracti*:

> 1. *There are actually parts in a continuum …*
>
> 2. *And these are actually infinite*, for the indefinite of Descartes is not in the thing but in the thinker. (L, 139)

This idea that the parts in the continuum are *actual* is the cornerstone of one of the main positions Leibniz will hold and develop throughout his struggle with the labyrinth. Furthermore, Leibniz argues against Descartes that these parts are "*actually infinite*" independent of our epistemological or sensual capacity to perceive this infinitude.[36] Moreover, the term 'actual' implies that the infinite division is not merely a characteristic of a geometrical continuum, but also of the real one. Accordingly, this idea of the infinitely many actual parts in the continuum is the basis of his claim that there are worlds within worlds to infinity, which is mentioned above.[37]

In fact, the idea that there are actually infinitely many parts in a continuum does not by itself necessarily come to mean that the continuum is *composed* solely or

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[36] In fact, Leibniz reaches to that conclusion through Cartesian demonstration of the division of matter through motion. (See Levey 1998, 53).

[37] See Garber 1982, 174.

ultimately *of* these actual parts, or these actual parts are the ultimate constituents of the continuum. But by reference to the course of the reasoning in the *Theoria Motus Abstracti* and to the explicit statements concerning the infinite division of the continuum in the *Theoria Motus Concreti*, it can be maintained that the continuum is infinitely divisible and is composed of infinitely many actual parts. Moreover, since those parts are actual, then the continuum is not only divisible to infinity, but also actually infinitely divided. Although it is not yet clear what 'actual parts' mean for Leibniz, these first two foundations differentiate this account from the earlier ones, where the status of parts were left uncertain. Furthermore, heretofore, being actually divided to infinity was not distinguished from being infinitely divisible either. Together with these novelties, on the other hand, Leibniz distinguishes himself from one of his major influences, Aristotle, who does not admit the actual infinite (Aristotle, 75-77), through another one, Descartes.

Nevertheless, Leibniz disagrees with Descartes on a critical point, as well, and their divergence stems from the knowledge of the infinite, which directs our attention again to the epistemological aspect of the problem. Descartes writes in his *Principia Philosophiae* that by motion, matter is divided to infinity. He arrives at this conclusion from his studies on the motion in plenum. In contrast to atomists, Descartes defends that motion is possible in a plenum, and shows this possibility by an argument similar to that of Aristotle's, assuming a world of contiguous bodies simultaneously moving within a circle. Certainly, the width of the spaces that a body is moving in can vary, and this difference is compensated by differing speeds. However, if the plenum is considered, this change of speed does not suffice by itself, because for Descartes, it is impossible for the bodies moving in a wider space to fill a narrower space, unless at least some part of these bodies adapt their shapes to the new medium. This means a displacement of some parts however vaguely, which is considered to be "genuine division" by Descartes (Descartes, 236-239). What Descartes describes is a division to infinity and it generates parts which are indefinitely many and indefinitely small. These particles are not comprehensible to our finite minds, for a finite cannot grasp the infinite. Hence, reasoning from the motion in the plenum, Descartes describes matter as actually *indefinitely* divided.

Departing to a certain extent from his ideas in the *Dissertatio* and in his letter to Thomasius, Leibniz does not prefer the term "indefinite," and by reasoning on 'infinite', he shows his disagreement with the idea that a finite mind cannot comprehend or at least reflect on it. Instead, over time he develops his own conception of the 'infinite', along with the notion of the *unassignable*, or as in the later terminology, *infinitesimal* magnitudes.[38]

3.1.1 Denial of Minima

It has been just argued that the first step of Leibniz's account for the continuum in the *Theoria Motus Abstracti* is that it is divided into infinitely many actual parts. Now, as a second step in the development of his account for the composition of the continuum, Leibniz distinguishes his position from the one based on Euclidean points, as well. Euclid defines points as "that which has no part, or has no magnitude" (LLC, 428, en. 9). It has always been very usual to think of a continuum as composed of such points, though it is more important how these points are elucidated and how the composition of the continuum from these points is justified.

There are two important philosophers, who made their own definitions of point which influenced the formation of Leibniz's early thoughts concerning the problem of the continuum to an important extent: Galileo and Hobbes. As I have briefly mentioned in the previous chapter, Galileo suggests a model for the continuum, similar to the one we derived from Leibniz's theory of motion in the letter to Thomasius. According to this model, continuum is infinitely divided into infinitely many "unquantifiable" indivisibles and likewise there are infinitely

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[38] Bassler thinks that Leibniz will even "ground his entire position in the *Theoria Motus Abstracti* (and later) on the orientation provided by his arguments concerning the infinite" (Bassler 1998a, 3).

many “unquantifiable” indivisible voids interspersed between indivisibles (Galileo, 33, 42-44).[39] Since these unquantifiable indivisibles are not quantified, and if this is interpreted as they have no quantities, these indivisibles can be considered as instantiations of Euclidean points, though Galileo’s approach is much more sophisticated and has metaphysical connotations.

On the other hand, Thomas Hobbes, who is another source of inspiration of Leibniz in the formation of the *Theoria Motus Abstracti*, makes an alternative definition against Euclidean point. Hobbes defines point as “that whose quantity is not considered, that is, that of which no quantity or part is computed in a demonstration; so that a point is not to be regarded as an indivisible, but as an undivided thing” (Hobbes quoted in LLC, 360). Hobbes thus notes that, by ‘point’, he does not mean something having *no* magnitude.

Likewise, in the *Theoria Motus Abstracti*, Leibniz maintains that there is nothing which does not have any magnitude or part. In other words, each part of a whole, however small, has parts as well. And accordingly, by his expression in the third foundation, Leibniz suggests that “[*t*]*here is no minimum in space or in a body*” (L, 139).

Thus the second step in the development of Leibniz’s account of the composition of the continuum in the *Theoria Motus Abstracti* turns out to be the denial of minima in the continuum. However, this denial seems dubious since Leibniz himself talks about infinitely small parts, which normally seem to be synonymous with ‘minima’. But he distinguishes his *unassignable* parts, that is, parts magnitudes of which do not correspond to an assignable quantity, from minima, which are devoid of magnitude, by equating the latter with Euclidean points having no magnitude.

[39] Galileo’s account of unquantifiable indivisibles is thoroughly examined in A. Mark Smith’s “Galileo’s Theory of Indivisibles: Revolution or Compromise?” which I have already mentioned. Eberhard Knobloch’s “Galileo and Leibniz: Different Approaches to Infinity,” on the other hand, is a very valuable study on the differences between Galileo and Leibniz’s conceptions of the ‘infinite’ and indivisibles, though it considers mainly the ideas of the later Leibniz who is more proficient in mathematics.

Leibniz sets forth two different justifications for his assertion that there are no minima. The first justification concerns the *position* of the supposed minima. Leibniz thinks that things are situated in space, which means that they are surrounded and touched by several different things which do not have any contact with one another. Thus, anything whatever small has "many faces," which are considered as "parts" by Leibniz (ibid.). Since minimum does not have any parts, it is not even situated, it has no position, i.e., it is nothing.

The second justification relies on the part-whole argument, which stems from Euclidean argument that a whole is always greater than its (proper) parts. But of course, this argument can be traced even back to Zeno's paradoxes. Leibniz frequently appeals to the part-whole argument, particularly against the idea of minima. Here, in a few words, Leibniz argues that if a minimum does not have magnitude, there arises an absurd consequence that a whole has as many minima as any of its parts however small (ibid.). Though in different forms, in his future writings this argument continues to be one of Leibniz's favorites to be used in connection with the continuity problem.[40]

3.1.2 Unextended Indivisibles with Magnitudes

In the fourth foundation, Leibniz's thesis gets more complicated and controversial, when he declares that "[*t*]*here are indivisibles or unextended beings*" (L, 139).[41] Thus, while he rejects minima as well as the composition of the continuum from Euclidean points, he still believes in a peculiar kind of indivisibles. His reason for this is that otherwise, the beginning and the end of a motion or a body would be unintelligible (ibid.).

In his proof, Leibniz takes a line as the representative of the beginning of a body, space, motion or time. Since this beginning, as a line, is extended, he begins to divide it. And for each time he divides the line, he designates the left side of the remaining line as the new "beginning." But this division goes on to infinity, for if

[40] See Beeley.

[41] In his translation, Arthur prefers "things" in place of "beings."

there are no indivisibles without extension, the remaining part will always be further divisible, since it has an extension. Therefore, for Leibniz, this seemingly unintelligible status of the beginning of a space, body, motion or time means that either there are no beginnings at all or there are unextended indivisibles, as the beginning of things, which nevertheless have magnitudes.

But prior to this, despite that 'beginning/ends' argument, there is still an apparent contradiction between the third and the fourth foundations, that is, between the denial of the minima and the adoption of unextended indivisibles. Leibniz rejects minima because of the fact that, that which has no magnitude, is no different than nothing. This means that his indivisibles, which actually exist, have magnitudes. On the other hand, they have no extension. So here the tension is that there are actually existing indivisibles simultaneously having magnitude but no extension. For Leibniz, this is because the magnitude under consideration is so small that it does not denote an extension. Or as Bassler suggests, Leibniz thinks that being unextended means being smaller than any *extended* thing. (Bassler 1995, 221-22, L, 140).

Hence Leibniz asserts that the beginning of a body and space is an indivisible point, of motion is an indivisible *conatus* (endeavor), and of time an indivisible instant. But since he rejects Euclidean points, he needs to redefine point as an instantiation of *unextended indivisibles*. In the fifth foundation, he maintains that "*a point is not which has no part*, nor that whose part is not considered" (LLC, 339). This argument in fact signifies the difference of Leibniz's conception of constituents of the continuum from that of Hobbes's, since Hobbes defined point as that the quantity of which is not considered (Arthur 2001, xxxiv).

According to Leibniz's definition, a point has parts but these parts are "indistant" from each other, and their magnitude is "inconsiderable, unassignable" (LLC, 340). When the magnitude of these parts is compared to another sensible magnitude, the ratio is infinitely small, or rather, it is smaller than any given ratio. So as to make it clear, this ratio is as that of 1 to infinity, but not as that of 0 to 1, and this is a very crucial distinction in Leibniz's thought.[42] Leibniz declares this conception of point to be the "foundation of the *method of Cavalieri*" invented by Bonaventura Cavalieri (L, 140).

Although Leibniz's points have parts and magnitudes, however small, as with unextended indivisibles, they have no extension. Because if they had extension, they could not be used to demonstrate the claim that bodies have a beginning and an end. On the other hand, Leibniz says that they have magnitude and parts, since in the third proposition, he clearly sets forth that which has no part or magnitude is no different than nothing. As Bassler nicely expresses, "though these entities have a "composite" structure in the sense that they have parts, *they do not have parts into which they, in turn, may be decomposed*" (Bassler 1995, 220-21).

Here surfaces a possible question: Would not the position (situation) of the parts of these points give rise to the same objection (concerning the beginning/end of the things) with respect to unextended indivisibles? Don't these indivisibles have parts which are supposed to have situations as well? Owing to the reason that the parts of the indivisibles are unable to be decomposed, and Leibniz says that they are indistant from each other, this question remains unanswered. Consequently, Leibniz's characterization of points and their parts appear to be one of the controversial aspects of his proposed solution for the composition of the continuum in the *Theoria Motus Abstracti*.

Even though here a metaphysical account of motion with seemingly physical connotations is expounded, the idea of the composition of the continuum still from unextended points *suggest*s that Leibniz considers the continuum primarily on a geometrical plane. In fact, this is apparent in his taking lines as representatives of bodies or motions. However, since he nevertheless consider the parts of the continuum as actual, it can be maintained that ultimately, in the *Theoria Motus Abstracti*, Leibniz tries to sustain the connection between

[42] See Bassler 1998a.

geometrical and physical accounts. And again, it is his metaphysical account of motion that bridges the two accounts.

Moreover, while arguing that the beginnings of bodies cannot be lines, since lines are extended, Leibniz also establishes a link between the indivisible beginnings and these extended lines, of which they are parts. Lines and points share the qualities of having a certain position and magnitude and parts, but they are still qualitatively different. Unlike an indivisible point, a line, being extended, is infinitely divisible. Furthermore, parts of an indivisible point are quite different than the parts of a line for two reasons. First, parts of an indivisible point cannot be decomposed, and second, these parts are "indistant" from each other. But how can extended lines be composed out of points qualitatively different from them? This is the foremost riddle of the labyrinth of the continuum, which is to be dealt in the following section.

3.1.3 Composition of the Continuum

The *Theoria Motus Abstracti* reveals the fundamental difficulty that has been lying dormant at the very heart of the labyrinth of the continuum: composition of a continuum from constituents that are not homogeneous with the continuous whole itself. The most basic example concerns the relation between points and lines. Composition of lines from points which have fundamental qualitative differences from the things they compose is a problem common to many similar accounts concerning the composition of the continuum.

This problem becomes particularly apparent when an abstract geometrical account is applied with respect to physical phenomena. Certainly, as the title itself suggests, in the *Theoria Motus Abstracti*, Leibniz upholds an "abstract theory of motion"; his handling of the issue differs from the *Theoria Motus Concreti* in certain respects. However, as is argued, the *Theoria Motus Abstracti* is nevertheless a metaphysical theory, which postulates a fundamental ontological structure. And this structure still grounds the ideas behind the account of concrete motion. Therefore, the metaphysical as well as the physical consequences of the account can be considered and interpreted on their own, providing that the bond between these accounts and the geometrical one is not ignored. For considering the theory to be solely geometrical may lead to a failure to perceive how Leibniz tries to connect the "abstract" to the "concrete." On the other hand, seeing these connections is also important in understanding Leibniz's later distinctions of the ideal and the real realms.

In any case, in the *Theoria Motus Abstracti*, while puzzling over the composition of the continuum, Leibniz does not only think of a schema of a possible solution, but an actually existing one. And perhaps the best evidence for this claim is Leibniz's assertion that there are "actual parts" in the continuum. However, there is still an ambiguity regarding those "actual parts" of which the continuum is supposed to be composed. Since indivisibles are the beginnings of certain bodies or motions, it can be said that there *are* extended bodies, or parts, and motions. If these extended ones are regarded as "actual", they can be understood as the actual parts that are the *constituents* of the continuum. Yet apparently they would not be basic, fundamental constituents. Moreover, in this case it would not be clear whether bodies or motions or both of them are the actual constituents of the continuum. If, on the other hand, those indivisibles are the fundamental constituents of the continuum, the previous argument that "'there are infinite actual parts in the continuum' comes to mean that 'the continuum is actually infinitely divided'" turns out to be false, for as their name suggests, indivisibles are not further divided.[43]

Richard Arthur remarks that Leibniz's indivisibles "are not explicitly identified with the actually infinite parts … i.e. they are not said to compose the continuum" (Arthur 2007a, 9-10). Similarly, Bradley Bassler notices that in the *Theoria Motus Abstracti*, Leibniz never says that the continuum is *composed* out of indivisibles, or that there are infinitely many of them (Bassler 1995, 221). But both of these

[43] This inconsonance between the ideas of infinite division and infinitely many indivisibles once again unveils the intricate character of the 'infinite' and its importance in the accounts of the composition of the continuum.

scholars see sufficient reasons for interpreting indivisibles as the constituents of the continuum. Arthur appeals to Leibniz's letters where Leibniz claims that his *Theoria Motus Abstracti* provides "a solution to the problem of the composition of the continuum in terms of points one greater than another" and he argues that this claim of Leibniz would make little sense if the points were not parts of the continuum" (Arthur 2007a, 10).[44] Likewise, Bassler believes that, although Leibniz does not explicitly say that the continuum is composed of indivisibles, or there are infinitely many indivisibles, both of these conclusions nevertheless follow from Leibniz's reasoning. This is a claim worth considering further. Bassler correctly reasons that there *should* be infinitely many indivisibles, if it is accepted that the continuum is divided to infinitely many extended parts. For these infinitely many parts *must* have as many indivisibles as their beginnings and ends. Obviously, his reasoning is also true if what is implied by "infinitely many actual parts in the continuum" *are* these indivisibles themselves. However, it is not so easy to claim that the continuum is *composed of* indivisibles without falling into inconsistencies. Although Bassler's argument makes sense when he says that the composition of the continuum from the "smallest" extended parts is impossible without contradicting Leibniz's proof for indivisible beginnings and ends, it is yet equally impossible to say that continuum is both infinitely divided and is composed of indivisibles.

Besides, one could put forward a kind of extreme argument which leaves Bassler's two conclusions undecided.[45] If these indivisibles have parts—and indeed they do according to Leibniz's own definition of a 'point'—those actual parts implied in the first foundation *may* be these unextended parts. In this case, although they are infinitely many, it does not follow that there are infinitely many indivisibles comprising these parts. One can think of a finite number of indivisibles having infinite number of unextended parts. Furthermore, since they

[44] This idea of comparable points will be examined below very soon.

[45] Bassler thinks of this possibility and considers it an "extremely implausible position" (Bassler 1998a, 9).

are still "parts," they can be said to constitute the indivisibles, which means that they are the actual fundamental constituents of the continuum. Nevertheless, I definitely do not think that this is what Leibniz has in mind...

3.2 The Theory of *Conatus*

Leaving these questions suspended at least for now, we can continue to follow the advancement of the *Theoria Motus Abstracti* from Leibniz's transition to the notion of *conatus*. Since the *Theoria Motus Abstracti* is a new theory of motion, its treatment differs from the earlier ones in an essential way. The notion of *conatus* plays the leading role in this change.

In the previous chapter, I tried to emphasize the bridging function that 'motion' played in those early writings. In this new account, this bridging function begins to be performed by the notion of *conatus*. As is indicated within the previous main section, *conatus* designates the beginning of the motion of indivisibles. In this sense, as it will be clearer, it is the metaphysical point where the geometrical account binds to the physical one. Furthermore, it is the "agent" that provides the connection between parts of a continuum, by means of the cohesion it brings about. And finally, though this particular aspect of *conatus* will be examined a bit later in this chapter, bodies and minds are brought together through this notion *conatus*. Now it is time to take a look at what *conatus* is, and how Leibniz places it in his *Theoria Motus Abstracti*.

3.2.1 A Striving for Motion

To begin with, in the *Theoria Motus Abstracti*, while unextended indivisible points are the beginnings of bodies in space, *conatuses* are considered to be the beginnings of motion. By Richard Arthur's definition, *conatus* is an "instantaneous tendency to move" (Arthur 2001, xxxiii, see also Carlin 372). Thus, in a sense, *conatus* is a kind of a potential to move, just as mobility is for primary matter of the letter to Thomasius. But this time, it is not that there is a kind of ideal mass at rest from which infinitely many motions are derived. Rather it appears to be that there are infinitely many *conatuses* which continuously instigate the motions of bodies.[46]

Furthermore, unlike the idea Leibniz recently considers in the fourth draft of his unpublished *De Rationibus Motus*, there are no unassignable gaps in motion. According to the account maintained in the *Theoria Motus Abstracti*, the motion propagated by *conatus* is "continuous or not interrupted by little intervals of rests" (L, 140). And once something is moved, it always moves until it is caused not to, or if it is at rest, it will stay at rest until it is caused to move.[47]

However, it is not perfectly clear if Leibniz does in no way allow for the possibility of things' being at rest. In foundation eight, possibility of rests is implied by the claim that a thing at rest can move when "a new cause of motion occurs" (ibid.). To the contrary, in the sixth foundation, paralleling his earlier view that a body at rest is no different than nothing, Leibniz notes that "the ratio of rest to motion is not that of a point to space but that of nothing to one" (ibid.). Thus on the one hand 'rest' seems to be a possible state of bodies, on the other hand, it has "no magnitude" and therefore, is a minimum. And according to Leibniz, minima do not exist in space or in a body. It is not explicitly stated that a minimum does not exist in motion either, but the reason for the impossibility of minima should be applicable to motion as well: Just as space and bodies, every motion has a magnitude, and having magnitude means having parts and it is absurd that in any part there are as many minima as there are in the whole.

