# THE APPLICATION AND EVALUATION OF FUNCTIONAL LINK NET TECHNIQUES IN FORECASTING ELECTRICITY DEMAND

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# ABSTRACT

THE APPLICATION AND EVALUATION OF FUNCTIONAL LINK NET TECHNIQUES IN FORECASTING ELECTRICITY DEMAND

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This thesis analyzes the application of functional link-net (FLN) method in forecasting electricity demand in Turkey. Current official forecasting model (MAED), which is employed by Turkish Electricity Transmission Company (TEİAŞ) and other methods are discussed. An emprical investigation and evaluation of using functional link nets is provided.

Keywords: Electricity Demand Forecasting, Nonlinearity, Functional Link Net (FLN)

# ÖZ

ELEKTRİK ENERJİSİ TALEP TAHMİNİNDE FONKSİYONEL BAĞ AĞLARI TEKNİĞİNİN UYGULANMASI VE DEĞERLENDİRİLMESİ

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Bu çalışma, Türkiye elektrik enerjisi talep tahmininde fonksiyonel bağ ağları tekniğinin deneysel bir uygulamasını incelemiştir. Resmi talep tahmin yöntemi (MAED) ve diğer güncel araştırmalar da incelenmiş ve farklı doğrusal olmayan modeller aracılığı ile yapay sinirsel ağların performansı ölçülmüş, TEİAŞ'ın modeli ile karşılaştırılmıştır.

Anahtar Kelimeler: Elektrik Enerjisi Talep Tahmini , Doğrusal Olmayan Modeller, Fonksiyonel Bağ Ağları (FLN)

To My Parents

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All my studies covered an important period of my life, I gratefully thank to my mother, my father, my sister and my spouse for the motivation, patience and the love they gave.

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# CHAPTER 1

# INTRODUCTION

The Turkish electricity sector has been operating under new rules and regulations since August 2006. The regulations are creating a new market for electricity that replaces government monopoly. Electrical energy is now a commercial commodity to be purchased and sold between parties. Accordingly, market players will predict future market behaviors, and therefore, electricity demand will have to be forecast by different distribution companies in different regions. This could cause problems in predicting the national demand in a reliable way.

Electricity demand projections are provided by The Turkish Electricity Transmission Company (TEİAŞ) on behalf of The Ministry of Energy and National Resources (MENR). These projections have been subject to error levels of up to 42 % over an 18 year horizon and 22% over 3 years. They rely on the use of a dated forecasting tool known as MAED (Model for Analysis of Energy Demand) first developed in 1977 and widely used in 40 different countries since then. This is clearly undesirable considering implementation of policy based on such projections.

Energy demand forecasting based on mathematical models is an established and mature area of research and various methods have been applied to this purpose. The literature is vast and widely dispersed and not always easy to get hold of since best-performing applications are often quartered in proprietary sources. Despite the maturity of the subject, new models and implementations are still constantly reported.

This study seeks to find out whether a more accurate forecasting tool,which is also easy to use in respect of data requirements, can be developed. We have been motivated especially by recent research on energy demand forecasting based on artificial neural networks (ANN), and particularly on the use of what is known as "functional link net" (FLN) mechanisms that are employed to improve forecast accuracy. Our concern in ANN models however, will not be to construct or to question the performance of ANNs, but will be confined to an empirical investigation of the potential of the FLN mechanism to improve demand forecasting, when it is implemented as a stand-alone application. Analysis of recent work in the area indicates that artificial neural networks with functional link nets perform well, but the mechanics are not clearly defined in the literature. One of our objectives is to acquire a better understanding of the mechanism by trying out and comparing different implementations of the concept in an experimental framework.

Foreasting demand for energy derives from complex interactions in human activity that cannot be determined completely. In applied economics, demand is assumed to be a function of an indeterminate number of “independent variables”, several of which if not all, are in fact interrelated to varying degrees. It is routinely assumed that the most important of these “determinants of demand” include the price of the good, the prices of related goods, income levels, the size of the consuming population, various expectations as to the future and other such factors. Forecasting methods traditionally belong to one of two broadly defined classes; those relying on an explicit association between the demand and its most significant determinants, together with a projection of those determinants into the future; and those that assume that the association is embedded in and therefore its effect can be captured from an analysis of historical demand, or time series. The first approach allows formulating our perceptions of the future in the form of different scenarios and thus reduces the disadvantage of having to assume that the future is already embedded in the past, a potential drawback to which the second approach seems to be more vulnerable. If the forecast horizon is long and energy investments are to be based on forecasts, relying on time-series projections quickly become questionable especially because most determinants of demand cannot be assumed to follow stationary paths. Functional Link Net models were first introduced by Pao (1988) in connection with ANN models as a device that enhances the performance of these models while simplifying their structure at the same time. The basic idea is to reduce the reliance of ANN models on so called hidden layers by providing nonlinear transformations of the determinants of demand which constitute the inputs to the network model. This idea is somewhat similar to the simultaneous use of various nonlinear transformations of the independent variables in linear regression models, but is taken up and pursued to further extent in a different framework than that of regression models.

This study is motivated by the use of FLN in ANN models and investigates the potential of the FLN concept independently when it is implemented for forecasting with models that seek to minimize forecast errors. The resulting implementation falls into the category of forecasting methods that rely on an explicit association between demand and its determinants, and therefore is suitable for use in investment planning for energy. Our investigation is exploratory and entirely computational, and is necessarily selective in the range of possible transformations that can be considered.

The FLN based model that we implement has the following characteristics: It relies on complex nonlinearizations of different specifications of the relation between forecast demand in a given period and its determinants in the same and previous periods, as well as forecasts for previous periods. Hence the equations are of a functional form in that the unknown – logarithmic—function appears on both sides of the equation. The model is formulated as an optimization model where the sum of squared errors is minimized and the nonlinear link between input and output variables at each time period are defined as constraints. All input variables are given, so decision variables are the constant coefficients of each transformation factor. Actual demand values from time series are included as input variables as are the gross domestic product (GDP) and the energy price index, as well as functions of these. Errors to be minimized are computed on the basis of proposed functional relations. We employ a systematic procedure to arrive at the final model specification by starting with a candidate list of possible transformations and constructing a simple model separately for each. The final selection of the transformations are based on the performance of each simple model as we select from among the best performing.

The performance of the models are compared against those from MAED projections produced by the Government and also those obtained from regression models that adopts similar nonlinear transformations of the independent variables that are used in the FLN models. In addition, we also use results produced by Akan & Tak(2006) employing a regression model with error correction steps. To ensure fair comparison we first use monthly demand data for two overlapping periods (1987-1996 and 1987-2000) to estimate model coefficients and once models are estimated, we project the GDP and energy price variables that will be used as inputs to forecasting for two periods (1997-2005 and 2001-2005). We then run the models on the basis of these projections to forecast demand for the same two periods. Comparisons are based on error calculations reflecting the difference between forecasts and observed demands.

The results are mixed but indicate the potential of FLN based models especially when lagged variables are included in the model specification.

The rest of this dissertation is arranged in three chapters. In the next chapter we briefly describe the Turkish energy system and discuss demand forecasting in general. In particular we describe the MAED model and discuss its performance. In the following chapter the FLN model is described and the forecasting methodology is explained in detail. Finally we provide a conclusion for our findings and indicate further possible research.

# CHAPTER 2

# THE TURKISH ELECTRICITY SUPPLY SYSTEM AND DEMAND FORECASTING

Forecasting models have to produce reliable results for the decision maker to take the right actions. Kumbaroğlu(2006) defines reliable models as:

- apply an appropriate modeling technique for the aim of forecasting
- have reasonable relationships between variables in the model
- assumptions are to be true or reasonable

Forecasting electricity demand has been the subject of much effort for both academic and professional purposes, and a very extensive volume of research has accumulated since the beginning period of the industry. This literature has been distributed over an extended range of resources that are not easy to access or assemble. Methods adopted clearly depend on the forecast horizon which is determined by the purpose of the study.

Forecasting electricity demand for operational purposes, such as for committing generating units over the coming week or scheduling power dispatch for the next day is necessarily quite a different problem than forecasting over the next year, which is important for fuel management and plant maintenance, or over the next several years which is necessary for investment purposes.

In this research, modeling techniques, the electricity consumption system in Turkey and necessary assumptions will be considered at the beginning with the previous studies in the literature. While doing this, it is not our aim to provide any exhaustive review of the entire literature but to provide a list of the principal methods of approach to forecasting energy demand for investment purposes. Our interest here will be confirmed to providing the backdrop that is necessary to place the present study into context. Hence research cited for this purpose is selected to represent a particular approach on forecasting.

## 2.1 TURKISH ELECTRICITY SUPPLY SYSTEM

An account of how the Turkish electricity supply system has developed through the years would provide a context and help better to understand the reason behind the importance of projecting electricity demand in an accurate way.

This part of the study will analyze electricity supply system in Turkey to understand the background of supply planning which is directly influenced by the accuracy of demand projections.

Turkish Electricity Supply System (ESS) has been growing with fast increasing demand as observed in most developing economies. The ESS has evolved over the years through four main periods: settlement, governmental monopoly, privatization and liberalization periods.

The Settlement Period started just before the beginning of the First World War. The first power plant of the Ottoman Empire was The Silahtarağa Thermal Power Plant commissioned in 1914. In 1923, total installed capacity was only 33 MW. With the foundation of Turkish Republic, installed capacity increased up to 126.2 MW and government created and charged Etibank with the responsibility of managing power supply and The General Directorate of Electric Power Resources Survey and Development Administration (EİEİ) was also founded. In 1939, municipalities were assigned for managing electricity sales in Turkey. In 1952, the first transmission lines were installed in İstanbul that marked the beginning of the interconnected system. Government involvement continued with State Hydraulic Works (DSİ) constructing three important power plants, Sarıyar HES, Seyhan HES and the Tunçbilek thermal power plant in1956. The Ministry of Energy and Natural Resources was founded in 1963.

Organizational restructuring led to a new period after year 1970 that can be called the central government monopoly period. The Turkish Electricity Authority (TEK) was formed to manage generation, transmission and distribution as a state monopoly except for a few local projects. Installed Capacity was 2234.9 MW at that time. Keban Hydropower Plant (HPP) was set up with half of the total national electricity generation capacity, 1330 MW in 1975. In 1980, installed capacity was increased to 5118.7 MW. In 1982, although TEK took all the rights of electricity generation from other organizations, such as municipalities and some other governmental associations, it did not last for a long time because Turkey entered to privatization period regarding electricity generation.

Privatization period started in 1984. Establishing Electricity Generation Plants or transfer of operating rights (TOOR) were given to private organizations. In 1993, TEK was separated into two companies: Turkish Electricity Generation & Transmission Company (TEAŞ) was formed for electricity generation and transmission and Turkish Electricity Distribution Company (TEDAŞ) was responsible from electricity distribution. Build-Operate-Transfer (B-O-T) Model was introduced in privatization period by the government of that time. In this model, electricity, generated by private companies, was bought by the government. Build – Operate (B-O) models were the next stage in the privatization period in 1996.

In 2001, liberalization period was initiated for supporting privatization. Formation of the Market was started with the Energy Market Law: Energy Market Regulatory Authority (EPDK) was formed and TEAŞ was separated into three companies: Turkish Electricity Generation Company (EÜAŞ), Turkish Electricity Trading and Contracting Company (TETAŞ) and Turkish Electricity Transmission Company

(TEİAŞ). Market formation went on with the decision of privatization of TEDAŞ in 2004. TEDAŞ was separated into 14 distribution companies. Privatization of these distribution companies still continues today. (TEDAŞ web site (2007), EMO Energy Commission Report (2006))

With these regulatory changes, demand forecasting will be done by private distribution companies and investments will be done according to these projections. Thus, electricity demand forecasting will be a more important tool with the increasing trend in the consumption whereas available resources are limited.

## 2.2 FORECASTING TECHNIQUES

Electrical energy is a strategic asset since it is the main resource for both household and industry. It is an important resource for all sectors, it cannot be stored and the requests for electrical energy changes during the day, so its procurement on time with best possible costs is a major problem of the suppliers. An effective planning can be done with a reliable forecast, thus, electricity demand planning is very important to secure affordable service levels. In this section, different forecasting methods in the literature were reviewed to understand the principles of forecasting.

Demand estimation methods mainly consider controllable and uncontrollable variables to predict on the outcome. Controllable variables may be price, advertisement, prices in distribution and the uncontrollable ones are the weather, inflation, GDP .

Past approaches have taken variables such as energy cost, alternative fuel cost, production levels (Donnelly (1987)) , and temperature in the case of energy demand or month-to-month energy consumption forecasts (Al-Shehri(2000)), into consideration among others. The number of facilities is also another variable with the assumption that two facilities will consume more energy than one facility with the same output, given the same technology and energy practices. Number of employees (population or working population is similar), transportation cost, electricity cost and fuel prices are some other examples that were used before. (Flores (2004))

Four main ways of obtaining demand information are defined by Dobbs (2000):

- qualitative demand information (guesstimates)
- historical data
- market survey information
- direct market experiments

Dobbs(2000) argues that guesstimates are not preferred if there is formal statistical information, since decision makers also consider past information when they make judgments on any scenario and historical data will help more if individual sales data for each product type exist. In addition, it is better to have the conditions apart from firm-specific data (general inflation, the measures of wealth or income) on that specific time period the historical data belongs to, so that these factors can also be considered. He emphasizes that *historical data are of greatest use in stable markets which feature little innovation or new product competition.*

On the other hand, market survey information may help finding how people behave in different scenarios, but it is less reliable than the direct observation of actual market behavior. Actual behavior can be discovered by direct market experiments such as the effect of speculations about energy crisis on energy consumption behavior, which is assumed to influence consumption in the short-term.

Energy Forecasting Methods are classified by Nasr (2002). His classification of forecasting methods is helpful to summarize all methods, so classification in this study follows his guideline, and summarizes better known methods in the literature.

### 2.2.1 Traditional Forecasting Methods

Time series and regression methods are generally defined as traditional methods. This section briefly defines methods and related work.

***Time Series Models***

Time series methods are often called naïve methods, since they only require the past values of the variable to be predicted. It is commonly used for operations planning applications. A pattern is to be defined to predict the future. These patterns could be a trend, seasonality, a cycle or randomness may also occur. The method is preferred where a dependent variable has a stable pattern. (Sweeney (1978))

Commonly used time series models are autoregressive (AR) and moving average (MA) models. AR( $p$ ) refers to the autoregressive model of order $p$ . The classical representation of forecasted value X at time t ($X_t$) according to AR( $p$ ) is

$$X_t = c + \sum_{i=1}^{p} a_i X_{t-i} + \varepsilon_t \qquad (2.1)$$

$X_t$ : forecasted value at time t

$\varepsilon_t$ : error value at time t

$c$ : constant

$p$ : order of autoregressive term

MA(q) model is defined as the moving average model of order q. Its representation can be formulized as:

$$X_t = \varepsilon_t + \sum_{i}^{q} b_i \varepsilon_{t-i} \qquad (2.2)$$

$X_t$ : forecasted value at time t

$b_i$ : weight for error lag i

$\varepsilon_t$ : error value at time t

q : order of moving average term

Box and Jenkins (1976) proposed a methodology that combines both models to produce ARMA (autoregressive moving average) model. This model is known as the cornerstone of stationary time series analysis. ARMA(*p*, *q*) refers to the model with *p* autoregressive terms and *q* moving average terms with the following expression:

$$X_t = \varepsilon_t + \sum_i a_i X_{t-i} + \sum_i b_i \varepsilon_{t-i} \qquad (2.3)$$

In an autoregressive model, stationarity is an important requirement for projecting without an undefined trend. Non stationary time series have a pronounced trend and do not have a constant long-run mean or variance. For non-stationary time-series, Box and Jenkins differentiate the series to get stationarity to which an ARMA model can be applied. The so called ARIMA model is classified in the Univariate Modelling part of the review.

The main steps of Box and Jenkins model are defined as model identification, estimation and validation. The identification step is to understand stationarity and the presence of seasonality by examining plots of the series, autocorrelation and partial correlation functions. In the second step, non-linear time series or maximum likelihood estimation procedures are used to estimate the model and finally diagnostic check is done by plotting the residuals to detect outliers and evidence of model fit. This procedure is applied in most of the studies in the literature.

***Regression Models***

This type of models is common for predicting economic phenomena such as the GNP and the GDP. Economic prediction models are typically of this type. (Nahmias (2001))

The OLS Regression Model is defined as below, where $a_i$'s are constants, $\varepsilon_t$ corresponds to error value at point t and $X_{it}$ is the input variable i at time t :

$$Y_t = a_0 + a_1 X_{1t} + a_2 X_{2t} + \ldots + a_n X_{nt} + \varepsilon_t \qquad (2.4)$$

This classical model applied in Akan and Tak (2006). They projected Turkey's electricity demand for 2001-2005 by an application of a regression model and improved it with error correction steps.

Al-Shehri (2000) used temperature, humidity and weather variation in his model to predict industrial energy demand. Data are classified on the price of electricity since the electricity tariff increases as the consumption increases. He proposed a model for the industrial electric energy consumption and its influencing parameters such as weather, demography, economic, fiscal factors and others in a fast-developing country. The fast growth rate and the fluctuating nature of energy consumption are the major factors in such systems. The model is based on a polynomial fit of the order 20 using the least-squares curve-fitting method.

$$P(x) = P_1 X^n + P_2 X^{n-1} + \ldots + P_n X + P_{n+1} \qquad (2.5)$$

where $P_i$ are the polynomial coefficients, $X$ represent the time in months, and $P(x)$ is the monthly industrial energy consumption. The paper shows that the polynomial fit provides a more accurate prediction and follows the seasonal variations more closely than the linear model.

In the literature, it is common to use the Cobb-Douglas form for demand function to avoid the error that the demand function is not linear so that taking logarithms of both sides will give a log-linear specification:

$$y_t = A_0\, x_{1t}^{\,a1} x_{2t}^{\,a2} \ldots x_{mt}^{\,am} e^{\varepsilon t} \tag{2.6}$$

In general, the larger the number of parameters involved, the higher the variance of estimators is likely to be. Large number of parameters it will cause the estimate to be far from the true value. İslam and Al-Alawi (1997) define candidate variables for long term prediction as population, GDP and all the variables defined for medium term predictions(weather variables, consumer index etc). They calculate the strength of the correlation between the candidate variables and the demand and eliminate the variables with less than 10% correlation.

A way of defining the variables is the *General to Specific Methodology*, where lagged variables may also have an influence on demand – demand at time t may also depend on past prices, $p_{t-1}$, $p_{t-2}$,… etc besides $p_t$.

There are several examples of using this method in energy consumption predictions. Donelly (1987) described a cross-sectional model that produces a long-term electricity consumption forecast for the diverse end-use sectors of industry:

$$QE= F( PE, PS,PC,Y ,X) \tag{2.7}$$

where QE is the electricity consumption in kWh (or any other energy unit) and it is a function of the electricity price per energy unit(PE), the price of a substitute good (PS), the price of a complementary good(PC), the consumer income (Y) and other factors(X). This generic model is based on the idea that both economic (e.g. energy cost, production level, GNP, income) and physical factors (e.g. weather) affect energy consumption.

Some energy forecasting models are defined according to the structure of the energy system in the country / region. Barakat and Rashed (1993) present a model for fast-developing areas and state that the classical techniques do not perform well for forecasting. To express in detail, they define fast growing areas by high economic growth, large surpressed demand and insufficient historical load data (annual growth rates in excess of 30% )

The reasons for their claim against classical methods are :

- High growth rates, high per capita GNP and subsidized energy tariffs.
- Huge and variable energy consumption levels during religious rituals, which create unstable and dynamic load characteristics.
- Highly varying weather conditions, climates having high temperature changes between seasons.
- Migration from rural to urban at an unpredictable rate.

Barakat and Rashed(1993) form a regression model by considering temperature changes (seasons), religious days. They separate the model according to temperature changes, in a monthly basis and assume that the high daily temperature values mean energy demand for cooling and the opposite is for heating, so they use absolute value of the difference between daily temperatures where base temperature is defined as $20^{o}$ C.

Aras (2004) applies the same idea to forecast residential consumption of natural gas in Turkey with an autoregressive model. Model is a combination of a multiple regression model and a first order autoregressive error model. Regression model is a function of time and temperature of the day value (temperature) and accounts for the trend and seasonal component of time series. The autoregressive model include random components that are to be created to diminish correlations between data series and he groups monthly data of the gas demand according to the level of heating need and then uses separate models for the two sets of data to observe the significance of climate in natural gas demand.

There are also examples of studies that is formed with mixed models to consider different aspects:

Energy Information Administration's (EIA) NEMS Model (National Energy Module System) is an example as a global energy model which represents the behavior of energy markets and their interactions with the U.S. economy (EIA, 2003). It reflects market economies, industry structure, energy policies and regulations. It consists of several modules that interact with each other through an integration module, thus achieving equilibrium.

Flores (2004) combines three models (Donelly(1987), Al-Shehri (2000), EIA (2001)) to reach a new industrial energy forecasting model: The multiple regression approach requires the data matrix to be complete (free of observation gaps) in order to build and evaluate the regression model, so encountered gaps are treated with time series' double exponential smoothing method with the smoothing constant $a$, around 0.7-0.9. More weight is given to the most recent observations and linear interpolation is used where missing data points exist.

The multivariate regression equation can be built by using the intercept and the coefficient values from the regression outputs. Once the equations are constructed, the time-series forecasts for each of the independent variables can be used to calculate the annual energy consumption forecasts for each model. This idea comes from the model NEMS defined in the upper paragraph. They execute a regression to define the relationship between factors and then forecast the independent variables by time series forecasting and apply this into the regression equation. Then the analysis proposes to evaluate the energy consumption savings, demand reduction, and environmental impact as consequences of implementing energy efficient practices. Energy savings is projected for the order of states' maquiladora industry for each year until 2010 using the energy forecast model, contrasting the current trend versus the implementation of the proposed regulation. Their comparison is to analyze complex changes in the factors which cannot be observed with the simplistic models such as the time series approach.

Another method is to combine data mining tools and the regression idea. Tsekouras et al (2003) define a data mining procedure to select the variables to put in a nonlinear model, and defined every combination of this variable set as candidates if variables correlation coefficients are high. The so-called regression coefficients are defined by considering the correlations between factors.

### 2.2.2 Recent Forecasting Methods

Recent researches are focused on more complicated forecasting methods such as genetic algorithms, artificial neural networks, ARIMA or similar methods.

***Multivariate Modeling With Cointegration Techniques***

An example for multivariate modeling is the Genetic Algorithm (GA). In this algorithm, data are represented in a way that is defined as chromosome structure and a fitness function is defined to compare their performance. In this algorithm, the way that the initial population is generated and the population size are very important. The aim is to reproduce a new generation from current parent chromosomes. The idea is that the best ones will survive and this continues until stopping criterion is reached.

Ozturk and Ceylan (2005) use GA for consumption using socio-economic indicators in Turkey. The reason for using a GA is that it may be easy to solve the nonlinear mathematical expressions by minimizing the sum of squared errors (SSE) between observed and estimated values. They give a list of total electricity consumption and socio-economic indicators of Turkey between 1980 & 2001. They use the 2/3 of data for model development and the rest for evaluation. The results are better than the Ministry's predictions in testing, so they concluded that the model MENR used is not suitable for Turkey: the electricity demand is shown higher than the real need in order to establish electric power plants that will use excess amount of natural gas.

***Univariate Modelling***

Autoregressive moving Average (ARIMA) Technique is a significant example for univariate modelling. The method searches for possible dependencies among values of the series from period to period. These types of models are known to have sophisticated mathematical techniques to estimate the model's parameters and need a substantial history of observations. On the other hand, under the right circumstances, these methods can significantly outperform the simpler ones. (Nahmias, 2001)

***Neural And Abductive Network Models***

To build a model for forecasting, a network is processed through three stages (Nasr, 2002):

(1) The training stage where the network is trained to predict future data based on past and present data.

(2) The testing stage where the network is tested to stop training or to keep in training.

(3) The evaluation stage where the network ceases training and is used to forecast future data and to calculate different measures of error.

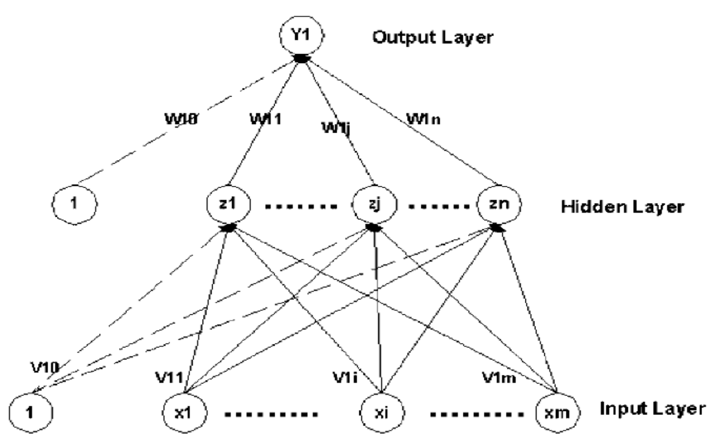



**Figure 1 Schematic Representation of the stages of an ANN model**

The training of the network by backpropagation consists of three stages:

_ The *feedforward* of the input training pattern.

_ The *calculation* and *backpropagation* of the associated error.
_ The *adjustment* of the *weights*.

During the feedforward phase, each input unit $X_i$ receives an input signal $x_i$ and broadcasts this signal to each of the hidden units $Z_1...Z_p$. The second stage of the backpropagation algorithm is the backpropagation of error. During this phase, the output unit $Y_1$ computes its difference (error) from actual values. Based on this error, an error term is computed. Then, error term is used to distribute the error on the output unit back to all units at the previous layer. The third phase of the training algorithm is to update the weights using the weight correction terms computed during the second phase. Therefore, by adjusting the weights, the network learns and improves its performance (Nasr, 2002).

Nasr (2002) uses different ANN models which are either univariate or multivariate and compared them for a better forecast. ANN is used to forecast natural gas consumption also in some papers (Hobbs et al., 1998; Brown, 1996; Aras, 2004).

Fu and Nguyen (2003) use dynamic functional-link net (FLN) and wavelet networks. They define an optimization model to minimize error between prediction and actual data, while they constrain with the nonlinear link between the dependent and independent variables, so that hidden layers are not required to sustain nonlinearity. They use the data of historical annual energies, population, ***gross*** state product (GSP), consumers price index (CPI) and electricity tariff and compare the mentioned network models with the classical recursive model to show the smaller error values with generated models.

***The Functional Link Net Structure***

A functional link net is a tool of ANN modelling where input signals are subjected to nonlinear transformation by creating tensors of the inputs that include polynomial terms and also other functional transformations. It is reported that the FLN device removes the need to define hidden layers between the input and the output nodes of the network and also provides for faster and more effective training. (Zhang et.all (2005), Klassen et.al (1988), Collister and Lahav (2004))

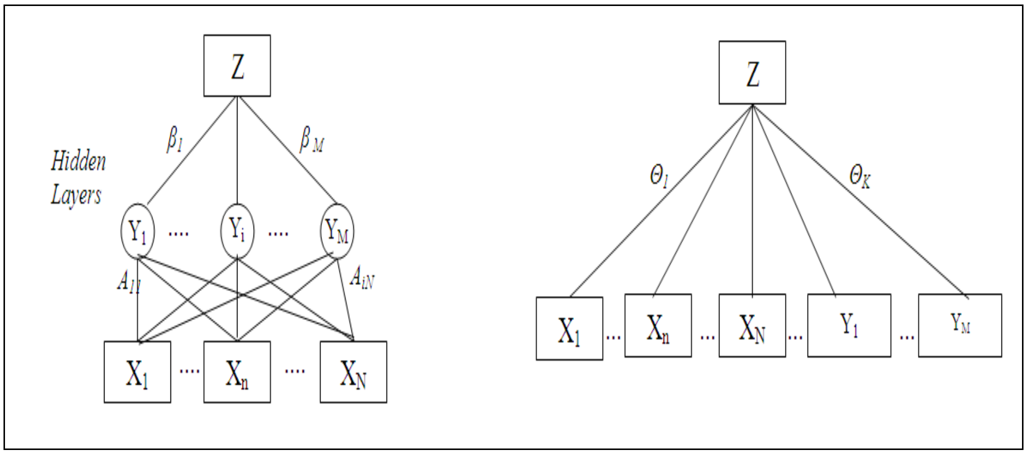



**Figure 2 Comparison of a single layer net and functional link-net architectures where $Y_i = G_i(A_iX + b_i)$**

Figure 2 shows a conventional ANN structure on the left and the FLN architecture on the right where the hidden nodes are eliminated. Here, inputs $Y_1$ to $Y_M$ would be the tensors of primary inputs $X_1$ to $X_N$ for example.

Neural networks are nonlinear where relationships between input and output are not known so that the relationship is learned at each step by training, then results are tested with another dataset (which correspond to the next time periods after training period) and evaluation is done for the future projection. So the first step is to define training and testing periods from the dataset.

To illustrate Figure 2, although there are different algorithms defined to train the network, backpropagation algorithm will be referred in the general structure of neural network:

An ANN structure consists of a number of layers of nodes. The first layer contains the input vectors, which in our application on electricity demand estimation are simply monthly GDP and Energy Price index. Input layer can be represented with $X_n$ : ($X_1$, $X_2$, .. $X_n$, .. $X_N$) where n shows each input vector. The final layer contains the outputs : usually one output is considered (Z). In our application, it will be electricity demand forecast value for the following year. As stated, input layer and output layer contains vectors with different magnitudes. Intervening layers are described as "hidden," and there is complete freedom over the number and size of hidden layers used. The nodes in a given layer are connected to all the nodes in adjacent layers. A particular network architecture can be denoted by $N_{in} : N_1 : N_2 : \ldots : N_{out}$ , where $N_{in}$ is the number of input nodes, $N_{out}$ is the number of output nodes, $N_1$ is the number of nodes in the first hidden layer, and so forth. To illustrate, 2:8:1 takes 2 inputs, has 8 nodes in a single hidden layer, and gives a single output.

