STRUCTURAL MODIFICATION WITH ADDITIONAL DEGREES OF FREEDOM IN LARGE SYSTEMS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

GÜVENÇ CANBALOĞLU

IN PARTIAL FULLFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
MECHANICAL ENGINEERING

JUNE 2009

Approval of the thesis:

STRUCTURAL MODIFICATION WITH ADDITIONAL DEGREES OF FREEDOM IN LARGE SYSTEMS

submitted by **GÜVENÇ CANBALOĞLU** in partial fulfillment of the requirements for the degree of **Master of Science in Mechanical Engineering Department, Middle East Technical University** by,

Prof. Dr. Canan Özgen ________
Dean, Graduate School of **Natural and Applied Sciences**

Prof. Dr. Suha Oral ________
Head of Department, **Mechanical Engineering**

Prof. Dr. H. Nevzat Özgüven ________
Supervisor, **Mechanical Engineering Dept., METU**

Dr. Halidun Fildiş ________
Co-Supervisor, **ASELSAN Inc.**

Examining Committee Members:

Prof. Dr. Yavuz Samim Ünlüsoy ______________________
Mechanical Engineering Dept., METU

Prof. Dr. H. Nevzat Özgüven ______________________
Mechanical Engineering Dept., METU

Prof. Dr. Yavuz Yaman ______________________
Aerospace Engineering Dept., METU

Asst. Prof. Dr. Yiğit Yazıcıoğlu ______________________
Mechanical Engineering Dept., METU

Dr. Halidun Fildiş ______________________
ASELSAN Inc.

Date: 26/06/2009

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name	: Güvenç CANBALOĞLU
Signature	:

ABSTRACT

STRUCTURAL MODIFICATION WITH ADDITIONAL DEGREES OF FREEDOM IN LARGE SYSTEMS

Canbaloğlu, Güvenç
M.S., Department of Mechanical Engineering
Supervisor : Prof. Dr. H. Nevzat Özgüven
Co-Supervisor : Dr. Halidun Fildiş

June 2009, 162 Pages

In the design and development stages of mechanical structures, it is important to predict the dynamic characteristics of modified structures. Since time and cost are critical in design and development stage, structural modification methods predicting the dynamic responses of modified structures from those of the original structure and modification properties are very important, especially for large systems.

In this thesis structural modification methods are investigated and an effective structural modification method for modifications with additional degrees of freedom is adapted to structures with distributed modifications and the performance of the method is investigated. A software program is developed in order to apply the structural modification method with additional degrees of freedom. In the software, the dynamic response of the modified structure is predicted by using the modal analysis results of ANSYS for the original structure and dynamic stiffness matrix of the modifying structure. In order to validate the approach used and the program developed, the dynamic analysis results obtained for modified structures by ANSYS are compared with those obtained by using the software. In order to investigate the performance of the structural modification method in real applications, the method is applied to a scaled aircraft model, and the results are compared with experimental results.

In order to demonstrate the importance of using the structural modification method with additional degrees of freedom for distributed modification, lumped and distributed models are used for a distributed modification and results are compared.

It is concluded in this study that using structural modification methods with additional degrees of freedom for a distributed modification increases the accuracy of the results, and it is observed that the method adapted is efficient for local modifications.

Keywords: Structural modification method, distributed modification, GARTEUR.

ÖZ

BÜYÜK SİSTEMLERDE EK SERBESTLİK DERECELİ YAPISAL DEĞİŞİKLİK

Canbaloğlu, Güvenç
Yüksek Lisans, Makine Mühendisliği Bölümü
Tez Yöneticisi : Prof. Dr. H. Nevzat Özgüven
Ortak Tez Yöneticisi : Dr. Halidun Fildiş

Haziran 2009, 162 Sayfa

Mekanik yapıların, tasarım ve geliştirme sürecinde, değişikliğe uğramış yapıların dinamik davranışlarını hesaplamak önem taşımaktadır. Tasarım ve geliştirme sürecinde, zaman ve maliyet kritik olduğundan, özellikle büyük sistemlerde, yapısal değişiklik yöntemleri, değişmiş yapının dinamik davranışının, esas yapının dinamik davranışı ile yapısal değişikliğin dinamik bilgileri kullanılarak belirlenmesi önemlidir.

Bu tezde yapısal değişiklik yöntemleri incelenmiş ve etkili bir ek serbestlik dereceli yapısal değişiklik yöntemi, dağıtılmış parametreli değişikliğe uğramış yapılara uyarlanmış ve yöntemin performansı incelenmiştir. Ek serbestlik dereceli yapısal değişiklik yöntemini uygulamak için bir program geliştirilmiştir. Programda, değişmiş yapının dinamik cevabı, esas yapı için ANSYS'te yapılmış olan analitik biçim analizi sonuçları ve değişikliğin dinamik direngenlik matrisi kullanılarak hesaplanmıştır. Kullanılan yöntemi ve geliştirilen programı doğrulamak için, değişmiş yapı için yapılmış olan analitik biçim analizi sonuçları, programdan elde

edilmiş sonuçlarla karşılaştırılmıştır. Yapısal değişiklik yönteminin, gerçek yapılardaki performansını incelemek için, yöntem ölçeklendirilmiş bir uçak modeli üzerinde uygulanmış ve sonuçlar deneysel sonuçlarla karşılaştırılmıştır.

Dağıtılmış yapısal değişiklik için ek serbestlik dereceli yapısal değişiklik yöntemi kullanmanın önemini göstermek amacıyla, dağıtılmış yapısal değişiklik için toplanmış parametreli ve dağıtılmış parametreli modeller kullanılmış ve sonuçlar karşılaştırılmıştır.

Bu tezde, dağıtılmış yapısal değişiklik için, ek serbestlik dereceli yapısal değişiklik yöntemlerinin sonuçların hassasiyetini arttırdığı sonucuna varılmıştır ve uyarlanan yöntemin bölgesel değişiklikler için etkili olduğu görülmüştür.

Anahtar Kelimeler: Yapısal değişiklik yöntemi, dağıtılmış parametreli değişiklik, GARTEUR.

To My Family…

ACKNOWLEDGEMENTS

I would like to express my sincere appreciation to my supervisor, Prof. Dr. H. Nevzat ÖZGÜVEN for his boundless help, excellent supervision and leading guidance from beginning to end of thesis work.

I would also like to express my sincere gratitude to my co-supervisor, Dr. Halidun FİLDİŞ, for his invaluable guidance throughout this study.

I would like to express my special thanks to Ahmet Levent AVŞAR for his discussion throughout this work.

The author would also like to thank his family for their endless love and support throughout his life.

Also I would like to thank to ASELSAN for giving me the opportunity to use the computational and testing capabilities.

Also I would like to thank to TÜBİTAK for giving me the financial support throughout this work.

TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

LIST OF SYMBOLS

$[B]$	: Dynamic Stiffness Matrix of Modified Structure
$[B_0]$	: Dynamic Stiffness Matrix of Original Structure
$[C]$	: Viscous Damping Matrix
$[D]$	: Dynamic Stiffness Matrix
$[D_0]$	: Dynamic Stiffness Matrix of Original Structure
$[D_{\text{mod}}]$	: Dynamic Stiffness Matrix of Modifying Structure
$\{F\}$, $\{f\}$	: Generalized Forcing Vector
i	: Unit Imaginary Number
$[I]$	: Identity Matrix
$[H]$	: Structural Damping Matrix
$[H_0]$	: Structural Damping Matrix of Original Structure
$[H_{\text{mod}}]$	: Structural Damping Matrix of Modifying Structure
k	: Stiffness
$[K]$	: Stiffness Matrix
$[K_c]$	: Condensed Stiffness Matrix of Modified Structure
$[K_{\text{mod}}]$	: Stiffness Matrix of Modifying Structure
$[K_0]$	: Stiffness Matrix of Original Structure
$[K_{0c}]$	: Condensed Stiffness Matrix of Original Structure
m	: Mass
$[M]$	: Mass matrix
$[M_0]$	: Mass Matrix of Original Structure

$[M_{\text{mod}}]$: Mass Matrix of Modifying Structure

$\{R\}$: Damping Force Vector

$\{x\}$: Generalized Displacement Vector

$[\Delta B]$: Dynamic Stiffness Matrix of Modification

$[\Delta C]$: Modification Viscous Damping Matrix

$[\Delta D]$: Modification Dynamic Stiffness Matrix

$[\Delta H]$: Modification Structural Damping Matrix

$[\Delta K]$: Modification Stiffness Matrix

$[\Delta M]$: Modification Mass Matrix

$[\alpha]$: Receptance Matrix of Original Structure

$[\delta]$: Dynamic Stiffness Matrix

$[\gamma]$: Receptance Matrix of Modified Structure

$\{\phi\}$: Mode Shape Vector

η : Modal Structural Damping Coefficient

ω : Frequency

LIST OF ABBREVIATIONS

FE	: Finite Element
FEA	: Finite Element Analysis
DOF	: Degree of Freedom
DMSM	: Dual Modal Space Modification Method
SDOF	: Single Degree of Freedom
FRF	: Frequency Response Function

CHAPTER 1

INTRODUCTION

1.1 Basics and Importance of Structural Modification

In the design stage of mechanical structures, the main objective is to satisfy the design requirements with the minimum possible cost and time. Since the mechanical structures are designed to be as strong as necessary, the overdesigned structures which were designed previously are replaced with their substitutes which satisfy the requirements with the minimum cost. However, in order to reduce the material usage thus the cost, the mechanical structures has less strength than the overdesigned structures; therefore the structural dynamics and strength of the structures become a critical issue. Considering the aerospace industry, mechanical structures of the aircrafts and the helicopters are designed according to the requirements which prevent the mechanical and aerodynamical problems that can be encountered and these design requirements are more severe compared to the ground-based structures; therefore, the structural and aerodynamical reliability of the structures are more critical.

Any modification applied on mechanical structures has an effect of changing the dynamic properties of the original structure; therefore the modified structures have to be reanalyzed, in order to obtain the new dynamic characteristics of them. After every modification, reanalyzing the whole structure is a very time consuming and cotly process. Especially for large ordered systems, after every structural modification, the whole analytical model of the new structure should be built or the structure has to be re-tested in order to ensure that the vibration characteristics of the structure are satisfied. However, this is a very costly and time-consuming operation. In order to understand the processes better, the following cycle shown in Figure 1.1 can be given [1].

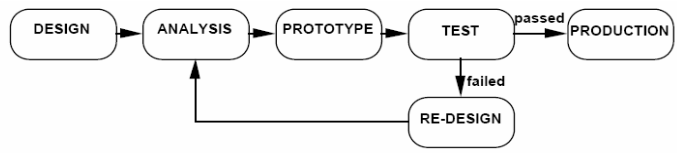


Figure 1.1 Simplified "Design to Production-Line" Cycle [1]

As seen in Figure 1.1, after any modification applied on the original structure or any re-design process, the whole process from analysis to testing has to be repeated. However by using structural dynamic modification techniques, the dynamic properties of the modified structure can be predicted from the dynamic characteristics of the original structure and the modification data. Therefore, after every modification introduced on the original structure, there is no need to construct the whole analytical model of the structure. Schematic view of the structural modification analysis is given in Figure 1.2.

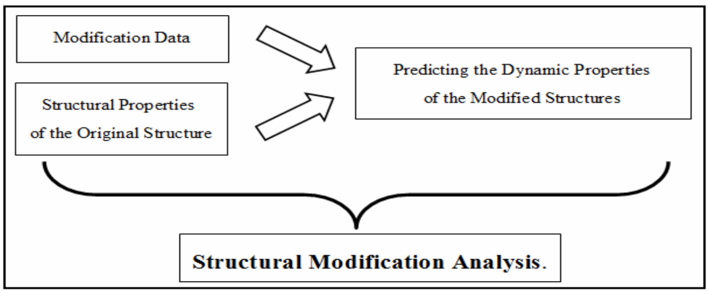


Figure 1.2 Schematic View of the Structural Modification Analysis

Due to the time and cost efficiency, structural dynamic modification techniques becomes more important in the design stages. Dynamic characteristics of the original structure are obtained experimentally or analytically. The finite element (FE) models of the original structure can be constructed or modal testing can be conducted on the original structure for obtaining dynamic behavior of the structure. In the analytical approaches, the frequency response functions (FRFs) or the system matrices of the original structure can be used in order to calculate the FRFs of the modified structure. In the experimental approaches, the measured FRFs or the estimated modal parameters can be used to estimate the FRFs of the modified structure.

1.2 Literature Survey

Structural modification methods focus on the change of dynamic behavior of a structure due to modifications in mass, stiffness and damping properties of the system. Global type of matrix changes due to these modifications were studied by O'Callahan and Avitabile [2]. In this work, O'Callahan and Avitabile [2] presented a structural modification procedure which uses complex (damped) modes obtained by the finite element analysis (FEA) or modal analysis. By this procedure global matrix changes in the mass, stiffness and damping were performed simultaneously. In later work, Wallack, et al. [3] presented the local structural modifications technique for the general matrix modifications by implementing the structural modification technique in one eigen solution.

The beam or rib types of modifications are extensively used in the designs, in order to increase the stiffness of the structures. Therefore these types of modifications were studied by different researchers. In structural modification procedure, generally the mass and stiffness matrices of a generalized beam element were used. O'Callahan and Chou [4] performed the beam modification procedure by using a local eigen value modification technique. Elliott and Mitchell [5] presented a method for analyzing the beam type structural modifications by combining the Dual Modal

Space Modification Method (DMSM) and Transfer Matrix Method. In a later work Tayeb and Williams [6] modeled straight, angled rib or beam like stiffeners and presented the usage of these elements in the structural dynamic modification analysis. Hutton [7] developed a FE based procedure in order to reanalyze the vibration response of the modified structure on which physical changes such as thickness and mass density are applied. In a further work, D'Ambrogio [8] studied the prediction of frequency response function of the modified structure subjected to modification in the form of rib and plate stiffeners causing flexural rigidity change and presented quasi-local characteristics of the additional dynamic stiffness matrix due to structural modification.

In order to apply the structural modification techniques, the usage of both theoretical and experimental data can be necessary. Wang, et al. [9] studied the effects of local modifications on the dynamic characteristics of the existing structures, by using experimentally obtained modal data of the original structure and the characteristics of the modification. Then, in order to derive the dynamic properties of the modified structure, Jones and Iberle [10] used the modal model derived from a set of measured FRFs of the original structure and directly measured FRF data of the original structure in structural modification procedure and compared these techniques. In later work Imregun, et al. [11] studied the usage of both measured and theoretical data in structural modifications by examining an alternative approach based on the FRF data. Salvini and Sestieri [12] developed a method for predicting the FRF of a given system from the experimentally determined FRF of the system itself subjected to different constraint which are any type involving either translational or rotational degree of freedoms (DOFs).

Structural dynamic modification problems can be divided into two categories: direct structural dynamic modifications and inverse structural dynamic modifications. Direct structural dynamic modification concentrates on the determination of modified structure characteristics due to modification on the original structure.

Conversely, inverse structural dynamic modification is an optimization procedure looking for necessary modifications in order to achieve the desired dynamic behavior. Kyprianou, et al. [13, 14] focused on inverse structural dynamic modifications. Li and He [15] presented a new approach for structural modifications required to change the dynamic characteristics of an undamped system. Furthermore, Park [16, 17] studied measured frequency response function based inverse structural dynamic modification in order to obtain necessary structural modifications. In a later work, Mottershead, et al. [18] presented an inverse method for assigning natural frequencies and nodes of normal modes of vibration by the addition of grounded springs and concentrated masses. The basic theory of direct structural modification using experimental FRF was presented by Crowley, et al. [19].

The Sherman-Morrison [20] and Woodbury [21] developed general matrix-update formulas which can be used in structural modification analysis. Özgüven [22] proposed a matrix inversion method in order to find receptances of locally damped structures from those of the corresponding undamped structure. Later a recursive solution algorithm was presented in order to avoid the matrix inversion [23]. In a further work [24], Özgüven presented an approach for reanalyzing a structure subjected to structural modification with or without additional DOF. In this method, by using FRF matrix of the original system and mass, stiffness and damping matrices of the modifying structure, the exact FRFs of the modified structure were estimated [24]. Şanlıtürk [25] used the same approach, but avoided matrix inversion by employing Sherman-Morrison method.

As alternative techniques in structural modifications, Bae, et al. [26] developed a technique called Successive Matrix Inversion for static analysis. By using this technique, the solutions of the any local modifications applied on a static FE model were obtained. Then in later study, Successive Matrix Inversion for static analysis was extended by Köksal, et al. [27] for the dynamic analysis of structures. In this method the FRF matrix of a modified structure was obtained by using the FRF matrix of the original structure and the modifying mass, stiffness and damping matrices. In order to avoid the matrix inversions in the equations, power series expansion method was used. Then in a further study, Köksal [28] studied the comparison of structural modification techniques and used these techniques in a real application of the modification of a jet aircraft. Mottershead et al. [29] also applied structural modification technique on the tailcone of the helicopter.

In structural modification, transfer function methods can also be used. These techniques were studied by Jingshuo, et al. [30]. In this work they presented a method for structural dynamic modification using transfer functions, and then in a later work [31], the concept of optimal modification was studied for structural dynamic modification using transfer matrix and the sensitivity analysis.

The modeling approach of distributed modifications in structural dynamic modifications was presented by D'Ambrogio and Sestieri [32]. Later, D'Ambrogio and Sestieri [33] studied the condensation and the expansion techniques in order to predict the dynamic effect of distributed modifications, since generally, dynamic properties of an original structure are identified by experimental techniques containing only translational DOF due to the difficulties in measuring rotational DOF, and structural information of modifying structure contains both rotational and translational DOF. Then in a further study, D'Ambrogio and Sestieri [34] extended the studies in order to obtain the dynamic characteristics of the modified structure subjected to distributed modifications by coupling the theoretical data and translational FRFs. Hang, et al. [35] focused on the distributed structural dynamics modification with additional DOFs by using the original relationship developed by Özgüven [24] and modeling method of the distributed modification developed by D'Ambrogio and Sestieri [32]. In a recent work, Canbaloglu and Özgüven [36] studied the structural modifications with additional DOFs for distributed modifications proposing a different approach for modeling distributed modification and applying it to a real structure by using the original relationship developed by Özgüven [24].

In structural modification problems, the rotational DOF is an important issue to be considered; therefore, Smiley [37] studied the necessity of rotational DOFs in the structural analysis and focused on the procedure of implementing rotational DOFs in structural modification analysis. Then in a later study, due to the effects of moment transfer in the connection points in structural modification procedures, generation of rotational DOFs from the existing translational DOFs by expansion techniques were studied by O'Callahan, et al. [38, 39]. In structural modification analysis, rotational DOFs are required for the beam and plate elements; therefore, O'Callahan and Avitabile [40] presented an approximate method using both translational and rotational DOFs in FE model in conjunction with the measured translational data in order to obtain a modal database of rotational and translational DOFs . In a further work, Mottershead, et al. [41] studied obtaining rotational receptances by using "T-block" approach in order to apply a pure moment to a structure so that the rotational receptances can be included in the structural modification analysis.

In structural dynamic modifications, truncation of modal data is another important issue, since the errors due to truncation have a considerable effect on the accuracy of the results. The modal model is usually truncated to finite number of modes, and this truncation leads to inaccurate results in structural dynamic modification due to limited number of modes used in the model. Braun and Ram [42] focused on this problem and presented the impossibility of obtaining exact solutions in structural modifications by using a truncated modal matrix. When the truncated modes were represented by the residual terms, these residual terms can be included in the structural dynamic modification technique and this technique was given by Sohaney [43]. In a later study, truncation effects and errors introduced due to truncation in the structural dynamic modification process were studied in detail by Avitabile, et al. [44] in order to present a better understanding of truncation effects. Bucher and

Braun [45] focused on the effect of truncation of the modal data in structural modifications and presented how to control and circumvent the effect of modal truncation. Canbaloglu and Özgüven [36] studied the truncation effect in structural dynamic modification with additional DOFs for distributed modifications.

1.3 Objective

The objective of this thesis is to obtain dynamic characteristics of a modified structure from those of the original structure and modification data, when modifications are distributed. This is accomplished by applying Özgüven's structural modification method with additional DOFs [24] for distributed modifications. It is also aimed in this study to develop a computer program that can apply structural modification method with additional DOFs to large structures and validate the results with different theoretical and experimental case studies. In this thesis, it is also intented to emphasize the importance and necessity of the structural dynamic modifications with additional DOFs.

1.4 Scope of the Thesis

Based on the objective of the study performed in this thesis, the outline of the thesis can be given as follows:

In Chapter 2, the theory of the structural dynamic modification methods with and without additional DOFs will be given. The modeling approaches for distributed modifications will also be explained in Chapter 2.

In Chapter 3, brief outline of the computer program developed in this thesis will be given. The computer program will be verified with different structural modification case studies in the same chapter.

In Chapter 4, in order to demonstrate the application of the program with a real case study, the modification will be applied on a test structure (GARTEUR SM-AG19). The results obtained from the computer program developed for structural modification with additional DOFs are compared and validated with experimental results.

In Chapter 5, the comparison of distributed and lumped modeling approaches for the modifications will be given. In order to show the importance of the distributed modeling by using additional DOFs when there are distributed modifications, different case studies will be presented.

In Chapter 6 the discussion, conclusions and recommendations for future work will be given.

CHAPTER 2

THEORY

2.1 Structural Modification without Additional Degrees of Freedom

2.1.1 Structural Modification by Using Sherman-Morrison Formula

Sherman-Morrison [20] formula is a simplified version of Woodbury's study [21]. Şanlıtürk [25] proposed a structural modification method which is based on Sherman-Morrison formula. In the method proposed by Şanlıtürk [25], direct inversion of the modified matrix using the information of the original matrix and modification is performed. Sherman-Morrison formula can be stated as follows;

Assume $[A]$ is a non-singular square matrix and $[A]^{-1}$ is the inverse of $[A]$ matrix. Modified matrix can be expressed in the form of following equation.

$$[A^*]=[A]+[\Delta A]=[A]+\{u\}\{v\}^T \tag{2.1}$$

Then the inverse of $[A^*]$ can be written as [20]:

$$[A^*]^{-1}=[A]^{-1}-\frac{\left([A]^{-1}\{u\}\right)\left(\{v\}^T[A]^{-1}\right)}{1+\{v\}[A]^{-1}\{u\}} \tag{2.2}$$

Sherman-Morrison formula is an extension of Woodbury formula where the modification matrix is given by the multiplication of two vectors. This modification matrix can be given as:

$$[\Delta A] = \{u\}\{v\}^T \tag{2.3}$$

The inverse of the modified matrix can be expressed by;

$$[A^*] = [A]^{-1} - [A]^{-1}[U]\left([I] + [V]^T[A]^{-1}[U]\right)[V]^T[A]^{-1} \tag{2.4}$$

Equation (2.4) is valid as long as $[A]_{NxN}$ is a square matrix. For n ≤ N, if $[A]$ and $\left([I] + [V]^T[A]^{-1}[U]\right)$ are invertible, any matrix $[\Delta A]$ can be written as the multiplication of two rectangular matrix $[U]_{Nxn}$ and $[V]_{nxN}$.

Although, the modifications are restricted to the cases where the modification can be written in the form of $\{u\}\{v\}^T$, the main advantage of the Shermon-Morisson formula is that, the inverse of the modified matrix can be calculated without any additional matrix inversion.

For a given structure, the equation of motion can be written as:

$$[M]\{\ddot{x}\} + i[H]\{x\} + [K]\{x\} = \{F\} \tag{2.5}$$

where $[M]$, $[H]$, $[K]$ are mass, structural damping and stiffness matrices of the structure respectively, $\{x\}$ is the vector of generalized coordinates, $\{F\}$ is the generalized forcing vector and i is the unit imaginary number. In the frequency domain, Equation (2.5) can be rearranged as:

$$\left([K]-\omega^2[M]+i[H]\right)\{x\}=\{F\} \tag{2.6}$$

$$\{x\}=\left([K]-\omega^2[M]+i[H]\right)^{-1}\{F\} \tag{2.7}$$

Then the receptance matrix is obtained as:

$$[\alpha]=\left([K]-\omega^2[M]+i[H]\right)^{-1} \tag{2.8}$$

For a structural modification problem, receptance matrix of the original structure can be obtained by the modal summation, after modal analysis. If this original receptance matrix is available, great simplification is introduced by the Sherman-Morrison formula. The receptance matrix of the modified structure can be written as follows:

$$[\gamma]=\left([K]+[\Delta K]-\omega^2\left([M]+[\Delta M]\right)+i\left([H]+[\Delta H]\right)\right)^{-1} \tag{2.9}$$

where $[\Delta M]$, $[\Delta H]$, $[\Delta K]$ are the mass, structural damping and stiffness matrices of modification respectively,

Using Equation (2.8), the following equation can be obtained.

$$[\gamma]=\left([\alpha]^{-1}+[\Delta D]\right)^{-1} \tag{2.10}$$

where dynamic stiffness matrix $[\Delta D]$ is expressed as:

$$[\Delta D]=[\Delta K]-\omega^2[\Delta M]+i[\Delta H] \tag{2.11}$$

Expressing the dynamic stiffness matrix $[\Delta D]$ as;

$$[\Delta D] = \{u\}\{v\}^T \tag{2.12}$$

The receptance matrix of the modified structure can be obtained by using the Sherman-Morrison formula in the following form

$$[\gamma] = [\alpha] - \frac{\left([\alpha]\{u\}\right)\left(\{v\}^T [\alpha]\right)}{1 + \{v\}^T [\alpha]\{u\}} \tag{2.13}$$

In this form, the formulation may not decrease the computational time, since the modification affects all the coordinates. However, when local modifications are introduced to the original structure, the method has a great advantage in terms of computational time for calculating receptance matrix of the modified structure.

As long as the modifications can be expressed in the form of $\{u\}\{v\}^T$, receptance matrix of the modified structure can be obtained by using Sherman-Morrison formula. When the modification and forcing coordinates are limited to a small number of coordinates (compared to total coordinates), and the responses are to be obtained at selected coordinates, Sherman-Morrison formula has a great advantage and it can provide substantial savings in computational time.

For the cases stated above, the coordinates can be partitioned as active and inactive coordinates. Both $\{x\}$ and $[\alpha]$ can be portioned as:

$$\{x\} = \begin{Bmatrix} \{x_i\} \\ \{x_a\} \end{Bmatrix} \qquad \{\alpha\} = \begin{bmatrix} [\alpha_{ii}] & [\alpha_{ia}] \\ [\alpha_{ai}] & [\alpha_{aa}] \end{bmatrix} \tag{2.14}$$

where subscripts i and a represent inactive and active coordinates, recpectively. If the matrices above are inserted in Equation (2.13) the following equation can be obtained.

$$\begin{bmatrix} \gamma_{ii} & \gamma_{ia} \\ \gamma_{ai} & \gamma_{aa} \end{bmatrix} = \begin{bmatrix} \alpha_{ii} & \alpha_{ia} \\ \alpha_{ai} & \alpha_{aa} \end{bmatrix} - \frac{\left(\begin{bmatrix} [\alpha_{ii}] & [\alpha_{ia}] \\ [\alpha_{ai}] & [\alpha_{aa}] \end{bmatrix} \begin{Bmatrix} \{0\} \\ \{u_a\} \end{Bmatrix} \right) \left(\begin{Bmatrix} \{0\} \\ \{v_a\} \end{Bmatrix}^T \begin{bmatrix} [\alpha_{ii}] & [\alpha_{ia}] \\ [\alpha_{ai}] & [\alpha_{aa}] \end{bmatrix} \right)}{1 + \begin{Bmatrix} \{0\} \\ \{v_a\} \end{Bmatrix}^T \begin{bmatrix} [\alpha_{ii}] & [\alpha_{ia}] \\ [\alpha_{ai}] & [\alpha_{aa}] \end{bmatrix} \begin{Bmatrix} \{0\} \\ \{u_a\} \end{Bmatrix}} \tag{2.15}$$

In the Equation (2.15) modifications are limited to active coordinates which are the coordinates where the responses are to be calculated. In Equation (2.15) also $\{u\}$ and $\{v\}$ vectors are partitioned as inactive and active coordinates.

For the cases where only the active coordinates are retained, still Sherman-Morrison formula can be used in order to obtain the receptance matrix of the modified structure. The following equation can be written for that case:

$$[\gamma_{aa}] = [\alpha_{aa}] - \frac{\left([\alpha_{aa}]\{u_a\}\right)\left(\{v_a\}^T [\alpha_{aa}]\right)}{1 + \{v_a\}^T [\alpha_{aa}]\{u_a\}} \tag{2.16}$$

As seen in Equation (2.16), it possible to perform the calculations by using the active coordinates alone. This brings great advantage, since the size of the matrix is much smaller than the total degrees of freedom of the structure

When the modification matrix can not be written as $\{u\}\{v\}^T$, then it is possible to decompose the modification matrix into sub modification matrices and the receptance matrix $[\alpha]$ can be calculated in n steps by considering one sub modification matrix at a time. The sub matrices can be expressed as

$$[\Delta K] = [\Delta K_1] + [\Delta K_2] + ... + [\Delta K_n] \tag{2.17}$$

where $[\Delta K_j] = \{u_j\}\{v_j\}^T$ (2.18)

By replacing the terms $[H]$ and $[\Delta H]$ with $\omega[C]$ and $\omega[\Delta C]$, respectively, all the equations given can be used when there is viscous damping instead of structural damping.

2.1.2 Matrix Inversion Method

The formulation is initially proposed by Özgüven [22] for the calculation of receptances of damped structures by using the receptances of undamped structures for a non-proportionally damped structure. In a later work, this method is extended to structural modification problems with and without additional degrees of freedom [24]. The formulation of Matrix Inversion Method is explained below.

For a structure, the equation of motion can be written as:

$$[M]\{\ddot{x}\} + i[H]\{x\} + [K]\{x\} = \{F\} \tag{2.19}$$

Steady response of the structure for a harmonic forcing at frequency ω is expressed as:

$$\{x\} = \left([K] - \omega^2[M] + i[H]\right)^{-1}\{F\} \tag{2.20}$$

By using Equation (2.20), the receptance matrix of the structure is given by

$$[\alpha] = \left([K] - \omega^2[M] + i[H]\right)^{-1} \tag{2.21}$$

If the structure is modified, then the receptance matrix of the modified system can be written as:

$$[\gamma]=\left([[K]+[\Delta K]]-\omega^2[[M]+[\Delta M]]+i[[H]+[\Delta H]]\right)^{-1} \tag{2.22}$$

where $[\Delta K]$, $[\Delta M]$ and $[\Delta H]$ represents the stiffness, mass and structural damping matrices of the modification, respectively.

By using Equation (2.22) the following equation can be written.

$$[\gamma]^{-1}=[\alpha]^{-1}+[D] \tag{2.23}$$

where $[D]$ is the dynamic stiffness matrix of the modification which can be expressed as:

$$[D]=[\Delta K]-\omega^2[\Delta M]+i[\Delta H] \tag{2.24}$$

If Equation (2.23) is pre-multiplied by $[\alpha]$ and post-multiplied by $[\gamma]$, the following equation can be obtained.

$$[\alpha]=[\gamma]+[\alpha][D][\gamma] \tag{2.25}$$

By simple matrix manipulations, receptance matrix of the modified structure can be expressed as:

$$[\gamma]=\left[[I]+[\alpha][D]\right]^{-1}[\alpha] \tag{2.26}$$

If the modifications are local, then the dynamic stiffness matrix can be partitioned as:

$$[D]=\begin{bmatrix}[D_{11}] & [0]\\ [0] & [0]\end{bmatrix} \tag{2.27}$$

By using the partitioned dynamic stiffness matrix, the receptance matrix of the modified structure can be written as [24]:

$$[\gamma_{11}]=\left[[I]+[\alpha_{11}][D_{11}]\right]^{-1}[\alpha_{11}] \tag{2.28}$$

$$[\gamma_{12}]^{T}=[\gamma_{21}]=[\alpha_{21}]\left[[I]-[D_{11}][\gamma_{11}]\right] \tag{2.29}$$

$$[\gamma_{22}]=[\alpha_{22}]-[\alpha_{21}][D_{11}][\gamma_{12}] \tag{2.30}$$

The important thing that should be pointed out is that, only a single matrix of an order equal to the number of coordinates related with structural modification is inverted in order to obtain the receptance matrix of the whole modified structure.

Instead of structural damping, if there is viscous damping $[C]$, $[H]$ and $[\Delta H]$ will be replaced by $\omega[C]$ and $\omega[\Delta C]$, respectively, in all the equations given above.

2.1.3 Extended Successive Matrix Inversion Method

Based on the Classical Successive Matrix Inversion Method which is used to obtain the modified structural responses by considering only modified portion of stiffness matrix [26], Extended Successive Matrix Inversion Method is developed [27]. In this method, the modified structural responses are obtained by considering only the modified portion of the dynamic stiffness matrix.

The equation of motion for an N degrees of freedom system can be written as:

$$[M]\{\ddot{x}\}+i[H]\{x\}+[K]\{x\}=\{F\} \tag{2.31}$$

The response of the system to a harmonic forcing at a frequency ω can be written as:

$$\{x\}=\left([K]-\omega^2[M]+i[H]\right)^{-1}\{F\} \tag{2.32}$$

Then the receptance matrix $[\alpha]$ of the system can be expressed as:

$$[\alpha]=\left[[K]-\omega^2[M]+i[H]\right]^{-1} \tag{2.33}$$

When the modification is introduced to the original structure, then the equation of motion of the modified system can be written as:

$$[M+\Delta M]\{\ddot{x}\}+i[H+\Delta H]\{x\}+[K+\Delta K]\{x\}=\{F\} \tag{2.34}$$

Making some manipulations, the following equation can be obtained.

$$\left[[K+\Delta K]-\omega^2[M+\Delta M]+i[H+\Delta H]\right]\{x\}=\{F\} \tag{2.35}$$

Then, the harmonic response of the modified system can be expressed as:

$$\{x\}=\left[[K+\Delta K]-\omega^2[M+\Delta M]+i[H+\Delta H]\right]^{-1}\{F\} \tag{2.36}$$

Then, the receptance matrix of the modified structure can be expressed as:

$$[\gamma]=\left[[K+\Delta K]-\omega^2[M+\Delta M]+i[H+\Delta H]\right]^{-1} \tag{2.37}$$

Pre-multiplying Equation (2.35) by $[\alpha]$, we obtain:

$$([I]-[P])\{x\}=\{F^{'}\} \tag{2.38}$$

where

$$\{F^{'}\}=[\alpha]\{F\} \tag{2.39}$$

$$[P]=-[\alpha]\left[[\Delta K]-\omega^{2}[\Delta M]+i[\Delta H]\right] \tag{2.40}$$

By using Equation (2.38) and Equation (2.39), the following equation can be obtained.

$$[\gamma]=([I]-[P])^{-1}[\alpha] \tag{2.41}$$

In this method, power series expansion is used for the inversion of the matrix in Equation (2.41) as successfully employed in Successive Matrix Inversion method [26]:

$$([I]-[P])^{-1}=[I]+[P]+[P]^{2}+[P]^{3}+... \tag{2.42}$$

Then $[T]$ matrix can be defined as:

$$[T]=[P]+[P]^{2}+[P]^{3}+... \tag{2.43}$$

For the matrix $[T]$ given above, each of the elements of $[T]$ can be expressed as:

$$T_{ij} = P_{ij}^{(1)} + P_{ij}^{(2)} + \ldots + P_{ij}^{(k)} + \ldots \tag{2.44}$$

where $P_{ij}^{(k)}$ represents the $(i,j)^{\text{th}}$ element of $\left[P\right]^{(k)}$.

By defining the k$^{\text{th}}$ recursion factor as:

$$r_{ij}^{(k)} = P_{ij}^{(k+1)} / P_{ij}^{(k)} \tag{2.45}$$

then, Equation (2.44) can be expressed as follows, if the recursion factor is constant through the expansion:

$$T_{ij} = P_{ij}\left(1 + r_{ij} + r_{ij}^{\ 2} + r_{ij}^{\ 3} + \ldots\right) \tag{2.46}$$

In Equation (2.46), by using the series expansion for the recursive terms, T_{ij} can be written as:

$$T_{ij} = P_{ij} / \left(1 - r_{ij}\right) \tag{2.47}$$

In order to eliminate the variability of the of the recursion factor, the modification matrix is decomposed into separate matrices, since the recursion factor is different through the series expansion. By decomposing, the following equation can be obtained.

$$[\Delta K] - \omega^2[\Delta M] + i[\Delta H] = \sum_{j=1}^{N}\left[[\Delta K^{(j)}] - \omega^2[\Delta M^{(j)}] + i[\Delta H^{(j)}]\right] \tag{2.48}$$

In Equation (2.48), $[\Delta K^{(j)}]$, $[\Delta M^{(j)}]$, $[\Delta H^{(j)}]$ represent the matrices that are composed of the jth columns of stiffness, mass and structural damping matrices, respectively, and zero columns except the jth columns.

The recursion factor is a constant for only one nonzero column of $[T]$, therefore the equation we can write

$$r = P_{jj} \tag{2.49}$$

Then Equation (2.47) can be expressed as:

$$T_{ij} = P_{ij} / (1 - r) \tag{2.50}$$

Defining matrix $[Y]$ and $[Z]$ as the dynamic stiffness matrix of the modification and original structure, respectively, the following equations can be written:

$$[Y] = \left[[\Delta K] - \omega^2 [\Delta M] + i[\Delta H] \right] \tag{2.51}$$

$$[Z] = \left[[K] - \omega^2 [M] + i[H] \right] \tag{2.52}$$

If the jth non-zero column of the structural modification is taken into consideration, then, the dynamic stiffness matrix can be expressed as:

$$\left[Z^{(j)} \right] = \left[Z^{(j-1)} \right] + \left[Y^{(j)} \right] \tag{2.53}$$

In Equation (2.53), $\left[Y^{(j)} \right]$ is a matrix which has the jth non-zero column of $[Y]$ matrix at its corresponding column, and zero columns elsewhere. $\left[Z^{(j-1)} \right]$ and

$\left[Z^{(j)}\right]$ represent the modified dynamic stiffness matrices in (j-1)$^{\text{th}}$ and j^{th} steps, respectively.

$\left[Z^{(0)}\right]$ denotes the initial $[Z]$ matrix. Also $\left[Z^{(j-1)}\right]^{-1}$ refers to $[\alpha]$, and $\left[Z^{(j)}\right]^{-1}$ refers to $[\gamma]$ in Equation (2.41).

From Equation (2.42) and Equation (2.43), the inverse of Equation (2.53) can be written as:

$$\left[Z^{(j)}\right]^{-1} = \left([I]+[T]\right)\left[Z^{(j-1)}\right]^{-1} \tag{2.54}$$

As seen in Equation (2.54), the modified FRF matrix can be calculated, by updating the matrices in Equation (2.54) for each nonzero column of the modification matrix. The sequence of the columns of the modification matrix used in the computation is not important, because each column of the modification matrix contributes to the dynamics of the system independently.

Furthermore, for a local modification, $[Y]$ will be a highly sparse matrix with many zero columns and rows that correspond to the coordinates at which there is no structural modification.

For all equations given above, if the system has viscous damping $[C]$, instead of structural damping, then $[H]$ and $[\Delta H]$ will be replaced by $\omega[C]$ and $\omega[\Delta C]$, respectively.

2.1.4 Özgüven's Recursive Solution Algorithm

In order to calculate the receptances of a modified structure from those of the original system and the modification matrices, Özgüven [23] developed a recursive solution algorithm. This method can be used for structural modifications problems. The derivation of the equations is given below [23] for a damping modification and obviously it can be generalized for any type of modification.

Consider the equations of motion of a structure which has *n* degrees of freedom. The equation of motion can be expressed as:

$$[M]\{\ddot{x}\}+i[H]\{x\}+[K]\{x\}=\{F\} \tag{2.55}$$

By solving the eigenvalue problem given below, the undamped modal data can be obtained.

$$[K]\{\phi\}=\omega^2[M]\{\phi\} \tag{2.56}$$

Considering Equation (2.55) , the internal damping of the structure can be replaced by a vector which is the set of equivalent forces that can be written in terms of the damping values and the displacement of the structure. Then the equation of motion takes the form:

$$[M]\{\ddot{x}\}+[K]\{x\}=\{F\}+\{R\} \tag{2.57}$$

where $\{R\}$ is a vector which represents damping forces given as:

$$\{R\}=-i[H]\{x\} \tag{2.58}$$

In Equation (2.57), there are two sets of external forces, one of which is expressed in terms of the unknown dynamic displacement $\{x\}$ of the structure, therefore it is possible to consider the damped structure as an undamped structure which has two sets of external forces one of which is given by Equation (2.58). By modal summation the undamped receptances of the structure defined by Equation (2.57), can be obtained.

