# BENEFITS OF VENDOR MANAGED INVENTORY POLICY IN A MANUFACTURER-RETAILER SUPPLY CHAIN

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# ABSTRACT


BENEFITS OF VENDOR MANAGED INVENTORY POLICY IN A MANUFACTURER-RETAILER SUPPLY CHAIN



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Vendor Managed Inventory (VMI) policy has been widely used in various supply chains due to the benefits such as lower inventory levels and costs of retailer, and less frequent stock outs. In this study, the benefits of VMI policy in a manufacturer-retailer setting are analyzed under three different scenarios (Traditional Decision Making, VMI agreement and Centralized Decision Making). A manufacturer that produces a particular product is considered and that product is sold to a retailer operating under known demand forecasts. Under Traditional Decision Making System, each party is responsible for its own costs. Under VMI, manufacturer controls the replenishment decisions of the retailer and solves a Constrained Two-Echelon Lot Sizing Problem with Backordering. Under Centralized Decision Making, manufacturer and retailer act like merged, the problem under consideration is Two-Echelon Single Item Lot Sizing with Backordering.

Through an extensive numerical study, three different scenarios' results are compared and the conditions beneficial under VMI are identified. Under VMI, a Lagrangean Relaxation algorithm is proposed to reduce solution time. In terms of computational effort, solution times of proposed algorithm and MIP model are compared.

Keywords: Constrained Lot Sizing, Vendor Managed Inventory, Dynamic Programming, Two-Echelon Supply Chain

# ÖZ

TEDARİKÇİ YÖNETİMLİ ENVANTER POLİTİKASININ
ÜRETİCİ -PERAKENDECİ TEDARİK ZİNCİRİ ALTINDA FAYDALARI

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Tedarikçi Yönetimli Envanter (TYE) anlaşması, düşük envanter seviyesi, düşük perakendeci maliyetleri ve az stok tükenmesi gibi faydalarından dolayı yaygın olarak uygulanmaktadır. Bu tezde, Bir İmalatçı-Bir Perakendeci içeren bir tedarik zincirinde TYE anlaşmasının faydaları üç ayrı senaryo (Geleneksel Karar Alma, TYE ve Merkezi Karar Alma) altında analiz edilmiştir. Bu tezde ele alınan imalatçı, tek ürün üretmekte ve bu ürünü, bilinen talep tahminleri ile çalışan bir perakendeciye satmaktadır. Geleneksel Karar Alma yaklaşımında, her bir firma sadece kendi maliyetinden sorumludur. TYE altında ise imalatçı, perakendecinin sipariş kararlarını yönetir ve problem "kısıtlar altına yok satmalı- iki kademeli parti miktarı belirlenmesi "dir. Merkezi karar alma sisteminde, imalatçı ve perakendeci tek bir firma gibi davranırlar. Burada ele alınan, "yok satmalı-iki kademeli parti miktarı belirleme" problemidir.

Kapsamlı bir sayısal analiz ile senaryoların sonuçları karşılaştırılmış ve TYE anlaşmasını teşvik eden durumlar belirlenmiştir. TYE altındaki problem, Lagrange gevşetme yöntemi algoritması ile çözülmüştür. Gerekli işlemsel çaba açısından, önerilen algoritma ile karışıktan sayılı modelin problemi çözme süreleri kıyaslanmıştır.

Anahtar Kelimeler: Kısıtlı Parti Büyüklüğü Belirleme, Tedarikçi Yönetimli Envanter, İki Kademeli Tedarik Zinciri, Dinamik Programlama

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# TABLE OF CONTENTS

APPENDICES

# LIST OF TABLES

TABLES

# LIST OF FIGURES

FIGURES

# CHAPTER 1

# INTRODUCTION

Vendor Managed Inventory (VMI) policy is a form of collaboration between a manufacturer and a retailer, where the manufacturer is flexible and has the liberty to plan its own production and dispatch schedule as long as agreed retailer's service levels are met. VMI enhances having the freedom to monitor customer demand and inventory information at the retailer. It is a partnership between a manufacturer and retailer, under which it is the manufacturer who decides when and in what quantity the retailer stock is replenished. It was popularized in the late 1980s by Wal-Mart and Procter&Gamble (Waller et al.,1999) and resulted in significant benefits, such as lower inventory levels, fewer stock-outs and increased sales. Successful VMI partnerships have been structured by other companies such as Dell (Kapuscinski et al., 2004).

Transportation decisions made by solely the retailer and independently do not necessarily consider its upstream business-partner's (manufacturer's) preferences. Her choices of the quantity and timing of transportation may create inflexibility in the manufacturer's operations, resulting in higher costs for it and the entire supply chain. It is therefore important to align the decisions in a supply chain, even when parties have different operational goals. In fact, performance of both parties depends not only on how well each member manages its operational processes, but also on how well the members coordinate their decisions (Achabal et al., 2000).

Two forms of coordination identified in the literature are vertical and virtual integration (Riemer et al., 2000). In *vertical integration* one party acquires the others or parties merge and it is usually called Centralized Decision Making System. However, that ends the independence of the firms, and can fail (Aviv and Federgruen, 1998) because of the behavioral difficulties in integrating the distinct organizational cultures and it is not a common application in industry. The second form of coordination is the *virtual integration* and harmonizes the parties' decisions by means of a business arrangement between them; while enabling more independency of the both parties. Thus, VMI is a means of virtual integration, kind of a partnership to coordinate replenishment decisions in a supply chain while maintaining the independence of chain members. In this partnership between a manufacturer and a retailer, it is the manufacturer that decides when and in what quantity the retailer's stock is replenished. With such an agreement, the manufacturer may be able to share the retailer's point-of-sale and inventory-level data. From the manufacturer's perspective, VMI can yield: Receiving (through electronic data interchange, fax or the internet) information on stock levels, sales, and any sales forecasts that have been made, generating replenishment orders as needed, sending dispatch advice (electronically) to the partner, and then the invoice (Intentia, 2000).

The decisions of replenishment, holding inventory, backordering, shipment have traditionally been made separately in the supply chain; however, their integration can have a significant impact on the overall system. A VMI system is a good example for the type of integration (Cetinkaya and Lee, 2006). In the VMI model a manufacturer observes and controls the inventory levels of the retailer, as opposed to the traditional approaches where the retailers track their own inventory and decide the time and amount of products to reorder. Thus, manufacturers can obtain a more uniform utilization of the transportation resources, which leads to lower distribution costs (Disney and Towill, 2003). It also offers the flexibility to choose the most preferred transportation mode. Some studies also confirm that retailers usually benefit from higher service levels and greater product availability due to the fact that the manufacturers can use existing inventory data at their retailer sites to predict the future demand more accurately (Achabal et al., 2000).

Many applications and studies discuss about the benefits of VMI but the studies discussing the conditions under which VMI is preferable for each member in a supply chain are scarce. In this thesis, we consider the benefits of VMI for both manufacturer and retailer under specific system parameters and define the conditions to motivate each party to join a VMI agreement. Dong and Xu (2002) state that despite the fact that it is the manufacturer taking control of replenishment decisions; retailer typically gains an increase in its profit through such a partnership under VMI, while the manufacturer's profit gain is less evident. Since the retailer sets the conditions of the partnership so that the performance measures (such as number of stock-outs, average inventory level) improve, the authors strongly state that it is generally the retailer VMI is most beneficial to. The authors conclude their study by stating other strategic benefits such as long-term partnership with the buyer and reductions in certain cost components, would make the VMI program sufficiently attractive to the manufacturer.

Discussion of "whether VMI is beneficial for the parties or not" depends on the agreement terms. Agreement terms are mainly defining the problem structure and they are imposed as constraints on the problem under VMI. In the VMI agreement, the retailer may require performance measures such as specific service level by imposing lower bound on the inventory level or upper bound on the backorder level. Similarly, due to the space constraints or to avoid high inventory costs, the retailer may limit the amount of replenishment from the manufacturer by imposing an upper bound on the inventory level or replenishment quantity. When the manufacturer cannot satisfy these performance measures, VMI agreement can charge a penalty cost to the manufacturer. Defining the terms of agreement for performance measures is a difficult task for the retailer to accomplish, due to the fact that the performance measures are core factors that define the problem environment. She should define the terms so that she is not worse off under VMI partnership.

In this study, there exists a supply chain consisting of a single manufacturer and a retailer. First, the traditional system is defined as Scenario 1, under which the manufacturer and the retailer operate independently. Secondly, vendor managed system is introduced as Scenario 2 and then they are compared in terms of decisions. Additionally, centralized decision making system is defined as Scenario 3, (system under which manufacturer and retailer act like merged) as a benchmark and system-wide costs are compared with Scenario 1 and Scenario 2. The benefits of the integration of the decisions of both parties become apparent during the comparison. Since the retailer sets the terms of the VMI agreement, she ensures via her performance measures to be never worse off under VMI.

### 1.1 Motivation for the study

In this study, three different scenarios are studied in detail; traditional system as Scenario 1; VMI agreement as Scenario 2 (VMI partnership) and Scenario 3 (centralized decision making). While doing these, our main motivation is to specify the cases under which VMI is preferable by the manufacturer and retailer. We also identify the conditions under which three scenarios behave similar and compare system-wide costs in order to make an analysis of integration efficiency. Many studies concentrate on the total supply chain costs; they either ignore the motivation for signing vendor managed inventory agreement for the parties, or focus only on the total supply chain benefits rather than the individuals' (Bertazzi et al.(2003), Bernstein and Federgruen (2003), Fry et al. (2001), Latifoğlu (2006)). There are few studies focusing only on the manufacturer's and/or retailer's benefits in a manufacturer and a retailer supply chain (Dong and Xu (2002), Gümüş (2006)). In this thesis, retailer faces deterministic demand. This thesis compares individuals' operating costs for Scenario 1 and 2, in order to identify the settings motivating VMI. The system-wide costs are also compared for Scenario 3 with 1 and 2 as a benchmark. Under Scenario 2, a less restricted policy of common VMI is allowed, where the manufacturer is only responsible from replenishment decisions, and opposite to the literature, retailer is responsible from inventory holding decisions at her site. In addition to this, restrictions on inventory and backorder levels are imposed as performance measures. Thus, our work contributes to the literature in many important ways. It analyzes benefits due to the vendor managed systems from both manufacturer's perspective and retailer's perspective under a less restrictive VMI agreement. It also identifies the conditions to make the manufacturer can be willing to join such an agreement. Additionally, we compare system-wide costs of the traditional system, VMI and centralized system as a benchmark. Below, each scenario is explained in detail.

### 1.1.1 Scenario 1 (Traditional System)

In Scenario 1, the retailer makes the ordering decisions and the manufacturer reacts. It considers a manufacturer and a retailer operating on their own where each party tries to minimize its own costs.

*Retailer's Problem under Scenario 1*

Scenario 1 involves a retailer who sells at her location a single product to a single customer for satisfying demands over a finite planning horizon. She has demand forecasts and operates based on these forecasts. The nature of the demand can be regarded as deterministic. Retailer decides on when to order from manufacturer, how much to keep inventory and how much to backorder. There is no capacity limit on order quantity and retailer can backorder demand. When she orders earlier than it is actually required, she holds inventory for the corresponding periods, on the other hand a late order may result in backordering costs. Order cost is assumed to include all administrative costs for processing the order, receiving costs and transportation costs associated with receiving the order from the manufacturer. Manufacturer is assumed to be in close proximity to the retailer thus, order cost can be assumed to occur as fixed cost and lead time of transportation is zero. There is not any inventory and backordered demand at the beginning and ending of planning horizon. Retailer's problem is to decide when and how many units to order to minimize total order (transportation cost), inventory holding and backorder costs over the finite horizon $T$. Under traditional system, retailer's optimal policy results in two performance measures' desired levels which are imposed as constraints to VMI system, average or total inventory level and average or total backorder level. Therefore, in the optimal ordering policy, two performance criteria's desired levels are recorded; average, (or total) inventory and average (or total) backorder level (can be regarded as also service level) through the planning horizon. Retailer's problem under Scenario 1 represents a typical retailer operating in the traditional case; deciding on her own in order to operate at a cost effective way for her company while tracking her two performance criteria. Retailer does not want to be worse off under VMI in terms of her performance measures. Hence, under VMI, retailer does not want to be worse off than in Scenario 1, manufacturer should take replenishment decisions so that performance measures are satisfied. Retailers' problem under Scenario 1 is in Uncapacitated Single Item Lot Sizing with Backordering context. (ULS-B)

*Manufacturer's Problem under Scenario 1*

Manufacturer produces a particular product at a unique location. Production capacity is assumed to be infinite. Single item is being produced to be sold to a single retailer that has demand forecasts during the planning horizon. Orders are placed according to these forecasts during the planning horizon. Manufacturer has to satisfy the demand of the retailer on time, based on the orders placed by the retailer. Backordering is not allowed at manufacturer's site. Demand parameter of manufacturer is the optimal order quantities of the retailer at each period. Manufacturer is located in close proximity to the retailer e.g. within the same vicinity of the same town. At each period, unit inventory holding cost and fixed production cost occurs at manufacturer's site. Starting and ending inventories of the planning horizon are assumed to be zero and unit holding cost of manufacturer is assumed to be always less than or equal to unit holding cost of retailer because of value addition during transportation and due to the profit gain Infinite raw material availability and production lead time being zero are assumed. Manufacturer has to decide when and how much to produce, he does not have the liberty of producing later than the retailer requires. Manufacturer's objective is to decide when and how many units to produce so as to minimize total production and inventory holding costs over the finite horizon without any shortages. Manufacturer's problem is basically in Uncapacitated Single Item Lot Sizing (ULS) context under Scenario 1.

Both manufacturer and retailer faces single echelon dynamic lot sizing problem under Scenario 1, and each problem is treated separately during the solution process since there is not any agreement between the parties in Traditional System.

### 1.1.2 Scenario 2 (VMI)

It is next assumed that the manufacturer is not content in simply reacting, and wants to get involved in retailer's order decisions. With VMI (Scenario 2), the manufacturer takes over the dispatch decisions and hence the issuing cost related to it (fixed cost of transportation) which might not be the same as what the retailer used to pay. Having the opportunity to review the retailer's demand forecasts, the manufacturer makes decisions regarding the quantity and timing of dispatch. Manufacturer's problem under VMI is to decide when and how much to dispatch to the retailer, when and how many units to produce to minimize total dispatch, production and inventory holding (of manufacturer) costs over finite planning horizon to satisfy deterministic dynamic end customer demands at the retailer; while two important performance measures of the retailer (total backorder level and total inventory level) are satisfied. These performance measures are imposed as two constraints to the model. This means, under VMI collaboration retailer sets two terms in the agreement. First condition is; retailer should not be worse than in Scenario 1 in terms of average or total backorder level. Second condition is; retailer should not be worse than in Scenario 1 in terms of average or total inventory level. These two performance measure constraints are complicating constraints for the VMI model. Under Scenario 2, manufacturer's problem is constrained two-echelon Uncapacitated Single Item Lot Sizing Problem with backordering (Constrained two-echelon ULS problem with backordering, i.e. multi-stage lot sizing).

### 1.1.3 Scenario 3 (Centralized System)

Scenario 3 is centralized decision making system. Manufacturer and retailer act like merged, no specific policy (like VMI) applies. Manufacturer and retailer behave like one single entity and try to minimize this single entity's total cost. In central decision making, both manufacturer and retailer are assumed to be controlled by the same corporate entity. The objective of the model is to decide when and how much to be produced by the manufacturer, when and how many units to be transported to the retailer, so as to minimize the sum of transportation, production, inventory holding and backorder costs at the retailer over the finite horizon to satisfy deterministic dynamic end customer demands. Scenario 3 is a benchmark for VMI in terms of total supply chain costs and results of Scenario 2 and 3 will be compared in order to see the effect of collaboration and centralization. Under Scenario 3, problem is two-echelon Uncapacitated Single Item Lot Sizing with backordering. (two-echelon ULS problem with backordering)

## 1.2 Thesis Outline

Each model designed under each scenario is described in detail in Chapter 3. In Chapter 4, an algorithm to solve constrained Two-Echelon ULS problem with Backordering is suggested. In Chapter 5, a numerical study is done so as to make benefit analyses for both parties and comparison of supply chain costs under different parameter settings. Optimal solutions are obtained for the manufacturer's and retailer's operating costs (total costs). Parties' individual and system-wide costs under three scenarios; traditional, VMI and centralized decision making are obtained. During computational experiment in Chapter 5, results are compared and performances of each model are presented. In Chapter 6, important conclusions are drawn and discussion about future work is made.

# CHAPTER 2

# LITERATURE REVIEW

The literature survey provided in this chapter relates to VMI agreements in general, and serves as a seed for Scenario 2. Two main categories of literature that have ties to our study are identified. The first concerns the VMI agreement structures and operational benefits obtained under VMI. The second category depicts Uncapacitated Single Item Lot Sizing (ULS) problem and its extensions through two-echelon dynamic lot sizing with backordering.

## 2.1 VMI

Traditionally, independent companies in a supply chain do not choose policies that optimize overall supply chain performance. Each firm tries to optimize its own objective. Collaboration in a supply chain yields better actions for chain members who align their decisions to achieve virtual integration. VMI is one example of virtual integration and there exist many different structures of VMI. In the literature, VMI is analyzed under different categorizations. Some studies are involved with single manufacturer- single retailer systems, while others assume a single manufacturer-multiple retailer supply chain. Single manufacturer-multiple retailer supply chains generally emphasize the savings obtained through shipment consolidation while a single manufacturer-single retailer supply chain seeks for savings due to centralized decision making or manufacturer’s increased flexibility.

Many authors study the optimal policy of the manufacturer and retailer under VMI agreement in a stochastic setting while some others study a deterministic environment. There are several distinguished terms of agreement policies that retailer reflects under VMI, such as lower and upper bounds on the inventory level, upper bound on replenishment quantities, required service level, required inventory turnover rates and so on. Moreover, VMI requires shifting some costs from the retailer to the manufacturer while retailer shares information with the manufacturer regarding the demand or inventory levels at her site. To illustrate, manufacturer can be responsible from transportation decisions and inventory of the retailer may not be invoiced till the usage (consignment stock policy). Depending on the terms of agreement, operational benefits can be obtained and many studies focus on the system-wide benefits of VMI and examine channel coordination (being close to centralized system) effect of VMI. Many studies consider traditional system and compare VMI and traditional system in terms of parties' benefits, production and transportation costs, effect of demand variance and other system parameters. Some studies focus on manufacturer's benefits under VMI while some other studies analyze both parties' cost savings under VMI.

To sum up, while reviewing existing studies about VMI, categorization under different subsections can be accomplished such as;

- Benefits of single manufacturer-single retailer and single manufacturer-multiple retailer supply chains.
- VMI agreements under deterministic and stochastic environments
- Terms of agreement structures (Possible performance measures of the retailer)
- Computing operational benefits under VMI (system-wide benefits, manufacturer's or retailer's benefits)

In this study, it is assumed that a single manufacturer- a single retailer supply chain operating in a deterministic environment. While discussing at each study according to the categorization made above, contributions of this thesis are detailed.

### 2.1.1 Previous Studies on VMI

In studies that assume a single manufacturer-single retailer supply chain, cost savings depend on many criteria. Fry, Kapuscinski and Olsen (2001) study the optimal policy of the manufacturer and the retailer under stochastic environment and compare system-wide costs under VMI and under traditional system. The authors study a VMI agreement that occurs in practice as (z,Z) type contract.
There is an upper limit on the inventory level of the retailer imposed as terms of VMI agreement and manufacturer is punished by a certain amount of cost if he exceeds this limit. The authors conclude that (*z,Z*) type VMI agreement performs better than traditional system in many settings, and VMI gives solutions close to a centralized model when demand variance is high.

There are studies which have single manufacturer-single retailer supply chain facing deterministic demand. Valentini and Zavanella (2003) describe the technique of consignment stock by a case study of a manufacturer providing parts to the automotive industry. In that example, the manufacturer manages the inventory of retailer using an (*s, S*) policy. Manufacturer ensures an available stock between a minimum level *s* and maximum level *S*. Authors conclude that consignment stock (CS) policy outperforms traditional systems (TS) and compare system-wide costs of two: CS and TS. Shah and Goh (2005) also consider a single manufacturer-single retailer supply chain with deterministic demand. Retailer decides on the range of inventory levels that the manufacturer should satisfy and different operating conditions are considered such as backordering, minimum and maximum specified inventory levels. The manufacturer pays a penalty for violating the minimum and maximum inventory levels determined by the retailer. There are fixed costs of production and transportation, the manufacturer's objective is to minimize cost by determining when and how much to dispatch to the retailer and when and how much to produce. The authors assume that demand is deterministic and constant over time. They suggest a structured hierarchical approach and propose an algorithm to deal with a numerical example. Jaruphongsa, Cetinkaya and Lee (2004) analyze a similar problem with demand time window considerations and name the model as "Two-Echelon dynamic lot sizing model with demand time windows and early and late delivery penalties". The authors study the optimality properties of the problem and provide a polynomial time algorithm for the cases with or without backordering.

In this thesis, single manufacturer-single retailer supply chain operating under a deterministic environment is studied. Manufacturer takes control of transportation decisions under a no consignment stock model. Each party is responsible for its own inventory holding costs and manufacturer establishes and manages the dispatching decisions of the retailer. Under VMI, transportation cost is shifted to manufacturer and backordering is allowed at retailer's site. Inventory level and backorder level should be at certain levels which are specified by the retailer as terms of agreement. Manufacturer should decide on when and in what quantity to dispatch while retailer is never worse off under VMI, in terms of her performance measures. Thus, in our model, manufacturer is allowed to be more flexible than the common VMI applications discussed in the literature because there are only maximum levels on the inventory and backorder levels rather than minimum-maximum levels and manufacturer is assumed to be only responsible from transportation cost rather than inventory holding cost of the retailer.

In VMI agreements with a single manufacturer and multiple retailers, the benefit constitutes of mainly of the savings in transportation because of order consolidation or savings due to coordination of retailer replenishments and service quality. To illustrate the flexibility of the manufacturer, assume that there is a non-critical delivery for retailer $i$ and it can be diverted for a day or two to enable a critical delivery to retailer $j$. Similarly, a smaller than usual replenishment to one customer may enable a larger than usual shipment to another customer. With the ability to balance the needs of all partners, the manufacturer can improve the system's performance without jeopardizing any individual retailer. (Achabal et al., 2000) Çetinkaya and Lee (2000) consider the case where the manufacturer uses $(s,S)$ policy for replenishing its inventory. Manufacturer faces random demands from a group of retailers and authors approach is to minimize total procurement, transportation, inventory carrying and backordering costs while satisfying demands in a stochastic environment. Çetinkaya and Lee (2000) synchronize inventory and transportation decisions. An analytical model enables determination of the optimal replenishment quantity and dispatch frequency. Their contribution is based on an idealized application of VMI, whereby the vendor has the autonomy of holding orders until a suitable dispatch time at which orders can be economically consolidated.

