CENTRALIZATION AND ADVANCE QUALITY INFORMATION IN REMANUFACTURING

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ABSTRACT

CENTRALIZATION AND ADVANCE QUALITY INFORMATION IN REMANUFACTURING

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In this study, value of quality information and the effects of centralization are investigated for a reverse supply chain consisting of a remanufacturer and a collector. Used products are collected and inspected to classify them into quality groups, then they are remanufactured to meet the demand of remanufactured products. The supply of collected products and demand of remanufactured products are both price-sensitive. The uncertain quality of the collected products is revealed by an inspection process. Two quality classes are considered, and the cost of remanufacturing depends on the quality class. The main decisions are on acquisition fee for the returns, the selling price for remanufactured products, and the transfer prices of inspected products between the collector and the remanufacturer. For this environment, centralized and decentralized settings are considered and different models that differ in availability of quality information when the pricing decisions are made are built. We explore the value of advance quality information and effects of centralization on the optimal prices and profits via a computational study.

Keywords: Remanufacturing, Value of Information, Centralization

ÖZ

YENİDEN İMAL ETME SÜRECİNDE MERKEZİLEŞME VE ERKEN KALİTE BİLGİSİ

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Calışmamızda bir üretici ve bir toplayıcıdan oluşan tersine tedarik zinciri için kalite bilgisinin değeri ve merkezileşmenin etkileri incelenmiştir. Kullanılmış ürünler tüketicilerden toplanmakta, kalite sınıflarına ayrılmak üzere kontrol edilmekte ve daha sonra yeniden imal edilen ürün talebini karşılamak üzere yeniden imal etme sürecine sokulmaktadır. Toplanan ürünlerin miktarı ve yeniden imal edilen ürünlere olan talep miktarı fiyata duyarlıdır. Toplanan ürünlerin belirsiz olan kaliteleri, yapılan kontrol işleminden sonra belirlenmektedir. İki kalite sınıfı var olduğu ve ürünleri yeniden imal etme maliyetinin bu kalite sınıflarına bağlı olduğu varsayılmaktadır. Sistemdeki temel kararlar; toplama ücreti, yeniden imal edilen ürünlerin satış fiyatı ve kontrol edilen ürünlerin toplayıcı ve üretici arasındaki el değiştirme fiyatı içindir. Bu ortamda, merkezi ve merkezi olmayan düzenler kabul edilmiş ve fiyatlandırma kararlarında kalite bilgisinin varlığına göre farklılaşan modeller kurulmuştur. Gerçekleştirilen sayısal çalışma ile erken kalite bilgisi ve merkezileşmenin, optimum fiyatlar ve kar tutarı üzerindeki etkileri incelenmiştir.

Anahtar Kelimeler: Yeniden İmal Etme, Bilginin Değeri, Merkezileşme

To my family

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CHAPTER 1

INTRODUCTION

Remanufacturing is a process of recovery in which used products are re-engineered to as-good-as-new products. It is different from other recovery processes in its completeness because remanufacturing meets the same customer expectation as the new products. It requires a full treatment process like manufacturing. The whole form of the processed item is preserved in remanufacturing.

Remanufacturing has been becoming familiar to us in various kinds of industries recently, due to reasons related to economics, legislations and corporate citizenship. Industries involved in remanufacturing activities are from electronics to automotive, paper to chemicals (Dekker et al. (2004)). Profit maximization gains more attention as the recycling and remanufacturing processes become a must due to the existing and upcoming governmental regulations and environmental urgency. Since the manufacturers are directed to remanufacturing via some regulations, they seek for the most effective way of the remanufacturing process, bringing them the maximum profit. Therefore, the studies investigating the profitability and effective ways of the remanufacturing activities gain importance.

Closed-loop supply chains, include additional reverse supply chain processes to the traditional forward supply chain activities such as product acquisition, transportation, testing, sorting, assignment of the most suitable recovery process and finally marketing the recovered products. Reverse supply chain management is the process of planning and optimizing the effective flow of materials, used products, finished goods and related information from the point of consumption to the point of source with the aim of recapturing value or proper disposal.

Studies in the literature on closed-loop supply chains include several aspects; inventory management, lot sizing, material planning, scheduling, pricing of recovered products, acquisition management, competition and coordination mechanisms among the supply chain actors and contract design.

In remanufacturing activities, used products are converted into recovered products by carrying out a series of operations and incurring the related remanufacturing cost. In real life not all of the used products on hand are at the same condition; some of them are at their end-of-life and some of them are in a better condition. Thus they require different remanufacturing operations and costs. The decisions of the remanufacturing quantities are made according to the quality of the products, prioritizing the better quality products in remanufacturing. The quality of the collected products is not known until they are inspected and classified, bringing an uncertainty in quality aspect. Uncertainty of quality gives rise to some problems in adjusting the usage quantities or determining the selling prices. It constitutes a risk on pricing decisions since quality is unknown until observing the quantities via inspection. Collection and inspection activities can be outsourced or undertaken by the remanufacturer itself. This brings the centralization and decentralization issue. Decentralization brings double marginalization effect due to the increase in the number of decision makers in the system.

In consideration of the extensive studies in literature and focusing on the quality aspect and centralization, our goal in this thesis is to assess the value of quality information and effect of centralization on pricing decisions in a closed-loop supply chain.

In this study, a pure product recovery system, consisting of a remanufacturer and a collector is considered. Used core products which are the single source of production for the remanufacturer are collected from users at an acquisition fee. The collected products are assumed to be of different qualities depending on their suitability for remanufacturing. Only two quality groups are considered: Superior and inferior quality. After collection, core products are inspected at a constant per unit inspection cost, in order to determine which quality group they are included in. The proportion of the superior quality products to total supply is uncertain. After the announcement of the quantities for each quality group, the inspected cores are transferred to the remanufacturer at needed quantities on transfer prices varying with the quality. These two quality groups differ in their remanufacturing costs. Superior quality products are assumed to be in a better condition than the inferior quality to be remanufactured, hence remanufacturing cost of superior quality groups is lower than that of inferior quality products. The next step is for the remanufacturer to determine the usage quantities from each quality group in remanufacturing process. The inspected used products can be salvaged for recycling, at a constant unit revenue. Recovered products are of single quality independent of the quality group of the inspected products used. Finally, recovered products are sold at a selling price to the customers.

We consider the case where supply of the used products and demand for the recovered products can be manipulated by the pricing decisions, meaning that both supply and demand are price sensitive. Supply is an increasing linear function of the acquisition fee and demand is a decreasing linear function of the selling price.

There are two main objectives of this study:

(i) to investigate the effects of centralization of remanufacturing operations

(ii) to assess the value of quality information.

In order to achieve the first objective, we consider centralized and decentralized supply chain configurations. In centralized setting, there is a single firm (remanufacturer) that carries out all operations; collection of used products, inspection, and remanufacturing. The basic decisions are the acquisition fee for used products and the selling price of the recovered product. Whereas, in the decentralized setting, collection and inspection of used products are performed by another firm, the collector. In addition to the activities and the decisions of the centralized system, this time, the inspected and classified cores are bought by the remanufacturer from the collector, hence the transfer prices of two quality classes should be determined. We consider three different cases for the underlying decision: (i) The transfer prices are exogenous (ii) The collector determines transfer prices before inspection (iii) The collector determines transfer prices after inspection.

In order to assess the value of quality information under centralized and decentralized settings, we consider several scenarios that differ in availability of quality information when the basic decisions are made. Notice that, in order to obtain the quality information, the collected products should be inspected. Hence the acquisition fee (consequently the number of products to be collected) is always determined under quality uncertainty. However, it is possible to use the quality information in the determination of transfer prices (when the system is decentralized) and the selling price of the recovered product (under both centralized and decentralized settings) by postponing the related decisions after the inspection process. Therefore, our different scenarios on the availability of quality information are constructed by changing the timing of pricing decisions.

The rest of the study is organized as follows: In Chapter 2, we present a literature review on closed loop supply chains concentrating on the related topics. These are pricing of recovered products, acquisition management for returns, effect of centralization of recovery decisions, uncertain return quality and value of information about quality or quantity of returns. Problem environment is also defined in Chapter 2. Chapter 3 includes a detailed model description of centralized setting, in which the remanufacturer is the only actor. Two models are constructed under centralized setting, differing in the availability of the quality information when the selling price is determined. Chapter 4 is on decentralized setting in which the collector is also included into the problem environment. The models constructed according to the availability of the quality information are described in detail in this chapter. Since analytical comparisons are not possible, a computational study is performed, details and results of which are discussed in Chapter 5. A comparison is made among the models of centralized and decentralized settings in order to detect the effect of quality information and centralization on pricing decisions and profit values. In Chapter 6, the study is concluded by summarizing the main findings.

CHAPTER 2

LITERATURE REVIEW AND PROBLEM DEFINITION

2.1 LITERATURE REVIEW

Closed-loop supply chain management is a widespread and rich research area for IE/OR field including studies on inventory management, lot sizing, material planning, scheduling, pricing of recovered products, competition and coordination among the supply chain actors and contract design.

Gupta and Gungor (1999), Dekker et al. (2004), Shaligram et al. (2009) and Guide and Van Wassenhove (2009) provide review studies on various aspects of closed loop supply chain management.

Gupta and Gungor (1999) present the development of research in environmentally conscious manufacturing and product recovery; studies related to life cycle of management of products, disassembly, material recovery, remanufacturing and pollution prevention are included.

The book edited by Dekker et al. (2004) is on quantitative models for closed-loop supply chains, and it provides the recently developed studies for collection and distribution management, inventory control, production planning and coordination mechanisms.

Shaligram et al. (2009) state that research and practice in reverse logistics are of all aspects, from collection of used products, their processing and finally to the outputs of processing, namely remanufactured products, pricing the outputs and waste material disposal. They indicate that mathematical modeling in reverse logistics is mainly focused on deterministic methods, having limited source on stochastic demand and supply figures.

Guide and Van Wassenhove (2009) provide an overview of business aspects of closed loop supply chains based on real life case studies. The main aspect focused on is profitable recovery processes from returned products. They approach the closed loop supply chain issue in perspective of an evolutionary structure. They reviewed the progress of this subject starting from individual activities in the reverse supply chain, establishing the key differences from traditional OR problems. Coordination and incentive alignment issues are followed by closed loop supply chain design issues, reaching to OR-based operations management integrating accounting and marketing issues. In this evolution, the conducted studies are reviewed and the significant issues the researchers should focus on in the future are mentioned.

In this thesis, we focus on pricing of recovered products and setting acquisition fees for returns for a recovery system where the level of quality of returns is uncertain. We study a number of different settings on the availability of quality information and decentralization of recovery related operations/decisions. In the literature review part, we restrict our attention to the studies that are most related to our work. Specifically we focus on studies about the following topics:

- Effects of centralization/decentralization of recovery decisions
- Pricing of recovered products
- Acquisition management for returns
- Value of quality/quantity information of returns
- Uncertain return quality

The reviewed are summarized with respect to the issues involved in Table 2.1. They are discussed in the following sections according to the main topic they are related to.

2.1.1 Effects of centralization/decentralization of recovery decisions

Reverse channel choice decision which addresses the problem of choosing the appropriate reverse channel structure for the collection of used products from customers and collaboration between different decision makers are important issues for closed-loop supply chains. Especially, when supply chain moves away from single decision maker model and some processes

Table 2.1: Reviewed studies and related topics

Studies reviewed	Effects of cent./decent. of recovery decisions	Pricing of recovered prod.s	Acquisition mang. for returns	Uncertain return quality	Value of quality/quantity information of returns
Aras et al. (2004)				+	
Bakal and Akcali (2006)		+	+	+	+
De Brito and Van der Laan (2009)		+			+
Ferrer (2003)		+			+
Ferrer and Ketzenberg (2004)		+			+
Ge et al. (2006)		+			+
Guide et al. (2003)		+	+	+	
Ketzenberg et al. (2006)					+
Karakayali et al. (2007)	+	+	+		
Mitra (2007)		+		+	
Savaskan et al. (2004)	+	+	+		
Savaskan and Wassenhove (2006)	+	+			
Sun et al. (2007)		+	+		
Tagaras and Zikopoulos (2008)				+	
Vadde et al. (2006)		+			
Vadde et al. (2007)		+			
Wang and Gao (2007)		+			
Zikopoulos and Tagaras (2007)				+	

like collection and inspection of used products are imposed to retailers or 3rd party collectors, coordination and exchange of information become more significant and inevitable. The overall goal is to maximize the gain of the system consisting of many decision makers at the same time respecting the existing legislative forces.

Savaskan et al. (2004) model three types of reverse channel structure that differ in the supply chain partner responsible for collection in a pure recovery system: Manufacturer, retailer or 3rd party collector. By adapting a game-theoretic approach, they relax the centralized planner assumption and model the independent decision-making process of each supply chain member in a single period. The manufacturer always acts as a Stackelberg leader of the system in all decentralized models, anticipating the reaction functions of the retailer or the 3rd party collector. When the "manufacturer collects", (s)he decides the wholesale price of the new products and the product return rate, while the retailer decides the retail price of the new products. In the "retailer collects" case, there is a single manufacturer and a single retailer. The retailer undertakes the collection activity, thus the decision of product return rate is on the retailer. The transfer price between the manufacturer and the retailer for the used products is determined by the manufacturer. In the "3rd party collects" case, the 3rd party collector decides the product return rate. The retailer only engages in the distribution process and decides the retail price while the manufacturer determines the wholesale price of new products and transfer price of the used products. All decision makers aim to maximize their own profit. Demand function is assumed to be linear and dependent on the retail price. The constructed models are compared in terms of the optimal wholesale price, the optimal retail price, the optimal product return rate, and the total supply chain profits. The results of the comparisons show that the "retailer collects" case provides the best results. This is related to the retailer's being closer to the customer since the retailer can efficiently reflect unit cost savings from remanufacturing to the final price of the product, and jointly optimize the investment in used-product collection. It is also stated that the manufacturer can further improve the profits of this case to the level of a centrally coordinated system by using a two-part tariff contract. On the contrary, 3rd party collector is stated to be preferable especially for his extra value adding product recovery services supplied, such as sorting, dismantling and quality monitoring which result in lower remanufacturing costs for the manufacturer. 3rd party's capability to impact the amount of value recovered from a remanufactured product makes outsourcing the collection activity economically justifiable.

Savaskan and Wassenhove (2006) focus on the economic trade-offs the manufacturer confronts while choosing an optimal reverse channel structure. A two echelon distribution system with a remanufacturer and two retailers is considered. They compare direct collection systems in which the manufacturer himself collects the used products and indirect collection systems in which retailers are used as collector parties in addition to sales activity. In addition to these decentralized settings, centralized settings are also considered for benchmarking purposes. Demand at each retailer is a function of his own and the other retailer's price. In decentralized direct collection model, the manufacturer decides on the wholesale price of the product and undertakes the collection effort in the reverse channel, by considering the effect of these decisions on the strategic behavior of the retailers. Then, each retailer chooses the price of the product while considering the competition from the other retailer. On the contrary, in decentralized indirect collection model, products are collected indirectly via the retailers and transferred to the manufacturer, at a constant per unit buy-back price. The manufacturer decides on the wholesale price and buy-back price. Each retailer determines the respective retail price and the collection quantity, taking the competition from the other retailer into consideration. In both decentralized models, manufacturer acts as a Stackelberg leader and the retailers are competing in a Bertrand pricing game.

In the direct and indirect centralized settings, pricing and product collection effort decisions are made by a central planner; therefore wholesale and buy-back prices are irrelevant here. As an extension, both homogenous and heterogeneous retailer cases are examined. Channel profits and return rate of products of the decentralized settings are compared. It is shown that reverse channel choice depends on the market and product properties as follows:

- For consumer products in which competition is an important determinant of prices, the retailer collecting system would be the preferred option for the manufacturer.
- For product categories for which the retailers have less impact on the prices, the manufacturer benefits from a direct collection system.

Another important result of the study is that the strategic interactions between competitive retailers provide increased profits when competition among them and impact of the retailers on the retail prices are intense. Savaskan and Wassenhove (2006) also attract attention to collaboration issue between manufacturer and nonidentical retailers, particularly when the market sizes of the retailers do not differ too much. By this way, the manufacturer can overcome the double marginalization problem in the forward channel due to the inclusion of the heterogenous retailers in the pricing decisions via two-part tariffs. Two-part tariffs adjust the wholesale price to each retailer's market size.

2.1.2 Pricing of recovered products

Vadde et al. (2007) state that pricing recovered products is a strategy to control inventory and raise the revenues in a remanufacturing environment with fluctuating demand and unpredictable discarded product returns. They consider a product recovery system which faces a legislative limit on the disposal quantity. Product recovery facilities (PRF) process discarded product returns and sell the recovered components. The good products separated from poor quality products among the discarded products are sent to a disassembly process to recover the reusable, recyclable and disposable components. The reusable components are then categorized as high grade components, which can be remanufactured, and scrap grade components, which can be recycled for their materials. Recyclable components are also classified as high and scrap quality; the high quality ones are relatively easier to recycle than the scrap quality. These four categories differ in their recovery options and costs. The PRF gives first priority to the sale of remanufactured components due to their better market value, and then to high quality recyclable components, scrap grade reusable components, and scrap quality recyclable components in turn. The disposable components are disposed of immediately after they are recovered, whereas the leftover scrap grade reusable and scrap quality recyclable components are disposed of at the end of the selling period.

Objective is to find the optimal prices of reusable and recyclable components while maximizing the profit. PRF operates in a monopolist environment. Inventory is kept for only recovered components. Demand for all component categories is deterministic and is a decreasing function of its price. Excess demand is lost. Yields of disassembly and sorting processes are deterministic.

Four scenarios are formed according to PRF's supply method for returns and type of discarded products. Discarded products are either single type or multi-type while PRF's supply method is either passively accepting or proactively acquiring the product returns. Multi-type discarded products can contain the same reusable, recyclable and disposable components. In the first and second scenarios, PRFs passively accept discarded product returns, having no influence on the quantity and timing of returns, whereas in the third and fourth scenarios, they proactively acquire the returns, by providing price incentives to customers who return their products, to minimize the uncertainty in the quantity and timing of returns. Single type discarded products are processed in the first and third scenarios and multi-type discarded products in the second and fourth scenarios. There is a market for all categories of components sold by PRF. In all scenarios, prices are posted only after the component yield is realized. Objective function is composed of revenue earned from sale of all categories of components, (cost of acquisition), cost to disassemble, holding cost, cost to process, cost to dispose. The resulting problem is a nonlinear optimization problem. The effects of product returns, disassembly yield, sorting yield, disposal limit, remanufacturing and processing cost, holding cost, disassembly cost, and disposal costs on the profits, prices, inventory levels, and disposal quantity are investigated for sensitivity analysis.

Wang and Gao (2007) consider the pricing decision in a mass customization environment. A custom-made production is examined in two stages: (i) Common parts production (ii) Individual production. They initially assume that two individual products are produced by common parts supplied from call-backs of old products. Demand for two individual products are linear functions of the retail prices. Demand of one type of individual product is reversely proportional to its own price and directly proportional to other product's price. Call-back cost is assumed to be a function of the call back ratio and consisting of fixed and variable costs.

According to the mode of call-back operation, two cases are considered:

1. Call-back by the manufacturer: In this case, the manufacturer buys the used products from customers directly at a fixed price, and first remanufactures the old products to satisfy demand and then purchase the materials to manufacture for the rest of the demand. The manufacturer determines the wholesale price of new products and call-back ratio.

2. Call-back by the retailer: In this case, the retailer collects old products from customers and sells them to the manufacturer. The manufacturer determines the wholesale price of the new product and the retailer determines the retail price and and call-back ratio.

The manufacturer is the leader and the retailer is the follower in both cases. The models constructed are solved to get the optimal wholesale prices, retail prices, call-back rate and exchange prices of call-back product with the objective of profit maximization for both the retailer and the manufacturer.

The model is extended to n product with n retailers case where each retailer sells a single type of product. The existence and the result of equilibrium solution for n product model is shown. A sensitivity analysis is conducted for different parameters and the main findings are as follows:

- As the unit cost of remanufacturing increases, the rate of recovery decreases resulting in lower profits for the retailer and the manufacturer.
- Call-back by retailer case is found to be better because the rate of recovery and thus profit of both players are higher than those in call-back by manufacturer case.
- Competition between the retailers in the model, which is extended to n products, decreases the sale price, increases demand and makes the profits higher for the manufacturer.

Sun et al. (2007) consider a reverse supply chain giving the responsibility of the collection process to third party firms which are named the recycling business in the study. In problem environment, uncertain supply of return products is considered. Recycling business buys back the used products from the end users. The returned quantity of used products is assumed to be random given the recycling price. The manufacturer then purchases the used products from the recycling business and remanufactures them into the serviceable components. Random yield occurs in remanufacturing process. Demand is also random. All salvaging revenues are zero but shortage costs exist for both the manufacturer and the recycling business. A Stackelberg game problem is studied where recycling business is the Stackelberg leader and determines the selling price before uncertain supply was realized and then the manufacturer decides the order quantity. The existence and uniqueness of Stackelberg equilibrium is proved. A numerical analysis is performed for the equilibrium point's behavior under varied parameters. It is revealed that the manufacturer's order quantity does not depend on the recycling business's uncertain supply, and the manufacturer's risk is larger than that with no supply uncertainty.

Dealing with end-of-life products bring more quality problems to remanufacturing process compared to regular production. Vadde et al. (2006) present pricing models to counter the prospect of product obsolescence for used products that can happen either gradually or suddenly. Reusable products start to become obsolete starting from the time they are introduced in the second hand market. Gradual obsolescence occurs over a long period of time while in contrast, sudden obsolescence can occur in a much shorter period of time. Obsolescence can cause demand drop or inventory accumulation. Pricing decisions of product recovery facilities (PRFs) for remanufactured/refurbished products should take the obsolescence into consideration. Monopolistic market, deterministic demand for reusable items, no replenishment for inventory in the selling horizon and no backlogging for excess demand are assumed. Cost of acquisition and production are out of interest.

The first model considers gradual obsolescence. The demand is assumed to be dependent on price and the obsolescence rate. The obsolescence rate describes the extent to which a product becomes obsolete in time and obsolescence rate at time t is assumed to be a continuous function and this rate increases with time. Unsold inventory is disposed within the pre-specified limit on disposal quantity due to the local environmental regulations. In a continuous time setting, the goal of the model is to find the price that maximizes the revenue over the selling horizon.

While studying pricing under sudden obsolescence, the demand is assumed to be a function of price only. The selling horizon is divided into T equal periods. The price is selected from a discrete set; p_1,p_2, ..., p_N. Obsolescence is modelled as a probability density function and it occurs at the end of the periods. PRF reviews the inventory level at the beginning of each period and chooses the price that maximizes the expected revenue from the beginning of that period to the end of the selling horizon. The problem is formulated as a dynamic programming model. A numerical study is conducted for both pricing models. It is shown that price, demand and revenue decrease at the same time as the gradual obsolescence parameter increases in the selling horizon. However, in the model with sudden obsolescence, the optimal price increases as the selling horizon progresses whereas the demand and revenue decreases.

2.1.3 Acquisition management for returns

In terms of the environment considered and decisions made, our study is most similar to the study of Karakayali et al. (2007). In this study, the authors focus on the decentralization of the pricing decisions of original equipment manufacturers (OEM), by outsourcing third party firms for collection and processing activities. Quantity of end-of-life products available and the demand for remanufactured parts are price-sensitive. A single type of product that can be classified into m different quality groups is assumed. Supply of used products in different quality groups is independent of each other, i.e., the acquisition price of one quality group does not influence the price, and, hence, the supply of another group. All quality groups are remanufactured into the same quality level. Wholesale prices of the used products incurred between the remanufacturer and the collector are dependent on the quality groups. Salvaging occurs for the unused collected products.

Optimal acquisition price for end-of-life products and optimal selling price of the remanufactured products are determined in the centralized and the decentralized models developed. Decentralization is considered for the decisions for collection and processing operations. The main aim of the study is to identify when and why the OEM would prefer to outsource the collection or the processing activities. The OEM gives leadership role to either remanufacturer or collector firm in different settings, to decide on wholesale price of the used products. The objective of the models is to specify the quantity of used products collected in order to maximize profits.

Regarding the impact of pricing decisions of the collector and the remanufacturer on the used product collection rates, it is shown through a numerical study that OEM would prefer to outsource the collection activity. In some cases, processing activity is also decided to be outsourced as the pricing behavior in the collector-driven channel induces a higher collection rate. It is also searched for how OEM can increase collection rate when the quantity of used products in preferred channel setting falls short of the collection rates mandated by the environmental regulation. The result is that OEM should make the investment needed to comply with the regulations by changing the system parameter(s) to drive the difference between realized and target rate to zero. Or OEM will have to pay the fines for not meeting the target collection rates.

Value of yield information on the profitability of remanufacturing is studied by Bakal and Akcali (2006). A remanufacturer is considered to acquire end-of-life vehicles (ELVs) from final owners, disassemble them into parts and sell remanufactured parts that are recovered in the secondary parts market. Recovery yield which is the fraction of parts that are remanufacturable is assumed to be random and depends on the acquisition price offered for the end-of-life vehicles. The realization of random yield is observed after an inspection process. Parts that are not remanufacturable are salvaged together with the body of the vehicle without the part, named the hulk. It is assumed that all remanufacturable parts are recovered into the same quality level. Both demand and supply functions are price sensitive, defined as deterministic linear functions.

The objective is to find the acquisition price for returned products and the selling price for remanufactured parts, maximizing the expected profit. Different single period models are formed according to the timing of pricing decisions for a single part: (i) Deterministic Pricing Model: The percentage of ELVs with remanufacturable parts is deterministic. (ii) Postponed Pricing Model: The remanufacturer has the opportunity to delay the determination of the selling price for the remanufactured parts until after the realization of the yield. (iii) Simultaneous Pricing Model: The remanufacturer has to determine the acquisition price for the ELVs and the selling price for the remanufactured parts simultaneously. (iv) Exogenous Acquisition Price: Acquisition price is exogenous, thus the remanufacturer has no control on the quantity and the quality of the items supplied. (v) Exogenous Selling Price: Selling price of the remanufactured parts is fixed, thus demand is constant and deterministic,on which the remanufacturer no longer has control. These models are analyzed via a computational study.

Main result of this study are as follows:

(i) Postponing sales price decision until observing the yield is always beneficial for the remanufacturer. the benefits increase with lower yield rates, higher variation and lower profit margins.

(ii) Perfect or at least partial information provides significant improvements in profits.

2.1.4 Value of quality/quantity information of returns

Value of information under different sources of uncertainty on pricing decisions is also studied in the literature. Since the uncertainty on the timing and the quantity of the return flows brings difficulty in inventory control, De Brito and Van der Laan (2009) investigate the impact of imperfect information on forecasting and inventory performances. A single product, single echelon, periodic review inventory system is considered. Each individual demand returns according to some return distribution. The production lead time is a fixed constant. Demands that cannot be satisfied are fully backordered with a penalty cost while overstocks are charged with a holding cost.

Four methods are developed in order to detect the performances over inventory related costs. Each method requires a different level of information for forecasting the lead time returns. Method *A* requires the overall return probability, i.e., the probability that a product is returned eventually. Method *B* requires additional information on previous demand per previous periods and knowledge of the complete return distribution. Method *C* makes use of the total amount of observed returned products in each previous period in addition to the requirements of method *B*. Method *D* needs to track the realized product returns from each past demand in addition to method *B*. Thus, method *D* makes optimal use of all relevant information, given perfect information. A simulation study is conducted to compare the performances of these methods over inventory related costs. Under perfect information, meaning that the information is realistic, method *D* is found to outperform all other methods as expected. However, under imperfect information on the overall return probability and imperfect information on the expected conditional time-to-return, the performances of the methods show that the most informed method, method *D*, does not necessarily lead to best performance. It is also emphasized that it is worth to further investigate the value of return information with respect to multiple criteria, like production scheduling and human resource management.

Ferrer (2003) studies optimal lot-sizing policies which are considered with value of information on return yield and time of the random yield's realization in the manufacturing site. He evaluates the effect of random production yields and supplier responsiveness in the remanufacturing facility. Remanufacturing facility meets its demand for parts by either buying directly from suppliers or disassembling and repairing used machines' parts. The focus of the study is on a single part which is accepted as more valuable than others in a single period. The recovery process is subject to random yield whose probability distribution is known before the disassembly and procurement quantities are determined. Disassembled parts to be repaired and repaired parts incur a holding cost for the manufacturer. The remanufacturing lead time and the supplier lead time are deterministic. Unsatisfied demand incurs a shortage cost. Cost function is composed of purchase cost, disassembly cost, repair cost, holding cost and shortage costs. The decisions are fraction of demand to be satisfied from the external supplier and the number of parts to be produced in the remanufacturing facility.

Four different scenarios are formed in order to observe the value of skilled operator, the value of speed and the value of information compared to the base case: (i) The hard way (base case): Remanufacturing lead time does not allow meeting potential shortages with last-minute orders from external suppliers. The proportion of parts to be procured from the external supplier and the number of machines to disassemble are defined at the beginning of the planning process. (ii) The value of the skilled operator: Disassembly identifies reparability yield. Only the parts with the potential to be successfully repaired go through the complete remanufacturing process. Under this strategy, when managers decide the number of machines to disassemble, or the number of new parts to procure from the new parts' supplier, the yield information is not available yet. (iii) The value of speed: Responsive supplier offers short lead time. The decisions on the number of machines to disassemble, and the number of new parts to procure from the new parts' supplier can be made sequentially. (iv) The value of information: Actual yield from cores is known. The disassembly and purchase decision is fully informed about the actual yield.

The performance of each scenario is evaluated under a wide range of parameters to illustrate the cost advantage of each process structure (process yield determined during disassembly, fast delivery or full information) and to observe the impact of yield variance. All combinations of parameters were examined in a full-factorial experiment and they are compared in terms of operating costs. A heuristic is also proposed to solve the extended lot-size model for multi-part items since the main model is dealing with a single part. It is reported that;

(i) Early information provides a significant reduction in operational costs.

(ii) Skilled labor force in disassembly operation is more advantageous than supplier responsiveness when yield variance is low.

(iv) The relative gain of having a responsive supplier increases significantly when the yield variance is high.

(v) The ability to learn the yield during disassembly process reduces the operating costs and more beneficial than developing a responsive supplier.

Ferrer and Ketzenberg (2004) investigate the impact of information on yield and lead time for lot-sizing models. The source of remanufactured parts is the inventory of cores that were previously collected. An unlimited supply of cores is assumed. Parts are released from cores by a disassembly process which has a constant cost per core and incur an inventory holding cost for parts in case of being stocked. Demand for finished products is assumed to be deterministic and there is a one-to-one relation between parts and finished products. The demand of parts for remanufacturing may be satisfied with new parts, remanufactured parts, or a mix of both part types depending on the relative costs of the available options. Unsatisfied demand is lost. There is a penalty cost for lost sale. Remanufacturing is subject to stochastic yield. The repair activity is assumed to be a Bernoulli trial. If the repair is not successful, then the part is discarded without cost.

The decision variables are the number of parts purchased from the supplier and the number of the parts remanufactured. Two factors affecting these decisions are the level of information about the remanufacturing yield and the supplier's lead time. Therefore a set of four models are formed including different combinations of yield information and lead times. Short or long lead times and early or late yield information are used in these combinations. If the lead time is short, then the decision to purchase new parts can be postponed until after the remanufacturing yield is realized. On the contrary, when the lead time is long, new parts must be ordered early because the remanufacturing yield is only realized when it is too late to place an order for new parts. When yield information comes early, then the part yield is known prior to the part recovery. In this case, the cost of repairs on bad parts can be avoided. When yield information comes late, then the part yield is known only after recovery, during the final testing phase of the part recovery station. Hence, in this case it is not possible to avoid the repair cost of unrecoverable parts. Markov decision process (MDP) is used to evaluate the defined infinite horizon, stochastic dynamic models. The computational study shows that:

(i) Investments in yield information can lead to significant cost savings while the benefits of lead time reduction are trivial.

(ii) It is worthwhile to invest in a process that is capable of identifying remanufacturing yields early to make more accurate disassembly and procurement decisions.

(iii) Cost of operating disassembly and remanufacturing facility is highly sensitive to the choice of parameter values.

(iv) The value of a short lead time increases with respect to the number of parts in a product. As a result, the value of a short lead time may be quite significant for complex products that are composed of a large number of parts.

Ketzenberg et al. (2006) study the value of information for a remanufacturer that faces uncertainty in demand, product return and product recovery (yield loss from the remanufacturing process). The remanufacturer satisfies demand either with new products or remanufactured products or a mix of both. Remanufacturing facility is not capacitated and all product returns are remanufactured. Any unsatisfied demand is lost and charged a shortage cost. Excess ending inventory is charged with holding cost. A single period model is formed to decide the quantity of new products to order with the objective of minimization of the total expected cost.

Base case is the scenario where the ordering decision is made prior to realizing demand, returns, and yield loss. The only known information in base case includes the sufficient statistics for each random variable and the relevant costs. Four other scenarios are formed by adding one or more items of information available prior to the ordering decision, representing improvements on the base case. The aim of the study is to evaluate the absolute and relative values of different types of information with respect to their improvements in total expected cost figures relative to the base case. The results show that the value of any specific information type is dependent to overall level of uncertainty in addition to the level of uncertainty that is attributed to the information for which it explains. It is stated that no dominance is clear in value among the different types of information. Therefore, to invest in more than one type of information is suggested even if one type of uncertainty dominates since the return on investment will be greater.

Ge et al. (2006) introduce the retailer in closed loop supply chain, not only responsible from the retail operation but also from collection operation. They focus on the information aspect in that the retailer always has more information about market than the manufacturer since the retailer is closer to the customers and determines the selling price, thus demand. Both demand and supply functions are price-sensitive. Customers' willingness to pay (valuation of new products) is assumed to be distributed uniformly in interval [0,1] and social environment consciousness is relatively low similar to real life. The closed-loop supply chain system can freely acquire enough used products if the social environment consciousness is high enough.

A benchmark centralized model is formed in which the manufacturer both collects and sells the products without a retailer in charge. The main model includes the retailer as the collector and seller. Principle-agent approach is followed where the manufacturer is the principle and retailer is the agent. The manufacturer offers the contract supplying the new product at a defined price and getting the end-of-life product at a defined fee. The retailer accepts the contracts if the profit he gains is larger than his reservation profit, the best outside opportunity for the retailer. Otherwise the retailer will not accept the contract and seek for an outside opportunity.

Contracts are introduced to provide the manufacturer to coordinate the manufacturer-retailer supply chain system under symmetric information and asymmetric information. Symmetric information implies that the manufacturer has access to the same information as the retailer. Different types of contracts are analyzed as various coordination mechanisms. Under symmetric information, two-part tariff is found to be effective to coordinate the closed loop supply chain. Moreover, cases under asymmetric information are examined according to the manufacturer's knowledge about the value of social environment consciousness. With pooling strategy, the manufacturer cannot observe the social environment consciousness and only know the prior probability of it, he provides a single contract to the retailer regardless of the social environment consciousness. With separating strategy, the manufacturer provides a menu of contracts to screen the information about social environment consciousness and the retailer chooses one of them based on his knowledge. The results of these strategies are compared with the benchmark model in which the manufacturer knows the value of the social environment consciousness. As a result, they show;

(i) When social environment consciousness gets higher, the retailer gains more profit.