However, this part-whole argument concerns the constituents of a continuum, and it is *conatus*, not 'rest', that can be considered as constituent of motion. The

[46] Garber (1982) writes that "… if the body contains (or even, perhaps, is *composed of*) an infinity of nonextended parts, each of these parts must *also* have its *own* conatus" (175)

[47] As Bassler indicates, this causality implies that the theory of abstract motion is not solely a geometrical or mathematical one: "… when the problem of the composition of the continuum is construed in a fuller sense to include problems of how motion comes to be and passes away—that is to say, when the dynamic problem of the generation of magnitude is addressed—then the continuum problem is an intrinsically physical, rather than a mathematical one" (Bassler 1998a, 16).

reason for this is not that 'rest' is not a motion, but '*conatus*' is; '*conatus'* is not a motion, either. According to the account presented in the *Theoria Motus Abstracti*, *conatus* is, at most, a striving for motion. One reason why rests cannot exist within a motion is because Leibniz takes every motion as a causally combined whole. As is indicated, if something moves, it continues to move until it is caused not to. Hence 'rest' implies a new cause, and thus cannot exist *within a* motion. Consequently, rest can only be considered as the end or the beginning of a motion, for what is the status of a body if it is not at rest when it no longer moves? However, rest is a minimum, if not a 'nothing', and the beginning and end of a motion is not a minimum, but an indivisible, which is in case of motion, a *conatus*.[48]

Conatus is qualitatively different from motion, in the sense that it is not a motion, just as a point is not an extension, though it is the beginning of any "extension." But in the *Theoria Motus Abstracti*, indivisibles seem not only to begin or end bodies, time or motion, they can also be interpreted as composing them. Because, being a tendency to move, *conatus* is, in a sense, the cause of motion. Furthermore, every motion has *conatus* as its part. If this interpretation that the *conatuses* compose motions is adopted, then it can be further argued that *conatus* not only composes motion, but also through the motion of the point, it abstractly constitutes space, and through the motion of point in space, it abstractly constitutes time. In this sense, *conatus* plays a unifying role between abstract and physical accounts of motion. Now it is time to elaborate how this composition occurs, in order to understand how the formulation of a new solution to the problem of the continuum is derived.

Leibniz says in the thirteenth foundation, that when a body is in motion, during the time of its *conatus*—which is an infinitely small time interval—one point of the body is "*in several places or points of space*, that is, it will fill a part of space greater than itself, or greater than it fill when it is at rest, or moving more slowly"

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[48] For a neat discussion of rests in the *Theoria Motus Abstracti*, see Bassler 1998a.

(LLC, 340). However, the space it fills when in motion is still unassignable, or for Leibniz, can still be considered as a point, because "in one instant no endeavour can traverse more than a point, or a smaller part of space than can be expounded, otherwise in a time it would traverse an infinite line" (ibid., 342).

But there still appears to be a qualitative difference between these two points, i.e., the point of the body, and the "point" it occupies while moving. As is indicated before, this difference is one of the key features of this account: The ratio between these two points is analogous to that of a point to a line or that of "an angle of tangential contact to a rectilinear angle" (L, 140). Nevertheless, it seems problematic that both the point and the space it fills when in motion are unassignable points, for the definition of unassignable is to be smaller than any given. This variety of unassignable "points" which are both smaller than any given suggests that there are different "kinds" of unassignable points. However, even if this is the case, Leibniz does not account for how these two kinds can be commensurable, that is, can have a ratio to one another. In fact, this is one of the difficulties that Leibniz tries to overcome in the account that is to be examined in Chapter 4.

The importance of Leibniz's ingenious idea that a point in motion occupies a larger space than it would fill at rest for the *Theoria Motus Abstracti* is momentous. This is not just a geometrical reasoning. Leibniz universalizes this idea and asserts that it applies to all that is moving: "*whatever moves, is never in one place when it moves*, nor indeed in one instant or least moment of time" (ibid.). Since there are no temporal minima, the space a body occupies is always greater than it would do when at rest. In a sense, this *is* the meaning of motion, or changing place. Just as a *conatus* itself, a body as a whole, which is propagated to move by *conatuses*, begins and ends its motion at different places during an instant. In Leibniz's words, "*No conatus without motion lasts longer than a* *moment except in minds*. For what is conatus in a moment is the motion of a body in time." (ibid., 141).[49]

Parenthetically, as Leibniz himself states, this seventeenth foundation is also his gateway to the mind-body problem. Leibniz here uses his theory to distinguish bodies and minds in an uncommon way. For Leibniz, bodies can be considered as "instantaneous minds," in the sense that they exist only instantaneously, and therefore lack memory (LLC, 341). Similarly, bodies lack the faculty of thought, and therefore they do not perceive their own actions and passions. This is because they cannot retain their *conatuses* for longer than a moment (ibid). Based on this argument, it is reasonable to say that one can find thoughts foreshadowing his future ideas concerning the categories of perception among different monads, such as the difference between apperceptions and *petites perceptions*.[50]

Thus in the *Theoria Motus Abstracti* the difference between bodies and minds is in a way defined with respect to *conatuses*. On the other hand, their connection is provided via *conatus*, too, in the sense that both have *conatus* as the origin of their actions. Furthermore, there are scholars who interpret these actions of minds as composed of motions. While distinguishing Leibniz's conception of *conatus* from that of Hobbes's, Konrad Moll indicates that

> Hobbes and Leibniz treated the problem of composition physically by adding and subtracting quantities or directions of motion, of endeavors in a geometrical way. But to Leibniz this is only one side of the problem of motion, for to him mental activity also is a sort of motion of endeavors. (Moll, 74-75)

Likewise, Loemker writes in his notes to the translation (of the *Theoria Motus Abstracti*), that via his account presented in the *Theoria Motus Abstracti*, Leibniz

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[49] To see that the same idea (that a moving point occupies a larger space than it would fill at rest) applies with respect to the *parts* of *a* body, see Leibniz's letter to Hobbes, particularly in L, 105-107)

[50] Daniel Garber (1982) similarly states that this "mentalization of the physical world and its grounding in an infinity of nonextended things" which, he thinks, are *conatuses*, are "*the first clear steps toward the monadology*" (176).

moves "beyond the mind-body dualism of Descartes to a position in which both are analyzed into elementary motions endowed with feeling" (L, 144).[51]

In any case, through *conatus*, Leibniz differentiates his metaphysics on account of his conception of active minds. Indeed, the significance of his theory of *conatus* with respect to the mind-body problem is that this theory bestows the active principle to individual minds themselves. As argued in Chapter 2, together with the revision of the letter to Thomasius, Leibniz cuts the link between motion and its cause, that is, God, and does not yet replace it with a satisfying principle that would provide at the same time autonomy to minds and the simplicity of mechanistic accounts. But Leibniz's use of *conatus* at this point becomes more explicit in his letter to Arnauld, which will be dealt with later in this chapter.[52]

3.2.2 Cohesion through Conatus

Another nifty consequence of the *conatus* theory pertains to the problem of cohesion. As emphasized before, cohesion is an indisputable face of the problem of the continuum, for which the theorists dealing with the problem have always had to give a reliable account which is also in harmony with their overall standpoint regarding the composition of the continuum. In this respect, Leibniz also gave different answers to the cohesion problem, while revising his theories concerning the problem of the continuum. As is seen under the subtitle of *Confessio*, Leibniz was not content with the atomistic solutions to cohesion suggested by the moderns. However, later he ceases to be satisfied with his own explanation that cohesion is ensured by God. Hence, in the *Theoria Motus Abstracti* he proposes a radically different solution. This original solution, which has an Aristotelian conception of continuity as its basis, proves to be one of the

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[51] In fact, there is a kind of overstatement here. As Bradley Bassler indicates, Leibniz neither explicitly states that *conatus* generates *motion* in minds, nor he attributes feeling to bodies. See Bassler 2002, 225; Bassler 1995, 245).

[52] Transition from the *Theoria Motus Abstracti* to the letter to Arnauld concerning the relations between motion and minds, and motion and bodies is wonderfully discussed in Bassler 2002: "Motion and Mind in the Balance: "The Transformation of Leibniz's Early Philosophy"".

most key characteristics of Leibniz's overall account of the continuum in the *Theoria Motus Abstracti*, and even of the account in his first Paris writings.

This time Leibniz explains cohesion of bodies by means of the notion of *conatus*. He asserts that through *conatuses*, colliding bodies *penetrate* into each other "*at the time of collision,* impulse, or impact" (LLC, 341). When a body collides with another body, in Leibniz's words, it "endeavors to move into the other's place, it begins to be in it, i.e. it begins to penetrate, or be united" (ibid.). Accordingly, boundaries of two bodies are situated at the same point of space. Thus, by their penetration into each other, their boundaries become one, and hence they become continuous in an Aristotelian sense. The conclusion differs from an older conclusion that can be derived from Leibniz's letter to Thomasius that discontinuity is a condition of having a diversity of bodies. And the new conclusion can be nicely summarized by Samuel Levey's monologue: "Does this mean that a variety of distinct actual parts can, if cohering, form a true continuum? Apparently that is what Leibniz means to defend …" (Levey 1999, 102).

To sum up, heretofore the *Theoria Motus Abstracti* tells us that motion is continuous and is generated by *conatuses*, which are related to motion as points are related to bodies and space. Continuous motion is never interrupted by rests. This means that *conatuses*, which do not last longer than a moment, which is infinitely small, follow one another in such a way that, there are no rests between any two of them.

Both in different bodies and within the same body, there may be *conatuses* towards different directions. Although there are *conatuses* in a body with opposite directions, that body can still be in motion, the magnitude of which is determined by the amount of the difference between these *conatuses*. Every motion means a change in place. When thought of at the level of *conatuses*, it can be said that each point of the body in motion strives to change its place, and penetrates into the spaces of the other points. During this reciprocal drive, boundaries of the bodies or the different parts of the same body become one. Hence bodies or parts cohere and are said to be continuous. In a nutshell, a continuous motion generated by the continuous propagation of *conatuses* result in the cohesion of bodies, and hence brings about a corporeal continuum. And this is Leibniz's solution to the continuity of things by his *conatus* theory in the *Theoria Motus Abstracti*.

3.3 The Second Formulation

Richard Arthur expresses this solution by means of a neat formulation, which he considers to be Leibniz's second theory of the continuum in his early years:

> The continuum is composed of indivisibles, defined as parts smaller than any assignable, with no gaps between them. Indivisibles have indistant parts, but no extension, and stand in the ratio of 1 to ∞ with the continuum they compose. Having parts, they have magnitude, so that any two indivisibles of the same order and dimension have a finite ratio. (Arthur 2007a, 8-9)

This formulation smoothly explains Leibniz's account of the composition of the continuum in the *Theoria Motus Abstracti*, and sets forth the differences of the new solution from the one illustrated by his Arthur's first formulation that is given in the previous chapter. However, this formulation is not complete in the sense that it does not give an account of cohesion within the continuum. Thus it would be more exhaustive if it included the cohesion through *conatuses*. This addition would also imply that in the *Theoria Motus Abstracti* composition of the continuum is provided only by means of motion of the actual parts.

3.3.1 Comparability of Points: The Key to the Solution

Although the very crucial characteristic of Leibniz's account, i.e., cohesion of *conatuses* by means of interpenetration is not involved in Arthur's formulation, he includes another distinctive idea that differentiates Leibniz's theory. As indicated above, Leibniz of the *Theoria Motus Abstracti* maintains that indivisibles have magnitudes and these magnitudes have a finite ratio with respect to each other. Later he indicates that his new definition of points was a very considerable advantage of his account, and was the key to the solution of the problem of the continuum:

> [the TMA] examines the reasons for abstract motions, and unfolds the wonderful nature of the continuum... so that as one endeavour is greater than another, so is one point greater than another, in which way I not only escaped from that whole labyrinth of the continuum, but also saved the Cavalierian geometry of indivisibles" (Leibniz quoted in Arthur 2007a, 10)[53]

Thus the characterization of points as unassignable indivisibles with magnitudes, does not only suggest that Leibniz's points, unlike minima, do not have to be equal with one another, but also makes it possible to compare these points on account of their having finite ratios with respect to each other.

What is more significant is that the idea of comparable points applies to *conatuses*, the basic units of motion. Accordingly, a variation in the velocity of motion, which is previously explained by the existence of interspersed rests, is now explained through the variations of the points, or *conatuses*. In fact, Leibniz argues that the inequality of points follows from the inequality of *conatuses* (L, 141). When one *conatus* is greater than another, it means that the motion generated by the first one will be faster than the other, and hence the first motion will cover a bigger space than the latter in a given time instant. I think this idea of comparing motions with respect to the *conatuses* instigating them supports the idea that *conatus* contains in a sense a potential to move. That means, a *conatus* with a bigger potential, would imply a bigger striving for motion, and therefore would stimulate a faster motion.

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[53] However, the same thing that holds for space points and *conatuses*, does not hold for instants, i.e., time units, which are all equal to each other. Leibniz needs time units to be uniform in order to render it possible to measure the motion. But his reasons, which are given in the foundation eighteen, are not much persuasive. In this sense, I agree with Bradley Bassler that Leibniz's decision that not the spatial points but the instants are uniform ultimately seems to be an arbitrary one. Bassler elucidates this problem in his 1998a paper in detail (see pp.13-18).

3.3.2 Applicability of the Solution to Phenomena

Now finally, it is time to discuss briefly the applicability of this theory of the continuum and motion to the world of phenomena. In the end, the *Theoria Motus Abstracti*, as its name suggests, is an abstract theory, and Leibniz admits that not all problems can be solved "by means of the abstract reasons for motions in bodies taken absolutely" (LLC, 343). However, he thinks that the difference between phenomena and what is suggested by the abstract account is mostly not *sensible*, and therefore, "suffices for the phenomena" (ibid.). Ultimately, it is not possible to distinguish solely by senses whether a body or motion is continuous or contiguous, or whether it is at rest or has a microscopic motion, just as senses cannot distinguish between an infinitangular polygon and a circle (ibid). Consequently, although Leibniz claims that the divergence is insensible and "suffices for the phenomena," it is possible to argue that these ideas are the precursors of Leibniz's inclusion of senses to his account as a means of distinguishing between the ideal-geometrical realm and real-physical realms in the near future.

Nevertheless, the concrete theory of motion Leibniz puts forth is mostly in accord with the abstract one, though there are some incongruities. In his physical theory, Leibniz rejects atoms, just as he denied minima or Euclidean points in the *Theoria Motus Abstracti*. Instead of atoms, he postulates sphere-shaped corpuscles in aether, which he calls *bullae*. Each of these corpuscles has a circular motion around its center, and this motion is again propagated by the *conatuses* that the corpuscles have. Owing to that circular motion, *bullae* have cohesiveness by which they cohere with one another. However, it seems that there is not an absolute continuum between bodies, since the ultimate particles are spheres, and there is always aether no matter how small between these spheres. Nevertheless, the very same aether that permeates the universe can be considered as the assurance of continuity, if it is noted that Leibniz does not deem aether as vacua. However, his statements in the *Theoria Motus Concreti* render these inferences regarding the continuity assured by aether doubtful. Because, in this theory of concrete motion, Leibniz states that "it is all the same whether you affirm or deny the *vacuum*, since I freely acknowledge that whatever is exhausted of air is filled with aether; in short, whether little empty spaces are left is irrelevant to the gist of our hypothesis" (LLC, 372). Moreover, as will be reflected in his *De Materia Prima* (On Primary Matter), his ideas about the aether change very soon.

To put it briefly, Leibniz introduces a fairly different account of continuity than he did in his letter to Thomasius. This difference may be examined from two main angles. First, unlike the account in his letter to Thomasius, where the continuum is only a potential, or an ideal whole, i.e. primary matter, and parts which have determinate figures, that is, bodies are discontinuous, in the *Theoria Motus Abstracti*, Leibniz tries to show that a continuum can be composed of actual parts. Hence, there can be and is an actual continuum in nature. Second, in contrast to the theory of motion he outlined in the letter, in his new theory, Leibniz maintains that motion is continuous, and does not involve any gaps or rests within it. More importantly, these two continua depicted in the *Theoria Motus Abstracti* are not independent from each other. Rather, the corporeal continuum is actualized through the cohesion rendered by the continuous motion instigated by *conatuses* as well as their interpenetration. Thus, Leibniz claims that there is an actual continuum, and the abstract theory behind this claim is the origin of his physical hypothesis which tries to confirm the existence of this actual continuity in the world of phenomena, where there are spherical bodies which have circular motion around their own axes (see Arthur 2001, xl). *Conatus*, and in turn the motion roused by it, is the bridge that binds the abstract-geometrical account with the physical one, under the same framework.

3.4 Towards a New Theory

Before moving to Paris, Leibniz writes a number of texts, most of which are drafts or letters. Some of those short pieces of writings written just after his *Theoria Motus Abstracti* reflect the standpoint Leibniz has arrived at in a concise way with a sharpened tone. For instance, his November letter to Antoine Arnauld is a very nice summary of his abstract theory of motion, as well as of the grounding ideas and objectives—such as the comparability of points, interpenetration of conatuses, and an Aristotelian conception of substance—of this theory. *De Materia Prima*, which is almost simultaneous with the theories of motion, rehashes earlier ideas in certain respects, even though it also declares some implicit ones more openly. Although the degree of convergence decreases substantially near the end of 1671, each of those writings more or less includes remarks or changes of thoughts implying that Leibniz is still not content with his theory. Accordingly, these series of gradual changes indicate that a new theory of the continuum and motion is on the way. Now before passing to this new theory, it would be better to concisely discuss this transition period.

3.4.1 Recapitulation of the Theory

3.4.1.1 De Materia Prima

De Materia Prima (On Primary Matter) signifies that Leibniz did not renounce his reconciliation project concerning an Aristotelian metaphysics and a modern methodology. As its title may suggest, in *De Materia Prima*, Leibniz once again talks about Aristotle's primary matter and compares it with Descartes' subtle matter. He indicates that both of them are infinitely divisible and lack form, which they acquire through motion. According to Leibniz, motion is imparted to both of them by a "mind." These motions, as a whole, and also in particular, are circular, similar to one in a vortex, which is reminiscent of his *bullae* maintained in the *Theoria Motus Concreti*. In *De Materia Prima*, Leibniz holds that continuity of matter is yielded by means of whirls impressing their motion upon the other whirls. The distinctive aspect of the Aristotelian account, which Leibniz tries to incorporate into his account of motion is that the "principle whirls" are bestowed with minds. (LLC, 344). In this way, Leibniz shows to us that he still holds his opinion that figure and motion is primarily brought about by means of an incorporeal principle, that is, mind—which he did not talk much about in the *Theoria Motus Abstracti*.

Here, paralleling the thought concerning empty space which he had exposed before, Leibniz explicitly argues that "*primary matter is nothing if it is at rest*" (ibid.). Leibniz claims that this is akin to the Scholastic claim that primary matter attains existence through form (ibid). Accordingly, Leibniz argues that primary matter is identical to space, which I think supports the argument that Leibniz considers primary matter to be ideal. On the other hand, the main reason behind the idea that primary matter is no different than empty space is not an abstract one: Leibniz seems to be using the argument "to be is to be perceived" by saying that "*whatever is not sensed is nothing. But that in which there is no variety is not sensed*" (ibid.). Thus, Leibniz begins to give even a more explicit and critical role to senses in distinguishing real things from the ideal ones.[54]

Leibniz nevertheless thinks that the world is plenum. However, it is composed of circular motions, since Leibniz thinks that, with motions in only one direction, it would be no different than rest (ibid). These circular motions, on the other hand, "mutually obstruct each other, or act upon one another" (ibid.). And through all these motions, bodies are formed. Although matter is "*actually divided into infinite part*s … [*a*]*ll bodies cohere to one another*" though they also "*separate from every other*, *although not without resistance* (ibid.). This last one also indicates the reason why in this small piece of writing, Leibniz explicitly denies atoms, by saying that "[t]here are no Atoms, i.e. bodies whose parts never separate," as well (ibid.). However, Leibniz' characterization of 'atom' as that "whose parts never separate" (ibid.), not as that which has no magnitude, i.e. parts is quite interesting. Because in the *Theoria Motus Abstracti*, Leibniz maintains that there are indivisibles which have parts, yet there is no mention about the possibility of the parts of an indivisible to separate.[55] This little piece of addition,

[54] See Bassler 2002, 226-227, where he likewise interprets Leibniz's argument as "the first explicit Leibnizian variation of … the *esse est percipi* theme."

[55] In her translation, Christia Mercer writes "whose parts are never divisible." Again the same reservation holds, since it is not mentioned in the *Theoria Motus Abstracti* that the parts of indivisibles are "divisible" (see Mercer 2001, 293).

therefore, can be considered as an indication of the change that is to take place with regard to Leibniz's idea of unassignable indivisibles.

