# EXPERIMENTAL INVESTIGATION ON SHARP CRESTED RECTANGULAR WEIRS

#### A THESIS SUBMITTED TO THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES OF MIDDLE EAST TECHNICAL UNIVERSITY

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE IN CIVIL ENGINEERING

AUGUST 2009

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#### ABSTRACT

## EXPERIMENTAL INVESTIGATION ON SHARP CRESTED RECTANGULAR WEIRS

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Sharp crested rectangular weirs used for discharge measurement purposes in open channel hydraulics are investigated experimentally. A series of experiments were conducted by measuring discharge and head over the weir for different weir heights for full width weir. It is seen that after a certain weir height, head and discharge relation does not change. Hence a constant weir height is determined. For that height; discharge and head over the weir are measured for variable weir width, starting from the full width weir to slit weir. Description of the discharge coefficient valid for the full range of weir widths and an empirical expression involving dimensionless flow variables is aimed. Experimental data obtained for this purpose and the results of the regression analysis performed are represented.

**Key Words:** Flow measurement, Sharp crested weir, Rectangular weir, Open channel flow

## DİKDÖRTGEN KESİTLİ KESKİN KENARLI SAVAKLAR ÜZERİNE DENEYSEL BİR ARAŞTIRMA

Şişman, H. Çiğdem Yüksek Lisans, İnşaat Mühendisliği Bölümü Tez Yöneticisi : Doç. Dr. A. Burcu Altan-Sakarya Ortak Tez Yöneticisi : Doç. Dr. İsmail Aydın

Açık kanal hidroliğinde debi ölçümü amacıyla kullanılan dikdörtgen kesitli, keskin kenarlı savaklar deneysel olarak incelenmiştir. Öncelikle tam açıklıkta çeşitli savak yüksekliklerinde deneyler yapılmış olup, bu deneylerde debi ve savak üstü su yükü ölçülmüştür. Bu deneyler sonucunda belirli bir savak yüksekliğinden sonra savak üstü su yükü ve debi ilişkisinde bir değişiklik olmadığı gözlenmiştir. Böylece sabit bir savak yüksekliği belirlenmiştir. Daha sonra belirlenen sabit savak yüksekliğinde tam açıklıklı savağa kadar değişken savak genişliği için debi ve savak üstü su derinliği ölçülmüştür. Böylece tüm savak genişliği için debi ve savak üstü su derinliği ölçülmüştür. Böylece tüm savak genişlikleri için geçerli olabilecek bir debi katsayısının tanımlanması ve boyutsuz akım parametreleri ile ilişkilendirilerek ampirik bir denklem ile ifade edilmesi amaçlanmıştır. Bu kapsamda elde edilen deneysel veriler ve uygulanan regresyon analizi sonuçları sunulmuştur.

Anahtar Kelimeler: Akım ölçümleri, Keskin kenarlı savak, Dikdörtgen kesitli savak, Açık kanal akımı

To My Family

#### ACKNOWLEDGMENTS

The author wishes to thank her supervisor Assoc. Prof. Dr. A. Burcu ALTAN-SAKARYA and co-supervisor Assoc. Prof. Dr. İsmail AYDIN for their guidance, advice and support throughout the research.

The author would also like express her deepest gratitude to her father Necmettin Şişman, mother Yasemin Şişman, her sister F. Didem Şişman and her grandmother Fidan Timurkaynak and also her uncles Hamza Şişman, Necati Şişman and her cousins A. Filiz Şişman, T. Çağrı Şişman and also Ali Emre Mutlu and her friends for their love, understanding, trust and support throughout her whole life.

Finally the author would also like to thank laboratory technicians; Turgut Ural, Cengiz Tufaner and Hüseyin Gündoğdu for their support and guidance throughout all experimental works.

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#### LIST OF SYMBOLS

- $A_T$ : Area of Tank
- b: Width of the Weir
- $b_c$ : Width of the Weir (Kindsvater and Carter (1957))
- b<sub>e</sub>: Effective Width of the Weir
- B: Width of the Channel
- B<sub>1</sub>: Width of the Channel (Kindsvater and Carter (1957))
- C: Calibration Constant
- $C_d$ : Discharge Coefficient
- C<sub>e</sub>: Effective Discharge Coefficient
- g: Gravitational Acceleration
- $\gamma$  : Specific Weight of Fluid
- h : Head on the Weir
- $h_e$ : Effective Head on the Weir
- $h_T$ : Water Height in the Tank
- $h_1$ : Head on the Weir (Kindsvater and Carter (1957))
- $H_1$ : Total Head at Section 1
- H<sub>2</sub>: Total Head at Section 1
- K<sub>b</sub>: Quantity Represents the Effect of Viscosity and Surface Tension
- $K_h$ : Quantity Represents the Effect of Viscosity and Surface Tension
- $\ell$ : Characteristic Length
- $\mu$  : Dynamic Viscosity of Fluid
- $\nu$ : Kinematic Viscosity of Fluid

- P: Weir Height
- P<sub>s1</sub>: Pressure at Section 1
- P<sub>s2</sub>: Pressure at Section 2
- P<sub>1</sub>: Weir Height (Kindsvater and Carter (1957))
- Q : Discharge
- R : Reynolds Number
- $\rho$  : Density of Fluid
- $\sigma$  : Surface Tension
- t : Time
- u : Average Velocity
- $u_T$ : Average Velocity in the Tank
- $u_1$ : Average Velocity at Section 1
- u<sub>2</sub>: Average Velocity at Section 2
- V : Characteristic Velocity
- W: Weber Number
- $z_1$ : Elevation at Section 1
- $z_2$ : Elevation at Section 2

## **CHAPTER 1**

#### INTRODUCTION

Measurement of discharge in open channels is one of the main concerns in hydraulic engineering. Sharp crested weirs (also called thin-plate weirs or notches) are used to measure discharge in open channels by using the principle of rapidly varied flow. They are extensively used in laboratories, industries, irrigation practice and also used as dam instrumentation device. Thus accurate flow measurement is very important.

In recent years, many researchers made studies in order to measure discharge over the weirs exactly. Some of these studies are experimental whereas some of them are theoretical. These studies may be categorized upon the type of the weir and limitations of the research. Section 1.1 gives brief information about recent studies.

Wide range of data is studied in present study experimentally. Initially, experiments are conducted for different weir heights in order to determine a constant weir height where head and discharge relation does not change. Then different weir openings are investigated from slit weir to full width weir. In Section 1.2 summary and scope of the present study are mentioned.

#### 1.1. Literature Survey

For many years sharp crested rectangular weirs have been investigated by many researchers. The common objective of these studies is to investigate the flow behaviour of weirs and to obtain a discharge coefficient which describes the real behaviour. Some of them are explained below briefly.

In 1929 **Rehbock** performed experiments with small discharges and concluded with a discharge coefficient equation of full width sharp crested weir (Franzini and Finnemore, 1997). Rehbock showed that discharge coefficient

depends on water height on the weir (*h*) and the ratio of the water head to the weir height (h/P). The details of this study are explained in Section 3.1 below.