For a typical coordinate *s,* the damping force on this coordinate can be written as:

$$R_s = -i\sum_{k=1}^{n} h_{sk} x_k \tag{2.59}$$

By using the definition of receptance, the dynamic displacement of the p^{th} coordinate can be expressed as:

$$x_p = \sum_{s=1}^{n} \alpha_{ps} (F_s + R_s) \tag{2.60}$$

$$x_p = \sum_{s=1}^{n} \alpha_{ps} F_s - i\sum_{s=1}^{n} \alpha_{ps} \sum_{k=1}^{n} h_{sk} x_k \tag{2.61}$$

In Equation (2.60) and Equation (2.61) both x_p and x_k represent the displacements in the damped system. Setting all external forces, except F_j to zero and dividing all term by F_j, the receptance γ_{pj} of the damped system can be calculated from Equation (2.61) as:

$$\gamma_{pj} = \alpha_{pj} - i\sum_{s=1}^{n} \alpha_{ps} \sum_{k=1}^{n} h_{sk} (x_k / F_j) \tag{2.62}$$

The term x_k / F_j in Equation (2.62) can be replaced by γ_{kj} by using the definition of the receptance. Then Equation (2.62) becomes:

$$\gamma_{pj} = \alpha_{pj} - i\sum_{s=1}^{n} \alpha_{ps} \sum_{k=1}^{n} h_{sk} \gamma_{kj} \tag{2.63}$$

Equation (2.63) is valid only for any p and j (p=1,2,…,n; j=1,2,…,n). Concentrating on only a single element h_{sk} of the damping matrix, while the rest of the damping elements are taken to be zero, Equation (2.63) can be written as:

$$\gamma_{pj} = \alpha_{pj} - i\alpha_{ps} h_{sk} \gamma_{kj} \qquad (p=1,2,\ldots,n;\, j=1,2,\ldots,n) \tag{2.64}$$

By taking $p=k$, γ_{kj} can be expressed as:

$$\gamma_{pj} = \alpha_{kj} / (1 + i\alpha_{ks} h_{sk}) \qquad (p=1,2,\ldots,n) \tag{2.65}$$

After the calculation of γ_{kj} (j=1,2,…,n) from Equation (2.65), the remaining receptance values γ_{pj} (p=1,2,…,k-1,k+1,…,n; j=1,2,…,n) can be obtained from the calculated values of γ_{kj} (j=1,2,…,n) by using Equation (2.64).

In the formulations given above, only one element of the damping matrix is considered therefore these formulations give only the receptances of the system that are composed of the undamped system and a single damping element h_{sk}. In order to obtain the receptances of the damped structure, the calculated receptances should be treated as new α values in Equation (2.64) and Equation (2.65). A new set of receptances can be obtained by considering another damping element of the original damping matrix $[H]$.

If this algorithm is repeated for all nonzero elements of $[H]$, the final receptance matrix $[\gamma]$ gives the receptances of the damped system.

For locally damped structures, the damping matrix can be written as the addition of proportional and non-proportional damping matrix and this brings great reduction in the computational effort. The damping matrix can be written as:

$$[H]=[H]_P+[H]_N \tag{2.66}$$

In Equation (2.66), $[H]_P$ represents the proportional part of the damping matrix and $[H]_N$ represents the non-proportional part of the damping matrix. Non-proportional damping matrix can be expressed as:

$$[H]_N=\begin{bmatrix}[H_{11}] & [0]\\ [0] & [0]\end{bmatrix} \tag{2.67}$$

By using Equation (2.65), γ_{kj} (j=1, 2,…, n) can be calculated and in order to find γ_{pj} for only m values of p (m is the order of the sub matrix $[H_{11}]$), Equation (2.64) can be used. Therefore, the final values of γ_{pj} (p=1,2,…,m; j=1,2,…,n) which include the effect of all the m^2 damping values can be obtained without computing the receptances corresponding to undamped coordinates. Since the number of damping elements is just m^2, then the number of recomputations of each receptance will be reduced from n^2 to m^2.

Receptances of undamped nodes, can be calculated as follows [23]:

$$\gamma_{pj}=\alpha_{pj}-i\sum_{s=1}^{n}\alpha_{ps}\sum_{k=1}^{n}h_{sk}\gamma_{kj} \tag{2.68}$$

for $p=m+1,\ldots,n$ and $j=p,\ldots,n$.

By considering one column of the damping matrix at a time, formulation can be further improved. If the k-th column of the damping matrix $[H_{11}]$ is considered, by using Equation (2.63) the following equation can be written.

$$\gamma_{pj} = \alpha_{pj} - \left(i\sum_{s=1}^{n} \alpha_{ps} \sum_{k=1}^{n} h_{sk} \right) \gamma_{kj} \tag{2.69}$$

For $p=k$, we can write

$$\gamma_{kj} = \alpha_{kj} \Big/ \left(1 + i\sum_{s=1}^{m} \alpha_{ks} h_{sk} \right) \qquad (j=1,2,\ldots,n) \tag{2.70}$$

Once α_{kj} ($j=1,2,\ldots,n$) are calculated which include the effect of the k-th column of the damping matrix, the remaining elements of $[\gamma_{11}]$ and $[\gamma_{12}]$ can be obtained from Equation (2.69) for $j=1,2,\ldots,n$ and $p=1,2,\ldots,k-1,k+1,\ldots,m$.

The final values of the upper mxn portion of $[\gamma]$ can be calculated by repeating this procedure m times (k=1, 2,…, m). Then the remaining elements of the receptance matrix can be obtained from Equation (2.68).

If damping matrix is replaced by any general dynamic stiffness matrix, all the formulations given above can be used for any structural modification problems:

Replacing $[H_{11}]$ matrix by

$$[\delta_{11}] = \left([K_{11}] - \omega^2 [M_{11}] + i[H_{11}] \right)^{-1} \tag{2.71}$$

where $[K_{11}]$, $[M_{11}]$ and $[H_{11}]$ are the stiffness, mass and hysteretic damping matrices of the modifying structure, respectively.

The same equations can be used for structural modification problems.

By using Equation (2.71), the elements of $[\delta_{11}]$ can be written as:

$$\delta_{sk} = k_{sk} - \omega^2 m_{sk} + ih_{sk} \tag{2.72}$$

In Equations (2.63) through (2.70), the damping terms ih_{sk} can be replaced by the corresponding elements δ_{sk}. Then the equations will be valid for structural modification problems. However, these equations can be used in the given forms only if there is no additional DOF due to the modifying structure.

2.1.5 Modeling of Distributed Modifications without Additional Degrees of Freedom

In structural modification problems, modeling distributed modifications is more difficult compared with lumped modifications. There are different approaches in the literature. Especially, W. D'Ambrogio, A. Sestieri extensively studied the distributed modifications and developed a modeling approach for distributed modifications. The method proposed by W. D'Ambrogio, A. Sestieri [34] will be given below.

Assuming a FE model is not available for the original structure, and original structure is only known experimentally, the FRF of a modified structure can be obtained by the relationship given below.

$$[H] = ([I] + [H_0][\Delta B])^{-1}[H_0] \tag{2.73}$$

In Equation (2.73) $[H]$ and $[H_0]$ represents the FRFs of the original and modified structures, respectively, $[I]$ is the identity matrix and $[\Delta B]$ is the dynamic stiffness matrix of modifications introduced into the original structure.

Defining the subscripts "b" and "a" as interface DOFS between original and modifying structure and original structure DOFs not on the interface "b", respectively, the following equation can be written:

$$[\Delta B]=\begin{bmatrix} [0]_{aa} & [0]_{ab} \\ [0]_{ba} & [\Delta B]_{bb} \end{bmatrix} \tag{2.74}$$

it can be shown that [H] is given by [34] :

$$\begin{bmatrix} [H]_{aa} & [H]_{ab} \\ [H]_{ba} & [H]_{bb} \end{bmatrix}=\begin{bmatrix} [H_0]_{aa} & [H_0]_{ab} \\ [H_0]_{ba} & [H_0]_{bb} \end{bmatrix}-\begin{bmatrix} [H_0]_{ab} \\ [H_0]_{bb} \end{bmatrix}\Big[[\Delta B]_{bb}[I]_{bb} \\ +[H_0]_{bb}[\Delta B]_{bb}\Big]^{-1}\Big[[H_0]_{ba}[H_0]_{bb}\Big] \tag{2.75}$$

In the above equation the order of the inverted matrix is b which is smaller than the total DOF of the structure (a+b).

The dynamic stiffness matrix of modifications $[\Delta B]$ appearing in Equation (2.74) can be written as:

$$[\Delta B]=[B]-[B_0] \tag{2.76}$$

As seen in Equation (2.76), $[\Delta B]$ is the difference between the dynamic stiffness matrices of the modified and original structures. This corresponds to the dynamic stiffness matrix of the modification only for lumped modifications; however for distributed modifications it must be computed by taking the difference of two matrices. As an example, let us consider the effect of a rib stiffener with given characteristics [34]. The bending stiffness of the whole system is certainly a function of the rib's elasticity, but it is also very much dependent on the characteristics of the original structure. Since doubling the thickness of a flexural beam does not double the whole bending stiffness, the stiffness of the modified structure is not just the sum of the stiffness of the two components. The stiffness increases eight times due to the effect of the cross section area moment.

If both of the FE models are available for original structure and the modified one, the difference between the two models, performed on the whole set of DOFs will generate a $[\Delta B]$ matrix which is given by

$$[\Delta B] = -\omega^2[\Delta M] + j\omega[\Delta C] + [\Delta K] \tag{2.77}$$

In Equation (2.77), non-zero elements do not extend beyond the interface DOFs between the original and modifying structures. This implies a local character of $[\Delta B]$, when the entire finite elements DOFs are considered [34].

For complex structures, a good FE model may not be available in several practical applications, then the original structure can be known only experimentally by measuring the FRFs. For the modifying structure, usually the FE model of the structure can be easily constructed. In order to obtain the dynamic characteristics of the modified structure, FRF of the original structure and finite element of the modifying structure should be coupled. In the FE model both translational and rotational DOFs may exist however in experiments, only translational DOFs are usually measured due to the difficulties encountered in measuring the rotational DOFs. Therefore in order to have consistent DOFs for the original and modifying structure, condensation procedures should be used. After elimination of rotational

DOFs, the matrix $\left[\Delta B\right]$, reduced to the translational DOFs, can be expressed by the equation given below.

$$\left[\Delta B\right]=\left[B_c\right]-\left[B_{0c}\right] \tag{2.78}$$

In Equation (2.78), $\left[B_c\right]$ and $\left[B_{0c}\right]$ are the condensed dynamic stiffness matrices of the modified and the original structures, respectively. In this equation $\left[B_c\right]$ and $\left[B_{0c}\right]$ become full matrices due to the condensation process, and the same holds for $\left[\Delta B\right]$ when reduced to the translational DOFs.

In order to compute $\left[\Delta B\right]$ approximately, it is useful to consider non-local character of the modification matrix $\left[\Delta B\right]$, when reduced to the translational DOFs. For simplicity, when ω=0 $\left[\Delta B\right]$ is given by:

$$\left[\Delta K\right]=\left[\Delta B(\omega=0)\right] \tag{2.79}$$

It is possible to write [34]:

$$\Delta K_{ij}=\left.\frac{\Delta F_i}{x_j}\right|_{x_i=0,i\neq j}$$

where ΔK_{ij} represents the additional force arising, as a result of the modification, on the i^{th} DOF for a displacement in *j*, when all the DOFs in $\left[\Delta K\right]$ are set to zero, excluding x_j. Since the DOFs involved in $\left[\Delta K\right]$ are only translations, there is no constraints in the rotational DOFs. Therefore, although all the displacements, except x_j are blocked, there is an additional force on every DOF due to the imposed displacement which propagates to the interface area through the unconstrained rotations.

However, the elements of $[\Delta K]$ tend to vanish when either *i* or *j* get far from the interface because additional forces are rapidly decreasing as either *i* or *j* gets farther from the connection area. The proof and further details are given in [34].

From Equation (2.78) for $\omega=0$, the following equation can be written.

$$[\Delta K]=[K_c]-[K_{0c}] \tag{2.80}$$

Due to the quasi-local character of $[\Delta K]$, the relation given below can be expressed when either *i* or *j* is far from the interface.

$$[K_c]\approx[K_{0c}] \tag{2.81}$$

This difference can be approximately estimated by modeling only a portion of the two considered structures which includes the interface DOFs. Similar situation holds for $[\Delta B]$.

2.2 Structural Modification with Additional Degrees of Freedom

2.2.1 Özgüven's Formulation

Özgüven [24] proposed a formulation for the structural modifications which introduce additional DOFs to the structure. In the formulation the receptance matrix of the modified structure $[\alpha]$ can be partitioned as follows (Figure 2.1):

• The coordinates which correspond to the original structure only (a)

• The coordinates which are connection coordinates between the original and the modifying structure (b)

• The coordinates which correspond to the modifying structure only (c)

Then the following equations can be written for the original and modifying structures:

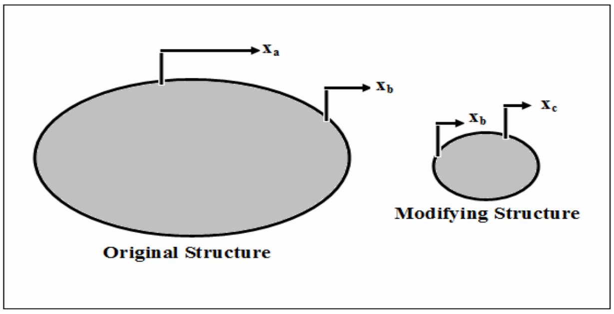


Figure 2.1 Original and Modifying Structure

$$[\alpha_0]^{-1} = \begin{bmatrix} \alpha_0^{aa} & \alpha_0^{ab} \\ \alpha_0^{ba} & \alpha_0^{bb} \end{bmatrix}^{-1} = [K_0] - \omega^2 [M_0] + i[H_0] \tag{2.82}$$

$$\begin{bmatrix} \alpha^{aa} & \alpha^{ab} & \alpha^{ac} \\ \alpha^{ba} & \alpha^{bb} & \alpha^{bc} \\ \alpha^{ca} & \alpha^{cb} & \alpha^{cc} \end{bmatrix}^{-1} = \begin{bmatrix} [\alpha_0]^{-1} & & 0 \\ & & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & & \\ 0 & & [D_{\text{mod}}] \end{bmatrix} \tag{2.83}$$

where $[\alpha_0]$ and $[\alpha]$ represent the receptance matrices of the original and modified structures, respectively. Pre-multiplying Equation (2.83) by

$$\begin{bmatrix} [\alpha_0] & & 0 \\ & & 0 \\ 0 & 0 & I \end{bmatrix} \tag{2.84}$$

and post-multiplying by $[\alpha]$ gives

$$\begin{bmatrix} \begin{bmatrix} \alpha_0^{aa} & \alpha_0^{ab} \\ \alpha_0^{ba} & \alpha_0^{bb} \end{bmatrix} & \begin{matrix} 0 \\ 0 \end{matrix} \\ \begin{matrix} 0 & \quad 0 \end{matrix} & I \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix} [\alpha] + \left[\begin{array}{c|c} 0 & \left[\alpha_0^{ab} \mid 0 \right].[D_{\text{mod}}] \\ \hline 0 & \multirow{2}{*}{$\begin{bmatrix} \alpha_0^{bb} & 0 \\ 0 & I \end{bmatrix}.[D_{\text{mod}}]$} \\ 0 & \end{array} \right] [\alpha] \tag{2.85}$$

After some matrix manipulations it is possible to write

$$\left[\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \alpha_0^{bb} & 0 \\ 0 & I \end{bmatrix}.[D_{\text{mod}}] \right] \begin{bmatrix} \alpha^{ba} \\ \alpha^{ca} \end{bmatrix} = \begin{bmatrix} \alpha_0^{ba} \\ 0 \end{bmatrix} \tag{2.86}$$

$$\left[\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \alpha_0^{bb} & 0 \\ 0 & I \end{bmatrix}.[D_{\text{mod}}] \right] \begin{bmatrix} \alpha^{bb} & \alpha^{bc} \\ \alpha^{cb} & \alpha^{cc} \end{bmatrix} = \begin{bmatrix} \alpha_0^{bb} & 0 \\ 0 & I \end{bmatrix} \tag{2.87}$$

$$\left[\alpha^{aa} \right] + \left[\alpha_0^{ab} \mid 0 \right] [D_{\text{mod}}] \begin{bmatrix} \alpha^{ba} \\ \alpha^{ca} \end{bmatrix} = \left[\alpha_0^{aa} \right] \tag{2.88}$$

$$\left[\alpha^{ab} \mid \alpha^{ac} \right] + \left[\alpha_0^{ab} \mid 0 \right] [D] \begin{bmatrix} \alpha^{bb} & \alpha^{bc} \\ \alpha^{cb} & \alpha^{cc} \end{bmatrix} = \left[\alpha_0^{ab} \mid 0 \right] \tag{2.89}$$

Then receptance submatrices of the modified system can be obtained as:

$$\begin{bmatrix} \alpha^{ba} \\ \alpha^{ca} \end{bmatrix} = \left[\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \alpha_0^{bb} & 0 \\ 0 & I \end{bmatrix} . [D_{\text{mod}}] \right]^{-1} \begin{bmatrix} \alpha_0^{ba} \\ 0 \end{bmatrix} \tag{2.90}$$

$$\begin{bmatrix} \alpha^{bb} & \alpha^{bc} \\ \alpha^{cb} & \alpha^{cc} \end{bmatrix} = \left[\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \alpha_0^{bb} & 0 \\ 0 & I \end{bmatrix} . [D_{\text{mod}}] \right]^{-1} \begin{bmatrix} \alpha_0^{bb} & 0 \\ 0 & I \end{bmatrix} \tag{2.91}$$

$$\left[\alpha^{aa} \right] = \left[\alpha_0^{aa} \right] - \left[\alpha_0^{ab} \;\vdots\; 0 \right] [D_{\text{mod}}] \begin{bmatrix} \alpha^{ba} \\ \alpha^{ca} \end{bmatrix} \tag{2.92}$$

$$\left[\alpha^{ab} \;\vdots\; \alpha^{ac} \right] = \left[\alpha_0^{ab} \;\vdots\; 0 \right] \left[[I] - [D] \begin{bmatrix} \alpha^{bb} & \alpha^{bc} \\ \alpha^{cb} & \alpha^{cc} \end{bmatrix} \right] \tag{2.93}$$

In the equations above, in order to calculate complete receptance matrix of the modified structure it is necessary to invert only a single matrix. The order of the matrix to be inverted is equal to the DOF of the modifying structure, which is usually much less than the total DOF of the system.

2.2.2 Extension of Özgüven's Formulation

Starting from the original relationship developed by Özgüven [24] for structural modifications and the description method for distributed modifications developed by D'Ambrogio and Sestieri [34], Hang [35] proposed a different method for the prediction of the FRFs for distributed structural modification with change in the DOFs. The theory of the proposed method is given below.

When structural modifications introduce additional DOFs into the system, the receptance FRF matrix of the modified structure $\left[H_1\right]$ can be partitioned into three parts as [35]:

(a) DOFs belonging to the original structure only (indicated by subscript “a”)

(b) Interface DOFs, through which the modifying part is connected to the original structure (indicated by subscript “b”)

(c) Passenger DOFs, belonging to the modifying structure only (indicated by subscript “c”)

Then the following equations can be written for the original and the modified structures:

$$[B_0]=[H_0]^{-1}=\begin{bmatrix} H_{0aa} & H_{0ab} \\ H_{0ba} & H_{0bb} \end{bmatrix}^{-1} \tag{2.94}$$

$$[B_1]=[H_1]^{-1}=\begin{bmatrix} H_{1aa} & H_{1ab} & H_{1ac} \\ H_{1ba} & H_{1bb} & H_{1bc} \\ H_{1ca} & H_{1cb} & H_{1cc} \end{bmatrix}^{-1}=\begin{bmatrix} [H_0]^{-1} & & 0 \\ & & 0 \\ 0 & 0 & 0 \end{bmatrix}+[\Delta B] \tag{2.95}$$

where $$[\Delta B]=[B_1]-[B_0]=\begin{bmatrix} 0 & 0 & 0 \\ 0 & \begin{bmatrix} \Delta B_{bb} & \Delta B_{bc} \\ \Delta B_{cb} & \Delta B_{cc} \end{bmatrix} & \\ 0 & & \end{bmatrix} \tag{2.96}$$

By matrix manipulation, the receptance matrix of the modified structure can be obtained as a function of the original receptance matrix and the delta dynamic stiffness matrix introduced by the modifying part. Then the receptance matrices of the modified structure can be given as:

$$\begin{aligned}[H_{1bb}] &= \left[[I]+[H_{0bb}][\Delta B_{bb}]-[H_{0bb}][\Delta B_{bc}][\Delta B_{cc}]^{-1}[\Delta B_{cb}]\right]^{-1}[H_{0bb}] \\ &= [\beta]^{-1}[H_{0bb}]\end{aligned} \tag{2.97}$$

$$[H_{1cb}] = -[\Delta B_{cc}]^{-1}[\Delta B_{cb}][H_{1bb}] = -[\alpha][H_{1bb}] \quad (2.98)$$

$$[H_{1ba}] = \Big[[I] + [H_{0bb}][\Delta B_{bb}] - [H_{0bb}][\Delta B_{bc}][\Delta B_{cc}]^{-1}[\Delta B_{cb}]\Big]^{-1}[H_{0ba}] \\ = [\beta]^{-1}[H_{0ba}] \quad (2.99)$$

$$[H_{1ca}] = -[\Delta B_{cc}]^{-1}[\Delta B_{cb}][H_{1ba}] = -[\alpha][H_{1ba}] \quad (2.100)$$

$$[H_{1bc}] = [H_{1cb}]^T \quad (2.101)$$

$$[H_{1ab}] = [H_{1ba}]^T \quad (2.102)$$

$$[H_{1ac}] = [H_{1ca}]^T \quad (2.103)$$

$$[H_{1cc}] = [\Delta B_{cc}]^{-1} - [\Delta B_{cc}]^{-1}[\Delta B_{cb}][H_{1bc}] = [\gamma] - [\alpha][H_{1bc}] \quad (2.104)$$

$$[H_{1aa}] = [H_{0aa}] - [H_{0ab}][\Delta B_{bb}][H_{1ba}] - [H_{0ab}][\Delta B_{bc}][H_{1ca}] \quad (2.105)$$

where $[\gamma] = [\Delta B_{cc}]^{-1}$ (2.106)

$$[\alpha] = [\Delta B_{cc}]^{-1}[\Delta B_{cb}] = [\gamma][\Delta B_{cb}] \quad (2.107)$$

$$[\beta] = [I] + [H_{0bb}][\Delta B_{bb}] - [H_{0bb}][\Delta B_{bc}][\Delta B_{cc}]^{-1}[\Delta B_{cb}] = [I] + [H_{0bb}][\Delta B_{bb}] - \\ [H_{0bb}][\Delta B_{bc}][\alpha] \quad (2.108)$$

In above equations two matrices should be inverted for the computation of the complete receptance matrix of the modified structure. The orders of matrix to be inverted are equal to the interface DOFs and the passenger DOFs, and these are much less than the total DOFs of the modified system.

2.2.3 Modeling Approach for Distributed Modifications with Additional Degrees of Freedom

In a structural modification problem the additional dynamic stiffness matrix due to structural modification is given by [32]:

$$[\Delta D] = [D] - [D_0] \tag{2.109}$$

In Equation (2.109) , $[D]$ and $[D_0]$ are the dynamic stiffness matrices of the modified and original structures, respectively.

For lumped modifications, $[\Delta D]$ corresponds directly to dynamic stiffness matrix of the modifying structure. However for distributed modifications, it has to be calculated by using Equation (2.109) which may not correspond to the dynamic stiffness matrix of the modifying structure. Dynamic stiffness matrices of the original and modified structures should be available in order to apply Equation (2.109). This requires availability of the FE models for original and modified structures. However, if these FE models were available, then there would be limited advantage of using structural modification method.

D'Ambrogio and Sestieri [34] overcame this drawback by using quasi-local characteristics of additional dynamic stiffness matrix due to structural modification $[\Delta D]$. Bounded region which covers the modifying area is modeled for both original and modified structures in order to obtain the additional dynamic stiffness matrix due to structural modification. However there are two drawbacks in the approach proposed by D'Ambrogio and Sestieri. These drawbacks are mainly due to the inaccurate modeling of the structure and quasi-local characteristics of the additional dynamic stiffness matrix due to structural modification.

In order to illustrate the first drawback, the beams in Figure 2.2 and Figure 2.3 can be considered. Beam model in Figure 2.2 is the original structure and it is modeled using beam elements. When it is modified by adding a shorter beam on it as shown in Figure 2.3, as the neutral planes of the thinner and thicker parts of the modified beam will not coincide, the FE model will not represent the real structure accurately. Modeling by using beam elements introduces certain errors independent from the quality of the mesh. D'Ambrogio and Sestieri discussed this error by modeling both original and modifying structures using beam and brick elements in the FE model [32].

The second error directly depends on the size of the bounded region which covers the modifying area. The error introduced to $\left[\Delta D\right]$ will be smaller, when the bounded region which covers the modifying area is larger. In this thesis, a different approach is used in application of structural modification technique, in order to eliminate these types of errors. When distributed modification is applied to an original structure in such a way that additional DOF is introduced, then it is not necessary to use Equation (2.109) in order to calculate the additional dynamic stiffness matrix due to structural modification, as the problem will be a structural coupling problem. In that case, the additional dynamic stiffness matrix due to structural modification will be the same as the dynamic stiffness matrix of the modifying structure which can directly be used in the structural modification method.

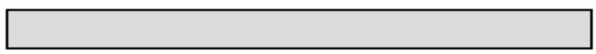


Figure 2.2 Original Beam

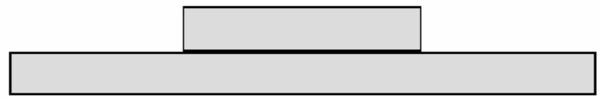


Figure 2.3 Modified Beam

Assuming the FE models of the original and modifying beams are available and they are modeled by using solid brick elements, distributed modifying beam structure which is modeled by FEM introduces additional DOF to the original structure. The finite element method provides $[K_0]$, $[M_0]$ and $[H_0]$ for the original structure, $[K_{\text{mod}}]$, $[M_{\text{mod}}]$ and $[H_{\text{mod}}]$ for the modifying structure. Then the dynamic stiffness matrices of the original and modifying structure are given by:

$$[D_0]=[K_0]-\omega^2[M_0]+i[H_0] \tag{2.110}$$

$$[D_{\text{mod}}]=[K_{\text{mod}}]-\omega^2[M_{\text{mod}}]+i[H_{\text{mod}}] \tag{2.111}$$

DOFs of the original and modifying structure can be divided as:

- DOFs which belong to original structure only (indicated by superscript a)
- DOFs at the connection points (indicated by superscript c)
- DOFs which belong to modifying structure only (indicated by superscript b)

Force displacement relations and displacement compatibility equations can be written as:

$$\{f_o\} = \begin{bmatrix} D_0^{aa} & D_0^{ac} \\ D_0^{ca} & D_0^{cc} \end{bmatrix} \begin{Bmatrix} x_0^a \\ x_0^c \end{Bmatrix} \tag{2.112}$$

$$\{f_{\text{mod}}\} = \begin{bmatrix} D_{\text{mod}}^{bb} & D_{\text{mod}}^{bc} \\ D_{\text{mod}}^{cb} & D_{\text{mod}}^{cc} \end{bmatrix} \begin{Bmatrix} x_{\text{mod}}^b \\ x_{\text{mod}}^c \end{Bmatrix} \tag{2.113}$$

$$\{x_0^c\} = \{x_{\text{mod}}^c\} = \{x^c\} \tag{2.114}$$

$$\{x_0^a\} = \{x^a\} \tag{2.115}$$

$$\{x_{\text{mod}}^b\} = \{x^b\} \tag{2.116}$$

Since the forces at the connection nodes are the sum of forces on the original and modifying parts, the force equation can be written as:

$$\{f_0^c\} + \{f_{\text{mod}}^c\} = \{f^c\} \tag{2.117}$$

Using Equation (2.112) and Equation (2.113), forces on the original and modifying structure can be written as:

$$\{f_0^c\} = \left[D_0^{ca}\right]\{x_0^a\} + \left[D_0^{cc}\right]\{x_0^c\} \tag{2.118}$$

$$\{f_{\text{mod}}^c\} = \left[D_{\text{mod}}^{cb}\right]\{x_{\text{mod}}^b\} + \left[D_{\text{mod}}^{cc}\right]\{x_{\text{mod}}^c\} \tag{2.119}$$

By first inserting Equation (2.115) and Equation (2.116) into Equation (2.118) and Equation (2.119), then combining with Equation (2.117) one obtains

$$\left\{ f^c \right\} = \left[D_0^{ca} \right]\left\{ x^a \right\} + \left[D_{\text{mod}}^{cb} \right]\left\{ x^b \right\} + \left[D_0^{cc} + D_{\text{mod}}^{cc} \right]\left\{ x^c \right\} \tag{2.120}$$

Note that;

$$\left\{ f^c \right\} = \left\{ f_0^c \right\} \tag{2.121}$$

$$\left\{ f^b \right\} = \left\{ f_c^b \right\} \tag{2.122}$$

Then one obtains

$$\left\{ f \right\} = \begin{Bmatrix} f^a \\ f^c \\ f^b \end{Bmatrix} = \begin{bmatrix} D_0^{aa} & D_0^{ac} & 0 \\ D_0^{ca} & \left[D_0^{cc} + D_{\text{mod}}^{cc} \right] & D_{\text{mod}}^{cb} \\ 0 & D_{\text{mod}}^{bc} & D_{\text{mod}}^{bb} \end{bmatrix} \begin{Bmatrix} x^a \\ x^c \\ x^b \end{Bmatrix} \tag{2.123}$$

Equation (2.123) represents the assembly of two matrices (the dynamic stiffness matrices of the original and modifying structures) and gives the dynamic stiffness matrix of the modified structure. Therefore for distributed modifications, if additional DOF is introduced to the structure, there is no need to use Equation (2.109); instead, dynamic stiffness matrix of the modifying structure can directly be used.

In this thesis, a new approach is proposed based on the modeling approach given above and the formulation given by Özgüven [24]. As given in [24], FRF matrix of a modified system can be partitioned as; DOFs which correspond to original structure only (superscript a), DOFs at connection points (superscript b), and DOFs that belong to modifying structure only (superscript c). Then Equation (2.90), (2.91), (2.92) and (2.93) are used to obtain the receptance matrix of the modified structure.

If additional DOF is introduced to the structure, without using Equation (2.109), the dynamic stiffness matrix of the modification can be directly taken as the dynamic stiffness matrix of the modifying structure for distributed modifications. Then using Equation (2.90), (2.91), (2.92) and (2.93), receptance matrix of the modified system can be calculated.

CHAPTER 3

VERIFICATION OF STRUCTURAL MODIFICATION PROGRAM

3.1 Structural Modification Program

The computer program named "Structural Modification with Additional Degrees of Freedom Program" is developed in MATLAB [49]. For the original structure, the program uses two different text files which include mode shape matrix information and natural frequency information, respectively. These two text files are extracted from ANSYS by using a macro file written in this study. The FRF matrix is calculated for the original structure by using Equation (3.1).

$$\alpha_{ij}(\omega) = \sum_{r=1}^{N} \frac{\Phi_{ir}\Phi_{jr}}{\omega_r^{\,2} - \omega^2 + i\eta\omega_r^{\,2}} \tag{3.1}$$

For the modifying structure, the program uses two files which have the stiffness and mass matrices of the modifying structure, respectively. These files are extracted from the result file of the ANSYS modal analysis which has an extension of " *.full ". However in order to have these stiffness and mass matrices, the file named " rdfull.f " which is in the ANSYS installation directory, should be compiled with Intel Fortran compiler to create the " rdfull.exe" file. Then by running this " rdfull.exe" file with the result file of the ANSYS modal analysis which has an extension of "*.full" in a separate folder, these stiffness and mass matrices are extracted. In the program, initial inputs which are the starting frequency, ending frequency, number of frequency points and structural damping coefficient should be defined by the user.

The number of nodes, the number of mode shapes to calculate FRF, connection nodes of the original structure and the number of DOF per node for the original structure should be defined by the user. For the modifying structure, number of nodes and connection nodes on modifying structure should be specified by the user. As an important point to be mentioned, the number of nodes, DOF per node and the number of mode shapes to calculate FRF, has to match with the information in the files created from the ANSYS. The FRF of the nodes specified by the user is calculated for the original and modified structure. In order to compare the results of the modified structure obtained from Özgüven's formulation [24], FRF can be calculated in the program which uses the direct ANSYS modal analysis result for the modified structure. While computing the FRFs in the program, firstly the text files that have the mode shape matrix and natural frequency information for the original structure are read, then the stiffness and mass matrix information is read from the file for the modifying structure. Original FRF matrix is obtained for the original structure, and dynamic stiffness matrix is obtained for the modifying structure. As a next step these matrices for the original and modifying structure are renumbered and the FRFs of modified structure are calculated. The FRF curves for the selected nodes are drawn on the graph for both original and modified structure. Furthermore, if it is required, the FRFs for the modified structure can also be calculated by using directly ANSYS modal analysis and the results can be displayed by the user.

3.2 Verification of the Program

In the program developed in this thesis, Özgüven's formulation [24] for the structural modification with additional degrees of freedom is used in order to calculate FRF of the modified structure. In order to compare validity of the results obtained from the program, the FRF is also calculated for the specified nodes by using ANSYS solution for the modified structure. In the following section, 5 different case studies are presented to illustrate the validity of the program developed. In the first and second case studies free-free plate models with 6 DOF per node will be used. In the third case study, a cantilevered plate will be used as a model. As a fourth case study, free-free beam structure having 3 DOF per node will be modified by attaching smaller beam under it. The final case study is the modification of an L-beam by attaching a rib on it. In these different case studies, program will be validated with models that have different boundary conditions and element types in the FE model.

3.2.1 Free-Free Plate

In this case study, a plate with dimensions 300mm x 300mm x 5mm is used as an original structure. Both the original plate and modifying plate which are shown in Figure 3.1 and Figure 3.2, respectively are modeled in ANSYS 11.0 by using SHELL 63 elements which has 3 translational and 3 rotational DOFs, yielding a total 6 DOF per node. The material used is aluminum which has the density of 2770 kg/m^3, Young's Modulus of 71 GPa and Poisson's ratio is taken as 0.33. The structural damping factor is taken as 0.01 for both of the structures.

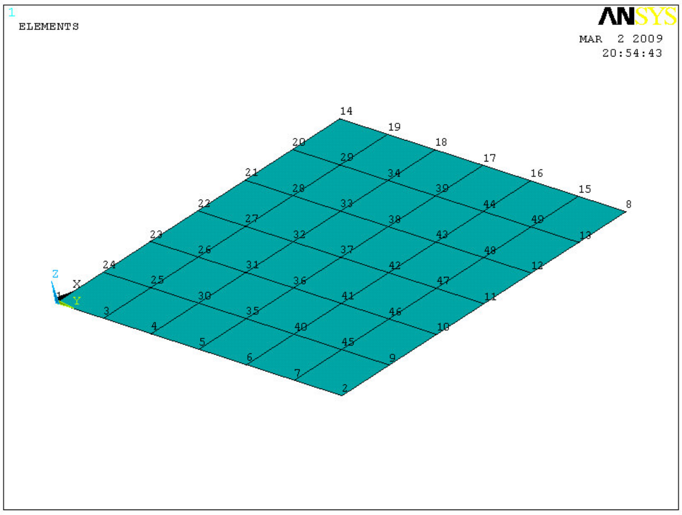


Figure 3.1 FE Model of the Original Plate

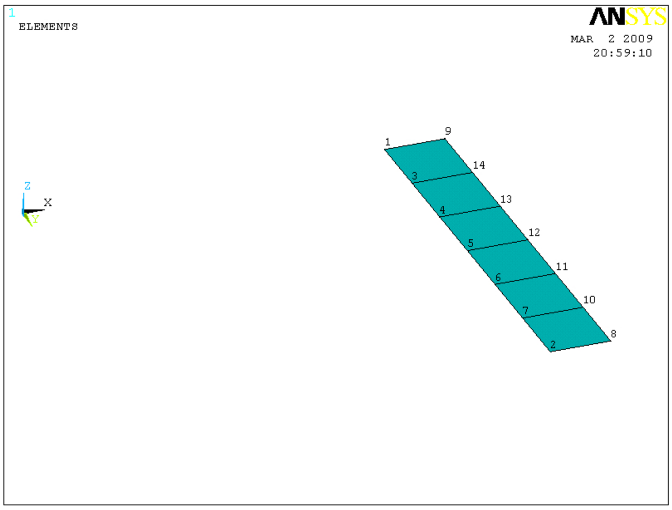


Figure 3.2 FE Model of the Modifying Plate

The geometrical and material properties of the original and modifying plates are given in Table 3.1. As shown in Table 3.2, original plate was divided into 36 elements and it has 49 nodes with 6 DOF per node yielding 394 total DOFs. The modifying plate was divided into 6 elements and it has 14 nodes with 6 DOF per node, giving 84 total DOFs.

Table 3.1 Geometrical and Material Properties of the Plates

	Original Plate	**Modifying Plate**
Young's Modulus (E)	71 GPa	71 GPa
Poisson's Ratio (ν)	0.33	0.33
Density (ρ)	2770 kg/m^3	2770 kg/m^3
Length	300mm	300mm
Width	300mm	50mm
Thickness	5mm	5mm

Table 3.2 FE Information of the Plate Models

	Original Plate	Modifying Plate
Number of Elements	36	6
Number of Nodes	49	14
DOF	294	84

There are 7 connection nodes between original and modifying structure. Connection nodes for the original structure are nodes 14, 19, 18, 17, 16, 15, 8 and corresponding connection nodes for the modifying structure are nodes 1, 3, 4, 5, 6, 7, 2. After the modification, the modified structure which is shown in Figure 3.3 was obtained.

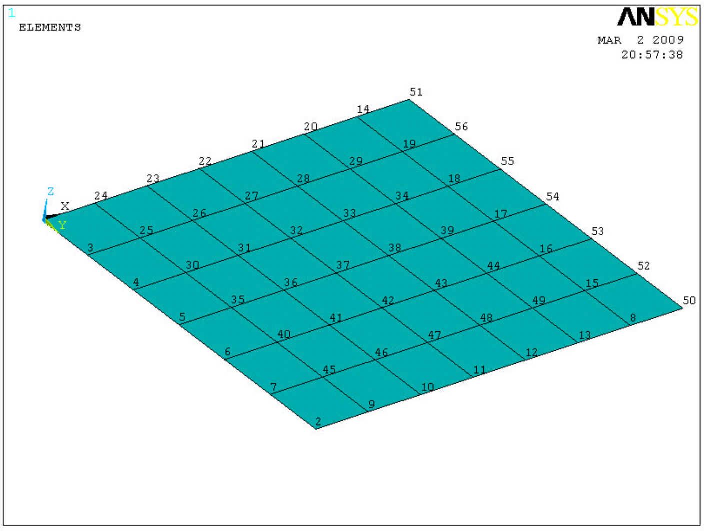


Figure 3.3 FE Model of the Modified Plate

The FRF values are calculated for the modified structure by the "Structural Modification with Additional Degrees of Freedom Program" and the results shown in Figure 3.4 are obtained for direct point FRF of node 2 of the original system in translational-z direction. As an option in the program, the calculated FRF by using Özgüven's formulation [24] can be compared with FRF calculated from ANSYS solution for the modified structure in order to see the accuracy of the method. As shown in Figure 3.5, both FRFs calculated from ANSYS solution for the modified structure and the one obtained from structural modification method match exactly except at higher frequencies. At higher frequencies there is a slight discrepancy in the FRF curves which is due to the number of modes used to calculate the FRF of the original structure as will be discussed in section 3.3.