3.4.1.2 Letter to Arnauld

Leibniz's letter to Arnauld is very expressive in the sense that the connections that have been traced in this chapter are articulated by Leibniz himself in a clear form. In this letter, it is seen that Leibniz has in mind a hierarchy when he says that "geometry, or the philosophy of position, is a step toward the philosophy of motion and of body and that the philosophy of motion is a step toward the science of mind" (L, 148). He further adds that this is the reason why he undertook a study of motion in the first place. In a similar fashion, heretofore in this study, I tried to argue that Leibniz tries to establish transitions from geometrical and abstract forms to the realm of metaphysics and physics—particularly in terms or by means of motion—and then constitute essentially similar bonds between bodies and minds. The essential stuff of this bond with regard to the *Theoria Motus Abstracti* was *conatus*, although Leibniz did not establish this bond thoroughly. In fact, this task is partly done in Leibniz's letter to Arnauld, which will be elucidated below.

Leibniz recapitulates his theory of motion for Antoine Arnauld, and here, for the first time, he explicitly states that the *essence* of body consists in motion, rather than on extension:

> … the essence of body consists rather in motion, since the concept of space involves nothing but magnitude and figure, or extension. (ibid.)

Thus, these two qualities, namely magnitude and figure, which were persistently considered to be the primary qualities of bodies throughout Leibniz's writings dealt with in Chapter 2, are now plainly taken to be unessential for bodies:

> The essence of body does not consist in extension, that is, in magnitude and figure, because empty space, even though extended, must necessarily be different from body. (ibid.)

Hence the distinctive quality of bodies is motion, and therefore, as also argued in *De Materia Prima*, "there is no body at rest, for such a thing would not differ from empty space" (ibid.). This is because motion provides bodies with cohesion and resistance, and Leibniz writes that "whatever at rest can be impelled and divided by motion, however small" and therefore would not have "cohesion of consistency" (ibid.).

In his letter to Arnauld, Leibniz also talks about the broad set of consequences brought forth by his conception of unextended points of the *Theoria Motus Abstracti*. First, as mentioned before, the new definition of point allows for a comparison between the magnitudes of different points, though the ratio is "incomparable to any sensible difference" (ibid.). Second, when the matter of concern is phoronomy, that is, the study of pure motion, the ratio of rest to motion is that of nothing to one, whereas the ratio of *conatus* to motion is that of a point to space. Third, within the time of the *conatus*, a body in motion may be "in many places or points of space" and "whatever moves is never in one place, not even in an infinitesimal instant" (ibid.). And finally, through *conatus*, bodies in motion mutually strive to penetrate into each other and their boundaries become common. As a result of this, a continuum in an Aristotelian sense is yielded (ibid.).

A crucial difference to note here is that Leibniz seems to distinguish his 'points' from 'indivisibles' as conceived by the fact that this particular conception of points depends highly on the notion of unextended indivisibles defended in the *Theoria Motus Abstracti*. In fact, this is admitted by Leibniz himself when he writes to Arnauld that "[i]n geometry I have demonstrated certain fundamental propositions on which depends a geometry of indivisibles, that is, a source of discoveries and demonstrations" (ibid., 149). Nevertheless, just below this sentence, he states that "Euclid was not wrong in speaking of parts of extension; that there are no indivisibles, yet there are non-extended beings" (ibid.). And again, however, after a few lines, he refers to the "phoronomy of indivisibles" from which he derived the outcomes concerning the relation between rest and motion, *conatus* and motion or point and space. It is possible to argue that here

Leibniz begins to differentiate his geometrical account from the metaphysical one, and hence, when motion of bodies is considered, he possibly denies the *existence* of indivisibles, though he makes use of them in his geometrical or abstract theory. Since at any rate, as is implied in the above-mentioned hierarchy, "the philosophy of motion and of body" is more substantial than geometry. However, this is not certain, given the significant functions indivisibles have in Leibniz's theory, such as their being beginnings and ends, or more notably, being indestructible husks of minds.

It has been already argued that the philosophy of minds has a higher and even more substantial status in that hierarchy of sciences and Leibniz believes that his ideas go beyond being a mere account of motion, when his inferences about minds are proved to follow the laws of motion. Just as he takes motion, which has *conatus* at its very basis, as the essence of bodies, he claims that "the true locus of our mind is a certain point or center" (ibid.). Therefore minds, which essentially consist in these indivisible points, are indestructible, and this is surely a reason why Leibniz kept indivisibles as a part of his account.[56]

In fact, Leibniz seems to think that minds, which reside in these points or centers—which I do not hesitate to call indivisibles—have something in common with bodies: They both consist of *conatuses*, and this is where those two worlds, the world of minds and the world of bodies, are brought together by Leibniz. Nevertheless, there is still an important difference here. Leibniz states that the motion of a body is in fact a composition of succeeding *conatuses*. Because of this mere sequence, bodies do not have recollection; as he has already stated in the *Theoria Motus Abstracti*, they are instantaneous minds (ibid.). On the other hand, minds are not a mere sequence of *conatuses*, but a harmony of them:

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[56] See Goldenbaum, 98-99, where she writes that "[t]he reason for Leibniz's quite particular definition of the point as an unextended thing, which has parts, is in my opinion a mere metaphysical-theological one: This new definition is useful for his task to explain the immortality of the mind, the possibility of the mystery of resurrection as well as of the possibility of transubstantiation. But such a definition of the point will be helpful too for Leibniz's draft of a theory of a mere abstract geometrical description of the rules of mechanical motion, which could be compatible with the Christian mysteries."

> As the body consists in a sequence of motions, so mind consists in a harmony of conatuses. The present motion of a body arises from the composition of preceding conatuses; the present conatus of a mind, that is, will, arises from the composition of preceding harmonies into a new one … (ibid.)

Thus, Leibniz's *conatuses* represent a striving for action. In case of a mere body this action is motion, and the continuity of this motion depends on the continuity of *conatuses*, since conatuses last only for an infinitely small time. In case of a mind, this action is thinking, and the characteristic of this thinking depends on the harmony that is composed by *conatuses*. Bradley Bassler has a nice interpretation regarding this harmony realized by *conatuses*, which is illustrated in a lucid way:

> Since mind can *preserve* conatus, it possesses the capability, presumably, to bring that conatus to bear in those cases where its effect would *not* be dissipative. It does this, precisely, by arranging conatus in such a way as to achieve the greatest harmony, or 'identity compensated by diversity'. Mind is the agent of the harmonization of forces, where forces are here conceived of conatus which comprise the building blocks of motion, and hence of physical existence. (Bassler 2002, 226)

As is indicated, Leibniz's use of *conatus* as a substitute for God's continuous creation of motion becomes more explicit in this letter to Arnauld. In this way, bodies are given a new principle of motion. And this principle, when sustained, reflects a state of mind, and thus, in a way, mind itself becomes the principle of action. A very similar interpretation is made by Daniel Garber, who writes, with respect to the writings between the *Theoria Motus Abstracti* and Leibniz's letter to Arnauld, that

> … the cause of motion is moved from God to bodies themselves; bodies are, it seems, the sources of their own motion, which derives from their own minds, and these minds together with their bodies constitute genuine substances. (Garber 1995, 277)

If this claim is admitted, it can be argued that by his theory of motion relying on the concept of *conatus*, Leibniz achieves what he has aspired from the beginning:

> If Leibniz wanted to understand the mind as the moving principle of the body and if he wanted to do so on the level of the new mechanical theory,

> he had to find an explanation for the relation of mind and body within a mechanical theory. He had to introduce the mind into the mechanical theory of motion itself. (Goldenbaum, 99)

By means of the theory of *conatuses* he developed to a new form, Leibniz finds a new principle of motion, and establishes a new version of Aristotelian substance. However, although it meets Leibniz demands of reconciliation of the philosophy of Aristotle with the philosophy of the moderns, the connection he establishes hereby between minds and bodies through *conatus* and motion in this form of the theory is not sophisticated much further in the future writings, and is replaced by a connection provided through sensations. Therefore, this account proves out to be only short-lived in consequence of Leibniz's changing view concerning *conatuses* and motion, which will be clear at the end of the Chapter 4.

3.4.2 Negating the Theory

Towards the end of the year 1671, the degree of divergence from the earlier theories of motion escalates. Although Leibniz does not give his motives for these changes of mind, it is possible to speculate certain reasons behind this divergence. One reason may be Leibniz's dissatisfaction with his account of minds and their relations with the realms of phenomena, since the bond provided by the concept of *conatus* between bodies and minds is not further elaborated and remains weak. Another one of them may be the tension between the physical and the geometrical accounts. The difficulties of explaining the actual motion solely by means of the geometrical account may easily be admitted. Furthermore, in this *Theoria Motus Abstracti* and *Theoria Motus Concreti*, Leibniz advances quite an original account which is not sophisticated enough to explain complicated phenomena regarding motion. To some extent, this explains why Leibniz inserts arguments based on our senses in some of his physical hypotheses. However, since most of these changes of mind belonging to this period disappear together with Leibniz's new theory, which is in fact very similar to the one he adopted in the *Theoria Motus Abstracti*, these small pieces of writings or drafts may be interpreted as Leibniz's experimental studies for alternative theories.

For instance, in one of those fragments, named "A Hypothesis of the System of the World," Leibniz argues that the world is not a plenum, and there are vacua situated in the interstices between the "globes" filling the world (LLC, 344-345). The reason why Leibniz denies that the world is a plenum rests on his assumption that a plenum is homogeneous, and has one center of motion. On the contrary, he believes that there is a plurality of those centers of motion, as there is a plurality of globes in motion. However, although they are in motion, these globes do not penetrate each other but only touch one another at a point. Since they do not penetrate each other, these globes have interstices between, which are vacua (ibid., 344).

On the other hand, bodies may or may not contain vacua within, though only atoms are included to the latter group. This means, Leibniz allows the existence of atoms, which are "indissoluble." On the other hand, there are other bodies which are "mundane, i.e. enduring for a while" or "momentaneous", i.e., existing only momentarily (ibid., 345). All these suggestions such as the existence of 'atoms', 'vacua', or 'moments' imply an approval of minima, even though they are mentioned in a seemingly "physical" hypothesis.

In another piece of writing named "On the Nature of Corporeal Things"[57], Leibniz uses his statement that there is a vacuum in order to point out to the distinction between body and space. Here it is seen that the definition of continuity changes. Now it is taken synonymous with contiguity:

> A continuum is a whole between any of whose parts other parts of the same thing are interjected.
>
> Something is interjected between two things if the sum of its distances from each of them is the distance apart of the two things. (ibid., 346)

In other words, the world is—or by Leibniz's expression, all worlds are—contiguous. If there were not contiguous worlds, parts of any world surrounded by

[57] The entire title is "On the Nature of Corporeal Things: A Specimen of Demonstrations from the Phenomena."

vacuum would dissipate, and there would be no resistance (ibid.). In a sense, Leibniz thinks that the continuum is a kind of whole, which does not dissolve thanks to the other wholes contiguous to it.

Of course, there are not only deviations from the earlier account. For instance, though it is not clear whether Leibniz considers vacuum as a kind of minimum, i.e., something which does not have parts, he argues that there are no minima in the continuum. Instead he indicates that there are unextended parts in the continuum, which is composed of infinite number of parts. By the way, Leibniz holds a broader conception of extension, which he defines as "the magnitude of the continuous" (ibid., 345). In this short piece of text, Leibniz also gives a definition of magnitude as the multiplicity of parts, which is indeed already implied in the *Theoria Motus Abstracti* (ibid.).

However, as is to be noticed, the general standpoint is quite different from that of the *Theoria Motus Abstracti* or *Theoria Motus Concreti*, particularly in the sense that no mention of *conatus* is made in these writing. As opposed to his earlier ideas that the boundaries become one when two things (*conatus* or bodies) cohere, and that "*whatever moves is never in one place while it moves*" (LLC, 341), he maintains that "[t]wo bodies cannot be in the same place" and that "the same body cannot be in several places" (ibid., 346).

As a result, by the late 1671, Leibniz indeed assumes a contiguity of bodies and motion, though he primarily reasons with respect to and from the phenomenal world. Nevertheless, all this account excluding the concept of *conatus* and bringing vacuum in turns out to be a stopgap that is to be abandoned within a short while.

CHAPTER 4

EARLY PARIS WRITINGS:
FROM MOTIONS TO SENSATIONS

It is commonly agreed that it was during his stay in Paris that Leibniz began to get seriously involved in mathematics. Although his initial questions and interest in mathematics were originally connected with the metaphysical problems Leibniz has so far dealt with and the standpoint he has reached, in time this new interest brought in changes to his metaphysical ideas. Furthermore, within this period, his main concern became mathematical studies, and the previous abstract and geometrical ground for metaphysics was later reinforced by these studies, especially by his studies on infinitesimal calculus. Therefore, the outcomes he arrived at through these studies determined by and large the destiny of the new arguments he raised to solve metaphysical problems. Accordingly, the definitions of some key concepts which concern this thesis, such as ‘continuum’, ‘infinite’ and ‘minimum’ continued to turn, yet around a new axis.

The influence of mathematics to Leibniz’s metaphysics beginning with the Paris period is comprehensively examined by numerous scholars. However, there is also the other side of the coin, which should not be ignored: Leibniz’s early metaphysical efforts are decisive for his mathematical studies. As Stuart Brown indicates, calculus provides a mathematical representation for the continuity of change and “Leibniz’s interest in calculus is thus not separate from his metaphysical interests, for instance, in the continuum" (Brown and Fox, 125).[58] In this respect, I think that especially Leibniz's struggle with the labyrinth of the continuum is essential to his development of infinitesimal calculus.

Of course, Leibniz's choice of focusing on mathematics is not arbitrary, and has several reasons, which may include the scholars Leibniz met, the new ideas or problems he encountered, or his feeling of lack of knowledge about mathematics. However, the swift success that comes up with revolutionary ideas and methods in mathematics is closely related with Leibniz's original outlook peculiar to his own. And the development of this outlook owes much to his search for a way out of the labyrinth of the continuum.

In this chapter, I will discuss some of the first writings of the Paris period, which may have played a considerable role in shaping Leibniz's thoughts on mathematics, geometry and the infinitesimal analysis. However, unsurprisingly, Leibniz's main motive while writing these texts is not a desire to develop a novel mathematical account. As is to be seen, these texts primarily deal with the problems that are metaphysical. Nevertheless, as regards some of these problems, such as the labyrinth of the continuum, Leibniz makes use of geometrical and arithmetical argumentations to demonstrate his conclusions. And again, with all its geometrical, physical, and above all, metaphysical aspects, the main concern of this chapter will be Leibniz's struggle with the problem of the composition of the continuum in his early Paris years.

The first writings of the Paris period begin in a way that follows the ideas of the *Theoria Motus Abstracti* to a considerable extent, above all concerning the ideas of cohesion. However, especially together with "De Minimo et Maximo," which is one of the early writings of the Paris period, Leibniz's account significantly

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[58] Likewise Joseph Hofmann writes that Leibniz's first mathematical achievements in Paris "originated in thoughts strongly influenced by considerations of logic and philosophy" (Hofmann, 14). Bassler also states that "Leibniz's first mathematical discoveries of the Paris period arose in conjunction with philosophical reflections which were a development of the pre-Paris metaphysics" (Bassler 1999, 161).

diverges from that of the *Theoria Motus Abstracti*, the reasons of which will be discussed in detail. I argue that in these writings of the Paris period, Leibniz carries his theory of the continuum composed of actual parts, and his theory of motion to its limits. At these limits, a new way of composing the continuum, and perhaps even a new perspective on metaphysics, seems to surface. And this will be discussed again along with what Richard Arthur puts forth as the formula of the third different theory concerning the problem of the continuum. However, as soon as Leibniz reaches these limits of his theory, he begins to depart from the view that there is an actual continuum, towards a general standpoint paralleling both his first and mature systematic thoughts that there is only an ideal continuum where the whole is prior to its parts, unlike the phenomenal contiguity where parts are prior to the whole. These limits also denote the beginning of Leibniz's intense studies on mathematics and calculus, and his partial renunciation of the concept of *conatus*, until he finds a new context to make use of it.

In a few words, in this chapter, I will first examine Leibniz's accounts of cohesion expounded in the texts named "A Demonstration of Incorporeal Substances" and "On the Cohesiveness of Bodies." I will point out that in these accounts, Leibniz once again makes use of *conatuses*, which he did not mention in a few writings written right after the *Theoria Motus Abstracti*. On the other hand, with regard to these texts, the change in Leibniz's conception of '*conatus*' will be discerned. Afterwards, I will examine "De Minimo et Maximo," in which a different conception of the composition of the continuum is maintained, and look at how the account that is yielded by this conception is differentiated from the previous accounts. This account will be discussed again along with what Richard Arthur puts forth as the formulation of the third different theory concerning the problem of the continuum. Finally, I will try to explain briefly how and why Leibniz departed from this account in his later Paris years.

4.1 Returning Back to Conatus

As is indicated at the end of the previous chapter, Leibniz's neglecting of *conatus* in his last writings before his Paris years does not last long. In a set of drafts which he called "A Demonstration of Incorporeal Substances," known to be written in the fall of 1672, Leibniz again grounds his theories of the continuum and cohesion upon the concept of *conatus*. It is clearly seen here that Leibniz retains his theory of continuity through cohesion in the *Theoria Motus Abstracti*, since both the main arguments and the argumentation itself are similar to the previous accounts explaining motion and cohesion in terms of *conatus*.

To recapitulate, Leibniz maintains that bodies impel each other through their motions, which means that they *endeavour* to enter one another's boundaries. In the *Theoria Motus Abstracti*, this endeavour to move is stimulated by *conatuses*. Now, in "A Demonstration of Incorporeal Substances," Leibniz defines *conatus* not only with respect to motion, but with respect to *action* in general[59]:

> An *endeavour* [conatus] is a part of an action smaller than any given part, or the beginning, middle, or end of an action—as an instant of time, and a point in space. (LLC, 3)

As is seen before, Leibniz considered the relation of a point to space, or of an instant to time, to be analogous to the relation of a *conatus* to motion, where these points, instants, and conatuses were, according to the Leibniz of the *Theoria Motus Abstracti*, indivisible beginnings or ends. However, it was in fact not clear how a *conatus* can be the *end* of a motion if it is a striving for motion. Yet here Leibniz repeats this argument and furthermore inserts a new idea that *conatus* can be a part in the middle of a motion. I believe that to some extent this can be understood if motion is considered to be composed of motions smaller than any given. In this way, a *conatus* may be in the middle, or at the end of that motion as a whole, stimulating an infinitesimal motion within the bigger motion. However,

[59]In fact, in Chapter 2, deriving from some implications Leibniz gives, I tried to interpret *conatus* as a striving for *action* in general. As Leibniz says, action of bodies is motion, whereas action of mind is thinking. And motion consists of succeeding *conatuses*, while mind consists of a harmony of *conatuses* (L, 149).

it is still not easy to understand how a *conatus* can designate the end of motion, unless i) all bodies are in motion, ii) every motion is continuous with another, and iii) every *conatus* designates both the end and the beginning of two continuous infinitesimal motions simultaneously.

On the other hand, there is still an easier way to understand the claim that *conatus* is "a part of an action … or the beginning, middle or end of an action" (ibid.): *Conatus* may be considered not merely as the indivisible unextended potential for action, or the indivisible limit of action, but as the infinitesimal part of action itself. Thus in case of bodies, *conatus* may stand for the actual constituents that compose the continuity of motion. In this sense, *conatuses* become homogeneous with motion, and as actual as a motion is. And as it will be seen, this is the stance where Leibniz is advancing towards.

However, before this advancement takes place, it is worthwhile to follow how Leibniz explains cohesion and continuity by bodies impelling one another. It is already clear that to have *conatus* for a body is to be in motion.[60] While in motion, bodies impel one another, which means, for Leibniz, that they "endeavour to move" one another (ibid.). On the other hand, Leibniz continues to define motion as being in two places at the same time: "[w]hatever moves is in motion at two places in a moment" (ibid., 5). Hence, if a body impels another while in motion, and if it is at two places in a moment while moving, that is to say, while impelling, that body already penetrates the other's place. Therefore, through the infinitesimal motions, a body immediately penetrates one another's place "by a part of it smaller than any given part" (ibid.). This reciprocal interaction renders their boundaries common. And in this way, bodies cohere and form a continuum. Besides, this is a continuum, because when bodies cohere, they become, by Leibniz's expression, "sympathetic", that is, "co-moving, i.e. one cannot be impelled without the other" (ibid.).

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[60] Of course, if the impact of that *conatus* is not compensated by another from the opposite direction.