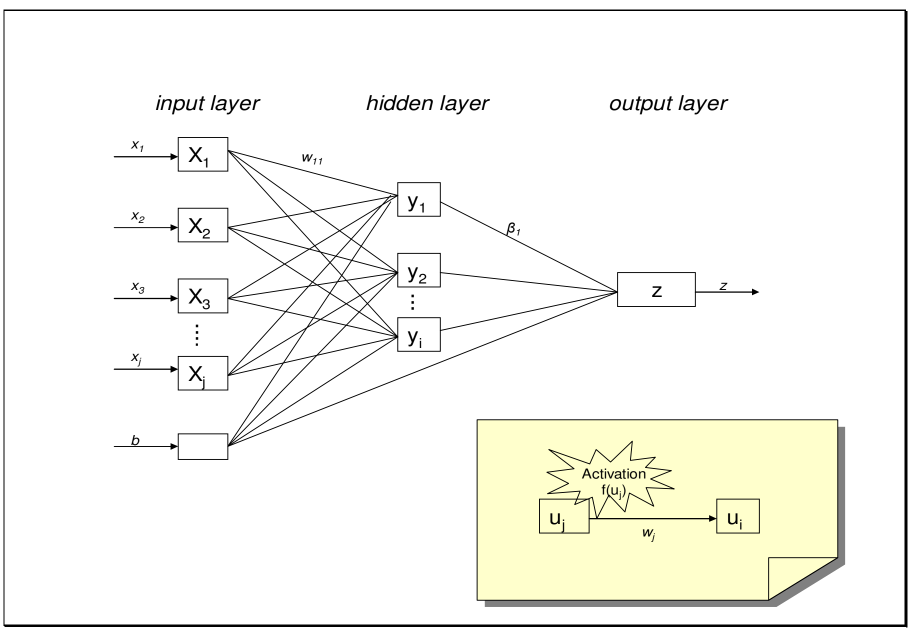



**Figure 3 Representation of ANN Structure**

For each node $u_i$, it is supposed that sum of various input signals activates the node and output signal is transferred from the node to various other nodes. Each connection carries a weight, $w_{ij}$, and all these weights are represented with vector W. For each node $u_i$, input signals can be shown as :

$$u_i = \sum_j w_{ij} f(u_j) \tag{2.8}$$

where $u_j$ represents nodes from the previous layer, $w_{ij}f(u_j)$ is the output signal from node $j$ to node $i$ and $f$ is the activation function. Activation function is usually nonlinear (eg. $f(u_j) = 1/[1+exp(-u_j)]$).

This relationship is represented in Figure 3. A bias node, b, can also be added to the algorithm to represent additional constant value in each layer. (Collister and Lahav (2004))

To illustrate, $x_j$ is the input signal for input node j and the node will be activated via function *f*. Output signal, $\hat{X}j$ , will be :

$$\hat{X}j = f(x_j) \tag{2.9}$$

*where f* is the activation function.

This output signal will reach hidden layer node i with weight $w_{ij}$ and then weighted sum of all signals will form input signal for this hidden layer as :

$$y_j = \sum_i w_{ij} \hat{X}_j + w_{bj} \tag{2.10}$$

where $w_{bj}$ is the weight of bias node.

Similarly, hidden layer will be activated with this input signals, so output signal from node i of the hidden layer will be:

$$\hat{Y}_i = f(y_i) \tag{2.11}$$

For a single hidden layer net, weighted sum of output signals of all hidden layer nodes and the bias node will equal to the input signal for the output layer. According to that, output of the model will be :

$$\hat{Z} = f(z) \quad where \quad z = \sum_i \beta_i \hat{Y}_i + w_b \tag{2.12}$$

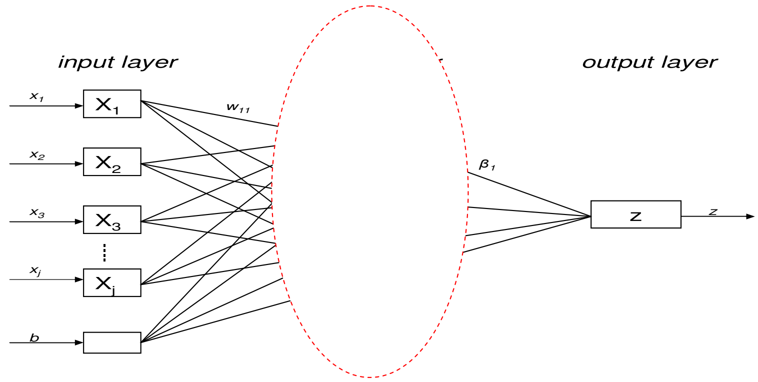
$\beta_i$ represents the weight constant between hidden layer i and output z , whereas $w_b$ is the weight constant between bias node and the output node. This stage is called the feedforward phase of the training.

The next stage is to train the model with initialized weights, and calculate the error of the model regarding the exact outcome. The error is then used to calculate weight correction terms and initial weights are changed according to the correction terms with the backpropagation method. Model will then be treated with corrected weights until weight values converge to an optimum point, so that testing to stop training ends. In other words, iterations will end when a local optimum point is reached.

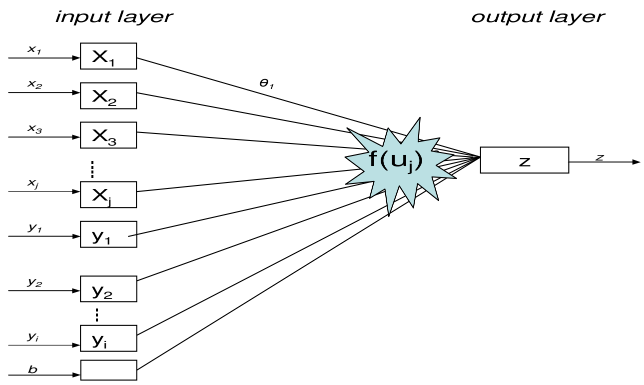
In this method, objective is to minimize (weighted) error between the projection and the actual values. Decision variables are both the weights (w, β) in the model and the hidden layer signals (Y). In addition, $Z_{k-1}$ values are also decision variables for $Z_k$ values in each iteration k. (Nasr et al, 2002)

The advantages of this neural network model are its nonlinearity and iteratively learning structure. Functional Link Net is introduced as the replacement of this hidden nodded structure, while it preserves the nonlinearity and iteratively learning mechanism and also simplifies the algorithm.

In this model, it is assumed that vector Y is to be introduced manually, with a defined nonlinear function (link) of input variables, X. Then, the decision variables will be weights of each variable in this nonlinear link and additionally, $Z_{k-1}$ values will still be the decision variables for $Z_k$ values in each iteration. Objective will not change. The way of using nonlinear terms with initially determined functions increased the actual number of inputs supplied comparing to the one-layer neural network (Wilamovski, 2007)




(Figure 4.a)




(Figure 4.b)

**Figure 4 A Schematic Representation of FLN algorithm**

As it is also presented in Figure 4, FLN algorithm is a very simple structure, objective is to minimize forecast error, with decision variables of constant values for each term in the input layer, but input layer is now enlarged with other derived variables from the initial inputs. Although optimization will help in an iterative manner to execute a mechanism for backpropagation, current classification of this model is still an important question: The model do not use an activation function, so without having signals, can we still name this model as an ANN model?

FLN Model was first defined by Yoh-Han Pao (1988). He suggests another way of utilizing higher order effects in neural nets for supervised learning via a nonlinear link, so that a nonlinear functional transform is carried out yielding to a new pattern vector in a larger space. Further researches are done then to show that learning rates are dramatically increased with the decreased number of recursions and simplified architectures. (Pao and Beer (1988), Klassen et all (1988),Zhang et.all (2005))

Fu (1994) shows the increased accuracy of mapping through the expansion of the basis set. Taylor series and Fourier series are given as examples where $x$ represent projected dataset, $x_0$ is any selected point in space, $a_n$, $b_n$, w are defined constant terms:

$$f(x) = \sum_{n=1}^{\infty} \frac{f^n(x_0)}{n!} x(x - x_0)^n \qquad \text{Taylor Series} \qquad (2.13)$$

$$f(x) = a_0 + \sum_n (a_n \cos nwx + b_n \sin nwx) \qquad \text{Fourier Series} \qquad (2.14)$$

These examples also give mathematical basis to the FLN, in which following functions are used to satisfy nonlinearity:

- $x, x^2, x^3, \ldots$
- x, sin Πx, cos Πx, sin 2Πx…
- $x_i, x_i x_j \ (j>i), \ldots$

Igelnik and Pao (1995) also analyze the random vector version of functional link nets (RVFL) with the Monte Carlo method and discuss the similarities for neural nets with hidden nodes. They find out that the RVFL is a universal approximator for continuous functions on bounded finite dimensional sets, and the RVFL is an efficient universal approximator with the rate of approximation error convergence to zero.

FLN is defined as a derived method from ANN architecture; however, it eliminates activation functions which are the key components for ANN algorithm. Thus, as it is stated above, this model is needed to be discussed in a structural way.

### 2.2.3 Other Techniques

Simulation is also another tool for complex models to select the right strategy. Sweeney (1978) reviews energy simulation and forecasting models. He states that if the user wants to do computer simulation experiments simulating the effects of energy consumption on economic growth or inflation rates, then econometric models are the appropriate analytical tools, but the forecasting accuracy of any econometric model is no better than the accuracy of the policy assumptions and assumptions about the firm's external environment which underline the model. The Model for Analysis of Energy Demand (MAED) is an example of projections by simulation. Another application is done by Hainoun et al(2006) for Syria.

There are also examples of some basic forecast models. Satman(2006) makes a projection by comparing the trends of global and national energy consumption and predict to have an increase in demand at the same ratio as the increase expected in the world.

### 2.2.4 Previous Studies of Projections In Turkey

Although there are several different methods in the literature, projection studies for energy demand in Turkey were limited and most of them are either done by the governmental organizations or reconciliations of the official forecast method. The most important problem is that there is no significant alternative that can be replaced with the governmental approach, while shortcomings of the official method are clearly identified for energy demand projections. Recent studies on demand projection of various types of energy can be summarized in Table 1.

**Table 1 Comparison of Recent Studies of Energy Demand in Turkey**

| Paper | Methodology | Details |
|---|---|---|
| Ediger, V. Ş. Akar, S. Uğurlu, B. (2006) *Forecasting production of fossil fuel sources in Turkey using a comparative regression and ARIMA model*. Energy Policy 34 3836-3846 | ARIMA method or Regression selected by a DSS | Improvement from a single model |
| Aras, H. Aras, N. (2004) *Forecasting Residental Natural Gas Demand*. Energy Sources, 26: 463-472 | Autoregressive time series model | Divided a year into two seasons - heating and cooling |
| Ediger, V. Ş. Tatlıdil, H.(2001) *Forecasting the primary energy demand in Turkey and analysis of cyclic patterns*. Energy Conversion and Management 43 473-487 | Analysis of Cyclic Patterns | Compared additional amount of energy consumption per year, total energy consumption and rates of energy consumption |
| Öztürk, H.K. Ceylan,H.(2005) *Forecasting total and industrial sector electricity demand based on genetic algoritm approach: Turkey case study*. Int. Jour. Energy Research 29:829-840 | Genetic Algorithm Electricity model (GAEM) | Based on minimizing sum of squared errors (SSE) / binary coding |
| Akan, Y. Tak, S. (2006) *Econometric Demand Analysis of Electrical Energy in Turkey*. | Econometric Analysis | Both general energy demand and sectoral energy demands |
| MAED Model by Ministry of Energy and Natural Resources (2006) | Simulation | Based on scenario analysis |

These articles are examples of similar approaches, so they are representative in terms of projection methods. All these studies are explained briefly to illustrated related methods in CHAPTER 2 .On the contrary, only Akan and Tak(2006)'s article and the MAED model shows an analysis on electricity demand in Turkey, so only the results of these two models can be comparable with our proposed model.

Liberalization in the electricity market will create different players in the area. Future supply projections and corresponding investments will not be properly planned without a reliable forecasting method and inaccurate projections will result in high energy supply costs.

MENR use Model for Analysis of Energy Demand (MAED) to project future demand, however, it is clear that error of MAED model is significantly increasing over years, reaching to 45% error in 15 years , or over 20% within only 3 years.

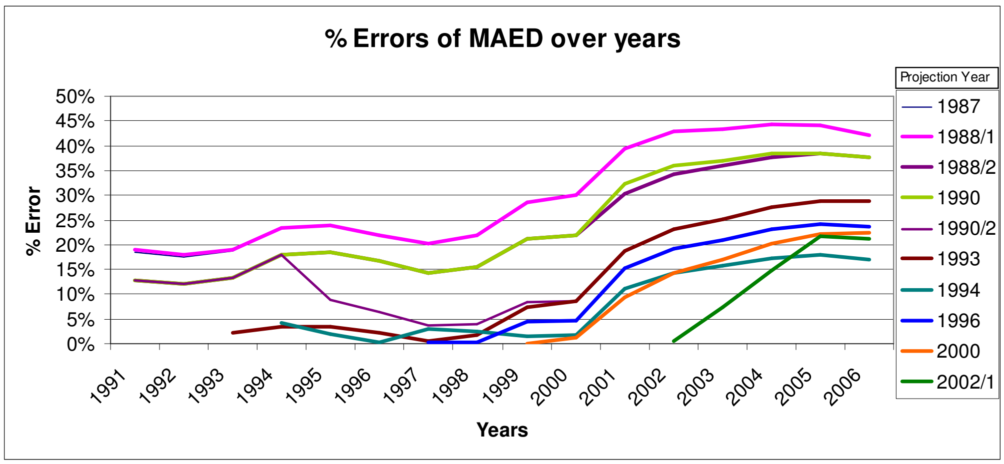



**Figure 5 Percent error of MAED over years**

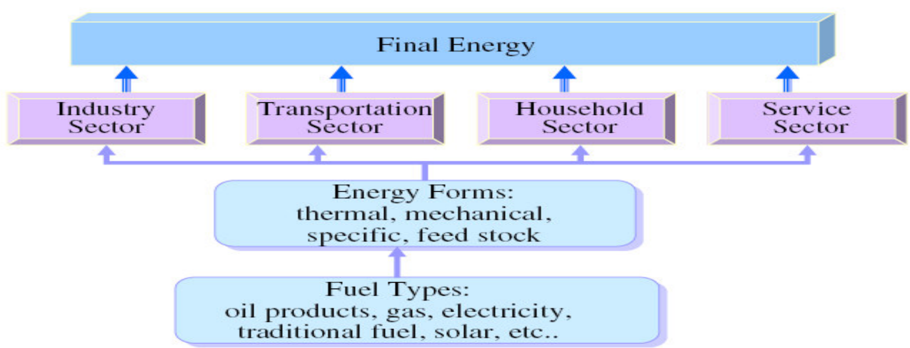
In Figure 5, current official electricity demand forecasting method is presented. The reader will notice high forecast errors in the previous projections, and discuss its consequences.

### *Turkey Official Electricity Demand Estimation : MAED (Model for Analysis of Energy Demand)*

The official use of mathematical modeling in energy forecasting and policy making by MENR was started in 1984. Before 1984, various best fit curves were used by State Planning Organization (SPO) and by the MENR. These forecast values predicted much higher values than the actual consumption. In 1984, The World Bank recommended the MENR use the simulation model MAED (Model for Analysis of Energy Demand)(Ediger and Tatlıgil (2002))

The methodology of the MAED model was originally developed by. B. Chateau and B. Lapillonne, and was presented as the MEDEE model. Since then the MEDEE model has been developed and adopted to be appropriate for modelling of various energy demand system. One such example is development of MEDEE-2 by B. Lapillonne for the needs of the International Institute for Applied Systems Analysis (IIASA), Laxenburg, Austria. While respecting the general structure of MEDEE-2, important modifications were introduced in MAED by the International Atomic Energy Agency (IAEA).

**Figure 6 MAED Bottom-Up Approach (Hainoun et al.(2006))**




In Figure 6, bottom-up approach of the MAED model is figured out. The MAED module is a scenario-based, simulation model that performs long-term energy and electricity demand forecasting by using a bottom-up approach starting with energy consumption levels for individual activities and end up with total future demand for fossil fuels, electricity, district heating, coke, and feedstock in each sector/sub-sector of the economy.

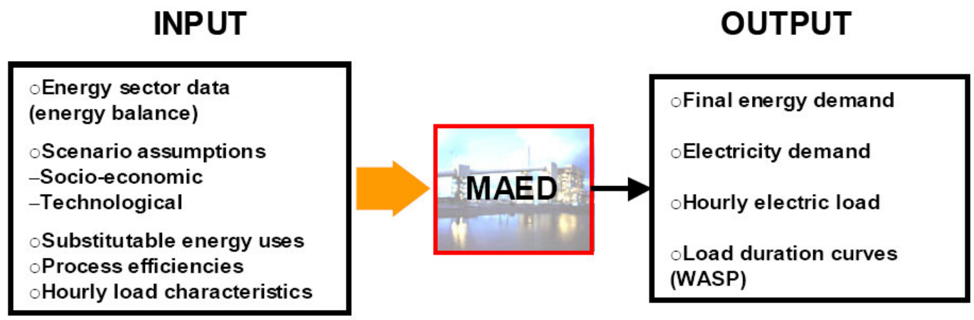



**Figure 7 Inputs and Outputs of MAED Model *(MAED Tutorial)***

In Figure 7, input and output relationships are summarized. To predict all outputs of the model, such as final energy demand, specifically electricity demand, hourly electric load and also load duration curves, main inputs are:

- *Energy Sector Data: A selected period is defined as "base year". This may correspond to a specific year or a time period. Data is used to* construct energy consumption patterns of defined "base year" to establish a base year energy balance.
- *Scenario Assumptions: Main assumptions are defined to* develope future scenarios, specific to a country's situation and objectives are generated based on social, economic and technological changes of the country.
- *Other Scenario Parameters:* Energy Demand is defined due to the relation type of end-use by taking into account market penetration, the efficiency of each alternative energy source and hourly load characteristics for electricity.

We will consider MAED Module 1 since we are not interested in the hourly electric load in this thesis. MAED Module 1 structure is explained in Figure 8 (IAEA, 2006):

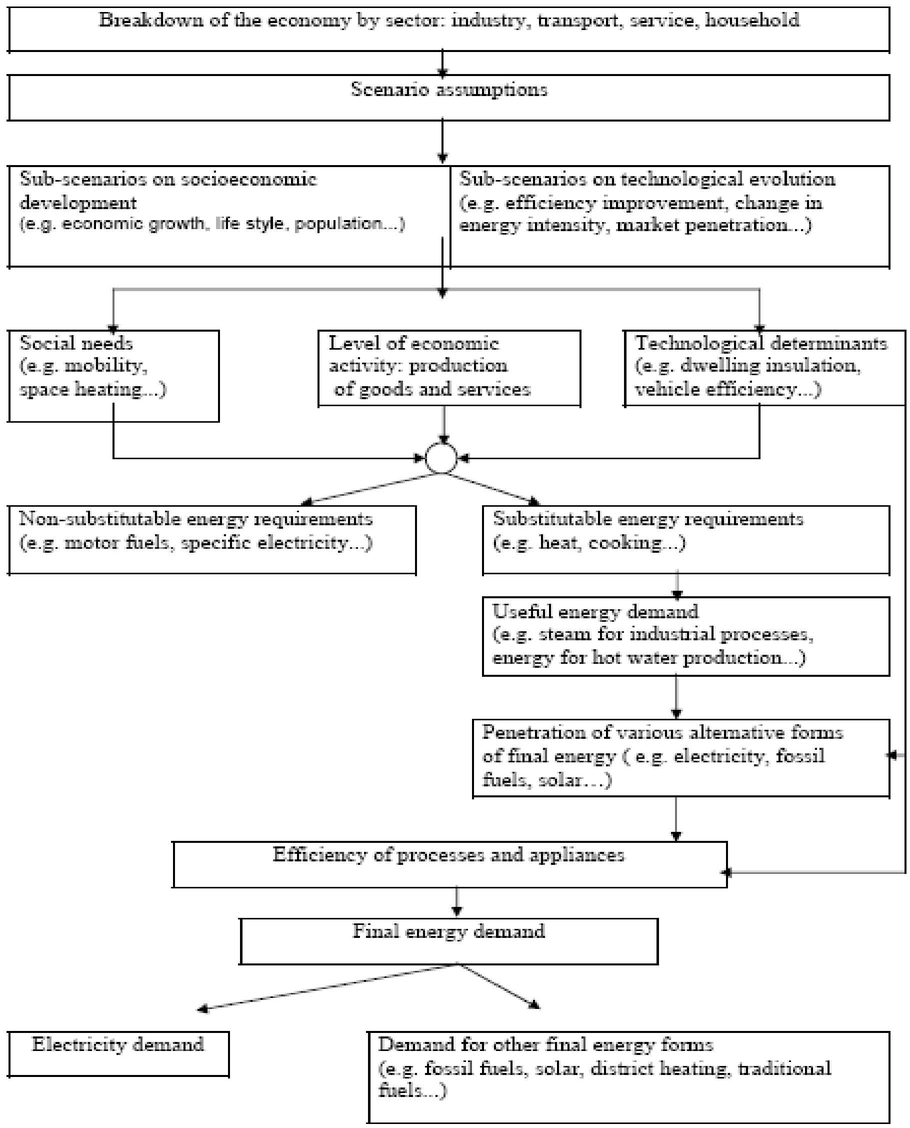



**Figure 8 Projection of Final Energy Demand by MAED *(MAED Tutorial)***

In the MAED/MEDEE approach a "scenario" is viewed as a consistent description of a possible long-term development pattern of a country, characterized mainly in terms of long-term direction of governmental socioeconomic policy. Following this approach, the planner can make assumptions about the possible evolution of the social, economic, and technological development pattern of a country that can be anticipated over the long term from current trends and governmental objectives

In summary the MAED Module 1 methodology comprises the following sequence of operations:

(1) disaggregation of the total energy demand of the country or region into a large number of end-use categories in a coherent manner;

(2) identification of the social, economic and technological parameters which affect each enduse category of the energy demand;

(3) establishing in mathematical terms the relationships which relate energy demand and the factors affecting this demand;

(4) developing (consistent) scenarios of social, economic and technological development for the given country;

(5) evaluation of the energy demand resulting from each scenario;

(6) selection among all possible scenarios proposed, the "most probable" patterns of development for the country.

According to the model, energy demand for any category of end use j in year t, $E_{j,t}$ is

$$E_{j,t} = \frac{E_{j,0}}{DP_{j,0}} CHSPEC_{j,t} \; DP_{j,t} \qquad (2.15)$$

where $DP_{j,t}$ is the value of driving factor for category of end use $j$ in year $t$ and $CHSPEC_{j,t}$ is the change of specific consumption for category $j$ in year $t$

Final energy demand for energy form $m$ in sector $h$ for category of end use $j$ in year $t$ is represented as:

$$E_{mhjt} = E_{hjt} \; \frac{MP_{mhjt}}{EFF_{mhjt}} \qquad (2.16)$$

where MP is the market penetration for energy form $m$ in sector $h$ for category of end use $j$ in year $t$ and EFF is the efficiency of energy form $m$ in sector $h$ for category of end use $j$ in year $t$.

Exact input variables and the calculations in the formation of output variables are explained in full detail in the MAED Tutorial.

MAED module uses data of a defined base year, which is supposed to have a stationary environment, and defines coefficients for the relations between input variables and the output variables according to this database. By defining some scenarios on the input variables, which means predicting the factors affecting energy demand, it concludes to a prediction of future energy demand according to those factors and coefficients.

*"All independent determining factors, i.e. the 'scenario', which constitute in a certain way the driving force of the model, are exogenously introduced"(*Altas et al(2006))

Assumptions used in a forecast model form the base for a prediction. For MAED model, analysis of past years' data such as trend, seasonality etc, is not considered in defining the independent factors (scenarios) but the targets that were defined by State Planning Organization (SPO) are assumed to be achieved, as a result of this, the model reflects the decision variables in a desired(targeted) trend. This assumption for the future projection of decision variables is a very important building block for the projection: the fluctuations from the targets will also hurt main assumption of the projection. Thus, in this study, this assumption will not be prefferred as a valid one, but a basic time series projection method will be used to project decision variables.

*Application of MAED Module In Turkey : A First Look*

TEİAŞ Projection Report (TEİAŞ (2007)) summarizes the projections of MAED with the exact demand. It is stated that although forecast of the electrical energy demand has a low precision in the long run, it is because of the assumed predictions of Turkey's Growth Rate etc. According to their explanation, since the target of the growth rate could not be achieved, their long run forecasts will not be as expected. The long run forecast errors are summarized in Figure 9.

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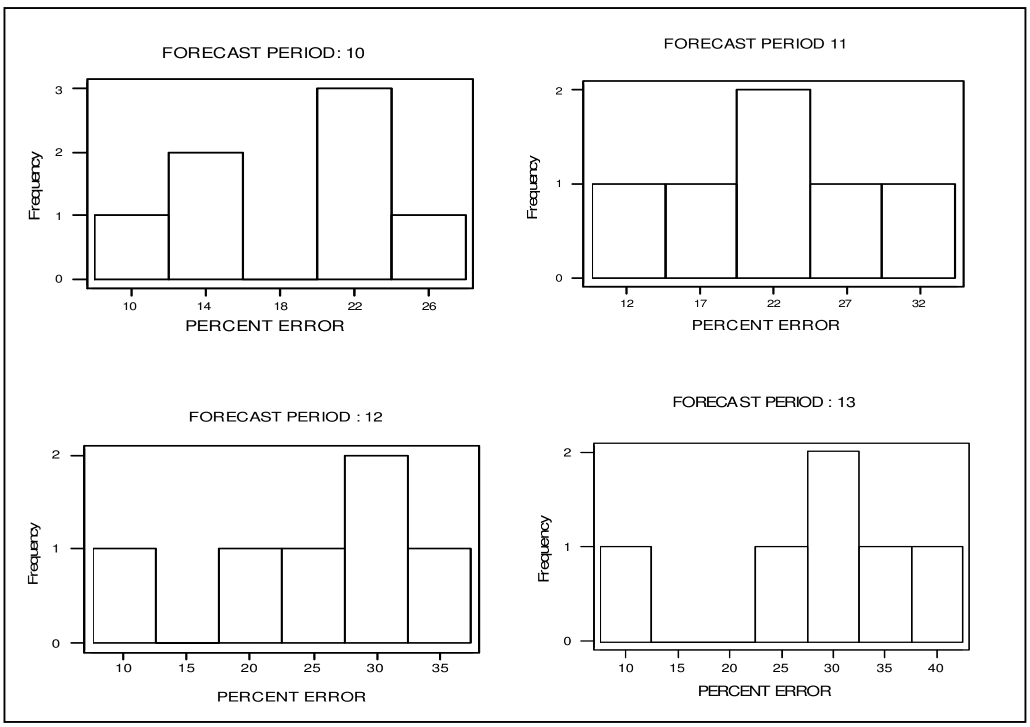


**Figure 9 Forecast Error Values of MAED for different forecast periods**

Mean of the MAED forecast error values is not close to 0 for any time period, it is always positive and error is not seem to be normally distributed (data are insufficient to check this in more detail). The histograms in Figure 9 represent the high positive error values for long run forecasts.

Since this error values do not have zero mean, it is concluded that the error terms do not fit to the OLS Regression Parameter Estimators' first principle (Dobbs (2000)) and it does not have the property of Unbiasedness, Efficiency or Inference. Other reasons for this error values can be defined such that the model does not include changes in the energy prices, substitute energy types and their relations with electricity, efficiency of electricity etc.(Kumbaroğlu (2006), Ediger and Tatlıdil (2002))

Ediger and Tatlıdil(2002) state that multiple regression method for the dataset used in MAED cannot be applied due to harmful multicollinearity, so they try to use simple regression technique to analyse the cyclic patterns of historical data curves, and conclude that Winter's exponential smoothing method is the best fitting method. They use the energy coefficient (energy consumption rate / GDP rate) to indicate the energy / economy interaction and they conclude that the additional amount of energy consumption per year is a better measure to observe energy demand performance than other variables since the differences from the previous years reflect the capability to consume and to meet its production / import.

Another report issued by governmental organizations is Turkey First National Communication Project (Altas et al.(2006)). This project is a detailed presentation of the official modeling tools that are used for MAED. In line with the decision variables' initialization assumption, the writer also mentions that according to the projection of GDP growth of SPO, although energy consumption has increased at an annual average rate of 3.7% since 1990, energy demand and pollution are expected to increase at a rate of 6% for the next 15 years.

Altas et al. (2006) state government's forecast model reference case assumptions as follows:

- No new domestic reserves of fossil fuels will be discovered;
- No limits are set on the import of crude oil, natural gas, or hard coal;
- No major changes are made in the country's energy pricing policies;
- No new major energy conservation or renewable resource programs are implemented; and
- The expansion of the electricity supply capacity is done on the basis of least cost over the whole system and over the whole planning horizon by considering Turkey's major policies

According to those assumptions, while electricity demand is increasing rapidly, they assume to use the same natural resources. However, as technology changes, new methods for using resources are coming up and the available resources (like coal, hydropower) are getting close to probable resource limits. In addition, the assumption of no new investments in the following 20 years for electricity supply system shows that government do not have any plans on energy generation, but at the same time, it is hard to occur in real life in a developing country with rapidly increasing energy demand.

# CHAPTER 3

# DEMAND FORECASTING WITH NONLINEAR TRANSFORMATIONS BASED ON FUNCTIONAL LINK NETS

This chapter describes our investigations into a specific modeling approach for forecasting medium-term electricity demand. The approach is based on minimizing an error function whose specification is motivated by the FLN device that has been proposed for enhancing learning in ANN architectures. We propose a generalized procedure for formulating the model in such a way that the performance of the final specification will be improved as much as possible. We report on our formulations and numerical results in detail. The results obtained from constructed models are compared with government forecasts and also with those from a multiple regression model we construct that has a similar specification to our models in the transformations of the independent variables.