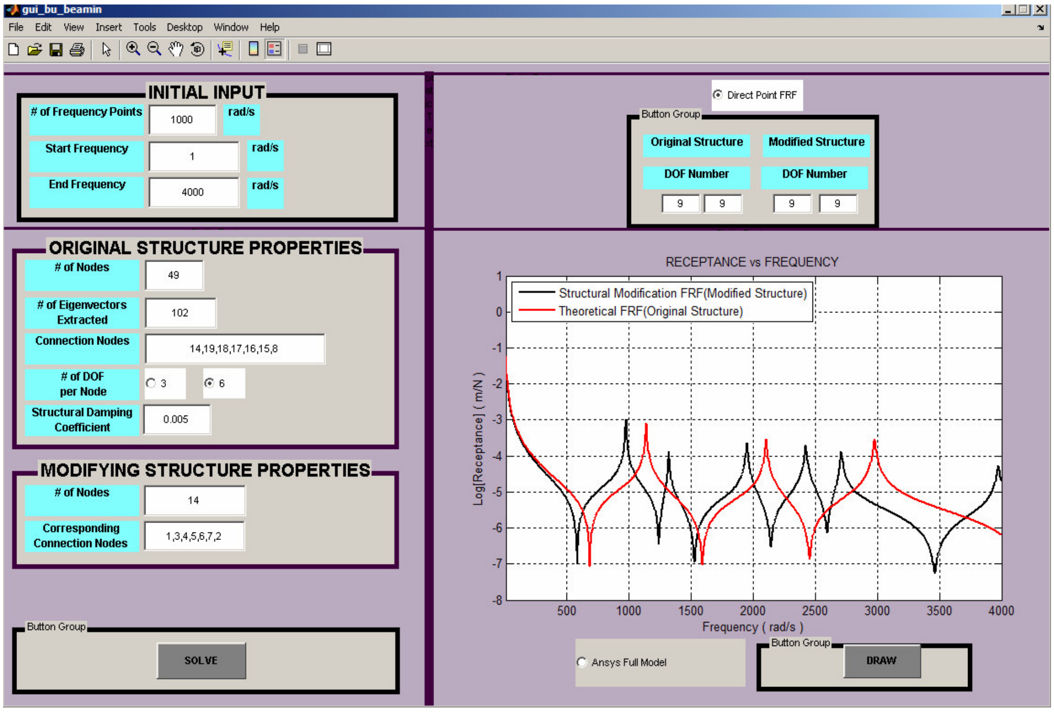


Figure 3.4 Direct Point FRF of Node 2 in Translational-z Direction for Original and Modified Structures

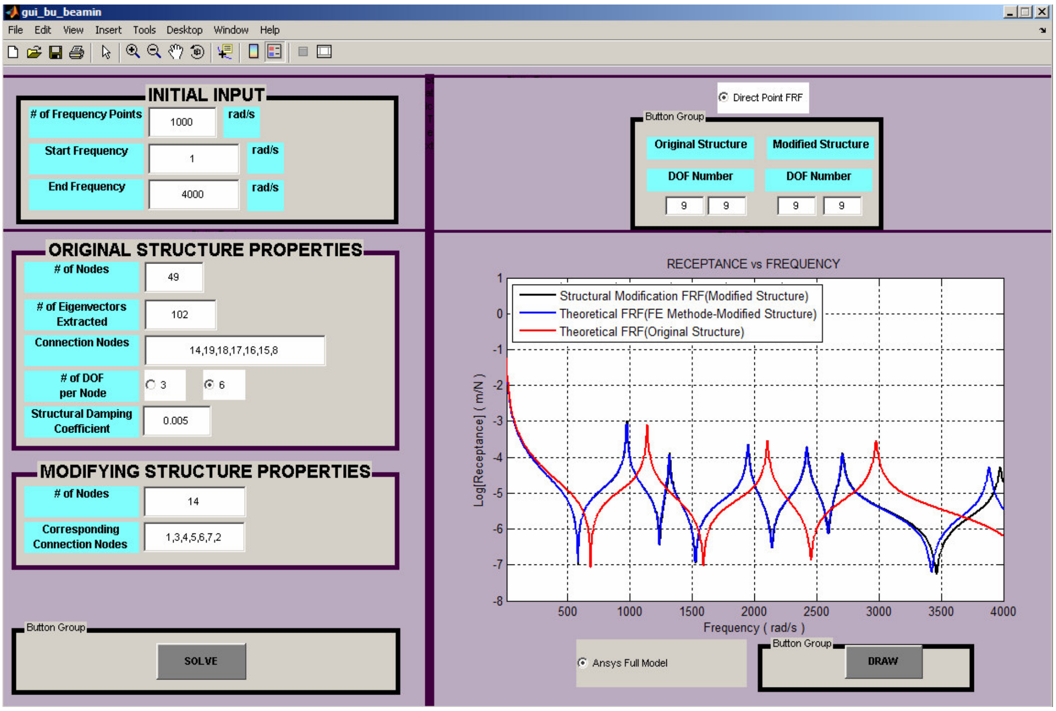


Figure 3.5 Comparison of Direct Point FRFs Obtained by Structural Modification and FEA for Node 2 of Modified Structure in Translational-z Direction

3.2.2 Free-Free Plate with Finer Mesh

In this case study, the same plates were used as the original structure and modifying structure, however finer mesh was used in the FE models of the original and modifying plate which are shown in Figure 3.6 and Figure 3.7, respectively. The material used is aluminum which has the density of 2770 kg/m^3, Young's Modulus of 71 GPa and Poisson's ratio is taken as 0.33. Both the geometrical and material properties of the original and modifying plates are shown in Table 3.3. As given in Table 3.4, original plate was divided into 144 elements and it has 169 nodes with 6 DOF per node giving 394 total DOFs. The modifying plate was divided into 24 elements and it has 39 nodes with 6 DOF per node yielding 234 total DOFs. The structural damping factor is taken as 0.005 for both structures.

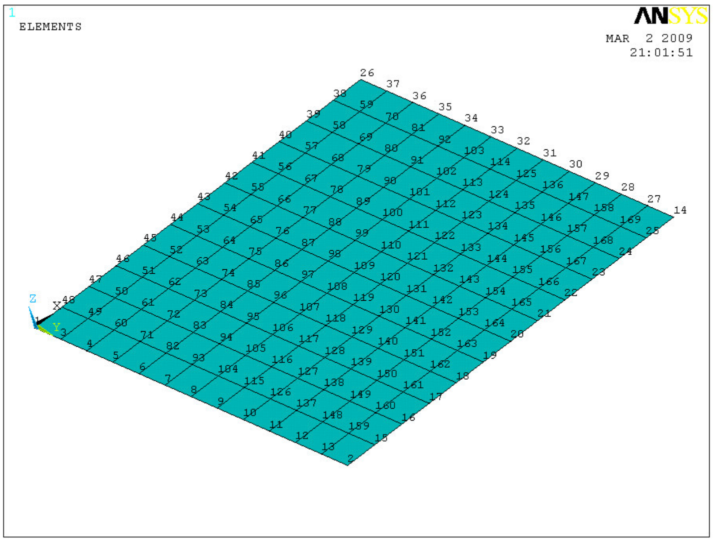


Figure 3.6 FE Model of the Original Plate

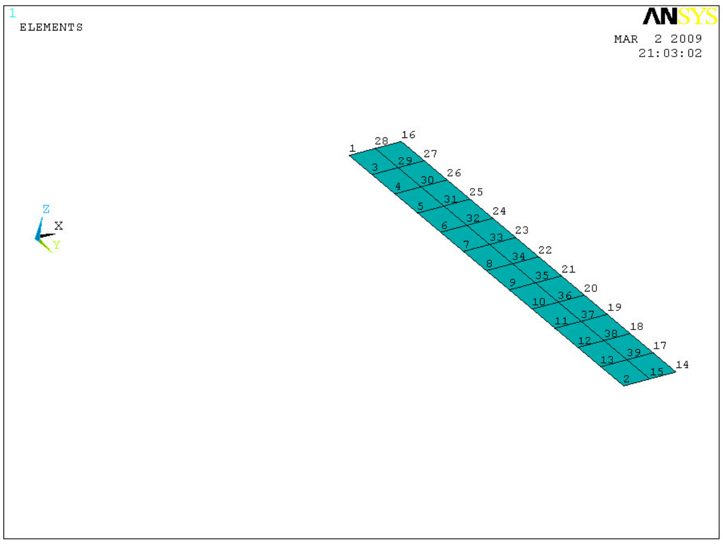


Figure 3.7 FE Model of the Modifying Plate

Table 3.3 Geometrical and Material Properties of the Plates

	Original Plate	**Modifying Plate**
Young's Modulus (E)	71 GPa	71 GPa
Poisson's Ratio (ν)	0.33	0.33
Density (ρ)	2770 kg/m^3	2770 kg/m^3
Length	300mm	300mm
Width	300mm	50mm
Thickness	5mm	5mm

Table 3.4 FE Information of the Plate Models

	Original Plate	**Modifying Plate**
Number of Elements	144	24
Number of Nodes	169	39
DOF	1014	234

There are 7 connection nodes between original and modifying structures. After the modification, the modified structure which is shown in Figure 3.8 was obtained.

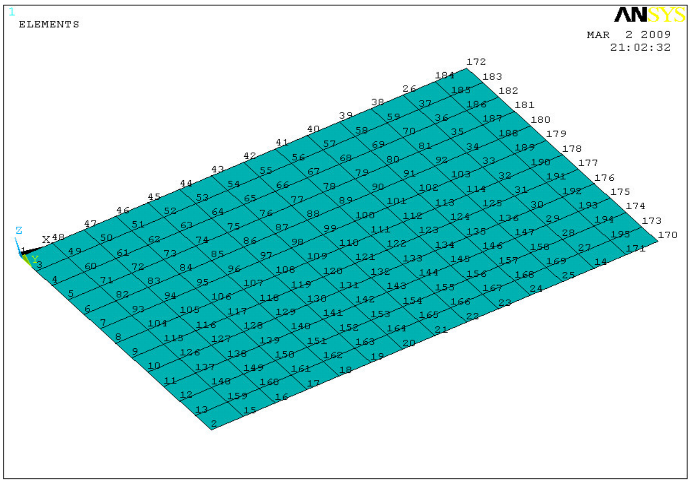


Figure 3.8 FE Model of the Modified Plate

The results shown in Figure 3.9 are obtained for direct point FRF of node 2 of the original system in translational-z direction. In order to demonstrate the validity of the method, FRF calculated from ANSYS solution for the modified structure is compared with the results obtained from structural modification program in Figure 3.10.

The structural damping coefficient of the structures can be adjusted in the program, without changing the FE model. In order to demonstrate this feature of the program, the same problem was solved for a different structural damping coefficient of 0.01. The results are shown in Figure 3.11 and Figure 3.12.

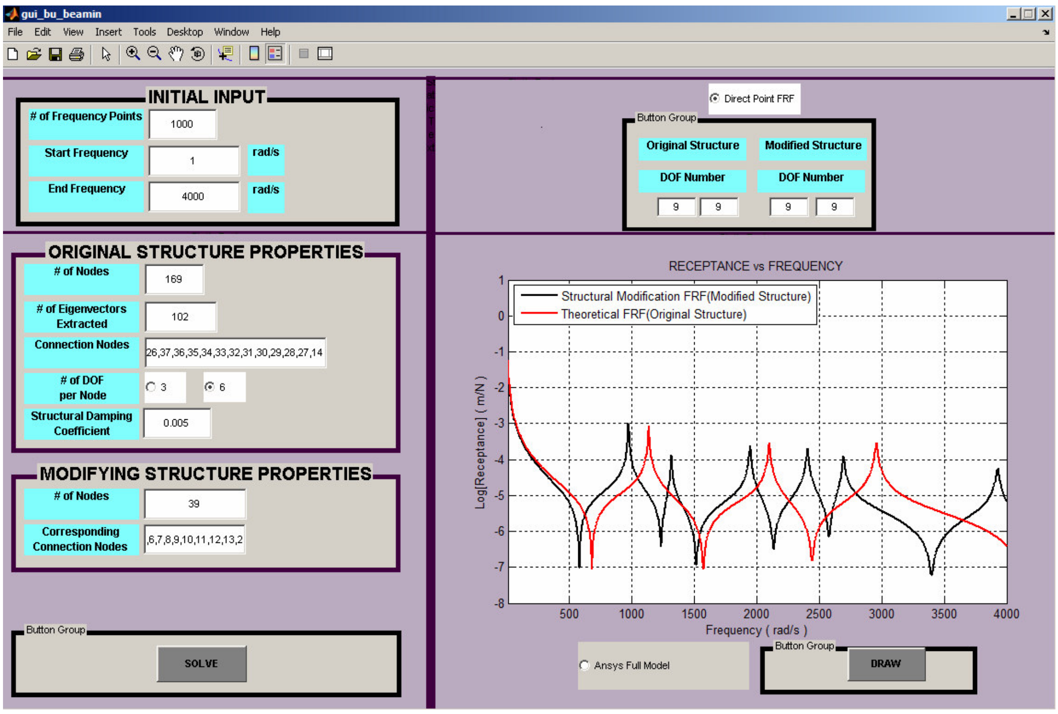


Figure 3.9 Direct Point FRF of Node 2 in Translational-z Direction for Original and Modified Structures

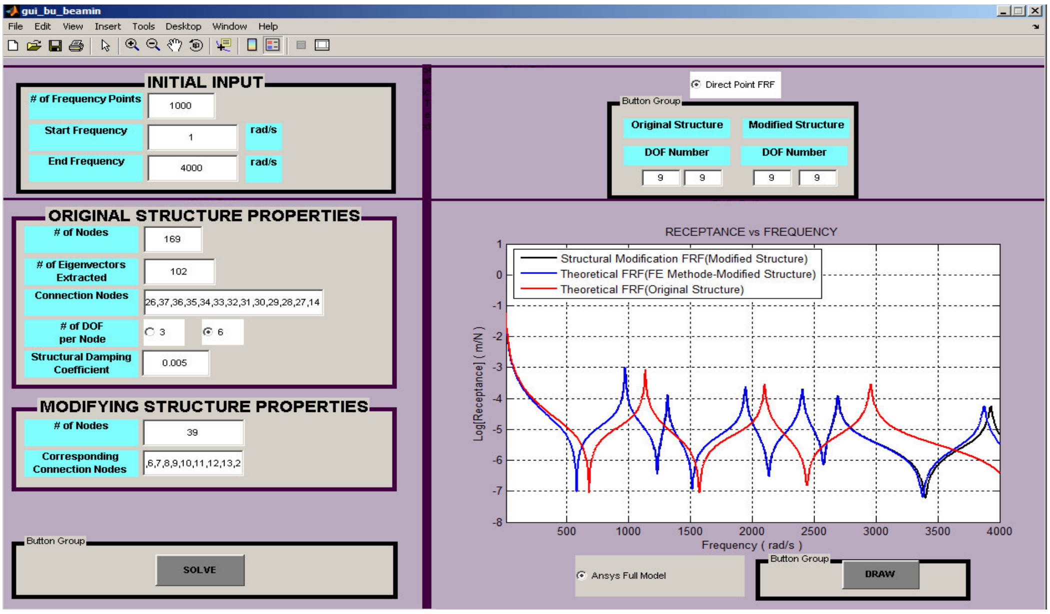


Figure 3.10 Comparison of Direct Point FRFs Obtained by Structural Modification and FEA for Node 2 in Translational-z Direction

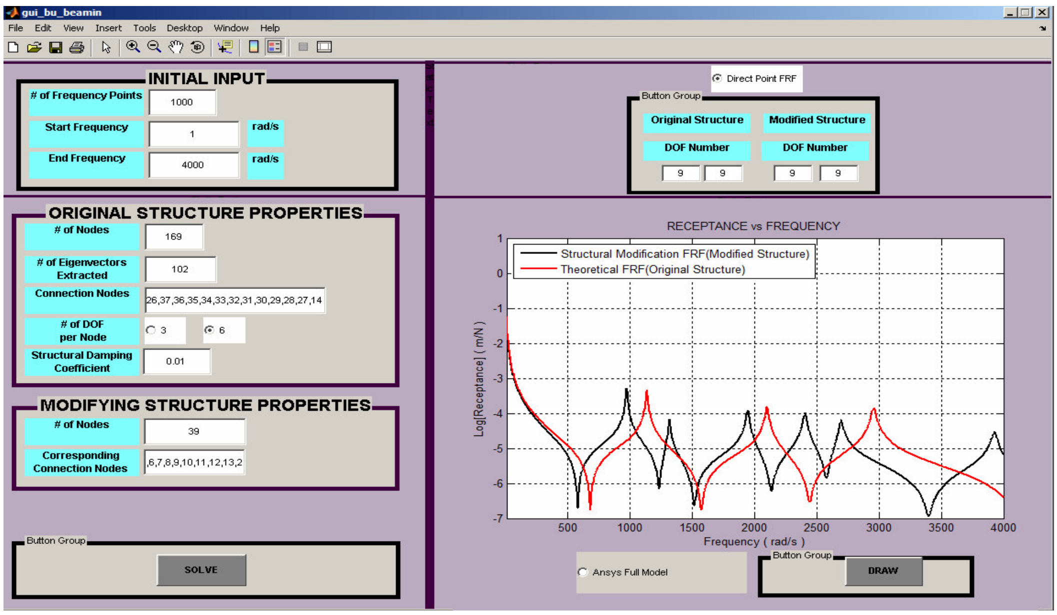


Figure 3.11 Direct Point FRF of Node 2 in Translational-z Direction for Original and Modified Structures

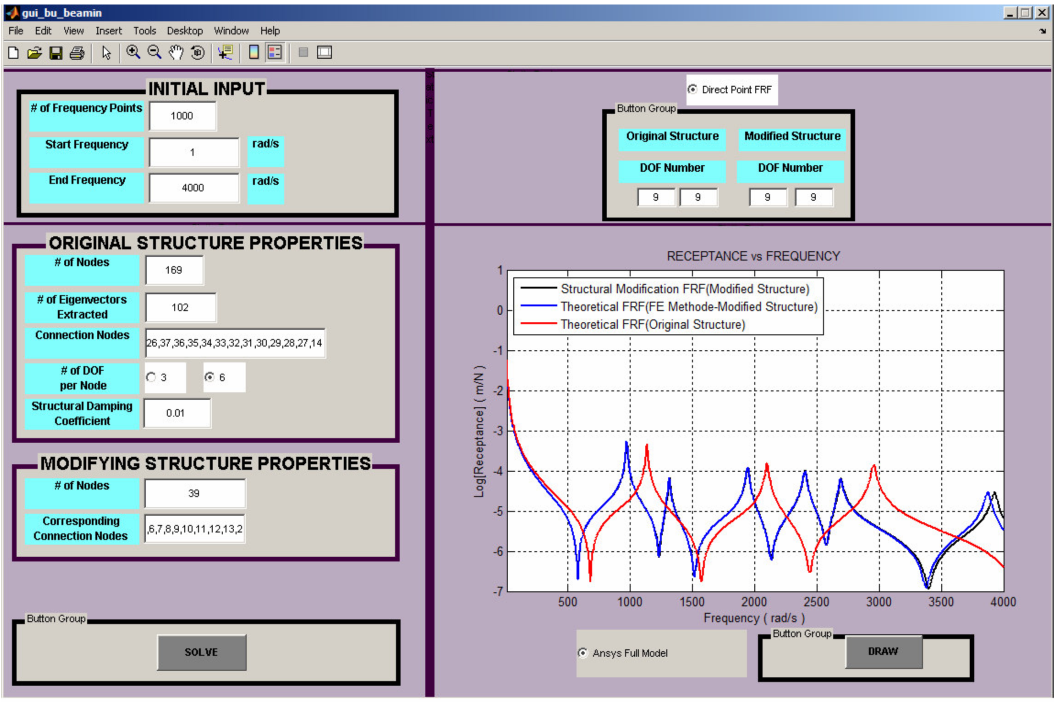


Figure 3.12 Comparison of Direct Point FRFs Obtained by Structural Modification and FEA for Node 2 in Translational-z Direction

3.2.3 Cantilevered Plate

In this case study, a cantilevered plate with dimensions 300mm x 300mm x 5mm is used as an original structure (Figure 3.13) and it is modified by attaching the smaller plate shown in Figure 3.14 to the free end of the plate. The material used is aluminum which has a density of 2770 kg/m^3, Young's Modulus of 71 GPa and Poisson's ratio is taken as 0.33. Both the geometrical and material properties of the original and modifying plates are shown in Table 3.5. As shown in Table 3.6, original plate was divided into 64 shell elements with 6 DOF per node yielding 488 total DOFs. The modifying structure was divided into 16 shell elements with 6 DOF per node giving 144 total DOFs. The structural damping factor is taken as 0.0075 for both structures.

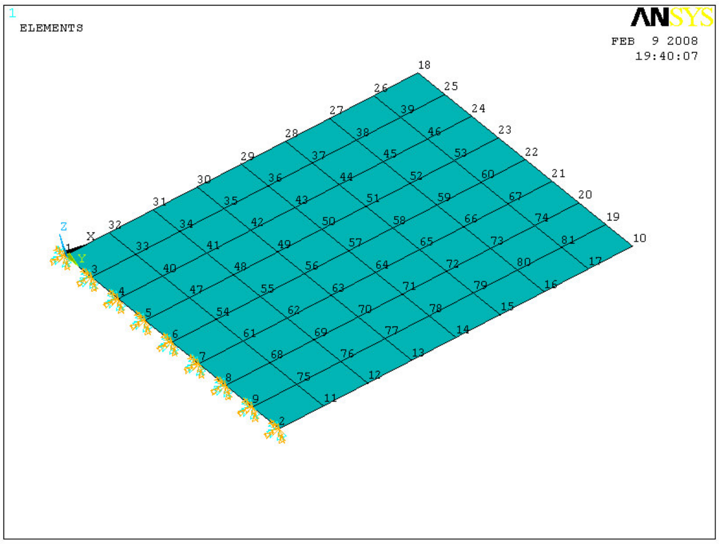


Figure 3.13 FE Model of the Original Plate

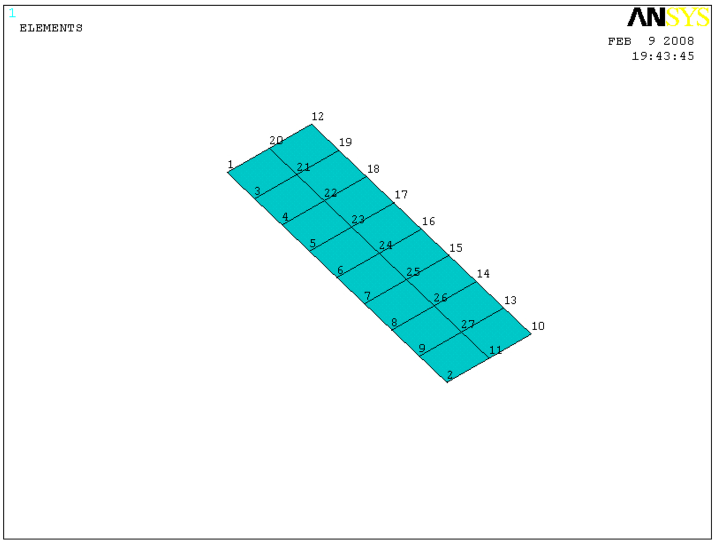


Figure 3.14 FE Model of the Modifying Plate

Table 3.5 Geometrical and Material Properties of the Plates

	Original Plate	Modifying Plate
Young's Modulus (E)	71 GPa	71 GPa
Poisson's Ratio (ν)	0.33	0.33
Density (ρ)	2770 kg/m^3	2770 kg/m^3
Length	300mm	300mm
Width	300mm	75mm
Thickness	2mm	2mm

Table 3.6 FE Information of the Plate Models

	Original Plate	Modifying Plate
Number of Elements	64	24
Number of Nodes	81	27
DOF	488	162

There are 9 connection nodes between original and modifying structures. After the modification, the modified structure shown in Figure 3.15 was obtained.

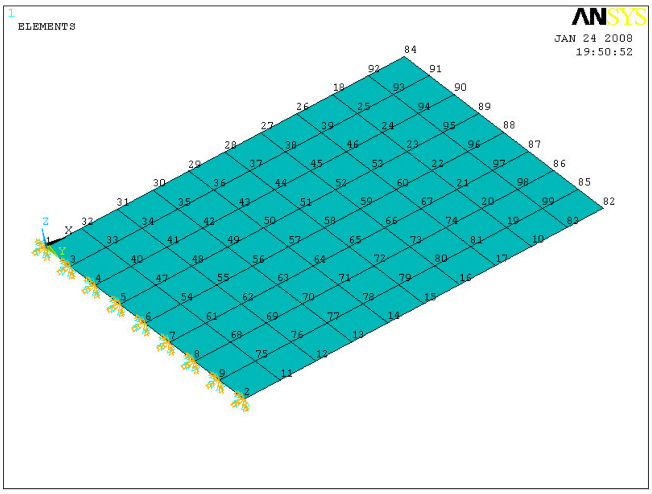


Figure 3.15 FE Model of the Modified Plate

In Figure 3.16, direct point FRFs of node 60 of the original and modified systems in translational-z direction are shown in the frequency range of 0-2000 rad/s. In order to show the validity of the method, FRFs calculated from ANSYS solution for the modified structure is compared with the results obtained from structural modification program in Figure 3.17. As can be seen in Figure 3.17, both FRFs calculated from ANSYS solution for the modified structure and the one obtained from structural modification method match for the first 3 modes. At higher frequencies the effect of truncation made in obtaining FRF of the original structure can be observed. Around the fourth and fifth modes there are discrepancies in the FRF curves, which are due to the number of modes used to calculate the FRF of the original structure.

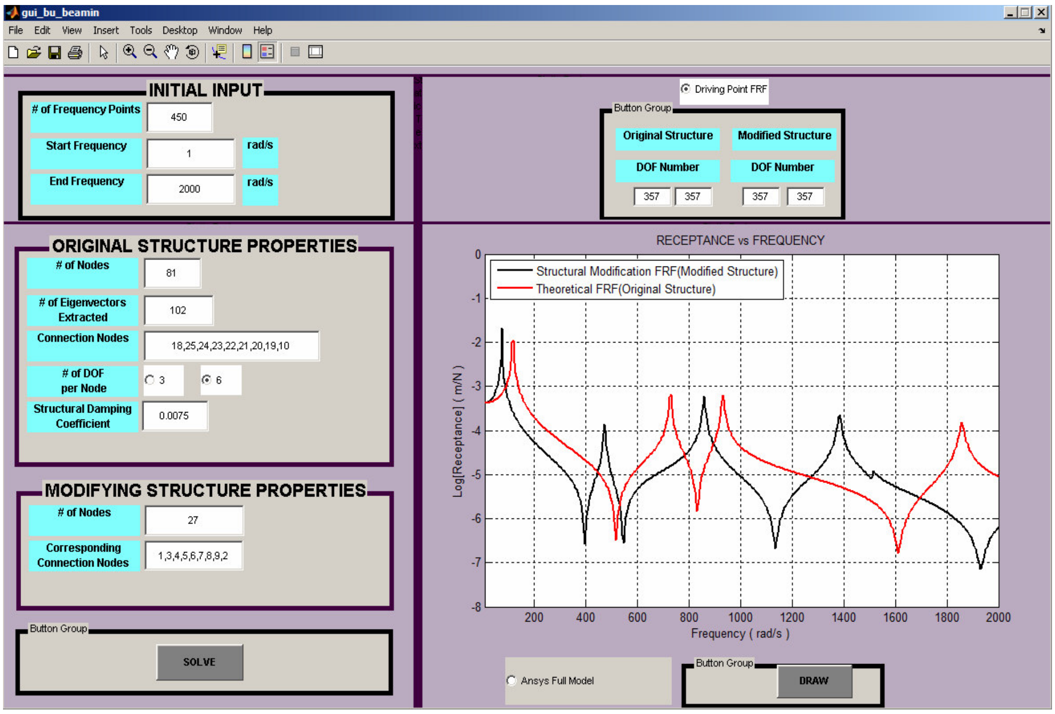


Figure 3.16 Direct Point FRF of Node 60 in Translational-z Direction for Original and Modified Structure

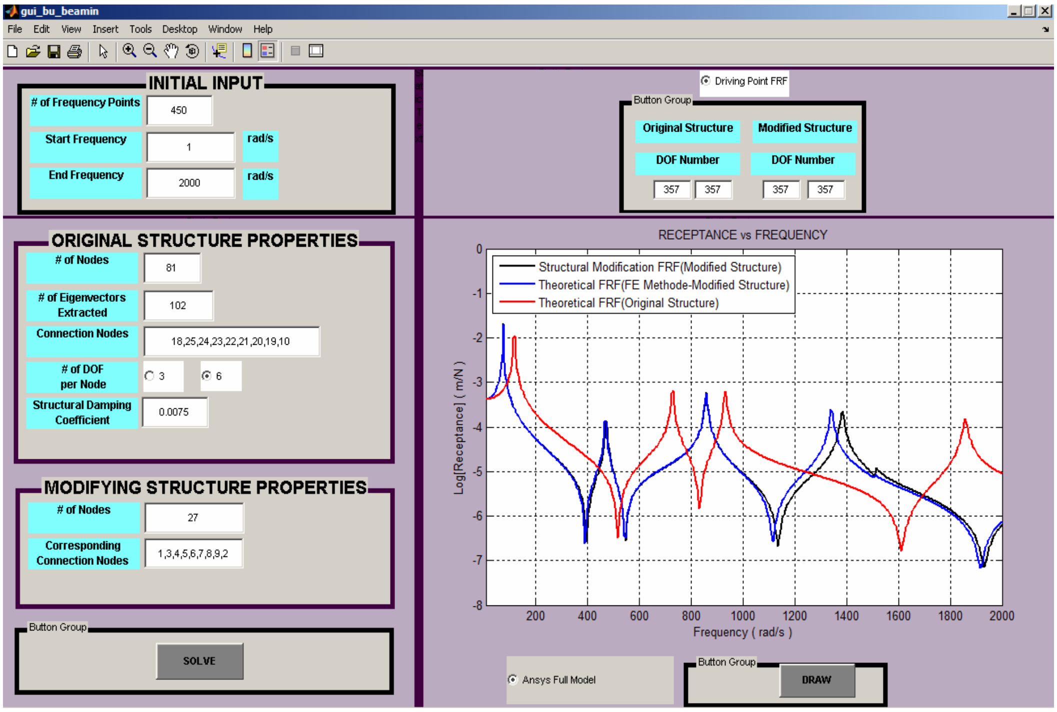


Figure 3.17 Comparison of Direct Point FRFs Obtained by Structural Modification and FEA for Node 60 in Translational-z Direction

3.2.4 Free-Free Beam

In this case study, a free-free beam shown in Figure 3.18 is modified by attaching a smaller beam under the original one. After the modification, the modified beam shown in Figure 3.19 was obtained. The material used is aluminum which has a density of 2770 kg/m^3, Young's Modulus of 71 GPa and Poisson's ratio is taken as 0.33. The original beam has dimensions 300mm x 300mm x 5mm. Both the geometrical and material properties of the original and modifying beams are shown in Table 3.7. Original beam was divided into 24 brick elements with 3 DOF per node yielding 723 total DOFs and the modifying structure was divided into 4 brick elements with 3 DOF per node giving 153 total DOFs, as shown in Table 3.8. The structural damping factor is taken as 0.01 for both structures.

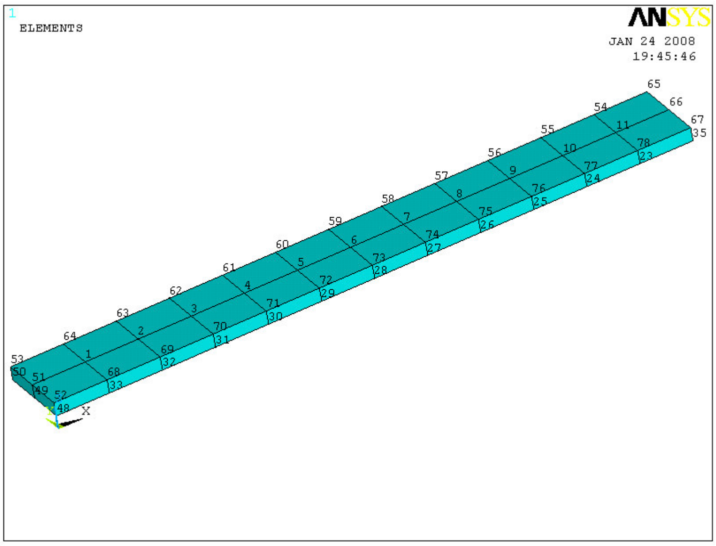


Figure 3.18 FE Model of the Original Beam

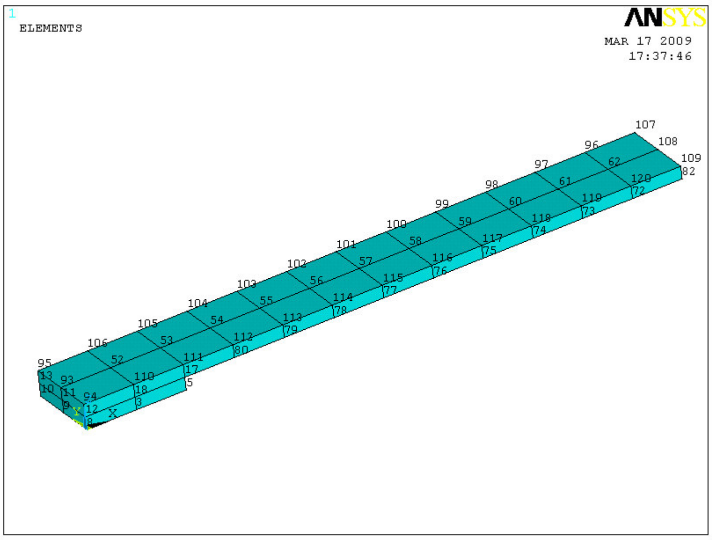


Figure 3.19 FE Model of the Modified Beam

Table 3.7 Geometrical and Material Properties of the Beams

	Original Plate	Modifying Plate
Young's Modulus (E)	71 GPa	71 GPa
Poisson's Ratio (ν)	0.33	0.33
Density (ρ)	2770 kg/m^3	2770 kg/m^3
Length	1200mm	200mm
Width	150mm	150mm
Thickness	25mm	25mm

Table 3.8 FE Information of the Beam Models

	Original Plate	Modifying Plate
Number of Elements	24	4
Number of Nodes	241	51
DOF	723	153

In Figure 3.20, direct point FRFs of node 61 of the original and modified system in translational-z direction are shown in the frequency range of 0-4000 rad/s. In the analysis, 1000 frequency points were used in the frequency range given above. In order to show the validity of the method, FRFs calculated from ANSYS solution for the modified structure are compared with the results obtained from structural modification program in Figure 3.21.

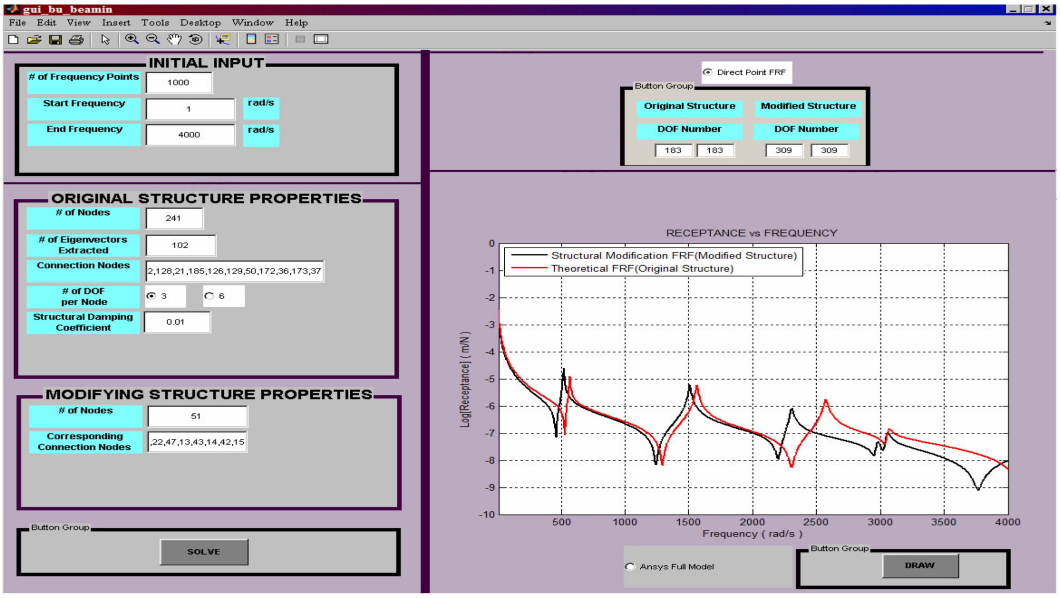


Figure 3.20 Direct Point FRF of Node 61 in Translational-z Direction for Original and Modified Structure

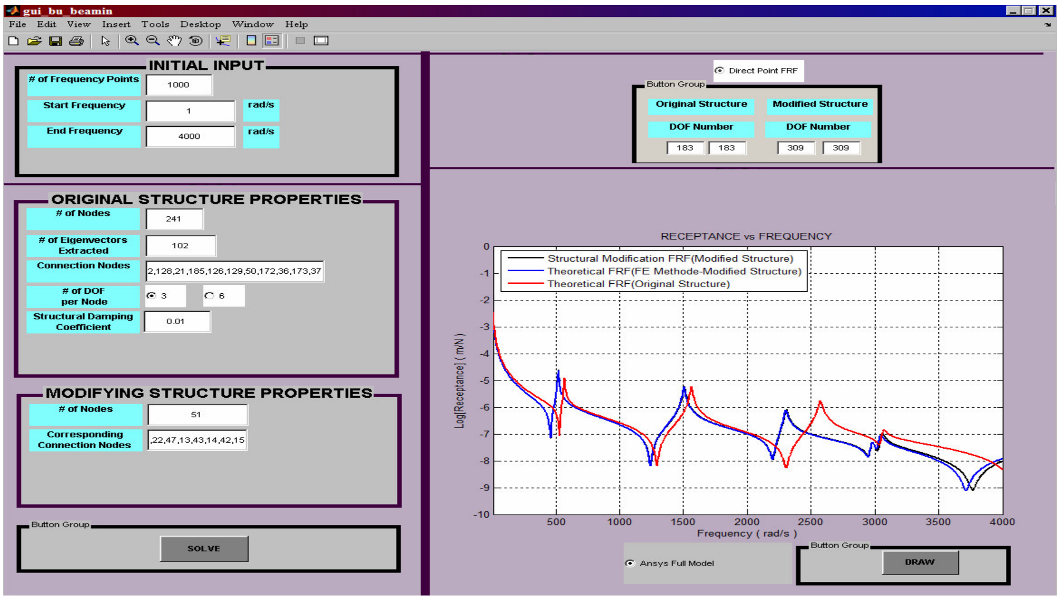


Figure 3.21 Comparison of Direct Point FRFs Obtained by Structural Modification and FEA for Node 61 in Translational-z Direction

3.2.5 Free-Free L-Beam

In this case study, a free-free L-beam shown in Figure 3.22 is modified by attaching a rib between the arms of it. The main objective of this modification is to increase bending stiffness of the structure so that natural frequencies of the bending modes increase. After the modification, the modified L-beam which is shown in Figure 3.23 was obtained. The material used is aluminum which has a density of 2770 kg/m^3, Young's Modulus of 71 GPa and Poisson's ratio is taken as 0.33. The vertical and horizontal parts of L-beam have the dimensions of 150mm x 120mm x 15mm and 200mm x 120mm x 20mm, respectively. The material properties of the original and modifying beams are shown in Table 3.9. Original L-beam was divided into 36 brick elements with 3 DOF per node yielding 960 total DOFs and the modifying structure was divided into 4 brick elements with 3 DOF per node giving 153 total DOFs as shown in Table 3.10. The structural damping factor is taken as 0.0075 for both structures.

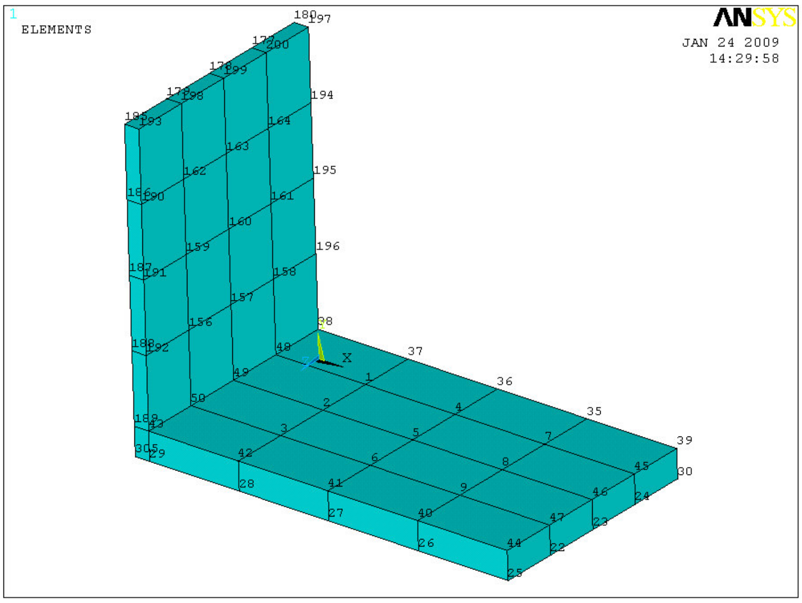


Figure 3.22 FE Model of the Original L-Beam

Table 3.9 Material Properties of the Beams

	Original L-Beam	Modifying Beam
Young's Modulus (E)	71 GPa	71 GPa
Poisson's Ratio (ν)	0.33	0.33
Density (ρ)	2770 kg/m^3	2770 kg/m^3

Table 3.10 FE Information of the Beam Models

	Original L-Beam	Modifying Beam
Number of Elements	36	4
Number of Nodes	320	51
DOF	960	153

In Figure 3.24, direct point FRFs of node 193 of the original and modified system in translational-x direction are shown in the frequency range of 0-10000 rad/s. In order to see the accuracy of the structural modification method in this application, FRF calculated from ANSYS solution for the modified structure is compared with the results obtained from structural modification program in Figure 3.25.