In another piece of writing of the same period, named "On the Cohesiveness of Bodies," Leibniz maintains a very similar position to the one explained above concerning the cohesion of bodies. In this writing, Leibniz makes an explicit definition of continuity in terms of cohesion: "Continuous things are contiguous ones with some cohesiveness" (ibid., 19).[61] His definition of 'contiguity', on the other hand, is similar to the one he used in his letter to Thomasius: things between which "there is no distance" are contiguous (ibid.). The idea is the same: in order to be continuous, bodies should be impelling, that is, endeavouring one another. However, this may not be sufficient, because bodies are incessantly under the influence of various *conatuses* from different directions, which may move them apart from each other. On the other hand, Leibniz thinks that, the fact that two contiguous bodies are resistant to the *conatuses* impelling them to move, in other words, their being at rest, does not render these bodies continuous. In fact, this discussion stems from Descartes' account of continuity which Leibniz directly confronts here. Simply put, Descartes thinks that what makes things hard and resistant is their parts' being at rest with respect to each other.[62] Accordingly, he claims that in case of a collision with another body, parts of a body at rest are more resistant than parts of a body having a motion towards the opposite direction. Leibniz finds Descartes' idea that 'a body at rest, or parts of a body at rest, will in no ways be impelled by any *conatus* impinging on it' unacceptable. He thinks, on the contrary, that "if there were no other endeavour extant in the body that would compensate for or moderate it, the body would break forth into motion" (LLC, 21). Besides, Leibniz has other grounds to oppose Descartes:

> If rest is the cause of cohesiveness, first of all no cohesive thing will be separable, since rest is infinitely different from motion; for if rest were to

[61] I should note that Richard Arthur translates '*consistentia*' as 'cohesiveness,' and '*cohaesio*' as 'cohesion'. Leibniz defines *consistentia corporum*, that is, the cohesiveness of bodies as "the quantity of force needed to destroy their contiguity" (LLC, 19). Arthur gives a persuasive explanation for his choice of the word 'cohesiveness' in the "Glossary" section of his Leibniz selection in *LLC*, 442-443. Briefly, he notes that according to the terminology of the time, *consistentia* came to mean the degree to which a body holds together (LLC, 380). Christia Mercer, on the other hand, translates '*consistentia*' as 'cohesion' as well (Mercer 2001, 79).

[62] See Descartes, 245: "… hard bodies are those whose particles are at rest relative to each other".

> resist more strongly than an opposite motion, it would resist infinitely more strongly. And secondly, all things will be equally cohesive, since all rests are equal. (ibid.)

Thus, Leibniz chooses to explain cohesion and resistance in terms of '*conatus*' and the basic aspects of his conception seems to persist when he says that *conatus*

> is the beginning of motion at a given moment. Therefore, it is the beginning of a change of place, i.e. of a transition from place to place, and therefore is in both places at the same time, since it cannot be in neither, i.e. nowhere. (ibid.)

However, one may assert that there is in fact a slight change in the "On the Cohesiveness of Bodies," since Leibniz for the first time attributes *conatus* a certain characteristic, which he indeed speaks of while referring to the motion of a body: to be in two places at the same time. Thus while indicating that *conatus* is the beginning of motion, Leibniz seems to consider *conatus* to be something already in motion, not just the beginning point of the motion. And if this interpretation is not mistaken, as is pointed out while examining "A Demonstration of Incorporeal Substances," it comes to mean that *conatus* stands for the infinitesimal part of motion, which actually compose motion together with other *conatuses*. Further, this interpretation suggests that *conatus* ceases to be an indivisible part of motion: the claim that it is at two places at the same time implies that it covers an extended magnitude. As I will argue later, this is a significant shift since it is rooted in the idea which makes Leibniz's new theory differ from the previous one of the *Theoria Motus Abstracti*: the rejection of indivisibles.

Before coming to that point of rupture, it should be noted that, this account of cohesion Leibniz suggests is not just a geometrical or an abstract one. Leibniz believes that all these ideas about cohesiveness and continuity due to *conatuses* are sustained for phenomena in nature. When a body, or part of a body, strives to move, it penetrates another body or part, which makes these parts or bodies continuous, and when they are continuous, they cannot be impelled without one another (ibid., 21-23). Leibniz thinks that from this "follows the propagations of motions in a plenum" and argues against the modern atomist accounts of cohesion that his account of cohesiveness can be used to explain phenomena "which cannot be accounted for by the pressure of the air" (ibid., 23).

4.2 "De Minimo et Maximo": Against Minima and Maxima

4.2.1 Renouncing Indivisibles

It can certainly be asserted that the definite rupture from the account of *Theoria Motus Abstracti* comes along with an explicit denial of indivisibles in Leibniz's "De Minimo et Maximo" (On Minimum and Maximum) written between late 1672 and early 1673.[63] On the one hand, this paper can be seen as a revision of the *Theoria Motus Abstracti*, though on the other hand, it shows itself as a new theory, the difference of which is grounded on the denial of indivisibles.

Here Leibniz takes indivisibles to be equivalent to minima, and begins the paper by stating that "[*t*]*here is no minimum, or indivisible, in space and body*" (LLC, 9). In order to prove this statement, Leibniz uses an argument similar to one of the arguments he used to reject minima before. In fact, this argument is a kind of proof he gives for the Euclidean axiom that the whole is greater than its parts.[64] Now before passing to his argument against indivisibles, it would be helpful to recall why Leibniz previously defended indivisibles against the idea of minima.

In the *Theoria Motus Abstracti*, Leibniz rejects minima owing to the fact that minima do not have any magnitude and therefore any space or body would have an equal number of minimal points with another one. This was problematic because it would come to mean that a whole would have an equal number points as one of its parts, or in the same way, a diagonal of a square would have equal

[63] The whole title is "De Minimo et Maximo; De Corporibus et Mentibus" (On Minimum and Maximum; On Bodies and Minds).

[64] This axiom is considered to be very basic for Leibniz's initial interest in mathematics, and decisive for his later thoughts, particularly concerning the nature of 'infinite' (cf. Aiton, 42; Arthur 2008, 4; Levey 1998). Eberhard Knobloch indicates that the universal validity of this axiom was indispensable for Leibniz "right from the start" (Knobloch, 94), and Bassler writes that this axiom is "the central tenet around which his philosophical and mathematical development was organized" (Bassler 1999, 163).

minima as one of its sides. As an alternative to those minimal points, Leibniz suggests to consider points to be indivisibles with magnitudes. Seemingly, Leibniz thought that his suggestion was not prone to the same question. One reason why he thought like that might be that if there are indivisibles with magnitudes, then the parts of a whole would have a different number of indivisible points than the whole. However, this might probably not be the case, because as is to be remembered, in the *Theoria Motus Abstracti*, Leibniz states that there are infinitely many actual parts in the continuum, and the common interpretation was that Leibniz meant indivisibles with infinitely many parts. Besides, even if one supposes that the infinitely many actual parts of the continuum that Leibniz speaks of are not the *indivisibles*, one still knows that there are infinitely many indivisibles in a continuum, since every actual part has an indivisible beginning.

Consequently, the idea that there are infinitely many indivisibles in any continuum appears to be incompatible with the thought that there might be a different number of Leibniz's indivisibles in a part than there are in a whole, unless a different conception of infinity which makes it possible to compare different infinities is set forth. Furthermore, even if the number of the indivisibles could be different in the part and in the whole, Leibniz does not give a satisfying explanation to how this happens given that these indivisibles are unextended.

In any case, in "De Minimo et Maximo," Leibniz explicitly expresses his discontent with his earlier idea of indivisibles. By means of the figure below, Leibniz tries to prove that indivisibles are minima and therefore should also be rejected.[65]

[65] I should acknowledge, in his Arthur 2007a, Richard Arthur has already used this figure together with exactly the same quotation from Leibniz, which *he* has translated into English. However, I did not want to paraphrase or divide Leibniz's demonstration in order to give a more comprehensible account.

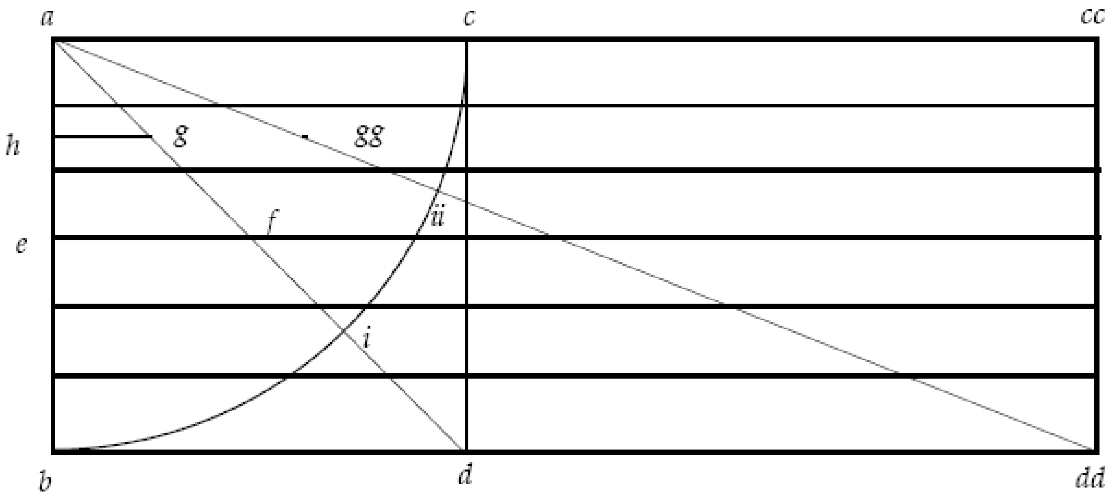

Figure 4.1 Leibniz's Refutation of Minima (from LLC, 11).

> … if there is an indivisible in space or body, there will also be one in the line *ab* … If there is one in the line *ab*, there will be indivisibles in it everywhere. Moreover, every indivisible point can be understood as the indivisible boundary of a line. So let us understand infinitely many lines, parallel to each other and perpendicular to *ab*, to be drawn from *ab* to *cd*. Now no point can be assigned in the transverse line or diagonal *ad* which does not fall on one of the infinitely many parallel lines extending perpendicularly from *ab*. For, if this is possible, let there exist some such point *g*: then a straight line *gh* may certainly be understood to be drawn from it perpendicular to *ab*. But this line *gh* must necessarily be one of all the parallels extending perpendicularly from *ab*. Therefore the point *g* falls—i.e. any assignable point will fall—on one of these lines. Moreover, the same point cannot fall on several parallel lines, nor can one parallel fall on several points. Therefore the line *ad* will have as many indivisible points as there are parallel lines extending from *ab*, i.e. as many as there are indivisible points in the line *ab*. Therefore there are as many indivisible points in *ad* as in *ab*. Let us assume in *ad* a line *ai* equal to *ab*. Now since there are as many points in *ai* as in *ab* (since they are equal), and as many in *ab* as in *ad*, as has been shown, there will be as many indivisible points in *ai* as in *ad*. Therefore there will be no points in the difference between *ai* and *ad*, namely, in *id*, which is absurd. (LLC, 9, 11)

Despite the ingenuity of the argument, it is dubious whether it disproves Leibniz's own indivisibles that he employed in the *Theoria Motus Abstracti*. As is discussed before, the distinctive aspect of these indivisibles, which distinguish them from minima, or specifically from Euclidean points, was the fact they have magnitudes and parts. However, it looks as if Leibniz ignored this particular feature of his previous account, since in “De Minimo et Maximo” he speaks of indivisibles as things lacking parts (LLC, 13). In order to understand Leibniz’s attitude, I think it would be helpful to do some reasoning before going further into “De Minimo et Maximo.” However, during this reasoning, let us have in mind that in his first years, Leibniz does not present merely geometrical accounts, but he thinks that these accounts can be utilized in explaining phenomena as well. At least, Leibniz thinks that (such as he did while writing the *Theoria Motus Abstracti*) there is no considerable discrepancy between the geometrical and physical/metaphysical accounts.

First of all, I think that for the Leibniz of the *Theoria Motus Abstracti*, the fact that there is an equal number of indivisible points in *ab* and *ad* would not be a problem, insofar as the points that compose these lines are different, owing to the fact that their magnitudes are different. But it seems here Leibniz thinks as if every indivisible point of two lines having the same length is equal in magnitude, since he takes “the line *ai* equal to *ab*.” If it can be argued that the indivisible points composing these lines have different magnitudes, just as it can be inferred from the *Theoria Motus Abstracti*, *ab* and *ai* may be considered as qualitatively different lines, because they may be composed of different parts, that is, parts with different magnitudes.

However, this is just an example of the fact that there are various problems with employing the thought that indivisibles may have different magnitudes in a geometrical demonstration. Perhaps in a geometrical account, it does not make much sense to compare different lines qualitatively, or talk about how two lines with equal length differ from each other by means of their constituents. However, Leibniz’s account of unextended indivisibles with magnitudes, his idea of points filling up spaces when in motion, and his making these ideas the fundamentals of his theory encourages one to raise objections. It is likely that Leibniz considered such problems while changing his account as well, though he did not discuss them in the *Theoria Motus Abstracti*.

One may speak of other problems stemming from the idea of differing magnitudes of indivisibles. For instance, if the indivisible points of a finite line can be different from those of another finite line, then there is no reason not to think that there can be points with different magnitudes *within* each of these lines. For each finite line can also be considered as composed of other finite lines, and these smaller finite lines can be different from each other, and accordingly, they can be composed of different indivisible points. Now if different finite lines are composed of points with different magnitudes, such as in the case of *ab* and *ad*, I believe that one should suggest a reason for this difference. For example, there should be an explanation if the indivisible points of *ai* are different from those of *id*, or of *ae* from those of *eb*. However, there seems to be no satisfactory explanation.

As is comprehensively discussed in the previous chapter, the foremost metaphysical implication of the idea that points have different magnitudes is that there are *conatuses* which instigate different motion. On the other hand, this metaphysical "implication" can also be considered as the very basis of this idea of comparable geometrical points, if the composition of lines is accounted for in terms of the points that traverse these lines.

In this respect, relying on Leibniz's ideas on the comparability of *conatuses* in terms of the motions they stimulate, Richard Arthur presents a nice analogy. The gist of that analogy is that if lines are considered as the motion of points through *conatuses*, it would be claimed that the point that forms the diagonal of a square has a "greater" *conatus* than that of a side, since these motions are proportional with the *conatuses*. Accordingly, even if the diagonal has equal number of indivisibles with its side, the magnitudes of the indivisibles would be different (Arthur 2001, xxxv).[66]

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[66] One may raise an objection to this analogy arguing that it seems to assume that lines are formed via just a single *conatus*, unless it is asserted that both the diagonal and the side, in this case, are composed of small lines, and all the small lines that compose the diagonal are longer than those which compose the side.

If one thinks in line with the analogy that Arthur presents, it can be assumed here that the side and the diagonal of a square are *ab* and *ad* respectively. In that case, in its simplest form, the reason why the indivisible points of *ab* and *ad* have different magnitudes is explained by the fact that *ad* is longer than *ab*. Using the theory of *conatus* and assuming that these lines are formed by the motion of indivisible point(s) in an equal finite interval of time, it is inferred that *ad* is composed of points with "greater" *conatuses*, or more simply, of "faster-moving" point(s).[67] However, one cannot likewise explain how the indivisible points of *ai* differ or do not differ from those of *id*.

However, to put it again, perhaps it is not fair to expect from a geometrical account to meet such objections which are indeed more related with the metaphysical implications of this geometrical account—*if* it is, of course, possible to distinguish what is geometrical from what is metaphysical in this particular theory of Leibniz. Nevertheless, I think it is not unexpected for a geometrical account to be simple to such an extent that it cannot provide answers to every particular question raised while analyzing the physical or metaphysical aspects of the theory as a whole. Even though it should be applicable to phenomena, as is to be expected, this applicability does not denote a perfect correspondence. Still, with regards to phenomena, magnitudes smaller than any given, and the motions stimulated by them can be compared. But it does not follow from this comparability that it can be proved for each geometrical line.

In passing, it is worth noting that the idea that the points of the side and the diagonal have different magnitudes does not necessarily denote that the number of their points is equal. Otherwise, this would come to mean that each and every line has the same number of points, which is quite absurd. As a matter of fact, it is

[67] One may raise an objection to the assumption that these two lines are formed in an "equal interval of time". I think this is not much different than assuming that they are "lines". In other words, this is still a geometrical, an abstract account, though it includes 'motion' in that account. And motion implies time. Just as it can be assumed, for example, that these lines are on the same plane—due to the fact that they do not intersect—one can also assume that these lines are formed within the same interval of time. I believe that it is a nice way to introduce motion into geometry.

Leibniz's assumption here that "the same point cannot fall on several parallel lines, nor can one parallel fall on several points" and therefore there are as many indivisible points in *ab* as in *ad* (ibid.). And if one is to argue that points of the side and the diagonal, which are of different magnitudes, are equal because of the very same reason that Leibniz maintains in "De Minimo et Maximo," that is, each of the indivisible points of the side falls to its correspondent point in the diagonal, it would again be problematic, since it cannot be explained how indivisible points with "greater" magnitudes would correspond to those with "smaller" magnitudes, if they have no extensions. If these indivisibles had extensions, one could accordingly argue that their number did not have to be equal since the limits of a "greater" point would correspond to more than one "smaller" point. Any point in the diagonal with greater extension and greater *conatus*, might correspond to more than one point on the side and this might not be problematic, as far as the parallel lines can differ just as the points do. After all, these lines may be formed out of different points. However, in the end, these points are *unextended*, and a possible effort to maintain similar arguments for indivisibles with magnitudes appears to be unpromising. The only possible thing to claim would be that each of the individual motions generated by the greater infinitesimal *conatuses* in the diagonal would correspond to more than one individual motion generated by the smaller infinitesimal *conatuses*, since motions generated with greater *conatuses* would cover a greater distance than motions generated with smaller *conatuses*.

Furthermore, each of these lines—the side and the diagonal—are continuous and thus they are composed of infinitely many indivisibles, as is discussed above. Accordingly, one may possibly object that there should be infinitely many correspondent points in *ad* as there are in *ab*, which seems to confirm Leibniz's argument against indivisibles, unless one would answer quoting Leibniz's own ideas from the *Dissertatio* that "one infinite is greater than other" (L, 75). In any case, all these questions uncover the need for giving a sound account of the infinite if one has to deal with the composition of the continuum. In fact, even defining the infinite becomes such a big difficulty that it becomes a new labyrinth on its own. This is the reason why Leibniz undertakes the task of contending with the problem of the infinite more intensely beginning with his Paris years and later on.

There is another crucial aspect of the *Theoria Motus Abstracti* that Leibniz seems to ignore here, which is Leibniz's own argument that indivisibles, or *conatuses*, share boundaries with one another, so that they become continuous.[68] Now when it is thought that the indivisibles are continuous in an Aristotelian sense, that is, if they share boundaries with one another, then their boundaries become "one." In this respect, the claim that each of the points on the side corresponds to a unique counterpart on the diagonal does not make sense. Since the boundary belongs to the indivisible, the parallel drawn to the "point" where boundaries overlap actually falls to two continuous "points" taken separately.[69] What is more, *conatuses* with the same direction are in this way compounded and form a whole. A similar argument may be defended for points. If each of those points penetrate into or share boundaries with each other, as a whole they form a line. And this line is different from the aggregate of the indivisible points of which it is formed.

As a result, I believe that Leibniz's argument is primarily a refutation of minima in the continuum, but not specifically of the indivisibles of the *Theoria Motus Abstracti*. This is mostly because Leibniz ignores the two important characteristics of the indivisibles of the *Theoria Motus Abstracti*: their having magnitudes, and their sharing boundaries. However, this is reasonable, because that conception of indivisibles has deep inconsistencies within itself. As is frequently indicated, the hardest thing to acknowledge with regard to indivisibles is their having magnitude and parts but no extension, which makes indivisibles in a sense already indistinguishable from minima. The reason why Leibniz does not make any mention of the magnitudes of indivisibles here may possibly be his

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[68] Bassler very briefly points out to this ignorance, as well (Bassler 1999, 165).

[69] Furthermore, these indivisible points may even penetrate each other, which makes the case more interesting yet more complicated. One can claim that with greater *conatuses*, there occurs a greater penetration. Accordingly, there may be equal number of indivisibles within the side and the diagonal, if the points of the diagonal have greater *conatuses*, and in turn a greater penetration.

awareness of the difficulty of that assumption. Without including *conatus* and motion, it is not easy to think of a magnitude of a point that is not extended. The fact that these magnitudes are comparable makes it even harder to imagine. The magnitude of a *conatus*, on the other hand, makes some sense, because it can be interpreted to indicate a potential to move faster than another with a smaller magnitude. Nevertheless, the magnitude of the motion of point or body moved through *conatus* at an instant, that is, the magnitude or space that the point or body traverses in an instant, is still unextended, which is not easier to understand.