**Kindsvater and Carter** made an extensive empirical investigation in 1957 (Bos, 1989). They introduced a number of discharge coefficient equations as a function of the water head on the weir over the weir height (h/P) and weir width over the channel width (b/B). This investigation is given in Section 3.2, since it is used to compare the present study.

**Kandaswamy and Rouse** (1957) obtained discharge coefficients on the basis of experimental results. The results of their study based on three different ranges, such that  $h/P \le 5$ , 5 < h/P < 15 and  $h/P \ge 15$ .

**Ramamurthy et al.** (1987) conducted experiments with a weir range of 0 < h/P < 10 and sill range of  $10 \le P/h \le \infty$ . Using momentum principle and experimental results, a relationship between discharge coefficient and parameter h/P (or P/h for sills) is obtained. And also velocity and pressure distributions in the region of nappe and on the weir face are investigated.

**Swamee** (1988) proposed a generalized weir equation for sharpcrested, narrow-crested, broad-crested and long-crested weirs by combining the equations obtained from previous works. The discharge coefficient equation, suggested by Swamee, depends on geometric characteristics of the weir, such as weir height, head on the weir and crest width.

**Aydin et al.** (2002) introduced the term slit weir which is suitable for measuring small discharges. At the end of their study they found a discharge coefficient equation in terms of Reynolds number. And in 2006 they improved the term slit weir and concluded with a discharge equation depending on Reynolds number and dimensionless number h/b. These two issues are drawn out below, in Section 3.3.

**Ramamurthy et al.** (2007) made an experimental investigation on "multislit weir" in order to extend the slit weir concept and measure not only very low discharge rates but also very high discharge rates accurately. They used three different multislit weir units (n=3,7 and 15) and weir opening of 5 mm. And they concluded that discharge coefficient depends on Reynolds number. But for large values of Reynolds number "inertial forces are high and viscous forces are negligible" which means that  $C_d$  does not depend on Reynolds number. They also showed that multislit weir can be used to measure wide range of discharge rates.

#### 1.2. Scope of the Study

In the present study, rectangular sharp crested weirs are investigated experimentally. Several experiments have been conducted with rectangular sharp crested weirs in laboratory. First of all, water surface profile investigation is made in order to determine the appropriate location for water head readings. Then, different weir heights of full width sharp crested rectangular weirs are investigated. Thus a constant weir height that is free from bottom boundary effect, is determined. Finally, the experimental study of different weir openings is made by keeping the weir height constant. Types of sharp crested rectangular weirs which are investigated vary from slit weir to full width weir.

In Chapter 2, the theoretical aspect of the subject is clarified. In Chapter 3 the earlier studies are presented and the ones that are used to compare with the present study are explained in detail. In Chapter 4, the present experimental setup and procedure are explained. The results of experimental study and comparison with previous studies are given in Chapter 5. Finally, in Chapter 6 conclusions of the deliberation are drawn out.

#### **CHAPTER 2**

## THEORETICAL CONSIDERATION

#### 2.1. Definition

The sharp crested weirs are vertical obstructions placed normal to the flow direction; hence water passes over the weir. The downstream edge of weir should be inclined at an angle of 60° or 45° a s can be seen from Figure 2.1. According to Chow (1959), the sharp crested weir is not only a measuring device for open-channel flow but also the simplest form of over-flow spillway. That is, the profile of the spillway was determined in conformity with the shape of the flow nappe over a sharp-crested weir.



Figure 2.1 The Cross-section Details of Sharp Crested Weirs

Figure 2.2 is a photograph of front view of the present experimental setup. The parameters which are used to describe the weir are; *B* is the channel width, *b* is the weir width, *P* is the weir height and *h* is the water head above the weir that is measured 3h - 4h away from the upstream of the weir.



Figure 2.2. The Parameters of Sharp Crested Rectangular Weirs

The weirs are categorized by the weir opening (*b*), such that fully contracted, partially contracted and full width weirs. (Bos, 1989)

a. Fully contracted weirs have a weir width (*b*) of smaller than channel width (*B*), so that the weir is apart from the bed and side effects of the flow.

b. Full width weirs have a weir width which extends to the channel width (b/B = 1.0). In literature this weir is frequently referred to as a rectangular suppressed weir or Rehbock weir.

c. Partially contracted weir is a type of contracted weir which is affected by the bed and walls slightly. The sharp crested weirs can be classified into three groups according to the geometry of weir: a) the rectangular weir, b) the V or triangular weir and c) special weirs, such as trapezoidal, circular or parabolic weirs (Figure 2.3) (Coxon (1959) and Henderson (1966)).



Figure 2.3. Types of Sharp Crested Weirs

In order to get exact results, the discharge should be measured when the nappe of weir is aerated (Franzini and Finnemore (1997) and Subramanya (1986)). That is the pressure for the upper and below nappe of fluid should be atmospheric. In Figure 2.4 and Figure 2.5 the difference between aerated and non-aerated nappe can be seen clearly, respectively. Figure 2.4 represents fully aerated flow. The upper and lower nappe of fluid can be seen clearly. Figure 2.5 is a type of non-aerated flow. The lower nappe of fluid clings to the weir, since the pressure of lower nappe is below the atmospheric pressure.

Experiments have been conducted by considering the aerated flow on the lower nappe. It is observed that for water height below 2 cm generally, water clings to the lower nappe which means non-aerated flow occurs. Thus, water head on the weir below 2 cm is not taken into consideration.



Figure 2.4 Aerated Nappe



Figure 2.5

Non-aerated Nappe

## 2.2. Discharge Equation

The theoretical discharge,  $Q_{ideal}$  for a rectangular sharp crested weir is found by assuming frictionless, parallel and horizontal flow with no loss (Henderson (1966) and Munson et al. (2002)). In Figure 2.6 schematic view of the flow over the weir is given.



Figure 2.6 Schematic View of Flow Over Weir

Bernoulli equation for flow along an arbitrary streamline A-B can be written as :

$$H_1 = H_2 \tag{2.1}$$

$$\frac{P_{s1}}{\gamma} + Z_1 + \frac{u_1^2}{2g} = \frac{P_{s2}}{\gamma} + Z_2 + \frac{u_2^2}{2g}$$
(2.2)

As can be seen from Figure 2.6,  $\frac{P_{s1}}{\gamma} + z_1 = y_1$ , where  $y_1$ =upstream flow depth. The pressure over the weir is atmospheric such that  $\frac{P_{s2}}{\gamma} = 0$ . Thus the Eqn. (2.2) can be written as:

$$y_1 + \frac{u_1^2}{2g} - z_2 = \frac{u_2^2}{2g}$$
(2.3)

Then the velocity on the weir equals to;

$$u_{2} = \sqrt{2g\left(y_{1} + \frac{u_{1}^{2}}{2g} - z_{2}\right)}$$
(2.4)

The discharge through an infinitesimal area element of depth  $\delta_z$  for a weir width *b* is shown below:

$$\delta Q_{ideal} = u_2 \cdot b \cdot \delta z \tag{2.5}$$

Introducing equation (2.4) into equation (2.5) and integrating  $Q_{\textit{ideal}}$  over z :

$$Q_{ideal} = \int_{0}^{h} \sqrt{2g\left(y_{1} + \frac{u_{1}^{2}}{2g} - z_{2}\right)} \cdot b \cdot \delta z$$
(2.6)

$$Q_{ideal} = \frac{2}{3} \cdot \sqrt{2g} \cdot b \cdot \left[ \left( h + \frac{u_1^2}{2g} \right)^{1.5} - \left( \frac{u_1^2}{2g} \right)^{1.5} \right]$$
(2.7)

The velocity head at section 1 can be assumed negligible. Hence the equation of theoretical discharge is expressed as :

$$Q_{ideal} = \frac{2}{3}\sqrt{2g}bh^{3/2}$$
 (2.8)

But the actual discharge, which depends on many parameters such as viscosity, surface tension, geometry of weir and so on, is given below.

$$Q_{actual} = C_d \frac{2}{3} \sqrt{2g} b h^{3/2}$$
(2.9)

where  $C_{d}$ =discharge coefficient which accounts for the accuracy of discharge.