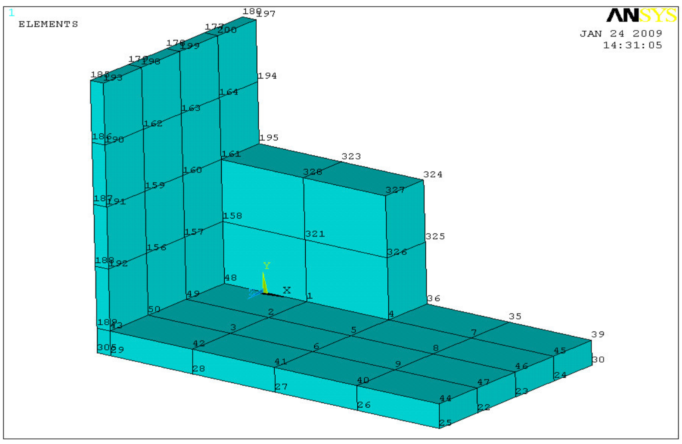


Figure 3.23 FE Model of the Modified L-Beam

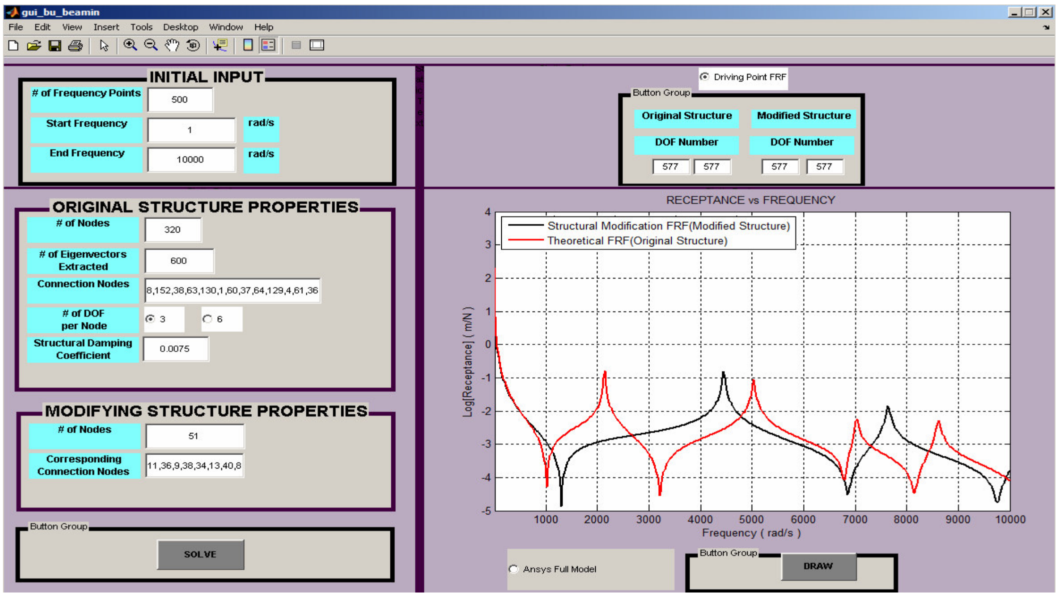


Figure 3.24 Direct Point FRF of Node 193 in Translational-x Direction for Original and Modified Structure

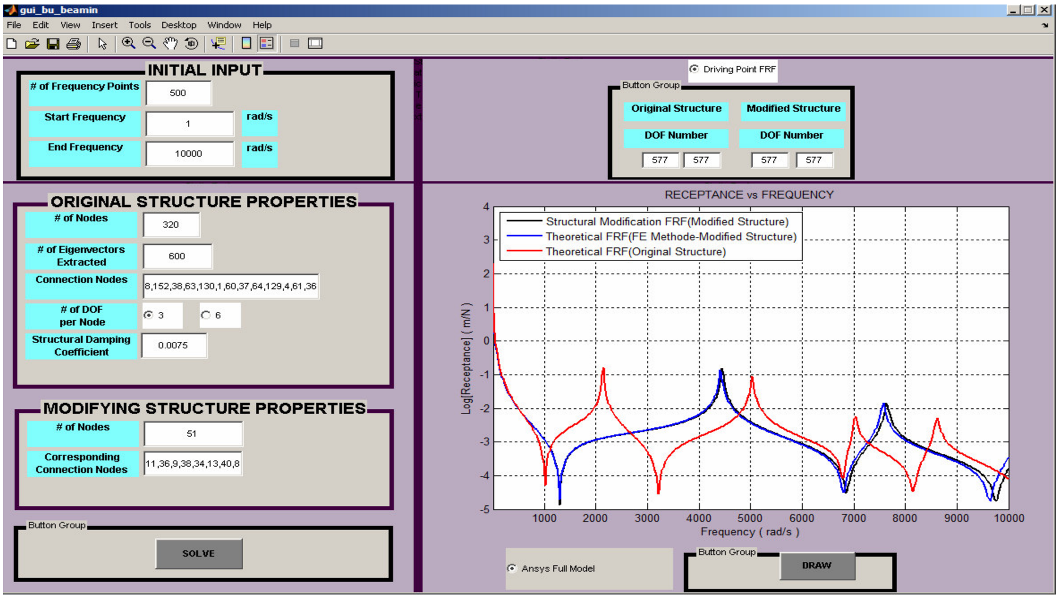


Figure 3.25 Comparison of Direct Point FRFs Obtained by Structural Modification and FEA for Node 193 in Translational-x Direction

3.3 The Effect of Modal Truncation on the Structural Modification Method

In this section, the effect of modal truncation made in the computation of the FRFs of the original structure on the accuracy of the structural modification method proposed is demonstrated by using the cantilever plate given in section 3.1.2. The same modification is applied to the original plate, and the results are obtained for the modified plate.

In order to show the effect of truncation, the FRFs of the modified plate are predicted by using different number of mode shapes in the calculation of FRFs of the original structure. The direct point FRFs of node 30 of the original and modified system in translational-z direction are calculated in the frequency range of 0-300 Hz by using 15, 30, 102 modes in turn. The results are shown in Figure 3.26 to Figure 3.28.

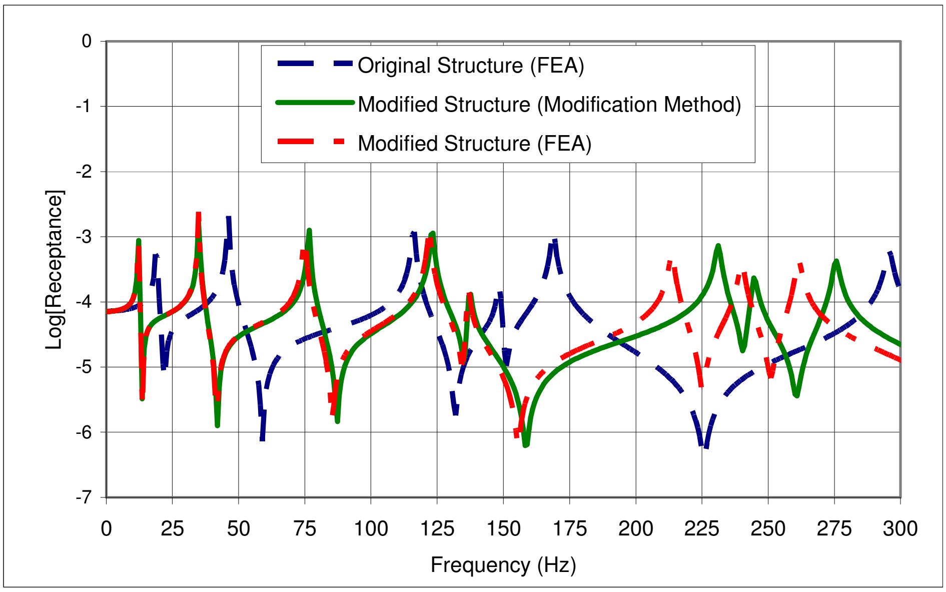


Figure 3.26 Comparison of FRFs at 30Z30Z (15 Modes are Used in the Calculation of FRFs of the Original Structure)

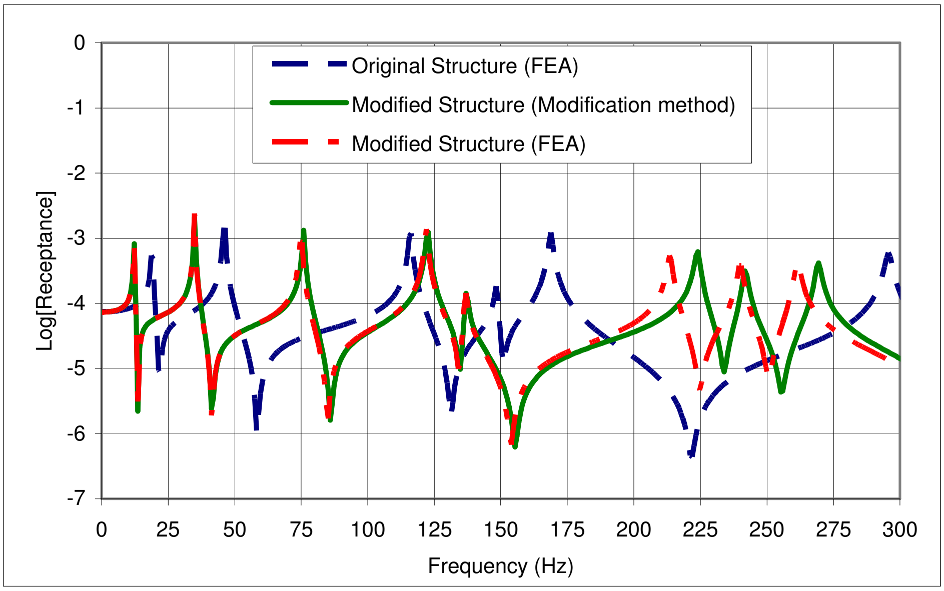


Figure 3.27 Comparison of FRFs at 30Z30Z (30 Modes are Used in the Calculation of FRFs of the Original Structure)

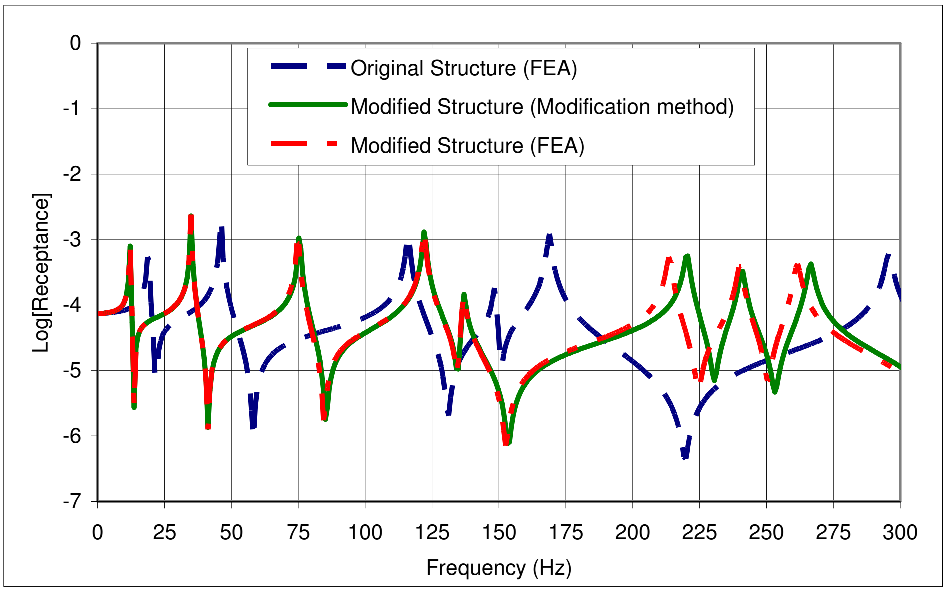


Figure 3.28 Comparison of FRFs at 30Z30Z (102 Modes are Used in the Calculation of FRFs of the Original Structure)

As can be seen from Figure 3.26 to Figure 3.28, the effect of truncation made in obtaining FRFs of the original structure can be observed at higher frequencies. The method is sensitive to the accuracy of the FRFs of the original structure and using higher number of the modes in the computation of the FRF of the original structure increases the accuracy of the FRFs predicted for modified structure. Although FRFs of the modified system is affected considerably, FRFs of the original structure are not affected much from truncation. Therefore, it is concluded that the effect of truncation is more pronounced for FRFs of the modified system than those of the original system (especially at higher frequencies).

CHAPTER 4

EXPERIMENTAL VALIDATION

4.1 Modal Test Theory

4.1.1 Introduction

A modal test is a test which is conducted in order to construct a mathematical model of the structure based entirely on measured vibration data [47]. In modal test, not only response is measured, but also the excitation is measured, so that, these quantities can be related to each other and relationship between them can be defined. When the mathematical model of the structure is constructed, the dynamic behavior or vibration behavior of the structure under any loading can be obtained. However the definition of mathematical model varies considerably from one application to other, it can be estimate of natural frequency and damping factor in one case and a full mass-spring dashpot model for the other [46].

The application areas of the modal test can differ. The most commonly used application is obtaining the dynamic behavior of the structure in order to compare the results with the corresponding data calculated from a finite element model. This application is needed in order to validate the theoretical model with the experimental results so that, the theoretical model can be used for predicting the response levels to different excitations. Modal test can also be used in order to construct a mathematical model of a structure which then may be used in a structural coupling. Another application of the modal test is to obtain a mathematical model which can be used to predict the effects of modifications to the original structure which may be referred to structural dynamic modifications application.

Modal test has 4 phases [47]:

- Test planning phase
- Measurement phase
- Analysis phase
- Modeling phase

4.1.1.1 Test Planning Phase

In modal testing it is vital to use correct equipment for different transduction, signal processing and analysis tasks and to measure all the necessary parameters. This means all the necessary quantities are included in parameter list and also all the unnecessary data are excluded from the list.

Essentially, a modal test includes the measurement of a set of response functions. These are usually measured as time-history records of various responses and excitation signals and they are often processed at source to obtain the FRFs. The structure is usually described by its FRF response model and it is fully defined by an FRF matrix. It is enough to measure a single row or column of the FRF matrix in order to obtain modal properties from measurements of responses. Based on the plan, two proper excitation points should be selected and a point FRF should be measured at these excitations points. Then the resonance frequencies on the two FRF curves should be compared in order to establish whether there are any modes present in one plot and absent from the other. The process of selecting and checking further excitation points should continue, until all modes have been identified.

After following these steps it is necessary to decide whether the excitation point chosen is suitable for exciting all modes or whether it is preferable to excite some modes from one point and others from other points. While selecting excitation locations, it is necessary to ensure that the excitation point is not at or close to a nodal line.

In test planning it is also important to choose response measurement locations. There are a number of considerations in the selection of convenient points that will provide clear visual interpretation of the resulting mode shapes, and in selection of DOFs which are necessary to have an unambiguous correlation between tests and analyze models. For the former one, with a uniform distribution of points that has a sufficiently fine mesh, the mode shapes can be displayed. However this choice is not necessarily the optimum set for the second case which may be a more quantitative application such as model validation, updating or modification. There are different procedures available to select proper measurement points that will satisfy the needs.

4.1.1.2 Measurement Phase

In the measurement phase, the main concern is to prepare the structure for test and measure the data which will be used to obtain the mathematical model of the structure and understand the dynamic behavior of the structure. Correct use of the equipment and installation of the transducers are very important in order to eliminate the systematic errors, because these types of errors are difficult to detect when they are compared to noise errors. Since these systematic errors are not easy to detect, when they are embedded into measured data, they may lead to serious errors in the construction of mathematical model of the structure. Once these errors are eliminated, the remaining part in the measurement phase is the measurement of excitation force and the resulting forces at the convenient points chosen in the test planning phase. The measured data will be displayed in the form of frequency response functions (FRFs) which are the ratios of responses to excitations.

4.1.1.3 Analysis Phase

In the analysis phase, the measured data are subjected to a process in order to determine the specific parameters of a mathematical model so that this model exhibits the same dynamic behavior as the measured in the modal test. The modal model is constructed by using the modal properties of the system which describe the dynamic characteristics of the structure. In this phase, curves are regenerated by curve fitting techniques which use the modal parameters. Many curve fitting methods operate on the response characteristic in the frequency domain, but there are also algorithms which perform the curve fitting in the time domain. The main concern is to obtain a curve which has the minimum discrepancy from the measured curve. For the analysis of the data, there are various algorithms; however the most powerful analysis methods are those which are performed on all FRF curves in a single computation within a wide frequency range. However the performance of these methods mostly depends on the consistency and uniformity across the complete set of measured data.

4.1.1.4 Modeling Phase

The main objective in this phase is to provide some insight into the validity and quality of the model which has been constructed. In the modeling phase the inconsistencies in the modal data are checked. There are also other checks which must be undertaken on the resulting model, such as verification that the modes are suitably real, and not complex.

4.1.2 Limitations and Sources of Errors in Modal Testing

In modal testing, in order to construct a mathematical model that will exhibit the same dynamic characteristics as the structure to be tested, knowledge of the errors and limitations in modal testing should be available. Due to different limitations and sources of errors in modal testing, there is always certain inaccuracy in the tests. These limitations and errors are random errors due to noise, limited number of measured degrees of freedom, poor modal analysis of experimental data, non-linear behavior of the structure or attached mechanical devices, systematic errors due to signal processing of the measured data, difficulty in measuring rotational degrees of freedom, systematic errors due to attachment of mechanical devices like springs, transducers and stingers to the structures. These limitations and errors can be divided into 3 groups [48]:

- Experimental data acquisition errors
- Signal processing errors and
- Modal analysis errors

These errors can be categorized within themselves, as given below [48]:

- Experimental data acquisition errors
    - Quality
        - Mechanical errors
            - Mass loading effect of transducers
            - Shaker-structure interaction
            - Supporting of the structure
        - Measurement noise
        - Nonlinearity
    - Quantity
        - Measuring enough points on the structure
        - Measuring enough degrees of freedom (i.e. Rotational DOFs)
- Signal processing errors
    - Leakage
    - Aliasing

- Effect of window functions
  - Effect of Discrete Fourier Transform
  - Effect of averaging

- Modal analysis errors :
  - Circle-Fit Modal Analysis
  - Line-Fit Modal Analysis
  - Global Modal Analysis

All these errors may lead to inaccurate and imprecise results in the modal test. If the mechanical errors are considered, these types errors are embedded into the data which are acquired. When shakers are used in modal test, the shaker dynamics may affect the total dynamics of the structure to be tested. In shaker testing axial force should be the only excitation of the structure therefore in order to provide this, a slim rod which is called stinger is used to attach the shaker to the structure. A stinger is stiff in the axial direction and flexible in the other directions.

Another important source of error is the mass loading effects of the transducers. In order to measure the dynamic force and response of a structure, in terms of FRFs, it is necessary to use accelerometers and force transducers; however usage of these equipments introduces changes to the structure due to the addition of masses. The input force excitation is partly spent on accelerating of the force transducer mass and also the accelerometer mass [46]. This is the main reason of mass-loading effects of transducers. Generally followed approach to resolve this problem is to use small accelerometers or force transducers in order to minimize the mass loading effect. Another approach is to employ mass-cancellation correction.

The boundary condition in modal test also plays an important role in the accuracy of the model constructed. In the free-free condition the test structure should be freely-supported in space and should not be attached at any of its coordinates; however it is not possible to provide a truly free-free support in practice. Therefore, this boundary condition is approximated by supporting the test structure on very soft springs such as light elastic cords or by hanging the structure with soft elastic bungee. However, especially for flexible structures, the elastic modes may interfere with rigid body modes and flexible modes may be affected from these rigid body modes. In the grounded condition the selected points of the structure should be fixed; but it is not possible to provide truly grounded condition in practical cases. All these approximations introduce certain errors to the mathematical model constructed.

4.2 Application of Structural Modification Method on Garteur SM-AG19 Model

In the previous part of this thesis structural modification method was applied to different theoretical models that were modeled by finite element method. In this section the structural modification method will be applied to a real structure. In order to apply the structural modification method to a real structure, GARTEUR SM-AG19 test structure which is used in the literature for modal testing [50], was constructed in Aselsan Inc. Microelectronics, Guidance and Electro Optics Division and modal test is conducted both on the original and modified GARTEUR SM-AG19 test structure. GARTEUR is the abbreviation of Group for Aeronautical Research and Technology in Europe and this GARTEUR SM-AG19 structure has been designed by a multinational research group. This structure has been designed in order to use the same experimental data for different investigations on structural dynamics and to have a common structure on which modal tests and modal analyses are conducted. In this part, in order to show the performance of the structural modification technique, GARTEUR SM-AG19 model was modified with beams under the wings which act as stiffeners causing increased flexural rigidity. The GARTEUR SM-AG19 model which was constructed in Aselsan Inc Microelectronics, Guidance and Electro Optics Division differs slightly from the original GARTEUR SM-AG19 model. In this model there was no viscoelastic tape and additional dampers, so damping characteristics of the GARTEUR SM-AG19 model used in this study is different from the original one. The general view of the GARTEUR SM-AG19 model constructed is shown in Figure 4.1.

Figure 4.1 General View of the GARTEUR SM-AG19 Model

In order to modify the original GARTEUR SM-AG19 model, two beams were attached under each wing. Two modal tests were performed: one on the original model which has no additional beam under the wings and one on the modified structure with additional beams under the wings. Firstly, in order to validate the finite element model of the original GARTEUR SM-AG19 model, the results obtained from the finite element method and modal test were compared. As a second step, in order to show the accuracy of the structural modification method, the results obtained from the finite element method for the modified GARTEUR SM-AG19 model and modal test were compared.

4.2.1 Modal Test Setup and Configuration

The overall mass is GARTEUR SM-AG19 model constructed is 41 kg and as a material, aluminum was used. The overall length of the structure is 1.5 m, the wing span is 2.0 m. The details of the dimensions of the model are shown in Figure 4.2

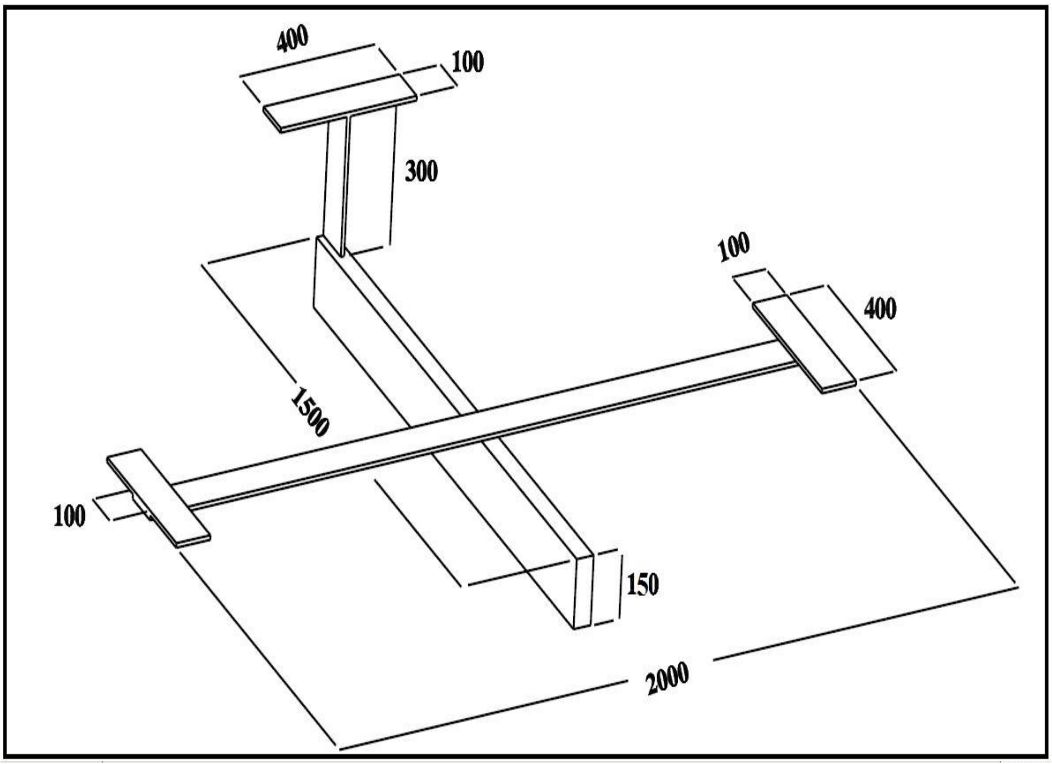


Figure 4.2 Dimensions of the GARTEUR SM-AG19 (All dimensions are in mm)

In the construction of GARTEUR SM-AG19 model, bolted joints are used. The details of the bolted joints are given in Figure 4.3.

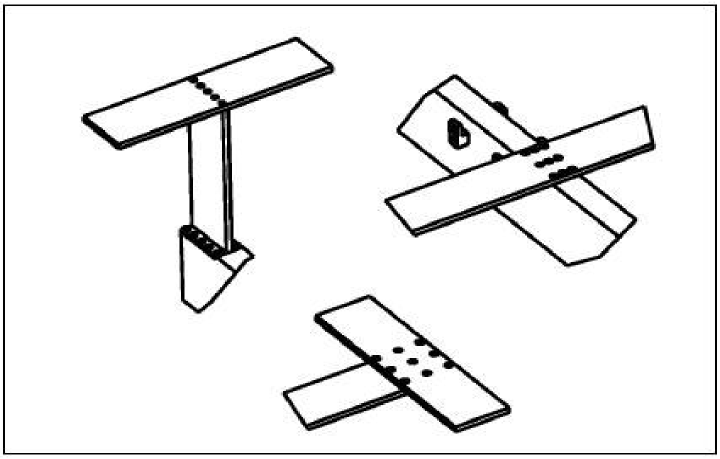


Figure 4.3 Details of Bolted Joints

Modal tests were performed with free-free boundary conditions. In order to provide free-free boundary condition, the test-bed was hung from the attachment point on the model to the metal cage support. As the attachment points, special hanging adaptor was designed on the test-bed. For the suspension, 4 elastic bungees were used. The details of the bungee, metal cage support and attachment point are shown in Figure 4.4 and Figure 4.5.

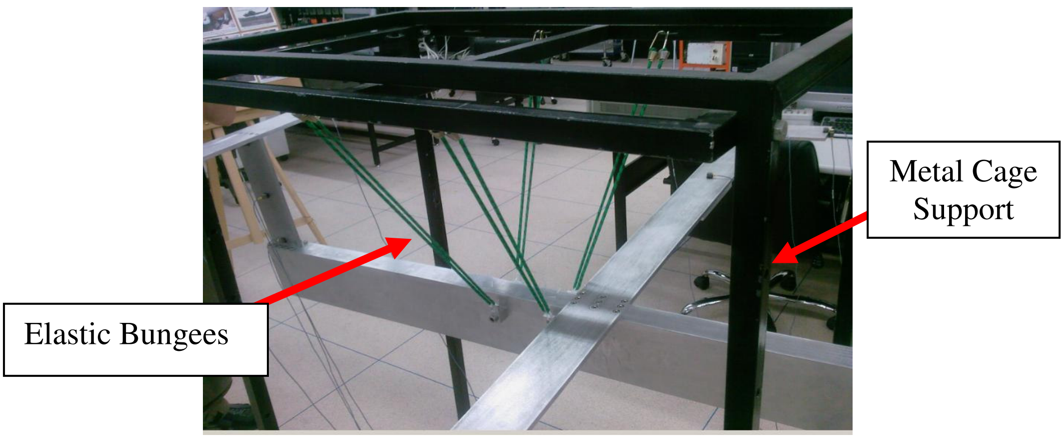


Figure 4.4 View of Bungee and Metal Cage Support

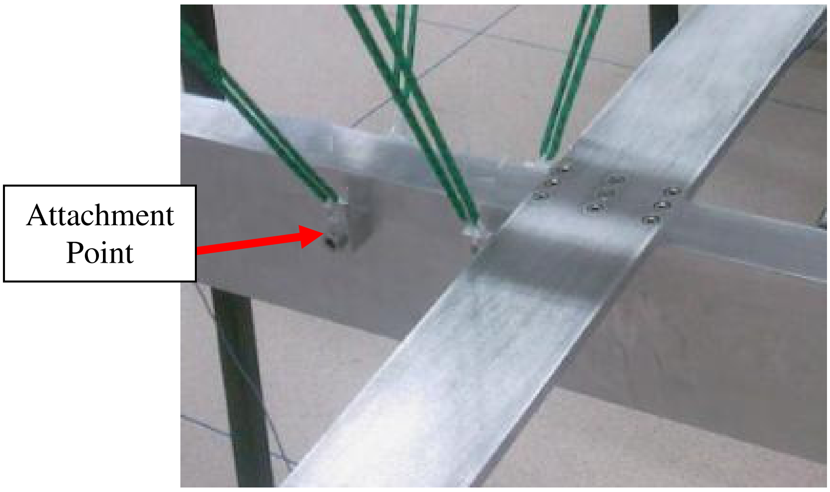


Figure 4.5 View of Attachment Point

The modal tests were performed by using modal impact hammer and accelerometers. In the experiment, total 16 accelerometers were used. The accelerometer positions of the experiment are shown in Figure 4.6. The directions and the nodes used for excitation and measurement are given in Table 1 and Table 2 respectively.

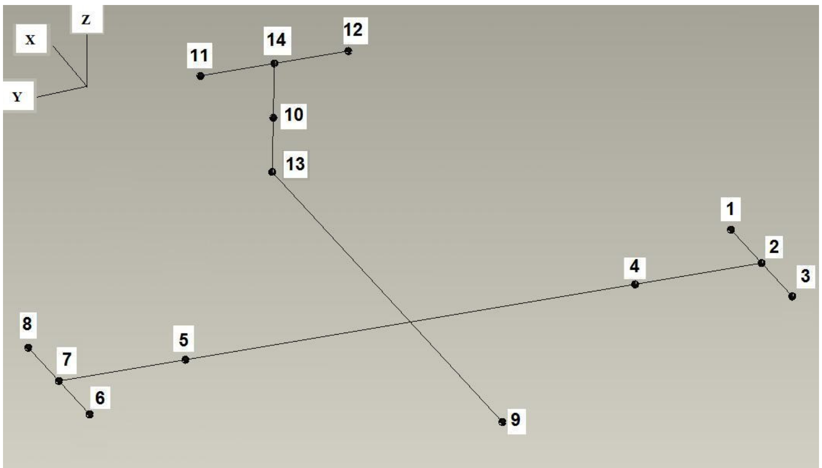


Figure 4.6 Accelerometer Positions

Table 4.1 Excitations Nodes and Directions

Excitation Nodes	Directions		
	x	y	z
1	√	√	√
2			√
3	√	√	√
4	√		√
5	√		√
6	√	√	√
7			√
8	√	√	√
9		√	
10		√	
11	√		√
12	√		√
13		√	
14		√	

Table 4.2 Measurement Nodes and Directions

Measurement Nodes	Directions		
	x	y	z
1			√
2			√
3	√		√
4			√
5			√
6	√		√
7			√
8			√
9		√	
10		√	
11			√
12			√
13		√	
14		√	

The only differences between the GARTEUR SM-AG19 model and modified system are the beams attached under the wings. In order to prevent parameters that may bring differences between the results of modal test of the original and modified models, the same accelerometer configurations and types were used for the original GARTEUR SM-AG19 model and modified system in the modal tests.

Modal test was conducted between 0-100 Hz and excitation in each DOF was performed for 5 averages. PULSE 11.0 software and Pulse Front-End 3560C were used in the modal tests. In the software, coherence values were monitored in the selected frequency range during the excitation, according to the coherence values, it was decided to repeat the hammer hit or not. Instrument and transducer properties are given in Table 4.3 and Table 4.4 respectively. In Figure 4.7, data acquisition system and modal hammer are shown.

Table 4.3 Instrumentation and Software Information

Instrumentation and Software	
Accelerometer	Bruel & Kjaer 4507 B Bruel & Kjaer 4508 B
Impact Hammer	Bruel & Kjaer 8200+2646
Analyzer	Pulse Front-End 3560C
Software	Pulse 11.0

Table 4.4 Transducer Properties

Transducer Type	Frequency Range 10%	Input Sensitivity
Accelerometer Bruel & Kjaer 4507 B	0.3-6k	10 mV/m/s^2
Accelerometer Bruel & Kjaer 4508 B	0.3-8k	10 mV/m/s^2

Figure 4.7 View of Data Acquisition System and Modal Hammer

4.2.2 Modal Test of Original Garteur SM-AG19 Model

The GARTEUR SM-AG19 model test setup and the measurement points on the test setup are shown in Figure 4.8 and Figure 4.9. Total 16 accelerometers were used in the modal test of original GARTEUR SM-AG19 model.

Figure 4.8 View of Measurement Points on the Left Wing and Vertical Stabilizer

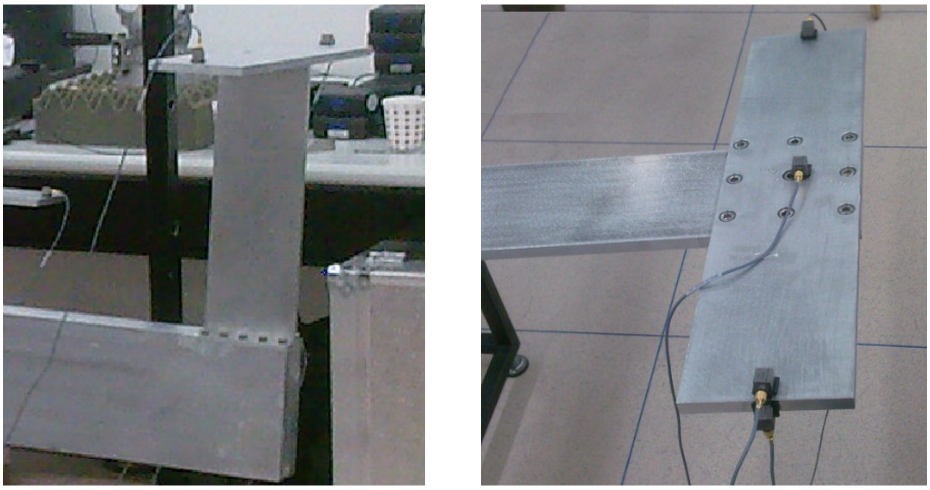


Figure 4.9 View of Measurement Points on the Vertical Stabilizer and Right Wing

The screen shot from the user interface of PULSE 11.0 software during the modal test is shown in Figure 4.10. In this figure the FRF measurement and coherence plot and measurement control windows can be seen. The first 6 natural frequencies of the original GARTEUR SM-AG19 model test setup are given in Table 4.5.

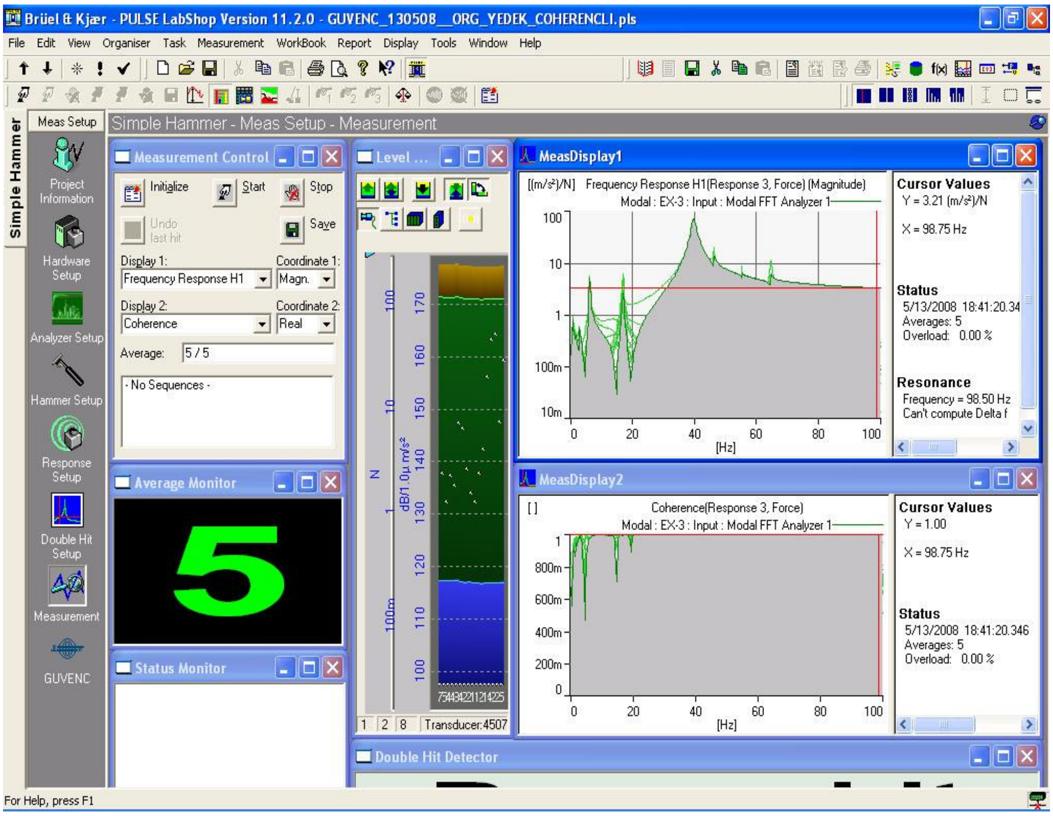


Figure 4.10 The Screen Shot from the User Interface of PULSE 11.0

Table 4.5 The First 6 Natural Frequencies (Excluding Rigid Body Modes) of the Original GARTEUR SM-AG19

Experimental Mode Number	Original Natural Frequencies (Hz)
1	6
2	16.75
3	38
4	39.5
5	39.75
6	46.25

The mode shapes of the original test-bed for the first six flexible modes in the frequency range of 0-50 Hz. are shown in Figure 4.11.

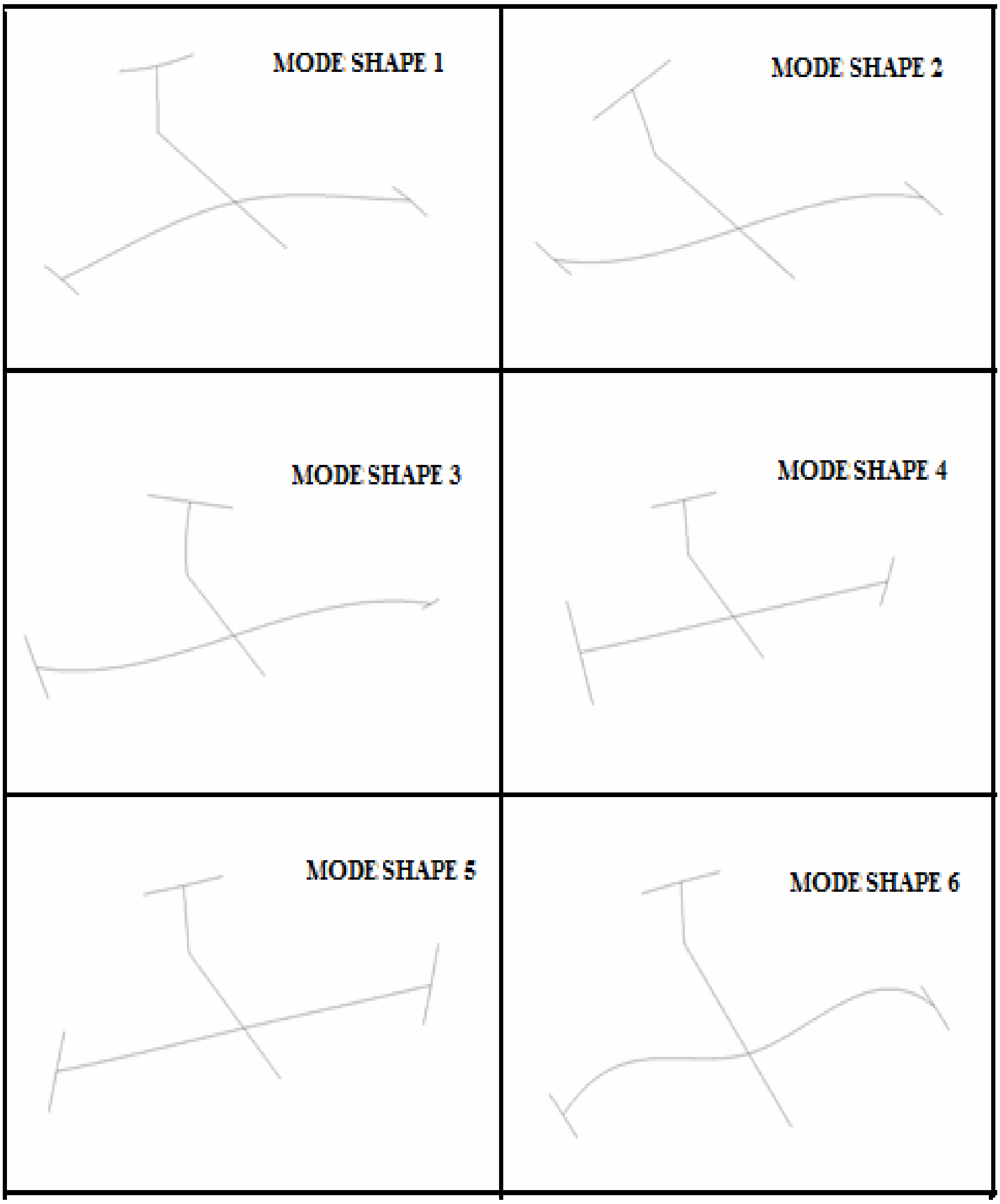


Figure 4.11 The Mode Shapes of the Original Test-bed

For the original test-bed, direct point FRF of node 3 in z-direction is given in Figure 4.12.

