STOCHASTIC INVENTORY MODELLING

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ABSTRACT

STOCHASTIC INVENTORY MODELLING

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In this master thesis study, new inventory control mechanisms are developed for the repairables in Nedtrain. There is a multi-item, multi echelon system with a continuous review and one for one replenishment policy and there are different demand supply options in each control mechanism. There is an aggregate mean waiting time constraint in each local warehouse and the objective is to minimize the total system cost. The base stock levels in each warehouse are determined with an approximation method. Then different demand supply options are compared with each other.

Keywords: Inventory control mechanism, base stock level, lateral and direct shipment

ÖZ

STOKASTİK ENVANTER MODELLEMESİ

Özkan, Erhun

Yüksek Lisans, Endüstri Mühendisliği Bölümü

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Bu tezde, Nedtrain'deki tamir edilebilir parçalar için yeni envanter kontrol mekanizmaları geliştirildi. Sürekli takip edilen ve teker teker ikmal edilen, çok parçalı, çok kademeli bir sistem incelenmiştir ve her bir kontrol mekanizmasında farklı bir talep karşılama seçeneği vardır. Her bir yerel envanter noktasında, talepler için bir bekleme süresi kısıtı vardır; ve amaç, toplam sistem maliyetini minimize etmektir. Yerel ve merkezi depodaki en iyi "base stock" (S-1,S) politikasını bulmaya yönelik bir sezgisel yöntem önerilip benzerleri ile karşılaştırılmıştır.

Anahtar Kelimeler: Envanter kontrol mekanizması, stok seviyesi, yanal ve direk nakliyat

To my family

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This master thesis project is the final result of my study in the double degree program of Eindhoven University of Technology and Middle East Technical University. It took almost one year to make this project and this is the result of all efforts and hard work that I spent. I would like to thank Nedtrain for giving me the opportunity to make this master study.

I could not do this project without the help of some specific people. Firstly, I would like to thank to Associate Prof. Yasemin Serin and Prof.dr.ir. Geert-Jan van Houtum for their supervision during my study. They oriented me in the best way with their great knowledge, experience, and enthusiasm. Secondly, I would like to thank to Bob Huisman for his supervision in Nedtrain. His great care, enthusiasm, knowledge and experience helped me a lot in my study. Making a maser study in a foreign country is always difficult for students. However, Bob minimized all of the problems that I encountered during my adaptation period in Nedtrain. Moreover, I would like to thank to Marcel Prakken for providing all kinds of information and data for me; and I would like to thank Joachim Arts for helping me in my analysis.

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LIST OF ABBREVIATIONS

CBT: The repair facility of the repairables and some main parts

COV: Coefficient of variation

FCFS: First come first served

NS: Nederlandse Spoorwegen

OB: Local maintenance point

Proplan: A software package which is used in the control of planned demand

R5: A software package which is used in the control of planned demand

RB: Overhaul point

RDC: Central warehouse

SB: Service point

SKU: Stock keeping unit

Xelus Software: The tool which is used in the control of all parts in the network of Nedtrain

LIST OF SYMBOLS

Indexes

I: The set of SKU-s

N_{loc}: The set of OB-s

$N = \{0\} \cup N_{loc}$: The set of all warehouses including RDC

Parameters

m_{in}: Demand for SKU i at OB n where $i \in I, n \in N_{loc}$

$t_{reg:}$ Transportation time between OB-s and RDC for all SKU-s

t_n^{RDC}: Direct shipment time from RDC to OB n for all SKU-s

t_n^{CBT}: Direct shipment time from CBT to OB n for all SKU-s

$t_{l,k}^{lat}$: Lateral transshipment time from OB k to OB l for all SKU where $k, l \in N_{loc}, l \neq k$

T_0: Mean replenishment lead time of RDC by CBT

μ_0: Replenishment rate of RDC by CBT

C_n^{RDC}: Direct shipment cost from RDC to OB n for all SKU i

C_n^{CBT}: Direct shipment cost from CBT to OB n for all SKU i

$C_{n,k}^{lat}$: Lateral transshipment cost from OB k to OB n for all SKU i

d_{RDC-n}: Distance between RDC and OB n (km).

d_{k-l}: Distance between OB k to OB l (km)

t_n^{RDC}: Time between OB n and RDC (hours)

t_l^k: Time between OB k to OB l (hours)

c_{driver}: Wage rate of a driver per hour

c_{usage}: Cost for renting a truck per km

c_{fuel}: Cost of fuel consumption by the truck

λ: uplift factor

W_n^{target}: Aggregate target waiting time value for all SKU i at OB n

h: Annual holding cost rate per SKU

p_i: Price of SKU i (€)

m_{i0}: Total demand for SKU i coming to the RDC

$\sigma_n = \left(\sigma_1(n), \sigma_2(n), \dots, \sigma_{(|N_{loc}|-1)}(n)\right)$: The pre-specified order of the OB-s for asking lateral transshipment

Variables

S_{in}: Base stock level for SKU i at warehouse n where $i \in I, n \in N$

W_{in}: Expected waiting time for a SKU i at OB n where $i \in I, n \in N_{loc}$

β_{in}: Fraction of demand for SKU i met by the stock directly at OB n

BW_{in}: Average backorder waiting time of SKU i at OB n

AW_{in}: Average waiting time of an order for SKU i at OB n

δ_{in}: Average demand rate for SKU i at warehouse n where $i \in I, n \in N$

$\alpha_{i,l,k}$: Fraction of demand for SKU i met by a lateral transshipment done from OB k to OB l where $i \in I,\ \ n, k, l \in N_{loc},\ \ l \neq k$

θ_{in}: Fraction of demand for SKU i met by a direct shipment from RDC to OB n

γ_{in}: Fraction of demand for SKU i met by a direct shipment from CBT to OB n

LT_{in}: Mean replenishment lead time of SKU i at OB n

μ_{in}: Expected replenishment rate of SKU i at OB n

C_{in}^{h}: Expected inventory holding cost for SKU i at OB n

C_{in}^{T}: Expected transportation cost for SKU i at OB n

IL_{in}: The inventory level of SKU i AT warehouse n where $i \in I, n \in N$

$E(W_{i0})$: Expected delay in RDC, for SKU i

$E(I_{in})$: Expected on hand inventory level for SKU i at warehouse n where $i \in I, n \in N$

$E(B_{in})$: Expected backorder level for SKU i at warehouse n where $i \in I, n \in N$

$\underline{S}$: $|I| \times |N|$ matrix consisting of all S_{in} where $i \in I, n \in N$

$Z(\underline{S})$: Total cost value for the solution $\underline{S}$

U_{ab}: $|I| \times |N|$ matrix having entries zero except for the cell (a,b) that has a volume of 1

$W_n(\underline{S})$: Aggregate mean waiting time for all SKU i, at OB n for the solution $\underline{S}$

$\widehat{M}_{in}$: Total demand rate including the lateral demands coming for SKU i at OB n when there is positive stock in OB n

$\widetilde{M}_{ilk}$: Lateral demand rate coming from OB l to OB k for SKU i when there is positive stock in the OB k where $i \in I,\ \ n, k, l \in N_{loc},\ \ l \neq k$

λ_{in}: Demand rate coming for SKU i at OB n when there is not any on-hand stock in the OB n

m_{i0}': Backorder rate SKU i at RDC in Model 3 and 4

$\bar{S}_i$: Total base stock level for SKU i at all of the OB-s

CHAPTER 1

INTRODUCTION AND RESEARCH ASSIGNMENT

1.1 Introduction

This master thesis project has been conducted in Nedtrain, which deals with the maintenance of rolling stock[1] (trains). Nedtrain gives great importance to achieve high service levels, because any delay in the maintenance of the rolling stocks affects the transportation of the whole Netherlands. Thus, Nedtrain has a very critical responsibility which concerns not only themselves but also a whole country. This fact is the main motivation behind this master thesis project.

In this study, control mechanism of the repairables is considered. There are thousands of different repairables used in the network of Nedtrain, which makes the control of them complicated and important; any improvement in the control of these parts will provide great benefit to the company.

In the control of repairables, there are different supply options used by Nedtrain. These are regular replenishments, lateral transshipments, and direct shipments. In this study, the effect of each supply option is analyzed. Four different models are constructed to minimize the total system costs subject to a target service level.

[1] Nedtrain uses the word 'Rolling Stock' for the word 'Train'. Thus instead of the word 'Train', 'Rolling Stock' will be used in this report

1.2 Company Description

Nedtrain is the company which is responsible for the maintenance of the rolling stock in the Netherlands. Although Nedtrain is founded officially in the early 90s, its origin extends to 19th century when the first railroad in the Netherlands was founded. In 1938; NS ("Nederlandse Spoorwegen"), parent company of Nedtrain, is founded by the merger of the two largest Dutch railway companies; in the early 90s, NS has been privatized and Nedtrain has been separated within NS as the company responsible for the maintenance operations of the rolling stock in the Netherlands. Currently, Nedtrain is one of the first class rolling stock maintenance companies in Europe. Approximately, 3500 employees work in the company and its annual revenue is around € 450 million in 2007.

1.3 Train Series and Parts Used in Nedtrain

There are fifteen different types of rolling stocks and approximately 3.000 rolling stock segments in Nedtrain. On the average a rolling stock has 4 segments, so Nedtrain has approximately 750 rolling stocks, currently.

Nedtrain uses three different parts in its network. These are consumables, repairables, and main parts.

- Consumables are the cheapest parts in the network. After used for a specific distance or time period, these parts are discarded. They are purchased and owned by Nedtrain and supplied by external suppliers.
- Repairables can be repaired and used in the supply chain for long times. They have two criticality classes (critical and non-critical). A repairable can contain consumables in its structure. These parts are repaired by the external suppliers or the repair facility of the repairables (CBT). They are purchased and owned by Nedtrain.
- Main parts, which are also repairables, are the largest parts in the network. They differ from repairables in terms of cost, technical importance (criticality), uniqueness, and ownership (they are owned by the clients of the Nedtrain) as well as in terms of their maintenance planning. A main part can contain both

repairables and consumables in its structure. When these type of parts break down, they are repaired at overhaul point which is in Haarlem (RB) or CBT.

1.4 Current Maintenance Operations

There are three different *types of maintenance* with respect to parts in Nedtrain:

- **Use based maintenance**: In this type of maintenance activity, a part is replaced after it is used for a specific time period or distance. This type of maintenance is planned preventive maintenance.
- **Condition based maintenance**: This type of maintenance activity is preventive maintenance with planned inspection and conditional replacement, in which a part is replaced after its performance falls below a specific level.
- **Failure based maintenance**: These are unplanned corrective maintenance activities, in which a part is replaced or repaired when it breaks down.

There are four different *types of maintenance actions* in Nedtrain:

- **First line service**: Every rolling stock is inspected, repaired and cleaned on a daily basis. Depending on the results of the inspection, condition and failure based maintenance can take place.
- **Short cyclical periodic maintenance**: These types of maintenance actions have longer interval periods and concerns planned preventive and corrective maintenance actions for repairables and consumables.
- **Failures**: When parts fail during the usage of the rolling stock, the criticality of the part is considered. If the part is critical, an extra arrival is done to the appropriate service location; otherwise the part is replaced or repaired in the next compulsory service.
- **Long term maintenance (Overhaul)**: Maintenance of repairables and main parts is done by long term maintenance. It is a use based maintenance activity. RB and CBT are the overhaul points in the supply chain of Nedtrain.

1.5 Logistics Structure of Nedtrain

There are more than 30 locations of Nedtrain. The headquarters is in Utrecht. There are different facilities of Nedtrain around the Netherlands. Maintenance operations are done in three different locations. These are: Service points (SB), local maintenance points (OB), and RB. In addition to these maintenance locations; a central stock distribution center (RDC), CBT and external suppliers exist in the supply chain of Nedtrain. The supply chain of Nedtrain can be seen in Figure 1.1.

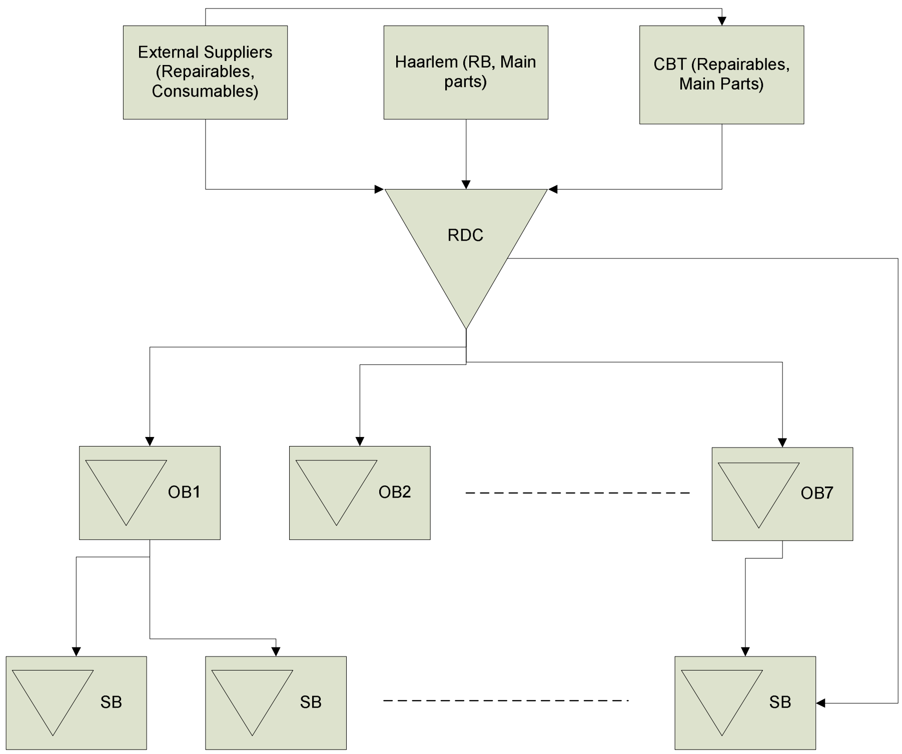

Figure 1.1: Supply Chain of Nedtrain

As it can be seen from Figure 1.1, there are four layers in the supply chain.

Service points (SB) form the lowest level. There are approximately 30 SB-s around the Netherlands. In these service locations, daily maintenance activities are done. At the end of each day, rolling stocks are cleaned and inspected in SB-s. However, only lower level maintenance operations can be done in these locations that are mostly unplanned (corrective). Only repairable and consumable parts are used in the maintenance activities of SB-s. The components are supplied by maintenance points (OB) mostly. However, in some cases the components are supplied directly from the central stock distribution center (RDC).

Nedtrain has Maintenance points (OB) in Onnen, Zwolle, Maastricht, Amsterdam, Watergraafsmeer, Leidschendam and Rotterdam. Each rolling stock should go to an OB for maintenance at specific time periods or after a specific distance is travelled. Every rolling stock series has its own maintenance period or distance. These type of services are planned maintenance activities in theory, because the amount of distance will be travelled can be estimated in terms of time or the visits of the rolling stocks can be scheduled by implementing fixed time period intervals. However, in practice, maintenance activities in the OB-s may become unplanned because of several reasons, which will be mentioned later. Both preventive and corrective maintenance actions are done in the OB-s. There is also a stock point which keeps main, repairable and consumable parts in each OB. These stock points are controlled centrally by the RDC.

RDC is a central stock point which supplies the stock points in OB-s and it supplies some of the SB-s directly. The stocks kept at the OB-s and RDC are controlled centrally in RDC by Xelus software. RDC is supplied by CBT, external suppliers and the component factory in Haarlem. RDC keeps the stocks of main, repairable and consumable parts.

The Overhaul point (RB) is the maintenance location in which refurbishment and overhauling of main parts are done. Each main part visits the RB, which is in Haarlem, every five to fifteen years and all of the components of the main parts are replaced with overhauled components. Maintenance done at RB is called long-cyclical maintenance and this type of maintenance is a planned operation because all components are replaced regardless of their condition. There is also a component factory in Haarlem which supplies approximately 80% of the main parts to the network.

CBT, which is in Tilburg, is the factory in which most of the repairables (approximately 75%) and a small amount of main parts (approximately 20%) are repaired. CBT supplies the repaired parts to RDC, and RDC distributes the parts to OB-s or SB-s.

There are also external suppliers in the network, which supply some of the repairables (25%) and all of the consumables parts. External suppliers also supply components to CBT.

Nedtrain initiated an applied research and development program in cooperation with several universities including Eindhoven University of Technology. The main question for this research and development program is given as:

"How to obtain and maintain the best combination of rolling stock, maintenance operations and supply chain, within the context of railway operations, to enable the customers to deliver competitive high quality services to their passengers?"

Nedtrain determined the management of repairables as a research area which has a large potential for improvement, and thus repairables will be considered in this master thesis project.

1.6 Repairables

As mentioned before, repairables can be repaired and used in the supply chain for long times. They have two criticality classes, critical and non-critical. Only failure of a critical repairable makes the rolling stock down. Repairables are repaired at the external suppliers or CBT. They are purchased and owned by Nedtrain.

There are 9356 different parts existing in the supply chain of Nedtrain (all of these parts cannot be considered as stock keeping units, because there is not any demand for some of them for years). 39% of repairables are considered critical, and 60% of them are considered as non-critical. The criticality level of the remaining 1% is not determined yet.

Repairables have a wide price range (€ 0,05 - € 265.300). The price distribution of the repairables for each criticality level can be seen in Table 1.1 below:

Table 1.1: Distribution of repairables with respect to price and criticality

Price range (€)	Critical (%)	Non-critical (%)	Unknown criticality level (%)	General
price < 5	2,3	3,9	23,8	3,5
5< price ≤ 10	0,1	0,7	1,6	0,5
10 < price ≤ 100	21,3	37,2	31,1	31,0
100< price ≤ 1000	55,6	46,9	18,0	49,9
1000< price ≤ 10.000	18,5	10,5	24,6	13,8
10.000 < price ≤ 100.000	2,0	0,8	0,8	1,3
price > 100.000	0,2	0,0	0,0	0,1
	100 (%)	100 (%)	100 (%)	100 (%)

As it can be seen, critical repairables have larger shares on higher price levels than non-critical ones.

Demands for the repairables are classified into planned and unplanned demand. Note that in planned demands, a part is replaced after a specific time period or distance. However, in practice this distinction does not work because of several factors. For instance; if the part fails earlier, the schedule of planned maintenance may be changed; or the arrival date of the rolling stocks to the maintenance locations can be changed at the last moment which changes the entire planned demand schedule.

When the planned and unplanned demands are aggregated, it is expected that the predictability of the demand increases, which can provide better forecast performance. Below, in Table 1.2 the change in the coefficient of variation (COV) can be seen when the planned and unplanned demand are aggregated. Note that '% of improved COV' column shows the improvement of the COV of aggregated demands with respect to the minimum of the COV of planned and unplanned demand.

Table 1.2: Improvement in coefficient of variance when planned and unplanned demands aggregated

Location	% of improved COV	% Average decrease in COV	% of products which has both planned and unplanned demand
RDC	82,76	8,91	5,89
OB1	82,35	8,50	14,86
OB2	68,08	1,68	19,94
OB3	71,19	3,32	20,65
OB4	76,67	7,70	21,05
OB5	70,83	5,11	15,85
OB6	74,26	5,26	12,16
OB7	80,00	11,59	8,06
Average	**75,77**	**6,51**	**14,81**

As it can be seen in Table 1.2; in general, 15% of the repairables have planned and unplanned demands and when they are aggregated, 76% of the time, coefficient of variance decreases, which increases predictability.

Nedtrain uses various forecasting methods in the control process of repairables. Currently forecasting for planned and unplanned demand is done separately. For the planned demand; software packages 'R5' and Proplan are used and these forecasts change over time, there is no frozen horizon. For the unplanned demands, the software package "Xelus system" is used. Currently, the Xelus system chooses the forecasting process automatically among five different forecasting methods (moving average, single exponential smoothing, double exponential smoothing, weighted average, and Croston's method) with respect to sigma ratio.