(ii) The manufacturer will prefer separating strategy to the pooling strategy under asymmetric information.

2.1.5 Uncertain return quality

One type of quality for the returned products is an incomplete and impractical assumption, because in real life, not all of the collected products are in same condition to be remanufactured, causing variable remanufacturing costs. In this context, Aras et al. (2004) analyze the stochastic nature of product returns. They present an approach for evaluating the cost-effectiveness of quality-based categorization of returned products. A joint manufacturing and remanufacturing system with three storage facilities is considered. The ”as good as new” products are stored in serviceable inventory, meaning that they are ready to meet the demand. The returned products are stored in one of the two remanufacturable inventories based on their quality condition. High quality and low quality returns are the two quality groups considered. The serviceable inventory is managed by a continuous review base stock policy which aims at keeping the serviceable inventory position at the base stock level. Thus, each time a demand is served from the serviceable inventory, returned products are remanufactured having the priority of high quality types. In the event that there is no high quality return in stock, the low quality returns are used as an alternative source for remanufacturing to keep the serviceable inventory level at the base stock level. A returned product is disposed if the associated remanufacturable inventory is at its disposal level.

The quality variability brings variabilities in remanufacturing costs, disposal costs and lead times. They study on potential cost savings related with the effective management of inventories under these uncertainties. By accepting customer demand and product returns as independent Poisson processes, they constructed a continuous-time Markov chain model. Comparisons are made according to the benchmark model which is a hybrid system without categorization according to the quality of returns. Due to their numerical studies, they conclude that categorization is cost-effective because it makes use of the additional quality information in the system and the improvement is shown to reach 10 percent. These savings are found to be higher when;

(i) the quality differences between return types are higher,

(ii) the return rate is higher relative to the demand rate,

(iii) the demand rate is lower,

(iv) the quality of both types is lower.

Value of sorting the returned products before disassembly is also discussed by Tagaras and Zikopoulos (2008). They study a supply chain consisting of a central remanufacturing facility and a number of collection sites, in an infinite horizon context. Central facility buys the returned products from the collection sites, and disassembles them into their remanufacturable parts at a constant disassembly cost per unit. Items that are not remanufacturable are disposed. The parts those are not disposed are remanufactured at a constant unit cost. Remanufactured units, incurring a holding cost, are then sold to customers to meet the market demand. Yield is assumed to be deterministic while demand is stochastic. In case of stockouts, demand is not lost but is completely backlogged incurring a shortage cost per unit.

In this setting, they investigate whether it is feasible to establish a sorting operation without going through the typically expensive disassembly process. This sorting process may reveal with some accuracy which units are indeed remanufacturable. Sorting process is evaluated according to the place sorting is performed: central or local sorting. Expected annual profit functions are maximized in order to find the optimal procurement decisions under each one of the possible system configurations: no sorting, central sorting and local sorting. They derive the conditions under which each one of the three alternative system structures is dominant. As a result, it is found technically feasible to establish a sorting operation that may show which units are remanufacturable with some accuracy, without going through the expensive disassembly process.

Zikopoulos and Tagaras (2007) investigate the effects of the quality of returned products on the profitability of remanufacturing facilities. Two collection sites and a refurbishing site who faces stochastic demand for refurbished products are considered in a single period setting. It is assumed that uncertain quality is determined just after the transportation of returned products from collection sites to the refurbishing site. Transportation, inspection, sorting and refurbishing are the main operations. All transported products are inspected, however, only quantity needed for refurbishing is used by refurbishing site. Demand for refurbished products is a continuous random variable. Shortage and disposal costs are incurred. Two collection sites generally have different quality levels of returned products. The proportions of the refurbishables in both sites are continuous random variables following a common distribution. Quality differences and correlation between supply sources are allowed by correlation in the yield distributions of the two collection sites.

Procurement and production lot sizes are decided with the objective of maximizing the expected total profit of the entire system. The model is analyzed in two stages according to the decisions made consecutively: First, the quantities to transport from the collection sites to the refurbishing center (procurement lot sizes) are determined and after the yield of the transported quantity is realized, the quantity to refurbish (production lot size) is determined by the refurbishing site. After numerical analysis, specific conditions under which single or dual sourcing is optimal are derived. It is found that the quality of returned products has a substantial effect on the profitability of the system. Two identical collection sites have positive effect on system's profitability compared to a single collection site because of the reduction of the yield variance.

Guide et al. (2003) focus on profitability of remanufacturing and pricing decisions, taking quantity and quality of product returns into consideration. The impact of quality-dependent acquisition prices on product returns and impact of selling prices on demand are included. They illustrate their framework on an application from the cellular telephone industry but they suggest the model could be used when the remanufacturing of a certain product will be initiated and terminated. Their model is for a single period and includes intermediaries maintaining a stock of used products supplied from final users. Intermediaries grade the returned products and sell them to the remanufacturer in different quality classes. The information about quality grades and procurement prices is shared. Within a certain quality class, all used products are assumed to have the same expected remanufacturing cost. It is assumed that acquisition price for one class does not influence the return rates in other classes. Although the returned products are from different quality classes, they are remanufactured into a single quality standard by the remanufacturer. Demand and return rate functions are assumed to be known.

They mainly focus on the market-driven recovery system and develop an economic analysis for calculating the profit-maximizing acquisition prices and the optimal selling price for remanufactured products. A heuristic is constructed for the cases when demand and return functions have complex shapes, but it is recommended to be used where the elasticities of the functions are roughly equal. Optimal prices are calculated from the model formed and sensitivity analysis is conducted on different parameters like remanufacturing cost, slope of the return function and slope of the demand function. It is stated that although the heuristic may not be very precise, it is advantageous for its simplicity.

Mitra (2007) considers the possibility of different quality levels for recovered products, hence he is interested in pricing differentiation for recovered products. In the problem environment considered, the manufacturer itself recovers the returned products. Demand is modelled as a function of price and availability. Availability is the awareness spread among customers about the recovered products. The higher the availability, the higher the demand for recovered products. Recovered products are classified into two quality classes, one is the remanufactured products, which are as-good-as new and the other is the refurbished products indicating lower quality. Probability of selling is expressed as decreasing linear functions of price and availability. Unsold remanufactured product can always be sold at the price of a refurbished product; however unsold refurbished products are disposed at a certain cost.

The objective of the model is to determine the prices for remanufactured and refurbished products separately in order to maximize the total revenue. A numerical study is carried out in the context of recycled cellular phones in India. A sensitivity analysis is performed to comment on the impacts of different parameters on revenue, such as maximum price of remanufactured products, disposal cost, availability of remanufactured and refurbished products, sensitivity parameters (sensitivity of selling probability to availability).

The aim of this thesis is to investigate the value of quality information and effect of centralization on pricing decisions and expected profit values. This study is different from the reviewed studies in literature in that value of quality information is examined for both the remanufacturer and the collector under centralized and decentralized settings. Thus value of quality information is discussed together with the effect of centralization for a remanufacturing environment. Besides, the relation of the remanufacturer and the collector is analyzed under uncertain yield and quality assumption.

2.2 PROBLEM DEFINITION

The notation used throughout the thesis is given in Table 2.2.

In this study, we consider a pure product recovery system consisting of a collector and a remanufacturer that receives used products and satisfies demand for recovered products. Supply of used products (cores) is the unique source of production (remanufacturing). Cores are collected and inspected by the collector. Then they are transferred to the remanufacturer. Core products are converted to recovered products by the remanufacturer to satisfy the external market demand. We focus on the case where both used product returns and recovered product demand can be manipulated by pricing decisions. Therefore, demand and supply quantities are deterministic but price sensitive. It is possible to increase demand for recovered products by decreasing the selling price. Similarly, core supply can be increased by increasing the acquisition fee.

Table 2.2: Used notations

f	Unit acquisition fee for the core products collected from the market.
ψ	Minimum supply quantity.
ϕ	Price sensitivity of the supply function.
$S(f)$	Total supply quantity of cores for a given $f \geq 0$ value, $S(f) = \psi + \phi f \geq 0$.
p	Unit selling price of the recovered product.
a	Potential market size, denoting the maximum demand quantity for recovered products.
b	Price sensitivity of the demand function.
$D(p)$	Demand for recovered products for a given $p \geq 0$ value, $D(p) = a - bp \geq 0$
X	Random variable representing the ratio of Type 1 products to total supply quantity of cores.
x	A realization of X.
$g(x)$	probability density function (p.d.f) of random variable X.
$G(x)$	Cumulative distribution function (c.d.f) of random variable X.
Q_1	Random variable representing quantity of Type 1 products in the total supply.
Q_2	Random variable representing quantity of Type 2 products in the total supply.
μ	Mean of the random variable X.
σ	Standard deviation of the random variable X.
d	Unit salvage value for cores.
c_I	Unit inspection cost for collected cores.
c_1	Unit remanufacturing cost for Type 1 products.
c_2	Unit remanufacturing cost for Type 2 products.
q_1	Amount of Type 1 products used in remanufacturing process.
q_2	Amount of Type 2 products used in remanufacturing process.
w_1	Unit transfer price of the Type 1 products.
w_2	Unit transfer price of the Type 2 products.

The collector collects core products from the supply market paying an acquisition fee per unit. It is assumed that there is no upper bound for supply of core products. Number of core products collected from supply market is a linear function of the acquisition fee, f, denoted as $S(f) = \psi + \phi f$, where $\psi, \phi > 0$. Parameter ψ represents the minimum supply quantity and ϕ represents the price sensitivity of the supply function with respect to the acquisition fee. Supply quantity is considered to be greater than or equal to zero.

All purchased cores are inspected in order to determine their state of quality. There is a constant inspection cost, c_I, per core. It is assumed that all cores are classified into two quality groups by inspection: (i) Type 1 products (superior quality) (ii) Type 2 products (inferior quality). Following the inspection process, the collector transfers the inspected cores to the remanufacturer. Remanufacturer pays unit transfer prices, w_1 for Type 1 products and w_2 for Type 2 products to buy the inspected cores in order to use in remanufacturing. The remanufacturer determines the quantity to order from each quality type. Although both types can be brought to the same recovered product by remanufacturing, their remanufacturing costs differ. Unit remanufacturing cost is c_1 and c_2 for Type 1 and Type 2 products, respectively. Remanufacturing costs include all costs related to the operations necessary to make the core usable and suitable for sale. Type 1 products are known to be in a better condition, thus unit remanufacturing cost of Type 1 products is lower than Type 2 products, i.e., $c_1 < c_2$. We focus on the case where the level of quality of returns is uncertain. To model the uncertainty, a random variable, X, representing the ratio of Type 1 products within the total core supply is considered. Note that, in all models developed, the underlying probability distribution function of the random variable is assumed to be known. The realization of this random variable, x, is observed after the inspection process. Thus, quantity of Type 1 products, Q_1, is $xS(f)$ and quantity of Type 2 products, Q_2, is $(1-x)S(f)$.

Finally, the recovered products are sold to the customers. Demand for recovered products is modelled by a deterministic linear function of selling price, $D(p) = a - bp$, where $a, b > 0$. Parameter a represents the potential market size and b represents the price sensitivity of the demand function with respect to the selling price. Demand quantity is considered to be greater than or equal to zero. Unsatisfied demand does not incur an additional shortage cost other than the lost profit margin. Total supply quantity can exceed demand quantity. In this case, supply surplus is not remanufactured and the surplus of inspected cores are salvaged at a constant unit revenue by the collector for recycling. The recycling (that is the recovery to material level) revenue depends usually on the weight of the products, we assume that it is identical for both quality types.

Figure 2.1 depicts the main activities in the system.

In this system, we aim to analyze the value of quality information and the effects of centralization. Section 2.2.1 discusses the centralized setting where the system is operated by a single firm, the remanufacturer, whereas in Section 2.2.2, we discuss the decentralized setting consisting of a collector and a remanufacturer.

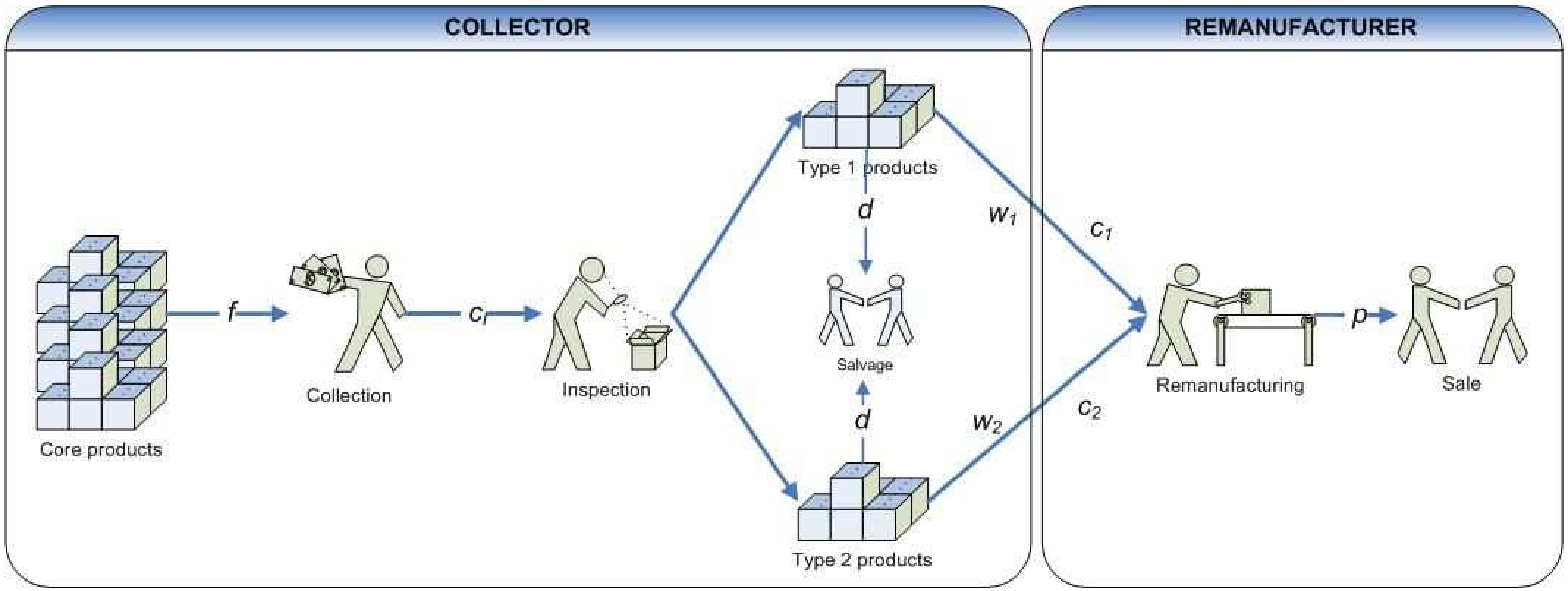


Figure 2.1: Problem Environment under the Decentralized Setting

2.2.1 Centralized Setting

In the centralized setting, there is a single firm, the remanufacturer. He carries out all operations from collection to the satisfaction of demand. Therefore, the acquisition fee of cores and the selling price of recovered products are determined by the remanufacturer and the transfer prices of cores are irrelevant. Depending on the sequence of events that results in different available information pieces, we consider two different centralized models: (i) Simultaneous pricing and (ii) Sequential pricing.

In the simultaneous pricing model (SMP), the remanufacturer determines the acquisition fee and the selling price simultaneously before the inspection process. Hence, the demand for recovered products is determined before the quality of returns is assessed. That is, there is no quality information prior to the pricing decisions. This type of decision can be due to timing constraints or market regulations.

In the sequential pricing model (SQP), the remanufacturer first determines the acquisition fee, f, hence the total supply quantity. After observing the quantities of both quality groups, Q_1 and Q_2, through the inspection, the remanufacturer determines the selling price, p, hence the demand, and the quantities to remanufacture from each type of quality, q_1 and q_2, simultaneously. The quantities to be remanufactured can be less than total supply of cores. In this case, supply surplus is salvaged with a unit revenue by the remanufacturer.

2.2.2 Decentralized Setting

In the decentralized setting, collection and inspection of the cores is performed by another firm, the collector. The acquisition fee is determined by the collector. The remanufacturer decides the quantity of each type to buy from the collector as well as the selling price of recovered products. Transfer prices of the collected and inspected products between collector and remanufacturer (w_1 and w_2) are either exogenous, or determined by the collector. When they are determined by the collector, we consider two different cases depending on the timing of the decision: (i) Before inspection and (ii) After inspection.

In exogenous pricing model (EP), transfer prices, w_1 and w_2, are assumed to be set by the market and are problem parameters. The sequence of events is as follows in exogenous pricing:

1. The collector decides acquisition fee of the cores, f, hence the supply quantity.
2. The collector inspects the cores and finds out the quantities of two quality groups, Q_1 and Q_2 available.
3. The remanufacturer decides the selling price of recovered products, p, hence the quantities of each quality to buy from the collector, q_1 and q_2. The collector salvages the unsold items at unit revenue, d.

When the collector sets the transfer prices, we consider two cases according to the timing of the decision of transfer prices. Transfer prices are determined either after the inspection process, therefore with quality information, or simultaneously with the acquisition fee, hence under quality uncertainty. This approach makes it possible to observe the effect of quality information in a decentralized setting.

In "collector's pricing before inspection" (CPBI) case, decision of transfer prices, w_1 and w_2, are before collection, thus there is uncertainty about quality. The sequence of events are as follows:

1. The collector decides transfer prices and acquisition fee together, thus the supply quantity.
2. The collector inspects the cores and finds out the quantities of two quality groups, Q_1 and Q_2 available.
3. The remanufacturer decides the selling price of recovered products, p, hence the quantities of each quality to buy from collector, q_1 and q_2. The collector salvages the unsold items at unit revenue, d.

In "collector's pricing after collection" (CPAI) case, decision of transfer prices, w_1 and w_2, are after collection, hence quality information is available during the collector's decision. The sequence of events are as follows:

1. The collector determines the acquisition fee, hence the supply quantity.

2. The collector inspects the cores and finds out the quantities of two quality groups, Q_1 and Q_2 available.

3. The collector decides transfer prices using the information about quality.

4. The remanufacturer decides the selling price of recovered products, p, hence the quantities of each quality to buy from collector, q_1 and q_2. The collector salvages the unsold items at unit revenue, d.

Our objective is to compare the decentralized setting to the centralized setting in order to observe the effects of decentralization. Furthermore, in both centralized and decentralized settings, we aim to characterize the value of quality information by considering different timings of decisions.

To analyze the effect of quality information, the centralized models, SMP and SQP are compared in their remanufacturer's profit values. The percentage differences are used in comparisons made. Similarly, in decentralized setting, CPBI and CPAI are compared in their collector's profit values since the quality information is beneficial for the decision of the collector. EP model in the decentralized setting is not used in comparisons since it is studied only as a preliminary model for the other decentralized models.

In addition to the effect of quality information, effect of centralization is also searched by comparing the centralized models to decentralized models. All pair comparisons are analyzed to search for the expected profit increase due to centralization. The situations in which a decentralized model outweigh the centralized model without quality information, i.e. SMP, are also searched for to investigate when the quality information is dominant to centralization.

CHAPTER 3

ANALYSIS OF CENTRALIZED SETTING

In this chapter, we consider the centralized setting consisting of only the remanufacturer. The remanufacturer collects used products from the market, inspects and sorts them. Then he remanufactures in order to satisfy the demand. The units inspected but not used in remanufacturing are salvaged.

In this setting, we analyze two models that results from different decision making sequences: In Section 3.1, we consider the case with simultaneous pricing, that is, acquisition price, f, and selling price, p, are determined by the remanufacturer at the same time before inspection. In Section 3.2, the remanufacturer postpones the selling price decision after the inspection operation, hence he has quality information while determining the selling price, p.

3.1 Simultaneous Pricing (SMP)

In simultaneous pricing model, we assume that the remanufacturer determines the acquisition fee of the cores and the selling price of the remanufactured products simultaneously before the inspection process. The sequence of decisions and events is as follows:

1. The remanufacturer determines the acquisition fee, f and selling price, p and he collects the corresponding returns.
2. The remanufacturer inspects all of the returns incurring unit inspection cost of c_I. The quantity available for both quality levels, Q_1 and Q_2 are observed.
3. The remanufacturer determines the number of units to remanufacture from both types of quality to satisfy the demand, q_1 ($\leq Q_1$) units of Type 1, and q_2 ($\leq Q_2$) units of Type

2 products. Cores that are not remanufactured, if any, are salvaged with unit revenue, d.

Since we do have a sequence of decisions, backward induction is used to characterize the optimal actions of the remanufacturer. First, we determine q_1 and q_2, for given Q_1,Q_2, f and p values. Then, we determine f and p by incorporating optimal q_1 and q_2.

Let $\Pi(q_1, q_2)$ denote the profit of the remanufacturer for given q_1 and q_2. The problem to determine q_1 and q_2, for given Q_1, Q_2, f and p values can be formulated as follows:

$$
\begin{aligned}
\text{Max} \quad \Pi(q_1, q_2) &= p(a - bp) + d(\psi + \phi f - (a - bp)) - c_1 q_1 - c_2 q_2 - (\psi + \phi f)(f + c_I) \\
\text{subject to} & \\
q_1 &\leq Q_1 && (3.1) \\
q_2 &\leq Q_2 \\
a - bp &= q_1 + q_2 && (3.2) \\
q_1, q_2, p, f &\geq 0
\end{aligned}
$$

In the optimal solution to the remanufacturer's problem shown above, q_2 can be positive only if the constraint (3.1) is binding since the remanufacturer would never utilize Type 2 items before depleting Type 1 items, as $c_1 < c_2$. Hence, optimal q_1 and q_2 can be determined as follows:

(i) When $Q_1 > a - bp \Rightarrow q_1 = a - bp$ and $q_2 = 0$ from the constraint (3.2).

(ii) When $Q_1 \leq a - bp \Rightarrow q_1 = Q_1$ and $q_2 = a - bp - Q_1$ from the constraint (3.2).

where $Q_1 = x(\psi + \phi f)$. Hence, considering the optimal q_1 and q_2 values, given Q_1, Q_2, f and p, the remanufacturer's problem to set f and p with profit $\Pi(f, p)$ can be formulated as follows:

$$
\begin{aligned}
\textbf{[SMP]}\ \text{Max}\quad \Pi(f,p) &= p(a-bp) + d(\psi+\phi f-(a-bp)) \\
&- c_1 \int_{\frac{a-bp}{\psi+\phi f}}^{1} (a-bp)g(x)dx - c_2 \int_{0}^{\frac{a-bp}{\psi+\phi f}} (a-bp)g(x)dx \\
&- (c_1-c_2) \int_{0}^{\frac{a-bp}{\psi+\phi f}} x(\psi+\phi f)g(x)dx - (\psi+\phi f)(f+c_I)
\end{aligned}
$$

subject to

$$
\begin{aligned}
a-bp &\leq \psi+\phi f \qquad (3.3) \\
p,f &\geq 0
\end{aligned}
$$

Lemma 3.1.1. $\Pi(f,p)$ *is jointly concave in* $p \geq 0$, $f \geq 0$.

Proof: First order and second order partial derivatives with respect to f and p are shown below:

$$
\begin{aligned}
\frac{\partial\Pi(f,p)}{\partial f} &= d\phi - \psi - 2\phi f - \phi c_I - (c_1-c_2)\left(\int_0^{\frac{a-bp}{\psi+\phi f}} x\phi g(x)dx\right) \\
\frac{\partial\Pi(f,p)}{\partial p} &= a - 2bp + bd + \left(\int_{\frac{a-bp}{\psi+\phi f}}^{1} bg(x)dx\right)c_1 + \left(\int_0^{\frac{a-bp}{\psi+\phi f}} bg(x)dx\right)c_2 \\
\frac{\partial^2\Pi(f,p)}{\partial f^2} &= -2\phi + \frac{(a-bp)^2\phi^2 g\left(\frac{a-bp}{\psi+\phi f}\right)(c_1-c_2)}{(\psi+\phi f)^3} < 0 \\
\frac{\partial^2\Pi(f,p)}{\partial p^2} &= -2b + \left(\frac{b^2 g\left(\frac{a-bp}{\psi+\phi f}\right)}{\psi+\phi f}\right)(c_1-c_2) < 0 \\
\frac{\partial^2\Pi(f,p)}{\partial f\partial p} &= \frac{\partial^2\Pi(f,p)}{\partial p\partial f} = \frac{b(a-bp)\phi g\left(\frac{a-bp}{\psi+\phi f}\right)}{(\psi+\phi f)^2}(c_1-c_2)
\end{aligned}
$$

Determinant of the Hessian matrix is given as follows:

$$
\left(\frac{\partial\Pi}{\partial f^2}\right)\left(\frac{\partial\Pi}{\partial p^2}\right) - \left(\frac{\partial\Pi}{\partial p\partial f}\right)^2 = 2\phi b\left(2 + \frac{c_1-c_2}{\psi+\phi f}g\left(\frac{c_1-c_2}{\psi+\phi f}\right)\left(b + \left(\frac{c_1-c_2}{\psi+\phi f}\right)^2\right)\right) > 0
$$

1. Second order partial derivatives are both less than zero.

2. Determinant of the Hessian matrix is greater than zero. □

Theorem 3.1.2. *Let* (f', p') *denote the solution to* $\frac{\partial \Pi(f,p)}{\partial f} = 0$, $\frac{\partial \Pi(f,p)}{\partial p} = 0$. *Then, the optimal solution of [SMP] is given by:*

$$(f^*, p^*) = \begin{cases} (f', p') & \text{if } a - bp' \le \psi + \phi f' \\ \left(\frac{\frac{a\phi}{b} - \frac{2\phi\psi}{b} - \psi - c_I\phi - (c_1 - c_2)\phi\mu - c_2\phi}{(\frac{2\phi^2}{b} + 2\phi)}, \frac{a - (\psi + \phi f^*)}{b} \right) & \text{otherwise} \end{cases}$$

Proof:

Since $\Pi(f, p)$ is concave (see Lemma 3.1.1), (f', p') is the global maximizer of $\Pi(f, p)$, if it satisfies the constraint (3.3). Otherwise, the constraint becomes binding, and the problem reduces to the following single-variable problem by using $p = \frac{a-(\psi+\phi f)}{b}$:

$$\begin{aligned} \text{Max} \quad \Pi(f) &= \frac{a(\psi + \phi f)}{b} - \frac{(\psi + \phi f)^2}{b} \\ &- (c_1 - c_2)(\psi + \phi f)\mu - c_2(\psi + \phi f) - (\psi + \phi f)(f + c_I) \qquad (3.4) \end{aligned}$$

subject to

$$f \ge 0$$

First and second order derivatives of $\Pi(f)$ are as follows:

$$\begin{aligned} \frac{d\Pi(f)}{df} &= \frac{a\phi}{b} - \frac{2\phi(\psi + \phi f)}{b} - \psi - 2\phi f - c_I\phi - (c_1 - c_2)\phi\mu - c_2\phi \\ \frac{d^2\Pi(f)}{df^2} &= \frac{-2\phi^2}{b} - 2\phi < 0 \end{aligned}$$

Since $\Pi(f)$ is concave, the optimal solution can be characterized by utilizing the first order condition:

$$\begin{aligned} f^* &= \frac{\frac{a\phi}{b} - \frac{2\phi\psi}{b} - \psi - c_I\phi - (c_1 - c_2)\phi\mu - c_2\phi}{(\frac{2\phi^2}{b} + 2\phi)} \\ p^* &= \frac{a - (\psi + \phi f^*)}{b} \end{aligned}$$

□

3.2 Sequential Pricing (SQP)

In this section, we consider the case that the remanufacturer decides acquisition fee of the used products and selling price of the recovered products sequentially. The order of events and decisions is defined as follows:

1. The remanufacturer determines the acquisition fee, f, and collects the corresponding returns.
2. The remanufacturer inspects all of the returns incurring unit inspection cost of c_I and observes the quantity available for both quality levels, Q_1 and Q_2.
3. The remanufacturer sets the selling price, p, remanufactures q_1 ($\leq Q_1$) units of Type 1, and q_2 ($\leq Q_2$) units of Type 2 products. Cores that are not remanufactured, if any, are salvaged with unit revenue, d.

Since there is a sequence of actions, backward induction is used to characterize the optimal actions of the remanufacturer. First, for given Q_1, Q_2 and f values, q_1, q_2 and p are determined. Then, we determine f by incorporating optimal q_1, q_2 and p.

The remanufacturer's problem to determine q_1, q_2 and p, for given Q_1, Q_2 and f values, with profit $\Pi(p, q_1, q_2)$ can be formulated as follows:

$$\begin{aligned}
\text{Max} \quad \Pi(p, q_1, q_2) &= p(a - bp) + d(\psi + \phi f - (q_1 + q_2)) - c_1 q_1 - c_2 q_2 \\
\text{subject to} & \\
q_1 &\leq Q_1 && (3.5) \\
q_2 &\leq Q_2 \\
a - bp &= q_1 + q_2 && (3.6) \\
q_1, q_2, p &\geq 0
\end{aligned}$$

In the optimal solution to the remanufacturer's problem above, Type 2 items will not be utilized until all Type 1 items are remanufactured since $c_1 < c_2$. Thus, optimal q_1, q_2 and p can be determined as follows:

(i) We first force $q_2 = 0$. then we have $q_1 = a - bp$ from constraint (3.6).

The problem to determine q_1, q_2 and p reduces to:

$$\begin{aligned} \text{Max} \quad \Pi(q_1) &= \frac{a-q_1}{b} q_1 + d(\psi + \phi f - q_1) - c_1 q_1 \\ \text{subject to} & \\ q_1 &\leq Q_1 \\ q_1 &\geq 0 \end{aligned} \tag{3.7}$$

The objective function (3.7) is concave in $q_1 \geq 0$, since the second order derivative with respect to q_1 is less than zero $\left(\frac{d^2\Pi}{dq_1^2} = \frac{-2}{b} < 0\right)$.

The optimal solution of the problem can be characterized utilizing the first order conditions and taking the constraint $q_1 \leq Q_1$ into account as follows:

$$(q_1^*, q_2^*, p^*) = \begin{cases} \left(\frac{a-b(d+c_1)}{2}, 0, \frac{a+b(d+c_1)}{2b}\right) & \text{if } Q_1 \geq \frac{a-b(d+c_1)}{2} \\ \left(Q_1, 0, \frac{a-Q_1}{b}\right) & \text{otherwise} \end{cases} \tag{3.8}$$

(ii) When $q_1 = Q_1 \Rightarrow q_2 \geq 0$ and $q_2 = a - bp - Q_1$ from constraint (3.6).

Hence the problem reduces to:

$$\begin{aligned} \text{Max} \quad \Pi(p) &= (p - d - c_2)(a - bp) + d(\psi + \phi f) + Q_1(c_2 - c_1) \\ \text{subject to} & \\ a - bp - Q_1 &\leq Q_2 \\ p &\geq 0 \end{aligned} \tag{3.9}$$

The objective function (3.9) is concave in $p \geq 0$, since the second order derivative with respect to p is less than zero $\left(\frac{d^2\Pi}{dp^2} = -2b < 0\right)$.

The optimal solution of the problem can be characterized utilizing the first order conditions and taking the constraint $a - bp - Q_1 \leq Q_2$ into account as follows:

$$(q_1^*, q_2^*, p^*) = \begin{cases} \left(Q_1, 0, \frac{a-Q_1}{b}\right) & \text{if } Q_1 \geq \frac{a-b(d+c_2)}{2} \\ \left(Q_1, \frac{a-b(d+c_2)}{2} - Q_1, \frac{a+b(d+c_2)}{2b}\right) & \text{if } Q_1 < \frac{a-b(d+c_2)}{2} \leq (\psi + \phi f) \\ \left(Q_1, Q_2, \frac{a-Q_1-Q_2}{b}\right) & \frac{a-b(d+c_2)}{2} > (\psi + \phi f) \end{cases} \tag{3.10}$$

Analyzing the results of two cases (Equations (3.8) and (3.10)) for the problem to determine q_1, q_2 and p, the optimal solution can be characterized as follows:

$$(q_1^*, q_2^*, p^*) = \begin{cases} \left(\frac{a-b(d+c_1)}{2}, 0, \frac{a+b(d+c_1)}{2b}\right) & \text{if } Q_1 \geq \frac{a-b(d+c_1)}{2} \\ \left(Q_1, 0, \frac{a-Q_1}{b}\right) & \text{if } \frac{a-b(d+c_2)}{2} \leq Q_1 < \frac{a-b(d+c_1)}{2} \\ \left(Q_1, \frac{a-b(d+c_2)}{2} - Q_1, \frac{a+b(d+c_2)}{2b}\right) & \text{if } Q_1 < \frac{a-b(d+c_2)}{2} \leq \psi + \phi f \\ \left(Q_1, Q_2, \frac{a-Q_1-Q_2}{b}\right) & \text{if } \frac{a-b(d+c_2)}{2} > \psi + \phi f \end{cases}$$

This solution shows that, when f is set such that $\frac{a-b(d+c_2)}{2} > \psi + \phi f$ by the remanufacturer, all of the available inspected products would be remanufactured. Whereas, when f is set such that $\frac{a-b(d+c_2)}{2} \leq \psi + \phi f$, the remanufactured units of the two quality levels depend on the quality realized; either all of Type 1 products are remanufactured and some (or none) of Type 2 products are remanufactured, or only some of Type 1 products are remanufactured.

