DESIGN OF A MECHANISM FOR OPENING HACTHBACK CAR BAGGAGE DOOR

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ABSTRACT

DESIGN OF A MECHANISM FOR OPENING HACTHBACK CAR BAGGAGE DOOR

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In this thesis, a new mechanism design for opening hatchback-car baggage door is introduced. Unlike the classical hinged doors used, this new design will include a mechanism so that the door will be opened vertically and thus occupy less space behind the car during the opening. In this fashion even the hatchback doors of closely parked cars may be opened. First, considering type synthesis, different mechanism types are investigated. In dimensional synthesis, with the help of Burmester theory, motion generation is applied. Using the circle and center point curves, considering link dimensions, transmission angle characteristics, branching and some order issues possible solutions that satisfy the position requirements are found. To actuate the mechanism, an appropriate gas-spring is sought. As a case study a prototype is manufactured and mounted on a sample hatchback car to check the mechanism performance.

Keywords: Mechanism Synthesis, Burmester Theory, Hatchback-Car, Baggage Door

ÖZ

BEŞ KAPILI ARAÇ BAGAJ KAPAĞININ AÇILMASI İÇİN MEKANİZMA TASARIMI

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Bu tezde, beş kapılı araçların bagaj kapağını açmak için yeni bir mekanizma tasarımı sunuldu. Klasik olarak kullanılan menteşeli kapıların aksine, bu tasarımda kapı yukarı doğru açılarak aracın arkasında açılma sırasında süpürülen hacmi en aza indirecektir. Bu sayede birbirine yakın park etmiş araçların bile bagaj kapağı açılabilecektir. İlk olarak, tip sentezi göz önünde bulundurularak farklı mekanizma tipleri araştırılmıştır. Boyut sentezinde, Burmester teorisi kullanılarak konum sentezi uygulanmıştır. Merkez ve çember eğrilerinden elde edilen olası çözümler; uzuv boyutları, bağlanma açısı,dallanma ve sıra kusurları düşünülürek uygun çözümler belirlenmiştir. Mekanizmayı hareket ettirmek için, uygun bir gazlı yay araştırılmıştır. Örnek çalışma olarak bir prototip üretilmiş ve beş kapılı bir araç üzerine montajı yapılarak uygulanabilirliği görülmüştür.

Anahtar Kelimeler: Mekanizma Sentezi, Burmester Teorisi, Beş Kapılı Araç, Bagaj Kapağı

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CHAPTER 1

INTRODUCTION

1.1 GENERAL

The aim of this thesis is to design an alternative baggage door opening mechanism for hatchback cars. In current baggage door designs for hatchback cars, usually the door is connected to the chassis by a revolute joint at the top and is rotated about a horizontal axis. In such a design, opening of the door requires a certain amount of space at the back of the car and the person opening the door must take a few steps backwards. The aim in this new design is to reduce the amount of space required at the back of the car.

A general design process of a mechanism starts with kinematic synthesis, and then kinematic, force and stress analysis follow the synthesis phase. These steps should be performed all together to obtain an acceptable solution. Kinematic synthesis is directly related with the motion. It is the process of designing a mechanism to accomplish a desired motional task. Kinematic synthesis can be divided two main sub branches; type synthesis and dimensional synthesis. Type synthesis searches type of the mechanism and dimensional synthesis determines significant dimensions that satisfy the best desired motion characteristics.

Obviously type synthesis must take place first. In type synthesis, the aim is to determine a suitable topological structure for the desired motion. That is, the number of links, number and type of joints and the connectivity information of the links are determined. There is no dimensional design in type synthesis. Also, the necessary degree of freedom of the mechanism and desirable configurations

should be fixed It is the most critical stage during the entire synthesis process, because of its dependence on the designer. Number synthesis (enumeration of mechanisms) is developed by Gruebler for a systematical procedure for the type synthesis [1].

In dimensional synthesis, significant dimensions and initial position of the mechanism are sought. Link lengths (distances between kinematic elements), gear ratios, cam-follower diameters, etc. are designed. To accomplish this work both graphical and analytical methods are available. For computers, implementation of the analytical methods is more appropriate due to easy formulization. With the help of high technology, more accurate results can be obtained in a short time.

Although there are several approaches for dimensional synthesis, use of prescribed position synthesis and optimization synthesis are generally accepted. In optimization synthesis, the error function defined with difference between real and desired outputs should be minimized. However prescribed position synthesis, determines mechanism dimensions that exactly satisfy some prescribed positions of the desired motion [2].

In finite position synthesis, a point on the moving plane which draws a circular arc on the fixed plane is called a circle point, and the corresponding center of this circular arc is called a center point. Aim of prescribed position synthesis is to find the center points and circle points for a given case in order to define the joints of a linkage.

Intuition and experience of the designer play major role in synthesis compared to other design stages. However, the problem requires the solution of mathematical systems as well. Even though calculation procedure does not provide a unique output, the designer will have to use his intuition and experience for the selection of the most suitable mechanism out of the possible combinations. As a whole it is necessary to use computer programs for synthesis with high developed computer

technology. The computer programs are capable of reaching the best solutions including user interaction both in analysis and synthesis.

Main aim of this thesis is to synthesize a new opening mechanism for hatchback trunk-lid. Mechanism should provide a motion such that required space for opening rear door is as small as possible. Obviously there are restrictions because of the nature of the problem. When the car chassis is already designed, there should be minimum modification on the chassis and the door structure. For obtaining better motion in better conditions some necessary modifications should be considered. Cost and reliability are always considered sub-design criteria.

1.2 STATE OF THE ART

Hatchback cars are commonly used in daily life. Classical approach to open trunk-lid of the car is the use of top-hinged mechanism. Commercial cars, especially vans and minivans can be considered in the hatchback category. For the opening mechanism of their rear door, some alternative solutions are introduced in the market.

In US 6,257,651 patent [15], displace ably mounted door (Figure 1.1) is introduced. Door consists of horizontal lamellas which are serially connected to each other. Using such a door structure necessitates a motor and a storage region to roll up the door. This causes an important reduction on the luggage volume. Also manufacturing a door in such a structure is complex and costly because both sliding and rotating lamella parts must be pushed in a lamella package. Also, from the aesthetic point of view, this solution is not preferred since there is no space for a window.

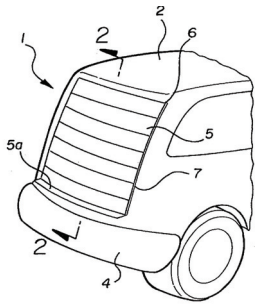


Figure 1.1: Displace ably mounted door [15]

Laterally two part door with simple hinges (Figure 1.2) is another common type of deck-lid structure. This time the user is limited in lateral space. Opening the doors extensively may cause problems for the other vehicles in the traffic. Also, possible wind load can cause trouble. Mostly, the doors are not symmetric because of the driver comfort; it prevents attaining larger visibility for driver. So, desired small enough space occupation can not be achieved.

Figure 1.2: Laterally two part door structure [16]

Upwardly folding door (Figure 1.3) with simple hinges is introduced in the US 6,068,327 patent [17]. This structure solves the space problem however it yields a

new hard task: door has two parts connected via pivots and to implement such a door is very hard and costly.

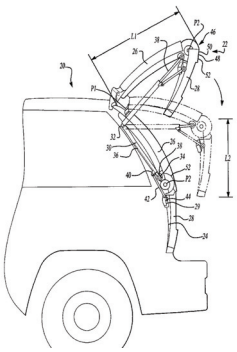


Figure 1.3: Upwardly folding door [17]

1.3 LITERATURE SURVEY

In multiple position synthesis, the basic theory is not significantly changed since early 19th century. It is first solved by Burmester [1-4] graphically. Graphical techniques tend to facilitate the development of intuition and geometric insight however, if handled manually, the accuracy is poor. Also in graphical techniques, use of an atlas of coupler curves, as was done by Hrones and Nelson [1,7] seems too easy and fast but it offers very low precision. Then, Freudenstein [2, 3] represented an analytical solution for restricted number of positions because of the mathematical constraints. By this method the problem is reduced to the simultaneous solution of a set of equations. So the result is more accurate and repeatable. In planar four bar mechanism synthesis, solution can only be obtained up to given five poses. More than five poses can be synthesized by using approximate poses. Angeles [4] has developed a method for approximate synthesis optimization. Erdman & Sandor [1-2] introduced a dyadic approach which is easy to apply numerically. Use of complex numbers in mechanism synthesis is initiated by these two researchers. Also they classified the problem such as motion

generation, path generation and function generation. Motion generation, can be defined as entire body guided through the prescribed motion. Function generation is the correlation of input and output links. Path generation is that a specified point on the link has a certain trajectory. This contributes simplicity for solution after definition of the problem. Planar four-bar mechanism can be considered as composed of two dyads (vector pairs). If synthesis is done for multi-loop mechanisms, the concept can be extended to triads (three vectors) [1]. Planar six-link mechanism is an example of a multi a loop mechanism. Soh [8] has developed a synthesis method for the six-link mechanism using 3R chains except for the Watt-II type. Meer et al. [9] introduced a method using homotopy to synthesize planar Stephenson mechanisms including circle point search.

With the help of high-developed computer technology, synthesis theory is adapted to many commercial software packages on the market such as LINCAGES [1] and WATT [10, 11]. These packages are helpful for designers, however, mostly only specific types of mechanisms are handled; especially the four-bar mechanism. There are lots of packages which have a graphical user interface and compatible with CAD environment. Demir [11], developed a package called CADSYN capable of synthesis and analysis of a planar four-bar mechanism in visual interactive CAD environment.

Burmester curve solution gives infinite number of solutions but this does not mean that all these mechanisms may be possible solution candidates. Branching problem, which implies that the mechanism satisfies same position in different configurations and order defects due to the non-sequential positions must be considered. Waldron [12] has developed a method that gives correct mechanism region from the Burmester curves.

Position synthesis theory is also handled as an optimization problem to obtain a desired path, i.e. the minimization of a proper objective function subject to constraints. Usually error between desired motion value and produced motion can

be defined as the objective function. Cabrera studied optimal synthesis of mechanisms using genetic algorithm. There are some other numerical techniques combined with optimization such as evolutionary algorithm [13] and Gauss constrained method [14].

Optimization techniques may be used for reducing the number of solutions to a desired one from infinite number of solutions. However, exact mathematical model is necessary and usually procedure can be finalized with local minima instead of a global minimum. In addition, the search algorithm does not give any information for developing intuition.

1.4 DESIGN PROCESS

In design, since the desired motion can be achieved in plane, planar mechanisms solutions are investigated. Linkage types of mechanisms are sought because of their simplicity and ease of application. Four-bar and six-link basic popular linkages are given priority. Then synthesis is done for both selected mechanism types. According to obtain results and considering design criteria one mechanism is chosen to continue design process. After that, to manufacture a prototype, detail design is started and links are given a shape regarding with test vehicle. Finally prototype mechanism is mounted on test vehicle to control the mechanism behavior in real environment

CHAPTER 2

FOUR-BAR MECHANISM DESIGN

2.1 KINEMATIC SYNTHESIS

In design, all joints are selected as revolute joints. Since it is simple and it can be easily obtained, also compared to prismatic joints energy loss due to friction is low. It is reliable and has a long operation life. Four link four revolute joint mechanism which is already known as fourbar is thought for design

In dimensional synthesis of the four-bar mechanism, four finitely separated position synthesis method is applied. This is given in Appendix A. Four positions of baggage-door with respect to vehicle body are specified inputs for synthesis. Closed position of door is kept same with current hinged door closed position however because of the problem definition full-open position is changed. It is important to open the door with less required volume on the rear as much as possible. The remaining two intermediate positions should be selected such that they are in between two limit positions and their distance to car-body does not exceed a desired value. In design process, designer could take these two intermediate steps as variable parameters, since Burmester theory is very sensitive to small changes on different finite positions, hence the designer should be very careful to select intermediate positions.

First a fixed reference frame (XY) with origin O is defined. It may be selected parallel to the side view of the vehicle. Then rear and top parts of the car body are represented by appropriate curves. Also, the door profile is composed by fitting a curve to critical points on the baggage door. It is illustrated on Figure 2.1. Links of

the mechanism must be in the space between the car body and the baggage door. If possible, present structure of both body and door are kept, however any acceptable modification that change the output in a desirable fashion should be considered. If the baggage door mechanism had to be designed prior to the design of the car body, the mechanism can be designed differently. However, in the case considered very little changes can be made on the car body. To do this, constraint regions must be defined so that the mechanism runs in this workspace. The region may be defined as a groove on the back of the car. This groove can be represented by a space whose edges are fitted to linear curves in the schematic model.

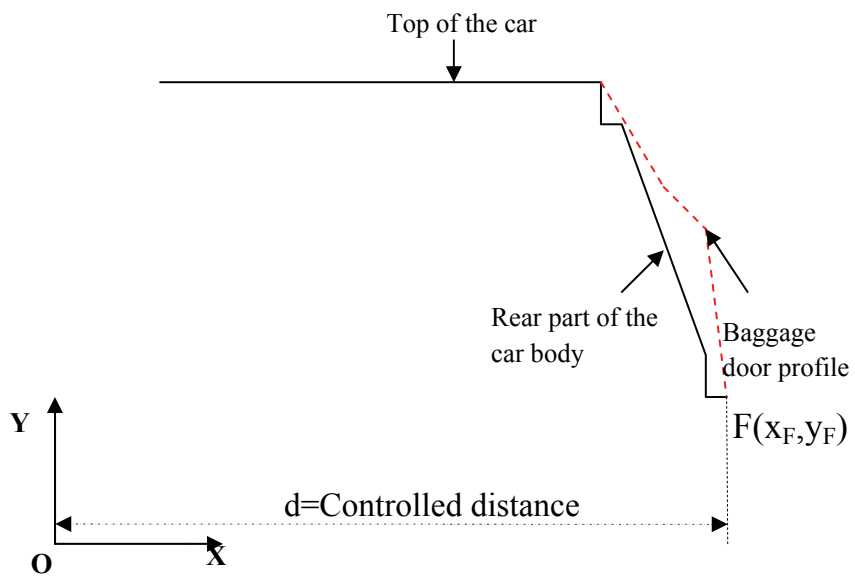


Figure 2.1: Schematic representation of the baggage door

Although all kinematic synthesis is based on a planar motion, parallel to the side view of the vehicle, in the detailed design process, mechanism members is thought

such that they can have parts out of design plane. In that phase, location problem of the links should be reconsidered from the three dimensional aspect.

In kinematic synthesis phase a computer code is written in Visual Basic® language and its output is ready to check whether motion is fully realizable or not and constraints are satisfied or not. The input parameters for the code are $\vec{\delta}_j$: displacement vector of moving reference frame for different positions with respect to the first position, α_j : rotation of moving reference frame for different positions with respect to the first orientation (Figure 2.2).

