# A THESIS SUBMITTED TO <br> THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES <br> OF MIDDLE EAST TECHNICAL UNIVERSITY 

BY

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# IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR <br> THE DEGREE OF MASTER OF SCIENCE <br> IN <br> AEROSPACE ENGINEERING 

## PARAMETRIC INVESTIGATION OF SPRAY CHARACTERISTICS USING INTERFEROMETRIC PARTICLE IMAGING TECHNIQUE

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# ABSTRACT <br> PARAMETRIC INVESTIGATION OF SPRAY CHARACTERISTICS USING INTERFEROMETRIC PARTICLE IMAGING TECHNIQUE 

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December 2009, 103 pages

Spray is an efficient tool in the usage whose primary objectives are to obtain droplets with increased liquid surface area and more dispersed liquid over a larger volume. The determination of spray characteristics has been a topic of extensive research recently. In the present investigation, the flow structure of a spray issuing from an oil burner nozzle was determined in a parametrical manner. The main tool in the experimental research is the Interferometric Particle Imaging (IPI) configuration. This method exploits the interference between light reflected from and refracted through individual transparent spray droplets which are illuminated by a laser light sheet in a wide angle forward-scatter region. Based on a scattering theory, the droplet diameter of spray particles can be related to the light pattern scattered from that particle. In addition, using double-framed images also enables the calculation of velocities associated with these particles. In this way, as a
representation of spray structure, the droplet size and velocity distributions were obtained prior to a change in the primary parameters of liquid flow e.g. surface tension, viscosity, density and the injection pressure. The evolution of spray characteristics in space were also examined by conducting measurements in different radial and axial locations relative to spray centerline.

Key-words: Characteristics of spray, flow parameters, Interferometric Particle Imaging (IPI), scattering theory, droplet distribution, spatial change

## öz

# SPREY KARAKTERISTiKLERININ PARÇACIK GÖRÜNTÜLEMELi Gỉişim ÖLÇÜM YÖNTEMi KULLANILARAK PARAMETRIK incelemesi 

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Aralık 2009, 103 sayfa

Spreyler kullanımdaki öncelikli amacın küçük sıvı parçacıklarının olabildiğince yüksek yüzey alanı/hacim oranına sahip olacak şekilde geniş bir hacime dağıtılması söz konusu olduğunda etkili araçlardır. Sprey karakteristiklerinin belirlenmesi son yıllarda artan sayıda araştırmaya konu olmaktadır. Bu çalışmada ise bir yakıt enjektöründen püskürtülen spreyin akış yapısı parametrik olarak incelenmiştir. Deneysel olan bu çalışmada ölçüm aracı olarak parçacık görüntülemeli girişim-ölçer düzeneği kullanılmışıır. Bu yöntem spreyi oluşturan ve ayırdedilebilen sıvı parçacıklarına güçlü ışık demetlerinin gönderilip, bu parçacıklardan yansıyan ve kırılan demetlerin oluşturduğu girişim desenlerini kullanmaktadır. Mie saçılıma teorisi temel alınarak sprey parçacıklarının boyutları bu parçacıklardan saçılmış ışık desenleriyle ilişkendirilebilmektedir. Ayrıca kısa zaman aralıklarıyla alınmış resim çiftleri
bu parçacıkların hızlarının da belirlenmesini sağlamaktadır. Bu yolla sprey akış yapısının göstergesi olarak parçacık boyut ve hız dağılımları, akış parametrelerinden sıvı yüzey gerilimi, viskozitesi, yoğunluğu ve enjeksiyon basınçlarından herbirinin değişimine karşılık elde edilmiştir. Bunların haricinde, spreye göre farklı açısal ve eksenel yönlerde ölçümler gerçekleştirilerek sprey karakteristiklerinin uzaysal değişimi de incelenmiştir.

Anahtar Kelimeler: Sprey karakteristikleri, akış parametreleri, Parçacık Görüntülemeli Girişim-Ölçüm (IPI), saçılma teorisi, parçacık dağılımları, uzaysal değişim

## ACKNOWLEDGEMENTS

I would like to express gratitude to my supervisor, Asst. Prof. Dr. Oğuz Uzol, and my co-supervisor, Prof. Dr. Serkan Özgen for their valuable support and encouragement throughout this work.

I appreciate Ahmet Uyar and Murat Ceylan for their efforts in the all stages of experimental study. I would like to thank to Prof. Dr. Nafiz Alemdaroğlu who shares the facilities of Aerodynamics Laboratory.

I am very thankful to Sinan Körpe and Barış Erdoğan who helped me a lot during the chemistry-related stages of the work.

Lastly, I want to express my love and gratitude to my family for understanding and love through the duration of my studies.

The support given through the grant 106M067 by The Scientific and Technological Research Council of Turkey (TUBITAK) to this thesis is greatly acknowledged.

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## LIST OF SYMBOLS

| AMD | Arithmetic Mean Diameter $(\mathrm{m})$ |
| :--- | :--- |
| SMD | Sauther Mean Diameter $(\mathrm{m})$ |
| PDF | Probability Density Function |
| $p_{i}$ | Light ray of order i |
| $E$ | Electric Field (N/c) |
| $c$ | Speed of light (m/s) |
| $\lambda$ | wavelength of the light $(\mathrm{m})$ |
| $\alpha_{p}$ | Polarizibility |
| $\theta_{z}$ | Incidence [ $\left.{ }^{\circ}\right]$ |
| $p$ | Acceleration of the charge $(\mathrm{N} / \mathrm{kg})$ |
| $\sigma$ | Differential scattering cross sections $\left(\mathrm{m}^{2} / \mathrm{rad}\right)$ |
| i | Angular intensity functions |
| $m$ | The complex refractive index |
| $n$ | The real refractive index |
| $k$ | The absorption coefficient $\left(\mathrm{m}^{-1}\right)$ |
| $\alpha$ | The size parameter |
| $U$ | Tangential Velocity [m/s] |
| $d$ | particle diameter $(\mathrm{m})$ |
| $\varepsilon$ | The angle of incidence $\left[^{\circ}\right]$ |
| $\beta$ | The angle of refraction[ $\left.{ }^{\circ}\right]$ |
| $\theta$ | The angle of deflection[ $\left.{ }^{\circ}\right]$ |
| $\delta$ | Phase difference $(\mathrm{m})$ |
| $N$ | Number of Fringes |
| $\theta$ | Scattering angle[ $\left.{ }^{\circ}\right]$ |


| $\alpha$ | Collecting angle[ $\left.{ }^{\circ}\right]$ |
| :--- | :--- |
| f | Focal number |
| $\sigma$ | Surface tension(dynes/cm) |
| $V M D$ | Volume Mean Diameter $(\mathrm{m})$ |
| $T$ | Temperature $\left({ }^{\circ} \mathrm{C}\right)$ |
| $P$ | Pressure $\left(\mathrm{N} / \mathrm{m}^{2}\right)$ |
| $D[1,0]$ | Arithmetic Mean Diameter $(\mathrm{m})$ |
| $D[2,0]$ | Surface Diameter $(\mathrm{m})$ |
| $D[3,0]$ | Volume Mean Diameter $(\mathrm{m})$ |
| $D[3,2]$ | Sauther Mean Diameter $(\mathrm{m})$ |
| $D[4,3]$ | Herdan Diameter $(\mathrm{m})$ |

Subscripts:

| VV | Both vertically polarized incident light and vertically <br> polarized scattered light with respect to the scattering |
| :--- | :--- |
|  | plane |
| $H H$ | Both horizontally polarized incident light and horizontally <br> polarized scattered light with respect to the scattering <br> plane |
| $D$ |  |
| $R$ | Diffracted Light |
| $X$ | Internal reflection Ray |
| $z$ | Externally reflected Ray |
|  | Polarization direction |

## CHAPTER 1

## INTRODUCTION

A liquid flow where a stream of small drops is dispersed into a relatively large volume is called a spray. Having necessary physical conditions such as high shear rates as a result of cavitation in the spray nozzle make ordinary jet flows efficient mixers in interaction with ambient air [1]. It is an efficient tool in the usage whose primary objectives are to obtain droplets with increased liquid surface area/volume rate and more dispersed liquid over a larger volume. For example, in the case of fuel injection in a combustion chamber, by increasing the surface area of individual fuel droplets, a spray enhances evaporation which consequently promotes high combustion efficiency while reducing the emissions of harmful materials [24]. Similar to this, better performance in related devices including sprays may be achieved by figuring out the structure of dispersed flow, and then making possible manipulations on the parameters of flow which are determined to be effective and adjustable.

The general configuration of experimental or numerical studies on sprays consists of a liquid ejection through a spray nozzle into stagnant air by the help of a water or oil pump. As a representation of spray characteristics, the droplet size and velocity distributions of spray particles are important quantities which can be used to calculate deterministic parameters of related flow like the Arithmetic Mean Diameter (AMD) or the Sauter Mean Diameter (SMD). The latter one, the Sauter Mean Diameter or the surface
area moment mean, is the most common tool that is determined by the distribution of particle sizes to foresee the action level in combustion applications where the active surface area has the biggest importance [5]. The size distribution or equivalently Probability Density Function (PDF) of droplets can be adjusted by changing primary parameters of liquid flow, e.g. surface tension, viscosity, density or by means of external control parameters like injection pressure.

Various models and theories have been developed in order to understand the effects of different parameters on the sizes of droplets which are broken up from the core of the flow and eventually constitute the spray [6-8]. Among them, the linear stability theory assuming linear evolution of droplets which means no interaction between individual particles in space and time can provide a qualitative determination of different parameters on the resulting spray particles. But, in reality, due to the non-linear interaction mechanism along the particles which are broken up especially around injector exit, the actual particle size distributions show discrepancy [9] compared to ones given by the theory and it is this reason that prevents the linear theory to give quantitative anticipations on spray flows. So, if the characteristics of a spray flow prior to a variable change are needed to be determined quantitatively, it is a must to conduct an experimental study in a parametrical manner.

In the experimental study of Snyder et al. [10], the size distributions of spray particles injected from a fan-spray atomizer were obtained by the help of a Malvern particle sizer for several liquid samples having different flow conditions. In a similar study [11], Snyder noted the presence of large drops at the edges of the spray with sizes several times larger compared to the central region. Also another type of spray namely hollow-cone spray was investigated by Okamoto et al. [12] in an experimental and numerical manner by paying a special attention to the liquid sheet formation at the
orifice. By comparing the numerical and experimental results, they commented on the effects of several parameters on the resulting spray. Mao et al. [13] measured the spray particle sizes generated by an air assist swirl atomizer by the help of a Malvern particle sizer in order to obtain droplet distributions and the Sauter mean diameters corresponding to different flow conditions.

In data acquisition, there are several experimental techniques, which rely on different principles by exploiting different physical properties of liquid particles. In general, these methods determine the characteristics of a spray flow by obtaining the size and velocity distributions of individual droplets.

The majority of measurements concerning spray flow are conducted using optical techniques. Based on the principle of Doppler shift effect of moving particles, Phase Doppler Anemometry (PDA) and Laser Doppler Anemometry (LDA) techniques provide point measurements to get information on the size and velocity of individual particles [14-15]. Although the measurements based on this method give very accurate results in the determination of velocity and size, the necessity to make large number of measurements [16] for meaningful results makes these methods an inconvenient option to analyze full flow field whose structure is complicated and three-dimensional, e.g. fuel spray. Another technique, namely Malvern particle sizer which measures the sizes of particles using the diffraction patterns of individual droplets is also not an appropriate method for complicated flows due to its insufficient spatial resolution [17-19]. The technique of high-speed photography in a magnified ambience is another optical method combining the abilities of both size and velocity determination when a double-pulsed illumination is used [20-21]. Although one can get both enough spatial and temporal resolution in this way, the price will be a very restricted field of view. In general, the above mentioned methods or some other derivatives lack to have at least one ability out of
two in spray measurements: to map a wide field of view of dispersed flow or to have a sufficient spatial resolution.