Repairables are stored in the OB-s and RDC. When a demand comes to an OB, the repairable is replaced by a ready for use one kept in stock; and the broken one is sent to either CBT or external suppliers. The stock kept in the OB-s is replenished by RDC which acts as a central stock point in the supply chain of repairables and it is supplied by CBT and external suppliers. All these inventory points are controlled centrally by the Xelus system. OB-s are replenished by a (s, Q) inventory controlling policy and CBT supplies the RDC with respect to a min-max policy. These min-max levels are calculated for the stocks kept in the whole network (in RDC and OB-s). The purpose of CBT is to keep the stocks between the min-max levels. However, current stock levels are outside the min-max stock levels for almost 76% of all the repairables (45% of time current stock is higher than max level, and 31% of time it is lower than min level). Hence the control process does not work properly and this situation results in a lot of extra, and probably unnecessary coordination efforts between the repairable planners and the CBT. Another problem occurs in the implementation of the control mechanism. According to a recent study done by Gordian (which is a consultancy company) in Nedtrain, employees do not implement the decisions of the Xelus in 80% of the time, which makes the control of repairables more complicated.

Currently, two different service levels are considered in the control of repairables. These are the network service level and local availability levels. Network service level is fill rate of repairables in the whole network; the fill rate is calculated by considering the total inventory kept in the whole network (RDC and all OB-s). Local availability is the availability of repairables in each OB. Network service level and local availability level are calculated as 98% and 93%, respectively in April 2009. Note that target network service level of Nedtrain is 99% for all products. Inventory turnover rate for the repairables is approximately 1.2 currently.

Some interesting situations are recognized in the control of repairables. Firstly, it is assumed that the demand of all repairables have a Normal distribution. However, generally most of the repairables have very low demand rates which cannot be modeled by a Normal distribution. To check this assumption, 127 different repairables (which are selected randomly from a random OB) are examined to understand their distribution. Firstly, the repairables are classified into their demand rate during one month (whether is this value is smaller than 10 or not; note that less than 3% of the demand has average demand more than or equal to 10). Then Anderson - Darling Normality test and Goodness of fit test for Poisson distribution are done to each of the selected repairables to understand their distribution. In addition to these tests, Kolmogorov - Smirnov Test for exponential distribution is done to understand whether the inter-arrival times between demands are exponentially distributed (In some cases Goodness of fit test for Poisson distribution cannot be done because enough demand classes cannot be formed, in these cases Kolmogorov - Smirnov Test for exponential distribution is used.). Results can be seen in Table 1.3, below:

Table 1.3: Distribution of demands of different types of repairables

	Average demand < 10				**Average Demand ≥ 10**		
Demand Type	**Poisson**	**Normal**	**None**	**Lack of data**	**Poisson**	**Normal**	**None**
Planned	2	0	21	37	0	0	16
Unplanned	33	1	12	14	0	10	11
Total	35	1	33	51	0	10	27

'Lack of data' column shows the demands which cannot be tested even by Kolmogorov - Smirnov Test. Since this type of repairables have very low demand (one or two in the observed period which is three years), it is impossible to test them. However, this type of parts is generally considered to have Poisson distribution in the literature.

According to the results when average demand per month is smaller than 10; 29% of the repairables have Poisson distributed and 42% of the repairables have very low demand in the last three years. If the repairables which have very low demand are assumed to have Poisson distributed, then 71% of the parts will have Poisson distribution when the average demand per month is smaller than 10.

For the unplanned demands, 55% of the demand has a Poisson distribution and 23% of the demand has very low demand. If the repairables which have very low demand are assumed to have Poisson distributed, then 78% of the parts will have Poisson distribution when the average demand per month is smaller than 10. For the planned demands, 35% of the demand does have neither Poisson nor Normal distribution but 62% of the demand has very low demand.

When average demand per month is more than or equal to ten, 27% of the repairables have normal distribution and the remaining ones do not fit to neither Normal nor Poisson distribution. In general, most of the repairables does not fit to Normal distribution, and the Poisson distribution can be a better alternative.

As mentioned earlier, repairables have a very wide price range (€ 0,05 - € 265.300). In the literature; special stochastic inventory control policies are implemented for expensive spare parts which have low demand rates. This kind of control policies can be a good research topic in Nedtrain. Moreover, Xelus system can implement advanced optimization techniques like METRIC, which is used in the control of spare parts which are expensive and have low demand rates.

Another interesting point is that although lateral transshipments are implemented when needed, inventory calculations are not done with respect to this factor. However, Xelus system is sophisticated planning software and it has an ability to consider pooling between stock points. However, it is not known how it deals with pooling.

1.7 Scope

The repairables considered in this thesis are the critical ones. Both planned and unplanned demands are considered in this study.

Almost all of the repairables can be purchased from suppliers and approximately only 1% of them are in the initial phase[2] with respect to their life cycle. Thus the repairables considered in this study are in the normal phase[3] with respect to the life cycle of spare parts.

The scope of this master thesis study is determined as the control mechanism of the repairables. Inputs for the control mechanism will be forecasted demand values. With respect to the target service levels; inventory levels, inventory holding and transportation cost values will be the outputs of the study.

Expected waiting time is used as the service level. There are two types of approaches which use expected waiting time for stock keeping units (SKU-s) as the service level in the literature: system and item approach. A system approach considers the aggregate service level for the repairables but item approach considers the target service level for

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[2] In the initial phase, a product appears on the market with components that have never been produced before. These components should be kept in stock as service parts, but historic demand figures are not available thus the basic problem is to estimate the quantities to be put in stock (see Fortuin [7]).

[3] In the normal phase, sales of the product have settled at a certain level. Demand for service parts can be predicted (see Fortuin [7]).

each SKU separately. It is possible to find studies which compare system approach and item approach in the literature of spare parts. Rustenburg et al. [25], Wong et al. [33], Wong et al. [34], Van Houtum et al. [13], and Kranenburg and van Houtum [17] compared system approach and item approach and they all found that system approach significantly outperforms the item approach. The reason of this result is that in the system approach, low service levels of expensive products are compensated by high service levels of cheap products. Because of this situation, system approach will be used in the service level constraint in each of the models

Below in Figure 1.2, the environment of the control system of the repairables in Nedtrain can be seen.

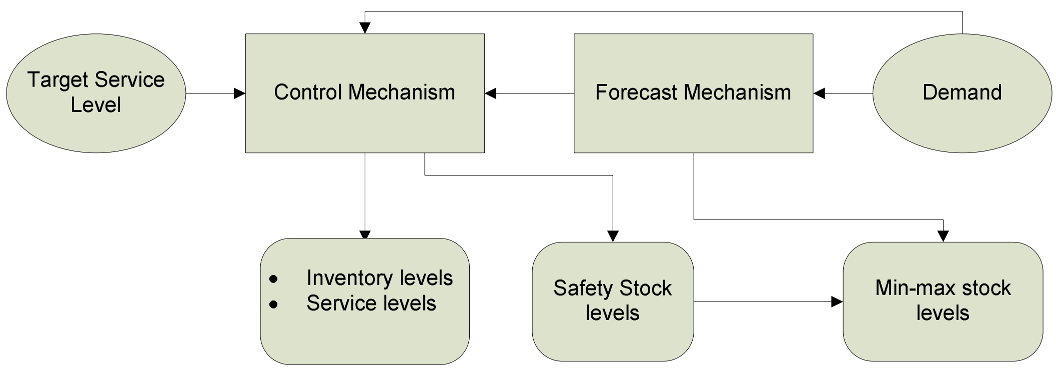

Figure 1.2: Control system of repairables in Nedtrain

An improvement in the control mechanism of repairables is believed to provide great benefits to Nedtrain. For instance, current value of the repairables kept in the stock is approximately € 18.7 million. If the inventory holding rate is assumed as 20% per year, then yearly inventory holding costs for the repairables becomes € 3.74 million which is a significant amount for Nedtrain (approximately % 0.83 of the total yearly revenue). Moreover; Nedtrain gives great importance to provide the best service to its customers, thus higher service levels will always support this goal.

However due to time restriction, following areas will be excluded from this study:

- Maintenance operations on CBT and SB-s
- Repair time for the broken parts in CBT

1.8 Problem Statement and Research Question

Currently, Nedtrain assumes normal distribution for the demand of every repairable and uses (s, Q) policy to control the inventory. This situation may cause poor performance; because repairables have different criticality levels, prices from each other and as shown before, Poisson distribution is more suitable to the repairables than the Normal distribution. Moreover; demand is distinguished as planned and unplanned demand, and forecasting is done separately for each type, but this situation decreases the predictability of the demand.

Also the inventory levels are mostly out of the min-max range which brings unnecessary coordination efforts between RDC and CBT. Lastly; current planning software, Xelus systems, has options like METRIC[4] or lateral transshipments that can be effectively used in the control process of repairables. Hence, the problem statement of the present study can be stated as

"In this study, the inventory control alternatives for repairables of Nedtrain will be evaluated in terms of the total costs under an appropriate service level constraint."

With respect to the problem statement, the research question is identified as:

"How can the inventory of the repairables be controlled with a specific service level target while minimizing total costs?"

For this research question, several sub questions are identified. These are:

- How can the repairables be classified?
- What is the optimal control policy for each class of the repairables?
- How can the results of these models be implemented?

Then the main project objectives are defined as:

- Delivering knowledge about repairables to Nedtrain

[4] METRIC is a multi-echelon technique for recoverable item control developed by Sherbrooke in 1968 (see Sherbrooke [27]).

- Providing a control mechanism tool which minimizes total cost of the system with respect to a target service level

In following chapters; firstly, the models constructed to deal with the research question, which evaluate different alternatives to improve the inventory control policy of Nedtrain, are described. Then, a review of the related literature is given. Then, solution procedures for the models are explained and validated. Afterwards, the numerical results and analyses are given and implementation issues of the findings are described; and lastly, conclusion and recommendation to Nedtrain for future research are given.

CHAPTER 2

MODELS

In this chapter, the models that are used to answer the research questions are described. Although Nedtrain also uses lateral[5] and direct[6] shipments in the maintenance of repairables, the control mechanism for the repairables is designed considering only normal replenishments. Moreover, Nedtrain has no insights about the effect of lateral and direct shipments. In order to analyze the effect of lateral and direct shipments, four different models are constructed using different control alternatives. These models are:

1. *Model 1*: A basic multi item, two echelon inventory control model with regular shipments only
2. *Model 2*: A multi item, two echelon model which also allows lateral transshipments between OB's in addition to regular shipments
3. *Model 3*: A multi item, two echelon model which also allows direct shipments from RDC or CBT to OB's in addition to regular shipments
4. *Model 4*: A multi item, two echelon model which allows both lateral and direct shipments in addition to regular shipments

The purpose of considering different models is to show Nedtrain the effects of different supply options by comparing them with the current policy.

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[5] Lateral transshipments occur between OB-s. For instance; when a demand comes to OB 1 and if the OB 1 does not have any stock on hand and if OB 2 has the demanded part in its stock, then the part is sent from OB 2 to OB 1 by a lateral transshipment.

[6] There are two types of direct shipments in the network of Nedtrain: Direct shipment from RDC to OB and from CBT to OB. A direct shipment occurs when a demand comes to an OB and if the OB does not have any stock on hand. In this case; if RDC has the demanded part in its stock, then the part is sent from RDC to OB by a direct shipment; otherwise an emergent repair is done in CBT and the ready-for-use part is sent from CBT to OB by a direct shipment.

The model descriptions consist of multiple components (demands, demand fulfillment, reorder policy, service levels, costs, assumptions etc). Several components are the same for all models. It will be explicitly denoted when a certain component only holds for specific models.

In the rest of the chapter; first repairables, their demand structure, and demand fulfillment process will be explained. Then; service levels and the model constraints will be given. Then; costs observed in the network and the objective function of the models will be described. Then assumptions made for the models will be mentioned. Lastly, the models will be formulated.

2.1 Repairables

There are 9356 different types of repairables in the supply chain of Nedtrain. There has not been any demand for some of these repairables within the last three years and these parts will be excluded from this study.

Repairables are classified in two criticality levels as critical and non-critical. This study considers only critical repairables. It is assumed that all are equally critical and failure of any repairable makes the rolling stock down. Only after its replacement the rolling stock is up again. Each critical repairable is a Stock- Keeping Unit (SKU). The set of SKU-s is denoted by I.

2.2 Demands

Several tests are done to understand the distribution of the demands of repairables and it is seen that Poisson processes are more suitable than Normal distribution for the repairables. Thus it is assumed that the demands occur according to Poisson processes. It is also assumed that for each SKU, the demand rate is stationary. The reason of this assumption is that the failure rates of the considered repairables are very low and the downtime of the rolling stocks are short in general, thus the decrease in demand rate is small, and thus it is reasonable to assume constant demand rates.

2.3 Network

Nedtrain has a multi-echelon structure in the control of repairables, as given in Figure 2.1.

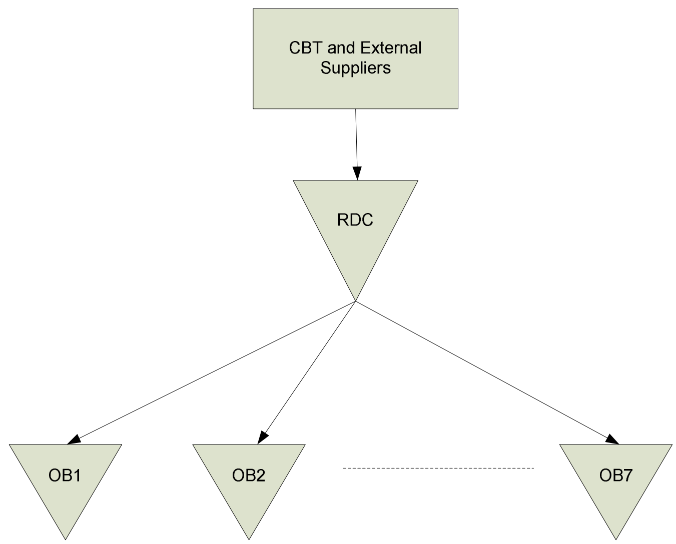

Figure 2.1: Echelon structure of Nedtrain for repairables

Seven OB-s (local warehouses) are in the lowest level of the supply chain of repairables. The set of OB-s is denoted by N_{loc}. Demands are assumed to come directly to the OB-s. For each SKU i at each OB n, demands occur according to a Poisson process with a constant rate $m_{in} \geq 0$ for $i \in I, n \in N_{loc}$.

Apart from the OB-s, there is a central warehouse, RDC; denoted by OB 0. Then N, the set of all warehouses, is $N = \{0\} \cup N_{loc}$. OB-s are replenished by the RDC.

In the highest level, there are CBT and External Suppliers; which repair the broken repairables. It is assumed that failed parts are repaired and returned after an exponential repair lead time.

2.4 Demand Fulfillment Process

Demand fulfillment process is different for each of the four models.

In Model 1; an incoming demand to an OB is fulfilled by the OB, if there is available ready-for-use part in stock as given in Figure 2.2. If the demanded part is not in stock, then it is backordered in the OB. Similarly, when an OB gives an order to RDC, RDC fulfills the order by its stock directly; if there is not any stock in RDC, then the demand is backordered in RDC. Thus in this model, backorders are allowed in both the OB-s and the RDC.

In Model 2; an incoming demand is fulfilled by the OB, if there is available ready-for-use part in stock. If the demanded part is not in stock, then other OB-s are checked. If any of the other OB-s has the demanded part in its stock, then a lateral transshipment is done from this OB to the one which cannot fulfill the demand, and the OB which sends the repairable gives an order to RDC (see Figure 2.2). An OB first checks the closest OB to itself for such a lateral transshipment requirement. If the closest one is also out of stock, then the second closest OB is checked and so on. However, if none of the OB-s has the demanded part, then the part is backordered in the OB where the demand initially came. Similarly, when an OB gives an order to RDC, RDC fulfills the order from its stock directly; if there is not any stock, and then the demand is backordered in RDC.

In Model 3; an incoming demand is fulfilled by the OB, if there is available ready-for-use part in stock. If the demanded part is not in the stock, then RDC makes a direct shipment to the OB, if it has the demanded part in its stock as given in Figure 2.2. If RDC is out of stock either, then CBT makes an emergent repair on the broken part and sends it urgently to the OB. Because of this demand fulfillment procedure, there is not any backorder in the OB-s, but backorder exists in RDC.

In Model 4; an incoming demand is fulfilled by the OB, if there is available ready-for-use part in stock. If the demanded part is not in the stock, then RDC makes a direct shipment to the OB directly, if it has the demanded part in its stock. If RDC does not have the demanded part in stock either, then other local warehouses are checked (with ‘the closest OB first’ order); and if the part is available in one of the other OB-s, then a lateral transshipment is done. If the demanded part is available neither in an OB nor in RDC, then CBT makes an emergent repair on the broken part and sends it urgently to the OB where the demand comes (see Figure 2.2). Because of this demand fulfillment procedure, there is not any backorder in the OB-s, but backorder exists in RDC.

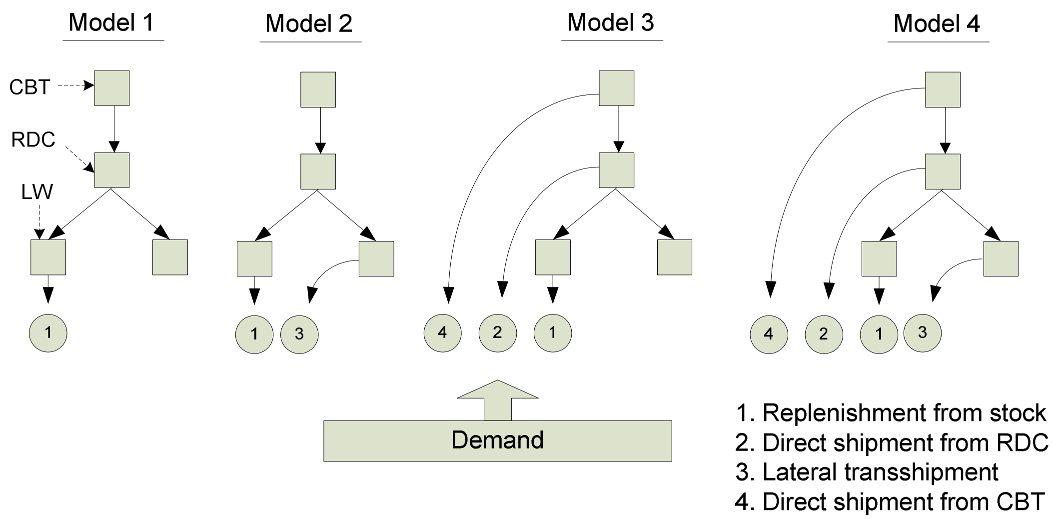

Figure 2.2: Nedtrain supply network with respect to each model

When an order is given by an OB to RDC, the orders for each SKU are aggregated and replenished together after a deterministic time period. Thus transportation time between OB-s and RDC for each SKU is same and denoted as t_{reg} for each SKU and OB.

For the direct shipments from RDC to OB-s, transportation time between OB n and RDC is denoted as t_n^{RDC} for every SKU i. For the lateral transshipments, transportation time for a lateral transshipment from OB k to OB l is denoted as $t_{l,k}^{lat}$ for every SKU i. For the direct shipment from CBT to OB-s, transportation time for an emergent supply of CBT to the OB n is denoted as t_n^{CBT} for every SKU i where $i \in I, n \in N_{loc}$. Note that the transportation times depend only to the distance travelled, so they do not depend on SKU-s.