Considering the optimal q_1, q_2 and p, for given Q_1, Q_2 and f values, shown above, the problem for the remanufacturer to set f is formulated as follows:

If the remanufacturer sets the acquisition fee such that $\frac{a-b(d+c_2)}{2} > (\psi + \phi f)$; q_1, q_2 and p are replaced with Q_1, Q_2 and $\frac{a-Q_1-Q_2}{b}$, respectively and his expected profit is given by (noting that $Q_1 = x(\psi + \phi f)$):

$$\begin{aligned} E[\Pi_1(f)] &= -(f + c_I)(\psi + \phi f) + \int_0^1 (p(a - bp) + d(\psi + \phi f - (q_1 - q_2)) - c_1 q_1 - c_2 q_2) g(x) dx \\ &= -(f + c_I)(\psi + \phi f) + \frac{a - bc_2}{b}(\psi + \phi f) - \frac{(\psi + \phi f)^2}{b} - \mu(\psi + \phi f)(c_1 - c_2) \end{aligned}$$

Whereas if he sets the acquisition fee such that $\frac{a-b(d+c_2)}{2} \leq (\psi + \phi f)$, his profit, given the realized values of $Q_1 = x(\psi + \phi f)$ and $Q_2 = (1 - x)(\psi + \phi f)$, is:

$$\Pi_2(f) = -(f + c_I + d)(\psi + \phi f) + \begin{cases} \frac{(a-b(d+c_1))^2}{4b} & x \geq \frac{a-b(d+c_1)}{2(\psi+\phi f)} \\ \frac{x(\psi+\phi f)(a-b(d+c_1))-(x(\psi+\phi f))^2}{b} & \frac{a-b(d+c_2)}{2(\psi+\phi f)} \leq x < \frac{a-b(d+c_1)}{2(\psi+\phi f)} \\ \frac{(a-b(d+c_2))^2}{4b} + x(\psi + \phi f)(c_2 - c_1) & x < \frac{a-b(d+c_2)}{2(\psi+\phi f)} \end{cases}$$

Taking expectation over $0 \leq x \leq 1$, we get:

$$\begin{aligned} E[\Pi_2(f)] &= -(f + c_I - d)(\psi + \phi f) + \int_{\frac{a-b(d+c_1)}{2(\psi+\phi f)}}^{1} \frac{(a - b(d + c_1))^2}{4b} g(x)dx \\ &+ \int_{\frac{a-b(d+c_2)}{2(\psi+\phi f)}}^{\frac{a-b(d+c_1)}{2(\psi+\phi f)}} \frac{x(\psi + \phi f)(a - b(d + c_1)) - (x(\psi + \phi f))^2}{b} g(x)dx \\ &+ \int_{0}^{\frac{a-b(d+c_2)}{2(\psi+\phi f)}} (\frac{(a - b(d + c_2))^2}{4b} + x(\psi + \phi f)(c_2 - c_1))g(x)dx \end{aligned}$$

Accordingly, the remanufacturer's problem to determine f, with expected profit $\Pi(f)$ can be stated as:

$$\textbf{[SQP]} \quad \text{Max} \quad \Pi(f)$$
$$\text{subject to}$$
$$f \geq 0$$

where

$$\Pi(f) = \begin{cases} E[\Pi_1(f)] & \psi + \phi f < \frac{a-b(d+c_2)}{2} \\ E[\Pi_2(f)] & \text{otherwise} \end{cases} \tag{3.11}$$

Note that $\Pi(f)$ is a piecewise function.

Lemma 3.2.1. $\Pi(f)$ *is continuously differentiable at* $\bar{f} = \frac{\frac{a-b(d+c_2)}{2} - \psi}{\phi}$.

Proof:

(i) The value of $E[\Pi_1(f)]$ and $E[\Pi_2(f)]$ are equal at $\bar{f}$, proving the continuity of $\Pi(f)$ as follows:

$$
\begin{aligned}
\Pi(\bar{f}) = E[\Pi_1(\bar{f})] = E[\Pi_2(\bar{f})] &= -\frac{a-b(d+c_2)}{2}(\frac{a-b(d+c_2)}{2\phi} - \frac{2\psi}{\phi} + c_I) \\
&+ \frac{a-b(d+c_2)}{2b}\frac{(a-bc_2-d)}{2}) \\
&- \mu\frac{a-b(d+c_2)}{2}(c_1-c_2)
\end{aligned}
$$

(ii) The value of the first order derivatives of $E[\Pi_1(f)]$ and $E[\Pi_2(f)]$, shown below, are equal at $\bar{f}$.

$$
\begin{aligned}
\frac{dE[\Pi_1(f)]}{df} &= \phi(c_2-c_1)\mu + \phi\left(\frac{a-2(\psi+\phi f)}{b} - c_2 - f - c_I\right) - (\psi+\phi f) \\
\frac{dE[\Pi_2(f)]}{df} &= \int_0^{\frac{a-b(d+c_2)}{2(\psi+\phi f)}} \phi(c_2-c_1)xg(x)dx \\
&+ \int_{\frac{a-b(d+c_2)}{2(\psi+\phi f)}}^{\frac{a-b(d+c_1)}{2(\psi+\phi f)}} \phi\left(\frac{(a-b(d+c_1))-2x(\psi+\phi f)}{b}\right)xg(x)dx \\
&- (\psi+\phi f) - \phi(c_I - d + f)
\end{aligned}
$$

By replacing f with $\bar{f} = \frac{\frac{a-b(d+c_2)}{2}-\psi}{\phi}$;

$$
\frac{dE[\Pi_2(f)]}{df}\Big|_{f=\bar{f}} - \frac{dE[\Pi_1(f)]}{df}\Big|_{f=\bar{f}} = 0 \tag{3.12}
$$

Hence, we can conclude that $\Pi(f)$ is differentiable at $f = \bar{f}$. □

Lemma 3.2.2. $\Pi(f)$ *is concave in* $f \geq 0$.

Proof: The second order derivatives of $E[\Pi_1(f)]$ and $E[\Pi_2(f)]$ with respect to $f \geq 0$ are shown below and they are always less than zero, hence we can conclude that both $\Pi_1(f)$ and $\Pi_2(f)$ are concave in $f \geq 0$.

$$\frac{d^2E[\Pi_1(f)]}{df^2} = -\phi^2 - 1 < 0$$

$$\frac{d^2E[\Pi_2(f)]}{df^2} = \int_{\frac{a-b(d+c_2)}{2(\psi+\phi f)}}^{\frac{a-b(d+c_1)}{2(\psi+\phi f)}} -\frac{2\phi^2}{b}x^2 g(x)dx - 2\phi < 0$$

Since $\Pi(f)$ is continuously differentiable at $f = \bar{f}$ (Lemma 3.2.1), the function $\Pi(f)$ is also concave in $f \geq 0$. □

Lemma 3.2.1 and 3.2.2 enable us to characterize the optimal solution to SQP.

Equality of the first order derivatives of $E[\Pi_1(f)]$ and $E[\Pi_2(f)]$ at$\bar{f}$ (Equation 3.12) means both parts of the remanufacturer's piecewise profit function ($E[\Pi_1(f)]$ and $E[\Pi_2(f)]$) behave in the same direction at $\bar{f}$. In other words; at $\bar{f}$, if $E[\Pi_1(f)]$ is increasing, then $E[\Pi_2(f)]$ is also increasing and if $E[\Pi_1(f)]$ is decreasing, then $E[\Pi_2(f)]$ is also decreasing. We know that $E[\Pi_1(f)]$ is valid when $f < \frac{\frac{a-b(d+c_2)}{2}-\psi}{\phi}$ and $E[\Pi_2(f)]$ is valid when $f > \frac{\frac{a-b(d+c_2)}{2}-\psi}{\phi}$.

Let f' and f'' denote the solution to ($\frac{dE[\Pi_1(f)]}{df} = 0$, $\frac{dE[\Pi_2(f)]}{df} = 0$), respectively. Then, we have the following theorem.

Theorem 3.2.3. *Let f^* denote the optimal solution to* ***[SQP]****. If $(\psi + \phi f') < \frac{a-b(d+c_2)}{2}$, $f^* = f'$. Otherwise, if $\frac{dE[\Pi_1(f)]}{df}|_{f=\bar{f}} = \frac{dE[\Pi_2(f)]}{df}|_{f=\bar{f}} = 0$, $f^* = \bar{f}$. Otherwise, $f^* = f''$.*

Proof: If $(\psi + \phi f') < \frac{a-b(d+c_2)}{2}$, we have $\frac{dE[\Pi_1(f)]}{df}|_{f=\bar{f}} < 0$. Then, by Equation (3.12) $\frac{dE[\Pi_2(f)]}{df}|_{f=\bar{f}} < 0$. Hence, f' is optimal. If $(\psi + \phi f') = \frac{a-b(d+c_2)}{2}$, then $\frac{dE[\Pi_1(f)]}{df}|_{f=\bar{f}} = \frac{dE[\Pi_2(f)]}{df}|_{f=\bar{f}} = 0$, then $f^* = \bar{f}$. Otherwise, if $(\psi + \phi f') > \frac{a-b(d+c_2)}{2}$, then $E[\Pi_1(f)]$ and $E[\Pi_2(f)]$ is increasing at $f = \bar{f}$; hence, $f^* = f''$. □

Corollary 3.2.4. *When the following conditions hold together, then the profits of simultaneous and sequential pricing models are equal:*

(i) $\frac{dE[\Pi_1(f)]}{df}|_{f=\bar{f}} = \frac{dE[\Pi_2(f)]}{df}|_{f=\bar{f}} < 0$ for sequential pricing. (f' is optimal)

(ii) $a - bp^* = \psi + \phi f^*$ for simultaneous pricing.

Therefore, it is not necessary to postpone the decision of the selling price under these conditions.

Proof: $E[\Pi_1(f)]$ in sequential pricing (3.11) when condition (i) holds and profit of the simultaneous pricing when condition (ii) holds (3.4) are equal. Profits are as follows:

For sequential pricing,

$$E[\Pi_1(f)] = -(f + c_I)(\psi + \phi f) + \frac{a - bc_2}{b}(\psi + \phi f) - \frac{(\psi + \phi f)^2}{b} - \mu(\psi + \phi f)(c_1 - c_2)$$

For simultaneous pricing,

$$\begin{aligned} \Pi(f) &= (\psi + \phi f)(f + c_I) + \frac{a(\psi + \phi f)}{b} - \frac{(\psi + \phi f)^2}{b} \\ &- (c_1 - c_2)(\psi + \phi f)\mu - c_2(\psi + \phi f) \end{aligned}$$

□

Simultaneous pricing model is solved as a single-variable problem when constraint (3.3) is binding, thus $a - bp = \psi + \phi f$. This equality means all of the supply of cores are remanufactured and sold with no salvaging. In sequential pricing, if the parts of the piecewise profit function, $E[\Pi_1(f)]$ and $E[\Pi_2(f)]$ are in the same direction at $f = \bar{f}$, then f' is found to be optimal. Therefore, when both conditions hold, there is no need to determine the selling price after the inspection process, since profit values and f, p, q_1 and q_2 are the same. This means that, quality information is ineffective on the pricing decisions and profit when the defined conditions hold for simultaneous and sequential pricing models.

CHAPTER 4

ANALYSIS OF DECENTRALIZED SETTING

In this chapter, we consider a decentralized supply chain consisting of a remanufacturer and a collector. The collector is responsible from collection and inspection of the core products and announces the quantities of two types of quality. The remanufacturer buys collected products from the collector at certain transfer prices to satisfy the demand. The unsold inspected units are salvaged by the collector. The transfer prices between the collector and the remanufacturer for different quality levels are denoted by w_1 and w_2.

In this setting, we analyze three models that result from different decision making sequences and structures, yielding different information availability situations. All of the models are Stackelberg games with collector as the leader and remanufacturer as the follower and they are explained in the following sections: In Section 4.1, we consider the case with exogenous transfer prices, that is, w_1 and w_2 are given to our model. In Section 4.2, the collector sets the transfer prices before inspection of the cores while in Section 4.3, we focus on the case where the collector has quality information prior to determining the transfer prices.

4.1 Exogenous Transfer Prices (EP)

In exogenous transfer prices model, we assume that the transfer prices between the collector and the remanufacturer for both quality levels are given. This model is analyzed in order to use in the analysis of the following two models of the decentralized setting: Collector's pricing before inspection and after inspection. The sequence of decisions and events is as follows:

1. The collector determines the acquisition fee, f, and collects the corresponding returns.

2. The collector inspects all of the returns incurring unit inspection cost of c_I and announces the quantity available for both quality levels, Q_1 and Q_2.

3. The remanufacturer sets the selling price, p, orders q_1 ($\leq Q_1$) units of Type 1 products, and q_2 ($\leq Q_2$) units of Type 2 products from the collector and pays w_1 and w_2 per unit, respectively. The collector salvages the unsold items with unit revenue, d.

Note that since we have two decision makers that make sequential decisions, the overall situation can be considered as a sequential game. Backward induction is used to characterize the optimal actions of the remanufacturer and the collector. "Backward induction is an iterative process for solving finite extensive form or sequential games. First, one determines the optimal strategy of the player who makes the last move of the game. Then, the optimal action of the next-to-last moving player is determined taking the last player's action as given. The process continues in this way backwards in time until all players' actions have been determined. Effectively, the Nash equilibrium of each subgame of the original game is determined."(Shor (2005))

Note that while determining the acquisition fee, hence the quantity of returns, the collector will incorporate the optimal decisions of the remanufacturer. Thus, first we determine p, q_1 and q_2 for the remanufacturer, given Q_1, Q_2 and f. Then we determine f for the collector by incorporating optimal p, q_1 and q_2.

It should also be noted that the quantity of the orders given by the remanufacturer depends on the aggregate cost of a remanufactured unit, the sum of transfer price and remanufacturing cost, $w_i + c_i$, $i = 1, 2$. If $w_1 + c_1$ is less than $w_2 + c_2$, the remanufacturer will prefer Type 1 items and order Type 2 items only if Type 1 items are depleted and vice versa. Hence, we will analyze the exogenous transfer prices model under two cases: (i) Type 1 is more economical (ii) Type 2 is more economical.

4.1.1 Type 1 is more economical

In this section, we consider the case where $w_1 + c_1 \leq w_2 + c_2$. We also assume that $a - b(c_1 + w_1) > 0$, to make the remanufacturing option profitable at least for more economical class of returns and prevent salvaging all of the supply.

The remanufacturer's problem (**[RP]**) to determine p, q_1 and q_2, for given Q_1 and Q_2 values, with profit $\Pi_R(p, q_1, q_2)$, can be formulated as follows:

$$
\begin{aligned}
\textbf{[RP]} \qquad\qquad \text{Max} \quad & \Pi_R(p, q_1, q_2) = p(a - bp) - (c_1 + w_1)q_1 - (c_2 + w_2)q_2 \\
& \text{subject to} \\
& q_1 \leq Q_1 \qquad (4.1) \\
& q_2 \leq Q_2 \\
& q_1 + q_2 = a - bp \qquad (4.2) \\
& p, q_1, q_2 \geq 0
\end{aligned}
$$

In the optimal solution to the remanufacturer's problem, q_2 can be positive only if constraint (4.1) is binding since the remanufacturer would never utilize Type 2 items before depleting Type 1 items (since $w_1 + c_1 \leq w_2 + c_2$). Hence, we can solve the problem by decomposing it to two cases: (i) $q_1 \leq Q_1$ and $q_2 = 0$ (ii) $q_1 = Q_1$ and $q_2 \geq 0$.

(i) When $q_1 \leq Q_1$ and $q_2 = 0$, we have $q_1 = a - bp$ from constraint (4.2). Hence, [RP] reduces to:

$$
\begin{aligned}
\text{Max} \quad \Pi_R(q_1) = \frac{a - q_1}{b} q_1 - (c_1 + w_1)q_1 \qquad (4.3) \\
\text{subject to} \\
0 \leq q_1 \leq Q_1
\end{aligned}
$$

The objective function (4.3) is concave in $q_1 \geq 0$, since the second order derivative with respect to q_1 is less than zero $\left(\frac{d^2\Pi}{dq_1^2} = \frac{-2}{b} < 0\right)$.

The optimal solution of the problem can be characterized utilizing the first order conditions and taking the constraint $q_1 \leq Q_1$ into account as follows:

$$
(q_1^*, q_2^*, p^*) = \begin{cases} \left(\frac{a-b(c_1+w_1)}{2}, 0, \frac{a+b(c_1+w_1)}{2b}\right) & \text{if } Q_1 \geq \frac{a-b(c_1+w_1)}{2} \\ \left(Q_1, 0, \frac{a-Q_1}{b}\right) & \text{otherwise} \end{cases} \qquad (4.4)
$$

(ii) When $q_1 = Q_1$ and $q_2 \geq 0$, we have $q_2 = a - bp - Q_1$ from constraint (4.2). Hence, [RP] reduces to:

$$\text{Max } \Pi_R(p) = p(a-bp) - (c_1+w_1)Q_1 - (c_2+w_2)(a-bp-Q_1) \tag{4.5}$$

subject to

$$a - bp - Q_1 \le Q_2$$

$$p \ge 0$$

The objective function (4.5) is concave in $p \ge 0$, since the second order derivative with respect to p is less than zero $\left(\frac{d^2\Pi}{dp^2} = -2b < 0\right)$.

The optimal solution of the problem can be characterized utilizing the first order conditions and taking the constraint $a - bp - Q_1 \le Q_2$ into account as follows:

$$(q_1^*, q_2^*, p^*) = \begin{cases} \left(Q_1, 0, \frac{a-Q_1}{b}\right) & \text{if } Q_1 \ge \frac{a-b(c_2+w_2)}{2} \\ \left(Q_1, \frac{a-b(c_2+w_2)}{2} - Q_1, \frac{a+b(c_2+w_2)}{2b}\right) & \text{if } Q_1 < \frac{a-b(c_2+w_2)}{2} \le (\psi+\phi f) \\ \left(Q_1, Q_2, \frac{a-Q_1-Q_2}{b}\right) & \frac{a-b(c_2+w_2)}{2} > (\psi+\phi f) \end{cases} \tag{4.6}$$

Analyzing the optimal solutions of two cases, (Equations (4.4) and (4.6)), the remanufacturer's optimal p, q_1 and q_2 decisions, for given values of Q_1, Q_2 and f, are:

$$(q_1^*, q_2^*, p^*) = \begin{cases} \left(\frac{a-b(c_1+w_1)}{2}, 0, \frac{a+b(c_1+w_1)}{2b}\right) & \text{if } Q_1 \ge \frac{a-b(c_1+w_1)}{2} \\ \left(Q_1, 0, \frac{a-Q_1}{b}\right) & \text{if } \frac{a-b(c_2+w_2)}{2} \le Q_1 < \frac{a-b(c_1+w_1)}{2} \\ \left(Q_1, \frac{a-b(c_2+w_2)}{2} - Q_1, \frac{a+b(c_2+w_2)}{2b}\right) & \text{if } Q_1 < \frac{a-b(c_2+w_2)}{2} \le (\psi+\phi f) \\ \left(Q_1, Q_2, \frac{a-Q_1-Q_2}{b}\right) & \text{if } \frac{a-b(c_2+w_2)}{2} > (\psi+\phi f) \end{cases} \tag{4.7}$$

The collector incorporates the optimal response of the remanufacturer given by Equation (4.7) into his decision making process to determine f.

Note that q_1 and q_2 depends on the realized quantities of Type 1 and Type 2 items and the relation between w_2 and f (see Equation (4.7)). If the collector sets the acquisition fee such that $\frac{a-b(c_2+w_2)}{2} > (\psi + \phi f)$, his expected profit is given by:

$$\begin{aligned} E[\Pi_1(f)] &= -(f+c_I)(\psi+\phi f) + \int_0^1 (q_1 w_1 + q_2 w_2) g(x) dx \\ &= (\psi+\phi f)(w_1\mu + w_2 - w_2\mu - f - c_I) \end{aligned} \tag{4.8}$$

Otherwise, if he sets the acquisition fee such that $\frac{a-b(c_2+w_2)}{2} \leq (\psi + \phi f)$, his profit, given the realized values of $Q_1 = x(\psi + \phi f)$ and $Q_2 = (1 - x)(\psi + \phi f)$, is:

$$
\begin{aligned}
\Pi_2(f) &= -(f + c_I + d)(\psi + \phi f) \\
&+ \begin{cases} \frac{a-b(c_1+w_1)}{2}(w_1 - d) & x \geq \frac{a-b(c_1+w_1)}{2(\psi+\phi f)} \\ x(\psi + \phi f)(w_1 - d) & \frac{a-b(c_2+w_2)}{2(\psi+\phi f)} \leq x < \frac{a-b(c_1+w_1)}{2(\psi+\phi f)} \\ x(\psi + \phi f)(w_1 - w_2) + (w_2 - d)\frac{a-b(c_2+w_2)}{2} & x < \frac{a-b(c_2+w_2)}{2(\psi+\phi f)} \end{cases}
\end{aligned} \tag{4.9}
$$

Taking expectation over $0 \leq x \leq 1$, we get:

$$
\begin{aligned}
E[\Pi_2(f)] &= -(f + c_I - d)(\psi + \phi f) + \int_{\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}}^{1} \frac{a - b(c_1 + w_1)}{2}(w_1 - d)g(x)dx \\
&+ \int_{\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}}^{\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}} x(\psi + \phi f)(w_1 - d)g(x)dx \\
&+ \int_{0}^{\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}} (x(\psi + \phi f)(w_1 - w_2) + (w_2 - d)\frac{a - b(c_2 + w_2)}{2})g(x)dx
\end{aligned} \tag{4.10}
$$

Then, the collector's problem to determine f, with expected profit $\Pi_C(f)$, can be stated as:

$$
\begin{aligned}
\textbf{[CP]} \quad & \text{Max} \quad \Pi_C(f) \\
& \text{subject to} \\
& \quad f \geq 0
\end{aligned}
$$

where

$$
\Pi_C(f) = \begin{cases} E[\Pi_1(f)] & \psi + \phi f < \frac{a-b(c_2+w_2)}{2} \\ E[\Pi_2(f)] & \text{otherwise} \end{cases} \tag{4.11}
$$

Note that $\Pi_C(f)$ is a piecewise function.

Lemma 4.1.1. *$\Pi_C(f)$ is continuous, but not differentiable at $f = \bar{f} = \frac{\frac{a-b(c_2+w_2)}{2} - \psi}{\phi}$.*

Proof:

(i) The values of $E[\Pi_1(f)]$ and $E[\Pi_2(f)]$ are equal at $\bar{f}$, proving the continuity of $\Pi_C(f)$:

$$
\begin{aligned}
\Pi_C(\bar{f}) &= E[\Pi_1(\bar{f})] = E[\Pi_2(\bar{f})] \\
&= \frac{a-b(c_2+w_2)}{2}\Big(-\frac{\frac{a-b(c_2+w_2)}{2}-\psi}{\phi} - c_I + w_1\mu - w_2\mu + w_2\Big)
\end{aligned}
$$

(ii) The value of the first order derivatives of $E[\Pi_1(f)]$ and $E[\Pi_2(f)]$, which are shown below, are not equal at $\bar{f}$.

$$
\begin{aligned}
\frac{dE[\Pi_1(f)]}{df} &= -\psi - \phi(2f + c_I - w_2) + \phi(w_1 - w_2)\mu \\
\frac{dE[\Pi_2(f)]}{df} &= - \quad \psi - 2\phi f - c_I\phi + d\phi + \phi(w_1 - w_2)\int_0^{\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}} xg(x)dx \\
&+ \quad \phi(w_1 - d)\int_{\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}}^{\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}} xg(x)dx
\end{aligned}
$$

$$
\frac{dE[\Pi_2(f)]}{df}\Big|_{f=\bar{f}} - \frac{dE[\Pi_1(f)]}{df}\Big|_{f=\bar{f}} = \phi(d - w_2) < 0. \tag{4.12}
$$

Hence, we can conclude that $\Pi_C(f)$ is not differentiable at $f = \bar{f}$. □

Lemma 4.1.2. $E[\Pi_1(f)]$ *and* $E[\Pi_2(f)]$ *are concave in* $f \geq 0$.

Proof: Second order derivatives of $E[\Pi_1(f)]$ and $E[\Pi_2(f)]$, are always less than zero as shown below:

$$
\begin{aligned}
\frac{d^2E[\Pi_1(f)]}{df^2} &= -2\phi \\
\frac{d^2E[\Pi_2(f)]}{df^2} &= - \quad 2\phi - \frac{\phi^2}{(\psi+\phi f)^3}(w_1 - d)\Big(\frac{a-b(c_1+w_1)}{2}\Big)^2 g\left(\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}\right) \\
&- \quad \frac{\phi^2}{(\psi+\phi f)^3}(w_2 - d)\Big(\frac{a-b(c_2+w_2)}{2}\Big)^2 g\left(\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}\right)
\end{aligned}
$$

Hence we can conclude that $E[\Pi_1(f)]$ and $E[\Pi_2(f)]$ are concave functions in $f \geq 0$. □

$\Pi_C(f)$ is found to be not differentiable at $f = \bar{f}$. However, we can still characterize the optimal solution to **[CP]** using Equation (4.12).

Let f' and f'' denote the solution to ($\frac{dE[\Pi_1(f)]}{df} = 0$, $\frac{dE[\Pi_2(f)]}{df} = 0$), respectively. Then, we have the following theorem:

Theorem 4.1.3. *Let f^* denote the optimal solution to **[CP]**. If $(\psi + \phi f') < \frac{a-b(c_2+w_2)}{2}$, $f^* = f'$. If $\frac{dE[\Pi_2(f)]}{df}|_{f=\bar{f}} < 0$, $f^* = \bar{f}$. Otherwise, $f^* = f''$.*

Proof: If $(\psi + \phi f') < \frac{a-b(c_2+w_2)}{2}$, we have $\frac{dE[\Pi_1(f)]}{df}|_{f=\bar{f}} < 0$. Then, by Equation 4.12, $\frac{dE[\Pi_2(f)]}{df}|_{f=\bar{f}} < 0$. Hence, f' is optimal. If $(\psi + \phi f') > \frac{a-b(c_2+w_2)}{2}$, then $\frac{dE[\Pi_1(f)]}{df}|_{f=\bar{f}} > 0$, that is, $E[\Pi_1(f)]$ is increasing at $f = \bar{f}$. In this case, if $\frac{dE[\Pi_2(f)]}{df}|_{f=\bar{f}} < 0$, then $f^* = \bar{f}$. Otherwise, $E[\Pi_2(f)]$ is increasing at $f = \bar{f}$; hence, $f^* = f''$.

Theorem 4.1.3 is depicted in Figure 4.1. If $E[\Pi_2(f)]$ is decreasing at $\bar{f}$, $E[\Pi_1(f)]$ is either decreasing more or increasing at $\bar{f}$. In (a), the case in which $E[\Pi_2(f)]$ is decreasing and $E[\Pi_1(f)]$ is decreasing at $\bar{f}$ is shown, hence f' is optimal. In (c), $E[\Pi_1(f)]$ is increasing, thus $\bar{f}$ is optimal. If $E[\Pi_2(f)]$ is increasing at $\bar{f}$, $E[\Pi_1(f)]$ is also increasing at $\bar{f}$ with a greater slope. In (b), this situation is shown with $f^* = \bar{f}$. (d) reveals the impossible case in which $E[\Pi_2(f)]$ is increasing and $E[\Pi_1(f)]$ is decreasing at $\bar{f}$.

4.1.2 Type 2 is more economical

In this section, we consider the case where $w_2 + c_2 < w_1 + c_1$. We also assume that $a - b(c_2 + w_2) > 0$, in order to make the remanufacturing option profitable at least for more economical class of returns and prevent salvaging all of the supply.

The remanufacturer's problem **[RP]** to determine p, q_1 and q_2, for given Q_1 and Q_2 values, with profit $\Pi_R(p, q_1, q_2)$, can be formulated as follows:

$$\textbf{[RP]} \qquad \text{Max} \quad \Pi_R(p, q_1, q_2) = p(a - bp) - (c_1 + w_1)q_1 - (c_2 + w_2)q_2$$

$$\text{subject to}$$

$$q_1 \leq Q_1$$

$$q_2 \leq Q_2 \tag{4.13}$$

$$q_1 + q_2 = a - bp \tag{4.14}$$

$$p, q_1, q_2 \geq 0$$

In the optimal solution to the remanufacturer's problem, q_1 can be positive only if constraint (4.13) is binding since the remanufacturer would never utilize Type 1 items before depleting

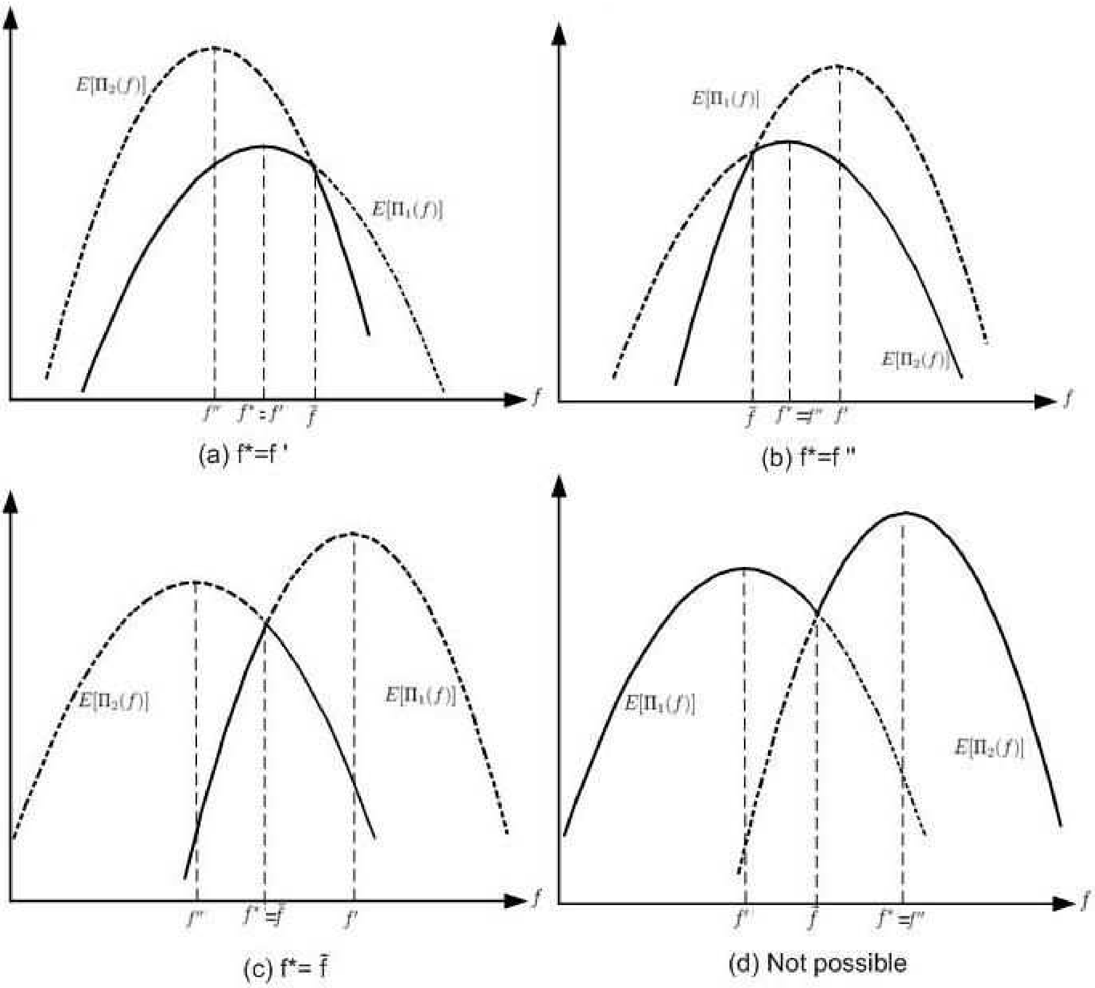


Figure 4.1: Expected Profit for given Unit Acquisition Fee (Summary for Theorem 4.1.3)

Type 2 items (since $w_2 + c_2 < w_1 + c_1$). Hence, we can solve the problem by decomposing it to two cases: (i) $q_2 \leq Q_2$ and $q_1 = 0$ (ii) $q_2 = Q_2$ and $q_1 \geq 0$.

(i) When $q_2 \leq Q_2$ and $q_1 = 0$, we have $q_2 = a - bp$ from constraint (4.14). Hence, [RP] reduces to:

$$\text{Max } \Pi_R(q_2) = \frac{a - q_2}{b} q_2 - (c_2 + w_2) q_2 \tag{4.15}$$

$$\text{subject to}$$

$$0 \leq q_2 \leq Q_2$$

The objective function (4.15) is concave in $q_2 \geq 0$, since the second order derivative with respect to q_2 is less than zero $\left(\frac{d^2\Pi}{dq_1^2} = \frac{-2}{b} < 0\right)$.

The optimal solution of the problem can be characterized utilizing the first order conditions and taking the constraint $q_2 \leq Q_2$ into account as follows:

$$(q_1^*, q_2^*, p^*) = \begin{cases} \left(0, \frac{a-b(c_2+w_2)}{2}, \frac{a+b(c_2+w_2)}{2b}\right) & \text{if } Q_2 \geq \frac{a-b(c_2+w_2)}{2} \\ \left(0, Q_2, \frac{a-Q_2}{b}\right) & \text{otherwise} \end{cases} \tag{4.16}$$

(ii) When $q_2 = Q_2$ and $q_1 \geq 0$, we have $q_1 = a - bp - Q_2$ from constraint (4.14). Hence, [RP] reduces to:

$$\text{Max} \quad \Pi_R(p) = p(a - bp) - (c_1 + w_1)(a - bp - Q_2) - (c_2 + w_2)Q_2 \tag{4.17}$$

$$\text{subject to}$$

$$a - bp - Q_2 \leq Q_1$$

$$p \geq 0$$

The objective function (4.17) is concave in $p \geq 0$, since the second order derivative with respect to p is less than zero $\left(\frac{d^2\Pi}{dp^2} = -2b < 0\right)$.

The optimal solution of the problem can be characterized utilizing the first order conditions and taking the constraint $a - bp - Q_2 \leq Q_1$ into account as follows:

$$(q_1^*, q_2^*, p^*) = \begin{cases} \left(0, Q_2, \frac{a-Q_2}{b}\right) & \text{if } Q_2 \geq \frac{a-b(c_1+w_1)}{2} \\ \left(\frac{a-b(c_1+w_1)}{2} - Q_2, Q_2, \frac{a+b(c_1+w_1}{2b}\right) & \text{if } Q_2 < \frac{a-b(c_1+w_1)}{2} \leq (\psi + \phi f) \\ \left(Q_1, Q_2, \frac{a-Q_1-Q_2}{b}\right) & \text{if } \frac{a-b(c_1+w_1)}{2} > (\psi + \phi f) \end{cases} \tag{4.18}$$

Analyzing the optimal solutions of two cases (Equations (4.16) and (4.18)), the remanufacturer's optimal p, q_1 and q_2 decisions, for given values of Q_1, Q_2 and f, are:

$$(q_1^*, q_2^*, p^*) = \begin{cases} \left(0, \frac{a-b(c_2+w_2)}{2}, \frac{a+b(c_2+w_2)}{2b}\right) & \text{if } Q_2 \geq \frac{a-b(c_2+w_2)}{2} \\ \left(0, Q_2, \frac{a-Q_2}{b}\right) & \text{if } \frac{a-b(c_1+w_1)}{2} \leq Q_2 < \frac{a-b(c_2+w_2)}{2} \\ \left(\frac{a-b(c_1+w_1)}{2} - Q_2, Q_2, \frac{a+b(c_1+w_1}{2b}\right) & \text{if } Q_2 < \frac{a-b(c_1+w_1)}{2} \leq (\psi + \phi f) \\ \left(Q_1, Q_2, \frac{a-Q_1-Q_2}{b}\right) & \text{if } \frac{a-b(c_1+w_1)}{2} > (\psi + \phi f) \end{cases} \tag{4.19}$$

The collector incorporates the optimal response of the remanufacturer given by Equation (4.19) into his decision making process to determine f.