Throughout this study, an optical method, namely Interferometric Particle Imaging (IPI), which exploits the interference between light reflected from and refracted through individual spray droplets under a coherent laser light illumination in a wide angle forward-scatter region is used as a measurement tool to investigate different states of spray prior to a change of different parameters. In this method, the output of a measurement is a set of two images showing spray particles captured in a focal and a de-focal plane. In the de-focal plane, some fringe patterns are formed due to the phase difference between the external reflection from the surface and the direct refraction through the droplet. Furthermore, based on a scattering theory, the fringe separation or the fringe number originating from a transparent spherical particle can be related to the droplet diameter of that particle. In this way, subsequent to some processing procedure on acquired data, the diameters of particles contained in an image become determined. Then, using large number of snapshots captured under the same conditions, a statistical statement of spray is obtained in terms of particle droplet distribution which eventually enables the calculation of Arithmetic and Sauter mean diameters for the associated case [22-25]. In this method, if only the droplet diameters are to be determined, the images can be single frame, but velocity information associated with these droplets needs both images to be double-framed because velocities are calculated by using the displacement vectors obtained from two snapshots of spray which were taken in a short time interval. It should also be noted that performing IPI data analysis requires calibration images to ensure overlap between the focused and de-focused images.

### 1.1 Objectives

In this study, the main objective is to investigate the influences of several parameters, e.g. surface tension, temperature -which is also able to alter several intrinsic parameters- and injection pressure on the onset of individual droplets constituting the spray flow experimentally by using IPI technique. The surface tension of water was reduced by adding small amounts of a surfactant while keeping the viscosity and density constant under same injection pressure and ambient temperature. In other cases, the water was heated by the help of a resistance heater as to give different values of viscosity, density and surface tension corresponding to each temperature value and then droplet distributions were obtained associated with these cases. In the last part, by changing injection pressure with the help of a by-pass mechanism, the resulting spray particles were observed and analyzed. The evolution of spray characteristics in space is another objective of this study and was examined by conducting measurements in different radial and axial locations relative to spray for different values of surface tension.

This thesis is presented in three parts. In Chapter 2, following a brief introduction on the fundamentals of the classical scattering theory, the Mie scattering theory and an extended geometrical approach to calculate droplet sizes are given in some details. After that, in the same Chapter, the data processing in IPI technique is explained in depth. The issues related with experimental setup and measurement details are given in Chapter 3. The experimental evidences are discussed in Chapter 4. The Appendix A and B are used to extend the advanced algebra beyond the scattering theory. The Appendix C summarizes the IPI particle detection algorithm and the additional results that are not commented in the Chapter 4 are presented in Appendix D.

## CHAPTER 2

## INTERFEROMETRIC PARTICLE IMAGING TECHNIQUE

This chapter presents the details of physical mechanism namely the scattering theory which is the basis of IPI technique. In the first part of this chapter, the fundamentals of scattering theory are briefly introduced through Equations 2.1-2.5. Then, an extended scattering theory to determine the intensity function of scattered light as developed by Gustav Mie is presented through Equations 2.6-2.23. Finally, by using the asymptotic expansions of Lorenz-Mie coefficients, the intensity function of scattered light is determined in an optical manner through Equations 2.24-2.38. Additionally, the constitution of interference fringes in terms of constructive and destructive interference is discussed through Equations 2.39-2.41. Finally, the technical aspects of the IPI processing procedure to identify particle diameters are explained in detail in the last part of this chapter.

### 2.1 Scattering Theory

When a transparent droplet with a spherical shape is exposed to a coherent light sheet, the incident light gets scattered in all directions into a reflection ray, $p_{0}$, and $n^{\text {th }}$ order refraction rays, $p_{1}, p_{2}, \ldots, p_{n}$.


Figure 2.1-Scattering of incident light from a small spherical particle

The intensity of the scattered light depends on the polarizability which is determined by the type of the transparent particle. However, more importantly, the light scattering has a direct dependence on particle size which makes it a valuable tool in particle sizing measurements. The light scattering from small particles can be explained assuming the incident light as an electromagnetic field which is described as:

$$
\begin{equation*}
E_{z}=E_{0} \cos \left(\frac{2 \pi c t}{\lambda}\right) \tag{2.1}
\end{equation*}
$$

where $E_{0}$ is the amplitude of the electric field, $c$ is the speed of light, and $\lambda$ is the wavelength of light. The subscript $z$ on $E$ means polarized light along the $z$ axis.

In case of a polarizable particle, the incident electric field will induce a dipole moment with a magnitude proportional to that field in the particle. The proportionality constant is called the polarizability $\alpha_{p}$. The higher a particle's polarizability the higher will be the magnitude of the dipole moment induced by a given electromagnetic field. The dipole moment is

$$
\begin{equation*}
p=\alpha_{p} E_{0} \cos \left(\frac{2 \pi c t}{\lambda}\right) \tag{2.2}
\end{equation*}
$$

The induced dipole moment will radiate light in all directions. Considering the radiated or scattered light at a distance $r$ from the origin along a line that makes an angle $\theta_{z}$ with the $z$ axis as in Figure 2.2.


Figure 2.2- Observation direction for light scattered off a particle at the origin in a direction that makes an angle $\theta_{z}$ with respect to the $z$ axis where the observation distance is $r$.

The scattered light field will be proportional to $\left(1 / c^{2}\right)\left(d^{2} p / d t^{2}\right)$. The second derivative of $p$ is the acceleration of the charge on the dipole moment. To include spatial effects, the scattered light is also proportional to $1 / r$ as electromagnetic fields drop as $1 / r$ and to $\sin \theta_{z}$ as to project of the dipole moment on the observation direction. Combining all these effects, the electric field for light scattered in the $\theta_{z}$ direction is

$$
\begin{equation*}
E_{s}=\frac{1}{r} \frac{1}{c^{2}} \frac{d^{2} p}{d t^{2}} \sin \theta_{z}=-\frac{1}{c^{2}} \alpha_{p} E_{0} \frac{4 \pi^{2} c^{2}}{r \lambda^{2}} \sin \theta_{z} \cos \left(\frac{2 \pi c t}{\lambda}\right) \tag{2.3}
\end{equation*}
$$

Because the equipment that measures scattered light is typically only sensitive to the intensity of light which is equal to the amplitude of the
square of electromagnetic field, squaring the amplitude of $E_{s}$ gives the scattered light intensity at $r$ and $\theta_{z}$ :

$$
\begin{equation*}
I_{s}=\alpha_{p}^{2} I_{0 z} \frac{16 \pi^{4}}{r^{2} \lambda^{4}} \sin ^{2} \theta_{z} \tag{2.4}
\end{equation*}
$$

where $I_{0 z}$ is the intensity of the polarized incident light.

$$
\begin{equation*}
I_{0 z}=E_{0}^{2} \tag{2.5}
\end{equation*}
$$

Gustav Mie [26] expanded this formulation for perfectly spherical particles by recasting Equation 2.4 in terms of the differential scattering cross sections, $\sigma$ ( $\mathrm{m}^{2} / \mathrm{rad}$ ), namely

$$
\begin{align*}
& I_{V V}=I_{0} \frac{1}{r^{2}} \sigma_{V V}^{\prime}  \tag{2.6}\\
& I_{H H}=I_{0} \frac{1}{r^{2}} \sigma_{H H}^{\prime} \tag{2.7}
\end{align*}
$$

In these two equations, the subscripts refer to the state of polarization of the incident and scattered light, respectively. The subscripts $V V$ refer to both vertically polarized incident light and vertically polarized scattered light with respect to the scattering plane. Similarly, the subscripts HH refer to both horizontally polarized incident light and horizontally polarized scattered light with respect to the scattering plane. For unpolarized incident light, the scattering is given by the following

$$
\begin{equation*}
I_{s c a t}=I_{0} \frac{1}{r^{2}} \sigma_{s c a t}^{\prime} \tag{2.8}
\end{equation*}
$$

where $\sigma_{s c a t}^{\prime}$ is the average of $\sigma_{V V}^{\prime}$ and $\sigma_{H H}^{\prime}$

These equations may also be reconsidered in terms of the rate of scattered energy into a defined solid angle [27], as shown in Figure 2.3.


Figure 2.3 - Angular scattering intensity

Based on the theory of Mie, the differential scattering cross sections are defined in terms of the angular intensity functions $i_{1}$ and $i_{2}$, as given by the expressions

$$
\begin{align*}
& \sigma_{V V}^{\prime}=\frac{\lambda^{2}}{4 \pi^{2}} i_{1}  \tag{2.9}\\
& \sigma_{H H}^{\prime}=\frac{\lambda^{2}}{4 \pi^{2}} i_{2} \tag{2.10}
\end{align*}
$$

the above two equations are averaged to define the differential scattering cross section for unpolarized incident light, which gives the relation

$$
\begin{equation*}
\sigma_{s c a t}^{\prime}=\frac{\lambda^{2}}{8 \pi^{2}}\left(i_{1}+i_{2}\right) \tag{2.11}
\end{equation*}
$$

In this formulation, the intensity functions are calculated from the infinite series given by

$$
\begin{align*}
& i_{1}=\left|\sum_{n=1}^{\infty} \frac{2 n+1}{n(n+1)}\left[a_{n} \pi_{n}(\cos \theta)+b_{n} \tau_{n}(\cos \theta)\right]\right|^{2}  \tag{2.12}\\
& i_{2}=\left|\sum_{n=1}^{\infty} \frac{2 n+1}{n(n+1)}\left[a_{n} \tau_{n}(\cos \theta)+b_{n} \pi_{n}(\cos \theta)\right]\right|^{2} \tag{2.13}
\end{align*}
$$

In Equations 2.12 and 2.13, the angular dependent functions $\pi_{n}$ and $T_{n}$ are expressed in terms of the Legendre polynomials by

$$
\begin{align*}
& \pi_{n}(\cos \theta)=\frac{P_{n}^{(1)}(\cos \theta)}{\sin \theta}  \tag{2.14}\\
& \tau_{n}(\cos \theta)=\frac{d P_{n}^{(1)}(\cos \theta)}{d \theta} \tag{2.15}
\end{align*}
$$

where the parameters $a_{n}$ and $b_{n}$ are defined as

$$
\begin{align*}
& a_{n}=\frac{\Psi_{n}(\alpha) \Psi_{n}^{\prime}(m \alpha)-m \Psi_{n}(m \alpha) \Psi_{n}^{\prime}(\alpha)}{\xi(\alpha) \Psi_{n}^{\prime}(m \alpha)-m \Psi_{n}(m \alpha) \xi_{n}^{\prime}(\alpha)}  \tag{2.16}\\
& b_{n}=\frac{m \Psi_{n}(\alpha) \Psi_{n}^{\prime}(m \alpha)-\Psi_{n}(m \alpha) \Psi_{n}^{\prime}(\alpha)}{m \xi(\alpha) \Psi_{n}^{\prime}(m \alpha)-\Psi_{n}(m \alpha) \xi_{n}^{\prime}(\alpha)} \tag{2.17}
\end{align*}
$$

The algorithms for computing the angular coefficients [Legendre functions $\pi_{n}(\cos \theta)$ and $\left.\tau_{n}(\cos \theta)\right]$ and scattering coefficients are given in Appendix A. It is worthwhile to note that the angular coefficients only depend on angle $\theta$, while the scattering coefficients depend on $m$ and diameter ( $d$ ) in which $m$ represents the complex refractive index and equals $n(1-i k)$ where $n$ is the
real refractive index and $k$ is the absorption coefficient. While the refractive index of a medium is a measure of how much the speed of light is reduced inside the medium, the absorption coefficient determines how far into a material light of a particular wavelength can penetrate before it is absorbed. For example, in a material with a low absorption coefficient, light is only poorly absorbed, and if the material is thin enough, it will appear transparent to that wavelength as in the current case with a water sample having a low absorption coefficient of about $10^{-3} \mathrm{~cm}^{-1}$ and a green light sheet having a wavelength of 532 nm .

