

DYNAMIC MODELING OF AN EXCAVATOR DURING DIGGING AND
SIMULATING THE MOTION

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SIMULATING THE MOTION**

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ABSTRACT

DYNAMIC MODELING OF AN EXCAVATOR DURING DIGGING AND SIMULATING THE MOTION

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The aim of this study is to perform the dynamic force analysis of a 3-degrees-of-freedom excavator during digging the soil and to simulate the motion on computer screen. Standard load calculations are done statically, therefore the effects of forces changing with time on the system cannot be observed. The dynamic analysis method used in the thesis is Recursive Newton – Euler Method and the numerical analysis method for simulation is 4th Order Runge – Kutta Method. After this study, the effects of sudden velocity changes; i.e, accelerational movements on construction machines, positions of bodies and dynamic forces on joints will be appointed and it will be possible to plan and control the motion.

Key Words: Excavator, Dynamic Analysis, Simulation

ÖZ

BİR EKSKAVATÖRÜN KAZI SIRASINDAKİ DİNAMİĞİNİ MODELLEME VE HAREKETİN SİMULASYONU

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Bu çalışmanın amacı 3 serbestlik dereceli bir ekskavatörün toprağı kazısı sırasındaki maruz kaldığı dinamik yüklerinin analiz edilmesi ve hareketin bilgisayarda simülasyonun gerçekleştirilmesidir. Standart yük hesaplamaları statik olarak yapılmaktadır ve bu nedenle zamana bağılı olarak değışen kuvvetlerin sistem üzerindeki etkisi görülememektedir. Bu tezde kullanılan dinamik analiz yöntemi Yenilemeli Newton – Euler Yöntemi ve simülasyon için yapılacak olan sayısal analizdeki yöntem ise 4. dereceden Runge – Kutta Yöntemidir. Bu çalışma sonrasında ani hız değışimlerinin (ivmeli hareketlerin) iş makinası üzerinde yarattığı etkiler, parçaların konumları ve mafsallardaki dinamik yükler saptanabilecek ve hareketin planlanması ile kontrolü mümkün olacaktır..

Anahtar Kelimeler: Ekskavatör, Dinamik Analiz, Simülasyon

To My Parents and My Sweetheart

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TABLE OF CONTENTS

ABSTRACT.....	iv
ÖZ.....	v
DEDICATION.....	vi
ACKNOWLEDGEMENTS.....	vii
TABLE OF CONTENTS.....	viii
LIST OF TABLES.....	xiii
LIST OF FIGURES.....	xv
NOMENCLATURE.....	xix
CHAPTER	
1. INTRODUCTION.....	1
2. LITERATURE SURVEY.....	5
2.1. Kinematics.....	5
2.2. Kinetics.....	6
2.2.1. Lagrange Formulation.....	7
2.2.2. Recursive Newton – Euler Formulation (RNEF)....	7
2.3. Simulation.....	9
2.4. Soil Modeling.....	10
2.5. Commercial Software for Dynamic Analysis of Mechanical Systems.....	11

3. METHODS OF ANALYSES.....	12
3.1. Kinematic Analysis.....	12
3.2. Dynamic Analysis.....	13
3.3. Numerical Analysis for Simulation.....	15
3.4. Computer Programs.....	16
4. DETERMINATION OF INPUT PARAMETERS.....	18
4.1. Theory of Cylinder Forces.....	18
4.2. Theory of Resistive Force of Soil.....	19
4.3. Digging Scenario.....	20
4.4. Test for Determining Cylinder Forces.....	25
4.5. Test for Determining Force on One of the Links.....	28
5. CASE STUDY.....	32
5.1. Test Results.....	32
5.1.1. Pressure Test Results.....	32
5.1.2. Strain Test Results.....	36
5.2. Iterations for Cohesion Factor of Soil.....	37
5.3. Results.....	42
5.3.1. Reaction Forces on Joints.....	43
5.3.2. Accelerations of Bodies.....	45
5.3.3. Velocities of Bodies.....	46
5.3.4. Positions of Bodies.....	47
5.4. Comparison with Commercial Software.....	48
5.4.1. Simulation Conditions on ADAMS.....	48

5.4.2. Results.....	51
6. DISCUSSIONS AND CONCLUSIONS.....	56
APPENDICES	
A. KINEMATIC ANALYSIS OF EXCAVATOR ATTACHMENTS WORKING IN TWO DIMENSIONS.....	59
A.1.Chassis and Boom.....	60
A.2.Arm.....	63
A.3.Bucket.....	67
B. EQUATIONS OF EXCAVATOR DYNAMICS.....	70
B.1.Multi-body Dynamics and Newton – Euler Formulation.....	70
B.1.1. Rotation Matrices.....	70
B.1.2. Inputs.....	71
B.1.3. Forward Dynamics.....	72
B.1.3.1. Positions, Rotational and Translational Velocities, Rotational and Translational Accelerations of Links.....	72
B.1.3.1.1. Chassis.....	73
B.1.3.1.2. Boom.....	74
B.1.3.1.3. Arm.....	77
B.1.3.1.4. Bucket.....	80
B.1.3.2. Velocities and Accelerations of Center of Gravities of Links.....	83
B.1.3.2.1. Chassis.....	83
B.1.3.2.2. Boom.....	84

B.1.3.2.3. Arm.....	85
B.1.3.2.4. Bucket.....	87
B.1.3.3. External Forces and Moments on Links.....	89
B.1.3.3.1. Inputs.....	89
B.1.3.3.2. Boom.....	90
B.1.3.3.3. Arm.....	91
B.1.3.3.4. Bucket.....	93
B.1.3.4. Actuating Forces and Points on Cylinders, Connecting Rods, Boom, Arm and Bucket.....	95
B.1.4. Backward Dynamics.....	98
B.1.4.1. Force, Torque Relations of Links.....	98
B.1.4.1.1. Bucket.....	99
B.1.4.1.2. Arm.....	100
B.1.4.1.3. Boom.....	101
B.1.4.2. First Step Expansions of Equations of Backward Dynamics.....	102
B.1.4.2.1. Bucket.....	102
B.1.4.2.2. Arm.....	103
B.1.4.2.3. Boom.....	106
B.1.4.3. Equations of Motion.....	110
C. SIMULATION EQUATIONS.....	112
C.1. Modified Equations o Motion.....	113
C.2. Solving Modified Equations of Motion by Using Numerical Method.....	116

C.2.1. Arranging Equations of Motion.....	116
C.2.2. Introduction to 4 th Order Runge – Kutta Method.....	120
C.2.3. Adaptation of Equations of Motion of a Three – Degrees – of – Freedom Excavator to 4 th Order Runge – Kutta Method.....	122
D. EXCAVATOR DYNAMICS SOFTWARE (EDS).....	127
D.1.Introduction to EDS.....	127
D.2.General View of EDS.....	128
D.2.1. Input Area.....	128
D.2.2. Running Simulation.....	129
D.2.3. Monitoring Results.....	130
D.2.3.1. Simulation Chart.....	130
D.2.3.2. Outputs.....	131
E. LOADCELL TEST.....	132
E.1. Method of the Test.....	132
REFERENCES.....	134

LIST OF TABLES

TABLES

Table 3.1 Algorithm for Manipulator Dynamics in RNEF.....	14
Table B.1 Positions, Rotational and Translational Velocities, Rotational and Translational Accelerations of Links.....	72
Table B.2 Initial Conditions: Positions, Rotational and Translational Velocities, Rotational and Translational Accelerations of Chassis.....	73
Table B.3 Positions, Rotational and Translational Velocities, Rotational and Translational Accelerations of Boom.....	74
Table B.4 Positions, Rotational and Translational Velocities, Rotational and Translational Accelerations of Arm.....	77
Table B.5 Positions, Rotational and Translational Velocities, Rotational and Translational Accelerations of Bucket.....	80
Table B.6 Velocities and Accelerations of Center of Gravities of Links.....	83
Table B.7 Velocities and Accelerations of Center of Gravities of Chassis.....	83
Table B.8 Velocities and Accelerations of Center of Gravities of Boom.....	84
Table B.9 Velocities and Accelerations of Center of Gravities of Arm.....	85
Table B.10 Velocities and Accelerations of Center of Gravities of Bucket.....	87
Table B.11 External Forces and Moments on Links.....	89
Table B.12 External Forces and Moments on Boom.....	90

Table B.13 External Forces and Moments on Arm.....	91
Table B.14 External Forces and Moments on Bucket.....	93
Table B.15 Force, Torque Relations of Links.....	98

LIST OF FIGURES

FIGURES

Figure 1.1 A typical rubber - tire type excavator.....	2
Figure 1.2 A typical crawler type excavator.....	2
Figure 2.1 Loader Mechanism.....	6
Figure 2.2 Upper Chassis - Starting Point of Forward Dynamics...	8
Figure 3.1 Number of Links.....	12
Figure 3.2 Sketch Drawing of an Open Chain Robot.....	14
Figure 3.3 ADAMS Front View of the Model	17
Figure 3.4 ADAMS Isometric View of the Model.....	17
Figure 4.1 Elements of a Hydraulic Actuator.....	18
Figure 4.2 Zelenin's Digging Notation.....	20
Figure 4.3 Bucket's Positions.....	21
Figure 4.4 Direction of Resistance of Soil.....	21
Figure 4.5 Start Position.....	22
Figure 4.6 Middle Position.....	23
Figure 4.7 End Position.....	24
Figure 4.8 Adaptors and Sensors.....	26
Figure 4.9 Cables and Hydrotechnik.....	26
Figure 4.10 Actuator Chambers.....	27

Figure 4.11 Initial Position in Testing.....	28
Figure 4.12 Strain Gauges on The Right – Hand Side Rod.....	29
Figure 4.13 Strain Gauges on The Left – Hand Side Rod.....	29
Figure 4.14 Strain Gauges and Cables.....	30
Figure 4.15 ESAM and Computer.....	31
Figure 5.1 Pressure Results of Boom Cylinder.....	32
Figure 5.2 Pressure Results of Arm Cylinder.....	33
Figure 5.3 Pressure Results of Arm Cylinder.....	33
Figure 5.4 Net Force of Boom Cylinder.....	34
Figure 5.5 Net Force of Arm Cylinder.....	35
Figure 5.6 Net Force of Bucket Cylinder.....	35
Figure 5.7 Strains on Connecting Rods.....	36
Figure 5.8 Initial Position of Attachments for Simulation.....	37
Figure 5.9 Final Position of Attachments for C=10000 Pa.....	38
Figure 5.10 Test and Simulation Compatibility for C=10000 Pa...	38
Figure 5.11 Final Position of Attachments for C=15000 Pa.....	39
Figure 5.12 Test and Simulation Compatibility for C=15000 Pa...	39
Figure 5.13 Final Position of Attachments for C=20000 Pa.....	40
Figure 5.14 Test and Simulation Compatibility for C=20000 Pa...	40
Figure 5.15 Final Position of Attachments for C=17000 Pa.....	41
Figure 5.16 Test and Simulation Compatibility for C=17000 Pa...	41
Figure 5.17 Time vs. Force Chart of the Joint between Boom Chassis.....	43

Figure 5.18 Time vs. Force Chart of the Joint between Arm and Boom	43
Figure 5.19 Time vs. Force Chart of the Joint between Bucket and Arm.....	44
Figure 5.20 Accelerations of Bodies (cohesion factor = 17000 Pa)	45
Figure 5.21 Accelerations of Bodies (cohesion factor = 20000 Pa)	45
Figure 5.22 Velocities of Bodies (cohesion factor = 17000 Pa).....	46
Figure 5.23 Velocities of Bodies (cohesion factor = 20000 Pa).....	46
Figure 5.24 Positions of Bodies (cohesion factor = 17000 Pa).....	47
Figure 5.25 Positions of Bodies (cohesion factor = 20000 Pa).....	47
Figure 5.26 Original Data and Trendline Compatibility for Boom Cylinders.....	49
Figure 5.27 Original Data and Trendline Compatibility for Arm Cylinder.....	50
Figure 5.28 Original Data and Trendline Compatibility for Arm Cylinder.....	51
Figure 5.29 Comparison of Results (Boom – Chassis Joint).....	52
Figure 5.30 Comparison of Results (Arm – Boom Joint).....	52
Figure 5.31 Comparison of Results (Bucket – Arm Joint).....	53
Figure 5.32 Bucket’s Middle Position on ADAMS	53
Figure 5.33 Time vs. Force by Using Different Cohesion Factors on Boom and Chassis Joint (Step Size=0,01)	54
Figure 5.34 Time vs. Force by Using Different Cohesion Factors on Boom and Chassis Joint (Step Size=0,002)	54
Figure A.1 Constant and Variables Lengths of Boom.....	60
Figure A.2 Input and Constant Angles of Boom.....	61

Figure A.3 Variable Angles.....	62
Figure A.4 Constant and Variables Lengths of Arm.....	63
Figure A.5 Input and Constant Angles of Arm.....	64
Figure A.6 Variable Angles of Arm.....	65
Figure A.7 Variable Angles of Arm Connecting Rods and Bucket (Detailed).....	66
Figure A.8 Constant and Variables Lengths of Bucket.....	67
Figure A.9 Input and Constant Angles of Bucket.....	68
Figure A.10 Variable Angles of Bucket.....	68
Figure A.11 Soil Resistance Direction.....	69
Figure B.1 Forces on Bucket Cylinder and Connecting Rods.....	95
Figure B.2 Forces on Bucket.....	99
Figure B.3 Forces on Arm.....	100
Figure B.4 Forces on Boom.....	101
Figure D.1 Input Area of EDS.....	128
Figure D.2 Running Simulation.....	129
Figure D.3 Simulation Chart.....	130
Figure E.1 Position for Loadcell Test.....	132

NOMENCLATURE

$F_{freedom}$: Degree – of – Freedom of Mechanism
l	: Number of Links
j	: Number of Joints
F_r	: Resistive Force of Soil
C	: Cohesion Factor
R	: Bucket Radius
ϕ	: Full Swing Angle of Bucket
Φ	: Increasing Swing Angle of Bucket
$F_{RodSide}$: Force on Rod Side of Hydraulic Actuator
$F_{CylinderSide}$: Force on Cylinder Side of Hydraulic Actuator
$^{Boom}F_{RodSide}$: Force on Rod Side of Hydraulic Actuator of Boom
$^{Boom}F_{CylinderSide}$: Force on Cylinder Side of Hydraulic Actuator of Boom
$^{Boom}F_{Net}$: Net Force on Hydraulic Actuator of Boom
$^{Arm}F_{RodSide}$: Force on Rod Side of Hydraulic Actuator of Arm
$^{Arm}F_{CylinderSide}$: Force on Cylinder Side of Hydraulic Actuator of Arm
$^{Arm}F_{Net}$: Net Force on Hydraulic Actuator of Arm
$^{Bucket}F_{RodSide}$: Force on Rod Side of Hydraulic Actuator of Bucket
$^{Bucket}F_{CylinderSide}$: Force on Cylinder Side of Hydraulic Actuator of Bucket
$^{Bucket}F_{Net}$: Net Force on Hydraulic Actuator of Bucket
$P_{RodSide}$: Pressure on Rod Side of Hydraulic Actuator
$A_{Cylinder}$: Cross-section Area of Cylinder Side of Hydraulic Actuator

A_{Rod}	: Cross-section Area of Rod of Hydraulic Actuator
$P_{CylinderSide}$: Pressure on Cylinder Side of Hydraulic Actuator
F_{link}	: Force on Link
$A_{cross-section}$: Cross-section Area of Link
E	: Elasticity of Link's Material
ε	: Strain
t	: Time
$F_{boom-cylinder}$: Trendline Force on Boom Cylinder
$F_{arm-cylinder}$: Trendline Force on Arm Cylinder
$F_{bucket-cylinder}$: Trendline Force on Bucket Cylinder
Li	: Length between points O_{i-1} & O_i
Lgi	: Length between points O_{i-1} & G_i
Lyz	: Length between points Y & Z
θ_i	: Input Angle Between Line x_{i-1} , Point O_{i-1} and Line x_i
λ_i	: Constant Angle Between Points G_{i+1} , O_i and O_{i+1}
β_1	: Constant Angle Between Points A, E and H
β_2	: Constant Angle Between Points I, A and C
β_3	: Constant Angle Between Points B, A and I
β_4	: Constant Angle Between Points I, C and A
β_5	: Constant Angle Between Points L, C and D
β_6	: Constant Angle Between Points J, C and L
β_7	: Constant Angle Between Points J, C and F
β_8	: Constant Angle Between Points P, D and N
ϕ_1	: Input Angle Between Line x_4 , Point N and Soil Resistance Direction F_{45}
ϕ_2	: Variable Angle Between Line x_3 , Points J and K

ϕ_3	:	Variable Angle Between Line x_3 , Points P and K
ϕ_4	:	Variable Angle Between Line x_3 , Points L and K
ϕ_5	:	Variable Angle Between Line x_2 , Points I and F
ϕ_6	:	Variable Angle Between Line x_2 , Points B and E
θ	:	Angular Position
$\dot{\theta}$:	Angular Velocity
$\ddot{\theta}$:	Angular Acceleration
\vec{p}	:	Translational Position Vector
$\vec{\omega}$:	Angular Velocity Vector
$\vec{v}, \vec{\dot{p}}$:	Translational Velocity Vectors
$\vec{\alpha}, \vec{\ddot{\omega}}$:	Angular Acceleration Vectors
$\vec{a}, \vec{\ddot{p}}$:	Translational Acceleration Vectors
\hat{A}	:	Rotation Matrix
m	:	Mass of Link
\vec{g}	:	Gravity Vector
\vec{F}	:	Force Vector on Joint
\hat{I}	:	Mass Moment of Inertia Matrix
\vec{M}	:	Moment Vector on Joint

CHAPTER 1

INTRODUCTION

Earth-moving machinery are machines designed to perform excavation, loading, transportation, drilling, spreading, compacting or trenching of earth, rock and other materials by their equipments or working tools [1].

An excavator is defined as “self-propelled machine on crawlers, wheels or legs, having an upper structure capable of a 360° swing with mounted equipment and which is primarily designed for excavating with a bucket, without movement of the undercarriage during the work cycle” [1]. A typical excavator consists of parts such as bucket, arm, boom, hydraulic cylinders, connecting rods, upper chassis, lower chassis, cab and travel train. The travel train can be either rubber-tire type (Figure 1.1) or crawler type (Figure 1.2).

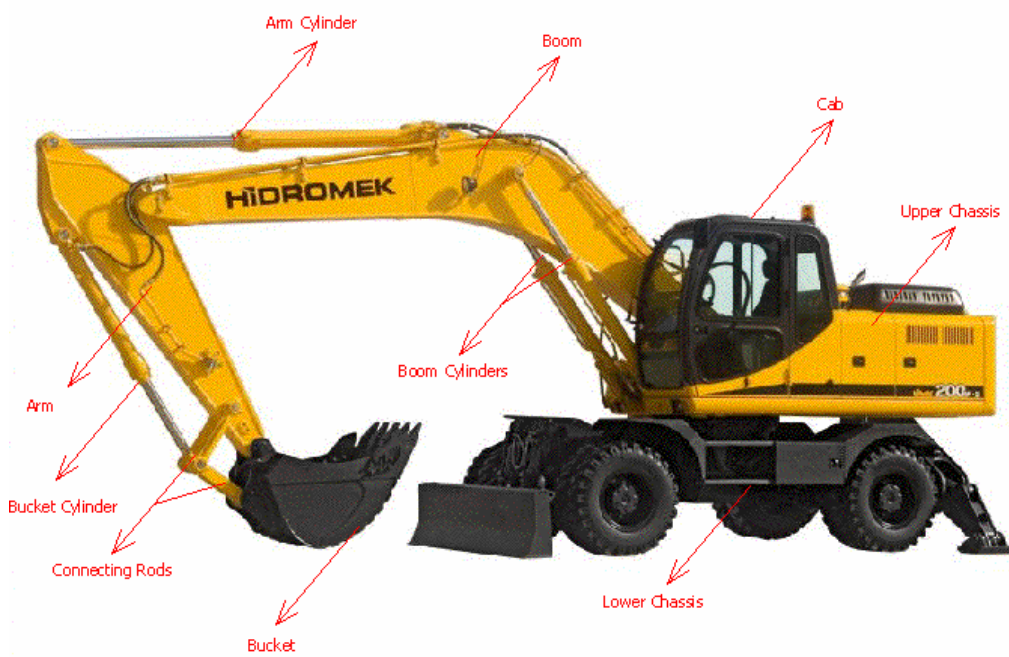


Figure 1.1 - A typical rubber - tire type excavator



Figure 1.2 - A typical crawler type excavator

Attachments are the moving parts of excavators. These are boom, arm, and bucket from chassis to end, respectively. Large forces can be obtained by using hydraulic oil in the hydraulic cylinders. A pair of hydraulic cylinders rotates the boom with respect to the upper chassis. The boom, arm and the bucket are serially connected and hydraulic cylinders also control their relative motion. Although there are four actuators, the attachment mechanism has 3 degrees of freedom since boom cylinders are parallel actuators. This is due to the excessive force requirement for boom's motion.

The aim of this thesis is to build a mathematical model of dynamics of the attachments during digging and to simulate the motion. Since there are 3 degrees of freedom, the mechanism can be considered as highly complex which requires a recursive method in order to model the dynamic behavior of boom, arm and bucket.

While formulating the equations of motion, attachments are considered as “open loop robotic system”. Therefore, this thesis mainly concerns about robot dynamics and therefore Recursive Newton – Euler Formulation (RNEF) is used.

Since there are 3 main links (boom, arm, bucket), 3x1 vector matrices and 3x3 coefficient matrices are formed. After applying RNEF; in other words, forward and inverse (backward) dynamics, the equations of motions of three main links are obtained. The simulation process follows this step. Simulation, which depends on a time interval, is the real time solution of dynamic equations. Since equations of motions are non-linear, second order differential equations; the simulation is performed by numerical analysis. Among different numerical methods, Runge – Kutta Method is chosen. Runge – Kutta Method is an iterative method that solves differential equations step by step. Therefore, an iteration algorithm is written on Visual Basic for Applications (VBA). Thus, by the pre-determined step size, a simulation process can be fulfilled within a time interval.

In order to run the simulation, resistive force of soil and actuating forces of hydraulic cylinders should be known. For these reasons, a soil model is built to find out the soil resistance and a test is performed to determine cylinder forces.

After finding out all necessary inputs, all dynamic outputs can be obtained by running the simulation. These outputs are dynamic forces on joints; accelerations, velocities and positions of bodies. So, thereby, force and motion characteristics of an excavator during digging motion are determined and these characteristics can be used in order to improve structure, design and control the motions of excavators.

CHAPTER 2

LITERATURE SURVEY

2.1. Kinematics

Söylemez [2] stated different methods of kinematical analysis and synthesis of basic mechanisms, such as; four-bar, slider-crank and inverted slider-crank. An excavator attachment mechanism consists of three inverted slider-crank mechanisms. Therefore, analyzing every piston-slider mechanism and connecting them to each other gives an exact result. By using the input parameters, fixed angles and fixed lengths of links, all positions of the bodies can be found out.

İpek [3] has analyzed the kinematics of the loader mechanism of Backhoe – Loaders. A loader mechanism (Figure 2.1) has two degrees of freedom and gives an idea about analyzing mechanisms, which are more complex than one degree of freedom ones. A loader is controlled by two piston-cylinder pairs for lifting the loader arm and tilting the bucket. Lifting cylinders connect the loader arm to the chassis of the machine and tilting cylinders connect two pairs of lever plate in order to adjust the position of the bucket.

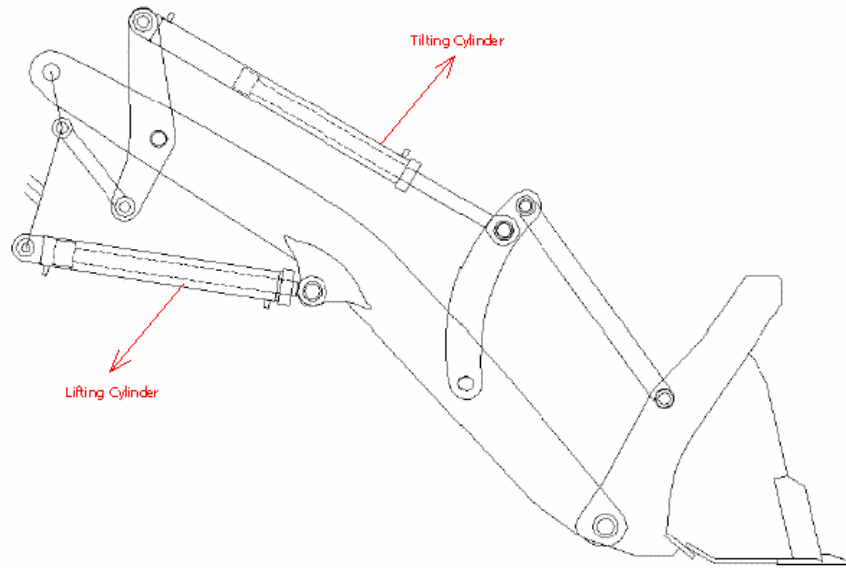


Figure 2.1 – Loader Mechanism

As in the case of loader, excavator mechanism can also be analyzed by assigning changing parameters as inputs and link positions as outputs.

Koivo [20] establishes a method of kinematic analysis of the excavators in his paper. The main purpose of the analysis is to determine the bucket position by using input parameters such as joint shaft angles or the lengths of the hydraulic cylinders.

2.2. Kinetics

After completing kinematic analysis, force analysis should be done. Depending on the needs, engineers can perform either static or dynamic force analyses. In this work, dynamic force analysis is taken into account; static force analysis is not considered. Then, kinetics of bodies is investigated. Kinetics is defined as the

branch of mechanics, which deals with the effects of forces changing the motion of bodies [4].

Simulating, controlling and planning of the motion of an excavator can be realized after building a mathematical dynamic model of the system. In order to achieve this, there are two methods: Lagrange's Formulation and Newton – Euler Formulation [15].

2.2.1. Lagrange Formulation

Lagrange's Formulation using Euler – Lagrange's Equations concludes the dynamic model of excavator in terms of all joint variables [5]. The initial step for Lagrange's Formulation is determining the kinetic and potential energies of all moving links. After Lagrange's energy function [16], in order to obtain the equations of motion of excavator, Euler – Lagrange Equations are applied. The major property of Lagrange's Formulation is that the resulting differential equations describe the motion in terms of the joint variables and the structural parameters of the manipulator [5].

2.2.2. Recursive Newton – Euler Formulation (RNEF)

RNEF is an alternative approach to model the robot dynamics. In this method, each link in succession is considered as free body and their dynamic behavior is determined by using Newton and Euler's Equations [18]. The most important advantage of this method is its computational simplicity. Besides, RNEF helps designer to understand the dynamic behavior of each link separately and gives information about the propagation of forces and torques through joints [5].

In RNEF, forces, torques, translational and rotational positions, velocities and accelerations are determined by an analysis starting from base link and proceeding

link by link until the end effector. This process is called forward dynamics [19]. In the case of excavators, forward dynamics starts from upper chassis (Figure 2.2), continues with boom, arm and ends with bucket. The bucket may be considered as the end effector of the excavator. In order to start with forward dynamics, a set of initial conditions and applied forces must be known as input. The first three sets of these initial conditions are the positions, velocities and accelerations of boom, arm and bucket and the remaining sets are the forces applied by hydraulic cylinders and resistive force of soil.

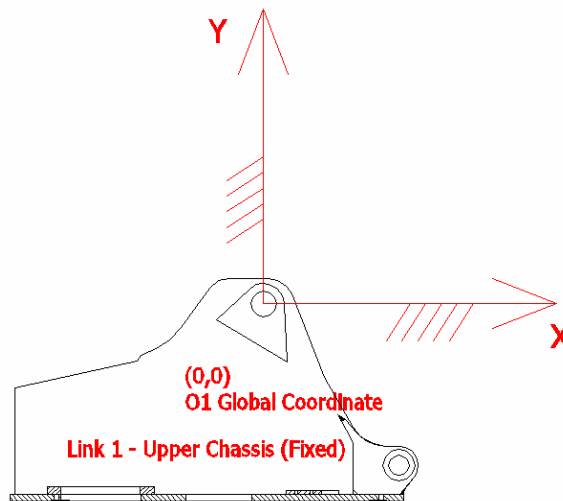


Figure 2.2 – Upper Chassis - Starting Point of Forward Dynamics

Inverse or backward dynamics is required in order to determine the generated forces and moments on joints [19]. Inverse dynamics starts from end effector (bucket) and ends at the base (upper chassis). By applying forward and inverse dynamics in succession, equations of motions of links can be determined.

In the previous section, Lagrange's Formulation is explained briefly. Due to its computational complexity compared with RNEF [5], Lagrange's Formulation is not preferred for the analysis in this work.

Vähä and Skibniewski [7] introduce a great view for dynamic model of an excavator. In their work, the steps of dynamic modeling of an excavator are explained basically. However, the motion is assumed as not only digging but also the rotary motion of the upper chassis is also considered. In other words, it is taken as 4-degrees-of-freedom excavator. In fact, it is not realistic since an excavator cannot dig the soil while the upperstructure swings. By taking this fact into account, Koivo et al [8] work on more specified condition of excavator's motion. They consider that an excavator's upperstructure cannot swing while attachments are working on soil. This leads a huge simplification that the analysis may be performed just in planar coordinate. Rotation of upperstructure is eliminated, i.e. the change of angle about vertical axis is fixed as zero and Denavit – Hartenberg Transformations [9] (Rotation matrices) are generated just for the motion of boom, arm and bucket. Beside this worth simplification, they have also corrected some mistakes on Vähä and Skibniewski's work.

Ha et al [21] introduces another aspect of dynamics of an excavator. In addition to dynamic modeling, control procedures are also explained. Feedback equations, Jacobian matrices and control vectors etc. are represented briefly. The future work of the thesis is automatic control of excavators and this paper [21] gives basic formulation for control systems.

2.3. Simulation

Simulation can be called as “imitation” of a real or a theoretical physical system [10]. Computer simulation is executing or imitating a model on a digital computer. The required information is the output result given by computer after

analyzing the execution. Simulation can be considered as natural activities of children [10]. As children understand the life and the world by imitating their environment, engineers and scientists also learn the characteristics of a physical object by simulating it as if it is real.

For simulation, a mathematical model of the physical object is required [10]. In most cases, objects that should be simulated may be too complex. So, generating the mathematical model of the physical object may be too difficult for engineers. In the case of an excavator, there are three bodies whose mathematical model should be constructed and analyzed. This means there should be three equations of motions that must be solved. However, these 2nd order non-linear differential equations are not so simple that one can solve them analytically; equations are solved by using numerical methods [11].

In the thesis, 4th Order Runge – Kutta Method is chosen among different types of numerical methods. Söylemez [12] stated that methods to determine the slope, which is used to reach the result step by step, are called Runge – Kutta Methods. For more accurate and correct results, 5th Order Runge – Kutta Method can be used, but for calculation simplicity 4th Order Runge – Kutta Method, which gives proper results if step sizes are small enough, is used.

2.4. Soil Modeling

Resistive force of soil is an unknown and it is also an input parameter of simulation. Bernold [14] mentioned in his paper about the works done about soil modeling in civil engineering and geotechnical sciences. Due to its simplicity and practicability, Zelenin's Method is chosen for this thesis among different methods for modeling the soil. Zelenin prefers a controlled motion of the bucket for determining soil resistance and gives necessary equation in his work. In order to apply Zelenin's equation, soil cohesion should be estimated.

2.5. Commercial Software for Dynamic Analysis of Mechanical Systems

ADAMS (Automatic Dynamic Analysis of Mechanical Systems) is one of the commercial computer programs of MSC Inc. ADAMS lets the analyzer build models of mechanical systems and simulate the full-motion of the complex behavior of mechanical assemblies. ADAMS also enables the engineer to analyze different design variations and select an optimum design [13].

CHAPTER 3

METHODS OF ANALYSES

In order to solve the digging motion and get the results, a mathematical model of excavator attachments is built, equations of motions of these attachments are found out and the motion is simulated. There are three main steps for building the model: kinematic analysis, dynamic analysis, and numerical analysis for simulation. For this purpose a computer program that contains all of these steps should be written.

3.1. Kinematic Analysis

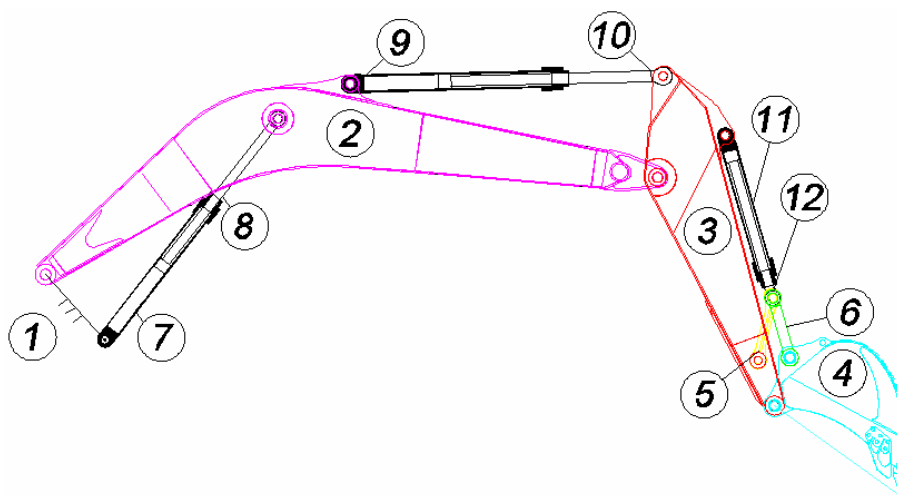


Figure 3.1 – Number of Links

The degree of freedom of mechanisms is found by using the following equation:

$$F_{freedom} = \lambda \cdot (l - j - 1) + \sum_{i=1}^j f_i \quad (3.1)$$

Number of Links (l) = 12 (Figure 3.1)

Number of Joints (j) = $12R + 3P = 15$

Degree-of-Freedom of All Joints ($\sum f_i$) = 15

Degree-of-Freedom of Space (λ) = 3

$$F_{freedom} = 3 \cdot (12 - 15 - 1) + 15 = 3 \quad (3.2)$$

From the calculation, it is seen that an excavator attachment mechanism has 3 degrees-of-freedom. In other words, there should be 3 input parameters in order to perform kinematic analysis. Generally, these parameters are lengths of hydraulic actuators but in this work, inputs are the angles of bodies with respect to previous bodies. Desired outputs of kinematic analysis are the positions of all links. All detailed kinematic analysis formulations of an excavator are given in Appendix A.

The program where the analyses are performed is Microsoft Excel. Excel is very useful software for parametric analyses. Since all variables are depending on each other, in other words, there are plenty of parametric equations; Excel becomes the best software choice for kinematic analysis. One of the most useful features of Excel is the ability of interaction with AutoCAD. So, with the help of a Visual Basic Application code, 2D view of an excavator can be transferred into Excel very rapidly.

3.2. Dynamic Analysis

For applying Recursive Newton – Euler Formulation, an algorithm [5] is followed (Table 3.1):

Table 3.1 – Algorithm for Manipulator Dynamics in RNEF

Forward Dynamics for Calculating the Position, Velocity and Acceleration of Link $j, j=1, \dots, N$	
1	Initialize the algorithm. Set $i=0$
2	Determine the rotational and translational velocities and accelerations for Link j where $j=i+1$
3	Compute the velocity and acceleration of the centroid of link j
4	Calculate the external force and moment exerted on link j
5	If $j < N$, increase i by one, that is, replace i by $i+1$, and return to step 2. If $j=N$, continue
Inverse (Backward) Dynamics for Computing the Generalized Torques (Equations of Motion) for Link $i=N, N-1, \dots, 1$	
6	Compute force and torque acting on link i
7	Decrease i by one, that is, replace i by $i-1$, and repeat from step 6 on until $i=1$
8	Calculate the generalized torque (equations of motion) of joint i for $i=1, \dots, N$

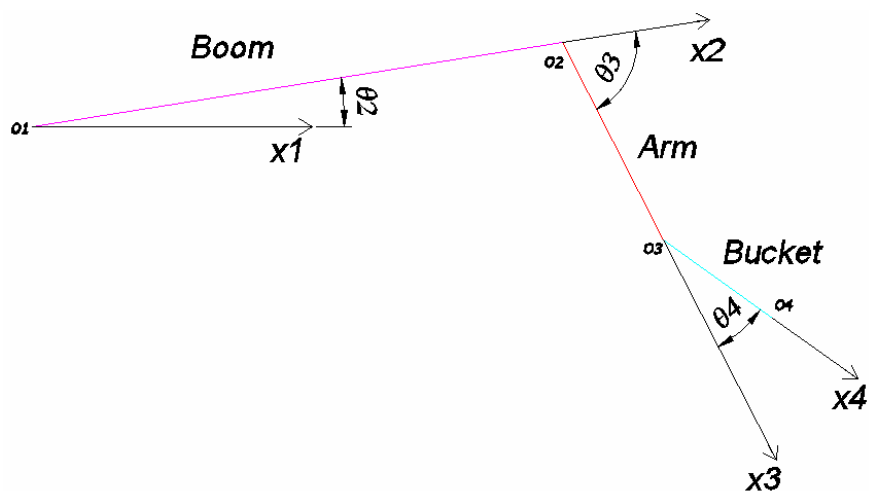


Figure 3.2 – Sketch Drawing of an Open Chain Robot

During the dynamic analysis, the excavator attachment system can be considered as an open chain robot (Figure 3.2) although it is a closed loop mechanism. The reasons of this consideration can be explained as followings:

- Compared to the weight of the boom, arm and bucket, the weights and the inertial effects of pistons, cylinders, connecting parts, hoses, pipes etc. are small and can be neglected in the analysis [6].
- The directions of forces exerted by hydraulic cylinders are always axial and these directions can be determined just by the kinematical analysis of the mechanism [6].