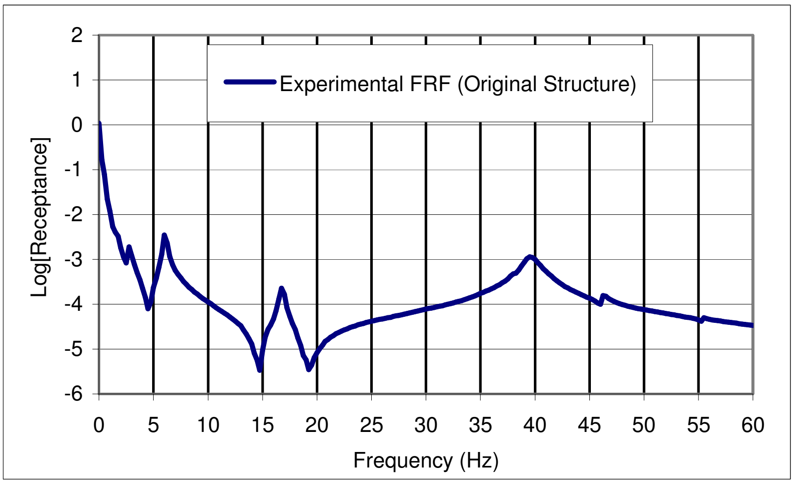


Figure 4.12 Experimental FRF of Node 3 in z-Direction for Original Test-Bed

When the structure is excited at node 3 in z-direction, screen shot of the coherence plot of the measurement at node 3 in z-direction is shown in Figure 4.13.

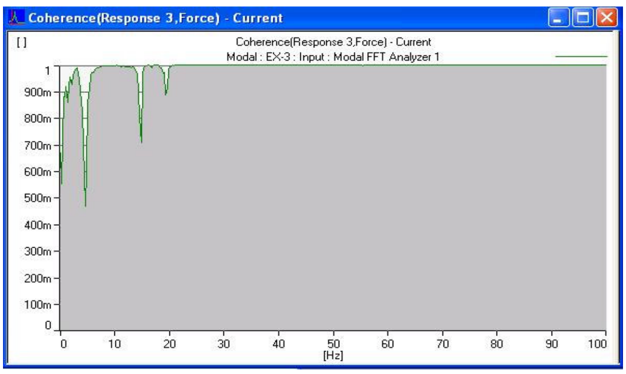


Figure 4.13 Coherence Plot of the Measurement at Node 3 in z-direction

4.2.3 Modal Test of Modified Garteur SM-AG19

GARTEUR SM-AG19 model was modified by attaching beams under the wings. The mass of each beam under the wing is 0.554 kg. The modifying beams under the wings can be seen in Figure 4.14.

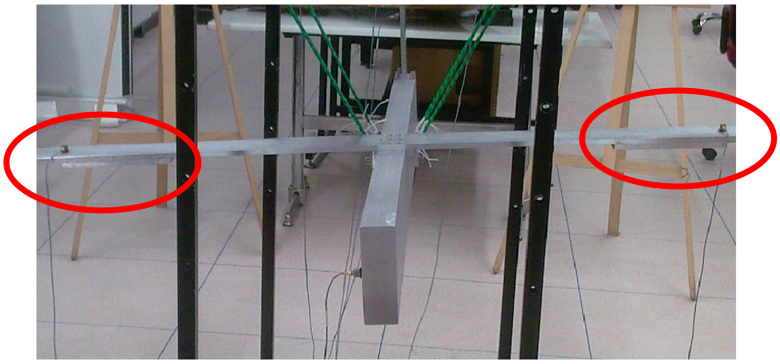


Figure 4.14 View of the Modifying Beams

As seen in Figure 4.15, the modifying beams were attached under the wings by bolted joints in order to provide rigid connections between the modifying beams and wings.

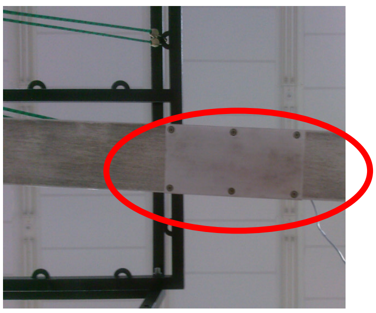


Figure 4.15 Bolted Joints on Modifying Beams

Natural frequencies of the modified GARTEUR SM-AG19 model are given in Table 4.6.

Table 4.6 The First 6 Natural Frequencies of the Modified GARTEUR SM-AG19 Model

Mode Number	**Modified Natural Frequencies (Hz)**
1	6.25
2	17.75
3	38.5
4	44
5	44.25
6	46.75

4.2.4 FE Model of Original and Modified Garteur SM-AG19 Model

In order to construct and analyze the finite element model of the test bed, ANSYS 11.0 was used. The test-bed was modeled using SOLID 186 elements which have 20 nodes per element. The FE model of the original GARTEUR SM-AG19 model (Figure 4.16) consists of 140 SOLID 186 elements and the finite element model has 1380 nodes with 3 translational DOF per node yielding total DOF of 4140. The FE model properties of the original GARTEUR SM-AG19 model are given in Table 4.7. The material properties assigned to finite element model of GARTEUR SM-AG19 model is given in Table 4.8.

Table 4.7 FE Model Properties

Element Type Used In FEM	SOLID 186
Number of Elements	140
Number of Nodes	1380
DOF	4140

Table 4.8 The Material Properties

Material Property	**Value**
Density (ρ)	2770 kg/m3
Poisson's Ratio (ν)	0.33
Modulus of Elasticity (E)	71 GPa

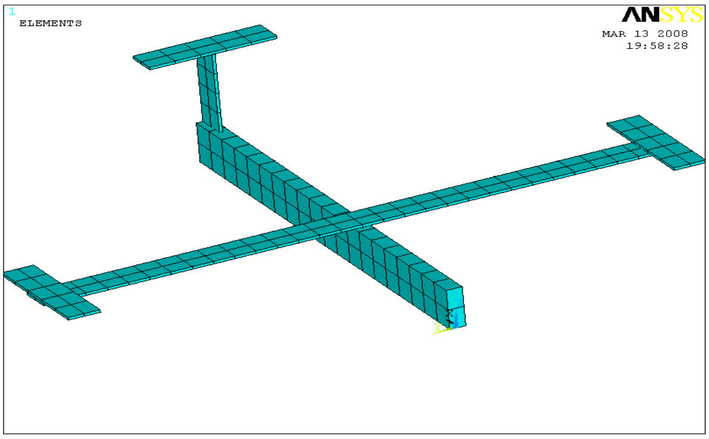


Figure 4.16 FE Model of Original GARTEUR SM-AG19 Model

The modifying beams attached under the wings are modeled by using SOLID 186 elements. The finite element properties of the modifying beam is given in Table 4.9 and the geometrical and material properties of the modifying beam are given in Table 4.10.

Table 4.9 FE Model Properties

Element Type Used In FEM	SOLID 186
Number of of Elements	6
Number of Nodes	70
DOF	210

Table 4.10 The Material Properties

Modulus of Elasticity (E)	71 GPa
Poisson's Ratio (ν)	0.33
Density (ρ)	2770 kg/m^3
Length	200mm
Width	100mm
Thickness	10mm

In the finite element model of the modified GARTEUR SM-AG19 model, same type of elements and material were used. The FE model of the modified GARTEUR SM-AG19 model is given in Figure 4.17. In the FE model, the modifying beams are rigidly connected to the original GARTEUR SM-AG19 model, therefore no additional stiffness was introduced to the interface nodes.

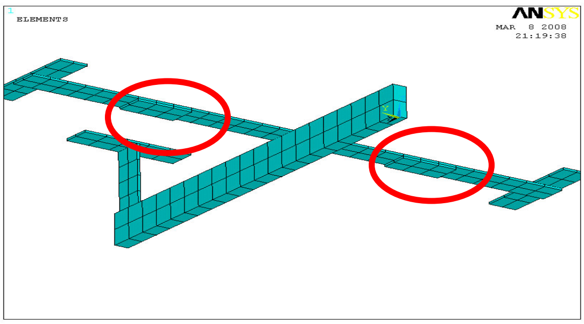


Figure 4.17 FE Model of the Modified GARTEUR SM-AG19 Model

The modal analysis of the original GARTEUR SM-AG19 model is performed by using ANSYS 11.0. Based on the results of modal analysis, first six mode shapes of and the model natural frequencies are given in Figure 4.18 and Table 4.11 respectively.

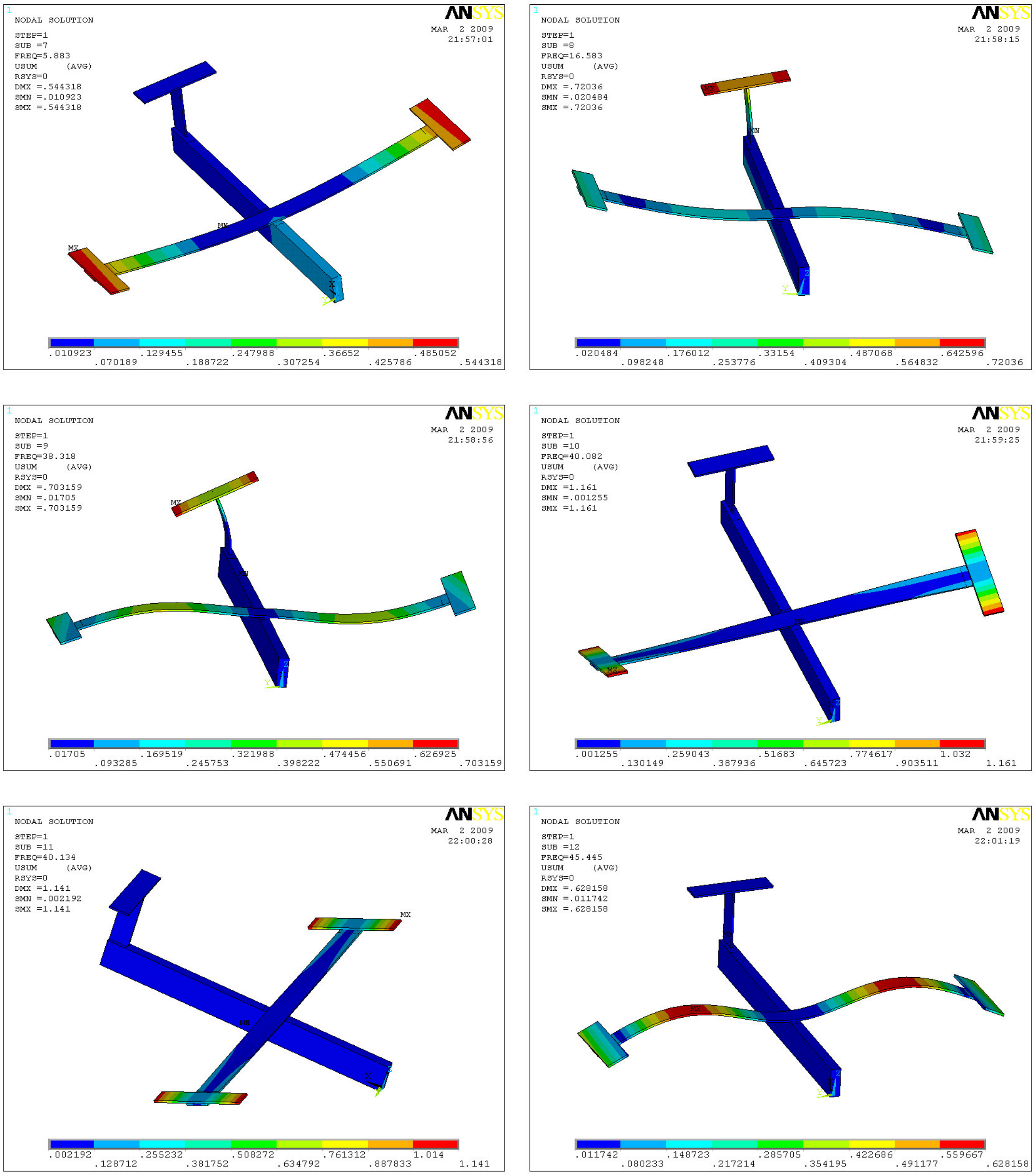


Figure 4.18 Mode Shapes of Original GARTEUR SM-AG19

Table 4.11 Natural Frequencies of Original GARTEUR SM-AG19

Mode Number	Natural Frequencies (Hz) (FEA)
1	5.883
2	16.583
3	38.318
4	40.082
5	40.134
6	45.445

The comparison of the natural frequencies of original GARTEUR SM-AG19 model calculated from FE model and those obtained experimentally are shown in Table 4.12 along with percentage errors in FEA results.

Table 4.12 Comparison of FEA and Experimental Results

Mode Number	Experimental Natural Frequencies (Hz)	FEA Natural Frequencies (Hz)	Error (%)
1	6	5.883	-1.95
2	16.75	16.583	-1.00
3	38	38.318	0.84
4	39.5	40.082	1.47
5	39.75	40.134	0.97
6	46.25	45.445	-1.74

As seen from Table 4.12, natural frequencies calculated from FEA are very close to those obtained experimentally. All the relative percentage errors in the natural frequencies are below 2 %.

4.2.5 Structural Modification on FE Model of Garteur SM-AG19 Model

In this section firstly, direct point FRF of node 217 in z-direction (transverse direction) in the FE model of the GARTEUR SM-AG19 model is compared with the corresponding FRF obtained experimentally. The position of the node 217 in the FE model is shown in Figure 4.19. Then the structural modification is applied to the original model by attaching beams under the wings.

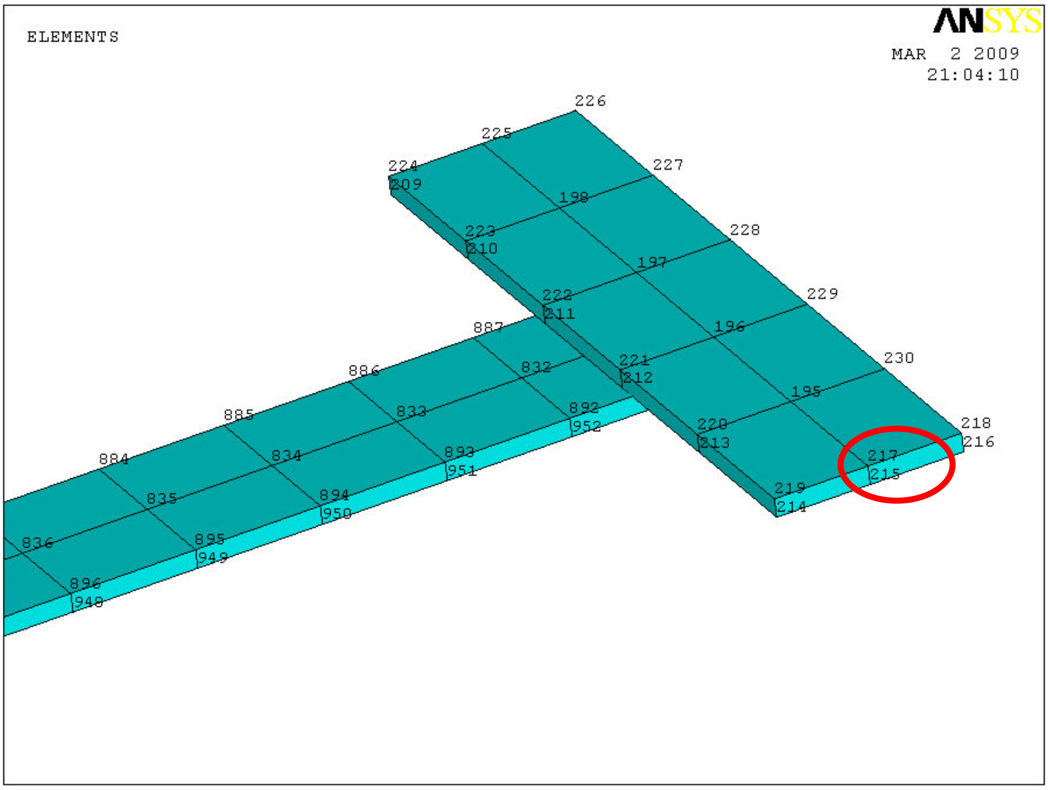


Figure 4.19 View of Node 217 of the FE Model of GARTEUR SM-AG19 Model

In the software developed in this thesis, the FRFs obtained from analysis of the original FE model were used in order to find the FRFs of the modified structure.

These FRFs were compared with those obtained experimentally. The connection nodes of the original model and the corresponding nodes of the modifying beams were set as inputs. Starting frequency was set as 1 rad/s and the ending frequency was selected as 380 rad/s. The structural damping coefficient was taken constant for all modes and selected as 0.032. Direct point FRF of node 217 in Z direction was selected as the required FRF, in order to compare it with the corresponding FRF obtained experimentally (Figure 4.20).

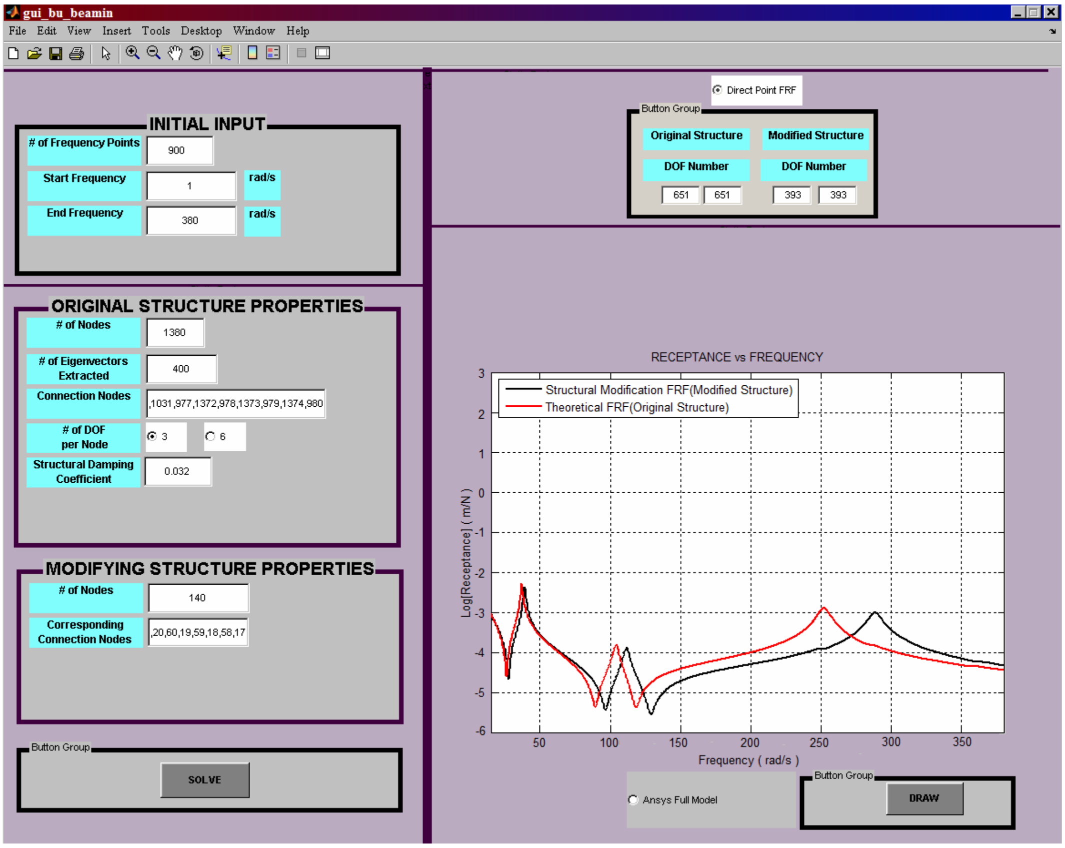


Figure 4.20 Direct Point FRF of Node 217 in z-Direction for Original and Modified GARTEUR SM-AG19 Model

By using the structural modification software, the driving FRF of the modified model was obtained for node 217 in Z direction. The comparison of FRFs is shown in Figure 4.21 to Figure 4.24. In these figures, Experimental FRF (Original Structure) represents FRF obtained experimentally for the original structure; Experimental FRF (Modified Structure) represents FRF obtained experimentally for the modified structure. Theoretical FRF (FEA-Original Structure) is FRF calculated from ANSYS 11.0 for the original structure, Structural Modification FRF (Modified Structure) is the FRF calculated from structural modification program by using 400 modes in the calculation of the FRFs for the original structure.

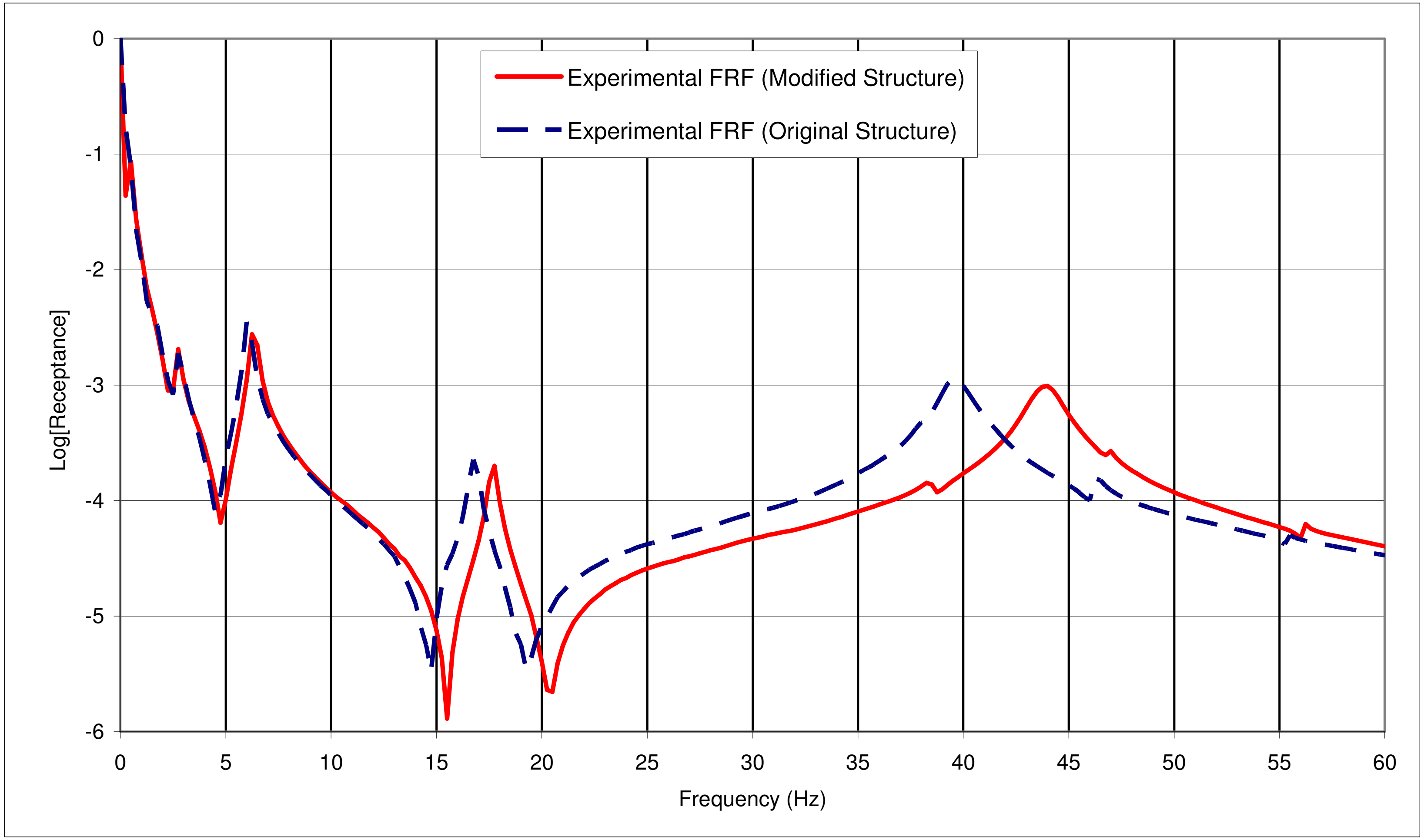


Figure 4.21 Experimental FRF at 3Z3Z for the Original and Modified GARTEUR SM-AG19 Model

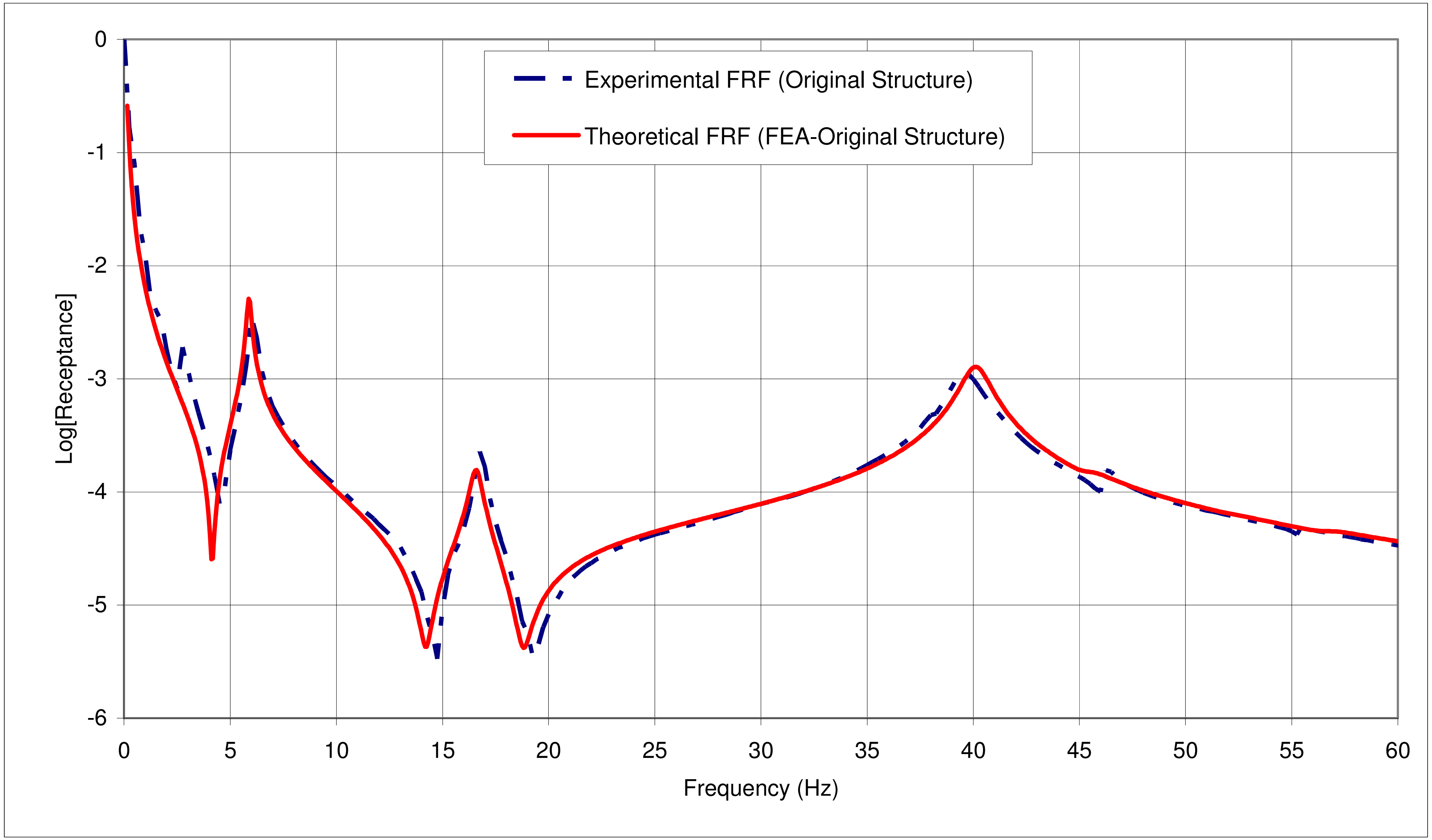


Figure 4.22 Theoretical (FEA) and Experimental FRF at 3Z3Z for Original GARTEUR SM-AG19 Model

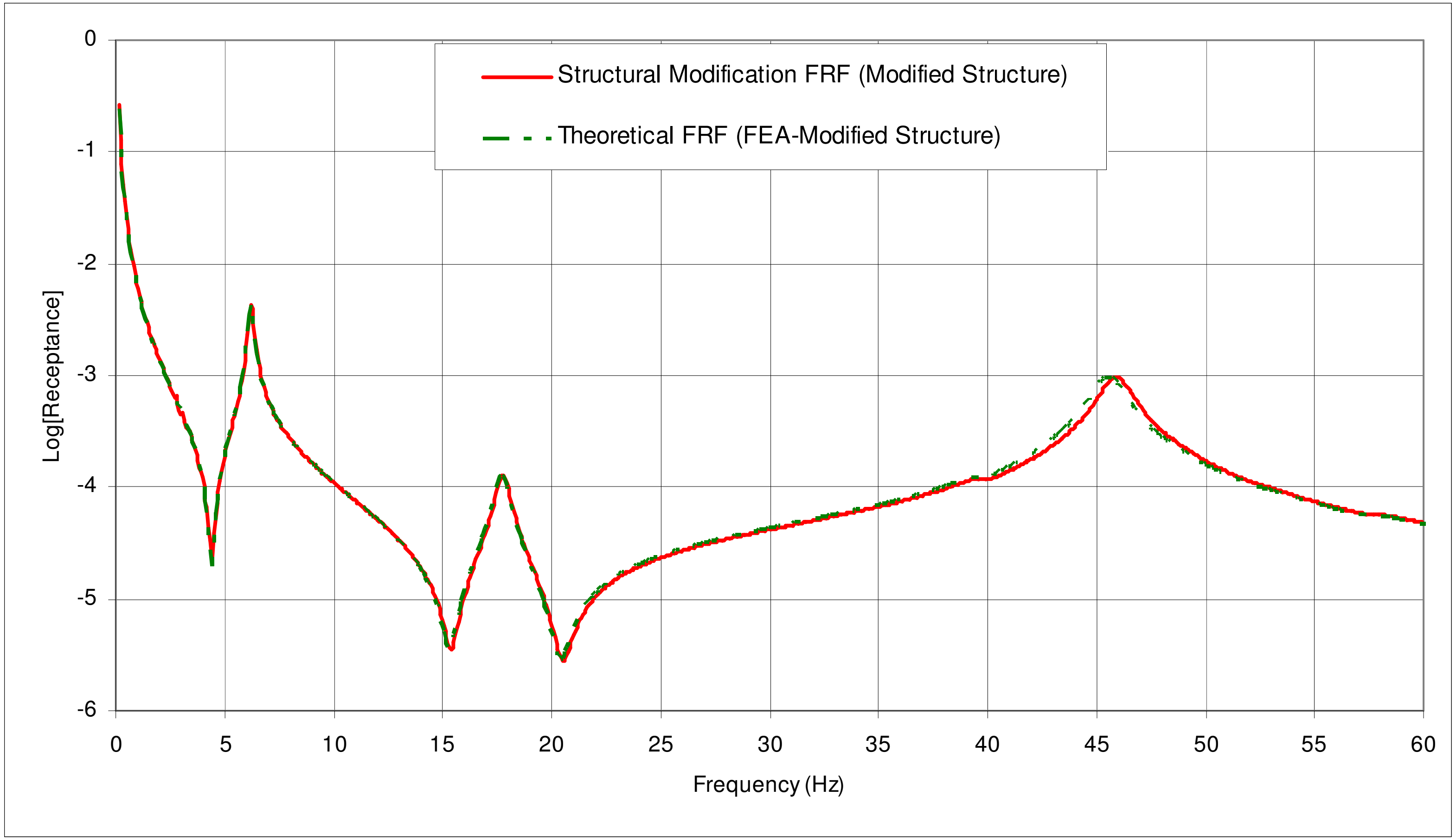


Figure 4.23 Theoretical (Structural Modification Method) and Theoretical (FEA) FRF at 3Z3Z for Modified GARTEUR SM-AG19 Model

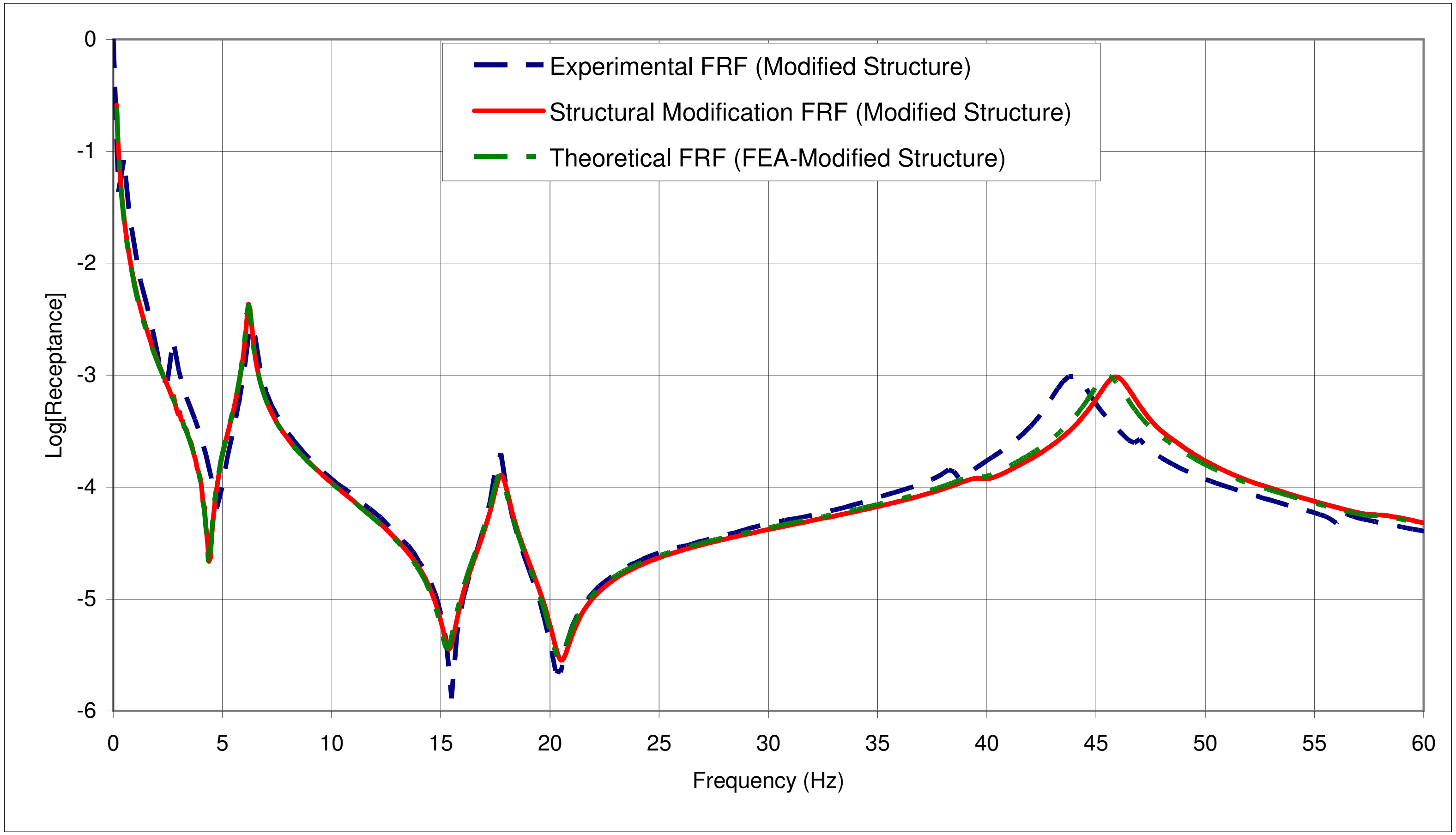


Figure 4.24 Theoretical (Structural Modification Method), Theoretical (FEA) and Experimental FRF at 3Z3Z for Modified GARTEUR SM-AG19

As seen from Figure 4.21, after the modification on the GARTEUR SM-AG19 model the dynamic characteristics of the model has changed and natural frequencies were shifted, therefore attaching beams under the wings has a considerable effect on the driving FRF of node 3 in z-direction. In Figure 4.22, which is given for the original structure, Theoretical FRF (FEA-Original Structure) is in good agreement with the Experimental FRF (Original Structure). The slight mismatch in the magnitudes of the FRFs at the resonances may be due to the constant loss factor used for all modes in the finite element model of the structure to represent the damping in the system. Also there is a mismatch in the first frequency which is the rigid body mode. In the FE analysis the boundary condition was modeled as free-free for the model; however in experiment free-free boundary condition of GARTEUR SM-AG19 model was obtained by flexible suspension bungees, which do not provide an exact free-free boundary conditions. Therefore, this approximation causes the rigid body mode of the test-model to be higher.

In Figure 4.23, Structural Modification FRF (Modified Structure) is compared with Theoretical FRF (FEA-Modified Structure) and a good agreement is obtained. In Figure 4.24, Theoretical FRFs are compared with Experimental FRF (Modified Structure). There is a good match between Structural Modification FRF (Modified Structure) and Experimental FRF (Modified Structure) curves. In the third elastic mode, there is a mismatch at the resonant frequency; this is mainly due to discrepancy between FE model and the test model of the original structure. Moreover the differences between the test and theoretical data are partly due to the effect of modal truncation made in the computation of the FRFs of the original model, on the accuracy of the structural modification method. In Figure 4.25, Theoretical (Structural Modification Method) FRF calculated by using 400 and 1200 modes in the calculation of the FRF of the original structure are compared in order to show the effect of truncation.

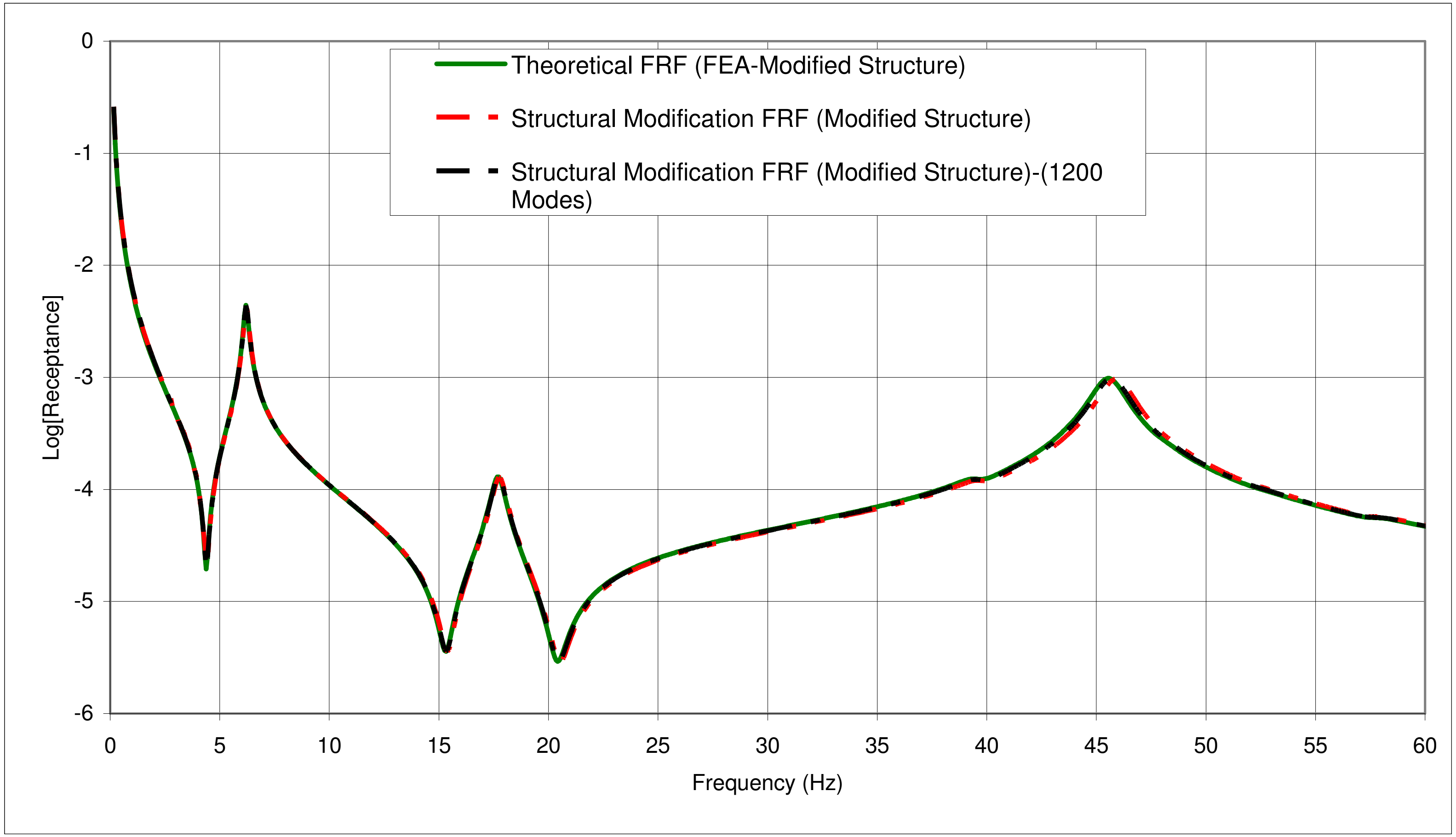


Figure 4.25 Theoretical (Structural Modification Method), Theoretical (FEA) and Theoretical (Structural Modification Method)-(1200 Modes) FRF at 3Z3Z for Modified GARTEUR SM-AG19 Model

4.3 Discussion of Mass Loading and Suspension Effect in Modal Testing

As given in the theory part of the modal test, the use of accelerometers and force transducers introduces changes to the structure due to the addition of masses. The response of a structure is affected from this mass loading affect of accelerometers and force transducers. This mass loading is important when the mass to be cancelled is of the same order as the apparent mass of the modes of the structure under the test. Furthermore, in modal testing, free-free boundary conditions are provided by using soft springs or soft bungees, therefore rigid body modes no longer have zero natural frequencies.

