BI-OBJECTIVE FACILITY LOCATION PROBLEMS IN THE PRESENCE OF PARTIAL COVERAGE

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submitted by **AHMET SİLAV** in partial fulfillment of the requirements for the degree of **Master of Science in Industrial Engineering Department, Middle East Technical University** by,

Prof. Dr. Canan Özgen ____________________
Dean, Graduate School of **Natural and Applied Sciences**

Prof. Dr. Nur Evin Özdemirel ____________________
Head of Department, **Industrial Engineering**

Assoc. Prof. Dr. Esra Karasakal ____________________
Supervisor, **Industrial Engineering Dept., METU**

Examining Committee Members:

Prof. Dr. Murat Köksalan ____________________
Industrial Engineering Dept., METU

Assoc. Prof. Dr. Esra Karasakal ____________________
Industrial Engineering Dept., METU

Prof. Dr. Meral Azizoğlu ____________________
Industrial Engineering Dept., METU

Assoc. Prof. Dr. Canan Sepil ____________________
Industrial Engineering Dept., METU

Prof. Dr. Sencer Yeralan ____________________
Agricultural and Biological
Engineering Dept., University of Florida

Date: 16.06.2009

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name: **Ahmet SİLAV**

Signature:

ABSTRACT

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Silav, Ahmet

M.S., Department of Industrial Engineering

Supervisor: Assoc. Prof. Esra Karasakal

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In this study, we propose a bi-objective facility location model that considers both partial coverage and service to uncovered demands. In this model, it is assumed that the demand nodes within the predefined distance of opened facilities are fully covered and after that distance the coverage level linearly decreases. The objectives are the maximization of the sum of full and partial coverage the minimization of the maximum distance between uncovered demand nodes and their closest opened facilities. We apply two existing Multi Objective Genetic Algorithms (MOGAs), NSGA-II and SPEA-II to the problem. We determine the drawbacks of these MOGAs and develop a new MOGA called modified SPEA-II (mSPEA-II) to avoid the drawbacks. In this method, the fitness function of SPEA-II is modified and the crowding distance calculation of NSGA-II is used. The performance of mSPEA-II is tested on randomly generated problems of different sizes. The results are compared with the solutions resulting from NSGA-II and SPEA-II. Our experiments show that mSPEA-II outperforms both NSGA-II and SPEA-II.

Keywords: Maximal Covering Location Problem, partial coverage, p-center, Multi-Objective Genetic Algorithm

ÖZ

KISMİ KAPSAMANIN OLDUĞU DURUMDA İKİ AMAÇLI YERLEŞİM PROBLEMLERİ

Silav, Ahmet
Yüksek lisans, Endüstri Mühendisliği Bölümü
Tez Yöneticisi: Doç Dr. Esra Karasakal

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Bu çalışmada, yerleşim problemleri için kısmi kapsamanın yanında kapsanmayan talep noktalarına verilen servisin de dikkate alındığı iki amaçlı bir model önerilmiştir. Açılmış servis noktalarından belirli uzaklığa kadar olan talep noktalarının tamamen kapsandığı, bu uzaklıktan sonra da kapsama derecesinin doğrusal olarak azaldığı varsayılmıştır. Birinci amaç, tam ve kısmi kapsama değerlerinin toplamını minimize etmektedir. İkinci amaç ise açılmış servis noktaları ve kapsanamayan talep noktaları arasındaki en büyük uzaklığı minimize etmektedir. Literatürdeki en bilinen iki Çok Amaçlı Genetik Algortima olan NSGA-II ve SPEA-II metotları probleme uygulanmıştır. Ayrıca bu iki metotun dezavantajları tespit edilmiş ve problemin çözümü için yeni bir genetik algoritma (mSPEA-II) önerilmiştir. mSPEA-II metodunda, SPEA-II'deki uygunluk (fitness) fonksiyonu revize edilmiş ve NSGA-II'nin kuboid uzaklık hesaplaması kullanılmıştır. mSPEA-II'nun performansı rassal olarak üretilmiş farklı büyüklükteki problemler üzerinde test edilmiştir. Sonuçlar NSGA-II ve SPEA-II'nun çözümleriyle karşılaştırılmış ve mSPEA-II metodunun diğerlerinden daha iyi sonuçlar verdiği belirlenmiştir.

Anahtar Kelimeler: Maksimum Kapsama Yerleşim Problemi, kısmi kapsama, p-merkez, Çok Amaçlı Genetik Algoritma

To My Precious Family

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CHAPTER 1

INTRODUCTION

In Maximal Covering Location Problems, a predefined number of facilities are located to cover the maximum number of demand points. In the classical approach, a demand node is fully covered if it is within a critical coverage distance of the facility, and not covered if outside the critical coverage distance. In many situations, such an assumption, which results in a sharp drop from fully covered to not covered, is not reasonable. Thus, we take into account partial coverage in location problems. We define another critical distance and assume that coverage level decreases while moving from the first critical distance to the second one.

Although maximizing the number of covered demand nodes is an important issue, uncovered demand should be considered in facility location problems, especially for emergency situations. Demand nodes outside the critical distance of opened facilities are to be serviced to provide maximal survival in emergency service cases. Thus, the distances between uncovered demand nodes and opened facilities become important. We extend the maximal covering location problem to incorporate "the p-center" concept into the uncovered demands. We add an objective function which minimizes the maximum of the distances between uncovered demand nodes and their nearest opened facilities to the maximal coverage location problem in the presence of partial coverage.

Four bi-objective models are formulated in this thesis, and their advantages and disadvantages are discussed. In all these models, the first objective is related to the coverage and the second is related to the service to uncovered demands. Among these four models, we concentrated on the model that has the objectives of maximizing the total coverage, including partial coverage, and minimizing the maximum distance between an uncovered demand nodes and their closest serving facilities. This model is NP-Hard. For small-sized problems, optimal solutions can be found. However, we were not able to generate a true Pareto front when the number of demand nodes and facility nodes increased. Therefore, we needed to use heuristics to estimate the true Pareto front for large-sized problems. We apply the two existing Multiobjective Genetic Algorithms (MOGAs), Non-Dominated Sorting GA (NSGA-II) and Strength Pareto Evolutionary Algorithm (SPEA-II) to our model. We analyzed the results of these two MOGAs and we determined that these MOGAs have some drawbacks related to the generation of the true Pareto front. Thus we discussed the drawbacks of these state of art MOGAs. To eliminate the drawbacks, we developed a MOGA based on SPEA-II and NSGA-II, and we called mSPEA-II.

The organization of the thesis is as follows: We summarize the literature on location coverage models and MOGA in Chapter 2. Theoretical information about the ε-constraint method, NSGA-II and SPEA-II is presented in Chapter 3. In Chapter 4, the model formulations and a discussion on the models are given. In Chapter 5, mSPEA-II is presented and computational results are given. Concluding remarks and further research suggestions are given in Chapter 6.

CHAPTER 2

DEFINITIONS AND LITERATURE REVIEW

In this chapter, we present some basic definitions concerning multi-objective optimization problems and summarize the literature on coverage problems. We review the subject of genetic algorithms and also give the fundamental studies of MOGAs. We finally present the application of MOGAs to location coverage problems.

2.1 DEFINITIONS

The general form of multi-objective optimization problems is as follows:

$$\begin{aligned} &\text{Maximize/Minimize} \quad f_m(x) \qquad m = 1, \ldots\ldots, M \\ &s.to \\ &x \in X \end{aligned}$$

where $f_m(x)$ are objective functions and each objective function is either to be minimized or maximized. A solution x is a vector of decision variables. X is the feasible decision space.

Definition 1. A solution x_2 dominates solution x_1 if in all objectives x_2 is not worse than x_1 and better than x_1 for at least one objective.

Definition 2. A solution is a nondominated solution if it is not dominated by any other solution.

Definition 3. The set of all nondominated solutions is called the Pareto-optimal set, Pareto-optimal Front or True Pareto Front.

2.2 COVERAGE PROBLEMS

Many types of location models have been developed using covering metrics in the last century. In these models, a demand node is defined as being covered if it is within the maximum service distance or time of a facility.

Set covering location problems were first studied by Toregas and Revelle (1971). They developed a model to find the minimum number of facility locations in which everyone is served in a maximal covering distance.

Church and Revelle (1974) proposed a new coverage model that eliminates the obligation to cover all demands and maximizes the amount of demand that can be covered by a fixed number of facility nodes. This model was developed to determine a set of facility locations that maximizes the total amount of demand serviced by the facilities within a predefined maximal coverage distance. This model is called the Maximal Coverage Location Problem (MCLP).

MCLP originally comes from the classical p-median problem which was introduced by Hakimi (1964). The objective is the minimization of total weighted travel distance between demand nodes and opened serving facilities. In the p-median model, each demand node is assigned to a single facility and the demand at the opened facility has to be served by itself.

In general, these models do not deal with the issue of crowding within the system. Especially in emergency situations, a facility can serve only one demand node at a time. A second call from another demand node may arise at the same time, and this demand node may not be covered. Hogan and Revelle (1986) considered the secondary coverage of a demand node, which is called backup coverage. In their formulation, another objective is defined which maximizes the amount of back-up coverage. Thus, stochastic nature of demand calls in areas of high demand was handled by backup coverage.

A basic underlying assumption of the location-covering models is that the facilities being opened are uncapacitated. Current and Storbeck (1988) formulated capacitated versions of the set covering problem and the maximal covering location problem. In these models, total demand assigned to a facility does not exceed the capacity of that facility.

Emergency service systems require that service be readily available to those who need it. Pirkul and Schilling (1988) integrated workload capacities and backup service in the siting of emergency service facilities. If the primary serving facility is busy or if its capacity is inadequate, a secondary facility is assigned. The objective function is formulated as minimizing both fixed and variable costs.

In order to deal with the problem of availability in emergency service, Daskin (1983) studied the maximal expected coverage location problem. He proposed a model in which probabilities are given to facility nodes that are not available at any given time. Church and Bianchi (1982) improved this model by combining the stochastic p-median problem and the expected coverage problem. They formulated a hybrid model that aims to optimize travel time and the availability of facility nodes.

Daskin et al. (1988) integrated the multiple, excess, backup and expected covering models with the concepts of both set coverage and maximal coverage. Revelle et al. (1996) proposed a maximal conditional covering model that maximizes the number of servers which are covered by another server within the supporting distance.

Revelle et al. (1996) proposed two models for the maximal conditional covering problem (MCCP) and multi-objective conditional covering problem (MOCCP). The objective of MCCP maximizes the number of demand nodes which are covered by another server within the supporting distance. This model does not allow supporting servers to be located at the same node as the primary nodes. In MOCCP, there are two maximization objectives. These are the maximization of the number of servers with supporting coverage, and the maximization of population with primary coverage. To solve this problem, a weighting method of multi-objective programming was used.

The maximal expected coverage relocation model was defined by Laporte et al. (2006). They intended to use a dynamic relocation strategy in order to respond to waiting demand nodes, and thus to maximize the expected covered demand. Erkut et al. (2007) provided a maximal survival location problem with probabilistic response times for emergency medical service stations.

Moore and Revelle (1982) modified the MCLP by taking into account the hierarchy of facilities. This model is called the Hierarchical Maximal Covering Location Problem (HMCLP). The problem's objective is the maximization of coverage with the facilities that provide hierarchically different service levels. This extension was developed to handle the problems which occurred in the planning of the health services. Serra et al. (1992) structured a model that uses the hierarchical concept to locate services. This model also allows for the location of new facilities, beside the relocation of existing ones.

A number of researchers have developed coverage models with more than one objective. Hogan and Revelle (1986) considered backup coverage beside the first coverage. The weights are assigned to each objective by using the noninferior set estimation (NISE) technique.

Storbeck and Vohra (1988) proposed a bi-objective coverage model that allows for trade-off between the first and secondary coverage. They combined these objectives in a single objective by using weights, and they estimated the compromised solutions.

Church et al. (1991) formulated a bi-objective maximal covering location model. The model takes into account the uncovered demands. The objectives are the maximization of total coverage and the minimization of the total weighted travel

distance of the demand not covered to reach its closest opened facility. The NISE method is used to generate an approximation of the nondominated solution set.
Rao et al. (1992) modeled a multi-objective facility location problem. They developed a methodology for locating a single facility considering three criteria. These criteria are the maximization of the minimum distance from the facility to demand points, the minimization of the maximum distance from the facilities to demand points, and the minimization of the sum of all transportation costs. A fuzzy goal programming approach was used to solve the problem.

Badri et al. (1998) proposed a multi-objective model for locating fire stations. They define eleven strategic objectives related to such aspects as monetary, demand, etc. These objectives are evaluated by the decision maker according to their importance. Sensitivity and scenario analyses are performed on the proposed model.

Araz et al. (2007) proposed a multi-objective coverage-based model. They considered the vehicle location problem for emergency services. The maximization of the first coverage of demands, maximization of backup coverage and minimization of the total travel distance between uncovered demands and opened facilities were stated as objectives. The aspiration levels of fuzzy goals were defined. An approach based on fuzzy goal programming and lexicographic linear programming was proposed in order to solve the problem.

In the above mentioned coverage problems, it is assumed that the demand node is covered if it is within the critical distance of an opened facility. In these models, the abrupt finish of coverage is assumed. The demand nodes are never covered beyond a certain distance. To model the real life situations more realistically, Krass et al. (2003) modeled a covering location problem using a gradual concept. The coverage provided by the facility is assumed to decrease gradually after a certain distance.

Karasakal and Karasakal (2004) introduced the partial coverage concept in a maximal covering location model. They expressed the difficulty of choosing the critical distance value, thus they formulated MCLP in the presence of partial coverage. They assumed that the demand point was partially covered up to a maximum critical distance, and fully covered within the minimum critical distance. They developed a solution approach based on Lagrangean relaxation.

2.3 MULTI-OBJECTIVE GENETIC ALGORITHM

2.3.1 A SHORT REVIEW OF GA

Genetic Algorithm (GA) is a meta-heuristic that imitates the theory of natural genetics. The first study of GA was conducted by John Holland as pointed out by Deb (2001). The algorithm of GA is quite different from other optimization techniques. GA borrows the working principle of natural genetics. All solutions corresponding to members are represented by binary or real valued strings that are called chromosomes. Each solution has a fitness value that shows its performance, and this fitness value determines the survival of a member in a population.

The algorithm starts with random generation of the initial population up to predefined size. These members are evaluated according to fitness values. Three main operators, namely reproduction, crossover and mutation, are used to create the new population. In the reproduction stage, the candidate solutions are selected as being parents. Thus, bad solutions are eliminated and duplicates of good solutions are put into the mating pool. In the crossover phase, two solutions are picked at random from the mating pool and, according to a predefined strategy, some portion of the chromosomes is exchanged between these members, and two new solutions are created. In order to maintain the diversity of the population, the gene(s) of the new solution’s chromosomes are changed with a given probability, and this operation is called mutation. After generating the offspring, they are evaluated with respect to their fitness values, and the parent population is updated. These steps are repeated until the stopping criteria are reached.

Over the last decade, GAs have been used to search for optimum solutions in several problems. Because of the population based working principle, GAs are adapted to generate nondominated solutions in multi-objective problems, and these techniques are called multi-objective genetic algorithms (MOGAs). MOGAs have become a popular research area in recent years. Deb categorizes MOGAs in two groups: Non-Elitist MOGAs and Elitist MOGAs.

Non-Elitist MOGAs:

"Non-Elitist MOGAs" do not preserve the good solutions in any iteration. These are the first and simplest MOGAs. Deb (2001) reports the first MOGA as a vector evaluated GA developed by David Schaffer. After the pioneering work of Schaffer, many new non-elitist MOGAs were proposed by researchers. The main MOGAs in the literature are as follows: VEGA, WBGA, RWGA, NSGA, NPGA, DSGA, and VOES. Even without an elite-preserving mechanism, these algorithms are well converged and disturbed in their studies.

Elitist MOGAs:

"Elitist MOGAs" are faster and better than the algorithms given above. Elitist MOGAs favor the elites of population and carry them to the next generation. The presence of elitism enhances the probability of creating better offspring. In multi-objective optimization, elitism is not as straightforward as in the single objective case because, in a single objective case, the elite solutions are obtained according to one objective function value. Thus, in elitist MOGAs some different strategies are used to graduate the solutions and determine the elite solutions. The first study of elitism in a multi-objective case was carried out by Rudolph (2001). However, he did not introduce the diversity preservation mechanism. DPGA, SPEA, PAES, PESA, NSGA-II, SPEA-II, e-MOEA, TDGA, SMS-EMOE are the fundamental elitist MOGAs in the literature.

2.3.2 APPLICATION OF MOGA IN LOCATION COVERAGE PROBLEMS

To our knowledge, there are a few studies that use MOGAs for multi-objective coverage problems. Li et al. (2005) used genetic algorithms in location search problems. The problem of selecting multiple sites is solved by using the evolutionary approach and geographical information. The objectives are the maximization of population coverage, the minimization of total transportation costs and the minimization of the proximity to roads. Each solution is represented in the space of N×N cells, where N refers to the number of facilities. The fitness values of solutions are found by the weighted sum of objective function values. Li et al. compared the results of GA with those of neighborhood search heuristic and simulated annealing, and found that GA outperformed the other methods.

Villegas et al. (2006) modeled a bi-objective uncapacitated facility location problem and used NSGA-II, PAES and an algorithm based on mathematical programming to solve this problem. They implemented this model on the Colombia coffee supply network. Each solution of the model is coded in a binary string where the *i-th* position indicates whether facility *i* is open. Binary tournament selection and a crossover pattern are used in GAs. The objectives of the model are the minimization of the total cost of assigning facilities to related demand nodes and fixed facility opening costs, and the maximization of coverage.

