PRICING DEFAULT AND PREPAYMENT RISKS OF FIXED-RATE MORTGAGES IN TURKEY: AN APPLICATION OF EXPLICIT FINITE DIFFERENCE METHOD

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ABSTRACT

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The mortgage system has been used for many years in many countries of the world. Although the system has undergone many changes over the passing years, the basics remain the same. So, it can be thought that the earlier systems form the basis of today's mortgage system even though it represents some differences in practice among the countries. However, this system is very new for Turkish financial market as compared with developed countries. The aim of this study is estimating the default and prepayment risk of mortgage contract and pricing the contract in emerging markets like Turkey.

In this study, a classical option pricing technique based on Cox, Ingersoll and Ross [8] is used in order to evaluate Turkish fixed-rate mortgages. In this methodology, the spot interest rate and the house price are used as state variables and it is assumed that the termination decision of mortgage is driven by a economic rationale. Under this framework, the model evaluates the embedded options, namely prepayment and default options, and the future payments which corresponds to the mortgage monthly payments. Another aim of this study is the pricing of mortgage insurance policy which has not been used yet in Turkish mortgage market but thought as potential derivative in this market. Therefore, the model used in the study also provides values for mortgage insurance policy.

The partial differential equation which is derived for the mortgage, its components and mortgage insurance policy does not have closed form solutions. To cope with this problem, an explicit finite difference method is used to solve the partial differential equation. Numerical results for the value of mortgage-related assets are determined under different economic scenarios. Results obtained in the basic economic scenario show that Turkish banks apply lower contract rates as compared with the optimal ones. This observation indicates that the primary mortgage market in Turkey is still in its infancy stage. Numerical results also suggest that it is beneficial for the lenders to have mortgage default insurance, especially for the high LTV ratio mortgages.

Keywords: FRMs, Default Option, Prepayment Option, Turkish Mortgage Market, Explicit Finite Difference Method

ÖZ

TÜRKİYE'DE SABİT FAİZLİ MORTGAGE KONTRATLARININ GERİ ÖDEMEME VE ERKEN ÖDEME OPSİYONLARININ FİYATLANDIRILMASI: AÇIK SONLU FARKLAR YÖNTEMİ UYGULAMASI

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Mortgage sistemi dünyanın birçok ülkesinde uzun yıllardan beri uygulanmaktadır. Her ne kadar sistem geçen yıllar içerisinde birçok değişikliğe uğramış olsada, sistemin temelleri aynı kalmıştır. Dolayısıyla, uygulama açısından ülkeler arasında farklılıklar görülsede, geçmişte uygulanan sistemler günümüz mortgage sisteminin temelini oluşturmaktadır. Ancak bu sistemin Türk finans piyasası içerisinde yerini alması diğer gelişmiş ülkelere kıyasla çok yenidir. Bu çalışmanın temel amacı sabit faizli mortgage kredilerinin Türkiye gibi enflasyonist ortamlarda geri ödenmeme ve vadesiden önce ödenme risklerinin hesaplanması ve kredilerin değerlerinin belirlenmesidir.

Bu çalışmada Türkiye'deki sabit faiz oranlı mortgage kontratlarının fiyatlamasında Cox, Ingersoll ve Ross [8]'un çalışmasını baz alan klasik opsiyon fiyatlama tekniği kullanılmaktadır. Model içerisinde faiz oranları ve ev fiyatları temel değişkenler olarak ele alınmış ve modelleme, mortgage kontratının sonlandırma kararının ekonomik gerekçeler nedeniyle alındığı varsayımı altında yapılmıştır. Bu çerçevede oluşturulan model erken ödeme ve geri ödenmeme opsiyonları ile ileri tarihli ödemeleri hesaplamaktadır. Bu çalışmanın bir diğer amacı da

Türkiye mortgage piyasasında henüz yerini almayan ancak potansiyel türev ürünü olarak düşünülen mortgage sigorta kontratlarının fiyatlandırılmasıdır.

Mortgage, mortgage bileşenlerini ve mortgage sigorta kontratlarını hesaplamak amacıyla oluşturulmuş kısmi diferansiyel denklemin kapalı form çözümleri bulunmamaktadır. Bu problemin üstesinden gelebilmek amacıyla, kısmi diferansiyel denklem çözümünde açık sonlu farklar metodu uygulanmaktadır. Mortgage bağımlı varlıklar için nümerik sonuçlar farklı ekonomik senaryolar altında elde edilmiştir. Temel ekonomik senaryo varsayımı altında elde edilen sonuçlar, Türk bankalarının mortgage kredilerinde olması gerekenden daha düşük faiz oranı uyguladığını göstermektedir. Bu gözlem ülkemizdeki birincil mortgage piyasasının henüz başlangıç aşamasında olduğunun da bir göstergesidir. Nümerik sonuçlar aynı zamanda mortgage sigorta poliçesinin özellikle yüksek LTV rasyolarının uygulandığı durumlarda kredi veren tarafa yarar sağladığını da göstermektedir.

Anahtar Kelimeler: Sabit Faiz Oranlı Mortgage Kredileri, Geri Ödenmeme Opsiyonu, Erken Ödeme Opsiyonu, Türkiye Mortgage Piyasası, Açık Sonlu Farklar Yöntemi

To my family who support me all through my life

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INTRODUCTION

1.1 An Overview to the Mortgage Markets

1.1.1 Mortgage Markets in Developed Economies

It is known that the mortgage system has been used for many years in many countries of the world. Although the system has undergone many changes over the passing years, the basics remain the same. So, it can be thought that the earlier systems form the basis of today's mortgage system even though it represents some differences in practice among the countries.

Today, in most developed economies, especially in the US and UK economies, the mortgage market is an important component of the capital markets. In the academic literature, there are many researchers sustaining this implication. The study of Warnock V. and Warnock F. [41] shows that the average mortgage debt-to-GDP ratio for the 2001-2005 period is 67.4 percent for the US economy and 66.6 percent for UK economy.

Jaffee and Renauld [25] state that mortgage development is likely to be a key factor in overall financial market development since an efficient mortgage market acts as a positive externality for other capital markets, creating pressure for higher efficiency in these markets. Actually, this explains why the mortgage market has an important role among the capital markets in developed economies.

In the United States, the development of mortgage market began in early 1900s. Before the Great Depression, residential mortgages in the United States were available only for a short term and featured bullet payments of principal at term. However, during the Great Depression, the value of the properties declined and consequently borrowers defaulted [21]. So, it can be said that lenders had no money to lend and borrowers had no money to pay during the Great Depression. At this point, the federal government had to intervene the housing finance market. For this purpose, the Home Owner's Loan Corporation (HOLC) was established in 1933. Subsequently, the Federal Housing Administration (FHA) and the Federal National Mortgage Association (Fannie Mae) were created in 1936 and 1938 respectively. While the purpose of FHA was to insure the lenders against defaulting borrowers, the mission of Fannie Mae was to provide liquidity and stability to the US housing and mortgage markets. In 1970, the Federal Home Loan Mortgage Corporation (Freddie Mac) was created to provide more funds for the growth mortgage market.

The United Kingdom mortgage market is one of the biggest mortgage markets in Europe and it offers a wide choice of mortgage products to borrowers. According to Douetil [14], between the mid-1970s and 1990, approximately 75 percent of all UK mortgages were on the endowment basis. In the endowment mortgage, borrower pays interest on the loan but the principal is repaid at the end of the mortgage. Moreover, Miles [35] states that at December 2003, the fixed rate mortgages accounted for 25 percent of the total mortgage loan in UK while the variable rate mortgages accounted for 35 percent.

With the growth in mortgage markets in 1980s, the secondary mortgage markets (SMMs) began to play an important role in developed economies. In SMMs, residential mortgages are sold by their originators to the financial investors. Therefore, SMM system provides more liquidity to the mortgage lenders. In the United States, the SMM is dominated by the federal agencies. These are Federal National Mortgage Association (Fannie Mae), Federal Home Loan Mortgage Corporation (Freddie Mac) and Government National Mortgage Association (Ginnie Mae). However, SMM also includes private companies which buy mortgages from the originators.

SMM systems have many different forms changing according to type of instrument and investors. Mortgage securitization is one of the most common form for the SMM. Jaffee and Renaud [24] give the definition of the mortgage securitization as the aggregation of individual mortgages into a security format, thus allowing mortgage assets to be sold more efficiently to capital market investors. By that aggregation, the risk on the individual loan is reduced. Mortgage securitization has a dominant role in the United States mortgage markets. However, this system is not preferred so much in the European markets like in the US markets.

1.1.2 Mortgage Markets in Emerging Economies

In both developed and emerging economies, the mortgage lenders are faced with different kinds of risks. Credit risk, interest rate risk and liquidity risk are the most important ones. However, these risks are more acute in emerging economies as compared with developed economies because of the volatile inflation and some political approaches which control the interest rates. For this reason, the development of mortgage systems in emerging economies is slow and inadequate.

Jaffee and Renauld [25] state that credit risk in other words default risk is usually measured by loan to value ratio (the ratio of the loan amount to the property value) and payment to income ratio (the ratio of annual mortgage payment to the borrower's annual income). Low loan to value ratios decrease the credit risk since the lender can recover the principal by selling the property in the case of default. However, property rights and foreclosure procedures are not well specified in emerging economies. Furthermore, estimation of property value is difficult in an inflationary environment. They also state that in most developed countries, the average income to property value ratio ranges from 1/4 to 1/3 while in most emerging economies this ratio tends to be 1/10 or lower. This indicates that mortgage loans are more risky in emerging economies rather than developed economies.

Interest rate risk is another type of risk which lenders faced with. In developed economies, interest rate risk can be eliminated by using other financial instruments. However, transactions have a high cost. Therefore, the adjustable rate mortgages are alternatively used to eliminate the interest rate risk. In emerging economies, the capital markets are not sufficient to eliminate whole risk and using the adjustable rate mortgages is more risky since the probability of default is higher. Moreover, high and more volatile real interest rates in emerging economies tend to increase the credit risk on adjustable rate mortgages.

The depositors in emerging economies are likely to value of liquidity. If the depositors withdraw their funds at the same time, the banks have to sell their assets at a discount to finance the deposit outflows. Government securities are good assets for this purpose, since they are traded in active and liquid markets. However, mortgages do not have short term maturities and they do not easily traded in SMMs. Furthermore, mortgage lenders incur a liquidity risk because of maturity mismatch between the funding sources and the mortgage loans. As is known, the funding sources generally have short maturities while the mortgage loans have significantly long maturities.

In conclusion, housing is a major purchase requiring long-term financing, and the factors that are associated with well-functioning housing finance systems are those that enable the provision of long-term finance. Across all countries, countries with stronger legal rights for borrowers and lenders, deeper credit information systems, and a more stable macroeconomic environment have deeper housing finance system. These same factors also help explain the variation in housing finance across emerging economies [41].

1.2 Turkish Mortgage Market

In Turkey, the housing credit sector began to develop in late 1990s. Basically, high inflation risk, economic uncertainty and the lack of a well-organized and deep enough mortgage market in Turkey have slowed down the growth of this sector since long term crediting under these conditions are very risky. However,in resent years, this sector has made up of about 10 percent of the country's Gross National Product (GNP), which has grown to 539.9 Billion TRY (US$ 381 Billion) in 2006. One of the reasons for this significant increase is the economic recovery in Turkish economy.

In fact, the inflation rate in Turkey has stabilized within a band of 15-20 within the last few years. The economy has been growing by around 6 per cent a year for the last five years, which is faster than many developed economies and most emerging markets. These recent improvements in Turkish economy enabled the Turkish finance sector to offer long-term funding at relatively cheap prices. According to the Banks Association of Turkey, mortgages represented only 7.64 per cent of the overall loan portfolio in 2003, whereas today mortgages have a share of 29.5 per cent.

The development of the mortgage system required new adjustments in Turkish housing finance system. For this purpose, the new housing finance system was created and carried into effect at 6 March 2007. The most important property of the new system is that it allows to create the secondary mortgage markets. By this way, banks and other financial institutions have a chance to offer low interest rates to their customers for the mortgage loans since funding the mortgage loans will be more cheaper.

Currently, Turkish banks offer a variety of mortgage products including Turkish Lira (TRY)-denominated fixed-rate, adjustable rate, and graduated payment mortgages and US Dollar, and, Euro-denominated mortgages. The most popular mortgage products are fixed rate mortgages (FRMs) with 60 to 120 month contract maturity, and the prevailing mortgage coupon rates range from 1.2 to and 1.53 percent in September, 2007. As the FRMs are popular mortgage products over the past few years, this study concentrates on pricing the fixed-rate mortgages based on structural option pricing models. In order to determine the basic FRM contract to price in our study, we collect information on the FRM contract details of eight deposit banks with the largest mortgage portfolios, namely, Finansbank, Oyak Bank, HSBC Bank, Akbank, Yapı Kredi Bank, Garanti Bank, Vakıf Bank, İş Bank. More specifically, we collect data for the contract maturity, coupon rate, loan-to-value (LTV) ratio, arrangement fee, prepayment penalty and the available insurance policies of these deposit banks.

Table 1.1 illustrates that, with the exception of Finansbank and Yapı Kredi Bank, the maximum Loan-to-Value (LTV) ratio is 75 for the FRMs. Finansbank and Yapı Kredi Bank originate FRMs with a maximum LTV of 95 to 100. The amount of upfront arrangement fee significantly varies among the banks. While Finansbank does not charge any arrangement fee, other banks may charge 1 to 5 of the loan amount as the arrangement and service fee. All the banks except for Yapi Kredi Bank charge a prepayment penalty of 2 of the outstanding loan balance at the time of prepayment. In terms of insurance policies, hazard and earthquake insurance is required by all lenders. This has been a requirement since 1999 and is provided by Turkish Catastrophe Insurance Pool (TCIP). Most of the lenders also require a life insurance policy that would remain in effect over the term of the mortgage. Such a policy would help to cover the full repayment of the loan in the event of borrower's death. Borrowers are required to renew their policy annually (at least during the term of the loan). Mortgage default insurance products are not prevalent in Turkey. The existing sectoral studies suggest that there is no urgent need for mortgage insurance as this will increase the cost of funds for borrowers.

1.3 Literature Review

In the academic literature, there is a vast number of studies using contingent claims approach to explain either prepayment or default or both of the mortgage termination behaviors of the borrowers. Among these studies, two basic approaches, namely reduced-form and structural models, are used for the valuation of mortgage. While the common goal of both modeling strategies is to account for all the embedded options in the mortgage contract they represent different features for the termination decision of the mortgage.

In the structural models, the termination of the embedded options in the mortgage are predictable from the information contained in the mortgage. For instance, in case of prepayment option, the prepayment decision of the borrower is directly related with the market value of the loan since the main aim of the borrower is minimizing the cost of the loan. If the borrower has a chance to refinance his/her loan at a favorable rate, then he/she would prefer to exercise prepayment option. In other words, a prepayment takes a place whenever the market value of the mortgage exceeds the market value of the refinancing loan. In the literature, this type of prepayment decision is called as the optimal prepayment.

The study of Findlay and Capozza [19] is one of the earliest studies which analyze only the prepayment options of the holders. In their study, they use both variable rate mortgage (VRM) and fixed rate mortgage (FRM) to analyze the prepayment behavior. Jackson and Kaserman [23] conclude that the equity position of the mortgage is the primary determinant of the default behavior of the borrower. Quigley and Van Order [37] use contingent claims approaches to model mortgage default and like Jackson and Kaserman, they also conclude that equity will still be a major factor in explaining default although exercise will also depend on the personal characteristics of the borrower. Erol and Patel [18] analyses default risk of wage-indexed payment mortgage (WIPM) in Turkey in comparison with other standard mortgage contracts originated in high inflationary economies.

The study of Cunningham and Capone [10] focuses on the household-specific choice of adjustable rate mortgage (ARM) termination and fixed rate mortgage (FRM) termination during periods of volatile interest rates and volatile house price changes. The results support that current equity net of moving costs dominates in the case of the default decision while mortgage and equity variables dominate in the case of prepayment.

Kau *et al.* [27, 29] emphasize the importance of the joint option which consists of prepayment and default options. In the joint option, there is an interaction between the prepayment and the default options in terms of their terminations. In other words, borrowers have the right to prepay the loan while ruling out the possibility of default and similarly, they also have the right to consider default by eliminating the possibility of prepayment. Therefore, contracts with only one of these options lead the borrower to behave differently from when both are present.

Kau *et al.* [28] evaluate the US adjustable rate mortgages (ARMs) with the embedded default and prepayment options. This study provide a theoretical valuation model for adjustable rate residential mortgages and uses numerical methods in order to price mortgage contract and the jointly exercise of the embedded options. Pereira-Azevedo [2, 3] use a similar framework for the mortgage valuation. However, in these studies, the authors focus on UK fixed rate mortgages (FRMs).

Dunsky and Ho [17] use a multinomial logit model to describe the mortgagors' behavior in dealing with the competing refinancing and default risks, and then utilizes a two factor arbitrage-free interest rate model to value the mortgages. The result identifies the competing effects of default and prepayment on the valuation of a mortgage loan.

Sharp *et al.* [40] develop a mortgage valuation model which includes the potential for early prepayment and for default. In the study, an improved finite-difference procedure is presented, together with a perturbation analysis based on the assumption of numerically small volatility of house price and interest rate, which leads to closed-form solutions and the results show that perturbation theory is a very efficient and effective tool in the solution of a contingent claims mortgage valuation model.

In the reduced-form approach, it is assumed that termination of the mortgage may occur for non-financial (personal) reasons, such as divorce, a new job, or death in the family. In other words, the termination of mortgage in this approach is an unpredictable stopping time. Therefore, the probability of termination should be taken into account in the mortgage valuation at each point time. For this purpose, the proportional hazards model, introduced by Cox [6] is used in modeling termination decision of the mortgage. The proportional hazards model assumes that, at each point in time during the mortgage contract period, the mortgage has a certain probability of termination condition on the survival of the mortgage. Since the termination decision is an unpredictable stopping time perfect hedging is not possible in such approach.

The early study, Schwartz and Torous [39], is an example of the reduced-form literature. In their study, they do not impose an optimal, value-minimizing call condition to price these securities. Rather, they assume that at each point in time there exists a probability of prepaying, this conditional probability depending upon the prevailing state of the economy. By integrating this prepayment function into the valuation framework, they provide a complete model to value mortgage-backed securities.

Deng *et al.* [11] present a unified model of the default and prepayment behavior of homeowners in a proportional hazard framework. The authors model uses the option-based approach to analyze default and prepayment, and considers these two interdependent hazards as competing risks. Deng [12] adopt a proportional hazard framework to analyze these competing and interdependent risks, in a model with time-varying covariates. The study of Deng *et al.* [13] models default and prepayment as dependent competing risks to effectively examine the joint nature of the put and call options.

Ciochetti *et al.* [5] examine the factors driving the borrower's decision to terminate commercial mortgage contracts with the lender through either prepayment or default. Using loan-level data, the authors estimate prepayment and default functions in a proportional hazard framework with competing risks. They conclude that high value of the put/call option is found to significantly reduce the call/put risk since the borrower forfeits both options by exercising one.

Kau *et al.* [30] extend the traditional hazard technique of estimating prepayment and default by allowing their baselines to be stochastic processes, rather than known paths of time, as is typically assumed. By working in the reduced form, this method offers an alternative to the empirical valuation of mortgages more easily implemented than the standard structural form approach of options pricing.

The recent study of Liao *et al.* [32] develop a reduced-form model which is able to value the mortgage without setting boundary conditions and derive a closed-form solution of the mortgage valuation equation under a general reduced-form model that embeds relevant economic variables.

This study uses the traditional option-based pricing model rather than reduced-form approach to price FRM contracts in an emerging economy. The main objective of this study is to price both the default risk and the prepayment risk of the FRMs, from the lenders' perspective, using the well-known option pricing model. This study also aims to price mortgage insurance policy which has not been used yet in Turkish mortgage market but thought as potential derivative in this market.

In the valuation framework, the partial differential equation which is derived for pricing the derivative asset does not have closed form solutions. In such cases, to cope with this problem, the analysts use other methods to approximate the value of the asset. The basic methods mostly used by the analysts are the Monte Carlo method (forward-pricing method) advocated by Boyle [4], finite difference approximation to the differential equation (backward-pricing method) suggested by Schwartz [38] and lattice (or tree) approach suggested by Cox *et al.* [7]. Despite the recent advances in forward pricing methods for pricing American options, backward pricing method is well established, and so has been used more extensively. Although it is computationally more complex, many researchers adopt backward pricing approach as the appropriate procedure to valuing mortgages with embedded default and prepayment options.

Given the specific details of the contract, the values of the financial assets embedded in a mortgage, namely default option, prepayment option and insurance product, are known at the expiry. Using appropriately small time steps, the partial differential equation derived for these assets can be used to work backwards from the final mortgage payment, calculating the asset values sequentially to the previous mortgage payment, then using that new set of terminal conditions to work back to a still earlier payment until eventually the origination of the contract is reached (Azevedo-Pereira *et al.* [2, 3]).

Table 1.1: Contract Details for FRMs Originated by the Largest Deposit Banks in Turkey (September, 2007)

Bank	Contract Maturity	Coupon Rate	*LTV* Ratio		Up-front Arrangement Fee	Prepayment Penalty	Insurance Services			
	(Months)		Min	Max			Hazard -Earthquake Insurance	Life Insurance	Accident Insurance	Mortgage Insurance
Finansbank	60-360	1.24%-1.29%	75%	95%-100%	No Fee	2.0%	Yes	Yes	Yes	No
Oyak Bank	60-144	1.35%	-	75%	Included in coupon rate	2.0%	Yes	Yes	Yes	No
HSBC Bank	60-360	1.30% -1.44%	5,000 TRY	75%	1%-2% of Loan Amount	2.0%	Yes	No	Yes	No
Akbank	60-240	1.21%-1.31%	-	75%	2%-3% of Loan Amount	2.0%	Yes	Yes	Yes	No
Yapı Kredi Bank	60-240	1.16%-1.33%	-	100%	1%-2% of Loan Amount	1.5%	Yes	Yes	Yes	No
Garanti Bank	60-240	1.34%	-	75%	1,000 TRY	2.0%	Yes	No	No	No
Vakıf Bank	1-240	1.30%	-	75%	$2\% of Loan Amount$	2.0%	Yes	Yes	No	No
İş Bank	49-120	1.46%-1.53%	-	-	1%-1.5% of Loan Amount	2.0%	Yes	Yes	Yes	No

CHAPTER 2

MORTGAGE AND MORTGAGE INSURANCE VALUATION

2.1 Contingent Claims Framework

2.1.1 Modeling the Term Structure of Interest Rate and House Prices

In mortgage valuation model, there are two state variables, namely, the spot interest rate, $r(t)$, and the value of underlying house, $H(t)$. According to assumptions in the model, the spot interest rate follows mean reverting square root diffusion process [8] and house price follows log-normal diffusion process [34]. The mean reverting square root process for the interest rate is given by the following equation:

$$dr = \kappa(\theta - r)dt + \sigma_r \sqrt{r} d\omega_r \tag{2.1}$$

where κ is the speed of adjustment in mean reverting process, θ denotes the steady state spot rate (average rate), σ_r represents the standard deviation of the interest rate disturbance and ω_r is the standardized Wiener process for the interest rate. Similarly, Equation 2.2 represents the log-normal diffusion process followed by the house prices,

$$\frac{dH}{H} = (\mu - \delta)dt + \sigma_H d\omega_H \tag{2.2}$$

where μ is the instantaneous average rate of return on house prices, δ denotes the per unit service flow, σ_H is the standard deviation of the house price disturbance, and ω_H represents the standardized Wiener process for house prices. Moreover, it is also assumed that there is a relation between two Wiener processes and this relation is represented as follows:

$$d\omega_r d\omega_H = \rho dt \tag{2.3}$$

where ρ is the instantaneous correlation coefficient between two Wiener processes.

In this study, mortgage prices are assumed to depend on the term structure of interest rates and house prices. Therefore, the mortgages are perceived as derivative assets. However, there are two other important factors which would affect the mortgage prices, namely, market price of risk associated with the spot interest rates and the house prices. However, since the state variable, H, is a traded asset the risk adjustment does not exist [2]. So, the only measurement of the investor's preferences would be the market price of risk associated with the spot interest rates. However, it should be noted that option pricing theory requires a risk-neutral economic environment. In a risk-neutral world, expected return on all securities is the risk free rate of interest, r, by reason of the fact that investors require no risk premium for their investments. In other words, market price of risk is null in risk-neutral environment.

It should be noted that a derivative can be valued in a risk neutral world by discounting expected payoff at the risk free interest rate. When we move from a risk-neutral world to a risk-averse world, both expected growth rate in the underlying variable and the discount rate that must be used for any payoff from the derivative change and these two changes always offset each other exactly [22]. Consequently, the price obtained in risk neutral world is also correct in real world.

Under this framework, it is known from standard arguments in finance that the value of any derivative asset, $F(r, H, t)$, depending only on the mentioned state variables and time must satisfy the following partial differential equation: (PDE) [8, 9, 27, 28, 29],

$$\begin{aligned} &\frac{1}{2}H^2\sigma_H^2\frac{\partial^2 F}{\partial H^2} + \rho H\sqrt{r}\sigma_H\sigma_r\frac{\partial^2 F}{\partial H\partial r} + \frac{1}{2}r\sigma_r^2\frac{\partial^2 F}{\partial r^2} \\ &\quad + \kappa(\theta - r)\frac{\partial F}{\partial r} + (r-\delta)H\frac{\partial F}{dH} + \frac{\partial F}{\partial t} - rF = 0. \end{aligned} \tag{2.4}$$

Path dependency is one of the most common problem in standard valuation procedures, especially ones using backward techniques. If the value of an asset depends on its previous values, then there will a problem in backward valuation procedure since the values are not available when they are required. In these cases, forward valuation procedures are more appropriate to use, since they do not create any path dependency problem. However, these procedures work well when no termination is allowed [26]. Mortgage valuation framework under this study allows two types of endogenous termination prior to maturity, namely, prepayment and default. Therefore, it is not possible to use forward valuation techniques. In this case, it should be used backward valuation procedures in which the path dependency problem are circumvented.

2.1.2 An Overview to the Mortgage Contract and Mortgage Insurance

2.1.2.1 General Notations

In this work, the following notations will be used for the valuation of mortgage contract and mortgage insurance:

n = the life of the mortgage in months,
L = amount of the loan,
c = the fixed coupon rate,
π = early termination penalty,
f = fraction for insurance coverage,
ξ = arrangement fee,
$\eta(i)$ = i^{th} payment date,
$OB(i)$ = outstanding balance after i^{th} payment date,
MP = monthly payment
$TD(t)$ = borrower's total dept at time t.

2.1.2.2 The Components of the Mortgage Value

In the mortgage valuation framework, the future payments that have to be paid by the borrower are not enough alone to determine the mortgage value. In order to determine the mortgage value, the value of options covered by the mortgage contract should also be known. In the next sections of the study, the following notations will be used for the components of the mortgage contract:

$V_B(r, H, t)$ = value of the mortgage to the borrower at time t for given r and H,
$A(r, t)$ = value of the remaining mortgage payments at time t for given r,
$D(r, H, t)$ = value of the default option at time t for given r and H,
$C(r, H, t)$ = value of the prepayment option at time t for given r and H,
$J(r, H, t)$ = value of the joint option at time t for given r and H.