In this section, the effects of mass loading of accelerometers and stiffness effect of elastic bungees on the modal test are investigated. In order to show these effects, the finite element models were constructed by adding the accelerometers as point masses and suspension bungees as springs.

4.3.1 Effect of Mass Loading of Accelerometers in Modal Testing

In this section, accelerometers are modeled as point masses in the FE model and they are lumped at the positions of measurement points. Based on the modal test performed on original GARTEUR SM-AG19 model, the added masses are given in Figure 4.26 and Table 4.13. Each accelerometer has a mass of 0.0048 kg.

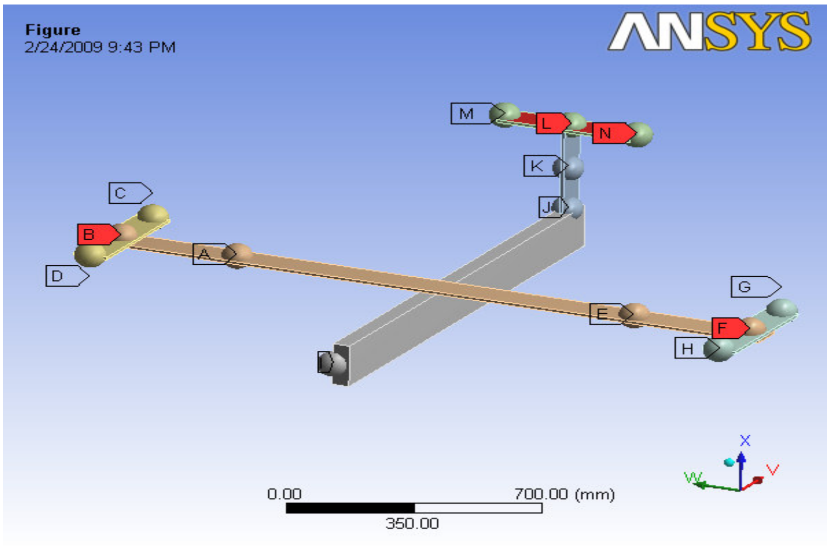


Figure 4.26 View of the Added Masses

Table 4.13 Location and Masses of Accelerometers

Measurement Nodes	Corresponding Node on FE model	Mass Added (kg)
1	G	0.0048
2	F	0.0048
3	H	0.0096
4	E	0.0048
5	A	0.0048
6	D	0.0096
7	B	0.0048
8	C	0.0048
9	I	0.0048
10	K	0.0048
11	M	0.0048
12	N	0.0048
13	J	0.0048
14	L	0.0048

The FE model constructed has the same mesh properties as that of the original GARTEUR SM-AG19 model, only difference being the mass added to the structure which represents mass loading effect of accelerometers. In Table 4.14, the first six natural frequencies are compared for the GARTEUR SM-AG19 model and mass loaded GARTEUR SM-AG19 model.

As seen in Table 4.14, adding accelerometers to the FE model of original GARTEUR SM-AG19 model has a very slight effect on the natural frequencies, especially for the first flexible modes. Therefore, for the test structure studied there is no considerable shift in the natural frequencies due to mass loading of accelerometers in the frequency range of interest.

Table 4.14 Comparison of Natural Frequencies

Mode Number	Natural Frequencies of Original Model (Hz)	Natural Frequencies of Original Model With Accelerometers (Hz)	Change with Respect to Natural Frequency of Original Model (%)
1	5.881	5.859	-0.37
2	16.568	16.495	-0.44
3	38.278	38.126	-0.40
4	40.069	39.354	-1.78
5	40.117	39.404	-1.78
6	45.377	45.312	-0.14

4.3.2 Suspension Effect in Modal Testing

In this section, the suspension bungees used in the modal test of GARTEUR SM-AG19 model were modeled as springs which have the same stiffness values of the suspension bungees. In modal test of GARTEUR SM-AG19 model the rigid body mode was found as 2.75 Hz. In order to find equivalent stiffness value of the bungees the following simple model which is shown in Figure 4.27 can be used.

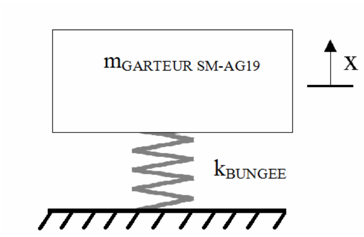


Figure 4.27 Simple Single Degree of Freedom (SDOF) Model

By using this simple model the stiffness of bungee can be found from the following equation.

$$\omega_{bouncing} = \sqrt{\frac{k_{BUNGEE}}{m_{\text{GARTEUR SM-AG19}}}} \tag{4.1}$$

$$\text{m}_{\text{GARTEUR SM-AG19}} = 40.857 \text{ kg} \tag{4.2}$$

$$\omega_{bouncing} = 2.75 \text{ Hz} \tag{4.3}$$

Then stiffness of bungee was found as

$$k_{BUNGEE} = 12198 \text{ N/m} \tag{4.4}$$

Using this stiffness value in Equation (4.4), the FE model of the GARTEUR SM-AG19 model with bungee was constructed. The FE model of the GARTEUR SM-

AG19 model with bungee is given in Figure 4.28. Based on this model, modal analysis was performed on the model. The first six flexible natural frequencies are compared for the GARTEUR SM-AG19 model and GARTEUR SM-AG19 model with bungees in Table 4.15.

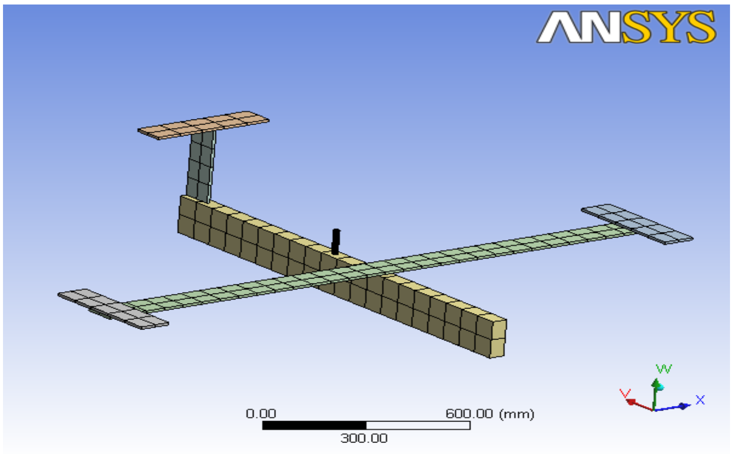


Figure 4.28 FE Model of the GARTEUR SM-AG19 Model with Bungee

Table 4.15 Comparison of Natural Frequencies

Mode Number	Natural Frequency of Original Model (Hz)	Natural Frequency of Original Model With Bungees (Hz)	Change with Respect to Natural Frequency of Original Model (%)
1	5.881	6.001	2.04
2	16.568	16.569	0.01
3	38.278	38.272	-0.02
4	40.069	40.077	0.02
5	40.117	40.125	0.02
6	45.377	45.38	0.01

It is seen from Table 4.15 that, addition of bungees into the finite element analysis, has a very small effect on the natural frequencies of the flexible modes. The effect of bungees is large only in the first mode as expected.

4.3.3 Combined Effect of Suspension and Mass Loading of Accelerometers in Modal Testing

In this section both the bungees and the accelerometers are modeled in the FE model of GARTEUR SM-AG19 model in order to study the combined effect of suspension and mass loading of accelerometers (Figure 4.29).

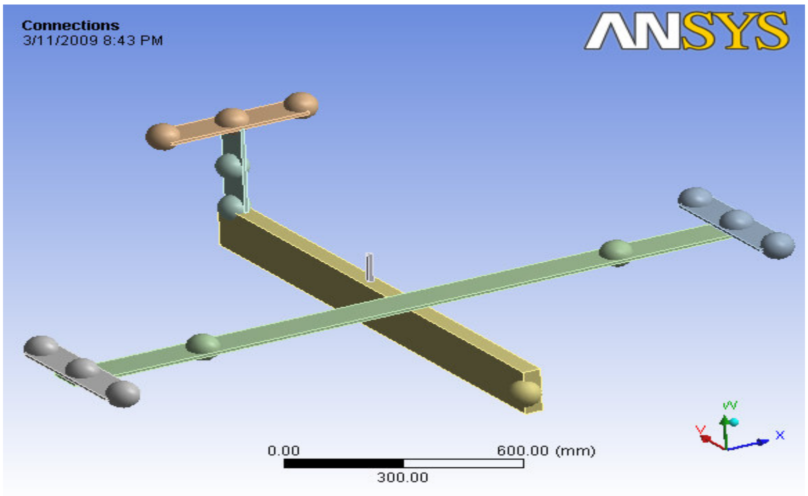


Figure 4.29 GARTEUR SM-AG19 Model with Accelerometers and Bungee

As in previous section, accelerometers were modeled as point masses and they were lumped at the positions of measurement points. Moreover, again the suspension bungees used in the modal test of GARTEUR SM-AG19 model were modeled as spring which has the same stiffness value of the suspension bungees. The same

SDOF model (Figure 4.27) was used in order to obtain the stiffness value of the suspension bungees. By using this simple model the stiffness of bungee can be found from the following equation.

$$\omega_{bouncing} = \sqrt{\frac{k_{BUNGEE}}{(m_{\text{GARTEUR SM-AG19}} + m_{\text{ACCELEROMETERS}})}} \tag{4.5}$$

$$\mathrm{m}_{\text{GARTEUR SM-AG19}} = 40.857 \text{ kg} \tag{4.6}$$

$$m_{\text{ACCELEROMETERS}} = 0.0768 \text{ kg} \tag{4.7}$$

$$\omega_{bouncing} = 2.75 \text{ Hz} \tag{4.8}$$

Then stiffness of bungees was found as

$$k_{BUNGEE} = 12221 \text{ N/m} \tag{4.9}$$

Using this stiffness value, the finite element model of the GARTEUR SM-AG19 model with suspension bungees and accelerometers was constructed. The first six flexible natural frequencies are compared for the GARTEUR SM-AG19 model and GARTEUR SM-AG19 model with bungees and accelerometers in Table 4.16.

As seen in Table 4.16, for the first mode, the effect of suspension bungees can easily be identified. However the changes in the natural frequencies are very small for the first six flexible modes.

Table 4.16 Comparison of Natural Frequencies

Mode Number	Natural Frequency of Original Model (Hz)	Natural Frequency of Original Model With Bungees and Accelerometers (Hz)	Change of Natural Frequency of Original Model With Bungees and Accelerometers with Respect to Natural Frequency of Original Model (%)	Experimental Natural Frequency of Original Model
1	5.881	5.975	1.6	6
2	16.568	16.492	-0.46	16.75
3	38.278	38.127	-0.39	38
4	40.069	39.361	-1.77	39.5
5	40.117	39.41	-1.76	39.75
6	45.377	45.321	-0.12	46.25

CHAPTER 5

IMPORTANCE OF USING ADDITIONAL DOF IN MODELING CONTINUOUS MODIFICATIONS

5.1 Introduction

When a modification is introduced to an original structure, the dynamic characteristics of the original structure change. The correct prediction of this change depends on the modification and the modeling approach used. When modification is in the form addition of rigid mass elements at certain locations then modification can be modeled as lumped elements without increasing the total DOF of the system. However when the modification is in the form of additional distributed mass then the modification can be modeled either as lumped or distributed, and the choice of the modeling approach may give different results for the modified structure. When the modification is modeled as distributed, the inertial effects of the modification are not neglected and furthermore the additional DOFs are introduced to the original structure. However, in the lumped modification approach, the inertial effects of the modifying structure are neglected and this may bring certain errors to the results. Furthermore, when the modification is in the form additional beam as in the case study in section 3.2.4, it is unavoidable to use distributed modification method, as the modification will also change the stiffness of the structure.

Considering the aircraft shown in Figure 5.1, the dynamic properties of the external payloads under the wings affect the dynamics characteristics of the aircraft. Therefore, while analyzing the whole aircraft with its external payloads, modeling the external payloads as lumped modifications leads to considerable errors. In order to demonstrate the importance of the choice of modeling approach for the modifications, two different case studies will be given in this chapter. In the first case study, the FE model of a beam is modified with a smaller beam attached under it and this modification is modeled both as lumped and distributed then the natural frequencies and FRFs of these models are compared. In the second case study, in order to demonstrate a modification under the wing of an aircraft, the FE model of GARTEUR SM-AG19 model is modified by attaching smaller beams under the wings. These beams are modeled first as lumped and then distributed, and the results are compared.

Figure 5.1 The Aircraft Model

5.2 Case Studies

5.2.1 Modification of a Beam Model

In this case study, the FE model of a beam is constructed by using ANSYS Workbench 11.0 (Figure 5.2). The original beam model has connection parts under it which are used to represent the interface structure for the modifying beam attached under the original one. After the FEA for the original beam, the natural frequencies shown in Table 5.1 are obtained. In order to compare the results obtained with different modeling approaches, the modifying part is modeled first as lumped and then distributed.

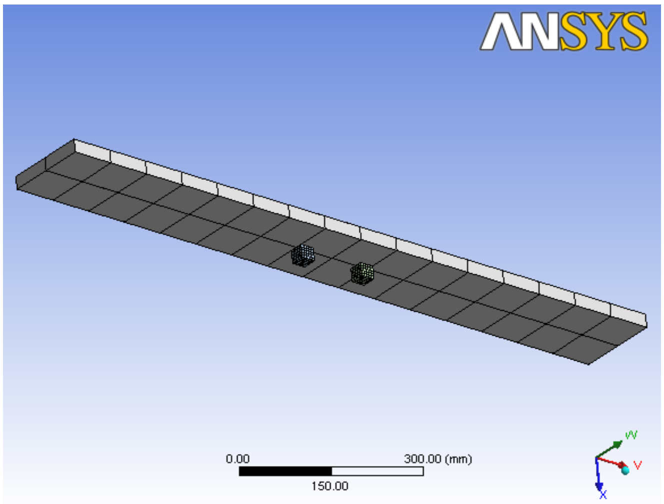


Figure 5.2 The FE Model of the Original Beam

Table 5.1 The Natural Frequencies of the Original Structure

Mode Number	Natural Frequencies of Original Model (Hz)
1	90.303
2	248.6
3	412.34
4	486.17
5	512.18
6	802.62
7	826.26

5.1.1.1 Distributed Model of the Modifying Beam

In this approach, the modifying beam is modeled as distributed therefore the inertial effects of the modifying beam are not neglected. Moreover, the additional DOFs are introduced to the original structure due to modeling approach of the structure. Based on this modeling approach, the FE model of the modified beam shown in Figure 5.3 is constructed. The natural frequencies of the modified beam are given in Table 5.2.

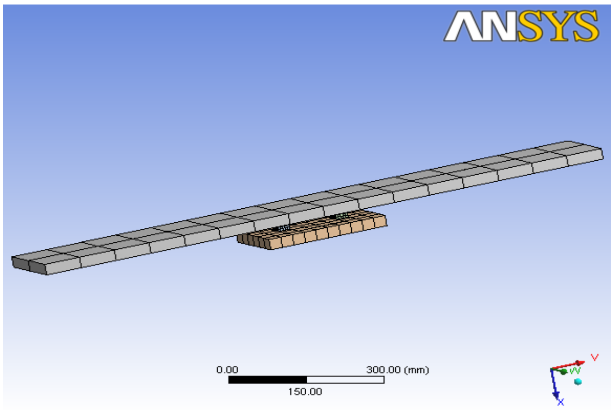


Figure 5.3 The FE Model of the Modified Beam

Table 5.2 The Natural Frequencies of Modified Model - Distributed Modeling

Mode Number	Natural Frequencies of Modified Model – Distributed Modeling (Hz)
1	87.963
2	235.7
3	419.63
4	458.74
5	471.06
6	613.29
7	696.92

5.1.1.2 Lumped Model of the Modifying Beam

In this approach, since the modifying beam is modeled as lumped, the inertial effects of the modifying beam are neglected. Also the approach of lumped model simulates the structural modifications without additional DOFs. In order to model the modifying beam as lumped, the parts shown in Figure 5.4 are used. Each of these parts has the half of the mass of the modifying beam. These parts are modeled as rigid by taking a very high value of modulus of elasticity and lower value for the poisson's ratio. The FE model of the modified beam shown in Figure 5.5 is constructed based on this modeling approach.

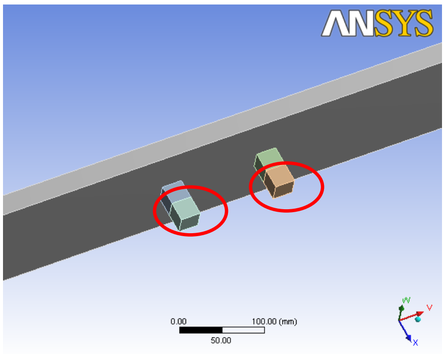


Figure 5.4 The Lumped Model of the Modifying Beam

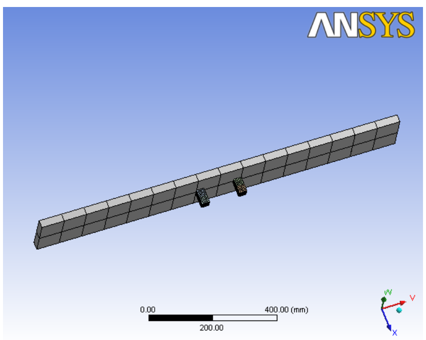


Figure 5.5 The FE Model of the Modified Beam

Performing the FEA, the natural frequencies are shown in Table 5.3 are obtained for the modified beam.

Table 5.3 The Natural Frequencies of Modified Model – Lumped Modeling

Mode Number	Natural Frequencies of Modified Model - Lumped Modeling (Hz)
1	81.468
2	237.89
3	409.07
4	440.78
5	459.44
6	715.03
7	722.19

5.1.1.3 Comparison of the Results

In this part, the natural frequencies and the corresponding mode shapes obtained by using different modification models are compared. Moreover the FRFs of the node 24 in x-direction (Figure 5.6) are obtained by using both models and they are compared.

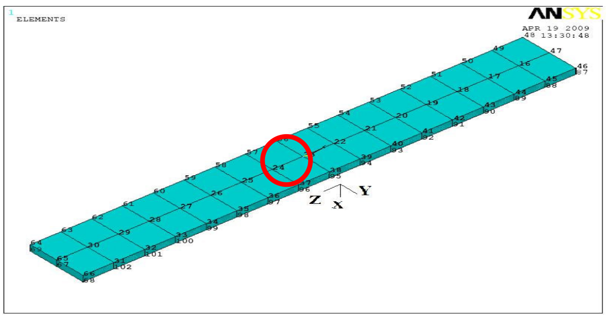


Figure 5.6 The View of Node 24

The mode shapes obtained by using lumped and distributed models are shown in Figure 5.7 and Figure 5.8.

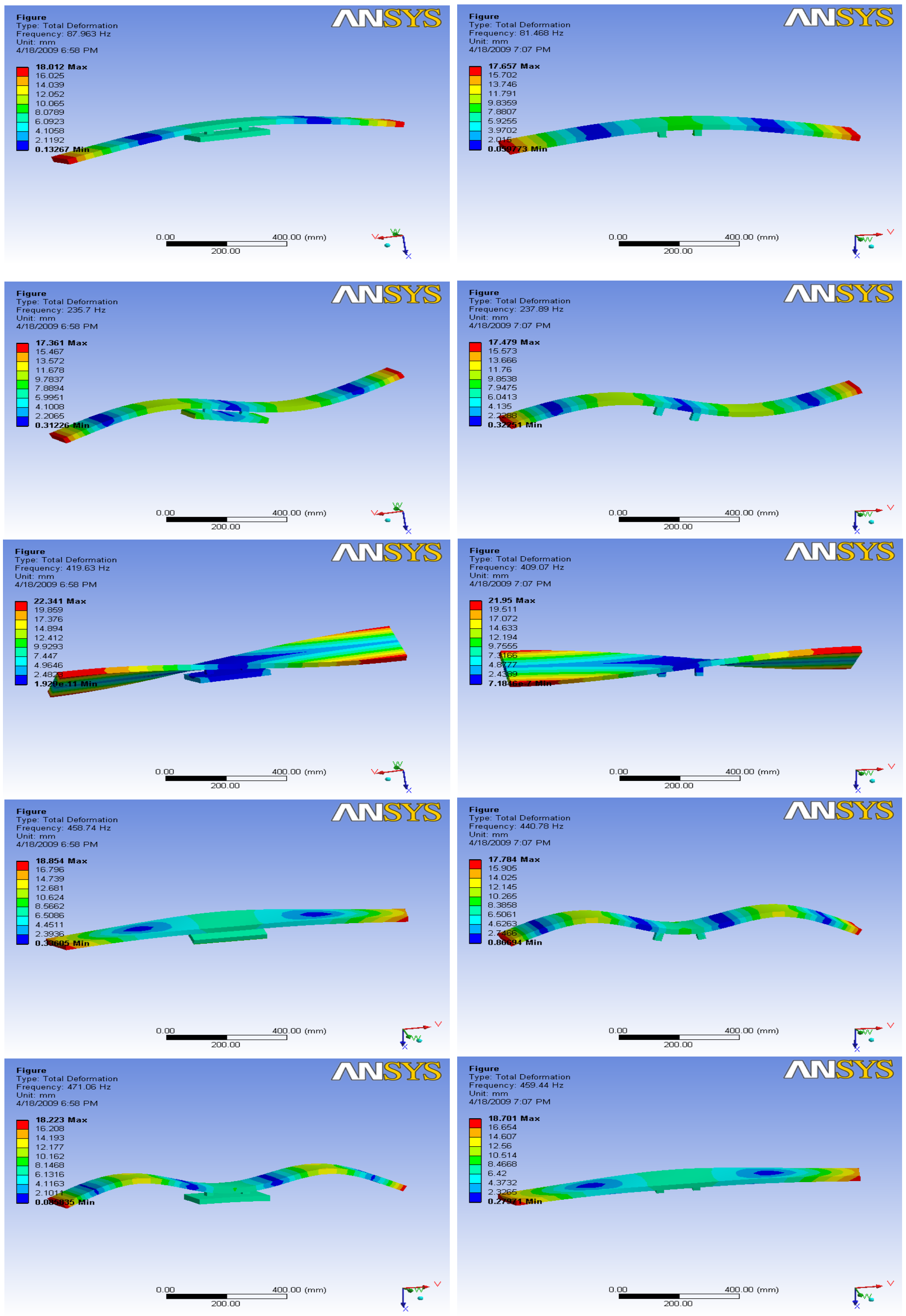


Figure 5.7 Comparison of the Mode Shapes (1st – 5th Mode Shapes)

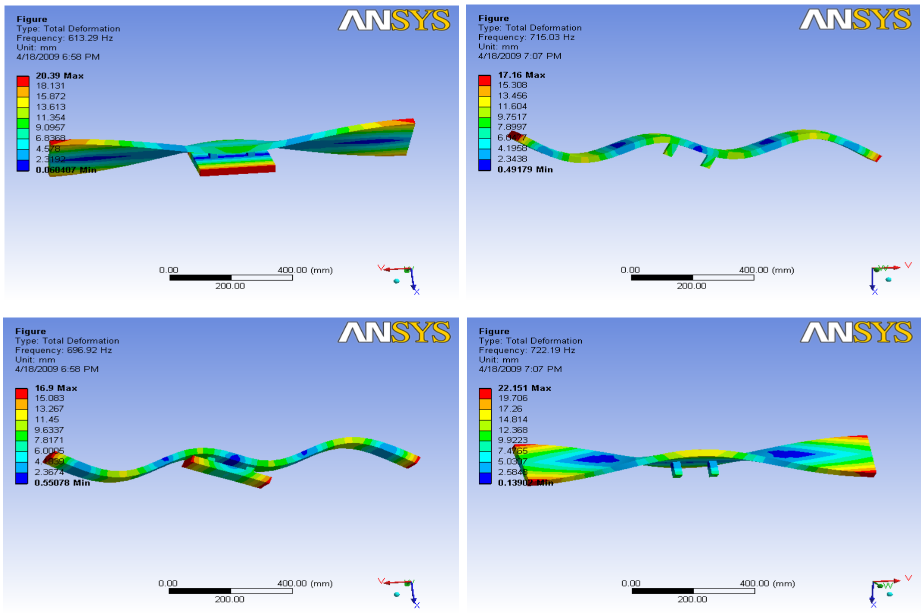


Figure 5.8 Comparison of the Mode Shapes (6th – 7th Mode Shapes)

The comparison of the natural frequencies is given in Table 5.4.

Table 5.4 Comparison of Natural Frequencies Obtained by Using Lumped and Distributed Modification Models

Mode Number	Natural Frequencies of Modified Model - Distributed Modeling (Hz)	Natural Frequencies of Modified Model – Lumped Modeling (Hz)	Change with Respect to Distributed Modeling (%)
1	87.963	81.468	-7.38
2	235.7	237.89	0.93
3	419.63	409.07	-2.52
4	458.74	440.78	-3.92
5	471.06	459.44	-2.47
6	613.29	715.03	16.59
7	696.92	722.19	3.63

When the natural frequencies in Table 5.4 are investigated, it is seen that, for the first seven natural frequencies except the second one, the lumped model introduces considerable errors; especially in the 1^{st} and 6^{th} natural frequencies these errors are larger. Furthermore as seen in Figure 5.7 and Figure 5.8, the 4^{th} and 5^{th} mode shapes and the 6^{th} and 7^{th} mode shapes switches.

In Figure 5.9, the comparison of FRFs obtained by using lumped and distributed modification models for node 24 in x-direction is given. In Figure 5.9, "Modified Structure (FEA)" represents the modified model with distributed modification and "Modified Structure with Lumped Model (FEA)" represents the modified model with lumped modification. As seen in Figure 5.9, there is a considerable discrepancy around the first natural frequency which can also be seen from the values inTable 5.4.

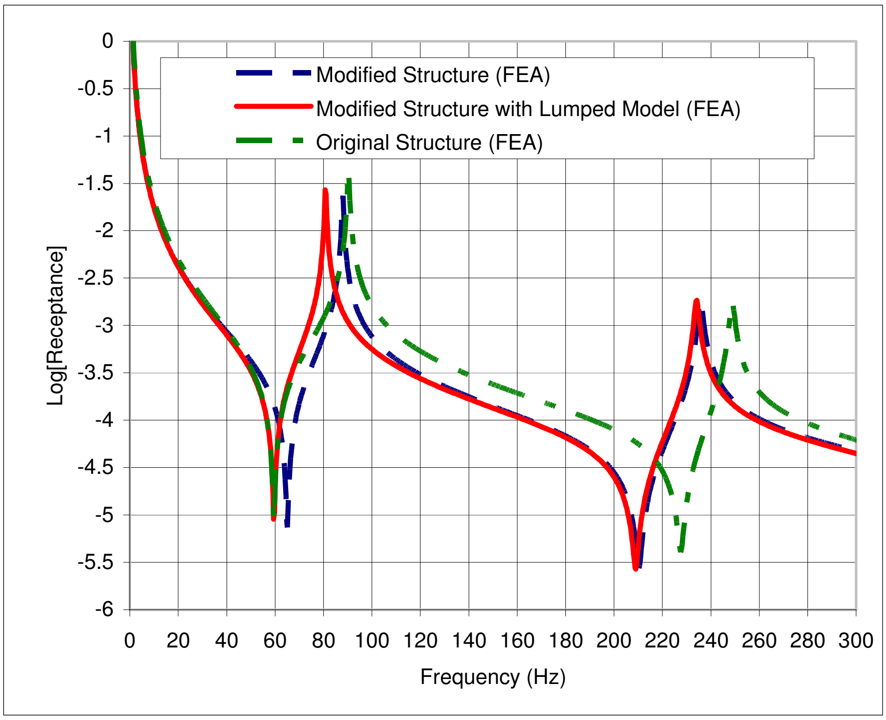


Figure 5.9 Comparison of FRFs for Node 24 in x-direction

5.2.2 Modification of a GARTEUR SM-AG19 Model

In this case study, the FE model of GARTEUR SM-AG19 model is constructed by using Ansys Workbench 11.0 (Figure 5.10). The GARTEUR SM-AG19 model has connections parts under it which is used to represent the interface structure for the modifying beam attached under the wings. These connection parts are modeled as rigid in the FE models. Performing the FEA for the original GARTEUR SM-AG19 model, the natural frequencies shown in Table 5.5 are obtained. As in the previous case study, two different modeling approaches which are distributed and lumped model approach are performed for the modifying structure. The natural frequencies and the FRFs obtained from these two different models are compared.

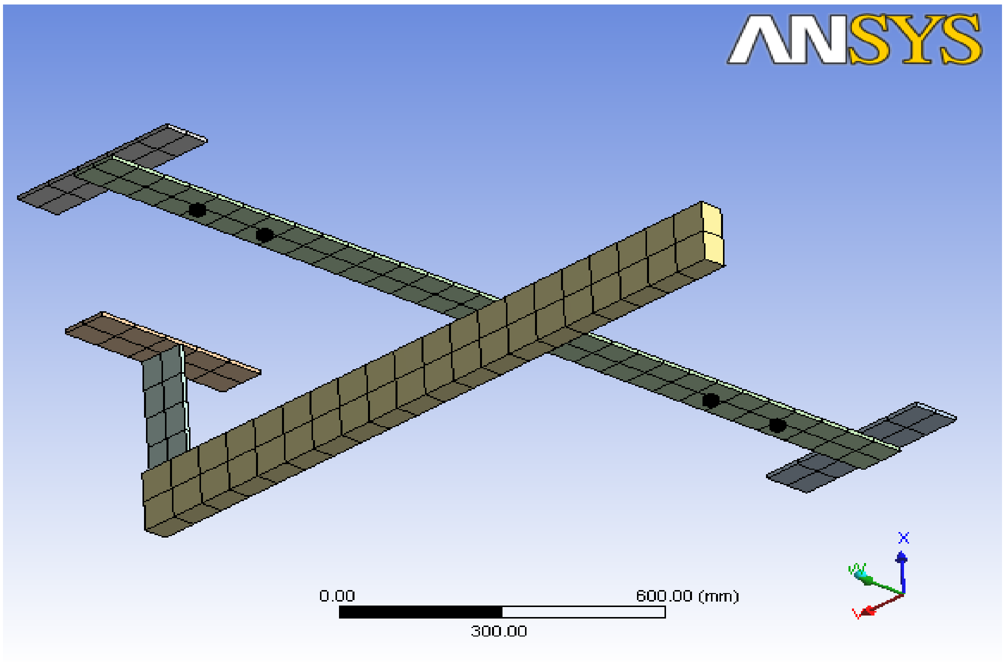


Figure 5.10 FE Model of the Original GARTEUR SM-AG19 Model

Table 5.5 The Natural Frequencies of the Original GARTEUR SM-AG19 Model

Mode Number	Natural Frequencies of Original Model (Hz)
1	5.91
2	16.81
3	38.89
4	42.96
5	43.03
6	46.81

5.2.2.1 Distributed Model of the Modifying Beams

In this approach, the modifying beams attached under the wings are modeled as distributed and additional DOFs are introduced to the original structure. Since the modifying beam is modeled as distributed, the inertial effects of the modifying beam are also included in the FEA of the modified structure. The FE model constructed for the modified GARTEUR SM-AG19 model is shown in Figure 5.11. The natural frequencies obtained from the FEA are also given in Table 5.6.

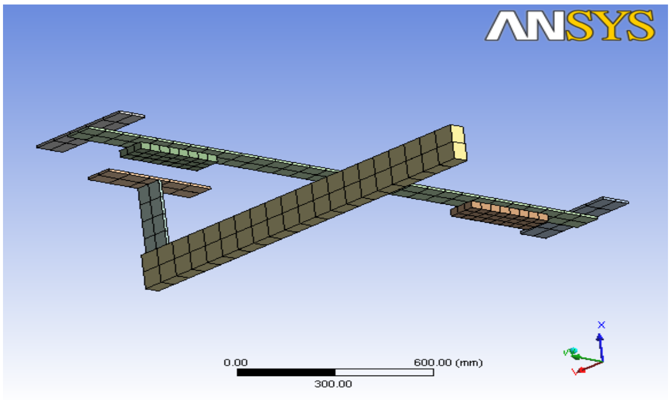


Figure 5.11 The FE Model of the Modified GARTEUR SM-AG19 Model

Table 5.6 The Natural Frequencies of Modified Model – Distributed Modeling

Mode Number	Natural Frequencies of Modified Model – Distributed Modeling (Hz)
1	4.62
2	15.83
3	30.61
4	33.83
5	34.32
6	35.31

5.2.2.2 Lumped Model of the Modifying Beams

In this approach, the modifying beams are modeled as lumped, thus the inertial effects of the modifying structure are not introduced to the original structure. In order to model the modifying beams as lumped, almost rigid parts each of which has the half of the mass of the modifying beam are attached on the original GARTEUR SM-AG19 model. The FE model constructed for the modified beam is shown in Figure 5.12. The natural frequencies obtained from the FEA are also given in Table 5.7.

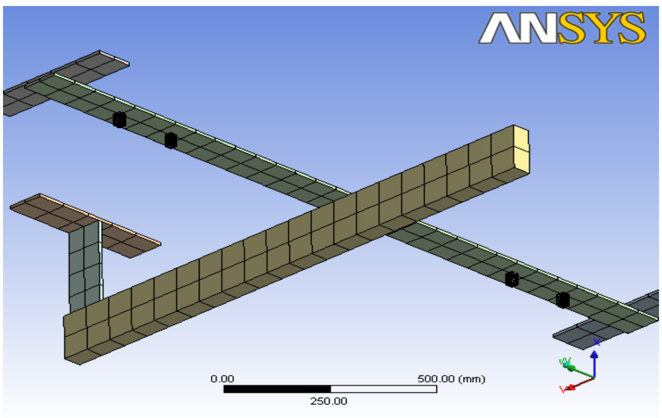


Figure 5.12 The FE Model of the Modified GARTEUR SM-AG19 Model

Table 5.7 The Natural Frequencies of Modified Model – Lumped Modeling

Mode Number	Natural Frequencies of Modified Model – Lumped Modeling (Hz)
1	4.59
2	15.27
3	27.87
4	27.97
5	35.55
6	36.04

5.2.2.3 Comparison of the Results

In this part, the natural frequencies and the FRFs of the node 131 in x-direction (Figure 5.13) obtained by using different modification models are compared. The comparison of the natural frequencies is given in Table 5.8.

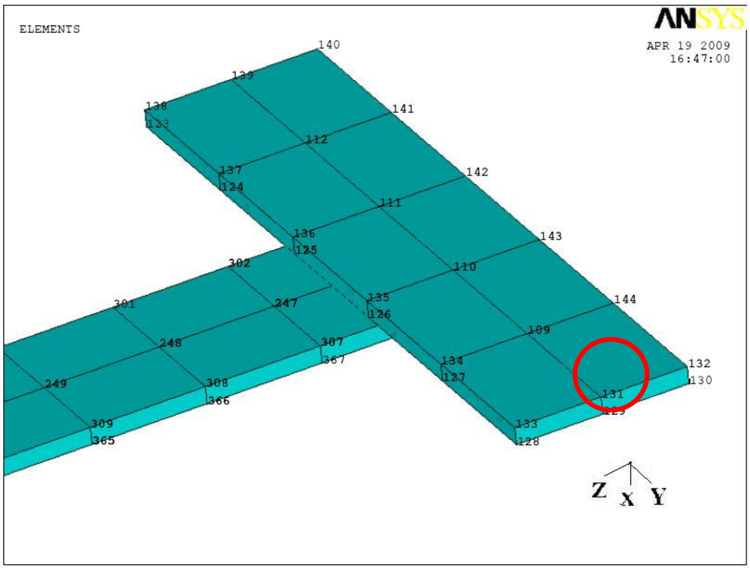


Figure 5.13 The View of Node 131

Table 5.8 Comparison of Natural Frequencies Obtained by Using Lumped and Distributed Modification Models

Mode Number	Natural Frequencies of Modified Model - Distributed Modeling (Hz)	Natural Frequencies of Modified Model - Lumped Modeling (Hz)	Change with Respect to Distributed Modeling (%)
1	4.62	4.59	-0.65
2	15.83	15.27	-3.54
3	30.61	27.87	-8.95
4	33.83	27.97	-17.32
5	34.32	35.55	3.58
6	35.31	36.04	2.07

From Table 5.8, it is observed that, for the first six natural frequencies except the first one, the lumped model introduces considerable errors; especially in the 3^{rd} and 4^{th} natural frequencies the lumped model leads to much larger errors.

In Figure 5.14, the comparison of FRFs obtained by using lumped and distributed modification models for node 131 in x-direction is given.

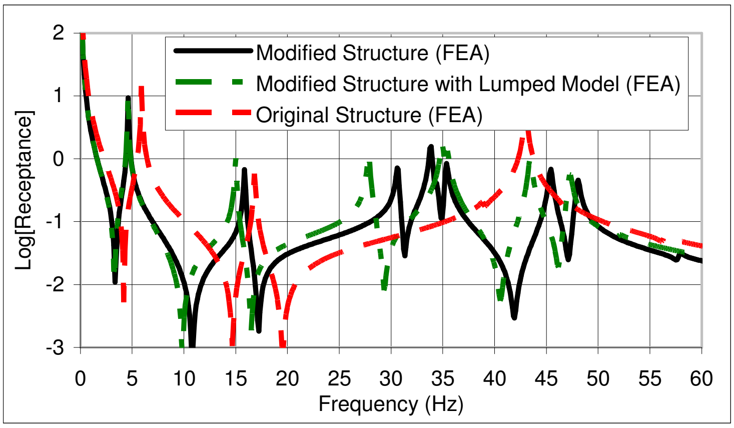


Figure 5.14 Comparison of FRFs of Node 131 in x-direction

As seen in Figure 5.14, between 0-50 Hz there is a considerable discrepancy between the FRF curves around natural frequencies except for the first natural frequency.

5.3 Conclusions

As shown in the case studies, modeling approach of the modifying structure leads to different results for the modified structures. Modeling distributed modifications as lumped, causes considerable errors in the natural frequencies and the FRFs of the modified structure. This emphasizes the choice of modeling approach for distributed modifications.

CHAPTER 6

RESULTS AND CONCLUSIONS

6.1 Summary of the Results and Conclusions

The main objective of this thesis is to obtain the dynamic characteristics of a modified structure from those of the original structure and the system matrices of distributed modifications by using structural modification methods with additional DOFs.

In this thesis, the theories of the structural dynamic modification methods with and without additional DOFs are studied in detail. The structural modification methods without additional degrees of freedom and the modeling approaches for distributed modifications without introducing additional DOFs to the original structure are given. When additional DOFs are introduced to the original structure for a distributed modification, then the modification problem becomes more complex. Therefore, structural modification techniques with additional DOFs should be used in order to solve such problems. The theoretical backgrounds of these methods are also explained in the scope of this thesis. Furthermore, since the modifications in real life are usually distributed, a modeling approach for distributed modifications is given.

In this thesis, Özgüven's structural modification method with additional DOFs [24] is applied to structures with distributed modifications. It is shown in this study that distributed structural modifications in the form of, for instance a stiffener to a plate, can successfully be treated as a structural modification problem with additional DOF.

In order to apply the structural dynamic modification method developed by Özgüven [24] for distributed modification problems, a computer program is developed in MATLAB. The computer program developed in MATLAB has a graphical user interface and is capable of solving structural dynamic modification problems with additional DOFs. The computer program uses natural frequencies and the modal vectors of the original structure, and mass and stiffness matrices of the modifying structure. The natural frequencies and the modal vectors for the original structure are extracted from the modal analysis results of the original structure performed in ANSYS. The mass and stiffness matrices of the modifying structure are obtained by using the ".full" modal analysis result file of ANSYS. The experimental results for the natural frequencies and the mode shape vectors of the original structure can also be used in the computer program developed in MATLAB. The details and user manual of the computer program is given Appendix A.

The computer program developed to solve the structural modification problems with additional DOFs is validated with different theoretical case studies. In the case studies, plate and beam models are used and they are all modeled in ANSYS. In order to show the capability of the computer program developed, different case studies with different boundary conditions, different damping values and different number of DOFs per node are solved and the results are validated by comparing them with FEA results of the modified system. An original plate model that has 6 DOF per node is modified with a smaller plate and the dynamic characteristic of the modified structure are obtained by using the computer program and they are compared with those of the FEA of the modified structure. Since the structural modification method with additional DOFs [24] is an exact method, a very good agreement is observed in the results. The method is also applied to a beam structure which has 3 DOFs per node. The FE model of the beam is modified with a smaller beam attached under it and FRF of the modified structure is obtained by the program. As in the previous case study a very good agreement is observed.

