ANALYSIS OF ANALOGY USE ON FUNCTION CONCEPT IN THE NINTH GRADE MATHEMATICS TEXTBOOK AND CLASSROOMS

EMEL ÜNVER

DECEMBER 2009

ANALYSIS OF ANALOGY USE ON FUNCTION CONCEPT IN THE NINTH GRADE MATHEMATICS TEXTBOOK AND CLASSROOMS

A THESIS SUBMITTED
TO
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
THE MIDDLE EAST TECHNICAL UNIVERSITY

BY

EMEL ÜNVER

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
SECONDARY SCIENCE AND MATHEMATICS EDUCATION

DECEMBER 2009

ANALYSIS OF ANALOGY USE ON FUNCTION CONCEPT IN THE NINTH GRADE MATHEMATICS TEXTBOOK AND CLASSROOMS

submitted by **EMEL ÜNVER** in partial fulfillment of the requirements for the degree of **Master of Science in Secondary Science and Mathematics Education, Middle East Technical University** by,

Prof. Dr. Canan Özgen _____
Dean, Graduate School of **Natural and Applied Sciences**

Prof. Dr. Ömer Geban _____
Head of Department, **Secondary Science and Mathematics Education**

Assoc. Prof. Dr. Behiye Ubuz _____
Supervisor, **Secondary Science and Mathematics Education Dept., METU**

Examining Committee Members:

Assoc. Prof. Dr. Soner Yıldırım _____
CEIT Dept., METU

Assoc. Prof. Dr. Behiye Ubuz _____
SSME Dept., METU

Assist. Prof. Dr. Esen Uzuntiryaki _____
SSME Dept., METU

Dr.Ufuk Yıldırım _____
SSME Dept., METU

Dr. İ. Elif Yetkin Özdemir _____
ELE Dept., Hacettepe University

Date: _____

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Emel Ünver

ABSTRACT

ANALYSIS OF ANALOGY USE ON FUNCTION CONCEPT IN THE NINTH GRADE MATHEMATICS TEXTBOOK AND CLASSROOMS

Ünver, Emel

M.S., Department of Secondary Science and Mathematics Education
Supervisor: Assoc.Prof. Dr Behiye UBUZ

December 2009, 64 pages

The primary objective of the current study was to examine how analogies are used on function concept in the ninth grade mathematics textbook and classrooms.

Using qualitative research procedure comprising textbook analysis and classroom observations, a picture was developed of how analogies used on function concept in ninth grade mathematics textbook and classrooms. One mathematics textbook, the primary source for observed classes was selected and analyzed for the study. Chapter entitled as "functions" in the selected textbook was closely examined for use of analogies therein. Subtitles of the chapter used in the analysis were identified according to the subtitles taught in observed classes. Moreover, the data were obtained from the observation of two teachers' 9^{th} grade mathematics clasess during the 7-week data collection period. Totally, twenty-five lessons were videorecorded.

Having determined which comparisons would be counted as analogies, each of the textbook and classroom analogies was classified according to an analogy classification framework including eight criteria, modified from that of Thiele and Treagust's (1994).

Analysis of the textbook suggested that all the analogies were enriched and functional, majority of them presented in both verbal and pictorial formats, most of them were advance organizers or post-synthesizers and some of them were embedded activators. However, none of them was explained completely and contained any limitations. On the other side, analysis of classroom analogies revealed that nearly all the analogies were functional, enriched or extended presented verbally as embedded activators, and all of them explained absolutely without any stated limitations.

Keywords: Function, analogy, textbook analysis, classroom observation

ÖZ

9. SINIF MATEMATİK DERS KİTABI VE SINIFLARINDA FONKSİYONLARDA KULLANILAN BENZETİMLERİN ANALİZİ

Ünver, Emel

Yüksek Lisans, Orta Öğretim Fen ve Matematik Alanları Eğitimi Bölümü
Tez Yöneticisi: Doç. Dr Behiye UBUZ

Aralık 2009, sayfa

Bu çalışmanın ana amacı, dokuzuncu sınıf seviyesinde fonksiyon konusunda yer alan benzetimlerin, matematik ders kitabında ve sınıflarında nasıl kullanıldığını incelemektir.

Ders kitabı analizi ve sınıf gözlemlerini içeren nitel araştırma yöntemleri kullanılarak, dokuzuncu sınıf seviyesinde fonksiyon konusunda benzetimlerin nasıl kullanıldığı gösterilmiştir. Gözlemlenen sınıflarda ana kaynak olarak kullanılan bir adet matematik ders kitabı, bu çalışma için seçilerek analiz edilmiştir olup seçilen kitabın, "fonksiyonlar" bölümünde benzetim kullanımı yakından incelenmiştir. Analizde, gözlemlenen sınıflarda anlatılan alt başlıklarla uyumlu olarak belirlenen bölüm alt başlıkları kullanılmıştır. Ayrıca, veriler iki öğretmenin dokuzuncu matematik sınıflarına ait yedi haftalık veri toplama süresince gözlemlenerek elde edilmiş olup, toplam yirmibeş ders kameraya kaydedilmiştir.

Hangi karşılaştırmaların benzetim sayılabileceği kararlaştırıldıktan sonra, kitap ve sınıf benzetimlerin herbiri Thiele & Treagust (1994) tarafından oluşturulan

benzetim sınıflandırma planında yapılan değişiklik sonucunda elde edilen sekiz categorilik bir plana göre sınıflandırılmıştır.

Kitapta yer alan benzetimlerin analizi; Bütün benzetimlerin fonksiyonel, zenginleştirilmiş, büyük bir çoğunluğunun sözel ve şekilsel formatlı olduğunu ve yine benzetimlerin büyük bir kısmının konunun başında veya sonunda olmakla beraber bir kısmının ise konunun anlatımı sırasında yer aldığını göstermektedir. Ancak, sonuçlar hiçbir benzetmenin tam olarak açıklanmadığını ve hiçbirinin sınırlılıklarından bahsedilmediğini sergilemektedir. Öte yandan, sınıf benzetimlerinin analizleri neredeyse benzetimlerin tamamının fonksiyonel, zenginleştirlmiş veya genişletilmiş olduğunu, sözel olarak konu ile birlikte sunulduğunu ve hepsinin hiçbir sınırlandırma belirtilmeksizin tam olarak açıklanmadığını ortaya koymaktadır.

Anahtar Kelimeler: Fonksiyon, benzetim, ders kitabı analizi, sınıf gözlemi

To My Parents, Elif & Gültekin

ACKNOWLEDGEMENTS

I would like to express my gratitude to my supervisor Assoc. Prof. Dr.Behiye Ubuz for directing my M.S program and providing guidance and inspiration for every step of the research and writing my thesis.

I would like to thank my committee members for their critique, feedback.

I am grateful to my friends Semirhan Gökçe and Eray Şentürk for their genuine enthusiasm and constructive comments. I also thank to Hakan Gençoğlu and Selçuk Şener for their helping to complete my data collection. I am indepted to the support of Burak Sezer. I am thanking him for sharing all the stress with me and encouraging me.

My sincere appreciation goes out to the students that participated in this study and especially to two participating teachers for giving so freely of their time.

Finally, I would like to note contributions and sacrifices from my family throughout the years. I appreciate the sacrifices they made to help me to get to where I am today.

Thank you all very much indeed.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

FIGURES

LIST OF ABBREVATIONS

ABBREVATIONS

F: Functional Analogy

F & S: Functional and Structural Analogy

V: Verbal Analogy

P: Pictorial Analogy

P-V: Pictorial and Verbal Analogy

AO: Advance Organizer

EA: Embedded Activator

PS: Post- Synthesizer

S: Simple Analogy

En: Enriched Analogy

Ex: Extended Analogy

AEx: Analog Explanation

ICS: Indication of Cognitive Strategy

NB: Analogies used by Neslihan in Class B

SS: Analogies used by Seçil in Class S

CHAPTER 1

INTRODUCTION

Functions are important in the development of mathematical knowledge and knowledge in other subject areas in school curriculum (Kwari, 2007). The notion of function is so abstract that presents many difficulties in its didactical metaphor (Kwari, 2007). The understanding of functions does not appear to be easy, because of the diversity of representations associated with this concept (Hitt, 1998). An essential goal of mathematics teaching is that students be able to pass from one presesentation to another without falling into contradictions (Hitt, 1998). Therefore, a substantial number of research studies have examined the role of different representations on the understanding and interpretation of functions (e.g., Eisenberg & Dreyfus, 1994; Even, 1998; Hitt, 1998; Leinhardt, Zaslavsky & Stein, 1990; Thomas, 2003; Zazkis, Liljedahl, & Gadowsky, 2003). These researches have focused on the students' abilities to transformations of functions and their properties from one system of representation to another. According to Kwari (1997), representations are useful for interpreting physical, social and mathematical phenemona. The representation of functions can be a context (physical objects, picture, diagram), verbal (words), numerical (list, table), geometric (graphs) and symbolic (equation, formula) (Kwari, 1997). Although analogies have a prominent place in the teaching and learning of functions (Kwari, 1997) because they have potential to make explanations of new material intelligible to students by comparing it to material that is already familiar, studies have not focused on analogies. Analogies can be referred as powerful representations in that they allow us to create similarities for a variety of purposes. Simply analogies are comparisons between two domains of knowledge – one familiar and one unfamiliar or less familiar.

These potentials of analogies make the use of analogies by teachers and the occurrence of analogies in textual material used by teachers and children of some

exterest. Given that analogies can support understanding, to what extent do authors provide them in textbooks? With what frequency are they included? What kinds of analogies are used? How mathematics teachers utilize analogies during classroom teachings? How often do teachers use analogies? Do they utilize same or different analogies? Do students offer any analogies related to the concept durng the class discussions? Do the classroom analogies and the textbook analogies are consistent? These questions are the focus of this study.

We believe that research regarding the use of analogies on function concept is needed so teachers be informed of findings of this study in order for them to understand how textbooks supplement analogy use on function concept, they will have an idea of analogy should be used during classroom teaching. Moreover, by the help of this study, textbook writers should be informed regarding to results of the research in order to design future textbooks. Furthermore, it is known that analogies are several roles can play in promoting meaningful learning; they can also help view a concept from new perspective (Orgill, 2003). The present study aims to answers to the following research questions: (1) How are analogies used in 9^{th} grade mathematics textbook? (2) How are analogies used by mathematics teachers in observed 9^{th} grade classes? (3) What are the similarities and differences between textbook and classroom analogies and their uses?

Definition of Terms

Concept of function: The definition of function simply uses the idea of univalence that for each element in the domain there is exactly one element in the codomain, with no other required properties of the correspondence. Arbitrariness and univalence are features of the concept of function. Arbitrariness of functions refers to both the relationship between the two sets on which the function is defined and the sets themselves. The arbitrary nature of the two sets means those functions do not have to be described on any specific set of objects. Arbitrariness is closely interrelated to an analytical judgment when an instance belongs to a concept family (Even, 1990).

Analogy: An analogy is a comparison between familiar and less fanmiliar domains of knowledge. In the literature, the familiar domain called as “vehicle”, “base”, or “analog”; and the less familiar domain called as “topic” or “target”.

Mapping: Mapping is described as the process of aligning and drawing between the analog and target objects (Richland, 2003).

CHAPTER 2

LITERATURE REVIEW

This chapter presents the review of literature relative to the learning and teaching function concept, the role of analogies in learning and teaching process, epistemology of mathematics teachers' content knowledge and teachers' knowledge of function and the importance of analogies in particular.

2.1. LEARNING AND TEACHING FUNCTION CONCEPT

The concept of function is one of most essential and fundamental concept carried throughout the full range of education from kindergarden to graduate school (Harel & Dubinsky, 1992; Selden & Selden, 1992). It appears in all areas of mathematics; arithmetic in the early grades, algebra in elementary and secondary school, and transformational geometry in high school (Haciomeroglu, 2006; Harel & Dubinsky, 1992). People involved in the calculus reform agree that any student hoping to learn something about calculus must has a strong conception of functions (Harel & Dubinsky, 1992). However, importance of gaining an extensive understanding of the function concept is much more than considering the use of functions in a standart beginning calculus course (Selden & Selden, 1992). This is since function concept occurred right through mathematics, going beyond calculus, and used in a variety of ways. Therefore, now it plays an essential and unifying role (Selden & Selden, 1992). Cooney & Wilson (1993) asserted that because almost all mathematics deals with relationships between quantities, there would be no actual understanding mathematics without functional thinking. In addition, polynomials, equations, ratio, propotion and variation, relationships stated in words, relationships in tabular representations of numerical facts (e.g tables used in graphic representation), and relationships symbolized by formulas all gave chances for stress in functional thinking (Cooney & Wilson, 1993). Functions used in the comparisons

of abstract mathematical structures; for instance, they are used to inform whether two sets have the same cardinality, whether one group is the homomorphic image of another. As well, they maybe used as elements of abstract mathematical structures such as vector spaces and operations on those, rings and groups (Selden & Selden, 1992; Carlson, 1995). Furthermore, other most frequent use of functions occurs in modeling real life situations (Conney & Wilson, 1993; Selden & Selden, 1992). In science, modeling motion with time as the independent variable (Selden & Selden, 1992), electrical fields (Carlson, 1995) and population density (Carlson, 1995; Selden & Selden, 1992).