In traditional systems, the independent parties in a supply chain do not choose policies that optimize overall supply-chain performance; common case is each party instead attempting to optimize its own objective. Coordination within a supply chain then mainly refers to finding the optimal actions for members who need to align their decisions to achieve optimal chain costs and many studies examine VMI as a supply chain coordinator. To illustrate, Bernstein and Federgruen (2003) study a constant-demand-rate VMI setting characterized as a partially centralized model (the retailer retains decision rights on pricing and sales target). The manufacturer determines a replenishment strategy for the entire supply chain. They show that channel coordination can be achieved under VMI. In their model, the manufacturer incurs all inventory holding costs including those at the retailer. In CI system as discussed before, ownership of the goods is transferred to the retailer only after they are used. Hence, the agreement Bernstein and Federgruen (2003) consider should be regarded as VMI and CI together, rather than pure VMI. Information sharing under VMI can be done through many means, retailer can share demand information, consumption rate and inventory levels such as in Bernstein and Federgruen (2003) at each period. In this thesis, under VMI retailer only shares demand information with the manufacturer. Fry, Kapuscinski and Olsen (2001) also compare system-wide costs and search the impact of VMI as a supply chain coordinator as discussed above. Bertazzi et al. (2003) consider an inventory routing problem in which items are distributed from a facility to a set of retailers. The costs considered are fixed and variable production, transportation and inventory costs at the parties. Optimal traditional system and VMI problems include vehicle routing problem also, making the problems harder to solve. The authors compare system-wide costs of VMI and traditional system and show that heuristically found VMI policies can dramatically reduce system costs. In stochastic environment, Toptal and Çetinkaya (2006) consider the coordination problem between a manufacturer and a retailer operating under generalized replenishment costs that include fixed costs as well as stepwise freight costs. The authors study the stochastic demand, single-period setting where the retailer must decide on the order quantity to satisfy random demand for a single item with a short product life cycle. Authors compare system-wide costs of centralized and traditional systems and identify the improvement rate results from channel coordination. Aviv and Federgruen (1998), aim of investigating impacts of information sharing in a two-echelon system while considering system-wide costs. They consider a single manufacturer and multiple retailers focusing on inventory and distribution cost performance measures. They assume a VMI agreement that leads to a fully centralized planning model where the manufacturer minimizes the system-wide total cost of inventory holding and distribution.

In this study, three types of scenarios of inventory control policies are treated in detail. First, traditional system is taken as Scenario 1, and manufacturer and retailer are assumed to act separately. Retailer determines an order quantity and passes it to the manufacturer. Both system-wide costs and parties' individual costs are calculated. Secondly, manufacturer and retailer are governed by VMI agreement (Scenario 2). System-wide costs and members costs are calculated. Finally, centralized system is examined as a benchmark and system-wide costs are calculated in this vertical integration. This study differs from the studies just discussed above in a way that while comparing individuals' costs under each scenario, also aims to identify any potential benefits in both vertical and virtual integration compared with traditional case and present system parameters that makes VMI a channel coordinator.

There are also studies that compare the supply chain members' operating costs and consider VMI is beneficial to which party, under what conditions. Studies that consider a single manufacturer and a single retailer environment, the question of when each participant prefers VMI to the traditional system or whether the manufacturer benefits from a VMI agreement is not addressed much as well as retailer's and manufacturer's individual benefits. Discussed before, Çetinkaya and Lee (2000) conclude that when inventory holding and dispatching costs decrease, savings of the manufacturer under VMI increase. This is not the most frequent case for VMI; Dong and Xu (2002) state that usually retailer gets significant benefits from VMI agreement. Authors conclude that in the long run, VMI can more likely to increase manufacturer's benefits than in the short run. Authors state that despite the fact that it is the manufacturer taking control of replenishment decisions, retailer typically gains an increase in its profit through such a partnership under VMI, while the manufacturer's profit gain is less evident. There are also studies that question whether VMI is beneficial for both the manufacturer and supply chain. Copacino (1993) confirms that a typical VMI program involves a manufacturer that monitors inventory levels at retailer's warehouses and assumes responsibility for replenishing that inventory to achieve specified targets through the use of highly automated electronic messaging systems but this design of the system is very important. According to Copacino (1993), a poorly designed VMI agreement can harm the manufacturer who ships more often to satisfy the inventory turns required at the retailer. Gümüş (2006) studies a supply chain composed of a single manufacturer and multiple retailers facing time-varying demand. The author models manufacturer's and retailer's dynamic lot sizing models as MIP and finds optimal values to their decision variables. Traditional system is taken as base case. Possible system-wide savings and parties' savings under VMI agreement are analyzed. Author states that centralized system is the most cost effective case.

In this study, "supply chain members benefiting from VMI agreement under which conditions" are identified. Under VMI, it is assumed that manufacturer can access to the end item demand information of retailer and has the freedom to decide when and how much to replenish while being charged by dispatch cost. Both manufacturer's and retailer's benefits in traditional and VMI systems are also compared together with system-wide costs. In this study, centralized decision making system is considered as a benchmark and supply chain costs of the scenarios are compared.

In this thesis, the circumstances all scenarios approach to each other are presented. Traditional system approaches to both VMI and centralized system in terms of total supply chain costs under different system parameters. Depending on the term of agreement and system parameters, VMI can behave like traditional system and therefore, making a partnership like VMI is beneficial in terms of system-wide costs. This thesis differs from existing studies in several aspects; under VMI, there is a partnership where the terms of the VMI agreement are defined so that the retailer is never worse off under VMI, and the manufacturer is responsible for transportation decisions. Additionally, main aim of analyzing VMI agreement is to specify the conditions that the manufacturer losses are minimized under vendor managed system. In our VMI model, we further allow for more flexible (less restricted) policies that aims to satisfy the desired performance measures; our VMI setting is a special case of VMI discussed in the literature. Note that these policies are less restrictive than the structured ones (for example min-max type) considered in the literature. Defining the situations in which supplier and/or manufacturer should prefer VMI agreement is not a common area of study. There are some studies comparing traditional and VMI environments as mentioned above. This study differs from the existing studies in that they do not guarantee retailer is better off under VMI while demand is deterministic. Additionally, we challenge the argument that "VMI is beneficial in terms of system-wide costs" and show that VMI can behave like Traditional System under specific system parameters. Each scenario is compared in terms of both supply chain members' costs and supply chain costs. Traditional, VMI and Centralized systems' decisions are also compared.

## 2.2 Studies on Uncapacitated Single Item Lot Sizing

The uncapacitated single item lot sizing model was first introduced by Wagner and Whitin (1959) who developed the classical dynamic programming (DP) algorithm to find solutions. Soon after, the basic model was extended in many directions to include backordering (Zangwill, 1959) and holding and production capacities (Florian and Klein, 1971). An important extension of classical single item lot sizing problem is backordering. Zangwill (1969) basically extended Wagner- Whitin model to permit backordering for single item model. Two dynamic economic lot size problems are analyzed by in this study, the first being single item case with backlogging and the second a multi-echelon model. In each model, the objective is to find a production schedule that minimizes the total production and inventory costs. Problem of minimizing a concave function is being considered and in the paper it is shown that both models are naturally represented by single source networks. The network formulations reveal the underlying structure of the models, and facilitate development of efficient dynamic programming algorithms for calculating the optimal production schedules.

In this study, Uncapacitated Lot Sizing (ULS) problem is dealt with/without backlogging for retailer and manufacturer correspondingly. Under scenario 1, manufacturer's problem is ULS and retailer's problem ULS with Backordering. Under scenario 2, manufacturer's problem is constrained ULS with Backordering and under scenario 3, problem under consideration is Two-Echelon ULS Problem with Backordering.

There are researchers who focus on multi stage lot sizing problems with backordering of one party. Lee, Çetinkaya and Jaruphongsa (2003) present a model for computing the parameters of an integrated inventory replenishment and outbound dispatch scheduling policy under dynamic demand considerations. The authors adopt a network approach to solve it and propose polynomial time algorithms using some optimal solution properties. This paper gives us a motivation in our studies and it is the most related study to our research. We adapted a special case of this network approach. The optimal decision is how often and in what quantities to replenish the stock at an upstream supply chain member (a warehouse), and how often to release an outbound shipment to a downstream supply-chain member (a distribution center). The problem is represented using a two-echelon dynamic lot-sizing model with pre-shipping and late shipping considerations, where outbound cargo capacity constraints are considered via a stepwise cargo cost function. Although their study is motivated by a third-party warehousing application, the underlying model is applicable in the general context of coordinating inventory and outbound transportation decisions. We applied an extension of their model to a two-echelon system in which a manufacturer and a retailer takes coordinated decisions. The authors' problem differs due to the stepwise cargo cost structure. Their study presents several structural properties of the problem and develops a polynomial time algorithm for computing the optimal solution.

Lee, Çetinkaya and Jaruphongsa (2003) consider a manufacturer who ships the final products to a third party warehouse that serves as a distribution center with deterministic time varying demand. So as to save cost while integrating transportation and replenishment policies, the manufacturer is interested in implementing a shipment consolidation routine. Thus, the demands at the distribution center can be shipped earlier or later than they are required. The third party warehouse cooperates with the manufacturer and implements the distribution policy specified by the manufacturer. As a result, the authors examined a two-stage serial system consisting of the third party warehouse and the distribution center. They have proven optimality conditions and they modeled the system in a network based approach. The authors reduced the network according to the phases of optimality conditions and defined the arc costs (i to j) as satisfying corresponding demand by a single replenishment during period (i, j) . A polynomial time algorithm is generated to calculate all arc costs and after all arc costs are calculated, optimal decision is found through a shortest path problem solving procedure via dynamic programming. The problem becomes easier when cargo capacity if the demand of a given period does not exceed the cargo capacity.

Lee et al. (1994) study two variants of the dynamic economic lot sizing problem. In the first one, the production is restricted to be a multiple of a fixed batch size in each period, and the costs are time varying, whereas in the second, a more general form of product order cost structure is assumed. Polynomial time algorithms are proposed for each case. Jin and Muriel (2005) study a system composed of one warehouse receiving a single product from a supplier and replenishing the inventory of n retailers with direct shipments. Retailers order goods from a warehouse whose inventory is in turn replenished by an external supplier. The transportation cost is characterized by a stepwise cost function. There is a fixed cost per truck, dispatched from supplier to warehouse and from warehouse to retailers. The holding costs at the warehouse and retailers are linear. The objective is to decide when and how many units to ship from supplier to warehouse and from warehouse to retailers so as to minimize total transportation and holding costs over the finite horizon without any shortages. This single warehouse-multi retailer problem is NP Hard. System is characterized by centralized and decentralized decision making and modeled. They used Lagrangean decomposition technique where the Lagrangean multipliers are updated via subgradient method. They compared small and medium instances and concluded that the gap between decentralized and centralized decision making decreases as the problem scale increases.

Some researchers investigate the polyhedral structure of the lot sizing problems. Pochet and Wolsey (2005) show that a good understanding of the polyhedral structure of single item lot sizing problems with backordering can be very useful in solving more complicated problems, involving multiple products and stages. Because the single item lot sizing polyhedron is contained as a fundamental substructure in those problems, investigating inventory bounds and fixed costs within the lot sizing context is meaningful from a practical point of view. There are valid inequalities imposed so as to decrease solution time and convex hull of the problem while reformulating the original problems' variables. Atamtürk and Küçükyavuz (2005) analyze lot sizing problem with inventory bounds and fixed costs while examining polyhedral structure of the problem. They consider two models, one with linear costs on inventory, the other with linear and fixed costs on inventory. For both models, they identify inequalities that make use of inventory bounds explicitly and give exact separation algorithms. They present computational experiments that show the effectiveness of the results in tightening the linear programming relaxations of the lot sizing problem with inventory bounds and fixed costs. Solyali and Süral (2008) consider a two-echelon supply chain in which a manufacturer replenishes a retailer facing deterministic time-varying demand of a single product over a finite time horizon. Manufacturer replenishes the retailer employing an order up to $S$ policy over $T$ periods. The authors transform this problem into an equivalent network problem where the nodes denote periods and arcs denote replenishment quantities. Then the network is extended to account for the integrated replenishments at both levels; they find the shortest path over the extended network and find the optimal solution to the original problem.

Similar to previous works, we deal with identifying optimal solutions and comparing the effectiveness of the models for two-echelon uncapacitated single item lot sizing problem. However, when compared to the previous studies, in this study a MIP model is provided for the specific problem under VMI policy that captures all decisions related with inventory management both at the retailers and at the manufacturers.  We also analyze and solve two-stage ULS problem with backordering (both with and without performance constraints) Furthermore, methods are proposed to identify both lower bounds and upper bounds on the optimal solution, together with the effectiveness comparison of obtaining optimal solutions via MIP solver CPLEX and obtaining near-optimal solutions via dynamic programming algorithm based Lagrangean Relaxation algorithm.

# CHAPTER 3

# PROBLEM CONTEXT AND MODEL DEFINITION

Including all the properties just defined in Chapter 1, three different scenarios are detailed and models for each scenario are defined in this chapter. First scenario is traditional system; second scenario is vendor managed system and third one is centralized decision making system.

In the problem of interest, a manufacturer sends its final products to a retailer that has to satisfy end-customer demand of single item. Manufacturer considers fixed cost of production and inventory holding cost while the retailer considers fixed cost of transportation, inventory holding cost and backordering cost. Retailer has two performance measures to track (i) Inventory level among planning horizon denoted by IL (ii) backorder level among planning horizon denoted by SL.  Total backorder level is interpreted as Service Level (1- $\beta$ ) because fraction of demands that stock out each period yields same solution as total stock out value.

*Scenario 1: Traditional System*

In traditional system, there are two independent decision makers. Each decision maker tries to minimize its' own operational costs. Retailer (R) decides when and how many units to order at each period while minimizing total transportation, inventory holding and total backorder costs over the horizon *T*. Retailer decides to order at certain periods, requiring product shipment from the manufacturer.

Manufacturer's (M) objective is to minimize inventory holding and total production cost while optimal ordering policy of the retailer is the demand sequence for manufacturer. Backordering is not allowed at manufacturer's site.

Traditional systems environment is visually illustrated in Figure 1.

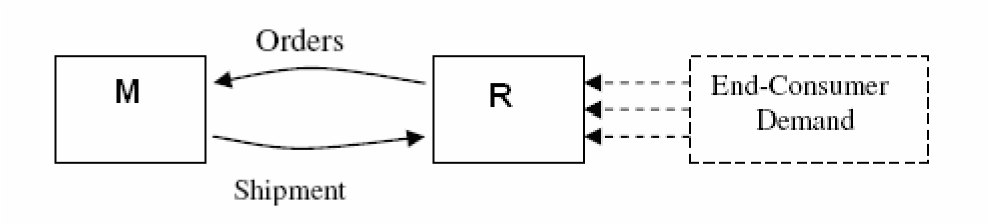


**Figure 1** Independent Decision Making in a Two-Echelon Supply Chain: Traditional System

Under Scenario 1, end-consumer demand information is not shared with the manufacturer and this situation is represented as a dotted line (Figure 1).

*Scenario 2: VMI*

Under Scenario 2, there is VMI agreement between two parties. Retailer shares end consumer demand information with the manufacturer and manufacturer decides when to produce and replenish the inventory at the retailer's site. As discussed before, retailer's total inventory level (IL) and backorder level (SL) are performance measures to guarantee that the retailer is never worse off in terms of her performance measures under VMI. These two are imposed as constraints to VMI system. Figure 2 visualizes system under VMI partnership.

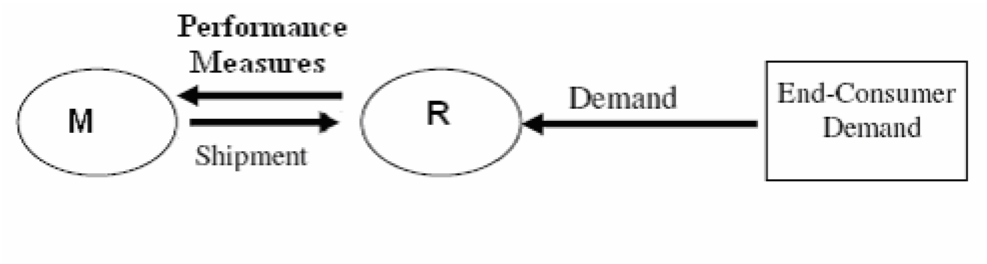


**Figure 2** VMI System

*Scenario 3: Centralized Decision Making*

Scenario 3 represents centralized decision making system and both retailer and manufacturer are part of one single entity. In centralized decision making, one member of the supply chain acquires the other or parties act like merged. The manufacturer and retailer belong to the same corporate entity that minimizes the system-wide costs. System-wide costs (simply obtained by adding up the cost function of each party) of Scenario 1, 2 and 3 are compared and main aim is to identify any potential benefits emerge in centralized decision making. Scenario 3 is taken as a benchmark and the fact that Scenario 3 is not a common application in industry rises from the result that it kills the independence of the firms. Figure 3 illustrates Scenario 3.

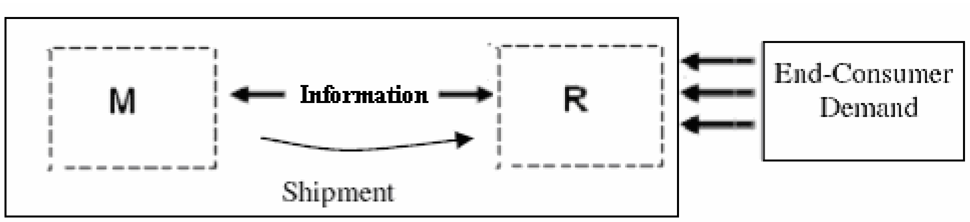


**Figure 3** Centralized Decision Making System

As in Figure 3, manufacturer and retailer are part of one closed system and faces end-consumer's demand as a single corporate identity.

### 3.1 Notation

To keep the notation compact and easy to follow, a single section is provided for each of the decision variables of the retailer's and the manufacturer's, although it may be used in multiple models.

**Indices:**

$t$=1… $T$ (Time Period)

**Parameters:**

| | | |
|---|---|---|
| $d_t^r$ | : | Demand of period $t$ at retailer's site |
| $h_t^r$ | : | Unit holding cost of retailer at period $t$ |
| $b_t^r$ | : | Unit backorder cost of retailer at period $t$ |
| $K_t^r$ | : | Fixed cost of transportation of retailer at period $t$ |
| $d_t^m$ | : | Demand of period $t$ at manufacturer's site (Optimal ordering policy of the retailer) |
| $h_t^m$ | : | Unit holding cost of manufacturer in period $t$ |
| $K_t^m$ | : | Fixed production cost of manufacturer at time $t$ |
| $M$ | : | Big number (which can be defined as $\sum_t \mathrm{D}_t^r$) |
| $SL$ | : | Maximum level on total backorder level over periods |
| $IL$ | : | Maximum level on total inventory level over periods |

**Discrete Decision Variables:**

$X_t^r$ : Quantity ordered in period $t$ in Scenario 1 (In Scenario 2 and 3; $X^r_t$ is quantity dispatched at time $t$)

$I_t^r$ : Inventory level at retailer's site at the end of period $t$

$E_t^r$ : Backorder level at retailer's site at the end of period $t$

$X_t^m$ : Quantity produced at manufacturer's site in period $t$

$I_t^m$ : End period inventory at manufacturer's site at the end of period $t$

**0-1 Variables:**

$$Y_t^r = \begin{cases} 1 & if \quad X_t^r > 0 \\ 0 & otherwise \end{cases} \quad \text{and} \quad Y_t^m = \begin{cases} 1 & if \quad X_t^m > 0 \\ 0 & otherwise \end{cases}$$

**Total Costs:**

*Traditional System: Scenario 1*

$TSC^m$ : Manufacturer's objective function value under traditional system

$TSC^r$ : Retailer's objective function value under traditional system

$TSC$ : System-wide costs under Scenario 1 ($TSC = TSC^r + TSC^m$)

*VMI: Scenario 2*

$VMI^m$ : Manufacturer's objective function value under VMI.

$VMI^r$ : Retailer's objective function value under VMI.

$L(u,k)$ : Relaxed problem's objective function value (Lower Bound on VMI optimal solution).

*VMI* : System-wide costs under Scenario 2 ($VMI = VMI^m + VMI^r$).

$VMI^{up}$ : Upper Bound on VMI optimal solution obtained by *LagRel* and *ObtainFeasible* Heuristic (if *LagRel* outputs infeasible solution).

$VMI^l$ : Relaxed VMI model solved in GAMS with solver CPLEX.

*Centralized System: Scenario 3*

*Cent* : System-wide costs under Centralized System

## 3.2 Scenario 1: Traditional System Model Definition

### 3.2.1 Retailer's Problem

Retailer's operating costs are inventory holding, backordering and fixed cost of transportation under Scenario 1. Retailer decides how much to order at each period to minimize the sum of its operating costs. Every time an order is placed, fixed transportation cost incurs.

**Objective:** Minimize sum of inventory, backorder and fixed order costs

**Model (Problem TS-R):**

$$\text{Minimize TSC}^r = \sum_t K_t{}^r Y_t^r + \sum_t h_t{}^r I_t^r + \sum_t b_t{}^r E_t^r \qquad (1)$$

s.t.

$$X_t^r - I_t^r + E_t^r + I_{t-1}^r - E_{t-1}^r = d_t^r \quad t=1\dots T \qquad (2)$$

$$X_t^r \le M.Y_t^r \quad t=1\dots T \qquad (3)$$

$$I^r{}_0 = E^r{}_0 = 0 \qquad (4)$$

$$X_t^r \in \mathrm{Z}^+,\ Y_t^r \in \{0,1\}, I_t^r, E_t^r \ge 0\ \forall t$$

The objective function and the constraints of the model can be described as follows;

(1) Total costs incurred by the retailer: Transportation cost + Inventory Holding

Cost + Backordering Cost. (2) Balances material flow among periods: End customer's demand should be satisfied/backordered by balancing the quantities backordered stocked and ordered. (3) Ensures binary variable gets a value of 1 if any order is given: If any order is given to the manufacturer in period *t*; $Y_t^r$ equals 1. (4) Ensures starting inventory and backorder levels are zero: There is not any inventory and/or unsatisfied demand at the beginning of the planning horizon.

There are two performance measures that retailer is interested in; total or average inventory (IL) and service level (SL). Retailer does not want to be worse off under VMI from the perspectives of IL and SL. Thus, retailer's optimal policy under Scenario 1 yields levels of these performance measures. While deciding on dispatch quantity and time, manufacturer should not exceed these levels under VMI.

Performance measures are defined as below. Values are obtained from retailer's optimal policy under Scenario 1.

- Total backorder level among periods: $\sum_t E_t^r = SL$
- Total inventory level among periods: $\sum_t I_t^r = IL$

Note that taking averages for each performance measure are also possible by simply dividing the equations by *T*. As discussed before, total backorder level is interpreted as Service Level (*SL*) because fraction of demands that stock out each period (1-$\beta$) yields same solution as total stockout value.

In the optimal policy of retailer in traditional system (optimal solution of TS-R), total backorder level equals to *SL* and total inventory level equals to *IL*. Problem is uncapacitated single item lot sizing problem with the extension backordering.

### 3.2.2 Manufacturer's Problem

Manufacturer's operating costs are fixed cost of production and inventory holding cost. Under Scenario 1, manufacturer also minimizes its costs based on the orders placed by the retailer. Retailer's optimal ordering policy constitutes manufacturer's demand for each period. Manufacturer decides how much to produce, he is forced to produce and ship as much as retailer demands. Backordering is not allowed at manufacturer's site.

