AN ANALYTICAL AND EXPERIMENTAL STUDY ON PILED RAFT FOUNDATIONS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

BEREN YILMAZ

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
CIVIL ENGINEERING

FEBRUARY 2010

Approval of the thesis:

AN ANALYTICAL AND EXPERIMENTAL STUDY ON PILED RAFT FOUNDATIONS

submitted by **BEREN YILMAZ** in partial fulfillment of the requirements for the degree of **Master of Science in Civil Engineering Department, Middle East Technical University** by,

Prof. Dr. Canan ÖZGEN __________________________
Dean, Graduate School of **Natural and Applied Sciences**

Prof. Dr. Güney ÖZCEBE __________________________
Head of Department, **Civil Engineering**

Prof. Dr. M. Ufuk ERGUN
Supervisor, **Civil Engineering Dept., METU** __________________________

Examining Committee Members:

Prof. Dr. Orhan EROL __________________________
Civil Engineering Dept., METU

Prof. Dr. M. Ufuk ERGUN __________________________
Civil Engineering Dept., METU

Prof. Dr. Erdal ÇOKÇA __________________________
Civil Engineering Dept., METU

Asst. Prof. Dr. Nejan HUVAJ SARIHAN __________________________
Civil Engineering Dept., METU

Yük. Müh. A. Mengüç ÜNVER __________________________
Argem Geoteknik Müh. Müş. Ltd. Şti.

Date: 03.02.2010

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name : Beren YILMAZ

Signature :

ABSTRACT

AN ANALYTICAL AND EXPERIMENTAL STUDY ON PILED RAFT FOUNDATIONS

YILMAZ, Beren

M.S., Department of Civil Engineering

Superviser: Prof. Dr. Mehmet Ufuk ERGUN

February 2010, 119 pages

Two different concepts and design procedures namely settlement reducing piles and piled raft foundations have been studied independently in this thesis.

A laboratory study is conducted on model rafts with differing number of model settlement reducing piles. Pile length, pile diameter, type of soil and size of raft are kept constant and settlements are measured under sustained loading. Remolded kaolin is consolidated under controlled stresses before tests are performed in model boxes. The tests are conducted under two sustained loadings of 75 kPa and 40 kPa. 0(raft), 16 and 49 number of piles are used. During the tests, all of the skin friction is mobilized. Several tests are conducted for each combination to see the variability. It is concluded that increasing the pile number beyond an optimum value is inefficient

as far as the amount of settlement is considered. Also an analytical procedure has been followed to calculate settlements with increasing number of piles.

In the second part of this thesis, finite element analyse have been performed on a piled raft foundation model, using Plaxis 3D Foundation Engineering software. This analyse are supported with analytical methods. The piled raft model is loaded with 450 kPa raft pressure. The studies are conducted in two sets in which different pile lengths are used; 25 m and 30 m respectively. The numbers of piles are increased from 63 to 143. All other parameters are kept constant. The results showed that again an optimum number of piles will be sufficient to reduce the settlement to the acceptable level. The analytical methods indicate a similar behavior. The comparison and results are presented in the study.

Keywords: piled raft systems, settlement reducing piles, model tests, settlement of pile groups

ÖZ

KAZIK-RADYE SİSTEMLERİN ANALİTİK VE DENEYSEL OLARAK İNCELENMESİ

YILMAZ, Beren

Yüksek Lisans, İnşaat Mühendisliği Bölümü

Tez Yöneticisi: Prof. Dr. Mehmet Ufuk ERGUN

Şubat 2010, 119 sayfa

Bu tez kapsamında, oturma azaltıcı kazıklar ve kazık-radye sistemleri olmak üzere, iki farklı konsept ve tasarım sistemi üzerinde, birbirlerinden bağımsız olarak çalışılmıştır.

Farklı sayıda oturma azaltıcı model kazık içeren modeller üzerinde laboratuvar deneyleri yapılmıştır. Kazık boyu, çapı, zemin tipi ve radye boyutları sabit bırakılmış ve oturmalar sürekli yükleme altında ölçülmüştür. Testlerin yapılmasından önce, yoğurulmuş kaolin tipi kil, model kutularda, kontrollü basınç altında konsolide edilmiştir. Deneylerde, 75 kPa ve 40 kPa olmak üzere, iki değerde sürekli yükleme yapılmıştır. Kazık sayıları 0, 16 ve 49 olarak kullanılmıştır. Deneyler sırasında, kazığın bütün yüzey sürtünmesi mobilize olmuştur. Çeşitliliği görmek adına, her kombinasyon için birkaç deney yapılmıştır. Sonuç olarak görülmüştür ki, kazık

sayısını optimum seviyeden yukarı çıkarmak, oturma açısından kullanılabilir bir etki yaratmamaktadır. Bununla birlikte, kazık sayısı arttıkça oturmayı hesaplamak için, bilinen ve kabul edilen bir analitik metod kullanılmıştır.

Tezin ikinci bölümünde, kazık-radye sistemi modeline, sonlu eleman çözümlemesi yapılmıştır. Bu amaçla Plaxis 3D Foundation Engineering programı kullanılmıştır. Bu çözümleme, analitik hesaplarla da desteklenmiştir. Kullanılan model 450 kPa ile yüklenmiştir. Çalışmalar, 25 m ve 30 m olmak üzere iki farklı kazık boyu dikkate alınarak, iki set halinde yapılmıştır. Çalışmalar sırasında, kazık sayıları 63' den 143' e kadar arttırılmıştır. Geri kalan tüm parametreler sabit tutulmuştur. Sonuçlar, benzer olarak, optimum sayıda kazık kullanmanın oturmayı kabul edilebilir seviyeye indirmede yeterli olacağını göstermektedir. Karşılaştırmalar ve sonuçların detaylı incelemesi tez içerisinde sunulmuştur.

Anahtar Kelimeler: kazık-radye sistemler, oturma azaltıcı kazıklar, model test, kazık gruplarında oturma

To my beautiful family

ACKNOWLEDGMENTS

I would like to express my gratitudes to my supervisor Prof. Dr. M. Ufuk Ergun for his vision, guidance, encouragement and support throughout my studies.

I would also like to thank Mr. Ali Bal and Mr. Ulaş Nacar for their technical support during the laboratory studies.

I would like to give my sincere thanks to Yeşim Ünsever and Abdullah Sandıkkaya for their support, hospitality and friendship. Also I would like to thank Volkan Kaltakçı for his comments and friendship throughout the study.

I am very grateful to my friends Zeynep Çekinmez, Sülen Kitapçıgil and Sibel Kerpiççi Kara for their support, understanding but most importantly beautiful friendship since I met them.

Also I wish special thanks to my friends Ceren Atalı and Berrin Efendioğlu for their precious friendship, encouragement and understanding throughout my life.

I would like to give my love to my beautiful, extended family for just being who they are. Finally, I wish to show my grateful love to my mom, dad and sister for believing in me no matter what.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

One of the most important aspects of a civil engineering project is the foundation system. Designing the foundation system carefully and properly, will surely lead to a safe, efficient and economic project overall. In other words, foundation system design is one of the most critical and important step when a civil engineering project is considered. Until quite recently, there were some separately used systems like shallow foundations such as rafts and deep foundations such as piles. However, lately the foundation engineers tend to combine these two separate systems. By combining these two systems, the foundation engineer will provide the necessary values for the design, obtain the required safety and also come out with a more economical solution.

The conventional pile design philosophy is based on that piles carry all the load and they are accepted as a group, no contribution is made by the raft to the ultimate load capacity. The new trend in the foundation engineering is combining raft foundations and pile foundations. The combined system can be based on different design philosophies which can be classified as follows:

1) Settlement reducing pile concept: In this philosophy, piles are only located to reduce the total settlement and they are designed to work at limiting equilibrium, in other words, for the piles, factor of safety values against bearing capacity is taken as unity.

2) Piled raft concept: This philosophy is one of the newly adopted concepts in which a significant portion of total load is carried by the raft contrarily to the conventional design. Piles are designed to work at 70-80% of the ultimate load capacity.

3) Differential settlement control: Placing piles under the raft strategically and of course in a limited number will enhance the ultimate load capacity of the foundation and decrease both the settlement and the differential settlement.

In this thesis, emphasize will be given to the first and second design philosophies presented above; namely the settlement reducing piles and the piled raft foundations. In the scope of this research, settlement analysis methods and the settlement behavior have been rewieved. Studies have been supported with known and applicable methods searched in the literature. The literature study covered many methods.

In the first part of the thesis, in order to investigate the behavior of settlement reducing piles, experimental studies have been conducted. Additionaly, an analytical procedure has been followed. The scope of the exterimental study was to observe the settlement behavior and investigate the effect of the number of piles inserted under the raft to the settlement of the system. The experiments have been carried out with simple models consisting of different number of model piles and a model raft. The soil beneath was medium clay. Conducting this experiment showed the settlement behavior and gave an idea about the real behavior.

In order to support the experimental studies, different analytical hand calculations were searched and described. Though, there are not many methods in the literature that enhance the main idea beneath this system, the search focused mainly on the methods which accept separate stiffnesses for the system, raft and piles. Factor of safety for the piles against bearing capacity is generally taken as unity. This idea of mobilizing full capacity of the piles is not common for most of the practices. However, some methods that follow these criteria are present. One of the methods was further analyzed and used for determination of the settlement behavior. The outcomes of the both experimental and analytical studies confirm each other in many aspects and support the idea that lies behind this system.

In the second part of the study, another new design concept of piled raft foundation was further investigated. In the foundation systems, piles are generally introduced to reduce settlement. However, while designing the raft pile systems, a detailed settlement analysis is rarely done. Usually in the conventional design, the contribution of the raft to the load carried by the system is ignored and only the necessary factor of safety value for the settlement is taken into consideration without detailed analysis. Since, designing a pile group should be focused on satisfying the settlement criteria, the main issue in a safe, efficient and economical design of piled rafts is determination of optimum number of piles for an acceptable settlement.

In order to study this concept, finite element analyses have been carried out for observing the settlement behavior of piled raft foundations. A common model was used; this model consisted of two sets in which pile lengths were different. For this purpose, Plaxis 3D Foundation software has been used. Also some analytical methods that have been introduced in the literature were applied to the aforementioned model. Once again, the results were compared and some interpretations were made.

In this thesis, emphasis is given to the effects of the number of piles to the settlement behavior of two newly adopted systems. The load distribution between piles and raft is very important and since piles are introduced to reduce settlements, detailed settlement analysis should be done. Thus, for every design a tolerable settlement value should be decided and design should be done accordingly. Considering these concepts, the settlement behavior has been investigated. It was realized that, when considering the settlement behavior, adding piles under the raft beyond an optimum number will not have any effect on settlement reduction. In other words, beyond this number, adding piles no longer reduces the settlements drastically.

To sum up, this thesis is aimed to clarify the effect of number of piles to the settlement behavior for newly adopted systems, the piled raft foundations and settlement reducing piles. Also the settlement analysis methods and design methods were further investigated. Finite element analysis was used to see the results obtained for complete analysis in case of piled raft foundation systems. Moreover, an experimental model study was conducted to observe the real behavior of settlement reducing piles as good as possible.

The detailed literature study and the methods used are presented in Chapter 2. Details of the experimental and analytical studies on settlement reducing piles are given in Chapter 3. The analytical and finite element analyses on piled raft foundation are studied in Chapter 4. Finally, the comparisons and results are presented in Chapters 5 and 6, respectively.

CHAPTER 2

LITERATURE REVIEW OF SETTLEMENT REDUCING PILES AND PILED RAFT FOUNDATIONS

2.1 Settlement Reducing Piles

In traditionally designed systems, due to limitations given in the regulations, generally piles are designed to carry the structural loads with high factor of safety values like 2 or 3. However, the recent designs accept piles as they are asissting to the system in terms of satisfying the criteria for total or differential settlement, thus they can be working in their full capacities whereas the system overall still possess a factor of safety of 2 or 3 (O'Neill, 2005). Such systems where piles are working at 80 or 90 percent or in some cases 100 percent of their ultimate capacity, i.e.accepting factor of safety of piles as unity, can be referred as the settlement reducing piles.

2.1.1 General Concepts About Settlement Reducing Piles

In more recent times, for solving the settlement problems, engineers started to use new systems based on the concept of settlement reducing piles. Thought this concept had been introduced many years ago, because of the strict limitations in the codes and the conventionality in the design, it has not been used throughout the history. However, with some new codes, this concept came into the stage where the adequate bearing capacity is provided by the raft and piles are used without safety factors just to eliminate the settlement problem (de Sanctis and Russo, 2008).

The idea behind applying no safety factors, i.e. taking factor of safety as unity, is that accepting that piles are working at their ultimate shaft frictions and there are very little or no end bearing capacity (Love, 2003). At the same time, this situation leads to accepting the fact that the behaviour of piled raft foundation systems highly influenced by the behaviour of the raft and that the stiffness of the raft should be considered (Castelli and Di Mauro, 2003).

Chosing an appropriate factor of safety is dependent upon to the tolerances for the acceptable settlement. Thus, the settlement characteristics of the project will determine the appropriate factors of safety. When large settlements can be tolerated, very small factors of safety (even unity) can be used. When large pile groups are considered, design should primarily satisfy the settlement considerations (Fleming et al., 1992).

Despite the above described facts, generally in conventional design procedures, the settlement used in the design falls far below the acceptable limits, thus ends up placing more piles than needed. It is more important than reducing settlements to a limit than diminish the deformations if an economic and safe design is desired (Horikoshi and Randolph, 1996).

The fact that today many design procedures are based on the capacity, can be a result of the belief that predicting the settlement behaviour is more difficult and less reliable. However, this is not true for pile foundations with recent techniques and bolder decisions. Moreover, unlike the capacity of piles, the settlement behaviour depends on the soil characteristics and the installation process become less effective (Randolph, 2003). Thus, an efficient settlement analysis leads to a safe and economical design.

2.1.2 Design of Settlement Reducing Piles

While designing the settlement reducing piles, some concepts should be decided paying special attention (Poulos, 2002). These issues are listed by Poulos as follows:

- Maximum settlement
- Differential settlement
- Ultimate load capacities for vertical, lateral and moment loadings

- Pile loads and moments
- Raft moments and shears

In the design process, for the settlement reducing piles special attention should be given to avoid the overdesign, since it can cause as much trouble as designing undercapacity (Love, 2003).

However, since piles are only there to reduce settlements, they do not contribute to the bearing capacity. Hence, the unpiled raft should be sufficient from the bearing capacity point of view and it should provide a factor of safety of at least 3 (De Sanctis et al.,2002).

Unfortunately, engineers have been designing the systems based on the principle that adequate number of piles should be placed in order to carry the structural weight. However, they should be thinking in terms of settlements and the placement should be done according to the acceptable settlement limits (De Sanctis et al.,2002).

While designing the settlement reducing piles, the load sharing between raft and piles should be taken into account since the piles are not placed to provide bearing capacity. However, the settlement behaviour of the raft directly changes when the piles are presented so, this load sharing becomes more important. This load sharing is based on the stiffnesses of the soil and raft and the pile settlements (Love, 2003).

The load sharing behaviour of the system highly depends on the working conditions of the piles, i.e. the factor of safety values applied to the piles. There are some detailed analysis of case histories in the literature which give reasonable explanations about the relation between load sharing and the load carrying performances of the piles and rafts. In the paper presented by O'Neill in 2005, some case histories were investigated. It has been seen that when piles are loaded to 50 percent of their ultimate capacities, the load carried by the raft drops to almost one-half of the total load whereas when they are loaded to 80 percent or above of their ultimate capacities, the piles carry a lower proportion of the load (O'Neill, 2005).

In order to optimize the design process of settlement reducing piles, some parameters need to be decided like the mandatory pile number, the deficient load capacity of each pile and the allowable settlement. At the mean time, the load sharing behaviour and the relative settlement relationship should be analysed in detail (Fioravante et al., 2008).

2.1.3 The Analytical Methods For Settlement Analysis of Settlement Reducing Piles

The analytical methods for analysing the settlement reducing piles should be based on the fact that no safety factors will be applied to the capacity of piles. Thus, they will be used just for the settlement reducing and they will have very little or no contribution to the bearing capacity. Accordingly, some of the conventional methods will not be suitable for analysing the settlement reducing piles. In case of such foundation systems, methods which consider the load sharing and provide opportunity to mobilize full capacities of piles, should be used. The methods which have been used and compared with the case histories are presented below.

2.1.3.1 Fleming et al., 1992

This method combines the stiffnesses of raft and foundation and introduces the stiffness of the piled raft foundation system. It is based on the principles and formulai presented by Fleming et al., 1992 (after Randolph, 1983). Settlement of the system is divided into two components as settlement due to load carried by the raft and settlement due to the load carried by the piles.

- Settlement due to the load carried by the raft,

$$sett = \frac{net\ load}{k_f} \qquad \textit{(1)}$$

$$net\ load = (q \times B \times B) - (n \times shaft\ cap \times perc\ of\ mobilization) \qquad \textit{(2)}$$

$$k_f = \frac{k_p + k_c(1 - 2\alpha_{cp})}{1 - \alpha_{cp}^2 k_c / k_p} \qquad \textbf{\textit{(3)}}$$

$$\alpha_{cp} = \frac{\ln\left(r_m/r_c\right)}{\ln\left(r_m/r_o\right)} \qquad \textbf{\textit{(4)}}$$

Where; r_o = pile radius.

$$r_c = \sqrt{\frac{B^2}{n\pi}} \qquad \textbf{\textit{(5)}}$$

$$r_m = 2.5L\rho(1 - \upsilon) \qquad \textbf{\textit{(6)}}$$

Where; B = width of the foundation,

n = number of piles in the group,

L = length of the piles,

ν = poissons ratio of the soil and

ρ = soil inhomogenity factor (ratio of $G_{L/2}$ to G_L)

$G_{L/2}$ and G_L are shear modulus at half length and full length of the pile respectively.

$$k_p = \frac{2\pi L G_L}{\ln\left(\frac{r_m}{r_o}\right)} \qquad \textbf{\textit{(7)}}$$

$$k_c = \frac{2G_L}{I(1 - \upsilon)}\sqrt{(B \times B)} \qquad \textbf{\textit{(8)}}$$

Where; I = influence factor for the raft.

- Settlement due to the load carried by the piles,

$$sett = (overall\ elastic\ sett) \times \alpha_{cp} \qquad \textbf{\textit{(9)}}$$

$$overall\ elastic\ sett = R_s \times (rel.slip\ btw.upper\ part\ of\ pile\ and\ soil) \qquad \textbf{\textit{(10)}}$$

Where; R_s = settlement ratio.

- Thus, the total settlement will be sum of the two settlements calculated above.

2.1.3.2 Clancy and Randolph, 1993

This method considers the interaction between pile and raft. Moreover, like the other methods, the overall stiffness of the piled raft system is considered. Thus, settlement is calculated as follows.

$$w_{pr} = \frac{total\ load}{k_{pr}} \tag{11}$$

$$k_{pr} = \frac{\left[k_p + k_r\left(1 - 2\alpha_{rp}\right)\right]}{\left[1 - \left({k_r}/{k_p}\right)\alpha_{rp}^2\right]} \tag{12}$$

$$\alpha_{rp} = 1 - \frac{\ln(n)}{\ln\left({2r_m}/{d}\right)} \tag{13}$$

$$\alpha_{pr} = \alpha_{rp}\frac{k_r}{k_p} \tag{14}$$

Where; n = number of piles and
d = diameter of pile.

$$k_p = \frac{2\pi L G_L}{\ln\left(\frac{r_m}{r_o}\right)} \tag{15}$$

$$k_r = \frac{2G_L}{I(1-\upsilon)}\sqrt{(B \times B)} \tag{16}$$

Where; I = influence factor for the raft.

$$r_m = 2.5L\rho(1-\upsilon) \tag{17}$$

Where; L = length of the piles,

ν = poissons ratio of the soil and

ρ = ratio of $G_{L/2}$ to G_L

Furthermore, the values of α_{rp} and α_{pr} can be calculated again in order to check with previously calculated values.

$$\alpha_{rp} = \frac{k_p}{P_p}\left(w_{pr} - \frac{P_r}{k_r}\right) \tag{18}$$

$$\alpha_{pr} = \frac{k_r}{P_r}\left(w_{pr} - \frac{P_p}{k_p}\right) \tag{19}$$

Where; P_r = load carried by the raft and

P_p = load carried by the piles.

2.1.3.3 De Sanctis et al., 2002

In the method presented, the system is classified in two groups as small and large rafts. The small piled rafts are those systems in which the raft alone is not adequate to fullfill the bearing capacity requirements. Also, the width of the raft is usually between 5 to 15 meters and is small compared to the length of the piles. On the other hand, the large piled raft foundations are those in which the piles are used only as settlement reducers and in general in such systems, the width of the raft is larger compared to the length of the piles.

Some curves have been presented in order to calculate the average settlement reduction which is the ratio of settlement of the raft with piles to the settlement of the unpiled raft. From these curves, it can be realized that the ratio is dependent upon some factors. These factors are described herein.

- The relative structural stiffness of the raft; K_{rs},

$$K_{rs} = \frac{4}{3}\frac{E_r\left(1-\upsilon_s^2\right)^2}{E_s\left(1-\upsilon_r^2\right)^2}\left(\frac{t}{B}\right)^2 \tag{20}$$

Where; t = thickness of the raft,

B = width of the raft,

ν_s = poissons ratio of the soil,

ν_r = poissons ratio of the raft,

E_r = modulus of elasticity of the raft and

E_s = modulus of elasticity of the soil.

- The ratio of the pile group area to the raft area; A_g / A,

$$A_g = \left[\left(\sqrt{n} - 1\right)s\right]^2 \tag{21}$$

$$A = B^2 \tag{22}$$

Where; n = number of piles in the group and

s = spacing of the piles.

- The average settlement of the unpiled raft; (De Sanctis et al., 2002, after Fraser and Wardle, 1976),

$$w_r = \frac{qB}{E_s} I_w \tag{23}$$

Where; q = load applied per metersquare and

I_w = influence coefficient.

- The ratio of the length of the piles to the width of the raft; L/B,

These parameters will be used to estimate the settlement reduction factor. For this aim, some curves had been formed by De Sanctis et al.(2002). These curves for the small and large piled rafts are given below in Figures 2.1 and 2.2 respectively.

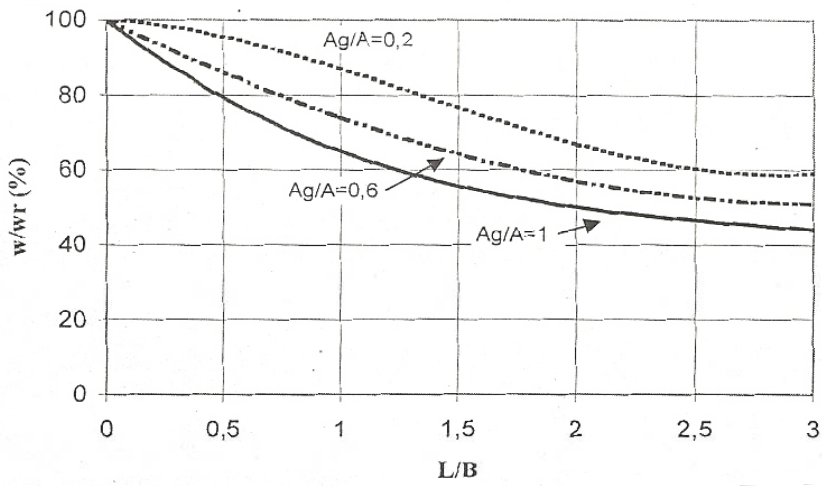


Figure 2.1 Average settlement reduction for small piled rafts(De Sanctis et al. , 2002)

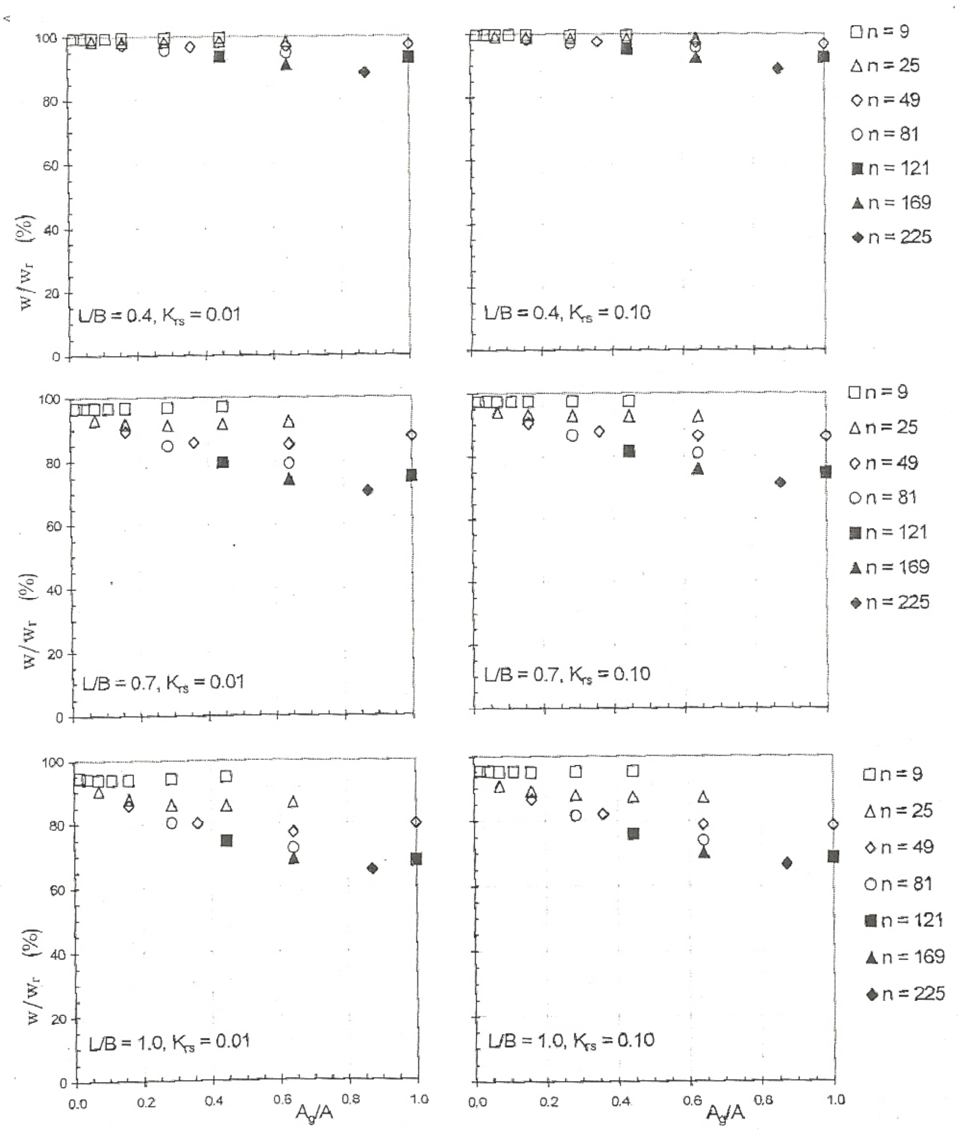


Figure 2.2 Average settlement reduction for large piled rafts (De Sanctis et al. , 2002)

2.1.4 Advantages of Settlement Reducing Piles

The concept of settlement reducing piles has some very useful benefits. For instance, since the piles are introduced as settlement reducers, the required raft thickness, for an acceptable vertical and differential settlement, is thinner. Moreover, this concept relies on placing piles at necessary locations and in necessary numbers. These two points results in a more economic design. (Love, 2003) However, this concept also provides the required safety and the desired behaviour. In other words, this system will give the opportunity to provide the most economic solution with satisfying the necessary requirements for the desired behaviour (De Sanctis et al.,2002 after Viggiani, 2000).