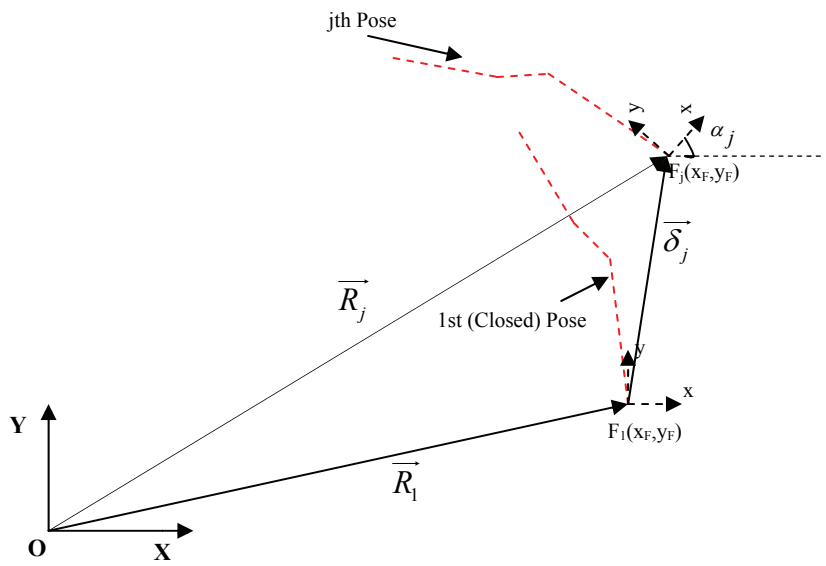


Figure 2.2: Input parameters for synthesis

The code may be divided into sub-codes, all of which have individual outputs. From the aspect of synthesis, the outputs are the coordinates of the fixed joints of the four-bar mechanism. There are nine parameters that represent a unique four-bar

mechanism. Kinematic analysis can be performed with these parameters as an input. With the help of analysis, branching and order problems can be checked, that is whether the mechanism has a motion that satisfy the four positions in the desired order and correctly or not. Another function is defined in the code to find the farthest point on the door throughout the motion for the synthesized four bar mechanism. Distance of this point between the origins of the fixed frames is represented as controlled output. The necessary maximum distance for the case where simple hinges are used can be referred as current maximum distance, as shown on Figure 2.3 Also the reduction ratio is defined as the ratio of difference of the controlled output and current maximum distance divided by the current maximum distance:

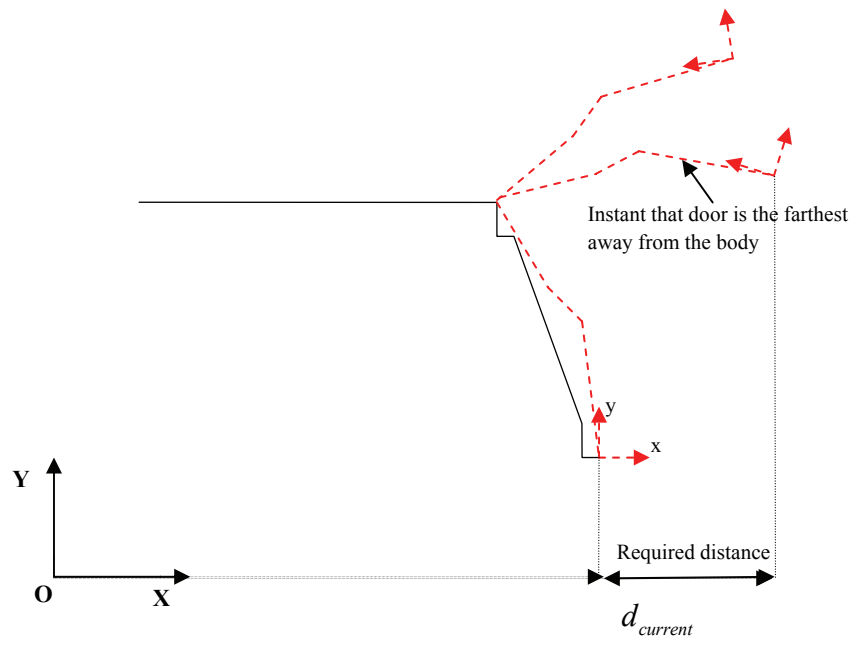


Figure 2.3: Necessary distances to open for top-hinged door

Reduction ratio: r

Current maximum distance: $d_{current}$

Controlled output: d

$$r = \frac{d_{current} - d}{d_{current}}$$

After specifying the door positions, the necessary inputs for synthesis can be found easily. For simplicity, the bottom edge of the door can be selected as the origin of the moving frame and the orientation of moving frame is selected such that it is initially parallel to the fixed frame. An example of four positions is given on Figure 2.4

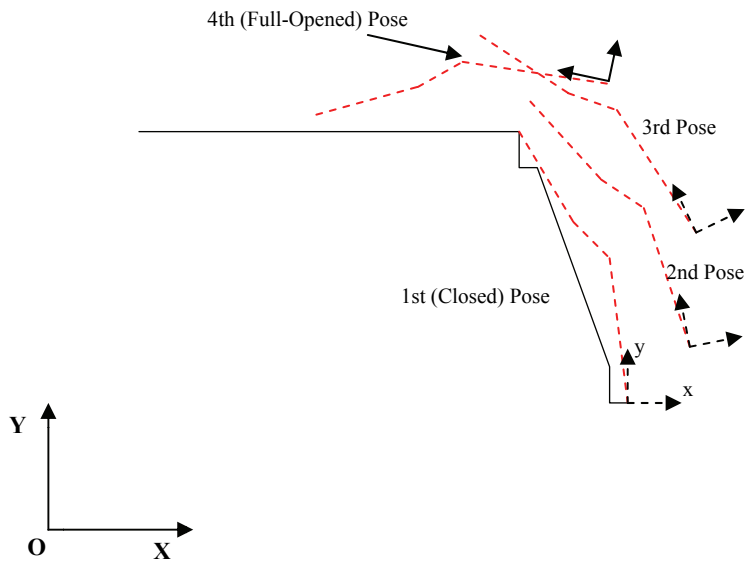


Figure 2.4 Example of four positions of door

Since the aim is to reduce the required distance for opening the door, the controlled output can be defined as the point of the door profile which has the

maximum horizontal distance with respect to the origin of the car body frame throughout the motion. This farthest point may not be fixed throughout the motion, so while mechanism runs this point must be found at every instant, to find the overall farthest distance. This output parameter must be checked and increase in the reduction ratio for the required distance should be sought.

2.2 KINEMATIC ANALYSIS

Consider the kinematic chain A_0ABB_0 shown on Figure 2.5. A_0B_0 is the fixed link. Given the coordinates of joints all link lengths can be found. Then, for a given θ_2, θ_3 and θ_4 angles are solved.[5] As a result, positions of all points can be determined throughout the motion. The coordinates of a coupler point P can be evaluated with the following procedure:

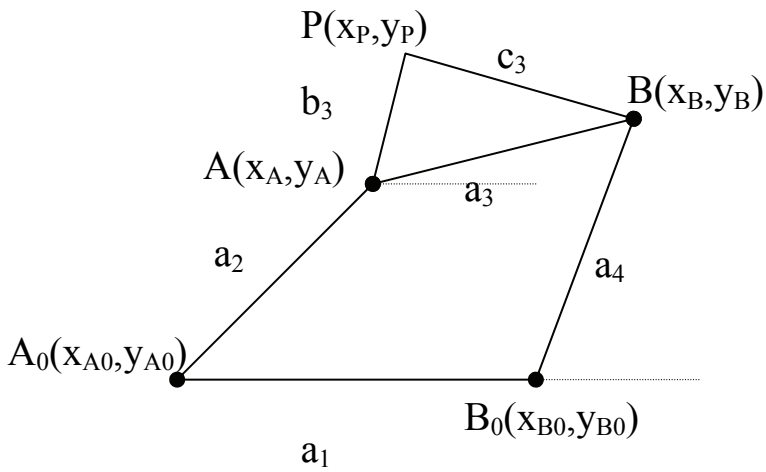


Figure 2.5: Joint positions of four bar

$$a_1 = \sqrt{(x_{A0} - x_{B0})\wedge 2 + (y_{A0} - y_{B0})\wedge 2} \qquad a_2 = \sqrt{(x_{A0} - x_A)\wedge 2 + (y_{A0} - y_A)\wedge 2}$$

$$a_3 = \sqrt{(x_A - x_B)\wedge 2 + (y_A - y_B)\wedge 2} \qquad a_4 = \sqrt{(x_B - x_{B0})\wedge 2 + (y_B - y_{B0})\wedge 2}$$

$$\alpha = \mathrm{acos}\left(\frac{b_3^{\,2} + a_3^{\,2} - c_3^{\,2}}{2 \cdot a_3 \cdot b_3}\right)$$

$$x_p = x_{A0} + a_2 \cdot \cos(\theta_2) + b_3 \cdot \cos(\theta_3 + k \cdot \alpha)$$

$$y_p = y_{A0} + a_2 \cdot \sin(\theta_2) + b_3 \cdot \sin(\theta_3 + k \cdot \alpha)$$

If P is above the line AB take k=1 else k=-1

The positions of the selected critical points are evaluated to find the point with maximum horizontal distance during the motion. The most farthest on the door could be used to compare the change in the reduction ratio among the possible mechanisms.

2.3 STATIC FORCE ANALYSIS

In application, the user controls the door motion manually by hand. So the mechanism experiences low accelerations. Because of this, all inertial forces on the links can be neglected. The mechanism and the door must be in equilibrium at least in two positions: open and closed positions; however for ease in application another balance position between the initial and last positions can be determined. With this extra balance point, while opening the door in the first phase user applies a force on the door up to this balance point and then the rest of the motion is achieved without any external user force. In the second phase gas springs exert force on the door. While closing the door, both the springs and the user change their role in both phases. Now, the user applies force to the door to close it up to the equilibrium position and then gas springs exert force to bring the door to the closed position.

There are two types of external forces on the mechanism weight of the door and gas-spring balancing force. The free body diagram of all components of the mechanism and gas-spring are shown on Figure 2.6

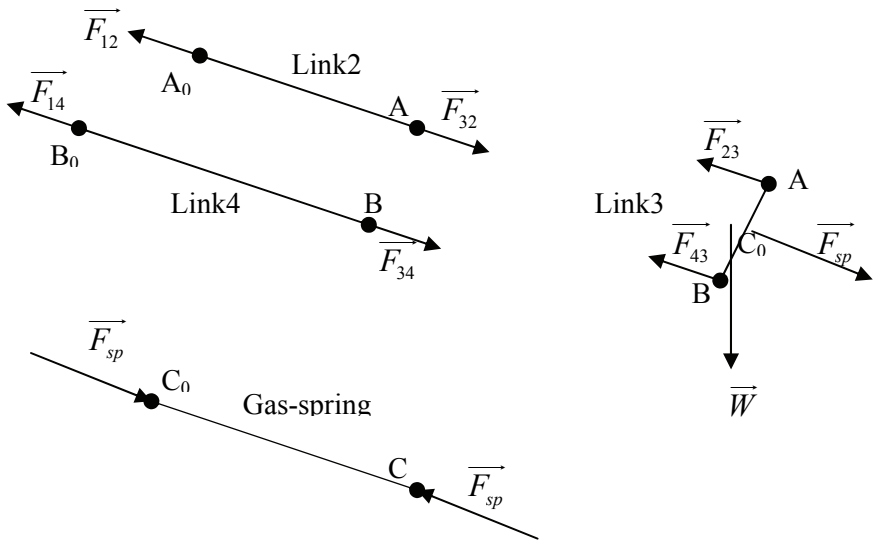


Figure 2.6: Free body diagrams of the links

Weights of links and gas-spring are so small compared to the weight of the door. Also friction causes very low lost energy. So both link weights and friction forces can be neglected. According to these assumptions link2, link4 and the gas spring become two force members. So $\overrightarrow{F_{12}}$ and $\overrightarrow{F_{32}}$ are collinear, opposite in direction and have the same magnitude. Similar properties are valid for $\overrightarrow{F_{13}}$ and $\overrightarrow{F_{34}}$. It is known from the 3rd law of Newton that joint forces are equal but in opposite direction. Then it is obvious that

$$\overrightarrow{F_{23}} = -\overrightarrow{F_{32}} = \overrightarrow{F_{12}}$$
$$\overrightarrow{F_{43}} = -\overrightarrow{F_{34}} = \overrightarrow{F_{13}}$$

For the fully open and closed position cases, the baggage door must be in equilibrium in both cases. In horizontal and vertical direction total net force is zero and also moment about point A_0 should be zero.

$$\sum \overrightarrow{F_x} = \vec{0}$$
$$\sum \overrightarrow{F_y} = \vec{0}$$
$$\sum \overrightarrow{M}_{A_0} = \vec{0}$$

There are three unknowns $\left|\overrightarrow{F_{23}}\right|, \left|\overrightarrow{F_{43}}\right|, \left|\overrightarrow{F_{sp}}\right|$ and three scalar linear equations. The system has a unique solution and can be solved using linear algebra.

Let the angle of $\overrightarrow{AA_0}$ vector be α_{32}, angle of $\overrightarrow{BB_0}$ vector be α_{34} and angle of $\overrightarrow{C_0C}$ vector be α_{sp}, also it is known that gravitational force is always downward and its angle can be taken as $\frac{3\pi}{2}$. After that three equilibrium equations are:

$$\left|\overrightarrow{F_{sp}}\right| \cdot \cos\alpha_{sp} + \left|\overrightarrow{F_{23}}\right| \cdot \cos\alpha_{32} + \left|\overrightarrow{F_{43}}\right| \cdot \cos\alpha_{34} = 0$$

$$\left|\overrightarrow{F_{sp}}\right| \cdot \sin\alpha_{sp} + \left|\overrightarrow{F_{23}}\right| \cdot \sin\alpha_{32} + \left|\overrightarrow{F_{43}}\right| \cdot \sin\alpha_{34} + \left|\overrightarrow{W}\right| \cdot \sin(\frac{3\pi}{2}) = 0$$

$$(\left|\vec{r}_{A0C_x}\right| \cdot \sin\alpha_{sp} - \left|\vec{r}_{A0C_y}\right| \cdot \cos\alpha_{sp}) \cdot \left|\overrightarrow{F_{sp}}\right| + (\left|\vec{r}_{A0B_x}\right| \cdot \sin\alpha_{34} - \left|\vec{r}_{A0B_y}\right| \cdot \cos\alpha_{34}) \cdot \left|\overrightarrow{F_{43}}\right| + \left|\overrightarrow{W}\right| \cdot \left|\vec{r}_{A0G_x}\right| \cdot \sin(\frac{3\pi}{2}) = 0$$

Let A be the coefficient matrix, B be vector of unknown forces and C be the forcing vector. It is concluded that $AB = C$ where

$$A = \begin{pmatrix} (\left|\vec{r}_{A0C_x}\right| \cdot \sin\alpha_{sp} - \left|\vec{r}_{A0C_y}\right| \cdot \cos\alpha_{sp}) & 0 & (\left|\vec{r}_{A0B_x}\right| \cdot \sin\alpha_{34} - \left|\vec{r}_{A0B_y}\right| \cdot \cos\alpha_{34}) \\ \cos\alpha_{sp} & \cos\alpha_{32} & \cos\alpha_{34} \\ \sin\alpha_{sp} & \sin\alpha_{32} & \sin\alpha_{34} \end{pmatrix}$$

$$B = \begin{pmatrix} \left|\overrightarrow{F_{sp}}\right| \\ \left|\overrightarrow{F_{23}}\right| \\ \left|\overrightarrow{F_{43}}\right| \end{pmatrix}, \; C = \begin{pmatrix} \left|\overrightarrow{W}\right| \cdot \left|\vec{r}_{A0G_x}\right| \\ 0 \\ \left|\overrightarrow{W}\right| \end{pmatrix}$$

B vector can be found with $B = A^{-1}C$.

2.4 GAS SPRING SELECTION

Gas springs are used commonly in automotive industry. They are especially used to support the weight of vehicle doors while they are open. There is compressed gas in a cylinder and with varying piston length and spring force is obtained via the gas pressure. It can be assumed that the spring has a linear force characteristic at a constant temperature. However compression and extension force behavior is different because of the friction. A typical force vs stroke curve is shown on Figure 2.7

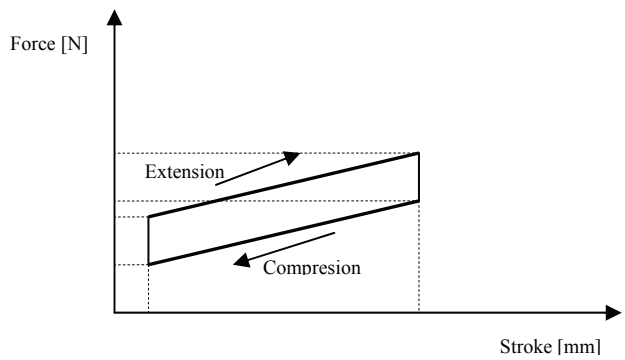


Figure 2.7: An example of Force vs. Stroke curve for a typical gas-spring

Using equilibrium equations, neglecting the reaction forces from the chassis at the extreme poses; necessary spring force values for the open and closed positions of the door can be obtained. These values determine the force limits of the gas-spring. Then, necessary spring forces at every instant are found and the required balancing force vs. stroke variation of the spring is obtained. Desired gas spring force

characteristic should intersect necessary spring force curve at a point that enough stroke is accessed to open and close door easily in two different phases.

The force applied by the user is modeled using the classical top hinged case. In the first position necessary force to open the door and in the last position necessary force to close the door are determined. The direction of the force can be assumed to be vertical at all times for simplicity. While opening it is upward, while closing it is downward.

Determination of positions of points C and C_0 in Figure 2.7 is directly related with the kinematics of the motion; their locations must not prevent the motion. However positions of these points are also related with force characteristic, because of this, C and C_0 positions should be determined bi-checking both kinematic and force characteristics.