In "De Minimo et Maximo," Leibniz denies any kind of minima or indivisibles, and time and motion are no exceptions. Since Leibniz thinks that he demonstrated the impossibility of the indivisible points of space, he accordingly argues that there are no indivisible or minimal time moments. This is because, in an indivisible time, with a uniform motion, only an indivisible of space would be traversed. If one argues that within an indivisible time an amount of space that is not a minimum is traversed, than it would follow that within a time that is not indivisible or minimum, "infinite divisible spaces would be traversed, and in some perceptible time, an infinite space would be traversed," and this would obviously not be a desired consequence (LLC, 11). Therefore, Leibniz asserts that time is not composed of minimal moments either.

In summary, it can be argued that the assumption of unextended but not minimal points is one of the main problems that Leibniz wanted to overcome with his new solution to the problem of the composition of the continuum. However, this was of course not the only reason to renounce indivisibles. As this last counterargument illustrates, it is hard to account for how a finite continuum is composed of indivisibles. For this yields, as Leibniz indicates, a ratio of the finite to the infinite, although both the indivisible and the line or motion or time interval are finite. In a broader sense, the composition of an entity from others which are qualitatively different than the thing they are said to compose is the difficulty that leads to the labyrinth of the continuum. Besides, as is discussed in the previous chapter, as their name suggests, indivisibles are also incompatible with the idea that the continuum is infinitely divided and composed of infinitely many parts. And in the *Theoria Motus Abstracti* Leibniz's indivisibles were apparently the ultimate constituents of this infinitely divided continuum. Thus, these were the problems concerning the indivisibles that Leibniz was to break free of by means of his new conception of the continuum that is illustrated in "De Minimo et Maximo."

4.2.2 The Monster of the Labyrinth: The Infinite

As its name implies, in "De Minimo et Maximo," Leibniz does not only have a problem with the minima, but also with the maxima. What Leibniz maintains by contesting maxima is the idea that there is nothing such as a largest or an infinite number, and an infinite whole corresponds to nothing. There he sets forth that "[*t*]*here is no maximum in things*, *or what is the same thing*, *the infinite number of all unities is not one whole*, *but is comparable to nothing*" (LLC, 13).

In fact, in his denial of maxima, Leibniz uses a very similar argument he used to refute indivisibles in this paper. The similarity stems again from the Euclidean part-whole axiom, which is the main idea lying under both arguments. But this time, denial of maxima is demonstrated not by means of a geometrical scheme, but through an arithmetical reasoning, which will be explained below. This argument, just as the one designed against minima, has at its background a need to confront the difficulties of forming a sound conception of the infinite. And this need is augmented more than ever by his deeper interest in Galileo.[70]

The refutation of maxima by means of an arithmetical analysis of numbers is directly associated with what he has recently read from Galileo. In the fall of 1672, Leibniz is known to have written some notes about Galileo's dialogue *Two New Sciences*.[71] Leibniz learns from this work that, as opposed to his belief in the *Dissertatio*, Galileo does not think that infinities are comparable. In the "First

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[70] cf. Bassler 1999, 168-169.

[71] These notes are included in LLC, 4-9.

Day" of his dialogue, through his main interlocutor, Salviati, Galileo declares that there are

> difficulties that derive from reasoning about infinites with our finite understanding, giving to them those attributes that we give to finite and bounded things. This, I think, is consistent, for I consider that the attributes of greater, lesser, and equal do not suit infinities, of which it cannot be said that one is greater, or less than, or equal to, another. (Galileo, 39-40)

The argument Galileo derives this conclusion from is the one that inspires Leibniz to argue against maxima and the infinite number. Galileo first indicates that it is customary to think that the aggregate of all numbers are more than, say squares, or roots of all numbers. But then he argues that it is not true, because every number is indeed the root of a square number (ibid., 40). Thus for each square number, there will correspond a root number, which comes to mean, for Galileo, that there are as many root numbers, as there are square numbers:

> I don't see how any other decision can be reached than to say that all the numbers are infinitely many; all their roots are infinitely many; that the multitude of squares is not less than that of all numbers, nor is the latter greater than the former. And in final conclusion, the attributes of equal, greater, and less have no place in infinite, but only in bounded quantities. (ibid., 41)

I think the similarity of this correspondence between root and square numbers to the one between the side and the diagonal is already noticeable. The difficulty is again comparing different sets of numbers, or "wholes," which have infinitely many numbers, or parts. Since these parts are infinitely many, there seems to be a one-to-one correspondence between the two infinities.

Now if there is such a correspondence, for Leibniz, it would come to mean that the whole is not greater than its part, which is absurd. However, the argument implies that Galileo does not directly allow such a conclusion. On the contrary, he thinks that these different infinities are not comparable. Nevertheless Leibniz thinks that, by the roots-squares example, Galileo comes to conclude that "in the infinite the whole is not greater than the part" which still violates the part-whole axiom (LLC, 9).

However, this does not mean that Leibniz is not aware of the difficulty Galileo's reasoning lays bare concerning the character of the infinite. In fact, in these notes, Leibniz begins to differentiate his conception of the infinite from Galileo's. Apparently, Leibniz thinks Galileo's argument fails because Galileo still speaks of an infinite number, or number of all numbers, and considers it to be a whole. And Leibniz is not wrong, since Galileo declares that the only number that can be considered the infinite is unity, that is, one (cf. Bassler 1998b, 857).

Galileo is not without a reason. Again reasoning from the same argument, he indicates that the ratio of the number of squares to the number of all numbers diminishes as the greater number of the comparison group is increased:

> up to one hundred there are ten squares, which is to say that one-tenth are squares; in ten thousand, only one-hundredth part are squares; in one million, only one one-thousandth. Yet in the infinite number, if one can conceive that, it must be said that there are as many squares as all numbers together. (Galileo, 41)

Galileo suggests that one departs from the infinite as one thinks of greater numbers. Thus, he reaches the conclusion that

> if any number may be called infinite, it is unity. And truly, in unity are those conditions and necessary requisites of the infinite number. I refer to those [conditions] of containing in itself as many squares as cubes, and as many as all the numbers [contained]. (ibid., 45)

Hence, Galileo asserts that the number of all numbers is one, or the unity of them. However, in a draft called "Accessio ad Arithmeticam Infinitorum," Leibniz opposes this view, by arguing that it does not satisfy the condition of containing all numbers, as it contains its powers and roots, etc:

> … Leibniz notices, the number of all numbers must be equal in size to the number of all even numbers, or all multiples of three, and so forth. These properties are not shared by number one, which, although it contains all its powers, does not contain all its multiples (Bassler 1998b, 858)

Leibniz's choice in place of unity, or one, is really interesting, nevertheless in line with his denial of maxima in "De Minimo et Maximo," where he writes that the infinite number is comparable to nothing. In “Accessio ad Arithmeticam Infinitorum,” he asserts that, if there is to be an infinite number, it should be zero, which satisfies all these conditions (ibid.). And indeed, zero contains all its roots, powers, and multiples within itself.

Yet, of course, the more important thing is not the proof that Leibniz sets forth to demonstrate that ‘zero is the number of all numbers’, but what Leibniz tries to suggest by this argumentation. In fact, Leibniz gives several arguments against the notion of “infinite number” in various writings of the Paris period, which I will not be able to examine within this study. Here, it is very important to see that, by making use of “zero,” which means for him “nothing,” and equating it with the infinite number, Leibniz comes to declare that there is *no* infinite number at all. As Knobloch neatly expresses, “only if we assume the existence of an actually given infinite number are we led to the Galilean paradox of the number sets” (Knobloch, 94).

Although Leibniz remains loyal to his ultimate objective to construct a universal language and reconciling mathematics with metaphysics with the aim of explaining the world, which he plainly articulates in his *Dissertatio*, by denying the infinite number, he reaches a point that is rather contrary to the ideas of the dissertation. In that early work, like Galileo, Leibniz spoke of infinite number of parts, or the number of all parts, as a whole. And since he thought at that time that the number of parts can be enumerated, it denoted for him a determinate unity. Accordingly, he believed that the infinite is something determinate, and comparable to other infinities. Furthermore, in the *Theoria Motus Abstracti*, Leibniz also talked about infinitely many actual parts of the continuum, though he did not make clear what he understood from the infinite.[72] However, by arguing now that the infinite number is zero, Leibniz seems to endanger his infinitely many actual parts, which is, though, not starkly felt in “De Minimo et Maximo.” It

[72] cf. Bassler 1999, 167: “In particular he does not make it clear whether this means simply that the number of parts is ‘greater than any given magnitude’, or whether he intends this actual infinity to be understood in some other sense.”

ill be seen that this problem will not only haunt Leibniz's account concerning the infinitely many actual parts, but also concerning any infinitesimal quantity he speaks of, and thus, the infinitesimals of his future calculus, since the character of infinitely many parts is directly related with the infinitesimal quantities.

Although Leibniz gives signals of assuming a rather new outlook in the following period of his Paris years, particularly concerning his conception of the infinite, this change does not take place all of a sudden. The task is so intricate, even for Leibniz, that it is very difficult to give a systematic account of his thoughts of the later years of his Paris period following "De Minimo et Maximo." But as Salviati in Galileo's dialogue says,

> let us remember that we are among infinities and indivisibles, the former incomprehensible to our finite understanding by reason of their largeness, and the latter by their smallness. Yet we see that human reason does not want to abstain from giddying itself about them. (Galileo, 34)

Now, at least one can say for now that in "De Minimo et Maximo," Leibniz sets himself free from these two, that is, infinities and indivisibles, by means of equating them with nothing:

> We therefore hold that two things are excluded from the realm of intelligibles: minimum, and maximum; the indivisible, or what is entirely *one*, and *everything*; what lacks parts, and what cannot be part of another. (LLC, 13)

This exclusion certainly opens the way for Leibniz to lay down a novel account of the continuum. However, this conclusion shows only that Leibniz completes the main negative part of his new idea. And now, it is time to see how he carries the *Theoria Motus Abstracti* to its limits, and what he posits, instead, as a new theory.

4.3 De Minimo et Maximo: The Continuum at its Limits

4.3.1 Infinitely Many Small Things

After his rejection of minima and the infinite number in “De Minimo et Maximo,” Leibniz puts forth his positive statements beginning with the one that is to ground the next formulation for the composition of the continuum:

> *There are in the continuum infinitely many small things*, *that is*, *things infinitely smaller than any given sensible thing*. (LLC, 13)

A couple of interpretative remarks need to be made in order to understand the nature of these infinitely many actual parts. First, this statement suggests that Leibniz replaces the indivisibles of the *Theoria Motus Abstracti*, which he equates with minima, with these smaller than any given, that is, infinitesimal, things. Yet, there is another perceptible difference between the accounts in the *Theoria Motus Abstracti*, and “De Minimo et Maximo.” Unlike in the *Theoria Motus Abstracti*, in “De Minimo et Maximo,” Leibniz does not say that these “things” are actual. This can be interpreted in different ways. One interpretation would hold that Leibniz takes these parts, and the continuum as well, not to be real, but ideal, or potentially divisible wholes. The other interpretation would take these infinitely many small things to be real as well as actual. I shall contend that in “De Minimo et Maximo,” the distinction between potential and actual continua is rendered rather indistinct, when compared to the previous accounts.

A second point of debate concerns whether Leibniz considers these infinitesimal things to be the ultimate constituents that *compose* the continuum. What I believe is that Leibniz has in mind that the infinitesimals compose the continuum. For if there are such things in the continuum, and there are no indivisibles, what else can a continuum be composed of? If there were assignable gaps, what we have would not be a continuum. And even if there were unassignable gaps in the continuum, it could not be argued that the continuum is *composed* of them. If the point is that, perhaps, Leibniz meant to say that a continuum is not something *composed*, but an ideal whole, which is prior to the parts in it, that is, parts of which are only potential, it would still not be wrong to say that the continuum is composed of those things. Besides, I still believe that for that time being, Leibniz does not have in mind a merely ideal continuum, though he first defines these things that are smaller than any given by apparently geometrical entities, namely points, lines, and surfaces, which are, as I will argue, at least analogous to real things.

Richard Arthur, on the other hand, remarks that in "De Minimo et Maximo," "Leibniz does not claim that these infinitesimals are actual parts of the continuum, as he had in the *Theoria Motus Abstracti* … his argument only demonstrates that there are such things in the continuum, not that they compose it" (LLC, Arthur's note 7, 379). In fact, in Chapter 3, I have mentioned Arthur's similar claims with regard to the *Theoria Motus Abstracti* itself. There Arthur pointed out that in the *Theoria Motus Abstracti*, "indivisibles or unassignables are not explicitly identified with the "actually infinite parts"" and therefore "they are not said to compose the continuum" (Arthur 2007a, 9-10). Nevertheless, he thought that this was Leibniz's intention (ibid, 3). As is to be remembered, he supported his claim by referring to Leibniz's retrospective comments. To recall again, in these comments Leibniz claims that he escapes the labyrinth of the continuum by postulating points that are greater than one another and Arthur accordingly argues that this "would make little sense if the points were not parts of the continuum" (ibid, 10, fn.13). Moreover, in the same page, he maintains that his claim is further confirmed with Leibniz's consideration of points as parts of space (ibid., 10). However, concerning this paper, Arthur does not quite allow for an interpretation that these infinitely small things are actual things which compose the continuum.

I cannot see why Arthur does not think that Leibniz's retrospective comment applies to the infinitely small things of "De Minimo et Maximo" as well, given that the conception of point, together with that of line and surface, defended in this paper is not different from the conception of point in the *Theoria Motus Abstracti* in these three senses: they are comparable to each other, they are parts of space or geometrical continua, and they are at least analogous to parts of physical things. Furthermore, as I will try to argue below, the infinitesimals of "De Minimo et Maximo" are more propitious than unextended indivisibles concerning the actual composition of the continuum.

To make sense of these infinitesimals which are neither minima nor indivisibles, let us first examine them as parts of the geometrical continuum by looking at how Leibniz defines 'points', 'lines' and 'surfaces:

> A point is of length, breadth, and depth infinitely smaller than any that can be sensed; a line is of breadth and depth infinitely smaller than any that can be sensed; a surface is of depth infinitely smaller than any that can be sensed. (ibid., 15)

Now Leibniz thinks that his definition of points, lines and surfaces follows from the premise that there are infinitely small things (ibid). In this respect, points, lines, and surfaces can be interpreted as being at least geometrically analogous to infinitely small actual things, if not identical to them. And it would not be wrong to claim that geometrical continua are composed of these infinitesimal things. Points, lines, and surfaces are not minima; they have infinitesimal quantities. And given that they are not indivisible, they are divisible, thus, extended, no matter how small they are.

However, there is a crucial point: Leibniz does not define a point with respect to a line, or a line with respect to the point. He underlines this point with the following words:

> … I do not wish to define a point as a line of length smaller than any given length, since a center should be conceived not as a line of length smaller than any given length, but as a figure smaller than any given figure, as, for example, the center of a circle should be conceived as a circle smaller than any given circle, the parts of which are angles. (ibid.)

It can be inferred from this passage that, for Leibniz, different dimensions are not actually transposable to one another.[73] Thus it seems that Leibniz thinks that the difference between points, lines, and surfaces is not merely a quantitative, but a

[73] This is discussed in Arthur 2007a, with reference to the differences between the interpretations of Pascal and Leibniz concerning Cavalieri's indivisibles

qualitative one. Now if assume that he thinks in that way, consequently, one can only argue that a continuous line is composed only of other continuous lines, a surface from other surfaces, and even a point, from other points. If adding infinitely many infinitesimal points does not compose a line, then it can be asserted that the relation of ultimate constituents with the continuum they are said to compose is different here than it was in the *Theoria Motus Abstracti*. In the *Theoria Motus Abstracti*, indivisible points, for instance, were said to compose bodies, and the ratio of a point to the body it composes were taken analogous to the ratio of a point to the line (LLC, 341). Now although the idea of the ratio of an infinitesimal to the continuum it composes is conserved—given that it is compared to the ratio of 1 to ∞, rather than to the ratio of 0 to 1—in "De Minimo et Maximo," if our assumption is sound, it appears to be that the ratio of *a point to the line* cannot be given in terms of the ratio of 1 to ∞.

So the main difference between the *Theoria Motus Abstracti* and "De Minimo et Maximo" can be summed as follows. In the *Theoria Motus Abstract* the continuum is heterogeneous in two senses: (i) the points constituting the continuum are heterogeneous with one another in that they have different magnitudes; (ii) the composition of the extended whole is different from the unextended indivisibles that compose it. In "De Minimo et Maximo," on the other hand, extended things are not composed of unextended ones. Thus the continuum of extended things is homogeneously composed.

In addition, even though the points of "De Minimo et Maximo" are extended as well, they do not compose lines. Here, the compositions of the different continua are homogeneous, that is, lines are composed of lines, while surfaces are composed of surfaces. In fact, it is implied that the infinitesimal points and infinitesimal lines are incomparable, or in Bassler's words, "they do not stand to each other in any finite ratio" (Bassler 1999, 174).[74] Here what Bassler signifies

[74] Bradley Bassler points out with respect to the *Theoria Motus Abstracti* that there are quantitatively heterogeneous elements in the continuum, which I have also mentioned. What I tried to call attention, on the other hand, was that these (heterogeneous) elements are qualitatively heterogeneous with the continuum they compose. Nevertheless, with reference to "De Minimo et

by the lack of the finite ratio must primarily be understood with respect to the ratio of different *kind* of infinitesimals to one another. That is to say, infinitesimals which have different dimensions, such as an infinitesimal point and a line do not have such a ratio. On the other hand, as in the *Theoria Motus Abstracti*, in "De Minimo et Maximo" it can easily be asserted that infinitesimals of the same kind, i.e., the same dimension, are comparable: "*There is one point smaller than another*" (LLC, 15)[75]

4.3.2 The Third Formulation: Continuity of Divisible Infinitesimals

The comparability of points provides the passage to the realm of metaphysics, motion, and the physical things, or bodies. As in the *Theoria Motus Abstracti*, here in "De Minimo et Maximo" the speeds of the motions of bodies are proportionate to the respective beginnings of their motions. The beginning of a faster motion traverses an infinitesimal line that is longer than the infinitesimal line traversed by the beginning of a slower motion, though the difference is smaller than any that can be sensed (ibid.).

Together with this account of lines, the motions of which are proportional to their *conatuses*, it is now possible to see the third formulation that Richard Arthur derives from Leibniz's early writings, which he suggests as the theory of the years between 1672 and 1675:

> a continuous line is composed of infinitely many infinitesimal lines, each of which is divisible and proportional to a generating motion at an instant (*conatus*). (Arthur 2007a, 16)

Maximo," the point Bassler emphasizes by "incomparability" in a sense parallels with my treatment of "heterogeneity" in this respect. Therefore, I should give credit to him. See Bassler 1999, 174.

[75] With respect to points, Leibniz explains this difference by the vertices of their angles, where angles signify the magnitude of points. A larger angle denotes a greater vertex, which means a greater point (LLC, 15). Leibniz does not only maintain that one point can be smaller than another, but also that "[*o*]*ne point can be infinitely smaller than another*" (ibid.). According to Leibniz, "any rectilinear angle, however small is greater than any angle of contact" i.e. the angle between the tangent and the periphery of the circle (ibid.; also see Arthur's 8th note in 379).

In fact, although it incorporates motion, this formulation concerns rather the geometrical aspect of the theory, as is obvious from its being constructed in terms of lines. Furthermore, as with the formulation given with respect to the theory in the *Theoria Motus Abstracti*, it does not incorporate Leibniz's account of cohesion. But since, the theory that is to be derived from "De Minimo et Maximo" embraces phenomena as well as geometrical entities, it should be applicable to the physical things, their continuity, and thus their cohesion, which I will try to discuss in the following section.

4.3.3 Bodies are Motions

The idea that lines are proportional to *conatuses* that generate motion is an indication of the fact that "De Minimo et Maximo" is not only a geometrical, but also a metaphysical account. In this respect, "De Minimo et Maximo" is close to the *Theoria Motus Abstracti* where the passage between geometrical and physical accounts is provided by means of motion, and in particular, through the concept of *conatus*, yielding a metaphysical account. *Conatuses* are considered to be beginnings of motions, which are illustrated in the above-given formulation by lines, though this time they are not indivisible beginnings.