## 3.1 MODEL FORMULATION

The model we construct to forecast Turkey's electricity demand will be compared with four different models:

- TEİAŞ's MAED model.
- A classical recursive model (CRM) with similar structure with FLN

- An autoregressive AR(p) model
- An OLS models with the same input variables

Our aim in general will be minimizing the total weighted error, by selecting changing coefficients of variables at each time step :

$$\mathrm{Err}(x_t, y_t) = \sum_t w_t (x_t - y_t)^2 \tag{3.1}$$

where $w_t$ is weight for time t, $x_t$ is the forecast value and $y_t$ is the realization for the forecast data at time t. To improve the model further, logarithm (log) or difference functions can be used. In this way, small variations in the dataset will be noticeable together with outliers since the magnitude of the change decreases by the logarithm function.

The FLN idea will be implemented within a recursive framework as suggested by Fu and Nguyen (2003). We first define function L as:

$$L(S_t) = \log(S_t) - \log(S_{t-1}) = \log(S_t / S_{t-1}) \tag{3.2}$$

where, in this case, $S_t$ is the forecast for period t.

Objective is redefined by taking difference of logarithm of each variable :

$$\mathrm{Min}\ z_t = \mathrm{Err}(\log(S_t), \log(E_t)) = \sum_t w_t (\log(S_t) - \log(E_t))^2 \tag{3.3}$$

such that

$$L(S_t) = b\, L(X_t) + c\, F(L(X_t)) \tag{3.4}$$

where

the function F represents the (FLN) transformations to be incorporated

b and c are constant coefficients to be estimated

$S_t$ : Forecast at time t

$E_t$ : Realization at time t

$X_t$ : Value of independent variable at time t ; such that $X_t=(G_t, P_t)$

$G_t$ : GDP

$P_t$ : Price index for energy

The equation (3.4) can be rewritten in the form:

$$\log(S_t) = a \log(S_{t-1}) + b\, L(X_t) + c\, F(L(X_t)) \tag{3.5}$$

where the coefficient a is defined to express the relationship between current and lagged forecasts.

Performance of selected nonlinear functions will be examined to observe best possible combination among those functions. The use of lagged terms makes the models recursive and nonlinear. Different variants of functional link net models are defined to find the best links to predict electricity demand.

Lagged terms of projected variable ($S_t$) used in the models are chosen in two different way in order to understand the effect of using lagged variables :

- Only $S_{t-1}$
- $S_{t-1}$ and $S_{t-2}$

Nonlinear functions (F), ie. the tensors of the set of independent variables to be compared on performance are:

- $x^2, x^3, x^4, x^5$
- $\sin \Pi x$ , $\sin^2 \Pi x$, $\sin^3 \Pi x$, $\sin^4 \Pi x$, $\sin^5 \Pi x$
- $x_1x_2$, $x_1^2x_2$, $x_1x_2^2$, $x_1^2x_2^2$, $x_1^3x_2^3$

These functions are defined arbitrarily to see effect of different functions. Chosen functions to compare for performance are power, sinus, crossproduct of variables and powers of those.

FLN Model will be trained with the selected variables in the next sections, for two different training periods and the model will be validated for the following time periods. Evaluation, third step of the neural network algorithm (Nasr(2002)), will be done by the comparison of MAPE results of other methods for electricity demand forecast in Turkey.

### 3.1.1 Data Sets

Data used in the models are monthly electrical energy demand from January 1987 to December 2005, monthly energy and fuel index and monthly GDP (Central Bank of Turkey(CBTR) web site (2006), World Energy Council Turkey National Comitee (WEC TNC) (2006)) The reason to choose these two variables (GDP and Price) is the previous analysis on correlation of these variables and energy demand. To illustrate, Fells (1991) states that 1% GDP increase requires 1.5% increase in energy use in an economy undergoing industrialization and 0.5% increase in an industrialized economy. Taylor(1975) incorporated several price elasticity studies for electricity demand and concluded that short run elasticity is -0.2 and long run elasticity is between -0.7 and -0.9. According to these previous studies, electricity demand changes with the changes in GDP and price is the other factor that electricity is influenced in the long run.

In TEİAŞ's web site, monthly energy transmitted from lines is given. This data do not cover only net electricity demand but it also includes inefficiency factors between transmission and consumption phases. Since this is the only monthly electricity consumption data, inefficiency factors are cleaned from the dataset by assuming a direct correlation between net electricity demand and transmitted amount during a year. The difference between net yearly electricity demand data and yearly nationwide transmitted electrical energy is interpolated to eliminate the monthly inefficiency factor from monthly data. For calculated data for monthly energy demand, see Appendix A.

Previous projections in the literature will be compared with the projections of the current model, but using actual values of dependent variables GDP and energy prices in the proposed projections would give better more accuracy to the projections and since previous projections did not have that information at the forecast year, those variables should also be forecasted to see performance without the effect of using realizations. Moreover, model performance will be understood better without this accuracy, as realizations of those variables will not be known when forecasting is done for future time periods. Thus, both GDP and Price index values will be projected for the validation periods and future projections with selected methods:

***Price Index :*** The trend on this data set is significant, so Holt's method is used to forecast this data set.

In Holt's method, objective is to minimize mean absolute deviation with defined base estimate $L_t$ and trend estimate $T_t$ by changing smoothing constants $\alpha$ and $\beta$. This method is summarized for forecasted value $x_t$ as:

$$x_t = a + bt + \varepsilon_t \tag{3.6}$$

$$L_t = \alpha x_t + (1-\alpha)(L_{t-1} + T_{t-1}) \tag{3.7}$$

$$T_t = \beta(L_t - L_{t-1}) + (1-\beta) T_{t-1} \tag{3.8}$$

where

$\varepsilon_t$ : error term for period t

a : base level at the beginning of period 1

b : per period trend

***GDP :*** This variable has high seasonality and trend in the data, so Winter's method, which is stated as good at reflecting both properties by Winston(1993), is used for forecast.

In Winter's method, a similar model with Holt's method is applied with an additional seasonality factor $S_t$:

$$x_{k,t} = ( L_{t-1} + tT_{t-1})\, S_{t+k-c} \tag{3.9}$$

$$L_t=\alpha \frac{x_t}{s_{t-c}} +(1-\alpha)( L_{t-1} + T_{t-1}) \tag{3.10}$$

$$T_t = \beta(L_t - L_{t-1}) + (1- \beta)\, T_{t-1} \tag{3.11}$$

$$S_t = \gamma \frac{x_t}{L_t} + (1- \gamma)\, S_{t-c} \tag{3.12}$$

where

c : number of periods in the length of the seasonal pattern

k: order of period in a season

**3.1.2 Performance Of Different FLN Transformations**

FLN model results depend on nonlinear components of input variables, so it is very important to select the right functions for each variable. To achieve this, an experimental approach will be applied to a group of different nonlinear components and the best performing ones will be referred for a larger base model. All these components will be compared with MAED model, the classical recursive model and autoregressive (AR) model which only consider past electricity consumption.

The general model (1) is applied with different nonlinear F(x) functions to see the performance of each. Adding second lag of electricity demand data is also another factor that will be considered during the analysis.

To illustrate, for the function $F(x) = x^2$, model (3.4) will be:

$$\text{Min } z_t = \text{Err}(\log(S_t), \log(E_t)) = ½ \sum_t w_t (\log(S_t) - \log(E_t))^2 \tag{3.13}$$

such that

$$\log(S_t) = a \log (S_{t-1}) + b\, L(x_t) + c\, (L(x_t))^2 + d \tag{3.14}$$

When 2nd lag term ($S_{t-2}$) is added, the equation becomes:

$$\text{Min } z_t = ½ \sum_t w_t (\log(S_t) - \log(E_t))^2 \tag{3.15}$$

such that

$$\log(S_t) = a \log(S_{t-1}) + b\, L(x_t) + c\, (L(x_t))^2 + e \log(S_{t-2}) + d \tag{3.16}$$

For the cases where x refers to GDP, b is restricted to be nonnegative, and for cases where x is referring to energy price index, b is restricted to be nonpositive to have a logical relationship between these variables and the decision variables.

To clarify more, the open form of proposed model (3.13) is explained as:

$$Minz_t = \tfrac{1}{2}\left[w_1(\log(S_1)-\log(E_1))^2 + w_2(\log(S_2)-\log(E_2))^2 + w_3(\log(S_3)-\log(E_3))^2 + .. + w_t(\log(S_t)-\log(E_t))^2\right] \tag{3.17}$$

such that

$$\log(S_1) = \log(E_1) \tag{3.18}$$

$$\log(S_2) = a \log(S_1) + b \log\left(\frac{x_2}{x_1}\right) + c\left(\log\left(\frac{x_2}{x_1}\right)\right)^2 + d \tag{3.19}$$

$$\log(S_3) = a \log(S_2) + b \log\left(\frac{x_3}{x_2}\right) + c\left(\log\left(\frac{x_3}{x_2}\right)\right)^2 + d \tag{3.20}$$

….

$$\log(S_t) = a \log(S_{t-1}) + b \log\left(\frac{x_t}{x_{t-1}}\right) + c\left(\log\left(\frac{x_t}{x_{t-1}}\right)\right)^2 + d \tag{3.21}$$

The model can also be expressed in a single objective function by substituting in all the constraints:

$$\text{Min } z_t = \tfrac{1}{2} \Big[ w_2\left(\mathrm{a}\log(\mathrm{E}_1) + \mathrm{b}\log\left(\frac{x_2}{x_1}\right) + \mathrm{c}\left(\log\left(\frac{x_2}{x_1}\right)\right)^2 + \mathrm{d} - \log(E_2)\right)^2 +$$

$$w_3\left(\mathrm{a}\left[\mathrm{a}\log(\mathrm{E}_1) + \mathrm{b}\log\left(\frac{x_2}{x_1}\right) + \mathrm{c}\left(\log\left(\frac{x_2}{x_1}\right)\right)^2 + d\right] + \mathrm{b}\log\left(\frac{x_3}{x_2}\right) + \mathrm{c}\left(\log\left(\frac{x_3}{x_2}\right)\right)^2 + d - \log(E_3)\right)^2$$

$$+ w_4\left(\mathrm{a}\left[\mathrm{a}\left[\mathrm{a}\log(\mathrm{E}_1) + \mathrm{b}\log\left(\frac{x_2}{x_1}\right) + \mathrm{c}\left(\log\left(\frac{x_2}{x_1}\right)\right)^2 + d\right] + \mathrm{b}\log\left(\frac{x_3}{x_2}\right) + \mathrm{c}\left(\log\left(\frac{x_3}{x_2}\right)\right)^2 + d\right] + \mathrm{b}\log\left(\frac{x_4}{x_3}\right) + \mathrm{c}\left(\log\left(\frac{x_4}{x_3}\right)\right)^2 + d - \log(E_4)\right)^2$$

$$+\ldots$$

$$+ w_t\left(\mathrm{a}^{t-1}\log(\mathrm{E}_1) + \mathrm{a}^{t-2}\,\mathrm{b}\log\left(\frac{x_2}{x_1}\right) + \mathrm{a}^{t-2}\mathrm{c}\left(\log\left(\frac{x_2}{x_1}\right)\right)^2 + d^{-2}d + \mathrm{a}^{t-3}\mathrm{b}\log\left(\frac{x_3}{x_2}\right) + \mathrm{a}^{t-3}\mathrm{c}\left(\log\left(\frac{x_3}{x_2}\right)\right)^2 + d^{-3}d + .. + \mathrm{b}\log\left(\frac{x_t}{x_{t-1}}\right) + \mathrm{c}\left(\log\left(\frac{x_t}{x_{t-1}}\right)\right)^2 + d - \log(E_t)\right)^2 \Big]$$

(3.22)

The model is nonlinear, nonlinearity in the constraints comes from the coefficient a, coefficient of lagged variable, but other coefficients of input variables (b, c and d) do not have a similar effect on the nonlinearity of the model since their powers are not effective in the model.

When we remove the transformations from the initial equation, the constraint of the model becomes:

$$\log(S_t) = \mathrm{a}\log(S_{t-1}) + \mathrm{d} \qquad (3.23)$$

with the same objective function with the model (3.13).

This model is similar to the autoregressive model AR(1) which is explained in section 2.2.1 . Thus, AR( $p$ ) model will be an important model that will be compared with our proposed model to observe effect of adding the input variables as other parameters. Similarly, if $S_{t-2}$ is added to the model, then the model will be similar to AR(2) model in terms of nonlinearity. Thus, in this analysis, the performances of FLN models will be compared with AR models to see the effect of FLN components.

Objective is to minimize total weighted error, so choosing appropriate weights will improve performance of the forecast for the projected years. Weights are chosen starting from 0.01 for each monthly value of year 1987, and increased up to 0.95 for monthly values of year 2005. By this way, weighted model will be focused on the recent years for projection. All weights are shown in Table 2

**Table 2 Monthly Weights Used In the Model For Each Year**

| Year | $W_t$ | Year | $W_t$ | Year | $W_t$ |
|---|---|---|---|---|---|
| 1987 | 0,01 | 1992 | 0,1 | 1997 | 0,55 |
| 1988 | 0,02 | 1993 | 0,2 | 1998 | 0,6 |
| 1989 | 0,03 | 1994 | 0,3 | 1999 | 0,65 |
| 1990 | 0,04 | 1995 | 0,4 | 2000 | 0,7 |
| 1991 | 0,05 | 1996 | 0,5 | | |

For a functional link net model, a set of nonlinear functions should be selected to cover relationships between variables. An experimental approach will be applied to select best performing functions among several alternatives. The chosen function pairs will form a unique model for the proposal.

The method to evaluate performance of each FLN models is tested with the dataset 1987-1996 where TEİAŞ's model MAED had 12.5% mean absolute percentage error (MAPE) value for the validation period 1997-2005. MAPE results smaller than MAED's performance for the same validation period will be shown in bold characters in the Table 4, Table 5 and Table 6.

Although decision variables of FLN model are the constant coefficients to be estimated, objective includes polynomial functional transformations that form the main architecture in Figure 4 and the recursive structure of the formulation forms a nonlinear link between iterations, hence the solution can converge on different local minima depending upon the initial point from which the solution algorithm starts

For each F(x) function, we need to define a method to see the performance: training the dataset with several different initial points can be an option, but a structured way of defining these initial points will be more effective to measure the local optimum results. Therefore, choosing the right initial points is very important. To see the performance of FLN model, results obtained from AR model can be used as initial points, so that it will be easier to explain the improvement that is gained by FLN model rather than using a simple autoregressive model.

If we will not take the nonlinear transformations in the model into account, then the model will become as:

$$\text{Min } z_t = \text{Err}(\log(S_t), \log(E_t)) = ½ \sum_t w_t (\log(S_t) - \log(E_t))^2 \tag{3.24}$$

subject to

$$\log(S_t) = a \log(S_{t-1}) + b\, L(x_t) + d \tag{3.25}$$

open form of the constraint will be :

$$\log(S_t) = \log(S_{t-1}) + b \log\left(\frac{x_t}{x_{t-1}}\right) + d \tag{3.26}$$

This equation is the same as :

$$\frac{S_t}{S_{t-1}} = \frac{X_t^b}{X_{t-1}^b} e^d \tag{3.27}$$

which is equal to Cobb-Douglas function which can simply be defined as :

$$\Delta S = A \Delta X^b \tag{3.28}$$

where A is a parameter.

By considering above model during evaluating performance of FLN model, it will be possible to comment on the nonlinear transformations and the difference they gain rather than linear transformations. The above model is called as classical recursive model and will be used for comparison.

To summarize, performance of a FLN function depends on selecting the right components from several alternatives. The selection is done in a structural way:

- A set of alternative nonlinear components are defined
- All these are tested with separate single models that include only the defined component
- Initialization of FLN model is done with the output of AR model.
- Performance of these FLN models are compared with AR, Regression, MAED and classical recursive models

All component functions used for the analysis are listed in Table 3.

**Table 3 Applied FLN Functions For Testing Performance**

| Variables | GDP | | P | | GDP & P | | GDP x P |
|---|---|---|---|---|---|---|---|
| Functions | X | sin πx | x | sin πx | x | sin πx | $x_1x_2$ |
| **Function -1** | X | sin πx | x | sin πx | x | sin πx | $x_1x_2$ |
| **Function -2** | $x^2$ | sin $\pi x^2$ | $x^2$ | sin $\pi x^2$ | $x^2$ | sin $\pi x^2$ | $x_1^2x_2$ |
| **Function -3** | $x^3$ | sin $\pi x^3$ | $x^3$ | sin $\pi x^3$ | $x^3$ | sin $\pi x^3$ | $x_1x_2^2$ |
| **Function -4** | $x^4$ | sin $\pi x^4$ | $x^4$ | sin $\pi x^4$ | $x^4$ | sin $\pi x^4$ | $x_1^2x_2^2$ |
| **Function -5** | $x^5$ | sin $\pi x^5$ | $x^5$ | sin $\pi x^5$ | $x^5$ | sin $\pi x^5$ | $x_1^3x_2^3$ |

Microsoft Excel solver is used to s this model, since solve spreadsheet solvers are easier to use for iterative calculations and data analysis is done via Minitab version 13.20.(2000)

The Microsoft Excel Solver tool uses the Generalized Reduced Gradient (GRG2) nonlinear optimization code developed by Leon Lasdon. The creators of the model mention that Excel supports a rich variety of operators and several hundred built-in functions, as well as user-written functions. In contrast, GAMS, AMPL, and similar modeling languages include only a small set of operators and functions sufficient for expressing linear, smooth nonlinear, and integer optimization models. On the other hand, they explain that although Excel solver is designed mainly for nonlinear models; it should be used with different starting points to avoid nonoptimal solutions which are seen in some models with starting points with zero values. In addition, they also mention the convergence criteria which can be customized with solver's options menu. The convergence precision can be changed and different optimal solutions can be found while convergence changes(Flystra, et al. 1998). The reader may find the initial criteria for the calculations done in this study in Figure 10.

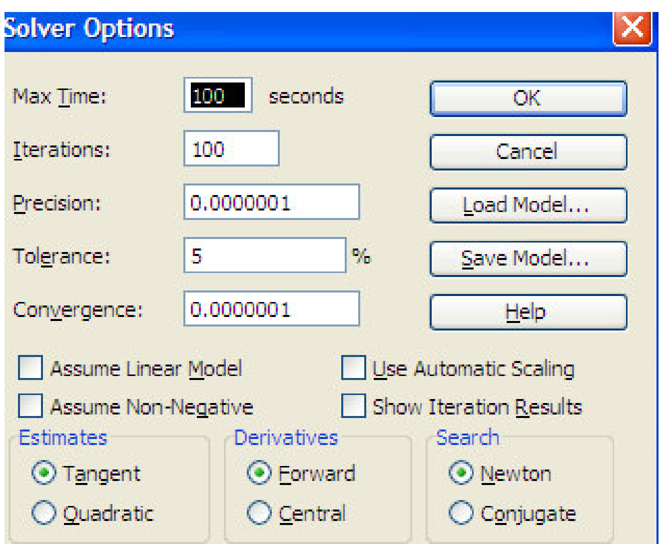


**Figure 10 Initial Criterias Used in Excel Solver**

We verify the accuracy of the Excel solver by constructing a small sample model in which all recursive substitutions are explicitly implemented such that the resulting static version can be solved with reliable solvers provided in the GAMS system. Capability of Excel solver is tested by comparing with different GAMS solvers with a small subset of the whole model (21). For this test, t is defined as 5 and x is defined both for GDP and P. GAMS representation for the proposed model is shown in Appendix B. Although initial assumptions and solvers are different in these two software, both models converge with the similar $S_t$ values and the objective values are close to each other. Thus, we can conclude that Excel is a reliable solver for optimization of these models. Table 4 shows the comparison of these models. Output results of the GAMS and Excel for this observation can be seen in Appendix C.

**Table 4 Comparison of Performances of Excel vs. GAMS**

| | Excel Solver | GAMS Solvers | | | | |
|---|---|---|---|---|---|---|
| | | CONOPT3 | COINIPOPT | MINOS | SNOPT | KNITRO |
| ***a*** | 0.12758 | -0.04229 | 50.71800 | 0.00061 | -0.00004 | -87.86780 |
| ***b*** | 0 | 0 | 14.25184 | 0 | 0 | 2.21887 |
| ***c*** | -3.00435 | -2.38485 | -10.51380 | 0 | 0 | -17.72030 |
| ***d*** | -1.14957 | -2.98583 | -171.31400 | -3.55681 | -3.55735 | -246.10600 |
| ***e*** | -0.02611 | -0.00474 | 6216.80800 | -266.53700 | -265.48300 | 784.14270 |
| ***f*** | 0 | 0 | 0 | 0 | 0 | 0 |
| ***g*** | 3.09867 | 3.70179 | -176.12800 | 3.54984 | 3.55208 | 316.51320 |
| ***$log(S_1)$*** | 3.524 | 3.525 | 3.537 | 3.525 | 3.525 | 3.557 |
| ***$log(S_2)$*** | 3.544 | 3.545 | 3.537 | 3.545 | 3.545 | 3.556 |
| ***$log(S_3)$*** | 3.547 | 3.544 | 3.539 | 3.544 | 3.544 | 3.561 |
| ***$log(S_4)$*** | 3.550 | 3.551 | 3.547 | 3.551 | 3.551 | 3.561 |
| ***$log(S_5)$*** | 3.551 | 3.551 | 3.556 | 3.551 | 3.551 | 3.556 |
| ***Weighted Error $\sum w_t[log(S_t)-log(E_t)]^2$*** | 0.00000038 | 0.00000027 | 0.00000123 | 0.00000026 | 0.00000026 | 0.00000853 |

Initial points will be the output we get from AR models, so next step is to calculate AR(1) and AR(2) models of electricity demand for the training data set (1987-

1996). These models are calculated in Minitab and summarized in Figure 11. Both are significant for 90% CI.

AR(1) and AR(2) are calculated, initial points of FLN models are defined according to these points and the models are trained with the defined functions.

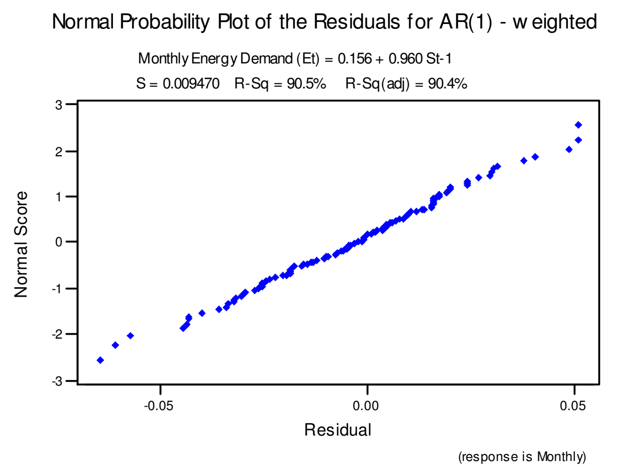


Figure 11(a)

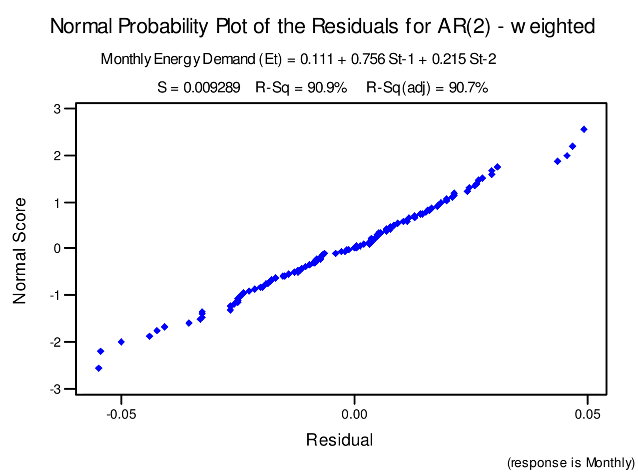


Figure 11(b)

**Figure 11 Results of weighted AR models for dataset 1987- 1996**

The error values and performances of the FLN functions for variables of GDP are summarized in Table 5 MAPE Results that are better than MAED Result (12.5%) are shown in bold.

**Table 5 Error and Performance of FLN functions where x is referred to GDP**

| Variable : GDP($x_1$), reference year 1996 | | | | |
|---|---|---|---|---|
| | **Weighted Error (zt)** | | **MAPE** | |
| | **St-1** | **St-1 & St-2** | **St-1** | **St-1 & St-2** |
| $\mathbf{X_1}$ | *0.004402* | *0.004100* | *18.99%* | *5.08%* |
| $\mathbf{X_1^2}$ | 0.004247 | 0.004089 | **8.31%** | **7.50%** |
| $\mathbf{X_1^3}$ | 0.004386 | 0.004100 | 17.54% | **6.89%** |
| $\mathbf{X_1^4}$ | 0.004398 | 0.004100 | 18.53% | **6.76%** |
| $\mathbf{X_1^5}$ | 0.004402 | 0.004100 | 18.82% | **6.74%** |
| $\mathbf{\sin \pi X_1}$ | 0.004409 | 0.004109 | 19.14% | **5.74%** |
| $\mathbf{\sin \pi X_1^2}$ | 0.004244 | 0.004018 | **10.84%** | **5.31%** |
| $\mathbf{\sin \pi X_1^3}$ | 0.004355 | 0.004099 | 14.88% | **6.99%** |
| $\mathbf{\sin \pi X_1^4}$ | 0.004391 | 0.004100 | 17.78% | **6.24%** |
| $\mathbf{\sin \pi X_1^5}$ | 0.004401 | 0.004100 | 18.74% | **6.76%** |
| ***AR*** | *0.005202* | *0.005099* | *23.28%* | *30.15%* |

Observing the results, it can be concluded that including second lag of electricity demand data, significant improvement is achieved for nonlinear functions.

**Table 6 Error and Performance of FLN functions where x is referred to P**

| Variable : Price ($x_2$), reference year 1996 | | | | |
|---|---|---|---|---|
| | Weighted Error ($z_t$) | | MAPE | |
| | $S_{t-1}$ | $S_{t-1}$ & $S_{t-2}$ | $S_{t-1}$ | $S_{t-1}$ & $S_{t-2}$ |
| **$X_2$** | *0.005200* | *0.004962* | *21.30%* | *15.81%* |
| **$X_2^2$** | 0.005131 | 0.004914 | 20.68% | **11.97%** |
| **$X_2^3$** | 0.005199 | 0.004962 | 21.42% | 16.08% |
| **$X_2^4$** | 0.005199 | 0.004962 | 21.72% | 15.82% |
| **$X_2^5$** | 0.005199 | 0.004962 | 21.79% | 15.80% |
| **$\sin \pi X_2$** | 0.004843 | 0.004959 | 39.70% | 17.04% |
| **$\sin \pi X_2^2$** | 0.005131 | 0.004913 | 20.83% | 14.20% |
| **$\sin \pi X_2^3$** | 0.005198 | 0.004873 | 20.87% | 15.07% |
| **$\sin \pi X_2^4$** | 0.005199 | 0.004962 | 21.57% | 15.90% |
| **$\sin \pi X_2^5$** | 0.005199 | 0.158113 | 21.76% | 15.81% |
| *AR* | *0.005202* | *0.005099* | *23.28%* | *30.15%* |

Table 6 represents the results for the functions of energy price index ($P_t$), and significant improvement for second order of electricity demand can be observed for this data set also. When those two results in Table 5 and Table 6 are compared, it can be concluded that GDP is a better predictor than Price. The results are in line with the previous studies in literature: as it is stated in section 3.1.1, 1% GDP increase requires 0.5-1.5% increase in energy use in an economy and short run price elasticity is -0.2 and long run price elasticity is between -0.7 and -0.9.

**Table 7 Error and Performance of FLN functions where x is referred to GDP&P**

| Variable : GDP ($X_2$) & Price ($X_2$), reference year 1996 | | | | |
|---|---|---|---|---|
| | Weighted Error ($z_t$) | | MAPE | |
| | $S_{t-1}$ | $S_{t-1}$ & $S_{t-2}$ | $S_{t-1}$ | $S_{t-1}$ & $S_{t-2}$ |
| **$X_1$, $X_2$ (CRM)** | 0.004398 | 0.004100 | 15.11% | **5.08%** |
| **$X_1^2$, $X_2^2$** | 0.004272 | 0.004114 | **4.37%** | **5.40%** |
| **$X_1^3$, $X_2^3$** | 0.004411 | 0.004100 | 13.83% | **6.58%** |
| **$X_1^4$, $X_2^4$** | 0.004400 | 0.004100 | 15.38% | **6.73%** |
| **$X_1^5$, $X_2^5$** | 0.004398 | 0.004100 | 15.83% | **6.73%** |
| **$\sin \pi X_1$, $\sin \pi X_2$** | 0.004403 | 0.004107 | 16.93% | **5.94%** |
| **$\sin \pi X_1^2$,$\sin \pi X_2^2$** | 0.004190 | 0.003868 | **7.78%** | 17.57% |
| **$\sin \pi X_1^3$,$\sin \pi X_2^3$** | 0.004279 | 0.003969 | **10.99%** | **8.00%** |
| **$\sin \pi X_1^4$,$\sin \pi X_2^4$** | 0.004406 | 0.004100 | 14.55% | **6.35%** |
| **$\sin \pi X_1^5$,$\sin \pi X_2^5$** | 0.004399 | 0.004100 | 15.68% | **6.74%** |
| **$X_1X_2$** | 0.004400 | 0.004096 | 17.09% | **6.16%** |
| **$X_1^2X_2$** | 0.004393 | 0.004100 | 17.87% | **6.75%** |
| **$X_1X_2^2$** | 0.004397 | 0.004100 | 16.63% | **6.76%** |
| **$X_1^2X_2^2$** | 0.004397 | 0.004397 | 16.54% | **6.72%** |
| **$X_1^3X_2^3$** | 0.004397 | 0.004397 | 16.52% | **6.71%** |
| ***AR*** | ***0.005202*** | ***0.005099*** | ***23.28%*** | ***30.15%*** |

The most significant result from Table 7 is the positive effect of $S_{t-2}$ on performance. In all cases, model performs better by adding $S_{t-2}$ to the model. In addition, all models show better results than AR model.

