INTERVAL PRIORITY WEIGHT GENERATION FROM INTERVAL COMPARISON MATRICES IN ANALYTIC HIERARCHY PROCESS

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ABSTRACT

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In this study, for the well-known Analytic Hierarchy Process (AHP) method a new approach to interval priority weight generation from interval comparison matrix is proposed. This method can be used for both inconsistent and consistent matrices. Also for the problems having more than two hierarchical levels a synthesizing heuristic is presented. The performances of the methods, interval generation and synthesizing, are compared with the methods that are already available in the literature on randomly generated matrices.

Keywords: Analytic Hierarchy Process, Interval Comparison Matrix, Multi Criteria Decision Making, Linear Programming

ÖZ

ANALİTİK HİYERARŞİ SÜRECİNDE ARALIK DEĞERLERDEN OLUŞAN KARŞILAŞTIRMA MATRİSLERİNDEN ÖNCELİK ARALIKLARI BELİRLEME

Öztürk, Ufuk
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Bu çalışmada, yaygın olarak kullanılan Analitik Hiyerarşi Süreci'nde aralık değerlerden oluşan karşılaştırma matrislerinden öncelik aralıkları belirlemek için yeni bir yaklaşım önerilmiştir. Önerilen metod hem tutarlı hem de tutarsız karşılaştırma matrisleriyle kullanılabilir. Ayrıca ikiden fazla hiyerarşik seviyenin olduğu durumlarda kullanılmak üzere yeni bir sentezleme yönteminden de bahsedilmiştir. Önerilen her iki yöntemin, aralık değer üretme ve sentezleme, performansları rassal üretilmiş matrisler yardımıyla literatürde kullanılmakta olan diğer yöntemlerle karşılaştırılmıştır.

Anahtar kelimeler: Analitik Hiyerarşi Yöntemi, Aralık Değerleri, Çok Kriterli Karar verme, Lineer Programlama

To My Family…

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TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

CHAPTER 1

INTRODUCTION

In real life, decision-makers, either organizations or people, face with many situations in which they have to make a decision among present alternatives. However, the most preferable alternative is not always easily selected among the set of alternatives. In these kind of situations the decision-maker determines the decision criteria which the final choice would be made according to. The problems in which among many alternatives the most preferred one has to be chosen according to a set of criteria is called multi criteria decision making (MCDM) problems. In literature there are many tools or methods available to support the decision making process. The Analytic Hierarchy Process (AHP) is the most common method used as the MCDM support tool. In literature there are some extensions to AHP. In this thesis, we study the interval AHP, which uses interval numbers instead of precise judgments in the comparison matrices.

We develop a one stage linear programming method that generates interval priority weights from interval comparison matrices. We assume that weights are multiplicative. For the normalization of the interval weights we modify the interval normalization constraint composed for the additive weights to handle the multiplicative case. The proposed method can be used for both consistent and inconsistent comparison matrices. We also propose two variations to the method. In the first variation, a second stage model is introduced besides the proposed method. In this second stage, the maximum error is aimed to be minimized. In the second variation, objective function of the one stage linear programming model is modified and the maximum error is minimized.

In the second part of the thesis, we propose a new method for synthesizing the obtained interval weights. For the multiplication case there is only one other method available in literature which is developed by Wang et al. (2005a). We test the performances of the proposed interval weight generation and synthesizing methods on randomly generated problems.

As it is expressed by Wang and Elhag (2006), because of the complexity and uncertainty of the real life, using interval judgments is more natural and easier than using precise judgments. Thus the interval comparison matrices and interval priority weights are used in our proposed methods. And according to the performance measures, the proposed methods have less error than the other methods available in the literature. In order to linearize and simplify the proposed methods a multiplication normalization constraint is developed. It is the first time a multiplication normalization constraint is used in an interval comparison weight generation method. This application can be a leverage point for the future studies. When the number of alternatives and criteria increases in the problem, the synthesis of the obtained interval weights become harder. In our approach, instead of solving a set of nonlinear programs, an algorithm is proposed which will give a quick ranking in a short period of time.

The organization of the thesis is as follows: In Chapter 2, the literature survey is presented. A brief history and major applications about AHP method is also mentioned in this chapter. In Chapter 3, the basic principles of the AHP method are presented. In Chapter 4, the interval AHP methods and methods relevant to our study are summarized. In Chapter 5, the proposed method, the assumptions made and some of the variations of the method are presented. In Chapter 6, firstly the performance measures are presented and the performances of the developed interval weight generation methods are tested and compared with the methods in the literature. And in the last chapter the conclusions and the future studies are discussed.

CHAPTER 2

LITERATURE SURVEY

AHP, which was introduced by Saaty (1980), enables the decision-maker to model the most complicated problems by using a hierarchy, formed by the sub-criteria levels and alternatives. Both qualitative criteria and quantitative criteria can easily be combined in the AHP. This method is based on the pairwise comparisons made by the decision-maker not only for the criteria but also for the alternatives according to the criteria described. In the comparison the 1-9 comparison scale is used.

In Belton and Gear's (1983) study, the rank reversal situation is discussed and a modification to the normalization procedure was suggested in order to prevent this rank reversal situation. However, later on Barzilai and Golany (1994) claimed that the reason for rank reversal was not the normalization procedure used in AHP. They suggested to use weighted-geometric-mean aggregation rule in order to prevent this rank reversal situation.

In some cases, since the problem faced is a complex one, the decision-maker cannot easily make a discrete comparison. Instead the decision-maker can use an interval value for the comparison of the alternatives. These kinds of AHPs are called Interval AHPs. These interval values are first introduced by Saaty and Vargas (1987) who suggested a simulation approach to obtain weights from interval comparison matrices. They also discussed Rank reversal situation. In many studies similar kind of simulation technique is suggested, (see, e.g., Hauser and Tadikamalla, 1996; Zhang et. al, 2003; Banuelas and Antony, 2004). Cox (2007) presented a complete enumeration method and compared the performance of this method with uniform and normal distributed simulation techniques.

Another method is suggested by Arbel (1989), in which the interval judgments made by the decision-maker are taken as linear constraints, and by solving a linear program the final precise weights are obtained. However, Kress (1991) showed that the linear program suggested by Arbel cannot be solved if the given comparisons are inconsistent. Later, Arbel and Vargas (1993) presented two approaches for the interval comparison matrices. In the first one, they described a simulation approach, assuming that all the interval judgments are uniform, and obtained the interval priority vectors after randomly sampling these interval judgments. In the second approach, they presented a mathematical model in which the feasible region can be derived by using the interval judgments. If there is a feasible region in the problem, they suggested a linear program to obtain the final precise priority weights. Haines (1998) examined the distribution of the alternatives developed by using the Arbel's (1989) method. The results of the uniform distribution and the distribution of random convex combinations of the vertices are investigated.

Bryson and Mouborin (1996) developed the Action Learning Evaluation Procedure in which both from interval to interval and point to point estimations were used. At the beginning of the procedure, interval comparisons of the decision maker is used to calculate the interval weights, which generally gave an ambiguous ranking, and then, in the following steps the interval comparisons of the decision maker is narrowed to obtain an unambiguous ranking.

Islam et al. (1997) presented a method called Lexicographic Goal Programming (LGP), which can obtain a crisp set of weights even when an inconsistent decision matrix is described by the decision-maker. However, Wang (2006) showed that the LGP has a faulty part: when the LGP is used both for upper and lower triangular decisions separately, the final rankings obtained are different. Sugihara et al. (2004) developed upper approximation (UA) and lower approximation (LA) methods. In the UA method, a model is proposed in which upper-bounds on the interval weight data are used. And lower-bounds are used in the LA method. The UA method can be solved both for consistent and inconsistent cases. However, the LA method can be solved only for consistent matrices. Arbel and Vargas (2007) presented a method in which Euclidian centers are used to find the final priority vector from given interval comparison matrix. The final priority vector can contain both precise and interval values. However, the problem has to be a consistent one in order to obtain a feasible solution.

In conventional AHP, additive normalization constraint is used while normalizing the final weights. After normalizing the final weights, the comparison of the alternatives can easily be done. Barzilai (1997) proposed a multiplicative normalization constraint instead of an additive constraint. In this constraint the multiplication of the weights are set to be 1. AHP models using this constraint are called Multiplicative Analytic Hierarchy Process (MAHP) models. Stam (2003), described some of the properties of MAHP and showed some of the differences between the additive and multiplicative AHP by using simulation experiments. Wang et al. (2005a) used this multiplicative normalization constraint in the two-stage logarithmic goal programming (TLGP) method in which interval priority weights can be obtained from given interval decision matrices. In the first stage of this method, the minimum objective value for the least inconsistent case is found. And in the second stage, by using this objective function value in a goal program, minimum and maximum interval values for each alternative is calculated. The goal program has to be run for each alternative present in the problem in order to obtain the final priority weight vector. This method can be used for both inconsistent and consistent matrix entries. Wang et al. (2005b) also described a method in which firstly a consistency check is done. For the consistent interval comparison matrices Arbel's (1989) preference programming method is suggested and for the inconsistent matrices an eigenvector-based non-linear program is built. Chandran et al. (2005) presented a two-stage linear programming approach and in the first stage the inconsistency of the given comparison matrix is minimized and then in the second stage of the method a crisp set of weights is obtained. This method can be used both for crisp and interval entries.

Sugihara and Tanaka (2001) described the normalization constraint which can be used with the interval comparison matrices. Wang and Elhag (2007) used this interval additive normalization constraint in their proposed approach based on goal programming (GP) and obtain the final interval weights of the alternatives. Entani and Tanaka (2007) also used the interval normalization constraint in their method. In this method, from given crisp comparison data, interval weights are obtained using upper approximation approach.

In this study, inspired from Chandran et al.'s method, we propose a method to generate interval weights from interval comparison matrices and compare the performance of this method with other interval weight generation methods which are namely Sugihara et al.'s (2004) UA and LA methods, Wang et al.'s (2005a) TLGP method, Wang and Elhag's (2007) GP. This method can handle both consistent and inconsistent judgments.

CHAPTER 3

ANALYTIC HIERARCHY PROCESS (AHP)

3.1 General Overview:

Analytic hierarchy process (AHP) (Saaty, 1980) is the one of the most widespread used MCDM approach. The logic behind AHP is forming a hierarchical structure in which top level of the hierarchy is the global objective of the system. By going down in the hierarchical structure a more specific hierarchical level is reached. And this decomposition is done until the most basic and manageable criterion level is reached. In this hierarchical structure both qualitative and quantitative criteria can be handled. This principle of AHP is one of the important properties that are used in MCDM since in most of the problems decision makers have to consider quantitative and qualitative criteria together.

3.2 Hierarchical Structures in AHP

A hierarchy, defined in dictionary, is an arrangement of items (objects, names, values, categories, etc.), in which the items are represented as being "above," "below," or "at the same level as" one another. In AHP, hierarchy is the structure in which all the alternatives, criteria, sub-criteria, goals are listed. A sample of the hierarchical structure is shown in Figure 3.1.

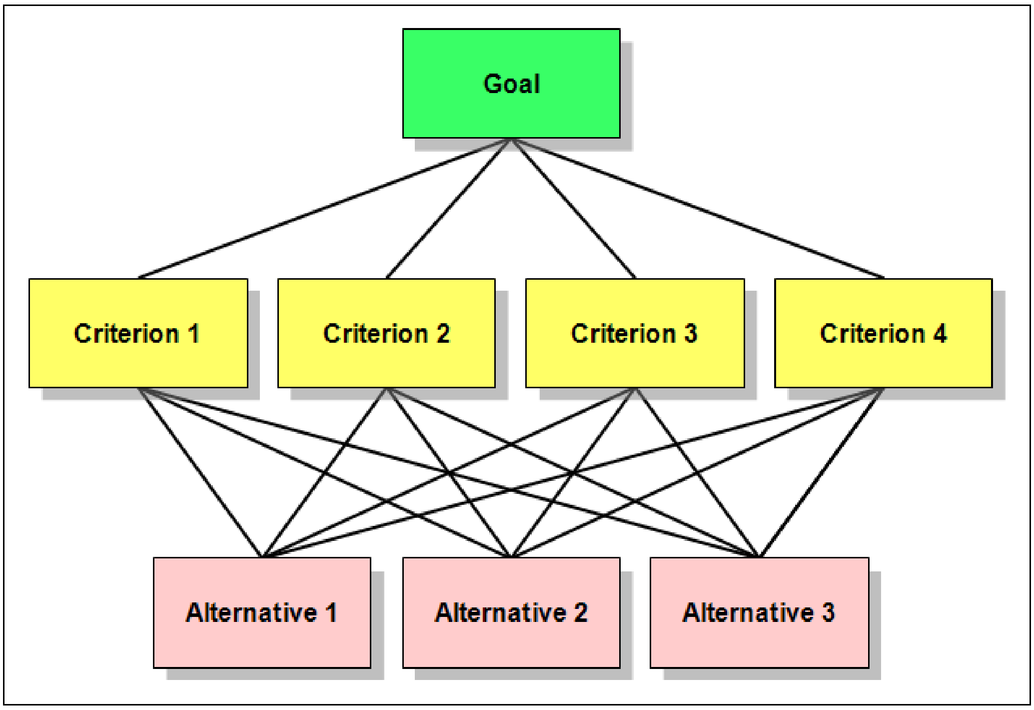

Figure 3.1 A sample of hierarchical structure in AHP [Wikipedia]

As shown in Figure 3.1, alternatives, criteria and goal are structured. The goal can further be broken down to sub-goals, sub-goals into criteria, criteria into subcriteria and etc. Each element of the hierarchical structure is written in different nodes and the arrows combining the alternatives, criteria and goal show the dependency among them. The structure of the hierarchy is not only dependent on the nature of the problem but also on the knowledge, profession, background, etc. of the decision-maker.

3.3 Establishing the Priorities

After constructing the hierarchical structure of the problem, what is left is setting priorities of the alternatives and criteria. At this point pairwise comparisons of the elements are made to obtain the priorities. An illustration of the pairwise comparison matrix is given in Figure 3.2:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

Figure 3.2 An illustration of pairwise comparison matrix

where in this matrix the following equations are valid:

- $a_{ij} = 1$ if $i = j$.
- $a_{ij} = \dfrac{1}{a_{ji}}$ $i, j = 1,2, \ldots ,n$

For the elements in the A comparison matrix the 1-9 scale is used to define the pairwise comparison of the alternatives/criteria. The judgments used in the 1-9 scale are explained in Table 3.1 (Saaty, 1980).

Table 3.1 1-9 scale

Intensity of Judgments	Definition	Explanation
1	Equal Importance	Two activities contribute equally to the objective
3	Weak importance one over another	Experience and judgment slightly favor one activity over another
5	Essential or Strong importance	Experience and judgment strongly favor one activity over another
7	Very strong or demonstrated importance	An activity is favored very strongly over another; its dominance demonstrated in practice

Intensity of Judgments	Definition	Explanation
9	Absolute importance	The evidence favoring one activity over another is of the highest possible order of affirmation
2, 4, 6, 8	Intermediate values between adjacent scale values	When compromise is needed
Reciprocals of above nonzero	If activity *i* has one of the above nonzero numbers assigned to it when compared with activity *j*, then *j* has the reciprocal value when compared with *i*	A reasonable assumption

After filling the pairwise comparison matrices according to the 1-9 scale in the system, the local priority weights are determined by using the Eigenvalue Method. The formulation used in this method is shown below:

$$Aw = \lambda_{\max} w \tag{3.1}$$

where w is the right eigenvector and $\lambda_{\max}$ is the maximum eigenvalue for the comparison matrix A.

The decision-maker makes pairwise comparisons for every alternative and criterion in the system and the question of consistency arises at this point. The full consistency of a comparison is described by the following way (Saaty, 1980): If A is 2 times more important than B, B is 3 times more important than

C, then A has to be 6 times more important than C. This relation has to be valid for every element in the system and can be formulated as follows:

$$a_{ij} = a_{ik} a_{kj} \qquad i, j = 1,2, \ldots, n \tag{3.2}$$

In the consistent case the calculated $\lambda_{\max}$ will be equal to n and the given comparison matrix A will have rank 1. The local priority weights of the alternatives can be found by normalizing one of the columns of the A matrix.

However, in real life it is sometimes hard to make a consistent comparison. If the given matrix is an inconsistent one, the consistency index of the matrix can be calculated by the following formula:

$$CI = \frac{\lambda_{\max} - n}{n - 1} \tag{3.3}$$

This consistency index shows the “closeness to consistency” for a matrix. The consistency ratio of the matrix, which measures whether the matrix is sufficiently consistent or not, is calculated by the following formula:

$$CR = \frac{CI}{RI} \tag{3.4}$$

where RI is the random index, which is calculated by the scientists in the Oak Ridge National Laboratory for randomly generated matrices using a sample size of 100. The RI increases as the order of the matrix increases. In Table 3.2 the RI of the matrices are given.

Table 3.2 Random indices (Saaty, 1980)

Order	1	2	3	4	5	6	7	8	9
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45

In general, if consistency ratio is less than 0.1 the matrix is said to be consistent enough. Otherwise, the entries that are given by the decision maker have to be revised until a satisfactory consistency index is obtained.

CHAPTER 4

PRIORITY WEIGHT GENERATION METHODS

In this chapter, the AHP methods which generate interval priority weights from interval comparison matrices are investigated. In order to explain the theory behind the investigated methods, some of the methods that generate priority weights and that use precise comparison matrix are also presented. As a result, the methods investigated in this chapter can be divided into three subgroups: interval weight generation methods from interval comparison matrix (interval-interval), precise weight generation methods from interval comparison matrix (interval-precise) and interval weight generation methods from precise comparison matrix (precise-interval).

4.1 Interval Weight Generation Methods From Precise Comparison Matrix (Precise-Interval)

4.1.1 Possibilistic Analytic Hierarchy Method

Possibilistic AHP for Crisp Data (PAHPC) method is presented by Sugihara (2004) in which interval priority weights are obtained from precise comparison matrices. This method is the foundation stone of the Lower/Upper Approximation Methods, presented also in the same paper (Sugihara, 2004).

Let us assume that A is the comparison matrix given by the decision-maker:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

The matrix A is said to be consistent if the elements of the matrix A satisfy the following condition:

$$a_{ij} = a_{ik} \times a_{kj} \qquad i, j = 1,2,\ldots,n \tag{4.1}$$

This condition is also called strong transitivity (Arbel, 1989). The weak transitivity (Arbel, 1989) relation is satisfied when in the order relation of the alternatives the following relation is satisfied: if A is preferred to B, and B is preferred to C, then, A is preferred to C. The matrix not satisfying the weak transitivity is called non-transitive which is also called inconsistent. In this method inconsistent comparison data is used in order to obtain the final weights.

Let us assume that the estimated interval weights are shown as $W_i = [\, w_i^L, w_i^U]$ where w_i^L is the lower and w_i^U is the upper bound of the interval weight W_i. The interval matrix can be obtained as follows:

$$W_{ij} = \left[\frac{w_i^L}{w_j^U}, \frac{w_i^U}{w_j^L} \right] \qquad i, j = 1,2,\ldots, n.\ i \neq j \tag{4.2}$$

When it is compared with conventional (precise) AHP, two new constraints are introduced in order to satisfy the normalization of the final interval priority weights. These constraints are explained in the following definition.

Definition 1 (Interval Normalization), (Sugihara and Tanaka, 2001): An interval weight set is normalized if and only if the following equations are satisfied:

$$\sum_i w_i^U - \max_j (w_j^U - w_j^L) \le 1, \qquad (4.3)$$

$$\sum_i w_i^L + \max_j (w_j^U - w_j^L) \ge 1, \qquad (4.4)$$

These constraints can also be written as follows:

$$w_j^U + \sum_{i=1, i \ne j}^{n} w_i^L \le 1, \qquad j = 1,2,\ldots,n. \qquad (4.5)$$

$$w_j^L + \sum_{i=1, i \ne j}^{n} w_i^U \ge 1, \qquad j = 1,2,\ldots,n, \qquad (4.6)$$

In order to show these definitions in an example let us assume that W_1 = [0.2, 0.8], W_2 = [0.2, 0.3] and W_3 = [0.1, 0.4]. The value 0.2 cannot be taken as W_1 since there is no such a combination of W_2 and W_3 which satisfies the equation (4.6). Also W_1 cannot be taken as 0.8 since there is no such a combination of W_2 and W_3 which satisfies the equation (4.5). The interval weights can be a normalized weight vector if W_1 = [0.5, 0.6], W_2 = [0.2, 0.4] and W_3 = [0.1, 0.2] which satisfies the equations (4.5) and (4.6) together.

So the following theorem can be written accordingly:

Theorem 1: If both of the equations (4.5) and (4.6) are satisfied, there is a weight vector w = (w_1, w_2, … , w_n) such that

$$\sum_i w_i = 1, \qquad i = 1,2,\ldots, n \qquad (4.5)$$

$$w_i^L \le w_i \le w_i^U \qquad i = 1,2,\ldots, n. \qquad (4.6)$$

In the PAHP for a given a_{ij} in order to find the interval weights W_i the following conditions has to be satisfied:

- The given pairwise comparison matrices have to be between the obtained upper and lower weights:

$$\frac{w_i^L}{w_j^U} \le a_{ij} \le \frac{w_i^U}{w_j^L} \quad \leftrightarrow \quad a_{ij} w_j^U \ge w_i^L \quad a_{ij} w_j^L \le w_i^U \qquad (4.7)$$

  where w_i^L is the lower weight and w_i^U is the upper weight of the ith alternative.

- The normalization constraints shown in Definition 1 have to be satisfied.

- The obtained weight has to be such that the difference of the upper weight and the lower weight has to be minimized. From this condition the following objective function can be written:

$$\min \sum_i (w_i^U - w_i^L) \qquad (4.8)$$

Considering the above conditions the following linear program can be written as follows:

$$\text{Min} \quad J = \sum_i (w_i^U - w_i^L)$$

subject to

$$a_{ij} w_j^U \ge w_i^L \qquad i, j = 1,2,\dots, n$$

$$a_{ij} w_j^L \ge w_i^U \qquad i, j = 1,2,\dots, n$$

$$w_j^U + \sum_{i=1, i \neq j}^{n} w_i^L \leq 1, \qquad j = 1,2,\ldots,n.$$

$$w_j^L + \sum_{i=1, i \neq j}^{n} w_i^U \geq 1, \qquad j = 1,2,\ldots,n,$$

$$w_i^U \geq w_i^L, \qquad i = 1,2,\ldots, n$$

$$w_i^L \geq \in \qquad i = 1,2,\ldots, n$$

If the given comparison matrix is a consistent one, the objective function value J becomes 0. The objective function value shows the consistency of the given comparison matrix.

4.2 Precise Weight Generation Methods From Precise Comparison Matrix (Precise-Precise)

4.2.1 Linear Programming Approach

Chandran et al. (2005) proposed Linear Programming (LP) approach in order to generate precise priority weights from precise comparison data. The method has two stages. In the first stage, the objective function value having the minimum inconsistency for the given comparison matrix is found. In the second stage, by using this consistency bound, a priority weight vector is calculated.

Let the following equation be valid for the given comparison data:

$$\frac{w_i}{w_j} = a_{ij}\varepsilon_{ij}, \qquad i, j = 1,2,\ldots, n \tag{4.9}$$

where a_{ij} is the given comparison matrix data, w_i and w_j are the priorty weights for the ith and jth alternative respectively, and ε_{ij} is the error estimate for this equation. If the given comparison data is consistent, ε_{ij} will be equal to 1.

Equation (4.9) is not a linear one. Thus natural logarithm of the both side of the equation is taken and the following equation is obtained:

$$\ln(w_i) - \ln(w_j) - \ln(\varepsilon_{ij}) = \ln(a_{ij}) \qquad i, j = 1,2,\dots, n \qquad (4.10)$$

After replacing the $\ln(w_i)$ with x_i, $\ln(w_j)$ with x_j and $\ln(\varepsilon_{ij})$ with y_{ij} equation (4.10) will become:

$$x_i - x_j - y_{ij} = \ln(a_{ij}) \qquad i, j = 1,2,\dots, n \qquad (4.11)$$

For the given comparison matrix A, the error parameter (ε_{ij}) for both upper triangle and lower triangle is calculated. In the first stage of the method since the inconsistency is minimized, the maximum error for the comparison i vs j and j vs i is used. The following constraints are introduced in order to satisfy this condition:

$$z_{ij} \geq y_{ij}, \qquad i, j = 1,2,\dots, n \qquad (4.12)$$

$$z_{ij} \geq y_{ji}, \qquad i, j = 1,2,\dots, n \qquad (4.13)$$

Since there are infinitely many solutions valid in the solution set, a variable has to be fixed. In this case w_1 is set to 1 with the following constraint:

$$x_1 = 0 \qquad (4.14)$$

Chandran et al. (2005) included the element dominance and row dominance (Saaty and Vargas, 1984) properties of the comparison matrices in the proposed LP. In the element dominance, if $a_{ij} > 1$ for a comparison, it implies $w_i \geq w_j$. In the row dominance $w_i \geq w_j$ condition is implied if there is relation between i

and j such that $a_{ik} \geq a_{jk}$ for all k. These properties can be put in the LP by including the following constraints:

$$x_i - x_j \geq 0, \qquad i, j = 1,2,..., n;\ a_{ij} > 1 \qquad (4.15)$$

$$x_i - x_j \geq 0, \qquad i, j = 1,2,..., n;\ a_{ik} \geq a_{jk},\ \text{for all } k \qquad (4.16)$$

In the first stage of the proposed LP it is aimed to minimize the total inconsistency. The following objective function is introduced:

$$\text{Minimize} \qquad \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} z_{ij} \qquad (4.17)$$

After combining all of the mentioned constraints, sign constraints and objective function together the first stage model is obtained.

<u>The First Stage Model</u>:

$$\text{Minimize} \qquad \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} z_{ij}$$

subject to

$x_i - x_j - y_{ij} = \ln(a_{ij}),$ $\quad i, j = 1,2,..., n,\ i \neq j,$

$z_{ij} \geq y_{ij},$ $\quad i, j = 1,2,..., n,$

$z_{ij} \geq y_{ji},$ $\quad i, j = 1,2,..., n,$

$x_1 = 0,$

$x_i - x_j \geq 0,$ $\quad i, j = 1,2,..., n;\ a_{ij} > 1$

$x_i - x_j \geq 0,$ $\quad i, j = 1,2,..., n;\ a_{ik} \geq a_{jk},$ for all k

$z_{ij} \geq 0,$ $\quad i, j = 1,2,..., n,$

x_i, y_{ij} URS, $\quad i, j = 1,2,..., n.$

The model above minimizes the sum of logarithms of positive errors in the natural logarithm space. If the logarithm transformation was not done, the objective function would be the product of the errors (ε_{ij}). If the given comparison matrix is a perfectly consistent one, the optimal objective function value will be equal to zero.

After obtaining the optimal objective function value in the first stage, this value is used in the second stage. In the second stage, due to the fact that there can be multiple optimal solutions, the maximum error parameter (ε_{ij}) value is minimized. This can be done by the following linear model:

<u>The Second Stage Model</u>:

Minimize $\quad z_{\max}$

subject to

$$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} z_{ij} = z^*,$$

$$x_i - x_j - y_{ij} = \ln(a_{ij}), \qquad i, j = 1,2,\ldots, n,\ i \neq j,$$

$$z_{ij} \geq y_{ij}, \qquad i, j = 1,2,\ldots, n,$$

$$z_{ij} \geq y_{ji}, \qquad i, j = 1,2,\ldots, n,$$

$$z_{\max} \geq z_{ij}, \qquad i, j = 1,2,\ldots, n,$$

$$x_1 = 0,$$

$$x_i - x_j \geq 0, \qquad i, j = 1,2,\ldots, n;\ a_{ij} > 1$$

$$x_i - x_j \geq 0, \qquad i, j = 1,2,\ldots, n;\ a_{ik} \geq a_{jk},\ \text{for all } k$$

$$z_{ij} \geq 0, \qquad i, j = 1,2,\ldots, n,$$

$$x_i, y_{ij}\ \text{URS}, \qquad i, j = 1,2,\ldots, n,$$

$$z_{\max} \geq 0.$$

In Chandran et al. (2005), an approach to obtain crisp data from interval comparison matrices is also suggested. Let us assume that given interval's lower bound is l_{ij} and upper bound is u_{ij}. They revised the first and second stage of the precise-precise case by writing the geometric mean of the given interval values instead of the a_{ij} entries. This makes the method usable for interval-precise cases. The applied revision can be formalized as follows:

$$a_{ij} = (l_{ij} \times u_{ij})^{1/2} \qquad i, j = 1,2,..., n. \qquad (4.18)$$

For the element dominance case the following revisions are made:

- When $l_{ij} > 1$ is given, this means that the lower bound for this comparison is l_{ij} and the following equation can be written instead of equation (4.15) :

  $$x_i - x_j \geq \ln l_{ij}, \qquad i, j = 1,2,..., n. \qquad (4.19)$$

- When $u_{ij} < 1$ is given, this means that the upper bound for this comparison is u_{ij} and the following equation can be written instead of equation (4.16) :

  $$x_i - x_j \leq \ln u_{ij}, \qquad i, j = 1,2,..., n. \qquad (4.20)$$

Also the suggested model can be used while handling the interval and precise comparison data together.

4.3 Interval Weight Generation Methods From Interval Comparison Matrix (Interval-Interval)

In this section the methods which generate interval weight from interval comparison matrices are described.

4.3.1 Lower/Upper Approximation Methods

Sugihara et al. (2004) developed lower/upper approximation methods to generate interval priority weights from interval comparison matrices.

Let us assume that given comparison data is:

$$\left[A_{ij}\right] = \left[a_{ij}^{L}, a_{ij}^{U}\right]$$

where a_{ij}^{L} is the lower bound, a_{ij}^{U} is the upper bound for the given comparison data. In this given matrix the reciprocal property (Saaty and Vargas, 1987) is present, which can be shown as follows:

$$a_{ij}^{L} = \frac{1}{a_{ji}^{U}}, \qquad a_{ij}^{U} = \frac{1}{a_{ji}^{L}}, \qquad i, j = 1,2,\ldots, n, \qquad i \neq j. \qquad (4.21)$$

where $[A_{ii}] = [1, 1]$.

The aim of this method is to find a lower bound and an upper bound for the given comparison matrix. This relation can be shown as follows:

<u>For Lower Approximation:</u>

$W_{ij*} \subseteq \left[A_{ij}\right]$ Lower Approximation

$$a_{ij}^{L} \le \frac{\underline{w_{i*}}}{\overline{w_{j*}}}, \qquad \frac{\overline{w_{i*}}}{\underline{w_{j*}}} \le a_{ij}^{U}$$

$$a_{ij}^{L}\,\overline{w_{j*}} \le \underline{w_{i*}}, \quad a_{ij}^{U}\,\underline{w_{j*}} \ge \overline{w_{i*}} \qquad (4.22)$$

where $\overline{w_{i*}}$ is the upper priority weight and $\underline{w_{i*}}$ is the lower priority weight of the *i*th alternative obtained after lower approximation. a_{ij}^U and a_{ij}^L are the given upper and lower entry for the *i* vs *j* comparison.

For Upper Approximation:

$W_{ij}^* \supseteq [A_{ij}]$ Upper Approximation

$$a_{ij}^L \geq \frac{\underline{w_i^*}}{\overline{w_j^*}}, \qquad \frac{\overline{w_i^*}}{\underline{w_j^*}} \geq a_{ij}^U$$

$$a_{ij}^L \overline{w_j^*} \geq \underline{w_i^*}, \quad a_{ij}^U \underline{w_j^*} \geq \overline{\underline{w_i^*}} \tag{4.23}$$

The relations (4.22) and (4.23) are the main constraints that will be used in the lower approximation and upper approximation models, respectively.

The relation between the given data, the upper approximation model and the lower approximation model can also be shown in the Figure 4.1:

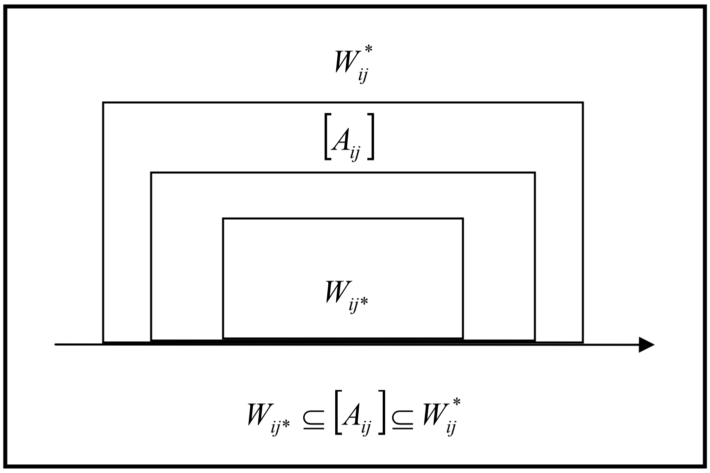

Figure 4.1 Upper and lower approximations (Sugihara et al., 2004)

By using the relations (4.22) and the normalization constraints (4.5) and (4.6) the Lower and Upper approximation models can be written as the follows:

<u>Lower Approximation Model:</u>

Maximize $\quad J_{*} = \sum_{i} (\overline{w_{i*}} - \underline{w_{i*}})$

subject to

$a^{L}_{ij}\,\overline{w_{j*}} \le \underline{w_{i*}}, \qquad i, j = 1,2,\ldots, n.$

$a^{U}_{ij}\,\underline{w_{j*}} \ge \overline{w_{i*}}. \qquad i, j = 1,2,\ldots, n.$

$\sum_{i\in\Omega-\{j\}}^{n} \underline{w_{i*}} + \overline{w_{j*}} \le 1, \qquad j = 1,2,\ldots,n.$

$\sum_{i\in\Omega-\{j\}}^{n} \overline{w_{i*}} + \underline{w_{j*}} \ge 1,, \qquad j = 1,2,\ldots,n,$

$\overline{w_{i*}} \ge \underline{w_{i*}}, \qquad i = 1,2,\ldots, n$

$\underline{w_{i*}} \ge \in \qquad i = 1,2,\ldots, n$

Upper Approximation Model:

Minimize $\quad J^{*} = \sum_{i} (\overline{w_i^{*}} - \underline{w_i^{*}})$

subject to

$a^{L}{}_{ij}\, \overline{w_j^{*}} \geq \underline{w_i^{*}}, \qquad i, j = 1,2,\ldots, n,$

$a^{U}{}_{ij}\, \underline{w_j^{*}} \geq \overline{w_{i^{*}}}, \qquad i, j = 1,2,\ldots, n,$

$\sum_{i \in \Omega - \{j\}}^{n} \underline{w_i}^{*} + \overline{w_j}^{*} \leq 1, \qquad j = 1,2,\ldots,n.$

$\sum_{i \in \Omega - \{j\}}^{n} \overline{w_i}^{*} + \underline{w_j}^{*} \geq 1,, \qquad j = 1,2,\ldots,n,$

$\overline{w_i^{*}} \geq \underline{w_i}^{*}, \qquad i, j = 1,2,\ldots, n,$

$\underline{w^{*}} \geq \in, \qquad i, j = 1,2,\ldots, n.$

In the upper approximation model, it is guaranteed that there exists an optimal solution. However, this is not the case for the lower approximation model. The comparison matrix has to be a consistent one in order to have an optimal solution in the lower approximation model (Wang and Elhag, 2006).

Comparison of the Interval Priorities:

After obtaining the interval priority weights, the interval priority weights have to be compared in order to prioritize the alternatives. However, the comparison of the interval weights is hard, when it is compared to precise data comparison. In Sugihara et al's (2004) study some preference relations between the interval weights are presented.

It is defined that the interval weight $W_i = [\underline{w_i}, \overline{w_i}]$ is preferred to $W_j = [\underline{w_j}, \overline{w_j}]$ if and only if the following relationships hold:

$$\underline{w_i} \geq \underline{w_j}\,, \qquad \overline{w_i} \geq \overline{w_j}\,.$$

The above relationship is also shown in Figure 4.2.

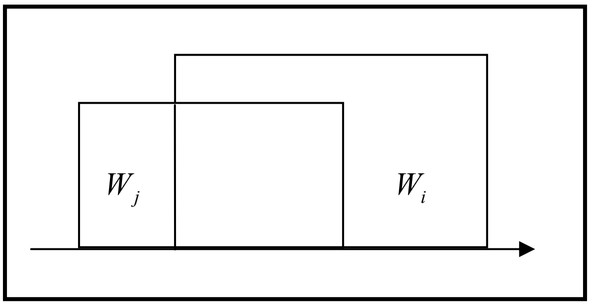

Figure 4.2 An example of $W_i \succ W_j$

A second relationship is described for the case where $W_i \supseteq W_j$ or $W_i \subseteq W_j$, the interval weight $W_i = [\underline{w_i}, \overline{w_i}]$ is preferred to $W_j = [\underline{w_j}, \overline{w_j}]$ if and only if:

$$\underline{w_i} + \overline{w_i} \geq \underline{w_j} + \overline{w_j}\,, \qquad \overline{w_i} - \underline{w_i} \leq \overline{w_j} - \underline{w_j}\,,$$

This means that the center of W_i is larger than the center of W_j while the width of W_i is narrower than the width of W_j.

4.3.2 Two Stage Logarithmic Goal Programming Method

Wang et al. (2005a) suggested a two stage logarithmic goal programming method in order to generate interval priority weights from interval comparison matrices. The method is applicable to both consistent and inconsistent matrices.

In the first stage, the objective function having the minimum inconsistency is found and then in the second stage, by using the objective function value found in the first stage, minimum and maximum interval values for each alternative is calculated. If there are n alternatives in the problem, the second stage goal program has to be solved $2n$ times in order to obtain the final interval priority weights.

Let us assume that lower bound is l_{ij} and upper bound is u_{ij} and the given interval comparison matrix is shown as:

$$A = \begin{bmatrix} 1 & [l_{12}, u_{12}] & \cdots & [l_{1n}, u_{1n}] \\ [l_{21}, u_{21}] & 1 & \cdots & [l_{2n}, u_{2n}] \\ \vdots & \vdots & \vdots & \vdots \\ [l_{n1}, u_{n1}] & [l_{n2}, u_{n2}] & \cdots & 1 \end{bmatrix}$$

where $l_{ij} = 1/u_{ji}$ and $u_{ij} = 1/l_{ji}$.

Definition 2 (Interval Consistency) (Wang et al., 2005a): Given an interval comparison matrix $A = (a_{ij})_{\mathrm{nxn}}$, with $l_{ij} \le a_{ij} \le u_{ij}$ and $a_{ii} = l_{ii} = u_{ii} = 1$ for i, j = 1, 2, … , n, if the following convex feasible region $S = \left\{ w = (w_1, w_2, ..., w_n) \mid l_{ij} \le w_i / w_j \le u_{ij}, \sum_{i=1}^{n} w_i = 1, w_i > 0 \right\}$ is not empty, then A is said to be a consistent interval comparison matrix. Otherwise, it is an inconsistent one.

Theorem 2 (Wang et al., 2005b): $A = (a_{ij})_{\mathrm{nxn}}$ is said to be a consistent interval comparison matrix if and only if the following inequality:

$$\max_k(l_{ik}l_{kj}) \le \min_k(u_{ik}u_{kj}), \qquad \text{for all } i, j, k = 1, \ldots, \text{n}.$$

is satisfied.

For a priority weight vector $w = (w_1, w_2, \ldots, w_n)$, there are two possible constraints available to be used for this weight vector. The first one is additive constraint which can be shown as:

$$\sum_{i=1}^{n} w_i = 1, \qquad i, j = 1, 2, \ldots, \text{n}. \tag{4.24}$$

And the multiplicative constraint can be shown as follows:

$$\prod_{i=1}^{n} w_i = 1, \qquad i, j = 1, 2, \ldots, \text{n}.$$

which is also equal to:

$$\sum_{i=1}^{n} \ln w_i = 0. \qquad i, j = 1, 2, \ldots, \text{n}. \tag{4.25}$$

In the method proposed by Wang et al. (2005) the multiplicative constraint is used.

The interval judgments given in the comparison matrix A can be written as the following:

$$l_{ij} \le w_i / w_j \le u_{ij}, \qquad i, j = 1,2, \ldots, \text{n}. \tag{4.26}$$

And in order to make the inequality (4.26) linear the natural logarithm of the both sides is taken and the following constraint is obtained:

$$\ln l_{ij} \le \ln w_i - \ln w_j \le \ln u_{ij}, \qquad i, j = 1,2, \dots ,n. \tag{4.27}$$

However, in the real life situations the above constraint is hardly satisfied since the decision makers mostly make inconsistent comparisons. Thus, in order to satisfy the constraint, two error terms are introduced, p_{ij} and q_{ij}:

$$\ln l_{ij} - p_{ij} \le \ln w_i - \ln w_j \le \ln u_{ij} + q_{ij}, \qquad i, j = 1,2, \dots ,n. \tag{4.28}$$

The error terms p_{ij} and q_{ij} are nonnegative numbers, but only one of them can be positive which can be expressed as:

$$p_{ij} q_{ij} = 0 \qquad i, j = 1,2, \dots ,n. \tag{4.29}$$

In the consistent case both of the error parameters are equal to zero. The degree of the inconsistency of the interval comparison matrix can be found by the sum of the p_{ij} and q_{ij} values:

$$J = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(p_{ij} + q_{ij}\right) \tag{4.30}$$

By combining the equations (4.25), (4.28), (4.29) and (4.30) the following goal programming model can be obtained:

Minimize $\quad J = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(p_{ij} + q_{ij}\right)$

subject to

$\ln w_i - \ln w_j + p_{ij} \leq \ln l_{ij}$ $\quad i, j = 1,2,\ldots,n,$

$\ln w_i - \ln w_j - q_{ij} \leq \ln u_{ij}$ $\quad i, j = 1,2,\ldots,n,$

$\sum_{i=1}^{n} \ln w_i = 0,$

$p_{ij} q_{ij} = 0,$ $\quad i, j = 1,2,\ldots,n,$

$p_{ij}, q_{ij} \geq 0$ $\quad i, j = 1,2,\ldots,n.$

Since, the logarithmic parameters are positive when $w_i \geq 1$ and negative when $w_i < 1$ the following positive variables are introduced in order to simplify the program:

$$x_i = \frac{\ln w_i + |\ln w_i|}{2}, \qquad i = 1,2,\ldots,n,$$

$$y_i = \frac{-\ln w_i + |\ln w_i|}{2}, \qquad i = 1,2,\ldots,n.$$

And $\ln w_i$ can be expressed as

$$\ln w_i = x_i - y_i, \qquad i = 1,2,\ldots,n.$$

And by adding the newly introduced parameters in to the first stage of the proposed model will become:

The First Stage Model:

Minimize $\quad J = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left(p_{ij} + q_{ij}\right)$

subject to

$x_i - y_i + y_j - x_j + p_{ij} \geq \ln l_{ij}$, i = 1,2,…,n-1, j = i+1,…,n,

$x_i - y_i + y_j - x_j - q_{ij} \leq \ln u_{ij}$, i = 1,2,…,n-1, j = i+1,…,n,

$\sum_{i=1}^{n} \ln w_i = 0$,

$x_i y_i = 0$, i = 1,2,…,n,

$x_i , y_i \geq 0$, i = 1,2,…,n,

$p_{ij} q_{ij} = 0$, i = 1,2,…,n-1, j = i+1,…,n,

$p_{ij} , q_{ij} \geq 0$ i = 1,2,…,n-1, j = i+1,…,n.

For the above model, if the optimal objective function value J is equal to 0, this means that the given matrix is an interval comparison matrix.

In the second stage of the proposed method, the obtained objective function value is used in order to find the upper and lower priority weights of the alternatives. The proposed second stage program is as follows:

Maximize/Minimize $\ln w_i = x_i - y_i$

subject to

$x_i - y_i + y_j - x_j + p_{ij} \geq \ln l_{ij}$, i = 1,2,…,n-1, j = i+1,…,n,

$x_i - y_i + y_j - x_j - q_{ij} \leq \ln u_{ij}$, i = 1,2,…,n-1, j = i+1,…,n,

$\sum_{i=1}^{n} \ln w_i = 0$,

$x_i y_i = 0$, i = 1,2,…,n,

$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left(p_{ij} + q_{ij} \right) = J^*$,

$x_i , y_i \geq 0$, i = 1,2,…,n,

$p_{ij} q_{ij} = 0$, i = 1,2,…,n-1, j = i+1,…,n,

$p_{ij}, q_{ij} \geq 0$ $\qquad$ i = 1,2,…,n-1, $\qquad$ j = i+1,…,n,

Synthesis of the Interval Priorities:

It is not a straightforward task to combine the obtained interval weights to calculate the final composite weights of the alternatives. In Wang et al. (2005a) a method for synthesizing the obtained interval weights is proposed. Consider that the obtained priority weights are shown in Table 4.1.