In this work, there are some other simplifications:

- All bodies and joints are considered as rigid
- No frictions on revolute and cylindrical joints
- No lateral loads from soil
- No hydraulic effects (friction, compressibility etc.)

All detailed dynamic analysis formulations of an excavator are given in Appendix B.

3.3. Numerical Analysis for Simulation

After completing dynamic modeling, three equations of motions are obtained. First one is the equation of motion of boom, second one is the equation of motion of arm and last one is the equation of motion of bucket. All equations are second order, non-linear, complex differential equations in the following form:

$$J(q) \cdot \ddot{q} + C(q) \cdot \dot{q}^2 = Q(q, \dot{q}) \quad (3.3)$$

where q, \dot{q}, \ddot{q} are denoting position, velocity and acceleration parameters of the equation, respectively [12].

Simulation runs if the equations of motions are solved simultaneously with respect to time. In order to solve equations simultaneously, 4th Order Runge – Kutta Numerical Method, which is a member of a family of single point methods [17] is used. Moments on joints are zero. Therefore, by equating moments to zero, numerical analysis can be performed.

All detailed numerical analysis formulations of digging motion are given in Appendix C.

3.4. Computer Programs

As mentioned before, in order to build a dynamic model of an excavator, a computer program should be written. In this thesis, by using Excel, Excavator Dynamics Software (EDS) is established (Appendix D). However, there are different commercial computer programs, which are being used by several companies for performing dynamic analysis. MSC. ADAMS (Figure 3.3 and Figure 3.4) is one of these programs. For verifying EDS and measuring the compatibility between two computer programs, the same model is built in ADAMS and the simulation runs with the same conditions.

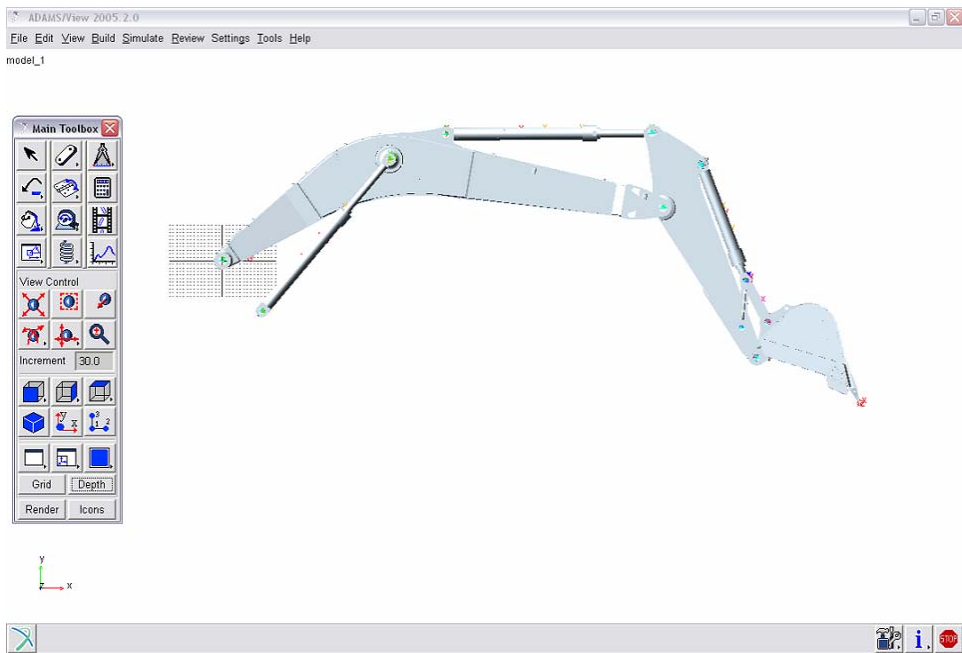


Figure 3.3 – ADAMS Front View of the Model

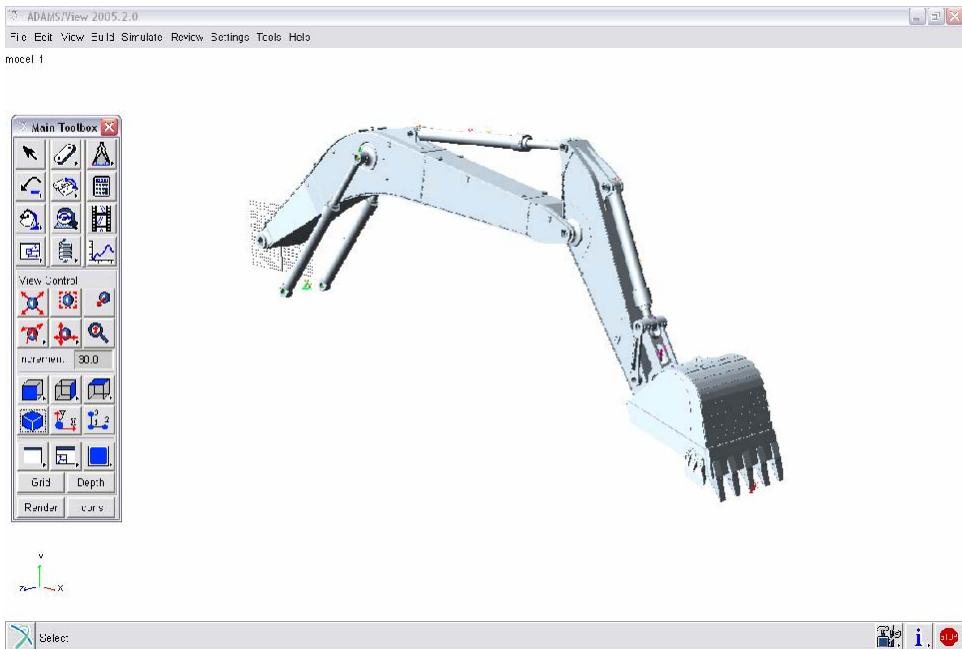


Figure 3.4 – ADAMS Isometric View of the Model

CHAPTER 4

DETERMINATION OF INPUT PARAMETERS

4.1. Theory of Cylinder Forces

Hydraulic cylinders are the actuators of excavator attachments. Therefore, all input forces are going to be exerted from these cylinders. In a 20-tons-excavator, there are 4 hydraulic cylinders; two of them are placed to move the boom, one of them is placed to move the arm and the remaining one is placed to move the bucket.

Cylinder – piston systems exert force linearly in the direction of axis of rod [6]. This force depends on four main parameters:

- Pressure on the cylinder side
- Pressure on the rod side
- Rod Diameter (Figure 4.1)
- Cylinder diameter (Figure 4.1)

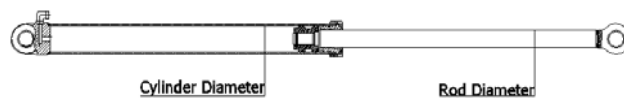


Figure 4.1 – Elements of a Hydraulic Actuator

When hydraulic cylinder is extending, it is under compressive load and hydraulic oil fills cylinder section of the hydraulic actuator. On the other hand, when hydraulic cylinder is retracting, it is under tensile load and hydraulic oil fills the rod section of the actuator.

Analytical determination of the hydraulic pressure during the motion requires the modeling of the hydraulic drive system as a whole. In this work, experimental determination is preferred.

In order to determine the cylinder forces, a test is performed. During this test, pressures of cylinder chambers are measured simultaneously. So, exerted force on cylinders can be calculated. These results are the inputs in order to simulate the digging motion.

4.2. Theory of Resistive Force of Soil

During digging motion (Figure 4.2), the resistive force, which is applied by soil, on the edge of the bucket can be determined by the Zelenin's Equation [14]:

$$F_r(\Phi) = 10 \cdot C \cdot R^{1.35} \cdot (\cos(\phi - \Phi) - \cos(\phi))^{1.35} \quad (4.1)$$

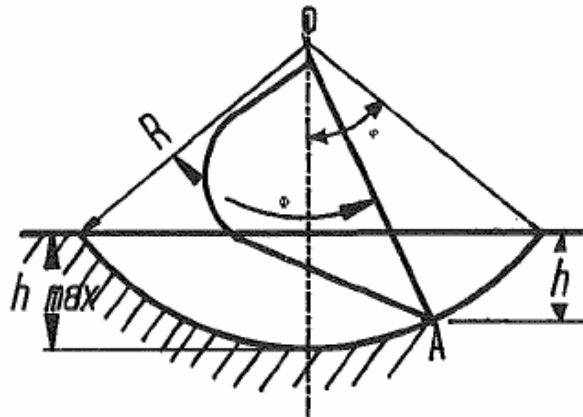


Figure 4.2 – Zelenin's Digging Notation

4.3. Digging Scenario

Bucket digs the soil by using either bucket cylinder or arm cylinder. Among infinitely many possibilities, in order to standardize the calculations, just one digging scenario is considered in this work. According to this scenario, boom and arm are standing immobile and bucket is going to turn about its joint axis. By this way, the formula derived by Zelenin is applicable. In this formula, there is one unknown: C , soil cohesion. Since pointing a specific cohesion is not a mechanical engineering concern, in this work, appropriate possible soil cohesion is going to be found out by an iterative process. Only bucket cylinder will be extracted to dig the soil. Bucket has a 1448 mm sweep radius and 108.9° (its half, 54.45°) sweep angle as shown in Figure 4.3. Besides the magnitude, resistance direction is considered as parallel to the bucket tip (Figure 4.4). The scenario begins with the excavator's position shown in Figure 4.5. Bucket is going to pass the stage shown in Figure 4.6 and process continues till the bucket gets out from soil (Figure 4.7)

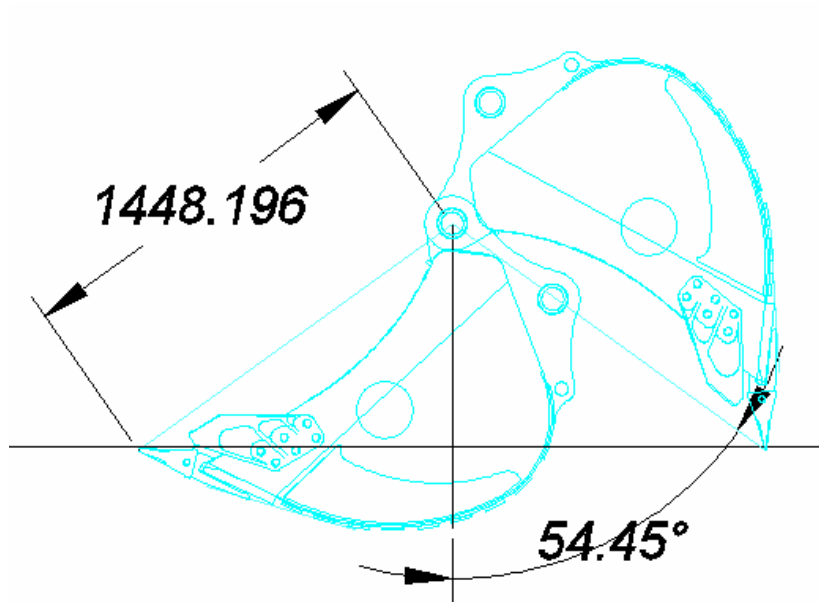


Figure 4.3 - Bucket's Positions

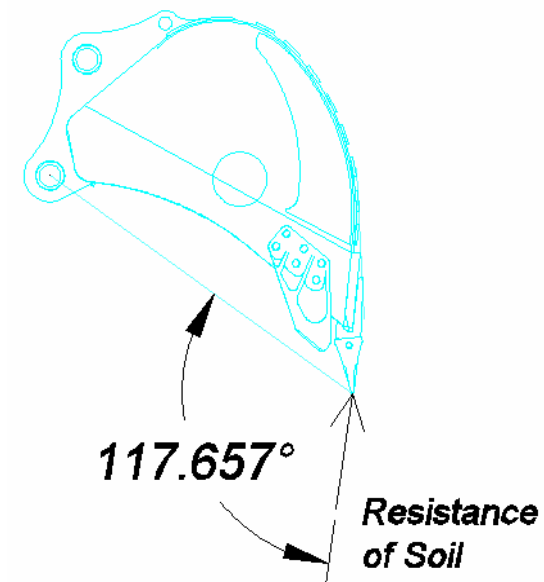


Figure 4.4 - Direction of Resistance of Soil

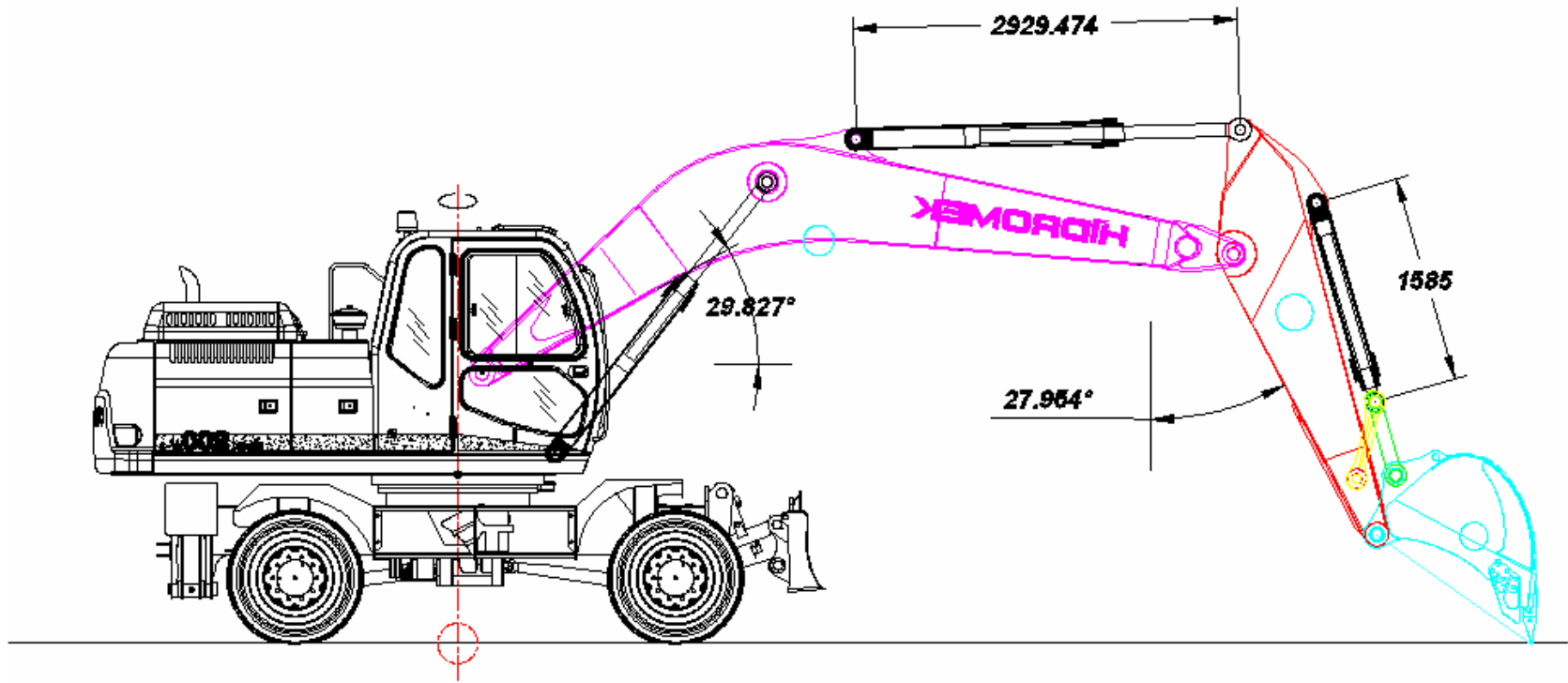


Figure 4.5 - Start Position

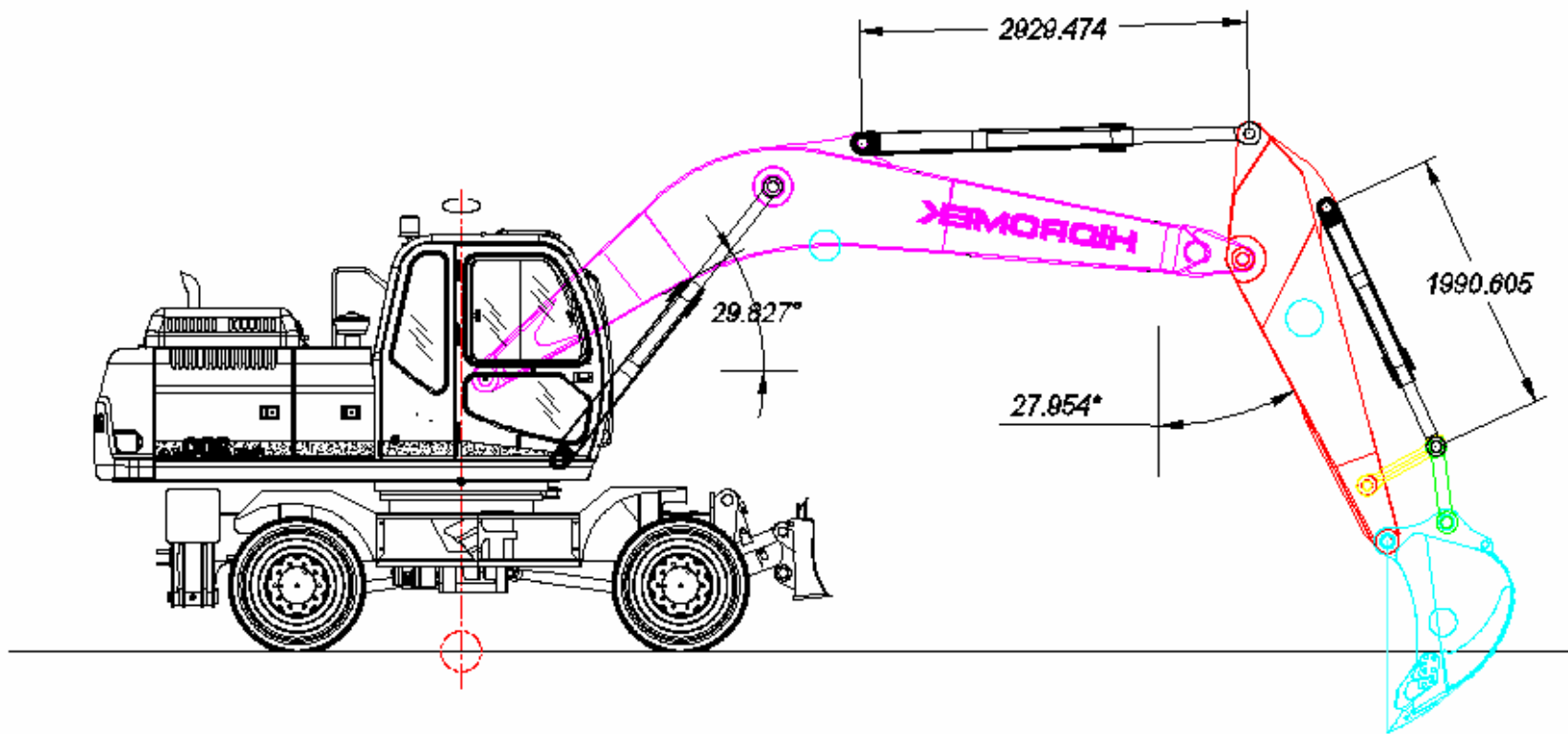


Figure 4.6 - Middle Position

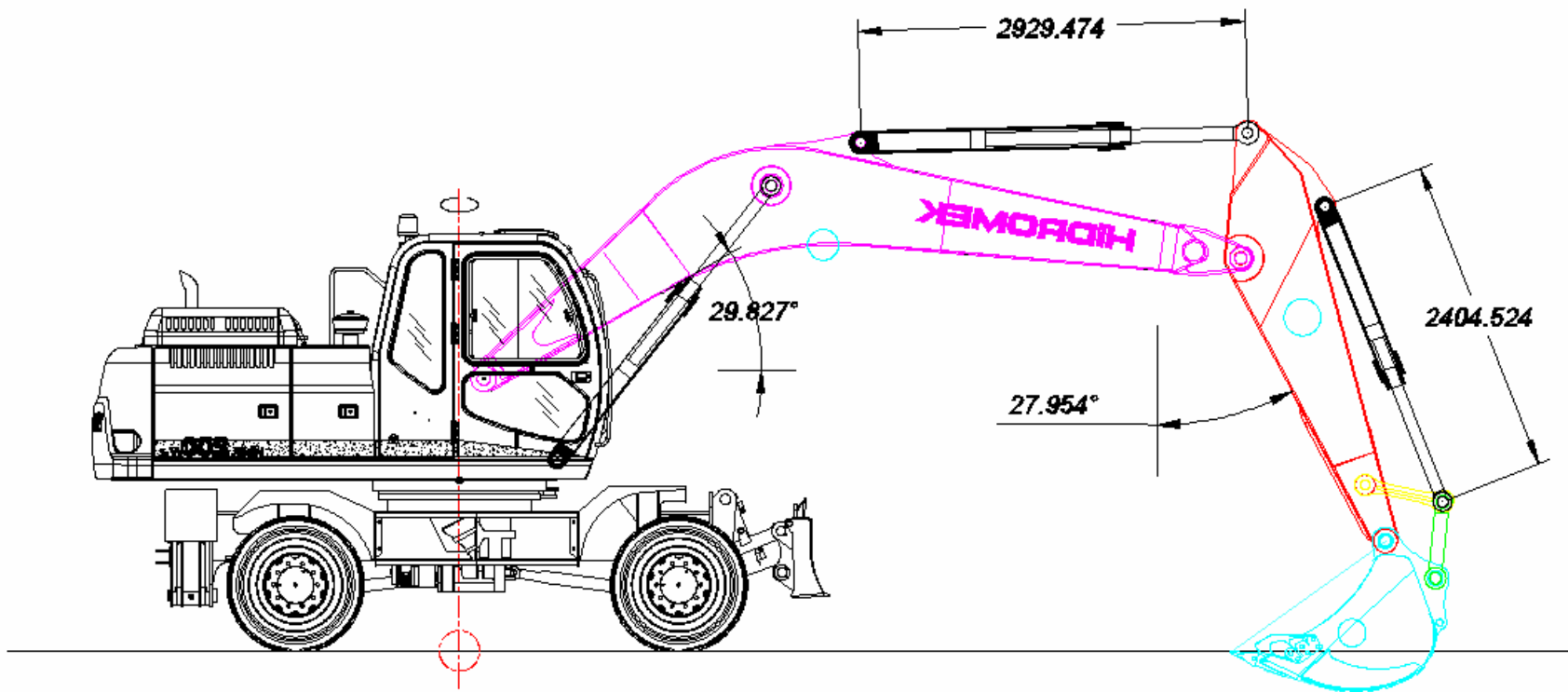


Figure 4.7 - End Position

4.4. Test for Determining Cylinder Forces

Hydraulic cylinders are operated by using pressurized hydraulic oil, which comes from oil tank through pipes and hoses with the help of pumps. According to resistance on the rod side of the actuator, pressure is going to change in order to move piston against resistance. For determining cylinder forces, first of all, pressures inside the actuator chambers are determined by testing. Then, forces are calculated and subtracted from each other for finding out resultant force exerted by hydraulic actuator.

For performing the test, the following apparatus are needed:

- Pressure measuring device (Hydrotechnik)
- Sensors
- Adaptors
- Cables

Adaptors are pieces, which are attached to hydraulic pipes on excavator. Sensors converting hydraulic pressures to electrical signals are connected to adaptors. Finally, Hydrotechnik, which monitors electrical signals as hydraulic pressures, is connected to sensors by cables (Figure 4.8 and Figure 4.9).

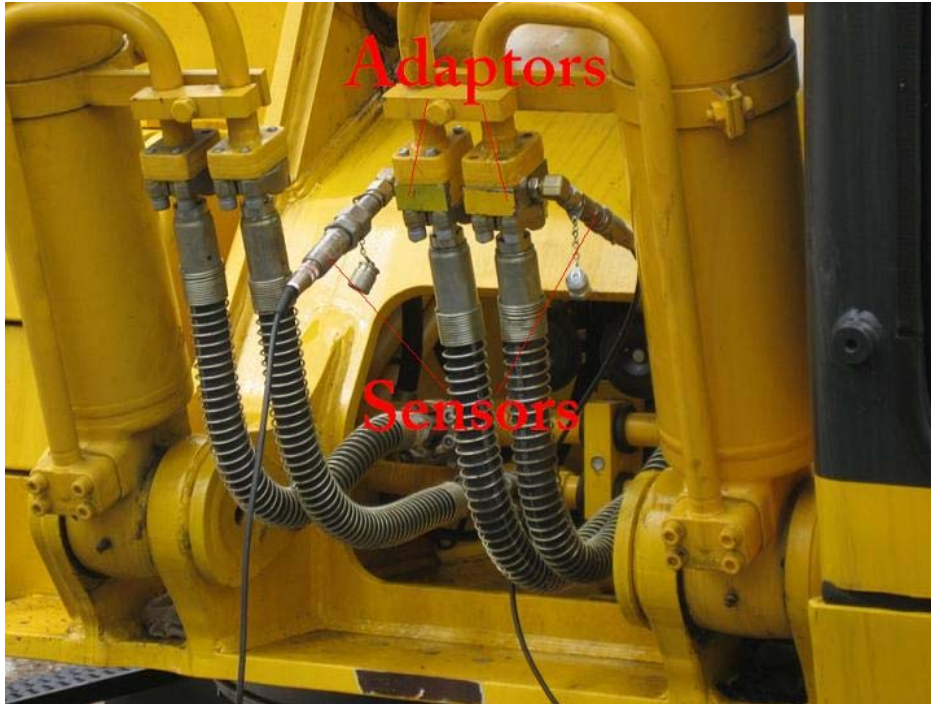


Figure 4.8 – Adaptors and Sensors

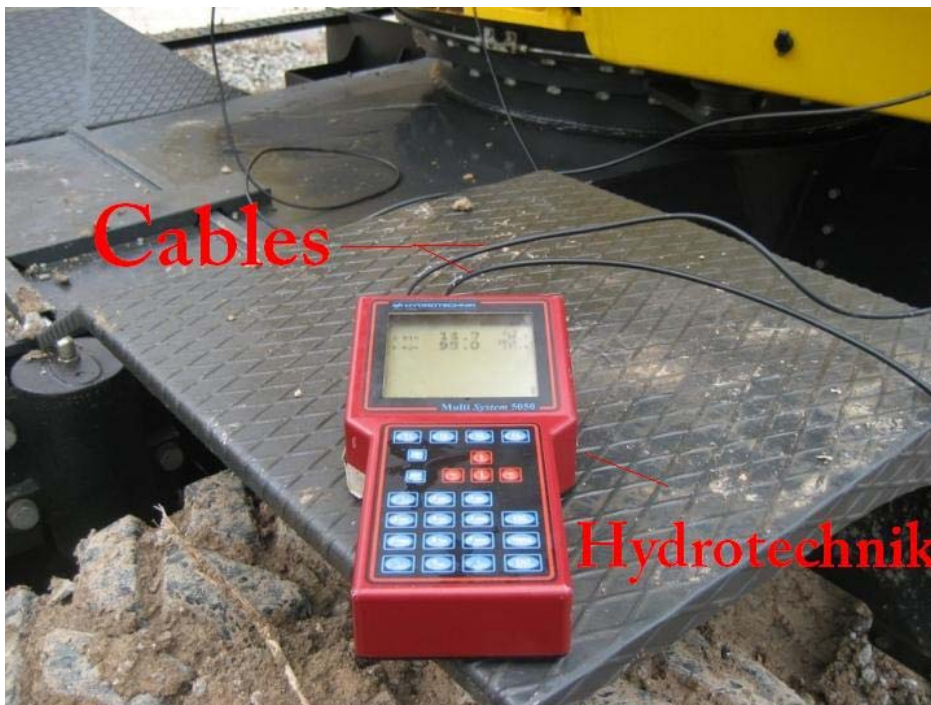


Figure 4.9 – Cables and Hydrotechnik

There are 6 pressure input parameters: two of them are on boom actuator (cylinder and rod chambers), two of them are on arm actuator (cylinder and rod chambers), two of them are on bucket actuator (cylinder and rod chambers) (Figure 4.10). For a perfect data accusation, there should be 6 adaptors, 6 sensors, 6 hydraulic cables and a Hydrotechnik with 6 input channels. However, due to lack of materials (only 3 sensors) and Hydrotechnik's property deficiency (4 input channels) test data cannot be collected in one time. To beat this handicap, test is performed several times with the same conditions. For example, firstly, sensors are connected to adaptors, which are attached to boom actuator's inputs and data are collected. Secondly, sensors are connected to adaptors, which are attached to arm actuator's inputs, and data are collected. Finally, the same procedure is done for bucket actuator. Every test is repeated several times in order to collect accurate data.

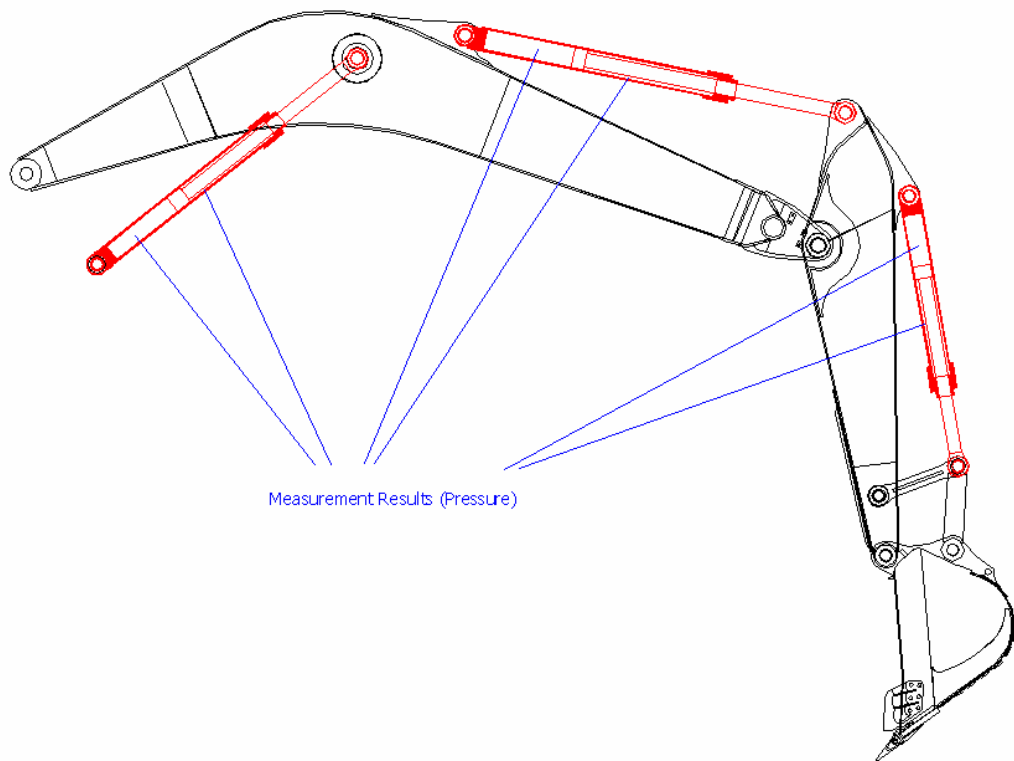


Figure 4.10 – Actuator Chambers

Test starts in determined position (Figure 4.11) for Zelenin's Formula and bucket digs the soil. At the same time pressure is measured from both chambers of boom actuator. After taking several data, same position is assured for collecting data from arm and bucket cylinders and several data are collected for them.



Figure 4.11 – Initial Position in Testing

4.5. Test for Determining Force on One of the Links

The aim of performing pressure test is to obtain forces in hydraulic cylinders and using this data in simulation as inputs. However, in order to verify the simulation, one output should also be tested and compared with the output obtained by the computer simulation. Among different options, strain measurements of connecting rods are selected for verifying the simulation. Therefore 4 strain gauges are used to measure force on connecting rods. Two of them are used for one rod (Figure 4.12) and two of them are used for its parallel rod (Figure 4.13).



Figure 4.12 – Strain Gauges on The Right – Hand Side Rod



Figure 4.13 - Strain Gauges on The Left – Hand Side Rod

Before the digging test, the verification of strain gauges should be completed. In order to do this, a loadcell test is performed for a selected position and results are compared with the theoretical calculations. If loadcell test results and theoretical calculations are compatible with each other, then it can be assumed that strain gauges are collecting data correctly. In Appendix E, the detailed procedure and results of load cell test is explained. To conclude, verification of strain gauges is completed and gauges are usable for digging test.

For performing the strain gauge test during digging the following apparatus are needed:

- Strain Gauges and Terminals
- Data Acquisition System (ESAM Hardware)
- Computer (ESAM Software)
- Cables

Strain gauges are connected to terminals, terminals are connected to ESAM Hardware through cables and ESAM is connected to computer (Figure 4.14 and Figure 4.15).



Figure 4.14 – Strain Gauges and Cables

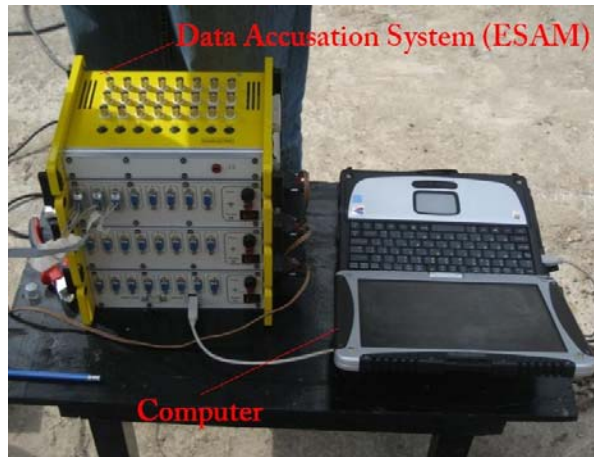


Figure 4.15 – ESAM and Computer

As in case of pressure testing, excavator should be in its standard initial position. After that several data are collected by repeating the same digging motion.