All these results are showing the local optimum points that are found by setting the same starting point which is the output for AR model. The results do not mean that they are reflecting the global optimum points, but these points are meaningful to compare with this methodology. For further analysis, we check whether there is a significant positive / negative effect when we combine two different components in one model. We need to ensure whether there is a correlation between performance results with one nonlinear component in Table 5 and Table 6 and corresponding results of similar models with the addition of both of these two components in Table 7. This correlation is examined by sampling:

- **Performance of using good performing components together in one model:**

The models that include $x^2$ as the nonlinear component perform better than the others with only 7.5 % MAPE where x refers GDP and 11.97% MAPE where x refers Price Index. Table 7 shows that using these two "good performers" in one model results with 6.05% MAPE for the validation dataset, This example shows that using two good performer components in a single model result with a better projection.

All similar results are evaluated by the same comparison. Thus, it is observed that in most cases using both components in one single model either improves the result than the two sub-models or the MAPE result do not worsen for the validation dataset. The only example that does not fit this generalization is the component $\sin\pi^2$ component, so we cannot claim that using two different components together in one single model will improve the performance, but the performance of the model do not worsen in most cases.

### 3.1.3 Building The FLN Model

As explained above, *"Functional Link Net uses non-linear functions which express the relationships between the output and input variables, so that it is defined as a neural network model that does not require hidden layers. The non-linearity is satisfied by multi-variable non-linear functions."* (Fu and Nguyen, 2003)
Different nonlinear functions are tested and evaluated to get more detailed information on FLN models, but it is also important to show the performance of a model that includes more than two nonlinear components. Increase in the number of components will help to identify the system more accurately with the help of nonlinear relationships among different variables. On the other hand, increasing number of variables may also increase complexity without any significant improvement over the performance, so number of variables that should be included in the model is also an important decision point. In this study, nonlinear components

will be selected from the variables with good MAPE results in the analysis in 3.1.2 . Second lag of the electricity demand data will also be included in the model since it significantly improves performance. Table 8 shows the selected variables.

**Table 8 Selected Variables for FLN**

| GDP ($X_1$) | P ($X_2$) | GDP ($X_1$)& P ($X_2$) |
|---|---|---|
| $X_1^5$ | $X_2^2$ | $X_1$ $X_2$ |
| sin π $X_1$ | sin π $X_2^2$ | |

Performance of the FLN model will be evaluated together with the performances of four models:

The first model is defined as the classical recursive model (CRM), and it is formed just to understand the nonlinearity effect of the FLN function. CRM is simply formed as the Cobb-Douglas model with the same model algorithm with FLN. The second model is TEİAŞ's forecasting tool, MAED. TEİAŞ's forecast errors for the same projection period will be used. The third model is an autoregressive model with the same nonlinear components. The last model is the regression model for the same input variables. Akan and Tak's econometric analysis (2006) results for (2001-2006) will also be added for consideration. Mean Absolute Percentage Error (MAPE) will be used as the performance measure over all.
While FLN results are tested with other projections, the results that are derived by training should be validated with a different defined time period. The forecast error values of other projections for the same training and validation periods will be used for the comparison. Two different test periods will be used in the analysis: 1987-1996 & 1987-2000.

The main reason for this is to observe performance of the model when an outlier change the dynamics of the current system. Year 2001 is known with an economic crisis, and it can also be observed in the GDP and price data starting from year 2000. So after year 2000, projections changed with the results of economical changes. If a forecast model is not operating with a reasonable error when future expectations do not realize, then it will not be effective in the long run although it performs well for the training dataset.

MAED model have high forecast error for the latter time periods, especially after 5 years from the projection date, forecast error increase dramatically as it is stated in Figure 5. So, projection performance for more than 5 years is an important criteria, together with the performance of the model during fluctuations.

To conclude, in the first projection, training dataset will be defined from 1987 to 1996 and validation dataset will be from 1997 to 2005 (covering 9 years in total). Next projection will be trained with the training dataset for years 1987 to 2000 and validation period will cover 2001-2005 (covering 5 years). This two different training will help us to see:

performance for longer time periods → validation set will be 1997 – 2005
performance with outliers → validation set will be 2001-2005 . Outlier year 2001 is the first forecasted year . Outlier year is shown in Figure 12.

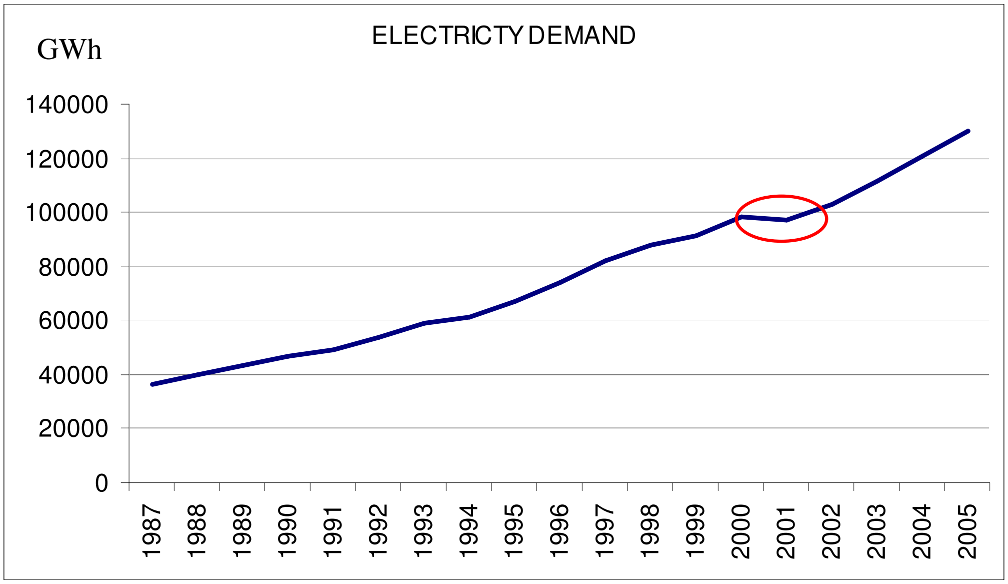


**Figure 12 Representation of Outlier Values of Electricity Demand in year 2000**

The model considers historical electrical energy consumption data, GDP and energy price index(P). It uses a dynamic model with an objective of minimizing total forecast error while constraining defined forecast model with constant parameters.

The dynamic model has an iterative procedure that aims to minimize total forecast error for all and constraints a recursive model to be satisfied with appropriate coefficient values for expected relations between dependent and independent variables. The model reaches to the objective where error values between all iterations are minimized and therefore best fit is observed.

Proposed model can be explained as follows:

Decision variables:

$S_{t-1}$ : First lag of the projected demand

a, b, c, d, e, f, g, k, m, n : related constants of the model

Objective:

$$\text{Min } z = ½ \sum w_t (\log S_t - \log E_t)^2 \tag{3.29}$$

Such that

$$\begin{aligned}\text{Log St} = {} & a \log S_{t-1} + b \log(GDP_t / GDP_{t-1}) + c \log(P_t / P_{t-1}) + d\, [\log(GDP_t / GDP_{t-1})]^2 \\ & + e\, [\log(P_t / P_{t-1})]^2 + k \sin[\prod \log(GDP_t / GDP_{t-1})]^2 + m \sin[\prod \log(P_t / P_{t-1})]^2 \\ & + n \log(GDP_t / GDP_{t-1}) \log(P_t / P_{t-1}) + f \log S_{t-2} + g \qquad (3.30) \\ & b \geq 0 \qquad c \leq 0\end{aligned}$$

Where t is time index, $S_t$ is the forecasted electrical energy demand at time t, and letters b to m represent constants.

## 3.2 MODEL APPLICATION

In this section, models defined in Part 3.1.3 are applied and testing was done to measure forecast performance of the models with two different training data sets defined.

The Classical Recursive Model is shown to be a tool to observe the performance of the functional link-net with the same structure for modeling but without nonlinearity in the transformations. Both models will be compared with TEİAŞ Projection Model (MAED) to see their performance against current model used in Turkey and two different OLS Regression model to see the difference of the FLN model with OLS models. The three stages of Network Forecast Modeling which was defined by Nasr (2002) are considered in the model application: Training, Testing and Evaluation.

### 3.2.1 Training and Validation

The objective is to evaluate a new way of forecasting National Electricity Demand Forecast. Therefore, a new model should be comparable with the current MAED results and should be easy to be used for next periods.

The model has three main variables: monthly GDP, energy price index and past electricity demand. If we use existing data for the projected years in the testing state, we will not be able to consider performance of our model without known independent variables for a current year, which will also not help us for projection of future demand where we will not have accurate values of these data. We should have an estimation method for these variables so that we can compare our model with the current method at the same given conditions. Forecasted GDP and Price data is shown in Figure 13. Detailed explanations for selected forecasting methods of GDP and Price Index can be found in section 3.1.1 and forecasted GDP and Price values can be found in Appendix D.

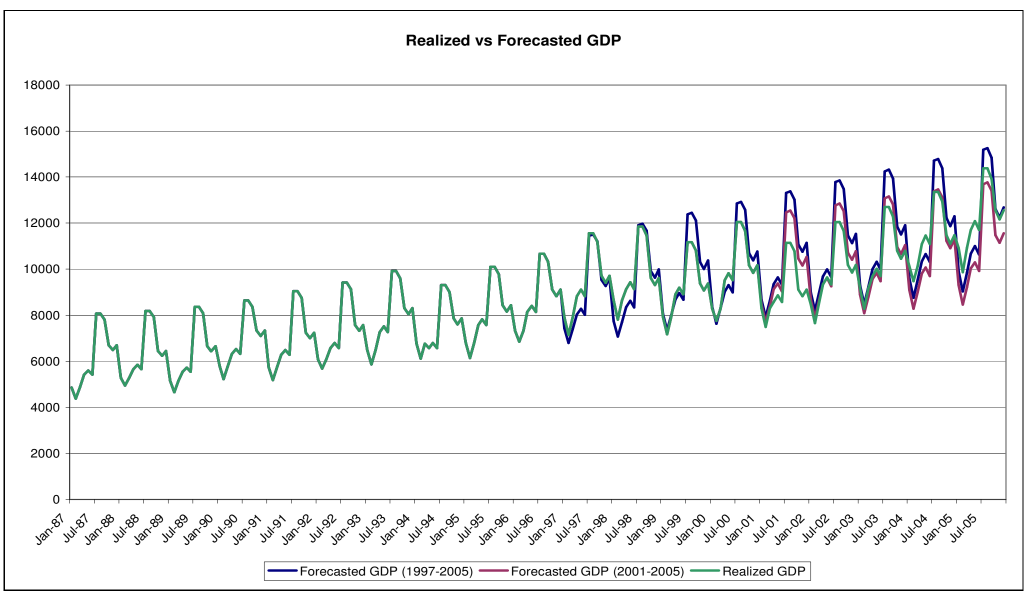


Figure 13(a)

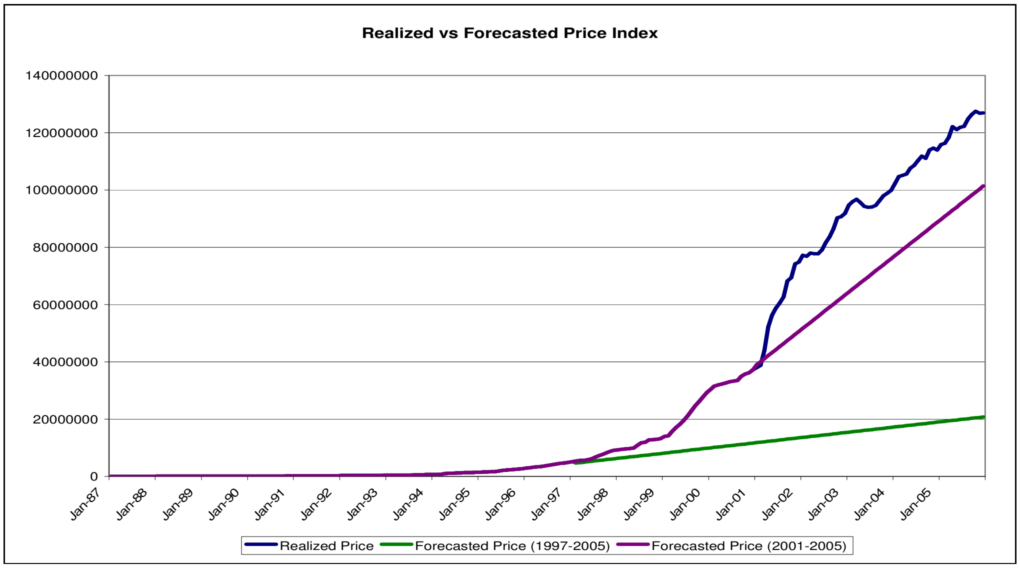


Figure 13(b)

**Figure 13 Realized vs Forecasted Values according to the Model**

### *Training The Classical Recursive Model*

Previously defined classical recursive model is formed as follows:

$$\text{Min } z = \tfrac{1}{2} \sum w_t (\log S_t - \log E_t)^2 \qquad (3.31)$$

Where

$$\text{Log } St = a \log S_{t-1} + b \log(GDP_t / GDP_{t-1}) + c \log(P_t / P_{t-1}) + f \log S_{t-2} + g \qquad (3.32)$$

$b \geq 0$ $\qquad\qquad$ $c \leq 0$

CRM is solved by Excel solver. Optimum points are found for the data sets (1987-1996) and (1987-2000) as in Table 9. Initial points are defined with the output of AR(2) model for the same datasets.

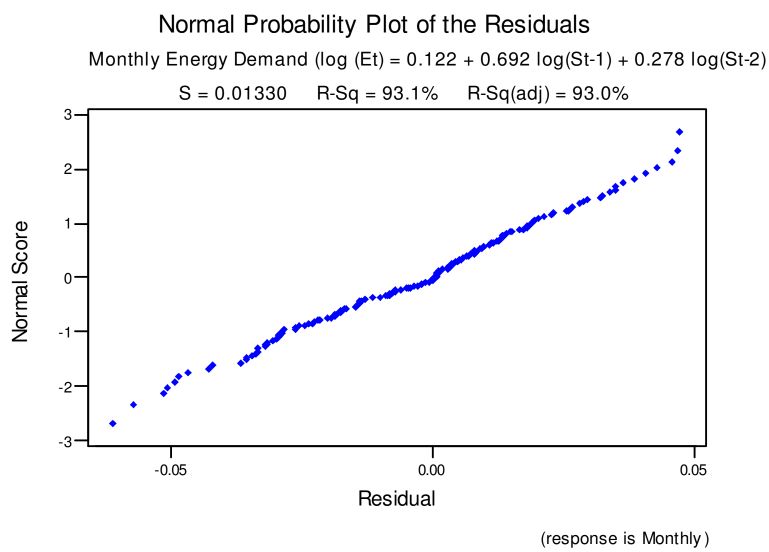


**Figure 14 AR(2) Model for training set (1987-2000)**

AR(2) Results dataset were defined in Figure 11. Results of weighted AR models for dataset 1987- 1996 are in Figure 14 for training set (1987-2000)

**Table 9 Optimum Results for Classical Recursive Model (CRM)**

| | Data Set (1987-1996) | Data Set (1987-2000) |
|---|---|---|
| a | **0.7444729** | **0.7023985** |
| b | **0.1809212** | **0.1706379** |
| c | 0 | 0 |
| f | 0.2427686 | 0.2804854 |
| g | 0.0523555 | 0.0691666 |
| Z | 0.004100 | 0.004112 |
| MAPE | 5.08% | 5.04% |

Comparison of the linear models with the real data can also be seen in Figure 15. . St(CRM -1996) represents projection of CRM forecast with the training set 1987-1996 and St(CRM -2000) represents projection of CRM forecast with the training set 1987-2000 correspondingly.

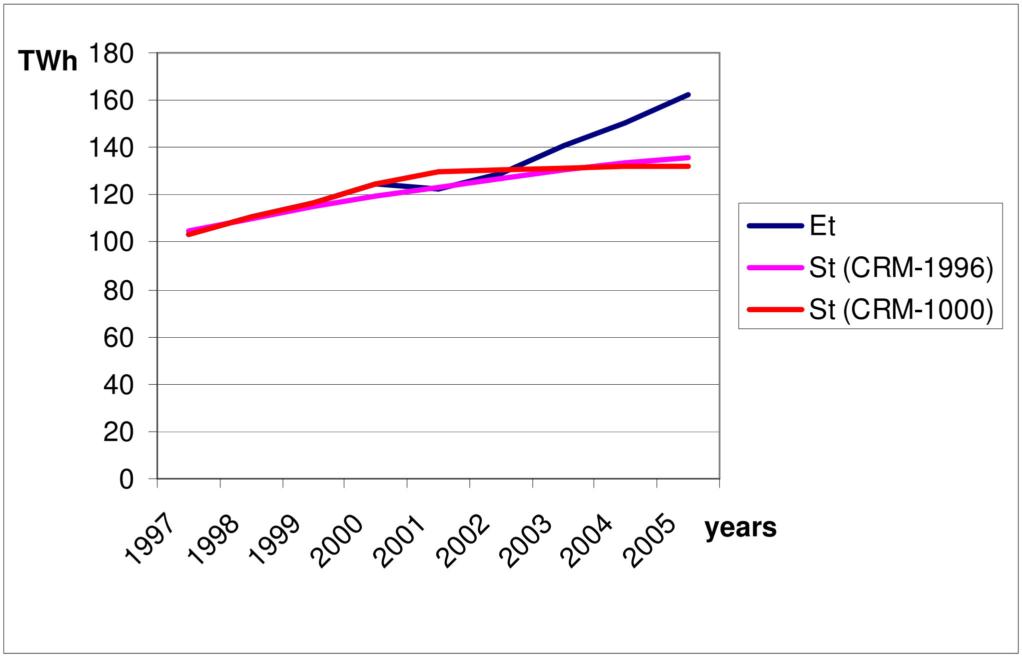


**Figure 15 Comparison of performance of CRM Models**

***Training The Functional Link Net Model***

In this section, proposed functional link net model will be examined for two different training datasets and the performance of the model will be examined for this model. Functional Link-Net Model is defined as follows:

$$\text{Min } z = ½ \sum w_t (\log S_t - \log E_t)^2 \qquad (3.33)$$

Such that

$$\begin{aligned} \text{Log } St = {} & a \log S_{t-1} + b \log(GDP_t / GDP_{t-1}) + c \log(P_t / P_{t-1}) + d [\log(GDP_t / GDP_{t-1})]^2 \\ & + e [\log(P_t / P_{t-1})]^2 + k \sin[\prod \log(GDP_t / GDP_{t-1})]^2 + m \sin[\prod \log(P_t / P_{t-1})]^2 \\ & + n \log(GDP_t / GDP_{t-1}) \log(P_t / P_{t-1}) + f \log S_{t-2} + g \qquad (3.34) \end{aligned}$$

where $b \geq 0$ and $c \leq 0$

Model is solved for two data sets to identify the performance in each case.

**Table 10** shows MAPE results for the validation data set in each case. (1997-2005 and 2001-2005)

**Table 10 Local Optimum Result for Proposed FLN Model**

| | Data Set (1987-1996) | Data Set (1987-2000) |
|---|---|---|
| *a* | 0.730412766 | 0.731810432 |
| *b* | 5.155507402 | 5.100235891 |
| *c* | 0 | 0 |
| *d* | 0.074246153 | 0.073476701 |
| *e* | 0.015274183 | -0.03357836 |
| *f* | 0.25954157 | 0.258180263 |
| *g* | 0.04144575 | 0.041279618 |
| *k* | -1.620780515 | -1.602692558 |
| *m* | -0.181715763 | -0.162677829 |
| *n* | 0.671624019 | 0.662562339 |
| *Weighted Error* $\sum w_t[log(S_t)-log(E_t)]^2$ | 0.00393124249 | 0.00393117742 |
| *MAPE* | 6.46% | 4.58% |

This method will also be compared with OLS model, so it is important to check normality of error. The null hypothesis of having normal probability of error functions for the proposed model for validation dataset 1997-2005 cannot be rejected for 90% CI and 2000-2005 cannot be rejected for 95% CI for the Kolmogorov Smirnov Normality Test. You can see histogram figures in Figure 16 and test results in Appendix E.

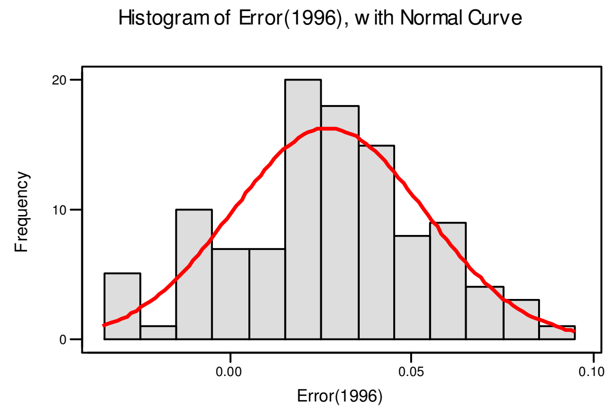


Figure 16(a)

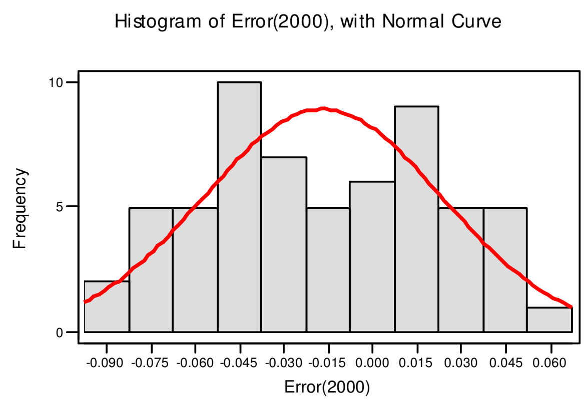


Figure 16(b)

**Figure 16 Histogram of Error Functions with Normal Curve**

### *Training The Regression Models*

Two different regression models will be compared with the proposed models:

- A similar model with the same input dataset with proposed FLN model
- A simple model with 3 basic input variables: lagged variables of demand, GDP and price index.

First model will be a regression model for the first constraint of the FLN model :

$$\text{Log St} = a \log S_{t-1} + b[\log(GDP_t / GDP_{t-1})] + c \log(P_t / P_{t-1}) + d [\log(GDP_t / GDP_{t-1})]^5$$
$$+ e [\log(P_t / P_{t-1})]^2 + k \sin[\prod \log(GDP_t / GDP_{t-1})] + m \sin[\prod \log(P_t / P_{t-1})]^2$$
$$+ n \log(GDP_t / GDP_{t-1}) \log(P_t / P_{t-1}) + f \log S_{t-2} + g \qquad (3.35)$$

Regression analysis is as follows for training dataset (1987-1996):

| Predictor | Coef | SE Coef | T | P |
|---|---|---|---|---|
| Constant | 0.1090 | 0.1045 | 1.04 | 0.299 |
| GDP lo | 20.313 | 9.293 | 2.19 | 0.031 |
| Energy P | 0.7662 | 0.2674 | 2.87 | 0.005 |
| [log(GDP | -902.9 | 538.9 | -1.68 | 0.097 |
| [log(Pt/ | 45.9 | 653.2 | 0.07 | 0.944 |
| sin Pi [ | -6.462 | 2.978 | -2.17 | 0.032 |
| sinPi[lo | -16.3 | 208.4 | -0.08 | 0.938 |
| [log(GDP | -2.983 | 2.753 | -1.08 | 0.281 |
| log(St-1 | 0.62752 | 0.08683 | 7.23 | 0.000 |
| log(St-2 | 0.34143 | 0.08676 | 3.94 | 0.000 |

S = 0.008398     R-Sq = 93.0%     R-Sq(adj) = 92.5%

$R^2$ value is above 90%, which means model is representative for the training dataset. Details can be found in Appendix F.

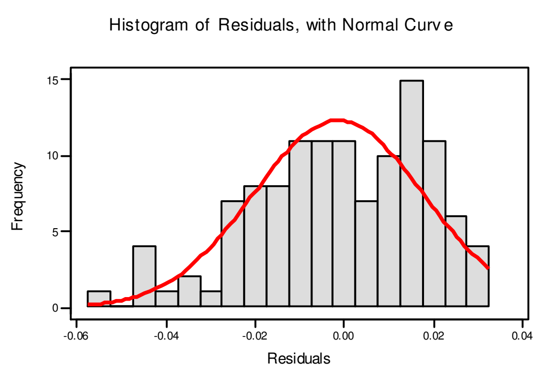


Figure 17(a)

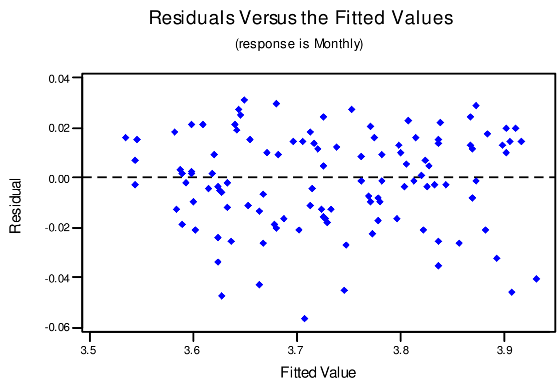


Figure 17(b)

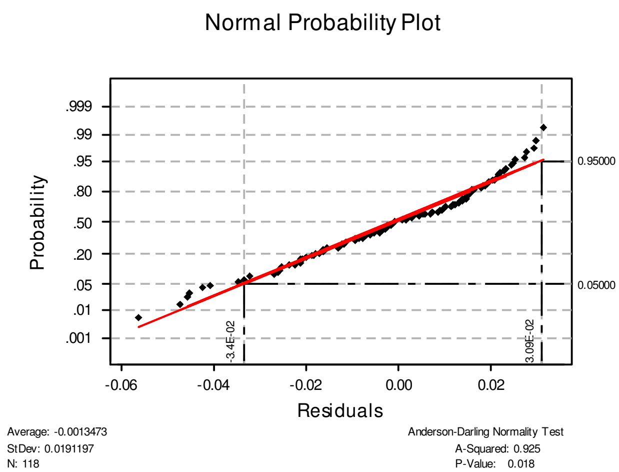


Figure 17(c)

**Figure 17 Output Analysis for Regression Model 1 (1987-1996)**

Figure 17 shows the output analysis for the training dataset 1987-1996. Residuals vs Fit Graphs show no bias for the error values and normal distribution assumption for the error values cannot be rejected for 0.95% CI according to normality check.

Regression analysis for training dataset (1987-2000) is as follows:

| Predictor | Coef | SE Coef | T | P |
|---|---|---|---|---|
| Constant | 0.01087 | 0.08227 | 0.13 | 0.895 |
| GDP lo | 7.29 | 10.76 | 0.68 | 0.499 |
| Energy P | 0.5188 | 0.2071 | 2.51 | 0.013 |
| [log(GDP | -3.2 | 620.0 | -0.01 | 0.996 |
| [log(Pt/ | -684.0 | 519.0 | -1.32 | 0.189 |
| sin Pi[l | -2.322 | 3.443 | -0.67 | 0.501 |
| sinPi [l | 216.7 | 165.4 | 1.31 | 0.192 |
| [log(GDP | -1.372 | 2.930 | -0.47 | 0.640 |
| log(St-1 | 0.69792 | 0.06798 | 10.27 | 0.000 |
| log(St-2 | 0.29819 | 0.06846 | 4.36 | 0.000 |

S = 0.01184    R-Sq = 94.8%    R-Sq(adj) = 94.5%

$R^2$ value is above 90%, which means model is representative for the training dataset. Detailed regression analysis result is in Appendix F.

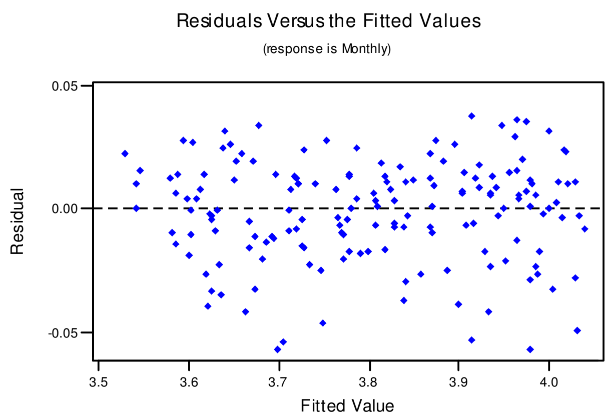


Figure 18(a)

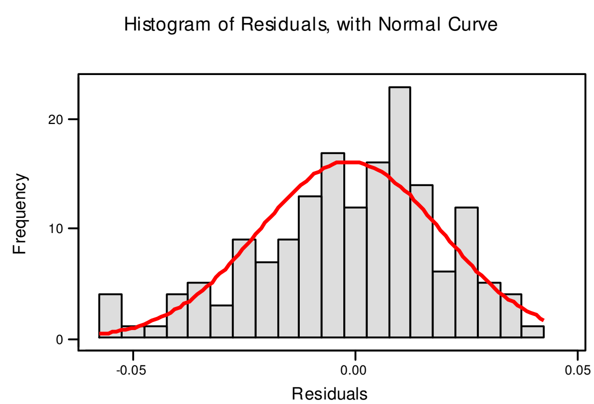


Figure 18(b)

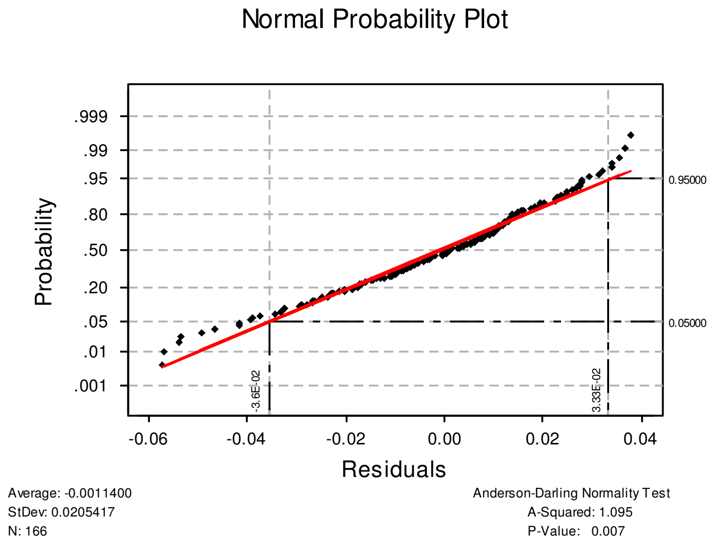


Figure 18(c)

**Figure 18 Output Analysis for Regression Model 1 (1987-2000)**

Figure 18 shows the output analysis for the training dataset 1987-2000. Residuals vs Fit Graphs show no bias for the error values and normal distribution assumption for the error values cannot be rejected for 0.95% CI according to normality check.