Gas springs can be obtained in standard form from the market. It is better to seek a force range consistent with standards rather than to find a specific force value. After determination of proper force value, if a force characteristic is desired to be modified, replacements of gas-spring problem can be reconsidered and then positions of points C and C_0 should be reevaluated considering the kinematic behavior.

CHAPTER 3

DESIGN OF SIX LINK MECHANISM

3.1 KINEMATIC SYNTHESIS OF SIX-LINK MECHANISMS

Using the Gruebler [1] equation possible link types for a mechanism can be determined for a given degree of freedom (dof) of the mechanism, number of links and joints:

$$F = 3 \cdot (l - j - 1) + \sum f_i$$

Number of links: $l = B + T + Q + P + H.....$

Number of joints: $j = \dfrac{2B + 3T + 4Q + 5P + 6H....}{2}$

Total joint degree of freedom: $\sum f_i = \dfrac{2B + 3T + 4Q + 5P + 6H....}{2}$

where B: number of binary links, T: number of ternary links, Q: number of quaternaries, P: number of pentagonal, H: number of hexagonal….

One dof six-link mechanisms which have seven revolute joints are sought. From the Gruebler equation

$$2B + 3T = 14$$

Also total number of links is equal to the six

$$B + T = 6$$

There are two linear equations with unknowns B and T and the solution is unique:

$$B = 4$$
$$T = 2$$

This means that to obtain single degree of freedom from the kinematic chain which has six links and seven revolute joints, there must be four binary links and two ternary links. Now order of connections of links is important. It can be divided into two categories according to replacement of the ternary links. When ternary links are adjacent it is called a Watt chain, otherwise it is called a Stephenson chain. With kinematic inversion, two different Watt mechanisms and three different Stephenson mechanisms can be obtained. These are shown on Figure 3.1 and Figure 3.2 [1-2].

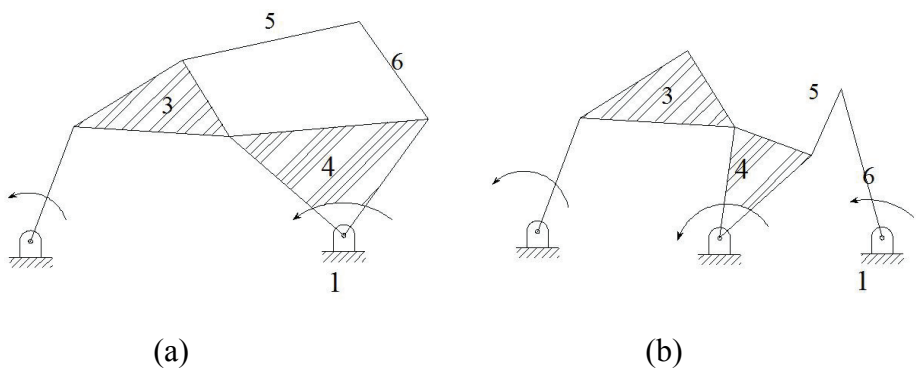


Figure 3.1: (a) Watt-I Type; (b) Watt-II Type [1]

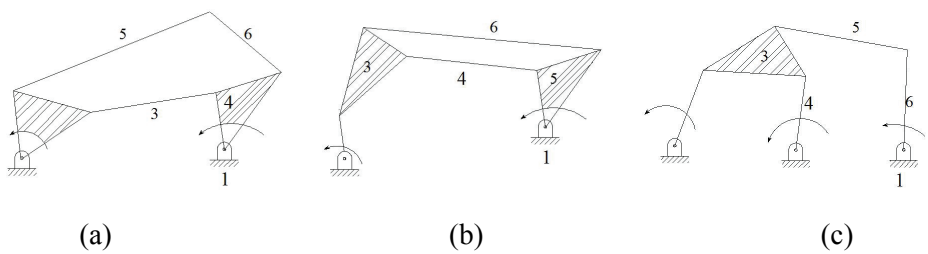


Figure 3.2: (a) Stephenson I Type (b) Stephenson II Type (c) Stephenson III Type [1]

3.2 SYNTHESIS OF WATT I LINKAGE

It can be seen in Figure 3.1 that link 1 is fixed and link 5 can be taken as the output link. Four possible configurations of the mechanism can be considered. These configurations are shown in Figure 3.3.

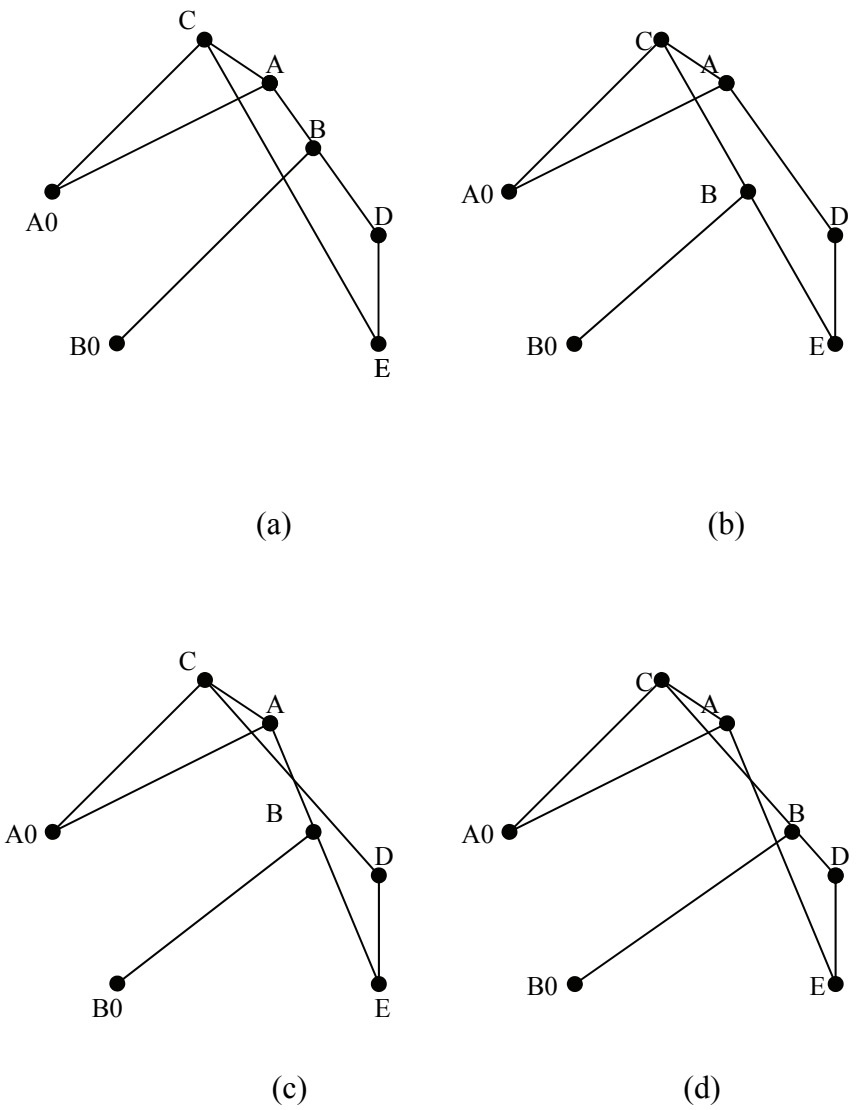


Figure 3.3: (a) $A_0ACEDBB_0$ chain (b) $A_0CADEBB_0$ chain (c) $A_0ACDEBB_0$ chain (d) $A_0CAEDBB_0$ chain

Since number of design parameters is increased compared to the four-bar case to simplify solving mathematical equations, three positions synthesis is used. That is three finite positions of the door are considered. In six link mechanism there are seven joints if all joint coordinates are determined, then the mechanism is synthesized. Together with a parameter associated with the initial pose of the mechanism, there are fourteen parameters which must be evaluated. All joint coordinates can be considered as design parameters; for instance (x_{A0}, y_{A0}). For three positions synthesis there are four scalar equations. Also it is assumed that one of the ternary links have collinear kinematic elements, which is implemented as another equation. So nine parameters are free to choose and five parameters are solved from the system of five scalar equations.

Because of similarity in synthesis only one of the above four chains are mentioned in detail. For instance $A_0ACEDBB_0$ chain showed on Figure 3.3 (a) can be considered. Input parameters for synthesis are initial pose of coupler link DE; A_0AD dyad for initial position and also one coordinate of point B at initial position. Output parameters are coordinates of points C and B_0 also the other coordinate of point B at initial position. So there are nine specified parameters, then remaining five unknown parameters are solved with five scalar equations from coupler link positions and equation of line which point B is on.

First $(x_{A0}, y_{A0}), (x_A, y_A), (x_{D,} y_D), (x_E, y_E)$ are specified. Also from the problem definition three positions of points D and E are obtained. They are already determined with the help of door profile. Using relative position concept and kinematic inversion position of point C is determined. This can be achieved by considering motion of link DE with respect to link A_0A. Since A_0 is fixed its coordinates are same through all three positions. In three finite positions coordinates of point A are determined with the help of point D. Initially determined points are represented on Figure 3.4.

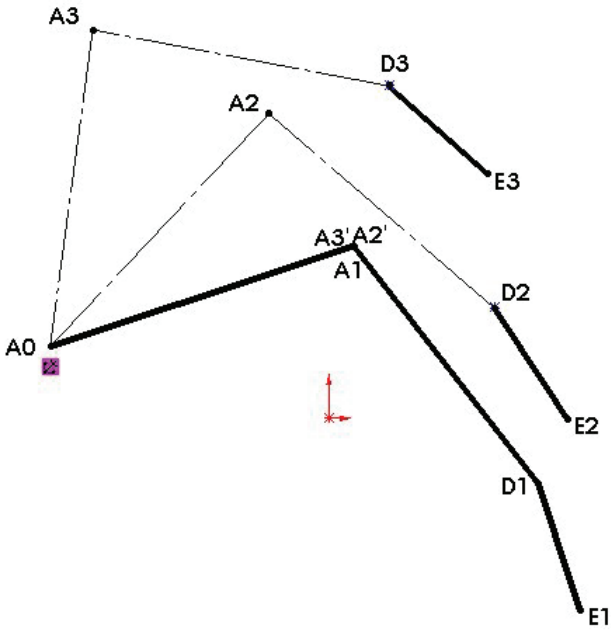


Figure 3.4: Three positions of link A_0A and link DE

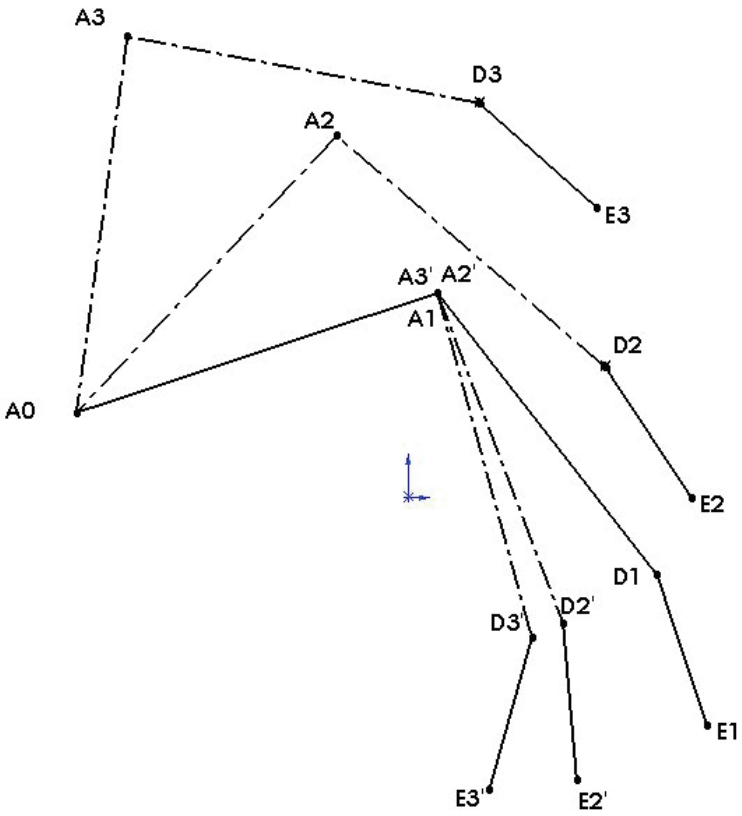


Figure 3.5: Relative positions of link DE with respect to link A_0A

During the kinematic inversion link A_0A is held fixed. A_0A_2 and A_0A_3 lines coincide with A_0A_1 by rotating A_0A about point A_0 with same angle in reverse direction. Relative position of line DE is achieved and called DE' (Figure 3.5). It is desired to connect point E with a corresponding point C to satisfy the motion. To do this, three position synthesis is used and a unique center point for points E1, E2', E3' is determined. Figure 3.6 shows how position of point C is determined.

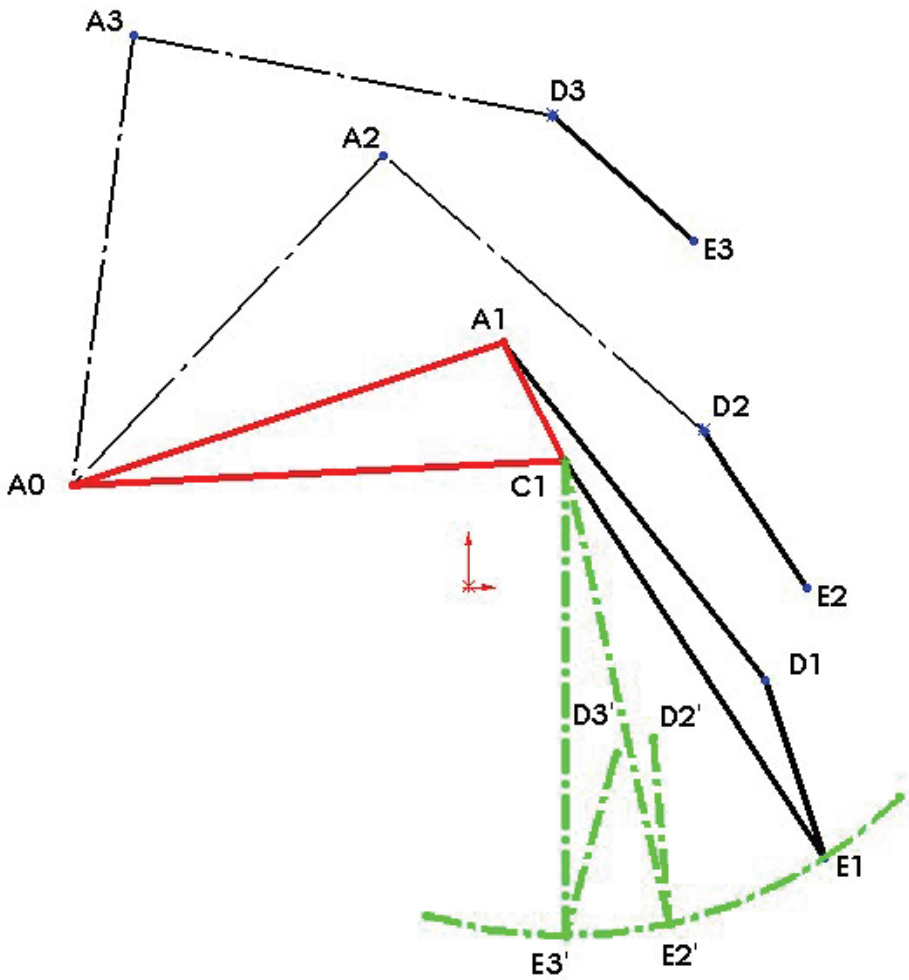


Figure 3.6: Position of point C

Selecting a point B on line AD all positions of point B can be acquired. With three position synthesis using B1, B2 and B3, point B_0 can be determined. It can be seen on Figure 3.7.