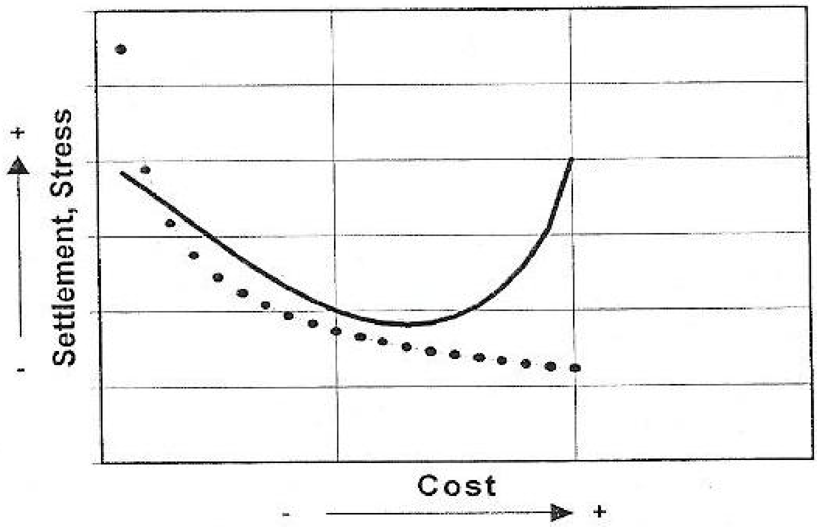


Figure 2.3 Typical pattern for performance vs. cost (De Sanctis et al. , 2002)

In Figure 2.3, it can be seen that for some situations when more money is being spent, settlement decreases continuously while for other situations no matter how much money is being spent, after a limit no benefit can be supplied. Moreover, it should not be forgotten that there are acceptable limits for the settlement; thus it is not necessary to diminish the settlement.

Another advantage of placing settlement reducing piles under the raft is from the differential settlement point of view. This concept certainly minimizes the differential settlement further in some situations, differential settlement diminishes

(De Sanctis et al.,2002). It should be noted that, while controlling the differential settlement the location of piles becomes also important.

Furthermore, since this concept involves thinner rafts and less pile, the construction time will be reduced and less workmanship will be needed.

On the other hand, it should be noted that the settlement reducing piles can also be used to minimize the differential settlement and total settlement for flexible rafts as well as the rigid rafts (Fioravante et al., 2008).

So, it can surely be concluded that, though it is a new concept, settlement reducing piles have a large range of application fields and it is a very beneficial concept due to the fact that it combines economy, safety and applicability.

2.2 Piled Raft Foundations

In foundation engineering, generally the most popular types of foundations used for high rise buildings or special structures are raft foundations or pile foundations. These systems when implemented alone, will fullfill the design requirements; however, in most cases they become oversafe and economically not efficient. Further more, in some cases when being used alone they can cause some important problems. On the other hand, when the conditions are suitable, these systems can be combined and one can have a more efficient, safe and economical design. Thus, piled raft foundation system is one of those combined systems.

2.2.1 General Concepts About Piled Raft Foundations

As stated by Poulos (2001), the behaviour of a piled raft foundation is affected by some factors like; the number of piles, the nature of loading, raft thickness and applied load level. Some researches have been made on piled raft foundations giving special attention to these effects.

When the number of piles is considered, it can be seen that increasing the number of piles not always brings the best solution and best performance. Thus, with an optimum number, the system will be more efficient. Increasing number beyond an optimum number does not always generate a big difference (Poulos, 2001).

The design of a piled raft foundation has three main stages as preliminary stage, detailed examination phase and detailed design phase (Poulos, 2001).

In a preliminary stage, usually the effects of pile number on load carrying capacity and settlement is observed. In order to see these effects, the performance of a raft foundation without piles needs to be analyzed. Using this analysis, it can be known if the raft alone satisfies the ultimate load capacity or not. This stage helps us to decide on the design philosophy.

In the detailed examination phase, the pile locations and some requirements are decided. In order to locate the piles, the load distribution under the raft with no piles underneath should be known. Generally, detailed analysis is not done for the load distribution, but it is accepted as uniform over the raft area. However, for this step, a detailed analysis needs to be done and the maximum loads under columns should be found. Then it can be decided under which columns, a pile is needed. This is decided by considering the exceedence of maximum moment, maximum shear in the raft or the maximum contact pressure below the raft.

Finally, in the third stage, a detailed analysis and confirmation is done for the location and number of piles, i.e optimum number and locations are decided. There are several methods for analysing the pile raft systems as stated by Poulos, 2001. These methods can be classified as follows:

1) "strip on springs" approach: Raft is modeled as a series of strip footings and piles are modeled as springs.
2) "plate on springs" approach: Raft is modeled as plate and piles are modeled as springs.

3) Boundary element methods: Both raft and piles are discretised and elastic theory is used.
4) Combined methods: Uses boundary element analysis for piles and finite element analysis for the raft.
5) Simplified finite element analysis
6) 3-D finite element analysis

2.2.2 Design of Piled Raft Foundations Based on Settlement Analysis

In order to understand the design methods and settlement analysis, the settlement behaviour of pile groups should be analysed. There are some factors which affect the settlement behaviour of piles in a group. These factors which are explained by Poulos (1993) can be listed and described as follows:

1) Lateral non-homogenity of soil

This non-homogenity creates variation in soil stiffness and this variation is especially important for bored piles. While assessing the settlement of piles, the most important geotechnical parameter is the soil stiffness, i.e. the Young's modulus(E_s) or the shear modulus(G_s). It should be noted that there can be different values of Young's modulus encountered in the pile shaft, just below the pile tip, well below the pile tip and between the piles. There are some useful correlations which can be used to calculate the modulus. Usually in those emprical correlations, some standart in-situ tests are used, like the standart penetration test or the cone penetration test. However, it is important to realise that these relations only give an approximate value and should not be relied on completely. Some of these correlations are developed by several researchers such as Hirayama, Poulos and Wroth. There are several factors which influence the soil stiffness, such as:

- Soil type
- Installation effects
- Stress level and system type
- Stress history

- Short-term and long-term conditions
- Initial stres level

2) Nonlinear pile response

Since the nonlinearity of the pile group response is severe than the nonlinearity of the single pile response, the nonlinear effects increase the settlement ratio which is the ratio of settlement of the piled raft to the settlement of unpiled raft. As the load level increases, the group settlement ratio also increases.

3) Short-term and long-term settlements

In clay the short-term settlements are calculated using the undrained soil modulus whereas, for long-term conditions drained modulus is used. There are successfull empirical relationships which relate the drained modulus to the undrained modulus. These formulations are acceptable but for normally consolidated clays, it may estimate the undrained modulus lower than reality. This leads to an underestimation of the consolidation settlement, thus the final settlement appear lower than the actual conditions. The ratio of the short-term to long-term settlement of a group is affected by the efficiency of piles in a group. Thus, this efficiency decreases with the decrease in the Poisson's ratio.

4) Shadowing effect in pile groups

This effect appears when the spacing is close, thus results in the overlapping of the failure zones of the rows of piles in a group. This overlapping, i.e. the shadowing effect will cause a reduction in lateral capacity and increase in the group deflection. This fact leads to setting some limitations for pile spacing.

Since the design will be based on the settlement analysis, the behaviour should be analysed very carefully.

Though the piles are generally used to reduce settlement, conventionally design process is based on the axial capacities. In the design, settlement analysis should be done properly with paying attention to the causes of the occured settlement, i.e. if the settlement is caused by external loads on piles or caused by things other than loads.

Moreover, the settlement analysis should take into account some important factors like load distribution in the pile, length of the zone above and below the neutral plane, drag load at the location of the neutral plane, pile shaft and toe resistance at long term equilibrium, etc.

The settlement based design should include a proper settlement analysis. There are numerous techniques which calculate the settlement of piled-raft foundations. They can be classified due to the models they use and due to the acceptances made while analysing. According to Poulos(1993), the analysis methods can be classified as follows:

1) Purely empirical techniques that relate settlement to that of a single pile
2) Simplified techniques that reduce the pile group system to an equivalent raft
3) Analytical methods that consider interaction between piles and the surrounding soil

Methods in the first category can be described briefly as; the methods in which interaction factors are used by superpositioning, the ones in which the load settlement curves are modified to cover the group effects and the settlement ratio methods. Interaction factors are ratios derived in order to relate settlement of single pile to that of a group of piles.

The second category methods are those in which the system is considered not seperately but as a whole group. They can sometimes be suitable for settlement calculation; however they are not suitable for determining the settlement distribution (Poulos, 2006). The equivalent raft method, the equivalent pier method and different versions of these methods can be considered as such. Furthermore, it should be noted that the traditional approaches based on the equivalent raft systems are currently replacing by the techniques which consider the interactions between piles, thus giving proper attention to the number of piles in a group (Fleming et al., 1992). The methods lay in the first and the third categories can be considered as such. Besides, the second category of the methods will not be in the scope of this research, since they have no contribution to the pile number-settlement relationship.

The importance of the third category comes from the fact that the pile-pile, pile-raft, raft-pile and raft-raft interactions are very important in the design and analysis of the piled raft foundation systems. Thus, if these effects are ignored, both the capacity evaluation and settlement prediction will be highly misleading.

Unfortunately, in practice, the analytical approaches used in the settlement analysis for the design process usually accept the piled raft system as an equivalent raft and do not consider the number of piles in the group. This tendency is generally due to the generalization that as the pile number increase, maximum settlement decreases. This is of course true, but this is true for some extend. Ideally and more logically there should be an optimum number of piles which beyond this value, no considerable reduction in settlement will occur. Thus by considering the settlement analysis methods which takes into account the number of piles, a more economical but still safe design could be possible.

2.2.3 The Analytical Methods For Settlement Analysis of Piled Raft Foundations

In this research, the methods considering the number of piles in the group will be analysed in order to correlate the number of piles with the settlement and to verify that there is an optimum number for piled raft systems. The known methods in the literature will be briefly discussed further in this section.

2.2.3.1 Methods Considering Interaction Factors

In the methods which consider the interaction factors to evaluate the group settlement, the main issue is about determining the interaction factors. Thus, the differences in the techniques come from the difference in obtaining the interaction factors.

There are several factors which the interaction factor depend upon. These factors, combined from the works of two previous researchers; Lee, 1993 and Poulos, 1993, are presented below:

i) Spacing between piles

ii) Length to diameter ratio of piles

iii) Stiffness of piles relative to soil

Regarding to above three factors, the effects are analysed by Lee (1993) and the results are discussed below.

Effects of pile spacing and stiffness of piles relative to soil (λ) is shown in Figures 2.4 and 2.5 for homogeneous soil and nonhomogeneous soil (Gibson soil) respectively for different length to pile radius ratios. These figures are prepared

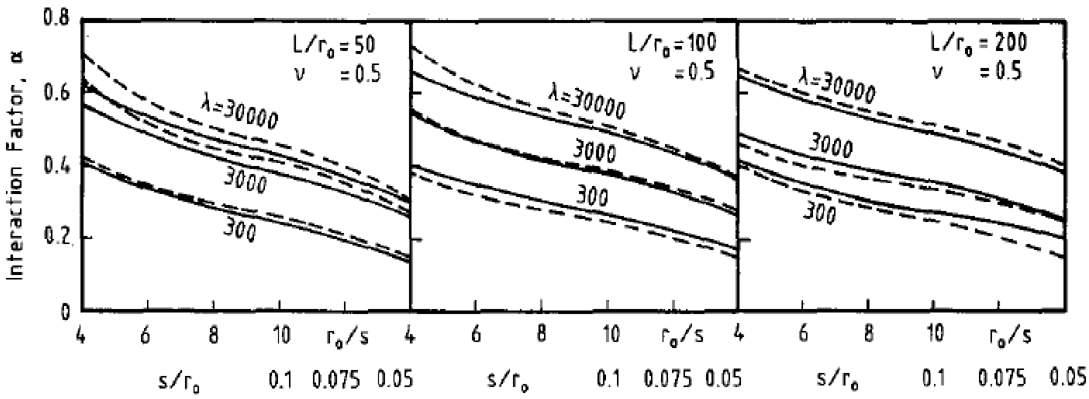


Figure 2.4 Effects of pile spacing and stiffness relative to soil for homogeneous soil (Lee, 1993)

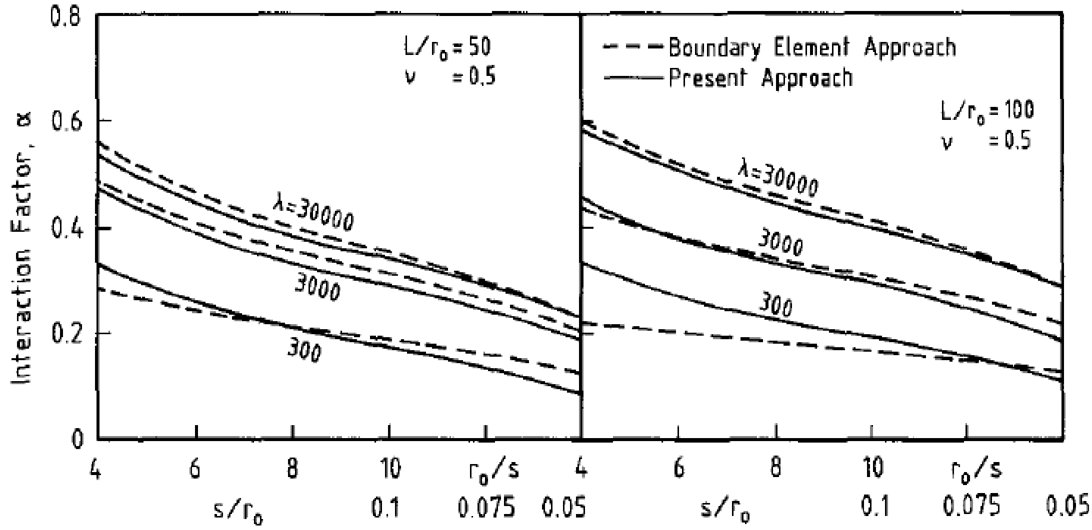


Figure 2.5 Effects of pile spacing and stiffness relative to soil for nonhomogeneous soil (Lee, 1993)

iv) Nature of the bearing stratum

As can be seen from Figure 2.6, interaction factor decreases as the stiffness of the bearing stratum with respect to the soil increases. Since when a hard layer is present at the base of the soil layer, the interaction factor decreases and subsequently there can be over-estimates of pile interactions for deep layers.

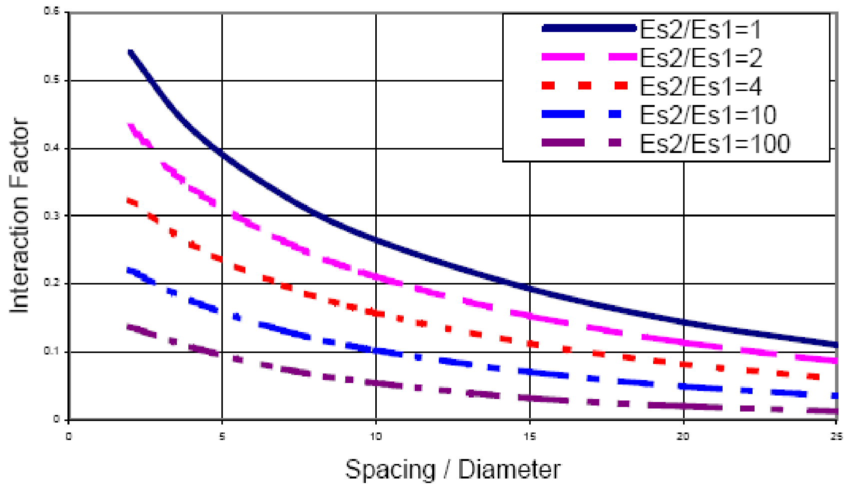


Figure 2.6 Influence of bearing stratum stiffness on interaction factors (Poulos, 2006)

v) Distribution of soil modulus with depth

As can be seen from Figure 2.7, in non-homogeneous soils the interaction factors are smaller than that of the uniform soil profiles. So, if the non-homogenity of the soil is not considered, the settlements will be over-estimated.

There are several methods to overcome the problem of the layered soil profiles, like Mindlin's equation and the other modified methods. The methods differ due to whether or not the soil gets stiffer with depth or gets softer with depth.

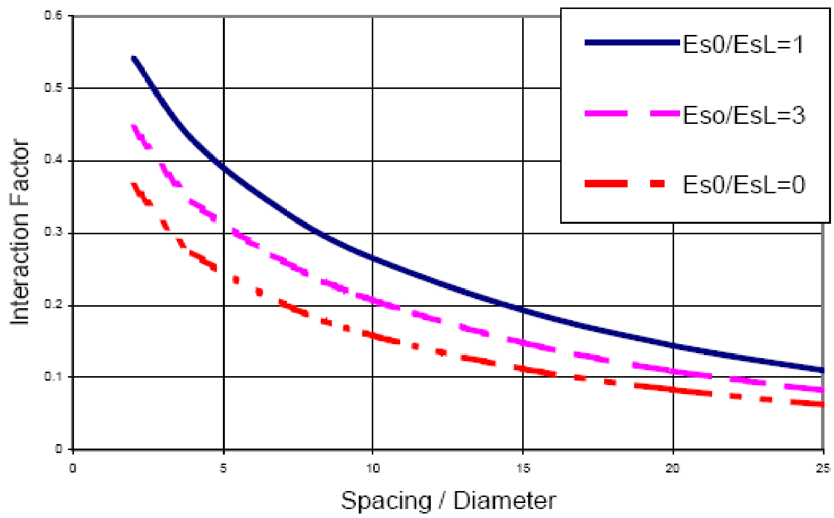


Figure 2.7 Influence of soil modulus distribution on interaction factors (Poulos, 2006)

vi) Type of loading

vii) Stiffer soil between the piles

Between the piles there exist small strain levels which lead a stiffer soil between piles than at the pile-soil interface. Thus, due to this stiffer soil, the interaction factor reduces significantly.

viii) Effects of similar and dissimilar piles

As displayed by Hewitt (1988), interaction factors are effected from the difference in diameter or length of the piles present.

If the piles i and j have different lengths, then;

$$\alpha_{ij} = \begin{cases} (\alpha_{ii} - \alpha_{jj})/2 & for\ L_i > L_j \\ \alpha_{jj} & for\ L_i < L_j \end{cases} \qquad \textit{(24)}$$

If the piles i and j have different diameters, then;

$$\alpha_{ij} = \alpha_{jj} \qquad \textit{(25)}$$

ix) Compressible underlaying layers

Although the compressible layers below the pile tips do not affect the settlement of a single pile, in a pile group this presence increases the settlement. This effect is especially important for the larger groups. If this effect is not taken into account, the calculated settlements will be much greater than that of having a continuous competent stratum.

x) The effect of applying the interaction factor on both the elastic and plastic component

It has been argued by several researchers that the interaction factor should only be applied to the elastic component, since plastic component does not transmit to the adjacent piles.

Also from Figure 2.8, the difference arises from applying the interaction factor only to elastic component can be seen. It is clear that if the interaction factor is applied to the total settlement, the settlement will be greater, thus over-estimated.

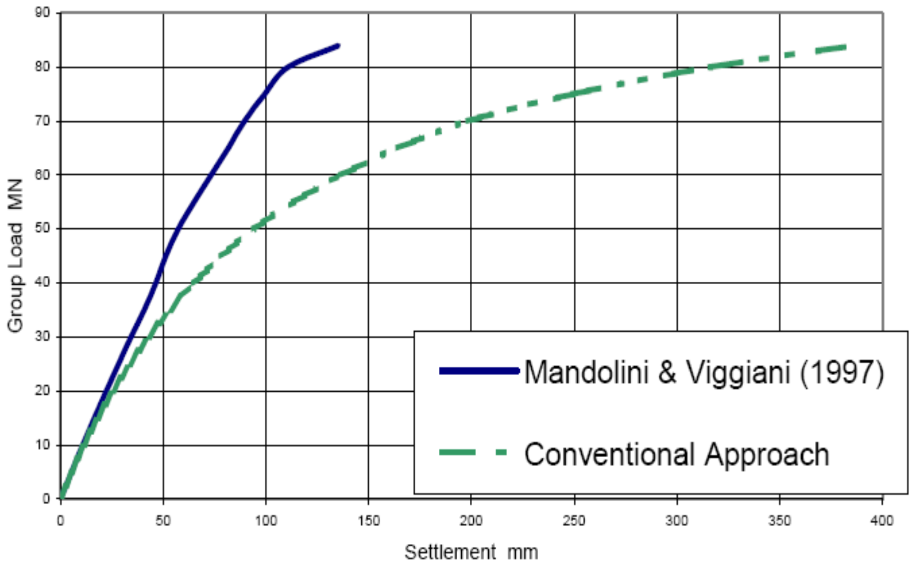


Figure 2.8 Effect of basis of analysis on group-load settlement behavior (Poulos, 2006)

The original approaches accept interaction factors from the plots of interaction factor, α versus ratio of pile spacing to diameter of pile, s/d. These graphical forms are generally taken from a complete analysis of two equally loaded piles, likewise the boundary element analysis. However, today also some closed form, empirical solutions are available to calculate the values of the interaction factors in a group. Some solutions developed by different researchers will be given below; also some methods will be explained in detail.

i) Mandolini and Viggiani, 1997
ii) Lee, 1993
iii) Modified calculation of interaction by Poulos, 1988
iv) Calculation of a pile group settlement based on the method proposed by Butterfield and Douglas(1981)

In this method, the stiffness of the pile group can simply be evaluated as follows.

$$K = fnk \tag{26}$$

where, K = the group stiffness
f = efficiency
n = number of piles in a group
k = individual pile stiffness
The efficiency can be calculated using equation 27.

(27)

where, e = the exponent which can be obtained by some charts (Appendix A)
As stated by Fleming et al., 1992; the exponent,e is depended upon some factors like;

- Length to diameter ratio of the pile
- λ, pile stiffness ratio (E_p / G_s)
- Spacing of the pile
- Slope of the stiffness increase with depth
- Poisson' s ratio

The settlement of a group can be determined using the group stiffness.

$$S = n\frac{P}{K} \qquad \textit{(28)}$$

where, S = the group settlement

P = average load per pile

The case histories show that this simple calculation is reasonable while determining the group settlement if the soil parameters are determined accurately.

2.2.3.2 Methods Considering Settlement Ratio

Interaction factors can also be used in another method for estimating pile group settlement, settlement ratio method. In a group, under a known load for each pile, settlement of the piles will be more than that of a single pile. This incraese will be addressed using a flexibility ratio, which is known as the settlement ratio (Mohan, 1988). Settlement of a group can be related to the settlement of a single pile using this ratio.

$$S_G = R_S S_{1av} \qquad \textit{(29)}$$

where, S_G = group settlement

R_S = settlement ratio

S_{1av} = single pile settlement under same average load

The settlement ratio can be evaluated by several means. One of them is using the interaction factor to calculate the settlement ratio. Also settlement ratio can be approximated by some analytical formulations, considering the number of piles in the group, for example the one Randolph proposed (Poulos,1993).

$$R_S = n^w \qquad \textit{(30)}$$

where, n = number of piles in the group
w = exponent (generally lies between 0.2 – 0.6)

It should also be noted that, settlement ratio should only be applied to the elastic component of the single pile settlement.

Thus, using this settlement ratio, the stiffness of the pile group can be calculated. Stiffness of the pile group is a function of the stiffnesses of individual piles, pile number and an efficiency parameter.

$$K = \eta_w . nk \qquad (31)$$

where, n = number of piles in a group
k = single pile stiffness
η_w = an efficiency factor (for no interaction, taken as unity)

This efficiency will either be accepted as the inverse of the settlement ratio or calculated using some empirical correlations. This group stiffness can further be used in the assessment of the overall settlement of the pile group.

2.2.3.3 Methods Considering Load Transfer Curves

The linear models used in the design of pile groups do not consider non-linear load-deformation characteristics of soil. In order to overcome this behaviour, the load-transfer curves can be used instead. These methods give the opportunity to run non-linear analysis. In this category of methods load-transfer functions are used to represent the relationship between the load at any point and the soil deformation at that point. However, this method should also be modified in order to be convenient to use in the group analysis, since it has limited use. These limitations can be stated as follows as stated in Basile (2003);

i) The value of the modulus of the subgrade reaction will depend on several parameters like soil properties, pile properties and loading conditions. Since no direct tests can give force-displacement relationships, engineering judgement is required. This leads to some errors.

ii) From the analysis, no information about the deformation pattern around the pile can be gathered. Thus, the interaction between piles cannot be found. So, evaluating the group effects is only possible by modifying the load-transfer curves for single piles.

iii) The effect of pile-head fixity to the results is not clear.

Regarding these limitations, it is clear that, in order to use these curves in the analysis of pile groups, some modifications should be done. Without these modifications this method can only be used to design the similar piles not to design the pile groups.

1) Shen et al., 2000

This approach consists of simple formulai based on the load-transfer curves which provides a quick estimation of settlement of symetrical and rectangular pile groups. The deformation along the shaft can be evaluated as follows:

$$w_z(z) = \sum_{j=1}^{k} \beta_{ij} \left(1 - \frac{z}{l}\right)^{j-1} \qquad (32)$$

for i = 1, 2, …….., n_p

where, β_{ij} = undetermined coefficients

z = depth below the pile head

l = pile length

$$[h]\{\beta\} = \{P_T\} \qquad (33)$$

$$[h] = [k_p] + [k_s][A] + [k_{sb}] \qquad (34)$$

where, [h] = coefficient matrix for pile-soil system

$[k_p]$, $[k_s]$, $[A]$ and $[k_{sb}]$ = matrices related to the stiffness of piles and soil

$\{P_T\}$ = vector related to the loads at pile heads.