However, at this point, it would be recalled that in the *Theoria Motus Abstracti*, Leibniz granted the possibility of the beginning of bodies or motions through indivisibles. This was the main reason behind Leibniz's preference for indivisibles in the *Theoria Motus Abstracti*, which was separate from the 'part-whole argument' he used to reject minima together with indivisibles in "De Minimo et Maximo." According to Leibniz, without indivisibles "the beginning and the end of a motion or a body would be unintelligible" (L, 139). As is detailed in Chapter 3, he tried to prove his argument via dividing a continuous line until the indivisible point which would represent the beginning is reached. Surprisingly, in "De Minimo et Maximo," he uses the same argument, but this time, not to justify indivisibles but to demonstrate infinitesimal things.

In fact, Leibniz's main proof for the existence of infinitesimal things is derived from this "beginnings" argument. He takes a line that is said to be traversed by a motion which, as expected, has a beginning. He designates some part of the traversed line from the left as the beginning of that motion. Then he begins to divide this part without cutting the beginning, which always remains at the left of the divided line. After a sequence of divisions, Leibniz arrives at his conclusion:

> … even if my hand is unable and my soul unwilling to pursue the division to infinity, it can nevertheless in general be understood at once that everything that can be cut off without cutting off the beginning does not involve the beginning. And since parts can be cut off to infinity (for the continuum, as others have demonstrated, is divisible to infinity), it follows that the beginning of the line, i.e. that which is traversed in the beginning of the motion, is infinitely small. (LLC, 13)

Thus Leibniz takes 'beginning' as a premise that cannot be denied, and given his rejection of indivisibles, he claims that those beginnings must be *infinitely small*.

However, Leibniz admits that, if the beginning of a space or body is an infinitely small line, or a point that is understood as an infinitely small line, it would not be the true beginning as far as it can still be divided, as one point or line can still be smaller than another infinitely small one. On the other hand, the option that assumes the existence of indivisibles or anything that lacks parts is unacceptable for Leibniz. Hence, it is exactly this point where Leibniz makes his distinctive move by inserting 'motion' and '*conatus*' into the account. He argues that an infinitely small line can be assigned as the beginning of a body, if that body is defined as something that moves:

> But if a body is understood as that which moves, then its beginning will be defined as an infinitely small line. For even if there exists another line smaller than it, the beginning of its motion can nonetheless be taken to be simply something that is greater than the beginning of some other slower motion. But the beginning of a body we define as the beginning of motion, itself, i.e. endeavour, since otherwise the beginning of the body would turn out to be an indivisible. (ibid., 17)

Thus, if one is to argue that there is always a smaller line, Leibniz's answer is that the smaller can be assigned as the beginning of a slower motion, while the greater is of a faster one, though they are both the constituents of the same continuous motion of the body. Accordingly, if there is any part of a body which does not move, its beginning must be an indivisible, which is not possible. Therefore, it can be maintained that, for the Leibniz of "De Minimo et Maximo," body is a set of smaller and greater motions propagated by *conatuses*, which is once again explicitly articulated by Leibniz:

> Hence it follows that there is no matter in body distinct from motion, since it would necessarily contain indivisibles, so that there is even less ground for a space distinct from matter. Hence it is finally understood that "*to be a body* is nothing other than *to move*." (ibid., 17)

Accordingly, there is no space in which no matter takes part, nor is there any matter which is not in motion, or in other words, which is not the motion itself. Thus, bodies are essentially motions, a view Leibniz has already maintained in his letter to Arnauld, as is discussed in Chapter 3. With this idea of equating bodies with motions, Leibniz clearly places 'motion' at the core of his account, and in line with his previous writings written just after the *Theoria Motus Abstracti*, he comes to deny the actuality of some of the notions he used before such as 'space' 'vacuum' or 'primary matter' in itself and at rest.

As a result, if there is a bodily continuum, it is composed of motions. These motions themselves are, on the other hand, instigated by *conatuses*, which are infinitesimal motions. *Conatuses* are nevertheless not indivisible; they are rather real parts of motion, as is indicated while examining his writings of the same period concerning cohesion. Neither are they just potentials; although they are infinitesimal, they are still extended parts of motions, that is to say, they are actual parts of motion. Nevertheless, in a sense, they can still be considered as potentials, since they are the instigators of motion and determine its speed. This is why I have claimed at the beginning of the previous section that the divergence between potential and actual becomes indistinct in "De Minimo et Maximo."

Furthermore, in a sense, these *conatuses* can be taken to be homogeneous with the motions generated by their stimulation, which saves the current account from the difficulty of accounting for the composition of the continuous whole from qualitatively different parts, if, of course, *conatuses* are understood as the ultimate constituents of the continuum.

In any case, continuum is composed of motions generated by *conatuses*. If continuum is composed of motions, then it comes to mean that there is a continuity of motion. And if motion is itself continuous, it would come to mean that *conatuses* are continuous, as well.[76] Thence, it can be claimed that continuum is in this sense ultimately composed of *conatuses*, infinitesimal parts of motions.

If bodily continuum is composed of infinitesimal motions, then one can certainly argue that at the beginning of "De Minimo et Maximo" what Leibniz means by infinitely many things in the continuum are these motions. Thus points, lines, and surfaces (defined in the previous section as the infinitesimals things in the continuum) turn out to be analogous to the infinitesimal parts of motions. Hence, I believe that this is a reasonable standpoint to hold, which renders it possible to state that the continuum is composed of infinitesimal motions. It can be argued that the gap between the geometrical and physical frameworks can be closed to the extent that the analogy between Leibniz's infinitely small geometrical entities and infinitesimal motions can be interpreted as bordering on identity.

I believe that Leibniz's metaphysical theory of *conatus* as the infinitely small but extended beginning of motion is what renders the transition between the geometrical and the physical accounts so smooth in "De Minimo et Maximo." If it is admitted that all of these parts are ultimately *conatuses*, owing to the fact that *conatuses* are infinitesimal motions, then these entities can be interpreted as pure geometrical representations of different kinds of *conatuses*. In this respect, I think their qualitative difference may be explained by reference to the characteristic or the principle of the motions they generate. But on the other hand, if it is maintained that they represent different kinds of actual *conatuses*, it may be

[76] Though the continuity of *conatuses* with one another should still be explained by their cohesion with one another. And Leibniz gives this explanation in his writings concerning cohesion, which has been already examined in this study.

difficult to account for the composition of the continuum. If they are, as has been argued, incomparable with each other, it would be hard to explain their cohesion with one another.

Notwithstanding this difficulty, if the account of cohesion Leibniz held at the beginning of the Paris period—which is presented in his writings “A Demonstration of Incorporeal Substances” and “On the Cohesiveness of Bodies” —is incorporated, Leibniz’s new theory becomes more interesting. If one can use “motions” instead of “bodies,” the cohesion of bodies can be explained by various motions’ endeavoring to enter one another’s place. Thus motions become continuous by means of cohering with one another. And this cohesion is ultimately carried out by *conatuses*. However, if one takes into account the idea in the *Theoria Motus Abstracti* that *conatuses* do not persist for any longer than an infinitesimal duration of time, then it can once again be inferred that there should also be a continuity of *conatuses* for the persistence of cohesion (cf. Arthur 1999, 116).

Although Leibniz still defends that *conatuses* are beginnings of motions, this account may eliminate the need to assign a beginning or an end to continuous motion. If all motions are continuous with one another, they share boundaries with one another. Then one may suggest two conclusions. First, one may say that motion is eternal, therefore there are no beginnings of the continuous motion in general. Second, every single *conatus* is both the beginning and the end of two consequent motions. Thus, while one *conatus* stimulates and becomes an infinitesimal part of motion, it also makes the previous infinitesimal motion cease. These two conclusions do not conflict one another and may simultaneously be maintained.

4.3.4 Concluding Remarks: The Limits of the Theory of *Conatuses*

Despite the fact that these are mostly speculations mainly inferred from a short piece of paper written by Leibniz, later some of these thoughts are even articulated by him, though it is difficult to argue that his entire theory is maintained in these writings. For instance, just before his ideas concerning various subjects of metaphysics becomes a flux of conflicting thoughts, in *De Materia*, *de Motu*, *de Minimis*, *de Continuo* penned in December 1675, Leibniz writes:

> (1) Every continuum is infinite. … (2) Change cannot cease, or, whatever is moved, will go on moving. And by the same token, it cannot begin. (3) Every body is in motion. For every body is movable, and whatever is movable has been moved. (4) To be in a place is to traverse a place, because a moment is nothing; and everybody is in motion. (LLC, 39, cf. DSR, 21).

Hence we arrive at what I choose to describe as the limits of Leibniz's early theory of motion. This theory is introduced in the *Theoria Motus Abstracti* and developed mainly in "De Minimo et Maximo." The critical move of "De Minimo et Maximo" is the negation of indivisibles that is defended in the *Theoria Motus Abstracti*. However, the theory in "De Minimo et Maximo" is radically different from the *Theoria Motus Abstracti* not because of this denial of indivisibles, but because it is grounded on the idea that motion is the essence of bodies, or more briefly that bodies are motions. Motions are infinitely many, and continuous, because they cohere with one another through *conatuses*, which can be understood as the ultimate constituents of motion.

The idea that bodies exist by means of endeavoring is also maintained in another essay of the same period called *Certain physical propositions*, where Leibniz states that "matter and motion or endeavor are the same, and that their differences are fictions just like between a subject and a characteristic attribute" (Leibniz quoted in Mercer 2001, 398). From the idea that *conatuses* are parts of motions, and therefore, they are motions, it can be derived that there is homogeneity in the continuum, which still allows for the diversity of motions, as long as motions are at the same time actual divisions. Later in 1675, this diversity of motions is more explicitly acknowledged by Leibniz when he indicates that every motion is "different from an infinity of others" (DSR, 13). In summary, unlike the *Theoria Motus Abstracti*, "De Minimo et Maximo" constitutes the continuum from parts that are not qualitatively different from the continuum they compose. Hence this account is saved from the main difficulty of the labyrinth of the continuum: accounting for the qualitative difference of the constituents of the continuum.

With this account, everything in nature is reduced to motions, or actions of *conatuses*. This homogeneous continuity is achieved by the bridging function of motion and *conatuses* serves to explain at the same time the cohesion within bodies and the cohesion between bodies, which together constitute a continuity of bodies and motion, explained in terms of motions and *conatuses*. This is Leibniz's conception of continuum in "De Minimo et Maximo": an actual continuum infinitely divided into continuous parts of motion. The idea underlying this entire dynamic account of continuum and rendering this account so innovative, in my humble opinion, was Leibniz's explanation of bodies in terms of motion, instead of a preference to remain faithful to the commonplace explanation of motion in terms of bodies. And I believe that is how Leibniz carries his theory of motion to its limits.

4.4 The End of the Theory: Nothing without Minds

There is another reason why I call this theory the limits of Leibniz's early as well as exceptional theory of motion and what is interpreted here as the third formulation of the composition of the continuum. But this reason concerns the other side of the limit, that is, the point beyond which this theory ceases to be a theory which is thoroughly grounded on "motion." I believe that Leibniz shifts to the other side of this limit, as soon as motion is portrayed as something that does not exist on its own, and the existence of bodies requires to be "sensed" by a mind.

I have deliberately postponed bringing up Leibniz's shift so far, with the intention of characterizing and presenting the thought of the early Paris period as one that is grounded on 'motion' and the concept of *conatus*, where the continuum is homogeneous with its constituents. Because in that particular form, the account of

"De Minimo et Maximo" could best be considered an exceptional account that is built on the *Theoria Motus Abstracti*.

Now given the account I have portrayed, it may seem quite surprising, but I maintain that the limit I have been speculating on can be interpreted to have been already surpassed within "De Minimo et Maximo," where Leibniz justifies the existence of bodies and motions through their being sensed. Now where exactly does Leibniz make this move? With respect to "De Minimo et Maximo," the last proposition given in this chapter before submitting some concluding remarks about the apparent theory of motion was that "*to be a body* is nothing other than *to move*" (LLC, 17). In the course of this essay, this proposition appeared to be innovative, in the sense that it was quite contrary to the established way of explaining motion through bodies. But the truth is, in no time at all, Leibniz revokes this traditional thought again: 'Motion' is always the motion of something:

> Since to be a body is to move, it must be asked what is to move. If it is to change place, then what is place? isn't this determined by reference to bodies? If to move is to be transferred from the vicinity of one body to another body, the question returns, what is body? Thus body will be inexplicable, that is, impossible, unless motion can be explained without body entering its definition. It is no good saying that to move is to change space, when we have concluded that there is no distinction between space and body. So what in the end are body and motion really, if we are to avoid this circle? (ibid.)

The key to Leibniz's answer to this question is the notion of mind, the absence of which would probably have surprised readers who are used to the indispensable role of this notion in other accounts of Leibniz. Nevertheless, the timing of its incorporation into the account is not unusual, given that Leibniz appeals to it whenever his account comes to a deadlock such as this explanatory circle.[77]

[77] In this sense, I find it meaningful to discuss whether Leibniz aims, from the very beginning, to arrive at the necessity of minds, and in particular, that of God, or he uses minds as a magic wand to appeal to whenever necessary.

As a matter of fact, Leibniz's theory is still innovative in the sense that he does not change his mind concerning his claim that bodies are motions. On the other hand, in order to avoid this explanatory circle of space, Leibniz deems it necessary to define body or motion in terms of being sensed by a mind:

> So what in the end are body and motion really, if we are to avoid this circle? What else, but being sensed by some mind. (ibid.)

Together with this proposition, bodies or motions swiftly become entities the existence of which depends on the existence of minds. This dependence does not stem from the reasons given in earlier accounts such as that bodies are nothing without their union with minds, or that their form or motion has its principle in an incorporeal. It stems from the claim that only things that are perceived can exist. Thus, in "De Minimo et Maximo," Leibniz employs another variation of the "*esse est percipi*" principle, which is mostly associated with George Berkeley, who was not born at that time.

In fact, as is to be remembered from the previous chapter, Bradley Bassler remarks that this principle has already been employed in *De Materia Prima* when Leibniz wrote that "*whatever is not sensed is nothing*" (ibid., 344, Bassler 2002, 226-227). Even this is not the first time Leibniz incorporated senses to the account. As is indicated from the very beginning, the problem of the continuum has always had an epistemological aspect. But especially together with the *Theoria Motus Concreti*, where Leibniz begins to speak of the sensible and insensible things, this epistemological aspect begins to get transformed from a question of knowledge into a question of sensation. Moreover, in "De Minimo et Maximo," it may be noticed that even in the definitions of what I have called the geometrical entities, that is, points, lines, and surfaces, Leibniz considers senses as a decisive factor. He defines these entities as things smaller than any that can be *sensed*. Finally, this decisiveness is amplified even to a greater extent, when Leibniz puts forth that bodies or motions do not exist unless they are sensed.

In spite of this decisive role that senses play, Leibniz does not make clear what he means by “to sense” or “to be sensed.” The only thing that is certain is that it is an activity of minds. Just as bodies, minds are also sensed by one another. On the other hand, Leibniz takes for granted that the absence of any particular mind does not affect the existence of things. Therefore, there should be a mind which senses everything. Moreover, such a mind must be independent of bodies, in order not to end in a new explanatory circle. Hence, Leibniz concludes, “*for the existence of bodies, it is certain that some mind immune from body is required, different from all the others we sense*” and it exists “*per se*” (LLC, 17). And this mind is, above all, God. Accordingly, Leibniz thinks that his argumentation in “De Minimo et Maximo” “vindicated the necessity of minds, but from among minds, only God’s” (ibid., 19).

Together with the addition of this condition of being sensed by mind, the earlier connection between minds and bodies through *conatus* appears to be ignored by Leibniz. In its stead, Leibniz places sensation as the connection between minds and phenomena. However, this time, existence of bodies becomes a mere matter of being perceived, which keeps one from speaking of a continuum between minds and bodies, since the actuality of bodies comes in question.

4.4.1 To be is to be Harmonious

I think Leibniz’s distinction of minds from bodies by means of what he called the harmony of *conatuses* was quite successful as a metaphysical attempt. Furthermore, it was in line with his aim to revive an Aristotelian conception of substance that is compatible with mechanistic physics. This account made it possible to assign each body its own principle of motion by what Christia Mercer calls a “mind-like substance” (Mercer 2001). As long as *conatus* was considered to be a striving, bodies and minds were made both comparable and different at once. They are comparable because both are reducible to actions stimulated by *conatuses*. The differing characteristics of the *conatuses* that stimulate the relevant actions, on the other hand, render them different. As will be recalled from the discussion of Leibniz's letter to Arnauld in the previous chapter, body consists in a sequence of motions whereas mind consists in a harmony of *conatuses*.

Although Leibniz does not speak of mind as the harmony of *conatuses* in "De Minimo et Maximo," the notion of *conatus* is not renounced all of a sudden, but it appears occasionally within the flux of Leibniz's upcoming thoughts and the centrality it had in the *Theoria Motus Abstracti* gradually ceases.[78] And the conception of minds as harmony of *conatuses* is substituted by a conception of the harmony of sensations granted by God. In an important part of *De Summa Rerum* written in February 1676, which is known as "On the Secrets of the Sublime," Leibniz argues that

> [p]articular minds exist, in sum, simply because the supreme being judges it harmonious that there should exist somewhere that which understands, or, is a kind of intellectual mirror or replica of the world. To exist is nothing other than to be harmonious; consistent sensations are the mark of existence. (DSR, 25).

At this point, it is possible to think that the more Leibniz departs from the accounts he maintained particularly in the *Theoria Motus Abstracti* and the beginning of Paris period, the closer he approaches to the main ideas of his mature system expounded in the *Monadology*. However, as it is argued in the previous chapter, there were very important ideas in the *Theoria Motus Abstracti*, such as the idea of individual substances with minds, which already foreshadowed the *Monadology*. Furthermore the *Monadology* was not only a continuation of certain accounts that have essential correlations with it, but rather a product of all the thoughts and writings that Leibniz revised, canceled, and developed. Thus, as is implicitly suggested throughout this thesis, the mature metaphysical system of Leibniz can be seen as a consequence of a developmental series of accounts and ideas, each of which progresses through surpassing the inconsistencies and contradictions within the previous ones. Therefore, I believe that it is more

[78] However, the notion of *appetite*, which is quite important for the account in *Monadology*, has its origins from the notion of *conatus*, which is also evident by Leibniz's equating these two (see LLC, p.335). Unfortunately, I will not be able to the connection of these early and late notions of Leibniz's thought within this study.

meaningful to see Leibniz's writings as a unity, constructed within quite a long deal of time.

However, in the end, what is known as Leibniz's mature "solution" to the labyrinth of the continuum, or to the problem of the composition of the continuum, was not the one that is outlined above with reference to "De Minimo et Maximo" and its conception of continuous infinitesimal motions. What made this conception exceptional was that it provided a perfect homogeneity, which would be one of the main desiderata concerning an account of continuity. This perfect homogeneity was provided by means of reducing the entire corporeal world to motions. In this respect, it may be argued that Leibniz could have easily incorporated minds into this account. As he previously maintained, the actions of minds were thoughts, while the actions of bodies were motions, though both thoughts and motions were fundamentally composed of *conatuses*. Moreover, Leibniz did not search further for a definition of 'sensation' as he did for 'motion'. Sensations could also be reducible to actions, because what is sensation, in the end, if it is not a kind of action?[79] But if Leibniz had reduced sensation to action, it would have again caused an explanatory circle, since "to act" would mean "to be sensed."[80]

Nevertheless, by reducing everything ultimately to actions, the characteristic of which is determined by *conatuses*, Leibniz might have sustained the perfect homogeneity as regards bodies and minds, or corporeals and incorporeals, as well. Therefore, he might have arrived at what I have illustrated in Chapter 2 as the

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[79] On the "Plenitude of the World" Leibniz writes that "... perceptions consist ... in an aggregate of infinitely many acts" (DSR, 85).

[80] In any case, it would have been better if Leibniz had given at least a further definition of sensation, given that he defined perhaps the most fundamental metaphysical term, i.e. existence, in terms of it: "simply that which is the cause of consistent sensations" (DSR, 25). Later Leibniz grounds this idea that existence is to be harmonious on his Principle of Perfection, or the Principle of the Best, which is given as the sufficient reason for the existents. (See for instance Brown and Fox, 179, or Russell, 33-39 among many). This principle is already evident in one of the late Paris writings called "On Existence": "So for things to exist is the same as for them to be understood by God to be best, i.e., the most harmonious. We have no idea of existence, other than we understand things to be sensed. Nor can there be any other idea of existence, since existence is included in the essence of necessary beings alone" (DSR, 113).

most comprehensive continuum. This conception would also be in harmony with his future idea that everything has some kind of a mind, though with different degrees of confusion.