The size parameter $\alpha$ is defined in terms of particle diameter, $d$, as

$$
\begin{equation*}
\alpha=\frac{2 \pi d}{\lambda} \tag{2.18}
\end{equation*}
$$

The Ricatti-Bessel functions $\Psi$ and $\xi$ are defined in terms of the half-integer-order Bessel function of the first kind $\left(J_{n+1 / 2}(z)\right)$, where

$$
\begin{equation*}
\Psi_{n}(z)=\left(\frac{\pi z}{2}\right)^{\frac{1}{2}} J_{n+1 / 2}(z) \tag{2.19}
\end{equation*}
$$

Equation 2.20 describes the parameter $\xi_{n}$

$$
\begin{equation*}
\zeta_{n}(z)=\left(\frac{\pi z}{2}\right)^{\frac{1}{2}} H_{n+1 / 2}(z)=\Psi_{n}(z)+i X_{n}(z) \tag{2.20}
\end{equation*}
$$

where $H_{n+1 / 2}(z)$ is the half-integer-order Hankel function of the second kind, where the parameter $X_{n}$ is defined in terms of the half-integer-order Bessel function of the second kind, $\mathrm{Y}_{n+1 / 2}(\mathrm{Z})$, namely

$$
\begin{equation*}
X_{n}(z)=-\left(\frac{\pi z}{2}\right)^{\frac{1}{2}} Y_{n+1 / 2}(z) \tag{2.21}
\end{equation*}
$$

Finally, the total extinction and scattering cross sections are expressed as

$$
\begin{align*}
& \sigma_{\text {ext }}=\frac{\lambda^{2}}{2 \pi} \sum_{n=0}^{\infty}(2 n+1) \operatorname{Re}\left\{a_{n}+b_{n}\right\}  \tag{2.22}\\
& \sigma_{\text {scat }}=\frac{\lambda^{2}}{2 \pi} \sum_{n=0}^{\infty}(2 n+1)\left(\left|a_{n}\right|^{2}+\left|b_{n}\right|^{2}\right) \tag{2.23}
\end{align*}
$$

Therefore, the intensity function of scattered light formulating by Equations 2.6 and 2.7 is determined.
van de Hulst [28] demonstrated that the Mie formulas admit asymptotically for $\alpha \rightarrow \infty$, a principle for localizing rays and of a fundamental separation of diffracted, refracted, and reflected light which is referred to as the opticallocalization principle (derivations are given in Appendix B). This approach asserts that the term of order $n$ in the partial wave expansion in Mie formulas corresponds approximately to a ray of distance ( $n+1 / 2$ )/k from the center of the particle, where $k$ is the wave number defined as $k=2 \pi / \lambda$. It is shown by van de Hulst that when $\alpha$ >> 1, the expansions in Equations 2.12 and 2.13 may be truncated at $n+1 / 2 \approx \alpha$, and the remaining finite sum is separated into two parts: one is independent of the nature of the particle and the other is dependent on it. For transparent spherical particles where the refractive index $m$ is real, those terms are calculated by van de Hulst in the near forward directions by using the asymptotic expansions of LorenzMie coefficients as $\alpha \rightarrow \infty$. Thus the scattered light field at point $P$ is shown to be composed of two parts [29]:
(1) A diffracted light field component independent of the nature of the particle. For the forward direction, the amplitude functions for the diffracted light field are derived asymptotically as $\alpha \rightarrow \infty$ as,

$$
\begin{equation*}
i_{D_{1}}=i_{D_{2}}=\frac{k^{2} d^{2}}{4} \frac{J_{1}[k d(\sin \theta) / 2]}{k d(\sin \theta) / 2} \tag{2.24}
\end{equation*}
$$

which are referred as real amplitude functions $A_{D}$, i.e., $A_{D 1,2}=i_{D 1}(\theta)=i_{D 2}(\theta)$, where subscripts $D, 2$, and 1 refer to the diffracted light, first plane-polarized and second plane polarized, respectively. $J$ is the Bessel function.
(2) Reflected and refracted rays dependent on the nature of the particle. Similarly the amplitude functions for reflected and refracted light fields can be derived when $\alpha \rightarrow \infty$ as,
$i_{1,2}(\theta)=\sum_{p, q} \frac{k d}{2} E_{1,2}\left[\frac{\sin (2 \varepsilon)}{2 \sin \theta\left|\frac{d \theta^{\prime}}{d \varepsilon}\right|}\right]^{1 / 2} \cdot \exp \left[i \delta+\frac{i \pi}{2}\left(p+1-\frac{1}{2} q-\frac{1}{2} s-2 l\right)\right]$
where $p$ is the number of internal reflections (for $p_{0}$ the ray is externally reflected, for $p=1$ the ray is refracted and leaves the particle without any internal reflections, etc.), $\varepsilon$ is the angle of incidence, and $\beta$ is the angle of refraction; $\varepsilon$ and $\beta$ are related to each other by Snell's law:

$$
\begin{equation*}
m=\sin \varepsilon / \sin \beta \tag{2.26}
\end{equation*}
$$

Figure 2.4 shows the optical geometry and the various components of the scattered light field.


Figure 2.4-Geometrical optics components of the scattered light

The angle of deflection $\theta^{\prime}$ is defined as

$$
\begin{equation*}
\theta^{\prime}=\pi-2 \varepsilon-p \pi+2 p \beta \tag{2.27}
\end{equation*}
$$

and $\theta$ 'is related to the scattering angle $\theta$ as

$$
\begin{equation*}
\theta^{\prime}=2 \pi l+q \theta \tag{2.28}
\end{equation*}
$$

where $q$ is either +1 or -1
$l$ is an integer which must be chosen so that $\theta$ lies between 0 and $\pi ; i$ is an integer defined by the sign of $\left(d \theta^{\prime} / d \varepsilon\right)$ which is -1 for $m>1$ and +1 for $m<1$ when $p<2$; and

$$
\begin{equation*}
\frac{d \theta^{\prime}}{d \varepsilon}=2 p \frac{\tan \beta}{\tan \varepsilon}-2 \tag{2.29}
\end{equation*}
$$

$E_{1}$ and $E_{2}$ are defined as

$$
\begin{align*}
& E_{1,2}=\gamma_{1,2} \text { for } p=0  \tag{2.30}\\
& E_{1,2}=\left(1-\gamma_{1,2}^{2}\right)\left(-\gamma_{1,2}\right)^{p-1} \text { for } p>0 \tag{2.31}
\end{align*}
$$

where

$$
\begin{align*}
& \gamma_{1}=(\cos \varepsilon-m \cos \beta) /(\cos \varepsilon+m \cos \beta)  \tag{2.32}\\
& \gamma_{2}=(m \cos \varepsilon-\cos \beta) /(m \cos \varepsilon+\cos \beta) \tag{2.33}
\end{align*}
$$

are Fresnel reflection coefficients. The phase difference $\delta$ is given by

$$
\begin{equation*}
\delta=k d(\cos \varepsilon-p m \cos \beta) \tag{2.34}
\end{equation*}
$$

The summation sign in Equation 2.25 signifies that all the scattered light field components should be summed up before calculating the scattered light intensity. This leads to the interference of several scattered light rays externally reflected ( $p_{0}$ in Figure 2.4), refracted with no internal reflections ( $p_{1}$ in Figure 2.4), etc.-. In the forward direction, only the diffracted, externally reflected $\left(p_{0}\right)$, and refracted without internally reflected $\left(p_{1}\right)$ components are of any significant amplitude, so that the summation in Equation 2.25 is carried out only for the value of $p$ of 0 and 1 .

The total scattered light intensity at point $P$ is derived as

$$
I_{\theta, \phi}=i_{2,1}(\theta) \cdot i^{*} 2,1(\theta)\left\{\begin{array}{c}
\cos ^{2} \varphi  \tag{2.35}\\
\sin ^{2} \varphi
\end{array}\right\}\left(1 / k^{2} r^{2}\right)
$$

Subscripts 2 and 1, as mentioned before, indicate the scattered light components plane-polarized in the second plane and first plane, respectively.

In reading Equation 2.35 the rule is that subscript $\theta$ corresponds to subscript 2 and $\cos ^{2} \varphi$, and subscript $\varphi$ corresponds to subscript 1 and $\sin ^{2} \varphi$.
$i_{2,1}^{*}(\theta)$ can be written as;

$$
\begin{align*}
& i_{2,1}(\theta) \cdot i_{2,1}^{*}(\theta)=A_{D 2,1}^{2}+A_{R 2,1}^{2}+A_{X 2,1}^{2}+2 A_{D 2,1} A_{R 2,1} \sin \delta_{R} \\
& -2 A_{D 2,1} A_{X 2,1} \sin \delta_{X}-2 A_{R 2,1} A_{X 2,1} \cos \left(\delta_{R}-\delta_{X}\right) \tag{2.36}
\end{align*}
$$

where subscripts $D, R$, and $X$ correspond to diffracted, refracted without internal reflection, and externally reflected geometrical optics rays, respectively. Real amplitude functions $A_{R}$ and $A_{x}$ are defined as

$$
\begin{align*}
& A_{R 1,2}=E_{R 1,2}(d / 2)\left(\frac{\sin 2 \varepsilon}{2 \sin \theta\left|\frac{d \theta^{\prime}}{d \varepsilon}\right|}\right)^{1 / 2} \cdot k  \tag{2.37}\\
& A_{X 1,2}=E_{X 1,2}(d / 2)\left(\frac{\sin 2 \varepsilon}{2 \sin \theta\left|\frac{d \theta^{\prime}}{d \varepsilon}\right|}\right)^{1 / 2} \cdot k \tag{2.38}
\end{align*}
$$

Therefore, by using the asymptotic expansions of Lorenz-Mie coefficients as $\alpha \rightarrow \infty$, the intensity function of scattered light is determined in an optical manner. This is a useful approach to calculate important quantities, as an example, the calculated values of the light intensity scattered from a droplet having $60 \mu \mathrm{~m}$ diameter is shown in Figure 2.5.


Figure 2.5 - Intensity distribution of the scattered-light from a droplet having $60 \mu \mathrm{~m}$ diameter

As can be seen from Figure 2.5, the scattered light has the most intense level at low scattering angles in which reflection ray from the droplet also becomes unimportant compared to the diffraction. This interval of interest is the region used in diffraction-based techniques like the Malvern droplet sizer.

As shown by Glantschnig et al. [30] that the dominant light rays directing towards the droplet are just the 0th order reflected and first-order refracted rays in the forward-scatter region. Figure 2.6 compares the intensities of reflection and refraction rays in case of a water droplet with a vertically polarized illumination. Two rays correspond to same value at approximately 70deg. When these two rays originating from a spherical droplet pass through a collecting lens and fall into a focal and a de-focal plane via an optical setup, a fringe interferogram within a circular edge is constituted in the de-focal plane due to the interference of these rays as shown in Figure 2.7.


Figure 2.6 - Light intensities of 0th order reflection and 1st order refraction rays for water ( $n=1.33$ )


Figure 2.7-Optical paths of Oth order reflection and 1st order refraction rays passing through a lens and projecting into two imaging planes

As we know that light is a traveling wave which is composed of an oscillating electric and magnetic field. The two light waves originating from the same source are coherent meaning that they have exactly the same wavelength and maintain a constant phase relative to each other. An interferogram is constituted when two coherent waves occur at the same place and same time. The light amplitude differences in an interferogram results from the algebraic sum of the individual amplitudes of $0^{\text {th }}$ order reflection ray and $1^{\text {st }}$ order refractions ray. Constructive interference occurs when the waves are exactly in phase so that their amplitudes are added. Destructive interference occurs when the waves are exactly out of phase (by one half wavelength) and their amplitudes subtract.