Inverse of matrix [h] gives the flexibility relationship of the piles.

$$[h]^{-1}\{P_T\} = \{\beta\} \quad \textbf{\textit{(35)}}$$

$$[f_t]\{P_T\} = \{w_t\} \quad \textbf{\textit{(36)}}$$

The stiffness relationship is as follows:

$$\{P_T\} = [f_t]^{-1}\{w_t\} \quad \textbf{\textit{(37)}}$$

For single piles:

$$[k_p] = \frac{E_p \pi r_o^2}{l}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & \frac{4}{3} \end{bmatrix} \quad \textbf{\textit{(38)}}$$

where, E_p = pile modulus

r_o = radius of the pile

$$[k_s] = \frac{G_t}{r_o \zeta}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \textbf{\textit{(39)}}$$

where, G_t = shear soil modulus at pile toe

$$\zeta = \ln\left(\frac{r_m}{r_0}\right) \quad \textbf{\textit{(40)}}$$

$$r_m = 2.5\rho l(1 - \nu_s) \quad \textbf{\textit{(41)}}$$

where, ρ = soil inhomogenity factor (ratio of shear soil modulus at pile middepth to that of the base)

ν_s = Poisson's ratio

$$[k_{sb}] = \frac{4r_o G_t}{1-\nu_s}\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix} \tag{42}$$

$$[A] = 2\pi r_o l\begin{bmatrix}\rho & \frac{1}{6}(4\rho-1) & \frac{1}{6}(3\rho-1)\\ \frac{1}{6}(4\rho-1) & \frac{1}{6}(3\rho-1) & \frac{1}{20}(8\rho-3)\\ \frac{1}{6}(3\rho-1) & \frac{1}{20}(8\rho-3) & \frac{1}{15}(5\rho-2)\end{bmatrix} \tag{43}$$

Thus, [h] can be evaluated as follows:

$$[h] = G_t r_o [H] \tag{44}$$

$$[h]^{-1} = \frac{1}{G_t r_o}[H]^{-1} \tag{45}$$

$$[H] = \begin{bmatrix}\frac{2\pi l\rho}{r_o\zeta}+\frac{4}{1-\nu_s} & \frac{\pi l(4\rho-1)}{3r_o\zeta} & \frac{\pi l(3\rho-1)}{3r_o\zeta}\\ \frac{\pi l(4\rho-1)}{3r_o\zeta} & \frac{\pi r_o\lambda}{l}+\frac{\pi l(3\rho-1)}{3r_o\zeta} & \frac{\pi r_o\lambda}{l}+\frac{\pi l(8\rho-3)}{10r_o\zeta}\\ \frac{\pi l(3\rho-1)}{3r_o\zeta} & \frac{\pi r_o\lambda}{l}+\frac{\pi l(8\rho-3)}{10r_o\zeta} & \frac{4\pi r_o\lambda}{3l}+\frac{2\pi l(5\rho-2)}{15r_o\zeta}\end{bmatrix} \tag{46}$$

where, λ = relative stiffness of piles

$$\lambda = \frac{E_p}{G_L} \tag{47}$$

$$\frac{1}{G_t r_o}[H]^{-1}\{P_T\} = \{\beta\} \tag{48}$$

$$\frac{1}{G_t r_o} f_t P_t = w_t \tag{49}$$

$$\frac{P_t}{G_t r_o w_t} = f_t^{-1} \tag{50}$$

$$[H]^{-1} = \left[\frac{2\pi l\rho}{r_o\zeta} + \frac{4}{1-\nu_s}\right]^{-1} \tag{51}$$

The pile load settlement ratio can be given as;

$$\frac{P_t}{G_t r_o w_t} = \left[\frac{2\pi l\rho}{r_o\zeta} + \frac{4}{1-\nu_s}\right] \tag{52}$$

For symetric pile groups:

$$[H] = \begin{bmatrix} \frac{2\pi l\rho}{r_o\zeta_1} + \frac{4}{(1-\nu_s)\zeta_2} & \frac{\pi l(4\rho-1)}{3r_o\zeta_1} & \frac{\pi l(3\rho-1)}{3r_o\zeta_1} \\ \frac{\pi l(4\rho-1)}{3r_o\zeta_1} & \frac{\pi r_o\lambda}{l} + \frac{\pi l(3\rho-1)}{3r_o\zeta_1} & \frac{\pi r_o\lambda}{l} + \frac{\pi l(8\rho-3)}{10r_o\zeta_1} \\ \frac{\pi l(3\rho-1)}{3r_o\zeta_1} & \frac{\pi r_o\lambda}{l} + \frac{\pi l(8\rho-3)}{10r_o\zeta_1} & \frac{4\pi r_o\lambda}{3l} + \frac{2\pi l(5\rho-2)}{15r_o\zeta_1} \end{bmatrix} \tag{53}$$

The load settlement ratio for two piles can be determined as described below.

$$\frac{P_t}{G_t r_o w_t} = \left[\frac{2\pi l\rho}{r_o\zeta_1} + \frac{4}{(1-\nu_s)\zeta_2}\right] \tag{54}$$

$$\zeta_1 = \zeta + \ln\left({}^{r_m}/_{s}\right) \tag{55}$$

$$\zeta_2 = 1 + {}^{2r_o}/_{\pi s} \tag{56}$$

If there are three symetric piles:

$$\zeta_1 = \zeta + 2\ln\left({}^{r_m}/_{s}\right) \tag{57}$$

$$\zeta_2 = 1 + {}^{4r_0}/_{\pi s} \qquad \textit{(58)}$$

If there are four symetric piles:

$$\zeta_1 = \zeta + 2\ln\left({}^{r_m}/_{s}\right) + \ln\left({}^{r_m}/_{\sqrt{2}s}\right) \qquad \textit{(59)}$$

$$\zeta_2 = 1 + {}^{4r_0}/_{\pi s} + {}^{2r_0}/_{\sqrt{2}\pi s} \qquad \textit{(60)}$$

Moreover, the above described procedure can be used to analyse the pile groups in elastic half space with the settlement ratios.

In the below figures, the inverse of the settlement ratio is plotted against the number of piles for different radii, spacing to radius ratio and Poisson's ratio. Figure 2.9 is determined under a "s/d" ratio of 6 and a Poisson's ratio of 0.5, whereas Figure 2.10 is determined under a "s/d" ratio of 10 and a Poisson's ratio of 0.2.

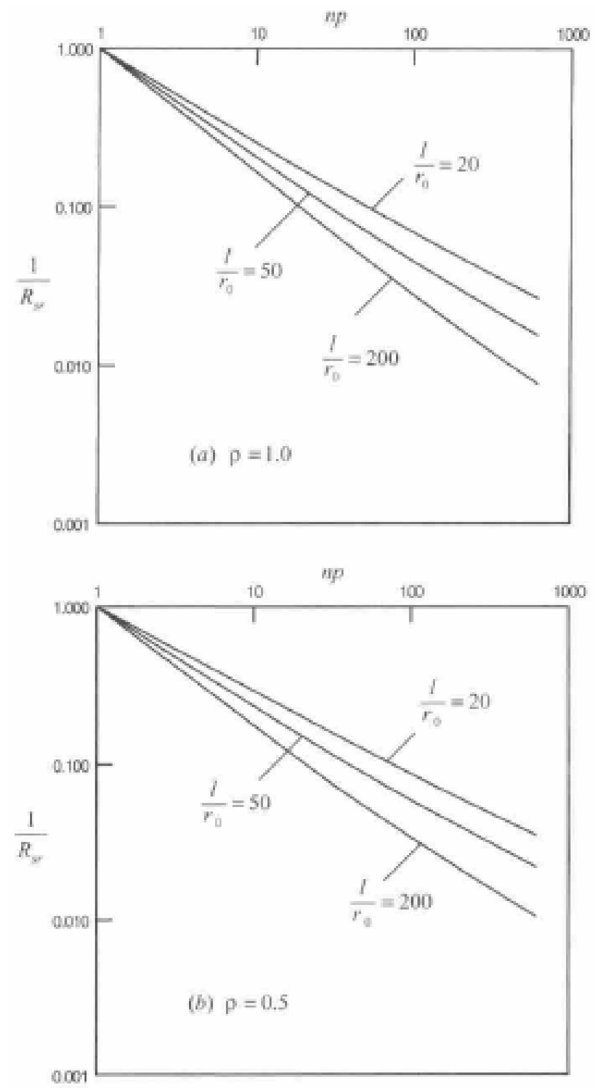


Figure 2.9 Graph of 1/R_{sr} versus number of piles for s/r_o=6, v_s=0.5 (Shen et al, 2000)

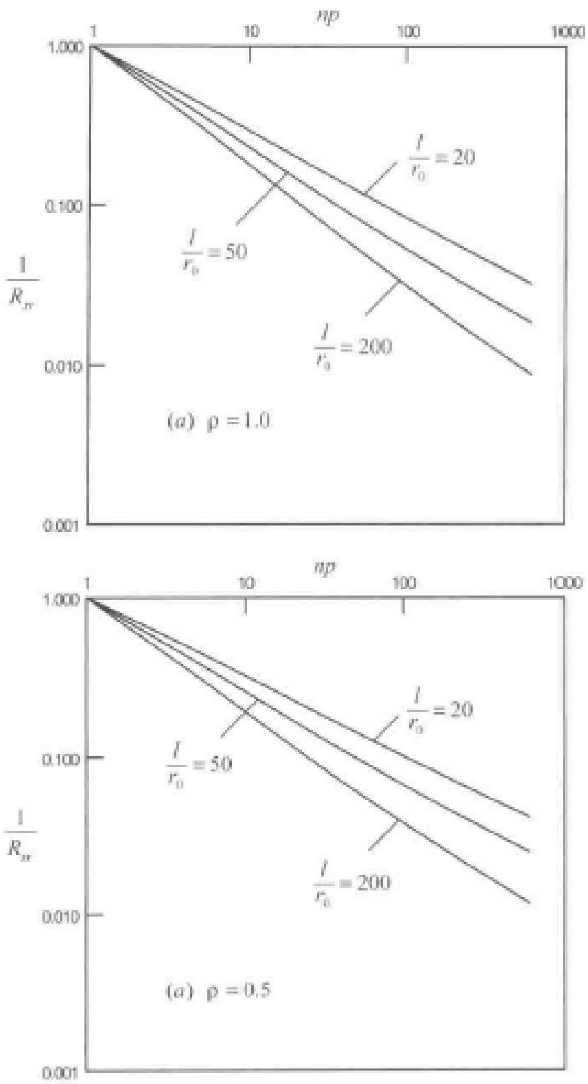


Figure 2.10 Graph of $1/R_{sr}$ versus number of piles for $s/r_o=10$, $v_s=0.2$ (Shen et al, 2000)

It can clearly be seen that the above relationship of the settlement ratio is a straight line. Thus, by determining the equation of this line, the behaviour of the group can easily be obtained. From the above described procedure using the load-transfer curves, the Equations 52 and 53 for single piles can be used to determine this equation.

$$R_{sr} = \left(n_p\right)^{\log_4 R_{sr4}} \tag{61}$$

where, R_{sr} = pile group settlement ratio

n_p = number of piles in the group

R_{sr4} = pile group settlement ratio for 2x2 pile group (for four symetric piles)

$$R_{sr4} = \frac{\frac{2\pi\rho l}{r_o\zeta} + \frac{4}{1-v_s}}{\frac{2\pi\rho l}{r_o\xi_1} + \frac{4}{(1-v_s)\zeta_2}} \qquad \textbf{\textit{(62)}}$$

However, r_m should be calculated differently.

$$r_m = 2.5l\rho(1-v_s) + 0.7s\ln\left({}^{l}/_{r_o}\right) \qquad \textbf{\textit{(63)}}$$

The ratio of the group settlement ratio for compressible pile groups to the group settlement ratio for rigid pile groups, i.e. "R_s / R_{sr}" can also be correlated with the number of piles. These relationships are shown in Figure 2.11 for different radii of the piles and for a spacing of 10 pile radius.

It should be noted that, for rectangle pile groups of the breadth-width ratio up to 4, the group can be analysed as square.

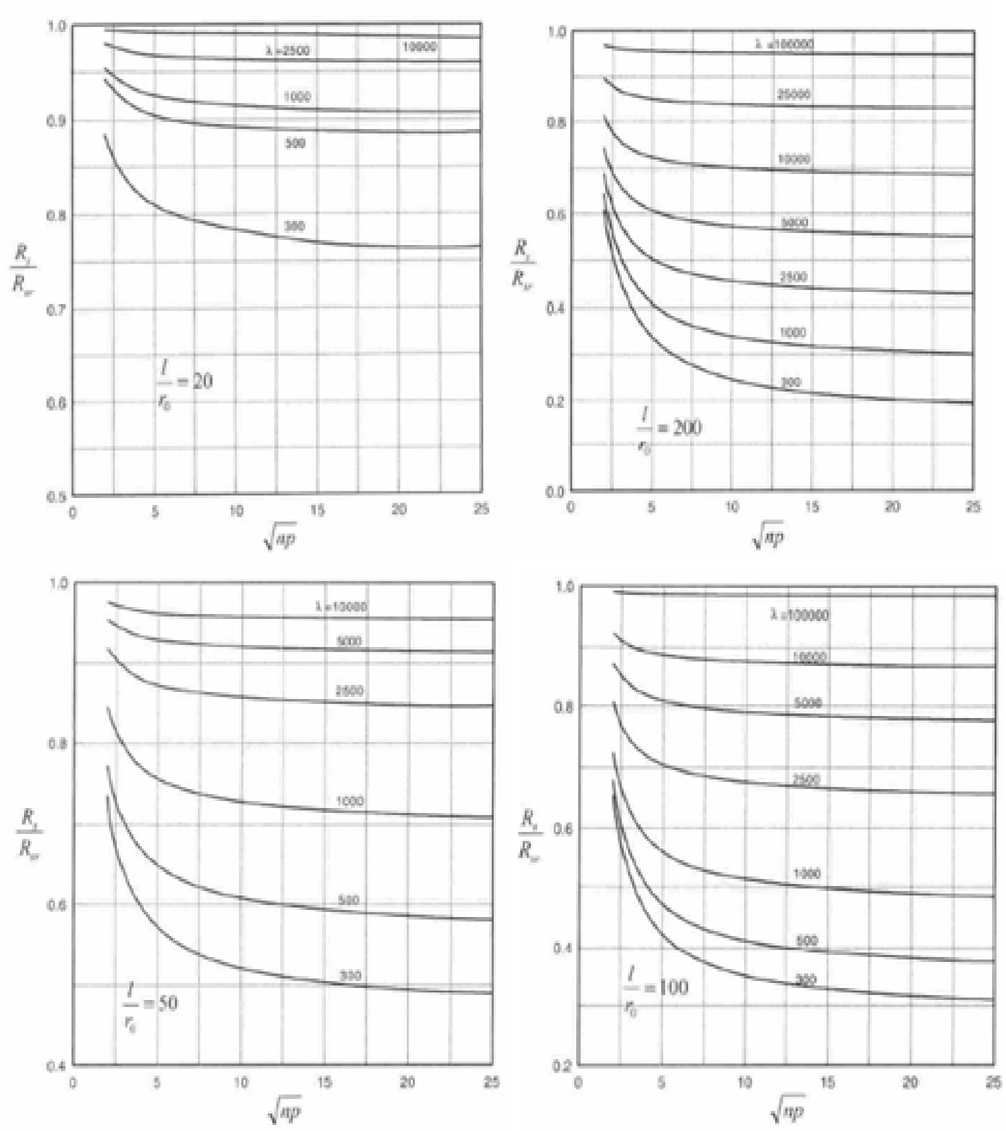


Figure 2.11 Normalized group settlement ratio versus number of piles (Shen et al, 2000)

2) A modified method presented by Shen and Teh, 2002

This method is an approach to calculate the group stiffness based on the load-transfer curves using a simple spreadsheet calculation. The calculations are very similar to the previous explained method by Shen et al., 2000. This method is advantageous since it does not use complex computer programs, it is not very time consuming and gives reasonable results. Below, the theory beneath this approach will be explained briefly.

In a group of piles, the displacement of a pile "i" can be given as:

$$w_i(z) = \sum_{j=1}^{k} \beta_{ij} \left(1 - \frac{z}{l}\right)^{j-1} \qquad \textbf{\textit{(64)}}$$

for i = 1,2,….,n_p

where, β_{ij} = undetermined coefficients

z = depth below the pile head

l = pile length

$$[h]\{\beta\} = \{P_T\} \qquad \textbf{\textit{(65)}}$$

$$[h] = [k_p] + [k_s][A] + [k_{sb}] \qquad \textbf{\textit{(66)}}$$

where, [h] = coefficient matrix for pile-soil system

$[k_p]$, $[k_s]$, [A] and $[k_{sb}]$ = matrices related to the stiffness of piles and soil

$\{P_T\}$ = vector related to the loads at pile heads.

By expressing the matrices related to the stiffness of piles and soil with their submatrices and making some adjustments, the Equation 65 can be written as:

$$\begin{bmatrix} [k_{pp}] + k_{ss11}[A_s] + k_{bb11}[B_s] & \cdots & k_{ss1np}[A_s] + k_{bb1np}[B_s] \\ k_{ss21}[A_s] + k_{bb21}[B_s] & \cdots & k_{ss2np}[A_s] + k_{bb2np}[B_s] \\ \vdots & \cdots & \vdots \\ k_{ssnp1}[A_s] + k_{bbnp1}[B_s] & \cdots & [k_{pp}] + k_{ssnpnp}[A_s] + k_{bbnpnp}[B_s] \end{bmatrix} \begin{Bmatrix} \{\beta_1\} \\ \{\beta_2\} \\ \vdots \\ \{\beta_{np}\} \end{Bmatrix} = \begin{Bmatrix} \{P_1\} \\ \{P_2\} \\ \vdots \\ \{P_{np}\} \end{Bmatrix} \qquad \textbf{\textit{(67)}}$$

The matrices [k_{sij}], [k_{sbij}] and [B_s] are given in Appendix B.

If the piles in the group assumed to be settled equally, the equation can be written as follows:

$$[h_p]\{\beta_p\} = \{P_p\} \quad (68)$$

$$[h_p] = \left[n_p[k_{pp}] + \sum_{i=1}^{n_p}\sum_{j=1}^{n_p} k_{ssij}[A_s] + \sum_{i=1}^{n_p}\sum_{j=1}^{n_p} k_{bbij}[B_s]\right] \quad (69)$$

$$\{\beta_p\} = \{\beta_{p1}, \beta_{p2}, \ldots, \beta_{pk}\}^t \quad (70)$$

$$\{P_p\} = \{P, P, \ldots, P\}^T \quad (71)$$

where, P = overall vertical load applied on the pile cap.
Equation 69 can also be written as:

$$[h_p] = \left[n_p\{k_{pp}\} + \alpha_p k_{ss}[A_s] + \alpha_{pb}\, k_{bb}[B_s]\right] \quad (72)$$

$$k_{ss} = {G_l}/{r_o\zeta} \quad (73)$$

$$\alpha_p = \sum_{i=1}^{n_p}\sum_{j=1}^{n_p} {k_{ssij}}/{k_{ss}} \quad (74)$$

$$\alpha_{pb} = \sum_{i=1}^{n_p}\sum_{j=1}^{n_p} {k_{bbij}}/{k_{bb}} \quad (75)$$

For the value of ζ, determination can be done using the below equation (Randolph and Wroth, 1978 and Fleming et al, 1992) :

$$\zeta = \ln\left({l}/{r_o}\,(0.25 + (2.5\rho(1 - \nu_s) - 0.25)\xi)\right) \quad (76)$$

where, $\xi = {}^{G_t}/_{G_b}$

For pile groups, ζ generally lies between 3-6.

Then the matrix $[h_p]$ becomes;

$$[h_p] = G_l r_o [H] \tag{77}$$

$$[H] = \begin{bmatrix} 2\mu_s\rho + \mu_b & \frac{\mu_s(4\rho - 1)}{3} & \frac{\mu_s(3\rho - 1)}{3} \\ \frac{\mu_s(4\rho - 1)}{3} & \mu_p + \frac{\mu_s(3\rho - 1)}{3} & \mu_p + \frac{\mu_s(8\rho - 1)}{10} \\ \frac{\mu_s(3\rho - 1)}{3} & \mu_p + \frac{\mu_s(8\rho - 1)}{10} & \frac{4}{3}\mu_p + \frac{2\mu_s(5\rho - 2)}{15} \end{bmatrix} \tag{78}$$

where, ρ = soil inhomogenity factor (ratio of soil shear modulus at the mid-depth to that at the base)

$\mu_s = \pi\alpha_p l \,/\, r_o \,/\, \zeta$

$\mu_b = 4\alpha_{pb}/(1 - \nu_s)/\xi$

$\mu_p = \pi\lambda n_p/l/r_o$

$\lambda = {}^{E_p}/_{G_l}$

The group stiffness ratio obtained from all the calculations is given below:

$$\frac{P}{G_l r_o w_t} = \frac{1}{\sum_{i=1}^{3}\sum_{j=1}^{3} H_{ij}^{-1}} \tag{79}$$

where, w_t = uniform settlement of pile head

H^{-1}_{ij} = coefficients of inverted matrix [H].

If the pile group is rigid, then the following relationships can be observed.

$$\frac{P}{G_l r_o w_t} = \frac{2\pi l \alpha_p}{r_o \zeta} + \frac{4\alpha_{pb}}{\xi(1 - \nu_s)} \tag{80}$$

$$\alpha_p = \sum_{i=1}^{n_p}\sum_{j=1}^{n_p} f_{sij}^{-1} \tag{81}$$

$$\alpha_{pb} = \sum_{i=1}^{n_p}\sum_{j=1}^{n_p} f_{sbij}^{-1} \tag{82}$$

The f^{-1}_{sij} and f^{-1}_{sbij} in the above equations are the coefficients of the matrices $[f_s]^{-1}$ and $[f_{sb}]^{-1}$. These matrices are the inverses of the matrices $[f_s]$ and $[f_{sb}]$ which are given in the Appendix C.

Another way of determining the factors α_p and α_{pb} is using the charts available for different values of spacing to diameter ratios. These charts are given in Figure 2.12.

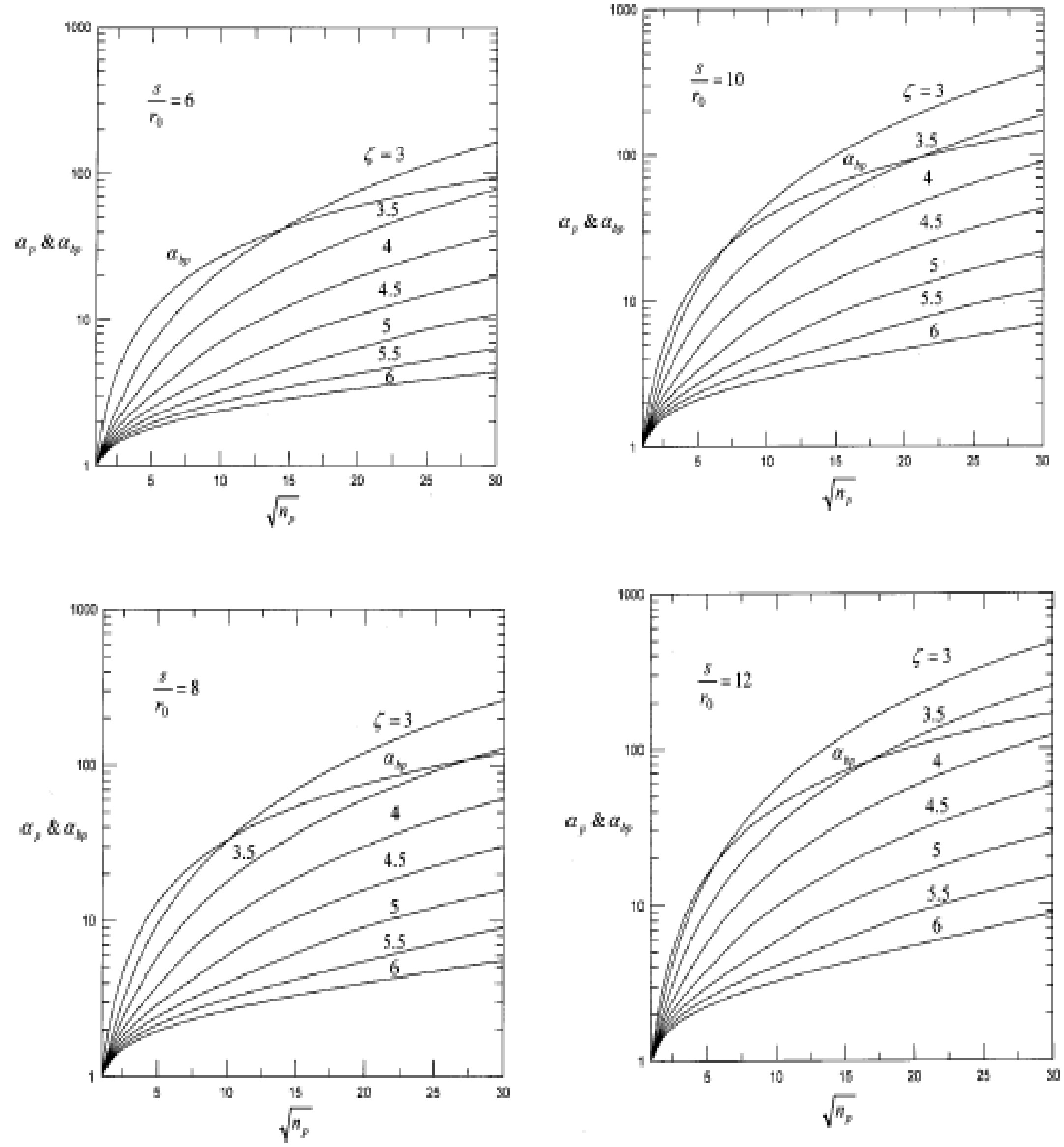


Figure 2.12 Coefficients α_p and α_{bp} versus number of piles (Shen and Teh, 2002)

2.2.3.4 Methods Based on Complete Analysis

Complete analysis generally means to consider each pile in the group in detail. Boundary element analysis, finite element analysis or some combined methods can be used for this kind of analysis. This kind of methods can be used to overcome the problems encountered in the load-transfer curve approaches and the interaction factor method. By this complete analysis, the piles having different length, diameter, stiffness or base and shaft resistance can be taken into consideration in detail. Also nonlinear soil-pile response and the pile interaction can be considered. Moreover, the load and bending moment distribution along the piles can easily be obtained. Although this type of analysis is more accurate and more detailed, it is very time consuming.

1) Finite element method

The finite element analysis determines the load transfer behaviour of the piles through the surrounding soil however it is not very applicable to pile groups. Moreover, this kind of analysis is very time consuming, the cost is very high and data preparation needs too much attention. So, the boundary element analyse are more preferable. However, recently with computer programs finite element analysis could be done in a simpler manner. Of course while using these kind of programs, the engineers should dominate to the rules and the acceptances of the program. Hence, he or she should always be in control and do not rely on the program completely.

2) Boundary element method

Unlike the finite element analysis, the boundary element analysis gives accurate solutions for pile groups, it is not very time consuming and it has a lower cost since it gives solutions using the boundary values. It gives special care to some critical locations like pile-soil interface.

