REAL-TIME-OPTIMIZATION OF DRILLING PARAMETERS DURING DRILLING OPERATIONS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

TUNA EREN

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY
IN
PETROLEUM AND NATURAL GAS ENGINEERING

FEBRUARY 2010

Approval of the thesis:

REAL-TIME-OPTIMIZATION OF DRILLING PARAMETERS DURING DRILLING OPERATIONS

submitted by **TUNA EREN** in partial fulfilment of the requirements for the degree of **Doctor of Philosophy in Petroleum and Natural Gas Engineering Department, Middle East Technical University** by,

Prof. Dr. Canan Özgen ________________
Dean, **Graduate School of Natural and Applied Sciences**

Prof. Dr. Mahmut Parlaktuna ________________
Head of Department, **Petroleum & Natural Gas Engineering**

Assoc. Prof. Dr. Mehmet Evren Özbayoğlu ________________
Supervisor, **Petroleum & Natural Gas Engineering Dept., METU**

Examining Committee Members:

Prof. Dr. Mahmut Parlaktuna ________________
Petroleum & Natural Gas Engineering Dept, METU

Assoc. Prof. Dr. Mehmet Evren Özbayoğlu ________________
Petroleum & Natural Gas Engineering Dept., METU

Prof. Dr. Nurkan Karahanoğlu ________________
Geological Engineering Department, METU

Prof. Dr. Mustafa Kök ________________
Petroleum & Natural Gas Engineering Dept, METU

Assist. Prof. Dr. Gürşat Altun ________________
Petroleum & Natural Gas Engineering Dept, İTÜ

Date: 01 / 02 / 2010

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name: Tuna Eren

Signature:

ABSTRACT

REAL-TIME-OPTIMIZATION OF DRILLING PARAMETERS DURING DRILLING OPERATIONS

Eren, Tuna

Ph.D., Petroleum and Natural Gas Engineering Department

Supervisor: Assoc. Prof. Mehmet Evren Özbayoğlu

February 2010, 145 pages

Real-time optimization of drilling parameters during drilling operations aims to optimize weight on bit, bit rotation speed for obtaining maximum drilling rate as well as minimizing the drilling cost. The process is considered to be formation specific. A statistical method such as multiple linear regression technique has been used for the drilling optimization. An extensive literature survey on drilling optimization was conducted for this research study. A model is developed for this purpose using actual field data collected through modern well monitoring and data recording systems, which predicts the rate of drilling penetration as a function of available parameters. The rate of penetration general equation is optimized for effective functions at each data point. In order to optimize the parameters in the field, a computer network is required to be developed. The computer network will keep the piped data directly from the data source, and continuously be collecting the new data to be fed. A database present at the central computer will be continuously calculating the developed model parameters by means of multiple regression technique and inform the team at the field. The field engineer will transmit the current drilling parameters back to the central computer, and the headquarters will determine the new model parameters and optimum drilling parameters by including the recently received information. Therefore, there will be a real-time-optimization process. It is considered that this technique is going to be widely used in future drilling activities since it could reduce drilling costs and minimize probability of encountering problems due to working with optimized parameters.

It has been found that drilling rate of penetration could be modelled in real-time environment as a function of independent drilling variables such as weight on bit, rotation speed of the string, drilling fluid weight, and formation characteristics. The ability to have the drilling rate of penetration with respect to depth characteristically with certain parameters for specific formations on real-time basis could bring new insights to the nature of drilling optimization studies. Any significant departure of the actual rate of penetration from the predicted rate of penetration trend could have important indications which could be detected beforehand in real-time. The study has also achieved one of its objectives, giving the optimized independent drilling parameters found following statistical synthesis.

Keywords: Real-time, drilling optimization, weight-on-bit, bit rotation, multiple regression.

ÖZ

SONDAJ ESNASINDA SONDAJ PARAMETRELERİNİN GERÇEK-ZAMANLI-OPTİMİZASYONU

Eren, Tuna
Doktora, Petrol ve Doğal Gaz Mühendisliği Bölümü
Tez Yöneticisi: Doç. Dr. Mehmet Evren Özbayoğlu

Şubat 2010, 145 sayfa

Sondaj esnasında gerçek-zamanlı sondaj parametrelerinin optimize edilmesi çalışmasının amacı, azami sondaj hızı ve asgari sondaj maliyeti elde etmek üzere matkaba uygulanan ağırlık, matkap dönüş hızı parametrelerini optimize etmektir. Optimizasyon işlemleri sondajı yapılmakta olan formasyonun özelliklerine özgü çalışmaktadır. Çoklu doğrusal regresyon gibi istatistiksel bir metod kullanılarak bir sondaj optimizasyon metodolojisi ortaya konulmuştur. Çalışma doğrultusunda sondaj optimizasyonu ile ilgili geniş kapsamlı bir literatür araştırması yapılmıştır. Bu amaçla, gelişmiş kuyu izleme ve bilgi depolama sistemlerinden faydalanılarak elde edilmiş olan saha verileri kullanılarak, sondaj ilerleme hızını mevcut parametreleri kullanarak tahmin eden bir model oluşturulmuştur. Oluşturulmuş olan ilerleme hızı denklemi etkisi altında olduğu fonksiyonlara bağlı olarak her noktada optimize edilecektir. Saha parametrelerinin optimizasyonu için bir bilgisayar ağı kurulacaktır. Bilgisayar ağı kaynağından direkt olarak iletilmiş olan verileri saklayacak, ve sürekli olarak yeni verileri toplayacaktır. Merkezi bilgisayarda bulunan bir veri tabanı sürekli olarak model parametrelerini çoklu regresyon tekniğini kullanarak hesaplayıp, sahada bulunan ekibe bilgi gönderecektir. Saha mühendisi, mevcut sondaj parametrelerini merkez bilgisayara gönderecek, merkez de bu parametreleri veri bankasına dahil edip, yeni model parametreleri ile birlikte optimum sondaj parametrelerini tespit ederek sahadaki mühendise geri bildirecektir. Böylece, gerçek-zamanlı-optimizasyon sağlanmış olacaktır. Önerilmiş olan bu tekniğin sondaj maliyetlerini düşürdüğü ve optimize edilmiş parametrelerle çalışmayı sağladığı için problemlerle karşılaşma ihtimalini düşürmesi sebebi ile geniş olarak ilerideki sondaj operasyonlarıda kullanılacağı düşünülmektedir.

Sondaj ilerleme hızının matkaba uygulanan ağırlık, dizi dönüş hızı, sondaj sıvısı yoğunluğu, formasyon karakteristikleri vs. gibi bağımsız sondaj parametrelerinin bir fonksiyonu olarak gerçek-zaman ortamında modellenebileceği bulunmuştur. Sondaj ilerleme hızının derinliğe göre karakteristiksel olarak belirli parametrelere ve formasyonlara bağlı olarak gerçek-zamanlı olarak biliniyor olması sondaj operasyonlarının doğasına yeni ufuklar kazandırabilir. Gerçek sondaj ilerleme hızının tahmin edilmiş olan eğilim dışına yönelecek olan davranışlar göstermesi gerçek.zamanlı şekilde önceden tespit edilebilecek önemli ipuçlarını ortaya koyabilir. Bu çalışma ayrıca bağımsız sondaj parametrelerinin istatistiksel sentez sonucu bulunmuş olan optimum değerlerini verdiği için ana amacını sağlamıştır.

Anahtar Kelimeler: Gerçek zamanlı, sondaj optimizasyonu, matkaba verilen ağırlık, matkap dönüş hızı, çoklu regresyon.

to

my

father

mother

&

two brothers

ACKNOWLEDGEMENTS

It has been a great experience working on a subject such as drilling optimization. I am grateful to my supervisor Assoc. Dr. Mehmet Evren Özbayoğlu for his guidance and sincere support all throughout the research. It has been a pleasure working in such a distinctive research with my supervisor who has contributed to drilling industry with important findings.

The feeling of a great honour has always been effective for being privileged enough to study at the department of Petroleum and Natural Gas Engineering, Middle East Technical University. It is always a compliment to be thankful to all of the instructors at the department, but especially to Prof. Dr. Parlaktuna, Prof. Dr. Kök for their guidance in the research, and particularly to Prof. Dr. Karahanoğlu for the constructive command of this study from statistical stand point. I would like to give my kind thanks to Assist. Dr. Gursat Altun for accepting to be a member of the committee, for the very important contributions to the study and being kind enough to attend to the defence meetings coming from Istanbul by organizing his schedule.

I would like to give very sincere thanks to my current manager Mr De Col (head of the Drilling Department) for his encouragement and support in this study. Special thanks also should be given to my current and previous employer, respectively Eni and TPAO for the encouragement to perform this study during the course of the research in the interval of 2004 – 2009. I would like to thank Eni E&P for giving permission to use the dataset given in this study.

It is a great opportunity to pay respect to my grandfather; Muhittin Eren, “*a pioneer in the oil-drilling business in Turkey,*” who has been the very source to endeavour in drilling industry.

My family members truly deserve appreciation as they have always been next to me. Their encouragement and support definitely increased the quality of the current study.

Finally the countless friends, administrative staff of our department and the colleagues not mentioned deserves the best regards and appreciation for their invaluable contribution and support towards the realization of this study. Their support has been among one of the most important incentives to perform this work.

A great establishment would have been achieved; in case this study could bring new insights to the drilling industry due to being one of the pioneering statistical methods in drilling optimization and in real-time basis.

TABLE OF CONTENTS

APPENDICES

LIST OF TABLES

LIST OF FIGURES

NOMENCLATURE

Roman

Symbol		**Unit, Field (SI)**
a_1	formation strength parameter	-
a_2	exponent of the normal compaction trend	-
a_3	under-compaction exponent	-
a_4	pressure differential exponent	-
a_5	bit weight exponent	-
a_6	rotary speed exponent	-
a_7	tooth wear exponent	-
a_8	hydraulic exponent	-
A_B	borehole area	[L^2], in^2 (mm^2)
B	fractional bearing wear	-
b	bearing constant	-
B_C	bit cost	$
C_b	bit cost	$
C_1	proportionality constant in penetration rate equation	-
C_f	cost per drilled interval	[$/L], $/ft ($/m)
C_{fd}	formation drillability parameter	-
C_r	daily rig rate	$
D	depth of borehole	[L], ft (m)
d_b	diameter of the bit	[L], in (mm)
d_n	equivalent bit nozzle diameter	[L], in (mm)
d_p	exponent in dimensionless Pi term	-
$\dot{D}$	bit tooth dullness, fraction of original tooth	-
E	rock hardness	[$M/(LT^2)$], psi (N/m^2)
E_s	bit specific energy	[$M/(LT^2)$], psi (N/m^2)

f_1	formation strength function	-
f_2	formation normal compaction function	-
f_3	formation compaction function	-
f_4	pressure differential of hole bottom function	-
f_5	bit diameter and weight function	-
f_6	rotary speed function	-
f_7	tooth wear function	-
f_8	hydraulic function	-
F-value	result of a method (*F-test*) comparing statistical models that have been fit to a data set	-
$\dot{F}$	rate of penetration	[L/T], ft/min (m/min)
F	distance drilled by bit	[L], ft (m)
F_a	axial bottom hole assembly weight component	$[ML/T^2]$, 1000 lbf (N)
F_n	normal bottom hole assembly weight component	$[ML/T^2]$, 1000 lbf (N)
g_p	pore pressure gradient of the formation,	$[M/L^3]$, ppg (sg)
h	bit tooth dullness, fractional tooth height worn away	-
H_1, H_2	constants for tooth geometry of bit types	-
h_t	heat transfer coefficient	$[ML^3(\Theta)]$, Btu/°Fft^{2}hr (Btu/°Km^{2}hr)
J_1	composite drilling parameter representing all but tooth wear	-
J_2	tooth wear composite function used to calculate fractional tooth wear	-
K	proportionality constant for rock strength effect	
L	tabulated function of $\overline{W}$ used in bearing life equation	
N	rotary speed	$[T^{-1}]$, rpm (-)
n	data point numbers used in regression analysis	-
Q ,q	volumetric flow rate	$[L^3/T]$, gpm (l/m)

R^2	regression coefficient	-
R	rate of penetration	-
r	residual error in the drilling ROP equation	-
S	drilling fluid parameter	
S_{xx}	matrix sums of squares and cross products of the drilling ROP parameters	-
S_{xy}	column matrix of the sums of cross products of dependent variable	-
t	time (usually bit rotating time)	[T], hours
t_b	bit drilling time	[T], hours
t_c	drill pipe connection time	[T], hours
t_t	round trip time	[T], hours
T	temperature at the bottom of the hole	[Θ], ^{o}F (K)
x	drilling rate of penetration independent parameter	-
x_2	normal compaction drilling parameter	-
x_3	under-compaction drilling parameter	-
x_4	pressure differential drilling parameter	-
x_5	bit weight drilling parameter	-
x_6	rotary speed drilling parameter	-
x_7	tooth wear drilling parameter	-
x_8	bit hydraulics drilling parameter	-
V	volume of removed rock	-
W	weight on bit	[ML/T^2], 1000 lbf (N)
W_e	vertical weight on bit component	[ML/T^2], 1000 lbf (N)
W/d	weight on bit per inch of bit diameter	[M/T^2], 1000 lbf/in (N/m)
$\overline{W}$	equivalent bit weight	[ML/T^2], 1000 lbf (N)
$(W/d)_{max}$	bit weight per diameter where teeth fails instantaneously	[M/T^2], 1000 lbf/in (N/m)
$(W/d)_t$	threshold bit weight at which the bit starts to drill	[M/T^2], 1000 lbf/in (N/m)

Greek

Symbol		**Unit, Field (SI)**
α	hole section inclination from vertical	degree
μ_s	bit specific coefficient of sliding friction	-
ν	drilling fluid kinematic viscosity	[M/LT], cp (Pa s)
Δp	differential pressure	[M/(LT^2)], psi (Pa)
ρ	drilling fluid's density	[M/T^3], ppg (kg/m^3)
ρ_c	equivalent circulating mud density at the hole bottom	[M/L^3], ppg (sg)
μ	apparent viscosity at 10,000 sec^{-1}	[M/LT], cp (Pa s)
τ_H	formation abrasiveness constant	[T], hours
τ_B	bearing constant	[T], hours
τ_y	yield stress	[M/LT^2], lbf/100ft^2(Pa)

Subscripts

Symbol		**Unit**
B	bit or borehole	-
f	fraction	-
i	index number for ith data point	-
j	index number for jth drilling rate of penetration equation coefficient	-
min	minimum	-
max	maximum	-
N	nozzle	-

Abbreviations

Symbol		**Unit**
BHA	bottom hole assembly	-
ECD	equivalent circulating density	[M/L^3], ppg (sg)

Fm	formation	-
FTP	file transfer protocol	-
IADC	International Association of Drilling Contractors	-
MSE	mechanical specific energy	$[M/(LT^2)]$, psi (Pa)
MD	measured depth	[L], ft (m)
MLU	Mud Logging Unit	-
MW	mud weight (Density)	$[M/L^3]$, ppg (sg)
MWD	measurements while drilling	-
OSC	operation support centre	-
Opt	optimum	-
PDC	polycrystalline diamond cutter	-
PPFG	pore pressure fracture gradient	-
ROP	rate of penetration	[L/T], ft/hr (m/hr)
RPM	revolution per minute	$[T^{-1}]$, rpm (-)
RT	real time	-
RTOC	real time operations centre	-
SD	standard deviation	-
SPP	standpipe pressure	$[M/(LT^2)]$, psi (Pa)
TD	total depth	[L], ft (m)
TDS	top drive system	-
TVD	true vertical depth	[L], ft (m)
WITS	wellsite information transfer specification	-
WITSML	wellsite information transfer standard markup language	-
WOB	weight on bit	$[ML/T^2]$, 1000 lbf (N)
WWW	world wide web	-

CHAPTER 1

INTRODUCTION

Future oilfield resource developments are subject to drill wells in cost efficient manners. For that reason future management of oilfield drilling operations will face new hurdles to reduce overall costs, increase performances and reduce the probability of encountering problems. Drilling wells for energy search from the ground has shown considerable technological advances in the recent years. Different methods from different disciplines are being used nowadays in drilling activities in order to obtain a safe, environmental friendly and cost effective well construction. Communication and computer technologies are among the most important disciplines which can contribute to drilling optimization. Large amount of data could be piped through different locations on the planet in reliable and time efficient manners. Advanced computer technologies are now being used in storing large amounts of data, and solving complex problems.

From the very early beginning of the drilling campaigns the operators have always been seeking to reduce the drilling costs mainly by increasing the drilling speed. In the drilling industry, the first well drilled in a new field (a wildcat well) generally will have the highest cost. With increasing familiarity to the area optimized drilling could be implemented decreasing costs of each subsequent well to be drilled until a point is reached at which there is no more significant improvement [1]. The relationship among drilling parameters are complex, the effort is to determine what combination of operating conditions result in minimum cost drilling [2]. The generally accepted convention for a proper planning of any drilling venture is to optimize operations and minimize expenditures [3]. Another essential aspect of the optimization is to enhance the technology and make the system effective [4]. Recently environmental friendly activities have also started to be common practice in certain locations, which in turn could be achieved by means of reducing the risks associated with having technical problems.

In recent years the increasing emphasis that is being paid by the oil and gas field operator companies towards working much efficiently at the rig sites are based on some important reasons. The most important of all are: cost and trouble free operations. During a peak in the cost of hydrocarbon resources, the rig supplier and oil field service provider contractor charges are increasing, pushing operators to work efficiently. Due to the complexity of the activities being offshore and/or being in the form clusters operators restraining themselves from causing a damage, which may result in destruction of more than one well due to their proximity between each other being very close. Directional techniques allowed drilling multiple wells from one location, thus eliminating construction of expensive structures for each well [5]. Due to the drilling requirements similarity of the wells located at close distances, collecting past data, and utilizing in a useful manner is considered to have an important impact on drilling cost reduction provided that optimum parameters are always in effect.

Major drilling variables considered to have an effect on drilling rate of penetration (ROP) are not fully comprehended and are complex to model [6]. For that very reason accurate mathematical model for rotary drilling penetration rate process has not so far been achieved. There are many proposed mathematical models which attempted to combine known relations of drilling parameters. The proposed models worked to optimize drilling operation by means of selecting the best bit weight and rotary speed to achieve the minimum cost. Considerable drilling cost reductions have been achieved by means of using the available mathematical models.

It is important to bear in mind that formation properties, which are uncontrollable are one of the most critical factors in drilling performance determination. Drilling fluid properties and bit types, though controllable are not in good drilling practice to change in ordinary bit runs. However, hydraulics, the weight applied to the bit and bit and rotary speed are among the controllable factors.

The scope of this study is to make use of the data from rig sites. The data available to the drilling engineer is mainly sourced from Mud Logging Units (MLU). It is also known that recently manufactured rigs are being equipped with powerful data import capabilities that make connection to third-party Wellsite Information Transfer Specification (WITS) data simple and reliable [7]. WITS is a communications format used for the transfer of a wide variety of wellsite data from one computer system to another. It is a recommended format by which Operating and Service companies involved in the Exploration and Production areas of the Petroleum Industry may exchange data in either an online or batch transfer mode [8]. New generation tool is Wellsite Information Transfer Standard Markup Language (WITSML) which is the standard transmission of wellsite data in a consistent form which would enable the integration of information from different suppliers [9,10]. The data could be piped in real-time, be processed and interpreted such as to recommend the optimum drilling parameters back in real-time as well. This cycle is easily achievable in today's technology by means of using advanced communication systems, and innovative computer technologies. In order to understand what has been done so far in regards to drilling optimization it is very important to see what has been performed in the recent history.

1.1. Drilling Optimization History

The history of rotary drilling goes back to the beginning of the 1900 [11]. During conception period the rotary drilling principle developed by the introduction of rotary bits, casing installation and cementing techniques, and developments in drilling fluids. During the development period, which took place after 20 years following Spindletop more powerful rigs, better bits, improved cementing and drilling fluid treatment techniques were introduced. In 1950s the scientific period took place with expansion in drilling research, better understanding of the hydraulic principles, significant improvements in bit technology, improved drilling fluid technology and most important of all optimized drilling. After 1970s rigs with full automation systems, closed-loop computer systems, with ability to control the drilling variables started to operate in oil and gas fields.

Figure I-1 gives the time line of drilling optimization history. One of the first attempts for the drilling optimization purpose was presented in the study of Graham and Muench in 1959 [12]. They analytically evaluated the weight on bit and rotary speed combinations to derive empirical mathematical expressions for bit life expectancy and for drilling rate as a function of depth, rotary speed, and bit weight. In 1963 Galle and Woods [13] produced graphs and procedures for field applications to determine the best combination of drilling parameters. One of the most important drilling optimization studies performed was in 1974 by Bourgoyne and Young [14]. They proposed the use of a linear drilling penetration rate model and performed multiple regression analysis to select the optimized drilling parameters. They used minimum cost formula, showing that maximum rate of penetration may coincide with minimum cost approach if the technical limitations were ignored. In the mid 1980s operator companies developed techniques of drilling optimization in which their field personnel could perform optimization at the site referring to the graph templates and equations. In 1990s different drilling planning approaches were brought to surface [3,15]. New techniques identified the best possible well construction performances. Later on "*Drilling the Limit*" optimization techniques were also introduced [16]. Towards the end of the millennium real-time monitoring techniques started to take place, e.g. drilling parameters started to be monitored from off locations. A few years later real-time operations/support centres started to be constructed. Some operators proposed advanced techniques in monitoring of drilling parameters at the rig site.

Following the early developments in rotary drilling system, ground-breaking developments in the latter years of the century took place. Highly inclined wells were drilled using rotary steerable; pressure controlled drilling techniques with acquisition of drilling parameters.

In recent years drilling parameters are easily acquired, stored and also transferred in real-time basis. Following the invent of the sophisticated and automated rig data acquisition microelectronic systems linked to computers, a range of drilling optimization and control services started to take place [17]. *Drill-off* tests performed to optimize drilling penetration rate and bit life [18] are now able to be conducted with advanced techniques using smart computer systems. The test is applied by the driller by means of applying a little bit of excessive weight, locking the brake to keep the string from running into the hole [19,20,21].

Despite all the advances in the communication and computer technologies the literature survey conducted did not reveal the utilization of drilling optimization technique by means of statistical synthesis in real-time basis. For that reason this study is decided to be performed.

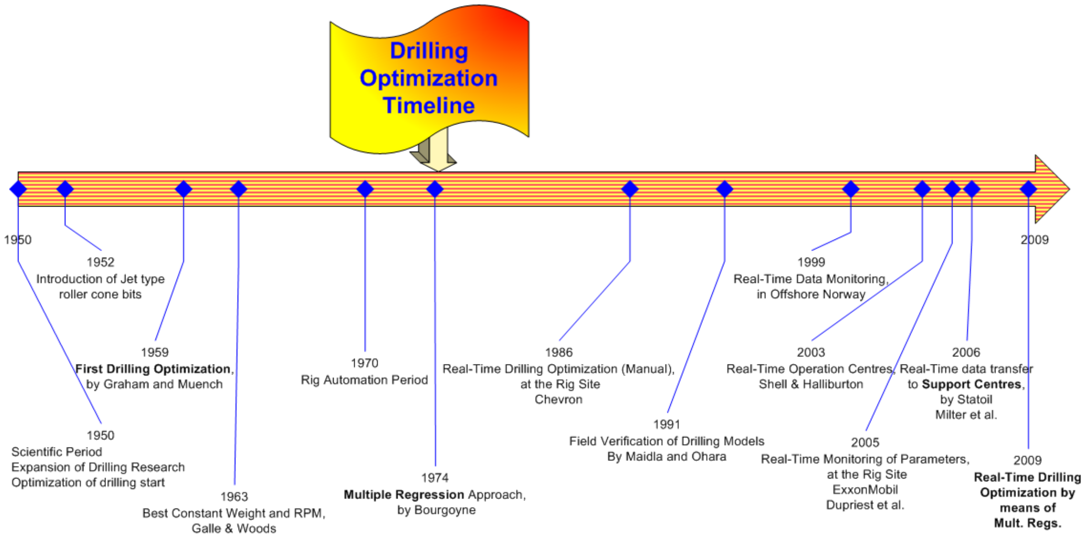


Figure I-1: Time line for drilling optimization.

1.2. Drilling Data Used for the Analysis

Necessary analyses for this research study performed using data belonging to directionally drilled wells in offshore environment in the Mediterranean Sea. The available data that has been provided by Eni E&P is similar to data that has been presented in the given references [22]. The wells were directionally drilled from fixed locations. The drilling objective was to reach production target levels with relatively high inclinations. **Figure I-2** gives brief information of directional wells processed for real-time drilling optimization. Results belonging to two wells have been presented in the study; the data of the third well have been used for the development of the model.

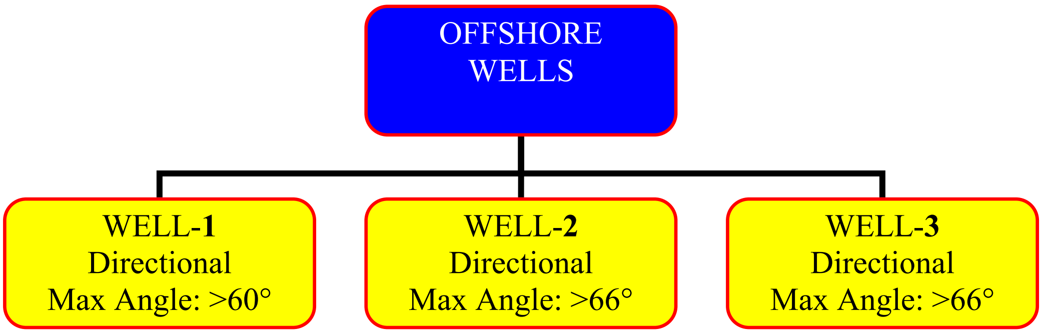
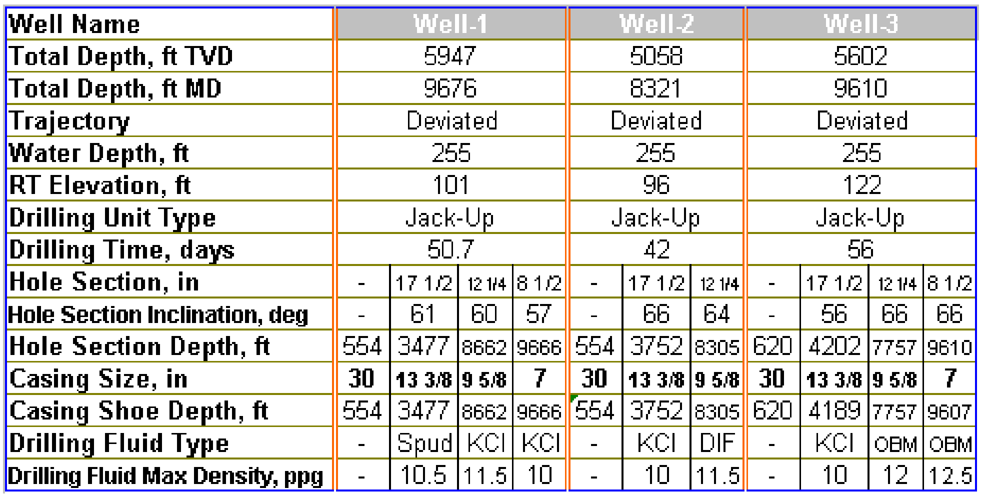


Figure I-2: Wells summary information.

Table I-1 gives the summary information of the three wells data of which is utilized in the scope of this research study. All of the wells are with a deviated directional profile with inclinations up to 60 degrees. The wells drilled in drilling times of between 30 to 75 days to total depths (TD) of up to 2400 m Measured Depth (MD).

Table I-1: Offshore wells info summary.



Well Name	Well-1				Well-2			Well-3			
Total Depth, ft TVD	5947				5058			5602			
Total Depth, ft MD	9676				8321			9610			
Trajectory	Deviated				Deviated			Deviated			
Water Depth, ft	255				255			255			
RT Elevation, ft	101				96			122			
Drilling Unit Type	Jack-Up				Jack-Up			Jack-Up			
Drilling Time, days	50.7				42			56			
Hole Section, in	-	17 1/2	12 1/4	8 1/2	-	17 1/2	12 1/4	-	17 1/2	12 1/4	8 1/2
Hole Section Inclination, deg	-	61	60	57	-	66	64	-	56	66	66
Hole Section Depth, ft	554	3477	8662	9666	554	3752	8305	620	4202	7757	9610
Casing Size, in	**30**	**13 3/8**	**9 5/8**	**7**	**30**	**13 3/8**	**9 5/8**	**30**	**13 3/8**	**9 5/8**	**7**
Casing Shoe Depth, ft	554	3477	8662	9666	554	3752	8305	620	4189	7757	9607
Drilling Fluid Type	-	Spud	KCl	KCl	-	KCl	DIF	-	KCl	OBM	OBM
Drilling Fluid Max Density, ppg	-	10.5	11.5	10	-	10	11.5	-	10	12	12.5

The wells were drilled from a single offshore location using jack up type drilling rigs. The three dimensional view of the wells is given in **Figure I-3**. Specific information regarding the database of the drilling parameters is given in Appendix A.

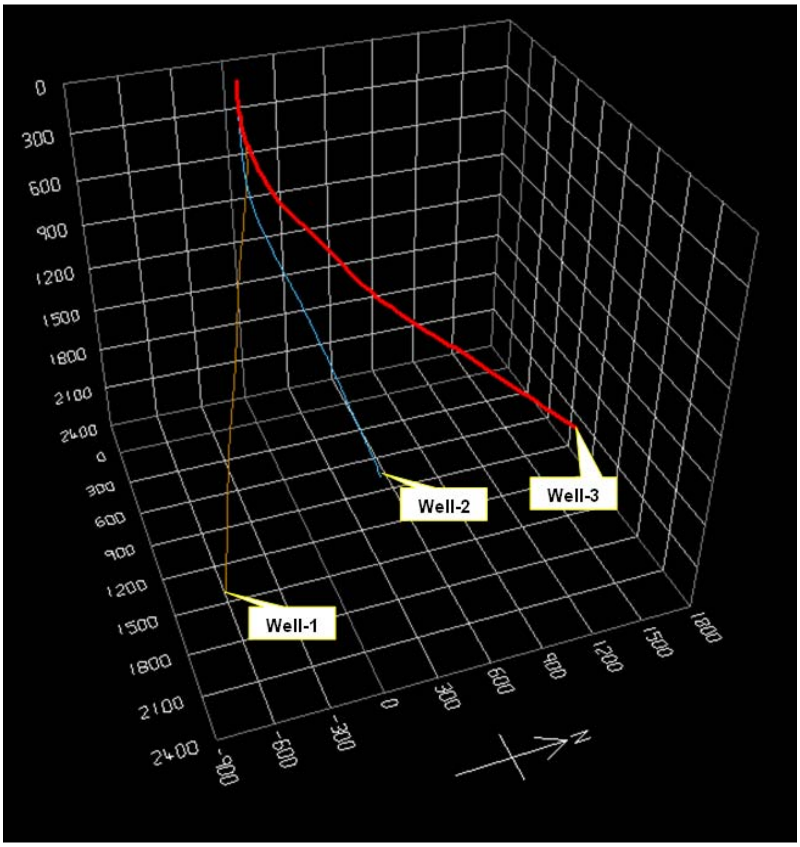


Figure I-3: Three dimensional deviated view of the wells.

Figure I-4 gives some pictures of drilling data acquisition system fixed inside mud logging unit and some of its sensors. The data from specific equipment/machinery is transmitted after being measured by means of sophisticated transducers/sensors. By means of a data acquisition unit (left picture) all of the required data is recorded for analysis. Hook-load sensor is shown (right picture) which is attached to the dead-line of the rig. The available data has been collected through the MLU which was the centre at which the drilling parameters were being acquired by means of the sensors placed at various points. Mud logging units are mainly instrumented for the following tasks [23], **Table I-2**.

Table I-2: Tasks of mud logging units [23].

Measure drilling parameters (ROP, WOB, RPM, Flow Rate).
Record properties of drilling cuttings.
Transmit and record acquired drilling data.
Provide drilling monitoring services, on site and/or at remote locations.

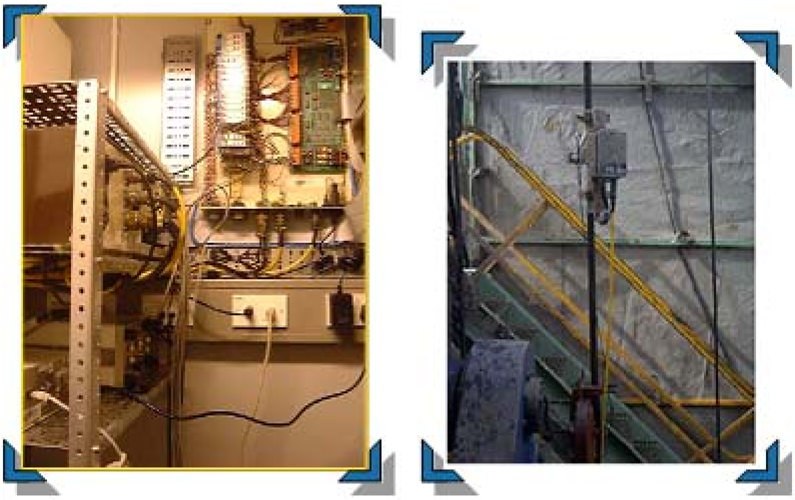


Figure I-4: Data acquisition unit picture from a Mud Logging Unit (Left), WOB sensor attached to the dead line of the drilling line (Right).

With the enhancement of the communication technologies and the use of computers, large amounts of data has been started to be piped from field locations in order to be analyzed. Actual data is the only source of information to make a recommendation to optimize the drilling operations. The parameters those of which could be collected from a drilling activity are as listed in **Table I-3**. Each parameter to be collected from the rig site is going to have an impact to the overall optimization process. Data reliability and accuracy is very important, all of the data collecting sensors should be accurately calibrated and be signalling the correct magnitude of measurement. The success of drilling optimization is closely related with the quality of the recorded drilling parameters.

Table I-3: Parameters from a drilling activity.

WOB	Drillstring properties
RPM	Casing details
Pump parameters	Drilling fluid properties
Depth	Torque
Inclination	Hook-load
Azimuth	LWD
ROP	MWD

The parameters recorded for drilling optimization are critically important to be representative of data they are meant to reflect. The brief description of the drilling parameters is deemed necessary to be explained.

WOB: It is the abbreviation for “Weight on Bit”. It represents amount of weight applied onto the bit, that is then transferred to the formation which in turn is the energy created together with string speed that advances drillstring. It is measured through the drilling line, usually by means of having attached a strain-gauge which measures the magnitude of the tension in the line itself, and gives the weight reading based on the calibration. This sensor measures a unique value, which is the overall weight (Hook-load) of the string including the weight of the block and Top Drive System (TDS). For all of these circumstances correct calibration is required in order to have proper reading for this drilling parameter.

RPM: This parameter stands for “revolution per minute”. It represents the rotational speed of the drillstring. With the invention of TDS; the reading is directly linked to the electronics of the unit itself. It is considered that the measurements for this parameter are accurate as long as the acquisition system set-up has been thoroughly made up.

Pump Parameters: The pump parameters are composed of the liner size in use, pump strokes, and the pump pressure. In case there are two pumps working simultaneously all of the data for two of the pumps should be acquired. With the electric pumps the stroke is transmitted in the same way as RPM. The pressure at the pump in case of having been acquired could be compared with the reliability of the standpipe pressure. Pump pressure should always be greater than the standpipe pressure. Use of flow meters could also be adapted for accurate flow rate measurements.

Depth: The value of depth, in other means the bit position is input in the MLU. The operator is the responsible for that; usually it is linked to the position of the block, by means of the sensors located at the crown block.

Inclination – Azimuth: These two parameters are in the responsibility of the directional driller. An efficient communication between the MLU and the Measurements While Drilling (MWD) unit is to the benefit of these two parameters which may be very important for Wellbore Stability considerations.

ROP: This parameter is the most important parameter, since all of the calculations in this study are based on estimations of Rate of Penetration (ROP). It is measured through the relative change of the position of the block in time. Accurate calibrations are very important in order to have a representative ROP parameter.

String – Casing Properties: The string and casing properties are very important when the frictional pressure losses are to be calculated.

Fluid Properties: Rheological properties and the density of the drilling fluids are also among the very important parameters to be recorded for optimization purposes. Usually the drilling fluid density is measured through calibrated MW sensors. Rheological properties on the other hand are still measured manually. Recent developments in regards to real-time pipe viscometers dictate alternative solutions. There are experimental studies performed in the laboratory using pipe-viscometers [24]. Continuous real-time viscometer probes placed on the flow line (which are reportedly under development) could facilitate data acquisition over the rheological properties of drilling fluids in real-time.

Torque: This parameter is the torque of the drillstring while it is rotating. It is measured by means of TDS systems. Previously the readings for this parameter were relative. This parameter is going to be significantly important for inclined and highly deviated wellbores, which is also related with the wellbore cleaning issues.

LWD: LWD stands for Logging While Drilling. Formation related parameters could be captured during drilling and be used in the optimization process. However in the scope of this study no LWD consideration has been taken into account.

1.3. Factors Affecting Rate of Penetration

The factors known to have an effect on rate of penetration are listed under two general classifications such as controllable and environmental. Controllable factors are the factors which can be instantly changed such as weight on bit, bit rotary speed, hydraulics. Environmental factors on the other hand are not controllable such as formation properties, drilling fluids requirements. The reason that drilling fluid is considered to be an environmental factor is due to the fact that a certain amount of density is required in order to obtain certain objectives such as having enough overpressure to avoid flow of formation fluids. Another important factor is the effect of the overall hydraulics to the whole drilling operation which is under the effect of many factors such as lithology, type of the bit, downhole pressure and temperature conditions, drilling parameters and mainly the rheological properties of the drilling fluid. Rate of penetration performance depends and is a function of the controllable and environmental factors. It has been observed that the drilling rate of penetration generally increases with decreased Equivalent Circulating Density (ECD).

Another important term controlling the rate of penetration is the cuttings transport. Ozbayoglu et al. 2004 [25], conducted extensive sensitivity analysis on cuttings transport for the effects of major drilling parameters, while drilling for horizontal and highly inclined wells. It was concluded that average annular fluid velocity is the dominating parameter on cuttings transport, the higher the flow rate the less the cuttings bed development. Drilling penetration rate and wellbore inclinations beyond 70° did not have any effect on the thickness of the cuttings bed development. Drilling fluid density did have moderate effects on cuttings bed development with a reduction in bed removal with increased viscosities. Increased eccentricity positively effected cuttings bed removal. Smaller the cuttings the difficult it is to remove the cuttings bed. It is clear that turbulent flow is better for bed development prevention.

One of the most important considerations in order to have an efficiently cuttings transported hole is to take into account the factors given in **Table I-4** [26].

Table I-4: Factors for efficient hole cleaning.

1	Hole angle
2	Fluid Velocity
3	Fluid Properties (rheological properties and density)
4	Cuttings Size, shape, and concentration
5	Annular size
6	Rate of pipe rotation and pipe eccentricity
7	Fluid flow regime (laminar or turbulent)

The main characteristics of rotary drilling penetration performance is not only the fracture of the rock on bottom, but also the removal of the fractured cuttings from the rock face in an instant and efficient manner to provide further fracturing and drilling progress [26]. Due to the complexity of understanding the rate of penetration mechanism of drilling operations, industry pioneers have adopted empirical approaches by quantifying the effects of the controllable parameters on ROP performance, more than the analytical model implementation for the understanding of rate of penetration in the industry of drilling.

It is reportedly known that time spent for the drilling of wells is composed by up to 30% “rotating time” of the total well construction time. Penetration rate optimization is consequently an important cost reduction consideration [27].

One important assumption of this study is the characteristic consideration of the formation being drilled. It has been assumed that all of the properties of the formations affecting rate of penetration, that is subject to optimization are macroscopically homogeneous and are with unique physical properties throughout the entire interval.

1.4. Assumptions for Drilling Conditions

The following assumptions also as given by Miska [28] are considered to have been satisfied during the course of a drilling activity so that the general relations given in this study are going to be functioning effectively:

- Bottom hole cleaning is effectively achieved.
- The bit and BHA assembly combination in use is one properly selected for the formation being drilled.
- The formation interval being drilled is considered to be homogeneous.
- The rig and auxiliary equipment are efficiently functioning.

1.5. Objective of This Study

The optimization of rate of penetration for the drilling activities is going to have direct effects on the cost reduction, together with elimination of problems. It has been reported that drilling optimization should be based on the accumulated and statistically processed empirical data rather than working with implicit relations [29].

The objective of this research study is to create a methodology which would achieve the following in real-time basis:

- Achieve the exponents of multiple regression specific to formation which are necessary to be used in the general rate of penetration equation such as formation strength parameter, normal compaction coefficient, bit weight and rotary speed [14].
- Have a rate of penetration vs. depth as a function of certain drilling parameters [30].
- Determine the optimum drilling parameters specifically belonging to the formation, in order to drill safely maximizing the rate of penetration and reducing the overall costs.