The value of the joint option at any point in time can be expressed as:

$$J(r, H, t) = C(r, H, t) + D(r, H, t) \tag{2.5}$$

So, the value of mortgage to the borrower at time t will be given as follows:

$$V_B(r,H,t) = A(r,H,t) - C(r,H,t) - D(r,H,t) = A(r,H,t) - J(r,H,t). \tag{2.6}$$

The value of the mortgage insurance is depend on the value of mortgage since the default decision is determined by the value of the mortgage. However, it is not a component of the mortgage as a result of the fact that the mortgage insurance provides the benefit just for the lender in the case of default. Therefore, it is a part of the mortgage value to the lender. For this reason, the mortgage value to the lender is different from the mortgage value to the borrower. This relation can be formalized by:

$$V_L(r,H,t) = V_B(r,H,t) + I(r,H,t) \tag{2.7}$$

where $I(r,H,t)$ is the value of the mortgage insurance at time t for given r and H.

At payment dates, there is a distinction between the value of an asset immediately before and immediately after each payment. The following notations will be used to determine this distinction,

$$F^-(r,H,t) = \text{value of the asset F immediately before the payment,}$$
$$F^+(r,H,t) = \text{value of the asset F immediately after the payment.}$$

2.2 Pricing the Fixed-Rate Mortgage and Mortgage Insurance

2.2.1 Valuation of Monthly Payments and Outstanding Balance[1]

The mortgage contract used in this work is a fixed rate mortgage. Therefore, the monthly payments remain the same during the life of the mortgage and each payment, MP, is given by the following equation:

$$MP = \frac{\left(\frac{c}{12}\right)\left(1+\frac{c}{12}\right)^n}{\left(1+\frac{c}{12}\right)^n - 1} OB(0) \tag{2.8}$$

where $OB(0)$ is the amount of debt at the origination of the loan.

The outstanding balance after each payment date, $\eta(i)$, is expressed as follows:

$$OB(i) = \frac{\left(\left(1+\frac{c}{12}\right)^n - \left(1+\frac{c}{12}\right)^i\right)}{\left(1+\frac{c}{12}\right)^n - 1} OB(0). \tag{2.9}$$

[1] See Appendix A for the details.

2.2.2 Valuation of Future Payments

The valuation of the promised payments is easier as compared with the other components of the mortgage. This valuation can be considered as a bond valuation. In a bond valuation process, the only element that affects its value is the term structure of interest rates for given cash flows. Additionally, using the fixed payments also makes this valuation easy since there is no need to adjust the payments at each payment date.

The valuation process differs between the maturity of the loan and other payment dates. At termination, the value of the promised payments is equivalent to MP, unpaid balance at that moment. So, the terminal condition for the loan is

$$A^{-}(r,t) = MP \qquad \text{for } t = \eta(n) \tag{2.10}$$

In each payment date, the value of the debt is decreased by the amount paid, MP. For this reason, for all other payment dates the terminal condition is given by the following equation:

$$A^{-}(r,t) = A_B^{+}(r,t) + MP \qquad \text{for } t = \eta(1), ..., \eta(n-1). \tag{2.11}$$

2.2.3 Mortgage Value

In the previous section, the mortgage value is represented as a function of the amortizing payments and the value of the joint option (see Equation 2.6). As is known, the value of the joint option is determined by r and H directly since these state variables have an important role in the evaluation of the options embedded in the joint option.

If the house price is less than the value of the remaining payments, a rational borrower will prefer to default since the amount obtained by selling the house will not cover all remaining payments. It can be said that, house prices have a direct impact on the default option value. However, in the prepayment option, the main factor that affects the value of option is the term structure of the interest rates. If the interest rates decrease, the borrower has a chance to pay the remaining payments by getting into debt at a lower interest rate. So, house price is not considered directly in the valuation of prepayment option. However, in the case of default, the prepayment option will be valueless automatically since the loan is terminated by the default option. This means that the prepayment option is affected by the house price indirectly. The same arguments are also hold for the default option. Namely, the exercise of the prepayment option makes the default option valueless. As a result of this relation between these two options, they can not be considered separately.

At termination, the borrower has two alternatives, to pay or to default. So, the value of mortgage at termination is given by:

$$V_B^-(r, H, t) = \max\{MP, H\} \qquad \text{for } t = \eta(n) \tag{2.12}$$

as the same value for the other payment dates is formalized as follows:

$$V_B^-(r, H, t) = \max\{(V_B^+(r, H, t) + MP), H\} \qquad \text{for } t = \eta(1), ..., \eta(n-1). \tag{2.13}$$

2.2.4 The Value of the Borrower's Total Debt in Case of Early Termination

At any point in time, the borrower's total debt equals the sum of the unpaid principal and the accrued interest on this amount. However, in the case of prepayment, the borrower should pay an additional amount required by the lender. Actually, this amount is a kind of penalty since the mortgage is terminated by the borrower before the maturity. The early termination penalty can be determined in different ways. For example, it can be required as a fixed amount or a percentage of the unpaid principle. However, in this study, the latter is used to calculate the borrower's total debt in case of early termination. So, the amount of the total debt will be given by:

$$TD(t) = (1 + \pi)\{1 + c(t - \eta(i))\}OB(i) \qquad \text{for } \eta(i) \le t \le \eta(i+1). \tag{2.14}$$

where π is the early termination penalty required by the lender.

2.2.5 Valuation of the Default Option

The default decision is assumed to occur when the value of the mortgage to the borrower, not the value of the remaining payments, exceeds the house price. In the case of default, the borrower not only loses the house but also loses the options that gives a change to terminate the mortgage before maturity. Therefore, in the default, both options will be valueless since the loan is terminated.

On payment dates, the value of the default option is reduced as a result of the reduction in promised payments. Moreover, if the house price exceeds the monthly payment at the maturity, the value of the default option will be equal to zero. So, the default value at the maturity of the loan and on other payments dates can be calculated by:

$$D^{-}(r, H, t) = \max\{0, (MP - H)\} \qquad \text{for } t = \eta(n) \tag{2.15}$$

and

$$D^{-}(r, H, t) = \begin{cases} D^{+}(r, H, t), & \\ \qquad\qquad \text{if } V^{-}(r, H, t) = V^{+}(r, H, t) + MP \quad \text{(no default)} & \\ A^{-}(r, H) - H, & \\ \qquad\qquad \text{if } V^{-}(r, H, t) = H \quad \text{(default)} & \end{cases} \tag{2.16}$$
$$\text{for } t = \eta(1), ..., \eta(n-1)$$

respectively.

2.2.6 Valuation of the Prepayment Option

At the maturity of the loan, the prepayment option is valueless since it can not provide any profit to the borrower. So, the terminal condition at maturity is given by:

$$C^{-}(r, H, t) = 0 \qquad \text{for } t = \eta(n). \tag{2.17}$$

The value of the prepayment option at any payment date, except the maturity, will be equal to its future value if defaulting does not occur at that time. Therefore, the value of the prepayment on other payment dates will be as follows:

$$C^{-}(r, H, t) = \begin{cases} C^{+}(r, H, t), & \\ \qquad\qquad \text{if } V^{-}(r, H, t) = V^{+}(r, H, t) + MP \quad \text{(no default)} & \\ 0, & \\ \qquad\qquad \text{if } V^{-}(r, H, t) = H \quad \text{(default)} & \end{cases} \tag{2.18}$$
$$\text{for } t = \eta(1), ..., \eta(n-1)$$

It should be noted that the value of the prepayment option is the only unknown in Equation 2.4. So, its value can be represented as a function of the mortgage value and the other components of the mortgage value. In other words, the value of the prepayment option can be calculated by $C = A - V_B - D$.

2.3 Pricing the Mortgage Insurance

As mentioned in Section 2.1.2.2, the mortgage insurance is not a part of the mortgage although its value is directly affected by the mortgage value. By the mortgage insurance, the mortgage lender is protected against the default risk since the insurer guarantees to pay a certain amount for the loss. The high level of this amount is determined by a fraction on which the mortgage lender and the insurer agreed. In the case of default, the mortgage lender will hold the house. Therefore, the total loss suffered by the lender will be equal to the difference between the borrower's total debt and the value of the house. Under this framework, the value of the mortgage insurance at the maturity is given by:

$$I^{-}(r, H, t) = \max\{0, \min\{MP - H, \phi MP\}\} \qquad \text{for } t = \eta(n) \tag{2.19}$$

where ϕ is the fraction determined by the insurance contract.

On other payment dates, the same argument holds. So, the terminal condition for the value of the mortgage insurance at any other payment date can be expressed by:

$$I^{-}(r, H, t) = \begin{cases} I^{+}(r, H, t), & \\ \qquad \text{if } V^{-}(r, H, t) = V^{+}(r, H, t) + MP \quad \text{(no default)} & \\ \max\{0, \min\{TD_t^{-} - H, \phi TD_t^{-}\}\}, & \\ \qquad \text{if } V^{-}(r, H, t) = H \quad \text{(default)} & \end{cases} \tag{2.20}$$
$$\text{for } t = \eta(1), ..., \eta(n-1)$$

where TD is the total debt of the borrower.

2.4 The Equilibrium Condition

The mortgage valuation under this study offers no-arbitrage opportunity. For this reason, the equilibrium condition, in other words, no-arbitrage condition, should be satisfied in mortgage valuation. To satisfy the equilibrium condition, the value of the mortgage to the lender must be equal to the amount lent. However, in some cases, the borrower pays an additional amount called arrangement fee at the origination of the mortgage. Therefore, this amount should also be taken into account in the equilibrium condition. The no-arbitrage condition for the mortgage is given by:

$$V_L(r(0), H(0), 0) = V_B(r(0), H(0), 0) + I(r(0), H(0), 0) = (1 - \xi)L \tag{2.21}$$

where ξ is the arrangement fee expressed as a percentage of the amount lent.

The main aim of this section is to find a contract rate which satisfy Equation 2.21. For this purpose, the secant iteration technique is used in the calculation of the equilibrium contract rates.

CHAPTER 3

NUMERICAL SOLUTION OF A TWO-STATE VARIABLE CONTINGENT CLAIMS MORTGAGE VALUATION MODEL USING THE EXPLICIT FINITE DIFFERENCE METHOD

3.1 Finite Difference Methodology

Finite difference methods approximate the solutions to differential equations by replacing the each partial derivative with a difference quotient. The difference quotients used in the method are derived from the Taylor's series expansions. The basic idea behind the finite difference methodology is dividing the domain of the problem into a grid and using the nodes obtained by this way to approximate the derivative terms.

3.1.1 Notations

Assume that $u(x)$ is a continuous function of a single variable x. Then, the function at any of the grid points is given by:

$$u(x_i) \equiv u(ih) \equiv u_i \qquad for \quad i = 0, 1, 2, \ldots$$

where h is the constant grid spacing in x dimension. In the equation, the integer i denotes the position of the mesh point along the x coordinate. Therefore, any point can be represented by the product of the integer i and the grid spacing h. When the time dimension is taken into consideration, the difference and differential equations at the point $x = ih$ and $t = nk$ will be denoted by U_i^n and u_i^n respectively.

In two-dimensional case, the functions for the grid points can be expressed as:

$$u(y_j, x_i) \equiv u(jl, ih) \equiv u_{j,i} \qquad for \quad j = 0, 1, 2, \ldots \quad \text{and} \quad i = 0, 1, 2, \ldots$$

where l and h are the grid spacings in y and x dimensions respectively. In this case, the difference and differential equations at the point $y = jl$, $x = ih$ and $t = nk$ will be represented by $U^n_{j,i}$ and $u^n_{j,i}$ respectively.

3.1.2 Finite Difference Approximations

Taylor series expansions have an important role in the finite difference methods since they are used in order to approximate the derivative expressions in differential equations. According to Taylor's theorem, a function $f(x)$ which is infinitely differentiable can be expressed as:

$$f(x+h) = f(x) + hf^{'}(x) + \frac{1}{2!}h^2 f^{''}(x) + \frac{1}{3!}h^3 f^{'''}(x) \cdots . \tag{3.1}$$

If the terms of order h^2 and higher are neglected, then the following equation is obtained:

$$f^{'}(x) = \frac{f(x+h) - f(x)}{h} + O(h) \tag{3.2}$$

where $O(h)$ is the order of truncation error determined by the size of largest term of the truncated series. So, the relation

$$f^{'}(x) \quad \approx \quad \frac{f(x+h) - f(x)}{h} \tag{3.3}$$

can be used to approximate first-order derivative of f at point x. This is the forward difference approximation. There are some other ways to approximate partial derivatives, namely, backward difference and central difference methods. The derivation of finite difference formulas for these models are similar to the derivation of forward difference formula. In the backward difference method, Taylor's theorem is applied to the function $f(x - h)$ with $h > 0$. Taylor's series expansion for $f(x - h)$ is

$$f(x-h) = f(x) - hf^{'}(x) + \frac{1}{2}h^2 f^{''}(x) - \frac{1}{3!}h^3 f^{'''}(x) + \cdots . \tag{3.4}$$

So, the backward difference formula will be

$$f^{'}(x) \quad \approx \quad \frac{f(x) - f(x-h)}{h}. \tag{3.5}$$

The central difference method is a technique in which the functions $f(x + h)$ and $f(x - h)$ are used together. Therefore, it can be considered as a combination of forward and backward difference methods. In the derivation of the central difference formula, Equations 3.1 and 3.4 are added and the obtained equation is solved for $f^{'}(x)$. Consequently, the difference formula for this method will be given by:

$$f^{'}(x) \approx \frac{f(x+h) - f(x-h)}{2h}. \tag{3.6}$$

The central difference formula has truncation errors of order $O(h^2)$. It should be noted that the accuracy of the approximations is proportional to the order of the error. Therefore, whenever possible, it is common to use central differences in finite difference algorithms [1].

The approximations to the higher-order derivatives are obtained in the same way. However, in this case, the order of truncation error will be higher.

Under this framework, the forward difference approximations to first-order partial derivatives of the function $u(x, y)$ are given by

$$u_{x|j,i} \approx \frac{U_{j,i+1} - U_{j,i}}{h} \tag{3.7}$$

$$u_{y|j,i} \approx \frac{U_{j+1,i} - U_{j,i}}{l}. \tag{3.8}$$

Equations 3.7 and 3.8 have truncation errors of $O(h)$ and $O(l)$ respectively.

Similarly, the backward approximations to the same derivatives will be represented by:

$$u_{x|j,i} \approx \frac{U_{j,i} - U_{j,i-1}}{h} \tag{3.9}$$

$$u_{y|j,i} \approx \frac{U_{j,i} - U_{j-1,i}}{l}. \tag{3.10}$$

where the orders of truncation errors are $O(h)$ and $O(l)$ respectively. Moreover, the following equations represent the central difference approximations to second-order partial derivatives:

$$u_{xx|j,i} \approx \frac{U_{j,i+1} - 2U_{j,i} + U_{j,i-1}}{h^2} \tag{3.11}$$

$$u_{yy|j,i} \approx \frac{U_{j+1,i} - 2U_{j,i} + U_{j-1,i}}{l^2}. \tag{3.12}$$

In this case, the truncation errors have orders $O(h^2)$ and $O(l^2)$ respectively.

Finally, the last extension required in Equation 2.4 is the mixed derivative terms. By using the Taylor's theorem, the following result is obtained:

$$u_{yx|j,i} \approx \frac{U_{j+1,i+1} - U_{j-1,i+1} - U_{j+1,i-1} + U_{j-1,i-1}}{4hl}. \tag{3.13}$$

The truncation error is of dimension $O(h^2) + O(l^2)$.

3.1.3 Numerical Solution

The original PDE expressed in Equation 2.4 has an infinite domain. However, working with infinite domains in numerical methods causes some problems in terms of infinite boundary conditions. Therefore, the state variables of interest rates and house prices should be transformed to cope with the problems caused by infinite boundary conditions. For this purpose, the infinite area $(0, \infty) \times (0, \infty)$ is mapped onto the unit square $(0, 1) \times (0, 1)$. By following the study of Azevedo-Pereira [1], the subsequent transformations are chosen for the state variables:

$$y = \frac{1}{1 + \psi r} \qquad \psi > 0 \tag{3.14}$$

$$x = \frac{1}{1 + \omega H} \qquad \omega > 0 \tag{3.15}$$

As can be seen from Equation 3.14, the variable y will have a value of 0 for $r = \infty$ and a value of 1 for $r = 0$. The same argument also holds for the house price transformation. This means that the transformed variable x will have a value of 0 for $H = \infty$ and a value of 1 for $H = 0$. Consequently, the following equations represent the inverse transformations of r and H dimensions respectively:

$$r = \frac{1 - y}{\psi y} \qquad \psi > 0 \tag{3.16}$$

$$H = \frac{1 - x}{\omega x} \qquad \omega > 0 \tag{3.17}$$

The values used for the scale factors ψ and ω affect the density of the points in the grid. As the values of ψ and ω increase, the number of points that correspond to small r and H values will increase. On the other hand, the smaller the values of ψ and ω, the less points that correspond to small r and H values will be in the solution grid. Therefore, the values for these scale factors should be chosen according to the possible values of the state variables in the market. In this study, the values of r ranging from 2%-3% and 20%-25% are required. For this reason, the value $\psi = 10$ was chosen in the interest rate transformation. Consequently, the middle point of the y grid corresponds to $r = 10\%$. Moreover, $\omega = 1$ was used in the house price transformation. In this case, the middle point in the x grid corresponds to the initial value of $H = 1$.

Wilmott *et al.* [42] state that parabolic PDEs should be solved in the forward dimension. For this purpose, the time variable was also transformed. The transformation for the time variable is given by:

$$\tau = T - t \tag{3.18}$$

where T is the maturity of the loan. So, the inverse transformation is

$$t = T - \tau. \tag{3.19}$$

The original function of the asset value can be expressed in terms of the new variables as follows:

$$W(x, y, \tau) = F(r(y), H(x), t(\tau)). \tag{3.20}$$

The derivatives of the original function should also be expressed in terms of the new variables. Equations 3.21 and 3.22 give the new forms of the derivatives in r dimension:

$$\frac{\partial F}{\partial r} = \frac{\partial W}{\partial y}\frac{\partial y}{\partial r} \tag{3.21}$$

$$\frac{\partial^2 F}{\partial r^2} = \frac{\partial^2 W}{\partial y^2}\left(\frac{\partial y}{\partial r}\right)^2 + \frac{\partial^2 y}{\partial r^2}\left(\frac{\partial W}{\partial y}\right) \tag{3.22}$$

The first and second derivatives with respect to H can be expressed in a similar way:

$$\frac{\partial F}{\partial H} = \frac{\partial W}{\partial x}\frac{\partial x}{\partial H} \tag{3.23}$$

$$\frac{\partial^2 F}{\partial H^2} = \frac{\partial^2 W}{\partial x^2}\left(\frac{\partial x}{\partial H}\right)^2 + \frac{\partial^2 x}{\partial H^2}\left(\frac{\partial W}{\partial x}\right). \tag{3.24}$$

The transformed version of the mixed derivative can be represented as follows:

$$\frac{\partial^2 F}{\partial H \partial r} = \frac{\partial^2 W}{\partial x \partial y}\left(\frac{\partial y}{\partial r}\right)\left(\frac{\partial x}{\partial H}\right). \tag{3.25}$$

Consequently, it is necessary to determine the derivatives of new state variables with respect to the original ones. The derivatives in r dimension can be written as:

$$\frac{\partial y}{\partial r} = -\psi y^2 \tag{3.26}$$

$$\frac{\partial^2 y}{\partial r^2} = 2\psi^2 y^3. \tag{3.27}$$

Similarly, the first and second derivatives in the H dimension can be expressed as:

$$\frac{\partial x}{\partial H} = -\omega x^2 \tag{3.28}$$

$$\frac{\partial^2 x}{\partial H^2} = 2\omega^2 x^3 \tag{3.29}$$

respectively.

After all these transformations and substitutions, the fundamental PDE (2.4) can be written in the following form:

$$
\begin{aligned}
&\frac{1}{2}H(x)^2\sigma_H^2\omega^2x^4\frac{\partial^2 W}{\partial x^2} + \rho H(x)\sqrt{r(y)}\sigma_H\sigma_r\psi\omega x^2y^2\frac{\partial^2 W}{\partial x\partial y}\\
&+\frac{1}{2}r(y)\sigma_r^2\psi^2y^4\frac{\partial^2 W}{\partial y^2} + \left\{H(x)^2\sigma_H^2\omega^2x^3 - \left[(r(y)-\delta)H(x)\omega x^2\right]\right\}\frac{\partial W}{\partial x}\\
&+\left\{r(y)\sigma_r^2\psi^2y^3 - \left[\kappa(\theta - r(y)) - \sigma_r\sqrt{r(y)}\lambda r\right]\psi y^2\right\}\frac{\partial W}{\partial y}\\
&-\frac{\partial W}{\partial \tau} - r(y)W = 0. \qquad (3.30)
\end{aligned}
$$

3.1.4 Finite Difference Representation of Fundamental Partial Differential Equation

In this section, the finite difference representation of the transformed PDE will be derived at the interior points and the boundary points. For this purpose, this section is divided into four parts, namely, the interior points, the lower boundaries, the upper boundaries and the corners of the grid.

3.1.4.1 Interior Points

As mentioned in Section 3.1.3, the domain of the PDE has been mapped onto the unit square. So, the new state space is uniformly represented as follows. For the transformed house price, unit space $[0, 1]$ is subdivided into I intervals such that $Ih = x_I = 1$ and $ih = x_i$. Similarly, unit space for the transformed interest rate variable is subdivided into J intervals such that $Jl = y_J = 1$ and $jl = y_j$. Moreover, for the time to maturity, the interval $[0, T]$ is subdivided into N intervals such that $Ns = \tau_N = 1$ and $ns = \tau_n$. So, the asset value, $F(r, h, t)$ will be approximated by $U^n_{j,i}$. In this study, 50 steps are used to for subdivision of the unit space. In other words, the numerical solution is obtained from the 50×50 grid where $h = l = 0.02$. In addition to this, 66 time steps are used to satisfy the numerical stability[1].

Using the notation, $U^n_{j,i}$, and difference approximations defined in Section 3.1.2, the trans-

[1] This is inline with the study of Kau *et al.* [28].

formed PDE (3.30) is approximated by the following difference equation:

$$
\begin{aligned}
&\frac{1}{2}H(x)^2\sigma_H^2\omega^2x^4\frac{U^n_{j,i+1}-2U^n_{j,i}+U^n_{j,i-1}}{h^2}\\
&+\rho H(x)\sqrt{r(y)}\sigma_H\sigma_r\psi\omega x^2y^2\frac{U^n_{j+1,i+1}-U^n_{j-1,i+1}-U^n_{j+1,i-1}+U^n_{j-1,i-1}}{4lh}\\
&+\frac{1}{2}r(y)\sigma_r^2\psi^2y^4\frac{U^n_{j+1,i}-2U^n_{j,i}+U_{j-1,i}}{l^2}\\
&+\left\{H(x)^2\sigma_H^2\omega^2x^3-\left[(r(y)-\delta)H(x)\omega x^2\right]\right\}\frac{U^n_{j,i+1}-U^n_{j,i-1}}{2h}\\
&+\left\{r(y)\sigma_r^2\psi^2y^3-\left[\kappa(\theta-r(y))-\sigma_r\sqrt{r(y)}\lambda_r\right]\psi y^2\right\}\frac{U^n_{j+1,i}-U^n_{j-1,i}}{2l}\\
&-\frac{U^{n+1}_{j,i}+U^n_{j,i}}{s}-r(y)U^n_{j,i}=0.
\end{aligned}
\tag{3.31}
$$

In this approximation, the central difference method is used for the space derivatives while the forward difference approximation is used for the time derivative. This difference between the time derivative and space derivatives is engendered by the impossibility of using central method in the time derivative approximation [42]. If Equation 3.31 is rearranged, it is possible to see the value of the asset at a certain time-step as a function of its own value at the previous time steps. The following equation gives this relation:

$$
\begin{aligned}
U^{n+1}_{j,i} &= \left\{1-H(x)^2\sigma_H^2\omega^2x^4\left(\frac{s}{h^2}\right)-r(y)\sigma_r^2\psi^2y^4\left(\frac{s}{l^2}\right)-r(y)s\right\}U^n_{j,i}\\
&+\frac{1}{2}H(x)^2\sigma_H^2\omega^2x^4\left(\frac{s}{h^2}\right)\left(U^n_{j,i+1}+U^n_{j,i-1}\right)\\
&+\frac{1}{2}r(y)\sigma_r^2\psi^2y^4\left(\frac{s}{l^2}\right)\left(U^n_{j+1,i}+U^n_{j-1,i}\right)\\
&+\left\{H(x)^2\sigma_H^2\omega^2x^3-\left[(r(y)-\delta)H(x)\omega x^2\right]\right\}\left(\frac{s}{2h}\right)\left(U^n_{j,i+1}-U^n_{j,i-1}\right)\\
&+\left\{r(y)\sigma_r^2\psi^2y^3-\left[\kappa(\theta-r(y))-\sigma_r\sqrt{r(y)}\lambda_r\right]\psi y^2\right\}\left(\frac{s}{2l}\right)\left(U^n_{j+1,i}-U^n_{j-1,i}\right)\\
&+\rho H(x)\sqrt{r(y)}\sigma_H\sigma_r\psi\omega x^2y^2\left(\frac{s}{4hl}\right)\left(U^n_{j+1,i+1}-U^n_{j-1,i+1}-U^n_{j+1,i-1}+U^n_{j-1,i-1}\right).
\end{aligned}
\tag{3.32}
$$

Moreover, there is an alternative representation of Equation 3.32 in which the coefficients of

$U^n_{j,i}$s are perfectly isolated. That is

$$
\begin{aligned}
U^{n+1}_{j,i} &= \left\{1 - H(x)^2\sigma_H^2\omega^2x^4\left(\frac{s}{h^2}\right) - r(y)\sigma_r^2\psi^2y^4\left(\frac{s}{l^2}\right) - r(y)s\right\}U^n_{j,i} \\
&+ \left\{\frac{1}{2}H(x)^2\sigma_H^2\omega^2x^4\left(\frac{s}{h^2}\right) + \left\{H(x)^2\sigma_H^2\omega^2x^3 - (r(y)-\delta)H(x)\omega x^2\right\}\left(\frac{s}{2h}\right)\right\}U^n_{j,i+1} \\
&+ \left\{\frac{1}{2}H(x)^2\sigma_H^2\omega^2x^4\left(\frac{s}{h^2}\right) - \left\{H(x)^2\sigma_H^2\omega^2x^3 - (r(y)-\delta)H(x)\omega x^2\right\}\left(\frac{s}{2h}\right)\right\}U^n_{j,i-1} \\
&+ \left\{\frac{1}{2}r(y)\sigma_r^2\psi^2y^4\left(\frac{s}{l^2}\right) + \left\{r(y)\sigma_r^2\psi^2y^3 - \left[\kappa(\theta - r(y)) - \sigma_r\sqrt{r(y)}\lambda_r\right]\psi y^2\right\}\left(\frac{s}{2l}\right)\right\}U^n_{j+1,i} \\
&+ \left\{\frac{1}{2}r(y)\sigma_r^2\psi^2y^4\left(\frac{s}{l^2}\right) - \left\{r(y)\sigma_r^2\psi^2y^3 - \left[\kappa(\theta - r(y)) - \sigma_r\sqrt{r(y)}\lambda_r\right]\psi y^2\right\}\left(\frac{s}{2l}\right)\right\}U^n_{j-1,i} \\
&+ \rho H(x)\sqrt{r(y)}\sigma_H\sigma_r\psi\omega x^2y^2\left(\frac{s}{4hl}\right)\left(U^n_{j+1,i+1} - U^n_{j-1,i+1} - U^n_{j+1,i-1} + U^n_{j-1,i-1}\right).
\end{aligned}
\tag{3.33}
$$

In the main PDE 3.30, the coefficients of the second derivative terms are always positive. However, the same argument is not true for the first derivative terms. In order to keep the errors associated with finite difference representation inside acceptable bounds, it is necessary to guarantee that all the U^n coefficients are positive [36]. So, the finite difference scheme in Equation 3.33 has some stability problems because of that reason. According to Morton and Mayers [36], there is an alternative way to avoid this problem. The general idea behind the method is using the forward or backward differences for the first derivative terms instead of using cental differences. In other words, a forward difference approximation is used when the coefficient of the first derivative term is positive and a backward difference approximation is used when the coefficient of the first derivative term is negative. The use of one-sided differences depending on the sign of the first derivative term is called 'upwind differencing' method.