4.4.2 Later Paris Writings: The Waning of the *Conatus* Theory

The choice of explaining the existence of motions by sensations puts Leibniz's pursuit of the conception of continuity on quite a different track. The direction of this track is determined by a transition from a *conatus*-based account of motion to a more sensation-based account that ultimately leads to his mature conception of the *Monadology*.

In what follows, I would like to try to decipher some of the reasons behind Leibniz's change of direction, in the hope that it would be of some help in understanding his motives. Many of these reasons can be inferred from the flux of thoughts of his late Paris period.[81] However, since these thoughts are fairly volatile, it becomes hard to distinguish why Leibniz renounces certain thoughts or embraces others. Given the erratic and multitudinous nature of the Paris writings, it is not possible to do full justice to and engage in a thorough treatment of all these writings within the scope of this thesis. Therefore the following discussion does not by any means claim to be comprehensive. I shall only try to identify few reasons behind the abandonment of the account based on actual infinitesimal *conatuses*, which are, in fact, all interrelated with one another.

The abandonment of the *conatus*-based account is instigated by Leibniz's growing concern over the reality of infinitesimals. After Leibniz directs his interest, on the one hand, to an account based on the harmony of sensations and minds, and on the other hand, to intense mathematical studies on the nature of the infinite, he begins to change his mind concerning the character of the infinitesimals. In fact, this is not really unexpected, given Leibniz's association of existence with being sensed, and his definition of infinitely small things with respect to senses. If to exist

[81] cf. Mercer 2001, 293

meant to be perceived, then it would not be reasonable to argue that infinitesimals, which are smaller than any that can be sensed are actual constituents of the continuum.

Accordingly, in the later Paris writings Leibniz begins to interpret infinitesimals not as actual things, but as *fictions*. For instance, in a work penned in Paris named *De Quadratura arithmetica*, Leibniz states that

> Nor does it matter whether there are such quantities [as infinities and infinitesimals] in the nature of things, for it suffices that they be introduced by a fiction, since they allow abbreviations of speech and thought in discovery as well as in demonstration. (Leibniz quoted in Arthur 2007b, 22)[82]

This idea that infinitesimals are fictions that help mathematical analyses and understanding phenomena survives in Leibniz's mature writings, as one of his letters to Des Bosses clearly shows:

> Philosophically speaking, I hold that there are no more infinitely small magnitudes than infinitely large ones, or that there are no more infinitesimals than infinituples. For I hold both to be fictions of the mind due to an abbreviated manner of speaking, fitting for calculation, as are also imaginary roots in algebra. (Arthur 2008, 3).

Richard Arthur associates the interpretation of infinitesimals as fictions with Leibniz's statement in "On Motion and Matter" written in April 1676 that "endeavours are true motions, not infinitesimal ones," and considers this idea as the mark of the collapse of the *conatus* theory of cohesion and the "actualist interpretation of infinitesimals of Leibniz's third theory" (Arthur 2007a, 21, Arthur 2001, xli). Since *conatuses* were no longer infinitely small motions, infinitely small interpenetration of bodies through these *conatuses* became implausible. Furthermore, according to Arthur, it turned out to be hard to account for the "curved trajectories of bodies" by means of finite *conatuses*, which could rather constitute polygons (ibid., 24).

[82] cf. Anapolitanos, pp.84-86. See Arthur 2008, for the whole story of Leibniz's consideration of infinitesimals as fictions.

It would be misleading to say, however, that Leibniz is consistent on the question of infinitesimals throughout the Paris period, since there are writings in which Leibniz speaks of infinitely small actual things. In these writings, one can see that Leibniz denies infinite numbers but still pursues to demonstrate the existence of infinitesimals in nature. To take only one example, in "On the Secrets of the Sublime," a very important but highly unpredictable writing of the Paris period, it can be inferred that Leibniz searches for a passage between geometrical and physical accounts: "… the hypothesis of infinities and of infinitely small things is admirably consistent and is successful in geometry, this also increases the probability that they really exist" (DRM, 29). Correspondingly, although Samuel Levey agrees that Leibniz takes infinitesimals as fictional entities, he also states that Leibniz "is not fully satisfied that they have been proved to be impossible" (Levey 1998, 55, fn. 9).

The second reason for Leibniz's abandoning his account of the continuum composed of infinitesimal *conatuses* I believe stems from the difficulties concerning the overall applicability of this account. The theory of 'body' equivalent to infinitely divisible infinitesimal motions was not easily applicable to the world of physical things. Above all, as their definition suggests, these infinitesimal motions were said to be smaller than any that can be sensed. Since to be is to be sensed for Leibniz, just like any infinitesimal entity, infinitesimal motions turned to be fictions with respect to Leibniz's new account. Moreover, provided that a physical account requires elements that are somehow given to senses, it would not make much sense to argue that there is a perfect compatibility between geometrical or metaphysical accounts and physical account.

Moreover, some very crucial features of the earlier theories based on *conatuses* such as interpenetration of the parts of a continuum, or body's being at two places while in motion were abandoned. Probably, Leibniz considered these features he had defended before to be contrary to the senses. As I have already argued, abandonment of the account of interpenetration of things was a direct consequence of considering infinitesimals as fictions.[83] I think not experiencing such interpenetrations in the nature of things may be one of the reasons that encouraged Leibniz to renounce this theory with respect to his sphere-shaped constituents of matter more easily.

His second earlier idea that a body is at two places at the same time in a given time instant was also explicitly denied in the late Paris years. Although in December 1675, Leibniz still describes motion being at more than one place in a finite interval of time[84], in his dialogue *Pacidius Philalethi*, written in the autumn of 1676, through one of the interlocutors, Leibniz states that to be in two places at the same time is as absurd as simultaneously being and not being in the same state (LLC, 167, see Arthur 2001, xli). Instead, Leibniz defended that motion should be defined as

> a state composed of the last moment of existing in some place and the first moment of existing, not in the same place, but in the next different place. Therefore … motion is … the aggregate of two momentaneous existences in two neighboring places. (ibid.)

Furthermore, things get more complicated when minds are incorporated into the account. For example, it would be really difficult to account for the reason why we do not see that individual minds, each of them being harmony of *conatuses* and in turn continuous with each other, coalesce with one another and become "one." A similar difficulty is expressed by Leibniz in one of the Paris texts, from which it can be inferred that minds would be much more volatile then they are now, if the same laws governed bodies and minds (DSR, 9).

Finally, I maintain that Leibniz rejection of indivisibles made this theory of infinitesimal motions hard to defend. As is indicated in numerous occasions, Leibniz had in mind to establish a metaphysics grounded on an Aristotelian conception of substances. Within the compass of this aim, in the *Theoria Motus*

[83] See Levey 1999, 102.

[84] "If something is moved with a speed than which no greater is intelligible, it will be everywhere at the same time" (DSR, 19).

Abstracti Leibniz spliced bodies and minds within an indivisible and thus indestructible individual substance. Thus, one of the advantages of indivisibles for Leibniz was that they provided an indestructible housing for minds. Consequentially, without indivisibles minds became homeless, and therefore ready to be destroyed, though it was the last thing Leibniz would expect from an incorporeal substance. Moreover, Leibniz might have also thought that this denial of indivisibles becomes an obstacle in front of the individuation of minds. Although an account of individuation of minds relying upon the harmony of *conatuses* would be a possible defense, a continuous striving of actions one over another would make it difficult to account for stable individuated minds.

Thus a return back to the idea of indivisibles is already perceivable in Leibniz's later Paris writings. In December 1675, he distinguishes space from minds by means of his statement that "whatever has separable parts cannot think" (ibid.). Likewise, in March 1676, he argues that minds are indivisible entities:

> … there is something in space that remains throughout changes, and this is eternal … Whatever is divisible, whatever is divided, is altered—or rather, is destroyed. Matter is divisible, therefore it is destructible, for whatever is divided is destroyed. … There is in matter, as there is in space, something eternal and indivisible … thought enters into the formation of matter, and there comes into existence a body which is one and unsplittable, i.e., an atom, of whatever size it may be, whenever it has a single mind. … From this it can easily be understood why no mind can be dissolved naturally; for if it could it would have been dissolved long ago. (ibid., 45, 47)

In fact this becomes the reason why in some of his Paris writings, Leibniz defended atoms and even vacuum. In "On the Plenitude of the World" while Leibniz maintains that there is a plenum of atoms, since there are no assignable vacuum, he appears to be implying the "unassignable gaps" he defended in his letter to Thomasius: "all emptiness collected into one have no greater ratio to any assignable space than the angle of contact has to a straight line" (DSR, 89). Parkinson thinks that this would come to mean "that as far as science is concerned, from there is no vacuum, but that for metaphysics, there is" (DSR, Parkinson's endnote 109, 123). In fact, a similar view is implied in "On the Secrets of the Sublime":

> A metaphysical vacuum is an empty place, no matter how small, provided that it is genuine and real. A physical plenum is consistent with a metaphysical vacuum which is unassignable. (ibid., 23, 25)

This physical plenum that Leibniz has in mind while writing "On the Secrets of the Sublime" is by his expression "a multitude of infinitely many points, that is, of bodies which are less than any which can be assigned" from which he derives those metaphysical voids (ibid., 23). On the other hand, what follows from this physical plenum of infinitely many points has very significant implications concerning Leibniz's mature view on the continuity of matter and space:

> Perhaps it follows from this that matter is divided into perfect points, i.e., into all the parts into which it can be divided. No absurdity follows from this. For it will follow that a perfect fluid is not continuous but is discrete, i.e., is a multitude of points. Therefore it does not follow from this that a continuum is composed of points, for liquid matter would not be a true continuum, even if space is a true continuum. From this, again, it is evident how much difference there is between space and matter. Matter alone can be explained by multitude without continuity. And indeed matter seems to be a discrete entity; for even if it is assumed to be solid, yet in so far as it is matter, when its cement ceases to exist (e.g. by motion or by something else) it will be reduced to a state of liquidity, i.e., of divisibility, from which it follows that it is composed of points, since it can be dissolved into points. I prove this by the internal motion of a solid. Matter is therefore discrete, not a continuous entity. It is only contiguous, and is united by motion or by a kind of mind. (ibid., 25)

One may easily perceive in these sentences a blend of several earlier thought and arguments concerning the problem of the continuum. Leibniz speaks of infinite division, unassignable vacuum, and cohesion by motion or mind. Richard Arthur rightly points out the influences of the ideas articulated by Galileo, Gassendi, and Descartes with regard to these particular thoughts of Leibniz (LLC, 387, Arthur's note 5). Nevertheless, these ideas, such as the actual infinite division of and discreteness of matter, potentially infinite divisibility of "ideal" space, will prove to be central in Leibniz's mature conception of the continuum.

I think these ideas of the later Paris period lay bare how decisive an effect Leibniz's preference of 'indivisibles' in line with a conception of substance grounded on minds has on his entire physical and metaphysical accounts. In fact, in the *Theoria Motus Concreti*, a conception of a "plenum" that consists of indestructible, indivisible spherical atoms, which have circular motions around their own axes, is already maintained. However, it seems that in these later writings Leibniz ceases to call infinitely divided corporeal parts 'a continuum', which may be related with the decline of the account of cohesion through *conatuses* and with it the idea of interpenetration between *conatuses*.

As is already mentioned, there is not a complete consistency between different writings of the Paris period. To the contrary, it is more like a flux of different thoughts, which Leibniz goes through as a result of many reasons some of which I have just mentioned.[85] However, from inside this flux, Leibniz extracts his infinitesimal calculus and many ideas that would play great roles in his future philosophy. Nevertheless, this should never be taken to mean that Leibniz does maintain a perfectly consistent standpoint thenceforth. Although not as frequently and drastically as in his early years, Leibniz goes on to change his ideas time after time, and continues to defend different thoughts which may seem to be contradicting with each other. Perhaps this is the only possible way that Leibniz could advance in the labyrinth.

[85] In fact, what we have covered so far with respect to Leibniz's writings between his *Dissertatio* and the beginning of the Paris period can also be considered to be a part of this flux of thoughts. Bothe Catherine Wilson and Christia Mercer think along these lines. Mercer indicates that throughout 1670s, "Leibniz's views about cohesion, continuity, motion, and matter are in flux. He tries out positions, rejects them, and then picks them up again. … Such frequent changes in ideas about the constituents of reality suggest that Leibniz's entire system is in flux" (Mercer 2001, 293). Wilson writes that between the *Confessio Philosophi* of 1671 and the famous *Discourse to Metaphysics*, there is a swirl of statements relating to minds, particles, God and vortices which are highly resistant to systematic interpretation (Wilson, 223).

CHAPTER 5

CONCLUSION: THE BRIDGING FUNCTION OF THE METAPHYSICS OF *CONATUS*

One thing that this thesis ascertains is the aptness of the term 'labyrinth' in explaining the intricate nature of the problem of the continuum. Going along with Leibniz in his search for a way out of the labyrinth, we have passed through a web of entangled paths, bridges, tunnels, all of which laid before us several new riddles to be unraveled. For this study, not the exit, but the labyrinth itself and Leibniz's efforts to find a way out were the main foci of interest. Accordingly, in this thesis, I tried to trace the paths Leibniz chose when moving forward in the labyrinth, keeping an eye on the motives behind his choices, and the consequences that are brought about by them. It is occasionally witnessed how a chosen path proved in the end to be an impasse. However, I deduced that each impasse contributed to Leibniz's subsequent pursuit of an exit from the labyrinth. Moreover, there were times in which it was me who lost the trail and became unable to follow Leibniz's reasoning. Consequently my quest in search of an understanding of Leibniz's solutions concerning the problem of the continuum became a labyrinth on its own. On the other hand, my perplexity encouraged me to make speculations regarding the nature of the labyrinth. Along with what Richard Arthur set forth as the phases in Leibniz's interpretation of infinitesimals and early attempts on the problem of the continuum, I have discussed three alternative solutions to the problem of the composition of the continuum that can be derivable from the early writings of Leibniz, and tried to examine them as exhaustively as my knowledge and the scope of this thesis allowed.

Leibniz's progress in the labyrinth can be considered as consisting of several subsequent moves. I deem it possible to establish connections between most of these moves. These connections suggest a common *modus operandi* that governs the progress in the labyrinth, which can be characterized as i) a negation of the earlier accounts (of the other philosophers or of Leibniz himself) and abandoning their characteristic ideas, ii) but at the same time preserving some other decisive ideas involved in these accounts, iii) and finally making use of them in a developed form in the following account. That is to say, in each new account Leibniz introduces, it is possible to perceive its characteristic difference from and similarity to an earlier account that he had taken into consideration. I believe that this governing *modus operandi* which I claim is involved in Leibniz's progress in the labyrinth may provide a perspective for the following conclusive remarks.

As easy as it is to perceive the influence of other philosophers and schools in the earliest writings of Leibniz, it is equally hard to speak of the finer distinctions he feels more and more forced to make as he encounters the difficulties of the labyrinth. Accordingly, the more Leibniz makes these distinctions, the more he approaches a new solution to the composition of the continuum. Furthermore, with respect to the writings that I have examined in this thesis, new accounts of motion accompany the formation of these new solutions, with motion gaining increased importance with each new account.

The mode of operating I have just mentioned above is apparent first of all in Leibniz's gradual differentiation of his position from that of other philosophers'. Thus this differentiation is not an outcome of an utter negation of the thoughts of others. Rather, in his early years, Leibniz often makes use of ideas originating from the philosophers that influenced him against one another in distinguishing his accounts from the others, while trying to address some of the mainly same questions they were occupied with.

In this thesis, I have often tried to accentuate two claims, which are intrinsically connected to one another. The broader claim is that Leibniz brings together the geometrical/ideal and real/physical accounts of the composition of the continuum within the framework of metaphysics, or what may be called the metaphysical account of the composition of the continuum. The second and the more specific claim is that the geometrical and physical accounts are partially coalesced into the metaphysical account through the unifying characteristic of the notion '*conatus*' and motion that is continuously instigated by *conatuses*. This coalescence comes more evidently to light as the role of motion in the account becomes more essential. In other words, *conatuses* and motion generated by them, function as a bridge between the geometrical/ideal and the physical/real accounts.

By these two claims I intend to point at the possibility of the success of the solutions to the composition of the continuum which are based upon a dynamical conception of the constituents of the continuum. Given the somewhat smooth and sheer transitions between the ideal/geometrical and real/physical accounts, my two claims also serve to indicate that, in Leibniz's early accounts, these domains do not easily lend themselves to isolation from each other, even though Leibniz begins to severe them from each other in his later Paris years, in connection with a relative waning of the idea of *conatuses*.

The bridging role of metaphysics of *conatus*, and the relationships and transitions between the above-mentioned accounts of the continuum, may be examined in three interdependent contexts. The first one involves the question that is at the core of the labyrinth: "what is a continuum composed of?" This question has been mostly examined with reference to the continuity of space, bodies, and motion. Although this context appears to be primarily a geometrical one, it was seen that the margins gradually waned. The second one concerns the question "how is a continuum composed of its constituents?" which is directly connected to the problem of cohesion, that is, the problem of how the parts of a continuum cohere so that they are the same parts of a continuous whole. Although this seems more of a concern of physical accounts, Leibniz's explanations stem from metaphysical grounds, and seem applicable even to the geometrical constituents of the continuum. The third context for examining the bridging role of metaphysics of *conatus* is the relation between bodies and minds, or corporeals and incorporeals. This relation is in fact the point where this thesis begins and ends.

5.1 The Composition of the Continuum

In the geometrical/ideal accounts of the continuum, the main concern is the composition of the geometrical or spatial continuum, which has at its center the question "what is a continuum composed of?" However, insofar as Leibniz does not differentiate bodies from mere extension, or even from mere matter (which he calls the "primary matter" in his April letter to Thomasius), continuity of bodies can be included within the borders of the geometrical account. In a manner of speaking, continuity of bodies *as* geometrical figures designate the borders of the geometrical account. On the other hand, on the other side of this border, there is the continuity of bodies *as* physical existents, and of their physical constituents.

Throughout this study, it is seen that Leibniz makes his distinctions between 'body,' 'space,' and 'matter' in stages. In Chapter 2, it is noted that, at first, he does not differentiate between bodies and mere matter, or between mere matter and space (or extension). In the *Dissertatio*, Leibniz treats the continuum as being composed of infinitely many bodies or parts, and therefore as infinitely divisible, although he does not distinguish (potentially) infinite divisibility from (actually) infinite division. Furthermore, he takes numbers to be analogous to bodies, which implies a bodily continuum composed of infinitely many atomic units, though he does not provide a satisfactory account for this composition. Thus in the *Dissertatio*, since Leibniz does not make these distinctions, it seems as if the geometrical and the physical composition of the continuum are matching. However, when the epistemological aspect of the problem is involved, which is unavoidable when the knowledge and conception of the 'infinite' is in question, the deviation between geometrical and physical, and therefore the ideal and real accounts, becomes visible.[86]

Together with his theological writings, Leibniz begins to distinguish bodies from mere space and extension by their motion, though the reality of motion remains highly in question. Further, in his April letter to Thomasius, Leibniz distinguishes between primary matter and bodies. While primary matter is an ideal, geometrical, infinitely divisible continuum that designates a potential for motion and division, bodies, which are generated by and divided through motion, are only contiguous. Thus, with reference to an Aristotelian conception of continuity, there is a physical or corporeal discontinuum in nature.[87]

Heretofore, motion only separates the geometrical/ideal and physical/real domains; it does not bridge them. Furthermore, strictly speaking, motion itself is not continuous either. In the letter to Thomasius and *De Rationibus motus*, what we have is a sort of discontinuous motion, interrupted by infinitesimal, or by Arthur's expression *unassignable*, "little rests."[88] One of the crucial points to note with respect to this conception of motion is the characterization of these rests, that is, their being infinitesimal, or "smaller than any given." Owing to this characterization, Leibniz thinks, in a sense, that motion is continuous, because these rests are "shorter" than any given. The frequency of these rests, on the other hand, determines the variations in the speed of the moving body, thus, according to Leibniz, they do not render motion discontinuous. Furthermore, in his letter to Thomasius, Leibniz accounts for motion by means of an explanation based on God's *continuous* creation, according to which God recreates bodies in each and every instant of their motion.