The study works with numerical correlations using the drilling parameters to observe their effects to the rate of penetration. The correlations are used to maximize the ROP during drilling operations.

CHAPTER 2

LITERATURE OVERVIEW

Optimization of drilling activities for oil and gas wells is an area for which numerous detailed research studies have been performed. Optimized drilling is a system of pre-selecting the magnitude of controllable drilling variables to maximize footage or minimize drilling cost [31]. It is considered that with the increasing demand to drill wells, the area of research on the optimization of the drilling operations is going to be one over which scientist will be working on.

The early research studies conducted to optimise the drilling rate concentrated on the optimum parameter selection considering the hydraulic maximization. The industry has been considering that most important factor to affect drilling rate is the bit selection [6], and this could be achieved by trial and error.

Most of the studies performed in the literature have foreseen static drilling optimization process. The drilling parameters were required to be investigated off-site for the optimization of the subsequent drilling events, due to lacking opportunity to transfer data in real-time basis. However, in today’s technology data could be transferred by means of advanced communications systems.

Recent studies on real-time drilling optimization have been an important reference for this study. It has been observed that some attempts have been made to achieve optimization in real-time basis, however among the reviewed references no source has been found working on statistical correlation in real-time environment to optimize the drilling process.

There are mainly two optimization methodologies; using analytical models such as the method of Galle and Woods [13], drill-off tests, and use of the numerical (statistical) models such as multiple regression analysis [14].

2.1. Drilling Optimization Studies

One of the most important early studies performed in regards to optimal drilling detection was by Bourgoyne and Young [14]. They constructed a linear penetration rate model and performed a multiple regression analysis of drilling data in order to select the bit weight, rotary speed, and bit hydraulics. In their analysis they included the effects of formation strength, formation depth, formation compaction, pressure differential across the hole bottom, bit diameter, bit weight, rotary speed, bit wear and bit hydraulics. They found that regressional analysis procedure can be used to systematically evaluate many of the constants in the penetration rate equation. They mentioned that multi-well data sourcing should be obtained for the regression constant evaluations. They used the data of a minimum of 25 wells to get a_1, a_2,..., a_8 constants. They concluded that the use of relatively simple drilling optimization equations can reduce drilling costs by about 10%. The equations defined by Bourgoyne and Young, is given in Appendix B.

Speer [32] was the first to propose a comprehensive method in 1958 for determining optimum drilling techniques. In his study empirical interrelationships of penetration rate, weight on bit rotary speed, hydraulic horse power and formation drillability were demonstrated. He combined five relationships in a single chart to determine optimum drilling technique from a minimum of field test data.

Garnier and Lingen [33] conducted laboratory drilling experiments with drag bits and roller bits at elevated mud, pore, and confining pressures on rocks differing in strength and permeability. Their findings supported that reduction in penetration rate owing to an increase in rock strength is controlled by the difference in between mud and pore pressure. They also mentioned that rocks at depth are less drillable than when brought to surface, due to the downhole conditions of pressure differential across the parted cuttings, which are forced downhole restrained to be lifted.

Graham and Muench [12] are one of the first researchers who conducted evaluations on drilling data to determine optimum weight on bit and rotary speed combination. They used a mathematical analysis method for drilling related costs to drill under optimum circumstances. Empirical mathematical expressions were derived for bit life expectancy and drilling rate as a function of depth, rotary speed and bit weight. The proposed mathematical relations contained constants representative of the respective formations existing in the area. Their study resulted in being able to propose optimum weight on bit and rotary speed by means of calculations under any drilling conditions in order to minimize total drilling costs.

Maurer [34] derived rate of penetration equation for roller-cone type of bits considering the rock cratering mechanisms. The equation was based on ‘perfect cleaning’ condition where all of the rock debris is considered to be removed between tooth impacts. A working relation between drilling rate, weight on bit and string speed was achieved assuming that the hole was subject perfect hole cleaning circumstances. It was also mentioned that the obtained relationships were a function of the drilling depth. The rate of drilling equation expressed as given in equation (2.1).

$$\frac{dF}{dt} = \frac{4}{\pi d_b^2} \frac{dV}{dt} \tag{2.1}$$

where, F is the distance drilled by bit, t is time, V is volume of rock removed, and d_b is diameter of the bit.

Galle and Woods [13] are one of the first researchers who investigated the effect of best constant bit weight and rotary speed for lowest cost; developing mathematical relations. Graphs and procedures for field applications were produced to determine: best combinations of constant weight and rotary speed. They assumed a relation for the wear rate as a function of time in relation to inverse ratio of bit weight to bit diameter. The given equation was limited with a load application of 10,000 lbf/in of bit diameter. They also published an equation giving a relation in between the tooth wear rate and rotary speed for only milled tooth bits designed for soft formations. In their graphs drilling cost, footage, drilling hours and condition of teeth and bearing of the dull bit may be calculated. The drilling costs were demonstrated to be reduced using the recommended combinations of the drilling parameters. They presented the drilling rate equation as given in (2.2) as a function of WOB and RPM.

$$\frac{dF}{dt} = C_{fd}\frac{\overline{W}^{k}\,r}{a^{p}} \tag{2.2}$$

where, C_{fd} is the formation drillability parameter;

$a = 0.028125h^2 + 6.0h + 1$

k = 1.0 (for most formations except very soft formations), 0.6 (for very soft formations)

p = 0.5 (for self-sharpening or chipping type bit tooth wear)

$$r_{hard-formations} = \left[e^{\frac{-100}{N^2}} N^{0.428} + 0.2N\left(1 - e^{\frac{-100}{N^2}}\right)\right] \tag{2.3}$$

$$r_{soft-formations} = \left[e^{N^2} N^{0.75} + 0.5N\left(1 - e^{\frac{-100}{N^2}}\right)\right] \tag{2.4}$$

$$\overline{W} = \frac{7.88WOB}{D_b} \tag{2.5}$$

Galle and Woods also defined Rate of Dulling and Bearing life equation respectively in (2.6) and (2.9).

$$\frac{dh}{dt} = \frac{1}{A_f}\frac{i}{am} \text{ (Rate of Dulling Equation)} \tag{2.6}$$

where,

$$i = N + 4.348 \times 10^{-5} N^3 \tag{2.7}$$

$$m = 1359.1 - 714.19 \log_{10} \overline{W} \tag{2.8}$$

$$B = S\frac{L}{N} \text{ (Bearing Life Equation)} \tag{2.9}$$

Bingham [35] proposed a rate of penetration equation based on laboratory data, equation (2.10). In their equation the threshold bit weight was assumed to be negligible and rate of penetration was a function of applied weight on the bit and rotary speed of the string. The bit weight exponent, a_5 was set to be determined experimentally through the prevailing conditions.

$$R = K\left(\frac{WOB}{d_b}\right)^{a_5} N \tag{2.10}$$

Eckel [36] performed microbit studies expressing the drilling rate exponentially as a function of pseudo bottom hole or near bit-nozzle Reynolds number. The relation introduced was reported to be independent of bit weight and speed, differential borehole pressure, and formation.

Young [37] performed development of onsite computer system to control bit weight and rotary speed. He introduced a minimum cost drilling terminology with four main equations; *drilling rate* as function of weight on bit and bit tooth height, *bit wearing rate* as a function of bit rotation speed, *bit tooth wear rate* and finally *drilling cost*. By integration of the introduced equations for the optimum weight on bit and rotary speed constants the best solutions are reported to be obtained.

Lummus [11] in his study mentioned that mud and hydraulics are two of the most important factors affecting the optimization of drilling operations. Application of drilling optimization could be possible when the capability of the rig is enough to provide adequate hydraulics. Rig is capable to provide necessary string rotation, and with the bottom hole assembly enough weight to the bit. Also the mud handling equipment should have enough power to respond to the requirements.

Wardlaw [38] gave the relationship between drilling efficiency and penetration rate controlled by rotary speed or weight on bit. He showed that the two controllable parameters are manipulated by differential pressure on bottom, mud characteristics, circulation rate, jet velocity and bit design. In his study charts were presented giving the relationship between penetration rate and different drilling parameters. The given charts were used for the determination of the optimum drilling parameters.

Reed [39] developed a variable weight-speed optimal drilling model which was being solved using a Monte Carlo scheme for the least cost per foot drilling. Linear (using the least squares technique) and curvilinear smoothing techniques were developed to re-order and smooth the paths as well as decrease the cost per foot at intermediate states of the Monte Carlo calculations. The developed method was proved valuable because of the rigor and the remarkable ease of constraint inclusion. It was reported that optimal drilling was desirable not only for weight and speed in short, but also for the hydraulics, bit selection, well design, mud treatment, solids separation.

Lummus [40] worked on importance of data acquisition and analysis of data for optimized drilling purposes. The study concentrated on the need to plan, maintain and appraise the drilling of wells. The important requirements for optimized drilling were cited to be, the data needed for computer input to calculate optimum values for the controllable drilling conditions. Daily data from the rig to determine how efficient the drilling optimization is and to provide basis to suggest required updates. Finally the data needed to design much more efficient future wells.

Wilson and Bentsen [41] investigated various drilling optimization procedures concentrating on optimization of weight on bit and rotary speed. With increasing complexity and data requirement three methods were developed. First method; Point Optimization: minimizes the cost per foot during a bit run. Second method; Interval Optimization: minimizes the cost of a selected interval. The third method; Multi-Interval Optimization: minimizes the cost of over a series of intervals. Authors concluded that their model could have act as a guide toward good drilling procedures and that the cost saving could have been considerable.

Tansev [42] conducted an heuristic approach involving the interaction of raw drilling data and performing regression techniques for optimized drilling. The controlling variables considered were weight on bit, rotary speed and bit hydraulics horse power. Cost minimization was achieved under a heuristic framework. It was observed that penetration rate and bit life can be obtained in a uniform drilling interval with the same bit. The introduced approach did not produce an absolute optimal plan, however provided a systematic framework that could be improved as drilling progressed. An online optimization was recommended either at the rig site or in communicating with a central processor.

Bizanti and Blick [43] conducted tests investigating the variables effecting the rate of cuttings removal from bottom hole. They presented dimensional analysis techniques. A regression analysis of the data indicated how the rate of cuttings removal was a function of the variables in terms of dimensionless parameters such as Reynolds number, Froude number, rotational Reynolds number. Following the regression analysis they presented the R^2 and *F-values* to describe the significance of the correlation. The curves they presented after their findings based on the developed regression equations could allow optimizing bottom hole cleaning, therefore improving rate of penetration minimizing drilling cost per foot.

Al-Betairi et al. [26] presented a case study for optimizing drilling operations in the Arabian Gulf area. The drilling model proposed by Bourgoyne and Young [14] was applied in their model with Statistical Analysis System was validated. They observed that for particular set of the coefficients of the model became sensitive. Severity of the multicollinearity effect on each parameter was observed to be inversely proportional to the influence of that parameter on the rate of penetration. The more the data points the reliable estimated drilling parameters became. The accuracy of estimating optimum WOB and rotary speed suffered due to the presence of multicollinearity in the proposed model.

Reza and Alcocer [44] developed a dynamic non-linear, multidimensional, dimensionless drilling model for deep drilling applications using Buckingham Pi theorem. Buckingham Pi theorem is a dimensional analysis theorem used to generate equations in dimensionless forms. The model consisted of three equations; rate of penetration, rate of bit dulling and rate of bearing wear. Their study reflected the effects of following variables on given three equations: weight on bit, rotary speed, bit diameter, bit nozzle diameter, bit bearing diameter, drilling fluid characteristics (density and viscosity), drilling fluid circulation rate, differential pressure, rock hardness, temperature and heat transfer coefficient. They defined the rate of penetration as given in equation (2.11) in a form of non-linear, multivariable equation.

$$\frac{\dot{F}}{Nd_p} = C_1\left[\frac{Nd_p{}^2}{\upsilon}\right]^a\left[\frac{Nd_p{}^3}{Q}\right]^b\left[\frac{Ed_p}{W}\right]^c\left[\frac{\Delta pd_p}{W}\right]^d \tag{2.11}$$

Where, *C₁, a, b, c,* and *d* are unknown parameters. In order to find the coefficients using the available data a linear regression analysis methodology was applied after taking the natural logarithm of both sides of the equation above. When the solution of the *rate of penetration* equation was written the following relation was reported to investigate the deep well drilling problems, equation (2.12).

$$\frac{\dot{F}}{Nd_p} = 0.33\left[\frac{Nd_p^{\,2}}{\upsilon}\right]^{0.43}\left[\frac{Nd_p^{\,3}}{Q}\right]^{-0.68}\left[\frac{Ed_p}{W}\right]^{-0.91}\left[\frac{\Delta pd_p}{W}\right]^{-0.15} \tag{2.12}$$

The general equation for the *rate of bit dulling* was obtained as in equation (2.13).

$$\frac{\dot{D}}{ND_b} = 0.001\left[\frac{Q}{ND_b^{\,3}}\right]^{0.56}\left[\frac{W}{ED_b^{\,2}}\right]^{0.26}\left[\frac{D_b}{Q}\right]^{-0.03} \tag{2.13}$$

The general equation for the *bit bearing life* was obtained as in equation (2.14).

$$\frac{\dot{B}}{N} = 0.05\left[\frac{th_t d_p}{WN}\right]^{0.51}\left[\frac{\upsilon}{Nd_p^{\,2}}\right]^{0.4}\left[\frac{Q}{Nd_p^{\,3}}\right]^{-0.5} \tag{2.14}$$

In the second part of their study Reza and Alcocer [45] mentioned that the exponents of the derived models are sensitive to unknowns, and there would be fluctuations from region to region and well to well. For that particular reason their finding was generalization of the model specifically for a region and in deep field/wells.

Warren [46] presented an ROP model that includes the effect of both the initial chip generation and cuttings-removal process. The rate of penetration equation they derived is formed of two terms, working only with perfect hole cleaning assumption. The first term defined the maximum rate supporting the WOB effect without tooth penetration rate, the second term on the other hand considering tooth penetration into the formation. The equation was found to fit the experimental data for both steel tooth and insert bit types.

Miska [28] presented three governing differential equations; rate of penetration, rate of teeth wear, and rate of bearings wear. It was concluded that the given equations could have been successfully used for predicting and optimization purposes provided that some major conditions were satisfied. Three major conditions could be listed as: bottom hole cleaning is adequate, rock-bit is properly selected to the formation drilled, and formation can be considered macroscope-homogeneous.

Wojtanowicz and Kuru [47] presented a new methodology of drilling process planning and control; which combined theory of single-bit control with an optimal multi-bit drilling program for a well. Dynamic drilling strategy was compared to conventional drilling optimization and typical field practices; the considerable cost-saving potential of 25 and 60% was respectively estimated. The method appeared to be the most cost effective for expensive and long-lasting PDC bits through better utilization of their performance and reduction in the number of bits needed for the hole.

Maidla and Ohara [30] tested a drilling model on offshore drilling data and compared the findings with the Bourgoyne and Young's model. Their objective was to be able to select bit, bit bearing, WOB and drillstring rotation to minimize drilling costs. Their finding was that ROP for successive wellbores in the same area could be predicted based on the coefficients calculated from the past drilling data, resulting in cost savings. They concluded that the drilling model performances depended on the quality of the data used to conduct the syntheses. They presented isocost, iso-ROP graphs for cost effective drilling activities.

Pessier and Fear [48] elaborated the Mechanical Specific Energy methodology which was developed by Teale [49]. They performed simulator tests in the computer and conducted laboratory tests to quantify and develop an energy balanced model for drilling of boreholes under hydrostatically pressurized conditions. They derived an equation for mechanical specific energy, equation (2.15). They found better identification methodologies (than WOB and ROP concentrated evaluation) for bearing problems of the drill bits which are more quick and reliable by continuously monitoring E_s and μ, equation (2.16).

$$E_s = WOB\left(\frac{1}{A_B} + \frac{13.33\mu_s N}{D_B ROP}\right) \tag{2.15}$$

$$\mu = 36\frac{T}{D_B WOB} \tag{2.16}$$

Cooper et al. [50] developed a drilling simulator program, which they believed aimed to be simple to understand and use. Their simulator contained properties in which drilling engineers could experiment changing effects of the operating parameters in order to optimize drilling operations. The simulator contained an algorithm which determines drilling rate of penetration and wear rate of the bit. The total time and cost are available together with cost per foot in total and for the bit in use during the drilling run.

Mitchell [51] in his book presented the purpose of optimal weight on bit and rotary speed selection. One of the most important reasons was given to be producing the lowest drilling cost per foot, **Figure II-1**. Also controlling the direction of the borehole and identifying over-pressured zones were among optimum parameters selection. He also mentioned the contouring method of selecting optimal weight and string speed.

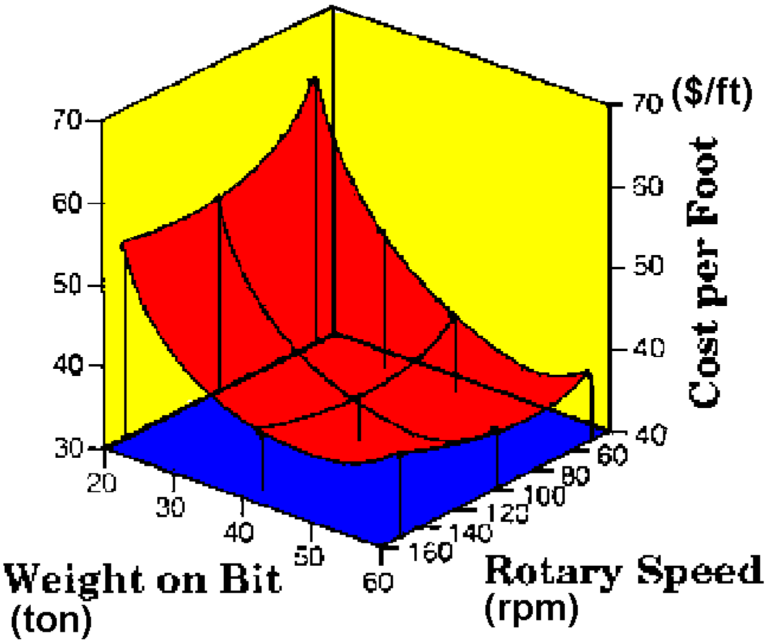


Figure II-1: Optimum bit weight and speed chart, after Mitchell [51].

Bernt [52] worked on three new approaches which considered non-physical variable product of flow rate and bit pressure drop to optimize the hydraulics in the drilling boreholes. Likewise the previously used common approaches of maximum horse power and jet impact force criteria the new optimization model aimed at providing sufficient flow rate to clean the cuttings in the wellbore.

Barragan et al. [27] worked on a research study based on heuristic approach (a method for finding minimum values) seeking drilling optimization using Monte Carlo Simulation and specially developed numerical algorithms. They analyzed penetration rate equations developed such as by Moore, Maurer, and Bingham. They considered five different drilling optimization methods (by means of WOB and RPM optimization), such as; single bit run for homogeneous and non-homogeneous formation cases, single bit run with homogeneous and non-homogeneous drilling interval, and finally optimization for each homogeneous formation and specific bit. They concluded that optimization of well phases would be more economical than optimization of single bit runs. They found that heuristic method could be suitable for optimization of directional drilling activities. They also found that optimization could be performed as the data was being acquired for the necessary modification of the following bit run.

Serpen [53] performed a computerized drilling optimization study. Computer programmes were developed for very common six different drilling optimization methods which mostly made use of graphs. Namely the methods covered were: Galle-Woods method, Constant Energy Drilling approach, drill-off tests approach, multiple regression approach, modified multiple regression approach, drilling hydraulics optimization. The study aimed to aid the drilling team working in the field in terms of being able to determine optimized drilling parameters, and also aid planning engineers to make effective estimations of parameters.

Dubinsky and Baecker [29] examined the dynamic behaviour of drill bit, developing a simulation system for various drilling conditions. They performed simulation of major drilling dynamic dysfunctions such as bit bounce, lateral vibrations, BHA/bit whirl, torque shocks, stick-slip and torsional oscillations. They concluded that the model for the on-line drilling optimization should be based on the accumulated and statistically processed practical experience.

Fear [54,55] quantified the controllable drilling parameters to observe the effects on the ROP performance based on formation similarity. Clear observation of the effect of the controllable drilling parameters was seen. Also bit drilling sections were subdivided into subsections to minimize ROP variation due to bit condition.

Samuel and Miska [4] presented an analytical study on the optimization of motor performance and drilling parameters. Comprehensive optimization of Positive Displacement Motors with the drilling parameters of roller cone and diamond bits was studied to be applicable. General optimal relations between rotational speed and weight on bit will help the user to prevent the operation of the motor beyond the capability of the motor.

Pereira [56] performed a study which developed and used a realistic solution to complex-multi-variable drilling problems by simultaneous optimization of the mechanical, hydraulic, and cuttings transport parameters in vertical and horizontal wells. Results confirmed that the use of an optimization method can significantly reduce the drilling costs.

Millheim and Gaebler [57] presented a new concept based on heuristics, utilizing unused data accumulations from 22 wells and processing them. They called their process as "*Virtual Experience Simulation"* (VES) for drilling. Their simulator was developed offering a special ergometric design compatible with specific drilling data sets in specific geographical and geological environment. The simulation is basically known to be a bridge between the knowledge contained in activated data sets and the ability to quickly learn the previously gained insights and experiences with statistical data evaluations. Very good ROP isomeric maps as well as 3D graphs were illustrated in the outcomes of their work.

Osgouei [58] conducted a study developing a comprehensive model to obtain a realistic solution for determining the rate of penetration for Insert and PDC bits. The scope of the study covered inclined and horizontal wells. The model provided an efficient tool for determining the combined effects of several variables on the rate of penetration within realistic technological constraints. Results also reflected that the use of an optimization technique can reduce drilling costs significantly.

Gelfgat et al. [31] published a book in which their investigation and analysis revealed that the process of optimized well drilling in the United States was based on statistical information from previously drilled wells or from individual bit runs, and prompt drilling parameter adjustments were allowed. They summarized their observations in **Table II-1** which they believed the efficiency in drilling industry especially in the United States of America were related to the organizational and technical issues.

Table II-1: Features believed to have effect on drilling efficiency [30].

1	A consistently high level of drilling technology and equipment provided by equipment and material supply companies.
2	Availability and smooth operation of instrumentation and recording equipment for drilling operations.
3	Exclusive use of the rotary drilling method that facilitates the optimization of drilling parameters within the acceptable range of rotational bit speed variation.
4	A reliable system of payment for drilling crews that is not dependent on drilling results.

They mentioned that rigs in various countries could not have had optimized drilling processes due to not having reliable working equipment to achieve or maintain high quality drilling operations. Old rigs also lacked reliable instrumentation; recording equipment together with well maintained tools or systems. The most widely used turbodrilling method did not produce rotational bit speed information. The official reports were significantly affected with the payment method and lacked objectivity; resulting in not being able to use them for optimization purposes.

Millheim [59] mentioned that "*Proactively managed drilling operations optimize company performance*". This would mean that drilling optimizations would also be based on proactive management of various factors.

Akgun [60] investigated the controllable drilling variables having effect on drilling rate. Mud weight, weight on bit, rotary speed, bit type and hydraulics are among the controllable drilling variables. Proper selection of the controllable variables is reported to significantly improve drilling rate. An upper drilling rate limit or "technical limit" concept has been introduced which can not be exceeded without risking the drilling operations safety. For example selected mud weight need not be less than the weight which would result in well kick and borehole collapse (wellbore stability). WOB and RPM parameters need to be at maximum possible values considering the minimum bit operational cost and drillstring stability. Flow rate need to be selected at an optimum value by considering bit hydraulics and hole cleaning.

Ozbayoglu and Omurlu [61] performed a study in which they mathematically optimized drilling parameters in order to reduce the drilling costs. They considered that weight on bit, rotation speed, bit type and wear, and bit hydraulics have direct impact on rate of penetration. An analytical drilling cost equation was defined considering a non-linear rate of penetration equation. Drilling parameters of the actual field data collected from the literature were optimized using the defined equation by means of certain mathematical models. They observed that drilling costs were reduced up to four times.

2.2. Real-Time Drilling Optimization Studies

One of the first real-time drilling optimization studies was performed by Simmons [62]. His findings were based on application of all possible means at the time their study was published in order to get the drilling parameters optimised at the rig site. It should be acknowledged that data transmission from drilling locations, usually for remotely allocated ones, has been started by the beginning of the new millennium. It was concluded that the combination of current technology and engineering, coupled with “real time” drilling optimization, will nearly always save on rotating hours, improve drilling efficiency, reduce possible formation damaging effects and ultimately save on overall drilling expenditures.

John et al. [63] introduced a web-based data delivery system which provided encrypted and high data transmission (as high as 10-20 times that of conventional FTP transfer rates) in real-time. They mentioned that a paradigm shift in the way the asset team and contractors utilize data for early decisions was created. It was demonstrated that timely decisions could be taken by means of involving best expertise from around the world following the transmission of pertinent data to and from the rig sites.

Ursem et al. [64] demonstrated how an operator and a service company implemented the use of latest technology within the scope of Real Time Operations Center (RTOC). They performed some pilot tests on exploration wells which revealed communications improved interventions and made the advices much more clear; limiting downtimes. They proved that critical decisions are multidisciplinary in origin and common ground is needed for all parties involved. They concluded that it was possible to influence unexpected outcomes in real time instead of relying on an expensive lesson to be learned.

Rommetveit et al. [65] developed a new and innovative drilling automation and monitoring system. The project was named as Drilltronics. All available surface and subsurface drilling data was utilized to optimize the drilling process. One of the introduced modules was “Bit Load Optimization Module” which modulated rotary speed and weight on bit and observed how respective changes effected the rate of penetration. They mentioned that preventing stick-slip occurrences by means of activating one of the introduced algorithms increased rate of penetration by 15% to 30%.

Mochizuki et al. [66] examined and categorized technology components in real time optimization to clarify the value of real time projects. They reviewed case histories and demonstrated the impact of real time optimization projects. They concluded that real time optimization projects are difficult in type because they involve people, technology and process components. Tools to assess the contributed value and technology components were suggested. Technology component categorization, use of spider diagrams, presenting the economic evaluation processes was introduced. Benchmarking and project justifications were also demonstrated as well as giving comparisons in relation to case histories and giving lessons learned.

Dupriest and Koederitz [67] effectively used Mechanical Specific Energy (MSE) concept in evaluating drilling efficiency of bits in real-time basis. They developed a system allowing the driller to continuously monitor MSE calculated through surface measurements alongside with other normal mechanical drilling logs. Bit balling type of occurrences was easily identifiable with the analysis.

Iversen et al. [68] developed an advanced monitoring and optimization for drilling operations, based on real-time data measurements. Together with many drilling monitoring technique the bit load optimization module offered WOB and RPM modulation by means of observing how small change in these parameters affected rate of penetration. The developed system has demonstrated the ability to maintain the drilling operation within critical limits and thereby, increase safety and reduce down-time during drilling operations.

Milter et al. [69] worked on real-time data transfer from offshore to land support centres for drilling, well intervention and production operations. The piped data was focused on quality by multi-disciplinary relevant personnel, not necessarily from a fixed location, but anywhere with high speed internet communication. The optimization was conducted based on the judgement of expert involved in the process based on his/her experience. They concluded that by means of the automatic surveillance by means of real-time data transmission the number of unforeseen events was reduced. The number of well shut-ins was decreased increasing the regularity in operations.

Tollefsen et al. [70] conducted a case study which allowed an operator to drill confidently in a very tight hydraulic envelope and eliminate running a string of casing, which resulted in a savings to the operator. LWD data has been followed on real time basis via satellite communication. The recommendations on mud weight were sent to the rig and appropriate action was taken accordingly. Real-time hydrodynamic monitoring succeeded in identifying fracture gradient details of the wellbore in subject.

Monden and Chia [71] showed that decision making point could be moved from the data acquisition point to the Operation Support Centres (OSC). It was mentioned that real time connectivity from the rigs to the offices was becoming the norm for many operations being performed recently. They concluded that, a “data-centric” approach to drilling optimization can improve drilling efficiency. It was shown that in many cases the value of the OSCs can be measured, which would have direct impact on time and cost considerations.

Remmert et al. [72] performed an approach to manage rate of penetration using real time, customized surveillance technology to continuously maximize both bit cutter efficiency and transmission of energy from rig floor to the bit. During the course of their application the improvements were made on; hydraulic horsepower redesign to reduce the effects of the cuttings at the cutting structure of the bit. They continuously monitored MSE and adjusted WOB and RPM to address downhole vibrations and reduce energy loss.

Strathman et al. [73] addressed the problem of not utilizing the well data in time rather than depth, and focusing only on individual well data instead of making comparisons between them. A system was rolled out in which data was stored in an historian and was stacked in vertical trends in time. A data capture system up to 200 parameters/well with data frequency of every 5 seconds simultaneously through 20 wells was able to be realized. A stored data of 3000 wells shall be achievable on-line through the developed system. Authors mentioned that costs were reduced due to allowing drilling experts being able to look more easily at multiple wells, and reduce the time required for reaction where necessary.

Iversen et. al. [74] adapted a system into the rig control mechanism in order to transfer signal from both surface and downhole sensors in real-time basis to and from the rig in the North Sea. The introduced system worked dynamically for well flow, drillstring mechanics, thermo-physical, solids transport, torque and drag models. Their test study proved that it is possible to achieve a system which can calculate parameters and verify the quality of the safeguard calculations. For instance the diagnosed wellbore stability issues and cuttings transport through trend analysis of friction between wellbore and drillstring using a torque and drag model. They concluded that the system may alleviate some of the challenges like fluid loss, stuck pipe, and pack-off tendencies, and suggestions for behaviour to be followed could be made in assisting to avoid them. They also mentioned that the calculation functionality of the system is a function of data quality as well as it is a function of correct system setup.

Iqbal [75] presented an algorithm for calculation and optimization procedure for drilling optimization using real time parameters for roller cutter insert type of bits. The methodology is composed of steps in which weight exponent in the given drilling rate of penetration was evaluated, optimum string revolution speed and WOB parameters were found using correlation or plots. The optimum parameter selection is made based on the basis of the least cost per foot relation. It was concluded in the study that exploratory wells' efficiency could also be improved using this technique where no proven information would be available.

2.3. Result of the Literature Overview

Many detailed and effective research studies has been performed in the area of drilling optimization most of them aiming to reduce the cost. In the early stages of the drilling industry, due to the nature of the communication systems not having been so advanced, studies were concentrated on data handling for post well analysis. Also the initial data analyses were required to be conducted on manually collected data.

Following the developments in data acquisition systems, and particularly advances in communication segment, data from the rig site(s) could have been transferred. Only following that, research studies started to handle the available data, and this initiated drilling optimization on much more reliable basis. The optimization attempts mainly concerned either on crew performance, concepts such as "Learning Curves", or concentrated on parameters which performed the best in an entire data segment. This sort of approaches did not taken into consideration many factors which could very well have an effect on the overall drilling rate of penetration optimization process.

Recent studies are observed to do drilling optimization in real-time, however among the investigated references there is no work working with statistical correlations in real-time environment.

In this study "*Multiple Regression*" technique is being used to optimize the drilling parameters on real-time basis. The multiple regression technique is used to linearly model the relationship between the dependent rate of penetration and the independent drilling parameters such as weight on bit, bit revolution, formation properties, and hydraulics considerations. Following the determination of the relation between the dependent and independent parameters drilling cost per foot equation is utilized to find the optimum parameters for optimum drilling.

CHAPTER 3

STATEMENT OF THE PROBLEM

The process is to collect the drilling data from different drilling sites on real-time basis, store them in a central computer. In order to run the process a data filtering is necessary. For each specific formation the stored data could be utilized in order to run the multiple regression technique to estimate the rate of penetration. Data before being processed is considered to give much accurate results if corrected for the weight on bit being applied to the bit considering the well inclination, and additional bit rotation due to the motor in the case of deviated wellbores. The determined optimum drilling parameters which when utilized are going to be reducing the drilling related costs. The determined optimum drilling parameters are required to be piped back to the rig site in order to be applied.

Benefit of multiple regression is the ability of being able to estimate the rate of penetration as a function of independent drilling parameters. Following the analysis of the drilling parameters database specifically for each formation, a relation between the drilling parameters and ROP trend could be determined and during actual drilling activity a predetermined ROP performance could be agreed between the parties (such as Operator and Contractor). ROP performance to be plotted as a function of the controllable variables with respect to depth will be monitored making sure any departure from the preset trend would have some indication of extraordinary happenings. Necessary actions would required to be taken in case of a departure from the predetermined ROP trend, which could be in the very general terms due to change in formation, characteristics of drilling parameters or technical reasons in the rig auxiliary system. Sensitivity analysis on the results of the multiple regression coefficients while data is being collected if performed in real-time should give coherent charts when in the same formation; their behaviour should vary in different formations.

CHAPTER 4

MODEL THEORY

Drilling rate of penetration model is defined in order to accomplish the unique objective of this study that is to conduct real-time analysis for drilling rate of penetration optimization. The study aims to optimize applied weight on bit and string rotation. Optimization is to be formation specific. Multiple linear regression technique is used for the methodology of optimization, which is a statistical approach. A computer program has been prepared to determine the coefficients of the drilling rate of penetration general equation; mathematically calculating rate of penetration as a function of controllable drilling and uncontrollable parameters. Similar computer programs are also reported in the literature [53] which were prepared for drilling operations however are known to be not have been utilized.

4.1. Introduction to the Methodology

The drilling rate of penetration model adapted for this study is a function of independent variables. Examples of the independent variables are parameters like; compaction effect, bit diameter, weight and rotary speed effect, tooth wear and bit hydraulics. All of the drilling parameters should be collected at the MLU and then piped to a central computer for storage and synthesis. The noisy data is going have a negative effect on the outcome of the process, for this reason the data to be piped as much noise free as possible. The methodology is to handle more than one rig data at a time, and concatenating all the data in a unique data set and running the statistical analysis. It is considered that the methodology is going to function even if the mud type, the drilling depths, and bit types are different, because all such parameters are taken into account within the model itself. Because the process is formation specific; for two wells data of which is being collected simultaneously the data process should be performed considering the data of two wells mutually for the sections across which the formation properties are the same. In case the lithologies of the two wells are totally different from each other the process should be run individually for each well.

Figure IV-1 gives the theory of the methodology application schematically. The process is to acquire drilling data at a rig site network, pipe the acquired data to the operation centre or central computer. The central computer will run the analysis and send the feed back to the rig site back again. The role of driller is at utmost importance for the implementation of the optimum drilling parameters determined following the process [76].

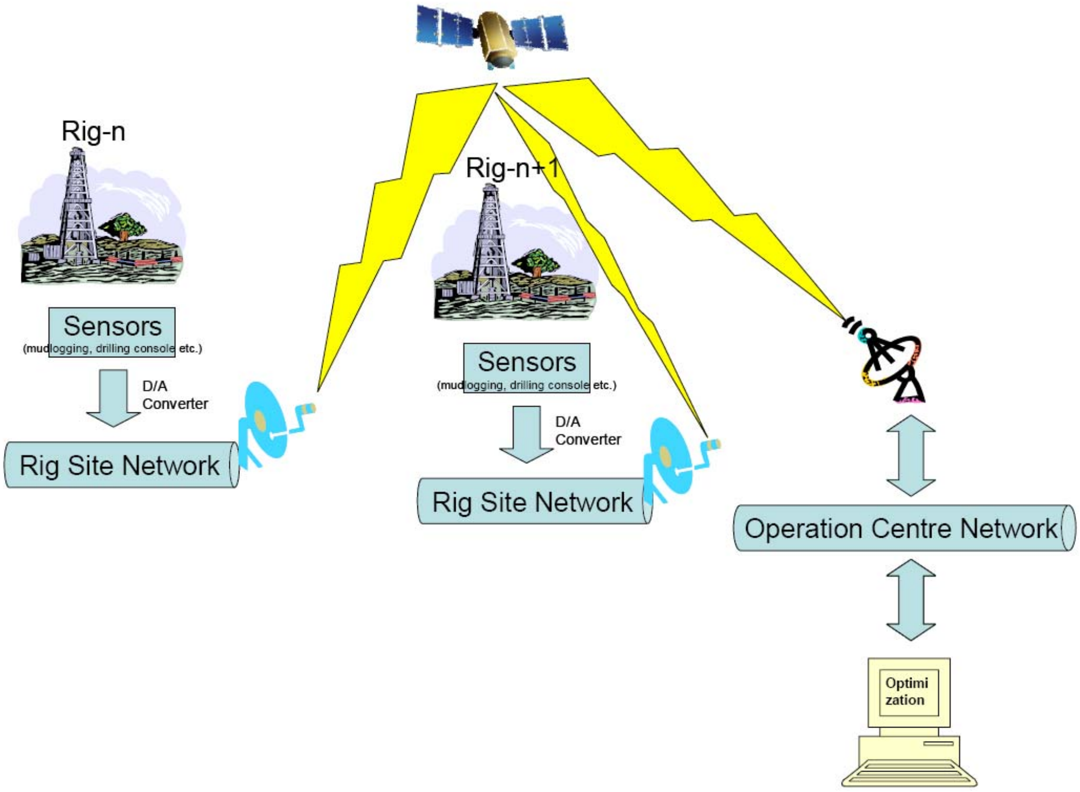


Figure IV-1: Drilling optimization theory drawing.

4.2. Rate of Penetration Models

There are three very common general drilling rate of penetration models in the industry, they are: Maurer [33], Galle and Woods [13] and Bourgoyne & Young [14] theories.

4.2.1. Maurer's Method

Maurer's [33] method was developed based on a theoretical penetration equation for roller cone bits as a function of WOB, RPM, bit size and rock strength. The developed equation was based on observations such as the amount the crater cutter can create, rock strength related considerations.

4.2.2. Galle & Woods' Method

Galle and Woods [13] investigated the best selection effect of WOB and RPM. They presented graphs for the best selection of the drilling parameters combination. They demonstrated that drilling costs are reduced in case of using their method.

4.2.3. Bourgoyne and Youngs' Method

Bourgoyne and Youngs' [14] method is the most important drilling optimization method since it is based on statistical synthesis of the past drilling parameters. A linear penetration model is being introduced and multiple regression analysis over the introduced rate of penetration equation is being conducted. For that reason this method is considered to be the most suitable method for the real-time drilling optimization.

4.3. Selected Drilling Rate of Penetration Parameters

The model proposed by Bourgoyne and Young [14] has been adopted for this study in order to derive equations to perform the ROP estimation using the available input data. This model has been selected because it is considered as one of the complete mathematical drilling models in use of the industry for roller-cone type of bits. The coefficients used in the proposed model are modified accordingly to be inline with the available data. Equation (4.1) gives the general linear rate of penetration equation which is a function of both controllable and uncontrollable drilling variables. When the multiple regression process is performed the model has been modified based on controllable parameters. **Table C-1** gives the drilling parameter items considered to have an effect on the rate of penetration.

$$\frac{dF}{dt} = e^{\left(a_1 + \sum_{j=2}^{8} a_j x_j\right)} \tag{4.1}$$

The normalization constants given in the general ROP equation are modified accordingly as a function of the data property when used as an input to the regression cycle. The coefficients should give accurate predictions for ROP; when modified normalization constants are used. The constants given in equation (4.1); a_1 through a_8 should be determined through the multiple regression analysis using the drilling data. In this study these coefficient are considered to be determined on real-time basis. They represent the effects of formation strength, compaction effect, pressure differential, bit weight, rotary speed, tooth wear and hydraulic exponent. The threshold weight on bit and bit diameter value is not a constant, it significantly may have varying magnitudes based on formation characteristics, and for this reason whole data trend is observed when this threshold value is determined as an input. The same value could easily be obtained from a drill-off test. The fractional tooth height calculation methodology is a function of reference abrasiveness constants in the same field, and is related to the time bit in use have operated. The open form of the general ROP equation for roller cone bit types is given in equation (4.2).

$$\frac{dF}{dt} = Exp\left(a_1 + a_2(8000 - D) + a_3(D^{0.69}(g_p - 9) + a_4 D(g_p - \rho_c) + a_5 Ln\left(\frac{\frac{w}{d_b} - 0.02}{4 - 0.02} \right) + a_6 Ln\left(\frac{N}{60} \right) + a_7(-h) + a_8 \frac{\rho q}{350 \mu d_n} \right) \tag{4.2}$$

The considered effects of the controllable and uncontrollable drilling variables on ROP are individually described below for each item. **Figure IV-2** gives the schematically represented general rate of penetration equation for roller-cone bit types.