CHAPTER 5

CASE STUDY

5.1. Test Results

5.1.1. Pressure Test Results

After eliminating improper data, pressure results are shown below:

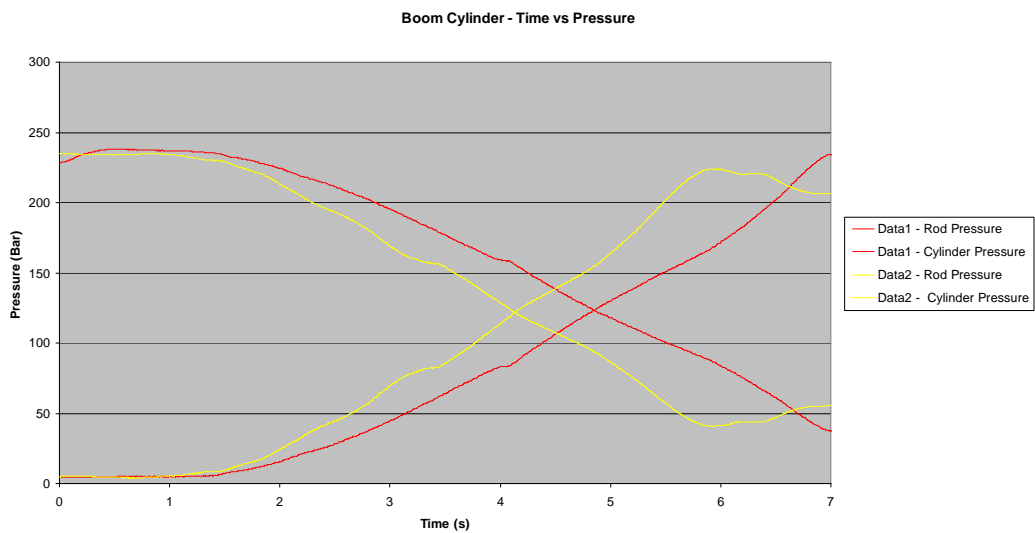


Figure 5.1 – Pressure Results of Boom Cylinder

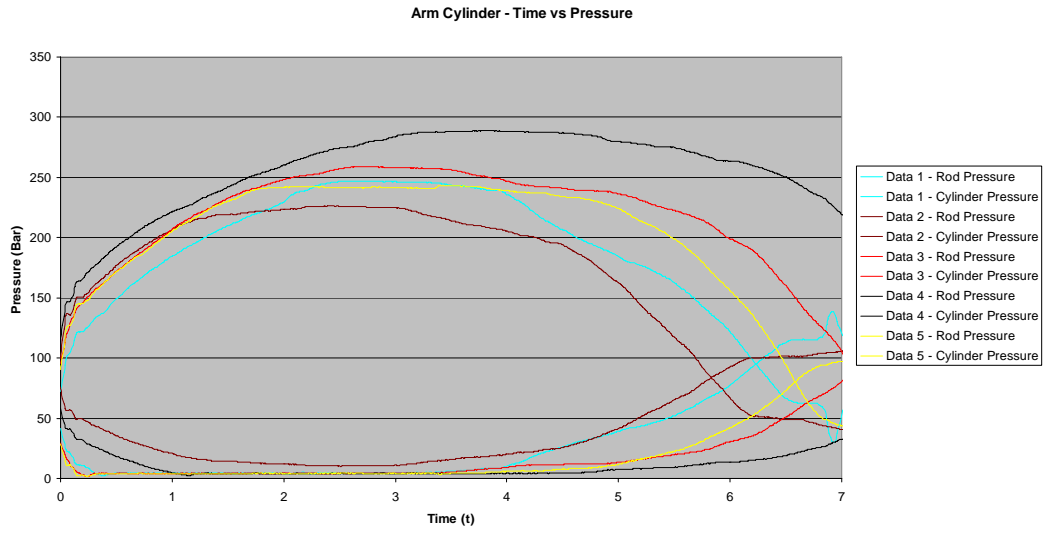


Figure 5.2 – Pressure Results of Arm Cylinder

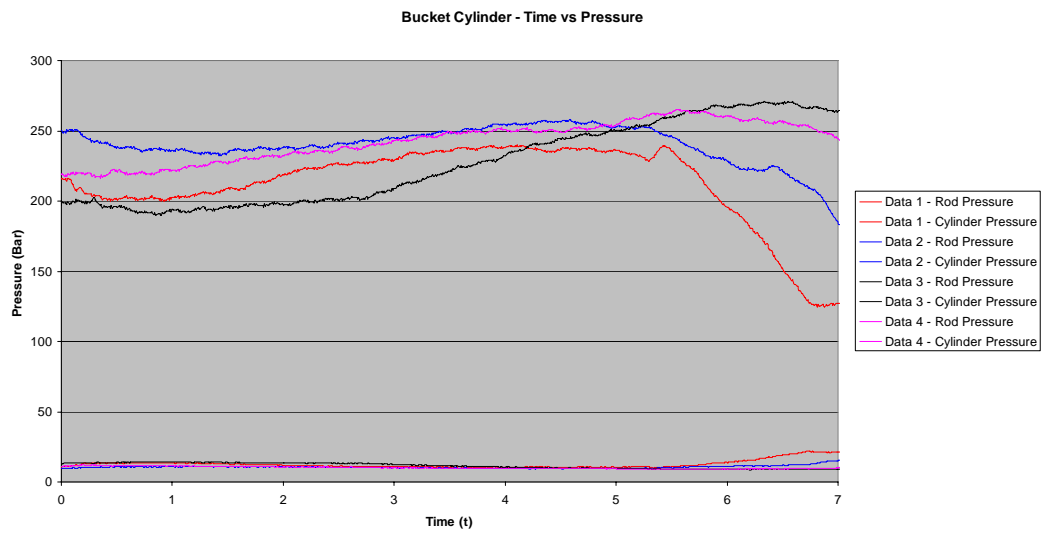


Figure 5.3 – Pressure Results of Arm Cylinder

By using following equations net forces on cylinders can be calculated

$$F_{RodSide} = P_{RodSide} \cdot (A_{Cylinder} - A_{Rod}) \quad (5.1)$$

$$F_{CylinderSide} = P_{CylinderSide} \cdot A_{Cylinder} \quad (5.2)$$

$$^{Boom} F_{Net} = ^{Boom} F_{RodSide} - ^{Boom} F_{CylinderSide} \quad (5.3)$$

$$^{Arm} F_{Net} = ^{Arm} F_{CylinderSide} - ^{Arm} F_{RodSide} \quad (5.4)$$

$$^{Bucket} F_{Net} = ^{Bucket} F_{RodSide} - ^{Bucket} F_{CylinderSide} \quad (5.5)$$

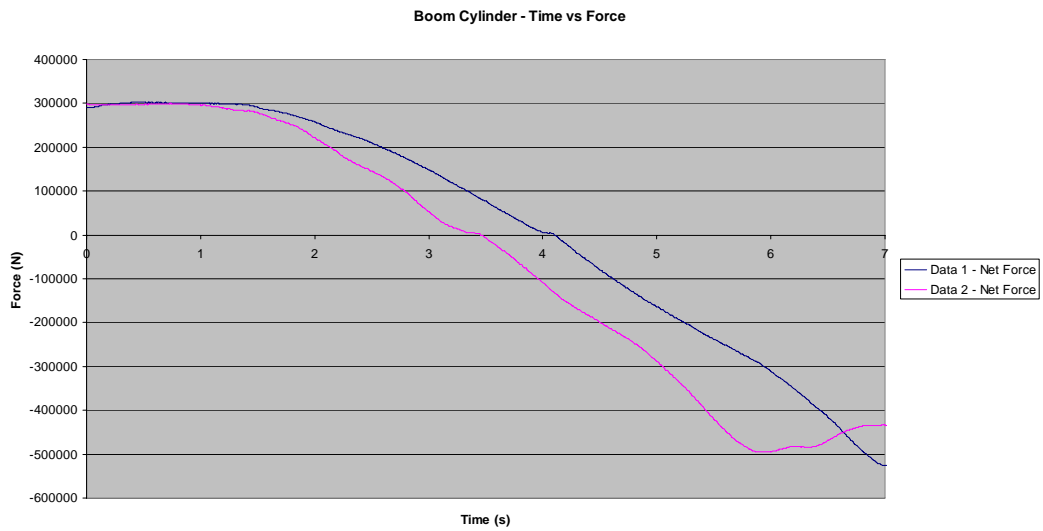


Figure 5.4 – Net Force of Boom Cylinder

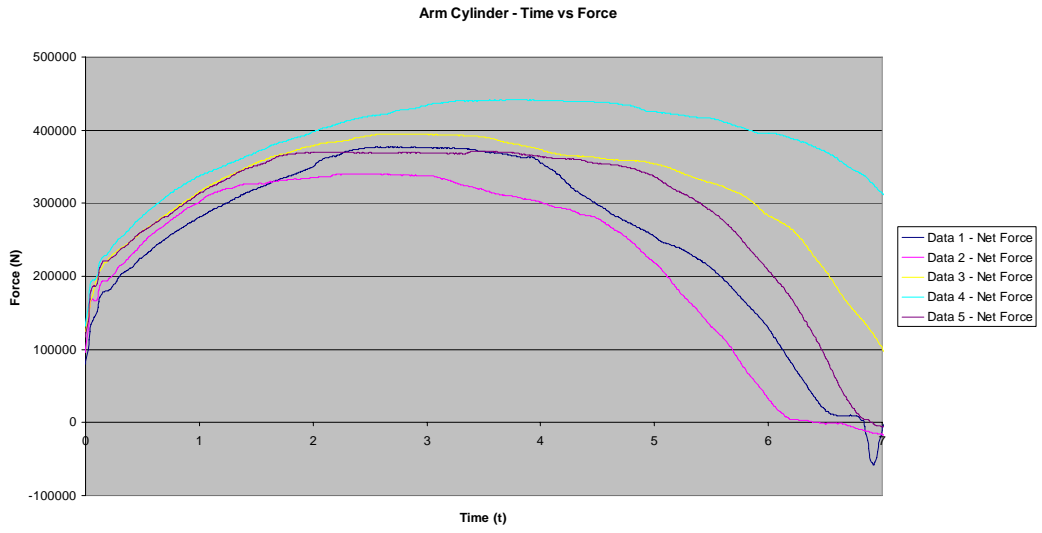


Figure 5.5 – Net Force of Arm Cylinder

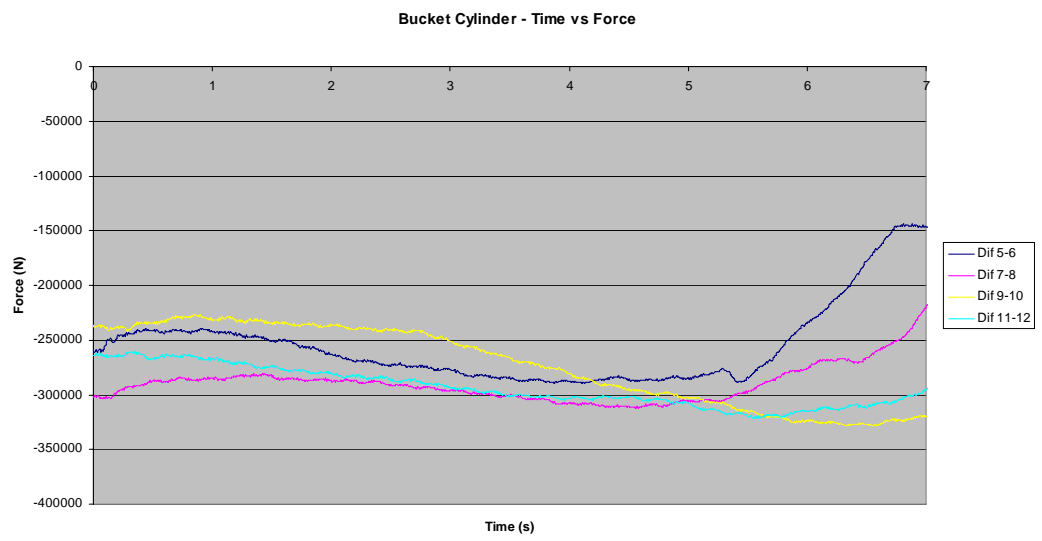


Figure 5.6 – Net Force of Bucket Cylinder

5.1.2. Strain Test Results

After eliminating improper data, strain results are shown below:

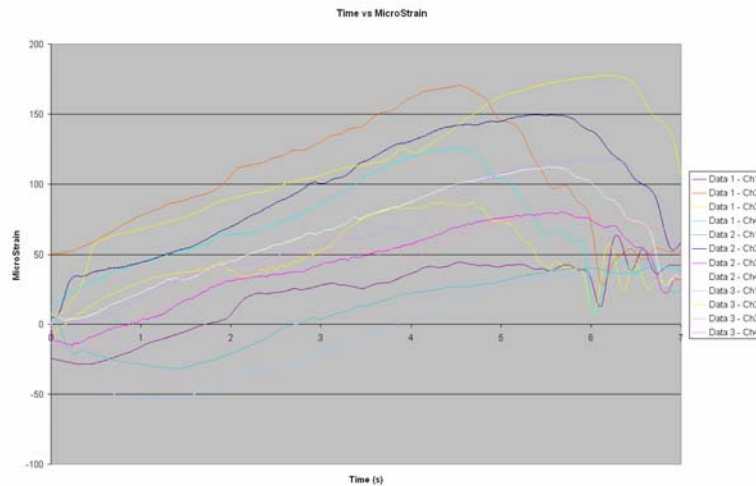


Figure 5.7 – Strains on Connecting Rods

The gauge connected to Channel 4 (Ch 4) gave the most coherent results in the loadcell test. Among the digging test outputs, the second data of gauge Ch4 (Data 2 – Ch 4) gave the most compatible result with the simulation results. Therefore, gauge shown by color white is taken into account during evaluation of the digging test. Digging motion takes almost 6 seconds. Between 6th and 7th seconds, the actuator stops and bucket starts to vibrate. Due to these vibrations, strain results are shown as oscillating in the graph. The conversion of strain data into force value is derived as following:

$$F_{\text{link}} = A_{\text{cross-section}} \cdot E \cdot \varepsilon \quad (5.6)$$

5.2. Iterations for Cohesion Factor of Soil

By using pressure data obtained from digging test, simulation is run in Excavator Dynamics Software (EDS). As mentioned before, digging motion takes almost 6 seconds and in every 0.01 second, one datum is collected. Therefore 600 data are used during simulation. There should be a VBA program, which enters test results and Zelenin Soil Model results into input boxes in EDS simultaneously. For using Zelenin's Formula, soil cohesion factor should be known. It can be estimated by iterations.

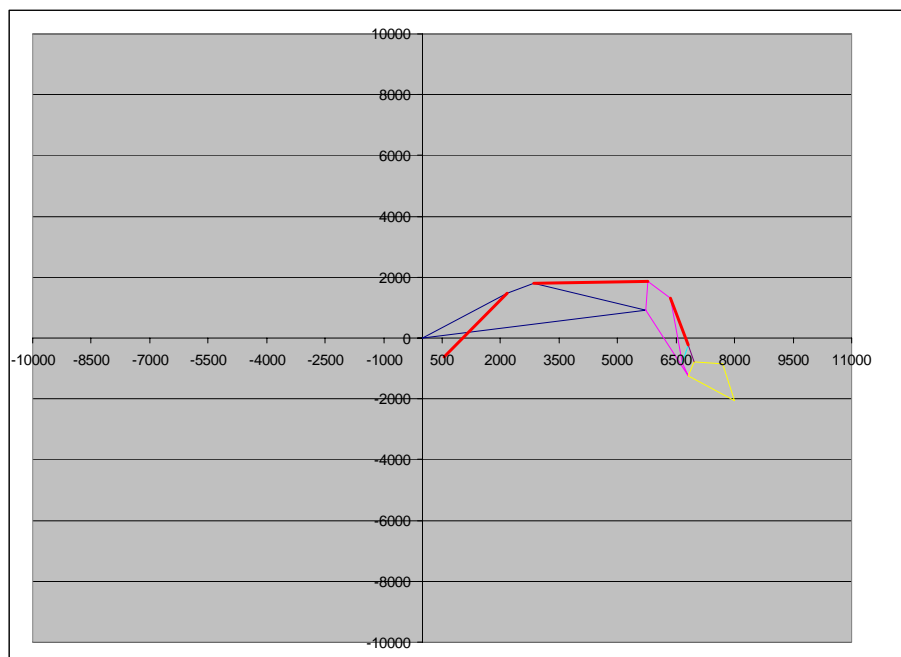


Figure 5.8 – Initial Position of Attachments for Simulation

$$F_r(\Phi) = 10 \cdot C \cdot 1448^{1.35} \cdot (\cos(54.45 - \Phi) - \cos(54.45))^{1.35} \quad (5.7)$$

- Iteration 1 - Cohesion factor $C=10000$ Pa

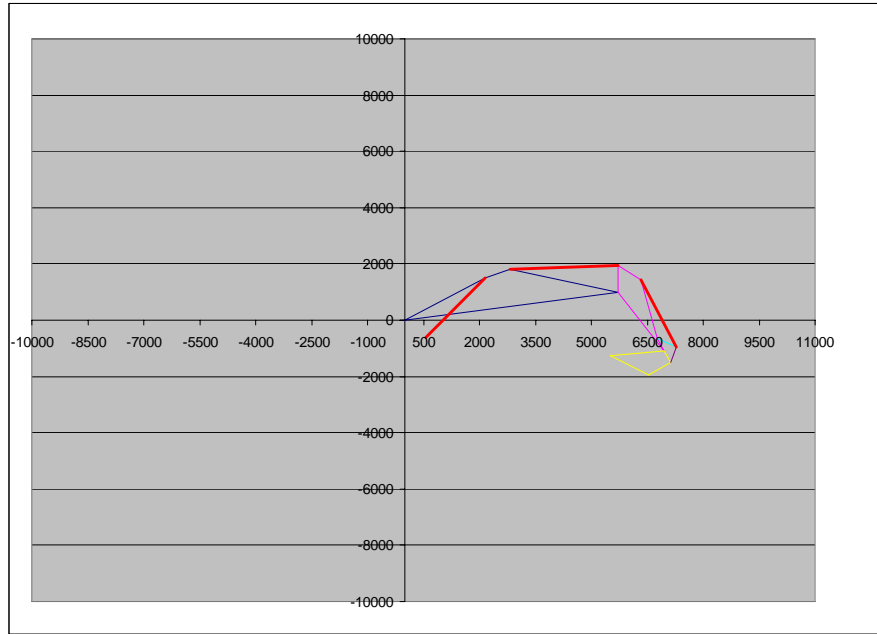


Figure 5.9 – Final Position of Attachments for $C=10000$ Pa

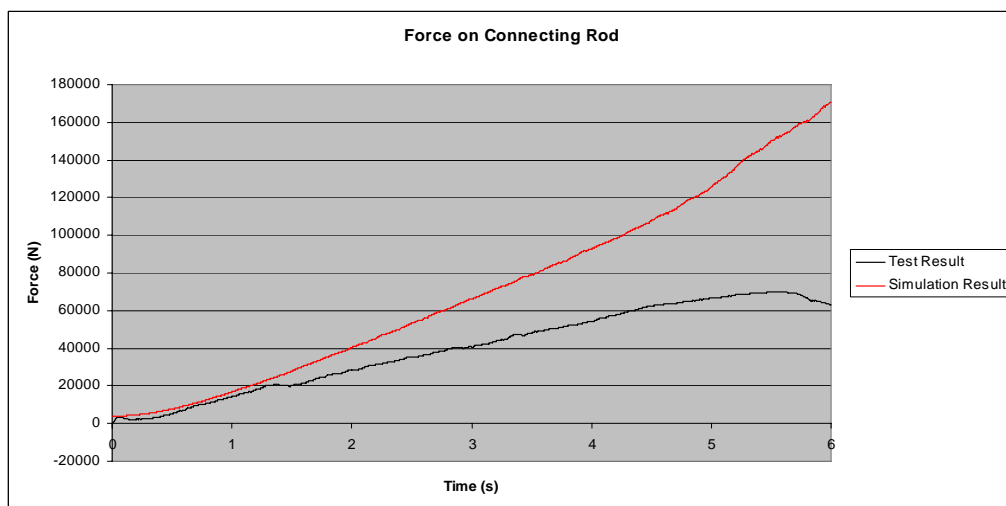


Figure 5.10 – Test and Simulation Compatibility for $C=10000$ Pa

- Iteration 2 - Cohesion factor $C=15000$ Pa

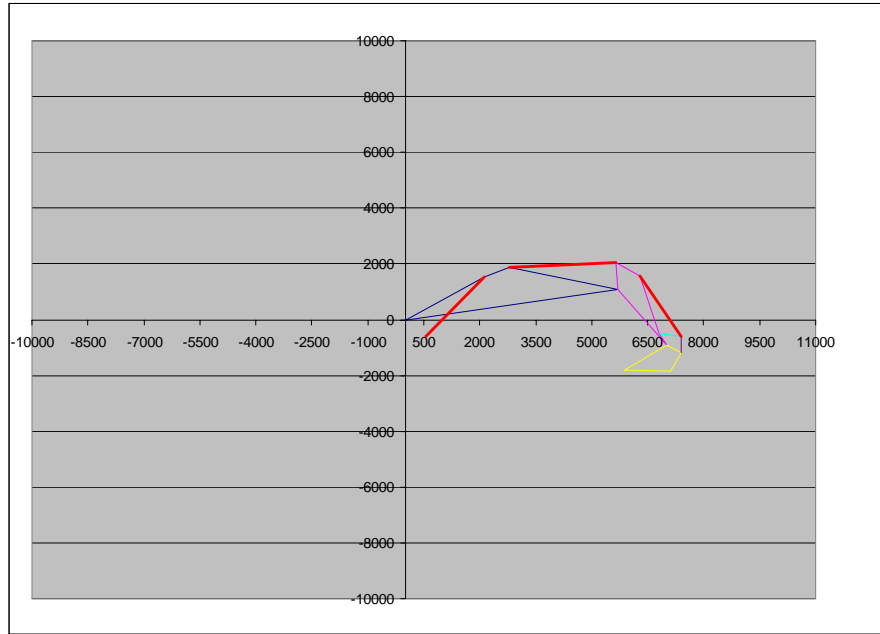


Figure 5.11 – Final Position of Attachments for $C=15000$ Pa

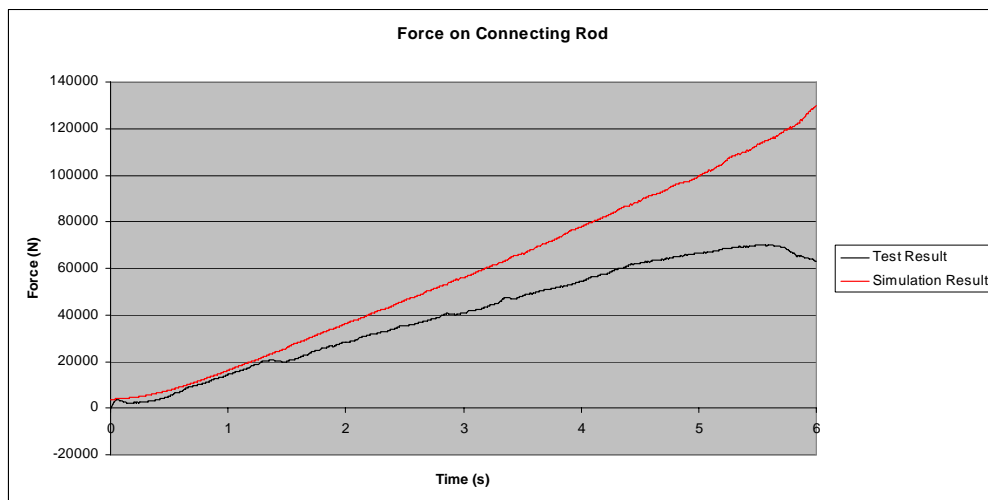


Figure 5.12 – Test and Simulation Compatibility for $C=15000$ Pa

- Iteration 3 - Cohesion factor $C=20000$ Pa

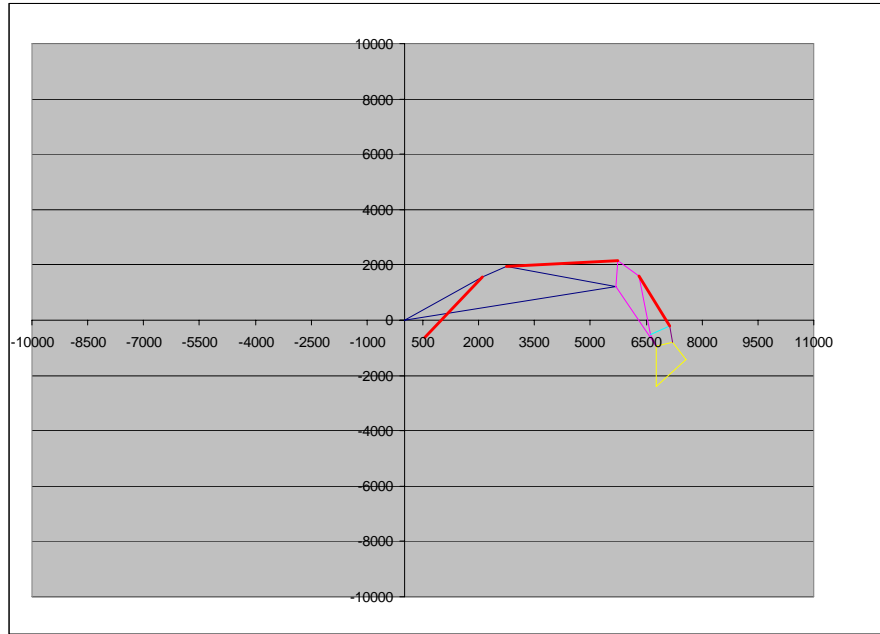


Figure 5.13 – Final Position of Attachments for $C=20000$ Pa

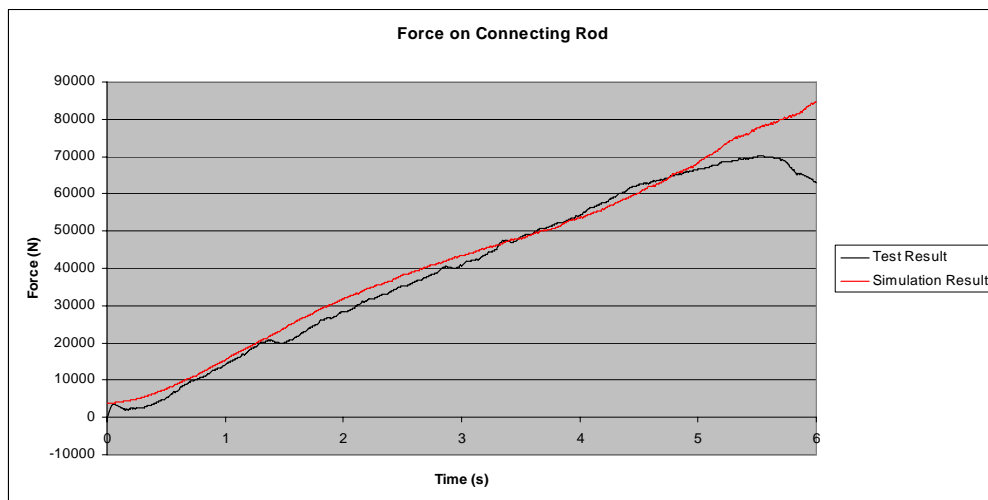


Figure 5.14 – Test and Simulation Compatibility for $C=20000$ Pa

- Iteration 3 - Cohesion factor $C=17000$ Pa

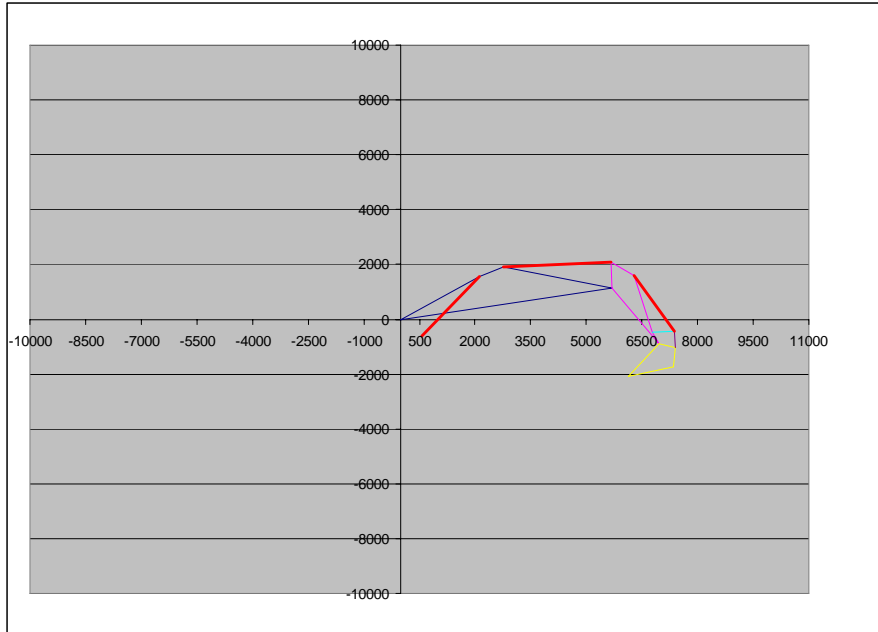


Figure 5.15 – Final Position of Attachments for $C=17000$ Pa

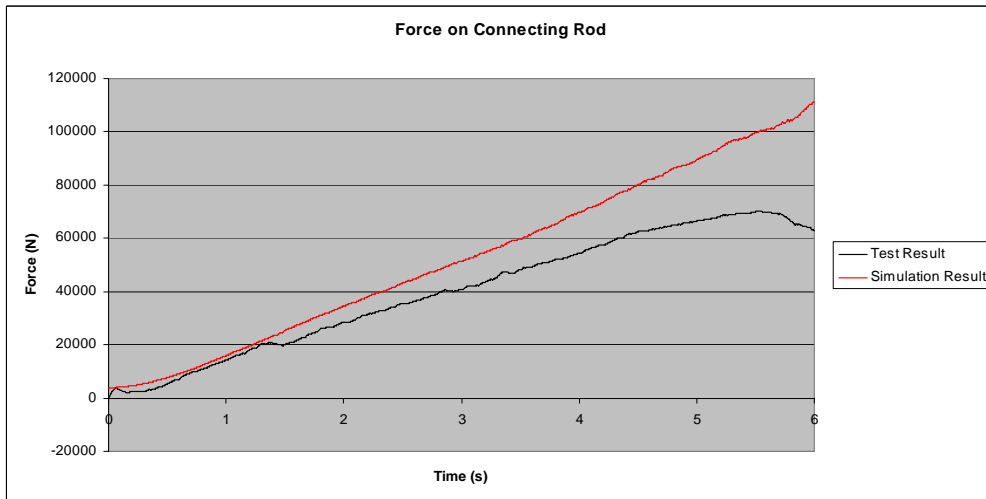


Figure 5.16 – Test and Simulation Compatibility for $C=17000$ Pa

5.3. Results

Samples, which are shown by using cohesions 17000 Pa and 20000 Pa, are the most appropriate results. Sample done by using 17000 Pa cohesion factor gives an excellent simulation result. On the other hand, sample done by using 20000 Pa cohesion factor gives much better force verification result than other sample. However, its simulation result is not as good as the sample done by using 17000 Pa cohesion factor. Therefore, it can be concluded that, the cohesion factor which can be applied to this problem is in anywhere between 17000 Pa and 20000 Pa. So, both results can be accepted and analyses are performed for both results. When a simulation runs in a dynamic analysis program, different outputs can be obtained. These are reaction forces on joints; accelerations, velocities and displacements of bodies.

Yellow lines on joint reaction forces tables indicate the forces on joints when excavator is in “bucket breakout” position.

5.3.1. Reaction Forces on Joints

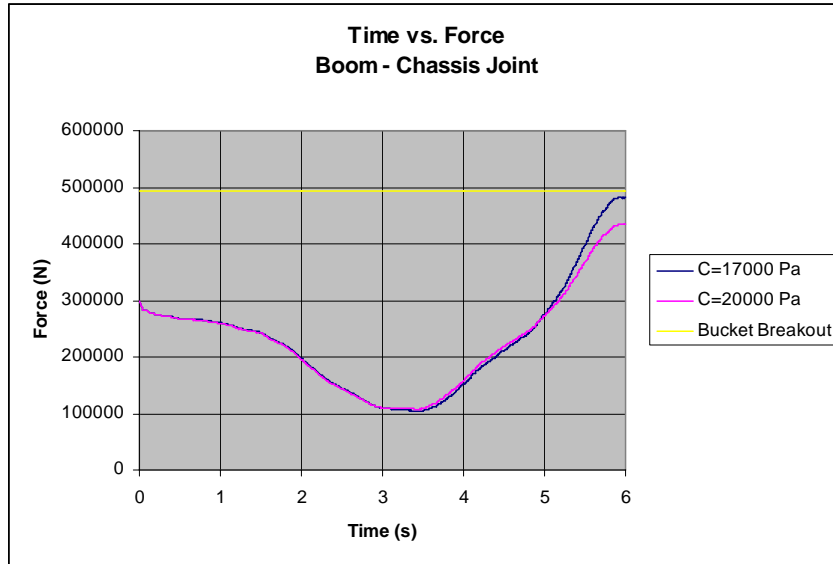


Figure 5.17 – Time vs. Force Chart of the Joint between Boom and Chassis

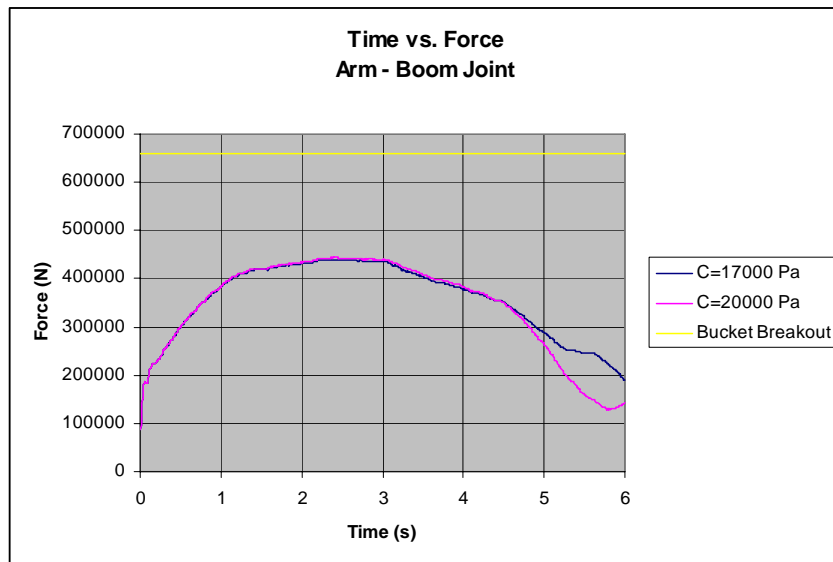


Figure 5.18 – Time vs. Force Chart of the Joint between Arm and Boom

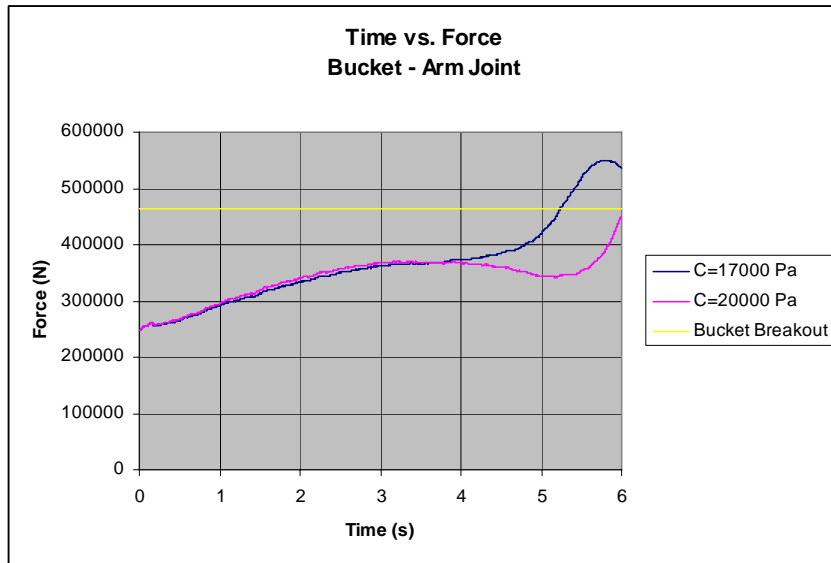


Figure 5.19 – Time vs. Force Chart of the Joint between Bucket and Arm

5.3.2. Accelerations of Bodies

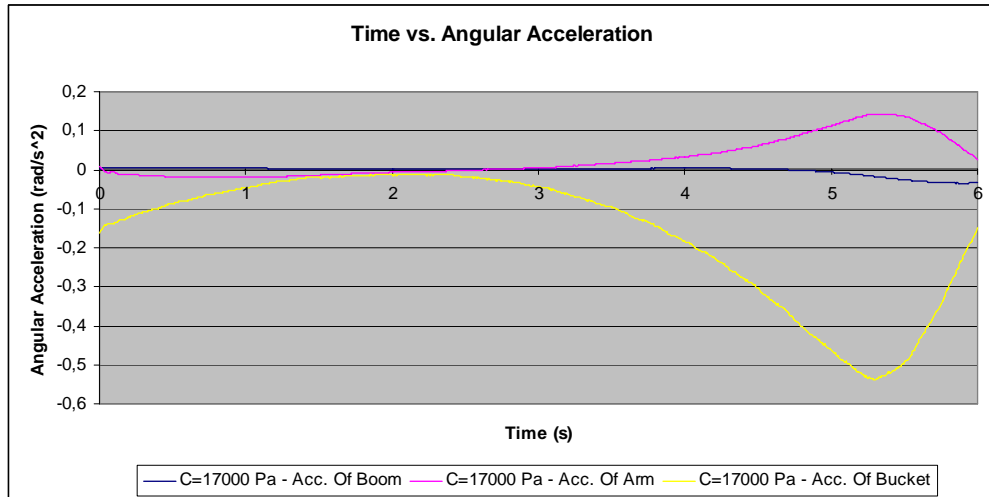


Figure 5.20 – Accelerations of Bodies (cohesion factor = 17000 Pa)

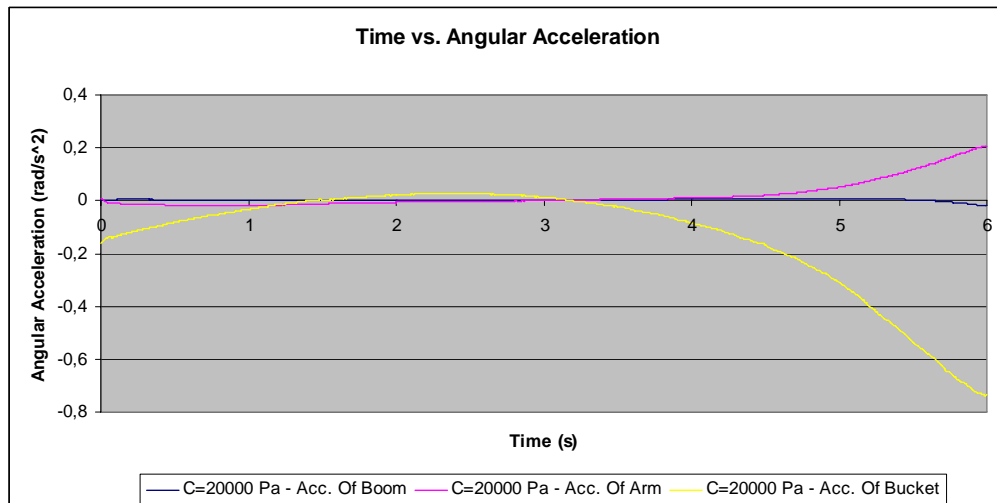


Figure 5.21 – Accelerations of Bodies (cohesion factor = 20000 Pa)

5.3.3. Velocities of Bodies

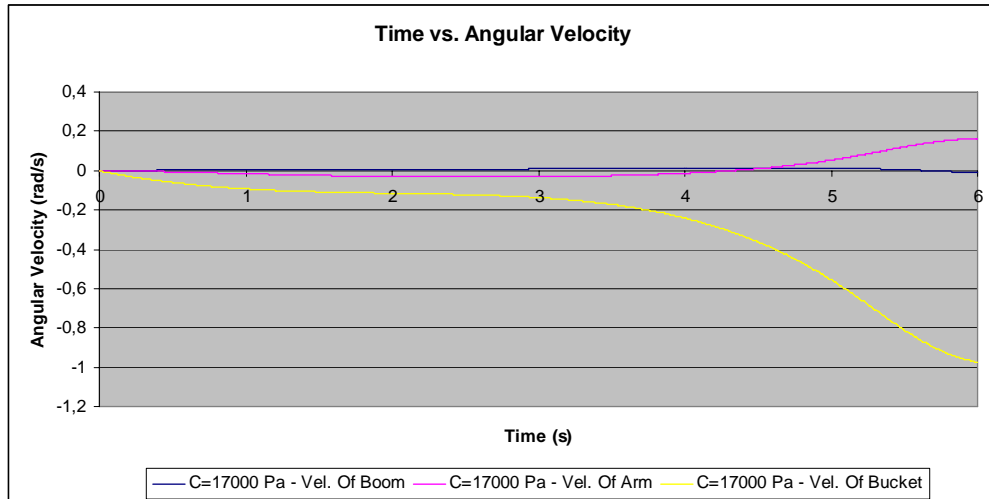


Figure 5.22 – Velocities of Bodies (cohesion factor = 17000 Pa)