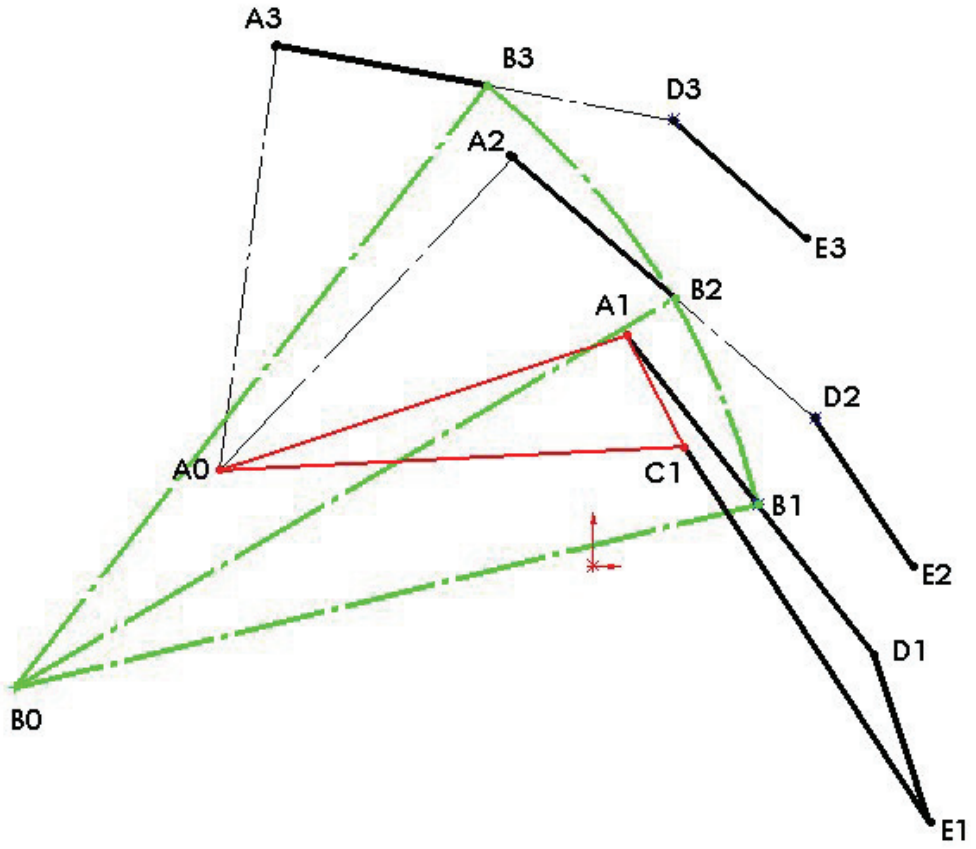


Figure 3.7: Position of point B

After determining all necessary joint positions, the six-link mechanism is synthesized. The other three configurations shown in Figure 3.3 may be synthesized similarly using relative position and three position synthesis only changing connecting points and placement of point B.

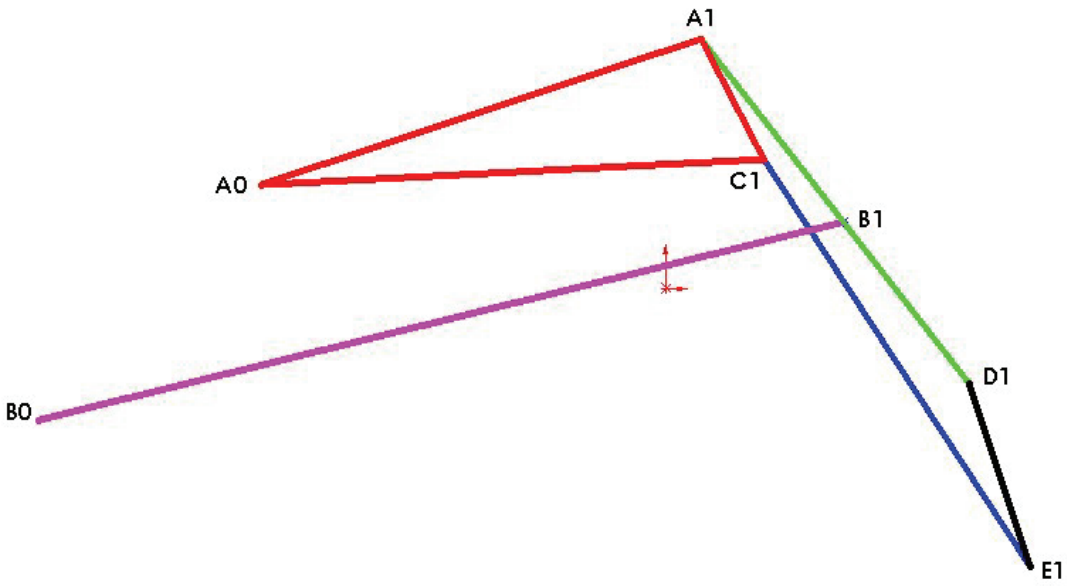


Figure 3.8: Six-link mechanisms

Final designed links are shown on Figure 3.8. Whether the mechanism works correctly or not should be checked. Kinematic analysis is done to ensure this. Then with the selected coupler point trajectory the minimum required distance to open is checked. This part of the procedure is the same with the four-bar mechanism synthesis.

CHAPTER 4

MECHANISM DESIGN FOR SAMPLE HATCBACK VEHICLE

4.1 MODELING CAR BODY AND BAGGAGE DOOR

In this study, a Renault Clio Hatchback vehicle is used to adapt a new mechanism for the baggage door. To test the mechanism both real and virtual environment chassis of the car and vehicle 3-D solid model are available. First, in the CAD environment the mechanism is assembled on the car.

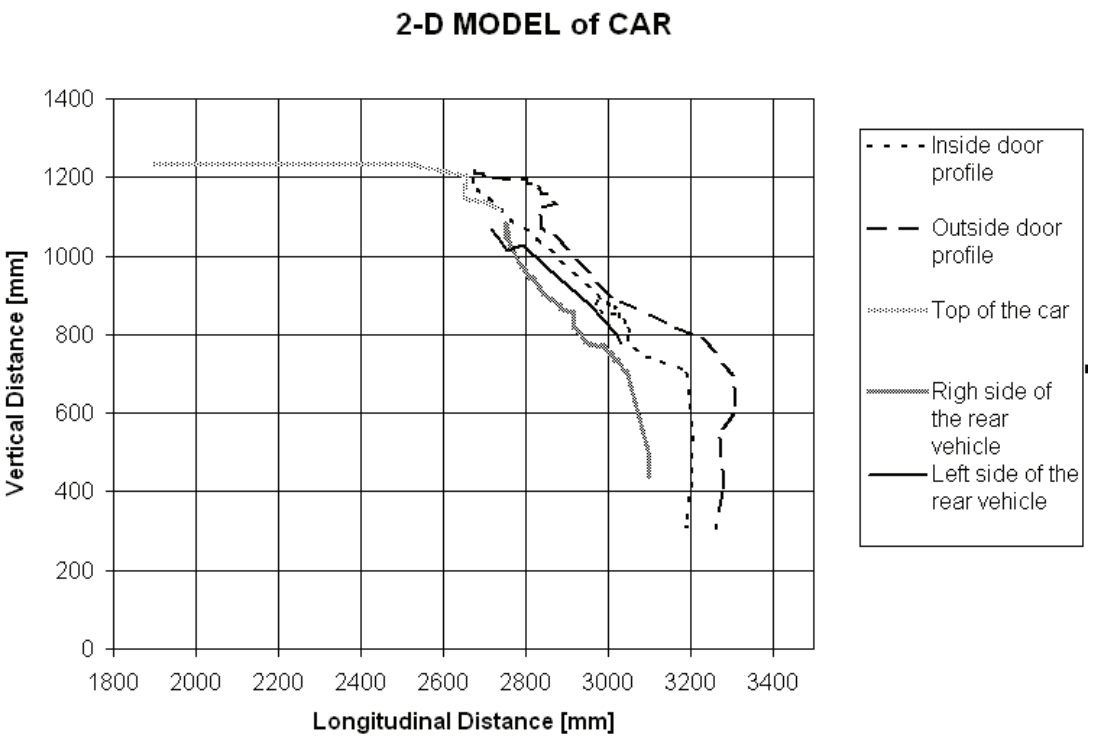


Figure 4.1: Modeling the car in working plane

Then with car chassis, the mechanism prototype is mounted on the body and the motion is tested. All used data are acquired from the 3D CAD model. A working plane should be defined parallel to the symmetry plane of the vehicle as the motion plane for the mechanism. Then, enough number of points are selected from the rear chassis and door profile of the vehicle. These points are projected into the working plane. The two profile curves actually define the workspace limits when the mechanism is in folded position.

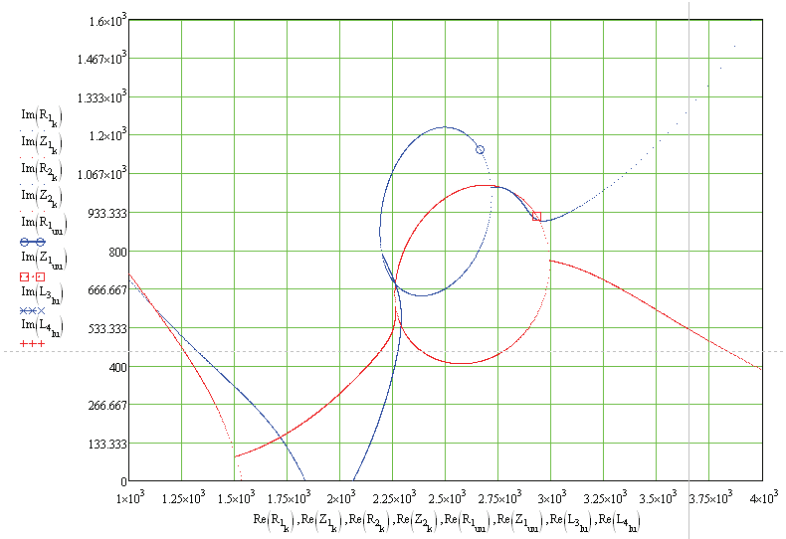
Figure 4.1 represents necessary constraints for the car body and the baggage door. Because of the unsymmetrical structure of rear of the car, it is identified with both right and left side curves.

The links of the mechanism must be between the chassis and the door profile curves. Joints of the mechanism on the baggage door should be between inside and outside profile of the door. These constraints are all considered in the folded position. Also while moving, all parts of the mechanism should not intersect the top profile and at final position the inside door profile should not intersect the top profile. All these curves are defined and represented as input parameters and used for motion constraint equations.

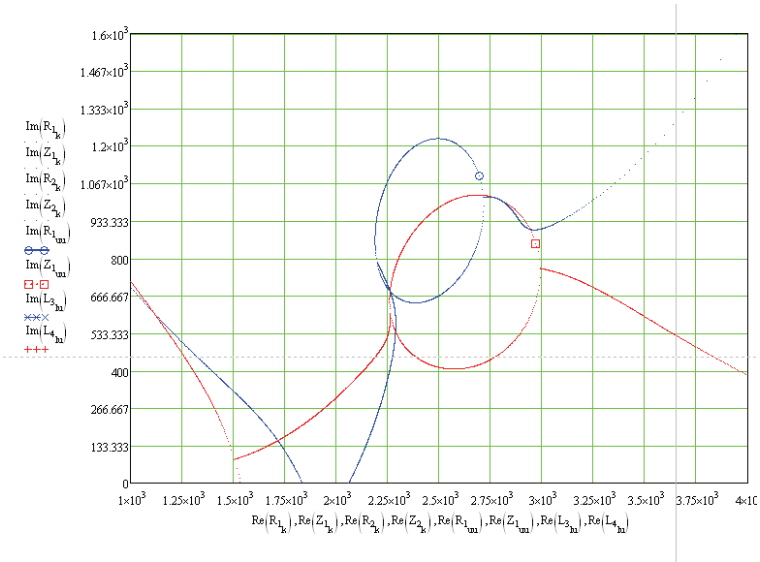
4.2 KINEMATIC SYNTHESIS OF A FOUR-BAR MECHANISM FOR A GIVEN HATCHBACK VEHICLE

In synthesis, the car body is considered fixed and the baggage door is taken as the coupler link. Then the relative motion of the door with respect to the car chassis is defined with four separate positions. Using four positions, it is possible to avoid infinite solutions, and the design can be controlled with one parameter. Circle points should be on the car body and center points on the door are simultaneously solved using Burmester curves and constraints are taken into consideration. Figure 4.2 represents sample Burmester curve pairs and selected circle and center

points respectively. The blue curve is the circle curve and the red one is the center curve.



(a)



(b)

Figure 4.2: (a) Circle points curve (b) Center points curve

Then kinematic analysis is performed to see whether the motion is satisfied or not and whether the required space behind the vehicle is small enough. If the solution is not satisfactory, the four positions of door parameters are changed and new solutions are obtained. Burmester curves are sensitive to small position variations. This method is iteratively applied until a satisfactory solution is obtained. In this procedure, priority is given to select circle points satisfying the constraint equations. However it is seen that the obtained mechanisms do not satisfy the desired motion with circle points in the constrained region. To accomplish more satisfactory motion, some regional constraints are broken; this means that a new design of car chassis is required. Table 4.1 gives the numerical values for sample solution four bar with respect to the ground reference frame.

Table 4.1: Numerical values of designed four bar mechanism

	X[mm]	Y[mm]
Center point1(A)	2930	919
Circle point1(A0)	2660	1149
Center point2(B)	2971	851
Circle point2(B0)	2695	1092

	Length[mm]	Ratio
Fixed Link	67	1,0
Crank Link	355	5,3
Coupler Link	79	1,2
Rocker Link	366	5,5

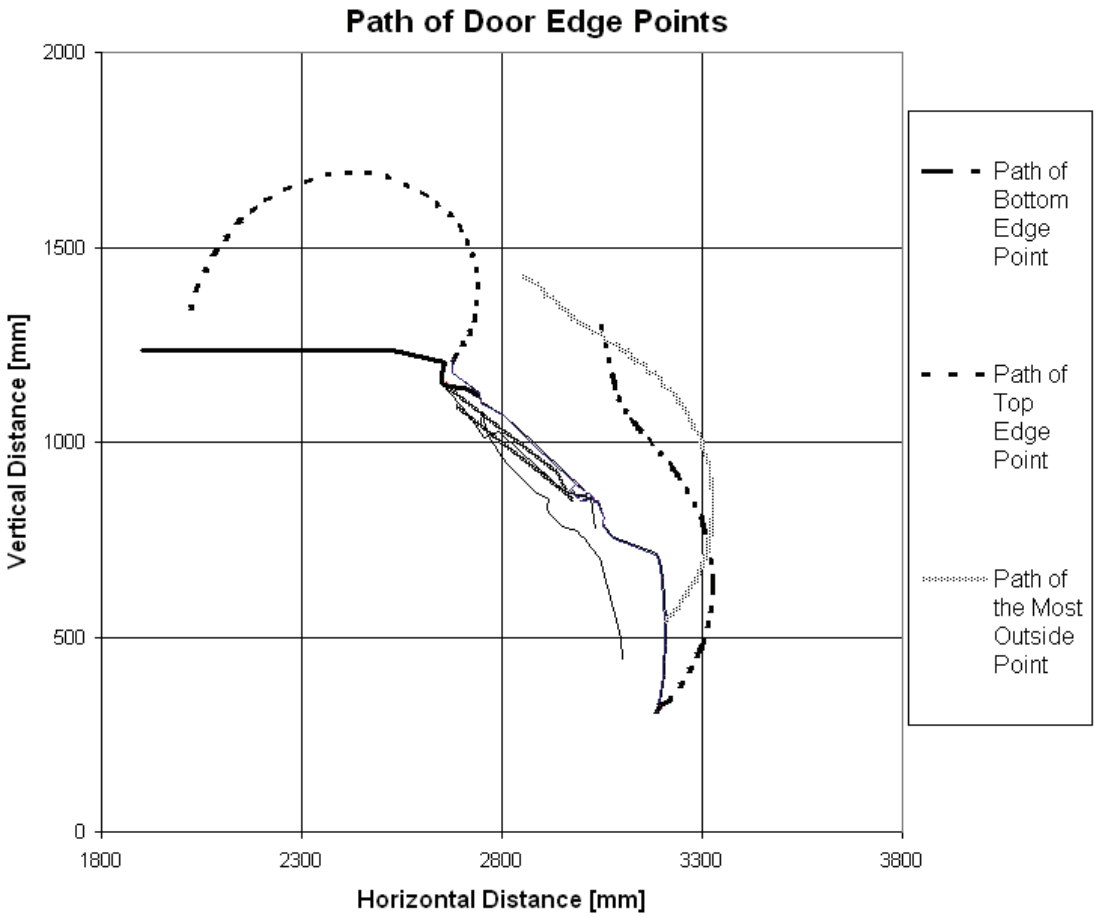


Figure 4.3: Extreme points of door through the motion

In Figure 4.3, paths of design points used as a parameter are given throughout the motion. The improvement on the necessary distance can be obtained with comparing the distance in the top hinged case. Figure 4.4 represents the extreme point path when hinged mechanism is used.