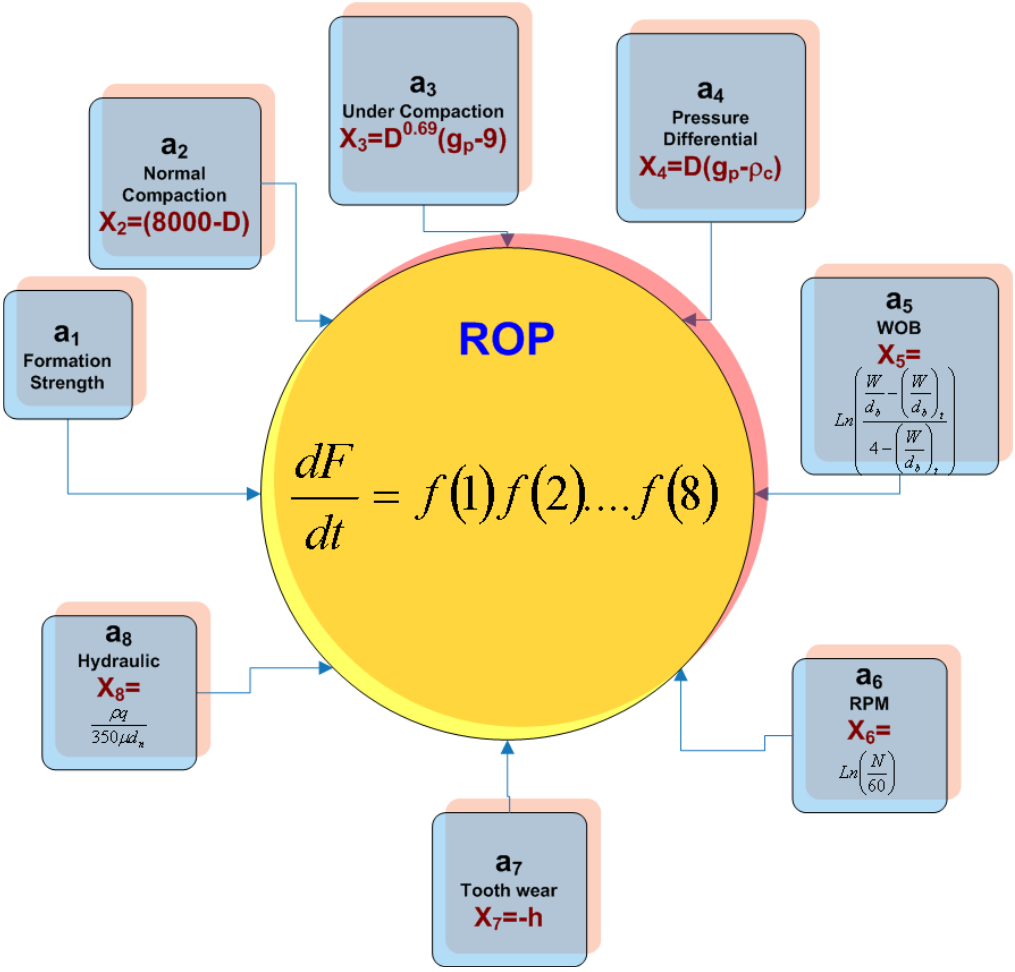


Figure IV-2: General rate of penetration equation.

4.3.1. Formation Strength Function

The coefficient for the effect of formation strength is represented by "a_1". It has been considered that the less the value for this constant, the less the penetration rate. This coefficient includes also the effects of parameters not mathematically modelled such as; the effects of drilled cuttings. Other factors which could be included for future considerations but known to be under this function could be drilling fluid details, solids content, efficiency of the rig equipment/material, crew experience, and service contractors' efficiency.

Factors for future considerations could be introduced as new functions for example the more the solids content in the mud the less the efficiency of the rate of penetration is to be expected. Other factors could also be introduced accordingly based on the effect they would have over the general rate of penetration.

The equation for the formation strength related effects are defined as in equation (4.3). The "f1" term is defined in the same unit as rate of penetration, for that reason it is called drillability of the formation of interest.

$$f_1 = e^{a_1} \tag{4.3}$$

4.3.2. Formation Compaction Function

There are two functions allocated for the consideration of the formation compaction over rate of penetration. The primary function for the effect of normal compaction trend is defined by "a_2". The primary effect of formation compaction considers an exponential decrease in penetration rate with increasing depth, as given in equation (4.4). In other means this function assumes increasing rock strength with depth due to normal compaction.

$$f_2 = e^{a_2 X_2} = e^{a_2(8000-D)} \tag{4.4}$$

The additional function considered to have an effect over the penetration rate in regards of the formation compaction is defined by coefficient "a_3". This function considers the effect of under compaction in abnormally pressured formations. In other means within over-pressured formations rate of penetration is going to show an increased behaviour. There is an exponential increase in penetration rate with increasing pore pressure gradient, equation (4.5).

$$f_3 = e^{a_3 X_3} = e^{a_3 D^{0.69}(g_p-9)} \tag{4.5}$$

4.3.3. Pressure Differential of Hole Bottom Function

The function for the pressure differential is defined by coefficient "a_4". Pressure differential of hole bottom function is considered to reduce penetration rate with decreasing pressure difference. Whenever the pressure differential between the hole bottom and the formation is zero, the effect of this function is going to be equal to 1 for the overall process, equation (4.6).

$$f_4 = e^{a_4 X_4} = e^{a_4 D(g_p-g_c)} \tag{4.6}$$

4.3.4. Bit Diameter and Weight Function

The function for the bit diameter and weight is defined by coefficient "a_5". The bit weight and bit diameter are considered to have a direct effect over penetration rate, equation (4.7). $\left(\frac{W}{d_b}\right)_t$ is the threshold bit weight, the reported values for this term are ranging from 0.6 to 2.0. In this study the magnitude for this term has been determined specifically based on the characteristics of the formation. The force at which fracturing begins beneath the tooth is called the *threshold* force. The given function is normalized for 4000 lbf per bit diameter.

$$f_5 = e^{a_5 X_5} = e^{a_5 Ln\left(\left(\frac{W}{d_b} - \left(\frac{W}{d_b} \right)_t \right) \Big/ \left(4 - \left(\frac{W}{d_b} \right)_t \right) \right)} \tag{4.7}$$

A modification to correct the weight applied on the bit to its vertical component was performed. Well trajectories of the deviated wells are with varying inclinations. The applied weight can be corrected provided that the section of the bottom hole assembly transferring weight to the bit all remain in a constant inclination. Considering the geometrical relation as given in **Figure IV-3**, the applied weight to the bit (*W*) could be corrected to its vertical component (W_e), equation (4.7). The load applications are always applicable when the bit is in contact with the formation, for that reason the drag force has been ignored.

$$W_e = \frac{W}{Cos(\alpha)} \tag{4.8}$$

where,

α is the hole section inclination angle from vertical.

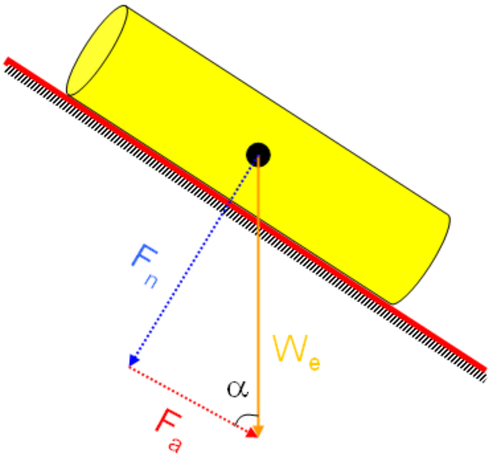


Figure IV-3: Schematics of a bottom hole assembly section.

4.3.5. Rotary Speed Function

The function for the rotary speed is defined by coefficient "a_6". Likewise the direct relation of bit weight on penetration rate the rotary speed is also set to have a similar relation, equation (4.8). The normalizing value to equalize the rotary speed function to 1 is taken to be an appropriate magnitude based on the actual rotation of the bit.

$$f_6 = e^{a_6 X_6} = e^{a_6 Ln\left(\frac{N}{60}\right)} \tag{4.9}$$

A modification to account for the additional bit rotation due to additional bit rotation generated by the motor has also been applied. The efficiency of the motors in use should be taken into consideration and additional rotation count calculation should be adequately performed. Technical specifications of drilling motor are necessary references for the account of additional bit rotation. There are references provided by service providers in the use of industry for drilling motors in which technical details could be found [77].

Figure IV-4 gives an example motor performance chart to determine the motor speed. The given chart indicates a 900 gpm of flow rate corresponds to a motor speed of about 160 rpm. When the additional motor speed corrections are performed they are referenced to the respective flow rate & motor speed and were accordingly interpolated.

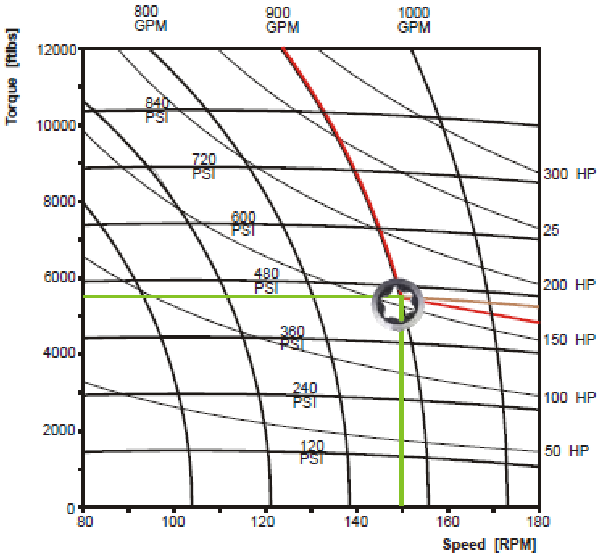


Figure IV-4: Motor performance chart [77].

4.3.6. Tooth Wear Function

The function for the tooth wear is defined by coefficient "a_7". The tooth wear function is calculated by means of determining the fractional tooth height, the more the tooth wear the less the penetration rate, equation (4.10). In order to calculate the respective tooth height, a bit record for similar bit type that has been used within the same formation is necessary. In case such information is not available a bit record belonging to a well with the most similar formation and drilling characteristics is recommended to be used in order to make the model usable.

$$f_7 = e^{a_7 X_7} = e^{a_7(-h)} \tag{4.10}$$

Appendix C gives the formation abrasiveness constant calculation methodology.

4.3.7. Hydraulic Function

The function for the hydraulic effect is defined by coefficient "a_8". The hydraulic function represents the effects of the bit hydraulics. Reynolds number function has been selected, as given in equation (4.11).

$$f_8 = e^{a_8 X_8} = e^{a_8 \frac{\rho q}{350 \mu d_n}} \tag{4.11}$$

One of the important problems in the real world is optimization [78]. Optimization is the process of choosing the best variable from a group of data set [79]. Simply, it means solving problems in which minimizing or maximizing a function systematically is necessary. Differentiation plays a key role in solving such real world problems.

The optimization approach introduces drilling parameters selection which considers reducing the drilling costs as a function of WOB and RPM.

CHAPTER 5

OPTIMIZATION OF DRILLING PARAMETERS

Whenever the cost of drilling activity is reduced, the whole process is considered to have been optimized. Optimization could be performed by means of adjusting the magnitude of two or more independent parameters. This could be achieved mainly by means of:

- Minimized cost per foot,
- Minimizing problems.

The cost of a drilling process could be minimized by means of working with optimized combination of controllable drilling parameters. Hole problems those of which are being generated due to inefficient parameter usage generally occurring at the rig sites could be avoided.

Drilling optimization is conducted considering that the rig equipment, bottom hole assembly (BHA), and the bit to be used are already the optimum selections in the scope of this real-time optimization study. In order to achieve the objective of minimum cost drilling the bit should be prevented from damages when run into the hole.

Mathematical model for the penetration rate could be written as a function of drilling parameters such as WOB/d_b, RPM, as given in equation (5.1). Also the bit tooth wear has been considered in the same equation for the optimization purposes.

Important controllable drilling parameters: WOB and RPM, which could be manipulated easily and promptly.

$$\frac{dF}{dt} = f(\frac{W}{d_b}, N, h) \tag{5.1}$$

The optimized *WOB* and *RPM* should lie within the operation window of the two independent drilling parameters, respectively as given in equations (5.2) and (5.3). Hole cleaning should always be adequate, in inappropriate hole cleaning conditions regrinding process is going to be observed due to the re-drilling of the unremoved cuttings.

$$WOB_{\min} \le WOB_{Opt} \le WOB_{\max} \tag{5.2}$$

$$N_{\min} \le N_{Opt} \le N_{\max} \tag{5.3}$$

Drilling cost per foot equation is as defined in equation (5.4). It has been defined to be a function of daily rig rate, bit cost, and timing required in the course of the bit runs. This equation is known to be the mostly applied drilling cost formula in the literature.

$$C_f = \frac{C_b + C_r\left(t_t + t_c + t_b\right)}{\Delta F} \tag{5.4}$$

where, C_f is the cost per drilled interval, C_r is the daily rig rate, C_b is the bit cost, t_t round trip time, t_c connection time, hr, and t_b bit drilling time. ΔF is the footage drilled with the bit in the use, ft.

Bit drilling time, t_b, (with respective to tooth wear), equation (C.3), and drilling interval, ΔF, (with respect to general drilling functions and tooth wear), equation

(C.12) when inserted into equation (4.4), after modifying the same, the cost per foot equation could be redefined.

$$C_f = \frac{C_r}{\left(J_1 e^{(-a_7 h)}\right) J_2 \tau_H \left(1 + H_2 h\right) dh} \left(\frac{C_b}{C_r} + t_t + t_c + J_2 \tau_H \int_0^{h_f} (1 + H_2 h) dh \right) \tag{5.5}$$

Calculus states that any differentiable equation when differentiated to the first order it would have been maximized. This statement could be written as below in order to optimize the drilling cost with respect to WOB.

$$f'\left(C_f\right) = \frac{\partial\left(C_f\right)}{\partial\left(W / d_b\right)} = 0 \tag{5.6}$$

The second derivative of the equation (5.6) would define whether the response of the function is a relative minimum or a maximum [80], such as,

$f''\left(C_f\right) > 0$, then $f\left(C_f\right)$ is a relative minimum (5.7)

$f''\left(C_f\right) < 0$, then $f\left(C_f\right)$ is a relative maximum (5.8).

When equation (5.5) is re-arranged, one gets the following;

$$C_f = \frac{C_r}{e^{(-a_7 h)}\left(1 + H_2 h\right) dh} \left(\frac{\frac{C_b}{C_r} + t_t + t_c}{J_1 J_2 \tau_H} + \frac{\int_0^{h_f} (1 + H_2 h) dh}{J_1} \right) \tag{5.9}$$

The detailed drilling cost equation could be written in a form including the equivalent forms of composite drilling, J_1, and tooth wear, J_2 parameters respectively from equations (D.8) and (D.2).

$$C_f = \frac{C_r}{e^{(-a_7 h)}(1 + H_2 h)dh} \left(\frac{\frac{C_b}{C_r} + t_t + t_c}{XY} + \frac{\int_0^{h_f} (1 + H_2 h)dh}{Z} \right) \tag{5.10}$$

Where;

$$X = \left(e^{a_1} e^{a_2 X_2} e^{a_3 X_3} e^{a_4 X_4} e^{a_5 Ln\left(\frac{A}{4 - \left(\frac{W}{d_b} \right)_t} \right)} e^{a_6 X_6} e^{a_8 X_8} \right)$$

$$A = \frac{W}{d_b} - \left(\frac{W}{d_b} \right)_t$$

$$Y = B\left(\frac{60}{N} \right)^{H1} \left(\frac{1}{1 + \frac{H_2}{2}} \right) \tau_H$$

$$B = \left[\frac{\left(\frac{W}{d_b} \right)_m - \left(\frac{W}{d_b} \right)}{\left(\frac{W}{d_b} \right)_m - 4} \right]$$

$$Z = \left(e^{a_1} e^{a_2 X_2} e^{a_3 X_3} e^{a_4 X_4} e^{a_5 Ln\left(\frac{\frac{W}{d_b} - \left(\frac{W}{d_b} \right)_t}{4 - \left(\frac{W}{d_b} \right)_t} \right)} e^{a_6 X_6} e^{a_8 X_8} \right)$$

Differentiating equation (5.10) to the first order and equalizing to zero with respect to WOB independent parameter would result in drilling cost optimization for WOB parameter.

$$f'\left(C_f\right) = \frac{\partial\left(C_f\right)}{\partial\left(W/d_b\right)} =$$

$$\frac{C_r}{e^{(-a_7 h)}Udh}\left[-\frac{\mathrm{N}}{\zeta}\frac{1}{\psi}\frac{1}{[B]}\frac{a_5}{(A)}+\frac{\mathrm{N}}{\zeta}\frac{1}{\psi}\frac{1}{\left(\left(\frac{W}{d_b}\right)_m-\left(\frac{W}{d_b}\right)\right)^2}\left(4-\left(\frac{W}{d_b}\right)_t\right)-\frac{\int_0^{h_f}Udh}{\theta}\frac{1}{\psi}\frac{a_5}{(A)}\right] \tag{5.11}$$

Where;

$$U = \left(1 + H_2 h\right)$$

$$\mathrm{N} = \left[\frac{C_b}{C_r} + t_t + t_c\right]$$

$$\zeta = \left(e^{a_1} e^{a_2 X_2} e^{a_3 X_3} e^{a_4 X_4} e^{a_6 X_6} e^{a_8 X_8}\right)\left(\left(\frac{60}{N}\right)^{H1}\right)\left(\left(\frac{1}{1+\frac{H_2}{2}}\right)\tau_H\right)$$

$$\psi = e^{a_5 Ln\left(\frac{\frac{W}{d_b}-\left(\frac{W}{d_b}\right)_t}{4-\left(\frac{W}{d_b}\right)_t}\right)}$$

$$\theta = \left(e^{a_1} e^{a_2 X_2} e^{a_3 X_3} e^{a_4 X_4} e^{a_6 X_6} e^{a_8 X_8}\right)$$

Rearranging equation (5.11) simplifies evaluation.

$$\frac{\partial\left(C_f\right)}{\partial\left(W/d_b\right)}=\left[\begin{array}{l}-\dfrac{\mathrm{N}}{\left(\left(\dfrac{60}{N}\right)^{H1}\right)\left(\left(\dfrac{1}{1+\dfrac{H_2}{2}}\right)\tau_H\right)}\dfrac{1}{\left[J_2\right]}\dfrac{a_5}{\left(\left(\dfrac{W}{d_b}\right)-\left(\dfrac{W}{d_b}\right)_t\right)}\\ +\dfrac{\mathrm{N}}{\left(\left(\dfrac{60}{N}\right)^{H1}\right)\left(\left(\dfrac{1}{1+\dfrac{H_2}{2}}\right)\tau_H\right)}\dfrac{1}{\left(\left(\dfrac{W}{d_b}\right)_m-\left(\dfrac{W}{d_b}\right)\right)^2}\left(4-\left(\dfrac{W}{d_b}\right)_t\right)\\ -\displaystyle\int_0^{h_f}Udh\dfrac{a_5}{\left(\left(\dfrac{W}{d_b}\right)-\left(\dfrac{W}{d_b}\right)_t\right)}\end{array}\right]=0 \tag{5.12}$$

Using the distribute law of mathematics, equation above can be re-written,

$$\frac{\partial\left(C_f\right)}{\partial\left(W/d_b\right)}=\left\{\frac{\left[\dfrac{C_b}{C_r}+t_t+t_c\right]}{\left(\left(\dfrac{60}{N}\right)^{H1}\right)\left(\left(\dfrac{1}{1+\dfrac{H_2}{2}}\right)\tau_H\right)}\left[a_5-\frac{\left(\left(\dfrac{W}{d_b}\right)-\left(\dfrac{W}{d_b}\right)_t\right)\left(4-\left(\dfrac{W}{d_b}\right)_t\right)}{\left(\left(\dfrac{W}{d_b}\right)_m-\left(\dfrac{W}{d_b}\right)\right)^2}\right]+a_5J_2\tau_H\int_0^{h_f}(1+H_2h)dh=0\right\} \tag{5.13}$$

Simplifying equation above results in having the relation below,

$$\left[\frac{C_b}{C_r}+t_t+t_c\right]\left[a_5-\frac{\left(\left(\dfrac{W}{d_b}\right)-\left(\dfrac{W}{d_b}\right)_t\right)}{\left(\left(\dfrac{W}{d_b}\right)_m-\left(\dfrac{W}{d_b}\right)\right)}\right]+a_5J_2\tau_H\int_0^{h_f}(1+H_2h)dh=0 \tag{5.14}$$

Equation (5.9) should also be differentiated as a function of rotary speed, N.

$$f'\left(C_f\right)=\frac{\partial\left(C_f\right)}{\partial\left(N\right)}=0 \tag{5.15}$$

The solution of the respective derivative for equation (5.15) is as give,

$$\left[\frac{C_b}{C_r}+t_t+t_c\right]\left(1-\frac{H_1}{a_6}\right)+J_2\tau_H\int_0^{h_f}(1+H_2h)dh=0 \tag{5.16}$$

The optimum equation for the weight for each diameter of bit size is as given below, equation (5.17).

$$\left[\frac{W}{d_b}\right]_{Opt}=\frac{a_5H_1\left(\frac{W}{d_b}\right)_{\max}+a_6\left(\frac{W}{d_b}\right)_t}{a_5H_1+a_6} \tag{5.17}$$

In a similar manner the optimum bit speed can be expressed in the following form (5.18), after being obtained using the equation (5.9).

$$[N]_{Opt}=60\left[\frac{\tau_H}{t_b}\frac{\left(\frac{W}{d_b}\right)_{\max}-\left(\frac{W}{d_b}\right)_{Opt}}{\left(\frac{W}{d_b}\right)_{\max}-4}\right] \tag{5.18}$$

CHAPTER 6

DATA PROCESS

Data processing is composed of bringing the data into a processable condition. Then the multiple regression technique is utilized on how to find the coefficients to estimate the drilling rate of penetration. This chapter is composed of the description of the data, the multiple regression technique and finally representation of the results. Also included is the data piping from the source. The results of the finding of the model used over the processed data are evaluated in this chapter. It has been observed that drilling rate of penetration is calculated with reasonable accuracy.

No data filtering process has been performed at any stage of the project. The motive behind this was to know what to expect about what sort of outcomes are going to be achieved in case the process was going to be applied to actual field cases.

6.1. Data Description

Drilling data available for the study was acquired from a field located in the Mediterranean Sea. The subsurface geology of the offshore environment is showing similarities throughout region. Nile Delta is formed of different formation classifications. The lithological specification of the formations is mainly dominated by means of shale and sandstone. **Table VI-1** gives the detailed table for the formations Lithology description [81]. The composition of the surface formations (Mit Ghamr and Wastani formations) is mainly sands with some shale intercalation in Wastani formation. Shale is the main component of the underlying formations (Kafr El Sheikh) with some sandstone intercalations [81].

Table VI-1: Formations lithology descriptions [81].

AGE	FORMATION	DESCRIPTION
Pleistocene	Mit Ghamr	Sands & Clay
	Wastani	Sand, Sand Stone & Shale
Pliocene	Kafr El-Sheikh	Shale & Sand Stone
Middle & Late Miocene	Rosetta	Salt & Anhydrite
	Wakar	Shale & Sand Stone
	Sidi Salem	Sand Stone & Shale
Early Miocene	Qantara	Marl, Limestone & Shale
Oligocene	Tineh	Marl & Limestone

The generalized stratigraphy of Egypt’s Nile Delta is is as given in **Figure VI-1** [82]. The main reservoir is present at shallow and deep depths. Shallow zones, less than 2200 m (7200 ft) belong to Kafr El Sheikh Formation of Pliocene age which is characterized by high porosity and permeability.

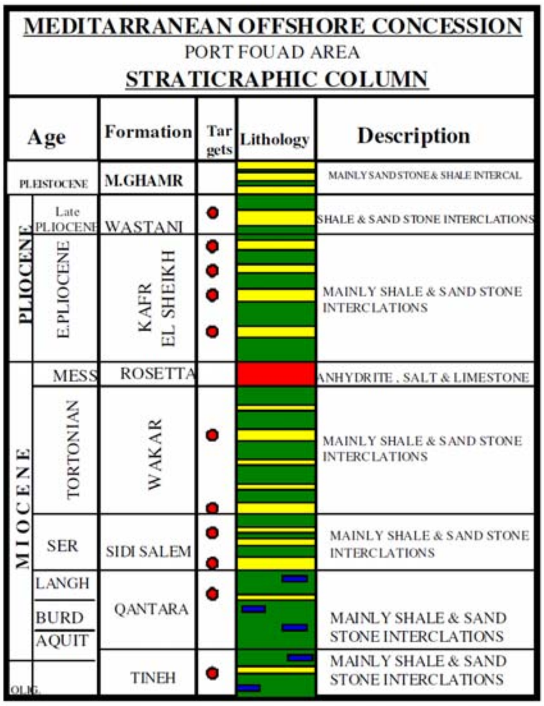


Figure VI-1: Stratigraphy of Port Fouad Marine Field [82].

The pore pressure fracture gradient of the Mediterranean Nile Delta Region is given in **Figure VI-2** [83]. Abnormal and variable pore pressures and resultant hole instability problems such as swelling shales, cavings, tight holes and eventual loss of circulations have been encountered in the Nile Delta wells [84]. More specifically, the overpressured zones originate through Kafr El Sheikh Formation primarily because of overburden and tectonic effects. The pressure gradually increases with depth and shale content.

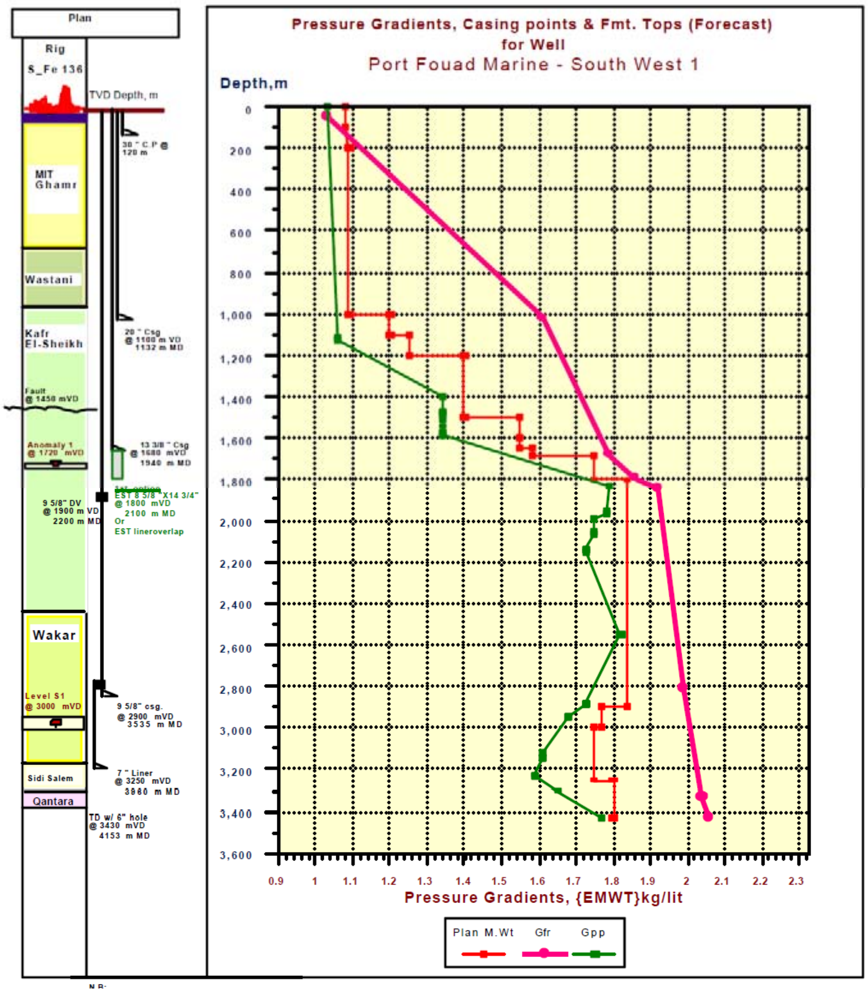


Figure VI-2: PPFG chart for Nile Delta Region [83].

Figure VI-3 gives the casing and formation top details of the wells used in this study. The well total depths are at about 6000 ft. The conductor pipe of the wells has been installed to a depth of about 100 m below the mud line. Surface casing of the wells have been installed to a depth after Formation-1. The production casing has been installed to a depth to allow a cased hole production from the reservoir. It can be observed that the second and the third well have similarity in terms of their lithological specifications.

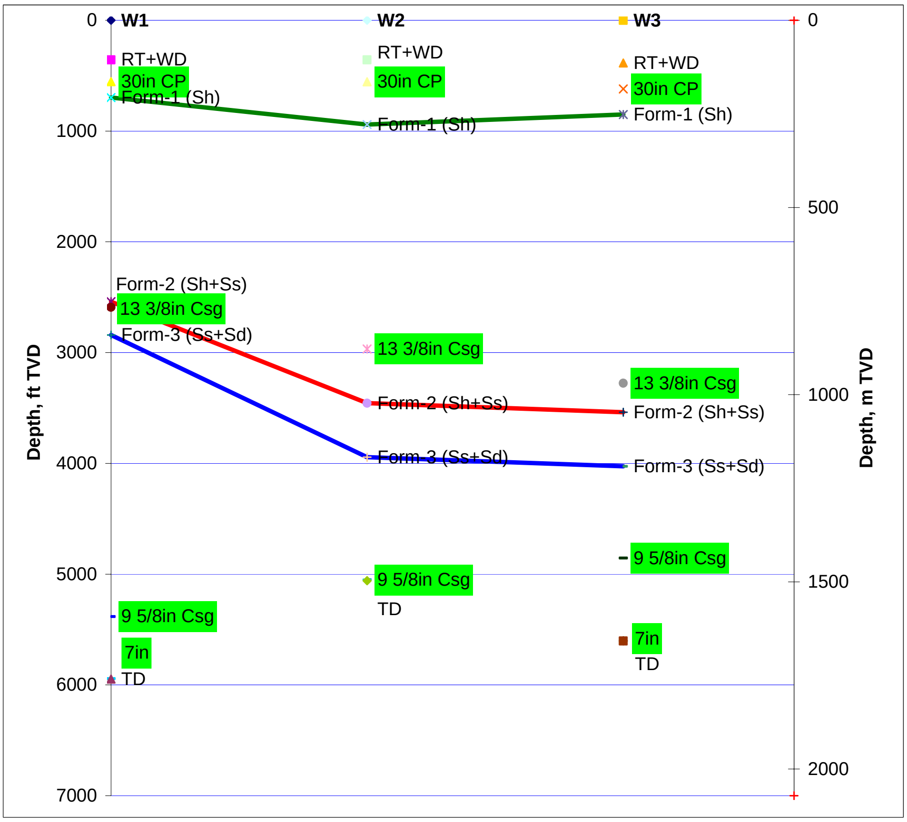


Figure VI-3: Lithological column/Casing points of the wells.

6.2. Real-Time Data Piping

Data piping is a very important step in the optimization process. The data following the acquisition should be piped as efficiently as possible to the central computer in which the optimization process is going to take place. The optimization cycle should work as in the following sequence.

- Data should be piped in real-time environment to a central computer right from the rig site,
- Central computer together with the information already existing in its database should process the data, and determine the optimized drilling parameters,
- The optimized drilling parameters should be piped back again to the drilling location in order to be applied.

The optimization process flow applied in this study was composed of using drilling parameters data collected at the mud logging units. **Figure VI-4** gives the flow chart of the process flow applied. The data was prepared accordingly in order to provide suitable means for the multiple regression application based on the defined general rate of penetration equation. Pore pressure, wellbore inclination, equivalent circulating density, mud rheological properties, and additional rotation of the bit due to the motor for each data point was calculated and included in the database. Once the required dataset was stored, multiple regression was run, and regression coefficients were determined. The determined coefficients are used in calculating the predicted rate of penetration and determine the optimum parameters.

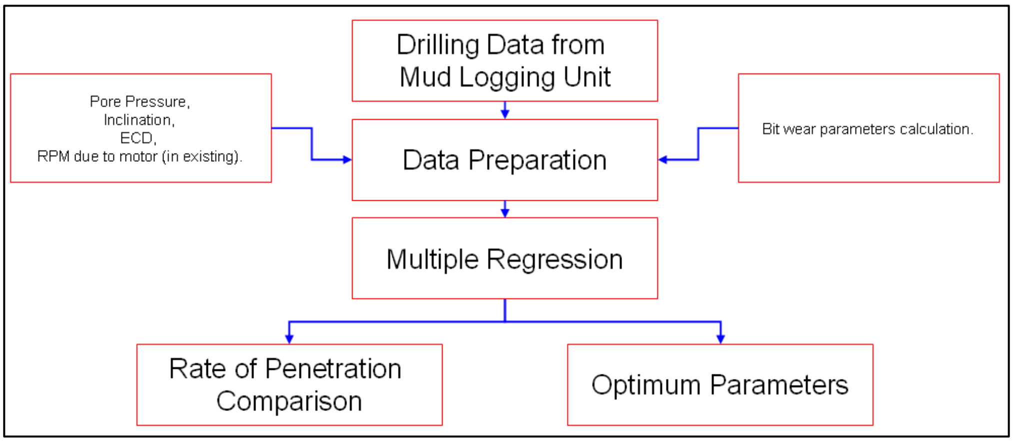


Figure VI-4: Process flow chart.

For the purpose of real-time data piping a macro code in excel has been written using “Visual Basic” language, **Figure VI-5**. The written code facilitates the handling of the data, and the data could easily be created, and then be placed on an internet server.

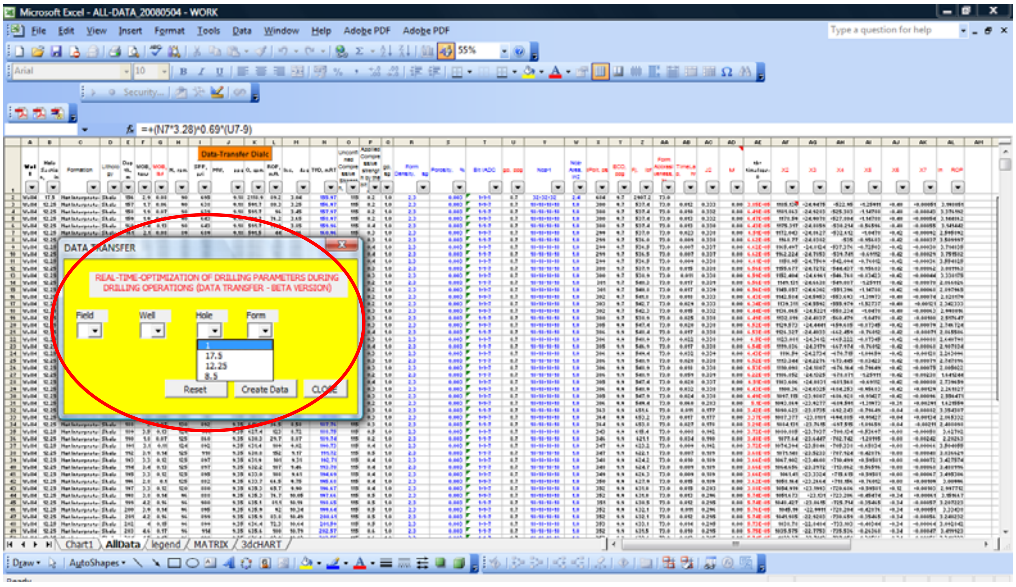


Figure VI-5: Data piping excel worksheet.

External data could be captured into the excel environment once it has been located anywhere over World Wide Web. The feature of excel which calls the data automatically is "Import Excel Data – New Web Query" as shown in **Figure VI-6**.

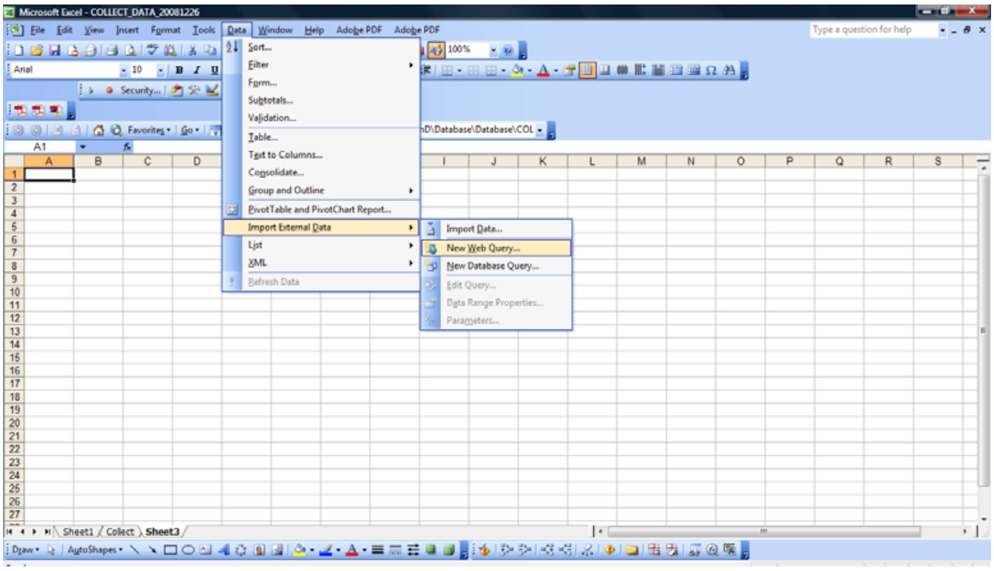


Figure VI-6: Data collection using Excel's "Import External Data" feature.

6.3. Data Process – Multiple Regression Technique

Multiple Regression solution is critically important for the success of drilling optimization. Because there are eight main equations, each time the data is regressed, eight unknown coefficients are solved. The process is used for constructing the drilling rate of penetration formula, predicting the ROP as a function of other independent variables.

A regression model that contains more than one regressor variable is called a multiple regression model [85]. Multivariate data analysis is characterization of an observation unit by several variables [87]. Multivariate analysis methods get affected for the changes in the magnitude of the several properties simultaneously. Multiple regression considers all possible interactions within combinations of variables as well as the variables themselves.

The drilling rate of penetration model in order to estimate the effect of drilling parameters on rate of penetration is given by equation (6.1). The given equation states that the value of a dependent variable, *dD/dt*, is exponentially equal to a constant term plus the sum of a series of independent variables and plus a random error (also called as residual error). The multiple regression solves the set of equations to find the "a" regression constants. Another definition would be, the given number of equations with their respective unknowns can be used to calculate $a_1,.....,a_n$.

$$\frac{dD}{dt} = \exp\left(a_1 + \sum_{j=2}^{8} a_j x_j\right) \tag{6.1}$$

The first step in determination of the coefficients is modifying equation (6.1). If the natural logarithm of both sides is taken, the following system yields;

$$\ln\frac{dD}{dt} = a_1 + \sum_{j=2}^{8} a_j x_j \tag{6.2}$$

For each data point of the equation (6.2), regression coefficients give a rate of change (or slope) in the magnitude of the dependent variable. This change in magnitude could be termed as the residual error, given in equation (6.3).

$$r_i = a_1 + \sum_{j=2}^{8} a_j x_j - \ln \frac{dD}{dt} \tag{6.3}$$

For each data point the residual error should be a minimum. The sum of the square of the residuals is a minimum, (6.4).

$$\sum_{i=}^{8} \quad r_i^{\ 2} \qquad \textit{is a minimum} \tag{6.4}$$

The relation in equation (6.5) should be satisfied in order to have the residuals sum a minimum.

$$\frac{\partial\left(\sum_{i=1}^{8} r_i^2\right)}{\partial\left(a_j\right)} = \sum_{i=1}^{n} 2r_i \frac{\partial r_i}{\partial a_j} = \sum_{i=1}^{n} 2r_i x_j = 0 \tag{6.5}$$

This can be achieved by selecting the regression coefficients accordingly. This can be achieved by applying a least-squares solution to a linear equation by solving a set of normal equations for the "a" coefficients. In order to solve the general equation given in (6.2), a least squares solution is applied by appropriately constructing a matrix. The matrix form is as given in equation (6.6).

$$S_{xy} = S_{xx} b \tag{6.6}$$

where, S_{xx} is the column matrix of the sums of cross products of dependent variable with (drilling parameters) $x_1, x_1^2, x_1^3, \ldots, x_1^m$; S_{xy} is a matrix of sums of squares and cross products of the drilling parameters, "*a*" coefficient estimates and the column matrix of unknown regression coefficients. The solution of the matrix is represented in equation (6.7).