Yang et al. (2007) proposed a model based on fuzzy multi-objective programming and a genetic algorithm. The developed method was applied to the fire station location problem. Five objectives were defined. The first objective was the minimization of the total setup and operating cost in fire stations, and the rest concerned the minimization of the longest distance from a fire station to any incident site. The achievement levels for fuzzy objectives were defined. The fuzzy multi-objective optimization model was converted into a single unified goal, and a genetic algorithm was applied to the problem.

CHAPTER 3

THEORETICAL BACKGROUND

In this chapter, we describe three methods: the Epsilon-constraint method, NSGA-II and SPEA-II. We briefly present the basic working principles of these methods.

3.1 EPSILON (ε) CONSTRAINT METHOD

The ε-constraint method is a multiobjective optimization technique, proposed by Haimes et al. (1971). This method converts the multiobjective problem into single objective and finds the nondominated solutions.

The algorithm starts with the construction of the payoff table. To find the ranges of each objective and construct the payoff table, the problem is solved independently for each objective. Next, except one objective, all other objectives are formulated as constraints and the following single objective optimization problem is obtained assuming that all objectives are maximization.

$$
\begin{aligned}
& Max\ f_i(x) \\
& s.t \\
& f_k(x) \geq \varepsilon_k, \quad k = 1, ..., p \ \ and \ \ k \neq i \\
& x \in S
\end{aligned}
$$

where S is the feasible region and p is the number of objectives.

In order to avoid finding the weakly efficient solutions, the objective function is modified by adding an auxiliary term, and for a multi-objective maximization problem, the model is follows:

$$Max\ f_i(x) + \rho . \sum_{k=1, k \neq i}^{p} f_k(x)$$

$$s.t$$

$$f_k(x) \geq \varepsilon_k, \quad k = 1, ..., p \ \ and \ \ k \neq i$$

$$x \in S$$

where ρ is a “sufficiently” small number.

For instance, a bi-objective problem, where one objective is maximization and the other is minimization, is formulated as follows:

$$\begin{array}{lcl} Min\ f_1(x) & & Min\ f_1(x) - \rho . f_2(x) \\ Max\ f_2(x) & \xrightarrow{\text{by } \varepsilon\text{-constraint method}} & s.t \\ s.t & & f_2(x) \geq K \\ x \in S & & x \in S \end{array}$$

The nondominated solutions are shown in Figure 1.

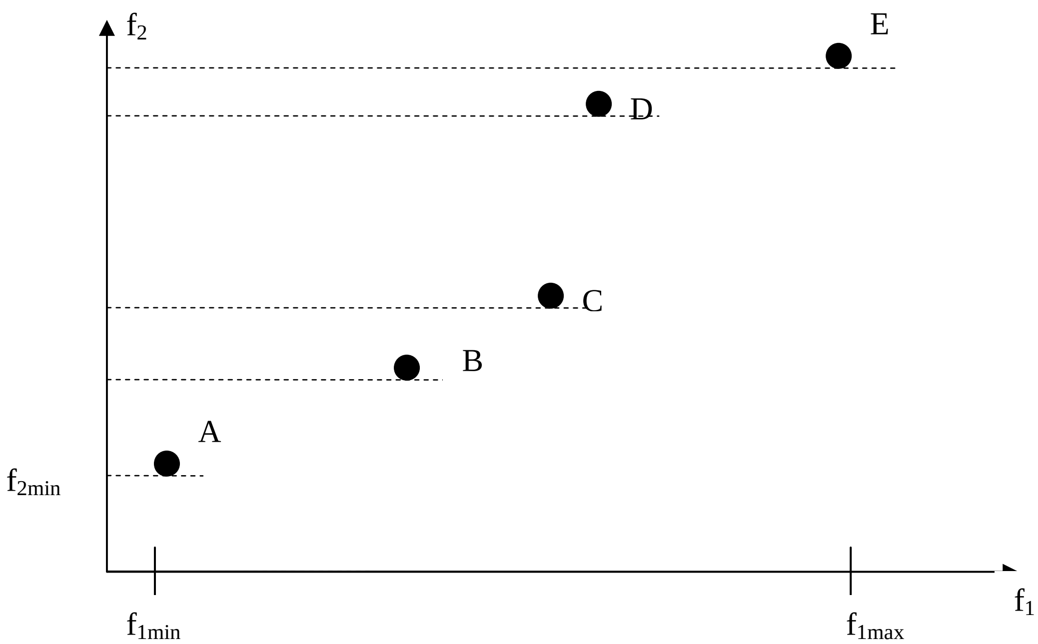

Figure 1 *True Pareto Front of an Example Problem*

“K” is taken as zero in the beginning. The single objective optimization problem is solved and the first found solution is A. After that the “K” is set to $f_{2min}+\rho$, where ρ is a very small constant, such as 0.001, and the problem is solved again. Solution B is generated. This technique is repeated and all the solutions in the Pareto Front are generated. If the same solution is found, then all nondominated solutions have been generated and the algorithm is finished.

3.2 NSGA-II

The Non-Dominated Sorting Genetic Algorithm (NSGA-II) proposed by Deb et al (2000) is an elitist multiobjective evolutionary algorithm. NSGA-II incorporates elitism by allowing the best solutions from the parent and offspring pools of chromosomes to be retained. It also incorporates a crowding distance calculation, to ensure that solutions are spread out along the Pareto Front, rather than converging to a single solution.

NSGA-II uses nondominated sorting to keep up good solutions instead of assigning a fitness value to each member. The diversity of the solutions is maintained by crowding distance calculations. Nondominated sorting is used to classify the entire population of parent and offspring.

There are different strategies to find the fronts of solutions. In the basic approach, to identify the solutions of the first nondominated front in a population, each solution is compared with every other solution in the population to determine if it is nondominated. If it is not dominated by any other solution, then it is in the first front. In order to find the next nondominated front members, the solutions which belong to previous fronts are discounted temporarily and the above procedure is repeated. The fronts obtained in an example problem which has minimization objectives are given in Figure 2.

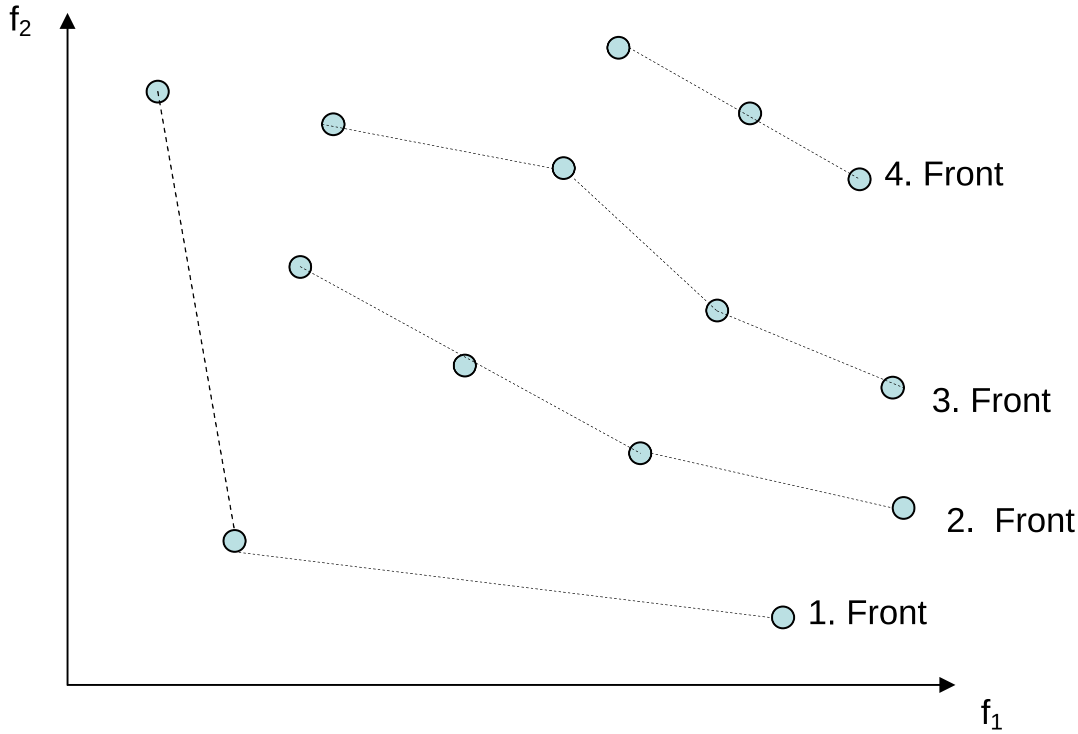

Figure 2 *The Fronts of Minimization Problem*

NSGA-II uses crowding distance in order to maintain a good spread of solutions in the solution set. The crowding distances are calculated for each front. To get an estimate of the density of solutions surrounding the solution *i* in the population, the average distance between two solutions on two sides of solution *i* along each objective is calculated. For all objectives, high crowding distance values are assigned to boundary solutions. For the intermediate solutions, the crowding distance values for each objective are calculated as follows.

$$d_{I_j^m} = \frac{(f_m^{I_{j+1}^m} - f_m^{I_{j-1}^m})}{(f_m^{\max} - f_m^{\min})}$$

where;

I_j denotes the solution index of the j^{th} member in the list which is obtained by sorting of objective m values in ascending order.

$f_m^{I_{j+1}^m}$ and $f_m^{I_{j-1}^m}$ are the neighbors of j^{th} member according to the m^{th} objective. $f_m^{\max}$ and $f_m^{\min}$ are the maximum and minimum values for the m^{th} objective.

The crowding distance value of one member in a front is measured by $\sum_{m=1}^{M} d_{I_j^m}$

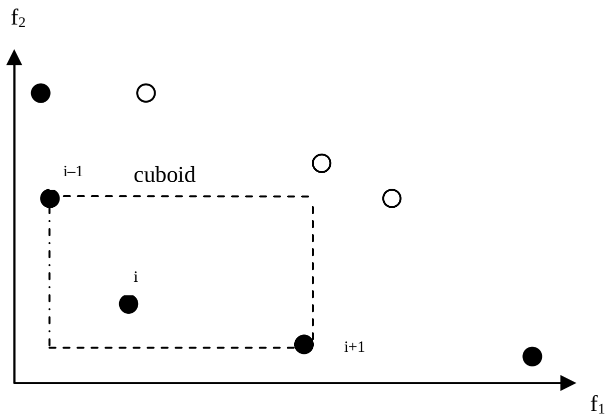

Figure 3 *The Cuboid Distances of a Solution in a Front*

This calculation is done for all intermediate members of a front and for all fronts. The average distance between a member and its neighbor is calculated for all objectives and this gives the crowding distance of a solution in its front.

The algorithm starts by generating random members and creating the initial population. Since there are no offspring at the beginning of the algorithm, nondominated sorting is applied only to the parent population.

Nondominated sorting is performed and crowding distance of each solution is calculated. Selection of members is made from beginning of the first front. Before filling up the mating pool, μ individuals are selected.

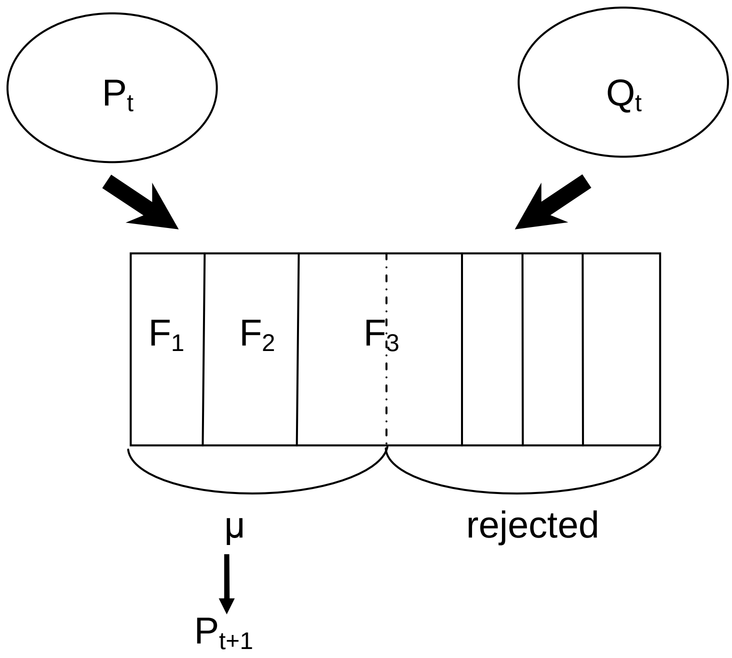

Figure 4 *The Creation of New Population*

In Figure 4, P_t and Q_t represent parent and offspring sets respectively. F_i represents the fronts and F_1 has nondominated solutions. Members of F_1 and F_2 are taken but the total number of individuals at hand is less than μ. By adding all the members of F_3, the set will have more than μ individuals. Thus, not all the solutions in F_3 are selected. For F_3, the crowding distance values determine which members are to be selected. The members having larger crowding distances are selected until μ members are chosen. Thus, a new population that has μ members is generated.

New population members are used to fill the mating pool. The crowded tournament selection strategy is used. Two members are selected; if they are in different fronts, then the member having the lower front number is chosen and put in the mating pool. If they are in the same front, then the member having the larger crowding distance is chosen and put into the mating pool.

Next, the crossover and mutation are applied and the new offspring' are generated. After obtaining an offspring population, the two populations are combined. The algorithm continues until the stopping criteria are reached. The flow chart of NSGA-II is presented in Figure 5.

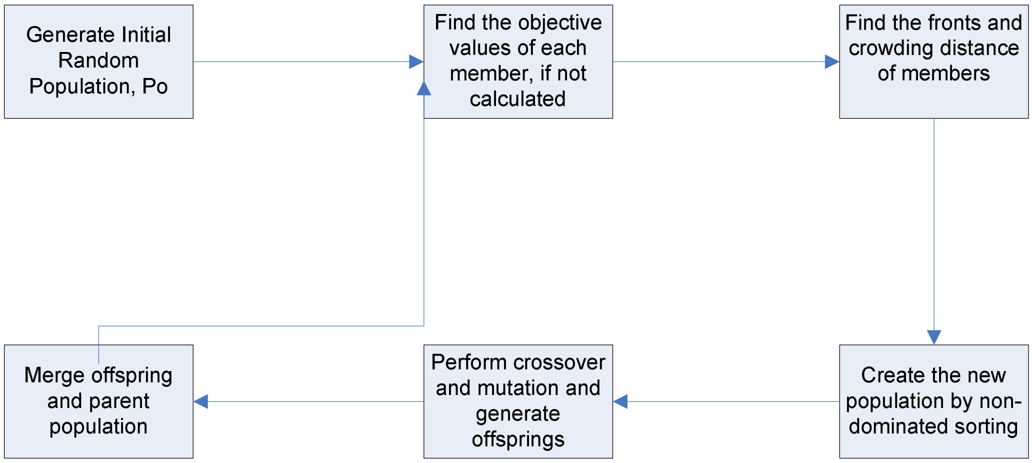

Figure 5 *The Flowchart of NSGA-II*

3.3 SPEA-II

Like NSGA-II, the Strength Pareto Evolutionary Algorithm (SPEA-II) proposed by Zitzler et al. (2002) is one of the most important multi-objective evolutionary algorithms that use the elitism approach. The SPEA-II algorithm was proposed to eliminate the possible weaknesses of its predecessor SPEA (Zitzler and Thiele, 1999). SPEA uses a regular population and external nondominated set. The fitness assignment is applied individually for these sets. If the number of nondominated solutions is greater than a predefined size then a clustering technique is used. SPEA-II uses a regular population and an archive (external) set. New generated offspring are stored in a regular set, and an archive set is used for storing the nondominated solutions. The fitness assignment, based on the domination concept and density estimation, is done for the union of these sets. The diversity of solutions is accomplished by the density estimation technique. This technique is an especially useful algorithm when most individuals do not dominate each other.

In the SPEA-II fitness assignment, not only is the domination of a solution considered, but also by which solution it is dominated. Firstly, the strength of each solution in the archive and regular population is calculated. The strength of a member *S(i)*, is defined as the number of solutions that it dominates.

$$S(i) = \left| \left\{ j \middle| j \in P_t + \overline{P_t} \wedge i \succ j \right\} \right|$$

where;

"$\succ$" corresponds to the Pareto dominance relation

$P_t + \overline{P_t}$: is the union of regular population and archive set

The greater strength means that it dominates more members in the population, so it may be a good solution. The raw fitness of a solution is calculated by summing up the strength of all the solutions that dominate the given solution. Thus, high raw fitness means that it is dominated by many individuals, so minimization of raw fitness, *R(i),* is important.

$$R(i) = \sum_{j \in P_t + \overline{P_t},\, j \succ i} S(j)$$

An illustrative example is given below when both objectives are minimization.

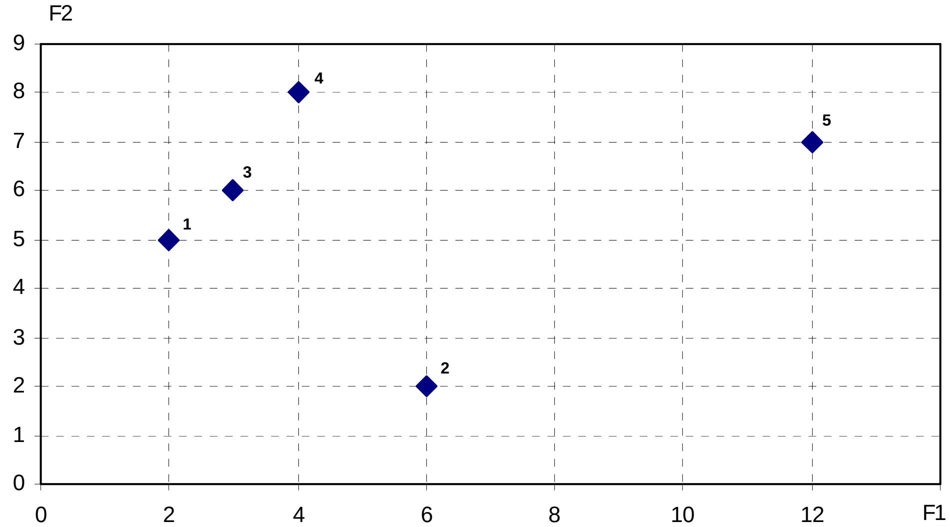

Figure 6 *Solutions in Objective Space*

The strength and raw fitness values for each solution are given in Table 1.