Table 4.1 Complete set of priority weights obtained

Alternative	Criterion 1	Criterion 2	…	Criterion m	Composite weight
	$[w_1^L, w_1^U]$	$[w_2^L, w_2^U]$	…	$[w_m^L, w_m^U]$	
A_1	$[w_{11}^L, w_{11}^U]$	$[w_{12}^L, w_{12}^U]$	…	$[w_{1m}^L, w_{1m}^U]$	$[w_{A_1}^L, w_{A_1}^U]$
A_2	$[w_{21}^L, w_{21}^U]$	$[w_{22}^L, w_{22}^U]$	…	$[w_{2m}^L, w_{2m}^U]$	$[w_{A_2}^L, w_{!A_2}^U]$
$\vdots$	$\vdots$	$\vdots$	…	$\vdots$	$\vdots$
A_n	$[w_{n1}^L, w_{n1}^U]$	$[w_{n2}^L, w_{n2}^U]$	…	$[w_{nm}^L, w_{nm}^U]$	$[w_{A_n}^L, w_{A_n}^U]$

In the proposed two stage logarithmic goal programming the multiplicative constraint is used, so synthesizing the interval priorities has to be done accordingly. Wang et al. (2005a) proposed a pair of nonlinear programming models which will help the decision maker to synthesize the obtained weights:

Minimize $\qquad w_{A_i}^L = \prod_{j=1}^{m} \left(w_{ij}^L\right)^{w_j}$

subject to

$W \in \Omega_W$.

Maximize $\quad w_{A_i}^U = \prod_{j=1}^{m} \left(w_{ij}^U\right)^{w_j}$

subject to

$W \in \Omega_W$.

where $W = (w_1, w_2, \ldots, w_n)$ is the weight vector.

$\Omega_W = \left\{ W = (w_1, w_2, \ldots, w_n) \mid w_j^L \le w_j \le w_j^U, \prod_{i=1}^{m} w_j = 1 \right\}$ is the feasible region and the lower and upper bound of the composite weight is $w_{A_i} = \left[w_{A_i}^L, w_{A_i}^U\right]$. The above non-linear program has to be solved for each alternative in order to obtain the final composite weights.

Comparison of the Interval Priorities:

As it is described in the previous method, Wang et al. (2005a) also suggested a comparison method for the interval priority weights. However, different than Sugihara et al.'s (2004) method, the degree of preference between the alternatives is calculated in the suggested method.

Let us assume that $a = [a_1, a_2]$ and $b = [b_1, b_2]$ are two interval weights. The possible relationship between these two interval weights are shown in Figure 4.4.

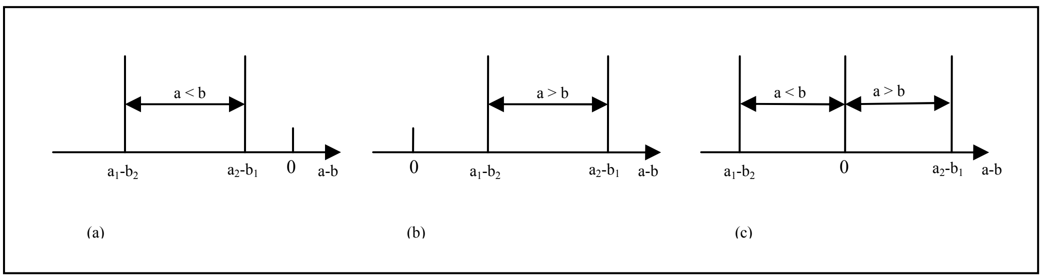

Figure 4.3 Possible relationships between two interval weights *a* and *b*.

The degree of preference of a over b is calculated by using the following formula:

$$P(a > b) = \frac{\max(0, a_2 - b_1) - \max(0, a_1 - b_2)}{(a_2 - a_1) + (b_2 - b_1)} \tag{4.31}$$

With the same logic the degree of preference of b over a is calculated by the following formula:

$$P(a > b) = \frac{\max(0, b_2 - a_1) - \max(0, b_1 - a_2)}{(a_2 - a_1) + (b_2 - b_1)} \tag{4.32}$$

Since a_2-b_1 and a_1-b_2 are the maximum and the minimum of a-b, the above equations can be written as follows:

$$P(a > b) = \frac{\max(0, \max(a - b)) - \max(0, \min(a - b))}{\max(a - b) - \min(a - b)} \tag{4.33}$$

$$P(b > a) = \frac{\max(0, \max(b - a)) - \max(0, \min(b - a))}{\max(b - a) - \min(b - a)} \tag{4.34}$$

From the above equations it is easy to see that $P(a > b)$ + $P(b > a)$ = 1. If $a = b$, then $a_1 = b_1$ and $a_2 = b_2$. The preference relation for this case will be $P(a > b)$ = $P(b > a)$ = 0.5.

If $P(a > b)$ > $P(b > a)$, then it is said that a is superior to b to the degree of $P(a > b)$. If $P(a > b)$ = $P(b > a)$, then a is said to be indifferent to b. And if

$P(b > a) > P(a > b)$, *a* is said to be inferior to *b* to the degree of $P(b > a)$. The described relations are tabulated in Table 4.2.

Table 4.2 Preference relations illustrations

Preference Relation	Illustration
$P(a > b) > P(b > a)$	$a \overset{P(a>b)}{\succ} b$
$P(a > b) = P(b > a)$	$a \sim b$
$P(b > a) > P(a > b)$	$a \overset{P(b>a)}{\prec} b$

There are some properties defined for this comparison method:

- $P(a > b) = 1$ if and only if $a_1 \geq b_2$
- If $a_1 \geq b_1$ and $a_2 \geq b_2$, then $P(a > b) \geq 0.5$ and $P(b > a) \leq 0.5$.
- If *b* is nested in *a*, ie. $a_1 \leq b_1$ and $a_2 \geq b_2$, then $P(a > b) \geq 0.5$ if and only if $\frac{a_1 + a_2}{2} \geq \frac{b_1 + b_2}{2}$.
- If $P(a > b) \geq 0.5$ and $P(b > c) \geq 0.5$, then $P(a > c) \geq 0.5$.

These properties help the decision-maker to make a complete ranking of the obtained interval priority weights. In the method proposed by Wang et al. (2005a) the following steps are followed while ranking the final weights:

1. By using the equations (4.33) and (4.34) the preference degree matrix has to be calculated:

$$
\begin{array}{c}
\\ P_D = \\ \\
\end{array}
\begin{array}{c|cccc}
 & w_1 & w_2 & \cdots & w_n \\
\hline
w_1 & - & p_{12} & \cdots & p_{1n} \\
w_2 & p_{21} & - & \cdots & p_{2n} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
w_n & p_{n1} & p_{n2} & \cdots & -
\end{array}
$$

2. After obtaining the degree of preference relations between the weights, a directed diagram has to be drawn for the preference relations satisfying $p_{ij} \geq 0.5$ relation. An example of the directed diagram is shown in Figure 4.4:

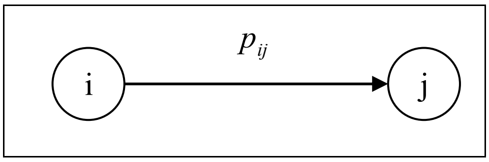

Figure 4.4 Preference relation of w_i and w_j on a directed diagram

3. From the drawn diagram, the complete preference rankings of the alternatives are obtained. In the case of complex relations between interval weights, a simple row-column elimination method can be used. Firstly, a row where all elements are greater than 0.5 is found. The corresponding alternative takes the first place in the ranking and the row and column of this alternative is eliminated from the matrix. In the remaining matrix a row again having all elements greater than zero is zero. And this alternative is ranked as the second one and the corresponding row and column of this alternative are eliminated. The iteration is repeated until a complete ranking is obtained.

4.3.3 Goal Programming Method

Third and the last interval priority weight generation method from the interval comparison data is the goal programming method proposed by Wang and Elhag (2007). The aim of this method is to obtain the final interval priority weights by solving only one linear model.

In this method the lower interval values and upper interval values are tabulated separately in the A_U and A_L matrix, respectively, where $A_L \leq A \leq A_U$. The A, A_U and A_L can be shown as follows:

$$A = \begin{bmatrix} 1 & [l_{12}, u_{12}] & \cdots & [l_{1n}, u_{1n}] \\ [l_{21}, u_{21}] & 1 & \cdots & [l_{2n}, u_{2n}] \\ \vdots & \vdots & \vdots & \vdots \\ [l_{n1}, u_{n1}] & [l_{n2}, u_{n2}] & \cdots & 1 \end{bmatrix}$$

$$A_L = \begin{bmatrix} 1 & l_{12} & \cdots & l_{1n} \\ l_{21} & 1 & \cdots & l_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ l_{n1} & l_{n2} & \cdots & 1 \end{bmatrix} \qquad A_U = \begin{bmatrix} 1 & u_{12} & \cdots & u_{1n} \\ u_{21} & 1 & \cdots & u_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ u_{n1} & u_{n2} & \cdots & 1 \end{bmatrix}$$

The values of the $a = [l_{ij}, u_{ij}] \approx [w_i^L, w_i^U]/[w_j^L, w_j^U]$ in the A matrix has to be normalized interval weight vector according to the normalization constraints (4.5) and (4.6).

If the relation $[l_{ij}, u_{ij}] \approx [w_i^L, w_i^U]/[w_j^L, w_j^U]$ is taken into account A matrix can be written as:

$$
A=\begin{bmatrix}
1 & \frac{[w_1^L, w_1^U]}{[w_2^L, w_2^U]} & \cdots & \frac{[w_1^L, w_1^U]}{[w_n^L, w_n^U]} \\
\frac{[w_2^L, w_2^U]}{[w_1^L, w_1^U]} & 1 & \cdots & \frac{[w_2^L, w_2^U]}{[w_n^L, w_n^U]} \\
\vdots & \vdots & \cdots & \vdots \\
\frac{[w_n^L, w_n^U]}{[w_1^L, w_1^U]} & \frac{[w_n^L, w_n^U]}{[w_2^L, w_2^U]} & \cdots & 1
\end{bmatrix}
$$

The division operation rule on interval numbers is described as:

$$[a_L, a_U]/[b_L, b_U] = [a_L / b_U, a_U / b_L] \tag{4.37}$$

By the help of this equation the A matrix can be written as follows:

$$
A=\begin{bmatrix}
1 & \left[\frac{w_1^L}{w_2^U}, \frac{w_1^U}{w_2^L}\right] & \cdots & \left[\frac{w_1^L}{w_n^U}, \frac{w_1^U}{w_n^L}\right] \\
\left[\frac{w_2^L}{w_1^U}, \frac{w_2^U}{w_1^L}\right] & 1 & \cdots & \vdots \\
\vdots & \vdots & \cdots & \vdots \\
\left[\frac{w_n^L}{w_1^U}, \frac{w_n^U}{w_1^L}\right] & \left[\frac{w_n^L}{w_2^U}, \frac{w_n^U}{w_2^L}\right] & \cdots & 1
\end{bmatrix}
$$

The A matrix can be separated into A_L and A_U matrices. And the following equations can be obtained from these matrices:

$$A_L W_U = W_U + (n-1)W_L \tag{4.38}$$

$$A_U W_L = W_L + (n-1)W_U \tag{4.39}$$

where W_L and W_U are the lower and upper priority weight vectors respectively.

Since there is uncertainty in the decision maker's judgments the equations above may not hold precisely. The deviation errors can be calculated by the following equations:

$$\mathrm{E} = (A_L - I)W_U - (n-1)W_L \tag{4.40}$$

$$\Gamma = (A_U - I)W_L - (n-1)W_U \tag{4.41}$$

where E and Γ are the error parameter vectors for each of the equations.

The following optimization model can be constructed:

Minimize $\quad J = \sum_{i=1}^{n} \left(|\varepsilon_i| + |\gamma_i| \right)$

subject to

$$(A_L - I)W_U - (n-1)W_L - \mathrm{E} = 0,$$

$$(A_U - I)W_L - (n-1)W_U - \Gamma = 0,$$

$$w_i^L + \sum_{j=1, j\neq i}^{n} w_j^U \geq 1, \qquad i, j = 1,2,\ldots,n,$$

$$w_i^U + \sum_{j=1, j\neq i}^{n} w_j^L \leq 1, \qquad i, j = 1,2,\ldots,n,$$

$$W_U - W_L \geq 0,$$

$$W_U, W_L \geq 0.$$

Let,

$$\varepsilon_i^+ = \frac{\varepsilon_i + |\varepsilon_i|}{2} \text{ and, } \varepsilon_i^- = \frac{-\varepsilon_i + |\varepsilon_i|}{2}, \qquad i=1,\ldots,n,$$

$$\gamma_i^+ = \frac{\gamma_i + |\gamma_i|}{2} \text{ and, } \gamma_i^- = \frac{-\gamma_i + |\gamma_i|}{2}, \qquad i=1,\ldots,n,$$

Based on ε_i^+ and ε_i^-, ε_i and $|\varepsilon_i|$ can be written as

$$\varepsilon_i = \varepsilon_i^+ - \varepsilon_i^-, \qquad i = 1,\ldots,n,$$

$|\varepsilon_i| = \varepsilon_i^{+} + \varepsilon_i^{-}$, $\quad i = 1,\ldots,n.$

And the optimization model will become:

$$\text{Minimize} \qquad J = \sum_{i=1}^{n} \left(\varepsilon_i^{+} + \varepsilon_i^{-} + \gamma_i^{+} + \gamma_i^{-} \right)$$

subject to

$$(A_L - I)W_U - (n-1)W_L - \mathrm{E}^{+} + \mathrm{E}^{-} = 0,$$

$$(A_U - I)W_L - (n-1)W_U - \Gamma^{+} + \Gamma^{-} = 0,$$

$$w_i^L + \sum_{j=1, j\neq i}^{n} w_j^U \geq 1, \qquad i, j = 1,2,\ldots,n,$$

$$w_i^U + \sum_{j=1, j\neq i}^{n} w_j^L \leq 1, \qquad i, j = 1,2,\ldots,n,$$

$$W_U - W_L \geq 0,$$

$$W_U, W_L, \Gamma^{+}, \Gamma^{-}, \mathrm{E}^{+}, \mathrm{E}^{-} \geq 0$$

<u>Synthesis of the Interval Priorities:</u>

Like in the previous method presented by Wang et al. (2005a), Wang and Elhag (2007) also suggested a method to synthesize the obtained weights assuming that weights are additive. Let us again assume that the obtained weights are shown in the Table 4.2. The following pair of linear programs is suggested by Bryson and Mobolurin (1995):

$$\text{Minimize} \qquad w_{A_i}^L = \sum_{j=1}^{m} w_{ij}^L w_j$$

subject to

$W \in \Omega_W$.

Maximize $\quad w_{A_i}^U = \sum_{j=1}^{m} w_{ij}^U w_j$

Subject to

$W \in \Omega_W$.

where $W = (w_1, w_2, \ldots, w_n)$ is the weight vector.

$\Omega_W = \left\{ W = (w_1, w_2, \ldots, w_n) \mid w_j^L \le w_j \le w_j^U, \sum_{j=1}^{m} w_j = 1, j = 1, \ldots, m \right\}$ is the feasible region and the lower and upper bound of the composite weight is $w_{A_i} = \left[w_{A_i}^L, w_{A_i}^U \right]$. The above linear program has to be solved for each alternative in order to obtain the final composite weights.

<u>Comparison of the Interval Priorities:</u>

After synthesizing the interval weights the comparison of the interval weights is made. In this study Wang and Elhag (2007) used the upper/lower approximation methods' comparison method developed by Sugihara et al. (2004).

CHAPTER 5

PROPOSED METHOD

In this chapter the proposed method that generates interval priority weights from interval comparison matrices is described. The proposed method is inspired from the two-stage linear programming method presented by Chandran et al. (2005). Firstly, the proposed model is explained. Then, two variations of the proposed model are described. Lastly, the synthesis and comparison of the obtained interval weights is presented.

5.1 Building The Model

Let the given interval comparison matrix A be as follows:

$$A = \begin{bmatrix} 1 & [l_{12}, u_{12}] & \dots & [l_{1n}, u_{1n}] \\ [l_{21}, u_{21}] & 1 & \dots & [l_{2n}, u_{2n}] \\ \vdots & \vdots & \vdots & \vdots \\ [l_{n1}, u_{n1}] & [l_{n2}, u_{n2}] & \dots & 1 \end{bmatrix}$$

In this interval comparison l_{ij} is the lower bound of the given comparison and u_{ij} is the upper bound of the comparison. The following relation is valid for all of the comparisons made in the matrix:

$$l_{ij} = 1/u_{ij} \qquad i, j = 1,2,\dots,n \qquad (5.1)$$

If it is assumed that for alternative i the interval priority weight is [w_i^L , w_i^U], then in a consistent comparison matrix the following equations will hold:

$$\frac{w_i^L}{w_j^U} = l_{ij} \qquad i, j = 1,2,\ldots,n. \tag{5.2}$$

$$\frac{w_i^U}{w_j^L} = u_{ij} \qquad i, j = 1,2,\ldots,n. \tag{5.3}$$

However, since in real life the decision maker can make inconsistent comparisons, the above equations will be satisfied by introducing error parameters $\in_{ij}$ and α_{ij} :

$$\frac{w_i^L}{w_j^U} = l_{ij} \in_{ij} \qquad i, j = 1,2,\ldots,n. \tag{5.4}$$

$$\frac{w_i^U}{w_j^L} = u_{ij}\alpha_{ij} \qquad i, j = 1,2,\ldots,n. \tag{5.5}$$

The error parameters will be equal to 1 if the given comparison is a consistent one. Equations (5.4) and (5.5) are not linear. So in order to linearise the equations the natural logarithm of both sides are taken:

$$\ln(w_i^L) - \ln(w_j^U) - \ln(\in_{ij}) = \ln(l_{ij}) \qquad i, j = 1,2,\ldots,n. \tag{5.6}$$

$$\ln(w_i^U) - \ln(w_j^L) - \ln(\alpha_{ij}) = \ln(u_{ij}) \qquad i, j = 1,2,\ldots,n. \tag{5.7}$$

After replacing $\ln(w_i^L)$ with x_i^L, $\ln(w_j^U)$ with x_j^U, $\ln(\in_{ij})$ with y_{ij} and $\ln(\alpha_{ij})$ with ω_{ij} equations (5.6) and (5.7) will become:

$$x_i^L - x_j^U - y_{ij} = \ln(l_{ij}) \qquad i, j = 1,2,\ldots,n. \tag{5.8}$$

$$x_i^U - x_j^L - \omega_{ij} = \ln(u_{ij}) \qquad i, j = 1,2,\ldots,n. \tag{5.9}$$

In the proposed model the objective function minimizes the sum of all the calculated errors which is shown below:

$$\text{Minimize} \qquad \sum_{i=1} \sum_{j=i+1} (z_{ij} + t_{ij}) \tag{5.10}$$

In the upper triangular matrix, the error of *i* vs *j* and in the lower triangular part, the error of *j* vs *i* is calculated. Thus, while calculating the errors both type of the errors is taken into account. However for a pair of alternatives only the largest error is counted. The following set of constraints is introduced in order to satisfy this situation:

$$z_{ij} \geq y_{ij} \qquad i, j = 1,2,\ldots,n;\ i < j \tag{5.11}$$

$$z_{ij} \geq y_{ji} \qquad i, j = 1,2,\ldots,n;\ i < j, \tag{5.12}$$

$$t_{ij} \geq \omega_{ij} \qquad i, j = 1,2,\ldots,n;\ i < j, \tag{5.13}$$

$$t_{ij} \geq \omega_{ji} \qquad i, j = 1,2,\ldots,n;\ i < j, \tag{5.14}$$

where z_{ij} and t_{ij} are the maximum of the error of *i* vs *j* and *j* vs *i* comparisons.

In the additive normalization for a given alternative the sum of the lower priority weight of that alternative and the upper priority of the rest of the alternatives must be greater than 1, which is formulated in equation (4.35). Also for a given alternative the sum of the upper priority weight of that alternative and the lower priority of the rest of the alternatives must be less than 1, which is formulated in equation (4.36). In the proposed method since the multiplicative constraint is used, the conventional normalization constraint is modified in the same logic which can be formulated as follows:

$$\prod_{\substack{j \neq i \\ j=1}} w_j^U \times w_i^L \geq 1 \tag{5.15}$$

$$\prod_{\substack{j \neq i \\ j=1}} w_j^L \times w_i^U \leq 1 \tag{5.16}$$

In order to linearise the above equations the natural logarithm of the both sides are taken:

$$\sum_{\substack{j \neq i \\ j=1}} \ln w_j^U + \ln w_i^L \geq 0 \tag{5.17}$$

$$\sum_{\substack{j \neq i \\ j=1}} \ln w_j^L + \ln w_i^U \leq 0 \tag{5.18}$$

By combining the above explained equations the following method is obtained:

Minimize $\quad \sum_{i=1} \sum_{j=i+1} (z_{ij} + t_{ij})$

subject to

$x_i^L - x_j^U - y_{ij} = \ln(l_{ij}) \qquad i, j = 1,2,\ldots,n$

$x_i^U - x_j^L - \omega_{ij} = \ln(u_{ij}) \qquad i, j = 1,2,\ldots,n$

$z_{ij} \geq y_{ij} \qquad i, j = 1,2,\ldots,n;\ i < j$

$z_{ij} \geq y_{ji} \qquad i, j = 1,2,\ldots,n;\ i < j$

$t_{ij} \geq \omega_{ij} \qquad i, j = 1,2,\ldots,n;\ i < j$

$t_{ij} \geq \omega_{ji} \qquad i, j = 1,2,\ldots,n;\ i < j$

$\sum_{\substack{j \neq i \\ j=1}} x_j^U + x_i^L \geq 0 \qquad i = 1,2,\ldots,n$

$\sum_{\substack{j \neq i \\ j=1}} x_j^L + x_i^U \leq 0 \qquad i = 1,2,\ldots,n$

$x_i^L \leq x_i^U \qquad i = 1,2,\ldots,n$

x_i^L, x_i^U URS $\qquad i = 1,2,\ldots,n$

y_{ij}, ω_{ij} URS $\qquad i, j = 1,2,\ldots,n$

$z_{ij}, t_{ij} \geq 0 \qquad\qquad i, j = 1,2,\ldots,n$

By solving the above linear program the required interval priority weights can be obtained. In this program, the decision-maker can use both precise and interval comparison data.

5.2 Variations Of The Proposed Method

In the proposed model, the objective function generates the interval priority weights from interval comparison matrices by minimizing the total inconsistency of the obtained weight data. In this chapter, two other variations of the proposed method is suggested.

5.2.1 First Variation (Introducing Second Stage)

As the first variation a second stage for the proposed method is introduced. In the second stage, by using the objective function value of the first stage the maximum error of the system is minimized. The following constraints are introduced to the program in order to calculate the maximum error parameter:

$$g_{\max} \geq z_{ij} \qquad i, j = 1,2,\ldots,n;\ i < j, \tag{5.19}$$

$$g_{\max} \geq t_{ij} \qquad i, j = 1,2,\ldots,n;\ i < j. \tag{5.20}$$

where $g_{\max}$ is the maximum error value.

And the objective function of the second stage program is:

$$\text{Minimize} \qquad g_{\max} \tag{5.21}$$

The second stage program can be written as follows:

Minimize $\quad g_{\max}$

subject to

$$
\begin{array}{lll}
x_i^L - x_j^U - y_{ij} = \ln(l_{ij}) & & i, j = 1,2,\ldots,n, \\
x_i^U - x_j^L - \omega_{ij} = \ln(u_{ij}) & & i, j = 1,2,\ldots,n, \\
z_{ij} \geq y_{ij} & & i, j = 1,2,\ldots,n;\ i < j, \\
z_{ij} \geq y_{ji} & & i, j = 1,2,\ldots,n;\ i < j, \\
t_{ij} \geq \omega_{ij} & & i, j = 1,2,\ldots,n;\ i < j, \\
t_{ij} \geq \omega_{ji} & & i, j = 1,2,\ldots,n;\ i < j, \\
\sum_{i=1} \sum_{j=i+1} (z_{ij} + t_{ij}) = z_{opt} & & \\
\sum_{\substack{j \neq i \\ j=1}} x_j^U + x_i^L \geq 0 & & i = 1,2,\ldots,n, \\
\sum_{\substack{j \neq i \\ j=1}} x_j^L + x_i^U \leq 0 & & i = 1,2,\ldots,n, \\
g_{\max} \geq z_{ij} & & i, j = 1,2,\ldots,n;\ i < j, \\
g_{\max} \geq t_{ij} & & i, j = 1,2,\ldots,n;\ i < j, \\
x_i^L \leq x_i^U & & i = 1,2,\ldots,n, \\
x_i^L, x_i^U & \text{URS} & i = 1,2,\ldots,n, \\
y_{ij}, \omega_{ij}, g_{\max} & \text{URS} & i, j = 1,2,\ldots,n, \\
z_{ij}, t_{ij} \geq 0 & & i, j = 1,2,\ldots,n,
\end{array}
$$

where z_{opt} is the optimal objective value of the first stage program.

5.2.2 Second Variation (Minimizing Maximum Error)

In the second variation of the proposed method, the objective function of the second stage that minimizes the maximum error parameter is used without restricting total inconsistency. The proposed method is shown below:

Minimize $g_{\max}$

subject to

$x_i^L - x_j^U - y_{ij} = \ln(l_{ij})$ $\quad$ $i, j = 1,2,\ldots,n,$

$x_i^U - x_j^L - \omega_{ij} = \ln(u_{ij})$ $\quad$ $i, j = 1,2,\ldots,n,$

$z_{ij} \geq y_{ij}$ $\quad$ $i, j = 1,2,\ldots,n;\ i < j,$

$z_{ij} \geq y_{ji}$ $\quad$ $i, j = 1,2,\ldots,n;\ i < j,$

$t_{ij} \geq \omega_{ij}$ $\quad$ $i, j = 1,2,\ldots,n;\ i < j,$

$t_{ij} \geq \omega_{ji}$ $\quad$ $i, j = 1,2,\ldots,n;\ i < j,$

$\sum_{\substack{j \neq i \\ j=1}} x_j^U + x_i^L \geq 0$ $\quad$ $i = 1,2,\ldots,n,$

$\sum_{\substack{j \neq i \\ j=1}} x_j^L + x_i^U \leq 0$ $\quad$ $i = 1,2,\ldots,n,$

$g_{\max} \geq z_{ij}$ $\quad$ $i, j = 1,2,\ldots,n;\ i < j,$

$g_{\max} \geq t_{ij}$ $\quad$ $i, j = 1,2,\ldots,n;\ i < j,$

$x_i^L \leq x_i^U$ $\quad$ $i = 1,2,\ldots,n,$

x_i^L, x_i^U URS $\quad$ $i = 1,2,\ldots,n,$

$y_{ij}, \omega_{ij}, g_{\max}$ URS $\quad$ $i, j = 1,2,\ldots,n,$

$z_{ij}, t_{ij} \geq 0$ $\quad$ $i, j = 1,2,\ldots,n,$

5.3 Synthesizing And Comparison Of The Obtained Weights

In this section a new procedure is presented in order to combine the obtained interval weights of an alternative for different criteria and find the final composite interval weight.

Let the problem have k criteria and n alternatives shown in Figure 5.1 and the interval weights of each alternative/criteria are given in Table 5.1.

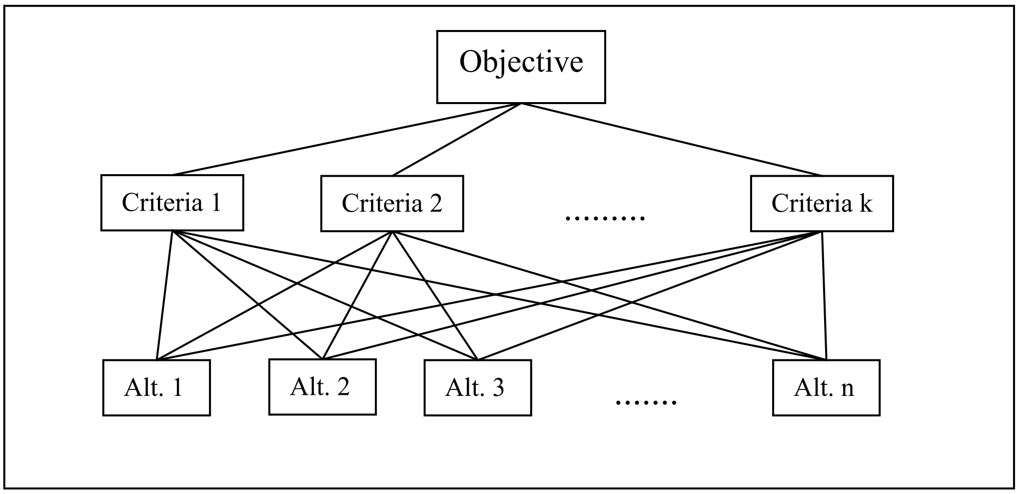

Figure 5.1 Hierarchy of criteria

Table 5.1 Complete set of weights

Alternative	Criteria 1	Criteria 2	…	Criteria k
	$\left[w_1^L, w_1^U\right]$	$\left[w_2^L, w_2^U\right]$	…	$\left[w_k^L, w_k^U\right]$
Alt. 1	$\left[w_{11}^L, w_{11}^U\right]$	$\left[w_{12}^L, w_{12}^U\right]$	…	$\left[w_{1k}^L, w_{1k}^U\right]$
Alt. 2	$\left[w_{21}^L, w_{21}^U\right]$	$\left[w_{22}^L, w_{22}^U\right]$	…	$\left[w_{3k}^L, w_{3k}^U\right]$
⋮	⋮	⋮	…	⋮
Alt. n	$\left[w_{n1}^L, w_{n1}^U\right]$	$\left[w_{n2}^L, w_{n2}^U\right]$	…	$\left[w_{nm}^L, w_{nm}^U\right]$

Since the weights are assumed to be multiplicative in the proposed methods, the composite weight of an alternative is obtained by multiplying the "effective weights" of that alternative for each criterion. The effective weight is defined as the weight of the alternative for a criterion to the power weight of that criterion and shown by the following formula:

$$w_{eff_{ij}} = \left(w_{ij}\right)^{w_j} \qquad (5.22)$$

where w_{ij} is the weight of the alternative i for the criterion j and w_j is the weight of the criterion j.

The minimum composite weight of an alternative is found by multiplying the minimum "effective weights" of that alternative for each criterion. Also with the same logic maximum composite weight is found as the product of the maximum "effective weights".

For a criterion $w_j = \left[w_j^L, w_j^U\right]$ and an alternative $w_{ij} = \left[w_{ij}^L, w_{ij}^U\right]$, there are four possible values for maximize/minimize the effective weight. This can be either $\left(w_{ij}^L\right)^{w_j^L}$, or $\left(w_{ij}^L\right)^{w_j^U}$,or $\left(w_{ij}^U\right)^{w_j^L}$,or $\left(w_{ij}^U\right)^{w_j^U}$. In order to determine a rule for the 1-9 scale, all possible alternatives are investigated in nine different regions which are shown in the following Table 5.2

Table 5.2 Regions defined for alternative and criterion values

Region 1	All base and power values are between 1/9 and 1
Region 2	Both base values are between 1/9 and 1, lower power value is between 1/9 and 1 and upper power value is greater than 1
Region 3	Both base values are between 1/9 and 1, both power values are greater than 1
Region 4	Lower base value is between 1/9 and 1 and upper base value is greater than 1 and both power values are between 1/9 and 1
Region 5	Lower base and power values are between 1/9 and 1 and upper base and power values are greater than 1
Region 6	Lower base value is between 1/9 and 1 and upper base value is greater than 1 and both power values are greater than 1
Region 7	Both base values are greater than 1 , both power values are between 1/9 and 1
Region 8	Both base values are greater than 1 , lower power value is between 1/9 and 1 and upper power value is greater than 1
Region 9	All base and power values are greater than 1

The base values defined in the table, are taken as alternatives' priority weights for a criterion and the power values defined in the table, are taken as the corresponding criterion's priority weights as in equation (5.22). For the nine regions shown in Table 5.2 a relation between criterion values $w_j = \left[w_j^L, w_j^U\right]$, alternative values $w_{ij} = \left[w_{ij}^L, w_{ij}^U\right]$, and the minimum/maximum possible values are searched.

For the first region both the alternative's and criterion's priority weights are between 1/9 and 1. The minimum possible value is obtained by $\left(w_{ij}^L\right)^{w_j^U}$ and the maximum possible value is obtained by $\left(w_{ij}^U\right)^{w_j^L}$.

For the rest of the regions the combinations of alternative and criterion bounds giving the maximum and minimum values are obtained is shown in Table 5.3.

Table 5.3 Maximum/Minimum effective weights of the regions

Region	Maximum	Minimum
Region 1	$\left(w_{ij}^{U}\right)^{w_j^L}$	$\left(w_{ij}^{L}\right)^{w_j^U}$
Region 2	$\left(w_{ij}^{U}\right)^{w_j^L}$	$\left(w_{ij}^{L}\right)^{w_j^U}$
Region 3	$\left(w_{ij}^{U}\right)^{w_j^L}$	$\left(w_{ij}^{L}\right)^{w_j^U}$
Region 4	$\left(w_{ij}^{U}\right)^{w_j^U}$	$\left(w_{ij}^{L}\right)^{w_j^U}$
Region 5	$\left(w_{ij}^{U}\right)^{w_j^U}$	$\left(w_{ij}^{L}\right)^{w_j^U}$
Region 6	$\left(w_{ij}^{U}\right)^{w_j^U}$	$\left(w_{ij}^{L}\right)^{w_j^U}$
Region 7	$\left(w_{ij}^{U}\right)^{w_j^U}$	$\left(w_{ij}^{L}\right)^{w_j^L}$
Region 8	$\left(w_{ij}^{U}\right)^{w_j^U}$	$\left(w_{ij}^{L}\right)^{w_j^L}$
Region 9	$\left(w_{ij}^{U}\right)^{w_j^U}$	$\left(w_{ij}^{L}\right)^{w_j^L}$

From Table 5.3 it can easily be seen that there are three different combinations of alternative and criterion weights from which maximum and minimum effective weights are obtained. In the first three regions the criterion weights change on the other hand, the alternative's weights are kept the same. The same situation is valid for Regions 4-5-6 and Regions 7-8-9. So it can be said that the maximum/minimum effective weight calculation is independent from the criterion weights values, i.e. it is dependent to the alternative's weight values. This can be shown as follows:

- If $w_{ij}^{L}, w_{ij}^{U} \geq 0,\ w_{ij}^{L}, w_{ij}^{U} \leq 1$

$\Rightarrow$ maximum $w_{eff_{ij}} = \left(w_{ij}^{U}\right)^{w_j^L}$, minimum $w_{eff_{ij}} = \left(w_{ij}^{L}\right)^{w_j^U}$

- If $w_{ij}^{L}, w_{ij}^{U} \geq 1$

$\Rightarrow$ maximum $w_{eff_{ij}} = \left(w_{ij}^{U}\right)^{w_j^U}$, minimum $w_{eff_{ij}} = \left(w_{ij}^{L}\right)^{w_j^L}$

- If $0 \le w_{ij}^{L} \le 1,\ \ w_{ij}^{U} \ge 1$

$\Rightarrow$ maximum $w_{eff_{ij}} = \left(w_{ij}^{U}\right)^{w_j^U}$, minimum $w_{eff_{ij}} = \left(w_{ij}^{L}\right)^{w_j^U}$

After multiplying the effective weights the minimum and maximum composite weights are obtained. In the next section the performance of the proposed synthesizing weight method and the Wang et al.'s (2005a) synthesizing method are compared. For the comparison of the interval priority weights Wang et al.'s (2005a) method is used because in this method the degree of preference of the alternatives are found. By the help of the degree of preference relation a decision-maker can observe the dominance level of an alternative instead of a complete ranking.

CHAPTER 6

COMPUTATIONAL EXPERIMENTS

In this chapter the performances of the proposed methods generating interval priority weights from interval comparison matrices are compared. In this chapter firstly, the performance measures used in the comparisons are described. Secondly, the methods are compared by using an example that is commonly used in the literature. Thirdly, the performance of the methods are tested on randomly generated matrices. And lastly, proposed interval matrix synthesizing methods are compared with a method developed by Wang et al. (2005a).

6.1 Performance Measures

Three methods explained in Chapter 4 and the proposed methods can generate interval priority weights from interval matrices. In order to determine the best performing method, some performance measures have to be defined. In literature, Wang and Elhag (2007) proposed the "fitted error" performance measure from which the comparison of the final interval weights are made.

In order to compare the weights using the fitted error performance measure, the obtained final weights have to be converted into the comparison matrix.

Let $W^L = \left(w_1^L, w_2^L, \ldots, w_n^L\right)$ be the lower and $W^U = \left(w_1^U, w_2^U, \ldots, w_n^U\right)$ be the upper priority weights obtained by using an interval priority weight generation method. These weights can be transformed into the comparison matrix by using the following equation:

$$\tilde{a}_{ij} = \left[\tilde{l}_{ij}, \tilde{u}_{ij}\right] = \left[w_i^L / w_j^U, w_i^U / w_j^L\right] \tag{6.1}$$

where, $\tilde{l}_{ij}$ and $\tilde{u}_{ij}$ is the lower and upper interval comparison elements. The comparison matrix $\tilde{A}$ will become:

$$\tilde{A} = \begin{bmatrix} 1 & \left[\dfrac{w_1^L}{w_2^U}, \dfrac{w_1^U}{w_2^L}\right] & \cdots & \left[\dfrac{w_1^L}{w_n^U}, \dfrac{w_1^U}{w_n^L}\right] \\ \left[\dfrac{w_2^L}{w_1^U}, \dfrac{w_2^U}{w_1^L}\right] & 1 & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots \\ \left[\dfrac{w_n^L}{w_1^U}, \dfrac{w_n^U}{w_1^L}\right] & \left[\dfrac{w_n^L}{w_2^U}, \dfrac{w_n^U}{w_2^L}\right] & \cdots & 1 \end{bmatrix}$$

As it is known that $\tilde{A}$ is not always equal to the interval comparison matrix A given by the decision maker, the difference between $\tilde{A}$ and A will be the error of the calculated priority weights. In Wang and Elhag's (2007) study, the fitted error is calculated as follows:

$$\text{Fitted Error} = \sum_{i=1}^{n}\sum_{j=1}^{n}\left[\left(l_{ij} - \tilde{l}_{ij}\right)^2 + \left(u_{ij} - \tilde{u}_{ij}\right)^2\right] \tag{6.2}$$

In the fitted error the square of the difference between the comparison matrices is summed. This measure penalizes large errors since errors are squared. We define another measure called absolute error which is calculated by summing absolute values of the errors. Absolute error is useful because it measures inconsistency in the same units as the comparison matrices.

Absolute Error = $\sum_{i=1}^{n}\sum_{j=1}^{n}\left[\left|l_{ij}-\tilde{l}_{ij}\right|+\left|u_{ij}-\tilde{u}_{ij}\right|\right]$ (6.3)

In order to compare the different dimensioned matrices average of the defined errors have to be calculated considering the number of entries. Thus, the following modifications are made in the equations (6.2) and (6.3):

$$\text{Fitted Error} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}\left[\left(l_{ij}-\tilde{l}_{ij}\right)^2+\left(u_{ij}-\tilde{u}_{ij}\right)^2\right]}{2n} \tag{6.4}$$

$$\text{Absolute Error} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}\left[\left|l_{ij}-\tilde{l}_{ij}\right|+\left|u_{ij}-\tilde{u}_{ij}\right|\right]}{2n} \tag{6.5}$$

where *n* is the number of alternatives present in the system.

6.2 Comparison Of The Methods On An Illustrative Example

The example used by Arbel and Vargas (1993), Haines (1998) and Wang et al. (2005b) chosen. The models, are solved in the CPLEX (for LP) and BARON solvers of the GAMS (v.23.0) software. The results are inserted to Excel by using the GAMS-Excel interface codes.

The comparison matrix is:

$$A = \begin{bmatrix} 1 & [2,5] & [2,4] & [1,3] \\ \left[\frac{1}{5},\frac{1}{2}\right] & 1 & [1,3] & [1,2] \\ \left[\frac{1}{4},\frac{1}{2}\right] & \left[\frac{1}{3},1\right] & 1 & \left[\frac{1}{2},1\right] \\ \left[\frac{1}{3},1\right] & \left[\frac{1}{2},1\right] & [1,2] & 1 \end{bmatrix}$$

According to Theorem 2 presented in Chapter 4 the above comparison matrix is a consistent matrix. So the lower approximation method also can be used. By solving the models the following results are obtained:

<u>Lower Approximation Method (LA) (Sugihara et al., 2004):</u>

The obtained weights are:

$w_1 = [0.435, 0.522]$, $w_2 = [0.174, 0.217]$, $w_3 = [0.130, 0.174]$,

$w_4 = [0.174, 0.174]$

And the ranking of the alternatives is:

$$A_1 \overset{100½}{\succ} A_2 \overset{100½}{\succ} A_4 \overset{100½}{\succ} A_3$$

<u>Upper Approximation Method (UA) (Sugihara et al., 2004):</u>

The obtained weights are:

$w_1 = [0.333, 0.417]$, $w_2 = [0.083, 0.278]$, $w_3 = [0.093, 0.167]$,

$w_4 = [0.139, 0.333]$

And the ranking of the alternatives is:

$$A_1 \overset{100\%}{\succ} A_4 \overset{64\%}{\succ} A_2 \overset{69\%}{\succ} A_3$$

Two Stage Logarithmic Goal Programming Method (TSGPM) (Wang et al., 2005a) :

Since it is a consistent matrix the objective function value of the first stage model is equal to 0. After solving the second stage the obtained weights are:

$w_1 = [1.682, 2.449]$, $w_2 = [0.760, 1.107]$, $w_3 = [0.500, 0.841]$,

$w_4 = [0.687, 1.000]$

And the ranking of the alternatives is:

$A_1 \overset{100\%}{\succ} A_2 \overset{63.6\%}{\succ} A_4 \overset{76.4\%}{\succ} A_3$

Goal Programming Method (GP) (Wang and Elhag, 2007):

The obtained weights are:

$w_1 = [0.354, 0.552]$, $w_2 = [0.170, 0.248]$, $w_3 = [0.083, 0.168]$,

$w_4 = [0.159, 0.230]$

And the ranking of the alternatives is:

$A_1 \overset{100\%}{\succ} A_2 \overset{60\%}{\succ} A_4 \overset{94\%}{\succ} A_3$

The Proposed Method (PM):

The obtained weights are:

$w_1 = [1.257, 2.121]$, $w_2 = [0.707, 1.414]$, $w_3 = [0.471, 0.707]$,

$w_4 = [0.707, 0.943]$

And the ranking of the alternatives is:

$A_1 \overset{100\%}{\succ} A_2 \overset{75\%}{\succ} A_4 \overset{100\%}{\succ} A_3$

The First Variation of The Proposed Method (PMV1):

The obtained weights are:

$w_1 = [1.500, 2.250]$, $w_2 = [0.750, 1.500]$, $w_3 = [0.500, 0.750]$,

$w_4 = [0.750, 1.000]$

And the ranking of the alternatives is:

$$A_1 \overset{100\%}{\succ} A_2 \overset{75\%}{\succ} A_4 \overset{100\%}{\succ} A_3$$

The Second Variation of The Proposed Method (PMV2):

The obtained weights are:

$w_1 = [1.354, 2.471]$, $w_2 = [0.692, 0.947]$, $w_3 = [0.442, 0.806]$,

$w_4 = [0.662, 0.968]$

And the ranking of the alternatives is:

$$A_1 \overset{100\%}{\succ} A_2 \overset{50.8\%}{\succ} A_4 \overset{78.5\%}{\succ} A_3$$

From the results found, it can easily be seen that the rankings are slightly different. And the error calculated for each method is reported in Table 6.1.

Table 6.1 Calculated errors of the proposed methods for 4x4 matrix

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Fitted Error	1.24	1.81	0.78	1.52	0.79	0.76	0.92
Absolute Error	1.05	1.14	0.97	1.15	0.73	0.70	1.15

In this 4x4 matrix the first variation of the proposed method has the minimum error in both of the performance measure. Also proposed method and two stage logarithmic programming method gave small fitted errors. Absolute error of the proposed method is also small. The second variation of the proposed method produces moderate errors for all alternatives rather than small errors for some alternatives and extremely large ones for the rest due to the objective function. Therefore, with respect to the fitted error it shows the fourth best performance. However, in terms of the absolute error performance measure it shows the worst performance.