**Objective**: Minimize total of inventory holding, fixed production costs.

**Model (Problem TS-M):**

$$\text{Minimize } \ \mathrm{TSC^m} = \sum_t K_t^{\,m} Y_t^{\,m} + \sum_t h_t^{\,m} I_t^m \tag{5}$$

s.t.

$$X_t^m - I_t^m + I_{t-1}^m = d_t^m = X_t^r \quad t\text{=1… } T \tag{6}$$

$$X_t^m \le M.Y_t^m \quad t\text{=1… } T \tag{7}$$

$$I^m{}_0 = 0 \tag{8}$$

$$X_t^m \in \mathrm{Z}^+,\ \ Y_t^m \ \in \{0,1\}, I_t^m \ge 0 \ \ \forall t$$

The objective function and the constraints of the model can be described as follows;

(5) Total costs incurred by the manufacturer: Inventory Holding + Fixed Production Cost, (6) Balances material flow among periods: Retailer's demand (Optimal order quantities obtained by solving TS-R) should be satisfied while balancing the quantities stocked and produced. (7) Ensures binary variable gets a value of 1 if production in the corresponding period is greater than zero: If there is any production in period $t$, $Y_t^m$ equals to 1. (8) Ensures beginning inventory levels are zero: There is not any inventory at the beginning of the planning horizon.

### 3.3 Scenario 2: Vendor Managed Inventory Policy Model Definition

As discussed in Chapters 1 and 2, VMI is a means of collaboration between retailer and manufacturer, where manufacturer can access many kinds of information. Having an access to the unknowns which are not shared in traditional case, eases to manage both retailer's and manufacturer's inventory and decisions.

Under Scenario 2, manufacturer can access to demand forecast of retailer. In the traditional system, retailer's optimal ordering policy yields optimal inventory levels and optimal backorder levels. These two levels are imposed as two constraints under VMI model because retailer should be better off under VMI. Thus, VMI system is in the context of Two-Echelon Constrained Uncapacitated Lot Sizing problem with Backordering (C-ULS-B).

Retailer delegates dispatch decision to the manufacturer but requires that total inventory level among periods should not exceeded IL and service level (SL) is satisfied.

- Total backorder level among periods should not exceed a pre-specified (specified in Scenario 1) service level SL. Thus; $\sum_t E^r{}_t \leq SL$
- Total inventory level among periods should not exceed a pre-specified (specified in Scenario 1) inventory level IL. Thus; $\sum_t I^r{}_t \leq IL$

**Objective**: Minimize sum of inventory holding cost of the manufacturer, fixed replenishment cost and fixed dispatch cost. Manufacturer takes control of dispatch decisions and in turn pays all of the dispatch cost.

**Model (VMI Problem: C-ULS-B):**

$$\text{Minimize VMI}^{\text{m}} = \sum_t K_t{}^r Y_t^r + \sum_t K_t{}^m Y_t^m + \sum_t h_t{}^m I_t^m \qquad (9)$$

s.t.

$$X_t^r - I_t^r + E_t^r + I_{t-1}^r - E_{t-1}^r = d_t^r \quad t=1\ldots T \qquad (10)$$

$$X_t^m - I_t^m + I_{t-1}^m = X_t^r \quad t=1\ldots T \qquad (11)$$

$$X_t^m \leq M.Y_t^m \quad t=1\ldots T \qquad (12)$$

$$X_t^r \leq M.Y_t^r \quad t=1\ldots T \qquad (13)$$

$$\sum_t E^r{}_t \leq SL \quad t=1\ldots T \qquad (14)$$

$$\sum_t I^r{}_t \leq IL \quad t=1\ldots T \qquad (15)$$

$$I^r{}_0 = E^r{}_0 = I^m{}_0 = 0 \tag{16}$$

$$X_t^m,\ X_t^r \in Z^+,\ Y_t^m,\ Y_t^r \in \{0,1\}, I_t^r, E_t^r, I_t^m \geq 0 \quad \forall t$$

The objective function and the constraints of the model can be described as follows;

(9) Total costs incurred by the manufacturer under VMI: Inventory Holding + Production Cost + Transportation Cost (10) Ensures the balance of material flow at retailer's site among periods: End customer's demand should be satisfied/ backordered by balancing the quantities backordered stocked and dispatched (11) Ensures the balance of material flow at manufacturer's site among periods: Retailer's demands should be satisfied while balancing the quantities stocked and produced (12) and (13) are the constraints ensuring that if any production/dispatch takes places, fixed cost occurs. (14) Assures that total amount of backorder at retailer's site must be smaller than a pre-specified amount. (15) Assures total amount of inventory among periods at retailer's site must be less than a pre-specified amount. (16) Ensures all beginning inventory and backorder levels are zero.

Both *SL* and *IL* limits are to be obtained from the optimal policy of retailer under Scenario 1 (Optimal solution of Problem TS-R).

### 3.4 Centralized System Model Definition

In this section, "Two-Echelon ULS Problem with Backordering (ULS-B)" is dealt. Manufacturer and retailer act like merged. They have one single objective and they follow a coordinated decision making policy. Their objective is minimizing sum of manufacturer's and retailer's total costs.

**Objective**: Minimize sum of $TSC^r$ and $TSC^m$

**Model (Centralized Problem: ULS-B):**

Minimize

$$Cent = \sum_t K_t{}^m Y_t^m + \sum_t h_t{}^m I_t^m + \sum_t K_t{}^r Y_t^r + \sum_t h_t^r I_t^r + \sum_t b_t{}^r E_t^r \tag{17}$$

s.t.

$$X_t^r - I_t^r + E_t^r + I_{t-1}^r - E_{t-1}^r = d_t^r \quad t=1\ldots T \tag{18}$$

$$X_t^m - I_t^m + I_{t-1}^m = X_t^r \quad t=1\ldots T \tag{19}$$

$$X_t^m \le M.Y_t^m \quad t=1\ldots T \tag{20}$$

$$X_t^r \le M.Y_t^r \quad t=1\ldots T \tag{21}$$

$$I^r{}_0 = E^r{}_0 = I^m{}_0 = 0 \tag{22}$$

$$X_t^m,\ X_t^r \in Z^+,\ Y_t^m,\ Y_t^r \in \{0,1\}, I_t^r, E_t^r, I_t^m \ge 0 \quad \forall t$$

The objective function and the constraints of the model can be described as follows;

(17) Total costs of retailer and manufacturer: Summation of objective function elements of traditional retailer and manufacturer's problems (18) Ensures the balance of material flow at retailer's site among periods: End-customer demand should be satisfied/backordered by balancing the quantities backordered stocked and dispatched (19) Ensures the balance of material flow at manufacturer's site among periods: End-consumer demand should be satisfied while balancing the quantities stocked and produced (20) and (21) are the constraints ensuring that if any production/dispatch takes places, fixed cost occurs. (22) Ensures that all beginning inventory and backorder levels are zero.

# CHAPTER 4

# A SOLUTION ALGORITHM FOR TWO-ECHELON C-ULS-B PROBLEM

In this chapter, an algorithm to solve Two-Echelon C-ULS-B is suggested. A polynomial time algorithm (Dynamic Programming Algorithm) has already been developed in order to solve optimally the Two-Echelon ULS-B and there exists a study about the algorithm in the literature. (Lee, Çetinkaya and Jaruphongsa, 2003) The main difference between Two-Echelon C-ULS-B and ULS-B is that two performance measures (total inventory and backorder level among planning horizon *T*) which are imposed as constraints to the C-ULS-B.

In Scenario 2, there are two complicating constraints imposed as retailer's performance measures to the model. Without these constraints, problem is in ULS-B context. ULS-B problem can be solved in polynomial time (Lee, Çetinkaya and Jaruphongsa, 2003). Imposing these constraints yields a harder problem to solve. Thus, main aim of this chapter is offering an algorithm to obtain a efficient feasible solution for C-ULS-B problem while improving solution times of the C-ULS-B problem. Two complicating constraints in C-ULS-B problem are relaxed to obtain a lower bound for the optimal solution while Lagrangean multipliers are updated using subgradient optimization technique. Relaxed model is also solved in GAMS 2.0.13.0 with solver CPLEX 7.0 but solution time is too high, thus a more efficient algorithm is proposed in this chapter.

For the remainder of the chapter, outline of the sections can be summarized as follows. In Section 4.1, complicating constraints of C-ULS-B are relaxed and model definition of relaxed C-ULS-B problem is made. In Section 4.2, subgradient algorithm is proposed to update the Lagrangean multipliers.

In Section 4.3, a Dynamic Programming algorithm is proposed in order to improve the optimal solution time of the MIP model under VMI (C-ULS-B).Solution times of MIP model and the proposed Dynamic Programming based Lagrangean Relaxation algorithm are compared in Chapter 5. In Section 4.4, ULS-B problem is discussed and polynomial time algorithm is suggested to solve the problem optimally.

## 4.1 Relaxed model of C-ULS-B

In this section, two complicating constraints of Scenario 2 are relaxed and a Dynamic Programming (DP) based Lagrangean Relaxation algorithm is developed.

**Parameters:**

Parameters are defined in Section 3.1.

Backorder level (14) and inventory level (15) constraints are relaxed. Lagrangean function is named as L (u,k) (Fisher,1981).

**Model (Problem VMI-R):**

$$\text{Minimize } L(u,k) = \sum_t Y_t^m K_t^m + \sum_t Y_t^r K_t^r + \sum_t h_t^m I_t^m + k\left(\sum_t I^r{}_t - IL\right) + u\left(\sum_t E^r{}_t - SL\right) \tag{23}$$

s.t.

$$X_t^r - I_t^r + E_t^r + I_{t-1}^r - E_{t-1}^r = d_t^r \quad t=1\ldots T \tag{24}$$

$$X_t^m - I_t^m + I_{t-1}^m = X_t^r \quad t=1\ldots T \tag{25}$$

$$X_t^m \le M.Y_t^m \quad t=1\ldots T \tag{26}$$

$$X_t^r \leq M.Y_t^r \quad t=1\ldots T \tag{27}$$

$$I^r{}_0 = E^r{}_0 = I^m{}_0 = 0 \tag{28}$$

$$X_t^m,\ X_t^r \in Z^+,\ Y_t^m,\ Y_t^r \in \{0,1\}, I_t^r, E_t^r, I_t^m \geq 0 \ \ \forall t,\ u,k \geq 0$$

Where u and k are Lagrangean multipliers for backorder and inventory level constraints correspondingly.

## 4.2 Subgradient Algorithm: How to update multipliers u and k

A formal definition for subgradient algorithm (Fisher, 1981) used in this section can be summarized step by step as follows:

**Table 1** Subgradient algorithm steps

*{Input}*

*An upper bound Lj (explained in Section 4.3)*

*Initial values* $u^0$ *and* $k^0$ $\{u^0 = 0, k^0 = 0\}$

*{Initialization}*

$\theta_0 := 2$

*{Subgradient iterations}*

**For** *j:=0,1,… do*

$\gamma^j := -SL + \left\{\sum_t E^r{}_t\right\}^j$ *// Gradient value is computed*

$\zeta^j := -IL + \left\{\sum_t I^r{}_t\right\}^j$ *// Gradient value is computed*

**Table 1** (continued)

$t_j := \theta_j \left(L^j - L(u,k)^j\right) \big/ \left((\gamma^j)^2 + (\zeta^j)^2\right)$ //where $t_j$ is stepsize

//$L^j$ is upperbound value updated (Section 4.3)

// $L(u,k)^j$ is objective value of VMI-R(Section 4.4)

$u^{j+1} := Max\{0, u^j + t_j \gamma^j\}$

$k^{j+1} := Max\{0, k^j + t_j \zeta^j\}$

**If** $| u^{j+1} - u^j | < 10^{-7}$ AND $| k^{j+1} - k^j | < 10^{-7}$ **Then**

Stop

**End If**

**If** no progress in more than N iterations **Then**

$\theta_{j+1} := \theta_j / 2$

**Else**

$\theta_{j+1} := \theta_j$

**End If**

$j := j+1$

**End For**

Number of iterations is limited to 50 ($N$=50) and the model converges whenever conditions above are satisfied. Subgradient algorithm is used to update multipliers $u$ and $k$. In order to obtain $L(u,k)^j$ at iteration j, Dynamic Programming algorithm discussed in Section 4.4 is called by the subgradient algorithm.

Algorithm can be summarized by the following steps:

1. If there is not any improvement in the objective value for the last five iterations, $\theta_j$ is divided by 2; in order to proceed with a tinier step size.
2. Gradient values for each constraint are calculated.
3. Upperbound is updated as discussed in Section 4.3.
4. Step size is calculated.
5. Lagrange multipliers are updated.
6. If the absolute values of the updated multipliers are both less than $10^{-7}$, algorithm converges.

### 4.3 How to update $L^j$

In subgradient algorithm, there is a dynamic upperbound revised at each iteration.

In Chapter 5, while doing computational experiment it is assumed that $h^r{}_t$, $h^m{}_t$, $b^r{}_t$, $K^m{}_t$ and $K^r{}_t$ are constant over time. Thus; $L^j$ is can be formulated as;

$$L^j = L(u,k)^j + b^r \,|\sum_t E_t^r - SL\,| + h^r \,|\sum_t I_t^r - IL\,| \qquad (29)$$

At each iteration, there is a dynamic upperbound and value of the upperbound changes with the solutions and objective value of the relaxed problem.

### 4.4 How to obtain $L(u,k)^j$

In order to obtain $L(u,k)^j$, a DP based Lagrangean Relaxation algorithm is proposed and named as *LagRel*.

General case of C-ULS-B problem is handled by Lee, Cetinkaya and Jaruphongsa in 2003 and a polynomial time algorithm is suggested to obtain optimal solutions. Lee, Cetinkaya and Jaruphongsa (2003)'s study is named as Generalized Problem for the rest of the thesis. The authors study two-echelon inventory replenishment and dispatch scheduling policy under deterministic dynamic demand and the problem they consider is in context of Two-Echelon ULS-B. The authors represent the problem using Two-Echelon Dynamic Lot Sizing Model. They present several structures of the optimal solutions and propose a polynomial time algorithm for computing optimal solution. The authors study same structure as in Scenario 3;

which is also named as ULS-B. Hence, Scenario 3 can also be optimally solved in polynomial time.

**Relaxed Problem of C-ULS-B: How to solve problem VMI-R**

As mentioned earlier, we consider a manufacturer who ships his final products to a retailer that serves end-customer. Retailer and manufacturer have VMI agreement, and manufacturer ensures that retailer has total inventory and backorder levels which are lower than pre-specified amounts. Manufacturer's problem is to find production and dispatch quantities that minimize costs under VMI partnership. When two complicating constraints are relaxed, C-ULS-B problem has a similar structure to Lee, Cetinkaya and Jaruphongsa (2003)'s study. If Generalized Problem and C-ULS-B are compared; similar structure can easily be noticed. Multipliers $u$ and $k$ stands for unit backorder and inventory holding costs where the constraints of both problems are same. In Section 4.4.1, C-ULS-B being a special case of Generalized Problem becomes apparent. In order to reduce the solution time, an algorithm to solve C-ULS-B is proposed and as two complicating constraints in C-ULS-B are relaxed, lower bound and upper bound on the optimal solution of VMI problem can be obtained. Lagrangean multipliers are updated by the subgradient algorithm and the relaxed problem is solved by the DP algorithm where the hybrid of Lagrangean Relaxation and DP algorithms is used to obtain efficient bounds on VMI optimal solution and algorithm is called *LagRel*. Therefore, *LagRel* algorithm obtains a lower bound for the relaxed problem that has an objective to minimize; fixed cost of production, fixed cost of transportation, holding cost of manufacturer and backorder and inventory levels multiplied by $u$ and $k$ respectively.

In brief, in order to solve VMI-R problem, *LagRel* algorithm is proposed which is a hybrid of Lagrangean Relaxation and Dynamic Programming algorithms. *LagRel* constitutes of Subgradient and DP algorithms. Subgradient algorithm calls DP algorithm which obtains a lower bound ($L(u,k)^j$) on the optimal objective value of C-ULS-B problem.

*Outline of subsections in Section 4.4*

In section 4.4.1, model definition of Generalized Problem is made. In this section, Two-Echelon C-ULS-B problem is also presented to be a special case of the Generalized Problem. Problem structure of the Two-Echelon C-ULS-B problem is also discussed in section 4.4.1. In section 4.4.2, optimality properties that ease the solution of the C-ULS-B problem are presented. In 4.4.3 network representation of C-ULS-B problem is defined. The pseudo code of the *LagRel* algorithm can be found in Appendix B.

### 4.4.1 Model Definition and Properties of Generalized Problem

Lee, Çetinkaya and Jaruphongsa (2003) examined a two-stage supply chain consisting of a Third Party Warehouse (TPW) and a Distribution Center (DC). They consider a manufacturer who ships the final products to a Third Party Warehouse serving a distribution center with deterministic time varying demand. The optimal decision is how often and in what quantities to replenish the stock at an upstream supply chain member (TPW), and how often to release an outbound shipment to a downstream supply-chain member (DC). The demands at the distribution center can be shipped earlier or later than they are required.

The model of authors' problem is also formulated in notation defined in Section 3.1. It is easier to recognize the resemblance when variables and parameters are defined in this notation. TPW can be regarded as manufacturer and DC can be regarded as retailer. Visualization of the problem context can be found in Figure 4. If Figures 3 and 4 are compared, the fact that centralized problem and generalized problem having the same structure is easily recognized.

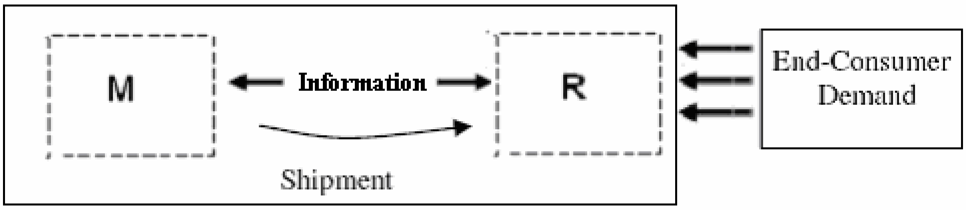


**Figure 4** Generalized Problem's Context

**Model Definition:**

Let T denote the length of planning horizon. For $t$=1,…,$T$, following defined.

**Notation:**

$d_t^r$ : denotes the demand at the DC at period $t$.

$X_t^m$ : denotes the replenishment quantity that is received at the TPW at period $t$.

$X_t^r$ : denotes the dispatch quantity at DC in period $t$.

$I_t^m$ : denotes the inventory level at the TPW at the end of period $t$.

$I_t^r$ : denotes the inventory level at the DC at the end of period $t$.

$E_t^r$ : denotes the backorder level at the DC at the end of period $t$

$h_t^m$ : denotes the unit holding cost at TPW in period $t$.

$h_t^r$ : denotes the cost of holding one unit in period $t$ at the DC.

It is assumed that $h_t^m \leq h_t^r$

$b_t^r$ : denotes the unit backordering cost in period $t$ at DC.

$K_t^m$ : denotes the fixed cost of a replenishment at the TPW in period *t*.

$P(X_t^r)$ : denotes the cost of dispatching units from the TPW to the DC in period $t$ .

Where;

$$P(X_t^r) = \begin{cases} 0 & X_t^r = 0 \\ S + nA & (n-1)W < X_t^r \leq nW \end{cases}$$

S is fixed cost for each delivery , W is the capacity of each cargo , A is the cost of delivering a cargo, n is number of cargos used.

**Model (Generalized Problem):**

$$\text{Minimize DP} = \sum_t K_t^m Y_t^m + \sum_t P(y_t) + \sum_t h_t^m I_t^m + \sum_t h_t^r I_t^r + \sum_t b_t^r E_t^r \quad (30)$$

s.t.

$$X_t^m - I_t^m + I_{t-1}^m = X_t^r \quad t=1\ldots T \quad (31)$$

$$X_t^r - I_t^r + E_t^r + I_{t-1}^r - E_{t-1}^r = d_t^r \quad t=1\ldots T \quad (32)$$

$$I^r_0 = I^r_T = E^r_0 = E^r_T = I^m_0 = I^m_T = 0 \quad (33)$$

$$I_t^r, E_t^r, I_t^m, X_t^r, X_t^m \geq 0 \quad \forall t \quad (34)$$

Where $Y_t^m$ =1 when $X_t^m$ > 0, 0 otherwise.

If the problem handled is as C-ULS-B (Two-Echelon Constrained ULS problem with Backordering) and compared with Generalized Problem, similar structure can easily be seen except per cargo cost. In C-ULS-B, transportation capacity is assumed to be very large thus only fixed cost of transportation is incurred. This is the only condition that makes C-ULS-B a special case of Generalized Problem. Moreover, if (23) and (29) are compared, it can easily be recognized that *u* and *k* stands for $b_t^r$ and $h_t^r$ correspondingly and constraints of the problems are same. Thus, a special case of Generalized Problem is handled while solving relaxed VMI problem. $L(u,k)^j$ is obtained by Dynamic Programming algorithm, a special case of the suggested algorithm in Lee, Cetinkaya and Jaruphongsa (2003) while *u* and *k* are updated by subgradient algorithm discussed in Section 4.2.

### 4.4.2 Optimality Properties of C-ULS-B

In this section, optimality conditions of Generalized Problem are discussed and additional properties emerged from Two-Echelon C-ULS-B problem characteristics are also expressed. After examining optimality properties, network representation of C-ULS-B problem (Scenario 3) and necessary definitions are made.

**Definitions**:

Period t is called:

- *Production (replenishment) period* if $X^m{}_t > 0$,
- *Dispatch period* if $X^r{}_t > 0$,
- *Manufacturer regeneration point* if $I_t^m = 0$,
- *Customer(retailer) regeneration point* if $I_t^r = 0$.

Optimal solution should satisfy the properties defined below. The following optimality properties are used to develop an efficient algorithm to solve the problem.

*Property 1*:

A-There exists an optimal solution such that $I^m{}_{t-1}.X^m{}_t=0$ for all $t=1,\ldots, T$ ( If we have production at period t ($X^m{}_t > 0$) then $I^m{}_{t-1} = 0$) (Wagner and Whitin,1959)

**Proof:** If $I^m{}_{t-1} > 0$ , there is production a previous period k such that $k<t$. Thus, production amount of period k can be reduced by $I^m{}_{t-1}$ and production amount of t can be increased by $I^m{}_{t-1}$. This can result in only reduction in inventory holding cost.

B- There exists an optimal solution such that $I^r{}_{t-1}.\ X^r{}_t = 0$ for all $t = 1,\ldots,T$ (If we have order at period t ($X^r{}_t > 0$) then $I^r{}_{t-1} = 0$. Additionally note that for the solution to be sensible, $I^r_t$ and $E^r_t$ are not both positive in the same period $t$) (Wagner and Whitin,1959)

**Proof:** This is an additional property emerged from the specialty that there is not transportation capacity limitation in the problem this thesis deals. Suppose that $I^r_{t-1}$ > 0. Then there exists a previous dispatch period named as period $k$. Dispatch amount at $k$ can be reduced by $I^r_{t-1}$ and the dispatch amount at period $t$ can be increased by the same amount. Doing so will reduce the total inventory holding cost at retailer and will not affect the dispatch schedule and the corresponding costs. Backordering is allowed at retailer's site, thus when an order is placed, either retailer does not have any inventory or she has backordered demand.

*Property 2:*

An optimal solution exists such that manufacturer produces only when an order is taken. This means, a production period is also called dispatch period.

(For a given t, $X^m_t > 0$ only if $X^r_t > 0$)

**Proof**: If not (i.e. $X^r_t = 0$), production at period t can be delayed to the next dispatch period. In this way, holding cost at the manufacturer's site decreases while all other costs do not change.

*Property 3:*

There exists an optimal solution such that for each $k=1,\ldots,T$, either

$X^m_k = X^r_k + X^r_{k+1} + \ldots + X^r_l$ for some $l \geq k$, or $X^m_k = 0$.