The sign of the coefficient of the first derivative terms changes across the grid and the PDE used in this study is two dimensional PDE. For this reason, there are four alternatives for the combination of the first derivative signs. Actually, under normal conditions, two of them will tend to occur. However, Matlab program code (see Appendix B) used to solve the problem was written by considering all alternatives.

It is possible to see an example of the application of this type of procedure in Equation 3.34. In this example, backward approach was used to approximate the first derivative term in house price dimension and the forward approach was used to approximate the first derivative term in interest rate dimension:

$$
\begin{aligned}
U_{j,i}^{n+1} &= \left\{1 - H(x)^2\sigma_H^2\omega^2x^4\left(\frac{s}{h^2}\right) - r(y)\sigma_r^2\psi^2y^4\left(\frac{s}{l^2}\right) - r(y)s\right\}U_{j,i}^n \\
&+\frac{1}{2}H(x)^2\sigma_H^2\omega^2x^4\left(\frac{s}{h^2}\right)U_{j,i+1}^n \\
&+\left\{H(x)^2\sigma_H^2\omega^2x^3 - (r(y)-\delta)H(x)\omega x^2\right\}\left(\frac{s}{2h}\right)U_{j,i}^n \\
&+\left\{\frac{1}{2}H(x)^2\sigma_H^2\omega^2x^4\left(\frac{s}{h^2}\right) - \left\{H(x)^2\sigma_H^2\omega^2x^3 - (r(y)-\delta)H(x)\omega x^2\right\}\left(\frac{s}{2h}\right)\right\}U_{j,i-1}^n \\
&+\left\{\frac{1}{2}r(y)\sigma_r^2\psi^2y^4\left(\frac{s}{l^2}\right) + \left\{r(y)\sigma_r^2\psi^2y^3 - \left[\kappa(\theta - r(y)) - \sigma_r\sqrt{r(y)}\lambda_r\right]\psi y^2\right\}\left(\frac{s}{2l}\right)\right\}U_{j+1,i}^n \\
&+\frac{1}{2}r(y)\sigma_r^2\psi^2y^4\left(\frac{s}{l^2}\right)U_{j-1,i}^n \\
&-\left\{r(y)\sigma_r^2\psi^2y^3 - \left[\kappa(\theta - r(y)) - \sigma_r\sqrt{r(y)}\lambda_r\right]\psi y^2\right\}\left(\frac{s}{2l}\right)U_{j,i}^n \\
&+\rho H(x)\sqrt{r(y)}\sigma_H\sigma_r\psi\omega x^2y^2\left(\frac{s}{4hl}\right)\left(U_{j+1,i+1}^n - U_{j-1,i+1}^n - U_{j+1,i-1}^n + U_{j-1,i-1}^n\right).
\end{aligned}
\tag{3.34}
$$

3.1.4.2 Upper Boundary Conditions

In the transformed version of the main PDE, the upper boundary conditions of the transformed variables correspond with the lower boundary conditions of the original state variables. Therefore, this section actually gives the details of lower boundary conditions in the original state variables.

House Price Dimension When $H = 0$, the value of the mortgage will be obviously greater than the value of the house. In this case, a national borrower will lead to default. Consequently, prepayment option will have no value and so,

$$C(r, 0) = 0. \tag{3.35}$$

Moreover, in case of default, the value of the mortgage will be equal to the value of the house and consequently, the value of the default will be equal to the value of the promised payments because of the relation represented in the Equation (2.6). So,

$$
\begin{aligned}
V_B(r, 0) &= H = 0 \\
D(r, 0) &= A(r, t).
\end{aligned}
\tag{3.36}
$$

Under these circumstances, the degenerate version of the main PDE (2.4) for the value of insurance is

$$\frac{1}{2}r\sigma_r^2\frac{\partial I}{\partial r^2} + \kappa(\theta - r)\frac{\partial I}{\partial r} + \frac{\partial I}{\partial t} - rI = 0. \tag{3.37}$$

Interest Rate Dimension When $r = 0$, there will be no discounting. Therefore, at any point in time, the value of interest rate in the next moment will be equal to $\kappa\theta s$ where s is the dimension of the time step. Consequently, the value of the promised payments on boundaries will be given by:

$$A(0, t) = A(\kappa\theta s, t + s). \tag{3.38}$$

In the valuation of the other assets, the degenerate form of the main PDE (2.4) will be used. The following equation represents this degenerate form:

$$\frac{1}{2}H^2\sigma_H^2\frac{\partial F}{\partial H^2} - \delta H\frac{\partial F}{\partial H} + \kappa\theta\frac{\partial F}{\partial r} + \frac{\partial F}{\partial t} = 0. \tag{3.39}$$

3.1.4.3 Lower Boundary Conditions

Like in the previous Section 3.1.4.2, the lower boundary conditions in the transformed variables correspond to the upper boundary conditions of the original state variables.

House Price Dimension When $H \to \infty$, the value of the default option will be equal to zero. However, the value of the prepayment option will take a value which is different than zero as a result of the relation represented in Equation 2.6. So, the degenerate form for the value of the prepayment option is

$$\frac{1}{2}r\sigma_r^2\frac{\partial C}{\partial r^2} + \kappa(\theta - r)\frac{\partial C}{\partial r} + \frac{\partial C}{\partial t} - rC = 0. \tag{3.40}$$

As mentioned before, the value of default option is zero under the condition that house price goes to infinity:

$$\lim_{H\to\infty} D(r, H) = 0. \tag{3.41}$$

So, by the Equation (2.6), the value of the mortgage contract will be given by:

$$\lim_{H\to\infty} V_B(r, H) = A(r) - \lim_{H\to\infty} C(r, H). \tag{3.42}$$

In addition, the value of the insurance policy will be valueless since there is no default. That is:

$$\lim_{H\to\infty} I(r, H) = 0. \tag{3.43}$$

Interest Rate Dimension As r goes to infinity, the value of any future payment will be equal to zero. Consequently, the values of all other assets will be valueless since all of them contain the term of future payments:

$$\lim_{r\to\infty} A(r) = 0 \tag{3.44}$$

$$\lim_{r\to\infty} V_B(r, H) = 0 \tag{3.45}$$

$$\lim_{r\to\infty} D(r, H) = 0 \tag{3.46}$$

$$\lim_{r\to\infty} C(r, H) = 0 \tag{3.47}$$

$$\lim_{r\to\infty} I(r, H) = 0. \tag{3.48}$$

3.1.4.4 Corners of the Grid

The corners of the grid are the points on which both state variables reach extreme values. In other words, they are the points corresponding to the cases in which $H = 0$ or $H = \infty$ and $r = 0$ or $r = \infty$.

Corners in the Upper Boundary of the Interest Rate Dimension The corners of the upper boundary of the interest rate dimension are points at which $r = 0$ and $H = 0$ or $H = \infty$. So, the value of any asset at point $r = 0$ and $H = 0$ is given by the following equation:

$$F(0, 0, t) = F(\kappa\theta s, 0, t + s). \tag{3.49}$$

Similarly, when $H \to \infty$, the value of the assets will be given by:

$$\lim_{H\to\infty} F(0, H, t) = F(\kappa\theta s, H, t + s). \tag{3.50}$$

Corners in the Lower Boundary of the Interest Rate Dimension In this case, the points corresponding to the case in which $r \to \infty$ will be considered. The argument is the same with the one mentioned in Section 3.1.4.3. This means that as r goes to infinity, the value of any future payment and the value of any asset which involve future payment will be equal to zero. Consequently, the values of the assets will be given by:

$$\lim_{r\to\infty} A(r) = 0 \tag{3.51}$$

$$\lim_{r\to\infty} V_B(r, H) = 0 \tag{3.52}$$

$$\lim_{r\to\infty} D(r, H) = 0 \tag{3.53}$$

$$\lim_{r\to\infty} C(r, H) = 0 \tag{3.54}$$

$$\lim_{r\to\infty} I(r, H) = 0. \tag{3.55}$$

3.2 Free Boundary

3.2.1 Prepayment Region

Prepayment can take place at any moment in time. For this reason, it is necessary to determine the prepayment region. A rational borrower will be averse the prepayment while the value of the total debt is greater than the mortgage value. In other words, there is no need to prepay if

$$V_B \leq TD. \tag{3.56}$$

So, the boundary of the prepayment region will be given by the following equality:

$$V_B = TD. \tag{3.57}$$

According to Merton [34], it is also required that the slopes of the functions V_B and TD should equal to each other at the boundary. The derivatives of TD with respect to the space variables are equal to zero since it is independent from the state variables. Therefore,

$$\frac{\partial TD}{\partial H} = 0 \tag{3.58}$$

$$\frac{\partial TD}{\partial r} = 0. \tag{3.59}$$

So, when $V_B = TD$, the following conditions will be hold:

$$\frac{\partial V_B}{\partial H} = \frac{\partial TD}{\partial H} = 0 \tag{3.60}$$

$$\frac{\partial V_B}{\partial r} = \frac{\partial TD}{\partial r} = 0 \tag{3.61}$$

Under this framework, the valuation equation will be expressed by:

$$\frac{\partial V_B}{\partial t} + \mathcal{L}V_B = 0 \qquad \text{if} \quad V_B < TD \tag{3.62}$$

$$V_B = TD \qquad \text{otherwise} \tag{3.63}$$

where $\mathcal{L}$ is the second-order linear operator in Equation 2.4. In case of $V_B = TD$,

$$\frac{\partial V_B}{\partial t} + \mathcal{L}V_B = \frac{\partial TD}{\partial t} + \mathcal{L}TD = (1+\pi)cO - rTD. \tag{3.64}$$

Therefore, the valuation equation can be written as:

$$\frac{\partial V_B}{\partial t} + \mathcal{L}V_B = \begin{cases} 0, & \text{if } V_B < TD \\ (1+\pi)cO - rTD, & \text{if } V_B = TD. \end{cases} \tag{3.65}$$

3.2.2 Default Region

It should be noted that default makes sense only on payment dates and it will also be taken into consideration outside the prepayment region since it is impossible to default the prepaid loan. Therefore, the terminal conditions given by Equations 2.16 and 2.17 are enough to determine the boundary conditions for the default region.

CHAPTER 4

ANALYSIS OF THE NUMERICAL SOLUTIONS

4.1 Basic Mortgage Contract

Table 4.1 represents the values of the parameters used in the construction of base case economic environment and the basic properties of the mortgage contract. As can be seen from the table, two different types of fixed rate mortgage contract have been used in this study. In the first type, the lender requires only the arrangement fee from the borrowers. However, in the second type, the lender also requires the early termination penalty in case of prepayment in addition to the arrangement fee. Both contract have the same maturity, 10 years, and the par value of the house has been determined as 100 000 TRY. It should be noted that these parameters have determined according to the data set obtained from different banks of Turkey.

In this section, the numerical solutions will be analyzed by using 3-D figures. The aim of this kind of procedure is to investigate the smoothness of the solutions and their consistency in economic sense. For an accurate analysis, we have focused on the center of (51×51) dimensional grid since working with whole grid may cause some misconceptions. Therefore, the figures have been constructed by a set of results corresponding to (41×41) nodes situated in the center of the grid.

Figures 4.1 and 4.2 represent the values of the promised payments for the mortgage contract without and with early termination penalty, respectively. In the model, the value of the future payments, $A(r,t)$, is expressed as a function of interest rates only. So, it is expected that changes in house prices do not have an effect on its value. This argument is also supported by figures. As can be seen from the figures, the value of the future payments is constant along the H dimension. Another observable feature in figures is that there is an inverse relationship with the interest rates since higher interest rates cause lower present values.

The value of the default option for both type of mortgage contract is represented in Figures 4.5 and 4.6. As it is seen in both figures, when the house prices decrease under certain level the probability of default increases and so this causes positive default values. For high house prices, the value of the default option is equal to zero since the default risk decreases as the house prices increase. According to figures, the highest level of the default value corresponds to the environment in which both state variables have small values.

The mortgage crisis in the US has began to seem in an economic environment in which low house prices and high interest rates exist. Therefore, the claim mentioned above can be seen to contradict with the main reason of the sub-prime mortgage crisis in the US. However, there is a distinction between two event. In the US, the most important reason for the crisis is that higher interest rates cause low house prices. So, the borrowers had some difficulties to refinance their credit. Higher interest rates especially affect adjustable rate mortgages rather than fixed rate mortgages. In this work, fixed rate mortgages have been studied. Therefore, using fixed rate mortgages rather than adjustable rate mortgages can be thought as a explanation of this contradiction.

Figures 4.9 and 4.10 represent the value of the mortgage insurance. The value of the mortgage insurance takes the value zero for high house prices since the probability of defaulting also decreases in this area. Moreover, the value of the default option increases as the value of the house price decreases. Consequently, the value of the mortgage insurance also decreases since its value is directly related with the case of defaulting. Another important property in figures is that interest rates do not have a crucial effect on the value of the mortgage insurance for low house prices since under these circumstances the default decision will be more superior as compared with the prepayment decision.

The value of the prepayment option is represented in Figures 4.7 and 4.8. As mentioned above, default decision terminating the prepayment option is more preferable decision in case of low house prices. Therefore, in figures, the value of the call option is equal to zero and it is not affected by the changes in interest rate dimension for low house prices. However, the value of the prepayment option begins to be affected by the interest rates when the house prices reach a certain value since there is no need to default when the house price is higher enough. At low level of interest rates, rational borrowers prefer to refinance their loan. For this reason, the value of the call option will be increase as the interest rates decrease.

Finally, Figures 4.3 and 4.4 represent the value of mortgage contract without and with early termination penalty, respectively. As mentioned in previous sections, the value of the mortgage contract depends on $A(r,t)$, $D(r,H,t)$ and $C(r,H,t)$. So, analyzing the solutions for the mortgage value is difficult because of its complex structure. At low levels of house price, increasing default value causes a decrease in the mortgage value since there is an inverse relationship between the value of the default option and the value of the mortgage contract. Moreover, the interest rate effect does not exist in this situation as a result of high default risk. Consequently, the value of the mortgage contract will increase as the house price increases. After a certain level of house prices, the effect of interest rate can be followed easily. The increase in the level of interest rates leads to decrease in the value of mortgage contract. Actually, higher interest rates decrease the value of future payments and the value of the prepayment option together. However, the decrease in the promised payments is superior. For this reason, the mortgage value calculated as ($V_B = A - D - C$) is decreasing at high levels of interest rate.

4.2 Equilibrium Contract Rates

In the equilibrium framework, the crucial point is that a mortgage contract can only be acceptable if there is no-arbitrage opportunity for both agents. In other words, the lender and the borrower is not able to make an immediate profit at the origination of the loan. The general equilibrium condition was given in Section 2.4. However, this condition does not hold for all type of mortgage contracts since some of them do not require any arrangement fee or mortgage insurance. For this reason, the general equilibrium condition has different forms adjusted to each type of mortgage contract. Adjusted equilibrium conditions are given below:

1) Mortgage Contract Without Arrangement Fee and Mortgage Insurance

$$V_B(r(0), H(0), 0) = L \tag{4.1}$$

In this type of mortgage contract, the mortgage value to the borrower should be equal to the amount lent at time zero. Therefore, the lender's position will be equal to $V_B(r(0), H(0), 0) - L$ at $t = 0$.

2) Mortgage Contract With Arrangement Fee and Without Mortgage Insurance

$$V_B(r(0), H(0), 0) = (1 - \xi)L \tag{4.2}$$

According to above equation, the value of the mortgage to the borrower should be equal to the amount obtained by subtracting arrangement fee. So, the lender's position will be $V_B(r(0), H(0), 0) = (1 - \xi)L$ at $t = 0$.

3) Mortgage Contract Without Arrangement Fee and With Mortgage Insurance

$$V_B(r(0), H(0), 0) + I(r(0), H(0), 0) = L \tag{4.3}$$

As can be seen from above equation, the value of the mortgage to the lender, $VB + I$, should be equal to amount lent. This indicates that the lender's position at $t = 0$ will be equal to $V_B(r(0), H(0), 0) + I(r(0), H(0), 0) = L$.

4) Mortgage Contract With Arrangement Fee and Mortgage Insurance

$$V_B(r(0), H(0), 0) + I(r(0), H(0), 0) = (1 - \xi)L \tag{4.4}$$

Actually, this type of mortgage contract can be called as 'full mortgage' and it can be seen that its equilibrium condition is exactly the same with the condition given by Equation 2.21. So, the lender's position at $t = 0$ will be equal to $V_B(r(0), H(0), 0) + I(r(0), H(0), 0) = (1 - \xi)L$.

Now, it is possible to calculate equilibrium contract rates for each type of mortgage contract described above.

Tables 4.2, 4.4 and 4.6 give the lender's position for each type of mortgage contract with different LTV (Loan to Value) ratios. However, these tables have been obtained by using mortgage contracts not requiring early termination penalty. To see the effect of early termination penalty on the equilibrium rates, the same processes have been also applied to the mortgages which require early termination penalty (Tables 4.3, 4.5 and 4.7). In all tables, it is also possible to see intervals in which equilibrium contract rates lie.

Tables 4.2 and 4.3 have been prepared under the assumption that LTV ratio is equal to 75% [1]. As can be seen from tables, the default value lies between 0 and 3 which indicates that mortgage contract with 75% LTV ratio has low default risk. Therefore, the mortgage insurance has also low values.

[1] According to data set obtained from some biggest banks of Turkey, most of the Turkish banks use 75% maximum loan-to-value ratio.

If we compare the lender's position in the second type of mortgage (mortgage contract with arrangement fee and mortgage insurance) with the lender's position in the fourth type of mortgage (mortgage contract with arrangement fee and without mortgage insurance), we will see that the equilibrium contract rates for both type of mortgages lie between 19.52% and 20%. In fact, this is the result of low insurance values. In other words, requiring mortgage insurance does not provide any benefit to lender since the probability of default is low for 75% LTV ratio.

According to above results, monthly optimal contract rate for 10 year mortgage loan with 75% LTV ratio should be lie between 1.63% and 1.67%. However, Turkish banks which give 10 year mortgage loan with 75% LTV ratio require contract rates lying between 1.26% and 1.53% in monthly basis. As considering the economic environment in the period 2002-2007, it can be said that Turkish banks applied lower contract rates as compared with the optimal ones. This indicates that in Turkey, conditions are not sufficient to construct an efficient primary mortgage market.

Tables 4.4 and 4.5 represent the results for 95% LTV ratio. Although most of the Turkish banks have determined their maximum LTV ratio as 75%, there are some banks which use 95% and 100% as a maximum level for LTV ratio. In both tables, the value of the default option changes between the values 0 and 1432. This result indicates that 95% LTV Ratio is considerably risky. Therefore, the mortgage insurance for 95% LTV ratio takes higher values.

If the lender's position in the second type of mortgage (mortgage contract with arrangement fee and mortgage insurance) is compared with the lender's position in the fourth type of mortgage (mortgage contract with arrangement fee and without mortgage insurance), it will be seen that the optimal contract rate for the mortgage contract requiring insurance lies between 19.00% and 19.44%. However, it takes a value between 19.44% and 20.00% for the contract not requiring any mortgage insurance. According to these results, lenders using 95% LTV ratio in their mortgage contract and having mortgage insurance can reach the same contract value by using lower contract rates. Furthermore, the value of the lender's position obtained by using 19.44% contract rate turns from negative to positive when the lender have a mortgage insurance. Consequently, having a mortgage insurance increase the value of the lender's position especially for higher LTV ratios.

The last study under basic economic scenario is referring the relationship between the components of mortgage contract. The main components which determine the structure of the contract are early termination penalty, arrangement fee and optimal contract rate. So, the banks need to determine these components accurately. Even though the trade-off calculations require some complex methods, the basic idea behind these methods is the optimization process. Table 4.8 represents the optimal contract rates for different early termination penalty and arrangement fee combinations. As can be seen from the table, banks can offer a lower contract rate if they demand higher arrangement fee or early termination penalty.

Turkish banks giving mortgage loan apply 2% early termination penalty and arrangement fees ranging from 1% to 3% of the loan. Finansbank is a good example to analyze the trade-off results in Turkish mortgage market since it offers a mortgage contract which is very similar to contract used in the construction of the trade-off table. Finansbank apply 1.27% contract rate in monthly basis for their mortgage contract which have 10 year maturity, 95% LTV ratio, 2% early termination penalty and no arrangement fee. However, according to Table 4.8, the optimal contract rate for the same mortgage contract is 20.09% in annual basis which is approximately equal to 1.67% in monthly basis. As mentioned before, the contract rates applied by Turkish banks are lower than the equilibrium contract rates.

4.3 Effects of Changes in Economic Environment

As noted in section 4.1, the economic environment used in mortgage valuation is described by a series of parameters. In this section, our aim is to get different economic environments by changing the values of these parameters and analyze the numerical results of mortgage valuation model obtained under new economic environments.

4.3.1 Volatility of State Variables

4.3.1.1 Interest Rate Volatility

Interest rate volatility affects all components of the mortgage contract since all of them are defined as a function of interest rate (see, Chapter 2). Table 4.9 and 4.10 represent the relationship between the interest rate volatility and mortgage components. As can be seen from the tables and figures, the value of the future payments increases with increases in interest rate volatility. Actually, this is an unexpected result. However, Pereira [1] explains this result by the fact that the gain obtained by a fall in the discount rate will exceeds the loss generated by an increase of the same magnitude in the discount rate and so an increase in the interest rate volatility increases the expected value of future payments due to the Jensen's inequality.

The options, C and D, are affected by the future payments (see, Chapter 2). So, it would be expected that their values tend to increase as the interest rate volatility increases. However, it should be also noted that the changes in interest rate volatility affects the sizes of both prepayment and default regions. In addition to this, the option values, C and D, are also related to each other. Under this framework, the value of the call option has a direct relationship with the changes in interest rate volatility. However, the same argument does not hold for the default option since the increase in the option value, D, does not continue for high levels of interest rate volatility. This can be explained by the fact that the changes in the value of the prepayment option has an effect on the default value more than the future payments value.

Increases in interest rate volatility induce higher levels of prepayment and consequently a reduction in the probability of defaulting. Moreover, the value of mortgage insurance is not affected by the value of future payments since its value is entirely determined by the unpaid principle. Due to these reasons, an increase in interest rate volatility, σ_r, will tend to increase the value of mortgage insurance.

4.3.1.2 House Price Volatility

Changes in house price volatility have significantly different effects on the components of the mortgage contract as compared with those induced by interest rate variation. Such a difference is mainly related with the valuation of future payments, A. As mentioned before, r is the only state variable affecting A and so the value of the future payments is entirely independent of house price price, H, and consequently its volatility, σ_H.

In that case, changes in the joint option, $C + D$, due to changes in house price volatility is the only determinant for the value of mortgage contract, V_B. As can be seen from Tables 4.11 and 4.12, the value of the default option, D, tend to increase as house price volatility increases. This can be explained by the fact that contract will reach the default region rather than prepayment or continuation regions in case of higher house price volatilities. Additionally, this fact also explains the decline in the value of the prepayment option, C. So, it can be obviously said that changes in house price volatility induce opposite effects in the components of the joint option. However, according to tables, the decreases in the prepayment option value will surpass the increases in the default option value. Consequently, there will be a decline in the value of the joint option as house price volatility increases. As mentioned in the previous chapters, the relation between the mortgage contract value and the joint option value is expressed by equation $V_B = A - (D + C)$. Under this condition, a decline in the joint option will produce an increase in the value of the contract and consequently there will be an inverse relationship between the mortgage contract value and house price volatility.

According to tables, the mortgage insurance has a direct relationship with the house price volatility. This is explained by the fact that the house price volatility impacts default much more directly than prepayment and mortgage insurance value is directly related with the probability of default.

4.3.1.3 Combined Effects Induced by the Volatilities of State Variables

Tables 4.13 and 4.14 give the numerical results for different combinations of LTV ratio, interest rate volatility and house price volatility.

As mentioned in Section 4.3.1.1, an increase in the interest rate volatility produce a direct effect in the value of future payments, A, and consequently default option, D, and prepayment option C. Additionally, the value of the mortgage to the borrower, V_B, and the insurance value is decreasing for higher interest rate volatilities except for the LTV ratio 100%. As a result, there will be a decline in the value of $V_B + I$. As is known, the value of the mortgage to the lender depends on the value of mortgage to the borrower and the mortgage insurance and in order to satisfy equilibrium condition, the contract rate should be increased to compensate the declines in V_B and I. An increase in the house price volatility induces an increase in the default value, D, and a decrease in the prepayment value, C. So, the total effect of the joint option on V_B is a reduction since the future payments does not have any effect on V_B. Moreover, mortgage insurance value, I, tends to increase for higher house price volatilities. However, the magnitude of increase in I is slightly dominating the magnitude of decline in V_B. This means that there will be a small increase in the value of the lender's position. Consequently, in order to compensate this small increase in the value of mortgage to lender, it is necessary to decrease the contract rate.

4.3.2 Different Levels of Loan-to-Value (LTV) Ratio

The analysis of this section will be given according Tables 4.13 and 4.14.

For a given house price, the amount of the loan will be increase as the LTV ratio increases. Therefore, under normal circumstances, the value of future payments, A, tends to increase since the amount of the loan increases.

In case of higher LTV ratios, it is expected that the probability of default increases since it is more possible to reach a situation in which the outstanding dept surpasses the value of the house. However, on the other hand, an increase in the LTV ratio will correspond to an increase in the total dept and consequently an extension in the prepayment region. It is obvious that an extension in the prepayment region makes the default area smaller. According to Tables 4.13 and 4.14, the default option value, D, has a direct relationship with LTV ratio and this results indicates that the former effect dominates the latter.

The relationship between the value of the prepayment option, C, and LTV ratio is not obvious like in the default option. The prepayment option value represents a direct relationship with the future payments value for low LTV ratios, however, for higher levels of LTV, its value decreases as a result of the great extension in the default region.

It is known that the mortgage insurance is directly related with the default option. Therefore, its direct relation with the LTV ratio is not a surprising result.

The changes in LTV ratio produce an increase in both V_B and I, and consequently require lower equilibrium contract rates. However, as the LTV ratio rises, there will be a significant growth in both V_B and I, and this causes a sharp decline in the equilibrium contract rate.