#### 2.3. Dimensional Analysis

The discharge passing over the weir is a function of several parameters (Figure 2.2), which is mathematically expressed by equation (2.10).

$$Q = f_1(h, b, B, P, \rho, \mu, g, \sigma)$$
 (2.10)

where *h*=head over the weir crest *b*=weir width *B*=channel width *P*=height of weir  $\rho$ =density of fluid  $\mu$ =dynamic viscosity of fluid *g*=gravitational acceleration  $\sigma$ =surface tension A dimensional analysis is performed to find a relation between the discharge coefficient and other parameters stated above. Below a mathematical expression of this relation is given.

$$\frac{Q}{g^{1/2}bh^{3/2}} = f_2\left(R, W, \frac{h}{b}, \frac{b}{B}, \frac{h}{P}\right)$$
(2.11)

Since the discharge equation can be expressed such that :

$$Q = C_d \frac{2}{3} \sqrt{2g} b h^{3/2}$$
 (2.12)

Thus the discharge coefficient equals to :

$$C_d = \frac{Q}{2/3(2g)^{1/2} bh^{3/2}}$$
(2.13)

Finally Eqn. (2.11) can be written as Eqn. (2.14). As can be seen, the discharge coefficient depends on Reynolds number, Weber number and geometry of weir and channel.

$$C_{d} = f_{3}\left(R, W, \frac{h}{b}, \frac{b}{B}, \frac{h}{P}\right)$$
(2.14)



The general definition of Reynolds number and Weber number are given in Eqn. (2.15) and Eqn. (2.16) respectively In most fluid mechanics problems, by means of dimensionless numbers, there will be a characteristic velocity, length and fluid property such as viscosity and density.. For special conditions, different characteristic length and velocity definitions can be used. In this study two different Reynolds number definitions are used and details of this topic is explained in following sections.

$$\mathbf{R} = \mathbf{V}\ell/\boldsymbol{\upsilon} \tag{2.15}$$

$$W = \frac{V^2 \ell \rho}{\sigma} = \frac{2ghb\rho}{\sigma}$$
(2.16)

## **CHAPTER 3**

## LITERATURE REVIEW

For sharp crested rectangular weirs, measuring discharge accurately is very important. And since the discharge, which is needed to be found, depends on many parameters such as viscosity, surface tension and geometry, it is difficult to calculate the exact value of discharge. So many researches have been made to find an accurate equation of discharge coefficient. Some of these researches are explained in Section 1.1.

In this chapter details of some of the previous studies, which are further used in order to compare the present study, are explained. The conclusions, limitations of the studies and suggested equations are drawn out. The details of studies, which are considered in this section, are listed below;

- Rehbock (1929)
- Kindsvater and Carter (1957)
- Aydın et al. (2002)
- Aydın et al (2006).

The comparison of previous works with the present study and results of the comparison are explained in Chapter 5 Results and Discussion. The graphs are illustrated in order to make the subject clearer. And also in Chapter 5, the percent difference between present study and previous studies study is given.

#### 3.1. Study of Rehbock (1929)

Rehbock (1929) made experiments of full width sharp crested weirs. And at the end of experimental works he concluded with a discharge coefficient equation which depends on water head on the weir (h) and weir height (P). The empirical equations of discharge and discharge coefficient are given in equations (3.1) and (3.2), respectively.

$$Q = C_d \frac{2}{3} \sqrt{2g} b h^{3/2}$$
 (3.1)

$$C_d = 0.611 + 0.08 \frac{h}{P} + \frac{1}{1000} h \tag{3.2}$$

Rehbock's formula has been found to be accurate within 0.5% for values of *P* from 0.33 to 3.3 ft (0.1 to 1.0 m) and for values of *h* from 0.08 to 2 ft (0.025 to 0.60 m) with the ratio h/P not greater than 1.0 (Franzini & Finnemore, 1997). The limitations of Rehbock's study are also listed in Table 3.1 below.

 Table 3.1
 Limitations of Rehbock's Experimental Study

0.10 m ≤	Ρ	<	1.00 m
0.025 m ≤	h	<	0.60 m
	h/P	<	1.00

#### 3.2. Study of Kindsvater and Carter (1957)

In 1957 Kindsvater and Carter made an extensive study about sharp crested rectangular weirs. They introduced a parameter of  $C_e$  (effective discharge coefficient) which is free from the surface tension and viscosity effects due to contraction of water at the weir (Eqn (3.4), (3.5) and (3.6)). The details of this study including results and limitations are explained in this section.

In order to be consistent with equations and graphs, the symbols, Kindsvater and Carter used, are kept in their original form. The channel width *B* is shown as  $B_1$ , weir width *b* is shown as  $b_c$ , weir height *P* is  $P_1$  and water head on the weir *h* is  $h_1$  in this section.

$$Q = C_e \frac{2}{3} \sqrt{2g} b_e h_e^{3/2}$$
(3.3)

$$C_{\rm e} = 0.602 + 0.075 h_1 / P_1$$
 (Full Width Weirs) (3.4)

$$b_e = b_c + K_b \tag{3.5}$$

$$h_e = h_1 + K_h \tag{3.6}$$

where  $h_e$ =effective water height on weir  $b_e$ =effective weir width  $C_e$ =effective discharge coefficient

The quantities  $K_b$  and  $K_h$  represent the combined effects of the several phenomena attributed to viscosity and surface tension (Eqn (3.5) and (3.6)). The constant positive value for  $K_h$ =0.001 m is recommended for all values of the ratios of  $b_c/B_1$  and  $h_1/P_1$ . Empirically defined values for  $K_b$  as a function of the ratio  $b_c/B_1$  are given in Figure 3.1.

Equation (3.3) is actually a different form of actual discharge equation of sharp crested weir (Eqn. (2.9)). The difference of equation (3.3) from equation (2.9) is that weir width and water head are presented as a function of surface tension and viscous effects, they are not variables of discharge coefficient function.