Table 1 *Strength and Raw Fitness Values of Solutions*

	1	***2***	***3***	***4***	***5***
S(i)	3	1	2	0	0
R(i)	0	0	3	5	6

Since solutions 1 and 2 are nondominated, their raw fitness values are 0. Solution 4 is dominated by the solutions 1 and 3, so its raw fitness value is the sum of the strength values of solution 1 (S(1)=3) and solution 3 (S(3)=2).

The density estimation technique is used in SPEA-II to differentiate the members that have identical fitness values. Actually, the value of the raw fitness for each solution provides diversification.

For each member in the union set, the distances to all other members are calculated and sorted in increasing order. k^th^ order distance value gives the σ_i^k value. The “k” is determined as the square root of the sample size. Since the regular population size is N and archive size is $\overline{N}$, $k = \sqrt{N + \overline{N}}$. For solution i, the density fitness value, D_i is;

$$D_i = \frac{1}{\sigma_i^k + 2}$$

The nearest neighborhood of a solution is not considered when calculating the distance in order to allow clustering to some extent. D_i is always less than 1, so density fitness values only help ordering solutions having identical raw fitness. Finally the fitness of a solution, *F(i)* is;

$$F(i) = R(i) + D(i)$$

The algorithm starts with a randomly generated population, in other words, a, regular population. SPEA-II proposes an archive set which tries to store nondominated individuals. At the beginning of the algorithm, the archive set Q_0 is empty.

The regular population, P_t and archive population Q_t are combined. The objective values are calculated, and then the raw fitness and density fitness are found. The nondominated solutions from the combined populations are obtained and copied to a new archive set, Q_{t+1}. SPEA-II has a predetermined archive size. Thus, during iteration, whether the new archive set size is the same as this predefined archive size has to be checked. Indeed, three situations may arise when filling up the archive.

If the nondominated front size is the same as the archive size, the new archive set is made up entirely of these solutions.

If the nondominated front size is smaller than the archive size, this means that some dominated members have to be selected. This can be done according to the fitness values. Members having small fitness values, which are greater than 1, are selected until the predefined archive size is achieved.

If the nondominated front size is greater than the archive size, then a truncation procedure is implemented. The σ_i^k values are calculated for all members in this temporary archive. This is very important because when calculating density fitness, all the members in the regular population and the archive set are considered. However, in the truncation procedure, σ_i^k values are calculated only for the members that are nondominated and in the temporary archive. This means that σ_i^k values are different in the first fitness calculation step and in the truncation step. The members that have the minimum σ^k distance are removed one-by-one from the archive until the archive size is reached.

After the new archive set is obtained, binary tournament selection is performed on these members. Two members are randomly selected and the member having the smaller fitness value is put into the mating pool. If the members have the same fitness values, then their σ_i^k values are compared. The member that has greater σ_i^k values is preferable. After the mating pool is filled, the crossover and mutation operations are applied and new offspring are generated. The new solutions are set as the new regular population and the algorithm iterates until one of the stopping criteria is reached. The flow chart of SPEA-II is presented in Figure 7.

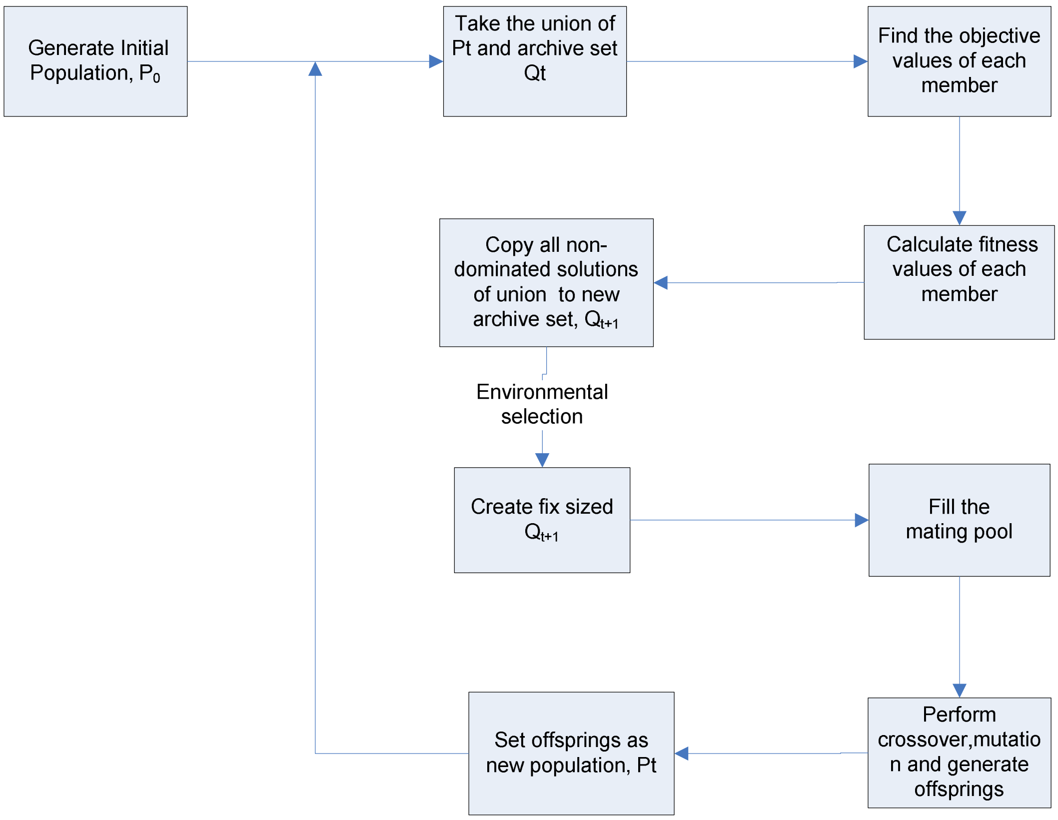

Figure 7 *The Flowchart of SPEA-II*

CHAPTER 4

BI-OBJECTIVE COVERAGE MODELS

In this chapter, we define four bi-objective coverage models. The formulations of the models are presented. We compare these models and discuss their weak and strong points.

4.1 MOTIVATION

There are many types of location models and studies using coverage objectives in the literature. In the classical approaches, demand is assumed to be covered if it is within S distance of an opened facility and is not covered if it is not within such a distance. These models assume that any demand point is either covered or not covered. This seems to be an unrealistic modeling assumption. For instance, a demand point is covered if the nearest opened facility distance to that demand point is S-ε and not covered if the nearest opened facility distance to that demand point is S+ε.

Facility location problems dealing with the coverage concept mainly focus on covering the demands, but satisfying the whole demand may not always be possible due to certain limitations. However, in real life, although a demand point is not within the coverage distance, it has to be served by one of the facility. Thus, we have to consider the uncovered demand and build a more realistic model.

In the literature, most of the maximal coverage location problems and their extensions that obtain information about the uncovered demand are generally modeled using a single objective. The models that take the uncovered demand into account have more than one objective. However, the objective functions of these models are represented as the weighted linear sum of objective functions. There are few studies for coverage problems that have more than one objective and generate a true Pareto front.

In this chapter, we formulate and solve four bi-objective coverage models. We find the Pareto-optimal sets and make comparisons.

4.2 MODEL FORMULATIONS

Model-1: ***P Median Maximal Coverage***

We firstly solve the bi-objective maximal coverage model presented in Church et al (1991). In this model, besides the coverage objective, an objective that minimizes the weighted travel distance of the demand not covered to reach its closest opened facility is formulated.

We use this model without any modification. They use the NISE method to solve the problem, whereas we use the ε-constraint method in order to find the Pareto set. We call this model the "P-Median Maximal Coverage (Model-1)".

The model is follows:
Model-1: *P Median Maximal Coverage*

$$Min\ Z_1 = \sum_{i \in I} \sum_{j \notin M_i} a_i . d_{ij} . x_{ij}$$

$$Min\ Z_2 = \sum_{i \in I} a_i . y_i$$

s.to

$$\sum_{j \in M_i} f_j + y_i \geq 1 \qquad \forall i \in I, \qquad (1)$$

$$\sum_{j \notin M_i} x_{ij} - y_i = 0 \qquad \forall i \in I, \qquad (2)$$

$$f_j - x_{ij} \geq 0 \qquad \forall i \in I,\ j \notin M_i, \qquad (3)$$

$$\sum_{j \in J} f_j = p, \qquad (4)$$

$$x_{ij} \in \{0,1\} \qquad \forall i \in I,\ j \in J,$$
$$y_i \in \{0,1\} \qquad \forall i \in I,$$
$$f_j \in \{0,1\} \qquad \forall j \in J,$$

where;

variables

$$x_{ij} = \begin{cases} 1 & \text{if a demand node i is not within S of its nearest facility, j} \\ 0 & \text{otherwise,} \end{cases}$$

$$y_i = \begin{cases} 1 & \text{if a demand node i is not within S of its nearest facility,} \\ 0 & \text{otherwise,} \end{cases}$$

$$f_j = \begin{cases} 1 & \text{if a facility is sited at j,} \\ 0 & \text{otherwise,} \end{cases}$$

parameters

a_i = demand at node i,

p = number of facilities to be sited,

I = set of demand nodes,

J = set of potential facility sites,

d_{ij} = shortest travel distance seperating node i from node j,

$M_i = \{ j \mid d_{ij} \le S \}$

S = maximal coverage distance,

The objectives of Model-1 are the minimization of total uncovered demand and the minimization of total weighted distance between uncovered demands and their nearest facilities.

Constraint set (1) ensures that if there is not any opened facility within the distance S for a demand node, then that demand node is an uncovered demand node and the variable y is forced to be 1. Constraint set (2) is formulated to assign the uncovered demand to an opened facility, because if the distance between an opened facility and a demand node i is less than S, then that demand node is a covered demand node and y_i is 0 so all the x_{ij} values are 0 for that demand node by the constraint set (2). However, if a demand node is not covered, then $y_i=1$ and constraint set (2) requires the demand node be assigned to a facility. Constraint set (3) ensures that an uncovered demand node will be assigned to a facility only if a facility is located at the site. Thus, if f_j value is 0, this means that the facility is not opened, so all the corresponding x_{ij} values are 0. If the f_j value is 1, then uncovered demand may be linked to this facility, so some x_{ij} values may be 1. Constraint (4) restricts number of opened facility.

Model-2: ***P-Center Maximal Coverage***

The p-center problem, also known as the minimax problem, is to locate p facilities and assign demand nodes to them in order to minimize the maximum distance between a demand node and the facility to which it is allocated.

In the literature, there are no studies that use the p-center concept for handling the uncovered demand in MCLPs. We construct a new bi-objective model, namely the "P center Maximal coverage model" (Model-2). The first objective of the model is to minimize the maximum distance between uncovered demands and their nearest opened facilities. The second objective is to minimize the total uncovered demand.

The underlying assumption is that an uncovered demand can probably be served by its nearest opened facility when it is needed. This objective is important, especially in emergency service facility location and coverage problems, because human life is the most important issue in emergency situations such as fires and those requiring an ambulance, even if there is only one person to be saved. Instead of using an objective that minimizes the total distance between uncovered demands and their nearest opened facility, as in Model-1, using an objective that minimizes the maximum distance between uncovered demand points and their nearest opened facility may be more reasonable.

The model is as follows:

Model-2: *P-Center Maximal Coverage*

$$Min\ Z_1 = A$$
$$Min\ Z_2 = \sum_{i \in I} a_i . y_i$$
s.to

Eq. (1)-(4)

$$\sum_{j \notin M_i} x_{ij} . d_{ij} \leq A \qquad \forall i \in I, \qquad (5)$$

Constraint set (5) calculates the maximum of the distances between uncovered demand nodes and their nearest facilities. The first objective minimizes the maximum distance between an uncovered demand node and the nearest facility to that node.

Model-3: ***P-Median Partial Coverage***

In Model-1 and Model-2, a demand node is assumed to be either fully covered or not. So the coverage decision variables are defined as binary integer variables. However, in some cases demand nodes can be partially covered according to distance between facilities and demand nodes (Krass et al., 2003; Karasakal and Karasakal, 2004)

It is more realistic to model the coverage level as a decreasing function of distance from an opened facility in some problems related to medical services, delivery problems, etc. Thus, we intended to consider this partial coverage concept in a new model and the “P-Median Partial Coverage Model” (Model-3) was constructed. Model-3 is the same as Model-1, except for the full coverage. Partial coverage, which is defined as a function of the distance of the demand point to the facility, is incorporated into the model.

Model-3: *P-Median Partial Coverage*

$$Min\ Z_1 = \sum_{i \in I} \sum_{j \notin M_i} a_i . d_{ij} . x_{ij}$$

$$Max\ Z_2 = \sum_{i \in I} \sum_{j \notin M_i} c_{ij} . s_{ij} . a_i$$

s.to

Eq. (1)-(4)

$$s_{ij} - f_j \leq 0 \qquad \forall i \in I,\ j \notin M_i, \qquad (6)$$

$$\sum_{j \in M_i} s_{ij} \leq 1 \qquad \forall i \in I, \qquad (7)$$

$$s_{ij} \in \{0,1\} \qquad \forall i \in I,\ j \in J,$$

where;

$$s_{ij} = \begin{cases} 1 & \text{if the demand at point i is either partially or fully covered,} \\ 0 & \text{otherwise,} \end{cases}$$

$$c_{ij} = \begin{cases} 1 & \text{if } d_{ij} \leq S, \\ f(d_{ij}) & \text{if } S \leq d_{ij} \leq T, (0 < f(d_{ij}) < 1), \\ 0 & \text{otherwise,} \end{cases}$$

$$M_i = \{ j \mid d_{ij} \leq T \}$$

T = maximal partial coverage distance,

S = maximal full coverage distance,

A new critical coverage distance, T is defined. To handle partial coverage, we assume that coverage level decreases linearly with the increase in the distance between the facility and demand point if the distance is between S and T and is zero after T.

This model maximizes the total coverage, including both partial and full coverage. A linear function is used to calculate the coverage levels according to distance (see Fig. 8).

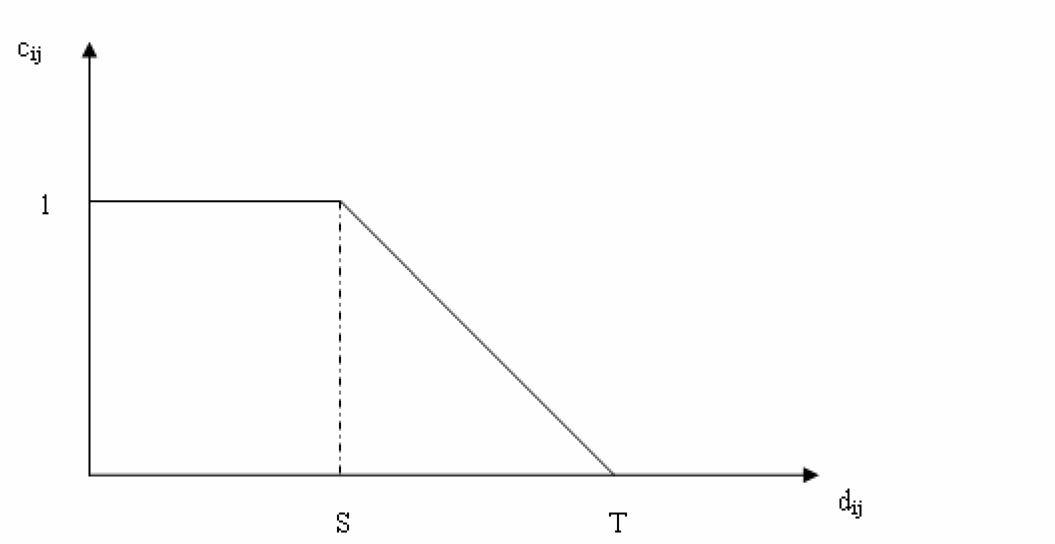

Figure 8 *Coverage Level Function*

In Model-1 and Model-2 the set of nodes j which can cover demand node i, M_i, is obtained according to the S maximum critical coverage distance value however in Model-3 and Model-4, the value M_i is determined with respect to T, whose value is greater than that of S. The other remarkable point is that all the x_{ij} and y_i values will be found according to T value. If a demand node is at distance between S and T to its nearest opened facility, then this demand is assumed to be partially covered in this model.

The constraint sets (6) and (7) are peculiar to the partial coverage concept. If the j^{th} facility is not opened, then all its s_{ij} values have to be 0, so the demand nodes will not be assigned to that facility, but if the j^{th} facility is opened than the values s_{ij} can be 0 or 1 and may get a facility assigned to it or not. This will be determined together with the constraint set (7), because constraint set (7) allows any demand point to be covered by at most one of the facility locations sited. The second objective is the maximization of total coverage, so if there is more than one facility covering a demand node, then this objective determines which of them will provide service. The first objective of this model is the same as in Model-1. We want to minimize the total uncovered weighted distance.

Model-4: ***P-Center Partial Coverage***

The “P-Center Partial Coverage Model” (Model-4) is formulated in the presence of partial coverage concept.

Like Model-2, Model-4 also minimizes the maximum distance to uncovered demand nodes.

Model-4: *P-Center Partial Coverage*

$$Min\ Z_1 = A$$

$$Max\ Z_2 = \sum_{i \in I} \sum_{j \notin M_i} c_{ij} . s_{ij} . a_i$$

s.to

Eq. (1)-(7)

The first objective minimizes the maximum distance between demand nodes and opened facilities. The second objective maximizes the total coverage including both partial and full coverage.