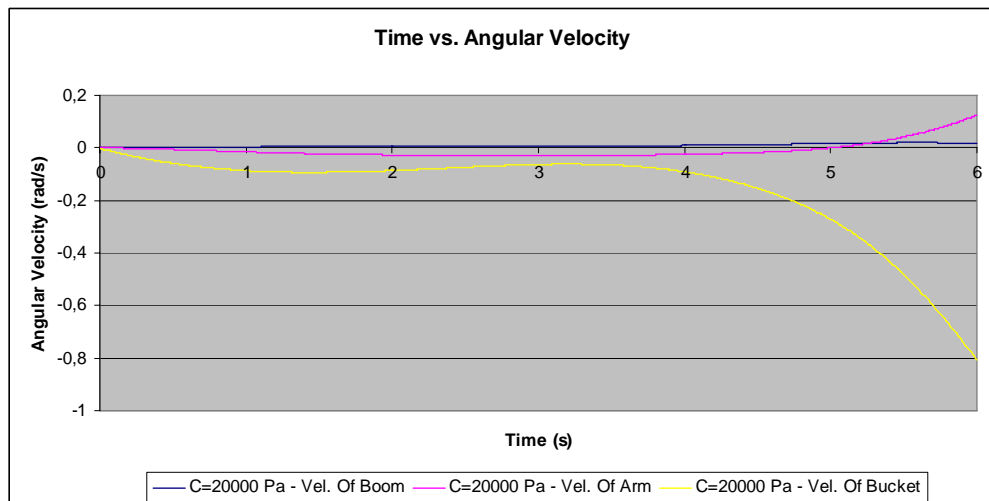


Figure 5.23 – Velocities of Bodies (cohesion factor = 20000 Pa)

5.3.4. Positions of Bodies

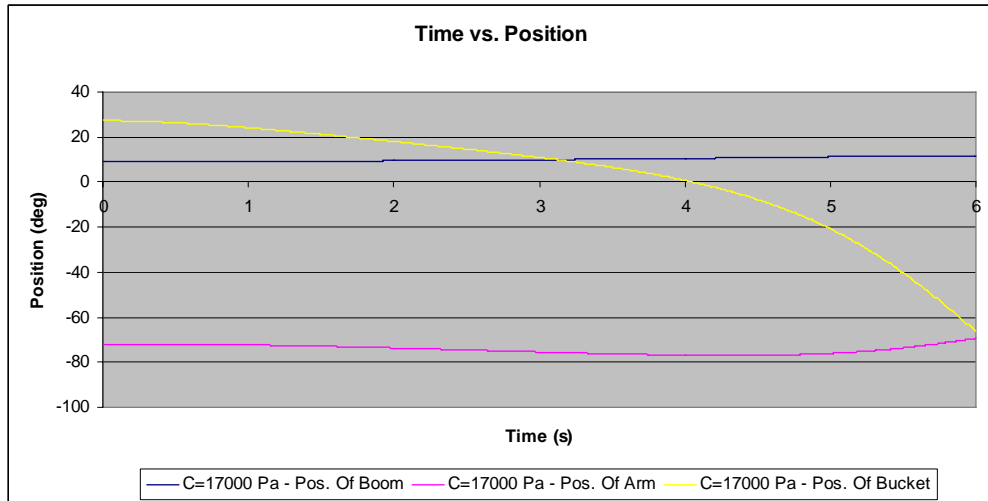


Figure 5.24 – Positions of Bodies (cohesion factor = 17000 Pa)

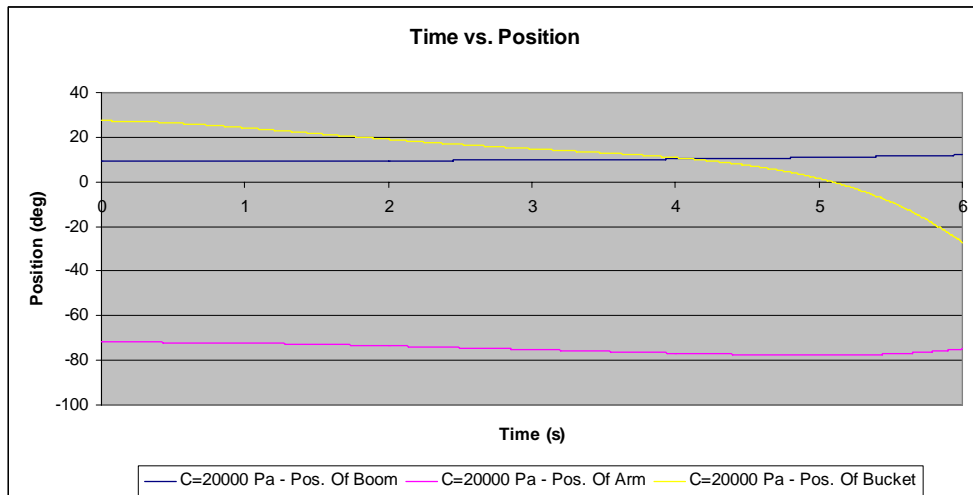


Figure 5.25 – Positions of Bodies (cohesion factor = 20000 Pa)

5.4. Comparison with a Commercial Software

After completing the analysis in EDS, the simulation is performed in a commercial dynamics program. By this way, the compatibility of EDS and the commercial program can be tested and a comparison can be done. For this purpose, MSC ADAMS is chosen as commercial program.

5.4.1. Simulation Conditions in ADAMS

As in case of EDS, ADAMS should use test results as input parameters. Thus, test results should be converted into proper equations and written to ADAMS as equations. The equation of soil reaction is known; however, input forces of cylinders are disorganized data. In order to obtain these equations, the “Add Trendline” option of Excel is used.

$$F_{boom-cylinder}(t) = 432.22 \cdot t^6 - 8025.2 \cdot t^5 + 55726 \cdot t^4 - 174147 \cdot t^3 + 203089 \cdot t^2 - 82794 \cdot t + 304693 \quad (5.8)$$

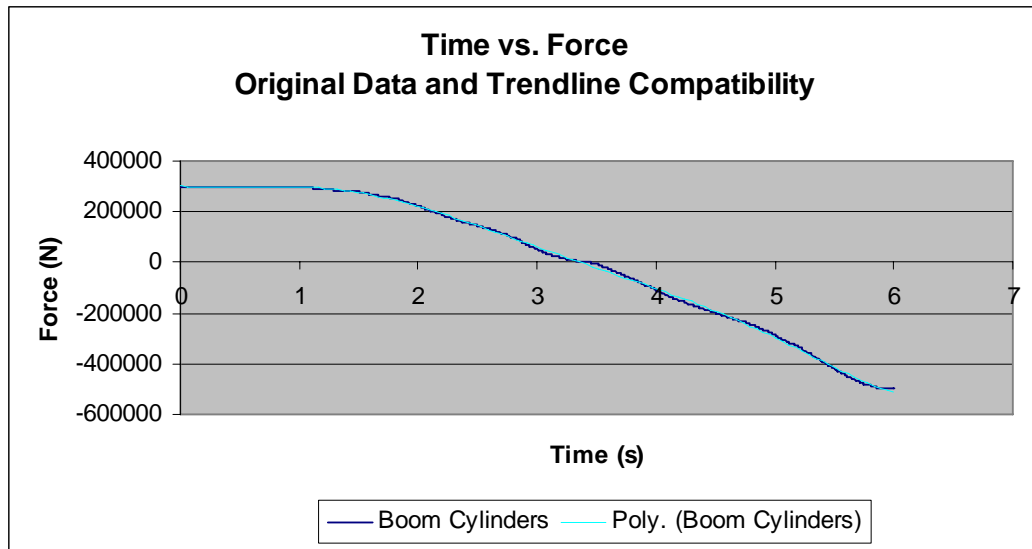


Figure 5.26 - Original Data and Trendline Compatibility for Boom Cylinders

$$F_{arm-cylinder}(t) = 55.87 \cdot t^6 - 740.01 \cdot t^5 + 990.47 \cdot t^4 + 21511 \cdot t^3 - 124023 \cdot t^2 + 261122 \cdot t + 143365 \quad (5.9)$$

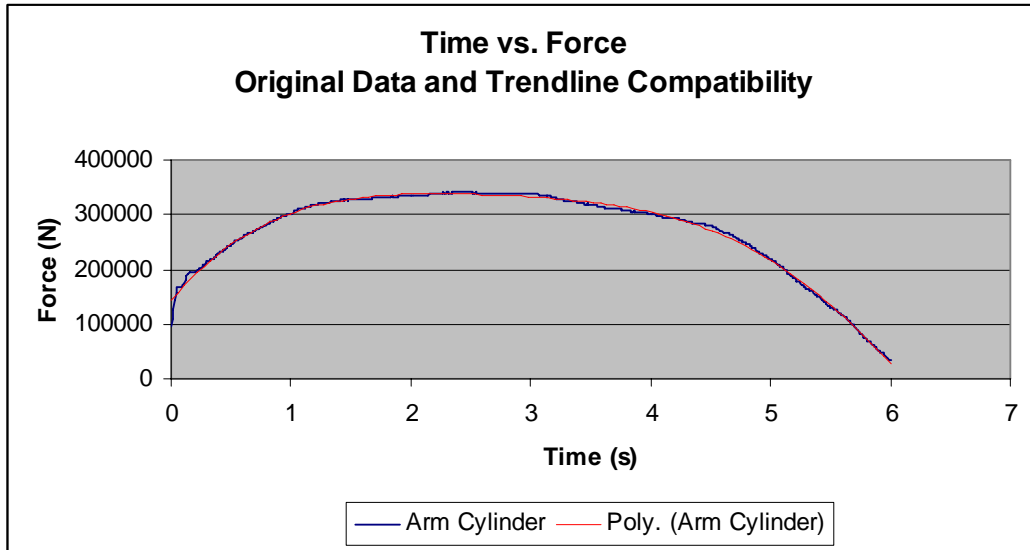


Figure 5.27 - Original Data and Trendline Compatibility for Arm Cylinder

$$F_{bucket-cylinder}(t) = -33.321 \cdot t^6 + 695.16 \cdot t^5 - 5343.7 \cdot t^4 + 20582 \cdot t^3 - 45741 \cdot t^2 + 51376 \cdot t - 305709 \quad (5.10)$$

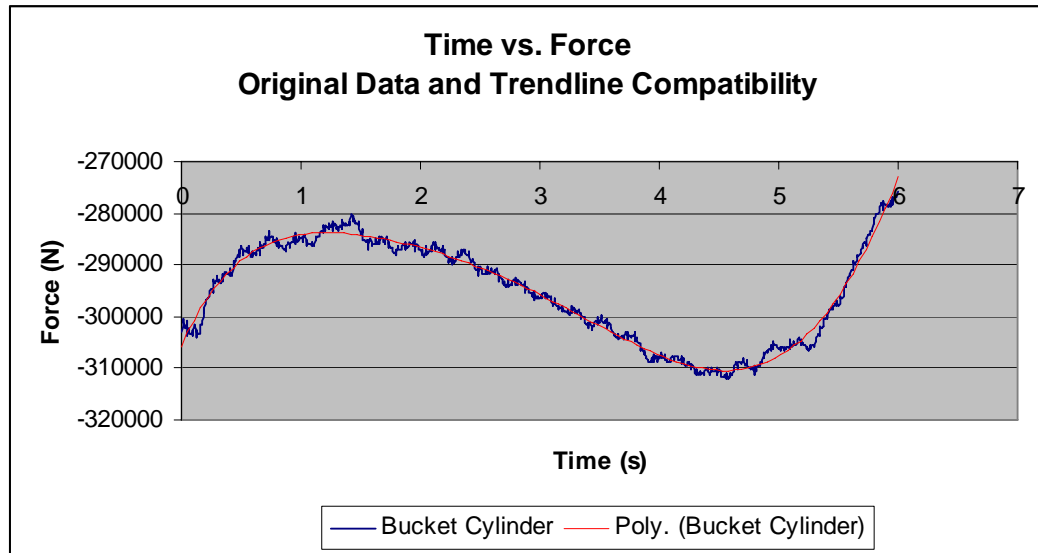


Figure 5.28 - Original Data and Trendline Compatibility for Arm Cylinder

Input parameters used in EDS and ADAMS are almost equal. In addition to that, start positions and mass properties are also same.

5.4.2. Results

Simulation is performed for 20000 Pa cohesion factor, results are tabulated and compared with results obtained from EDS.

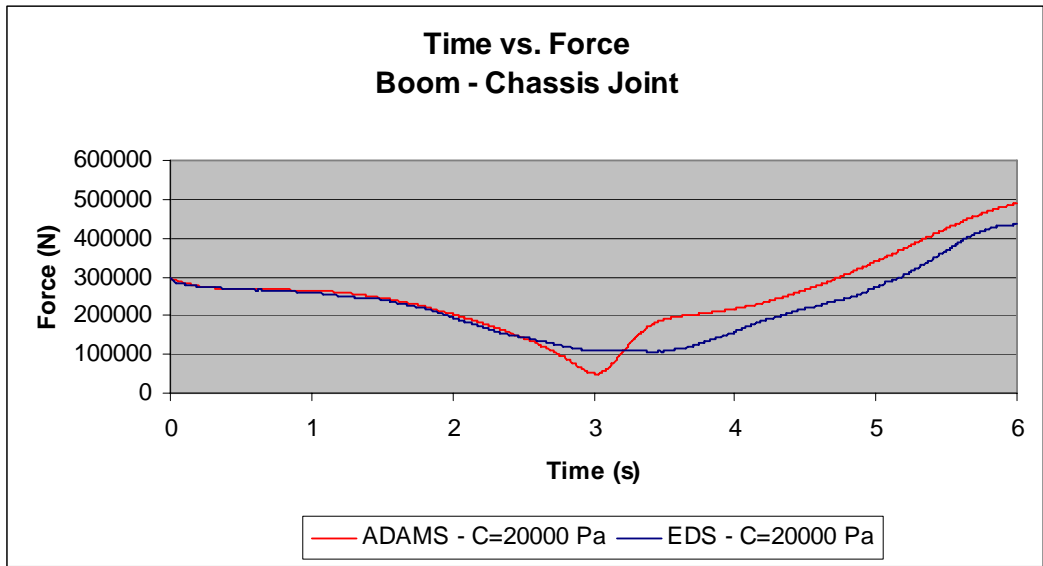


Figure 5.29 – Comparison of Results (Boom – Chassis Joint)

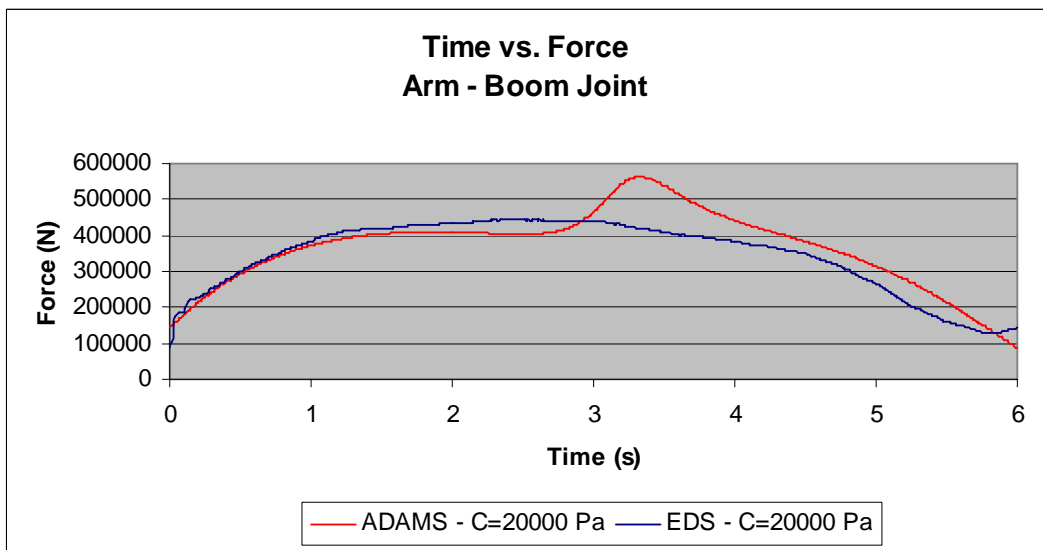


Figure 5.30 – Comparison of Results (Arm – Boom Joint)

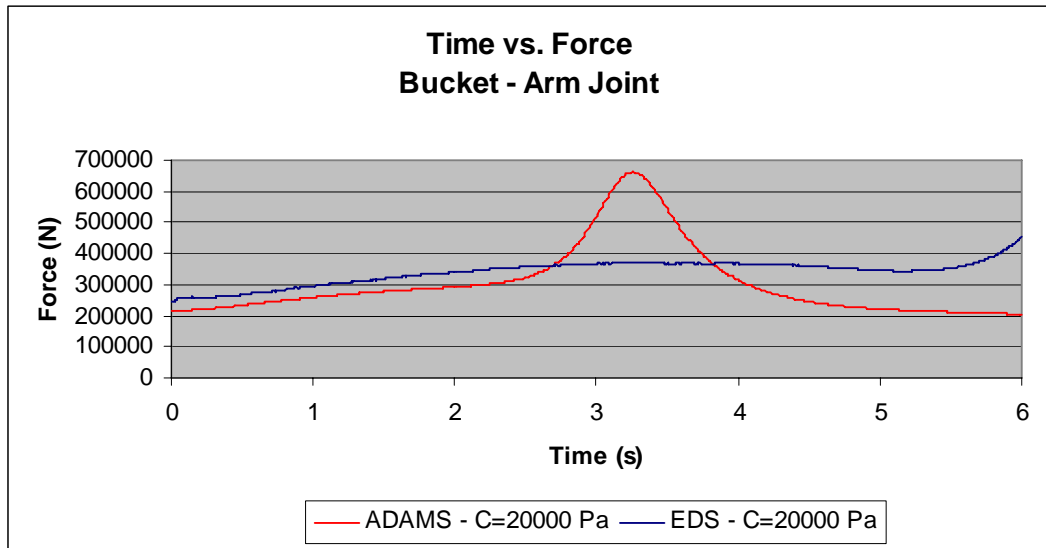


Figure 5.31 – Comparison of Results (Bucket – Arm Joint)

There are differences between ADAMS and EDS results. Especially, there is a peak on ADAMS results when bucket is in middle position as shown in Figure 5.32.



Figure 5.32 - Bucket's Middle Position on ADAMS

In order to overcome this problem, different cohesion factors are experimented on ADAMS. The results are tabulated on Figure 5.33 and Figure 5.34

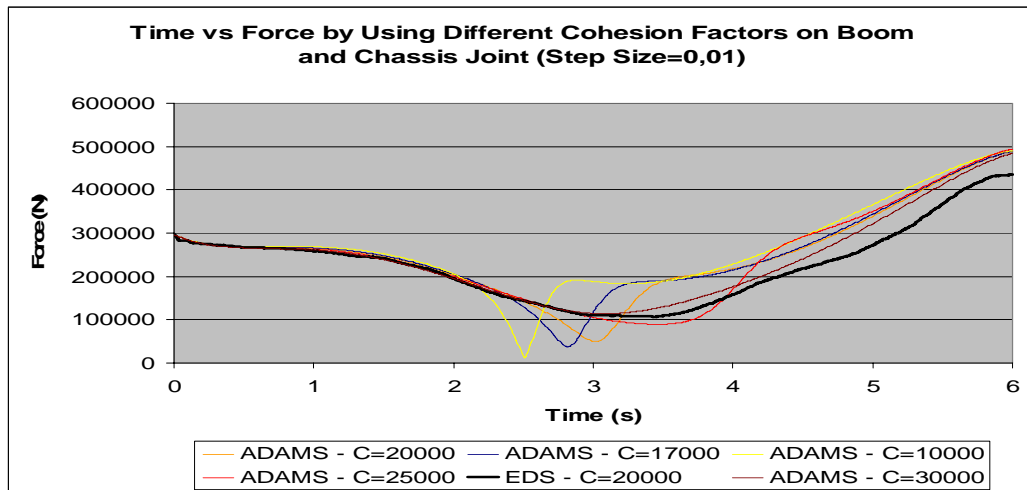


Figure 5.33 - Time vs. Force by Using Different Cohesion Factors on Boom and Chassis Joint (Step Size=0,01)

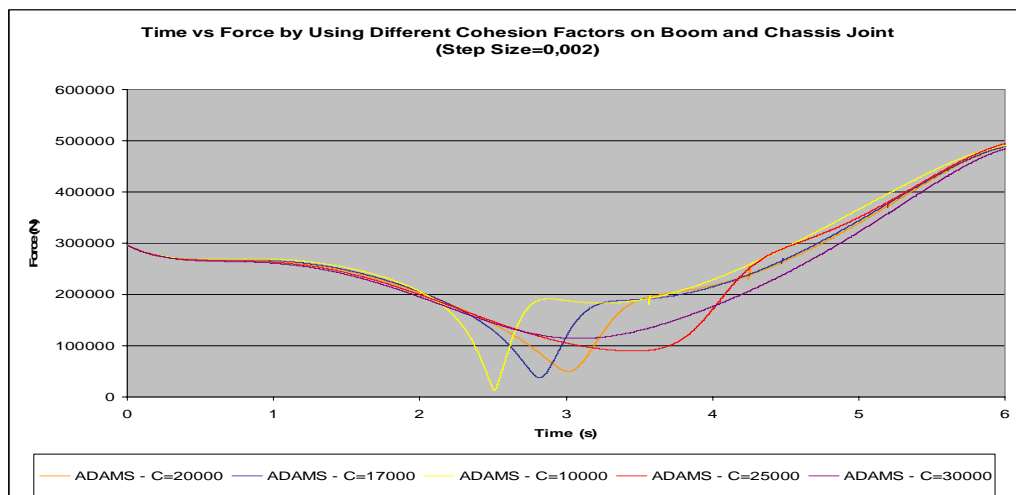


Figure 5.34 - Time vs. Force by Using Different Cohesion Factors on Boom and Chassis Joint (Step Size=0,002)

From the graphs, it is understood that ADAMS gives results with peaks. In order to understand why these peaks are formed, the algorithm of ADAMS should be investigated.

CHAPTER 6

DISCUSSION AND CONCLUSIONS

In this work, a mathematical model of an excavator is built and a dynamic analysis is performed during digging motion. In order to achieve this process a computer program is written that contains kinematic analysis of mechanisms, dynamic analysis of bodies and numerical analysis for simulation of the model.

The first step of building a mathematical model is kinematic analysis. Excavators are complex mechanisms with 3 degrees of freedom. Therefore, with three input parameters, all variables of mechanisms are formulated. Secondly, dynamic analysis is performed by using Recursive Newton – Euler Formulation. So, for three bodies, three equations of motions are written. Finally, simulation is run by solving equations of motions. Since these are 2nd order, non-linear, differential equations, a numerical method is used. For this purpose, 4th Order Runge – Kutta Method is chosen.

After building mathematical model of an excavator, inputs for simulation are determined. Three actuator forces are determined by a controlled test. Soil reaction's magnitude and direction, which are the remaining inputs, are determined by Zelenin's Equation.

In order to verify the simulation, another test is performed. In this test, strain gauges are glued on one of the links and forces are measured during digging motion. Also a loadcell test is done to verify the strain gauges.

For simulation, there is one unknown remaining, C , cohesion factor of soil. For fixing proper cohesion factor values, several iterations are made. After simulations, final position of bucket and force on the link on which strain gauges are glued are observed and compared with the desired conditions.

After fixing cohesion factor value and running simulation, all results of motion phenomena can be observed. Forces on joints, accelerations, velocities and position changes of bodies are plotted.

Dynamic analysis means, analyzing a moving object with respect to time. Therefore, analyzing an object dynamically gives an idea about its motion and affecting forces with changing time. First advantage of knowing force and motion characteristics of an object is that engineers can predict dynamic forces, which are formed due to accelerated motion, before the motion. So, if required, enhancements can be done on the body and stresses on the structure may be decreased. Second advantage of dynamic analysis is that motion planning of a mechanism can be done. Designing a manipulator's path in a time interval can be achieved by knowing its acceleration and velocity. Finally a computer aided control system for mechanical systems can be established. A computer controlled excavator can perform much more accurate operations than a manually controlled one. In recent years, a trend in this direction is seen in the literature [22]

The work and Excavator Dynamics Software (EDS) can be improved by doing following additions and changes: combining with a hydraulic model, adding frictions, adding inertial effects of connecting rods, cylinder – pistons, hoses and pipes etc. and working with an accurate soil model.

EDS is an Excel based program. That means, the construction machines company that will use EDS is not going to pay extra fee other than Office programs. Besides, EDS is a flexible program that users can do any change in their model. EDS is an expert in modeling digging motion, since 3-D effects are not considered in its background. Therefore, results will be out of noise coming from 3 dimensional effects. However, because of its flexibility, another user can develop EDS for 3-D applications very simply. All codes, equations, matrices etc. are exhibited in the EDS. In other words, it is an open source code. The flexibility of Excel provides improvements on codes. In addition to that, users can see all processes simultaneously. On the other hand, one can export a solid model from a 3-D modeling program into ADAMS, so there will not be any extra effort in order to obtain mass and mass moments of inertia information. Visual aesthetic of models in ADAMS are much more beautiful than models in EDS.

If simulation time or step size increases then error accumulation grows and simulation may not give proper results at the end. An accurate result also depends on the method used in numerical analysis. Different numerical analysis methods used in EDS and ADAMS may cause different result.

This work does not include a strength analysis of parts. Therefore, there is no need to work on a hard soil and the data acquisition test is performed in a random environment for collecting real data.

APPENDIX A

KINEMATIC ANALYSIS OF EXCAVATOR ATTACHMENTS WORKING IN TWO DIMENSIONS

A detailed formulation of kinematic analysis of excavators is explained in this appendix.

A.1.Chassis and Boom

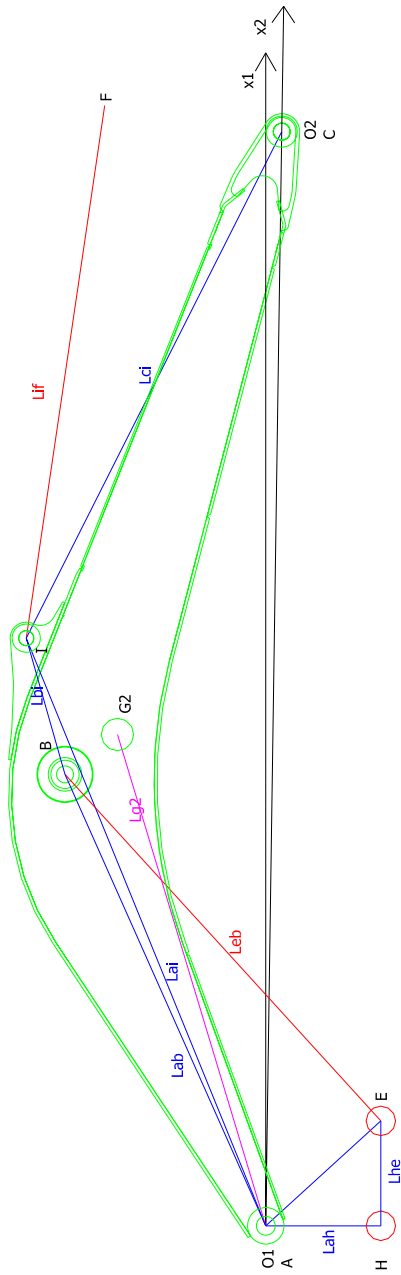


Figure A.1 – Constant and Variables Lengths of Boom

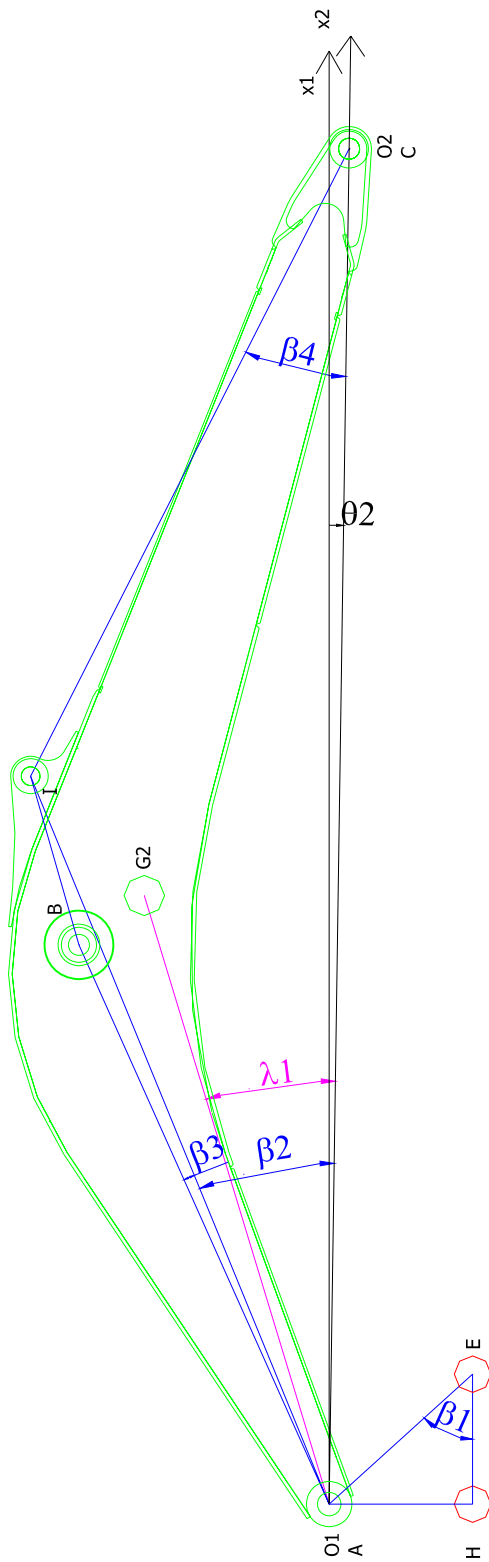


Figure A.2 – Input and Constant Angles of Boom

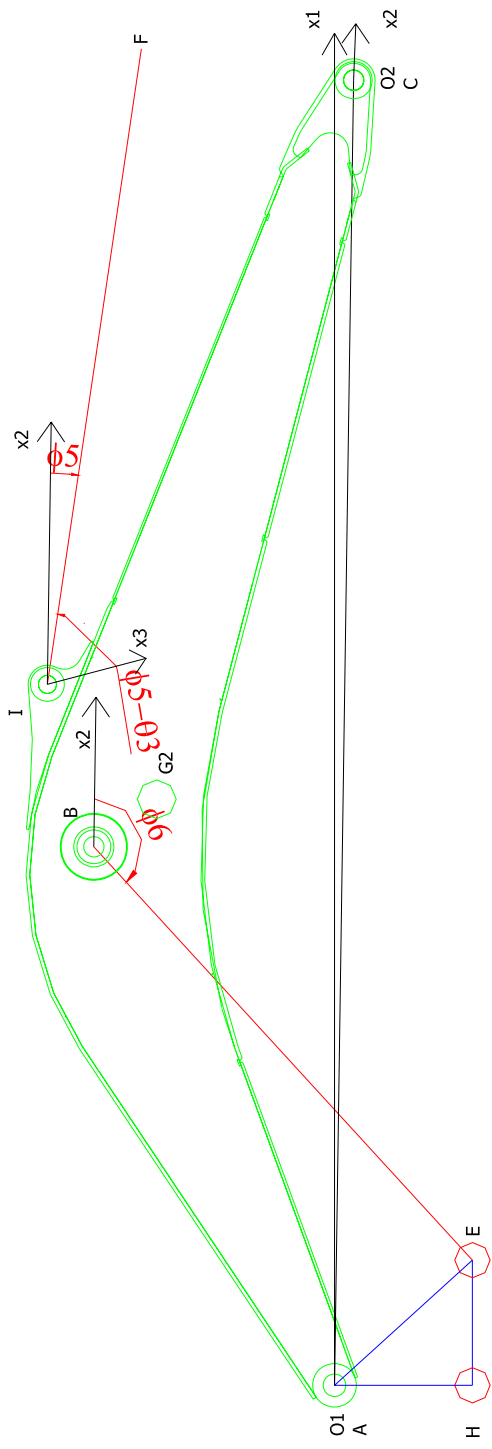


Figure A.3 – Variable Angles

$$Leb = \sqrt{(Lah^2 + Lhe^2) + Lab^2 - 2 \cdot \sqrt{Lah^2 + Lhe^2} \cdot Lab \cdot \cos(\beta_1 + \theta_2 + \beta_2 + \beta_3)} \quad (\text{A.1})$$

$$\phi_5 = \arctan\left(\frac{L2 \cdot \sin(\theta_2) + Lcf \cdot \sin(\theta_2 + \theta_3 + \beta_5 + \beta_6 + \beta_7) - Lai \cdot \sin(\theta_2 + \beta_2)}{L2 \cdot \cos(\theta_2) + Lcf \cdot \cos(\theta_2 + \theta_3 + \beta_5 + \beta_6 + \beta_7) - Lai \cdot \cos(\theta_2 + \beta_2)}\right) - \theta_2 \quad (\text{A.2})$$

$$\phi_6 = -\pi - (\theta_2 - \arctan\left(\frac{Lah + Lab \cdot \sin(\theta_2 + \beta_2 + \beta_3)}{-Lhe - Lab \cdot \cos(\theta_2 + \beta_2 + \beta_3)}\right)) \quad (\text{A.3})$$