Figure 3.1 The value of  $K_b$  with respect to  $b_c/B_1$  (Bos, 1989)

The effective discharge coefficient depends on  $b_c/B_1$  ratio and  $h_1/P_1$  ratio, which is listed in Table 3.2 and the graph of this relation, can be seen in Figure 3.2.

b₀/B₁	C <sub>e</sub>
1.00	0.602 + 0.075 <i>h</i> <sub>1</sub> / <i>P</i> <sub>1</sub>
0.90	$0.599 + 0.064 h_1/P_1$
0.80	0.597 + 0.045 <i>h</i> <sub>1</sub> / <i>P</i> <sub>1</sub>
0.70	$0.595 + 0.030 h_1/P_1$
0.60	0.593 + 0.018 <i>h</i> <sub>1</sub> / <i>P</i> <sub>1</sub>
0.50	0.592 + 0.011 <i>h</i> <sub>1</sub> / <i>P</i> <sub>1</sub>
0.40	0.591 + 0.0058 <i>h</i> <sub>1</sub> / <i>P</i> <sub>1</sub>
0.30	$0.590 + 0.0020 h_1/P_1$
0.20	0.589 - 0.0018 <i>h</i> <sub>1</sub> / <i>P</i> <sub>1</sub>
0.10	0.588 - 0.0021 <i>h</i> <sub>1</sub> / <i>P</i> <sub>1</sub>
0.00	0.587 - 0.0023 <i>h</i> <sub>1</sub> / <i>P</i> <sub>1</sub>

Table 3.2Effective Discharge Coefficient as a Function of  $b_c/B_1$ and  $h_1/P_1$ 

The slope of effective discharge coefficient ( $C_e$ ) curve is negative for  $b_c/B_1 < 0.3$  and positive for  $0.3 \le \frac{b_c}{B_1} \le 1.0$ . The  $C_e$  value changes from 0.78 to 0.57.



Figure 3.2 Graph of  $C_e$  versus  $h_1/P_1$  for values of  $b_c/B_1$ (Bos, 1989)

The limits of application for partially contracted and full width weirs are listed below (Bos, 1989):

- i. The recommended minimum head over the weir is 0.03 m.
- ii. The upper limit for  $h_1/P_1$  is 2.0 where the minimum  $P_1$  should be 0.10 m.
- iii. The width of weir should be greater than 0.15 m.
- iv. To avoid non-aerated flow tailwater level should be at least 0.05 m below crest level.

And finally the limitations of fully contracted sharp crested weir are given in Table 3.3.

B <sub>1</sub> -b <sub>c</sub>	≥	4h₁		
$h_1/P_1$	≤	0.5		
h₁/b <sub>c</sub>	≤	0.5		
0.07 m	≤	h <sub>1</sub>	<	0.60 m
b <sub>c</sub>	≥	0.30 m		
P <sub>1</sub>	≥	0.30 m		

Table 3.3Limitations of a fully contracted sharp crestedrectangular weir (Bos, 1989)

#### 3.3. Concept of Slit Weir

In 2002 Aydın et al. introduced the term "slit weir". This type of weir is a narrow rectangular sharp crested weir, efficient to measure small discharges accurately. At the end of the study, they found an empirical equation, which depends on Reynolds number (Eqn (3.7)). The ranges of data are listed below:

- *b* (m) = 0.005, 0.01, 0.015, 0.02, 0.03, 0.04, 0.05, 0.075
- P(m) = 0.04, 0.08, 0.16
- $Q(m^3/s) = 0.00003 0.005$

As a result the discharge coefficient is:

$$C_d = 0.562 + 11.354 / R^{0.5} \tag{3.7}$$

where

$$R = Q/h\upsilon \tag{3.8}$$

The root mean square error in predicting discharge using Eqn. (3.7) is calculated as 0.0096 by Aydın et. al (2002). And also 80 % of the data is within the ±1 % of the value predicted by Eqn. (3.7).

The graph of discharge coefficient versus Reynolds number for experimental data and the data obtained by substituting the measured data in Eqn. (3.7) is shown in Figure 3.4.



Figure 3.4 Graph of  $C_d$  versus R with both data points and equation line (Aydın et al., 2002)

In 2006 Aydın et al. improved their study and at the end of their experimental study, an expression of discharge coefficient is given (Eqns. (3.9) and (3.10)).

- b (m) = 0.005, 0.0075, 0.010, 0.015, 0.020, 0.030, 0.040, 0.050, 0.060,0.075
- *P*(m) = 0.04, 0.08, 0.16
- $Q(m^3/s) = 0.00001 0.00421$

As a result the discharge coefficient is

$$C_{d} = 0.562 + \frac{10\left\{1 - \exp\left[-\left(2h/b\right)^{2}\right]\right\}^{-1}}{R^{0.45}}$$
(3.9)

For 
$$h/b > 2$$
:  
 $C_d = 0.562 + 10/R^{0.45}$ 
(3.10)

where

$$R = \sqrt{(2gh)b/v} \tag{3.11}$$

For h/b>2, the experimental data is grouped around one curve, but for smaller h/b values,  $C_d$  changes for different weir widths. Therefore Eqn. (3.9) will be simplified for h/b>2 and Eqn. (3.10) will be used for this range. The relative error for 89 % of the total experimental data is calculated as  $\pm 2$  % by Aydın et. al (2006).



Figure 3.5 Graph of  $C_d$  versus R for different weir openings (*b*) with both data points and equation lines (Aydın et al., 2006)

## **CHAPTER 4**

# **EXPERIMENTAL STUDIES**

## 4.1. The Experimental Setup

The experimental setup includes 6.0 m long 32 cm width of fiberglass rectangular channel, an entrance structure and a tank as can be seen from the figure below (Figure 4.1 and Figure 4.2).





Front View of Experimental Setup


Figure 4.2 Side View of Experimental Setup

The discharge is controlled by a valve at the entrance of channel (Figure 4.3). The water enters the channel through a 20 cm diameter of vertical pipe, then passes through screens which regulate the flow and reduce the surface waves (Figures 4.4 and 4.5).



Figure 4.3

Valve and Entrance Structure



dimension are in cm

Figure 4.4 The Schematic Plan View of Setup



Figure 4.5 The Schematic Profile View of Setup

After an entrance structure the water passes through a fiberglass rectangular channel, which has a channel height of 45 cm. The point gauge is located 1.20 m before the weir in order to get rid of drawdown effects. In literature, the effective measurement point is considered as 3- 4*h* away from the weir location (Subramanya (1986) and Franzini and Finnemore (1997)). Considering the maximum *h* for present study, which is smaller than 28 cm, the point gauge location should be  $4 \times 0.28 = 1.12m$ . So the selected point gauge location is appropriate for this study. Point gauge accuracy is 0.1 mm along the centerline of the approach channel (Figure 4.6).