Lastly, replenishment lead time (repair + transportation) of RDC by CBT is assumed to be exponentially distributed and its mean is T_0 for each SKU i where $i \in I$. Detailed information about the transportation times can be seen in Appendix A.

2.5 Reorder Policy

Since Nedtrain aggregates its orders and replenishes them together to each OB periodically, it is more suitable to implement a *one for one replenishment policy* in the control of repairables because batching the SKU demands at OB's does not bring any benefit in terms of the order or transportation cost of in the long term. Also Nedtrain has the ability to check its stock levels at each day. Hence, a *continuous review base stock (S, S-1) policy* is considered for each SKU in this study.

2.6 Service Levels

Nedtrain uses availability as the service level. Although availability is suitable as service level measure in Model 1, in Model 2, 3 and 4, lateral or direct shipment options are considered and expected waiting time per repairable is used as the service level measure rather than availability, because these alternatives decrease the expected waiting time for demanded repairable; availability is now distributed and does not make much sense.

Let W_{in} denote the expected waiting time for SKU i at OB n (used in all models) for $i \in I, n \in N_{loc}$. Additionally, β_{in} denotes the fraction of the demand for SKU i met by the stock in the OB n directly, θ_{in} denotes the fraction of the demand for SKU i at OB n met by direct shipments from RDC, $\alpha_{i,l,k}$ denotes the fraction of the demand coming for SKU i at OB l met by lateral transshipments from OB k; and γ_{in} denotes the fraction of the demand for SKU i at OB n met by direct shipments from CBT where $i \in I,\ n, k, l \in N_{loc},\ l \neq k$. Note that; β_{in} will be used in Model 2, 3, and 4; θ_{in} and γ_{in} will be used in Model 3 and 4; and $\alpha_{i,l,k}$ will be used in Model 2 and 4.

The expected waiting time, W_{in}, is computed differently in Model 1, 2, 3 and 4 as follows:

In Model 1:

$$W_{in} = \frac{E(B_{in})}{m_{in}} \qquad \forall\ i \in I, n \in N_{loc}$$

where, $E(B_{in})$ denotes the expected backorder level for SKU i at OB n where $i \in I, n \in N_{loc}$.

In Model 2:

$$W_{in} = \beta_{in}.0 + \sum_{k\in N_{loc}, k\neq n} \left(\alpha_{i,n,k}. t_{n,k}^{lat}\right) + \left(1 - \beta_{in} - \sum_{k\in N_{loc}} \alpha_{i,n,k}\right). BW_{in} \quad \forall\, i \in I, \quad \forall\; n \in N_{loc}$$

where BW_{in} denotes the average waiting time for a backorder for SKU i at OB n where $i \in I, n \in N_{loc}$.

In Model 3:

$$W_{in} = \beta_{in}.0 + \theta_{in}.\; t_n^{RDC} + \gamma_{in}.\; t_n^{CBT} \quad \forall\; i \in I, n \in N_{loc}$$

In Model 4:

$$W_{in} = \beta_{in}.0 + \theta_{in}.\; t_n^{RDC} + \sum_{k\in N_{loc}, k\neq n} \left(\alpha_{i,n,k}. t_{n,k}^{lat}\right) + \gamma_{in}.\; t_n^{CBT} \quad \forall\; i \in I, n \in N_{loc}$$

The service level constraint represented by the aggregate waiting time will be $\sum_{i\in I}\left(\frac{m_{in}}{\sum_{i\in I} m_{in}}. W_{in}\right) \leq \; W_n^{target} \quad \forall\; n \in N_{loc}$

2.7 Costs

Costs in the control of repairables are inventory holding cost and transportation cost as shown in Figure 2.3. Only the maintenance of broken repairables is studied here, the purchase cost is excluded. Backorder or waiting costs are also ignored because expected waiting time is constrained.

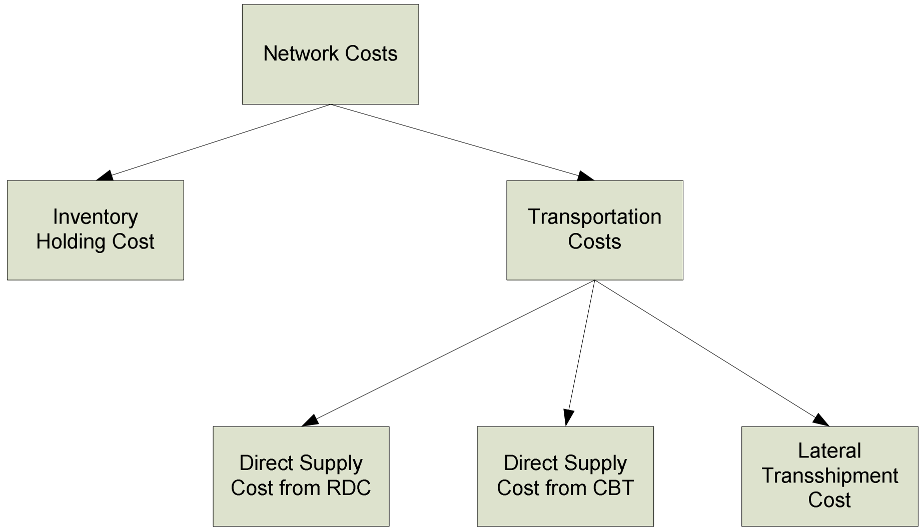

Figure 2.3: Cost components in the network of repairables

2.7.1 Transportation Costs

Transportation costs include the costs of normal (regular) supplies, direct shipment costs from RDC or CBT to the OB-s and lateral transshipment costs. However; since Nedtrain makes the regular supplies every day no matter what the amount of the orders is, cost of the regular supplies is fixed and independent of the order size. For that reason, costs of regular replenishments are excluded, thus transportation cost in Model 1 is assumed 0. More information about transportation costs can be seen in Appendix A.

For SKU i; direct shipment cost from RDC to OB n is denoted as C_n^{RDC}; direct shipment cost from CBT to OB n is denoted as C_n^{CBT}; and lateral transshipment cost from OB k to OB l is denoted as $C_{l,k}^{lat}$ where $i \in I,\ n,k,l \in N_{loc},\ l \neq k$.

$$C_n^{RDC} = d_{RDC-n}.\left(c_{usage} + c_{fuel}\right) + t_n^{RDC}.c_{driver} \qquad \forall\, i \in I, n \in N_{loc}$$

$$C_{l,k}^{lat} = d_{k-l}.\left(c_{usage} + c_{fuel}\right) + t_l^{k}.c_{driver} \qquad \forall\, i \in I, k \in N_{loc},\ l \in N_{loc}\ \ l \neq k$$

$$C_n^{CBT} = \ C_n^{RDC}.\lambda \qquad \forall\, i \in I, n \in N_{loc}$$

where

d_{RDC-n}: Distance between RDC and OB, $n \in N_{loc}$ (km).

d_{k-l}: Distance between OB, $k \in N_{loc}$ and OB, $l \in N_{loc}$ (km)

t_n^{RDC}: Time between OB, $n \in N_{loc}$ and RDC (hours)

t_l^k: Time between OB, $k \in N_{loc}$ and OB, $l \in N_{loc}$ (hours)

c_{driver}: Wage rate of a driver per hour

c_{usage}: Cost for renting a truck per km

c_{fuel}: Cost of fuel consumption per km by the truck

λ: uplift factor

An uplift factor is used to calculate the direct shipments from CBT to an OB. Since CBT and RDC are both in Tilburg, the cost of direct shipment from RDC to an OB should be very close to the cost of direct shipment from CBT to an OB. However for a direct shipment from CBT to an OB; in addition to the shipment cost of the repairable from CBT to the OB, there is some cost for the emergent shipment of the broken repairable from the OB to CBT and quick repair of the broken part.

Hence, total transportation cost denoted as C_{in}^T for SKU i at OB n for $i \in I, n \in N_{loc}$ is:

For Model 2:

$$C_{in}^T = \sum_{k \in N_{loc}, n \neq k} C_{n,k}^{lat} . m_{in} . \alpha_{i,n,k} \qquad \forall\, i \in I, n \in N_{loc}$$

For Model 3:

$$C_{in}^T = C_n^{RDC} . m_{in} . \theta_{in} + C_n^{CBT} . m_{in} . \gamma_{in} \qquad \forall\, i \in I, n \in N_{loc}$$

For Model 4:

$$C_{in}^T = C_n^{RDC} . m_{in} . \theta_{in} + C_n^{CBT} . m_{in} . \gamma_{in} + \sum_{k \in N_{loc}, n \neq k} C_{n,k}^{lat} . m_{in} . \alpha_{i,n,k} \quad \forall\, i \in I, n \in N_{loc}$$

2.7.2 Inventory Holding Costs

Inventory holding costs reflect the opportunity costs of the money invested. The most obvious holding cost components are the cost of equipment, materials, and labor to operate the space; insurance expenses; security costs; interest on the money invested in the inventory and space, and other direct expenses.

Let C_{in}^{h} denote the inventory holding cost for SKU i at warehouse n for $i \in I, n \in N$. Then inventory holding cost for each of the four models is

$$C_{in}^{h} = h.p_i.S_{in} \quad \forall\, i \in I, n \in N$$

h: Annual holding cost rate per SKU (It is assumed 20% in this study)

p_i: Price of SKU, $i \in I$ (€)

S_{in}: Base stock level of SKU i at warehouse n for $i \in I, n \in N$

2.8 Assumptions

The assumptions done so far are:

1. Demands directly come to the OB-s according to a Poisson process for each SKU i at OB n.
2. Demand rates for each SKU i at OB n are assumed to be stationary.
3. CBT is assumed to make the repair of all repairables. Thus External Suppliers are excluded from the study.
4. Repair lead time of each SKU i is independent and has exponential distribution, and mean repair lead time is same for each SKU i.
5. Fixed order costs are assumed zero.
6. Repairables do not have any condemnation.
7. A one-for-one replenishment policy is used for the all OB-s.
8. All repairables are equally critical, failure of any repairable makes the rolling stock down, and the replacement of the repairables make the rolling stock up again.

9. Transportation times and costs are same for each SKU i

for $i \in I, n \in N_{loc}$.

2.9 Model Formulation

All four models determine the base stock levels of the OB-s to minimize the total system costs under a waiting time constraint. Each model considers a different set of transshipment possibilities.

2.9.1 Model 1

In Model 1; only normal replenishments are considered.

Input parameters:

m_{in}: Average demand rate of SKU i at OB n for $i \in I, n \in N_{loc}$

t_{reg}: Regular transportation time between RDC and OB-s

T_0: Mean replenishment lead time of RDC by CBT for all SKU i

h: Annual holding cost rate per SKU i

p_i: Price of SKU i (€)

Outputs:

S_{in}: Base stock level of SKU i at warehouse n for $i \in I, n \in N$

W_{in}: Expected waiting time of SKU i in OB n for $i \in I, n \in N_{loc}$

Model 1:

$$\min \quad \sum_{i \in I} \sum_{n \in N} C_{in}^{h}$$

such that

$$\sum_{i\in I}\left(\frac{m_{in}}{\sum_{i\in I} m_{in}}.W_{in}\right) \le \; W_n^{target} \qquad \forall \; n \in N_{loc}$$

$$S_{in} \ge 0 \; and \; integer \quad \forall \; i \in I, n \in N$$

where

$$C_{in}^h = h.p_i.S_{in} \quad \forall \; i \in I, n \in N$$

$$W_{in} = \frac{E(B_{in})}{m_{in}} \quad \forall \; i \in I, n \in N_{loc}$$

2.9.2 Model 2

In Model 2; regular replenishments and lateral transshipments are considered.

Input parameters:

m_{in}: Average demand rate of SKU i at OB n for $i \in I, n \in N_{loc}$

t_{reg}: Regular transportation time between RDC and OB-s

T_0: Mean replenishment lead time of RDC by CBT for all SKU i

$t_{l,k}^{lat}$: Lateral transshipment time between OB k and OB l for all SKU i

h: Annual holding cost rate per SKU i

p_i: Price of SKU i for $i \in I$ (€)

$C_{l,k}^{lat}$: Average lateral transshipment cost between OB k and OB l for all SKU i

Outputs:

S_{in}: Base stock level of SKU i at warehouse n for $i \in I, n \in N$

W_{in}: Expected waiting time of SKU i in OB n for $i \in I, n \in N_{loc}$

Model 2:

$$\min \sum_{i\in I}\left(\sum_{n\in N} C_{in}^{h} + \sum_{n\in N_{loc}} C_{in}^{T}\right)$$

such that

$$\sum_{i\in I}\left(\frac{m_{in}}{\sum_{i\in I} m_{in}}.W_{in}\right) \le W_n^{target} \qquad \forall\ n \in N_{loc}$$

$$S_{in} \ge 0\ and\ integer \quad \forall\ i \in I, n \in N$$

where

$$W_{in} = \beta_{in}.0 + \sum_{k\in N_{loc}, k\neq n} (\alpha_{i,n,k}.t_{n,k}^{lat}) + \left(1 - \beta_{in} - \sum_{k\in N_{loc}} \alpha_{i,n,k}\right).BW_{in} \quad \forall\ i \in I, \forall\ n \in N_{loc}$$

$$C_{in}^{h} = h.p_i.S_{in} \quad \forall\ i \in I, n \in N$$

$$C_{in}^{T} = \sum_{k\in N_{loc}, n\neq k} C_{n,k}^{lat}.m_{in}.\alpha_{i,n,k} \qquad \forall\ i \in I, n \in N_{loc}$$

2.9.3 Model 3

In Model 3; regular replenishments and direct shipments are considered.

Input parameters:

m_{in}: Average demand rate of SKU i at OB n for $i \in I, n \in N_{loc}$

t_{reg}: Regular transportation time between RDC and OB-s

T_0: Mean replenishment lead time of RDC by CBT for all SKU i

t_n^{RDC}: Average direct shipment time between RDC and OB n for all SKU i

t_n^{CBT}: Average direct shipment lead time between CBT and OB n for all SKU i

h: Annual holding cost rate per SKU i

p_i: Price of SKU i for $i \in I$ (€)

C_n^{RDC}: Average direct shipment cost of an OB n by RDC for all SKU i

C_n^{CBT}: Average direct shipment cost of an OB n by CBT for all SKU i

Outputs:

S_{in}: Base stock level of SKU i at warehouse n for $i \in I, n \in N$

W_{in}: Expected waiting time of SKU i in OB n for $i \in I, n \in N_{loc}$

Model 3:

$$\min \quad \sum_{i \in I} \left(\sum_{n \in N} C_{in}^h + \sum_{n \in N_{loc}} C_{in}^T \right)$$

such that

$$\sum_{i \in I} \left(\frac{m_{in}}{\sum_{i \in I} m_{in}} . W_{in} \right) \leq \; W_n^{target} \qquad \forall \; n \in N_{loc}$$

$$S_{in} \geq 0 \; and \; integer \quad \forall \, i \in I, n \in N$$

where

$$W_{in} = \; \beta_{in}.0 + \theta_{in}. \; t_n^{RDC} + \gamma_{in}. \; t_n^{CBT} \qquad \forall \; i \in I, n \in N_{loc}$$

$$C_{in}^h = h.p_i.S_{in} \qquad \forall \, i \in I, n \in N$$

$$C_{in}^T = \; C_n^{RDC}.m_{in}. \, \theta_{in} + \; C_n^{CBT}.m_{in}. \, \gamma_{in} \qquad \forall \, i \in I, n \in N_{loc}$$

2.9.4 Model 4

In Model 4; regular replenishments, lateral and direct shipments are considered.

Input parameters:

m_{in}: Average demand rate of SKU i at OB n for $i \in I, n \in N_{loc}$

t_{reg}: Regular transportation time between RDC and OB-s

T_0: Mean replenishment lead time of RDC by CBT for all SKU i

$t_{l,k}^{lat}$: Lateral transshipment time between OB k and OB l for all SKU i

t_n^{RDC}: Average direct shipment time between RDC and OB n for all SKU i

t_n^{CBT}: Average direct shipment lead time between CBT and OB n for all SKU i

h: Annual holding cost rate per SKU i

p_i: Price of SKU i for $i \in I$ (€)

$C_{l,k}^{lat}$: Average lateral transshipment cost between OB k and OB l for all SKU i

C_n^{RDC}: Average direct shipment cost of an OB n by RDC for all SKU i

C_n^{CBT}: Average direct shipment cost of an OB n by CBT for all SKU i

Outputs:

S_{in}: Base stock level of SKU i at warehouse n for $i \in I, n \in N$

W_{in}: Expected waiting time of SKU i in OB n for $i \in I, n \in N_{loc}$

Model 4:

$$\min \quad \sum_{i \in I} \left(\sum_{n \in N} C_{in}^{h} + \sum_{n \in N_{loc}} C_{in}^{T} \right)$$

such that

$$\sum_{i \in I} \left(\frac{m_{in}}{\sum_{i \in I} m_{in}} . W_{in} \right) \leq \; W_n^{target} \qquad \forall \; n \in N_{loc}$$

$$S_{in} \geq 0 \; and \; integer \quad \forall\, i \in I, n \in N$$

where

$$W_{in} = \beta_{in}.0 + \theta_{in}.\; t_n^{RDC} + \sum_{k \in N_{loc}, k \neq n} \left(\alpha_{i,n,k}.\, t_{n,k}^{lat} \right) + \gamma_{in}.\; t_n^{CBT} \quad \forall\; i \in I, n \in N_{loc}$$

$$C_{in}^{h} = h.\, p_i.\, S_{in} \quad \forall\, i \in I, n \in N$$

$$C_{in}^{T} = C_n^{RDC}.\, m_{in}.\; \theta_{in} + C_n^{CBT}.\, m_{in}.\; \gamma_{in} + \sum_{k \in N_{loc}, n \neq k} C_{n,k}^{lat}.\, m_{in}.\, \alpha_{i,n,k} \quad \forall\, i \in I, n \in N_{loc}$$

2.10 Literature Review

This part briefly explains the literature related to the four models.

2.10.1 Model 1

This model deals with a two echelon, multi item system with a continuous review and one for one replenishment policy. Sherbrooke [28] developed a multi echelon model for recoverable item control (METRIC) for the systems with continuous review base stock policy. This method is capable of calculating expected backorder level for each of the local warehouses with respect to given base stock values. At some point, Sherbrooke replaced the real stochastic lead time with its mean, which makes METRIC an approximate evaluation method. Graves [11] developed exact and approximate evaluation procedures for multi echelon systems with continuous review base stock policy. In the approximation method, Graves fits the first two moments of the demands with negative binomial distribution. Rustenburg [25] et al. generalized Graves' exact and approximate evaluation method for two echelon, multi indenture systems. Wong et al. [34] presented four different heuristics to optimize the base stock levels with respect to a target waiting time constraint in a two echelon spare parts system. They used these heuristic methods with METRIC and Graves' exact and approximate evaluation methods. In Model 1; a greedy algorithm, which is one of the heuristic methods presented by Wong et al. [35] is used with the METRIC of Sherbrooke. The reason why

METRIC is used in Model 1 is that this method fits perfectly to the demand structure of repairables (Poisson demands) and to the echelon structure of Nedtrain (a multi echelon model). The reason why METRIC is used instead of Graves' Approximation Method is that the software that Nedtrain uses in the control of repairables has the ability to use METRIC method, which will make the implementation process much easier.