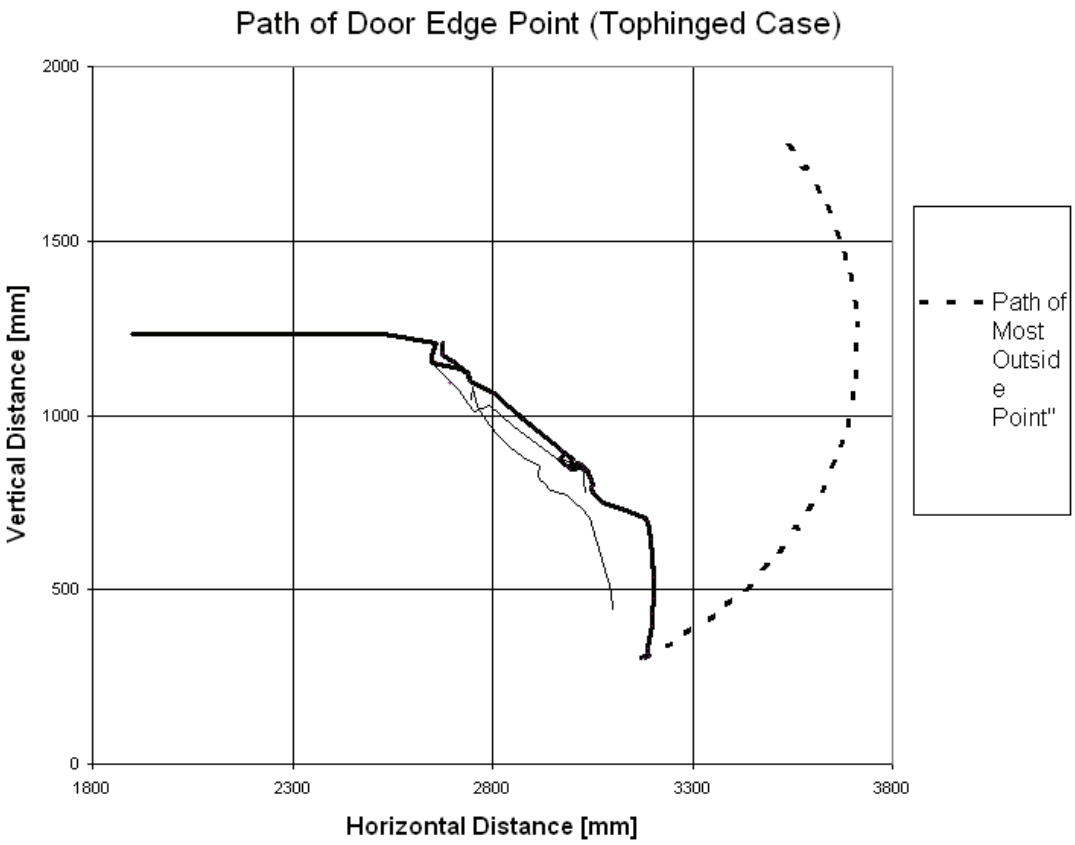


Figure 4.4: Top hinged door, path of the most outside point

With a sample four bar mechanism the door can be opened in 99 mm however; traditional top hinged door requires 505 mm to open it from back of the car body. It can be seen that the distance for opening the door is reduced about 80% of the original value.

4.3 KINEMATIC SYNTHESIS OF A SIX-LINK MECHANISM FOR A GIVEN HATCHBACK VEHICLE

To simplify the process, three position synthesis is used. First, three positions of the door are specified as input. These positions can be changed to obtain better results. Figure 4.5 shows sample three positions of the door.

Figure 4.5: Selected three positions of baggage door

Figure 4.6: Initially specified joint positions

First three links are specified. Fixed and three moving joints are selected. Two moving joints are chosen near to the door profile

Relative positions of point E (See Figure 4.6) are obtained with respect to link2 (Link A_0A in Figure 4.6), so a center point can be selected for point E. Methodology already presented on Chapter 3 is applied. It is called point C and represented on Figure 4.7 with obtained five links and two dof sub mechanism.

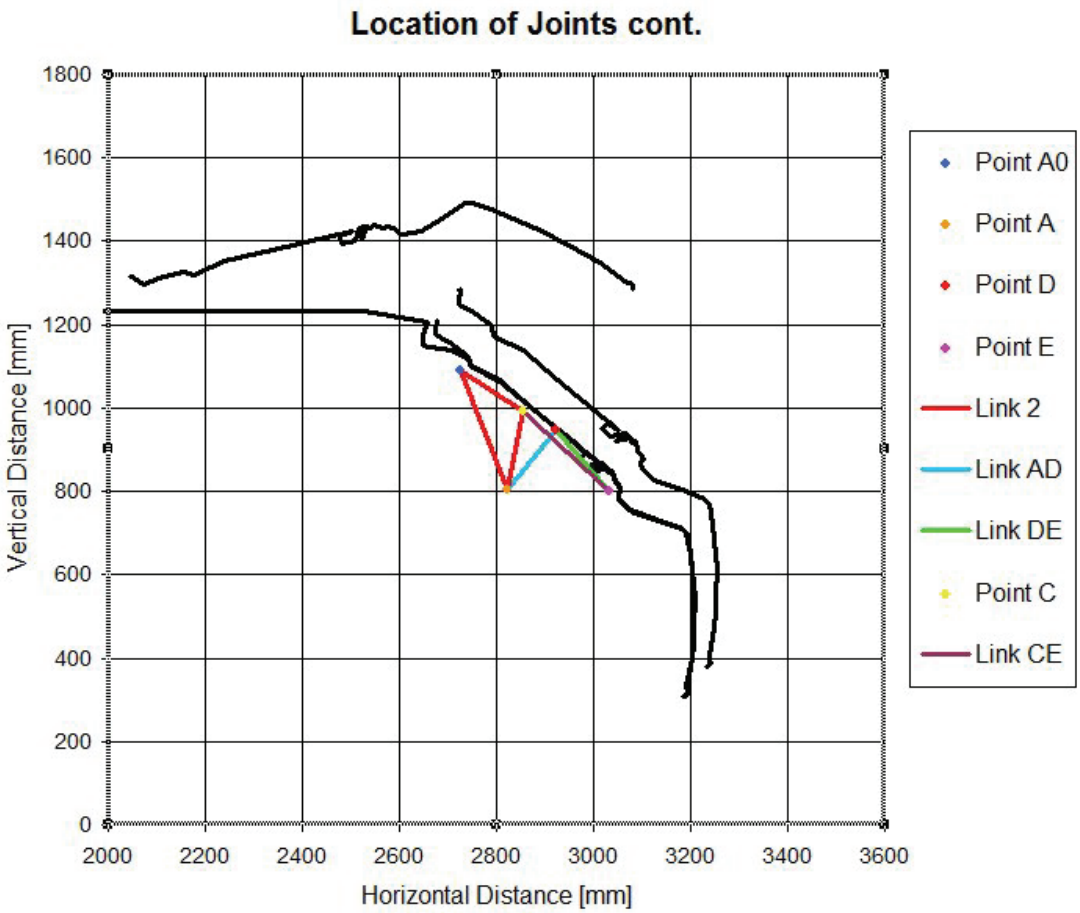


Figure 4.7: Sub-mechanism - five links and five revolute joints

With addition of one more link and two more revolute joints, mechanism synthesis is completed. One end of this additional link is pivoted to ground; the other end

should be located on link5 or link3. In this design to satisfy motion requirements new link is connected with link3. For ease of manufacture, the joint is located collinearly with the two other joints on link3. Using the methodology in Chapter 3 position of point B_0 is determined. The final design of links is shown on Figure 4.9. Table 4.2 gives the numerical results of six-link mechanism.

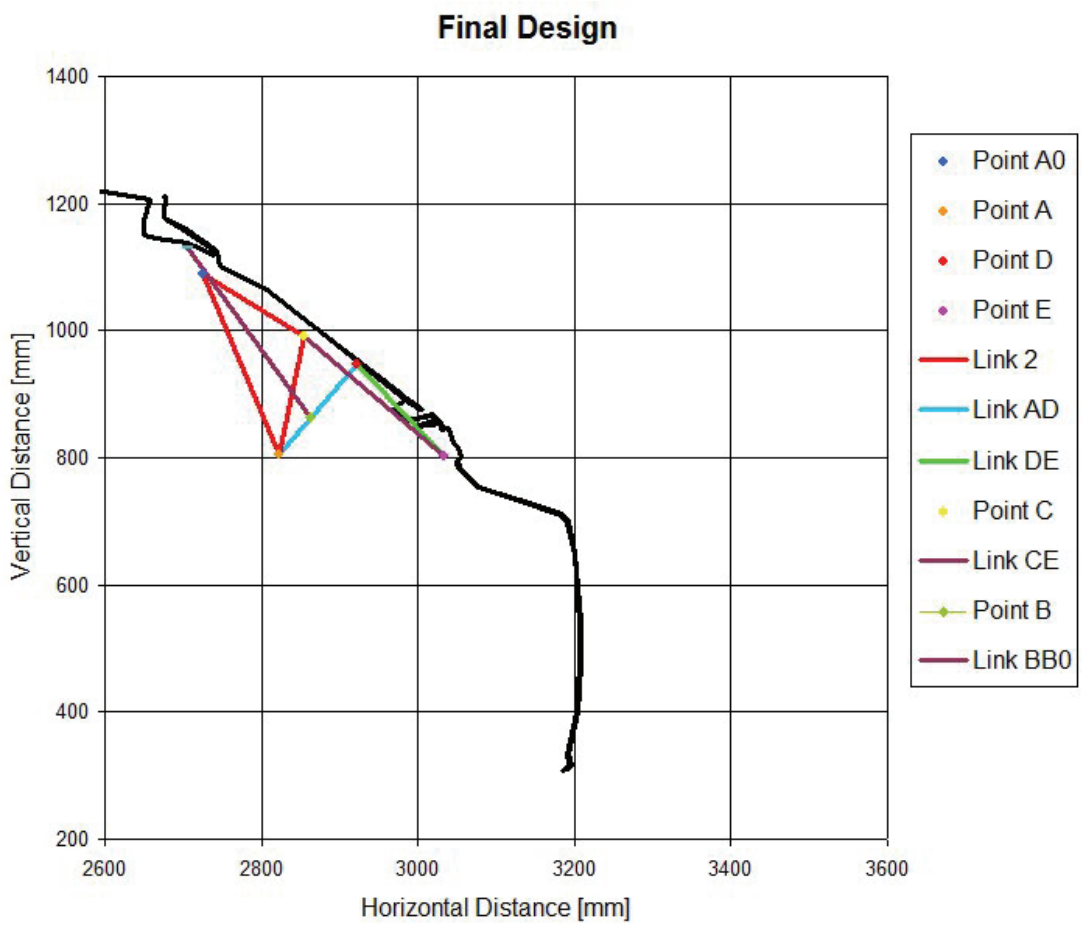


Figure 4.8: Designed six-link mechanism

Table 4.2: Position of joint and link lengths of six-link mechanism

(a) Joints' position

	X [mm]	Y [mm]
Fixed Joint (A0)	2726	1088
Joint (A)	2822	805
Joint (E)	3033	802
Joint (D)	2922	946
Joint (B)	2863	863
Joint (C)	2855	990
Fixed Joint (B0)	2706	1131

(b) Link Lengths

	Length [mm]	Ratio
Fixed Link (A0B0)	47	1.0
Link2-1 (A0A)	299	6.3
Link2-2 (A0C)	162	3.4
Link2-3 (AC)	188	4.0
Link3 (AD)	173	3.7
Link4 (BB0)	310	6.6
Link5 (EC)	259	5.5
b3 (AD)	71	1.5
c3 (DB)	102	2.2
Link6 (DE)	182	3.9

The maximum horizontal distance is evaluated taking the farthest point from the vehicle chassis. Also top edge point curve is drawn to control the motion. Finally, necessary distance to open door is measured as 77 mm. If it is compared with the hinged case the gain is found as about 85 %.

4.5 DETAILED DESIGN OF THE MECHANISM

From this point on, because of cost and placement problems, only the four bar mechanism design is considered. After determining the position of joints and effective link lengths, a prototype of the mechanism is manufactured and mounted on the sample car to test the motion of the mechanism. The 3D model of the vehicle is obtained in CATIA® environment. All links and parts are designed in CATIA® environment considering chassis profile of the car.

Although the motion is planar, because of the crossing of links and rear structure of the car, links should be bent out of plane on their profile. Thickness of the links is taken as large as possible to fit the mechanism in the available empty space. The thickness is selected as 4 mm. In shape design, bending of the links is avoided as much as possible. However, it is necessary at least one bending in both links for robustness.

To satisfy motion characteristics, the chassis geometry is changed. There is a slot of 40x80 mm on both corners of the car body. In addition to this, rear side of the chassis near the rubber gasket is changed to enlarge the space for the mechanism. It is extended widely about 1 cm on both side. Three moving links are shown on the Figure 4.9, Figure 4.10,Figure 4.11. The figures are not scaled. Since link1 is ground frame it is not shown, however there is a special profile on top corner of the car for smoothness of chassis structure (Figure 4.12).