**Table 11 Regression Results for Regression model 1**

| | Data Set (1987-1996) | Data Set (1987-2000) |
|---|---|---|
| *a* | 0.62752 | 0.69792 |
| *b* | 20.313 | 7.29 |
| *c* | 0.7662 | 0.5188 |
| *d* | -902.9 | -3.2 |
| *e* | 45.9 | -684 |
| *f* | 0.34143 | 0.29819 |
| *g* | 0.109 | 0.01087 |
| *k* | -6.462 | -2.322 |
| *m* | -16.3 | 216.7 |
| *n* | -2.983 | -1.372 |
| *Weighted Error* $\sum w_t[\log(S_t)-\log(E_t)]^2$ | 0.003656 | 0.003865 |
| *MAPE* | 41.77% | 10.09% |

Table 11 shows the results for regression results of the Regression model 1.When we compare both results, it is seen that although regression gives better results in the training dataset, MAPE results for the validation dataset is worse than all other models, so it can be concluded that with the increased number of decision variables, model is overfitting for the expected output for future projections. Graphical representation of projection results are shown in
Figure 19

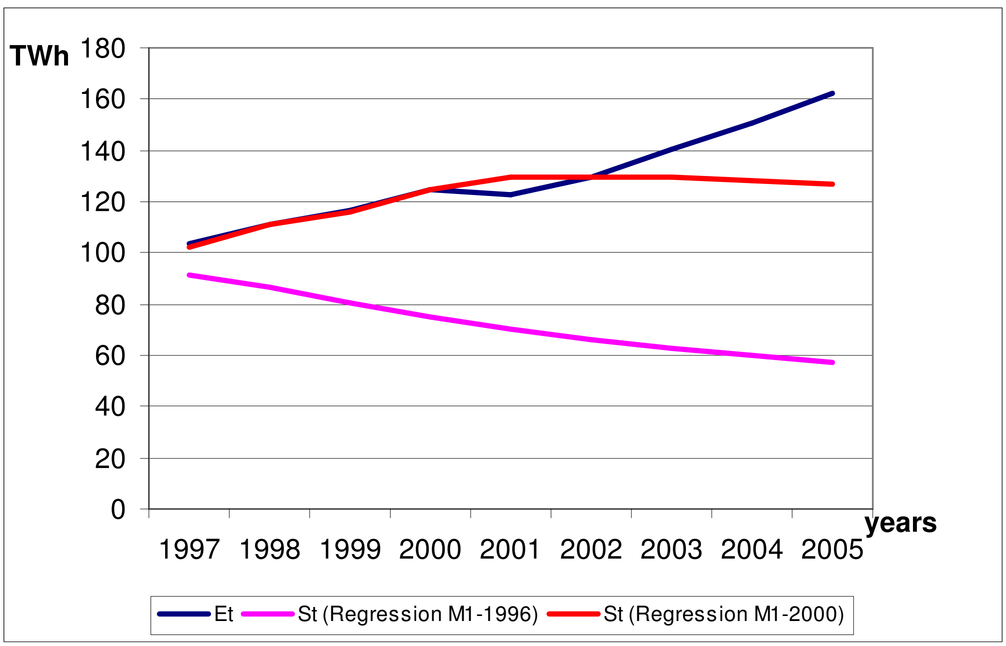


**Figure 19 Representation of Projections of Regression Model 1**

Second model will be a regression model for a simple set of input variables :

$$\text{Log } St = a \log S_{t-1} + b[\log(GDP_t / GDP_{t-1})] + c \log(P_t / P_{t-1}) + f \log S_{t-2} + g \qquad (3.36)$$

Regression analysis is as follows for training dataset (1987-1996):

```
Predictor        Coef     SE Coef        T       P
Constant       0.1301      0.1092     1.19   0.236
GDP   lo      0.15642     0.04012     3.90   0.000
Energy P     -0.04549     0.09899    -0.46   0.647
log(St-1      0.96699     0.02854    33.88   0.000

S = 0.009306    R-Sq = 91.0%     R-Sq(adj) = 90.7%
```

$R^2$ value is above 90%, which means model is representative for the training dataset. Detailed regression analysis result is in Appendix G.

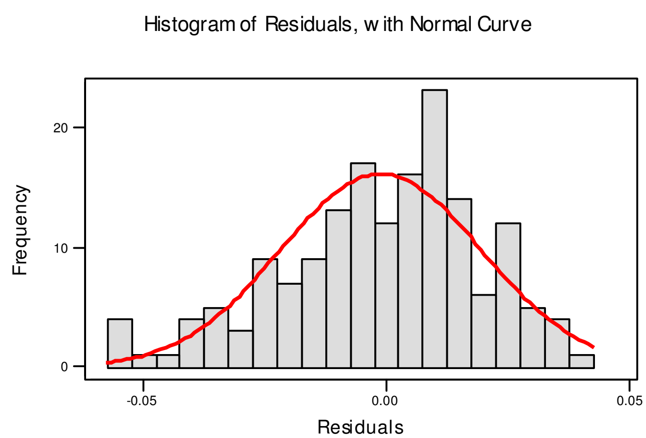


Figure 20(a)

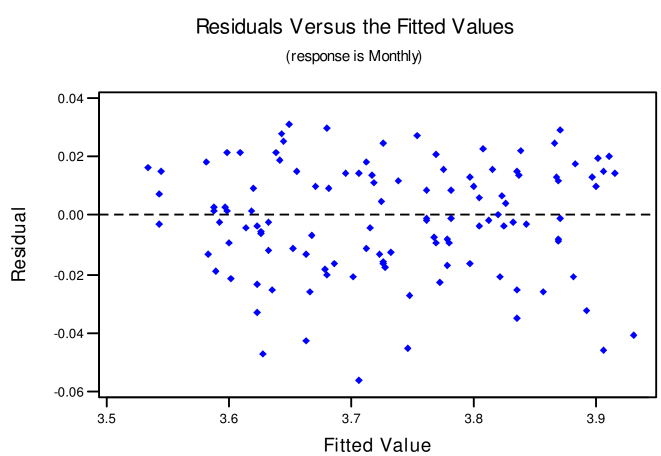


Figure 20(b)

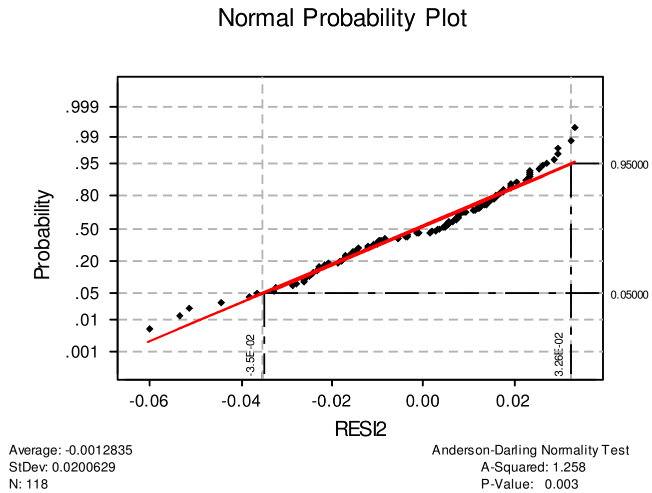


Figure 20(c)

**Figure 20 Output Results of Regression Model 2 for training dataset (1987-1996)**

Figure 20 shows the output analysis for the training dataset 1987-1996. Residuals vs Fit Graphs show no bias for the error values and normal distribution assumption for the error values cannot be rejected for 0.95% CI according to normality check.

When we apply the same model for training dataset (1987-2000), regression results are as following:

| Predictor | Coef | SE Coef | T | P |
|---|---|---|---|---|
| Constant | 0.07489 | 0.08277 | 0.90 | 0.367 |
| GDP lo | 0.15739 | 0.03107 | 5.07 | 0.000 |
| Energy P | 0.0441 | 0.1188 | 0.37 | 0.711 |
| log(St-1 | 0.69648 | 0.07032 | 9.90 | 0.000 |
| log(St-2 | 0.28501 | 0.07048 | 4.04 | 0.000 |

S = 0.01242 R-Sq = 94.0% R-Sq(adj) = 93.9%

$R^2$ value is above 90%, which means model is representative for the training dataset. Detailed regression analysis result is in Appendix G.

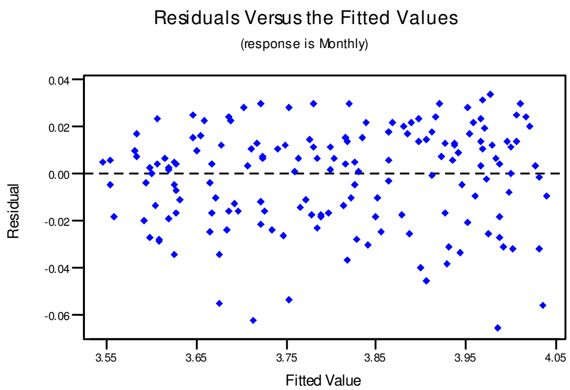


Figure 21(a)

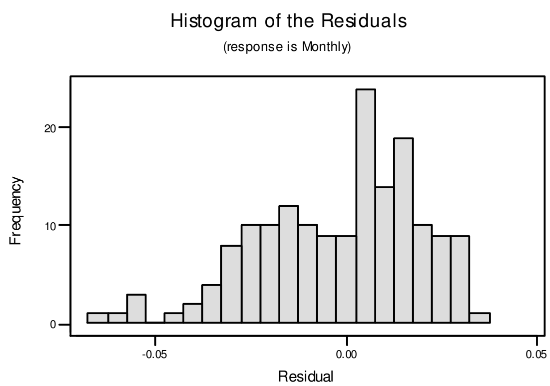


Figure 21(b)

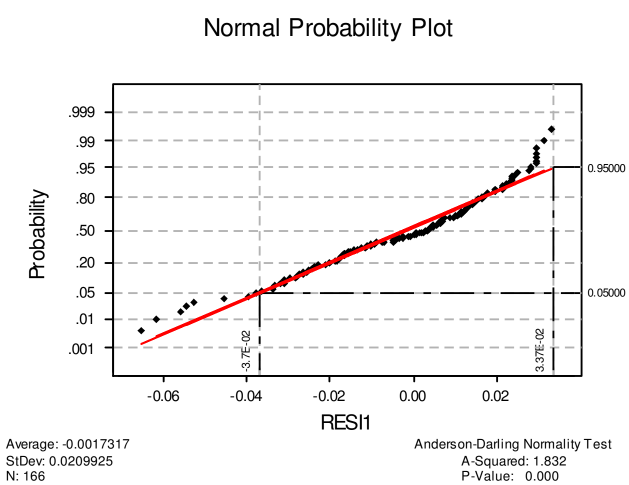


Figure 21(c)

**Figure 21 Output Results of Regression Model 2 for training dataset (1987-2000)**

Figure 21 shows the output analysis for the training dataset 1987-2000. Residuals vs Fit Graphs show no bias for the error values and normal distribution assumption for the error values cannot be rejected for 0.95% CI according to normality check.

**Table 12 Regression Results for Regression Model 2**

| | **Data Set (1987-1996)** | **Data Set (1987-2000)** |
|---|---|---|
| *a* | **0.69346** | 0.69648 |
| *b* | **0.16511** | 0.15739 |
| *c* | **-0.00703** | 0.0441 |
| *d* | **0** | 0 |
| *e* | **0** | 0 |
| *f* | **0.29094** | 0.28501 |
| *g* | **0.0636** | 0.07489 |
| *k* | **0** | 0 |
| *m* | **0** | 0 |
| *n* | **0** | 0 |
| *Weighted Error* $\sum w_t[log(S_t)-log(E_t)]^2$ | **0.00412087** | 0.004147 |
| *MAPE* | 6.31% | 8.22% |

Regression Model 2 does not contain nonlinear transformations but this simple structure allows us better forecasting performance than the previous model whereas training error is worse than the first model. Graphical representation of projection results are shown in Figure 22.

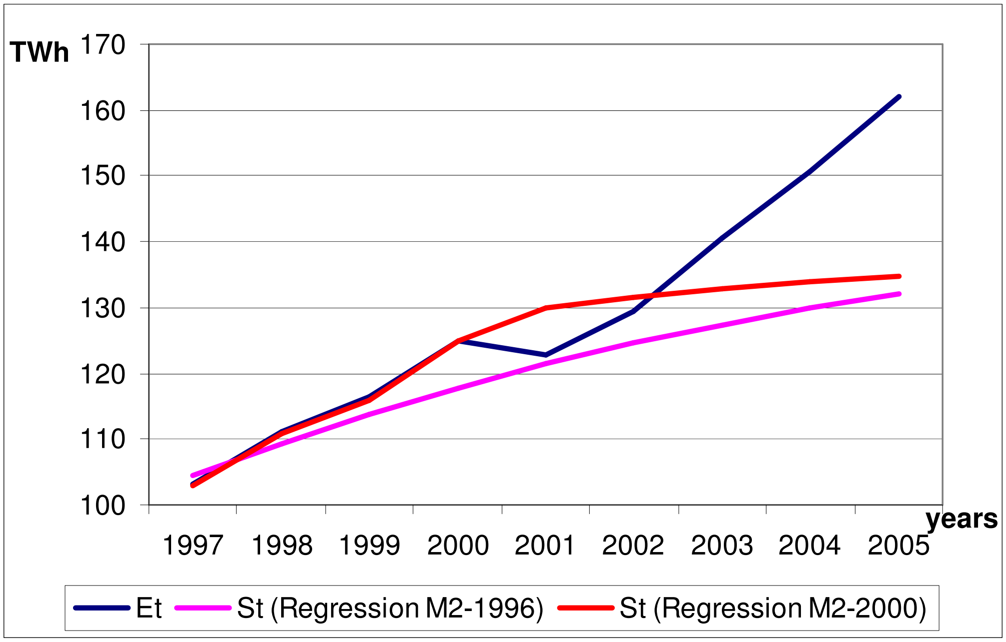


**Figure 22 Representation of Projections of Regression Model 2**

The results of this model will be compared in the next section with the FLN model.

### 3.2.2 Evaluation

The performance results of all models are summarized in Table 13. All other models are compared with FLN model to observe similarities / differences between each other.

AR(2) model is the model that is proposed to find initial points for FLN model, and it considers 1st and 2nd lag of projected demand. CRM model is a similar to FLN on the optimizing structure, but it do not include nonlinear transformations. Regression model 2 is the regression model that includes 1st and 2nd lag of projected demand, GDP and price index values. MAED and Econometric model are two different model that has been done in Turkey to project demand for electricity.

Table 13 Comparison of AR(2), Regression, CRM, FLN Model, MAED and Econometric Model with yearly percent errors

| Years | Actual Energy Demand (TWh) | Absolute Percent Errors of Projections | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | AR(2) | | Regression Model 2 | | CRM | | MAED | | Econometric Model (Akan & Tak - 2006) | New FLN Model | |
| | | 1996 | 2000 | 1996 | 2000 | 1996 | 2000 | 1996 | 2000 | 2000 | 1996 | 2000 |
| 1997 | 103 | 4.5% | | 1.1% | | 1.3% | - | 0.2% | - | | 2.8% | - |
| 1998 | 111 | 15.5% | | 1.5% | | 1.0% | - | 0.2% | - | | 3.2% | - |
| 1999 | 116 | 22.4% | | 2.2% | | 1.5% | - | 4.4% | - | | 5.4% | - |
| 2000 | 125 | 29.7% | | 5.7% | | 4.6% | - | 4.7% | - | | 4.6% | - |
| 2001 | 123 | 30.0% | 5.5% | 1.0% | 6.0% | 0.4% | 5.54% | 15.2% | 9.4% | 10.9% | 12.8% | 8.0% |
| 2002 | 129 | 34.6% | 0.4% | 3.7% | 1.8% | 1.9% | 0.81% | 19.4% | 14.2% | 9.3% | 12.6% | 7.0% |
| 2003 | 141 | 40.5% | 8.6% | 9.4% | 5.4% | 7.4% | 6.67% | 21.2% | 17.0% | 9.9% | 8.7% | 2.6% |
| 2004 | 151 | 45.0% | 14.9% | 13.8% | 11.1% | 11.6% | 12.52% | 23.1% | 20.1% | 8.7% | 5.8% | 0.8% |
| 2005 | 162 | 49.2% | 21.0% | 18.5% | 16.8% | 16.1% | 18.28% | 24.1% | 22.3% | 7.3% | 2.3% | 4.5% |
| MAPE | | 30.15% | 10.05% | 6.3% | 8.22% | 5.08% | 8.76% | 12.50% | 16.59% | 9.22% | 6.46% | 4.58% |

In most cases, forecasts that are done after year 1996 have poorer performance after year 2001, which is obviously because of the outlier year 2001. In 1996, there is less visibility to foresee the fluctuation. AR(2) model has the worst performance among all, and could not respond to the increase in base demand. MAED model is also affected similarly, forecast error definitely increases each year. FLN and CRM are somehow different: Their iterative nonlinear structure significantly fits and MAPE results are definitely lower than others, absolute errors do not continuously increase with time. MAPE Result of FLN model between years 1997-2005 is 6.46%, which is slightly higher than MAPE result of CRM.

Figure 23 represents the graphical representation of projections between years 1997-2005.

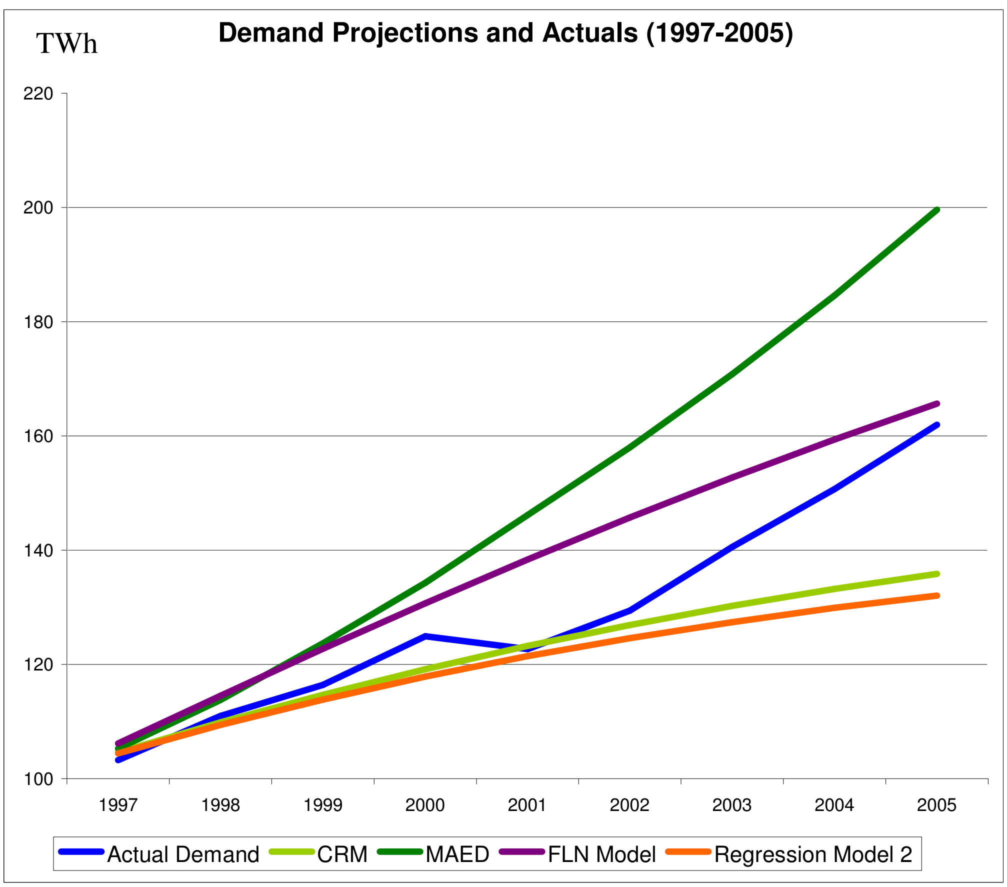


**Figure 23 Demand Projections and Actuals (1997 – 2005)**

Forecasts after year 2000 are more important considering the significant change in demand after that year. In this case, MAED performs even worse than the AR(2) model, because AR model depends only on previous demand values, so an outlier changes all projections and error values varies with higher error values. For the validation period 2001-2005, FLN MAPE result is the best result among all, with 4.58%. Econometric model of Akan & Tak is also better than MAED, and it is also better than a regression model because the model is improved with an error correction term. It can also be seen from

Figure 24.

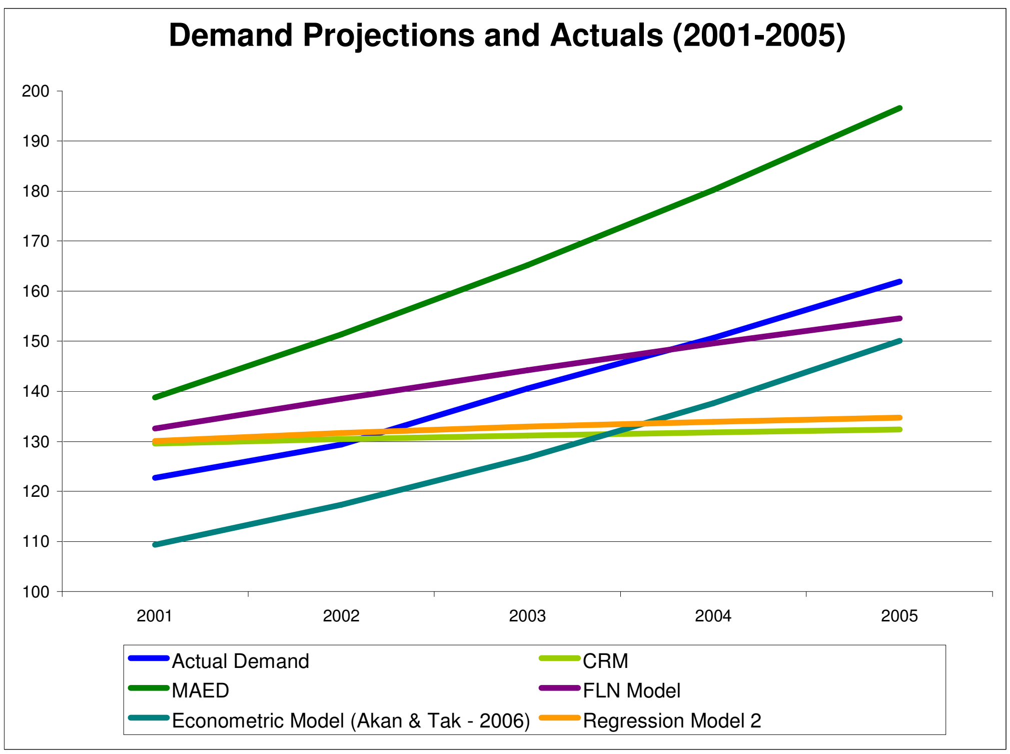


**Figure 24 Demand Projection and Actuals (2001-2005)**

All these case based analysis shows that FLN model is performing better than other similar models for electricity demand projection. Its nonlinear structure allows better fits than others, but for different initial points model converge to different objective values. Regression model 2 is closely related with CRM, which is similar to each other with the same input values. Thus, we can conclude that the most effective component of FLN model. On the other hand, FLN model is improved version of CRM with the same structure but also using the nonlinear transformations. Since using the same nonlinear transformations in a regression model do not improve the model but worsen the forecast performance by overfitting, we can conclude that the exact improvement of forecast performance comes from nonlinear dynamic structure of the model with constraits a and k.

The most important result of this analysis is all analysis shows local optimum points for FLN model, which means there can be better performing solutions. Better objective values can be found by trying different initial points. To illustrate, CRM model could be selected as another step at initialization stage, local optimum points of CRM model can be defined as initial points, and better results may be obtained by this way. Further studies on those topics will deepen the work.

# CHAPTER 4

# CONCLUSION

Electricity is the most important secondary energy that is not easily substitutable and is expensive to produce. Its management calls for reliable and valid demand forecasting since investment lead times are long. Although electricity demand forecasting is a long established research area the models that are used in Turkey and on which investment programs are based exhibit systematic errors and cannot be relied on except for the very short run. This research investigates the potential of the FLN concept that was introduced in connection with demand forecasting based on artificial neural networks. The FLN models are implemented as a stand alone forecasting tool in which total weighted sum of squared errors are minimized under the constraint of nonlinear links between dependent and independent variables.

An experimental approach is adopted to try out and identify the best performing nonlinear transformations of the independent variables using simple models of one transformation each, and a more complex and final model is formulated from good performers. The initial assumption that forecast accuracy would increase by adding different nonlinear components was born out in general. For majority of the cases, forecast results either improved or accuracy was sustained.

Nonlinearities prevent the assurance of a global optimum and good initial points are important for the search algorithm in producing better results on training datasets, so initial points were selected using a simple autoregressive AR($p$) model. Forecast performances were evaluated after training with a given dataset and validation with other datasets. Two different forecasting periods were taken into account: (1996-2005) was used to see long term performance of the forecasting tool, and (2001-2005) was used to see the effect of outlier data (2001 was a year of economic crisis in Turkey) at the beginning of the forecast horizon .

MAED is the model used to produce official projections and it is analysed in detail in this research. MAED is a scenario based simulation model and its performance has been criticized in several articles mainly in that it is prone to produce systematic errors and tends to overestimate the demand more as forecasts extend into the future. The FLN models have been seen to perform significantly better than MAED, and they do not exhibit systematic bias with advancing time periods.

FLN models were also compared with an econometric model developed by Akan and Tak(2006) although their model moves from a different set of assumptions than adopted in this research. Other models that use similar scenario assumptions and structures included:

- AR(p) : An auto regressive model using only the past few realizations of demand as input variables. Although these projections provided good starting points for the training dataset, they did not perform so well with the validation dataset, especially after the outlier year 2001.
- Regression : Regression with the same independent variables and their transformations that are used in the FLN models helps to see whether the performance of FLN would be replicated. Even though both models use the same input variables and objectives, there is a difference between them arising from the recursive nature of the FLN model. The results are similar to those from the AR(p) models; although regression model performs better than the AR(p) model for the training dataset, it performs worse with the validation dataset.
- Classical Recursive Model (CRM): The general approach of this model is the same as that of FLN, only this model does not use the nonlinear transformations between input and output variables. CRM embodies the

recursive nature of the FLN that is based on lagged relationships but linear links are assumed between output and input variables. CRM is found to perform better than any other models that are analysed, but FLN improves the forecast accuracy with nonlinear links.

We can conclude overall that the FLN structure definitely has the potential to produce better forecasts than several more conventional methods as demonstrated by our empirical investigations.

Methodology of defining and evaluating FLN model is formed to understand the mechanics, but the approach can also be improved with further analysis. This model converges to local optimum solutions, so all models used same initial points to compare in a logical way, however there can be better local optimum points and they can be found out by changing convergence criteria or initialization assumptions. Using different initialization methods can be analyzed in future works. FLN models are defined by modifying / reconstructing of ANN models, so comparing FLN models with definite ANN methods should also be an important topic to be covered in the literature.