Since, notion of function is a most essential one for mathematics; many mathematics educators thought that there was a requirement for stress on functional thinking in school mathematics (Cooney & Wilson, 1993). The idea of including functions in school mathematics come from the German mathematician Felix Klein in 1904. After his call, functions included into analytic geometry and calculus in high school curriculum (Cooney & Wilson, 1993; Mesa, 2004). In 1920s, emphasize of functional thinking in every area of secondary mathematics is recommended. As early as 1921, the National Committee on Mathematical Requirements of the Mathematical Association of America suggested that the study of functions have the central focus in secondary mathematics (Cooney & Wilson, 1993). The NCTM Curriculum and Evaluation Standarts (1989) called for starting activities related with functions as early as fourth grade and carrying on up to high school curriculum. Arguments about the function concept have produced a gradual evolution up to reach its recognized meaning. The most noteworthy evolution in the mathematical definition of function occurred before 20^{th} century and the notion of a function became central in mathematics of the 20^{th} century. Moreover, this evolution brought an evolution in pedagogical philosophy. This pedagogical evolution included drastic alterations of the importance of the function concept and its teaching (Cooney & Wilson, 1993).

Research in the field of functions supports that mathematical development of the function concept closely related with development on the teaching functions in

school mathematics. The notion of function evolved from being a numerical entity (Babylonian tables), an equation (Leibniz and Euler), an arbitrary correspondence between numerical intervals (Dirichlet) and lastly, a correspondence between any pair that do not need to be numerical sets (Luzin). Then, it was accepted as a change or as a variable depending on other variables in 19th century up to middle of 20th century. In this historical development of the function concept, Peter Gustav Lejeune Dirichlet was defined functions as arbitrary correspondances. In 1939, Nicolas Bourbaki introduced the ordered pair definition of function we use nowadays and referred as the Bourbaki approach (Hansson, 2006; Selden & Selden, 1992). With this definition, changes occurred in school curricula definition by two sets A and B with a rule which assings accurately on member of B (the codomain) to each member of A (the domain). Although, this definition was similar to Dirichlet's, it is different from Dirichlet's definition in the domain and codomain no longer is restricted to sets of numbers. Many researchers in mathematics education (Akkoc & Tall, 2003; Eisenberg, 1991; Even, 1993; Sierpinska, 1992; Tall, 1992; Vinner & Dreyfus, 1989) found this definition was extremely abstract, particularly for secondary students. Even though, literature emphasize that a less abstract notion of functions as rules more easily grasped than more abstract notion of function as sets of ordered pairs (Eisenberg 1992; Selden & Selden, 1992), Bourbaki's definition has remained dominant in mathematics and has affected teaching and learning of functions at secondary school.

When looking at the historical development of the functions, they have seen first as dependence relations defining real life phenomena, next as algebraic expressions, then as arbitrary correspondances and lastly as sets of ordered pairs. The historical evolution satisfies that function concept is a complex and multifaceted and its richness permeates almost all fields of mathematics. Moreover, since its complex nature and various uses, functional situations provide themselves to a diversity of representations, including equations, graphs, tables, and verbal descriptions (Lloyd & Wilson, 1998). Besides, the function concept has numerous synonyms such as

mapping, operator, transform which used in various forms of representations in different contexts (Hansson, 2006).

In the school curriculum, function is an advance topic that is classically not discovered in details until the secondary level (Lloyd & Wilson, 1998). Indeed, research on students' understanding functions supports that students do not understand functions sufficiently although they performed many operations on functions and solved many problems by the help of functions (Polaki, 2005). In the light of long historical development of the function and discussions within the mathematical world concerning its nature, it appears normal for secondary students or higher-level students have problems to grasp functions (Gagatsis, Elia, & Mousoulides, 2006; Jones, 2006).

There are many teaching and learning problems regarding the concept of function bounded up in the complexity of its history (Iaderosa & Malara, 2001). Literature supports that students appear to have similar concerns regarding what is and is not a function as early mathematicians. Numerous studies (Breidenbach, Dubinsky, Hawks & Nichols, 1992; Carlson, 1998; Eisenberg & Dreyfus, 1994; Even, 1990, 1993, 1998; Romberg, Carpenter & Fennema, 1993; Thomas, 2003; Thompson, 1994; Vinner & Dreyfus, 1989; Williams, 1998) performed concerning conceptual knowledge of function of students verifying common contradiction between students' conception of functions and the definitions of functions. Dreyfus, Artigue, Eisenberg and Wheeler (1990) stated that difficulties might be derived inconsistencies between the formal definition of function and the concept definition that a student remembers.

In classes, students are frequently demonstrated the Bourbaki (ordered pair) definition or some version of the Dirichlet (correspondence) definition of function (Selden & Selden, 1992). Vinner advised that there is an inevitable argument between the structure of mathematics and the cognitive process of concept gaining.

Michelsen (2001) wanted students participating in a one of their research to reflect the complexity of the concept of function and the related difficulties

experience in learning functions. 70, 10th grade Danish students wrote an essay about their conceptions of functions and the utility of the concept. When thinking functions as rules, students obviously had difficulties of assigning logical meanings to them and they asked for expressing instances of the usefulness of the functions.

Kaput (1994) described the gap with the difference between mathematical functions that defined by algebraic formulas and empirical functions that explain everyday phenomena. The problem of transferring formal mathematical knowledge to a new context is one of the biggest rigors in education. Moreover, it is known that it is complex for students to apply concepts, ideas learned in the school mathematics in a new situation in or out of school (Kaput, 1994).

2.2 THE ROLE OF ANALOGIES IN LEARNING AND TEACHING PROCESS

Although, there are many definitions of analogy in the literature (Glynn, 1991; Treagust et al., 1992; Duit et al., 2001), basically, an analogy is a comparison between familiar and unfamiliar domains of knowledge. However, saying just an analogy is a comparison might be an oversimplification. Actually, it is a comparison between two domains and this comparison is a special kind of comparison that is defined by its purpose and type of information it relates. Gentner (1989) offered a working definition:

> An analogy is a mapping of knowledge from one domain (the base) into another (the target), which conveys that a system of relations that holds among the base objects also holds among the target objects (p.201).

Glynn, Duit and Thiele (1995) and Metsela and Gynn (1996) stated that an analogy is drawn by identifying similarities between two concepts. In this way, ideas can be transferred from a familiar concept to an unfamiliar one. In the literature, the familiar domain is referred to as the “vehicle”, “base”, “source”, or “analog” domain; and the less familiar domain, or the domain to be learned, is referred to as the

"target" domain. For the purpose of this study, the familiar concept is called the analog and the unfamiliar one is called the target. Both the analog and the target have features (also called attributes). If the analog and the target share common or similar features, an analogy can be drawn between them. A systematic comparison, verbally or visually, between the features of the analog and target is called a mapping (Glynn et al., 1995).

There is a need to clearly define to what an analogy is so that it is not confused with illustrations and examples (Glynn et al., 1995). The analog requires the selection of a student world analog to assist in the explanation of the content specific target or topic (Glynn et al., 1995). No analog maps onto the target perfectly; if the target and analog were identical, they would be the same concept. Every analog breaks down at some point, and no two analogs are similar. For these reasons, teachers and authors should attempt to suggest several analogs to students. Each analog has its corresponding and noncorresponding features; some analogs will be better for some purposes than others will (Glynn et al., 1995).

Just as they did in Galileo's time, scientisits still make frequent use of analogies. In fact, throught the history of science, analogies have played an important role in explanation, insight, and discovery (Glynn et al., 1995). It is not surprising; therefore, that teachers and textbook authors routinely use analogies to explain complex concepts to students. Often, teachers and authors are unaware that they are using analogies and they do it automatically. Throught their lessons, especially when responding to students' questions, teachers regularly preface their explanations with everyday expressions such as: "It's just like, it's the same as, it's no different than, think of it as". In textbooks, authors use more formal expressions like "Similarly, likewise, along related lines, in comparison to, and in contrast with". These expressions are all ways of saying, "Let me give you an analogy" (Glynn et al., 1995).

Analogical reasoning has long been believed to play a central role in mathematics learning and problem solving; however, little is known about how

analogy is used in everyday instructional context (Richland, Holyoak, & Stigler, 2004). Despite the indications that analogy may be an important component of human thinking and learning, currently, little is known about analogy as an instructional device in everyday practice. Traditional studies of analogical reasoning require participants to complete a formal higher order relationship such that "a" is related to "b" in the same way as "c" is related to "d" typically notated as a:b::c:d (Goswami & Brown, 1989).

In spite of the extensive use of analogy in chemistry teaching, there have been comparatively few studies about actual use of analogy during instruction. Duit (1991) suggests this is probably true for the sciences in general: as he puts it "little is known about how analogies are used in the classroom" (p. 659). Research into the use of analogy in chemistry and science teaching has centred on two themes: the prevalence of analogy in curriculum materials such as textbooks, and in classroom practice and students' use of analogy or analog models.

Curtis and Reigeluth (1984) developed a classification system to survey the presence of analogies in 26 science textbooks ranging from elementary to post-secondary levels. Each analog was inspected regarding the following features: (a) analogical relationship, (b) presentational format, (c) content condition, (d) position in the text, (e) level of enrichment. Since the system was limited for some analogies that did not fit well into the existing classification system, keeping main features of the criteria, an adapted classification system formed by Thiele and Treagust (1994). The new classification system was named as "Analogy Classification Framework" and it was used to examine 10 high school chemistry textbooks. Each analogy was scrutinized concerning the following characteristics shown in Table 2.1, three of which (c, d, and e) were used before by Curtis and Reigeluth (1984).

Table 2.1 Analogy Classification Framework of Thiele & Treagust (1994)

a) The *content* of the target concept – what aspect of chemistry is being considered by the target concept;
b) The *location* of the analogy through the textbook – at what stage of the curriculum is the

analogy being presented;

c) The *analogical relationship* between analogy and target – whether the analog and target share functional or structural-functional attributes;

d) The *presentational format* – whether the analog verbal, pictorial or pictorial-verbal?

e) The *condition* or level of abstraction of the analog and target concepts – whether they have an abstract or concrete cognitive level;

f) The *position* of the analog relevant to the target – whether it is before, during, or after the presentation of the target, or whether it is presented in the margin;

g) The *level of enrichment* – to what extent is the mapping between analog and target domains done by the author;

h) The *pre-topic orientation* – is there evidence of further *analog explanation* of the analog domain and/or have the authors included any *strategy identification* that will indicate that the text has an analogical nature;

i) The presence of any stated *limitations* or warning which highlights to the students where possible mis-matches may occur.

Thiele and Treagust (1994) added the benchmark, "content area of the target domain" in order to offer information regarding in which chemistry content areas most frequently analogies used. Further, they inserted criterion for deciding where the analogy was located with respect to its progress through the textbook as a whole. They divided each textbook into ten parts according to their page numbers and assigned each analogy to one of those parts.

Curtis and Reigeluth (1984) defined two major analogical relationships: (1) structural, and (2) functional. When analog and the target domains having similar appearance (e.g size, shape or color) or similar object attributes, they stated that the analog and the target have a structural relationship. On the other hand, a functional relationship defined as the analog and the target domains share similar functions or behaviours. Moreover, they defined a third type that merges both relationships and

called it structural- functional relationship. Thile and Treagust (1994) also included these three types of analogical relationships into their classification scheme.

Analogies identified in two formats that are verbal and pictural-verbal. Verbal analogies defined by analogies including just written text where analogy is clarified in words alone. On the other side, pictorial- verbal analogies described as written analogies supported by pictures of the analog domain. These pictures may be a drawing, maybe a photograph (Curtis and Reigeluth, 1984).

Curtis and Reigeluth (1984), Thiele and Treagust (1994) scrutinized that the analog and the target domains of each of the analogies can be organized as either concrete or abstract under the benchmark of "condition". They used three possible combitanations: (1) concrete-concrete: both analog and the target domains are of a concrete nature; (2) abstract- abstract: both analog and the target domains are of an abstract nature; (3) concrete- abstract: the analog is of a concrete and the target is of an abstract nature. Since, Curtis and Reigeluth (1984) stressed that an analogy should offer a bridge from familir to unfamiliar concept or from simple to difficult content, they did not include the abstract-concrete combination, which could be a fourth possible one.

The position of the analog in relationship to the target might differ (Curtis & Reigeluth, 1984). The analog domain can be introduced before presentation of the target domain as an advance organizer or following a complete description of the target domain as a post synthesizer. On the other hand, the analog might be presented during the instruction, at a point where the content became difficult. In that position, it acts as an embedded activator (Curtis & Reigeluth, 1984; Thiele and Treagust, 1994).