Note that q_1 and q_2 depends on the realized quantities of Type 1 and Type 2 items and the relation between w_1 and f (see Equation (4.19)). If the collector sets the acquisition fee such that $\frac{a-b(c_1+w_1)}{2} > (\psi + \phi f)$, his expected profit is given by:

$$\begin{aligned} E[\Pi_1(f)] &= -(f + c_I)(\psi + \phi f) + \int_0^1 (q_1 w_1 + q_2 w_2) g(x) dx \\ &= (\psi + \phi f)(w_1 \mu + w_2 - w_2 \mu - f - c_I) \end{aligned} \tag{4.20}$$

Otherwise, if he sets the acquisition fee such that $\frac{a-b(c_1+w_1)}{2} \leq (\psi + \phi f)$, his profit, given the realized values of $Q_1 = x(\psi + \phi f)$ and $Q_2 = (1 - x)(\psi + \phi f)$, is:

$$\begin{aligned} \Pi_2(f) &= -(f + c_I + d)(\psi + \phi f) \\ &+ \begin{cases} \frac{a-b(c_2+w_2)}{2}(w_2 - d) & x \geq \frac{a-b(c_2+w_2)}{2(\psi+\phi f)} \\ x(\psi + \phi f)(w_2 - d) & \frac{a-b(c_1+w_1)}{2(\psi+\phi f)} \leq x < \frac{a-b(c_2+w_2)}{2(\psi+\phi f)} \\ x(\psi + \phi f)(w_2 - w_1) + (w_1 - d)\frac{a-b(c_1+w_1)}{2} & x < \frac{a-b(c_1+w_1)}{2(\psi+\phi f)} \end{cases} \end{aligned} \tag{4.21}$$

Taking expectation over $0 \leq x \leq 1$, we get

$$
\begin{aligned}
E[\Pi_2(f)] &= -(f + c_I - d)(\psi + \phi f) + \int_{\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}}^{1} \frac{a - b(c_2 + w_2)}{2}(w_2 - d)g(x)dx \\
&+ \int_{\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}}^{\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}} x(\psi + \phi f)(w_2 - d)g(x)dx \\
&+ \int_{0}^{\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}} (x(\psi + \phi f)(w_2 - w_1) + (w_1 - d)\frac{a - b(c_1 + w_1)}{2})g(x)dx \qquad (4.22)
\end{aligned}
$$

Then, the collector's problem to determine f, with expected profit $\Pi_C(f)$, can be stated as:

$$
\begin{array}{lll}
\textbf{[CP]} & \text{Max} & \Pi_C(f) \\
& \text{subject to} & \\
& & f \geq 0
\end{array}
$$

where

$$
\Pi_C(f) = \begin{cases} E[\Pi_1(f)] & \psi + \phi f < \frac{a-b(c_1+w_1)}{2} \\ E[\Pi_2(f)] & \text{otherwise} \end{cases} \qquad (4.23)
$$

Note that $\Pi_C(f)$ is a piecewise function.

Lemma 4.1.4. $\Pi_C(f)$ *is continuous, but not differentiable at* $f = \bar{f} = \frac{\frac{a-b(c_1+w_1)}{2} - \psi}{\phi}$.

Proof:

(i) The values of $E[\Pi_1(\bar{f})]$ and $E[\Pi_2(\bar{f})]$ are equal at $\bar{f}$, proving the continuity of $\Pi_C(f)$:

$$
\begin{aligned}
\Pi_C(\bar{f}) &= E[\Pi_1(\bar{f})] = E[\Pi_2(\bar{f})] \\
&= \frac{a - b(c_1 + w_1)}{2}\left(-\frac{\frac{a-b(c_1+w_1)}{2} - \psi}{\phi} - c_I + w_1\mu - w_2\mu + w_2\right)
\end{aligned}
$$

(ii) The value of the first order derivatives of $E[\Pi_1(f)]$ and $E[\Pi_2(f)]$, which are shown below, are not equal at $\bar{f}$.

$$\frac{dE[\Pi_1(f)]}{df} = -\psi - \phi(2f + c_I - w_2) + \phi(w_1 - w_2)\mu$$

$$\begin{aligned}\frac{dE[\Pi_2(f)]}{df} = & - \psi - 2\phi f - c_I\phi + d\phi + \phi(w_2 - w_1)\int_{1-\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}}^{1} (1-x)g(x)dx \\ & + \phi(w_2 - d)\int_{1-\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}}^{1-\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}} (1-x)g(x)dx\end{aligned}$$

$$\frac{dE[\Pi_2(f)]}{df}\,|_{f=\bar{f}} - \frac{dE[\Pi_1(f)]}{df}\,|_{f=\bar{f}} = \phi(d - w_1) < 0. \tag{4.24}$$

Hence, we can conclude that $\Pi_C(f)$ is not differentiable at $f = \bar{f}$. □

Lemma 4.1.5. *$E[\Pi_1(f)]$ and $E[\Pi_2(f)]$ are concave in $f \geq 0$.*

Proof: Second order derivatives of $E[\Pi_1(f)]$ and $E[\Pi_2(f)]$ are always less than zero as shown below:

$$\begin{aligned}\frac{d^2E[\Pi_1(f)]}{df^2} &= -2\phi \\ \frac{d^2E[\Pi_2(f)]}{df^2} &= - 2\phi - \frac{\phi^2}{(\psi+\phi f)^3}(w_1 - d)(\frac{a-b(c_1+w_1)}{2})^2 g\left(1 - \frac{a-b(c_1+w_1)}{2(\psi+\phi f)}\right) \\ & - \frac{\phi^2}{(\psi+\phi f)^3}(w_2 - d)(\frac{a-b(c_2+w_2)}{2})^2 g\left(1 - \frac{a-b(c_2+w_2)}{2(\psi+\phi f)}\right)\end{aligned}$$

Hence we can conclude that $E[\Pi_1(f)]$ and $E[\Pi_2(f)]$ are concave functions in $f \geq 0$. □

$\Pi_C(f)$ is found to be not differentiable at $f = \bar{f}$. However, we can still characterize the optimal solution to **[CP]** using Equation (4.24).

Let f' and f'' denote the solution to ($\frac{dE[\Pi_1(f)]}{df} = 0$, $\frac{dE[\Pi_2(f)]}{df} = 0$), respectively. Then, we have the following theorem:

Theorem 4.1.6. *Let f^* denote the optimal solution to **[CP]**. If $(\psi + \phi f') < \frac{a-b(c_2+w_2)}{2}$, $f^* = f'$. If $\frac{dE[\Pi_2(f)]}{df}|_{f=\bar{f}} < 0$, $f^* = \bar{f}$. Otherwise, $f^* = f''$.*

Proof: Proof of Theorem 4.1.6 is similar to the proof of Theorem 4.1.3, since the difference of the first order derivatives of $E[\Pi_1(f)]$ and $E[\Pi_2(f)]$ are less than zero at $\bar{f}$ for both cases. □

4.2 Collector's Pricing Before Inspection (CPBI)

In this section, we assume that the transfer prices between the collector and the remanufacturer for both quality levels are determined by the collector before the inspection process. The sequence of decisions and events is as follows:

1. The collector determines the acquisition fee, f and transfer prices, w_1 and w_2 simultaneously and collects the corresponding returns.
2. The collector inspects all of the returns incurring unit inspection cost of c_I and announces the quantity available for both quality levels, Q_1 and Q_2.
3. The remanufacturer sets the selling price, p, orders $q_1(\leq Q_1)$ units of Type 1, and $q_2(\leq Q_2)$ units of Type 2 products from the collector, and pays w_1 and w_2 per unit, respectively. The collector salvages the unsold items at unit revenue, d.

Backward induction is used to characterize the optimal actions of the remanufacturer and the collector. Note that while determining the acquisition fee and transfer prices, hence the quantity of returns, the collector will incorporate the optimal decisions of the remanufacturer on selling price. Hence, first we determine q_1, q_2 and p for the remanufacturer, for given Q_1, Q_2, f, w_1 and w_2 values. Then, we determine f, w_1 and w_2 for the collector by incorporating optimal q_1, q_2 and p.

We solve two sub-problems dividing the feasible region into two parts according to the realized aggregate cost of a remanufactured unit ($w_i + c_i, i = 1, 2$): (i) Type 1 is more economical (ii) Type 2 is more economical.

4.2.1 Type 1 is more economical

In this section, we consider the case where $c_1 + w_1 \leq c_2 + w_2$ holds. Hence the remanufacturer first buys Type 1 products and then considers Type 2 products only when Type 1 is not sufficient. We also assume that $a - b(c_1 + d) \geq 0$, since otherwise the remanufacturer would never order from the collector.

The remanufacturer's problem is to determine q_1, q_2 and p, for given Q_1, Q_2, f, w_1 and w_2 values as in exogenous pricing model. The only difference from EP is in transfer prices; they are also determined by the collector instead of being exogenous in this model. Therefore, optimal solution to the remanufacturer's problem ([RP]) (Equation 4.7) and collector's problem (4.11) with piecewise profit functions (4.8) and (4.10) in Section 4.1 when Type 1 is more economical is also valid in this case.

The collector's problem to determine f, w_1 and w_2, with expected profit $\Pi_C(f, w_1, w_2)$, can be stated as:

$$\begin{array}{lll} \textbf{[CP]} & \text{Max} & \Pi_C(f, w_1, w_2) \\ & \text{subject to} & \\ & & f, w_1, w_2 \geq 0 \end{array}$$

where

$$\Pi_C(f) = \begin{cases} E[\Pi_1(f, w_1, w_2)] & \psi + \phi f < \frac{a-b(c_2+w_2)}{2} \\ E[\Pi_2(f, w_1, w_2)] & \text{otherwise} \end{cases} \tag{4.25}$$

$$E[\Pi_1(f, w_1, w_2)] = (\psi + \phi f)(w_1\mu + w_2 - w_2\mu - f - c_I)$$

$$\begin{aligned} E[\Pi_2(f, w_1, w_2)] = & -(f + c_I - d)(\psi + \phi f) + \int_{\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}}^{1} \frac{a - b(c_1 + w_1)}{2}(w_1 - d)g(x)dx \\ & + \int_{\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}}^{\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}} x(\psi + \phi f)(w_1 - d)g(x)dx \\ & + \int_{0}^{\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}} (x(\psi + \phi f)(w_1 - w_2) + (w_2 - d)\frac{a - b(c_2 + w_2)}{2})g(x)dx \end{aligned}$$

Lemma 4.2.1. *$E[\Pi_1(f, w_1, w_2)]$ is jointly concave in $f \geq 0, w_1 \geq 0, w_2 \geq 0$.*

Proof: First order and second order partial derivatives of $E[\Pi_1(f, w_1, w_2)]$ with respect to the three decision variables, f, w_1 and w_2, are found as follows:

$$
\begin{aligned}
\frac{\partial E[\Pi_1(f,w_1,w_2)]}{\partial f} &= -\psi - \phi(2f + c_I - w_2) + \phi(w_1 - w_2)\mu \\
\frac{\partial E[\Pi_1(f,w_1,w_2)]}{\partial w_1} &= (\psi + \phi f)\mu \qquad (4.26) \\
\frac{\partial E[\Pi_1(f,w_1,w_2)]}{\partial w_2} &= (\psi + \phi f)(1-\mu) \qquad (4.27) \\
\frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial f^2} &= -2\phi < 0 \\
\frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_1^2} &= \frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_2^2} = 0 \\
\frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial f d w_1} &= \frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_1 \partial f} = \phi\mu \\
\frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial f d w_2} &= \frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_2 \partial f} = \phi - \phi\mu \\
\frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_1 d w_2} &= \frac{\partial^2 E[\Pi_1(f)]}{\partial w_2 d w_1} = 0
\end{aligned}
$$

The Hessian matrix is:

$$
H = \begin{pmatrix}
\frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial f^2} & \frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial f \partial w_1} & \frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial f \partial w_2} \\
\frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_1 d f} & \frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_1^2} & \frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_1 \partial w_2} \\
\frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_2 d f} & \frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_2 \partial w_1} & \frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_2^2}
\end{pmatrix} = \begin{pmatrix} A & B & C \\ B & D & 0 \\ C & 0 & F \end{pmatrix}
$$

Determinant of the Hessian matrix is found to be equal to zero, as shown below:

$$
\begin{aligned}
|\,\mathrm{H}\,| &= ADF + BC0 + CB0 - A0^2 - B^2F - C^2D \\
&= ADF - B^2F - C^2D \\
&= -2\phi 00 - (\phi\mu)^2 0 - (\phi - \phi\mu)^2 0 \\
&= 0
\end{aligned}
$$

$E[\Pi_1(f, w_1, w_2)]$ is jointly concave in $f \geq 0$, $w_1 \geq 0$ and $w_2 \geq 0$ since;

- $\frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial f^2} < 0$
- $\frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_1^2} = 0$
- $\frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_2^2} = 0$

- $\mid H \mid = 0$.

$\square$

Lemma 4.2.2. *w_1 and w_2 should be set the following maximum values in order to maximize $E[\Pi_1(f, w_1, w_2)]$:*

$$\begin{aligned} (w_1)_{max} &= \frac{a - 2(\psi + \phi f)}{b} - c_1 \\ (w_2)_{max} &= \frac{a - 2(\psi + \phi f)}{b} - c_2 \end{aligned}$$

Proof: $\Pi_1(f, w_1, w_2)$ is an increasing function of both w_1 and w_2 since first order derivative functions with respect to w_1 and w_2 (Equations (4.26) and (4.27)) are always greater than zero. So the collector sets the maximum values for these decision variables to maximize his profit.

Maximum value of w_2 is found from $\frac{a-b(c_2+w_2)}{2} > \psi + \phi f$ which is the interval for w_2 and f at which $E[\Pi_1(f, w_1, w_2)]$ is valid in Equation (4.25):

$$\begin{aligned} \frac{a - b(c_2 + w_2)}{2} &> \psi + \phi f \\ w_2 &< \frac{a - 2(\psi + \phi f)}{b} - c_2 \\ (w_2)_{\max} &= \frac{a - 2(\psi + \phi f)}{b} - c_2 \end{aligned} \tag{4.28}$$

Whereas, maximum value of w_1 is found by plugging the maximum value of w_2 into $c_1 + w_1 \leq c_2 + w_2$, the condition required for Type 1 to be more economical:

$$\begin{aligned} c_1 + w_1 &\leq c_2 + w_2 \\ c_1 + w_1 &\leq c_2 + (\frac{a - 2(\psi + \phi f)}{b} - c_2) \\ w_1 &\leq \frac{a - 2(\psi + \phi f)}{b} - c_1 \\ (w_1)_{\max} &= \frac{a - 2(\psi + \phi f)}{b} - c_1 \end{aligned} \tag{4.29}$$

$\square$

Maximum values of w_1 and w_2 are used to reduce $E[\Pi_1(f, w_1, w_2)]$ to a single-variable function as follows:

$$
\begin{aligned}
E[\Pi_1(f, w_1, w_2)] &= (\psi + \phi f)(w_1\mu + w_2 - w_2\mu - f - c_I) \\
E[\Pi_1(f)] &= (\psi + \phi f)((c_2 - c_1)\mu - c_2 - f - c_I + \frac{a - 2(\psi + \phi f)}{b})
\end{aligned}
$$

$E[\Pi_1(f)]$ is concave in $f \geq 0$ since the second order derivative with respect to f is less than zero. $\left(\frac{d^2E[\Pi_1(f)]}{df^2} = -2\phi - 4\frac{\phi^2}{b} < 0\right)$. Hence, optimal f for the first piece of the collector's profit function, $E[\Pi_1(f)]$, can be found by using the first order equation as follows:

$$
\begin{aligned}
\frac{dE[\Pi_1(f)]}{df} &= -\psi - \frac{2\phi\psi}{b} + \phi(c_2 - c_1)\mu - \phi(c_2 + 2f + c_I) \\
&+ \phi\frac{a - 2(\psi + 2\phi f)}{b} = 0 \\
f^* &= \frac{-\psi - \frac{4\phi\psi}{b} + \frac{\phi a}{b} + \phi(c_2 - c_1)\mu - \phi(c_2 + c_I)}{2\phi + 4\frac{\phi^2}{b}} \qquad (4.30)
\end{aligned}
$$

By plugging Equation (4.30) into Equations (4.29) and (4.28), the optimal w_1 and w_2 can be obtained.

For the second piece of the collector's profit function, $E[\Pi_2(f, w_1, w_2)]$, which is valid when $\frac{a-b(c_2+w_2)}{2} < \psi + \phi f$, the first order partial derivatives with respect to f, w_1 and w_2 are found as follows:

$$
\begin{aligned}
\frac{\partial E[\Pi_2(f, w_1, w_2)]}{\partial f} &= - \quad \psi - 2\phi f - c_I\phi + d\phi + \phi(w_1 - w_2)\int_0^{\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}} xg(x)dx \\
&+ \quad \phi(w_1 - d)\int_{\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}}^{\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}} xg(x)dx \qquad (4.31)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E[\Pi_2(f, w_1, w_2)]}{\partial w_1} &= \int_0^{\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}} x(\psi + \phi f)g(x)dx \\
&+ \int_{\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}}^{1} \frac{a - b(c_1 + 2w_1 - d)}{2}g(x)dx \qquad (4.32)
\end{aligned}
$$

$$\frac{\partial E[\Pi_2(f,w_1,w_2)]}{\partial w_2} = \int_0^{\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}} \frac{a-b(c_2+2w_2-d)}{2} g(x)dx - \int_0^{\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}} x(\psi+\phi f)g(x)dx \tag{4.33}$$

To show the concavity of the profit function $E[\Pi_2(f,w_1,w_2)]$, the second order partial derivatives are found as:

$$\begin{aligned}
\frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial f^2} &= -\frac{\phi^2}{(\psi+\phi f)^3}\left((w_1-d)\left(\frac{a-b(c_1+w_1)}{2}\right)^2 g\left(\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}\right)\right. \\
&- \left.(w_2-d)\left(\frac{a-b(c_2+w_2)}{2}\right)^2 g\left(\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}\right)\right) - 2\phi \\
\frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial w_1^2} &= \frac{-b^2(w_1-d)}{4(\psi+\phi f)} g\left(\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}\right) \\
&- \int_{\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}}^{1} bg(x)dx
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial w_2^2} &= \frac{b^2(w_2-d)}{4(\psi+\phi f)} g\left(\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}\right) \\
&- \int_0^{\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}} bg(x)dx
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial f \partial w_1} = \frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial w_1 \partial f} &= \frac{-b\phi(w_1-d)(a-b(c_1+w_1))}{4(\psi+\phi f)^2} g\left(\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}\right) \\
&+ \int_0^{\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}} \phi x g(x)dx
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial f \partial w_2} = \frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial w_2 \partial f} &= \frac{b\phi(w_2-d)(a-b(c_2+w_2))}{4(\psi+\phi f)^2} g\left(\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}\right) \\
&- \int_0^{\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}} \phi x g(x)dx
\end{aligned}$$

$$\frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial w_1 \partial w_2} = \frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial w_2 \partial w_1} = 0$$

Hessian matrix is found for $E[\Pi_2(f, w_1, w_2)]$ as follows:

$$\begin{pmatrix} \frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial f^2} & \frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial f \partial w_1} & \frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial f \partial w_2} \\ \frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial w_1 \partial f} & \frac{d^2 E[\Pi_2(f,w_1,w_2)]}{\partial w_1^2} & \frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial w_1 \partial w_2} \\ \frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial w_2 \partial f} & \frac{d^2 E[\Pi_2(f,w_1,w_2)]}{\partial w_2 \partial w_1} & \frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial w_2^2} \end{pmatrix} = \begin{pmatrix} A & B & C \\ B & D & 0 \\ C & 0 & F \end{pmatrix}$$

Determinant of the Hessian matrix is found as:

$$\begin{aligned} |\,\mathrm{H}\,| &= ADF + BC0 + CB0 - A0^2 - B^2F - C^2D \\ &= ADF - B^2F - C^2D \end{aligned}$$

We couldn't show the joint concavity of $E[\Pi_2(f, w_1, w_2)]$ since the expression for the determinant on Hessian is very complicated.

An exhaustive search algorithm is used to determine the optimal w_1, w_2 and f values in the computational study. The combination of w_1, w_2 and f that gives the maximum collector's expected profit is determined. (The details of the procedure can be seen in Appendix A.)

Finally, for [CP] to determine w_1, w_2 and f, the optimal solutions of $E[\Pi_1(f, w_1, w_2)]$ and $E[\Pi_2(f, w_1, w_2)]$ are compared and the one with the best expected profit is selected as the approximate optimal solution of [CP] for the case when Type 1 is more economical.

4.2.2 Type 2 is more economical

In this section, we consider the case where $c_2 + w_2 < c_1 + w_1$ holds. Hence the remanufacturer first buys Type 2 products and then considers Type 1 products only when Type 2 is not sufficient. We also assume that $a - b(c_1 + d) \geq 0$, since otherwise the remanufacturer would never order from the collector.

The remanufacturer's problem is to determine q_1, q_2 and p, for given Q_1, Q_2, f, w_1 and w_2 values as in exogenous pricing model. The only difference is in transfer prices; they are also determined by the collector instead of being exogenous in this model. Therefore, optimal solution to the remanufacturer's problem ([RP]) (Equation 4.19) and collector's problem (4.23) with piecewise profit functions (4.20 and 4.22) in Section 4.1 when Type 2 is more economical is also valid in this case.

The collector's problem to determine f, w_1 and w_2, with expected profit $\Pi_C(f, w_1, w_2)$, can be stated as:

$$\begin{array}{lll} \textbf{[CP]} & \text{Max} & \Pi_C(f, w_1, w_2) \\ & \text{subject to} & \\ & & f, w_1, w_2 \geq 0 \end{array}$$

where

$$\Pi_C(f) = \begin{cases} E[\Pi_1(f, w_1, w_2)] & \psi + \phi f < \frac{a-b(c_1+w_1)}{2} \\ E[\Pi_2(f, w_1, w_2)] & \text{otherwise} \end{cases} \tag{4.34}$$

$$E[\Pi_1(f, w_1, w_2)] = (\psi + \phi f)(w_1\mu + w_2 - w_2\mu - f - c_I)$$

$$\begin{aligned} E[\Pi_2(f, w_1, w_2)] &= -(f + c_I - d)(\psi + \phi f) + \int_0^{1-\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}} \frac{a - b(c_2 + w_2)}{2}(w_2 - d)g(x)dx \\ &+ \int_{1-\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}}^{1-\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}} (1 - x)(\psi + \phi f)(w_2 - d)g(x)dx \\ &+ \int_{1-\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}}^{1} ((1 - x)(\psi + \phi f)(w_2 - w_1) + (w_1 - d)\frac{a - b(c_1 + w_1)}{2})g(x)dx \end{aligned}$$

Lemma 4.2.3. $E[\Pi_1(f, w_1, w_2)]$ *is jointly concave in* $f \geq 0, w_1 \geq 0, w_2 \geq 0$.

Proof: First order and second order partial derivatives of $E[\Pi_1(f, w_1, w_2)]$ with respect to the three decision variables, f, w_1 and w_2 are found as follows:

$$\begin{aligned} \frac{\partial E[\Pi_1(f, w_1, w_2)]}{\partial f} &= -\psi - \phi(2f + c_I - w_2) + \phi(w_1 - w_2)\mu \\ \frac{\partial E[\Pi_1(f, w_1, w_2)]}{\partial w_1} &= (\psi + \phi f)\mu \end{aligned} \tag{4.35}$$

$$\begin{aligned}
\frac{\partial E[\Pi_1(f,w_1,w_2)]}{\partial w_2} &= (\psi+\phi f)(1-\mu) \qquad (4.36)\\
\frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial f^2} &= -2\phi\\
\frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_1^2} &= \frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_2^2} = 0\\
\frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial f \partial w_1} &= \frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_1 \partial f} = \phi\mu\\
\frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial f \partial w_2} &= \frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_2 \partial f} = \phi - \phi\mu\\
\frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_1 \partial w_2} &= \frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_2 \partial w_1} = 0
\end{aligned}$$

The Hessian matrix is:

$$\begin{pmatrix}
\frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial f^2} & \frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial f \partial w_1} & \frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial f \partial w_2}\\
\frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_1 \partial f} & \frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_1^2} & \frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_1 \partial w_2}\\
\frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_2 \partial f} & \frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_2 \partial w_1} & \frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_2^2}
\end{pmatrix} = \begin{pmatrix} A & B & C\\ B & D & 0\\ C & 0 & F \end{pmatrix}$$

Determinant of the Hessian matrix is found to be equal to zero, as shown below:

$$\begin{aligned}
\mid \mathrm{H} \mid &= ADF + BC0 + CB0 - A0^2 - B^2F - C^2D\\
&= ADF - B^2F - C^2D\\
&= -2\phi 00 - (\phi\mu)^2 0 - (\phi - \phi\mu)^2 0\\
&= 0 \qquad (4.37)
\end{aligned}$$

$E[\Pi_1(f,w_1,w_2)]$ is jointly concave in $f \geq 0$, $w_1 \geq 0$ and $w_2 \geq 0$ since;

- $\frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial f^2} < 0$
- $\frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_1^2} = 0$
- $\frac{\partial^2 E[\Pi_1(f,w_1,w_2)]}{\partial w_2^2} = 0$
- $\mid H \mid = 0.$

□

Lemma 4.2.4. *w_1 and w_2 should be set the following maximum values in order to maximize $E[\Pi_1(f, w_1, w_2)]$:*

$$
\begin{aligned}
(w_1)_{max} &= \frac{a - 2(\psi + \phi f)}{b} - c_1 \\
(w_2)_{max} &= \frac{a - 2(\psi + \phi f)}{b} - c_2
\end{aligned}
$$

Proof: $\Pi_1(f, w_1, w_2)$ is an increasing function of both w_1 and w_2 since first order derivative functions with respect to w_1 and w_2 (Equations (4.35) and (4.36)) are always greater than zero. So the collector sets the maximum values for these decision variables to maximize his profit.

Maximum value of w_2 is found from $\frac{a-b(c_1+w_1)}{2} > \psi + \phi f$ which is the interval for w_1 and f at which $E[\Pi_1(f, w_1, w_2)]$ is valid in Equation (4.34):

$$
\begin{aligned}
\frac{a - b(c_1 + w_1)}{2} &> \psi + \phi f \\
w_1 &< \frac{a - 2(\psi + \phi f)}{b} - c_1 \\
(w_1)_{\text{max}} &= \frac{a - 2(\psi + \phi f)}{b} - c_1
\end{aligned}
\tag{4.38}
$$

Whereas, maximum value of w_2 is found by plugging the maximum value of w_1 into $c_2 + w_2 < c_1 + w_1$, the condition required for Type 2 to be more economical:

$$
\begin{aligned}
c_2 + w_2 &\leq c_1 + w_1 \\
c_2 + w_2 &\leq c_1 + (\frac{a - 2(\psi + \phi f)}{b} - c_1) \\
w_2 &\leq \frac{a - 2(\psi + \phi f)}{b} - c_2 \\
(w_2)_{\text{max}} &= \frac{a - 2(\psi + \phi f)}{b} - c_2
\end{aligned}
\tag{4.39}
$$

Maximum values of w_1 and w_2 are used to reduce $E[\Pi_1(f, w_1, w_2)]$ to a single-variable function as follows:

$$
\begin{aligned}
E[\Pi_1(f, w_1, w_2)] &= (\psi + \phi f)(w_1\mu + w_2 - w_2\mu - f - c_I) \\
E[\Pi_1(f)] &= (\psi + \phi f)((c_2 - c_1)\mu - c_2 - f - c_I + \frac{a - 2(\psi + \phi f)}{b})
\end{aligned}
$$

$E[\Pi_1(f)]$ is concave in $f \geq 0$ since the second order derivative with respect to f is less than zero. $\left(\frac{d^2E[\Pi_1(f)]}{df^2} = -2\phi - 4\frac{\phi^2}{b} < 0\right)$. Hence, optimal f for the first piece of the collector's profit function,$E[\Pi_1(f)]$, can be found by using the first order partial equation as follows:

$$
\begin{aligned}
\frac{\partial E[\Pi_1(f)]}{\partial f} &= -\psi - \frac{2\phi\psi}{b} + \phi(c_2 - c_1)\mu - \phi(c_2 + 2f + c_I) \\
&+ \phi\frac{a - 2(\psi + 2\phi f)}{b} = 0 \\
f^* &= \frac{-\psi - \frac{4\phi\psi}{b} + \frac{\phi a}{b} + \phi(c_2 - c_1)\mu - \phi(c_2 + c_I)}{2\phi + 4\frac{\phi^2}{b}} \qquad (4.40)
\end{aligned}
$$

By plugging Equation (4.40) into Equations (4.38) and (4.39), the optimal w_1 and w_2 can be obtained.

For the second piece of the collector's profit function, $E[\Pi_2(f, w_1, w_2)]$, which is valid when $\frac{a-b(c_1+w_1)}{2} < \psi + \phi f$, first order partial derivatives with respect to f, w_1 and w_2 are found as follows:

$$
\begin{aligned}
\frac{\partial E[\Pi_2(f, w_1, w_2)]}{\partial f} &= -\psi - 2\phi f - c_I\phi + d\phi \\
&+ \phi\frac{a - b(c_2 + w_2)}{2(\psi + \phi f)^2}(w_2 - d)(\frac{a - b(c_2 + w_2)}{2} - (\psi + \phi f))g(\frac{a - b(c_2 + w_2)}{2(\psi + \phi f)}) \\
&+ \phi\frac{a - b(c_1 + w_1)}{2(\psi + \phi f)^2}(w_1 - d)((\psi + \phi f) - \frac{a - b(c_1 + w_1)}{2})g(\frac{a - b(c_1 + w_1)}{2(\psi + \phi f)}) \\
&+ \phi(w_2 - w_1)\int_0^{\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}}(1 - x)g(x)dx \\
&+ \phi(w_2 - d)\int_{\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}}^{\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}}(1 - x)g(x)dx \qquad (4.41)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial E[\Pi_2(f, w_1, w_2)]}{\partial w_1} &= \frac{b(w_1 - d)}{2}g(\frac{a - b(c_1 + w_1)}{2(\psi + \phi f)}) \\
&- \int_0^{\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}}((1 - x)(\psi + \phi f) - \frac{a - b(c_1 + 2w_1 - d)}{2})g(x)dx \qquad (4.42)
\end{aligned}
$$

$$\begin{aligned}
\frac{\partial E[\Pi_2(f,w_1,w_2)]}{\partial w_2} &= -\frac{b(w_2-d)}{2}g(\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}) \\
&+ b(w_2-d)\frac{(a-b(c_2+w_2)}{2(\psi+\phi f)}g(\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}) \\
&+ \int\limits_{\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}}^{1} \frac{a-b(c_2+2w_2-d)}{2}g(x)dx \\
&+ \int\limits_{0}^{\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}} (1-x)(\psi+\phi f)g(x)dx \qquad (4.43)
\end{aligned}$$

To show the concavity of the profit function, $E[\Pi_2(f,w_1,w_2)]$, the second order partial derivatives are found as for $E[\Pi_1(f,w_1,w_2)]$ and they are provided in Appendix B.

Hessian matrix is found for $E[\Pi_2(f,w_1,w_2)]$ as follows:

$$\begin{pmatrix} \frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial f^2} & \frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial f \partial w_1} & \frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial f \partial w_2} \\ \frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial w_1 \partial f} & \frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial w_1^2} & \frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial w_1 \partial w_2} \\ \frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial w_2 \partial f} & \frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial w_2 \partial w_1} & \frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial w_2^2} \end{pmatrix} = \begin{pmatrix} A & B & C \\ B & D & 0 \\ C & 0 & F \end{pmatrix}$$

Determinant of the Hessian matrix is found as:

$$\begin{aligned}
\mid H \mid &= ADF + BC0 + CB0 - A0^2 - B^2F - C^2D \\
&= ADF - B^2F - C^2D
\end{aligned}$$

We couldn't show the joint concavity of $E[\Pi_2(f,w_1,w_2)]$ since the expression for the determinant on Hessian is very complicated.

An exhaustive search algorithm is used to determine the optimal w_1, w_2 and f values in the computational study. The combination of w_1, w_2 and f that gives the maximum collector's expected profit is determined. (The details of the procedure can be seen in Appendix A.)

Finally, for [CP] to determine w_1, w_2 and f, the optimal solutions of $E[\Pi_1(f,w_1,w_2)]$ and $E[\Pi_2(f,w_1,w_2)]$ are compared and the one with the best expected profit is selected as the approximate optimal solution of [CP] for the case when Type 2 is more economical.

To find the overall approximate optimal solution to the model CPBI, two cases are analyzed: (i) $c_1 + w_1 \leq c_2 + w_2$ (ii) $c_2 + w_2 < c_1 + w_1$. Optimal results found for (i) and (ii) are compared and the one giving the greater collector's profit value is selected as the optimal solution to the model CPBI. The details of the algorithm implemented in Matlab is provided in Appendix A.

4.3 Collector's Pricing After Inspection (CPAI)

In this section, we assume that the transfer prices between the collector and the remanufacturer for both quality levels are determined by the collector after the inspection process. The sequence of decisions and events is as follows:

1. The collector determines the acquisition fee, f, for the returned products and collects the corresponding returns.
2. The collector inspects all of the returns incurring unit inspection cost of c_I and announces the quantity available for both quality levels, Q_1 and Q_2.
3. The collector determines the transfer prices w_1 and w_2.
4. The remanufacturer sets the selling price, p, orders q_1 ($\leq Q_1$) units of first quality, and q_2 ($\leq Q_2$) units of second quality, and pays w_1 and w_2 per unit, respectively. The collector salvages the unsold items at unit revenue, d.

Note that while determining the acquisition fee and transfer prices, hence the quantity of returns, the collector will incorporate the optimal decisions of the remanufacturer. Hence, backward induction is used to characterize the optimal actions of the remanufacturer and the collector.

First, we determine q_1, q_2 and p for the remanufacturer, for given Q_1, Q_2, f, w_1 and w_2 values. Then, we determine w_1 and w_2 for the collector by incorporating the optimal q_1, q_2 and p and for given f, Q_1 and Q_2 values. Finally f is determined for the collector by incorporating the optimal values of other decision variables.

Similar to Section 4.2, the problem is analyzed in two sub-problems dividing the feasible region into two parts according to the realized aggregate cost of a remanufactured unit ($w_i + c_i, i = 1, 2$): (i) Type 1 products are more economical (ii) Type 2 products are more economical.

4.3.1 Type 1 is more economical

In this section, we consider the case where $c_1 + w_1 \leq c_2 + w_2$ is assumed.