The result obtained from one example cannot be generalized. Thus, in the following section the performance of the methods are tested on randomly generated matrices.

6.3 Comparison Of The Methods On Randomly Generated Matrices

In our computational study firstly, ten 5x5 matrices are generated using Excel random number generation tool (see Appendix A). Then, for each comparison matrix by using the above methods the interval priority weights are generated. Lastly, the performance of each method is calculated in order to determine the best performing one. One of the randomly generated matrices and its results are shown below.

First 5x5 Matrix (Inconsistent):

The randomly generated matrix is as follows:

Table 6.2 First Randomly Generated 5x5 Matrix

	alt1		alt2		alt3		alt4		alt5	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
alt1	1	1	2	5	1/8	1/4	1/7	1/3	1/2	2
alt2	1/5	1/2	1	1	2	8	1/5	4	1/5	3
alt3	4	8	1/8	1/2	1	1	4	4	1/7	2
alt4	3	7	1/4	5	1/4	1/4	1	1	1/2	6
alt5	1/2	2	1/3	5	1/2	7	1/6	2	1	1

The randomly generated matrix is an inconsistent one so the lower approximation method cannot be applied. By using this matrix the following priority weights are obtained for the weight generation methods:

Table 6.3 Calculated interval weights for 5x5 matrix1

	LA		UA		TSGPM		GP		PM		PMV1		PMV2	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
alt1	Infeasible		0.029	0.231	0.370	0.758	0.039	0.039	0.684	1.595	1.146	1.146	0.785	1.138
alt2			0.046	0.248	0.315	2.551	0.050	0.590	0.419	2.392	0.458	1.834	1.047	1.806
alt3			0.031	0.233	1.480	3.031	0.220	0.228	0.299	4.187	1.450	1.450	1.038	1.792
alt4			0.058	0.260	0.500	1.760	0.144	0.144	1.047	4.785	0.573	2.292	0.742	1.194
alt5			0.043	0.231	0.740	1.516	0.001	0.540	0.797	2.093	0.573	2.292	0.916	1.580

In the table, one of the weights is written in red because when the goal programming method is applied the lower bound of alternative 5 is found as 0. But while converting the priority weights into comparison matrices this 0 value leads to an undefined entry. Thus an epsilon value is taken as 0.001 which is less than the lowest lower bound of the alternative weights. From these priority weights the following error values are found for the methods:

Table 6.4 Calculated errors for 5x5 matrix1

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Fitted Error	-	33.44	27.12	416.18	47.77	20.68	24.25
Absolute Error	-	6.78	6.39	16.27	6.46	5.54	6.66

For this 5x5 matrix the table shows that the first variation of the proposed method has the best performance among the other methods for both performance criteria. Two stage logarithmic programming and second variation of the proposed method also show good performance. The maximum error value is found in the goal programming method. When the error values for this method are compared with the other methods' error values, goal programming method has the highest error. Since the priority weights' ratio is used while calculating the comparison matrix $\tilde{A}$, some of the entries are found to be higher than the maximum allowable value for the comparison, which is 9. The calculated $\tilde{A}$ matrix is shown below:

Table 6.5 Calculated comparison matrix for the goal programming method

	Alt1		Alt2		Alt3		Alt4		Alt5	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Alt1	1	1	0.07	0.77	0.17	0.18	0.27	0.27	0.07	3.87
Alt2	1.29	15.24	1	1	0.22	2.68	0.35	4.09	0.09	58.97
Alt3	5.70	5.89	0.37	4.55	1	1	1.53	1.58	0.41	22.76
Alt4	3.72	3.72	0.24	2.88	0.63	0.65	1	1	0.27	14.40
Alt5	0.26	13.95	0.02	10.78	0.04	2.45	0.07	3.75	1	1

The values greater than 9 are highlighted with red. After observing this situation in the matrices, a smoothing operation has been done to the calculated comparison matrices. For all of the methods, numbers greater than 9 are taken as 9 and the fitted and absolute errors are calculated again. With the smoothing operation the following errors are calculated:

Table 6.6 Calculated errors for 5x5 matrix1 after smoothing operation

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Fitted Error	-	33.44	26.05	36.54	26.29	20.68	24.25
Absolute Error	-	6.78	6.33	8.06	5.42	5.54	6.66

As it can be seen from the table above, after the smoothing operation the worst performing method is again the goal programming method. But this time the error values are significantly smaller than the non-smoothed calculations. For the fitted errors the best performing one is the first variation of the proposed method. However, in the absolute error calculations the best performing method is the proposed method.

Other 5x5 Matrices (Inconsistent):

These calculations are made for the other nine 5x5 matrices (see Appendix B). The obtained interval weights are tabulated in appendix part and the performance criteria for non-smoothed and smoothed case are shown below:

Table 6.7 Error values of 5x5 inconsistent matrices

			LA	UA	TSGPM	GP	PM	PMV1	PMV2
Fitted Error	Non-smoothed	Average	-	47.33	42.50	558.35	29.84	26.55	46.58
		Std. Dev.	-	12.93	20.60	653.25	11.15	10.51	40.35
	Smoothed	Average	-	46.03	30.29	34.49	27.03	24.24	30.89
		Std. Dev.	-	11.48	9.50	11.54	8.69	7.69	12.27
Absolute Error	Non-smoothed	Average	-	8.48	7.91	18.04	6.07	5.79	8.26
		Std. Dev.	-	1.47	1.86	8.81	1.51	1.53	1.69
	Smoothed	Average	-	8.34	6.99	7.75	5.89	5.62	7.50
		Std. Dev.	-	1.34	1.55	1.90	1.52	1.40	2.02

The 5x5 randomly generated matrices show that for both cases (smoothed and non-smoothed) the first variation of the proposed method and the proposed method shows the best performances in both of the performance criteria. Standard deviations of these two methods are small, this means their performances for all matrices are almost same. When both of the methods' absolute error values are compared it can be seen there is little difference between them. If the fitted error values are compared, first variation of the proposed method showed better performance than proposed method.

In the non-smoothed cases the goal programming method showed the worst performance among the other methods for both of the performance criteria. However, the situation changes in the smoothed case. In the smoothed cases, upper approximation method has the worst performance. But again goal programming has an average error value of 7.75 which is still worse than other methods.

In order to use the lower approximation method the matrix has to be consistent, which is in this case is the eighth matrix. In this matrix the lower approximation method generated higher error when it is compared with the three proposed methods and the two stage logarithmic programming method. But an exact judgment cannot be done according to the results of a matrix.

<u>5x5 Consistent Matrices:</u>

Ten 5x5 consistent matrix are generated according to the Theorem 2 explained in the Chapter 4. In the Excel random number generator tool, to make the generated numbers in accordance with the theorem, the lower interval values are randomly generated from the numbers $l = \left\{ \frac{1}{9}, \frac{1}{8}, \frac{1}{7}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2} \right\}$ and the upper interval values are generated from the number set $u = \{1,2,3,4,5,6,7,8,9\}$. The following results are obtained for the 5x5 consistent matrices:

Table 6.8 Error values of 5x5 consistent matrices

			LA	UA	TSGPM	GP	PM	PMV1	PMV2
Fitted Error	Non-smoothed	Average	20.27	28.82	96.95	1399.21	13.81	10.78	14.48
		Std. Dev.	4.41	7.90	77.46	2788.83	3.49	3.39	4.73
	Smoothed	Average	20.27	27.83	27.41	14.81	12.06	10.59	13.19
		Std. Dev.	4.41	8.09	8.53	5.19	3.17	3.48	3.29
Absolute Error	Non-smoothed	Average	4.48	5.86	12.15	17.10	3.43	3.31	4.67
		Std. Dev.	0.60	1.04	4.92	19.53	0.67	0.66	0.73
	Smoothed	Average	4.48	5.77	6.45	4.33	3.27	3.28	4.47
		Std. Dev.	0.60	1.05	1.13	0.90	0.70	0.67	0.60

From the above results it can be seen that for the consistent matrices the best performing models are again the first variation of the proposed method and the proposed method. In the non-smoothed case for the fitted error measure the worst performing model is goal programming. Eventhough consistent matrices were used, the error values of the goal programming method are much higher than the other methods'. After the smoothing operation the worst performing model for the fitted error is upper approximation and for the absolute error is two stage logarithmic programming.

<u>10x10 and 15x15 Matrices:</u>

In this part performance of the methods are tested on randomly generated inconsistent and consistent 10x10 and 15x15 matrices are investigated (see Appendix C). The results are shown in the following tables:

Table 6.9 Error values of 10x10 inconsistent matrices

			LA	UA	TSGPM	GP	PM	PMV1	PMV2
Fitted Error	Non-smoothed	Average	-	129.05	82.00	31448.64	60.73	60.18	91.41
		Std. Dev.	-	23.16	17.24	12920.88	15.71	14.38	17.62
	Smoothed	Average	-	128.44	76.02	110.07	58.82	59.56	89.49
		Std. Dev.	-	23.20	15.71	24.29	14.31	14.74	19.54
Absolute Error	Non-smoothed	Average	-	22.06	17.86	237.55	14.40	14.48	19.78
		Std. Dev.	-	2.63	2.19	68.63	2.31	2.44	2.40
	Smoothed	Average	-	21.99	17.35	20.98	14.25	14.42	19.63
		Std. Dev.	-	2.64	2.05	3.06	2.21	2.46	2.56

Table 6.10 Error values of 10x10 consistent matrices

			LA	UA	TSGPM	GP	PM	PMV1	PMV2
Fitted Error	Non-smoothed	Average	73.09	74.60	161.13	11255.68	27.49	27.36	39.55
		Std. Dev.	8.02	9.91	136.50	3708.11	2.98	3.01	8.82
	Smoothed	Average	73.09	74.01	54.99	43.63	27.24	27.02	36.91
		Std. Dev.	8.02	9.93	12.63	7.76	2.77	2.72	4.88
Absolute Error	Non-smoothed	Average	15.76	15.23	22.05	112.73	8.79	8.79	11.52
		Std. Dev.	1.26	1.33	9.47	29.76	0.56	0.53	1.09
	Smoothed	Average	15.76	15.16	13.88	12.03	8.76	8.75	11.25
		Std. Dev.	1.26	1.33	1.55	1.04	0.54	0.51	0.88

Table 6.11 Error values of 15x15 inconsistent matrices

			LA	UA	TSGPM	GP	PM	PMV1	PMV2
Fitted Error	Non-smoothed	Average	-	223.90	137.87	29416.98	94.34	93.68	151.73
		Std. Dev.	-	12.77	16.52	19403.51	8.96	8.68	14.94
	Smoothed	Average	-	223.90	130.51	179.83	93.99	93.36	151.69
		Std. Dev.	-	12.77	12.88	15.57	8.99	8.78	14.94
Absolute Error	Non-smoothed	Average	-	37.06	29.68	308.75	23.66	23.66	32.40
		Std. Dev.	-	1.35	1.61	72.31	1.42	1.42	1.86
	Smoothed	Average	-	37.06	29.10	33.47	23.57	23.58	32.36
		Std. Dev.	-	1.35	1.63	1.63	1.42	1.43	1.86

Table 6.12 Error values of 15x15 consistent matrices

			LA	UA	TSGPM	GP	PM	PMV1	PMV2
Fitted Error	Non-smoothed	Average	123.96	130.34	130.54	10285.12	41.65	41.21	63.63
		Std. Dev.	11.32	8.83	39.32	11320.33	5.00	5.13	7.37
	Smoothed	Average	123.96	130.34	77.68	64.06	41.61	41.17	62.95
		Std. Dev.	11.32	8.83	11.79	7.79	5.00	5.13	7.17
Absolute Error	Non-smoothed	Average	26.71	25.99	25.68	126.33	14.31	14.27	18.43
		Std. Dev.	1.42	0.97	3.46	40.73	1.06	1.04	1.20
	Smoothed	Average	26.71	25.99	20.73	17.86	14.27	14.23	18.33
		Std. Dev.	1.42	0.97	1.60	1.13	1.06	1.05	1.18

From the tables above for the 10x10 and 15x15 matrices, it can easily be seen that the best performing methods are proposed method and the first variation of the proposed method. Errors and standard deviations are significantly small compared to the other methods' magnitudes of the errors. But this situation is not valid for the second variation of the proposed method. The second variation's standard deviation is higher than the other proposed ones. In some of the cases second variation has error values close to the proposed method and first variation method but mostly its error values are slightly higher.

The goal programming method has the worst performance when the non-smoothed cases are considered. After the smoothing operation the lower approximation and upper approximation methods have the maximum error values. Also the standard deviations of lower approximation, two stage logarithmic goal programming and goal programming methods are moderately higher than those of the proposed methods.

To summarize, the proposed method and the first variation of the proposed method show the best performance for the interval comparison matrices in the computational experiments. When the dimension of the matrices increases, the difference of the error calculations of the methods become more visible. The error values of the 5x5 matrices are smaller than those of 10x10 and 15x15. This means that the error values increase as the dimensions of the matrix increase. The second variation of the proposed method showed an inconsistent performance. In some cases the error values remain small but still the performance of the second variation is fairly lower than the other proposed methods.

6.4 Comparison Of The Sythesizing Methods

After obtaining the interval priority weights of the comparison matrices, the results have to be combined in order to obtain the final composite weights of the alternatives. Wang et al. (2005a) used a synthesizing method for the multiplicative AHP methods. We develop a method to synthesize the obtained interval weights. In this section these two mentioned methods are compared on several examples.

Comparison on An Example:

Synthesizing methods are illustrated on an example problem taken from Wang et al. (2005a).

A person is interested in investing his money to any one of the four portfolios: bank deposit (BD), debentures (DB), government bonds (GB), and shares (SH). Out of these portfolios he has to choose only one based upon four criteria: return (Re), risk (Ri), tax benefits (Tb), and liquidity (Li). The hierarchical structure of the problem is shown in Figure 6.1.

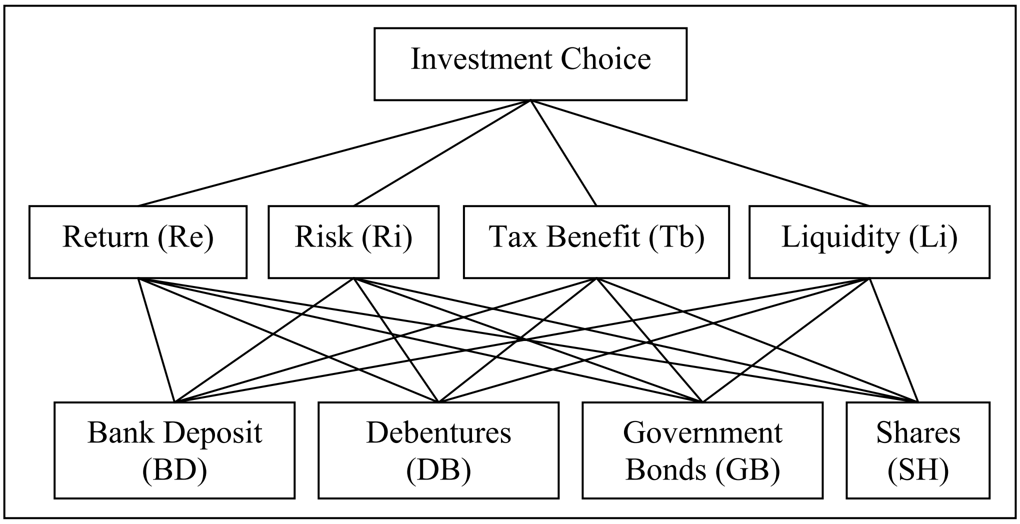

Figure 6.1 Hierarchy structure of the example (Wang et al., 2005a)

And the obtained final interval weights of the criteria/alternatives and the composite weights of the alternatives are tabulated in Table 6.30.

Table 6.13 Obtained global and local weights of the alternatives and criteria by using Wang et al.'s synthesizing method (Wang et al., 2005a)

Portfolio	Re	Ri	Tb	Li	Global priority
	[2.5718, 4.2426]	[1.2910, 2.0933]	[0.4777, 0.7747]	[0.2357, 0.3888]	
BD	[0.5774, 0.7401]	[2.4719, 4.0536]	[0.4855, 0.4855]	[3.0274, 3.8337]	[0.2321, 10.3091]
DB	[1.4142, 1.8841]	[1.2209, 1.9168]	[0.4855, 0.4855]	[1.1892, 1.5651]	[1.9283, 45,1463]
GB	[0.2268, 0.3021]	[0.5774, 0.9306]	[2.9130, 2.9130]	[0.5046, 0.6389]	[0.0008, 0.0817]
SH	[3.0214, 4.2426]	[0.2193, 0.3618]	[1.4565, 1.4565]	[0.3195, 0.4495]	[0.5508, 137.2216]

The obtained local weights are used in our proposed method, steps of which are shown below:

Global Weight of BD:

For Re criterion the local interval weight of BD is $[0.5774, 0.7401]$ and the interval weight of the criteria Re is $[2.5718, 4.2426]$. The given value satisfies the properties of case 1 of the proposed method in which, while calculating the maximum global weight $\left(w_{ij}^{U}\right)^{w_j^L}$ formula is used. And while calculating the minimum global weight $\left(w_{ij}^{L}\right)^{w_j^U}$ formula is used. In this case $\left(w_{ij}^{U}\right)^{w_j^L}$ is equal to $(0.7401)^{2.5718}$ and $\left(w_{ij}^{L}\right)^{w_j^U}$ is equal to $(0.5774)^{4.2426}$. These calculations are repeated for local weights of BD for each criterion. Then, minimum and maximum weights of BD are calculated.

Minimum BD $= (0.7401)^{2.5718} \times (4.0536)^{2.0933} \times (0.4855)^{0.4777} \times (3.8337)^{0.3888}$
$= 0.2321$

Maximum BD $= (0.5774)^{4.2426} \times (2.4719)^{1.2910} \times (0.4855)^{0.7746} \times (3.0274)^{0.2357}$
$= 10.3095$

The same calculations are made for the other alternatives and the following results are obtained:

Table 6.14 Obtained global of the alternatives and criteria by using proposed synthesizing method

	Min	Max	Upper Deviation (%)	Lower Deviation (%)
BD	0.2321	10.3095	0.0183	0.0098
DB	1.8778	48.3509	2.6193	0.0710
GB	0.0007	0.0864	6.6355	0.0576
SH	0.5507	137.2219	0.0107	0.0000

As a result, it can be seen that the obtained global weights are close to each other in both of the methods. The difference between the methods is Wang et al.'s method is an exact method which requires the solution of two non-linear models having $\prod_{i=1}^{m} w_j = 1$ constraint, and the proposed method is a heuristic that finds relaxed interval weights without using a normalization constraint.

Also the performances of the methods are compared on randomly generated matrices (see Appendix D). We assume that there are five alternatives and five criteria in the problem and we generated five problems randomly.

Using both of the synthesizing methods the following results are obtained.

Table 6.15 Obtained global weights by using the synthesizing methods

		Wang et al.		Proposed		Upper	Lower
		Lower	Upper	Lower	Upper	Deviation (%)	Deviation (%)
Problem 1	Alt1	0.2191	22.7275	0.2113	49.4883	3.5525	117.7461
	Alt2	0.0008	32.3366	0.0005	32.5030	37.3246	0.5144
	Alt3	0.0553	185.0444	0.0553	303.9950	0.0000	64.2822
	Alt4	0.0355	122.6730	0.0332	209.4311	6.4205	70.7230
	Alt5	0.0117	21.3055	0.0117	29.0379	0.0000	36.2930
Problem 2	Alt1	0.0398	36.5868	0.0328	36.5868	17.6465	0.0000
	Alt2	0.0008	27.2242	0.0008	27.2242	0.0000	0.0000
	Alt3	0.1781	63.7921	0.0887	75.9426	50.2093	19.0471
	Alt4	0.0192	209.1730	0.0192	209.1730	0.0000	0.0000
	Alt5	0.0013	25.5640	0.0010	26.4148	23.3854	3.3281
Problem 3	Alt1	0.0007	31.6029	0.0002	31.6029	75.8883	0.0000
	Alt2	0.0006	12.7042	0.0001	17.8011	75.3458	40.1194
	Alt3	0.0009	9.0931	0.0004	12.6980	54.2446	39.6438
	Alt4	0.00019	23.2277	0.00004	26.7516	80.6553	15.1710
	Alt5	0.00011	12.9208	0.00003	14.5263	74.4798	12.4253
Problem 4	Alt1	0.0009	11.2780	0.0001	39.0538	84.5110	246.2832
	Alt2	0.0016	66.5126	0.0005	66.5126	69.4113	0.0000
	Alt3	0.0270	15.3668	0.0030	48.5695	88.7042	216.0679
	Alt4	0.0259	13.9959	0.0066	35.6913	74.4457	155.0120
	Alt5	0.0230	30.4689	0.0230	66.4315	0.0000	118.0306
Problem 5	Alt1	0.0182	63.6762	0.0129	63.6762	29.2536	0.0000
	Alt2	0.0007	171.2502	0.0002	289.4520	69.7651	69.0229
	Alt3	0.0182	132.2915	0.0076	137.8372	58.0536	4.1920
	Alt4	0.0217	146.6669	0.0217	203.1249	0.0000	38.4940
	Alt5	0.0051	77.4377	0.0038	163.1183	25.2413	110.6445

From the above results, it can be seen that the proposed heuristic showed large deviation from the Wang et al.'s synthesizing method. Only the second and third problems showed a reasonable deviation. All other problems have at least one entry having more than 100% percent deviation. But the affect of these deviations in the final ranking has to be investigated.

By using the degree of preference relation (Appendix D) the following rankings are obtained.

Table 6.16 Final rankings of the alternatives

Rankings									
Problem Set 1		Problem Set 2		Problem Set 3		Problem Set 4		Problem Set 5	
Wang et al.	Prop.	Wang et al.	Prop.	Wang et al.	Prop.	Wang et al.	Prop.	Wang et al.	Prop.
alt3	alt3	alt4	alt4	alt1	alt1	alt2	alt2	alt2	alt2
60.1%	59.2%	76.6%	73.4%	57.6%	54.2%	68.6%	50.1%	53.9%	58.8%
alt4	alt4	alt3	alt3	alt4	alt4	alt5	alt5	alt4	alt4
79.2%	80.9%	63.6%	67.5%	64.3%	60.0%	66.5%	57.8%	52.6%	55.5%
alt2	alt1	alt1	alt1	alt5	alt2	alt3	alt3	alt3	alt5
58.6%	60.5%	57.4%	57.4%	50.4%	55.1%	52.3%	55.4%	63.1%	54.2%
alt1	alt2	alt2	alt2	alt2	alt5	alt4	alt1	alt5	alt3
51.9%	52.8%	51.6%	50.8%	58.3%	53.4%	55.4%	52.2%	54.9%	68.4%
alt5	alt5	alt5	alt5	alt3	alt3	alt1	alt4	alt1	alt1

The results above show that the obtained global interval values in the proposed synthesizing method generate wider interval values than the synthesizing method proposed by Wang et al. (2005a) which is expected. The difference in the logic of two methods is the inclusion of the normalization constraint $\prod_{i=1}^{m} w_j = 1$. The final ranking of the alternatives showed that the best alternatives are same when this five problem set is considered. But in the rest of the alternatives some rank reversal situations are observed. This means that the deviation caused by the exclusion of the normalization constraint has caused the rank reversals in the problems. The problem set 2 is the only problem set that the rank reversal is not observed. In the other problem sets this rank reversal is observed only once. This rank reversal situation is observed in the cases where the degree of preference values are close to %50. For example, in the problem set 3, alternative 5 preferred to alternative 2 with a pereference degree of 50.4 % in Wang et al.'s method, however in proposed method, alternative 2 preferred to alternative 5 with a preference degree of 55.1%.

CHAPTER 7

CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORK

In this study, we develop an approach to generate interval priority weights from inconsistent and consistent interval comparison matrices. The proposed method minimizes the total error of the weights from the given comparison matrix. This method is inspired from the study performed by Chandran et al. (2005). And we also propose two more variations for the proposed method. In the first variation a second stage program is introduced which minimizes the maximum error, by using the first stage's objective function. And in the second variation, the proposed method's objective function is formulated as the minimization of the maximum error instead of the minimization of the total error.

In literature there are three important methods available which generate interval priority weights from interval comparison matrix and only one of them assumes that there is a multiplicative relation between the priority weights of the alternatives. All of these methods are mentioned and explained in detail in Chapter 4 of this study. Our proposed method also assume the multiplicative relation between the priority weights

The performances of the proposed methods and the other methods are compared according to the defined performance measure. In the computational experiments, the proposed method and the first variation of the proposed method showed the best performances in all of the randomly generated matrices. When these two methods are compared, the first variation showed a better performance as the dimension of the matrices increases. Since in the first variation it is required to solve two models and in the proposed method only one method is required, in the problems having many comparison matrices the proposed method can be used.

After obtaining the interval priority weights a new synthesizing algorithm is also developed. In this algorithm it is aimed to find a composite global weight which takes all the possible regions into account by relaxing the product of the priority weights equal to one constraint. As it is expected the obtained global weight interval is wider than the interval found by the method proposed by Wang et al. (2005a). The proposed method gives decision maker the opportunity to obtain the global interval weights just using an algorithm instead of solving a nonlinear program for each alternative twice. However, the proposed synthesizing method showed some deviation from the Wang et al. (2005a)'s method which leads rank reversal of the alternatives.

As a future work synthesizing methods can be compared on different hierarchically structured problems and the rank reversal of the alternatives can be investigated. The proposed methods can be applied to a real life problem and the results obtained can be compared with the other models.

The proposed interval judgments can be used in investment projects since the input resources and output of the project can only be forecasted in interval values. So an application to the project selection case can be done.

In some of the situations the decision-maker will be a group or an organization. An application of the proposed method to the group decision making situations can be done.

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APPENDIX A

RANDOMLY GENERATED MATRICES

A.1 5x5 Inconsistent Matrix Generation

Table A.1: 5x5 inconsistent matrix1

		alt1		alt2		alt3		alt4		alt5	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 1	alt1	1	1	2	5	1/8	1/4	1/7	1/3	1/2	2
	alt2	1/5	1/2	1	1	2	8	1/5	4	1/5	3
	alt3	4	8	1/8	1/2	1	1	4	4	1/7	2
	alt4	3	7	1/4	5	1/4	1/4	1	1	1/2	6
	alt5	1/2	2	1/3	5	1/2	7	1/6	2	1	1

Table A.2: 5x5 inconsistent matrix2

		alt1		alt2		alt3		alt4		alt5	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 2	alt1	1	1	1/2	9	1/8	4	1/4	2	1/9	1/4
	alt2	1/9	2	1	1	1/6	8	1/9	1/7	1	6
	alt3	1/4	8	1/8	6	1	1	1/2	7	1/6	1
	alt4	1/2	4	7	9	1/7	2	1	1	1/4	1
	alt5	4	9	1/6	1	1	6	1	4	1	1

Table A.3: 5x5 inconsistent matrix3

		alt1		alt2		alt3		alt4		alt5	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 3	alt1	1	1	2	8	1	3	1/7	3	5	7
	alt2	1/8	1/2	1	1	1/8	1/8	1/9	1/9	1/8	2
	alt3	1/3	1	8	8	1	1	1/8	1/5	1/7	1/6
	alt4	1/3	7	9	9	5	8	1	1	2	9
	alt5	1/7	1/5	1/2	8	6	7	1/9	1/2	1	1

Table A.4: 5x5 inconsistent matrix4

Matrix 4		alt1		alt2		alt3		alt4		alt5	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
	alt1	1	1	1/7	9	1	2	5	9	1/3	1/2
	alt2	1/9	7	1	1	1	1	1/2	8	1/7	7
	alt3	1/2	1	1	1	1	1	1/6	8	3	3
	alt4	1/9	1/5	1/8	2	1/8	6	1	1	1/8	7
	alt5	2	3	1/7	7	1/3	1/3	1/7	8	1	1

Table A.5: 5x5 inconsistent matrix5

Matrix 5		alt1		alt2		alt3		alt4		alt5	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
	alt1	1	1	1/9	1/6	4	9	5	6	1/8	1/3
	alt2	6	9	1	1	1/6	6	1/5	1/4	1/6	6
	alt3	1/9	1/4	1/6	6	1	1	1/7	1/5	1/3	4
	alt4	1/6	1/5	4	5	5	7	1	1	1/4	6
	alt5	3	8	1/6	6	1/4	3	1/6	4	1	1

Table A.6: 5x5 inconsistent matrix6

Matrix 6		alt1		alt2		alt3		alt4		alt5	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
	alt1	1	1	5	5	1/7	8	4	6	1/5	6
	alt2	1/5	1/5	1	1	6	7	1/6	5	1/8	1/7
	alt3	1/8	7	1/7	1/6	1	1	1/4	9	1/3	2
	alt4	1/6	1/4	1/5	6	1/9	4	1	1	4	6
	alt5	1/6	5	7	8	1/2	3	1/6	1/4	1	1

Table A.7: 5x5 inconsistent matrix7

Matrix 7		alt1		alt2		alt3		alt4		alt5	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
	alt1	1	1	1/9	4	7	9	1/3	3	1/5	1/3
	alt2	1/4	9	1	1	1/6	1/2	1/7	3	3	7
	alt3	1/9	1/7	2	6	1	1	1/7	2	5	6
	alt4	1/3	3	1/3	7	1/2	7	1	1	1/7	1/4
	alt5	3	5	1/7	1/3	1/6	1/5	4	7	1	1

Table A.8: 5x5 inconsistent matrix8

Matrix 8		alt1		alt2		alt3		alt4		alt5	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
	alt1	1	1	1/3	3	1/2	1	1/4	1/3	1/9	6
	alt2	1/3	3	1	1	1/3	1/2	1/9	3	1/8	6
	alt3	1	2	2	3	1	1	1/5	9	1/2	9
	alt4	3	4	1/3	9	1/9	5	1	1	1/5	9
	alt5	1/6	9	1/6	8	1/9	2	1/9	5	1	1

Table A.9: 5x5 inconsistent matrix9

Matrix 9		alt1		alt2		alt3		alt4		alt5	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
	alt1	1	1	1/3	1	1	7	4	8	6	8
	alt2	1	3	1	1	3	7	1/9	1/4	7	9
	alt3	1/7	1	1/7	1/3	1	1	1	2	1/7	2
	alt4	1/8	1/4	4	9	1/2	1	1	1	1/9	1/7
	alt5	1/8	1/6	1/9	1/7	1/2	7	7	9	1	1

Table A.10: 5x5 inconsistent matrix10

Matrix 10		alt1		alt2		alt3		alt4		alt5	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
	alt1	1	1	1/8	1/7	1	4	5	6	6	6
	alt2	7	8	1	1	1/9	1/3	1/8	1/4	1/9	1
	alt3	1/4	1	3	9	1	1	1/4	4	1/6	5
	alt4	1/6	1/5	4	8	1/4	4	1	1	1/4	3
	alt5	1/6	1/6	1	9	1/5	6	1/3	4	1	1

A.2 5x5 Consistent Matrix Generation

Table A.11 5x5 consistent matrix1

Matrix 1		alt1		alt2		alt3		alt4		alt5	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
	alt1	1	1	1/5	8	1/7	4	1/5	1	1/9	3
	alt2	1/8	5	1	1	1/6	9	1/2	2	1/2	3
	alt3	1/4	7	1/9	6	1	1	1/8	6	1/2	1
	alt4	1	5	1/2	2	1/6	8	1	1	1/9	1
	alt5	1/3	9	1/3	2	1	2	1	9	1	1

Table A.12 5x5 consistent matrix2

		alt1		alt2		alt3		alt4		alt5	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 2	alt1	1	1	1/7	5	1/4	6	1/5	1	1/9	6
	alt2	1/5	7	1	1	1/8	8	1/2	7	1/4	9
	alt3	1/6	4	1/8	8	1	1	1/4	9	1/7	1
	alt4	1	5	1/7	2	1/9	4	1	1	1/8	1
	alt5	1/6	9	1/9	4	1	7	1	8	1	1

Table A.13 5x5 consistent matrix3

		alt1		alt2		alt3		alt4		alt5	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 3	alt1	1	1	1/3	3	1/8	3	1/4	9	1/6	5
	alt2	1/3	3	1	1	1/2	8	1/9	6	1/7	6
	alt3	1/3	8	1/8	2	1	1	1/6	2	1/5	2
	alt4	1/9	4	1/6	9	1/2	6	1	1	1/9	6
	alt5	1/5	6	1/6	7	1/2	5	1/6	9	1	1

Table A.14 5x5 consistent matrix4

		alt1		alt2		alt3		alt4		alt5	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 4	alt1	1	1	1/4	9	1/5	5	1/4	2	1/6	5
	alt2	1/9	4	1	1	1/2	8	1/4	4	1/7	7
	alt3	1/5	5	1/8	2	1	1	1/4	9	1/3	9
	alt4	1/2	4	1/4	4	1/9	4	1	1	1/3	9
	alt5	1/5	6	1/7	7	1/9	3	1/9	3	1	1

Table A.15 5x5 consistent matrix5

		alt1		alt2		alt3		alt4		alt5	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 5	alt1	1	1	1/7	3	1/2	6	1/9	6	1/3	8
	alt2	1/3	7	1	1	1/9	6	1/3	3	1/3	3
	alt3	1/6	2	1/6	9	1	1	1/2	7	1/8	1
	alt4	1/6	9	1/3	3	1/7	2	1	1	1/7	4
	alt5	1/8	3	1/3	3	1	8	1/4	7	1	1

Table A.16 5x5 consistent matrix6

		alt1		alt2		alt3		alt4		alt5	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 6	alt1	1	1	1/2	3	1/6	5	1/9	6	1/8	8
	alt2	1/3	2	1	1	1/3	8	1/4	8	1/4	2
	alt3	1/5	6	1/8	3	1	1	1/2	4	1/3	5
	alt4	1/6	9	1/8	4	1/4	2	1	1	1/6	9
	alt5	1/8	8	1/2	4	1/5	3	1/9	6	1	1

Table A.17 5x5 consistent matrix7

		alt1		alt2		alt3		alt4		alt5	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 7	alt1	1	1	1/9	4	1/7	3	1/5	6	1/9	5
	alt2	1/4	9	1	1	1/9	3	1/9	2	1/9	6
	alt3	1/3	7	1/3	9	1	1	1/9	9	1/4	9
	alt4	1/6	5	1/2	9	1/9	9	1	1	1/3	8
	alt5	1/5	9	1/6	9	1/9	4	1/8	3	1	1

Table A.18 5x5 consistent matrix8

		alt1		alt2		alt3		alt4		alt5	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 8	alt1	1	1	1/2	5	1/7	3	1/6	4	1/4	5
	alt2	1/5	2	1	1	1/5	9	1/6	7	1/5	6
	alt3	1/3	7	1/9	5	1	1	1/7	4	1/4	7
	alt4	1/4	6	1/7	6	1/4	7	1	1	1/2	3
	alt5	1/5	4	1/6	5	1/7	4	1/3	2	1	1

Table A.19 5x5 consistent matrix9

		alt1		alt2		alt3		alt4		alt5	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 9	alt1	1	1	1/7	3	1/4	9	1/8	3	1/5	7
	alt2	1/3	7	1	1	1/2	9	1/4	5	1/8	2
	alt3	1/9	4	1/9	2	1	1	1/3	9	1/6	1
	alt4	1/3	8	1/5	4	1/9	3	1	1	1/6	3
	alt5	1/7	5	1/2	8	1	6	1/3	6	1	1

Table A.20: 5x5 consistent matrix10

Matrix 10		alt1		alt2		alt3		alt4		alt5	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
	alt1	1	1	1/4	1	1/7	3	1/2	9	1/6	7
	alt2	1	4	1	1	1/2	9	1/2	1	1/8	1
	alt3	1/3	7	1/9	2	1	1	1/5	2	1/9	9
	alt4	1/9	2	1	2	1/2	5	1	1	1/4	3
	alt5	1/7	6	1	8	1/9	9	1/3	4	1	1

A.3 10x10 Inconsistent Matrix Generation

Table A.21: 10x10 inconsistent matrix1

Matrix 1	alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
alt1	1	1	1/2	4	1/3	7	2	4	1/7	1/4	1/2	8	6	9	1/6	8	3	5	8	8
alt2	1/4	2	1	1	1/9	1/2	1/6	3	6	9	1/4	9	1/7	1/7	1/6	9	1/5	1	1/5	7
alt3	1/7	3	2	9	1	1	1/8	1/7	1/8	4	1/4	9	1/7	9	1/9	1/8	1/3	4	1/6	5
alt4	1/4	1/2	1/3	6	7	8	1	1	1/3	2	1/3	7	4	6	1	5	1/7	4	1/9	5
alt5	4	7	1/9	1/6	1/4	8	1/2	3	1	1	2	8	1/3	9	1/5	1	1/9	1/3	7	9
alt6	1/8	2	1/9	4	1/9	4	1/7	3	1/8	1/2	1	1	1/9	4	6	7	2	3	1/9	1/8
alt7	1/9	1/6	7	7	1/9	7	1/6	1/4	1/9	3	1/4	9	1	1	1/9	1/5	1/6	1/3	1/7	4
alt8	1/8	6	1/9	6	8	9	1/5	1	1	5	1/7	1/6	5	9	1	1	1/7	9	1/8	9
alt9	1/5	1/3	1	5	1/4	3	1/4	7	3	9	1/3	1/2	3	6	1/9	7	1	1	1/6	7
alt10	1/8	1/8	1/7	5	1/5	6	1/5	9	1/9	1/7	8	9	8	9	1/4	7	1/7	6	1	1

Table A.22: 10x10 inconsistent matrix2

Matrix 2	alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
alt1	1	1	1/8	1/5	1/3	2	2	3	1/7	1/3	1/7	9	1/9	7	1/7	9	1/8	1/3	1/7	4
alt2	5	8	1	1	1/4	4	1/5	1/2	7	8	2	7	3	8	1/3	9	1/6	8	1/7	4
alt3	1/2	3	1/4	4	1	1	1/2	9	1/7	4	1/6	1	1/5	1	1/7	1/7	1/9	1	1/5	8
alt4	1/3	1/2	2	5	1/9	2	1	1	1	6	2	6	1/9	4	1/3	6	1/9	8	1/8	5
alt5	3	7	1/8	1/7	1/4	7	1/6	1	1	1	1/3	2	1/8	1/2	1	5	1/7	2	1/5	8
alt6	1/9	7	1/7	1/2	1	6	1/6	1/2	1/2	3	1	1	1/5	1	1/7	6	1/6	1/3	1/6	6
alt7	1/7	9	1/8	1/3	1	5	1/4	9	2	8	1	5	1	1	1/3	4	1/6	1/4	1/4	1/4
alt8	1/9	7	1/9	3	7	7	1/6	3	1/5	1	1/6	7	1/4	3	1	1	1	3	1/5	8
alt9	3	8	1/8	6	1	9	1/8	9	1/2	7	3	6	4	6	1/3	1	1	1	2	2
alt10	1/4	7	1/4	7	1/8	5	1/5	8	1/8	5	1/6	6	1/6	6	4	4	1/2	1/2	1	1

Table A.23 10x10 inconsistent matrix3

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 3	alt1	1	1	5	5	1/6	1/4	1/8	7	1/9	1	5	8	1/5	5	1/8	1/3	1/2	7	1/9	1/2
	alt2	1/5	1/5	1	1	1/4	9	3	5	1/3	9	1/9	7	1/7	1/5	1/7	1/5	4	9	4	4
	alt3	4	6	1/9	4	1	1	1/3	6	1/3	7	3	8	1/4	1	2	9	4	9	1/8	1
	alt4	1/7	8	1/5	1/3	1/6	3	1	1	1/9	3	1/7	1/5	3	3	8	8	1	1	4	8
	alt5	1	9	1/9	3	1/7	3	1/3	9	1	1	8	9	1/4	8	1/7	1/5	1/5	1	1/6	2
	alt6	1/8	1/5	1/7	9	1/8	1/3	5	7	1/9	1/8	1	1	1/9	2	6	9	1/7	1/3	1/4	6
	alt7	1/5	5	5	7	1	4	1/3	1/3	1/8	4	1/2	9	1	1	1/9	1/7	2	6	1/3	8
	alt8	3	8	5	7	1/9	1/2	1/8	1/8	5	7	1/9	1/6	7	9	1	1	1/4	1/4	1/6	1/5
	alt9	1/7	2	1/9	1/4	1/9	1/4	1	1	1	5	3	7	1/6	1/2	4	4	1	1	3	5
	alt10	2	9	1/4	1/4	1	8	1/8	1/4	1/2	6	1/6	4	1/6	4	1/8	3	1/5	1/3	1	1

Table A.24 10x10 inconsistent matrix4

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 4	alt1	1	1	1/7	9	1/3	0.5	1/7	1/6	1/3	5	1/6	2	1/9	9	1/7	7	1/6	1/5	5	9
	alt2	1/9	7	1	1	3	4	1/9	2	1/8	4	1/5	1/2	1	8	1/6	4	3	3	1/8	6
	alt3	2	3	1/4	1/3	1	1	1/2	2	5	8	9	9	1/7	8	1/7	4	1/8	4	1/6	1
	alt4	6	7	1/2	9	1/2	2	1	1	1/9	1/4	1/6	7	1/7	1/3	3	6	1/3	4	1/8	1/6
	alt5	1/5	3	1/4	8	1/8	1/5	4	9	1	1	7	8	1/2	2	1/7	7	1/7	6	1/3	3
	alt6	1/2	6	2	5	1/9	1/9	1/7	6	1/8	1/7	1	1	1/2	7	1/5	9	1/9	1/7	1	1
	alt7	1/9	9	1/8	1	1/8	7	3	7	1/2	2	1/7	2	1	1	1/5	1/2	1/9	6	1	3
	alt8	1/7	7	1/4	6	1/4	7	1/6	1/3	1/7	7	1/9	5	2	5	1	1	1/8	9	5	7
	alt9	5	6	1/3	1/3	1/4	8	1/4	3	1/6	7	7	9	1/6	9	1/9	8	1	1	1	6
	alt10	1/9	1/5	1/6	8	1	6	6	8	1/3	3	1	1	1	1	1/3	1	1/6	1	1	1

Table A.25 10x10 inconsistent matrix5

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 5	alt1	1	1	1/8	7	2	6	1/5	2	1/7	2	1/3	9	6	8	1/5	4	1/2	2	1/5	4
	alt2	1/7	8	1	1	1/7	4	6	8	1/5	1/4	1/3	1/3	3	4	1/6	7	1/7	1/6	1/4	1/3
	alt3	1/6	1/2	1/4	7	1	1	1/7	1/5	1/7	8	4	7	2	3	4	6	1/7	1/4	1/2	6
	alt4	1/2	5	1/8	1/6	5	7	1	1	1	9	1/9	7	1/3	8	1/8	8	1/7	1/5	1	5
	alt5	1/2	7	4	5	1/8	7	1/9	1	1	1	1/6	5	1/4	9	1/5	4	1/7	1/5	1/7	1/5
	alt6	1/9	3	3	3	1/7	1/4	1/7	9	1/5	6	1	1	2	7	1/9	1/4	4	5	1/5	7
	alt7	1/8	1/6	1/4	1/3	1/3	1/2	1/8	3	1/9	4	1/7	1/2	1	1	1	5	2	6	1	3
	alt8	1/4	5	1/7	6	1/6	1/4	1/8	8	1/4	5	4	9	1/5	1	1	1	1/5	7	1/6	1
	alt9	1/2	2	6	7	4	7	5	7	5	7	1/5	1/4	1/6	1/2	1/7	5	1	1	2	6
	alt10	1/4	5	3	4	1/6	2	1/5	1	5	7	1/7	5	1/7	5	1/3	1	1/6	1/2	1	1

Table A.26 10x10 inconsistent matrix6

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 6	alt1	1	1	1/8	1/6	1/9	1/8	4	7	1/4	1/3	1/8	1/7	1/8	7	1/2	4	7	8	7	9
	alt2	6	8	1	1	1/6	1/3	1/8	4	1/5	1/5	1/9	1/4	6	8	1/4	4	1/4	4	1	7
	alt3	8	9	3	6	1	1	1/6	1	7	8	1/8	1/7	1/3	2	1/3	3	1/3	5	1/4	1/3
	alt4	1/7	1/4	1/4	8	1	6	1	1	9	9	1/4	1	1/9	4	1/6	1/5	1/5	1/5	1/3	1/3
	alt5	3	4	5	5	1/8	1/7	1/9	1/9	1	1	1/9	1/5	1/5	5	1/9	1/3	5	8	1/6	1/4
	alt6	7	8	4	9	7	8	1	4	5	9	1	1	1/7	9	1/6	1/4	2	8	1/5	3
	alt7	1/7	8	1/8	1/6	1/2	3	1/4	9	1/5	5	1/9	7	1	1	1/5	9	1/2	2	2	2
	alt8	1/4	2	1/4	4	1/3	3	5	6	3	9	4	6	1/9	5	1	1	2	3	1/8	1/3
	alt9	1/8	1/7	1/4	4	1/5	3	5	5	1/8	1/5	1/8	1/2	1/2	2	1/3	1/2	1	1	1/2	2
	alt10	1/9	1/7	1/7	1	3	4	3	3	4	6	1/3	5	1/3	5	1/2	1/2	1/2	2	1	1