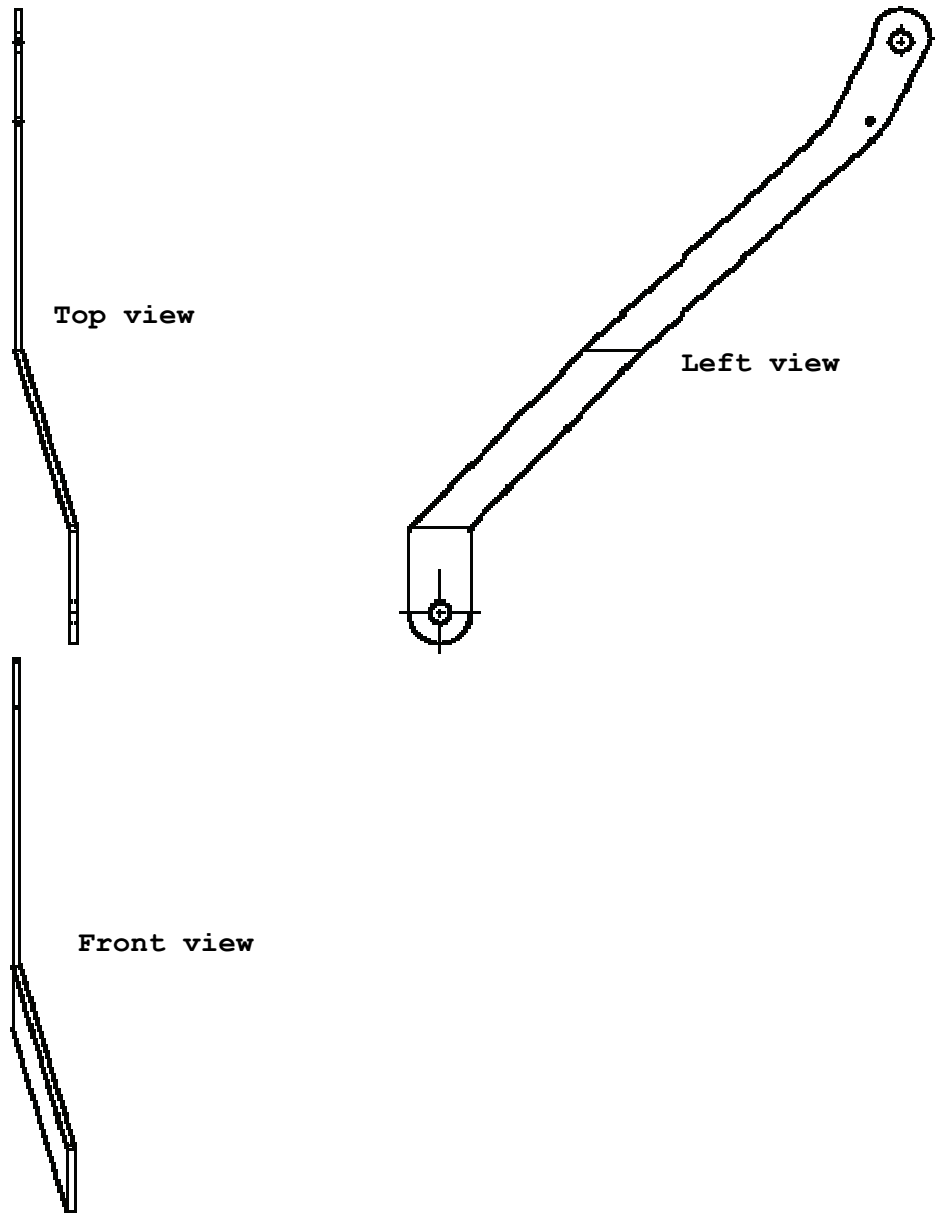


Figure 4.9: Crank link

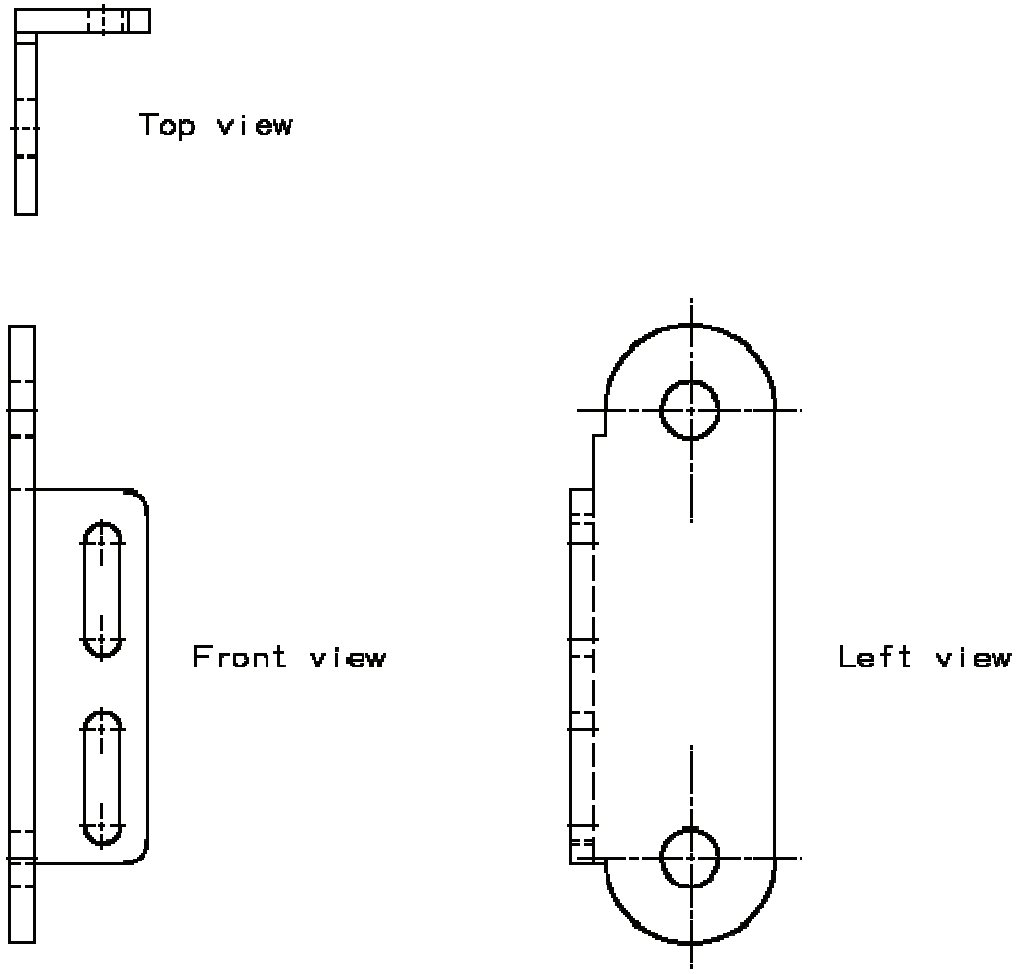


Figure 4.10: Coupler link (attached to the door)

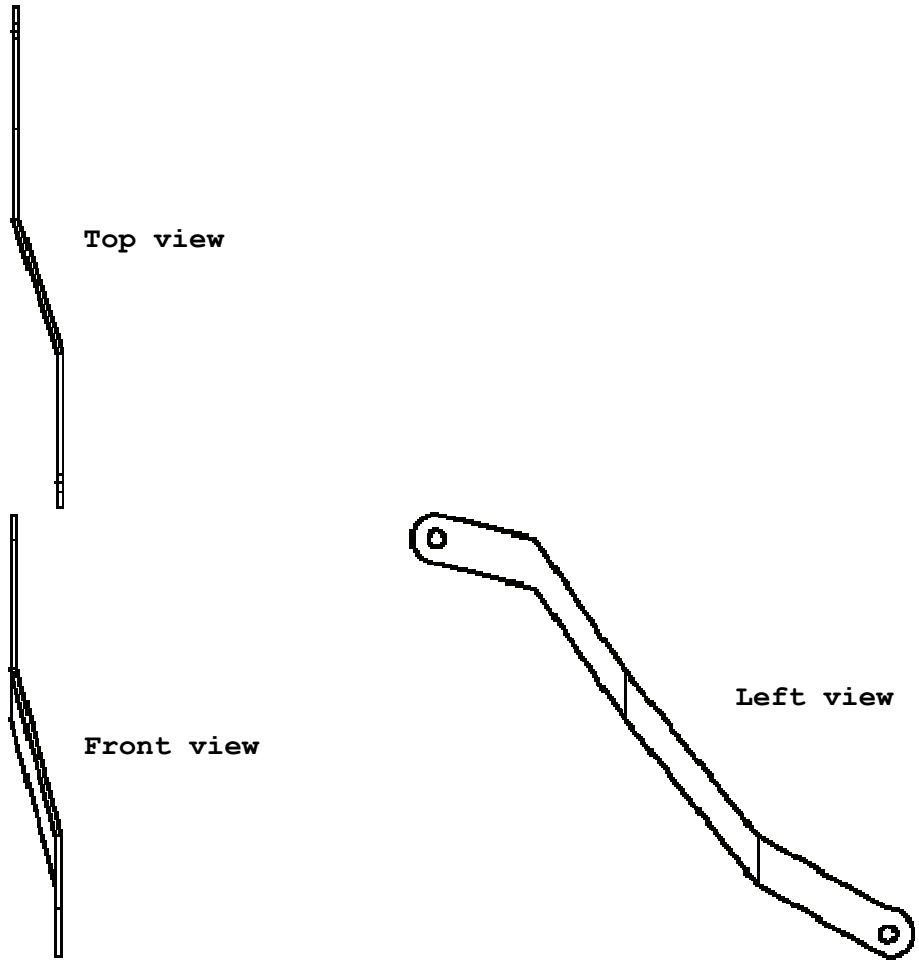


Figure 4.11: Rocker link

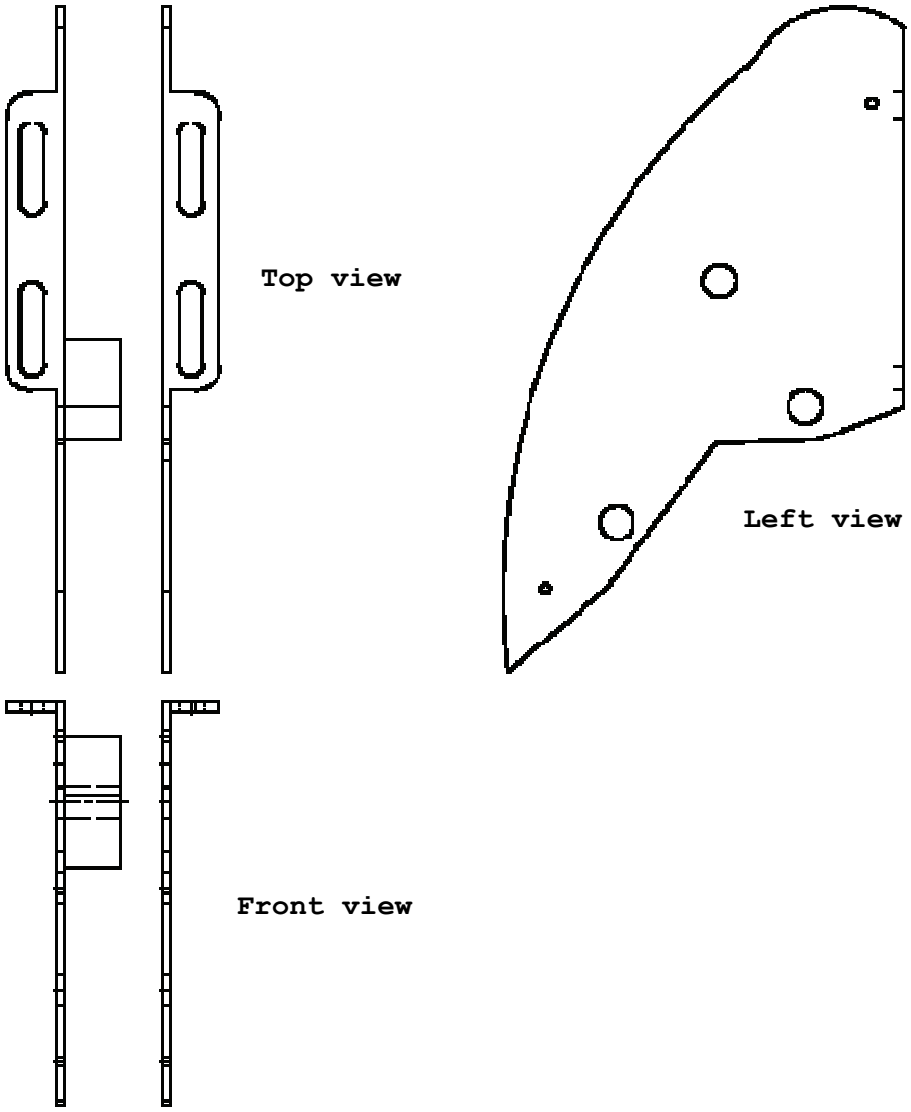
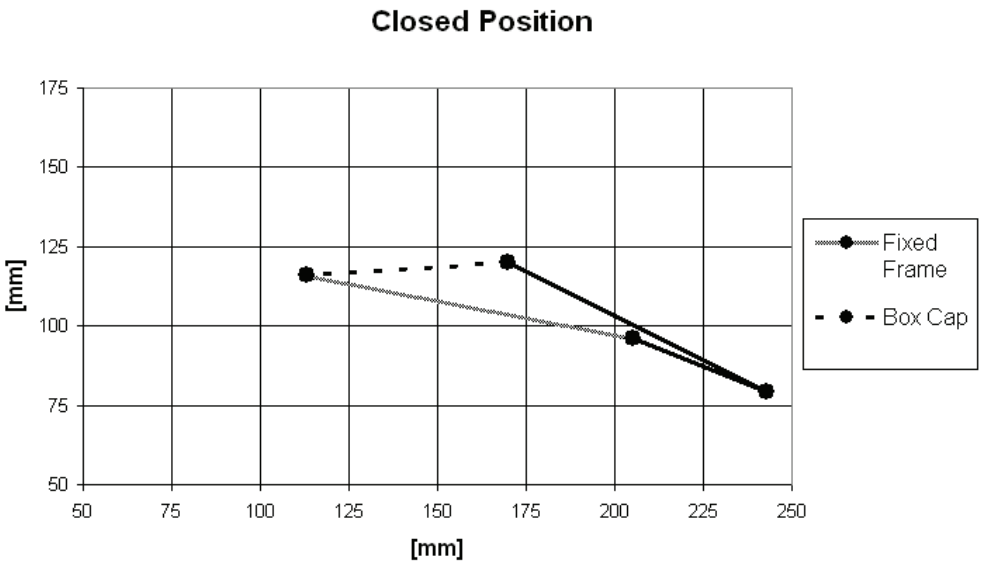


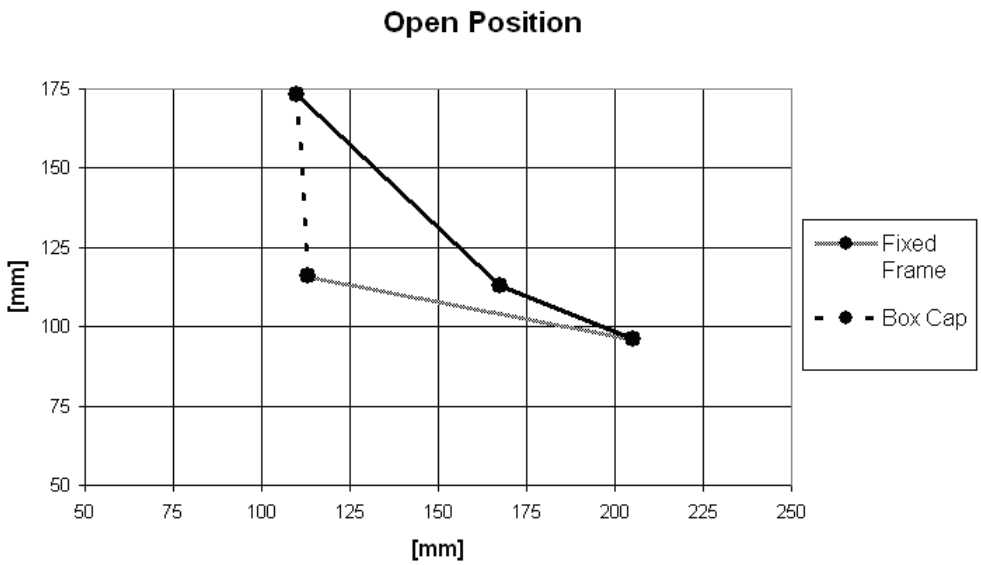
Figure 4.12: Casement for the top corner

With this box shown in Figure 4.12, also, any kind of water, especially rain can be directed towards out of the car. A channel is designed for separating outside and inside of the car. To prevent permanently open area on top of the car, a new small mechanism is designed for covering that slot (Figure 4.13). It is designed considering a cap as the rocker link and the coupler link is jointed with the crank link of the main door mechanism. Links are synthesized with multi-position

synthesis again similarly to the main mechanism. The mechanism for box-cap moves simultaneously with the main mechanism. Figure 4.13 represents open and closed positions of this mechanism.



(a)



(b)

Figure 4.13: Initial (a) and final (b) positions of the box-cap mechanism

These links also have some different profiles because of the constraints. Links are designed to move the mechanism conveniently. There are two new links and the other links are main mechanism members. The new links are given in the Figure 4.14 and Figure 4.15.

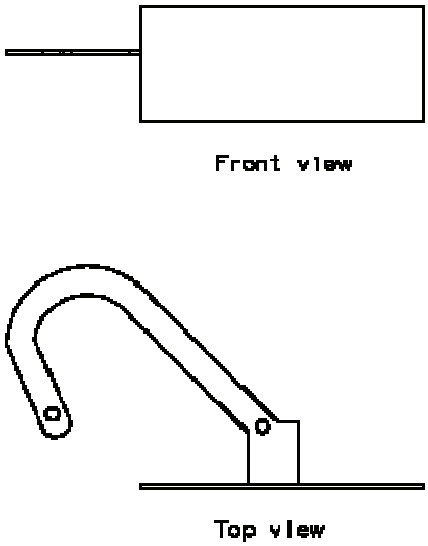


Figure 4.14: Box mechanism rocker link

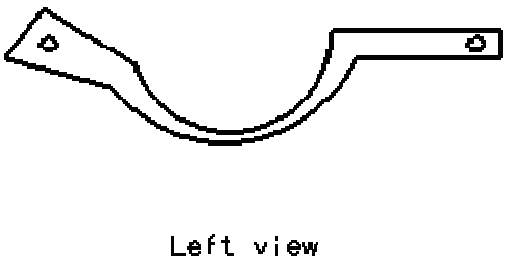


Figure 4.15: Box mechanism coupler link

4.6 DESIGN OF GAS SPRING FOR SAMPLE HATCHBACK VEHICLE

After the design of the mechanism, it is necessary to determine how to actuate the mechanism. In this design, the coupler link, which is the door itself, is thought as actuated member with manual user hand force. Relatively high load is necessary to move the mechanism for a common user. To overcome this problem and simplify user responsibility, gas springs are adapted to the mechanism. It helps during both opening and closing the door. While it is extending, opening is achieved by the gas spring after a certain balance position. During closing, motion is achieved without any external force after that balance point.

In the design of the gas spring, how much force is supplied by gas spring and where the gas spring is mounted are the main questions. Considering two design criteria, gas spring characteristics and dimensions are determined. Since market supplies a limited range for gas springs, this constraint effects the selection of the gas spring.