**Proof:** Property 3 follows from properties 1 and 2.

*Property 4:*

Each period's demand is solely satisfied with only one dispatch.

**Proof:** Dispatch cost increases when each period's demand is not satisfied by only one dispatch. Suppose at $k$, there are two dispatches to satisfy the demand. Since there is no capacity limit on transportation, eliminating one dispatch and increasing the other dispatch quantity decreases total transportation cost.

*Property 5:*

There exists an optimal solution such that if *l-1* and *m* are two consecutive customer regeneration points, then there is one dispatch period between *l* and *m*. This means, a dispatch cannot be used solely to satisfy the backorders.

**Proof:** Proof is based on Theorem 2 in Lee, Çetinkaya and Jaruphongsa (2003). In Theorem 2, it is concluded that there is only one less than truck load period between *l* and *m*. In our case, all dispatch periods are less than truck load period because there is not capacity limit on transportation.

### 4.4.3 Network Representation of Two-Echelon C-ULS-B

(j,b) denotes an arc between nodes *j* and *b* such that *b>j*. We create nodes for each period (j) for *j=1,...,T* and also create a dummy node (0). We intend to construct the network in which the shortest path from node (0) to node (j) gives the minimum cost solution for satisfying demands from periods *i* to *j* by a single production between *i* and *j* where there exist optimal dispatch schedule from periods *i* to *j*. Figure 5 shows the network representation of the problem for *T*=4.

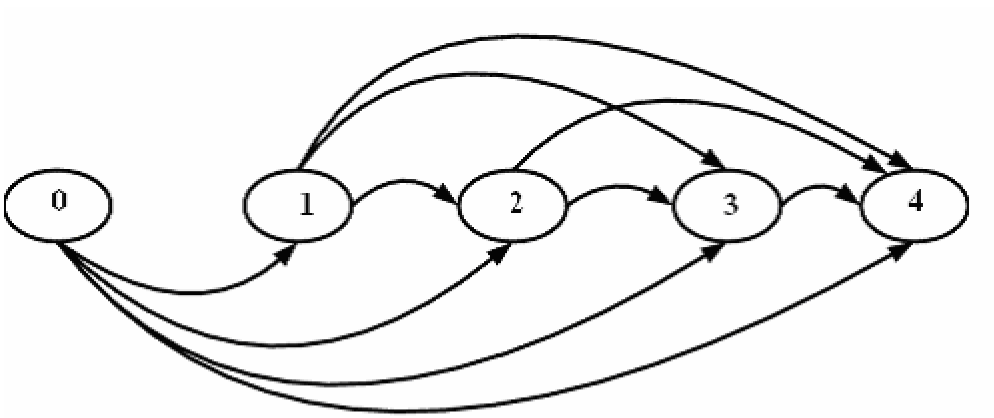


**Figure 5** Network Representation of ULS Problem for T=4

**A polynomial time algorithm for calculating all arc costs**

To find arc cost C(i,j), arc $(i,j)_k$ is defined as the problem of finding the minimum total cost solution for satisfying the demands of periods $i+1$ to $j$, ($D_{i+1}, D_{i+2,....,}D_j$) by a production period at $k$. $C_k(i,j)$ is also defined as the corresponding minimum total cost where $i<k\leq j$. $C_k(i,j)$ is defined as $\infty$ for other values of $k$

$C(i,j) = Min \{ C_k(i,j) : i < k \leq j\}$

Thus, $C_k(i,j)$ is to be found for all possible $i, k, j$. A subproblem is defined where $0\leq s<t<T$ with $1\leq k\leq t$ as the problem of finding the minimum total cost solution for using a production at period $k$ to satisfy the demands of periods $i+1$ to $j$.

In the customer subproblem $(s, t)$, since the replenishment period is fixed to be at $k$ , the problem is to find a dispatch schedule for satisfying the demands $s+1$ to $t$. The minimum total cost of the customer subproblem $(s, t)$ is denoted by $g_k (s, t)$ and it includes manufacturer inventory holding cost, dispatch costs, backorder and inventory costs but it does not include the fixed cost of production at the manufacturer. In brief, the minimum cost solution of the $(i, j)_k$ problem may be made up of several subproblem solutions where $i\leq s< t \leq j$ and $1 \leq k \leq t$.

If there is no retailer regeneration point in the optimal solution of the (i,j) problem then;

$$C_k (i, j) = K^m + g_k (i, j)$$

In general;

If $g_k (s, t)$ values for all $0 \leq s < t \leq T, 1 \leq k \leq t$ exist, $C_k (i, j)$ can be found by using the recursive equation given below.

For $i=0...T$

$j = i+1,...,T$

$k = i+1,....,j$

$C_k (i, j) = Min \{K^m + g_k (i, j); min\{C_k(i, t) + g_k(t, j): k \leq t <j\}$

All possible $g_k(s,t)$ should be computed for all $0\leq s< t \leq T, 1 \leq k \leq t$ as a first step. Then all values of $C_k (i, j)$ *for* $0 \leq i< j \leq T$ and $i < k \leq j$ can be found.

To summarize the calculation steps, shortest path between i=0 and j=3 is investigated. (Figure 6)

Description of the calculation steps and their order of complexity are briefly given below.

- Generate the nodes and construct the network.(T(T-1)/2 arcs ~$O(T^2)$)
- Generated arcs and definitions are:
    - C(0,1) stands for satisfying first periods' demand by a production at the beginning of first period.
    - C(0,2) stands for satisfying first and second periods' demands by a production between periods 1 and 2 (k=1 or 2).
    - C(0,3) stands for satisfying first, second and third periods' demands by a production between periods 1 and 3 (k=1,2 or 3)
    - C(1,2) stands for satisfying second periods' demand by a production at the second period (k=2)
    - C(2,3) stands for satisfying third periods' demand by a production at the third period (k= 3)
    - C(0,3) stands for satisfying second and third periods' demands by a production between periods 2 and 3 (k=1,2 or 3)
- To be able to calculate each $C_k(i,j)$, all possible $g_k(i,j)$ values should have been obtained.($O(T^2)$)
    - To illustrate, while calculating $C_k(0,3)$;
        - $C_k(0,3)$= Min $\{K^m + g_k(0,3); \min\{C_k(0,t) + g_k(t,3):k\leq t<3\}$
- For calculating each arc cost, all possible production periods are determined and minimum cost solution for each arc is selected as arc cost. (O(T))
    - To illustrate, while calculating C(0,3) ; k $\in$ ( 1,2,3)
    - C((0,3)) = Min $\{ C_k(0,3) : 0< k \leq 2\}$

Thus, complexity of the algorithm is O $(T^5)$.

The assumption of $I^r_0 = E^r_0 = I^m_0 = 0$ eliminates many nodes on the network and makes the solution of shortest path problem easier. If beginning of planning horizon inventory and backorder levels are allowed to be greater than zero, at each period there should be many nodes representing possible beginning of period inventory values and backorder values. Therefore, if the assumption regarding beginning period inventory and backorder levels is not considered, complexity of the problem would be higher.

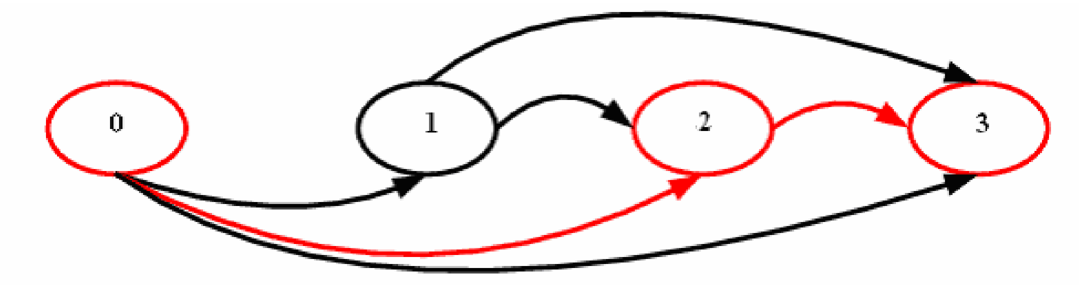


**Figure 6** Shortest Path of T=3; (0,2,3) represents shortest path

In summary, Lagrangean Relaxation based algorithm is proposed to obtain a near-optimal solution for C-ULS-B problem. C-ULS-B problem points a similar structure and it is a special case of Generalized Problem, to which a polynomial time algorithm (Dynamic Programming Algorithm) is proposed by Lee, Cetinkaya and Jaruphongsa in 2003. While solving Two-Echelon C-ULS-B problem, two complicating constraints of the problem are relaxed and Lagrangean multipliers *u* and *k* are updated by subgradient algorithm, $L(u,k)^j$ is obtained by Dynamic Programming algorithm and thus combination of Lagrangean relaxation and Dynamic programming algorithms is proposed to obtain bounds on the optimal solution and named as *LagRel*.

Special case of Lee, Cetinkaya and Jaruphongsa (2003)'s study is defined in Section 4.4.1 and similarities between Generalized Problem and C-ULS-B are established.

In Section 4.4.2, optimality conditions of C-ULS-B are presented. In Section 4.4.3, network representation of C-ULS-B problem is discussed and calculation of arc costs are explained.

## 4.5 How to convert infeasibility to feasibility: Upper Bound Heuristic

When relaxation algorithm converges, infeasible solutions are obtained during experimental analyses. Steps of the algorithm used to convert infeasible solution into feasible solution is proposed are discussed as below.

*ObtainFeasible* algorithm can be summarized by the following steps;

1- Decision variable values of the infeasible solution are given as inputs to the algorithm.

   a. Production Quantities ($X^{m}{}_{t}$)

   b. Order Quantities($X^{r}{}_{t}$)

   c. Desired Backorder Level (*SL*)

   d. Desired Inventory Level (*IL*)

   e. $I_t^m, I_t^r, E_t^r$ are calculated by *ObtainFeasible*

2- The algorithm runs until the feasible solution is obtained and cost effectiveness cannot be improved, there is a loop in which total inventory and total backorder values are compared with the desired levels *IL* and *SL*. Whenever the desired values are obtained, algorithm focuses on decreasing the manufacturer cost, by keeping solution in feasible region.

3- Infeasibility gaps are checked. ($\sum_t E_t^r - SL$ and $\sum_t I_t^r - IL$)

4- If there is more infeasibility in Inventory Constraint (i.e. $|\sum_t I_t^r - IL| > |\sum_t E_t^r - SL|$)

Go to Step 5, else step 7. (We start with the worst one, this forces algorithm to obtain feasibility faster)

5- If $\sum_t I_t^r > IL$

a. Max ($X_t^r$- $D_t$) value is found among periods and call this period k

b. **If** possible;

update $X_t^r$ and $X_t^m$ as $X_t^r = X_t^r - (X_t^r - D_t)$ and $X_t^m = X_t^m - (X_t^r - D_t)$

c. **Else** while going back from k to k-1, k-2...i update $X_t^r$ and $X_t^m$ till ($X_t^r$- $D_t$) is zero.

6- Infeasibility gaps are checked. ($\sum_t E_t^r - SL$ and $\sum_t I_t^r - IL$) , if feasible go to step 8 else step 4.

7- If $\sum_t E_t^r > SL$

a. The backorder value of the period causes highest backorder (period *k*) is added to $X_t^r$ and $X_t^m$. (Max ($D_t$ - $X_t^r$) value is found among periods and call this period k)

b. Infeasibility gaps are checked. ($\sum_t E_t^r - SL$ and $\sum_t I_t^r - IL$)

8- Algorithm runs at most by a certain number of iterations (in our case the maximum number of iteration is 1000); this is controlled by counting the number of iterations. (*During the experimental study this condition has never been reached but there should be a condition to avoid infinite loops*)

9- Cost effectiveness is checked.

a. Is any improvement on the solution achievable? Algorithm focuses on decreasing the manufacturer's cost.

i. It pulls the solution to feasible region, heuristically algorithm forces inventory or backorder level to increase as soon as the performance measures are satisfied. If, in step 5, inventory level is decreased more than necessary ($\sum_t I_t^r < IL$) then total inventory level is forced to approach to *IL* while reducing total cost. If total cost can be reduced while increasing inventory level till *IL*, values of decision variables are updated accordingly. If it cannot be reduced then algorithm continues with step 10.

ii. If backorder level is decreased more than necessary ($\sum_t E_t^r$ <SL) then backorder level is increased and tried to reach to *SL* while reducing total cost. If total cost can be reduced while increasing backorder level till *SL*, values of decision variables are updated accordingly.

10- Algorithm stops if there cannot be any improvement on the cost *and* inventory level is reached *and* backorder level is reached. $VMI^{up}$, (manufacturer's cost obtained by *ObtainFeasible*) value is recorded.

Since this algorithm is only a heuristic, it does not guarantee optimality. Pseudo code of the algorithm is presented in Appendix A.

**Example 4.1**:

Infeasible solution's outputs are taken as inputs to *ObtainFeasible.*

Infeasible solutions result is presented to track the changes in costs during *ObtainFeasible* heuristic.

*Infeasible results:*

Objective Value: 2287

T=12

Parameters are taken as constant among the planning horizon *T*.

$K^r$: 50, $K^m$: 500, $h^m$: 1, $h^r$: 3, $b^r$: 1

**Inputs** to *ObtainFeasible*

$D_t$: [81 54 69 15 93 160 39 57 90 55 55 64]

$X^r_t$: [81 54 69 0 108 160 39 0 147 55 55 64]

$X^m_t$: [204 0 0 0 307 0 0 0 321 0 0 0]

*IL: 0*

*SL: 54*

$E^{r}_{t}$ ,$I_t^m$ and $I_t^r$ are calculated by *ObtainFeasible* algorithm. Steps are numbered according to the steps discussed above.

**Table 2** Explanation of Example 4.1

| Step# | Explanation |
|---|---|
| **Step 0** | Total backorder level of infeasible solution is 72, inventory level is 0 according to the inputs $D_t$ , $X^{r}_{t}$ and $X^{m}_{t.}$ |
| **Step 4** | Find the reason of infeasibility: Backorder Level<br>72 > 54<br>Record: Backorder level: INFEASIBLE, Inventory level: FEASIBLE |
| **Step 7 a** | Find the period of maximum ($D_t$-$X^r$) (i.e.the period that creates maximum backorder)<br>$D_8$-$X_8^r$=57 is the maximum value, k = 8<br>Update production and order quantities<br>$X_t^r = X_t^r + (D_t - X_t^r)$<br>$X_8^r = X_8^r$+57 =0+57=57<br>$X_t^m = X_t^m + (D_t - X_t^r)$<br>$X_8^m = X_8^m$ +57=0+57=57 |
| **Step 7 b** | Total Inventory = 228, Total Backorder = 15 |
| **Step 8** | Backorder level: FEASIBLE, Inventory level: INFEASIBLE |
| **Step 4** | Find the reason of infeasibility: Inventory Level<br>228>0 |

**Table 2** (continued)

| | |
|---|---|
| **Step 6** | Find the period of maximum ($X_t^r - D_t$) (i.e. the period that creates maximum inventory)<br>$X_8^r - D_8$=57 is the maximum value, $k$=8 |
| **Step 6a** | Update production and order quantities (while they are greater than zero[1])<br>$X_9^r = X_9^r - (X_t^r - D_t)$, $X_9^r = X_9^r$ -57 =147-57=90<br>$X_9^m = X_9^m - (X_t^r - D_t)$, $X_9^m = X_9^m$ -57=321-57=264 |
| **Step 6b** | Total Inventory= 0, Total Backorder= 15 |
| **Step 6c** | Backorder level: FEASIBLE, Inventory level: FEASIBLE<br>Go to step 9 to reduce cost. |
| **Step 9** | Check Cost Effectiveness, Manufacturer's Objective: 2.787 |
| **Step 10.a.ii** | Check if there can be any improvement on the cost<br>Find feasibility gaps and compare them<br>$IL-\sum_t I^r{}_t$ =0-0=0, $SL-\sum_t E^r{}_t$ =54-15=39, 39>0<br>Focus on increasing the backorder level till 54.<br>Find minimum $X^r_t$, (<>0), Min($X^r_t$) = $X^r_7$= 39<br>Check demands till zero demand (if any) or till the first period; find the smallest demand. Find $X^r_5$=108; k=5.<br>Check $(T-k)*X^r_7 <= SL-\sum_t E^r{}_t$ , Is (12-5)*39<=39<br>Make no change |
| **Step 11** | Feasibility is achieved, record total cost as 2787, algorithm stops. |

1 If $X_t^m - (X_t^r - D_t) < 0$ ; check the previous periods and find the nearest period k that creates amount of ($X_k^r - D_t$) positive gap, and reduce the production of period k by ($X_t^r - D_t$) - $X_t^m$, decrease the production of period t by $X_t^m$. In summary, subtract the positive gap by searching previously on the periods so as to minimize inventory cost.

In Chapter 5, computational analysis is done and discussed. In order to evaluate the lower bound (*L(u,k)*) and upperbound (Objective value of feasible solution) performances; optimal objective values and objective values of Lagrangean relaxation model (solved in GAMS with solver CPLEX and solved by *LagRel* Algorithm) are compared in Chapter 5 and Appendix C. Moreover, for the experiments that produced infeasible solutions, performance of the *ObtainFeasible* heuristic is also evaluated in Chapter 5. It is shown that *LagRel* algorithm improves solution time by 1/3 on the average.

### 4.6 Model Definition and Properties of ULS-B

Two-Echelon ULS-B problem (Scenario 3) is also solved via Dynamic Programming algorithm and optimal solutions are obtained in polynomial time. If objective terms of Generalized and Centralized Problems are checked, it is easily figured out that they are same except variable cost of transportation. Constraints of both problems are same. Thus, ULS-B is a special case of Generalized Problem and can be solved optimally in polynomial time.

# CHAPTER 5

# COMPUTATIONAL EXPERIMENTS AND RESULTS

Computational experiments are conducted in order to analyze how the system parameters affect manufacturer's and retailer's savings and system-wide costs of VMI, Traditional and Centralized system.

Main initiatives for establishing computational experiments are;

- Identify the conditions manufacturer saves under VMI agreement.
- Identify the conditions that increase retailer's savings under VMI.
- Compare system-wide costs of VMI, Traditional and Centralized System.
- Assess the performance of the proposed solution algorithm and compare computational times of MIP and *LagRel*.

The effect of the following parameter changes for both short term ($T$=12) and long term ($T$=40) experiments are analyzed. For short term experiments length of the planning horizon is assumed to be 12 periods and for short term it is assumed to be at least 40 periods. While effect of a specific parameter change is being analyzed, remaining parameter values and costs are averaged and presented. To illustrate, when effect of demand variance is being studied, remaining parameters ($b^r$, $h^r$, $h^m$, $K^m$ and $K^r$) are assumed not to have any effect on the cost. Therefore, cost average is taken over the remaining parameters in order to see the influence of demand variance.

*Parameters:*

- Demand faced by the retailer is assumed to be normally distributed. Corresponding distributions are: N ($\mu$=100, $\sigma$=10) and N ($\mu$=100, $\sigma$ =70). For each distribution, 10 different demand sets are generated.
- Backorder cost is constant per period and $b^r \in \{1, 6, 15\}$
- Unit holding cost of retailer is constant per period and $h^r = 3$
- Unit holding cost of manufacturer is constant per period and $h^m \in \{1,3\}$, thus $h^r \geq h^m$.
- Fixed cost of production at manufacturer's site is constant per period and

  $K^m \in \{150, 500, 1000, 2500\}$
- Fixed cost of transportation at retailer's site is constant per period and

  $K^r \in \{50, 150\}$
- Ratio of fixed cost of production to dispatch changes. During the experiments following set of parameters are taken into account.
  - When $K^r \in \{50\}$, $K^m \in \{150, 500, 1000, 2500\}$
  - When $K^r \in \{150\}$, $K^m \in \{150, 500, 1000\}$

For this purpose, parameter set is generated as in Table 3.

**Table 3** Parameter Set for Computational Experiments

| Parameter | Value Set |
|---|---|
| $T$ | Short Term: 12 periods, Long Term: 40 Periods |
| $d_t^r$ | N(100,$10^2$), N(100,$70^2$) |
| $h^r$ ; $h^m$ | {3}; {1,3} |
| $K^m$ | {150,500,1000,2500} |
| $K^r$ | {50,150} |
| $b^r$ | {1,6,15} |

In the following sub-sections, results of short term and long term experiments are presented. MIP is used to solve Scenario 1 (problems TS-M and TS-R) and Scenario 3 (ULS-B). C-ULS-B is also solved by MIP model and relaxed model (VMI-R) is solved by both proposed by *LagRel* Algorithm and in GAMS with solver CPLEX (for only short term analyses) in order to compare the solution times. All runs are made on a computer which has Intel® Pentium® M Processor 1600 MHz, 1.5 GHz 512 MB RAM. All MIP models are solved in GAMS 2.0.13.0 with solver CPLEX 7.0. *LagRel* algorithm is solved in MATLAB 7.6.0.324.

### 5.1 Experiments under Short Planning Horizon

In this section, planning horizon is 12 periods. 20 demand sets are generated; 10 of which is generated with low variance (N(100,$10^2$)) and 10 with high variance (N(100,$70^2$)). For each demand set, 42 runs are made for 42 different set of parameters for solving each problem. In brief, 840 runs are made during experimental analyses in this sub-section.

- Experiments are made to identify how the system parameters affect the benefits of both manufacturer and retailer under VMI. Analyses made while comparing individual parties' costs are;

- The effect of demand variance.
- The effect of production cost and transportation cost ($K^m$ and $K^r$)
- The effect of unit backorder cost

- Experiments are made to identify how the system parameters affect the system-wide costs under VMI, Centralized System and Traditional System. Analyses while comparing system-wide costs are;
  - The effect of production cost and transportation cost ($K^m$ and $K^r$)
  - The effect of unit backorder cost

Short term and long term analyses resulted in similar conclusions. Because of this reason, only long term analyses results' are presented in this chapter. Short term analyses' figures and discussion is presented in Appendix C.

### 5.2 Experiments under Long Planning Horizon

In this section, 40 periods or longer periods are assumed to reflect long term planning horizon. Given sets of parameters in Section 5.1, single set of demand with low variance and single set of demand with high variance are generated for 40 periods. 84 instances for 40-period problem are generated.

#### 5.2.1 VMI and Traditional System Comparison

In this section both retailer's and manufacturer's costs are examined and how the parameter settings affect retailer's and manufacturer's benefits is identified. The expressions below explain the terms which are compared in this chapter.

$TSC^r$: Retailer's cost under traditional system

$$= \sum_t K_t^r Y_t^r + \sum_t h_t^r I_t^r + \sum_t b_t^r E_t^r$$

$VMI^r$: Retailer's cost under VMI

$$= \sum_t h_t^r I_t^r + \sum_t b^r E_t^r$$

$TSC^{m:}$ Manufacturer's cost under traditional system

$$=\sum_t K_t^{\ m} Y_t^m + \sum_t h_t^{\ m} I_t^m$$

$VMI^m$: Manufacturer's optimal cost under VMI

$$= \sum_t K_t^{\ r} Y_t^r + \sum_t K_t^m Y_t^m + \sum_t h_t^{\ m} I_t^m$$

*TSC:* System-wide costs under traditional system

$= TSC^m + TSC^r$

*VMI:* System-wide costs under VMI

$= VMI^m + VMI^r$

Retailer's saving under VMI = $\%Saving^r = {(TSC^r - VMI^r)}/{TSC^r}$

Manufacturer's saving under VMI= $\%Saving^m = {(TSC^m - VMI^m)}/{TSC^m}$

Cent: System-wide costs under centralized system.

$$= \sum_t K_t^{\ r} Y_t^r + \sum_t h_t^r I_t^r + \sum_t b_t^{\ r} E_t^r + \sum_t K_t^{\ m} Y_t^m + \sum_t h_t^{\ m} I_t^m$$

#### 5.2.1.1 Effect of $K^r$ and unit backorder cost $b^r$

*Retailer's Perspective:*

Any change in $K^m$ value does not affect retailer's cost under traditional system because retailer and manufacturer decide independently under Scenario 1. In addition to this, $K^m$ does not affect retailer's cost under VMI. This result emerges from the fact that two complicating constraints are binding in the optimal solution of C-ULS-B. When the constraints are binding, VMI behaves like Traditional System and $K^m$ does not affect retailer's operating costs under VMI. As $K^m$ changes, retailer's cost under VMI (which is holding and backordering costs) does not change and is equal to those values under traditional system. Thus, solely effect of $K^r$ and $b^r$ will be analyzed in this section.