Figure 2.8- Interference of 1 st order refraction and 0th order reflection rays

The interference of laser light after it passed through a transparent particle is shown in Figure 2.8. The angles for constructive interference (bright fringes) can be found by applying the condition:

$$
\begin{equation*}
d \cdot \sin \theta=m \lambda \quad(m=0,1,2,3, \ldots) \tag{2.39}
\end{equation*}
$$

Whereas the angles for destructive interference (dark fringes) can be found by applying the condition:

$$
\begin{equation*}
d \cdot \sin \theta=\left(m+\frac{1}{2}\right) \lambda \quad(m=0,1,2,3, \ldots) \tag{2.40}
\end{equation*}
$$

If the interference pattern is viewed on a plane with a distance $L$ from the particle, the particle diameter can be found from the equation:

$$
\begin{equation*}
d=\frac{\lambda \cdot m \cdot L}{y} \quad(m=0,1,2,3, \ldots) \tag{2.41}
\end{equation*}
$$

where $y$ is the distance from the center of the interference pattern to the $m^{\text {th }}$ bright line in the pattern. This formula applies as long as $\theta$ is small (i.e., $y$ is small compared to $L$ ).

Therefore, using the geometrical approach, it was shown that the fringe pattern is actually a consequence of phase difference between reflected and refracted rays and it is that phase difference which determines the interfringe space or fringe number in an interferogram. Furthermore, calculations based on the optical geometry using the phase difference yields a relation between fringe number and droplet diameter. The mathematical relationship between particle diameter $d$ and the number of fringes $N$ was formulated by Hesselbacher [31] (without derivation) as:

$$
\begin{equation*}
d=\frac{2 \lambda N}{\alpha} \cdot \frac{1}{\cos (\theta / 2)+\frac{m \sin (\theta / 2)}{\sqrt{m^{2}-2 m \cdot \cos (\theta / 2)+1}}} \tag{2.42}
\end{equation*}
$$

where $\lambda$ is the wavelength of the laser light sheet and $m$ is the real part of the refractive index of the working fluid. $\theta$ is the scattering angle whereas $\alpha$ is the collecting angle. It should be noted that the measured diameter is not dependent on the intensity of illumination or the magnification of the lenses.

There are 5 variables in Equation 2.42 to determine droplet diameter where the scattering and collecting angles are fixed after the positioning of cameras. The refractive index is known as water is employed as a working fluid throughout experiments. The illumination light is a green laser sheet whose wavelength is also known. Therefore all the variables except fringe number are fixed. The remaining parameter, $N$, for an individual pattern, needs to be identified in order to calculate the particle size. The identification is conducted through an IPI processing.

### 2.2 IPI Processing

IPI processing determines the size of of spherical, transparent particles through the fringe patterns observed in a de-focused image. To determine diameter only, the image can be single frame while velocity information needs both images to be double-framed. To accurately determine the position of a particle, two overlapping images are required; a focused image and a de-focused image. Performing IPI data analysis requires calibration images to ensure overlap between the focused and de-focused images. Calibration has to be performed before IPI analysis and is done via acquiring a set of the target image by both of the cameras as shown in Figure 2.9.


Figure 2.9 - Calibration images captured by camera A and camera B

The target used in the calibration is a dot matrix type which has the dimesions of $200 \times 200 \mathrm{~mm}$ having 5 mm spacing between black dots on white background. The diameter of zero marker is 2.7 mm , main marker is 2.0 mm , and axis marker is 13 mm . Following the image acquisition for calibration, a method namely Imaging Model Fit is applied on the images to describe the mapping of points from subject to the image plane by determining model parameters through analysis of two calibration images having a resolution of $1344 \times 1024$ pixels $^{2}$ - which is same as actual measurement images- as shown in Figure 2.10.


Figure 2.10 - Image Model Fits for the calibration images shown in Figure 2.9

After the acquisition of images containing spray particles by two cameras as shown in Figure 2.11 and Figure 2.12, they are pre-processed using a high pass filter to remove the low frequency information in order to enhance weak fringes or fringes that show intensity variation. The Laplacian $5 x 5$ is found to be a useful filter for enhancing fringes and applied to all images which will be processed in the particle sizing.


Figure 2.11-An instantaneous snapshot of spray particles captured in a defocal plane


Figure 2.12-An instantaneous snapshot of spray particles captured in the focal plane


Figure 2.13 - The effect of Laplacian $5 x 5$ fitering on the image shown in
Figure 2.11


Figure 2.12

A Laplacian filter forms a basis for edge detection method. It is used to compute the second derivatives of intensities of an image, which measure the rate at which the first derivative change. This helps to determine if a change in adjacent pixel values is an edge or a continuous progression. Additionaly, the image is sharpened when contrast is enhanced between areas with little variation in brightness or darkness by the help of this filter. By combining these effects, it acts as a high-pass filter which tends to retain the high frequency information within an image while reducing the low frequency information.

Following the image capturing and filtering, they are processed under a particle sizing algorithm as described by Glover et al. [22] which is presented in Appendix C to determine the size of individual particles in the whole field using the fringe patterns of detected droplets one of which is shown as magnified in Figure 2.15.


Figure 2.15-A fringe pattern of a spray particle which is processed in the algorithm in Appendix C.

Using Equation 2.42 in conjunction with the fringe number which is evaluated by the algorithm, the diameters of the detected droplets are identified. The resulting data then is displayed in graphical form as shown in Figure 2.16.


Figure 2.16 - Representation of validated and invalidated particles detected
in the images shown in Figure 2.13 and 2.14 after IPI processing

The graphical representation displays the data as circles over each particle detected. The positions and the diameters of the circles coincide with the ones on the de-focused image. While green circles are indicating validated particles, the red ones indicate invalid particles. The invalid particles are one of those who are detected by just one of the cameras but not by the other one, who have a circular edge with a very weak interference fringe or an edge without a fringe, and those whose fringe patterns overlap with each other as to yield a complex pattern that is impossible to analyze. In the case of IPI processing for the images shown in Figure 2.13 and 2.14, a total
number of 158 particles can be detected in which 8 of them are validated giving a validation ratio of $5.1 \%$.

Apart from the graphical representation of the related size information, the data is also presented in tabular form displaying the diameter of the particle corresponding to its position in the image. In this way, once the IPI datasets are processed, following post-processing procedures can be applied in order to obtain diameter histogram. The resulting diameter histogram of validated particles contained in image shown in Figure 2.16 is represented in Figure 2.17.


Figure 2.17 - Diameter histogram of validated particles contained in the image shown in Figure 2.16

Taking higher number of measurements and obtaining their IPI processing results in the same region of the spray flow under same conditions, the summation of the individual diameter histograms constitute a statistically convergent droplet distribution for the related spray state. An example in
terms of the particle distribution function (PDF) is given in Figure 2.18 which is obtained by using 300 image sets and resulting IPI processing data.


Figure 2.18 - The particle droplet distribution (pdf) of a spray portion using 300 data-sets of IPI processing

Using the IPI processing results showing the droplet diameters corresponding to their positions in the image it is also possible to calculate the minimum diameter that can be measured in this technique. For example, considering the particle which is magnified in Figure 2.15 having a total number of 14 fringes, the IPI processing identifies its diameter as $52 \mu m$ meaning that had a particle be such small that it would contain only one fringe in its edge and assuming that it would have the same diameter/fringe ratio as above mentioned particle, the minimum measurable size can be calculated as $d_{\text {min }}=52 \mu \mathrm{~m} / 14$ fringes $=3.71 \mu \mathrm{~m} /$ fringe .

## CHAPTER 3

## EXPERIMENTAL SETUP and MEASUREMENT DETAILS

### 3.1 Test Facility

The experiments were conducted in a facility as represented in Figure 3.1. It consists of a 10 liter water reservoir with a 2200 W resistance heater, a water pump which can provide a constant flow rate of 70 liter per hour, a bypass mechanism which transfers some amount of flow coming from the pump back into the reservoir to regulate the flow rate and the injection pressure, and an oil burner type spray nozzle having a capability to inject liquid at a maximum flow rate of 30 liter per hour at 6.9 bar.


Figure 3.1-Schematics of experimental set-up

In the experiment, the light source is a double-pulsed Nd:YAG laser which generates a laser sheet having a wavelength of $\lambda=532 \mathrm{~nm}$ with a maximum power of $200 \mathrm{~mJ} / \mathrm{pulse}$ at a repetition rate of 15 Hz . The laser sheet thickness was 5 mm in the observation region. The CCD sensors in the two cameras are 12-bit grayscale sensors within a spatial resolution of $1344 \times 1024$ pixels $^{2}$. The measurement area has a length of 52.5 mm in horizontal direction and 40.0 mm in vertical direction. The magnification is 7.5:1. The scattering angle was set at $\theta=80 d e g$ as it gives a high visibility of interference pattern in the polarization case of present arrangement. The collecting angle was adjusted at $\alpha=14 d e g$ as it corresponds to an f number of 2.8. The positioning of cameras relative to spray can be seen in Figure 3.2. The lenses used were Nikon Micro Nikkor with focal length 60 mm and an f number that can be changed between: 2.8-32.


Figure 3.2 - Positioning of cameras relative to spray

### 3.2 Choosing Collecting Angle or Aperture Size Based on the Validation Ratio

Improving the image quality and getting higher validation ratios via image manupilation is a technique based on software which is applicable after the image acquisition. But reliable data and higher validation rates can also be achieved by adjusting the optical receiving equipment in an appropriate way before the measurements.

One of the parameters determining the size information of a droplet is the collecting angle $\alpha$, as shown in Equation 2.42. Collecting angle in IPI represents the angle of light cone as the apex of cone is the illuminated particle and the round base of cone is its projection into the CCD sensor.

The optical equipment employed in this study allows the adjustment of collecting angle via aperture rings integrated on the macro objective lenses as represented in Figure 3.3. As in the case of commercial cameras, the term of aperture refers to the size of opening in the lens which determines the angle or amount of light falling into the sensor. The size of opening is controlled by an adjustable diaphram of overlapping blades. It is known as a photography phrase that successive apertures halve the amount of incoming light. To achieve this, the diaphragm reduces the aperture diameter by a factor 1.4 (square root of 2 ) so that the aperture surface is halved each successive step as shown in Figure 3.4.


Figure 3.3- Components of receiving optics


Figure 3.4- Comparison of successive aperture values

Changing the value of aperture will cause a variation in the exposure time and the depth of field of an image. But, because the focal length, CCD sensors, illumination intensity or distance between illuminated spray and cameras remain unchanged during the experiments, a variation in aperture will only affect the depth of field (DOF) of the resulting image.

Depth of field is a term which refers to the areas of the photograph both in front and behind the main focus point which remain sharp or in focus. A
larger aperture corresponding to smaller $f$ number, e.g. $f / 2.8$, has a shallow depth of field. Anything behind or in front of the main focus point will appear blurred. A smaller aperture corresponding to larger f number, e.g. f/32, has a greater depth of field. Objects within a certain range behind or in front of the main focus point will also appear sharp.

Considering the illuminated spray particles in conjunction with the facts discussed above, the aperture setting will cause two main differences in taken images.

First of all, using a large $f$ number will induce a narrow collecting angle. Taking into consideration the relation between particle diameter and fringe number, Equation 2.29, the variables of present case to calculate diameter are the collecting angle and the fringe number. Keeping the diameter of particle constant, reducing the value of collecting angle will also cause a reduction in the number of fringes. Because, the same particle can be visualized within a smaller edge and with a smaller number of fringes which, therefore, makes it possible to detect more particles in an image.