4.3.3 Effects of Spot Rate

Tables 4.15, 4.16, 4.17, 4.18, 4.19 and 4.20 represent the numerical results for different levels of spot rate. It should be noted that for 75% LTV ratio, $V_B - (1 - \xi)L$ is used to find the equilibrium contract rates since the US mortgage insurance system require no insurance for

LTV ratios which are less than 80%. According to numerical results, V_B and I tend to decrease for higher levels of spot rate, $r(0)$. Obviously, this effect produces a decline in both $V_B - (1 - \xi)L + I$ and $V_B - (1 - \xi)L$ and consequently in order to satisfy equilibrium condition, it is necessary to increase the coupon rate. In other words, there is a direct relationship between the level of the initial spot rate and the contract rate under the assumption that long term average of the interest rate, θ is constant.

The prepayment option value, C, increases as the spot rate increases. This can be explained by the fact that higher levels of spot rate make the prepayment region larger. Under normal conditions, there will be a reduction in the default option value because of the extension in the prepayment region. However, higher coupon rates tend to increase the value of the future payments, A. As is known, the value of the default option has a direct relation with the value of A and so default option value tend to increase for higher levels of initial spot rate since the effect of A dominates the effects of prepayment region.

As mentioned in Section 4.3.1.1, an increase in the interest rate volatility produces a decline in V_B and I, and consequently $V_B - (1-\xi)L + I$ and $V_B - (1-\xi)L$ for all types of yield curves. As a result, in order to reach equilibrium condition, the coupon rate should be increased. This means that the coupon rate tends to increase and to reach the levels of θ for higher levels of interest rate volatility.

Higher levels of house price volatility have opposite effects in the evolution of D and C, and this produces an increase in V_B since the overall effect in the joint option tends to be a reduction. Additionally, the mortgage insurance value, I represents a direct relationship with higher levels of house price volatility. Therefore, these two effect produce a reduction in coupon rate as a result of the increase in $V_B - (1 - \xi)L + I$ and $V_B - (1 - \xi)L$.

4.3.4 Effects of Correlation Coefficient Between Two State Variables

Tables 4.21 and 4.22 represent the effects induced by changes in the correlation coefficient between two state variables, ρ. It is normally expected that there is a direct relationship between the state variables r and H. This means that in case of an increase in r, the probability of default tends to decrease due to the increase in H. However, a decline in r with low house prices will increase the probability of default. As a consequence, this makes the prepayment region smaller. Therefore, an increase in ρ tends to increase the default option value while decreasing the prepayment option value.

As is known, under normal conditions, the mortgage insurance, I, move in a direct relationship with D. Therefore, it will also have a direct relation with changes in ρ.

Changes in D, C and I well compensate each other. Therefore, the value of $V_B - (I - \xi)L + I$ remains almost the same and this induces slight movements, mostly no movements, in coupon rates to satisfy the equilibrium.

Table 4.1: Base Values

PARAMETERS	CONTRACT	
	Repayment Mortgage With Arrangement Fee and Without Early Termination Penalty	**Repayment Mortgage With Arrangement Fee and Early Termination Penalty**
1. Economic Environment		
* Spot interest rate, $r(0)$	10%	10%
* Long term average of interest rate, θ	24%	24%
* Speed of reversion, κ	56%	56%
* House service flow, δ	4%	4%
* Correlation coefficient, ρ	0	0
2. Contract		
* Maturity, η	120 months	120 months
* Value of house at origination, H	100 000 TRY	100 000 TRY
* Arrangement fee, ξ	2%	2%
*Early termination penalty, π	-	2%

Table 4.2: Changes in the Contract Rate ($LTV = 75\%$)

(Mortgage Without Early Termination Penalty)

Contract Rate	Future Payments	Mortgage	Default	Prepayment	Insurance	Value of the Mortgage to the Lender			
(c)	(A)	(V)	(D)	(C)	(I)	$VB - L$	$VB - (1-\xi)L$	$VB - L + I$	$VB - (1-\xi)L + I$
16.00%	66464	64805	0	1659	0	−10195	−8695	−10195	−8695
17.00%	68958	67234	0	1724	1	−7766	−6266	−7766	−6266
18.00%	71492	69698	1	1794	1	−5302	−3802	−5301	−3801
19.00%	74066	72165	1	1900	2	−2835	−1335	−2833	−1333
19.57%	75557	73498	2	2057	2	−1502	−2	−1500	0
20.00%	76678	74338	2	2338	2	−662	838	−660	840
21.00%	79327	74995	0	4332	0	−5	1495	−5	1495
22.00%	82011	75000	0	7012	0	−0	1500	−0	1500
23.00%	84730	74994	0	9736	0	−6	1494	−6	1494

The following parameters were used in the constraction of this table: the arrangement fee (ξ) is 2%; the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the interest rate volatility (σ_r) is 12%; the house price volatility (σ_H) is 9%; the house service flow (δ) is 4%; LTV ratio is 75% and the correlation coefficient (ρ) is 0.

Table 4.3: Changes in the Contract Rate (LTV = 75%)

(Mortgage With Early Termination Penalty)

Contract Rate	Future Payments	Mortgage	Default	Prepayment	Insurance	Value of the Mortgage to the Lender			
(c)	(A)	(V)	(D)	(C)	(I)	$VB-L$	$VB-(1-\xi)L$	$VB-L+I$	$VB-(1-\xi)L+I$
16.00%	66464	64806	0	1658	0	−10194	−8694	−10194	−8694
17.00%	68958	67237	0	1721	1	−7763	−6263	−7762	−6262
18.00%	71492	69707	1	1785	1	−5293	−3793	−5292	−3792
19.00%	74066	72208	1	1857	2	−2792	−1292	−2790	−1290
19.52%	75412	73497	2	1913	3	−1503	−3	−1501	−1
20.00%	76678	74656	3	2019	3	−344	1156	−341	1159
21.00%	79327	76417	3	2907	2	1417	2917	1419	2919
22.00%	82011	76499	0	5512	0	1499	2999	1499	2999
23.00%	84730	76489	0	8241	0	1489	2989	1489	2989

The following parameters were used in the constraction of this table: the arrangement fee (ξ) is 2%; the early termination penalty (π) is 2%; the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (θ) is 56%; the interest rate volatility (σ_r) is 12%; the house price volatility (σ_H) is 9%; the house service flow (δ) is 4%; LTV ratio is 75% and the correlation coefficient (ρ) is 0.

Table 4.4: Changes in the Contract Rate ($LTV = 95\%$)

(Mortgage Without Early Termination Penalty)

Contract Rate	Future Payments	Mortgage	Default	Prepayment	Insurance	Value of the Mortgage to the Lender			
(c)	(A)	(V)	(D)	(C)	(I)	$VB-L$	$VB-(1-\xi)L$	$VB-L+I$	$VB-(1-\xi)L+I$
16.00%	84188	82067	30	2092	91	−12933	−11033	−12842	−10942
17.00%	87347	85117	71	2158	174	−9883	−7983	−9708	−7808
18.00%	90557	88175	170	2212	322	−6825	−4925	−6503	−4603
19.00%	93817	91179	372	2265	488	−3821	−1921	−3333	−1433
19.47%	95382	92508	571	2303	592	−2492	−592	−1900	−0
20.00%	97126	93780	820	2526	673	−1220	680	−547	1353
21.00%	100481	94995	763	4723	442	−5	1895	437	2337
22.00%	103881	94988	0	8893	0	−12	1888	−12	1888
23.00%	107324	94988	0	12336	0	−12	1888	−12	1888

The following parameters were used in the constraction of this table: the arrangement fee (ξ) is 2%; the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the interest rate volatility (σ_r) is 12%; the house price volatility (σ_H) is 9%; the house service flow (δ) is 4%; LTV ratio is 95% and the correlation coefficient (ρ) is 0.

Table 4.5: Changes in the Contract Rate ($LTV = 95\%$)

(Mortgage With Early Termination Penalty)

Contract Rate	Future Payments	Mortgage	Default	Prepayment	Insurance	Value of the Mortgage to the Lender			
(c)	(A)	(V)	(D)	(C)	(I)	$VB - L$	$VB - (1 - \xi)L$	$VB - L + I$	$VB - (1 - xi)L + I$
16.00%	84188	82068	30	2090	91	−12932	−11032	−12840	−10940
17.00%	87347	85121	71	2154	174	−9879	−7979	−9704	−7804
18.00%	90557	88186	170	2201	322	−6814	−4914	−6492	−4592
19.00%	93817	91223	376	2218	491	−3777	−1877	−3287	−1387
19.44%	95267	92510	576	2181	600	−2490	−590	−1890	10
20.00%	97126	94042	904	2180	724	−958	942	−234	1666
21.00%	100481	96217	1432	2832	789	1217	3117	2006	3906
22.00%	103881	96898	944	6039	396	1898	3798	2294	4194
23.00%	107324	96886	0	10438	0	1886	3786	1886	3786

The following parameters were used in the constraction of this table: the arrangement fee (ξ) is 2%; the early termination penalty (π) is 2%; the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the interest rate volatility (σ_r) is 12%; the house price volatility (σ_H) is 9%; the house service flow (δ) is 4%; LTV ratio is 95% and the correlation coefficient (ρ) is 0.

Table 4.6: Changes in the Contract Rate (LTV=100%)

(Mortgage Without Early Termination Penalty)

Contract Rate	Future Payments	Mortgage	Default	Prepayment	Insurance	Value of the Mortgage to the Lender			
(c)	(A)	(V)	(D)	(C)	(I)	$VB-L$	$VB-(1-\xi)L$	$VB-L+I$	$VB-(1-\xi)L+I$
16.00%	88619	86342	103	2175	317	−13658	−11658	−13341	−11341
17.00%	91944	89491	235	2218	568	−10509	−8509	−9941	−7941
18.00%	95323	92571	585	2166	1077	−7429	−5429	−6351	−4351
19.00%	98755	95401	1278	2076	1699	−4599	−2599	−2900	−900
19.30%	99793	96185	1546	2062	1815	−3815	−1815	−2000	0
20.00%	88619	86342	103	2175	317	−13658	−11658	−13341	1706
21.00%	91944	89491	235	2218	568	−10509	−8509	−9941	2702
22.00%	95323	92571	585	2166	1077	−7429	−5429	−6351	2864
23.00%	98755	95401	1278	2076	1699	−4599	−2599	−2900	2947

The following parameters were used in the constraction of this table: the arrangement fee (ξ) is 2%; the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the interest rate volatility (σ_r) is 12%; the house price volatility (σ_H) is 9%; the house service flow (δ) is 4%; LTV ratio is 100% and the correlation coefficient (ρ) is 0.

Table 4.7: Changes in the Contract Rate ($LTV = 100\%$)

(Mortgage With Early Termination Penalty)

Contract Rate	Future Payments	Mortgage	Default	Prepayment	Insurance	Value of the Mortgage to the Lender			
(c)	(A)	(V)	(D)	(C)	(I)	$VB - L$	$VB - (1 - \xi)L$	$VB - L + I$	$VB - (1 - \xi)L + I$
16.00%	88619	86343	103	2173	317	−13657	−11657	−13340	−11340
17.00%	91944	89495	235	2214	568	−10505	−8505	−9937	−7937
18.00%	95323	92582	585	2156	1078	−7418	−5418	−6341	−4341
19.00%	98755	95436	1281	2037	1701	−4564	−2564	−2863	−863
19.28%	99723	96186	1545	1992	1814	−3814	−1814	−2000	−0
20.00%	102238	97884	2801	1553	1921	−2116	−116	−195	1805
21.00%	105770	98942	6221	607	1944	−1058	942	886	2886
22.00%	109348	99091	9456	802	1999	−909	1091	1089	3089
23.00%	112973	99104	12657	1212	2064	−896	1104	1168	3168

The following parameters were used in the constraction of this table: the arrangement fee (ξ) is 2%; the early termination penalty (π) is 2%; the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the interest rate volatility (σ_r) is 12%; the house price volatility (σ_H) is 9%; the house service flow (δ) is 4%; LTV ratio is 100% and the correlation coefficient (ρ) is 0.

Table 4.8: Trade-Off Between Arrangement Fee, Early Termination Penalty and Contract Rate

Prepayment Penalty	Arrangement Fee (ξ)			
(π)	0.00	0.01	0.02	0.03
0.00	21.93%	19.83%	19.47%	19.15%
0.01	20.14%	19.77%	19.45%	19.14%
0.02	20.09%	19.75%	19.44%	19.13%
0.05	20.05%	19.73%	19.43%	19.13%

The following parameters were used in the constraction of this table: the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the interest rate volatility (σ_r) is 12%; the house price volatility (σ_H) is 9%; the house service flow (δ) is 4%; LTV ratio is 95% and the correlation coefficient (ρ) is 0.

Table 4.9: Interest Rate Variation

(Mortgage Without Early Termination Penalty)

Interest Rate Volatility	Future Payments	Mortgage	Default	Prepayment	Insurance
(σ_r)	(A)	(V)	(D)	(C)	(I)
6.0%	90147	90503	143	0	436
9.0%	90318	89513	159	645	384
12.0%	90557	88175	170	2212	322
15.0%	90870	86506	167	4197	253
18.0%	91232	84572	151	6509	190

The following parameters were used in the constraction of this table: the arrangement fee (ξ) is 2%; the contract rate (c) is 18%; the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the house price volatility (σ_H) is 9%; the house service flow (δ) is 4%; LTV ratio is 95% and the correlation coefficient (ρ) is 0.

Table 4.10: Interest Rate Variation

(Mortgage With Early Termination Penalty)

Interest Rate Volatility (σ_r)	Future Payments (A)	Mortgage (V)	Default (D)	Prepayment (C)	Insurance (I)
6.0%	90147	90504	143	0	436
9.0%	90318	89517	160	641	385
12.0%	90557	88186	170	2201	322
15.0%	90870	86522	167	4180	254
18.0%	91232	84591	153	6488	192

The following parameters were used in the constraction of this table: the arrangement fee (ξ) is 2%; the contract rate (c) is 18%; the early termination penalty (π) is 2%; the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the house price volatility (σ_H) is 9%; the house service flow (δ) is 4%; LTV ratio is 95% and the correlation coefficient (ρ) is 0.

Table 4.11: House Price Variation

(Mortgage Without Early Termination Penalty)

House Price Volatility (σ_H)	Future Payments (A)	Mortgage (V)	Default (D)	Prepayment (C)	Insurance (I)
3.0%	90557	86798	2	3756	4
6.0%	90557	87335	35	3186	64
9.0%	90557	88175	170	2212	322
12.0%	90557	89206	470	881	920
15.0%	90557	90279	940	0	1868

The following parameters were used in the constraction of this table: the arrangement fee (ξ) is 2%; the contract rate (c) is 18%; the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the interest rate volatility (σ_r) is 12%; the house service flow (δ) is 4%; LTV ratio is 95% and the correlation coefficient (ρ) is 0.

Table 4.12: House Price Variation

(Mortgage With Early Termination Penalty)

House Price Volatility (σ_H)	Future Payments (A)	Mortgage (V)	Default (D)	Prepayment (C)	Insurance (I)
3.0%	90557	86804	2	3750	4
6.0%	90557	87343	35	3179	64
9.0%	90557	88186	170	2201	322
12.0%	90557	89225	471	861	920
15.0%	90557	90321	941	0	1871

The following parameters were used in the constraction of this table: the arrangement fee (ξ) is 2%; the contract rate (c) is 18%; the early termination penalty (π) is 2%; the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the interest rate volatility (σ_r) is 12%; the house service flow (δ) is 4%; LTV ratio is 95% and the correlation coefficient (ρ) is 0.

Table 4.13: Combined Effects of Changes in *LTV* Ratios and House Price and Interest Rate Volatilities

(Mortgage Without Early Termination Penalty)

LTV		Eq. Contract Rate		Future Payments		Mortgage		Default		Prepayment		Insurance	
		(c)		(A)		(V)		(D)		(C)		(I)	
		$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$
75%	$\sigma_r = 12\%$	19.57%	19.14%	75557	74436	73498	73481	2	13	2057	941	2	20
	$\sigma_r = 15\%$	20.21%	19.79%	77508	76387	73496	73479	3	18	4009	2891	2	19
95%	$\sigma_r = 12\%$	19.47%	18.97%	95381	93728	92507	91946	571	822	2303	960	592	1158
	$\sigma_r = 15\%$	20.11%	19.61%	97823	96158	92505	91955	847	1181	4471	3022	592	1145
100%	$\sigma_r = 12\%$	19.30%	18.77%	99793	97945	96185	95304	1546	1833	2062	808	1815	2700
	$\sigma_r = 15\%$	19.93%	19.39%	102360	100463	96210	95311	2230	2562	3920	2590	1800	2693

The following parameters were used in the constraction of this table: the arrangement fee (ξ) is 2%; the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the house service flow (δ) is 4%; and the correlation coefficient (ρ) is 0.

Table 4.14: Combined Effects of Changes in LTV Ratios and House Price and Interest Rate Volatilities

(Mortgage With Early Termination Penalty)

LTV		Eq. Contract Rate (c)		Future Payments (A)		Mortgage (V)		Default (D)		Prepayment (C)		Insurance (I)	
		$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$
75%	$\sigma_r = 12\%$	19.52%	19.08%	75412	74282	73497	73478	2	14	1913	789	3	22
	$\sigma_r = 15\%$	20.10%	19.68%	77213	76089	73489	73477	3	21	3720	2591	3	22
95%	$\sigma_r = 12\%$	19.44%	18.94%	95260	93626	92504	91943	575	819	2181	864	600	1159
	$\sigma_r = 15\%$	20.03%	19.54%	97559	95937	92492	91946	856	1185	4212	2806	602	1155
100%	$\sigma_r = 12\%$	19.28%	18.75%	99723	97878	96186	95303	1545	1819	1992	756	1814	2698
	$\sigma_r = 15\%$	19.89%	19.35%	102218	100307	96222	95305	2235	2540	3761	2462	1781	2690

The following parameters were used in the constraction of this table: the arrangement fee (ξ) is 2%; the early termination penalty (π) is 2%; the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the house service flow (δ) is 4% and the correlation coefficient (ρ) is 0.

Table 4.15: Combined Effects of Changes in Spot Rates and House Price and Interest Rate Volatilities ($LTV = 75\%$)

(Mortgage Without Early Termination Penalty)

Spot Rate		Eq. Contract Rate		Future Payments		Mortgage		Default		Prepayment		Insurance	
		(c)		(A)		(V)		(D)		(C)		(I)	
		$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$
12%	$\sigma_r = 12\%$	20.22%	19.80%	75655	74557	75655	74557	2	13	2156	1045	2	18
	$\sigma_r = 15\%$	20.91%	20.49%	77704	76603	77704	76603	3	18	4201	3094	2	17
15%	$\sigma_r = 12\%$	21.54%	21.11%	76017	74942	73499	73500	2	14	2515	1429	2	15
	$\sigma_r = 15\%$	22.32%	21.90%	78291	77216	73500	73491	3	17	4789	3708	2	13
18%	$\sigma_r = 12\%$	22.76%	22.34%	76521	75493	73495	73496	2	15	3023	1982	2	13
	$\sigma_r = 15\%$	23.65%	23.25%	79027	78024	73500	73500	3	18	5525	4506	1	11
21%	$\sigma_r = 12\%$	24.50%	24.11%	77568	76636	73496	73494	3	16	4070	3126	1	10
	$\sigma_r = 15\%$	25.49%	25.13%	80272	79378	73498	73498	3	21	6771	5859	1	9

The following parameters were used in the constraction of this table: the arrangement fee (ξ) is 2%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the house service flow (δ) is 4%; LTV ratio is 75% and the correlation coefficient (ρ) is 0.

Table 4.16: Combined Effects of Changes in Spot Rates and House Price and Interest Rate Volatilities ($LTV = 75\%$)

(Mortgage With Early Termination Penalty)

Spot Rate		Eq. Contract Rate		Future Payments		Mortgage		Default		Prepayment		Insurance	
		(c)		(A)		(V)		(D)		(C)		(I)	
		$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$
12%	$\sigma_r = 12\%$	20.13%	19.70%	75411	74304	73492	73499	2	15	1917	790	2	21
	$\sigma_r = 15\%$	20.75%	20.32%	77281	76172	73497	73499	3	21	3781	2652	2	20
15%	$\sigma_r = 12\%$	21.34%	20.90%	75528	74422	73493	73494	3	16	2033	912	2	17
	$\sigma_r = 15\%$	22.03%	21.60%	77551	76454	73500	73500	4	22	4047	2932	2	16
18%	$\sigma_r = 12\%$	22.46%	22.02%	75793	74709	73500	73500	3	16	2291	1193	2	15
	$\sigma_r = 15\%$	23.22%	22.80%	77953	76888	73497	73497	4	23	4453	3368	2	14
21%	$\sigma_r = 12\%$	24.00%	23.58%	76356	75342	73497	73497	3	19	2856	1826	2	12
	$\sigma_r = 15\%$	24.87%	24.47%	78758	77782	73492	73500	4	23	5262	4259	1	11

The following parameters were used in the constraction of this table: the arrangement fee (ξ) is 2%; the early termination penalty (π) is 2%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the house service flow (δ) is 4%; LTV ratio is 75% and the correlation coefficient (ρ) is 0.

Table 4.17: Combined Effects of Changes in Spot Rates and House Price and Interest Rate Volatilities ($LTV = 95\%$)

(Mortgage Without Early Termination Penalty)

Spot Rate		Eq. Contract Rate		Future Payments		Mortgage		Default		Prepayment		Insurance	
		(c)		(A)		(V)		(D)		(C)		(I)	
		$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$
12%	$\sigma_r = 12\%$	20.12%	19.62%	95497	93857	92513	91980	615	867	2370	1010	584	1111
	$\sigma_r = 15\%$	20.80%	20.30%	98059	96422	92515	92005	914	1259	4630	3157	583	1095
15%	$\sigma_r = 12\%$	21.42%	20.93%	95928	94353	92522	92059	716	985	2689	1308	572	1038
	$\sigma_r = 15\%$	22.20%	21.70%	98767	97170	92537	92073	1036	1440	5194	3657	563	1019
18%	$\sigma_r = 12\%$	22.64%	22.16%	96553	95068	92528	92113	830	1124	3194	1831	571	979
	$\sigma_r = 15\%$	23.51%	23.04%	99648	98158	92536	92132	1195	1630	5917	4395	562	969
21%	$\sigma_r = 12\%$	24.34%	23.92%	97758	96476	92525	92184	1024	1378	4209	2915	570	915
	$\sigma_r = 15\%$	25.33%	24.92%	101191	99908	92542	92197	1390	1928	7259	5783	553	901

The following parameters were used in the constraction of this table: the arrangement fee (ξ) is 2%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the house service flow (δ) is 4%; LTV ratio is 95% and the correlation coefficient (ρ) is 0.

Table 4.18: Combined Effects of Changes in Spot Rates and House Price and Interest Rate Volatilities ($LTV = 95\%$)

(Mortgage With Early Termination Penalty)

Spot Rate		Eq. Contract Rate		Future Payments		Mortgage		Default		Prepayment		Insurance	
		(c)		(A)		(V)		(D)		(C)		(I)	
		$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$
12%	$\sigma_r = 12\%$	20.06%	19.56%	95275	93675	92500	91980	617	862	2157	834	594	1119
	$\sigma_r = 15\%$	20.68%	20.19%	97651	96063	92500	91992	916	1265	4235	2806	593	1112
15%	$\sigma_r = 12\%$	21.28%	20.79%	95456	93912	92517	92053	700	955	2239	905	582	1047
	$\sigma_r = 15\%$	21.97%	21.49%	98016	96473	92524	92059	1023	1415	4469	2999	575	1038
18%	$\sigma_r = 12\%$	22.39%	21.92%	95779	94316	92522	92111	779	1058	2478	1147	573	986
	$\sigma_r = 15\%$	23.15%	22.69%	98529	97075	92529	92113	1141	1561	4859	3401	573	982
21%	$\sigma_r = 12\%$	23.92%	23.50%	96488	95199	92522	92181	916	1239	3050	1779	572	919
	$\sigma_r = 15\%$	24.80%	24.39%	99525	98253	92528	92181	1295	1783	5702	4288	563	918

The following parameters were used in the constraction of this table: the arrangement fee (ξ) is 2%; the early termination penalty (π) is 2%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the house service flow (δ) is 4%; LTV ratio is 95% and the correlation coefficient (ρ) is 0.

Table 4.19: Combined Effects of Changes in Spot Rates and House Price and Interest Rate Volatilities (LTV = 100%)

(Mortgage Without Early Termination Penalty)

Spot Rate		Eq. Contract Rate		Future Payments		Mortgage		Default		Prepayment		Insurance	
		(c)		(A)		(V)		(D)		(C)		(I)	
		σ_H = 9%	σ_H = 12%	σ_H = 9%	σ_H = 12%	σ_H = 9%	σ_H = 12%	σ_H = 9%	σ_H = 12%	σ_H = 9%	σ_H = 12%	σ_H = 9%	σ_H = 12%
12%	σ_r = 12%	19.96%	19.40%	99952	98062	96231	95332	1647	1910	2075	820	1771	2669
	σ_r = 15%	20.63%	20.08%	102622	100720	96251	95368	2375	2687	3996	2666	1756	2641
15%	σ_r = 12%	21.27%	20.70%	100466	98534	96320	95421	1877	2102	2269	1010	1677	2571
	σ_r = 15%	22.04%	21.46%	103416	101467	96342	95474	2832	2992	4242	3002	1650	2528
18%	σ_r = 12%	22.49%	21.94%	101129	99319	96360	95546	2217	2360	2552	1413	1638	2457
	σ_r = 15%	23.34%	22.78%	104332	102463	96374	95574	3449	3323	4509	3567	1626	2400
21%	σ_r = 12%	24.21%	23.67%	102477	100744	96437	95652	2707	2801	3334	2291	1560	2350
	σ_r = 15%	25.19%	24.66%	106037	104321	96430	95713	4128	4026	5478	4582	1570	2280

The following parameters were used in the constraction of this table: the arrangement fee (ξ) is 2%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the house service flow (δ) is 4%; LTV ratio is 100% and the correlation coefficient (ρ) is 0.

Table 4.20: Combined Effects of Changes in Spot Rates and House Price and Interest Rate Volatilities (LTV = 100%)

(Mortgage With Early Termination Penalty)

Spot Rate		Eq. Contract Rate		Future Payments		Mortgage		Default		Prepayment		Insurance	
		(c)		(A)		(V)		(D)		(C)		(I)	
		$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$	$\sigma_H = 9\%$	$\sigma_H = 12\%$
12%	$\sigma_r = 12\%$	19.92%	19.37%	99820	97951	96235	95342	1634	1887	1951	722	1768	2655
	$\sigma_r = 15\%$	20.55%	20.01%	102369	100471	96257	95368	2421	2652	3691	2451	1743	2628
15%	$\sigma_r = 12\%$	21.18%	20.61%	100144	98250	96336	95451	1841	2052	1967	747	1666	2558
	$\sigma_r = 15\%$	21.89%	21.32%	102909	100977	96363	95496	2784	2947	3763	2533	1630	2509
18%	$\sigma_r = 12\%$	22.31%	21.76%	100554	98750	96379	95575	2146	2256	2029	919	1622	2416
	$\sigma_r = 15\%$	23.09%	22.56%	103515	101749	96388	95646	3350	3281	3777	2823	1610	2347
21%	$\sigma_r = 12\%$	23.90%	23.34%	101495	99686	96481	95676	2503	2606	2511	1404	1526	2323
	$\sigma_r = 15\%$	24.79%	24.25%	104753	103000	96460	95742	3844	3776	4449	3482	1540	2256

The following parameters were used in the constraction of this table: the arrangement fee (ξ) is 2%; the early termination penalty (π) is 2%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the house service flow (δ) is 4%; LTV ratio is 100% and the correlation coefficient (ρ) is 0.