There are several computer programs which uses the boundary element analysis with several other methods have been developed by several researches. These are listed by Fleming et al (1992). Some examples are given below:

- DEPIG which is developed by Poulos in 1990. Uses a simplified boundary element method analysis and interaction factors.
- MPILE, originally named PIGLET (developed by Randolph, 1980). Uses a semi-emprical method with analytical solutions and interaction factors.
- PGROUP which is developed by Banerjee and Driscoll in 1976. Uses a linear elastic analysis.
- GEPAN which is developed by Xu and Poulos in 2000. Uses a linear analysis.
- PGROUPN which is developed by Basile in 1999. Uses a non-linear boundary element analysis.

2.2.4 Advantages of Piled Raft Foundations

In many cases, especially when the raft alone does not satisfy the settlement and differential settlement criteria but have an adequate load carrying capacity, using a piled raft foundation instead of a conventional piled foundation, has many advantages. Although, the conventional approaches are easier to deal with, when applicable pile rafts, give a more convenient and economic solution. Moreover, these systems are more successful at soil profiles consisting relatively stiff clays or relatively dense sands and structures which has a high slenderness ratio.

Advantages of using a piled raft foundation can be listed as given by El-Mossallamy (2002) :

- Heave will be minimised
- The required limits for differential settlements, settlements and tilting will be satisfied
- If some regions of the foundation is subjected to different loads, this system will minimize the differential settlement

- If eccentric loading or difficult subsoil conditions arise a risk of foundation tilting, piled raft foundation system will decrease this tilting
- Since the new design requires placing piles at strategical locations, the raft stresses and moments will be reducted.

CHAPTER 3

EXPERIMENTAL AND ANALYTICAL STUDIES ON SETTLEMENT REDUCING PILES

3.1 The Experimental Studies on Settlement Reducing Piles

3.1.1 Scope of The Experimental Study

In this research, in order to support the analytical studies, a laboratory study is to be conducted on model systems. The laboratory study will give an approximation about the settlement behavior of pile groups with increasing number of piles.

In this study, it is intended to show that, increasing the number of piles do not cause an excessive reduction in total settlement. Regardance of the conventional design procedures, by increasing the number of piles, the optimization in the design could not be satisfied. Thus, beyond an optimum number, settlement behavior will tend to become steady. Expressed in a different way; although, adding piles to the raft solves the settlement problem, the reduction in the amount of settlement does not continue steadily; thus, reduction stops at an optimum number of piles. The experiments are intended to show this behavior of settlement reducing piles, accordingly in the experiments, the number of piles is augmented whereas all the other parameters like pile length, pile diameter, type of soil, size of raft will remain constant.

In the experiments, the raft will be modeled with aluminum footings whose dimensions are 50 x 50 x 10 mm and the piles will be modeled with brass nails with rasped sides which are 2 mm in diameter and 75 mm in length. In the series of tests, the model footings with different number of brass nails inserted beneath will be subjected to the sustained loading and settlements will be measured. Because of the dimensions of the model footing and the related limitations, at most 49 brass nails can be inserted in a 7x7 square pattern.

Another important point that should be addressed is that, the given load is high, resulting in the failure of the piles. In other words, in the designed model, under the given load, the piles had been yielded and the capacity of piles is fully mobilized. So, the model can be said to represent a system which is constituted of a raft with settlement reducing piles.

3.1.2 Experimental Setup

In the experimental studies, the soil sample is prepared and consolidated under a certain pressure. Then the model system is prepared and it is loaded to a decided pressure. The settlements occurring under that pressure is measured using special equipments.

In such a procedure, the elements constitute the model can be listed as follows:

- Plexiglas box
- Geotextiles
- Commercial type of kaolin clay
- Brass nails
- Aluminum footing

The necessary equipments to build up the setup and proceed the experiments can be itemized as follows:

- Loading jack for consolidation process
- Timber templates for insertion of nails
- Load hangers
- Displacement dials
- Data acquisition system consists of a software that record readings, a data logger and a computer

The above given entries will further be explained in this section.

3.1.2.1 Plexiglas Boxes and Geotextiles

In order to place the kaolin clay, the boxes manufactured by Kul (2003) was used. The plexiglas box has inside diameters of 20 x 20 x 20 cm, the wall thickness is 1 cm. Before placing the kaolin clay, the geotextiles, which are used for drainage purposes and to prevent drying of soil, were laid. The plexiglas box and the geotextiles can be seen in below Figures 3.1 and 3.2.

Figure 3.1 Plexiglas box and geotextiles

Figure 3.2 Plexiglas box covered with geotextiles

It should be noted that the dimensions of the plexiglas boxes are well integrated with the vertical stress distribution since the width of the model footing will be 5 cm and the effective pressures distribution is extended down to 2 or 3 times width, i.e. 10-15 cm. Thus, the model can be said to be an elastic half space.

3.1.2.2 Commercial Type of Kaolin Clay

In the tests remolded kaolin is being used. Remolded kaolin has lower liquid limit and lower activity, thus it is favored in most cases to the other types of clay. These properties which provide avoidance from swelling, shrinkage and some other problems enable kaolin type of clay as a preferable material for model studies. In order to perform the study consciously, the sample is tested in the laboratory for some typical material properties. The laboratory study and their results will further be explained in the Section 3.1.3.1.

It should also be noted that the kaolin type of clay used for the experiments had been derived from remolding of the kaolin powder, respecting the desired water content. In the following experimental study, the water content is desired to be 40%. After remolding process, the specimens are allowed to rest in the humidity room for at least five or six days.

3.1.2.3 Brass Nails and Aluminium Footing

For modeling the raft with settlement reducing piles, brass and aluminum is used. The schematic representation for a sample case with 16 piles is shown in Figure 3.3.

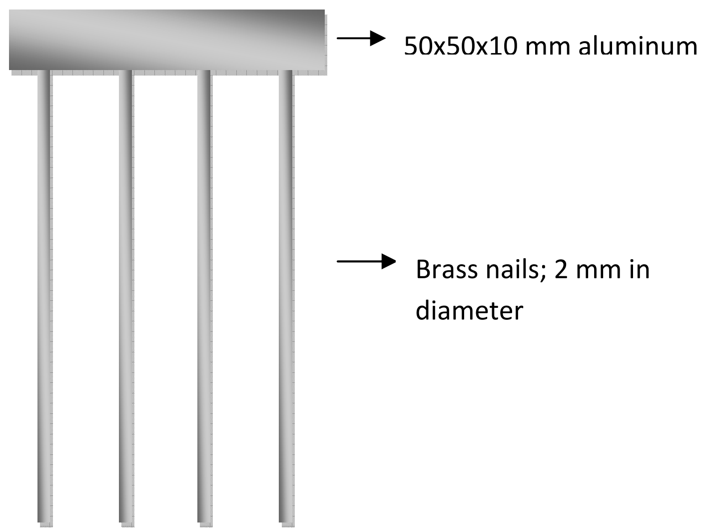


Figure 3.3 Schematic representation of the simple case

The piles are modeled with brass nails, 2 mm in diameter and 75 mm in length. It should also be mentioned here that, the length of the nails are consistent with the recommendations, i.e. nail length is 1,5 times the width of the footing. The nails have rasped edges to enhance friction. In further analysis, the modulus of elasticity of brass is accepted as $1x10^8$ kPa (Engin, 2005 after Kul, 2003). The brass nails used in the experimental study is shown in figure 3.4.

Figure 3.4 Brass nails

The raft in the system is modeled by an aluminum footing which has dimensions of 50 x 50 x10 mm. In further analysis, the modulus of elasticity of aluminum is accepted as $69x10^6$ kPa. In order to place the displacement dials to the system, a metal sheet is welded at the top portion of the footing. The aluminum footing is shown in Figure 3.5.

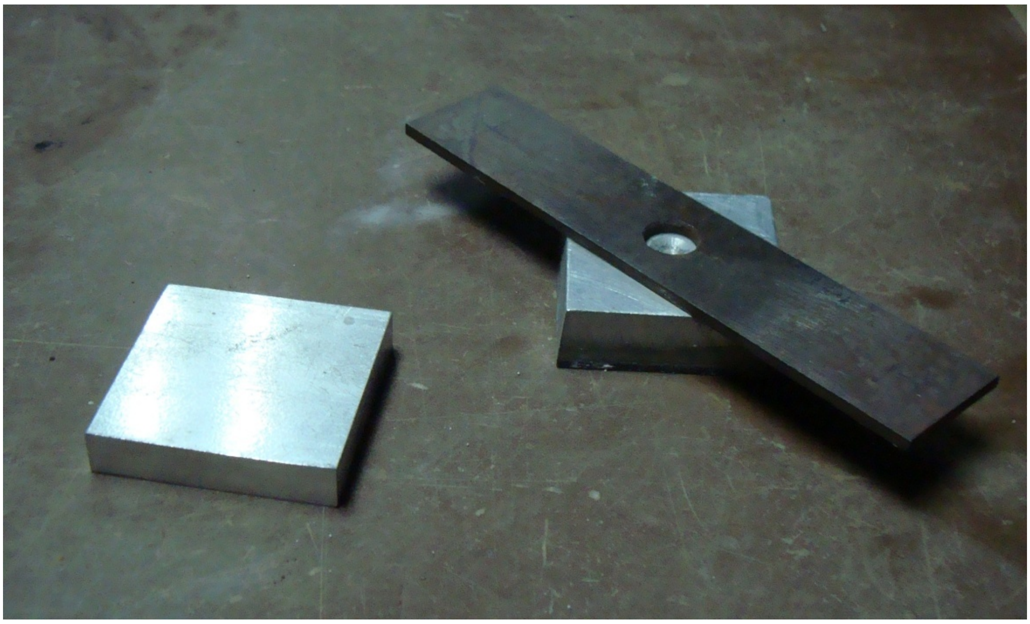


Figure 3.5 Aluminum footing

3.1.2.4 Loading Jack

The cured kaolin clay is placed in the plexiglas boxes and covered according to the procedure. Then the samples are consolidated by means of the loading jack connected to the loading frames. The consolidation pressure was 102 kPa. The frame system and the loading jack are shown in Figures 3.6 and 3.7 respectively.

3.1.2.5 Timber Templates

After the consolidation process, the model foundation system is prepared by inserting the nails and placing the footing. In the purpose of inserting nails, timber templates are used. These templates were prepared by grooving one side of the template in the designed pattern; each groove was 2 mm wide. The reason behind using these templates was that they provide a correct and proper insertion of nails in the desired pattern. The timber templates are shown in Figure 3.8.

Figure 3.6 Frame system

Figure 3.7 Loading jack

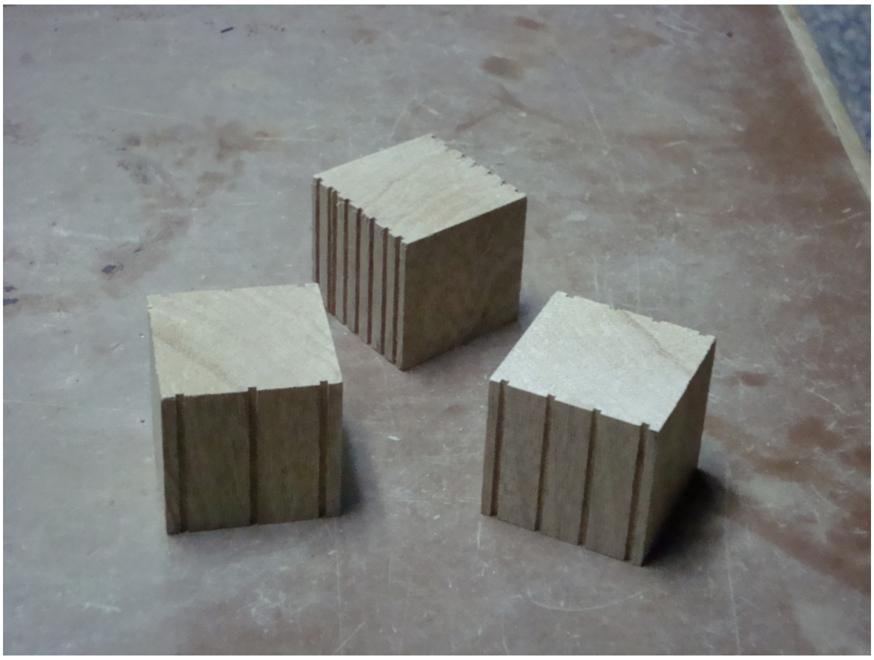


Figure 3.8 Timber templates

3.1.2.6 Load Hangers and Displacement Dials

When the preparation of the soil model is completed, the system was suspended to the pressures of 75 and 40 kPa and the displacements have been measured. An apparatus was prepared to maintain the desired circumstance. Constituents of the apparatus are shown in Figure 3.9.

Figure 3.9 Constituents of the apparatus

The displacements have been measured by linear variable differential transformers (herein after referred as lvdt). Lvdt is an electrical transformer used to measure linear displacements. The lvdt's used had been calibrated prior to the experimental study.
A schematic representation of the system is given in Figure 3.10.

3.1.2.7 Data Acquisition System

In order to deal with the readings taken from the lvdt's, a data acquisition system was used. This system included a 16-channel data logger (ADU), a computer and software (DADU) that arrange and record the readings. The system is shown in Figure 3.11.

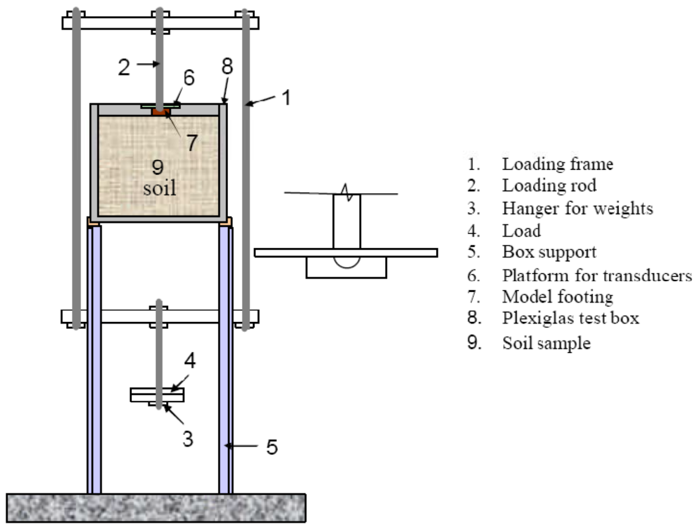


Figure 3.10 Schematic representation of the system (Engin, 2005 after Kul, 2003)

Figure 3.11 Data acquisition system

3.1.3 Laboratory Testing

3.1.3.1 Laboratory Testing to Determine The Properties of Kaolin Clay

The properties of the kaolin type of clay had been determined with some standard laboratory tests. Moreover, for each sample box some experiments were conducted before and after the loading, to ensure that the boxes prepared were consisted with each other. These experiments will further be explained in this chapter. Thus, it should be noted that while performing each experiment, TS 1900 was taken as a basis.

1) **Specific Gravity Test**

The specific gravity of the sample had been determined by the specific gravity test, in which sample was crashed into small parts, dried for one day; then smashed and mixed with distilled water. The result of the test is given in Table 3.1.

2) **Atterberg Limits Tests**

The liquid limit, plastic limit and the plasticity index of the sample had been determined using Atterberg limit tests. The sample had been rested for a sufficient time, thus became homogeneous and mature. Then sample was being tested. The results are given in Table 3.1. Thus, according to the USCS (Unified Soil Classification System), soil is classified as CL, i.e. low plasticity clay.

3) **Hydrometer Analysis**

The grain size distribution of the sample had been determined with the hydrometer analysis since it was a fine grained soil. The test was conducted with a dried, sieved and smashed sample. The grain size distribution graph is given in Figure 3.12. Thus, according to the USCS (Unified Soil Classification System), soil is classified as CL, i.e. low plasticity clay.

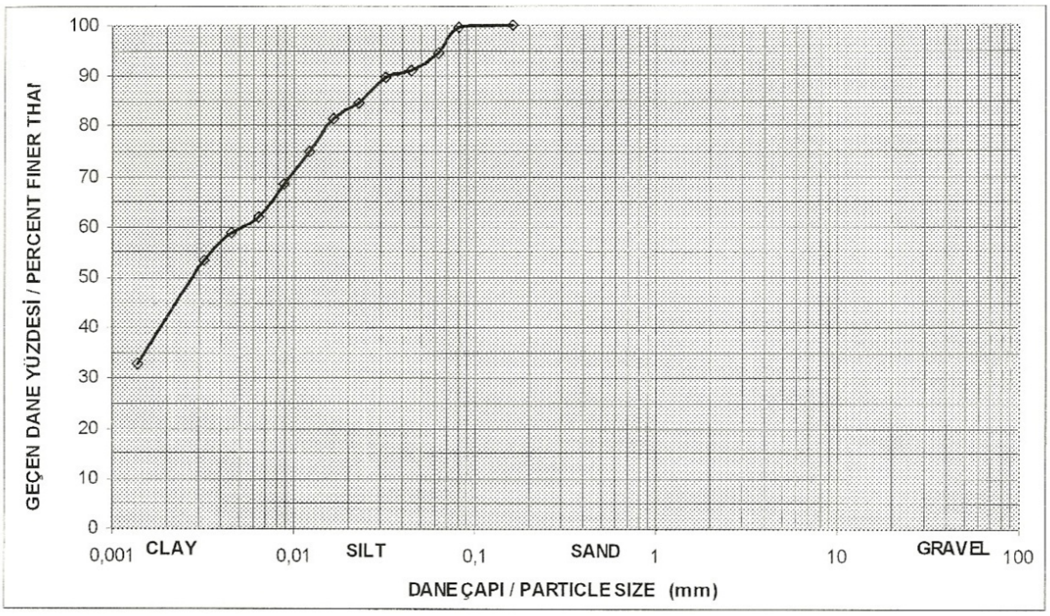


Figure 3.12 Grain size distribution graph

4) Consolidation Test

The compression index, coefficient of volume compressibility and coefficient of vertical consolidation have been determined using the consolidation test. The sample was consolidated with the consolidation apparatus under different pressures. The results are shown in Table 3.1.

Table 3.1 Results of the standart laboratory tests on kaolin clay

NAME OF THE EXPERIMENT	RESULTS OF THE EXPERIMENT
Specific gravity test	G_s = 2.60
Atterberg limits	LL = 42.5 % , PL = 24 % , I_p = 18.5 %
Consolidation test	c_c = 0.70
	m_v values are given in table 3.2, e versus log σ graph and c_v versus σ graph are given in figures 3.13 and 3.14, respectively.

Table 3.2 m_v values obtained in consolidation test

Consolidation pressure (kPa)	**m_v (m^2/kN)**
0-50	0.0004571
50-100	0.0003468
100-200	0.0002353
200-400	0.0001205
400-800	0.0001080
800-1600	0.0000565

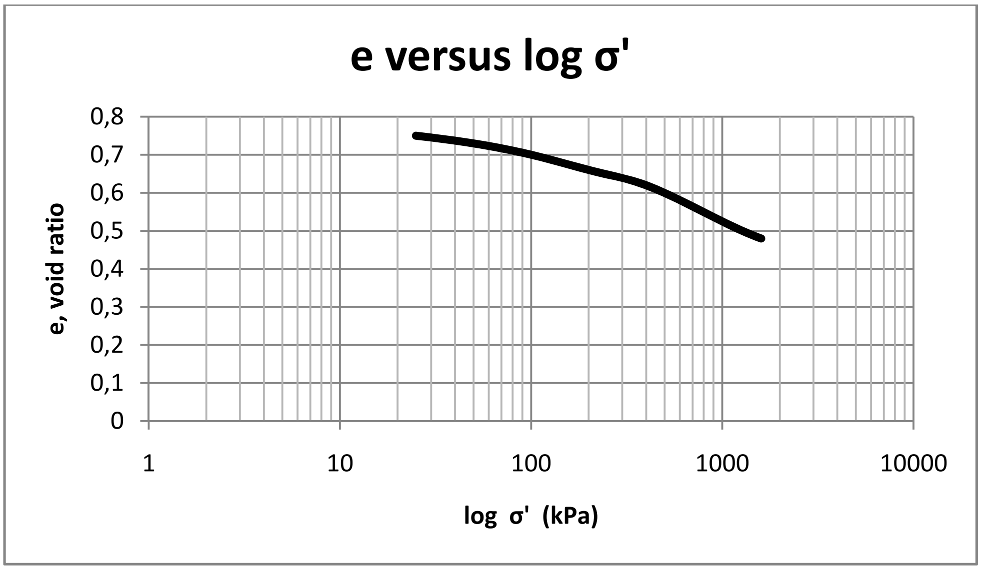


Figure 3.13 e versus log σ graph

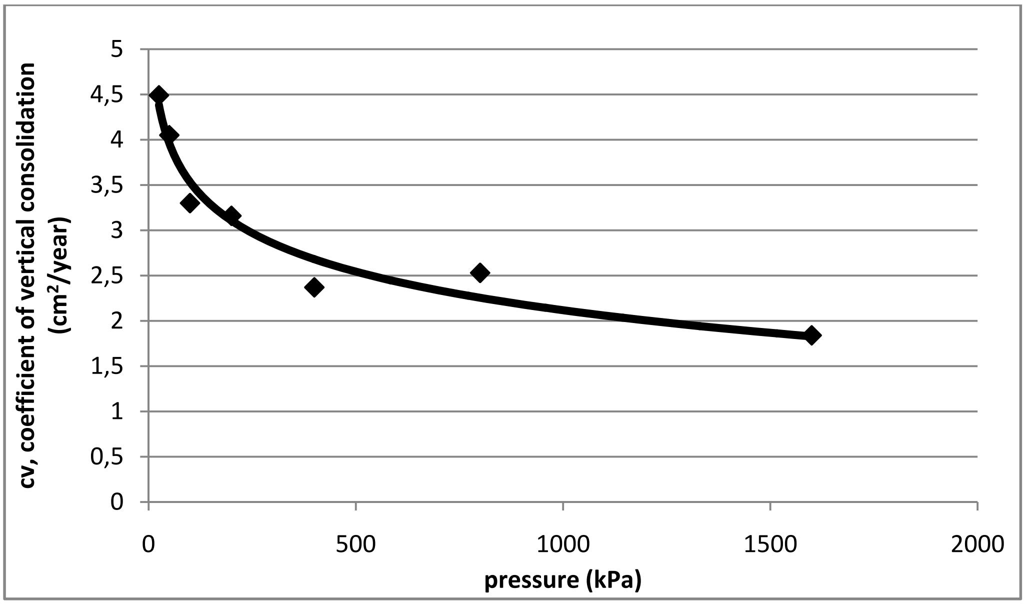


Figure 3.14 c_v versus σ graph

5) Vane Shear Test

After the samples in the boxes had been consolidated, prior to the placement of the model foundation system, all the boxes were tested using vane shear apparatus. The sufficiency of the samples prepared, was decided according to the results of these tests.

6) Triaxial Compression Test and Unconfined Compression Test

Right after the application of the loading, the sample soils under the model footings were further tested in order to sustain the consistency of the experiments. These tests also confirmed the suitability of the boxes with each other. The triaxial tests were performed unconsolidated undrained.

7) Moisture Content Determination

Moreover, after the loading process, the moisture contents of each clay filled box were determined from the samples taken from different parts of the boxes. These tests were done in order to check the homogeneity of the boxes and control if the soil was dried more than expected or not.

It should further be noted that with the last mentioned three tests, the soil specimens which the model systems had been applied, was checked to be sufficient or not. The efficiency and appropriateness of the experiments were guaranteed.

3.1.3.2 Laboratory Testing Program For Model Piled Raft

1) Preparation Of the Foundation Model

In order to achieve a reliable result, a series of experiments needed to be conducted. Since the experimental study was aimed to focus on only one variable, other properties should be kept constant including the properties and behavior of the soil samples. This standardization was achieved by performing the same methods from the first step, remolding, until the last step.

For preparing the soil sample, firstly the kaolin was remolded to the desirable water content, in this study this value was 40%. The remolding process was conducted with the mixer until the sample was homogeneous and the desired water content was achieved. The preparation is shown in Figure 3.15.

Figure 3.15 Remolding process

After the remolding process had been done, the samples are put into plastic bags, which provided maintenance of the samples' water content. Then they were placed in the humidity room and kept there until they were prepared for consolidation. This resting period was at least five or six days.

2) The Testing Procedure

There were 16 boxes prepared and tested throughout the study. The testing procedure was same for all boxes and for all pressures. The steps can be summarized as follows.

i) The plexiglas box, 20 x 20 x 20 cm in dimensions was covered with geotextiles to prevent drying of soil during testing and stabilize the drainage conditions.

ii) The sample was taken from the humidity room and placed in the plexiglas box covered with geotextiles. In the boxes, replacement of clay was done layer by layer and special attention was given not to create any air voids in between. Then the box was closed with a plexiglas cap.

iii) The prepared box filled with clay was then placed into the loading frame system for consolidation. The piston was arranged to press the cap. The loading jack was fixed to give a pressure of 102 kPa approximately on the sample. The boxes stay under consolidation for about three weeks. During the consolidation process, the procedure was controlled by the dial gages fixed at the cap of the boxes. Moreover, continuous moisturizing was applied to prevent drying of the sample.

iv) After the consolidation period was completed, the box was taken from the loading jack and prepared for the model testing. The cap of the box was opened and first 3 cm of soil was removed assuming that this portion will be disturbed. Then the surface was smoothed and leveled. The surface was covered with a thin nylon sheet that has a square cut in the middle. This nylon sheet prevents drying of surface during the experiment.

v) In order to insert the model system in desired pattern, templates are prepared from cardboard. The patterns used will be described further in this section. Using the cardboard and timber templates, brass nails which 2 mm in diameter and 75 mm in length were inserted. The insertion was made slowly, in a continuous and steady manner.

vi) Above the brass nails, the model aluminum raft which has dimensions of 5 x 5 x 1 cm was placed. The box was then put into the testing apparatus.

vii) In the testing apparatus, the loads corresponding to the desired pressures were hung to the load hangers. Lvdt's were placed at two opposite corners of the thin metal sheet welded at the top of the aluminum footing. For further checking, a dial gage was also fixed to this thin metal sheet.

viii) The loading road was then released and simultaneously the software was started. The recordings were taken more frequently for the first hours, then the intervals between the readings were increased. The testing period had been changed from five to ten days, regarding to the testing model.