The four constraint sets (1), (2), (3), and (4) are common in all four models. These four constraints determine which facility or facilities will be opened, which demand nodes are covered or uncovered, and which uncovered demand nodes are linked to which facility. Constraint set (5) provides the minimization of the maximum distance to the first objective. The constraint sets (6) and (7) provide calculation of the coverage value.

4.3 COMPARISON OF MODELS

In order to illustrate the differences between partial coverage/maximal coverage and p-center/p-median objectives, we formulate four models and evaluate these models on 10 randomly generated problem data. Each problem has 200 demand nodes and 10 alternative facilities and the number of facilities to be opened, *p*, is taken as 3. 200

demand nodes and 10 facility coordinates are generated according to uniform distribution and the maximal coverage distance, *S*, and partial coverage distance, *T*, are determined as 10 and 20 units respectively. The demand quantities for 200 demand nodes are also generated according to uniform distribution.

Since we have bi-objective problems, the ε-constraint method is used in order to find all nondominated solutions. We transformed bi-objective models to single objective, and formulated the second objective as constraints.

So the objectives of the new models are as follows:

Model-1:

$$Min\ Z_1 = \sum_{i \in I} \sum_{j \notin M_i} a_i . d_{ij} . x_{ij} + p. \sum_{i \in I} a_i . y_i$$

s.to

Eq. (1)-(4)

$$\sum_{i \in I} a_i . y_i \leq \varepsilon \text{ (new constraint)}$$

Model-2:

$$Min\ Z_1 = A + p. \sum_{i \in I} a_i . y_i$$

s.to

Eq. (1)-(5)

$$\sum_{i \in I} a_i . y_i \leq \varepsilon \text{ (new constraint)}$$

Model-3:

$$Min\ Z_1 = \sum_{i \in I} \sum_{j \notin M_i} a_i . d_{ij} . x_{ij} - p. \sum_{i \in I} \sum_{j \notin M_i} c_{ij} . s_{ij} . a_i$$

s.to

Eq. (1)-(4), Eq. (6)-(7)

$$\sum_{i \in I} \sum_{j \notin M_i} c_{ij} . s_{ij} . a_i \geq \varepsilon \text{ (new constraint)}$$

Model-4:

$$Min\ Z_1 = A - p.\sum_{i \in I} \sum_{j \notin M_i} c_{ij}.s_{ij}.a_i$$

$s.to$

Eq. (1)-(7)

$$\sum_{i \in I} \sum_{j \notin M_i} c_{ij}.s_{ij}.a_i \geq \varepsilon \text{ (new constraint)}$$

We solve the problems by using the GAMS 2.0 Cplex solver. Each time we find a solution, we change the value of the new constraint right hand side iteratively, and in the end we find the Pareto optimal set for all these problems.

For Model-1 and Model-2 solutions, the partial coverage values, total weighted distance between uncovered demands and their nearest facilities (p-median values) and maximum of the distances between uncovered demand nodes and their nearest facilities (p-center values) are calculated in order to be able to compare the results with those of other models in the presence of partial coverage.

In Tables 2, 3, 4 and 5 only one randomly generated problem's results according to four models are presented. The results of all ten problems are given in Appendix A.

Table 2 *Model-1: P-Median Maximal Coverage Results*

facilities	*total cov.*	*full cov.*	*partial cov.*	*p-median*	*p-center*
4,5,8	3716	2342	1374	164422	63.4
3,4,10	3856	2516	1340	177214	69.4
4,5,10	3876	2734	1142	195598	69.4

Table 3 *Model-2: P-Center Maximal Coverage Results*

facility	*total cov.*	*full cov.*	*partial cov.*	*p-median*	*p-center*
1,8,10	4020	2161	1859	156868	54.8
1,3,10	3731	2423	1308	178756	55.0
5,8,10	3613	2432	1181	180758	63.4
4,5,10	3876	2734	1142	195598	69.4

Table 4 *Model-3: P-Median Partial Coverage Results*

facility	*total cov.*	*full cov.*	*partial cov.*	*p-median*	*p-center*
1,4,9	3679	1951	1728	146141	55.4
1,4,8	3979	2071	1908	151205	54.8
1,8,10	4020	2161	1859	156868	54.8
4,8,10	4023	2254	1769	165927	70.1
1,4,10	4199	2463	1736	169019	69.3

Table 5 *Model-4: P-Center Partial Coverage Results*

facility	*total cov.*	*full cov.*	*partial cov.*	*p-median*	*p-center*
1,8,10	4020	2161	1859	156868	54.8
1,4,10	4199	2463	1736	169019	69.3

According to the results of Model-1, the best solution according to the p-median objective is to site facilities at the 4th, 5th and 8th nodes. When partial coverage is considered, Model-3 chooses the 1st, 4th and 9th nodes as the best locations with respect to p-median objective. As seen in Tables 4 and 5, the best result is to site at the 1st, 4th and 10th nodes according to total coverage. Tables 2 and 3 show that opening facilities at the 4th, 5th and 10th nodes maximizes full coverage. To minimize the maximum distance between uncovered demand nodes and the nearest opened facilities, the facilities have to be sited at the 1st, 8th and 10th nodes.

As seen from the first problem, the results change when partial coverage is considered. The Pareto fronts of ten problems are generated for each model. For each problem's model solution, the maximum, minimum, average values and number of solutions in the Pareto front are calculated, and for ten problems, the average values are calculated. The results are given in Table 6.

Table 6 shows that the full coverage values of Model-1 and Model-2 are greater than those of Model-3 and Model-4. However, the total coverage values are vice versa because, the first two models do not consider partial coverage while selecting the facilities. Another interesting point is that the average p-median value of Model-1 is worse than that of Model-4, even if Model-1 focuses on p-median, while Model-4 focuses on p-center. This is also due to the maximal partial coverage distance.

Computational results show that incorporating partial coverage, defining p-median and p-center objectives have substantial effect on solutions of the problems. We argue that Model-3 and Model-4 are useful especially in location problems where coverage decreases gradually after some distance.

Table 6 ***Average Results of Four Model***

		total coverage			full coverage			partial coverage			p-median			p-center		
	no. sol.	**min**	**avg**	**max**	**min**	**avg**	**max**	**min**	**avg**	**max**	**min**	**avg**	**max**	**min**	**avg**	**max**
Model-1	4	4608.5	4836.9	5023.8	2141.2	2312	2442	2190	2525	2859	177235	198279	222315	58.4	64.5	71.9
Model-2	4	4451.4	4702.5	5052.4	2126	2271	2442	2121	2432	2804	184710	208136	227265	56.7	63.5	71.9
Model-3	2	5109.9	5125.6	5147.1	1978.8	2088	2182	2942	3038	3155	174759	178939	183889	58.4	61.0	64.0
Model-4	2	4816.4	5056	5235	1856	2019	2198	2876	3037	3282	176204	184545	193824	56.6	60.0	63.7

4.4 PROPOSED MODEL

In this section, we show the effect of p-median and p-center objectives in the presence of partial coverage on an illustrative example problem. The demand and facility nodes are located in a 20×20 km area. Nodes shown in Figure 9 are both demand nodes and potential facility sites. It is assumed that only one facility can be located.

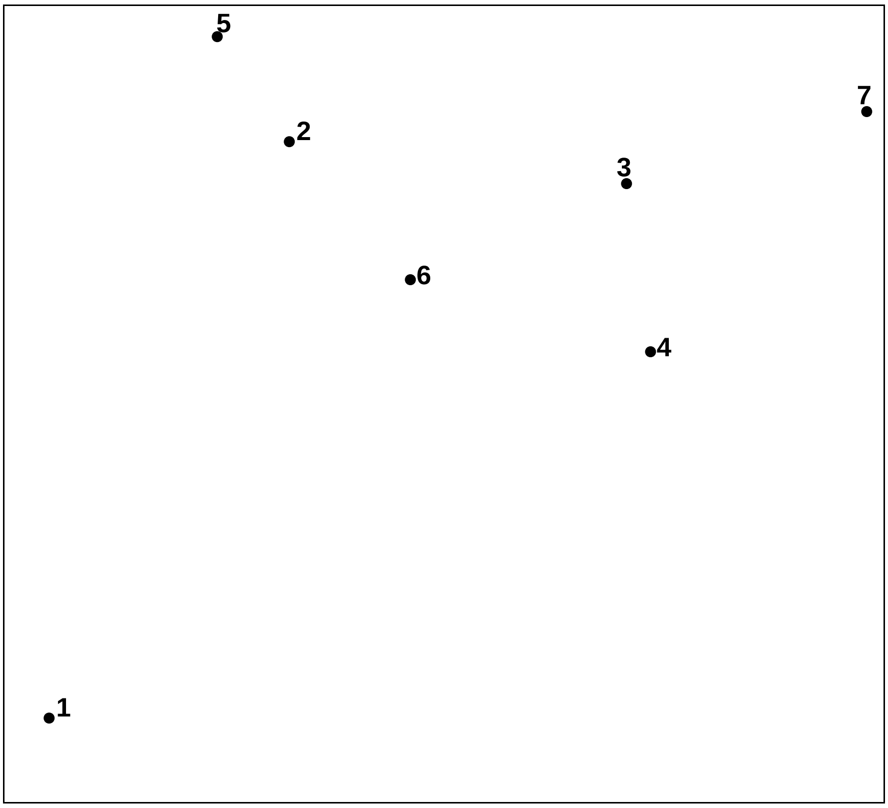

Figure 9 *An Example Problem*

The critical maximal coverage and partial coverage distances are 7 and 10 km respectively. The total demand is 400 units and demands of the nodes are given in Table 7.

Table 7 *Demand Quantities*

No	*Demand Quantities*
1	5
2	20
3	70
4	100
5	25
6	30
7	150

By solving Model-3 and Model-4, the following results are obtained.

Table 8 *Model-3 Results*

P-MEDIAN AND PARTIAL COV.			
facility	**total cov.**	**p-median**	**p-center**
3	362.00	371.83	20.510

Model-3 finds only one solution that selects the 3rd node for the location of a facility. This solution is the best according to both p-median and coverage objective. The maximum distance between uncovered demands and nearest facilities is 20.518 km. There are only two uncovered demands which are 1 and 5. The maximum distance comes from the 1st demand point.

Table 9 *Model-4 Results*

P-CENTER AND PARTIAL COV.			
facility	**total cov.**	**p-median**	**p-center**
6	231.65	1887.00	15.000
4	269.90	661.00	18.025
3	362.00	371.83	20.518

Model-4 finds three nondominated solutions. The best solution according to the p-center objective is to open a facility at node 6. If the 6th node is selected, then there will be two uncovered demand node, 1 and 7. The maximum distance, which is 15 km, comes from the 1st demand point.

There is a difference of 5 km encountered when selecting the 3rd and 6th facilities in this small area. Model-4 focuses on being close to all demand nodes whatever the amount of their demand.

This illustrative example shows that p-center problem that focuses on the uncovered demand nodes open the centralized facility as expected. So the maximum time to reach any uncovered demand point is minimized. P-median considers the total distance to the uncovered demand nodes. In real life situations, a demand node is generally served from its nearest facility. Thus, considering distance between demand node and nearest facility would be more reasonable . The incorporation of p-center objective into partial coverage problem ensures that emergency services are provided to the demand points in the shortest possible time even there is one person to be saved. Thus, Model-4 seems more suitable for the provision of maximal survival rates in emergency service location problems; thus, in the next part of the thesis, we will focus on this model.

CHAPTER 5

GENETIC ALGORITHM

In this chapter, the drawbacks of NSGA-II and SPEA-II are given. To avoid these drawbacks, a genetic algorithm, called mSPEA-II is proposed. The parameter setting of mSPEA-II is performed. The computational results and the discussion of the three methods are presented.

5.1 MOTIVATION FOR PROPOSED GA

We determined our model as p-center partial coverage model (Model-4) in the previous chapter. This model can be solved by ε-constraint method using GAMS for small problems. However, as the number of demand nodes, potential facility sites and facilities to be sited increases, the computational complexity of the problem increases. For instance, if we have 100 potential facility sites and we have to choose three of them, then the number of feasible solutions will be 970,200. In real life problems, the number of potential facility sites and selected facilities may be much higher. If we have 1000 potential facility sites and have to choose 20 of them, then the number of feasible solutions will be 8.25×10^{59}.

A middle-size problem with 500 demand nodes, 200 facility nodes and 5 selected facilities was solved using GAMS to find one efficient solution, and the run time was 6.5 hours. The true Pareto front is unknown. For example, if it has 100 efficient solutions then the run time will be approximately 650 hours for a middle-size problem. Thus, it is not possible to find a true Pareto front for middle or large size problems. In this chapter, multi-objective evolutionary algorithms are studied to generate the true Pareto front of Model-4. The state-of-the-art algorithms NSGA-II and SPEA-II are used. Also, a genetic algorithm, based on SPEA-II (mSPEA-II) is developed.

The NSGA-II algorithm does not use a fitness assignment. Nondominated sorting is applied and the front of each solution is found. Thus, the sorting of members may cause worse results, because in a front, some members may be dominated by many members of the other front, and others may be dominated by a few members. Giving the same priority to these solutions may not be correct. A simple bi-objective problem where objectives are to be minimized is depicted in Figure 10.

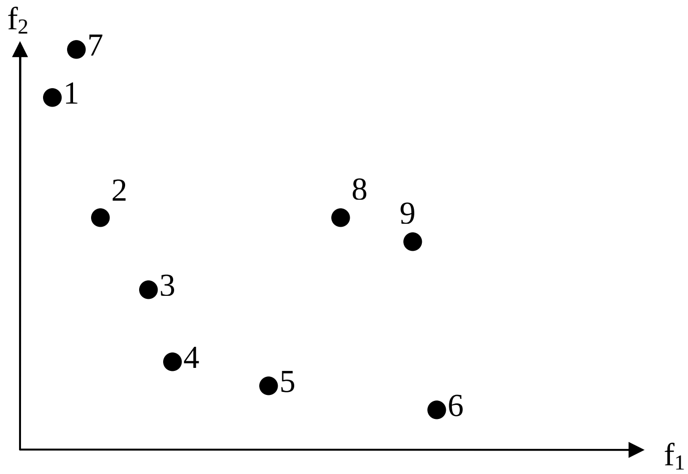

Figure 10 *A Sample Population*

There are two fronts. Solutions 7 and 9 are in the second front. Solution 7 is dominated only by Solution 1. Solution 9 is dominated by three solutions, 3, 4 and 5. Since they are boundary solutions in the second front, their crowding distances are the same and very big. So, in a selection situation, they have the same importance. However, solution 7 should have more priority with respect to solution 9, because, according to f_1 values, Solution 7 should be kept in the population. Which member dominates which member in a population is not considered while comparing the solutions in NSGA-II. Thus, difficulty in converging to the true Pareto-optimal front may arise.

In SPEA-II, the fitness value for a solution is the total of raw fitness and density fitness. SPEA-II fitness assignment is quite a good technique. Information regarding which solution dominates which solution is kept and the strength value of each member calculated. The density fitness is calculated to differentiate those solutions which have the same raw fitness values. Also, density distance values are used in a truncation operation when the actual archive is greater than the fixed archive size. Calculating and sorting all the distances is of complexity O (M^2logM). The worst run-time complexity of the truncation operator is O (M^3) and, on average, it will be lower than O (M^2logM). Actually, the computational complexity of SPEA-II is considerably greater than that of NSGA-II.

In SPEA-II, the fitness assignment already provides diversification. Density fitness values are always between zero and one. It does not change the order of the solutions. It only makes a difference in the fitness values that are the same. Two solutions in a population may have the same fitness in two different cases.

If two members are dominated by the same members, then the sum of their strength values will be equal. For instance, Figure 11 shows an illustrative multi-objective minimization problem population. Solutions 6 and 7 are dominated by same solutions 2, 3 and 4, and the raw fitness values of solutions 6 and 7 are the same.

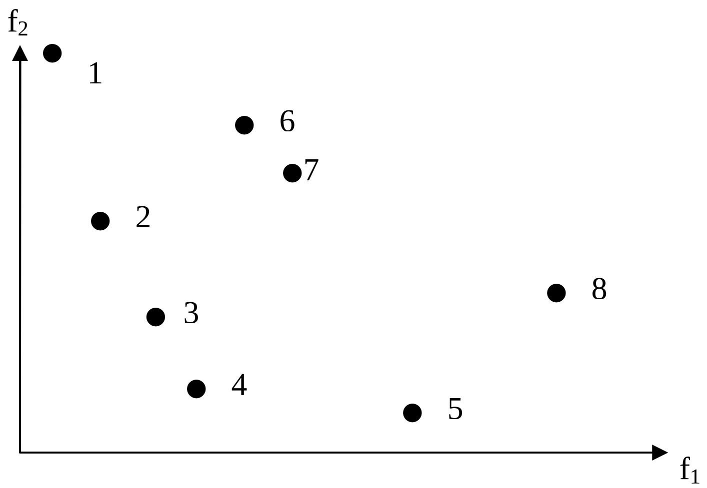

Figure 11 *The Solutions in a Population*

The second situation is when two solutions are dominated by different solutions, but the sum of the strength values of these dominating solutions are the same. The sample population which is composed of archive and regular populations in an iteration is displayed in Figure 12. The strength and raw fitness values are shown in Table 10.