Since FRF matrices of the original structure are used in the structural modification method, the validity of the method strongly depends on accuracy of the FRFs of the original structure. The FRFs of the original structure are obtained by using the modal summation of the modal responses and the accuracy of the FRFs depends on the modal truncation or the number of modes used in this summation. In order to show the sensitivity of the method, FE of a beam is constructed in ANSYS and then it is modified by attaching a smaller beam under it. While performing structural modification method analysis in the computer program, the number of modes used in the calculation of the FRFs of the original structure is varied, and the results for each case are compared with those obtained from FEA of the modified structure. Due to the modal truncation in the calculation of the FRFs of the original structure, discrepancies are observed in the FRFs calculated at higher frequencies when insufficient number of modes is used in calculating the FRFs of the original structure. It is observed that, using higher number of the modes in the computation of the FRFs of the original structure increases the accuracy of the FRFs predicted for modified structure, and the effect of truncation is more pronounced on the FRFs of the modified structure compared with that on the FRFs of the original structure.

Although the structural modification method with additional DOFs [24] is validated with different theoretical case studies, in real life the applications are more complex. Therefore in order to apply and validate the method when applied to a real structure, the GARTEUR SM-AG19 structure designed by a multinational research group (Group for Aeronautical Research and Technology in Europe) was constructed. This GARTEUR SM-AG19 structure is being used extensively in the field of modal testing in literature [50]. The GARTEUR SM-AG19 structure is modified by attaching beams under the wings of it. Modal tests are conducted on the original and modified GARTEUR SM-AG19 structures and the FRFs of both structures are experimentally obtained. The experimental FRFs of the original and modified structures are compared in order to see the effect of the modification under the wings of the structure. It is observed that the natural frequencies of the structure have changed due to the modifications applied to the structure. Since the structural modification method uses the modal data of the original structure, the FE model of the original GARTEUR SM-AG19 structure is also obtained and FEA is performed in ANSYS 11.0. In order to see the accuracy of the FE model of the original GARTEUR SM-AG19 structure, the FRFs obtained theoretically and experimentally are compared. It is observed that there is a good agreement between Theoretical FRF (FEA-Original Structure) and the Experimental FRF (Original Structure). However, due to the constant loss factor used for all modes in the FE model of the structure to represent the damping in the system, there is a slight mismatch in the magnitudes of the FRFs at some resonances. In the FE model, boundary conditions for the model were taken as free-free; however in the experiment free-free boundary condition of GARTEUR SM-AG19 model was obtained by flexible suspension bungees, which do not provide exact free-free boundary conditions. Therefore it is observed that there is a mismatch in the first frequency which corresponds to the rigid body mode. The flexible suspension bungees used to provide free-free boundary conditions lead to non-zero natural frequency for the rigid body mode of the test-model.

After the comparison of the Theoretical FRF (FEA-Original Structure) and the Experimental FRF (Original Structure), the FRFs of the modified structure are obtained by using the computer program developed. It is observed that Structural Modification FRF (Modified Structure) is in good agreement with the Experimental FRF (Modified Structure). However in the third elastic mode, there is a discrepancy at the resonant frequency which is due to the discrepancy between FE model results and the experimental results of the original structure. Furthermore, it is believed that, the differences between two results are partly due to the effect of modal truncation made in the computation of the FRFs of the original model.

Although there are some discrepancies between Structural Modification FRF (Modified Structure) and the Experimental FRF (Modified Structure), from the results of GARTEUR SM-AG19 model it is concluded that the structural modification method with additional DOFs is an effective method for predicting the dynamic response of a modified structure.

Since most of the modifications in real life are distributed and introduce additional DOFs to the original structures, using lumped parameter approaches for even mass modifications may lead to incorrect results. Therefore, as the ultimate goal of this thesis the importance of structural dynamic modifications with additional DOFs is shown. In aircraft structures, for instance modeling an external payload under the aircraft as a lumped mass may cause serious errors. When the modifications are modeled as lumped masses, the inertial effects of the modifications are neglected, therefore dynamic effects of the modification can not be totally introduced to the original structure.

In order to show the effect of these two modeling approaches on the dynamic response predictions of the modified structure, two different case studies are performed. In both case studies modifications are modeled in ANSYS Workbench. In this case studies, the modifications are modeled first as lumped and then as distributed, and the differences between the results are studied. Firstly, the FE model of a beam is modified by attaching a smaller beam under it, and the modification is modeled first as lumped and then as distributed. The natural frequencies, mode shapes and the FRFs obtained from these models are compared. It is observed that, there is a considerable difference in the natural frequencies especially at certain modes. Furthermore, it is seen that the order of the mode shapes are different in the modified models. In the second case study, a more realistic model, GARTEUR SM-AG19 model, is used as the original structure. The FE model of the GARTEUR SM-AG19 model is modified by attaching an external payload to the connection parts under the wings. As in the first case study, the modification is modeled first as lumped and then as distributed. The natural frequencies and FRFs are compared for both of modified models. It is observed that there is a considerable discrepancy between the natural frequencies of the modified models. Also there are differences between the FRF curves of the modified models. It is concluded that lumped modifications introduce considerable errors to the predictions for modified models. Therefore, in order to obtain more accurate predictions for the modified structures, the modifications should be modeled as distributed and structural dynamic modifications with additional DOFs should be used in order to obtain the dynamic response of modified structures.

As a summary, it is concluded in this study that an effective structural dynamic modification method with additional DOFs can successfully be used for structural dynamic modification problems with distributed modifications. A computer program that is compatible with ANSYS and capable of applying structural dynamic modification with additional DOFs for large systems is developed in MATLAB. The predicted results obtained by using the program developed are validated with different theoretical case studies. A real case application for structural modification is demonstrated by using GARTEUR SM-AG19 model. As a last point, the importance of distributed modifications and the structural dynamic modification methods with additional DOFs are shown.

6.2 Recommendations for Future Work

The computer program developed in this thesis has not a stand-alone executable file, therefore in order to have a stand alone executable file the program can be written by using different visual programming codes. Also the graphical user interface of the program can also be improved by using different visual programming codes.

In order to predict the FRFs of the modified GARTEUR SM-AG19 model, the computer program uses the FEA results of the original structure and system matrices of the modifying structure. However as an alternative approach, the modal test results of the original GARTEUR SM-AG19 model may be used. Thus, whenever the original structure is available, rather than the response predicted from FE model of the original structure, more accurate experimental results can be used. In this case modal expansion techniques should be used in order to have consistent DOFs with the FE of the modifying structure.

Since most of the assembly interfaces include joints in the real life applications and these joints have non-linearity, non-linearity in the joints can be studied and this can be included in the structural dynamic modification analysis. That is, the structural dynamic modification method with additional DOFs may be extended to modifications involving non-linearity to have a better prediction for the modified structures.

REFERENCES

[1] Imamovic N., Validation of Large Structural Dynamic Models Using Modal Test Data, PhD Thesis, Imperial College of Science, Technology & Medicine, London, 1998.

[2] O'Callahan J., Avitabile P., A Structural Modification Procedure Using Complex Modes, Proceedings of the 1st International Modal Analysis Conference, Orlando, Florida, 1982.

[3] Wallack P., Skoog P., Richardson M., Simultaneous Structural Dynamic Modification (S^2DM), Proceedings of the 6th International Modal Analysis Conference, Kissimmee, Florida, 1988.

[4] O'Callahan J. C., Chou C. M., Structural Dynamic Modification Using Beam Mass and Stiffness Matrices, Proceedings of the 3rd International Modal Analysis Conference, Orlando, Florida, 1985.

[5] Elliott K. B., Mitchell L. D., Structural Modification Utilizing Beam Elements, Proceedings of the 5th International Modal Analysis Conference, London, England, 1987.

[6] Tayeb M. M., Williams E.J., Rib Stiffeners for Use in Structural Dynamic Modifications, Proceedings of the 6th International Modal Analysis Conference, Kissimmee, Florida, 1988.

[7] Hutton S. G., Vibration Reanalysis of Modified Structures Using Finite Elements, Proceedings of the 7th International Modal Analysis Conference, Las Vegas, Nevada, 1989.

[8] D'Ambrogio W., Consistent Modeling of Continuous Structural Dynamic Modifications, Proceedings of the 9th International Modal Analysis Conference, Florence, Italy, 1991.

[9] Wang B. P., Clark G., Chu F. H., Structural Dynamic Modification Using Modal Analysis Data, Proceedings of the 3rd International Modal Analysis Conference, Orlando, Florida, 1985.

[10] Jones R., Iberle K., Structural Modification: A Comparison of Techniques, Proceedings of the 4th International Modal Analysis Conference, Los Angeles, California, 1986.

[11] Imregun M., Robb D. A., Ewins D.J., Structural Dynamic Modification and Coupling Dynamic Analysis Using Measured FRF Data, Proceedings of the 5th International Modal Analysis Conference, London, England, 1987.

[12] Salvini P., Sestieri A., Predicting the New Frequency Response Function of a Structure When Adding Constraints, Proceedings of the 9th International Modal Analysis Conference, Florence, Italy, 1991.

[13] Kyprianou A., Mottershead J. E., Ouyang H., Structural Modification. Part 2: Assignment of Natural Frequencies and Antiresonances by an Added Beam, Journal of Sound and Vibration, 284(1–2), pp.267–281, 2005.

[14] Kyprianou A., Mottershead J. E., Ouyang H., Assignment of Natural Frequencies by an Added Mass and One or More Springs, Mechanical Systems and Signal Processing, 18(2), pp.263–289 , 2004.

[15] Li T., He J., Local Structural Modification Using Mass and Stiffness Changes, Engineering Structures, 21(11), pp.1028-1037, 1999.

[16] Park Y. H., Park Y. S., Structural Modification Based on Measured Frequency Response Functions: An Exact Eigenproperties Reallocation, Journal of Sound and Vibration, 237(3), pp.411–426, 2000.

[17] Park Y. H., Park Y. S., Structure Optimization to Enhance Its Natural Frequencies Based on Measured Frequency Response Functions, Journal of Sound and Vibration, 229(5), pp.1235–1255, 2000.

[18] Mottershead J. E., Mares C., Friswell M. I., An Inverse Method for The Assignment of Vibration Nodes, Mechanical Systems and Signal Processing, 15(1), pp.87–100, 2001.

[19] Crowley J. H., Rocklin G. T., Klosterman A. L., Vold H., Direct Structural Modification Using Frequency Response Functions, Proceedings of the 2nd International Modal Analysis Conference Orlando, Florida, 1984.

[20] Sherman J., Morrison W. J., Adjustment of an Inverse Matrix Corresponding to Changes in the Elements of a Given Column or Given Row of the Original Matrix, Ann. Math. Statist., 20, pp.621, 1949.

[21] Woodbury M., Inverting Modified Matrices, Memorandum Report 42, Statistical Research Group, Princeton University, Princeton, NJ, 1950.

[22] Özgüven H. N., Determination of Receptances of Locally Damped Structures, Proceedings of the Second International Conference on Recent Advances in Structural Dynamics, v.2, pp.887-892, Southampton, England, April 9-13, 1984.

[23] Özgüven H. N., A New Method for Harmonic Response of Non-proportionally Damped Structures Using Undamped Modal Data, Journal of Sound and Vibration, v.117, n.2, pp.313-328, 1987.

[24] Özgüven H. N., Structural Modifications Using Frequency Response Functions, Mechanical Systems and Signal Processing, v.4, n.1, pp.53-63, 1990.

[25] Şanlıtürk K. Y., An Efficient Method for Linear and Nonlinear Structural Modifications, Proceedings of ESDA 2002: 6th Biennial Conference on Engineering Systems Design and Analysis, ESDA2002/APM-028, Istanbul, Turkey, 2002.

[26] Bae H., Grandhi, R. V., Canfield, R. A., Successive Matrix Inversion Method for Reanalysis of Engineering Structural Systems, AIAA Journal, 42, 8, pp.1529, 2004.

[27] Köksal S., Cömert M. D., Özgüven H. N., Reanalysis of Dynamic Structures Using Successive Matrix Inversion Method, Proceedings of the 24th International Modal Analysis Conference, St. Louis, Missouri, 2006.

[28] Köksal S., Development and Comparative Evaluation of a New Structural Modification Method In Application to Aircraft Structures, MS Thesis, Middle East Technical University, Ankara, 2006.

[29] Mottershead J. E., Tehrani M. G., Stancioiu D., James S., Shahverdi H., Structural Modification of a Helicopter Tailcone, Journal of Sound and Vibration, 298 (1-2), p. 366–384, 2006.

[30] Jingshuo Z., Jin Q., Zongfang T. Yaodong C., Yueming S., Naiyan L., Structural Dynamic Modification Using Transfer Functions Part I Theories, Proceedings of the 5th International Modal Analysis Conference, London, England, 1987.

[31] Jingshuo Z., Jin Q., Zongfang T. Yaodong C., Yueming S., Naiyan L., Structural Dynamic Modification Using Transfer Functions Part II Applications, Proceedings of the 5th International Modal Analysis Conference, London, England, 1987.

[32] D'Ambrogio W., Sestieri, A. Using Distributed Modifications to Change the Dynamic Behavior of Structures, Proceedings of the 17th International Modal Analysis Conference, Kissimmee, Florida, 1999.

[33] D'Ambrogio W., Sestieri A., Predicting The Effect of Distributed Structural Modifications by Alternative Techniques, Proceedings of the 19th International Modal Analysis Conference, Kissimmee, Florida, 2001.

[34] D'Ambrogio W., Sestieri A., Coupling Theoretical Data and Translational FRFs to Perform Distributed Structural Modification, Mechanical Systems and Signal Processing, 15(1), pp.157–172, 2001.

[35] Hang H., Shankar K., Lai J., Distributed Structural Dynamic Modifications Without Rotational Degrees of Freedom, Proceedings of the 25th International Modal Analysis Conference, Orlando, Florida, 2007.

[36] Canbaloglu G., Özgüven H. N., Structural Modifications with Additional DOF-Applications to Real Structures, Proceedings of the 27th International Modal Analysis Conference, Orlando, Florida, 2009.

[37] Smiley R. G., Rotational Degrees of Freedom in Structural Modification, Proceedings of the 2nd International Modal Analysis Conference, Orlando, Florida, 1984.

[38] O'Callahan J. C., Lieu I. W., Chou C. M., Determination of Rotational Degrees of Freedom for Moment Transfer in Structural Modifications, Proceedings of the 3rd International Modal Analysis Conference, Orlando, Florida, 1985.

[39] Avitabile P., O'Callahan J. C., Chou C. M., Kalkunte V., Expansion of Rotational Degrees of Freedom for Structural Dynamic Modifications, Proceedings of the 5th International Modal Analysis Conference, London, England, 1987.

[40] O'Callahan J. C., Avitabile P., Lieu I. W., Madden R., An Efficient Method Determining Rotational Degrees of Freedom from Analytical and Experimental Modal Data, Proceedings of the 4th International Modal Analysis Conference, Los Angeles, California, 1986.

[41] Mottershead J. E., Kyprianou A., Ouyang H., Structural Modification. Part 1: Rotational Receptances, Journal of Sound and Vibration, 284(1-2), pp.249–265, 2005.

[42] Braun S., Ram Y. M., On Structural Modification in Truncated System, Proceedings of the 5th International Modal Analysis Conference, London, England, 1987.

[43] Sohaney R. C., Bonnecase D., Residual Mobilities and Structural Dynamic Modifications, Proceedings of the 7th International Modal Analysis Conference, Las Vegas, Nevada, 1989.

[44] Avitabile P., O'Callahan J., Pechinsky F., Understanding Structural Dynamic Modification Truncation, Proceedings of the 8th International Modal Analysis Conference, Kissimmee, Florida, 1990.

[45] Bucher I., Braun S., Structural Modifications: How to Circumvent the Effect of Modal Truncation, Proceedings of the 11th International Modal Analysis Conference, Kissimmee, Florida, 1993.

[46] Ewins D.J. Modal Testing: Theory Practice and Appliacation, Research Studies Press Ltd., Baldock, Hertfordshire, England, 2000.

[47] Ewins D.J. Basics and state-of-the-art of modal testing, Sadhana-Academy Proceedings in Engineering Sciences, Vol. 25, Part 3, pp. 207-220, 2000.

[48] Ashory M. R., High Quality Modal Testing Methods, PhD Thesis, Imperial College of Science, Technology & Medicine, London, 1999.

[49] MATLAB R2007b Help Manual.

[50] Link M., Friswell M., Working Group 1: Generation of Validated Structural Dynamic Models- Results of a Benchmark Study Utilizing the Garteur SM-AG19 Test-Bed, Mechanical Systems and Signal Processing, 17(1), pp. 9-20, 2003.

APPENDIX A

USER MANUAL

In this appendix, the user manual for the program developed in order to apply the Structural Modification Method with Additional DOFs is given.

A.1 Computer Program Developed for Structural Modification Method with Additional DOFs

In this thesis, a computer program is developed in order to apply Özgüven's structural modification method with additional DOFs to the structural modification problems. The program is developed by using the MATLAB graphical user interface. The general view of the graphical user interface can be seen in Figure A.1.

Figure A.1 The General View of the Graphical User Interface

A.1.1 Definition of the Files Used in the MATLAB Program

The computer program developed in MATLAB uses some files that are extracted from the modal analysis performed in ANSYS 11.0. Furthermore, in order to extract these files from ANSYS 11.0, some macro files and ".exe" files are used. The definitions of these files are given below.

- ***FREQ_ORG.txt*****:** The text file that contains the natural frequencies of the original structure.

- ***MODAL_ORG.txt*****:** The text file that contains the mode shape vectors of the original structure.

- ***MASS_MODIF.matrix*****:** The file that contains the mass matrix information of the modifying structure.

- ***STIFFNESS_MODIF.matrix*****:** The file that contains the stiffness matrix information of the modifying structure.

- ***FREQ.txt*****:** The text file that contains the natural frequencies of the modified structure.

- ***MODAL.txt*****:** The text file that contains the mode shape vector of the modified structure

- ***ModalDataExport_Org.txt:*** Macro file that should be read by the ANSYS 11.0 after the modal analysis of the original structure.

- ***ModalDataExport.txt:*** Macro file that should be read by the ANSYS 11.0 after the modal analysis of the modified structure.

- ***userprog.exe:*** In order to extract the ***"STIFFNESS_MODIF.matrix"*** and ***"MASS_MODIF.matrix"***, this file should be run with "file.full" file which is obtained after the modal analysis performed in ANSYS 11.0 for the modifying structure.

A.1.2 The Use of MATLAB Program

Before running the computer program developed in MATLAB, the following steps should be performed.

- In order to extract the ***"FREQ_ORG.txt"*** and ***"MODAL_ORG.txt"*** files from the ANSYS 11.0, ***"ModalDataExport_Org.txt"*** macro file should be read from ANSYS after the modal analysis of the original structure.

- In order to extract the ***"FREQ.txt"*** and ***"MODAL.txt"*** files from the ANSYS 11.0, ***"ModalDataExport.txt"*** macro file should be read from ANSYS after the modal analysis of the modified structure.

- By running the ***"userprog.exe"*** file, ***"STIFFNESS_MODIF.matrix"*** and ***"MASS_MODIF.matrix"*** *files* should be extracted from the *"file.full"* file which is obtained after the modal analysis performed in ANSYS 11.0 for the modifying structure.

Since all these extracted files are read by the program written in MATLAB, these files should be in the same folder with the source codes of MATLAB program. After performing the steps given above, the computer program developed in MATLAB can be run. In order to show the application of the program, the free-free plate given in section 3.2.1 is used and step-by-step, the application of the program is shown below:

Set the number of frequency points that will be used in the analysis (Figure A.2).

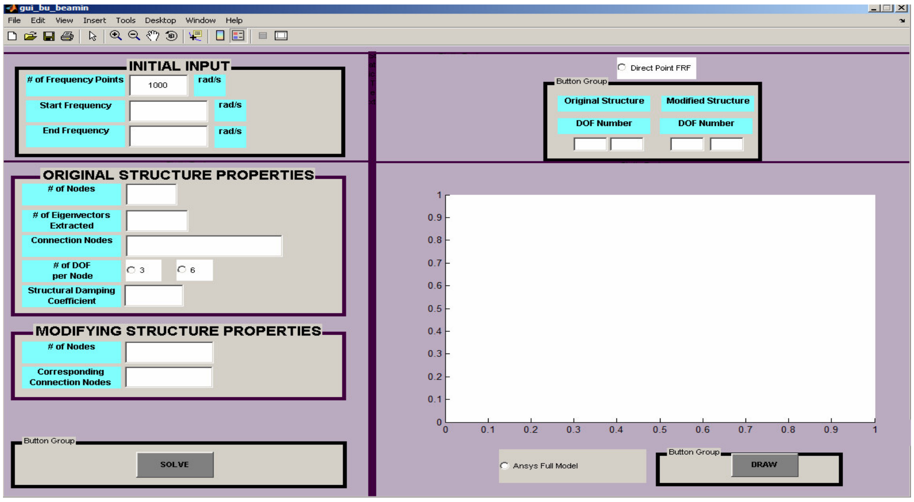


Figure A.2 Setting the Number of Frequency Points

Set the start and end frequencies for the analysis (Figure A.3).

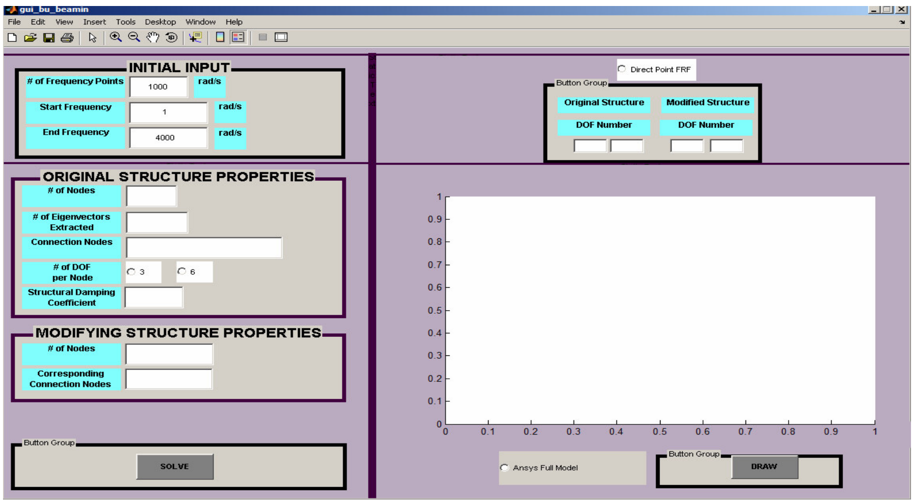


Figure A.3 Setting the Start and End Frequency

Set the number of nodes for the original structure (Figure A.4).

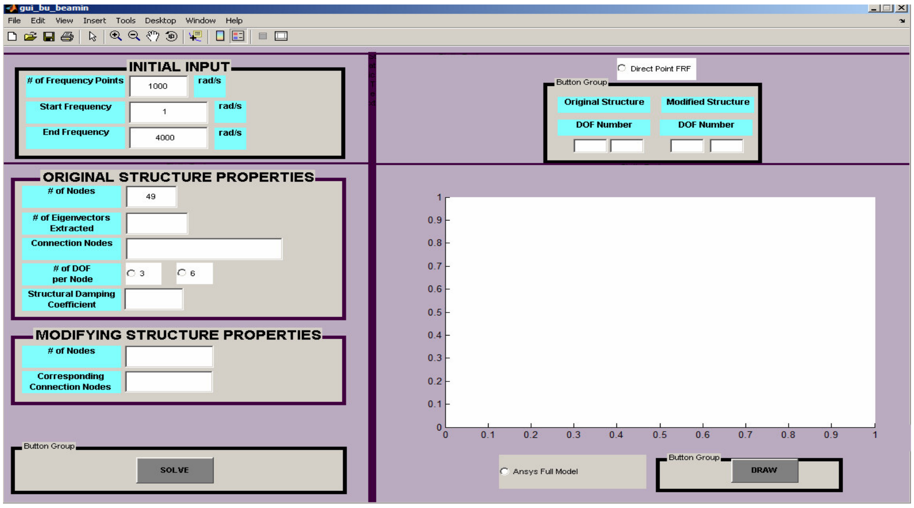


Figure A.4 Setting the Number of Nodes of the Original Structure

Set the number of eigenvectors extracted from the FE program (Figure A.5).

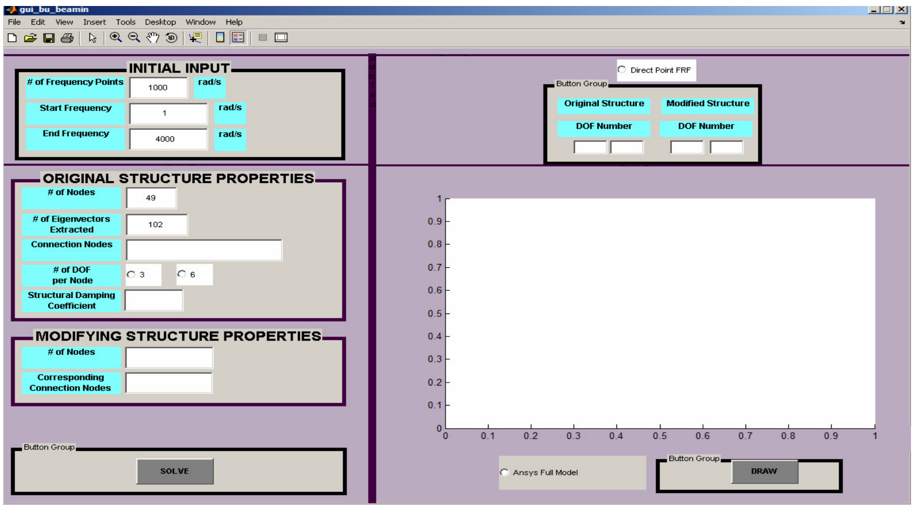


Figure A.5 Setting the Number of Eigenvectors Extracted

Set the connection nodes on the original structure (Figure A.6).

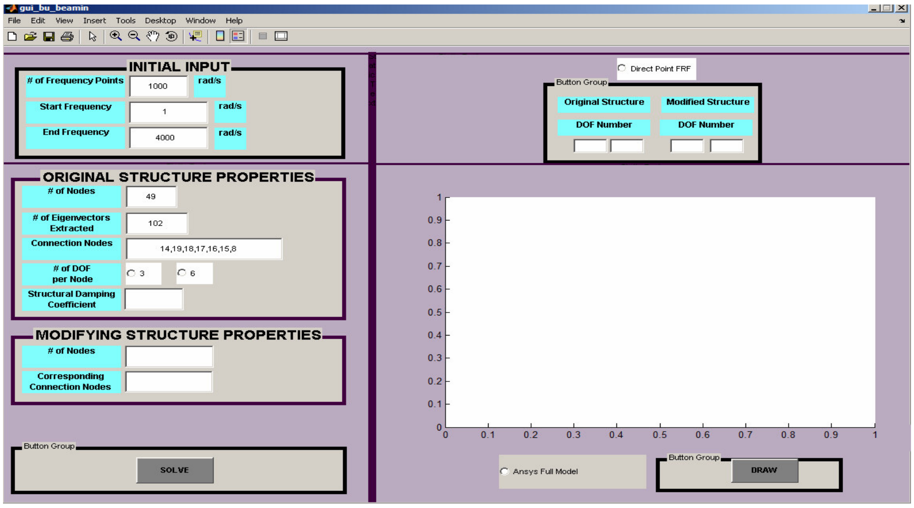


Figure A.6 Setting the Connection Nodes on the Original Structure

Select the number of DOF per node for the original structure (Figure A.7).

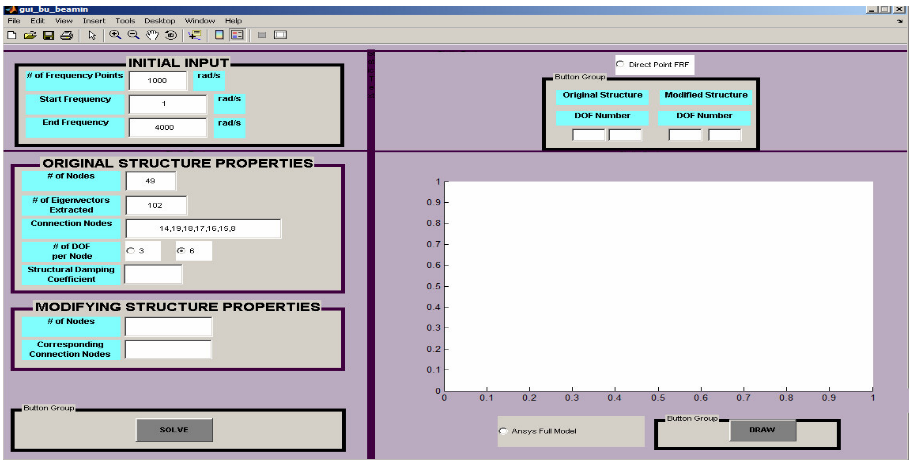


Figure A.7 Selecting the Number of DOF per Node for the Original Structure

Set the structural damping coefficient (Figure A.8).

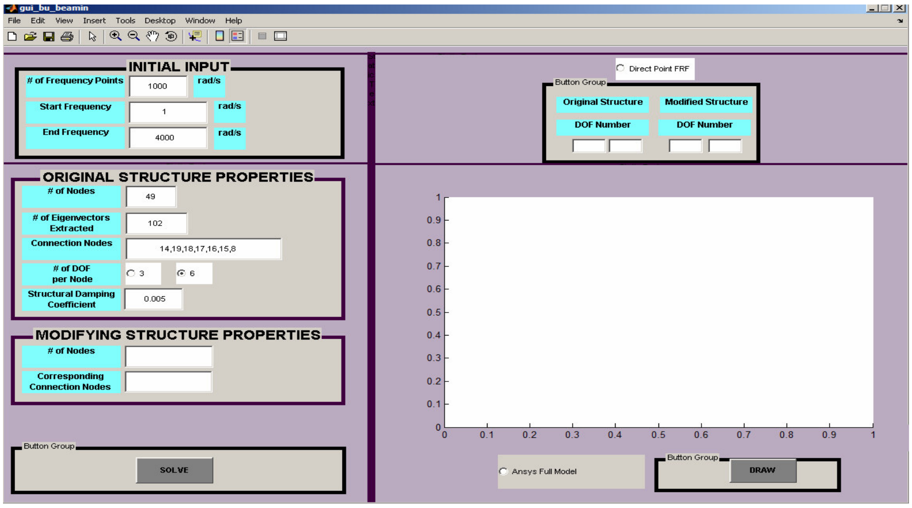


Figure A.8 Setting the Structural Damping Coefficient

Set the number of nodes for the modifying structure (Figure A.9).

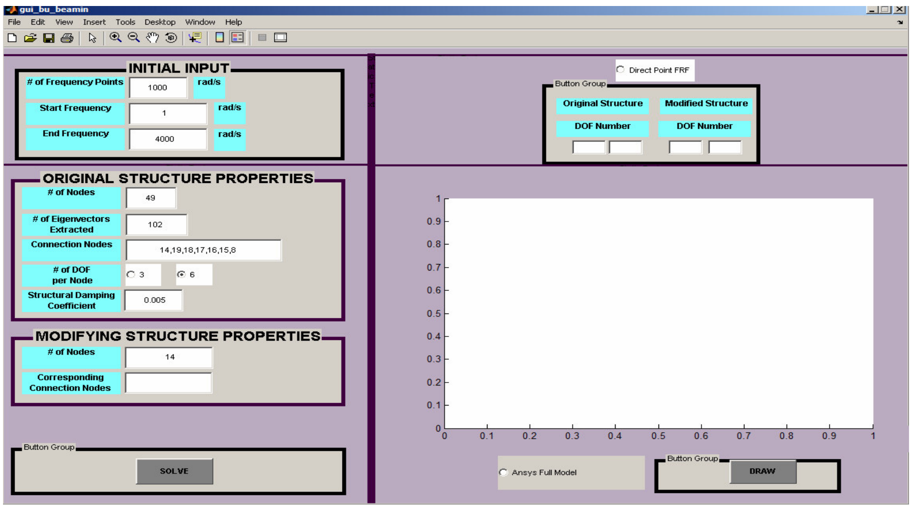


Figure A.9 Setting the Number of Nodes of the Modifying Structure

Set the corresponding connection nodes on the modifying structure (Figure A.10).

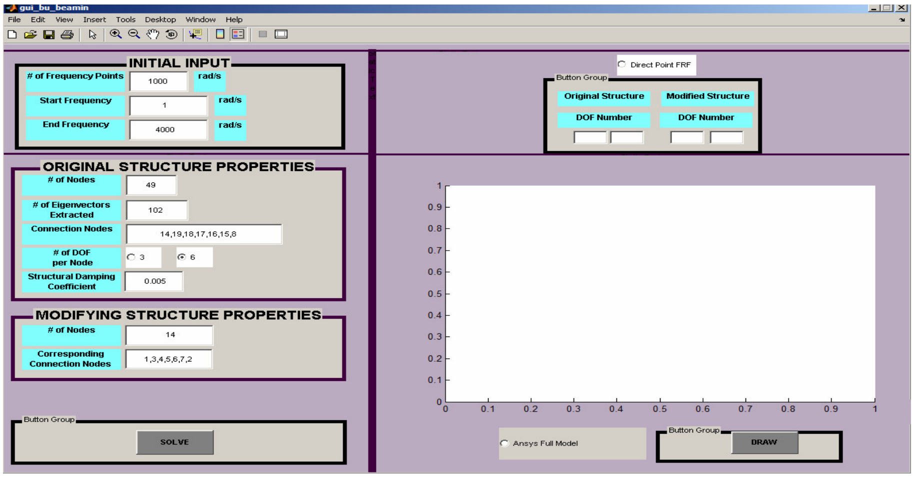


Figure A.10 Setting the Connection Nodes on the Modifying Structure

Select the “Direct Point FRF” radio button, if direct point FRFs will be calculated (Figure A.11). With this option all direct point FRFs are written in an Excell file.

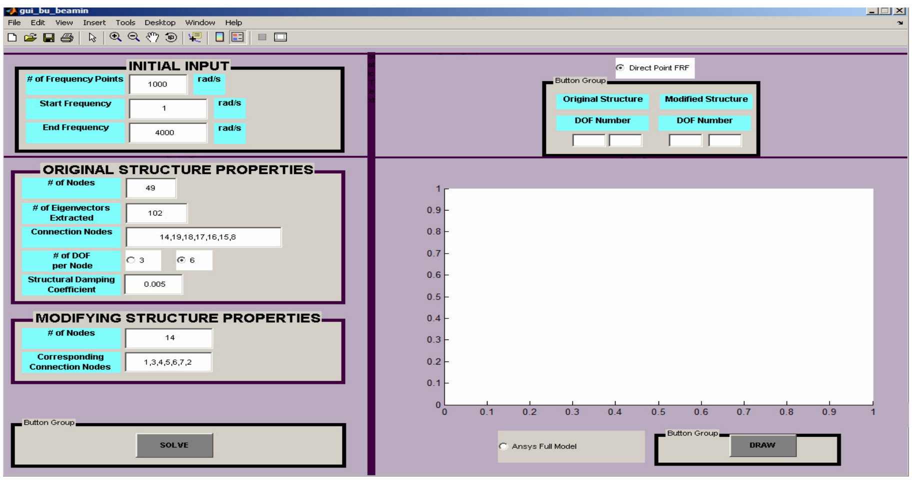


Figure A.11 Selecting the “Direct Point FRF” Radio Button

Enter the indexes of the FRF to be calculated (Figure A.12).

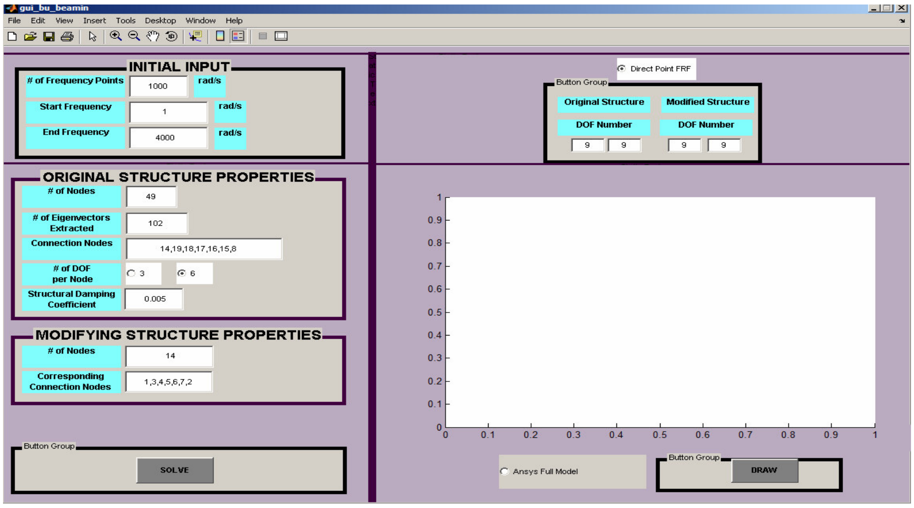


Figure A.12 Entering the Indexes of the FRF to be Calculated

Press the "SOLVE" button and wait until the solution is performed (Figure A.13).

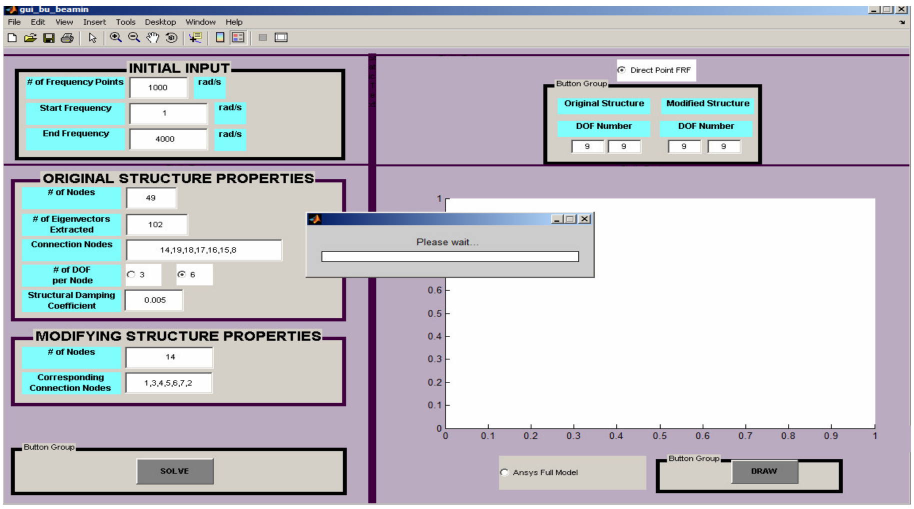


Figure A.13 Pressing the "SOLVE" Button

Press the “DRAW” button to see the FRF curves calculated (Figure A.14).

Figure A.14 Pressing the “DRAW” Button

Select the “Ansys Full Model” radio button and press “DRAW” Button, in order to compare the results with the FEA of the modified structure. (Figure A.15).

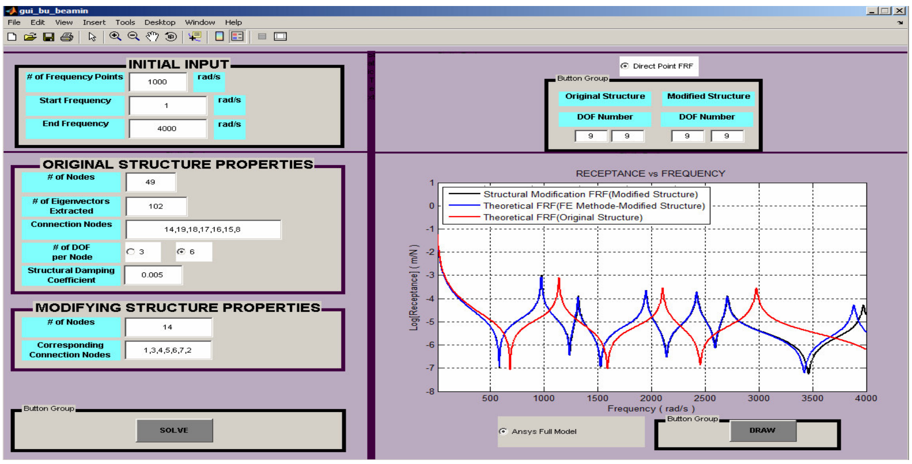


Figure A.15 Selecting the “Ansys Full Model” Radio Button

APPENDIX B

CONFERENCE PAPER

Structural Modifications with Additional DOF - Applications to Real Structures *

Güvenç Canbaloğlu
ASELSAN INC.
Microelectronics, Guidance and Electro Optics Division
06750, Akyurt, Ankara, Turkey

H. Nevzat Özgüven
Department of Mechanical Engineering, Middle East Technical University
06531, Ankara, Turkey

Nomenclature

$[D]$	Dynamic stiffness matrix
$[K]$	Stiffness matrix
$[M]$	Mass matrix
$[H]$	Structural damping matrix
$\{f\}$	Force vector
$\{x\}$	Displacement vector
$[\alpha]$	Receptance matrix

* *Published in the Proceedings of the 27th International Modal Analysis Conference, Orlando, Florida, 2009. (The typos found after submitting the paper are corrected in this version)*

Subscripts

0	Original structure
mod	Modifying structure

ABSTRACT

In structural dynamics, it is important to obtain the dynamic properties of a modified structure from those of the original structure, especially for large systems. In this study, an effective structural modification method for modifications with additional degrees of freedom is applied to a real structure with distributed modifications in order to investigate the performance of the method. In this method, which has been proposed in an earlier study by one of the authors of this paper, the frequency response functions (FRFs) of the modified structure are calculated from those of the original system and system matrices of the modifying structure. The performance of the method is investigated by applying it to GARTEUR SM-AG19 model with modifications in the form of beams attached under the wings acting as stiffeners causing flexural rigidity. The receptances calculated by using the structural modification method are compared with measured ones. A very good agreement is observed between predicted and measured results, and it is concluded that the structural reanalysis method proposed can be successfully and efficiently used for structures with distributed modifications.