3) The Testing Schedule

The three different model systems used in the experiments can be listed as follows:

- The raft foundation alone
- The raft foundation consisting of 16 piles (Figure 3.16)
- The raft foundation consisting of 49 piles (Figure 3.17)

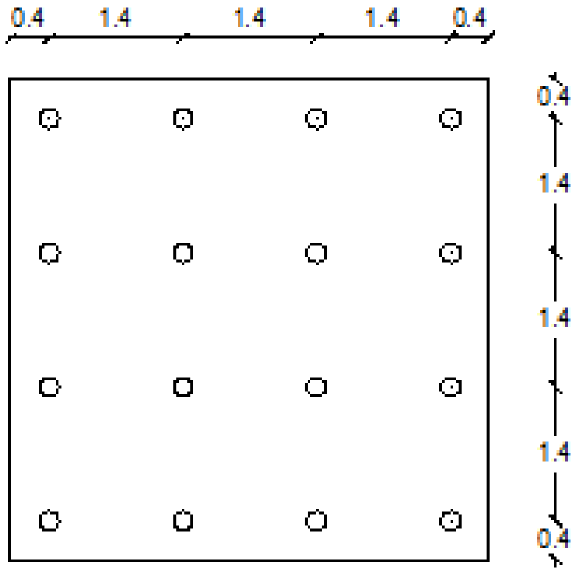


Figure 3.16 Pattern for 16 piles (all in cm)

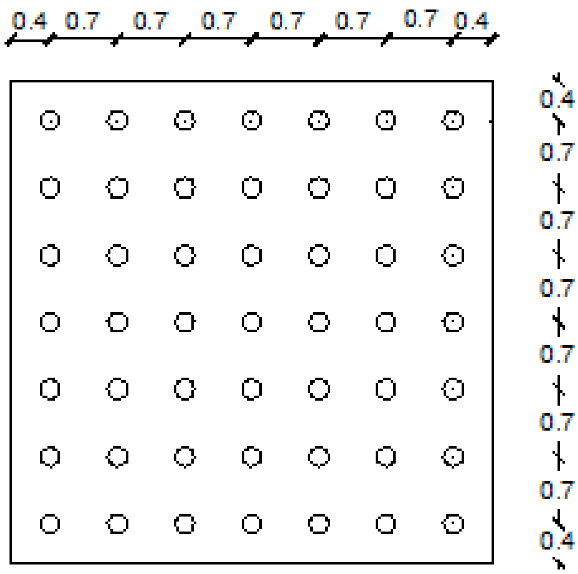


Figure 3.17 Pattern for 49 piles (all in cm)

During the study, there were 16 boxes prepared in order to achieve the best result. The tests were conducted for the same pattern at the same pressure until consistent results had been determined. In the tests, two pressures have been applied to the model aluminium raft, i.e. 40 kPa and 75 kPa.

3.1.4 Results of The Experimental Study

As mentioned before a series of experiments had been conducted to see the effect of number of piles on the settlement behavior. Since only the number of piles effect was the point of concern, all other parameters kept constant, such as length of the piles, width of foundation, soil properties, pile diameter etc.

In order to ensure the reliability of the results, the soil parameters were frequently checked. Some laboratory tests were performed to each sample, both before and after the application of the desired loading to the sample. Below in Table 3.3, the results of these tests are given for each box.

Table 3.3 Results of the laboratory tests performed on samples

		Before loading stage		After loading stage		
Box	Description	Moisture content	Vane test result	Moisture content	Triaxial test result (c)	Unconfined comp. test result (q_u)
B5	7x7 (49 piles)	40%	c = 28.17 kPa	30%	37.0 kPa	55.5 kPa
B6	no pile	40%	c = 29.54 kPa	30%	43.7 kPa	56.8 kPa
B7	4x4 (16 piles)	40%	c = 33.23 kPa	31.6%	32.0 kPa	36.5 kPa
B8	no pile	40%	c = 24.81 kPa	32.4%	27.4 kPa	30 kPa
B9	4x4 (16 piles)	40%	c = 30.77 kPa	31.3%	30.4 kPa	35.2 kPa
B10	7x7 (49 piles)	40%	c = 26.25 kPa	30.4%	39.6 kPa	45.9 kPa
B11	4x4 (16 piles)	40%	c = 39.11 kPa	31.1%	28.5 kPa	36.6 kPa
B12	no pile	40%	c = 24.82 kPa	32.3%	18.7 kPa	14.0 kPa
B13	4x4 (16 piles)	40%	c = 25.02 kPa	31.8%	22.9 kPa	19.8 kPa
B14	no pile	40%	c = 24.61 kPa	32.4%	13.8 kPa	9.8 kPa
B15	4x4 (16 piles)	40%	c = 23.79 kPa	32.7%	N/A	N/A
B16	no pile	40%	c = 21.95 kPa	33.1%	N/A	N/A

The settlements of the raft measured for each box is given in Table 3.4. Please note that, the results presented herein exclude the first four boxes, since they can be accepted to serve for the calibration purposes.

Table 3.4 Settlements obtained for each box

Box	Description	Load	Measured settlement
B5	7x7 (49 piles)	75 kPa	1.86 mm
B6	no pile	75 kPa	2.37 mm
B7	4x4 (16 piles)	75 kPa	2.30 mm
B8	no pile	75 kPa	2.74 mm
B9	4x4 (16 piles)	75 kPa	2.05 mm
B10	7x7 (49 piles)	75 kPa	1.80 mm
B11	4x4 (16 piles)	75 kPa	2.11 mm
B12	no pile	75 kPa	2.55 mm
B13	4x4 (16 piles)	75 kPa	2.09 mm
B14	no pile	75 kPa	2.77 mm
B15	4x4 (16 piles)	40 kPa	1.29 mm
B16	no pile	40 kPa	1.91 mm

The consistency of the results can be seen in the Figure 3.18. Since, the experimental data is consistent with each other, the average values can be taken as representative of the behavior. The graphs given in Figures 3.19 and 3.20, respectively for 40 kPa and 75 kPa, show the settlement behavior and pile number relationship.

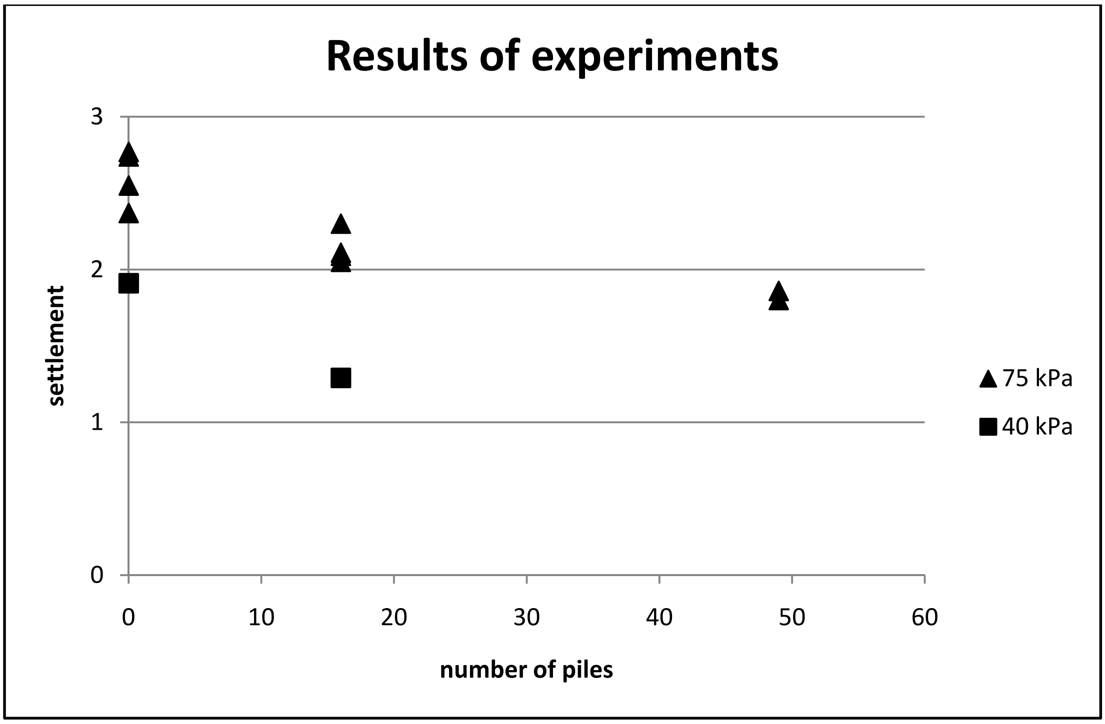


Figure 3.18 Consistency of the results

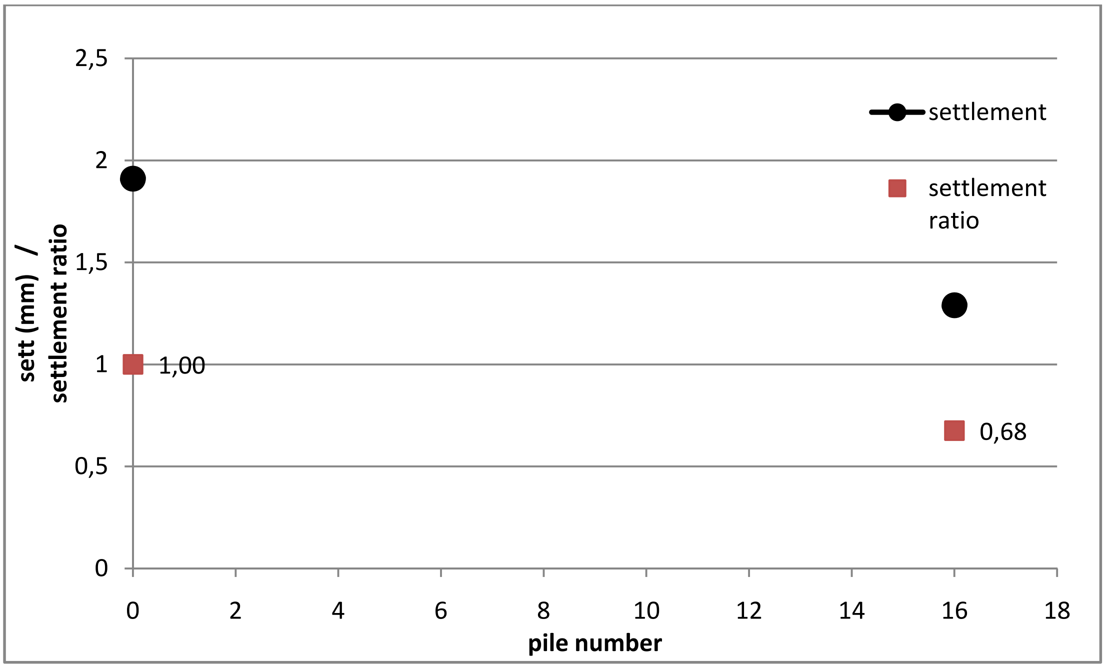


Figure 3.19 Pile number versus settlement graph at 40 kPa pressure

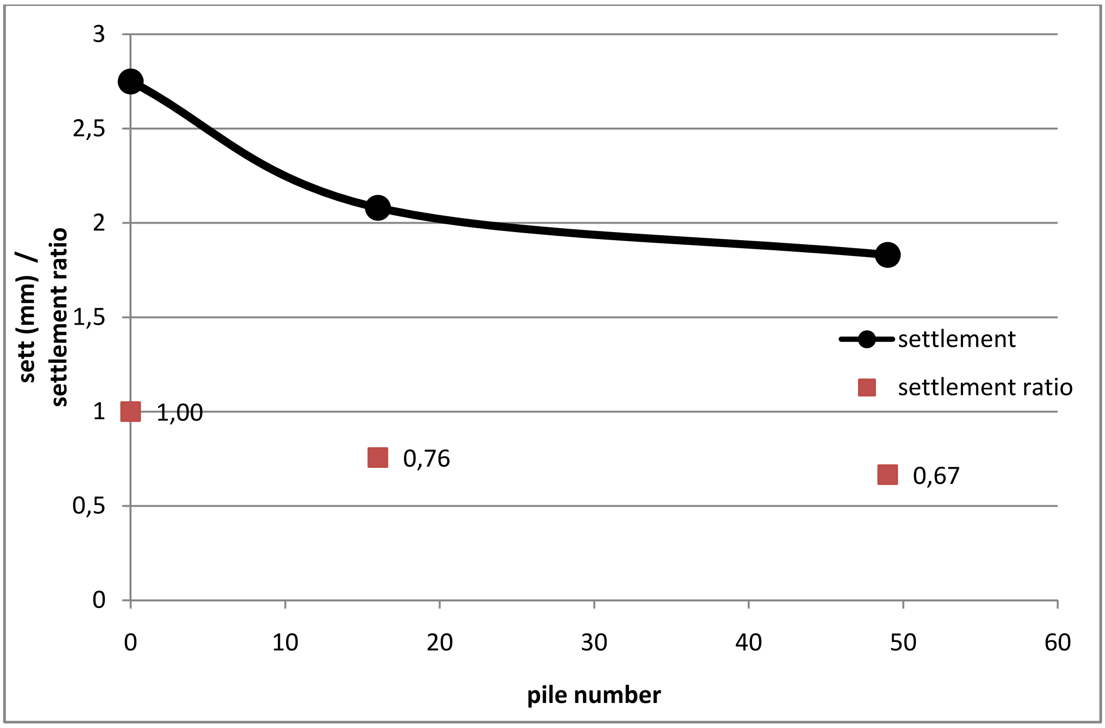


Figure 3.20 Pile number versus settlement graph at 75 kPa pressure

By observing Figure 3.20, reasonable and applicable results can be obtained. It can clearly be seen that increasing number of piles does not necessarily decrease the settlement at a considerable amount, thus from a point the behavior tend to turn into a steady phase.

When the dimensions of the raft is considered, the models used for the experiments can be said to be the ones which have the maximum number of piles, i.e. 49 piles and the average number of piles, i.e. 16 piles. The raft without piles can be accepted as a reference data, however the values obtained are very useful for interpretation of the test data. Thus, considering the results of these series of experiments, it can be said that after an optimum point, placing more piles below the raft does not have a considerable influence on the settlement. When raft is adequate for capacity, addition of piles has a significant positive effect from settlement point of view. However, adding piles in excess numbers is unnecessary.

It should further be noted that, in the experimental study, the model piles had been yielded, so the factor of safety values for model piles were most likely very close to unity. The model piles in the system behaved like settlement reducing piles.

These results gathered from the experimental study will be compared with the analytical studies on settlement reducing piles which will further be described in the next section.

3.2 The Analytical Studies on Settlement Reducing Piles

As mentioned before, though the concept of pile foundations has first been developed to solve settlement problems, capacity based design was the general trend. Because of that, settlement analysis did not receive any attention, thus the present methods of settlement analysis are generally not very sophisticated. However, nowadays engineers have begun to realize the importance and advantages of the settlement based design methods. So, the settlement reducing piles become one of the recent trends. In order to analyze these kinds of systems, analysis procedures which take into account some important aspects should be used. These aspects can be summarized as follows:

- The load sharing between raft and piles should be taken into account since the bearing capacity problem can only be solved by the raft and piles carry a lower portion of the total load.
- The factor of safety values for the piles, the raft and the system should be taken into account separately, since factor of safety for piles will be taken as unity.
- Separate stiffness values for piles, raft and the system should be taken into account since each element of the system has different functions and contributions to the system.
- The analysis method should permit the mobilization of the full capacity for the piles in the system.

There have not been too many methods of analysis which satisfy the required aspects and suitable for settlement reducing piles. However, this kind of analysis can easily be coped with boundary element approaches which are available with some codes written by some researchers. Though there are a few analytical approaches that can be managed with simple hand calculations.

Some methods in the literature which can be used in settlement analysis for these systems were discussed in Chapter 2. These can be listed as follows:

- Fleming et al., 1992
- Clancy and Randolph, 1993
- De Sanctis, 2002

In this chapter, method proposed by Clancy and Randolph will be used. The results will be checked according to the behavior obtained from the experimental study.

3.2.1 Clancy and Randolph, 1993

The model that has been used in this chapter has dimensions proportional with the model used in the experiments, the materials and soil properties remain same as the experimental study.

There have been three systems analyzed; the raft foundation alone, raft with 16 piles, raft with 49 piles and raft with 81 piles. The patterns for 16 piles and 49 piles are shown in Figures 3.21 and 3.22.

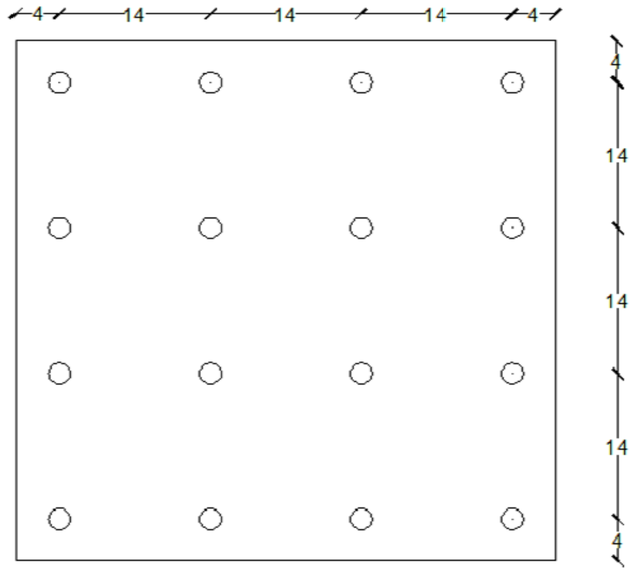


Figure 3.21 Pattern for 16 piles (all in m)

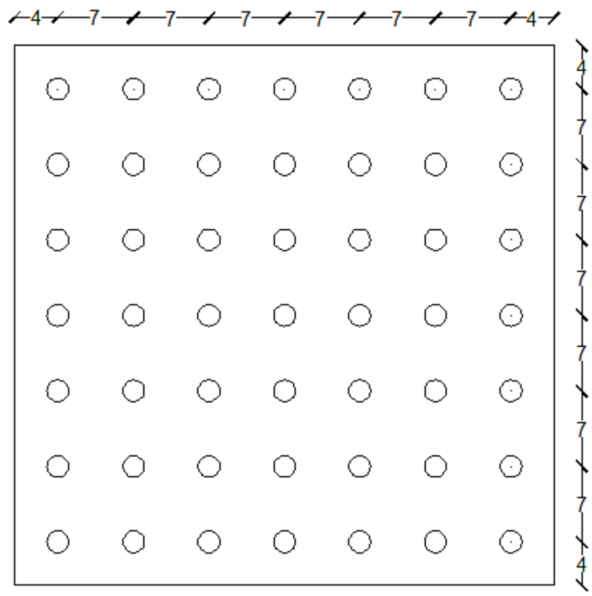


Figure 3.22 Pattern for 49 piles (all in m)

The raft is 50 x 50 x 10 m in dimension and piles are 2 m in diameter and 75 m in length. The material properties of raft and soil are kept constant. Also properties and classification of soil have been kept constant.

The idea behind the method is calculating an overall stiffness for the system derived from the separate stiffness of both pile and raft. The calculations are made using the formulae given in Section 2.1.3.2 as Equations 11 to 17. These formulae have also been presented below.

$$r_m = 2.5L\rho(1-\upsilon)$$

$$k_p = \frac{2\pi L G_L}{\ln\left(\frac{r_m}{r_o}\right)}$$

$$k_r = \frac{2G_L}{I(1-\upsilon)}\sqrt{(B\times B)}$$

$$\alpha_{rp} = 1 - \frac{\ln(n)}{\ln\left({}^{2r_m}/_{d}\right)}$$

$$k_{pr} = \frac{\left[k_p + k_r\left(1-2\alpha_{rp}\right)\right]}{\left[1-\left({}^{k_r}/_{k_p}\right)\alpha_{rp}^2\right]}$$

$$w_{pr} = \frac{total\ load}{k_{pr}}$$

In above formulation, ρ value is calculated by dividing shear modulus at the half length of the pile to the shear modulus at the total length of the pile and I is the influence factor for the raft which is taken from the related tables given in Appendix D.

The values used in the settlement calculation are showed in Table .3.5.

The settlement calculation of the models based on the mentioned method is presented in Table 3.6. Also settlement of the unpiled raft values are presented in mentioned table as w_{ur}.

Table 3.5 Properties of the model

Raft	**B (m)**	50
	t (m)	10
	ν_r	0.35
Pile	**I (cap)**	0.82
	d (m)	2
	L (m)	75
	r_o (m)	1
	r_m (m)	80.96591
Soil	**G_L (kPa)**	8250
	ρ	0.86
	ν_s	0.5

Table 3.6 Settlement calculation of the model

Press. (kPa)	n (# of piles)	k_r	k_p	α_{rp}	α_{pr}	k_{pr}	w_{pr} (cm)	w_{ur} (cm)
75 kPa	raft alone	1266416.51	884773.77	-	-	-	14.8	14.8
	16 (4x4)	1266416.51	884773.77	0.369	0.528	1511062	12.4	14.8
	49 (7x7)	1266416.51	884773.77	0.114	0.164	1897177	9.9	14.8
	81 (9x9)	1266416.51	884773.77	-1E-04	-1E-04	2151433	8.7	14.8
40 kPa	raft alone	1266416.51	884773.77	-	-	-	7.9	7.9
	16 (4x4)	1266416.51	884773.77	0.369	0.528	1511062	6.6	7.9
	49 (7x7)	1266416.51	884773.77	0.114	0.164	1897177	5.3	7.9
	81 (9x9)	1266416.51	884773.77	-1E-04	-1E-04	2151433	4.6	7.9

For further checking of the values of α_{rp} and α_{pr}, two other formulae can be used. These were also given in Section 2.1.3.2 with Equations 18 and 19.

$$\alpha_{rp} = \frac{k_p}{P_p}\left(w_{pr} - \frac{P_r}{k_r}\right)$$

$$\alpha_{pr} = \frac{k_r}{P_r}\left(w_{pr} - \frac{P_p}{k_p}\right)$$

The load sharing between raft and piles is given by Fleming et al., 1992.

$$\frac{P_r}{P_r+P_p} = \frac{k_r(1-\alpha_{rp})}{k_p+k_r(1-2\alpha_{rp})} \qquad \textbf{\textit{(83)}}$$

Thus, the recalculation of the α_{rp} and α_{pr} values is given in Table 3.7.

As seen from the table below, the values are nearly the same with the values found formerly. So, the values have been crosschecked.

Table 3.7 Recalculation of the α values

		$P_r / (P_r + P_p)$	P_r (kN)	P_p (kN)	α_{rp}	α_{pr}
75 kPa	16 piles	0.657	123160.22	64339.78	0.368	0.527
	49 piles	0.602	112968.41	74531.59	0.116	0.165
	81 piles	0.589	110437.50	77062.50	-0.002	-0.001
40 kPa	16 piles	0.657	65685.45	34314.55	0.364	0.525
	49 piles	0.602	60249.82	39750.18	0.121	0.170
	81 piles	0.589	58900.00	41100.00	-0.011	-0.010

3.2.2 Results of The Settlement Analysis of Settlement Reducing Piles

Since the concept of settlement reducing piles is recently developed, there are a few methods and some codes which can be considered suitable for this kind of analysis. In this study, the analyses have been made using only the method presented by Clancy and Randolph, 1993.

The resulting settlements calculated by aforementioned method have already been given in Table 3.6. The figures presenting the relationship between the settlement and pile number are given in Figures 3.23 and 3.24 for pressures of 75 kPa and 40 kPa respectively.

In these figures, it can be seen that though adding piles to the raft decreases settlement to the desired amount, there can be seen a tendency to turn into a straight line.

However, it can be seen that this mentioned behavior is not very similar to the behavior derived at the experimental study. The comparison and discussion related to the settlement behaviors obtained from both experimental and analytical studies will be given further in Chapter 5.

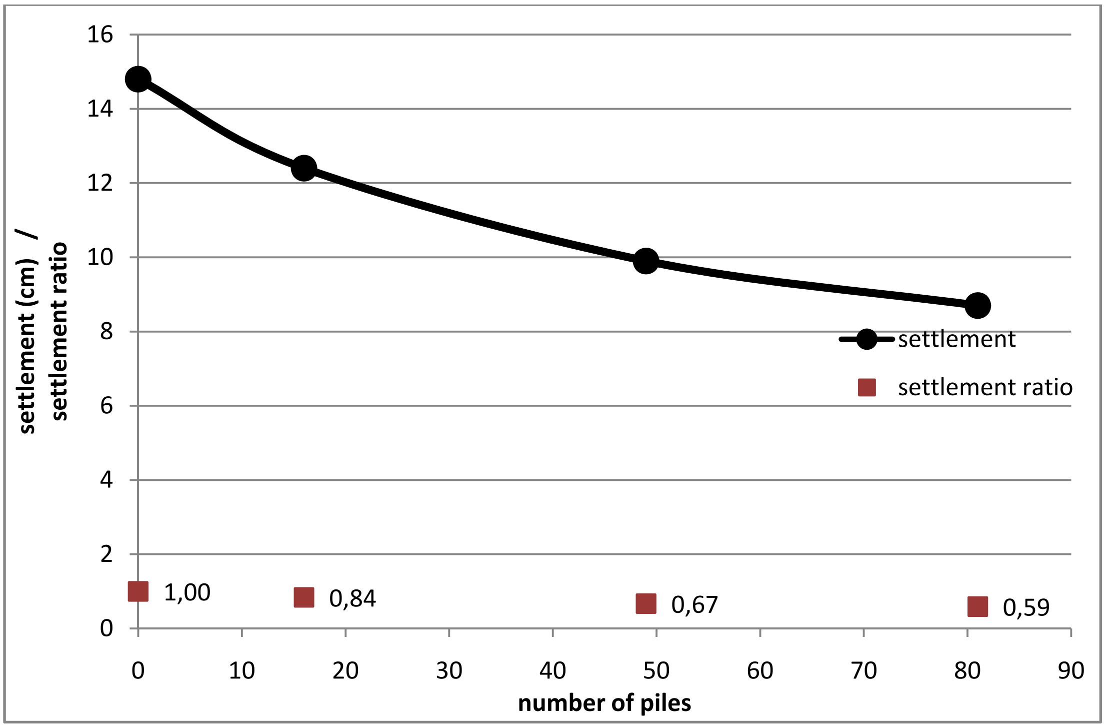


Figure 3.23 Settlement versus pile number relationship for 75 kPa pressure

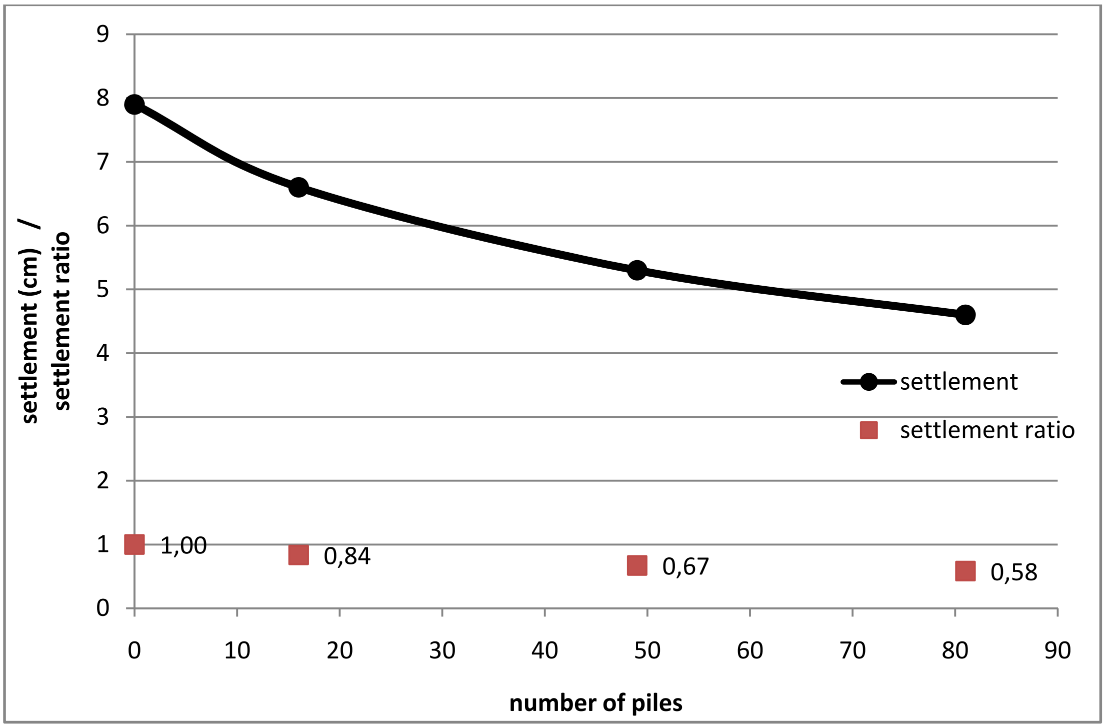


Figure 3.24 Settlement versus pile number relationship for 40 kPa pressure

CHAPTER 4

SETTLEMENT ANALYSIS OF PILED RAFT FOUNDATIONS

4.1 The Piled Raft Foundation Models Used For The Analysis

In this chapter of the aforementioned study, the settlement of piled raft will be analyzed using some simple hand calculations and a finite element analysis software. The main goal of the research is showing the effect of pile number to the settlement behavior of the piled raft foundations.