Table 4.21: Correlation Coefficient Variation

(Mortgage Without Early Termination Penalty)

Rho	Eq.Contract Rate	Future Payments	Default	Prepayment	Insurance
(ρ)	(c)	(A)	(D)	(C)	(I)
−0.20	19.47%	95378	406	2373	499
−0.15	19.47%	95380	450	2353	525
−0.10	19.47%	95378	493	2334	551
−0.05	19.47%	95374	528	2321	568
0.00	19.48%	95383	572	2303	592
0.05	19.47%	95382	613	2284	616
0.10	19.48%	95384	646	2274	629
0.15	19.48%	95394	687	2259	652
0.20	19.48%	95394	728	2240	674

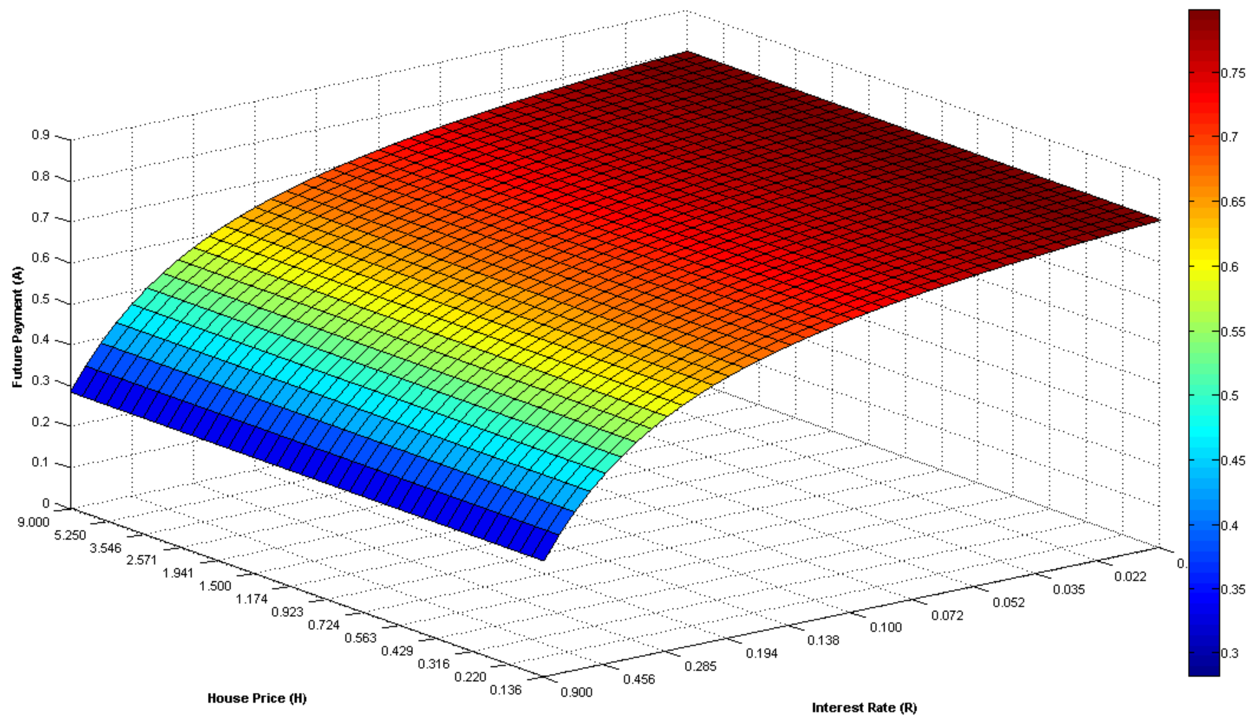
The following parameters were used in the constraction of this table: the arrangement fee (ξ) is 2%; the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the interest rate volatility (σ_r) is 12%; the house price volatility (σ_H) is 9%; the house service flow (δ) is 4%; and LTV ratio is 95%.

Table 4.22: Correlation Coefficient Variation

(Mortgage With Early Termination Penalty)

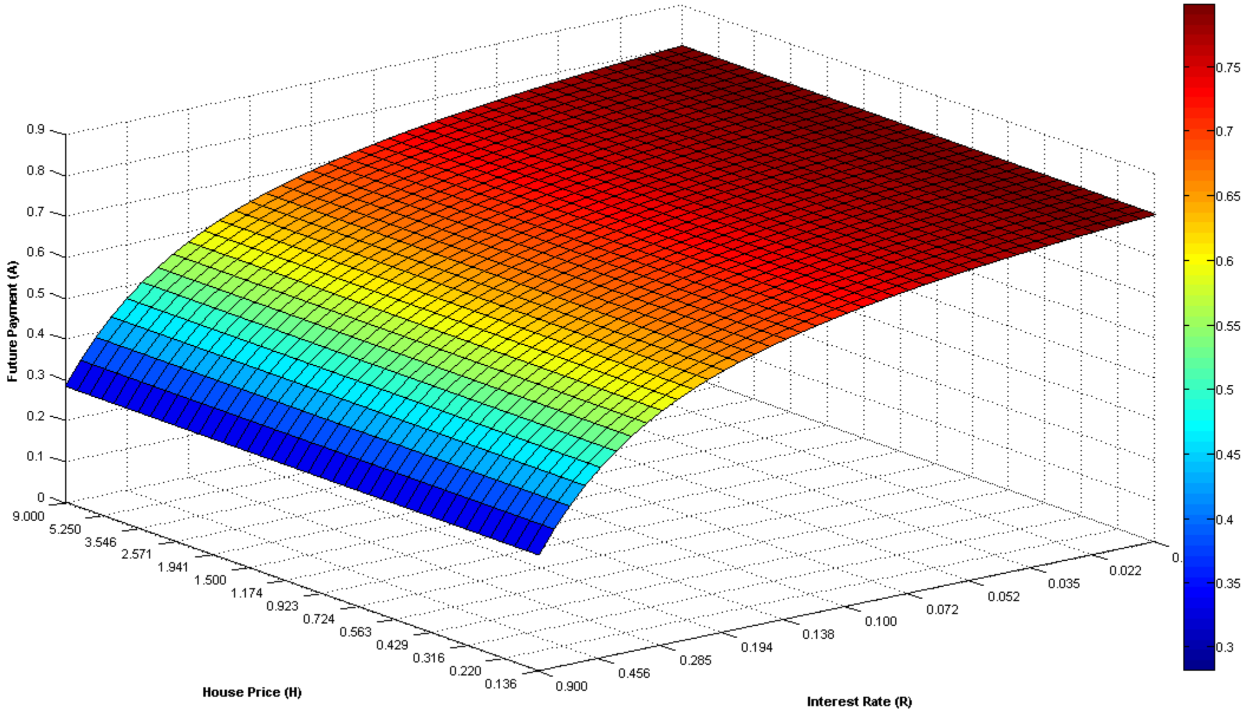
Rho	Eq.Contract Rate	Future Payments	Default	Prepayment	Insurance
(ρ)	(c)	(A)	(D)	(C)	(I)
−0.20	19.43%	95235	409	2238	507
−0.15	19.44%	95252	454	2222	534
−0.10	19.44%	95255	496	2207	555
−0.05	19.44%	95254	534	2196	576
0.00	19.44%	95257	575	2181	600
0.05	19.44%	95261	617	2166	620
0.10	19.44%	95273	652	2158	638
0.15	19.44%	95275	691	2145	660
0.20	19.44%	95281	733	2130	682

The following parameters were used in the constraction of this table: the arrangement fee (ξ) is 2%; the early termination penalty (π) is 2%; the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the interest rate volatility (σ_r) is 12%; the house price volatility (σ_H) is 9%; the house service flow (δ) is 4%; and LTV ratio is 95%.



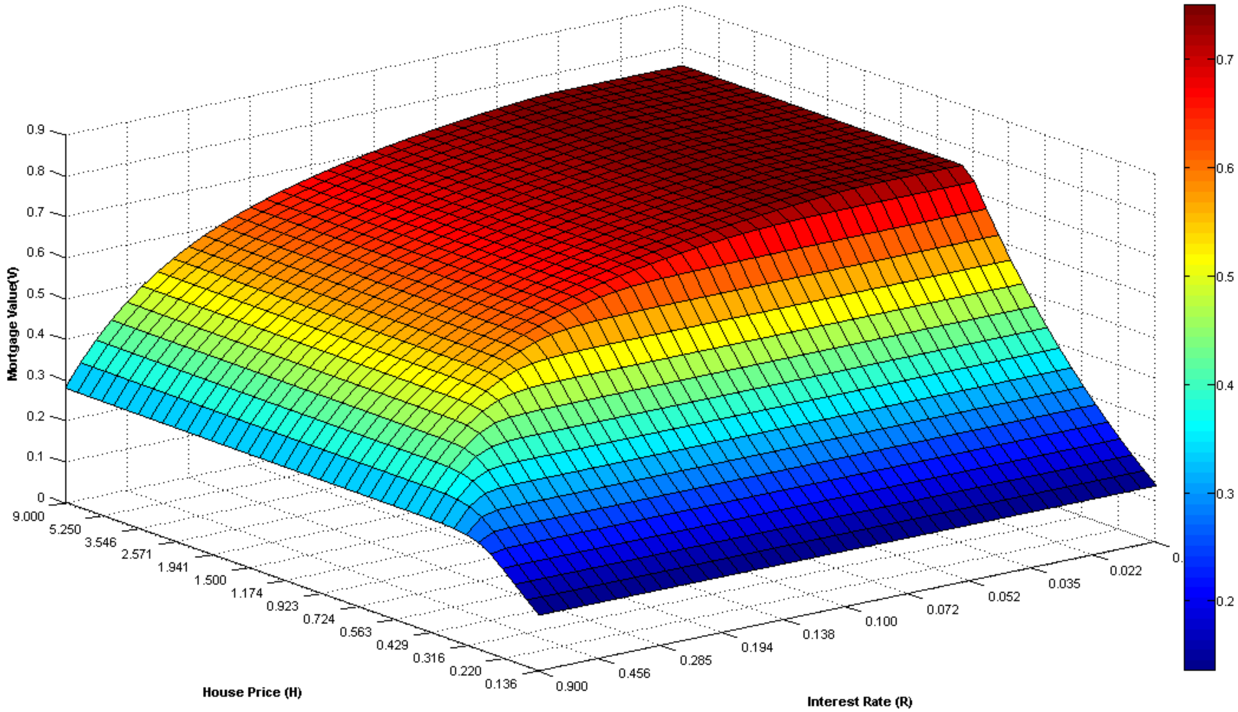
The following parameters were used in the constraction of the figure: the arrangement fee (ξ) is 2%; the contract rate (c) is 18%; the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the interest rate volatility (σ_r) is 12%; the house price volatility (σ_H) is 9%; the house service flow (δ) is 4%; LTV ratio is 95% and the correlation coefficient (ρ) is 0.

Figure 4.1: Value of Future Payments (A)
(Mortgage Without Early Termination Penalty)



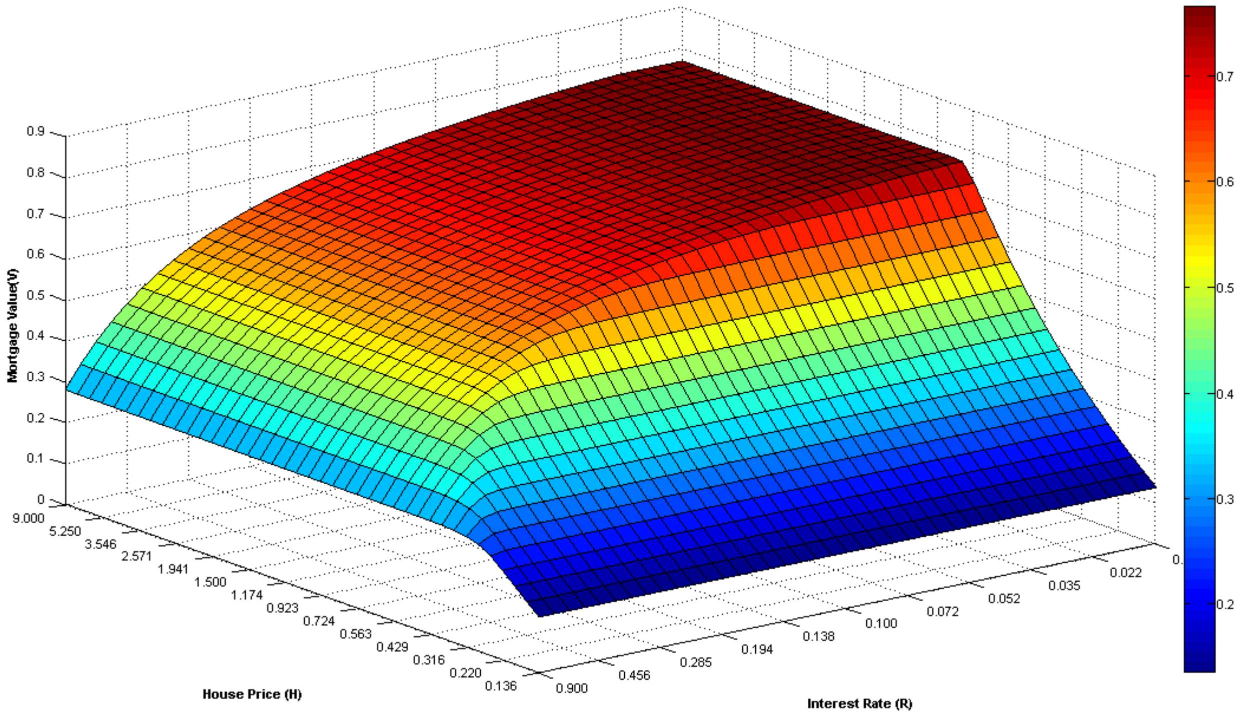
The following parameters were used in the constraction of the figure: the arrangement fee (ξ) is 2%; the contract rate (c) is 18%; the early termination penalty (π) is 2%; the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the interest rate volatility (σ_r) is 12%; the house price volatility (σ_H) is 9%; the house service flow (δ) is 4%; LTV ratio is 95% and the correlation coefficient (ρ) is 0.

Figure 4.2: Value of Future Payments (A)
(Mortgage With Early Termination Penalty)



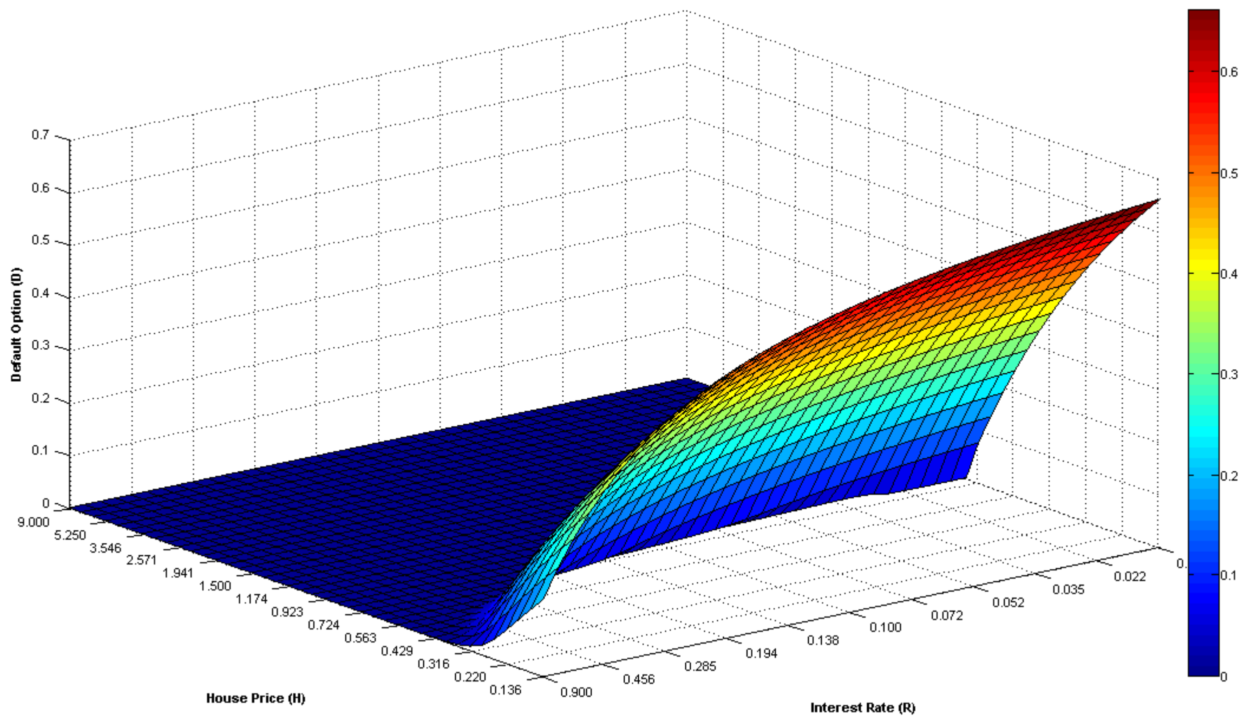
The following parameters were used in the constraction of the figure: the arrangement fee (ξ) is 2%; the contract rate (c) is 18%; the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the interest rate volatility (σ_r) is 12%; the house price volatility (σ_H) is 9%; the house service flow (δ) is 4%; LTV ratio is 95% and the correlation coefficient (ρ) is 0.

Figure 4.3: Value of Mortgage (V)
(Mortgage Without Early Termination Penalty)



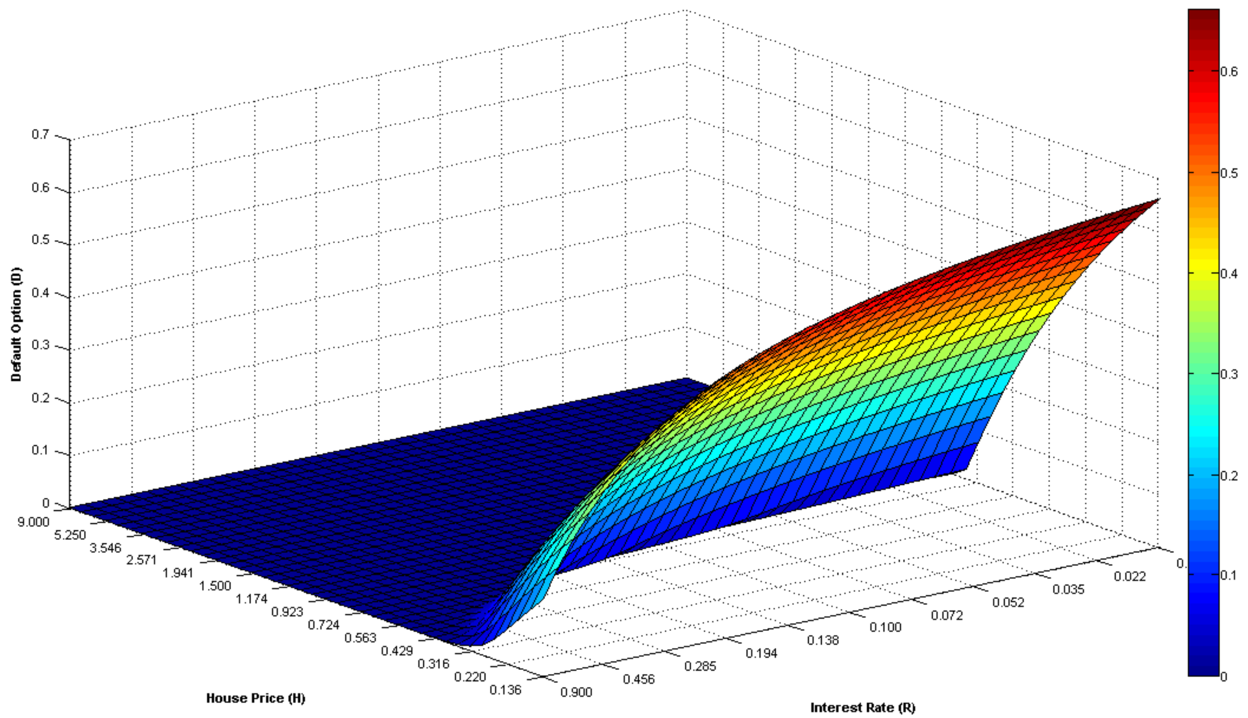
The following parameters were used in the constraction of the figure: the arrangement fee (ξ) is 2%; the contract rate (c) is 18%; the early termination penalty (π) is 2%; the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the interest rate volatility (σ_r) is 12%; the house price volatility (σ_H) is 9%; the house service flow (δ) is 4%; LTV ratio is 95% and the correlation coefficient (ρ) is 0.

Figure 4.4: Value of Mortgage (V)
(Mortgage With Early Termination Penalty)



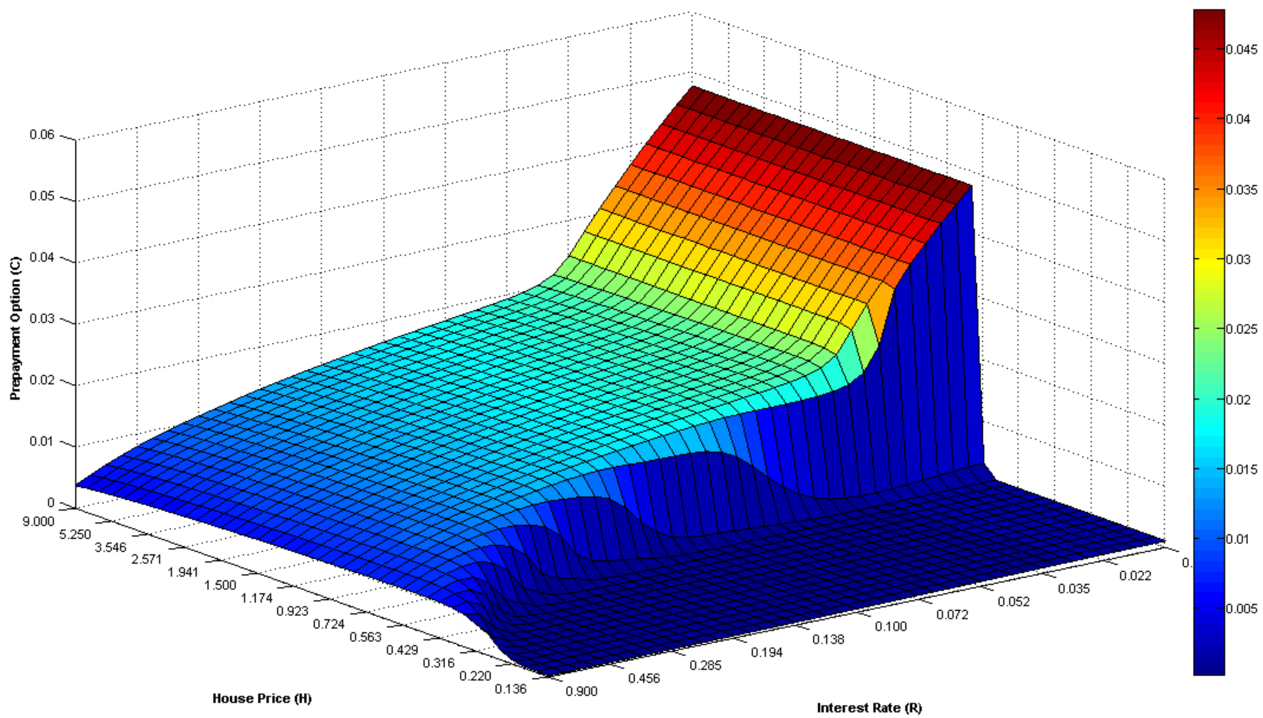
The following parameters were used in the constraction of the figure: the arrangement fee (ξ) is 2%; the contract rate (c) is 18%; the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the interest rate volatility (σ_r) is 12%; the house price volatility (σ_H) is 9%; the house service flow (δ) is 4%; LTV ratio is 95% and the correlation coefficient (ρ) is 0.

Figure 4.5: Value of Default Option (D)
(Mortgage Without Early Termination Penalty)



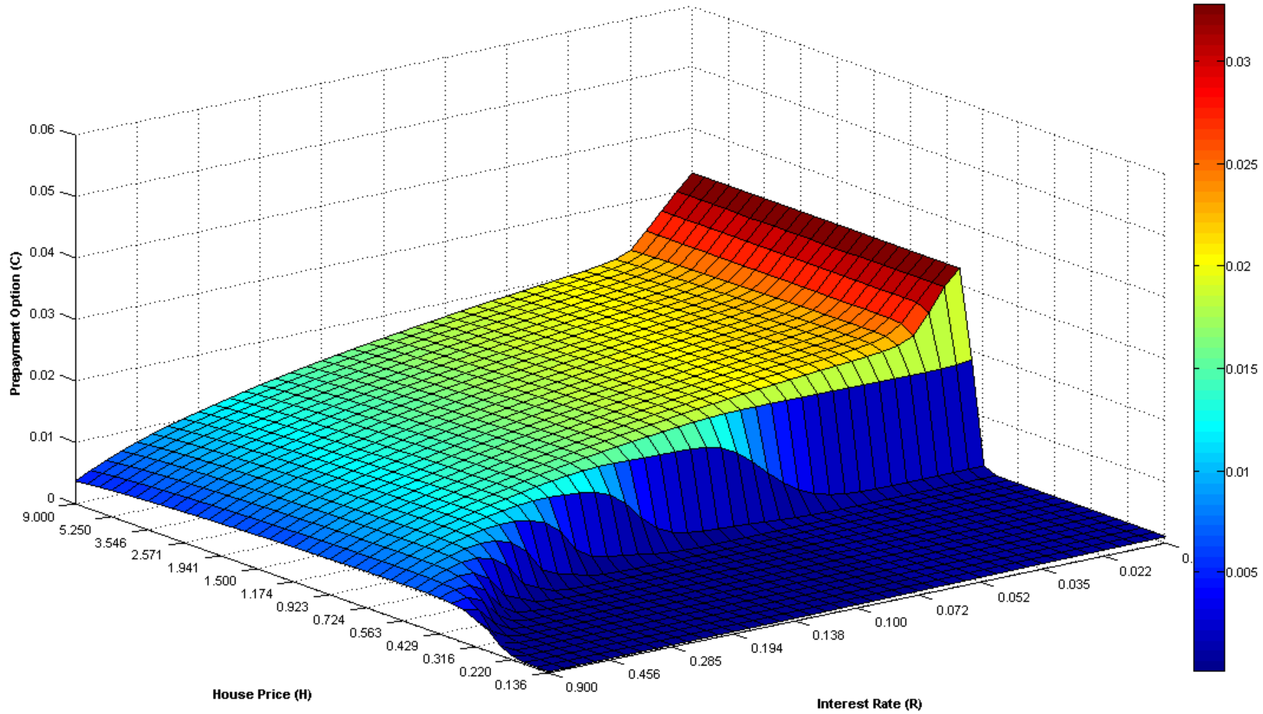
The following parameters were used in the constraction of the figure: the arrangement fee (ξ) is 2%; the contract rate (c) is 18%; the early termination penalty (π) is 2%; the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the interest rate volatility (σ_r) is 12%; the house price volatility (σ_H) is 9%; the house service flow (δ) is 4%; LTV ratio is 95% and the correlation coefficient (ρ) is 0.