2.10.2 Model 2

This model uses lateral transshipments between local warehouses and has two echelon, multi item system with a continuous review and one for one replenishment policy. Wong et al. [33] considered lateral transshipments in a single echelon, multi item system with a continuous review base stock policy. They assumed that the central warehouse has an ample capacity which makes the system single echelon. They also used emergent shipments from the central warehouse when all of the local warehouses are out of stock. They used Markov process description in their solution procedure. Wong et al. [34] considered the same system with Wong et al. [33] but they used different solution methods. Kranenburg and van Houtum [17] considered lateral transshipments in a single echelon, multi item system with a continuous review base stock policy. Like Wong et al. [33], they assumed that the central warehouse has an ample capacity and used emergent shipments from the central warehouse when all of the local warehouses are out of stock; but different from Wong et al. [33], they also considered partial pooling[7] in addition to full pooling. They developed an approximate evaluation method by assuming that the overflow demands occur according to Poisson process. They minimized the total inventory and transportation costs with respect to a target waiting time constraint and used a greedy heuristic to determine the base stock levels. In Model 2, approximate evaluation method of Kranenburg and van Houtum [17] is integrated to a two echelon system and unlike Kranenburg and van Houtum [17], emergent shipments from an ample plant is not used in this model. Axsäter [3] also considered the system of Model 2. But the difference between Axsäter [3] and our model is that we use pre-defined lateral

[7] In the partial pooling case, only some of the local warehouses share their inventory for lateral transshipments.

orders unlike Axsäter [3], which uses a random order; and unlike our system Axsäter [3] uses lateral transshipments also for meeting backordered demands.

2.10.3 Model 3

Model 3 deals with a two echelon, multi item system with a continuous review and one for one replenishment policy. When a local warehouse is out of stock and if the central warehouse has stock on hand, then a direct shipment is done from the central warehouse to the local warehouse; and if even the central warehouse has not stock on hand, then a direct shipment is done from the repair facility, which has ample repair capacity, to the local warehouse.

Muckstadt and Thomas [21] is the only study in the literature which considers exactly the same system of Model 3. They used an approximate evaluation method to calculate the service levels and total cost values for given base stock levels. In addition to this method, a new approximate evaluation method is developed for Model 3.

2.10.4 Model 4

Model 4 considers the same system with Model 3 with one significant difference that a lateral transshipment is done between the local warehouses when the local warehouse and central warehouse is out of stock. Thus, before making a direct shipment from the repair facility, lateral transshipment is done, if it is possible.

There is not any study which deals with the same system considered in Model 4. However, Alfredsson and Verrijdt [2] considered the closest system to Model 4. The difference between their model and Model 4 is that if a local warehouse is out of stock, Model 4 first checks the central warehouse for a direct shipment, then it checks the other local warehouses for lateral transshipment, and lastly it makes a direct shipment from the repair facility; but in Alfredsson and Verrijdt [2], it first checks the other local warehouses for lateral transshipment, then it checks the central warehouse for a direct shipment, and lastly it makes a direct shipment from the repair facility. In the solution procedure of Alfredsson and Verrijdt [2], they aggregate the local warehouses and calculate the fraction of demand satisfied by a direct delivery from central warehouse and plant, then they calculate the fraction of demand satisfied by each local warehouse directly from stock and lateral transshipments. However, because of the above difference, the solution procedure of Alfredsson and Verrijdt [2] cannot be implemented in Model 4. Thus a new solution procedure is developed.

CHAPTER 3

SOLUTION PROCEDURES

In this chapter, the solution procedures to obtain the solutions of the models described in Chapter 2 and their validation process is explained. The solution methods employed here are approximate procedures that do not necessarily compute the optimal solutions. They consist of two parts. The first part calculates the expected waiting time and total cost values for each SKU i at OB n for given base stock levels. The second part, a greedy procedure, finds feasible base stock levels with respect to the target waiting time constraint by minimizing the total cost of the system. This part does not necessarily find the optimal solution. Such an approximate solution method is preferred rather than seeking the optimal solution, because it is computationally much more efficient.

3.1 Solution Procedure for Model 1

As mentioned before; in Model 1, the METRIC approximation of Sherbrooke is used with a greedy algorithm to minimize the total inventory cost with respect to a target waiting time constraint. The METRIC and the greedy algorithm are described below:

3.1.1 METRIC

METRIC means 'Multi Echelon Technique for Recoverable Item Control'. This method first considers the central warehouse (RDC) and calculates the expected delay in RDC and with respect to this value, it considers the local warehouses (OB-s), and calculates the expected waiting time for each OB.

For the central warehouse (RDC):

Let

- m_{i0} denote the total demand rate coming to the RDC for SKU i where $i \in I$
- IL_{i0} denote the inventory level of SKU i in RDC
- $E(I_{i0})$ denote the expected on hand inventory level of SKU i in RDC
- $E(B_{i0})$ denote the expected backorder level of SKU i in RDC
- $E(W_{i0})$ denote the expected delay of SKU i in RDC

Since base stock policy is used in the OB-s, then $m_{i0} = \sum_{n \in N_{loc}} m_{in}$.

By Palm's theorem (see Silver et al. [29]),

$$P\{IL_{i0} = x\} = \frac{(m_{i0}.T_0)^{S_{i0}-x}.e^{-m_{i0}.T_0}}{(S_{i0} - x)!} \tag{1}$$

Then;

$$E(I_{i0}) = \sum_{x=1}^{S_{i0}} x.P\{IL_{i0} = x\} \; ; \; E(B_{i0}) = \sum_{x=-\infty}^{-1} x.P\{IL_{i0} = x\} \tag{2}$$

or equivalently,

$$E(B_{i0}) = E(I_{i0}) - E(IL_{i0}) = E(I_{i0}) - (S_{i0} - m_{i0}.T_0) \tag{3}$$

Then by the Little's law,

$$E(W_{i0}) = E(B_{i0})/m_{i0} \qquad (4).$$

Because of FCFS policy in CBT and Poisson demand, $E(W_{i0})$ is same for all OB-s.

For the local warehouses (OB-s):

Let:

- LT_{in} denote the replenishment lead time for SKU i at OB n for $i \in I, n \in N_{loc}$
- IL_{in} denote the inventory level of SKU i at OB n
- $E(I_{in})$ denote the expected on hand inventory level of SKU i at OB n
- $E(B_{in})$ denote the expected backorder level of SKU i at OB n

Then

$$LT_{in} = t_{reg} + E(W_{i0}) \tag{5}$$

In this point, it is assumed that the replenishment lead time for SKU i at OB n is independent of other SKU-s, and its mean is LT_{in}. Then by using Palm's theorem,

$$P\{IL_{in} = x\} = \frac{(m_{in}.LT_{in})^{S_{in}-x}.e^{-m_{in}.LT_{in}}}{(S_{in} - x)!} \tag{6}$$

$$E(I_{in}) = \sum_{x=1}^{S_{in}} x.P\{IL_{in} = x\} \tag{7}$$

$$E(B_{in}) = E(I_{in}) - (S_{in} - m_{in}.LT_{in}) \tag{8}$$

Then by the Little's law, for SKU i at OB n

$$W_{in} = E(B_{in})/m_{in} \qquad (9).$$

Consequently, METRIC finds the expected waiting time, W_{in}, for each SKU i at OB n for given base stock levels where $i \in I, n \in N_{loc}$.

This method is not needed to be validated in this study because it has been already validated in the literature (See Graves [11], Wong et al. [35]).

3.1.2 Greedy Algorithm for Model 1

This heuristic method is developed to determine the feasible base stock levels feasible. It is the same greedy algorithm that is used in Wong et al. [35]. The algorithm is used to find a feasible solution with a cost as low as possible. The basic idea of this heuristic is to add units of stock in an iterative way. It starts with 0 base stock levels and at each iteration, one unit of an SKU i is added in a warehouse n such that the largest decrease in distance to the set of feasible solutions per extra unit of additional cost is gained. The procedure is terminated when a feasible solution is obtained.

For the algorithmic description, following notation is introduced. Let $\underline{S}$ denote $|I| \times |N|$ matrix consisting of all current S_{in} where $i \in I, n \in N$; $Z(\underline{S})$ denotes the total cost of the solution $\underline{S}$; U_{ab} denotes $|I| \times |N|$ matrix having entries zero except for the cell (a,b) that has a value of 1; and $W_n(\underline{S})$ denotes the aggregate mean waiting time over all SKU i, at OB n for the current solution $\underline{S}$.

The procedure starts with setting all base stock levels to zero for all SKU-s and warehouses ($\underline{S} = 0$). For each solution $\underline{S}$, the distance to the set of feasible solutions is defined as $\sum_{n \in N_{loc}} \left(W_n(\underline{S}) - W_n^{target} \right)^+$ where $(x)^+ = max\{0, x\}$. At each iteration; the ratio

$$r_{in} = \Delta W_{in} / \Delta Z_{in} \tag{10}$$

is calculated where

$$\Delta W_{in} = \sum_{n \in N_{loc}} \left\{ \left(W_n(\underline{S}) - W_n^{target} \right)^+ - \left(W_n(\underline{S} + U_{in}) - W_n^{target} \right)^+ \right\} \tag{11}$$

$$\Delta Z_{in} = Z(\underline{S} + U_{in}) - Z(\underline{S}) \tag{12}$$

for each combination of $i \in I, n \in N$.

Note that, in Model 1, the total cost will increase at each iteration as base stocks increase because only the inventory holding cost is considered. Thus r_{in} is always greater than or equal to 0.

$\Delta W_{i^*n^*}$ where $i^* \in I, n^* \in N_{loc}$ are only dependent on the base stock level of $S_{i^*n^*}$ and S_{i^*0}. These values are not subject to change if a base stock level of another SKU or the same SKU in different OB is increased. Similarly, ΔW_{i^*0} are only dependent to the base stock level of $S_{i^*n^*}$ where $i^* \in I, n^* \in N$; these values are not subject to change if a base stock level for another SKU is increased. Computation time can be saved if only the results that change are updated. A formal description of the greedy procedure is given next.

Greedy Procedure for Model 1

Step 1: Set the initial solution $\underline{S} = 0$; calculate $W_n(0)$ for all warehouses

Step 2: For all combinations $i \in I$ and $n \in N$: Calculate ΔW_{in} , ΔZ_{in} , and r_{in}.

Step 3: Let i^* and n^* be defined as $r_{i^*n^*} = \max\ r_{in}$. Set $\underline{S} = \underline{S} + U_{i^*n^*}$. If $W_n(\underline{S}) \leq W_n^{target}$ for all $n \in N_{loc}$, end; otherwise go to *Step 2*.

3.2 Solution Procedure for Model 2

As mentioned before, in Model 2, approximation method of Kranenburg and van Houtum [17] is integrated to a two echelon system and a greedy algorithm is developed to minimize the total inventory and transportation cost with respect to a target waiting time constraint. The approximate evaluation method and the greedy algorithm are described below.

3.2.1 Approximate Evaluation Method for Model 2

This method consists of two main steps. In the first step, for given base stock levels, expected waiting time in the central warehouse (RDC) is calculated and replenishment lead times for each of the OB-s are calculated. In the second step, fill rate of each local warehouse and the fraction of demands supplied by lateral transshipment between the local warehouses are calculated by an iterative procedure, then service level and cost values can be calculated.

Calculating the expected delay of RDC:

This step is the same with the procedure used to find the expected delay of RDC in METRIC.

Because of the base stock policy used in the OB-s, $m_{i0} = \sum_{n \in N_{loc}} m_{in}$.

By Palm's theorem (see Silver et al. [29]),

$$P\{IL_{i0} = x\} = \frac{(m_{i0}.T_0)^{S_{i0}-x}.e^{-m_{i0}.T_0}}{(S_{i0}-x)!} \tag{13}$$

$$E(I_{i0}) = \sum_{x=1}^{S_{i0}} x.P\{IL_{i0} = x\}\ ;\ E(B_{i0}) = \sum_{x=-\infty}^{-1} x.P\{IL_{i0} = x\} \tag{14}$$

or equivalently,

$$E(B_{i0}) = E(I_{i0}) - E(IL_{i0}) = E(I_{i0}) - (S_{i0} - m_{i0}.T_0) \quad (15)$$

Then by the Little's law,

$$E(W_{i0}) = E(B_{i0})/m_{i0} \quad (16)$$

Because of FCFS policy in CBT and Poisson demand, $E(W_{i0})$ is same for all OB-s.

Calculating fill rates and the fraction of demands supplied by lateral transshipments:

Some additional variables are defined in this step. These are:

$\widehat{M}_{in}$: Total demand rate including the lateral demands coming for SKU i at OB n when there is positive stock in OB n

$\widetilde{M}_{ilk}$: Lateral demand rate coming from OB l to OB k for SKU i when there is positive stock in the OB k for $i \in I,\ \ n, k, l \in N_{loc},\ \ l \neq k$

$\sigma_n = \left(\sigma_1(n), \sigma_2(n), \ldots, \sigma_{(|N_{loc}|-1)}(n)\right)$: The pre-specified order of the OB-s for asking lateral transshipment. For instance in a network containing three local warehouses, $\sigma_1 = (2,3)$; this means that when local warehouse 1 is out of stock, it will first check local warehouse 2 and then local warehouse 3 for lateral transshipment (See Appendix B).

λ_{in}= Demand rate coming for SKU i at OB n when there is no on-hand stock at OB n

Then for each SKU i at OB n, the mean replenishment lead time is

$$LT_{in} = t_{reg} + E(W_{i0}) \quad (17)$$

In this point, it is assumed that the replenishment lead time for SKU i at OB n is exponentially distributed and its mean is LT_{in}. Then the total demand rates for SKU i at OB n and lateral demand rates for SKU i from OB n to the other OB-s respectively are

$$\widehat{M}_{in} = m_{in} + \sum_{x=1,x\neq n}^{|N_{loc}|} \widetilde{M}_{ixn} \tag{18}$$

$$\widetilde{M}_{in\sigma_1(n)} = (1-\beta_{in}).m_{in} \tag{19}$$

$$\widetilde{M}_{in\sigma_x(n)} = \left(1-\beta_{i\sigma_{x-1}(n)}\right).\widetilde{M}_{in\sigma_{x-1}(n)} \qquad 1 < x \leq |N_{loc}| - 1 \tag{20}$$

From the equations above, it is seen that $\widehat{M}_{in}$ is dependent to β_{in} (See Appendix C, for a simple example about the (18)-(20)). In order to use a Markov process to calculate β_{in}, it is assumed that total demand rate coming for SKU i at OB n have a Poisson process, which also means that the overflow demands arrive according to a Poisson process.

When there is not any on-hand stock for SKU i at OB n, then the demand rate coming for SKU i at OB n is

$$\lambda_{in} = m_{in}. \prod_{x=1,x\neq n}^{|N_{loc}|} (1-\beta_{ix}) \tag{21}$$

Equation (21) states that an OB has backorder, if all of the OB-s are out of stock. Otherwise, even if an OB does not have any stock on hand, lateral transshipment can be done from other OB-s. It is assumed that the demand coming to OB n, when the OB has backorder, is a Poisson process, and the event that there is positive stock for SKU i at every OB n is independent of each other. Let the replenishment rate of a single SKU i at OB n by RDC be

$$\mu_{in} = 1/\, LT_{in} \tag{22}$$

Then if $\widehat{M}_{in}$ and λ_{in} are known, $\{X_{in}(t),\ t > 0\}$ is a Continuous Time Markov Chain (CTMC) with a rate diagram given in Figure 3.1. Let $X_{in}(t)$ be the number of SKU-s in replenishment for SKU i at OB n at time t. Since backorder exists in each OB n, number of states is infinite.

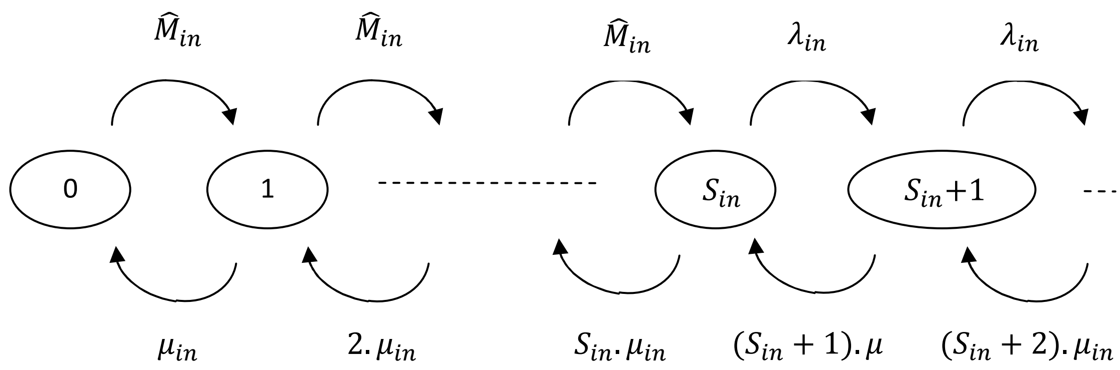

Figure 3.1: Rate diagram of the Markov process describing the number of SKU-s in replenishment in Model 2

If $\widehat{M}_{in}$ and λ_{in} is known, then β_{in} can be calculated. Let π_x denote the steady state probabilities for the CTMC shown in Figure 3.1, where $0 \leq x \leq +\infty$ and x is integer (Calculation of π_x can be seen in Appendix D). Then,

$$\beta_{in} = \sum_{x=0}^{S_{in}-1} \pi_x \tag{23}$$

If all β_{in} values are known, then all $\widehat{M}_{in}$ values can be calculated from (18)-(20) and all λ_{in} can be calculated from (21). Thus, an iterative procedure is used to find the $\widehat{M}_{in}$, λ_{in} and β_{in} for all SKU i at OB n for $i \in I, n \in N_{loc}$.

Iterative algorithm for Model 2

Let ϵ =10^{-6}. For each SKU i:

Step 1: Assume no lateral transshipment exists between all of the OB-s and $\lambda_{in} = 0$ for each OB n. Then $\widehat{M}_{in} = m_{in}$; and calculate β_{in} for each OB n by using (23).

Step 2: Using β_{in}, calculate the $\widehat{M}_{in}$ and λ_{in} for one OB by using (18)-(21). Then calculate β_{in} for the same OB.

Step 3: Repeat *Step 2* for each OB n.

Step 4: Repeat *Step 2* and *Step 3* until $\widehat{M}_{in}$ does not change more than ϵ for each OB n where $n \in N_{loc}$.

The variables $\widehat{M}_{in}$, λ_{in} and β_{in} converge in all cases considered in this study. Figure 3.2 3.3, and 3.4 show the convergence of $\widehat{M}_{i1}$, λ_{i1} and β_{i1} for a setting (see instance 1 in Table 3.1 to find the details of the setting).