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# APPENDIX A

# MONTHLY ELECTRICITY DEMAND GENERATED FROM TRANSMITTED ENERGY

**Table 14 Monthly Electricity Demand Generated From Monthly Transmitted Energy**

| | 1987 | 1988 | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 | 1996 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Jan** | 2943 | 3287 | 3452 | 3849 | 3919 | 4495 | 4987 | 5285 | 5606 | 6338 |
| **Feb** | 2711 | 3096 | 3159 | 3492 | 3757 | 4208 | 4706 | 4855 | 5159 | 5618 |
| **March** | 2884 | 3264 | 3432 | 3762 | 4194 | 4535 | 4684 | 5053 | 5337 | 6260 |
| **April** | 2846 | 3106 | 3396 | 3293 | 3595 | 3900 | 4515 | 4708 | 5238 | 5605 |
| **May** | 2880 | 3058 | 3382 | 3692 | 4015 | 4217 | 4571 | 4594 | 4962 | 5559 |
| **Jun** | 2958 | 3164 | 3542 | 3801 | 3729 | 4010 | 4464 | 4772 | 5451 | 5838 |
| **Jul** | 3246 | 3312 | 3643 | 3842 | 4391 | 4685 | 5213 | 5197 | 5856 | 6545 |
| **Aug** | 3076 | 3574 | 3913 | 4208 | 4493 | 4914 | 5260 | 5392 | 6093 | 6653 |
| **Sep** | 3211 | 3443 | 3747 | 4216 | 4254 | 4651 | 5061 | 5283 | 5684 | 6076 |
| **Oct** | 3304 | 3478 | 3805 | 4349 | 4270 | 4670 | 5051 | 5188 | 5700 | 6355 |
| **Nov** | 3240 | 3393 | 3778 | 4219 | 4266 | 4701 | 5218 | 5373 | 5944 | 6517 |
| **Dec** | 3398 | 3547 | 3872 | 4097 | 4399 | 4998 | 5506 | 5702 | 6364 | 6793 |
| | **1997** | **1998** | **1999** | **2000** | **2001** | **2002** | **2003** | **2004** | **2005** | |
| **Jan** | 6995 | 7648 | 7736 | 8732 | 8954 | 9547 | 9868 | 10294 | 10756 | |
| **Feb** | 6206 | 7073 | 7333 | 8488 | 8083 | 7803 | 8626 | 9697 | 10365 | |
| **March** | 7150 | 7832 | 7609 | 8462 | 7586 | 8600 | 9705 | 10026 | 10907 | |
| **April** | 6424 | 6349 | 7019 | 7322 | 7433 | 8205 | 8481 | 9382 | 10118 | |
| **May** | 6406 | 6790 | 7182 | 7544 | 7428 | 7911 | 8432 | 9346 | 10207 | |
| **Jun** | 6325 | 6862 | 7175 | 7676 | 7601 | 8180 | 8880 | 9607 | 10104 | |
| **Jul** | 7110 | 7652 | 8012 | 8614 | 8592 | 9246 | 9976 | 10852 | 11572 | |
| **Aug** | 7030 | 7804 | 7819 | 8576 | 8681 | 9195 | 10193 | 10876 | 11888 | |
| **Sep** | 6673 | 7182 | 7229 | 7890 | 7795 | 7983 | 8992 | 10197 | 10692 | |
| **Oct** | 6856 | 7047 | 7434 | 8011 | 7770 | 8088 | 8979 | 9943 | 10844 | |
| **Nov** | 7044 | 7380 | 8044 | 8329 | 8401 | 8623 | 9068 | 9734 | 10733 | |
| **Dec** | 7665 | 8085 | 8613 | 8651 | 8747 | 9568 | 10568 | 11188 | 11615 | |

# APPENDIX B

# REPRESENTATION OF PROPOSED MODEL WITH GAMS

Model (21) can be solved for t=5 similarly in GAMS with the following algorithm :

```
Sets
t time  periods /0,1,2,3,4,5/
index(t)   /1,2,3,4,5/
opening(t)  /0/;
Parameters
Energy(index)   Logarithm of electricity demand in month t
/
1          3.52495080776062
2          3.54679859971171
3          3.54214981646263
4          3.54634212612522
5          3.55588912114912
 /

GDP(index)  Difference of logarithm of GDP in month t from logarithm of GDP in»
 month t-1
/
1          -0.0442036624601352
2           0.0442036624601352
3           0.048001126762693
4           0.0142404392694866
5          -0.0142404392694866
/

P(index)  Difference of logarithm of P in month t from logarithm of P in month»
 t-1
/
1          0.008681544107596470000
2          0.0007686771302175540000
3          0.0004942450918399290000
4          0.0000000000000000000000
5          0.0000000000000000000000
/

w(index)  weight  for  optimization
/
1         0.01
2         0.01
3         0.01
4         0.01
5         0.01
/;

Variables
Error
a
d
e
f

Positive Variables
S(t)      logarithm of electricity demand at t
b
Negative Variables
c;

Equations
Objective defining error  function
Demand1 demand projection at 1
Demand2 demand projection at 2
Demand3 demand projection at 3
```

```
Demand4 demand projection at 4
Demand5 demand projection at 5
Initial
;

Objective.. error=e= (w("1")*sqr(S("1")-Energy("1"))+w("2")*sqr(S("2")
-Energy("2"))+w("3")*sqr(S("3")-Energy("3"))+
w("4")*sqr(S("4")-Energy("4"))+w("5")*sqr(S("5")-Energy("5")))/2    ;

Demand1.. S("1")=e= a*S("0") + b*GDP("1")+c*P("1")+d*sqr(GDP("1"))
+ e*sqr(P("1"))+f   ;
Demand2.. S("2")=e= a*S("1") + b*GDP("2")+c*P("2")+d*sqr(GDP("2"))
+ e*sqr(P("2"))+f   ;
Demand3.. S("3")=e= a*S("2") + b*GDP("3")+c*P("3")+d*sqr(GDP("3"))
+ e*sqr(P("3"))+f   ;
Demand4.. S("4")=e= a*S("3") + b*GDP("4")+c*P("4")+d*sqr(GDP("4"))
+ e*sqr(P("4"))+f   ;
Demand5.. S("5")=e= a*S("4") + b*GDP("5")+c*P("5")+d*sqr(GDP("5"))
+ e*sqr(P("5"))+f   ;

Initial.. S("0")=e=3.554001289065530   ;




model trial/all/;
solve trial using nlp minimizing error;
option error:8, a:8,b:8,c:8,d:8,e:8,f:8;
display error.l, a.l,b.l,c.l,d.l,e.l ,f.l;



```

For the whole model, following short explanation can be used in GAMS:

```
Sets
t time periods /0,1,2,3,4,5/
index(t)  /1,2,3,4,5/
opening(t) /0/;
Parameters
Energy(index)   Logarithm of electricity demand in month t
/
1          3.52495080776062
2          3.54679859971171
3          3.54214981646263
4          3.54634212612522
5          3.55588912114912
 /

GDP(index) Difference of logarithm of GDP in month t from logarithm of GDP in»
 month t-1
/
1          -0.0442036624601352
2           0.0442036624601352
3           0.048001126762693
4           0.0142404392694866
5          -0.0142404392694866
/

P(index) Difference of logarithm of P in month t from logarithm of P in month»
 t-1
/
1          0.0086815441075964700000
2          0.0007686771302175540000
3          0.0004942450918399290000
4          0.0000000000000000000000
5          0.0000000000000000000000
/

w(index) weight for optimization
/
1         0.01
2         0.01
3         0.01
4         0.01
5         0.01
/;

Variables
Error
a
d
e
f

Positive Variables
S(t)      logarithm of electricity demand at t
b
Negative Variables
c;

Equations
Objective defining error function
Demand(t) demand projection at t
Initial;

```

```
Objective..  error=e= sum(index,  w(index)*sqr(S(index)-Energy(index)))/2      ;
Demand(index-1)..  S(index)=e=  a*S(index-1)  + b*GDP(index)+c*P(index)+  d*sqr(G»
DP(index))+  e*sqr(P(index))+f    ;
Initial..  S("0")=e=3.554001289065530   ;




model trial/all/;
solve trial using nlp minimizing  error;
option error:8,  a:8,b:8,c:8,d:8,e:8,f:8;
display error.l,  a.l,b.l,c.l,d.l,e.l  ,f.l;



```

# APPENDIX C

# EXCEL AND GAMS OUTPUT COMPARISON SUMMARY

## Excel Solver Results

| Date | Monthly Energy Demand (log ($E_t$)) | GDP log($GDP_t/GDP_{t-1}$) | Energy Price Index log($P_t/P_{t-1}$) | [log($GDP_t/GDP_{t-1}$)]$^2$ | [log($P_t/P_{t-1}$)]$^2$ | log($S_t$) | log($S_{t-1}$) | log($S_{t-2}$) | $w_t$ | Error [log($S_t$)-log($E_t$)]$^2$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 01.01.87 | 3.55400 | | | | | | | | | |
| 02.01.87 | 3.52495 | -0.04420 | 0.00868 | 0.00195 | 0.00008 | 3.524 | 3.554 | | 0.01 | 0.0000014 |
| 03.01.87 | 3.54680 | 0.04420 | 0.00077 | 0.00195 | 0.00000 | 3.544 | 3.525 | 3.55 | 0.01 | 0.0000089 |
| 04.01.87 | 3.54215 | 0.04800 | 0.00049 | 0.00230 | 0.00000 | 3.547 | 3.547 | 3.52 | 0.01 | 0.0000238 |
| 05.01.87 | 3.54634 | 0.01424 | 0.00000 | 0.00020 | 0.00000 | 3.550 | 3.542 | 3.55 | 0.01 | 0.0000160 |
| 06.01.87 | 3.55589 | -0.01424 | 0.00000 | 0.00020 | 0.00000 | 3.551 | 3.546 | 3.54 | 0.01 | 0.0000252 |

| | |
|---|---|
| a | **0.127578801** |
| b | 0 |
| c | -3.004346819 |
| d | -1.149573907 |
| e | -0.026112222 |
| f | 0 |
| g | 3.098668152 |
| $\sum w_t$[log($S_t$)-log($E_t$)]$^2$ | 0.00000038 |

| Monthly Actual vs Forecast | |
|---|---|
| $E_t$ (KWh) | $S_t$ (KWh) |
| 3522.075 | 3498.013947 |
| 3484.575 | 3523.958888 |
| 3518.375 | 3550.898491 |
| 3596.575 | 3555.274249 |

**Figure 25 Representation of Excel Interface**

**GAMS with CONOPT3 Solver**

```
 S O L V E      S U M M A R Y

     MODEL   trial              OBJECTIVE  Error
     TYPE    NLP                DIRECTION  MINIMIZE
     SOLVER  CONOPT             FROM LINE  86

**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      2 LOCALLY OPTIMAL
**** OBJECTIVE VALUE               0.0000

 RESOURCE USAGE, LIMIT          0.547      1000.000
 ITERATION COUNT, LIMIT        20         10000
 EVALUATION ERRORS              0             0

    C O N O P T 3   version 3.14S
    Copyright (C)   ARKI Consulting and Development A/S
                    Bagsvaerdvej 246 A
                    DK-2880 Bagsvaerd, Denmark

 Using default options.

    The model has 13 variables and 7 constraints
    with 43 Jacobian elements, 15 of which are nonlinear.
    The Hessian of the Lagrangian has 5 elements on the diagonal,
    5 elements below the diagonal, and 7 nonlinear variables.

 ** Optimal solution. Reduced gradient less than tolerance.

  CONOPT time Total                            0.219 seconds

 Workspace          =     0.04 Mbytes
    Estimate        =     0.04 Mbytes
    Max used        =     0.02 Mbytes

---- VAR S  logarithm of electricity demand at t

     LOWER     LEVEL     UPPER    MARGINAL

0     .       3.554     +INF       .
1     .       3.525     +INF       .
2     .       3.545     +INF       .
3     .       3.544     +INF  4.4854E-9
4     .       3.551     +INF       .
5     .       3.551     +INF  2.7116E-8
```

```
----    88 VARIABLE Error.L            =   0.00000027
          VARIABLE a.L              =  -0.04229088
          VARIABLE b.L              =   0.00000000
          VARIABLE c.L              =  -2.38485357
          VARIABLE d.L              =  -2.98583428
          VARIABLE e.L              =  -0.00474105
          VARIABLE f.L              =   3.70179271
```

**GAMS with COINIPOPT Solver**

```
         S O L V E     S U M M A R Y

    MODEL   trial              OBJECTIVE  Error
    TYPE    NLP                DIRECTION  MINIMIZE
    SOLVER  COINIPOPT          FROM LINE  86

**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      2 LOCALLY OPTIMAL
**** OBJECTIVE VALUE               0.0000

 RESOURCE USAGE, LIMIT          0.188      1000.000
 ITERATION COUNT, LIMIT        10         10000
 EVALUATION ERRORS              0             0

GAMS/CoinIpopt NLP Solver (IPOPT Library 3.4dev, using MUMPS Library 4.7.3)

Number of objective function evaluations             = 11
Number of objective gradient evaluations             = 11
Number of equality constraint evaluations            = 11
Number of inequality constraint evaluations          = 0
Number of equality constraint Jacobian evaluations   = 11
Number of inequality constraint Jacobian evaluations = 0
Number of Lagrangian Hessian evaluations             = 10
Total CPU secs in IPOPT (w/o function evaluations)   =      0.188
Total CPU secs in NLP function evaluations           =      0.000

---- VAR S  logarithm of electricity demand at t

    LOWER     LEVEL     UPPER    MARGINAL

0      .       3.554     +INF  4.216E-12
1      .       3.537     +INF  4.340E-12
2      .       3.537     +INF  4.365E-12
3      .       3.539     +INF  4.201E-12
4      .       3.547     +INF  4.446E-12
5      .       3.556     +INF  5.722E-10


----    88 VARIABLE Error.L            =   0.00000123
          VARIABLE a.L              =  50.71800011
```

```
VARIABLE b.L          =  14.25184011
VARIABLE c.L          =  -1.05138E+1
VARIABLE d.L          =  -1.71314E+2
VARIABLE e.L          =  6.216808E+3
VARIABLE f.L          =  -1.76128E+2
```

**GAMS with MINOS Solver**

```
   S O L V E     S U M M A R Y

    MODEL   trial            OBJECTIVE  Error
    TYPE    NLP              DIRECTION  MINIMIZE
    SOLVER  MINOS               FROM LINE  86

**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      2 LOCALLY OPTIMAL
**** OBJECTIVE VALUE              0.0000

 RESOURCE USAGE, LIMIT          0.109      1000.000
 ITERATION COUNT, LIMIT        21          10000
 EVALUATION ERRORS              0            0

GAMS/MINOS       May  1, 2008 22.7.2 WIN 3906.4799 VIS x86/MS Windows
M I N O S  5.51     (Jun 2004)

 Work space allocated           --    0.78 Mb

 EXIT - Optimal Solution found, objective:       0.2589486E-06

    LOWER     LEVEL     UPPER    MARGINAL

0     .       3.554     +INF      .
1     .       3.525     +INF  6.1530E-9
2     .       3.545     +INF      .
3     .       3.544     +INF  -4.984E-8
4     .       3.551     +INF      .
5     .       3.551     +INF  -3.03E-11

                 LOWER     LEVEL     UPPER    MARGINAL

---- VAR b              .        .       +INF  1.2695E-6
---- VAR c            -INF       .        .    -6.037E-9


**** REPORT SUMMARY :        0     NONOPT
                       0 INFEASIBLE
                       0  UNBOUNDED
                       0     ERRORS
```

```
----     88 VARIABLE Error.L             =   0.00000026
           VARIABLE a.L                =   0.00060855
           VARIABLE b.L                =   0.00000000
           VARIABLE c.L                =   0.00000000
           VARIABLE d.L                =  -3.55681293
           VARIABLE e.L                =  -2.66537E+2
           VARIABLE f.L                =   3.54983826
```

**<u>GAMS with SNOPT Solver</u>**

```
     S O L V E      S U M M A R Y

     MODEL   trial              OBJECTIVE  Error
     TYPE    NLP                DIRECTION  MINIMIZE
     SOLVER  SNOPT                 FROM LINE  86

**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      2 LOCALLY OPTIMAL
**** OBJECTIVE VALUE                0.0000

 RESOURCE USAGE, LIMIT          0.203      1000.000
 ITERATION COUNT, LIMIT         7         10000
 EVALUATION ERRORS              0             0

SNOPT 7.2-4      May  1, 2008 22.7.2 WIN 3906.4799 VIS x86/MS Windows

 Work space allocated           --    0.21 Mb

 EXIT - Requested accuracy could not be achieved, objective:       0.2589217E-06

 Work space used by solver      --    0.06 Mb


---- VAR S  logarithm of electricity demand at t

     LOWER     LEVEL     UPPER    MARGINAL

0     .       3.554     +INF       .
1     .       3.525     +INF  2.683E-10
2     .       3.545     +INF       .
3     .       3.544     +INF  -2.79E-11
4     .       3.551     +INF       .
5     .       3.551     +INF  1.2865E-9
```

```
----     88 VARIABLE Error.L            =   0.00000026
           VARIABLE a.L                 =  -0.00004362
           VARIABLE b.L                 =   0.00000000
           VARIABLE c.L                 =   0.00000000
           VARIABLE d.L                 =  -3.55734767
           VARIABLE e.L                 =  -2.65483E+2
           VARIABLE f.L                 =   3.55207583
```

**GAMS with KNITRO Solver**

```
           S O L V E      S U M M A R Y

     MODEL   trial              OBJECTIVE  Error
     TYPE    NLP                DIRECTION  MINIMIZE
     SOLVER  KNITRO             FROM LINE  86

**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      2 LOCALLY OPTIMAL
**** OBJECTIVE VALUE                0.0000


KNITRO           May  1, 2008 22.7.2 WIN 3906.4799 VIS x86/MS Windows


---- VAR S  logarithm of electricity demand at t

     LOWER     LEVEL     UPPER    MARGINAL

0      .       3.554     +INF       EPS
1      .       3.557     +INF  4.048E-11
2      .       3.556     +INF  4.235E-11
3      .       3.561     +INF  3.064E-11
4      .       3.561     +INF       EPS
5      .       3.556     +INF  2.6082E-8


----     88 VARIABLE Error.L            =   0.00000853
           VARIABLE a.L                 =  -8.78678E+1
           VARIABLE b.L                 =   2.21886846
           VARIABLE c.L                 =  -1.77203E+1
           VARIABLE d.L                 =  -2.46106E+2
           VARIABLE e.L                 =  7.841427E+2
           VARIABLE f.L                 =  3.165132E+2
```

# APPENDIX D

# GDP & ENERGY PRICE INDEX FORECAST RESULTS

**Table 15 Energy Price Index Forecasts for (1997-2005)**

| S | 18646.8 | | PRICE FORECASTING (1997-2005) | |
|---|---|---|---|---|
| G | 10 | | | |
| Date | Monthly Energy Demand (Et) | price (alpha &beta 0.1) | S-price | G-price |
| Jan-87 | 3581 | 18647 | 18656 | 10 |
| Feb-87 | 3349 | 19023 | 18701 | 13 |
| Mar-87 | 3522 | 19057 | 18749 | 17 |
| Apr-87 | 3485 | 19079 | 18797 | 20 |
| May-87 | 3518 | 19079 | 18843 | 23 |
| Jun-87 | 3597 | 19079 | 18887 | 25 |
| Jul-87 | 3884 | 19606 | 18981 | 32 |
| Aug-87 | 3714 | 19949 | 19107 | 41 |
| Sep-87 | 3848 | 20854 | 19318 | 58 |
| Oct-87 | 3941 | 21495 | 19588 | 79 |
| Nov-87 | 3878 | 21889 | 19890 | 102 |
| Dec-87 | 4035 | 24648 | 20457 | 148 |
| Jan-88 | 3981 | 26247 | 21169 | 205 |
| Feb-88 | 3790 | 29169 | 22153 | 282 |
| Mar-88 | 3958 | 30776 | 23270 | 366 |
| Apr-88 | 3800 | 31964 | 24468 | 449 |
| May-88 | 3752 | 32365 | 25662 | 524 |
| Jun-88 | 3858 | 34087 | 26976 | 603 |
| Jul-88 | 4006 | 36011 | 28422 | 687 |
| Aug-88 | 4268 | 38002 | 29998 | 776 |
| Sep-88 | 4137 | 38877 | 31584 | 857 |
| Oct-88 | 4172 | 41026 | 33300 | 943 |
| Nov-88 | 4087 | 43983 | 35216 | 1040 |
| Dec-88 | 4241 | 49017 | 37533 | 1168 |
| Jan-89 | 4193 | 52787 | 40109 | 1309 |
| Feb-89 | 3896 | 53736 | 42649 | 1432 |
| Mar-89 | 4173 | 53223 | 44995 | 1523 |

**Table 15 Energy Price Index Forecasts for (1997-2005) (cont'd)**

| | | | | |
|---|---|---|---|---|
| Apr-89 | 4136 | 55480 | 47415 | 1613 |
| May-89 | 4122 | 58894 | 50014 | 1711 |
| Jun-89 | 4285 | 59579 | 52511 | 1790 |
| Jul-89 | 4388 | 64614 | 55332 | 1893 |
| Aug-89 | 4661 | 66740 | 58177 | 1988 |
| Sep-89 | 4493 | 68541 | 61003 | 2072 |
| Oct-89 | 4552 | 69755 | 63743 | 2139 |
| Nov-89 | 4525 | 78192 | 67113 | 2262 |
| Dec-89 | 4620 | 81532 | 70591 | 2384 |
| Jan-90 | 4741 | 81867 | 73863 | 2472 |
| Feb-90 | 4372 | 82201 | 76922 | 2531 |
| Mar-90 | 4651 | 82764 | 79785 | 2564 |
| Apr-90 | 4167 | 84473 | 82561 | 2585 |
| May-90 | 4579 | 86515 | 85283 | 2599 |
| Jun-90 | 4691 | 87302 | 87824 | 2593 |
| Jul-90 | 4734 | 87847 | 90161 | 2568 |
| Aug-90 | 5112 | 98370 | 93293 | 2624 |
| Sep-90 | 5120 | 106207 | 96946 | 2727 |
| Oct-90 | 5257 | 120497 | 101755 | 2935 |
| Nov-90 | 5123 | 127130 | 106934 | 3160 |
| Dec-90 | 4997 | 128660 | 111950 | 3345 |
| Jan-91 | 4822 | 130391 | 116805 | 3496 |
| Feb-91 | 4652 | 133194 | 121591 | 3625 |
| Mar-91 | 5110 | 138005 | 126495 | 3753 |
| Apr-91 | 4483 | 146531 | 131876 | 3916 |
| May-91 | 4922 | 154700 | 137683 | 4105 |
| Jun-91 | 4623 | 159381 | 143547 | 4281 |
| Jul-91 | 5316 | 169159 | 149961 | 4494 |
| Aug-91 | 5422 | 180411 | 157051 | 4754 |
| Sep-91 | 5172 | 183194 | 163944 | 4968 |
| Oct-91 | 5188 | 189641 | 170984 | 5175 |
| Nov-91 | 5184 | 197679 | 178311 | 5390 |
| Dec-91 | 5324 | 212499 | 186581 | 5678 |
| Jan-92 | 5608 | 239723 | 197006 | 6153 |
| Feb-92 | 5307 | 248546 | 207697 | 6607 |
| Mar-92 | 5650 | 249223 | 217796 | 6956 |
| Apr-92 | 4983 | 260962 | 228373 | 7318 |
| May-92 | 5316 | 267619 | 238883 | 7637 |
| Jun-92 | 5098 | 279731 | 249842 | 7969 |
| Jul-92 | 5808 | 296111 | 261641 | 8352 |
| Aug-92 | 6048 | 316405 | 274634 | 8816 |
| Sep-92 | 5772 | 329664 | 288072 | 9279 |
| Oct-92 | 5792 | 348103 | 302426 | 9786 |
| Nov-92 | 5825 | 358636 | 316854 | 10250 |
| Dec-92 | 6136 | 381556 | 332550 | 10795 |
| Jan-93 | 6204 | 386804 | 347691 | 11229 |
| Feb-93 | 5908 | 388297 | 361858 | 11523 |

**Table 15 Energy Price Index Forecasts for (1997-2005) (cont'd)**

| | | | | |
|---|---|---|---|---|
| Mar-93 | 5885 | 397161 | 375759 | 11761 |
| Apr-93 | 5706 | 398884 | 388656 | 11875 |
| May-93 | 5765 | 419650 | 402443 | 12066 |
| Jun-93 | 5652 | 452296 | 418287 | 12444 |
| Jul-93 | 6442 | 469812 | 434639 | 12835 |
| Aug-93 | 6492 | 508007 | 453527 | 13440 |
| Sep-93 | 6282 | 546685 | 474939 | 14237 |
| Oct-93 | 6271 | 574606 | 497719 | 15091 |
| Nov-93 | 6448 | 600722 | 521601 | 15970 |
| Dec-93 | 6751 | 621668 | 545981 | 16811 |
| Jan-94 | 6708 | 635427 | 570056 | 17538 |
| Feb-94 | 6244 | 670744 | 595909 | 18369 |
| Mar-94 | 6458 | 689726 | 621823 | 19124 |
| Apr-94 | 6086 | 1014200 | 678272 | 22856 |
| May-94 | 5962 | 1072137 | 738229 | 26566 |
| Jun-94 | 6154 | 1106116 | 798928 | 29980 |
| Jul-94 | 6613 | 1134518 | 859468 | 33036 |
| Aug-94 | 6824 | 1194022 | 922656 | 36051 |
| Sep-94 | 6706 | 1227814 | 985617 | 38742 |
| Oct-94 | 6604 | 1249585 | 1046882 | 40994 |
| Nov-94 | 6804 | 1287102 | 1107798 | 42986 |
| Dec-94 | 7159 | 1401231 | 1175830 | 45491 |
| Jan-95 | 7177 | 1457508 | 1244939 | 47853 |
| Feb-95 | 6695 | 1513204 | 1314833 | 50057 |
| Mar-95 | 6886 | 1573759 | 1385777 | 52146 |
| Apr-95 | 6780 | 1587532 | 1452883 | 53642 |
| May-95 | 6483 | 1658721 | 1521745 | 55164 |
| Jun-95 | 7009 | 1873688 | 1606586 | 58131 |
| Jul-95 | 7446 | 2110621 | 1709308 | 62590 |
| Aug-95 | 7701 | 2211191 | 1815828 | 66983 |
| Sep-95 | 7261 | 2280370 | 1922567 | 70959 |
| Oct-95 | 7278 | 2395258 | 2033699 | 74976 |
| Nov-95 | 7540 | 2553615 | 2153169 | 79426 |
| Dec-95 | 7993 | 2632954 | 2272631 | 83429 |
| Jan-96 | 8075 | 2828729 | 2403327 | 88156 |
| Feb-96 | 7301 | 2938592 | 2536194 | 92627 |
| Mar-96 | 7992 | 3138711 | 2679810 | 97726 |
| Apr-96 | 7287 | 3372588 | 2837041 | 103676 |
| May-96 | 7238 | 3430886 | 2989735 | 108578 |
| Jun-96 | 7537 | 3676049 | 3156087 | 114356 |
| Jul-96 | 8298 | 3848596 | 3328257 | 120137 |
| Aug-96 | 8414 | 4091093 | 3512664 | 126564 |
| Sep-96 | 7794 | 4337968 | 3709102 | 133551 |
| Oct-96 | 8094 | 4507153 | 3909104 | 140196 |
| Nov-96 | 8267 | 4648227 | 4109193 | 146186 |
| Dec-96 | 8564 | 4847048 | 4314546 | 152102 |
| Jan-97 | | | 4466648 | |

**Table 15 Energy Price Index Forecasts for (1997-2005) (cont'd)**

| | | | | |
|---|---|---|---|---|
| Feb-97 | | | 4618750 | |
| Mar-97 | | | 4770853 | |
| Apr-97 | | | 4922955 | |
| May-97 | | | 5075058 | |
| Jun-97 | | | 5227160 | |
| Jul-97 | | | 5379262 | |
| Aug-97 | | | 5531365 | |
| Sep-97 | | | 5683467 | |
| Oct-97 | | | 5835570 | |
| Nov-97 | | | 5987672 | |
| Dec-97 | | | 6139775 | |
| Jan-98 | | | 6291877 | |
| Feb-98 | | | 6443979 | |
| Mar-98 | | | 6596082 | |
| Apr-98 | | | 6748184 | |
| May-98 | | | 6900287 | |
| Jun-98 | | | 7052389 | |
| Jul-98 | | | 7204491 | |
| Aug-98 | | | 7356594 | |
| Sep-98 | | | 7508696 | |
| Oct-98 | | | 7660799 | |
| Nov-98 | | | 7812901 | |
| Dec-98 | | | 7965003 | |
| Jan-99 | | | 8117106 | |
| Feb-99 | | | 8269208 | |
| Mar-99 | | | 8421311 | |
| Apr-99 | | | 8573413 | |
| May-99 | | | 8725516 | |
| Jun-99 | | | 8877618 | |
| Jul-99 | | | 9029720 | |
| Aug-99 | | | 9181823 | |
| Sep-99 | | | 9333925 | |
| Oct-99 | | | 9486028 | |
| Nov-99 | | | 9638130 | |
| Dec-99 | | | 9790232 | |
| Jan-00 | | | 9942335 | |
| Feb-00 | | | 10094437 | |
| Mar-00 | | | 10246540 | |
| Apr-00 | | | 10398642 | |
| May-00 | | | 10550744 | |
| Jun-00 | | | 10702847 | |
| Jul-00 | | | 10854949 | |
| Aug-00 | | | 11007052 | |
| Sep-00 | | | 11159154 | |
| Oct-00 | | | 11311257 | |
| Nov-00 | | | 11463359 | |
| Dec-00 | | | 11615461 | |

**Table 15 Energy Price Index Forecasts for (1997-2005) (cont'd)**

| | | | | |
|---|---|---|---|---|
| Jan-01 | | | 11767564 | |
| Feb-01 | | | 11919666 | |
| Mar-01 | | | 12071769 | |
| Apr-01 | | | 12223871 | |
| May-01 | | | 12375973 | |
| Jun-01 | | | 12528076 | |
| Jul-01 | | | 12680178 | |
| Aug-01 | | | 12832281 | |
| Sep-01 | | | 12984383 | |
| Oct-01 | | | 13136486 | |
| Nov-01 | | | 13288588 | |
| Dec-01 | | | 13440690 | |
| Jan-02 | | | 13592793 | |
| Feb-02 | | | 13744895 | |
| Mar-02 | | | 13896998 | |
| Apr-02 | | | 14049100 | |
| May-02 | | | 14201202 | |
| Jun-02 | | | 14353305 | |
| Jul-02 | | | 14505407 | |
| Aug-02 | | | 14657510 | |
| Sep-02 | | | 14809612 | |
| Oct-02 | | | 14961714 | |
| Nov-02 | | | 15113817 | |
| Dec-02 | | | 15265919 | |
| Jan-03 | | | 15418022 | |
| Feb-03 | | | 15570124 | |
| Mar-03 | | | 15722227 | |
| Apr-03 | | | 15874329 | |
| May-03 | | | 16026431 | |
| Jun-03 | | | 16178534 | |
| Jul-03 | | | 16330636 | |
| Aug-03 | | | 16482739 | |
| Sep-03 | | | 16634841 | |
| Oct-03 | | | 16786943 | |
| Nov-03 | | | 16939046 | |
| Dec-03 | | | 17091148 | |
| Jan-04 | | | 17243251 | |
| Feb-04 | | | 17395353 | |
| Mar-04 | | | 17547455 | |
| Apr-04 | | | 17699558 | |
| May-04 | | | 17851660 | |
| Jun-04 | | | 18003763 | |
| Jul-04 | | | 18155865 | |
| Aug-04 | | | 18307968 | |
| Sep-04 | | | 18460070 | |
| Oct-04 | | | 18612172 | |
| Nov-04 | | | 18764275 | |

**Table 15 Energy Price Index Forecasts for (1997-2005) (cont'd)**

| | | | | |
|---|---|---|---|---|
| Dec-04 | | | 18916377 | |
| Jan-05 | | | 19068480 | |
| Feb-05 | | | 19220582 | |
| Mar-05 | | | 19372684 | |
| Apr-05 | | | 19524787 | |
| May-05 | | | 19676889 | |
| Jun-05 | | | 19828992 | |
| Jul-05 | | | 19981094 | |
| Aug-05 | | | 20133196 | |
| Sep-05 | | | 20285299 | |
| Oct-05 | | | 20437401 | |
| Nov-05 | | | 20589504 | |
| Dec-05 | | | 20741606 | |