For the analysis made for investigating the settlement behavior of the piled raft foundations, a simple model has been used with increasing pile number. Thus, a residential building with no basement will be supported by a piled raft foundation. The load transferred to the base is 450 kPa.

The model consists of a rectangular raft with dimensions of 24 x 28 m and thickness of 2 m. The diameter of piles is 1 m. The analyses have been made in two sets; for the first set pile length has been 25 m, whereas for the second set it has been 30 m. Table 4.1 shows the pile numbers considered for each set. Please note that, the marked values are applied only for the finite element analyses.

Table 4.1 Pile numbers considered for each set

Set	Pile length	Number of piles
1	L = 25 m	143 (13x11)
		120 (12x10)
		99 (11x9)
		80 (10x8) *
2	L = 30 m	120 (12x10)
		99 (11x9)
		80 (10x8)
		63 (9x7) *

At both sets for all of the different models considered pile spacing is kept as 2.25 m. Moreover, for the whole case; the ultimate shaft capacity of the piles is taken as 197.92 kN / m and the ultimate end bearing capacity is taken as 989 kN.

The modulus of elasticity for the piles and the raft has been accepted as $3x10^7$ kPa; Poisson's ratio value is 0.1 and the unit weight is 2400 kg / m^3.

The soil beneath the desired construction is normally consolidated medium clay which has an increasing modulus of elasticity with depth. The properties of the soil medium are tabulated below in Table 4.2. The properties are decided according to the commonly used cases for soil conditions under this type of buildings. Please note that it is assumed that no water table is encountered.

Table 4.2 Properties of the soil used in the analysis

c_u	undrained shear strength	140 kPa
c'	appearent cohesion	5 kPa
E_u	undrained modulus of elasticity	40 MPa
E	modulus of elasticity	30 MPa at the surface, increasing linearly with 500 kPa for 1 m depth
Φ'	friction angle	25^{o}
v_u	undrained Poisson's ratio	0,35
v	Poisson's ratio	0,20
m_v	coefficient of volume compressibility	0.0005 cm^2 / kg
γ	unit weight of soil	20 kN / m^3
α	skin friction factor	0.45^{o}

4.2 Analytical Studies For Settlement Calculation of Piled Raft Foundations

The general concepts and methods regarding to the settlement based design have been explained in detail in Section 2.2.

As mentioned before, the methods that can be used for analysis can be classified as follows:

1) Purely empirical techniques that relate settlement of the group to that of a single pile
2) Simplified techniques that reduce the pile group system to an equivalent raft
3) Analytical methods that consider interaction between piles and the surrounding soil

In this thesis and further in this chapter, the methods fall into the first and third categories have been used for the analysis. This is because the methods fall into the second category, do not give ideas about the settlement behavior and the settlement versus pile number relationship, although they can predict settlement.

4.2.1 Settlement Ratio Method

The method used herein was proposed by Butterfield and Douglas (1981) and one of the applications of the method is presented in Ergun (1995).

This method is based on the belief that settlement of the pile group is related with pile number and stiffness of the pile group. The pile group settlement can be derived from an efficiency factor which considers the effect of the number of piles in the group, the soil properties (like Poisson's ratio) and dimensional constant of the raft.

The formulation will be given below.

$$S = n\frac{P}{K} \tag{84}$$

where, S = the group settlement

P = average load per pile

n = number of piles in a group

K = the group stiffness

$$K = fnk \tag{85}$$

where, f = efficiency

k = individual pile stiffness

(86)

where, e = the exponent which can be obtained by some charts (given in Appendix A)

The exponent,e is depended upon some factors like;

- Length to diameter ratio of the pile

- λ, pile stiffness ratio (E_p / G_s)
- Spacing of the pile
- Slope of the stiffness increase with depth
- Poisson' s ratio

Throughout the study, the exponential "e" has been calculated as given in Table 4.3. Using the calculated values for e, settlements of the model piled raft for different pile numbers have been calculated. The calculated settlement values for single pile are given as w_1. Calculations are tabulated in Table 4.4.

Table 4.3 Calculation for the exponential e

		n	s (m)	d (m)	v	G_S (kPa)	E_P (kPa)	λ	ρ	e
L=25 m	**13x11**	143	2.25	1	0.2	18750	3E+07	1600	0.853	0.68
	10x12	120	2.25	1	0.2	18750	3E+07	1600	0.853	0.68
	9x11	99	2.25	1	0.2	18750	3E+07	1600	0.853	0.68

		n	s (m)	d (m)	v	G_S (kPa)	E_P (kPa)	λ	ρ	e
L=30 m	**10x12**	120	2.25	1	0.2	18750	3E+07	1600	0.833	0.682
	9x11	99	2.25	1	0.2	18750	3E+07	1600	0.833	0.682
	8x10	80	2.25	1	0.2	18750	3E+07	1600	0.833	0.682

Table 4.4 Settlement calculation for the settlement ratio analysis

Length	n	P (load per pile)	s (m)	e	f (n^{-e})	w_1 (m)	k (kN/m)	K	S (m)	S (cm)
25 m	143	2340.25	2.25	0.68	0.0342	0.00353	662421.9	3242218	0.103	10.32
	120	2788.80	2.25	0.68	0.0386	0.00421	662421.9	3065298	0.109	10.92
	99	3380.36	2.25	0.68	0.044	0.0051	662421.9	2882292	0.116	11.61
30 m	120	2788.80	2.25	0.682	0.0382	0.00363	767528.4	3517824	0.095	9.513
	99	3380.36	2.25	0.682	0.0435	0.0044	767528.4	3309074	0.101	10.11
	80	4183.20	2.25	0.682	0.0504	0.00545	767528.4	3092268	0.108	10.82

4.2.2 Modified Method By Shen and Teh, 2002

This method is based on the principles of the load transfer curves. The non-linear load-deformation characteristics of the soil can be modeled with this type of methods. Using this method, settlement behavior observation can be overcome with simple spread sheet calculations.

The formulae used for this method had been explained in detail at Section 2.2.3.3, from Equation 64 to Equation 82.

Some of these equations are also stated below.

$$w_i(z) = \sum_{j=1}^{k} \beta_{ij}\left(1 - \frac{z}{l}\right)^{j-1}$$

$$[h]\{\beta\} = \{P_T\}$$

$$[h_p]\{\beta_p\} = \{P_p\}$$

$$[h_p] = G_l r_o [H]$$

$$[H] = \begin{bmatrix} 2\mu_s\rho + \mu_b & \dfrac{\mu_s(4\rho - 1)}{3} & \dfrac{\mu_s(3\rho - 1)}{3} \\ \dfrac{\mu_s(4\rho - 1)}{3} & \mu_p + \dfrac{\mu_s(3\rho - 1)}{3} & \mu_p + \dfrac{\mu_s(8\rho - 1)}{10} \\ \dfrac{\mu_s(3\rho - 1)}{3} & \mu_p + \dfrac{\mu_s(8\rho - 1)}{10} & \dfrac{4}{3}\mu_p + \dfrac{2\mu_s(5\rho - 2)}{15} \end{bmatrix}$$

$$\mu_s = \pi\alpha_p l/r_o/\zeta$$

$$\mu_b = 4\alpha_{pb}/(1 - \nu_s)/\xi$$

$$\mu_p = \pi\lambda n_p/l/r_o$$

$$\frac{P}{G_l r_o w_t} = \frac{1}{\sum_{i=1}^{3}\sum_{j=1}^{3} H_{ij}^{-1}}$$

$$\frac{P}{G_l r_o w_t} = \frac{2\pi l \alpha_p}{r_o \zeta} + \frac{4\alpha_{pb}}{\xi(1-\nu_s)}$$

$$\alpha_p = \sum_{i=1}^{n_p}\sum_{j=1}^{n_p} f_{sij}^{-1}$$

$$\alpha_{pb} = \sum_{i=1}^{n_p}\sum_{j=1}^{n_p} f_{sbij}^{-1}$$

The calculations are given in tables 4.5 and 4.6 for 25 m piles and 30 m piles, respectively. Settlements obtained are tabulated below as Table 4.7.

Table 4.5 Settlement calculations of 25 m long piles for load transfer curve analysis

n_p	l/r_o	ρ	λ	v_s	s/r_o	ξ
143	50	0.853	1694.12	0.2	4.5	1
ζ	α_p	α_{pb}	μ_s	μ_b	μ_p	
4.45	4.203	27.012	148.3608	135.06	15221.6	
[H] =	388.1636	119.2821	77.09818	$P/G_l r_o w_t$ =		
	119.2821	15298.69	16227.46	↓		
	77.09818	16227.46	20340.26			
$[H]^{-1}$ =	0.002588	-6.4E-05	4.09E-05	383.2789219		
	-6.4E-05	0.000427	-0.00034			
	4.09E-05	-0.00034	0.00032			

n_p	l/r_o	ρ	λ	v_s	s/r_o	ξ
120	50	0.853	1694.12	0.2	4.5	1
ζ	α_p	α_{pb}	μ_s	μ_b	μ_p	
4.45	3.874	24.483	136.7475	122.42	12773.4	
[H] =	355.7063	109.945	71.06313	$P/G_l r_o w_t$ =		
	109.945	12844.43	13700.5	↓		
	71.06313	13700.5	17072.45			
$[H]^{-1}$ =	0.002826	-8.1E-05	5.31E-05	350.8570716		
	-8.1E-05	0.000543	-0.00044			
	5.31E-05	-0.00044	0.000408			

n_p	l/r_o	ρ	λ	v_s	s/r_o	ξ
99	50	0.853	1694.12	0.2	4.5	1
ζ	α_p	α_{pb}	μ_s	μ_b	μ_p	
4.45	3.54	21.965	124.9577	109.83	10538	
[H] =	323.0029	100.466	64.93637	$P/G_l r_o w_t$ =		
	100.466	10602.96	11385.23	↓		
	64.93637	11385.23	14088.44			
$[H]^{-1}$ =	0.003115	-0.00011	7.18E-05	318.2000431		
	-0.00011	0.000717	-0.00058			
	7.18E-05	-0.00058	0.000538			

Table 4.6 Settlement calculations of 30 m long piles for load transfer curve analysis

n_p	l/r_o	ρ	λ	v_s	s/r_o	ξ
120	60	0.833	1600	0.2	4.5	1
ζ	α_p	α_{pb}	μ_s	μ_b	μ_p	
4.605	3.532	24.483	144.5747	122.42	10053.1	
[H] =	363.2764	112.3827	72.23914	$P/G_l r_o w_t$ = ↓		
	112.3827	10125.34	11009.3			
	72.23914	11009.3	13445.86			
$[H]^{-1}$ =	0.002775	-0.00013	9.4E-05	356.933723		
	-0.00013	0.000906	-0.00074			
	9.4E-05	-0.00074	0.000681			

n_p	l/r_o	ρ	λ	v_s	s/r_o	ξ
99	60	0.833	1600	0.2	4.5	1
ζ	α_p	α_{pb}	μ_s	μ_b	μ_p	
4.605	3.263	21.965	133.5637	109.83	8293.8	
[H] =	332.3422	103.8235	66.73735	$P/G_l r_o w_t$ = ↓		
	103.8235	8360.542	9177.182			
	66.73735	9177.182	11096.96			
$[H]^{-1}$ =	0.003041	-0.00019	0.00014	325.9925974		
	-0.00019	0.001309	-0.00108			
	0.00014	-0.00108	0.000984			

n_p	l/r_o	ρ	λ	v_s	s/r_o	ξ
80	60	0.833	1600	0.2	4.5	1
ζ	α_p	α_{pb}	μ_s	μ_b	μ_p	
4.605	3.02	19.46	123.6171	97.3	6702.06	
[H] =	303.246	96.09167	61.76733	$P/G_l r_o w_t$ = ↓		
	96.09167	6763.832	7519.655			
	61.76733	7519.655	8971.77			
$[H]^{-1}$ =	0.003349	-0.00032	0.000247	296.8214261		
	-0.00032	0.002199	-0.00184			
	0.000247	-0.00184	0.001653			

Table 4.7 Settlement values for load transfer curve analysis

Set	Pile length	Pile number	P (kN)	G_l (kPa)	r_o (m)	$P/G_l r_o w_t$	w_t (m)	w_t (cm)
1	25 m	143	302400	17708,33	0,5	383,2789	0,089109	8,91
		120	302400	17708,33	0,5	350,8571	0,097343	9,73
		99	302400	17708,33	0,5	318,2	0,107333	10,73
2	30 m	120	302400	17708,33	0,5	356,9337	0,095686	9,57
		99	302400	17708,33	0,5	325,9926	0,104767	10,48
		80	302400	17708,33	0,5	296,8214	0,115064	11,51

4.2.3 Results of The Analytical Studies on Piled Raft Foundations

1) Settlement Ratio Method (Butterfield and Douglas, 1981)

The graphical results obtained are given in Figure 4.1.

Analyzing the figure, it can be seen that for both pile lengths the settlement decreases as pile number increases as expected. However, the rate of this decrease in settlement reduces as the pile number increases, in other words, the effect of increasing the pile number at the settlement behavior gets more inactive. This issue points out itself even more in the case of longer piles.

2) Load Transfer Curves (Shen and The, 2002)

The graphical results obtained are given in Figure 4.2.

When this figure is analyzed, a similar relationship with Figure 4.1 can be seen. The decrease rate reduces in this figure too; however this is not very noticeable. Though, a tendency to straighten the curve can be seen at the closing. Another point to mention is that the settlement behavior is almost the same even if the lengths of the piles are different.

The results of the simple hand calculations regarding to the piled raft foundations have further been checked with finite element analysis software. This analysis will be explained later in this chapter. The discussion between the results of different methods and finite element analysis will be given in Chapter 5.

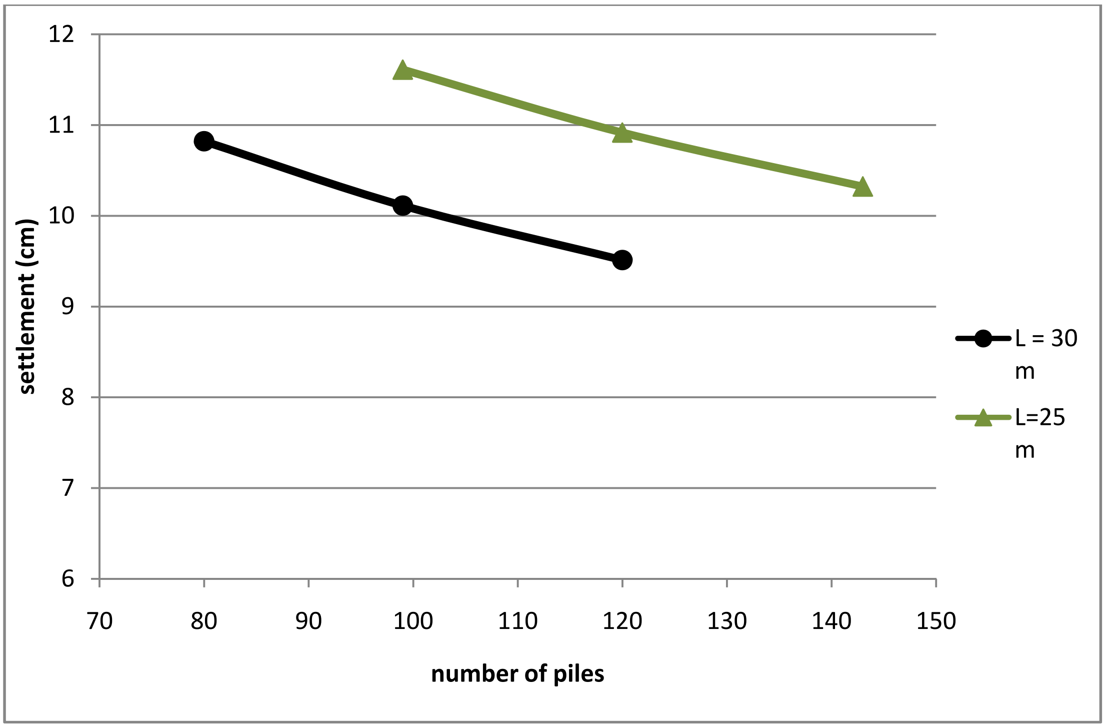


Figure 4.1 Graphical results for settlement ratio method

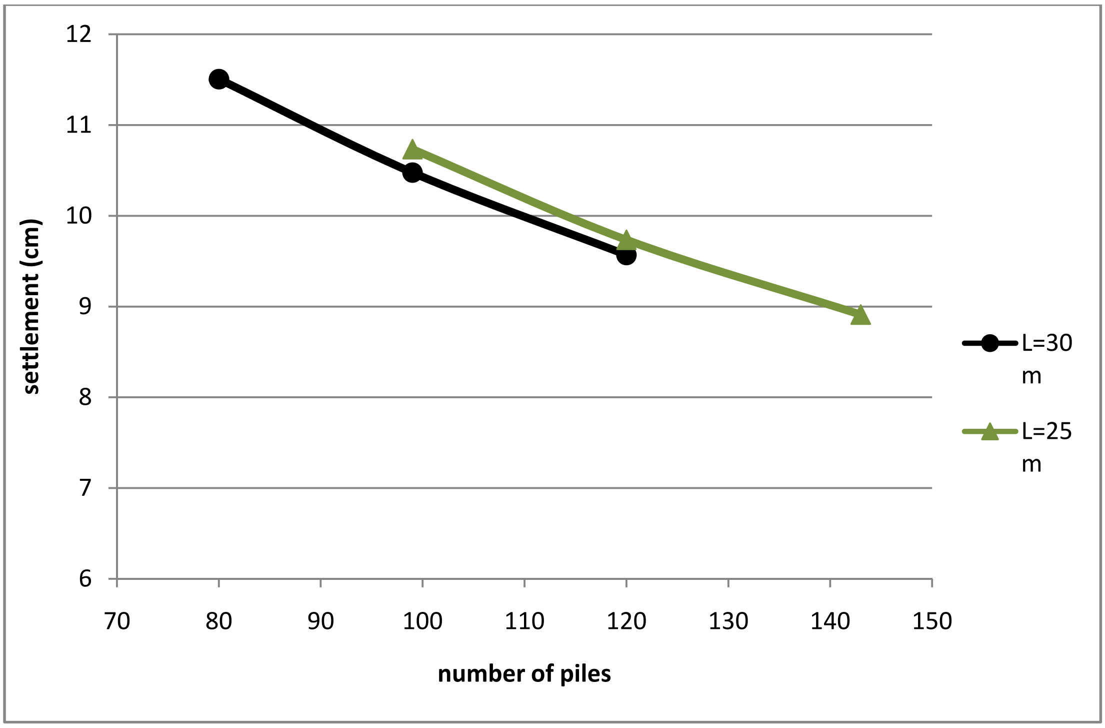


Figure 4.2 Graphical results for load transfer curve analysis

4.3 Finite Element Analysis For Estimating Settlement of Piled Raft Foundations

Finite element analysis is one of the complete analysis methods. Some problems encountered in previously mentioned methods can easily be overcome by complete analysis. Other type of complete analysis is boundary element analysis. The complete analysis can be achieved by some special software and codes.

Comparing to all other methods mentioned, finite element analysis is the most time consuming one and it needs more money and much effort for data preparation than the others. However, this kind of analysis helps to determine the load transfer behavior and the settlement distribution.

In the aforementioned study, in order to check and support the behavior derived from simple hand calculations, a finite element analysis software has been used.

4.3.1 Analysis Using Plaxis 3D Foundation

This software is a 3 dimensional finite element analysis program which all types of foundation structures can be modeled and analyzed with. Special foundation types and elements can be modeled easily. Program is one of the most user-friendly programs in the market since the input procedures and modeling steps can be done with graphical interfaces. It also has a wide range of professional output facilities.

In the scope of this study, the model presented previously in this chapter at section 4.1, has been analyzed using this software. All the parameters entered for modeling the foundation system have already been given in Table 4.2.

In the analysis, Mohr-Coulomb failure criteria had been used and drained analysis had been made since in the soil profile no water table is encountered.

There have been eight models analyzed in two sets as given in Table 4.1. In order to serve as an example, the model designed for 120 piles with lengths of 25 m after meshing is given in Figure 4.3.

After running the program, the output graphs obtained for sets 1 and 2, are given below in Figures 4.4 to 4.11.