Table A.27 10x10 inconsistent matrix7

Matrix 7	alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
alt1	1	1	1/6	3	1/6	1/6	1/9	6	1/5	8	1/7	4	1/3	3	1/8	9	1/9	1/6	1/8	1/6
alt2	1/3	6	1	1	1	7	1/9	1/7	1/5	6	1/9	8	1/2	7	1/3	8	4	9	1/6	2
alt3	6	6	1/7	1	1	1	1/4	2	1/9	1/5	1/5	6	1/9	3	1/3	3	1/4	8	1/9	7
alt4	1/6	9	7	9	1/2	4	1	1	1/6	5	1/8	3	1/5	7	1/3	5	9	9	1/6	9
alt5	1/8	5	1/6	5	5	9	1/5	6	1	1	1/7	5	1/7	8	1/5	3	1/5	1/2	1/2	1/2
alt6	1/4	7	1/8	9	1/6	5	1/3	8	1/5	7	1	1	1/7	6	1/5	5	1/4	1	3	3
alt7	1/3	3	1/7	2	1/3	9	1/7	5	1/8	7	1/6	7	1	1	1/3	7	1/8	7	1/3	4
alt8	1/9	8	1/8	3	1/3	3	1/5	3	1/3	5	1/5	5	1/7	3	1	1	1/9	1/5	2	8
alt9	6	9	1/9	1/4	1/8	4	1/9	1/9	2	5	1	4	1/7	8	5	9	1	1	1/3	5
alt10	6	8	1/2	6	1/7	9	1/9	6	2	2	1/3	1/3	1/3	1/3	1/4	3	1/5	3	1	1

Table A.28 10x10 inconsistent matrix8

Matrix 8	alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
alt1	1	1	1/5	1/2	1/9	1/7	1/3	2	5	8	2	8	1/5	1	1/9	3	1/6	6	1/4	6
alt2	2	5	1	1	1/9	1/5	1/9	1/3	1/8	1/8	1/7	2	1	7	1/2	5	2	7	6	8
alt3	7	9	5	9	1	1	1/5	1/2	1	6	1/4	1/3	1/7	1/4	1/9	1/9	1/6	4	3	7
alt4	1/2	3	3	9	2	5	1	1	4	7	1/2	9	1/7	1	2	8	1/7	2	1/7	4
alt5	1/8	1/5	8	8	1/6	1	1/7	1/4	1	1	1	8	5	6	1/2	4	1/5	1/2	1/2	3
alt6	1/8	1/2	1/2	7	3	4	1/9	2	1/8	1	1	1	1/6	1/2	1/5	7	8	9	5	7
alt7	1	5	1/7	1	4	7	1	7	1/6	1/5	2	6	1	1	6	8	1/7	1/3	3	5
alt8	1/3	9	1/5	2	9	9	1/8	1/2	1/4	2	1/7	5	1/8	1/6	1	1	1	8	1/9	1/9
alt9	1/6	6	1/7	1/2	1/4	6	1/2	7	2	5	1/9	1/8	3	7	1/8	1	1	1	5	7
alt10	1/6	4	1/8	1/6	1/7	1/3	1/4	7	1/3	2	1/7	1/5	1/7	1/5	1/5	1/3	1/7	1/5	1	1

Table A.29 10x10 inconsistent matrix9

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 9	alt1	1	1	1/5	7	2	4	1/6	6	1/4	6	1/9	1/3	1/2	7	1/8	7	1/2	6	1/8	1/6
	alt2	1/7	5	1	1	4	5	3	5	1/7	6	1/7	3	1/8	4	1/3	3	1/5	1/3	1/2	4
	alt3	1/4	1/2	1/5	1/4	1	1	1/6	8	2	2	1/4	9	1/4	2	1/3	4	1/9	1/3	1/4	1/2
	alt4	1/6	6	1/5	1/3	1/8	6	1	1	1/8	1/2	1/2	1	1/9	1/6	1/6	3	1/6	7	1/5	3
	alt5	1/6	4	1/6	7	1/2	1/2	2	8	1	1	1/6	3	1/6	7	1/5	1	1/2	3	1/4	4
	alt6	3	9	1/3	7	1/9	4	1	2	1/3	6	1	1	7	9	4	6	1/3	7	1/8	1/2
	alt7	1/7	2	1/4	8	1/2	4	6	9	1/7	6	1/9	1/7	1	1	1	4	1/3	9	1/6	5
	alt8	1/7	8	1/3	3	1/4	3	1/3	6	1	5	1/6	1/4	1/4	1	1	1	1/3	1/2	1	3
	alt9	1/6	2	3	5	3	9	1/7	6	1/3	2	1/7	3	1/9	3	2	3	1	1	3	9
	alt10	6	8	1/4	2	2	4	1/3	5	1/4	4	2	8	2	8	1/5	6	1/9	1/3	1	1

Table A.30 10x10 inconsistent matrix10

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 10	alt1	1	1	1/6	7	1/6	1/2	1/2	3	1	2	2	6	3	4	1/3	9	1	2	1/5	7
	alt2	1/7	6	1	1	4	4	1/8	7	1/4	2	1/2	9	1/8	5	1	8	1/6	1/2	1/2	6
	alt3	2	6	1/4	1/4	1	1	1/4	7	1/3	6	6	7	6	6	1/6	3	1/2	1	1/7	8
	alt4	1/3	2	1/7	8	1/7	4	1	1	1/3	9	1/8	1/7	1/4	1/3	1/8	3	1/2	3	1	5
	alt5	1/2	1	1/2	4	1/6	3	1/9	3	1	1	4	5	6	8	1/2	4	1/4	1/4	1/2	9
	alt6	1/6	1/2	1/9	2	1/7	1/6	7	8	1/5	1/4	1	1	1/3	2	1/7	1/5	1/9	5	1/6	1/2
	alt7	1/4	1/3	1/5	8	1/6	1/6	3	4	1/8	1/6	1/2	3	1	1	1/3	4	1/7	1/4	1/8	1/3
	alt8	1/9	3	1/8	1	1/3	6	1/3	8	1/4	2	5	7	1/4	3	1	1	1/2	2	1/7	3
	alt9	1/2	1	2	6	1	2	1/3	2	4	4	1/5	9	4	7	1/2	2	1	1	1	4
	alt10	1/7	5	1/6	2	1/8	7	1/5	1	1/9	2	2	6	2	6	3	8	1/4	1	1	1

A.4 10x10 Consistent Matrix Generation

Table A.31 10x10 consistent matrix1

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 1	alt1	1	1	1/5	2	1/6	9	1/6	6	1/5	6	1/7	5	1/9	9	1/7	5	1/8	9	1/9	4
	alt2	1/2	5	1	1	1/5	5	1/5	7	1/9	8	1/3	8	1/8	3	1/5	7	1/8	1	1/2	7
	alt3	1/9	6	1/5	5	1	1	1/5	4	1/3	3	1/9	4	1/8	8	1/4	7	1/6	9	1/5	2
	alt4	1/6	6	1/7	5	1/4	5	1	1	1/7	7	1/8	4	1/8	5	1/4	2	1/3	2	1/8	8
	alt5	1/6	5	1/8	9	1/3	3	1/7	7	1	1	1/3	4	1/3	5	1/6	5	1/6	8	1/8	2
	alt6	1/5	7	1/8	3	1/4	9	1/4	8	1/4	3	1	1	1/8	3	1/3	8	1/9	4	1/6	7
	alt7	1/9	9	1/3	8	1/8	8	1/5	8	1/5	3	1/3	8	1	1	1/3	9	1/2	3	1/6	4
	alt8	1/5	7	1/7	5	1/7	4	1/2	4	1/5	6	1/8	3	1/9	3	1	1	1/3	4	1/6	4
	alt9	1/9	8	1	8	1/9	6	1/2	3	1/8	6	1/4	9	1/3	2	1/4	3	1	1	1/5	3
	alt10	1/4	9	1/7	2	1/2	5	1/8	8	1/2	8	1/7	6	1/7	6	1/4	6	1/3	5	1	1

Table A.32 10x10 inconsistent matrix2

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 2	alt1	1	1	1/7	2	1/7	7	1/3	4	1/7	2	1/9	1	1/8	6	1/7	4	1/3	8	1/6	4
	alt2	1/2	7	1	1	1/3	1	1/4	3	1/8	1	1/6	2	1/4	3	1/6	1	1/6	7	1/2	6
	alt3	1/7	7	1	3	1	1	1/4	8	1/5	7	1/8	5	1/3	5	1/7	4	1/9	2	1/4	9
	alt4	1/4	3	1/3	4	1/8	4	1	1	1/5	8	1/5	3	1/5	5	1/9	7	1/2	6	1/7	4
	alt5	1/2	7	1	8	1/7	5	1/8	5	1	1	1/2	2	1/7	1	1/5	6	1/3	4	1/2	5
	alt6	1	9	1/2	6	1/5	8	1/3	5	1/2	2	1	1	1/8	1	1/9	4	1/4	7	1/9	4
	alt7	1/6	8	1/3	4	1/5	3	1/5	5	1	7	1	8	1	1	1/5	9	1/8	3	1/3	9
	alt8	1/4	7	1	6	1/4	7	1/7	9	1/6	5	1/4	9	1/9	5	1	1	1/8	4	1/6	1
	alt9	1/8	3	1/7	6	1/2	9	1/6	2	1/4	3	1/7	4	1/3	8	1/4	8	1	1	1/6	8
	alt10	1/4	6	1/6	2	1/9	4	1/4	7	1/5	2	1/4	9	1/4	9	1/9	3	1/8	6	1	1

Table A.33 10x10 consistent matrix3

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 3	alt1	1	1	1/9	2	1/4	9	1/8	4	1/9	2	1/7	5	1/3	1	1/2	7	1/2	6	1/2	3
	alt2	1/2	9	1	1	1/9	9	1/4	1	1/8	5	1/4	8	1/5	8	1/9	1	1/9	8	1/9	7
	alt3	1/9	4	1/9	9	1	1	1/9	6	1/7	7	1/3	6	1/5	7	1/2	7	1/6	2	1/2	1
	alt4	1/4	8	1	4	1/6	9	1	1	1/4	1	1/4	5	1/2	7	1/6	2	1/2	2	1/6	7
	alt5	1/2	9	1/5	8	1/7	7	1	4	1	1	1/2	9	1/4	3	1/9	7	1/2	5	1/5	6
	alt6	1/5	7	1/8	4	1/6	3	1/5	4	1/9	2	1	1	1/5	6	1/4	4	1/6	9	1/5	1
	alt7	1	3	1/8	5	1/7	5	1/7	2	1/3	4	1/6	5	1	1	1/2	8	1/9	8	1/9	5
	alt8	1/7	2	1	9	1/7	2	1/2	6	1/7	9	1/4	4	1/8	2	1	1	1/5	3	1/6	1
	alt9	1/6	2	1/8	9	1/2	6	1/2	2	1/5	2	1/9	6	1/8	9	1/3	5	1	1	1/4	8
	alt10	1/3	2	1/7	9	1	2	1/7	6	1/6	5	1	5	1	5	1/5	9	1/8	4	1	1

Table A.34 10x10 consistent matrix4

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 4	alt1	1	1	1/8	4	1/4	4	1/5	5	1/2	4	1/2	3	1/9	5	1/3	1	1/2	8	1/5	9
	alt2	1/4	8	1	1	1/2	1	1/6	1	1/8	3	1/8	3	1/3	5	1/2	2	1/5	9	1/4	5
	alt3	1/4	4	1	2	1	1	1/9	3	1/8	9	1/6	9	1/6	5	1/6	5	1/9	3	1/3	5
	alt4	1/5	5	1	6	1/3	9	1	1	1/2	4	1/3	1	1/2	6	1/7	4	1/8	2	1/2	4
	alt5	1/4	2	1/3	8	1/9	8	1/4	2	1	1	1/9	3	1/4	8	1/2	7	1/9	8	1/6	7
	alt6	1/3	2	1/3	8	1/9	6	1	3	1/3	9	1	1	1/9	2	1/2	3	1/2	2	1/7	1
	alt7	1/5	9	1/5	3	1/5	6	1/6	2	1/8	4	1/2	9	1	1	1/6	2	1/3	3	1/2	3
	alt8	1	3	1/2	2	1/5	6	1/4	7	1/7	2	1/3	2	1/2	6	1	1	1/9	6	1/8	6
	alt9	1/8	2	1/9	5	1/3	9	1/2	8	1/8	9	1/2	2	1/3	3	1/6	9	1	1	1/3	1
	alt10	1/9	5	1/5	4	1/5	3	1/4	2	1/7	6	1	7	1	7	1/3	2	1	3	1	1

Table A.35 10x10 consistent matrix5

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 5	alt1	1	1	1/7	1	1/3	2	1/7	8	1/2	1	1/9	5	1/5	7	1/4	7	1/6	7	1/5	1
	alt2	1	7	1	1	1/9	2	1/8	1	1/9	1	1/8	1	1/7	8	1/7	7	1/3	4	1/5	9
	alt3	1/2	3	1/2	9	1	1	1/3	6	1/9	8	1/6	7	1/5	9	1/9	7	1/3	6	1/4	8
	alt4	1/8	7	1	8	1/6	3	1	1	1/9	8	1/8	7	1/4	1	1/3	7	1/2	5	1/4	5
	alt5	1	2	1	9	1/8	9	1/8	9	1	1	1/7	3	1/2	6	1/5	8	1/2	7	1/5	2
	alt6	1/5	9	1	8	1/7	6	1/7	8	1/3	7	1	1	1/7	3	1/5	4	1/4	6	1/2	8
	alt7	1/7	5	1/8	7	1/9	5	1	4	1/6	2	1/3	7	1	1	1/9	1	1/3	7	1/9	6
	alt8	1/7	4	1/7	7	1/7	9	1/7	3	1/8	5	1/4	5	1	9	1	1	1/6	8	1/8	1
	alt9	1/7	6	1/4	3	1/6	3	1/5	2	1/7	2	1/6	4	1/7	3	1/8	6	1	1	1/7	4
	alt10	1	5	1/9	5	1/8	4	1/5	4	1/2	5	1/8	2	1/8	2	1/6	9	1/4	7	1	1

Table A.36 10x10 consistent matrix6

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 6	alt1	1	1	1/7	7	1/6	2	1/9	6	1/7	8	1/6	2	1/9	9	1/8	7	1/9	6	1/3	7
	alt2	1/7	7	1	1	1/6	9	1/8	7	1/9	1	1/4	2	1/3	1	1/4	3	1/6	5	1/2	2
	alt3	1/2	6	1/9	6	1	1	1/3	4	1/9	6	1/3	1	1/3	6	1/9	2	1/4	7	1/9	3
	alt4	1/6	9	1/7	8	1/4	3	1	1	1/5	7	1/8	9	1/6	3	1/4	6	1/6	1	1/7	9
	alt5	1/8	7	1	9	1/6	9	1/7	5	1	1	1/6	8	1/2	3	1/7	1	1/2	8	1/4	2
	alt6	1/2	6	1/2	4	1	3	1/9	8	1/8	6	1	1	1/4	5	1/4	3	1/7	9	1/8	2
	alt7	1/9	9	1	3	1/6	3	1/3	6	1/3	2	1/5	4	1	1	1/4	5	1/2	7	1/6	3
	alt8	1/7	8	1/3	4	1/2	9	1/6	4	1	7	1/3	4	1/5	4	1	1	1/4	9	1/7	5
	alt9	1/6	9	1/5	6	1/7	4	1	6	1/8	2	1/9	7	1/7	2	1/9	4	1	1	1/4	8
	alt10	1/7	3	1/2	2	1/3	9	1/9	7	1/2	4	1/2	8	1/2	8	1/3	6	1/8	4	1	1

Table A.37 10x10 consistent matrix7

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 7	alt1	1	1	1/5	4	1/8	2	1/8	5	1/2	6	1/5	7	1/6	2	1/8	4	1/5	3	1/3	9
	alt2	1/4	5	1	1	1/9	5	1/6	8	1/2	3	1/2	1	1/9	3	1/4	5	1/6	1	1/3	1
	alt3	1/2	8	1/5	9	1	1	1/6	9	1/2	1	1/8	5	1/2	5	1/2	4	1/7	1	1/6	3
	alt4	1/5	8	1/8	6	1/9	6	1	1	1/5	9	1/2	3	1/7	8	1/3	5	1/4	4	1/6	5
	alt5	1/6	2	1/3	2	1	2	1/9	5	1	1	1/8	3	1/7	6	1/8	2	1/8	6	1/3	8
	alt6	1/7	5	1	2	1/5	8	1/3	2	1/3	8	1	1	1/9	6	1/4	8	1/7	9	1/6	4
	alt7	1/2	6	1/3	9	1/5	2	1/8	7	1/6	7	1/6	9	1	1	1/9	7	1/7	3	1/9	3
	alt8	1/4	8	1/5	4	1/4	2	1/5	3	1/2	8	1/8	4	1/7	9	1	1	1/8	2	1/4	3
	alt9	1/3	5	1	6	1	7	1/4	4	1/6	8	1/9	7	1/3	7	1/2	8	1	1	1/9	7
	alt10	1/9	3	1	3	1/3	6	1/5	6	1/8	3	1/4	6	1/4	6	1/3	9	1/7	9	1	1

Table A.38 10x10 consistent matrix8

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 8	alt1	1	1	1/4	2	1/4	1	1/4	3	1/7	9	1/3	4	1/7	5	1/2	1	1/3	3	1/2	3
	alt2	1/2	4	1	1	1/2	1	1/8	9	1/8	6	1/6	7	1/5	4	1/9	9	1/3	7	1/8	4
	alt3	1	4	1	2	1	1	1/6	8	1/6	9	1/7	3	1/9	7	1/4	1	1/2	1	1/6	5
	alt4	1/3	4	1/9	8	1/8	6	1	1	1/8	8	1/3	1	1/9	2	1/6	2	1/6	6	1/8	7
	alt5	1/9	7	1/6	8	1/9	6	1/8	8	1	1	1/9	4	1/5	2	1/4	7	1/5	8	1/2	6
	alt6	1/4	3	1/7	6	1/3	7	1	3	1/4	9	1	1	1/3	3	1/9	9	1/7	7	1/6	6
	alt7	1/5	7	1/4	5	1/7	9	1/2	9	1/2	5	1/3	3	1	1	1/6	2	1/9	5	1/6	1
	alt8	1	2	1/9	9	1	4	1/2	6	1/7	4	1/9	9	1/2	6	1	1	1/5	6	1/3	9
	alt9	1/3	3	1/7	3	1	2	1/6	6	1/8	5	1/7	7	1/5	9	1/6	5	1	1	1/3	7
	alt10	1/3	2	1/4	8	1/5	6	1/7	8	1/6	2	1/6	6	1/6	6	1	6	1/7	3	1	1

Table A.39 10x10 consistent matrix9

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 9	alt1	1	1	1/3	6	1/9	9	1/6	5	1/8	2	1/6	3	1/9	1	1/8	7	1/5	5	1/5	9
	alt2	1/6	3	1	1	1/4	5	1/3	5	1/5	2	1/4	2	1/4	7	1/9	1	1/4	8	1/4	8
	alt3	1/9	9	1/5	4	1	1	1/3	2	1/2	4	1/4	6	1/6	8	1/2	1	1/9	3	1/6	6
	alt4	1/5	6	1/5	3	1/2	3	1	1	1/2	8	1/3	1	1/7	8	1/3	7	1/6	2	1/7	3
	alt5	1/2	8	1/2	5	1/4	2	1/8	2	1	1	1/2	7	1/6	4	1/6	5	1/2	2	1/6	1
	alt6	1/3	6	1/2	4	1/6	4	1	3	1/7	2	1	1	1/5	3	1/6	4	1/8	8	1/6	1
	alt7	1	9	1/7	4	1/8	6	1/8	7	1/4	6	1/3	5	1	1	1/5	8	1/4	9	1/7	7
	alt8	1/7	8	1	9	1	2	1/7	3	1/5	6	1/4	6	1/8	5	1	1	1/7	8	1/5	9
	alt9	1/5	5	1/8	4	1/3	9	1/2	6	1/2	2	1/8	8	1/9	4	1/8	7	1	1	1/5	2
	alt10	1/9	5	1/8	4	1/6	6	1/3	7	1	6	1	6	1	6	1/7	7	1/2	5	1	1

Table A.40 10x10 consistent matrix10

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 10	alt1	1	1	1/5	6	1/4	4	1/6	9	1/2	4	1/2	2	1/8	4	1/2	8	1/8	3	1/8	8
	alt2	1/6	5	1	1	1/3	3	1/6	4	1/9	2	1/6	7	1/8	4	1/3	4	1/5	2	1/3	1
	alt3	1/4	4	1/3	3	1	1	1/5	9	1/7	5	1/5	5	1/4	9	1/4	8	1/2	7	1/9	8
	alt4	1/9	6	1/4	6	1/9	5	1	1	1/6	8	1/9	9	1/5	8	1/7	1	1/6	4	1/8	1
	alt5	1/4	2	1/2	9	1/5	7	1/8	6	1	1	1/8	1	1/5	7	1/8	9	1/4	5	1/7	6
	alt6	1/2	2	1/7	6	1/5	5	1/9	9	1	8	1	1	1/3	8	1/4	2	1/2	2	1/6	8
	alt7	1/4	8	1/4	8	1/9	4	1/8	5	1/7	5	1/8	3	1	1	1/3	1	1/5	6	1/7	8
	alt8	1/8	2	1/4	3	1/8	4	1	7	1/9	8	1/2	4	1	3	1	1	1/9	4	1/6	4
	alt9	1/3	8	1/2	5	1/7	2	1/4	6	1/5	4	1/2	2	1/6	5	1/4	9	1	1	1/2	3
	alt10	1/8	8	1	3	1/8	9	1	8	1/6	7	1/8	6	1/8	6	1/8	7	1/3	2	1	1

A.5 15x15 Inconsistent Matrix Generation

Table A.41 15x15 inconsistent matrix1

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10		alt11		alt12		alt13		alt14		alt15	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 1	alt1	1	1	3	9	3	7	1	6	1/3	1/2	1/6	4	1/2	7	1/7	1/2	4	9	6	9	2	8	1/4	3	1/8	6	1/2	6	1/8	6
	alt2	1/9	1/3	1	1	1/8	1/3	1/4	5	6	9	9	9	1/3	5	2	5	1/3	3	1	3	1/5	1	1/6	3	4	7	1/3	7	1/6	1/2
	alt3	1/7	1/3	3	8	1	1	1	8	1/8	1/2	7	8	2	8	4	9	1/3	4	1/6	2	1/8	1/3	3	9	1/8	3	1/9	6	1	7
	alt4	1/6	1	1/5	4	1/8	1	1	1	1/9	1	1/9	4	1/4	9	1/7	7	4	8	1/5	1/3	5	7	1/5	1/2	1/7	1/5	1/2	1	1/9	1/2
	alt5	2	3	1/9	1/6	2	8	1	9	1	1	1/4	1	1/4	1	1/6	1/4	1/5	4	1/9	1/3	1/8	3	6	7	1/4	1/2	1/4	1/3	1/7	5
	alt6	1/4	6	1/9	1/9	1/8	1/7	1/4	9	1	4	1	1	7	9	1/4	2	1/7	6	1/5	7	1/7	1/4	1/2	6	1/3	6	1/9	3	1/3	7
	alt7	1/7	2	1/5	3	1/8	1/2	1/9	4	1	4	1/9	1/7	1	1	1/6	4	1/9	1/4	2	5	1/3	4	1/9	1/3	1/3	1/3	1/5	5	1/5	2
	alt8	2	7	1/5	1/2	1/9	1/4	1/7	7	4	6	1/2	4	1/4	6	1	1	1/3	2	1	8	1/7	3	1/8	1/6	1/2	5	1/4	7	1/9	1/7
	alt9	1/9	1/4	1/3	3	1/4	3	1/8	1/4	1/4	5	1/6	7	4	9	1/2	3	1	1	2	3	5	8	1/5	4	1/6	1/4	2	7	1	5
	alt10	1/9	1/6	1/3	1	1/2	6	3	5	3	9	1/7	5	1/7	5	1/5	1/2	1/3	1/2	1	1	1/6	9	1/9	1/5	1/2	4	1/4	3	6	8
	alt11	1/8	1/2	1	5	3	8	1/7	1/5	1/3	8	4	7	1/4	3	1/3	7	1/8	1/5	1/9	6	1	1	1/8	1/6	1/7	1/5	1/2	2	1/6	2
	alt12	1/3	4	1/3	6	1/9	1/3	1/7	1/6	1/7	1/6	1/6	2	3	9	6	8	1/4	5	5	9	6	8	1	1	1/9	1	1/2	5	1/3	1/2
	alt13	1/6	8	1/7	1/4	1/6	8	2	7	2	4	1/6	3	3	3	1/5	2	4	6	1/4	2	1/2	2	1	9	1	1	1/4	8	1/6	1/4
	alt14	1/6	2	1/7	3	1/6	9	1	2	3	4	1/3	9	1/5	5	1/7	4	1/7	2	1/3	4	1/2	2	1/2	5	1	4	1	1	4	8
	alt15	1/6	8	2	6	1/7	1	2	9	1/5	7	1/7	3	1/2	5	7	9	1/5	5	1/8	1/6	1/2	6	2	3	4	6	1/8	1/4	1	1

Table A.42 15x15 inconsistent matrix2

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10		alt11		alt12		alt13		alt14		alt15	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 2	alt1	1	1	7	8	1/8	2	1/4	4	1/3	1/2	1/9	1/7	4	9	1/6	6	2	4	1/2	3	1	4	1	6	1/8	1/3	3	5	1/2	1
	alt2	1/8	1/7	1	1	5	6	1/4	1/2	1/8	8	1/4	8	5	6	1/5	7	1/2	5	1/6	7	9	9	1/5	1	6	8	3	8	1/7	9
	alt3	1/2	8	1/6	1/5	1	1	1/7	5	7	9	1/2	3	1/5	5	1/6	6	1/9	5	1/8	1/5	1/4	1/3	5	8	1/5	1/4	1	3	6	7
	alt4	1/4	4	2	4	1/5	7	1	1	1/5	6	1/9	5	1/2	1/2	1/8	2	1/7	6	7	9	1/2	6	1/5	7	1/5	1	3	7	1/4	6
	alt5	2	3	1/8	8	1/9	1/7	1/6	5	1	1	1/7	7	9	9	1/2	4	1/8	9	1/8	3	3	6	1/8	2	1/8	1/7	1/4	2	1/7	3
	alt6	7	9	1/8	4	1/3	2	1/5	9	1/7	7	1	1	1/5	9	3	5	1/6	1/2	6	8	1/2	1/2	1/6	9	6	7	1/8	1/5	1/4	7
	alt7	1/9	1/4	1/6	1/5	1/5	5	2	2	1/9	1/9	1/9	5	1	1	1/9	9	1/3	4	1	6	1/2	5	1/2	5	1/2	7	1/6	1/5	1/7	5
	alt8	1/6	6	1/7	5	1/6	6	0.5	8	1/4	2	1/5	1/3	1/9	9	1	1	1/8	1/2	1/6	1/5	1/6	1/5	1/6	8	1	4	1/8	5	1/5	1/2
	alt9	1/4	1/2	1/5	2	1/5	9	1/6	7	1/9	8	2	6	1/4	3	2	8	1	1	1/5	1/4	1/6	9	1/6	1	1/9	1/8	1/6	3	1/5	1/2
	alt10	1/3	2	1/7	6	5	8	1/9	1/7	1/3	8	1/8	1/6	1/8	1/6	1/6	1	4	5	1	1	4	6	4	9	1/9	1	1/7	1/5	3	3
	alt11	1/4	1	1/9	1/9	3	4	1/6	2	1/6	1/3	2	2	1/5	2	5	6	1/9	6	1/6	1/4	1	1	1/9	6	1/4	1/3	1/4	4	1/8	8
	alt12	1/6	1	1	5	1/8	0.2	1/2	8	1/2	8	1/9	6	1/5	2	1/8	6	1	6	1/9	1/4	1/6	9	1	1	1/6	1/2	1/6	7	1/8	1
	alt13	3	8	1/8	1/6	1/3	5	1/3	5	7	8	1/7	1/6	1/7	2	1/4	1	8	9	1	9	1/4	4	2	6	1	1	3	7	1/5	9
	alt14	1/5	1/3	1/8	1/3	1/3	1	1/7	1/3	1/2	4	5	8	5	6	1/5	8	1/3	9	5	7	1/4	4	1/6	7	1/7	1	1	1	1/8	5
	alt15	1	2	1/9	7	1/7	1/6	1/6	4	1/3	7	1/7	4	1/5	7	2	5	4	5	1/3	1/3	1/8	8	1	8	1/9	5	1/5	8	1	1

Table A.43 15x15 inconsistent matrix3

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10		alt11		alt12		alt13		alt14		alt15	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 3	alt1	1	1	1/9	1/2	1/4	1	1/4	1	1/9	1/8	1/9	3	1/9	5	1/7	1/5	8	8	1/2	9	1/8	1/2	1/9	2	1/5	8	1	8	4	8
	alt2	2	9	1	1	1/8	1/6	1	7	4	9	4	9	1/9	1/4	1/3	2	3	9	1/6	1/5	1/7	1/3	1/6	5	1/7	2	1/9	1/6	1	9
	alt3	1	4	6	8	1	1	1/7	2	1/6	2	1/2	6	5	6	1/6	1/2	1	7	1/9	1/3	1/9	2	1/9	7	1/5	6	1/9	7	2	7
	alt4	1	4	1/7	1	1/2	7	1	1	1/8	2	1/5	5	1/8	9	1/7	3	1/4	1/4	1/2	2	3	9	1/9	7	1/3	2	1/5	5	1/3	2
	alt5	8	9	1/9	1/4	1/2	6	1/2	8	1	1	2	6	1/5	2	1/4	1/4	6	9	1/8	9	1/3	9	6	7	1/4	5	1/8	7	1/4	8
	alt6	1/3	9	1/9	1/4	1/6	2	1/5	5	1/6	1/2	1	1	1/3	2	1	4	3	4	1/7	8	7	8	1/8	1/7	1/9	4	1/4	2	1/4	3
	alt7	1/5	9	4	9	1/6	1/5	1/9	8	1/2	5	1/2	3	1	1	6	8	1/6	5	1/4	7	3	9	1/2	2	1/7	1/3	1/3	3	1/9	1/8
	alt8	5	7	1/2	3	2	6	1/3	7	4	4	1/4	1	1/8	1/6	1	1	6	9	1/2	4	1/9	7	1/3	3	1/3	9	1/3	2	1/2	9
	alt9	1/8	1/8	1/9	1/3	1/7	1	4	4	1/9	1/6	1/4	1/3	1/5	6	1/9	1/6	1	1	1/6	1/3	2	5	1/4	5	1/4	4	1/3	8	1/7	9
	alt10	1/9	2	5	6	3	9	1/2	2	1/9	8	1/8	7	1/8	7	1/7	4	3	6	1	1	1/9	1/8	1/6	1/4	1/5	6	4	8	1/8	1/7
	alt11	2	8	3	7	1/2	9	1/9	1/3	1/9	3	1/8	1/7	1/9	1/3	1/7	9	1/5	1/2	8	9	1	1	1/2	1/2	1/3	8	1/9	1	1/9	7
	alt12	1/2	9	1/5	6	1/7	9	1/7	1/6	1/7	1/6	7	8	1/2	2	1/3	3	1/5	4	4	6	2	2	1	1	1/8	2	1	7	1/8	7
	alt13	1/8	5	1/2	7	5	9	1/5	1/2	1/5	4	1/4	9	3	7	1/9	3	1/4	4	1/6	5	1	9	1/2	8	1	1	1/8	1/4	1/7	1/5
	alt14	1/8	1	6	9	1/7	9	1/5	5	1/7	8	1/2	4	1/3	3	1/2	3	1/8	5	1/8	1/4	1/9	1	1	7	1	8	1	1	1/7	9
	alt15	1/8	1/4	1/9	1	1/7	1/2	0.5	3	1/8	4	1/3	4	8	9	1/9	2	1/8	7	7	8	1/7	9	1/7	8	5	7	1/9	7	1	1

Table A.44 15x15 inconsistent matrix4

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10		alt11		alt12		alt13		alt14		alt15	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 4	alt1	1	1	1/7	1/5	1/8	8	1/7	4	1/4	1/2	1/2	3	1/2	5	1/9	1/4	1/5	1/3	1/7	5	3	8	1/2	6	1/5	9	1/8	1/7	1/9	1/7
	alt2	5	7	1	1	1/2	4	1	9	1/7	1/4	1/5	7	3	5	1/6	2	1	7	1/9	1/2	5	5	1/9	7	1/5	1	6	7	2	9
	alt3	1/8	8	1/4	2	1	1	1	5	1/9	1/7	3	7	2	3	5	6	4	6	1/7	1/5	1/8	1/4	1/3	1/3	3	5	7	9	1/9	9
	alt4	1/4	7	1/9	1	1/5	1	1	1	7	9	1/3	9	1/2	9	1/6	2	2	5	1/7	6	1	4	2	9	1	6	1/4	1/2	1/2	2
	alt5	2	4	4	7	7	9	1/9	1/7	1	1	4	5	1/9	1	1/6	9	1/6	1/4	5	8	1/6	9	1/4	9	6	7	2	6	1/3	7
	alt6	1/3	2	1/7	5	1/7	1/3	1/9	3	1/5	1/4	1	1	1/3	1/2	1/7	1/2	1/6	4	3	7	1/3	5	1/6	3	1/3	1/3	1/8	1/5	1/4	5
	alt7	1/5	2	1/5	1/3	1/3	1/2	1/9	2	1	9	2	3	1	1	1/8	1/2	1/9	1/7	1/6	1/4	1/3	9	1/8	1/2	1/2	5	1/6	1/3	1	5
	alt8	4	9	1/2	6	1/6	1/5	1/2	6	1/9	6	2	7	2	8	1	1	5	7	1/4	5	1/4	7	1/2	4	1/6	1	1/2	8	1/7	4
	alt9	3	5	1/7	1	1/6	1/4	1/5	1/2	4	6	1/4	6	7	9	1/7	1/5	1	1	1/3	3	1/2	4	1/7	5	1/8	3	3	5	1/7	1
	alt10	1/5	7	2	9	5	7	1/6	7	1/8	1/5	1/7	1/3	1/7	1/3	4	6	1/3	3	1	1	1/5	7	1/3	1/3	1/4	9	2	6	1/3	7
	alt11	1/8	1/3	1/5	1/5	4	8	1/4	1	1/9	6	1/5	3	1/9	3	1/7	4	1/4	2	1/7	5	1	1	1/4	2	5	8	4	9	1/4	1
	alt12	1/6	2	1/7	9	3	3	1/9	4	1/9	4	1/3	6	2	8	1/4	2	1/5	7	3	3	1/2	4	1	1	5	7	1/2	7	6	7
	alt13	1/9	5	1	5	1/7	1/3	1/6	4	1/7	1/6	3	3	1/5	2	1	6	1/3	8	1/9	4	1/9	1/4	1/7	1/5	1	1	1/9	1/6	1/4	1/3
	alt14	7	8	1/7	1/6	1/9	1/7	2	4	1/6	1/2	5	8	3	6	1/8	2	1/3	4	1/6	1/2	4	9	1/2	7	1	6	1	1	1/7	9
	alt15	7	9	1/9	1/2	1/9	9	1/2	2	1/7	3	1/5	4	1/5	1	1/4	7	1/5	1	1/7	3	1	4	1/7	1/6	3	4	1/9	7	1	1

Table A.45 15x15 inconsistent matrix5

Matrix 5	alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10		alt11		alt12		alt13		alt14		alt15	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
alt1	1	1	1/4	6	1/9	1/6	1	5	1/8	1/7	1/7	3	5	5	3	4	3	5	1/4	1	2	8	1/2	4	1/6	5	1/9	1/2	1/2	6
alt2	1/6	4	1	1	1/7	9	1/8	7	1/2	4	1/7	5	1/9	1/9	1/5	5	1/5	7	2	8	4	7	7	9	1/7	1/5	1/6	3	1/6	1/4
alt3	6	9	1/9	7	1	1	1/6	9	1/5	4	7	9	1	4	1/7	3	2	8	2	7	1/2	8	1/6	8	1/2	7	1/5	1/4	1/9	1/4
alt4	1/5	1	1/7	8	1/9	6	1	1	1/4	1/2	1/9	7	4	4	1/8	1	1/9	5	2	4	6	9	1/6	9	1/7	1/6	8	9	1/4	3
alt5	7	8	1/4	2	1/4	5	2	4	1	1	4	5	1/6	4	1/3	4	1/5	1/4	1/9	8	1/5	8	1/3	3	1/5	9	1/6	1/5	1	2
alt6	1/3	7	1/5	7	1/9	1/7	1/7	9	1/5	1/4	1	1	2	5	1/9	7	1	9	1/8	4	3	5	1/8	4	4	7	1/9	8	1/9	1/8
alt7	1/5	1/5	9	9	1/4	1	1/4	1/4	1/4	6	1/5	1/2	1	1	4	9	1/8	8	2	7	3	5	1/3	3	1/3	1	1/3	5	1/8	1/5
alt8	1/4	1/3	1/5	5	1/3	7	1	8	1/4	3	1/7	9	1/9	1/4	1	1	1/3	7	1/8	4	7	8	3	8	1/4	3	1/3	1/2	4	4
alt9	1/5	1/3	1/7	5	1/8	1/2	1/5	9	4	5	1/9	1	1/8	8	1/7	3	1	1	1/4	8	1/4	5	1/7	5	1/3	1/2	1/2	3	1/9	5
alt10	1	4	1/8	1/2	1/7	1/2	1/4	1/2	1/8	9	1/4	8	1/4	8	1/7	1/2	1/8	4	1	1	3	7	1/7	1/5	1/6	1/3	2	2	1/6	5
alt11	1/8	1/2	1/7	1/4	1/8	2	1/9	1/6	1/8	5	1/5	1/3	1/5	1/3	1/8	1/7	1/5	4	1/7	1/3	1	1	1/8	1/2	3	7	1/4	1	4	4
alt12	1/4	2	1/9	1/7	1/8	6	1/3	3	1/3	3	1/4	8	1/3	3	1/8	1/3	1/5	7	5	7	2	8	1	1	1/4	4	6	9	1/4	1
alt13	1/5	6	5	7	2	5	1/8	6	1/9	5	1/7	1/4	1	3	1/3	4	2	3	3	6	1	4	1/4	4	1	1	1/4	5	5	5
alt14	2	9	1/3	6	4	5	1/9	1/8	5	6	1/8	9	1/5	3	2	3	2	6	1/2	1/2	1/4	1	6	9	1	4	1	1	1/6	3
alt15	1/6	2	4	6	4	9	1/3	4	1/2	1	8	9	5	8	1/4	1/4	1/5	1/3	1/5	6	1/4	1/4	1	4	1/5	1/5	1/3	6	1	1

Table A.46 15x15 inconsistent matrix6

Matrix 6	alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10		alt11		alt12		alt13		alt14		alt15	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
alt1	1	1	1/7	6	1/7	2	7	9	1/9	1/9	1/5	1/2	1/4	4	6	7	1/3	4	1/8	9	1/6	1/6	1/5	8	1/6	1/5	4	5	1/5	8
alt2	1/6	7	1	1	7	8	3	7	1/2	3	1/5	8	2	7	4	4	1	4	1/7	9	5	7	1/5	7	1	8	5	9	3	6
alt3	1/2	7	1/8	1/7	1	1	1	2	1/8	1/2	1/3	5	1/8	4	1	4	1/4	9	5	7	1/8	8	1/5	2	1/5	1/5	1/8	1/2	3	7
alt4	1/9	1/7	1/7	1/3	1/2	1	1	1	1/2	6	1/7	1	1/5	1/3	1/9	1/6	2	7	6	6	4	6	1/9	9	1/4	5	4	8	4	8
alt5	9	9	1/3	2	2	8	1/6	2	1	1	1/4	1	2	7	1/2	5	1/7	6	1/8	6	1/7	2	1/6	1	1/2	6	1/8	1/5	1/3	6
alt6	2	5	1/8	5	1/5	3	1	7	1	4	1	1	6	7	8	8	3	7	1/5	5	1/7	1/7	1/5	9	1/5	5	1/8	9	1/5	6
alt7	1/4	4	1/7	1/2	1/4	8	3	5	1/7	1/2	1/7	1/6	1	1	3	7	1	7	1/7	1/6	1/5	4	1/4	6	1/9	6	1/3	4	1/4	7
alt8	1/7	1/6	1/4	1/4	1/4	1	6	9	1/5	2	1/8	1/8	1/7	1/3	1	1	3	7	1/8	3	1/8	6	1/7	9	1/6	9	1/2	2	1/5	4
alt9	1/4	3	1/4	1	1/9	4	1/7	1/2	1/6	7	1/7	1/3	1/7	1	1/7	1/3	1	1	1/9	9	1/6	1/3	1/3	4	1/5	4	1/2	8	1/4	9
alt10	1/9	8	1/9	7	1/7	1/5	1/6	1/6	1/6	8	1/5	5	1/5	5	6	7	1/9	9	1	1	1/9	1/5	1/9	1/6	1/6	5	1/3	2	1/2	5
alt11	6	6	1/7	1/5	1/8	8	1/6	1/4	1/2	7	7	7	1/4	5	1/6	8	3	6	5	9	1	1	1/7	9	1	5	1/8	7	8	9
alt12	1/8	5	1/7	5	1/2	5	1	6	1	6	1/9	5	1/6	4	1/9	7	1/4	3	6	9	1/9	7	1	1	1/2	2	1/5	8	1/9	9
alt13	5	6	1/8	1	5	8	1/5	1/4	1/6	2	1/5	5	1/6	9	1/9	6	1/4	5	1/5	6	1/7	8	1/2	2	1	1	1/5	1/4	1/8	5
alt14	1/5	1/4	1/9	1/5	2	8	1/8	1/4	5	8	1/9	8	1/4	3	1/2	2	1/8	6	1/2	3	1/8	7	1/5	8	1	4	1	1	1/3	1/2
alt15	1/8	5	1/6	1/3	1/7	1/3	1/8	1/4	1/6	3	1/6	5	1/7	4	1/4	5	1/5	4	1/5	2	1/9	1/8	1/9	9	1/5	8	2	3	1	1