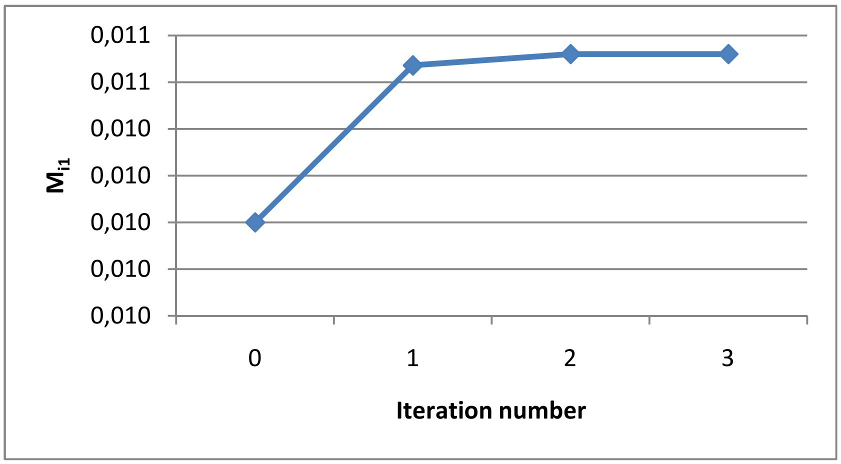

Figure 3.2: $\widehat{M}_{i1}$ at each iteration in a setting in Model 2

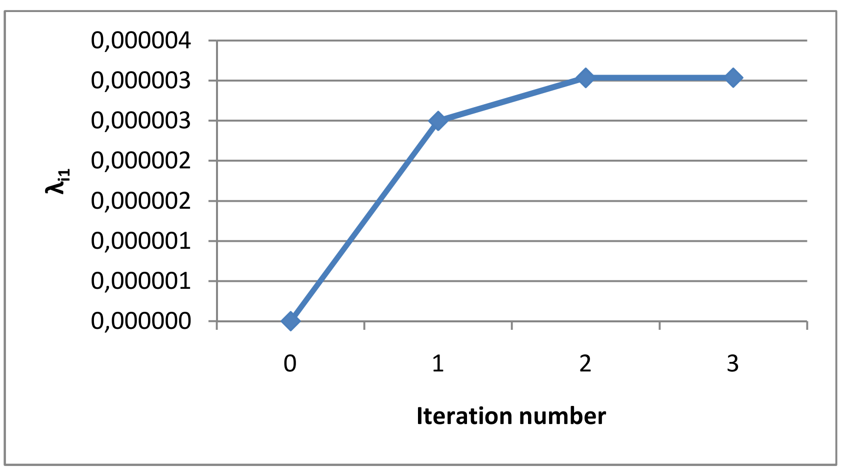

Figure 3.3: λ_{i1} at each iteration in a setting in Model 2

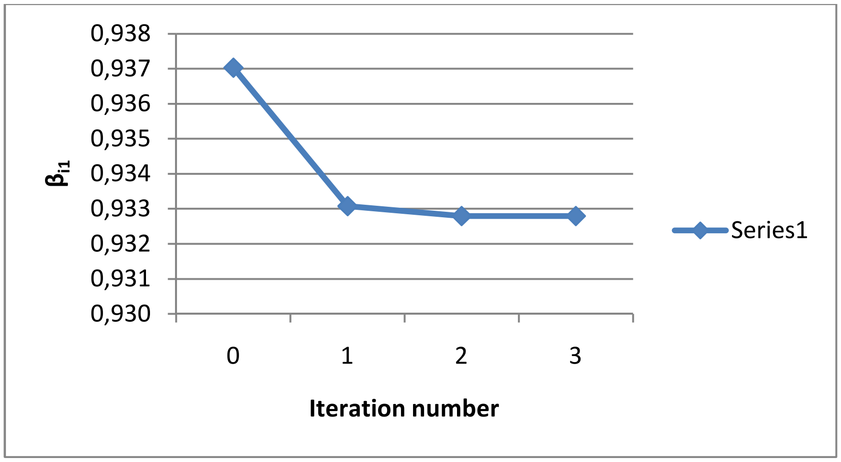

Figure 3.4: β_{i1} at each iteration in a setting in Model 2

As can be seen in Figure 3.2 and 3.3, the initial value of $\widehat{M}_{in}$, is equal to m_{in}, direct demand for that OB, because it is assumed that initially no lateral transshipment exists for each OB n and the initial value of λ_{in} is 0. Note that m_{in} is the minimum and maximum value for $\widehat{M}_{in}$ and λ_{in} respectively. In this case the corresponding β_{in} for each OB n becomes the largest. In the following iteration, $\widehat{M}_{in}$ increases because β_{in} is greater than 0, which makes the lateral demand rate coming from the other OB-s greater than 0, and λ_{in} becomes greater than 0 because of (21). As $\widehat{M}_{in}$ and λ_{in} increase β_{in} decreases for every OB n. Then, $\widehat{M}_{in}$ and λ_{in} will again increase (see iteration number 2 in Figure 3.2 and 3.3), because fillrate of the other OB-s decreases, which increases the lateral demand rates coming to each OB n and decreases the lateral supplies done by the other OB-s . Afterwards, β_{in} for each OB n will decrease (see iteration number 2 in Figure 3.4). Hence, at each iteration $\widehat{M}_{in}$ and λ_{in} will increase and β_{in} will decrease for each OB n. Since $\widehat{M}_{in}$ and λ_{in} are increasing and bounded by the total and direct demand rate coming to the system for SKU i respectively and β_{in} is decreasing and always nonnegative, the algorithm converges.

When the algorithm stops; $\alpha_{i,l,k}$, the fraction of the demand coming for SKU i at OB l met by lateral transshipments from OB k for $i \in I,\ n, k, l \in N_{loc}, l \neq k$, can be calculated as

$$\alpha_{i,l,k} = \frac{\beta_{ik}.\widetilde{M}_{ilk}}{m_{il}} \quad \forall\ i \in I, l \in N_{loc}, k \in N_{loc}\ l \neq k \tag{24}$$

Let AW_{in} denote the average waiting time of an order for SKU i at OB n, and δ_{in} denote the average demand rate for SKU i at OB n where $i \in I, n \in N_{loc}$. Then AW_{in} can be calculated by using Little's Law,

$$AW_{in} = E(B_{in})/\delta_{in} \tag{25}$$

where

$$\delta_{in} = \left(\sum_{x=0}^{S_{in}-1} \pi_x\right).\widehat{M}_{in} + \left(\sum_{x=S_{in}}^{+\infty} \pi_x\right).\lambda_{in} \tag{26}$$

Then, BW_{in}, the expected backorder waiting time of SKU i at OB n is

$$BW_{in} = \frac{AW_{in}}{\sum_{x=S_{in}}^{+\infty} \pi_x} \tag{27}$$

Detailed calculation of $E(B_{in})$ and δ_{in} can be seen in Appendix E.

Consequently, the approximate evaluation method used in Model 2, calculates the fillrates (β_{in}) and fraction of demand met by lateral transshipments ($\alpha_{i,l,k}$) for each SKU i at each OB n for $i \in I,\ n, k, l \in N_{loc}, l \neq k$.

3.2.2 Numerical Experiments for the Approximate Evaluation Method of Model 2

In order to validate the approximate evaluation method used in Model 2, the model is simulated using ARENA Software. The results obtained from the simulation runs are considered as the exact values. The values obtained from the algorithm are compared with the exact values.

To test the performance of approximation algorithm, 44 different instances are formed. In each instance, there are 4 local warehouses. The mean replenishment lead time of the central warehouse, T_0, is assumed 15 days and the transportation time between the central warehouse and the local warehouses, t_{reg}, is assumed 3 days which are the values used by Alfredson and Verrijt [2]. Pre-specified lateral demand orders are determined as $\sigma_1 = (2,3,4)$, $\sigma_2 = (3,4,1)$, $\sigma_3 = (4,1,2)$, and $\sigma_4 = (1,2,3)$. Moreover, 20 of the instances are symmetrical that consist of identical local warehouses (same demand rate, transportation times and base stock levels); and the remaining 24 instances are asymmetric. Remaining parameters used in the calculations are

$$t_{n,k}^{lat} = 0{,}3\ days, C_{n,k}^{lat} = €\,125, p_i = €\,5.000, h = 20\% \qquad \forall\, n, l \in N_{loc}, l \neq n$$

Five different quantities of Model 2 are approximated and compared with the simulation. These are

β_{in}: Fillrate of SKU i at OB n for $i \in I, n \in N_{loc}$

$\alpha_{i,l,k}$: Fraction of the demand coming for SKU i at OB l met by lateral transshipments from OB k for $i \in I,\ n, k, l \in N_{loc}, l \neq k$

W_{in}: Expected waiting time for SKU i at OB n

BW_{in}: Expected backorder waiting time for SKU i at OB n

and total yearly cost value (€) of the system.

Table 3.1 shows the results of the symmetric instances.

Table 3.1: Comparison of the approximations with the exact values of the symmetric instances of Model 2

				β_{in}		$\alpha_{i,n,\sigma_1(n)}$		$\alpha_{i,n,\sigma_2(n)}$		$\alpha_{i,n,\sigma_3(n)}$		W_{in} (days)		BW_{in} (days)		Total Yearly Cost (€)	
Inst.	m_{in}	S_{i0}	S_{in}	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.
1	0,01	1	1	0,93	0,93	0,06	0,06	0,01	0,00	0,00	0,00	0,03	0,02	8,32	0,00	5.121	5.123
2		2	1	0,96	0,96	0,03	0,04	0,00	0,00	0,00	0,00	0,01	0,01	6,97	0,00	6.067	6.067
3	0,04	1	1	0,55	0,52	0,19	0,25	0,09	0,12	0,05	0,06	1,10	0,16	8,46	0,69	7.398	8.115
4		2	1	0,70	0,68	0,16	0,22	0,06	0,07	0,03	0,02	0,45	0,09	7,18	0,13	7.820	8.248
5		1	2	0,90	0,91	0,08	0,08	0,01	0,01	0,00	0,00	0,04	0,03	6,36	0,00	9.692	9.630
6		2	2	0,95	0,96	0,04	0,04	0,01	0,00	0,00	0,00	0,02	0,01	5,66	0,00	10.360	10.292
7	0,08	1	1	0,20	0,15	0,13	0,13	0,10	0,11	0,07	0,09	4,69	2,72	9,19	5,08	9.339	9.840
8		2	1	0,30	0,23	0,16	0,18	0,10	0,14	0,07	0,10	3,02	1,17	7,91	2,94	10.783	12.084
9		2	2	0,72	0,73	0,16	0,20	0,06	0,05	0,02	0,01	0,33	0,08	5,85	0,08	13.507	13.862
10		2	3	0,92	0,93	0,06	0,06	0,01	0,00	0,00	0,00	0,03	0,02	4,69	0,00	15.134	14.951
11		3	1	0,42	0,34	0,18	0,22	0,10	0,15	0,06	0,10	1,83	0,42	6,87	1,41	11.853	13.854
12		6	1	0,68	0,66	0,18	0,23	0,06	0,08	0,03	0,03	0,32	0,10	4,54	0,09	13.902	14.801
13	0,16	1	1	0,03	0,02	0,03	0,02	0,03	0,02	0,02	0,02	10,06	9,68	11,23	10,56	7.234	6.851
14		2	1	0,04	0,03	0,04	0,03	0,03	0,03	0,03	0,03	8,58	8,07	10,03	9,08	9.024	8.452
15		2	2	0,20	0,14	0,13	0,12	0,09	0,11	0,07	0,09	3,62	2,22	7,00	3,95	18.470	19.317
16		3	1	0,07	0,04	0,05	0,04	0,05	0,04	0,04	0,04	7,16	6,50	8,93	7,63	11.013	10.262
17		3	2	0,27	0,20	0,15	0,16	0,10	0,13	0,07	0,10	2,71	1,27	6,34	2,76	20.229	22.289
18		6	1	0,19	0,11	0,12	0,10	0,08	0,09	0,06	0,08	3,52	2,40	6,29	3,66	17.585	17.540
19		6	2	0,53	0,49	0,17	0,25	0,09	0,13	0,05	0,07	0,92	0,16	4,84	0,41	22.829	26.955
20		6	3	0,80	0,84	0,12	0,14	0,04	0,02	0,01	0,00	0,16	0,05	3,96	0,01	23.099	22.761

Table 3.2 shows the parameter settings and Table 3.3, 3.4 and 3.5 show the results of the asymmetric instances.

Table 3.2: Parameter settings used in the asymmetric instances

Instance	m_{i1}	m_{i2}	m_{i3}	m_{i4}	t_n^{reg}	S_{i0}	S_{i1}	S_{i2}	S_{i3}	S_{i4}
1	0,01	0,02	0,03	0,04	3	1	1	1	1	1
2						2	1	1	1	1
3						2	1	1	2	2
4	0,04	0,08	0,12	0,16	3	1	1	1	1	1
5						2	1	1	1	1
6						2	1	1	2	2
7						3	1	1	2	2
8	0,08	0,12	0,16	0,2	3	1	1	1	1	1
9						2	1	1	1	1
10						2	1	1	2	2
11						3	2	2	2	2
12						3	2	2	3	3
13						6	3	3	3	3
14						6	3	3	4	4
15						6	1	2	3	4
	m_{i1}	t_1^{reg}	t_2^{reg}	t_3^{reg}	t_4^{reg}	S_{i0}	S_{i1}	S_{i2}	S_{i3}	S_{i4}
16	0,02	1	2	3	4	1	1	1	1	1
17						1	2	2	2	2
18						2	1	1	1	1
19	0,1	1	2	3	4	2	1	1	1	1
20						1	2	2	2	2
21						2	2	2	2	2
22	0,16	1	2	3	4	2	1	1	1	1
23						1	2	2	2	2
24						2	2	2	2	2

Table 3.3: Comparison of the approximations with the exact values of the asymmetric instances of Model 2

	β_{i1}		β_{i2}		β_{i3}		β_{i4}		$\alpha_{i,1,2}$		$\alpha_{i,1,3}$		$\alpha_{i,1,4}$		$\alpha_{i,2,1}$		$\alpha_{i,2,3}$	
Inst.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.
1	0,79	0,79	0,79	0,79	0,74	0,73	0,69	0,66	0,13	0,17	0,04	0,03	0,01	0,01	0,02	0,01	0,13	0,15
2	0,89	0,90	0,88	0,89	0,85	0,84	0,81	0,79	0,08	0,09	0,02	0,01	0,00	0,00	0,01	0,00	0,09	0,10
3	0,94	0,94	0,90	0,89	0,98	0,99	0,97	0,98	0,05	0,05	0,01	0,01	0,00	0,00	0,00	0,00	0,10	0,11
4	0,16	0,12	0,14	0,10	0,11	0,08	0,09	0,06	0,10	0,09	0,06	0,06	0,04	0,04	0,09	0,10	0,08	0,07
5	0,23	0,16	0,21	0,14	0,18	0,12	0,14	0,10	0,13	0,12	0,07	0,09	0,04	0,06	0,09	0,11	0,11	0,10
6	0,33	0,24	0,29	0,22	0,43	0,37	0,39	0,34	0,14	0,17	0,16	0,22	0,07	0,13	0,05	0,08	0,25	0,29
7	0,44	0,35	0,39	0,32	0,54	0,51	0,50	0,47	0,15	0,21	0,15	0,23	0,06	0,10	0,04	0,06	0,27	0,34
8	0,07	0,06	0,06	0,04	0,04	0,03	0,03	0,02	0,05	0,04	0,03	0,03	0,02	0,02	0,06	0,05	0,04	0,03
9	0,10	0,07	0,08	0,06	0,06	0,04	0,05	0,03	0,06	0,05	0,04	0,04	0,03	0,03	0,06	0,06	0,05	0,04
10	0,14	0,09	0,12	0,07	0,20	0,14	0,18	0,12	0,08	0,07	0,12	0,12	0,07	0,09	0,05	0,06	0,15	0,13
11	0,41	0,33	0,41	0,34	0,38	0,31	0,33	0,27	0,18	0,22	0,09	0,14	0,05	0,08	0,08	0,11	0,16	0,21
12	0,54	0,50	0,51	0,49	0,60	0,60	0,56	0,56	0,17	0,24	0,11	0,15	0,05	0,06	0,04	0,05	0,23	0,31
13	0,90	0,95	0,91	0,94	0,87	0,90	0,82	0,84	0,07	0,05	0,02	0,00	0,00	0,00	0,01	0,00	0,06	0,05
14	0,94	0,97	0,92	0,94	0,95	0,97	0,92	0,94	0,04	0,03	0,01	0,00	0,00	0,00	0,00	0,00	0,06	0,06
15	0,58	0,57	0,73	0,74	0,82	0,85	0,88	0,93	0,26	0,32	0,10	0,10	0,03	0,02	0,01	0,00	0,18	0,22
16	0,85	0,85	0,84	0,84	0,82	0,82	0,80	0,80	0,10	0,13	0,03	0,02	0,01	0,00	0,01	0,01	0,11	0,13
17	0,99	0,99	0,98	0,99	0,98	0,99	0,98	0,98	0,01	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,02	0,01
18	0,93	0,94	0,92	0,92	0,90	0,90	0,88	0,88	0,05	0,06	0,01	0,00	0,00	0,00	0,00	0,00	0,06	0,07
19	0,23	0,16	0,21	0,15	0,19	0,13	0,18	0,12	0,12	0,12	0,08	0,10	0,05	0,08	0,08	0,10	0,12	0,11
20	0,50	0,47	0,49	0,46	0,47	0,44	0,45	0,41	0,19	0,24	0,09	0,13	0,05	0,07	0,06	0,08	0,18	0,24
21	0,61	0,61	0,60	0,59	0,57	0,57	0,55	0,54	0,17	0,23	0,07	0,09	0,03	0,04	0,05	0,05	0,17	0,23
22	0,06	0,04	0,05	0,04	0,05	0,03	0,04	0,03	0,04	0,03	0,03	0,03	0,03	0,02	0,04	0,04	0,04	0,03
23	0,19	0,14	0,17	0,12	0,16	0,11	0,15	0,10	0,12	0,11	0,08	0,09	0,06	0,07	0,08	0,09	0,11	0,10
24	0,25	0,18	0,23	0,17	0,22	0,15	0,20	0,14	0,14	0,14	0,09	0,11	0,06	0,08	0,08	0,11	0,13	0,13

Table 3.4: Comparison of the approximations with the exact values of the asymmetric instances of Model 2

	$\alpha_{i,2,4}$		$\alpha_{i,3,1}$		$\alpha_{i,3,2}$		$\alpha_{i,3,4}$		$\alpha_{i,4,1}$		$\alpha_{i,4,2}$		$\alpha_{i,4,3}$		W_{i1} **(days)**	
Instance	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.
1	0,04	0,04	0,06	0,07	0,02	0,02	0,14	0,18	0,20	0,26	0,06	0,06	0,02	0,01	0,28	0,06
2	0,02	0,01	0,03	0,03	0,01	0,00	0,10	0,13	0,15	0,19	0,03	0,02	0,01	0,00	0,08	0,03
3	0,00	0,00	0,00	0,00	0,00	0,00	0,02	0,01	0,02	0,02	0,00	0,00	0,00	0,00	0,02	0,02
4	0,05	0,05	0,11	0,11	0,07	0,08	0,06	0,05	0,14	0,12	0,09	0,09	0,05	0,06	5,36	2,54
5	0,06	0,07	0,12	0,13	0,08	0,10	0,09	0,08	0,16	0,14	0,10	0,11	0,06	0,08	3,93	1,49
6	0,11	0,17	0,08	0,10	0,04	0,07	0,16	0,21	0,14	0,16	0,08	0,11	0,10	0,15	2,40	0,49
7	0,10	0,16	0,06	0,09	0,04	0,06	0,16	0,23	0,14	0,18	0,07	0,11	0,08	0,12	1,51	0,23
8	0,02	0,02	0,06	0,05	0,04	0,04	0,03	0,02	0,07	0,06	0,04	0,04	0,03	0,03	7,88	6,14
9	0,03	0,03	0,08	0,07	0,05	0,05	0,04	0,03	0,09	0,07	0,06	0,05	0,04	0,04	6,52	4,75
10	0,09	0,10	0,06	0,07	0,04	0,05	0,11	0,11	0,09	0,08	0,06	0,06	0,09	0,11	5,14	3,22
11	0,08	0,12	0,13	0,17	0,07	0,11	0,14	0,19	0,22	0,24	0,12	0,16	0,06	0,10	1,59	0,33
12	0,09	0,11	0,07	0,09	0,04	0,04	0,16	0,22	0,17	0,22	0,08	0,11	0,06	0,07	0,86	0,15
13	0,02	0,01	0,03	0,02	0,01	0,00	0,08	0,09	0,14	0,16	0,03	0,01	0,01	0,00	0,06	0,02
14	0,01	0,00	0,01	0,00	0,00	0,00	0,04	0,03	0,06	0,05	0,01	0,00	0,00	0,00	0,03	0,01
15	0,05	0,03	0,01	0,01	0,01	0,00	0,13	0,13	0,04	0,04	0,03	0,02	0,02	0,01	0,34	0,13
16	0,03	0,02	0,04	0,03	0,01	0,00	0,12	0,15	0,14	0,17	0,03	0,03	0,01	0,00	0,12	0,05
17	0,00	0,00	0,00	0,00	0,00	0,00	0,02	0,01	0,02	0,02	0,00	0,00	0,00	0,00	0,00	0,00
18	0,01	0,01	0,02	0,01	0,00	0,00	0,08	0,09	0,10	0,11	0,01	0,01	0,00	0,00	0,03	0,02
19	0,08	0,09	0,11	0,12	0,07	0,09	0,11	0,10	0,15	0,14	0,10	0,11	0,06	0,08	3,72	2,15
20	0,09	0,13	0,11	0,15	0,06	0,08	0,18	0,23	0,22	0,27	0,11	0,14	0,06	0,07	1,09	0,20
21	0,08	0,09	0,10	0,12	0,05	0,05	0,18	0,23	0,21	0,28	0,09	0,11	0,04	0,04	0,64	0,12
22	0,03	0,02	0,05	0,04	0,04	0,03	0,03	0,03	0,05	0,04	0,04	0,03	0,03	0,03	7,07	6,46
23	0,08	0,08	0,10	0,11	0,07	0,09	0,10	0,09	0,13	0,12	0,09	0,10	0,06	0,08	3,78	2,53
24	0,08	0,10	0,11	0,13	0,08	0,10	0,12	0,12	0,16	0,16	0,10	0,12	0,07	0,09	2,86	1,56