Figure 4.6: Value of Default Option (D)
(Mortgage With Early Termination Penalty)



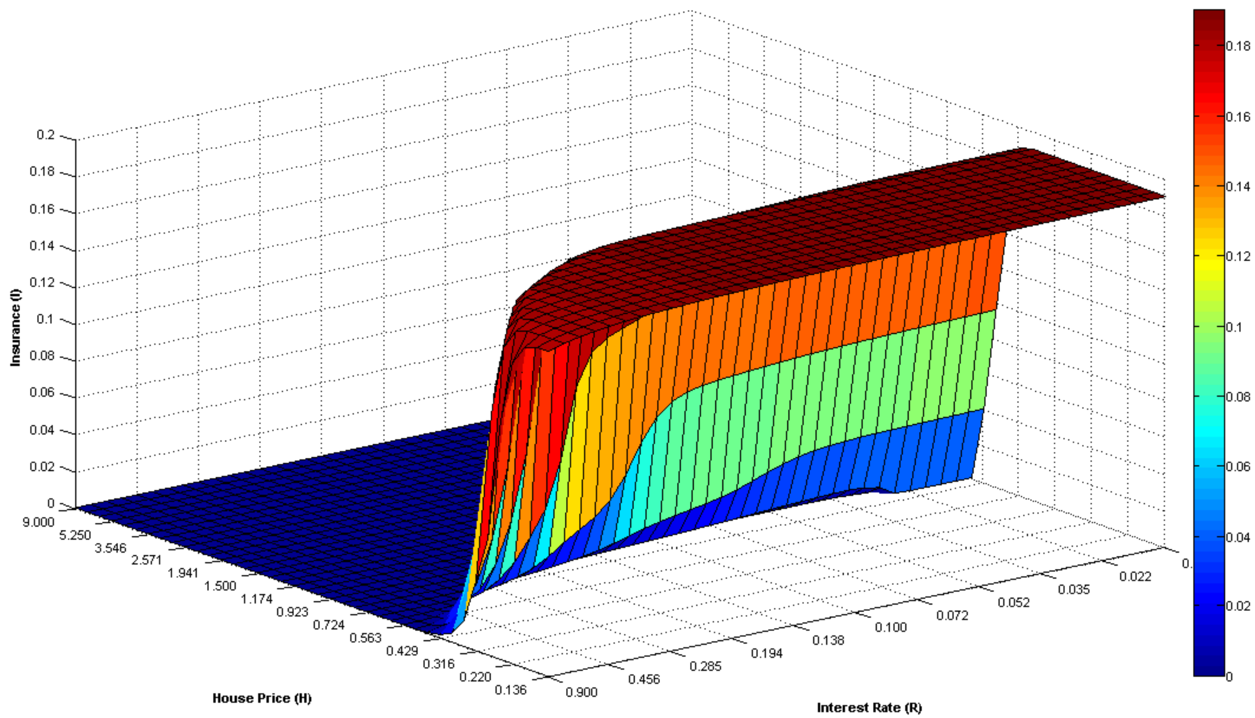
The following parameters were used in the constraction of the figure: the arrangement fee (ξ) is 2%; the contract rate (c) is 18%; the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the interest rate volatility (σ_r) is 12%; the house price volatility (σ_H) is 9%; the house service flow (δ) is 4%; LTV ratio is 95% and the correlation coefficient (ρ) is 0.

Figure 4.7: Value of Prepayment Option (C)
(Mortgage Without Early Termination Penalty)



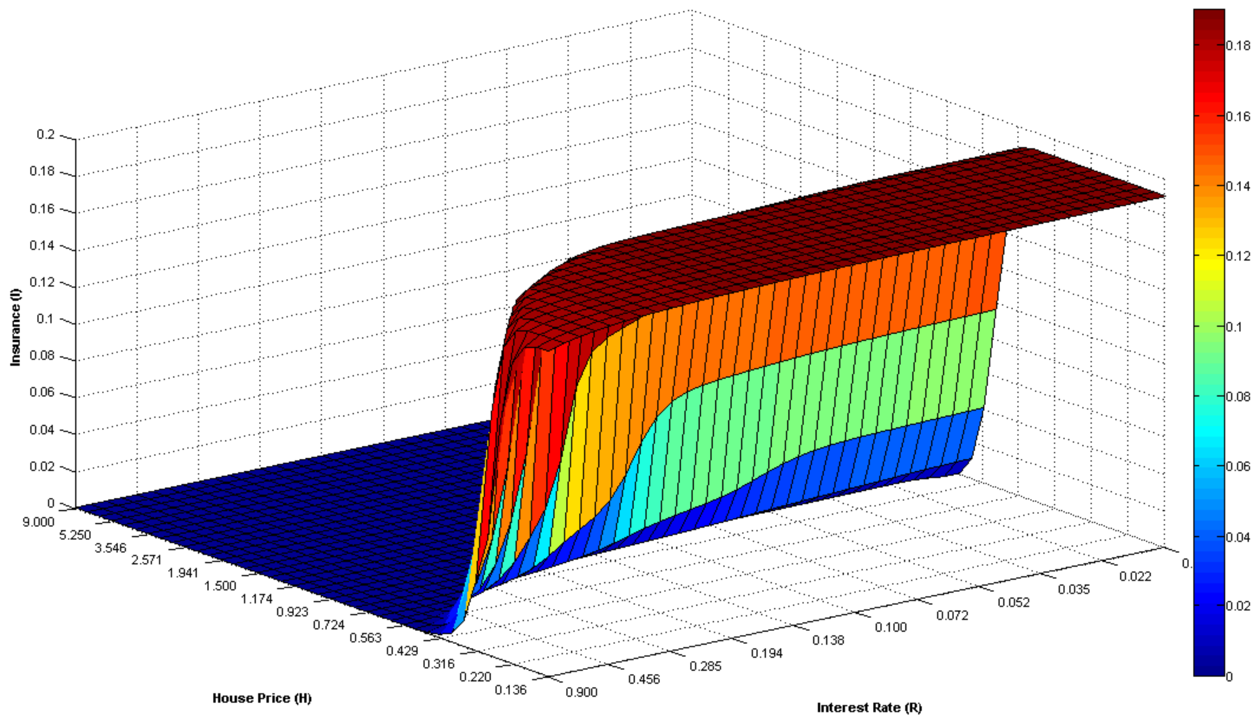
The following parameters were used in the constraction of the figure: the arrangement fee (ξ) is 2%; the contract rate (c) is 18%; the early termination penalty (π) is 2%; the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the interest rate volatility (σ_r) is 12%; the house price volatility (σ_H) is 9%; the house service flow (δ) is 4%; LTV ratio is 95% and the correlation coefficient (ρ) is 0.

Figure 4.8: Value of Prepayment Option (C)
(Mortgage With Early Termination Penalty)



The following parameters were used in the constraction of the figure: the arrangement fee (ξ) is 2%; the contract rate (c) is 18%; the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the interest rate volatility (σ_r) is 12%; the house price volatility (σ_H) is 9%; the house service flow (δ) is 4%; LTV ratio is 95% and the correlation coefficient (ρ) is 0.

Figure 4.9: Value of Insurance (I)
(Mortgage Without Early Termination Penalty)



The following parameters were used in the constraction of the figure: the arrangement fee (ξ) is 2%; the contract rate (c) is 18%; the early termination penalty (π) is 2%; the spot interest rate ($r(0)$) is 10%; the long term average of the interest rate (θ) is 24%; the speed of reversion (κ) is 56%; the interest rate volatility (σ_r) is 12%; the house price volatility (σ_H) is 9%; the house service flow (δ) is 4%; LTV ratio is 95% and the correlation coefficient (ρ) is 0.

Figure 4.10: Value of Insurance (I)
(Mortgage With Early Termination Penalty)

CHAPTER 5

CONCLUSION

This study develops a model for the valuation of fixed-rate mortgages in emerging markets like Turkey. During the evaluation process, the default and prepayment risk of mortgage contract are also estimated. In addition to these, the model used in this study also prices the mortgage insurance policy which has not been used yet in Turkish mortgage market but thought as a potential derivative in this market.

In the preliminary stage of the study, we collect information on the FRM contract details of eight deposit banks with the largest mortgage portfolios in Turkey, namely, Finansbank, Oyak Bank, HSBC Bank, Akbank, Yapı Kredi Bank, Garanti Bank, Vakıf Bank, İş Bank. More specifically, we collect data for the contract maturity, coupon rate, loan-to-value (LTV) ratio, arrangement fee, prepayment penalty and the available insurance policies of these deposit banks.

The work starts with a brief summary of the mortgage markets which is categorized according to the development level of the countries and an overview of the Turkish mortgage market. Subsequently, a brief review and discussion of the literature on mortgage valuation is present, giving special emphasis to term structure modeling and the modeling of mortgage components.

Considering the studies mentioned in the literature review, a decision was made to employ a contingent claims framework. Both early termination options, namely, prepayment and default option are taken into consideration in the valuation process. It should also be noted that in the model, suboptimal prepayment behavior is not considered. In other words, it is assumed that the termination decision of mortgage is driven by a economic rationale.

A framework based on Cox, Ingersoll and Ross [8] equilibrium model is used in order to evaluate fixed-rate mortgages and mortgage related assets. In this methodology, the spot interest rate and the house price are used as state variables. The corresponding partial differential equation which is derived for the mortgage, its components and mortgage insurance policy does not have closed form solutions. Consequently, to cope with this problem, an explicit finite difference method is used to solve the partial differential equation. In this valuation process, the value of the monthly payments, the value of the borrower's debt in case of early termination, the equilibrium condition and the terminal condition on each of the payment date are determined. The explicit finite difference method is applied using backward solution techniques due to the fact that the mortgage related assets is present at the termination of the mortgage contract.

In the numerical method, the original PDE is transformed and its original infinite domain is mapped into a unit square. After this process, the common boundary conditions and the prepayment free boundary are formulated. However, since the free boundary creates some difficulties in working, the problem is converted in a non-linear PDE with a fixed boundary. Subsequently, difference equations for the first and second derivative components are generated and to satisfy the stability of the algorithm 'upwind diffrencing' scheme is used in the approximation process. Finally, in order to solve PDE, the Matlab code is developed.

Numerical results for the value of mortgage-related assets are determined under different economic scenarios. In this implementation, it is seen that every economic scenario leads to different equilibrium contract rates and different values for the mortgage related assets. Results obtained in the basic economic scenario show that Turkish banks apply lower contract rates as compared with the optimal ones. This observation indicates that the primary mortgage market in Turkey is still in its infancy stage. Numerical results also suggest that it is beneficial for the lenders to have mortgage default insurance, especially for the high LTV ratio mortgages.

REFERENCES

[1] Azevedo-Pereira, J. A., *Fixed Rate Mortgage Valuation Using A Contingent Claims Approach*, Ph.D. Thesis submitted to the University of Manchester, 1997.

[2] Azevedo-Pereira, J. A., Newton, D. P. and D. A. Paxson, *UK Fixed Rate Repayment Mortgage and Mortgage Indemnity Valuation*, Real Estate Economics, 30(2):185-211, 2002.

[3] Azevedo-Pereira, J. A., Newton, D. P. and D. A. Paxson, *Fixed Rate Endowment Mortgage and Mortgage Indemnity Valuation*, Journal of Real Estate Finance and Economics, 26(2/3):197-221, 2003.

[4] Boyle, P. P., *Options: A Monte Carlo Approach*, Journal of Financial Economics, 4:323-338, 1977.

[5] Ciochetti B. A., Deng, Y., Gao, B. and R. Yao, *The Termination of Commercial Mortgage Contracts through Prepayment and Default: A Proportional Hazard Approach with Competing Risks*, 30(4):595-633, 2002.

[6] Cox, D. R., *Regression Models and Life-Tables*, Journal of the Royal Statistical Society. Series B (Methodological), 34(2):187-220, 1972.

[7] Cox, J. C., Ross S. A. and M. Rubinstein *Option Pricing: A Simplified Approach*, Journal of Financial Economics, 7:229-264, 1979.

[8] Cox, J. C., Ingersoll, J. E. and S. A. Ross, *An Intertemporal General Equilibrium Model of Asset Prices*, Econometrica, 53(2):363-384, 1985.

[9] Cox, J. C., Ingersoll, J. E. and S. A. Ross, *A Theory of the Term Structure of Interest Rates*, Econometrica, 53(2):385-407, 1985.

[10] Cunningham, D. F. and C. A. Capone, Jr., *The Relative Termination Experience of Adjustable to Fixed-Rate Mortgages*, The Journal of Finance, 45(5):1687-1703, 1990.

[11] Deng, Y., Quigley, J. M. and R. Van Order, *Mortgage Default and Low Downpayment Loans: The Costs of Public Subsidy*, Regional Science and Urban Economics, 26(3-4):263-285, 1996.

[12] Deng, Y., *Mortgage Termination: An Empirical Hazard Model with a Stochastic Term Structure*, Journal of Real Estate Finance and Economics, 14(3):309-331, 1997.

[13] Deng, Y., Quigley, J. M. and R. Van Order, *Mortgage Terminations, Heterogeneity and the Exercise of Mortgage Options*, Econometrica, 68(2):275-307, 2000.

[14] Douetil, D. J., *The Interrelationship Between the Mortgage and Insurance Industries in the United Kingdom*, Housing Policy Debate, 5(3), 1994.

[15] Dunn, K. B. and J. J. McConnell, *Valuation of GNMA Mortgage-Backed Securities*, The Journal of Finance, 36(3):599-616, 1981.

[16] Dunn, K. B. and J. J. McConnell, *A Comparison of Alternative Models for Pricing GNMA Mortgage-Backed Securities*, The Journal of Finance, 36(2):471-484, 1981.

[17] Dunsky, R. M. and T. S. Y. Ho, *Valuing Fixed Rate Mortgage Loans with Default and Prepayment Options*, Journal of Fixed Income, 16(4):7-31, 2007.

[18] Erol, I. and K. Patel, *Default risk of wage-indexed payment mortgage in Turkey*, Journal of Housing Economics, 14:271-293,2005.

[19] Findlay III, M. C. and D. R. Capozza, *The Variable-Rate Mortgage and Risk in the Mortgage Market: An Option Theory Perspective*, Journal of Money, Credit and Banking, 9(2):356-364, 1977.

[20] Green, J. and J.B. Shoven, *The Effects of Interest Rates on Mortgage Prepayments*, Journal of Money, Credit and Banking, 18(1):41-59, 1986.

[21] Green, R. K. and S. M. Wachter, *The American Mortgage in Historical and International Context*, Journal of Economic Perspectives, 19(4):93-114, 2005.

[22] Hull, J. C., *Options, Futures, and Other Derivative Securities*, Prentice-Hall International, 3rd edition, 1997.

[23] Jackson, J. R. and D. L. Kaserman, *Default Risk on Home Mortgage Loans: A Test of Competing Hypotheses*, The Journal of Risk and Insurance, 47(4):678-690, 1980.

[24] Jaffee, D. M. and B. Renaud, *Securitization in European Mortgage Markets*, Paper presented at the First International Real Estate Conference, Stockholm, Sweden, 1995.

[25] Jaffee, D. M. and B. Renaud *Strategies to Develop Mortgage Markets in Transition Economies*, In Financial Sector Reform and Privatisation in Transition Economies, Ed. J. Doukas, V. Murinde and C. Wihlborg.

[26] Kau, J. B., Keenan, D. C., Muller, W. J. and J.F. Epperson, *The Valuation and Analysis of Adjustable Rate Mortgages*, Management Science, 36(12):1417-1431, 1990.

[27] Kau, J. B., Keenan, D. C., Muller W. J. and J. F. Epperson, *An General Valuation Model for Fixed-Rate Residential Mortgages*, Journal of Money, Credit and Banking, 24(3):279-299, 1992.

[28] Kau, J. B., Keenan, D. C., Muller, W. J. and J. F. Epperson, *Option Theory and Floating-Rate Securities with a Comparison of Adjustable- and Fixed-Rate Mortgages*, The Journal of Business, 66(4):595-618, 1993.

[29] Kau, J. B., Keenan, D. C., Muller, W. J. and J. F. Epperson, *The Valuation at Origination of Fixed-Rate Mortgages with Default and Prepayment*, Journal of Real Estate Finance and Economics, 11(1):5-36, 1995.

[30] Kau, J. B., Keenan, D. C. and A. A. Smurov, *Reduced Form Mortgage Pricing as an Alternative to Option-Pricing Models*, The Journal of Real Estate Finance and Economics, 33(3):183-196, 2006.

[31] Lea, M. J., *Efficiency and Stability of Housing Finance Systems:A Comparison of the United Kingdom and the United States*, Housing Policy Debate, 5(3), 1994.

[32] Liao, S. L., Tsai, M. S. and S. L. Chiang, *Closed-Form Mortgage Valuation Using Reduced-Form Model*, Real Estate Economics, 36(2):313-347, 2008.

[33] Longstaff, F. A. and E. S. Schwartz, *A simple approach to valuing risky fixed and floating rate debt*, Journal of Finance 50(3):789-819.

[34] Merton, R. C., *Theory of Rational Option Pricing*, The Bell Journal of Economics and Management Science, 4(1):141-183, 1973.

[35] Miles, D., *The UK Mortgage Market: Taking a Longer-Term View*, Working Paper, UK Treasury, 2004.

[36] Morton, K. W. and D. F. Mayers, *Numerical solutions of Partial Diffrential Equations*, Cambridge University Press, 1994.

[37] Quigley, J. M. and R. Van Order, *Explicit Tests of Contingent Claims Models of Mortgage Default*, Journal of Real Estate Finance and Economics, 11:99-117, 1995.

[38] Schwartz, E. S., *The Valuation of Warrants: Implementing A New Approach*, Journal of Financial Economics, 4(1):79-93, 1977.

[39] Schwartz, E. S. and W. N. Torous, *Prepayment and the Valuation of Mortgage-Backed Securities*, The Journal of Finance, 44(2):375-392, 1989.

[40] Sharp, N. J., Newton, D. P. and P. W. Duck, *An Improved Fixed-Rate Mortgage Valuation Methodology with Interacting Prepayment and Default Options*, Journal of Real Estate Finance and Economics, 36:307-342, 2008.

[41] Warnock V. C. and F. E. Warnock, *Markets and Housing Finance*, Journal of Housing Economics, 17:239-251, 2008.

[42] Wilmott, P., Dewynne, J. and S. Howison, *Option Pricing: Mathematical Models and Computation*, Oxford Financial Press, 1993.

Appendix A

Derivation of the Formulas for the Valuation of the Monthly Payments and the Outstanding Balance

A.1 Formula for the Value of the Monthly Payments

The basic rule used to find the monthly payments is shown below:

$$OB(0) = \left(\frac{1}{1+\frac{c}{12}}\right) MP + \left(\frac{1}{1+\frac{c}{12}}\right)^2 MP + \ldots + \left(\frac{1}{1+\frac{c}{12}}\right)^n MP. \tag{A.1}$$

To get a simple formula for $OB(0)$, the following equation is obtained:

$$\begin{aligned} OB(0) - \left(\frac{1}{1+\frac{c}{12}}\right) OB(0) = &\left(\left(\frac{1}{1+\frac{c}{12}}\right) MP + \left(\frac{1}{1+\frac{c}{12}}\right)^2 MP + \ldots + \left(\frac{1}{1+\frac{c}{12}}\right)^n + MP\right) \\ &-\left(\left(\frac{1}{1+\frac{c}{12}}\right)^2 MP + \left(\frac{1}{1+\frac{c}{12}}\right)^3 MP + \ldots + \left(\frac{1}{1+\frac{c}{12}}\right)^{n+1} MP\right). \end{aligned} \tag{A.2}$$

Obviously, all terms except the first and the last one cancel out each other and it follows that

$$OB(0) - \left(\frac{1}{1+\frac{c}{12}}\right) OB(0) = \left(\frac{1}{1+\frac{c}{12}}\right) MP - \left(\frac{1}{1+\frac{c}{12}}\right)^{n+1} MP \tag{A.3}$$

or equivalently,

$$OB(0) = \frac{\left(1 - \left(1+\frac{c}{12}\right)^{-n}\right)}{\frac{c}{12}} MP. \tag{A.4}$$

If MP is written in terms of $OB(0)$, then

$$MP = \frac{\frac{c}{12}}{\left(1 - \left(\frac{1}{1+\frac{c}{12}}\right)^n\right)} OB(0). \tag{A.5}$$

It is also possible to write Equation A.5 as

$$MP = \frac{\left(\frac{c}{12}\right)\left(1+\frac{c}{12}\right)^n}{\left(1+\frac{c}{12}\right)^n - 1} OB(0) \tag{A.6}$$

A.2 Formula for the Value of the Outstanding Balance

The outstanding balance value immediately after a payment date, $\eta(i)$ can be expressed as following:

$$OB(i) = \left(OB(0) - \frac{1 - \left(1 + \frac{c}{12}\right)^{-i}}{\frac{c}{12}} MP \right) \left(1 + \frac{c}{12}\right)^{i}. \tag{A.7}$$

Then, by substituting MP, it is possible to get

$$OB(i) = \left(OB(0) - \left(\frac{1 - \left(1 + \frac{c}{12}\right)^{-i}}{\frac{c}{12}} \right) \left(\frac{\left(\frac{c}{12}\right)\left(1 + \frac{c}{12}\right)^{n}}{\left(1 + \frac{c}{12}\right)^{n} - 1} OB(0) \right) \right) \left(1 + \frac{c}{12}\right)^{i} \tag{A.8}$$

or equivalently,

$$OB(i) = \left(\frac{OB(0)\left(1 + \frac{c}{12}\right)^{n} - OB(0) - \left(1 + \frac{c}{12}\right)^{n} OB(0) + \left(1 + \frac{c}{12}\right)^{n-i} OB(0)}{\left(1 + \frac{c}{12}\right)^{n} - 1} \right) \left(1 + \frac{c}{12}\right)^{i}. \tag{A.9}$$

The first and third terms cancel out each other and consequently,

$$OB(i) = \frac{\left(\left(1 + \frac{c}{12}\right)^{n} - \left(1 + \frac{c}{12}\right)^{i}\right) OB(0)}{\left(1 + \frac{c}{12}\right)^{n} - 1}. \tag{A.10}$$