In addition, Curtis and Reigeluth (1984) described the levels of an analogy as: (1) simple: states the relationship between analog and target domains by using connectors " is like a.." or " maybe compared to", without no further description, (2) enriched: states the grounds of analogical relationship between the analog and target and sometimes contains the limitations of the relationship, (3) extended: contains

several analogs or several attributes of one analog used to describe one target. Figure 3.3 formed by Curtis and Reigeluth (1984) summaries the level of enrichment.

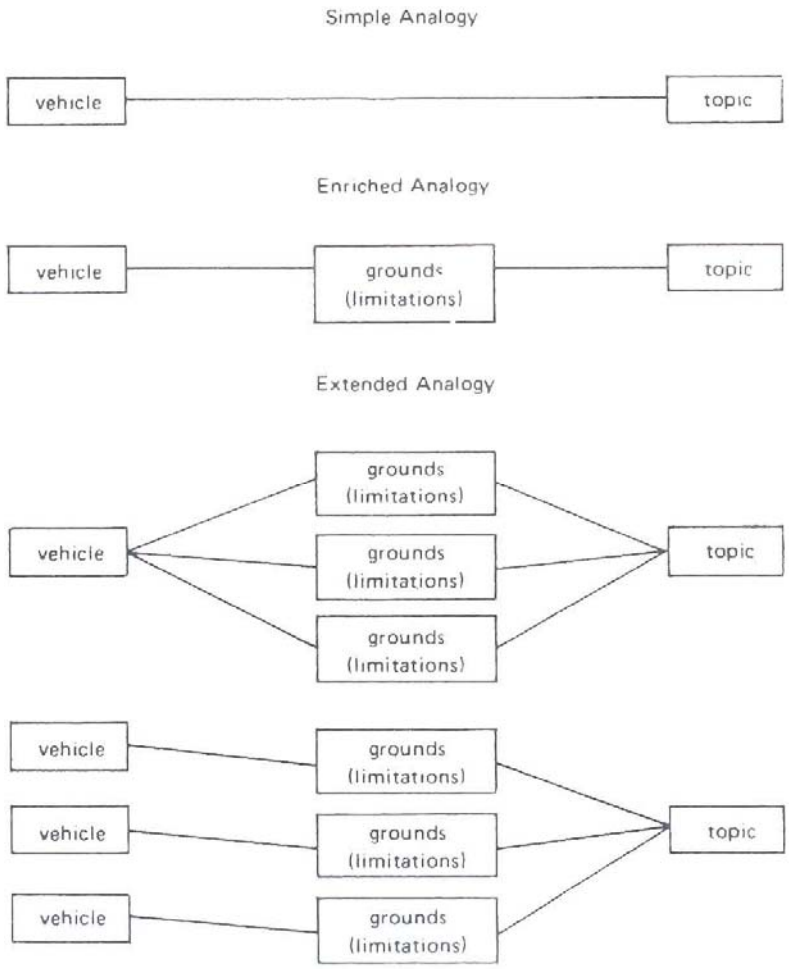

Figure 2.1 Three levels of enrichment for analogies (adapted from Curtis and Reigeluth, 1984, p. 111)

Curtis and Reigeluth (1984) mentioned that textbook writers suppose that the learners are familiar with the used analog and the coginitive strategy of analogical thinking. However, in case of the learners are unfamiliar with the analog or they are familiar but find the analog is so complex; there will be a need of explaining the analog domain (Curtis & Reigeluth, 1984). Moreover, Thiele and Treagust (1994) argued that analog clarification is required because it guarantees that the learner is dealing with the appropriate features while analogical transfer occurs. This

explanation might be a simple expression of just a few words through a paragraph, which describes the related analog attributes (Thiele & Treagust, 1991).

In addition, Curtis and Reigeluth (1984) mentioned that identifying analogy as a kind of cognitive strategy is used technique by textbook writters. It was affirmed that by the help of strategy identification, the learners would have a chance of being alerted regarding the comparison between the analog and the target domains. Moreover, in the case of strategy identification, learners specifically would be aware of the unfamiliar term (Curtis & Reigeluth, 1984).

Differently, although an expressed limitation still thought to be an example of enrichment, a further classification criterion was inserted to the analogy classification sytem of Curtis and Reigeluth (1984)'s by Thiele and Treagust (1994). Afterwards, they checked for each analogy whether they comprised of: (1) a general declaration of the limitation of analogy use; or (2) a declaration of the unshared attributes in the analogy.

2.3 THE ROLE OF TEXTBOOKS IN LEARNING AND TEACHING PROCESS

Textbooks are basic sources that shape the way of teaching and learning mathematics. They are integral parts of mathematicians' and mathematics teachers' daily lives. Teachers use them to prepare their lesson plans, to teach, to search for a formula or a theorem, and to prepare exam questions. They develop their lesson plans directly from the exercises and activities from the txetbooks, and frequently, some of the teachers' own understanding of the concepts they teach are derived from the textbooks (Jones, 2006). On the other hand, students use them to study and to solve homework questions (Kajander & Lovric, 2009). In this case, information that presented in each textbook is vitally important to students' understanding of the concepts (Jones, 2006). Despite the common presence of textbooks in schooling and vital educative practices, a small number of research studies have focused on mathematics textbooks in relation to their mathematical content (Mesa, 2004;

Conklin, 2004). This is even more surprising considering the centrality of textbooks in education and that they decide what school mathematics is (Cochran, 2008).

Although they are not enough, there have been attempts at evaluating textbooks, there are studies discovering the relationship between textbooks and mostly K-12 curriculum. In addition, there are searches using content analysis to explore the ways in which textbooks are used in classrooms and beyond. However, again not many mathematics education researchers have taken close examine what is in the textbooks by searching for how the material is presented and what kind of learning may be implied (Kajander & Lovric, 2009).

It is apperant as obtained from the literature that written analogies may play a role in making text more accessible to students and they can be effective and useful learning tools if they are presented in a clear way. Analogies have been utilized by teachers to explain concepts in text and orally. Textbook analogies have potential advantage of being an omnipresent resource for students, students can consult when the teacher is not available to make new information more understandable (Orgill, 2003). Analogies are often involved in textbooks since some students need presentations of different concepts in order to learn them meaningfully (Thiele, Venville, & Treagust, 1995). Since, textbook authors can allow thought and time to constructing them, textual analogies have also potential of being more explicit then oral ones (Orgill, 2003).

Although, there are potential advantages of using analogies in textbooks, there are potential problems of using analogies in textbooks (Duit, 1991). Curtis and Reigeluth (1984) mentioned that text analogies are very different from oral ones since there is no mechanism for instant feedback or modification for individual students. For this reason, written analogies must be presented with obvious explanations (Curtis & Reigeluth, 1984). However, studies of Curtis and Reigeluth (1984), Thiele and Treagust (1994) suggests that analogies are no often described clearly. Duit (1991) claimed that over half of the analogies in textbooks are not explained and just a few of them explained completely. Textbook authors expected

that students should explain textbook analogies by themselves or teachers should explain each of them to students (Thiele & Treagust, 1995). For this this reason, most of the time textbook authors do not state limitations of any analogy they present in their textbooks (Thiele & Treagust, 1995). On the contrary these taught, Bean, Searles, & Cowen (1990) declare that the utilization of the textbook analogies indicate that students do not employ analogies except utterly told them to take advantage of them. Curtis and Reigeluth (1984), Thiele and Treagust (1994) and Thiele and Treagust (1995) mentioned that analogies were identified by using the word "analogy" or "analogous". Nevertheless, this strategy does not promise that the students will use the analogy as a learning tool.

2.4 EPISTEMOLOGY OF MATHEMATICS TEACHERS' CONTENT KNOWLEDGE

Shulman (2004) stated that content knowledge refers to "the amount and organization of knowledge per se in the mind of the teacher" (p. 201). However, this does not mean that teachers should only be able to define the accepted truths of the domain to students. They must also be capable of explaining why a particular statement is necessary to consider or be demonstrated, why it is worth knowing, and how it relates to other statements, both within the discipline and without, both in theory and practice (Shulman, 2004). A teacher must understand that there are a variety of ways of organizing the discipline. The teacher should recognize alternative forms of organization and the pedagogical grounds for selecting one under different circumstances. The teacher should also understand the syntax of the discipline. For Shulman (2004), to think properly about content knowledge requires more than knowledge of the facts or concepts of a domain. Shulman (2004) stated that

> The teacher need not only understand that something is so; the teacher must further understand why it is so, on what grounds its demand can be asserted…The teacher requires to understand why a given topic is particularly central to a discipline whereas another may be somewhat peripheral (p. 202).

2.5 TEACHERS' KNOWLEDGE OF FUNCTION AND THE IMPORTANCE OF ANALOGIES IN PARTICULAR

Developing an understanding of the function concept includes a comprehension of its network of relations. Gaining knowledge of the relationships and being able to use functions in different contexts is a learning process that requires a longer period. It is therefore, appropriate to introduce the concept of function in school mathematics and to gradually expand the students' knowledge of functions, their applications, representations and relations to other concepts, and successively make the students able to handle functions in a more flexible way (Hansson, 2006).

After Felix Klein (1905) engaged in a curriculum reform, the concept of function considered as a unifying concept in mathematics. Although, parallel developments were actualized on curriculums of other countries, the outcomes of the reform was not visible (Sierpinska & Lerman, 1996). Cooney and Wilson (1993) mentioned that the reason of this case maybe arised from the teachers' conceptions of functions.

To deal with the concept of function succesfully in their practice, it is important for mathematics teachers to have a well-developed conceptual knowledge of functions, including the concept's significance in mathematics and relationships to other concepts (Cooney & Wilson, 1993; Eisenberg, 1992; Even, 1993; Thomas, 2003; Vollrath, 1994).

Understanding of the function concept includes a comprehension of its network of relations. Gaining knowledge of the relationships and being able to utilize functions in different contexts is a learning process that requires a longer period. It is; therefore, appropriate to introduce the concept of function in school mathematics, to slowly expanding the students' knowledge of functions, their applications, representations and relations to other concepts, and successively make the students able to handle functions in a more flexible way (Hansson, 2006).

CHAPTER 3

METHOD

The primary objective of the current study was to examine how analogies are used on function concept at 9th grade. In this chapter, a description of the research design and methodology are presented. It includes the rationale for selecting the methodology, data sources, analysis of analogies used in mathematics textbook, description of classification frameworks for textbook and classroom analogies and analysis of analogies in mathematics classrooms.

3.1 Research Design

Three general questions that concentrated on and directed the research concerning the use of analogies in mathematics classrooms:

1) How are analogies used in 9th grade mathematics textbook?

2) How are analogies used by mathematics teachers in observed 9th grade classes?

3) What are the similarities and differences between textbook and classroom analogies and their uses?

The nature of these questions suggested that a qualitative research design would make possible to gather and analyze data through suitable methods to respond the questions. Qualitative research is an approach enabling for extensive explanation in natural settings (Creswell, 2003).

A two-faced research approach to answer the questions proposed: (1) content analysis of a textbook to understand how analogies are used in mathematics textbooks on function concept and (2) case study using classroom observations to

understand how analogies used in mathematics classrooms on function concept. As seen, a different methodology was employed to address each facets of the research mentioned. Each methodology will be discussed in depth in the following parts.

3.2 Data Sources

The research utilized a qualitative case study using naturalistic classroom observations and content analysis of a textbook as the main sources of data.

The research sample included two volunteered high school mathematics teachers (Neslihan and Seçil) teaching at a public urban high school in Istanbul, and their 9^{th} grade students. Seçil and Neslihan had started their careers at secondary schools and from that, they had been teaching in secondary classes for 10 and 14 years, respectively. Both of them earned a B.S degree in mathematics.

Study was conducted with two classes, class B of Neslihan and class S of Seçil. There were approximately 25 students in each class. Both of these clasess included almost same numbered male and female students and again roughly, number of female and the number of male students were equal in each class. According to first mathematics mid-terms results of 9^{th} grade classes, both of the teachers mentioned that, their ninth grade students represented a range of achievement levels. However, informally, by the teachers' own assessments, relative to the rest of their classes not observed for the study, observed classes B and S contain higher achiever students.

All the members of the population are not equal data sources (Wiersma, 2000); however, selected participants were believed to be information-rich cases. There were a few high schools in Istanbul whose administrations were informed about the conducted research before data collection phase. All school administrations were willing to involve the research however; one of them positioned in an easily accessible place for the researcher was selected. There were eight mathematics teachers at the school. The selection of the two teachers was based on the following criteria: (1) they were eager to have their class observed during a few months; and

(2) they were helpful to the researcher in providing easy access to their classrooms. Before the sampling process both of the teachers stated that use of analogies could encourage student understanding of the function concept and they added they utilize analogies very frequently whilst their teaching.

One mathematics textbook was examined to determine the nature of analogy use in the area of functions. Since it was commonly used and the primary source for observed classes, described textbook was chosen to be analyze. The textbook was published in 2008 by the Turkish Ministry of Education for 9^{th} grade students and is generally used as textbook in variety of schools.