Table A.47 15x15 inconsistent matrix7

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10		alt11		alt12		alt13		alt14		alt15	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 7	alt1	1	1	1/2	2	1/4	9	1/3	9	1/5	1/2	1/4	3	1/4	8	1/3	3	6	8	2	8	8	8	1/9	8	1/2	2	1	3	1/8	1
	alt2	1/2	2	1	1	1/5	1	2	4	1/9	5	5	8	1/5	1/3	1/8	5	1/7	1/2	5	6	1/8	4	1/3	7	1/8	6	1/9	1/7	1/6	4
	alt3	1/9	4	1	5	1	1	1/9	9	1/2	1/2	4	5	2	6	1/2	2	1/7	1/3	1/8	1/3	1/7	1/2	1/6	1/2	1/8	6	1/8	1/3	1/8	1/2
	alt4	1/9	3	1/4	1/2	1/9	9	1	1	1/4	4	1/8	1	2	7	1/6	4	1/9	7	1/9	5	5	8	1/7	1/6	1/9	4	4	7	1/5	1/3
	alt5	2	5	1/5	9	2	2	1/4	4	1	1	1/5	4	1/6	1/5	1/3	1	1/4	0.5	4	7	1/9	8	1/8	5	1/7	1/7	1/3	1	1/3	7
	alt6	1/3	4	1/8	1/5	1/5	1/4	1	8	1/4	5	1	1	2	6	1/3	2	1/4	8	5	6	1/7	1/2	1/9	9	1/4	1/3	1/3	2	1	8
	alt7	1/8	4	3	5	1/6	1/2	1/7	1/2	5	6	1/6	1/2	1	1	1/8	2	3	4	7	8	6	7	1/9	1	1/7	8	1/2	6	1/4	1
	alt8	1/3	3	1/5	8	1/2	2	1/4	6	1	3	1/2	3	1/2	8	1	1	1/6	3	1/8	9	5	7	1/5	1/4	1/8	7	1/5	9	5	8
	alt9	1/8	1/6	2	7	3	7	1/7	9	2	4	1/8	4	1/4	1/3	1/3	6	1	1	1/8	8	1	8	1/6	1	1/9	5	1/9	8	1/2	8
	alt10	1/8	1/2	1/6	1/5	3	8	1/5	9	1/7	1/4	1/6	1/5	1/6	1/5	1/8	1/7	1/8	8	1	1	6	7	1/9	5	1/2	9	2	3	1/5	8
	alt11	1/8	1/8	1/4	8	2	7	1/8	1/5	1/8	9	2	7	1/7	1/6	1/7	1/5	1/8	1	1/7	1/6	1	1	3	9	1/4	2	1/5	2	4	7
	alt12	1/8	9	1/7	3	2	6	1/5	8	1/5	8	1/9	9	1	9	4	5	1	6	1/5	9	1/9	1/3	1	1	2	9	1/8	2	1/9	1/8
	alt13	1/2	2	1/6	8	8	8	1/7	1/4	7	7	3	4	1/8	7	1/7	8	1/5	9	1/9	2	1/2	5	1/9	1/2	1	1	1/8	8	1/9	1/5
	alt14	1/3	1	7	9	3	8	1/7	1/4	1	3	1/2	3	1/6	2	1/9	5	1/9	9	1/3	1/2	1/5	2	1/8	2	1	8	1	1	1/7	1/5
	alt15	1	8	1/4	6	2	8	3	5	1/7	3	1/8	1	1	4	1/8	1/5	1/8	1/2	1/8	5	1/7	1/4	8	9	5	9	5	7	1	1

Table A.48 15x15 inconsistent matrix8

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10		alt11		alt12		alt13		alt14		alt15	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 8	alt1	1	1	1/9	9	1/5	1/2	1/9	1/6	1/6	1	3	4	3	3	1/7	1	1/9	3	1/6	9	8	9	1/3	2	1/5	8	3	6	1/9	9
	alt2	1/9	9	1	1	1/3	3	1/7	1/4	1/4	5	6	6	4	7	1/7	3	1/7	8	7	9	1/2	7	7	9	1/8	9	4	7	2	9
	alt3	2	5	1/3	3	1	1	1/5	1/4	1/8	1/4	3	5	1/4	1/3	1/8	1/5	3	5	1/8	5	1/4	8	1/6	5	1/4	3	1	5	3	4
	alt4	6	9	4	7	4	5	1	1	1/4	1/4	1/6	3	2	5	1/9	5	1/4	8	1	8	1/6	1	3	5	1	7	1	7	1/7	5
	alt5	1	6	1/5	4	4	8	4	4	1	1	1/8	4	4	8	3	8	1/4	7	1/7	1/4	1/2	8	1/9	6	1/7	1/5	3	9	9	9
	alt6	1/4	1/3	1/6	1/6	1/5	1/3	1/3	6	1/4	8	1	1	1/2	9	3	8	1/7	8	4	9	3	8	3	5	1/6	7	1/3	6	1/3	1/2
	alt7	1/3	1/3	1/7	1/4	3	4	1/5	1/2	1/8	1/4	1/9	2	1	1	1/9	1/3	5	8	6	6	1/5	4	5	7	5	7	1/5	1/4	1/7	1
	alt8	1	7	1/3	7	5	8	1/5	9	1/8	1/3	1/8	1/3	3	9	1	1	1/4	3	5	9	1	8	1/8	1/7	1/4	4	1	5	4	4
	alt9	1/3	9	1/8	7	1/5	1/3	1/8	4	1/7	4	1/8	7	1/8	1/5	1/3	4	1	1	1/5	6	1/4	1/3	1/9	4	1/9	1/6	1/3	9	1/4	3
	alt10	1/9	6	1/9	1/7	1/5	8	1/8	1	4	7	1/9	1/4	1/9	1/4	1/6	1/6	1/6	5	1	1	1/7	1/6	1/5	1	1/5	4	1/6	1/4	2	6
	alt11	1/9	1/8	1/7	2	1/8	4	1	6	1/8	2	1/8	1/3	1/4	5	1/8	1	3	4	6	7	1	1	1/9	9	8	9	1/8	9	1/7	1/5
	alt12	1/2	3	1/9	1/7	1/5	6	1/6	9	1/6	9	1/5	1/3	1/7	1/5	7	8	1/4	9	1	5	1/9	9	1	1	4	9	1/3	4	1/7	8
	alt13	1/8	5	1/9	8	1/5	1/3	1	1	5	7	1/7	6	1/7	1/5	1/4	4	6	9	1/4	5	1/9	8	1/9	1/4	1	1	1/6	5	3	5
	alt14	1/6	1/3	1/7	1/4	1/5	1	1/7	1	1/9	1/3	1/6	3	4	5	1/5	1	1/9	5	4	6	1/8	9	1/3	4	1/5	1	1	1	1/2	7
	alt15	1/9	9	1/9	1/2	1/4	1/3	1/5	7	1/9	1/9	2	3	1	7	1/4	1/4	1/8	4	1/6	1/2	5	7	1/8	7	1/5	1/3	1/7	2	1	1

Table A.49 15x15 inconsistent matrix9

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10		alt11		alt12		alt13		alt14		alt15	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 9	alt1	1	1	2	9	1/4	5	3	7	1/3	8	1/5	2	1/6	6	1/8	1/5	1/7	6	1/8	5	1	7	1	4	6	7	1/9	2	1/5	6
	alt2	1/9	1/2	1	1	1/5	4	1/6	1/2	1/4	3	1/5	1/5	1/9	4	1/5	2	1	5	1/8	1/5	3	4	1/7	1	1/6	4	1/9	3	1/8	8
	alt3	1/5	4	1/4	5	1	1	1/8	1	1/9	1/3	7	9	1/8	3	3	6	1/2	8	1/3	4	1/4	8	1/4	8	5	8	1/8	4	1/2	5
	alt4	1/7	1/3	2	6	1	8	1	1	1/6	5	1/9	1/4	1/8	1	1/8	4	1/5	1/2	1/5	1/2	1/8	7	1/9	1/7	5	6	4	9	1/9	5
	alt5	1/8	3	1/3	4	3	9	1/5	6	1	1	1/5	9	1/4	8	1/6	1/4	1/3	3	1/2	4	3	7	1	3	3	7	1/6	1/6	1/7	4
	alt6	1/2	5	5	5	1/9	1/7	4	9	1/9	5	1	1	1/3	3	2	3	1/6	1	1/6	1	1/8	6	1/8	1/5	1	8	1/5	5	1/9	8
	alt7	1/6	6	1/4	9	1/3	8	1	8	1/8	4	1/3	3	1	1	1/3	7	1/3	7	1/7	3	1/3	2	6	6	1/8	1/6	1/3	2	1/3	7
	alt8	5	8	1/2	5	1/6	1/3	1/4	8	4	6	1/3	1/2	1/7	3	1	1	5	6	3	6	2	9	1/2	6	1/2	1	1/3	1	1/5	4
	alt9	1/6	7	1/5	1	1/8	2	2	5	1/3	3	1	6	1/7	3	1/6	1/5	1	1	1/2	6	6	6	1/8	4	1/7	1/2	1/6	1	1/5	1
	alt10	1/5	8	5	8	1/4	3	2	5	1/4	2	1	6	1	6	1/3	7	1/6	2	1	1	1/9	5	2	4	2	4	1/8	1/4	7	8
	alt11	1/7	1	1/4	1/3	1/8	4	1/7	8	1/7	1/3	1/6	8	1/2	3	1/9	1/2	1/6	1/6	1/5	9	1	1	1/4	1	1/5	9	1/7	3	1/5	2
	alt12	1/4	1	1	7	1/8	4	1/3	1	1/3	1	5	8	1/6	1/6	1/6	2	1/4	8	1/4	1/2	1	4	1	1	1/9	1	1/3	4	1/9	1
	alt13	1/7	1/6	1/4	6	1/8	8	1/9	1/5	1/7	1/3	1/8	1	6	8	1	2	2	7	1/4	1/2	1/3	7	1	9	1	1	1/9	3	1/8	6
	alt14	1/2	9	1/3	9	1/4	8	1/9	1/4	6	6	1/5	5	1/2	3	1	3	1/2	6	4	8	1/7	3	1/3	4	1/3	1	1	1	1	1
	alt15	1/6	5	1/8	8	1/5	2	1/5	9	1/4	7	1/8	9	1/7	3	1/4	5	5	5	1/8	1/7	1/2	5	1	9	1/6	8	1	1	1	1

Table A.50 15x15 inconsistent matrix10

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10		alt11		alt12		alt13		alt14		alt15	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 10	alt1	1	1	1	5	1/2	1/2	1/7	3	1/6	7	3	7	5	7	1/7	1/6	3	6	1/5	1	1/8	4	2	6	1	7	7	8	1/8	6
	alt2	1/5	1	1	1	1/6	8	3	5	1/3	4	1/7	1/4	1/6	1/4	1/5	1/4	1/5	1/3	5	8	2	2	1/5	9	1	7	1/9	9	1/9	1/7
	alt3	2	2	1/8	6	1	1	1/8	2	1/3	1/2	2	4	1/3	3	1/4	8	1/8	9	1/5	6	1/8	9	1/3	1	1/8	4	4	9	1/5	8
	alt4	1/3	7	1/5	1/3	1/2	8	1	1	1/9	1/5	1/8	4	1/9	3	1/8	6	8	9	1/5	1	1/7	1/2	1/6	1/2	4	9	5	8	1/9	1/7
	alt5	1/7	6	1/4	3	2	3	5	9	1	1	1/2	9	1/4	9	1/9	5	1/2	6	1/6	1/2	1/7	7	1/9	1/3	1	7	1/7	4	1/7	6
	alt6	1/7	1/3	4	7	1/4	1/2	1/4	8	1/9	2	1	1	1/3	4	1/3	7	5	8	1/3	6	3	3	1/6	9	1/5	9	1/3	4	1/2	1
	alt7	1/7	1/5	4	6	1/3	3	1/3	9	1/9	4	1/4	3	1	1	1/6	9	1/7	1/7	1/9	1	1/9	8	3	3	1/7	5	1/8	2	1/7	1/3
	alt8	6	7	4	5	1/8	4	1/6	8	1/5	9	1/7	3	1/9	6	1	1	1/7	1/4	1/3	3	1/7	1/4	3	8	1/6	1/5	1/7	1/5	1/8	1/3
	alt9	1/6	1/3	3	5	1/9	8	1/9	1/8	1/6	2	1/8	1/5	7	7	4	7	1	1	1/7	1/2	8	8	1/9	2	1/4	3	1/7	1	1/4	8
	alt10	1	5	1/8	1/5	1/6	5	1	5	2	6	1/6	3	1/6	3	1	9	2	7	1	1	5	7	5	7	1/4	1/4	1/3	1	1/6	2
	alt11	1/4	8	1/2	1/2	1/9	8	2	7	1/7	7	1/3	1/3	1/8	9	4	7	1/8	1/8	1/7	1/5	1	1	1/4	1/4	1/5	7	1/4	5	1/4	6
	alt12	1/6	1/2	1/9	5	1	3	3	9	3	9	1/9	6	1/3	1/3	1/8	1/3	1/2	9	1/7	1/5	4	4	1	1	3	5	1/7	7	1/5	1/3
	alt13	1/7	1	1/7	1	1/4	1/4	1/5	1/4	1/7	1	1/9	5	1/5	7	5	6	1/3	4	4	4	1/5	4	1/5	1/3	1	1	1/2	5	1/5	5
	alt14	1/8	1/7	1/9	9	1/9	1/4	1/8	1/5	1/4	7	1/4	3	1/2	8	5	7	4	7	1	3	1/4	5	1/7	7	1/5	1	1	1	1/8	1/4
	alt15	1/6	8	7	9	1/8	5	7	9	1/6	7	1	2	3	7	3	8	1/7	4	1/2	6	1/6	4	3	5	1/5	5	4	8	1	1

A.6 15x15 Consistent Matrix Generation

Table A.51 15x15 consistent matrix1

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10		alt11		alt12		alt13		alt14		alt15	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 1	alt1	1	1	1/7	6	1/9	5	1/2	3	1/9	5	1/5	3	1/6	4	1/5	5	1/8	5	1/4	8	1/3	5	1/3	7	1/6	1	1/4	1	1/7	5
	alt2	1/6	7	1	1	1/7	7	1/9	7	1/6	2	1/9	7	1/4	3	1/6	3	1/8	9	1/6	4	1/6	8	1/6	2	1/9	6	1/7	5	1/9	6
	alt3	1/5	9	1/7	7	1	1	1/7	2	1/3	1	1/8	4	1/6	1	1/2	2	1/5	8	1/2	8	1/5	9	1/6	9	1/9	4	1/6	6	1/9	5
	alt4	1/3	2	1/7	9	1/2	7	1	1	1/3	5	1/2	6	1/9	1	1/4	2	1/6	6	1/7	4	1/7	1	1/7	1	1/3	1	1/6	8	1/8	4
	alt5	1/5	9	1/2	6	1	3	1/5	3	1	1	1/6	1	1/9	1	1/5	2	1/9	4	1/5	1	1/6	6	1/6	2	1/9	5	1/7	5	1/8	7
	alt6	1/3	5	1/7	9	1/4	8	1/6	2	1	6	1	1	1/7	5	1/4	6	1/7	3	1/7	6	1/2	4	1/4	9	1/5	1	1/3	3	1/6	4
	alt7	1/4	6	1/3	4	1	6	1	9	1	9	1/5	7	1	1	1/3	5	1/6	5	1/4	5	1/3	2	1/6	8	1/9	4	1/8	1	1/8	3
	alt8	1/5	5	1/3	6	1/2	2	1/2	4	1/2	5	1/6	4	1/5	3	1	1	1/5	1	1/4	9	1/6	5	1/5	6	1/3	1	1/2	3	1/4	4
	alt9	1/5	8	1/9	8	1/8	5	1/6	6	1/4	9	1/3	7	1/5	6	1	5	1	1	1/9	8	1/5	6	1/4	1	1/3	6	1/3	5	1/9	2
	alt10	1/8	4	1/4	6	1/8	2	1/4	7	1	5	1/6	7	1/6	7	1/5	4	1/8	9	1	1	1/2	5	1/2	1	1/6	5	1/5	3	1/6	4
	alt11	1/5	3	1/8	6	1/9	5	1	7	1/6	6	1/4	2	1/2	3	1/5	6	1/6	5	1/5	2	1	1	1/3	6	1/6	1	1/7	6	1/8	5
	alt12	1/7	3	1/2	6	1/9	6	1/2	6	1/2	6	1/9	4	1/8	6	1/6	5	1	4	1	2	1/6	3	1	1	1/3	3	1/9	4	1/7	5
	alt13	1	6	1/6	9	1/4	6	1	6	1/5	9	1	5	1/4	9	1	3	1/6	3	1/5	6	1/6	7	1/3	3	1	1	1/5	5	1/2	8
	alt14	1	4	1/5	7	1/6	6	1/8	6	1/5	7	1/3	3	1	8	1/3	2	1/5	6	1/3	5	1/7	6	1/9	4	1	5	1	1	1/4	3
	alt15	1/5	7	1/6	9	1/5	9	1/4	8	1/7	8	1/4	6	1/3	8	1/4	4	1/2	6	1/4	6	1/5	8	1/5	7	1/8	2	1/3	4	1	1

Table A.52 15x15 consistent matrix2

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10		alt11		alt12		alt13		alt14		alt15	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 2	alt1	1	1	1/4	2	1/9	9	1/4	4	1/2	9	1/5	5	1/6	6	1/4	1	1/7	4	1/6	8	1/4	7	1/6	6	1/5	4	1/5	7	1/5	4
	alt2	1/2	4	1	1	1/9	6	1/8	1	1/2	1	1/2	7	1/3	3	1/2	6	1/4	6	1/2	3	1/3	8	1/3	8	1/6	7	1/6	8	1/7	9
	alt3	1/9	9	1/6	9	1	1	1/7	3	1/8	8	1/7	9	1/3	5	1/4	2	1/6	7	1/4	2	1/4	9	1/4	9	1/4	1	1/3	1	1/5	5
	alt4	1/4	4	1	8	1/3	7	1	1	1/9	3	1/5	3	1/9	7	1/5	1	1/7	8	1/5	7	1/8	5	1/4	9	1/6	9	1/9	1	1/6	1
	alt5	1/9	2	1	2	1/8	8	1/3	9	1	1	1/8	5	1/3	5	1/8	9	1/5	3	1/8	1	1/9	4	1/5	9	1/2	6	1/8	8	1/9	2
	alt6	1/5	5	1/7	2	1/9	7	1/3	5	1/5	8	1	1	1/9	6	1/7	2	1/2	8	1/2	2	1/8	7	1/5	7	1/2	9	1/8	4	1/7	8
	alt7	1/6	6	1/3	3	1/5	3	1/7	9	1/5	3	1/6	9	1	1	1/8	7	1/7	5	1/6	4	1/8	6	1/5	1	1/3	8	1/7	6	1/6	3
	alt8	1	4	1/6	2	1/2	4	1	5	1/9	8	1/2	7	1/7	8	1	1	1/7	7	1/8	6	1/8	6	1/7	5	1/9	4	1/5	4	1/3	6
	alt9	1/4	7	1/6	4	1/7	6	1/8	7	1/3	5	1/8	2	1/5	7	1/7	7	1	1	1/7	7	1/4	5	1/5	5	1/5	1	1/9	6	1/9	3
	alt10	1/8	6	1/3	2	1/2	4	1/7	5	1	8	1/2	2	1/2	2	1/4	6	1/7	7	1	1	1/7	8	1/6	7	1/6	2	1/4	6	1/2	9
	alt11	1/7	4	1/8	3	1/9	4	1/5	8	1/4	9	1/7	8	1/6	8	1/6	8	1/5	4	1/8	7	1	1	1/3	9	1/2	6	1/7	5	1/6	7
	alt12	1/6	6	1/8	3	1/9	4	1/9	5	1/9	5	1/7	5	1	5	1/5	7	1/5	5	1/7	6	1/9	3	1	1	1/3	3	1/6	7	1/8	4
	alt13	1/4	5	1/7	6	1	3	1/9	9	1/6	2	1/9	2	1/8	3	1/4	9	1	5	1/2	6	1/5	7	1/3	3	1	1	1/3	1	1/9	4
	alt14	1/7	5	1/8	6	1	3	1	9	1/8	8	1/4	8	1/6	7	1/4	5	1/6	6	1/6	4	1/7	5	1/6	7	1	3	1	1	1/3	2
	alt15	1/4	5	1/9	7	1/5	5	1	6	1/2	9	1/8	7	1/3	6	1/6	3	1/3	2	1/9	2	1/7	6	1/4	8	1/4	9	1/2	3	1	1

Table A.53 15x15 consistent matrix3

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10		alt11		alt12		alt13		alt14		alt15	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 3	alt1	1	1	1/5	7	1/4	6	1/2	3	1/4	9	1/4	8	1/7	4	1/2	5	1/2	4	1/3	2	1/4	9	1/2	1	1/6	6	1/8	9	1/8	7
	alt2	1/7	5	1	1	1/3	1	1/2	9	1/9	3	1/9	8	1/6	5	1/3	9	1/7	5	1/4	1	1/5	9	1/3	5	1/3	2	1/8	2	1/6	8
	alt3	1/6	4	1	3	1	1	1/9	9	1/7	6	1/2	8	1/9	7	1/5	2	1/3	6	1/4	1	1/5	5	1/9	5	1/5	9	1/7	5	1/4	8
	alt4	1/3	2	1/9	2	1/9	9	1	1	1/9	8	1/7	1	1/3	7	1/2	7	1/7	6	1/6	1	1/7	5	1/3	4	1/5	8	1/7	9	1/6	1
	alt5	1/9	4	1/3	9	1/6	7	1/8	9	1	1	1/7	4	1/4	3	1/6	3	1/6	5	1/3	6	1/3	2	1/9	7	1/3	2	1/2	9	1/4	4
	alt6	1/8	4	1/8	9	1/8	2	1	7	1/4	7	1	1	1/2	5	1/8	5	1/7	2	1/9	6	1/5	7	1/5	7	1/8	1	1/5	3	1/6	3
	alt7	1/4	7	1/5	6	1/7	9	1/7	3	1/3	4	1/5	2	1	1	1/8	1	1/5	3	1/8	5	1/7	5	1/6	5	1/2	2	1/7	4	1/6	6
	alt8	1/5	2	1/9	3	1/2	5	1/7	2	1/3	6	1/5	8	1	8	1	1	1/2	4	1/2	2	1/2	4	1/5	4	1/4	5	1/4	4	1/9	2
	alt9	1/4	2	1/5	7	1/6	3	1/6	7	1/5	6	1/2	7	1/3	5	1/4	2	1	1	1/3	5	1/3	1	1/5	4	1/8	4	1/6	5	1/5	7
	alt10	1/2	3	1	4	1	4	1	6	1/6	3	1/6	9	1/6	9	1/5	8	1/5	3	1	1	1/3	7	1/7	1	1/9	4	1/6	7	1/8	3
	alt11	1/9	4	1/9	5	1/5	5	1/5	7	1/2	3	1/7	5	1/5	7	1/4	2	1	3	1/7	3	1	1	1/7	3	1/7	4	1/9	8	1/7	1
	alt12	1	2	1/5	3	1/5	9	1/7	9	1/7	9	1/7	5	1/5	6	1/4	5	1/4	5	1	7	1/3	7	1	1	1/8	3	1/9	1	1/7	6
	alt13	1/6	6	1/2	3	1/9	7	1/8	7	1/2	3	1	8	1/2	2	1/5	4	1/4	8	1/4	9	1/8	9	1/3	8	1	1	1/4	9	1/8	1
	alt14	1/9	8	1/2	8	1/5	7	1/9	7	1/9	2	1/3	5	1/4	7	1/4	4	1/5	9	1/7	6	1/9	8	1/9	1	1	4	1	1	1/9	4
	alt15	1/7	8	1/8	6	1/8	4	1	6	1/4	4	1/3	6	1/6	6	1/2	9	1/7	7	1/3	8	1	7	1/6	7	1	8	1/4	9	1	1

Table A.54 15x15 consistent matrix4

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10		alt11		alt12		alt13		alt14		alt15	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 4	alt1	1	1	1/2	6	1/3	4	1/3	6	1/9	7	1/6	5	1/2	9	1/7	1	1/4	7	1/4	8	1/9	8	1/2	4	1/8	8	1/2	3	1/7	3
	alt2	1/6	2	1	1	1/5	9	1/7	3	1/6	4	1/3	1	1/7	3	1/7	9	1/7	1	1/6	1	1/2	6	1/6	9	1/6	8	1/5	2	1/5	1
	alt3	1/4	3	1/9	5	1	1	1/6	5	1/9	4	1/3	8	1/9	6	1/9	7	1/6	6	1/2	3	1/6	7	1/7	5	1/5	3	1/7	7	1/4	1
	alt4	1/6	3	1/3	7	1/5	6	1	1	1/4	8	1/6	5	1/9	3	1/7	4	1/6	9	1/8	8	1/6	3	1/7	1	1/9	5	1/8	6	1/8	4
	alt5	1/7	9	1/4	6	1/4	9	1/8	4	1	1	1/5	9	1/4	4	1/2	3	1/3	3	1/2	6	1/8	7	1/6	2	1/9	3	1/5	1	1/3	7
	alt6	1/5	6	1	3	1/8	3	1/5	6	1/9	5	1	1	1/2	7	1/4	4	1/7	5	1/5	2	1/6	1	1/9	2	1/2	6	1/3	2	1/3	9
	alt7	1/9	2	1/3	7	1/6	9	1/3	9	1/4	4	1/7	2	1	1	1/5	5	1/3	2	1/8	8	1/6	7	1/8	1	1/7	9	1/2	5	1/3	8
	alt8	1	7	1/9	7	1/7	9	1/4	7	1/3	2	1/4	4	1/5	5	1	1	1/2	4	1/8	9	1/3	6	1/4	2	1/4	1	1/9	8	1/7	9
	alt9	1/7	4	1	7	1/6	6	1/9	6	1/3	3	1/5	7	1/2	3	1/4	2	1	1	1/8	6	1/4	6	1/2	6	1/6	5	1/4	8	1/4	2
	alt10	1/8	4	1	6	1/3	2	1/8	8	1/6	2	1/2	5	1/2	5	1/8	8	1/6	8	1	1	1/2	2	1/7	2	1/4	4	1/9	3	1/4	4
	alt11	1/8	9	1/6	2	1/7	6	1/3	6	1/7	8	1	6	1/7	6	1/6	3	1/6	4	1/2	2	1	1	1/7	5	1/6	7	1/5	5	1/5	9
	alt12	1/4	2	1/9	6	1/5	7	1/2	6	1/2	6	1/2	9	1	8	1/2	4	1/6	2	1/2	7	1/5	7	1	1	1/3	1	1/8	6	1/9	6
	alt13	1/8	8	1/8	6	1/3	7	1/5	8	1/3	9	1/6	2	1/9	7	1	4	1/5	6	1/4	4	1/5	5	1	3	1	1	1/8	2	1/9	3
	alt14	1/3	2	1/2	5	1/7	7	1/6	8	1	5	1/2	3	1/5	2	1/8	9	1/8	4	1/3	9	1/5	5	1/8	6	1	8	1	1	1/9	2
	alt15	1/3	7	1	5	1	4	1/4	8	1/7	3	1/9	3	1/8	3	1/9	7	1/2	6	1/4	4	1/9	5	1/6	9	1/3	9	1/2	9	1	1

Table A.55 15x15 consistent matrix5

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10		alt11		alt12		alt13		alt14		alt15	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 5	alt1	1	1	1/6	9	1/6	9	1/3	3	1/4	5	1/2	5	1/8	8	1/8	4	1/2	3	1/7	2	1/6	7	1/3	4	1/7	4	1/7	6	1/4	4
	alt2	1/9	6	1	1	1/2	3	1/7	7	1/9	3	1/7	9	1/8	1	1/3	2	1/6	3	1/4	8	1/6	9	1/3	5	1/9	8	1/8	4	1/7	9
	alt3	1/9	6	1/3	2	1	1	1/4	9	1/9	2	1/2	1	1/4	9	1/4	6	1/6	8	1/3	4	1/9	9	1/2	4	1/5	2	1/3	2	1/9	3
	alt4	1/3	3	1/7	7	1/9	4	1	1	1/8	2	1/2	6	1/8	1	1/4	1	1/8	8	1/7	8	1/5	6	1/5	4	1/7	3	1/4	6	1/3	1
	alt5	1/5	4	1/3	9	1/2	9	1/2	8	1	1	1/3	3	1/4	3	1/8	8	1/8	3	1/2	7	1/5	9	1/4	9	1/7	1	1/3	4	1/2	5
	alt6	1/5	2	1/9	7	1	2	1/6	2	1/3	3	1	1	1/2	3	1/4	7	1/6	4	1/3	5	1/2	6	1/2	2	1/8	2	1/6	5	1/8	6
	alt7	1/8	8	1	8	1/9	4	1	8	1/3	4	1/3	2	1	1	1/9	2	1/5	7	1/9	8	1/8	1	1/2	6	1/9	5	1/3	1	1/4	7
	alt8	1/4	8	1/2	3	1/6	4	1	4	1/8	8	1/7	4	1/2	9	1	1	1/4	4	1/7	5	1/2	9	1/9	2	1/5	3	1/3	9	1/5	4
	alt9	1/3	2	1/3	6	1/8	6	1/8	8	1/3	8	1/4	6	1/7	5	1/4	4	1	1	1/3	5	1/7	3	1/6	4	1/3	5	1/3	3	1/2	4
	alt10	1/2	7	1/8	4	1/4	3	1/8	7	1/7	2	1/5	3	1/5	3	1/8	9	1/5	3	1	1	1/5	4	1/4	1	1/4	9	1/6	9	1/3	7
	alt11	1/7	6	1/9	6	1/9	9	1/6	5	1/9	5	1/6	2	1	8	1/9	2	1/3	7	1/4	5	1	1	1/7	5	1/8	8	1/9	8	1/2	2
	alt12	1/4	3	1/5	3	1/4	2	1/9	4	1/9	4	1/2	2	1/6	2	1/2	9	1/4	6	1	4	1/5	7	1	1	1/3	2	1/8	2	1/8	6
	alt13	1/4	7	1/8	9	1/2	3	1/3	4	1	7	1/2	8	1/5	9	1/3	5	1/5	3	1/9	4	1/8	9	1/2	3	1	1	1/7	6	1/5	3
	alt14	1/6	7	1/4	8	1/2	3	1/6	4	1/4	3	1/5	6	1	3	1/9	3	1/3	4	1/9	6	1/9	8	1/8	2	1	7	1	1	1/8	7
	alt15	1/4	4	1/9	7	1/3	9	1	3	1/5	2	1/6	8	1/7	4	1/4	5	1/4	8	1/7	3	1/2	2	1/6	8	1/3	5	1/7	8	1	1

Table A.56 15x15 consistent matrix6

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10		alt11		alt12		alt13		alt14		alt15	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 6	alt1	1	1	1/8	8	1/2	8	1/5	6	1/6	5	1/5	8	1/2	7	1/5	2	1/7	8	1/8	1	1/7	9	1/9	9	1/4	7	1/3	7	1/9	3
	alt2	1/8	8	1	1	1/5	1	1/7	1	1/4	9	1/9	8	1/6	4	1/4	1	1/4	8	1/9	2	1/2	2	1/5	3	1/5	4	1/6	5	1/5	9
	alt3	1/8	2	1	5	1	1	1/6	2	1/9	9	1/4	1	1/3	8	1/7	4	1/2	8	1/8	3	1/6	1	1/6	2	1/7	1	1/8	1	1/5	1
	alt4	1/6	5	1	7	1/2	6	1	1	1/4	6	1/8	9	1/4	1	1/8	7	1/7	9	1/8	4	1/7	2	1/7	2	1/5	6	1/9	2	1/6	8
	alt5	1/5	6	1/9	4	1/9	9	1/6	4	1	1	1/3	5	1/8	1	1/8	9	1/4	2	1/2	9	1/7	2	1/6	9	1/3	4	1/6	1	1/8	7
	alt6	1/8	5	1/8	9	1	4	1/9	8	1/5	3	1	1	1/9	9	1/8	1	1/8	6	1/8	8	1/3	4	1/7	1	1/4	3	1/2	9	1/2	6
	alt7	1/7	2	1/4	6	1/8	3	1	4	1	8	1/9	9	1	1	1/7	2	1/9	2	1/2	7	1/4	9	1/3	9	1/2	2	1/8	2	1/9	4
	alt8	1/2	5	1	4	1/4	7	1/7	8	1/9	8	1	8	1/2	7	1	1	1/5	8	1/6	2	1/7	9	1/2	5	1/6	4	1/5	2	1/4	7
	alt9	1/8	7	1/8	4	1/8	2	1/9	7	1/2	4	1/6	8	1/2	9	1/8	5	1	1	1/6	7	1/4	9	1/7	6	1/8	2	1/6	4	1/2	7
	alt10	1	8	1/2	9	1/3	8	1/4	8	1/9	2	1/8	8	1/8	8	1/7	2	1/7	6	1	1	1/5	2	1/3	3	1/4	8	1/6	5	1/9	1
	alt11	1/9	7	1/2	2	1	6	1/2	7	1/2	7	1/4	3	1/9	4	1/9	7	1/9	4	1/2	5	1	1	1/3	7	1/7	2	1/3	4	1/3	4
	alt12	1/9	9	1/3	5	1/2	6	1/9	6	1/9	6	1	7	1/9	3	1/5	2	1/6	7	1/3	3	1/7	3	1	1	1/2	7	1/4	9	1/5	7
	alt13	1/7	4	1/4	5	1	8	1/6	9	1/4	3	1/3	4	1/2	2	1/4	6	1/2	8	1/8	4	1/4	3	1/7	2	1	1	1/3	7	1/8	6
	alt14	1/7	3	1/5	6	1	8	1/2	9	1	6	1/9	2	1/2	8	1/2	5	1/4	4	1/5	6	1/3	4	1/4	9	1	3	1	1	1/4	1
	alt15	1/3	9	1/9	5	1	5	1/8	6	1/7	8	1/6	2	1/4	9	1/7	4	1/7	7	1	9	1/4	3	1/7	5	1/6	8	1	4	1	1

Table A.57 15x15 consistent matrix7

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10		alt11		alt12		alt13		alt14		alt15	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 7	alt1	1	1	1/4	4	1/7	4	1/9	4	1/5	8	1/4	8	1/3	6	1/4	7	1/8	4	1/9	9	1/9	2	1/4	4	1/7	7	1/2	4	1/2	1
	alt2	1/4	4	1	1	1/9	4	1/3	3	1/2	5	1/2	8	1/9	5	1/8	4	1/7	5	1/6	7	1/5	9	1/4	1	1/7	8	1/6	5	1/2	4
	alt3	1/4	7	1/4	9	1	1	1/5	2	1/4	8	1/7	1	1/6	6	1/7	7	1/4	2	1/6	7	1/4	6	1/3	7	1/5	7	1/7	9	1/4	3
	alt4	1/4	9	1/3	3	1/2	5	1	1	1/3	5	1/4	8	1/4	1	1/5	9	1/3	4	1/4	7	1/4	6	1/4	2	1/5	6	1/2	3	1/8	8
	alt5	1/8	5	1/5	2	1/8	4	1/5	3	1	1	1/2	7	1/5	4	1/4	7	1/7	2	1/6	2	1/3	4	1/5	4	1/3	1	1/2	7	1/3	4
	alt6	1/8	4	1/8	2	1	7	1/8	4	1/7	2	1	1	1/2	7	1/8	6	1/7	4	1/3	9	1/3	6	1/9	1	1/6	3	1/8	5	1/4	7
	alt7	1/6	3	1/5	9	1/6	6	1	4	1/4	5	1/7	2	1	1	1/2	8	1/5	1	1/8	7	1/6	6	1/3	9	1/5	8	1/8	5	1/2	2
	alt8	1/7	4	1/4	8	1/7	7	1/9	5	1/7	4	1/6	8	1/8	2	1	1	1/5	2	1/2	8	1/4	2	1/5	2	1/9	6	1/8	7	1/8	1
	alt9	1/4	8	1/5	7	1/2	4	1/4	3	1/2	7	1/4	7	1	5	1/2	5	1	1	1/6	7	1/2	5	1/9	6	1/6	4	1/5	6	1/2	4
	alt10	1/9	9	1/7	6	1/7	6	1/7	4	1/2	6	1/9	3	1/9	3	1/7	8	1/7	6	1	1	1/7	2	1/9	2	1/4	4	1/4	7	1/4	5
	alt11	1/2	9	1/9	5	1/6	4	1/6	4	1/4	3	1/6	3	1/6	6	1/2	4	1/5	2	1/2	7	1	1	1/9	4	1/9	4	1/5	8	1/2	6
	alt12	1/4	4	1	4	1/7	3	1/4	5	1/4	5	1	9	1/9	3	1/2	5	1/6	9	1/2	9	1/4	9	1	1	1/6	1	1/9	5	1/3	7
	alt13	1/7	7	1/8	7	1/7	7	1/6	2	1	3	1/3	6	1/8	5	1/6	9	1/4	6	1/4	4	1/8	5	1	6	1	1	1/5	8	1/5	3
	alt14	1/4	2	1/5	6	1/9	7	1/3	2	1/7	2	1/5	8	1/5	8	1/7	8	1/6	4	1/7	4	1/5	8	1/9	5	1	5	1	1	1/5	6
	alt15	1	2	1/4	2	1/3	4	1/8	8	1/4	3	1/7	4	1/2	2	1	8	1/4	9	1/5	4	1/6	2	1/7	3	1/3	5	1/6	5	1	1

Table A.58 15x15 consistent matrix8

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10		alt11		alt12		alt13		alt14		alt15	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 8	alt1	1	1	1/9	3	1/8	1	1/8	6	1/4	2	1/4	3	1/7	6	1/9	6	1/8	4	1/3	2	1/4	3	1/9	8	1/5	6	1/6	8	1/8	8
	alt2	1/3	9	1	1	1/2	5	1/5	8	1/7	6	1/2	8	1/6	2	1/5	2	1/5	3	1/8	8	1/7	6	1/8	5	1/9	2	1/2	4	1/7	8
	alt3	1	8	1/5	2	1	1	1/2	2	1/6	9	1/6	2	1/9	2	1/2	6	1/2	7	1/4	9	1/9	9	1/2	5	1/9	7	1/7	1	1/6	6
	alt4	1/6	8	1/8	5	1/2	2	1	1	1/4	8	1/9	7	1/9	3	1/9	4	1/3	8	1/7	9	1/4	3	1/5	4	1/2	2	1/3	4	1/2	6
	alt5	1/2	4	1/6	7	1/9	6	1/8	4	1	1	1/7	4	1/5	9	1/3	5	1/2	6	1/3	3	1/6	8	1/5	4	1/9	5	1/5	7	1/7	5
	alt6	1/3	4	1/8	2	1/2	6	1/7	9	1/4	7	1	1	1/7	5	1/5	7	1/6	5	1/6	4	1/3	4	1/4	2	1/9	3	1/5	6	1/8	9
	alt7	1/6	7	1/2	6	1/2	9	1/3	9	1/9	5	1/5	7	1	1	1/2	3	1/7	4	1/3	9	1/2	2	1/5	6	1/2	5	1/5	7	1/9	4
	alt8	1/6	9	1/2	5	1/6	2	1/4	9	1/5	3	1/7	5	1/3	2	1	1	1/8	3	1/8	8	1/2	4	1/8	8	1/9	5	1/4	2	1/2	1
	alt9	1/4	8	1/3	5	1/7	2	1/8	3	1/6	2	1/5	6	1/4	7	1/3	8	1	1	1/7	5	1/4	7	1/5	9	1/3	9	1/7	1	1/6	6
	alt10	1/2	3	1/8	8	1/9	4	1/9	7	1/3	3	1/4	6	1/4	6	1/9	3	1/5	7	1	1	1/4	5	1/2	6	1/3	9	1/6	4	1/3	7
	alt11	1/3	4	1/6	7	1/9	9	1/3	4	1/8	6	1/4	3	1/2	2	1/4	2	1/7	4	1/5	4	1	1	1/6	8	1/4	6	1/9	1	1/9	3
	alt12	1/8	9	1/5	8	1/5	2	1/4	5	1/4	5	1/2	4	1/6	5	1/8	8	1/9	5	1/6	2	1/8	6	1	1	1/4	1	1/8	5	1/5	7
	alt13	1/6	5	1/2	9	1/7	7	1/2	3	1/5	9	1/3	9	1/5	2	1/5	9	1/9	3	1/9	3	1	9	1	4	1	1	1/3	8	1/3	6
	alt14	1/8	6	1/4	2	1	7	1/4	3	1/7	5	1/6	5	1/7	5	1/2	4	1	3	1/4	6	1/9	1	1/8	5	1	3	1	1	1/3	8
	alt15	1/8	8	1/8	7	1/6	6	1/6	2	1/5	7	1/9	8	1/4	9	1	2	1/6	4	1/7	3	1/3	9	1/7	5	1/6	3	1/8	3	1	1

Table A.59 15x15 consistent matrix9

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10		alt11		alt12		alt13		alt14		alt15	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 9	alt1	1	1	1/4	9	1/9	4	1/9	3	1/7	3	1/6	1	1/5	6	1/6	6	1/3	6	1/6	3	1/8	4	1/7	1	1/5	7	1/6	5	1/8	6
	alt2	1/9	4	1	1	1/3	6	1/6	9	1/2	6	1/3	9	1/3	1	1/5	4	1/7	2	1/2	8	1/3	9	1/5	3	1/7	3	1/9	9	1/6	6
	alt3	1/4	9	1/6	3	1	1	1/2	5	1/4	3	1/2	9	1/4	4	1/8	1	1/7	2	1/2	4	1/2	9	1/3	7	1/6	9	1/5	5	1/4	3
	alt4	1/3	9	1/9	6	1/5	2	1	1	1/5	2	1/7	2	1/9	3	1/7	4	1/7	9	1/8	6	1/2	5	1/9	5	1/8	2	1/4	2	1/4	1
	alt5	1/3	7	1/6	2	1/3	4	1/2	5	1	1	1/6	6	1/4	3	1/5	4	1/2	5	1/7	3	1/3	8	1/3	2	1/4	7	1/6	7	1/4	2
	alt6	1	6	1/9	3	1/9	2	1/2	7	1/6	6	1	1	1/4	6	1/3	3	1/9	2	1/5	5	1/8	9	1/7	3	1/8	4	1/5	2	1/5	6
	alt7	1/6	5	1	3	1/4	4	1/3	9	1/3	4	1/6	4	1	1	1/9	4	1/2	6	1/5	2	1/8	5	1/3	7	1/9	5	1/2	9	1/5	5
	alt8	1/6	6	1/4	5	1	8	1/4	7	1/4	5	1/3	3	1/4	9	1	1	1/6	4	1/6	8	1/6	9	1/2	3	1/4	2	1/5	9	1/7	2
	alt9	1/6	3	0.5	7	1/2	7	1/9	7	1/5	2	1/2	9	1/6	2	1/4	6	1	1	1/9	1	1/2	3	1/4	6	1/6	6	1/3	2	1/8	2
	alt10	1/3	6	1/8	2	1/4	2	1/6	8	1/3	7	1/5	5	1/5	5	1/2	5	1	9	1	1	1/9	9	1/8	7	1/6	6	1/6	5	1/4	9
	alt11	1/4	8	1/9	3	1/9	2	1/5	2	1/8	3	1/9	8	1/5	8	1/9	6	1/3	2	1/9	9	1	1	1/4	2	1/9	6	0.2	7	1/9	1
	alt12	1	7	1/3	5	1/7	3	1/2	3	1/2	3	1/3	7	1/7	3	1/3	2	1/6	4	1/7	8	1/2	4	1	1	1/9	2	1/9	1	1/4	3
	alt13	1/7	5	1/3	7	1/9	5	1/2	4	1/7	4	1/4	8	1/5	9	1/2	4	1/6	6	1/6	6	1/7	5	0.5	9	1	1	1/2	2	1/9	3
	alt14	1/5	6	1/9	9	1/5	5	1/2	4	1/7	6	1/2	5	1/9	2	1/9	5	1/2	6	1/5	6	0.2	7	1/9	1	1	2	1	1	1/8	1
	alt15	1/6	8	1/6	6	1/3	4	1	4	1/2	4	1/6	5	1/5	5	1/2	7	1/2	9	1/9	4	1	9	1/3	4	1/3	9	1	8	1	1

Table A.60 15x15 consistent matrix10

		alt1		alt2		alt3		alt4		alt5		alt6		alt7		alt8		alt9		alt10		alt11		alt12		alt13		alt14		alt15	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 10	alt1	1	1	1/6	4	1/5	3	1/2	3	1/7	9	1/5	1	1/7	7	1/3	1	1/6	5	1/3	2	1/9	5	1/9	8	1/3	2	1/6	9	1/4	1
	alt2	1/4	6	1	1	1/6	4	1/6	8	1/2	8	1/4	7	1/6	1	1/8	2	1/2	7	1/5	2	1/6	4	1/7	6	1/2	4	1/6	9	1/6	9
	alt3	1/3	5	1/4	6	1	1	1/4	3	1/3	1	1/8	1	1/3	7	1/4	4	1/3	1	1/8	3	1/4	1	1/9	6	1/5	7	1/8	8	1/3	4
	alt4	1/3	2	1/8	6	1/3	4	1	1	1/6	8	1/7	8	1/4	8	1/4	8	1/2	4	1/2	9	1/4	4	1/3	8	1/3	4	1/6	1	1/8	5
	alt5	1/9	7	1/8	2	1	3	1/8	6	1	1	1/9	3	1/2	3	1/9	5	1/3	1	1/6	2	1/3	5	1/3	2	1/3	7	1/3	3	1/6	8
	alt6	1	5	1/7	4	1	8	1/8	7	1/3	9	1	1	1/9	7	1/4	6	1/6	7	1/9	2	1/4	6	1/9	1	1/9	6	1/5	9	1/5	3
	alt7	1/7	7	1	6	1/7	3	1/8	4	1/3	2	1/7	9	1	1	1/4	4	1/9	9	1/7	8	1/4	4	1/4	4	1/4	6	1/9	8	1/3	1
	alt8	1	3	1/2	8	1/4	4	1/8	4	1/5	9	1/6	4	1/4	4	1	1	1/3	6	1/4	3	1/2	8	1/7	3	1/6	6	1/3	6	1/5	2
	alt9	1/5	6	1/7	2	1	3	1/4	2	1	3	1/7	6	1/9	9	1/6	3	1	1	1/2	5	1/8	7	1/9	9	1/2	3	1/2	6	1/9	3
	alt10	1/2	3	1/2	5	1/3	8	1/9	2	1/2	6	1/2	9	1/2	9	1/8	7	1/5	2	1	1	1/4	6	1/3	4	1/2	3	1/2	4	1/6	7
	alt11	1/5	9	1/4	6	1	4	1/4	4	1/5	3	1/6	4	1/4	4	1/8	2	1/7	8	1/6	4	1	1	1/3	2	1/8	9	1/6	3	1/4	9
	alt12	1/8	9	1/6	7	1/6	9	1/2	3	1/2	3	1	9	1/4	4	1/3	7	1/9	9	1/4	3	1/2	3	1	1	1/2	8	1/6	4	1/7	5
	alt13	1/2	3	1/4	2	1/7	8	1/4	6	1/7	3	1/6	9	1/6	4	1/6	6	1/3	2	1/3	2	1/3	6	1/8	2	1	1	1/5	4	1/9	8
	alt14	1/9	6	1/9	6	1/8	8	1	6	1/3	3	1/9	5	1/8	9	1/6	3	1/6	2	1/4	2	1/6	3	1/6	4	1	5	1	1	1/4	8
	alt15	1	4	1/9	6	1/4	3	1/5	8	1/8	6	1/3	5	1	3	1/2	5	1/3	8	1/7	6	1/9	4	1/5	7	1/8	9	1/8	4	1	1