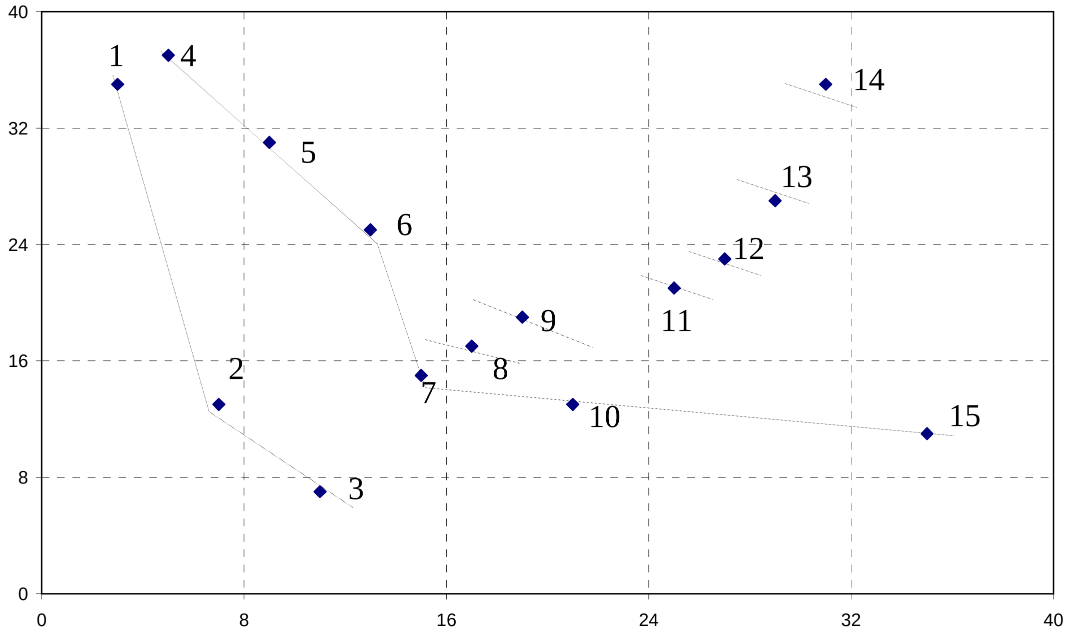

Figure 12 *Union of Archive and Regular Population in an Iteration*

Table 10 *Strength and Raw Fitness Values*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
S(i)	2	10	10	0	1	2	6	5	4	4	3	2	1	0	0
R(i)	0	0	0	2	10	20	20	26	31	20	39	42	16	55	10

As seen in Table 10, the raw fitness values of solutions 5 and 15 are the same. The density fitness values are calculated according to the k^{th} order distance value. The total population size is 15, so $k=\sqrt{15}\approx 4$. In the distance matrix for solution 5, the fourth distance is calculated as the distance between solution 9 and itself. For solution 15, the fourth distance value is calculated as the distance between solution

13 and itself. To find these distance values for density fitness calculations, the computational complexity increases. There are eight fronts in this population. Solutions 5 and 15 are in the same front. However, by using the density estimation technique, the distances of different front members are taken into consideration.

In both situations, a different strategy should be used to differentiate the fitness values of solutions, both to decrease computational complexity and keep good solutions in order to maintain the diversity and convergence with the true Pareto front.

5.2 PROPOSED GA: The Modified SPEA-II (mSPEA-II)

In the new proposed GA, instead of density fitness, *D(i)* , the objective fitness values *minO(i)*, are calculated. For each minimization objective, the solutions are sorted in ascending order. *minO(i)* is set to the minimum rank among all objectives. For instance, the objective values for a bi-objective minimization problem are given in Table 11.

Table 11 *Objective Function Values of Solutions*

	f_1	f_2
1	2	8
2	5	7
3	8	2
4	3	20
5	6	9
6	10	3
7	30	4
8	12	5

These solutions are sorted according to both objectives, which are shown in Table 12.

Table 12 *Solutions Sorted According to Each Objective*

Sol.	f_1	*Sol.*	f_2
1	2	**3**	2
4	3	**6**	3
2	5	**7**	4
5	6	**8**	5
3	8	**2**	7
6	10	**1**	8
8	12	**5**	9
7	30	**4**	20

As seen in Table 12, *minO(7)*=3, because it is in the seventh rank for objective one, f_1, and in the third rank for objective two, f_2, so the minimum is 3.

A fitness value for each solution is determined as follows:

$$F_1(i) = R(i) + \frac{\min \mathrm{O}(i)}{\text{no. of solution+1}} \qquad (8)$$

The fitness value of a solution is the sum of raw fitness, which is same as in SPEA-II, and objective fitness which is always less than 1. Two solutions that have the same raw fitness values are differentiated by their objective fitness values. Thus, more importance is given to boundary solutions. For some objectives, a solution may have worse objective values, but if it has a better objective value for at least one objective, then its objective fitness will be small and that solution can be kept. This method also decreases the run time of the algorithm.

SPEA-II has an archive of fixed size. If the nondominated solutions of the regular - and archive populations is less than this fixed size, then the dominated solutions are filled up to this size. The dominated solutions are selected according to fitness values which are greater than 1, because, all nondominated solutions have fitness values less than 1. In this selection, if two solutions have the same raw fitness values, then the objective fitness values determine the selection, but if the nondominated solutions are greater than the fixed archive size, then some members have to be discarded. In SPEA-II, this truncation is done according to density distance values. In our proposed algorithm, we do not calculate density distance values, in order to decrease the run time. As mentioned above, we calculate the objective fitness values. However this may not work well when all the solutions in the archive set are nondominated. An illustrative example problem is shown in Figure 13.

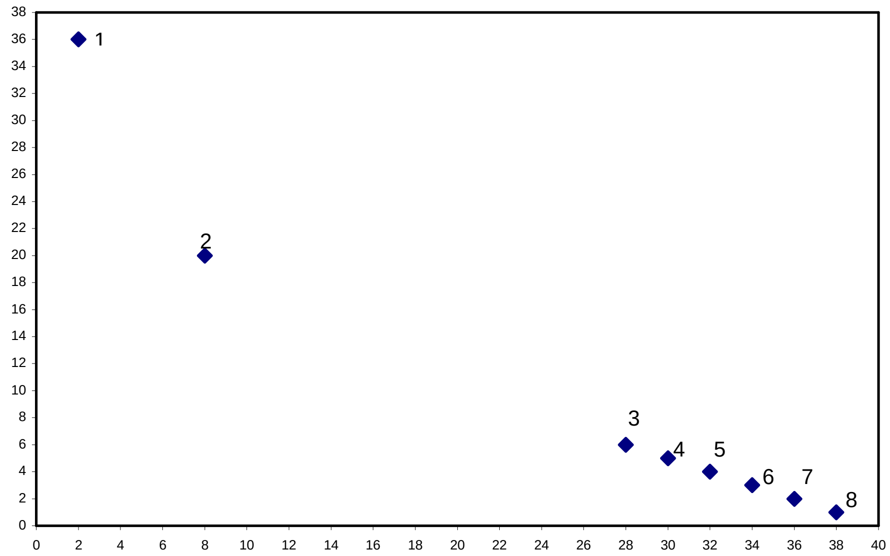

Figure 13 *Illustrative Example When All Solutions are Nondominated*

In the above simple problem, all eight solutions are nondominated. The *minO(7)*=1, *minO(2)*=1. However, solution 2 is responsible for a big region in order to maintain convergence. Therefore, selecting solutions to discard by *minO(i)* or objective fitness could avoid the convergence to the true Pareto front. Our proposition is that if the algorithm is in a truncation operation, this means all the solutions are nondominated, and then the crowding distances can be calculated for each solution as in NSGA-II. The crowding distance, which is the average distance between two solutions on two sides of solution *i* along each objective, is calculated. For all objectives, crowding distance values set to the infinite values for boundary solutions. So the new fitness assignment is made in this step.

The fitness is calculated as the total of raw fitness and crowding fitness.

$$F_2(i) = R(i) + \frac{1}{\mathrm{C(i)+1}} \qquad (9)$$

where C(i) is the cuboid distance of *i*.

These two strategies are used for calculating fitness values. The question is, which of the fitness values for each solution is used when filling up the mating pool, because we calculate different fitness values in respect of actual archive size. In this step the strategy is:

-if the archive size is less than the predefined fixed archive size, then the fitness values that are calculated as the sum of raw and objective fitness are used for binary tournament

-if the archive size is greater than the fixed archive size, then the fitness values that are calculated as the sum of raw and crowding fitness are used for binary tournament.

The flow chart of this new algorithm is given in Figure 14. The pseudo code of mSPEA-II is given in Appendix B.

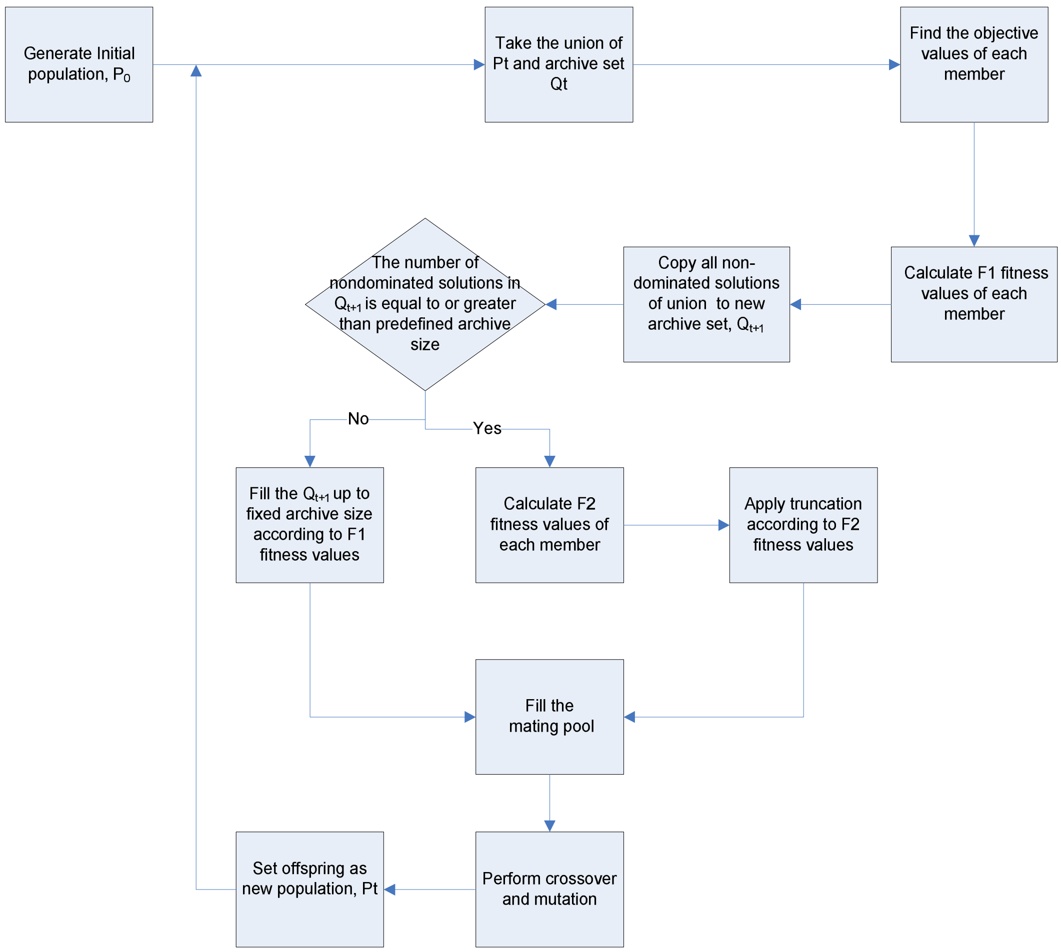

Figure 14 *Flowchart of mSPEA-II*

5.3 PARAMETER SETTINGS

Each solution to the model was encoded in a real parameter that indicates the facility to be opened.

5	*9*	*7*

Figure 15 *Representation of a Solution*

The number of opened facilities determines the size of the string. In Figure 15, the number of facilities to be opened is 3. The opened facilities are 5, 7 and 9. The amount of demand of each demand node, the coordinates of the facility and demand nodes are inputted to the program, coded in Java. The program creates the matrix of distances between demand and facility nodes. The Euclidean distances are considered.

Each member of the population is evaluated with respect to two objectives; total coverage and p-center. For instance, for facilities 5, 7 and 9, the demand nodes within the distances *S* and *T* are found and demand nodes are assigned facilities with highest coverage levels. The amount of demand is multiplied by the coverage level to calculate coverage of the demand node. (If it is fully covered, then coverage level is 1). The sum of the coverage of all demand nodes gives the total coverage.

The p-center objective is the maximum distance between an opened facility and a demand node. So, in the distance matrix, for facilities 5, 7 and 9, the uncovered demands according to distance *T* are determined. For demand node *i*, the minimum distance from the opened facilities 5, 7 and 9 tells us how far the demand node is from an opened facility. This distance is calculated for each demand node, and the maximum value of these distances gives the p-center objective value.

The objectives are normalized in all MOGAs. For SPEA-II, the distances in objective space between members are calculated for the density estimation technique. In NSGA-II and mSPEA-II, crowding distances are also calculated for the niching mechanism. The range of objectives is different, so scaling has to be done. Objective function values are divided by the estimated maximum possible values in the Pareto front.

In the crossover mechanism, a pattern is used. For example, the {1,2,2,1} pattern means taking the first and fourth genes of the first member, and the second and third genes of the second member. However, after crossover, we may need to repair the mechanism.

In a population, two parents and offspring are shown in Figure 16. The pattern is {2, 1, 2}.

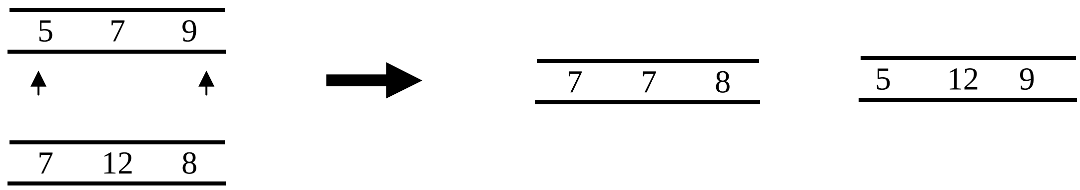

Figure 16 *Crossover Mechanism*

The first offspring genes are 7, 7 and 8. Thus, this means that Facility 7 is opened twice. This is an infeasible solution, so one of the genes should be replaced randomly by another gene that is neither 7 nor 8.

In the mutation mechanism, only one gene is selected for mutation. All the genes are not potentially mutable. A gene is randomly selected, and it is mutated with a probability. After mutation, the member is again checked for repair.

In filling up the mating pool, binary tournament selection is used.

For parameter setting, experimental runs are done on small size problems. Five different problems are randomly generated, and four different parameters are used. For each parameter and each problem, five runs are done. Thus, a total of 100 runs are performed.

All five problems are of the same size. The characteristics of the problems are displayed in Table 13.

Table 13 *Problem Characteristics*

No. of Facility Node	50
No. of Demand Node	200
Side length of Region	200
Demand Amount Interval	0-500
No.of facilities to be opened *(p)*	6
Partial Coverage Distance *(T)*	20
Full Coverage Distance *(S)*	10

All coordinates are generated from a uniform distribution between 0-200 units. All the demand amounts are generated from a uniform distribution between 0 and 500. The partial coverage distance is 10% of the region size, and the full coverage distance is 5% of the region size, which are 20 and 10 respectively.

All these problems are firstly solved by the ε-constraint method using GAMS, and the true Pareto front is obtained. After that, the parameters are compared according to the Hyper Volume Ratio (HVR). Then, for different parameter settings, mSPEA-II is run and the results obtained from mSPEA-II are compared with the true Pareto front based on the Hyper Volume Ratio (HVR).

HVR is the extension of the hypervolume metric (Zitzler and Thiele, 1998). Hypervolume metric calculates the total objective space dominated by a given set of nondominated solutions with respect to a predefined reference point. This metric evaluates both closeness and diversity of results. The reference can be determined as the nadir point or worse.

$$\mathrm{HV} = volume\left(\bigcup_{i=1}^{|Q|} v_i\right)$$

where; Q is the nondominated front and v_i is the objective space dominated by solution i with respect to a reference point, W. Figure 17 shows the nondominated solutions' hypervolumes when the first objective is to be minimized and the second objective is to be maximized.

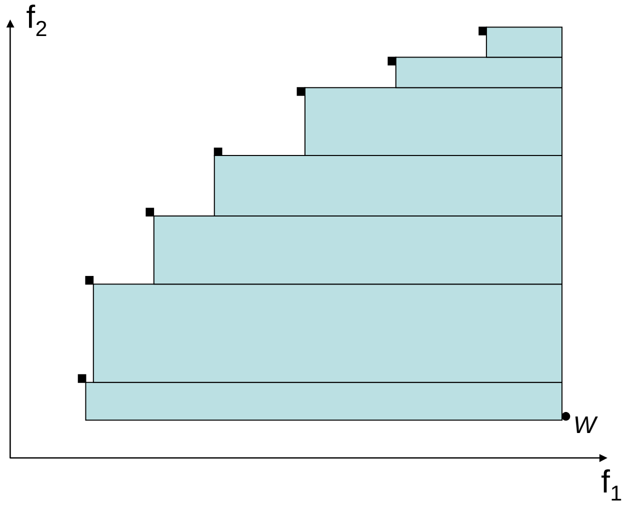

Figure 17 *Enclosed Area by NonDominated Solutions*

HVR measures the ratio of the region enclosed by the nondominated set, $HV_{Approximation}$, and the region enclosed by the true Pareto front, $HV_{Optimal}$. The Hypervolume and Hypervolume Ratio metrics depend on the selected reference point.

$$HVR = \frac{HV_{Approximation}}{HV_{Optimal}} \qquad (10)$$

In all runs some of the parameters are set equal. These are:

- Pattern is {1,2,1,2,1,2}
- Population and Archive Size are {50,50}

__Table 14__ Parameters in Problems

	Iteration size	*Mutation probability*
Parameter I	200	0.1
Parameter II	100	0.1
Parameter III	200	0.3
Parameter IV	100	0.3

After these settings are implemented, runs are done to choose one of the parameters. The HVR results of all these runs are depicted in Table 15.