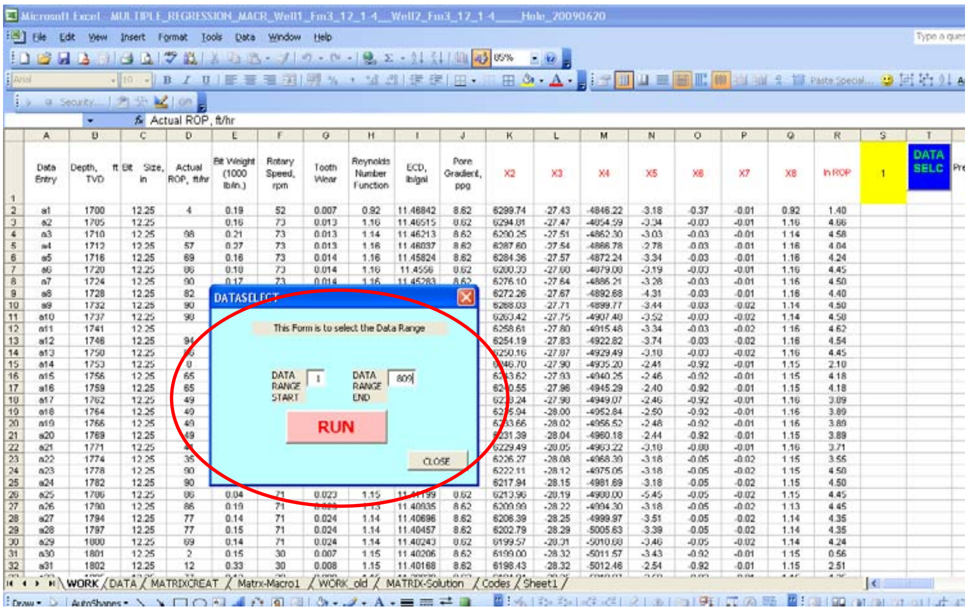
$$a = S^{-1}{}_{xx} S_{xy} \tag{6.7}$$

Matrix equation (6.8) gives the matrix relation for the determination of the "*a*" coefficients.

$$\begin{bmatrix} n & \sum_{i=1}^{n} x_{i1} & \sum_{i=1}^{n} x_{i2} & . & . & \sum_{i=1}^{n} x_{ik} \\ \sum_{i=1}^{n} x_{i1} & \sum_{i=1}^{n} x_{i1}^{2} & \sum_{i=1}^{n} x_{i1}x_{i2} & \sum_{i=1}^{n} x_{i1}x_{i3} & . & \sum_{i=1}^{n} x_{i1}x_{ik} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \sum_{i=1}^{n} x_{ik} & \sum_{i=1}^{n} x_{ik}x_{i1} & \sum_{i=1}^{n} x_{ik}x_{i2} & \sum_{i=1}^{n} x_{ik}x_{i3} & . & \sum_{i=1}^{n} x_{ik}^{2} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ . \\ . \\ a_k \end{bmatrix} = . \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_{i1}y_i \\ \sum_{i=1}^{n} x_{i2}y_i \\ . \\ . \\ \sum_{i=1}^{n} x_{ik}y_i \end{bmatrix} \tag{6.8}$$

The given matrix is solved for the only unknown "*a*" coefficients, which are used to be inserted in the drilling rate of penetration general equation.

Microsoft Excel has been used to process that data. Visual Basic statements are used for the Microsoft Excel worksheet functions. Appendix D gives the written code for the multiple regression process. This way the program has been utilized to solve the constants a_1, a_2... a_8 of the general equation (4.2) in a fast and error free approach. **Figure VI-7** gives the interface for the automated process. The user can select the start and the end of the range over which multiple regression is required to be performed.



Actual ROP, ft/hr

This Form is to select the Data Range

DATA RANGE START: 1

DATA RANGE END: 809

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1	Data Entry	Depth, ft TVD	Bit Size, in	Actual ROP, ft/hr	Bit Weight (1000 lb/in.)	Rotary Speed, rpm	Tooth Wear	Reynolds Number Function	ECD, lb/gal	Pore Gradient, ppg	X2	X3	X4	X5	X6	X7	X8	ln ROP	1
2	a1	1700	12.25	4	0.19	52	0.007	0.92	11.46842	8.62	6299.74	-27.43	-4846.22	-3.18	-0.37	-0.01	0.92	1.40	
3	a2	1705	12.25		0.16	73	0.013	1.16	11.46515	8.62	6294.81	-27.47	-4854.59	-3.34	-0.03	-0.01	1.16	4.66	
4	a3	1710	12.25	98	0.21	73	0.013	1.14	11.46213	8.62	6290.25	-27.51	-4862.30	-3.03	-0.03	-0.01	1.14	4.58	
5	a4	1712	12.25	57	0.27	73	0.013	1.16	11.46037	8.62	6287.60	-27.54	-4866.78	-2.78	-0.03	-0.01	1.16	4.04	
6	a5	1716	12.25	69	0.16	73	0.014	1.16	11.45824	8.62	6284.36	-27.57	-4872.24	-3.34	-0.03	-0.01	1.16	4.24	
7	a6	1720	12.25	86	0.18	73	0.014	1.16	11.4556	8.62	6280.33	-27.60	-4879.08	-3.19	-0.03	-0.01	1.16	4.45	
8	a7	1724	12.25	90	0.17	73	0.014	1.16	11.45283	8.62	6276.10	-27.64	-4886.21	-3.28	-0.03	-0.01	1.16	4.50	
9	a8	1728	12.25	82							6272.26	27.67	-4892.68	-4.31	-0.03	0.01	1.16	4.40	
10	a9	1732	12.25	90							6268.03	-27.71	-4899.77	-3.44	-0.03	-0.02	1.14	4.50	
11	a10	1737	12.25	98							6263.42	-27.75	-4907.40	-3.52	-0.03	-0.02	1.14	4.58	
12	a11	1741	12.25								6258.61	-27.80	-4915.48	-3.34	-0.03	-0.02	1.16	4.62	
13	a12	1746	12.25	94							6254.19	27.83	-4922.82	-3.74	-0.03	-0.02	1.16	4.54	
14	a13	1750	12.25	[illegible]							6250.16	-27.87	-4929.49	-3.10	-0.03	-0.02	1.16	4.45	
15	a14	1753	12.25	8							[illegible]	-27.90	-4935.20	-2.41	-0.92	-0.01	1.15	2.10	
16	a15	1756	12.25	65							[illegible]	-27.93	-4940.25	-2.46	-0.92	-0.01	1.15	4.18	
17	a16	1759	12.25	65							[illegible]	27.96	-4945.29	2.40	-0.92	0.01	1.15	4.18	
18	a17	1762	12.25	49							[illegible]	-27.98	-4949.07	-2.46	-0.92	-0.01	1.16	3.89	
19	a18	1764	12.25	49							[illegible]	-28.00	-4952.84	-2.50	-0.92	-0.01	1.16	3.89	
20	a19	1766	12.25	49							[illegible]	-28.02	-4956.52	-2.48	-0.92	-0.01	1.16	3.89	
21	a20	1769	12.25	49							6231.39	28.04	-4960.18	2.44	-0.92	0.01	1.15	3.89	
22	a21	1771	12.25	[illegible]							6229.49	-28.05	-4963.22	-3.18	-0.88	-0.01	1.16	3.71	
23	a22	1774	12.25	35							6226.27	-28.08	-4968.30	-3.18	-0.05	-0.02	1.15	3.55	
24	a23	1778	12.25	90							6222.11	-28.12	-4975.05	-3.18	-0.05	-0.02	1.15	4.50	
25	a24	1782	12.25	90							6217.94	28.15	-4981.69	3.18	-0.05	0.02	1.15	4.50	
26	a25	1786	12.25	86	0.04	71	0.023	1.15	[illegible]	8.62	6213.96	-28.19	-4988.00	-5.45	-0.05	-0.02	1.15	4.45	
27	a26	1790	12.25	86	0.19	71	[illegible]	[illegible]	11.40835	8.62	6209.99	-28.22	-4994.30	-3.18	-0.05	-0.02	1.13	4.45	
28	a27	1794	12.25	77	0.14	71	0.024	1.14	11.40696	8.62	6206.39	-28.25	-4999.97	-3.51	-0.05	-0.02	1.14	4.35	
29	a28	1797	12.25	77	0.15	71	0.024	1.14	11.40457	8.62	6202.79	-28.29	-5005.63	-3.39	-0.05	-0.02	1.14	4.35	
30	a29	1800	12.25	69	0.14	71	0.024	1.14	11.40243	8.62	6199.57	-28.31	-5010.60	-3.46	-0.05	-0.02	1.14	4.24	
31	a30	1801	12.25	2	0.15	30	0.007	1.15	11.40206	8.62	6199.00	-28.32	-5011.57	-3.43	-0.92	-0.01	1.15	0.56	
32	a31	1802	12.25	12	0.33	30	0.008	1.15	11.40168	8.62	6198.43	28.32	-5012.46	-2.54	-0.92	0.01	1.15	2.51	

Figure VI-7: Multiple regression process.

Figure VI-8 gives the multiple regression process cycle. The first step in the process is to have the “X” parameters calculated for each data point. The next step is to accordingly collate the calculated “X” parameters in order to create the matrix. In the scope of this study a matrix of 8x8 is being created. Once the matrix has been calculated the same can be solved and the “a” coefficients could be determined. For each progressive inclusion of the data points the process steps are repeated until the final data point. The final data point is going to be the one that is considered to be the most accurate.

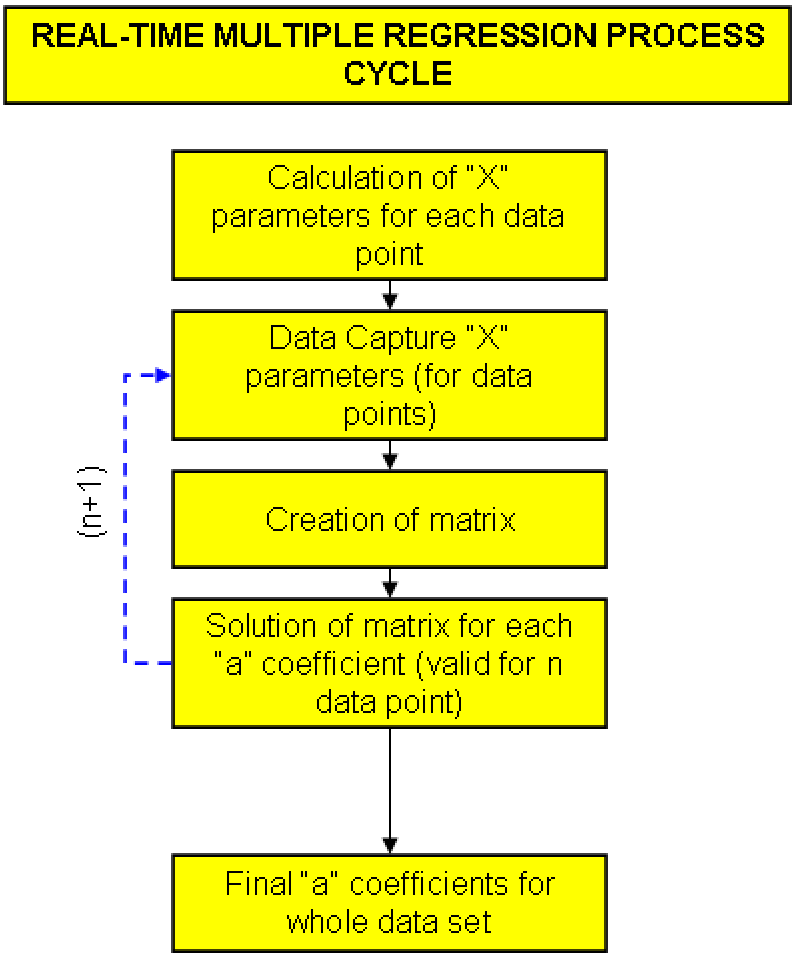


Figure VI-8: Multiple Regression Process Cycle.

It is worthwhile to mention that the multiple regression analysis procedure should produce meaningful results not only the general rate of penetration equation has been defined in an appropriate manner but also to the range of the values of the drilling parameters $X_2, \ldots, X_8$.

An example solution for multiple regression is given in Appendix E. The given example is giving the solution of the multiple regressions for each data point provided in the reference by Bourgoyne and Young [13].

CHAPTER 7

RESULTS & DISCUSSION

Results give multiple regression outputs specific to the respective formations within the subject borehole diameter. The results for different optimization runs are as summarized in the following sections. The runs were individually performed for the same formation types, and provided that the data were belonging to equally sized bit diameters.

7.1. Presentation of the Results

The results are presented in three fold from data process point of view. First; no correction to the dataset, second; correction of WOB due to inclination, third interpolation together with WOB correction and additional bit rotation due to the motor. The code prepared in visual basic for interpolations is as given in Appendix F. The WOB term for each data point was corrected considering the wellbore inclination at the depth where the dataset was acquired, the input WOB magnitude was converted into the vertical component of the applied weight on bit. The main motive of interpolation is to eliminate the effect of noisy data and reduce the number of data input. In the first and second case there are about 900 data points, however in the last case the number of data points is reduced to less than 200. An example to interpolation is given in **Figure VII-1**; the chart includes the actual rotary speed magnitudes as well as the points over which it has been asked to interpolate. The interpolation is performed by means of a macro prepared using Microsoft Excel. The main principle of interpolation is to get the magnitude in terms of range, between the nearest actual two points at which interpolation is required to be done.

There differences in the data range are the range of WOB in the second case, and additional bit rotation in the third. The WOB magnitudes are relatively higher in the second case, because the vertical component of the same has been considered as input. The additional bit rotations are higher in magnitudes as well due to the effect of the mud motor.

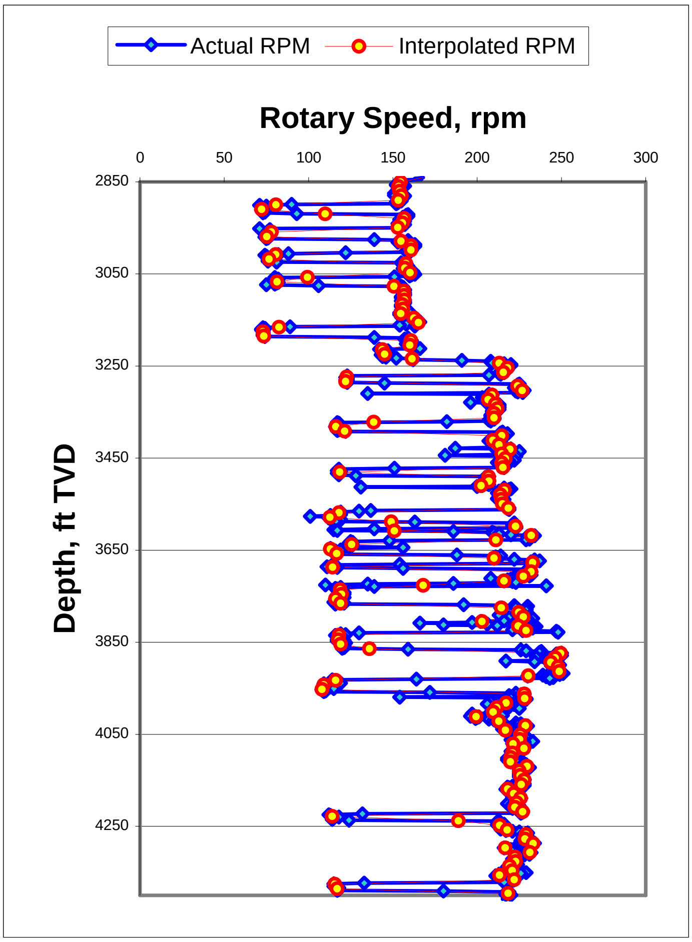
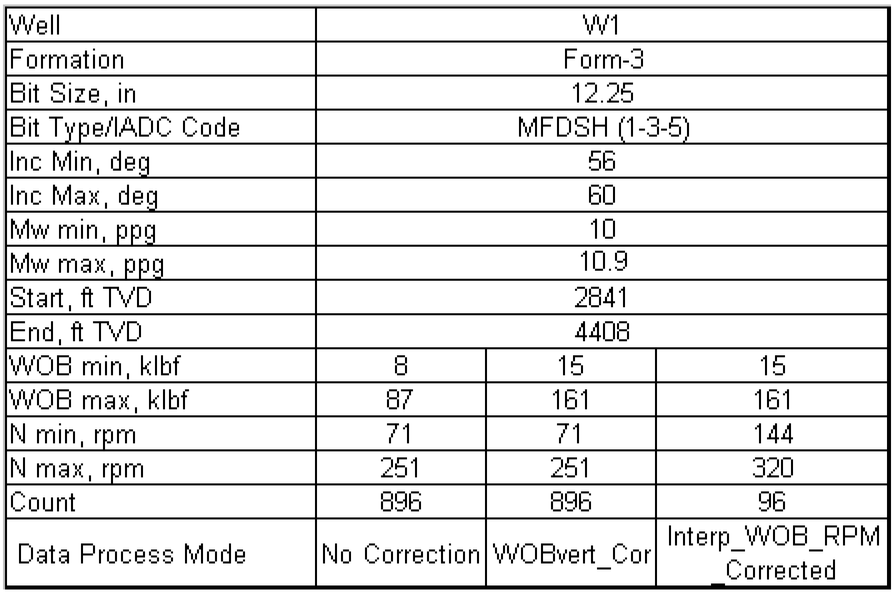


Figure VII-1: Data interpolation example chart.

The details of the performed data runs are as summarized in **Table VII-1**, such as wellbore deviation, drilling fluid density, the weight applied to the bit and the rotation speed. The given dataset are belonging to Well-1 and formation-3. The bit type in use was a milled tooth in type. The general operating parameters for weight on bit magnitudes are between 15 klbs and 55 klbs, and 50 to 250 rpm for rotary speed [86]. The bit was designed for steerable mud motor applications in very soft low compressive strength, unconsolidated formations such as clays, marls, soft shales, halites gypsums and sands.

The reported weight on bit ranges are the magnitudes recorded through mud logging system, for this reason the instrument readings are included in the database, some ranges could be extremely large due to scale input. This sort of data ranges could be attributed to noise.

Table VII-1: Performed data runs for Well-1 12 ¼" Hole Formation-3.

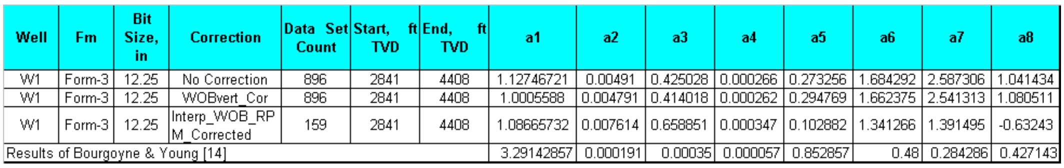


Well	W1		
Formation	Form-3		
Bit Size, in	12.25		
Bit Type/IADC Code	MFDSH (1-3-5)		
Inc Min, deg	56		
Inc Max, deg	60		
Mw min, ppg	10		
Mw max, ppg	10.9		
Start, ft TVD	2841		
End, ft TVD	4408		
WOB min, klbf	8	15	15
WOB max, klbf	87	161	161
N min, rpm	71	71	144
N max, rpm	251	251	320
Count	896	896	96
Data Process Mode	No Correction	WOBvert_Cor	Interp_WOB_RPM_Corrected

The coefficients of multiple regression for Well-1, Formation-3 are observed in the following sensitivity graphs, respectively from **Figure VII-2** to **Figure VII-9**. The charts are drawn for respective coefficient and data point. The data points are given with progressively increasing depth data.

The results of the "a" coefficients are given in **Table VII-2**. The given results are representing the total number of data points for each observation. It is observed that the coefficient magnitudes are showing similarities for the first two cases, and are observed to have different magnitudes in the third case. Also the “a” coefficient results are compared with the average of the “a” coefficients with that of Bourgoyne and Young [14] data of which has been taken in shale, offshore Louisiana area. The coefficients are expected to be different than the ones belonging to the data used for this study, since the drilling locations and the drilling techniques machinery used are different.

Table VII-2: The results of the "a" coefficients.



Well	Fm	Bit Size, in	Correction	Data Set Count	Start, ft TVD	End, ft TVD	a1	a2	a3	a4	a5	a6	a7	a8
W1	Form-3	12.25	No Correction	896	2841	4408	1.12746721	0.00491	0.425028	0.000266	0.273256	1.684292	2.587306	1.041434
W1	Form-3	12.25	WOBvert_Cor	896	2841	4408	1.0005588	0.004791	0.414018	0.000262	0.294769	1.662375	2.541313	1.080511
W1	Form-3	12.25	Interp_WOB_RPM_Corrected	159	2841	4408	1.08665732	0.007614	0.658851	0.000347	0.102882	1.341266	1.391495	-0.63243
Results of Bourgoyne & Young [14]							3.29142857	0.000191	0.00035	0.000057	0.852857	0.48	0.284286	0.427143

The results of the multiple regression analysis indicate that the coefficients are giving fluctuating trends in initial sections of the data, however with the progressive data sections the coefficients are starting to consistently converge and a follow a trend based on their response.

Figure VII-2 gives the formation strength effect “a_1” coefficient with data points. Because this coefficient is under the influence of other factors not considered, the modelling of the same is presented with varying behaviour. It is generally observed that with increasing data points the final magnitude is getting much more stabilized. The final magnitudes for this coefficient are in the range of +/- 1.072 through the outcome of the analyzed data.

The trend for coefficient of the datasets is showing similar behaviour for the case of no correction and WOB vertical correction. On the other side the dataset with interpolated data input is following a different trend than the other two.

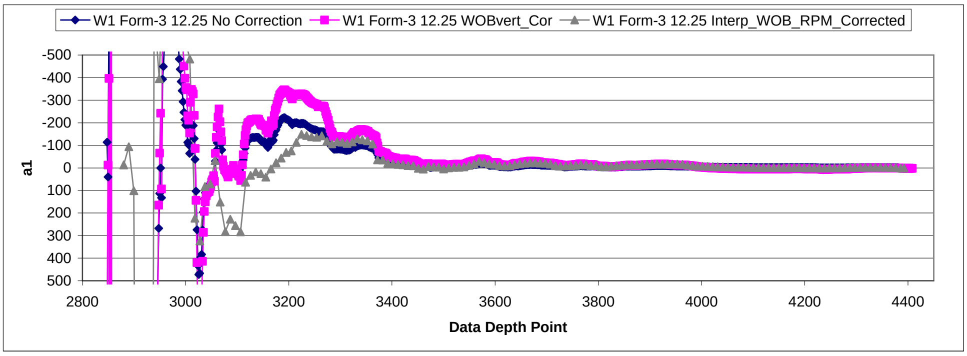


Figure VII-2: a_1 coefficient vs data points.

Figure VII-3 gives the "a_2" coefficient magnitudes which is representing the formation compaction effect. The range of the magnitudes for this coefficient is lying in between +/- 0.006. This range of magnitude is considered to have a statistical significance. A coherent data trend is observed among all the cases. This could be attributed to the fact that the compaction effect in the formation of investigation is not very dominant.

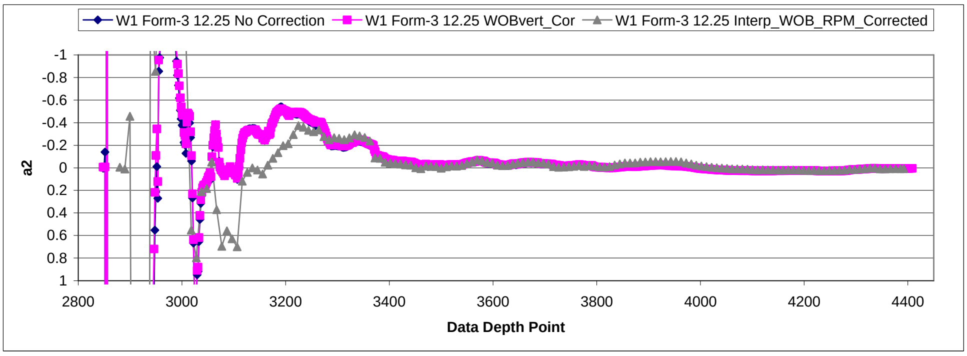


Figure VII-3: a_2 coefficient vs data points.

Figure VII-4 gives the "a_3" coefficient magnitudes which is representing the under-compaction effect in abnormally pressured formations. A coherent trend is observed among all three cases throughout the data range. The ultimate data points are in the range of +/-0.499.

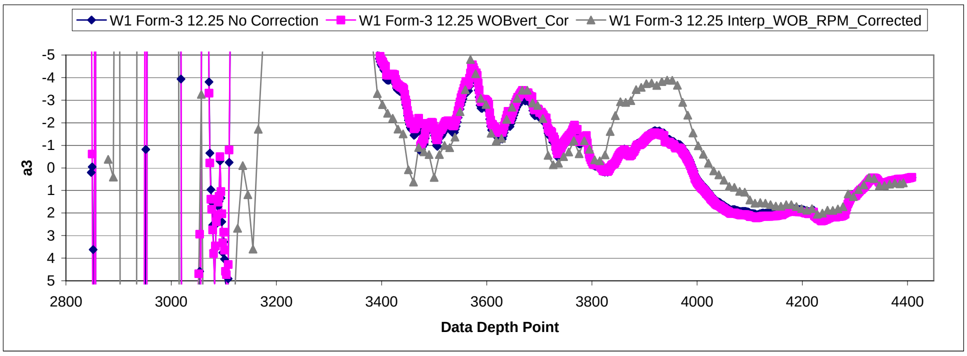


Figure VII-4: a_3 **coefficient vs data points.**

Figure VII-5 gives the "a_4" coefficient magnitudes which represents the pressure differential of the bottom hole. A coherent trend between the first two cases is observed all throughout the range; however a slightly lower trend at certain parts is seen in the third case when the data interpolation was applied. The final data magnitudes for this coefficient are at about +/-0.0003.

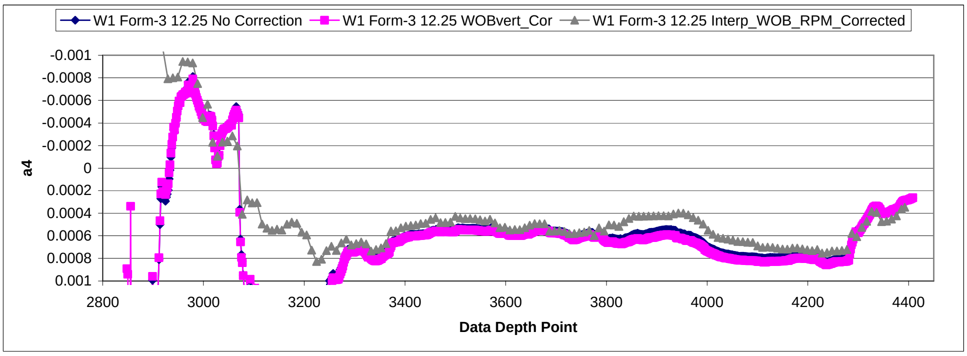


Figure VII-5: a_4 **coefficient vs data points.**

Figure VII-6 gives the "a_5" coefficient magnitudes which represents the bit diameter and applied weight on bit. The weight on bit effect is a controllable drilling parameter, and does have a direct effect on the rate of penetration. The trend is observed show fluctuating behaviour, and getting stabilized with increased data points. The interpolated case is observed to follow a slightly higher trend as compared to the other two. The final data magnitudes for this coefficient are within the range of about +/- 0.224.

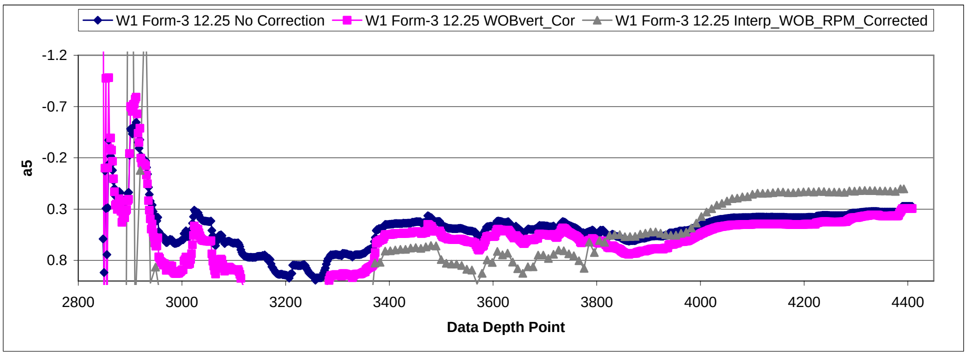


Figure VII-6: a_5 **coefficient vs data points.**

Figure VII-7 gives the "a_6" coefficient magnitudes which represents the rotary speed. This coefficient is observed to show an early stabilized trend in the first two cases. The third case is following a trend that is higher as compared to the other two. It is important to highlight that the third case is with additional rotation of the bit due to the motor. The coefficients are observed to be at a magnitude of about +/- 1.563.

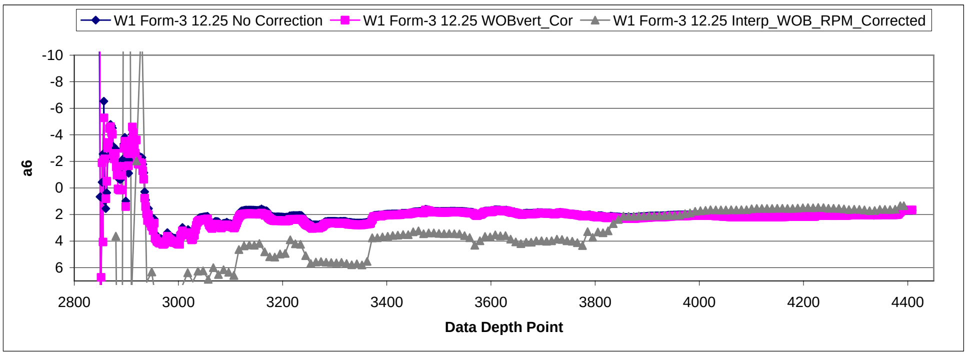


Figure VII-7: a_6 **coefficient vs data points.**

Figure VII-8 gives the "a_7" coefficient magnitudes which represents the tooth wear. The trend for all three cases is observed to show varying occurrences. The final magnitude for this coefficient is about +/- 2.173.

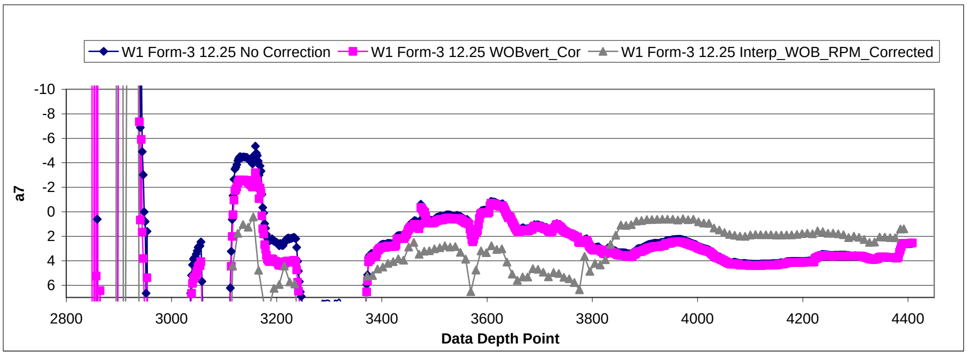


Figure VII-8: a_7 coefficient vs data points.

Figure VII-9 gives the "a_8" coefficient magnitude which represents the hydraulic function. The two cases other than interpolation are showing a similar behaviour whereas the third case is following a slightly lower trend. The ultimate magnitude for this coefficient is about+/- 0.497.

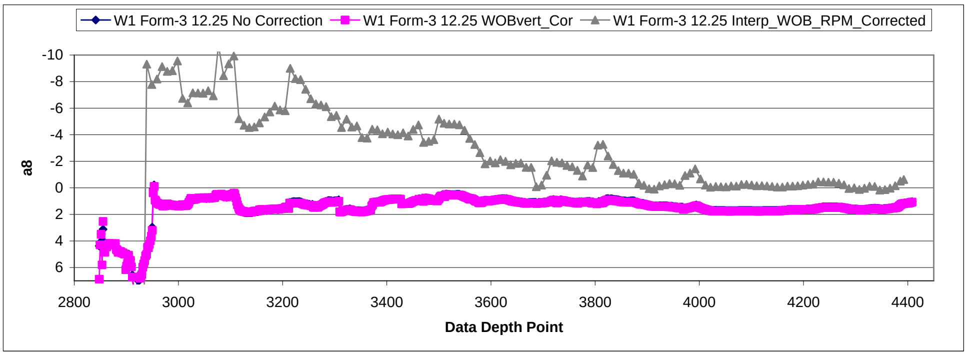


Figure VII-9: a_8 coefficient vs data points.

7.1.1 ROP Performance Result

One of the outputs for the data analysis is observed vs. predicted rate of penetration comparison chart, as given in **Figure VII-10**. The chart includes three different data sets for the same database. The available data section is composed of about 900 data points. The first set of data is the one without any correction. The second data set is the one only with weight on bit correction. There is a close similarity in the coefficient of determination magnitude (R^2) between the first and the second observations. The third data set is the interpolated form of data. The number of dataset has been reduced to about 150, by means of interpolating the data points at a regular depth interval of about every 3 meters. All parameters have been sampled accordingly to have the most representative magnitude in reference to what the actual data readings were. Also the bit rotation correction due to the mud motor has been performed in the third data group. The coefficient of determination is with a magnitude of almost two fold in reference to latter.

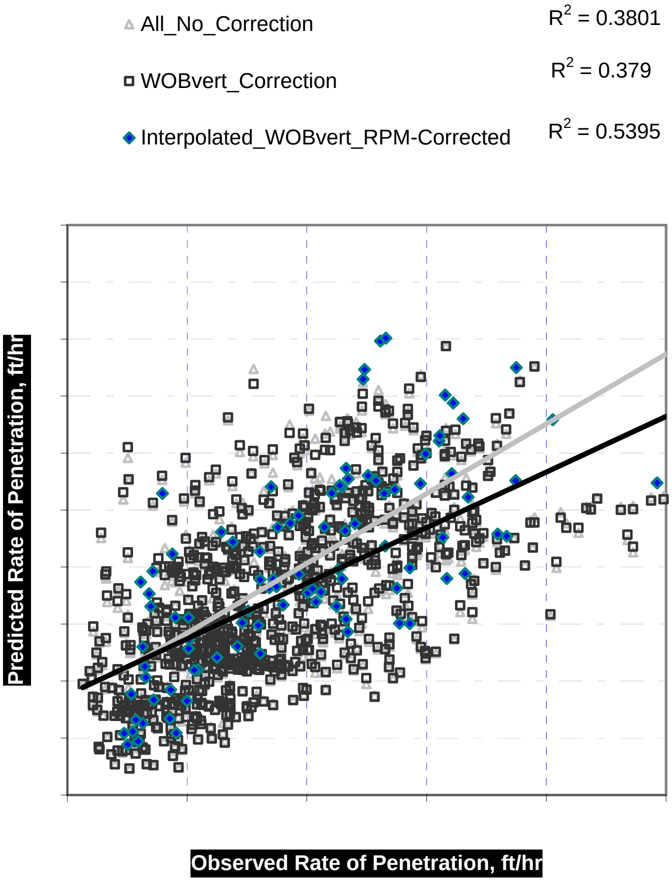


Figure VII-10: Multiple regression output, observed vs. predicted ROP for Well-1, 12 ¼" Hole, Fm3.

Figure VII-11 gives error histogram chart for three data groups belonging to Well-1, Formation-3 and 12 ¼" hole section. It is observed that the absolute error range for the first two data groups are with an average of about 46% for no correction and

WOB correction data groups. For interpreted WOB and RPM correction group the error average is observed to be less at a magnitude of about 37%.

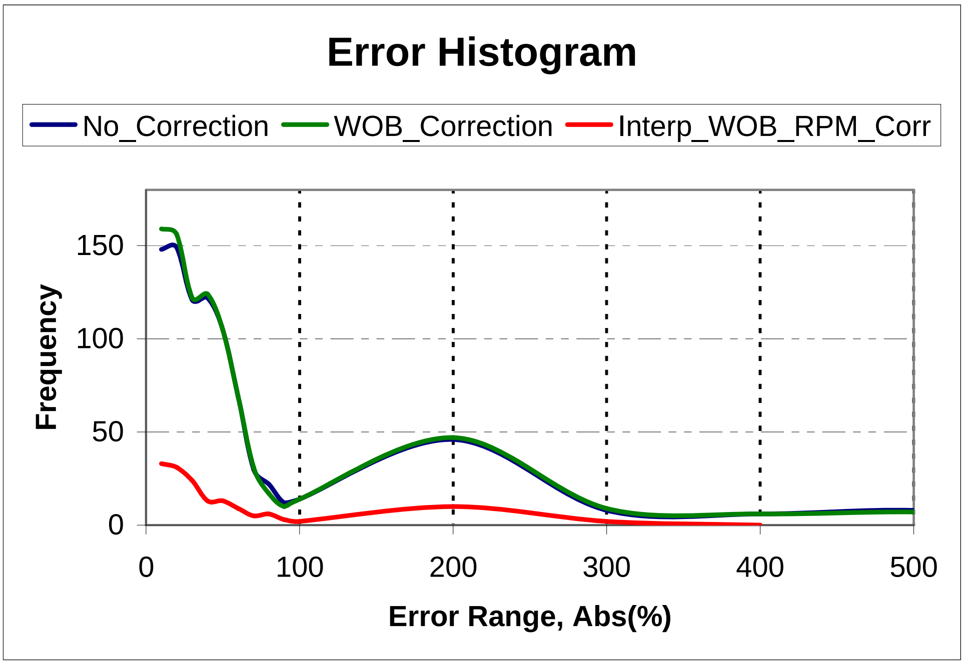
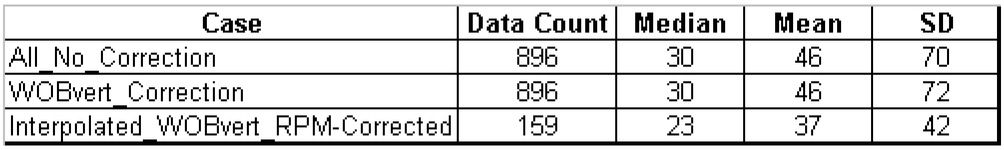


Figure VII-11: Error histogram for the date set provided for Well-1, 12 ¼" Hole, Fm3.

Table VII-3 gives the statistical information summary for the analysed data. The mean absolute error for the third data group is lower for the other two data observations. This indicates that least error margin is for the last data group. Standard Deviation (SD) for the same data group is about 42, whereas the first two data groups are with relatively large SD magnitudes.

Table VII-3: Statistical information summary for Well-1, 12 1/4" Hole and Formation-3.



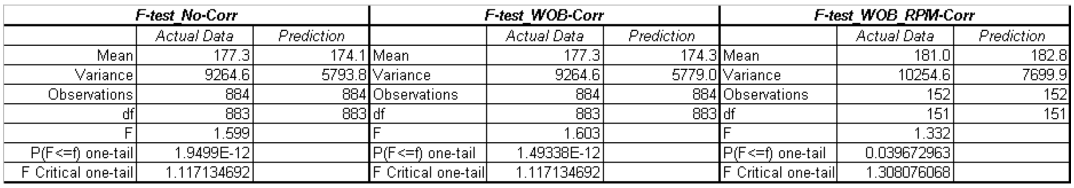
Case	Data Count	Median	Mean	SD
All_No_Correction	896	30	46	70
WOBvert_Correction	896	30	46	72
Interpolated_WOBvert_RPM-Corrected	159	23	37	42

In order to test the similarity between two sample variance *F-test* is used to compare two population variances [89]. The appropriate test of hypothesis is also termed as

Variance-ratio test. The *F-test* figures out how significantly the estimated rate of penetration is different from the actual magnitudes. **Table VII-4** gives the results of the *F-tests* conducted to compare the three cases of datasets. The results are achieved using the statistical analysis tool add-in "*F-Test* Two Sample for Variances" of Microsoft Excel [90]. The following descriptions are the guidelines to interpret the outcomes of the test:

- A value of *F* close to 1 provides evidence that the underlying population variances are equal.
- If *f*<1;
  - "P(F <= f) one-tail" gives the probability of observing a value of the F-statistic less than f when population variances are equal,
  - "F Critical one-tail" gives the critical value less than 1 for the chosen significance level, Alpha (The alpha level is "in the range 0...1" a significance level related to the probability of having a type I error (rejecting a true hypothesis)). The magnitude of Alpha used here is 0.05.
- If *f*>1;
  - "P(F <= f) one-tail" gives the probability of observing a value of the F-statistic greater than f when population variances are equal,
  - "F Critical one-tail" gives the critical value greater than 1 for Alpha.

Table VII-4: The results of the F-tests.



F-test_No-Corr			*F-test_WOB-Corr*			*F-test_WOB_RPM-Corr*		
	Actual Data	*Prediction*		*Actual Data*	*Prediction*		*Actual Data*	*Prediction*
Mean	177.3	174.1	Mean	177.3	174.3	Mean	181.0	182.8
Variance	9264.6	5793.8	Variance	9264.6	5779.0	Variance	10254.6	7699.9
Observations	884	884	Observations	884	884	Observations	152	152
df	883	883	df	883	883	df	151	151
F	1.599		F	1.603		F	1.332	
P(F<=f) one-tail	1.9499E-12		P(F<=f) one-tail	1.49338E-12		P(F<=f) one-tail	0.039672963	
F Critical one-tail	1.117134692		F Critical one-tail	1.117134692		F Critical one-tail	1.308076068	

F-test has revealed that that the magnitude in WOB_RPM correction case is the one which is close to 1.0. This means that the magnitude of errors between the actual and predicted rate of penetration is relatively small when compared to other two case.

The rate of penetration predictions are plotted in **Figure VII-12**. The scales are plotted with increasing magnitudes from top to bottom and left to right. The additional data set given in this graph in addition to **Figure VII-10** is the interpolated actual rate of penetration. The best match is observed in the data set in which the data has been interpolated, vertical weight on bit correction and additional rotation on the bit due to motor that has been taken into consideration.