Two portative pieces of side plates are located above the weir. They are used to have contracted weirs for different weir openings (Figure 4.8). After water passes the weir, it drops into a  $1x1x1 \text{ m}^3$  of a tank where the discharge can be measured volumetrically. The details of this measurement are explained in Section 4.2.



portative side plates

Figure 4.8 Side Plates of Weir for Different Weir Openings

## 4.2. Pressure Transducer, Amplifier and Calibration

The discharge measurements are made by a pressure transducer which measures the pressure change in  $1x1 \text{ m}^2$  tank after the weir (Figures 4.9 and 4.10). The pressure transducer (Figure 4.11) transforms the pressure change data to voltage and then amplifier transmits the voltage change with respect to time graph to the computer. The graph transferred to the computer can be seen from Figure 4.12; the x-axis of the graph is duration and y-axis is voltage. The slope of this graph represents the pressure change with respect to time which can be accepted as average velocity in the tank ( $u_T$ ). In order to

obtain the exact discharge, a calibration is made by measuring the discharge in the tank with a piezometric tube and stop watch. After that constant C is obtained (Eqn. 4.1). Finally the discharge can be calculated by dividing the slope of pressure change graph by constant C. As a result, discharge can be obtained by equation (4.2) shown below.

$$\frac{dh_{\tau}}{dt} = \frac{d(P/\gamma)}{dt} = y/x = slope = u_{\tau} \times C$$
(4.1)

Pressure change

$$Q = \frac{\frac{dh_{\tau}}{dt}}{C} \times A_{\tau}$$
(4.2)

where  $A_{T}$ = area of tank  $h_{T}$ = water depth in the tank

The variables y and x in Eqn. (4.1) are the symbols of linear equation of data (Series 1) in Figure 4.12. The variable y represents for the y–axis of graph (voltage) and variable x represents for the x–axis of the graph (time). As explained before, the slope of linear equation in Figure 4.12 is simply shown as "y/x".



Figure 4.9 The Amplifier and Computer



Figure 4.10

Amplifier



Figure 4.11 Pressure Transducer



Figure 4.12 Sample of Graph Obtained From Electronic Device

## 4.3. Water Surface Profile

In order to decide the point gauge location, a water surface profile study was made for different discharge values. At the section locations shown in Figure 4.13, water depth is recorded for different discharges. And a graph is used to illustrate water surface in channel (Figure 4.14). The x-axis of the graph represents the distance from the weir and y—axis represents the water height from the bottom of the channel. As can be seen water surface is almost stationary after 1.00 m from the weir. So as stated in Section 2.1, to be on the safe side, the point gauge is located 1.20 m upstream from the weir. The width of the sharp crested weir is 32 cm (full width). Finally the water depth readings for corresponding sets of different discharge are listed in Table 4.1.







Figure 4.14

Water Surface for Different Discharges

Table 4.1	Water Depth for Different Discharge Conditions
-----------	--

		Q <sub>1</sub>	Q <sub>2</sub>	Q <sub>3</sub>	$Q_4$	Q <sub>5</sub>	$Q_6$	Q <sub>7</sub>	Q <sub>8</sub>
Point	Distance From Weir (cm)	Water Depth From the Bottom of Channel (cm)							
1	270	20.55	19.50	18.30	17.30	16.00	14.60	13.30	10.50
2	220	20.55	19.40	18.30	17.30	15.90	14.50	13.20	10.50
3	170	20.50	19.40	18.30	17.30	15.90	14.50	13.20	10.50
4	140	20.50	19.40	18.30	17.30	15.90	14.50	13.20	10.50
5	110	20.50	19.40	18.30	17.30	15.90	14.50	13.20	10.50
6	80	20.45	19.40	18.30	17.30	15.90	14.50	13.20	10.50
7	60	20.45	19.30	18.20	17.30	15.90	14.50	13.10	10.40
8	40	20.45	19.30	18.20	17.20	15.80	14.40	13.10	10.40
9	30	20.40	19.30	18.20	17.20	15.80	14.40	13.10	10.40
10	20	20.35	19.30	18.15	17.10	15.80	14.40	13.10	10.30
11	10	20.25	19.20	18.10	17.00	15.80	14.40	13.10	10.30

# **CHAPTER 5**

# **RESULTS AND DISCUSSION**

#### 5.1. Introduction

In this chapter, the results of experiments and comparison of the results with previous works are discussed in detail.

First of all, in section 5.1.1 experiments for full width sharp crested weir of different weir height are discussed. The weir height that is free from bottom boundary effects is chosen to be the constant weir height for the rest of the study. Then after determining a fixed weir height, experiments are continued for different weir openings from full width to slit weir. By changing weir width, the flow characteristics are observed and a discharge equation is tried to be found. The details of the second part of the experimental work are drawn out in sections 5.1.2, 5.2 and 5.3.

#### 5.1.1. Experimental Works for Different Weir Heights

The selection of constant weir height is accomplished after making several experimental studies with different weir heights. Experiments are carried out for 5 different weir heights; 2, 4, 6, 8 and 10 cm for full width openings. In Figure 5.1; P10, P8, P6, P4 and P2 correspond to weir heights of 10, 8, 6, 4 and 2 cm, respectively. And each symbol represents the different data group. As can be seen from Figure 5.1, after P = 6 cm, there seems no change in the variations of Q with *h*, compared to 8 and 10 cm. However, 2 and 4 cm of weir heights differ from each and all other. Hence, it can be concluded that for the range of the experiments that were carried out, the selection of the weir height to be 10 cm will make the effect of bottom boundary diminish. So, the discharge coefficient,  $C_d$  will become independent of *h*/*P*.



Figure 5.1 Relationship Between Discharge (Q) and Head on the Weir (*h*) for All Collected Data of Different Weir Heights

ω

#### 5.1.2. Experimental Works for Different Weir Openings

Experiments are continued after determining a constant weir height of 10 cm. Different weir openings are investigated hydraulically and the results of this study are explained below.

If two figures (Figure 5.2 and Figure 5.3) are compared regarding the data points, Figure 5.2 shows that as the discharge increases for a specified weir width, the water head over the weir obtained are not reliable due to fluctuation in the channel. For each weir width (*b*), the outlier data differ from each other. In other words; each weir width has its own measurable spectrum, which gives more reliable results. In Figure 5.3 and throughout the whole calculations the outlier data points are not considered. The data considered are shown in Figure 5.3 and the range of the experimental data is listed in Table 5.1.

The weir width ranges were selected in order to cover a spectrum from slit weir to full width weir. Weir height is constant throughout the whole experiments, which is 10 cm as can be seen from Table 5.1. Discharge is changing from 0.00023 m<sup>3</sup>/s (0.23 lt/s) to 0.05204 m<sup>3</sup>/s (52.04 lt/s) which covers a wide spectrum. Water head on the weir is not below 2 cm in order to get rid of aeration problem and get reliable discharge measurements. In addition to measured variables, calculated values such as *h/b*, *h/P*, *b/B* are also shown in Table 5.1.