A.2. Arm

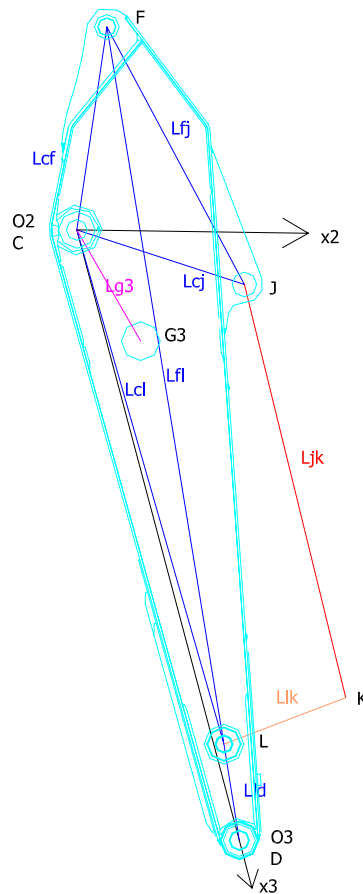


Figure A.4 – Constant and Variables Lengths of Arm

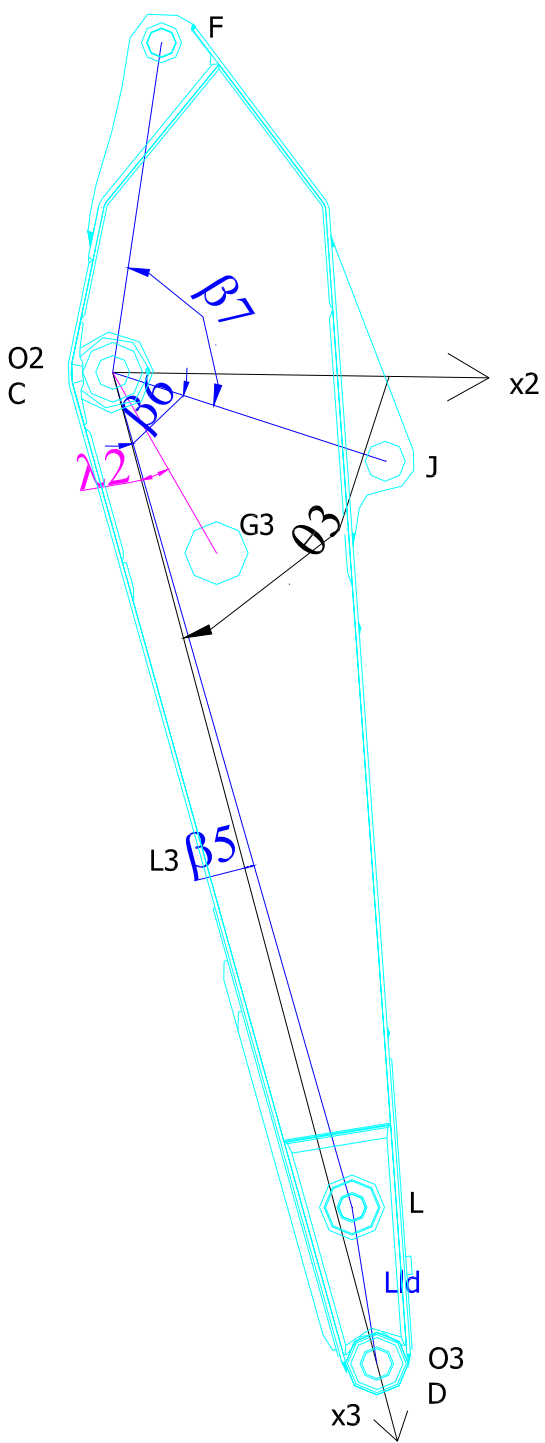


Figure A.5 – Input and Constant Angles of Arm

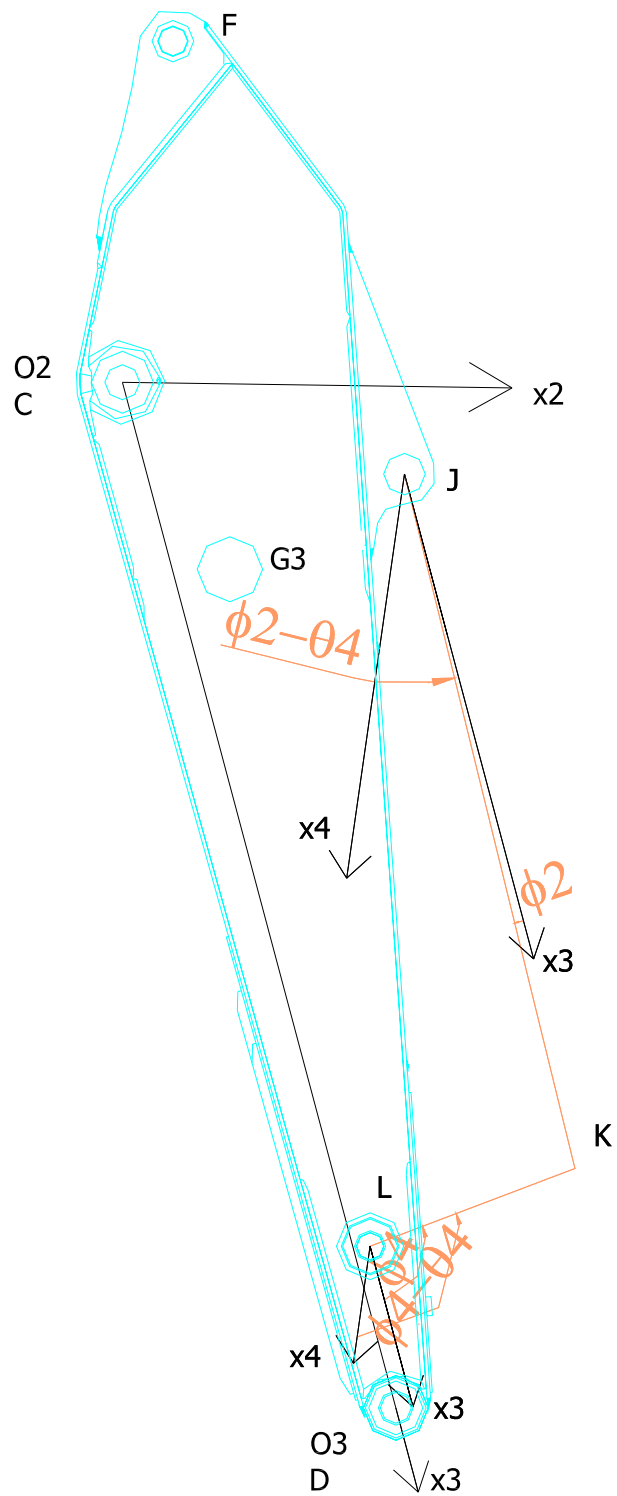


Figure A.6 – Variable Angles of Arm

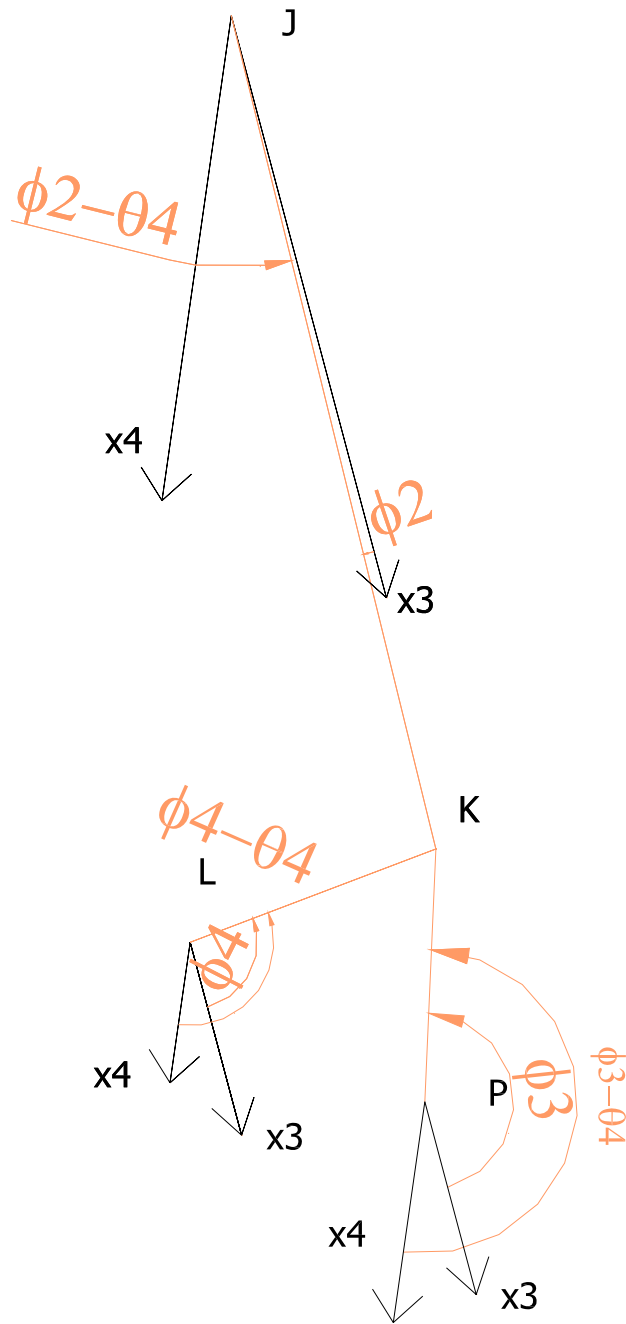


Figure A.7 – Variable Angles of Arm Connecting Rods and Bucket (Detailed)

$$Lci = \sqrt{Lci^2 + Lcf^2 - 2 \cdot Lci \cdot Lcf \cdot \cos(\pi - (\beta_4 + \theta_3 + \beta_5 + \beta_6 + \beta_7))} \quad (\text{A.4})$$

$$\phi_2 = \arccos\left(\frac{Lcj^2 + Ljk^2 - (Lcl^2 + Llk^2 - 2 \cdot Lcl \cdot Llk \cdot \cos(\pi - \phi_4 + \beta_5))}{2 \cdot Lcj \cdot Ljk}\right) + \beta_5 + \beta_6 - \pi \quad (\text{A.5})$$

$$\phi_3 = \pi + \beta_8 + \theta_4 - \arccos\left(\frac{Lpk^2 + Ldp^2 - (Llk^2 + Lld^2 - 2 \cdot Llk \cdot Lld \cdot \cos(\phi_4 + \arccos(\frac{Lld^2 + L3^2 - Lcl^2}{2 \cdot Lld \cdot L3})))}{2 \cdot Lpk \cdot Ldp}\right) \quad (\text{A.6})$$

$$\phi_4 = \arccos\left(\frac{(Lld^2 + Ldp^2 - 2 \cdot Lld \cdot Ldp \cdot \cos(\pi - \arccos(\frac{Lld^2 + L3^2 - Lcl^2}{2 \cdot Lld \cdot L3}) + \beta_8 + \theta_4)) + Lld^2 - Ldp^2}{2 \cdot \sqrt{(Lld^2 + Ldp^2 - 2 \cdot Lld \cdot Ldp \cdot \cos(\pi - \arccos(\frac{Lld^2 + L3^2 - Lcl^2}{2 \cdot Lld \cdot L3}) + \beta_8 + \theta_4)) \cdot Lld}}\right) \quad (\text{A.7})$$

A.3. Bucket

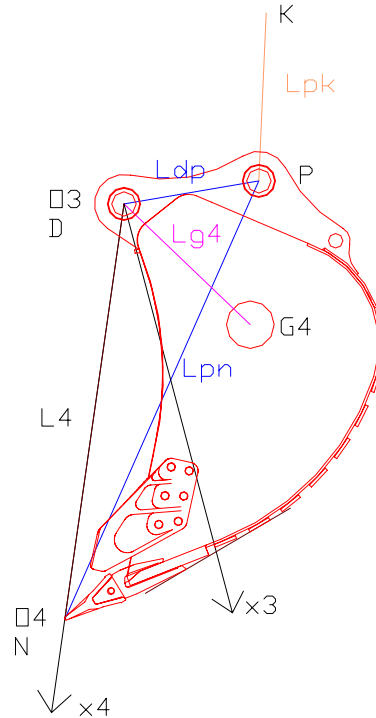


Figure A.8 – Constant and Variables Lengths of Bucket

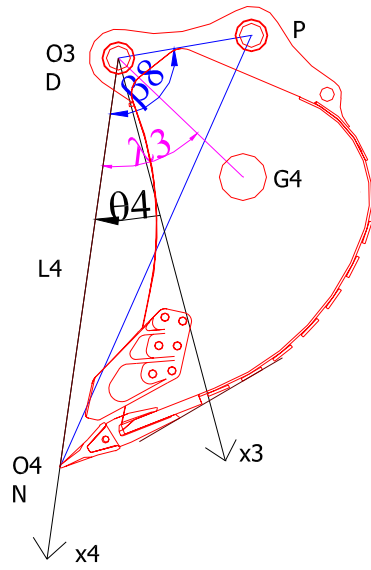


Figure A.9 – Input and Constant Angles of Bucket

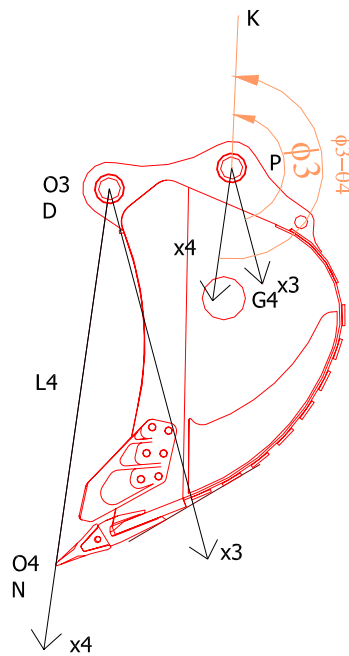


Figure A.10 – Variable Angles of Bucket

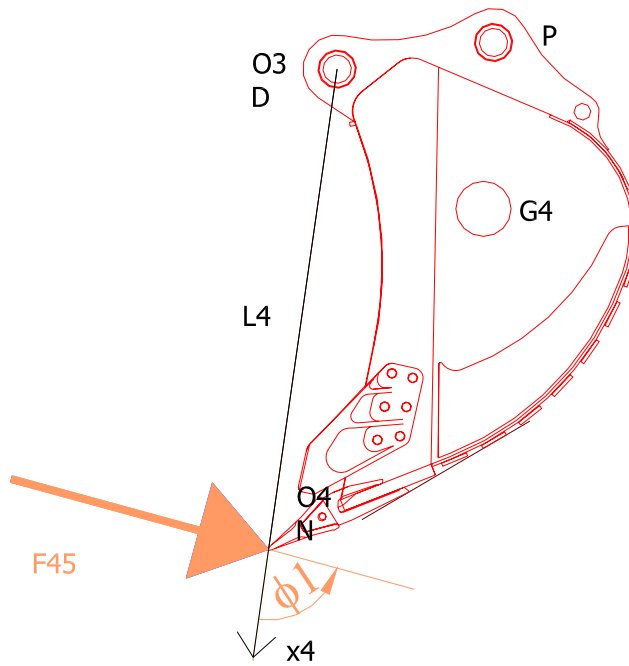


Figure A.11 – Soil Resistance Direction

$$Lci = \frac{\sqrt{(Lcl^2 + Lcj^2 - 2 \cdot Lcl \cdot Lcj \cdot \cos(\beta_6)) + Llk^2}}{-2 \cdot \sqrt{Lcl^2 + Lcj^2 - 2 \cdot Lcl \cdot Lcj \cdot \cos(\beta_6)} \cdot Llk} \cdot \cos(\pi + \beta_5 - \phi_4 - \arccos(\frac{(Lcl^2 + Lcj^2 - 2 \cdot Lcl \cdot Lcj \cdot \cos(\beta_6)) + Lcl^2 - Lcj^2}{2 \cdot Lcl \cdot Lcj})) \quad (A.8)$$

ϕ_1 (Input)

APPENDIX B

EQUATIONS OF EXCAVATOR DYNAMICS

A detailed formulation of dynamic modeling of excavators is explained in this appendix.

B.1. Multi-body Dynamics and Newton-Euler Formulation

B.1.1. Rotation Matrices

$$A_1^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_2^1 = \begin{bmatrix} \cos(\theta_2) & \sin(\theta_2) & 0 \\ -\sin(\theta_2) & \cos(\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2 = \begin{bmatrix} \cos(\theta_3) & \sin(\theta_3) & 0 \\ -\sin(\theta_3) & \cos(\theta_3) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_4^3 = \begin{bmatrix} \cos(\theta_4) & \sin(\theta_4) & 0 \\ -\sin(\theta_4) & \cos(\theta_4) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_2^0 = A_1^0 \cdot A_2^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos(\theta_2) & \sin(\theta_2) & 0 \\ -\sin(\theta_2) & \cos(\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta_2) & \sin(\theta_2) & 0 \\ -\sin(\theta_2) & \cos(\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_3^0 = A_1^0 \cdot A_2^1 \cdot A_3^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos(\theta_2) & \sin(\theta_2) & 0 \\ -\sin(\theta_2) & \cos(\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos(\theta_3) & \sin(\theta_3) & 0 \\ -\sin(\theta_3) & \cos(\theta_3) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_3^0 = \begin{bmatrix} \cos(\theta_2 + \theta_3) & \sin(\theta_2 + \theta_3) & 0 \\ -\sin(\theta_2 + \theta_3) & \cos(\theta_2 + \theta_3) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_4^0 = A_1^0 \cdot A_2^1 \cdot A_3^2 \cdot A_4^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos(\theta_2) & \sin(\theta_2) & 0 \\ -\sin(\theta_2) & \cos(\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos(\theta_3) & \sin(\theta_3) & 0 \\ -\sin(\theta_3) & \cos(\theta_3) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos(\theta_4) & \sin(\theta_4) & 0 \\ -\sin(\theta_4) & \cos(\theta_4) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A_4^0 = \begin{bmatrix} \cos(\theta_2 + \theta_3 + \theta_4) & \sin(\theta_2 + \theta_3 + \theta_4) & 0 \\ -\sin(\theta_2 + \theta_3 + \theta_4) & \cos(\theta_2 + \theta_3 + \theta_4) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

B.1.2. Inputs

Angles

$${}^2\bar{\theta}_2 = \begin{bmatrix} 0 \\ 0 \\ \theta_2 \end{bmatrix} \quad {}^3\bar{\theta}_3 = \begin{bmatrix} 0 \\ 0 \\ \theta_4 \end{bmatrix} \quad {}^4\bar{\theta}_4 = \begin{bmatrix} 0 \\ 0 \\ \theta_4 \end{bmatrix}$$

Angular Velocities

$${}^2\bar{\dot{\theta}}_2 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix} \quad {}^3\bar{\dot{\theta}}_3 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_4 \end{bmatrix} \quad {}^4\bar{\dot{\theta}}_4 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_4 \end{bmatrix}$$

Angular Accelerations

$${}^2\bar{\ddot{\theta}}_2 = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_2 \end{bmatrix} \quad {}^3\bar{\ddot{\theta}}_3 = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_3 \end{bmatrix} \quad {}^4\bar{\ddot{\theta}}_4 = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_4 \end{bmatrix}$$

B.1.3. Forward Dynamics

B.1.3.1. Positions, Rotational and Translational Velocities, Rotational and Translational Accelerations of Links

Table B.1 – Positions, Rotational and Translational Velocities, Rotational and Translational Accelerations of Links

<p>Position Relation, $i=0,\dots,N-1$</p> ${}^{i+1}\vec{p}_{0(i+1)} = A_{i+1}^0 \cdot \vec{p}_{0(i+1)}$
<p>Angular Velocity Relations, $i=0,\dots,N-1$</p> ${}^{i+1}\vec{\omega}_{0i} = A_{i+1}^i \cdot {}^i\vec{\omega}_{0i}$ ${}^{i+1}\vec{\omega}_{0(i+1)} = {}^{i+1}\vec{\omega}_{0i} + \vec{\dot{\theta}}_{i+1}$ $\vec{\omega}_{0(i+1)} = [A_{i+1}^0]^{-1} \cdot {}^{i+1}\vec{\omega}_{0(i+1)}$
<p>Linear Velocity Relations, $i=0,\dots,N$</p> ${}^{i+1}\vec{v}_{0i} = A_{i+1}^i \cdot {}^i\vec{v}_{0i}$ ${}^{i+1}\vec{p}_{i+1}^* = A_{i+1}^0 \cdot (\vec{p}_{0(i+1)} - \vec{p}_{0i})$ ${}^{i+1}\vec{v}_{0(i+1)} = {}^{i+1}\vec{\dot{p}}_{0(i+1)} = {}^{i+1}\vec{v}_{0i} + {}^{i+1}\vec{\omega}_{0(i+1)} \times {}^{i+1}\vec{p}_{i+1}^*$ $\vec{v}_{0(i+1)} = [A_{i+1}^0]^{-1} \cdot {}^{i+1}\vec{v}_{0(i+1)}$
<p>Angular Acceleration Relations, $i=0,\dots,N-1$</p> ${}^{i+1}\vec{\alpha}_{0i} = A_{i+1}^i \cdot {}^i\vec{\alpha}_{0i}$ ${}^{i+1}\vec{\alpha}_{0(i+1)} = {}^{i+1}\vec{\alpha}_{0i} + \vec{\ddot{\theta}}_{i+1} + {}^{i+1}\vec{\omega}_{0i} \times \vec{\dot{\theta}}_{i+1}$ $\vec{\alpha}_{0(i+1)} = [A_{i+1}^0]^{-1} \cdot {}^{i+1}\vec{\alpha}_{0(i+1)}$
<p>Linear Acceleration Relations, $i=0,\dots,N-1$</p> ${}^{i+1}\vec{a}_{0i} = A_{i+1}^i \cdot {}^i\vec{a}_{0i}$ ${}^{i+1}\vec{a}_{0(i+1)} = {}^{i+1}\vec{a}_{0i} + {}^{i+1}\vec{\alpha}_{0(i+1)} \times {}^{i+1}\vec{p}_{i+1}^* + {}^{i+1}\vec{\omega}_{0(i+1)} \times ({}^{i+1}\vec{\omega}_{0(i+1)} \times {}^{i+1}\vec{p}_{i+1}^*)$ $\vec{p}_{0(i+1)} = [A_{i+1}^0]^{-1} \cdot {}^{i+1}\vec{a}_{0(i+1)}$

B.1.3.1.1. Chassis (i=0)

Table B.2 – Initial Conditions: Positions, Rotational and Translational Velocities, Rotational and Translational Accelerations of Chassis

Position	$\bar{p}_{01} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	${}^1\bar{p}_{01} = \hat{A}_1^0 \cdot \bar{p}_{01} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$
Angular Velocity	$\bar{\omega}_{01} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	${}^1\bar{\omega}_{01} = \hat{A}_1^0 \cdot \bar{\omega}_{01} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$
Translational Velocity	$\dot{\bar{p}}_{01} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	${}^1\dot{\bar{v}}_{01} = {}^1\dot{\bar{p}}_{01} = \hat{A}_1^0 \cdot \dot{\bar{p}}_{01} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$
Angular Acceleration	$\ddot{\bar{\omega}}_{01} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	${}^1\ddot{\bar{\alpha}}_{01} = {}^1\ddot{\bar{\omega}}_{01} = \hat{A}_1^0 \cdot \ddot{\bar{\omega}}_{01} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$
Translational Acceleration	$\ddot{\bar{p}}_{01} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	${}^1\ddot{\bar{a}}_{01} = {}^1\ddot{\bar{p}}_{01} = \hat{A}_1^0 \cdot \ddot{\bar{p}}_{01} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Table B.3 – Positions, Rotational and Translational Velocities, Rotational and Translational Accelerations of Boom

Position	$\bar{p}_{01} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\bar{p}_{02} = \bar{p}_{01} + \begin{bmatrix} L2 \cdot \cos(\theta_2) \\ L2 \cdot \sin(\theta_2) \\ 0 \end{bmatrix}$	${}^2\bar{p}_{02} = \hat{A}_2^0 \cdot \bar{p}_{02} = \begin{bmatrix} L2 \\ 0 \\ 0 \end{bmatrix}$	$\bar{p}_{02} = \begin{bmatrix} L2 \cdot \cos(\theta_2) \\ L2 \cdot \sin(\theta_2) \\ 0 \end{bmatrix}$
Angular Velocity	${}^1\bar{\omega}_{01} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	${}^2\bar{\omega}_{01} = \hat{A}_2^1 \cdot {}^1\bar{\omega}_{01} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ ${}^2\bar{\omega}_{02} = {}^2\bar{\omega}_{01} + {}^2\bar{\theta}_2 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix}$	$\bar{\omega}_{02} = [\hat{A}_2^0]^{-1} \cdot {}^2\bar{\omega}_{02} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix}$

B.1.3.1.2. Boom (i=1)

Table B.3 (cont'd) – Positions, Rotational and Translational Velocities, Rotational and Translational Accelerations of Boom

Angular Acceleration	${}^1\bar{\omega}_{01} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	${}^2\bar{\alpha}_{01} = \hat{A}_2^1 \cdot {}^1\bar{\alpha}_{01} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ ${}^2\bar{\alpha}_{02} = {}^2\bar{\omega}_{02} = {}^2\bar{\alpha}_{01} + {}^2\ddot{\theta}_2 + {}^2\bar{\omega}_{01} \times {}^2\bar{\theta}_2 = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_2 \end{bmatrix}$	$\bar{\omega}_{02} = [\hat{A}_2^0]^{-1} \cdot {}^2\bar{\omega}_{02} = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_2 \end{bmatrix}$
	Translational Velocity	${}^1\bar{p}_{01} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	${}^2\bar{v}_{01} = \hat{A}_2^1 \cdot {}^1\bar{v}_{01} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ ${}^2\bar{p}_2 = \hat{A}_2^0 \cdot (\bar{p}_{02} - \bar{p}_{01}) = \begin{bmatrix} L2 \\ 0 \\ 0 \end{bmatrix}$ ${}^2\bar{v}_{02} = {}^2\bar{p}_{02} = {}^2\bar{v}_{01} + {}^2\bar{\omega}_{02} \times {}^2\bar{p}_2 = \begin{bmatrix} 0 \\ L2 \cdot \dot{\theta}_2 \\ 0 \end{bmatrix}$

Table B.3 (cont'd) – Positions, Rotational and Translational Velocities, Rotational and Translational Accelerations of Boom

Translational Acceleration	${}^1\ddot{\vec{p}}_{01} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	${}^2\vec{a}_{01} = A_2^1 \cdot {}^1\vec{a}_{01} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ ${}^2\vec{a}_{02} = {}^2\ddot{\vec{p}}_{02} = {}^2\vec{a}_{01} + {}^2\vec{\alpha}_{02} \times {}^2\vec{p}_2^* + {}^2\vec{\omega}_{02} \times ({}^2\vec{\omega}_{02} \times {}^2\vec{p}_2^*)$ $= \begin{bmatrix} -L2 \cdot (\dot{\theta}_2)^2 \\ L2 \cdot \ddot{\theta}_2 \\ 0 \end{bmatrix}$	$\ddot{\vec{p}}_{02} = [\hat{A}_2^0]^{-1} \cdot {}^2\ddot{\vec{p}}_{02} = \begin{bmatrix} -L2 \cdot \begin{bmatrix} \cos(\theta_2) \cdot (\dot{\theta}_2)^2 \\ + \sin(\theta_2) \cdot \ddot{\theta}_2 \end{bmatrix} \\ -L2 \cdot \begin{bmatrix} \sin(\theta_2) \cdot (\dot{\theta}_2)^2 \\ - \cos(\theta_2) \cdot \ddot{\theta}_2 \end{bmatrix} \\ 0 \end{bmatrix}$
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Table B.4 – Positions, Rotational and Translational Velocities, Rotational and Translational Accelerations of Arm

Position	$\bar{p}_{02} = \begin{bmatrix} L2 \cdot \cos(\theta_2) \\ L2 \cdot \sin(\theta_2) \\ 0 \end{bmatrix}$ $\bar{p}_{03} = \bar{p}_{02} + \begin{bmatrix} L3 \cdot \cos(\theta_2 + \theta_3) \\ L3 \cdot \sin(\theta_2 + \theta_3) \\ 0 \end{bmatrix}$	${}^3\bar{p}_{03} = \hat{A}_3^0 \cdot \bar{p}_{03} = \begin{bmatrix} L2 \cdot \cos(\theta_3) + L3 \\ -L2 \cdot \sin(\theta_3) \\ 0 \end{bmatrix}$	$\bar{p}_{03} = \begin{bmatrix} L2 \cdot \cos(\theta_2) + L3 \cdot \cos(\theta_2 + \theta_3) \\ L2 \cdot \sin(\theta_2) + L3 \cdot \sin(\theta_2 + \theta_3) \\ 0 \end{bmatrix}$
Angular Velocity	${}^2\bar{\omega}_{02} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix}$	${}^3\bar{\omega}_{02} = \hat{A}_3^2 \cdot {}^2\bar{\omega}_{02} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix}$ ${}^3\bar{\omega}_{03} = {}^3\bar{\omega}_{02} + {}^3\bar{\dot{\theta}}_3 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix}$	$\bar{\omega}_{03} = [\hat{A}_3^0]^{-1} \cdot {}^3\bar{\omega}_{03} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix}$

B.1.3.1.3. Arm (i=2)

Table B.4 (cont'd) – Positions, Rotational and Translational Velocities, Rotational and Translational Accelerations of Arm

Translational Velocity	${}^2\bar{p}_{02} = \begin{bmatrix} 0 \\ L2 \cdot \dot{\theta}_2 \\ 0 \end{bmatrix}$	${}^3\bar{v}_{02} = \hat{A}_3^2 \cdot {}^2\bar{p}_{02} = \begin{bmatrix} L2 \cdot \sin(\theta_3) \cdot \dot{\theta}_2 \\ L2 \cdot \cos(\theta_3) \cdot \dot{\theta}_2 \\ 0 \end{bmatrix}$ ${}^3\bar{p}_3^* = \hat{A}_3^0 \cdot (\bar{p}_{03} - \bar{p}_{02}) = \begin{bmatrix} L3 \\ 0 \\ 0 \end{bmatrix}$ ${}^3\bar{v}_{03} = {}^3\bar{p}_{03} = {}^3\bar{v}_{02} + {}^3\bar{\omega}_{03} \times {}^3\bar{p}_3^*$ $= \begin{bmatrix} L2 \cdot \sin(\theta_3) \cdot \dot{\theta}_2 \\ L2 \cdot \cos(\theta_3) \cdot \dot{\theta}_2 + L3 \cdot (\dot{\theta}_2 + \dot{\theta}_3) \\ 0 \end{bmatrix}$	$\bar{p}_{03} = [\hat{A}_3^0]^{-1} \cdot {}^3\bar{p}_{03}$ $= \begin{bmatrix} -L2 \cdot \sin(\theta_2) \cdot \dot{\theta}_2 - L3 \cdot \sin(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3) \\ L2 \cdot \cos(\theta_2) \cdot \dot{\theta}_2 + L3 \cdot \cos(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3) \\ 0 \end{bmatrix}$
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Table B.4 (Cont'd) – Positions, Rotational and Translational Velocities, Rotational and Translational Accelerations of Arm

Angular Acceleration	${}^2\bar{\omega}_{02} = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_2 \end{bmatrix}$	${}^3\bar{\alpha}_{02} = \hat{A}_3^2 \cdot {}^2\bar{\alpha}_{02} = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_2 \end{bmatrix}$ ${}^3\bar{\alpha}_{03} = {}^3\bar{\omega}_{03} = {}^3\bar{\alpha}_{02} + {}^3\ddot{\theta}_3 + {}^3\bar{\omega}_{03} \times {}^3\ddot{\theta}_3$ $= \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_2 + \ddot{\theta}_3 \end{bmatrix}$	$\bar{\omega}_{03} = \left[\hat{A}_3^0 \right]^{-1} \cdot {}^3\bar{\omega}_{03} = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_2 + \ddot{\theta}_3 \end{bmatrix}$
Translational Acceleration	${}^2\bar{p}_{02} = \begin{bmatrix} -L2 \cdot (\dot{\theta}_2)^2 \\ L2 \cdot \ddot{\theta}_2 \\ 0 \end{bmatrix}$	${}^3\bar{a}_{02} = \hat{A}_3^2 \cdot {}^2\bar{a}_{02}$ $= \begin{bmatrix} L2 \cdot (\sin(\theta_3) \cdot \ddot{\theta}_2 - \cos(\theta_3) \cdot (\dot{\theta}_2)^2) \\ L2 \cdot (\cos(\theta_3) \cdot \ddot{\theta}_2 + \sin(\theta_3) \cdot (\dot{\theta}_2)^2) \\ 0 \end{bmatrix}$ ${}^3\bar{a}_{03} = {}^3\bar{p}_{03} = {}^3\bar{a}_{02} + {}^3\bar{\alpha}_{03} \times {}^3\bar{p}_3^* + {}^3\bar{\omega}_{03} \times ({}^3\bar{\omega}_{03} \times {}^3\bar{p}_3^*)$ $= \begin{bmatrix} L2 \cdot (\sin(\theta_3) \cdot \ddot{\theta}_2 - \cos(\theta_3) \cdot (\dot{\theta}_2)^2) - L3 \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2 \\ L2 \cdot (\cos(\theta_3) \cdot \ddot{\theta}_2 + \sin(\theta_3) \cdot (\dot{\theta}_2)^2) + L3 \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) \\ 0 \end{bmatrix}$	$\bar{p}_{03} = \left[\hat{A}_3^0 \right]^{-1} \cdot {}^3\bar{p}_{03}$ $= \begin{bmatrix} \left(-L2 \cdot (\sin(\theta_2) \cdot \ddot{\theta}_2 + \cos(\theta_2) \cdot (\dot{\theta}_2)^2) \right) \\ \left(-L3 \cdot (\sin(\theta_2 + \theta_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) \right. \\ \left. + \cos(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2 \right) \\ \left(L2 \cdot (\cos(\theta_2) \cdot \ddot{\theta}_2 - \sin(\theta_2) \cdot (\dot{\theta}_2)^2) \right) \\ \left(+L3 \cdot (\cos(\theta_2 + \theta_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) \right. \\ \left. - \sin(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2 \right) \\ 0 \end{bmatrix}$

Table B.5 – Positions, Rotational and Translational Velocities, Rotational and Translational Accelerations of Bucket

Position	$\bar{p}_{03} = \begin{bmatrix} L2 \cdot \cos(\theta_2) + L3 \cdot \cos(\theta_2 + \theta_3) \\ L2 \cdot \sin(\theta_2) + L3 \cdot \sin(\theta_2 + \theta_3) \\ 0 \end{bmatrix}$ $\bar{p}_{04} = \bar{p}_{03} + \begin{bmatrix} L4 \cdot \cos(\theta_2 + \theta_3 + \theta_4) \\ L4 \cdot \sin(\theta_2 + \theta_3 + \theta_4) \\ 0 \end{bmatrix}$	${}^4\bar{p}_{04} = \hat{A}_4^0 \cdot \bar{p}_{04}$ $= \begin{bmatrix} L2 \cdot \cos(\theta_3 + \theta_4) + L3 \cdot \cos(\theta_4) + L4 \\ -L2 \cdot \sin(\theta_3 + \theta_4) - L3 \cdot \sin(\theta_4) \\ 0 \end{bmatrix}$	$\bar{p}_{04} = \begin{bmatrix} L2 \cdot \cos(\theta_2) + L3 \cdot \cos(\theta_2 + \theta_3) + L4 \cdot \cos(\theta_2 + \theta_3 + \theta_4) \\ L2 \cdot \sin(\theta_2) + L3 \cdot \sin(\theta_2 + \theta_3) + L4 \cdot \sin(\theta_2 + \theta_3 + \theta_4) \\ 0 \end{bmatrix}$
Angular Velocity	${}^3\bar{\omega}_{03} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix}$	${}^4\bar{\omega}_{03} = \hat{A}_4^3 \cdot {}^3\bar{\omega}_{03} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix}$ ${}^4\bar{\omega}_{04} = {}^4\bar{\omega}_{03} + {}^4\dot{\theta}_4 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4 \end{bmatrix}$	$\bar{\omega}_{04} = [\hat{A}_4^0]^{-1} \cdot {}^4\bar{\omega}_{04} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4 \end{bmatrix}$

Table B.5 (cont'd)– Positions, Rotational and Translational Velocities, Rotational and Translational Accelerations of Bucket

Translational Velocity	${}^3\bar{p}_{03} = \begin{bmatrix} L2 \cdot \sin(\theta_3) \cdot \dot{\theta}_2 \\ L2 \cdot \cos(\theta_3) \cdot \dot{\theta}_2 + L3 \cdot (\dot{\theta}_2 + \dot{\theta}_3) \\ 0 \end{bmatrix}$	${}^4\bar{v}_{03} = \hat{A}_4^3 \cdot {}^3\bar{p}_{03}$ $= \begin{bmatrix} L2 \cdot \sin(\theta_3 + \theta_4) \cdot \dot{\theta}_2 + L3 \cdot \sin(\theta_4) \cdot (\dot{\theta}_2 + \dot{\theta}_3) \\ L2 \cdot \cos(\theta_3 + \theta_4) \cdot \dot{\theta}_2 + L3 \cdot \cos(\theta_4) \cdot (\dot{\theta}_2 + \dot{\theta}_3) \\ 0 \end{bmatrix}$ ${}^4\bar{p}_4^* = \hat{A}_4^0 \cdot (\bar{p}_{04} - \bar{p}_{03}) = \begin{bmatrix} L4 \\ 0 \\ 0 \end{bmatrix}$ ${}^4\bar{v}_{04} = {}^4\bar{v}_{04} = {}^4\bar{v}_{03} + {}^4\bar{\omega}_{04} \times {}^4\bar{p}_4^*$ $= \begin{bmatrix} L2 \cdot \sin(\theta_3 + \theta_4) \cdot \dot{\theta}_2 + L3 \cdot \sin(\theta_4) \cdot (\dot{\theta}_2 + \dot{\theta}_3) \\ L2 \cdot \cos(\theta_3 + \theta_4) \cdot \dot{\theta}_2 + L3 \cdot \cos(\theta_4) \cdot (\dot{\theta}_2 + \dot{\theta}_3) \\ +L4 \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4) \\ 0 \end{bmatrix}$	$\bar{p}_{04} = [\hat{A}_4^0]^{-1} \cdot {}^4\bar{p}_{04}$ $= \begin{bmatrix} -L2 \cdot \sin(\theta_2) \cdot \dot{\theta}_2 - L3 \cdot \sin(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3) \\ -L4 \cdot \sin(\theta_2 + \theta_3 + \theta_4) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4) \\ L2 \cdot \cos(\theta_2) \cdot \dot{\theta}_2 + L3 \cdot \cos(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3) \\ L4 \cdot \cos(\theta_2 + \theta_3 + \theta_4) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4) \\ 0 \end{bmatrix}$
Angular Acceleration	${}^3\bar{\omega}_{03} = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_2 + \ddot{\theta}_3 \end{bmatrix}$	${}^4\bar{\alpha}_{03} = \hat{A}_4^3 \cdot {}^3\bar{\omega}_{03} = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_2 + \ddot{\theta}_3 \end{bmatrix}$ ${}^4\bar{\alpha}_{04} = {}^4\bar{\omega}_{04} = {}^4\bar{\alpha}_{03} + {}^4\ddot{\theta}_4 + {}^4\bar{\omega}_{04} \times {}^4\bar{\theta}_4$ $= \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4 \end{bmatrix}$	$\bar{\omega}_{04} = [\hat{A}_4^0]^{-1} \cdot {}^4\bar{\omega}_{04} = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4 \end{bmatrix}$

Table B.5 (cont'd)– Positions, Rotational and Translational Velocities, Rotational and Translational Accelerations of Bucket