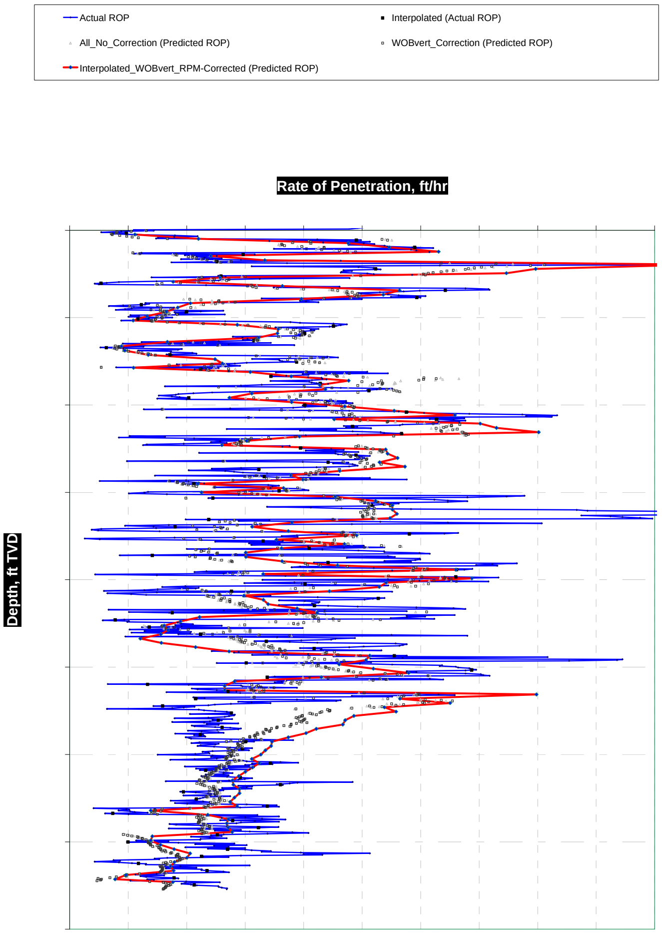


Figure VII-12: Rate of penetration comparison vs. depth for Well-1 12 1/4" hole-Fm3.

The optimized weight on bit and bit rotation are given and compared with the actual occurrences respectively in **Figure VII-13** and **Figure VII-14**.

The optimized magnitudes are calculated based on interpolated and corrected data groups and plotted in red colour in the charts. They are the recommended magnitudes for one of the independent weight on bit drilling parameter. The optimized weight on bit calculation for the given dataset is the vertical component output, the actual rate of penetration occurrences are also given in the same chart. Whenever the calculated optimum weight on bit is going to be applied the vertical component is required to be transferred into axial form using the well inclination. It is observed that generally applying a lower weight on bit throughout the section drilled would have resulted in optimization of drilling process.

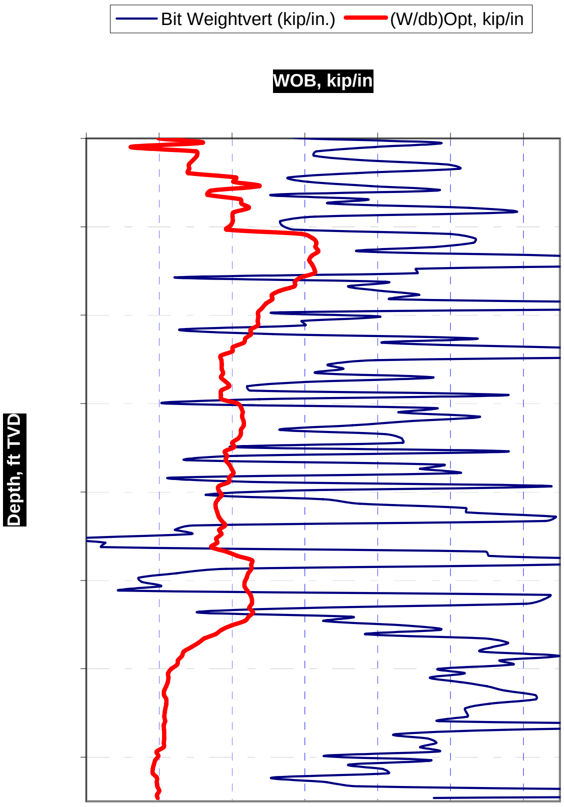


Figure VII-13: Optimized WOB for Well-1 12 1/4" hole-Fm3.

Bit rotation optimization trend was calculated to have higher magnitudes in reference to what was actually applied. However with the introduction of more data points the trend is getting less in magnitude and getting closer to actually applied bit rotation speeds. The most recent optimum weight on bit and bit rotation are the recommended values for the field and formation in subject. It is observed that both weight on bit and bit rotary speed optimization in the initial sections of the charts are with varying trends, however with inclusion of more data the trends are getting stable.

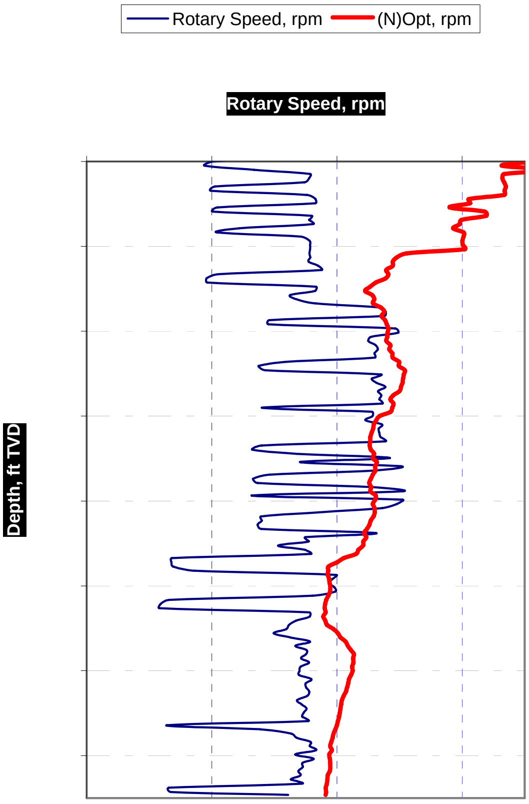


Figure VII-14: Optimized bit rotation per minute for Well-1 12 1/4" hole-Fm3.

7.1.2 ROP Performance Monitoring

In this section an example chart is given in which rate of penetration of the actual occurrences and the predicted ones are compared. In order to create this chart a multiple regression has been run and the results presented are achieved. Rate of penetration based on the drilling parameters and general equation of penetration rate for Well-3 in formation-1 and 17 ½” hole section is as given **Figure VII-15**. The predicted rate of penetration performance is accompanied with a magnitude band which is slightly above and below in reference to the predicted range. The performance band is as shown with ±30% window of the predicted performance which is highlighted to be green in colour. When the actual performance is monitored it is observed that it generally it tends to stay within the defined range. This type of rate of penetration monitoring methodology could be adapted for field use for real time applications.

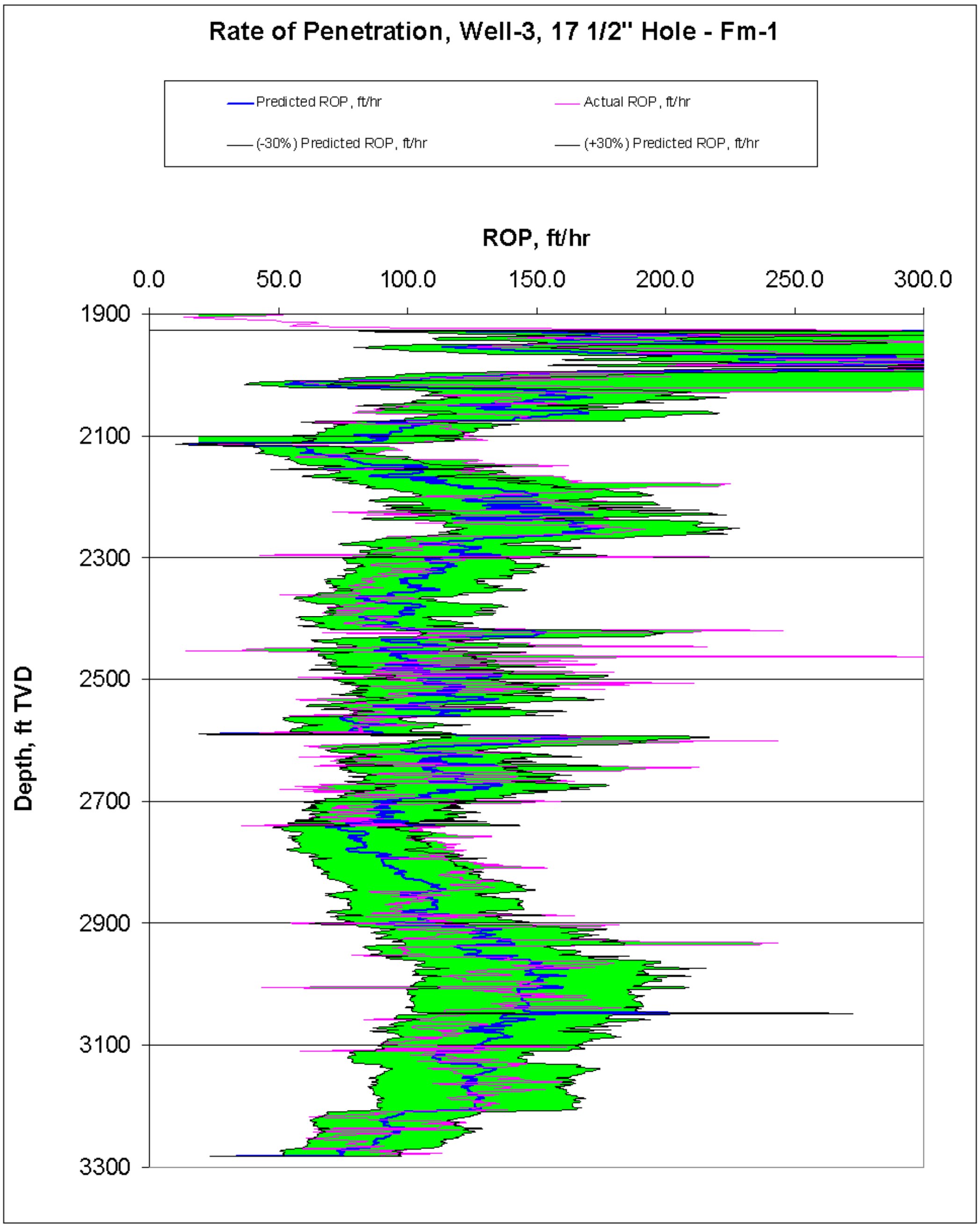


Figure VII-15: Rate of penetration for Well-3 17 ½" hole-Fm1.

7.2. Cost Considerations

One of the foremost objective of this study is to be able to reduce costs of drilling operations. It is known that rotation times in drilling operations accounts up to 30% of the well cost [55]. No actual field tests have been performed; however it has been demonstrated that rotation times during drilling activities could significantly be reduced following the selection of the optimized parameters.

Figure VII-16 gives the rate of penetration comparison for the actual and optimized case using the data after Bourgoyne and Young [14]. The available data is assumed to belong to the same field in order to make necessary comparisons. Optimized drilling rate is achieved following the utilization of the optimized drilling parameters; weight on bit and rotation speed in the equation used for the calculation of the predicted rate of penetration. The performance of the optimized drilling rate is showing a much more efficient trend.

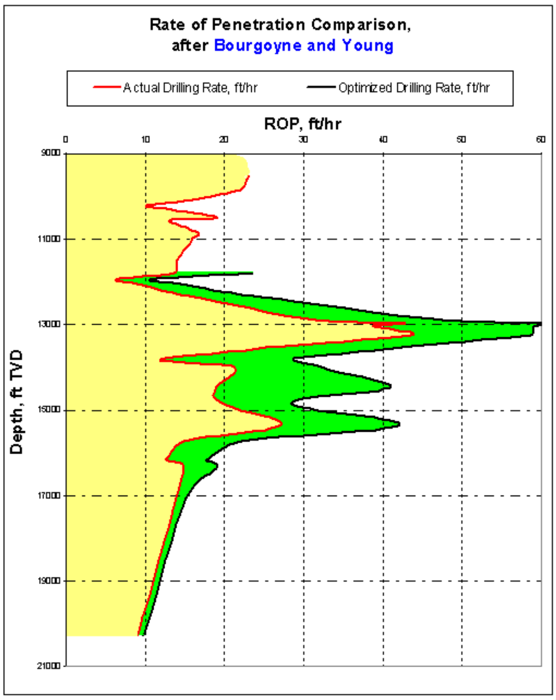


Figure VII-16: Rate of penetration comparison actual vs. optimized case for Bourgoyne and Youngs' data [14].

Chart in **Figure VII-17** gives the drilling rotation time for the data set provided in reference [14]. The drilling rotation time for the optimized line is reasonably less as compared to actual case. The optimized drilling rotation time is calculated to be 27% less than the actual performance.

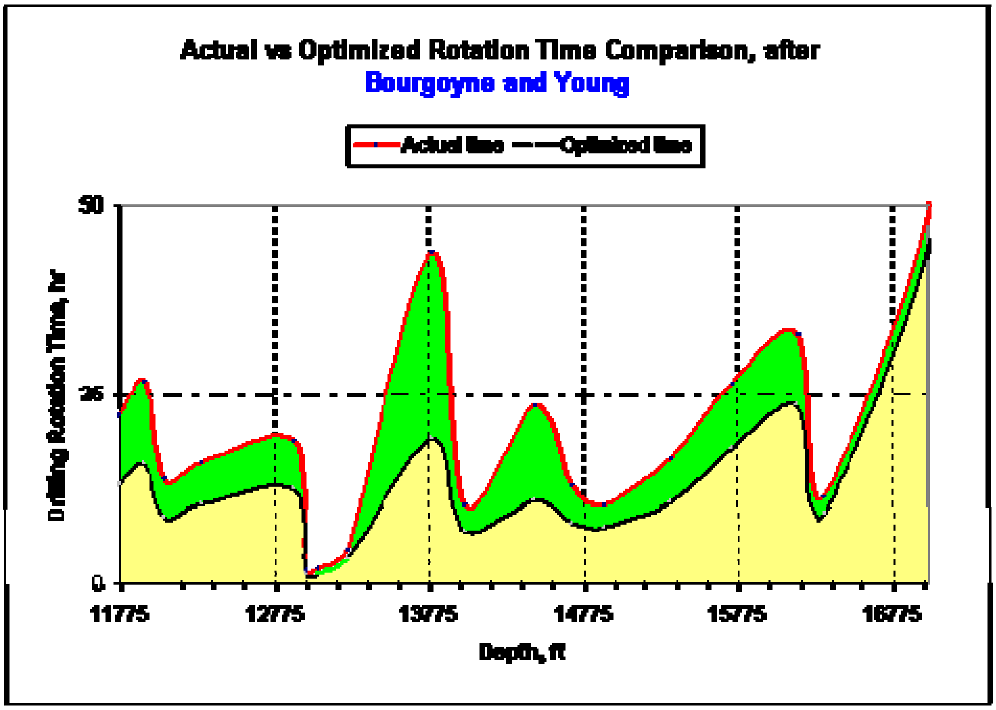


Figure VII-17: Actual vs. optimized drilling time comparison for Bourgoyne and Young data [14].

Figure VII-18 gives the actual and optimized drilling cost vs depth for the data provided in **Figure VII-17**. Common cost per foot values in the use of industry were preferred to construct the given cost per foot comparison [90]. Bit cost, B_C for each bit was taken at a cost of 2500$, daily rig rate, C_r, was considered to be 900$/hr. The time fro tripping, t_t was considered to be 3 hours for 5000 ft of depth, and 6 hours for 10,000 ft. The trip time for each respective given depth was calculated accordingly. It has been reported that drilling costs could be reduced up to approximately 10% [14].

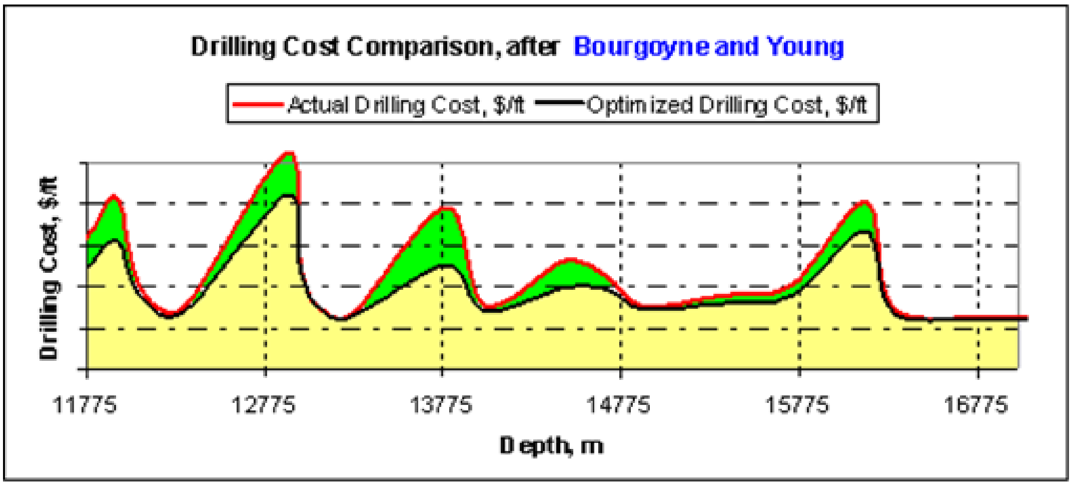


Figure VII-18: Cost comparison for actual and optimized data for Bourgoyne and Young data [14].

Optimized rate of penetration for Well-1 12 ¼” hole section across Formation-3 is as plotted in **Figure VII-19**. Three ROP occurrences are given in the same chart, they are actual, predicted and optimized rate of penetrations. Two different colour shadings are differentiating the difference in rate of penetration in case optimum drilling parameters were used. Green shading shows the additional ROP speed saving. The “a” coefficients calculated through rate of penetration prediction at each individual data point has been used when inserting the optimized weight on bit and bit rotation speed into the general equation and getting the optimized response of rate of penetration. It is observed that the rate of penetration response in the optimized case is expectedly more efficient as compared to actual case. Having a rate of penetration trend that is greater in magnitude than the actual occurrences would mean a lower cost drilling.

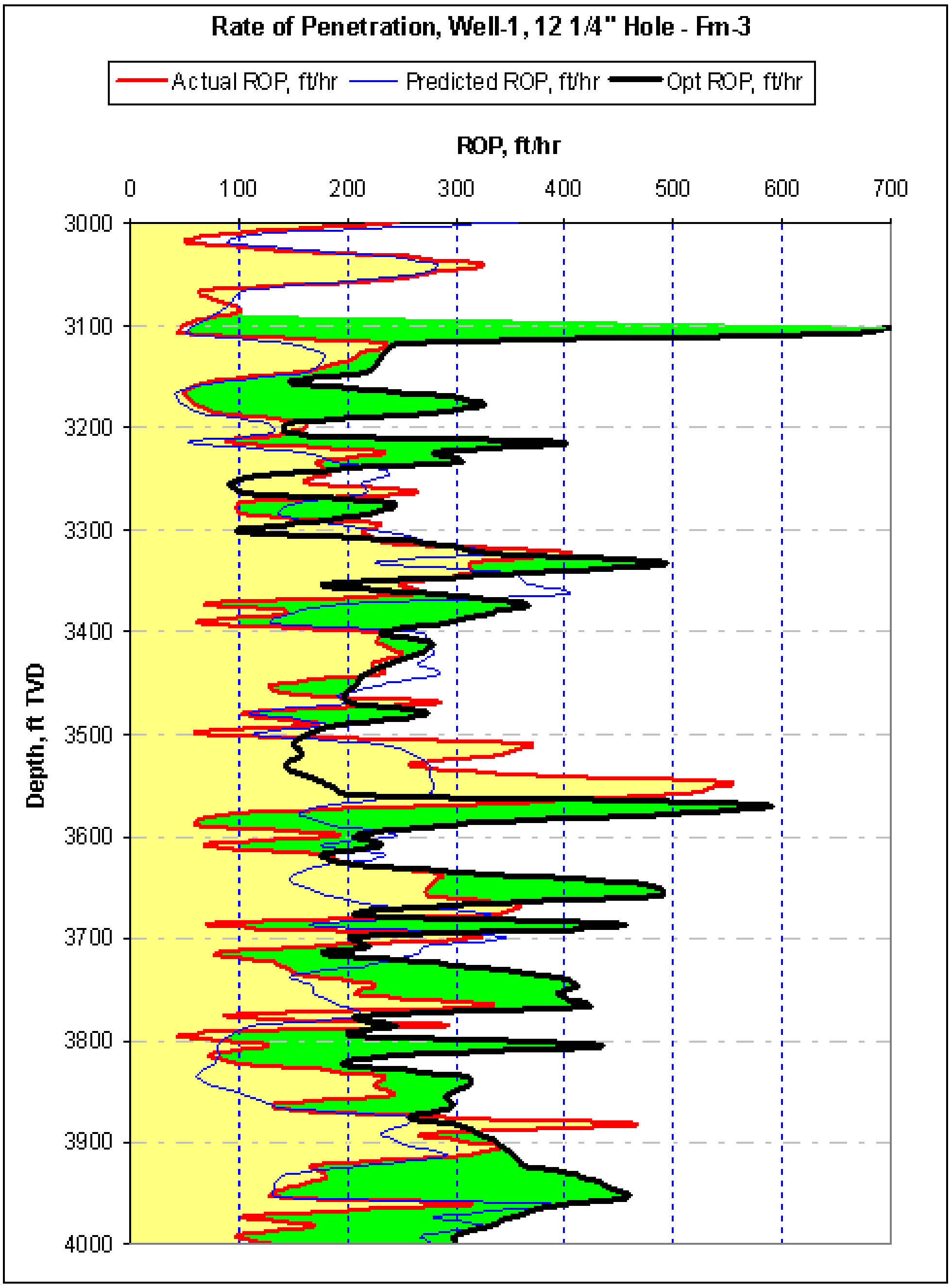


Figure VII-19: Optimized rate of penetration for Well-1 12 1/4" hole-Fm3.

Actual vs optimized rotation time comparison for Well-1 12 ¼" hole section Formation-3 is given in **Figure VII-20**. The optimized rotation time is observed to be less than the actual rotation time by a percentage of about 38%, as shown with the green shading. This should mean that when using the optimum drilling parameters the time required to drill the same hole section is going to be less.

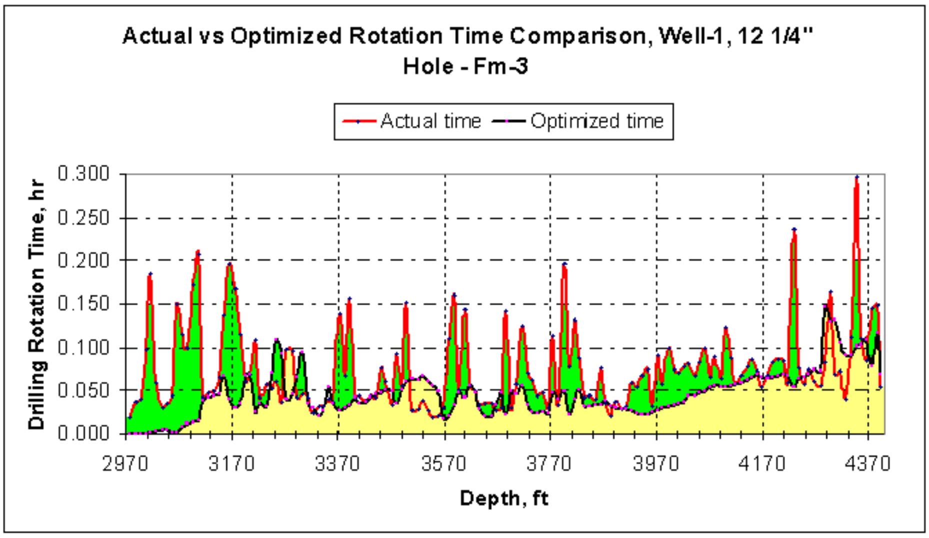


Figure VII-20: Actual vs. optimized drilling time comparison for Well-1 12 1/4" hole-Fm3.

Drilling cost comparison in between the actual and optimized case is as given in **Figure VII-21**. The overall cost considering similar cost figures as given in the previous case is with an amount that is approximately 22 % less, as shown with the green shading.

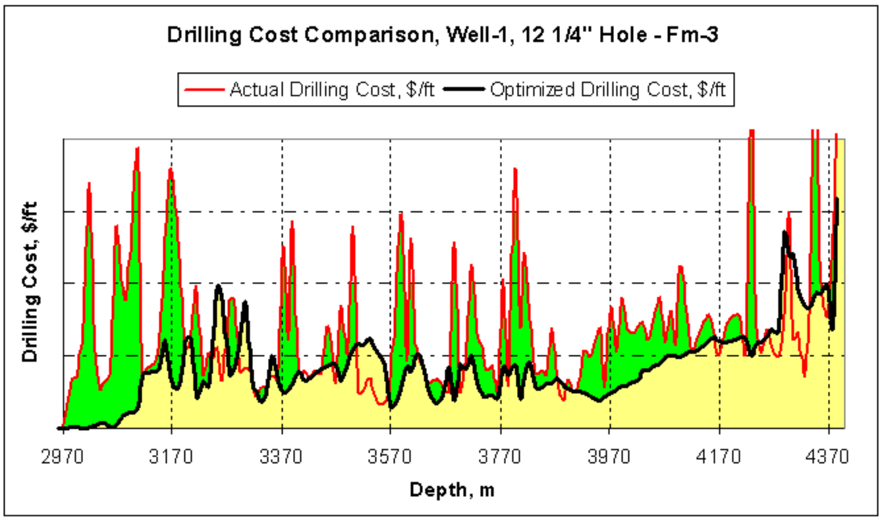


Figure VII-21: Cost comparison for actual and optimized data for Well-1 12 1/4" hole-Fm3.

The real-time optimization process has tried to demonstrate that when the optimized drilling parameters are used, it is expected that drilling costs during actual drilling course could be less by a significant magnitude.

7.3. Discussion

One most important observation considered to have an impact to the quality of the results on real-time drilling optimization studies is the data quality. The data should be as consistent as possible. Noisy data distracts the processability of the data. Data filtering is always deemed as necessary for the data to be processed. Data within the dataset considered to be noisy should be eliminated, and filtered before the application of the multiple regression. This filtering process can be performed before the data is transmitted from the rig site, data transmission engineer could perform task. Data interpolation could be implemented in order to avoid spiky data. Alternatively during real-time operations just beforehand of data transmission, the data could be sampled at certain intervals likewise the reporting/transmission methodology of directional drilling information.

With increased data points in a database the regression constants gradually become better and more accurate. Consequently the rate of penetration predictions get more accurate.

The observed drilling rate vs. predicted drilling rate graphs revealed that reasonable results were achievable. Knowing the dependent variable's function the predicted value could be determined by means of using the optimized drilling variables and assuming the optimized rate of penetration results without actually performing drilling activity.

Sensitivity results for the regression coefficients revealed important outcomes. The initial sections for all coefficients were composed of spiky trends, however with the introduction of more data, the trend is observed to get consistent and give better results. In this study eight functions are used, however this number could be increased, which would certainly take into consideration different effects believed to have an influence in drilling operations. Examples to additional functions those of which could be included in the scope of this study are torque, cuttings content, friction factor between the bottom hole assembly and hole section.

Even though there is no possibility to test the actual rate of penetration following the determination and adaptation of the controllable drilling parameters an estimation of the same has been given in this study. The estimates were determined using the most recent coefficients of the multiple regression process. It is observed that total rotation times to drill the same distance were significantly less, and consequently the drilling costs were less.

Rate of penetration vs depth graphs are plotted to show the differences between the predicted and actual ROP performances for certain formation and wellbore diameters. The achieved results gave consistent outputs. It is known that small data volumes would give less accurate ROP performance predictions as compared to data with large volumes. The predicted ROP values gave significant magnitudes in reference to the actual performance. The reason for this achievement is the efficiency of the multiple regression process. The process had considered the actual occurrences and determined rate of penetration based on the general rate of penetration equation.

CHAPTER 8

CONCLUSION

A real-time drilling optimization methodology was developed and demonstrated applicable to achieve optimum controllable drilling parameters. The foremost task of this study is to optimize the drilling parameters. The data used in the scope of this study was belonging to wells drilled in Mediterranean Offshore. The optimum drilling parameters are the parameters which are determined through the multiple regression technique to give minimum drilling cost. Results indicated that drilling costs were able to be reduced in real-time provided that optimum controllable parameters are applied.

Multiple regression technique proved that it could be functioning efficiently during the actual drilling activities, as well as for well planning and drilling scenario construction purposes. ROP performance predictions could be used by means of utilizing of the regression coefficients specific to the formations. It has been observed that with more data quantity the regression coefficient quality achievement has been improved. In the world of drilling optimization technology the rarely sought right answer is predicting rate of penetration beforehand realizing the drilling, and recommending the optimized parameters for the drilling process. Modern well monitoring equipment are required to be used in order to collect the necessary drilling data and to perform multiple regression which determines the constants defined in the general rate of penetration equation.

The study of Bourgoyne and Young [14] and the study presented here are based on analysis of drilling data collected from different drilling locations. For this very reason it should be expected that the coefficients of the general rate of penetration equation following multiple regression are not going to be similar when the drilling locations are different. Some of the normalization constants were determined specifically for the data content that was available. It has been observed that the results when different normalization constants other than the ones given in the study were used for f_2 and f_3 functions of the general rate of penetration, accurate results were still achieved.

Formation finger printing approach is the type of terminology which could be adapted for the optimization of the drilling operations. Each formation while being drilled could be identified based on their respective set of regression coefficients such as $a_1, a_2, \ldots, a_8$ as has been defined in this study. The more the data volome used the more accurate the coefficients turned out.

Formation specific regression exponents respectively are going to have a relative weight over the general rate of penetration magnitude from one condition to another, for the circumstances that each individually have an effect for. Especially coefficient a_1 is an important parameter which is believed to have the effect of different effects such as crew efficiency, rig properties, equipment quality. All these effects could be considered as separate functions for future applications of this study. Eventually the definition of each formation in terms of regression coefficients could be achieved, and performance of desired drilling rate could be compared based on the coefficients together with simultaneous drilling optimization. Respectively the other exponents are going to get a magnitude as a matter of the effect for the parameter they are representing.

Following the determination of the coefficients of the general rate of penetration the predicted magnitudes of the same could be constructed when used with the defined magnitudes of the independent drilling parameters. It has been demonstrated that the operator and the contractor could agree upon the predetermined ROP performance window and realize the monitoring of the drilling activity based on the actual response.

Corrections were applied to the data set during the analysis, which is believed to be performed during the course of real-time optimization of the drilling parameters. The applied weight on bit for each data point was converted into the vertical component taking the wellbore inclination into consideration. Weight on bit correction eliminates the discrimination of the applied weight due to the directional nature of the wellbore trajectory. The additional bit rotation due to the motor in the case of deviated wellbores was included in the calculations of the bit revolution. These two corrections are believed to have facilitated data processes, and accurate results were achieved. The statistical findings and *F-test* also have indicated that the data ended up with better results with the subject corrections.

The spiky trend of the data was eliminated by means of applying the interpolation. The available data were interpolated and pertinent samplings were performed. The multiple regression was performed to the data following interpolations. The results indicate that much more accurate predictions of the rate of penetration were achieved. The dependent drilling parameters; weight on bit and string revolution were optimized with more accuracy following the interpolations.

The time for rotation and consequently the time required for drilling have observed to have been reduced, even though no actual field test has been performed. The drilling cost during the actual drilling case is assumed to have been reduced by about 22%.

CHAPTER 9

RECOMMENDATIONS

This study would be invaluable if torque considerations were added to the Bourgoyne and Young's approach. Cuttings transport if included as a function into the general rate of penetration would be very useful.

The methodology should be modified to be suitable for the application of the given methodology for PDC bits. Use of the PDC bits during drilling activities in recent years has shown significant increases even with directional wells. Generation of a model which can effectively work for the optimization of the drilling operations is going to be an important contribution.

Charts for weight on bit optimization have shown similar results as the rotary speed optimization case. The data should be stacked considering a moving average in order to have better results.

It is strongly recommended that whenever drilling parameters are being collected utmost attention should be given to make sure that the piped data is representative of what was actually the case during the course of drilling activity.

An important recommendation is the reporting of the results. Whenever multiple regression is being conducted for certain drilling data, and provided that the data is belonging to a deviated wellbore, the vertical component of the optimized weight on bit should be calculated, which is going to be a function of the inclination of the wellbore trajectory in subject. Consideration of the downhole drag forces and the detailed geometry of the bottom hole assembly is going to significantly contribute to the results of the analyses.

One of the most critical tasks in the scope of this study was the determination of the tooth wear on the bit in use. An efficient mechanism could largely contribute to the methodology in case the instant actual tooth wear on the bit could be acquired in real-time. Developing mechanisms transmitting actual tooth wear directly from the bit will be extremely important.

REFERENCES

[1] Azar J.J., and Samuel G.R., "*Drilling Engineering*", PennWell Corporation, Tulsa, OK, **2007**

[2] http://www.ipims.com/data/pe33/E1856.asp?Code=yy&UserID=xx, Petroleum Engineering, Drilling Engineering, Drilling Optimization, Practical Guidelines, visited on 04/04/**2009** 00.34

[3] Carden, R.S., Grace, R.D., and, Shursen, J.L.,: "*Drilling Practices*," Petroskills-OGCI, Course Notes, Tulsa, OK, **2006**, pp 1-8

[4] Samuel G.R., and Miska S., "*Optimization of Drilling Parameters with the Performance of Multilobe Positive Displacement Motor (PDM),*" SPE 47791, IADC/SPE Asia Pacific Drilling Conference, Jakarta, Indonesia, September **1998**

[5] Short J.A., "*Introduction to Directional and Horizontal Drilling,*" PennWell Publishing Company, Tulsa, OK, **1993**, pp 3

[6] Bourgoyne A.T. Jr., Millheim K.K., Chenevert M.E., and Young F.S., "*Applied Drilling Engineering*", Society of Petroleum Engineers Text Book Series, Vol.1, Richardson, TX, **1986**

[7] Drilling data management with server-based systems and proprietary software. *World Oil,* Vol. 229 No.12, pp 78, December **2008**

[8] http://home.sprynet.com/~carob/, Wellsite Information Transfer Specification, visited on 13/09/**2009** 13.30

[9] http://www.witsml.org/witsml, Wellsite Information Transfer Standard Markup Language, visited on 21/11/**2009** 21.30

[10] Pickering J.G., Kreijger J., Grovik L.O., Franssens D., and Doniger A., "*WITSML "Comes of Age" for the Global Drilling and Completions Industry,*" SPE 124347-MS, SPE Annual Technical Conference and Exhibition, New Orleans, Louisiana, 4-7 October **2009**

[11] Lummus J.L., "*Drilling Optimization,*" SPE 2744, SPE 40th Annual California Fall Meeting, San Francisco, November **1969**

[12] Graham J.W. and Muench N.L., "*Analytical Determination of Optimum Bit Weight and Rotary Speed Combinations,*" SPE 1349-G, Fall Meeting of the Society of Petroleum Engineers, Dallas, TX, October **1959**

[13] Galle E.M and Woods A.B., "*Best Constant Weight and Rotary Speed for Rotary Rock Bits,*" Drill. And Prod. Prac., API **1963**, pp 48-73

[14] Bourgoyne A.T. Jr., Young F.S., "*A Multiple Regression Approach to Optimal Drilling and Abnormal Pressure Detection*", SPE 4238, August **1974**

[15] Bond D.F., Scott P.W., Page P.E., and Windham T.M., "*Applying Technical Limit Methodology for Step Change in Understanding and Performance,*" SPE 51181, IADC/SPE Drilling Conference, New Orleans, April **1998**

[16] Schreuder J.C. and Sharpe P.J., "*Drilling The Limit – A Key to Reduce Well Costs,*" SPE 57258, Asia Pacific Improved Oil Recovery Conference, Malaysia, October **1999**

[17] Bradley H.B., "*Petroleum Engineering Handbook,*" Society of Petroleum Engineers, Richardson, TX, **1987**, Chapter 52 (Mud Logging), pp- 52-1

[18] Devereux S., "*Practical Well Planning and Drilling Manual,*" PennWell Publishing Company, Tulsa, OK, **1998**, pp 412

[19] Baker Hughes Inteq: "Drilling Engineering Workbook", Training Development, Houston, TX, **1995**, pp 9-18

[20] Lyons W.C., "*Standard Handbook of Petroleum & Natural Gas Engineering,*" Gulf Professional Publishing, Houston, Texas, VOLUME-I **1996**, pp 1022

[21] Lyons W.C., and Plisga G.J., "*Standard Handbook of Petroleum & Natural Gas Engineering,*" Gulf Professional Publishing, **2005**, pp 4 (363 - 366)

[22] Eren T., and Ozbayoglu E., "*Real Time Optimization of Drilling Parameters During Drilling Operations,*" SPE Paper 129126, SPE Oil and Gas India Conference and Exhibition, Mumbai, India, 20–22 January **2010**

[23] Rabia H., "Well Engineering & Construction," Entrac Consulting Ltd, **2002**, pp 25

[24] Cayeux E., Dvergsnes E.W., and Iversen F., "*Real-Time Optimization of the Drilling Process – Challenges in Industrialization,*" SPE/IADC 119650, SPE/IADC Drilling Conference and Exhibition, Amsterdam, The Netherlands, March **2009**

[25] Ozbayoglu M.E., Miska S.Z., Reed T., and Takach N., "*Analysis of the Effects of Major Drilling Parameters on Cuttings Transport Efficiency for High-Angle Wells in Coiled Tubing Drilling Operations,*" SPE 89334, SPE/IcoTA CT Conf. and Exhb., Houston, TX, March **2004**

[26] Al-Betairi E.A., Moussa M., and Al-Otaibi S., "*Multiple Regression Approach to Optimize Drilling Operations in the Arabian Gulf Area,*" SPE 13694, Middle East Oil Symposium, Bahrain, March **1985**

[27] Barragan R.V., Santos O.L.A., Maidla E.E., "*Optimization of Multiple Bit Runs,*" SPE 37644, SPE/IADC Drilling Conference, Amsterdam, March **1997**

[28] Stefan Miska., "*Developments in Petroleum Engineering*", Gulf Publishing Company, (Collected works of Arthur Lubinski), **1988**,Vol-2, pp 266-275

[29] Dubinsky V.S., and Baecker D.R., "*An Interactive Drilling Dynamics Simulator for Drilling Optimization and Training,*" SPE 49205, SPE Annual Technical Conference, New Orleans, September **1998**

[30] Maidla E.E., and S. Ohara, "*Field Verification of Drilling Models and Computerized Selection of Drill Bit, WOB, and Drillstring Rotation,*" SPE Drilling Engineering, SPE Paper 19130, September **1991**, pp 189-195

[31] Gelfgat Y.A., Gelfgat M. Y., and Lopatin Y.S., "*Advanced Drilling Solutions Lessons From the FSU,*" PennWell Corporation, Tulsa, OK, **2003**, pp 199 - 290

[32] Speer J.W., "*A Method for Determining Optimum Drilling Techniques,*" Drill. And Prod. Prac., API **1958**, pp 25

[33] Garnier A.J. and van Lingen N.H.., "*Phenomena Affecting Drilling Rates at Depth,*" SPE 1097-G, Annual Fall Meeting of SPE, Houston, TX, May **1959**

[34] Maurer W.C., "*The 'Perfect-Cleaning' Theory of Rotary Drilling,*" Journal of Pet. Tech, November **1962**

[35] Bingham M.G., "*A New Approach to Interpreting Rock Drillability,*" re-printed from Oil and Gas Journal, April **1965**

[36] Eckel J.R., "*Microbit Studies of the Effect of Fluid Properties and Hydraulics on Drilling Rate, II,*" SPE 2244, SPE Annual Fall Meeting, Houston, October **1968**

[37] Young F.S.Jr., "*Computerized Drilling Control,*" SPE 2241, SPE 43rd Annual Fall Meeting, Houston, October **1968**

[38] Wardlaw H.W.R., "*Drilling Performance Optimization and Identification of Overpressure Formations,*" SPE 2388, The Uni. Of Texas Austin, TX, **1969**

[39] Reed R.L, "*A Monte Carlo Approach to Optimal Drilling,*" SPE 3513, SPE Annual Fall Meeting, New Orleans, October **1971**

[40] Lummus J.L., "*Acquisition and Analysis of Data for Optimized Drilling,*" SPE 3716, Journal of Petroleum Technology, November **1971**

[41] Wilson D.C., and Bentsen R.G., "*Optimization Techniques for Minimizing Drilling Costs,*" SPE 3983, 47th SPE Annual Fall Meeting, San Antonio, October **1972**