APPENDIX B

INTERVAL WEIGHTS OBTAINED FOR THE MATRICES

B.1 5x5 Inconsistent Matrix Generation

Table B.1 Obtained interval weights for 5x5 inconsistent matrix1

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 1	alt1	Infeasible		0.029	0.231	0.370	0.758	0.039	0.039	0.684	1.595	1.146	1.146	0.785	1.138
	alt2			0.046	0.248	0.315	2.551	0.050	0.590	0.419	2.392	0.458	1.834	1.047	1.806
	alt3			0.031	0.233	1.480	3.031	0.220	0.228	0.299	4.187	1.450	1.450	1.038	1.792
	alt4			0.058	0.260	0.500	1.760	0.144	0.144	1.047	4.785	0.573	2.292	0.742	1.194
	alt5			0.043	0.231	0.740	1.516	0.010	0.540	0.797	2.093	0.573	2.292	0.916	1.580

Table B.2 Obtained interval weights for 5x5 inconsistent matrix2

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 2	alt1	Infeasible		0.028	0.238	0.379	0.559	0.053	0.288	0.413	0.825	0.251	1.419	0.736	0.773
	alt2			0.026	0.237	0.287	0.871	0.069	0.079	0.481	1.886	0.335	0.876	0.368	1.012
	alt3			0.030	0.240	0.871	1.888	0.022	0.595	0.236	2.888	0.355	2.007	0.499	3.092
	alt4			0.034	0.245	1.516	2.237	0.220	0.220	0.413	1.651	0.710	1.004	1.655	1.655
	alt5			0.040	0.250	1.516	2.237	0.010	0.390	1.651	1.651	1.262	2.258	0.722	1.546

Table B.3 Obtained interval weights for 5x5 inconsistent matrix3

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 3	alt1	Infeasible		0.047	0.284	1.634	2.753	0.204	0.446	0.704	2.816	0.812	3.014	1.942	4.291
	alt2			0.034	0.074	0.221	0.403	0.010	0.061	0.352	0.352	0.377	0.406	0.159	0.351
	alt3			0.039	0.274	0.504	0.848	0.032	0.071	0.453	0.704	0.639	0.639	0.614	0.614
	alt4			0.095	0.332	2.328	3.920	0.316	0.658	3.168	3.620	3.197	3.876	2.966	6.554
	alt5			0.037	0.274	0.435	1.473	0.106	0.106	0.402	1.584	0.431	1.599	0.805	1.778

Table B.4 Obtained interval weights for 5x5 inconsistent matrix4

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 4	alt1	Infeasible		0.034	0.244	1.041	1.888	0.260	0.260	0.770	2.598	1.080	2.160	1.573	1.573
	alt2			0.027	0.237	1.256	2.074	0.074	0.527	0.429	2.333	0.945	1.920	0.720	2.672
	alt3			0.039	0.249	1.256	2.074	0.059	0.141	0.289	2.333	1.080	1.080	0.891	1.247
	alt4			0.027	0.237	0.189	0.344	0.029	0.139	0.333	2.309	0.240	2.520	0.156	1.297
	alt5			0.034	0.244	0.573	2.010	0.010	0.386	1.000	1.000	0.360	1.920	0.764	1.224

Table B.5 Obtained interval weights for 5x5 inconsistent matrix5

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 5	alt1	Infeasible		0.027	0.234	0.380	1.687	0.214	0.214	0.264	1.897	0.930	0.930	0.915	1.050
	alt2			0.039	0.246	0.439	2.440	0.031	0.582	0.295	1.476	0.588	1.764	1.099	1.437
	alt3			0.026	0.233	0.218	0.556	0.010	0.010	0.211	0.984	0.294	1.176	0.575	1.150
	alt4			0.039	0.246	1.168	3.404	0.138	0.143	0.443	1.476	1.247	1.764	1.003	1.050
	alt5			0.041	0.248	0.864	3.491	0.062	0.612	0.246	1.771	0.294	4.989	0.549	1.725

Table B.6 Obtained interval weights for 5x5 inconsistent matrix6

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 6	alt1	Infeasible		0.034	0.244	2.923	4.536	0.077	0.384	0.500	2.965	2.333	3.270	0.737	3.126
	alt2			0.031	0.241	0.585	0.907	0.058	0.058	0.552	0.552	0.552	0.552	0.950	0.950
	alt3			0.030	0.240	0.277	0.494	0.010	0.307	0.371	2.223	0.409	2.016	0.391	1.021
	alt4			0.027	0.236	0.839	1.719	0.038	0.345	0.247	1.482	0.224	1.635	0.731	1.187
	alt5			0.039	0.249	0.573	1.259	0.170	0.213	1.112	1.112	1.008	1.226	1.179	1.179

Table B.7 Obtained interval weights for 5x5 inconsistent matrix7

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 7	alt1	Infeasible		0.027	0.238	0.242	2.518	0.058	0.389	0.581	1.140	0.739	1.388	1.177	1.634
	alt2			0.035	0.245	0.530	2.612	0.004	0.335	0.249	1.140	0.347	3.556	0.876	2.629
	alt3			0.026	0.237	0.850	3.898	0.164	0.164	1.493	1.493	0.812	0.925	0.940	0.940
	alt4			0.035	0.245	0.341	1.303	0.011	0.343	0.380	1.742	0.463	2.218	0.392	1.177
	alt5			0.035	0.245	0.324	2.491	0.101	0.101	2.661	2.661	1.013	1.013	0.876	1.052

Table B.8 Obtained interval weights for 5x5 inconsistent matrix8

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 8	alt1	0.123	0.123	0.026	0.237	0.500	1.246	0.086	0.086	0.426	0.799	0.426	0.799	0.605	0.605
	alt2	0.082	0.082	0.027	0.238	0.290	0.871	0.038	0.174	0.266	1.278	0.266	1.278	0.481	0.961
	alt3	0.164	0.247	0.048	0.259	0.758	1.888	0.193	0.327	0.959	1.917	1.065	1.917	1.442	2.883
	alt4	0.370	0.493	0.029	0.240	1.589	4.193	0.114	0.413	0.426	2.397	0.426	2.397	0.763	3.027
	alt5	0.055	0.260	0.027	0.238	0.290	2.297	0.010	0.508	0.213	2.131	0.213	2.131	0.197	3.126

Table B.9 Obtained interval weights for 5x5 inconsistent matrix9

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 9	alt1	Infeasible		0.076	0.280	2.281	3.301	0.290	0.377	1.486	3.468	2.052	3.591	0.836	1.115
	alt2			0.025	0.229	2.352	3.511	0.298	0.471	3.034	3.034	2.693	4.040	0.976	1.708
	alt3			0.033	0.236	0.445	0.850	0.010	0.173	0.433	1.011	0.513	0.898	0.429	0.750
	alt4			0.025	0.229	0.347	0.642	0.040	0.040	0.433	0.433	0.449	0.513	0.897	1.154
	alt5			0.025	0.229	0.357	0.533	0.112	0.112	0.506	1.180	0.449	0.786	1.061	1.061

Table B.10 Obtained interval weights for 5x5 inconsistent matrix10

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 10	alt1	Infeasible		0.029	0.235	2.440	4.743	0.051	0.259	2.766	2.766	2.825	2.825	0.924	0.924
	alt2			0.027	0.233	0.211	0.541	0.057	0.130	0.307	0.553	0.314	0.565	0.895	1.023
	alt3			0.041	0.247	0.967	2.724	0.075	0.282	0.615	2.766	0.565	2.825	0.975	1.114
	alt4			0.039	0.245	0.591	1.516	0.043	0.251	0.692	2.459	0.706	2.260	0.975	1.114
	alt5			0.039	0.245	0.407	0.790	0.078	0.286	0.553	2.766	0.565	2.825	1.114	1.114

B.2 5x5 Consistent Matrix Generation

Table B.11 Obtained interval weights for 5x5 consistent matrix1

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 1	alt1	0.063	0.156	0.031	0.233	0.229	1.320	0.002	0.286	0.212	1.559	0.287	1.079	0.217	1.151
	alt2	0.156	0.313	0.029	0.231	0.574	2.402	0.057	0.240	0.318	2.338	0.335	2.081	0.352	1.864
	alt3	0.156	0.313	0.026	0.227	0.500	2.237	0.027	0.311	0.260	1.909	0.231	2.011	0.352	1.243
	alt4	0.156	0.313	0.031	0.233	0.500	1.821	0.123	0.123	0.623	1.061	0.432	1.436	0.470	1.151
	alt5	0.313	0.313	0.077	0.279	0.871	2.702	0.152	0.324	0.996	0.996	0.694	1.340	0.621	1.726

Table B.12 Obtained interval weights for 5x5 consistent matrix2

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 2	alt1	0.070	0.077	0.027	0.241	0.240	1.516	0.019	0.276	0.271	1.602	0.294	1.545	0.178	1.469
	alt2	0.153	0.488	0.031	0.245	0.435	4.872	0.050	0.465	0.320	1.896	0.309	2.060	0.266	2.203
	alt3	0.088	0.279	0.031	0.245	0.257	2.297	0.010	0.335	0.237	1.402	0.258	1.589	0.228	1.799
	alt4	0.077	0.279	0.027	0.242	0.339	2.091	0.081	0.081	0.207	0.837	0.294	0.618	0.542	0.888
	alt5	0.279	0.613	0.027	0.242	0.678	4.580	0.129	0.256	0.837	1.659	0.320	2.132	0.664	1.599

Table B.13 Obtained interval weights for 5x5 consistent matrix3

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 3	alt1	0.175	0.316	0.030	0.242	0.297	3.064	0.025	0.248	0.349	1.494	0.359	1.541	0.237	1.509
	alt2	0.105	0.526	0.027	0.238	0.305	3.866	0.056	0.278	0.299	1.793	0.308	1.849	0.299	1.267
	alt3	0.105	0.175	0.030	0.242	0.268	2.169	0.058	0.126	0.349	0.598	0.308	0.616	0.282	1.064
	alt4	0.088	0.526	0.027	0.239	0.274	4.193	0.043	0.318	0.299	2.091	0.264	1.849	0.299	1.509
	alt5	0.088	0.526	0.040	0.251	0.308	4.522	0.053	0.305	0.299	2.091	0.308	2.157	0.326	2.074

Table B.14 Obtained interval weights for 5x5 consistent matrix4

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 4	alt1	0.124	0.355	0.040	0.252	0.291	3.314	0.055	0.265	0.306	1.292	0.339	1.222	0.216	1.297
	alt2	0.178	0.497	0.028	0.240	0.304	3.895	0.066	0.257	0.306	1.808	0.339	1.885	0.306	1.835
	alt3	0.130	0.355	0.030	0.242	0.291	3.728	0.058	0.292	0.258	1.528	0.244	1.697	0.216	1.297
	alt4	0.178	0.497	0.027	0.239	0.349	3.565	0.083	0.184	0.452	1.223	0.471	1.357	0.306	1.835
	alt5	0.071	0.391	0.027	0.239	0.204	3.178	0.002	0.304	0.258	1.356	0.189	1.832	0.177	1.059

Table B.15 Obtained interval weights for 5x5 consistent matrix5

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 5	alt1	0.154	0.462	0.027	0.240	0.359	3.650	0.056	0.329	0.560	1.464	0.490	1.470	0.345	1.794
	alt2	0.154	0.462	0.027	0.240	0.415	3.277	0.041	0.276	0.488	1.281	0.490	1.345	0.270	1.404
	alt3	0.077	0.154	0.030	0.243	0.280	2.112	0.068	0.160	0.213	1.281	0.245	1.406	0.317	1.099
	alt4	0.154	0.154	0.035	0.247	0.258	2.169	0.010	0.282	0.244	1.464	0.245	1.537	0.270	1.404
	alt5	0.154	0.462	0.030	0.243	0.488	3.471	0.091	0.225	0.366	1.708	0.448	1.470	0.496	1.337

Table B.16 Obtained interval weights for 5x5 consistent matrix6

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 6	alt1	0.094	0.472	0.027	0.232	0.265	3.519	0.026	0.310	0.185	1.480	0.220	1.760	0.290	1.537
	alt2	0.157	0.189	0.058	0.264	0.370	2.862	0.075	0.250	0.370	1.974	0.440	1.113	0.249	1.194
	alt3	0.157	0.472	0.033	0.239	0.354	3.245	0.080	0.193	0.296	1.110	0.352	1.320	0.307	1.537
	alt4	0.118	0.315	0.033	0.239	0.244	3.482	0.023	0.307	0.247	1.665	0.293	1.760	0.187	1.266
	alt5	0.094	0.472	0.027	0.232	0.291	3.565	0.059	0.223	0.185	1.480	0.220	1.760	0.290	1.900

Table B.17 Obtained interval weights for 5x5 consistent matrix7

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 7	alt1	0.118	0.369	0.027	0.243	0.204	3.245	0.021	0.218	0.164	1.091	0.196	1.091	0.209	1.399
	alt2	0.092	0.332	0.027	0.243	0.203	3.178	0.038	0.176	0.218	1.091	0.218	1.091	0.220	1.180
	alt3	0.125	0.549	0.027	0.243	0.315	5.515	0.056	0.343	0.364	1.963	0.364	1.963	0.293	2.099
	alt4	0.166	0.591	0.027	0.243	0.315	5.036	0.060	0.315	0.364	1.963	0.364	1.963	0.371	1.659
	alt5	0.074	0.499	0.027	0.243	0.215	3.959	0.038	0.235	0.218	1.454	0.218	1.454	0.220	1.244

Table B.18 Obtained interval weights for 5x5 consistent matrix8

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 8	alt1	0.160	0.479	0.034	0.241	0.312	3.129	0.073	0.223	0.233	1.582	0.277	1.383	0.461	1.219
	alt2	0.096	0.319	0.040	0.248	0.266	3.471	0.050	0.297	0.308	2.093	0.277	1.660	0.244	1.499
	alt3	0.160	0.479	0.028	0.235	0.288	3.965	0.021	0.365	0.233	1.582	0.237	1.423	0.250	1.983
	alt4	0.120	0.439	0.035	0.243	0.339	3.764	0.071	0.266	0.396	1.395	0.346	1.660	0.348	1.935
	alt5	0.146	0.239	0.034	0.241	0.276	2.759	0.059	0.193	0.349	0.930	0.277	1.107	0.397	1.133

Table B.19 Obtained interval weights for 5x5 consistent matrix9

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 9	alt1	0.107	0.321	0.030	0.243	0.246	3.446	0.041	0.360	0.231	1.458	0.226	1.806	0.331	1.623
	alt2	0.107	0.429	0.031	0.244	0.349	3.630	0.073	0.257	0.461	1.458	0.451	1.505	0.283	1.384
	alt3	0.143	0.214	0.027	0.240	0.233	2.169	0.052	0.135	0.162	0.922	0.201	0.903	0.276	1.063
	alt4	0.107	0.429	0.027	0.239	0.284	3.104	0.014	0.270	0.292	1.844	0.301	1.806	0.253	1.238
	alt5	0.214	0.536	0.035	0.247	0.488	4.282	0.101	0.297	0.615	1.750	0.602	1.806	0.496	1.657

Table B.20 Obtained interval weights for 5x5 consistent matrix10

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 10	alt1	0.143	0.143	0.034	0.248	0.401	1.246	0.066	0.370	0.303	2.307	0.347	1.785	0.243	1.744
	alt2	0.143	0.143	0.030	0.244	0.530	1.585	0.164	0.164	0.627	0.627	0.681	1.009	0.765	1.007
	alt3	0.057	0.286	0.027	0.240	0.272	2.000	0.028	0.292	0.279	2.122	0.231	1.362	0.255	1.830
	alt4	0.143	0.286	0.028	0.241	0.660	2.091	0.126	0.160	0.627	0.989	0.520	1.155	0.442	0.671
	alt5	0.143	0.514	0.027	0.240	0.758	4.193	0.014	0.616	0.330	2.508	0.385	2.079	0.463	3.325

B.3 10x10 Inconsistent Matrix Generation

Table B.21 Obtained interval weights for 10x10 inconsistent matrix1

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 1	alt1	Infeasible		0.016	0.114	1.900	3.253	0.130	0.130	0.477	1.390	0.517	1.378	1.036	1.332
	alt2			0.013	0.114	0.295	0.750	0.003	0.127	0.248	1.251	0.295	1.034	0.680	0.994
	alt3			0.013	0.114	0.258	0.453	0.022	0.089	0.199	1.430	0.197	1.418	0.636	0.930
	alt4			0.013	0.101	1.519	2.458	0.001	0.109	0.348	1.112	0.345	1.103	0.969	1.417
	alt5			0.013	0.114	1.609	2.871	0.012	0.136	0.358	1.324	0.354	1.575	0.906	0.930
	alt6			0.013	0.089	0.828	1.477	0.001	0.124	0.166	0.834	0.172	0.827	0.680	0.870
	alt7			0.013	0.114	0.246	0.398	0.001	0.001	0.199	0.795	0.184	0.788	0.848	0.848
	alt8			0.013	0.114	1.563	2.634	0.074	0.198	0.222	1.788	0.221	1.772	0.906	0.930
	alt9			0.013	0.114	0.649	1.112	0.077	0.085	0.278	1.241	0.276	1.286	0.848	1.240
	alt10			0.013	0.114	0.460	2.211	0.003	0.127	0.199	1.490	0.197	1.544	0.848	1.062

Table B.22 Obtained interval weights for 10x10 inconsistent matrix2

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 2	alt1	Infeasible		0.013	0.111	0.323	0.542	0.010	0.057	0.192	0.954	0.185	0.919	0.578	1.309
	alt2			0.016	0.115	1.823	2.758	0.110	0.189	0.964	1.770	0.529	1.892	1.085	1.309
	alt3			0.013	0.111	0.445	0.665	0.010	0.120	0.253	0.744	0.244	0.717	0.335	0.757
	alt4			0.012	0.111	0.890	1.331	0.036	0.219	0.318	1.518	0.431	1.462	0.508	1.150
	alt5			0.014	0.111	0.407	1.097	0.037	0.037	0.246	1.341	0.237	1.292	0.651	0.935
	alt6			0.012	0.088	0.445	0.665	0.010	0.030	0.253	0.562	0.244	0.710	0.510	1.155
	alt7			0.014	0.113	0.622	0.930	0.047	0.076	0.379	1.265	0.365	1.218	0.862	0.862
	alt8			0.012	0.111	0.491	2.364	0.010	0.111	0.268	1.341	0.258	1.292	0.878	0.878
	alt9			0.014	0.113	2.518	3.764	0.127	0.169	0.744	1.719	0.717	1.656	0.651	1.473
	alt10			0.014	0.113	1.340	2.874	0.010	0.176	0.238	1.341	0.230	1.292	0.571	1.258

Table B.23 Obtained interval weights for 10x10 inconsistent matrix3

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 3	alt1	Infeasible		0.013	0.101	0.533	0.771	0.078	0.096	0.381	1.410	0.388	1.436	0.973	1.442
	alt2			0.013	0.114	0.605	1.661	0.001	0.149	0.571	2.937	0.646	1.508	0.997	1.477
	alt3			0.013	0.114	2.195	3.266	0.044	0.193	0.571	2.284	0.646	2.585	1.095	1.622
	alt4			0.013	0.114	1.036	1.780	0.007	0.122	0.627	0.979	0.563	1.131	1.216	1.385
	alt5			0.013	0.114	0.549	0.890	0.001	0.149	0.326	1.713	0.369	1.939	1.246	1.442
	alt6			0.013	0.114	0.282	0.445	0.041	0.041	0.211	0.564	0.215	0.574	0.912	1.182
	alt7			0.013	0.114	2.013	2.748	0.001	0.149	0.282	1.903	0.287	1.939	0.973	1.081
	alt8			0.013	0.114	1.143	2.221	0.034	0.034	0.598	0.598	0.517	0.718	0.997	1.154
	alt9			0.013	0.089	1.007	1.374	0.066	0.066	0.783	0.783	0.798	0.798	0.608	0.901
	alt10			0.014	0.114	0.336	0.458	0.001	0.149	0.317	0.846	0.323	0.862	0.779	1.154

Table B.24 Obtained interval weights for 10x10 inconsistent matrix4

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 4	alt1	Infeasible		0.012	0.112	0.420	0.560	0.001	0.120	0.220	1.202	0.248	1.116	0.807	0.941
	alt2			0.012	0.100	1.632	3.511	0.001	0.155	0.264	0.962	0.253	0.992	0.584	1.022
	alt3			0.014	0.114	0.902	1.613	0.098	0.104	0.280	1.079	0.319	0.992	1.148	1.148
	alt4			0.013	0.112	0.758	1.630	0.001	0.068	0.481	1.079	0.496	1.158	0.822	0.822
	alt5			0.014	0.114	1.236	1.671	0.059	0.093	0.240	1.683	0.248	1.736	0.893	1.077
	alt6			0.012	0.112	0.303	0.410	0.001	0.001	0.337	1.321	0.389	1.266	0.501	0.876
	alt7			0.012	0.112	0.615	0.824	0.001	0.180	0.195	1.362	0.220	1.256	0.674	1.180
	alt8			0.012	0.112	0.686	1.537	0.001	0.280	0.240	1.683	0.248	1.736	0.668	1.170
	alt9			0.012	0.112	2.110	2.827	0.096	0.186	0.270	1.751	0.289	1.984	0.893	1.337
	alt10			0.012	0.101	0.615	0.824	0.079	0.096	0.582	0.582	0.579	0.579	0.718	1.108

Table B.25 Obtained interval weights for 10x10 inconsistent matrix5

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 5	alt1	Infeasible		0.014	0.113	1.484	2.679	0.001	0.169	0.294	1.392	0.294	1.392	0.766	2.883
	alt2			0.016	0.115	0.324	0.597	0.046	0.046	0.245	0.785	0.245	0.785	0.766	0.951
	alt3			0.016	0.113	0.684	1.286	0.090	0.090	0.232	1.450	0.232	1.450	0.759	0.832
	alt4			0.012	0.111	0.783	1.598	0.001	0.118	0.232	1.532	0.232	1.532	0.766	0.824
	alt5			0.012	0.111	0.516	1.241	0.001	0.109	0.232	1.226	0.232	1.226	0.590	0.936
	alt6			0.013	0.111	0.571	1.628	0.015	0.143	0.245	1.392	0.245	1.392	0.549	1.070
	alt7			0.012	0.111	0.260	0.593	0.001	0.123	0.196	0.680	0.196	0.680	0.287	0.824
	alt8			0.014	0.113	0.531	1.563	0.001	0.168	0.242	1.471	0.242	1.471	0.664	0.961
	alt9			0.016	0.113	2.393	4.407	0.145	0.145	1.624	1.624	1.624	1.624	0.885	0.885
	alt10			0.016	0.086	0.872	1.686	0.057	0.057	0.306	0.981	0.271	0.981	0.759	2.853

Table B.26 Obtained interval weights for 10x10 inconsistent matrix6

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 6	alt1	Infeasible		0.013	0.117	0.579	1.175	0.059	0.059	0.595	3.699	0.291	1.812	1.022	1.022
	alt2			0.013	0.108	0.976	1.464	0.013	0.123	0.334	3.008	0.259	1.165	0.985	1.122
	alt3			0.015	0.119	0.716	0.987	0.090	0.090	0.881	1.903	0.303	1.456	1.115	1.262
	alt4			0.013	0.117	0.422	0.502	0.001	0.001	0.638	2.114	0.291	1.036	0.981	1.075
	alt5			0.013	0.117	0.444	0.796	0.001	0.001	0.528	0.528	0.259	0.364	0.800	0.800
	alt6			0.017	0.121	3.795	4.521	0.197	0.206	2.642	4.756	1.820	2.330	1.262	1.262
	alt7			0.013	0.118	0.422	0.502	0.060	0.255	0.528	4.756	0.259	2.330	1.028	1.204
	alt8			0.013	0.117	2.108	2.512	0.018	0.213	0.925	2.642	0.485	1.294	0.981	1.433
	alt9			0.015	0.065	0.639	0.829	0.001	0.062	0.761	1.321	0.291	0.910	0.733	1.071
	alt10			0.013	0.105	1.265	1.507	0.001	0.186	1.915	1.915	0.777	1.214	0.833	1.071

Table B.27 Obtained interval weights for 10x10 inconsistent matrix7

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 7	alt1	Infeasible		0.012	0.112	0.181	0.369	0.001	0.001	0.220	0.949	0.665	2.660	0.554	0.819
	alt2			0.013	0.112	0.629	1.928	0.061	0.110	0.277	1.505	0.739	4.789	0.928	0.928
	alt3			0.012	0.100	0.517	1.627	0.021	0.127	0.277	0.922	0.931	2.660	0.712	0.825
	alt4			0.014	0.114	1.895	4.637	0.057	0.195	0.277	1.793	0.931	5.321	0.941	3.740
	alt5			0.014	0.112	0.426	1.137	0.001	0.118	0.237	1.186	0.665	3.326	0.598	0.928
	alt6			0.014	0.113	1.170	2.468	0.017	0.155	0.237	1.537	0.665	4.656	0.403	1.603
	alt7			0.014	0.112	0.622	3.115	0.011	0.148	0.256	1.660	0.776	4.656	0.436	1.731
	alt8			0.012	0.101	0.296	0.850	0.001	0.107	0.307	0.830	0.887	2.794	0.247	0.982
	alt9			0.012	0.112	1.152	2.430	0.058	0.058	0.465	1.186	1.478	3.725	0.712	0.722
	alt10			0.013	0.112	0.506	1.736	0.052	0.120	0.359	0.922	0.745	4.434	0.813	0.928

Table B.28 Obtained interval weights for 10x10 inconsistent matrix8

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 8	alt1	Infeasible		0.012	0.110	0.764	1.460	0.001	0.060	0.610	3.887	0.327	1.765	0.717	0.921
	alt2			0.012	0.110	1.310	1.917	0.073	0.073	0.597	2.613	0.474	1.158	0.715	0.891
	alt3			0.012	0.110	0.550	0.760	0.081	0.081	0.914	1.728	0.371	1.038	0.779	0.779
	alt4			0.016	0.114	2.122	2.580	0.055	0.196	2.259	4.180	0.882	1.853	1.166	1.593
	alt5			0.014	0.101	0.540	0.746	0.003	0.144	0.597	1.190	0.294	0.635	0.869	1.186
	alt6			0.013	0.111	0.808	1.245	0.016	0.156	0.574	3.658	0.294	1.482	0.953	0.953
	alt7			0.016	0.110	1.560	2.402	0.040	0.180	3.442	3.442	1.765	1.765	1.053	1.438
	alt8			0.012	0.110	1.061	1.290	0.063	0.110	0.523	1.194	0.232	0.949	0.847	0.847
	alt9			0.012	0.110	1.061	1.290	0.001	0.141	0.648	3.658	0.389	1.482	0.877	0.986
	alt10			0.014	0.112	0.212	0.258	0.001	0.001	0.523	0.907	0.212	0.588	0.921	0.986

Table B.29 Obtained interval weights for 10x10 inconsistent matrix9

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 9	alt1	Infeasible		0.013	0.117	0.635	1.118	0.040	0.085	0.268	2.008	0.265	1.984	0.380	1.080
	alt2			0.015	0.117	0.995	2.155	0.001	0.127	0.287	1.338	0.283	1.323	0.760	0.760
	alt3			0.013	0.117	0.439	0.712	0.001	0.048	0.335	0.574	0.372	0.567	0.314	1.274
	alt4			0.013	0.091	0.297	0.494	0.001	0.088	0.167	1.004	0.165	0.992	0.596	0.596
	alt5			0.019	0.104	0.557	0.925	0.001	0.158	0.335	1.338	0.331	1.323	0.260	1.053
	alt6			0.013	0.117	1.828	3.000	0.018	0.151	0.446	2.008	0.441	1.984	0.948	3.241
	alt7			0.013	0.117	0.574	1.231	0.090	0.101	0.287	1.338	0.283	1.488	0.567	0.908
	alt8			0.017	0.104	0.535	0.853	0.001	0.091	0.335	1.004	0.331	0.992	0.178	0.722
	alt9			0.013	0.117	1.481	2.458	0.018	0.177	0.382	1.004	0.378	0.992	0.570	2.310
	alt10			0.013	0.104	1.634	2.639	0.132	0.132	0.335	1.338	0.331	1.323	0.968	1.258

Table B.30 Obtained interval weights for 10x10 inconsistent matrix10

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 10	alt1	Infeasible		0.018	0.114	1.103	1.590	0.054	0.134	1.072	3.217	1.057	3.033	1.452	4.105
	alt2			0.014	0.110	0.955	1.948	0.001	0.175	0.460	4.136	0.433	4.077	0.793	2.558
	alt3			0.015	0.110	1.815	2.460	0.068	0.165	1.182	3.217	1.019	3.316	1.239	1.314
	alt4			0.014	0.110	0.405	1.072	0.001	0.175	0.630	3.446	0.582	3.398	0.365	1.178
	alt5			0.012	0.108	1.086	1.566	0.051	0.125	1.608	2.068	1.586	2.228	0.867	0.939
	alt6			0.012	0.108	0.267	0.385	0.013	0.013	0.460	0.460	0.453	0.529	1.244	1.275
	alt7			0.014	0.110	0.303	0.410	0.007	0.007	0.536	0.536	0.553	0.650	0.578	0.958
	alt8			0.013	0.108	0.908	1.230	0.035	0.131	0.517	3.217	0.557	3.171	0.962	1.735
	alt9			0.022	0.110	1.815	2.460	0.099	0.099	2.144	2.757	2.600	2.600	1.690	1.690
	alt10			0.012	0.108	0.703	2.072	0.064	0.151	0.689	3.574	0.680	3.370	0.586	1.309

B.4 10x10 Consistent Matrix Generation

Table B.31 Obtained interval weights for 10x10 consistent matrix1

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 1	alt1	0.039	0.228	0.012	0.112	0.180	3.728	0.001	0.140	0.178	1.203	0.181	1.304	0.247	1.226
	alt2	0.114	0.114	0.013	0.112	0.291	3.028	0.030	0.114	0.214	1.403	0.217	1.522	0.259	0.765
	alt3	0.064	0.193	0.012	0.112	0.211	3.726	0.001	0.130	0.200	1.069	0.217	1.087	0.247	1.226
	alt4	0.060	0.120	0.014	0.114	0.231	3.635	0.021	0.102	0.200	1.069	0.217	1.087	0.247	1.226
	alt5	0.093	0.193	0.014	0.114	0.235	3.570	0.024	0.124	0.179	1.195	0.215	1.101	0.225	1.119
	alt6	0.080	0.270	0.012	0.112	0.223	4.258	0.023	0.107	0.241	1.247	0.261	1.268	0.226	0.938
	alt7	0.090	0.280	0.012	0.112	0.319	5.313	0.028	0.123	0.239	1.603	0.220	1.739	0.323	1.602
	alt8	0.060	0.240	0.012	0.087	0.244	3.702	0.024	0.088	0.239	1.076	0.220	1.087	0.259	1.170
	alt9	0.114	0.180	0.012	0.112	0.430	3.912	0.036	0.097	0.269	0.950	0.313	0.815	0.323	1.529
	alt10	0.096	0.228	0.014	0.112	0.309	4.069	0.031	0.116	0.269	1.434	0.272	1.565	0.259	1.170

Table B.32 Obtained interval weights for 10x10 consistent matrix2

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 2	alt1	0.077	0.077	0.012	0.111	0.210	1.943	0.001	0.099	0.202	0.887	0.196	0.941	0.330	1.162
	alt2	0.067	0.067	0.014	0.097	0.313	1.762	0.033	0.049	0.300	0.719	0.294	0.706	0.220	0.775
	alt3	0.090	0.202	0.012	0.111	0.381	3.344	0.026	0.121	0.257	1.864	0.249	1.764	0.389	1.162
	alt4	0.111	0.232	0.012	0.111	0.265	3.326	0.025	0.121	0.240	1.198	0.235	1.176	0.301	1.058
	alt5	0.074	0.148	0.014	0.112	0.421	2.888	0.038	0.092	0.431	1.109	0.392	1.176	0.373	1.314
	alt6	0.077	0.148	0.012	0.111	0.346	2.927	0.035	0.092	0.399	1.198	0.353	1.307	0.387	1.314
	alt7	0.148	0.269	0.012	0.111	0.571	3.862	0.040	0.129	0.240	1.198	0.235	1.176	0.438	1.168
	alt8	0.067	0.067	0.012	0.111	0.374	5.133	0.024	0.151	0.222	1.797	0.221	1.742	0.352	1.162
	alt9	0.101	0.222	0.014	0.112	0.355	3.505	0.010	0.142	0.202	1.775	0.196	1.764	0.332	1.168
	alt10	0.067	0.134	0.012	0.111	0.228	2.670	0.004	0.137	0.222	1.213	0.235	1.176	0.387	1.162

Table B.33 Obtained interval weights for 10x10 consistent matrix3

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 3	alt1	0.087	0.087	0.012	0.111	0.448	1.908	0.033	0.082	0.329	0.767	0.190	0.946	0.335	1.063
	alt2	0.043	0.043	0.012	0.111	0.287	1.625	0.002	0.137	0.192	1.752	0.187	1.708	0.289	1.071
	alt3	0.087	0.174	0.012	0.111	0.418	2.518	0.001	0.135	0.219	1.533	0.213	1.494	0.281	0.949
	alt4	0.087	0.087	0.018	0.111	0.406	2.548	0.043	0.107	0.383	1.314	0.274	1.308	0.446	1.029
	alt5	0.087	0.261	0.012	0.111	0.500	4.481	0.041	0.130	0.350	1.533	0.342	1.494	0.365	1.355
	alt6	0.043	0.174	0.012	0.111	0.227	2.297	0.001	0.103	0.219	1.035	0.213	1.009	0.314	1.167
	alt7	0.087	0.174	0.012	0.100	0.530	2.349	0.030	0.107	0.219	1.095	0.213	1.067	0.377	1.080
	alt8	0.043	0.174	0.012	0.111	0.358	2.564	0.036	0.094	0.259	0.876	0.252	0.854	0.380	1.167
	alt9	0.087	0.174	0.012	0.111	0.356	2.564	0.007	0.123	0.287	1.314	0.280	1.281	0.351	1.304
	alt10	0.174	0.174	0.014	0.112	0.652	2.892	0.052	0.116	0.256	1.690	0.249	1.647	0.414	1.286

Table B.34 Obtained interval weights for 10x10 consistent matrix4

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 4	alt1	0.083	0.083	0.013	0.112	0.418	2.000	0.035	0.128	0.395	1.244	0.393	1.249	0.286	1.406
	alt2	0.083	0.083	0.014	0.114	0.364	1.473	0.032	0.107	0.301	1.244	0.286	1.249	0.240	0.938
	alt3	0.083	0.167	0.012	0.112	0.438	2.670	0.001	0.139	0.205	1.580	0.250	1.428	0.349	1.048
	alt4	0.083	0.083	0.014	0.112	0.574	1.943	0.047	0.091	0.474	1.264	0.428	1.450	0.313	1.048
	alt5	0.083	0.167	0.012	0.112	0.326	2.462	0.001	0.150	0.213	1.641	0.181	1.999	0.349	1.717
	alt6	0.083	0.167	0.012	0.112	0.616	2.083	0.042	0.076	0.415	0.948	0.416	1.088	0.349	1.048
	alt7	0.083	0.167	0.014	0.114	0.364	2.259	0.001	0.131	0.249	0.948	0.250	0.857	0.299	1.470
	alt8	0.083	0.167	0.012	0.098	0.536	2.232	0.041	0.112	0.316	1.185	0.363	1.180	0.469	0.729
	alt9	0.083	0.167	0.013	0.112	0.385	2.169	0.018	0.124	0.261	1.505	0.286	1.428	0.313	1.406
	alt10	0.167	0.167	0.012	0.100	0.758	2.670	0.053	0.092	0.316	0.948	0.286	1.088	0.469	1.250

Table B.35 Obtained interval weights for 10x10 consistent matrix5

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 5	alt1	0.082	0.082	0.013	0.103	0.306	1.231	0.027	0.074	0.671	4.438	0.188	1.160	0.246	0.984
	alt2	0.082	0.082	0.013	0.114	0.342	1.414	0.024	0.082	0.740	2.898	0.184	1.136	0.343	1.024
	alt3	0.054	0.245	0.013	0.114	0.306	2.881	0.027	0.156	0.986	6.340	0.276	1.657	0.322	1.638
	alt4	0.082	0.095	0.013	0.114	0.443	2.446	0.032	0.129	1.057	4.438	0.276	1.243	0.362	0.946
	alt5	0.082	0.163	0.014	0.116	0.461	2.267	0.040	0.117	1.040	5.072	0.291	1.326	0.348	1.448
	alt6	0.095	0.286	0.016	0.118	0.461	4.957	0.031	0.157	0.906	5.918	0.237	1.657	0.382	1.947
	alt7	0.095	0.095	0.013	0.114	0.584	2.972	0.015	0.116	0.704	4.734	0.221	1.105	0.322	0.910
	alt8	0.095	0.095	0.013	0.114	0.652	3.776	0.001	0.127	0.634	4.529	0.166	1.289	0.320	0.984
	alt9	0.048	0.163	0.014	0.079	0.204	2.421	0.001	0.083	0.725	2.959	0.189	0.829	0.202	1.028
	alt10	0.095	0.190	0.013	0.114	0.364	3.128	0.029	0.087	0.888	4.227	0.249	1.105	0.335	1.255

Table B.36 Obtained interval weights for 10x10 consistent matrix6

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 6	alt1	0.049	0.147	0.012	0.112	0.191	3.888	0.001	0.161	0.162	1.430	0.161	1.910	0.841	3.458
	alt2	0.074	0.111	0.013	0.112	0.276	2.024	0.027	0.070	0.278	0.834	0.273	0.837	0.800	2.090
	alt3	0.074	0.074	0.012	0.087	0.282	2.748	0.030	0.091	0.185	0.989	0.220	0.969	0.614	2.490
	alt4	0.066	0.066	0.014	0.114	0.224	2.625	0.006	0.166	0.238	1.458	0.242	1.453	0.841	2.824
	alt5	0.111	0.111	0.014	0.114	0.412	3.104	0.033	0.116	0.208	1.320	0.239	1.339	0.909	3.735
	alt6	0.074	0.221	0.013	0.112	0.401	3.669	0.036	0.089	0.247	1.250	0.286	1.091	0.884	3.202
	alt7	0.111	0.221	0.012	0.112	0.395	2.930	0.037	0.078	0.278	0.989	0.323	1.146	0.742	3.050
	alt8	0.111	0.265	0.016	0.112	0.552	3.933	0.037	0.121	0.278	1.296	0.279	1.291	1.326	5.451
	alt9	0.066	0.221	0.012	0.112	0.284	3.326	0.001	0.125	0.165	1.456	0.167	1.453	1.002	4.121
	alt10	0.111	0.147	0.013	0.112	0.418	2.824	0.034	0.144	0.330	1.668	0.382	1.674	0.884	3.634

Table B.37 Obtained interval weights for 10x10 consistent matrix7

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 7	alt1	0.075	0.150	0.014	0.115	0.312	2.699	0.013	0.116	0.672	3.585	0.248	1.133	0.227	1.018
	alt2	0.075	0.075	0.013	0.102	0.336	2.056	0.031	0.061	0.896	2.017	0.283	0.637	0.227	0.720
	alt3	0.075	0.075	0.014	0.115	0.301	1.866	0.035	0.087	0.907	3.585	0.306	1.062	0.301	0.810
	alt4	0.075	0.300	0.013	0.113	0.301	4.590	0.001	0.141	0.717	5.378	0.227	1.700	0.227	1.661
	alt5	0.075	0.150	0.013	0.102	0.375	2.144	0.028	0.085	0.645	3.320	0.212	1.007	0.361	1.146
	alt6	0.100	0.150	0.013	0.113	0.385	3.362	0.028	0.145	0.756	4.780	0.260	1.388	0.341	1.146
	alt7	0.075	0.150	0.013	0.113	0.240	2.991	0.001	0.140	0.672	4.425	0.212	1.487	0.208	1.519
	alt8	0.075	0.150	0.013	0.113	0.264	2.862	0.025	0.090	0.645	3.585	0.222	1.041	0.298	1.362
	alt9	0.075	0.300	0.013	0.101	0.552	5.098	0.040	0.147	1.344	5.163	0.425	1.700	0.321	2.291
	alt10	0.075	0.225	0.013	0.113	0.393	3.547	0.029	0.130	1.195	4.537	0.347	1.562	0.285	1.191

Table B.38 Obtained interval weights for 10x10 consistent matrix8

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 8	alt1	0.076	0.076	0.016	0.112	0.391	1.644	0.037	0.061	0.372	0.637	0.367	0.630	0.383	0.894
	alt2	0.051	0.076	0.012	0.112	0.286	1.762	0.001	0.136	0.212	1.486	0.210	1.469	0.184	1.057
	alt3	0.076	0.076	0.012	0.112	0.475	1.888	0.043	0.076	0.260	1.062	0.257	1.049	0.383	0.833
	alt4	0.076	0.076	0.012	0.100	0.214	2.352	0.001	0.107	0.195	1.438	0.193	1.421	0.216	1.149
	alt5	0.076	0.228	0.012	0.112	0.390	4.280	0.001	0.155	0.212	1.561	0.210	1.543	0.255	1.469
	alt6	0.076	0.228	0.012	0.112	0.328	3.088	0.033	0.138	0.260	1.274	0.257	1.259	0.416	1.787
	alt7	0.114	0.152	0.013	0.112	0.317	3.266	0.032	0.110	0.312	1.062	0.309	1.049	0.328	1.249
	alt8	0.076	0.152	0.012	0.112	0.591	2.491	0.040	0.130	0.404	1.171	0.400	1.157	0.324	1.864
	alt9	0.076	0.152	0.014	0.114	0.509	3.104	0.028	0.125	0.212	1.321	0.210	1.469	0.302	1.149
	alt10	0.152	0.152	0.012	0.100	0.634	2.670	0.030	0.116	0.212	1.561	0.210	1.543	0.453	0.906

Table B.39 Obtained interval weights for 10x10 consistent matrix9

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 9	alt1	0.062	0.062	0.013	0.113	0.225	2.392	0.001	0.130	0.185	1.455	0.185	1.437	0.209	1.016
	alt2	0.077	0.123	0.013	0.102	0.276	2.107	0.026	0.106	0.264	1.455	0.274	1.215	0.186	0.903
	alt3	0.092	0.123	0.013	0.113	0.336	2.622	0.028	0.111	0.277	1.108	0.243	1.095	0.238	1.155
	alt4	0.062	0.062	0.016	0.116	0.322	2.094	0.030	0.103	0.291	0.831	0.287	1.110	0.250	1.022
	alt5	0.092	0.123	0.015	0.100	0.360	2.192	0.036	0.079	0.277	0.970	0.274	0.972	0.289	0.899
	alt6	0.062	0.185	0.014	0.102	0.404	2.731	0.035	0.062	0.277	0.873	0.266	0.901	0.362	0.899
	alt7	0.062	0.185	0.014	0.114	0.336	3.717	0.031	0.136	0.242	1.662	0.243	1.642	0.360	1.749
	alt8	0.123	0.185	0.014	0.115	0.418	3.797	0.012	0.142	0.208	1.636	0.205	1.594	0.410	1.343
	alt9	0.062	0.185	0.013	0.113	0.312	3.100	0.001	0.130	0.255	1.056	0.243	1.095	0.315	1.528
	alt10	0.185	0.308	0.013	0.113	0.728	4.447	0.036	0.131	0.277	1.662	0.370	1.458	0.319	1.547