<p>Translational Acceleration</p>	${}^3\ddot{\vec{p}}_{03} = \begin{bmatrix} L2 \cdot (\sin(\theta_3) \cdot \ddot{\theta}_2 \\ -\cos(\theta_3) \cdot (\dot{\theta}_2)^2 \\ -L3 \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2 \\ L2 \cdot (\cos(\theta_3) \cdot \ddot{\theta}_2 \\ +\sin(\theta_3) \cdot (\dot{\theta}_2)^2 \\ +L3 \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) \\ 0 \end{bmatrix}$	${}^4\ddot{\vec{a}}_{03} = \hat{A}_4^3 \cdot {}^3\ddot{\vec{a}}_{03}$ $= \begin{bmatrix} L2 \cdot (\sin(\theta_3 + \theta_4) \cdot \ddot{\theta}_2 - \cos(\theta_3 + \theta_4) \cdot (\dot{\theta}_2)^2 \\ -L3 \cdot (\cos(\theta_4) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2 - \sin(\theta_4) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3)) \\ L2 \cdot (\cos(\theta_3 + \theta_4) \cdot \ddot{\theta}_2 + \sin(\theta_3 + \theta_4) \cdot (\dot{\theta}_2)^2 \\ +L3 \cdot (\sin(\theta_4) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2 + \cos(\theta_4) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3)) \\ 0 \end{bmatrix}$ ${}^4\ddot{\vec{a}}_{04} = {}^4\ddot{\vec{p}}_{04} = {}^4\ddot{\vec{a}}_{03} + {}^4\ddot{\vec{a}}_{04} \times {}^4\vec{p}_4^* + {}^4\ddot{\vec{\omega}}_{04} \times ({}^4\vec{\omega}_{04} \times {}^4\vec{p}_4^*)$ $= \begin{bmatrix} L2 \cdot (\sin(\theta_3 + \theta_4) \cdot \ddot{\theta}_2 - \cos(\theta_3 + \theta_4) \cdot (\dot{\theta}_2)^2 \\ -L3 \cdot (\cos(\theta_4) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2 - \sin(\theta_4) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3)) \\ -L4 \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)^2 \\ L2 \cdot (\cos(\theta_3 + \theta_4) \cdot \ddot{\theta}_2 + \sin(\theta_3 + \theta_4) \cdot (\dot{\theta}_2)^2 \\ +L3 \cdot (\sin(\theta_4) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2 + \cos(\theta_4) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3)) \\ +L4 \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) \\ 0 \end{bmatrix}$	$\ddot{\vec{p}}_{04} = [\hat{A}_4^0]^{-1} \cdot {}^4\ddot{\vec{p}}_{04}$ $= \begin{bmatrix} -L2 \cdot (\sin(\theta_2) \cdot \ddot{\theta}_2 + \cos(\theta_2) \cdot (\dot{\theta}_2)^2 \\ -L3 \cdot (\sin(\theta_2 + \theta_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) \\ +\cos(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2 \\ -L4 \cdot (\sin(\theta_2 + \theta_3 + \theta_4) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) \\ +\cos(\theta_2 + \theta_3 + \theta_4) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)^2) \\ L2 \cdot (\cos(\theta_2) \cdot \ddot{\theta}_2 - \sin(\theta_2) \cdot (\dot{\theta}_2)^2) \\ +L3 \cdot (\cos(\theta_2 + \theta_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) \\ -\sin(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) \\ +L4 \cdot (\cos(\theta_2 + \theta_3 + \theta_4) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) \\ -\sin(\theta_2 + \theta_3 + \theta_4) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)^2) \\ 0 \end{bmatrix}$
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B.1.3.2. Velocities and Accelerations of Center of Gravities of Links

Table B.6 – Velocities and Accelerations of Center of Gravities of Links

<p>Velocity Relations, $i=0, \dots, N-1$</p> ${}^{i+1}\vec{p}_{(i+1)G}^* = A_{i+1}^0 \cdot (\vec{p}_{0(i+1)G} - \vec{p}_{0(i+1)})$ ${}^{i+1}\vec{v}_{0(i+1)G} = {}^{i+1}\vec{p}_{0(i+1)G} = {}^i\vec{v}_{0(i+1)} + {}^i\vec{\omega}_{0(i+1)} \times {}^i\vec{p}_{(i+1)G}^*$ $\vec{v}_{0iG} = [A_{i+1}^0]^{-1} \cdot {}^{i+1}\vec{p}_{0(i+1)G}$	
<p>Linear Acceleration Relations, $i=0, \dots, N-1$</p> ${}^{i+1}\vec{a}_{0(i+1)G} = {}^{i+1}\vec{p}_{0(i+1)G} = {}^{i+1}\vec{a}_{0(i+1)} + {}^{i+1}\vec{\alpha}_{0(i+1)} \times {}^{i+1}\vec{p}_{(i+1)G}^* + {}^{i+1}\vec{\omega}_{0(i+1)} \times ({}^{i+1}\vec{\omega}_{0(i+1)} \times {}^i\vec{p}_{(i+1)G}^*)$ $\vec{a}_{0iG} = [A_{i+1}^0]^{-1} \cdot {}^{i+1}\vec{p}_{0(i+1)G}$	

B.1.3.2.1. Chassis ($i=0$)

Table B.7 – Velocities and Accelerations of Center of Gravities of Chassis

Position	$\vec{p}_{01G} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	
Velocity	${}^1\vec{p}_{1G}^* = \hat{A}_1^0 \cdot (\vec{p}_{01G} - \vec{p}_{01}) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ ${}^1\vec{v}_{01G} = {}^1\vec{p}_{01G} = {}^1\vec{v}_{01} + {}^1\vec{\omega}_{01} \times {}^1\vec{p}_{1G}^* = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	$\vec{v}_{01G} = \vec{p}_{01G} = [\hat{A}_1^0]^{-1} \cdot {}^1\vec{v}_{01G} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$
Acceleration	${}^1\vec{a}_{01G} = {}^1\vec{p}_{01G}$ $= {}^1\vec{a}_{01} + {}^1\alpha_{01} \times {}^1\vec{p}_{1G}^* + {}^1\vec{\omega}_{01} \times ({}^1\vec{\omega}_{01} \times {}^1\vec{p}_{1G}^*) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	$\vec{a}_{01G} = \vec{p}_{01G} = [\hat{A}_1^0]^{-1} \cdot {}^1\vec{a}_{01G} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Table B.8 – Velocities and Accelerations of Center of Gravities of Boom

Position	$\bar{p}_{02G} = \bar{p}_{01} + \begin{bmatrix} Lg2 \cdot \cos(\theta_2 + \lambda_1) \\ Lg2 \cdot \sin(\theta_2 + \lambda_1) \\ 0 \end{bmatrix}$	$\bar{p}_{02G} = \begin{bmatrix} Lg2 \cdot \cos(\theta_2 + \lambda_1) \\ Lg2 \cdot \sin(\theta_2 + \lambda_1) \\ 0 \end{bmatrix}$
Velocity	${}^2\bar{p}_{2G}^* = \hat{A}_2^0 \cdot (\bar{p}_{02G} - \bar{p}_{02}) = \begin{bmatrix} Lg2 \cdot \cos(\lambda_1) - L2 \\ Lg2 \cdot \sin(\lambda_1) \\ 0 \end{bmatrix}$ ${}^2\bar{v}_{02G} = {}^2\bar{\dot{p}}_{02G} = {}^2\bar{v}_{02} + {}^2\bar{\omega}_{02} \times {}^2\bar{p}_{2G}^* = \begin{bmatrix} -\dot{\theta}_2 \cdot Lg2 \cdot \sin(\lambda_1) \\ \dot{\theta}_2 \cdot Lg2 \cdot \cos(\lambda_1) \\ 0 \end{bmatrix}$	$\bar{v}_{02G} = \bar{\dot{p}}_{02G} = \left[\hat{A}_2^0 \right]^{-1} \cdot {}^2\bar{v}_{02G} = \begin{bmatrix} -Lg2 \cdot \sin(\theta_2 + \lambda_1) \cdot \dot{\theta}_2 \\ Lg2 \cdot \cos(\theta_2 + \lambda_1) \cdot \dot{\theta}_2 \\ 0 \end{bmatrix}$
Acceleration	${}^2\bar{a}_{02G} = {}^2\bar{\ddot{p}}_{02G} = {}^2\bar{a}_{02} + {}^2\alpha_{02} \times {}^2\bar{p}_{2G}^* + {}^2\bar{\omega}_{02} \times ({}^2\bar{\omega}_{02} \times {}^2\bar{p}_{2G}^*)$ $= \begin{bmatrix} -Lg2 \cdot (\sin(\lambda_1) \cdot \ddot{\theta}_2 + \cos(\lambda_1) \cdot (\dot{\theta}_2)^2) \\ Lg2 \cdot (\cos(\lambda_1) \cdot \ddot{\theta}_2 - \sin(\lambda_1) \cdot (\dot{\theta}_2)^2) \\ 0 \end{bmatrix}$	$\bar{a}_{02G} = \bar{\ddot{p}}_{02G} = \left[\hat{A}_2^0 \right]^{-1} \cdot {}^2\bar{a}_{02G}$ $= \begin{bmatrix} -Lg2 \cdot (\sin(\theta_2 + \lambda_1) \cdot \ddot{\theta}_2 + \cos(\theta_2 + \lambda_1) \cdot (\dot{\theta}_2)^2) \\ Lg2 \cdot (\cos(\theta_2 + \lambda_1) \cdot \ddot{\theta}_2 - \sin(\theta_2 + \lambda_1) \cdot (\dot{\theta}_2)^2) \\ 0 \end{bmatrix}$

B.1.3.2.2. Boom (i=1)

Table B.9 – Velocities and Accelerations of Center of Gravities of Arm

Position	$\bar{p}_{03G} = \bar{p}_{02} + \begin{bmatrix} Lg3 \cdot \cos(\theta_2 + \theta_3 + \lambda_2) \\ Lg3 \cdot \sin(\theta_2 + \theta_3 + \lambda_2) \\ 0 \end{bmatrix}$	$\bar{p}_{03G} = \begin{bmatrix} L2 \cdot \cos(\theta_2) + Lg3 \cdot \cos(\theta_2 + \theta_3 + \lambda_2) \\ L2 \cdot \sin(\theta_2) + Lg3 \cdot \sin(\theta_2 + \theta_3 + \lambda_2) \\ 0 \end{bmatrix}$
Velocity	${}^3\bar{p}_{3G}^* = \hat{A}_3^0 \cdot (\bar{p}_{03G} - \bar{p}_{03}) = \begin{bmatrix} Lg3 \cdot \cos(\lambda_2) - L3 \\ Lg3 \cdot \sin(\lambda_2) \\ 0 \end{bmatrix}$ ${}^3\bar{v}_{03G} = {}^3\bar{p}_{03G} = {}^3\bar{v}_{03} + {}^3\bar{\omega}_{03} \times {}^3\bar{p}_{3G}^*$ $= \begin{bmatrix} -Lg3 \cdot \sin(\lambda_2) \cdot (\dot{\theta}_2 + \dot{\theta}_3) + L2 \cdot \sin(\theta_3) \cdot \dot{\theta}_2 \\ Lg3 \cdot \cos(\lambda_2) \cdot (\dot{\theta}_2 + \dot{\theta}_3) + L2 \cdot \cos(\theta_3) \cdot \dot{\theta}_2 \\ 0 \end{bmatrix}$	$\bar{v}_{03G} = \bar{p}_{03G} = [\hat{A}_3^0]^{-1} \cdot {}^3\bar{v}_{03G}$ $= \begin{bmatrix} -Lg3 \cdot \sin(\theta_2 + \theta_3 + \lambda_2) \cdot (\dot{\theta}_2 + \dot{\theta}_3) - L2 \cdot \sin(\theta_2) \cdot \dot{\theta}_2 \\ Lg3 \cdot \cos(\theta_2 + \theta_3 + \lambda_2) \cdot (\dot{\theta}_2 + \dot{\theta}_3) + L2 \cdot \cos(\theta_2) \cdot \dot{\theta}_2 \\ 0 \end{bmatrix}$

B.1.3.2.3. Arm (i=2)

Table B.9 (cont'd) – Velocities and Accelerations of Center of Gravities of Arm

Acceleration	${}^3\bar{a}_{03G} = {}^3\bar{\ddot{p}}_{03G} = {}^3\bar{a}_{03} + {}^3\alpha_{03} \times {}^3\bar{p}_{3G}^* + {}^3\bar{\omega}_{03} \times ({}^3\bar{\omega}_{03} \times {}^3\bar{p}_{3G}^*)$ $= \begin{bmatrix} \left[\begin{array}{l} -Lg3 \cdot \left[\sin(\lambda_2) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) + \cos(\lambda_2) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2 \right] \\ +L2 \cdot (\sin(\theta_3) \cdot \ddot{\theta}_2 - \cos(\theta_3) \cdot (\dot{\theta}_2)^2) \end{array} \right] \\ \left[\begin{array}{l} Lg3 \cdot \left[\cos(\lambda_2) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) - \sin(\lambda_2) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2 \right] \\ +L2 \cdot (\cos(\theta_3) \cdot \ddot{\theta}_2 + \sin(\theta_3) \cdot (\dot{\theta}_2)^2) \end{array} \right] \\ 0 \end{bmatrix}$	$\bar{a}_{03G} = \bar{\ddot{p}}_{03G} = [\hat{A}_3^0]^{-1} \cdot {}^3\bar{a}_{03G}$ $= \begin{bmatrix} \left[\begin{array}{l} -Lg3 \cdot \left[\sin(\theta_2 + \theta_3 + \lambda_2) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) + \cos(\theta_2 + \theta_3 + \lambda_2) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2 \right] \\ -L2 \cdot (\sin(\theta_2) \cdot \ddot{\theta}_2 + \cos(\theta_2) \cdot (\dot{\theta}_2)^2) \end{array} \right] \\ \left[\begin{array}{l} Lg3 \cdot \left[\cos(\theta_2 + \theta_3 + \lambda_2) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) - \sin(\theta_2 + \theta_3 + \lambda_2) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2 \right] \\ -L2 \cdot (-\cos(\theta_2) \cdot \ddot{\theta}_2 + \sin(\theta_2) \cdot (\dot{\theta}_2)^2) \end{array} \right] \\ 0 \end{bmatrix}$
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Table B.10 – Velocities and Accelerations of Center of Gravities of Bucket

Position	$\bar{p}_{04G} = \bar{p}_{03} + \begin{bmatrix} Lg4 \cdot \cos(\theta_2 + \theta_3 + \theta_4 + \lambda_2) \\ Lg4 \cdot \sin(\theta_2 + \theta_3 + \theta_4 + \lambda_2) \\ 0 \end{bmatrix}$	$\bar{p}_{04G} = \begin{bmatrix} L2 \cdot \cos(\theta_2) + L3 \cdot \cos(\theta_2 + \theta_3) + Lg4 \cdot \cos(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \\ L2 \cdot \sin(\theta_2) + L3 \cdot \sin(\theta_2 + \theta_3) + Lg4 \cdot \sin(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \\ 0 \end{bmatrix}$
Velocity	${}^4\bar{p}_{4G}^* = \hat{A}_4^0 \cdot (\bar{p}_{04G} - \bar{p}_{04}) = \begin{bmatrix} Lg4 \cdot \cos(\lambda_3) - L4 \\ Lg4 \cdot \sin(\lambda_3) \\ 0 \end{bmatrix}$ ${}^4\bar{v}_{04G} = {}^4\bar{p}_{04G} = {}^4\bar{v}_{04} + {}^4\bar{\omega}_{04} \times {}^4\bar{p}_{4G}^*$ $= \begin{bmatrix} \begin{pmatrix} -Lg4 \cdot \sin(\lambda_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4) \\ +L2 \cdot \sin(\theta_3 + \theta_4) \cdot \dot{\theta}_2 + L3 \cdot \sin(\theta_4) \cdot (\dot{\theta}_2 + \dot{\theta}_3) \end{pmatrix} \\ \begin{pmatrix} Lg4 \cdot \cos(\lambda_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4) \\ +L2 \cdot \cos(\theta_3 + \theta_4) \cdot \dot{\theta}_2 + L3 \cdot \cos(\theta_4) \cdot (\dot{\theta}_2 + \dot{\theta}_3) \end{pmatrix} \\ 0 \end{bmatrix}$	$\bar{v}_{04G} = \bar{p}_{04G} = \left[\hat{A}_4^0 \right]^{-1} \cdot {}^4\bar{v}_{04G}$ $= \begin{bmatrix} \begin{pmatrix} -Lg4 \cdot \sin(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4) \\ -L2 \cdot \sin(\theta_2) \cdot \dot{\theta}_2 - L3 \cdot \sin(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3) \end{pmatrix} \\ \begin{pmatrix} Lg4 \cdot \sin(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4) \\ +L2 \cdot \sin(\theta_2) \cdot \dot{\theta}_2 + L3 \cdot \sin(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3) \end{pmatrix} \\ 0 \end{bmatrix}$

B.1.3.2.4. Bucket (i=3)

Table B.10 (cont'd) – Velocities and Accelerations of Center of Gravities of Bucket

Acceleration	${}^4\bar{a}_{04G} = {}^4\bar{p}_{04G} = {}^4\bar{a}_{04} + {}^4\alpha_{04} \times {}^4\bar{p}_{4G} + {}^4\bar{\omega}_{04} \times ({}^4\bar{\omega}_{04} \times {}^4\bar{p}_{4G})$ $= \begin{bmatrix} -Lg4 \cdot (\sin(\lambda_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) + \cos(\lambda_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)^2) \\ +L3 \cdot (\sin(\theta_4) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) - \cos(\theta_4) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) \\ +L2 \cdot (\sin(\theta_3 + \theta_4) \cdot \ddot{\theta}_2 - \cos(\theta_3 + \theta_4) \cdot (\dot{\theta}_2)^2) \\ Lg4 \cdot (\cos(\lambda_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) - \sin(\lambda_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)^2) \\ +L3 \cdot (\cos(\theta_4) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) + \sin(\theta_4) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) \\ +L2 \cdot (\cos(\theta_3 + \theta_4) \cdot \ddot{\theta}_2 + \sin(\theta_3 + \theta_4) \cdot (\dot{\theta}_2)^2) \\ 0 \end{bmatrix}$	$\bar{a}_{04G} = \bar{p}_{04G} = [\hat{A}_4^0]^{-1} \cdot {}^4\bar{a}_{04G}$ $= \begin{bmatrix} -Lg4 \cdot (\sin(\theta_2 + \theta_3 + \theta_4 + \lambda_2) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) + \cos(\theta_2 + \theta_3 + \theta_4 + \lambda_2) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)^2) \\ -L3 \cdot (\sin(\theta_2 + \theta_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) + \cos(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) \\ -L2 \cdot (\sin(\theta_2) \cdot \ddot{\theta}_2 + \cos(\theta_2) \cdot (\dot{\theta}_2)^2) \\ Lg4 \cdot (\cos(\theta_2 + \theta_3 + \theta_4 + \lambda_2) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) - \sin(\theta_2 + \theta_3 + \theta_4 + \lambda_2) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)^2) \\ L3 \cdot (\cos(\theta_2 + \theta_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) - \sin(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) \\ L2 \cdot (\cos(\theta_2) \cdot \ddot{\theta}_2 - \sin(\theta_2) \cdot (\dot{\theta}_2)^2) \\ 0 \end{bmatrix}$
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B.1.3.3. External Forces and Moments on Links

Table B.11 – External Forces and Moments on Links

<p style="text-align: center;">Force Relations, $i=1, \dots, N-1$</p> $\vec{g}_{0(i+1)} = \begin{bmatrix} 0 \\ -9.81 \\ 0 \end{bmatrix}$ ${}^{i+1}\vec{F}_0^{i+1} = m_{i+1} \cdot \left[A_{i+1}^0 \cdot (\vec{p}_{0(i+1)G} + \vec{g}_{0(i+1)}) \right]$ $\vec{F}_0^{i+1} = \left[A_{i+1}^0 \right]^{-1} \cdot {}^{i+1}\vec{F}_0^{i+1}$
<p style="text-align: center;">Moment Relations, $i=1, \dots, N-1$</p> ${}^{i+1}\hat{I}_{0(i+1)} = \hat{A}_{i+1}^0 \cdot \hat{I}_{0(i+1)} \cdot \left[\hat{A}_{i+1}^0 \right]^{-1}$ ${}^{i+1}\vec{H}_{0(i+1)} = {}^{i+1}I_{0(i+1)} \cdot {}^{i+1}\vec{\omega}_{0(i+1)}$ ${}^{i+1}\vec{M}_{0(i+1)}^c = {}^{i+1}I_{0(i+1)} \cdot {}^{i+1}\vec{\alpha}_{0(i+1)}$ ${}^{i+1}\vec{M}_0^{i+1} = {}^{i+1}\vec{M}_{0(i+1)}^c + {}^{i+1}\vec{\omega}_{0(i+1)} \times {}^{i+1}\vec{H}_{0(i+1)}$ $\vec{M}_0^{i+1} = \left[A_i^0 \right]^{-1} \cdot {}^{i+1}\vec{M}_0^{i+1}$

B.1.3.3.1. Inputs

$$\hat{I}_{02} = \begin{bmatrix} I_{x2} & 0 & 0 \\ 0 & I_{y2} & 0 \\ 0 & 0 & I_{z2} \end{bmatrix}$$

$$\hat{I}_{03} = \begin{bmatrix} I_{x3} & 0 & 0 \\ 0 & I_{y3} & 0 \\ 0 & 0 & I_{z3} \end{bmatrix}$$

$$\hat{I}_{04} = \begin{bmatrix} I_{x4} & 0 & 0 \\ 0 & I_{y4} & 0 \\ 0 & 0 & I_{z4} \end{bmatrix}$$

Table B.12 - External Forces and Moments on Boom

<p style="text-align: center;">Force</p>	${}^2\bar{F}_0^2 = m_2 \cdot \left[\hat{A}_2^0 \cdot (\bar{p}_{02G} + \bar{g}_{02}) \right]$ $= \begin{bmatrix} -m_2 \cdot (Lg_2 \cdot (\sin(\lambda_1) \cdot \ddot{\theta}_2 + \cos(\lambda_1) \cdot (\dot{\theta}_2)^2) - g_2 \cdot \sin(\theta_2)) \\ m_2 \cdot (Lg_2 \cdot (\cos(\lambda_1) \cdot \ddot{\theta}_2 - \sin(\lambda_1) \cdot (\dot{\theta}_2)^2) + g_2 \cdot \cos(\theta_2)) \\ 0 \end{bmatrix}$	$\bar{F}_0^2 = \left[\hat{A}_2^0 \right]^{-1} \cdot {}^2\bar{F}_0^2$ $= \begin{bmatrix} -m_2 \cdot [Lg_2 \cdot (\sin(\lambda_1 + \theta_2) \cdot \ddot{\theta}_2 + \cos(\lambda_1 + \theta_2) \cdot (\dot{\theta}_2)^2)] \\ m_2 \cdot [Lg_2 \cdot (\cos(\lambda_1 + \theta_2) \cdot \ddot{\theta}_2 - \sin(\lambda_1 + \theta_2) \cdot (\dot{\theta}_2)^2) + g_2] \\ 0 \end{bmatrix}$
<p style="text-align: center;">Moment</p>	${}^2\hat{I}_{02} = \hat{A}_2^0 \cdot \hat{I}_{02} \cdot \left[(\hat{A}_2^0)_R \right]^{-1}$ $= \begin{bmatrix} I_{x_2} \cdot \cos(\theta_2)^2 + I_{y_2} \cdot \sin(\theta_2)^2 & -I_{x_2} \cdot \cos(\theta_2) \cdot \sin(\theta_2) + I_{y_2} \cdot \sin(\theta_2) \cdot \cos(\theta_2) & 0 \\ -I_{x_2} \cdot \cos(\theta_2) \cdot \sin(\theta_2) + I_{y_2} \cdot \sin(\theta_2) \cdot \cos(\theta_2) & I_{x_2} \cdot \sin(\theta_2)^2 + I_{y_2} \cdot \cos(\theta_2)^2 & 0 \\ 0 & 0 & I_{z_2} \end{bmatrix}$ ${}^2\bar{H}_{02} = {}^2\hat{I}_{02} \cdot {}^2\bar{\omega}_{02}$ $= \begin{bmatrix} 0 \\ 0 \\ I_{z_2} \cdot \dot{\theta}_2 \end{bmatrix}$ ${}^2\bar{M}_{02}^c = {}^2\hat{I}_{02} \cdot {}^2\bar{\alpha}_{02}$ $= \begin{bmatrix} 0 \\ 0 \\ I_{z_2} \cdot \ddot{\theta}_2 \end{bmatrix}$ ${}^2\bar{M}_0^2 = {}^2\bar{M}_{02}^c + {}^2\bar{\omega}_{02} \times {}^2\bar{H}_{02}$ $= \begin{bmatrix} 0 \\ 0 \\ I_{z_2} \cdot \ddot{\theta}_2 \end{bmatrix}$	$\bar{M}_0^2 = \left[\hat{A}_2^0 \right]^{-1} \cdot {}^2\bar{M}_0^2 = \begin{bmatrix} 0 \\ 0 \\ I_{z_2} \cdot \ddot{\theta}_2 \end{bmatrix}$

Table B.13 - External Forces and Moments on Arm

Force	${}^3\bar{F}_0^3 = m_3 \cdot \left[\hat{A}_3^0 \cdot (\bar{\ddot{p}}_{03G} + \bar{g}_{03}) \right]$ $= \begin{bmatrix} -m_3 \cdot \left[\begin{array}{l} Lg_3 \cdot (\sin(\lambda_2) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) + \cos(\lambda_2) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) \\ -L2 \cdot (\sin(\theta_3) \cdot \ddot{\theta}_2 - \cos(\theta_3) \cdot (\dot{\theta}_2)^2) - g_3 \cdot \sin(\theta_2 + \theta_3) \end{array} \right] \\ m_3 \cdot \left[\begin{array}{l} Lg_3 \cdot (\cos(\lambda_2) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) - \sin(\lambda_2) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) \\ +L2 \cdot (\cos(\theta_3) \cdot \ddot{\theta}_2 + \sin(\theta_3) \cdot (\dot{\theta}_2)^2) + g_3 \cdot \cos(\theta_2 + \theta_3) \end{array} \right] \\ 0 \end{bmatrix}$	$\bar{F}_0^3 = \left[\hat{\lambda}_3^0 \right]^{-1} \cdot {}^3\bar{F}_0^3$ $= \begin{bmatrix} \left[\begin{array}{l} Lg_3 \cdot (\sin(\theta_2 + \theta_3 + \lambda_2) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) \\ + \cos(\theta_2 + \theta_3 + \lambda_2) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) \\ +L2 \cdot (\sin(\theta_2) \cdot \ddot{\theta}_2 + \cos(\theta_2) \cdot (\dot{\theta}_2)^2) \end{array} \right] \\ m_3 \cdot \left[\begin{array}{l} Lg_3 \cdot (\cos(\theta_2 + \theta_3 + \lambda_2) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) \\ - \sin(\theta_2 + \theta_3 + \lambda_2) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) \\ +L2 \cdot (\cos(\theta_2) \cdot \ddot{\theta}_2 - \sin(\theta_2) \cdot (\dot{\theta}_2)^2) \\ +g_3 \end{array} \right] \\ 0 \end{bmatrix}$
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B.1.3.3.3. Arm (i=2)

Table B.13 (cont'd) - External Forces and Moments on Arm

Moment	${}^3\hat{I}_{03} = \hat{A}_3^0 \cdot \hat{I}_{03} \cdot [\hat{A}_3^0]^{-1}$ $= \begin{bmatrix} I_{x3} \cdot \cos(\theta_2 + \theta_3)^2 + I_{y3} \cdot \sin(\theta_2 + \theta_3)^2 & -I_{x3} \cdot \cos(\theta_2 + \theta_3) \cdot \sin(\theta_2 + \theta_3) + I_{y3} \cdot \sin(\theta_2 + \theta_3) \cdot \cos(\theta_2 + \theta_3) & 0 \\ -I_{x3} \cdot \cos(\theta_2 + \theta_3) \cdot \sin(\theta_2 + \theta_3) + I_{y3} \cdot \sin(\theta_2 + \theta_3) \cdot \cos(\theta_2 + \theta_3) & I_{x3} \cdot \sin(\theta_2 + \theta_3)^2 + I_{y3} \cdot \cos(\theta_2 + \theta_3)^2 & 0 \\ 0 & 0 & I_{z3} \end{bmatrix}$ ${}^3\bar{H}_{03} = {}^3\hat{I}_{03} \cdot {}^3\bar{\omega}_{03}$ $= \begin{bmatrix} 0 \\ 0 \\ I_{z3} \cdot (\dot{\theta}_2 + \dot{\theta}_3) \end{bmatrix}$ ${}^3\bar{M}_{03}^c = {}^3\hat{I}_{03} \cdot {}^3\bar{\alpha}_{03}$ $= \begin{bmatrix} 0 \\ 0 \\ I_{z3} \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) \end{bmatrix}$ ${}^3\bar{M}_0^3 = {}^3\bar{M}_{03}^c + {}^3\bar{\omega}_{03} \times {}^3\bar{H}_{03}$ $= \begin{bmatrix} 0 \\ 0 \\ I_{z3} \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) \end{bmatrix}$	$\bar{M}_0^3 = [\hat{A}_3^0]^{-1} \cdot {}^3\bar{M}_0^3$ $= \begin{bmatrix} 0 \\ 0 \\ I_{z3} \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) \end{bmatrix}$
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Table B.14 - External Forces and Moments on Bucket

Force	${}^4\bar{F}_0^4 = m_4 \cdot \left[\hat{A}_4^0 \cdot (\bar{p}_{04G} + \bar{g}_{04}) \right]$ $= m_4 \cdot \begin{bmatrix} Lg4 \cdot (\sin(\lambda_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) + \cos(\lambda_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)^2) \\ -L3 \cdot (\sin(\theta_4) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) - \cos(\theta_4) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) \\ -L2 \cdot (\sin(\theta_3 + \theta_4) \cdot \ddot{\theta}_2 - \cos(\theta_3 + \theta_4) \cdot (\dot{\theta}_2)^2) - g_4 \cdot \sin(\theta_2 + \theta_3 + \theta_4) \\ Lg4 \cdot (\cos(\lambda_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) - \sin(\lambda_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)^2) \\ L3 \cdot (\cos(\theta_4) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) + \sin(\theta_4) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) \\ +L2 \cdot (\cos(\theta_3 + \theta_4) \cdot \ddot{\theta}_2 + \sin(\theta_3 + \theta_4) \cdot (\dot{\theta}_2)^2) + g_4 \cdot \cos(\theta_2 + \theta_3 + \theta_4) \\ 0 \end{bmatrix}$	$\bar{F}_0^4 = \left[\hat{A}_4^0 \right]^{-1} \cdot {}^4\bar{F}_0^4$ $= m_4 \cdot \begin{bmatrix} Lg4 \cdot (\sin(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) \\ + \cos(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)^2) \\ +L3 \cdot (\sin(\theta_2 + \theta_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) \\ + \cos(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) \\ +L2 \cdot (\sin(\theta_2) \cdot \ddot{\theta}_2 + \cos(\theta_2) \cdot (\dot{\theta}_2)^2) \\ Lg4 \cdot (\cos(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) \\ - \sin(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)^2) \\ +L3 \cdot (\cos(\theta_2 + \theta_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) \\ - \sin(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) \\ +L2 \cdot (\cos(\theta_2) \cdot \ddot{\theta}_2 - \sin(\theta_2) \cdot (\dot{\theta}_2)^2) + g_4 \\ 0 \end{bmatrix}$
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B.1.3.3.4. Bucket (i=3)

Table B.14 (cont'd) - External Forces and Moments on Bucket

<p>Moment</p>	${}^4\hat{I}_{04} = \hat{A}_4^0 \cdot \hat{I}_{04} \cdot [\hat{A}_4^0]^{-1}$ $= \begin{bmatrix} I_{x4} \cdot \cos(\theta_2 + \theta_3 + \theta_4)^2 + I_{y4} \cdot \sin(\theta_2 + \theta_3 + \theta_4)^2 & -I_{x4} \cdot \cos(\theta_2 + \theta_3 + \theta_4) \cdot \sin(\theta_2 + \theta_3 + \theta_4) + I_{y4} \cdot \sin(\theta_2 + \theta_3 + \theta_4) \cdot \cos(\theta_2 + \theta_3 + \theta_4) & 0 \\ -I_{x4} \cdot \cos(\theta_2 + \theta_3 + \theta_4) \cdot \sin(\theta_2 + \theta_3 + \theta_4) + I_{y4} \cdot \sin(\theta_2 + \theta_3 + \theta_4) \cdot \cos(\theta_2 + \theta_3 + \theta_4) & I_{x4} \cdot \sin(\theta_2 + \theta_3 + \theta_4)^2 + I_{y4} \cdot \cos(\theta_2 + \theta_3 + \theta_4)^2 & 0 \\ 0 & 0 & I_{z4} \end{bmatrix}$ ${}^4\bar{H}_{04} = {}^4\hat{I}_{04} \cdot {}^4\bar{\omega}_{04}$ $= \begin{bmatrix} 0 \\ 0 \\ I_{z4} \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4) \end{bmatrix}$ ${}^4\bar{M}_{04}^c = {}^4\hat{I}_{04} \cdot {}^4\bar{\alpha}_{04}$ $= \begin{bmatrix} 0 \\ 0 \\ I_{z4} \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) \end{bmatrix}$ ${}^4\bar{M}_0^4 = {}^4\bar{M}_{04}^c + {}^4\bar{\omega}_{04} \times {}^4\bar{H}_{04}$ $= \begin{bmatrix} 0 \\ 0 \\ I_{z4} \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) \end{bmatrix}$	$\bar{M}_0^4 = [\hat{A}_4^0]^{-1} \cdot {}^4\bar{M}_0^4$ $= \begin{bmatrix} 0 \\ 0 \\ I_{z4} \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) \end{bmatrix}$
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B.1.3.4. Actuating Forces and Points on Cylinders, Connecting Rods, Boom, Arm and Bucket

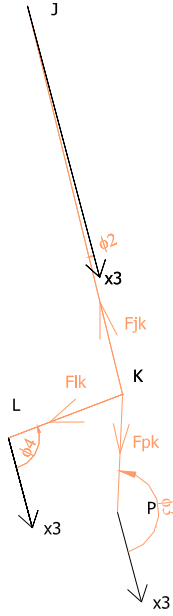


Figure B.1 – Forces on Bucket Cylinder and Connecting Rods

$$\vec{F}_{JK} + \vec{F}_{PK} + \vec{F}_{LK} = 0$$

Let us evaluate the vector sum on x3-y3 coordinate system:

$$-F_{JK} \cdot \cos(\phi_2) - F_{LK} \cdot \cos(\phi_4) - F_{PK} \cdot \cos(\phi_3) = 0$$

$$-F_{JK} \cdot \sin(\phi_2) - F_{LK} \cdot \sin(\phi_4) - F_{PK} \cdot \sin(\phi_3) = 0$$

Since force created by bucket cylinder is known it is assumed as input. Therefore,

$$F_{PK} = F_{JK} \cdot \left(\frac{\sin(\phi_2) - \cos(\phi_2) \cdot \tan(\phi_4)}{-\sin(\phi_3) + \cos(\phi_3) \cdot \tan(\phi_4)} \right) \quad (\text{B.1})$$

$$F_{LK} = F_{JK} \cdot \left(\frac{\sin(\phi_2) - \cos(\phi_2) \cdot \tan(\phi_3)}{-\sin(\phi_4) + \cos(\phi_4) \cdot \tan(\phi_3)} \right) \quad (\text{B.2})$$

All actuating forces and positions:

$$\bar{F}_{a(4,1)} = \bar{F}_{PK} = \begin{bmatrix} F_{PK} \cdot \cos(\theta_2 + \theta_3 + \phi_3) \\ F_{PK} \cdot \sin(\theta_2 + \theta_3 + \phi_3) \\ 0 \end{bmatrix} \quad (\text{B.3})$$

$$\bar{F}_{a(3,1)} = \bar{F}_{LK} = \begin{bmatrix} F_{LK} \cdot \cos(\theta_2 + \theta_3 + \phi_4) \\ F_{LK} \cdot \sin(\theta_2 + \theta_3 + \phi_4) \\ 0 \end{bmatrix} \quad (\text{B.4})$$

$$\bar{F}_{a(3,2)} = \bar{F}_{JK} = \begin{bmatrix} F_{JK} \cdot \cos(\theta_2 + \theta_3 + \phi_2) \\ F_{JK} \cdot \sin(\theta_2 + \theta_3 + \phi_2) \\ 0 \end{bmatrix} \quad (\text{B.5})$$

$$\bar{F}_{a(3,3)} = -\bar{F}_{a(2,1)} = \bar{F}_{IF} = \begin{bmatrix} F_{IF} \cdot \cos(\theta_2 + \phi_5 + \pi) \\ F_{IF} \cdot \sin(\theta_2 + \phi_5 + \pi) \\ 0 \end{bmatrix} \quad (\text{B.6})$$

$$\bar{F}_{a(2,2)} = \bar{F}_{EB} = \begin{bmatrix} F_{EB} \cdot \cos(\theta_2 + \phi_6) \\ F_{EB} \cdot \sin(\theta_2 + \phi_6) \\ 0 \end{bmatrix} \quad (\text{B.7})$$

$$\vec{P}_{0(4,1)} = \vec{p}_{0P} = \begin{bmatrix} L2 \cdot \cos(\theta_2) + L3 \cdot \cos(\theta_2 + \theta_3) + Ldp \cdot \cos(\theta_2 + \theta_3 + \theta_4 + \beta_8) \\ L2 \cdot \sin(\theta_2) + L3 \cdot \sin(\theta_2 + \theta_3) + Ldp \cdot \sin(\theta_2 + \theta_3 + \theta_4 + \beta_8) \\ 0 \end{bmatrix} \quad (\text{B.8})$$

$$\vec{P}_{0(3,1)} = \vec{p}_{0L} = \begin{bmatrix} L2 \cdot \cos(\theta_2) + Lcl \cdot \cos(\theta_2 + \theta_3 + \beta_5) \\ L2 \cdot \sin(\theta_2) + Lcl \cdot \sin(\theta_2 + \theta_3 + \beta_5) \\ 0 \end{bmatrix} \quad (\text{B.9})$$

$$\vec{P}_{0(3,1)} = \vec{p}_{0J} = \begin{bmatrix} L2 \cdot \cos(\theta_2) + Lcj \cdot \cos(\theta_2 + \theta_3 + \beta_5 + \beta_6) \\ L2 \cdot \sin(\theta_2) + Lcj \cdot \sin(\theta_2 + \theta_3 + \beta_5 + \beta_6) \\ 0 \end{bmatrix} \quad (\text{B.10})$$

$$\vec{P}_{0(3,3)} = \vec{p}_{0F} = \begin{bmatrix} L2 \cdot \cos(\theta_2) + Lcf \cdot \cos(\theta_2 + \theta_3 + \beta_5 + \beta_6 + \beta_7) \\ L2 \cdot \sin(\theta_2) + Lcj \cdot \sin(\theta_2 + \theta_3 + \beta_5 + \beta_6 + \beta_7) \\ 0 \end{bmatrix} \quad (\text{B.11})$$

$$\vec{P}_{0(2,1)} = \vec{p}_{0I} = \begin{bmatrix} Lai \cdot \cos(\theta_2 + \beta_2) \\ Lai \cdot \sin(\theta_2 + \beta_2) \\ 0 \end{bmatrix} \quad (\text{B.12})$$

$$\vec{P}_{0(2,2)} = \vec{p}_{0B} = \begin{bmatrix} Lab \cdot \cos(\theta_2 + \beta_2 + \beta_3) \\ Lab \cdot \sin(\theta_2 + \beta_2 + \beta_3) \\ 0 \end{bmatrix} \quad (\text{B.13})$$