Table 3.5: Comparison of the approximations with the exact values of the asymmetric instances of Model 2

	W_{i2} (days)		W_{i3} (days)		W_{i4} (days)		BW_{i1} (days)		BW_{i2} (days)		BW_{i3} (days)		BW_{i4} (days)		Total Yearly Cost (€)	
Instance	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.
1	0,29	0,06	0,30	0,08	0,31	0,10	8,07	0,05	8,61	0,10	8,58	0,09	8,15	0,07	6.088	6.243
2	0,09	0,03	0,10	0,05	0,11	0,06	6,80	0,01	7,37	0,01	7,14	0,01	6,77	0,01	6.682	6.744
3	0,03	0,03	0,01	0,00	0,01	0,01	8,78	0,00	8,08	0,00	5,20	0,00	4,91	0,00	8.200	8.185
4	5,98	4,20	6,48	5,31	6,90	6,07	8,33	3,65	9,28	6,08	10,05	7,70	10,69	8,82	9.535	9.369
5	4,35	2,53	4,69	3,26	4,96	3,77	7,30	2,46	8,08	4,27	8,71	5,53	9,20	6,41	11.384	11.635
6	2,62	0,77	1,88	0,55	2,01	0,66	7,79	1,38	8,48	2,49	6,09	1,75	6,49	2,15	14.008	16.008
7	1,64	0,30	1,16	0,21	1,23	0,24	7,03	0,60	7,60	1,10	5,42	0,81	5,73	0,96	14.712	17.124
8	8,61	7,71	9,23	8,78	9,75	9,56	9,46	7,11	10,33	8,92	11,08	10,17	11,71	11,07	8.211	7.738
9	7,10	6,05	7,58	6,97	8,01	7,64	8,42	5,77	9,15	7,36	9,78	8,47	10,32	9,29	10.146	9.541
10	5,57	4,16	4,08	2,82	4,39	3,32	8,61	4,95	9,33	6,43	6,83	4,35	7,36	5,13	14.343	14.476
11	1,73	0,43	1,84	0,52	1,93	0,58	5,38	0,89	5,86	1,35	6,24	1,71	6,54	1,94	19.990	23.074
12	0,92	0,16	0,72	0,12	0,76	0,14	5,68	0,29	5,99	0,41	4,74	0,36	4,93	0,39	20.932	23.453
13	0,06	0,02	0,07	0,03	0,09	0,05	3,70	0,00	3,98	0,00	3,99	0,00	3,90	0,00	21.232	20.774
14	0,03	0,02	0,02	0,01	0,03	0,02	4,01	0,00	3,84	0,00	3,15	0,00	3,13	0,00	21.666	21.175
15	0,22	0,08	0,16	0,04	0,12	0,02	6,87	0,02	4,78	0,01	3,70	0,01	3,11	0,02	20.671	20.697
16	0,13	0,05	0,14	0,05	0,15	0,06	7,40	0,02	8,05	0,02	8,55	0,02	8,85	0,02	5.589	5.634
17	0,00	0,00	0,01	0,00	0,01	0,01	6,88	0,00	7,12	0,00	7,72	0,00	7,50	0,00	9.064	9.049
18	0,04	0,02	0,05	0,03	0,05	0,04	5,75	0,00	6,68	0,00	7,22	0,00	7,59	0,00	6.319	6.329
19	4,08	2,44	4,45	2,73	4,78	3,02	7,09	3,76	7,79	4,28	8,48	4,80	9,14	5,33	11.177	11.706
20	1,20	0,22	1,28	0,23	1,35	0,24	5,78	0,78	6,34	0,87	6,75	0,94	7,16	1,00	15.403	17.405
21	0,70	0,12	0,75	0,13	0,79	0,14	5,17	0,28	5,71	0,31	6,11	0,34	6,47	0,35	15.649	17.133
22	7,75	7,17	8,42	7,91	9,12	8,65	8,43	7,37	9,24	8,19	10,05	9,02	10,87	9,88	9.363	8.739
23	4,12	2,82	4,45	3,12	4,78	3,42	6,58	4,08	7,17	4,55	7,75	5,04	8,34	5,54	16.882	17.150
24	3,13	1,75	3,40	1,95	3,65	2,15	5,90	2,95	6,47	3,34	7,02	3,73	7,55	4,12	18.882	20.048

According to the results, it is seen that the approximate evaluation method underestimates the W_{in} values in each instance, because it underestimates the BW_{in} values. Average absolute percentage deviation in the total yearly cost are 5.71% and 5.24% for the symmetric and asymmetric cases respectively.

The computation time for the approximate evaluation method is quite short. Average computation time of one instance for the approximate evaluation method is smaller than 3 milliseconds in a Intel Dual Core 3GHz computer.

3.2.3 Greedy Algorithm for Model 2

This heuristic method is developed to determine any feasible base stock levels for the aggregate waiting time constraint for Model 2. The idea is the same with that in Model 1 with some minor differences. The current procedure does not stop after a feasible solution is found but continues and computes r_{in} is differently. The reason of these differences will be explained later.

The procedure starts with setting all base stock levels to zero, $\underline{S} = 0$ for all SKU-s and warehouses. For each $\underline{S}$, the distance to the set of feasible solutions is defined as $\sum_{n \in N_{loc}} \left(W_n(\underline{S}) - W_n^{target} \right)^+$ where $(x)^+ = max\{0, x\}$. At each iteration the ratio

$$r_{in} = \begin{cases} \dfrac{\Delta W_{in}}{\Delta Z_{in}} & if\ \Delta Z_{in} > 0 \\ \Delta W_{in}.\Delta Z_{in} & if\ \Delta Z_{in} \leq 0 \end{cases} \tag{28}$$

is calculated where

$$\Delta W_{in} = \sum_{n \in N_{loc}} \left\{ \left(W_n(\underline{S}) - W_n^{target} \right)^+ - \left(W_n(\underline{S} + U_{in}) - W_n^{target} \right)^+ \right\} \tag{29}$$

$$\Delta Z_{in} = Z(\underline{S} + U_{in}) - Z(\underline{S}) \tag{30}$$

for all $i \in I$ and $n \in N$.

The total cost may increase or decrease at each iteration because the total inventory cost always increases but total transportation cost may decrease when the base stock level increases. For that reason, r_{in} in (28) is defined to consider also the situations in which the base stock level is increased and the total cost decreases.

$\Delta W_{i^*n^*}$, change in the waiting time value of SKU i^* at OB n^*, for $i^* \in I, n^* \in N$ is only dependent on the base stock level of $S_{i^*n^*}$. These values are not subject to change if a base stock level of another SKU increases. Computation time can be saved if only the results that change are updated. A formal description of the greedy procedure is given next.

Greedy Procedure for Model 2

Step 1: Set the initial solution $\underline{S} = 0$; calculate $W_n(0)$ for all warehouses

Step 2: For all combinations $i \in I$ and $n \in N$, calculate ΔW_{in} , ΔZ_{in} , and r_{in}.

Step 3: If $min\{\Delta Z_{in}\}$ is greater than 0, let i^* and n^* be defined as $r_{i^*n^*} = \max\ r_{in}$; otherwise let i^* and n^* be defined as $r_{i^*n^*} = \min\ r_{in}$. Set $\underline{S} = \underline{S} + U_{i^*n^*}$. If $W_n(\underline{S}) \leq W_n^{target}$ for all $n \in N_{loc}$, go to *Step 4*; otherwise go to *Step 2*.

Step 4: For all combinations $i \in I, n \in N$: Calculate ΔZ_{in}.

Step 5: Let i^* and n^* be defined as $\Delta Z_{i^*n^*} = \min\ \Delta Z_{i^*n^*}$. If $\Delta Z_{i^*n^*} < 0$, then set $\underline{S} = \underline{S} + U_{i^*n^*}$ and go *to Step 4*; otherwise end.

In some cases although the target waiting time values are met by every OB n, increasing the base stock level may decrease the total cost because this can decrease the lateral transshipment rates between OB-s, which in turn decreases total transportation costs. Thus, the greedy algorithm used in Model 2, checks the possible decrease in the total cost once the target waiting time values are met by every OB n.

3.3 Solution Procedure for Model 3

Two different methods are analyzed In Model 3: The approximate evaluation method of Muckstadt and Thomas [21], and a new approximate evaluation method. Both of them calculate the service levels and transportation costs for given base stock levels. Then a greedy algorithm is developed to minimize the total inventory and transportation cost with respect to a target waiting time constraint. The approximate evaluation methods and the greedy algorithm are described below.

3.3.1 The approximate evaluation method of Muckstadt and Thomas

This method first considers the central warehouse (RDC) and calculates the expected delay and fillrate of RDC, and then it calculates the fillrates of the OB-s.

Calculating the expected delay of RDC:

This step is the same with the procedure used to find the expected delay of RDC in METRIC.

Because of the base stock policy used in the OB-s, $m_{i0} = \sum_{n \in N_{loc}} m_{in}$.

By Palm's theorem (see Silver et al. [29]),

$$P\{IL_{i0} = x\} = \frac{(m_{i0}.T_0)^{S_{i0}-x}.e^{-m_{i0}.T_0}}{(S_{i0}-x)!} \tag{31}$$

$$E(I_{i0}) = \sum_{x=1}^{S_{i0}} x.P\{IL_{i0} = x\} \; ; \; E(B_{i0}) = \sum_{x=-\infty}^{-1} x.P\{IL_{i0} = x\} \tag{32}$$

or equivalently,

$$E(B_{i0}) = E(I_{i0}) - E(IL_{i0}) = E(I_{i0}) - (S_{i0} - m_{i0}.T_0) \tag{33}$$

Then by the Little's law,

$$E(W_{i0}) = E(B_{i0})/m_{i0} \tag{34}$$

Because of FCFS policy in CBT and Poisson demand, $E(W_{i0})$ is same for all OB-s.

Let π_x denote the steady state probabilities for the $M/G/\infty$ queue in RDC, where $0 \leq x < +\infty$ and x is integer and representing the number of parts in replenishment. Then,

$$\pi_x = \left(\frac{1}{x!}\right).(m_{i0}.T_0)^x.e^{-(m_{i0}.T_0)} \qquad 0 \leq x < +\infty \tag{35}$$

$$\beta_{i0} = \sum_{x=0}^{S_{i0}-1} \pi_x \qquad (36)$$

Calculating the fill rates of the OB-s:

When demand comes and if the OB-s are out of stock, then direct shipments are done from RDC or CBT to the OB-s; and because of this demand fulfillment process, there is not any backorder in any OB n. Then for each SKU i at OB n, the mean replenishment lead time is

$$LT_{in} = t_{reg} + E(W_{i0}) \qquad (37)$$

At this point, it is assumed that the replenishment lead time for SKU i at OB n is exponentially distributed and its mean is LT_{in}. Then β_{in} can be calculated by using the Erlang Loss Probability for each SKU i at OB n. Let $L(c, p)$ denotes this probability, where c represents the number of servers and p the offered load. Then

$$L(c,p) = \frac{p^c/_{c!}}{\sum_{x=0}^{c} p^x/_{x!}} \qquad (38)$$

$$\beta_{in} = 1 - L\big(S_{in}, (\, m_{in}.\ LT_{in})\big) \qquad (39)$$

Finally, θ_{in} which denotes the fraction of demand for SKU i met by direct shipment from RDC to OB n and γ_{in} which denotes the fraction of demand for SKU i met by direct shipment from CBT to OB n for $i \in I, n \in N_{loc}$, are

$$\theta_{in} = \beta_{i0}.(1 - \beta_{in}) \qquad (40)$$

$$\gamma_{in} = (1 - \beta_{i0}).(1 - \beta_{in}) \qquad (41)$$

3.3.2 A New Approximate Evaluation Method for Model 3

Like Muckstadt and Thomas [21], this method calculates the service levels and cost for given base stock levels. It finds the expected delay in RDC, $E(W_{i0})$, and fill rates of each OB n, β_{in}, iteratively.

Calculating the expected delay of RDC:

When the inventory amount in RDC is greater than 0, because of the base stock policy used in the OB-s, $m_{i0} = \sum_{n \in N_{loc}} m_{in}$. However when the inventory amount in RDC is lower than or equal to 0; the probability that OB n requests a replenishment order from RDC is β_{in}. Because, with $(1-\beta_{in})$ probability, the OB n will be out of stock and there will be a direct shipment from CBT to the OB. Thus, $m_{in}.\beta_{in}$ is the demand rate to RDC for each OB n, when RDC is out of stock. Let m'_{i0} denotes the demand rate coming to RDC for SKU i, when the inventory amount in RDC is lower than or equal to 0 where $i \in I, n \in N_{loc}$. Then,

$$m'_{i0} = \sum_{n \in N_{loc}} m_{in}.\beta_{in} \tag{42}$$

In (42), the events that there is positive stock for SKU i at every OB n for all $n \in N_{loc}$ are assumed to be independent. Let the replenishment rate of RDC by CBT for each SKU i be

$$\mu_0 = 1/\,T_0 \tag{43}$$

and $\bar{S}_i$ be

$$\bar{S}_i = \sum_{n \in N_{loc}} S_{in} \qquad \forall\, i \in I \tag{44}$$

Because of the direct shipments from CBT to the OB-s, the inventory amount in RDC will be between $-\bar{S}_i$ and S_{i0}. There can be backorder in RDC, only when there is on-hand stock in at least one OB, thus $-\bar{S}_i$ is the minimum inventory amount or maximum number of backorders that can occur in RDC.

If all β_{in} are known and the demand coming to RDC for SKU i, when the inventory amount in RDC is lower than or equal to 0, is a Poisson process; then $X_{in}(t)$, the stock level for SKU i at RDC at time t, is a Continuous Time Markov Chain (CTMC) with a rate diagram given in Figure 3.5.

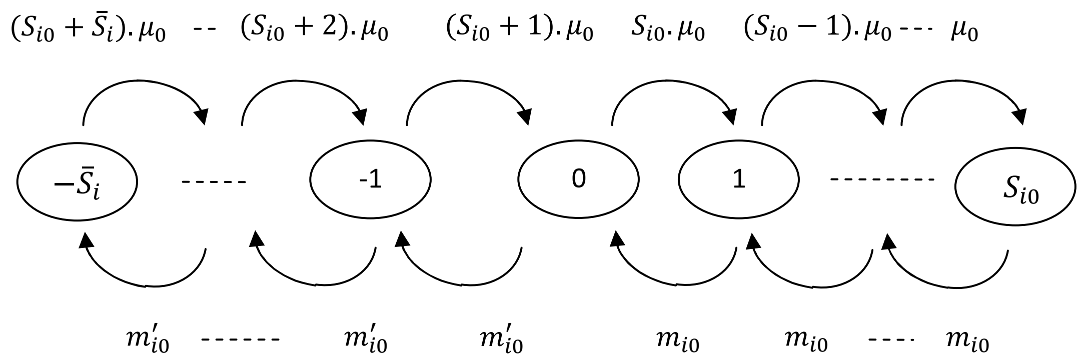

Figure 3.5: Rate diagram of the Markov process describing the stock level in RDC in Model 3

Let π_x denote the steady state probabilities for the continuous time Markov chain shown in Figure 3.5, where $0 \leq x \leq +\infty$ and x is integer. Then the steady state probabilities can be calculated as

$$\pi_x = \begin{cases} (S_{i0} - x + 1).\dfrac{\mu_0}{m'_{i0}}.\pi_{x-1} & -\bar{S}_i < x \leq 0 \\ (S_{i0} - x + 1).\dfrac{\mu_0}{m_{i0}}.\pi_{x-1} & 0 < x \leq S_{i0} \end{cases} \tag{45}$$

Then

$$\beta_{i0} = \sum_{x=1}^{S_{i0}} \pi_x \tag{46}$$

$$E(B_{i0}) = \sum_{x=-\bar{S}_i}^{-1} (-x).\pi_x \tag{47}$$

Let δ_{i0} be the average demand rate coming to RDC for each SKU i. Then,

$$\delta_{i0} = \left(\sum_{x=-\bar{S}_i+1}^{0} \pi_x\right).m'_{i0} + \left(\sum_{x=1}^{S_{i0}} \pi_x\right).\ m_{i0} \tag{48}$$

Note that (48) is an approximation because m'_{i0} is dependent to π_x. Then, expected delay at RDC can be calculated by the Little's law.

$$E(W_{i0}) = E(B_{i0})/\delta_{i0} \quad (49)$$

Because of FCFS policy in CBT and Poisson demand, $E(W_{i0})$ is same for all OB-s.

<u>*Calculating the fill rates*</u>:

For each SKU i at OB n, the mean replenishment lead time is

$$LT_{in} = t_{reg} + E(W_{i0}) \quad (50)$$

In this point, it is assumed that the replenishment lead time for SKU i at OB n is exponentially distributed and its mean is LT_{in}. Because of the direct shipments, there is not any backorder in each OB n. Then β_{in} can be calculated by Erlang Loss Probability for each SKU i at OB n.

$$\beta_{in} = 1 - L\big(S_{in}, (\, m_{in}.\ LT_{in})\big) \quad (51)$$

Thus β_{in} is dependent to $E(W_{i0})$, and vice versa. Then, an iterative solution procedure is used to find the $E(W_{i0})$ and β_{in} for every SKU i at every OB n for $i \in I, n \in N_{loc}$.

Iterative algorithm for Model 3

Let ϵ =10^{-6}. For each SKU i:

Step 1: Assume no delay occurs in RDC, which means $E(W_{i0}) = 0$. Then $LT_{in} = t_{reg}$, and calculate β_{in} for every OB n.

Step 2: Using β_{in}, calculate the $E(W_{i0})$. Then calculate β_{in} for every OB n.

Step 3: Repeat *Step 2* until $E(W_{i0})$ does not change more than ϵ.

The variables $E(W_{i0})$ and β_{in} converge in all cases considered in this study. Figure 3.6 and 3.7 show convergence of $E(W_{i0})$ and β_{in} for a setting.