As the focus is on analogy use on the function concept, only the chapter related to function concept was concerned and other topic coverage were not undertaken. However, not all the subtopics of the chapter were included in the analysis, just subtopics that were consistent with the subtopics taught in the classes were selected. Upon close examination of the chapter, it was realized that there was a section called “operations”. Although, in the book, “operations” were related with the subject of functions, it was decided not to include into analysis since (1) both of the teachers did not include this subject to their function teaching and (2) they expressed that they would teach this section separately after finishing functions. Therefore, just subtopics of functions consistent with the subtopics utilized in the classroom teachings, became the basis for this study. This meant that not including the entire chapter of functions in the book, but accepting only subtopics and eliminating section not related with functions.

The subtopics involve the function concept, graph of function, types of functions, one to one function, into and onto function, identity function, constant function, linear function, inverse function and compound function. These subtopics within the chapter in the selected textbook was closely examined for use of analogies therein. The unit of analysis under these subtopics were introduction parts, activities, texts or pictures given under the title of “interesting”, and all exercises. Introduction parts are parts that are presented afterwards the title of the subtopic. These parts

contain may be a textual explanation, a picture, a photo, a graph, a diagram or maybe a textual explanation with a picture or pictures or maybe any kind of these mentioned visuals. Activities are parts that contain textual explanations, sometimes photos or pictures used for to relate students' existing knowledge with what they are planned to learn. Generally, these parts start with a sentence "let's imagine that… For instance, analogy named as "countries social security system" was detected in the activities part (pg. 87). In the textbook, there are parts under the title of "interesting" containing written or pictorial instances for the described subtopics. For example, photos of old and new types of calculators regarding with the function concept (pg. 87) and an anectode related with constant function (pg. 98) were given under the titles of "interesting". Furthermore, there are parts as "exercises" which include questions and their answers related with the explained subtopics. A total of 20 analogies were identified from the textbook. In Appendix A, every section title and corresponding page numbers analyzed in the textbook were listed.

End-of-section texts under titles in the book as "measurement and assessment" were excluded from the counts since; they were not used during classroom teachings and exercises under these parts were not given students as assignments.

3.3 Classification of Analogies Used in Textbook and Classrooms

Analogies recognized in the textbook and in the classrooms were classified along with an extended classification system of Thiele and Treagust's (1994) based on the classification scheme devised by Curtis and Reigeluth' study (1984). The classification system named as the Analogy Classification Framework (as shown in Table 2.1) includes nine criteria, six of which (c- h) previously formed by Curtis and Reigeluth (1984).

Following development of the initial categorization scheme, further studies used it for analyses of social science textbooks, high school chemistry and biology textbooks and college biochemistry textbooks. Neither the analogy classification system of Curtis and Reigeluth (1984), nor the analogy classification framework of

Thiele and Tregust were used for examining any mathematics textbooks. Because mathematics is exterior of the area of science and only the function concept would be analysed, the chosen framework was limited for analogies identified for the current study. Although the framework mainly contained previously formed criteria, some modifications to Thiele and Treagust's (1994) coding system were made to concentrate on analogies more easily, one criterion (e of Table 2.1) omitted and other criteria modified for the purpose of this study. Since the nature of mathematics, mathematics textbooks always employ concrete analogs explaining abstract target concepts. For this reason, it was thought that there was no need to include this criterion into the analogy classification framework that was adapted for this study.

Although the use of modified analogy classification framework to analyze classroom analogies was quite consistent with the analysis of the textbook analogies, a few adaptations of scheme made it for suitable for the analysis of classroom analogies. Ultimately, coding schemes including eight criteria shown in Table 3.1 for textbook analogies and Table 3.2 for classroom analogies were obtained and the logic behind each of the facet described in turn in the following sections.

3.3.1 Analysis of Analogies Used in Mathematics Textbook

After identifying the unit of analyses, these identified units were read completely to identify comparisons between function concepts and concepts with which students could be expected to come into contact with in their daily lives. All such comparisons marked without considering that they were analogies or not. Studying all the comparisons would be time consuming so, comparisons conforming analogy definition stated for the current study were considered analogies. In deciding whether a comparison should be labeled as an analogy or not, any judgement made about the quality of the analogy. Examining each analogy, analog and target concepts given in Table 4.4 were determined. Next, a preliminary categorization of individual analogies was made based on a framework described for the current study. While examining all the analogies, they were assigned to a certain categories discussed in this section. Then, by rereading the initial categorization, some changes

were made on the framework and finally the framework shown in Table 3.1 was obtained. In case of analogies that seen multiple times in the function chapter, the first appearance of the analogy was categorized and it was referenced to the subsequent appearances of the analogy. For instance, analogy of "function machine" first used for the target "one to one function" then it was used for the other target concepts, "identity function, constant function, linear function, inverse function and composite function", respectively (as shown in Table 4.4).

The coding system that employed for this study, the reasoning behind each facet and the modifications of the scheme are presented below:

Table 3.1 Analogy Classification Framework used to classify textbook analogies found in the current study.

a) The content of the target concept- Are there specific subtopics of the function chapter attempted to teach with analogies? Are there specific subtopics of the function chapter tend not to be taught with analogies?

b) The location of the analogy through the chapter- Is the analogy presented at the beginning of the chapter, in the middle of the chapter, at the end of the chapter?

c) The analogy relationship between analogy and target- Do the analog and target share functional or structural-functional features?

d) The presentational format- Is the analogy presented verbally, pictorially and verbally or pictorially?

e) The position of the analog relevant to the target- Is it before as an "advance organizer", during as an "embedded activator" or after as a "post-synthesizer" the presentation of the target? Is it presented in the margin?

f) The level of enrichment of the analogy- what extent is the mapping of shared attributes? Is the analogy "simple", "enriched" or "extended"?

g) The pre-topic orientation- Do the analogies include explanation of the analog? Is there an identification of analogy as a cognitive strategy?

h) Limitations- Is there any warning regarding the limitations of the analogy?

As Thiele and Treagust (1994) did, the benchmark of the content area of the target domain included into the framework in order to decide in which subtopics of functions attempted to teach with analogies.

By the criterion of "the location in the chapter", it was intended to decide where the analogy was located with respect to its progress through the chapter. Briefly, this criterion was used for to decide whether analogies presented at the beginning, in the middle or at the end of the chapter. To decide analogies location in the chapter, whole function chapter divided into three parts by page numbers and each analogy was assigned to one of those parts.

In the current study, as Curtis and Reigeluth (1984) and Thiele and Treagust (1994) did, analog and target domains share similar functions or behaviours were said to have similar "function". Moreover, analogies whose analog and target domains share both similar function and similarities in external features were identified as having similar "structure-function". The original classification framework presented by Thiele and Treagust (1994) identified a third type of relationship. They said that two domains have similar "structure" when they contained similar appearances (same size, shape or color) or similar object attributes. However, this type was not included in the classification framework modified for the present study since having a taught of comparisons only based on similar object features or external characteristics cannot be accept as an analogy. This is a point, which is different from the original classification framework.

Analogies including only written texts were accepted as verbal analogies on the other hand all diagrams, graphs, pictures and photos without any written text that highlights the analog was accepted as pictorial analogy. Morevover, written analogies supported by pictures of the analog domain were identified as pictorial-verbal analogies. Here, to preclude confusion, (1) pictorial presentation that compares the analog to the target domains or (2) pictorial presentations of the analog domains were coded as pictorial-verbal analogies.

In addition, an analogy was counted as “embedded activator” if the analogy was presented in selected parts of the chapter with the primary discussion of the target concept. In addition, an analogy considered to be an “advance organizer” when it was presented before the primary discussion of the target concept. Finally, an analogy accepted as “post-synthesizer” if it was presented following the discussion of the target concept. Under the category of position, a subcategory marginalized was inserted to decide whether analogies were positioned in the margin of the text or not. Analogies that are advance organizers, embedded activators and post-synthesizers all were coded as positioned in the body part.

For the aim of this study, an analogy counted as “enriched” if there is an analogy statement explaining the relationship of analog and the target, even if an incomplete explanation. All analogies with no explanation were accepted as “simple”. They were generally composed of three main parts: the analog, target and a connector such as “like, is/are like, similar, similarly, think of, as, just as, is/are”. The enriched analogy is an extension of the simple analogy in which the grounds for the analogy relationship is made precise. On the other hand, while analyzing textbooks it was realized that some of the analogies were repeated several times. At this point, as older studies did, analogies which were used multiple times in the whole chapter were wanted to differ from others, so a third type was added to benchmark of level of enrichment. Here, analogical statements applied to several targets accepted as “extended analogy”. Moreover, if an analogy was used several times throught the chapter without considering that was a simple or an enriched one coded as “extended”.

Whether or not the analog explained in any detail or the presence of the indication of the use of analogy as a cognitive strategy were examined under the category of “pre-topic orientation” as Curtis and Reigeluth (1984) and Thiele and Treagust (1994) did. At this point, it did not mean to search for a completely defined analog however, search for maybe a simple phrase containing a few words that explain relevant analog attributes. Furthermore, in this study, seeked for analogies engaging any words “an analogy” or “analogous” to intend of strategy identification.

Here, following possible cases were thought : (1) the analog domain of the analogy explained, (2) there is an indication of cognitive strategy by stressing the word of "analogy" or "analogous", (3) Both analog of the domain explained and the word of "analogy" or "analogous" was used, (4) there is neither an explaination of the analog not a strategy identification.

Finally, under the category of "limitations", each analogy was inspected to find whether they included a general statement of the limitation of analogy or any statement concerning particularly to the unshared features in the analogy.

The two coders (My supervisor who had prior experience and engagement with this study and me) further scrutinized each analogy independently with an original agreement of 87 % for the 20 analogies (8 criteria × 20 analogies) classification. 23 analogies from me and 20 analogies from her were identified, however, when the disaggrements were reviewed, the remaining (13 %) were agreed upon following consensus discussion, so discussion resulted in 100 % agreement. She was provided with subset of coding from each content area. For the graph of function, types of functions, into and onto function data, agreement on codes was 100 %. On the codes for the concept of function, one to one function, identity function, constant function, linear function, composite function and inverse function, agreements on codes were 77 %, 83 %, 83 %, 89 %, 83 %, 75 %, 83 %, respectively. When the disagreements were reviewed, discussion resulted in 100 % agreement.

3.3.2 Analysis of Analogies in Mathematics Classrooms

The study involved a case study approach with naturalistic observation. The data were obtained from the observation of two teachers' 9^{th} grade mathematics classes during a 7-week data collection period: 18 December 2008- 11 February 2009. Classroom observations were prosecuted for two consecutive lessons (per lesson lasting 40 minutes) as scheduled in the teachers' programmes. Classes were observed three days (Tuesdays, Wednesdays and Thursdays) in every weeks of that period. Some days, lessons could not be conducted because of mid-term examinations or vacations. Moreover, sometimes no lessons would be held for a

whole week. During the third week of January and first week of February, students were on school holidays (see Table C). Eventually, twenty-five (thirteen of Neslihan's and twelve of Seçil's) lessons were videorecorded. Subtopics observed regarding functions included: function concept, graph of function, types of functions (one to one, onto and into, identity, constant, zero, linear), composite function and inverse function (see Table 4.7).

Since being the researcher of the current study, I completed data collection phase. During classroom observations, I had an observer- participant role. At classrooms, I primarily acted as an observer just for gathering data and interacted casually and nondirectly wih students. During observations, I sat at a desk placed in the back of the room, opposite the teachers' table.

Evertson and Green (1986) mentioned that an observer can has a negative effect on the validity or the reliability of the collected data. Researchers should be aware of possible observer effects and should take precautions to avoid or minimize them (Evertson & Green, 1986). For current study, to avoid arousing student and teachers' curiosity and to satisfy that the teachers and the students behave naturally, I made several visits to classes before starting data collection. Moreover, this preparation period, I videorecorded all the lessons to students and the teachers are accustomed to the video recording before starting actual one.

During each class period, field notes were taken about the subtopics taught in the classrooms and noted down regarding to followings: (1) any comparisons that might be accepted as analogies, (2) any teaching methods referred as "an analogy" during the class periods without judging they actually refer an analogy or not, and (3) under what conditions identified comparisons were used. In addition, following each class period, impressions of the analogies used in the clasess were noted down. Instead, during the observation, I wrote more qualitative classroom observation notes to form a database for analysis, and lesson recordings. Although, it was known that all lessons would be transcribed by watching the video-recordings, field notes were taken. Frick and Semmel (1978) mentioned a kind of observer reliability called intra-

observer reliability, which is the observer is consistent in her observational codings. This type of reliability can be established by observer twice code a video or audiotape of events similar to to those that she will be asked to observe in the field (Frick & Semmel, 1978). Therefore, I took field notes to be sure which of the comparisons will be coded as analogies.