Figure 5.2 Relationship Between Discharge (Q) and Head on the Weir (*h*) for All Collected Data

ω



Figure 5.3 Relationship Between Discharge (Q) and Head on the Weir (h) for Collected Data Excluding Outliers

Table 5.1	Measured and Calculated Ranges of Experimental Process	
-----------	--	--

b (m)	P (m)	Q <sub>min</sub> (m³/s)	Q <sub>max</sub> (m³/s)	h <sub>min</sub> (m)	h <sub>max</sub> (m)	h/b <sub>min</sub>	h/b <sub>max</sub>	h/P <sub>min</sub>	h/P <sub>max</sub>	b/B
0.02	0.10	0.00023	0.00578	0.0317	0.2796	1.59	13.98	0.32	2.80	0.0625
0.04	0.10	0.00025	0.01071	0.0217	0.2732	0.54	6.83	0.22	2.73	0.1250
0.06	0.10	0.00043	0.01624	0.0229	0.2772	0.38	4.62	0.23	2.77	0.1875
0.08	0.10	0.00067	0.02145	0.0262	0.2760	0.33	3.45	0.26	2.76	0.2500
0.10	0.10	0.00071	0.02687	0.0233	0.2757	0.23	2.76	0.23	2.76	0.3125
0.12	0.10	0.00081	0.03130	0.0217	0.2695	0.18	2.25	0.22	2.70	0.3750
0.14	0.10	0.00086	0.03780	0.0208	0.2739	0.15	1.96	0.21	2.74	0.4375
0.16	0.10	0.00134	0.04348	0.0256	0.2730	0.16	1.71	0.26	2.73	0.5000
0.18	0.10	0.00146	0.04418	0.0253	0.2515	0.14	1.40	0.25	2.52	0.5625
0.20	0.10	0.00165	0.04673	0.0256	0.2421	0.13	1.21	0.26	2.42	0.6250
0.22	0.10	0.00140	0.04963	0.0209	0.2323	0.09	1.06	0.21	2.32	0.6875
0.24	0.10	0.00129	0.04871	0.0179	0.2140	0.07	0.89	0.18	2.14	0.7500
0.26	0.10	0.00187	0.05204	0.0224	0.2078	0.09	0.80	0.22	2.08	0.8125
0.28	0.10	0.00153	0.05074	0.0181	0.1897	0.06	0.68	0.18	1.90	0.8750
0.30	0.10	0.00211	0.05069	0.0220	0.1810	0.07	0.60	0.22	1.81	0.9375
0.32	0.10	0.00223	0.05183	0.0214	0.1721	0.07	0.54	0.21	1.72	1.0000

The relationship between  $C_d$  and h/b is illustrated in Figure 5.4 for the entire set of collected data. For each set of experiments, the characteristics of flow are similar up to width opening (*b*) of 8 cm. For weir opening s of 2, 4, 6 and 8 cm; each data set has a decreasing tendency. When weir width exceeds 8 cm, the data tends to increase systematically for each *b*. Thus it is suitable to divide the whole data and consider them into two groups such as slit and contracted weir. Throughout all measurements, the weir opening (*b*) equals to 2 cm, 4 cm, 6 cm and 8 cm are considered as slit weir

In order to explain the difference between slit weir and contracted weir clearly, Figure (5.5) is given below. It is obvious that b=10 cm has a different tendency than the other weir openings. It is because, for slit weir, weir openings are smaller compared to the water head on the weir, thus water head (*h*) plays an important role. And also for slit weir, since the weir opening is so small, side effects are negligible. On the other hand side effects should be considered for contracted and full width weirs.

As a conclusion the investigation of sharp crested weirs is continued in two parts in present study. Below the detailed information about the study is given for slit weir and contracted weir separately.



Figure 5.4  $C_d$  versus h/b for All Width Openings

4



Figure 5.5 Difference between Slit and Contracted Weir

4

## 5.2. Slit Weir

A sharp crested weir having a weir width between 2 cm and 8 cm is considered as a slit weir. It should be mentioned that 8 cm of weir width in a 32 cm wide channel corresponds to 1/4 of the channel width. It can be concluded that a weir of  $b \le B/4$  is considered as slit weir. Below the present slit weir data is shown in Figure 5.6 and Figure 5.7. Figure 5.6 represents the relationship between  $C_d$  and h/b and Figure 5.7 represents the relationship between  $C_d$  and  $R_{slit}$ .

The Reynolds number considered for slit weir is given in Eqn (3.11). For slit weirs, the characteristic velocity is accepted as Torichelli velocity which is  $\sqrt{2gh}$  and weir width (*b*) is used for length parameter.

$$R_{\rm slit} = \frac{\sqrt{(2gh)b}}{\nu} \tag{3.11}$$









Figure 5.7  $C_d$  versus  $R_{slit}$  for Slit Weir

In Figure 5.8 the relationship between discharge coefficient and Weber number (*W*) is shown. Weber number equation is given in equation (5.1) below. The characteristic velocity is accepted as Torichelli velocity which is  $\sqrt{2gh}$ , and representative length is weir width (*b*).

$$W = \frac{\left(\sqrt{2gh}\right)^2 b\rho}{\sigma} = \frac{2ghb\rho}{\sigma}$$
(5.1)



Figure 5.8 C<sub>a</sub>

## $C_d$ versus W for Slit Weir

#### 5.2.1. Comparison with Kindsvater and Carter (1957)

Slit weir data obtained from present work are compared with the equation proposed by Kindsvater and Carter (1957). The comparison is illustrated in Figure 5.9. The data points represent the data obtained by experimental study and lines represent the Kindsvater and Carter's equation (Eqn (3.3)). The limits of Eqn. (3.3), that is explained in Section 3.2, are not taken into consideration, since the limits of present study for slit weir does not match the limits of Kindsvater and Carter's study. Nevertheless, the difference between present study and Eqn. (3.3) is negligible, as explained below.



Figure 5.9 Comparison of Slit Weir Data with Kindsvater and Carter (1957)

As can be seen from Figure 5.10, the difference between measured discharge and calculated discharge is not significant. The error with respect to measured discharge is calculated by Eqn. (5.2). The overall percent error is around 2 % and 73.38 % of all slit data have error smaller than 2%. Moreover, 96.77 % of data have error smaller than 3 %.

% Error=
$$\frac{Q_{exp} - Q_{calc}}{Q_{exp}} \times 100$$
 (5.2)

where  $Q_{exp}$  = discharge measured by experimental works  $Q_{calc}$  = discharge calculated by Kindsvater and Carter formula (Eqn 3.3)



Figure 5.10 Percent Error with respect to Experimental Discharge

#### 5.2.2. Comparison with Aydın et al. (2006)

Slit weir data obtained from present work are compared with the data obtained in 2006 (Aydın et al.). The discharge coefficient equation used to compare the data is given by Eqn. (3.9). This equation is also explained in Section 3.4. The results of this study are shown in Figure 5.11.

In Figure 5.11, b2, b4, b6 and b8 represent the present data points for weir openings of 2 cm, 4 cm, 6 cm and 8 cm, respectively. And also bb2, bb4 and bb6 represent the data points for weir openings of 2 cm, 4 cm, 6 cm, respectively, obtained by Aydın et al. (2006). Finally Eqn.(3.9)-b2, Eqn.(3.9)-b4, Eqn.(3.9)-b6, Eqn.(3.9)-b8 represents the equation obtained by using Eqn. (3.9) for weir openings of 2 cm, 4 cm, 6 cm and 8 cm, respectively.