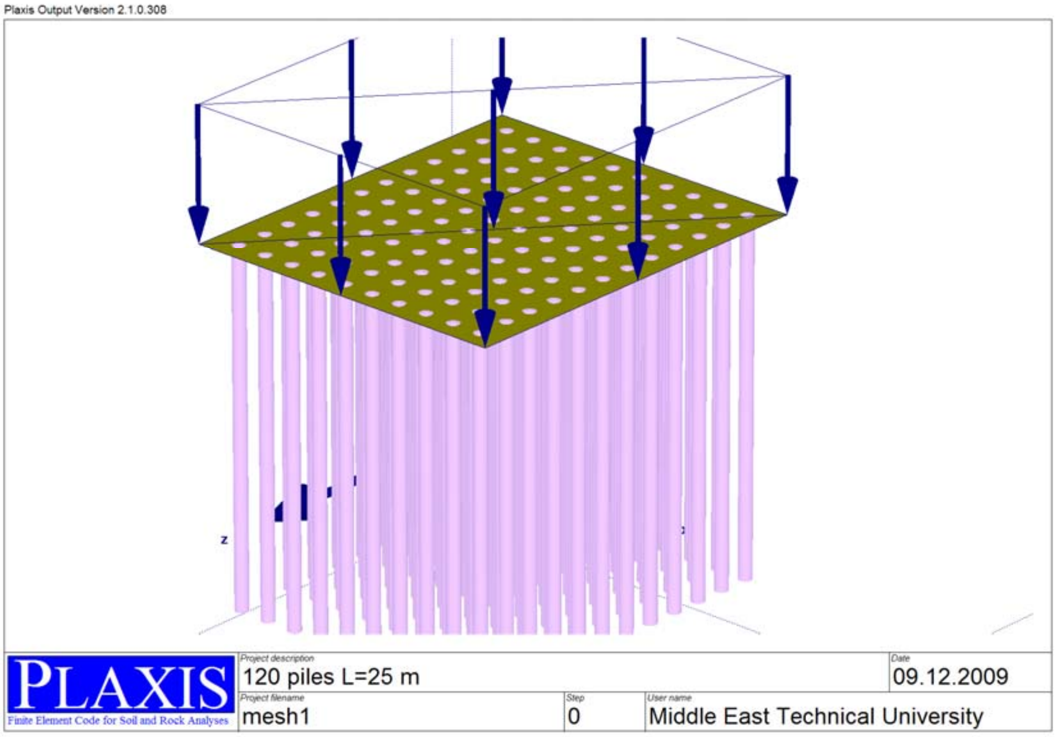


Figure 4.3 Model designed for 120 piles in Plaxis 3D Foundation

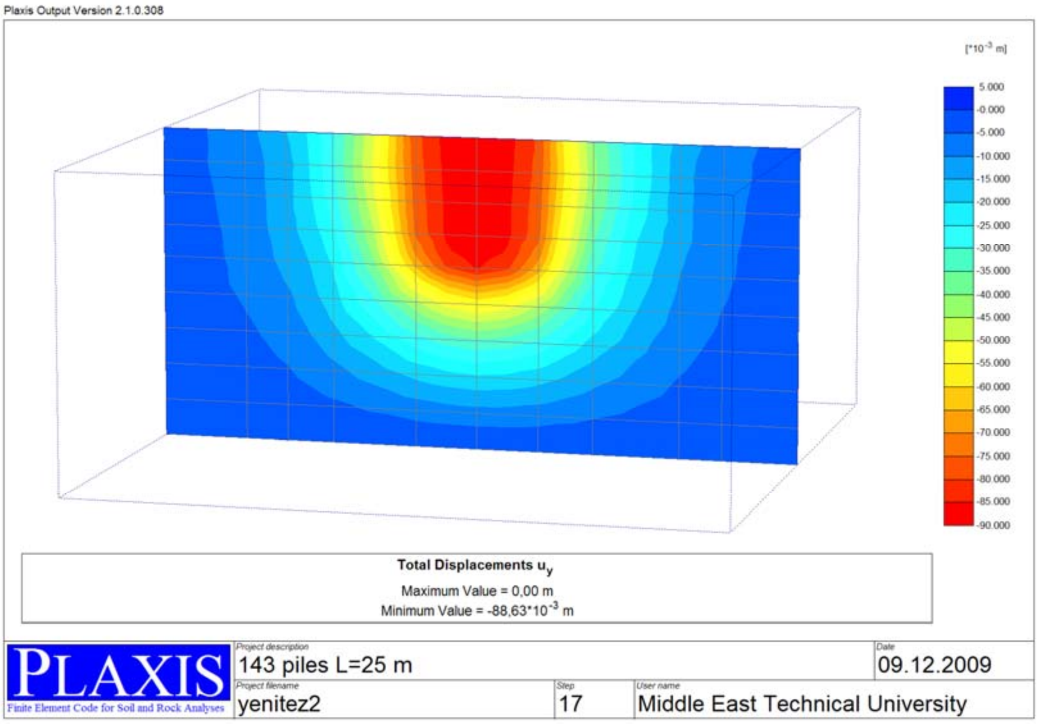


Figure 4.4 Output graph for 143 piles L=25 m

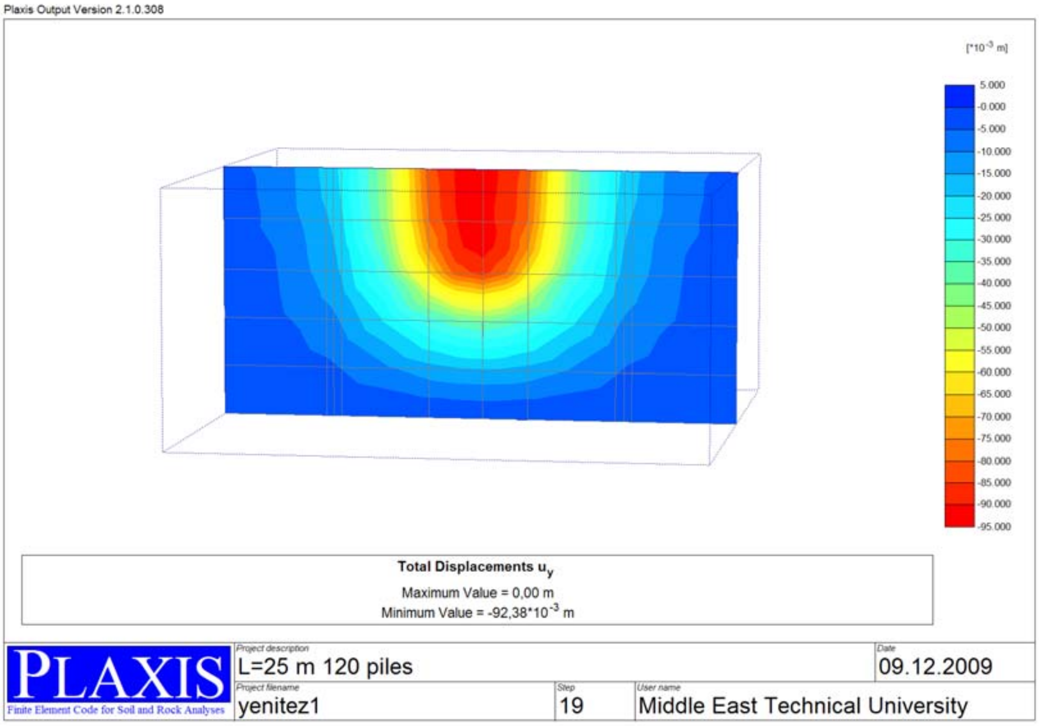


Figure 4.5 Output graph for 120 piles L=25 m

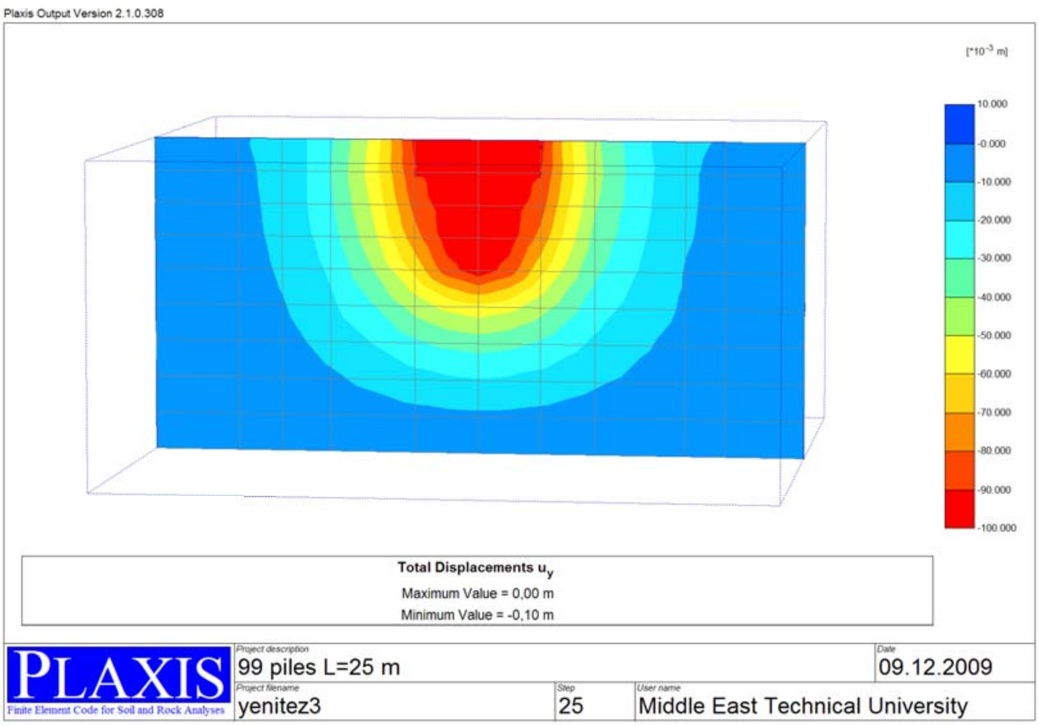


Figure 4.6 Output graph for 99 piles L=25 m

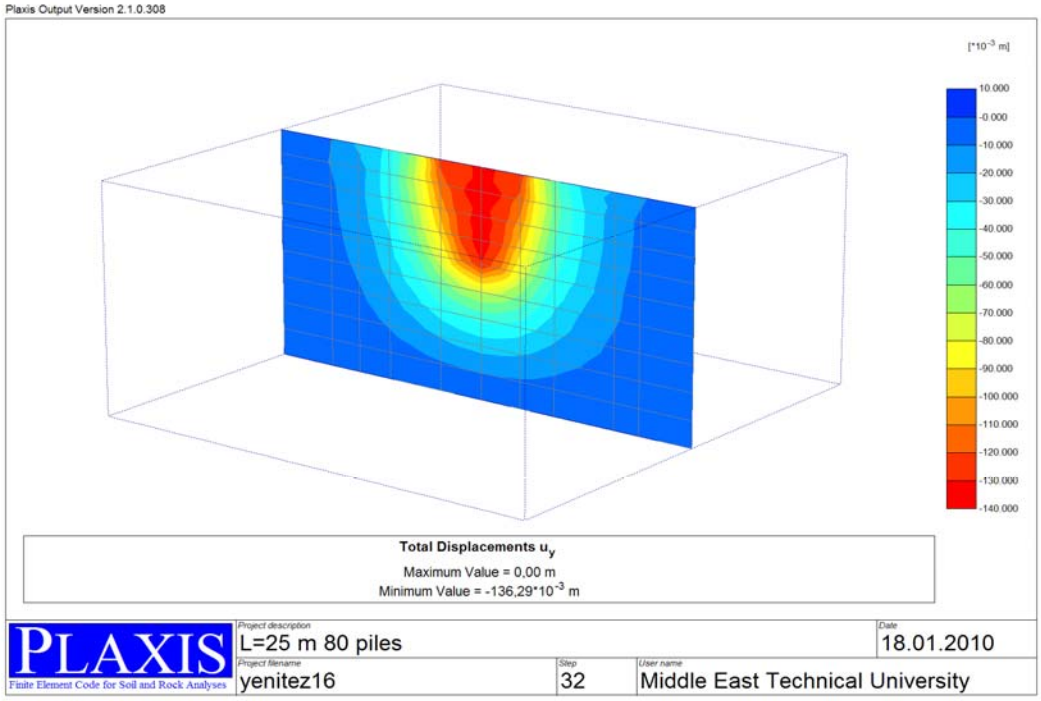


Figure 4.7 Output graph for 80 piles L=25

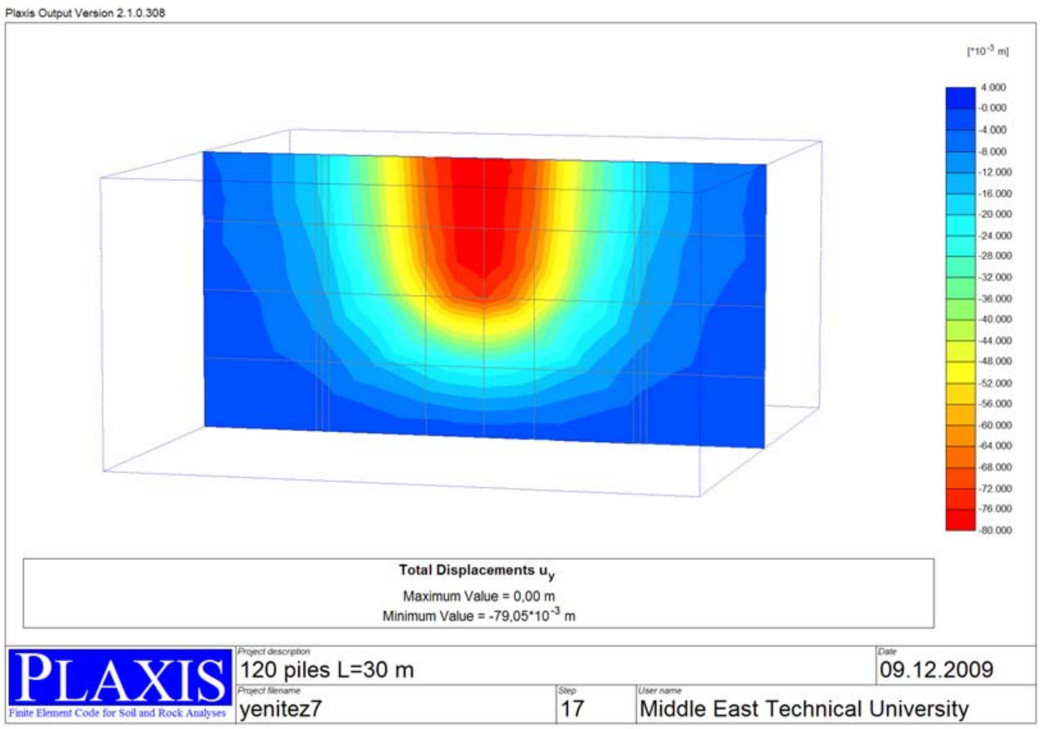


Figure 4.8 Output graph for 120 piles L=30 m

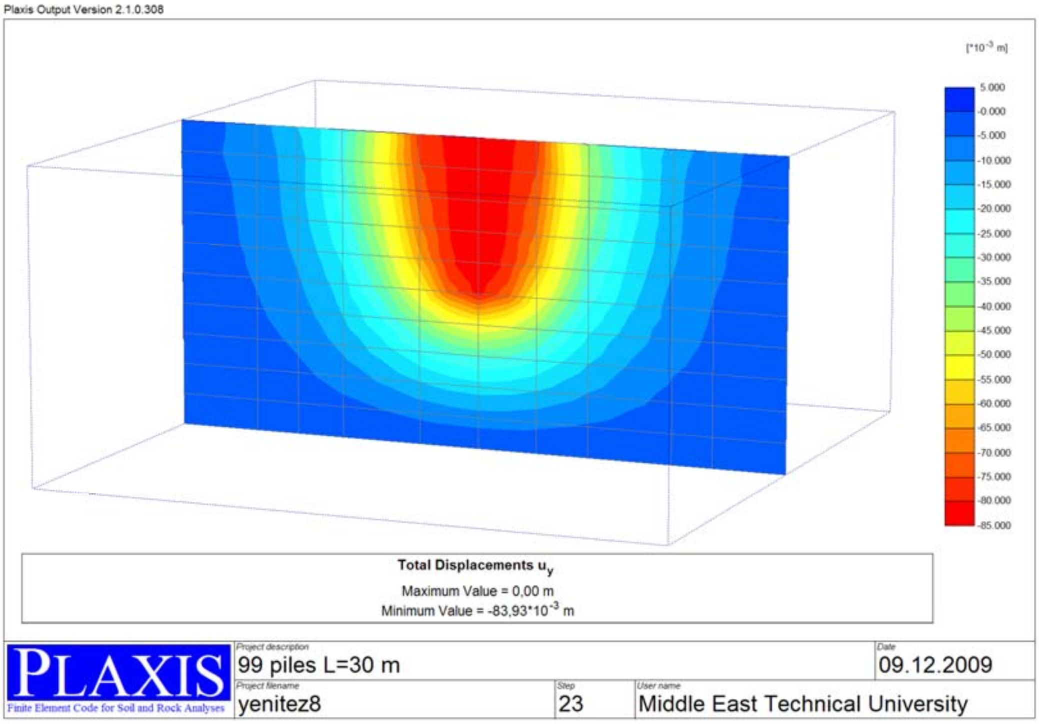


Figure 4.9 Output graph for 99 piles L=30 m

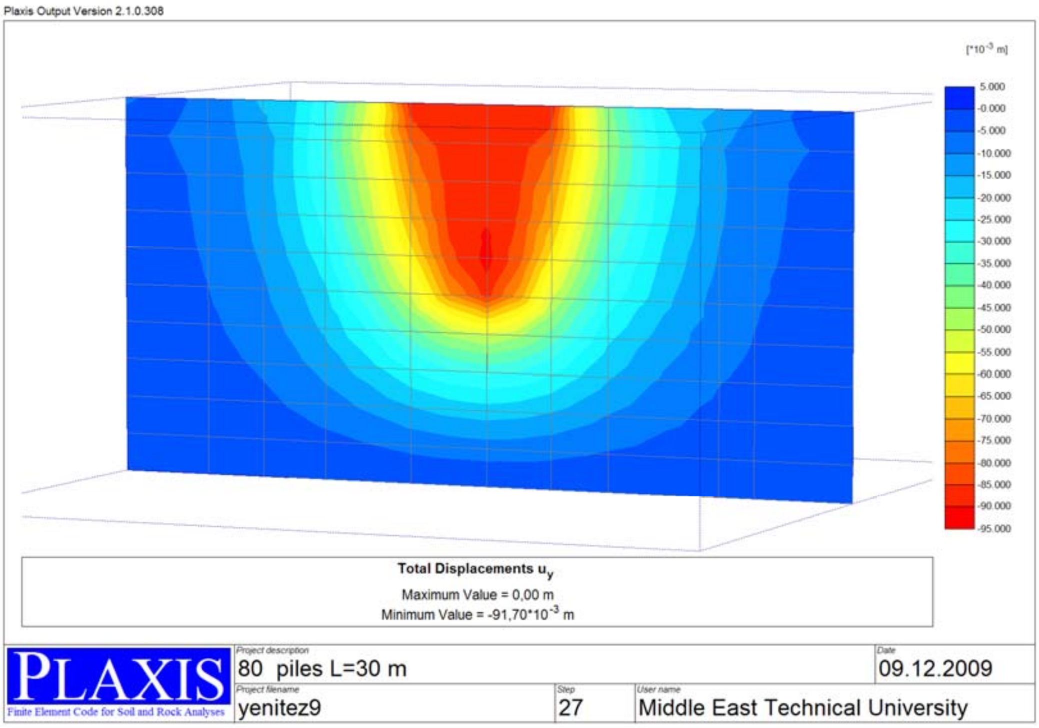


Figure 4.10 Output graph for 80 piles L=30 m

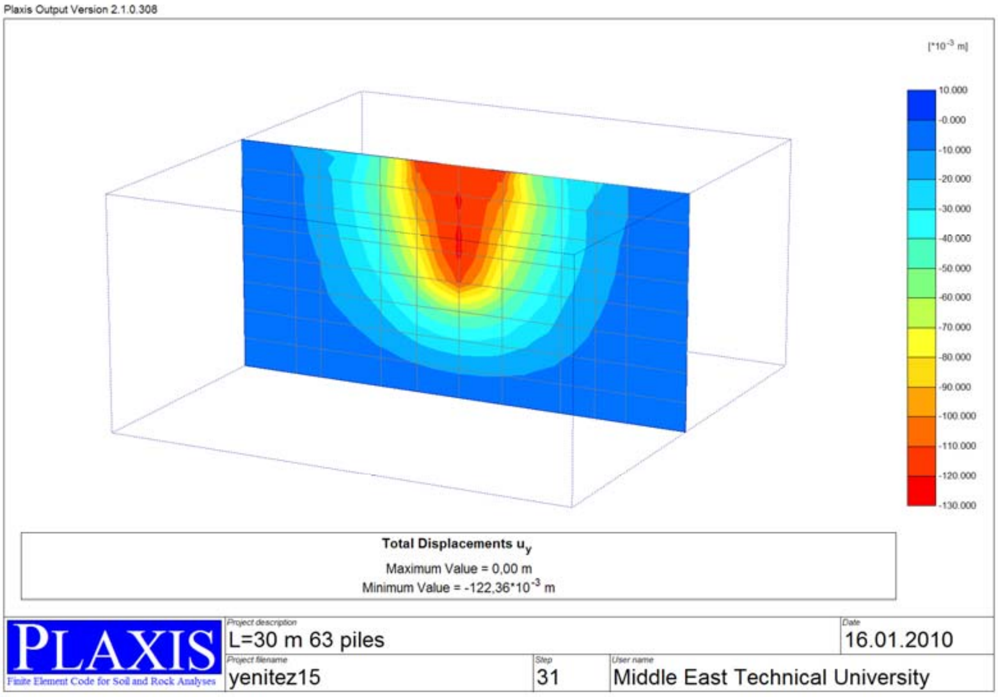


Figure 4.11 Output graph for 63 piles L=30 m

4.3.2 Results of The Finite Element Analysis of The Piled Raft Foundations

The results obtained from 3 dimensional finite element analysis software Plaxis 3D Foundation is given below as Table 4.8.

Table 4.8 Results of the analysis in Plaxis 3D Foundation

PILE LENGTH	# OF PILES	MAX. SETT. (CM)
L = 25 m	143	8.9
	120	9.3
	99	10.0
	80	13.6
L = 30 m	120	7.9
	99	8.4
	80	9.2
	63	12.2

The graphical representation obtained for both set is given as Figure 4.12. Analyzing this figure, it can easily be seen that the reduction in settlement decreases when pile number increases. In other words, increasing the pile number more and more will not always mean that settlements decrease significantly. So, it should not be forgotten that, realizing there is an optimum number of piles which can fulfill the design requirements; an economic and still safe design can be accomplished. Thus, observing the results of the analysis it is clear that beyond an optimum number, the settlement values no further decrease.

As mentioned before, the finite element studies will further be checked against the simple hand calculation results in Chapter 5 of this thesis. Thus, a detailed and more general conclusion will be reached.

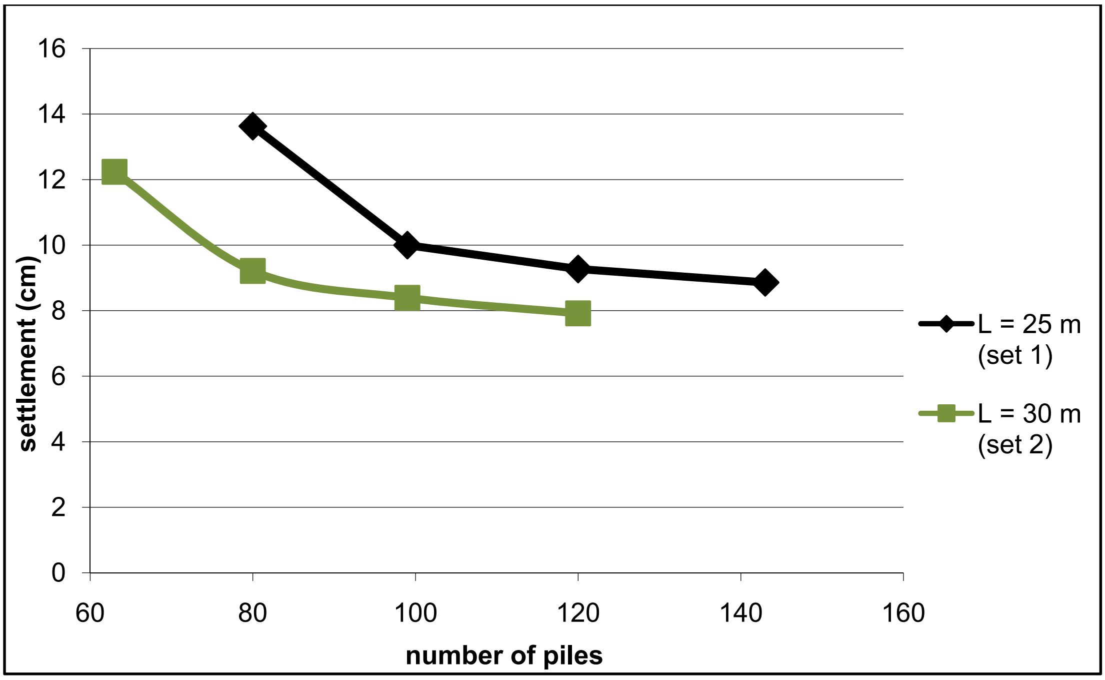


Figure 4.12 Graphical representation of the results for the Plaxis 3D Foundation

CHAPTER 5

DISCUSSION OF RESULTS

5.1 General Concepts

In some cases, bearing capacity problem can easily be solved by using raft foundations. However, when settlement values are beyond the tolerable limits, raft foundation can be used together with pile foundations. Nowadays, with increasing technology and some sophisticated engineering design and decisions, the best solution appears to be combined foundations in which piles are used together with rafts. This solution gives safe, economical and efficient design.

In this thesis, the effect of pile number to the settlement behavior of piled rafts and settlement reducing piles had been researched. For both of the systems, it is very clear that beyond an optimum number of piles, settlement no longer decreases. So, using an excessive number of piles below the raft will not achieve the safest solution. In the aforementioned study, besides analytical simple hand calculations, experimental works were conducted to see the effect for settlement reducing piles and finite element analyse were done to see the effect for piled raft foundations.

In this chapter, the results obtained from all the studies will be discussed separately for settlement reducing piles and piled raft foundations in two parts.

5.2 Discussion of Results For Settlement Reducing Piles

In cases when raft foundation alone satisfies the bearing capacity criteria, piles may be needed for settlement requirements. Since these piles have no benefit to the bearing capacity, their full capacities can be mobilized and they can be permitted to yield. In other words, the factor of safety against bearing capacity failures for piles in the system can be taken almost unity. Thus, these piles will only serve as settlement reducers. In such systems, since piles yield, the major portion of the load is carried by the raft. So, the load distribution between piles and the raft becomes important.

There are few methods which can manage this kind of analysis. Such methods should consider the load distribution between piles and raft and the stiffness for each element in the system should be taken separately.

In the experimental studies conducted, the piles had been yielded resulting in such a behavior same as the settlement reducing piles. In the experimental studies, a system consists of a raft model 50 x 50 x 10 mm in dimensions was used together with model piles which are 2 mm in diameter. The soil medium was kaolin type of clay on which all necessary laboratory tests had been conducted to specify and classify. The classification was determined as low plasticity clay with G_s = 2,60. Tests were conducted at two pressure levels namely 75 kPa and 40 kPa. Under 75 kPa pressure; raft alone, raft with 16 piles and raft with 49 piles were tested, whereas under 40 kPa only raft alone and raft with 16 piles were tested. The observed settlements are in the range of 2.70 to 1.80 mm under 75 kPa pressure, while the range become 1.91 to 1.29 mm under 40 kPa pressure. The settlement behavior observed clearly demonstrates that settlement decreases with increasing pile number only up to an optimum number, beyond that number decrement in the settlement becoming very small that it no longer posses any importance.

As described earlier there is an analytical approach used for settlement reducing piles. Instead of using model raft and model pile diameters in mm, they are scaled and system is modeled keeping the pressure units same. The raft dimensions are 50 x 50 m. The pile diameter is 2 m and 16, 49 and 81 piles are analyzed. The applied pressures were 75 and 40 kPa. The undrained shear strength of the soil medium is kept as around 30 kPa and the coefficient of volume compressibility was around 0.0001 to 0.0004 m^2/kN. The method used was proposed by Clancy and Randolph in 1993. The importance of this method is that the stiffness of the system as a whole is calculated using separate stiffness of pile and raft. The calculated settlements were in the range of 14.8 to 8.7 cm under 75 kPa, whereas under 40 kPa pressure they were in the range of 7.9 to 4.6 cm. In the results observed from this method, the effect of inserting piles on the settlement of foundations can be observed. An idea about the settlement behavior can also be hold.

Although experimental and analytical results cannot be compared directly, trends in settlement reduction are interesting. A parameter called settlement reduction factor, ρ was used in the figures. This parameter is calculated by dividing settlement of the raft with piles to the settlement of the raft alone. This parameter shows for each pile number, how much the settlement is reduced when using that number of piles. As this ratio gets closer to unity, this shows that, the settlement of the raft including piles gets closer to that of the raft alone. Obviously, a smaller value for this ratio is better but, it should not be forgotten that there are acceptable limits for settlements, thus decreasing this ratio to very small values has no benefits at the same time it increases the money and time that should be spend. Moreover, it is obvious that beyond a limit, further decreasing this ratio has a very small effect on settlement behavior.

In Table 5.1, ρ values for the experimental studies and analytical calculations are given. The graphical representation is shown in Figure 5.1.

Table 5.1 ρ values for the experimental studies and analytical

	number of piles	**ρ**
Clancy and Randolph' 93	0	1
	16	0.838
	49	0.668
	81	0.588
experimental studies	0	1
	16	0.756
	49	0.665

It can be seen that, placing an optimum number of piles under the raft or placing the maximum number of piles that is possible, does not give much different settlement values. So, placing an optimum number of piles can be efficient enough in many cases. However, it should be noted that, in the analytical approach, this behavior is not clear. This can be because of the differences in the limitations of the method.

In the practical hand calculations, design should be made considering that piles are not placed to cover the bearing capacity, thus they can yield under given loads. In other words, they can be mobilized with full capacity and work only to reduce settlements to the tolerable limits.