After fieldwork phase of the research was completed, a set of field notes and visual data were gathered. All those data need to be analyzed and interpreted. For this purpose, lessons were transcribed completely and at the same time field notes used as supplementary sources. Data involved whole class teaching with teacher-centered instruction. The first step of data analysis was to identify the comparisons and teaching techniques that would be coded as analogies. Neslihan and Seçil offered all the detected comparisons, students did not offer any comparisons related to the concept of function during class discussions. By the same way of detection of textbook analogies, comparisons conforming analogy definition stated for the current study were considered as analogies. However, after reading transcripts, some comparisons and teaching techniques different from textbook analogies were detected. Sometimes, teachers used techniques, which were useful in promoting conceptual transfer, however, which did not satisfy analogy definition. They utilized these types of transfers mostly while solving problems. For example, Neslihan used this technique while solving the following question related with composite functions:

If $\begin{matrix} f(x)=3x-4 \\ g(x)=2x+3 \end{matrix}$ then, what is the value of $fog(x)$?

As solving the question, first she wrote, $(3x-4)\circ(2x+3)$.

> Neslihan: $(2x+3)$ is guest of $(3x-4)$. It is coming to $(3x-4)$'s home from door side and after coming $(3x-4)$ is saying: "welcome welcome and you can sit down". Then, $(2x+3)$ is trying to sit on the 4's place then 4 is saying, "It's not your place, you will sit near the place of x ", then $(2x+3)$ is sitting the place of x.

After that, she wrote $3.(2x+3)-4$.

Neslihan: This is the case, $(2x+3)$ came to as a quest, finally the result is equal to $6x-5$.

Since, these types of comparisons or teaching techniques were not in the focus of this study, they were not accepted as analogies. After distinguishing analogies from all potential analogies, analog and target domains of them were identified, such as "blanket- onto concept" and "function machine- constant function". The following step of the analysis included detailed inspection of the analogies by categorizing each of them according to the adapted analogy classification framework, which was described in detail before. Although use of the adapted scheme to analyze classroom analogies was consistent with the analysis of textbook analogies at some points, it was not sufficient for analogies that were present in the classrooms. Thus, a few adaptations were made to make it appropriate for the analysis of observed analogies. The coding system that employed for classroom analogies, the reasoning behind each element of the scheme and the adaptations of the scheme are as following:

Table 3.2 Analogy Classification Framework used to classify classroom analogies

a) The content of the target concept- Are there specific subtopics of function concept attempted to teach with analogies? Are there specific subtopics of function concept tended not to be taught with analogies?
b) The location of the analogy in the class- Is the analogy presented: (1) at the beginning, (2) in the middle or (3) at the end of the teaching of the function concept?
c) The analogy relationship between analogy and target- Do the analog and target domains of analogies fromed by the teachers share functional or structural-functional features?
d) The presentational format- Do the teachers presented analogies: (1) verbally, (2) pictorially or (3) verbally and pictorially?
e) The position of the analog relevant to the target- Is it before as an "advance organizer", during as an "embedded activator" or after as a "post-synthesizer" the main discussion of

the target domain?

f) The level of enrichment of the analogy- Are the analogie, employed in the classes by the teachers, unexplained ("simple"), explained to some extent ("enriched") or used more than once ("extended") over function concept?

g) The pre-topic orientation- Do teachers explain analog concept? Do the teachers introduce their analogies by using the word "analogy" in their classes?

h) Limitations- Do teachers state the limitations of the analogies?

By the criterion of the content of the target domain, it was intended to determine the concepts that teachers teach with analogies in their clasess and how the concepts were used in clasess compare to those in the textbook.

One of the adaptations was related with the benchmark of "location of the analogy". For categorizing the location of the analogy in the class, each class was divided into class hours. As mentioned before, respectively 13 and 12 class-hours of Neslihan and Seçil were observed. To satisfy consistence with analysis of textbook analogies, the location of analogies in the clasess determined by dividing each class chronologically into three parts. Then each of the analogies allocated to one of the three parts. For example, an analogy detected in the third class-hour of Seçil was allocated in the first part. Since, every part contained (13/3= 4.3) hours.

Similarly did for textbook analogies, under the categorization of analogical relationships, it was searched for that Neslihan and Seçil tended to use analogies whose analog and target domains share relational or both relational and external similarities.

Under the criterion of presentational format, it was wanted to decide that teachers presented analogies only verbally or they included pictures to their analogies. Here, if a teacher draws any diagram, scheme, graph or a picture accepted as pictorial analogy.

In the case of classroom analogies, an analogy considered to be an embedded activator if it was presented with the main discussion of the target concept. In addition, if the analogy was utilized before or after the main discussion of a target, it was thought the analogy to be an advance organizer or a post-synthesizer, respectively.

Ideally, analogies would be absolutely explained as they are used, however, this rarely comes true (Orgill, 2003). In this study, analogies, which were partially explained, accepted as "enriched", and which was completely unexplained accepted as "simple". Analogies used more then once when teaching the function concept were accepted as "extended".

By the benchmark of pre-topic orientation, as did while examining the textbook analogies, here it was searched for whether teachers explained the analog domains of the analogies or not and it was seeked for any strategies that stated with word of "analog" or "analogies". Here, same coding system used for the textbook analogies was employed. Furthermore, under a separate categorization, it was examined, whether the teachers stated the limitations of the analogies to their students or not.

Completed occurrences of analogy were used as the unit of analysis for the current study. In order to maximize reliability, analogy identification, subset of codes was assessed with the assist of an independent coder, my supervisor who had prior experience and engagement with this study. She was provided with a subset of the data from each content area. She was provided with individual excerpts of the transcript that had been coded. This was presented with the related portion of video that she could review as many times as necessary. For the function concept, composite function and inverse function data, agreement on codes was 100 %. On codes types of functions, agreement was 83 %. When the disagreements were reviewed, discussion resulted in 100 % agreement.

CHAPTER 4

RESULTS

4. 1 Textbook Analogy Categorization

A total of 20 analogies were recognized from the function chapter of the selected textbook. Table 4.1 offers an overview of textbook analogies by summarizing the distribution through the different categories. In Table B1, a summary table of analogies showing their analog and target domains, analogical relationship, presentational format, position in the text, level of enrichment, pre-topic orientation and limitations is given.

Table 4.1 Analogy Classification Framework and the number of analogies in each category

Category	n	%	
Analogical Relationship			
Functional (F)	20	100.0	
Both Functional and Structural (F & S)	0	0.0	% = 100.0
Presentational Format			
Verbal (V)	2	10.0	
Pictorial (P)	8	40.0	
Pictorial-Verbal (P-V)	10	50.0	% = 100.0
Position in the Text			
Advanced Organizer (AO)	7	35.0	
Embedded Activator (EA)	4	20.0	
Post-synthesizer (PS)	8	40.0	
Marginalised (M)	1	5.0	% = 100.0
Level of Enrichment			
Simple (S)	3	15.0	
Enriched (En)	10	50.0	
Extended (Ex)	7	35.0	% = 100.0
Pre-topic Orientation			
Analog Explanation (AEx)	11	55.0	
Indication of Cognitive Strategy (ICS)	0	0.0	
Both of Analog Explanation & Strategy Identification (B)	0	0.0	
None of Pre-topic Orientation (N)	9	45.0	% = 100.0
Limitations			
None (N)	20	100.0	
Specific (S)	0	0.0	% = 100.0

4.1.1 Target Concepts

The content of the target concept was labeled into 10 categorizations. Table 4.2 provides data suggesting that a significant proportion of the analogies (5, 25.0 %) related with “Function Concept” which is compared to calculator, countries social security system, granting scholarship, compliance clothing, crops and their place of growth, part and whole relationship, stores and types of sold products. After examining all analogies, it can be deduced that only one analogy, “function machine” is repeated for almost all targets except from function concept, graph of functions, into and onto functions.

Table 4.2 Analysis of the frequency of analogy use compared to target content area

Content of the target concept	n	%
Function concept	5	25.0
Graph of function	2	10.0
Types of functions	1	5.0
One to one function	3	15.0
Into and onto function	1	5.0
Identity function	1	5.0
Constant function	3	15.0
Linear function	1	5.0
Composite function	2	10.0
Inverse function	1	5.0
Totals	20	100.0

4.1.2 Analogy Location in the Textbook

Thiele and Treagust (1994) determined analogy location in the textbooks by dividing the whole books into 10 parts, and then placed the analogies in these parts according to their page numbers. In a similar way, the function chapter splited, but

into three parts by page numbers and allocated each analogy into one of those parts. Table 4.3 suggests that the analogies were tended to use at the beginning of the chapter. In fact, the number of analogies decreases from the beginning to the end of the textbook.

Table 4.3 Analysis of the position of the analogies in the chapter

Location	n	Page numbers
1	13	87-97
2	4	98-107
3	3	108-117

4.1.3 Analogical Relationship of Analog and Target Domains

In the textbook surveyed, all the analogical relationships were functional (20, 100.0 %), this means that no shared a structural or both structural and functional relationship. Table 4.4 suggests that the analog and target domains do not have the same physical appearance. For example, almost all subtopics of functions described the functions as being a function machine. The intention is to support the understanding of functional relationship. In this analogy, just take only one target, think that one to one function is the target and the function machine is the analog. A function is a process converting every input to an output and a function machine obtains an output for every input entered to the machine.

Table 4.4 Analog and the target concept relations

Analog (Familiar Domain)	Target (Unfamiliar Domain)
Calculator	Function concept
Countries social security system	Function concept
Granting scholarship	Function concept
Compliance clothing	Function concept
Crops and their place of growth	Function concept

Part and whole relationship	graph of function
Stores and types of sold products	graph of function
Function machine	Types of function, one to one function, identity function, constant function, linear function, inverse function, composite function
Vehicles where they employed	one to one function
Stores and things found in those stores	one to one function
offspring-maturity of creatures	onto and into functions
much love chocolate to see nothing else	constant function
Animals' habitats	constant function
Plane travel not to be arrived at once a time	composite function

4.1.4 Presentational Format

As illustrated in Table 4.1, the majority of the analogies were in a combination of pictures and words as pictorial-verbal analogies (10, 50.0 %). Only 2 (10.0 %) of the identified analogies had a verbal and the remainder (8, 40.0 %) had a pictorial presentation of the analog.

Most of the pictorial-verbal analog illustrations were presented in section prefaces. Those that were not presented in section prefaces were depicted in the body of the sections under activities. In addition, the pictorial analog depictions were given under example parts. Further analysis revealed that identified analogies are rarely in marginalized position. As Table 4.5 shows, just one verbal analogy was in a marginalized position.

Table 4.5 Frequencies of marginalized in the chapter

	Marginalised	Body	Total
Verbal (V)	1	1	2
Pictorial (P)	0	8	8
Pictorial-Verbal (P-V)	0	10	10
Total	1	19	20

4.1.5 Position of the Analog Relative to the Target

An inspection of the 20 analogies recognized in the current study revealed that the most (7, 35.0 %) were presented as advance organizer (see Table 4.1). It is remarkable that many of the analogies were presented as an analogical advance organizer. An example of one of those analogies is given below in Figure 4.1

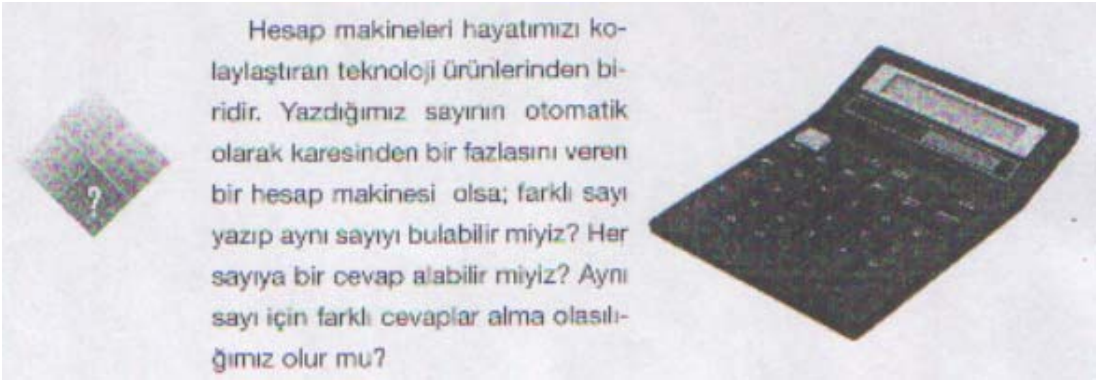

Figure 4.1 An analogy presented as advance organizer (adapted from MEB p.87)

On the other hand, 4 (20 %) analogies in the surveyed textbook were positioned as embedded activators; appearing during, rather than before or after the instruction on the target. Here (Figure 4.2) is one of them:

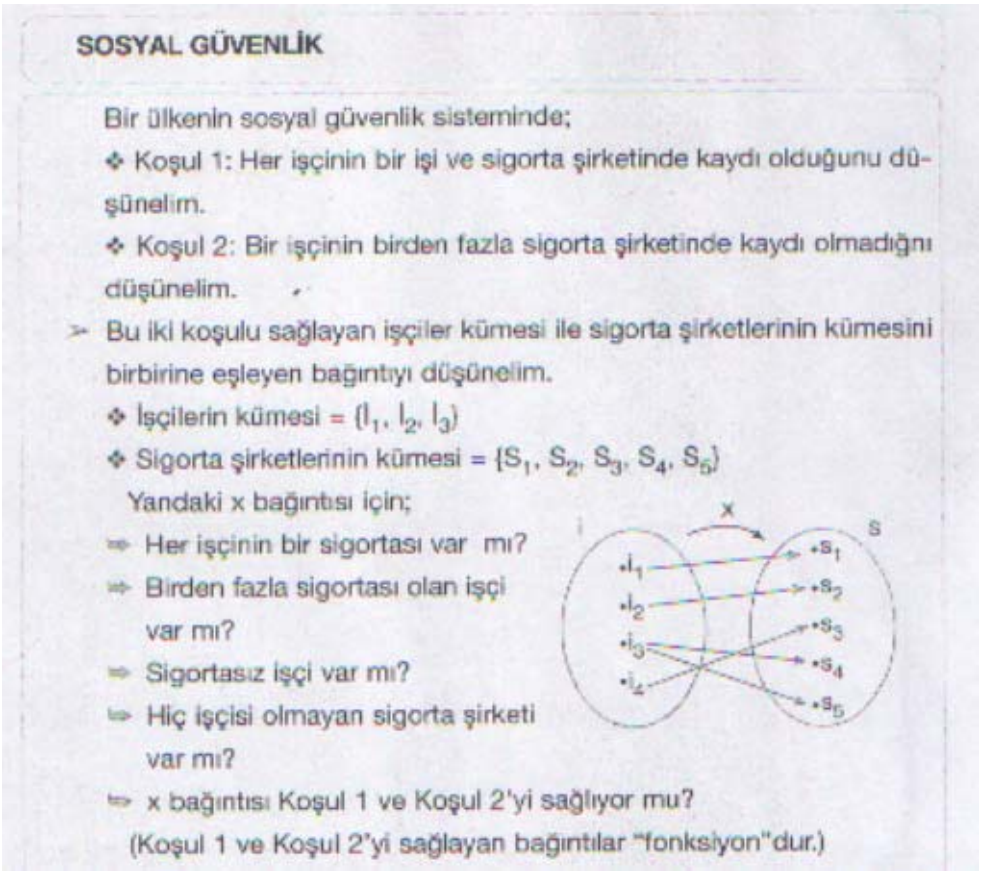

Figure 4.2 An analogy presented as embedded activator (adapted from MEB p. 87)

In current study, 8 (40 %) of the analogies found were post synthesizers as much as analogies positioned as advance organizers. An illustration is included below in Figure 4.3.

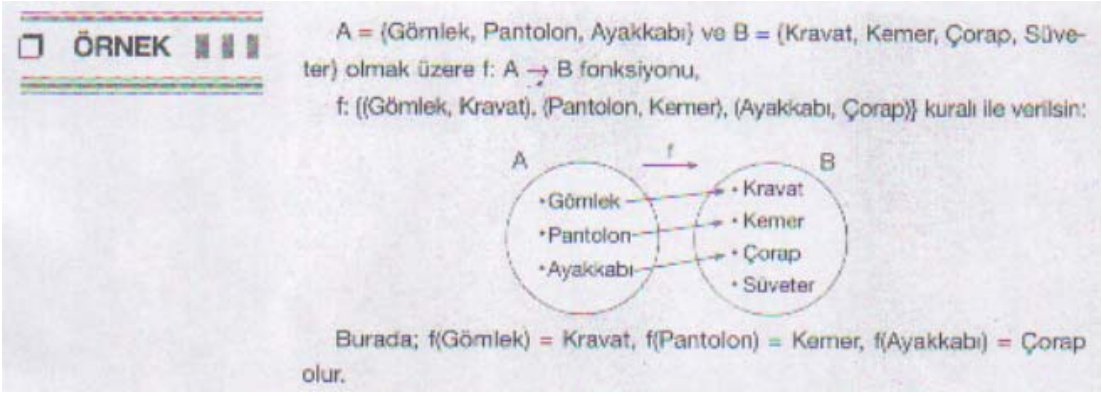

Figure 4.3 An analogy presented as post synthesizer (adapted from MEB p. 88)

Under this categorization, four types of analog position relative to target were considered. Position types of advance organizer, embedded activator and post synthesizer were taught when analogies positioned in the body part of the text, however an anaogy was given in the margin, labeled as marginalized analogy (1, 5.0 %).

4.1.6 Level of Enrichment

Half of the analogies that found in the function concept were enriched. However, this number does not show completely explained analogies. Instead, it indicates that analogies included at least one explicit sign of the reason of the analog and the target comparison. In addition, 7 (35 %) of the analogies found in the chapter were used multiple times for several targets labeled as “extended”. For instance, analogy whose analog, “function machine” matched with the targets “types of functions, one to one function, identity function, constant function, linear function, composite function and inverse function” counted as extended analogy in the current study. Only 3 (15 %) of the analogies identified as simple.

4.1.7 Pre-Topic Orientation

The analog clarification attempts to satisfy that the students are dealing with the correct attributes of analogical transfer (Thiele & Treagust, 1994). However, Table 4.1 depicts that just 11 (55 %) of the analogies had some analog explanation. In the whole chapter, no analogies included identification of the analogy using the words “analogy” or “analogous”.

4.1.8 Limitations

Out of 20 analogies recognized in the chapter, none of the analogies included specific warnings or limitations (see Table 4.1)

4.2 Classroom Analogy Categorization

A total of 32 analogies were recognized from the observation of two clasess. The number of analogies varied noticeably, Neslihan used totally, 11 analogies

whilst Seçil utilized 21 analogies. The analysis of analogies observed at Seçil's and Neslihan's clasess are illustrated in Table 4.6 that depicts separately the results. These tables offer an overview of observed analogies by summarizing the distribution through the different categories. In Table 2B (see Appendix B), summary table of classroom analogies showing their analog and target domains, analogical relationship, presentational format, position in the text, level of enrichment, pre-topic orientation and limitations is given.

Table 4.6 Analogy Classification Framework and the number of analogies in each category

Category	Neslihan	%	Seçil	%	Total	%
Analogical Relationship						
Functional (F)	9	28.1	21	65.6	30	93.8
Both Functional and Structural (F & S)	2	6.3	0	0.0	2	6.3
Presentational Format						
Verbal (V)	11	34.4	17	53.1	28	87.5
Pictorial (P)	0	0.0	0	0.0	0	0.0
Pictorial-Verbal (P-V)	0	0.0	4	12.5	4	12.5
Position of Analog Relative to Target						
Advanced Organizer (AO)	2	6.25	2	6.25	4	12.5
Embedded Activator (EA)	7	21.9	10	31.2	17	53.1
Post-synthesizer (PS)	2	6.25	9	28.15	11	34.4
Level of Enrichment						
Simple (S)	1	3.1	1	6.3	2	9.4
Enriched (En)	5	9.4	10	34.4	15	43.8
Extended (Ex)	5	21.8	10	25.0	15	46.8
Pre-topic Orientation						
Analog Explanation (AEx)	9	28.1	15	46.9	24	75.0
Indication of Cognitive Strategy (ICS)	0	0.0	0	0.0	0	0.0
Both of Analog Explanation & Strategy Identification (B)	1	3.1	0	0.0	1	3.1
None of Pre-topic Orientation (N)	1	3.1	6	18.8	7	21.9
Limitations						
None (N)	11	34.4	21	65.6	32	100.0
Specific (S)	0	0.0	0	0.0	0	0.0
Total Number of Analogies	11	34.4	21	65.6	32	100.0

4.2.1 Target Concepts

The target concepts covered by analogies in observed clasess were a little different from those in the textbook. In each class used at least one analogy about

function concept (4, 12.5 %), one to one function (3, 9.4 %), onto function (4, 12.5 %), into function (3, 9.4 %) functions, constant function (4, 12.5 %), composite (5, 15.6%) and inverse function (4, 12.5 %) as used in the textbook (see Table 4.2). Not Neslihan, however Seçil utilized analogies corresponding to identity function, linear function and graph of function were taught with analogies as done in the textbook. There were not many analogies regarding these concepts.

Although there were many similarities in the target concepts that used in the textbook and the classrooms, the target concepts of partial function and zero function covered only in the classrooms. In fact, just an analogy corresponding to partial function taught by Neslihan and just one analogy regarding zero function taught by Seçil.

Table 4.7 Analysis of the frequency of analogy use compared to target content area

	Neslihan		Seçil		Totals	
Content of the target concept	n	%	n	%	n	%
Function concept	2	6.3	2	6.3	4	12.5
Partial function	1	3.1	0	0.0	1	3.1
Graph of function	1	3.1	0	0.0	1	3.1
One to one function	1	3.1	2	6.3	3	9.4
Onto function	2	6.3	2	6.3	4	12.5
Into function	1	3.1	2	6.3	3	9.4
Identity function	0	0.0	1	3.1	1	3.1
Constant function	2	6.3	2	6.3	4	12.5
Zero function	0	0.0	1	3.1	1	3.1
Linear function	0	0.0	1	3.1	1	3.1
Composite function	1	3.1	4	12.5	5	15.6
Inverse function	1	3.1	3	9.4	4	12.5
Totals	11	34.4	21	65.6	32	100.0

4.2.2 Analogy Location in the Mathematics Classes

Data shown in Table 4.8 suggests that the analogies tend to be used more often in the middle stages. However, textbook analysis suggested that more analogies were found at the beginning of the the textbook. Orgill (2003) mentioned that difference of analogy location in the textbook and the classes might be caused by their different usage or the different purposes in the two settings (Orgill, 2003).

Table 4.8 Analysis of the position of the analogies in clasess

Location	n	%
1	7	21.9
2	16	50.0
3	9	28.1

4.2.3 Analogical Relationship of Analog and Target Domains

In observed clasess, of the 32 analogies that were scrutinized in the present research (see Table 4.6), nearly all (93.8%) were classified as functional, with just 2 (6.3 %) being both structural and functional. Patty analogy for function concept and song called “shattered” analogy for partial function were identified as structural and functional. Besides, the taught of the analog patty and the target function sharing similar functions, it was also taught that there was a similarity between their external features since a function contains x in it like f(x) and also the patty contains different ingredients in it. In addition, the analog, song called “shattered and the target partial functions had similar functions because of their dictionary meaning, they accepted as they had external similarity of their names. This proportion is parallel to the proportion of analogies used in the textbook. However, all the analogies identified in the textbook shared functional similarities not shared any structural in other words external features.

Although there were similarities in the target concepts covered analogies in the textbook and the classes, for most parts, the analogies used to explain these target concepts were not consistent with those used in the classes (see Table 4.9). Exception of the function machine analogy for identity, constant, linear and composite functions, all other analogies utilized in classes different from those utilized in textbook.

Table 4.9 Analog and the target concept relations that observed in Class B

Analog (Familiar Domain)	Target (Unfamiliar Domain)
A mother's sending her children to sightseeing	Function concept, onto and into functions, one to one function, inverse function
Patty including different ingredients	Function concept
A popular song name of "Shattered"	Partial Function
Blanket used for warmth	Onto function
Siblings going somewhere all together	Constant function
Knowing who will win the big lottery reward before drawings	Constant function
Introducing a friend to your buddy then they become two buddies and they ignore you	Composite function

Table 4.10 Analog and the target concept relations that observed in Class S

Analog (Familiar Domain)	Target (Unfamiliar Domain)
Travelling somewhere	Function concept, one to one function, onto function, constant function,
Official marriage	Function concept
Preference of a menu	One to one function
Selecting somethings from a list	Onto function
Ordering drinks from a list	Into function

Function machine	Into function, identity function, constant function, zero function, linear function, composite function
Pairing up boys and girls for dance contest	Graph of function
Going somewhere and turning back to first place	Inverse function
Different ways of calling the relationship between mother and her child	Inverse function
Scattering money to someone and then taking back them	Inverse function
Process of obtaining carpet from wool	Composite function
Process of obtaining plank from tree	Composite function
Travelling somewhere giving break then continuing the trip	Composite function

4.2.4 Presentational Format

Most (87.5%) of the analogies observed in the classes were presented verbally. This is not similar to presentational format used in textbook, where half of the analogies were pictorial- verbal. Data shown in Table 4.6 maintain that the analogies tend not to be presented in a just pictorial format. Further, analysis revealed that teachers preferred to accompany pictorial analogies by some verbal explanations.

4.2.5 Position of the Analog Relative to the Target

The majority of the analogies identified in the textbook were presented as advance organizers or post synthesizers and a few were presented as embedded activators after the introduction of the target domain just before conclusions about the target.

Analogy use in classes was a bit different from analogy use in the textbook. Despite the fact that majority (53.1 %) were presented as embedded activators, only 4 out of 32 analogies were presented advance organizers and 11(34.4 %) analogies were presented as post synthesizers.

In the textbook, almost half of the analogies were presented before and almost other half of them was presented after the main discussion of the target. However, in classes teachers tended to use analogies with main discussion of the target concept.