Since the remanufacturer determines q_1, q_2 and p, for given Q_1, Q_2, f, w_1 and w_2 values in both exogenous transfer prices model and collector's pricing after inspection model, the optimal solution to the remanufacturer's problem in Section 4.1 when Type 1 is more economical (Equation (4.7)) is also valid in this case. Hence optimal decision of the remanufacturer is characterized as follows:

$$(q_1^*, q_2^*, p^*) \quad = \quad \begin{cases} \left(\frac{a-b(c_1+w_1)}{2}, 0, \frac{a+b(c_1+w_1)}{2b}\right) & \text{if } Q_1 \geq \frac{a-b(c_1+w_1)}{2} \\ \left(Q_1, 0, \frac{a-Q_1}{b}\right) & \text{if } \frac{a-b(c_2+w_2)}{2} \leq Q_1 < \frac{a-b(c_1+w_1)}{2} \\ \left(Q_1, \frac{a-b(c_2+w_2)}{2} - Q_1, \frac{a+b(c_2+w_2)}{2b}\right) & \text{if } Q_1 < \frac{a-b(c_2+w_2)}{2} \leq (\psi + \phi f) \\ \left(Q_1, Q_2, \frac{a-Q_1-Q_2}{b}\right) & \text{if } \frac{a-b(c_2+w_2)}{2} > (\psi + \phi f) \end{cases}$$

The collector incorporates the optimal response of the remanufacturer's problem given in Equation (4.7) into his decision making process to determine the transfer prices, w_1 and w_2, for given f, Q_1 and Q_2 values. Since the remanufacturer's problem has a solution for q_1, q_2 and p in four distinct intervals, collector's problem is also studied according to these intervals separately.

The collector's profit function is: $w_1 q_1 + w_2 q_2 + d(Q_1 + Q_2 - q_1 - q_2)$. Since the decision of the acquisition fee is prior to the decision of the transfer prices, total acquisition cost and total inspection cost are not included in the profit function.

Theorem 4.3.1. *The optimal w_1 and w_2 for the collector when Type 1 is more economical are characterized as follows:*

$$(w_1^*, w_2^*) = \begin{cases} \left(\frac{a-b(c_1-d)}{2b}, \frac{a-b(2c_2-c_1-d)}{2b}\right) & \text{if } Q_1 \geq \frac{a-b(c_1+d)}{4} \\ \left(\frac{a-2Q_1}{b} - c_1, \frac{a-2Q_1}{b} - c_2\right) & \text{if} \frac{a-b(c_2+d)}{4} \leq Q_1 < \frac{a-b(c_1+d)}{4} \\ \left(\frac{a-b(2c_1-c_2-d)}{2b}, \frac{a-b(c_2-d)}{2b}\right) & \text{if} Q_1 < \frac{a-b(c_2+d)}{4} \leq \psi + \phi f \\ \left(\frac{a-bc_1-2(\psi+\phi f)}{b}, \frac{a-bc_2-2(\psi+\phi f)}{b}\right) & \text{if} \frac{a-b(c_2+d)}{4} > \psi + \phi f \end{cases} \tag{4.44}$$

Proof: Collector's problem to determine transfer prices is studied according to the intervals in (4.7) separately as follows:

When $Q_1 \geq \frac{a-b(c_1+w_1)}{2}$;

The profit function can be expressed using the corresponding optimal q_1 and q_2 expressions as follows:

$$\begin{aligned}
\Pi_1(w_1, w_2) &= w_1 q_1 + w_2 q_2 + d(Q_1 + Q_2 - q_1 - q_2) \\
&= w_1 \frac{a - b(c_1 + w_1)}{2} + w_2 0 + d(\psi + \phi f - \frac{a - b(c_1 + w_1)}{2} - 0) \\
\Pi_1(w_1) &= (w_1 - d)\frac{a - b(c_1 + w_1)}{2} + d(\psi + \phi f)
\end{aligned}$$

Collector's problem reduces to:

$$\begin{aligned}
\text{Max } \Pi_1(w_1) &= (w_1 - d)\frac{a - b(c_1 + w_1)}{2} + d(\psi + \phi f) \\
\text{subject to} & \\
Q_1 &\geq \frac{a - b(c_1 + w_1)}{2} && (4.45) \\
w_1 + c_1 &\leq w_2 + c_2 && (4.46) \\
w_1 &\geq 0
\end{aligned}$$

The objective function is concave in $w_1 \geq 0 \left(\frac{d^2\Pi_1}{dw_1^2} = -b < 0\right)$, the optimal solution can be characterized utilizing the first order conditions and taking the constraint $Q_1 \geq \frac{a-b(c_1+w_1)}{2}$ into account as follows:

$$(w_1^*, w_2^*) = \begin{cases} \left(w_1 = \frac{a-b(c_1-d)}{2b}, w_2 \geq \frac{a-b(2c_2-c_1-d)}{2b}\right) & \text{if } Q_1 \geq \frac{a-b(c_1+d)}{4} \\ \left(w_1 = \frac{a-2Q_1}{b} - c_1, w_2 \geq \frac{a-2Q_1}{b} - c_2\right) & \text{otherwise} \end{cases} \quad (4.47)$$

When $\frac{a-b(c_2+w_2)}{2} \leq Q_1 < \frac{a-b(c_1+w_1)}{2}$;

The profit function can be expressed as follows:

$$\begin{aligned}
\Pi_2(w_1, w_2) &= w_1 q_1 + w_2 q_2 + d(Q_1 + Q_2 - q_1 - q_2) \\
&= w_1 Q_1 + w_2 * 0 + d(Q_1 + Q_2 - Q_1 - 0) \\
\Pi_2(w_1) &= w_1 Q_1 + d Q_2
\end{aligned}$$

Collector's problem reduces to:

$$
\begin{aligned}
\text{Max } \Pi_2(w_1) &= w_1 Q_1 + d Q_2 \\
\text{s.to} & \\
\frac{a - b(c_2 + w_2)}{2} &\leq Q_1 < \frac{a - b(c_1 + w_1)}{2} && (4.48) \\
w_1 + c_1 &\leq w_2 + c_2 && (4.49) \\
w_1 &\geq 0
\end{aligned}
$$

The objective function is linearly increasing in $w_1 \geq 0 \left(\frac{d^2\Pi_2}{dw_1^2} = 0 \right)$, the optimal solution can be characterized utilizing the constraints of the model and the result is same with the second piece of the piecewise optimal solution to the collector's problem when $Q_1 \geq \frac{a-b(c_1+w_1)}{2}$, shown in (4.47). Therefore, we can combine it with Equation (4.47).

When $Q_1 < \frac{a-b(c_2+w_2)}{2} \leq (\psi + \phi f)$;
The profit function can be expressed by replacing w_1 with $w_2 + c_2 - c_1$ as Type 1 is more economical:

$$
\begin{aligned}
\Pi_3(w_1, w_2) &= w_1 q_1 + w_2 q_2 + d(Q_1 + Q_2 - q_1 - q_2) \\
&= w_1 Q_1 + w_2 \left(\frac{a - b(c_2 + w_2)}{2} - Q_1 \right) + d(Q_1 + Q_2 - \frac{a - b(c_2 + w_2)}{2}) \\
&= (w_1 - w_2) Q_1 + (w_2 - d) \frac{a - b(c_2 + w_2)}{2} + d(\psi + \phi f) \\
\Pi_3(w_2) &= (c_2 - c_1) Q_1 + (w_2 - d) \frac{a - b(c_2 + w_2)}{2} + d(\psi + \phi f)
\end{aligned}
$$

Collector's problem reduces to:

$$
\begin{aligned}
\text{Max } \Pi_3(w_2) &= (c_2 - c_1) Q_1 + (w_2 - d) \frac{a - b(c_2 + w_2)}{2} + d(\psi + \phi f) \\
\text{s.to} & \\
Q_1 &< \frac{a - b(c_2 + w_2)}{2} \leq (\psi + \phi f) && (4.50) \\
w_2 &\geq 0
\end{aligned}
$$

The objective function is concave in $w_2 \geq 0 \left(\frac{d^2\Pi_3}{dw_2^2} = -b < 0 \right)$, the optimal solution can be characterized utilizing the first order conditions and taking the constraint $Q_1 < \frac{a-b(c_2+w_2)}{2} \leq (\psi + \phi f)$ into account as follows:

$$(w_1^*, w_2^*) = \begin{cases} \left(\frac{a-b(2c_1-c_2-d)}{2b}, \frac{a-b(c_2-d)}{2b}\right) & \text{if } Q_1 < \frac{a-b(c_2+d)}{4} \text{ and } \frac{a-b(c_2+d)}{4} \leq \psi + \phi f \\ \left(\frac{a-2Q_1}{b} - c_1, \frac{a-2Q_1}{b} - c_2\right) & \text{if } Q_1 > \frac{a-b(c_2+d)}{4} \text{ and } \frac{a-b(c_2+d)}{4} \leq \psi + \phi f \\ \left(\frac{a-2(\psi+\phi f)}{b} - c_1, \frac{a-2(\psi+\phi f)}{b} - c_2\right) & \text{if } Q_1 < \frac{a-b(c_2+d)}{4} \text{ and } \frac{a-b(c_2+d)}{4} > \psi + \phi f \end{cases} \tag{4.51}$$

When $\frac{a-b(c_2+w_2)}{2} > (\psi + \phi f)$;

The profit function can be expressed as:

$$\begin{aligned} \Pi_4(w_1, w_2) &= w_1 q_1 + w_2 q_2 + d(Q_1 + Q_2 - q_1 - q_2) \\ &= w_1 Q_1 + w_2 Q_2 + d(Q_1 + Q_2 - Q_1 - Q_2) \\ &= w_1 Q_1 + w_2 Q_2 \end{aligned}$$

Collector's problem reduces to:

$$\begin{aligned} \text{Max } \Pi_4(w_1, w_2) &= w_1 Q_1 + w_2 Q_2 \\ \text{s.to} & \\ \frac{a - b(c_2 + w_2)}{2} &> \psi + \phi f \qquad (4.52) \\ w_1 + c_1 &\leq w_2 + c_2 \qquad (4.53) \\ w_1, w_2 &\geq 0 \end{aligned}$$

The objective function is concave in $w_1 \geq 0$, $w_2 \geq 0$ $\left(\frac{d^2\Pi_4}{dw_1^2} = 0 \text{and} \frac{d^2\Pi_4}{dw_2^2} = 0\right)$, the optimal solution can be characterized utilizing the constraints of the model and the result is same with the third piece of the piecewise optimal solution to the collector's problem when $Q_1 < \frac{a-b(c_2+w_2)}{2} \leq (\psi + \phi f)$, shown in (4.51). Therefore, we can combine it with Equation (4.51).

We can finally characterize the collector's solution for w_1 and w_2 when Type 1 is more economical as in (4.44) by combining the piecewise solutions of four distinct parts:

$$(w_1^*, w_2^*) = \begin{cases} \left(\frac{a-b(c_1-d)}{2b}, \frac{a-b(2c_2-c_1-d)}{2b}\right) & \text{if } Q_1 \geq \frac{a-b(c_1+d)}{4} \\ \left(\frac{a-2Q_1}{b} - c_1, \frac{a-2Q_1}{b} - c_2\right) & \text{if} \frac{a-b(c_2+d)}{4} \leq Q_1 < \frac{a-b(c_1+d)}{4} \\ \left(\frac{a-b(2c_1-c_2-d)}{2b}, \frac{a-b(c_2-d)}{2b}\right) & \text{if} Q_1 < \frac{a-b(c_2+d)}{4} \leq \psi + \phi f \\ \left(\frac{a-bc_1-2(\psi+\phi f)}{b}, \frac{a-bc_2-2(\psi+\phi f)}{b}\right) & \text{if} \frac{a-b(c_2+d)}{4} > \psi + \phi f \end{cases}$$

□

4.3.2 Type 2 is more economical

In this section, we consider the case where $c_2 + w_2 < c_1 + w_1$ is assumed.

Since the remanufacturer determines q_1, q_2 and p, for given Q_1, Q_2, f, w_1 and w_2 in both the "exogenous transfer prices" model and the "collector's pricing after inspection" model, the optimal solution to the remanufacturer's problem in Section 4.1 when Type 2 is more economical (Equation 4.19) is also valid in this case. Hence the optimal decision of the remanufacturer is characterized as follows:

$$(q_1^*, q_2^*, p^*) = \begin{cases} \left(0, \frac{a-b(c_2+w_2)}{2}, \frac{a+b(c_2+w_2)}{2b}\right) & \text{if } Q_2 \geq \frac{a-b(c_2+w_2)}{2} \\ \left(0, Q_2, \frac{a-Q_2}{b}\right) & \text{if } \frac{a-b(c_1+w_1)}{2} \leq Q_2 < \frac{a-b(c_2+w_2)}{2} \\ \left(\frac{a-b(c_1+w_1)}{2} - Q_2, Q_2, \frac{a+b(c_1+w_1}{2b}\right) & \text{if } Q_2 < \frac{a-b(c_1+w_1)}{2} \leq (\psi + \phi f) \\ \left(Q_1, Q_2, \frac{a-Q_1-Q_2}{b}\right) & \text{if } \frac{a-b(c_1+w_1)}{2} > (\psi + \phi f) \end{cases}$$

The collector incorporates the optimal response of the remanufacturer's problem given in Equation (4.19) into his decision making process to determine the transfer prices, w_1 and w_2, for given f, Q_1 and Q_2 values. Since the remanufacturer's problem has a solution for q_1, q_2 and p in four distinct intervals, collector's problem is also studied according to these intervals separately.

The collector's profit function is: $w_1q_1 + w_2q_2 + d(Q_1 + Q_2 - q_1 - q_2)$. Since the decision of the acquisition fee is prior to the decision of the transfer prices, total acquisition cost and total inspection cost are not included in the profit function.

Theorem 4.3.2. *Optimal w_1 and w_2 for the collector when Type 2 is more economical are characterized as follows:*

$$(w_1^*, w_2^*) = \begin{cases} \left(\frac{a-b(2c_1-c_2-d)}{2b}, \frac{a-b(c_2-d)}{2b}\right) & \text{if } Q_2 \geq \frac{a-b(c_2+d)}{4} \\ \left(\frac{a-2Q_2}{b} - c_1, \frac{a-2Q_2}{b} - c_2\right) & \text{if} \frac{a-b(c_1+d)}{4} \leq Q_2 < \frac{a-b(c_2+d)}{4} \\ \left(\frac{a-b(c_1-d)}{2b}, \frac{a-b(2c_2-c_1-d)}{2b}\right) & \text{if} Q_2 < \frac{a-b(c_1+d)}{4} \leq \psi + \phi f \\ \left(\frac{a-bc_1-2(\psi+\phi f)}{b}, \frac{a-bc_2-2(\psi+\phi f)}{b}\right) & \text{if} \frac{a-b(c_1+d)}{4} > \psi + \phi f \end{cases} \quad (4.54)$$

Proof: Proof is similar to the proof of Theorem 4.3.1 and is provided in the Appendix C. $\square$

Incorporating the decision of transfer prices and optimal decisions of the remanufacturer, collector's problem to determine f is analyzed next. Results in sections 4.3.1 and 4.3.2 are used for collector's problem to determine the acquisition fee.

The collector's profit function is defined as follows:

$$w_1q_1 + w_2q_2 + d(Q_1 + Q_2 - q_1 - q_2) - (f + c_I)(\psi + \phi f); \tag{4.55}$$

The procedure to find the optimal results for CPAI can be summarized as follows:

For a given f value, the expected profit of the collector is calculated as follows:

- For each value of x;
  - Obtain the optimal w_1 and w_2 values from equations (4.44) and (4.54).
  - Find the corresponding expected collector's profit with Equation (4.55) for both sub-problems: (i) $c_1 + w_1 \leq c_2 + w_2$ and (ii) $c_2 + w_2 < c_1 + w_1$.
  - Select the one giving the best collector's profit value from (i) and (ii).
- Determine the expected profit for the collector at the given f value.

In order to find the optimal acquisition fee, f, the above procedure is repeated exhaustively in the relevant range and the f value that gives the highest profit is selected.

The details of the algorithm implemented in Matlab is provided in Appendix D.

CHAPTER 5

COMPUTATIONAL STUDY

In this chapter, we analyze the effects of quality information and centralization on the pricing decisions and profit levels.

Since the models, we presented in Chapter 3 and Chapter 4, do not facilitate any analytical comparisons, we perform a computational analysis to quantify the value of decision making with quality information and effects of centralization. Computational study is performed through the algorithms implemented in Matlab 7.0 to find the optimal solutions. The model that considers exogenous transfer prices is not included in the computational study as its primary purpose is to facilitate the analysis of the other decentralized models.

This chapter is organized as follows: In Section 5.1, we pose the research questions that we are interested in and introduce the related performance measures. Section 5.2 analyzes the effects of problem parameters on the pricing decisions and expected profits. In section 5.3, we quantify the value of quality information and effects of centralization considering various combinations of problem parameters.

5.1 PERFORMANCE MEASURES AND RESEARCH QUESTIONS

The major research questions that we pose and hope to provide answers for, through the computational analysis can be summarized as follows:

1. In the centralized setting, what are the effects of delaying the decision of the selling price on the number of cores collected, number of cores remanufactured and expected profit?
2. In the decentralized setting, what are the effects of delaying the decision of the transfer

prices on the number of cores collected by the collector, number of cores remanufactured and the expected profit of the collector and the remanufacturer?

3. What are the effects of centralization on the number of cores collected, number of cores remanufactured and expected profit?

4. Noting that the decision of the selling price is always delayed in the decentralized setting, are there any cases where decentralized system provides higher profits than the centralized system with simultaneous decision making?

Note that questions 1 and 2 relate to the value of quality information whereas question 3 is about the effect of centralization. On the other hand, the answer to question 4 will reveal whether the value of quality information could overcome the detrimental effects of decentralization.

In seeking the answers to the above questions, we start with a sensitivity analysis to characterize how the optimal solutions are effected by the changes in problem parameters in Section 5.2. Then we perform a full factorial design in Section 5.3 to analyze the results with a number of different parameter sets. Performance measures other than the number of cores collected, number of cores remanufactured and profit, include the percentage improvement in expected profits due to centralization and quality information. More specifically;

- For question 1, we compare sequential pricing to simultaneous pricing in the centralized setting (SQP versus SMP) and calculate $(\frac{(\Pi_{SQP}-\Pi_{SMP})}{\Pi_{SMP}}100)$ for the remanufacturer's profit.

- For question 2, we compare collector's pricing before inspection to after inspection in the decentralized setting (CPBI versus CPAI) and calculate $(\frac{(\Pi_{CPAI}-\Pi_{CPBI})}{\Pi_{CPBI}}100)$ for the collector's profit.

- For question 3, we compare sequential pricing in the centralized setting to collector's pricing cases in the decentralized setting (SQP versus CPBI/ CPAI) and calculate $(\frac{(\Pi_{SQP}-\Pi_{CPBI})}{\Pi_{CPBI}}100)$ and $(\frac{(\Pi_{SQP}-\Pi_{CPAI})}{\Pi_{CPAI}}100)$ for the system profit.

- For question 4, we compare simultaneous pricing in the centralized setting to collector's pricing cases in the decentralized setting (SMP versus CPBI/ CPAI) and calculate $(\frac{(\Pi_{SMP}-\Pi_{CPBI})}{\Pi_{CPBI}}100)$ and $(\frac{(\Pi_{SMP}-\Pi_{CPAI})}{\Pi_{CPAI}}100)$ for the system profit.

In Table 5.1, comparisons made are summarized where ”+” sign is used to indicate which subject the question relates to.

Table 5.1: Comparison of the models and expected results of the comparisons.

Comparison	Research question	Value of Quality Information	Effect of Centralization
SMP vs SQP	1	+ (for remanufacturer)	
CPBI vs CPAI	2	+ (for collector)	
SQP vs CPBI	3		+
SQP vs CPAI	3		+
SMP vs CPBI	4	+	+
SMP vs CPAI	4	+	+

Note that throughout our computational study, we restrict ourselves to the case where the fraction of Type 1 cores is Normally distributed. While selecting the parameters of Normal random variable (mean, μ and standard deviation, σ), the probability that the random variable is in interval [0,1] is kept in 0.99957 and 1. The functions including p.d.f. and c.d.f. of Normal random variable that appear in the expected profit or optimality conditions that are derived in Chapter 3 and 4 are calculated according to the derivations given in Appendix E.

5.2 ANALYSIS OF PARAMETER SENSITIVITY

Sensitivity analysis is conducted for some of the parameters used in models according to the base parameter set defined. Optimal prices, quantities and profits are evaluated while each parameter is increased and others kept constant. To see the effects of interactions between different parameters on optimal results, various constant values of other parameters are also used while the selected parameter is being increased. Base parameter set is selected inspiring from the paper by Bakal and Akcali (2006) and the results of pilot runs, such that effect of quality information is high. The base parameter set used in the sensitivity analysis is provided in Table 5.2.

Table 5.2: Base parameter set values.

a	b	d	c_I	c_1	c_2	ψ	ϕ	μ	σ
200	5	10	5	10	30	15	6	0.4	0.12

Table 5.3 summarizes the behavior of the reported performance measure under the respective optimal solution according to an increase in the defined parameter. Uparrow shows an increase in the related variable while downarrow shows a decrease and rightarrow shows no change and finally question marks show that no consistent behavior is observed for that solution. Transfer prices, collector's and remanufacturer's profit values are only valid for the decentralized setting. Other results are valid for both the centralized and the decentralized settings. CPAI is not taken into consideration when evaluating transfer prices since its results are dependent on the realized values of x, the quality ratio.

We start with the analysis of market size for remanufactured items. Recall that the demand is assumed to be linearly decreasing in price and it is formulated as $a - bp$, where a represents the potential market size. Overall results for the problem instances considered are provided in Appendix F. a is changed between 100 and 300 with a stepsize of 20. We present our findings with respect to an increase in a for the centralized and the decentralized settings separately below:

For the centralized setting;

- In the optimal solutions for centralized models, the optimal acquisition fee, f, and the optimal selling price, p, show an increasing behavior as the potential market size increases. An increase in the potential market size causes an increase in supply to gain more from this potential and supply is increased via increased acquisition fee. The increase in the selling price is also reasonable since when potential demand is more, higher prices are offered to gain more profit.
- Supply and demand quantities increase as the potential market size increases since the system tries to take benefit of this potential.
- The optimal system profit increases. Figure 5.1 shows the behavior of the system profit for SMP with increasing a, under various parameter values. The only observation significant for the behavior of the profit in a is the case with smaller b, price sensitivity of

Table 5.3: Summary of the parameter sensitivity for all parameters

Increase in	OPTIMAL SOLUTION								
	Acq. fee: f	**Sell. Price: p**	**Tr. Price: w_1**	**Tr. Price: w_2:**	**Supply**	**Demand**	**System Prof.**	**Coll.'s Prof.**	**Remanuf.'s Prof.**
a	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
d	$\uparrow$	$\uparrow$	$\downarrow\uparrow$	$\downarrow\uparrow$	$\rightarrow\uparrow$	$\downarrow$	$\rightarrow\uparrow$	??	??
c_2	$\downarrow\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
ψ	$\downarrow$	$\downarrow$	$\downarrow$	??	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	??
μ	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$

the demand function. That is when the demand is less sensitive to the price, the increase in the optimal profit values is steeper compared to the base case.

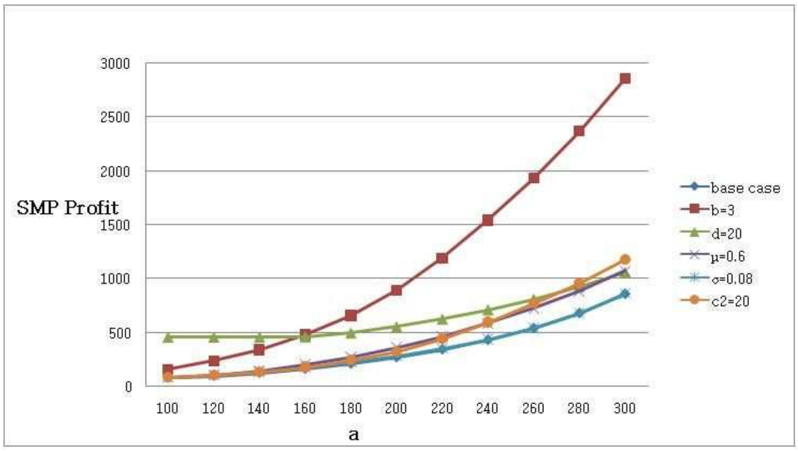
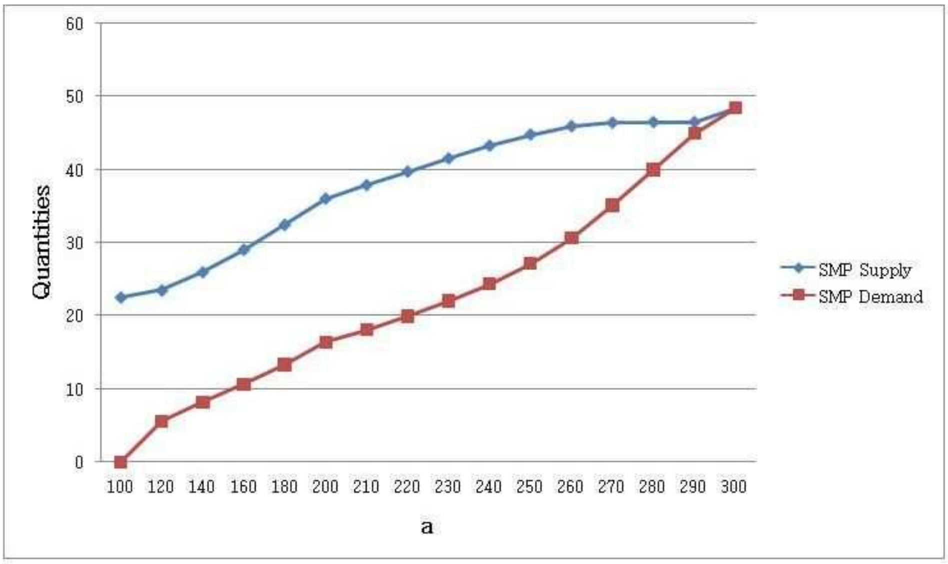


Figure 5.1: SMP Profit when a increases

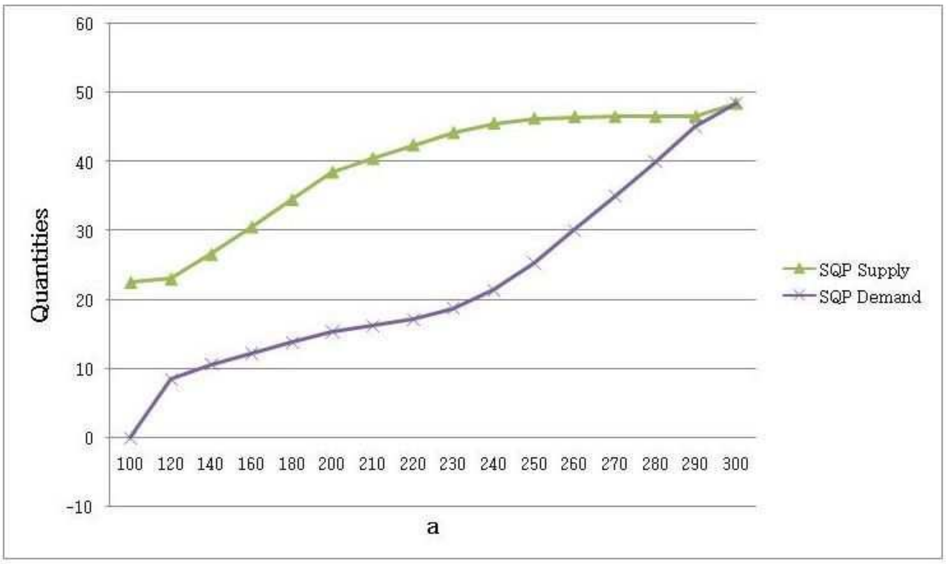
- For relatively lower values of a, supply is larger than the demand for all parameter sets used. In such cases, the remanufacturer aims to satisfy demand which is relatively small, from superior quality items since they are more profitable. As a increases, supply and demand become equal at a threshold value of $a = 300$ under the base parameter set. We observe a similar threshold value for different parameter sets as well. The graphs showing the supply and demand for the centralized models under the base parameter set can be seen in Figure 5.2.

- The prices and profits of SMP and SQP also get equal when supply and demand become equal. It means if all of the supply quantity is sold in the market, there is no benefit of postponing the pricing decision after observing the quantity. For all of the parameter sets considered, there is a threshold value of a after which optimal results of SMP and SQP are identical. Therefore it can be stated that "If the potential market size is 'sufficiently large', there is no value of quality information".

- When a increases, the percentage benefit of quality information begins at zero and increases, and then again goes to zero. Figure 5.3 shows the behavior of the percent-



(i) Supply and demand quantities for SMP



(ii) Supply and demand quantities for SQP

Figure 5.2: Supply and demand for centralized setting when a increases

age improvement of SQP over SMP when a increases under various parameter values. When a, the potential market size, is very small, there is almost no demand, hence effects of quality information is not significant. As a increases, the potential to sell the recovered products increase, thus the quality information starts to be more effective and critical to increase profit. When a increases further, all of the cores will be remanufactured and sold at a higher price, resulting in a less significant improvement by quality information.

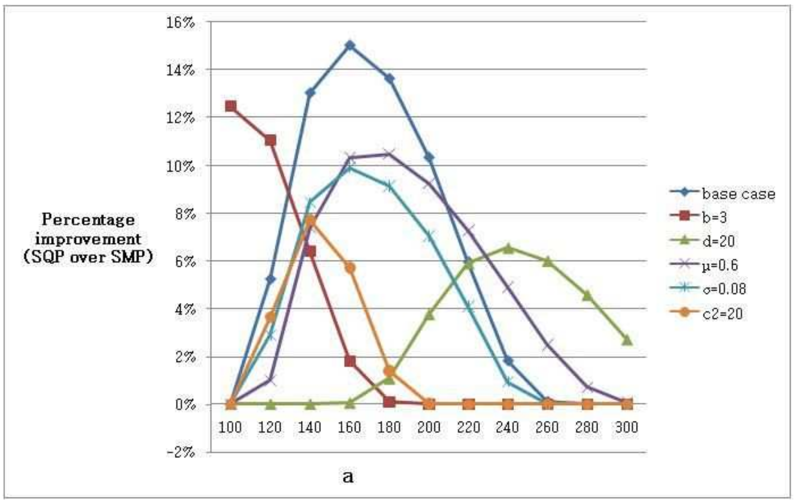


Figure 5.3: Percentage difference in profit of SMP and SQP when a increases

For the decentralized setting;

- The behaviors of the optimal acquisition fee, the optimal selling price, supply and demand quantities as a increases are the same with the centralized models.
- The optimal transfer prices increase in a. Profits for the collector and the remanufacturer also increase, resulting in an increase in the system profit.
- As in the centralized models, we observe a threshold value of a after which supply equals to demand for the decentralized models as well. However these threshold values are greater than the centralized models for the same parameter sets. Figure 5.4 shows the supply and demand for decentralized models.

- The observation that says prices and profits of SMP and SQP also get equal when supply and expected demand become equal is valid for the decentralized models, CPBI and CPAI. However, the expected profit of the collector for CPBI and CPAI become almost the same.