Appendix B

Matlab Code for Mortgage Valuation

```
function [H R OldA OldV OldD OldC OldI]=MortgageValuation(InputData)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%'InputData' is the matrix of external parameters in the valuation of      %
%fixed rate repayment mortgage. The rows of the matrix are composed as      %
%following;                                                                 %
%                                                                           %
%       InputData(1) = contract maturity                                    %
%       InputData(2) = time step per month                                  %
%       InputData(3) = long-term mean reversion rate of interest rate       %
%       InputData(4) = long-term mean of interest rate                      %
%       InputData(5) = interest rate volatility                             %
%       InputData(6) = market price of risk                                 %
%       InputData(7) = house service flow                                   %
%       InputData(8) = house price volatility                               %
%       InputData(9) = scale factor for interest rate transformation        %
%       InputData(10)= scale factor for house price transformation          %
%       InputData(11)= correlation coefficient                              %
%       InputData(12)= early termination penalty                            %
%       InputData(13)= loan to value ratio                                  %
%       InputData(14)= coupon rate                                          %
%       InputData(15)= fraction(for insurance)                              %
%       InputData(16)= number of grid points in x dimension                 %
%       InputData(17)= number of grid points in y dimension                 %
%       InputData(18)= arrangement fee                                      %
%                                                                           %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
%%%%%%%%%%%%%%%%%%%%%
%Internal Parameters%
%%%%%%%%%%%%%%%%%%%%%

%In the following part,"cn" is the counter for months, "cs" is the counter
%for time steps, "cx" is the counter for house price dimension and "cy" is
%the counter for interest rate dimension. Moreover "...inc" represents the
%increment for the variable "..."
cnmin=1;
cnmax=InputData(1);
cninc=1;

s=1/(12*InputData(2));          %time increment
csmin=1;
csmax=csmin+InputData(2);
csinc=1;

cxmin=1;
cxmax=cxmin+InputData(16);
cxinc=1;

cymin=1;
cymax=cymin+InputData(17);
cyinc=1;

%"x" is the transformed variable in the house price dimension. So, the grid
%will be linear in x but not in H (house price)
xmin=0;
xmax=1;
xinc=1/InputData(16);
x=xmin:xinc:xmax;

%"y" is the transformed variable in the interest rate dimention. The grid
%is linear in y but not in R (interest rate)
ymin=0;
```

```
ymax=1;
yinc=1/InputData(17);
y=ymin:yinc:ymax;

cxhalf=cxmin+round(InputData(16)/2);
cyhalf=cymin+round(InputData(17)/2);

%%%%%%%%%%%%%%%%%%%%%%%%%%
%Preliminary Calculations%
%%%%%%%%%%%%%%%%%%%%%%%%%%

%The value of house price
H(cxmax)=0;
H(cxmin)=inf;

for cx=cxmin+1:cxmax-1
    H(cx)=(1-x(cx))/(InputData(10)*x(cx));
end

%The value of interest rate
R(cymax)=0;
R(cymin)=inf;

for cy=cymin+1:cymax-1
    R(cy)=(1-y(cy))/(InputData(9)*y(cy));
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Final Terminal Conditions%
%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Valuation of monthly payment
M=(((InputData(14)/12)*(1+(InputData(14)/12))^cnmax)/...
    ((1+(InputData(14)/12))^cnmax-1))*InputData(13);

%Borrower's total debt
TD=M;
```

```
for cy=cymin:cymax
    NewA(cy)=M;
    OldA(cy)=NewA(cy);
end

for cy=cymin:cymax
    for cx=cxmin+1:cxmax
        OldV(cy,cx)=min(H(cx),M);
        OldC(cy,cx)=0;
        OldD(cy,cx)=max(0,M-H(cx));
        OldI(cy,cx)=max(0,min(TD-H(cx),InputData(15)*TD));
    end
        %H is infinite(cx=cxmin)
        OldD(cy,cxmin)=0;
        OldC(cy,cxmin)=0;
        OldV(cy,cxmin)=min(TD,OldA(cy)-OldD(cy,cxmin)-OldC(cy,cxmin));
        OldI(cy,cxmin)=0;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% The Beginning Of The Last Month%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

cn=cnmin;
for cs=csmin:csmax

    %Valuation of outstanding balance
    TermOB1=(1+InputData(14)/12)^cnmax;
    TermOB2=(1+InputData(14)/12)^(cnmax-1);
    TermOB3=((1+InputData(14)/12)^cnmax)-1;
    OB(cn)=((TermOB1-TermOB2)/TermOB3)*InputData(13);

    %Borrower's total debt
    TD=(1+InputData(14)*((csmax-cs)/(InputData(2)*12)))*OB(cn);

    %PDE solution for A(r)
```

```
for cy=cymin+1:cymax-1
    Term1=1-R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
        (y(cy)^4)*(s/(yinc^2))-R(cy)*s;
    Term2=0.5*R(cy)*(InputData(5)^2)*(InputData(9)^2)...
        *(y(cy)^4)*(s/(yinc^2));
    Term3=(R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
        (y(cy)^3)-(InputData(3)*(InputData(4)-R(cy))-...
        InputData(6)*R(cy))*InputData(9)*(y(cy)^2))*(s/yinc);

    EQ1=(R(cy)*(InputData(5)^2)*(InputData(9)^2)*(y(cy)^3)-...
        (InputData(3)*(InputData(4)-R(cy))-InputData(6)*R(cy))*...
        InputData(9)*(y(cy)^2));

    if EQ1<0
       NewA(cy)=Term1*OldA(cy)+Term2*(OldA(cy+1)+OldA(cy-1))+...
           Term3*(OldA(cy)-OldA(cy-1));
    else
       NewA(cy)=Term1*OldA(cy)+Term2*(OldA(cy+1)+OldA(cy-1))+...
           Term3*(OldA(cy+1)-OldA(cy));
    end
end

%Boundary Conditions

%The Value of A(r) at the corners
NewA(cymin)=0;
NewA(cymax)=OldA(cymax-1);

%%%Corners of the grid%%%

%Corner of the grid in which H=0,R=0%
NewV(cymax,cxmax)=OldV(cymax-1,cxmax);
NewC(cymax,cxmax)=OldC(cymax-1,cxmax);
NewD(cymax,cxmax)=OldD(cymax-1,cxmax);
NewI(cymax,cxmax)=OldI(cymax-1,cxmax);
OldV(cymax,cxmax)=NewV(cymax,cxmax);
OldC(cymax,cxmax)=NewC(cymax,cxmax);
```

```
OldD(cymax,cxmax)=NewD(cymax,cxmax);
OldI(cymax,cxmax)=NewI(cymax,cxmax);

%Corner of the grid in which H=inf,R=0%
NewV(cymax,cxmin)=OldV(cymax-1,cxmin);
NewC(cymax,cxmin)=OldC(cymax-1,cxmin);
NewD(cymax,cxmin)=OldD(cymax-1,cxmin);
NewI(cymax,cxmin)=OldI(cymax-1,cxmin);
OldV(cymax,cxmin)=NewV(cymax,cxmin);
OldC(cymax,cxmin)=NewC(cymax,cxmin);
OldD(cymax,cxmin)=NewD(cymax,cxmin);
OldI(cymax,cxmin)=NewI(cymax,cxmin);

%Corner of the grid in which H=inf,R=inf%
NewV(cymin,cxmin)=0;
NewC(cymin,cxmin)=0;
NewD(cymin,cxmin)=0;
NewI(cymin,cxmin)=0;
OldV(cymin,cxmin)=NewV(cymin,cxmin);
OldC(cymin,cxmin)=NewC(cymin,cxmin);
OldD(cymin,cxmin)=NewD(cymin,cxmin);
OldI(cymin,cxmin)=NewI(cymin,cxmin);

%Corner of the grid in which H=0,R=inf%
NewV(cymin,cxmax)=0;
NewC(cymin,cxmax)=0;
NewD(cymin,cxmax)=0;
NewI(cymin,cxmax)=0;
OldV(cymin,cxmax)=NewV(cymin,cxmax);
OldC(cymin,cxmax)=NewC(cymin,cxmax);
OldD(cymin,cxmax)=NewD(cymin,cxmax);
OldI(cymin,cxmax)=NewI(cymin,cxmax);

%%%Edges of the grid%%%

%H=0, R varies%
for cy=cymin+1:cymax-1
```

```
    NewC(cy,cxmax)=0;
    NewD(cy,cxmax)=NewA(cy);
    NewV(cy,cxmax)=0;

    Term4=1-R(cy)*(InputData(5)^2)*(InputData(9)^2)*(y(cy)^4)*...
        (s/(yinc^2))-R(cy)*s;
    Term5=0.5*R(cy)*(InputData(5)^2)*(InputData(9)^2)*(y(cy)^4)*...
        (s/(yinc^2));
    Term6=(R(cy)*(InputData(5)^2)*(InputData(9)^2)*(y(cy)^3)-...
        (InputData(3)*(InputData(4)-R(cy))-InputData(6)*R(cy))*...
        InputData(9)*(y(cy)^2))*(s/yinc);

    EQ2=R(cy)*(InputData(5)^2)*(InputData(9)^2)*(y(cy)^3)-...
        (InputData(3)*(InputData(4)-R(cy))-InputData(6)*R(cy))*...
        InputData(9)*(y(cy)^2);

    if EQ2<0
       NewI(cy,cxmax)=Term4*OldI(cy,cxmax)+Term5*(OldI(cy+1,cxmax)+...
           OldI(cy-1,cxmax))+Term6*(OldI(cy,cxmax)-OldI(cy-1,cxmax));
    else
       NewI(cy,cxmax)=Term4*OldI(cy,cxmax)+Term5*(OldI(cy+1,cxmax)+...
           OldI(cy-1,cxmax))+Term6*(OldI(cy+1,cxmax)-OldI(cy,cxmax));
    end

    OldC(cy,cxmax)=NewC(cy,cxmax);
    OldD(cy,cxmax)=NewD(cy,cxmax);
    OldV(cy,cxmax)=NewV(cy,cxmax);
    OldI(cy,cxmax)=NewI(cy,cxmax);
end

%R=0, H varies%
for cx=cxmin+1:cxmax-1
    Term7=1-(H(cx)^2)*(InputData(8)^2)*(InputData(10)^2)*...
        (x(cx)^4)*(s/(xinc^2));
    Term8=0.5*(H(cx)^2)*(InputData(8)^2)*(InputData(10)^2)*...
        (x(cx)^4)*(s/(xinc^2));
    Term9=((H(cx)^2)*(InputData(8)^2)*(InputData(10)^2)*...
```

```
        (x(cx)^3)+InputData(7)*H(cx)*InputData(10)*(x(cx)^2))*(s/xinc);
    Term10=-InputData(3)*InputData(4)*InputData(9)*(y(cy)^2)*(s/yinc);

    NewV(cymax,cx)=Term7*OldV(cymax,cx)+Term8*(OldV(cymax,cx+1)+...
        OldV(cymax,cx-1))+Term9*(OldV(cymax,cx+1)-OldV(cymax,cx))+...
        Term10*(OldV(cymax,cx)-OldV(cymax-1,cx));
    OldV(cymax,cx)=NewV(cymax,cx);
    NewD(cymax,cx)=Term7*OldD(cymax,cx)+Term8*(OldD(cymax,cx+1)+...
        OldD(cymax,cx-1))+Term9*(OldD(cymax,cx+1)-OldD(cymax,cx))+...
        Term10*(OldD(cymax,cx)-OldD(cymax-1,cx));
    OldD(cymax,cx)=NewD(cymax,cx);
    NewC(cymax,cx)=max(0,NewA(cymax)-OldV(cymax,cx)-OldD(cymax,cx));
    OldC(cymax,cx)=NewC(cymax,cx);
    NewI(cymax,cx)=Term7*OldI(cymax,cx)+Term8*(OldI(cymax,cx+1)+...
        OldI(cymax,cx-1))+Term9*(OldI(cymax,cx+1)-OldI(cymax,cx))+...
        Term10*(OldI(cymax,cx)-OldI(cymax-1,cx));
    OldI(cymax,cx)=NewI(cymax,cx);
end

%H=inf, R varies%
for cy=cymin+1:cymax-1
    Term11=1-R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
        (y(cy)^4)*(s/(yinc^2))-R(cy)*s;
    Term12=0.5*R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
        (y(cy)^4)*(s/(yinc^2));
    Term13=(R(cy)*(InputData(5)^2)*(InputData(9)^2)*(y(cy)^3)-....
        (InputData(3)*(InputData(4)-R(cy))-InputData(6)*R(cy))*...
        InputData(9)*(y(cy)^2))*(s/yinc);

    EQ3=R(cy)*(InputData(5)^2)*(InputData(9)^2)*(y(cy)^3)-...
        (InputData(3)*(InputData(4)-R(cy))-InputData(6)*R(cy))*...
        InputData(9)*(y(cy)^2);

    if EQ3<0
        NewC(cy,cxmin)=Term11*OldC(cy,cxmin)+...
            Term12*(OldC(cy+1,cxmin)+OldC(cy-1,cxmin))+...
            Term13*(OldC(cy,cxmin)-OldC(cy-1,cxmin));
```

```
    else
        NewC(cy,cxmin)=Term11*OldC(cy,cxmin)+...
            Term12*(OldC(cy+1,cxmin)+OldC(cy-1,cxmin))+...
            Term13*(OldC(cy+1,cxmin)-OldC(cy,cxmin));
    end

    OldC(cy,cxmin)=NewC(cy,cxmin);
    NewV(cy,cxmin)=NewA(cy)-OldC(cy,cxmin);
    NewD(cy,cxmin)=0;
    NewI(cy,cxmin)=0;

    OldV(cy,cxmin)=NewV(cy,cxmin);
    OldD(cy,cxmin)=NewD(cy,cxmin);
    OldI(cy,cxmin)=NewI(cy,cxmin);
end

 %R=inf, H varies%
 for cx=cxmin+1:cxmax-1
     NewC(cymin,cx)=0;
     NewD(cymin,cx)=0;
     NewV(cymin,cx)=0;
     NewI(cymin,cx)=0;
     OldC(cymin,cx)=NewC(cymin,cx);
     OldD(cymin,cx)=NewD(cymin,cx);
     OldV(cymin,cx)=NewV(cymin,cx);
     OldI(cymin,cx)=NewI(cymin,cx);
 end

 %The main algorithm to solve the main PDE for V(H,r),D(H,r) and I(H,r)

 %In the following part, 'state' is related with the prepayment region
 %state=1 is used for the boundary of the prepayment region and state=2
 %is used for the interior points of the prepayment region.

 cx_first=cxmin;
 cy_first=cymax;
```

```
for cx=cxmin+1:cxmax-1
    state=1;
    for cy=cymin+1:cymax-1
        Term14=1-(H(cx)^2)*(InputData(8)^2)*(InputData(10)^2)*...
            (x(cx)^4)*(s/(xinc^2))-R(cy)*(InputData(5)^2)*...
            (InputData(9)^2)*(y(cy)^4)*(s/(yinc^2))-R(cy)*s;
        Term15=0.5*(H(cx)^2)*(InputData(8)^2)*(InputData(10)^2)*...
            (x(cx)^4)*(s/(xinc^2));
        Term16=((H(cx)^2)*(InputData(8)^2)*(InputData(10)^2)*...
            (x(cx)^3)-(R(cy)-InputData(7))*H(cx)*InputData(10)*...
            (x(cx)^2))*(s/xinc);
        Term17=0.5*R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
            (y(cy)^4)*(s/(yinc^2));
        Term18=(R(cy)*(InputData(8)^2)*(InputData(9)^2)*...
            (y(cy)^3)-(InputData(3)*(InputData(4)-R(cy))-...
            InputData(6)*R(cy))*InputData(9)*(y(cy)^2))*(s/yinc);
        Term19=InputData(11)*H(cx)*sqrt(R(cy))*InputData(8)*...
            InputData(5)*InputData(9)*InputData(10)*(x(cx)^2)*...
            (y(cy)^2)*(s/(4*xinc*yinc));

        EQ4=(H(cx)^2)*(InputData(8)^2)*(InputData(10)^2)*...
            (x(cx)^3)-(R(cy)-InputData(7))*...
            H(cx)*InputData(10)*(x(cx)^2);
        EQ5=R(cy)*(InputData(5)^2)*(InputData(9)^2)*(y(cy)^3)-...
            (InputData(3)*(InputData(4)-R(cy))-InputData(6)*...
            R(cy))*InputData(9)*(y(cy)^2);

        if     EQ4>=0 && EQ5>=0
               NewV(cy,cx)=Term14*OldV(cy,cx)+...
                   Term15*(OldV(cy,cx+1)+OldV(cy,cx-1))+...
                   Term16*(OldV(cy,cx+1)-OldV(cy,cx))+...
                   Term17*(OldV(cy+1,cx)+OldV(cy-1,cx))+...
                   Term18*(OldV(cy+1,cx)-OldV(cy,cx))+...
                   Term19*(OldV(cy+1,cx+1)-OldV(cy+1,cx-1)-...
                   OldV(cy-1,cx+1)+OldV(cy-1,cx-1));
        elseif EQ4<0 && EQ5>=0
               NewV(cy,cx)=Term14*OldV(cy,cx)+...
```

```
            Term15*(OldV(cy,cx+1)+OldV(cy,cx-1))+...
            Term16*(OldV(cy,cx)-OldV(cy,cx-1))+...
            Term17*(OldV(cy+1,cx)+OldV(cy-1,cx))+...
            Term18*(OldV(cy+1,cx)-OldV(cy,cx))+...
            Term19*(OldV(cy+1,cx+1)-OldV(cy+1,cx-1)-...
            OldV(cy-1,cx+1)+OldV(cy-1,cx-1));
elseif EQ4>=0 && EQ5<0
       NewV(cy,cx)=Term14*OldV(cy,cx)+...
            Term15*(OldV(cy,cx+1)+OldV(cy,cx-1))+...
            Term16*(OldV(cy,cx+1)-OldV(cy,cx))+...
            Term17*(OldV(cy+1,cx)+OldV(cy-1,cx))+...
            Term18*(OldV(cy,cx)-OldV(cy-1,cx))+...
            Term19*(OldV(cy+1,cx+1)-OldV(cy+1,cx-1)-...
            OldV(cy-1,cx+1)+OldV(cy-1,cx-1));
else
       NewV(cy,cx)=Term14*OldV(cy,cx)+...
            Term15*(OldV(cy,cx+1)+OldV(cy,cx-1))+...
            Term16*(OldV(cy,cx)-OldV(cy,cx-1))+...
            Term17*(OldV(cy+1,cx)+OldV(cy-1,cx))+...
            Term18*(OldV(cy,cx)-OldV(cy-1,cx))+...
            Term19*(OldV(cy+1,cx+1)-OldV(cy+1,cx-1)-...
            OldV(cy-1,cx+1)+OldV(cy-1,cx-1));
end

%Borrower's total debt in case of early termination
TD=(1+InputData(12))*(1+InputData(14)*((csmax-cs)/...
    (InputData(2)*12)))*OB(cn);

%Continuation region in terms of prepayment
if NewV(cy,cx)<TD
   OldV(cy,cx)=NewV(cy,cx);
    if      EQ4>=0 && EQ5>=0
            NewD(cy,cx)=Term14*OldD(cy,cx)+...
                Term15*(OldD(cy,cx+1)+OldD(cy,cx-1))+...
                Term16*(OldD(cy,cx+1)-OldD(cy,cx))+...
                Term17*(OldD(cy+1,cx)+OldD(cy-1,cx))+...
                Term18*(OldD(cy+1,cx)-OldD(cy,cx))+...
```

```
                Term19*(OldD(cy+1,cx+1)-OldD(cy+1,cx-1)-...
                OldD(cy-1,cx+1)+OldD(cy-1,cx-1));
            OldD(cy,cx)=NewD(cy,cx);
            NewC(cy,cx)=max(0,NewA(cy)-OldV(cy,cx)-...
                OldD(cy,cx));
            OldC(cy,cx)=NewC(cy,cx);
            NewI(cy,cx)=Term14*OldI(cy,cx)+...
                Term15*(OldI(cy,cx+1)+OldI(cy,cx-1))+...
                Term16*(OldI(cy,cx+1)-OldI(cy,cx))+...
                Term17*(OldI(cy+1,cx)+OldI(cy-1,cx))+...
                Term18*(OldI(cy+1,cx)-OldI(cy,cx))+...
                Term19*(OldI(cy+1,cx+1)-OldI(cy+1,cx-1)-...
                OldI(cy-1,cx+1)+OldI(cy-1,cx-1));
            OldI(cy,cx)=NewI(cy,cx);
        elseif  EQ4<0 && EQ5>=0
            NewD(cy,cx)=Term14*OldD(cy,cx)+...
                Term15*(OldD(cy,cx+1)+OldD(cy,cx-1))+...
                Term16*(OldD(cy,cx)-OldD(cy,cx-1))+...
                Term17*(OldD(cy+1,cx)+OldD(cy-1,cx))+...
                Term18*(OldD(cy+1,cx)-OldD(cy,cx))+...
                Term19*(OldD(cy+1,cx+1)-OldD(cy+1,cx-1)-...
                OldD(cy-1,cx+1)+OldD(cy-1,cx-1));
            OldD(cy,cx)=NewD(cy,cx);
            NewC(cy,cx)=max(0,NewA(cy)-OldV(cy,cx)-...
                OldD(cy,cx));
            OldC(cy,cx)=NewC(cy,cx);
            NewI(cy,cx)=Term14*OldI(cy,cx)+...
                Term15*(OldI(cy,cx+1)+OldI(cy,cx-1))+...
                Term16*(OldI(cy,cx)-OldI(cy,cx-1))+...
                Term17*(OldI(cy+1,cx)+OldI(cy-1,cx))+...
                Term18*(OldI(cy+1,cx)-OldI(cy,cx))+...
                Term19*(OldI(cy+1,cx+1)-OldI(cy+1,cx-1)-...
                OldI(cy-1,cx+1)+OldI(cy-1,cx-1));
            OldI(cy,cx)=NewI(cy,cx);
        elseif  EQ4>=0 && EQ5<0
            NewD(cy,cx)=Term14*OldD(cy,cx)+...
                Term15*(OldD(cy,cx+1)+OldD(cy,cx-1))+...
```

```
            Term16*(OldD(cy,cx+1)-OldD(cy,cx))+...
            Term17*(OldD(cy+1,cx)+OldD(cy-1,cx))+...
            Term18*(OldD(cy,cx)-OldD(cy-1,cx))+...
            Term19*(OldD(cy+1,cx+1)-OldD(cy+1,cx-1)-...
            OldD(cy-1,cx+1)+OldD(cy-1,cx-1));
        OldD(cy,cx)=NewD(cy,cx);
        NewC(cy,cx)=max(0,NewA(cy)-OldV(cy,cx)-...
            OldD(cy,cx));
        OldC(cy,cx)=NewC(cy,cx);
        NewI(cy,cx)=Term14*OldI(cy,cx)+...
            Term15*(OldI(cy,cx+1)+OldI(cy,cx-1))+...
            Term16*(OldI(cy,cx+1)-OldI(cy,cx))+...
            Term17*(OldI(cy+1,cx)+OldI(cy-1,cx))+...
            Term18*(OldI(cy,cx)-OldI(cy-1,cx))+...
            Term19*(OldI(cy+1,cx+1)-OldI(cy+1,cx-1)-...
            OldI(cy-1,cx+1)+OldI(cy-1,cx-1));
        OldI(cy,cx)=NewI(cy,cx);
    else
        NewD(cy,cx)=Term14*OldD(cy,cx)+...
            Term15*(OldD(cy,cx+1)+OldD(cy,cx-1))+...
            Term16*(OldD(cy,cx)-OldD(cy,cx-1))+...
            Term17*(OldD(cy+1,cx)+OldD(cy-1,cx))+...
            Term18*(OldD(cy,cx)-OldD(cy-1,cx))+...
            Term19*(OldD(cy+1,cx+1)-OldD(cy+1,cx-1)-...
            OldD(cy-1,cx+1)+OldD(cy-1,cx-1));
        OldD(cy,cx)=NewD(cy,cx);
        NewC(cy,cx)=max(0,NewA(cy)-OldV(cy,cx)-...
            OldD(cy,cx));
        OldC(cy,cx)=NewC(cy,cx);
        NewI(cy,cx)=Term14*OldI(cy,cx)+...
            Term15*(OldI(cy,cx+1)+OldI(cy,cx-1))+...
            Term16*(OldI(cy,cx)-OldI(cy,cx-1))+...
            Term17*(OldI(cy+1,cx)+OldI(cy-1,cx))+...
            Term18*(OldI(cy,cx)-OldI(cy-1,cx))+...
            Term19*(OldI(cy+1,cx+1)-OldI(cy+1,cx-1)-...
            OldI(cy-1,cx+1)+OldI(cy-1,cx-1));
        OldI(cy,cx)=NewI(cy,cx);
```

```
    end

%Prepayment region
elseif state==1
    Term20=1-(H(cx)^2)*(InputData(8)^2)*(InputData(10)^2)*...
        (x(cx)^4)*(s/(xinc^2))-R(cy)*(InputData(5)^2)*...
        (InputData(9)^2)*(y(cy)^4)*(s/(yinc^2))-R(cy)*s;
    Term21=0.5*(H(cx)^2)*(InputData(8)^2)*...
        (InputData(10)^2)*(x(cx)^4)*(s/(xinc^2));
    Term22=((H(cx)^2)*(InputData(8)^2)*(InputData(10)^2)*...
        (x(cx)^3))*(s/xinc);
    Term23=0.5*R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
        (y(cy)^4)*(s/(yinc^2));
    Term24=R(cy)*(InputData(8)^2)*(InputData(9)^2)*...
        (y(cy)^3)*(s/yinc);
    Term25=InputData(11)*H(cx)*sqrt(R(cy))*InputData(8)*...
        InputData(5)*InputData(9)*InputData(10)*...
        (x(cx)^2)*(y(cy)^2)*(s/(4*xinc*yinc));
    Term26=-(1+InputData(12))*InputData(14)*OB(cn)*s+...
        R(cy)*TD*s;

    NewV(cy,cx)=Term20*OldV(cy,cx)+...
        Term21*(OldV(cy,cx+1)+OldV(cy,cx-1))+...
        Term22*(OldV(cy,cx+1)-OldV(cy,cx))+...
        Term23*(OldV(cy+1,cx)+OldV(cy-1,cx))+...
        Term24*(OldV(cy,cx)-OldV(cy-1,cx))+...
        Term25*(OldV(cy+1,cx+1)-OldV(cy+1,cx-1)-...
        OldV(cy-1,cx+1)+OldV(cy-1,cx-1))+Term26;
    OldV(cy,cx)=NewV(cy,cx);
    NewD(cy,cx)=Term20*OldD(cy,cx)+....
        Term21*(OldD(cy,cx+1)+OldD(cy,cx-1))+...
        Term22*(OldD(cy,cx+1)-OldD(cy,cx))+...
        Term23*(OldD(cy+1,cx)+OldD(cy-1,cx))+...
        Term24*(OldD(cy,cx)-OldD(cy-1,cx))+...
        Term25*(OldD(cy+1,cx+1)-OldD(cy+1,cx-1)-...
        OldD(cy-1,cx+1)+OldD(cy-1,cx-1));
    OldD(cy,cx)=NewD(cy,cx);
```

```
        NewC(cy,cx)=max(0,NewA(cy)-OldV(cy,cx)-OldD(cy,cx));
        OldC(cy,cx)=NewC(cy,cx);
        NewI(cy,cx)=Term20*OldI(cy,cx)+...
            Term21*(OldI(cy,cx+1)+OldI(cy,cx-1))+...
            Term22*(OldI(cy,cx+1)-OldI(cy,cx))+...
            Term23*(OldI(cy+1,cx)+OldI(cy-1,cx))+...
            Term24*(OldI(cy,cx)-OldI(cy-1,cx))+...
            Term25*(OldI(cy+1,cx+1)-OldI(cy+1,cx-1)-...
            OldI(cy-1,cx+1)+OldI(cy-1,cx-1));
        OldI(cy,cx)=NewI(cy,cx);
        state=2;
        cx_first=max(cx,cx_first);
        cy_first=min(cy,cy_first);
    else
        Term20=1-(H(cx)^2)*(InputData(8)^2)*(InputData(10)^2)*...
            (x(cx)^4)*(s/(xinc^2))-R(cy)*(InputData(5)^2)*...
            (InputData(9)^2)*(y(cy)^4)*(s/(yinc^2))-R(cy)*s;
        Term21=0.5*(H(cx)^2)*(InputData(8)^2)*...
            (InputData(10)^2)*(x(cx)^4)*(s/(xinc^2));
        Term22=((H(cx)^2)*(InputData(8)^2)*(InputData(10)^2)*...
            (x(cx)^3))*(s/xinc);
        Term23=0.5*R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
            (y(cy)^4)*(s/(yinc^2));
        Term24=R(cy)*(InputData(8)^2)*(InputData(9)^2)*...
            (y(cy)^3)*(s/yinc);
        Term25=InputData(11)*H(cx)*sqrt(R(cy))*InputData(8)*...
            InputData(5)*InputData(9)*InputData(10)*(x(cx)^2)*...