**Table 16 Energy Price Index Forecasts for (2001-2005)**

| S | 18646.8 | | PRICE FORECASTING (2001-2005) | |
|---|---|---|---|---|
| G | 10 | | | |
| Date | Monthly Energy Demand (Et) | price (alpha &beta 0.1) | S-price | G-price |
| Jan-87 | 3581 | 18647 | 18656 | 10 |
| Feb-87 | 3349 | 19023 | 18701 | 13 |
| Mar-87 | 3522 | 19057 | 18749 | 17 |
| Apr-87 | 3485 | 19079 | 18797 | 20 |
| May-87 | 3518 | 19079 | 18843 | 23 |
| Jun-87 | 3597 | 19079 | 18887 | 25 |
| Jul-87 | 3884 | 19606 | 18981 | 32 |
| Aug-87 | 3714 | 19949 | 19107 | 41 |
| Sep-87 | 3848 | 20854 | 19318 | 58 |
| Oct-87 | 3941 | 21495 | 19588 | 79 |
| Nov-87 | 3878 | 21889 | 19890 | 102 |
| Dec-87 | 4035 | 24648 | 20457 | 148 |
| Jan-88 | 3981 | 26247 | 21169 | 205 |
| Feb-88 | 3790 | 29169 | 22153 | 282 |
| Mar-88 | 3958 | 30776 | 23270 | 366 |
| Apr-88 | 3800 | 31964 | 24468 | 449 |
| May-88 | 3752 | 32365 | 25662 | 524 |
| Jun-88 | 3858 | 34087 | 26976 | 603 |
| Jul-88 | 4006 | 36011 | 28422 | 687 |
| Aug-88 | 4268 | 38002 | 29998 | 776 |
| Sep-88 | 4137 | 38877 | 31584 | 857 |
| Oct-88 | 4172 | 41026 | 33300 | 943 |
| Nov-88 | 4087 | 43983 | 35216 | 1040 |
| Dec-88 | 4241 | 49017 | 37533 | 1168 |
| Jan-89 | 4193 | 52787 | 40109 | 1309 |
| Feb-89 | 3896 | 53736 | 42649 | 1432 |
| Mar-89 | 4173 | 53223 | 44995 | 1523 |
| Apr-89 | 4136 | 55480 | 47415 | 1613 |
| May-89 | 4122 | 58894 | 50014 | 1711 |
| Jun-89 | 4285 | 59579 | 52511 | 1790 |
| Jul-89 | 4388 | 64614 | 55332 | 1893 |
| Aug-89 | 4661 | 66740 | 58177 | 1988 |
| Sep-89 | 4493 | 68541 | 61003 | 2072 |
| Oct-89 | 4552 | 69755 | 63743 | 2139 |
| Nov-89 | 4525 | 78192 | 67113 | 2262 |
| Dec-89 | 4620 | 81532 | 70591 | 2384 |
| Jan-90 | 4741 | 81867 | 73863 | 2472 |
| Feb-90 | 4372 | 82201 | 76922 | 2531 |
| Mar-90 | 4651 | 82764 | 79785 | 2564 |
| Apr-90 | 4167 | 84473 | 82561 | 2585 |
| May-90 | 4579 | 86515 | 85283 | 2599 |

**Table 16 Energy Price Index Forecasts for (2001-2005) (cont'd)**

| | | | | |
|---|---|---|---|---|
| Jun-90 | 4691 | 87302 | 87824 | 2593 |
| Jul-90 | 4734 | 87847 | 90161 | 2568 |
| Aug-90 | 5112 | 98370 | 93293 | 2624 |
| Sep-90 | 5120 | 106207 | 96946 | 2727 |
| Oct-90 | 5257 | 120497 | 101755 | 2935 |
| Nov-90 | 5123 | 127130 | 106934 | 3160 |
| Dec-90 | 4997 | 128660 | 111950 | 3345 |
| Jan-91 | 4822 | 130391 | 116805 | 3496 |
| Feb-91 | 4652 | 133194 | 121591 | 3625 |
| Mar-91 | 5110 | 138005 | 126495 | 3753 |
| Apr-91 | 4483 | 146531 | 131876 | 3916 |
| May-91 | 4922 | 154700 | 137683 | 4105 |
| Jun-91 | 4623 | 159381 | 143547 | 4281 |
| Jul-91 | 5316 | 169159 | 149961 | 4494 |
| Aug-91 | 5422 | 180411 | 157051 | 4754 |
| Sep-91 | 5172 | 183194 | 163944 | 4968 |
| Oct-91 | 5188 | 189641 | 170984 | 5175 |
| Nov-91 | 5184 | 197679 | 178311 | 5390 |
| Dec-91 | 5324 | 212499 | 186581 | 5678 |
| Jan-92 | 5608 | 239723 | 197006 | 6153 |
| Feb-92 | 5307 | 248546 | 207697 | 6607 |
| Mar-92 | 5650 | 249223 | 217796 | 6956 |
| Apr-92 | 4983 | 260962 | 228373 | 7318 |
| May-92 | 5316 | 267619 | 238883 | 7637 |
| Jun-92 | 5098 | 279731 | 249842 | 7969 |
| Jul-92 | 5808 | 296111 | 261641 | 8352 |
| Aug-92 | 6048 | 316405 | 274634 | 8816 |
| Sep-92 | 5772 | 329664 | 288072 | 9279 |
| Oct-92 | 5792 | 348103 | 302426 | 9786 |
| Nov-92 | 5825 | 358636 | 316854 | 10250 |
| Dec-92 | 6136 | 381556 | 332550 | 10795 |
| Jan-93 | 6204 | 386804 | 347691 | 11229 |
| Feb-93 | 5908 | 388297 | 361858 | 11523 |
| Mar-93 | 5885 | 397161 | 375759 | 11761 |
| Apr-93 | 5706 | 398884 | 388656 | 11875 |
| May-93 | 5765 | 419650 | 402443 | 12066 |
| Jun-93 | 5652 | 452296 | 418287 | 12444 |
| Jul-93 | 6442 | 469812 | 434639 | 12835 |
| Aug-93 | 6492 | 508007 | 453527 | 13440 |
| Sep-93 | 6282 | 546685 | 474939 | 14237 |
| Oct-93 | 6271 | 574606 | 497719 | 15091 |
| Nov-93 | 6448 | 600722 | 521601 | 15970 |
| Dec-93 | 6751 | 621668 | 545981 | 16811 |
| Jan-94 | 6708 | 635427 | 570056 | 17538 |
| Feb-94 | 6244 | 670744 | 595909 | 18369 |
| Mar-94 | 6458 | 689726 | 621823 | 19124 |
| Apr-94 | 6086 | 1014200 | 678272 | 22856 |

**Table 16 Energy Price Index Forecasts for (2001-2005) (cont'd)**

| | | | | |
|---|---|---|---|---|
| May-94 | 5962 | 1072137 | 738229 | 26566 |
| Jun-94 | 6154 | 1106116 | 798928 | 29980 |
| Jul-94 | 6613 | 1134518 | 859468 | 33036 |
| Aug-94 | 6824 | 1194022 | 922656 | 36051 |
| Sep-94 | 6706 | 1227814 | 985617 | 38742 |
| Oct-94 | 6604 | 1249585 | 1046882 | 40994 |
| Nov-94 | 6804 | 1287102 | 1107798 | 42986 |
| Dec-94 | 7159 | 1401231 | 1175830 | 45491 |
| Jan-95 | 7177 | 1457508 | 1244939 | 47853 |
| Feb-95 | 6695 | 1513204 | 1314833 | 50057 |
| Mar-95 | 6886 | 1573759 | 1385777 | 52146 |
| Apr-95 | 6780 | 1587532 | 1452883 | 53642 |
| May-95 | 6483 | 1658721 | 1521745 | 55164 |
| Jun-95 | 7009 | 1873688 | 1606586 | 58131 |
| Jul-95 | 7446 | 2110621 | 1709308 | 62590 |
| Aug-95 | 7701 | 2211191 | 1815828 | 66983 |
| Sep-95 | 7261 | 2280370 | 1922567 | 70959 |
| Oct-95 | 7278 | 2395258 | 2033699 | 74976 |
| Nov-95 | 7540 | 2553615 | 2153169 | 79426 |
| Dec-95 | 7993 | 2632954 | 2272631 | 83429 |
| Jan-96 | 8075 | 2828729 | 2403327 | 88156 |
| Feb-96 | 7301 | 2938592 | 2536194 | 92627 |
| Mar-96 | 7992 | 3138711 | 2679810 | 97726 |
| Apr-96 | 7287 | 3372588 | 2837041 | 103676 |
| May-96 | 7238 | 3430886 | 2989735 | 108578 |
| Jun-96 | 7537 | 3676049 | 3156087 | 114356 |
| Jul-96 | 8298 | 3848596 | 3328257 | 120137 |
| Aug-96 | 8414 | 4091093 | 3512664 | 126564 |
| Sep-96 | 7794 | 4337968 | 3709102 | 133551 |
| Oct-96 | 8094 | 4507153 | 3909104 | 140196 |
| Nov-96 | 8267 | 4648227 | 4109193 | 146186 |
| Dec-96 | 8564 | 4847048 | 4314546 | 152102 |
| Jan-97 | 8793 | 5032727 | 4523256 | 157763 |
| Feb-97 | 7940 | 5313765 | 4744294 | 164091 |
| Mar-97 | 8960 | 5499337 | 4967480 | 170000 |
| Apr-97 | 8176 | 5546990 | 5178431 | 174095 |
| May-97 | 8157 | 5782123 | 5395486 | 178391 |
| Jun-97 | 8069 | 6144709 | 5630960 | 184100 |
| Jul-97 | 8918 | 6672635 | 5900817 | 192675 |
| Aug-97 | 8831 | 7278776 | 6212021 | 204528 |
| Sep-97 | 8445 | 7717210 | 6546615 | 217535 |
| Oct-97 | 8643 | 8187444 | 6906479 | 231768 |
| Nov-97 | 8846 | 8746541 | 7299077 | 247851 |
| Dec-97 | 9517 | 9153756 | 7707610 | 263919 |
| Jan-98 | 9616 | 9231161 | 8097492 | 276515 |
| Feb-98 | 8999 | 9523232 | 8488930 | 288008 |
| Mar-98 | 9813 | 9606680 | 8859912 | 296305 |

**Table 16 Energy Price Index Forecasts for (2001-2005) (cont'd)**

| | | | | |
|---|---|---|---|---|
| Apr-98 | 8223 | 9703856 | 9210981 | 301781 |
| May-98 | 8696 | 9941670 | 9555653 | 306070 |
| Jun-98 | 8773 | 10840301 | 9959581 | 315856 |
| Jul-98 | 9620 | 11695542 | 10417448 | 330057 |
| Aug-98 | 9783 | 11901956 | 10862950 | 341602 |
| Sep-98 | 9116 | 12662176 | 11350314 | 356178 |
| Oct-98 | 8972 | 12793175 | 11815160 | 367045 |
| Nov-98 | 9328 | 12932764 | 12257261 | 374550 |
| Dec-98 | 10084 | 13086676 | 12677298 | 379099 |
| Jan-99 | 9846 | 13951996 | 13145957 | 388055 |
| Feb-99 | 9421 | 14198561 | 13600467 | 394701 |
| Mar-99 | 9712 | 15617458 | 14157396 | 410923 |
| Apr-99 | 9090 | 17058088 | 14817297 | 435821 |
| May-99 | 9262 | 18209765 | 15548782 | 465388 |
| Jun-99 | 9254 | 19450750 | 16357828 | 499753 |
| Jul-99 | 10137 | 21126490 | 17284472 | 542442 |
| Aug-99 | 9934 | 22908958 | 18335119 | 593263 |
| Sep-99 | 9312 | 24671602 | 19502704 | 650695 |
| Oct-99 | 9528 | 26061616 | 20744221 | 709777 |
| Nov-99 | 10171 | 27618580 | 22070456 | 771423 |
| Dec-99 | 10772 | 29203126 | 23478004 | 835036 |
| Jan-00 | 10980 | 30381229 | 24919858 | 895717 |
| Feb-00 | 10723 | 31459466 | 26379965 | 952156 |
| Mar-00 | 10695 | 31965047 | 27795414 | 998486 |
| Apr-00 | 9495 | 32336971 | 29148207 | 1033916 |
| May-00 | 9729 | 32639263 | 30427837 | 1058488 |
| Jun-00 | 9867 | 33034774 | 31641170 | 1073972 |
| Jul-00 | 10856 | 33229771 | 32766605 | 1079119 |
| Aug-00 | 10815 | 33488858 | 33810037 | 1075550 |
| Sep-00 | 10093 | 34983024 | 34895330 | 1076524 |
| Oct-00 | 10220 | 35752329 | 35949902 | 1074329 |
| Nov-00 | 10555 | 36198897 | 36941698 | 1066076 |
| Dec-00 | 10894 | 37190100 | 37926006 | 1057899 |
| Jan-01 | | | 38983905 | |
| Feb-01 | | | 40041804 | |
| Mar-01 | | | 41099703 | |
| Apr-01 | | | 42157601 | |
| May-01 | | | 43215500 | |
| Jun-01 | | | 44273399 | |
| Jul-01 | | | 45331298 | |
| Aug-01 | | | 46389197 | |
| Sep-01 | | | 47447096 | |
| Oct-01 | | | 48504995 | |
| Nov-01 | | | 49562894 | |
| Dec-01 | | | 50620793 | |
| Jan-02 | | | 51678692 | |
| Feb-02 | | | 52736590 | |

**Table 16 Energy Price Index Forecasts for (2001-2005) (cont'd)**

| | | | | |
|---|---|---|---|---|
| Mar-02 | | | 53794489 | |
| Apr-02 | | | 54852388 | |
| May-02 | | | 55910287 | |
| Jun-02 | | | 56968186 | |
| Jul-02 | | | 58026085 | |
| Aug-02 | | | 59083984 | |
| Sep-02 | | | 60141883 | |
| Oct-02 | | | 61199782 | |
| Nov-02 | | | 62257681 | |
| Dec-02 | | | 63315580 | |
| Jan-03 | | | 64373478 | |
| Feb-03 | | | 65431377 | |
| Mar-03 | | | 66489276 | |
| Apr-03 | | | 67547175 | |
| May-03 | | | 68605074 | |
| Jun-03 | | | 69662973 | |
| Jul-03 | | | 70720872 | |
| Aug-03 | | | 71778771 | |
| Sep-03 | | | 72836670 | |
| Oct-03 | | | 73894569 | |
| Nov-03 | | | 74952467 | |
| Dec-03 | | | 76010366 | |
| Jan-04 | | | 77068265 | |
| Feb-04 | | | 78126164 | |
| Mar-04 | | | 79184063 | |
| Apr-04 | | | 80241962 | |
| May-04 | | | 81299861 | |
| Jun-04 | | | 82357760 | |
| Jul-04 | | | 83415659 | |
| Aug-04 | | | 84473558 | |
| Sep-04 | | | 85531456 | |
| Oct-04 | | | 86589355 | |
| Nov-04 | | | 87647254 | |
| Dec-04 | | | 88705153 | |
| Jan-05 | | | 89763052 | |
| Feb-05 | | | 90820951 | |
| Mar-05 | | | 91878850 | |
| Apr-05 | | | 92936749 | |
| May-05 | | | 93994648 | |
| Jun-05 | | | 95052547 | |
| Jul-05 | | | 96110446 | |
| Aug-05 | | | 97168344 | |
| Sep-05 | | | 98226243 | |
| Oct-05 | | | 99284142 | |
| Nov-05 | | | 100342041 | |
| Dec-05 | | | 101399940 | |

**Table 17 Calculated GDP Projections by Winter's Method**

| Date | Forecasted GDP (1997-2005) | Forecasted GDP (2001-2005) | Realized GDP |
|---|---|---|---|
| Jan-87 | 4859 | 4859 | 4859 |
| Feb-87 | 4389 | 4389 | 4389 |
| Mar-87 | 4859 | 4859 | 4859 |
| Apr-87 | 5427 | 5427 | 5427 |
| May-87 | 5608 | 5608 | 5608 |
| Jun-87 | 5427 | 5427 | 5427 |
| Jul-87 | 8070 | 8070 | 8070 |
| Aug-87 | 8070 | 8070 | 8070 |
| Sep-87 | 7810 | 7810 | 7810 |
| Oct-87 | 6704 | 6704 | 6704 |
| Nov-87 | 6487 | 6487 | 6487 |
| Dec-87 | 6704 | 6704 | 6704 |
| Jan-88 | 5291 | 5291 | 5291 |
| Feb-88 | 4950 | 4950 | 4950 |
| Mar-88 | 5291 | 5291 | 5291 |
| Apr-88 | 5660 | 5660 | 5660 |
| May-88 | 5849 | 5849 | 5849 |
| Jun-88 | 5660 | 5660 | 5660 |
| Jul-88 | 8192 | 8192 | 8192 |
| Aug-88 | 8192 | 8192 | 8192 |
| Sep-88 | 7928 | 7928 | 7928 |
| Oct-88 | 6446 | 6446 | 6446 |
| Nov-88 | 6238 | 6238 | 6238 |
| Dec-88 | 6446 | 6446 | 6446 |
| Jan-89 | 5161 | 5161 | 5161 |
| Feb-89 | 4661 | 4661 | 4661 |
| Mar-89 | 5161 | 5161 | 5161 |
| Apr-89 | 5542 | 5542 | 5542 |
| May-89 | 5726 | 5726 | 5726 |
| Jun-89 | 5542 | 5542 | 5542 |
| Jul-89 | 8363 | 8363 | 8363 |
| Aug-89 | 8363 | 8363 | 8363 |
| Sep-89 | 8093 | 8093 | 8093 |
| Oct-89 | 6656 | 6656 | 6656 |
| Nov-89 | 6441 | 6441 | 6441 |
| Dec-89 | 6656 | 6656 | 6656 |
| Jan-90 | 5788 | 5788 | 5788 |
| Feb-90 | 5228 | 5228 | 5228 |
| Mar-90 | 5788 | 5788 | 5788 |
| Apr-90 | 6330 | 6330 | 6330 |
| May-90 | 6541 | 6541 | 6541 |
| Jun-90 | 6330 | 6330 | 6330 |

**Table 16 Calculated GDP Projections by Winter's Method (Cont'd)**

| | | | |
|---|---|---|---|
| Jul-90 | 8643 | 8643 | 8643 |
| Aug-90 | 8643 | 8643 | 8643 |
| Sep-90 | 8364 | 8364 | 8364 |
| Oct-90 | 7340 | 7340 | 7340 |
| Nov-90 | 7104 | 7104 | 7104 |
| Dec-90 | 7340 | 7340 | 7340 |
| Jan-91 | 5735 | 5735 | 5735 |
| Feb-91 | 5180 | 5180 | 5180 |
| Mar-91 | 5735 | 5735 | 5735 |
| Apr-91 | 6285 | 6285 | 6285 |
| May-91 | 6494 | 6494 | 6494 |
| Jun-91 | 6285 | 6285 | 6285 |
| Jul-91 | 9044 | 9044 | 9044 |
| Aug-91 | 9044 | 9044 | 9044 |
| Sep-91 | 8752 | 8752 | 8752 |
| Oct-91 | 7240 | 7240 | 7240 |
| Nov-91 | 7006 | 7006 | 7006 |
| Dec-91 | 7240 | 7240 | 7240 |
| Jan-92 | 6081 | 6081 | 6081 |
| Feb-92 | 5689 | 5689 | 5689 |
| Mar-92 | 6081 | 6081 | 6081 |
| Apr-92 | 6579 | 6579 | 6579 |
| May-92 | 6799 | 6799 | 6799 |
| Jun-92 | 6579 | 6579 | 6579 |
| Jul-92 | 9428 | 9428 | 9428 |
| Aug-92 | 9428 | 9428 | 9428 |
| Sep-92 | 9124 | 9124 | 9124 |
| Oct-92 | 7576 | 7576 | 7576 |
| Nov-92 | 7332 | 7332 | 7332 |
| Dec-92 | 7576 | 7576 | 7576 |
| Jan-93 | 6494 | 6494 | 6494 |
| Feb-93 | 5865 | 5865 | 5865 |
| Mar-93 | 6494 | 6494 | 6494 |
| Apr-93 | 7274 | 7274 | 7274 |
| May-93 | 7517 | 7517 | 7517 |
| Jun-93 | 7274 | 7274 | 7274 |
| Jul-93 | 9922 | 9922 | 9922 |
| Aug-93 | 9922 | 9922 | 9922 |
| Sep-93 | 9602 | 9602 | 9602 |
| Oct-93 | 8311 | 8311 | 8311 |
| Nov-93 | 8043 | 8043 | 8043 |
| Dec-93 | 8311 | 8311 | 8311 |
| Jan-94 | 6773 | 6773 | 6773 |
| Feb-94 | 6117 | 6117 | 6117 |
| Mar-94 | 6773 | 6773 | 6773 |
| Apr-94 | 6577 | 6577 | 6577 |
| May-94 | 6797 | 6797 | 6797 |

**Table 17 Calculated GDP Projections by Winter's Method (Cont'd)**

| | | | |
|---|---|---|---|
| Jun-94 | 6577 | 6577 | 6577 |
| Jul-94 | 9313 | 9313 | 9313 |
| Aug-94 | 9313 | 9313 | 9313 |
| Sep-94 | 9012 | 9012 | 9012 |
| Oct-94 | 7864 | 7864 | 7864 |
| Nov-94 | 7610 | 7610 | 7610 |
| Dec-94 | 7864 | 7864 | 7864 |
| Jan-95 | 6800 | 6800 | 6800 |
| Feb-95 | 6142 | 6142 | 6142 |
| Mar-95 | 6800 | 6800 | 6800 |
| Apr-95 | 7573 | 7573 | 7573 |
| May-95 | 7825 | 7825 | 7825 |
| Jun-95 | 7573 | 7573 | 7573 |
| Jul-95 | 10108 | 10108 | 10108 |
| Aug-95 | 10108 | 10108 | 10108 |
| Sep-95 | 9782 | 9782 | 9782 |
| Oct-95 | 8429 | 8429 | 8429 |
| Nov-95 | 8157 | 8157 | 8157 |
| Dec-95 | 8429 | 8429 | 8429 |
| Jan-96 | 7324 | 7324 | 7324 |
| Feb-96 | 6851 | 6851 | 6851 |
| Mar-96 | 7324 | 7324 | 7324 |
| Apr-96 | 8141 | 8141 | 8141 |
| May-96 | 8412 | 8412 | 8412 |
| Jun-96 | 8141 | 8141 | 8141 |
| Jul-96 | 10674 | 10674 | 10674 |
| Aug-96 | 10674 | 10674 | 10674 |
| Sep-96 | 10330 | 10330 | 10330 |
| Oct-96 | 9121 | 9121 | 9121 |
| Nov-96 | 8826 | 8826 | 8826 |
| Dec-96 | 9121 | 9121 | 9121 |
| Jan-97 | 7438 | 7907 | 7907 |
| Feb-97 | 6790 | 7142 | 7142 |
| Mar-97 | 7395 | 7907 | 7907 |
| Apr-97 | 8027 | 8827 | 8827 |
| May-97 | 8287 | 9122 | 9122 |
| Jun-97 | 8017 | 8827 | 8827 |
| Jul-97 | 11450 | 11564 | 11564 |
| Aug-97 | 11517 | 11564 | 11564 |
| Sep-97 | 11211 | 11191 | 11191 |
| Oct-97 | 9542 | 9718 | 9718 |
| Nov-97 | 9266 | 9405 | 9405 |
| Dec-97 | 9608 | 9718 | 9718 |
| Jan-98 | 7748 | 8651 | 8651 |
| Feb-98 | 7072 | 7814 | 7814 |
| Mar-98 | 7701 | 8651 | 8651 |
| Apr-98 | 8358 | 9136 | 9136 |

**Table 17 Calculated GDP Projections by Winter's Method (Cont'd)**

| | | | |
|---|---|---|---|
| May-98 | 8627 | 9441 | 9441 |
| Jun-98 | 8345 | 9136 | 9136 |
| Jul-98 | 11917 | 11848 | 11848 |
| Aug-98 | 11985 | 11848 | 11848 |
| Sep-98 | 11665 | 11465 | 11465 |
| Oct-98 | 9927 | 9621 | 9621 |
| Nov-98 | 9638 | 9311 | 9311 |
| Dec-98 | 9993 | 9621 | 9621 |
| Jan-99 | 8057 | 7945 | 7945 |
| Feb-99 | 7353 | 7177 | 7177 |
| Mar-99 | 8006 | 7945 | 7945 |
| Apr-99 | 8689 | 8908 | 8908 |
| May-99 | 8967 | 9205 | 9205 |
| Jun-99 | 8673 | 8908 | 8908 |
| Jul-99 | 12383 | 11172 | 11172 |
| Aug-99 | 12453 | 11172 | 11172 |
| Sep-99 | 12119 | 10812 | 10812 |
| Oct-99 | 10312 | 9380 | 9380 |
| Nov-99 | 10011 | 9078 | 9078 |
| Dec-99 | 10378 | 9380 | 9380 |
| Jan-00 | 8367 | 8290 | 8290 |
| Feb-00 | 7635 | 7756 | 7756 |
| Mar-00 | 8312 | 8290 | 8290 |
| Apr-00 | 9019 | 9512 | 9512 |
| May-00 | 9307 | 9829 | 9829 |
| Jun-00 | 9001 | 9512 | 9512 |
| Jul-00 | 12850 | 12053 | 12053 |
| Aug-00 | 12921 | 12053 | 12053 |
| Sep-00 | 12572 | 11664 | 11664 |
| Oct-00 | 10697 | 10173 | 10173 |
| Nov-00 | 10383 | 9844 | 9844 |
| Dec-00 | 10763 | 10173 | 10173 |
| Jan-01 | 8676 | 8474 | 8300 |
| Feb-01 | 7916 | 7708 | 7497 |
| Mar-01 | 8617 | 8381 | 8300 |
| Apr-01 | 9350 | 9130 | 8579 |
| May-01 | 9647 | 9382 | 8865 |
| Jun-01 | 9329 | 9038 | 8579 |
| Jul-01 | 13317 | 12466 | 11145 |
| Aug-01 | 13389 | 12546 | 11145 |
| Sep-01 | 13026 | 12220 | 10786 |
| Oct-01 | 11082 | 10462 | 9121 |
| Nov-01 | 10756 | 10161 | 8827 |
| Dec-01 | 11148 | 10538 | 9121 |
| Jan-02 | 8986 | 8684 | 8487 |
| Feb-02 | 8198 | 7899 | 7666 |
| Mar-02 | 8923 | 8589 | 8487 |

**Table 17 Calculated GDP Projections by Winter's Method (Cont'd)**

| | | | |
|---|---|---|---|
| Apr-02 | 9680 | 9355 | 9339 |
| May-02 | 9987 | 9613 | 9650 |
| Jun-02 | 9656 | 9260 | 9339 |
| Jul-02 | 13784 | 12772 | 12040 |
| Aug-02 | 13856 | 12854 | 12040 |
| Sep-02 | 13480 | 12519 | 11652 |
| Oct-02 | 11467 | 10717 | 10184 |
| Nov-02 | 11129 | 10409 | 9855 |
| Dec-02 | 11534 | 10794 | 10184 |
| Jan-03 | 9295 | 8895 | 9170 |
| Feb-03 | 8480 | 8090 | 8283 |
| Mar-03 | 9228 | 8796 | 9170 |
| Apr-03 | 10011 | 9580 | 9699 |
| May-03 | 10327 | 9845 | 10022 |
| Jun-03 | 9984 | 9482 | 9699 |
| Jul-03 | 14250 | 13078 | 12696 |
| Aug-03 | 14324 | 13161 | 12696 |
| Sep-03 | 13934 | 12818 | 12287 |
| Oct-03 | 11852 | 10972 | 10801 |
| Nov-03 | 11501 | 10656 | 10453 |
| Dec-03 | 11919 | 11050 | 10801 |
| Jan-04 | 9605 | 9105 | 10135 |
| Feb-04 | 8761 | 8281 | 9481 |
| Mar-04 | 9534 | 9003 | 10135 |
| Apr-04 | 10341 | 9806 | 11091 |
| May-04 | 10667 | 10076 | 11460 |
| Jun-04 | 10312 | 9704 | 11091 |
| Jul-04 | 14717 | 13383 | 13363 |
| Aug-04 | 14792 | 13468 | 13363 |
| Sep-04 | 14388 | 13116 | 12932 |
| Oct-04 | 12237 | 11227 | 11476 |
| Nov-04 | 11874 | 10903 | 11106 |
| Dec-04 | 12304 | 11306 | 11476 |
| Jan-05 | 9914 | 9316 | 10926 |
| Feb-05 | 9043 | 8472 | 9869 |
| Mar-05 | 9840 | 9211 | 10926 |
| Apr-05 | 10672 | 10031 | 11706 |
| May-05 | 11007 | 10307 | 12096 |
| Jun-05 | 10640 | 9927 | 11706 |
| Jul-05 | 15184 | 13689 | 14386 |
| Aug-05 | 15260 | 13775 | 14386 |
| Sep-05 | 14842 | 13415 | 13922 |
| Oct-05 | 12622 | 11483 | 12561 |
| Nov-05 | 12246 | 11151 | 12156 |
| Dec-05 | 12689 | 11562 | 12561 |