__Table 15__ HVR Results

	Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	**Average**
Parameter Set I	0.954	0.841	0.869	0.923	0.925	**0.902**
Parameter Set II	0.883	0.835	0.846	0.863	0.908	**0.867**
Parameter Set III	0.911	0.917	0.871	0.926	0.932	**0.911**
Parameter Set IV	0.941	0.872	0.827	0.908	0.931	**0.896**

The average values of each run and parameter are calculated. As seen in Table 15, the best average value, 0.911, is obtained with the third parameter set. Thus, for small size problems, this parameter will be used in computational experiments. Iteration size is set to 200, and the mutation probability is set to 0.3 in the computational experiments. For big size problems, the iteration size is increased relatively. The HVR values obtained as the iteration size is increased for this problem set are depicted in Figure 18.

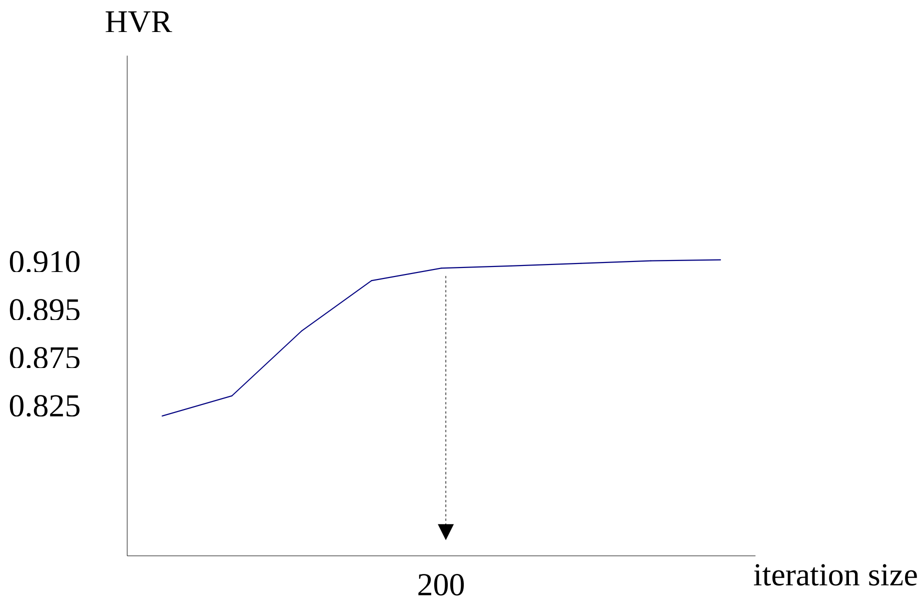

Figure 18 *HVR Values for Different Iteration Size*

5.4 COMPUTATIONAL RESULTS

5.4.1 PROBLEM GENERATION

To test the performance of the mSPEA-II for different size coverage problems, six problem sets are generated. In each problem set, there are ten randomly generated problems. We run each problem 5 times with different seeds. Number of demand and facility nodes changes between problem sets. For each problem set, iteration size is determined by taking into account the parameter setting values.

The problem set properties are given in Table 16. The parameters of the second problem set are determined according to the best results of the previous section. For the third, fourth, fifth and sixth problem sets, the iteration size is increased as the size of problems increases, and for the first problem set, iteration size is decreased. The comparisons are made according to these iteration sizes.

In Table 16, the region length value shows the side length of the plane where facility and demand nodes are generated. For instance, the x-y coordinates are generated from a uniform distribution between 0 and 200 in the first four problem sets. Demand amounts are generated from a uniform distribution between 0 and 500 for all problem sets. Full coverage and partial coverage distance are taken as 5% and 10% of the region size, respectively.

__Table 16__ Problem Sets Characteristics

	Problem Set 1	Problem Set 2	Problem Set 3	Problem Set 4	Problem Set 5	Problem Set 6
No. of Demand Nodes	100	200	250	500	1000	1500
No. of Facility Nodes	25	50	75	100	150	200
No. of Facility to be sited	3 and 5	5 and 7	5 and 7	5 and 7	5 and 7	5 and 7
Iteration Size	100	200	500	1000	1500	2000
Full Cov. Dis.	10	10	10	10	20	20
Partial Cov. Dis.	20	20	20	20	40	40
Region Length	200	200	200	200	400	400
Crossover Pattern	1,2,1,2,1,..	1,2,1,2,1,..	1,2,1,2,1,..	1,2,1,2,1,..	1,2,1,2,1,..	1,2,1,2,1,..
Mutation Prob.	0.3	0.3	0.3	0.3	0.3	0.3
Arch/Pop. Size	50	50	50	50	50	50
Demand Amount Interval	0-500	0-500	0-500	0-500	0-500	0-500

5.4.2 PERFORMANCE METRICS

To compare the performance of the algorithms, three metrics are used for the first, second, third and fourth test problems. These metrics are the Hyper Volume Ratio (HVR), Inverted Generational Distance (IGD) and Percentage of Found Solutions. For the fifth and sixth test problems, one additional metric, Set Coverage Metric (C(A,B)) is also used.

The Inverted Generational Distance (Bosman and Thiernes, 2003) calculates the average Euclidean distance between nondominated solutions and their closest true nondominated front member. Hence, an algorithm with a small IGD is better.

$$\text{IGD} = \frac{1}{|Q|}\sum_{i=1}^{Q}\left(\min_{j\in P}\left\|z_i - z_j\right\|_2\right) \qquad (11)$$

where Q denotes the nondominated set generated by GA and P denotes the true Pareto front set and $\left\|z_i - z_j\right\|_2$ represents the Euclidean distance between z_i and z_j.

The third metric that we use is the percentage of solutions generated by MOGA in the true Pareto front. Since, for the last three problem sets, we do not know the true Pareto front, the estimated Pareto front is created by combining the nondominated solutions of all algorithms. Thus, for the last three problem sets, this metric gives us the contribution of that algorithm when generating the estimated Pareto front.

Set coverage metric C (A, B), calculates the fraction of B dominated by A.

$$\text{C(A,B)} = \frac{\left|\{b \in B\}\middle|\exists a \in A : a \succ b\right|}{|B|} \qquad (12)$$

where A denotes Pareto Front generated by algorithm A and B denotes Pareto front generated by algorithm B. C (A, B) =1 means all members in Algorithm B’s Pareto front are dominated by the A algorithm. C (A, B) =0 represents the situation in which no individual in B is dominated by A. The domination operator is not a symmetric operator. Thus C (A, B) is not necessarily equal to 1- C (A, B).

The set coverage metric is used for only the fifth and sixth problem sets. Since we do not know the true Pareto front in these sets, this measure is used to compare the algorithms' Pareto optimal sets with each other.

5.4.3 RESULTS

Small size problems are solved by the ε-constraint method. The true Pareto optimal front is found for these problem sets. The first three problem sets can be solved exactly without violating the time limit. However, the other three problem sets cannot be solved within 120 minutes. (Actually they cannot be solved within 6 hours, but 120 minutes is taken as a constant reference). For medium and large size problems, all run results for the three EAs are combined and an estimated Pareto front is generated by combining the nondominated results of the three EAs.

For each problem and each p value (number of facilities to be opened), NSGA-II, SPEA-II and mSPEA-II are run five times using different seeds. For each run, the HVR, IGD and percentage of found solutions metrics are calculated. The average metric values and standard deviations of problems are given in Table 17. For each problem set, the average values of problems runs are given in Appendix C.

Table 17 *Average Metric Values*

		NSGA-II						SPEA-II						mSPEA-II					
		HVR		IGD		% of found sol.		HVR		IGD		% of found sol.		HVR		IGD		% of found sol.	
		mean	st.dev	mean	st.dev	mean	st.dev	mean	st.dev	mean	st.dev	mean	st.dev	mean	st.dev	mean	st.dev	mean	st.dev
P.Set 1	p=3	0.9895	0.0174	0.0004	0.0017	0.8826	0.1124	0.9898	0.0162	0.0014	0.0044	0.8586	0.1776	0.9936	0.0092	0.0003	0.0005	0.8862	0.0940
	p=5	0.9689	0.0369	0.0025	0.0043	0.6178	0.2965	0.9769	0.0267	0.0026	0.0042	0.6470	0.2274	0.9911	0.0135	0.0005	0.0008	0.8244	0.1462
P.Set 2	p=5	0.9746	0.0243	0.0035	0.0037	0.4402	0.3306	0.9752	0.0198	0.0029	0.0040	0.3760	0.2545	0.9876	0.0121	0.0008	0.0012	0.6996	0.1669
	p=7	0.9663	0.0260	0.0050	0.0053	0.2278	0.1885	0.9710	0.0219	0.0039	0.0042	0.2258	0.1603	0.9815	0.0135	0.0014	0.0016	0.4400	0.1716
P.Set 3	p=5	0.9608	0.0308	0.0040	0.0050	0.4326	0.2768	0.9725	0.0200	0.0017	0.0024	0.5402	0.2453	0.9831	0.0160	0.0013	0.0016	0.5936	0.1946
	p=7	0.9513	0.0300	0.0064	0.0067	0.2418	0.2190	0.9665	0.0174	0.0032	0.0042	0.3208	0.1628	0.9783	0.0154	0.0009	0.0015	0.5478	0.1445
P.Set 4	p=5	0.9596	0.0227	0.0027	0.0024	0.3084	0.2090	0.9762	0.0154	0.0032	0.0066	0.3874	0.1480	0.9842	0.0148	0.0007	0.0008	0.6312	0.1759
	p=7	0.9598	0.0215	0.0052	0.0041	0.1220	0.1506	0.9788	0.0132	0.0026	0.0025	0.3160	0.1800	0.9854	0.0090	0.0011	0.0010	0.6164	0.1368
P.Set 5	p=5	0.9473	0.0330	0.0030	0.0023	0.2328	0.1855	0.9649	0.0333	0.0014	0.0016	0.4150	0.2122	0.9741	0.0329	0.0008	0.0016	0.6820	0.1433
	p=7	0.9471	0.0224	0.0048	0.0035	0.0900	0.0911	0.9701	0.0234	0.0020	0.0022	0.2956	0.1838	0.9785	0.0183	0.0009	0.0010	0.5066	0.1574
P.Set 6	p=5	0.9632	0.0194	0.0034	0.0038	0.1924	0.1432	0.9831	0.0128	0.0018	0.0037	0.5272	0.2011	0.9876	0.0121	0.0005	0.0006	0.7252	0.1704
	p=7	0.9527	0.0234	0.0068	0.0054	0.0666	0.0938	0.9785	0.0132	0.0018	0.0016	0.2632	0.1425	0.9846	0.0106	0.0009	0.0008	0.4676	0.1675

The range of HVR values is roughly between 0.95 and 1. A point a little worse than the nadir point is selected as the reference point. The HVR value depends on the chosen reference point. For all problem sets, the average HVR values of mSPEA-II are greater than those of the other algorithms. Also, the standard deviations of the HVR values of mSPEA-II are less than those of the other algorithms. This means that mSPEA-II approximated the true Pareto front better than the others and the HVR values are more consistent than the others, due to the lower standard deviation. The IGD results are normalized values. To find the distances, scaling is performed since the range of objective functions differs greatly. The IGD average values of mSPEA-II are better than those of the others. This shows that the generated nondominated solutions of mSPEA-II are closer to the true Pareto front than those of the other algorithms. The third metric shows the percentage of found solutions. For small size problems, such as in problem set 1 and p=3, the values are close to each other for all algorithms. However, when the problem size increases, the difference is more obvious. For instance, on average, mSPEA-II found 54.78%, SPEA-II found 32.08% and NSGA-II found 24.18% of true Pareto front solutions. For problem set 4, 5 and 6, this metric gives the contribution of that algorithm to the estimated Pareto front, since the true Pareto fronts are unknown for these problem sets. In these problem sets, mSPEA-II still outperforms the other algorithms on average according to the number of found solution metrics.

An illustration of nondominated solutions found by the MOGAs and the generated true Pareto front are depicted in Figure 19. This figure is the result of the true Pareto front in the tenth problem of Problem Set 3. The number of facilities to be opened is five. The shown nondominated fronts are the first run results of MOGAs. Some solutions of the true Pareto front cannot be generated by any of the MOGAs.

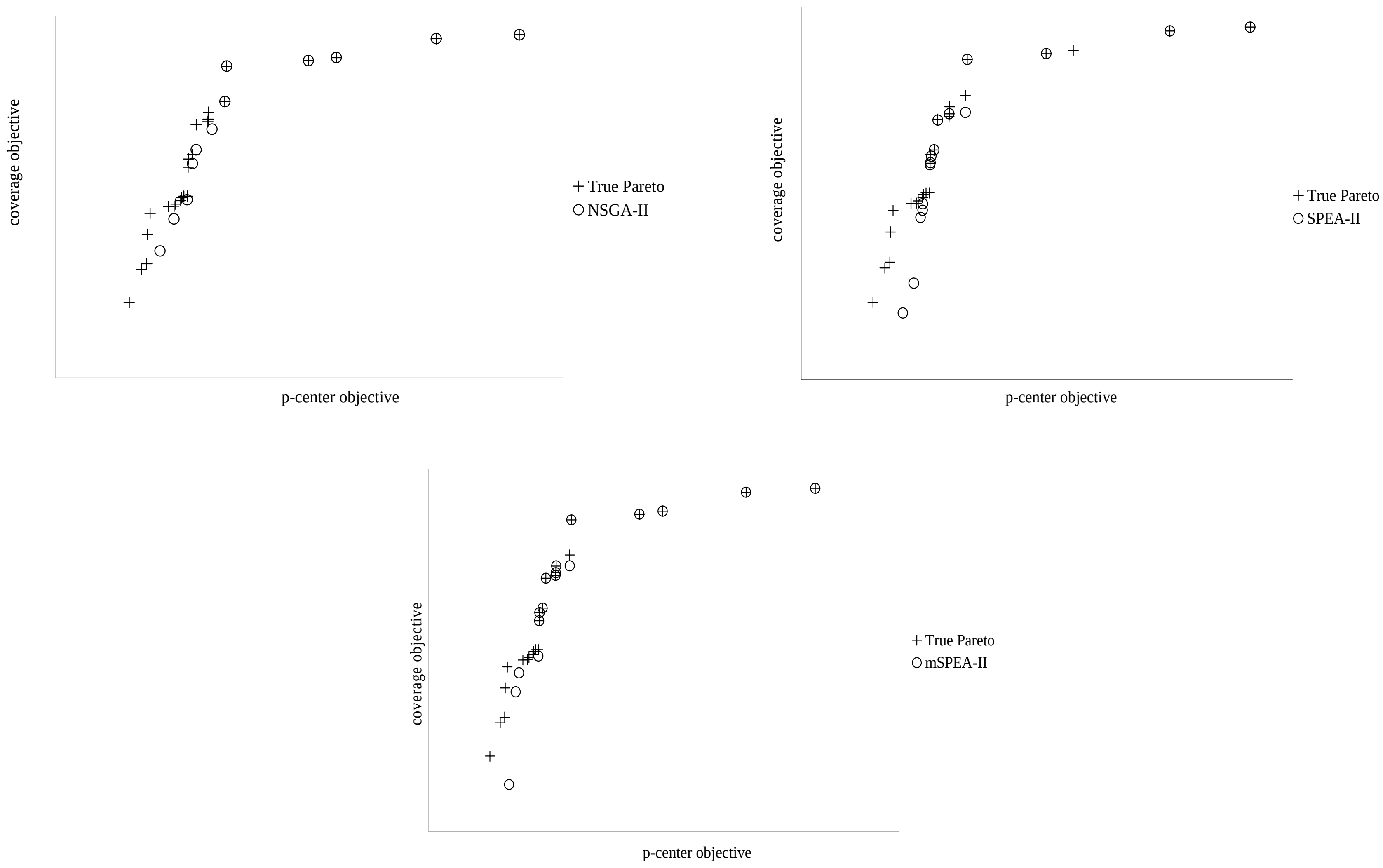

Figure 19 *Non-Dominated Fronts of Each Method*

The set coverage values for problem sets 5 and 6 are presented in Table 18.

Table 18 *Average Set Coverage Metric Values*

		P=5		
		mSPEA-II	NSGA-II	SPEA-II
P.Set 5	mSPEA-II	-	0.31	0.16
	NSGA-II	0.02	-	0.04
	SPEA-II	0.03	0.15	-
		mSPEA-II	NSGA-II	SPEA-II
P.Set 6	mSPEA-II	-	0.28	0.10
	NSGA-II	0.01	-	0.01
	SPEA-II	0.03	0.23	-

		P=7		
		mSPEA-II	NSGA-II	SPEA-II
P.Set 5	mSPEA-II	-	0.70	0.26
	NSGA-II	0.00	-	0.04
	SPEA-II	0.10	0.64	-
		mSPEA-II	NSGA-II	SPEA-II
P.Set 6	mSPEA-II	-	0.82	0.30
	NSGA-II	0.01	-	0.02
	SPEA-II	0.11	0.75	-

In the C (A, B) metric, “A” corresponds to a row of the table and “B” corresponds to a column of the table. This metric compares two algorithms. In problem set 5, while the number of selected facilities is 7 (p=7), mSPEA-II dominates more members in NSGA-II and SPEA-II compared to the case where the number of selected facilities is 5. This shows that if the string size increases, or the number of selected facilities increases, the performance of the algorithm improves. For instance, on average mSPEA-II dominates 82% of NSGA-II members and 30% of SPEA-II members in problem set 6 and p=7. Also SPEA-II dominates 75% of NSGA-II members on average in this problem set. For the same problem set NSGA-II dominates about 1% of mSPEA-II and 3% of SPEA-II. Computational experiments show that mSPEA-II outperforms the other algorithms and the second best performing algorithm is SPEA-II.

Figure 20 shows the estimated true Pareto Front of the fifth problem of Problem Set 4 . The number of facilities to be opened is seven (p=7). There are 14 nondominated solutions in the estimated true Pareto front. This front is created by combining all run results of the three MOGAs. mSPEA-II generates all of them, while SPEA-II generates ten of them and NSGA-II only four. The performance metric values are calculated by considering this estimated front.