However, the actual impact of aperture value on the resulting images arises from its influence on depth of field. As discussed above, a greater depth of field corresponding to a smaller aperture assists to get deeper focusing area in an image. Considering the illuminated spray particles associated with the depth of field concept, it means that when the aperture is set to a large number yielding a deeper focus, all of the particles within a laser illumination, which has a 5 mm thickness, will be recorded in the image. Recording that many particles will prevent to achieve acceptable validation ratios because of excessive amount of overlapping fringe patterns originated in different depths of spray. For this reason, the aperture number of receiving optics should be kept as small as possible. However, it shouldn't be ignored that using smaller $f$ numbers may yield to spherical
aberration which causes substantial curvature of fringes depending on the quality of macro lens. In this manner, it is obvious that sufficiently small aperture numbers has to be chosen in order to get an image with distinguishable fringes within the limits of acceptable spherical aberration. In this study, three different aperture sizes, $f / 2.8, f / 5.6$ and $f / 11$, corresponding to the collecting angles of $14^{0}, 7,5^{\circ}$, and $3,8^{\circ}$ respectively, were tested to qualify the most suitable one giving the highest validation ratio. The results of this comparison are shown in Table 3.1. In each case, arithmetic means of validation rate were calculated from 100 images.

| $f$ <br> number | Number of <br> Images | Average <br> Validation <br> Ratio |
| :---: | :---: | :---: |
| 2.8 | 100 | 20.6 |
| 5.6 | 100 | 7.2 |
| 11 | 100 | 1.1 |

Table 3.1-Comparison of averaged validation ratios calculated over 100 images corresponding to 3 different aperture numbers

### 3.3 Flow Conditions

Adjustment of surface tension was accomplished by the addition of a Sodium Oleate $\left(\mathrm{C}_{18} \mathrm{H}_{33} \mathrm{NaO}_{2}\right)$ solution as a surfactant. By the addition of different amounts on the level of ppm, five samples of spray liquids having different surface tensions were prepared. Among them, water without the addition of surfactant has the highest surface tension with 72 dynes/cm whereas the reduced values are $68,62,58$ and 51 dynes $/ \mathrm{cm}$ at $25^{\circ} \mathrm{C}$. It should also be noted that the variation in the viscosities of these liquids don't exceed $0.5 \%$ which can be assumed to be negligible considering the possible measurement errors. Table 3.2 summarizes the fluid properties and flow conditions for these liquids.

| $\sigma$ <br> $($ dynes $/ \mathrm{cm})$ | T <br> $(\mathrm{K})$ | $\rho$ <br> $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ | P <br> $(\mathrm{bar})$ | $\mu$ <br> $\left(\mathrm{Ns} / \mathrm{m}^{2}\right)$ <br> $\left(\mathrm{x} 10^{-3}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| 72 | 298 | 997.04 | 1.1 | 0.900 |
| 68 | 298 | 997.04 | 1.1 | 0.900 |
| 62 | 298 | 997.04 | 1.1 | 0.900 |
| 58 | 298 | 997.04 | 1.1 | 0.900 |
| 51 | 298 | 997.04 | 1.1 | 0.900 |

Table 3.2 - Flow conditions of measurements corresponding to different liquid surface tensions

By the help of a resistance heater, 7 water samples were obtained having temperature values of $25,35,45,60,70,80$ and $90^{\circ} \mathrm{C}$. The corresponding values of viscosity, surface tension and density for each temperature value were comprised by the help of thermodynamic tables. In each case of measurements related with temperature change, the spray was re-circulated for a long time before the acquisition until the whole circulating system, including pipes, pump and injector, reaches thermal equilibrium. Conditions of flow related to temperature change are given in Table 3.3.

| T <br> $\left({ }^{\circ} \mathrm{C}\right)$ | $\mathrm{P}(\mathrm{bar})$ | n | $\sigma$ <br> $($ dynes $/ \mathrm{cm})$ | $\boldsymbol{\mu}$ <br> $\left(\mathrm{Ns} / \mathrm{m}^{2}\right)$ <br> $\left(\mathrm{x} 10^{-3}\right)$ | $\rho$ <br> $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 25 | 1.1 | 1.332 | 72.0 | 0.900 | 997 |
| 35 | 1.1 | 1.331 | 70.4 | 0.715 | 994 |
| 45 | 1.1 | 1.329 | 68.8 | 0.600 | 990 |
| 60 | 1.1 | 1.327 | 66.2 | 0.467 | 983 |
| 70 | 1.1 | 1.325 | 64.4 | 0.404 | 978 |
| 80 | 1.1 | 1.322 | 62.6 | 0.355 | 972 |
| 90 | 1.1 | 1.320 | 60.8 | 0.315 | 965 |

Table 3.3-Flow conditions of measurements corresponding to different liquid temperatures

Seven spray liquids at 7 different temperatures have different corresponding values of surface tension, viscosity and density values but same injection pressure, i.e. $P=1.1$ bar. The changes in the liquid parameters as a function of temperature are shown in Figure 3.5, 3.6 and 3.7.


Figure 3.5 - The change of surface tension of water as a function of temperature


Figure 3.6 - The change of water viscosity as a function of temperature


Figure 3.7 - The change of water density as a function of temperature

In Figure 3.5, where the variation in surface tension of water is plotted as a function of temperature, there is about a $15 \%$ linear drop in surface tension along the interested interval, whereas the viscosity value in Figure 3.6 at $90^{\circ} \mathrm{C}$ decreases $65 \%$ compared to the one at $25^{\circ} \mathrm{C}$, having a 2.5 times difference between the extreme values. In Figure 3.7, where the variation in water density is plotted as a function of temperature, there is a relatively small change - a maximum difference of only $3 \%$ - opposing the variations in other two parameters. Therefore, it can be concluded that, along the current interval, a change in temperature will cause a major impact only on the water viscosity which makes it the main factor determining the particle size distributions.

The effects of three different spray injection pressures, namely 1.1, 1.3 and 1.5 bar, which are regulated by a by-pass mechanism were examined on the spray formation. The flow conditions for these flows are shown in Table 3.4 .

| P <br> $(\mathrm{bar})$ | T <br> $(\mathrm{K})$ | $\sigma$ <br> $($ dynes $/ \mathrm{cm})$ | $\mu$ <br> $\left(\mathrm{Ns} / \mathrm{m}^{2}\right)$ <br> $\left(\mathrm{x} 10^{-3}\right)$ | $\rho$ <br> $\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| 1.1 | 298 | 72 | 0.900 | 997.04 |
| 1.3 | 298 | 72 | 0.900 | 997.04 |
| 1.5 | 298 | 72 | 0.900 | 997.04 |

Table 3.4-Flow conditions of measurements corresponding to different injection pressures

## CHAPTER 4

## RESULTS

After deciding on the appropriate experimental settings as described in the previous Chapter, a set of experiments were conducted to obtain droplet size distributions for different values of surface tension, temperature and injection pressure at 8 different measurement locations within a spray as shown in Figure 4.1. This figure is a wide angle image of the spray under experimental conditions having the dimensions of 400 mm in vertical and 250 mm in horizontal directions. Within this image, the exact measurement locations having the dimensions of $52,5 \mathrm{~mm}$ in horizontal and 40 mm in vertical directions are represented with white rectangles. The midpoints of the measurement areas are located at $y=15,55,95,135,175 \mathrm{~mm}$ and radial locations of $r=0$ and 50 mm relative to the injector. In other experiments related with temperature and pressure variations, the measurements were taken only at a single location which was positioned at $r=0$ and $y=95 \mathrm{~mm}$. In each of the measurements, 300 pairs of images were captured in order to obtain reliable statistical information on the spray formation.


Figure 4.1 - Measurement stations for the investigation of axial evolution of spray droplets for different values of surface tension

The probability density functions of the droplet diameter for water, $\sigma=72 d y n e s / c m$, are shown in Figure 4.2.


Figure 4.2 - Droplet size distributions in 8 measurement locations for

$$
\sigma=72 d y n e s / c m
$$

At the measurement location closest to the injector exit, located at $r=0$, $y=15 \mathrm{~mm}$, the droplet distribution having a single peak value around $10 \mu \mathrm{~m}$ shows a relatively uni-modal characteristic compared to the other ones along the center of the spray. Taking into consideration that the employed injector is a type that converts the incoming liquid flow into a spray by the
help of internal cavitation effect and not by means such as air-assisted tools like co-axial air flow, there is relatively less deformation of liquid particles around the exit of injector acting by the governing forces due to the surrounding air flow on those particles. As a result, only a limited amount of interaction between particles occurs as to yield another "source" of peak in droplet size spectrum, and hence a uni-modal distribution is constituted. But as they are decelerated and deformed because of the acting aerodynamic forces, more interaction and collisions occur to yield a coalescence between droplets and thereby a bi-modal form of droplet distribution is constituted along the axial and radial directions as can be seen downstream of $y=55 \mathrm{~mm}$. In Figure 4.2, at a measurement location of $r=0$ and $y=55 \mathrm{~mm}$, it is observed that the particle diameter distribution is bi-modal as the most weighted diameter is around $10 \mu \mathrm{~m}$ and the second peak value is weighted around $70 \mu \mathrm{~m}$. Downstream of this location, along the spray axis, the primary peak value gets reduced and the secondary one becomes more weighted at same rate. Further downstream, at $r=0, y=135 \mathrm{~mm}$, the droplet spectrum becomes quasi bi-modal with a transformed shape differing from the former one by a diminished peak around $20 \mu \mathrm{~m}$ and a main peak weighting at $70 \mu \mathrm{~m}$, and beyond this region, at $r=0, y=175 \mathrm{~mm}$, the shape of the distribution remains unchanged meaning that almost all of the particles originating from injector are evolved and have taken new shapes downstream of nozzle by the action of aerodynamic forces. Then as a result of deceleration and less interaction, they finally come to equilibrium.

On the other hand, in the axial direction along the radial position of $r=50 \mathrm{~mm}$, the evolution of droplet distribution is different from the one which is located along $r=0$. In the three measurement locations, beginning from the $r=50 \mathrm{~mm}$, $y=55 \mathrm{~mm}$ and extending towards the $r=50 \mathrm{~mm}, y=135 \mathrm{~mm}$, all the droplet size distributions have a uniform characteristics which have a bi-modal shape whose highest peak is around $70 \mu \mathrm{~m}$, and the secondary peak is around
$20 \mu m$. Besides, the evolution of these distributions along the radial direction is also important.

For instance, when the two measurement results at $r=0$ and $r=50 \mathrm{~mm}$ along an axial position of $y=55 \mathrm{~mm}$ are compared, the one locating at radially away from the injector, at $r=50 \mathrm{~mm}$, has a bi-modal characteristic in which the weighted diameter is around $70 \mu \mathrm{~m}$, whereas the other one along the centerline of the injector, at $r=0$, has a much smaller weighted diameter around $10 \mu \mathrm{~m}$ with a distribution of bi-modal form. The mechanism behind this fact may be explained by higher net forces exerted on particles which are away from the injector exit as a result of their longer displacement. Because the particles that are carried along the outer region of spray undergo more deformation and faster deceleration compared to the ones that move shorter distances. Similarly, when the two observation region along $y=95 \mathrm{~mm}$ are compared, it can be seen that while the distribution shape at $r=100 \mathrm{~mm}$ remains unchanged relative to upper one, the one at $r=0$ becomes evolved as the primary peak of upper distribution weighting at $15 \mu \mathrm{~m}$ is diminished and a secondary peak weighting at $70 \mu \mathrm{~m}$ appears, and considering the shape, it can be said that it becomes so similar to its neighbor at $r=50 \mathrm{~mm}$. Further downstream, at two different radial measurement locations along $y=135 \mathrm{~mm}$, the results correspond to each other pointing out that the size distributions of spray particles become isotropic.

In determining the structure of spray flow, there are some other deterministic quantities which can be calculated using droplet size distributions e.g. the Arithmetic Mean Diameter (AMD) and the Sauter Mean Diameter (SMD).