Comparison of New Experimental Data, Old Experimental Data (Aydın et al. (2006)) and Slit Weir Equation (Eqn. (3.9))

Different from previous study made in 2006 by Aydın et al., the study is enhanced and measured discharge values become greater than the previous study. Hence, greater Reynolds numbers are obtained in the present study. Considering the relation between  $C_d$  and Reynolds number; the old data for *b* is 2 cm, the maximum Reynolds number is approximately 45000; for *b* is 4 cm, it is 70000 and for *b* is 6 cm maximum Reynolds number is around 90000. On the other hand, for the present study Reynolds number approaches to 140000 for *b* equals to 6 cm. Thus different behavior of  $C_d$  as a function of *R* is observed. It is seen that as *R* gets larger, for different weir widths, the discharge coefficient approaches to different values asymptotically rather than one asymptotic value as obtained before by Aydın et al. (2006). So an improved study is needed for rectangular slit weirs. The percent error is calculated by Eqn. (5.2),  $Q_{calc}$  is the discharge calculated by Eqn (3.9)and  $Q_{exp}$  is the present measured discharge by experimental works. As can be seen from the Figure 5.12, the error with respect to measured discharge is acceptable since the maximum error is around 8 %.

The overall percent error is between 4 % to -3 %, disregarding few data points. And also 62.25 % of all slit data have error smaller than 2 %, whereas 92.31 % of all slit data have error smaller than 3 %. Besides that, the study made by Aydın et al. (2006) has relative error within 2 % for 89 % of the total data.



Figure 5.12 Percent Error with respect to Experimental Discharge and Eqn. (3.6)

## 5.3. Contracted Weir

The sharp crested weirs having widths greater or equal to 10 cm are considered as contracted weirs as explained in Section 5.1.

For contracted weirs the definition of Reynolds number is revised and a new form of Reynolds number is used. In the definition of Reynolds number (R) for contracted weirs, the square root of flow area is accepted as a characteristic length for contracted weir. Because for this type of weirs, both weir width and water head are important.

$$R_{\text{contracted}} = \frac{\sqrt{(2gh)}\sqrt{(bh)}}{v}$$
(5.3)

#### 5.3.1. Comparison with Rehbock (1929)

As stated, Rehbock made a study for full width sharp crested weirs. Therefore the comparison is made only for full width weir data obtained. Below the present data and Rehbock study is compared (Fig. 5.13).





The percent error given in Figure 5.16 is calculated by Eqn. (5.2),  $Q_{calc}$  is the discharge calculated by Eqn (3.1)and  $Q_{exp}$  is the present measured discharge by experimental works. As can be seen from the Figure 5.14, the relative error is between +18 % and – 2 %. In addition to this, 44.12 % of all contracted data have error smaller than 5%. Moreover, 88.24 % of data have error smaller than 10 %.



Figure 5.14 Percent Error with respect to Experimental Discharge and Eqn. (3.1)

#### 5.3.2. Comparison with Kindsvater and Carter (1957)

The comparison of Kindsvater and Carter's study with the present contracted and full width weir is given in this section.

The equations for each weir opening that Kindsvater and Carter have suggested, is drawn as lines and experimental data is represented as points, in Figure 5.15. For large weir openings such as full width, Kindsvater and Carter equation deviates from experimental data. But for weir openings smaller than 24 cm, two studies gets close to each other. The limits of Eqn. (3.3) are not considered, however, as explained below the difference between present study of contracted weir data and Eqn. (3.3) is acceptable.



Figure 5.15 Comparison of Contracted Weir Data with Eqn. (3.3)

The percent error is calculated by Eqn. (5.2),  $Q_{calc}$  is the discharge calculated by Eqn (3.3) and  $Q_{exp}$  is the present measured discharge by experimental works. The relative percent error is between -1 % and 13 %, and 81.27 % of all contracted data have error smaller than 4 %, whereas 92.41 % of all contracted data have error smaller than 6 %.





## 5.4. Present Study

For contracted and full width weirs a different approach is used and a discharge equation is obtained without a discharge coefficient but with a number of different coefficients. Regression analysis is performed for contracted and full width weirs. During regression analysis, the discharge (Q) over the weir area ( $b \cdot h$ ) as a function of h/b is considered as the objective function and three best fit variables ( $c_1$ ,  $c_2$  and  $c_3$ ) are tried to be found. The dimensions of variables,  $c_1$ ,  $c_2$  and  $c_3$ , are in m/s.

$$\frac{Q}{bh} = c_1 + c_2 \left(\frac{h}{b}\right) + c_3 \left(\frac{h}{b}\right)^{3/2}$$
(5.4)

where;

$$\boldsymbol{c}_{1} = \boldsymbol{e}_{1} + \boldsymbol{e}_{2} \left(\frac{\boldsymbol{b}}{\boldsymbol{B}}\right) + \boldsymbol{e}_{3} \left(\frac{\boldsymbol{b}}{\boldsymbol{B}}\right)^{2}$$
(5.5)

$$\boldsymbol{c}_2 = \boldsymbol{f}_1 + \boldsymbol{f}_2 \left(\frac{\boldsymbol{b}}{\boldsymbol{B}}\right) + \boldsymbol{f}_3 \left(\frac{\boldsymbol{b}}{\boldsymbol{B}}\right)^2 \tag{5.6}$$

$$c_3 = g_1 + g_2 \left(\frac{b}{B}\right) + g_3 \left(\frac{b}{B}\right)^2$$
(5.7)

At the end of regression analysis, the constants  $e_1$ ,  $e_2$ ,  $e_3$ ,  $f_1$ ,  $f_2$ ,  $f_3$ ,  $g_1$ ,  $g_2$ ,  $g_3$  are determined and listed below in Table 5.2.

<i>e</i> 1 (m/s) :	0.246
<i>e</i> <sub>2</sub> (m/s) :	-0.058
<i>e</i> <sub>3</sub> (m/s) <i>:</i>	0.010
<i>f</i> <sub>1</sub> (m/s) :	0.144
<i>f</i> <sub>2</sub> (m/s) :	0.540
<i>f</i> <sub>3</sub> (m/s) :	1.434
<i>g</i> ₁(m/s) <i>:</i>	-0.059
<i>g</i> <sub>2</sub> (m/s) :	0.185
<i>g</i> <sub>3</sub> (m/s) :	-1.122

# Table 5.2Values of Constants as a Result of RegressionAnalysis

The variables  $c_1$ ,  $c_2$  and  $c_3$  with respect to *b/B* ratio are given in Figures 5.17, 5.18 and 5.19, respectively. As can be seen from the graphs,  $c_1$  has almost linear relationship with *b/B*, whereas  $c_2$  and  $c_3$  has a parabolic relationship with *b/B*.



Figure 5.17  $c_1$  versus b/B



Figure 5.18

c₂ versus b∕B



Figure 5.19  $c_3$  versus b/B

Figure 5.20 expresses the relation between discharge and water head over the weir both for experimental data and calculated values using Eqn. (5.4). The lines represent the discharge values calculated by Eqn. (5.4) with respect to observed water head over the weir during experiments.



Figure 5.20 Q and *h* Relation for Comparison of Measured with Eqn. (5.4)

S

Figure 5.21 gives the comparison of experimental data and calculated data as  $C_d$  versus h/b relation. The relation between  $C_d$  and R;  $C_d$  and W can be observed by Figure 5.22 and Figure 5.23, respectively. As can be seen in Figure 5.22 and 5.23; it is very difficult to fix an equation of discharge coefficient as a function of Reynolds number and/or Weber number. Weber number for contracted weir is also calculated by using equation (5.1) like slit weir.