Table B.40 Obtained interval weights for 10x10 consistent matrix10

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 10	alt1	0.109	0.218	0.014	0.112	0.379	3.713	0.029	0.113	0.261	1.336	0.855	4.692	0.176	1.567
	alt2	0.073	0.109	0.013	0.087	0.287	2.392	0.006	0.092	0.223	0.731	0.782	2.566	0.264	1.115
	alt3	0.109	0.218	0.013	0.112	0.317	4.512	0.026	0.145	0.292	1.462	0.938	5.132	0.248	1.410
	alt4	0.036	0.109	0.012	0.112	0.179	2.259	0.001	0.115	0.167	1.462	0.586	5.132	0.111	0.991
	alt5	0.055	0.109	0.014	0.114	0.243	2.896	0.009	0.127	0.234	1.279	0.766	4.490	0.198	0.991
	alt6	0.109	0.218	0.012	0.112	0.409	3.713	0.042	0.086	0.292	1.462	1.026	4.692	0.330	1.058
	alt7	0.073	0.109	0.012	0.112	0.243	2.713	0.004	0.122	0.183	1.169	0.641	3.830	0.142	0.991
	alt8	0.109	0.218	0.013	0.112	0.390	3.282	0.043	0.079	0.183	1.169	0.641	4.105	0.330	0.991
	alt9	0.109	0.218	0.016	0.114	0.401	3.207	0.034	0.106	0.365	1.002	1.283	3.519	0.496	1.487
	alt10	0.109	0.218	0.014	0.114	0.376	4.095	0.037	0.132	0.213	1.637	0.748	5.362	0.372	2.379

B.5 15x15 Inconsistent Matrix Generation

Table B.41 Obtained interval weights for 15x15 inconsistent matrix1

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
	alt1			0.0113	0.0900	2.3039	3.4209	0.0276	0.1254	1.4504	5.8018	1.4362	5.8167	0.9642	1.5746
	alt2			0.0100	0.0900	0.8839	1.1881	0.0689	0.0689	0.9670	2.9009	0.9575	2.9084	1.2856	1.2856
	alt3			0.0100	0.0900	1.7253	2.3762	0.0225	0.1203	1.3054	4.6414	1.2926	4.6535	1.0755	1.7563
	alt4			0.0100	0.0900	0.8217	1.1180	0.0010	0.0560	0.5802	2.4174	0.5817	2.3936	0.8571	1.2958
	alt5			0.0100	0.0900	0.4109	0.5941	0.0010	0.0654	0.7832	0.7834	0.7181	1.0906	0.8571	1.3224
	alt6			0.0100	0.0900	0.4451	0.7336	0.0316	0.0787	0.6216	5.0765	0.5984	5.2352	1.0497	1.0497
	alt7			0.0100	0.0700	0.3208	0.4158	0.0010	0.0238	0.5802	2.9009	0.5817	2.8725	0.8164	0.8164
Matrix 1	alt8	Infeasible		0.0100	0.0800	0.4548	0.6013	0.0010	0.0508	0.7252	3.4811	0.7181	3.4901	0.8571	1.1020
	alt9			0.0100	0.0900	0.8376	1.5487	0.0360	0.0554	0.9670	3.9162	0.9695	3.8779	1.1903	1.1903
	alt10			0.0100	0.0900	0.5075	1.1207	0.0010	0.0462	0.7252	2.9009	0.7479	2.9084	0.8571	1.1428
	alt11			0.0100	0.0800	0.8217	1.1737	0.0010	0.0569	0.7252	1.9339	0.7479	1.9389	0.7170	1.1709
	alt12			0.0100	0.0900	0.8626	1.1881	0.0594	0.0594	0.9790	4.8348	0.9695	4.8473	1.0497	1.0497
	alt13			0.0129	0.0900	0.9433	1.2173	0.0536	0.0904	1.2087	4.0612	1.2465	4.0718	1.2856	1.2856
	alt14			0.0113	0.0900	1.6236	2.1300	0.0010	0.0978	0.9670	2.9009	0.9695	2.8725	0.8214	1.3414
	alt15			0.0113	0.0900	1.6839	2.2634	0.0045	0.1023	0.9670	4.3513	0.9695	4.4874	1.0497	1.0497

Table B.42 Obtained interval weights for 15x15 inconsistent matrix2

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
	alt1			0.0100	0.0900	0.5639	1.8043	0.0614	0.0614	0.5308	0.9683	0.5845	0.8724	1.0637	1.0637
	alt2			0.0113	0.0900	1.4648	4.4237	0.0587	0.1145	0.3631	1.3795	0.3272	1.3741	0.8990	1.2108
	alt3			0.0100	0.0900	0.7507	2.9551	0.0010	0.0549	0.2190	1.1496	0.2290	1.1451	0.7928	1.2536
	alt4			0.0100	0.0900	0.9448	3.4098	0.0010	0.1008	0.2427	1.4524	0.2513	1.4612	0.9828	0.9828
	alt5			0.0100	0.0900	0.7698	2.4547	0.0010	0.0765	0.2080	1.2133	0.2147	1.2563	0.8919	0.9025
	alt6			0.0113	0.0900	1.0963	4.5205	0.0255	0.1263	0.2934	1.5089	0.2922	1.5029	0.9344	1.2013
	alt7			0.0100	0.0900	0.2791	0.8028	0.0010	0.0747	0.2299	1.0949	0.2290	1.0905	0.7897	0.7897
Matrix 2	alt8	Infeasible		0.0100	0.0900	0.3532	1.1812	0.0010	0.0900	0.1916	1.3138	0.1908	1.3087	0.6400	1.1785
	alt9			0.0100	0.0900	0.2197	0.6516	0.0010	0.0329	0.2421	0.7262	0.2435	0.7384	0.4862	0.8954
	alt10			0.0100	0.0900	0.4183	1.7965	0.0418	0.0418	0.4044	1.0616	0.2908	1.1689	0.7866	0.9589
	alt11			0.0100	0.0800	0.2914	1.2379	0.0010	0.0262	0.2190	0.5868	0.2181	0.5845	0.7928	0.7959
	alt12			0.0100	0.0900	0.3675	1.0400	0.0010	0.0633	0.1677	1.1496	0.1670	1.1451	0.6375	1.1739
	alt13			0.0113	0.0900	0.6934	1.9860	0.0879	0.0879	1.0616	1.4524	0.8906	1.7176	0.8990	1.1939
	alt14			0.0100	0.0900	0.5720	1.7888	0.0457	0.0457	0.2075	1.0616	0.2454	0.8906	0.8059	1.4840
	alt15			0.0100	0.0800	0.5905	1.9651	0.0031	0.1039	0.2421	1.4559	0.2435	1.4358	0.7928	0.8091

Table B.43 Obtained interval weights for 15x15 inconsistent matrix3

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
	alt1			0.0100	0.0900	0.4045	0.5547	0.0010	0.0313	0.6216	1.7982	0.6292	1.6435	1.0136	1.0438
	alt2			0.0100	0.0900	0.8091	1.1093	0.0295	0.0295	0.8654	4.4956	0.8389	4.6224	0.9764	1.2959
	alt3			0.0100	0.0800	0.8624	1.3183	0.0633	0.0695	0.8654	4.1959	0.8389	4.3143	0.9481	1.2584
	alt4			0.0100	0.0900	1.3798	2.0548	0.0007	0.1105	1.0384	3.4966	1.0067	3.5953	0.9720	1.2901
	alt5			0.0100	0.0900	1.6946	3.1528	0.0010	0.1098	1.2238	5.5945	1.2783	5.6628	1.0750	1.4269
	alt6			0.0100	0.0900	0.8247	1.3964	0.0010	0.0527	0.9179	2.7973	0.9438	2.8762	1.1198	1.1198
	alt7			0.0100	0.0900	0.8767	1.7253	0.0002	0.0703	1.1014	3.4966	0.9438	4.3142	1.1072	1.1072
Matrix 3	alt8	Infeasible		0.0100	0.0900	1.6920	2.4990	0.0010	0.0941	1.7307	4.8952	1.6779	5.0334	0.8848	1.1743
	alt9			0.0100	0.0900	0.2697	0.3698	0.0010	0.0010	0.6993	0.8654	0.7191	2.8762	0.7865	1.0438
	alt10			0.0100	0.0900	0.8091	1.1093	0.0010	0.0660	0.8654	4.1959	0.8389	4.3143	0.7865	1.0438
	alt11			0.0100	0.0900	0.4512	0.6719	0.0619	0.0713	0.6993	4.8952	0.7191	5.0336	1.0136	1.3180
	alt12			0.0113	0.0900	0.7308	1.3183	0.0661	0.0661	0.8991	5.1921	0.9245	5.0336	1.0056	1.3347
	alt13			0.0100	0.0900	0.7033	1.1797	0.0234	0.1332	0.6993	4.3268	0.7191	4.1946	0.9615	1.2762
	alt14			0.0100	0.0900	0.7169	1.2025	0.0010	0.1109	0.7417	5.1921	0.8089	5.0336	1.0750	1.4269
	alt15			0.0100	0.0900	0.8561	2.4463	0.0010	0.0945	0.6993	4.8952	0.7191	5.0334	1.0750	1.4269

Table B.44 Obtained interval weights for 15x15 inconsistent matrix4

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 4	alt1	Infeasible		0.0100	0.0900	0.2726	0.3730	0.0010	0.0010	0.2211	1.0471	0.2340	1.0421	0.6699	0.6699
	alt2			0.0100	0.0900	1.4408	2.2267	0.0673	0.0673	1.1054	1.6926	1.0695	1.6378	0.7105	1.3660
	alt3			0.0100	0.0900	1.1640	1.9845	0.0010	0.0677	0.5584	1.6753	0.5403	1.6210	0.6454	0.8298
	alt4			0.0100	0.0900	1.1959	2.0389	0.0200	0.1130	0.3224	1.4360	0.3120	1.3894	1.1697	1.9008
	alt5			0.0100	0.0900	1.7356	3.1430	0.0981	0.0981	1.0971	1.8990	1.0617	1.8372	0.7666	1.2879
	alt6			0.0100	0.0700	0.3364	0.4768	0.0010	0.0398	0.2418	0.6382	0.2340	0.6947	0.5697	1.0952
	alt7			0.0100	0.0900	0.5529	0.7566	0.0278	0.0710	0.2418	0.6448	0.2340	0.6239	0.8952	0.8952
	alt8			0.0100	0.0900	1.0453	1.4817	0.0361	0.1291	0.7180	1.9344	0.6947	1.8718	0.7744	0.9293
	alt9			0.0100	0.0900	0.6377	1.0892	0.0010	0.0928	0.2872	1.1054	0.2779	1.0695	0.8131	1.1510
	alt10			0.0100	0.0900	0.7432	2.4065	0.0010	0.0901	0.2374	1.5476	0.2297	1.6378	0.9857	1.1817
	alt11			0.0100	0.0900	0.4257	0.7116	0.0010	0.0930	0.2763	0.7254	0.2674	0.7019	0.5697	1.0952
	alt12			0.0100	0.0900	2.0760	2.9016	0.0320	0.1196	0.4145	1.6753	0.4680	1.6210	0.8057	1.5489
	alt13			0.0100	0.0800	0.3102	0.5405	0.0010	0.0010	0.2393	0.3224	0.2316	0.4680	0.4390	0.8440
	alt14			0.0100	0.0900	1.6703	2.3510	0.0431	0.0431	0.3165	1.2896	0.3062	1.2479	0.7105	0.7105
	alt15			0.0100	0.0900	0.6143	1.0474	0.0010	0.0683	0.2713	1.1702	0.2779	1.0695	0.6085	0.8204

Table B.45 Obtained interval weights for 15x15 inconsistent matrix5

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 5	alt1	Infeasible		0.0100	0.0800	1.4499	1.5675	0.0219	0.0856	0.3706	1.4641	0.3692	1.4584	0.6768	1.0280
	alt2			0.0100	0.0900	0.7500	0.9030	0.0440	0.0576	0.2440	1.4824	0.2431	1.4766	0.9269	0.9269
	alt3			0.0100	0.0900	1.2733	1.3699	0.0657	0.0657	0.2762	1.6113	0.2752	1.6051	0.8085	1.3859
	alt4			0.0100	0.0900	1.4499	1.5675	0.0084	0.0921	0.2928	1.2706	0.2917	1.2657	0.7908	0.9446
	alt5			0.0100	0.0900	1.4831	1.7856	0.0010	0.0836	0.3706	1.3811	0.3913	1.3758	0.9167	0.9167
	alt6			0.0100	0.0900	0.6366	0.6849	0.0010	0.0704	0.2762	1.7318	0.2752	1.7250	0.5355	0.9066
	alt7			0.0113	0.0900	0.6366	0.6849	0.0407	0.0407	0.3464	0.9207	0.3450	0.9172	1.0627	1.0627
	alt8			0.0100	0.0900	1.4499	1.5675	0.0024	0.0861	0.4092	1.4732	0.4051	1.4675	0.9269	1.6542
	alt9			0.0100	0.0900	0.6366	0.6849	0.0010	0.0836	0.2118	1.2201	0.2109	1.2153	0.6068	1.0829
	alt10			0.0113	0.0900	0.6366	0.6849	0.0010	0.0689	0.2302	0.8471	0.2293	0.8438	0.7650	1.2693
	alt11			0.0100	0.0700	0.2122	0.2283	0.0010	0.0010	0.1841	0.1841	0.1834	0.1834	0.7760	0.7760
	alt12			0.0100	0.0900	0.4833	0.5225	0.0010	0.0600	0.3254	1.1118	0.3057	1.1740	0.8085	1.0395
	alt13			0.0100	0.0700	1.9099	2.0548	0.0205	0.1042	0.4911	1.3811	0.4696	1.3758	0.7996	1.0510
	alt14			0.0100	0.0900	2.8998	3.1350	0.0080	0.0916	0.4235	1.2706	0.4219	1.2657	0.7760	0.7760
	alt15			0.0133	0.0900	1.0044	1.7184	0.0936	0.0936	0.4235	1.1713	0.4219	1.1667	1.0760	1.0829

Table B.46 Obtained interval weights for 15x15 inconsistent matrix6

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 6	alt1	Infeasible		0.0100	0.0900	0.4357	1.1338	0.0473	0.0635	0.2661	1.1826	0.2747	1.0300	0.8804	1.1161
	alt2			0.0129	0.0900	2.3999	3.3717	0.1072	0.1157	1.1175	1.8625	1.1772	1.9227	1.4538	2.5785
	alt3			0.0100	0.0900	0.6164	1.0761	0.0640	0.0640	0.3353	1.2670	0.3461	1.0987	0.8160	1.1456
	alt4			0.0100	0.0900	1.1906	1.6083	0.0010	0.0210	0.2910	1.1175	0.3004	1.1772	0.9793	0.9931
	alt5			0.0100	0.0900	1.6249	2.2111	0.0010	0.1000	0.2794	1.1175	0.2943	1.1536	1.2721	1.2721
	alt6			0.0100	0.0900	1.7999	2.3935	0.0613	0.0796	0.3649	1.4667	0.3461	1.5450	1.2721	1.2721
	alt7			0.0100	0.0800	0.8224	1.2948	0.0211	0.0603	0.2956	1.0643	0.2747	1.0987	1.1288	1.2888
	alt8			0.0113	0.0900	0.3172	0.4369	0.0010	0.0010	0.2235	0.5588	0.2307	0.5886	0.7546	1.1587
	alt9			0.0100	0.0900	0.5906	0.7793	0.0010	0.0395	0.2095	0.9313	0.2207	0.9614	0.6446	1.1433
	alt10			0.0100	0.0900	0.2907	0.3776	0.0010	0.1113	0.1863	1.5017	0.1962	1.2017	0.9793	1.2888
	alt11			0.0100	0.0900	2.4765	3.2674	0.0198	0.0965	0.2661	1.5964	0.2747	1.6824	1.1307	1.9332
	alt12			0.0100	0.0900	1.7580	2.2834	0.0010	0.1158	0.1774	1.6763	0.1831	1.7305	0.9793	1.1161
	alt13			0.0100	0.0900	0.9166	1.9430	0.0010	0.1112	0.2328	1.1175	0.2403	1.1772	0.7254	1.2865
	alt14			0.0100	0.0800	0.3612	0.6564	0.0010	0.0884	0.2661	0.8869	0.2472	0.9156	0.8055	1.3569
	alt15			0.0100	0.0900	0.3071	0.4052	0.0010	0.0491	0.1863	0.8381	0.1923	0.8829	0.9793	1.1161

Table B.47 Obtained interval weights for 15x15 inconsistent matrix7

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 7	alt1	Infeasible		0.0100	0.0900	1.3050	1.6849	0.0506	0.1039	0.4381	1.1715	0.4356	1.0890	0.8730	1.2901
	alt2			0.0100	0.0800	0.6446	0.8323	0.0010	0.0505	0.1984	1.3017	0.2074	1.2100	0.6610	0.7156
	alt3			0.0100	0.0900	0.3146	0.3964	0.0010	0.0010	0.1984	0.2264	0.2074	0.3319	0.5288	0.7814
	alt4			0.0100	0.0900	1.7978	2.2655	0.0010	0.1170	0.1940	1.3799	0.1997	1.3793	0.6919	1.0225
	alt5			0.0100	0.0900	0.6291	0.7928	0.0010	0.0526	0.3450	0.7762	0.3448	0.7986	0.5884	0.8695
	alt6			0.0100	0.0900	0.6509	1.5894	0.0010	0.0884	0.3471	1.5524	0.3630	1.3793	0.7128	1.0534
	alt7			0.0100	0.0800	1.7761	2.1844	0.0735	0.0735	0.2587	0.9918	0.2371	1.0372	0.8730	1.2901
	alt8			0.0113	0.0900	1.8420	2.3213	0.0578	0.1148	0.3905	1.3142	0.3630	1.3068	0.8730	1.2901
	alt9			0.0100	0.0900	1.2736	1.6445	0.0178	0.0811	0.2479	1.3885	0.2593	1.4520	0.7845	1.1594
	alt10			0.0100	0.0900	0.3552	0.4369	0.0280	0.0280	0.2170	1.5187	0.2299	1.3793	0.6724	0.9938
	alt11			0.0113	0.0900	0.3640	0.4586	0.0153	0.0153	0.2170	1.3885	0.2069	1.2738	0.5884	0.8695
	alt12			0.0100	0.0900	1.3905	2.1261	0.0067	0.1336	0.2760	1.9526	0.2759	1.8150	0.7699	0.9860
	alt13			0.0100	0.0900	0.6767	0.8528	0.0010	0.1269	0.2170	1.5868	0.2017	1.6595	0.8730	1.0419
	alt14			0.0100	0.0900	0.6446	0.8425	0.0503	0.0503	0.2893	1.0414	0.2548	1.0345	0.8730	1.2901
	alt15			0.0113	0.0900	1.7761	2.1844	0.0010	0.0911	0.3742	1.0349	0.4367	1.1494	0.9899	0.9899

Table B.48 Obtained interval weights for 15x15 inconsistent matrix8

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 8	alt1	Infeasible		0.0100	0.0900	1.1723	1.4896	0.0010	0.1043	0.6266	2.4365	0.3171	0.9719	0.8233	1.0695
	alt2			0.0100	0.0900	3.3789	3.8486	0.0889	0.0889	2.0304	5.2212	0.8099	2.1868	0.9652	1.0108
	alt3			0.0100	0.0800	1.1263	1.2829	0.0010	0.0550	1.0152	3.1330	0.5062	1.2683	0.8888	0.9010
	alt4			0.0100	0.0900	1.6967	2.0263	0.0088	0.1131	1.1279	4.0609	0.4756	1.6199	0.7900	1.2901
	alt5			0.0100	0.0900	3.3789	3.8486	0.0072	0.1115	2.0304	4.5116	0.8099	1.9025	1.0108	1.0108
	alt6			0.0129	0.0900	0.8484	1.0131	0.0598	0.0888	0.7895	3.7597	0.3171	1.5854	1.0585	1.3066
	alt7			0.0100	0.0900	0.8447	0.9622	0.0348	0.0348	0.8122	0.8122	0.3240	0.3448	0.7900	1.2901
	alt8			0.0113	0.0900	1.1866	1.5078	0.0660	0.0815	2.3686	3.4808	0.9512	1.4579	0.8753	0.8753
	alt9			0.0100	0.0900	0.3814	0.4699	0.0010	0.1043	0.4767	4.1187	0.2025	1.6551	0.5586	0.9122
	alt10			0.0100	0.0800	0.2176	0.2544	0.0010	0.0010	0.5801	0.9869	0.2430	0.3963	0.6706	0.8753
	alt11			0.0100	0.0900	0.5148	1.3218	0.0010	0.0651	0.4700	4.0609	0.1982	1.6199	0.6466	0.8359
	alt12			0.0100	0.0900	0.5700	0.7358	0.0200	0.0671	0.7519	3.8071	0.3171	1.4579	0.7544	1.1200
	alt13			0.0100	0.0900	1.0929	1.2774	0.0252	0.1004	0.5371	3.9163	0.2430	1.5854	0.6738	0.9506
	alt14			0.0100	0.0900	0.8484	1.0131	0.0010	0.0398	0.6266	2.3686	0.2642	0.9512	0.8233	1.3444
	alt15			0.0100	0.0900	0.3754	0.4276	0.0010	0.0496	0.7833	1.2532	0.3171	0.5285	0.9122	0.9122

Table B.49 Obtained interval weights for 15x15 inconsistent matrix9

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 9	alt1	Infeasible		0.0100	0.0900	2.1497	2.6362	0.0511	0.0687	1.0263	4.3633	1.0218	4.4694	1.3128	1.8533
	alt2			0.0100	0.0800	0.3583	0.4394	0.0010	0.0366	0.6060	2.6390	0.6208	2.7032	0.6106	1.2751
	alt3			0.0100	0.0900	0.7617	2.0667	0.0004	0.0919	0.6787	5.2780	0.6758	5.4063	0.9655	2.0161
	alt4			0.0100	0.0900	0.7166	0.8787	0.0010	0.0690	0.5387	3.6361	0.5518	3.7245	0.8540	1.1279
	alt5			0.0113	0.0900	1.0957	1.7167	0.0454	0.0454	0.8797	3.2321	0.9011	3.0654	1.1640	1.1640
	alt6			0.0100	0.0900	1.1121	2.1233	0.0010	0.0748	0.7917	3.9585	0.8109	4.0547	0.8540	1.0296
	alt7			0.0113	0.0900	0.7166	0.8787	0.0126	0.1041	0.8797	5.0905	0.9011	5.0683	1.0862	1.1640
	alt8			0.0114	0.0900	0.9177	1.3022	0.0912	0.0912	1.3195	3.0301	1.3516	3.1038	1.2413	2.4025
	alt9			0.0113	0.0700	0.4768	1.0488	0.0010	0.0791	0.7272	2.6390	0.7240	2.7032	1.1586	1.8533
	alt10			0.0100	0.0800	1.7914	2.1968	0.0873	0.0873	2.1112	3.3937	1.6894	4.3251	1.1956	1.7460
	alt11			0.0100	0.0900	0.3583	0.4394	0.0010	0.0741	0.6597	1.9792	0.6758	2.0274	0.6903	1.4415
	alt12			0.0100	0.0900	0.5478	0.8620	0.0010	0.0328	0.8484	2.6390	0.8447	2.7032	1.0576	1.3514
	alt13			0.0100	0.0900	0.9067	1.2709	0.0010	0.0324	0.6597	3.3937	0.6758	3.7245	0.9355	1.2751
	alt14			0.0100	0.0900	1.0748	1.3181	0.0010	0.1247	1.6968	3.9585	1.6895	4.0547	0.9355	1.5828
	alt15			0.0100	0.0900	1.0748	1.3181	0.0332	0.0795	0.7272	4.8481	0.7449	4.9660	0.6106	1.2751

Table B.50 Obtained interval weights for 15x15 inconsistent matrix10

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 10	alt1	Infeasible		0.0113	0.0800	1.9473	2.7716	0.0828	0.0828	2.7497	4.2503	2.8597	4.0678	0.9795	1.1767
	alt2			0.0100	0.0900	0.3769	0.5280	0.0010	0.0508	0.8501	2.7497	0.8007	2.9056	0.9135	1.2155
	alt3			0.0100	0.0900	1.6660	2.5796	0.0010	0.1260	0.7471	4.0479	0.7626	4.0670	0.9795	1.2155
	alt4			0.0100	0.0900	0.4222	0.8929	0.0010	0.0927	0.5060	2.0531	0.5084	2.2877	0.9950	1.1194
	alt5			0.0100	0.0900	1.0098	1.5569	0.0010	0.1246	0.7333	4.2503	0.7264	4.3585	0.9898	1.3171
	alt6			0.0100	0.0900	1.3253	1.9554	0.0626	0.0626	0.9777	3.5420	1.0168	3.5587	1.1646	1.5496
	alt7			0.0100	0.0900	0.7010	1.1117	0.0010	0.0218	0.6110	2.9330	0.6226	2.9056	0.8457	1.1253
	alt8			0.0100	0.0900	0.2470	0.4280	0.0010	0.0729	0.5060	2.9330	0.5084	3.0503	0.8871	1.1194
	alt9			0.0100	0.0800	0.6906	1.3712	0.0427	0.0427	0.7084	1.8216	0.7264	1.7434	0.9898	0.9898
	alt10			0.0113	0.0900	1.9953	2.8745	0.0737	0.0737	1.1807	4.3995	1.1862	4.3585	0.8235	1.0957
	alt11			0.0100	0.0900	0.4418	0.6518	0.0010	0.0782	0.7590	3.5420	0.7264	3.5587	0.7400	0.9846
	alt12			0.0100	0.0900	1.0079	1.8275	0.0010	0.0599	0.8555	3.0360	0.8717	2.9056	1.0748	1.1194
	alt13			0.0100	0.0800	0.4165	0.6449	0.0010	0.0848	0.6072	3.0360	0.5811	2.9056	1.3377	1.7800
	alt14			0.0100	0.0900	0.4165	0.6449	0.0010	0.0526	0.7333	3.5420	0.7117	3.5587	1.0748	1.1136
	alt15			0.0100	0.0900	2.3345	3.5290	0.0651	0.1000	3.6431	4.2773	3.5587	4.3585	1.0691	1.4226

B.6 15x15 Consistent Matrix Generation

Table B.51 Obtained interval weights for 15x15 consistent matrix1

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 1	alt1	0.0357	0.1071	0.0100	0.0800	0.3080	2.4634	0.0010	0.0686	0.2262	0.9894	0.2315	0.9646	0.1904	0.8548
	alt2	0.0268	0.1071	0.0100	0.0900	0.1975	2.6808	0.0010	0.0859	0.1759	1.3852	0.1736	1.4404	0.2426	1.0892
	alt3	0.0268	0.0536	0.0100	0.0900	0.3008	1.5277	0.0151	0.0781	0.1979	1.4841	0.2058	1.2153	0.2290	1.0280
	alt4	0.0536	0.0536	0.0100	0.0900	0.2678	1.8734	0.0177	0.0533	0.1979	0.9235	0.2058	0.9453	0.2423	0.9138
	alt5	0.0536	0.0536	0.0100	0.0900	0.3299	1.5999	0.0010	0.0553	0.1847	0.6596	0.1921	0.7716	0.3427	0.8548
	alt6	0.0536	0.1071	0.0129	0.0900	0.4368	2.3394	0.0195	0.0724	0.2770	1.1308	0.2778	1.1575	0.3238	1.4538
	alt7	0.0536	0.1071	0.0100	0.0900	0.4319	2.6093	0.0259	0.0742	0.2474	1.3569	0.2411	1.3890	0.3427	1.0280
	alt8	0.0536	0.0536	0.0117	0.0900	0.3927	1.8882	0.0221	0.0603	0.3298	0.9235	0.3205	0.9603	0.3631	0.8644
	alt9	0.0536	0.0536	0.0100	0.0900	0.4437	3.0079	0.0011	0.0869	0.1855	1.4072	0.1929	1.3890	0.3427	1.0280
	alt10	0.0536	0.0536	0.0100	0.0900	0.4237	2.4915	0.0190	0.0702	0.2309	1.0554	0.2363	1.0417	0.2849	1.0280
	alt11	0.0536	0.1071	0.0100	0.0700	0.3602	2.9048	0.0149	0.0669	0.1979	1.0993	0.2026	1.0417	0.3427	1.0990
	alt12	0.0536	0.1071	0.0100	0.0900	0.6300	3.5070	0.0257	0.0659	0.2356	1.1081	0.2894	1.1112	0.3427	1.5386
	alt13	0.1071	0.1071	0.0117	0.0900	0.6546	3.0521	0.0314	0.0839	0.3694	1.3852	0.3704	1.3890	0.5129	1.0280
	alt14	0.1071	0.1071	0.0100	0.0900	0.7180	3.1965	0.0285	0.0721	0.2770	1.1308	0.2778	1.1575	0.3427	1.5386
	alt15	0.0357	0.2143	0.0113	0.0900	0.3313	4.8449	0.0175	0.0919	0.2309	1.5831	0.2401	1.6205	0.3663	1.6448

Table B.52 Obtained interval weights for 15x15 consistent matrix2

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 2	alt1	0.0487	0.0688	0.0100	0.0900	0.2766	2.5889	0.0161	0.0704	0.2324	0.9821	0.2374	1.0320	0.2920	1.1459
	alt2	0.0650	0.0650	0.0100	0.0900	0.3730	1.5572	0.0231	0.0842	0.3274	1.1365	0.3392	1.1775	0.3395	1.0186
	alt3	0.0487	0.0688	0.0100	0.0900	0.2950	2.3056	0.0155	0.0659	0.2357	1.2277	0.2442	1.2720	0.2940	1.0186
	alt4	0.0650	0.0688	0.0100	0.0900	0.3982	2.1427	0.0010	0.0759	0.1964	1.1621	0.2035	0.9498	0.3395	1.0186
	alt5	0.0650	0.0650	0.0100	0.0900	0.4488	2.0548	0.0003	0.0729	0.1660	1.4143	0.1990	1.2312	0.3395	1.1459
	alt6	0.0344	0.1300	0.0100	0.0900	0.3443	2.8144	0.0169	0.0722	0.1964	1.1621	0.2035	1.2041	0.2804	1.1002
	alt7	0.0344	0.0344	0.0100	0.0900	0.3082	2.7858	0.0135	0.0787	0.1937	1.0476	0.2007	1.0176	0.2596	1.0186
	alt8	0.0688	0.1300	0.0100	0.0800	0.4630	2.8396	0.0237	0.0692	0.1937	1.0775	0.2007	1.0855	0.3820	0.9054
	alt9	0.0650	0.0688	0.0100	0.0700	0.3117	2.4146	0.0132	0.0719	0.1886	1.3154	0.1954	1.3629	0.2558	1.0037
	alt10	0.0650	0.0688	0.0100	0.0900	0.5196	2.3603	0.0245	0.0644	0.2134	1.1621	0.2212	1.2041	0.3820	1.0549
	alt11	0.0573	0.1949	0.0100	0.0900	0.2287	3.4581	0.0010	0.0913	0.1746	1.4941	0.1759	1.5481	0.2998	1.1762
	alt12	0.0344	0.1720	0.0100	0.0700	0.3431	3.2967	0.0010	0.0783	0.1660	0.9821	0.1720	1.0176	0.3395	1.2790
	alt13	0.0688	0.0688	0.0100	0.0900	0.4402	2.6484	0.0235	0.0629	0.2455	0.9429	0.2580	0.9769	0.3395	1.1459
	alt14	0.0688	0.1462	0.0100	0.0900	0.6084	4.2067	0.0253	0.0648	0.2245	1.1621	0.2293	1.1872	0.3820	1.4988
	alt15	0.0731	0.1300	0.0100	0.0900	0.5296	3.1476	0.0222	0.0682	0.2455	1.1786	0.2544	1.2211	0.3395	1.0186

Table B.53 Obtained interval weights for 15x15 consistent matrix3

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 3	alt1	0.0489	0.0977	0.0113	0.0900	0.4353	2.8520	0.0202	0.0751	0.2542	1.3075	0.2718	1.2579	0.2943	1.0283
	alt2	0.0326	0.0489	0.0100	0.0900	0.3138	2.0710	0.0166	0.0655	0.1868	1.1485	0.1797	1.1648	0.2892	1.0106
	alt3	0.0489	0.0489	0.0100	0.0900	0.3743	2.4828	0.0185	0.0727	0.1838	1.1485	0.1864	1.1648	0.3369	1.1370
	alt4	0.0489	0.0489	0.0100	0.0900	0.2678	1.4473	0.0010	0.0765	0.1723	1.2251	0.1747	1.2424	0.2892	1.0106
	alt5	0.0489	0.1466	0.0100	0.0900	0.3440	3.1502	0.0170	0.0751	0.1930	1.1485	0.1941	1.1980	0.3369	1.1772
	alt6	0.0489	0.0733	0.0100	0.0900	0.3130	1.7080	0.0010	0.0719	0.1634	1.1622	0.1797	1.0871	0.3369	1.1370
	alt7	0.0489	0.0489	0.0100	0.0900	0.2690	2.1277	0.0081	0.0728	0.2144	0.9188	0.2174	0.8697	0.2892	1.0106
	alt8	0.0489	0.0977	0.0100	0.0800	0.4003	2.4250	0.0241	0.0531	0.3308	0.9188	0.3261	0.9318	0.3369	1.1772
	alt9	0.0391	0.0733	0.0100	0.0700	0.3005	2.5667	0.0172	0.0662	0.2297	1.0720	0.2330	1.0871	0.2874	1.0044
	alt10	0.0489	0.0977	0.0100	0.0900	0.4663	2.9613	0.0269	0.0571	0.2541	1.0337	0.2405	1.0483	0.3790	1.0283
	alt11	0.0733	0.0977	0.0100	0.0900	0.3541	3.0662	0.0010	0.0548	0.1838	0.9188	0.1864	0.8985	0.3348	1.1370
	alt12	0.0977	0.0977	0.0100	0.0900	0.7043	3.5722	0.0205	0.0778	0.2297	1.2863	0.2330	1.3045	0.3428	1.0106
	alt13	0.0733	0.0977	0.0100	0.0900	0.4251	2.2015	0.0226	0.0775	0.2584	1.2863	0.2621	1.3045	0.3790	1.1370
	alt14	0.0977	0.0977	0.0100	0.0900	0.4574	2.9950	0.0055	0.0820	0.2144	1.3782	0.2174	1.3977	0.3369	1.0283
	alt15	0.0977	0.1954	0.0100	0.0900	0.5763	4.9725	0.0272	0.0984	0.2953	1.2863	0.2995	1.3045	0.3790	1.3243

Table B.54 Obtained interval weights for 15x15 consistent matrix4

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 4	alt1	0.0504	0.0672	0.0100	0.0900	0.3832	2.1520	0.0195	0.0761	0.2825	1.2914	0.2899	1.2100	0.1843	1.1060
	alt2	0.0504	0.0504	0.0114	0.0900	0.2657	1.5999	0.0151	0.0469	0.2152	0.5650	0.2017	0.6482	0.4608	1.1060
	alt3	0.0504	0.0672	0.0100	0.0800	0.2486	2.8700	0.0010	0.0774	0.1883	1.3183	0.1933	1.2276	0.3687	0.9831
	alt4	0.0252	0.1008	0.0100	0.0900	0.1875	3.2761	0.0010	0.0810	0.1614	1.1299	0.1681	1.1596	0.1383	1.1060
	alt5	0.0504	0.1008	0.0100	0.0900	0.3401	2.5064	0.0178	0.0719	0.2197	1.3183	0.2315	1.2626	0.4148	1.1060
	alt6	0.0504	0.0504	0.0100	0.0900	0.4285	2.3790	0.0200	0.0628	0.2260	0.9685	0.2319	1.0083	0.3687	1.2443
	alt7	0.0504	0.1008	0.0100	0.0900	0.3876	2.0548	0.0166	0.0636	0.2197	1.3183	0.2255	1.3528	0.1843	1.1060
	alt8	0.0672	0.1008	0.0100	0.0900	0.4368	2.3394	0.0121	0.0853	0.2563	1.1299	0.2630	1.1596	0.3687	1.1060
	alt9	0.0504	0.1345	0.0100	0.0800	0.3580	3.2217	0.0204	0.0710	0.2690	1.1299	0.2521	1.1596	0.3687	1.0753
	alt10	0.0504	0.1008	0.0100	0.0800	0.3336	2.7638	0.0207	0.0625	0.2197	1.1299	0.2104	1.1596	0.3687	1.2904
	alt11	0.0504	0.1008	0.0113	0.0900	0.4791	2.8845	0.0041	0.0851	0.1883	1.3183	0.1933	1.3528	0.4148	1.2640
	alt12	0.1008	0.1008	0.0100	0.0900	0.4829	2.3155	0.0254	0.0787	0.2637	1.3183	0.3025	1.2100	0.3687	1.1060
	alt13	0.1008	0.1008	0.0100	0.0900	0.5809	2.5889	0.0010	0.0739	0.2260	1.0985	0.2319	1.0522	0.3687	1.2904
	alt14	0.1008	0.1008	0.0100	0.0900	0.6371	2.9348	0.0242	0.0639	0.2637	1.0761	0.2706	1.0083	0.3687	1.1060
	alt15	0.0672	0.1513	0.0100	0.0900	0.5484	3.6315	0.0239	0.0809	0.2825	1.2914	0.2899	1.3444	0.3687	1.2904

Table B.55 Obtained interval weights for 15x15 consistent matrix5

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 5	alt1	0.0811	0.1622	0.0100	0.0900	0.3076	2.9869	0.0165	0.0782	0.2563	1.1524	0.2560	1.1512	0.1851	1.3139
	alt2	0.0541	0.0541	0.0100	0.0900	0.2472	2.1596	0.0010	0.0829	0.1645	1.4406	0.1644	1.4390	0.1788	0.9685
	alt3	0.0541	0.0541	0.0100	0.0900	0.3348	2.0000	0.0175	0.0637	0.2812	0.8997	0.2809	0.8988	0.2391	0.9685
	alt4	0.0541	0.0541	0.0100	0.0900	0.3042	1.7687	0.0154	0.0482	0.2107	1.1247	0.2105	1.1235	0.1564	0.9685
	alt5	0.0541	0.0541	0.0100	0.0900	0.3310	2.6002	0.0217	0.0770	0.3239	1.4058	0.3236	1.4044	0.2086	0.9685
	alt6	0.0541	0.1081	0.0100	0.0700	0.4091	2.2538	0.0229	0.0510	0.2499	0.9717	0.2458	0.9861	0.3228	0.9385
	alt7	0.0541	0.0541	0.0100	0.0800	0.4387	2.4808	0.0225	0.0657	0.2280	1.2956	0.2277	1.2943	0.3228	0.7533
	alt8	0.0541	0.1622	0.0100	0.0900	0.4171	3.5652	0.0219	0.0811	0.2812	1.1247	0.2809	1.1235	0.3228	0.9685
	alt9	0.0541	0.1622	0.0113	0.0800	0.2754	4.0285	0.0019	0.0800	0.2429	1.1247	0.2465	1.1235	0.1564	1.1105
	alt10	0.0811	0.0811	0.0100	0.0900	0.2652	2.2715	0.0010	0.0875	0.2249	0.8997	0.2247	0.8988	0.2825	0.9685
	alt11	0.0541	0.1081	0.0100	0.0900	0.5115	3.4365	0.0010	0.0884	0.1645	1.5376	0.1644	1.5360	0.3228	1.8771
	alt12	0.0811	0.1081	0.0100	0.0900	0.4867	2.7327	0.0224	0.0529	0.2881	0.8429	0.2878	0.8420	0.3228	1.4348
	alt13	0.0541	0.0541	0.0100	0.0900	0.4222	2.8745	0.0221	0.0819	0.2591	1.4058	0.2589	1.4044	0.3228	1.3182
	alt14	0.0541	0.1622	0.0100	0.0900	0.5809	3.3019	0.0231	0.0650	0.2214	1.3162	0.2150	1.3148	0.3228	1.8771
	alt15	0.0541	0.1081	0.0100	0.0900	0.4014	3.0177	0.0196	0.0847	0.2812	1.1517	0.2809	1.1505	0.3228	1.2514

Table B.56 Obtained interval weights for 15x15 consistent matrix6

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 6	alt1	0.0452	0.0905	0.0100	0.0900	0.3213	2.1936	0.0071	0.0897	0.2203	1.5876	0.2160	1.5555	0.2317	1.0991
	alt2	0.0452	0.0452	0.0100	0.0900	0.2758	1.2360	0.0010	0.0597	0.1985	1.0368	0.2000	1.0082	0.2895	0.9769
	alt3	0.0452	0.0452	0.0100	0.0900	0.3292	1.4869	0.0185	0.0259	0.2205	0.4405	0.2222	0.4321	0.3256	0.9769
	alt4	0.0452	0.0905	0.0100	0.0900	0.3108	2.1689	0.0161	0.0720	0.2101	1.3871	0.2160	1.3333	0.3256	1.0991
	alt5	0.0452	0.0905	0.0100	0.0900	0.2783	1.8734	0.0010	0.0903	0.1985	1.1677	0.2222	0.9722	0.2317	1.0991
	alt6	0.0452	0.0452	0.0100	0.0900	0.4888	2.6186	0.0167	0.0722	0.1985	1.1664	0.1944	1.1666	0.3256	1.0991
	alt7	0.0905	0.0905	0.0100	0.0900	0.4598	2.5111	0.0010	0.0747	0.2481	0.9331	0.2222	1.0208	0.3664	1.3902
	alt8	0.0754	0.1809	0.0100	0.0900	0.6015	3.7570	0.0261	0.0763	0.2941	1.5435	0.3025	1.5555	0.3664	1.7378
	alt9	0.0452	0.0905	0.0100	0.0900	0.2875	2.5198	0.0010	0.0889	0.1944	1.4707	0.1944	1.5123	0.3277	1.5543
	alt10	0.0905	0.0905	0.0100	0.0900	0.3924	2.4061	0.0199	0.0688	0.1985	1.6538	0.2000	1.6666	0.3664	0.8684
	alt11	0.0503	0.0905	0.0100	0.0700	0.4663	2.3603	0.0222	0.0714	0.3559	0.9923	0.3659	1.0000	0.3256	0.9769
	alt12	0.0452	0.1508	0.0100	0.0900	0.5441	3.7675	0.0196	0.0820	0.2521	1.3230	0.2520	1.3333	0.3664	0.7646
	alt13	0.0452	0.0905	0.0113	0.0900	0.4192	2.4403	0.0202	0.0775	0.2919	0.8811	0.2520	1.0000	0.3256	1.3026
	alt14	0.0905	0.0905	0.0100	0.0900	0.5743	2.9231	0.0312	0.0677	0.3308	1.1907	0.3333	1.2000	0.3664	0.9769
	alt15	0.0905	0.0905	0.0100	0.0900	0.7043	3.8802	0.0227	0.0730	0.2101	1.2606	0.2160	1.2703	0.3256	1.0991