B.1.4. Backward Dynamics

B.1.4.1. Force, Torque Relations of Links

Table B.15 – Force, Torque Relations of Links

<p>Force Relations, $i=N-1, \dots, 1$</p> ${}^{i+1}\vec{F}_{i,(i+1)} = {}^{i+1}\vec{F}_{(i+1),(i+2)} - {}^{i+1}\vec{F}_0^{i+1} + \sum_k {}^{i+1}\vec{F}_{a(i+1,k)}$ $\vec{F}_{i,(i+1)} = [A_{(i+1)}^0]^{-1} \cdot {}^{i+1}\vec{F}_{i,(i+1)}$
<p>Torque Relations, $i=N-1, \dots, 1$</p> ${}^{i+1}\vec{M}_{i,(i+1)} = -{}^{i+1}\vec{M}_{i,(i+1)} - \left(\sum_k ({}^{i+1}\vec{p}_{0((i+1),k)} - {}^{i+1}\vec{p}_{0i}) \times {}^{i+1}\vec{F}_{a((i+1),k)} \right) + ({}^{i+1}\vec{p}_{0((i+1)G)} - {}^{i+1}\vec{p}_{0i}) \times {}^{i+1}\vec{F}_0^{i+1} + {}^{i+1}\vec{M}_0^{i+1}$ $\vec{M}_{i,(i+1)} = [A_{i+1}^0]^{-1} \cdot {}^{i+1}\vec{M}_{i,(i+1)}$

B.1.4.1.1. Bucket (i=3)

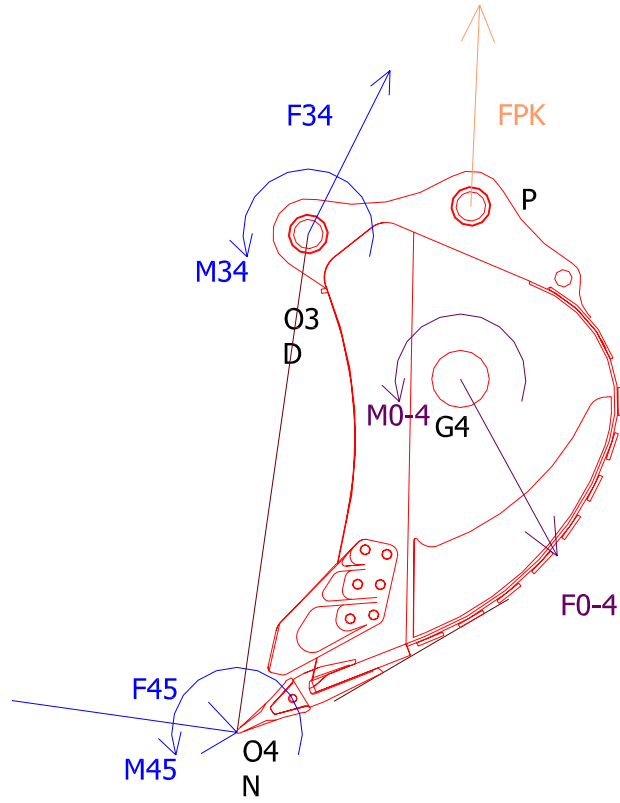


Figure B.2 – Forces on Bucket

$${}^4\vec{F}_{34} = -{}^4\vec{F}_0^4 + {}^4\vec{F}_{45} + {}^4\vec{F}_{PK} \quad (\text{B.14})$$

$$\begin{aligned} {}^4\vec{M}_{34} = & -{}^4\vec{M}_{45} - ({}^4\vec{p}_{04} - {}^4\vec{p}_{03}) \times {}^4\vec{F}_{45} - ({}^4\vec{p}_{0P} - {}^4\vec{p}_{03}) \times {}^4\vec{F}_{PK} \\ & + ({}^4\vec{p}_{04G} - {}^4\vec{p}_{03}) \times {}^4\vec{F}_0^4 + {}^4\vec{M}_0^4 \end{aligned} \quad (\text{B.15})$$

B.1.4.1.2. Arm

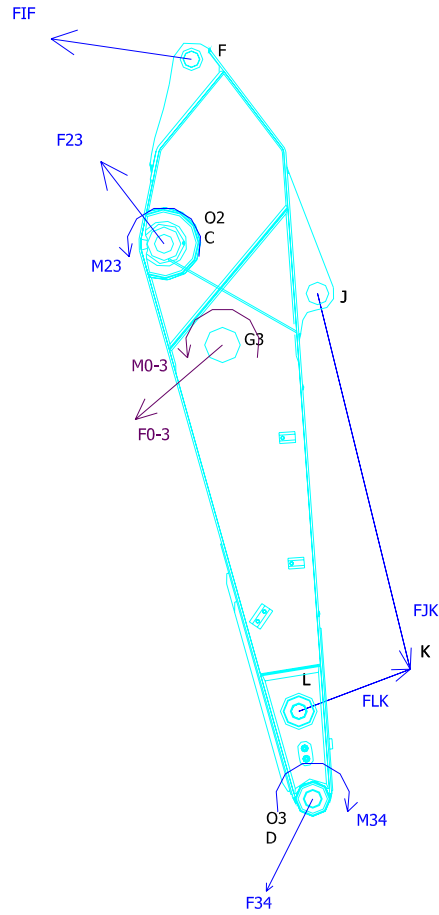


Figure B.3 – Forces on Arm

$${}^3\vec{F}_{23} = -{}^3\vec{F}_0^3 + {}^3\vec{F}_{34} + {}^3\vec{F}_{LK} + {}^3\vec{F}_{JK} + {}^3\vec{F}_{FI} \quad (\text{B.16})$$

$$\begin{aligned} {}^3\vec{M}_{23} = & -{}^3\vec{M}_{34} - ({}^3\vec{p}_{03} - {}^3\vec{p}_{02}) \times {}^3\vec{F}_{34} - ({}^3\vec{p}_{0L} - {}^3\vec{p}_{02}) \times {}^3\vec{F}_{LK} - ({}^3\vec{p}_{0J} - {}^3\vec{p}_{02}) \times {}^3\vec{F}_{JK} \\ & - ({}^3\vec{p}_{0F} - {}^3\vec{p}_{02}) \times {}^3\vec{F}_{FI} + ({}^3\vec{p}_{03G} - {}^3\vec{p}_{02}) \times {}^3\vec{F}_0^3 + {}^3\vec{M}_0^3 \end{aligned} \quad (\text{B.17})$$

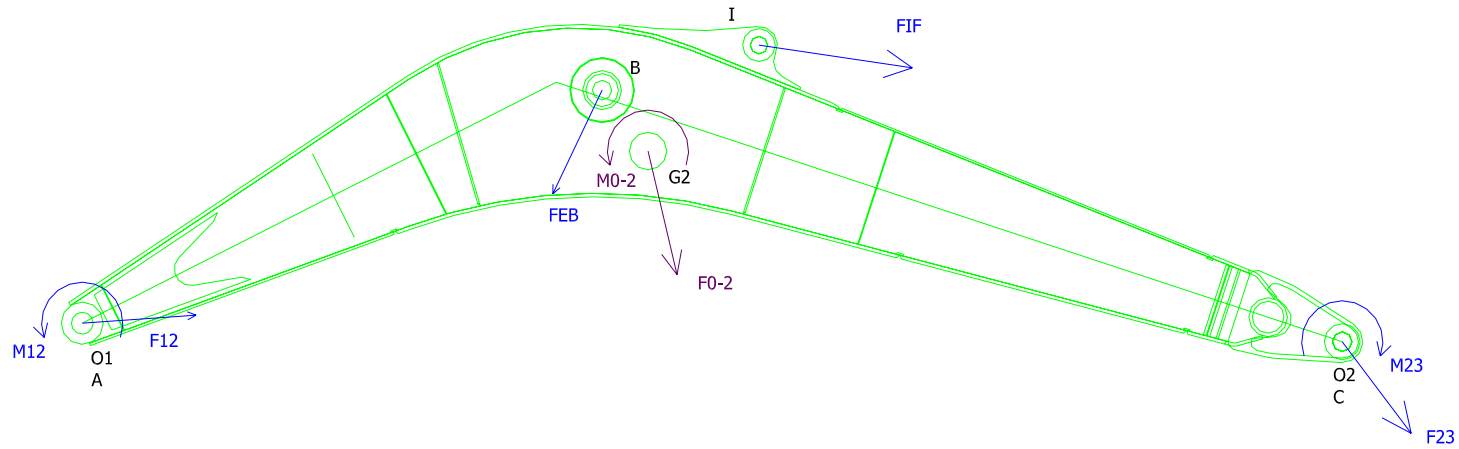


Figure B.4 – Forces on Boom

$${}^2\vec{F}_{12} = -{}^2\vec{F}_0^2 + {}^2\vec{F}_{23} - {}^2\vec{F}_{IF} + {}^2\vec{F}_{BE} \quad (B.18)$$

$${}^2\vec{M}_{12} = -{}^2\vec{M}_{23} - ({}^2\vec{p}_{02} - {}^2\vec{p}_{01}) \times {}^2\vec{F}_{23} + ({}^2\vec{p}_{0I} - {}^2\vec{p}_{01}) \times {}^2\vec{F}_{IF} - ({}^2\vec{p}_{0B} - {}^2\vec{p}_{01}) \times {}^2\vec{F}_{EB} + ({}^2\vec{p}_{02G} - {}^2\vec{p}_{01}) \times {}^2\vec{F}_0^2 + {}^2\vec{M}_0^2 \quad (B.19)$$

B.1.4.2. First Step Expansions of Equations of Backward Dynamics

B.1.4.2.1. Bucket

$$\begin{aligned}
 \bar{M}_{34} = & -\bar{M}_{45} - \begin{bmatrix} L4 \cdot \cos(\theta_2 + \theta_3 + \theta_4) \\ L4 \cdot \sin(\theta_2 + \theta_3 + \theta_4) \\ 0 \end{bmatrix} \times \begin{bmatrix} F_{45} \cdot \cos(\theta_2 + \theta_3 + \theta_4 + \phi_1) \\ F_{45} \cdot \sin(\theta_2 + \theta_3 + \theta_4 + \phi_1) \\ 0 \end{bmatrix} - \begin{bmatrix} Ldp \cdot \cos(\theta_2 + \theta_3 + \theta_4 + \beta_8) \\ Ldp \cdot \sin(\theta_2 + \theta_3 + \theta_4 + \beta_8) \\ 0 \end{bmatrix} \times \begin{bmatrix} F_{PK} \cdot \cos(\theta_2 + \theta_3 + \phi_3) \\ F_{PK} \cdot \sin(\theta_2 + \theta_3 + \phi_3) \\ 0 \end{bmatrix} \\
 & + \begin{bmatrix} Lg4 \cdot \cos(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \\ Lg4 \cdot \sin(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \\ 0 \end{bmatrix} \times \begin{bmatrix} -m_4 \cdot \left(Lg4 \cdot (\sin(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) + \cos(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)^2) \right. \\ \left. + L3 \cdot (\sin(\theta_2 + \theta_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) + \cos(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) + L2 \cdot (\sin(\theta_2) \cdot \ddot{\theta}_2 + \cos(\theta_2) \cdot (\dot{\theta}_2)^2) \right) \\ m_4 \cdot \left(Lg4 \cdot (\cos(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) - \sin(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)^2) \right. \\ \left. + L3 \cdot (\cos(\theta_2 + \theta_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) - \sin(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) + L2 \cdot (\cos(\theta_2) \cdot \ddot{\theta}_2 - \sin(\theta_2) \cdot (\dot{\theta}_2)^2) - g_4 \right) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ I_{z4} \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) \end{bmatrix} \quad (B.20)
 \end{aligned}$$

$$\begin{aligned}
 \bar{M}_{23} = & -\bar{M}_{45} - \begin{bmatrix} L4 \cdot \cos(\theta_2 + \theta_3 + \theta_4) \\ L4 \cdot \sin(\theta_2 + \theta_3 + \theta_4) \\ 0 \end{bmatrix} \times \begin{bmatrix} F_{45} \cdot \cos(\theta_2 + \theta_3 + \theta_4 + \phi_1) \\ F_{45} \cdot \sin(\theta_2 + \theta_3 + \theta_4 + \phi_1) \\ 0 \end{bmatrix} - \begin{bmatrix} Ldp \cdot \cos(\theta_2 + \theta_3 + \theta_4 + \beta_8) \\ Ldp \cdot \sin(\theta_2 + \theta_3 + \theta_4 + \beta_8) \\ 0 \end{bmatrix} \times \begin{bmatrix} F_{PK} \cdot \cos(\theta_2 + \theta_3 + \phi_3) \\ F_{PK} \cdot \sin(\theta_2 + \theta_3 + \phi_3) \\ 0 \end{bmatrix} \\
 & + \begin{bmatrix} Lg4 \cdot \cos(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \\ Lg4 \cdot \sin(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \\ 0 \end{bmatrix} \times \begin{bmatrix} -m_4 \cdot \left(Lg4 \cdot (\sin(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) + \cos(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)^2) \right. \\ \left. + L3 \cdot (\sin(\theta_2 + \theta_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) + \cos(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) + L2 \cdot (\sin(\theta_2) \cdot \ddot{\theta}_2 + \cos(\theta_2) \cdot (\dot{\theta}_2)^2) \right) \\ Lg4 \cdot (\cos(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) - \sin(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)^2) \\ \left. + L3 \cdot (\cos(\theta_2 + \theta_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) - \sin(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) + L2 \cdot (\cos(\theta_2) \cdot \ddot{\theta}_2 - \sin(\theta_2) \cdot (\dot{\theta}_2)^2) - g_4 \right) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ I_{z4} \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
& + \begin{bmatrix} L3 \cdot \cos(\theta_2 + \theta_3) \\ L3 \cdot \sin(\theta_2 + \theta_3) \\ 0 \end{bmatrix} \times \left[\begin{array}{l} -m_4 \cdot \left(\begin{array}{l} Lg4 \cdot (\sin(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) + \cos(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)^2) \\ + L3 \cdot (\sin(\theta_2 + \theta_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) + \cos(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) + L2 \cdot (\sin(\theta_2) \cdot \ddot{\theta}_2 + \cos(\theta_2) \cdot (\dot{\theta}_2)^2) \end{array} \right) \\ m_4 \cdot \left(\begin{array}{l} Lg4 \cdot (\cos(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) - \sin(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)^2) \\ + L3 \cdot (\cos(\theta_2 + \theta_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) - \sin(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) + L2 \cdot (\cos(\theta_2) \cdot \ddot{\theta}_2 - \sin(\theta_2) \cdot (\dot{\theta}_2)^2) - g_4 \end{array} \right) \\ 0 \end{array} \right] - \begin{bmatrix} F_{45} \cdot \cos(\theta_2 + \theta_3 + \theta_4 + \phi_1) \\ + F_{PK} \cdot \cos(\theta_2 + \theta_3 + \phi_3) \\ F_{45} \cdot \sin(\theta_2 + \theta_3 + \theta_4 + \phi_1) \\ + F_{PK} \cdot \sin(\theta_2 + \theta_3 + \phi_3) \end{bmatrix}
\end{aligned}$$

$$- \begin{bmatrix} Lcl \cdot \cos(\theta_2 + \theta_3 + \beta_5) \\ Lcl \cdot \sin(\theta_2 + \theta_3 + \beta_5) \\ 0 \end{bmatrix} \times \begin{bmatrix} F_{LK} \cdot \cos(\theta_2 + \theta_3 + \phi_4) \\ F_{LK} \cdot \sin(\theta_2 + \theta_3 + \phi_4) \\ 0 \end{bmatrix} - \begin{bmatrix} Lcj \cdot \cos(\theta_2 + \theta_3 + \beta_5 + \beta_6) \\ Lcj \cdot \sin(\theta_2 + \theta_3 + \beta_5 + \beta_6) \\ 0 \end{bmatrix} \times \begin{bmatrix} F_{JK} \cdot \cos(\theta_2 + \theta_3 + \phi_2) \\ F_{JK} \cdot \sin(\theta_2 + \theta_3 + \phi_2) \\ 0 \end{bmatrix} - \begin{bmatrix} Lcf \cdot \cos(\theta_2 + \theta_3 + \beta_5 + \beta_6 + \beta_7) \\ Lcf \cdot \sin(\theta_2 + \theta_3 + \beta_5 + \beta_6 + \beta_7) \\ 0 \end{bmatrix} \times \begin{bmatrix} F_{IF} \cdot \cos(\theta_2 + \phi_5) \\ F_{IF} \cdot \sin(\theta_2 + \phi_5) \\ 0 \end{bmatrix}$$

$$\begin{aligned}
& + \begin{bmatrix} Lg3 \cdot \cos(\theta_2 + \theta_3 + \lambda_2) \\ Lg3 \cdot \sin(\theta_2 + \theta_3 + \lambda_2) \\ 0 \end{bmatrix} \times \begin{bmatrix} -m_3 \cdot \begin{bmatrix} Lg3 \cdot (\sin(\theta_2 + \theta_3 + \lambda_2) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) + \cos(\theta_2 + \theta_3 + \lambda_2) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) \\ +L2 \cdot (\sin(\theta_2) \cdot \ddot{\theta}_2 + \cos(\theta_2) \cdot (\dot{\theta}_2)^2) \end{bmatrix} \\ m_3 \cdot \begin{bmatrix} Lg3 \cdot (\cos(\theta_2 + \theta_3 + \lambda_2) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) - \sin(\theta_2 + \theta_3 + \lambda_2) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) \\ +L2 \cdot (\cos(\theta_2) \cdot \ddot{\theta}_2 - \sin(\theta_2) \cdot (\dot{\theta}_2)^2) - g_3 \\ 0 \end{bmatrix} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ I_{z3} \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) \end{bmatrix} \quad (\text{B.21})
\end{aligned}$$

$$\begin{aligned}
 \bar{M}_{12} = -\bar{M}_{45} &= - \begin{bmatrix} L4 \cdot \cos(\theta_2 + \theta_3 + \theta_4) \\ L4 \cdot \sin(\theta_2 + \theta_3 + \theta_4) \\ 0 \end{bmatrix} \times \begin{bmatrix} F_{45} \cdot \cos(\theta_2 + \theta_3 + \theta_4 + \phi_1) \\ F_{45} \cdot \sin(\theta_2 + \theta_3 + \theta_4 + \phi_1) \\ 0 \end{bmatrix} - \begin{bmatrix} Ldp \cdot \cos(\theta_2 + \theta_3 + \theta_4 + \beta_8) \\ Ldp \cdot \sin(\theta_2 + \theta_3 + \theta_4 + \beta_8) \\ 0 \end{bmatrix} \times \begin{bmatrix} F_{PK} \cdot \cos(\theta_2 + \theta_3 + \phi_3) \\ F_{PK} \cdot \sin(\theta_2 + \theta_3 + \phi_3) \\ 0 \end{bmatrix} \\
 &+ \begin{bmatrix} Lg4 \cdot \cos(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \\ Lg4 \cdot \sin(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \\ 0 \end{bmatrix} \times \begin{bmatrix} -m_4 \cdot \begin{bmatrix} Lg4 \cdot (\sin(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) + \cos(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)^2) \\ +L3 \cdot (\sin(\theta_2 + \theta_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) + \cos(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) + L2 \cdot (\sin(\theta_2) \cdot \ddot{\theta}_2 + \cos(\theta_2) \cdot (\dot{\theta}_2)^2) \end{bmatrix} \\
 m_4 \cdot \begin{bmatrix} Lg4 \cdot (\cos(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) - \sin(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)^2) \\ +L3 \cdot (\cos(\theta_2 + \theta_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) - \sin(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) + L2 \cdot (\cos(\theta_2) \cdot \ddot{\theta}_2 - \sin(\theta_2) \cdot (\dot{\theta}_2)^2) - g_4 \end{bmatrix} \\
 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ I_{z4} \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
& + \begin{bmatrix} L3 \cdot \cos(\theta_2 + \theta_3) \\ L3 \cdot \sin(\theta_2 + \theta_3) \\ 0 \end{bmatrix} \times \begin{bmatrix} -m_4 \cdot \begin{bmatrix} Lg4 \cdot (\sin(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) + \cos(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)^2) \\ +L3 \cdot (\sin(\theta_2 + \theta_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) + \cos(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) + L2 \cdot (\sin(\theta_2) \cdot \ddot{\theta}_2 + \cos(\theta_2) \cdot (\dot{\theta}_2)^2) \end{bmatrix} - \begin{bmatrix} F_{45} \cdot \cos(\theta_2 + \theta_3 + \theta_4 + \phi_1) \\ +F_{PK} \cdot \cos(\theta_2 + \theta_3 + \phi_3) \end{bmatrix} \\
m_4 \cdot \begin{bmatrix} Lg4 \cdot (\cos(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) - \sin(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)^2) \\ +L3 \cdot (\cos(\theta_2 + \theta_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) - \sin(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) + L2 \cdot (\cos(\theta_2) \cdot \ddot{\theta}_2 - \sin(\theta_2) \cdot (\dot{\theta}_2)^2) - g_4 \end{bmatrix} - \begin{bmatrix} F_{45} \cdot \sin(\theta_2 + \theta_3 + \theta_4 + \phi_1) \\ +F_{PK} \cdot \sin(\theta_2 + \theta_3 + \phi_3) \end{bmatrix} \\
0 \end{bmatrix}
\end{aligned}$$

$$- \begin{bmatrix} Lcl \cdot \cos(\theta_2 + \theta_3 + \beta_5) \\ Lcl \cdot \sin(\theta_2 + \theta_3 + \beta_5) \\ 0 \end{bmatrix} \times \begin{bmatrix} F_{LK} \cdot \cos(\theta_2 + \theta_3 + \phi_4) \\ F_{LK} \cdot \sin(\theta_2 + \theta_3 + \phi_4) \\ 0 \end{bmatrix} - \begin{bmatrix} Lcj \cdot \cos(\theta_2 + \theta_3 + \beta_5 + \beta_6) \\ Lcj \cdot \sin(\theta_2 + \theta_3 + \beta_5 + \beta_6) \\ 0 \end{bmatrix} \times \begin{bmatrix} F_{JK} \cdot \cos(\theta_2 + \theta_3 + \phi_2) \\ F_{JK} \cdot \sin(\theta_2 + \theta_3 + \phi_2) \\ 0 \end{bmatrix} - \begin{bmatrix} Lcf \cdot \cos(\theta_2 + \theta_3 + \beta_5 + \beta_6 + \beta_7) \\ Lcf \cdot \sin(\theta_2 + \theta_3 + \beta_5 + \beta_6 + \beta_7) \\ 0 \end{bmatrix} \times \begin{bmatrix} F_{IF} \cdot \cos(\theta_2 + \phi_5) \\ F_{IF} \cdot \sin(\theta_2 + \phi_5) \\ 0 \end{bmatrix}$$

$$\begin{aligned}
& + \begin{bmatrix} Lg_3 \cdot \cos(\theta_2 + \theta_3 + \lambda_2) \\ Lg_3 \cdot \sin(\theta_2 + \theta_3 + \lambda_2) \\ 0 \end{bmatrix} \times \begin{bmatrix} -m_3 \cdot \begin{bmatrix} Lg_3 \cdot (\sin(\theta_2 + \theta_3 + \lambda_2) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) + \cos(\theta_2 + \theta_3 + \lambda_2) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) \\ +L_2 \cdot (\sin(\theta_2) \cdot \ddot{\theta}_2 + \cos(\theta_2) \cdot (\dot{\theta}_2)^2) \end{bmatrix} \\ m_3 \cdot \begin{bmatrix} Lg_3 \cdot (\cos(\theta_2 + \theta_3 + \lambda_2) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) - \sin(\theta_2 + \theta_3 + \lambda_2) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) \\ +L_2 \cdot (\cos(\theta_2) \cdot \ddot{\theta}_2 - \sin(\theta_2) \cdot (\dot{\theta}_2)^2) - g_3 \end{bmatrix} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ I_{z_3} \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
& + \begin{bmatrix} L2 \cdot \cos(\theta_2) \\ L2 \cdot \sin(\theta_2) \\ 0 \end{bmatrix} \times \left[\begin{array}{l} -m_3 \cdot \left[\begin{array}{l} Lg3 \cdot (\sin(\theta_2 + \theta_3 + \lambda_2) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) + \cos(\theta_2 + \theta_3 + \lambda_2) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) \\ + L2 \cdot (\sin(\theta_2) \cdot \ddot{\theta}_2 + \cos(\theta_2) \cdot (\dot{\theta}_2)^2) \end{array} \right] \\ \\ m_3 \cdot \left[\begin{array}{l} Lg3 \cdot (\cos(\theta_2 + \theta_3 + \lambda_2) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) - \sin(\theta_2 + \theta_3 + \lambda_2) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) \\ + L2 \cdot (\cos(\theta_2) \cdot \ddot{\theta}_2 - \sin(\theta_2) \cdot (\dot{\theta}_2)^2) + g_3 \end{array} \right] \end{array} \right] \\
& - m_4 \cdot \left[\begin{array}{l} Lg4 \cdot (\sin(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) + \cos(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)^2) \\ + L3 \cdot (\sin(\theta_2 + \theta_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) + \cos(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) \\ + L2 \cdot (\sin(\theta_2) \cdot \ddot{\theta}_2 + \cos(\theta_2) \cdot (\dot{\theta}_2)^2) \end{array} \right] \\
& - m_4 \cdot \left[\begin{array}{l} Lg4 \cdot (\cos(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4) - \sin(\theta_2 + \theta_3 + \theta_4 + \lambda_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4)^2) \\ + L3 \cdot (\cos(\theta_2 + \theta_3) \cdot (\ddot{\theta}_2 + \ddot{\theta}_3) - \sin(\theta_2 + \theta_3) \cdot (\dot{\theta}_2 + \dot{\theta}_3)^2) \\ + L2 \cdot (\cos(\theta_2) \cdot \ddot{\theta}_2 - \sin(\theta_2) \cdot (\dot{\theta}_2)^2) - g_4 \end{array} \right] \\
& 0 \\
& \left[\begin{array}{l} F_{45} \cdot \cos(\theta_2 + \theta_3 + \theta_4 + \phi) \\ + F_{PK} \cdot \cos(\theta_2 + \theta_3 + \phi_3) \\ + F_{LK} \cdot \cos(\theta_2 + \theta_3 + \phi_4) \\ + F_{JK} \cdot \cos(\theta_2 + \theta_3 + \phi_2) \\ + F_{IF} \cdot \cos(\theta_2 + \phi_3) \\ F_{45} \cdot \sin(\theta_2 + \theta_3 + \theta_4 + \phi) \\ + F_{PK} \cdot \sin(\theta_2 + \theta_3 + \phi_3) \\ + F_{LK} \cdot \sin(\theta_2 + \theta_3 + \phi_4) \\ + F_{JK} \cdot \sin(\theta_2 + \theta_3 + \phi_2) \\ + F_{IF} \cdot \sin(\theta_2 + \phi_3) \end{array} \right] \\
& + \begin{bmatrix} Lai \cdot \cos(\theta_2 + \beta_2) \\ Lai \cdot \sin(\theta_2 + \beta_2) \\ 0 \end{bmatrix} \times \begin{bmatrix} F_{IF} \cdot \cos(\theta_2 + \phi_3) \\ F_{IF} \cdot \sin(\theta_2 + \phi_3) \\ 0 \end{bmatrix} - \begin{bmatrix} Lab \cdot \cos(\theta_2 + \beta_2 + \beta_3) \\ Lab \cdot \sin(\theta_2 + \beta_2 + \beta_3) \\ 0 \end{bmatrix} \times \begin{bmatrix} F_{EB} \cdot \cos(\theta_2 + \phi_6) \\ F_{EB} \cdot \sin(\theta_2 + \phi_6) \\ 0 \end{bmatrix} + \begin{bmatrix} Lg2 \cdot \cos(\theta_2 + \lambda_1) \\ Lg2 \cdot \sin(\theta_2 + \lambda_1) \\ 0 \end{bmatrix} \times \left[\begin{array}{l} -m_2 \cdot [Lg2 \cdot (\sin(\lambda_1 + \theta_2) \cdot \ddot{\theta}_2 + \cos(\lambda_1 + \theta_2) \cdot (\dot{\theta}_2)^2)] \\ m_2 \cdot [Lg2 \cdot (\cos(\lambda_1 + \theta_2) \cdot \ddot{\theta}_2 - \sin(\lambda_1 + \theta_2) \cdot (\dot{\theta}_2)^2) - g_2] \\ 0 \end{array} \right] + \begin{bmatrix} 0 \\ 0 \\ I_{z2} \cdot \ddot{\theta}_2 \end{bmatrix} \quad (B.22)
\end{aligned}$$

$$\begin{bmatrix} M_{12} \\ M_{23} \\ M_{34} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} I_{z_2} + Lg 2^2 \cdot m_2 + L2^2 \cdot m_4 + L2^2 \cdot m_3 \\ + L3 \cdot m_4 \cdot L2 \cdot \cos(\theta_3) \\ + Lg 4 \cdot m_4 \cdot L2 \cdot \cos(\theta_3 + \theta_4 + \lambda_3) \\ + Lg 3 \cdot m_3 \cdot L2 \cdot \cos(\theta_3 + \lambda_2) \end{bmatrix} & \begin{bmatrix} I_{z_3} + Lg 3^2 \cdot m_3 + L3^2 \cdot m_4 \\ + L3 \cdot m_4 \cdot L2 \cdot \cos(\theta_3) \\ + Lg 3 \cdot m_3 \cdot L2 \cdot \cos(\theta_3 + \lambda_2) \\ + Lg 4 \cdot m_4 \cdot L3 \cdot \cos(\theta_4 + \lambda_3) \end{bmatrix} & \begin{bmatrix} I_{z_4} + Lg 4^2 \cdot m_4 \\ + L2 \cdot m_4 \cdot Lg 4 \cdot \cos(\theta_3 + \theta_4 + \lambda_3) \\ + L3 \cdot m_4 \cdot Lg 4 \cdot \cos(\theta_4 + \lambda_3) \end{bmatrix} \\ \begin{bmatrix} L3 \cdot m_4 \cdot L2 \cdot \cos(\theta_3) \\ + Lg 4 \cdot m_4 \cdot L2 \cdot \cos(\theta_3 + \theta_4 + \lambda_3) \\ + Lg 3 \cdot m_3 \cdot L2 \cdot \cos(\theta_3 + \lambda_2) \end{bmatrix} & \begin{bmatrix} I_{z_3} + Lg 3^2 \cdot m_3 + L3^2 \cdot m_4 \\ + Lg 4 \cdot m_4 \cdot L3 \cdot \cos(\theta_4 + \lambda_3) \end{bmatrix} & \begin{bmatrix} I_{z_4} + Lg 4^2 \cdot m_4 \\ + L3 \cdot m_4 \cdot Lg 4 \cdot \cos(\theta_4 + \lambda_3) \end{bmatrix} \\ Lg 4 \cdot m_4 \cdot L2 \cdot \cos(\theta_3 + \theta_4 + \lambda_3) & Lg 4 \cdot m_4 \cdot L3 \cdot \cos(\theta_4 + \lambda_3) & I_{z_4} + Lg 4^2 \cdot m_4 \end{bmatrix} \cdot \begin{bmatrix} \ddot{\theta}_2 \\ \ddot{\theta}_2 + \ddot{\theta}_3 \\ \ddot{\theta}_2 + \ddot{\theta}_3 + \ddot{\theta}_4 \end{bmatrix}$$

B.1.4.3. Equations of Motion

$$\begin{aligned}
& \begin{bmatrix} L3 \cdot m_4 \cdot \cos(\theta_2 + \theta_3) \\ +L2 \cdot m_4 \cdot \cos(\theta_2) + L2 \cdot m_3 \cdot \cos(\theta_2) \\ +Lg4 \cdot m_4 \cdot \cos(\theta_2 + \theta_3 + \theta_4 + \lambda_3) + Lg3 \cdot m_3 \cdot \cos(\theta_2 + \theta_3 + \lambda_2) + Lg2 \cdot m_2 \cdot \cos(\theta_2 + \lambda_1) \end{bmatrix} \\
& \begin{bmatrix} L3 \cdot m_4 \cdot \cos(\theta_2 + \theta_3) \\ +Lg4 \cdot m_4 \cdot \cos(\theta_2 + \theta_3 + \theta_4 + \lambda_3) + Lg3 \cdot m_3 \cdot \cos(\theta_2 + \theta_3 + \lambda_2) \end{bmatrix} \\
& Lg4 \cdot m_4 \cdot \cos(\theta_2 + \theta_3 + \theta_4 + \lambda_3)
\end{aligned} \cdot g - \begin{bmatrix} Ldp \cdot \sin(-\theta_4 - \beta_8 + \phi_3) \\ +L3 \cdot \sin(\phi_3) \\ +L2 \cdot \sin(\theta_3 + \phi_3) \end{bmatrix} \cdot F_{PK} - \begin{bmatrix} Lcl \cdot \sin(-\beta_5 + \phi_4) \\ +L2 \cdot \sin(\theta_3 + \phi_4) \\ Lcl \cdot \sin(-\beta_5 + \phi_4) \\ 0 \end{bmatrix} \cdot F_{LK}
\end{aligned}$$

III

$$\begin{aligned}
& \begin{bmatrix} Lcj \cdot \sin(-\beta_5 - \beta_6 + \phi_2) \\ +L2 \cdot \sin(\theta_3 + \phi_2) \end{bmatrix} \cdot F_{JK} - \begin{bmatrix} Lcf \cdot \sin(-\theta_3 - \beta_5 - \beta_6 - \beta_7 + \phi_5) \\ +L2 \sin(\phi_5) - Lai \cdot \sin(-\beta_2 + \phi_5) \end{bmatrix} \cdot F_{IF} - \begin{bmatrix} Lab \cdot \sin(-\beta_2 - \beta_3 + \phi_6) \\ 0 \\ 0 \end{bmatrix} \cdot F_{EB} - \begin{bmatrix} L4 \cdot \sin(\phi_1) \\ +L3 \cdot \sin(\theta_4 + \phi_1) \\ +L2 \cdot \sin(\theta_3 + \theta_4 + \phi_1) \\ L4 \cdot \sin(\phi_1) \\ +L3 \cdot \sin(\theta_4 + \phi_1) \\ L4 \cdot \sin(\phi_1) \end{bmatrix} \cdot F_{45} - \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot M_{45} \quad (B.23)
\end{aligned}$$

APPENDIX C

SIMULATION EQUATIONS

A detailed numerical analysis of excavator's motion equations is explained in this appendix.