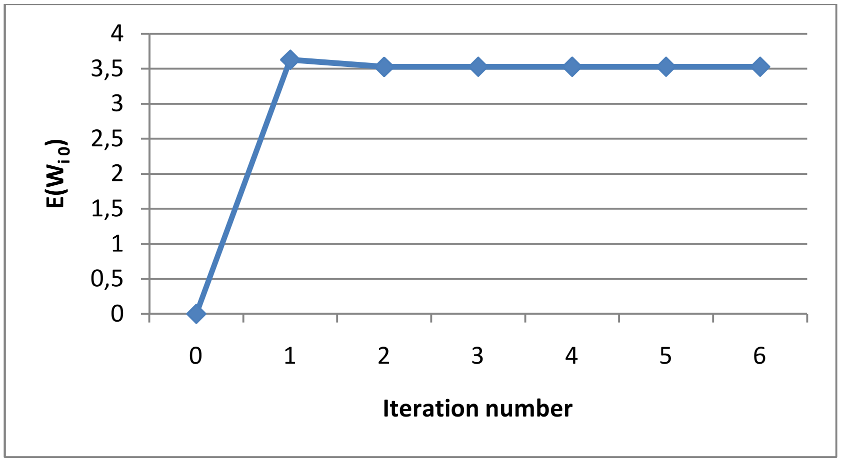

Figure 3.6: $E(W_{i0})$ at each iteration in instance 1 of Model 3

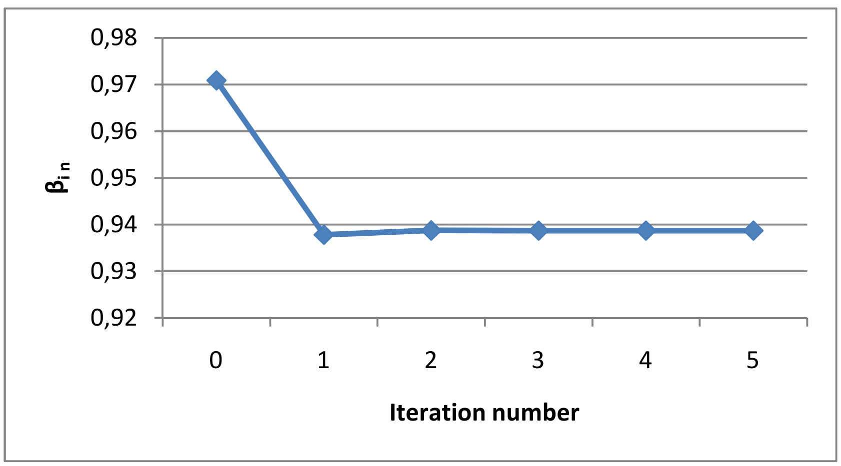

Figure 3.7: β_{in} at each iteration in instance 1 of Model 3

As it can be seen from Figure 3.6, the initial value of $E(W_{i0})$ is 0. In this case the corresponding β_{in} becomes the largest because the lower the expected delay in RDC, the lower the lead times and the higher the fillrate of each OB. In the following iteration, $E(W_{i0})$ becomes the largest, because β_{in} of each OB is the largest and the higher the fillrate values of the OB-s, the higher the number of replenishment orders from RDC and the higher the delay for these orders in RDC. Afterwards, β_{in} become the lowest as seen in iteration number 1 in Figure 3.7, because $E(W_{i0})$ is the largest. Then, $E(W_{i0})$ becomes the second lowest and greater than 0 as seen in iteration number 2 in Figure 3.6. Note that in order $E(W_{i0})$ to be 0, each β_{in} value should be 0, but that is not the case. Then; β_{in} becomes the second largest foe each OB, because the current $E(W_{i0})$ is the second lowest as seen in iteration number 2 in Figure 3.7. Hence; at each odd numbered iteration, $E(W_{i0})$ and β_{in} will increase and decrease respectively, at each even numbered iteration, $E(W_{i0})$ and β_{in} will decrease and increase respectively; and at each iteration, the difference between two consecutive $E(W_{i0})$ and β_{in} values will decrease. Thus, sooner or later these values converge.

The algorithm is robust with respect to the initial value of $E(W_{i0})$. For all possible initial values of $E(W_{i0})$, it converges to the same value. However, the choice of the initial value of $E(W_{i0})$ affects the number of iterations done, which affects the computation time. Generally, the closer the initial value is to the converged value, the shorter the computation time. However; in our cases, the change in the computation time was insignificant.

After convergence, θ_{in} which denotes the fraction of demand for SKU i met by direct shipment from RDC to OB n and γ_{in} which denotes the fraction of demand for SKU i met by direct shipment from CBT to OB n for $i \in I, n \in N_{loc}$, are

$$\theta_{in} = \beta_{i0}.(1 - \beta_{in}) \tag{52}$$

$$\gamma_{in} = (1 - \beta_{i0}).(1 - \beta_{in}) \tag{53}$$

Consequently, the new approximate evaluation method used in Model 3, calculates the fillrates (β_{in}); the fraction of demand for SKU i met by direct shipment from RDC to OB n (θ_{in}); and the fraction of demand for SKU i met by direct shipment from CBT to OB n (γ_{in}) for each SKU i at each OB n for $i \in I, n \in N_{loc}$.

3.3.3 Numerical Experiments for the Approximate Evaluation Methods of Model 3

In order to validate the approximate evaluation methods used in Model 3, the model is simulated using ARENA Software. The results obtained from the simulation runs are considered as the exact values. The values obtained from the algorithms are compared with the exact values.

To test the performance of the two approximation algorithms, same instances used to test Model 2 are used. Five different quantities of Model 3 are approximated and compared with the simulation. These are

$E(W_{i0})$: Expected delay in RDC SKU i

β_{i0}: Fillrate of SKU i at RDC

β_{in}: Fillrate of SKU i at OB n for $i \in I, n \in N_{loc}$

θ_{in}: Fraction of demand for SKU i met by direct shipment done from RDC to OB n

γ_{in}: Fraction of demand for SKU i met by direct shipment done from CBT to OB n

Table 3.6 shows the results of the symmetric instances of Model 3.

Table 3.6: Comparison of the approximations with the exact values of the symmetric instances of Model 3

				$E(W_{i0})$			β_{i0}			β_{in}			θ_{in}			γ_{in}		
Inst.	m_{in}	S_{i0}	S_{in}	Exact	Appr.	M. & T.	Exact	Appr.	M. & T.	Exact	Appr.	M. & T.	Exact	Appr.	M. & T.	Exact	Appr.	M. & T.
1	0,01	1	1	3,27	3,53	3,72	0,59	0,55	0,55	0,94	0,94	0,94	0,00	0,03	0,03	0,06	0,03	0,03
2		2	1	0,56	0,64	0,67	0,90	0,88	0,88	0,97	0,96	0,96	0,02	0,03	0,03	0,02	0,00	0,00
3	0,04	1	1	7,41	7,61	9,32	0,18	0,14	0,09	0,71	0,70	0,67	0,00	0,04	0,03	0,29	0,26	0,30
4		2	1	3,44	3,81	4,99	0,46	0,36	0,31	0,79	0,79	0,76	0,02	0,08	0,07	0,19	0,14	0,17
5		1	2	8,87	8,99	9,32	0,11	0,10	0,09	0,92	0,93	0,92	0,00	0,01	0,01	0,08	0,06	0,07
6		2	2	4,63	4,82	4,99	0,33	0,32	0,31	0,96	0,96	0,96	0,00	0,01	0,01	0,04	0,02	0,03
7	0,08	1	1	9,13	9,03	11,90	0,07	0,05	0,01	0,51	0,51	0,46	0,00	0,02	0,00	0,49	0,47	0,54
8		2	1	5,55	5,69	8,92	0,21	0,14	0,05	0,59	0,59	0,51	0,01	0,06	0,02	0,39	0,35	0,46
9		2	2	7,75	7,89	8,92	0,08	0,08	0,05	0,83	0,83	0,81	0,00	0,01	0,01	0,17	0,16	0,18
10		2	3	8,57	8,63	8,92	0,05	0,06	0,05	0,94	0,95	0,94	0,00	0,00	0,00	0,06	0,05	0,05
11		3	1	3,32	3,62	6,25	0,38	0,26	0,14	0,66	0,65	0,57	0,03	0,09	0,06	0,30	0,26	0,36
12		6	1	0,57	0,75	1,31	0,82	0,71	0,65	0,78	0,77	0,74	0,12	0,16	0,17	0,10	0,07	0,09
13	0,16	1	1	10,24	9,82	13,44	0,02	0,02	0,00	0,32	0,33	0,28	0,00	0,01	0,00	0,68	0,66	0,72
14		2	1	7,14	6,87	11,88	0,07	0,05	0,00	0,38	0,39	0,30	0,01	0,03	0,00	0,61	0,58	0,70
15		2	2	9,71	9,65	11,88	0,01	0,01	0,00	0,59	0,60	0,54	0,00	0,01	0,00	0,41	0,40	0,46
16		3	1	5,09	4,97	10,32	0,16	0,09	0,00	0,44	0,44	0,32	0,01	0,05	0,00	0,55	0,51	0,68
17		3	2	7,77	7,77	10,32	0,03	0,03	0,00	0,64	0,65	0,58	0,00	0,01	0,00	0,36	0,34	0,42
18		6	1	1,84	2,02	5,84	0,49	0,29	0,08	0,56	0,55	0,41	0,08	0,13	0,05	0,36	0,31	0,54
19		6	2	3,62	3,92	5,84	0,21	0,18	0,08	0,78	0,77	0,71	0,01	0,04	0,02	0,22	0,19	0,27
20		6	3	4,79	5,03	5,84	0,12	0,12	0,08	0,89	0,90	0,88	0,00	0,01	0,01	0,11	0,09	0,11

Figure 3.8 – 3.11 show the β_{i0}, β_{in}, θ_{in}, and γ_{in} of the two approximate methods and of the simulation for the symmetric instances. Note that the instances of the exact results are ranked from smallest.

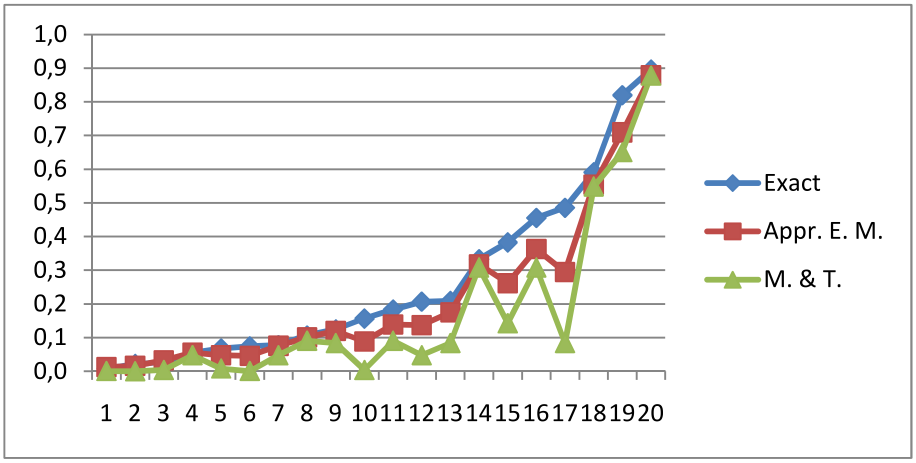

Figure 3.8: β_{i0} in the symmetric instances in Model 3

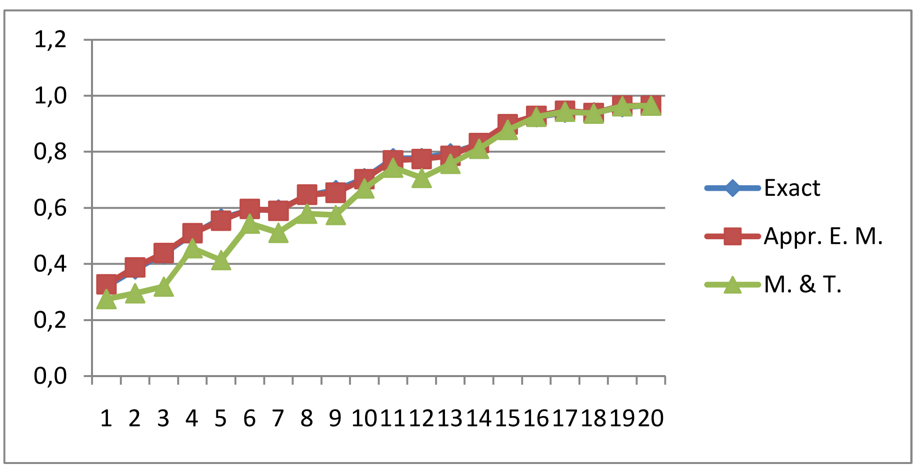

Figure 3.9: β_{in} in the symmetric instances in Model 3

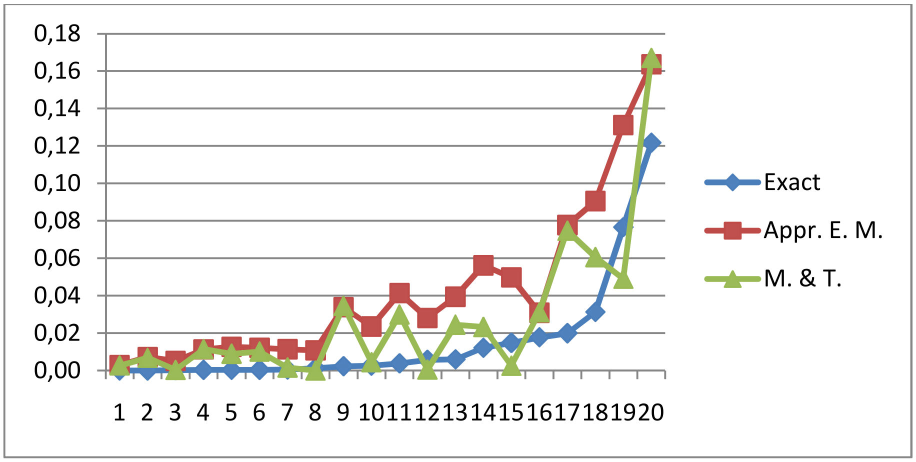

Figure 3.10: θ_{in} in the symmetric instances in Model 3

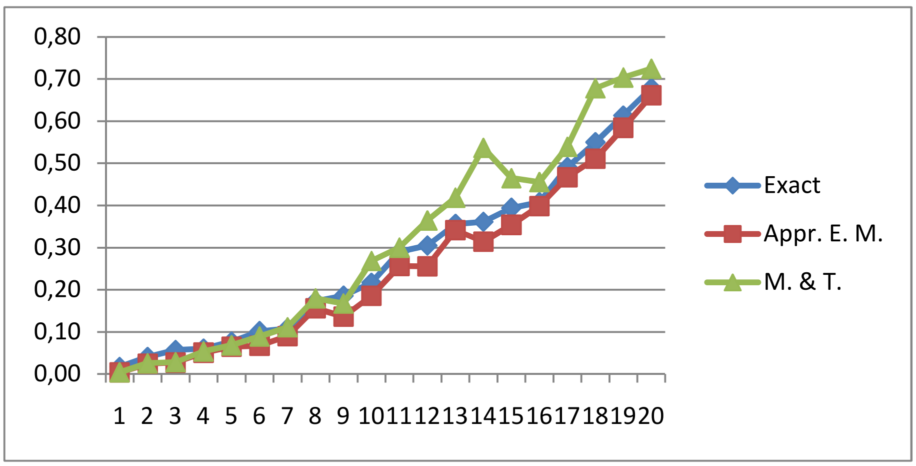

Figure 3.11: γ_{in} in the symmetric instances in Model 3

According to the results; the new approximate evaluation method performs better than the method of Muckstadt and Thomas in every variable except θ_{in}.

The new approximate evaluation method and the method of Muckstadt and Thomas underestimate β_{i0} in 85%, and 100% of the instances respectively. The average absolute percentage deviations for β_{i0} of these methods are 20% and 53% respectively. However, these values become 0,82% and 8,62% for β_{in} respectively. Thus the new approximate evaluation method estimates β_{in} quite accurately. However, the method of Muckstadt and Thomas underestimate β_{in} in 85% of the instances.

Another interesting point is that the new approximate evaluation method overestimates θ_{in} and underestimates γ_{in} in every instance. In (52) and (53), the events that there is positive stock for SKU i at RDC and OB n are assumed independent, although these events are dependent. θ_{in} and γ_{in} calculated by (52) and (53) with the exact β_{i0} and β_{in} and compared with the exact values to see the reason of the deviation. Figure 3.12 and 3.13 show this comparison. Note that the instances of the exact results are ranked from the smallest to the largest.

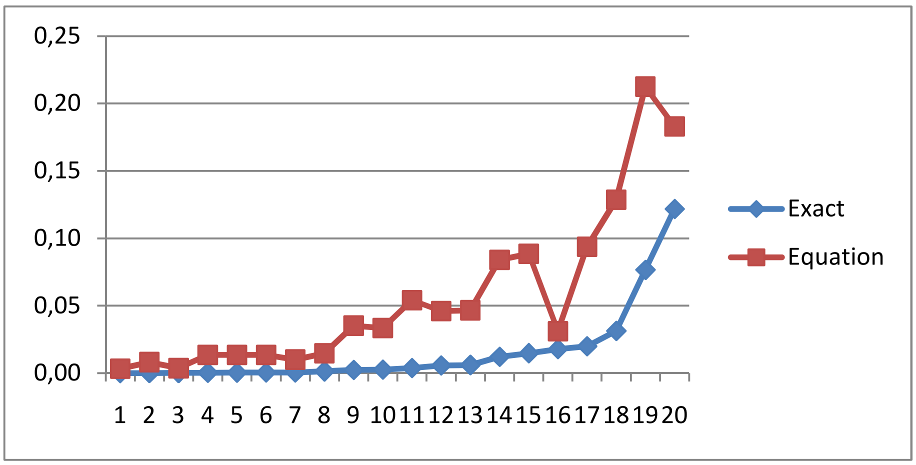

Figure 3.12: Comparison of the exact θ_{in} with the ones calculated by (52) and (53) with the exact β_{i0} and β_{in} in Model 3

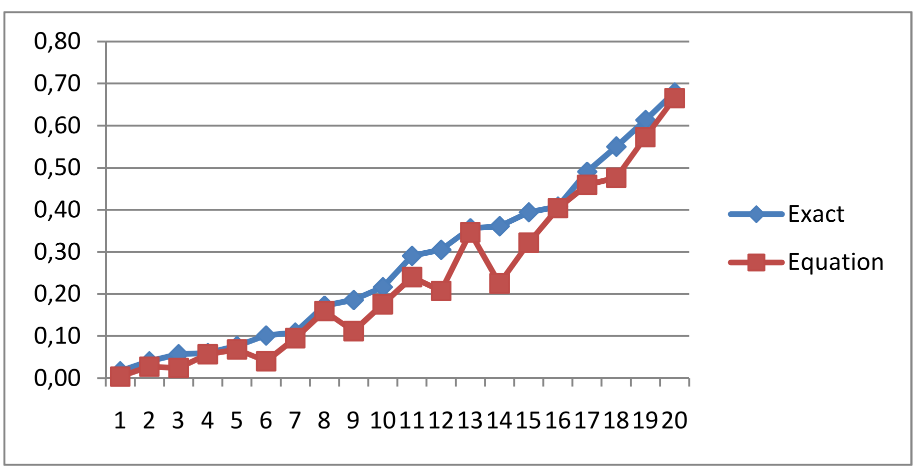

Figure 3.13: Comparison of the exact γ_{in} with the ones calculated by (52) and (53) with the exact β_{i0} and β_{in} in Model 3

As it can be seen from Figure 3.12 and 3.13, (52) overestimates θ_{in} and (53) underestimates γ_{in}. Exact θ_{in} is equal to the probability that there is positive stock for SKU i at RDC given that OB n is out of stock for SKU i. However in (52), θ_{in} is calculated by just multiplying the probability that there is positive stock for SKU i at RDC with the probability that OB n is out of stock for SKU i. Intuitively, it is seen that when there is positive stock in RDC, there is more likely positive stock in an OB either, because OB-s are replenished by the RDC. Thus, when there is positive stock in RDC, it is less likely that an OB is out of stock. However, (52) does not consider this situation and as a result it overestimates the θ_{in}. Underestimation of θ_{in} by (53) can be explained in a similar way.