Since there are no high accelerations, only static force analysis is done. All joint forces and the necessary gas-spring force are evaluated at certain discrete points throughout the motion. In calculations, the coupler link is thought to be in equilibrium. Obtained required gas spring force values are used to determine force values on the two extreme positions. However since the position of gas-spring also effects the force characteristics the extreme values can be varied. Figure 4.16 shows gas-spring force values with respect to angular positions of the door for the designed gas spring connection points.

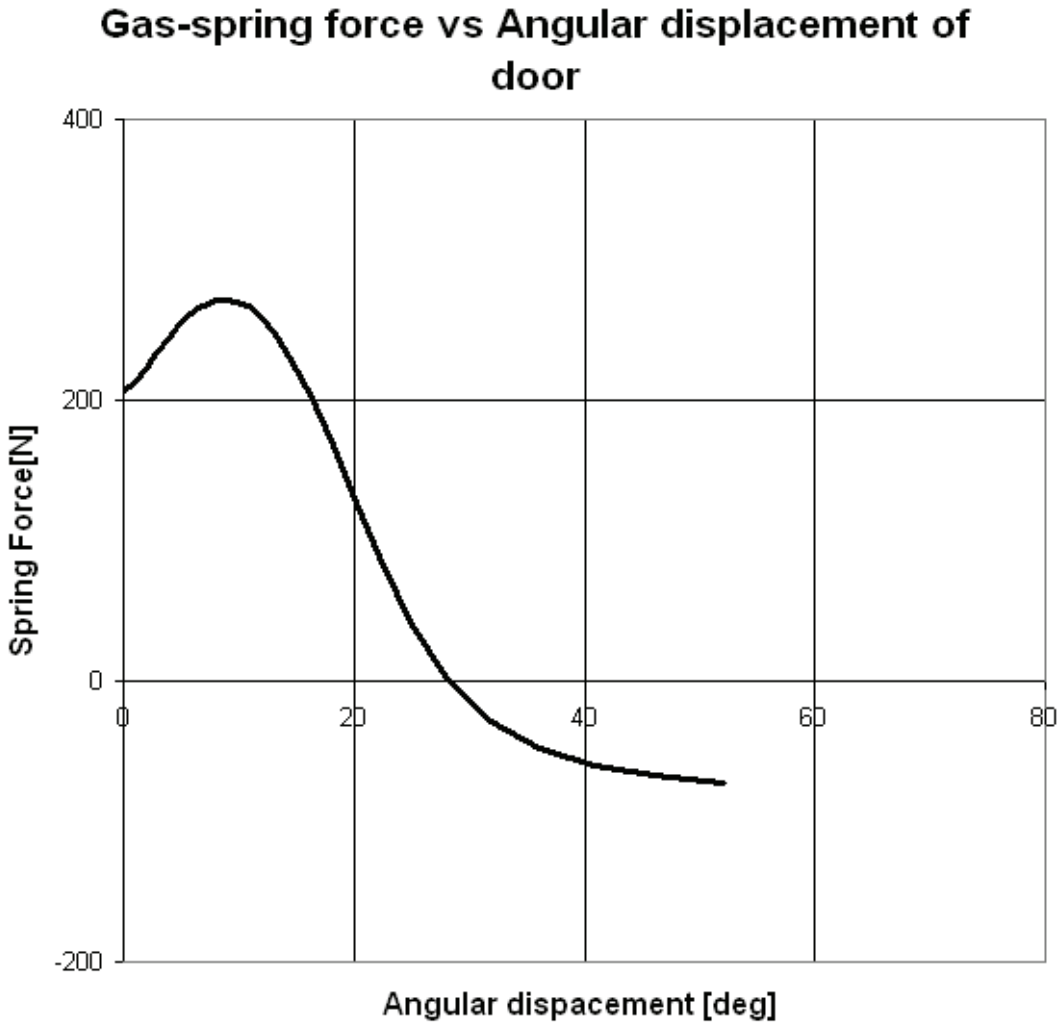


Figure 4.16: Balancing gas-spring force

Placement of the gas spring is determined with both kinematic and dynamic motion characteristics simultaneously. First coordinates of two points are evaluated according to specified motion. Then with that position value necessary external hand force is evaluated and it is compared with desired acceptable maximum hand force. This procedure is iteratively done until an acceptable external hand force is obtained. Figure 4.17 represent required hand force to balance the mechanism throughout the motion.

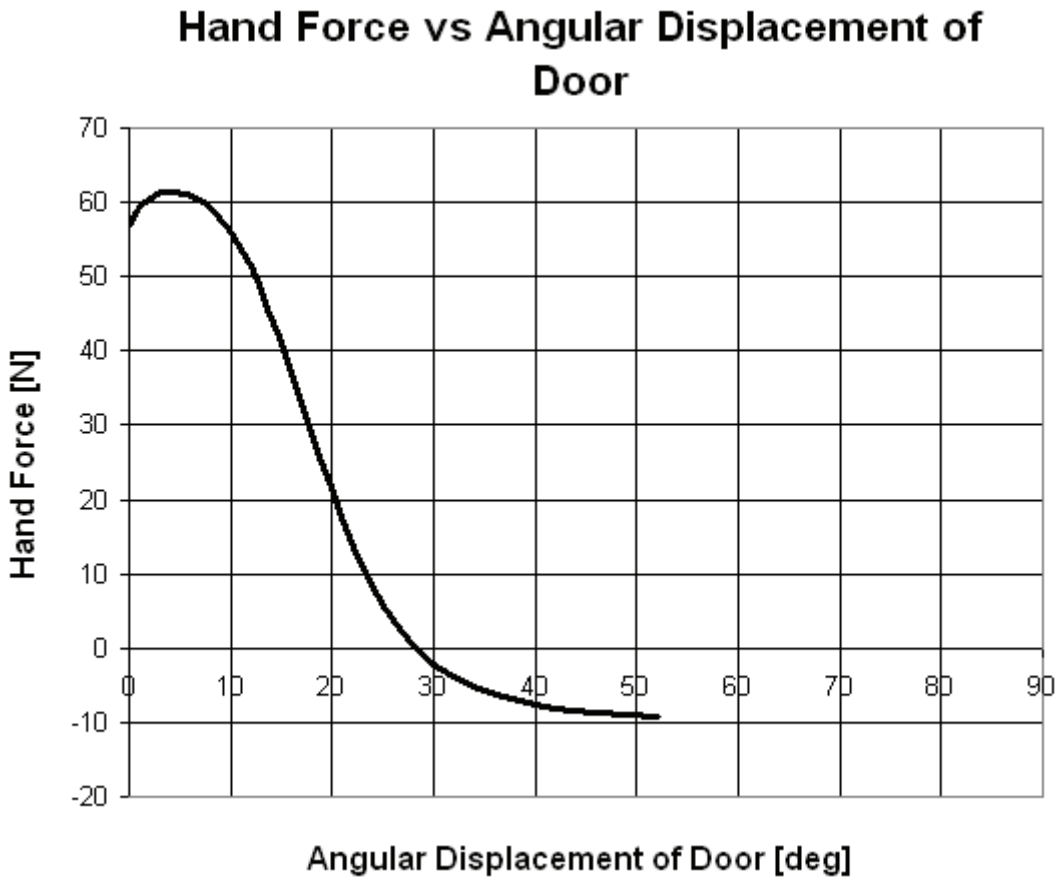


Figure 4.17: Balancing hand force with respect to angular displacement of the door

In calculation process, acceptable hand force value is evaluated considering the force values for the classical hinged door. In that case hand force trend throughout the motion are referenced for new design. This is shown on the Figure 4.18.

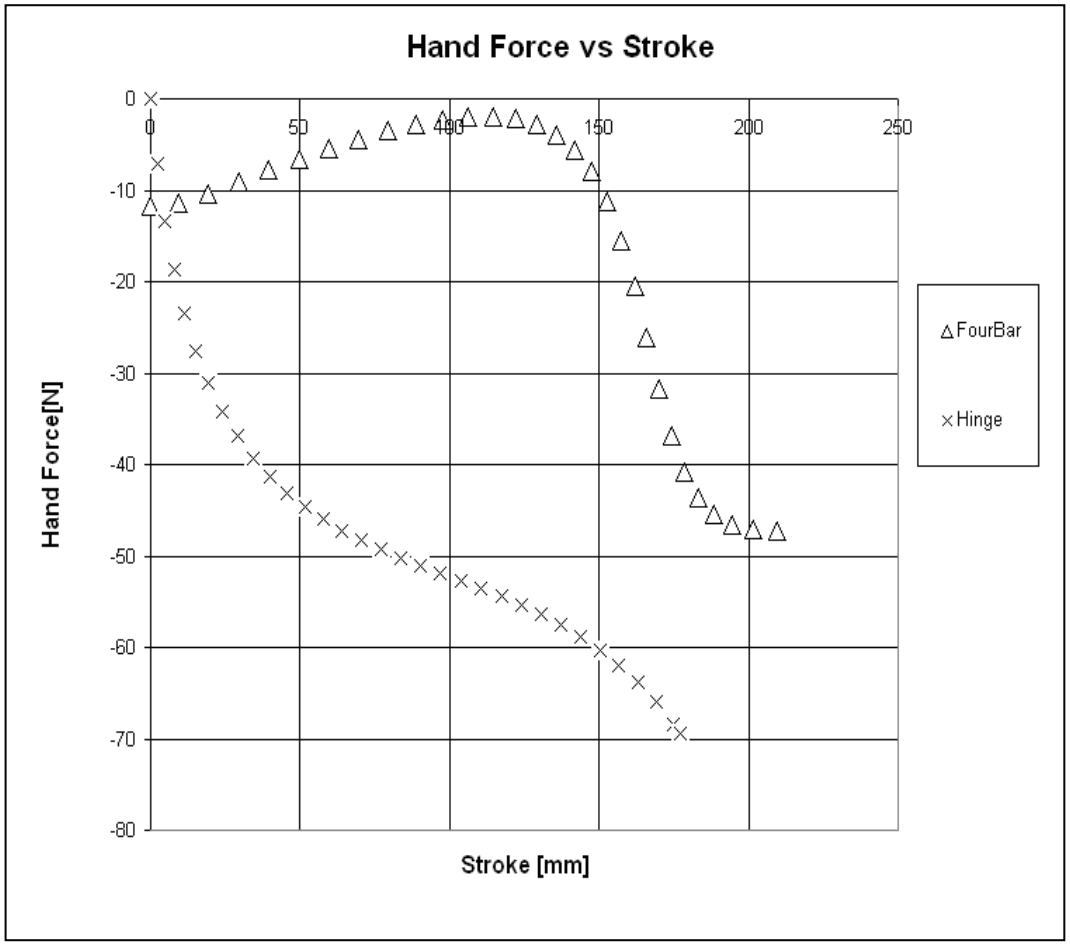


Figure 4.18: Hand force in two different designs

The above criterion reduces the possible range for fitting the gas-spring. Moreover, while deciding on the spring mounting positions chassis constraints are also important. Although, through the design process, any change on the chassis is not desirable, to satisfy both kinematic and dynamic constraints, the rear inside profile of the chassis is modified. Table 4.3 gives information about the final design of the gas spring.

Table 4.3: Specifications of the gas-spring

Parameter	Value
L1 [Length-Compressed]	340 mm
L2[Length-Extended]	560 mm
Stroke	220 mm
D1[Diameter of Tube]	18 mm
D2[Diameter of Piston]	8 mm
Fmax[Maximum Force]	300 N

4.7 IMPLEMENTATION

As mentioned previous chapter, two alternative linkages solution are searched for Renault Clio Hatchback model, however only one of them, four bar solution is used for implementation. This application is thought for proof of concept, so it is not considered as final design including links' detailed strength analysis. Although six-link mechanism solution gives good results of necessary distance to open door. Placement of mechanism on the car-body causes more changes on the chassis. That is, chassis should be redesigned according to new mechanism dimensions. Huge alteration is avoided because of difficulty in application process.

Door itself, car-body and four bar mechanism can be separated each other through the process. Mechanism members are manufactured using ST-55 steel. Since links have inflection points, first link's profile is cut and with press machines it is bended from specified points. To check the manufacturing error of bending in dimensions, a template is prepared. Then links are assembled and mechanism is shown on Figure 4.19 and Figure 4.20.

Figure 4.19: Assembly of Four Bar (Side view)

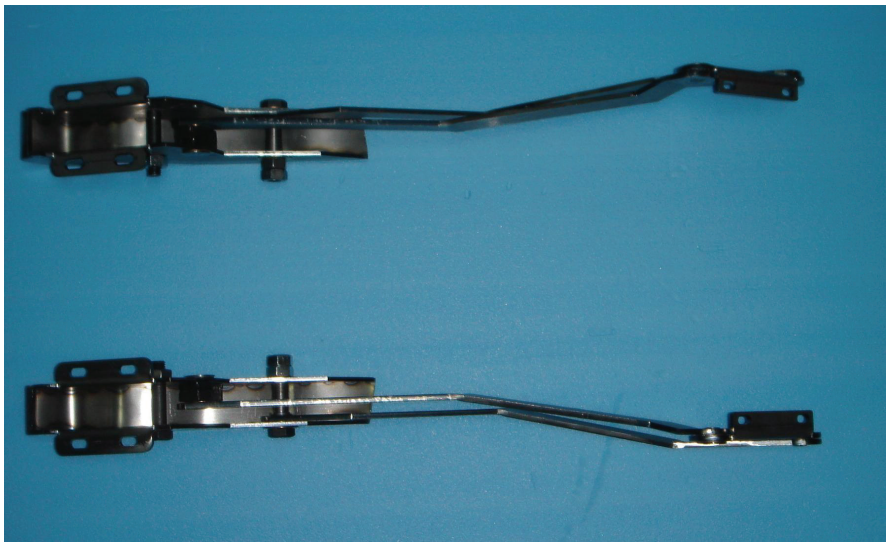


Figure 4.20: Assembly of Four Bar (Top view)

Door is dismounted and a small part is inserted to door mounting location. It is represented on Figure 4.21

Figure 4.21: Change in rear door

Slotted region is created on top corner of rear car, it can be shown on Figure 4.23.

Figure 4.22: Rear corner of the vehicle (Top view)

As mentioned above some changes are done rear side of the vehicle to close the slot and empty regions a box from the fiber-glass is adapted to car-body. This is especially important for disposing rain water from the car.

Finally, with selected gas spring, mechanism is mounted both on the door and the car body. Figure 4.24,25,26,27 show various points of view of mechanism including open and closed positions.

Figure 4.23: Back view of the vehicle in full open door position

Figure 4.24: Side view of the vehicle in closed door position

Figure 4.25: Front view of the vehicle zoomed

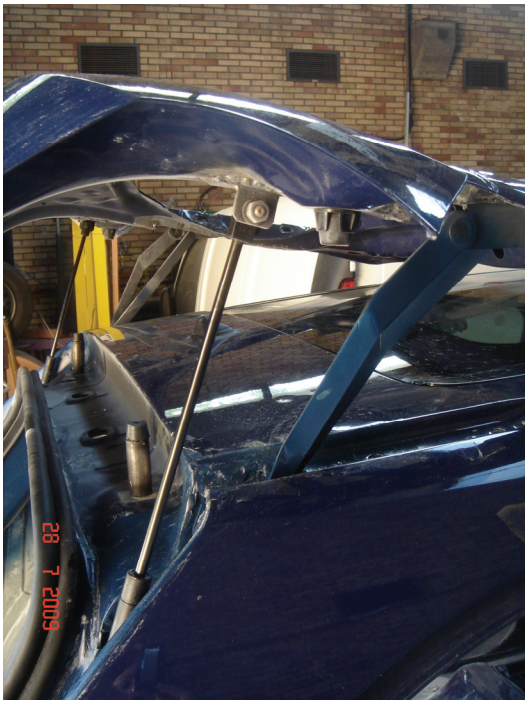


Figure 4.26: Side view of the vehicle zoomed

CHAPTER 5

DISCUSSION AND CONCLUSION

In this study, a novel concept is introduced for the opening mechanism for hatchback baggage doors. Definition of problem is based on the necessary space to open the baggage door. Aim of the study is achieved with decreasing required opening distance comparing to the traditional case. Simplicity and durability are taken into account during the linkage designs. Among the four and six link mechanisms, the four bar mechanism is selected for application since its cost and placement parameters are more convenient. With the help of the test vehicle, obtained design results are tested with a four bar mechanism and it is observed that there is almost 400 mm reduction on the required horizontal distance.