Figure 5.21  $C_d$  and h/b Relation for Comparison of Measured Data with Those Calculated by Eqn. (5.4)

S



Figure 5.22 *C<sub>d</sub>* and *R<sub>contracted</sub>* Relation for Comparison of Measured Data with Those Calculated by Eqn. (5.4)



Figure 5.23  $C_d$  and W Relation for Comparison of Measured Data with Those Calculated by Eqn. (5.4)

σ

The percent error is calculated by Eqn. (5.2),  $Q_{calc}$  is the discharge calculated by Eqn (5.4)and  $Q_{exp}$  is the present measured discharge by experimental works.

For all data points the percent error is below 6 % and it is an acceptable value. In addition to this neglecting few data points; the overall percent error is between 2 % to -2 %. And also 89.90 % of all contracted data have error smaller than 1 %, whereas 99.24 % of all data have error smaller than 2 %.



Figure 5.24 Percent Error with respect to Experimental Discharge and Eqn. (5.4)

## **CHAPTER 6**

## CONCLUSION

In the present study, an empirical approach is used for the investigation of rectangular sharp crested weirs. As indicated in the previous chapters, experiments are made with a 32 cm width of fiberglass channel and a rectangular weir for different weir openings from full width weir to slit weir of 2 cm minimum opening.

The conclusions of the analysis of the experimental data are listed below:

- i. For all weir openings below 2 cm of water head over the weir, nonaerated flow is observed. Thus the minimum water head is 2 cm.
- ii. Water surface profile experiments are conducted to determine the effective measurement location of point gauge, and it is concluded that the drawdown effect of sharp crested weir is negligible 1.20 m upstream from the weir location which is greater than 3-4*h* as given in literature.
- Full width sharp crested rectangular weirs are investigated by changing the weir height and it is concluded that bottom boundary effect is negligible for *P* equals to 6, 8 10 cm. Weir height of 2 and 4 cm is not efficient to investigate the sharp crested rectangular weirs. As a result weir height of 10 cm is chosen for the rest of the experiments.

- iv. Sharp crested rectangular weirs should be considered in two parts such as; slit weir and contracted weir.
- v. Slit weir experimental data results show that the proposed discharge coefficient equation by Aydın et al. (2006) is reliable. But the equation (Eqn. 3.9) should be improved for larger values of discharge.
- vi. For contracted and full width weirs a discharge equation is derived which is a function of *h*, *b* and *h/b* and three coefficients such as  $c_1$ ,  $c_2$  and  $c_3$ . These three different coefficients are also functions of *b/B*. The equations, which are used to describe the discharge through the contracted and full width weirs, are given below:

$$\frac{\mathsf{Q}}{bh} = \mathbf{c}_1 + \mathbf{c}_2 \left(\frac{h}{b}\right) + \mathbf{c}_3 \left(\frac{h}{b}\right)^{3/2}$$

$$c_{1} = e_{1} + e_{2} \left(\frac{b}{B}\right) + e_{3} \left(\frac{b}{B}\right)^{2}$$
$$c_{2} = f_{1} + f_{2} \left(\frac{b}{B}\right) + f_{3} \left(\frac{b}{B}\right)^{2}$$
$$c_{3} = g_{1} + g_{2} \left(\frac{b}{B}\right) + g_{3} \left(\frac{b}{B}\right)^{2}$$

<i>e</i> 1 (m/s) :	0.246
<i>e</i> <sub>2</sub> (m/s) :	-0.058
e <sub>3</sub> (m/s) :	0.010
<i>f</i> <sub>1</sub> (m/s) :	0.144
<i>f</i> <sub>2</sub> (m/s) :	0.540
<i>f</i> <sub>3</sub> (m/s) <i>:</i>	1.434
$g_{\scriptscriptstyle 1}({ m m/s})$ :	-0.059
<i>g</i> <sub>2</sub> (m/s) :	0.185
<i>g</i> <sub>3</sub> (m/s) :	-1.122

vii. A regression analysis is performed in order to find the values of the coefficients, explained above. The table of the results is given as:

viii. The relation of discharge coefficient and Reynolds number and the relation of discharge coefficient and Weber number are not considered for contracted weirs. Since the relation of  $C_d$  versus R and  $C_d$  versus W result in complex equations, those are difficult to use for real life applications.

## REFERENCES

Aydın, İ., Ger, A. M. and Hınçal, O. (2002). "Measurement of Small Discharges in Open Channels by Slit Weir." Journal of Hydraulic Engineering, Vol. 128, No. 2, 234-237.

Aydın, İ., Altan-Sakarya, A. B. and Ger, A. M. (2006). "Performance of Slit Weir." Journal of Hydraulic Engineering, Vol. 132, No. 9, 987-989.

Bos, M. G. (1989). "Discharge Measurement Structures." International Institute for Land Reclamation and Improvement, Third Edition, Wageningen, The Netherlands.

Chow, V. T. (1959). "Open Channel Hydraulics." McGraw-Hill Book Company Inc., Newyork.

Coxon, W. F. (1959). "Flow Measurement and Control." Heywood & Company Ltd., London.

Franzini, J. B. and Finnemore, E. J. (1997). "Fluid Mechanics with Engineering Applications." McGraw-Hill Company Inc.

Henderson, F. M. (1966). "Open Channel Flow." Prentice-Hall Inc.

Hınçal, O. (2000). "Discharge Coefficient for Slit Weirs." MSc. Thesis Department of Civil Engineering, Middle East Technical University, Ankara, Turkey. Kandaswamy, P. K. and Rouse, H. (1957) "Characteristics of Flow Over Terminal Weirs and Sills." Journal of of Hydraulics Division, Vol. 83, No. 4, August, 1-13.

Kinsdvater, C. E. and Cater, R. W. (1957). "Discharge Characteristics of Rectangular Thin-Plate Weirs." Journal of Hydraulics Division, Vol. 83, No. 6, December, 1-36.

Munson, B. R., Young, D. F. and Okiishi, T. H. (2002). "Fundamentals of Fluid Mechanics." John Wiley & Sons Inc., Newyork, USA.

Subramanya, K. (1986). "Flow in Open Channels." McGraw-Hill Publishing Company Limited

Swamee, P. K., (1988). "Generalized Rectangular Weir Equations." Journal of Hydraulic Engineering, Vol. 114, No. 8, 945-949.

Ramamurthy, A. S., Tim, U. S. and Rao, M. V. J. (1987). "Flow over Sharp Crested Plate Weirs." Journal of Irrigation and Drainage Engineering, Vol. 113, No. 2, 163-172.

Ramamurthy, A. S., Qu, J., Zhai, C. and Vo, D. (2007). "Multislit Weir Characteristics." Journal of Irrigation and Drainage Engineering, Vol. 133, No. 2, 198-200.

Rehbock, T. (1929). "Discussion of 'Precise Measurements'." By K. B. Turner. Trans., ASCE, Vol. 93, 1143-1162.