[42] Tansev E., "*A Heuristic Approach to Drilling Optimization,*" SPE 5546, SPE Annual Fall Meeting, Dallas, TX, October **1975**

[43] Bizanti M. and Blick E.F., "*Fluid Dynamics of Wellbore Bottom Hole Cleaning,*" SPE 12888, December **1983**

[44] Reza M.R. and Alcocer C.F., "*A Unique Computer Simulation Model Well Drilling: Part I – The Reza Drilling Model,*" SPE 15108, SPE 56th California Regional Meeting of SPE, Oakland, CA, April **1986**

[45] Reza M.R. and Alcocer C.F., "*A Unique Computer Simulation Model Well Drilling: Part II – The Reza Drilling Model,*" SPE 15108, SPE 56th California Regional Meeting of SPE, Oakland, CA, April **1986**

[46] Warren T.M., "*Penetration-Rate Performance of Roller–Cone Bits,*" SPE 13259, SPE Annual Technical Conference, Houston, March **1986**

[47] Wojtanowicz A.K., and Kuru.E, "*Minimum-Cost Well Drilling Strategy Using Dynamic Programming,*" Journal of Energy Resources Technology, February **1990**

[48] Pessier R.C. and Fear M.J., "*Quantifying Common Drilling Problems With Mechanical Specific Energy and a Bit-Specific Coefficient of Sliding Friction*", SPE 24584, Washington DC, October **1992**

[49] Teale R., "*The Concept of Specific Energy in Rock Drilling*", International J. Rock Mech. Mining Sci (**1965**) 2, pp 57-73

[50] Cooper G.A., Cooper A.G. and Bihn G., "*An Interactive Drilling Simulator for Teaching and Research,*" SPE 30213, Petroleum Computer Conference, Houston, TX, June **1995**

[51] Mitchell B., "*Advanced Oilwell Drilling Engineering Handbook & Computer Programs,*" The Society of Petroleum Engineers of AIME, Richardson, TX, **1995**, pp 522 - 550

[52] Bernt S.A., "*Modern Well Design*", Rogaland University Center, Stavanger, Norway, Published by A.A. Balkerna, pp 22-26, **1996**

[53] Serpen U., "*Sondaj Optimizasyonu için Bilgisayar Programları Geliştirilmesi*", TPAO Genel Müdürlüğü, Sondaj Grup Başkanlığı, December, **1997**

[54] Fear M.J., "*How To Improve Rate of Penetration in Field Operations,*" SPE 35107, IADC/SPE Drilling Conference, New Orleans, March **1996**

[55] Fear M.J., "*How to Improve Rate of Penetration in Field Operations,*" SPE 55050, IADC/SPE Drilling Conference, New Orleans, November **1998**

[56] Pereira J.J., "*Comprehensive Optimization of Drilling Parameters for Horizontal Wells,*" Ph.D. Thesis, The Graduate School, University of Tulsa, OK, **1999**

[57] Millheim K.K. and Gaebler T., "*Virtual Experience Simulation for Drilling – The Concept,*" SPE 52803, SPE/IADC Drilling Conference, Amsterdam, Holland, March **1999**

[58] Osgouei R.E., "*Rate of Penetration Estimation Model for Directional and Horizontal Wells,*" Ms.S. Thesis, The Graduate School, Middle East Technical University, **2007**

[59] Millheim K., "*Proactively Managed Drilling Operations Optimize Company Performance,*" Oil & Gas Journal, September 17 **2001**

[60] Akgun F., "*How to Estimate the Maximum Achievable Drilling Rate without Jeopardizing Safety,*" SPE 78567, Abu Dhabi Inter. Pet. Exh. And Conf., October **2002**

[61] Ozbayoglu M.E. and Omurlu C., "*Minimization of Drilling Cost by Optimization of the Drilling Parameters,*" 15th International Petroleum and Natural Gas Congress and Exhibition of Turkey, 11-13 May **2005**

[62] Simmons E.L., "*A Technique for Accurate Bit Programming and Drilling Performance Optimization,*" IADC/SPE 14784, Drilling Conference, Dallas, TX, February **1986**

[63] John Z., Ahsan A., and Reid I., "*Optimized Decision Making Through Real Time Access to Drilling and Geological Data from Remote Wellsites,*" SPE 77855, SPE Asia Pacific Oil and Gas Conference and Exhibition, Melbourne, Australia, October **2002**

[64] Ursem L.J., Williams J.H., Pellerin N.M., and Kaminski D.H., "*Real Time Operations Centers; The people aspects of Drilling Decision Making,*" SPE/IADC 79893, SPE/IADC Drilling Conference, Amsterdam, Netherlands, February **2003**

[65] Rommetveit R., Bjorkevoll K.S., Halsey G.W., Larsen H.F., Merlo A., Nossaman L.N., Sweep M.N., Knut M.S., and Inge S., "*Drilltronics: An Integrated System for Real-Time Optimization of the Drilling Process,*" IADC/SPE 87124, IADC/SPE Drilling Conference, Dallas, Texas, 2-4 March **2004**

[66] Mochizuki S., Saputelli L.A., Kabir C.S. et al., "*Real Time Optimization: Classification and Assessment,*" SPE 90213, SPE Annual Technical Conference and Exhibition, Houston, TX, September **2004**

[67] Dupriest F.E., and Koederitz W., "*Maximizing Drill Rates with Real-Time Surveillance of Mechanical Specific Energy,*" IADC/SPE 92194, Drilling Conference, Amsterdam, February **2005**

[68] Iversen, F.P., Cayeux, E., Dvergsnes, E.W., Gravdal, J.E., Vefring, E.H., "*Monitoring and Control of Drilling Utilizing Continuously Updated Process Models,*" SPE 99207, IADC/SPE Drilling Conference, Miami, Florida, February **2006**

[69] Milter J., Bergjord O.G., Hoyland K., and Rugland B., "*Use of Real-Time Data at the Statfjord Field Anno 2005,*" SPE 99257, SPE Intelligent Energy Conference and Exhibition, Amsterdam, The Netherlands, April **2006**

[70] Tollefsen E., Goobie R.B., Noeth S., Sayers C., Boere L.den, et al., "*Optimize Drilling and Reduce Casing Strings Using Remote Real-Time Well Hydraulic Monitoring,*" SPE 103936, First International Oil Conference and Exhibition, Cancun, Mexico, September **2006**

[71] Monden T., and Chia C.R., "*Operation Support Centers – Real Time Drilling Optimization and Risk Mitigation,*" SPE 110950, SPE Saudi Arabia Technical Symposium, Dhahran, May **2007**

[72] Remmert S.M., Witt J.W., and Dupriest F.E., "*Implementation of ROP Management Process in Qatar North Field,*" SPE 105521, SPE/IADC Drilling Conference, Amsterdam, February **2007**

[73] Strathman M., Elley D., Meirerhoefer N., "*Time-Based Real-Time-Drilling-Operations Excellence Delivered,*" SPE 107303, SPE Digital Energy Conference and Exhibition, Houston, TX, April **2007**

[74] Iversen F.P, Cayeux E., Dvergsnes E.W, Ervik R., Byrkjeland M., Welmer W., A. Merlo et. al., "*Offshore Field Test of a New Integrated System for Real-Time Optimization of the Drilling Process,*" IADC/SPE 112744, IADC/SPE Drilling Conference held in Orlando, Florida, USA, 4-6 March **2008**

[75] Iqbal F., "*Drilling Optimization Technique - Using Real Time Parameters,*" SPE 114543, SPE Russian Oil & Gas Technical Conference and Exhibition, Moscow, Russia, 28-30 October **2008**

[76] Reinhold W.B., Close D.A., "*Drilling Optimization-The Driller's Role,*" SPE 29365, SPE Drill& Compl 12 (1), **1997, pp** 5-12

[77] "*Baker Hughes – INTEQ Navi-Drill*^{TM} *Motor Handbook,*" 11th Edition, Rev A, March **2009**

[78] Gill G.S., "*The Calculus Bible,*" http://www.math.byu.edu/Math/CalculusBible/, visited on June **2009**

[79] http://en.wikipedia.org/wiki/Optimization_(mathematics), visited on 13 June **2009**

[80] Wrede R., and Spiegel M.R., "*Theory and Problems of Advanced Calculus,*" McGraw-Hill Companies Inc, **2002**, pp 73

[81] Hussein N.K., Alsayed, N. A., Seif, M.A. Abu, Shalaby, S.A., "*Drilling Operations in the Mediterranean Concessions: Challanges for Drilling Engineers and Rig Personnel,*" SPE 29866, Middle East Oil Show, Bahrain, 11-14 March **1995**

[82] Badri M.A., Sayers C., and Hussein R.A., "*Pore Pressure Prediction Data Using Seismic Velocities and Log Data in the Offshore Nile Delta, Egypt,*" SPE 68195-MS, SPE Middle East Oil Show, Bahrain, 17-20 March **2001**

[83] Hassan S., Klimentos T., Badri M., Sengul M., and Zeid A., "*Optimizing Drilling Performance By Wellbore Stability Evaluation And Directional Drilling Practices,*" SPE 57575-MS, SPE/IADC Middle East Drilling Technology Conference, Abu Dhabi, United Arab Emirates, 8-10 November **1999**

[84] Hyatt P., Little I., Hanafi A., Sinor A., and Herran G., "*Ream While Drilling Technology Cuts $1 Million in Nile Delta Well,*" SPE/IADC 39248, SPE/IADC Middle East Drilling Technology Conference, Bahrain, 23-25 November **1997**

[85] Montgomery D.C., and Runger G.C., "*Applied Statistics and Probability for Engineers*" Third Edition, John Wiley & Sons, Inc, USA, **2003**, pp 482

[86] "*Smith Tool Drill Bits,*" http://www.smith.com/Datasheets/dsDisplay.aspx?BitID=4636&page=b8be3750-db3f-4767-81db-858d74dbdf17, visited on January **2010**

[87] Davis J.C., "*Statistics and Data Analysis in Geology*" Third Edition, John Wiley & Sons, USA, Inc, **2002**, pp 461

[88] "*Microsoft Visual Basic 6.3,*" Version 1024, Copyright (c) 1987-**2001** Microsoft Corp.

[89] DeCoursey W.J., "*Statistics and Probability for Engineering Applications with Microsoft® Excel,*" Elsevier Science, USA, **2003**

[90] "*Microsoft ® Office Excel 2003,*" (11.8231.8221) SP3 Part of Microsoft Office Professional Edition 2003 Copyright (c) 1985-**2003** Microsoft Corporation.

[91] Lapeyrouse N.J., "*Formulas and Calculations for Drilling, Production, and Workover,*" Gulf Professional Publishing, Elsevier Science, USA, **2002**

[92] "*Online Matrix Calculator,*" http://www.bluebit.gr/matrix-calculator/linear_equations.aspx, powered by NET Matrix Library, visited on June **2009**

[93] Donati F., Ferrara P., Maestrami M., Oppelt J., and Kruger S., "*From Lean to Extreme-Lean Well Profile: Field Experience in the Mediterranean Sea,*" SPE 112953, SPE North Africa Technical Conference and Exhibition, Marrakech Morocco, 12-14 March **2008**

[94] "MATLAB R2008a" symbolic toolbox, Version 7.6.0.324, February 10, **2008**

APPENDIX A

MEDITERRANEAN SEA FIELD AND DATABASE SPECIFIC INFORMATION

Data used in this study is belonging to wells drilled in offshore Mediterranean Sea. The map of the Mediterranean Sea is as given in **Figure A-1** [92].

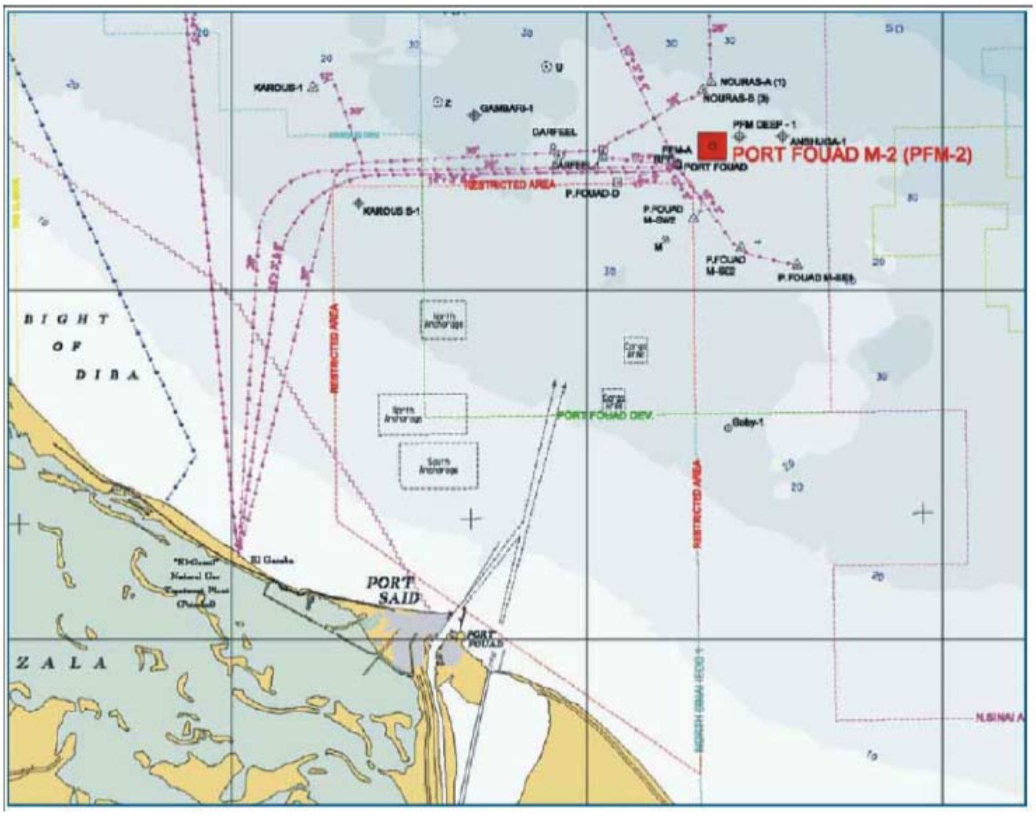


Figure A-1: Map of Offshore Nile Delta, Mediterranean Sea [92].

A typical directional trajectory for the wells drilled in Nile Delta is given in **Figure A-2** [83]. Offset wells in the Mediterranean Sea often experience with borehole stability and stuck pipe problems, especially over intervals of higher angle of deviation over the Kafr El Sheikh formation.

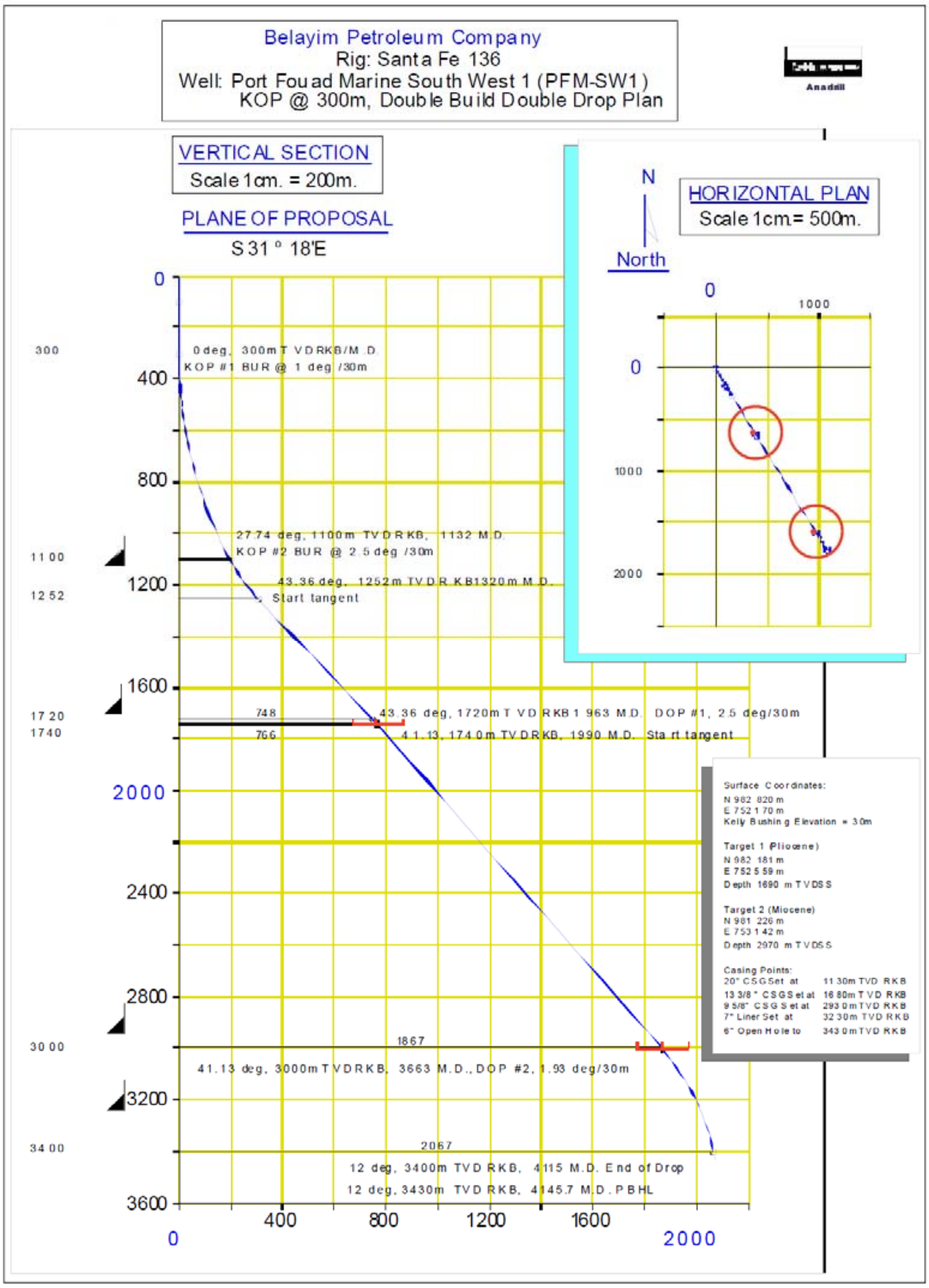


Figure A-2: A typical directional trajectory for Nile Delta Region [83].

Specific information regarding the investigated drilling data through the scope of this research study is provided in this section. The total number of data set for the scope of this research study is about 9500 observations. **Figure A-3** gives the number of available data set for each formation investigated.

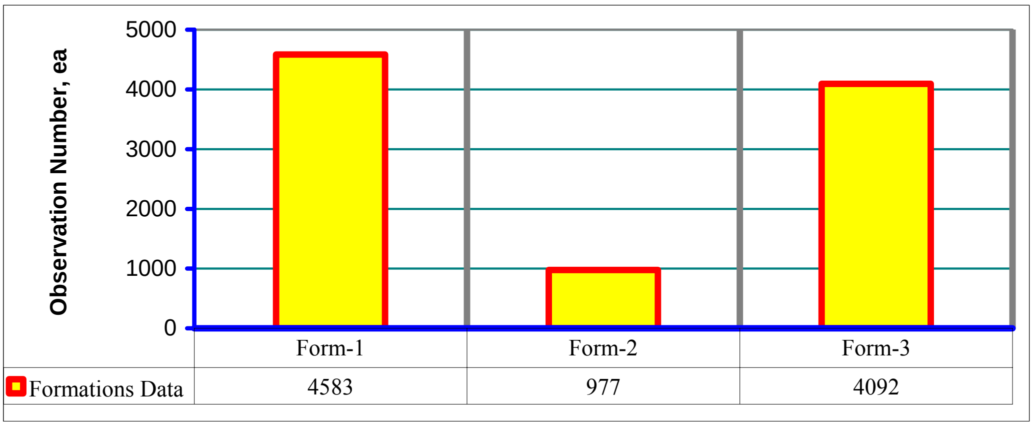
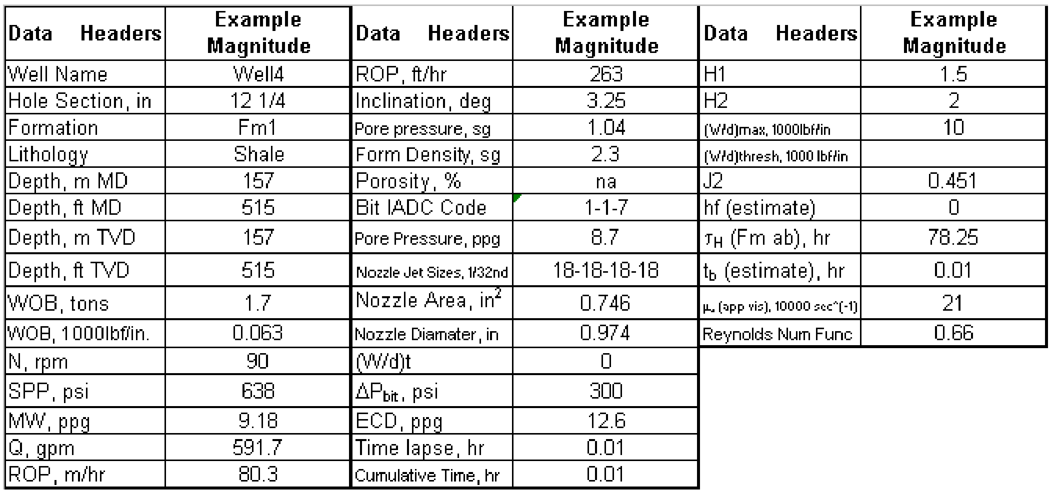


Figure A-3: Number of observations for the formations.

Each observation data set is mainly comprised of the information header magnitude as given in **Table A-1**. There are approximately 40 data headers before the multiple regression takes place.

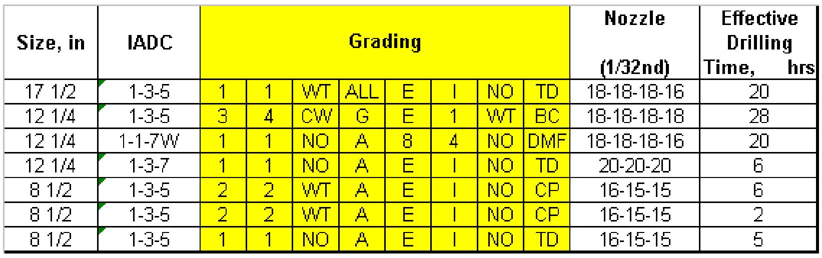
Table A-1: Example magnitude of a data header.



Data Headers	Example Magnitude	Data Headers	Example Magnitude	Data Headers	Example Magnitude
Well Name	Well4	ROP, ft/hr	263	H1	1.5
Hole Section, in	12 1/4	Inclination, deg	3.25	H2	2
Formation	Fm1	Pore pressure, sg	1.04	(W/d)max, 1000lbf/in	10
Lithology	Shale	Form Density, sg	2.3	(W/d)thresh, 1000 lbf/in	
Depth, m MD	157	Porosity, %	na	J2	0.451
Depth, ft MD	515	Bit IADC Code	1-1-7	hf (estimate)	0
Depth, m TVD	157	Pore Pressure, ppg	8.7	τ_H (Fm ab), hr	78.25
Depth, ft TVD	515	Nozzle Jet Sizes, 1/32nd	18-18-18-18	t_b (estimate), hr	0.01
WOB, tons	1.7	Nozzle Area, in^2	0.746	μ_∞ (app vis), 10000 sec^(-1)	21
WOB, 1000lbf/in.	0.063	Nozzle Diamater, in	0.974	Reynolds Num Func	0.66
N, rpm	90	(W/d)t	0		
SPP, psi	638	ΔP_{bit}, psi	300		
MW, ppg	9.18	ECD, ppg	12.6		
Q, gpm	591.7	Time lapse, hr	0.01		
ROP, m/hr	80.3	Cumulative Time, hr	0.01		

Some of the bit records of investigated wells are as given in **Table A-2**. The given bit records include bit size, type, IADC code, depth in-out and the final IADC grading together with the nozzles used during the bit run.

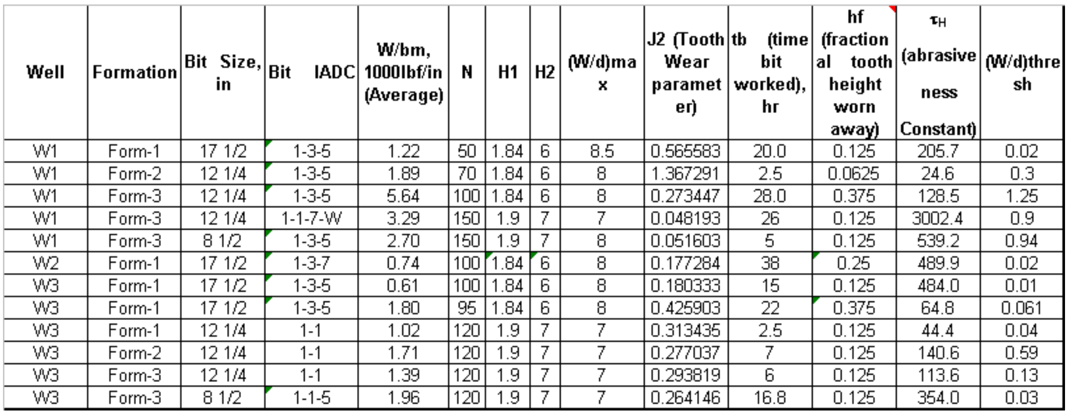
Table A-2: Bit record summary for Well-1.



Size, in	IADC	Grading								Nozzle (1/32nd)	Effective Drilling Time, hrs
17 1/2	1-3-5	1	1	WT	ALL	E	I	NO	TD	18-18-18-16	20
12 1/4	1-3-5	3	4	CW	G	E	1	WT	BC	18-18-18-18	28
12 1/4	1-1-7W	1	1	NO	A	8	4	NO	DMF	18-18-18-16	20
12 1/4	1-3-7	1	1	NO	A	E	I	NO	TD	20-20-20	6
8 1/2	1-3-5	2	2	WT	A	E	I	NO	CP	16-15-15	6
8 1/2	1-3-5	2	2	WT	A	E	I	NO	CP	16-15-15	2
8 1/2	1-3-5	1	1	NO	A	E	I	NO	TD	16-15-15	5

The abrasiveness constants for some bit records are as given in **Table A-3**.

Table A-3: Abrasiveness constant for the performed data runs.



Well	Formation	Bit Size, in	Bit IADC	W/bm, 1000lbf/in (Average)	N	H1	H2	(W/d)max	J2 (Tooth Wear parameter)	tb (time bit worked), hr	hf (fractional tooth height worn away)	τ_H (abrasiveness Constant)	(W/d)thresh
W1	Form-1	17 1/2	1-3-5	1.22	50	1.84	6	8.5	0.565583	20.0	0.125	205.7	0.02
W1	Form-2	12 1/4	1-3-5	1.89	70	1.84	6	8	1.367291	2.5	0.0625	24.6	0.3
W1	Form-3	12 1/4	1-3-5	5.64	100	1.84	6	8	0.273447	28.0	0.375	128.5	1.25
W1	Form-3	12 1/4	1-1-7-W	3.29	150	1.9	7	7	0.048193	26	0.125	3002.4	0.9
W1	Form-3	8 1/2	1-3-5	2.70	150	1.9	7	8	0.051603	5	0.125	539.2	0.94
W2	Form-1	17 1/2	1-3-7	0.74	100	1.84	6	8	0.177284	38	0.25	489.9	0.02
W3	Form-1	17 1/2	1-3-5	0.61	100	1.84	6	8	0.180333	15	0.125	484.0	0.01
W3	Form-1	17 1/2	1-3-5	1.80	95	1.84	6	8	0.425903	22	0.375	64.8	0.061
W3	Form-1	12 1/4	1-1	1.02	120	1.9	7	7	0.313435	2.5	0.125	44.4	0.04
W3	Form-2	12 1/4	1-1	1.71	120	1.9	7	7	0.277037	7	0.125	140.6	0.59
W3	Form-3	12 1/4	1-1	1.39	120	1.9	7	7	0.293819	6	0.125	113.6	0.13
W3	Form-3	8 1/2	1-1-5	1.96	120	1.9	7	7	0.264146	16.8	0.125	354.0	0.03

APPENDIX B

EQUATIONS DEFINED BY BOURGOYNE AND YOUNG, 1974 [14]

Bourgoyne and Young equations as given in their study are summarized in this section. The introduced model has found greater acceptance in the drilling industry due to its wide approach. The following general rate of penetration equation has been stated which is a function of various drilling parameters, equation (B.1).

$$\frac{dF}{dt} = \exp\left(a_1 + \sum_{j=2}^{8} a_j x_j\right) \tag{B.1}$$

the drilling parameters, x_2 to x_8 are calculated with their respective equation, x_1 is the dummy variable, equal to 1 for every observation. The coefficients a_i are calculated accordingly by means of multiple regression, and should be unique for a certain set of data. The constants a_i and parameters x_i are related to each other in the definition of rate of penetration equation.

Drilling parameter items considered to have an effect on the rate of penetration are as summarized in **Table B-1**.

Table B-1: Drilling parameters having an effect on rate of penetration.

1	Formation strength
2	Formation depth
3	Formation compaction
4	Pressure differential across the hole bottom
5	Bit diameter and bit weight
6	Rotary speed
7	Bit wear
8	Bit hydraulics

Each given effect in **Table B-1** is expanded below. Formation strength effect is defined by equation (B.2).

$$x_1 = 1 \tag{B.2}$$

Equation (B.2) includes the effects not have been modelled yet, such as drilled solids effect.

Formation depth effect is defined by equation (B.3).

$$x_2 = 10000 - D \tag{B.3}$$

Formation compaction effect is defined by equation (B.4).

$$x_3 = D^{0.69}\left(g_p - 9.0\right) \tag{B.4}$$

Differential pressure across the hole bottom effect is defined by equation (B.5).

$$x_4 = D(g_p - \rho_c) \tag{B.5}$$

Bit diameter and bit weight effect is defined by equation (B.6).

$$x_5 = \ln\left[\frac{\frac{W}{d_b} - \left(\frac{W}{d_b}\right)_t}{4.0 - \left(\frac{W}{d_b}\right)_t}\right] \tag{B.6}$$

Rotary speed effect is defined by equation (B.7).

$$x_6 = \ln\left[\frac{N}{60}\right] \tag{B.7}$$

Tooth wear effect is defined by equation (B.8).

$$x_7 = -h \tag{B.8}$$

Bit hydraulics effect is defined by equation (B.9).

$$x_8 = \frac{\rho q}{350 \mu d_n} \tag{B.9}$$

The bit tooth wear model defined by equation (B.10).

$$\frac{dh}{dt} = \frac{H_3}{\tau_H}\left(\frac{N}{60}\right)^{H1}\left[\frac{\left(\frac{W}{d}\right)_m - 4}{\left(\frac{W}{d}\right)_m - \left(\frac{W}{d}\right)}\right]\left(\frac{1 + \frac{H_2}{2}}{1 + H_2 h}\right) \tag{B.10}$$

The bit bearing wear model defined by equation (B.11).

$$\frac{dB}{dt} = \frac{1}{\tau_B}\left(\frac{N}{60}\right)\left[\frac{W}{4d}\right]^b \tag{B.11}$$

APPENDIX C

FORMATION ABRASIVENESS CONSTANT CALCULATION [6]

The methodology for the calculation of the formation abrasiveness constant is given in this appendix.

Instantaneous tooth wear could be calculated by means of finding the abrasiveness constant for a known bit record in the subject formation. Formation abrasiveness constant is a parameter when reached the bit in use will become inefficient to drill ahead.

The instantaneous tooth wear equation is given in terms of the relation in equation (C.1). It has been defined by the combination of tooth geometry, bit weight and rotary speed.

$$\frac{dh}{dt}=\frac{1}{\tau_H}\left(\frac{N}{60}\right)^{H1}\left[\frac{\left(\frac{W}{d_b}\right)_m-4}{\left(\frac{W}{d_b}\right)_m-\left(\frac{W}{d_b}\right)}\right]\left(\frac{1+\frac{H_2}{2}}{1+H_2h}\right) \tag{C.1}$$

where, τ_H is formation abrasiveness constant, hours, h_f fractional tooth wear, H_1 and H_2 are tooth geometry constants. The recommended tooth-wear parameter constants for roller cone cutter bits are as given in **Table C-1**, [6]. These parameters should be based on general field experience and drilling practices observed in field applications. A particular study could be conducted to update these parameters.

Table C-1: Recommended tooth-wear parameters for roller cone bits [6].

Bit Class	H_1	H_2	$\left(\frac{W}{D}\right)_{\max}$
1 – 1 to 1 - 2	1.90	7.0	7.0
1 – 3 to 1 - 4	1.84	6.0	8.0
2 – 1 to 2 – 2	1.80	5.0	8.5
2 - 3	1.76	4.0	9.0
3 – 1	1.70	3.0	10.0
3 – 2	1.65	2.0	10.0
3 – 3	1.60	2.0	10.0
4 – 1	1.50	2.0	10.0

Please note that the tooth wear formula given above is going to be normalized at 60 rpm of bit rotation speed and a constant bit weight of 4,000 lbf/in. The normalization magnitudes are selected accordingly for the specific conditions in the scope of this study.

In order to be able to calculate the formation abrasiveness constant a tooth wear parameter is required to be introduced, which is basically the reciprocal of the some of the given terms in the composite tooth wear equation (C.1). The tooth wear parameter is symbolized as J_2, equation (C.2).

$$J_2 = \left[\frac{\left(\frac{W}{d_b}\right)_m - \left(\frac{W}{d_b}\right)}{\left(\frac{W}{d_b}\right)_m - 4} \right] \left(\frac{60}{N}\right)^{H1} \left(\frac{1}{1+\frac{H_2}{2}}\right) \tag{C.2}$$

If both sides of equation (C.1) is written in a terms of J_2, the following relation is achieved when integrated with equation (C.2):

$$\int_0^{t_b} dt = J_2 \tau_H \int_0^{h_f} (1 + H_2 h) dh \tag{C.3}$$

when equation (C.3) is integrated, the following relation yields, equation (C.4).

$$t_b = J_2 \tau_H \left(h_f + H_2 \frac{h_f^2}{2} \right) \tag{C.4}$$

The formation abrasiveness constant could then be written as given in equation (C.5).

$$\tau_H = \frac{t_b}{J_2 \left(h_f + H_2 \frac{h_f^2}{2} \right)} \tag{C.5}$$

Formation abrasiveness constants for each formation are given in Appendix A.

Once formation abrasiveness constant is known, a t_b, time of bit rotation as a function of predefined constants and tooth wear as a fraction, could be calculated, solving equation (C.4).

An arbitrary h_f value could first be selected and until a tooth wear fraction is iterated, the selection of h_f should be determined, provided that the bit rotating time that is back calculated equal to the actual bit rotation time that is available in the database.

C.1 Bit Footage in terms of final tooth wear

It is important to calculate the tooth wear function f_7 especially for bits with substantial wear. This calculation could be calculated by means of introducing a composite drilling variable which could account all other functions but tooth wear. When equation (4.1) is written in an open form the following relation is achieved:

$$\frac{dF}{dt} = \left(f_1 f_2 f_3 f_4 f_5 f_6 f_7 f_8\right) \tag{C.6}$$

If a composite drilling parameter, J_1 is defined in the form as in equation (C.7).

$$J_1 = \left(f_1 f_2 f_3 f_4 f_5 f_6 f_8\right) \tag{C.7}$$

The composite drilling parameter could also be written in an open form, equation (C.8).

$$J_1 = e^{a_1} e^{a_2 X_2} e^{a_3 X_3} e^{a_4 X_4} e^{a_5 Ln\left(\frac{\frac{W}{d_b} - \left(\frac{W}{d_b}\right)_t}{4 - \left(\frac{W}{d_b}\right)_t}\right)} e^{a_6 X_6} e^{a_8 X_8} \tag{C.8}$$

The general penetration rate equation (C.6) can be rearranged,

$$\frac{dF}{dt} = \left(J_1 f_7\right) \tag{C.9}$$

when tooth wear function is written in an open form,

$$dF = \left(J_1 e^{(-a_7 h)}\right) dt \tag{C.10}$$

If both sides of equation (C.3) are re-written, the following relation between bit drilling time and respective tooth wear yields.

$$dt = J_2 \tau_H \left(1 + H_2 h\right) dh \tag{C.11}$$

When equation (C.11) is substituted into the penetration equation of (C.10), the following relation is obtained. This equation gives the drilled interval as a function of general drilling functions also having the tooth wear relation in the right hand side.

$$dF = \left(J_1 e^{(-a_7 h)}\right) J_2 \tau_H \left(1 + H_2 h\right) dh \tag{C.12}$$

Integration of the equation above as a function of bit tooth wear, results in having the interval of a drilling section drilled defined by final tooth wear observed. The integration has been performed using the MATLAB Program, [93].

$$\Delta F = J_1 J_2 \tau_H \left((-1) \frac{e^{-a_7 h}}{a_7} - \frac{H_2 \left(e^{-a_7 h} + a_7 h e^{-a_7 h}\right)}{a_7^2} \right) \tag{C.13}$$