Thus, Leibniz considers bodies to be corresponding to assignable instants in and of motion, insofar as bodies are distinguished from mere matter by their motion. And between these instantaneous bodies, there are infinitesimal rests, or so to

[86] See Section 2.1

[87] See Sections 2.2 & 2.3

[88] See Section 2.4.1.

speak, infinitesimal or unassignable gaps. Hence, Arthur's first formulation illustrating the solution to the composition of the continuum is introduced: "the continuum is composed of assignable points separated by unassignable gaps" (Arthur 2007a, 5). As a result, although in this particular account, motion is not the unifying means between the ideal and real realms, it is the notion by means of which a formulation of a common explanation of, or solution to, the composition of the continuum is derived. Furthermore, this common explanation comes to suggest that there is a transition from the account of the spatial continuum to the account of continuity of motion. However, since bodies are identified with beings at instants, that is, some kind of "assignable points," the solution can still be considered a geometrical one.[89]

In the *Theoria Motus Abstracti*, examined mainly in Chapter 3, a new conception of the continuum, and together with it, a novel account of motion, are proposed. However, in the *Theoria Motus Abstracti*, not all the earlier ideas in connection with the problem of the continuum are negated. On the contrary, although these new accounts are quite different from the older ones, they nevertheless stand on ideas the seeds of which are to be found in the earlier accounts.

One of these ideas that are preserved in the *Theoria Motus Abstracti* is the infinite divisibility of the continuum. However, in the *Theoria Motus Abstracti*, Leibniz maintains that the continuum is not only potentially divisible, but also actually divided. With this move, Leibniz distinguishes his position from i) that of Aristotle's, in the sense that, he welcomes actual infinite division; and ii) that of Descartes', in the sense that he thinks not that the divided parts are indefinitely many, but actually infinitely many.[90]

The second idea that is preserved is the notion of "infinitesimal," that is "smaller than any given." However, this time, infinitesimals are not gaps or rests, but the actual constituents of the continuum: unextended indivisibles with magnitudes smaller than any given. Accordingly, this time, Leibniz has a conception of the

[89] See Section 2.4

[90] See Section 3.1

continuum as composed of actual indivisibles, which are infinitely many. However, Leibniz differentiates his indivisibles from minima, that is, from minimal points, which he associates with Euclidean points, which has no part and magnitude. In this respect, Leibniz adopts a sort of Hobbesian point, but denies that the point is "undivided," and that its magnitude is "not considered."[91] As a result, the skeleton of the second solution to the composition of the continuum, formulated by Arthur has been formed: "The continuum is composed of indivisibles, defined as parts smaller than any assignable, with no gaps between them" (Arthur 2007a, 8).

Leibniz demonstrates the existence of indivisibles on the basis of the idea that otherwise the beginning and the end of a line, body, time, or motion would be unintelligible.[92] Hence Leibniz declares that the beginning of a body and space is an indivisible point, of time an indivisible instant, and of motion an indivisible *conatus* (endeavour), that is, an infinitesimal striving for motion. Therefore, from the demonstration of indivisibles, Leibniz's account brings in *conatuses* as the actual constituents of the continuous motion.

In connection with this "beginnings" argument, another idea that was implied while discussing *De Rationibus motus* in Chapter 2 may be considered to be preserved in the *Theoria Motus Abstracti*. As has just been indicated, in *De Rationibus motus*, Leibniz explains the variations in the speed by the frequency of infinitesimal, or unassignable, rests. In the *Theoria Motus Abstracti*, the variations in the speed is explained once again in terms of infinitesimals, though this time infinitesimals are not "rests," since Leibniz argues that there are no rests in motion. Besides, this time it is not the frequency, but the magnitudes of infinitesimals, which determine the speed of the motion of bodies. Infinitesimal *conatuses*, which are the beginnings of bodies, have different magnitudes, and the bigger this magnitude, the faster the motion of the body is. In other words, the speeds of the motions of bodies are proportionate to the respective beginnings of

[91] See Section 3.1.2.

[92] See Section 3.1.2 for Leibniz's argumentation.

their motions. Therefore, indivisible points as well as *conatuses* are comparable, and their magnitudes have a finite ratio with respect to one another.

Hence, in the *Theoria Motus Abstracti*, the role of 'motion' is considerably augmented and *conatuses* begin to take a central role in Leibniz's accounts of the composition of the continuum. When *conatuses*, understood as beginning *points* of motion, are incorporated into the account as infinitesimal constituents, motion becomes a significant element of the geometrical account. As one might say, geometry acquires dynamism through *conatuses* and the motions stimulated by them. On the other hand, (although the physical account based upon *conatuses* is not yet mentioned in these concluding remarks) the geometrical/ideal account approaches the border of physical/real accounts, with their difference becoming more indistinct as metaphysics of *conatuses* bridges these two accounts. In his theory of "concrete motion," which accompanies the *Theoria Motus Abstracti*, Leibniz maintains that the continuum is infinitely divisible into infinitely many actual parts. These actual parts are stated to be sphere-shaped indivisible corpuscles each of which has a circular motion around its center. Now what makes the physical account approach the above-mentioned border through the mediation of metaphysics is that the motion of these indivisible yet infinitesimal physical spheres is again propagated by the *conatuses* that the corpuscles have.

At this point, I would like to point out an important ground of the claim that the geometrical/ideal account approaches the border of physical/real accounts and that metaphysics performs the bridging function. Through the idea of the comparability of *conatuses*, Leibniz incorporates causality into his abstract/geometrical account. As is indicated, he explains the variety in the magnitudes of the continua, by the magnitudes, or potentials, of their beginnings, which is particularly explained in terms of *conatuses*. And the fact that there is such a causal relationship between the geometrical/spatial continua and its constituents is evident when Leibniz argues that from the inequality of *conatuses*, the inequality of points follows (L, 141). And this is a very important step in Leibniz's growing conception of bodies in terms of the motion of its parts.

As is indicated within Section 3.1, Leibniz's statement that the parts of the continuum are actual already suggests that what Leibniz speaks of is not a mere geometrical continuum. This does not mean that Leibniz constructs the geometrical continuum from the actual points, but that geometrical account begins to get transformed into a physical one. In the same way, one may also claim that the divergence between the geometrical and physical accounts is reduced, if these indivisible points of the "geometrical continuum" can be taken as analogous to the sphere-shaped actual corpuscles of his concrete theory. However, there is also something that precludes the smoothness of the transition between the geometrical, physical, and the metaphysical accounts: In the *Theoria Motus Abstracti* the ultimate constituents of the continuum are not homogenous with the continuum they compose. There are indivisible points, but they compose extended lines; there are indivisible corpuscles, but they compose infinitely divisible bodies; and above all, there are *conatuses*, which compose motions, although they are not "motions" themselves. Furthermore, if it is acknowledged that the *conatuses*, or the indivisible points, first and foremost signify the beginnings of motion and bodies, but not the motions or bodies themselves, and that the magnitudes of these motions or bodies are proportionate to these indivisibles, *conatuses* and other indivisibles can be interpreted as *potentials* to generate motions or through the motion of points, extensions, rather than *actual* parts of motions or bodies. I believe that this is a point that is not much elucidated in the *Theoria Motus Abstracti* and it is "De Minimo et Maximo" that establishes a nearly perfect homogeneity of the constituents with the continuum they compose.

In Chapter 4, I tried to examine some of the early Paris writings, among which "De Minimo et Maximo" was considerably important. I believe, and have claimed, that this piece of writing developed the theory of motion and the conception of the continuum Leibniz introduced in the *Theoria Motus Abstracti* to its limits. In "De Minimo et Maximo," many ideas of the *Theoria Motus Abstracti*, such as the denial of minima, comparability of points, and infinite division of the continuum are preserved. However, there are crucial changes, first of which is Leibniz's identifying indivisibles with minima, and therefore negating them. What follows this negation is his idea of the composition of the continuum from extended but still infinitesimal "things", which are smaller than any given, and which are infinitely many.[93] Since these constituents are extended, the difficulty of accounting for the composition of the continuum from things that are heterogeneous with the continuum they compose is thus overcome in "De Minimo et Maximo." Hence Arthur submits the third formulation of the composition of the continuum: "a continuous line is composed of infinitely many infinitesimal lines, each of which is divisible and proportional to a generating motion at an instant (*conatus*)" (Arthur 2007a, 16).

However, what makes "De Minimo et Maximo" a theory at its limits, and radically different from the *Theoria Motus Abstracti* is not the negation of indivisibles, but Leibniz's explicit statement that motion is the essence of bodies, that is, "bodies are nothing other than to move" (LLC, 17). Furthermore, although Leibniz still retains the idea that *conatuses*, or infinitesimal things, are the beginnings of motions or bodies (which appears to be the same in this case), they are now infinitesimal parts of motions, which are homogeneous with the continuum they are a part of. Hence, through his metaphysics of *conatuses*, which are not any more mere potentials, but rather actual parts of motions, Leibniz saves his account from the difficulty of accounting for the qualitative difference of the constituents of the continuum, and sets forth a conception of continuum that is composed of infinitely many continuous but diverse motions.

On account of the claim that *conatuses* are actual parts of motions, in sections 4.3.3 and 4.3.4, I maintained that what Leibniz implies by infinitely many things in the continuum that are smaller than any given are these infinitesimal motions. From this perspective, my interpretation was that the infinitely many small things, i.e. points, lines, and surfaces that Leibniz speaks of are geometrical analogies to the infinitesimal parts of motions. Accordingly, I argued that the divergence between the geometrical and physical frameworks can be reduced to the extent that this analogy between Leibniz's infinitely small geometrical entities and

[93] See Section 4.3.1.

infinitesimal motions can be interpreted as bordering on identity. Finally, I hold that this accord between geometrical/ideal and physical/real accounts is once more provided in terms of the infinitesimal motions, namely *conatuses*. However, in order to substantiate this claim, I should first consider the way we have taken in the labyrinth in the second context I have named above: the question of how the composition of the continuum occurs from the ultimate constituents, or so to speak, the problem of cohesion.

5.2 The Problem of Cohesion

If the composition of the continuum is at the center of the labyrinth, then the problem of cohesion is the circle surrounding this center. In this sense, cohesion of parts within a continuum should unavoidably be accounted for in giving a sound conception of continuity. Before Leibniz, other philosophers had given various accounts for this problem. At the very beginning, Leibniz differentiates his position from the others concerning the problem of cohesion, as he suggests the need for a further explanation concerning cohesion *within* the constituents.

The moderns were trying to explain cohesion without resorting to other incorporeal entities. While, for instance, some of them thought that the reason for the cohesion between parts is the pressure of the air, or what is worse, "nature's abhorrence of vacuum," some others suggested that atoms are held together by means of some sort of hooks and rings. However, Leibniz thought that none of these explanations could account for the cohesion within the ultimate constituents.[94]

The first alternative Leibniz proposed against these explanations is the greatest incorporeal principle, that is, God. But before long I think Leibniz realized that the real question concerning the problem of cohesion was not "*by whom*" but "*how* the cohesion is actualized." There is a possible interpretation that Leibniz later associated cohesion with impenetrability and resistance.[95] At first he spoke

[94] See Section 2.2.1.

[95] See Section 2.3.1.

of impenetrability as one of the primary qualities of body. However, since his use of impenetrability was inconsistent with his idea of infinite division, soon after he switched to the notion of 'resistance' instead of 'impenetrability'. According to him, the cause of the resistance of bodies was their being in motion. Therefore, he differentiated body from mere space, and concluded that a body at rest is no different than a vacuum.[96] However, since the principle of motion was not in bodies, but is derived from an incorporeal principle, this account of cohesion by resistance is not a well-founded one. Furthermore, neither impenetrability nor resistance is sufficient to explain how the parts of a body cohere so that they constitute a certain individual body, because all bodies were, in principle, resistant, insofar as they are in motion.

Leibniz finds the answer he desires again in the theory of *conatuses*. This is somewhat a revolutionary solution, though it has as its basis an Aristotelian conception of continuity. This explanation of cohesion suggests that since *conatuses* are infinitesimal strivings for motions, they strive to enter one other's boundaries. Accordingly, bodies which are stimulated by *conatuses*, impel each other, and within infinitesimal durations of time, they penetrate one another so that their borders become one. This is how Leibniz explains cohesion, and together with it, the continuity of bodies with one another, as well as the cohesion and continuity within a particular body. As long as motion is ceaselessly instigated by *conatuses*, continuity persists.[97]

As a result, in the *Theoria Motus Abstracti*, and also in the early Paris writings that deals with the problem of cohesion, Leibniz brings together the continuity of bodies with the continuity of motion. This is also how the cohesion and continuity of indivisible geometrical points (of a line or a body as a geometrical figure) is explained. Through the motion and interpenetration of points, the metaphysical account grounded upon *conatus* and motion once again turns a geometrical account into a dynamical one which borders on the physical account. Likewise, in

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[96] See Section 2.3.1.

[97] See Section 3.2.2.

what Leibniz calls the concrete theory of motion, sphere-shaped corpuscles (*bullae*) cohere with one another and within themselves by means of their circular motions (see Arthur 2001, xl).[98]

In the writings examined in Chapter 4, Section 4.1, a quite similar account of cohesion to the one maintained in the *Theoria Motus Abstracti* is presented. However, since in the beginning of the Paris period, Leibniz considers *conatus* not as a mere potential, or as an unextended indivisible beginning of motion, this time *conatuses* are almost perfectly homogeneous with the motions they compose. Furthermore, in these writings, Leibniz accentuates his definitions of motion as being in two places at an infinitesimal time instant. This has the consequence that through the infinitesimal motions, namely *conatuses*, bodies immediately penetrate one another's place "by a part of it smaller than any given part" (LLC, 3). Thus their boundaries become "one" and how they become "sympathetic," i.e., co-moving, which means that "one cannot be impelled without the other" (ibid.). This idea suggests that there is a "kind of" harmony between *conatuses*, which make them co-moving, and in turn, a cohesive, continuous whole. Furthermore, through *conatus*, being the infinitesimal motion itself, Leibniz advances from points' sharing boundaries to the interfusion of motions in the physical world, as he says that from this account of co-moving bodies, "follows the propagations of motions in a plenum" (ibid., 23).

5.3 Bridging the Unbridgeable: Bodies and Minds

The terminus in our examination of Leibniz's struggle with the labyrinth from the standpoint of the relation between geometrical, physical and the metaphysical accounts of the continuum is the context described by the relation between bodies and minds, or corporeals and incorporeals. In fact, I intend to bring these

[98] Although there is a considerable correlation between the physical and geometrical accounts, one may still hold that there is a divergence between them. In fact, as is explained in Section 3.3.2, Leibniz thinks that this divergence between the physical things and what is suggested by the abstract account is mostly not sensible and "suffices for the phenomena" (LLC, 343). In addition, here, Leibniz begins to incorporate senses into the account in the way that presages his mature conception of continuity. He states that senses ultimately cannot distinguish whether a body or motion is continuous or contiguous, or whether it is at rest or has an infinitesimal motion, just as they cannot distinguish between an infinitangular polygon and a circle (ibid).

concluding remarks to an end, by looking at the transformation of the relation between these the most fundamental metaphysical categories, which lie beneath the labyrinth of the continuum.

In this respect, it is first of all worth reminding that from the very beginning, Leibniz has a project of reconciling scholastic/religious and modern/mechanical thoughts under the title of a new sort of Aristotelian philosophy. This project was decisive for his overall attitude towards metaphysics in general, and for his dealing with the labyrinth of the continuum, in particular.

The first occasion in which the relation between body and mind, or corporeals and incorporeals is observed in this thesis was in Section 2.1, where Leibniz's *Dissertatio* was examined. There Leibniz established a mover-moved relation between bodies and minds, or between God and the whole of corporeal world. Although this was essentially a separation, it was critical that this separation was brought about by 'motion.'

Leibniz's distinction was inspired by a sort of Scholastic interpretation of Aristotelian prime-mover argument. In the writings following the Dissertatio, Leibniz's aspiration to a sort of Aristotelian metaphysics intensified. Leibniz denied the pure mechanism of the moderns, which could not sufficiently account for the primary qualities of bodies. This was because he thought that they totally disregard incorporeals or reduce them to mere mechanical entities, which did not allow for the free-will or autonomy of minds, and therefore, for an Aristotelian conception of individual substances. He argued against the moderns by claiming that the primary qualities of bodies (magnitude, figure, and motion) do not follow from the nature of "body." However, he preserved in his philosophy the idea of the moderns that phenomena should be explained principally by these primary qualities of bodies, and these explanations should be kept as simple as possible. In harmony with these "positive" aspects of modern philosophy, Leibniz negated the needlessly complicated explanations of the Scholastics and their effort to account for phenomena in terms of some ghostly incorporeals.

As is just indicated, at the beginning, Leibniz kept God, the infinite incorporeal principle, as the direct cause of motion of bodies. However, along with his aspiration to an Aristotelian conception of substance, he wanted to give some autonomy to minds with respect to motion. As a result, he preserved the idea of an incorporeal principle, but in place of God, he put the concurrent mind, which corresponded to bodies and formed together with them individual substances as a union of body and mind. Therefore, without minds, the ontological status of bodies and motions became questionable. On the other hand, body taken *together with* a concurrent mind was considered to be a substance, which was in harmony with Leibniz's intentions. Nevertheless, together with the *Confessio*, Leibniz distinguished the actions of minds and bodies, which were said to be thinking and moving, respectively. However, all the mechanism of motions and change in connection with bodies were thenceforth founded upon minds.[99]

Although in his *De Transsubstantione*, Leibniz moved toward an Aristotelian conception of substance, in his letter to Thomasius, he receded to a radical position by his idea of continuous creation of a body in motion by God, and therefore rendered motion completely dependent upon its creator, i.e., God. Thus, this was again a conclusion which Leibniz would not be in favor of, at least for a long time. And after a short time, Leibniz abandoned his continuous creation hypothesis. On the other hand, this abandonment rendered motion once again unfounded, until Leibniz found a new account to ground motion, bodies, and minds, all at once.[100]

Needless to say, this account, introduced in the *Theoria Motus Abstracti*, is based on the theory of *conatuses*. By means of this theory which he developed to a new form, Leibniz finds a new principle of motion, and establishes a new version of Aristotelian substance. Preserving the earlier distinction between the actions of minds and bodies as thoughts and motions, respectively, Leibniz reduced all actions to a combination of *conatuses*. He argued that, bodies are "instantaneous

[99] See Section 2.2.2, and Arthur 2001, xxix.

[100] See Sections 2.3.2 and 2.3.3.

minds," since they cannot retain their *conatuses* no longer than a moment, and therefore, as opposed to minds, they lack memory.[101]

Leibniz's reducing of bodies and minds to different kinds of combinations of *conatuses* becomes more evident in his letter to Arnauld. There Leibniz characterizes minds by "harmony of *conatuses*," bodies by a "sequence of *conatuses*." Furthermore, since mind has as its bases *conatuses*, that is, indivisible and therefore indestructible constituents, Leibniz comes to establish indestructible husks for minds. Furthermore, as Daniel Garber states, the cause of motion of bodies becomes no more in God, but in their own minds, which "together with their bodies constitute genuine substances (Garber 1995, 277). Hence, from an account of indivisible geometrical points, Leibniz arrives at the composition of minds, that is, harmony of *conatuses*. And this constructs my final point with regard to the bridging role of '*conatus*' and a metaphysics based on it.[102]

However, although it meets Leibniz's demands of reconciliation of the philosophy of Aristotle with the philosophy of the moderns, with a conception of substances that is reminiscent of his future monads, the connection he establishes hereby between minds and bodies through *conatus* and motion is later, in the end of "De Minimo et Maximo" replaced by a connection provided through sensations.

This shift is inaugurated by Leibniz's definition of motion in terms of "being sensed by a mind." Together with the end of "De Minimo et Maximo," Leibniz defends a variation of the "*esse est percipi*" principle, arguing that "things which are perceived by a mind exist." Accordingly, Leibniz states that in this case, a mind absolutely independent from bodies is required. Thus, he believes that he demonstrated the existence of God, by this argument based on sensations.

As a result, the earlier connection between minds and bodies through *conatus* ceases to be maintained, together with Leibniz's addition to his account of motion the condition of being sensed by mind. In place of *conatuses*, on the other hand,

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[101] See Section 3.2.1.

[102] See Section 3.4.1.

Leibniz makes use of sensations as the connection between minds and phenomena. In his later Paris writings, even existence is explained in terms of the harmony of sensations which is *principally* provided by God.

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