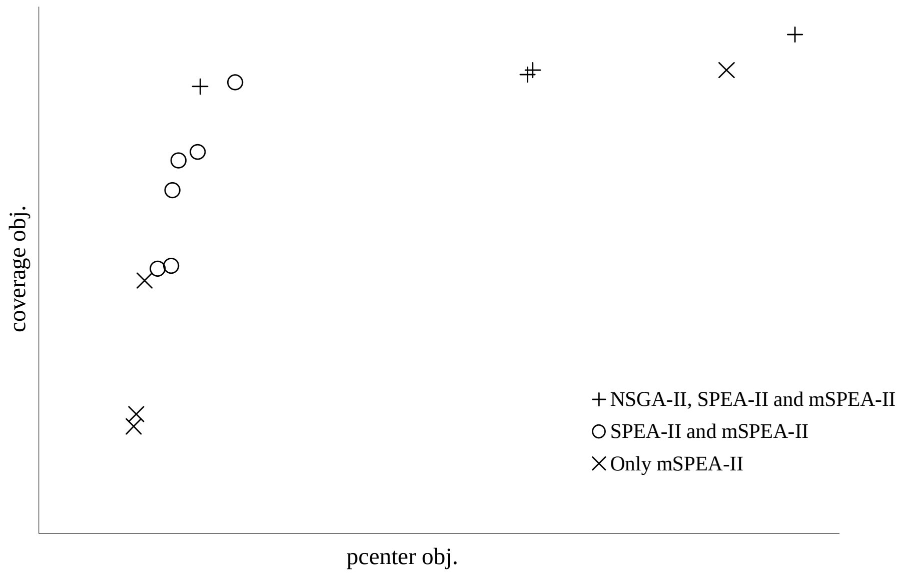

Figure 20 *Estimated Pareto Front of a Problem*

We test the following hypothesis at 99% significance level in order to check whether a significant difference exists between mSPEA-II and the other two algorithms. In a statistical test, only the Hypervolume metric is used. The Wilcoxon non-parametric statistical test is performed, and p-values are calculated to determine the better performing algorithm.

$$H_0 : \mu_{hvr}^{P} = \mu_{hvr}^{O}$$
$$H_1 : \mu_{hvr}^{P} \neq \mu_{hvr}^{O}$$

where *P* corresponds to the proposed algorithm (mSPEA-II) and *O* refers to the other algorithm.

μ_{hvr}^{P} is the mean of mSPEA-II and μ_{hvr}^{O} is the mean of NSGA-II or SPEA-II .

Table 19 *Wilcoxon Test Results of HVR*

		Competitor	p-value	Winner
Problem Set 1	p=3	NSGA-II	0.236	None
		SPEA-II	0.437	None
	p=5	NSGA-II	0	mSPEA-II
		SPEA-II	0	mSPEA-II
Problem Set 2	p=5	NSGA-II	0	mSPEA-II
		SPEA-II	0	mSPEA-II
	p=7	NSGA-II	0.001	mSPEA-II
		SPEA-II	0.001	mSPEA-II
Problem Set 3	p=5	NSGA-II	0	mSPEA-II
		SPEA-II	0	mSPEA-II
	p=7	NSGA-II	0	mSPEA-II
		SPEA-II	0	mSPEA-II
Problem Set 4	p=5	NSGA-II	0	mSPEA-II
		SPEA-II	0.002	mSPEA-II
	p=7	NSGA-II	0	mSPEA-II
		SPEA-II	0.003	mSPEA-II
Problem Set 5	p=5	NSGA-II	0	mSPEA-II
		SPEA-II	0	mSPEA-II
	p=7	NSGA-II	0	mSPEA-II
		SPEA-II	0	mSPEA-II
Problem Set 6	p=5	NSGA-II	0	mSPEA-II
		SPEA-II	0.004	mSPEA-II
	p=7	NSGA-II	0	mSPEA-II
		SPEA-II	0.002	mSPEA-II

Table 19 shows the p-values. For all problem sets except problem set 1, the p value is less than 0.005. Only in problem set 1 (p=3) is there not a statistically significant difference. mSPEA-II performs significantly better than the other algorithms in all problem types. Actually, the size of problem set 1 is very small, and in the predetermined iteration size, the results of all the algorithms are close to each other. The average values of mSPEA-II are slightly better than those of the other algorithms, but statistically we could not say there is a winner.

Table 20 shows the run times of the algorithms for each problem set and the number of facilities to be opened. For problem sets 4, 5 and 6, the true Pareto fronts cannot be generated in a reasonable time. For each problem set, mSPEA-II and NSGA-II run times are fairly close to each other. SPEA-II run times are greater than those of the other two algorithms for all problems.

Table 20 *Average Run Times of Algorithms(sec)*

		True Pareto Front	NSGA-II	SPEA-II	mSPEA-II
P.Set 1	p=3	54.7	1	1	1
	p=5	49.9	1	1	1
P.Set 2	p=5	494.5	2	3	2
	p=7	670.6	2	3	2
P.Set 3	p=5	2827.7	6	10	6
	p=7	2037.4	8	13	8
P.Set 4	p=5	-	18	26	18
	p=7	-	24	32	24
P.Set 5	p=5	-	44	53	42
	p=7	-	53	64	51
P.Set 6	p=5	-	83	100	77
	p=7	-	108	124	107

We choose archive size as 50 for each problem and for each MOGAs. In all problem sets, the generated non-dominted solutions are less than 50 for each MOGAs. Thus in SPEA-II, the truncation operation according to density fitness value is not used in these problems. Thus, the computational complexity of SPEA-II is not significantly higher than NSGA-II and mSPEA-II as it expected. Only the distance calculation of each solution increases the SPEA-II run times as compared to NSGA-II and mSPEA-II.

CHAPTER 6

CONCLUSIONS AND FURTHER RESEARCH

MCLP is one of the basic models in the area of facility location, and many studies and extensions have been done based on this model. In this study, we formulated four bi-objective coverage models that take into account partial coverage and service to uncovered demands. We discussed the weak and strong points of the models. We argued that the model that maximizes coverage, including partial coverage, and minimizes the maximum distance between uncovered demands and their nearest facilities is the most reasonable one for emergency service location problems where all demand nodes are not covered by facilities.

We proposed a MOGA, called mSPEA-II, to solve our model. In this algorithm, we modify the fitness assignment of SPEA-II and use the crowding distance calculation of NSGA-II in order to eliminate the drawbacks of NSGA-II and SPEA-II in reaching the true Pareto front. We experiment with several parameters of the mSPEA-II to improve its performance. We tested the performance of mSPEA-II on randomly generated problems and compared the results with those of NSGA-II and SPEA-II. Our experimental results show that the mSPEA-II algorithm performs better than the others. For small problems, the results of these MOGAs are close to each other, but on average mSPEA-II performs better. Also, the run time performance of mSPEA-II is better than that of SPEA-II, and approximately the same as that of NSGA-II. This is due to the decrease in distance calculation and the elimination of the truncation operator of SPEA-II.

For future work, this bi-objective model can be extended to handle capacity restrictions on the facilities. The bi-objective location coverage model with capacitated facilities would be more reasonable for real life applications.

We used MOGAs to generate the true Pareto front. Other meta-heuristics such as simulated annealing and Tabu search can be applied to solve the model. Another research direction may be to develop a preference based MOGA to find the best solution for the DM.

Another future research might be implementation of mSPEA-II to other models. One could try to test performance of the mSPEA-II for multi-objective problems that have more than two objectives.

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APPENDIX A

FOUR MODELS RESULTS

Table 21 *Model-1 Results*

		MODEL 1					
		tot. cov.	**fu. cov.**	**par.cov**	**p-med**	**p-cen**	**no.sol.**
P.1	**Min**	3716	2342	1142	164422	63.4	3
	Average	3816	2530.7	1285.3	179078	67.4	
	Max	3876	2734	1374	195598	69.4	
P.2	**Min**	3630	2530	1100	154179	53.9	2
	Average	3634.5	2534	1100.5	154505.5	53.9	
	Max	3639	2538	1101	154832	53.9	
P.3	**Min**	3335	2231	796	147815	50.1	5
	Average	3443.8	2428.6	1015.2	190850.2	65.6	
	Max	3621	2539	1390	245494	87.0	
P.4	**Min**	2928	1319	1120	194089	61.6	4
	Average	3282.3	1942	1340.3	225736.8	67.8	
	Max	3517	2225	1609	257808	73.9	
P.5	**Min**	3722	2416	1226	174386	51.1	2
	Average	3791.5	2456	1335.5	202431	60.8	
	Max	3861	2496	1445	230476	70.5	
P.6	**Min**	3606	2411	941	195785	67.3	3
	Average	3763.7	2505	1258.7	215945	70.6	
	Max	3921	2665	1482	252986	73.6	
P.7	**Min**	3981	2703	1278	170725	57.4	2
	Average	3994	2712.5	1281.5	171531	57.4	
	Max	4007	2722	1285	172337	57.4	
P.8	**Min**	3444	1726	1087	154571	55.6	5
	Average	3566.4	2062	1504.4	165534.8	61.1	
	Max	3671	2449	1894	175683	73.9	
P.9	**Min**	2724	1701	804	241078	69.1	6
	Average	2879.7	1861.3	1018.3	277508.5	81.1	
	Max	3030	1920	1251	316838	92.8	
P.10	**Min**	2995	2034	865	175303	53.9	4
	Average	3111	2088.5	1022.5	199663.5	59.3	
	Max	3280	2130	1246	221095	66.6	

AVG	**3528.3**	**2312.1**	**1216.2**	**198278.4**	**64.5**	**3.6**

Table 22 Model-2 Results

		MODEL 2					
		tot. cov.	**fu. cov.**	**par.cov**	**p-med**	**p-cen**	**no.sol.**
P.1	**Min**	3613	2161	1142	156868	54.8	4
	Average	3810	2437.5	1372.5	177995	60.6	
	Max	4020	2734	1859	195598	69.4	
P.2	**Min**	3639	2538	1101	154832	53.9	1
	Average	3639	2538	1101	154832	53.9	
	Max	3639	2538	1101	154832	53.9	
P.3	**Min**	3146	2417	710	161726	50.1	5
	Average	3348.8	2469.6	879.2	207365	69.4	
	Max	3464	2539	1047	245494	87	
P.4	**Min**	2551	1752	751	224610	55.6	5
	Average	2995.4	2000.4	995	264262.2	65.16	
	Max	3345	2225	1272	295146	73.9	
P.5	**Min**	3263	1790	1226	174386	48.3	3
	Average	3615.3	2234	1381.3	201668.7	56.6	
	Max	3861	2496	1473	230476	70.5	
P.6	**Min**	3606	2411	941	195785	67.3	4
	Average	3723.5	2527.5	1196	236667	70.1	
	Max	3841	2665	1353	252986	73.6	
P.7	**Min**	3691	2317	1285	172337	56.7	2
	Average	3849	2519.5	1329.5	178030	57.1	
	Max	4007	2722	1374	183723	57.4	
P.8	**Min**	3318	1931	1087	167307	55.8	5
	Average	3475.6	2173.2	1302.4	171905	62.1	
	Max	3671	2449	1512	176461	73.9	
P.9	**Min**	2724	1712	804	253946	69.1	5
	Average	2840	1824.4	1015.6	278360	78.5	
	Max	2981	1920	1191	316838	92.8	
P.10	**Min**	2600	1598	865	185306	52.7	4
	Average	2941	1979.5	961.5	210279	57.3	
	Max	3159	2130	1072	221095	66.6	

AVG	**3423.8**	**2270.4**	**1153.4**	**208136.4**	**63.1**	**3.8**

Table 23 Model-3 Results

		MODEL 3					
		tot. cov.	fu. cov.	par.cov	p-med	p-cen	no.sol.
P.1	Min	3679	1951	1728	146141	54.76	5
	Average	3980	2180	1800	157832	60.87	
	Max	4199	2463	1908	169019	70.06	
P.2	Min	3539	1913	1100	149729	53.86	6
	Average	3706.8	2243.8	1463	168884.2	63.7	
	Max	3942	2538	1871	202069	82.16	
P.3	Min	3621	2231	2924	147815	50	1
	Average	3621	2231	2924	147815	50	
	Max	3621	2231	2924	147815	50	
P.4	Min	2928	1320	1185	194089	61.6	3
	Average	3261.3	1877	1384.3	215046.3	65.7	
	Max	3517	2157	1608	226440	73.9	
P.5	Min	3666	2293	1373	173777	51.1	2
	Average	3763.5	2354.5	1409	174081.5	51.1	
	Max	3861	2416	1445	174386	51.1	
P.6	Min	3764	2415	1482	195785	67.3	2
	Average	3842.5	2427	2226.5	197424.5	69.1	
	Max	3921	2439	2971	199064	70.8	
P.7	Min	3981	2703	1278	170725	57.4	2
	Average	3994	2712.5	1281.5	171531	57.4	
	Max	4007	2722	1285	172337	57.4	
P.8	Min	3620	1726	1550	154571	56.6	2
	Average	3708	1986	1722	161831	65.2	
	Max	3796	2246	1894	169091	73.9	
P.9	Min	2952	1694	1256	241078	69.7	2
	Average	2991	1695	1296	244208	71	
	Max	3030	1696	1336	247338	72.3	
P.10	Min	3102	1735	1185	173880	60.8	3
	Average	3185.7	1895.3	1290.3	174419	60.9	
	Max	3280	2034	1440	175303	61.1	
	AVG	**3605.4**	**2160.2**	**1679.7**	**181307.3**	**61.5**	**2.8**

***Table 24** Model-4 Results*

		MODEL 4					
		tot. cov.	**fu. cov.**	**par.cov**	**p-med**	**p-cen**	**no.sol.**
P.1	**Min**	4020	2161	1736	156868	54.8	2
	Average	4109.5	2312	1797.5	162943.5	62.1	
	Max	4199	2463	1859	169019	69.4	
P.2	**Min**	3639	1972	1101	154832	53.9	3
	Average	3808	2344	1464	178255.3	68.4	
	Max	3942	2538	1871	202069	82.2	
P.3	**Min**	3621	2231	2924	147815	50	1
	Average	3621	2231	2924	147815	50	
	Max	3621	2231	2924	147815	50	
P.4	**Min**	2551	1004	1185	224610	56	4
	Average	2995.3	1658.8	1336.5	255088.8	63.2	
	Max	3517	2157	1547	295146	73.9	
P.5	**Min**	3263	1790	1445	174386	48.3	2
	Average	3562	2103	1758	187265	49.7	
	Max	3861	2416	2071	200144	51.1	
P.6	**Min**	3764	2415	1482	195785	67.3	2
	Average	3842.5	2427	2226.5	197424.5	69.1	
	Max	3921	2439	2971	199064	70.8	
P.7	**Min**	3691	2317	1285	172337	56.7	2
	Average	3849	2519.5	1329.5	178030	57.1	
	Max	4007	2722	1374	183723	57.4	
P.8	**Min**	3620	1726	1550	154571	56.6	2
	Average	3708	1986	1722	161831	65.2	
	Max	3796	2246	1894	169091	73.9	
P.9	**Min**	2952	1694	1256	241078	69	2
	Average	2991	1695	1296	244208	70.7	
	Max	3030	1696	1336	247338	72.3	
P.10	**Min**	2600	1598	1002	175303	52.7	3
	Average	3013	1904.3	1108.7	192791.3	55.8	
	Max	3280	2081	1246	217765	60.8	
	AVG	**3549.9**	**2118.1**	**1696.3**	**190565.2**	**61.1**	**2.3**

APPENDIX B

THE PSEUDO CODE OF mSPEA-II

```
Load Demands Coordinates
    for i to number of demand node, repeat
            take the coordinates[x,y]

Load Facilities Cordinates
    for i to number of facility node, repeat
            take the coordinates[x,y]

Initialize DemandFacility Distance_Matrix
    for i=0 to DF_Matrix.length, repeat
            for j=0 to DF_Matrix[i].length, repeat
                    calculate distance [i,j]

Generate Initial Population
    for i=1 to population size, repeat
            for j=0 to No.of facilites to be opened, repeat
                    generate distinct opened facility ID←random
                    uniform[0,facility size]
next population← initial population

for i=1 to iteration_size, repeat
    Calculate Objective Values of next_population
            for i=1 to population size, repeat
                    calculate totalcoverage objective
                    calculate pcenter objective

Unite next_population and archive population
            generate distinct list
                    remove same members
                    set united_list
Copy nondominated solutions of united_list to archive_list

    if archivelist_size= predefined size
        set archivelist

    else if archivelist_size<predefined size
        Calculate F1 fitness
                    for i=1 to united_list size, repeat
                            calculate strength and raw fitness
                            calculate D(i)
       fill dominated solutions(F1>1) up to predefined size
```

```
else
      Calculate F2 fitness
                    for i=1 to united_list size, repeat
                          calculate strength and raw fitness
                          calculate minO(i)

      apply truncation according to F2 values.


Fill the mating pool
             case where the actual archive size less than predefined size
                    for i=1 to pop.size, repeat
                       tournament selection according to F1 values
             case where the actual archive size greater than predefined size
                    for i=1 to pop.size, repeat
                       tournament selection according to F2 values

Perform crossover
             for i=0 to pop.size
                    select parent 1 and 2
                    use pattern
                    generate two children
Perfom mutation
             for i=0 to offspring/pop size
                    choose a gene
                    change facility ID←random, with mut.prob.
Repair the members
             for i=0 to pop.size
                    for j=0 to string_size
                           find the same facility ID
                           replace with different ID

Finalize
             set newpopulation ←offspring
```