1. INTRODUCTION

During the design of all mechanical structures it is required to fulfill certain mechanical criteria. However, any modification applied to a structure has an effect of changing the structural properties such as resonant frequencies, mode shapes and deformation distribution. When, for instance, an aircraft is modified by attaching external payload to it, the dynamic behavior of the aircraft changes and this change can be critical as it may cause serious vibration problems; hence, dynamic behavior of the modified aircraft has to be predicted in the design stage. In order to predict the dynamic behavior accurately, the finite element (FE) model of the modified structure can be constructed. However it may be very difficult and time-consuming to construct a new FE model for every modification. Therefore it will be more practical to predict the dynamic behavior of the modified structure by using dynamic response information of the original structure and dynamic data of the modifying structure.

Structural modification methods focus on the change of dynamic behavior of a structure due to modifications in mass, stiffness and damping properties of the system. Kyprianou, et al. [1] divided the structural dynamic modification problems into two categories: inverse structural dynamic modifications and direct structural dynamic modifications. Direct structural dynamic modification concentrates on the determination of modified structure characteristics due to modification on the original structure. Conversely, inverse structural dynamic modification is an optimization procedure looking for necessary modifications in order to achieve the desired dynamic behavior. Kyprianou, et al. [1, 2] focused on inverse structural dynamic modification. Li and He [3] presented a new approach for structural modifications required to change the dynamic characteristics of an undamped system. Furthermore, Park [4, 5] studied measured frequency response function based inverse structural dynamic modification in order to obtain necessary structural modifications. In a later work, Mottershead, et al. [6] presented an inverse method for assigning natural frequencies and nodes of normal modes of vibration by the addition of grounded springs and concentrated masses. In the direct structural dynamic modification research area different studies were conducted on lumped and distributed structural modifications with or without additional degrees of freedom (DOF). For lumped modification problems, Özgüven [7] proposed a matrix inversion method in order to find receptances of locally damped structures from those of the corresponding undamped structure. Later a recursive solution algorithm was presented in order to avoid the matrix inversion [8]. In a further work [9], Özgüven generalized this approach for reanalyzing a structure subjected to structural modification with or without additional DOF. In this method, the exact FRFs of the modified structure are calculated by using FRF matrix of the original system and mass, stiffness and damping matrices of the modifying structure [9]. Şanlıtürk [10] used the same approach, but avoided matrix inversion by employing Sherman-Morrison method. However, this approach, like many others in this category, is for modifications without additional DOF. In the direct structural dynamic modification, the number of studies on distributed structural modification problems is limited due to the difficulties in coupling continuous modifying structures with a structure. D'Ambrogio [11] studied the prediction of frequency response function of the modified structure subjected to modification in the form of rib and plate stiffeners causing flexural rigidity change and presented quasi-local characteristics of the additional dynamic stiffness matrix due to structural modification. Since dynamic properties of an original structure are identified by experimental techniques containing only translational DOF due to the difficulties in measuring rotational DOF, and structural information of modifying structure contains both rotational and translational DOF, reduction or expansion techniques have to be applied in order to obtain consistent dynamic properties for both of the structures. In a later work, D'Ambrogio and Sestieri [12-14] proposed a modeling approach for the distributed modifications and introduced different techniques to have consistent degrees of freedom for original and modifying structures. The difficulty introduced by rotational DOF in structural dynamics modification has been investigated by different researchers. Avitabile, et al. [15] developed and presented a different technique to determine rotational DOF to be used in structural dynamic modification problems. Hang, et al. [16] focused on the distributed structural dynamics modification with additional DOF by using the original relationship developed by Özgüven [9] and modeling method of the distributed modification developed by D'Ambrogio and Sestieri [12-14].

In this paper, using the original formulation of Özgüven [9], an approach is presented for predicting the dynamic response of a structure with distributed modifications from the response of the original structure itself and dynamic flexibility matrix of the modifying structure. In this approach the frequency response function of the original structure can be obtained either experimentally from modal testing or theoretically by using finite element method (FEM), and the modifying structure is modeled in such a way that consistent DOF are present at the connection nodes. The method proposed is validated by different case studies. The effect of modal truncation made in calculating FRFs of the original structure on the accuracy of the predicted FRFs is also investigated. In order to demonstrate the performance of the method when used for real structures, the scaled aircraft test structure GARTEUR SM-AG19 [17] is modified by attaching beams acting as stiffeners under the wings, and theoretically calculated FRFs are compared with experimentally measured ones.

2. THEORY

2.1. Modeling Approach for Distributed Modifications

In a structural modification problem the additional dynamic stiffness matrix due to structural modification is given by

$$[\Delta D] = [D] - [D_0] \tag{1}$$

where $[D]$ and $[D_0]$ are the dynamic stiffness matrices of the modified and original structures, respectively. For lumped modifications $[\Delta D]$ corresponds directly to dynamic stiffness matrix of the modifying structure. However for distributed modifications, it has to be calculated by using Eq. (1) which may not correspond to the dynamic stiffness matrix of the modifying structure. In order to apply Eq. (1), the dynamic stiffness matrices of the original and modified structures should be available. This requires the computation of the FE models for original and modified structures. However, if such FE models were available, the advantage of using structural modification method would be limited. D'Ambrogio and Sestieri [12] overcame this drawback by using quasi-local characteristics of additional dynamic stiffness matrix due to structural modification, $[\Delta D]$. In order to obtain the additional dynamic stiffness matrix due to structural modification, bounded region which covers the modifying area is modeled for both original and modified structures.

The approach proposed by D'Ambrogio and Sestieri has two drawbacks which are due to the inaccurate modeling of the structure and quasi-local characteristics of the additional dynamic stiffness matrix due to structural modification. The former error is due to not representing the modified structure accurately: Let us consider the beam in Figure 1, which is modeled using beam elements; if it is modified by adding a shorter beam on it as shown in Figure 2, as the neutral planes of the thinner and thicker parts of the modified beam will not coincide, the finite element model will not represent the real structure accurately. Modeling by using beam elements introduces certain errors independent from the quality of the mesh. D'Ambrogio and Sestieri discussed this error by modeling both original and modifying structures using beam and brick elements in the FE model [12]. The latter error directly depends on the size of the bounded region covering the modifying area. When the size of the bounded region which covers the modifying area is larger, the error introduced to $[\Delta D]$ will be smaller. In order to avoid such errors, a different approach for the application of the structural modification technique is used in this paper. If distributed modification is applied to an original structure in such a way that additional DOF is introduced, then it is not necessary to use Eq. (1) in order to calculate the additional dynamic stiffness matrix due to structural modification, as the problem will be a structural coupling problem. In that case, the additional dynamic stiffness matrix due to structural modification will be equal to the dynamic stiffness matrix of the modifying structure which can directly be used in the structural modification method.

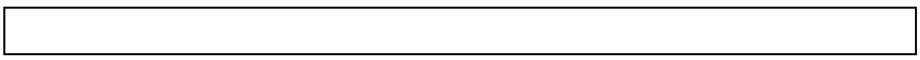


Figure 1. *Original Beam*

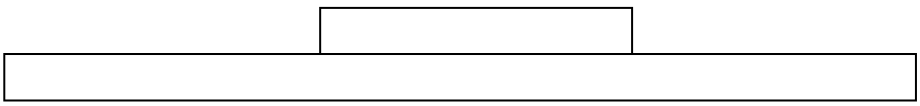


Figure 2. *Modified Beam*

For instance, the dynamic stiffness matrices of the original and modifying structures for the modified beam in Figure 2 are given by

$$[D_0]=[K_0]-\omega^2[M_0]+i[H_0] \quad (2)$$

$$[D_{\mathrm{mod}}]=[K_{\mathrm{mod}}]-\omega^2[M_{\mathrm{mod}}]+i[H_{\mathrm{mod}}] \quad (3)$$

where $[K_0]$, $[M_0]$ and $[H_0]$ represent stiffness mass and structural damping matrices of the original structure, and similarly $[K_{\mathrm{mod}}]$, $[M_{\mathrm{mod}}]$ and $[H_{\mathrm{mod}}]$ are stiffness, mass and structural damping matrices of the modifying structure They all can be obtained directly from the FE models of the original and modifying beams. When additional DOF are introduced to the original structure, the dynamic stiffness matrix of the modified structure can be obtained by assembling the dynamic stiffness matrices of the original and modifying structures.

Similarly, for distributed modifications, if additional DOF are introduced to the original structure there is no need to use Eq. (1); instead, additional dynamic stiffness matrix due to structural modification can directly be obtained from the FE model of the modifying structure.

2.2. Structural Modification Method with Additional DOF

In this section, the formulation given by Özgüven [9] is summarized. FRF matrix of a modified system can be partitioned as; DOFs which correspond to original structure only (superscript a), DOFs at connection points (superscript b), and DOFs that belong to modifying structure only (superscript c). Then the following equations can be written for original and modifying structures.

$$[\alpha_0]^{-1} = \begin{bmatrix} \alpha_0^{aa} & \alpha_0^{ab} \\ \alpha_0^{ba} & \alpha_0^{bb} \end{bmatrix}^{-1} = [K_0] - \omega^2 [M_0] + i[H_0] \tag{4}$$

$$\begin{bmatrix} \alpha^{aa} & \alpha^{ab} & \alpha^{ac} \\ \alpha^{ba} & \alpha^{bb} & \alpha^{bc} \\ \alpha^{ca} & \alpha^{cb} & \alpha^{cc} \end{bmatrix}^{-1} = \begin{bmatrix} \begin{bmatrix} \alpha_0^{aa} & \alpha_0^{ab} \\ \alpha_0^{ba} & \alpha_0^{bb} \end{bmatrix}^{-1} & & 0 \\ & & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \begin{bmatrix} D_{\text{mod}}^{bb} & D_{\text{mod}}^{bc} \\ D_{\text{mod}}^{cb} & D_{\text{mod}}^{cc} \end{bmatrix} & \\ 0 & & \end{bmatrix} \tag{5}$$

where $[\alpha_0]$ and $[\alpha]$ represent the receptance matrices of the original and modified structure, respectively. Pre-multiplying Eq. (5) by

$$\begin{bmatrix} \begin{bmatrix} \alpha_0^{aa} & \alpha_0^{ab} \\ \alpha_0^{ba} & \alpha_0^{bb} \end{bmatrix} & & 0 \\ & & 0 \\ 0 & 0 & I \end{bmatrix} \tag{6}$$

and post-multiplying by $[\alpha]$ gives

$$\begin{bmatrix} \begin{bmatrix} \alpha_0^{aa} & \alpha_0^{ab} \\ \alpha_0^{ba} & \alpha_0^{bb} \end{bmatrix} & & 0 \\ & & 0 \\ 0 & 0 & I \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix} [\alpha] + \left[\begin{array}{c|c} 0 & \begin{pmatrix} \alpha_0^{ab} & 0 \end{pmatrix} . [D_{\text{mod}}] \\ \hline 0 & \begin{pmatrix} \alpha_0^{bb} & 0 \\ 0 & I \end{pmatrix} . [D_{\text{mod}}] \\ 0 & \end{array}\right] [\alpha] \tag{7}$$

After some matrix manipulations the receptance submatrices of the modified system can be obtained as

$$\begin{bmatrix} \alpha^{ba} \\ \alpha^{ca} \end{bmatrix} = \left[\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \alpha_0^{bb} & 0 \\ 0 & I \end{bmatrix} . [D_{\text{mod}}] \right]^{-1} \begin{bmatrix} \alpha_0^{ba} \\ 0 \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} \alpha^{bb} & \alpha^{bc} \\ \alpha^{cb} & \alpha^{cc} \end{bmatrix} = \left[\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \alpha_0^{bb} & 0 \\ 0 & I \end{bmatrix} . [D_{\text{mod}}] \right]^{-1} \begin{bmatrix} \alpha_0^{bb} & 0 \\ 0 & I \end{bmatrix} \tag{9}$$

$$[\alpha^{aa}] = [\alpha_0^{aa}] - [\alpha_0^{ab} \mid 0][D_{\text{mod}}] \begin{bmatrix} \alpha^{ba} \\ \alpha^{ca} \end{bmatrix} \tag{10}$$

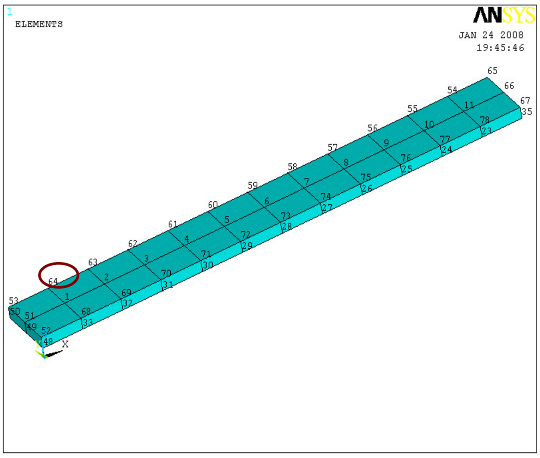
$$[\alpha^{ab} \mid \alpha^{ac}] = [\alpha_0^{ab} \mid 0] \left[[I] - [D] \begin{bmatrix} \alpha^{bb} & \alpha^{bc} \\ \alpha^{cb} & \alpha^{cc} \end{bmatrix} \right] \tag{11}$$

It should be noted that the order of the matrix to be inverted is equal to the DOF of the modifying structure, which is usually much less than then the total DOF of the structure.

3. NUMERICAL EXAMPLES

3.1. Free-Free Beam

The aim of this example is to demonstrate the accuracy of the structural modification method when applied to distributed systems. In this example a free-free beam shown in Figure 3 is modified by attaching a smaller beam under the original one. Original beam was divided into 24 brick elements (20 nodes per element) with 3 DOF per node yielding total DOF of 723. The modifying structure was divided into 4 brick elements with 3 DOF per node yielding total DOF of 153. The geometrical and material properties of the original and modifying beams are given in Table 1. The predicted direct point FRF for the modified beam at node 64 in Z direction is shown in Figure 5. The FE model of the modified structure is also constructed, and the FRF of the same coordinate is obtained and compared with that obtained by the structural modification method (Figure 5).



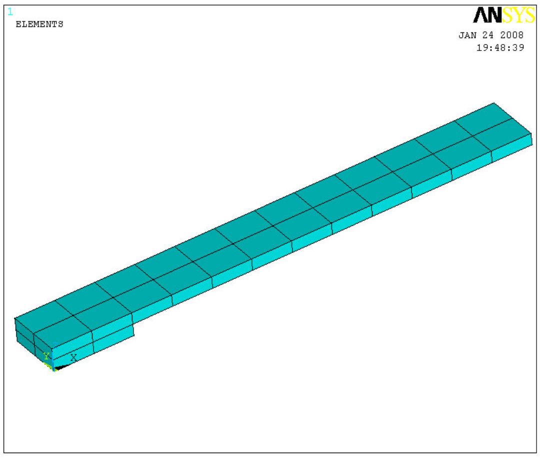


Figure 3. *The FE model of the original beam* **Figure 4.** *The FE model of the modified beam*

Table 1. *Geometrical and material properties of the original and modifying beams*

	Original Beam	**Modifying Beam**
Young's Modulus	71 GPa	71 GPa
Poisson's Ratio	0.33	0.33
Density	2770 kg/m^3	2770 kg/m^3
Length	1200mm	200mm
Width	150mm	150mm
Thickness	25mm	25mm

As can be seen from Figure 5, the predicted FRF with structural modification technique matches exactly with the FRF calculated from finite element analysis (FEA) of the modified structure, which is an expected result as the method is an exact one.

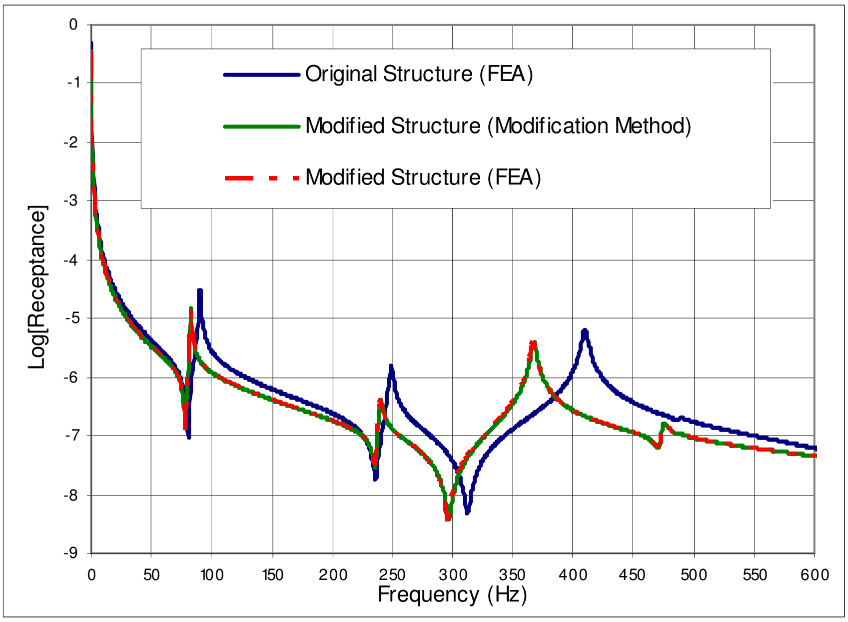
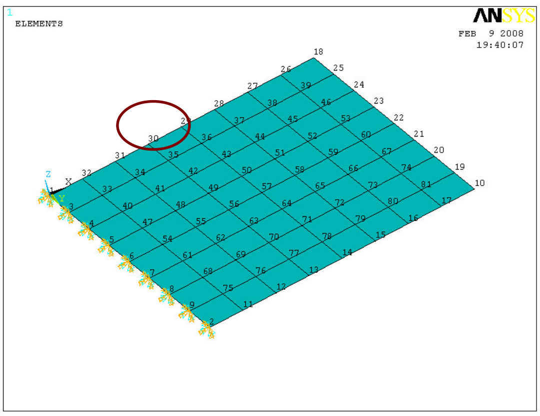


Figure 5. *Comparison of FRF at 64Z64Z*

3.2. Cantilever Plate

In this second example, the effect of modal truncation made in the computation of the FRFs of the original structure on the accuracy of the structural modification method proposed is demonstrated. In this example, the cantilever plate shown in Figure 6 is modified by attaching the plate shown in Figure 7 to the free edge of the original structure. Original plate was divided into 64 shell elements with 6 DOF per node yielding total DOF of 488. The modifying structure was divided into 16 shell elements with 6 DOF per node giving total DOF of 144. The geometrical and material properties of the original and modifying plates are given in Table 2. The direct point FRFs at node 30 in Z direction are predicted for the modified structure by employing the method proposed and by using different number of modes in calculating frequency response function of the original structure. The results are shown in Figure 9 to 11. The effect of truncation made in obtaining FRF of the original structure can be observed in higher frequencies. Using higher number of the modes in the computation of the FRF of the original structure increases the accuracy of the FRF predicted for modified structure.



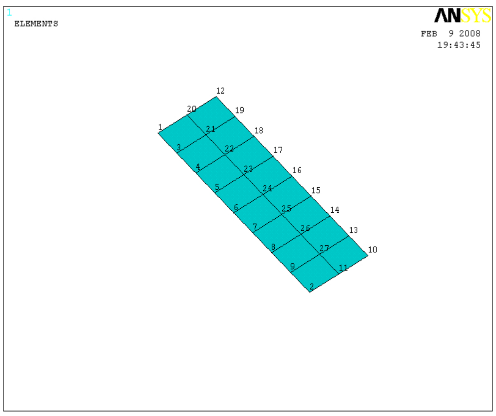


Figure 6. *The FE model of the original plate* **Figure 7.** *The FE model of the modifying plate*

Table 2. *Geometrical and material properties of the original and modifying plates*

	Original Beam	Modifying Beam
Young's Modulus	71 GPa	71 GPa
Poisson's Ratio	0.33	0.33
Density	2770 kg/m^3	2770 kg/m^3
Length	300mm	75mm
Width	300mm	300mm
Thickness	2mm	2mm

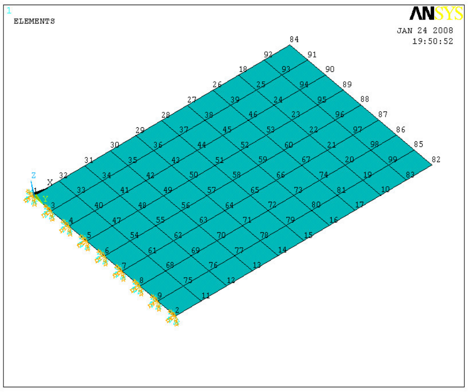


Figure 8. *The FE model of the modified plate*

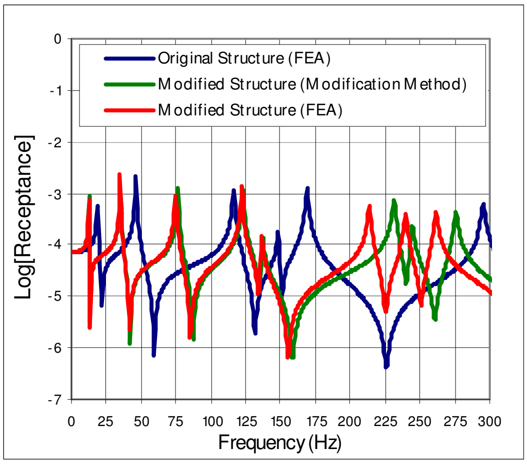


Figure 9. *Comparison of FRFs at 30Z30Z (15 modes are used in the calculation of FRFs original structure)*

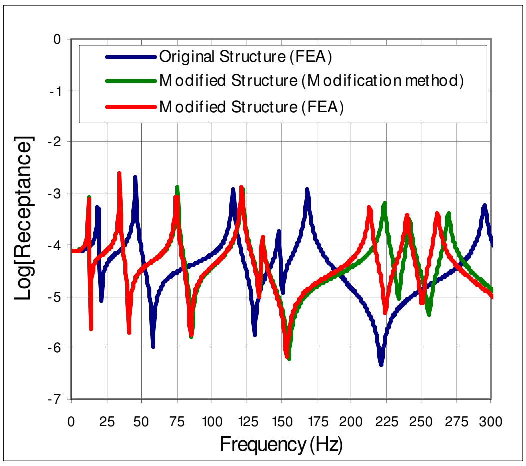


Figure 10. *Comparison of FRFs at 30Z30Z (30 modes are used in the calculation of FRFs original structure)*

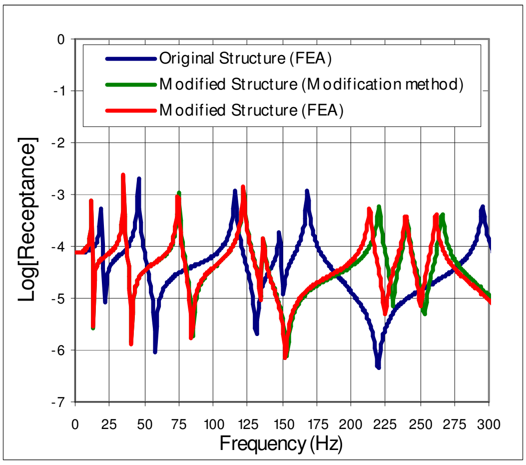


Figure 11. *Comparison of FRFs at 30Z30Z (102 modes are used in the calculation of FRFs of the original structure)*

3.3. GARTEUR SM-AG19 Model

In this part, in order to show the performance of the structural modification technique, GARTEUR SM-AG19 model (Figure 12) is modified with beams under the wings which act as stiffeners causing flexural rigidity (Figure 13). The GARTEUR SM-AG19 model constructed differs slightly from the original GARTEUR SM-AG19 model. Viscoelastic tape which is placed on the upper surface of the wings in the original model is not used in the present study. In order to apply the structural modification technique, the finite element models of the original and modifying structures are constructed. The solid brick elements are used in the finite element model of GARTEUR SM-AG19 model which has 1380 nodes with 3 DOF per node yielding total DOF of 4140. Two beams each having dimensions of the 200mmx100mmx10mm are used as modifying structures. They are modeled with solid brick elements and each beam has 70 nodes with 3 DOF per node resulting total DOF of 210 for each modifying beam.

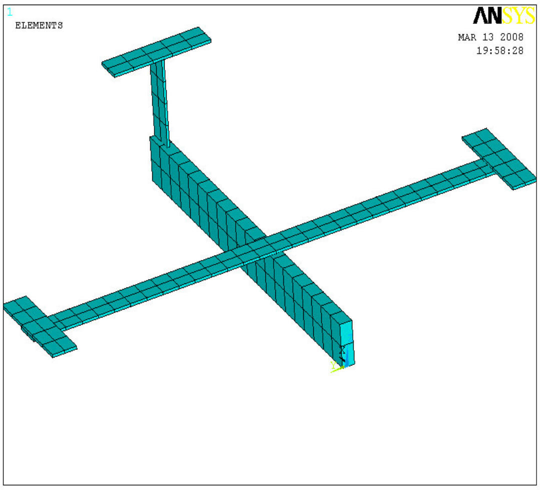


Figure 12. *FE model of GARTEUR SM-AG19*

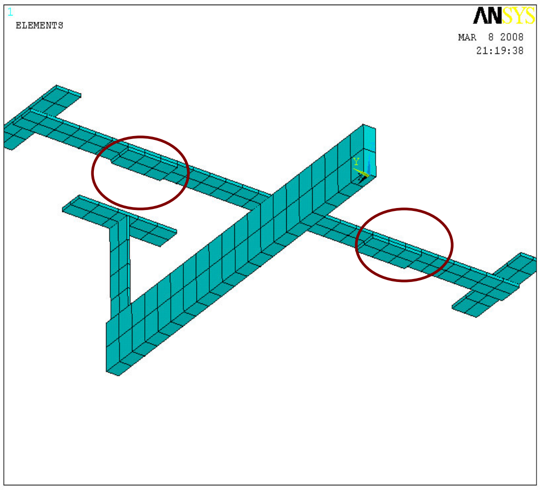


Figure 13. *FE model* of modified GARTEUR SM-AG19

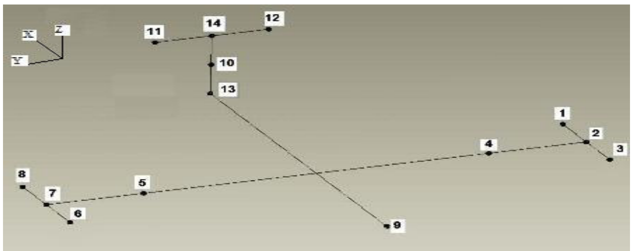


Figure 14. *Accelerometer positions*

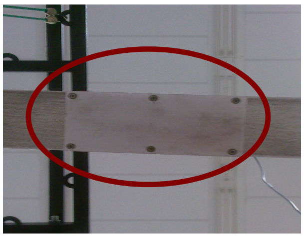


Figure 15. *Modifying beam*

Figure 16. *View of the experimental structure*

Modal test is conducted on the original and modified GARTEUR SM-AG19 model by using an impact hammer. During the test, the same measurement and excitation equipment is used for both original and modified structures. Accelerometer positions are shown in Figure 14. In Figures 15 and 16 modifying beam and the general view of the experimental structure are given, respectively. Direct point FRFs measured at point 3 in Z direction (Figure 14) for both original and modified structures are given in Figure 17.

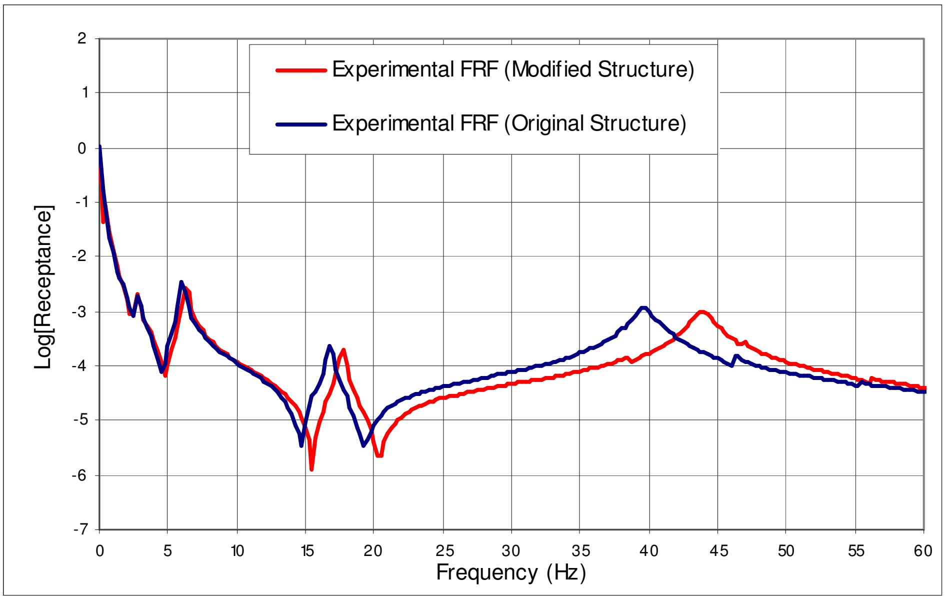


Figure 17. *Experimental FRF at 3Z3Z for the original and modified GARTEUR SM-AG19*

In Figure 18, direct point FRF of point 3 in Z direction obtained from the FE model is compared with that obtained experimentally in order to see the accuracy of the FE model of GARTEUR SM-AG19 model. As can be seen from Figure 18 they are in good agreement. The mismatch in the magnitudes of the FRF at the resonances may be attributed to the constant loss factor used for all modes in the finite element model of the structure to represent the damping in the system. The FRF of the modified structure is obtained by using the structural modification method proposed, and it is compared with the experimentally measured one (Figure 19). Again, a good agreement is observed between the FRF calculated by using the structural modification method and experimentally measured FRF. The discrepancy around the third mode may be due to the slight differences between the theoretical and experimental FRFs of the original structure. The truncation made in the calculation of the FRF of the original structure can be one of the reasons for such slight differences.

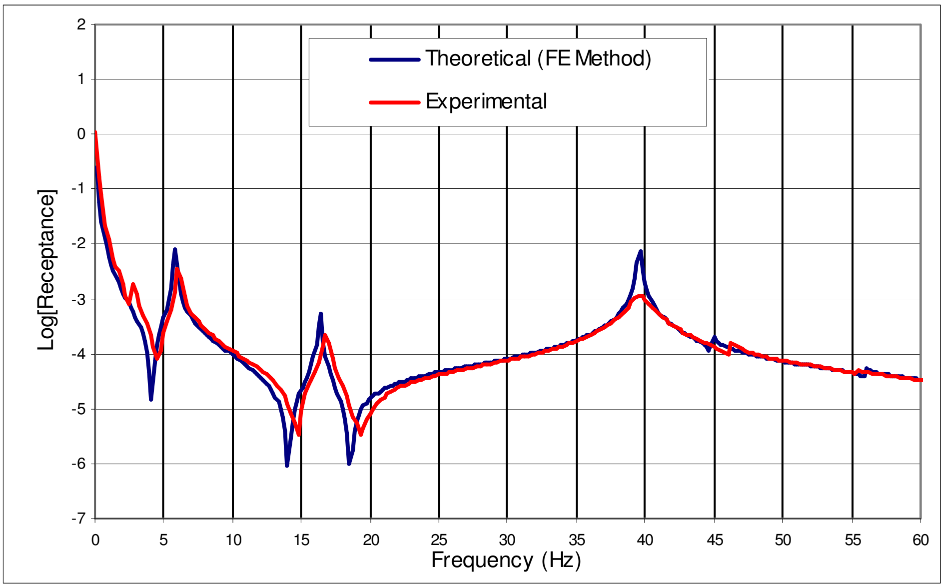


Figure 18. *Theoretical (FE method) and experimental FRF at 3Z3Z for original GARTEUR SM-AG19*

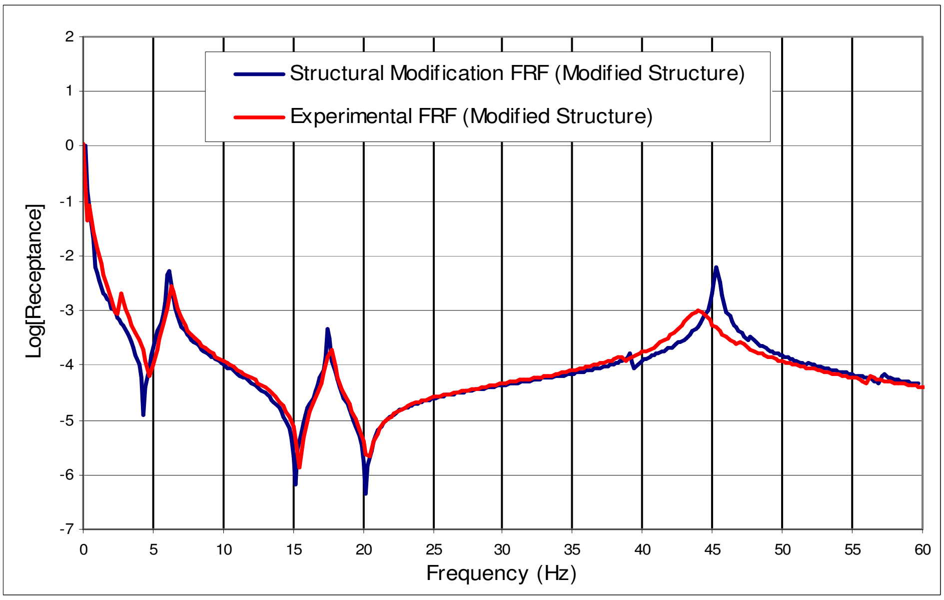


Figure 19. *Theoretical (structural modification method) and experimental FRF at 3Z3Z for modified GARTEUR SM-AG19*

The numerical examples given above show that the structural modification technique with additional DOF is an effective method for analyzing distributed modifications. Main advantage of the structural modification method employed is that there is no need to use Eq. (1) for the calculation of the additional dynamic stiffness matrix due to structural modification. Since additional DOF are introduced in modification, the structural modification problem turns into a structural coupling problem and therefore dynamic stiffness matrix of the modifying structure can directly be used in the structural modification method. Moreover, since modification introduces additional DOF to the original structure, a more accurate model is obtained for the modified structure.

4. CONCLUSIONS

In structural dynamic modification applications, difficulties may arise when modification is distributed. The main difficulty is due to the rotational degrees of freedom. Obtaining the additional dynamic stiffness matrix due to distributed structural modification is also another difficulty in structural modification methods. Although there are some approaches to overcome such difficulties by making simplifications and thus introducing inaccuracies; using structural modification methods with additional degrees of freedom, which is equivalent to treating the problem as a structural coupling problem, overcomes at least the later difficulty mentioned above. Then, there is no need to partially model the modified and the original structure in order to calculate the additional dynamic stiffness matrix due to distributed structural modification, as proposed in some previous studies [12, 16]. In this paper, the method proposed in an earlier study by one of the authors for structural dynamic modifications with additional degrees of freedom [9] is applied to structures with distributed modifications, the main objective being to investigate the performance of the method when used for real structures with distributed dynamic modifications. In the method proposed, the frequency response functions of the modified structure are calculated from those of the original structure and the system matrices of the modifying structure.

Firstly, the validity of the approach proposed is demonstrated by applying it to a beam problem: Theoretically calculated FRFs are compared with those obtained from the FE analysis of the modified structure, and a perfect match was observed as expected, since the method yields exact FRFs when exact values of the FRFs for the original structure are used. It was shown in the second case study that the accuracy of the predictions strongly depends on the accuracy of the FRFs used for the original structure. In order to study the effect of modal truncation made in calculating the FRFs of the original structure on the accuracy of the method, a cantilever plate which was modified by attaching a smaller plate to the free edge was modeled by using FEM. It is observed that the performance of the structural modification technique increases when the number of modes included in the computation of FRFs of the original structure is increased.

The performance of the method when applied to a real structure is also investigated by applying it to GARTEUR SM-AG19 model with modifications in the form of beams attached under the wings acting as stiffeners causing flexural rigidity. The receptances calculated by using the structural modification method are compared with experimentally measured ones. A very good agreement is observed between the predicted and measured results. The discrepancies in the magnitudes of the FRFs at resonances are attributed to the constant loss factor used for all modes in the finite element model of the original structure to represent the damping of the system. It is concluded in this study that the structural reanalysis method proposed can be successfully and efficiently used for structures with distributed modifications, and thus the problems encountered in approach such as the one suggested by D’Ambrogio and Sestieri for distributed structural modification problems can be avoided.

ACKNOWLEDGEMENT

The authors would like to thank ASELSAN Inc. Microelectronics, Guidance and Electro Optics Division for supporting this work.

REFERENCES

[1] **Kyprianou A., Mottershead J. E., Ouyang H.**, "Structural modification. Part 2: Assignment of natural frequencies and antiresonances by an added beam", Journal of Sound and Vibration, 2005, 284(1–2), p.267–281.

[2] **Kyprianou A., Mottershead J. E., Ouyang H.**, "Assignment of natural frequencies by an added mass and one or more springs", Mechanical Systems and Signal Processing, 2004, 18(2), p.263–289.

[3] **Li T., He J.**, "Local structural modification using mass and stiffness changes", Engineering Structures, 1999, 21(11), p.1028-1037.

[4] **Park Y. H., Park Y. S.**, "Structural modification based on measured frequency response functions: An exact eigen properties reallocation," Journal of Sound and Vibration, 2000, 237(3), p.411–426.

[5] **Park Y. H., Park Y. S.**, "Structure optimization to enhance its natural frequencies based on measured frequency response functions", Journal of Sound and Vibration, 2000, 229(5), p.1235–1255.

[6] **Mottershead J. E., Mares C., Friswell M. I.**, "An inverse method for the assignment of vibration nodes", Mechanical Systems and Signal Processing, 2001, 15(1), p.87–100.

[7] **Özgüven, H. N**., "Determination of receptances of locally damped structures", Proceedings of the Second International Conference on Recent Advances in Structural Dynamics, 1984, 2, Southampton, England, p.887-892.

[8] **Özgüven, H. N**., "A new method for harmonic response of non-proportionally damped structures using undamped modal data", Journal of Sound and Vibration, 1987, 117(2), p.313-328.

[9] **Özgüven, H. N**., "Structural modifications using frequency response functions", Mechanical Systems and Signal Processing, 1990, 4(1), p.53-63.

[10] **Şanlıtürk, K. Y**., "An efficient method for linear and nonlinear structural modifications", Proceedings of ESDA2002: 6th Biennial Conference on Engineering Systems Design and Analysis, ESDA2002/APM-028, 2002, Istanbul, Turkey.

[11] **W. D'Ambrogio**, "Consistent modeling of continuous structural dynamic modifications", Proceedings of the 9th International Modal Analysis Conference, 1991, Florence, Italy.

[12] **W. D'Ambrogio, A. Sestieri**, "Using distributed modifications to change the dynamic behavior of structures", Proceedings of the 17th International Modal Analysis Conference, 1999, Kissimmee, Florida.

[13] **W. D'Ambrogio, A. Sestieri**, "Coupling theoretical data and translational FRFs to perform distributed structural modification", Mechanical Systems and Signal Processing, 2001, 15(1), p.157–172.

[14] **W. D'Ambrogio, A. Sestieri**, "Predicting the effect of distributed structural modifications by alternative techniques", Proceedings of the 19th International Modal Analysis Conference, 2001, Kissimmee, Florida.

[15] **Avitabile P., O'Callahan J.C., Lieu I.W., Madden R.**, "An efficient method of determining rotational degrees of freedom from analytical and experimental modal data", Proceedings of the 4th International Modal Analysis Conference, 1986, Los Angeles, California.

[16] **Hang H., Shankar K., Lai J.**, "Distributed structural dynamic modifications without rotational degrees of freedom", Proceedings of the 25th International Modal Analysis Conference, 2007, Orlando, Florida, USA.

[17] **Link M., Friswell M., "**Working group 1: Generation of validated structural dynamic models- results of a benchmark study utilizing the Garteur SM-AG19 test-bed", Mechanical Systems and Signal Processing, 2003, 17(1), p.9-20.