*Effect of $K^r$*

Figure 7 shows the effect of increase in $K^r$ on retailer's cost under Scenario 1 and Scenario 2. In Figure 7 (a) retailer operates under Traditional System and in Figure 7 (b) retailer operates under VMI.

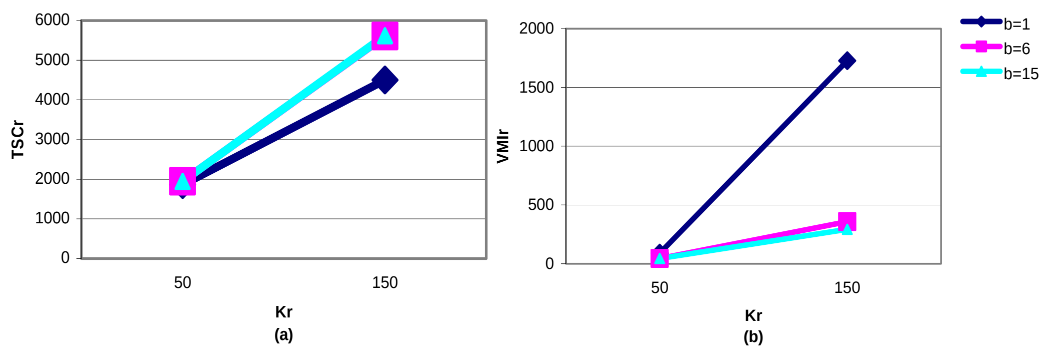


**Figure 7** Effect of $K^r$ under Scenario 1(a) and Scenario 2(b)

Comparison of Figure 7 (a) and (b) implies; retailer's cost under Scenario 1 and Scenario 2 follows similar pattern as $K^r$ increases. *Under Scenario 1*, retailer's operating cost is smallest when $K^r$ is equal to 50 (Figure 7 (a)). When transportation cost is small, retailer orders demand quantity from the manufacturer. Thus, backorder and inventory costs at retailer's site are minimized when $K^r$ is small. *Under Scenario 2* in Figure 7 (b), retailer's total cost is simply total of backordering and inventory holding costs of Scenario 1. Thus, when $K^r$ = 50, inventory and backorder levels are also minimized under Scenario 2.

*Under Scenario 1,* when $K^r$ increases from 50 to 150, ordering frequency of retailer decreases and backordering cost, inventory holding cost and fixed cost of transportation increases under Scenario 1. Thus, total cost of retailer increases (Figure 7 (a)).

*Under Scenario 2*, retailer's total cost is simply inventory and backorder costs which are retailer's performance measures that should be satisfied by the manufacturer. These performance measures are imposed as manufacturer's constraints to C-ULS-B problem. Right hand sides of service level and inventory level constraints are defined by TS-R. Therefore, retailer's costs are same as backordering and inventory holding costs of retailer under Scenario 1. In Figure 7 (b), as $K^r$ is increased, dispatch and production frequency decreases, total production cost decreases while backorder cost, inventory holding cost and total transportation cost increases. Thus, total operating cost of retailer under Scenario 2 (sum of backordering and inventory holding) increases as $K^r$ is increased as under Scenario 1.

In brief, the observations from retailer's perspective can be summarized as follows;

- Increase in fixed production cost ($K^m$) does not affect retailer's cost under Traditional Case and VMI.
- Retailer's costs are minimized when fixed transportation cost ($K^r$) is small under Traditional Case and VMI.
- Dispatch frequency increases under VMI.

*Effect of $b^r$ when $K^r = 50$*

In order to discuss the effect of unit backorder cost more efficiently, costs obtained under demand set with low and high variance are differentiated because behavior differs as demand variance differs.

> *Under demand forecast generated with low variance when $K^r = 50$;*
>
> As backorder cost increases, optimal policy of the retailer under Scenario 1 does not change because average backorder level and inventory level are near zero for $b^r \in \{1, 6, \text{and } 15\}$.
>
> *Under demand forecast generated with high variance when $K^r = 50$;*
>
> Retailer's total costs are equal to each other when $b^r = 6$ or 15, but not equal for $b^r = 1$ under Scenario 1.

On the average (average of all experiments including both demands with low variance and high variance), when $K^r = 50$ under traditional system as in Figure 7 (a) , retailer's cost decreases slightly as backorder cost decreases from 6 to 1 because total backorder cost increases while inventory holding cost and transportation cost decrease. Total retailer's cost decreases at a very small amount because sum of total backordering, transportation and inventory holding costs decrease. This decrease is assumed to be negligible. When $b^r \in \{6, 15\}$, retailer's cost remain same because inventory and backorder levels are zero for each time period. When $K^r = 50$ under VMI as in Figure 7 (b), retailer's cost slightly increases as unit backorder cost decreases from 6 to 1 because total backorder cost increases while inventory cost decreases. However, increase in backorder cost is much more than decrease in inventory cost. Retailer's total operating cost increases slightly under VMI as unit backorder cost change from 6 to 1. This increase is assumed to be negligible. (Figure 7 (b)) Moreover, retailer's total cost is smallest when $K^r = 50$ independent from the unit backorder cost change of retailer; retailer's operating cost is nearly zero in Figure 7 (b) since inventory and backorder levels are almost zero.

To summarize, when $K^r = 50$ as in Figure 7 (a) and (b) , retailer's cost is not affected much from a change in unit backorder cost because transportation frequency is very high and average inventory and backorder levels are very low. However, there is a slight decrease in retailer's cost under Scenario 1 (Figure 7 (a)) when unit backorder cost decreases to 1. There is a slight increase in retailer's cost under Scenario 2 (Figure 7 (b)) when unit backorder cost decreases to 1. These changes in retailer's cost are negligible and following conclusion is made;

- When $K^r = 50$, change in $b^r$ does not affect retailer's costs under both scenarios.

*Effect of $b^r$ when $K^r = 150$*

In order to discuss the effect of $b^r$ more efficiently, costs obtained under demand set with low and high variance are examined separately.

*Under demand forecast generated with low variance when $K^r = 150$;*

Under Scenario 1 as backorder cost increases from 6 to 15, optimal policy of the retailer does not change since backorder level and inventory level are all equal to zero for $b^r \in \{6, 15\}$.

*Under demand forecast generated with high variance when $K^r$ = 150;*

As backorder cost increases from 6 to 15, optimal policy of the retailer changes for demand set generated with high variance because, backorder level decreases to zero while inventory level remains nearly same when $b^r$ = 15.

As $b^r$ is increased from 1 to 6, total backorder cost decreases while dispatch frequency increases. On the total, retailer's total cost increases as $b^r$ is increased from 1 to 6 *under Scenario 1* (Figure 7 (a)). *Under Scenario 2*, retailer's cost includes solely backorder and inventory cost terms. As unit backorder cost increases, average backorder level increases and inventory level decreases, therefore total cost of the retailer decreases (Figure 7 (b)). There is a big decrease when $K^r$ = 150 because, when $K^r$ = 150 average inventory and backorder level is much more than in $K^r$ = 50 and retailer's cost is very influenced from unit backorder cost change. When $K^r$ is small, retailer's cost does not change much as backorder cost increase because dispatch and production frequency and quantities becomes similar to demand pattern as $K^r$ gets smaller. However when $K^r$ is larger, retailer's cost is more influenced by the change in $b^r$ because as fixed cost of transportation increases, average backorder and inventory levels increase due to the fact that dispatch frequency decreases.

There is an important difference between in Figure 7 (a) and (b). In (a), when unit backorder cost increases from 1 to 6, retailer's cost also increases. This means, under traditional system retailer benefits more when backorder cost is decreased. This is because under traditional system, retailer is responsible from when to order. When the trade off between holding inventory and backordering is considered, she decides on the order quantity and time according to the fixed cost of transportation. As unit backorder cost increases, retailer's frequency of transportation increases although backordering cost and inventory holding cost decreases, resulting in an increase in retailer's cost. In (b), opposite situation occurs to Scenario 1. When unit backorder cost increases, operating cost of the retailer decreases because corresponding backorder and inventory values decrease. This means, under VMI, retailer benefits more when backorder cost is increased.

To summarize the discussion above, observations can be drawn as follows;

- Under Traditional System as $b^r$ is increased, dispatch frequency increases, thus transportation cost and inventory cost increases while total backorder cost decreases.
- Under VMI, as $b^r$ is increased, dispatch frequency also increases but she does not pay transportation cost. Total of backorder and inventory cost decreases, consequently retailer's total cost decreases.

In brief, retailer favors increase in unit backorder cost under VMI, while opposite decision is taken under Traditional System.

Retailer's operating costs under Traditional System and VMI are compared. In order to analyze the change in percentage saving of the retailer under VMI, Figure 8 is presented as below.

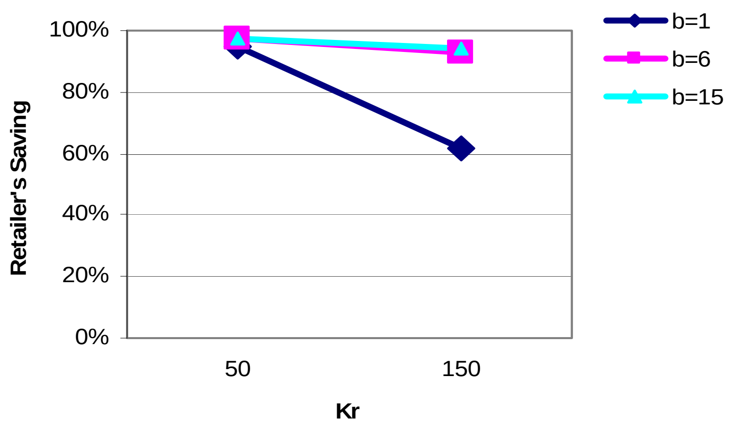


**Figure 8** Percentage Savings of Retailer under VMI

Retailer benefits for all cases under VMI as in Figure 8. There are not any instances that retailer is worse off under VMI. Her percentage saving decreases as backorder cost decreases when $K^r$ =150 because *when* $K^r$ *= 50*, average inventory and backorder levels are less. Thus retailer's savings are not affected much by the change in backorder cost when $K^r$ = 50. *When* $K^r$ *=150*, total backorder and inventory cost decreases as unit backorder cost of the retailer increases. Since transportation cost is charged to manufacturer under VMI, retailer's cost is solely total inventory and backorder cost. Thus, retailer's savings become less as backorder cost is decreased when fixed cost of transportation is increased.

If manufacturer and retailer would share the fixed cost of transportation under VMI rather than manufacturer paying it, depending on the portion of share, Figure 7 (b) would be different. The gap between lines of $b^r = 1$ and $b^r = 6$ or 15 would decrease. For example, when %90 of fixed cost of transportation is charged to retailer, operating cost of retailer would increase when unit backordering cost switches from 1 to 6 or 15. Scenario 1 approaches to Scenario 2 as the transportation cost portion of retailer increases.

Main observations of both short term and long term analyses from retailer's perspective can be summarized as follows;

- Retailer is always better off under VMI.
- Value of $K^m$ does not have any affect on retailer's cost under both Scenarios. Thus, $K^m$ does not affect retailer's percentage saving under VMI.
- Under VMI, as $b^r$ increases, retailer's costs decrease for both $K^r = 50$ and $K^r = 150$.
    - Retailer's cost slightly decreases for $K^r = 50$, because average inventory and backorder levels are very small. On the other hand, retailer's cost decrease very much as $b^r$ increases, when $K^r = 150$.
- Under Traditional System as $b^r$ increases, retailer's cost increase for both $K^r = 50$ and $K^r = 150$.
    - Amount of increase is much more in case where $K^r = 150$ because average inventory and backorder levels are much more than in case where $K^r = 50$.
- VMI is more beneficial for the retailer when $K^r = 50$ and $b^r = 15$ because retailer's cost is almost zero under VMI.
- Dispatch frequency increases under VMI for the unique set of parameters.

*Manufacturer's Perspective:*

In this part, affect of fixed cost of production, unit backorder cost of retailer and fixed cost of transportation on manufacturer's total costs under both scenarios is studied. Since fixed cost of transportation is charged to manufacturer under VMI, cost changes for each $K^r$ value are examined independently.

*Case 1:* $K^r = 50$

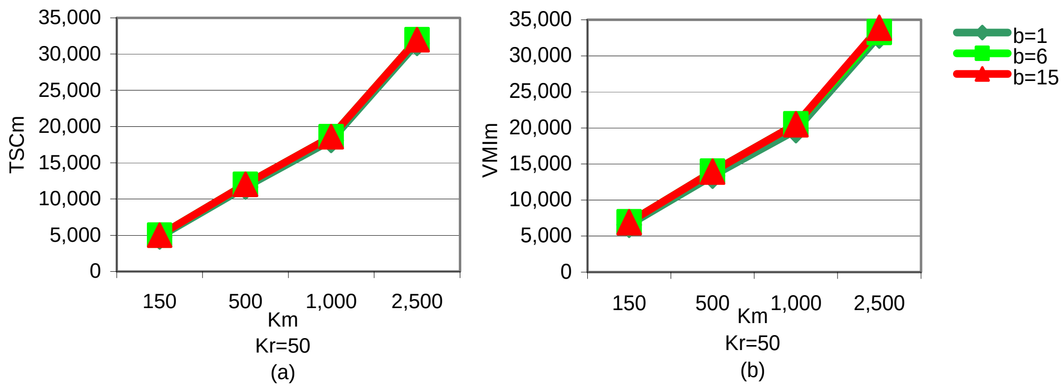


**Figure 9** Manufacturer's Cost under Scenario 1 (a) and Scenario 2 (b) when $K^r = 50$

Manufacturer's cost increases under both scenarios as $K^m$ increases when $K^r = 50$ as in Figure 9 (a) and (b). Under each scenario, manufacturer's operating costs are same when $b^r \in \{6, 15\}$.There is a very small cost difference between cases $b^r = 1$ and $b^r \in \{6, 15\}$. *Under Scenario 1,* as backorder cost decreases to 1, since total backordered amount increases and inventory level increases, retailer's optimal ordering frequency decreases and total production cost decreases. Thus, total cost of manufacturer slightly decreases in Figure 9 (a) when $b^r$=1. *Under Scenario 2*, when unit backorder cost increases from 1 to 6, transportation frequency of manufacturer increases and total production cost of manufacturer also increases. Therefore, total cost of manufacturer increases when unit backorder cost of the retailer increases from 1 to 6. However, any change in unit backorder cost does not have much influence on manufacturer's cost under either Scenario 1 or 2. That is why under each scenario, manufacturer's production and dispatch sequence is very high and holding inventory is not a decision. As discussed before, when $K^r$ is 50, dispatch frequency is very high and average inventory and backorder levels are low. Under traditional system, manufacturer has to satisfy the orders of retailer at the time they are required, thus production frequency depends only on the amount of fixed production cost. Under VMI, when $K^r$ is 50, average inventory and backorder levels are low and dispatch frequency is very high. Only difference between $TSC^m$ and $VMI^m$ is transportation cost, because inventory and backorder levels are very low and frequency of transportation stays same as $b^r$ changes. Although frequency of dispatch is high, total transportation cost is low when $K^r$ =50, compared to total production cost. As a result, manufacturer's cost is not affected much from unit backorder cost change of the retailer when $K^r$ =50 (Figure 9).

*Case 2: $K^r$=150*

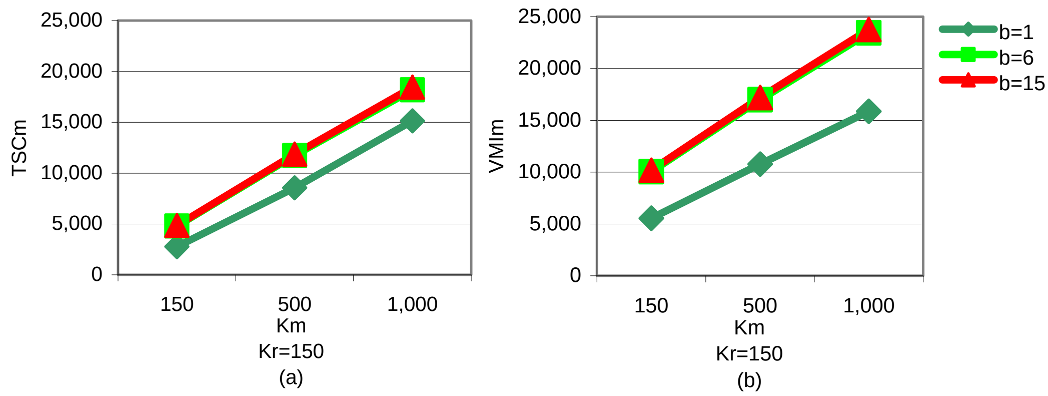


**Figure 10** Manufacturer's Cost under Scenario 1(a) and Scenario 2 (b) when $K^r$ =150

As in Figure 10 (a) and (b), manufacturer cost increases as $K^m$ increases for $b^r \in \{1, 6, \text{and } 15\}$.Amount of cost increase is very small when $b^r$ increases from 6 to 15 because dispatch, production decisions are very similar. Total cost of manufacturer increases when unit backorder cost of the retailer increases from 1, as in Figure 9. However, amount of increase is higher than $K^r$ = 50 (Figure 9) for several reasons. When $K^r$ = 50, manufacturer's production frequency is higher than in $K^r$ = 150. Consequently, backorder level is much more than the backorder level when $K^r$ =50 while all other parameter values do not change when $K^r$ = 150.As $K^r$ gets higher and $b^r$ gets smaller, manufacturer cost decrease because production frequency decreases and manufacturer manages dispatch and production decisions better. (Figure 10 (a) and (b))

*Under traditional case* (Figure 10 (a)), when unit backorder cost increases from 1 to 6 (or 6 to 15), total cost of manufacturer increases. When unit backorder cost of the retailer increases, targeted backorder and inventory levels under the terms of VMI agreement gets smaller (*SL* and *IL* constraints become tighter). Frequency of production increases and this situation results in an increase in total cost of manufacturer when there is an increase in $b^r$. While amount of increase when $b^r$ switches from 1 to 6 (Case 1) is considerable, amount of increase when $b^r$ switches from 6 to 15 (Case 2) is negligible. In Case 1, amount of decrease in backorder level and increase in inventory level is very much while in Case 2, differences are very small. (Figure 10 (a))

*Under VMI* (Figure 10 (b)), as backorder cost decreases to 1, amount of decrease in total cost is very high in comparison with traditional case Figure 10 (a). This result emerges from the fact that under VMI, manufacturer pays transportation cost and retailer's total cost under VMI only includes backordering and inventory holding costs.

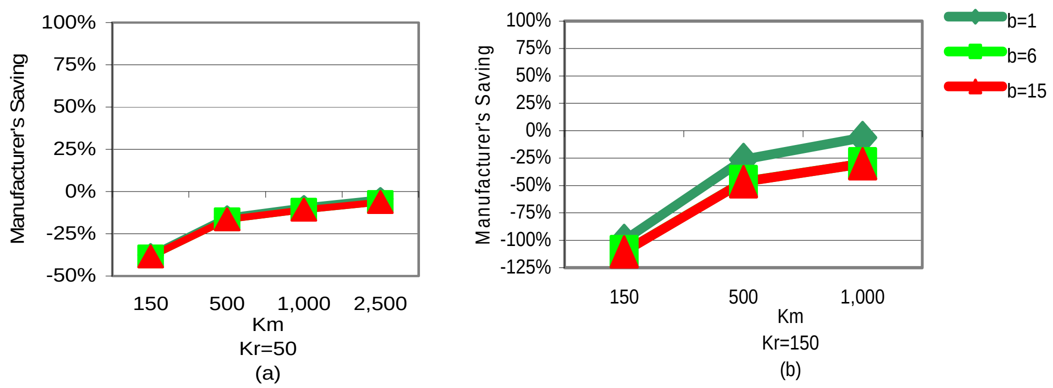


**Figure 11** Manufacturer's Savings under VMI

In Figure 11 (a) and (b), manufacturer's percentage savings under VMI are analyzed. If Figure 11 (a) and (b) are examined, it can be easily concluded that manufacturer cannot decrease his costs under VMI. Manufacturer's loss is almost zero when $K^r = 50$, $b^r = 1$ and $K^m = 2500$ because fixed cost of production dominates fixed cost of transportation. This means, production frequency is minimized while dispatch frequency is maximized. *Under both scenarios*, when dispatch frequency is high, manufacturer follows similar policy under traditional system and VMI. When backorder level is low, dispatch frequency is high and production frequency is very low due to high fixed cost. As fixed cost of production is high and fixed cost of transportation and unit backorder cost of the retailer is low, total cost of manufacturer under traditional system approaches to total cost of manufacturer under VMI. Since $K^r$ is charged to the manufacturer under Scenario 2, effect of $K^m$ dominates influence of $K^r$ when $K^m = 2500$ and $K^r = 50$. In this case, production frequency is low and dispatch frequency is high. Thus manufacturer holds inventory and retailer does not hold any inventory or she does not backorder. Production cost becomes the most important cost element. Therefore, traditional system and VMI system does not differ much.

Manufacturer's losses are very high when $K^r = 150$, $K^m = 150$ and $b^r = 1$ in Figure 11 (b). Backorder level is high under traditional system, frequency of dispatch and frequency of replenishment is high and dispatch and production periods are same. Under VMI, transportation cost is charged to the manufacturer. Transportation cost is high and under VMI, it is the main reason that yields an increase in manufacturer's losses.

Observations on analyses from manufacturer's perspective are;

- Manufacturer is not better off under VMI.
- When $K^r = 50$, any change in $b^r$ does not change manufacturer's cost much.
- As $K^m$ dominates $K^r$, manufacturer's costs under VMI approaches costs under Traditional System.

#### 5.2.1.2 Effect of demand pattern and $h^m$

*Retailer's Perspective:*

In this section, effect of demand variance and change in holding cost of manufacturer on retailer's costs are discussed.