APPENDIX D

MULTIPLE REGRESSION ANALYSIS CODE

```
Private Sub
CommandButton1_Click()
'this command selects the data range
and runs the multiple regression for
calculation  of the "a"s
Dim rangestart, rangeend, i, sira As
Integer

rangestart = TextBox1.Value + 1

For i = rangestart To TextBox2.Value
+ 1 - 4

rangeend = i + 4

' " & rangestart & "
' " & rangeend & "
'==========================
' THIS CODE OPENS DATA
SHEET AND CLEANS ALL OF
THE CONTENTS
   Sheets("DATA").Select
   Range("A2").Select
   Range(Selection,
Selection.End(xlToRight)).Select
   Range(Selection,
Selection.End(xlDown)).Select
   Selection.ClearContents
‘=========================
' THIS CODE COPIES THE
SELECTED RANGE AND PASTES
IT INTO THE DATA SHEET
  Sheets("WORK").Select
Range("K" & rangestart & ":R" &
rangeend & "").Select
Selection.Copy
   Application.CutCopyMode = False
   Selection.Copy
   Sheets("DATA").Select
   Range("A2").Select


Selection.PasteSpecial
Paste:=xlPasteValues,
Operation:=xlNone, SkipBlanks _
       :=False, Transpose:=False
'==========================
'
' getx2 Macro
' Macro recorded 31/05/2009 by Tuna
'


' THE OBJECTIVE OF THIS CODE
IS TO TRANSFER THE DATA
FROM
'DATA INTO MATRIXCREAT
SHEET 20090531 2113 CET

Dim num As Integer

'==========================
Sheets("MATRIXCREAT").Select
   Columns("A:BM").Select
   Range("BM1").Activate
   Selection.ClearContents
      Range("A2:BM3500").Select
   Selection.Interior.ColorIndex =
xlNone
   Selection.Font.ColorIndex = 0
‘=========================
Sheets("DATA").Select
num = Range("I1").Value
```

```
  Sheets("DATA").Select
  Range("a1:A" & num & "").Select
  Range(Selection,
Selection.End(xlDown)).Select
  Selection.Copy
  Sheets("MATRIXCREAT").Select
  Range("A1").Select
  Selection.PasteSpecial
Paste:=xlPasteValues,
Operation:=xlNone, SkipBlanks _
    :=False, Transpose:=False
'===========================
  Sheets("DATA").Select
  Range("b1:b" & num & "").Select
  Range(Selection,
Selection.End(xlDown)).Select
  Selection.Copy
  Sheets("MATRIXCREAT").Select
  Range("b1").Select
  Selection.PasteSpecial
Paste:=xlPasteValues,
Operation:=xlNone, SkipBlanks _
    :=False, Transpose:=False
'===========================
  Sheets("DATA").Select
  Range("b1:b" & num & "").Select
  Range(Selection,
Selection.End(xlDown)).Select
  Selection.Copy
  Sheets("MATRIXCREAT").Select
  Range("b1").Select
  Selection.PasteSpecial
Paste:=xlPasteValues,
Operation:=xlNone, SkipBlanks _
    :=False, Transpose:=False
'===========================
  Sheets("DATA").Select
  Range("c1:c" & num & "").Select
  Range(Selection,
Selection.End(xlDown)).Select
  Selection.Copy
  Sheets("MATRIXCREAT").Select
  Range("c1").Select
  Selection.PasteSpecial
Paste:=xlPasteValues,
Operation:=xlNone, SkipBlanks _
    :=False, Transpose:=False
'===========================
  Sheets("DATA").Select
  Range("d1:d" & num & "").Select
  Range(Selection,
Selection.End(xlDown)).Select
  Selection.Copy
  Sheets("MATRIXCREAT").Select
  Range("d1").Select
  Selection.PasteSpecial
Paste:=xlPasteValues,
Operation:=xlNone, SkipBlanks _
    :=False, Transpose:=False
'===========================
  Sheets("DATA").Select
  Range("e1:e" & num & "").Select
  Range(Selection,
Selection.End(xlDown)).Select
  Selection.Copy
  Sheets("MATRIXCREAT").Select
  Range("e1").Select
  Selection.PasteSpecial
Paste:=xlPasteValues,
Operation:=xlNone, SkipBlanks _
    :=False, Transpose:=False
'===========================
  Sheets("DATA").Select
  Range("f1:f" & num & "").Select
  Range(Selection,
Selection.End(xlDown)).Select
  Selection.Copy
  Sheets("MATRIXCREAT").Select
  Range("f1").Select
  Selection.PasteSpecial
Paste:=xlPasteValues,
Operation:=xlNone, SkipBlanks _
    :=False, Transpose:=False
'===========================
  Sheets("DATA").Select
  Range("g1:g" & num & "").Select
  Range(Selection,
Selection.End(xlDown)).Select
  Selection.Copy
  Sheets("MATRIXCREAT").Select
  Range("g1").Select
  Selection.PasteSpecial
Paste:=xlPasteValues,
Operation:=xlNone, SkipBlanks _
    :=False, Transpose:=False
'===========================
  Sheets("DATA").Select
```

```
   Range("h1:h" & num & "").Select
   Range(Selection,
Selection.End(xlDown)).Select
   Selection.Copy
   Sheets("MATRIXCREAT").Select
   Range("h1").Select
   Selection.PasteSpecial
Paste:=xlPasteValues,
Operation:=xlNone, SkipBlanks _
      :=False, Transpose:=False
'==========================
Selection.End(xlUp).Select
' SUMMATION TOOLBOX
   Range("A1").Select
   Selection.End(xlDown).Select
   Range("A" & num + 2 & "").Select
   ActiveCell.FormulaR1C1 =
"=SUM(R[-" & num & "]C:R[-1]C)"
   Range("A" & num + 2 & "").Select
   With Selection.Interior
      .ColorIndex = 6
      .Pattern = xlSolid
   End With
   Selection.Font.ColorIndex = 3

'==========================
'THE FIRST ROW
IDENTIFICATION IS BEING
WRITTEN IN THE FOLLOWING
COMMANDS
   Range("I1").Select
   ActiveCell.FormulaR1C1 = "X2^2"
'I3
   ActiveCell.Offset(1, 0).Activate
   ActiveCell.FormulaR1C1 = "=RC[-
8]^2"
   ActiveCell.Copy
   Range("I2:I" & num + 1 &
"").Select
   ActiveSheet.Paste
'==========================
   Range("J1").Select
   ActiveCell.FormulaR1C1 =
"X2X3"   'J3
   ActiveCell.Offset(1, 0).Activate
   ActiveCell.FormulaR1C1 = "=RC[-
9]*RC[-8]"
   ActiveCell.Copy
   Range("j2:j" & num + 1 &
"").Select
   ActiveSheet.Paste
'==========================
   Range("K1").Select
   ActiveCell.FormulaR1C1 =
"X2X4"   'K3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
10]*RC[-8]"
   ActiveCell.Copy
   Range("K2:K" & num + 1 &
"").Select
   ActiveSheet.Paste
'==========================
   Range("L1").Select
   ActiveCell.FormulaR1C1 =
"X2X5"   'L3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
11]*RC[-8]"
   ActiveCell.Copy
   Range("L2:L" & num + 1 &
"").Select
   ActiveSheet.Paste
'==========================
   Range("M1").Select
   ActiveCell.FormulaR1C1 =
"X2X6"   'M3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
12]*RC[-8]"
   ActiveCell.Copy
   Range("M2:M" & num + 1 &
"").Select
   ActiveSheet.Paste
'==========================
   Range("N1").Select
   ActiveCell.FormulaR1C1 =
"X2X7"   'N3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
13]*RC[-8]"
   ActiveCell.Copy
   Range("N2:N" & num + 1 &
"").Select
   ActiveSheet.Paste
'==========================
   Range("O1").Select
```

```
   ActiveCell.FormulaR1C1 =
"X2X8"   'O3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
14]*RC[-8]"
   ActiveCell.Copy
   Range("O2:O" & num + 1 &
"").Select
   ActiveSheet.Paste
'==========================
   Range("P1").Select
   ActiveCell.FormulaR1C1 =
"X3X2"   'P3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
14]*RC[-15]"
   ActiveCell.Copy
   Range("P2:P" & num + 1 &
"").Select
   ActiveSheet.Paste
'==========================
   Range("Q1").Select
   ActiveCell.FormulaR1C1 =
"X3X3"   'Q3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
15]*RC[-15]"
   ActiveCell.Copy
   Range("Q2:Q" & num + 1 &
"").Select
   ActiveSheet.Paste
'==========================
   Range("R1").Select
   ActiveCell.FormulaR1C1 =
"X3X4"   'R3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
16]*RC[-15]"
   ActiveCell.Copy
   Range("R2:R" & num + 1 &
"").Select
   ActiveSheet.Paste
'==========================
   Range("S1").Select
   ActiveCell.FormulaR1C1 =
"X3X5"   'S3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
17]*RC[-15]"
   ActiveCell.Copy
   Range("S2:S" & num + 1 &
"").Select
   ActiveSheet.Paste
'==========================
   Range("T1").Select
   ActiveCell.FormulaR1C1 =
"X3X6"   'T3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
18]*RC[-15]"
   ActiveCell.Copy
   Range("T2:T" & num + 1 &
"").Select
   ActiveSheet.Paste
'==========================
   Range("U1").Select
   ActiveCell.FormulaR1C1 =
"X3X7"   'U3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
19]*RC[-15]"
   ActiveCell.Copy
   Range("U2:U" & num + 1 &
"").Select
   ActiveSheet.Paste
'==========================
   Range("V1").Select
   ActiveCell.FormulaR1C1 =
"X3X8"   'V3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
20]*RC[-15]"
   ActiveCell.Copy
   Range("V2:V" & num + 1 &
"").Select
   ActiveSheet.Paste
'==========================
   Range("W1").Select
   ActiveCell.FormulaR1C1 =
"X4X2"   'W3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
20]*RC[-22]"
   ActiveCell.Copy
   Range("W2:W" & num + 1 &
"").Select
   ActiveSheet.Paste
‘=========================
```

```
   Range("X1").Select
   ActiveCell.FormulaR1C1 =
"X4X3"  'X3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
21]*RC[-22]"
   ActiveCell.Copy
   Range("X2:X" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
   Range("Y1").Select
   ActiveCell.FormulaR1C1 =
"X4X4"  'Y3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
22]*RC[-22]"
   ActiveCell.Copy
   Range("Y2:Y" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
=
   Range("Z1").Select
   ActiveCell.FormulaR1C1 =
"X4X5"  'Z3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
23]*RC[-22]"
   ActiveCell.Copy
   Range("Z2:Z" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
   Range("AA1").Select
   ActiveCell.FormulaR1C1 =
"X4X6"  'AA3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
24]*RC[-22]"
   ActiveCell.Copy
   Range("AA2:AA" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
   Range("AB1").Select
   ActiveCell.FormulaR1C1 =
"X4X7"  'AB3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
25]*RC[-22]"
   ActiveCell.Copy
   Range("AB2:AB" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
   Range("AC1").Select
   ActiveCell.FormulaR1C1 =
"X4X8"  'AC3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
26]*RC[-22]"
   ActiveCell.Copy
   Range("AC2:AC" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
   Range("AD1").Select
   ActiveCell.FormulaR1C1 =
"X5X2"  'AD3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=+RC[-
26]*RC[-29]"
   ActiveCell.Copy
   Range("AD2:AD" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
   Range("AE1").Select
   ActiveCell.FormulaR1C1 =
"X5X3"  'AE3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=+RC[-
27]*RC[-29]"
   ActiveCell.Copy
   Range("AE2:AE" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
   Range("AF1").Select
   ActiveCell.FormulaR1C1 =
"X5X4"  'AF3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=+RC[-
28]*RC[-29]"
   ActiveCell.Copy
   Range("AF2:AF" & num + 1 &
"").Select
```

```
   ActiveSheet.Paste
‘==========================
   Range("AG1").Select
   ActiveCell.FormulaR1C1 =
"X5X5"   'AG3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=+RC[-
29]*RC[-29]"
   ActiveCell.Copy
   Range("AG2:AG" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
   Range("AH1").Select
   ActiveCell.FormulaR1C1 =
"X5X6"   'AH3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=+RC[-
30]*RC[-29]"
   ActiveCell.Copy
   Range("AH2:AH" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
   Range("AI1").Select
   ActiveCell.FormulaR1C1 =
"X5X7"   'AI3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=+RC[-
31]*RC[-29]"
   ActiveCell.Copy
   Range("AI2:AI" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
   Range("AJ1").Select
   ActiveCell.FormulaR1C1 =
"X5X8"   'AJ3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=+RC[-
32]*RC[-29]"
   ActiveCell.Copy
   Range("AJ2:AJ" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
   Range("AK1").Select
   ActiveCell.FormulaR1C1 =
"X6X2"   'AK3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
32]*RC[-36]"
   ActiveCell.Copy
   Range("AK2:AK" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
   Range("AL1").Select
   ActiveCell.FormulaR1C1 =
"X6X3"   'AL3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
33]*RC[-36]"
   ActiveCell.Copy
   Range("AL2:AL" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
   Range("AM1").Select
   ActiveCell.FormulaR1C1 =
"X6X4"   'AM3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
34]*RC[-36]"
   ActiveCell.Copy
   Range("AM2:AM" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
   Range("AN1").Select
   ActiveCell.FormulaR1C1 =
"X6X5"   'AN3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
35]*RC[-36]"
   ActiveCell.Copy
   Range("AN2:AN" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
   Range("AO1").Select
   ActiveCell.FormulaR1C1 =
"X6X6"   'AO3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
36]*RC[-36]"
   ActiveCell.Copy
```

```
   Range("AO2:AO" & num + 1 &
"").Select
   ActiveSheet.Paste
‘=========================
   Range("AP1").Select
   ActiveCell.FormulaR1C1 =
"X6X7"   'AP3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
37]*RC[-36]"
   ActiveCell.Copy
   Range("AP2:AP" & num + 1 &
"").Select
   ActiveSheet.Paste
‘=========================
   Range("AQ1").Select
   ActiveCell.FormulaR1C1 =
"X6X8"   'AQ3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
38]*RC[-36]"
   ActiveCell.Copy
   Range("AQ2:AQ" & num + 1 &
"").Select
   ActiveSheet.Paste
‘=========================
   Range("AR1").Select
   ActiveCell.FormulaR1C1 =
"X7X2"   'AR3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
38]*RC[-43]"
   ActiveCell.Copy
   Range("AR2:AR" & num + 1 &
"").Select
   ActiveSheet.Paste
‘=========================
   Range("AS1").Select
   ActiveCell.FormulaR1C1 =
"X7X3"   'AS3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
39]*RC[-43]"
   ActiveCell.Copy
   Range("AS2:AS" & num + 1 &
"").Select
   ActiveSheet.Paste
‘=========================
   Range("AT1").Select
   ActiveCell.FormulaR1C1 =
"X7X4"   'AT3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
40]*RC[-43]"
   ActiveCell.Copy
   Range("AT2:AT" & num + 1 &
"").Select
   ActiveSheet.Paste
‘=========================
   Range("AU1").Select
   ActiveCell.FormulaR1C1 =
"X7X5"   'AU3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
41]*RC[-43]"
   ActiveCell.Copy
   Range("AU2:AU" & num + 1 &
"").Select
   ActiveSheet.Paste
‘=========================
   Range("AV1").Select
   ActiveCell.FormulaR1C1 =
"X7X6"   'AV3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
42]*RC[-43]"
   ActiveCell.Copy
   Range("AV2:AV" & num + 1 &
"").Select
   ActiveSheet.Paste
‘=========================
   Range("AW1").Select
   ActiveCell.FormulaR1C1 =
"X7X7"   'AW3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
43]*RC[-43]"
   ActiveCell.Copy
   Range("AW2:AW" & num + 1 &
"").Select
   ActiveSheet.Paste
‘=========================
   Range("AX1").Select
   ActiveCell.FormulaR1C1 =
"X7X8"   'AX3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
44]*RC[-43]"
```

```
   ActiveCell.Copy
   Range("AX2:AX" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
   Range("AY1").Select
   ActiveCell.FormulaR1C1 =
"X8X2"   'AY3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=+RC[-
44]*RC[-50]"
   ActiveCell.Copy
   Range("AY2:AY" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
   Range("AZ1").Select
   ActiveCell.FormulaR1C1 =
"X8X3"   'AZ3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=+RC[-
45]*RC[-50]"
   ActiveCell.Copy
   Range("AZ2:AZ" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
   Range("BA1").Select
   ActiveCell.FormulaR1C1 =
"X8X4"   'BA3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
46]*RC[-50]"
   ActiveCell.Copy
   Range("BA2:BA" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
   Range("BB1").Select
   ActiveCell.FormulaR1C1 =
"X8X5"   'BB3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=+RC[-
47]*RC[-50]"
   ActiveCell.Copy
   Range("BB2:BB" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
   Range("BC1").Select
   ActiveCell.FormulaR1C1 =
"X8X6"   'BC3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=+RC[-
48]*RC[-50]"
   ActiveCell.Copy
   Range("BC2:BC" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
   Range("BD1").Select
   ActiveCell.FormulaR1C1 =
"X8X7"   'BD3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=+RC[-
49]*RC[-50]"
   ActiveCell.Copy
   Range("BD2:BD" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
   Range("BE1").Select
   ActiveCell.FormulaR1C1 =
"X8X8"   'BE3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=+RC[-
50]*RC[-50]"
   ActiveCell.Copy
   Range("BE2:BE" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
   Range("BF1").Select
   ActiveCell.FormulaR1C1 =
"lnROP"   'BF3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
50]"
   ActiveCell.Copy
   Range("BF2:BF" & num + 1 &
"").Select
   ActiveSheet.Paste
‘==========================
   Range("BG1").Select
   ActiveCell.FormulaR1C1 =
"lnROPX2"   'BG3
   ActiveCell.Offset(1, 0).Activate
```

```
ActiveCell.FormulaR1C1 = "=RC[-
51]*RC[-58]"
   ActiveCell.Copy
   Range("BG2:BG" & num + 1 &
"").Select
   ActiveSheet.Paste
‘=========================
   Range("BH1").Select
   ActiveCell.FormulaR1C1 =
"lnROPX3"   'BH3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
52]*RC[-58]"
   ActiveCell.Copy
   Range("BH2:BH" & num + 1 &
"").Select
   ActiveSheet.Paste
‘=========================
   Range("BI1").Select
   ActiveCell.FormulaR1C1 =
"lnROPX4"   'BI3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
53]*RC[-58]"
   ActiveCell.Copy
   Range("BI2:BI" & num + 1 &
"").Select
   ActiveSheet.Paste
‘=========================
   Range("BJ1").Select
   ActiveCell.FormulaR1C1 =
"lnROPX5"   'BJ3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
54]*RC[-58]"
   ActiveCell.Copy
   Range("BJ2:BJ" & num + 1 &
"").Select
   ActiveSheet.Paste
‘=========================
   Range("BK1").Select
   ActiveCell.FormulaR1C1 =
"lnROPX6"   'BK3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
55]*RC[-58]"
   ActiveCell.Copy
   Range("BK2:BK" & num + 1 &
"").Select
   ActiveSheet.Paste
‘=========================
   Range("BL1").Select
   ActiveCell.FormulaR1C1 =
"lnROPX7"   'BL3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
56]*RC[-58]"
   ActiveCell.Copy
   Range("BL2:BL" & num + 1 &
"").Select
   ActiveSheet.Paste
‘=========================
   Range("BM1").Select
   ActiveCell.FormulaR1C1 =
"lnROPX8"   'BM3
   ActiveCell.Offset(1, 0).Activate
ActiveCell.FormulaR1C1 = "=RC[-
57]*RC[-58]"
   ActiveCell.Copy
   Range("BM2:BM" & num + 1 &
"").Select
   ActiveSheet.Paste
‘=========================


  Range("A" & num + 2 & "").Select
   Selection.Copy
   Range("B" & num + 2 & ":BM" &
num + 2 & "").Activate   ' HERE THE
SUMMATION IS BEING COPY
PASTED TO Bxx TO Hxx
   ActiveSheet.Paste
   Application.CutCopyMode = False

 Range("BM1").Select
   Range("BP1").Select   ' MATRIX
TITLE LINE IS BEING
GENERATED HERE
   ActiveCell.FormulaR1C1 = "n"
   ActiveCell.Offset(1, 0).Activate
   ActiveCell.FormulaR1C1 =
"=COUNT(C[-67])-1"
   ActiveCell.Offset(0, 1).Activate
'X2 column row1
   ActiveCell.FormulaR1C1 = "=+R["
& num & "]C[-68]"  ' num is the
number of data
   ActiveCell.Offset(0, 1).Activate
'X3 column row1
```

```
   ActiveCell.FormulaR1C1 = "=+R[" & num & "]C[-68]"
       ActiveCell.Offset(0, 1).Activate 'X4 column row1
   ActiveCell.FormulaR1C1 = "=+R[" & num & "]C[-68]"
         ActiveCell.Offset(0, 1).Activate 'X5 column row1
   ActiveCell.FormulaR1C1 = "=+R[" & num & "]C[-68]"
         ActiveCell.Offset(0, 1).Activate 'X6 column row1
   ActiveCell.FormulaR1C1 = "=+R[" & num & "]C[-68]"
         ActiveCell.Offset(0, 1).Activate 'X7 column row1
   ActiveCell.FormulaR1C1 = "=+R[" & num & "]C[-68]"
         ActiveCell.Offset(0, 1).Activate 'X8 column row1
   ActiveCell.FormulaR1C1 = "=+R[" & num & "]C[-68]"

    Range("Bp3").Select 'matrix Line-2 Column-1
ActiveCell.FormulaR1C1 = "=R[-1]C[1]"
    Range("Bp4").Select 'matrix Line-3 Column-1
ActiveCell.FormulaR1C1 = "=R[-2]C[2]"
       Range("Bp5").Select 'matrix Line-4 Column-1
ActiveCell.FormulaR1C1 = "=R[-3]C[3]"
          Range("Bp6").Select 'matrix Line-5 Column-1
ActiveCell.FormulaR1C1 = "=R[-4]C[4]"
Range("Bp7").Select 'matrix Line-6 Column-1
ActiveCell.FormulaR1C1 = "=R[-5]C[5]"
   Range("Bp8").Select 'matrix Line-7 Column-1
ActiveCell.FormulaR1C1 = "=R[-6]C[6]"
      Range("Bp9").Select 'matrix Line-8 Column-1
ActiveCell.FormulaR1C1 = "=R[-7]C[7]"

      Range("BQ1").Select
   ActiveCell.FormulaR1C1 = "X2"
    Range("BQ3").Select 'matrix Line-2 Column-2
 ActiveCell.FormulaR1C1 = "=R[" & num - 1 & "]C[-60]"
   Range("Br3").Select 'matrix Line-2 Column-3
ActiveCell.FormulaR1C1 = "=R[" & num - 1 & "]C[-60]"
   Range("Bs3").Select 'matrix Line-2 Column-4
ActiveCell.FormulaR1C1 = "=R[" & num - 1 & "]C[-60]"
   Range("bt3").Select 'matrix Line-2 Column-5
ActiveCell.FormulaR1C1 = "=R[" & num - 1 & "]C[-60]"
   Range("bu3").Select 'matrix Line-2 Column-6
ActiveCell.FormulaR1C1 = "=R[" & num - 1 & "]C[-60]"
   Range("bv3").Select 'matrix Line-2 Column-7
ActiveCell.FormulaR1C1 = "=R[" & num - 1 & "]C[-60]"
   Range("bw3").Select 'matrix Line-2 Column-8
ActiveCell.FormulaR1C1 = "=R[" & num - 1 & "]C[-60]"

   Range("Br1").Select
   ActiveCell.FormulaR1C1 = "X3"
   Range("BQ4").Select 'matrix Line-3 Column-2
   ActiveCell.FormulaR1C1 = "=R[" & num - 2 & "]C[-53]"
     Range("Br4").Select 'matrix Line-3 Column-3
   ActiveCell.FormulaR1C1 = "=R[" & num - 2 & "]C[-53]"
    Range("Bs4").Select 'matrix Line-3 Column-4
   ActiveCell.FormulaR1C1 = "=R[" & num - 2 & "]C[-53]"
```

```
    Range("Bt4").Select 'matrix Line-3 Column-5
   ActiveCell.FormulaR1C1 = "=R[" & num - 2 & "]C[-53]"
  Range("Bu4").Select 'matrix Line-3 Column-6
   ActiveCell.FormulaR1C1 = "=R[" & num - 2 & "]C[-53]"
  Range("Bv4").Select 'matrix Line-3 Column-7
   ActiveCell.FormulaR1C1 = "=R[" & num - 2 & "]C[-53]"
  Range("Bw4").Select 'matrix Line-3 Column-8
   ActiveCell.FormulaR1C1 = "=R[" & num - 2 & "]C[-53]"

   Range("BS1").Select
   ActiveCell.FormulaR1C1 = "X4"
    Range("BQ5").Select 'matrix Line-4 Column-2
   ActiveCell.FormulaR1C1 = "=R[" & num - 3 & "]C[-46]"
       Range("Br5").Select 'matrix Line-4 Column-3
   ActiveCell.FormulaR1C1 = "=R[" & num - 3 & "]C[-46]"
          Range("Bs5").Select 'matrix Line-4 Column-4
   ActiveCell.FormulaR1C1 = "=R[" & num - 3 & "]C[-46]"
      Range("Bt5").Select 'matrix Line-4 Column-5
   ActiveCell.FormulaR1C1 = "=R[" & num - 3 & "]C[-46]"
      Range("Bu5").Select 'matrix Line-4 Column-6
   ActiveCell.FormulaR1C1 = "=R[" & num - 3 & "]C[-46]"
      Range("Bv5").Select 'matrix Line-4 Column-7
   ActiveCell.FormulaR1C1 = "=R[" & num - 3 & "]C[-46]"
      Range("Bw5").Select 'matrix Line-4 Column-8
   ActiveCell.FormulaR1C1 = "=R[" & num - 3 & "]C[-46]"

    Range("BT1").Select
   ActiveCell.FormulaR1C1 = "X5"
      Range("BQ6").Select 'matrix Line-5 Column-2
   ActiveCell.FormulaR1C1 = "=R[" & num - 4 & "]C[-39]"
          Range("Br6").Select 'matrix Line-5 Column-3
   ActiveCell.FormulaR1C1 = "=R[" & num - 4 & "]C[-39]"
   Range("Bs6").Select 'matrix Line-5 Column-4
   ActiveCell.FormulaR1C1 = "=R[" & num - 4 & "]C[-39]"
   Range("Bt6").Select 'matrix Line-5 Column-5
   ActiveCell.FormulaR1C1 = "=R[" & num - 4 & "]C[-39]"
   Range("Bu6").Select 'matrix Line-5 Column-6
   ActiveCell.FormulaR1C1 = "=R[" & num - 4 & "]C[-39]"
   Range("Bv6").Select 'matrix Line-5 Column-7
   ActiveCell.FormulaR1C1 = "=R[" & num - 4 & "]C[-39]"
   Range("Bw6").Select 'matrix Line-5 Column-8
   ActiveCell.FormulaR1C1 = "=R[" & num - 4 & "]C[-39]"

   Range("BU1").Select
   ActiveCell.FormulaR1C1 = "X6"
Range("BQ7").Select 'matrix Line-6 Column-2
 ActiveCell.FormulaR1C1 = "=R[" & num - 5 & "]C[-32]"
Range("Br7").Select 'matrix Line-6 Column-3
 ActiveCell.FormulaR1C1 = "=R[" & num - 5 & "]C[-32]"
   Range("Bs7").Select 'matrix Line-6 Column-4
 ActiveCell.FormulaR1C1 = "=R[" & num - 5 & "]C[-32]"
   Range("Bt7").Select 'matrix Line-6 Column-5
 ActiveCell.FormulaR1C1 = "=R[" & num - 5 & "]C[-32]"
```

```
   Range("Bu7").Select 'matrix Line-6
Column-6
 ActiveCell.FormulaR1C1 = "=R[" &
num - 5 & "]C[-32]"
 Range("Bv7").Select 'matrix Line-6
Column-7
 ActiveCell.FormulaR1C1 = "=R[" &
num - 5 & "]C[-32]"
 Range("Bw7").Select 'matrix Line-6
Column-8
 ActiveCell.FormulaR1C1 = "=R[" &
num - 5 & "]C[-32]"

   Range("BV1").Select
   ActiveCell.FormulaR1C1 = "X7"
   Range("BQ8").Select 'matrix Line-
7 Column-2
ActiveCell.FormulaR1C1 = "=R[" &
num - 6 & "]C[-25]"
      Range("Br8").Select 'matrix
Line-7 Column-3
ActiveCell.FormulaR1C1 = "=R[" &
num - 6 & "]C[-25]"
        Range("Bs8").Select 'matrix
Line-7 Column-4
ActiveCell.FormulaR1C1 = "=R[" &
num - 6 & "]C[-25]"
         Range("Bt8").Select 'matrix
Line-7 Column-5
ActiveCell.FormulaR1C1 = "=R[" &
num - 6 & "]C[-25]"
         Range("Bu8").Select 'matrix
Line-7 Column-6
ActiveCell.FormulaR1C1 = "=R[" &
num - 6 & "]C[-25]"
         Range("Bv8").Select 'matrix
Line-7 Column-7
ActiveCell.FormulaR1C1 = "=R[" &
num - 6 & "]C[-25]"
         Range("Bw8").Select 'matrix
Line-7 Column-8
ActiveCell.FormulaR1C1 = "=R[" &
num - 6 & "]C[-25]"

   Range("BW1").Select
   ActiveCell.FormulaR1C1 = "X8"
      Range("BQ9").Select 'matrix
Line-8 Column-2
 ActiveCell.FormulaR1C1 = "=R[" &
num - 7 & "]C[-18]"
         Range("Br9").Select 'matrix
Line-8 Column-3
 ActiveCell.FormulaR1C1 = "=R[" &
num - 7 & "]C[-18]"
   Range("Bs9").Select 'matrix Line-8
Column-4
 ActiveCell.FormulaR1C1 = "=R[" &
num - 7 & "]C[-18]"
Range("Bt9").Select 'matrix Line-8
Column-5
 ActiveCell.FormulaR1C1 = "=R[" &
num - 7 & "]C[-18]"
  Range("Bu9").Select 'matrix Line-8
Column-6
 ActiveCell.FormulaR1C1 = "=R[" &
num - 7 & "]C[-18]"
     Range("Bv9").Select 'matrix Line-
8 Column-7
 ActiveCell.FormulaR1C1 = "=R[" &
num - 7 & "]C[-18]"
        Range("Bw9").Select 'matrix
Line-8 Column-8
 ActiveCell.FormulaR1C1 = "=R[" &
num - 7 & "]C[-18]"

   Range("BX1").Select
   ActiveCell.FormulaR1C1 = "a
values"

   Range("By1").Select
   ActiveCell.FormulaR1C1 =
"LnROP"
  Range("By2").Select ' 'lnROP
column line-1
ActiveCell.FormulaR1C1 = "=R[" &
num & "]C[-19]"
 Range("By3").Select ' 'lnROP
column line-2
ActiveCell.FormulaR1C1 = "=R[" &
num - 1 & "]C[-18]"
  Range("By4").Select ' 'lnROP
column line-3
ActiveCell.FormulaR1C1 = "=R[" &
num - 2 & "]C[-17]"
 Range("By5").Select ' 'lnROP
column line-4
```

```
ActiveCell.FormulaR1C1 = "=R[" &
num - 3 & "]C[-16]"
 Range("By6").Select ' 'lnROP
column line-5
ActiveCell.FormulaR1C1 = "=R[" &
num - 4 & "]C[-15]"
 Range("By7").Select ' 'lnROP
column line-6
ActiveCell.FormulaR1C1 = "=R[" &
num - 5 & "]C[-14]"
 Range("By8").Select ' 'lnROP
column line-7
ActiveCell.FormulaR1C1 = "=R[" &
num - 6 & "]C[-13]"
 Range("By9").Select ' 'lnROP
column line-8
ActiveCell.FormulaR1C1 = "=R[" &
num - 7 & "]C[-12]"

'  " & num & "
'  " & num + 2 & "

'Cells (row, column)
Range("B53:H53").Select
"ActiveCell.Offset(1, 0).Activate
‘==========================

matriks   ' RUNS THE matriks
PROCEDURE
============================
=======
Sheets("WORK").Select

‘==========================

' THIS PROCEDURE CAPTURES
THE "a" COEFFICIENTS FROM
THE MATRIX-MACRO1
WORKSHEET
   Range("W" & rangeend &
"").Select

   ActiveCell.FormulaR1C1 =
"='Matrx-Macro1'!R1C10"   'a1
 ActiveCell.Offset(0, 1).Activate    '
MOVE
    ActiveCell.FormulaR1C1 =
"='Matrx-Macro1'!R2C10"   'a2
```

```
 ActiveCell.Offset(0, 1).Activate    '
MOVE
     ActiveCell.FormulaR1C1 =
"='Matrx-Macro1'!R3C10"   'a3
 ActiveCell.Offset(0, 1).Activate    '
MOVE
     ActiveCell.FormulaR1C1 =
"='Matrx-Macro1'!R4C10"   'a4
 ActiveCell.Offset(0, 1).Activate    '
MOVE
     ActiveCell.FormulaR1C1 =
"='Matrx-Macro1'!R5C10"   'a5
 ActiveCell.Offset(0, 1).Activate    '
MOVE
     ActiveCell.FormulaR1C1 =
"='Matrx-Macro1'!R6C10"   'a6
 ActiveCell.Offset(0, 1).Activate    '
MOVE
     ActiveCell.FormulaR1C1 =
"='Matrx-Macro1'!R7C10"   'a7
      ActiveCell.Offset(0, 1).Activate
' MOVE
     ActiveCell.FormulaR1C1 =
"='Matrx-Macro1'!R8C10"   'a8

   Range("W" & rangeend & ":Ad" &
rangeend & "").Select
   Selection.Copy
   Selection.PasteSpecial
Paste:=xlPasteValues,
Operation:=xlNone, SkipBlanks _
       :=False, Transpose:=False
‘==========================
Range("s1").Select
‘==========================
Next i
‘==========================


End Sub

Private Sub
CommandButton2_Click()
UserForm1.Hide
End Sub



Private Sub TextBox2_Change()

End Sub
```

```
Private Sub UserForm_Initialize()
TextBox1.Value = 1
TextBox2.Value = 5 ' in case a data range of 1-4 is selected the multiple regression is not working
End Sub

Private Sub matriks()

'DET SOLUTION

Dim det As Single

Sheets("Matrx-Macro1").Activate

Cells(24, 1).Value = Cells(1, 1).Value
Cells(24, 2).Value = Cells(1, 2).Value
Cells(24, 3).Value = Cells(1, 3).Value
Cells(24, 4).Value = Cells(1, 4).Value
Cells(24, 5).Value = Cells(1, 5).Value
Cells(24, 6).Value = Cells(1, 6).Value
Cells(24, 7).Value = Cells(1, 7).Value
Cells(24, 8).Value = Cells(1, 8).Value

Cells(25, 1).Value = Cells(2, 1).Value
Cells(25, 2).Value = Cells(2, 2).Value
Cells(25, 3).Value = Cells(2, 3).Value
Cells(25, 4).Value = Cells(2, 4).Value
Cells(25, 5).Value = Cells(2, 5).Value
Cells(25, 6).Value = Cells(2, 6).Value
Cells(25, 7).Value = Cells(2, 7).Value
Cells(25, 8).Value = Cells(2, 8).Value

Cells(26, 1).Value = Cells(3, 1).Value
Cells(26, 2).Value = Cells(3, 2).Value
Cells(26, 3).Value = Cells(3, 3).Value
Cells(26, 4).Value = Cells(3, 4).Value
Cells(26, 5).Value = Cells(3, 5).Value
Cells(26, 6).Value = Cells(3, 6).Value
Cells(26, 7).Value = Cells(3, 7).Value
Cells(26, 8).Value = Cells(3, 8).Value

Cells(27, 1).Value = Cells(4, 1).Value
Cells(27, 2).Value = Cells(4, 2).Value
Cells(27, 3).Value = Cells(4, 3).Value
Cells(27, 4).Value = Cells(4, 4).Value
Cells(27, 5).Value = Cells(4, 5).Value
Cells(27, 6).Value = Cells(4, 6).Value
Cells(27, 7).Value = Cells(4, 7).Value
Cells(27, 8).Value = Cells(4, 8).Value

Cells(28, 1).Value = Cells(5, 1).Value
Cells(28, 2).Value = Cells(5, 2).Value
Cells(28, 3).Value = Cells(5, 3).Value
Cells(28, 4).Value = Cells(5, 4).Value
Cells(28, 5).Value = Cells(5, 5).Value
Cells(28, 6).Value = Cells(5, 6).Value
Cells(28, 7).Value = Cells(5, 7).Value
Cells(28, 8).Value = Cells(5, 8).Value

Cells(29, 1).Value = Cells(6, 1).Value
Cells(29, 2).Value = Cells(6, 2).Value
Cells(29, 3).Value = Cells(6, 3).Value
Cells(29, 4).Value = Cells(6, 4).Value
Cells(29, 5).Value = Cells(6, 5).Value
Cells(29, 6).Value = Cells(6, 6).Value
Cells(29, 7).Value = Cells(6, 7).Value
Cells(29, 8).Value = Cells(6, 8).Value

Cells(30, 1).Value = Cells(7, 1).Value
Cells(30, 2).Value = Cells(7, 2).Value
Cells(30, 3).Value = Cells(7, 3).Value
Cells(30, 4).Value = Cells(7, 4).Value
Cells(30, 5).Value = Cells(7, 5).Value
Cells(30, 6).Value = Cells(7, 6).Value
Cells(30, 7).Value = Cells(7, 7).Value
Cells(30, 8).Value = Cells(7, 8).Value

Cells(31, 1).Value = Cells(8, 1).Value
Cells(31, 2).Value = Cells(8, 2).Value
Cells(31, 3).Value = Cells(8, 3).Value
Cells(31, 4).Value = Cells(8, 4).Value
Cells(31, 5).Value = Cells(8, 5).Value
Cells(31, 6).Value = Cells(8, 6).Value
Cells(31, 7).Value = Cells(8, 7).Value
Cells(31, 8).Value = Cells(8, 8).Value

Cells(13, 2).Value = "=MDETERM(R24C1:R31C8)"

det = Cells(13, 2).Value

'DET SOLUTION

Dim DETA1 As Single

Cells(24, 1).Value = Cells(1, 9).Value
```

```
Cells(24, 2).Value = Cells(1, 2).Value
Cells(24, 3).Value = Cells(1, 3).Value
Cells(24, 4).Value = Cells(1, 4).Value
Cells(24, 5).Value = Cells(1, 5).Value
Cells(24, 6).Value = Cells(1, 6).Value
Cells(24, 7).Value = Cells(1, 7).Value
Cells(24, 8).Value = Cells(1, 8).Value

Cells(25, 1).Value = Cells(2, 9).Value
Cells(25, 2).Value = Cells(2, 2).Value
Cells(25, 3).Value = Cells(2, 3).Value
Cells(25, 4).Value = Cells(2, 4).Value
Cells(25, 5).Value = Cells(2, 5).Value
Cells(25, 6).Value = Cells(2, 6).Value
Cells(25, 7).Value = Cells(2, 7).Value
Cells(25, 8).Value = Cells(2, 8).Value

Cells(26, 1).Value = Cells(3, 9).Value
Cells(26, 2).Value = Cells(3, 2).Value
Cells(26, 3).Value = Cells(3, 3).Value
Cells(26, 4).Value = Cells(3, 4).Value
Cells(26, 5).Value = Cells(3, 5).Value
Cells(26, 6).Value = Cells(3, 6).Value
Cells(26, 7).Value = Cells(3, 7).Value
Cells(26, 8).Value = Cells(3, 8).Value

Cells(27, 1).Value = Cells(4, 9).Value
Cells(27, 2).Value = Cells(4, 2).Value
Cells(27, 3).Value = Cells(4, 3).Value
Cells(27, 4).Value = Cells(4, 4).Value
Cells(27, 5).Value = Cells(4, 5).Value
Cells(27, 6).Value = Cells(4, 6).Value
Cells(27, 7).Value = Cells(4, 7).Value
Cells(27, 8).Value = Cells(4, 8).Value

Cells(28, 1).Value = Cells(5, 9).Value
Cells(28, 2).Value = Cells(5, 2).Value
Cells(28, 3).Value = Cells(5, 3).Value
Cells(28, 4).Value = Cells(5, 4).Value
Cells(28, 5).Value = Cells(5, 5).Value
Cells(28, 6).Value = Cells(5, 6).Value
Cells(28, 7).Value = Cells(5, 7).Value
Cells(28, 8).Value = Cells(5, 8).Value

Cells(29, 1).Value = Cells(6, 9).Value
Cells(29, 2).Value = Cells(6, 2).Value
Cells(29, 3).Value = Cells(6, 3).Value
Cells(29, 4).Value = Cells(6, 4).Value
Cells(29, 5).Value = Cells(6, 5).Value
Cells(29, 6).Value = Cells(6, 6).Value
Cells(29, 7).Value = Cells(6, 7).Value
Cells(29, 8).Value = Cells(6, 8).Value

Cells(30, 1).Value = Cells(7, 9).Value
Cells(30, 2).Value = Cells(7, 2).Value
Cells(30, 3).Value = Cells(7, 3).Value
Cells(30, 4).Value = Cells(7, 4).Value
Cells(30, 5).Value = Cells(7, 5).Value
Cells(30, 6).Value = Cells(7, 6).Value
Cells(30, 7).Value = Cells(7, 7).Value
Cells(30, 8).Value = Cells(7, 8).Value

Cells(31, 1).Value = Cells(8, 9).Value
Cells(31, 2).Value = Cells(8, 2).Value
Cells(31, 3).Value = Cells(8, 3).Value
Cells(31, 4).Value = Cells(8, 4).Value
Cells(31, 5).Value = Cells(8, 5).Value
Cells(31, 6).Value = Cells(8, 6).Value
Cells(31, 7).Value = Cells(8, 7).Value
Cells(31, 8).Value = Cells(8, 8).Value

Cells(14, 2).Value =
"=MDETERM(R24C1:R31C8)"
A1 = Cells(14, 2).Value / det
Cells(1, 10).Value = A1

'DET SOLUTION

Dim DETA2 As Single

Cells(24, 1).Value = Cells(1, 1).Value
Cells(24, 2).Value = Cells(1, 9).Value
Cells(24, 3).Value = Cells(1, 3).Value
Cells(24, 4).Value = Cells(1, 4).Value
Cells(24, 5).Value = Cells(1, 5).Value
Cells(24, 6).Value = Cells(1, 6).Value
Cells(24, 7).Value = Cells(1, 7).Value
Cells(24, 8).Value = Cells(1, 8).Value

Cells(25, 1).Value = Cells(2, 1).Value
Cells(25, 2).Value = Cells(2, 9).Value
Cells(25, 3).Value = Cells(2, 3).Value
Cells(25, 4).Value = Cells(2, 4).Value
Cells(25, 5).Value = Cells(2, 5).Value
Cells(25, 6).Value = Cells(2, 6).Value
Cells(25, 7).Value = Cells(2, 7).Value
Cells(25, 8).Value = Cells(2, 8).Value
```

```
Cells(26, 1).Value = Cells(3, 1).Value
Cells(26, 2).Value = Cells(3, 9).Value
Cells(26, 3).Value = Cells(3, 3).Value
Cells(26, 4).Value = Cells(3, 4).Value
Cells(26, 5).Value = Cells(3, 5).Value
Cells(26, 6).Value = Cells(3, 6).Value
Cells(26, 7).Value = Cells(3, 7).Value
Cells(26, 8).Value = Cells(3, 8).Value

Cells(27, 1).Value = Cells(4, 1).Value
Cells(27, 2).Value = Cells(4, 9).Value
Cells(27, 3).Value = Cells(4, 3).Value
Cells(27, 4).Value = Cells(4, 4).Value
Cells(27, 5).Value = Cells(4, 5).Value
Cells(27, 6).Value = Cells(4, 6).Value
Cells(27, 7).Value = Cells(4, 7).Value
Cells(27, 8).Value = Cells(4, 8).Value

Cells(28, 1).Value = Cells(5, 1).Value
Cells(28, 2).Value = Cells(5, 9).Value
Cells(28, 3).Value = Cells(5, 3).Value
Cells(28, 4).Value = Cells(5, 4).Value
Cells(28, 5).Value = Cells(5, 5).Value
Cells(28, 6).Value = Cells(5, 6).Value
Cells(28, 7).Value = Cells(5, 7).Value
Cells(28, 8).Value = Cells(5, 8).Value

Cells(29, 1).Value = Cells(6, 1).Value
Cells(29, 2).Value = Cells(6, 9).Value
Cells(29, 3).Value = Cells(6, 3).Value
Cells(29, 4).Value = Cells(6, 4).Value
Cells(29, 5).Value = Cells(6, 5).Value
Cells(29, 6).Value = Cells(6, 6).Value
Cells(29, 7).Value = Cells(6, 7).Value
Cells(29, 8).Value = Cells(6, 8).Value

Cells(30, 1).Value = Cells(7, 1).Value
Cells(30, 2).Value = Cells(7, 9).Value
Cells(30, 3).Value = Cells(7, 3).Value
Cells(30, 4).Value = Cells(7, 4).Value
Cells(30, 5).Value = Cells(7, 5).Value
Cells(30, 6).Value = Cells(7, 6).Value
Cells(30, 7).Value = Cells(7, 7).Value
Cells(30, 8).Value = Cells(7, 8).Value

Cells(31, 1).Value = Cells(8, 1).Value
Cells(31, 2).Value = Cells(8, 9).Value
Cells(31, 3).Value = Cells(8, 3).Value
Cells(31, 4).Value = Cells(8, 4).Value
Cells(31, 5).Value = Cells(8, 5).Value
Cells(31, 6).Value = Cells(8, 6).Value
Cells(31, 7).Value = Cells(8, 7).Value
Cells(31, 8).Value = Cells(8, 8).Value

Cells(14, 2).Value =
"=MDETERM(R24C1:R31C8)"
A2 = Cells(14, 2).Value / det
Cells(2, 10).Value = A2

'DET SOLUTION

Dim DETA3 As Single

Cells(24, 1).Value = Cells(1, 1).Value
Cells(24, 2).Value = Cells(1, 2).Value
Cells(24, 3).Value = Cells(1, 9).Value
Cells(24, 4).Value = Cells(1, 4).Value
Cells(24, 5).Value = Cells(1, 5).Value
Cells(24, 6).Value = Cells(1, 6).Value
Cells(24, 7).Value = Cells(1, 7).Value
Cells(24, 8).Value = Cells(1, 8).Value

Cells(25, 1).Value = Cells(2, 1).Value
Cells(25, 2).Value = Cells(2, 2).Value
Cells(25, 3).Value = Cells(2, 9).Value
Cells(25, 4).Value = Cells(2, 4).Value
Cells(25, 5).Value = Cells(2, 5).Value
Cells(25, 6).Value = Cells(2, 6).Value
Cells(25, 7).Value = Cells(2, 7).Value
Cells(25, 8).Value = Cells(2, 8).Value

Cells(26, 1).Value = Cells(3, 1).Value
Cells(26, 2).Value = Cells(3, 2).Value
Cells(26, 3).Value = Cells(3, 9).Value
Cells(26, 4).Value = Cells(3, 4).Value
Cells(26, 5).Value = Cells(3, 5).Value
Cells(26, 6).Value = Cells(3, 6).Value
Cells(26, 7).Value = Cells(3, 7).Value
Cells(26, 8).Value = Cells(3, 8).Value

Cells(27, 1).Value = Cells(4, 1).Value
Cells(27, 2).Value = Cells(4, 2).Value
Cells(27, 3).Value = Cells(4, 9).Value
Cells(27, 4).Value = Cells(4, 4).Value
Cells(27, 5).Value = Cells(4, 5).Value
Cells(27, 6).Value = Cells(4, 6).Value
Cells(27, 7).Value = Cells(4, 7).Value
Cells(27, 8).Value = Cells(4, 8).Value
```

```
Cells(28, 1).Value = Cells(5, 1).Value
Cells(28, 2).Value = Cells(5, 2).Value
Cells(28, 3).Value = Cells(5, 9).Value
Cells(28, 4).Value = Cells(5, 4).Value
Cells(28, 5).Value = Cells(5, 5).Value
Cells(28, 6).Value = Cells(5, 6).Value
Cells(28, 7).Value = Cells(5, 7).Value
Cells(28, 8).Value = Cells(5, 8).Value

Cells(29, 1).Value = Cells(6, 1).Value
Cells(29, 2).Value = Cells(6, 2).Value
Cells(29, 3).Value = Cells(6, 9).Value
Cells(29, 4).Value = Cells(6, 4).Value
Cells(29, 5).Value = Cells(6, 5).Value
Cells(29, 6).Value = Cells(6, 6).Value
Cells(29, 7).Value = Cells(6, 7).Value
Cells(29, 8).Value = Cells(6, 8).Value

Cells(30, 1).Value = Cells(7, 1).Value
Cells(30, 2).Value = Cells(7, 2).Value
Cells(30, 3).Value = Cells(7, 9).Value
Cells(30, 4).Value = Cells(7, 4).Value
Cells(30, 5).Value = Cells(7, 5).Value
Cells(30, 6).Value = Cells(7, 6).Value
Cells(30, 7).Value = Cells(7, 7).Value
Cells(30, 8).Value = Cells(7, 8).Value

Cells(31, 1).Value = Cells(8, 1).Value
Cells(31, 2).Value = Cells(8, 2).Value
Cells(31, 3).Value = Cells(8, 9).Value
Cells(31, 4).Value = Cells(8, 4).Value
Cells(31, 5).Value = Cells(8, 5).Value
Cells(31, 6).Value = Cells(8, 6).Value
Cells(31, 7).Value = Cells(8, 7).Value
Cells(31, 8).Value = Cells(8, 8).Value

Cells(14, 2).Value =
"=MDETERM(R24C1:R31C8)"
A3 = Cells(14, 2).Value / det
Cells(3, 10).Value = A3

'DET SOLUTION

Dim DETA4 As Single

Cells(24, 1).Value = Cells(1, 1).Value
Cells(24, 2).Value = Cells(1, 2).Value
Cells(24, 3).Value = Cells(1, 3).Value
Cells(24, 4).Value = Cells(1, 9).Value
Cells(24, 5).Value = Cells(1, 5).Value
Cells(24, 6).Value = Cells(1, 6).Value
Cells(24, 7).Value = Cells(1, 7).Value
Cells(24, 8).Value = Cells(1, 8).Value

Cells(25, 1).Value = Cells(2, 1).Value
Cells(25, 2).Value = Cells(2, 2).Value
Cells(25, 3).Value = Cells(2, 3).Value
Cells(25, 4).Value = Cells(2, 9).Value
Cells(25, 5).Value = Cells(2, 5).Value
Cells(25, 6).Value = Cells(2, 6).Value
Cells(25, 7).Value = Cells(2, 7).Value
Cells(25, 8).Value = Cells(2, 8).Value

Cells(26, 1).Value = Cells(3, 1).Value
Cells(26, 2).Value = Cells(3, 2).Value
Cells(26, 3).Value = Cells(3, 3).Value
Cells(26, 4).Value = Cells(3, 9).Value
Cells(26, 5).Value = Cells(3, 5).Value
Cells(26, 6).Value = Cells(3, 6).Value
Cells(26, 7).Value = Cells(3, 7).Value
Cells(26, 8).Value = Cells(3, 8).Value

Cells(27, 1).Value = Cells(4, 1).Value
Cells(27, 2).Value = Cells(4, 2).Value
Cells(27, 3).Value = Cells(4, 3).Value
Cells(27, 4).Value = Cells(4, 9).Value
Cells(27, 5).Value = Cells(4, 5).Value
Cells(27, 6).Value = Cells(4, 6).Value
Cells(27, 7).Value = Cells(4, 7).Value
Cells(27, 8).Value = Cells(4, 8).Value

Cells(28, 1).Value = Cells(5, 1).Value
Cells(28, 2).Value = Cells(5, 2).Value
Cells(28, 3).Value = Cells(5, 3).Value
Cells(28, 4).Value = Cells(5, 9).Value
Cells(28, 5).Value = Cells(5, 5).Value
Cells(28, 6).Value = Cells(5, 6).Value
Cells(28, 7).Value = Cells(5, 7).Value
Cells(28, 8).Value = Cells(5, 8).Value

Cells(29, 1).Value = Cells(6, 1).Value
Cells(29, 2).Value = Cells(6, 2).Value
Cells(29, 3).Value = Cells(6, 3).Value
Cells(29, 4).Value = Cells(6, 9).Value
Cells(29, 5).Value = Cells(6, 5).Value
Cells(29, 6).Value = Cells(6, 6).Value
Cells(29, 7).Value = Cells(6, 7).Value
```

```
Cells(29, 8).Value = Cells(6, 8).Value

Cells(30, 1).Value = Cells(7, 1).Value
Cells(30, 2).Value = Cells(7, 2).Value
Cells(30, 3).Value = Cells(7, 3).Value
Cells(30, 4).Value = Cells(7, 9).Value
Cells(30, 5).Value = Cells(7, 5).Value
Cells(30, 6).Value = Cells(7, 6).Value
Cells(30, 7).Value = Cells(7, 7).Value
Cells(30, 8).Value = Cells(7, 8).Value

Cells(31, 1).Value = Cells(8, 1).Value
Cells(31, 2).Value = Cells(8, 2).Value
Cells(31, 3).Value = Cells(8, 3).Value
Cells(31, 4).Value = Cells(8, 9).Value
Cells(31, 5).Value = Cells(8, 5).Value
Cells(31, 6).Value = Cells(8, 6).Value
Cells(31, 7).Value = Cells(8, 7).Value
Cells(31, 8).Value = Cells(8, 8).Value

Cells(14, 2).Value =
"=MDETERM(R24C1:R31C8)"
A4 = Cells(14, 2).Value / det
Cells(4, 10).Value = A4

'DET SOLUTION

Dim DETA5 As Single

Cells(24, 1).Value = Cells(1, 1).Value
Cells(24, 2).Value = Cells(1, 2).Value
Cells(24, 3).Value = Cells(1, 3).Value
Cells(24, 4).Value = Cells(1, 4).Value
Cells(24, 5).Value = Cells(1, 9).Value
Cells(24, 6).Value = Cells(1, 6).Value
Cells(24, 7).Value = Cells(1, 7).Value
Cells(24, 8).Value = Cells(1, 8).Value

Cells(25, 1).Value = Cells(2, 1).Value
Cells(25, 2).Value = Cells(2, 2).Value
Cells(25, 3).Value = Cells(2, 3).Value
Cells(25, 4).Value = Cells(2, 4).Value
Cells(25, 5).Value = Cells(2, 9).Value
Cells(25, 6).Value = Cells(2, 6).Value
Cells(25, 7).Value = Cells(2, 7).Value
Cells(25, 8).Value = Cells(2, 8).Value

Cells(26, 1).Value = Cells(3, 1).Value
Cells(26, 2).Value = Cells(3, 2).Value
Cells(26, 3).Value = Cells(3, 3).Value
Cells(26, 4).Value = Cells(3, 4).Value
Cells(26, 5).Value = Cells(3, 9).Value
Cells(26, 6).Value = Cells(3, 6).Value
Cells(26, 7).Value = Cells(3, 7).Value
Cells(26, 8).Value = Cells(3, 8).Value

Cells(27, 1).Value = Cells(4, 1).Value
Cells(27, 2).Value = Cells(4, 2).Value
Cells(27, 3).Value = Cells(4, 3).Value
Cells(27, 4).Value = Cells(4, 4).Value
Cells(27, 5).Value = Cells(4, 9).Value
Cells(27, 6).Value = Cells(4, 6).Value
Cells(27, 7).Value = Cells(4, 7).Value
Cells(27, 8).Value = Cells(4, 8).Value

Cells(28, 1).Value = Cells(5, 1).Value
Cells(28, 2).Value = Cells(5, 2).Value
Cells(28, 3).Value = Cells(5, 3).Value
Cells(28, 4).Value = Cells(5, 4).Value
Cells(28, 5).Value = Cells(5, 9).Value
Cells(28, 6).Value = Cells(5, 6).Value
Cells(28, 7).Value = Cells(5, 7).Value
Cells(28, 8).Value = Cells(5, 8).Value

Cells(29, 1).Value = Cells(6, 1).Value
Cells(29, 2).Value = Cells(6, 2).Value
Cells(29, 3).Value = Cells(6, 3).Value
Cells(29, 4).Value = Cells(6, 4).Value
Cells(29, 5).Value = Cells(6, 9).Value
Cells(29, 6).Value = Cells(6, 6).Value
Cells(29, 7).Value = Cells(6, 7).Value
Cells(29, 8).Value = Cells(6, 8).Value

Cells(30, 1).Value = Cells(7, 1).Value
Cells(30, 2).Value = Cells(7, 2).Value
Cells(30, 3).Value = Cells(7, 3).Value
Cells(30, 4).Value = Cells(7, 4).Value
Cells(30, 5).Value = Cells(7, 9).Value
Cells(30, 6).Value = Cells(7, 6).Value
Cells(30, 7).Value = Cells(7, 7).Value
Cells(30, 8).Value = Cells(7, 8).Value

Cells(31, 1).Value = Cells(8, 1).Value
Cells(31, 2).Value = Cells(8, 2).Value
Cells(31, 3).Value = Cells(8, 3).Value
Cells(31, 4).Value = Cells(8, 4).Value
Cells(31, 5).Value = Cells(8, 9).Value
Cells(31, 6).Value = Cells(8, 6).Value
```

```
Cells(31, 7).Value = Cells(8, 7).Value
Cells(31, 8).Value = Cells(8, 8).Value

Cells(14, 2).Value =
"=MDETERM(R24C1:R31C8)"
A5 = Cells(14, 2).Value / det
Cells(5, 10).Value = A5

'DET SOLUTION

Dim DETA6 As Single

Cells(24, 1).Value = Cells(1, 1).Value
Cells(24, 2).Value = Cells(1, 2).Value
Cells(24, 3).Value = Cells(1, 3).Value
Cells(24, 4).Value = Cells(1, 4).Value
Cells(24, 5).Value = Cells(1, 5).Value
Cells(24, 6).Value = Cells(1, 9).Value
Cells(24, 7).Value = Cells(1, 7).Value
Cells(24, 8).Value = Cells(1, 8).Value

Cells(25, 1).Value = Cells(2, 1).Value
Cells(25, 2).Value = Cells(2, 2).Value
Cells(25, 3).Value = Cells(2, 3).Value
Cells(25, 4).Value = Cells(2, 4).Value
Cells(25, 5).Value = Cells(2, 5).Value
Cells(25, 6).Value = Cells(2, 9).Value
Cells(25, 7).Value = Cells(2, 7).Value
Cells(25, 8).Value = Cells(2, 8).Value

Cells(26, 1).Value = Cells(3, 1).Value
Cells(26, 2).Value = Cells(3, 2).Value
Cells(26, 3).Value = Cells(3, 3).Value
Cells(26, 4).Value = Cells(3, 4).Value
Cells(26, 5).Value = Cells(3, 5).Value
Cells(26, 6).Value = Cells(3, 9).Value
Cells(26, 7).Value = Cells(3, 7).Value
Cells(26, 8).Value = Cells(3, 8).Value

Cells(27, 1).Value = Cells(4, 1).Value
Cells(27, 2).Value = Cells(4, 2).Value
Cells(27, 3).Value = Cells(4, 3).Value
Cells(27, 4).Value = Cells(4, 4).Value
Cells(27, 5).Value = Cells(4, 5).Value
Cells(27, 6).Value = Cells(4, 9).Value
Cells(27, 7).Value = Cells(4, 7).Value
Cells(27, 8).Value = Cells(4, 8).Value

Cells(28, 1).Value = Cells(5, 1).Value
Cells(28, 2).Value = Cells(5, 2).Value
Cells(28, 3).Value = Cells(5, 3).Value
Cells(28, 4).Value = Cells(5, 4).Value
Cells(28, 5).Value = Cells(5, 5).Value
Cells(28, 6).Value = Cells(5, 9).Value
Cells(28, 7).Value = Cells(5, 7).Value
Cells(28, 8).Value = Cells(5, 8).Value

Cells(29, 1).Value = Cells(6, 1).Value
Cells(29, 2).Value = Cells(6, 2).Value
Cells(29, 3).Value = Cells(6, 3).Value
Cells(29, 4).Value = Cells(6, 4).Value
Cells(29, 5).Value = Cells(6, 5).Value
Cells(29, 6).Value = Cells(6, 9).Value
Cells(29, 7).Value = Cells(6, 7).Value
Cells(29, 8).Value = Cells(6, 8).Value

Cells(30, 1).Value = Cells(7, 1).Value
Cells(30, 2).Value = Cells(7, 2).Value
Cells(30, 3).Value = Cells(7, 3).Value
Cells(30, 4).Value = Cells(7, 4).Value
Cells(30, 5).Value = Cells(7, 5).Value
Cells(30, 6).Value = Cells(7, 9).Value
Cells(30, 7).Value = Cells(7, 7).Value
Cells(30, 8).Value = Cells(7, 8).Value

Cells(31, 1).Value = Cells(8, 1).Value
Cells(31, 2).Value = Cells(8, 2).Value
Cells(31, 3).Value = Cells(8, 3).Value
Cells(31, 4).Value = Cells(8, 4).Value
Cells(31, 5).Value = Cells(8, 5).Value
Cells(31, 6).Value = Cells(8, 9).Value
Cells(31, 7).Value = Cells(8, 7).Value
Cells(31, 8).Value = Cells(8, 8).Value

Cells(14, 2).Value =
"=MDETERM(R24C1:R31C8)"
A6 = Cells(14, 2).Value / det
Cells(6, 10).Value = A6

'DET SOLUTION

Dim DETA7 As Single

Cells(24, 1).Value = Cells(1, 1).Value
Cells(24, 2).Value = Cells(1, 2).Value
Cells(24, 3).Value = Cells(1, 3).Value
Cells(24, 4).Value = Cells(1, 4).Value
Cells(24, 5).Value = Cells(1, 5).Value
```

```
Cells(24, 6).Value = Cells(1, 6).Value
Cells(24, 7).Value = Cells(1, 9).Value
Cells(24, 8).Value = Cells(1, 8).Value

Cells(25, 1).Value = Cells(2, 1).Value
Cells(25, 2).Value = Cells(2, 2).Value
Cells(25, 3).Value = Cells(2, 3).Value
Cells(25, 4).Value = Cells(2, 4).Value
Cells(25, 5).Value = Cells(2, 5).Value
Cells(25, 6).Value = Cells(2, 6).Value
Cells(25, 7).Value = Cells(2, 9).Value
Cells(25, 8).Value = Cells(2, 8).Value

Cells(26, 1).Value = Cells(3, 1).Value
Cells(26, 2).Value = Cells(3, 2).Value
Cells(26, 3).Value = Cells(3, 3).Value
Cells(26, 4).Value = Cells(3, 4).Value
Cells(26, 5).Value = Cells(3, 5).Value
Cells(26, 6).Value = Cells(3, 6).Value
Cells(26, 7).Value = Cells(3, 9).Value
Cells(26, 8).Value = Cells(3, 8).Value

Cells(27, 1).Value = Cells(4, 1).Value
Cells(27, 2).Value = Cells(4, 2).Value
Cells(27, 3).Value = Cells(4, 3).Value
Cells(27, 4).Value = Cells(4, 4).Value
Cells(27, 5).Value = Cells(4, 5).Value
Cells(27, 6).Value = Cells(4, 6).Value
Cells(27, 7).Value = Cells(4, 9).Value
Cells(27, 8).Value = Cells(4, 8).Value

Cells(28, 1).Value = Cells(5, 1).Value
Cells(28, 2).Value = Cells(5, 2).Value
Cells(28, 3).Value = Cells(5, 3).Value
Cells(28, 4).Value = Cells(5, 4).Value
Cells(28, 5).Value = Cells(5, 5).Value
Cells(28, 6).Value = Cells(5, 6).Value
Cells(28, 7).Value = Cells(5, 9).Value
Cells(28, 8).Value = Cells(5, 8).Value

Cells(29, 1).Value = Cells(6, 1).Value
Cells(29, 2).Value = Cells(6, 2).Value
Cells(29, 3).Value = Cells(6, 3).Value
Cells(29, 4).Value = Cells(6, 4).Value
Cells(29, 5).Value = Cells(6, 5).Value
Cells(29, 6).Value = Cells(6, 6).Value
Cells(29, 7).Value = Cells(6, 9).Value
Cells(29, 8).Value = Cells(6, 8).Value

Cells(30, 1).Value = Cells(7, 1).Value
Cells(30, 2).Value = Cells(7, 2).Value
Cells(30, 3).Value = Cells(7, 3).Value
Cells(30, 4).Value = Cells(7, 4).Value
Cells(30, 5).Value = Cells(7, 5).Value
Cells(30, 6).Value = Cells(7, 6).Value
Cells(30, 7).Value = Cells(7, 9).Value
Cells(30, 8).Value = Cells(7, 8).Value

Cells(31, 1).Value = Cells(8, 1).Value
Cells(31, 2).Value = Cells(8, 2).Value
Cells(31, 3).Value = Cells(8, 3).Value
Cells(31, 4).Value = Cells(8, 4).Value
Cells(31, 5).Value = Cells(8, 5).Value
Cells(31, 6).Value = Cells(8, 6).Value
Cells(31, 7).Value = Cells(8, 9).Value
Cells(31, 8).Value = Cells(8, 8).Value

Cells(14, 2).Value =
"=MDETERM(R24C1:R31C8)"
A7 = Cells(14, 2).Value / det
Cells(7, 10).Value = A7

'DET SOLUTION

Dim DETA8 As Single

Cells(24, 1).Value = Cells(1, 1).Value
Cells(24, 2).Value = Cells(1, 2).Value
Cells(24, 3).Value = Cells(1, 3).Value
Cells(24, 4).Value = Cells(1, 4).Value
Cells(24, 5).Value = Cells(1, 5).Value
Cells(24, 6).Value = Cells(1, 6).Value
Cells(24, 7).Value = Cells(1, 7).Value
Cells(24, 8).Value = Cells(1, 9).Value

Cells(25, 1).Value = Cells(2, 1).Value
Cells(25, 2).Value = Cells(2, 2).Value
Cells(25, 3).Value = Cells(2, 3).Value
Cells(25, 4).Value = Cells(2, 4).Value
Cells(25, 5).Value = Cells(2, 5).Value
Cells(25, 6).Value = Cells(2, 6).Value
Cells(25, 7).Value = Cells(2, 7).Value
Cells(25, 8).Value = Cells(2, 9).Value

Cells(26, 1).Value = Cells(3, 1).Value
Cells(26, 2).Value = Cells(3, 2).Value
Cells(26, 3).Value = Cells(3, 3).Value
Cells(26, 4).Value = Cells(3, 4).Value
```

```
Cells(26, 5).Value = Cells(3, 5).Value
Cells(26, 6).Value = Cells(3, 6).Value
Cells(26, 7).Value = Cells(3, 7).Value
Cells(26, 8).Value = Cells(3, 9).Value

Cells(27, 1).Value = Cells(4, 1).Value
Cells(27, 2).Value = Cells(4, 2).Value
Cells(27, 3).Value = Cells(4, 3).Value
Cells(27, 4).Value = Cells(4, 4).Value
Cells(27, 5).Value = Cells(4, 5).Value
Cells(27, 6).Value = Cells(4, 6).Value
Cells(27, 7).Value = Cells(4, 7).Value
Cells(27, 8).Value = Cells(4, 9).Value

Cells(28, 1).Value = Cells(5, 1).Value
Cells(28, 2).Value = Cells(5, 2).Value
Cells(28, 3).Value = Cells(5, 3).Value
Cells(28, 4).Value = Cells(5, 4).Value
Cells(28, 5).Value = Cells(5, 5).Value
Cells(28, 6).Value = Cells(5, 6).Value
Cells(28, 7).Value = Cells(5, 7).Value
Cells(28, 8).Value = Cells(5, 9).Value

Cells(29, 1).Value = Cells(6, 1).Value
Cells(29, 2).Value = Cells(6, 2).Value
Cells(29, 3).Value = Cells(6, 3).Value
Cells(29, 4).Value = Cells(6, 4).Value
Cells(29, 5).Value = Cells(6, 5).Value
Cells(29, 6).Value = Cells(6, 6).Value
Cells(29, 7).Value = Cells(6, 7).Value
Cells(29, 8).Value = Cells(6, 9).Value

Cells(30, 1).Value = Cells(7, 1).Value
Cells(30, 2).Value = Cells(7, 2).Value
Cells(30, 3).Value = Cells(7, 3).Value
Cells(30, 4).Value = Cells(7, 4).Value
Cells(30, 5).Value = Cells(7, 5).Value
Cells(30, 6).Value = Cells(7, 6).Value
Cells(30, 7).Value = Cells(7, 7).Value
Cells(30, 8).Value = Cells(7, 9).Value

Cells(31, 1).Value = Cells(8, 1).Value
Cells(31, 2).Value = Cells(8, 2).Value
Cells(31, 3).Value = Cells(8, 3).Value
Cells(31, 4).Value = Cells(8, 4).Value
Cells(31, 5).Value = Cells(8, 5).Value
Cells(31, 6).Value = Cells(8, 6).Value
Cells(31, 7).Value = Cells(8, 7).Value
Cells(31, 8).Value = Cells(8, 9).Value

Cells(14, 2).Value =
"=MDETERM(R24C1:R31C8)"
A8 = Cells(14, 2).Value / det
Cells(8, 10).Value = A8
End Sub
```