4.2.6 Level of Enrichment

Ideally, analogies should be absolutely clarified when they are once utilized (Orgill, 2003). However, in fact unexplained (simple) analogies or incompletely explained (enriched) analogies are common (Orgill, 2003). In the observed classes, only 14(43.8%) of the analogies were enriched while 3 (9.4 %) were simple. Nearly half of the analogies that were presented in the classrooms were repeated throught the teaching of function concept, slightly more than the 35 % of analogies that were used more than once over the function chapter of the textbook.

4.2.7 Pre-Topic Orientation

Out of the 32 observed analogies, 24 (75%) of the analogies had some analog explanation which is more than detected in the textbook. Moreover, as Table 4.6 suggests that no analogies involved a strategy identification however, just 1 (3.1 %) analogy included a statement identifying the strategy as “an analogy” with the explanation of the analog. Glynn et al. (1989) alleged that if the strategy identification employed more often, then this would be as effective as adding of a warning, which direct students to the correct cognitive procedure. In general, analogies whose domains being compaired and relationships between their analog and target domains clarified, used more frequently than in the textbook.

4.2.8 Limitations

Teachers of observed classes also failed to affirm the limitations of the analogies they utilized. Harrison & Treagust (1993) asserted that in some cases students do not appreciate enough about a target concept or what the limitations of an analogy. In such cases, teachers have to state the limitation of the analogies they used.

CHAPTER V

INTERPRETATIONS AND IMPLICATIONS

5.1 Discussions of General Results

As part of the current study, it was intended to determine the similarities and differences between analogies used in the selected textbook and those used in the classrooms. Results demonstrates that similar numbers of analogies utilized by Seçil when compared to the number of textbook analogies. However, comparatively, Neslihan used fewer analogies (See Tables 4.1 and 4.6). Examining all detected analogies detaily, it deduced that Neslihan used completely different analogies compared to analogies used in textbook. The examination of Seçil's analogies reveal that she formed analogies different as well as similar analogies compared to textbook analogies. Detail inspection illustrates that there is only one common analogy between analogies of Seçil and analogies of the textbook. The commonly used analogy was "function machine"that was utilized for different targets both in textbook and in the classroom: into function, identity function, constant function, zero function, linear function and composite function. Analogies that were different than the textbook analogies were used mostly spontaneously by the teachers. All these results support the idea of while textbook is the basic source for Seçil, this case is not valid for Neslihan.

Moreover, results of the study show that while some analogies included in the textbook and in the classrooms were extended to different targets, some were not extended. Classroom analogies both of the teachers repeated some of the analogies more than one times. Neslihan used the analogy of "a mother's sending her childreto sightseeing" for function concept, onto function, into function, one to one function, inverse function. Seçil used analogies of "travelling somewhere" for one to one function and onto function, and "function machine" for into function, identity function, constant function, zero function, linear function, composite function.

Similarly, the analogy of "function machine" repeated for many times in the textbook the same targets that Seçil used them.

The results for the location of the analogies in the textbook differed from those for the classrooms. As Table 4.3 and 4.8 suggests that the analogies tended to be placed more frequently in the earlier stages of the textbook. However, classroom teachers tended to use analogies at the middle stages of the instruction of the functions. Thiele and Treagust (1994) asserted that detection of analogies at the beginning phases supports that authors tried to show analogies as friendly strategies. Orgill (2003) metioned that this common sense implies that students can require analogies to familiar concepts to aid them at first become familiar with the new topics presented at the beginng of the text and during the lectures. Moreover, when students become familiar with the concepts at the end of a semester or at the end of the textbook, they may not need as many analogies to daily-life objects and by the help of their old experiences help them to compherend the concepts. Maybe, instead they can reference new information to information that they have learned before in the course or in the textbook (Orgill, 2003).

Furthermore, results reveal that all the textbook analogies were functional. Similarly, although almost all classroom analogies were also functional, two analogies formed by Neslihan were both functional and structural analogies. Curtis and Reigeluth (1984) stated that structural-functional analogy could offer the most powerful relationship in the all three types as long as there were broad grounds and few limitations between the analog and target domains. Analogies, "patty including different ingredients", "a popular song name of shattered", used by Neslihan were both functional and structural analogies. However, relations between analog and target domains of the analogies were not explained explicitely. Besides, solely structural relationship may be the weakest, because it offers only ground between analog and target domains, also it is possible that it offers greay many differences. In a case of employment of such analogies, it can be searched for whether students are capable of determining which analog and target domains share just structural or external similarities. If they do not understand that the analog and the target domains

share only external resemblances, they can think functional properties of analog to target domains and develop misconceptions. In the current study, I did not found analogies with shared only "structure".

The overwhelming majority of the analogies observed in the classes were presented verbally. On the other hand, majority of the textbook analogies were presented verbally and pictorially. Orgill (2003) stressed that an analogy with accurate visual presentation can support analogical transfer (Orgill, 2003). Moreover, it is alluded by the accurate visual presentation that one depicting the analog traits which can be transferred to the target domain (Orgill, 2003). While examing both of the teachers' analogy uses during the clasess, there is a bit difference. Although, Neslihan used only oral analogies, Seçil included graphs with her analogies. Moreover, Neslihan wanted her students to act out analogies. In the current study, I examined three types of analogy presentation format which were pictorial, verbal or both pictorial and verbal. However, after detecting Neslihan's different analogy presentation format, I thought that maybe it would be added as a fourth subcategory under the category of presentation format.

Although classroom analogies were tended to use as an advance organizer, results of classroom analogies satisfies that Neslihan and Seçil were mostly used analogies as embedded activators. Moreover, results show that Seçil used more analogies after the main discussion of the target concept. In addition, it was observed that Neslihan preferred to use analogies mostly before or during the main discussion of the target concept. After observations, she stated that she wanted to take students attention to new concept by using this technique.

Approximately half of the analogies I found in the textbook were enriched to some degree and only one analogy "function machine" was extended. Moreover, Both of the teachers also did not use commonly simple analogies and they used mostly extended analogies and they used analogies, which were enriched in some degree. The results of the textbook analogies show that only three of them were used as a simple version. When examining that it can be easily seen that those analogies

were easily grasped analogies by the students because they presented with pictures, so maybe the textbook writers did not need to explain the mapping between the analog and the target domains of the analogies or they expected teachers to explain those analogies. However, Thiele and Treagust (1994) mentioned that contrary to textbook authors' expectations, teachers do not explain analogies used in the textbooks. At this point, textbook authors should not rely on teacher to clarify their analogies and they must offer explained analogies in the textbooks (Orgill, 2003).

Curtis and Reigeluth (1984) asserted that in written texts, simple analogies are rarely utilized since they need learners to provide a clarification of analogical relationships involving all the similarities and the differences between the analog and the target domains. When these similarities and differences are not ready for the learner, learners might compel to understand. Therefore, to avoid this ambiguity, simple analogies should be exploited in cases where the relationship between analog and the target domains is apparent and is not required to be explained (Curtis & Reigeluth, 1984). However, any analogies observed in the classes were not carried out these models' offerings. Approximately three out of four analogies in each of the classes that observed included at least one part of the clarification of the analog domain. Sometimes, explanations of the analogs were just verbal and sometimes they could be visual illustrations provided with verbally.

In general, analogies whose domains being compaired and relationships between their analog and target domains clarified, used more frequently than in the textbook. Orgill (2003) stressed that to support healthy analogical transfer between analog and the target domains; teachers should make clear the analogies they used. Moreover, analogies help students to construct conceptual links between old knowledge and the new information. In case of their mistaken use, students can form incorrect thoughts concerning target concepts (Orgill, 2003). In the current study teachers explained the analogies however, their effects on the students were not observed.

Thiele and Treagust (1994) argued that teachers frequently use analogies spontaneously and randomly. In the present study, Seçil used mostly textbook analogies which were planned before the lesson, However, they mentioned that an analogy explanation attempts to satisfy that the students are dealing with the correct attributes of analogical transfer (Thiele & Treagust, 1994). Some authors advised teachers should use guidelines for teaching with analogies for utilizing analogies more effectively (Orgill, 2003). Gudelines majorly cited in the analogy literature: the Teaching-With-Analogies (TWA) (Glynn, 1996), FAR (Focus, Action, Reflection) model (Treagust, Harrison, & Venville, 1998), and the General Model of Analogy Teaching (GMAT) (Zeitoun, 1984). According to these three guidelines suggested identifying an analogy as an "analogy" and stating the limitations of it when explaining an analogy. In the current study,

5.2 Limitations and Weaknesses

Limitation of the present study was that the study was limited to the unit of function concept.

5.3 Implications of the Study

By the help of the results of this study, teachers will be informed of findings of this study in order for them to understand how textbooks supplement analogy use on function concept, they will have an idea of analogy should be used during classroom teaching. Moreover, by the help of this study, textbook writers should be informed regarding to results of the research in order to design future textbooks.

5.4 Recommendations for Future Research

In many ways, this study forms a base for further research regarding how analogies are utilized in mathematics classes and textbooks. Following directions for future work are proposed by the current study.

- Can a collection of well-developed analogies be constructed?
- How do students react to the utilization of multiple analogies for the same topic?
- What are the possible outcomes of utilizing different analogies in the classroom versus the textbook?
- Do students think about textbook analogies?
- At what point do students question the validity of teachers' analogies?
- At what point do teachers develop a correct understanding of what analogies are and how they should be utilized in classes?
- What kind of training would aid teachers use analogies more effectively?

More studies are required to understand more fully the mental progression that students employ when using analogies. Further research is required on how students use analogies when in learning multifaceted mathematics subjects.

REFERENCES

Breidenbach, D., Dubinsky, E., Hawks, J., & Nichols, D. (1992). Development of the process conception of function. Educational Studies in Mathematics, 23(3), 247-285.

Carlson, M. P. (1995). A cross-sectional investigation of the development of the function concept. Unpublished Ph.D., University of Kansas, United States -- Kansas.

Cooney, T. J., & Wilson, M.R. (1993). Teachers' Thinking about Functions: Historical and Research Perspective. In T. Romberg, E. Fennema, & T. Carpenter (Eds.), Integrating research on the graphical representation of functions (pp.131-158). Hilsdale NJ: Lawrence Erlbaum.

Creswell, J.W. (2003). Research design: Qualitative, quantitative, and mixed method approaches. London: Sage.

Curtis, R. V., & Reigeluth, C. M. (1984). The use of analogies in written text. Instructional Science, 13(2), 99-117.

Eisenberg, T. (1991). Functions and associated learning difficulties. In D. Tall (Ed.), Advanced Mathematical Thinking (pp. 140-152). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Eisenberg, T., & Dreyfus, T. (1994). On understanding how students learn to visualize function transformations. In E. Dubinsky, Schoenfeld, A., & Kaput, J. (Ed.), Research in collegiate mathematics education 1 (Vol. 4, pp. 45-68). Providence, RI: American Mathematical Society.

Even, R. (1990). Subject matter knowledge for teaching and the case of functions. Educational Studies in Mathematics, 21(6), 521-544.

Even, R. (1993). Subject-Matter Knowledge and Pedagogical Content Knowledge: Prospective Secondary Teachers and the Function Concept (Vol. 24, pp. 94-116): National Council of Teachers of Mathematics.

Even, R. (1998). Factors involved in linking representations of functions. Journal of Mathematical Behavior, 17, 105-121.

Evertson, C. M., & Green, J. L. (1986). Observation as inquiry and method. In M.C. Wittrock (Ed.), Handbook of Research on Teaching (3rd ed., pp.162-213). New York: Macmillian.

Frick, T., & Semmel, M. I. (1978). Observer agreement and reliabilities of classroom observational measures. *Review of Educational Research*, 48, 157-184.

Gagatsis, A., Elia, I., & Mousoulides, N. (2006). Are registers of representations and problem solving processes on functions compartmentalized in students' thinking? Relime, Número Especial, 197–224.

Glynn, S. M., Britton, B. K., Semrud-Clikeman, M. & Muth. K. D. (1989). In J. A. Glover, R.R. Ronning & C. R. Reynoldss. Eds., *Handbook of Creavity: Assesment, Theory, and Research* (pp. 383-398). New York: Plenum.

Glynn, S. (1996). Teaching with Analogies: Building on the Science Textbook (National Reading Research Center). Reading Teacher, 49(6), 490-492.

Glynn, S. M., Duit, R. & Thiele, R. B. (1995). Teaching science with analogies: A strategy for constructing knowledge. In S. M. Glynn & R. Duit (Eds.), Learning science in the schools: Research reforming practice (pp. 247-276). Mahwah, NJ: Erlbaum.

Giordano, P. (2008). Learning the concept of function: Guess My Rule activities with Dr. Robert B. Davis. Unpublished Ed.D., Rutgers The State University of New Jersey - New Brunswick, United States -- New Jersey.

Haciomeroglu, G. (2006). Prospective secondary teachers' subject matter knowledge and pedagogical content knowledge of the concept of function. Unpublished Ph.D., The Florida State University, United States -- Florida.

Hansson, O. (2004). Preservice teachers' view on the concept of function: A study including the utilization of concept maps. Unpublished dissertation. Lulea University of Technology.