            (y(cy)^2)*(s/(4*xinc*yinc));
        Term26=-(1+InputData(12))*InputData(14)*OB(cn)*s+...
            R(cy)*TD*s;

        NewV(cy,cx)=Term20*OldV(cy,cx)+...
            Term21*(OldV(cy,cx+1)+OldV(cy,cx-1))+...
            Term22*(OldV(cy,cx+1)-OldV(cy,cx))+...
            Term23*(OldV(cy+1,cx)+OldV(cy-1,cx))+...
            Term24*(OldV(cy,cx)-OldV(cy-1,cx))+...
            Term25*(OldV(cy+1,cx+1)-OldV(cy+1,cx-1)-...
```

```
                OldV(cy-1,cx+1)+OldV(cy-1,cx-1))+Term26;
            OldV(cy,cx)=NewV(cy,cx);
            NewD(cy,cx)=0;
            OldD(cy,cx)=NewD(cy,cx);
            NewC(cy,cx)=max(0,NewA(cy)-OldV(cy,cx)-OldD(cy,cx));
            OldC(cy,cx)=NewC(cy,cx);
            NewI(cy,cx)=0;
            OldI(cy,cx)=NewI(cy,cx);
            state=2;
        end

        OldA(cy)=NewA(cy);
    end
end

if cy_first<=cymax
    for cy=cy_first:cymax-1
        OldV(cy,cxmin)=TD;
        OldC(cy,cxmin)=max(0,NewA(cy)-OldV(cy,cxmin)-OldD(cy,cxmin));
    end
end

if cx_first>cxmin
    for cx=cxmin:cx_first
        OldV(cymax,cx)=TD;
        OldD(cymax,cx)=0;
        OldC(cymax,cx)=max(0,NewA(cymax)-OldV(cymax,cx)-...
            OldD(cymax,cx));
        OldI(cymax,cx)=0;
    end
end
OldA(cymin)=NewA(cymin);
OldA(cymax)=NewA(cymax);
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%The Remaining Months of the Contract%
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for cn=cnmin+1:cnmax

    %Valuation of outstanding balance
    TermOB4=(1+InputData(14)/12)^cnmax;
    TermOB5=(1+InputData(14)/12)^(cnmax-cn);
    TermOB6=((1+InputData(14)/12)^cnmax)-1;
    OB(cn)=((TermOB4-TermOB5)/TermOB6)*InputData(13);

    %Borrower's total debt
    TD=OB(cn-1)+M;

    %Terminal Conditions
    for cy=cymin:cymax
        NewA(cy)=OldA(cy)+M;
        OldA(cy)=NewA(cy);
    end

    for cy=cymin:cymax
        for cx=cxmin+1:cxmax
            % 'VBLP' represents the value of the contract just before
            %the last payment date
            VBLP=OldV(cy,cx)+M;
            NewV(cy,cx)=min(VBLP,H(cx));
            OldV(cy,cx)=NewV(cy,cx);

            %Continuation region in terms of default
            if OldV(cy,cx)==VBLP
               NewD(cy,cx)=OldD(cy,cx);
               NewC(cy,cx)=OldC(cy,cx);
               NewI(cy,cx)=OldI(cy,cx);
               OldD(cy,cx)=NewD(cy,cx);
               OldC(cy,cx)=NewC(cy,cx);
               OldI(cy,cx)=NewI(cy,cx);
            %Default region (V=H)
            else
```

```
            NewC(cy,cx)=0;
            NewD(cy,cx)=OldA(cy)-H(cx);
            NewI(cy,cx)=max(0,min(TD-H(cx),InputData(15)*TD));
            OldD(cy,cx)=NewD(cy,cx);
            OldC(cy,cx)=NewC(cy,cx);
            OldI(cy,cx)=NewI(cy,cx);
        end
    end
    %H is infinite(cx=cxmin)
    NewV(cy,cxmin)=OldV(cy,cxmin)+M;
    NewD(cy,cxmin)=0;
    NewC(cy,cxmin)=OldA(cy)-NewV(cy,cxmin)-NewD(cy,cxmin);
    NewI(cy,cxmin)=0;
    OldV(cy,cxmin)=NewV(cy,cxmin);
    OldD(cy,cxmin)=NewD(cy,cxmin);
    OldC(cy,cxmin)=NewC(cy,cxmin);
    OldI(cy,cxmin)=NewI(cy,cxmin);
end

for cs=csmin:csmax

    %Valuation of outstanding balance
    TermOB4=(1+InputData(14)/12)^cnmax;
    TermOB5=(1+InputData(14)/12)^(cnmax-cn);
    TermOB6=((1+InputData(14)/12)^cnmax)-1;
    OB(cn)=((TermOB4-TermOB5)/TermOB6)*InputData(13);

    %Borrower's total debt
    TD=(1+InputData(14)*((csmax-cs)/(InputData(2)*12)))*OB(cn);

    %PDE solution for A(r)
    for cy=cymin+1:cymax-1
        Term1=1-R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
            (y(cy)^4)*(s/(yinc^2))-R(cy)*s;
        Term2=0.5*R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
            (y(cy)^4)*(s/(yinc^2));
        Term3=(R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
```

```
        (y(cy)^3)-(InputData(3)*(InputData(4)-R(cy))-...
        InputData(6)*R(cy))*InputData(9)*(y(cy)^2))*(s/yinc);

    EQ1=(R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
        (y(cy)^3)-(InputData(3)*(InputData(4)-R(cy))-...
        InputData(6)*R(cy))*InputData(9)*(y(cy)^2));

    if EQ1<0
       NewA(cy)=Term1*OldA(cy)+Term2*(OldA(cy+1)+OldA(cy-1))+...
           Term3*(OldA(cy)-OldA(cy-1));
    else
       NewA(cy)=Term1*OldA(cy)+Term2*(OldA(cy+1)+OldA(cy-1))+...
           Term3*(OldA(cy+1)-OldA(cy));
    end
end

%Boundary Conditions

%The Value of A(r) at the corners
 NewA(cymin)=0;
 NewA(cymax)=OldA(cymax-1);

%%%Corners of the grid%%%

%Corner of the grid in which H=0,R=0%
NewV(cymax,cxmax)=OldV(cymax-1,cxmax);
NewC(cymax,cxmax)=OldC(cymax-1,cxmax);
NewD(cymax,cxmax)=OldD(cymax-1,cxmax);
NewI(cymax,cxmax)=OldI(cymax-1,cxmax);
OldV(cymax,cxmax)=NewV(cymax,cxmax);
OldC(cymax,cxmax)=NewC(cymax,cxmax);
OldD(cymax,cxmax)=NewD(cymax,cxmax);
OldI(cymax,cxmax)=NewI(cymax,cxmax);

%Corner of the grid in which H=inf,R=0%
NewV(cymax,cxmin)=OldV(cymax-1,cxmin);
NewC(cymax,cxmin)=OldC(cymax-1,cxmin);
```

```
NewD(cymax,cxmin)=OldD(cymax-1,cxmin);
NewI(cymax,cxmin)=OldI(cymax-1,cxmin);
OldV(cymax,cxmin)=NewV(cymax,cxmin);
OldC(cymax,cxmin)=NewC(cymax,cxmin);
OldD(cymax,cxmin)=NewD(cymax,cxmin);
OldI(cymax,cxmin)=NewI(cymax,cxmin);

%Corner of the grid in which H=inf,R=inf%
NewV(cymin,cxmin)=0;
NewC(cymin,cxmin)=0;
NewD(cymin,cxmin)=0;
NewI(cymin,cxmin)=0;
OldV(cymin,cxmin)=NewV(cymin,cxmin);
OldC(cymin,cxmin)=NewC(cymin,cxmin);
OldD(cymin,cxmin)=NewD(cymin,cxmin);
OldI(cymin,cxmin)=NewI(cymin,cxmin);

%Corner of the grid in which H=0,R=inf%
NewV(cymin,cxmax)=0;
NewC(cymin,cxmax)=0;
NewD(cymin,cxmax)=0;
NewI(cymin,cxmax)=0;
OldV(cymin,cxmax)=NewV(cymin,cxmax);
OldC(cymin,cxmax)=NewC(cymin,cxmax);
OldD(cymin,cxmax)=NewD(cymin,cxmax);
OldI(cymin,cxmax)=NewI(cymin,cxmax);

%%%Edges of the grid%%%

%H=0, R varies%
for cy=cymin+1:cymax-1
    NewC(cy,cxmax)=0;
    NewD(cy,cxmax)=NewA(cy);
    NewV(cy,cxmax)=0;

    Term4=1-R(cy)*(InputData(5)^2)*(InputData(9)^2)*(y(cy)^4)*...
        (s/(yinc^2))-R(cy)*s;
```

```
    Term5=0.5*R(cy)*(InputData(5)^2)*(InputData(9)^2)*(y(cy)^4)*...
        (s/(yinc^2));
    Term6=(R(cy)*(InputData(5)^2)*(InputData(9)^2)*(y(cy)^3)-...
        (InputData(3)*(InputData(4)-R(cy))-InputData(6)*R(cy))*...
        InputData(9)*(y(cy)^2))*(s/yinc);

    EQ2=R(cy)*(InputData(5)^2)*(InputData(9)^2)*(y(cy)^3)-...
        (InputData(3)*(InputData(4)-R(cy))-InputData(6)*R(cy))*...
        InputData(9)*(y(cy)^2);

    if EQ2<0
       NewI(cy,cxmax)=Term4*OldI(cy,cxmax)+...
           Term5*(OldI(cy+1,cxmax)+OldI(cy-1,cxmax))+...
           Term6*(OldI(cy,cxmax)-OldI(cy-1,cxmax));
    else
       NewI(cy,cxmax)=Term4*OldI(cy,cxmax)+...
           Term5*(OldI(cy+1,cxmax)+OldI(cy-1,cxmax))+...
           Term6*(OldI(cy+1,cxmax)-OldI(cy,cxmax));
    end

    OldC(cy,cxmax)=NewC(cy,cxmax);
    OldD(cy,cxmax)=NewD(cy,cxmax);
    OldV(cy,cxmax)=NewV(cy,cxmax);
    OldI(cy,cxmax)=NewI(cy,cxmax);
end

%R=0, H varies%
for cx=cxmin+1:cxmax-1
    Term7=1-(H(cx)^2)*(InputData(8)^2)*(InputData(10)^2)*...
        (x(cx)^4)*(s/(xinc^2));
    Term8=0.5*(H(cx)^2)*(InputData(8)^2)*(InputData(10)^2)*...
        (x(cx)^4)*(s/(xinc^2));
    Term9=((H(cx)^2)*(InputData(8)^2)*(InputData(10)^2)*...
        (x(cx)^3)+InputData(7)*H(cx)*InputData(10)*...
        (x(cx)^2))*(s/xinc);
    Term10=-InputData(3)*InputData(4)*InputData(9)*...
        (y(cy)^2)*(s/yinc);
```

```
    NewV(cymax,cx)=Term7*OldV(cymax,cx)+...
        Term8*(OldV(cymax,cx+1)+OldV(cymax,cx-1))+...
        Term9*(OldV(cymax,cx+1)-OldV(cymax,cx))+...
        Term10*(OldV(cymax,cx)-OldV(cymax-1,cx));
    OldV(cymax,cx)=NewV(cymax,cx);
    NewD(cymax,cx)=Term7*OldD(cymax,cx)+...
        Term8*(OldD(cymax,cx+1)+OldD(cymax,cx-1))+...
        Term9*(OldD(cymax,cx+1)-OldD(cymax,cx))+...
        Term10*(OldD(cymax,cx)-OldD(cymax-1,cx));
    OldD(cymax,cx)=NewD(cymax,cx);
    NewC(cymax,cx)=max(0,NewA(cymax)-OldV(cymax,cx)-...
        OldD(cymax,cx));
    OldC(cymax,cx)=NewC(cymax,cx);
    NewI(cymax,cx)=Term7*OldI(cymax,cx)+...
        Term8*(OldI(cymax,cx+1)+OldI(cymax,cx-1))+...
        Term9*(OldI(cymax,cx+1)-OldI(cymax,cx))+...
        Term10*(OldI(cymax,cx)-OldI(cymax-1,cx));
    OldI(cymax,cx)=NewI(cymax,cx);
end

%H=inf, R varies%
for cy=cymin+1:cymax-1
    Term11=1-R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
        (y(cy)^4)*(s/(yinc^2))-R(cy)*s;
    Term12=0.5*R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
        (y(cy)^4)*(s/(yinc^2));
    Term13=(R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
        (y(cy)^3)-(InputData(3)*(InputData(4)-R(cy))-...
        InputData(6)*R(cy))*InputData(9)*(y(cy)^2))*(s/yinc);

    EQ3=R(cy)*(InputData(5)^2)*(InputData(9)^2)*(y(cy)^3)-...
        (InputData(3)*(InputData(4)-R(cy))-InputData(6)*R(cy))*...
        InputData(9)*(y(cy)^2);

    if EQ3<0
       NewC(cy,cxmin)=Term11*OldC(cy,cxmin)+...
```

```
           Term12*(OldC(cy+1,cxmin)+OldC(cy-1,cxmin))+...
           Term13*(OldC(cy,cxmin)-OldC(cy-1,cxmin));
    else
       NewC(cy,cxmin)=Term11*OldC(cy,cxmin)+...
           Term12*(OldC(cy+1,cxmin)+OldC(cy-1,cxmin))+...
           Term13*(OldC(cy+1,cxmin)-OldC(cy,cxmin));
    end

    OldC(cy,cxmin)=NewC(cy,cxmin);
    NewV(cy,cxmin)=NewA(cy)-OldC(cy,cxmin);
    NewD(cy,cxmin)=0;
    NewI(cy,cxmin)=0;

    OldV(cy,cxmin)=NewV(cy,cxmin);
    OldD(cy,cxmin)=NewD(cy,cxmin);
    OldI(cy,cxmin)=NewI(cy,cxmin);
end

%R=inf, H varies%
for cx=cxmin+1:cxmax-1
    NewC(cymin,cx)=0;
    NewD(cymin,cx)=0;
    NewV(cymin,cx)=0;
    NewI(cymin,cx)=0;
    OldC(cymin,cx)=NewC(cymin,cx);
    OldD(cymin,cx)=NewD(cymin,cx);
    OldV(cymin,cx)=NewV(cymin,cx);
    OldI(cymin,cx)=NewI(cymin,cx);
end

%The main algorithm to solve the main PDE for V(H,r),D(H,r)
%and I(H,r)

%In the following part, 'state' is related with the prepayment
%region state=1 is used for the boundary of the prepayment region
%and state=2 is used for the interior points of the prepayment
%region.
```

```
cx_first=cxmin;
cy_first=cymax;

for cx=cxmin+1:cxmax-1
    state=1;
    for cy=cymin+1:cymax-1
        Term14=1-(H(cx)^2)*(InputData(8)^2)*(InputData(10)^2)*...
            (x(cx)^4)*(s/(xinc^2))-R(cy)*(InputData(5)^2)*...
            (InputData(9)^2)*(y(cy)^4)*(s/(yinc^2))-R(cy)*s;
        Term15=0.5*(H(cx)^2)*(InputData(8)^2)*(InputData(10)^2)*...
            (x(cx)^4)*(s/(xinc^2));
        Term16=((H(cx)^2)*(InputData(8)^2)*(InputData(10)^2)*...
            (x(cx)^3)-(R(cy)-InputData(7))*H(cx)*InputData(10)*...
            (x(cx)^2))*(s/xinc);
        Term17=0.5*R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
            (y(cy)^4)*(s/(yinc^2));
        Term18=(R(cy)*(InputData(8)^2)*(InputData(9)^2)*...
            (y(cy)^3)-(InputData(3)*(InputData(4)-R(cy))-...
            InputData(6)*R(cy))*InputData(9)*(y(cy)^2))*(s/yinc);
        Term19=InputData(11)*H(cx)*sqrt(R(cy))*InputData(8)*...
            InputData(5)*InputData(9)*InputData(10)*(x(cx)^2)*...
            (y(cy)^2)*(s/(4*xinc*yinc));

        EQ4=(H(cx)^2)*(InputData(8)^2)*(InputData(10)^2)*...
            (x(cx)^3)-(R(cy)-InputData(7))*H(cx)*...
            InputData(10)*(x(cx)^2);
        EQ5=R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
            (y(cy)^3)-(InputData(3)*(InputData(4)-R(cy))-...
            InputData(6)*R(cy))*InputData(9)*(y(cy)^2);

        if     EQ4>=0 && EQ5>=0
               NewV(cy,cx)=Term14*OldV(cy,cx)+...
                   Term15*(OldV(cy,cx+1)+OldV(cy,cx-1))+...
                   Term16*(OldV(cy,cx+1)-OldV(cy,cx))+...
                   Term17*(OldV(cy+1,cx)+OldV(cy-1,cx))+...
                   Term18*(OldV(cy+1,cx)-OldV(cy,cx))+...
```

```
            Term19*(OldV(cy+1,cx+1)-OldV(cy+1,cx-1)-...
            OldV(cy-1,cx+1)+OldV(cy-1,cx-1));
elseif EQ4<0 && EQ5>=0
        NewV(cy,cx)=Term14*OldV(cy,cx)+...
            Term15*(OldV(cy,cx+1)+OldV(cy,cx-1))+...
            Term16*(OldV(cy,cx)-OldV(cy,cx-1))+...
            Term17*(OldV(cy+1,cx)+OldV(cy-1,cx))+...
            Term18*(OldV(cy+1,cx)-OldV(cy,cx))+...
            Term19*(OldV(cy+1,cx+1)-OldV(cy+1,cx-1)-...
            OldV(cy-1,cx+1)+OldV(cy-1,cx-1));
elseif EQ4>=0 && EQ5<0
        NewV(cy,cx)=Term14*OldV(cy,cx)+...
            Term15*(OldV(cy,cx+1)+OldV(cy,cx-1))+...
            Term16*(OldV(cy,cx+1)-OldV(cy,cx))+...
            Term17*(OldV(cy+1,cx)+OldV(cy-1,cx))+...
            Term18*(OldV(cy,cx)-OldV(cy-1,cx))+...
            Term19*(OldV(cy+1,cx+1)-OldV(cy+1,cx-1)-...
            OldV(cy-1,cx+1)+OldV(cy-1,cx-1));
else
        NewV(cy,cx)=Term14*OldV(cy,cx)+...
            Term15*(OldV(cy,cx+1)+OldV(cy,cx-1))+...
            Term16*(OldV(cy,cx)-OldV(cy,cx-1))+...
            Term17*(OldV(cy+1,cx)+OldV(cy-1,cx))+...
            Term18*(OldV(cy,cx)-OldV(cy-1,cx))+...
            Term19*(OldV(cy+1,cx+1)-OldV(cy+1,cx-1)-...
            OldV(cy-1,cx+1)+OldV(cy-1,cx-1));
end

%Borrower's total debt in case of early termination
TD=(1+InputData(12))*(1+InputData(14)*((csmax-cs)/...
    (InputData(2)*12)))*OB(cn);

%Continuation region in terms of prepayment
if NewV(cy,cx)<TD
   OldV(cy,cx)=NewV(cy,cx);
   if      EQ4>=0 && EQ5>=0
           NewD(cy,cx)=Term14*OldD(cy,cx)+...
```

```
            Term15*(OldD(cy,cx+1)+OldD(cy,cx-1))+...
            Term16*(OldD(cy,cx+1)-OldD(cy,cx))+...
            Term17*(OldD(cy+1,cx)+OldD(cy-1,cx))+...
            Term18*(OldD(cy+1,cx)-OldD(cy,cx))+...
            Term19*(OldD(cy+1,cx+1)-OldD(cy+1,cx-1)-...
            OldD(cy-1,cx+1)+OldD(cy-1,cx-1));
        OldD(cy,cx)=NewD(cy,cx);
        NewC(cy,cx)=max(0,NewA(cy)-OldV(cy,cx)-...
            OldD(cy,cx));
        OldC(cy,cx)=NewC(cy,cx);
        NewI(cy,cx)=Term14*OldI(cy,cx)+...
            Term15*(OldI(cy,cx+1)+OldI(cy,cx-1))+...
            Term16*(OldI(cy,cx+1)-OldI(cy,cx))+...
            Term17*(OldI(cy+1,cx)+OldI(cy-1,cx))+...
            Term18*(OldI(cy+1,cx)-OldI(cy,cx))+...
            Term19*(OldI(cy+1,cx+1)-OldI(cy+1,cx-1)-...
            OldI(cy-1,cx+1)+OldI(cy-1,cx-1));
        OldI(cy,cx)=NewI(cy,cx);
elseif  EQ4<0 && EQ5>=0
        NewD(cy,cx)=Term14*OldD(cy,cx)+...
            Term15*(OldD(cy,cx+1)+OldD(cy,cx-1))+...
            Term16*(OldD(cy,cx)-OldD(cy,cx-1))+...
            Term17*(OldD(cy+1,cx)+OldD(cy-1,cx))+...
            Term18*(OldD(cy+1,cx)-OldD(cy,cx))+...
            Term19*(OldD(cy+1,cx+1)-OldD(cy+1,cx-1)-...
            OldD(cy-1,cx+1)+OldD(cy-1,cx-1));
        OldD(cy,cx)=NewD(cy,cx);
        NewC(cy,cx)=max(0,NewA(cy)-OldV(cy,cx)-...
            OldD(cy,cx));
        OldC(cy,cx)=NewC(cy,cx);
        NewI(cy,cx)=Term14*OldI(cy,cx)+...
            Term15*(OldI(cy,cx+1)+OldI(cy,cx-1))+...
            Term16*(OldI(cy,cx)-OldI(cy,cx-1))+...
            Term17*(OldI(cy+1,cx)+OldI(cy-1,cx))+...
            Term18*(OldI(cy+1,cx)-OldI(cy,cx))+...
            Term19*(OldI(cy+1,cx+1)-OldI(cy+1,cx-1)-...
            OldI(cy-1,cx+1)+OldI(cy-1,cx-1));
```

```
        OldI(cy,cx)=NewI(cy,cx);
elseif  EQ4>=0 && EQ5<0
        NewD(cy,cx)=Term14*OldD(cy,cx)+...
            Term15*(OldD(cy,cx+1)+OldD(cy,cx-1))+...
            Term16*(OldD(cy,cx+1)-OldD(cy,cx))+...
            Term17*(OldD(cy+1,cx)+OldD(cy-1,cx))+...
            Term18*(OldD(cy,cx)-OldD(cy-1,cx))+...
            Term19*(OldD(cy+1,cx+1)-OldD(cy+1,cx-1)-...
            OldD(cy-1,cx+1)+OldD(cy-1,cx-1));
        OldD(cy,cx)=NewD(cy,cx);
        NewC(cy,cx)=max(0,NewA(cy)-OldV(cy,cx)-...
            OldD(cy,cx));
        OldC(cy,cx)=NewC(cy,cx);
        NewI(cy,cx)=Term14*OldI(cy,cx)+...
            Term15*(OldI(cy,cx+1)+OldI(cy,cx-1))+...
            Term16*(OldI(cy,cx+1)-OldI(cy,cx))+...
            Term17*(OldI(cy+1,cx)+OldI(cy-1,cx))+...
            Term18*(OldI(cy,cx)-OldI(cy-1,cx))+...
            Term19*(OldI(cy+1,cx+1)-OldI(cy+1,cx-1)-...
            OldI(cy-1,cx+1)+OldI(cy-1,cx-1));
        OldI(cy,cx)=NewI(cy,cx);
else
        NewD(cy,cx)=Term14*OldD(cy,cx)+...
            Term15*(OldD(cy,cx+1)+OldD(cy,cx-1))+...
            Term16*(OldD(cy,cx)-OldD(cy,cx-1))+...
            Term17*(OldD(cy+1,cx)+OldD(cy-1,cx))+...
            Term18*(OldD(cy,cx)-OldD(cy-1,cx))+...
            Term19*(OldD(cy+1,cx+1)-OldD(cy+1,cx-1)-...
            OldD(cy-1,cx+1)+OldD(cy-1,cx-1));
        OldD(cy,cx)=NewD(cy,cx);
        NewC(cy,cx)=max(0,NewA(cy)-OldV(cy,cx)-...
        OldD(cy,cx));
        OldC(cy,cx)=NewC(cy,cx);
        NewI(cy,cx)=Term14*OldI(cy,cx)+...
            Term15*(OldI(cy,cx+1)+OldI(cy,cx-1))+...
            Term16*(OldI(cy,cx)-OldI(cy,cx-1))+...
            Term17*(OldI(cy+1,cx)+OldI(cy-1,cx))+...
```

```
            Term18*(OldI(cy,cx)-OldI(cy-1,cx))+...
            Term19*(OldI(cy+1,cx+1)-OldI(cy+1,cx-1)-...
            OldI(cy-1,cx+1)+OldI(cy-1,cx-1));
        OldI(cy,cx)=NewI(cy,cx);
  end

%Prepayment region
elseif state==1
    Term20=1-(H(cx)^2)*(InputData(8)^2)*...
        (InputData(10)^2)*(x(cx)^4)*(s/(xinc^2))-...
        R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
        (y(cy)^4)*(s/(yinc^2))-R(cy)*s;
    Term21=0.5*(H(cx)^2)*(InputData(8)^2)*...
        (InputData(10)^2)*(x(cx)^4)*(s/(xinc^2));
    Term22=((H(cx)^2)*(InputData(8)^2)*...
        (InputData(10)^2)*(x(cx)^3))*(s/xinc);
    Term23=0.5*R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
        (y(cy)^4)*(s/(yinc^2));
    Term24=R(cy)*(InputData(8)^2)*(InputData(9)^2)*...
        (y(cy)^3)*(s/yinc);
    Term25=InputData(11)*H(cx)*sqrt(R(cy))*...
        InputData(8)*InputData(5)*InputData(9)*...
        InputData(10)*(x(cx)^2)*(y(cy)^2)*...
        (s/(4*xinc*yinc));
    Term26=-(1+InputData(12))*InputData(14)*OB(cn)*s+...
        R(cy)*TD*s;

    NewV(cy,cx)=Term20*OldV(cy,cx)+...
        Term21*(OldV(cy,cx+1)+OldV(cy,cx-1))+...
        Term22*(OldV(cy,cx+1)-OldV(cy,cx))+...
        Term23*(OldV(cy+1,cx)+OldV(cy-1,cx))+...
        Term24*(OldV(cy,cx)-OldV(cy-1,cx))+...
        Term25*(OldV(cy+1,cx+1)-OldV(cy+1,cx-1)-...
        OldV(cy-1,cx+1)+OldV(cy-1,cx-1))+Term26;
    OldV(cy,cx)=NewV(cy,cx);
    NewD(cy,cx)=Term20*OldD(cy,cx)+...
        Term21*(OldD(cy,cx+1)+OldD(cy,cx-1))+...
```

```
        Term22*(OldD(cy,cx+1)-OldD(cy,cx))+...
        Term23*(OldD(cy+1,cx)+OldD(cy-1,cx))+...
        Term24*(OldD(cy,cx)-OldD(cy-1,cx))+...
        Term25*(OldD(cy+1,cx+1)-OldD(cy+1,cx-1)-...
        OldD(cy-1,cx+1)+OldD(cy-1,cx-1));
    OldD(cy,cx)=NewD(cy,cx);
    NewC(cy,cx)=max(0,NewA(cy)-OldV(cy,cx)-OldD(cy,cx));
    OldC(cy,cx)=NewC(cy,cx);
    NewI(cy,cx)=Term20*OldI(cy,cx)+...
        Term21*(OldI(cy,cx+1)+OldI(cy,cx-1))+...
        Term22*(OldI(cy,cx+1)-OldI(cy,cx))+...
        Term23*(OldI(cy+1,cx)+OldI(cy-1,cx))+...
        Term24*(OldI(cy,cx)-OldI(cy-1,cx))+...
        Term25*(OldI(cy+1,cx+1)-OldI(cy+1,cx-1)-...
        OldI(cy-1,cx+1)+OldI(cy-1,cx-1));
    OldI(cy,cx)=NewI(cy,cx);
    state=2;
    cx_first=max(cx,cx_first);
    cy_first=min(cy,cy_first);
else
    Term20=1-(H(cx)^2)*(InputData(8)^2)*...
        (InputData(10)^2)*(x(cx)^4)*(s/(xinc^2))-...
        R(cy)*(InputData(5)^2)*(InputData(9)^2)*...
        (y(cy)^4)*(s/(yinc^2))-R(cy)*s;
    Term21=0.5*(H(cx)^2)*(InputData(8)^2)*...
        (InputData(10)^2)*(x(cx)^4)*(s/(xinc^2));
    Term22=((H(cx)^2)*(InputData(8)^2)*...
        (InputData(10)^2)*(x(cx)^3))*(s/xinc);
    Term23=0.5*R(cy)*(InputData(5)^2)*...
        (InputData(9)^2)*(y(cy)^4)*(s/(yinc^2));
    Term24=R(cy)*(InputData(8)^2)*(InputData(9)^2)*...
        (y(cy)^3)*(s/yinc);
    Term25=InputData(11)*H(cx)*sqrt(R(cy))*...
        InputData(8)*InputData(5)*InputData(9)*...
        InputData(10)*(x(cx)^2)*(y(cy)^2)*...
        (s/(4*xinc*yinc));
    Term26=-(1+InputData(12))*InputData(14)*OB(cn)*s+...
```

```
                R(cy)*TD*s;

            NewV(cy,cx)=Term20*OldV(cy,cx)+...
                Term21*(OldV(cy,cx+1)+OldV(cy,cx-1))+...
                Term22*(OldV(cy,cx+1)-OldV(cy,cx))+...
                Term23*(OldV(cy+1,cx)+OldV(cy-1,cx))+...
                Term24*(OldV(cy,cx)-OldV(cy-1,cx))+...
                Term25*(OldV(cy+1,cx+1)-OldV(cy+1,cx-1)-...
                OldV(cy-1,cx+1)+OldV(cy-1,cx-1))+Term26;
            OldV(cy,cx)=NewV(cy,cx);
            NewD(cy,cx)=0;
            OldD(cy,cx)=NewD(cy,cx);
            NewC(cy,cx)=max(0,NewA(cy)-OldV(cy,cx)-OldD(cy,cx));
            OldC(cy,cx)=NewC(cy,cx);
            NewI(cy,cx)=0;
            OldI(cy,cx)=NewI(cy,cx);
            state=2;
        end

        OldA(cy)=NewA(cy);
    end
end

if cy_first<=cymax
   for cy=cy_first:cymax-1
       OldV(cy,cxmin)=TD;
       OldC(cy,cxmin)=max(0,NewA(cy)-OldV(cy,cxmin)-...
           OldD(cy,cxmin));
   end
end

if cx_first>cxmin
   for cx=cxmin:cx_first
       OldV(cymax,cx)=TD;
       OldD(cymax,cx)=0;
       OldC(cymax,cx)=max(0,NewA(cymax)-OldV(cymax,cx)-...
           OldD(cymax,cx));
```

```
            OldI(cymax,cx)=0;
        end
    end
    OldA(cymin)=NewA(cymin);
    OldA(cymax)=NewA(cymax);
  end
end
```