C.1. Modified Equations of Motion

$$\begin{bmatrix} M_{12} \\ M_{23} \\ M_{34} \end{bmatrix} = \begin{bmatrix} \left(\begin{array}{l} I_{z2} + Lg2^2 \cdot m_2 + L2^2 \cdot m_4 + L2^2 \cdot m_3 \\ + L3 \cdot m_4 \cdot L2 \cdot \cos(\theta_{23} - \theta_2) \\ + Lg4 \cdot m_4 \cdot L2 \cdot \cos(\theta_{234} - \theta_2 + \lambda_3) \\ + Lg3 \cdot m_3 \cdot L2 \cdot \cos(\theta_{23} - \theta_2 + \lambda_2) \end{array} \right) & \left(\begin{array}{l} I_{z3} + Lg3^2 \cdot m_3 + L3^2 \cdot m_4 \\ + L3 \cdot m_4 \cdot L2 \cdot \cos(\theta_{23} - \theta_2) \\ + Lg3 \cdot m_3 \cdot L2 \cdot \cos(\theta_{23} - \theta_2 + \lambda_2) \\ + Lg4 \cdot m_4 \cdot L3 \cdot \cos(\theta_{234} - \theta_{23} + \lambda_3) \end{array} \right) & \left(\begin{array}{l} I_{z4} + Lg4^2 \cdot m_4 \\ + L2 \cdot m_4 \cdot Lg4 \cdot \cos(\theta_{234} - \theta_2 + \lambda_3) \\ + L3 \cdot m_4 \cdot Lg4 \cdot \cos(\theta_{234} - \theta_{23} + \lambda_3) \end{array} \right) \\ \left(\begin{array}{l} L3 \cdot m_4 \cdot L2 \cdot \cos(\theta_{23} - \theta_2) \\ + Lg4 \cdot m_4 \cdot L2 \cdot \cos(\theta_{234} - \theta_2 + \lambda_3) \\ + Lg3 \cdot m_3 \cdot L2 \cdot \cos(\theta_{23} - \theta_2 + \lambda_2) \end{array} \right) & \left(\begin{array}{l} I_{z3} + Lg3^2 \cdot m_3 + L3^2 \cdot m_4 \\ + Lg4 \cdot m_4 \cdot L3 \cdot \cos(\theta_{234} - \theta_{23} + \lambda_3) \end{array} \right) & \left(\begin{array}{l} I_{z4} + Lg4^2 \cdot m_4 \\ + L3 \cdot m_4 \cdot Lg4 \cdot \cos(\theta_{234} - \theta_{23} + \lambda_3) \end{array} \right) \\ Lg4 \cdot m_4 \cdot L2 \cdot \cos(\theta_{234} - \theta_2 + \lambda_3) & Lg4 \cdot m_4 \cdot L3 \cdot \cos(\theta_{234} - \theta_{23} + \lambda_3) & I_{z4} + Lg4^2 \cdot m_4 \end{bmatrix} \cdot \begin{bmatrix} \ddot{\theta}_2 \\ \ddot{\theta}_{23} \\ \ddot{\theta}_{234} \end{bmatrix}$$

$$\begin{aligned}
& \begin{bmatrix} (L3 \cdot m_4 \cdot L2 \cdot \sin(\theta_{23} - \theta_2) \\ + Lg4 \cdot m_4 \cdot L2 \cdot \sin(\theta_{234} - \theta_2 + \lambda_3) \\ + Lg3 \cdot m_3 \cdot L2 \cdot \sin(\theta_{23} - \theta_2 + \lambda_2) \end{bmatrix} \begin{bmatrix} (-L2 \cdot m_3 \cdot Lg3 \cdot \sin(\theta_{23} - \theta_2 + \lambda_2) \\ -L3 \cdot m_4 \cdot L2 \cdot \sin(\theta_{23} - \theta_2) \\ + Lg4 \cdot m_4 \cdot L3 \cdot \sin(\theta_{234} - \theta_{23} + \lambda_3) \end{bmatrix} \begin{bmatrix} (-L2 \cdot m_4 \cdot Lg4 \cdot \sin(\theta_{234} - \theta_2 + \lambda_3) \\ -L3 \cdot m_4 \cdot Lg4 \cdot \sin(\theta_{234} - \theta_{23} + \lambda_3) \end{bmatrix} \\
+ & \begin{bmatrix} (L3 \cdot m_4 \cdot L2 \cdot \sin(\theta_{23} - \theta_2) \\ + Lg4 \cdot m_4 \cdot L2 \cdot \sin(\theta_{234} - \theta_2 + \lambda_3) \\ + Lg3 \cdot m_3 \cdot L2 \cdot \sin(\theta_{23} - \theta_2 + \lambda_2) \end{bmatrix} \begin{bmatrix} Lg4 \cdot m_4 \cdot L3 \cdot \sin(\theta_{234} - \theta_{23} + \lambda_3) \\ Lg4 \cdot m_4 \cdot L3 \cdot \sin(\theta_{234} - \theta_{23} + \lambda_3) \\ 0 \end{bmatrix} \begin{bmatrix} -L3 \cdot m_4 \cdot Lg4 \cdot \sin(\theta_{234} - \theta_{23} + \lambda_3) \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} (\dot{\theta}_2)^2 \\ (\dot{\theta}_{23})^2 \\ (\dot{\theta}_{234})^2 \end{bmatrix}
\end{aligned}$$

114

$$\begin{aligned}
- & \begin{bmatrix} (L3 \cdot m_4 \cdot \cos(\theta_{23}) + L2 \cdot m_4 \cdot \cos(\theta_2) + L2 \cdot m_3 \cdot \cos(\theta_2) \\ + Lg4 \cdot m_4 \cdot \cos(\theta_{234} + \lambda_3) + Lg3 \cdot m_3 \cdot \cos(\theta_{23} + \lambda_2) + Lg2 \cdot m_2 \cdot \cos(\theta_2 + \lambda_1) \\ (L3 \cdot m_4 \cdot \cos(\theta_{23}) + Lg4 \cdot m_4 \cdot \cos(\theta_{234} + \lambda_3) + Lg3 \cdot m_3 \cdot \cos(\theta_{23} + \lambda_2)) \\ Lg4 \cdot m_4 \cdot \cos(\theta_{234} + \lambda_3) \end{bmatrix} \cdot g - \begin{bmatrix} (Ldp \cdot \sin(-\theta_{234} + \theta_{23} - \beta_8 + \phi_3) \\ + L3 \cdot \sin(\phi_3) \\ + L2 \cdot \sin(\theta_{23} - \theta_2 + \phi_3) \\ (Ldp \cdot \sin(-\theta_{234} + \theta_{23} - \beta_8 + \phi_3) \\ + L3 \cdot \sin(\phi_3) \\ Ldp \cdot \sin(-\theta_{234} + \theta_{23} - \beta_8 + \phi_3) \end{bmatrix} \cdot F_{PK} - \begin{bmatrix} (Lcl \cdot \sin(-\beta_5 + \phi_4) \\ + L2 \cdot \sin(\theta_{23} - \theta_2 + \phi_4) \\ Lcl \cdot \sin(-\beta_5 + \phi_4) \\ 0 \end{bmatrix} \cdot F_{LK}
\end{aligned}$$

$$\begin{aligned}
& - \begin{bmatrix} \left(\begin{array}{l} Lcj \cdot \sin(-\beta_5 - \beta_6 + \phi_2) \\ +L2 \cdot \sin(\theta_{23} - \theta_2 + \phi_2) \end{array} \right) \\ Lcj \cdot \sin(-\beta_5 - \beta_6 + \phi_2) \\ 0 \end{bmatrix} \cdot F_{JK} - \begin{bmatrix} \left(\begin{array}{l} Lcf \cdot \sin(-\theta_{23} + \theta_2 - \beta_5 - \beta_6 - \beta_7 + \phi_5) \\ +L2 \sin(\phi_5) - Lai \cdot \sin(-\beta_2 + \phi_5) \end{array} \right) \\ Lcf \cdot \sin(-\theta_{23} + \theta_2 - \beta_5 - \beta_6 - \beta_7 + \phi_5) \\ 0 \end{bmatrix} \cdot F_{IF} - \begin{bmatrix} Lab \cdot \sin(-\beta_2 - \beta_3 + \phi_6) \\ 0 \\ 0 \end{bmatrix} \cdot F_{EB} - \begin{bmatrix} \left(\begin{array}{l} L4 \cdot \sin(\phi_1) \\ +L3 \cdot \sin(\theta_{234} - \theta_{23} + \phi_1) \\ +L2 \cdot \sin(\theta_{234} - \theta_2 + \phi_1) \end{array} \right) \\ \left(\begin{array}{l} L4 \cdot \sin(\phi_1) \\ +L3 \cdot \sin(\theta_{234} - \theta_{23} + \phi_1) \end{array} \right) \\ L4 \cdot \sin(\phi_1) \end{bmatrix} \cdot F_{45} - \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot M_{45} \quad (C.1)
\end{aligned}$$

C.2. Solving Modified Equations of Motion by Using Numerical Method

C.2.1. Arranging Equations of Motion

General view of modified equations of motion:

$$\begin{aligned}
 0 = & A(\theta_2, \theta_{23}, \theta_{234}) \cdot \begin{bmatrix} \ddot{\theta}_2 \\ \ddot{\theta}_{23} \\ \ddot{\theta}_{234} \end{bmatrix} + B(\theta_2, \theta_{23}, \theta_{234}) \cdot \begin{bmatrix} (\dot{\theta}_2)^2 \\ (\dot{\theta}_{23})^2 \\ (\dot{\theta}_{234})^2 \end{bmatrix} \\
 & - C(\theta_2, \theta_{23}, \theta_{234}) \cdot g - D(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{PK} \\
 & - E(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{LK} - F(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{JK} \\
 & - G(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{IF} - H(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{EB} \\
 & - J(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{45} - K(\theta_2, \theta_{23}, \theta_{234}) \cdot M_{45} \tag{C.2}
 \end{aligned}$$

where

$$\theta_{23} = \theta_2 + \theta_3$$

$$\theta_{234} = \theta_2 + \theta_3 + \theta_4$$

$$\dot{\theta}_{23} = \dot{\theta}_2 + \dot{\theta}_3$$

$$\dot{\theta}_{234} = \dot{\theta}_2 + \dot{\theta}_3 + \dot{\theta}_4$$

$$A = \begin{bmatrix} \begin{pmatrix} I_{z2} + Lg2^2 \cdot m_2 + L2^2 \cdot m_4 + L2^2 \cdot m_3 \\ +L3 \cdot m_4 \cdot L2 \cdot \cos(\theta_{23} - \theta_2) \\ +Lg4 \cdot m_4 \cdot L2 \cdot \cos(\theta_{234} - \theta_2 + \lambda_3) \\ +Lg3 \cdot m_3 \cdot L2 \cdot \cos(\theta_{23} - \theta_2 + \lambda_2) \end{pmatrix} & \begin{pmatrix} I_{z3} + Lg3^2 \cdot m_3 + L3^2 \cdot m_4 \\ +L3 \cdot m_4 \cdot L2 \cdot \cos(\theta_{23} - \theta_2) \\ +Lg3 \cdot m_3 \cdot L2 \cdot \cos(\theta_{23} - \theta_2 + \lambda_2) \\ +Lg4 \cdot m_4 \cdot L3 \cdot \cos(\theta_{234} - \theta_{23} + \lambda_3) \end{pmatrix} & \begin{pmatrix} I_{z4} + Lg4^2 \cdot m_4 \\ +L2 \cdot m_4 \cdot Lg4 \cdot \cos(\theta_{234} - \theta_2 + \lambda_3) \\ +L3 \cdot m_4 \cdot Lg4 \cdot \cos(\theta_{234} - \theta_{23} + \lambda_3) \end{pmatrix} \\
 \begin{pmatrix} L3 \cdot m_4 \cdot L2 \cdot \cos(\theta_{23} - \theta_2) \\ +Lg4 \cdot m_4 \cdot L2 \cdot \cos(\theta_{234} - \theta_2 + \lambda_3) \\ +Lg3 \cdot m_3 \cdot L2 \cdot \cos(\theta_{23} - \theta_2 + \lambda_2) \end{pmatrix} & \begin{pmatrix} I_{z3} + Lg3^2 \cdot m_3 + L3^2 \cdot m_4 \\ +Lg4 \cdot m_4 \cdot L3 \cdot \cos(\theta_{234} - \theta_{23} + \lambda_3) \end{pmatrix} & \begin{pmatrix} I_{z4} + Lg4^2 \cdot m_4 \\ +L3 \cdot m_4 \cdot Lg4 \cdot \cos(\theta_{234} - \theta_{23} + \lambda_3) \end{pmatrix} \\
 Lg4 \cdot m_4 \cdot L2 \cdot \cos(\theta_{234} - \theta_2 + \lambda_3) & Lg4 \cdot m_4 \cdot L3 \cdot \cos(\theta_{234} - \theta_{23} + \lambda_3) & I_{z4} + Lg4^2 \cdot m_4
 \end{bmatrix}$$

$$B = \begin{bmatrix} \begin{pmatrix} L3 \cdot m_4 \cdot L2 \cdot \sin(\theta_{23} - \theta_2) \\ +Lg4 \cdot m_4 \cdot L2 \cdot \sin(\theta_{234} - \theta_2 + \lambda_3) \\ +Lg3 \cdot m_3 \cdot L2 \cdot \sin(\theta_{23} - \theta_2 + \lambda_2) \end{pmatrix} & \begin{pmatrix} -L2 \cdot m_3 \cdot Lg3 \cdot \sin(\theta_{23} - \theta_2 + \lambda_2) \\ -L3 \cdot m_4 \cdot L2 \cdot \sin(\theta_{23} - \theta_2) \\ +Lg4 \cdot m_4 \cdot L3 \cdot \sin(\theta_{234} - \theta_{23} + \lambda_3) \end{pmatrix} & \begin{pmatrix} -L2 \cdot m_4 \cdot Lg4 \cdot \sin(\theta_{234} - \theta_2 + \lambda_3) \\ -L3 \cdot m_4 \cdot Lg4 \cdot \sin(\theta_{234} - \theta_{23} + \lambda_3) \end{pmatrix} \\ \begin{pmatrix} L3 \cdot m_4 \cdot L2 \cdot \sin(\theta_{23} - \theta_2) \\ +Lg4 \cdot m_4 \cdot L2 \cdot \sin(\theta_{234} - \theta_2 + \lambda_3) \\ +Lg3 \cdot m_3 \cdot L2 \cdot \sin(\theta_{23} - \theta_2 + \lambda_2) \end{pmatrix} & Lg4 \cdot m_4 \cdot L3 \cdot \sin(\theta_{234} - \theta_{23} + \lambda_3) & -L3 \cdot m_4 \cdot Lg4 \cdot \sin(\theta_{234} - \theta_{23} + \lambda_3) \\ Lg4 \cdot m_4 \cdot L2 \cdot \sin(\theta_{234} - \theta_2 + \lambda_3) & Lg4 \cdot m_4 \cdot L3 \cdot \sin(\theta_{234} - \theta_{23} + \lambda_3) & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} L3 \cdot m_4 \cdot \cos(\theta_{23}) + L2 \cdot m_4 \cdot \cos(\theta_2) + L2 \cdot m_3 \cdot \cos(\theta_2) + Lg4 \cdot m_4 \cdot \cos(\theta_{234} + \lambda_3) + Lg3 \cdot m_3 \cdot \cos(\theta_{23} + \lambda_2) + Lg2 \cdot m_2 \cdot \cos(\theta_2 + \lambda_1) \\ L3 \cdot m_4 \cdot \cos(\theta_{23}) + Lg4 \cdot m_4 \cdot \cos(\theta_{234} + \lambda_3) + Lg3 \cdot m_3 \cdot \cos(\theta_{23} + \lambda_2) \\ Lg4 \cdot m_4 \cdot \cos(\theta_{234} + \lambda_3) \end{bmatrix}$$

$$D = \begin{bmatrix} Ldp \cdot \sin(-\theta_{234} + \theta_{23} - \beta_8 + \phi_3) + L3 \cdot \sin(\phi_3) + L2 \cdot \sin(\theta_{23} - \theta_2 + \phi_3) \\ Ldp \cdot \sin(-\theta_{234} + \theta_{23} - \beta_8 + \phi_3) + L3 \cdot \sin(\phi_3) \\ Ldp \cdot \sin(-\theta_{234} + \theta_{23} - \beta_8 + \phi_3) \end{bmatrix}$$

$$E = \begin{bmatrix} Lcl \cdot \sin(-\beta_5 + \phi_4) + L2 \cdot \sin(\theta_{23} - \theta_2 + \phi_4) \\ Lcl \cdot \sin(-\beta_5 + \phi_4) \\ 0 \end{bmatrix}$$

$$F = \begin{bmatrix} Lcj \cdot \sin(-\beta_5 - \beta_6 + \phi_2) + L2 \cdot \sin(\theta_{23} - \theta_2 + \phi_2) \\ Lcj \cdot \sin(-\beta_5 - \beta_6 + \phi_2) \\ 0 \end{bmatrix}$$

$$G = \begin{bmatrix} Lcf \cdot \sin(-\theta_{23} + \theta_2 - \beta_5 - \beta_6 - \beta_7 + \phi_5) + L2 \sin(\phi_5) - Lai \cdot \sin(-\beta_2 + \phi_5) \\ Lcf \cdot \sin(-\theta_{23} + \theta_2 - \beta_5 - \beta_6 - \beta_7 + \phi_5) \\ 0 \end{bmatrix}$$

$$H = \begin{bmatrix} Lab \cdot \sin(-\beta_2 - \beta_3 + \phi_6) \\ 0 \\ 0 \end{bmatrix}$$

$$J = \begin{bmatrix} L4 \cdot \sin(\phi_1) + L3 \cdot \sin(\theta_{234} - \theta_{23} + \phi_1) + L2 \cdot \sin(\theta_{234} - \theta_2 + \phi_1) \\ L4 \cdot \sin(\phi_1) + L3 \cdot \sin(\theta_{234} - \theta_{23} + \phi_1) \\ L4 \cdot \sin(\phi_1) \end{bmatrix}$$

$$K = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

There are two matrix equations, which are going to be solved numerically.

Matrix equation 1:

$$\begin{bmatrix} \dot{\theta}_2 \\ \dot{\theta}_{23} \\ \dot{\theta}_{234} \end{bmatrix} = \begin{bmatrix} \frac{d}{dt} \theta_2 \\ \frac{d}{dt} \theta_{23} \\ \frac{d}{dt} \theta_{234} \end{bmatrix}$$

Matrix equation 2:

$$\begin{bmatrix} \ddot{\theta}_2 \\ \ddot{\theta}_{23} \\ \ddot{\theta}_{234} \end{bmatrix} = [A(\theta_2, \theta_{23}, \theta_{234})]^{-1} \cdot (-B(\theta_2, \theta_{23}, \theta_{234}) \cdot \begin{bmatrix} (\dot{\theta}_2)^2 \\ (\dot{\theta}_{23})^2 \\ (\dot{\theta}_{234})^2 \end{bmatrix} \\ + C(\theta_2, \theta_{23}, \theta_{234}) \cdot g + D(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{PK} + E(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{LK} \\ + F(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{JK} + G(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{IF} + H(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{EB} \\ + J(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{45} + K(\theta_2, \theta_{23}, \theta_{234}) \cdot M_{45})$$

From matrix equation 2, an “f” matrix function should be obtained in order to use in 4th Order Runge – Kutta Numerical Method Solving.

$$f(\theta_2, \theta_{23}, \theta_{234}, \dot{\theta}_2, \dot{\theta}_{23}, \dot{\theta}_{234}) = [A(\theta_2, \theta_{23}, \theta_{234})]^{-1} \cdot (-B(\theta_2, \theta_{23}, \theta_{234}) \cdot \begin{bmatrix} (\dot{\theta}_2)^2 \\ (\dot{\theta}_{23})^2 \\ (\dot{\theta}_{234})^2 \end{bmatrix} \\ + C(\theta_2, \theta_{23}, \theta_{234}) \cdot g + D(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{PK} \\ + E(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{LK} + F(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{JK} \\ + G(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{IF} + H(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{EB} \\ + J(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{45} + K(\theta_2, \theta_{23}, \theta_{234}) \cdot M_{45}) \quad (C.3)$$

C.2.2. Introduction to 4th Order Runge – Kutta Method

For one-degree-of-freedom-mechanisms, Runge – Kutta Method is applied as following:

$$0 = \Delta(\theta) \cdot \ddot{\theta} + \Gamma(\theta) \cdot (\dot{\theta})^2 - \Psi(\theta) \quad (\text{C.4})$$

$$\dot{\theta} = \frac{d}{dt}\theta \quad \text{so:}$$

$$0 = \Delta(\theta) \cdot \frac{d}{dt}\dot{\theta} + \Gamma(\theta) \cdot (\dot{\theta})^2 - \Psi(\theta)$$

Therefore there are two direct and one indirect first order linear differential equations:

$$\frac{d}{dt}\theta = \dot{\theta}$$

$$\frac{d}{dt}\dot{\theta} = \Delta(\theta)^{-1} \cdot (-\Gamma(\theta) \cdot (\dot{\theta})^2 + \Psi(\theta))$$

$$f(\theta, \dot{\theta}) = \Delta(\theta)^{-1} \cdot (-\Gamma(\theta) \cdot (\dot{\theta})^2 + \Psi(\theta)) \quad (\text{C.5})$$

For each step:

$${}_{i+1}\theta = {}_i\theta + \varphi_1 \cdot h \quad (\text{C.6})$$

$${}_{i+1}\dot{\theta} = {}_i\dot{\theta} + \varphi_2 \cdot h \quad (\text{C.7})$$

where

$$\varphi_1 = (k_{11} + 2 \cdot k_{12} + 2 \cdot k_{13} + k_{14})/6$$

$$\varphi_2 = (k_{21} + 2 \cdot k_{22} + 2 \cdot k_{23} + k_{24})/6$$

h : step size

Where

$$k_{11} = {}_i\dot{\theta}$$

$$k_{21} = f({}_i\theta, {}_i\dot{\theta})$$

$$k_{12} = {}_i\dot{\theta} + 0.5 \cdot k_{21} \cdot h$$

$$k_{22} = f({}_i\theta + 0.5 \cdot k_{11} \cdot h, {}_i\dot{\theta} + 0.5 \cdot k_{21} \cdot h)$$

$$k_{13} = {}_i\dot{\theta} + 0.5 \cdot k_{22} \cdot h$$

$$k_{23} = f({}_i\theta + 0.5 \cdot k_{12} \cdot h, {}_i\dot{\theta} + 0.5 \cdot k_{22} \cdot h)$$

$$k_{14} = {}_i\dot{\theta} + k_{23} \cdot h$$

$$k_{24} = f({}_i\theta + k_{13} \cdot h, {}_i\dot{\theta} + k_{23} \cdot h)$$

C.2.3. Adaptation of Equations of Motion of a Three-Degree-of-Freedom-Excavator to 4th Order Runge – Kutta Method

$$\begin{aligned}
 0 = & A(\theta_2, \theta_{23}, \theta_{234}) \cdot \begin{bmatrix} \ddot{\theta}_2 \\ \ddot{\theta}_{23} \\ \ddot{\theta}_{234} \end{bmatrix} + B(\theta_2, \theta_{23}, \theta_{234}) \cdot \begin{bmatrix} (\dot{\theta}_2)^2 \\ (\dot{\theta}_{23})^2 \\ (\dot{\theta}_{234})^2 \end{bmatrix} \\
 & - C(\theta_2, \theta_{23}, \theta_{234}) \cdot g - D(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{PK} - E(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{LK} \\
 & - F(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{JK} - G(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{IF} - H(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{EB} \\
 & - J(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{45} - K(\theta_2, \theta_{23}, \theta_{234}) \cdot M_{45}
 \end{aligned}$$

$$\begin{bmatrix} \frac{d}{dt} \theta_2 \\ \frac{d}{dt} \theta_{23} \\ \frac{d}{dt} \theta_{234} \end{bmatrix} = \begin{bmatrix} \dot{\theta}_2 \\ \dot{\theta}_{23} \\ \dot{\theta}_{234} \end{bmatrix}$$

$$\begin{aligned}
 \begin{bmatrix} \frac{d}{dt} \dot{\theta}_2 \\ \frac{d}{dt} \dot{\theta}_{23} \\ \frac{d}{dt} \dot{\theta}_{234} \end{bmatrix} &= [A(\theta_2, \theta_{23}, \theta_{234})]^{-1} \cdot (-B(\theta_2, \theta_{23}, \theta_{234}) \cdot \begin{bmatrix} (\dot{\theta}_2)^2 \\ (\dot{\theta}_{23})^2 \\ (\dot{\theta}_{234})^2 \end{bmatrix} \\
 &+ C(\theta_2, \theta_{23}, \theta_{234}) \cdot g + D(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{PK} \\
 &+ E(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{LK} + F(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{JK} \\
 &+ G(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{IF} + H(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{EB} \\
 &+ J(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{45} + K(\theta_2, \theta_{23}, \theta_{234}) \cdot M_{45})
 \end{aligned}$$

$$\begin{aligned}
\begin{bmatrix} f(\theta_2, \dot{\theta}_2) \\ f(\theta_{23}, \dot{\theta}_{23}) \\ f(\theta_{234}, \dot{\theta}_{234}) \end{bmatrix} &= [A(\theta_2, \theta_{23}, \theta_{234})]^{-1} \cdot (-B(\theta_2, \theta_{23}, \theta_{234}) \cdot \begin{bmatrix} (\dot{\theta}_2)^2 \\ (\dot{\theta}_{23})^2 \\ (\dot{\theta}_{234})^2 \end{bmatrix} \\
&+ C(\theta_2, \theta_{23}, \theta_{234}) \cdot g + D(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{PK} \\
&+ E(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{LK} + F(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{JK} \\
&+ G(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{IF} + H(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{EB} \\
&+ J(\theta_2, \theta_{23}, \theta_{234}) \cdot F_{45} + K(\theta_2, \theta_{23}, \theta_{234}) \cdot M_{45})
\end{aligned}$$

For each step:

$$\begin{bmatrix} {}_{i+1}\theta_2 \\ {}_{i+1}\theta_{23} \\ {}_{i+1}\theta_{234} \end{bmatrix} = \begin{bmatrix} {}_i\theta_2 \\ {}_i\theta_{23} \\ {}_i\theta_{234} \end{bmatrix} + \begin{bmatrix} \varphi_{21} \\ \varphi_{31} \\ \varphi_{41} \end{bmatrix} \cdot h \quad (\text{C.8})$$

$$\begin{bmatrix} {}_{i+1}\dot{\theta}_2 \\ {}_{i+1}\dot{\theta}_{23} \\ {}_{i+1}\dot{\theta}_{234} \end{bmatrix} = \begin{bmatrix} {}_i\dot{\theta}_2 \\ {}_i\dot{\theta}_{23} \\ {}_i\dot{\theta}_{234} \end{bmatrix} + \begin{bmatrix} \varphi_{22} \\ \varphi_{32} \\ \varphi_{42} \end{bmatrix} \cdot h \quad (\text{C.9})$$

where

$$\bar{\varphi} = \begin{bmatrix} \varphi_{21} \\ \varphi_{22} \\ \varphi_{31} \\ \varphi_{32} \\ \varphi_{41} \\ \varphi_{42} \end{bmatrix} = \bar{k} \cdot \begin{bmatrix} 1/6 \\ 2/6 \\ 2/6 \\ 1/6 \end{bmatrix}$$

h : step size

where

$$\bar{k} = \begin{bmatrix} k_{211} & k_{212} & k_{213} & k_{214} \\ k_{221} & k_{222} & k_{223} & k_{224} \\ k_{311} & k_{312} & k_{313} & k_{314} \\ k_{321} & k_{322} & k_{323} & k_{324} \\ k_{411} & k_{412} & k_{413} & k_{414} \\ k_{421} & k_{422} & k_{423} & k_{424} \end{bmatrix}$$

where

$$k_{211} = {}_i\dot{\theta}_2$$

$$k_{221} = f({}_i\theta_2, {}_i\dot{\theta}_2)$$

$$k_{212} = {}_i\dot{\theta}_2 + 0.5 \cdot k_{221} \cdot h$$

$$k_{222} = f({}_i\theta_2 + 0.5 \cdot k_{211} \cdot h, {}_i\dot{\theta}_2 + 0.5 \cdot k_{221} \cdot h)$$

$$k_{213} = {}_i\dot{\theta}_2 + 0.5 \cdot k_{222} \cdot h$$

$$k_{223} = f({}_i\theta_2 + 0.5 \cdot k_{212} \cdot h, {}_i\dot{\theta}_2 + 0.5 \cdot k_{222} \cdot h)$$

$$k_{214} = {}_i\dot{\theta}_2 + k_{223} \cdot h$$

$$k_{224} = f({}_i\theta_2 + k_{213} \cdot h, {}_i\dot{\theta}_2 + k_{223} \cdot h)$$

$$k_{311} = {}_i\dot{\theta}_{23}$$

$$k_{321} = f({}_i\theta_{23}, {}_i\dot{\theta}_{23})$$

$$k_{312} = {}_i\dot{\theta}_{23} + 0.5 \cdot k_{321} \cdot h$$

$$k_{322} = f({}_i\theta_{23} + 0.5 \cdot k_{311} \cdot h, {}_i\dot{\theta}_{23} + 0.5 \cdot k_{321} \cdot h)$$

$$k_{313} = {}_i\dot{\theta}_{23} + 0.5 \cdot k_{322} \cdot h$$

$$k_{323} = f({}_i\theta_{23} + 0.5 \cdot k_{312} \cdot h, {}_i\dot{\theta}_{23} + 0.5 \cdot k_{322} \cdot h)$$

$$k_{314} = {}_i\dot{\theta}_{23} + k_{323} \cdot h$$

$$k_{324} = f({}_i\theta_{23} + k_{313} \cdot h, {}_i\dot{\theta}_{23} + k_{323} \cdot h)$$

$$k_{411} = {}_i\dot{\theta}_{234}$$

$$k_{421} = f({}_i\theta_{234}, {}_i\dot{\theta}_{234})$$

$$k_{412} = {}_i\dot{\theta}_{234} + 0.5 \cdot k_{421} \cdot h$$

$$k_{422} = f({}_i\theta_{234} + 0.5 \cdot k_{411} \cdot h, {}_i\dot{\theta}_{234} + 0.5 \cdot k_{421} \cdot h)$$

$$k_{413} = {}_i\dot{\theta}_{234} + 0.5 \cdot k_{422} \cdot h$$

$$k_{423} = f({}_i\theta_{234} + 0.5 \cdot k_{412} \cdot h, {}_i\dot{\theta}_{234} + 0.5 \cdot k_{422} \cdot h)$$

$$k_{414} = {}_i\dot{\theta}_{234} + k_{423} \cdot h$$

$$k_{424} = f({}_i\theta_{234} + k_{413} \cdot h, {}_i\dot{\theta}_{234} + k_{423} \cdot h)$$

$$\begin{bmatrix} {}_{i+1}\theta_2 \\ {}_{i+1}\theta_3 \\ {}_{i+1}\theta_4 \end{bmatrix} = \begin{bmatrix} {}_i\theta_2 \\ {}_i\theta_{23} \\ {}_i\theta_{234} \end{bmatrix} + \begin{bmatrix} \varphi_{21} \\ \varphi_{31} \\ \varphi_{41} \end{bmatrix} \cdot h - \begin{bmatrix} 0 \\ {}_{i+1}\theta_2 \\ {}_{i+1}\theta_{23} \end{bmatrix} \quad (\text{C.10})$$

$$\begin{bmatrix} {}_{i+1}\dot{\theta}_2 \\ {}_{i+1}\dot{\theta}_3 \\ {}_{i+1}\dot{\theta}_4 \end{bmatrix} = \begin{bmatrix} {}_i\dot{\theta}_2 \\ {}_i\dot{\theta}_{23} \\ {}_i\dot{\theta}_{234} \end{bmatrix} + \begin{bmatrix} \varphi_{22} \\ \varphi_{32} \\ \varphi_{42} \end{bmatrix} \cdot h - \begin{bmatrix} 0 \\ {}_{i+1}\dot{\theta}_2 \\ {}_{i+1}\dot{\theta}_{23} \end{bmatrix} \quad (\text{C.11})$$

APPENDIX D

EXCAVATOR DYNAMICS SOFTWARE (EDS)

D.1. Introduction to EDS

EDS is a computer program, which is composed of Microsoft Excel and Visual Basic for Applications. The high availability, cheapness, flexibility and simplicity of Office programs encouraged for writing the code for excavator dynamics on Excel and VBA.

EDS is the whole summary of all chapters mentioned before. The basics of this software are listed below:

- Hydraulic excavator having 3 attachments
- 2 dimensional motions
- Newton – Euler Method for dynamic modeling
- 4th Order Runge – Kutta Method for solving motion equations
- Determination of cylinder and soil reaction forces by real time testing

Before using EDS, user must have some information about attachments; such as, the masses and moments of inertia. User can find masses either by measuring the weights of attachments or by using a 3-D modeling program. Similarly, moments of inertia can be found either by hand calculating or by using a 3-D modeling program. It is proposed that for both determination of masses and moments of inertia, a 3-D modeling program is going to give a good approximate result and it will be too easy for user.

Transferring 2-D view of an excavator to Excel can be done either by hand or by using AutoCAD – Excel Transferring Code.

D.2. General View of EDS

D.2.1. Input Area

$M_{1,2}$	0		
$M_{2,3}$	0		
$M_{3,4}$	0		
θ_2	θ_3	θ_4	
0	0	0	
θd_2	θd_3	θd_4	
θdd_2	θdd_3	θdd_4	
Body	Mass	Inertia	
2 (Boom)			
3 (Arm)			
4 (Bucket)			
ϕ_b	$F_{4,5}$	$M_{4,5}$	
0			
F_{EB}	F_{If}	F_{JK}	
End Time	Step Size	$h - \Delta t$	
		#DIV/0!	
STEP	0		
TIME	0		

Gravity

Free
 Bucket Breakout
 Arm Breakout
 Zelenin Method

Figure D.1 – Input Area of EDS

Blue colored areas are left for user in order to be filled (Figure D.1).

D.2.2. Running Simulation



Figure D.2 – Running Simulation

After entering all necessary values and building the model, simulation can be performed (Figure D.2). User should use “Reset” button firstly. In fact, before every operation “Reset” button should be used.

User can either use “Run” button or scroll bar in order to run the simulation. The difference between “Run” button and scroll bar is that, “Run” button performs simulation operation until the given “End Time” value in steps entered on “Step Size” cell. However, by using scroll bar, user is not limited until “End Time”. Every click on up or down button makes simulation go one step further. With the given step size, simulation can go on continuously. Happenings about physical system can be investigated for every step.

D.2.3. Monitoring Results

D.2.3.1. Simulation Chart

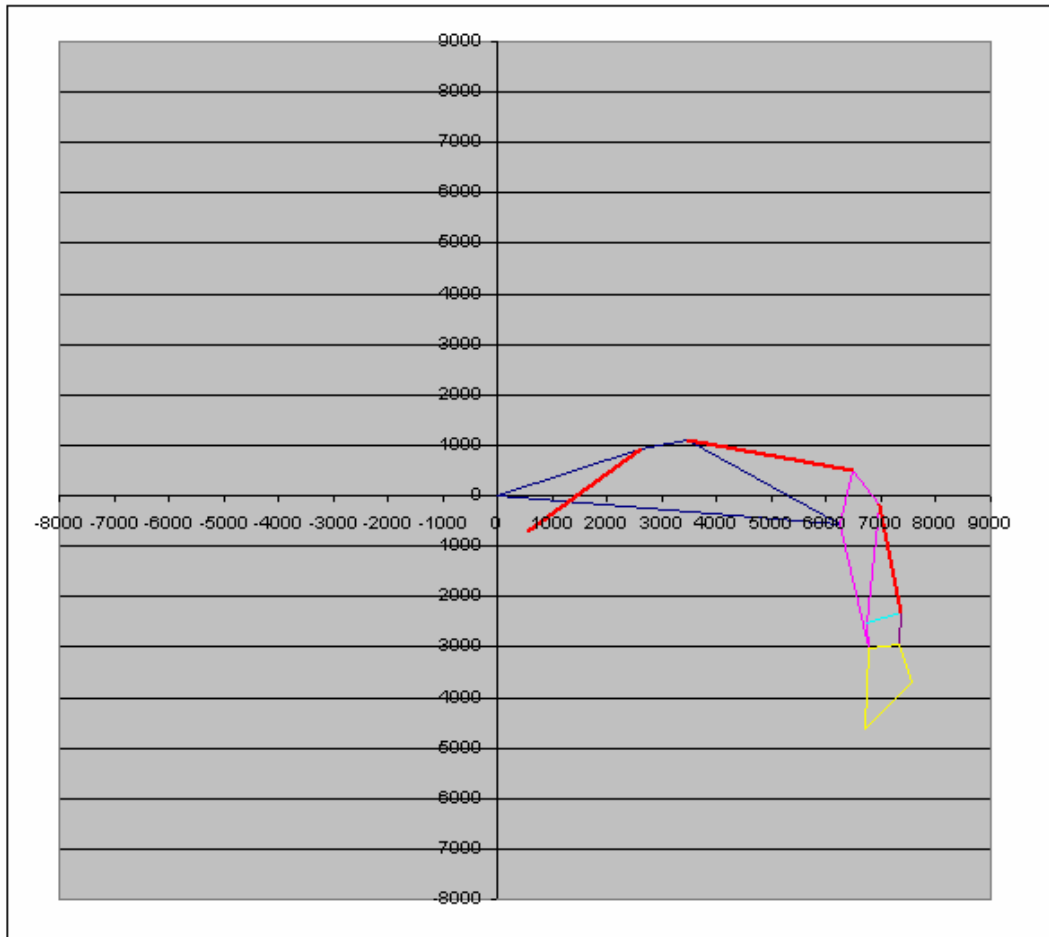


Figure D.3 – Simulation Chart

Chart shown in Figure D.3 is the visual object of EDS. The dark blue lines indicate boom, purple lines indicate arm, yellow lines indicate bucket, brown and light blue lines indicate connection rods and red lines indicate hydraulic cylinders. When simulation process starts, the motion of the system can be seen as near as its real behavior.

D.2.3.2. Outputs

Users can obtain values of major physical phenomena during simulation which can be listed as below:

- All forces created or exerted on every joint
- Positions of links
- Velocities of links
- Accelerations of links

APPENDIX E

LOADCELL TEST

E.1. Method of the Test

In order to verify the measurement results of strain gauges, a loadcell test is done before the digging test. Four strain gauges are attached to the chosen link and connected to ESAM. Arm and bucket is in position shown in Figure E.1

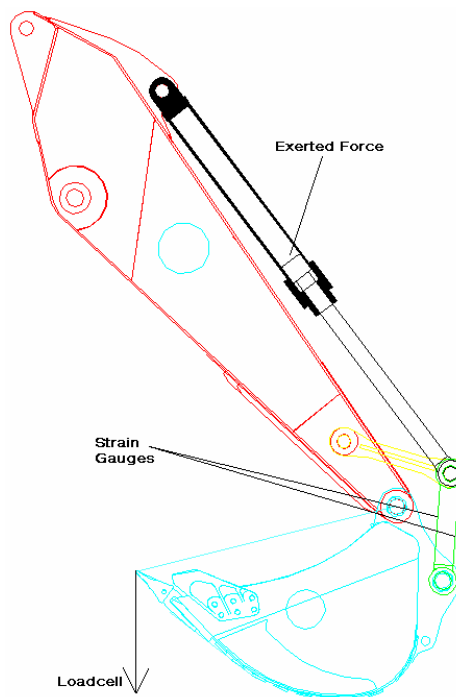


Figure E.1 – Position for Loadcell Test

Force is exerted on bucket cylinder whereas ESAM records gauge and loadcell results. On the other hand, an analytical static force analysis is performed for verifying test results.

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