APPENDIX C

RUN RESULTS OF PROBLEM SETS

Table 25 *Problem Set-1 Results*

	P=3								
	NSGA-II			SPEA-II			Proposed		
	HVR	IGD	No.of.sol	HVR	IGD	No.of.sol	HVR	IGD	No.of.sol
P.1	0.9944	0.0005	85.71%	0.9910	0.0006	80.00%	0.9902	0.0011	85.71%
P.2	0.9989	0.0001	98.00%	0.9988	0.0002	98.00%	0.9970	0.0001	96.00%
P.3	1.0000	0.0000	100.00%	1.0000	0.0000	100.00%	1.0000	0.0000	100.00%
P.4	0.9919	0.0000	87.27%	0.9887	0.0006	76.36%	0.9940	0.0002	89.09%
P.5	0.9795	0.0000	85.71%	0.9889	0.0076	88.57%	0.9884	0.0001	85.71%
P.6	0.9961	0.0002	85.00%	0.9997	0.0001	90.00%	0.9930	0.0000	80.00%
P.7	0.9873	0.0004	86.31%	0.9737	0.0002	91.28%	0.9953	0.0000	87.80%
P.8	0.9702	0.0026	81.36%	0.9767	0.0038	65.53%	0.9914	0.0000	86.16%
P.9	0.9906	0.0002	84.52%	0.9858	0.0007	84.13%	0.9926	0.0005	90.75%
P.10	0.9865	0.0002	88.36%	0.9913	0.0004	84.81%	0.9942	0.0005	84.59%
AVG	0.9895	0.0004	88.22%	0.9894	0.0014	85.87%	0.9936	0.0003	88.58%

	P=5								
	NSGA-II			SPEA-II			Proposed		
	HVR	IGD	No.of.sol	HVR	IGD	No.of.sol	HVR	IGD	No.of.sol
P.1	0.9996	0.0001	94.29%	0.9800	0.0002	82.86%	0.9902	0.0001	94.29%
P.2	0.9626	0.0015	61.67%	0.9541	0.0020	63.33%	0.9816	0.0009	80.00%
P.3	0.9890	0.0016	75.00%	0.9936	0.0029	72.50%	0.9984	0.0002	77.50%
P.4	0.9860	0.0009	77.14%	0.9895	0.0034	60.00%	0.9874	0.0018	68.57%
P.5	0.9035	0.0071	10.00%	0.9511	0.0049	50.00%	1.0000	0.0000	100.00%
P.6	0.9760	0.0021	41.43%	0.9839	0.0007	58.57%	0.9794	0.0004	65.71%
P.7	0.9604	0.0041	54.94%	0.9741	0.0058	50.73%	0.9931	0.0010	72.36%
P.8	0.9725	0.0037	68.51%	0.9767	0.0009	77.87%	0.9960	0.0000	96.36%
P.9	0.9777	0.0002	74.94%	0.9848	0.0006	78.20%	0.9878	0.0002	82.46%
P.10	0.9614	0.0039	59.54%	0.9811	0.0042	52.57%	0.9972	0.0000	87.43%
AVG	0.9689	0.0025	61.74%	0.9769	0.0026	64.66%	0.9911	0.0005	82.47%

Table 26 *Problem Set-2 Results*

	P=5								
	NSGA-II			SPEA-II			Proposed		
	HVR	IGD	No.of.sol	HVR	IGD	No.of.sol	HVR	IGD	No.of.sol
P.1	0.9952	0.0011	71.43%	0.9786	0.0021	25.71%	0.9903	0.0005	65.71%
P.2	0.9326	0.0049	13.33%	0.9521	0.0038	33.33%	0.9782	0.0023	62.67%
P.3	0.9687	0.0040	13.85%	0.9884	0.0021	33.85%	0.9964	0.0007	60.00%
P.4	0.9926	0.0059	85.71%	0.9746	0.0015	68.57%	1.0000	0.0000	100.00%
P.5	0.9878	0.0031	26.15%	0.9850	0.0005	44.62%	0.9888	0.0000	78.46%
P.6	0.9794	0.0044	56.67%	0.9506	0.0091	23.33%	0.9747	0.0004	66.67%
P.7	0.9801	0.0022	56.52%	0.9752	0.0016	43.74%	0.9892	0.0005	75.15%
P.8	0.9547	0.0054	16.54%	0.9850	0.0025	31.38%	0.9830	0.0012	54.97%
P.9	0.9764	0.0019	45.97%	0.9858	0.0017	27.97%	0.9897	0.0014	65.63%
P.10	0.9781	0.0023	53.98%	0.9768	0.0045	43.35%	0.9857	0.0006	69.91%
AVG	0.9746	0.0035	44.02%	0.9752	0.0029	37.58%	0.9876	0.0008	69.92%

	P=7								
	NSGA-II			SPEA-II			Proposed		
	HVR	IGD	No.of.sol	HVR	IGD	No.of.sol	HVR	IGD	No.of.sol
P.1	0.9527	0.0131	10.00%	0.9560	0.0083	20.00%	0.9893	0.0032	32.50%
P.2	0.9551	0.0047	24.71%	0.9731	0.0049	18.82%	0.9736	0.0021	49.41%
P.3	0.9536	0.0055	11.11%	0.9701	0.0030	16.67%	0.9874	0.0003	45.56%
P.4	0.9726	0.0021	28.33%	0.9587	0.0068	11.67%	0.9663	0.0023	31.67%
P.5	0.9675	0.0052	7.69%	0.9824	0.0014	18.46%	0.9989	0.0004	58.46%
P.6	0.9846	0.0017	24.29%	0.9740	0.0035	11.43%	0.9902	0.0013	32.86%
P.7	0.9562	0.0061	17.25%	0.9795	0.0031	30.88%	0.9743	0.0026	53.94%
P.8	0.9738	0.0016	42.79%	0.9771	0.0030	43.68%	0.9780	0.0007	43.35%
P.9	0.9655	0.0066	31.80%	0.9650	0.0024	24.14%	0.9752	0.0008	37.90%
P.10	0.9812	0.0038	30.10%	0.9739	0.0019	30.28%	0.9823	0.0006	54.45%
AVG	0.9663	0.0050	22.81%	0.9710	0.0039	22.60%	0.9815	0.0014	44.01%

Table 27 *Problem Set-3 Results*

	P=5								
	NSGA-II			SPEA-II			Proposed		
	HVR	IGD	No.of.sol	HVR	IGD	No.of.sol	HVR	IGD	No.of.sol
P.1	0.9591	0.0020	26.15%	0.9530	0.0031	29.23%	0.9724	0.0018	36.92%
P.2	0.9837	0.0018	44.62%	0.9857	0.0011	46.15%	0.9908	0.0009	61.54%
P.3	0.9961	0.0001	83.08%	0.9984	0.0002	84.62%	0.9980	0.0015	83.08%
P.4	0.9414	0.0040	16.00%	0.9667	0.0014	36.80%	0.9769	0.0008	50.40%
P.5	0.9336	0.0066	25.33%	0.9599	0.0020	66.67%	0.9698	0.0000	73.33%
P.6	0.9976	0.0008	82.50%	0.9986	0.0011	95.00%	0.9967	0.0024	80.00%
P.7	0.9241	0.0104	27.28%	0.9616	0.0020	44.48%	0.9855	0.0010	47.25%
P.8	0.9781	0.0030	71.56%	0.9815	0.0010	55.82%	0.9926	0.0010	68.33%
P.9	0.9678	0.0036	35.07%	0.9731	0.0011	42.53%	0.9832	0.0014	49.27%
P.10	0.9263	0.0082	20.45%	0.9464	0.0044	39.34%	0.9652	0.0010	42.67%
AVG	0.9608	0.0040	43.20%	0.9725	0.0017	54.06%	0.9831	0.0012	59.28%

	P=7								
	NSGA-II			SPEA-II			Proposed		
	HVR	IGD	No.of.sol	HVR	IGD	No.of.sol	HVR	IGD	No.of.sol
P.1	0.9468	0.0027	24.62%	0.9613	0.0038	23.08%	0.9499	0.0022	44.62%
P.2	0.9344	0.0141	10.91%	0.9641	0.0060	38.18%	0.9800	0.0006	54.55%
P.3	0.9871	0.0004	61.43%	0.9870	0.0019	50.00%	0.9890	0.0007	65.71%
P.4	0.9358	0.0095	12.38%	0.9507	0.0026	33.33%	0.9670	0.0010	42.86%
P.5	0.9269	0.0078	21.25%	0.9711	0.0036	35.00%	0.9855	0.0004	71.25%
P.6	0.9626	0.0039	18.75%	0.9605	0.0046	26.25%	0.9842	0.0015	47.50%
P.7	0.9448	0.0090	11.70%	0.9485	0.0031	22.03%	0.9699	0.0014	42.61%
P.8	0.9695	0.0040	35.34%	0.9764	0.0020	29.65%	0.9893	0.0003	59.33%
P.9	0.9378	0.0087	13.85%	0.9699	0.0024	25.84%	0.9838	0.0011	56.89%
P.10	0.9672	0.0036	31.62%	0.9756	0.0024	38.58%	0.9841	0.0000	62.17%
AVG	0.9513	0.0064	24.18%	0.9665	0.0032	32.20%	0.9783	0.0009	54.75%

Table 28 *Problem Set-4 Results*

	P=5								
	NSGA-II			**SPEA-II**			**Proposed**		
	HVR	**IGD**	**No.of.sol**	**HVR**	**IGD**	**No.of.sol**	**HVR**	**IGD**	**No.of.sol**
P.1	0.9695	0.0019	41.43%	0.9934	0.0010	54.29%	0.9699	0.0004	85.71%
P.2	0.9637	0.0041	27.27%	0.9816	0.0004	47.27%	0.9808	0.0001	58.18%
P.3	0.9552	0.0012	14.78%	0.9753	0.0006	32.17%	0.9807	0.0008	40.87%
P.4	0.9833	0.0012	60.00%	0.9679	0.0159	32.00%	0.9939	0.0008	72.00%
P.5	0.9620	0.0031	27.27%	0.9757	0.0015	36.36%	0.9930	0.0000	76.36%
P.6	0.9679	0.0035	26.25%	0.9841	0.0017	48.75%	0.9943	0.0015	67.50%
P.7	0.9452	0.0055	36.45%	0.9776	0.0025	31.98%	0.9783	0.0006	47.27%
P.8	0.9683	0.0025	35.17%	0.9859	0.0013	38.79%	0.9908	0.0007	68.52%
P.9	0.9443	0.0010	26.46%	0.9608	0.0018	33.81%	0.9776	0.0011	54.04%
P.10	0.9366	0.0026	13.20%	0.9593	0.0057	32.56%	0.9825	0.0006	60.09%
AVG	0.9596	0.0027	30.83%	0.9762	0.0032	38.80%	0.9842	0.0007	63.05%

	P=7								
	NSGA-II			**SPEA-II**			**Proposed**		
	HVR	**IGD**	**No.of.sol**	**HVR**	**IGD**	**No.of.sol**	**HVR**	**IGD**	**No.of.sol**
P.1	0.9808	0.0030	5.71%	0.9890	0.0007	34.29%	0.9909	0.0005	72.86%
P.2	0.9575	0.0088	14.29%	0.9745	0.0025	35.71%	0.9855	0.0002	70.00%
P.3	0.9564	0.0049	8.33%	0.9874	0.0026	19.17%	0.9872	0.0014	53.33%
P.4	0.9500	0.0087	6.15%	0.9778	0.0024	29.23%	0.9796	0.0020	50.77%
P.5	0.9633	0.0053	8.89%	0.9863	0.0036	35.56%	0.9761	0.0016	73.33%
P.6	0.9508	0.0051	20.00%	0.9801	0.0036	28.18%	0.9852	0.0019	55.45%
P.7	0.9447	0.0048	12.84%	0.9653	0.0029	26.84%	0.9774	0.0010	41.07%
P.8	0.9642	0.0054	6.23%	0.9780	0.0010	32.32%	0.9942	0.0005	75.01%
P.9	0.9646	0.0032	16.73%	0.9862	0.0021	35.29%	0.9922	0.0013	56.28%
P.10	0.9656	0.0025	22.25%	0.9633	0.0047	39.50%	0.9860	0.0010	67.75%
AVG	0.9598	0.0052	12.14%	0.9788	0.0026	31.61%	0.9854	0.0011	61.59%

__Table 29__ Problem Set-5 Results

	P=5								
	NSGA-II			SPEA-II			Proposed		
	HVR	IGD	No.of.sol	HVR	IGD	No.of.sol	HVR	IGD	No.of.sol
P.1	0.9469	0.0033	15.65%	0.9694	0.0004	37.39%	0.9915	0.0001	80.00%
P.2	0.9770	0.0013	40.00%	0.9754	0.0012	52.31%	0.9810	0.0002	75.38%
P.3	0.9781	0.0037	14.74%	0.9898	0.0011	4.21%	0.9970	0.0002	77.89%
P.4	0.8763	0.0023	45.71%	0.8750	0.0014	52.86%	0.8834	0.0006	62.86%
P.5	0.9766	0.0012	40.00%	0.9870	0.0010	66.15%	0.9757	0.0000	73.85%
P.6	0.9410	0.0031	5.00%	0.9812	0.0019	47.50%	0.9811	0.0004	61.25%
P.7	0.9353	0.0065	11.32%	0.9646	0.0020	23.52%	0.9756	0.0013	43.41%
P.8	0.9357	0.0033	9.57%	0.9722	0.0009	45.87%	0.9887	0.0001	70.60%
P.9	0.9590	0.0026	36.39%	0.9750	0.0008	52.37%	0.9774	0.0015	75.52%
P.10	0.9472	0.0025	14.18%	0.9594	0.0032	32.64%	0.9891	0.0008	60.10%
AVG	0.9473	0.0030	23.26%	0.9649	0.0014	41.48%	0.9741	0.0005	68.09%

	P=7								
	NSGA-II			SPEA-II			Proposed		
	HVR	IGD	No.of.sol	HVR	IGD	No.of.sol	HVR	IGD	No.of.sol
P.1	0.9676	0.0039	16.80%	0.9875	0.0010	50.40%	0.9933	0.0002	69.60%
P.2	0.9455	0.0069	4.71%	0.9552	0.0027	11.76%	0.9721	0.0008	56.47%
P.3	0.9446	0.0068	3.81%	0.9623	0.0018	14.29%	0.9775	0.0011	42.86%
P.4	0.9423	0.0064	6.15%	0.9785	0.0019	19.23%	0.9761	0.0014	34.62%
P.5	0.9625	0.0021	10.00%	0.9774	0.0005	45.71%	0.9811	0.0001	61.43%
P.6	0.9380	0.0045	3.81%	0.9768	0.0017	25.71%	0.9776	0.0008	43.81%
P.7	0.9320	0.0029	15.29%	0.9480	0.0042	18.87%	0.9589	0.0012	42.52%
P.8	0.9679	0.0031	10.69%	0.9793	0.0013	45.26%	0.9926	0.0003	66.84%
P.9	0.9390	0.0065	12.28%	0.9658	0.0030	27.22%	0.9743	0.0020	45.96%
P.10	0.9318	0.0054	5.57%	0.9701	0.0020	37.23%	0.9813	0.0011	41.89%
AVG	0.9471	0.0048	8.91%	0.9701	0.0020	29.57%	0.9785	0.0009	50.60%

Table 30 *Problem Set-6 Results*

	P=5								
	NSGA-II			SPEA-II			Proposed		
	HVR	IGD	No.of.sol	HVR	IGD	No.of.sol	HVR	IGD	No.of.sol
P.1	0.9773	0.0027	20.00%	0.9922	0.0008	52.31%	0.9980	0.0000	92.31%
P.2	0.9858	0.0011	30.67%	0.9975	0.0003	82.67%	0.9889	0.0007	74.67%
P.3	0.9578	0.0036	11.85%	0.9770	0.0016	40.74%	0.9866	0.0003	60.74%
P.4	0.9564	0.0012	31.43%	0.9865	0.0006	73.33%	0.9704	0.0001	76.19%
P.5	0.9702	0.0073	16.47%	0.9909	0.0067	47.06%	0.9993	0.0000	77.65%
P.6	0.9659	0.0020	12.94%	0.9800	0.0005	45.29%	0.9795	0.0007	64.71%
P.7	0.9400	0.0064	6.24%	0.9681	0.0009	33.64%	0.9840	0.0015	38.03%
P.8	0.9588	0.0023	13.65%	0.9822	0.0043	72.60%	0.9945	0.0002	88.39%
P.9	0.9579	0.0041	19.89%	0.9767	0.0012	43.84%	0.9843	0.0008	72.31%
P.10	0.9619	0.0029	27.74%	0.9803	0.0010	36.51%	0.9904	0.0003	79.81%
AVG	0.9632	0.0034	19.09%	0.9831	0.0018	52.80%	0.9876	0.0005	72.48%

	P=7								
	NSGA-II			SPEA-II			Proposed		
	HVR	IGD	No.of.sol	HVR	IGD	No.of.sol	HVR	IGD	No.of.sol
P.1	0.9496	0.0041	13.33%	0.9854	0.0016	38.67%	0.9923	0.0006	53.33%
P.2	0.9763	0.0046	6.00%	0.9932	0.0012	27.00%	0.9951	0.0006	53.00%
P.3	0.9566	0.0063	6.36%	0.9787	0.0023	21.82%	0.9837	0.0016	37.27%
P.4	0.9473	0.0114	5.00%	0.9858	0.0025	25.00%	0.9727	0.0004	70.00%
P.5	0.9429	0.0070	3.16%	0.9604	0.0024	16.84%	0.9851	0.0011	28.42%
P.6	0.9255	0.0079	0.63%	0.9852	0.0020	18.75%	0.9776	0.0015	33.13%
P.7	0.9476	0.0069	4.58%	0.9679	0.0019	13.01%	0.9828	0.0012	40.01%
P.8	0.9750	0.0018	17.67%	0.9738	0.0015	37.67%	0.9846	0.0006	55.71%
P.9	0.9437	0.0106	5.33%	0.9754	0.0015	28.54%	0.9848	0.0009	42.43%
P.10	0.9630	0.0074	4.22%	0.9789	0.0010	34.77%	0.9874	0.0005	54.08%
AVG	0.9527	0.0068	6.63%	0.9785	0.0018	26.21%	0.9846	0.0009	46.74%