We next consider the effects of salvage value, d. Overall results for the problem instances considered are provided in Appendix F. d is changed between 0 and 60 with a stepsize of 5. We observe the following with respect to an increase in d for the centralized and the decentralized settings presented separately below:

For the centralized setting;

- The optimal acquisition fee, f, and the optimal selling price, p, increase in d, however selling price stops increasing at some value of d.
- Supply increases while demand shows a decreasing trend. The behavior of supply and demand for centralized models under base parameter set is shown in Figure 5.5. Supply and demand are equal until some threshold value of d ($d \approx 5$ for the base case) after which supply exceeds demand. In general, there is a certain threshold unit salvage value beyond which supply exceeds demand and thus the value of quality information is observed. With small values of d, there is no chance of making profit through salvaging. Therefore, remanufacturer tries to use all of the supply in remanufacturing. However, as salvage revenue increases, salvaged quantity also increases since salvaging becomes profitable and the risk for profit caused by the quality uncertainty decreases relatively. For very large values of d, demand is zero since salvaging becomes more profitable.
- Profit increases in d in the centralized models. Figure 5.6 shows the behavior of the SMP's profit with increasing d, under various parameter values. Different parameter values do not show a significant effect on the general trend of the profit as d increases.
- When d increases, value of quality information starts to increase from zero and then

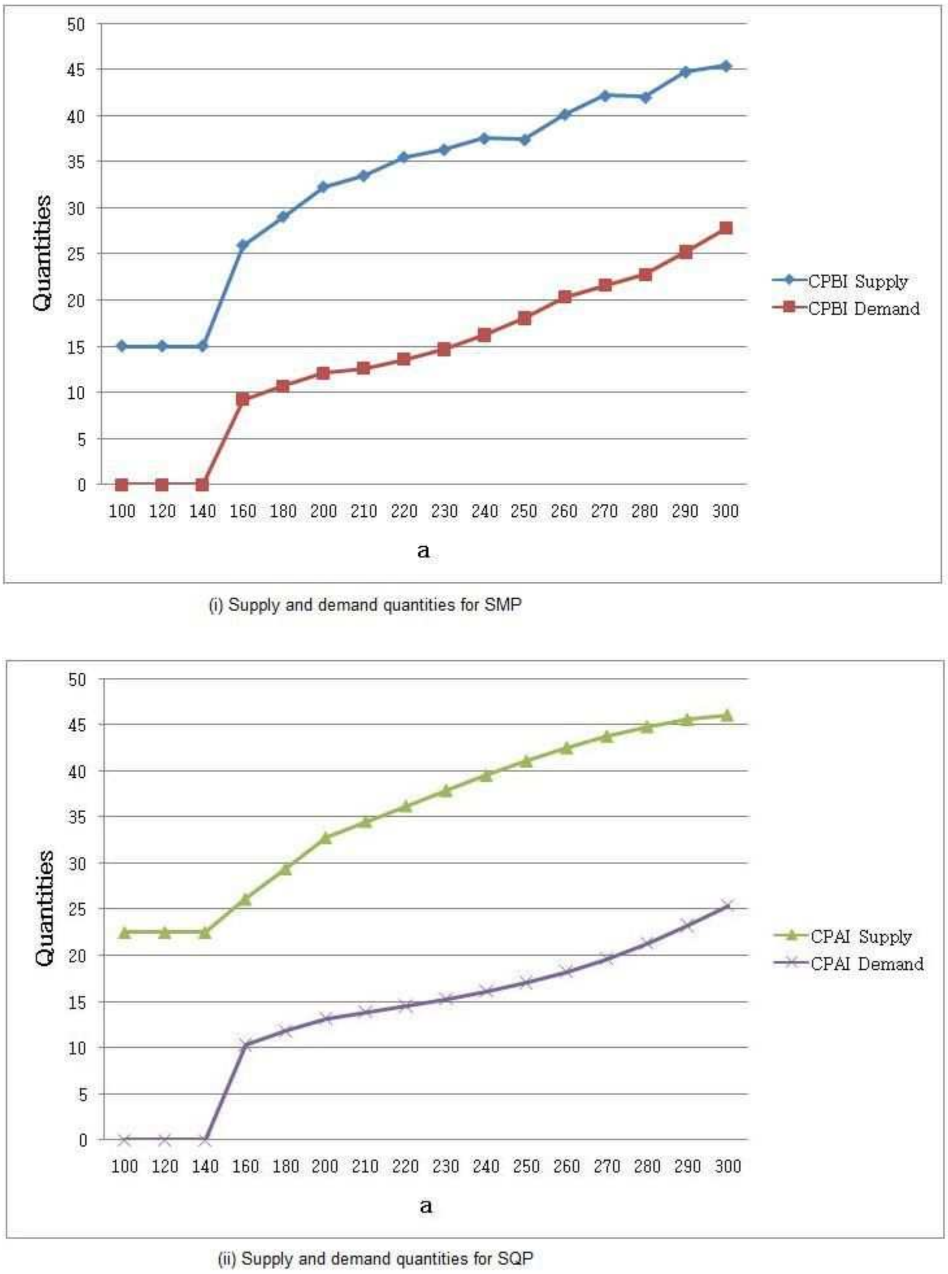
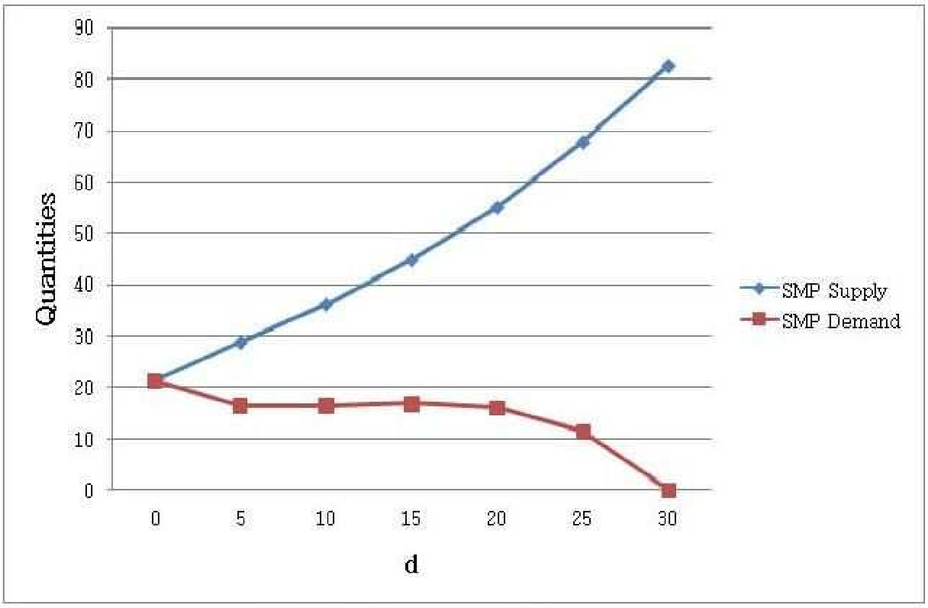
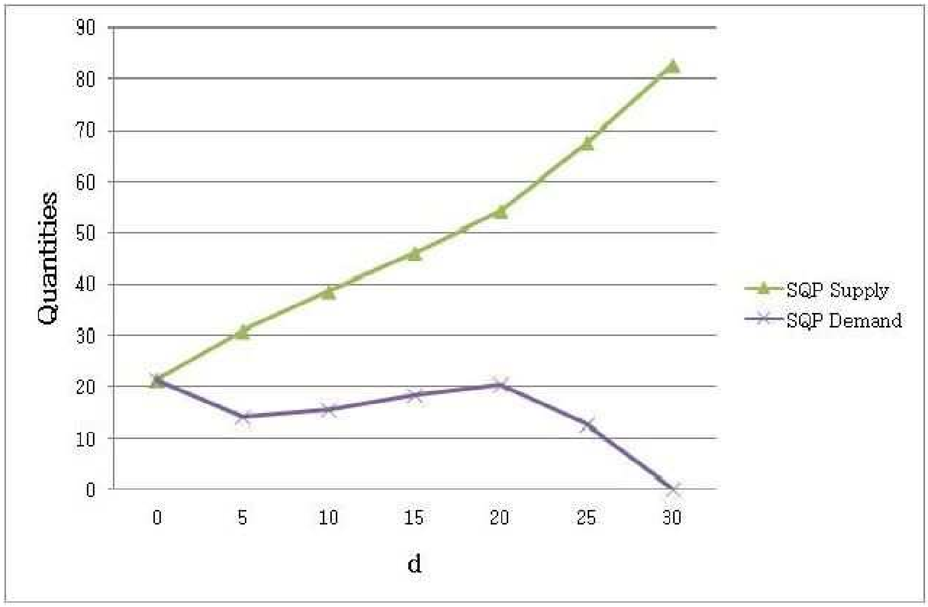


Figure 5.4: Supply and demand for decentralized setting when *a* increases



(i) Supply and demand quantities for SMP



(ii) Supply and demand quantities for SQP

Figure 5.5: Supply and demand for centralized setting when *d* increases

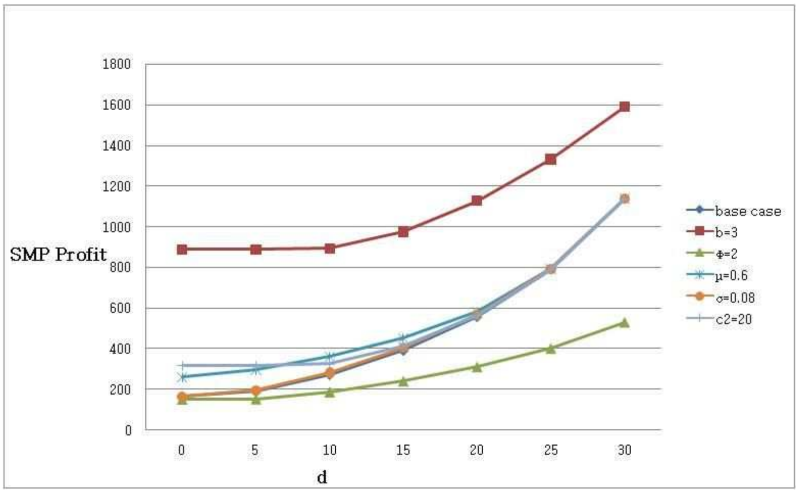


Figure 5.6: SMP Profit when d increases

decrease to zero again. Figure 5.7 shows the behavior of percentage differences for SMP and SQP when d increases under various parameter values. Effect of increase in d brings benefit to the remanufacturer at first since remanufacturing is on focus, but as the value of d is increased more, demand gets smaller and the quality information gets ineffective since salvaging gets profitable.

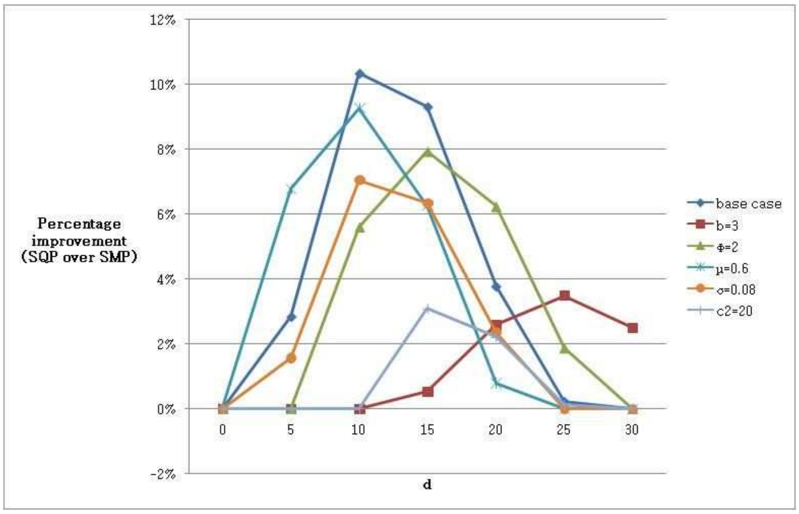


Figure 5.7: Percentage difference in profit of SMP and SQP when d increases

For the decentralized setting;

- The optimal acquisition fee, and optimal selling price shows the same increasing behavior as in centralized models.

- Supply increases while demand shows a decreasing trend. The behavior of supply and demand for decentralized models under base parameter set is shown in Figure 5.8.

- Supply always exceeds demand for the decentralized models under the parameter sets that we consider.

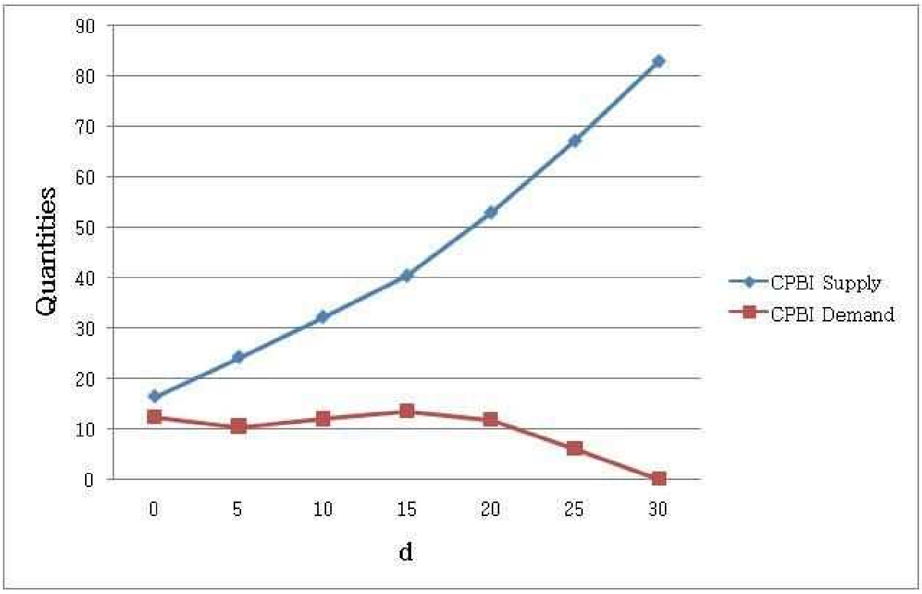
- In the decentralized setting, the collector's optimal profit increases while remanufacturer's profit generally decreases. Since the increase in collector's profit is larger than the decrease in remanufacturer's profit, total system profit increases. Transfer prices, w_1 and w_2 follow an increasing trend as salvage revenue increases. As the collector benefits from the salvaging, he makes the transfer prices higher.

Next, we consider the cost of remanufacturing Type 2 items, c_2. Overall results for the problem instances considered are provided in Appendix F. c_2 is changed between 10 and 35 with a stepsize of 5, holding c_1 constant. Observations are listed with respect to an increase in c_2 for the centralized and the decentralized settings separately below:

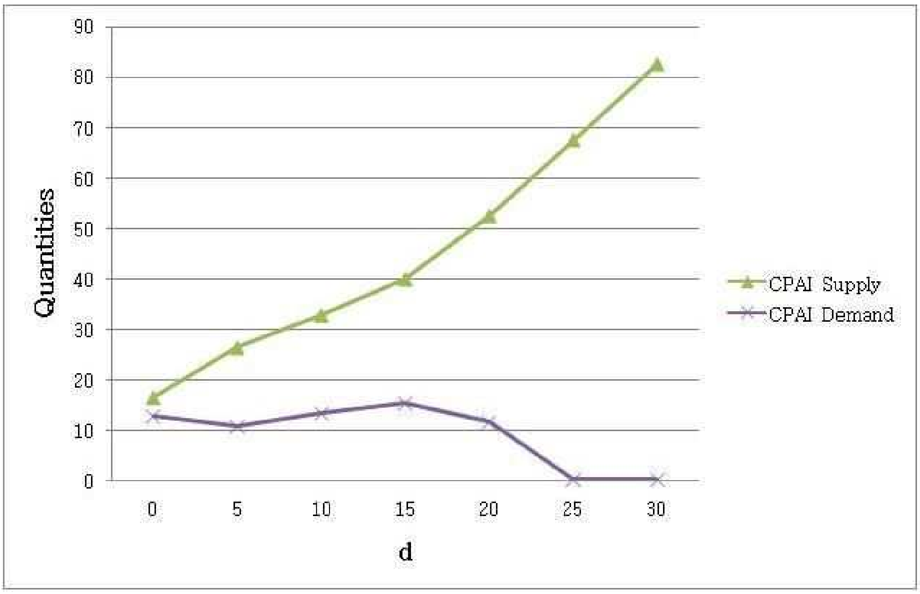
For the centralized setting:

- The optimal selling price, p increases, thus the demand is always decreasing. Whereas, the optimal acquisition fee, f, and supply first decrease and then increase while c_2 is increased.

- For relatively lower values of c_2, all cores collected are remanufactured and hence demand is equal to supply. In such cases, an increase in c_2 results in a decrease in supply and demand. As c_2 further increases, the remanufacturer starts to give up Type 2 items and aims to collect more in order to use Type 1 items in remanufacturing. Hence, supply starts to increase while demand continues to decrease, causing an increase in the salvaging quantity. The behavior of supply and demand for centralized models under



(i) Supply and demand quantities for CPBI



(ii) Supply and demand quantities for CPAI

Figure 5.8: Supply and demand for decentralized setting when *d* increases

base parameter set is shown in Figure 5.9. This behavior is observed in all parameter sets except the sets with higher salvage revenue. The effect of salvage revenue on supply hides the general trend when remanufacturing cost of Type 2 items increase.

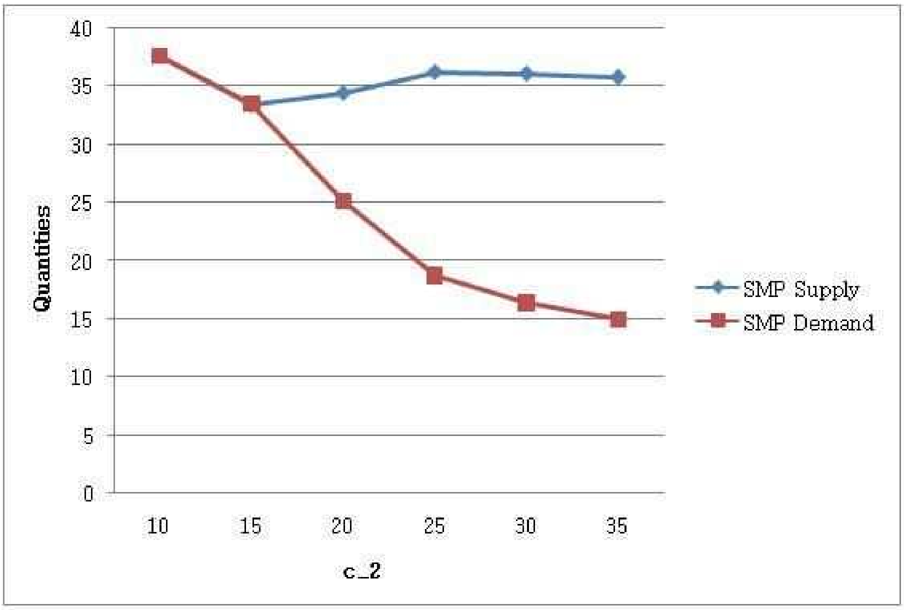
- An increase in unit remanufacturing cost of Type 2 items causes the system profits to decrease. Figure 5.10 shows the behavior of the SMP's system profit with increasing c_2, under various parameter values.
  - The decrease in profit is steeper compared to the base case when the price sensitivity of demand, b, is smaller. Smaller price sensitivity decreases the chance for manipulation of prices and as a result, profit values become more dependent on costs.
  - For greater value of d, the decrease in system profit due to an increase in c_2, is ignorable compared to the base case. When d is large, more cores are collected. As supply of cores increase, the remanufacturer only remanufactures Type 1 items. Therefore cost of Type 2 items becomes irrelevant.

- The percentage profit improvement due to the quality information increases continuously with an increase in c_2 since in this case the remanufacturing cost difference between the two quality groups gets larger, making quality information more important.

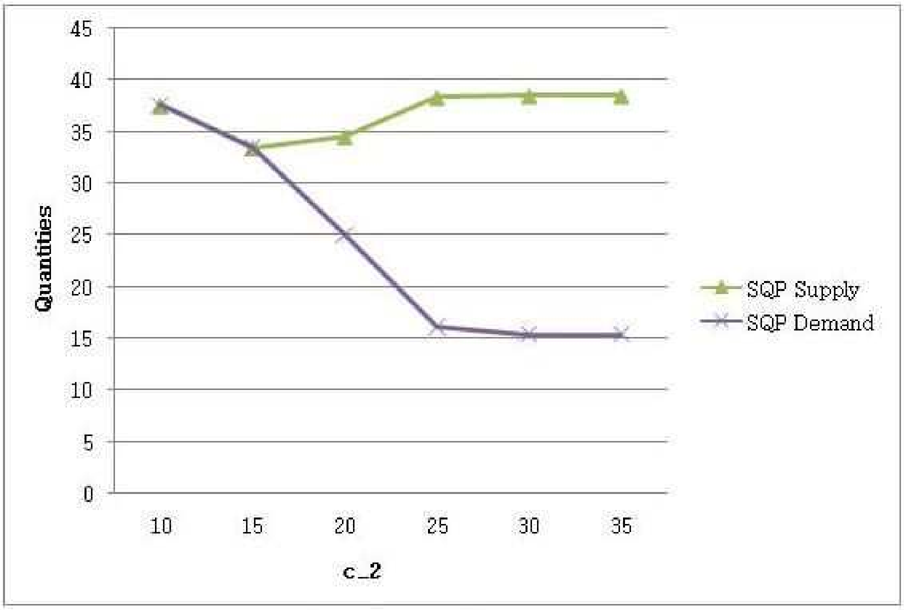
For the decentralized setting;

- The behaviors are the same for the optimal selling price, p and the optimal acquisition fee, f, thus supply and demand.
- The behavior of supply and demand for decentralized models are shown in Figure 5.11, following the similar trend with the centralized models .

- An increase in unit remanufacturing cost of Type 2 items causes the collector's and remanufacturer's profits to decrease, hence the system profit gets smaller.
- In decentralized models, optimal w_2 always decreases while optimal w_1 generally increases. Decrease in w_2 can be explained as an attempt to sell Type 2 products since



(i) Supply and demand quantities for SMP



(ii) Supply and demand quantities for SQP

Figure 5.9: Supply and demand for centralized setting when c_2 increases

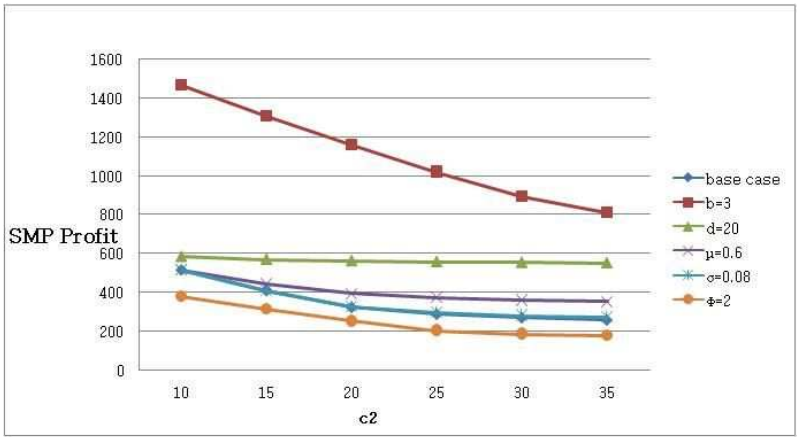


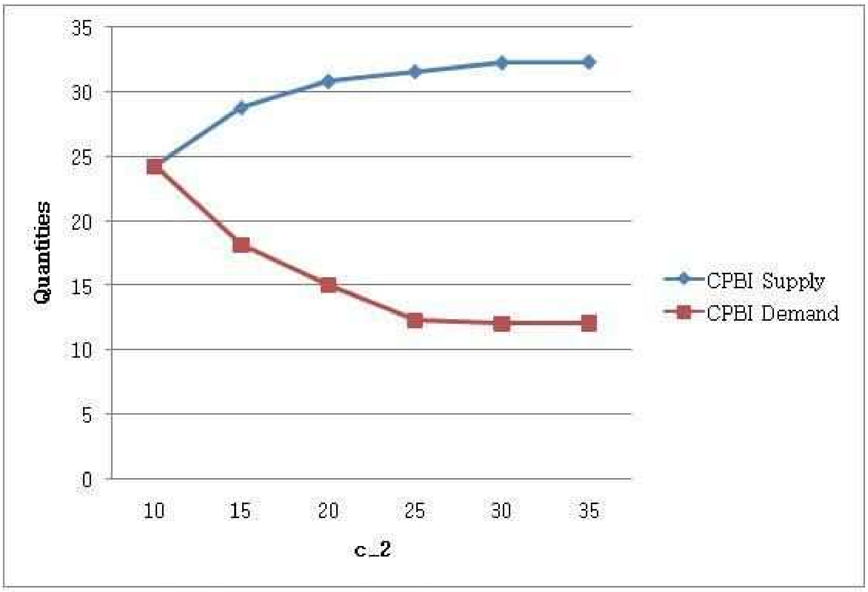
Figure 5.10: SMP Profit when c_2 increases

they are getting more expensive to remanufacture, the transfer price is decreased in order to increase the chance to sell them. Similarly, as Type 2 products are getting less attractive, the remanufacturer tends to buy more Type 1 products, then the collector tends to increase the price of Type 1.

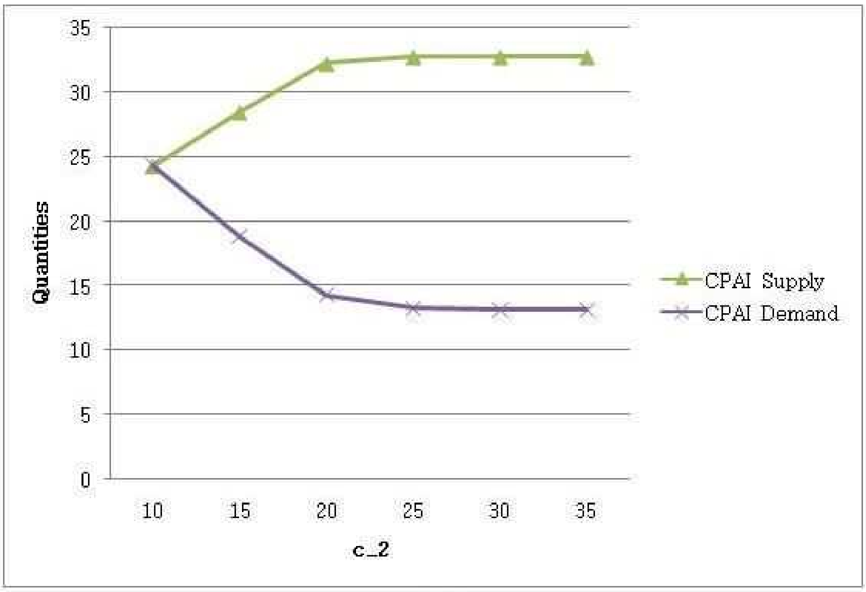
Recall that the supply is assumed to be linearly increasing in acquisition fee and it is formulated as $\psi + \phi f$, where ψ represents the minimum supply quantity. Overall results for the problem instances considered are provided in Appendix F. ψ is changed between 15 and 30 with a stepsize of 3. The observations due to the increase in minimum supply quantity are listed below for the centralized and the decentralized settings separately:

For the centralized setting;

- The optimal acquisition fee, f, and the optimal selling price, p, decrease in ψ. As the potential supply quantity increases, it brings a parallel increase in the optimal supply. Thus, although lower acquisition fee is set, supply gets higher. As a result, demand increases as well.
- Both supply and demand quantities increase in centralized models. Increase in supply outweighs the increase in demand, hence salvage quantities also increase as ψ is increased.
- The expected system profit increases, since high volumes of supply is obtained with low



(i) Supply and demand quantities for CPBI



(ii) Supply and demand quantities for CPAI

Figure 5.11: Supply and demand for decentralized setting when c_2 increases

acquisition fee, providing lower acquisition costs. The revenue gained from the increase in demand also provides increased profits for both SMP and SQP. Figure 5.12 shows the behavior of the SMP's system profit with increasing ψ, under various parameter values. Varying parameter values does not make a significant effect on the general trend of profit increases in ψ.

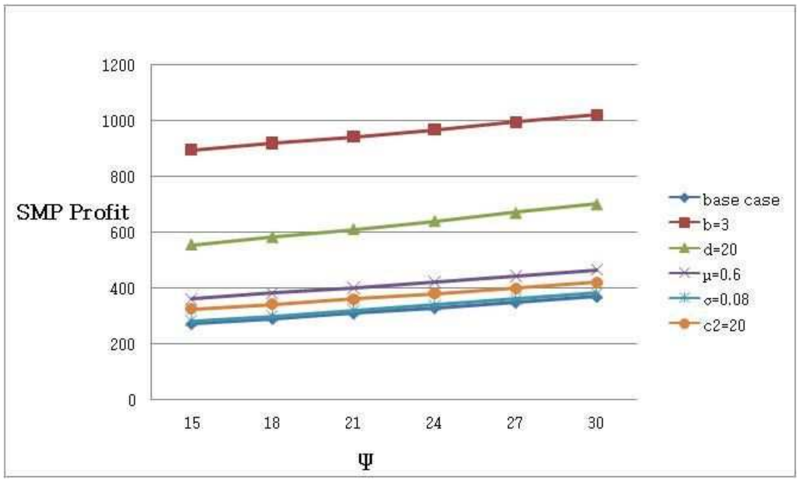


Figure 5.12: SMP Profit when ψ increases

- When ψ, the minimum supply quantity, increases, the effect of quality information also decreases due to the increase in supply quantity, hence number of Type 1 products.

For the decentralized setting:

- The optimal acquisition fee, the optimal selling price, supply and demand quantities follow the same behavior with an increase in ψ.
- The expected system profit increases while the collector's and the remanufacturer's profits are generally increasing in ψ.
- In decentralized models, optimal transfer price of Type 1 products, w_1, decreases and the optimal transfer price of Type 2 products, w_2, generally decreases. As supply in-

creases in ψ together with demand, the quantity of Type 1 also increases and the collector tries to sell more of Type 1 products by decreasing its price.

- When ψ, the minimum supply quantity, increases, the effect of quality information decreases due to the increase in supply quantity, thus Type 1 products. However, for small b or small c_2 values, the effect of quality information shows an increasing behavior but the percentage improvement starts at low values for low values of ψ. The same behavior is valid but ignorable in the centralized setting. Percentage differences for CPBI and CPAI in decentralized setting can be seen in Figure 5.13.

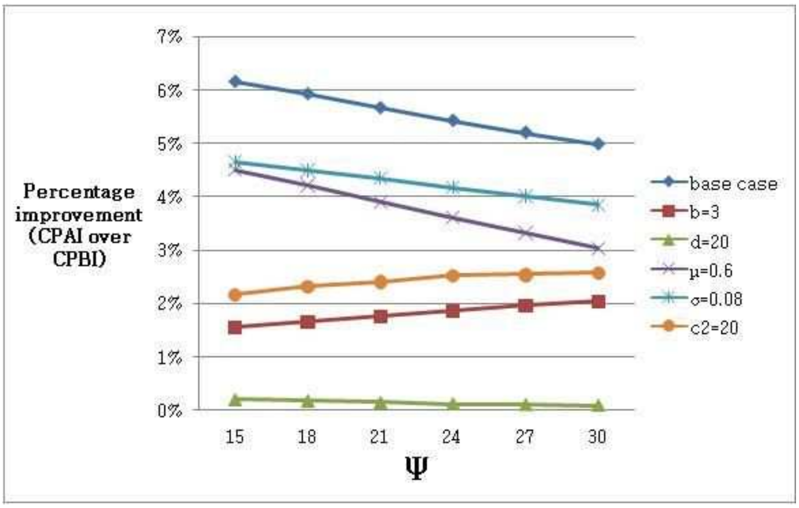


Figure 5.13: Percentage difference in profit of CPBI and CPAI when ψ increases

μ is the expected ratio of Type 1 products among total supply. Increase in μ means increase in the ratio of Type 1 products, a consequent decrease in the ratio of Type 2 products. Overall results for the problem instances considered are provided in Appendix F. μ is changed between 0.4 and 0.6 with a stepsize of 0.05. The observed results when μ increases are summarized below for the centralized and the decentralized settings separately:

For the centralized setting;

- The optimal acquisition fee, f, increases while the optimal selling price, p, decreases in centralized models.

- Both optimal supply and demand quantities increase with increasing μ parallel to the behaviors of f and p.

- System profits show an increasing behavior since the ratio of Type 1 products which are more profitable is increasing. Figure 5.14 shows the behavior of the SMP's system profit with increasing μ, under various parameter values.

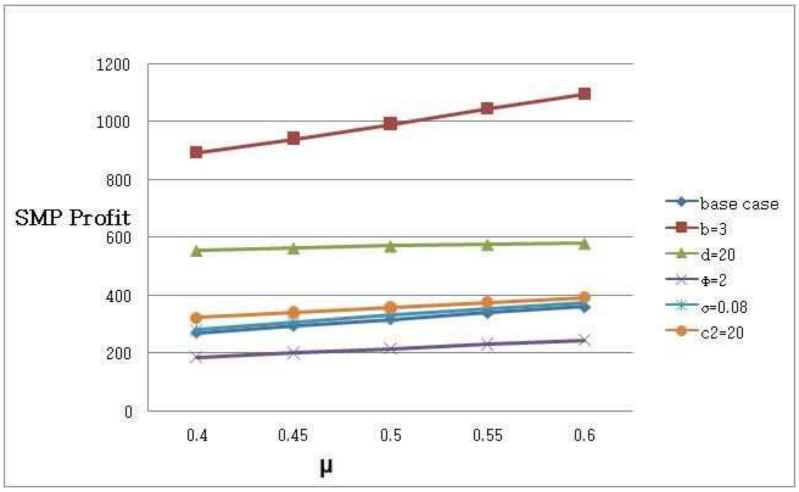


Figure 5.14: SMP Profit when μ increases

  - For smaller price sensitivity of demand, b, the increase in profit is steeper compared to the base case.
  - When remanufacturing cost of Type 2 products, c_2, is smaller, expected system profits are almost the same with the base parameter set since as the ratio of Type 1 items increase, the remanufacturing cost of Type 2 items, c_2, becomes ineffective.
  - When salvage revenue, d, is greater, increase in profit due to the increase in μ is ignorable. As salvaging becomes profitable with higher salvage revenue, decrease in uncertainty becomes almost ineffective for the profit values.

- When μ increases, the effect of quality information decrease slightly. Figure 5.15 shows the behavior of percentage differences for SMP and SQP when μ increases under various parameter values. As μ is increased, uncertainty decreases (since we keep σ constant, the coefficient of variation decreases), resulting in a decreasing effect of quality information. Whereas, when b or c_2 are smaller, effect of quality information shows an increasing behavior as μ is increased.

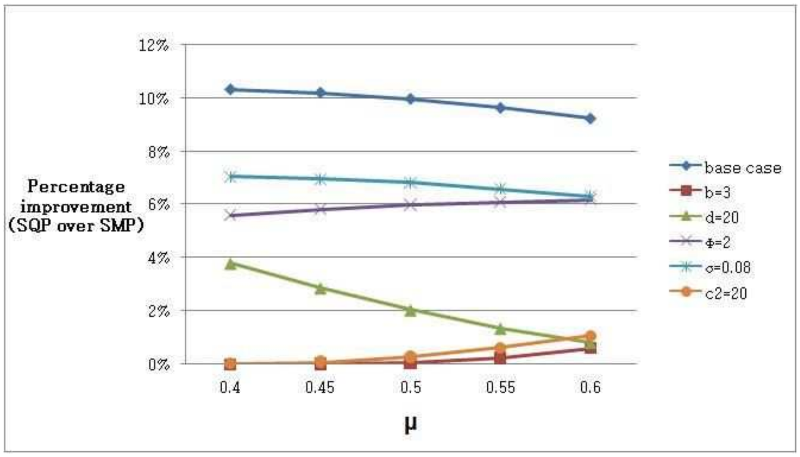


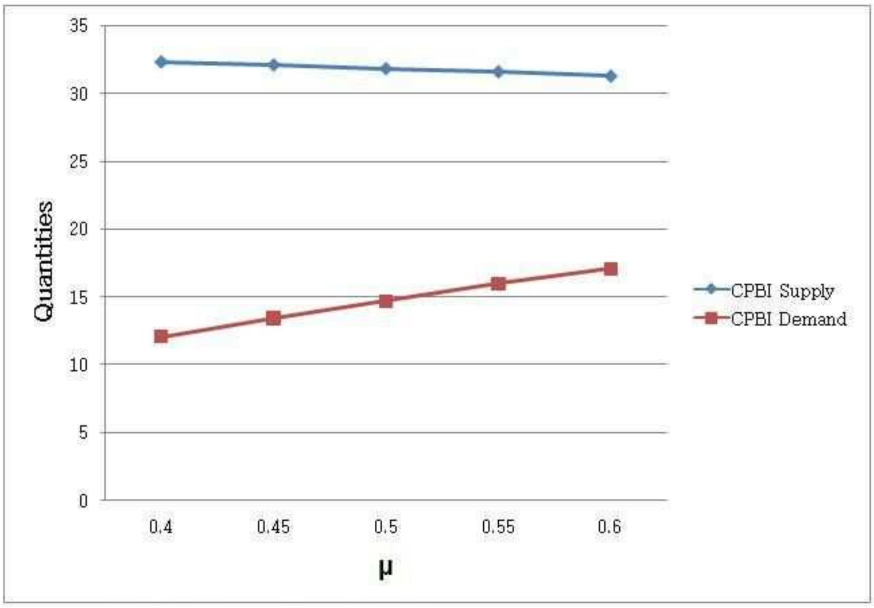
Figure 5.15: Percentage difference in profit of SMP and SQP when μ increases

For the decentralized setting;

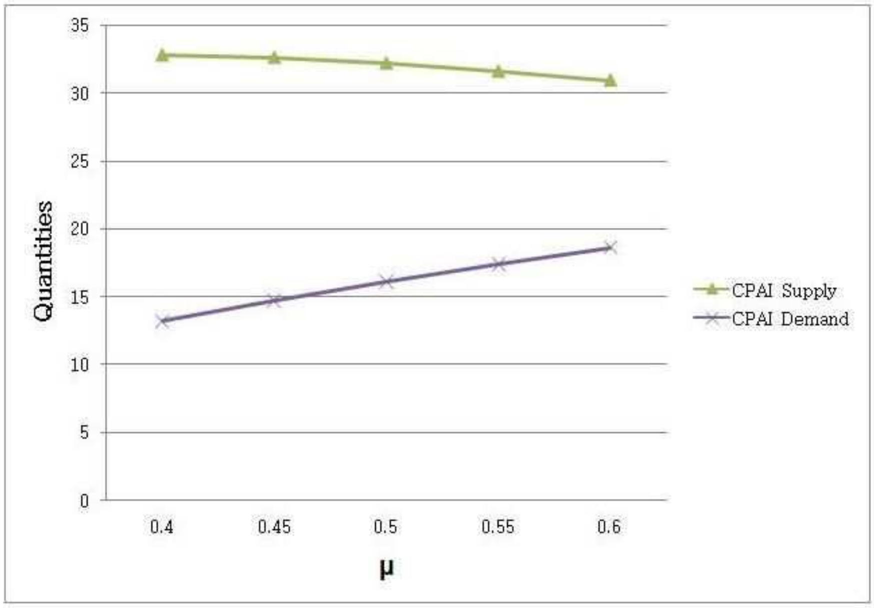
- The optimal acquisition fee, f, generally decreases while the optimal selling price, p, decreases in centralized models. Although increase in f is also observed, there is a decreasing trend in general.