In the case of basic averaging; the Arithmetic mean diameter $\mathrm{D}[1,0]$, is calculated by multiplying the diameter sizes with the number of
coressponding particle counts then summing them all and finally divided by the total number of particles that were measured:

$$
\begin{equation*}
A M D=D[1,0]=\frac{\sum_{i=1}^{N} d_{i} \cdot n_{i}}{\sum_{i=1}^{N} n_{i}} \tag{4.1}
\end{equation*}
$$

The Sauter Mean Diameter or the surface area moment mean is the most common tool that is determined by the particle sizes to foresee the action level in combustion applications where the active surface area has the biggest importance and is calculated as

$$
\begin{equation*}
\mathrm{SMD}=D[3,2]=\frac{\sqrt[3]{\sum_{i=1}^{N} d_{i}^{3} \cdot n_{i}^{3}}}{\sqrt{\sum_{i=1}^{N} d_{i}^{2} \cdot n_{i}^{2}}} \tag{4.2}
\end{equation*}
$$

The variations in the Arithmetic and the Sauter mean diameter of droplets using equations 4.1 and 4.2 along the axial and radial directions are shown in Figures 4.3 and 4.5, respectively.

Apart from AMD and SMD, there are other mean diameters with different meanings that can be calculated using droplet size distributions. For instance;

If the particle sizing information is needed in conjunction with studies concerning catalisation, the important term will be the surface areas of particles because the higher surface area means the higher activity of the catalyst. Taking into account that the surface area of a sphere is $4 \pi r^{2}$, the related averaging is:

$$
\begin{equation*}
\text { Surface Diameter }=D[2,0]=\frac{\sqrt{\sum_{i=1}^{N} d_{i}^{2} \cdot n_{i}^{2}}}{\sum_{i=1}^{N} n_{i}} \tag{4.3}
\end{equation*}
$$

In the investigation of sprays comparing the volume fraction or equivalently the mass of spray particles, the Volume Mean Diameter (VMD) is a useful tool in which the mean cubes of diameters are identified because volume of a sphere equals $\frac{4}{3} \pi r^{3}$.

$$
\begin{equation*}
V M D=D[3,0]=\frac{\sqrt[3]{\sum_{i=1}^{N} d_{i}^{3} \cdot n_{i}^{3}}}{\sum_{i=1}^{N} n_{i}} \tag{4.4}
\end{equation*}
$$

Another kind of mean diameter that is preferred in academic studies is the volume moment mean or the Herdan diameter of the particle which is identical to the weight equivalent mean if density is constant, which is defined as

$$
\begin{equation*}
\text { Herdan Diameter }=D[4,3]=\frac{\sqrt[4]{\sum_{i=1}^{N} d_{i}^{4} \cdot n_{i}^{4}}}{\sqrt[3]{\sum_{i=1}^{N} d_{i}^{3} \cdot n_{i}^{3}}} \tag{4.5}
\end{equation*}
$$

The variations in these mean diameters along axial and radial directions for all measurements are not commented in this chapter but given in Appendix D.


Figure 4.3 - Change in the Arithmetic and the Sauter mean droplet diameters for $\sigma=72 d y n e s / c m$ along axial direction at two different radial positions


Figure 4.4 - Measurement stations for the investigation of radial evolution of spray droplets


Figure 4.5 - Change in the Arithmetic and the Sauter mean droplet diameters for $\sigma=72$ dynes/cm along radial direction at three different axial positions

As can seen in Figure 4.3, along the axial direction of spray for $r=0$, the increments in the calculated Arithmetic and Sauter mean diameters are up to $50 \%$ and $40 \%$, respectively. On the other hand, along the axial direction for the measurement locations at the outer region of spray corresponding to a different radial position at $r=50 \mathrm{~mm}$, there seems no notable variation in the mean diameters obtained from size distributions. The variations in the mean droplet diameters along the radial direction are shown in Figure 4.5. In observation regions near the injector exit, e.g. $y=55 \mathrm{~mm}$, the variation in the Arithmetic mean diameter is $50 \%$ whereas the variation for the Sauter mean diameter is just $10 \%$, but as move away from the injector axially, the variation magnitude gets smaller, for example at $y=175 \mathrm{~mm}$, while the change in the Arithmetic mean is about $20 \%$, the change in the Sauter mean is less than $5 \%$, and eventually, at a location far away enough from injector, at $y=260 \mathrm{~mm}$, the mean diameters in the radial direction don't change at all.

Repeating the experiments for 4 different values of surface tension, which are obtained by the addition of a surfactant, at 8 measurement locations
yield similar droplet distributions and mean diameter values based on the evolution in axial and radial directions. For this reason, considering the similarities, the results which have the same characteristics with the one having a surface tension of $\sigma=72 d y n e s / c m$ are represented in Appendix D through the Figures D.3-D.14. The calculated Arithmetic and Sauter mean diameters corresponding to different surface tensions using these droplet distributions are shown in Figure 4.6 and 4.7. When analyzed, it can be concluded that the influence of surface tension on the onset of spray droplets is limited in the range tested because there seems no regular tendency for the variation of mean diameter values resulting from the change in the surface tension and there is only a slight difference between the extreme points of resulting mean diameters which is less than $10 \%$.

There are several possible explanations behind this constant, unchanging behaviour for mean droplet diameters within the current range of surface tension. Firstly, the range is a little bit too narrow to be able to observe a considerable change in resulting droplet diameters. For example, when compared with a similar study conducting by Snyder et al. [11] in which two different liquid samples with similar viscosities but different surface tensions of 68 and 20 dynes/cm were injected through a spray nozzle, only a \%20 change could be observed for the resulting droplet diameters. Considering the current study in which the surface tension is reduced from 72 to 51 dynes $/ \mathrm{cm}$, it is understandable not to obtain such a difference in the resulting diameters.

Another possible reason is the mass transfer along the interface between liquid and air due to the surface tension gradient which is known as the Marangoni effect. Since a liquid with a high surface tension pulls more strongly on the surrounding liquid than one with a low surface tension, the presence of a gradient in surface tension will cause the liquid to flow away from regions of low surface tension which in turn causes the surfactant to
stay at the surface. So that, the actual surface tension on the droplet may not be the same as the bulk of the fluid and therefore the measured diameters are independent of the surface tension.


Figure 4.6 - Variation of the Arithmetic mean diameter with surface tension in 8 measurement locations





Figure 4.7 - Variation of the Sauter mean diameter with surface tension in 8 measurement locations

The measurements related to the effect of temperature change on resulting droplet diameters were conducted at a single measurement location along the centerline of the injector, at $y=95 \mathrm{~mm}, r=0$. The probability density functions for droplet diameters of water spray at 7 different temperatures are shown in Figure 4.8. The Arithmetic and the Sauter mean diameters which are calculated using these distributions are shown in Figure 4.9 and 4.10, respectively.


Figure 4.8 - Droplet size distributions for water at 7 different temperatures at a measurement location of $r=0$ and $y=95 \mathrm{~mm}$


Figure 4.9 - Change in the Arithmetic mean droplet diameters for water at 7 different temperatures at a measurement location of $r=0$ and $y=95 \mathrm{~mm}$


Figure 4.10-Change in the Sauter mean droplet diameters for water at 7 different temperatures at a measurement location of $r=0$ and $y=95 \mathrm{~mm}$

As can be seen from these figures, all the probability density functions plotted in Figure 4.8 have very similar distributions. Actually, the calculated mean diameters from these plots also show small variations. For example, the calculated Sauter mean diameter remains virtually constant; having maximum and minimum values of $70.1 \mu \mathrm{~m}$ and $69.4 \mu \mathrm{~m}$, respectively, the Arithmetic mean diameter shows a little increament -about 3\%- with temperature. Therefore, concerning the experimental uncertainties on measurements, a variation in the temperature and hence the viscosity has a limited effect on the onset of spray particles under the experimental conditions and measurement interval of interest.

Similar to the measurements related to the temperature change, the effect of injection pressure on resulting droplet diameters were conducted at a single measurement location along the centerline of the injector, at $y=95 \mathrm{~mm}$ and $r=0$. The probability density functions for droplet diameters of water spray at 3 different pressure values, $P=1.1,1.3$ and 1.5 bar are shown in Figure 4.11. The related Arithmetic and Sauter mean diameters are shown in Figure 4.12 and 4.13 , respectively. In this case, a regular decreasing tendency in the mean droplet diameters with increasing injection pressure is obvious, while the Arithmetic mean drops by about $10 \%$, the Sauter mean drops by $15 \%$. It is also remarkable that the variation is linear in the measurement interval.


Figure 4.11 - Droplet size distributions for water under 3 different injection pressures of $1.1,1.3$ and 1.5 bar at $25^{\circ} \mathrm{C}$ at a measurement location of $\mathrm{r}=0$ and $y=95 \mathrm{~mm}$


Figure 4.12 - Change in the Arithmetic mean droplet diameters for water under 3 different injection pressures of 1.1, 1.3 and 1.5 bar at $25^{\circ} \mathrm{C}$ at a measurement location of $r=0$ and $\mathrm{y}=95 \mathrm{~mm}$


Figure 4.13 - Change in the Sauter mean droplet diameters for water under 3 different injection pressures of 1.1, 1.3 and 1.5 bar at $25^{\circ} \mathrm{C}$ at a measurement location of $r=0$ and $y=95 \mathrm{~mm}$

## CHAPTER 5

## CONCLUSIONS

An experimental study was conducted in order to investigate the effects of different parameters on the spray characteristics of an oil burner injector. Spray characteristics were evaluated by means of measured droplet distributions, and calculated quantities of the Arithmetic and the Sauter mean diameters. The droplet diameters of spray formation corresponding to different flow parameters were measured by the Interferometric Particle Imaging technique.

Within the current measurement range, the results demonstrate that the droplet sizes are independent of the liquid surface tension and the temperature. On the other hand, an increment in the injection pressure linearly reduced the resulting droplet size.

Additionally, the evolution of spray characteristics in space were also examined by conducting measurements at different radial and axial locations relative to spray and it was observed that at the centerline of injector, the initial uni-modal distribution of droplet diameters measured around the injector exit gradually becomes bi-modal along the axial and radial directions as a result of interaction between injected droplets and ambient air. In the measurement edge of the spray, the droplet sizes have very similar distributions as if they become direction-independent or isotropic.

As a future work, in order to understand the aerodynamic forces and interactions between spray particles, there are two main issues that should be investigated in detail:

- Determination of air flow around the spray
- Velocity calculation of identified particles and their correlations with their sizes


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## APPENDIX A

## 1. ANGULAR COEFFICIENTS

The Legendre functions $\pi_{n}(\cos \theta)$ are computed by means of the recurring relation

$$
\begin{align*}
& \pi_{n}(\cos \theta)=2 \cos \theta \pi_{n-1}(\cos \theta)-\pi_{n-2}(\cos \theta) \\
& +\left[\cos \theta \pi_{n-1}(\cos \theta)-\pi_{n-2}(\cos \theta)\right] \frac{1}{n-1} \tag{A.1}
\end{align*}
$$

When $n$ becomes big, the third term decreases to zero, allowing a more perfect behavior of the recurrent algorithm.