Results are satisfactory in terms of the main purpose. However, to obtain more powerful solutions, some changes on the car body are required. This means that design of such a mechanism can be handled together with the body design so that maximal space in the baggage can be acquired. Especially, placement of the gas spring can be reconsidered according to different car-body profiles, since its maximum force value is also affected with mounting positions.

There is a small box cap mechanism to protect car body when mechanism is in the folded position, with that subsystem the space on the top corner of the body is covered and entrance of any rain water in the car is prevented.

Also a six link mechanism may be considered concerning variations on the body; though the space occupied by the mechanism members is expected to be larger in

this case. The six link mechanism is seemingly more complex and more costly; however for some body profiles acceptable solutions can be obtained.

In motion of the mechanism dynamic characteristics are very hard to match with common human habits. It is different from the usual opening operation with the top hinged mechanism. This is especially due to the kinematic properties of the desired motion; however this effect can be reduced with selection of an assistant actuator. In the case study design presented in Chapter 4 the gas-spring is mounted on the door itself because of location constraints, however connection of crank or rocker link improves the actuator effect and the user is active in very small time.

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APPENDIX A

SYNTHESIS THEORY

A.1 Multiply Separated Position Synthesis

According to the synthesis method, mechanism passes exactly from some given precision positions, however the motion between these precision points is not guaranteed. Both finite and infinitesimal displacements can be considered. Tesar [11] introduced a notation such that finitely separated positions are shown with a hyphen placed between two positions, such as P-P, and infinitesimally separated positions are shown with nothing in between the positions, i.e. PP.

A mechanism can be considered as a combination of vector pairs called *dyads,* each of all realize the motion independently through the prescribed motion. To obtain the mechanism, the dyads should be combined after the synthesis. In a four-bar mechanism, two dyads must be synthesized independently. In Figure A.1 the dyads for a four-bar mechanism are shown.

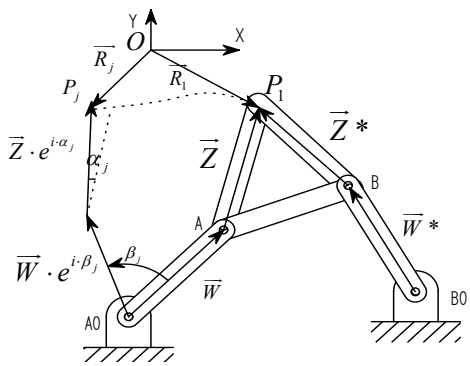

Figure A.1: Dyads shown on the four bar

$\vec{Z}$ and $\vec{W}$ pair is the first dyad of the four-bar mechanism and the other dyad is composed of $\vec{Z}*$ and $\vec{W}*$. To synthesize the first dyad, let P be a point on the coupler link and its motion from initial point P_1 to the jth position of point P may be defined with $\overrightarrow{OP_1} = \vec{R_1}$ and $\overrightarrow{OP_j} = \vec{R_j}$ vectors. All rotation angles are measured assuming counter clockwise direction as positive with respect to the fixed frame. β_j angle is rotation of $\vec{W}$ vector; similarly α_j angle represents rotation of $\vec{Z}$ vector.

Having defined the finitely separated motion specifying α_1, α_j, $\vec{R_1}$, $\vec{R_j}$, with the help of $A_0AP_1OP_jA_jA_0$ vector polygon, all vectors, which are $\vec{Z}$, $\vec{W}$, $\vec{R_1}$, $\vec{R_j}$, $\vec{Z} \cdot e^{i \cdot \alpha_j}$ and $\vec{W} \cdot e^{i \cdot \beta_j}$ are summed up and the loop closure equation can be stated as:

$$\vec{W} \cdot e^{i \cdot \beta_j} + \vec{Z} \cdot e^{i \cdot \alpha_j} - \vec{R}_j + \vec{R_1} - \vec{Z} - \vec{W} = \vec{0}$$

$$\vec{\delta}_j = \vec{R}_j - \vec{R}_1$$

Substituting $\vec{\delta_j}$ expression into the first equation:

$$\vec{W} \cdot (e^{i \cdot \beta_j} - 1) + \vec{Z} \cdot (e^{i \cdot \alpha_j} - 1) = \vec{\delta}_j \qquad \text{where j = 1, 2,3…}$$

This equation is known as standard-form equation [1].

When standard-form equation is obtained, unknown vectors $\vec{Z}$ and $\vec{W}$ can be solved. Same procedure can be applied for the second dyad to synthesize $\vec{Z}*$ and $\vec{W}*$ vectors. If both vector pairs are synthesized, all pivot points for the four-bar mechanism are determined.

To be more physically meaningful, fixed pivot coordinate vector $\vec{R}$ and moving coordinate vector $-\vec{Z}$ may be used instead of $\vec{Z}$ and $\vec{W}$ vectors.

$$\vec{R} = -\vec{Z} - \vec{W}$$

Now standard-form equation becomes:

$$\vec{R}\cdot(1-e^{i\cdot\beta_j})+\vec{Z}\cdot(e^{i\cdot\alpha_j}-e^{i\cdot\beta_j})=\vec{\delta_j} \text{ where j = 1, 2, 3...}$$

To obtain infinitesimally separated positions derivative of standard equation should be taken with respect to the angular displacement. For motion generation synthesis derivatives can be taken with respect to rotation of coupler plane (α). The values of rotation displacements α and β are evaluated at finitely prescribed position at which infinitesimally prescribed position occurs. Then equation for infinitesimally prescribed position becomes

$$\vec{R}\cdot\frac{d(1-e^{i\cdot\beta_j})}{d\alpha}\bigg|_{\substack{\alpha=\alpha_j\\ \beta=\beta_j}}+\vec{Z}\cdot\frac{d(e^{i\cdot\alpha_j}-e^{i\cdot\beta_j})}{d\alpha}\bigg|_{\substack{\alpha=\alpha_j\\ \beta=\beta_j}}=\frac{d\vec{\delta_j}}{d\alpha}\bigg|_{\substack{\alpha=\alpha_j\\ \beta=\beta_j}} \text{ where j=1,2,3...}$$

Tesar introduced a notation which combines finitely and infinitesimally separated equations

$$\vec{R}\cdot\frac{d^k(1-e^{i\cdot\beta_j})}{d\alpha^k}\bigg|_{\substack{\alpha=\alpha_j\\ \beta=\beta_j}}+\vec{Z}\cdot\frac{d^k(e^{i\cdot\alpha_j}-e^{i\cdot\beta_j})}{d\alpha^k}\bigg|_{\substack{\alpha=\alpha_j\\ \beta=\beta_j}}=\frac{d^k\vec{\delta_j}}{d\alpha^k}\bigg|_{\substack{\alpha=\alpha_j\\ \beta=\beta_j}}$$

Equation can be written as

$$\vec{R}\cdot(\sigma_l-b_l)+\vec{Z}\cdot(a_l-b_l)=\vec{\delta_l}$$

where $\sigma_l=1$ for $k=0$ $\sigma_l=0$ for $k\neq 0$, $a_l=\frac{d^k}{d\alpha^k}e^{i\cdot\alpha}\Big|_{\alpha=\sigma_j\alpha_j}$,

$$b_l=\frac{d^k}{d\alpha^k}e^{i\cdot\beta}\Big|_{\beta=\sigma_j\beta_j},\ \vec{\delta_l}=\frac{d^k\vec{\delta_j}}{d\alpha^k}\Big|_{\vec{\delta_j}=\sigma_l\vec{\delta_j}}$$

According to number of prescribed positions (finitely or infinitesimally) different solutions can be obtained. Maximum number of solutions is given in Table A.1.

There are three different tasks of synthesis and in each task the necessary specified parameters and unknowns may change. However, number of specified parameters and unknowns is independent of the type of the synthesis.

Table A.1: In the standard-form equation, maximum number of solutions for different prescribed positions

Number of Positions	Number of Scalar Equations	Number of Scalar Unknowns	Number of free choices	Number of solutions
2	2	5	5-2=3	∞^3
3	4	6	6-4=2	∞^2
4	6	7	7-6=1	∞^1
5	8	8	8-8=0	∞^0 (Finite)

Motion generation concerns both translational and rotational displacement of a body. Inputs for this problem are $\vec{\delta}$ and α for finitely separated positions. Also $\frac{d^k \vec{\delta_j}}{d\alpha^k}$ may be considered for infinitesimally separated positions. In this study for different cases three and four finitely separated position synthesis are investigated. The parameters to solve the equation are shown in
Table A.2

Table A.2: Parameter classification for three and four prescribed positions

Number of Prescribed Position	Input Parameters	Free Parameter Choice	Parameters to be Solved for
Three P-P-P	$\alpha_2, \vec{\delta_2}, \alpha_3, \vec{\delta_3}$	(β_2, β_3) or $\vec{R}$ or $\vec{Z}$	$\beta_2, \beta_3, \vec{R}, \vec{Z}$
Four P-P-P-P	$\alpha_2, \vec{\delta_2}, \alpha_3, \vec{\delta_3}, \alpha_4, \vec{\delta_4}$	β_2 or β_3 or β_4	$\beta_2, \beta_3, \beta_4, \vec{R}, \vec{Z}$

A.1.1 Three Finitely Separated Positions for Motion Generation

In standard-form equations, there are four independent scalar equations and six scalar unknowns. In order to solve the system of equations, two inputs must be specified. Designer has an option to select these two free parameters according to the nature of the problem. The most suitable two scalar parameter among (β_2, β_3), $\vec{R}$, $\vec{Z}$ can be chosen:

- β_2, β_3 specification, that is rotation of crank or output link vectors
- Center point specification, that is ground pivot location at initial position ($\vec{R}$ vector)
- Circle point specification, that is moving pivot location at initial position ($-\vec{Z}$ vector)

In the first case, β_2, β_3 as a free choice yields two linear complex equations in two complex unknowns. Solutions for $\vec{R}$, $\vec{Z}$ vectors are:

$$\vec{R} = \frac{-\delta_2 \cdot (e^{i\alpha_3} - e^{i\beta_3}) - \delta_3 \cdot (e^{i\beta_2} - e^{\alpha_2})}{(e^{i\beta_2} - 1) \cdot (e^{i\alpha_3} - 1) - (e^{i\beta_3} - 1) \cdot (e^{i\alpha_2} - 1)}$$

$$\vec{Z} = \frac{\delta_3 \cdot (e^{i\beta_2} - 1) - \delta_2 \cdot (e^{i\beta_3} - 1)}{(e^{i\beta_2} - 1) \cdot (e^{i\alpha_3} - 1) - (e^{i\beta_3} - 1) \cdot (e^{i\alpha_2} - 1)}$$

To obtain circle-point and center-point circles, one of the angular displacements can be considered as a variable parameter ranging from 0 to 2π. Then both curves can be generated analytically.

In the second case $\vec{R}$ vector as a free choice yields three linear complex equations in two complex unknowns:

$$\vec{W} + \vec{Z} = -\vec{R}$$
$$\vec{W} \cdot e^{i \cdot \beta_2} + \vec{Z} \cdot e^{i \cdot \alpha_2} = \vec{\delta_2} - \vec{R}$$
$$\vec{W} \cdot e^{i \cdot \beta_3} + \vec{Z} \cdot e^{i \cdot \alpha_3} = \vec{\delta_3} - \vec{R}$$

Now unknown vectors are $\vec{W}, \vec{Z}$. There is a solution if the determinant of augmented matrix of this system is zero.

$$\left| \begin{pmatrix} 1 & 1 & -\vec{R} \\ e^{i \cdot \beta_2} & e^{i \cdot \alpha_2} & \vec{\delta_2} - \vec{R} \\ e^{i \cdot \beta_3} & e^{i \cdot \alpha_3} & \vec{\delta_3} - \vec{R} \end{pmatrix} \right| = 0$$

$$\vec{D_1} + \vec{D_2} \cdot e^{i \cdot \beta_2} + \vec{D_3} \cdot e^{i \cdot \beta_3} = 0$$

where

$$\vec{D_1} = (\vec{\delta_3} - \vec{R}) \cdot e^{i \cdot \alpha_2} - (\vec{\delta_3} - \vec{R}) \cdot e^{i \cdot \alpha_3}$$
$$\vec{D_2} = -\vec{R} \cdot e^{i \cdot \alpha_3} - (\vec{\delta_3} - \vec{R})$$
$$\vec{D_3} = (\vec{\delta_2} - \vec{R}) + \vec{R} \cdot e^{i \cdot \alpha_2}$$

Since all $\vec{D}$ vector values are known β_2 and β_3 values can be evaluated. There are two sets of solutions, one being trivial: $\beta_2 = \alpha_2, \beta_3 = \alpha_3$.

In third case, i.e. circle point specification, the solution procedure is similar to the center point specification. This time $\vec{Z}$ vector is known then $\vec{R},\vec{W}$ vectors are evaluated.

A.1.2 Four Finitely Separated Position for Motion Generation

There are 6 scalar equations in 7 scalar unknowns. To solve the equation one of the β values must be specified.

$$\begin{pmatrix} e^{i\cdot\beta_2}-1 & e^{i\cdot\alpha_2}-1 \\ e^{i\cdot\beta_3}-1 & e^{i\cdot\alpha_3}-1 \\ e^{i\cdot\beta_4}-1 & e^{i\cdot\alpha_4}-1 \end{pmatrix} \cdot \begin{pmatrix} \vec{W} \\ \vec{Z} \end{pmatrix} = \begin{pmatrix} \vec{\delta_2} \\ \vec{\delta_3} \\ \vec{\delta_4} \end{pmatrix}$$

Since there are 2 complex unknowns for three complex equations, solution exists if and only if the rank of the augmented matrix is 2. That is, determinant of the augmented matrix must be zero. The determinant is:

$$\Delta_1 + \Delta_2 \cdot e^{i\cdot\beta_2} + \Delta_3 \cdot e^{i\cdot\beta_3} + \Delta_4 \cdot e^{i\cdot\beta_4} = 0$$

where

$$\Delta_1 = -\Delta_2 - \Delta_3 - \Delta_4$$

$$\Delta_2 = \left| \begin{pmatrix} e^{i\alpha_3}-1 & \vec{\delta_3} \\ e^{i\alpha_4}-1 & \vec{\delta_4} \end{pmatrix} \right|$$

$$\Delta_3 = -\left| \begin{pmatrix} e^{i\alpha_2}-1 & \vec{\delta_2} \\ e^{i\alpha_4}-1 & \vec{\delta_4} \end{pmatrix} \right|$$

$$\Delta_4 = \left| \begin{pmatrix} e^{i\alpha_2}-1 & \vec{\delta_2} \\ e^{i\alpha_3}-1 & \vec{\delta_3} \end{pmatrix} \right|$$

This equation is called the compatibility equation. Since the compatibility equation is transcendental, an implicit solution procedure is available.

Let $\Delta = \Delta_1 + \Delta_2 \cdot e^{i \cdot \beta_2}$ (.1)

$$\beta_{4_1} = \arg(t) \quad \beta_{4_2} = \arg(u) \quad (.2)$$

$$\beta_{3_1} = \arg(\frac{-\Delta - \Delta_4 \cdot e^{i \cdot \beta_{4_1}}}{\Delta_3}), \beta_{3_2} = \arg(\frac{-\Delta - \Delta_4 \cdot e^{i \cdot \beta_{4_2}}}{\Delta_3})$$

where $t = \dfrac{-Q_1 + \sqrt{Q_1^2 - 4 \cdot Q_2 \cdot Q_3}}{2 \cdot Q_2}, u = \dfrac{-Q_1 - \sqrt{Q_1^2 - 4 \cdot Q_2 \cdot Q_3}}{2 \cdot Q_2}$

$$Q_1 = \Delta \cdot \bar{\Delta} + \Delta_4 \cdot \bar{\Delta}_4 - \Delta_3 \cdot \bar{\Delta}_3$$

$$Q_2 = \Delta_4 \cdot \bar{\Delta}$$

$$Q_3 = \Delta \cdot \bar{\Delta}_4$$

For a given β_2 value, two sets of β_3 and β_4 values are obtained. In both set, the solutions are in one to one correspondence.

After determining each β value, equations can be solved with linear solution method for $\vec{W}$ and $\vec{Z}$ unknown vectors.

Let $\vec{k} = \vec{R} - \vec{Z}$ and $\vec{m} = \vec{k} - \vec{W}$ for circle and center point vectors. If a β_2 value is varied in steps from 0 to 2π, each value of β_2 yields two sets of points called Burmester point pairs. And these point pairs compose circle and center point curves. Also, these two different solution sets correspond to two different branches of the mechanism.