- Optimal supply and demand quantities show the same behavior as f and p, respectively. Since as the ratio of Type 1 items increases, the remanufacturer can easily have more of Type 1 although the collected supply is less. Supply and demand quantities in decentralized models under base parameter set can be seen in Figure 5.16.

- System profits show an increasing behavior since the ratio of Type 1 products which are more profitable is increasing. Both the remanufacturer's and the collector's profit increases since the amount of Type 1 products increase in the system, providing more earnings to the collector and less cost to the remanufacturer.



(i) Supply and demand quantities for CPBI



(ii) Supply and demand quantities for CPAI

Figure 5.16: Supply and demand for centralized setting when μ increases

- In decentralized models, transfer prices w_1 always decreases and w_2 generally decreases. The amount of Type 1 products in system increases and the decrease in the acquisition fee and selling price is reflected to the transfer prices as well.
- The same observation is valid for the effect of quality information as explained in the centralized setting.

5.3 ANALYSIS OF FULL FACTORIAL DESIGN

A full factorial experiment is designed for the parameter values given in Table 5.4. The aim of the full factorial experiment is to detect the value of quality information and effect of centralization over different models under determined values of parameters. Price sensitivity of supply and demand functions, unit salvage revenue, unit remanufacturing cost of Type 2 items, mean and standard deviation of the ratio of Type 1 items among total supply are considered at low and high values in this design. All combinations of these parameter values (totally 216 problem instances) are solved for the centralized and decentralized settings to analyze the results for defined research questions in Section 5.1.

Table 5.4: Parameter values used for full factorial design

a	b	d	c_I	c_1	c_2	ψ	ϕ	μ	σ
200	3	10	5	10	20	15	2	0.4	0.08
	5	20			30		4	0.5	0.1
							6	0.6	0.12

5.3.1 Value Of Quality Information

The quality of the inspected products is assumed to be uncertain in the problem setting. Quality classes of the collected cores are determined after the inspection process. Therefore, the pricing models in which some of the pricing decisions are taken after the inspection process use the quality information which enables the decision maker to adjust the usage quantities according to the known quantities on hand, thus increasing the profit values. The effect of quality information is analyzed by comparing the models those differ in their pricing sequences in centralized and decentralized settings, with or without the realized quality information.

5.3.1.1 SMP versus SQP:

In the centralized setting, the only decision maker, the remanufacturer, determines the selling price of the recovered products either before or after the inspection process. SMP and SQP models differ only in the quality information. In SMP, the remanufacturer makes pricing decision under uncertainty about quality while in SQP, the remanufacturer determines the selling price with quality information.

When the results of the full factorial experiment is analyzed for the centralized setting, it is observed that the optimal profit for SQP is always greater than or equal to the profit for SMP. The maximum, minimum and average values of percentage improvement of SQP over SMP over all problem instances are provided in Table 5.5. It means that, in a centralized setting, postponing the decision of selling price just after the observation of the quantities of the quality groups provides the remanufacturer to gain greater profit due to having the quality information.

Table 5.5: Percentage improvements in centralized and decentralized settings for different types of profit.

	MAX	**MIN**	**AVG**
% system profit improvement on SQP over SMP	10.3%	0%	1.7%
% system profit improvement on CPAI over CPBI	5.3%	-3.4%	0.8%
% collector's profit improvement on CPAI over CPBI	**6.2%**	**0.0%**	**1.3%**
% remanufacturer's profit improvement on CPAI over CPBI	15.7%	-95.4%	-3.4%

The maximum value of percentage improvement is observed for the set in which the remanufacturing cost of Type 2, c_2, is high, price sensitivity of demand, b, is high, price sensitivity of supply, ϕ, is high, salvage revenue, d is low. Moreover mean is low and standard deviation is high. The results are analyzed according to these parameters separately to see if these observations can be generalized for the value of information:

- The remanufacturing cost of Type 2 products, c_2, is either 20 or 30 in the full factorial design, 108 of the problem instances have $c_2 = 30$ and the remaining 108 have $c_2 = 20$.

The averages, minimum and maximum values are evaluated separately for the two levels of c_2 and shown in Table 5.6. The difference can be seen between high and low values of the remanufacturing cost of Type 2 products. Higher remanufacturing cost provides a greater value of quality information. Improvements in centralized models get smaller when cost gets smaller. Type 1 products become more important for the remanufacturer, since remanufacturing Type 2 products becomes more expensive whereas the other remains fixed. As the products differ significantly in their costs, quality uncertainty gets more significant and the value of information becomes more significant. Moreover, the pairwise comparisons between the problem sets that only differ in their c_2 value show us that there is no case in which lower remanufacturing cost outweighs the one with higher remanufacturing cost in terms of the percentage improvements. All 108 comparisons show $c_2 = 30$ results with more value of information.

Table 5.6: Percentage profit improvement of SQP over SMP for different c_2 values

	$c_2 = 30$	c_2=20
max	10.3%	3.1%
min	0.0%	0.0%
avg	2.9%	0.5%

- Table 5.7 shows the results for the high and low values of salvage revenue indicating that percentage improvements are better with lower d values when the maximum values of improvements are compared. However the averages are found to be equal and it is seen that at 20 of 108 comparisons, $d = 10$ is dominating the $d = 20$ case. We cannot directly comment on the effect of d on the value of quality information. In addition, the results in the parameter sensitivity show us that value of information increases with increase in d, following downward direction for very high values of d. Salvaging becomes profitable and quality information becomes irrelevant for very large salvage revenues.

- Price sensitivity of demand function, b is either 5 or 3 in the computations. Maximum and average values of improvements with higher b values are better as seen in Table 5.8. It means that value of information is significant when the demand quantities can

Table 5.7: Percentage profit improvement of SQP over SMP for different d values

	d=20	d=10
max	6.2%	10.3%
min	0.0%	0.0%
avg	1.7%	1.7%

be easily manipulated with selling price values. With smaller b, the existence of the quality information will not affect the demand quantity much through the change in selling price since demand is not so flexible. Moreover when the improvements are compared in pairwise, only in 7 of 108 comparisons revealed that smaller b has caused greater improvement.

Table 5.8: Percentage profit improvement of SQP over SMP for different b values

	b=5	b=3
max	10.3%	3.3%
min	0.0%	0.0%
avg	3.0%	0.4%

- Mean of the random variable expressing the ratio of Type 1 products among total supply, μ, has three possible values in our computations. As can be seen in Table 5.9, lower mean values show significant effect of quality information. Increase in μ with constant σ means decrease in uncertainty, resulting in more Type 1 products among total supply. $\mu = 0.6$ indicates the case that is affected less from the quality uncertainty, thus value of information is lower in improvement amounts. In 16 among 72 pairwise comparisons in terms of the improvements, (only $\mu = 0.6$ and $\mu = 0.4$ are compared for 72 sets) $\mu = 0.4$ is dominant. Therefore, we cannot conclude with an exact result for the relation between μ and the value of information.

Table 5.9: Percentage profit improvement of SQP over SMP for different μ values

	μ=0.6	μ=0.5	μ=0.4
max	9.2%	10.0%	10.3%
min	0.0%	0.0%	0.0%
avg	1.6%	1.7%	1.8%

- Price sensitivity of supply quantity, ϕ, has three possible values in the computations. Table 5.10 shows that higher ϕ values result in higher improvements in profit in case of quality information. Higher ϕ indicates that manipulation of the supply quantity by changing the prices becomes easier. Quality information requires the adjustment of supply and demand quantities through the pricing decisions, therefore higher ϕ brings significant value of information, although 15 of 72 total pairwise comparisons show that $\phi = 2$ dominates $\phi = 6$ in terms of percentage improvement in profit.

Table 5.10: Percentage profit improvement of SQP over SMP for different ϕ values

	$\phi = 6$	$\phi = 4$	$\phi = 2$
max	10.3%	8.7%	6.2%
min	0.0%	0.0%	0.0%
avg	1.8%	1.8%	1.5%

- Standard deviation of the ratio of Type 1 products, σ, has three values used in computations. Among them, higher deviations bring increased uncertainty, hence more significant value of information. In all of the 72 pairwise comparisons, $\sigma = 0.12$ dominates $\sigma = 0.08$ in case of improvement in profit.

Table 5.11: Percentage profit improvement of SQP over SMP for different σ values

	σ=0.12	σ=0.1	σ=0.08
max	10.3%	8.7%	7.0%
min	0.0%	0.0%	0.0%
avg	2.0%	1.7%	1.4%

There are cases in which quality information is ineffective in 39 problem instances, having percentage improvement of zero. In all of these cases, salvage quantity is found to be zero, showing that all of the supply is remanufactured. In general, we can say that when there is no salvaging in either SMP or SQP, the effect of quality information diminishes. It is a reasonable observation since all of the supply quantity is remanufactured and sold to meet the demand and thus quality information is ineffective in pricing decisions.

5.3.1.2 CPBI versus CPAI:

In decentralized setting, the collector determines the transfer prices of the inspected products either before or after the inspection process. CPBI and CPAI models differ only in the quality information. In CPBI, the collector makes the pricing decision before the inspection, hence under uncertainty about quality; while in CPAI, the collector determines the transfer prices after the inspection, thus with quality information.

Results of the full factorial experiment reveal that the collector's optimal profit in CPAI is observed to be greater than or equal to the collector's profit in CPBI. The maximum, minimum and average values of percentage improvement of CPAI over CPBI over all problem instances are provided in Table 5.5. System profit and the remanufacturer's profit values do not show a consistent behavior unlike the collector's profit. Some cases in which CPBI is dominant to CPAI can also be observed. However, the collector's profit generally shows the value of quality information since the quality information has an impact on the collector's optimal decision in the decentralized setting.

It can be stated that, similar to the centralized setting, postponing the decision of transfer prices just after the announcement of the quantities of the quality groups provides the collector to gain greater profit due to having the quality information.

In parameter sets at which value of quality information is observed, it can be stated that this value is generally greater for centralized setting compared to the decentralized setting. This relation can be seen in Appendix G. When the percentage difference is not zero for SQP versus SMP comparison, the improvement of SQP over SMP is higher than the improvement of CPAI over CPBI. This result is obtained from the maximum, minimum and average values found. The comparisons of the percentage improvements for all parameter sets do not support this observation, since in 43 of 216 comparisons, improvement in decentralized models are better than the improvement in the centralized models.

The effect of quality information is analyzed when the system parameters are changed individually for decentralized models and the results are provided in Appendix G.

For the decentralized setting, the maximum value of percentage improvement in the comparison of CPBI with CPAI is observed in the same problem instance which gives the maximum value of percentage improvement in the comparison of SMP with SQP in the centralized setting. Therefore, the results in case of value of information are similar to the centralized setting, thus details are not given here.

5.3.2 Effect of Centralization

Centralized and decentralized settings are differentiated in their decision makers. When the only decision maker is the remanufacturer, the decisions and processes are centralized. However, when decentralization occurs, the collector collects the cores and sells them to the remanufacturer after inspection. Due to the transfer prices between the remanufacturer and the collector, double marginalization occurs in decentralization. This brings the expected beneficial effect of centralization. The effect of centralization is analyzed by comparing the centralized models, SQP and SMP, to two models of the decentralized setting separately.

5.3.2.1 SQP versus CPBI/CPAI:

In SQP, the remanufacturer takes the advantage of pricing the remanufactured products after inspection with quality information on hand. Similarly, in CPAI, the remanufacturer makes his decision under quality information, but in this case the collector also decides the transfer prices after inspection with quality information on hand. Since quality information is available in both models, the only difference is the user of this information due to centralization of the pricing decisions. This brings the effect of centralization. The comparisons are made over the percentage improvements in total system profit values for all problem instances since in the decentralized setting, profit is shared between the remanufacturer and the collector.

The results reveal that total system profits are always better in SQP compared to CPAI, with a percentage difference varying between 0.3 and 22 percent and the average is 8.3 percent. The comparison values for SQP with CPAI are given in Appendix H. Similar results are obtained for the comparison of SQP with CPBI, since in CPBI, neither centralized decision making nor quality information for the collector is available. Whereas, the percentage improvements are greater between SQP and CPBI compared to the relation between SQP and CPAI, changing between 1.7 to 25.9 percent with an average of 9.2 percent. In this way, centralized decision making is proven to be better from the overall system perspective and centralization brings profit advantage to the remanufacturing system.

Different from the value of quality information, the percentage improvement of profit showing the effect of centralization has generally superior values when c_2 is low and b is low. However high ϕ and low d are similar for the centralization effect. Different parameter values are analyzed distinctly to examine the relation between these parameters and the effect of centralization. Appendix H gives details of the percentage improvement comparisons with focus of centralization.

- The averages, minimum and maximum values are evaluated separately for the possible two values for c_2, the remanufacturing cost of Type 2 products. It is observed that for small values of c_2, percentage improvements are better, resulting in a more significant effect of centralization. This result is unlike the observation made when assessing the value of information. The comparison values for CPAI is shown in Appendix H. The similar observation is valid for the comparison with CPBI having relatively larger improvement values since in CPBI there is neither centralization nor the value of quality information for the collector, thus a poorer model in both perspectives. The value of c_2 that makes the value of information more significant and effect of centralization higher are not identical.

- The results for the high and low values of salvage revenue indicate that percentage improvements are better with lower d values when the maximum values and averages of improvements are compared as given in Appendix H. The number of cases in which $d = 10$ dominates the result of $d = 20$ is 80 among total of 108, a considerable number to support our observation. In addition, for very high values of d, benefit of centralization goes to its minimum values.

- Maximum, minimum and average values of improvements with smaller b values are better as seen in Appendix H. It means that effect of centralization is significant when the demand quantities can not be easily manipulated with selling price values. This result is unlike the result for the value of information, having significant values for higher b. Moreover when the improvements are compared in pairwise, only 22 of 108 comparisons revealed that higher b has caused greater improvement for the effect of centralization.

- Mean of the random variable expressing the ratio of Type 1 products among total supply, μ, is examined for its three possible values in our computations. As can be seen in Appendix H, significance of the effect of centralization cannot be exactly derived. Lower mean values seem to dominate higher values since 17 of 72 pairwise comparisons (only $\mu = 0.6$ and $\mu = 0.4$ are compared in 72 sets) show that $\mu = 0.4$ results better in centralization effects measured in percentage improvement of profit over CPBI. However, maximum, minimum and average values of improvements do not support the same result.

- Price sensitivity of supply quantity, ϕ, has three possible values in the computations. The comparison values for ϕ, shown in Appendix H easily reveal that higher ϕ values result in more significant effect of centralization similar to the observation in value of quality information. Higher ϕ values result in higher improvements in profit via centralization. Higher ϕ indicates that manipulation of the supply quantity by changing the prices becomes easier. 7 of 72 total pairwise comparisons show that $\phi = 2$ dominates $\phi = 6$ in terms of percentage improvement in profit, which is not a reasonable number of cases to alter our observation.

- Standard deviation of the ratio of Type 1 products, σ, has three values used in computations. Among them, maximum, minimum and average values of improvements in profit does not show a significant difference in case of centralization. However, the cases in which $\sigma = 0.12$ dominates the $\sigma = 0.08$, are high in number, namely in 71 of 72 total comparisons, $\sigma = 0.08$ is found to be dominated by $\sigma = 0.12$.

As a general observation, it can be stated that supply and demand values are always higher in centralized models compared to the decentralized models. This observation is a result of benefit of centralization. Centralization of the pricing decisions compensates the amount lost due to double marginalization, resulting in higher supply and demand quantities.

5.3.2.2 SMP versus CPBI/CPAI:

The decision of the selling price is always delayed in the decentralized models, however they vary with collector's pricing decisions. Whereas, in SMP there is no quality information used and selling price is determined simultaneously with the acquisition fee. Therefore, it is expected to see some situations in which decentralized models can outweigh the centralized model, SMP, in case of the percentage improvements.

The comparison between centralized and decentralized settings is made over SMP in this section. It is observed that there are cases in which decentralized models are better in system profit values (with a minimum of -7.3 percent) for both CPBI and CPAI. The percentage improvements are shown in Appendix H. These situations are observed mostly in the sets having lower d and higher b and σ, similar to the conditions under which value of information is more significant. Different from the value of information, ϕ and c_2 are at their lower values when decentralized models outweigh SMP.

The improvement in profit reaches up to 25.9 percent over CPBI. The pairwise comparisons show that in 23 and 35 of total 216 cases, CPBI and CPAI are better than SMP in profit, respectively. These numbers are reasonable to indicate that it is possible to increase expected profits by decentralization.

Different parameter values are analyzed distinctly to explore the relation between these parameters and the outweighed effect of centralization by value of information.

- The averages, minimum and maximum values are evaluated separately for the possible two values for c_2, the remanufacturing cost of Type 2 products. It is observed that for maximum and average values of improvements seen in Appendix H, $c_2 = 20$ seems to be better. However minimum values of improvements show that more negative values exist in the cases with higher c_2. Since negative values for the improvements on SMP over CPBI/CPAI are the searched results, it is concluded that for smaller c_2, the dominance of the quality information over the effect of centralization gets more significant.
- The results for the high and low values of salvage revenue indicating that percentage improvements are better with lower d values when the maximum, minimum values and averages of improvements are compared, given in Appendix H. Number of negative improvement values is also taken into account in this comparison. The observation in d in this case is similar to the effect of quality information explained, and effect of centralization over SQP.
- Maximum and average values of improvements with smaller b values are better as seen in Appendix H. Whereas, since the negative values of the improvements over SMP are

looked for, minimum values of improvements are considered. Higher b gives negative minimum values, and when the distribution of the negative values among $b = 3$ and $b = 5$ are investigated, the result is that for higher values of b, the value of information outweighs the effect of centralization.

- Mean of the random variable expressing the ratio of Type 1 products among total supply, μ, is examined for its three possible values in our computations. As can be seen in Appendix H, negative improvement values are mostly placed under smaller μ values although the maximum and average values address other values of μ. Lower mean values result in highly dominated effect of centralization by the quality information.

- Price sensitivity of supply quantity, ϕ, has three possible values in the computations. The comparison values for ϕ, shown in Appendix H address the higher ϕ values for having significant improvement values. However, distribution of the negative improvement values are searched and the result is smaller ϕ. Therefore when price sensitivity of the supply quantity is low, value of information becomes dominant to centralization.

- Observation for the standard deviation of the ratio of Type 1 products, σ, is similar to the result for value of quality information. Higher standard deviation value results in greater number of cases in which improvement is found to be negative, showing the dominance of the value of information on centralization.

As a reply to the research question 4 in Section 5.1, large majority of these situations occur when the effect of quality information is observed at its higher values and the effect of centralization over SQP is investigated at its lower values. It shows that when the effect of quality information on profit is dramatic, effect of quality information can outweigh the effect of centralization.

CHAPTER 6

CONCLUSION

In remanufacturing environments, the collected products are observed to be from various qualities, depending on their suitability to be remanufactured. Quality variability brings a difference in their remanufacturing costs and transfer prices. Therefore determining the selling prices under uncertainty of quality affects the ordering quantities, remanufacturing quantities, and thus the system profit negatively. However, with available quality information prior to pricing, prices can be adjusted with the available known quantities from all types of quality, making system profits greater.

The owner of the collection and inspection activities bring the centralization issue forward. The remanufacturer can either collect the products himself or outsources the collection and inspection activities to a collector firm. Decentralization brings double marginalization effect due to the increase in the number of decision makers in the system.

In this study, we have considered a pure product recovery system consisting of a remanufacturer and a collector. Used core products are collected from users at an acquisition fee. The collected products are assumed to be of two quality groups considered: Superior quality and inferior quality groups. After collection, core products are inspected in order to determine which quality group they are included in. The proportion of the superior quality products to total supply is uncertain. After the announcement of the quantities for each quality group, the inspected cores are transferred to the remanufacturer at needed quantities on transfer prices varying with the quality. These two quality groups differ in their remanufacturing costs. Then the remanufacturer determines the usage quantities from each quality group in remanufacturing process. The unused inspected products those are not remanufactured can be disposed at a unit salvage revenue. Recovered products are of single quality independent of the quality group of the inspected products used. They are sold at a fixed selling price to the customers finally. Both supply and demand are assumed to be price sensitive. Supply is an increasing linear function of the acquisition fee and demand is a decreasing linear function of the selling price. The objective is to find the optimal acquisition fee of collected core products and selling price of the recovered products maximizing the expected profit.

In this problem setting, we constructed different models under centralized and decentralized settings according to the availability of the quality information after or before the pricing decisions. The objective is maximizing the expected profit. For these models, the optimal solutions are derived. Then a computational study is conducted in order to explore the value of quality information and effects of centralization on the optimal solutions.

Our numerical analysis reveal the following findings:

- In both centralized and decentralized settings, postponing the decision of selling price just after the observation of the quantities of the quality groups provides greater profit due to having the quality information.
- Value of information is found to be sensitive to the remanufacturing costs and when the difference between the cost of remanufacturing various types of quality increases, the value of information gets more significant.
- Value of information is also significant when the price sensitivity of the demand is higher.
- Value of information does not exist for very high values of the potential market size, meaning that the demand is high enough, thus not effected from the uncertain quality.
- If all of the supply quantity is sold in the market, meaning no salvaging is made, there is no benefit of postponing the pricing decision after observing the quantity.
- In general, there is a certain threshold unit salvage value beyond which supply exceeds demand and thus the value of quality information is observed.
- When mean of the random variable denoting the ratio of Type 1 products in the total supply increases, the expected quantity of Type 1 products increases and thus the effect of quality information decrease slightly. Higher values of standard deviation of this random variable results in more significant value of information.

- Improvement maintained by the quality information in centralized setting is generally greater than the improvement in decentralized setting.
- Centralization of pricing decisions is explored to be profitable since it prevents the double marginalization effect. This result is important for the remanufacturer companies in decision of outsourcing the collection activity or not.
- As a general observation, supply and demand values are higher in centralized models compared to the decentralized models. This observation is a result of benefit of centralization. Centralization of the pricing decisions compensates the amounts lost due to double marginalization, resulting in higher supply and demand quantities.
- Effect of centralization is detected to be higher for the problem instances in which the value of quality information has less effect.
- There are cases in which the value of quality information outweighs the benefit of centralization, generally when the effect of information is found to be dramatic.
- Centralization/decentralization decision should be analyzed in depth together with the potential information systems to ascertain the right configuration for the remanufacturing environment, maximizing the overall profit.

In future studies, inventory holding can be included in the model, making the problem multiperiod. Number of quality groups can also be increased or varying salvage revenues can be considered for different quality groups. Under the decentralized setting, a model in which the remanufacturer determines the transfer prices can also be examined in order to see the advantages or disadvantages of the change in the decision maker for the decentralized setting.

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APPENDIX A

ALGORITHMS TO FIND THE OPTIMAL SOLUTION FOR CPBI

To find the optimal w_1, w_2 and f for the collector in CPBI for $E[\Pi_1(f, w_1, w_2)]$, which is common for two cases when Type 1 or Type 2 is more economical, the following procedure is used:

- The point where $\frac{dE[\Pi_1(f)]}{df}$ is close to zero is searched for.
- For an increment of 0.5, for f values starting from zero, $\frac{dE[\Pi_1(f)]}{df}$ is calculated
- When absolute value of derivative function is less than 0.000001, run is stopped.
- Related f value is accepted as the optimal f for $E[\Pi_1(f, w_1, w_2)]$.
- Corresponding w_1 and w_2 values are found by $w_1 = ((a - 2 * (psi + phi * f_4opt))/b) - c_1$ and $w_2 = ((a - 2 * (psi + phi * f_4opt))/b) - c_2$.

However, when the second piece of the collector's profit function, $E[\Pi_2(f, w_1, w_2)]$, is evaluated, the cases in which Type 1 is more economical and Type 2 is more economical are analyzed separately since $E[\Pi_2(f, w_1, w_2)]$ values are not common.

To find the optimal w_1, w_2 and f for the collector in CPBI for $E[\Pi_2(f, w_1, w_2)]$ when Type 1 is more economical, the following procedure is used:

- w_1, w_2 and f are changed gradually within their intervals and related profit is calculated for each set.

- The maximum of the calculated profit values are determined and corresponding w_1, w_2 and f are stated as optimal for this part of the profit. The intervals used for the variables are as follows:

  – w_2 is increased from 0 to $(a/b) - c_2$ with a stepsize of $(((a/b) - c_2) - 0)/100$.

  – w_1 is increased from 0 to $c_2 + w_2 - c_1$ with a stepsize of $((c_2 + w_2 - c_1) - 0)/100$.

  – f is increased from $(((a - b(c_2 + w_2))/2) - psi)/phi$ to $max(w_1, w_2)$with a stepsize of $(max(w_1, w_2) - (((a - b(c_2 + w_2))/2) - psi)/phi)/100$.

To find the optimal w_1, w_2 and f for the collector in CPBI for $E[\Pi_2(f, w_1, w_2)]$ when Type 2 is more economical, the following procedure is used:

- w_1, w_2 and f are changed gradually within their intervals and related profit is calculated for each change.

- The maximum of these profit values are determined and corresponding w_1, w_2 and f are stated as optimal for this part of the profit. The intervals used for the variables are as follows:

  – w_1 is increased from 0 to $(a/b) - c_1$ with a stepsize of $(((a/b) - c_1) - 0)/100$.

  – w_2 is increased from 0 to $c_1 + w_1 - c_2$ with a stepsize of $((c_1 + w_1 - c_2) - 0)/100$.

  – f is increased from $(((a-b(c_1+w_1))/2)-psi)/phi$ with a stepsize of $(max(w_1, w_2)-(((a - b(c_1 + w_1))/2) - psi)/phi)/100$.

Note that when the stepsizes are less than 0.01, they are set to 0.01 for w_1, w_2 and f.

Finally, among the determined three profit values, the maximum is selected and the related f, w_1 and w_2 are concluded as the approximate optimal solutions.

APPENDIX B

SECOND ORDER DERIVATIVES FOR CPBI

Second order partial derivatives of $E[\Pi_2(f, w_1, w_2)]$ are as follows under the case when Type 2 is more economical:

$$\begin{aligned}
\frac{\partial^2 E[\Pi_2(f, w_1, w_2)]}{\partial f^2} &= -2\phi \\
&- (w_2 - d) g\left(\frac{a - b(c_2 + w_2)}{2(\psi + \phi f)}\right)\left(\frac{\phi^2(a - b(c_2 + w_2))^2}{2(\psi + \phi f)^3} - \frac{\phi^2(a - b(c_2 + w_2))}{2(\psi + \phi f)^2}\right) \\
&- (w_2 - d) g'\left(\frac{a - b(c_2 + w_2)}{2(\psi + \phi f)}\right)\left(\frac{\phi(a - b(c_2 + w_2))^2}{4(\psi + \phi f)^2} - \frac{\phi(a - b(c_2 + w_2))}{2(\psi + \phi f)}\right) \\
&+ (w_1 - d) g\left(\frac{a - b(c_1 + w_1)}{2(\psi + \phi f)}\right)\left(\frac{\phi^2(a - b(c_1 + w_1))^2}{2(\psi + \phi f)^3} - \frac{\phi^2(a - b(c_1 + w_1))}{2(\psi + \phi f)^2}\right) \\
&+ (w_1 - d) g'\left(\frac{a - b(c_1 + w_1)}{2(\psi + \phi f)}\right)\left(\frac{\phi(a - b(c_1 + w_1))^2}{4(\psi + \phi f)^2} - \frac{\phi(a - b(c_1 + w_1))}{2(\psi + \phi f)}\right) \\
&+ \frac{\phi^2(a - b(c_1 + w_1))}{(\psi + \phi f)^2}(w_1 - d)\left(1 - \frac{a - b(c_1 + w_1)}{2(\psi + \phi f)}\right) g\left(\frac{a - b(c_1 + w_1)}{2(\psi + \phi f)}\right) \\
&- \frac{\phi^2(a - b(c_2 + w_2))}{(\psi + \phi f)^2}(w_2 - d)\left(1 - \frac{a - b(c_2 + w_2)}{2(\psi + \phi f)}\right) g\left(\frac{a - b(c_2 + w_2)}{2(\psi + \phi f)}\right)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 E[\Pi_2(f, w_1, w_2)]}{\partial w_1^2} &= \frac{b}{2} g\left(\frac{a - b(c_1 + w_1)}{2(\psi + \phi f)}\right) + \frac{b^2(w_1 - d)}{4(\psi + \phi f)} g'\left(\frac{a - b(c_1 + w_1)}{2(\psi + \phi f)}\right) \\
&- \left(\frac{-b}{2}\left(1 - \frac{a - b(c_1 + w_1)}{2(\psi + \phi f)}\right) - \frac{a - b(c_1 + 2w_1 - d)}{2}\right) g\left(\frac{a - b(c_1 + w_1)}{2(\psi + \phi f)}\right) \\
&- \int_0^{\frac{a - b(c_1 + w_1)}{2(\psi + \phi f)}} b g(x) dx
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial w_2^2} &= \frac{-b}{2} g\left(\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}\right) - \frac{b^2(w_2-d)}{4(\psi+\phi f)} g'\left(\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}\right) \\
&+ \frac{b}{(\psi+\phi f)}\left(\frac{a-b(c_2+2w_2-d)}{2}\right) g\left(\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}\right) + \int_{\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}}^{1} -bg(x)dx \\
&- \frac{b}{2}\left(1-\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}\right) g\left(\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}\right) \\
&- b(w_2-d)\frac{a-b(c_2+w_2)}{2(\psi+\phi f)} g'\left(\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}\right)
\end{aligned}
$$

$$
\frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial w_1 \partial w_2} = \frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial w_2 \partial w_1} = 0
$$

$$
\begin{aligned}
\frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial f \partial w_1} &= \frac{-\phi}{4(\psi+\phi f)^2}\left(-2b(a-b(c_1+w_1))(w_1-d)-(a-b(c_1+w_1))^2\right) g\left(\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}\right) \\
&+ \frac{\phi}{2(\psi+\phi f)}(a-b(c_1+w_1))^2(w_1-d) g'\left(\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}\right) \\
&+ \frac{b\phi(w_1-d)}{2(\psi+\phi f)}\left(1-\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}\right) - \int_0^{\frac{a-b(c_1+w_1)}{2(\psi+\phi f)}} \phi(1-x)g(x)dx
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 E[\Pi_2(f,w_1,w_2)]}{\partial f \partial w_2} &= \frac{\phi}{4(\psi+\phi f)^2}\left(-2b(a-b(c_2+w_2))(w_2-d)-(a-b(c_2+w_2))^2\right) g\left(\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}\right) \\
&- \frac{\phi}{2(\psi+\phi f)}(a-b(c_2+w_2))^2(w_2-d) g'\left(\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}\right) \\
&- \frac{b\phi(w_2-d)}{2(\psi+\phi f)}\left(1-\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}\right) + \int_0^{\frac{a-b(c_2+w_2)}{2(\psi+\phi f)}} \phi(1-x)g(x)dx
\end{aligned}
$$

APPENDIX C

DETAILED PROOF OF THEOREM 4.3.2

Collector's problem to determine transfer prices is studied according to the intervals in (4.54) separately as follows: When $Q_2 \geq \frac{a-b(c_2+w_2)}{2}$;

The profit function is can be expressed using the corresponding optimal q_1 and q_2 expressions as follows:

$$\begin{aligned}
\Pi_1(w_1, w_2) &= w_1 q_1 + w_2 q_2 + d(Q_1 + Q_2 - q_1 - q_2) \\
&= w_1 0 + w_2 \frac{a - b(c_2 + w_2)}{2} + d(\psi + \phi f - 0 - \frac{a - b(c_2 + w_2)}{2}) \\
\Pi_1(w_2) &= (w_2 - d)\frac{a - b(c_2 + w_2)}{2} + d(\psi + \phi f)
\end{aligned}$$

Collector's problem reduces to:

$$\begin{aligned}
\text{Max } \Pi_1(w_2) &= (w_2 - d)\frac{a - b(c_2 + w_2)}{2} + d(\psi + \phi f) \\
\text{s.to} & \\
Q_2 &\geq \frac{a - b(c_2 + w_2)}{2} && \text{(C.1)} \\
w_2 + c_2 &\leq w_1 + c_1 && \text{(C.2)} \\
w_2 &\geq 0
\end{aligned}$$

The objective function is concave in $w_2 \geq 0$ $\left(\frac{d^2\Pi_1}{dw_2^2} = -b < 0\right)$, the optimal solution can be characterized utilizing the first order conditions and taking the constraint $Q_2 \geq \frac{a-b(c_2+w_2)}{2}$ into account as follows:

$$(w_1^*, w_2^*) = \begin{cases} \left(w_1 \geq \frac{a-b(2c_1-c_2-d)}{2b}, w_2 = \frac{a-b(c_2-d)}{2b}\right) & \text{when } Q_2 \geq \frac{a-b(c_2+d)}{4} \\ \left(w_1 \geq \frac{a-2Q_2}{b} - c_1, w_2 = \frac{a-2Q_2}{b} - c_2\right) & \text{otherwise} \end{cases} \tag{C.3}$$

When $\frac{a-b(c_1+w_1)}{2} \leq Q_2 < \frac{a-b(c_2+w_2)}{2}$;

The profit function can be expressed as follows:

$$\begin{aligned} \Pi_2(w_1, w_2) &= w_1 q_1 + w_2 q_2 + d(Q_1 + Q_2 - q_1 - q_2) \\ &= w_1 * 0 + w_2 * Q_2 + d(Q_1 + Q_2 - 0 - Q_2) \\ \Pi_2(w_2) &= w_2 Q_2 + dQ_1 \end{aligned}$$

Collector's problem reduces to:

$$\begin{aligned} \text{Max } \Pi_2(w_2) &= w_2 Q_2 + dQ_1 \\ \text{s.to} & \\ \frac{a - b(c_1 + w_1)}{2} &\leq Q_1 < \frac{a - b(c_2 + w_2)}{2} && \text{(C.4)} \\ w_2 + c_2 &\leq w_1 + c_1 && \text{(C.5)} \\ w_2 &\geq 0 \end{aligned}$$

The objective function is linearly increasing in $w_2 \geq 0$ $\left(\frac{d^2\Pi_2}{dw_2^2} = 0\right)$, the optimal solution can be characterized utilizing the constraints of the model and the result is same with the second piece of the piecewise optimal solution to the collector's problem when $Q_2 \geq \frac{a-b(c_2+w_2)}{2}$, shown in (C.3). Therefore, we can combine it with Equation (C.3).