# APPENDIX E

# NORMALITY ASSUMPTION FOR ERROR FUNCTION OF PROPOSED FLN MODEL

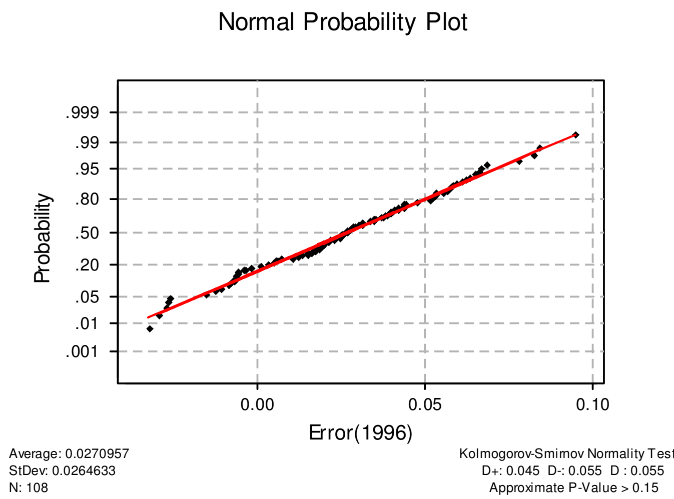


**Figure 26 Analysis of Error Function of Validation Dataset (1997-2005)**

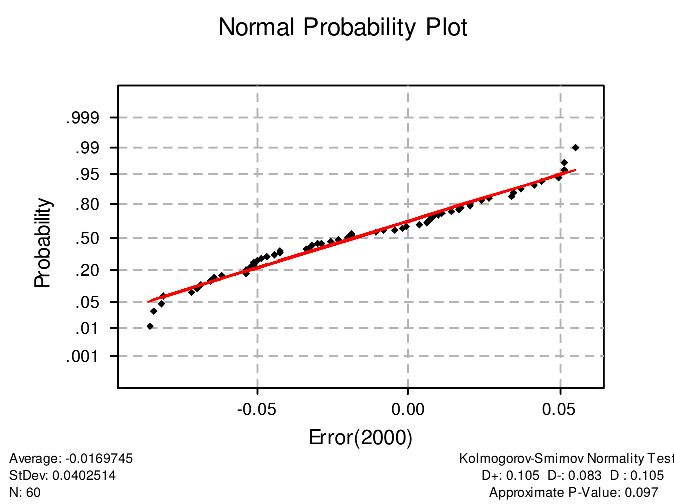


**Figure 27 Analysis of Error Function of Validation Dataset (2001-2005)**

# APPENDIX F

# REGRESSION ANALSIS FOR REGRESSION MODEL 1

Regression Analysis for the proposed model after 2000:

**Regression Analysis: Monthly Energy versus GDP (logd); price (logd); ...**

```
Weighted analysis using weights in wt

The regression equation is
Monthly Energy Demand (Et) = 0.0099 + 0.0699 GDP (logd) + 0.240 price
(logd) P
           - 46 GDP_sq4 + 2.2 Price_SQ3 + 0.146 Sinpi_sq - 0.30 SinpiP_sq
           + 0.320 Sinpi_3 - 0.0036 St-2

Predictor        Coef     SE Coef          T        P
Constant      0.00991     0.08255       0.12    0.905
GDP (log      0.06987     0.09010       0.78    0.439
price (l       0.2400      0.6310       0.38    0.704
GDP_sq4         -46.3       101.5      -0.46    0.649
Price_SQ         2.18       73.37       0.03    0.976
Sinpi_sq       0.1456      0.1124       1.30    0.197
SinpiP_s       -0.300       1.741      -0.17    0.863
Sinpi_3        0.3202      0.5205       0.62    0.539
St-2         -0.00356     0.02095      -0.17    0.865


S = 0.01284     R-Sq = 16.8%     R-Sq(adj) = 12.6%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         8   0.0052422   0.0006553      3.97    0.000
Residual Error   157   0.0258877   0.0001649
Total            165   0.0311298

No replicates. Cannot do pure error test.
Source       DF      Seq SS
GDP (log      1   0.0036422
price (l      1   0.0000041
GDP_sq4       1   0.0009331
Price_SQ      1   0.0003579
Sinpi_sq      1   0.0002334
SinpiP_s      1   0.0000077
Sinpi_3       1   0.0000590
St-2          1   0.0000048

Unusual Observations
```

| Obs | GDP (log | Monthly | Fit | SE Fit | Residual | St Resid |
|---|---|---|---|---|---|---|
| 29 | 0.179 | 0.010000 | 0.056681 | 0.036193 | -0.046681 | -0.72 X |
| 83 | -0.089 | 0.000000 | -0.006291 | 0.009837 | 0.006291 | 0.30 X |
| 86 | -0.013 | -0.030000 | -0.029614 | 0.023442 | -0.000386 | -1.12 X |
| 89 | 0.151 | 0.030000 | 0.046085 | 0.013346 | -0.016085 | -0.83 X |
| 100 | -0.014 | 0.030000 | 0.000328 | 0.010608 | 0.029672 | 1.71 X |
| 101 | 0.125 | 0.030000 | 0.037930 | 0.010487 | -0.007930 | -0.46 X |
| 108 | -0.029 | -0.040000 | -0.001889 | 0.003117 | -0.038111 | -2.13R |
| 109 | 0.029 | 0.040000 | 0.003930 | 0.003708 | 0.036070 | 2.03R |
| 110 | 0.046 | -0.040000 | 0.008000 | 0.004452 | -0.048000 | -2.73R |
| 113 | 0.118 | 0.040000 | 0.033302 | 0.007445 | 0.006698 | 0.40 X |
| 120 | -0.044 | -0.040000 | -0.001207 | 0.003629 | -0.038793 | -2.29R |
| 121 | 0.044 | 0.050000 | 0.005514 | 0.004024 | 0.044486 | 2.64R |
| 122 | 0.048 | -0.040000 | 0.004075 | 0.004662 | -0.044075 | -2.64R |
| 133 | 0.044 | 0.040000 | 0.003319 | 0.004491 | 0.036681 | 2.30R |
| 134 | 0.024 | -0.080000 | -0.000581 | 0.003955 | -0.079419 | -4.93R |
| 137 | 0.113 | 0.040000 | 0.032071 | 0.006837 | 0.007929 | 0.53 X |
| 143 | -0.083 | -0.010000 | -0.003737 | 0.008349 | -0.006263 | -0.46 X |
| 146 | 0.050 | -0.030000 | 0.008478 | 0.005267 | -0.038478 | -2.56R |
| 149 | 0.098 | 0.040000 | 0.025853 | 0.006571 | 0.014147 | 0.98 X |
| 158 | 0.060 | -0.050000 | 0.007510 | 0.005057 | -0.057510 | -3.97R |
| 161 | 0.103 | 0.040000 | 0.023286 | 0.007101 | 0.016714 | 1.23 X |

R denotes an observation with a large standardized residual
X denotes an observation whose X value gives it large influence.

Durbin-Watson statistic = 2.92

Lack of fit test
Possible interactions with variable Sinpi_3 (P-Value = 0.066)
Overall lack of fit test is significant at P = 0.066

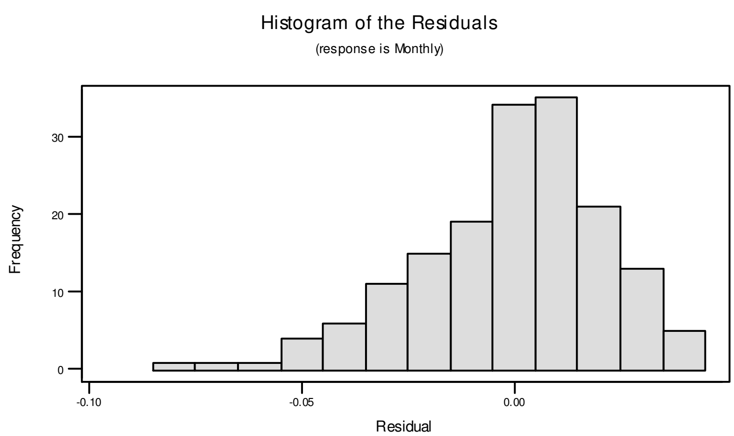


Figure 28(a)

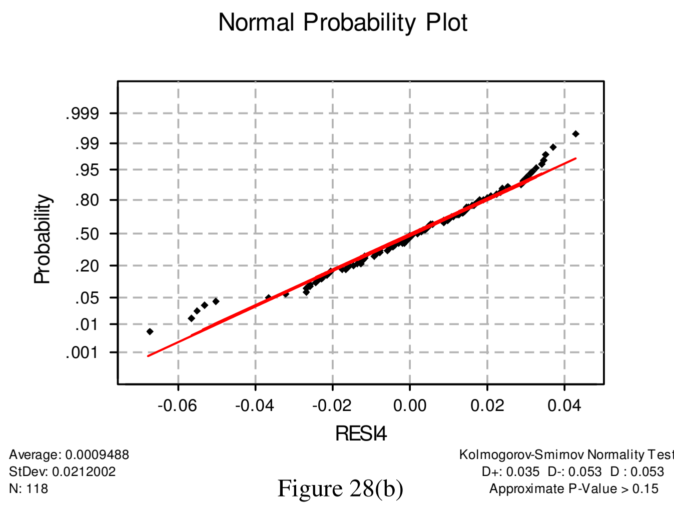


Figure 28(b)

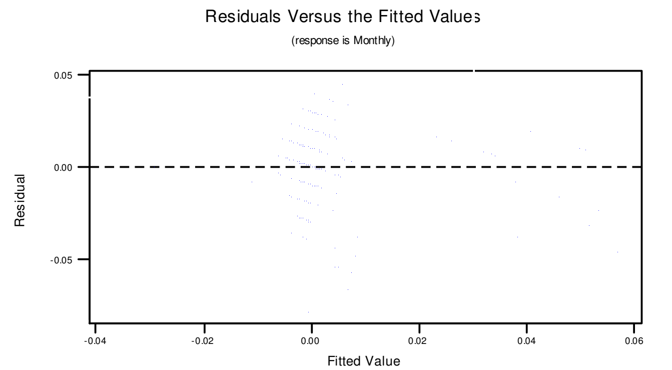


Figure 28(c)

**Figure 28 Error Analysis of Regression Model 1 at year 2000**

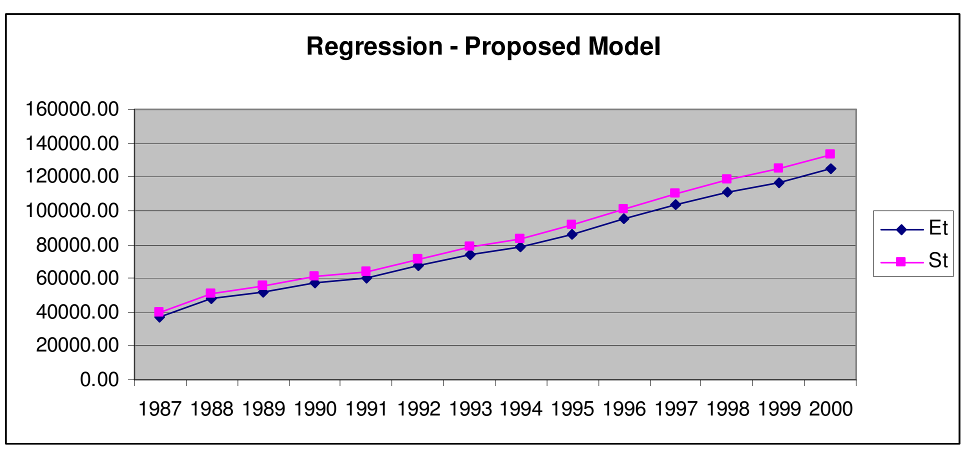

**Figure 29 Graphical Representation of Training Set Results of Regression Model 1 at 2000**

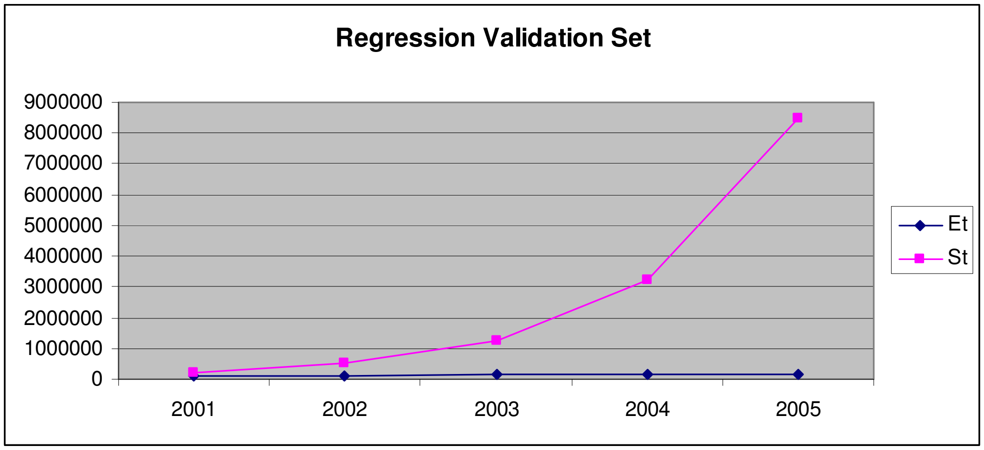


**Figure 30 Graphical Representation of Validation Set Results of Regression Model 1 at 2000**

For residuals showing that H0 : normal distribution → fail to reject with p >0.15

Regression Analysis for the proposed model after 1996 :

### Results for: proposed_96.MTW

### Regression Analysis: Monthly Ener versus GDP (logd); price (logd); ...

```
Weighted analysis using weights in wt

The regression equation is
Monthly Energy Demand logdiff(E = - 0.091 + 0.164 GDP (logd)
           - 0.149 price (logd) P + 73.7 GDP_sq4 - 72.6 Price_SQ3
           + 0.041 Sinpi_sq + 1.35 SinpiP_sq - 0.322 Sinpi_3 + 0.0232 St-2

Predictor        Coef     SE Coef          T        P
Constant      -0.0907      0.1089      -0.83    0.407
GDP (log      0.16376     0.09228       1.77    0.079
price (l      -0.1487      0.6906      -0.22    0.830
GDP_sq4         73.71       94.55       0.78    0.437
Price_SQ       -72.60       67.05      -1.08    0.281
Sinpi_sq       0.0410      0.1217       0.34    0.736
SinpiP_s        1.348       1.656       0.81    0.417
Sinpi_3       -0.3220      0.4875      -0.66    0.510
St-2          0.02325     0.02894       0.80    0.424

S = 0.008540    R-Sq = 25.7%     R-Sq(adj) = 20.3%
```

Analysis of Variance

| Source | DF | SS | MS | F | P |
|---|---|---|---|---|---|
| Regression | 8 | 0.00275115 | 0.00034389 | 4.72 | 0.000 |
| Residual Error | 109 | 0.00795002 | 0.00007294 | | |
| Total | 117 | 0.01070117 | | | |

No replicates. Cannot do pure error test.

| Source | DF | Seq SS |
|---|---|---|
| GDP (log | 1 | 0.00178747 |
| price (l | 1 | 0.00002649 |
| GDP_sq4 | 1 | 0.00029118 |
| Price_SQ | 1 | 0.00051445 |
| Sinpi_sq | 1 | 0.00003764 |
| SinpiP_s | 1 | 0.00002775 |
| Sinpi_3 | 1 | 0.00001911 |
| St-2 | 1 | 0.00004707 |

Unusual Observations

| Obs | GDP (log | Monthly | Fit | SE Fit | Residual | St Resid |
|---|---|---|---|---|---|---|
| 29 | 0.179 | 0.010000 | 0.069238 | 0.027647 | -0.059238 | -1.45 X |
| 62 | 0.034 | -0.050000 | 0.003054 | 0.005111 | -0.053054 | -2.00R |
| 83 | -0.089 | 0.000000 | -0.002630 | 0.008814 | 0.002630 | 0.20 X |
| 86 | -0.013 | -0.030000 | -0.029888 | 0.015592 | -0.000112 | -0.75 X |
| 89 | 0.151 | 0.030000 | 0.038069 | 0.009327 | -0.008069 | -0.65 X |
| 100 | -0.014 | 0.030000 | 0.014268 | 0.008699 | 0.015732 | 1.52 X |
| 101 | 0.125 | 0.030000 | 0.042071 | 0.009308 | -0.012071 | -1.23 X |
| 103 | -0.014 | -0.030000 | -0.002569 | 0.003210 | -0.027431 | -2.09R |
| 106 | 0.014 | 0.030000 | 0.001778 | 0.003255 | 0.028222 | 2.15R |
| 108 | -0.029 | -0.040000 | -0.003030 | 0.003599 | -0.036970 | -3.21R |
| 109 | 0.029 | 0.040000 | 0.010185 | 0.004482 | 0.029815 | 2.66R |
| 110 | 0.046 | -0.040000 | 0.012532 | 0.004800 | -0.052532 | -4.74R |
| 113 | 0.118 | 0.040000 | 0.024051 | 0.008015 | 0.015949 | 1.77 X |
| 115 | -0.014 | -0.030000 | 0.001561 | 0.003689 | -0.031561 | -2.74R |
| 116 | -0.054 | 0.020000 | -0.003545 | 0.004493 | 0.023545 | 2.10R |

R denotes an observation with a large standardized residual
X denotes an observation whose X value gives it large influence.

Durbin-Watson statistic = 2.96

Lack of fit test
Possible interactions with variable GDP_sq4 (P-Value = 0.044)
Overall lack of fit test is significant at P = 0.044

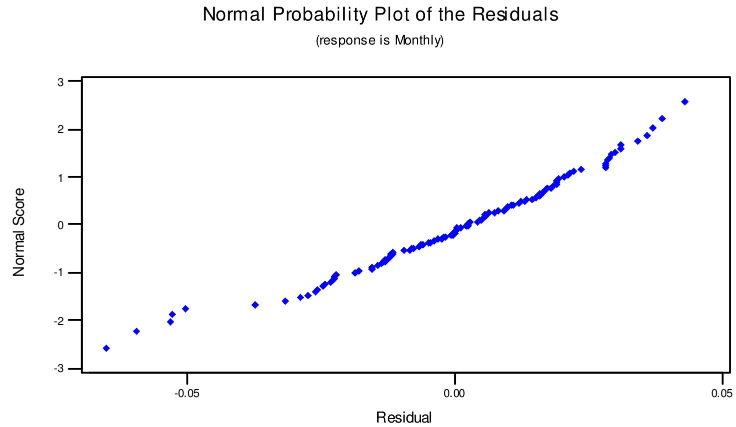


Figure 31(a)

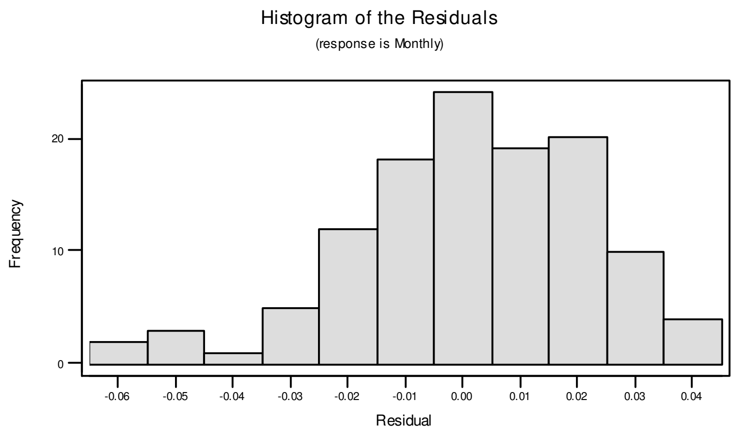


Figure 31(b)

**Figure 31 Error Analysis of Regression Model 1 at year 1996**

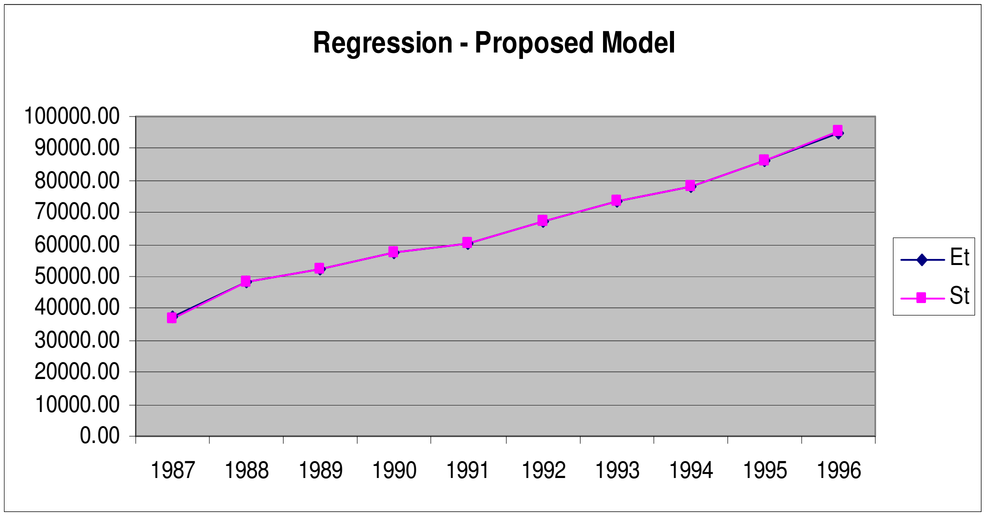


**Figure 32 Graphical Representation of Training Set Results of Regression Model 1 at 1996**

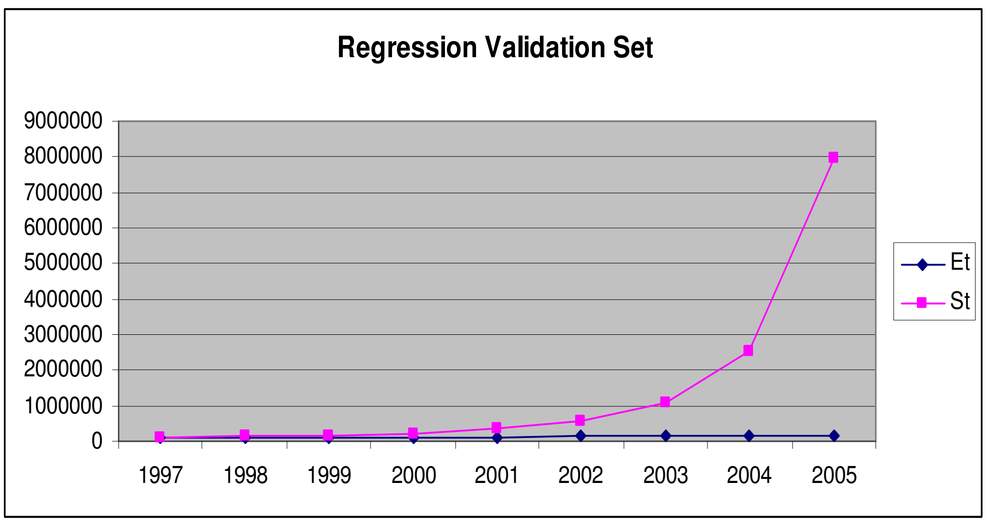


**Figure 33 Graphical Representation of Validation Set Results of Regression Model 1 at 1996**

# APPENDIX G

# REGRESSION ANALSIS FOR REGRESSION MODEL 2

Regression Analysis for the Regression model2 after 1996:

```
The regression equation is
Monthly Energy Demand (log (Et) = 0.0371 + 0.142 GDP   log(GDPt/GDPt-1)
           + 0.078 Energy Price Index   log(Pt/Pt- + 0.585 log(St-1)
           + 0.405 log(St-2)

118 cases used 2 cases contain missing values

Predictor        Coef     SE Coef          T        P
Constant      0.03709     0.06666       0.56    0.579
GDP   lo      0.14184     0.03233       4.39    0.000
Energy P       0.0776      0.1003       0.77    0.441
log(St-1      0.58536     0.07916       7.39    0.000
log(St-2      0.40544     0.07901       5.13    0.000

S = 0.02013     R-Sq = 96.5%     R-Sq(adj) = 96.4%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         4     1.26506     0.31626    780.63    0.000
Residual Error   113     0.04578     0.00041
Total            117     1.31084

Source       DF      Seq SS
GDP   lo      1     0.00101
Energy P      1     0.02774
log(St-1      1     1.22564
log(St-2      1     0.01067

Unusual Observations
Obs   GDP   lo    Monthly         Fit      SE Fit    Residual    St Resid
 40      0.039    3.62000     3.66736     0.00349    -0.04736       -2.39R
 52      0.040    3.65000     3.70440     0.00371    -0.05440       -2.75R
 64      0.034    3.70000     3.74683     0.00302    -0.04683       -2.35R
 88     -0.013    3.78000     3.81917     0.01469    -0.03917       -2.85RX
110     -0.029    3.86000     3.90424     0.00384    -0.04424       -2.24R

R denotes an observation with a large standardized residual
X denotes an observation whose X value gives it large influence.
```

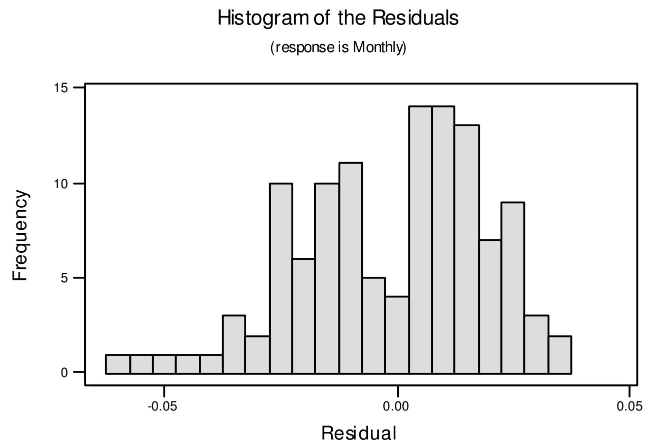


**Figure 34 Error Analysis of Regression Model 2 at year 1996**

## Regression Analysis for the Regression model 2 after 2000:

```
Weighted analysis using weights in wt

The regression equation is
Monthly Energy Demand (log (Et) = 0.0749 + 0.157 GDP   log(GDPt/GDPt-1)
           + 0.044 Energy Price Index   log(Pt/Pt- + 0.696 log(St-1)
           + 0.285 log(St-2)

166 cases used 2 cases contain missing values
                        or had zero weight

Predictor        Coef     SE Coef          T        P
Constant      0.07489     0.08277       0.90    0.367
GDP   lo      0.15739     0.03107       5.07    0.000
Energy P       0.0441      0.1188       0.37    0.711
log(St-1      0.69648     0.07032       9.90    0.000
log(St-2      0.28501     0.07048       4.04    0.000

S = 0.01242     R-Sq = 94.0%     R-Sq(adj) = 93.9%

Analysis of Variance

Source            DF          SS          MS         F        P
Regression         4    0.392515    0.098129    636.22    0.000
Residual Error   161    0.024832    0.000154
Total            165    0.417348

Source       DF      Seq SS
GDP   lo      1    0.000320
Energy P      1    0.052753
log(St-1      1    0.336920
log(St-2      1    0.002522
```

Unusual Observations

| Obs | GDP lo | Monthly | Fit | SE Fit | Residual | St Resid |
|---|---|---|---|---|---|---|
| 88 | -0.013 | 3.78000 | 3.81691 | 0.01759 | -0.03691 | -2.58RX |
| 103 | 0.125 | 3.87000 | 3.86425 | 0.00633 | 0.00575 | 0.31 X |
| 110 | -0.029 | 3.86000 | 3.90584 | 0.00209 | -0.04584 | -2.63R |
| 112 | 0.046 | 3.86000 | 3.89991 | 0.00385 | -0.03991 | -2.33R |
| 117 | -0.014 | 3.89000 | 3.92818 | 0.00239 | -0.03818 | -2.19R |
| 122 | -0.040 | 3.90000 | 3.93124 | 0.00634 | -0.03124 | -2.02RX |
| 124 | 0.036 | 3.91000 | 3.94374 | 0.00391 | -0.03374 | -2.07R |
| 127 | 0.155 | 3.95000 | 3.93743 | 0.00507 | 0.01257 | 0.79 X |
| 136 | 0.036 | 3.92000 | 3.98568 | 0.00353 | -0.06568 | -4.20R |
| 137 | 0.014 | 3.94000 | 3.94488 | 0.00540 | -0.00488 | -0.32 X |
| 139 | 0.155 | 3.98000 | 3.96673 | 0.00514 | 0.01327 | 0.87 X |
| 148 | 0.036 | 3.96000 | 3.99127 | 0.00276 | -0.03127 | -2.06R |
| 151 | 0.155 | 4.01000 | 3.99608 | 0.00526 | 0.01392 | 0.96 X |
| 153 | -0.012 | 3.97000 | 4.00216 | 0.00263 | -0.03216 | -2.12R |
| 155 | -0.013 | 4.01000 | 3.97666 | 0.00223 | 0.03334 | 2.19R |
| 157 | -0.094 | 4.04000 | 4.01017 | 0.00407 | 0.02983 | 2.09R |
| 160 | 0.035 | 3.98000 | 4.03617 | 0.00309 | -0.05617 | -3.87R |
| 163 | 0.155 | 4.04000 | 4.01565 | 0.00538 | 0.02435 | 1.76 X |
| 165 | -0.012 | 4.00000 | 4.03154 | 0.00304 | -0.03154 | -2.17R |

R denotes an observation with a large standardized residual
X denotes an observation whose X value gives it large influence.

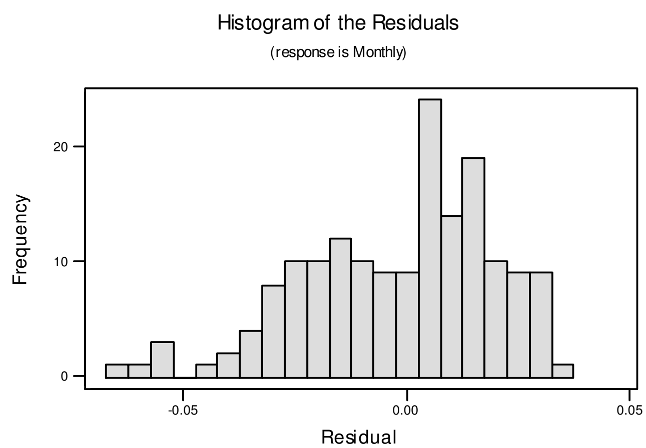


**Figure 35 Error Analysis of Regression Model 2 at year 2000**