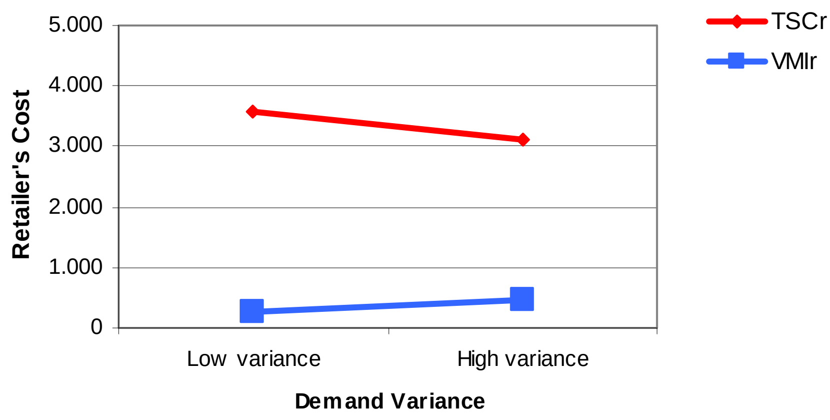


**Figure 12** Effect of demand variance to Retailer's Cost

In Figure 12, effect of demand variance to retailer's costs under VMI and Traditional System is examined. Increase in unit holding cost of the manufacturer does not affect the cost of retailer under traditional system because of independent decision making. Because of this, only figure related with demand variance change is presented. (Figure 12) Under traditional system, demand forecast with low variance creates more operational costs for the retailer (Figure 12). When demand forecast variance is high from one period to the other, retailer's average inventory level increases while average backorder level decreases. As backorder level decreases, backorder cost decreases while inventory holding cost of retailer increases. Total of backorder and inventory costs increase under high variance. Frequency of dispatch decreases resulting in a decrease in transportation cost. Thus under Scenario 1 retailer's cost decrease when demand variance is high. Under VMI, there is an opposite behavior because total of backorder and inventory costs increase when there is high variance under Scenario 2.

*Manufacturer's Perspective:*

In this part, effect of demand variance and change in holding cost of manufacturer on manufacturer's costs are discussed.

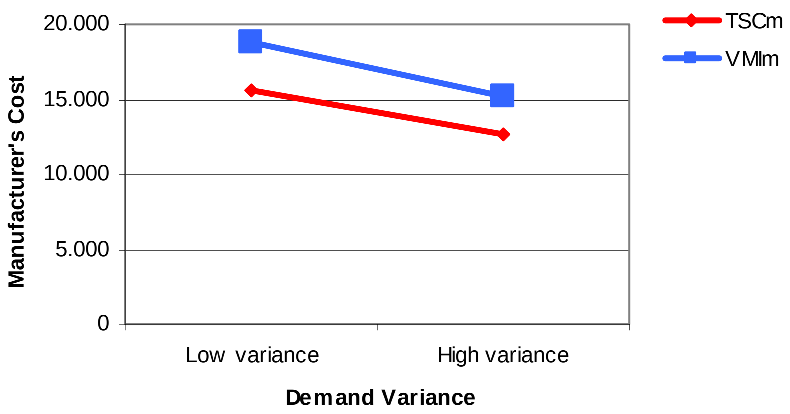


**Figure 13** Effect of demand variance on Manufacturer's Cost

Manufacturer's costs increase when $h^m$ is increased under both scenarios; as a result, solely relationship between total cost and demand variance is examined in Figure 13. When demand variance is high under Traditional System, manufacturer's cost becomes less .Similarly, as demand variance decreases under VMI; manufacturer's losses increase (Figure 13). Manufacturer holds more inventory under demand forecast with high variance and this decreases the frequency of production, decreasing total production cost under both scenarios.

Observations which can be drawn are as follows;

- Retailer's cost increase when demand variance is high under VMI.
- Manufacturer costs are less when demand variance is high under VMI.

### 5.2.2 System-wide costs comparison

*Under Traditional System*, each party takes independent decisions and system-wide costs are simply summation of manufacturer's and retailer's optimal operating costs. Manufacturer minimizes fixed cost of production and inventory holding cost while satisfying retailer's orders. Retailer minimizes fixed cost of transportation, inventory holding and backordering costs. *Under VMI System*; model's objective is to minimize operating cost of manufacturer where manufacturer's costs are fixed cost of production, transportation and holding cost while satisfying retailer's performance measures. Manufacturer is the decision maker. Retailer's costs are inventory holding cost and backorder cost. Retailer's costs do not affect the objective of the problem as soon as performance measures are satisfied. *Under Centralized System*, model's objective is to minimize system-wide costs; total of fixed cost of production, transportation, holding cost of manufacturer and retailer, and backorder cost. Manufacturer and retailer merges and act like a single entities, decision maker is this single entity. Thus, objectives of each Scenario are different as below.

**Objective of Scenario 1:** Minimize $TSC^r$ and $TSC^m$ independently and obtain $TSC$.

Where

$$TSC^r = \sum_t K^r Y_t^r + \sum_t h_t^r I_t^r + \sum_t b_t^r E_t^r$$

$$TSC^m = \sum_t K^m Y_t^m + \sum_t h_t^m I_t^m$$

and $TSC = TSC^r + TSC^m$

**Objective of Scenario 2:** Minimize $VMI^m$ and obtain $VMI$.

Where

$$VMI^m = \sum_t K^r Y_t^r + \sum_t K^m Y_t^m + \sum_t h_t^m I_t^m$$

$$VMI^r = \sum_t h_t^r I_t^r + \sum_t b_t^r E_t^r$$

And $VMI = VMI^r + VMI^m$.

**Objective of Scenario 3:**

$$Cent = \sum_t K^r Y_t^r + \sum_t K^m Y_t^m + \sum_t h_t^r I_t^r + \sum_t h_t^m I_t^m + \sum_t b_t^r E_t^r$$

Differences between system-wide costs are compared where**;**

$$\%Difference\_TSC = (TSC - Cent) / Cent$$

$$\%Difference\_VMI = (VMI - Cent) / Cent$$

Centralized system-wide costs are always better than Traditional system-wide costs and VMI system-wide costs due to the objective function structures. Centralized and VMI systems are different both in terms of constraints and objectives. Since Scenario 3 is benchmark for Scenarios 1 and 2, VMI and Traditional system-wide cost differences are compared with centralized system-wide costs. Centralized System is expected to produce better results because of integrated objective function, since it is taken as a theoretical benchmark, conditions under which remaining two scenarios approach to Scenario 3 are investigated. Figures numbered from 14 to 16 are presented in order to check Traditional system and VMI system behavior from system costs perspective, while considering the influences of $b^r$, $K^r$ and $K^m$.

In Case 1, unit backorder cost equals to 1.In Case 2, unit backorder cost equals to 6 and in Case 3 unit backorder cost equals to 15. All cases are visually presented in the Figures from 14 to 16.

*Case 1: $b^r$=1*

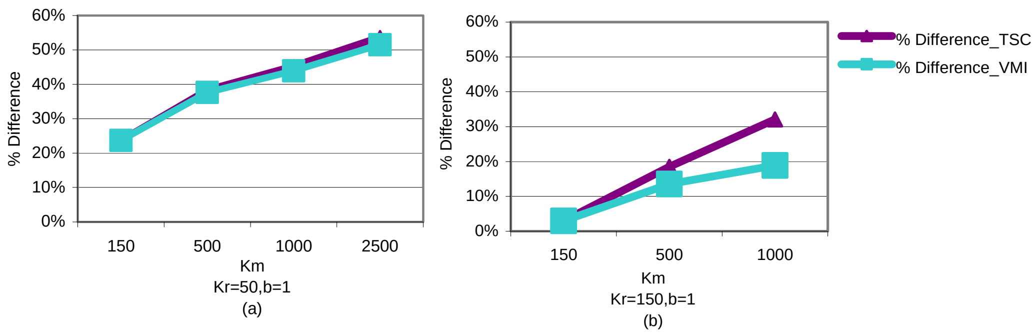


**Figure 14** Percentage Cost Difference between Scenario 1-3(a) and Scenario 1-3 (b) for $b^r = 1$, $K^r$ = 50 or 150

When $K^r$ is 50, VMI system behaves like traditional system because dispatch frequency is very high and average backorder and inventory levels are low. As fixed cost of production increases, centralized system becomes more beneficial in terms of supply chain costs because of integrated production and dispatch decisions. Under centralized system, dispatch and production frequency decreases, average backorder level increases, decreasing system-wide costs (Figure 14 (a)) Since retailer should not be worse off in terms of performance measures under VMI, traditional system and VMI optimal policies are similar. However, under centralized system, there is not any performance measure imposed as a constraint. Consequently, backorder and inventory level can be higher than retailer's optimal policy under traditional system, decreasing system-wide costs.

When $K^r$ is 150, as $K^m$ increases, system-wide costs' percentage difference increase. Under VMI, system savings increase as $K^m$ increase because average backorder and inventory levels increase when $K^r$=150. When $K^m$ = 1000, operating under traditional system becomes costly than operating under VMI. This is because as fixed production cost increases; total production cost decreases and dispatch frequency increases. VMI agreement becomes advantageous from system-wide costs' perspective. (Figure 14(b))

*Case 2: $b^r$=6*

When unit backorder cost increases from 1, system-wide costs of VMI and traditional system are nearly same because average inventory and backorder levels are very small due to high $b^r$. Thus, as production and dispatch frequency increases, VMI costs are same as traditional system costs (Figure 15 (a))

As fixed cost of production increases, centralized system gets more favorable because of integrated production, dispatch and inventory decisions (Figure 15 (b)).

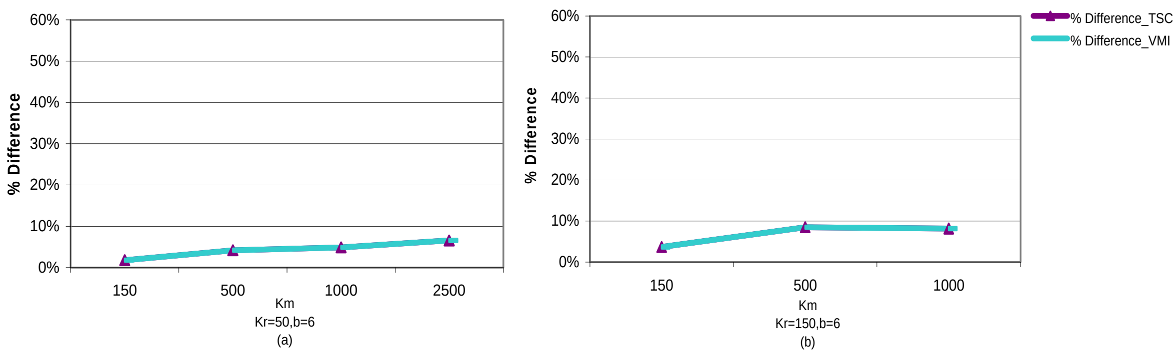


**Figure 15** Percentage Cost Difference between Scenario 1-2 (a) and Scenario 1-3 (b) for $b^r = 6$, $K^r$ = 50 or 150

*Case 3: $b^r$ =15*

As in case 2, Traditional system costs are same as VMI costs when $b^r$ = 15 (Figure 16 (a) and (b))

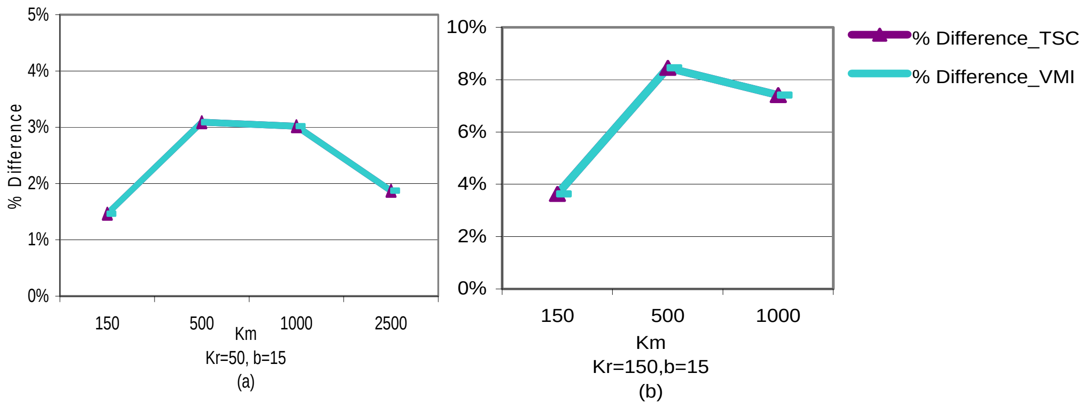


**Figure 16** Percentage Cost Difference between Scenario 1-3 (a) and Scenario 2-3 (b) for $b^r = 15$, $K^r = 50$ or $150$

As unit backorder cost is low, VMI and traditional system behaves opposite to centralized. As backorder cost decreases, backorder level increases and retailer's service level performance measure becomes unbinding under VMI.

Backorder level of VMI increases but this is not an objective of VMI. As soon as *SL* level is satisfied, VMI does not aim to minimize backorder level. This is opposite case of centralized system because Scenario 3's objective is to minimize system-wide costs. When case 1, case 2 and case 3 are compared (Figures 14,15 and 16) percentage difference of system-wide costs between Scenario 1-3 and Scenario 2-3 decrease as unit backorder cost of the retailer increases.

Important conclusions which can be drawn from system-wide costs comparison of Scenario 1, 2 and 3 both in short term and long term are as follows;

- As unit backorder cost of retailer increases, three scenarios' system-wide costs approach to each other.
- Decrease in unit backorder cost results in counter behavior in Scenario 3 and the other Scenarios.
- When $K^r = 50$, VMI system-wide costs approach to Traditional System costs as unit backorder cost increase.

## 5.3 Comparison of Computational Times

VMI model is solved in GAMS 2.0.13.0 with solver CPLEX 7.0. Lagrangean Relaxation algorithm (*LagRel*) discussed in Chapter 4, as hybrid of Lagrangean Relaxation and Dynamic Programming algorithms, is coded in MATLAB 7.6.0.324. 40, 50 and 60-period problems are studied and performance of the suggested Lagrangean Relaxation algorithm is analyzed.

If 40, 50 and 60-period problem's time requirements are compared for demand sets with low variance; computational time can be observed as in Table 4.

**Table 4** Time Requirements Comparison When Demand Variance is Low

| | **Total CPU Time (sec)** | | | **Average CPU Time Per instance (sec)** | | |
|---|---|---|---|---|---|---|
| | **40-Period** | **50-Period** | **60-Period** | **40-Period** | **50-Period** | **60-Period** |
| VMI | 1.322,88 | 3.360,749 | 8.174,13 | 31,50 | 80,02 | 194,62 |
| *LagRel* | 468,231 | 1.293,77 | 2.991,12 | 11,15 | 30,80 | 71,22 |

Lagrangean Relaxation algorithm's computational time decreases in comparison with VMI model as planning horizon length increases under demand forecast with low variance.

If 40, 50 and 60-period problem's time requirements are compared for demand sets with high variance; computational time can be observed as in Table 5.

**Table 5** Time Requirements Comparison When Demand Variance is High

| | Total CPU Time (sec) | | | Average CPU Time Per instance (sec) | | |
|---|---|---|---|---|---|---|
| | **40-Period** | **50-Period** | **60-Period** | **40-Period** | **50-Period** | **60-Period** |
| VMI | 1.951,44 | 8.682,99 | 18.322,40 | 46,46 | 206,74 | 436,25 |
| *LagRel* | 1.456,87 | 3.870,70 | 8.722,00 | 34,69 | 92,16 | 207,67 |

As demand variance gets higher, computational times of models approach to each other. *LagRel* algorithm performs better under demand forecast with low variance and solution time is improved by almost 3 times. *LagRel* algorithm performs better than VMI under demand forecast with high variance also, but solution time is improved by 2 times.

## 5.4 Upper and Lower Bound Evaluation

As discussed in Chapter 4, two performance measures for evaluating bound performance are taken into consideration.

Performance measure-1:

$Z^{*}-L(u,k))/Z^{*}$= (Optimal Objective Value of VMI-Lower Bound) /Optimal Objective Value

Performance measure-2:

$(VMI^{up}-Z^{*})/Z^{*}$= (Upper Bound-Optimal Objective Value)/Optimal Objective Value

- Upper Bound can be obtained by
  - Obtained by *LagRel* solution
  - Obtained by *ObtainFeasible* heuristic for infeasible results of the *LagRel* Model

*40-Period Results:*

Out of 84 runs, *LagRel* produces 9 infeasible results. 8 feasible but not optimal (very near optimal, 0,01% deviation at most) and 67 optimal solutions. Infeasible and feasible objective values are recorded and their average deviation from the optimal objective value is only 0,1%. Remaining results are optimal. Table 6 shows the upper bound and lower bound efficiency obtained by *LagRel* Algorithm.

**Table 6** Bound Evaluation for 40-Period results

| | (Z*- L(u,k))/Z* | (VMI$^{up}$-Z*)/ Z* |
|---|---|---|
| **Max.Deviation** | 1,3% | 84,8% |
| **Min.Deviation** | 0% | 0% |
| **Average** | 0,1% | 6,6% |

*50-Period Results:*

Out of 84 runs, 11 runs produced infeasible results. 8 runs produced feasible results. Infeasible and feasible objective values are recorded and their average deviation from the optimal objective value is only 0,1% on the average. Remaining results are optimal. Table 7 shows the upper bound and lower bound efficiency obtained by *LagRel* Algorithm for 50 periods.

**Table 7** Bound Evaluation for 50-Period results

| | ***(Z*- L(u,k))/Z**** | ***(VMI$^{up}$-Z*)/ Z**** |
|---|---|---|
| **Max.Deviation** | 5,16% | 91% |
| **Min.Deviation** | 0% | 0% |
| **Average** | 0,001% | 5,89% |

*60-Period Results:*

Out of 84 runs, 8 runs produced infeasible results. 18 runs produced feasible results. Infeasible and feasible objective values are recorded and on the total, their average deviation from the optimal objective value is around zero on the average. Remaining results are optimal. Table 8 shows the upper bound and lower bound efficiency obtained by *LagRel* Algorithm for 60 periods.

**Table 8** Bound Evaluation for 60-Period results

| | ***(Z*- L(u,k))/Z**** | ***(VMI$^{up}$-Z*)/ Z**** |
|---|---|---|
| **Max.Deviation** | 23,6% | 61% |
| **Min.Deviation** | 0% | 0% |
| **Average** | 0,67% | 3,56% |

In Appendix D, the process steps while doing experimental design is detailed and explained.

# CHAPTER 6

# SUMMARY AND CONCLUSIONS

In this thesis, we consider a single manufacturer and a single retailer supply chain under a deterministic demand setting. First, Traditional System (Scenario 1) is defined under which the manufacturer and the retailer operate independently and each party tries to minimize its own costs, and then introduce the vendor managed system (Scenario 2) and they are compared. Under Scenario 2, manufacturer is empowered to control dispatch decisions and total transportation cost is charged to the manufacturer. He has to satisfy the performance measures of the retailer and maximum total inventory and backorder levels are defined that manufacturer should operate within. Constraints are imposed to VMI system as terms of VMI agreement set by the retailer so that she ensures via her performance measures to be never worse off under VMI. Additionally, centralized decision making system (Manufacturer and Retailer act like merged) is defined as a benchmark (Scenario 3) and compare system-wide costs if Scenario 3 with Scenario 1 and Scenario 2. Under Scenario 1, Retailer's problem is ULS problem with backordering (TS-R) and manufacturer 's problem is in context of ULS problem (TS-M). They are formulated as MIP models and solved optimally in GAMS 2.0.13.0 with the solver CPLEX 7.0. Under Scenario 2, VMI system is in the context of constrained ULS problem with backordering (named as C-ULS-B) and a MIP model for VMI is also formulated. Since there are two complicating constraints imposed in VMI (limiting total backorder and inventory levels), both complicating constraints in C-ULS-B is relaxed and VMI-R is obtained. VMI-R is solved by Lagrangean Relaxation Algorithm in GAMS with solver CPLEX for short term planning horizon. Even for short term planning, Lagrangean Relaxation solved with CPLEX is not efficient in terms of computational times. Thus, a dynamic programming based Lagrangean Relaxation algorithm (*LagRel* Algorithm) is suggested for solving VMI-R in the long term analyses during this thesis studies. *LagRel* Algorithm outputs infeasible solutions for some of parameter settings, thus a heuristic to convert infeasible solution into a feasible solution (*ObtainFeasible* Algorithm) is proposed. 840 short term instances and 252 long term instances in the presence of high demand variance and low demand variance are generated and results of these instances are compared in terms of the manufacturer's costs, retailer's costs, system-wide costs and decisions. In addition to this, performances of MIP model and *LagRel* algorithm proposed for Scenario 2 are compared both in terms of upper bound and lower bound deviation and computational time requirements.

It is shown that retailer is always better off under VMI while manufacturer is always worse off in terms of operational costs. However, there are conditions that manufacturer's losses are minimized under VMI. When the fixed production cost is very high in comparison with the fixed transportation cost and unit backorder cost is very low, manufacturer's costs are nearly same as his costs under Traditional System. While manufacturer's losses are minimized, any increase in fixed production cost has no effect on the retailer's costs under VMI because two complicating constraints are binding in the optimal solution of VMI. All instances during experimental study produce binding cases.

Under VMI, when unit backorder cost is increased retailer's cost decreases as opposite to retailer's cost behavior under Traditional System. The retailer is very beneficial under VMI when fixed cost of transportation is low and unit backorder cost is high because under VMI retailer's costs are solely total backorder and inventory costs. When fixed transportation cost is low and unit backorder cost is very high, frequency of transportation is very high yielding in zero inventory and backorders. Retailer's cost is almost zero under this case. Manufacturer benefits from high demand variance under VMI while retailer's costs increase when demand variance is high. Manufacturer holds more inventory under demand forecast with high variance and this decreases the frequency of production, decreasing total production cost under both Scenarios. As fixed production cost dominates fixed transportation cost, both manufacturer's costs and retailer's costs under VMI approaches costs under Traditional System because of increased dispatch frequency and zero inventory-backorders. When system-wide costs are compared of scenarios 1,2 and 3, many important observations are obtained. Percentage cost differences between Scenario 1 and Scenario 3, Scenario 2 and Scenario 3 are compared. Under all cases, our results show that centralized decision making outperforms VMI and Traditional Decision Making in terms of supply chain costs. When fixed cost of transportation is low, VMI system behaves like traditional system because dispatch frequency is very high and average backorder and inventory levels are low. As fixed cost of production increases, centralized system becomes more beneficial because of integrated production and dispatch decisions. Under centralized system, dispatch and production frequency decreases, average backorder level increases, decreasing system wide costs. When unit backorder cost of the retailer increases, all scenarios system-wide costs approach each other.C-ULS-B is solved in GAMS with the solver CPLEX to obtain optimal solution. As two complicating constraints in C-ULS-B are relaxed, VMI-R is obtained. In short term experiments, VMI-R is solved by Lagrangean Relaxation algorithm in GAMS with solver CPLEX. The solution times are too high, thus a more efficient algorithm is studied. It is already known that two-stage ULS problem with backordering can be solved optimally by a polynomial time algorithm. (Lee, Çetinkaya and Jaruphongsa (2003)) A *LagRel* Algorithm is proposed to solve Constrained ULS problem with Backordering and obtain lower bound and upper bound on the optimal solution. 12-Period problem as short term, 40-50 and 60-Period problems to assess long term performance of *LagRel* Algorithm are solved. *LagRel* reduces computational time requirement of optimal solution by 3 times on the average. When demand variance is low, *LagRel* algorithm performance increases.