When $Q_2 < \frac{a-b(c_1+w_1)}{2} \leq (\psi + \phi f)$;

The profit function can be expressed by replacing w_2 with $w_1 + c_1 - c_2$ as Type 2 is more economical:

$$\begin{aligned}
\Pi_3(w_1,w_2) &= w_1q_1 + w_2q_2 + d(Q_1+Q_2-q_1-q_2)\\
&= w_1\left(\frac{a-b(c_1+w_1)}{2} - Q_2\right) + w_2Q_2 + d(Q_1+Q_2-\frac{a-b(c_1+w_1)}{2})\\
&= (w_2-w_1)Q_2 + (w_1-d)\frac{a-b(c_1+w_1)}{2} + d(\psi+\phi f)\\
\Pi_3(w_1) &= (c_1-c_2)Q_1 + (w_1-d)\frac{a-b(c_1+w_1)}{2} + d(\psi+\phi f)
\end{aligned}$$

Collector's problem reduces to:

$$\begin{aligned}
\text{Max } \Pi_3(w_1) &= (c_1-c_2)Q_1 + (w_1-d)\frac{a-b(c_1+w_1)}{2} + d(\psi+\phi f)\\
\text{s.to} &\\
Q_2 &< \frac{a-b(c_2+w_2)}{2} \leq (\psi+\phi f) \qquad \text{(C.6)}\\
w_1 &\geq 0
\end{aligned}$$

The objective function is concave in $w_1 \geq 0$ $\left(\frac{d^2\Pi_3}{dw_1^2} = -b < 0\right)$, the optimal solution can be characterized utilizing the first order conditions and taking the constraint $Q_2 < \frac{a-b(c_1+w_1)}{2} \leq (\psi+\phi f)$ into account as follows:

$$(w_1^*, w_2^*) = \begin{cases} \left(\frac{a-b(c_1-d)}{2b}, \frac{a-b(2c_2-c_1-d)}{2b}\right) & \text{if} Q_2 < \frac{a-b(c_1+d)}{4} \text{ and } \frac{a-b(c_1+d)}{4} \leq \psi+\phi f\\ \left(\frac{a-2Q_2}{b} - c_1, \frac{a-2Q_2}{b} - c_2\right) & \text{if } Q_2 > \frac{a-b(c_1+d)}{4} \text{ and } \frac{a-b(c_1+d)}{4} \leq \psi+\phi f\\ \left(\frac{a-2(\psi+\phi f)}{b} - c_1, \frac{a-2(\psi+\phi f)}{b} - c_2\right) & \text{if } Q_2 < \frac{a-b(c_1+d)}{4} \text{ and } \frac{a-b(c_1+d)}{4} > \psi+\phi f \end{cases} \qquad \text{(C.7)}$$

When $\frac{a-b(c_1+w_1)}{2} > (\psi+\phi f)$;

The profit function can be expressed as:

$$\begin{aligned}
\Pi_4(w_1,w_2) &= w_1q_1 + w_2q_2 + d(Q_1+Q_2-q_1-q_2) - (f+c_I)(\psi+\phi f)\\
&= w_1Q_1 + w_2Q_2 + d(Q_1+Q_2-Q_1-Q_2)\\
&= w_1Q_1 + w_2Q_2
\end{aligned}$$

Collector's problem reduces to:

$$
\begin{aligned}
\text{Max } \Pi_4(w_1, w_2) &= w_1 Q_1 + w_2 Q_2 \\
\text{s.to} & \\
\frac{a - b(c_1 + w_1)}{2} &> \psi + \phi f && \text{(C.8)} \\
w_2 + c_2 &\leq w_1 + c_1 && \text{(C.9)} \\
w_1, w_2 &\geq 0
\end{aligned}
$$

The objective function is concave in $w_1 \geq 0$, $w_2 \geq 0$ $\left(\frac{d^2\Pi_4}{dw_1^2} = 0 \text{and} \frac{d^2\Pi_4}{dw_2^2} = 0\right)$, the optimal solution can be characterized utilizing the constraints of the model and the result is same with the third piece of the piecewise optimal solution to the collector's problem when $Q_2 < \frac{a-b(c_1+w_1)}{2} \leq (\psi + \phi f)$, shown in (C.7). Therefore, we can combine it with Equation (C.7).

We can finally characterize the collector's solution for w_1 and w_2 when Type 2 is more economical as in (4.54) by combining the piecewise solutions of four distinct parts:

$$
(w_1^*, w_2^*) = \begin{cases}
\left(\frac{a-b(2c_1-c_2-d)}{2b}, \frac{a-b(c_2-d)}{2b}\right) & \text{if } Q_2 \geq \frac{a-b(c_2+d)}{4} \\
\left(\frac{a-2Q_2}{b} - c_1, \frac{a-2Q_2}{b} - c_2\right) & \text{if} \frac{a-b(c_1+d)}{4} \leq Q_2 < \frac{a-b(c_2+d)}{4} \\
\left(\frac{a-b(c_1-d)}{2b}, \frac{a-b(2c_2-c_1-d)}{2b}\right) & \text{if} Q_2 < \frac{a-b(c_1+d)}{4} \leq \psi + \phi f \\
\left(\frac{a-bc_1-2(\psi+\phi f)}{b}, \frac{a-bc_2-2(\psi+\phi f)}{b}\right) & \text{if} \frac{a-b(c_1+d)}{4} > \psi + \phi f
\end{cases}
$$

APPENDIX D

ALGORITHM TO FIND THE OPTIMAL SOLUTION FOR CPAI

Numeric analysis for CPAI is performed in Matlab with following basic algorithm:

- Change f between 0 and $a/b + d$, increasing by 0.01.
- Change x between 0 and 1, increasing by 0.005.
- With this f and x value, determine profit functions for the two cases:

  (i) When Type 1 is more economical:

$$
\begin{aligned}
Q_1 &= x(\psi + \phi f) \\
Q_2 &= (1 - x)(\psi + \phi f) \\
\text{if} Q_1 &\geq (a - b(c_1 + d))/4 \\
w_1 &= (a - b(c_1 - d))/(2b) \\
w_2 &= (a - b(2c_2 - c_1 - d))/(2b) \\
q_1 &= (a - b(c_1 + w_1))/2 \\
q_2 &= 0 \\
\text{else if} Q_1 &< (a - b(c_1 + d))/4 \text{and} Q_1 >= (a - b(c_2 + d))/4 \\
w_1 &= (a - 2Q_1)/b - c_1 \\
w_2 &= (a - 2Q_1)/b - c_2 \\
q_1 &= Q_1 \\
q_2 &= 0
\end{aligned}
$$

$$
\begin{aligned}
\text{else if} Q_1 &< (a-b(c_2+d))/4 \text{and} (a-b(c_2+d))/4 <= (\psi+\phi f)\\
w_1 &= (a-b(2c_1-c_2-d))/(2b)\\
w_2 &= (a-b(c_2-d))/(2b)\\
q_1 &= Q_1\\
q_2 &= (a-b(c_2+w_2))/2-Q_1\\
\text{else if}(a-b(c_2+d))/4 &> (\psi+\phi f)\\
w_1 &= (a-bc_1-2(\psi+\phi f))/b\\
w_2 &= (a-bc_2-2(\psi+\phi f))/b\\
q_1 &= Q_1\\
q_2 &= Q_2\\
\text{Profit} &= w_1q_1+w_2q_2+d(Q_1+Q_2-q_1-q_2)-(f+c_I)(\psi+\phi f)
\end{aligned}
$$

(ii) When Type 2 is more economical:

$$
\begin{aligned}
Q_1 &= x(\psi+\phi f)\\
Q_2 &= (1-x)(\psi+\phi f)\\
\text{if} Q_2 &\geq (a-b(c_2+d))/4\\
w_1 &= (a-b(2c_1-c_2-d))/(2b)\\
w_2 &= (a-b(c_2-d))/(2b)\\
q_1 &= 0\\
q_2 &= (a-b(c_2+w_2))/2\\
\text{else if} Q_2 &< (a-b(c_2+d))/4 \text{and} Q_2 >= (a-b(c_1+d))/4\\
w_1 &= (a-2Q_2)/b-c_1\\
w_2 &= (a-2Q_2)/b-c_2\\
q_1 &= 0\\
q_2 &= Q_2
\end{aligned}
$$

$$
\begin{aligned}
\text{else if} Q_2 &< (a-b(c_1+d))/4 \text{and} (a-b(c_1+d))/4 <= (\psi+\phi f) \\
w_1 &= (a-b(c_1-d))/(2b) \\
w_2 &= (a-b(2c_2-c_1-d))/(2b) \\
q_1 &= (a-b(c_1+w_1))/2-Q2 \\
q_2 &= Q_2 \\
\text{else if}(a-b(c_1+d))/4 &> (\psi+\phi f) \\
w_1 &= (a-bc_1-2(\psi+\phi f))/b \\
w_2 &= (a-bc_2-2(\psi+\phi f))/b \\
q_1 &= Q_1 \\
q_2 &= Q_2 \\
\text{Profit} &= w_1q_1+w_2q_2+d(Q_1+Q_2-q_1-q_2)-(f+c_I)(\psi+\phi f)
\end{aligned}
$$

- Select the maximum of the determined profit values for the two cases.
- Find expected profit for each f value numerically.
- Select the best profit giving f value as the near optimal solution for the collector's problem.

APPENDIX E

FUNCTIONS OF NORMAL DISTRIBUTION

In this appendix, how the standard normal distribution functions are used for expressing expectations for normal r.v. with mean μ and standard deviation σ is provided. (Hadley and Whitin (1963))

Since x is defined as normally distributed in our problem; ($x \sim Norm(\mu, \sigma)$) is changed to standard normal distribution z as follows:

$\Phi(z)$ is cumulative distribution function (cdf) and $\Omega(z)$ is probability density function (pdf) of standard normal distribution $z \sim Norm(0, 1)$.

When $\frac{x-\mu}{\sigma} = u$; then $x = \mu + u\sigma$.

$$\begin{aligned}
\int_{r}^{\infty} g(x)dx &= \int_{\frac{r-\mu}{\sigma}}^{\infty} \Omega(u)du \\
&= \bar{\Phi}(\frac{r-\mu}{\sigma}) \\
&= (1 - \Phi(\frac{r-\mu}{\sigma})) \\
\text{Therefore: } \int_{-\infty}^{r} g(x)dx &= \Phi(\frac{r-\mu}{\sigma}))
\end{aligned}$$

Similarly;

$$\begin{aligned}\int_{r}^{\infty} xg(x)dx &= \int_{\frac{r-\mu}{\sigma}}^{\infty} (\mu + u\sigma)\Omega(u)du \\ &= \mu(1 - \Phi(\frac{r-\mu}{\sigma})) + \sigma\Omega(\frac{r-\mu}{\sigma})\end{aligned}$$

Therefore;

$$\begin{aligned}\int_{0}^{r} xg(x)dx &= \mu - \int_{r}^{\infty} xg(x)dx \\ &= \mu - (\mu(1 - \Phi(\frac{r-\mu}{\sigma})) + \sigma\Omega(\frac{r-\mu}{\sigma})) \\ &= \mu\Phi(\frac{r-\mu}{\sigma}) - \sigma\Omega(\frac{r-\mu}{\sigma})\end{aligned}$$

In similar way and also by using integration by parts;

$$\begin{aligned}\int_{r}^{\infty} x^2 g(x)dx &= \int_{\frac{r-\mu}{\sigma}}^{\infty} (\mu + u\sigma)^2\Omega(u)du \\ &= \int_{\frac{r-\mu}{\sigma}}^{\infty} (\mu^2 + 2\mu\sigma + u^2\sigma^2)\Omega(u)du \\ &= \mu^2(1 - \Phi(\frac{r-\mu}{\sigma})) + 2\mu\sigma\Omega(\frac{r-\mu}{\sigma}) + \sigma^2(1 - \Phi(\frac{r-\mu}{\sigma})) + \sigma^2(\frac{r-\mu}{\sigma})\Omega(\frac{r-\mu}{\sigma})\end{aligned}$$

Integration by parts is used for;

$$\int_{\frac{r-\mu}{\sigma}}^{\infty} (\sigma^2 u^2)\Omega(u)du$$

where $k = u, dv = u\Omega(u)du$

and $dk = du$ and $v = -\Omega(u)$

$$\int_{\frac{r-\mu}{\sigma}}^{\infty}(\sigma^2u^2)\Omega(u)du = \sigma^2(-u\Omega(u)_{\frac{r-\mu}{\sigma}}^{\infty} - \int_{\frac{r-\mu}{\sigma}}^{\infty}(-\Omega(u)du))$$
$$= \sigma^2(1-\Phi(\frac{r-\mu}{\sigma})) + \sigma^2(\frac{r-\mu}{\sigma})\Omega(\frac{r-\mu}{\sigma})$$

To determine;

$$\int_{-\infty}^{r} x^2g(x)dx = \int_{-\infty}^{\infty} x^2g(x)dx - \int_{r}^{\infty} x^2g(x)dx$$

where $\int_{-\infty}^{\infty} x^2g(x)dx$ is found by;

$$\int_{-\infty}^{\infty}(x-\mu)^2g(x)dx = Var(x)$$
$$\int_{-\infty}^{\infty}(x^2-2\mu x+\mu^2)g(x)dx = \sigma^2$$
$$\int_{-\infty}^{\infty} x^2g(x)dx - 2\mu\int_{-\infty}^{\infty} xg(x)dx + \mu^2 = \sigma^2$$
$$\int_{-\infty}^{\infty} x^2g(x)dx - \mu^2 = \sigma^2$$
$$\int_{-\infty}^{\infty} x^2g(x)dx = \mu^2+\sigma^2$$

Therefore;

$$\int_{-\infty}^{r} x^2g(x)dx = (\mu^2+\sigma^2) - (\mu^2(1-\Phi(\frac{r-\mu}{\sigma})) + 2\mu\sigma\Omega(\frac{r-\mu}{\sigma})$$
$$+ \sigma^2(1-\Phi(\frac{r-\mu}{\sigma})) + \sigma^2(\frac{r-\mu}{\sigma})\Omega(\frac{r-\mu}{\sigma}))$$

APPENDIX F

THE OPTIMAL RESULTS FOR THE PARAMETER SENSITIVITY ANALYSIS

Table F.1: Optimal results of centralized models for the problem instances considered when a increases

SMP						
a	f	p	Profit	Supply	Demand	Salvage
100	1,25	20,00	84,37	22,50	0,00	22,50
120	1,41	22,88	98,02	23,47	5,58	17,88
140	1,83	26,37	125,65	25,96	8,15	17,81
160	2,34	29,88	163,16	29,04	10,62	18,42
180	2,91	33,34	210,92	32,45	13,31	19,14
200	3,51	36,73	270,11	36,05	16,36	19,69
220	4,12	40,01	342,47	39,73	19,93	19,80
240	4,71	43,13	430,66	43,27	24,36	18,91
260	5,16	45,87	539,74	45,95	30,65	15,30
280	5,25	48,00	680,37	46,50	40,00	6,49
300	5,57	50,32	859,26	48,41	48,41	0,00

SQP						
a	f	p	Profit	Supply	Demand	Salvage
100	1,25	20,00	84,37	22,50	0,00	22,50
120	1,33	22,31	103,15	23,00	8,45	14,55
140	1,92	25,88	142,01	26,55	10,62	15,93
160	2,59	29,56	187,66	30,51	12,21	18,31
180	3,25	33,24	239,66	34,48	13,79	20,69
200	3,91	36,92	298,01	38,45	15,38	23,07
220	4,57	40,57	362,87	42,39	17,16	25,24
240	5,09	43,72	438,50	45,53	21,40	24,13
260	5,24	45,99	540,34	46,46	30,04	16,42
280	5,25	48,00	680,37	46,50	40,00	6,50
300	5,57	50,32	859,26	48,41	48,41	0,00

Table F.2: Optimal results of decentralized models for the problem instances considered when *a* increases

CPBI								
a	f	p	w1	w2	supply	demand	Salvage	Collector's profit
100	0,00	20,00	0,00	0,00	15,00	0,00	15,00	0,00
120	0,00	24,00	0,00	0,00	15,00	0,00	15,00	0,00
140	0,00	28,00	0,00	0,00	15,00	0,00	15,00	0,00
160	1,82	30,16	17,79	1,96	25,95	9,21	16,73	154,13
180	2,34	33,87	20,96	5,88	29,05	10,66	18,39	194,11
200	2,88	37,59	24,19	8,80	32,29	12,05	20,24	239,29
220	3,42	41,28	27,45	10,50	35,51	13,58	21,94	289,63
240	3,75	44,76	30,92	11,88	37,53	16,19	21,34	348,46
260	4,19	47,93	33,86	13,86	40,13	20,35	19,78	422,36
280	4,50	51,45	36,90	16,90	41,98	22,75	19,23	508,09
300	5,07	54,45	38,90	18,90	45,40	27,75	17,65	605,43

CPAI								
a	f	p	w1	w2	supply	demand	Salvage	Collector's profit
100	2,96	37,36	24,75	4,75	32,76	13,18	19,58	254,02
120	3,25	39,22	26,47	6,47	34,50	13,89	20,61	280,93
140	3,53	41,08	28,19	8,20	36,18	14,58	21,60	309,21
160	3,81	42,94	29,89	9,90	37,86	15,32	22,54	338,90
180	4,08	44,77	31,57	11,58	39,48	16,14	23,34	370,15
200	4,34	46,58	33,18	13,19	41,04	17,11	23,93	403,18
220	4,58	48,34	34,71	14,72	42,48	18,28	24,20	438,33
240	4,79	50,06	36,15	16,16	43,74	19,69	24,05	476,06
260	4,96	51,73	37,48	17,49	44,76	21,36	23,40	516,85
280	5,09	53,34	38,71	18,72	45,54	23,28	22,26	561,22
300	5,17	54,92	39,86	19,87	46,02	25,42	20,60	609,65

Table F.3: Optimal results of centralized models for the problem instances considered when *d* increases

SMP						
d	f	p	Profit	Supply	Demand	Salvage
0	1,02	35,77	163,81	21,14	21,14	0,00
5	2,27	36,73	189,91	28,62	16,34	12,27
10	3,51	36,73	270,11	36,05	16,36	19,69
15	4,94	36,66	388,43	44,66	16,72	27,94
20	6,66	36,81	554,10	54,94	15,96	38,98
25	8,78	37,76	788,72	67,68	11,22	56,46
30	11,25	40,00	1134,35	82,50	0,00	82,50

SQP						
d	f	p	Profit	Supply	Demand	Salvage
0	1,02	35,77	163,81	21,14	21,14	0,00
5	2,60	37,23	195,29	30,61	13,84	16,77
10	3,91	36,92	298,01	38,45	15,38	23,07
15	5,15	36,33	424,53	45,90	18,36	27,54
20	6,51	35,92	574,97	54,06	20,38	33,68
25	8,75	37,52	790,49	67,51	12,38	55,13
30	11,25	40,00	1134,37	82,50	0,00	82,50

Table F.4: Optimal results of decentralized models for the problem instances considered when d increases

CPBI										
d	f	p	w1	w2	supply	demand	Salvage	Collector's profit	Remanuf.'s profit	System profit
0	0,25	37,50	25,00	5,00	16,53	12,50	4,03	107,86	31,25	139,11
5	1,56	37,88	25,06	6,10	24,34	10,61	13,72	152,76	28,49	181,25
10	2,88	37,59	24,19	8,80	32,29	12,05	20,24	239,29	39,39	278,68
15	4,24	37,28	24,00	10,00	40,44	13,60	26,84	355,29	43,43	398,72
20	6,32	37,62	25,16	9,60	52,94	11,88	41,06	520,64	29,06	549,70
25	8,69	38,76	27,51	9,90	67,14	6,22	60,92	774,93	7,75	782,68
30	11,33	39,95	29,90	9,90	82,95	0,25	82,70	1134,29	0,01	1134,30

CPAI										
d	f	w_1	w_2	price	Supply	Demand	Salvage	Collector's profit	Remanuf.'s profit	System profit
0	0,25	24,99	5,00	37,48	16,50	12,58	3,92	107,89	31,24	139,13
5	1,90	25,71	5,72	37,85	26,40	10,76	15,64	165,37	24,56	189,93
10	2,96	24,75	4,75	37,36	32,76	13,18	19,58	254,02	37,42	291,44
15	4,16	23,93	3,94	36,96	39,96	15,22	24,74	363,31	48,57	411,88
20	6,25	25,86	5,87	37,70	52,50	11,48	41,02	521,65	28,51	550,16
25	8,75	37,49	17,49	39,98	67,50	0,08	67,42	899,61	0,00	899,61
30	11,25	39,98	19,99	39,98	82,50	0,08	82,42	1383,78	0,00	1383,78

Table F.5: Optimal results of centralized models for the problem instances considered when c_2 increases

SMP						
c2	f	p	Profit	Supply	Demand	Salvage
10	3,75	32,50	515,63	37,50	37,50	0,00
15	3,07	33,32	409,26	33,41	33,41	0,00
20	3,24	34,98	323,33	34,43	25,08	9,35
25	3,53	36,26	288,12	36,18	18,70	17,49
30	3,51	36,73	270,11	36,05	16,36	19,69
35	3,46	37,01	258,01	35,77	14,96	20,81

SQP						
c2	f	p	Profit	Supply	Demand	Salvage
10	3,75	32,50	515,63	37,50	37,50	0,00
15	3,07	33,32	409,26	33,41	33,41	0,00
20	3,25	35,00	323,37	34,50	25,00	9,49
25	3,88	36,78	298,72	38,30	16,08	22,22
30	3,91	36,92	298,01	38,45	15,38	23,07
35	3,91	36,92	298,01	38,45	15,38	23,07

Table F.6: Optimal results of decentralized models for the problem instances considered when c_2 increases

CPBI								
c2	f	p	w1	w2	Supply	Demand	Salvage	Collector's profit
10	1,54	35,15	20,29	20,29	24,26	24,26	0,00	333,64
15	2,30	36,38	22,75	17,75	28,80	18,13	10,67	275,64
20	2,64	37,00	24,00	14,00	30,84	15,00	15,84	251,03
25	2,77	37,54	24,27	11,10	31,60	12,31	19,29	239,82
30	2,88	37,59	24,19	8,80	32,29	12,05	20,24	239,29
35	2,89	37,58	24,15	4,10	32,34	12,09	20,25	239,26

CPAI								
c2	f	p	w1	w2	Supply	Demand	Salvage	Collector's profit
10	1,54	35,14	20,30	20,30	24,24	24,31	-0,07	333,50
15	2,24	36,23	22,48	17,49	28,44	18,84	9,60	275,89
20	2,87	37,14	24,29	14,30	32,22	14,32	17,90	256,46
25	2,96	37,35	24,72	9,73	32,76	13,24	19,52	254,09
30	2,96	37,36	24,75	4,75	32,76	13,18	19,58	254,02
35	2,96	37,36	24,75	-0,24	32,76	13,18	19,58	254,03

Table F.7: Optimal results of centralized models for the problem instances considered when ψ increases

SMP						
Ψ	f	p	Profit	Supply	Demand	Salvage
15	3,51	36,73	270,11	36,05	16,36	19,69
18	3,22	36,64	288,45	37,30	16,81	20,49
21	2,93	36,55	307,42	38,56	17,26	21,29
24	2,64	36,46	327,01	39,82	17,71	22,11
27	2,35	36,37	347,24	41,08	18,16	22,92
30	2,06	36,28	368,09	42,35	18,60	23,75

SQP						
Ψ	f	p	Profit	Supply	Demand	Salvage
15	3,91	36,92	298,01	38,45	15,38	23,07
18	3,62	36,82	317,54	39,69	15,88	23,82
21	3,32	36,73	337,70	40,93	16,37	24,56
24	3,03	36,63	358,48	42,17	16,87	25,30
27	2,74	36,53	379,88	43,41	17,37	26,05
30	2,44	36,43	401,89	44,65	17,86	26,79

Table F.8: Optimal results of decentralized models for the problem instances considered when ψ increases

CPBI								
Ψ	f	p	w1	w2	Supply	Demand	Salvage	Collector's profit
15	2,88	37,59	24,19	8,80	32,29	12,05	20,24	239,29
18	2,43	37,57	24,14	9,80	32,61	12,16	20,45	255,53
21	2,18	37,48	23,98	9,60	34,11	12,62	21,49	272,43
24	1,92	37,38	23,78	9,00	35,49	13,09	22,40	289,85
27	1,66	37,29	23,62	8,80	36,94	13,54	23,40	307,81
30	1,39	37,20	23,45	8,60	38,36	13,98	24,38	326,34

CPAI								
Ψ	f	p	w1	w2	Supply	Demand	Salvage	Collector's profit
15	2,96	37,36	24,75	4,75	32,76	13,18	19,58	254,02
18	2,64	37,28	24,57	4,58	33,84	13,61	20,23	270,67
21	2,32	37,19	24,40	4,41	34,92	14,04	20,88	287,85
24	1,99	37,11	24,24	4,25	35,94	14,45	21,49	305,57
27	1,67	37,03	24,07	4,08	37,02	14,87	22,15	323,81
30	1,35	36,94	23,90	3,91	38,10	15,29	22,81	342,58

Table F.9: Optimal results of centralized models for the problem instances considered when μ increases

SMP						
µ	f	p	Profit	Supply	Demand	Salvage
0,4	3,51	36,73	270,11	36,05	16,36	19,69
0,45	3,66	36,36	294,02	36,98	18,19	18,79
0,5	3,79	36,00	317,11	37,71	20,00	17,71
0,55	3,88	35,64	339,23	38,28	21,79	16,48
0,6	3,95	35,29	360,27	38,68	23,55	15,13

SQP						
µ	f	p	Profit	Supply	Demand	Salvage
0,4	3,91	36,92	298,01	38,45	15,38	23,07
0,45	4,05	36,47	324,04	39,28	17,67	21,60
0,5	4,14	36,01	348,73	39,85	19,93	19,93
0,55	4,20	35,58	371,94	40,21	22,12	18,09
0,6	4,23	35,16	393,56	40,36	24,22	16,15

Table F.10: Optimal results of decentralized models for the problem instances considered when μ increases

CPBI								
µ	f	p	w1	w2	Supply	Demand	Salvage	Collector's profit
0,4	2,88	37,59	24,19	8,80	32,29	12,05	20,24	239,29
0,45	2,84	37,32	23,78	9,00	32,07	13,39	18,68	253,64
0,5	2,80	37,07	23,41	8,90	31,80	14,65	17,15	266,39
0,55	2,76	36,81	22,99	9,10	31,58	15,93	15,64	277,61
0,6	2,71	36,59	22,67	8,70	31,26	17,03	14,23	287,41

CPAI								
µ	f	p	w1	w2	Supply	Demand	Salvage	Collector's profit
0,4	2,96	37,36	24,75	4,75	32,76	13,18	19,58	254,02
0,45	2,93	37,07	24,14	4,14	32,58	14,67	17,91	268,57
0,5	2,86	36,79	23,57	3,57	32,16	16,07	16,09	281,04
0,55	2,76	36,53	23,07	3,07	31,56	17,35	14,21	291,59
0,6	2,65	36,29	22,60	2,61	30,90	18,57	12,33	300,33

APPENDIX G

PERCENTAGE IMPROVEMENTS ON THE VALUE OF INFORMATION

Table G.1: Percentage improvements over c_2 indicating value of information

	c_2=30			c_2=20			# of Comparison on c_2=30 to c_2=20	
	max	min	avg	max	min	avg	Worse	Better
% profit improvement of SQP over SMP	10,3%	0,0%	2,9%	3,1%	0,0%	0,5%	0	108
% collector's profit improvement of CPAI over CPBI	6,2%	0,0%	2,0%	3,2%	-0,1%	0,6%	0	108

Table G.2: Percentage improvements over d indicating value of information

	d=20			d=10			# of Comparison on d=20 to d=10	
	max	min	avg	max	min	avg	Worse	Better
% profit improvement of SQP over SMP	6,2%	0,0%	1,7%	10,3%	0,0%	1,7%	20	88
% collector's profit improvement of CPAI over CPBI	3,7%	0,0%	0,9%	6,2%	-0,1%	1,7%	42	66

Table G.3: Percentage improvements over b indicating value of information

	b=5			b=3			# of Comparison on b=5 to b=3	
	max	min	avg	max	min	avg	Worse	Better
% profit improvement of SQP over SMP	10,3%	0,0%	3,0%	3,3%	0,0%	0,4%	7	101
% collector's profit improvement of CPAI over CPBI	6,2%	0,0%	1,7%	3,2%	-0,1%	0,9%	21	87

Table G.4: Percentage improvements over μ indicating value of information

	μ=0.6			μ=0.5			μ=0.4			# of Comparison on μ=0.6 to μ=0.4	
	max	min	avg	max	min	avg	max	min	avg	Worse	Better
% profit improvement of SQP over SMP	9,2%	0,0%	1,6%	10,0%	0,0%	1,7%	10,3%	0,0%	1,8%	16	56
% collector's profit improvement of CPAI over CPBI	5,0%	-0,1%	1,2%	5,5%	0,0%	1,3%	6,2%	0,0%	1,3%	12	60

Table G.5: Percentage improvements over ϕ indicating value of information

	Φ=6			Φ=4			Φ=2			# of Comparison on Φ=6 to Φ=2	
	max	min	avg	max	min	avg	max	min	avg	Worse	Better
% profit improvement of SQP over SMP	10,3%	0,0%	1,8%	8,7%	0,0%	1,8%	6,2%	0,0%	1,5%	15	57
% collector's profit improvement of CPAI over CPBI	6,2%	0,0%	1,5%	5,6%	0,0%	1,4%	4,3%	-0,1%	1,1%	12	60

Table G.6: Percentage improvements over σ indicating value of information

	σ=0.12			σ=0.1			σ=0.08			# of Comparison on σ=0.12 to σ=0.08	
	max	min	avg	max	min	avg	max	min	avg	Worse	Better
% profit improvement of SQP over SMP	10,3%	0,0%	2,0%	8,7%	0,0%	1,7%	7,0%	0,0%	1,4%	0	72
% collector's profit improvement of CPAI over CPBI	6,2%	-0,1%	1,5%	5,5%	0,0%	1,3%	4,7%	0,0%	1,1%	0	72

APPENDIX H

PERCENTAGE IMPROVEMENTS ON EFFECT OF CENTRALIZATION

Table H.1: Percentage improvements over c_2 indicating effects of centralization

	c_2=30			c_2=20			# of Comparison on c_2=30 to c_2=20	
	max	min	avg	max	min	avg	Worse	Better
% profit improvement of SQP over CPBI	18,3%	1,7%	8,0%	25,9%	1,7%	10,3%	49	59
% profit improvement of SQP over CPAI	18,7%	0,3%	6,8%	22,0%	0,4%	9,9%	60	48
% profit improvement of SMP over CPBI	18,3%	-3,7%	5,1%	25,9%	-0,7%	9,8%	84	24
% profit improvement of SMP over CPAI	18,7%	-7,3%	3,9%	22,0%	-2,4%	9,3%	69	39

Table H.2: Percentage improvements over d indicating effects of centralization

	d=20			d=10			# of Comparison on d=20 to d=10	
	max	min	avg	max	min	avg	Worse	Better
% profit improvement of SQP over CPBI	14,4%	1,7%	7,5%	25,9%	1,7%	10,8%	80	28
% profit improvement of SQP over CPAI	13,7%	0,4%	6,7%	22,0%	0,3%	9,9%	62	46
% profit improvement of SMP over CPBI	13,7%	-3,7%	5,8%	25,9%	-3,3%	9,1%	72	36
% profit improvement of SMP over CPAI	13,7%	-3,0%	5,0%	22,0%	-7,3%	8,2%	69	39

Table H.3: Percentage improvements over b indicating effects of centralization

	b=5			b=3			# of Comparison on b=5 to b=3	
	max	min	avg	max	min	avg	Worse	Better
% profit improvement of SQP over CPBI	15,0%	1,7%	6,4%	25,9%	4,7%	11,9%	86	22
% profit improvement of SQP over CPAI	12,6%	0,3%	4,8%	22,0%	3,7%	11,9%	102	6
% profit improvement of SMP over CPBI	14,3%	-3,7%	3,0%	25,9%	4,6%	11,9%	104	4
% profit improvement of SMP over CPAI	11,8%	-7,3%	1,4%	22,0%	3,6%	11,8%	107	1

Table H.4: Percentage improvements over μ indicating effects of centralization

	μ=0.6			μ=0.5			μ=0.4			# of Comparison on μ=0.6 to μ=0.4	
	max	min	avg	max	min	avg	max	min	avg	Worse	Better
% profit improvement of SQP over CPBI	18,6%	3,3%	9,4%	25,9%	2,4%	9,1%	25,5%	1,7%	8,9%	17	55
% profit improvement of SQP over CPAI	21,8%	1,3%	8,6%	22,0%	0,7%	8,2%	21,6%	0,3%	8,1%	18	54
% profit improvement of SMP over CPBI	18,6%	-1,7%	7,8%	25,9%	-2,6%	7,4%	25,5%	-3,7%	7,1%	19	53
% profit improvement of SMP over CPAI	21,8%	-4,5%	7,0%	22,0%	-5,9%	6,5%	21,6%	-7,3%	6,4%	18	54

Table H.5: Percentage improvements over ϕ indicating effects of centralization

	Φ=6			Φ=4			Φ=2			# of Comparison on Φ=6 to Φ=2	
	max	min	avg	max	min	avg	max	min	avg	Worse	Better
% profit improvement of SQP over CPBI	25,9%	4,5%	11,0%	18,3%	3,7%	9,6%	13,1%	1,7%	6,9%	7	65
% profit improvement of SQP over CPAI	22,0%	2,1%	9,4%	18,7%	1,1%	8,7%	13,1%	0,3%	6,8%	17	55
% profit improvement of SMP over CPBI	25,9%	-3,1%	9,2%	18,3%	-3,3%	7,8%	13,1%	-3,7%	5,4%	8	64
% profit improvement of SMP over CPAI	22,0%	-7,3%	7,6%	18,7%	-6,8%	7,0%	13,1%	-4,9%	5,3%	24	48

Table H.6: Percentage improvements over σ indicating effects of centralization

	σ=0.12			σ=0.1			σ=0.08			# of Comparison on σ=0.12 to σ=0.08	
	max	min	avg	max	min	avg	max	min	avg	Worse	Better
% profit improvement of SQP over CPBI	25,5%	2,2%	9,3%	25,9%	1,9%	9,2%	25,9%	1,7%	9,0%	4	68
% profit improvement of SQP over CPAI	21,9%	0,4%	8,4%	22,0%	0,4%	8,3%	22,0%	0,3%	8,3%	1	71
% profit improvement of SMP over CPBI	25,5%	-3,7%	7,2%	25,9%	-3,0%	7,5%	25,9%	-2,3%	7,6%	19	53
% profit improvement of SMP over CPAI	21,9%	-7,3%	6,4%	22,0%	-6,0%	6,6%	22,0%	-4,6%	6,9%	19	53