APPENDIX E

MULTIPLE REGRESSION EXAMPLE

An example is considered to be useful and included as an appendix to help the reader understand how the calculations have been performed. An example solution for the data provided in Bourgoyne and Youngs' paper [14] is given. The available data is composed of 30 points, as given in **Table E-1**. The given data are belonging to different bit records from different wells drilled with various drilling parameters.

Table E-1: Data set for data after Bourgoyne and Young [14].

Data Entry	Depth, ft	Bit Number	Drilling Rate, ft/hr	Bit Weight (1000 lb/in.)	Rotary Speed, rpm	Tooth Wear	Reynolds Number Function	ECD, lb/gal	Pore Gradient, ppg
1	9515	7	23	2.58	113	0.77	0.964	9.5	9.0
2	9830	8	22	1.15	126	0.38	0.964	9.5	9.0
3	10130	9	14	0.81	129	0.74	0.827	9.6	9.0
4	10250	11	10	0.95	87	0.15	0.976	9.7	9.0
5	10390	12	16	1.02	78	0.24	0.984	9.7	9.0
6	10500		19	1.69	81	0.61	0.984	9.7	9.1
7	10575		13	1.56	81	0.73	0.984	9.7	9.2
8	10840	13	16.6	1.63	67	0.38	0.932	9.8	9.3
9	10960		15.9	1.83	65	0.57	0.878	9.8	9.4
10	11060		15.7	2.03	69	0.72	0.878	9.8	9.5
11	11475	15	14	1.69	77	0.2	0.887	10.3	9.5
12	11775	18	13.5	2.31	58	0.12	0.852	11.8	10.1
13	11940	21	6.2	2.26	67	0.2	0.976	15.3	12.4
14	12070	22	9.6	2.07	84	0.08	0.993	15.7	13
15	12315		15.5	3.11	69	0.4	1.185	16.3	14.4
16	12900	23	31.4	2.82	85	0.42	1.15	16.7	15.9
17	12975	24	42.7	3.48	77	0.17	1.221	16.7	16.1
18	13055		38.6	3.29	75	0.29	1.161	16.8	16.2
19	13250		43.4	2.82	76	0.43	1.161	16.8	16.2
20	13795	25	12.5	1.6	81	0.56	0.272	16.8	16.2
21	14010	26	21.1	1.04	75	0.46	0.201	16.8	16.2
22	14455	28	19	1.76	64	0.16	0.748	16.9	16.2
23	14695		18.7	2	76	0.27	0.819	17.1	16.2
24	14905	29	20.2	2.35	75	0.33	0.419	17.2	16.4
25	15350	30	27.1	2.12	85	0.31	1.29	17	16.5
26	15740		14.8	2.35	78	0.81	0.802	17.3	16.5
27	16155	32	12.6	2.47	80	0.12	0.67	17.9	16.5
28	16325		14.9	3.76	81	0.5	0.532	17.5	16.6
29	17060	34	13.8	3.76	65	0.91	0.748	17.6	16.6
30	20265	40	9	3.4	60	0.01	0.512	17.7	16

Multiple Regression technique has been applied in order to predict the ROP values. The procedure of solution is as defined in “Section 6.3”, however schematically a flow chart has been illustrated in **Figure E-1**. The minimum number of data set for a multiple regression to solve a matrix of 8x8 is 5. The coefficients for the first 5 data set is first solved, and then the loop is repeated in order to solve coefficients this time including one more set of data e.g., 5+1=6 data. The loop is continued until the number of requested data sets is processed.

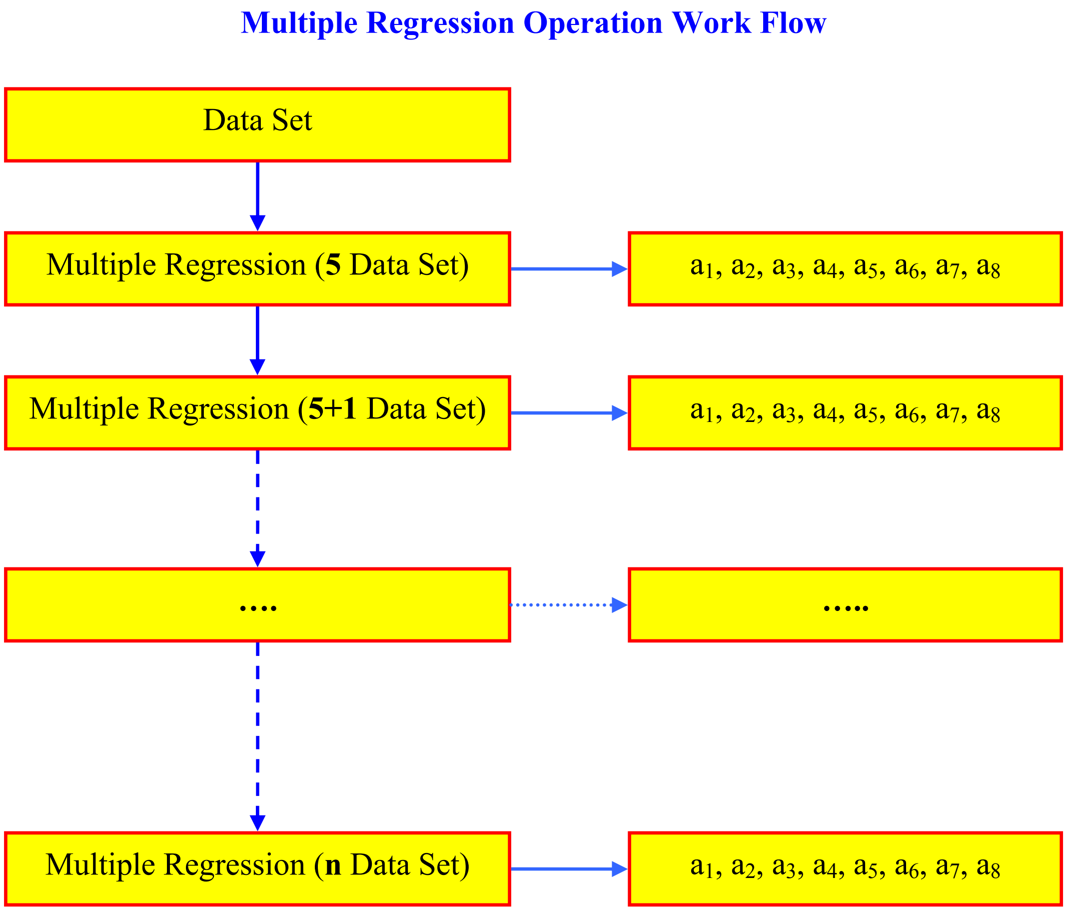


Figure E-1: Multiple regression workflow.

An example matrix is as given in **Table E-2**. It should be inferred that the number of data inputs to the construction of this matrix is 30, the upper leftmost number. The solution of the coefficients are as give in the rightmost column respectively from a_1, a_2, …, a_8.

A macro program in the Visual Basic Module [88] of Excel [90] has been written in order to perform all of the necessary calculations which return the matrix determinant of the defined array.

Table E-2: An example matrix, its equivalent and solutions of coefficients.

Matrix constructed with 30 data inputs								Ln (ROP)	“a” Coefficients
30	-88560	93471.3203	-369695.115	-26.35778	-7.43477	-12.04	25.97	84.9779155	3.359494485
-88560	450821950	-450311512	1411809496	52857.14	27863.81	32443.55	-68119.8	-248801.26	0.000080761
93471.32	-450311511.5	490330657.4	-1390789267	-55465.58	-27524.53	-34867.25	74583.46	272081.389	0.000132413
-369695.1	1411809496	-1390789267	6882496371	260813.7	110157.2	118135.9	-311257.9	-979933.23	0.000041084
-26.35778	52857.13725	-55465.5826	260813.7142	33.90434	5.002399	10.79968	-22.1088	-72.977011	0.271954573
-7.43477	27863.80759	-27524.5336	110157.2277	5.002399	2.846352	2.603319	-6.228022	-20.60145	0.314436477
-12.04	32443.55	-34867.2542	118135.9193	10.79968	2.603319	6.5702	-10.26883	-34.438513	0.467403443
25.97	-68119.8	74583.46462	-311257.87	-22.1088	-6.228022	-10.26883	24.55605	74.8467387	0.357728752

In order to make sure the macro code was functioning properly the results were compared with the solutions obtained from “*Online Matrix Calculator Systems of Linear Equations*” available from a reliable source in World Wide Web [91].

Four different solutions are as compared in Rate of Penetration vs depth graph **Figure E-2**. The given results are as summarized as following, **Table E-3**.

Table E-3: Results of the Bourgoyne and Young data set comparison.

ROP Result	**Definition**
Drilling Rate	This is the actual rate as has been reported in the data set.
Predicted ROP	This is the predicted rate of penetration with the most accurate “a” coefficients.
Predicted ROP (Internet)	This is the predicted rate of penetration in which the matrix has been solved using the program available in the WWW [91].
Predicted ROP with instant data	This is the predicted rate of penetration with progressive data expansion.

It should be observed that the predicted ROP results of which achieved using the specific macro program and the results obtained using the internet matrix solver are giving an exact match. On the other hand the results of the predicted ROP with instant data is showing some spiky results in the initial phase, however it is then catching a good trend and as should be expected giving an exact match for the final data with the predicted ROP data set. This is due to the reason that the final progressive data expansion with the instant data line is with the same number of dataset with the whole dataset.

All of the ROP prediction trends are showing close match with the results reported in the original reference.

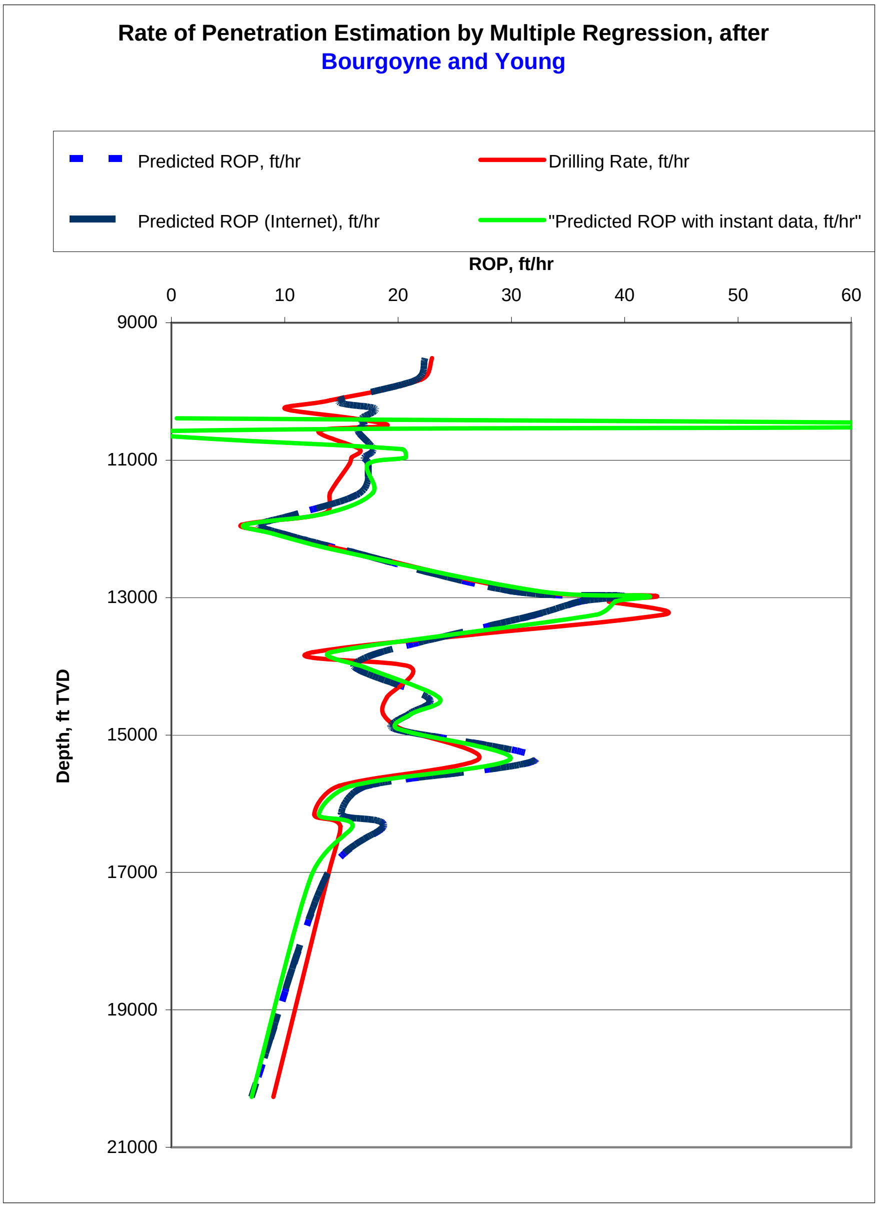


Figure E-2: ROP chart with actual and predicted results.

APPENDIX F

INTERPOLATION CODE

```
Private Sub CommandButton1_Click()

" Interpolation Program Code

Dim num, num2, sayi, bul1, bul2, cvp, interpsayi As Long

      Range("H2").Select
    ActiveCell.FormulaR1C1 = "=COUNT(C[-7])"
    Range("I2").Select
    ActiveCell.FormulaR1C1 = "=COUNT(C[-4])"

   Range("h2").Select
   sayi = Range("h2").Value

     Range("i2").Select
   interpsayi = Range("i2").Value
   For num2 = 2 To interpsayi + 1

   Range("e" & num2).Select
   bul1 = Range("e" & num2).Value ' the number that is required to be interpolated

   For num = 2 To sayi + 1

      If Range("a" & num).Value > bul1 Then
bul1 = num - 1
Exit For
End If
Next num
bul2 = bul1 + 1
cvp = Range("b" & bul1).Value + (Range("e" & num2).Value - Range("a" &
bul1).Value) * (Range("b" & bul2).Value - Range("b" & bul1).Value) / (Range("a"
& bul2).Value - Range("a" & bul1).Value)
Range("f" & num2).Select
Range("f" & num2).Value = cvp
Next num2
End Sub
```

CURRICULUM VITAE

PERSONAL INFORMATION

Surname, Name: Eren, Tuna
Nationality: Turkish (TC)
Date and Place of Birth: 16 May 1978, Midyat
Marital Status: Single
Phone: +90 312 241 25 38
email: e122413@metu.edu.tr

EDUCATION

Degree	**Institution**	**Year of Graduation**
MSc	METU Pet. & Nat. Gas Eng. Dept.	2004
BSc	Hacettepe Uni, Mining Engineering Dept.	2000
High School	TED Batman Petroleum College, Batman	1996

WORK EXPERIENCE

Year	**Place**	**Enrolment**
2007- Present	Eni E&P	Drilling & Comp. Eng.
2004-2007	TPAO	Drilling Engineer
2003-2004	METU Pet. & Nat. Gas Eng. Dept.	Project Assistant
2001-2003	Schlumberger	Field Engineer

FOREIGN LANGUAGES

English, Italian, Arabic.

PUBLICATIONS

1. Eren T., and Ozbayoglu E., "*Real Time Optimization of Drilling Parameters During Drilling Operations*" SPE Paper 129126, SPE Oil and Gas India Conference and Exhibition, Mumbai, India, 20–22 January 2010
2. Ozbayoglu E., Akin S., Eren T., "*Image Processing Techniques in Foam Characterization*", Energy Sources, Part A 29:1175-1185, 2007, Taylor & Francis Group, LLC
3. Eren T., Ozkale A., Ozer C., Kirbiyik S., "*A Case Study: Comparison of the Theoretical and Actual Drilling Hydraulic Pressures for Wells in Turkey*", 17th International Petroleum and Natural Gas Congress and Exhibition of Turkey, 29-31 May 2007, Turkey
4. Eren T., Ozbayoglu E., Akin S., "*Analysis of the influence of bubble size and texture on foam characterization*", 12th Multiphase Production Technology Conference, 25-28 May 2005 Barcelona, Spain
5. Ozbayoglu E., Akin S., Eren T, Gucuyener I.H., Kok M.V.,"*Rheological Characterization of Foam Using Image Processing Techniques*", 15th International Petroleum and Natural Gas Congress and Exhibition of Turkey, 11-13 May 2005, Turkey
6. Ozbayoglu E., Akin S., Eren T., "*Foam Characterization Using Image Processing Techniques*" SPE Paper 93860, SPE Western Regional Meeting, 30 March – 1 April 2005, Irvine, CA, USA

HOBBIES

Driving, travelling. Electronics. Computer technologies. Reading.