Additionally, upper bound and lower bound performances are discussed for C-ULS-B. First, 12-Period problem in order to reflect short term behavior of the models is examined. Lower bounds obtained by solving C-ULS-B deviate from the optimal objective value only 0.1% on the average. Maximum deviation is 3.72% and many runs obtained optimal solution. Upper bounds obtained by *ObtainFeasible* and *LagRel* algorithms deviate from the optimal solution 0.03% on the average. Secondly a long term analysis is done. First, 40-Period problem is analyzed. Infeasible and feasible objective values are recorded and their average deviation from the optimal objective value is only 0,13%. On the average, deviation of lower bound from optimal objective value is only 0.1%. Upper bound deviates 6.6% from the optimal objective value on the average. Secondly, 50-Period problem is examined. Lower bounds obtained by solving VMI-R deviate from the optimal objective value only 0.001% on the average. Upper bounds obtained by *LagRel* and *ObtainFeasible* Algorithms, deviate from optimal solution at a rate %5.89. Finally, 60-Period problem is studied. There is a 0,67% deviation from optimal solution in lower bounds on the average and upper bound deviation from optimal solution is solely %3.56. Therefore, *LagRel* and *ObtainFeasible* algorithms produce very good bounds on the optimal solution.

Future research can extend the analyses here in many directions. In this thesis, production and transportation capacities are assumed to be unbinding and uncapacitated case is studied. A natural question to consider is how to deal with capacities in each scenario. If capacities would be considered, problems become harder to solve. Additionally, C-ULS-B problem could be studied to solve via solely Dynamic Programming algorithm. Another direction which can be extended is manufacturer and retailer transportation cost shares under VMI. Retailer's benefits and manufacturer's losses are directly related with transportation cost, consequently percentage savings would be affected by the share portion. Moreover, constraints (12) and (13) could be relaxed and a solution algorithm for these could be analyzed and suggested.Pochet and Wolsey (2005) study a good understanding of polyhedral structure of single item lot sizing problems with backordering, convex hull of C-ULS-B can be studied. Moreover, there can be inventory lower bound in inventory performance measure of the retailer, which would change VMI benefits of the retailer and manufacturer accordingly.

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# APPENDIX A

# PSEUDOCODE OF *OBTAINFEASIBLE*

ALGORITHM *ObtainFeasible*

**<u>Inputs</u>: Demand, Order, Production, Parameters,** Inventory_Level, Backorder_Level

**<u>Outputs</u>:** Total Inventory, Total Backorder, **Retailers_Inventory**, **Backorder**, Retailers_Cost, Manufacturers_Cost

```
Compute Retailers_Inventory, Manufacturers_Inventory,Backorder

Compute Retailers_Cost, Manufacturers_Cost

prev_TOTALINV ← SUM(Retailers_Inventory)

prev_TOTALBO ← SUM(Backorder)

TOTALENV ← SUM(Retailers_Inventory)//find total inventory

TOTALBO ← SUM (Backorder)//find total backorder

Count ← 0//counter to avoid infinite loop, counts number of iterations
```

```
While((TOTALENV> Inventory_Level) OR (TOTALBO > Backorder_Level))
   Count ← Count + 1
   If(Count>1000) Then // at most 1000 attempts to find a feasible solution
      Break the While Loop
   End If
   If((TOTALINV-Inventory_Level)>(TOTALBO-Backorder_Level)) Then //more infeasibility in inventory
      [value index] ← MAX(Order–Demand)//to find period that creates maximum inventory
      If(value<=Order(index) AND value<=Production(index)) Then // Order and Production are ≥0
         Order(index)←Order(index)-value //to decrease inventory
         Production(index)←Production(index)-value
      Elseif(value<=Order(index) AND value>Production(index))
         While(~(value<=Order(index) AND value<=Production(index)))
            For d←index to 1 d ← d-1
               While(Production(d)>Order(d) AND Production(index)~=value)
                  Production(d) ← Production(d)-1
                  Production(index) ← Production(index)+1
                  If(Production(index)=value) Then
```

```
25                      Break the While Loop
26                    End If // if statement in line 24
27                  End While // while loop starting from line 21
28                End For // for loop starting from line 20
29              End While // while loop starting from line 19
30              Order(index)←Order(index)-value
31              Production(index)←Production(index)-value
32            End If // if statement in line 15
33        Else
34            [value index]←max(Demand-Order) // to find period that
          creates maximum backorder
35            Order(index)←Order(index)+value
36            Production(index)←Production(index)+value
37        End If // if statement in line 13
38    End While // while loop starting from line 8
39    Compute Retailers_Inventory, Manufacturers_Inventory,Backorder
40    Compute Retailers_Cost, Manufacturers_Cost
41    If(TOTALINV-Inventory_Level)>(TOTALBO-Backorder_Level)) Then
42        While(TOTALINV <= Inventory_Level AND
      TOTALBO<=Backorder_Level)// condition satisfied while
      decreasing manufacturer's cost
43            [value index]←MIN(Order(Order>0))
```

```
    For index←PERIOD to 1
        If(Order(index)=value) Then
            Break the For Loop
        End If
    End For
    k←index-1
    While(k>0)
        If(Production(k)~=0 AND Order(k)~=0) Then
            Break the While Loop
        End If
        k←k-1
    End While
    If(k>0 AND ((index-k)*Order(index)<=( Inventory_Level-TOTALINV))) Then
        While(TOTALINV <= Inventory_Level AND Production(index) > 0 AND (index-k)*Order(index) <= (Inventory_Level-TOTALINV)))
            Production(index)←Production(index)-1
            Order(index)←Order(index)-1
            Production(k)←Production(k)+1
            Order(k)←Order(k)+1
Compute Retailers_Inventory, Manufacturers_Inventory,Backorder
```

```
            TOTALENV← SUM(Retailers_Inventory)
            TOTALBO ← SUM (Backorder)
          End While
       End If
Compute Retailers_Inventory, Manufacturers_Inventory,Backorder
       TOTALENV ← SUM(Retailers_Inventory)//find total inventory
       TOTALBO ← SUM (Backorder)//find total backorder
       Compute Retailers_Cost, Manufacturers_Cost
       If Manufacturers_Cost is not improved Then
           Break the While Loop // while loop in line 42
       End If
   End While // while loop in line 42
Else
   ENTER ← 1 // to start cost improvement loop
   While(TOTALINV<=Inventory_Level AND TOTALBO<=Backorder_Level AND ENTER)
       [value index]←MIN(Order(Order>0))
       For index←PERIOD to 1
          If(Order(index)=value) Then
              Break the For Loop
          End If
```

```
83      End For
84      k←index-1
85      While(k>0)
86          If(Production(k)~=0 AND Order(k)~=0) Then
87              Break the While Loop
88          End If
89          k←k-1
90      End While // while loop in line 85
91      If(k>0 AND((PERIOD-k)*Order(index)<=(Backorder_Level-TOTALBO))) Then
92          While(TOTALBO<=Backorder_Level AND Order(index)>0 AND ((PERIOD-k) * Order(index)<=(Backorder_Level-TOTALBO)))// while solution is still in feasible region
93              Order(index)←Order(index)-1
94  Compute Retailers_Inventory, Manufacturers_Inventory,Backorder
95              TOTALENV ← SUM(Retailers_Inventory)
96              TOTALBO ← SUM (Backorder)
97          End While // while loop in line 92
98      Else
99          ENTER ←0 //calculation will result infeasible solution stop the improvement-loop
100     End If // if statement in line 91
101     Compute Retailers_Cost, Manufacturers_Cost
```

```
        If Manufacturers_Cost is not improved Then
            Break the While Loop //quit from improvement loop
        End If
    End While // while loop in line 77
    While(TOTALENV<=Inventory_Level AND TOTALBO<=Backorder_Level)
        [values index]←SORT(Production)
        CONTINUE1←1 // variable to control
        For i←1 to PERIOD
            If(values(i)=0) Then
                Continue //continue with next period
            End If
            For j←index(i)+1 to PERIOD
                If(Production(j)=0) Then
                    Continue //continue with next period
                Elseif((j-index(i))*values(i)<=(Backorder_Level-TOTALBO))
                    If(SUM(Order(1 to index(i)))>=Production(index(i))) Then
                        Production(j) ← Production(j) + Production(index(i))
                        Order(j) ← Order(j)+ Production(index(i))
                        temp←Production(index(i))
```

```
121                     Production(index(i))←0;
122                     For d←index(i) to 1
123                        If(Order(d)>=temp) Then
124                           Order(d)←Order(d)-temp
125                           Break the For Loop
126                        Else
127                           temp←temp-Order(d)
128                           Order(d)←0
129                        End If // if statement in line 123
130                     End For // for loop in line 122
131                  End If // if statement in line 117
132                  CONTINUE1←0 // in order to exit For loop also
133                  Break the For Loop // for loop in line 113
134               End If // if statement in line 114
135            End For // for loop in line 113
136            If(CONTINUE1=0) Then
137               Break the For Loop // for loop in line 109
138            End If
139         End For // for loop in line 109
140   Compute Retailers_Inventory, Manufacturers_Inventory,Backorder
141         TOTALENV ← SUM(Retailers_Inventory)//find total
```

```
    inventory
142        TOTALBO ← SUM (Backorder)//find total backorder
143        If Total Backorder Level is not improved Then
144           Break the While Loop
145        End If
146    End While // while loop in line 106
147 End If // if statement in line 41
148 Return Total Inventory, Total Backorder, Retailers_Inventory,
    Backorder, Retailers_Cost, Manufacturers_Cost
```

# APPENDIX B

# PSEUDOCODE OF *LAGREL*

DP Algorithm: Complexity O (n^5) n: number of period

**Inputs:** End Item Demand, Backorder Level, Inventory Level, Parameters ($h^r$, $h^m$, $K^r$, $K^m$, $b^r$)

**Description of functions:**

Critical notes about notation: Array variables are written in bold, sub-routine names are written in italic.

Compute_Gs () and ComputeCs () are two important functions; Gs is subproblem, Cs is main problem.

K is production period that satisfies demands i+1 through j.

The **Selected_I** matrix is used to keep the column indices of cells in the minimum-cost path (e.g. if the cost value of the cell (3, 2, 6) gives the minimum-cost value in the calculation of cell (3, 3, 6) then **Selected_I** (3, 3, 6) should be 2.

**Outputs**: Retailers Total Cost under VMI, Manufacturers Total Cost under VMI, bound (L (u,k)), Production Quantities, Dispatch quantities, Inventory Levels, Backorder Levels for each period

*ALGORITHM_ LagRel* (Demand, Backorder_Level, Envanter_Level, Parameters)

```
Initialize()
Compute_Gs(Demand, Parameters:KR,HM,BC,HR)
Compute_Cs(G,Parameters:KM)
For i←1 to PERIOD+1
   For j←1 to PERIOD+1
   Arc_Costs(i,j) ← C(1,i,j)
   Production_PERIODs(i,j) ← 1
      For k←2 to PERIOD
         If(Arc_Costs(i,j) > C(k,i,j)) Then
            Arc_Costs(i,j) ← C(k,i,j)
            Production_PERIODs(i,j) ← k
          End If
       End For
    End For
End For
Graph←Build_Graph(Arc_Costs)
[path_cost, PATH]←Find_Shortest_Path(Graph)
{ Beginning of Production Qty Calculations }
For i←1 to length(PATH)-1
    Production Qtys ← SUM(Demand(PATH(i)+1 to PATH(i+1)))
End For
{ End of Production Calculations }
```

```
counti←1
{ Beginning of Dispatch Qty Calculations }
For kk← length(PATH) to 2,   kk←kk-1
   DISPATCHES(1,counti)← DQ(Production_PERIODs (kk-1)+1, PATH(kk-1), PATH(kk))
    t←PATH(kk)
    temp←DISPATCHES(1,counti)
    While(1)//do forever, unless the conditions in the loops are FALSE
        temp←temp-Demand(t)
        If(temp≤0) Then
            Break the While Loop // if there are no more demand to dispatch
        Else
            t←t-1
            If(t≤0) Then
                Break the While Loop // if all dispatch quantities are calculated
            End If
        End If
    End While
    a ← t-1
    counti ← counti+1
    While(1)// do forever, unless the conditions in the loops are FALSE
        If(SELECTED_I(Production_PERIODs(kk-1)+1,PATH(kk-1),a)=0) Then
```

```
            Break the While Loop // if the beginning of the path is reached
        End If
        DISPATCHES (1,counti)← DQ (Production_PERIODs(kk-1)+1, PATH (kk-1),a)
        temp← DISPATCHES (1,counti)
        t←a
        While(1)// do forever, unless the conditions in the loops are FALSE
            temp←temp-Demand(t) //temporary variable to hold remaining demands
            If(temp≤0) Then
                break // if there are not more to dispatch
            Else
                t←t-1
                If(t≤0) Then
                    Break the While Loop // loop reaches to first period
                End If
            End If
        End While
        a← t-1
        counti ← counti+1
    End While
End For
{ End of Dispatches Calculations }
```

```
61 Calculate Retailers_Inventory, Manufacturers_Inventory and
   Backorder.
62 RetailersCost ← 0
63 ManufacturersCost ← 0
64 bound ← 0
65 For i←1 to PERIOD
66    RetailersCost ← RetailersCost +(BC*Backorder(i) +
   HR*Retailers_Inventory(I))
67    ManufacturersCost ← ManufacturersCost +
   HM*Manufacturers_Inventory(i)+ bv1*KM + bv2*KR
68    bound ← bound + U*(Backorder(i)-
      BackorderLevel(i))+KK*(Retailers_Inventory(i)-
      Retailers_InventoryLevel(i))+(HM*Manufacturers_Inventory(
      i))+(KM*bv1)+(KR*bv2)
69 End For
70 COST ← ManufacturersCost + RetailersCost
71 Return RetailersCost, ManufacturersCost, COST, bound,
   Retailers_Inventory, Manufacturers_Inventory Backorder,
   Order, Production
```

Variables in (66), bv1 and bv2, are the binary variables.

If the order in a period is greater than zero **Then** bv1 will be 1 for this period. If the production in a period is greater than zero bv2 will be 1 for this period. Otherwise they are 0.

*Initialize ()*

```
G(k:1 to PERIOD, i:1 to PERIOD+1, j:1 to PERIOD+1) ← Inf
//Because of dummy node
C(k:1 to PERIOD, i:1 to PERIOD+1, j:1 to PERIOD+1) ← Inf //
Because of dummy node
```

```
Arc_Costs(i:1 to PERIOD+1, j:1 to PERIOD+1)←-1 // Because of dummy node
DQ (i:1 to PERIOD+1, j:1 to PERIOD+1, k:1 to PERIOD+1) ← -1 //holds amount of dispatches
```

*Compute_Gs* (**Demand**, *Parameters:* $K^R$, $h^m$, $b^r$, $h^r$, $u$, $k$)

```
For j ← 2 to PERIOD+1 //
   For i ← 1 to j-1 //
      For k ← 2 to j
         If(i+1~=k) Then
            MING ← Inf
            M ← -1
            For m ← k to j // send demand of periods-i+1 to period-j at kth period.
               Backorder ← 0
               For den ← 1 to m-(i+1) // there may be backorder due to demand from i+1 to k
                  Backorder ← Backorder + Demand(m-den-1)*den //Demand splitting is not allowed {Property 4}
               End For
               Inventory ← 0
               For den ← 1 to j-m
                  Inventory ←Inventory+Demand(den+m-1)*den //Demand splitting is not allowed
```

```
{Property 4}
                End For
                manufInventory ← TOTALs(i,j-1) *(m-k)
                DQ(k,i,j) ← TOTALs(i,j-1) // a dispatch cannot be made to fill solely backorders
{Property 5}
                If((HM * manufInventory + U * Backorder + K * Inventory + bv * KR)≤MING) Then
                   MING ← HM * manufInventory + U * Backorder + K * Inventory + bv * KR
                   M ← m
                End If
             End For
             G(k,i,j) ← MING
             MIN_loc(k,i,j) ← M-1
          Else
             manufInventory ← SUM(Demand(i to j-1))*(i+1-k) //Demand splitting is not allowed
{Property 4}
             temp ← 1 to (j-k)
             Inventory ← SUM (Demand(k to j-1).*temp) // element-by-element multiplication
{Property 4}
             G(k,i,j) ← HM * manufInventory + K * Inventory + bv * KR
             DQ(k,i,j) ← TOTALs(i,j-1) // a dispatch cannot
```

```
be made to fill solely backorders
{Property 5}
            MIN_loc(k,i,j)  ←  I
          End If
        End For
      End For
    End For
    Return G, MIN_loc
```

Binary variable bv is 1 when the total order is from i+1 to j is greater than zero.

```
Compute_Cs (G,Parameters:KM)
For i←1 to PERIOD+1 // 1 ≤ i < j ≤ PERIOD+1
  For j←i+1 to PERIOD+1 // 1 ≤ i < j ≤ PERIOD+1
    For k←i+1 to j // i < k ≤ j
      MIN_VALUE ← KM + G(k,i,j)
      I←I
      For v←k to j-1
        If((C(k,i,v)+ G(k,v,j))≤ MIN_VALUE) Then
          MIN_VALUE ← (C(k,i,v)+ G(k,v,j))
          I ← v
        End If
      End For
      SELECTED_I(k,i,j) ← MIN_loc(k,I,j)//store indices
```

```
where the value has calculated
          temp←0 //temporary variable to calculate arc costs
          For den←I+1 to j
             temp←temp + Demand(den-1) //Demand splitting is not
allowed
{Property 4}
          End For
          DQ(k,i,j) ← temp // a dispatch cannot be made to
fill solely backorders
{Property 5}
          C(k,i,j)← MIN_VALUE
       End For
    End For
End For
Return DQ, Arc_Costs, SELECTED_I, C
```

# APPENDIX C

# EXPERIMENTS UNDER SHORT PLANNING HORIZON

Since long term and short term experiments yield similar results, short term experiment results are presented here.

### C.1. VMI and Traditional System Comparison

In this section both retailer's and manufacturer's costs are examined and the conditions where both manufacturer and retailer are willing to join a VMI agreement are identified under short term planning. Parameter set is shown in Chapter 5, Table 3.

The expressions below explain the terms which are compared in this section.

$TSC^{r}$: Retailer's cost under traditional system

$$= \sum_t K_t^{\,r} Y_t^r + \sum_t h_t^r I_t^r + \sum_t b_t^{\,r} E_t^r$$

$VMI^{r}$: Retailer's cost under VMI

$$= \sum_t h_t^r I_t^r + \sum_t b^r E_t^r$$

$TSC^{m}$: Manufacturer's cost under traditional system

$$= \sum_t K_t^m Y_t^m + \sum_t h_t^m I_t^m$$

$VMI^m$: Manufacturer's optimal cost under VMI

$$= \sum_t K_t^r Y_t^r + \sum_t K_t^m Y_t^m + \sum_t h_t^m I_t^m$$

$TSC$: System-wide costs under traditional system

$= TSC^m + TSC^r$

$VMI$: System-wide costs under VMI

$= VMI^m + VMI^r$

Retailer's saving under VMI = $\%Saving^r = {(TSC^r - VMI^r)}/{TSC^r}$

Manufacturer's saving under VMI= $\%Saving^m = {(TSC^m - VMI^m)}/{TSC^m}$

### C.1.1 Effect of $K^r$ and $b^r$

*Retailer's Perspective:*

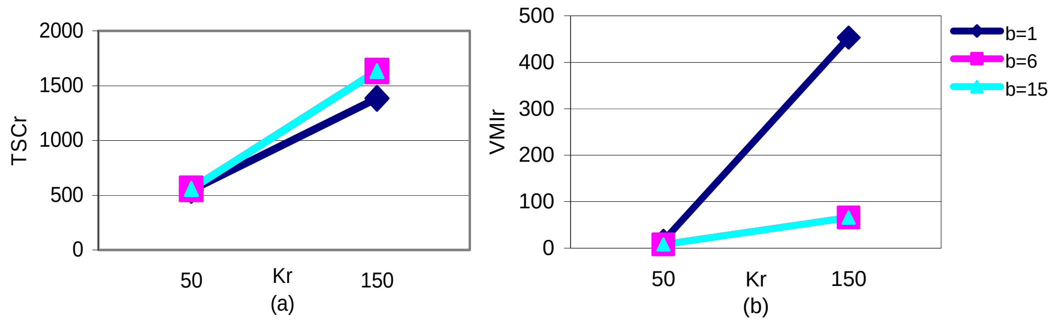


**Figure 17** Effect of $K^r$ under Scenario 1(a) and Scenario 2(b)

In Chapter 5, a long term analysis is discussed under same parameter setting with short term planning horizon. Similar results are obtained, thus for 12 period analyses, discussions are briefly presented. Observations from short term planning horizon results can be summarized as follows;

- Under Traditional System as $b^r$ is increased, dispatch frequency increases, thus transportation cost and inventory cost increases while total backorder cost decreases. Therefore; Retailer's total costs increase as unit backorder cost increase under Scenario 1 (a).
- Under VMI, as $b^r$ is increased, dispatch frequency also increases but she does not pay transportation cost. Total of backorder and inventory cost decreases thus retailer's total cost decreases. Therefore; Retailer's total costs decrease as unit backorder cost increase under Scenario 2.(b)

Retailer's saving percentage under VMI is shown in Figure 18.

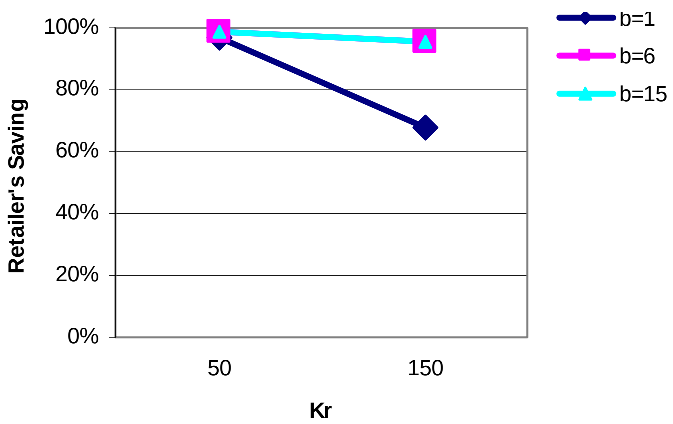


**Figure 18** Percentage Savings of Retailer under VMI

If manufacturer and retailer would share the fixed cost of transportation under VMI rather than manufacturer paying it, depending on the portion of share, Figure 18 would be different. The gap between lines of $b^r$ = 1 and $b^r$ = 6 or 15 would decrease. For example, when %90 of fixed cost of transportation is charged to retailer, operating cost of retailer would increase when unit backordering cost switches from 1 to 6 or 15. Scenario 1 approaches to Scenario 2 as the transportation cost portion of retailer increases.

Observations of both short term and long term analyses from retailer's perspective are as follows;

- Retailer is always better off under VMI.
- Value of $K^m$ does not have any effect on retailer's cost under both scenarios. Thus, $K^m$ does not affect retailer's percentage saving under VMI.
- Under VMI, as $b^r$ increases, retailer's cost decrease for both $K^r$ = 50 and $K^r$ = 150.
- Under Traditional System as $b^r$ increases, retailer's cost increase for both $K^r$ = 50 and $K^r$ =150.
- VMI is more beneficial for the retailer when $K^r$ = 50 and $b^r$ = 15 because retailer's cost is almost zero under VMI.

*Manufacturer's Perspective:*

In this part, effect of fixed cost of production on manufacturer's total costs under both scenarios is studied. Since fixed cost of transportation is charged to manufacturer under VMI, cost changes for each $K^r$ value are examined independently. Similar results are obtained to the results of long term planning horizon.