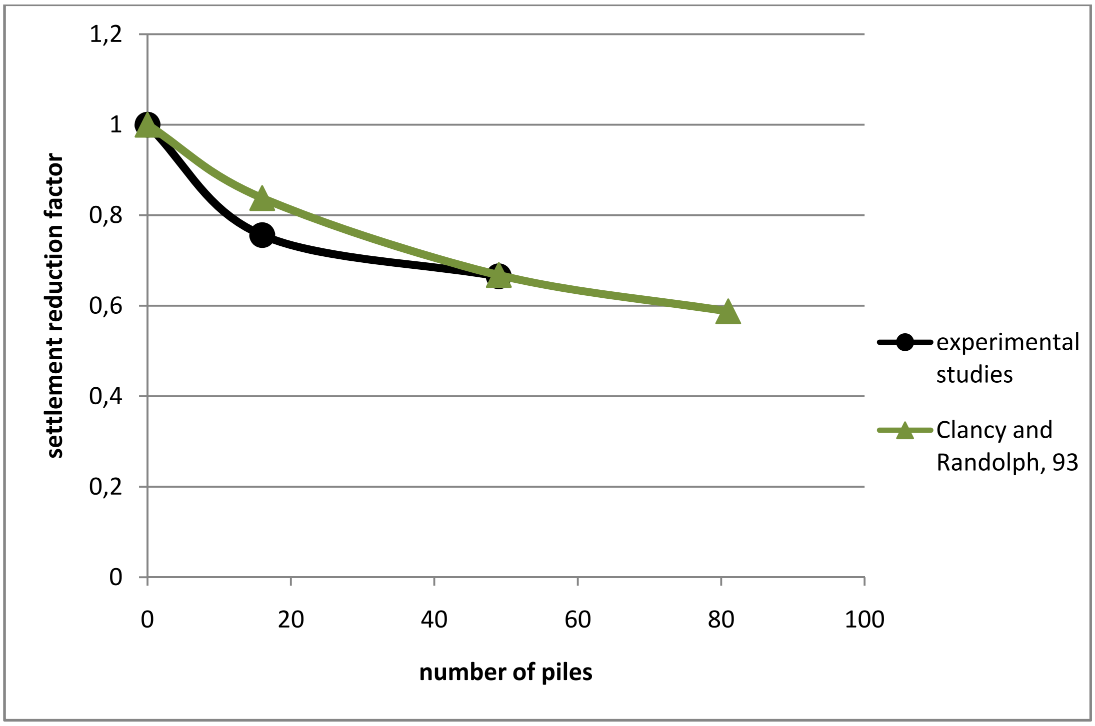


Figure 5.1 Graphical representation for ρ values for the experimental studies and analytical calculations

5.3 Discussion of Results For Piled Raft Foundations

In the past, pile foundations were used mostly for providing bearing capacity. This common use lead to design procedures based on bearing capacities of piles where a simple check is done for settlement. However, recently, the effects of the piles on the settlement of foundations have been receiving attention. Thus, the concept of piled resisted rafts where bearing capacity criteria is satisfied by raft mostly, the settlement criteria is satisfied by piles. It should not be forgotten that when analyzing this kind of foundations, the pile number should be taken into consideration. This is necessary because usually higher factor of safety values are used for piles and no attention is given to the tolerable settlements.

Since the scope of this thesis is analyzing the settlement behavior of piled foundations considering the pile number, only methods considering pile number have been used. As mentioned before, the methods that have been used for settlement prediction of pile groups can be classified in three categories; the methods that relate group settlement to that of a single pile, the ones which consider system as an equivalent raft and finally the methods which take into account the interaction between piles and soil.

In this thesis, two methods which fall into the first category and a finite element analysis program which makes a complete analysis and cover the interaction between piles and soil have been used.

One of the methods used was the method proposed by Butterfield and Douglas in 1981. This method calculates the group stiffness by considering the single pile stiffness, number of piles and an efficiency factor which is dependent to spacing of piles, Poisson's ratio, etc. The raft is 24 x 28 x 2 m. Two sets of piled raft foundation models have been used. One set comprised of piles which 25 m in length whereas for the other set pile lengths were 30 m. In the first set there were three patterns having 143, 120 and 99 piles. In the second set again three patterns were used with 120, 99 and 80 piles. The spacing is kept as 2.25 m for all cases. The soil is medium clay with increasing modulus of elasticity. The resulting settlements were in the range of

10.32 to 11.61 cm for set 1 and in the range of 9.51 to 10.82 cm for set 2. Further analyzing these results given in Figure 4.2, it can be seen that for both sets the curve that shows the pile number settlement relationship behaves less steep in the second portion. In other words, the reduction in settlement behavior becomes less effective as pile number increases, so excessive numbers of piles is not necessary for an efficient design.

Another analytical approach used was based on load transfer curves. It was proposed by Shen and Teh in 2002. This method permits non-linear deformation analysis. In this method, same models have been used. There still were two sets each have three different patterns. Dimensions of the raft were still 24 x 28 x 2 m. The patterns, spacing and properties of soil and materials remain same. The first set, in which pile lengths are taken as 25 m, the results came out in the range of 8.91 to 10.73 cm. In the second range where the pile lengths were 30 m, the range was between 9.57 and 11.51 cm. From Figure 4.2, the graphical results show that the settlement difference becomes smaller as pile number increases. In the graphs, the tendency of the curve to straighten can be seen clearly.

In order to support the analytical calculations, a finite element analysis was made using Plaxis 3D Foundation software. The model, its dimensions, patterns and soil and material properties were kept exactly the same. In the software, the Mohr Coulomb failure criterion has been used and drained analysis was activated. Pile numbers are 143, 120, 99 and 80 for 25 m length; 120, 99, 80 and 63 for 30 m length. The results show a settlement range of 8.86 to 13.63 cm in case of piles with 25 m lengths. As in the case of 30 m long piles, second set, the settlement range become 7.92 to 12.24 cm. The settlement ranges were very small. Also, from Figure 4.12, analyzing the curves derived from the finite element analysis clearly demonstrates that, the curve representing the relationship between settlement and the pile number loses its steep slope as pile number increases. This issue becomes even clearer in the case of shorter piles.

The all three methods used in this thesis for estimating the settlement behavior of the piled raft foundations analyzes the same models. This allows comparison between their results and discussion of the findings. The combined results evaluated from settlement ratio method, load-transfer curves and finite element analysis are given below in Table 5.2. The graphical representations of the results obtained from all methods are given in Figures 5.2 and 5.3 respectively, for pile lengths of 25 m and 30 m. When this data are analyzed it can be seen that in general, finite element analysis gives lower results. This difference appears more strict for longer piles, thus for the case of 25 m piles, the calculated values from the load transfer curve methods were very close to the finite element analysis. Among the methods used in the thesis for piled raft foundations, the most detailed and time consuming method is finite element analysis. In other words, it can be said that the most sophisticated analysis gives the smaller results. In general, the load transfer curve analysis and settlement ratio method give very similar results. However, the results appear to be overestimated when compared to the results gathered from Plaxis 3D Foundation software.

In all methods considered it is observed that adding more piles under the raft does not reduce the settlement significantly. Therefore increased number of piles under rafts will be overdesign and uneconomical. There should be an optimum number determined for the projects considering the tolerable settlements. Also it should not be forgotten that the methods should encourage more boulder decisions for these kinds of foundation systems since the piles should not carry the whole load.

Finally, the insensitivity of settlements calculated in all methods to number of piles is interesting and may be further examined by laboratory testing and field testing.

Table 5.2 Results obtained for piled raft foundations

		number of piles	settlement (cm)
Set 1 (L = 25 m)	Butterfield and Douglas' 81	143	10,32
		120	10,92
		99	11,61
	Shen and Teh' 02	143	8,91
		120	9,73
		99	10,73
	Plaxis 3D Foundation	143	8,86
		120	9,27
		99	10,00
		80	13,63
Set 2 (L = 30 m)	Butterfield and Douglas' 81	120	9,51
		99	10,11
		80	10,82
	Shen and Teh' 02	120	9,57
		99	10,48
		80	11,51
	Plaxis 3D Foundation	120	7,92
		99	8,39
		80	9,21
		63	12,24

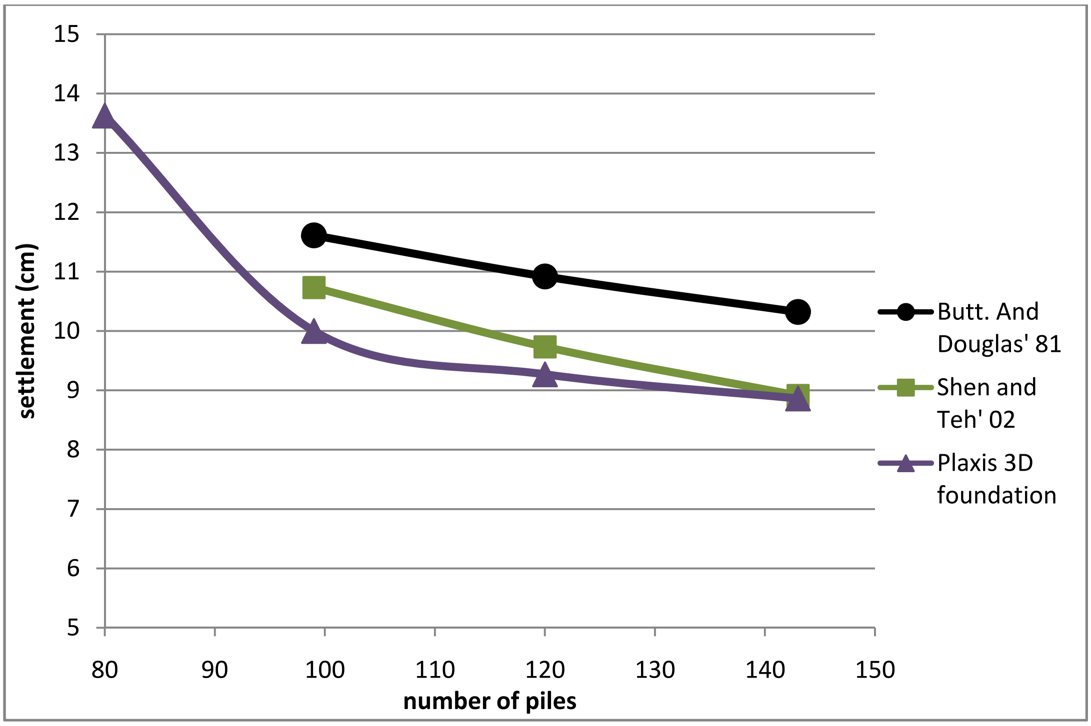


Figure 5.2 Graphical representation of the combined results for 25 m piles

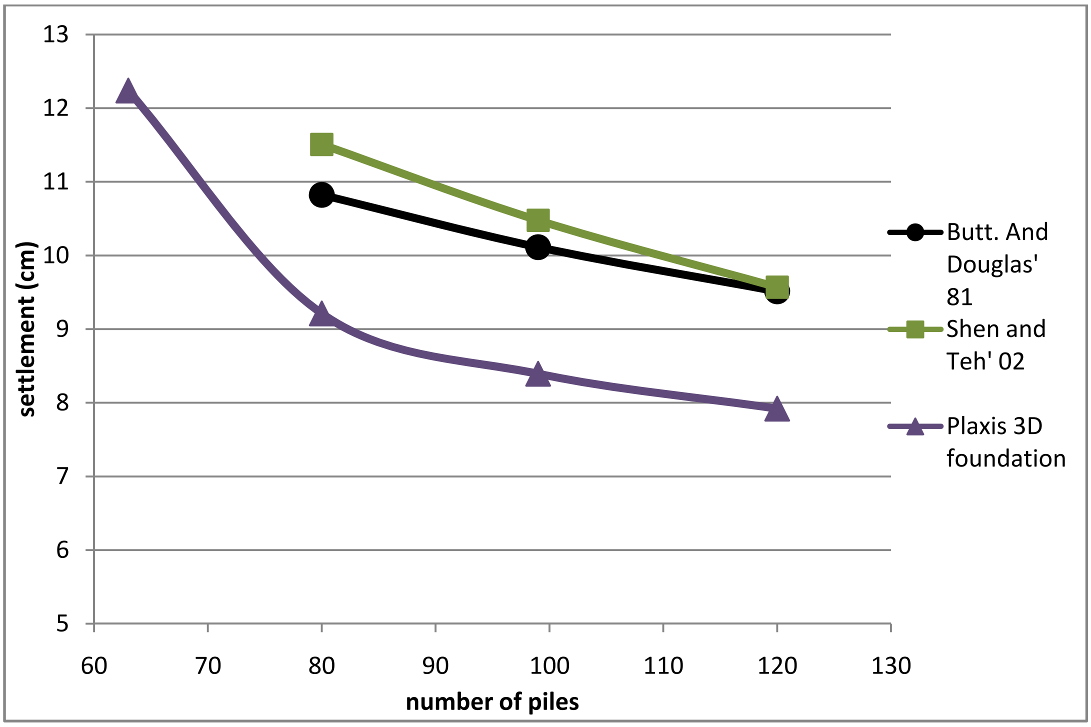


Figure 5.3 Graphical representation of the combined results for 30 m piles

CHAPTER 6

SUMMARY AND CONCLUSION

In this thesis, two recent concepts have been examined independently; settlement reducing piles and piled raft foundations.

A raft may be adequate in terms of bearing capacity but calculated settlements may exceed the tolerable values. In such cases, piles may be introduced under the raft foundation. These piles are limited in number so that they are continuously at the limit state with factor of safety of one. This concept is known as settlement reducing piles. In the aforementioned thesis, experimental studies had been conducted to observe the settlement behavior of such systems. The experiments have been done in model systems which consist of an aluminum model raft which has dimensions of 50 x 50 x 10 mm and brass model piles of 2 mm in diameter. The properties of the prepared clay soil are kept constant, pile number has been changed from the case with no piles, to the cases with 16 and 49 piles. In the studies, it is clearly seen that introducing piles under the raft reduces the settlement considerably. However, another important result has come out from the experiments. As the pile number increases further, the decrease in the settlement gets smaller. In other words, there is no significant effect of increasing pile number as far as the settlement is concerned. There exist an optimum number of piles that beyond this value, the settlement no longer decreases significantly. Moreover, in order to support the experimental studies, some analytical studies have also been carried out. Though, it is well realized that there are not many methods in the literature which can be used for settlement analysis of settlement reducing piles. This may be due to the fact that this system is a newly adopted system. In this thesis, an analytical settlement analysis proposed by Clancy and Randolph (1993) has been used. The simple hand calculations give similar results with the experimental studies. However, when the results are compared using settlement reduction factor, it is observed that the above mentioned behavior is not clear, though in the experiments this behavior is very clear.

In the second part of the thesis, analyses of piled raft systems have been made by three different methods. A 30 storey building supported by a piled raft has been analyzed. The system consists of a rectangular raft which has dimensions of 24 x 28 m and a thickness of 2 m. All the parameters besides the number of piles have been kept constant. The results have been obtained in two sets differ in length of piles. In the first set, piles are 25 m long whereas in the second set their length is 30 m. The pile numbers are 143, 120, 99 and 80 for the first set; 120, 99, 80 and 63 for the second set. Three methods are used for the settlement analysis. Simple hand calculations have been made with two methods proposed by Butterfield and Douglas in 1981 and Shen and Teh in 2002. The first method simulates the settlement ratio methods and the second method simulates the load transfer curves. Moreover, a finite element analysis has been conducted using Plaxis 3D foundation software. When the results are compared, it is observed that the finite element analysis gives the lowest values of settlement. The hand calculations give marginally higher results. Change of settlement with different pile numbers is similar in all three methods. The effect of pile number on settlement behavior is important. Addition of piles beyond a certain optimum number does not have a significant effect in the reduction of the settlement.

Based on experimental studies and the analytical calculations, calculations can be summarized as follows:

- For the combined systems like piled raft foundations and settlement reducing piles, the design is based on a specified maximum allowable settlement.
- For every design case, the optimum number of piles are determined by a trial and error procedure based on the tolerable settlement.
- The load distribution between pile and raft should be analyzed carefully and the results should be reflected to the settlement analysis.

- If settlement reducing piles concept is made use of, factor of safety against bearing capacity of piles can be taken close to or equal to unity. This fact will not mean that the system becomes unsafe.
- Increasing pile number does not mean that the reduction in settlement further increase, thus beyond some point the settlement curve tends to behave as a straight line. In other words, for every design there is an optimum number of piles that should be placed under the raft. This means economy and shorter construction time.

REFERENCES

- Basile, F., 2003, Analysis and design of pile groups, Numerical Analysis and Modelling in Geomechanics, ed. by John W. Bull, Spon Press, UK, pp.278-316

- Birand, A., 2001, Kazıklı Temeller, Teknik Yayınevi, Ankara

- Butterfield, R. and Douglas, R.A., 1981, Flexibility coefficients for the design of piles and pile groups, CIRIA Technical Notes, 108

- Castelli, F., and Di Mauro, F., 2003, Numerical analysis of piled raft foundations, BGA International Conference on Foundations, Thomas Telford Publishing, UK, pp.187-196

- Clancy, P., and Randolph, M.F., 1993, An approximate analysis procedure for piled raft foundations, International Journal for Numerical and Analytical Methods in Geomechanics, Vol.17, pp.849-869

- Das, M.B:, 1983, Advanced Soil Mechanics, Hemisphere Publishing Co., McGraw Hill, London

- De Sanctis, L., and Russo, G., 2008, Analysis and performance of piled rafts designed using innovative criteria, Journal of Geotechnical and Geoenvironmental Engineering, Vol.134, No.8, pp.1118-1128

- De Sanctis, L., Mandolini, A., Russo, G., and Viggiani, C., 2002, Some remarks on the optimum design of piled rafts, Deep Foundations 2002: An International Perspective on Theory, Design, Construction and Performance, ASCE, Orlando, Vol.1, pp.405-425

- El-Mossallamy, Y., 2002, Deep Foundations 2002: An International Perspective on Theory, Design, Construction and Performance, ASCE, Orlando, Vol.1, pp.426-439

- Engin, H.K., 2005, An experimental study of vertical and inclined soil nails under footings as settlement reducers, Middle East Technical University, Turkey

- Ergun, M.U., 1995, Predicted and observed settlement of a piled raft foundation, Eleventh African Regional Conference on Soil Mechanics and Foundation Engineering, Cairo, Egypt, Vol.3, pp 49-61

- Ergun, M.U., Kul, İ., and Engin, H.K., 2008, Use of inclusions under footings-a model study, Foundations Proceedings of the Second British Geotechnical Association International Conference on Foundations, IHS BRE Press, Great Britain, Vol.2, pp.1325-1334

- Erol, O., and Çokça, E., 2009, Sığ temeller, Zemin Mekaniği ve Temel Mühendisliği, İller Bankası Genel Müdürlüğü Kanalizasyon Dairesi Başkanlığı, Ankara, pp.4_1-4_67

- Fioravante, V., Giretti, D., and Jamiolkowski, M., 2008, Physical modelling of raft on settlement reducing piles, Proceedings of From Research to Practise in Geotechnical Engineering Congress, pp.206-239

- Fleming, W.G.K., Weltman, A.J., Randolph, M.F., and Elson, W.K., 1992, Piling Engineering, 2nd edition, Halsted Press, New York and Toronto, pp.179-214

- Gök, S., 2007, Kazıklı radye temellerin tasarımı, Ph.D Thesis, İstanbul Technical University, Turkey

- Horikoshi, K., and Randolph, M.F:, 1996, Centrifuge modelling of piled raft foundations on clay, Géotechnique, Vol.46, No.4,pp.741-752

- Kul, İ., 2003, Use of nails as settlement reducers under footings-a model study, Ph.D Thesis, Middle East Technical University, Turkey

- Lee, C.Y., 1993, Settlement of pile groups-practical approach, Journal of Geotechnical Engineering, Vol.119, No.9, pp.1449-1460

- Love, J.P., 2003, Use of settlement reducing piles to support a raft structure, Geotechnical Engineering, Iss.156, pp.177-181

- Mandolini, A., and Viggiani, C., 1997, Settlement of piled foundations, Géotechnique, Vol.47, No.4, pp.791-816

- Mohan, D., 1988, Pile Foundations, A. A: Balkema, Rotterdam, pp.110-116

- O'Neill, M.W., 2005, Piled rafts in geotechnical practice-an international perspective, Advances in Designing and Testing Deep Foundations, ASCE, USA, pp.70-81

- Plaxis 3D Foundation Reference Manual, Version 2, 2007

- Plaxis 3D Foundation Tutorial Manual, Version 2, 2007

- Poulos, H.G., 1975, Settlement of isolated foundations, Proceedings of the Symposium on Soil Mechanics - Recent Developments, Sydney, Australia, pp. 181 - 212.

- Poulos, H.G., 1988, Modified calculation of pile-group settlement interaction, Journal of Geotechnical Engineering, Vol.114, No.6, pp.697-705

- Poulos, H.G., 1993, Settlement prediction for bored pile groups, Deep Foundations on Bored and Auger Piles, Millpress Science Publishers, Netherlands, pp.103-115

- Poulos, H.G., 2001, Piled raft foundations: design and applications, Géotechnique, Vol.51, No.2, pp.95-113

- Poulos, H.G., 2002, Simplified design procedure for piled raft foundations, Deep Foundations 2002: An International Perspective on Theory, Design, Construction and Performance, ASCE, Orlando, Vol.1, pp.441-458

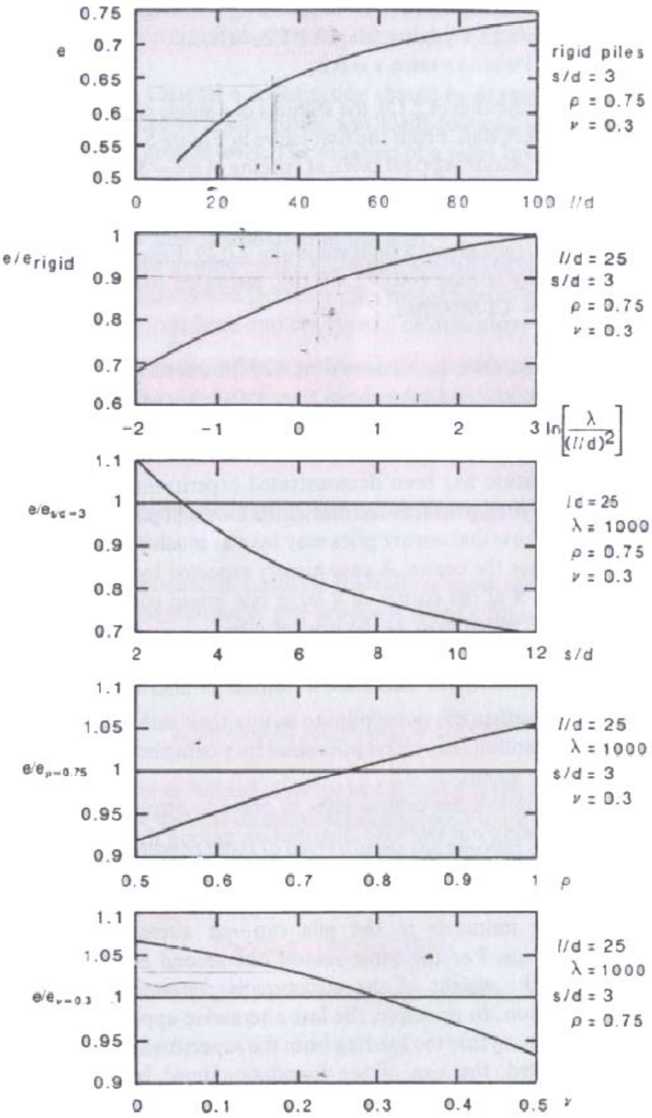
- Poulos, H.G., 2006, Pile group settlement estimation-research to practice, Geotechnical Special Publications, Coffey Geosciences, Australia, pp.1-21

- Randolph, M.F., 2003, Science and empricism in pile foundation design, Géotechnique, Vol.53, No. 10, pp.847-875

- Shen, W.Y., and Teh, C.I., 2002, Practical solution for group stiffness analysis of piles, Journal of Geotechnical and Geoenvironmental Engineering, Vol.128, No.8, pp.692-698

- Shen, W.Y., Chow, Y.K: and Yong, K.Y., 2000, Practical methods for settlement analysis of pile groups, Journal of Geotechnical and Geoenvironmental Engineering, Vol.126, No.10, pp.890-897

APPENDIX A



e
rigid piles
s/d = 3
ρ = 0.75
ν = 0.3
l/d
e/e rigid
l/d = 25
s/d = 3
ρ = 0.75
ν = 0.3
ln[λ/(l/d)²]
e/e s/d=3
l/d = 25
λ = 1000
ρ = 0.75
ν = 0.3
s/d
e/e ρ=0.75
l/d = 25
λ = 1000
s/d = 3
ν = 0.3
ρ
e/e ν=0.3
l/d = 25
λ = 1000
s/d = 3
ρ = 0.75
ν

APPENDIX B

$$[k_{pp}] = \frac{E_p \pi r_0^2}{\ell}\begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \frac{(i-1)(j-1)}{i+j-3} & \vdots \\ & \vdots & \\ 0 & \cdots & \frac{(k-1)(k-1)}{2k-3} \end{bmatrix}$$

$$[A_s] = 2\pi r_0 \ell \begin{bmatrix} \rho & \cdots & \frac{1+(2\rho-1)k}{(1+k)k} \\ \vdots & \frac{1+(2\rho-1)(i+j-1)}{(i+j)(i+j-1)} & \vdots \\ \frac{1+(2\rho-1)k}{(k+1)k} & \cdots & \frac{1+(2\rho-1)(2k-1)}{2k(2k-1)} \end{bmatrix}$$

$$[\boldsymbol{B_S}] = \begin{bmatrix} \mathbf{1} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix}$$

APPENDIX C

$$[f_s]=\begin{bmatrix} 1-\dfrac{\ln(s_{11}/r_0)}{\zeta} & \cdots & 1-\dfrac{\ln(s_{1np}/r_0)}{\zeta} \\ \vdots & \ddots & \vdots \\ 1-\dfrac{\ln(s_{np1}/r_0)}{\zeta} & \cdots & 1-\dfrac{\ln(s_{npnp}/r_0)}{\zeta} \end{bmatrix}$$

$$[f_{sb}]=\begin{bmatrix} \dfrac{1}{s_{11}/r_0} & \dfrac{2/\pi}{s_{12}/r_0} & \cdots & \dfrac{2/\pi}{s_{1np}/r_0} \\ \dfrac{2/\pi}{s_{21}/r_0} & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \dfrac{2/\pi}{s_{np1}/r_0} & \cdots & \cdots & \dfrac{1}{s_{npnp}/r_0} \end{bmatrix}$$

where s_{ij}=the pile spacing and $s_{ij}=r_0$ when $i=j$.

APPENDIX D

<table>
<tr><th rowspan="3">Shape</th><th colspan="4">Interaction Factors (I_c)</th></tr>
<tr><th colspan="3">Elastic foundations</th><th rowspan="2">Rigid foundations</th></tr>
<tr><th>Center</th><th>Corner</th><th>Average</th></tr>
<tr><td>Circle</td><td>1</td><td>0.64</td><td>0.85</td><td>0.79</td></tr>
<tr><td>Rectangle
L/B</td><td></td><td></td><td></td><td></td></tr>
<tr><td>1.0</td><td>1.122</td><td>0.561</td><td>0.946</td><td>0.82</td></tr>
<tr><td>1.5</td><td>1.358</td><td>0.679</td><td>1.148</td><td>1.06</td></tr>
<tr><td>2.0</td><td>1.532</td><td>0.766</td><td>1.300</td><td>1.20</td></tr>
<tr><td>3.0</td><td>1.783</td><td>0.892</td><td>1.527</td><td>1.42</td></tr>
<tr><td>4.0</td><td>1.964</td><td>0.982</td><td>1.694</td><td>1.58</td></tr>
<tr><td>5.0</td><td>2.105</td><td>1.052</td><td>1.826</td><td>1.70</td></tr>
<tr><td>10.0</td><td>2.540</td><td>1.270</td><td>2.246</td><td>2.10</td></tr>
<tr><td>100.0</td><td>4.010</td><td>2.005</td><td>3.693</td><td>3.47</td></tr>
</table>