Harel, G., & Dubinsky, E. (1992). The Concept of Function: Aspects of Epistemology and Pedagogy. MAA Notes (Vol. 25).

Harrison, A. G. & Treagust, D. F. (1993). Teaching with analogies: A case study in grade-10 optics. Journal of Research in Science Teaching, 30, 1291-1307.

Hitt, F. (1998). Difficulties in the articulation of different representations linked to the concept of function. The Journal of Mathematical Behavior, 17(1), 123-134.

Jones, M. (2006). Demystifying functions: The historical and pedagogical difficulties of the concept of the function. Rose-Hulman Undergraduate Math Journal, 7(2), 1-20.

Kajander, A., & Lovric, M. (2009). Mathematics Textbooks and Their Potential Role in Supporting Misconceptions. International Journal of Mathematical Education in Science and Technology, 40(2), 173-181.

Kaput, J. (1994). The representational roles of technology in connecting mathematics with authentic experience. In Biehler, R. et. al. (eds.) Mathematics didactics as a scientific discipline.

Kwari, R. (2007). An investigation into the development of the function concept through a problem-centred approach by Form 1 pupils in Zimbabwe. Unpublished M. Ed., University of South Africa (South Africa), South Africa.

Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, Graphs, and Graphing: Tasks, Learning, and Teaching. Review of Educational Research, 60(1), 1-64.

Lloyd, G. M., & Wilson, M. (1998). Supporting Innovation: The Impact of a Teacher's Conceptions of Functions on His Implementation of a Reform Curriculum. Journal for Research in Mathematics Education, 29(3), 248-274.

Mesa, V. (2004). Characterizing Practices Associated with Functions in Middle School Textbooks: An Empirical Approach. Educational Studies in Mathematics, 56(2), 255-286.

Michelsen, C. (2006). Functions: a modelling tool in mathematics and science. ZDM, 38(3), 269-280.

National Council of Teachers of Mathematics NCTM (1989). Curriculum and evaluation standards for school mathematics Virginia, USA.

Norman, A. (1992). Teachers' mathematical knowledge of the concept of function. In G. Harel & E. Dubinsky (Eds.), The concept of function. Aspects of epistemology and pedagogy MAA Notes, 25 (pp. 215-232). Washington, DC: Mathematical Association of America

Orgill, M. (2003). Playing with a double-edged sword: Analogies in biochemistry. Unpublished Ph.D., Purdue University, United States -- Indiana.

Richland, L. E. (2003). Analogy in classroom mathematics: Implications for interaction, memory, and induction. Unpublished Ph.D., University of California, Los Angeles, United States -- California.

Richland, L. E., Holyoak, K. J., & Stigler, J. W. (2004). Analogy Use in Eighth-Grade Mathematics Classrooms. Cognition and Instruction, 22(1), 37.

Selden, A., &. Selden, J. (1992). Research perspectives on conceptions of function: Summary and overview. In G. Harel & E. Dubinsky (Ed.), The concept of function: Aspects of epistemology and pedagogy, MAA Notes, 25 (pp. 1-16). Washington, DC: Mathematical Association of America

Sfard, A. (1992). Operational origins of mathematical objects and the quandary of reification – The case of function. In G. Harel & E. Dubinsky (Eds.), The concept of function: Aspects of epistemology and pedagogy, MAA Notes, 25 (pp. 59-84). Washington, DC: Mathematical Association of America.

Shulman, L., S. (2004). The wisdom of practice: essays on teaching, learning and learning to teach. San Francisco, CA: Jossey-Bass.

Sierpinska, A. (1992). On understanding the notion of function. In G. Harel & E. Dubinsky (Eds.), The concept of function: Aspects of epistemology and pedagogy (pp. 25-58). Washington, DC: Mathematical Association of America.

Tall, D. (1992). The transition to advanced mathematical thinking: Functions, limits, infinity, proof. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning.(pp.495-511). Toronto, Canada: Macmillian.

Tall, D. (1996). Functions and calculus. In A.J. Bishop et al. (Eds), International Handbook of Mathematics Education (pp. 289–325). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Thiele, R. B., & Treagust, D. F. (1991). Using Analogies To Aid Understanding in Secondary Chemistry Education.

Thiele, R. B., & Treagust, D. F. (1994). The nature and extent of analogies in secondary chemistry textbooks. Instructional Science, 22(1), 61-74.

Thomas, M. (2003). The role of representation in teacher understanding of function. In N. A. Pateman, B. J. Dougherty, & J. Zilliox (Eds.), Proceedings of the 27th Conference of the International Group for the Psychology of Mathematics Education held jointly with the 25th Conference of PME-NA (Vol. 4, pp. 291-298). Honolulu: University of Hawai'i.

Vinner, S., & Dreyfus, T. (1989). Images and Definitions for the Concept of Function (Vol. 20, pp. 356-366): National Council of Teachers of Mathematics.

Wiersma, W. (2000). Research methods in education: An introduction (7th ed.). Massachusetts: Allyn & Bacon.

Zazkis, R., Liljedahl, P., & Gadowsky, K. (2003). Conceptions of function translation: obstacles, intuitions, and rerouting. *The Journal of Mathematical Behavior, 22*(4), 435-448.

APPENDIX A

TITLES IN THE TEXTBOOK

Table A: Section numbers and corresponding page numbers of the function chapter

Section numbers and titles		Page numbers
3.4	Function Concept	87-90
3.4.1	Graph of Functions	91-92
3.5	Types of Functions	94
3.5.1	One to one Function	94-95
3.5.2	Into and onto Function	96
3.5.3	Identity Function	97
3.5.4	Constant Function	98
3.5.5	Linear Function	99-100
3.7	Composite Function	110-112
3.7.1	Inverse Function	112-117
	Total pages	23

APPENDIX B

SUMMARY TABLES OF TEXTBOOK AND CLASSROOM ANALOGY CLASSIFICATIONS

Table B1. Summary table of textbook analogy classification

Analog	Target	Analogical Relationship	Presentational format	Position of Analog Relative to Target	Level of enrichment	Pre-topic orientation	Limitations	Total 20
		Functional (F) Both Functional and Structural (F & S)	Verbal (V) Pictorial (P) Pictorial-Verbal (P-V)	Advanced Organizer (AO) Embedded Activator (EA) Post-synthesizer (PS) Marginalised (M)	Simple (S) Enriched (En) Extended (Ex)	Analog Explanation (AEx) Indication of cognitive Strategy (ICS) Both (B), None (N)	None (N) Specific (S)	
Calculator	Function concept	F	P-V	AO	S	AEx	N	5
Countries social security system		F	P-V	EA	En	AEx	N	
Granting scholarship		F	P-V	EA	En	AEx	N	
Compliance clothing		F	P	PS	En	N	N	
Crops and their place of growth		F	P	PS	En	N	N	
Part and whole relationship	Graph of function	F	P	PS	En	N	N	2
Stores and types of sold products		F	P	PS	En	N	N	
Function machine	Types of functions	F	V	AO	Ex	AEx	N	1

Table B1 Continued

Function machine	One to one function	F	P-V	EA	Ex	AEx	N	3
Vehicles where they employed		F	P	PS	En	N	N	
Stores and things found in those stores		F	P	PS	En	N	N	
Offspring-maturity of creatures	Into and onto function	F	P	PS	En	N	N	1
Function machine	Identity function	F	P-V	EA	Ex	AEx	N	1
much love chocolate to see nothing else		F	V	M	S	N	N	
Function machine	Constant function	F	P-V	EA	Ex	AEx	N	3
Animals' habitats		F	P	PS	S	N	N	
Function machine	Linear function	F	P-V	EA	Ex	AEx	N	1
Plane travel not to be arrived at a once a time	Composite function	F	P-V	AO	En	AEx	N	2
Function machine		F	P-V	EA	Ex	AEx	N	
Function machine	Inverse Function	F	P-V	EA	Ex	AEx	N	1

Table B2. Summary table of analogy classification for analogies used by Neslihan

Analog	Target Concepts	Analogical Relationship	Presentational Format	Position of Analog Relative to Target	Level of Enrichment	Pre-topic Orientation	Limitations	Total 11
		Functional (F) Both Functional and Structural (F & S)	Verbal (V) Pictorial (P) Pictorial-Verbal (P-V)	Advanced Organizer (AO) Embedded Activator (EA) Post-synthesizer (PS)	Simple (S) Enriched (En) Extended (Ex)	Analog Explanation (AEx) Indication of Cognitive Strategy (ICS) Both (B), None (N)	None (N) Specific (S)	
A mother's sending her children to sightseeing	Function Concept	F	V	EA	Ex	AEx	N	2
Patty including different ingredients		F & S	V	PS	En	B	N	
A popular song name of "Shattered"	Partial Function	F & S	V	AO	S	N	N	1
A mother's sending her children to sightseeing	Onto function	F	V	EA	Ex	AEx	N	2
Blanket used for warmth		F	V	EA	En	AEx	N	
A mother's sending her children to sightseeing	Into Function	F	V	EA	Ex	AEx	N	1
A mother's sending her children to sightseeing	One to one function	F	V	EA	Ex	AEx	N	1
Siblings going somewhere all together	Constant function	F	V	EA	En	AEx	N	2
Knowing who will win the big lottery reward before drawings		F	V	PS	En	AEx	N	
A mother's sending her children to sightseeing	Inverse function	F	V	AO	Ex	AEx	N	1
Introducing a friend to your buddy then they become two buddies and they ignore you	Composite function	F	V	EA	En	AEx	N	1

Table B3 Summary table of analogy classification for analogies used by Seçil

Analog	Target Concepts	Analogical Relationship	Presentational Format	Position of Analog Relative to Target	Level of Enrichment	Pre-topic Orientation	Limitations	Total 21
		Functional (F) Both structural and functional (F & S)	Verbal (V) Pictorial (P) Pictorial-Verbal (P-V)	Advanced Organizer (AO) Embedded Activator (EA) Post-synthesizer (PS)	Simple (S) Enriched (En) Extended (Ex)	Analogy Explanation (AEx) Indication of cognitive Strategy (ICS) Both (B), None (N)	None (N) Specific (S)	
Travelling somewhere	Function concept	F	P-V	PS	Ex	N	N	2
Official marriage		F	V	PS	En	AEx	N	
Travelling somewhere	One to one function	F	V	PS	Ex	AEx	N	2
Preference of a menu		F	P-V	PS	En	AEx	N	
Travelling somewhere	Onto function	F	V	PS	Ex	N	N	2
Selecting somethings from a list		F	V	PS	En	N	N	
Ordering drinks from a list	Into function	F	V	EA	En	AEx	N	2
Function machine		F	V	EA	Ex	N	N	
Function mchine	Identity function	F	V	EA	Ex	AEx	N	1
Function machine	Constant function	F	V	EA	Ex	AEx	N	2
Travelling to same place		F	V	EA	Ex	AEx	N	
Function machine	Zero Function	F	V	EA	Ex	AEx	N	1
Function machine	Linear function	F	V	EA	Ex	N	N	1
Pairing up boys and girls for dance contest	Graph of function	F	V	EA	En	AEx	N	1
Going somewhere and turning back to first place	Inverse function	F	V	EA	En	AEx	N	3
Different ways of calling the relationship between mother and her child		F	V	PS	En	AEx	N	

Table B3 Continued

Scattering money to someone and then taking back them		F	V	PS	En	AEx	N	
Process of obtaining carpet from wool	Composite function	F	P-V	AO	En	AEx	N	4
Process of obtaining plank from tree		F	V	AO	En	AEx	N	
Function machine		F	P-V	EA	Ex	AEx	N	
Travelling somewhere giving break then continuing the trip		F	V	PS	S	N	N	

APPENDIX C
ARCHIEVED VIDEOTAPED DATA

Table C Archieved videotaped data

Weeks	Tuesday		Wednesday		Thursday	
	Date	Videotapes	Date	Videotapes	Date	Videotapes
1					18.11. 2008	N-B1, N-B2 S-S1, S-S2
2	22.11.2008	N-B3	24.11.2008	S-S3, S-S4	25.11.2008	N-B4, N-B5 S-S5, S-S6
3	29.11.2008	N-B6, N-B7	31.11.2008	S-S7, S-S8	01.01.2009	Vacation (The day after New Year's)
4	05.01.2009	N-B8, N-B9	07. 01.2009	Mid-term examination of	08.01.2009	Mid-term examination of Mathematics
5	12.01.2009	N-B10, N-B11	14.01.2009	S-S9, S-S10	15.01.2009	Mid-term examination of English
6	19.01.2009	N-B12, N-B13	21.01.2009	Last days of the first semester, students did not attend the class	22.01.2009	Last days of the first semester, students did not attend the class
7	26.01.2009	Holiday	28.01.2009	Holiday	31.01.2009	Holiday
8	02.02.2009	Holiday	04.02.2009	Holiday	05.02.2009	Holiday
9	09.02.2009	There was no lesson, because of the first days of the new semester	11.02.2009	S-S11, S-S12		