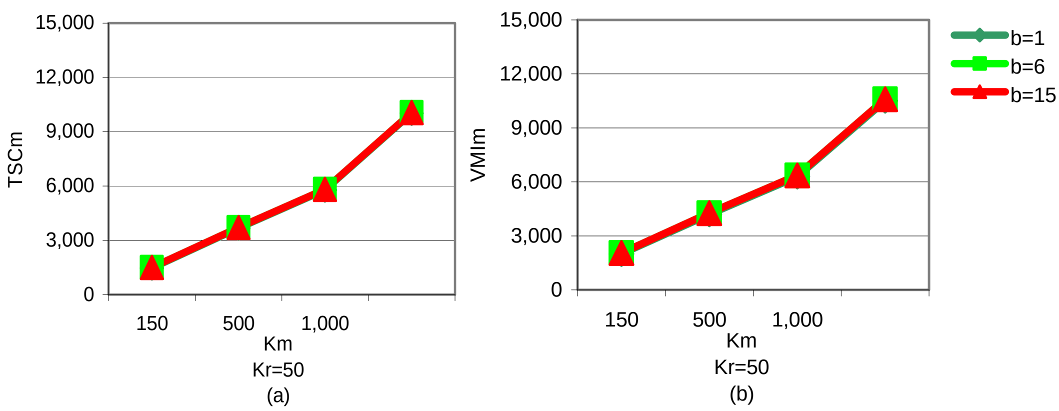


**Figure 19** Manufacturer's Cost under Scenario 1(a) and Scenario 2 (b) when $K^r$ =50

Manufacturer's cost increases under both scenarios as $K^m$ increases when $K^r = 50$ as in Figure 19 (a) and (b). Under each scenario, manufacturer's operating costs are same when $b^r \in \{6, 15\}$. Under Scenario 1, as backorder cost decreases to 1, since backordered amount increases, retailer's optimal ordering frequency decreases and total production cost decreases. Thus, total cost of manufacturer decreases as in Figure 19 (a). Under Scenario 2, when backorder cost increases from 1 to 6, dispatch frequency increases and total production cost of manufacturer also increases. Thus, total cost of manufacturer increases when unit backorder cost of the retailer increases from 1 to 6. However, any change in unit backorder cost does not have much effect on manufacturer's cost under either scenario 1 or 2. That is why under scenarios, manufacturer production and dispatch sequence is very high and holding inventory is not a decision.

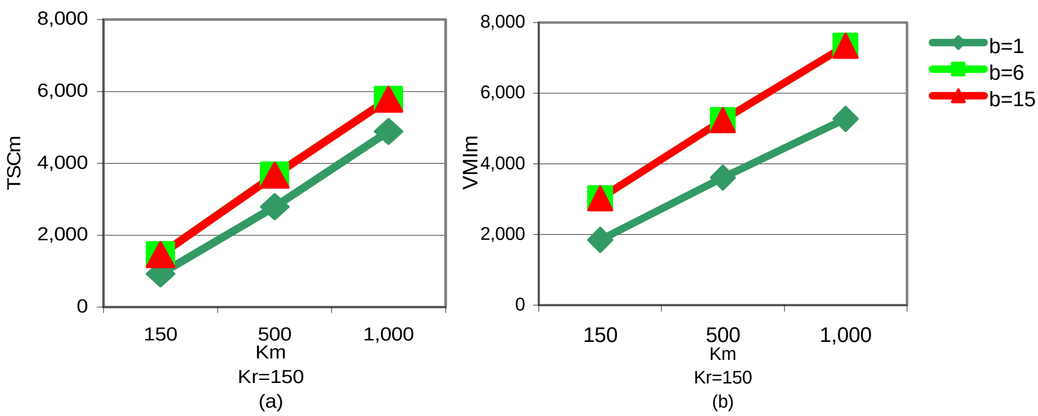


**Figure 20** Manufacturer's Cost under Scenario 1 (a) and Scenario 2 (b) when $K^r = 150$

As in Figure 20 (a) and (b), manufacturer cost increases as $K^m$ increases. Under each scenario, manufacturer's operating costs are same when $b^r \in \{6, 15\}$. Total cost of manufacturer increases when unit backorder cost of the retailer increases from 1 to 6, as in case $K^r = 50$. However, amount of increase is higher than $K^r = 50$ for several important reasons. When $K^r = 50$, manufacturer's production frequency is higher than in $K^r$ = 150.Consequently, when $K^r$ = 150, backorder level is much more than when $K^r$ = 50 while all other parameter values do not change. Under traditional case, less frequent dispatch means less frequent production since manufacturer has to produce according to the retailer's optimal ordering policy. Thus, amount of increase in $TSC^m$ as backorder cost increases when $K^r$ = 150 is much more than amount of increase when $K^r$ = 50. Under VMI, as backorder cost decreases to 1 (b), amount of decrease in total cost is very high in comparison with traditional case (a). This condition emerges from the fact that under VMI, manufacturer pays transportation cost and retailer's total cost under VMI only includes backordering and inventory holding costs. Any increase in backorder and inventory level under VMI causes an unexpected amount of change in retailer's total cost.

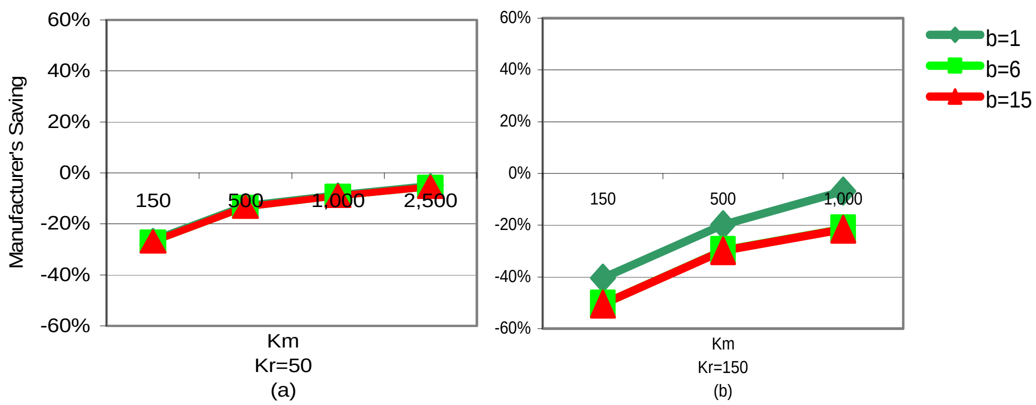


**Figure 21** Manufacturer's Savings under VMI

If Figure 21 (a) and (b) are examined, it can be concluded that manufacturer cannot decrease his costs under VMI. Manufacturer's loss is almost zero when $K^r$ = 50, $b^r$ = 1 and $K^m$ = 2500. As fixed cost of production is high and fixed cost of transportation and unit backorder cost of the retailer is low, total cost of manufacturer under traditional system approaches to total cost of manufacturer under VMI. Since $K^r$ is charged to the manufacturer under Scenario 2, effect of $K^m$

dominates influence of $K^r$ when $K^m$ = 2500 and $K^r$ =50. In this case, production frequency is low and dispatch frequency is high. Thus manufacturer holds inventory and retailer does not hold any inventory or she does not backorder. Production cost becomes important cost element. Therefore, traditional system and VMI system does not differ much.

Manufacturer's loss is very high when $K^r$ = 150, $K^m$ = 150 and $b^r$ = 1. Backorder and inventory level is very low under traditional system. Thus, under VMI, inventory and backorder performance measure levels should be zero, requiring a high dispatch frequency. Transportation cost is high and is charged to the manufacturer under VMI, increasing manufacturer's losses.

Observations on analyses from manufacturer's perspective are;

- Manufacturer is not better off under VMI.
- When $K^r$ = 50, any change in $b^r$ does not change manufacturer's cost much.
- As $K^m$ dominates $K^r$, manufacturer's costs under VMI approaches costs under Traditional System.

### C.1.2. Effect of demand pattern and $h^m$

*Retailer's Perspective:*

In order to see the influence of manufacturer holding cost and demand variation, all corresponding values are averaged out and the relation between holding cost ratio of manufacturer and retailer is analyzed for each distinguished demand variation.

Increase in unit holding cost of the manufacturer does not affect the cost of retailer under traditional system because of independent decision making. Because of this, only Figure 22 which is related with demand variance change is presented as below.

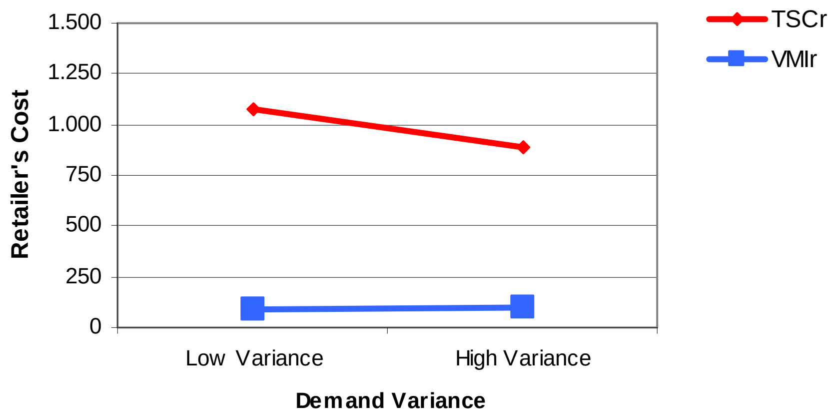


**Figure 22** Effect of demand variance to Retailer's Cost

Increase in unit holding cost of the manufacturer does not affect the cost of retailer under traditional system because of independent decision making. Because of this, only figure related with demand variance change is presented. (Figure 22) Under traditional system, demand forecast with low variance creates more operational costs for the retailer (Figure 22). When demand forecast variance is high from one period to the other, retailer's average inventory level increases while average backorder level decreases. As backorder level decreases, backorder cost decreases while inventory holding cost of retailer increases. Total of backorder and inventory costs increase under high variance. Frequency of dispatch decreases resulting in a decrease in transportation cost. Thus under Scenario 1 retailer's cost decrease when demand variance is high. Under VMI, there is an opposite behavior because total of backorder and inventory costs increase when there is high variance under Scenario 2.

Two important conclusions which can be drawn are as follows;

- Increase in demand variance and $h^m$, does not influence retailer's cost under VMI.
- Manufacturer losses are minimized when demand variance is high under VMI.

*Manufacturer's Perspective:*

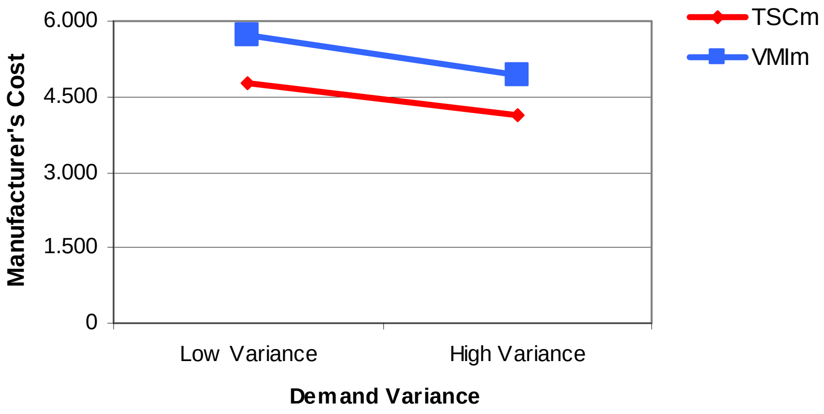


**Figure 23** Effect of demand variance on Manufacturer's Cost

Manufacturer's costs increase when $h^m$ increase under both scenarios so relationship between solely total cost and demand variance is examined. When demand variance is high under Traditional System, manufacturer's cost becomes less. Meanwhile, as demand variance decreases under VMI, manufacturer's losses increase. Manufacturer holds more inventories under demand forecast with high variance and this decreases the frequency of production, decreasing fixed production cost under both scenarios.

Observations drawn for short term planning are as follows;

- Retailer's cost increase when demand variance is high under VMI.
- Retailer's cost decrease when demand variance is high under Traditional System.
- Manufacturer costs are less when demand variance is high under both VMI and Traditional System

### C.2. System-Wide Costs Comparison

Under Traditional System, each party takes independent decisions and system-wide costs are directly summation of manufacturer's and retailer's optimal operating costs. Manufacturer minimizes fixed cost of production and inventory holding cost while satisfying retailer's orders. Retailer minimizes fixed cost of transportation, inventory holding and backordering costs. Under VMI System; model's objective is to minimize operating cost of manufacturer where manufacturer's costs are fixed cost of production, transportation and holding cost while satisfying retailer's performance measures. Manufacturer is the decision maker. Retailer's costs are inventory holding cost and backorder cost. Retailer's costs do not affect the objective of the problem as soon as performance measures are satisfied. Under Centralized System, model's objective is to minimize system-wide costs; total of fixed cost of production, transportation, holding cost of manufacturer and retailer, and backorder cost. Manufacturer and retailer merges and act like a single entities, decision maker is this single entity. Thus, objectives of each scenario are different as below.

**Objective of Scenario 1:** Minimize $TSC^r$ and $TSC^m$ independently and obtain TSC.

Where

$$TSC^r = \sum_t K^r Y_t^r + \sum_t h_t^r I_t^r + \sum_t b_t^r E_t^r$$

$$TSC^m = \sum_t K^m Y_t^m + \sum_t h_t{}^m I_t^m$$

and $TSC = TSC^r + TSC^m$

**Objective of Scenario 2:** Minimize $\text{VMI}^m$ and obtain VMI.

Where

$$VMI^m = \sum_t K^r Y_t^r + \sum_t K^m Y_t^m + \sum_t h_t{}^m I_t^m$$

$$VMI^r = \sum_t h_t^r I_t^r + \sum_t b_t^r E_t^r$$

and $VMI= VMI^r + VMI^m$

**Objective of Scenario 3:**

$$Cent=\sum_t K^r Y_t^r + \sum_t K^m Y_t^m + \sum_t h_t^r I_t^r + \sum_t h_t^m I_t^m + \sum_t b_t^r E_t^r$$

Differences between system-wide costs are compared where**;**

$$\%Difference\_TSC = (TSC - Cent) / Cent$$

$$\%Difference\_VMI = (VMI - Cent) / Cent$$

Since scenario 3 is benchmark for scenarios 1 and 2, VMI and Traditional system-wide cost differences are compared with centralized system-wide costs. Figures from 24 to 26 are presented in order to check Traditional system and VMI system behavior from system costs perspective in short term, while considering the effects of $b^r$, $K^r$ and $K^m$.

*Case 1: $b^r=1$*

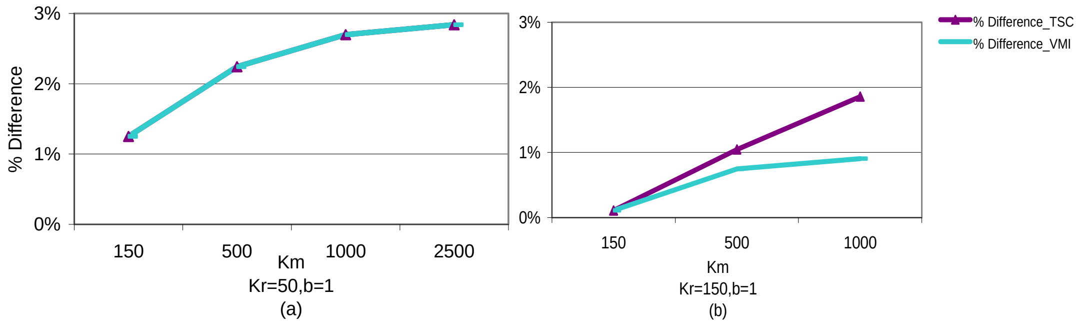


**Figure 24** Percentage Cost Difference between Scenario 1-3 (a) and Scenario 2-3 (b) for $b^r = 1$, $K^r =$ 50 or 150

*Case 2: $b^r$=6*

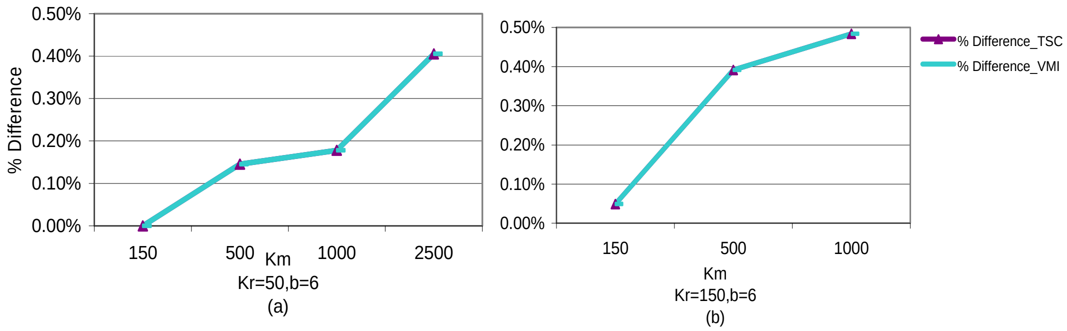


**Figure 25** Percentage Cost Difference between Scenario 1-3 (a) and Scenario 2-3 (b) for $b^r$ = 6, $K^r$ = 50 or 150

*Case 3: $b^r$=15*

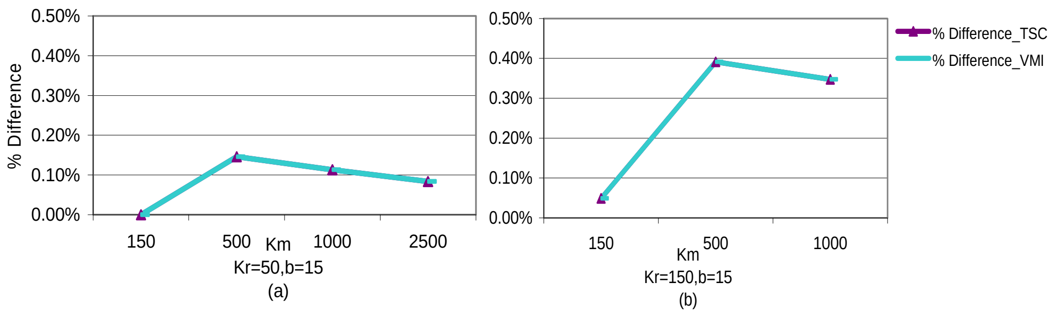


**Figure 26** Percentage Cost Difference between Scenario 1-3 (a) and Scenario 2-3(b) for $b^r$ = 15, $K^r$ =50 or 150

Similar discussion as in long term analyses (Chapter 5) is valid for each figure. Important conclusions which can be drawn from system-wide costs comparison of Scenario 1, 2 and 3 both in short term and long term are as follows; (Figures 24, 25, 26)

- Increase in unit backorder cost of retailer converges three scenarios
- Decrease in unit backorder cost results in counter behavior in scenario 3 and the other scenarios.
- When $K^{r}$ = 50, VMI system-wide costs approach to Traditional System costs as unit backorder cost increase.

**C.3. Lower Bound and Upper Bound Evaluation on Optimal Solution**

All problems are solved in GAMS 2.0.13.0 with solver CPLEX 7.0. Relaxed constrained ULS problem is also solved in GAMS with solver CPLEX. In order to evaluate the performance of relaxed model, optimal objective values and objective values of relaxed model are compared. Moreover, for the experiments that produced infeasible solutions, performance of the *ObtainFeasible* is also evaluated.

As discussed in Chapter 4, two performance measures for evaluating bound performance are taken into consideration.

**Performance measure-1:** $Z^{*}-L(u,k)) / Z^{*}$

= (Optimal Objective Value of VMI-Lower Bound) /Optimal Objective Value

**Performance measure-2:** $(VMI^{up} - Z^{*}) / Z^{*}$

= (Upper Bound-Optimal Objective Value)/Optimal Objective Value

Upper Bound can be;

- Obtained by Relaxed VMI model's solution
- Obtained by *ObtainFeasible* heuristic for infeasible results of the *LagRel* Model

**Table 9** Bound evaluation for 12-Period results

| | ***(Z*- L(u,k))/Z**** | ***(VMI$^{up}$-Z*)/Z**** |
|---|---|---|
| **Max.Deviation** | 3,72% | 19,32% |
| **Min.Deviation** | 0% | 0% |
| **Average** | 0,001% | 0,03% |

Relaxed VMI deviates 0,001% from the optimal solution on the average. In 836 runs out of 840, Relaxed Model obtains the optimal solution. Out of 840 runs, only 1 infeasible solution is obtained. Thus, 40-Period results for upper bound evaluation would be much more conclusive.

### C.4. Computational Time Requirements

C-ULS-B problem is relaxed and solved in GAMS (VMI$^{l}$), VMI is C-ULS problem solved optimally in GAMS and *LagRel* problem is solved in MATLAB.

**Table 10** Computational Time Requirements

| **Model** | **# of Calls** | **Total CPU Time (sec)** | **CPU Time per instance (sec)** |
|---|---|---|---|
| VMI$^{l}$ | 840 | 2.513 | 2,99 |
| VMI | 840 | 446 | 0,53 |
| *LagRel* | 840 | 115 | 0,13 |

As Table 10 shows, Lagrangean Relaxation Model coded in GAMS with solver CPLEX, is not sufficient. Thus, in long term analyses, this model solved in GAMS, is not considered.

# APPENDIX D

# HOW TO MAKE SO MANY RUNS

In brief, many runs are recorded in order to check the behavior of the models. It is a hard and time consuming process to succeed in so many runs for long periods, without making any mistake.

During short term experiments 840 runs are made and each parties' costs, performance measures under each scenario are recorded. During long term experiments, 84 runs are made.

These processes would take long time if steps below are done manually.

For each parameter set, following steps are done.

**1-** Initialize parameter set

**2-** Solve traditional retailer's problem

   a. All parameters are updated

   b. Solve the TS-R

   c. Record the objective value and optimal backorder and inventory levels

   d. Record optimal order quantities

**3-** Solve traditional manufacturer's problem

a. All parameters are updated

b. Demand: Copy optimal order quantities of the retailer obtained in step 2

c. Solve the TS-M

d. Record the objective value

**4-** Solve VMI model

a. All parameters are updated

a. Two complicating constraint limits are updated according to step 2.b

b. Solve the C-ULS-B MIP model

c. Record optimal retailer and manufacturer costs ,objective value

**5-** Solve Centralized Model

a. All parameters are updated

b. Solve the ULS-B MIP model

c. Record optimal retailer and manufacturer costs

**6-** Solve Lagrangean Relaxation Model in Gams (For small time bucket problem; T=12)

a. All parameters are updated

b. Two complicating constraint limits are updated according to step 2.b

c. Subgradient algorithm step size calculation is updated according to step 2.b

d. Solve the problem

e. Record retailer and manufacturer costs

f. Record objective value

g. Record upper bound

h. Record inventory and backorder levels

i. Record *ObtainFeasible* results if Lagrangean produces infeasibility

i. Upper bound

ii. Backorder Level

iii. Inventory Level

**7-** Solve *LagRel* in MATLAB (For big time bucket problem; T=40,50,60)

a. All parameters are updated

b. Two complicating constraint limits are updated according to step 2.b

c. Subgradient algorithm step size calculation is updated according to step 2.b

d. Solve the problem

e. Record retailer and manufacturer costs

f. Record objective value

g. Record upper bound

h. Record inventory and backorder levels

i. Record *ObtainFeasible* results if Hybrid produces infeasibility

i. Upper bound

ii. Backorder Level

iii. Inventory Level

RETURN TO STEP 1

Thus, 420 runs are made in short term and 252 runs are made in long term analyses and all steps are repeated accordingly. For long term analyses all steps are repeated as number of runs excluding relaxed VMI model solved in GAMS with solver CPLEX. This process would take months if all runs are done manually. A macro is coded in MATLAB for all these steps. All records are written in excel automatically to a specific cell.