Table B.57 Obtained interval weights for 15x15 consistent matrix7

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 7	alt1	0.0563	0.1127	0.0100	0.0900	0.4014	2.4501	0.0168	0.0671	0.2296	1.2244	0.2265	1.2078	0.7434	2.8345
	alt2	0.0563	0.0563	0.0100	0.0900	0.3407	2.0158	0.0171	0.0726	0.2411	1.1661	0.2382	1.1485	0.7798	2.9735
	alt3	0.0423	0.0563	0.0114	0.0900	0.2624	1.8508	0.0151	0.0842	0.2332	1.2438	0.2297	1.2250	1.1912	3.0660
	alt4	0.0563	0.0563	0.0113	0.0900	0.3937	2.4395	0.0210	0.0725	0.3061	1.2244	0.3020	1.2078	0.7810	2.9778
	alt5	0.0563	0.0563	0.0113	0.0700	0.3764	2.1742	0.0188	0.0513	0.2449	0.9329	0.2416	0.9188	0.9912	3.1283
	alt6	0.0563	0.0563	0.0100	0.0900	0.3562	1.9914	0.0037	0.0686	0.1530	1.2244	0.1641	1.1115	1.0423	2.9735
	alt7	0.0563	0.0845	0.0113	0.0900	0.4961	2.6758	0.0204	0.0768	0.2332	1.2244	0.2297	1.2078	1.0123	3.0925
	alt8	0.0563	0.1127	0.0100	0.0800	0.1935	2.2538	0.0010	0.0624	0.1749	1.0713	0.1750	1.0420	0.7434	2.8345
	alt9	0.0845	0.1690	0.0100	0.0800	0.5590	4.2351	0.0257	0.0773	0.3061	1.1661	0.3020	1.1485	1.0513	4.0085
	alt10	0.0282	0.1127	0.0100	0.0900	0.2499	3.1638	0.0010	0.0776	0.1749	1.3993	0.1701	1.4177	0.7220	2.7529
	alt11	0.0563	0.1690	0.0100	0.0900	0.3250	3.3896	0.0182	0.0702	0.2073	1.2053	0.2042	1.1909	0.7798	2.9735
	alt12	0.0563	0.0563	0.0100	0.0900	0.5097	2.2441	0.0252	0.0771	0.3061	1.2244	0.3020	1.2078	1.0108	3.2164
	alt13	0.0563	0.0563	0.0113	0.0900	0.6132	2.7393	0.0207	0.0761	0.2411	1.1661	0.1812	1.5098	1.0934	2.3859
	alt14	0.0563	0.1127	0.0100	0.0900	0.6422	2.8689	0.0038	0.0813	0.2041	1.2244	0.2025	1.2329	0.9926	3.7848
	alt15	0.1127	0.1127	0.0113	0.0900	0.5000	2.5889	0.0244	0.0624	0.2915	0.7142	0.2871	0.7247	0.9636	2.4135

Table B.58 Obtained interval weights for 15x15 consistent matrix8

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 8	alt1	0.0431	0.0861	0.0100	0.0800	0.2238	1.7080	0.0010	0.0699	0.1621	1.2106	0.1583	1.1688	0.2729	1.1941
	alt2	0.0431	0.1148	0.0100	0.0900	0.3622	3.1256	0.0168	0.0658	0.1946	1.2052	0.1900	1.2721	0.3168	1.3862
	alt3	0.0861	0.0861	0.0100	0.0900	0.3590	2.0460	0.0221	0.0634	0.2410	1.0423	0.2216	1.2665	0.3980	0.9288
	alt4	0.0574	0.1722	0.0100	0.0900	0.3348	3.4430	0.0186	0.0662	0.2583	0.9728	0.2533	0.9499	0.2956	0.9288
	alt5	0.0431	0.1579	0.0100	0.0900	0.2491	4.1165	0.0103	0.0852	0.2009	1.3619	0.2120	1.3298	0.3096	1.1028
	alt6	0.0431	0.0861	0.0100	0.0900	0.2915	3.1332	0.0152	0.0757	0.1946	1.3451	0.1855	1.3298	0.2873	0.8867
	alt7	0.0574	0.1722	0.0100	0.0900	0.3937	3.5866	0.0213	0.0888	0.2690	1.1673	0.2770	1.1398	0.3676	1.4080
	alt8	0.0574	0.0574	0.0100	0.0900	0.4255	2.5288	0.0178	0.0607	0.2432	0.8609	0.2375	0.8905	0.2310	0.8867
	alt9	0.0431	0.0861	0.0113	0.0900	0.2875	1.9775	0.0156	0.0703	0.2600	1.2999	0.2533	1.2665	0.2643	1.1565
	alt10	0.0526	0.1292	0.0100	0.0900	0.3322	3.7170	0.0186	0.0813	0.2600	1.1375	0.2533	1.1131	0.2956	1.2930
	alt11	0.0861	0.0861	0.0100	0.0900	0.6250	2.3687	0.0072	0.0637	0.2275	1.0400	0.2226	1.0132	0.3096	1.1941
	alt12	0.0431	0.0861	0.0100	0.0900	0.2602	2.0071	0.0010	0.0682	0.1946	1.2160	0.1900	1.1873	0.2985	1.1494
	alt13	0.0861	0.0861	0.0100	0.0900	0.6250	2.3687	0.0251	0.0845	0.3243	1.1622	0.3166	1.1398	0.3855	1.1941
	alt14	0.0861	0.0861	0.0100	0.0900	0.6250	2.3687	0.0278	0.0564	0.2432	0.9728	0.2375	0.9499	0.3980	0.9288
	alt15	0.0574	0.1148	0.0100	0.0900	0.4456	3.6186	0.0010	0.0749	0.1737	1.2971	0.1696	1.2609	0.2956	1.1028

Table B.59 Obtained interval weights for 15x15 consistent matrix9

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 9	alt1	0.0313	0.0521	0.0100	0.0900	0.1952	2.0317	0.0010	0.0810	0.1745	1.2564	0.6045	4.3528	0.3633	0.9684
	alt2	0.0521	0.0521	0.0100	0.0900	0.3227	3.0079	0.0111	0.0807	0.1999	1.4394	0.7255	5.2233	0.3228	0.9827
	alt3	0.0521	0.0521	0.0113	0.0900	0.3832	2.5003	0.0212	0.0640	0.3141	1.1451	1.0882	4.3528	0.3324	1.4526
	alt4	0.0521	0.1042	0.0100	0.0900	0.2378	2.5090	0.0142	0.0563	0.1799	0.7996	0.6529	2.9018	0.2495	1.0900
	alt5	0.0521	0.1042	0.0129	0.0800	0.3236	3.3712	0.0197	0.0605	0.3141	0.9619	1.0882	4.0115	0.3276	1.1465
	alt6	0.0521	0.1042	0.0100	0.0900	0.2978	2.7542	0.0010	0.0688	0.1804	1.0470	0.8706	3.6273	0.3228	0.9403
	alt7	0.0521	0.1563	0.0100	0.0900	0.4189	4.3056	0.0215	0.0820	0.2863	1.2564	1.0447	4.3528	0.3276	0.9827
	alt8	0.0521	0.1563	0.0129	0.0900	0.4559	3.1861	0.0198	0.0838	0.2405	1.0470	1.0029	3.6273	0.4842	0.9827
	alt9	0.0521	0.0781	0.0100	0.0900	0.3211	2.2284	0.0196	0.0641	0.2094	1.2024	0.8023	3.6273	0.2421	0.9684
	alt10	0.0781	0.1042	0.0100	0.0900	0.3986	3.5903	0.0154	0.0964	0.2094	1.3742	0.7255	5.2233	0.3228	1.4104
	alt11	0.0313	0.1042	0.0100	0.0900	0.2072	2.4441	0.0010	0.0689	0.1599	1.0495	0.5804	4.1786	0.2394	1.0460
	alt12	0.0781	0.0781	0.0100	0.0900	0.5022	3.0390	0.0239	0.0481	0.3490	0.9423	1.2091	3.2646	0.3276	0.9827
	alt13	0.0781	0.0781	0.0100	0.0900	0.3470	2.4875	0.0184	0.0820	0.2290	1.2564	0.8706	4.3528	0.3276	0.9827
	alt14	0.0781	0.0781	0.0100	0.0900	0.3986	2.6370	0.0208	0.0588	0.2513	1.1451	0.8706	4.3528	0.3276	0.9827
	alt15	0.1042	0.2083	0.0100	0.0900	0.5508	4.9016	0.0299	0.0856	0.3817	1.3960	1.4509	4.8364	0.4701	2.0542

Table B.60 Obtained interval weights for 15x15 consistent matrix10

		LA		UA		TSGPM		GP		PM		PMV1		PMV2	
		Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
Matrix 10	alt1	0.0550	0.0550	0.0100	0.0900	0.2763	1.9552	0.0167	0.0605	0.2297	0.8502	0.2233	0.8931	0.1676	0.9361
	alt2	0.0550	0.0550	0.0113	0.0900	0.2904	2.3338	0.0170	0.0813	0.2126	1.4174	0.2233	1.3396	0.3779	0.9929
	alt3	0.0550	0.0550	0.0100	0.0800	0.2566	1.6309	0.0174	0.0494	0.2756	0.9818	0.2820	1.0182	0.1951	0.8526
	alt4	0.0550	0.1101	0.0113	0.0900	0.3401	2.3155	0.0188	0.0887	0.2834	1.1024	0.2820	1.1281	0.3669	0.9089
	alt5	0.0550	0.0550	0.0100	0.0800	0.3816	2.1860	0.0199	0.0568	0.2834	0.8268	0.2820	0.8249	0.2884	0.8826
	alt6	0.0550	0.0550	0.0100	0.0900	0.3862	3.1367	0.0026	0.0806	0.2025	1.3501	0.1914	1.4545	0.3167	0.9361
	alt7	0.0550	0.0826	0.0100	0.0900	0.3760	2.7542	0.0010	0.0780	0.1929	1.1456	0.2115	1.1281	0.3359	0.9641
	alt8	0.0550	0.1651	0.0113	0.0900	0.3819	3.6342	0.0220	0.0703	0.2454	1.1420	0.2545	1.1281	0.3167	1.0226
	alt9	0.0550	0.1101	0.0100	0.0900	0.4791	2.5309	0.0226	0.0638	0.2025	1.1573	0.2078	1.1483	0.3167	1.0226
	alt10	0.0550	0.1101	0.0100	0.0900	0.4014	3.0687	0.0254	0.0707	0.3807	1.2311	0.3760	1.2374	0.2986	1.4860
	alt11	0.0550	0.1101	0.0113	0.0900	0.3615	3.2505	0.0176	0.0723	0.2131	1.1336	0.2273	1.1281	0.2884	1.2209
	alt12	0.0550	0.1651	0.0100	0.0900	0.4884	4.0036	0.0246	0.0818	0.2864	1.2272	0.2820	1.2374	0.3359	1.5726
	alt13	0.0367	0.1101	0.0100	0.0900	0.2618	2.3522	0.0010	0.0681	0.2273	0.8502	0.2424	0.8460	0.1966	1.0982
	alt14	0.1101	0.1101	0.0100	0.0900	0.4402	3.4135	0.0010	0.0708	0.1903	1.2692	0.1914	1.3396	0.3075	1.7178
	alt15	0.0826	0.1651	0.0100	0.0900	0.4961	3.6216	0.0189	0.0849	0.2454	1.6366	0.2475	1.6115	0.3261	1.7305

APPENDIX C

THE DATA OBTAINED FROM THE COMPARISON OF THE METHODS

Table C.1 Fitted error values of 5x5 inconsistent matrices (non-smoothed case)

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Matrix 1	-	33.44	27.12	416.18	47.77	20.68	24.25
Matrix 2	-	44.02	30.55	513.69	24.44	22.47	31.80
Matrix 3	-	34.38	28.09	501.78	18.56	19.03	158.53
Matrix 4	-	40.25	27.49	326.46	31.67	19.59	28.84
Matrix 5	-	47.04	55.53	722.36	31.60	45.17	40.18
Matrix 6	-	50.08	43.97	245.38	31.28	36.60	35.70
Matrix 7	-	58.99	75.59	2335.21	43.28	31.13	37.60
Matrix 8	26.48	37.90	22.46	187.37	9.02	9.12	17.51
Matrix 9	-	76.25	35.76	302.10	33.43	34.55	46.49
Matrix 10	-	50.98	78.44	32.94	27.31	27.16	44.85
Average	-	47.33	42.50	558.35	29.84	26.55	46.58
Std. Dev.	-	12.93	20.60	653.25	11.15	10.51	40.35

Table C.2 Fitted error values of 5x5 inconsistent matrices (smoothed case)

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Matrix 1	-	33.44	26.05	36.54	26.29	20.68	24.25
Matrix 2	-	43.25	30.55	46.12	24.44	22.47	31.80
Matrix 3	-	34.33	18.57	20.03	18.40	18.86	17.34
Matrix 4	-	39.87	25.28	33.19	31.67	19.59	20.46
Matrix 5	-	45.87	33.46	43.70	31.60	29.26	40.18
Matrix 6	-	49.38	37.04	31.72	28.58	30.11	35.70
Matrix 7	-	57.82	49.59	48.89	40.06	31.13	37.60
Matrix 8	26.48	35.66	16.52	11.79	8.51	8.61	10.28
Matrix 9	-	69.73	33.42	39.98	33.43	34.55	46.49
Matrix 10	-	50.98	32.47	32.94	27.31	27.16	44.85
Average	-	46.03	30.29	34.49	27.03	24.24	30.89
Std. Dev.	-	11.48	9.50	11.54	8.69	7.69	12.27

Table C.3 Absolute error values of 5x5 inconsistent matrices (non-smoothed case)

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Matrix 1	-	6.78	6.39	16.27	6.46	5.54	6.66
Matrix 2	-	8.24	7.10	18.68	5.28	5.11	7.94
Matrix 3	-	6.72	6.49	16.65	4.64	4.87	10.61
Matrix 4	-	7.67	7.08	15.61	7.95	4.89	6.40
Matrix 5	-	8.61	9.91	24.44	6.91	7.88	9.41
Matrix 6	-	9.03	9.24	14.70	7.42	7.67	8.32
Matrix 7	-	9.99	11.54	39.84	7.54	7.68	9.01
Matrix 8	5.98	7.42	5.94	10.92	3.16	3.16	5.24
Matrix 9	-	11.44	6.53	15.26	6.24	5.93	9.28
Matrix 10	-	8.89	8.85	7.99	5.13	5.17	9.69
Average	-	8.48	7.91	18.04	6.07	5.79	8.26
Std. Dev.	-	1.47	1.86	8.81	1.51	1.53	1.69

Table C.4 Absolute error values of 5x5 inconsistent matrices (smoothed case)

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Matrix 1	-	6.78	6.33	8.06	5.42	5.54	6.66
Matrix 2	-	8.17	7.10	8.80	5.28	5.11	7.94
Matrix 3	-	6.65	5.27	5.59	4.51	4.74	5.21
Matrix 4	-	7.63	6.44	7.15	7.95	4.89	5.48
Matrix 5	-	8.46	8.31	9.73	6.91	7.08	9.41
Matrix 6	-	8.98	8.51	7.87	7.12	7.11	8.32
Matrix 7	-	9.90	9.96	10.42	7.37	7.68	9.01
Matrix 8	5.98	7.19	4.85	3.86	2.94	2.94	4.04
Matrix 9	-	10.76	6.26	8.01	6.24	5.93	9.28
Matrix 10	-	8.89	6.84	7.99	5.13	5.17	9.69
Average	-	8.34	6.99	7.75	5.89	5.62	7.50
Std. Dev.	-	1.34	1.55	1.90	1.52	1.40	2.02

Table C.5 Fitted error values of 5x5 consistent matrices (non-smoothed case)

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Matrix 1	25.95	29.89	18.29	7585.18	16.75	12.91	14.92
Matrix 2	22.36	32.77	77.06	292.41	14.76	9.20	19.73
Matrix 3	13.01	25.65	110.59	22.84	8.82	8.25	11.07
Matrix 4	18.05	26.60	120.14	5648.17	11.13	11.12	16.54
Matrix 5	25.17	34.82	52.01	177.13	15.99	15.01	14.49
Matrix 6	18.84	21.58	80.69	21.78	17.21	7.74	17.14
Matrix 7	16.56	26.24	294.46	23.66	12.49	9.39	9.36
Matrix 8	16.55	16.49	94.14	18.36	8.12	5.19	6.99
Matrix 9	20.76	28.54	98.54	92.45	17.84	13.52	12.18
Matrix 10	25.43	45.63	23.53	110.16	15.01	15.49	22.41
Average	20.27	28.82	96.95	1399.21	13.81	10.78	14.48
Std. Dev.	4.41	7.90	77.46	2788.83	3.49	3.39	4.73

Table C.6 Fitted error values of 5x5 consistent matrices (smoothed case)

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Matrix 1	25.95	26.00	15.76	20.53	14.72	12.91	14.92
Matrix 2	22.36	32.59	15.89	16.68	14.70	9.20	15.79
Matrix 3	13.01	25.12	30.27	12.27	8.82	8.25	11.07
Matrix 4	18.05	25.65	36.17	11.08	11.13	10.62	15.60
Matrix 5	25.17	34.62	27.51	20.14	15.99	15.01	14.49
Matrix 6	18.84	18.76	39.01	14.96	11.99	7.74	16.29
Matrix 7	16.56	26.24	26.25	6.54	7.09	7.99	8.76
Matrix 8	16.55	16.49	36.47	10.42	8.12	5.19	6.99
Matrix 9	20.76	27.99	28.46	12.67	13.01	13.52	12.18
Matrix 10	25.43	44.89	18.32	22.86	15.01	15.49	15.78
Average	20.27	27.83	27.41	14.81	12.06	10.59	13.19
Std. Dev.	4.41	8.09	8.53	5.19	3.17	3.48	3.29

Table C.7 Absolute error values of 5x5 consistent matrices (non-smoothed case)

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Matrix 1	5.44	5.71	5.72	56.95	4.59	4.18	4.87
Matrix 2	4.69	6.38	10.49	15.05	3.78	3.49	4.88
Matrix 3	3.29	5.50	14.19	4.70	2.78	2.71	4.38
Matrix 4	4.18	5.70	14.81	49.83	3.30	3.30	5.16
Matrix 5	5.13	6.68	9.26	11.88	3.74	3.86	4.82
Matrix 6	4.29	4.52	12.21	5.31	2.88	2.62	5.36
Matrix 7	4.11	5.37	22.72	4.33	3.29	3.15	3.78
Matrix 8	4.39	4.47	13.47	4.79	2.28	2.20	3.47
Matrix 9	4.49	6.32	12.80	7.62	3.57	3.44	4.12
Matrix 10	4.83	7.96	5.81	10.49	4.08	4.16	5.83
Average	4.48	5.86	12.15	17.10	3.43	3.31	4.67
Std. Dev.	0.60	1.04	4.92	19.53	0.67	0.66	0.73

Table C.8 Absolute error values of 5x5 consistent matrices (smoothed case)

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Matrix 1	5.44	5.46	5.22	5.22	4.39	4.18	4.87
Matrix 2	4.69	6.36	5.26	4.96	3.77	3.49	4.36
Matrix 3	3.29	5.39	6.87	3.83	2.78	2.71	4.38
Matrix 4	4.18	5.60	7.39	3.65	3.30	3.20	4.89
Matrix 5	5.13	6.66	6.84	5.64	3.74	3.86	4.82
Matrix 6	4.29	4.33	7.69	4.34	2.54	2.62	5.24
Matrix 7	4.11	5.37	5.46	2.77	2.69	2.95	3.56
Matrix 8	4.39	4.47	8.09	3.73	2.28	2.20	3.47
Matrix 9	4.49	6.25	6.70	3.95	3.16	3.44	4.12
Matrix 10	4.83	7.87	5.02	5.17	4.08	4.16	4.96
Average	4.48	5.77	6.45	4.33	3.27	3.28	4.47
Std. Dev.	0.60	1.05	1.13	0.90	0.70	0.67	0.60

Table C.9 Fitted error values of 10x10 inconsistent matrices (non-smoothed case)

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Matrix 1	-	132.61	98.17	22542.40	65.44	65.84	124.04
Matrix 2	-	105.15	65.32	39527.90	46.42	46.46	78.82
Matrix 3	-	162.43	97.85	26786.24	86.63	80.25	105.31
Matrix 4	-	135.89	91.45	56937.65	63.40	63.28	101.80
Matrix 5	-	113.45	78.78	33853.62	53.19	53.61	78.68
Matrix 6	-	154.34	83.90	45048.48	80.09	81.35	99.34
Matrix 7	-	104.61	99.59	20960.43	49.66	51.89	87.92
Matrix 8	-	159.79	91.90	23632.60	74.63	71.55	97.85
Matrix 9	-	108.59	62.65	31764.68	45.09	44.88	76.86
Matrix 10	-	113.63	50.41	13432.38	42.72	42.69	63.48
Average	-	129.05	82.00	31448.64	60.73	60.18	91.41
Std. Dev.	-	23.16	17.24	12920.88	15.71	14.38	17.62

Table C.10 Fitted error values of 10x10 inconsistent matrices (smoothed case)

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Matrix 1	-	132.61	92.59	124.06	63.69	64.53	124.04
Matrix 2	-	104.36	64.65	75.51	46.39	46.26	78.82
Matrix 3	-	162.43	96.62	135.53	82.64	79.50	105.31
Matrix 4	-	135.10	88.91	113.12	63.40	63.28	101.80
Matrix 5	-	111.92	66.63	106.81	53.19	53.61	78.11
Matrix 6	-	153.43	82.68	149.36	70.69	81.35	99.34
Matrix 7	-	104.23	67.28	99.46	49.66	51.89	83.43
Matrix 8	-	158.89	88.62	119.53	74.63	71.55	97.85
Matrix 9	-	108.59	61.83	105.87	41.19	40.98	64.33
Matrix 10	-	112.83	50.37	71.48	42.72	42.69	61.89
Average	-	128.44	76.02	110.07	58.82	59.56	89.49
Std. Dev.	-	23.20	15.71	24.29	14.31	14.74	19.54

Table C.11 Absolute error values of 10x10 inconsistent matrices (non-smoothed case)

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Matrix 1	-	21.84	20.13	213.03	14.29	14.24	23.82
Matrix 2	-	19.35	15.73	268.38	12.63	12.56	18.24
Matrix 3	-	25.55	20.31	218.62	18.49	17.99	22.04
Matrix 4	-	22.53	19.08	366.15	15.23	15.29	21.15
Matrix 5	-	20.66	17.67	272.61	13.98	14.01	19.01
Matrix 6	-	25.38	18.58	278.93	16.68	18.12	21.01
Matrix 7	-	18.72	18.83	180.08	12.87	12.95	18.85
Matrix 8	-	25.55	18.99	199.15	16.54	16.54	20.64
Matrix 9	-	19.67	15.83	265.29	11.86	11.79	17.20
Matrix 10	-	21.30	13.49	113.24	11.43	11.36	15.78
Average	-	22.06	17.86	237.55	14.40	14.48	19.78
Std. Dev.	-	2.63	2.19	68.63	2.31	2.44	2.40

Table C.12 Absolute error values of 10x10 inconsistent matrices (smoothed case)

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Matrix 1	-	21.84	19.14	21.80	14.20	14.13	23.82
Matrix 2	-	19.23	15.58	16.44	12.61	12.50	18.24
Matrix 3	-	25.55	20.11	24.42	18.07	17.84	22.04
Matrix 4	-	22.44	18.94	20.96	15.23	15.29	21.15
Matrix 5	-	20.51	16.97	21.06	13.98	14.01	18.92
Matrix 6	-	25.29	18.35	25.50	16.05	18.12	21.01
Matrix 7	-	18.66	16.50	19.68	12.87	12.95	18.53
Matrix 8	-	25.46	18.71	22.55	16.54	16.54	20.64
Matrix 9	-	19.67	15.77	21.57	11.56	11.49	16.31
Matrix 10	-	21.21	13.47	15.86	11.43	11.36	15.67
Average	-	21.99	17.35	20.98	14.25	14.42	19.63
Std. Dev.	-	2.64	2.05	3.06	2.21	2.46	2.56

Table C.13 Fitted error values of 10x10 consistent matrices (non-smoothed case)

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Matrix 1	56.89	62.01	515.19	10154.27	22.02	23.09	29.43
Matrix 2	75.01	80.11	181.56	6062.34	31.57	30.13	42.48
Matrix 3	73.47	81.25	64.20	12548.14	29.21	30.35	45.45
Matrix 4	73.81	95.36	33.52	16873.51	28.67	29.20	36.82
Matrix 5	81.78	67.08	100.68	11203.31	29.31	27.49	37.89
Matrix 6	79.38	71.07	157.90	11284.98	30.65	31.58	39.57
Matrix 7	65.63	78.26	150.50	11831.30	26.22	25.91	33.81
Matrix 8	83.87	77.60	76.54	16569.76	27.56	27.89	36.41
Matrix 9	72.97	66.31	114.36	10539.71	24.96	24.15	32.76
Matrix 10	68.13	66.92	216.89	5489.44	24.68	23.76	60.87
Average	73.09	74.60	161.13	11255.68	27.49	27.36	39.55
Std. Dev.	8.02	9.91	136.50	3708.11	2.98	3.01	8.82

Table C.14 Fitted error values of 10x10 consistent matrices (smoothed case)

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Matrix 1	56.89	61.26	73.96	31.82	22.02	23.08	29.43
Matrix 2	75.01	79.49	61.05	41.10	31.37	30.13	42.48
Matrix 3	73.47	80.82	43.31	57.29	29.21	30.35	45.45
Matrix 4	73.81	94.72	33.52	53.31	28.67	29.20	36.82
Matrix 5	81.78	66.44	49.32	39.23	28.60	26.69	37.67
Matrix 6	79.38	70.68	59.60	37.23	29.16	29.10	39.57
Matrix 7	65.63	77.53	59.56	44.04	26.22	25.91	31.70
Matrix 8	83.87	77.22	42.76	41.53	27.56	27.89	36.09
Matrix 9	72.97	65.48	57.20	40.63	24.96	24.15	32.55
Matrix 10	68.13	66.50	69.60	50.13	24.57	23.74	37.37
Average	73.09	74.01	54.99	43.63	27.24	27.02	36.91
Std. Dev.	8.02	9.93	12.63	7.76	2.77	2.72	4.88

Table C.15 Absolute error values of 10x10 consistent matrices (non-smoothed case)

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Matrix 1	13.47	13.72	45.22	102.94	7.69	7.78	9.93
Matrix 2	16.45	16.39	23.76	69.74	9.14	9.07	11.78
Matrix 3	15.30	15.81	14.82	138.74	9.35	9.39	12.72
Matrix 4	15.70	18.00	11.16	158.60	9.28	9.16	11.13
Matrix 5	17.01	13.91	18.30	106.00	9.07	9.06	11.60
Matrix 6	16.62	14.49	22.75	109.43	9.34	9.33	12.14
Matrix 7	14.50	15.75	22.57	110.87	8.44	8.43	10.42
Matrix 8	17.71	15.55	14.92	153.01	8.85	8.90	11.52
Matrix 9	15.87	14.41	19.85	103.83	8.57	8.61	10.51
Matrix 10	14.99	14.31	27.15	74.18	8.19	8.17	13.50
Average	15.76	15.23	22.05	112.73	8.79	8.79	11.52
Std. Dev.	1.26	1.33	9.47	29.76	0.56	0.53	1.09

Table C.16 Absolute error values of 10x10 consistent matrices (smoothed case)

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Matrix 1	13.47	13.64	16.00	10.55	7.69	7.75	9.93
Matrix 2	16.45	16.32	14.69	11.56	9.12	9.07	11.78
Matrix 3	15.30	15.76	12.61	14.05	9.35	9.39	12.72
Matrix 4	15.70	17.94	11.16	13.11	9.28	9.16	11.13
Matrix 5	17.01	13.80	13.81	11.70	8.98	8.96	11.57
Matrix 6	16.62	14.44	13.94	11.09	9.22	9.09	12.14
Matrix 7	14.50	15.67	14.71	11.99	8.44	8.43	10.16
Matrix 8	17.71	15.50	11.90	11.79	8.85	8.90	11.43
Matrix 9	15.87	14.31	14.53	11.54	8.57	8.61	10.49
Matrix 10	14.99	14.25	15.43	12.93	8.15	8.16	11.13
Average	15.76	15.16	13.88	12.03	8.76	8.75	11.25
Std. Dev.	1.26	1.33	1.55	1.04	0.54	0.51	0.88

Table C.17 Fitted error values of 15x15 inconsistent matrices (non-smoothed case)

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Matrix 1	-	219.05	121.77	18569.69	93.11	94.01	139.41
Matrix 2	-	227.88	160.34	21452.60	98.07	98.78	155.71
Matrix 3	-	236.78	142.60	81981.38	104.85	104.16	175.07
Matrix 4	-	244.00	117.34	24505.73	94.81	94.87	136.21
Matrix 5	-	213.96	155.64	15845.42	87.32	86.85	148.65
Matrix 6	-	209.69	129.17	25994.57	85.88	83.86	156.05
Matrix 7	-	230.45	144.40	22275.15	104.10	100.78	154.83
Matrix 8	-	229.10	156.97	19441.61	101.69	100.45	166.49
Matrix 9	-	201.95	117.15	37526.17	76.49	76.53	124.73
Matrix 10	-	226.14	133.35	26577.49	97.04	96.51	160.12
Average	-	223.90	137.87	29416.98	94.34	93.68	151.73
Std. Dev.	-	12.77	16.52	19403.51	8.96	8.68	14.94

Table C.18 Fitted error values of 15x15 inconsistent matrices (smoothed case)

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Matrix 1	-	219.05	121.42	166.48	92.56	93.32	139.38
Matrix 2	-	227.88	116.70	169.18	98.04	98.43	155.68
Matrix 3	-	236.78	142.32	218.55	104.81	104.12	175.04
Matrix 4	-	244.00	115.05	186.53	94.78	94.83	136.17
Matrix 5	-	213.96	148.56	174.76	87.18	86.71	148.62
Matrix 6	-	209.69	127.35	165.05	85.30	83.29	156.02
Matrix 7	-	230.45	144.37	185.46	103.40	100.74	154.80
Matrix 8	-	229.10	142.10	175.28	100.83	99.67	166.46
Matrix 9	-	201.95	117.11	173.66	75.97	76.03	124.70
Matrix 10	-	226.14	130.15	183.39	97.01	96.48	160.09
Average	-	223.90	130.51	179.83	93.99	93.36	151.69
Std. Dev.	-	12.77	12.88	15.57	8.99	8.78	14.94

Table C.19 Absolute error values of 15x15 inconsistent matrices (non-smoothed case)

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Matrix 1	-	37.04	28.50	249.51	23.59	23.65	30.80
Matrix 2	-	37.31	30.26	283.61	23.99	24.16	33.08
Matrix 3	-	37.30	30.50	479.84	25.32	25.41	34.43
Matrix 4	-	39.99	27.36	300.48	23.70	23.67	30.78
Matrix 5	-	36.42	31.85	234.79	22.86	22.81	32.72
Matrix 6	-	35.38	28.81	318.68	22.09	22.02	32.91
Matrix 7	-	37.05	31.07	262.91	25.40	25.29	32.20
Matrix 8	-	37.65	31.31	258.94	24.36	24.32	34.69
Matrix 9	-	35.06	27.35	357.02	20.84	20.89	28.66
Matrix 10	-	37.39	29.82	341.75	24.43	24.39	33.70
Average	-	37.06	29.68	308.75	23.66	23.66	32.40
Std. Dev.	-	1.35	1.61	72.31	1.42	1.42	1.86

Table C.20 Absolute error values of 15x15 inconsistent matrices (smoothed case)

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Matrix 1	-	37.04	28.41	32.20	23.48	23.53	30.77
Matrix 2	-	37.31	27.25	32.32	23.96	24.08	33.04
Matrix 3	-	37.30	30.36	37.30	25.29	25.38	34.40
Matrix 4	-	39.99	27.13	35.01	23.67	23.63	30.74
Matrix 5	-	36.42	31.48	33.46	22.81	22.77	32.69
Matrix 6	-	35.38	28.38	31.65	21.94	21.87	32.88
Matrix 7	-	37.05	31.04	33.24	25.27	25.25	32.17
Matrix 8	-	37.65	30.25	33.25	24.13	24.12	34.66
Matrix 9	-	35.06	27.32	32.78	20.77	20.82	28.63
Matrix 10	-	37.39	29.39	33.47	24.40	24.36	33.66
Average	-	37.06	29.10	33.47	23.57	23.58	32.36
Std. Dev.	-	1.35	1.63	1.63	1.42	1.43	1.86

Table C.21 Fitted error values of 15x15 consistent matrices (non-smoothed case)

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Matrix 1	102.36	121.55	111.88	8213.92	32.36	31.26	49.96
Matrix 2	133.49	121.86	112.39	42311.36	41.37	40.44	68.84
Matrix 3	133.91	134.12	113.25	6489.51	42.05	41.39	67.23
Matrix 4	125.88	131.78	112.82	6301.75	43.36	42.73	71.41
Matrix 5	127.77	137.42	131.44	6818.91	42.54	42.63	68.86
Matrix 6	140.92	139.78	90.62	8929.46	50.72	50.38	73.07
Matrix 7	124.54	112.12	110.69	4258.50	35.93	36.25	59.03
Matrix 8	118.43	135.43	156.64	6233.87	42.55	42.77	60.56
Matrix 9	111.79	135.72	230.17	6490.92	39.99	39.00	57.09
Matrix 10	120.46	133.57	135.53	6803.05	45.65	45.26	60.22
Average	123.96	130.34	130.54	10285.12	41.65	41.21	63.63
Std. Dev.	11.32	8.83	39.32	11320.33	5.00	5.13	7.37

Table C.22 Fitted error values of 15x15 consistent matrices (smoothed case)

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Matrix 1	102.36	121.55	63.47	55.91	32.33	31.22	49.93
Matrix 2	133.49	121.86	74.97	62.32	41.34	40.40	68.81
Matrix 3	133.91	134.12	72.60	69.37	42.02	41.36	67.20
Matrix 4	125.88	131.78	70.87	66.73	43.32	42.70	71.27
Matrix 5	127.77	137.42	89.21	66.01	42.44	42.52	63.31
Matrix 6	140.92	139.78	68.82	76.26	50.69	50.35	73.03
Matrix 7	124.54	112.12	64.96	52.93	35.84	36.17	58.99
Matrix 8	118.43	135.43	83.07	64.30	42.52	42.74	60.52
Matrix 9	111.79	135.72	96.14	54.31	39.96	38.97	57.05
Matrix 10	120.46	133.57	92.71	72.40	45.62	45.22	59.38
Average	123.96	130.34	77.68	64.06	41.61	41.17	62.95
Std. Dev.	11.32	8.83	11.79	7.79	5.00	5.13	7.17

Table C.23 Absolute error values of 15x15 consistent matrices (non-smoothed case)

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Matrix 1	24.42	25.59	23.67	133.35	12.19	12.13	16.52
Matrix 2	28.19	24.54	24.15	233.70	14.12	14.10	19.21
Matrix 3	28.13	26.37	23.86	110.95	14.61	14.54	19.05
Matrix 4	27.09	25.95	24.02	109.93	14.79	14.76	19.86
Matrix 5	26.71	26.85	26.66	118.63	14.60	14.60	18.95
Matrix 6	28.53	26.33	22.08	140.83	15.54	15.44	20.20
Matrix 7	27.47	24.12	23.05	84.89	12.85	12.89	17.75
Matrix 8	25.67	26.84	28.01	106.49	14.75	14.78	17.85
Matrix 9	25.07	26.80	33.78	106.40	14.23	14.07	17.23
Matrix 10	25.80	26.54	27.51	118.17	15.42	15.38	17.70
Average	26.71	25.99	25.68	126.33	14.31	14.27	18.43
Std. Dev.	1.42	0.97	3.46	40.73	1.06	1.04	1.20

Table C.24 Absolute error values of 15x15 consistent matrices (smoothed case)

	LA	UA	TSGPM	GP	PM	PMV1	PMV2
Matrix 1	24.42	25.59	18.81	16.99	12.15	12.09	16.48
Matrix 2	28.19	24.54	20.60	17.67	14.09	14.07	19.18
Matrix 3	28.13	26.37	19.79	18.50	14.58	14.51	19.02
Matrix 4	27.09	25.95	19.97	18.34	14.76	14.73	19.79
Matrix 5	26.71	26.85	22.55	17.97	14.55	14.55	18.33
Matrix 6	28.53	26.33	19.54	19.84	15.50	15.40	20.16
Matrix 7	27.47	24.12	18.87	16.26	12.81	12.85	17.72
Matrix 8	25.67	26.84	21.35	18.13	14.71	14.75	17.81
Matrix 9	25.07	26.80	22.78	16.20	14.19	14.03	17.19
Matrix 10	25.80	26.54	22.99	18.72	15.39	15.34	17.57
Average	26.71	25.99	20.73	17.86	14.27	14.23	18.33
Std. Dev.	1.42	0.97	1.60	1.13	1.06	1.05	1.18

APPENDIX D

THE DATA USED FOR SYNTHESIZING METHOD COMPARISON

Table D.1 Problem set 1 used for synthesizing method comparison

	Local Interval Priority Weights of Alternatives									
	Criteria 1[2.333, 3.270]		Criteria 2 [0.552,0.552]		Criteria 3[0.409, 2.016]		Criteria 4[0.224,1.635]		Criteria 5[1.008,1.226]	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
alt1	1.146	1.146	0.458	1.834	1.450	1.450	0.573	2.292	0.573	2.292
alt2	0.251	1.419	0.335	0.876	0.355	2.007	0.710	1.004	1.262	2.258
alt3	0.812	3.014	0.377	0.406	0.639	0.639	3.197	3.876	0.431	1.599
alt4	1.080	2.160	0.945	1.920	1.080	1.080	0.240	2.520	0.360	1.920
alt5	0.930	0.930	0.588	1.764	0.294	1.176	1.247	1.764	0.294	4.989

Table D.2 Problem set 2 used for synthesizing method comparison

	Local Interval Priority Weights of Alternatives									
	Criteria 1[0.271, 1.602]		Criteria 2 [0.320,1.900]		Criteria 3[0.237, 1.402]		Criteria 4[0.207,0.837]		Criteria 5[0.837,1.659]	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
alt1	0.739	1.388	0.347	3.556	0.812	0.925	0.463	2.218	1.013	1.013
alt2	0.426	0.799	0.266	1.278	1.065	1.917	0.426	2.397	0.213	2.131
alt3	2.052	3.591	2.693	4.040	0.513	0.898	0.449	0.513	0.449	0.786
alt4	2.825	2.825	0.314	0.565	0.565	2.825	0.706	2.260	0.565	2.825
alt5	0.212	1.559	0.318	2.338	0.260	1.909	0.623	1.061	0.996	0.996

Table D.3 Problem set 3 used for synthesizing method comparison

	Local Interval Priority Weights of Alternatives									
	Criteria 1[0.233, 3.270]		Criteria 2 [0.308, 2.093]		Criteria 3[0.233, 1.582]		Criteria 4[0.386, 1.395]		Criteria 5[0.349, 0.930]	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
alt1	0.349	1.494	0.299	1.793	0.349	0.598	0.299	2.091	0.299	2.091
alt2	0.306	1.292	0.306	1.808	0.258	1.528	0.452	1.223	0.258	1.356
alt3	0.560	1.464	0.488	1.281	0.213	1.281	0.244	1.464	0.366	1.708
alt4	0.185	1.480	0.370	1.974	0.296	1.110	0.247	1.665	0.185	1.480
alt5	0.164	1.091	0.218	1.091	0.364	1.963	0.364	1.963	0.218	1.454

Table D.4 Problem set 4 used for synthesizing method comparison

	Local Interval Priority Weights of Alternatives									
	Criteria 1[1.198, 1.198]		Criteria 2 [0.379, 1.705]		Criteria 3[1.198, 1.198]		Criteria 4[0.682, 1.894]		Criteria 5[0.599, 2.696]	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
alt1	0.231	1.458	0.461	1.458	0.162	0.922	0.292	1.844	0.615	1.750
alt2	0.303	2.307	0.627	0.627	0.279	2.122	0.627	0.989	0.330	2.508
alt3	0.373	0.779	0.431	1.179	1.364	1.364	0.370	2.157	0.539	2.157
alt4	0.596	1.191	0.357	1.191	1.103	1.103	1.021	1.787	0.357	1.986
alt5	0.355	1.066	1.625	1.800	0.450	1.625	0.395	0.812	1.000	2.700

Table D.5 Problem set 5 used for synthesizing method comparison

Local Interval Priority Weights of Alternatives										
	Criteria 1[0.408, 1.750]		Criteria 2 [0.292, 0.907]		Criteria 3[1.361, 1.361]		Criteria 4[0.227, 1.458]		Criteria 5[0.350, 2.041]	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
alt1	0.515	4.488	0.978	1.911	0.428	0.616	0.643	0.643	0.511	2.298
alt2	0.481	3.607	0.419	2.675	0.205	2.568	0.321	1.607	0.292	1.313
alt3	0.534	2.885	0.611	1.630	1.284	2.996	0.383	2.250	0.328	0.875
alt4	0.721	3.005	0.815	3.354	0.514	2.568	1.607	1.607	0.328	1.167
alt5	1.202	1.924	0.382	4.891	0.428	1.027	0.268	3.214	0.438	1.459

Table D.6 Preference relation of the problem set 1 for Wang et al.'s method

	alt1	alt2	alt3	alt4	alt5
alt1	1	0.4144	0.1093	0.1563	0.5186
alt2	0.5856	1	0.1485	0.2084	0.6027
alt3	0.8907	0.8515	1	0.6014	0.8970
alt4	0.8437	0.7916	0.3986	1	0.8522
alt5	0.4814	0.3973	0.1030	0.1478	1

Table D.7 Preference relation of the problem set 1 for proposed method

	alt1	alt2	alt3	alt4	alt5
alt1	1	0.6051	0.1400	0.1912	0.6319
alt2	0.3949	1	0.0964	0.1342	0.5281
alt3	0.8600	0.9036	1	0.5921	0.9130
alt4	0.8088	0.8658	0.4079	1	0.8783
alt5	0.3681	0.4719	0.0870	0.1217	1

Table D.8 Preference relation of the problem set 2 for Wang et al.'s method

	alt1	alt2	alt3	alt4	alt5
alt1	1	0.5737	0.3635	0.1488	0.5890
alt2	0.4263	1	0.2977	0.1151	0.5157
alt3	0.6365	0.7023	1	0.2338	0.7153
alt4	0.8512	0.8849	0.7662	1	0.8912
alt5	0.4110	0.4843	0.2847	0.1088	1

Table D.9 Preference relation of the problem set 2 for proposed method

	alt1	alt2	alt3	alt4	alt5
alt1	1	0.5737	0.3247	0.1488	0.5810
alt2	0.4263	1	0.2633	0.1151	0.5075
alt3	0.6753	0.7367	1	0.2664	0.7426
alt4	0.8512	0.8849	0.7336	1	0.8879
alt5	0.4190	0.4925	0.2574	0.1121	1

Table D.10 Preference relation of the problem set 3 for Wang et al.'s method

	alt1	alt2	alt3	alt4	alt5
alt1	1	0.7133	0.7766	0.5764	0.7098
alt2	0.2867	1	0.5828	0.3536	0.4958
alt3	0.2234	0.4172	1	0.2813	0.4131
alt4	0.4236	0.6464	0.7187	1	0.6426
alt5	0.2902	0.5042	0.5869	0.3574	1

Table D.11 Preference relation of the problem set 3 for proposed method

	alt1	alt2	alt3	alt4	alt5
alt1	1	0.6397	0.7134	0.5416	0.6851
alt2	0.3603	1	0.5837	0.3996	0.5507
alt3	0.2866	0.4163	1	0.3219	0.4664
alt4	0.4584	0.6004	0.6781	1	0.6481
alt5	0.3149	0.4493	0.5336	0.3519	1

Table D.12 Preference relation of the problem set 4 for Wang et al.'s method

	alt1	alt2	alt3	alt4	alt5
alt1	1	0.1450	0.4227	0.4457	0.2698
alt2	0.8550	1	0.8123	0.8261	0.6858
alt3	0.5773	0.1877	1	0.5234	0.3351
alt4	0.5543	0.1739	0.4766	1	0.3146
alt5	0.7302	0.3142	0.6649	0.6854	1

Table D.13 Preference relation of the problem set 4 for proposed method

	alt1	alt2	alt3	alt4	alt5
alt1	1	0.3699	0.4457	0.5225	0.3701
alt2	0.6301	1	0.5779	0.6508	0.5002
alt3	0.5543	0.4221	1	0.5764	0.4222
alt4	0.4775	0.3492	0.4236	1	0.3494
alt5	0.6299	0.4998	0.5778	0.6506	1

Table D.14 Preference relation of the problem set 5 for Wang et al.'s method

	alt1	alt2	alt3	alt4	alt5
alt1	1	0.2711	0.3249	0.3027	0.4513
alt2	0.7289	1	0.5641	0.5386	0.6886
alt3	0.6751	0.4359	1	0.4742	0.6308
alt4	0.6973	0.4614	0.5258	1	0.6545
alt5	0.5487	0.3114	0.3692	0.3455	1

Table D.15 Preference relation of the problem set 5 for proposed method

	alt1	alt2	alt3	alt4	alt5
alt1	1	0.1803	0.3160	0.2386	0.2808
alt2	0.8197	1	0.6774	0.5876	0.6396
alt3	0.6840	0.3226	1	0.4042	0.4580
alt4	0.7614	0.4124	0.5958	1	0.5546
alt5	0.7192	0.3604	0.5420	0.4454	1