The first terms of the recurrence are

$$
\begin{equation*}
\pi_{1}(\cos \theta)=1 \quad \pi_{2}(\cos \theta)=3 \cos \theta \tag{A.2}
\end{equation*}
$$

The Legendre functions $\tau_{n}(\cos \theta)$ are deduced from the $\pi_{n}(\cos \theta)$ by means of

$$
\begin{equation*}
\tau_{n}(\cos \theta)=n \cos \theta \pi_{n}(\cos \theta)-(n+1) \pi_{n-1}(\cos \theta) \tag{A.3}
\end{equation*}
$$

## 2. SCATTERING COEFFICIENTS

Equations 3.16 and 3.17 are rewritten under the following forms:

$$
\begin{align*}
& a_{n}=\frac{\Psi_{n}(\alpha) \Psi_{n}^{\prime}(\mu)-m \Psi_{n}^{\prime}(\alpha)}{\xi(\alpha) A_{n}(\mu)-m \xi_{n}^{\prime}(\alpha)}  \tag{A.4}\\
& b_{n}=\frac{m \Psi_{n}(\alpha) A_{n}(\mu)-\Psi_{n}^{\prime}(\alpha)}{m \xi_{n}(\alpha) A_{n}(\mu)-\xi_{n}^{\prime}(\alpha)} \tag{A.5}
\end{align*}
$$

where

$$
\begin{equation*}
A_{n}(\mu)=\frac{\Psi_{n}^{\prime}(\mu)}{\Psi_{n}(\mu)} \tag{A.6}
\end{equation*}
$$

Furthermore we have

$$
\begin{equation*}
\frac{\Psi_{n}^{\prime}(\mu)}{\Psi_{n}(\mu)}=-\frac{n}{\mu}+\frac{J_{n-1 / 2}(\mu)}{J_{n+1 / 2}(\mu)} \tag{A.7}
\end{equation*}
$$

The Lentz algorithm enables computing the ratio $J_{n-1 / 2}(\mu) / J_{n+1 / 2}(\mu)$ without using any recurring relation. According to Lentz:

$$
\begin{equation*}
\frac{J_{n-1 / 2}(\mu)}{J_{n+1 / 2}(\mu)}=2(n+1 / 2) \mu^{-1}+\frac{1}{-2(n+3 / 2) \mu^{-1}}+\frac{1}{2(n+5 / 2) \mu^{-1}}+\cdots \tag{A.8}
\end{equation*}
$$

where

$$
\begin{equation*}
a_{1}+\frac{1}{a_{2}}+\frac{1}{a_{3}}+\ldots \tag{A.9}
\end{equation*}
$$

designates a continued fraction:

$$
\begin{equation*}
a_{1}+\frac{1}{a_{2}+\frac{1}{a_{3}+\ldots}} \tag{A.10}
\end{equation*}
$$

Here we have

$$
\begin{equation*}
a_{m}=(-1)^{m+1} 2(n+m-1 / 2) \mu^{-1} \tag{A.11}
\end{equation*}
$$

If the process under the denominator is limited, at the order $p$,

$$
\begin{equation*}
\left|a_{1}, a_{2}, \ldots, a_{p}\right|=a_{1}+\frac{1}{a_{2}}+\ldots+\frac{1}{a_{p}} \tag{A.12}
\end{equation*}
$$

Then, according to Lentz,

$$
\begin{equation*}
\frac{J_{n-1 / 2}(\mu)}{J_{n+1 / 2}(\mu)}=\frac{\left|a_{1}\right| \ldots\left|a_{q-1}, \ldots, a_{1}\right|\left|a_{q}, \ldots, a_{1}\right|}{\left|a_{2}\right| \ldots\left|a_{q-1}, \ldots, a_{2}\right|\left|a_{q}, \ldots, a_{2}\right|} \tag{A.13}
\end{equation*}
$$

where $q$ is determined by the condition that the particular $q$ th numerator and denominator are identical to the number of digits desired.

When the order $n$ is smaller than the integer part of $\alpha$, a recurring algorithm is used.

$$
\begin{align*}
& \Psi_{n+1}(\alpha)=\frac{2 n}{\alpha} \Psi_{n}(\alpha)-\Psi_{n-1}(\alpha)  \tag{A.14}\\
& \Psi_{n}^{\prime}(\alpha)=-\frac{n}{\alpha} \Psi_{n}(\alpha)+\Psi_{n-1}(\alpha) \tag{A.15}
\end{align*}
$$

with the initial functions

$$
\begin{align*}
& \Psi_{0}(\alpha)=\sin \alpha  \tag{A.16}\\
& \Psi_{1}(\alpha)=\frac{\sin \alpha}{\alpha}-\cos \alpha \tag{A.17}
\end{align*}
$$

The recurring relations for $\zeta_{n}(\alpha)$ and $\zeta_{n}^{\prime}(\alpha)$ are the same as for $\Psi_{n}(\alpha)$ and $\Psi_{n}^{\prime}(\alpha)$, respectively, with

$$
\begin{align*}
& \zeta_{0}(\alpha)=\sin \alpha+i \cos \alpha  \tag{A.18}\\
& \zeta_{1}(\alpha)=\left(\frac{\sin \alpha}{\alpha}-\cos \alpha\right)+i\left(\frac{\cos \alpha}{\alpha}+\sin \alpha\right) \tag{A.19}
\end{align*}
$$

When the order $n$ is higher than the integer part of a, these recurrent relations are not satisfactory as far as the function $\Psi_{n}(\alpha)$ is concerned.

In this case, again using the Lentz algorithm by writing

$$
\begin{equation*}
\Psi_{n+1}(\alpha)=\Psi_{n}(\alpha) \cdot\left[\frac{\Psi_{n+1}(\alpha)}{\Psi_{n}(\alpha)}\right] \tag{A.20}
\end{equation*}
$$

When the order $n_{c}$ is equal to the integer part of $\alpha, \Psi_{n_{c}}(\alpha)$ is computed by recurring algorithms. Then $\Psi_{n_{c}+1}(\alpha)$ and the higher-order functions are computed by means of Equation A. 20 where the ratio of functions is determined from the Lentz algorithm. From Equation 3.19:

$$
\begin{equation*}
\frac{\Psi_{n+1}(\alpha)}{\Psi_{n}(\alpha)}=\frac{J_{n+3 / 2}(\alpha)}{J_{n+1 / 2}(\alpha)} \tag{A.21}
\end{equation*}
$$

and the ratio $J_{n+3 / 2}(\alpha) / J_{n+1 / 2}(\alpha)$ is computed from Equation A. 13

## 3. Amplitude Functions

The angular and scattering coefficients have to be assambled in order to compute the amplitude functions (Equations 2.12 and 2.13). But the summation must be carried out up to a finite order N .

Different criterions can be used to determine N . The following criterion is used in the present work:

$$
\begin{equation*}
\operatorname{Min}\left[\operatorname{Re}\left(a_{n}\right), \operatorname{Re}\left(b_{n}\right), \operatorname{Im}\left(a_{n}\right)-\operatorname{Im}\left(b_{n}\right)\right]<10^{-30} \tag{A.22}
\end{equation*}
$$

where the function $\operatorname{Min}\left(a_{i}\right)$ designates the smaller argument among the arguments $\mathrm{a}_{\mathrm{i}} . \operatorname{Re}(z)$ and $\operatorname{Im}(z)$ designate the real part and the imaginary part of $z$.

## APPENDIX B

## 1. Derivations of the Amplitude Terms in the Geometrical Optics Light Scattering Theory

## a. Externally Reflected Light

Integer constants defined in Equation 2.25 are given as

$$
\begin{equation*}
p=0 ; q=+1 ; l=0 ; s=-1 \tag{B.1}
\end{equation*}
$$

so that the relationship between $\theta^{\prime}$ and $\theta$ becomes

$$
\begin{equation*}
\theta^{\prime}=\theta=\pi-2 \varepsilon \tag{B.2}
\end{equation*}
$$

It follows that

$$
\begin{align*}
& \left|\frac{d \theta^{\prime}}{d \varepsilon}\right|=2  \tag{B.3}\\
& \frac{\sin (2 \varepsilon)}{\sin \theta}=1 \tag{B.4}
\end{align*}
$$

The phase term becomes

$$
\begin{equation*}
\delta_{x}=k d \sin \theta / 2 \tag{B.5}
\end{equation*}
$$

and with the help of Snell's law, Equations 2.30 and 2.31, it can be shown that

$$
\begin{align*}
& E_{x_{1}}=\left\{\sin (\theta / 2)-\left[m^{2}-1+\sin ^{2}(\theta / 2)\right]^{1 / 2}\right\} /\{\sin (\theta / 2)+  \tag{B.6}\\
& {\left[m^{2}-1+\sin ^{2}(\theta / 2)\right]^{1 / 2}} \\
& E_{x_{2}}=\left\{m^{2} \sin (\theta / 2)-\left[m^{2}-1+\sin ^{2}(\theta / 2)\right]^{1 / 2}\right\} /\left\{m^{2} \sin (\theta / 2)+\right.  \tag{B.7}\\
& {\left[m^{2}-1+\sin ^{2}(\theta / 2)\right]^{1 / 2}}
\end{align*}
$$

## b. Refracted Light with no Internal Reflections

The integer constants become

$$
\begin{align*}
& p=1 ; q=-1 ; l=0 ; s=-1  \tag{B.8}\\
& \theta^{\prime}=-\theta=2 \beta-2 \varepsilon \tag{B.9}
\end{align*}
$$

With the help of Snell's law it can be shown that

$$
\begin{align*}
& \sin \varepsilon=m \sin (\theta / 2) /\left[m^{2}+1-2 m \cos (\theta / 2)\right]^{1 / 2}  \tag{B.10}\\
& \sin \beta=m \sin (\theta / 2) /\left[m^{2}+1-2 m \cos (\theta / 2)\right]^{1 / 2}
\end{align*}
$$

so that

$$
\begin{gather*}
\left.\frac{\sin (2 \varepsilon)}{2 \sin \theta}=(m / 2)[m \cos (\theta / 2)-1] /\left\{m^{2}+1-2 m \cos (\theta / 2)\right] \cos (\theta / 2)\right\}  \tag{B.11}\\
\left|\frac{d \theta^{\prime}}{d \varepsilon}\right|=2\left[m^{2}+1-2 m \cos (\theta / 2)\right] /[m(m-\cos (\theta / 2)] \tag{B.12}
\end{gather*}
$$

the phase difference is

$$
\begin{equation*}
\delta_{R}=-k \cdot d\left[m^{2}+1-2 m \cos (\theta / 2)\right]^{1 / 2} \tag{B.13}
\end{equation*}
$$

From Equations 2.31 and 2.33 :

$$
\begin{align*}
& E_{R_{1}}=4 m[m \cos (\theta / 2)-1][m-\cos (\theta / 2)] /\left(m^{2}-1\right)^{2}  \tag{B.14}\\
& E_{R_{2}}=E_{R_{1}} / \cos ^{2}(\theta / 2) \tag{B.15}
\end{align*}
$$

## 2. Derivation of the Scattered Light Intensity

Using van de Hulst formulation for an incident plane wave of unit intensity, the scattered light field at point $P$ becomes

$$
\begin{align*}
& e_{\theta}=(-i / k r) i_{z}(\theta) \cos \varphi \cdot \exp (-i k r+i \varpi t)  \tag{B.16}\\
& e_{\varphi}=(-i / k r) i_{z}(\theta) \sin \varphi \cdot \exp (-i k r+i \varpi t) \tag{B.17}
\end{align*}
$$

where

$$
\begin{align*}
& i_{2}(\theta)=i_{D_{2}}(\theta)+i_{R_{2}}(\theta)+i_{X_{2}}(\theta)  \tag{B.18}\\
& i_{1}(\theta)=i_{D_{1}}(\theta)+i_{R_{1}}(\theta)+i_{X_{1}}(\theta) \tag{B.19}
\end{align*}
$$

where

$$
\begin{align*}
& i_{D_{1,2}}=A_{D_{1,2}}  \tag{B.20}\\
& i_{R_{1,2}}=-i A_{R_{1,2}} \exp \left(i \delta_{R}\right)  \tag{B.21}\\
& i_{X_{1,2}}=-i A_{X_{1,2}} \exp \left(i \delta_{X}\right) \tag{B.22}
\end{align*}
$$

The total scattered light intensity for a unit incident intensity at point $P$ is written as

$$
\begin{equation*}
I_{\theta, \varphi}=e_{\theta, \varphi} \cdot e_{\theta, \varphi}^{*} \tag{B.23}
\end{equation*}
$$

It is from this relationship that Equations 2.35 and 2.36 can (B.7) be derived.

## APPENDIX C

## IMAGE PROCESSING ALGORITHM



Figure C. 1 - Particle sizing algorithm for IPI [22]


Figure C. 2 - Reduced, negated image for ellipse identification


Figure C. 3 - Edge-direction data after Canny edge detection


Figure C. 4 - Ellipses identified with nonoverlapping, rectangular areas across which the fringe spatial frequency is measured

## APPENDIX D

## RESULTS (CONTINUED)

| Measurement <br> Position: <br> Axial, Radial <br> $(\mathbf{i n ~ m m})$ | $\mathbf{D}[\mathbf{1 , 0 ]}$ <br> $(\boldsymbol{\mu m})$ | $\mathbf{D}[\mathbf{2 , 0 ]}]$ <br> $(\boldsymbol{\mu m})$ | $\mathbf{D}[\mathbf{3 , 0 ]}$ <br> $(\boldsymbol{\mu m})$ | $\mathbf{D}[\mathbf{3 , 2}]$ <br> $(\boldsymbol{\mu m})$ | $\mathbf{D}[4,3]$ <br> $(\boldsymbol{\mu m})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 5 , 0}$ | 31.15 | 36.64 | 41.50 | 53.23 | 60.91 |
| $\mathbf{5 5 , 0}$ | 42.80 | 48.94 | 53.50 | 63.92 | 68.74 |
| $\mathbf{9 5 , 0}$ | 52.69 | 55.01 | 60.78 | 68.10 | 71.20 |
| $\mathbf{1 3 5 , 0}$ | 60.34 | 63.16 | 65.24 | 69.63 | 72.19 |
| $\mathbf{1 7 5 , 0}$ | 60.78 | 63.52 | 65.56 | 69.84 | 72.38 |
| $\mathbf{5 5 , 5 0}$ | 60.97 | 63.87 | 65.99 | 70.44 | 72.89 |
| $\mathbf{9 5 , 5 0}$ | 62.22 | 64.52 | 66.33 | 69.96 | 72.10 |
| $\mathbf{1 3 5 , 5 0}$ | 62.44 | 64.52 | 66.30 | 69.93 | 72.01 |

Table D.1- Mean droplet diameters for $\sigma=72$ dynes $/ \mathrm{cm}$ at 8 measurement locations


Figure D. 1 - Change in mean droplet diameters for $\sigma=72 \mathrm{dynes} / \mathrm{cm}$ along axial direction at two different radial positions


Figure D. 2 - Change in mean droplet diameters for $\sigma=72$ dynes/cm along radial direction at three different axial positions


Figure D. 3 - Droplet size distributions in 8 measurement locations for $\sigma=68 \mathrm{dynes} / \mathrm{cm}$

| Measurement <br> Position: <br> Axial, Radial <br> (in mm) | $\mathbf{D}[\mathbf{1 , 0 ]}$ <br> $(\boldsymbol{\mu m})$ | $\mathbf{D}[\mathbf{2 , 0}]$ <br> $(\boldsymbol{\mu m})$ | $\mathbf{D}[\mathbf{3 , 0} \mathbf{0}$ <br> $(\boldsymbol{\mu m})$ | $\mathbf{D}[\mathbf{3 , 2}]$ <br> $(\boldsymbol{\mu m})$ | $\mathbf{D}[\mathbf{4 , 3}]$ <br> $(\boldsymbol{\mu m})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 5 , 0}$ | 31.76 | 36.66 | 41.07 | 51.54 | 58.92 |
| $\mathbf{5 5 , 0}$ | 42.87 | 48.74 | 53.16 | 63.24 | 68.12 |
| $\mathbf{9 5 , 0}$ | 54.12 | 56.52 | 61.08 | 67.92 | 70.93 |
| $\mathbf{1 3 5 , 0}$ | 60.51 | 63.03 | 65.35 | 69.48 | 71.84 |
| $\mathbf{1 7 5 , 0}$ | 61.24 | 63.48 | 65.64 | 69.74 | 72.01 |
| $\mathbf{5 5 , 5 0}$ | 64.39 | 66.33 | 67.81 | 70.89 | 72.85 |
| $\mathbf{9 5 , 5 0}$ | 61.96 | 64.33 | 66.08 | 69.73 | 71.94 |
| $\mathbf{1 3 5 , 5 0}$ | 62.09 | 64.41 | 66.13 | 69.71 | 71.86 |

Table D. 2 - Mean droplet diameters for $\sigma=68 d y n e s / c m$ at 8 measurement locations


Figure D. 4 - Change in mean droplet diameters for $\sigma=68$ dynes $/ \mathrm{cm}$ along axial direction at two different radial positions


Figure D. 5 - Change in mean droplet diameters for $\sigma=68 \mathrm{dynes} / \mathrm{cm}$ along radial direction at three different axial positions


Figure D. 6 - Droplet size distributions in 8 measurement locations for $\sigma=62 d y n e s / c m$

| Measurement <br> Position: <br> Axial, Radial <br> (in mm) | $\mathbf{D}[\mathbf{1 , 0 ]}$ <br> $(\boldsymbol{\mu m})$ | $\mathbf{D}[\mathbf{2 , 0}]$ <br> $(\boldsymbol{\mu m})$ | $\mathbf{D}[\mathbf{3 , 0} \mathbf{0}$ <br> $(\boldsymbol{\mu m})$ | $\mathbf{D}[\mathbf{3 , 2}]$ <br> $(\boldsymbol{\mu m})$ | $\mathbf{D}[\mathbf{4 , 3}]$ <br> $(\boldsymbol{\mu m})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 5 , 0}$ | 31.13 | 36.47 | 41.27 | 52.85 | 60.55 |
| $\mathbf{5 5 , 0}$ | 38.64 | 44.87 | 49.77 | 61.22 | 66.93 |
| $\mathbf{9 5 , 0}$ | 56.75 | 60.46 | 63.01 | 68.44 | 71.09 |
| $\mathbf{1 3 5 , 0}$ | 61.23 | 63.84 | 65.69 | 69.54 | 71.68 |
| $\mathbf{1 7 5 , 0}$ | 62.48 | 64.77 | 66.42 | 69.83 | 71.84 |
| $\mathbf{5 5 , 5 0}$ | 62.11 | 64.33 | 66.14 | 69.58 | 71.77 |
| $\mathbf{9 5 , 5 0}$ | 62.45 | 64.67 | 66.32 | 69.74 | 71.83 |
| $\mathbf{1 3 5 , 5 0}$ | 62.88 | 65.03 | 66.60 | 69.88 | 71.88 |

Table D. 3 - Mean droplet diameters for $\sigma=62 d y n e s / c m$ at 8 measurement locations


Figure D. 7 - Change in mean droplet diameters for $\sigma=62$ dynes/cm along axial direction at two different radial positions


Figure D. 8 - Change in mean droplet diameters for $\sigma=62$ dynes $/ \mathrm{cm}$ along radial direction at three different axial positions


Figure D. 9 - Droplet size distributions in 8 measurement locations for $\sigma=58 \mathrm{dyne} / \mathrm{cm}$

| Measurement <br> Position: <br> Axial, Radial <br> (in mm) | $\mathbf{D}[\mathbf{1 , 0 ]}$ <br> $(\boldsymbol{\mu m})$ | $\mathbf{D}[\mathbf{2 , 0 ]}$ <br> $(\boldsymbol{\mu m})$ | $\mathbf{D}[\mathbf{3 , 0} \mathbf{0}$ <br> $(\boldsymbol{\mu m})$ | $\mathbf{D}[\mathbf{3 , 2}]$ <br> $(\boldsymbol{\mu m})$ | $\mathbf{D}[\mathbf{4 , 3}]$ <br> $(\boldsymbol{\mu m})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 5 , 0}$ | 30.47 | 35.14 | 39.41 | 49.56 | 56.65 |
| $\mathbf{5 5 , 0}$ | 47.87 | 53.18 | 57.08 | 65.77 | 70.08 |
| $\mathbf{9 5 , 0}$ | 58.54 | 61.72 | 64.02 | 68.88 | 71.55 |
| $\mathbf{1 3 5 , 0}$ | 58.79 | 62.00 | 64.28 | 69.11 | 71.70 |
| $\mathbf{1 7 5 , 0}$ | 60.90 | 63.61 | 65.58 | 69.71 | 72.03 |
| $\mathbf{5 5 , 5 0}$ | 60.98 | 63.76 | 65.81 | 70.10 | 72.45 |
| $\mathbf{9 5 , 5 0}$ | 61.65 | 64.21 | 66.09 | 70.02 | 72.19 |
| $\mathbf{1 3 5 , 5 0}$ | 61.57 | 64.07 | 65.90 | 69.74 | 71.93 |

Table D. 4 - Mean droplet diameters for $\sigma=58$ dynes/cm at 8 measurement locations


Figure D.10- Change in mean droplet diameters for $\sigma=58 \mathrm{dynes} / \mathrm{cm}$ along axial direction at two different radial positions


Figure D. 11 - Change in mean droplet diameters for $\sigma=58$ dynes $/ \mathrm{cm}$ along radial direction at three different axial positions


$$
Y=95 \mathrm{~mm}
$$







Figure D. 12 - Droplet size distributions in 8 measurement locations for $\sigma=51$ dyne/cm

| Measurement <br> Position: <br> Axial, Radial <br> (in mm) | $\mathbf{D}[\mathbf{1 , 0 ]}$ <br> $(\boldsymbol{\mu m})$ | $\mathbf{D}[\mathbf{2 , 0}]$ <br> $(\boldsymbol{\mu m})$ | $\mathbf{D}[\mathbf{3 , 0}]$ <br> $(\boldsymbol{\mu m})$ | $\mathbf{D}[\mathbf{3 , 2}]$ <br> $(\boldsymbol{\mu m})$ | $\mathbf{D}[\mathbf{4 , 3}]$ <br> $(\boldsymbol{\mu m})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 5 , 0}$ | 30.46 | 35.29 | 39.62 | 49.92 | 57.04 |
| $\mathbf{5 5 , 0}$ | 45.09 | 50.78 | 54.91 | 64.21 | 68.48 |
| $\mathbf{9 5 , 0}$ | 55.67 | 59.51 | 62.21 | 67.99 | 70.86 |
| $\mathbf{1 3 5 , 0}$ | 58.57 | 61.74 | 63.99 | 68.74 | 71.23 |
| $\mathbf{1 7 5 , 0}$ | 58.91 | 62.22 | 64.57 | 69.53 | 72.09 |
| $\mathbf{5 5 , 5 0}$ | 59.56 | 62.84 | 65.41 | 69.95 | 71.91 |
| $\mathbf{9 5 , 5 0}$ | 61.15 | 63.86 | 65.69 | 69.54 | 71.56 |
| $\mathbf{1 3 5 , 5 0}$ | 60.87 | 63.72 | 65.36 | 69.36 | 71.28 |

Table D. 5 - Mean droplet diameters for $\sigma=51$ dynes/cm at 8 measurement locations


Figure D. 13 - Change in mean droplet diameters for $\sigma=51$ dynes/cm along axial direction at two different radial positions


Figure D. 14 - Change in mean droplet diameters for $\sigma=51$ dynes/cm along radial direction at three different axial positions


Figure D. 15 - Variation of $\mathrm{D}[2,0]$ with surface tension in 8 measurement locations


Figure D. 16 - Variation of $\mathrm{D}[3,0]$ with surface tension in 8 measurement locations


Figure D. 17 - Variation of $D[4,3]$ with surface tension in 8 measurement locations


Figure D. 18 - Change in mean droplet diameters for water at 7 different temperatures at a measurement location of $r=0$ and $y=95 m m$


Figure D. 19 - Change in mean droplet diameters for water under 3 different injection pressures of $1.1,1.3$ and 1.5 bar at $25^{\circ} \mathrm{C}$ at a measurement location of $r=0$ and $y=95 \mathrm{~mm}$