Table 3.7, 3.8 and 3.9 show the results of the asymmetric instances of Model 3. Parameters settings used in the asymmetric instances can be seen in Table 3.2.

Table 3.7: Comparison of the approximations with the exact values of the asymmetric instances of Model 3

	$E(W_{i0})$			β_{i0}			β_{i1}			β_{i2}			β_{i3}		
Inst.	Exact	Appr.	M. & T.	Exact	Appr.	M. & T.	Exact	Appr.	M. & T.	Exact	Appr.	M. & T.	Exact	Appr.	M. & T.
1	5,77	6,16	7,23	0,32	0,26	0,22	0,92	0,92	0,91	0,85	0,85	0,83	0,79	0,78	0,77
2	2,02	2,36	2,81	0,66	0,58	0,56	0,95	0,95	0,95	0,91	0,90	0,90	0,87	0,86	0,85
3	2,53	2,69	2,81	0,59	0,56	0,56	0,95	0,95	0,95	0,90	0,90	0,90	0,98	0,99	0,99
4	9,34	9,18	12,51	0,06	0,04	0,00	0,66	0,67	0,62	0,50	0,51	0,45	0,40	0,41	0,35
5	5,88	5,96	10,05	0,17	0,10	0,02	0,73	0,74	0,66	0,58	0,58	0,49	0,49	0,48	0,39
6	7,61	7,69	10,05	0,07	0,06	0,02	0,70	0,70	0,66	0,54	0,54	0,49	0,73	0,74	0,68
7	5,26	5,50	7,70	0,17	0,14	0,06	0,75	0,75	0,70	0,61	0,60	0,54	0,79	0,80	0,73
8	9,98	9,64	13,21	0,03	0,02	0,00	0,49	0,50	0,44	0,39	0,40	0,34	0,33	0,33	0,28
9	6,79	6,61	11,43	0,10	0,06	0,00	0,55	0,57	0,46	0,46	0,46	0,37	0,39	0,39	0,30
10	8,44	8,38	11,43	0,03	0,03	0,00	0,52	0,52	0,46	0,42	0,42	0,37	0,62	0,63	0,55
11	7,22	7,31	9,66	0,05	0,05	0,01	0,83	0,84	0,80	0,74	0,75	0,69	0,66	0,66	0,60
12	8,11	8,19	9,66	0,03	0,03	0,01	0,82	0,83	0,80	0,72	0,72	0,69	0,81	0,82	0,79
13	3,92	4,19	4,79	0,21	0,19	0,16	0,98	0,98	0,98	0,95	0,95	0,95	0,91	0,92	0,90
14	4,32	4,51	4,79	0,18	0,17	0,16	0,97	0,98	0,98	0,94	0,95	0,95	0,96	0,97	0,97
15	3,62	3,89	4,79	0,24	0,21	0,16	0,66	0,64	0,62	0,84	0,84	0,82	0,91	0,92	0,90
16	5,28	5,62	6,26	0,37	0,32	0,30	0,89	0,88	0,87	0,87	0,87	0,86	0,86	0,85	0,84
17	6,13	6,21	6,26	0,32	0,30	0,30	0,99	0,99	0,99	0,99	0,99	0,99	0,98	0,99	0,99
18	1,60	1,85	2,05	0,72	0,67	0,66	0,95	0,95	0,94	0,93	0,93	0,93	0,92	0,91	0,91
19	6,28	6,25	10,05	0,14	0,09	0,02	0,58	0,58	0,48	0,55	0,55	0,45	0,52	0,52	0,43
20	11,49	11,47	12,51	0,01	0,01	0,00	0,74	0,74	0,72	0,72	0,72	0,70	0,70	0,70	0,68
21	8,62	8,68	10,05	0,04	0,04	0,02	0,80	0,81	0,78	0,78	0,78	0,75	0,76	0,76	0,73
22	7,28	6,98	11,88	0,06	0,04	0,00	0,43	0,44	0,33	0,40	0,41	0,31	0,38	0,39	0,30
23	12,15	12,05	13,44	0,00	0,00	0,00	0,58	0,59	0,55	0,56	0,56	0,53	0,54	0,54	0,51
24	9,82	9,73	11,88	0,01	0,01	0,00	0,64	0,65	0,59	0,62	0,62	0,57	0,59	0,59	0,54

Table 3.8: Comparison of the approximations with the exact values of the asymmetric instances of Model 3

	β_{i3}			θ_{i1}			θ_{i2}			θ_{i3}			θ_{i4}		
Inst.	Exact	Appr.	M. & T.	Exact	Appr.	M. & T.	Exact	Appr.	M. & T.	Exact	Appr.	M. & T.	Exact	Appr.	M. & T.
1	0,74	0,73	0,71	0,00	0,02	0,02	0,00	0,04	0,04	0,00	0,06	0,05	0,01	0,07	0,06
2	0,84	0,82	0,81	0,01	0,03	0,03	0,02	0,06	0,06	0,03	0,08	0,08	0,04	0,10	0,11
3	0,97	0,98	0,98	0,01	0,03	0,03	0,02	0,06	0,06	0,00	0,01	0,01	0,00	0,01	0,01
4	0,34	0,34	0,29	0,00	0,01	0,00	0,00	0,02	0,00	0,00	0,02	0,00	0,00	0,02	0,00
5	0,42	0,41	0,32	0,00	0,03	0,01	0,01	0,04	0,01	0,01	0,05	0,01	0,02	0,06	0,01
6	0,65	0,65	0,59	0,00	0,02	0,01	0,00	0,03	0,01	0,00	0,02	0,01	0,00	0,02	0,01
7	0,72	0,72	0,65	0,01	0,03	0,02	0,01	0,05	0,03	0,00	0,03	0,02	0,00	0,04	0,02
8	0,28	0,28	0,24	0,00	0,01	0,00	0,00	0,01	0,00	0,00	0,01	0,00	0,00	0,01	0,00
9	0,34	0,34	0,26	0,00	0,03	0,00	0,01	0,03	0,00	0,01	0,04	0,00	0,01	0,04	0,00
10	0,56	0,56	0,48	0,00	0,01	0,00	0,00	0,02	0,00	0,00	0,01	0,00	0,00	0,01	0,00
11	0,59	0,59	0,52	0,00	0,01	0,00	0,00	0,01	0,00	0,00	0,02	0,00	0,00	0,02	0,00
12	0,75	0,75	0,71	0,00	0,01	0,00	0,00	0,01	0,00	0,00	0,01	0,00	0,00	0,01	0,00
13	0,87	0,88	0,86	0,00	0,00	0,00	0,00	0,01	0,01	0,00	0,02	0,02	0,00	0,02	0,02
14	0,94	0,95	0,95	0,00	0,00	0,00	0,00	0,01	0,01	0,00	0,00	0,00	0,00	0,01	0,01
15	0,95	0,96	0,95	0,03	0,08	0,06	0,00	0,03	0,03	0,00	0,02	0,02	0,00	0,01	0,01
16	0,84	0,84	0,83	0,00	0,04	0,04	0,00	0,04	0,04	0,00	0,05	0,05	0,01	0,05	0,05
17	0,98	0,98	0,98	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,01
18	0,90	0,90	0,89	0,01	0,04	0,04	0,01	0,05	0,05	0,02	0,06	0,06	0,03	0,07	0,07
19	0,49	0,49	0,42	0,00	0,04	0,01	0,01	0,04	0,01	0,01	0,04	0,01	0,01	0,05	0,01
20	0,68	0,68	0,66	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
21	0,74	0,74	0,71	0,00	0,01	0,00	0,00	0,01	0,00	0,00	0,01	0,00	0,00	0,01	0,01
22	0,35	0,36	0,28	0,00	0,02	0,00	0,00	0,03	0,00	0,00	0,03	0,00	0,01	0,03	0,00
23	0,52	0,52	0,49	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
24	0,57	0,57	0,52	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01	0,00

Table 3.9: Comparison of the approximations with the exact values of the asymmetric instances of Model 3

	γ_{i1}			γ_{i2}			γ_{i3}			γ_{i4}		
Inst.	Exact	Appr.	M. & T.	Exact	Appr.	M. & T.	Exact	Appr.	M. & T.	Exact	Appr.	M. & T.
1	0,08	0,06	0,07	0,15	0,11	0,13	0,20	0,16	0,18	0,25	0,20	0,23
2	0,04	0,02	0,02	0,08	0,04	0,05	0,10	0,06	0,07	0,13	0,07	0,08
3	0,04	0,02	0,02	0,08	0,04	0,05	0,02	0,01	0,01	0,02	0,01	0,01
4	0,34	0,32	0,38	0,50	0,48	0,55	0,59	0,57	0,65	0,66	0,64	0,71
5	0,27	0,24	0,34	0,41	0,37	0,50	0,50	0,47	0,60	0,56	0,53	0,66
6	0,30	0,28	0,34	0,45	0,43	0,50	0,27	0,25	0,32	0,35	0,33	0,41
7	0,24	0,22	0,28	0,38	0,35	0,43	0,21	0,18	0,25	0,28	0,24	0,33
8	0,51	0,49	0,56	0,61	0,59	0,66	0,67	0,66	0,72	0,72	0,70	0,76
9	0,44	0,41	0,53	0,54	0,50	0,63	0,60	0,57	0,70	0,65	0,62	0,74
10	0,48	0,46	0,53	0,58	0,56	0,63	0,38	0,36	0,45	0,44	0,43	0,52
11	0,17	0,15	0,20	0,26	0,24	0,31	0,34	0,32	0,40	0,41	0,39	0,47
12	0,18	0,17	0,20	0,28	0,27	0,31	0,19	0,17	0,21	0,25	0,24	0,28
13	0,02	0,01	0,02	0,05	0,04	0,05	0,09	0,07	0,08	0,13	0,10	0,12
14	0,03	0,02	0,02	0,06	0,04	0,05	0,04	0,02	0,02	0,06	0,04	0,04
15	0,31	0,28	0,32	0,15	0,12	0,16	0,08	0,06	0,08	0,05	0,03	0,04
16	0,11	0,08	0,09	0,13	0,09	0,10	0,14	0,10	0,11	0,15	0,11	0,12
17	0,01	0,01	0,01	0,01	0,01	0,01	0,02	0,01	0,01	0,02	0,01	0,01
18	0,04	0,02	0,02	0,05	0,02	0,03	0,06	0,03	0,03	0,07	0,03	0,04
19	0,42	0,38	0,52	0,45	0,41	0,54	0,47	0,44	0,56	0,50	0,46	0,57
20	0,26	0,25	0,28	0,28	0,28	0,30	0,30	0,30	0,32	0,32	0,32	0,34
21	0,20	0,18	0,22	0,22	0,21	0,24	0,24	0,23	0,27	0,26	0,25	0,29
22	0,57	0,54	0,67	0,59	0,56	0,69	0,62	0,59	0,70	0,64	0,61	0,72
23	0,42	0,41	0,45	0,44	0,44	0,47	0,46	0,46	0,49	0,48	0,48	0,51
24	0,36	0,35	0,41	0,38	0,38	0,43	0,41	0,40	0,46	0,43	0,43	0,48

Like in the symmetric cases, the new approximate evaluation method performs better than the method of Muckstadt and Thomas in every variable except θ_{in} in the asymmetric instances.

The new approximate evaluation method and the method of Muckstadt and Thomas underestimate β_{i0} in 75%, and 100% of the instances respectively. The average absolute percentage deviations for β_{i0} of these methods are 23% and 59% respectively. However, these values become 0,80% and 7,21% for β_{in} respectively. Thus the new approximate evaluation method estimates β_{in} quite accurately. However, the method of Muckstadt and Thomas underestimate β_{in} in 89% of the instances.

Like in the symmetric instances, the new approximate evaluation method overestimates θ_{in} and underestimates γ_{in} in every asymmetric instance with the same reason mentioned before. In the asymmetric instances, θ_{in} and γ_{in} calculated by (52) and (53) with the exact β_{i0} and β_{in} are greater and lower than the exact values respectively.

The computation time for the approximate evaluation methods are quite short. Average computation time of one instance for the method of Muckstadt and Thomas and the new approximate evaluation method is smaller than 1 and 4 milliseconds respectively in a Intel Dual Core 3GHz computer.

3.3.3.1 Cost and Service Level Validation

Total cost values and the expected waiting time values are calculated to analyze the accuracy of the approximation methods. Following parameter setting is used in the computations:

$$t_n^{RDC} = 0{,}3\ days,\ \ t_n^{CBT} = 2{,}1\ days,\ \ p_i = €\,5.000,\ \ h = 20\%,\ \ C_n^{RDC} = €\,125$$

$$C_n^{CBT} = €\,375 \qquad \forall\, n \in N_{loc}$$

Table 3.10 shows the total yearly cost and aggregate waiting time values for all OB-s for the simulation and the other two methods for the symmetric instances.

Table 3.10: Total yearly cost and aggregate waiting time values for all OB-s for the symmetric instances of Model 3

	Total yearly cost values (€)			**W_{in} (days)**		
Instances	Exact	Appr. E. M.	M. & T.	Exact	Appr. E. M.	M. & T.
1	5.318	5.212	5.219	0,12	0,07	0,07
2	6.126	6.080	6.080	0,04	0,02	0,02
3	11.390	10.919	11.791	0,61	0,55	0,64
4	10.213	9.556	10.215	0,40	0,31	0,37
5	10.679	10.473	10.547	0,16	0,14	0,15
6	10.890	10.620	10.649	0,09	0,05	0,06
7	26.519	25.791	28.688	1,03	0,99	1,13
8	23.434	22.322	26.704	0,83	0,76	0,98
9	17.559	17.011	18.010	0,36	0,33	0,38
10	16.633	16.267	16.403	0,13	0,11	0,11
11	20.812	19.514	23.853	0,65	0,56	0,78
12	16.223	15.327	16.359	0,25	0,19	0,24
13	64.478	63.262	68.465	1,43	1,39	1,52
14	59.876	57.992	67.655	1,29	1,24	1,48
15	45.713	45.075	49.921	0,86	0,84	0,96
16	55.603	53.205	66.471	1,16	1,09	1,42
17	42.211	41.253	47.731	0,75	0,72	0,88
18	43.858	41.352	58.459	0,78	0,70	1,14
19	33.142	31.437	38.238	0,46	0,40	0,57
20	27.506	26.216	28.044	0,23	0,19	0,24

Average absolute percentage deviations in the total yearly cost values for the new approximate evaluation method and the method of Muckstadt and Thomas are 3,5% and 8,1% respectively. Another interesting point is that the new approximate evaluation method underestimates the total cost and waiting time values. This result is predictable because the new method estimates β_{in} accurately, but it overestimates θ_{in} and underestimates γ_{in}. Since it is more costly and time consuming to make direct shipment from the CBT, the new approximate evaluation method underestimates the expected waiting time and the total cost values.

Table 3.11 shows the total yearly cost and aggregate waiting time values for each OB for the simulation and the other two methods for the asymmetric instances.

Average absolute percentage deviations in the total yearly cost values for the new approximate evaluation method and the method of Muckstadt and Thomas are 3,1% and 7% respectively. Again, it is seen that the new approximate evaluation method underestimates the total cost and waiting time values because it estimates β_{in} accurately, but it overestimates θ_{in} and underestimates γ_{in}.

Table 3.11: Total yearly cost and aggregate waiting time values for each OB for the asymmetric instances of Model 3

	Total Yearly Cost (€)			W_{i1} **(days)**			W_{i2} **(days)**			W_{i3} **(days)**			W_{i4} **(days)**		
Inst.	Exact	Appr.	M. & T.	Exact	Appr.	M. & T.	Exact	Appr.	M. & T.	Exact	Appr.	M. & T.	Exact	Appr.	M. & T.
1	7.747	7.393	7.678	0,17	0,14	0,16	0,32	0,25	0,29	0,43	0,35	0,40	0,53	0,44	0,49
2	7.520	7.148	7.257	0,09	0,05	0,06	0,16	0,10	0,11	0,23	0,15	0,16	0,28	0,19	0,21
3	8.495	8.323	8.333	0,09	0,06	0,06	0,17	0,11	0,11	0,03	0,01	0,01	0,05	0,02	0,02
4	36.474	35.661	39.394	0,71	0,67	0,80	1,05	1,00	1,16	1,25	1,21	1,36	1,38	1,34	1,49
5	32.781	31.561	37.929	0,57	0,51	0,71	0,87	0,80	1,06	1,06	0,99	1,26	1,19	1,13	1,40
6	26.746	25.940	29.591	0,63	0,60	0,71	0,95	0,92	1,06	0,57	0,53	0,67	0,74	0,70	0,86
7	24.071	22.966	26.962	0,51	0,47	0,60	0,80	0,75	0,92	0,44	0,38	0,53	0,58	0,52	0,70
8	55.054	53.994	58.755	1,08	1,04	1,19	1,28	1,24	1,39	1,41	1,38	1,52	1,51	1,48	1,60
9	50.766	49.085	57.821	0,93	0,87	1,12	1,13	1,07	1,33	1,27	1,21	1,46	1,36	1,31	1,56
10	43.163	42.168	48.158	1,01	0,97	1,12	1,21	1,18	1,33	0,80	0,76	0,94	0,93	0,90	1,08
11	35.619	34.761	40.070	0,35	0,32	0,42	0,55	0,51	0,65	0,71	0,68	0,84	0,85	0,82	0,99
12	30.351	29.754	32.797	0,38	0,36	0,42	0,59	0,57	0,65	0,39	0,37	0,45	0,51	0,50	0,60
13	24.559	23.385	24.406	0,05	0,03	0,04	0,11	0,08	0,10	0,19	0,15	0,18	0,27	0,22	0,26
14	23.606	22.593	22.875	0,05	0,04	0,04	0,12	0,09	0,10	0,07	0,05	0,05	0,12	0,09	0,10
15	25.212	23.894	25.653	0,65	0,61	0,70	0,32	0,27	0,33	0,18	0,13	0,18	0,11	0,07	0,10
16	6.458	6.198	6.303	0,24	0,18	0,20	0,27	0,20	0,22	0,30	0,22	0,24	0,32	0,25	0,27
17	9.179	9.113	9.114	0,03	0,01	0,01	0,03	0,02	0,02	0,04	0,02	0,02	0,04	0,03	0,03
18	6.699	6.482	6.507	0,09	0,05	0,05	0,12	0,06	0,07	0,14	0,08	0,08	0,16	0,09	0,10
19	31.230	29.857	36.058	0,87	0,81	1,09	0,94	0,87	1,13	1,00	0,93	1,17	1,04	0,98	1,21
20	25.022	24.704	25.941	0,55	0,54	0,59	0,59	0,58	0,63	0,64	0,62	0,67	0,68	0,67	0,71
21	22.687	22.104	23.979	0,42	0,39	0,47	0,47	0,44	0,51	0,51	0,48	0,56	0,55	0,53	0,60
22	59.052	57.078	66.949	1,19	1,13	1,41	1,25	1,19	1,45	1,30	1,24	1,48	1,34	1,29	1,51
23	48.545	48.145	50.798	0,88	0,87	0,94	0,93	0,92	0,98	0,97	0,96	1,02	1,01	1,01	1,06
24	44.768	44.056	48.871	0,75	0,73	0,86	0,81	0,79	0,91	0,86	0,84	0,96	0,91	0,89	1,00

3.3.4 Greedy Algorithm for Model 3

This heuristic method is developed to determine feasible base stock levels for the aggregate waiting time constraint for Model 3. The idea is the same with that in Model 2 with some minor differences. The current procedure computes distance to the set of feasible solutions and r_{in} differently. Both the new approximate evaluation method and the one of Muckstadt and Thomas can be used with this greedy algorithm.

The proposed procedure starts with setting all base stock levels to zero, $\underline{S} = 0$ for all SKU-s and warehouses. At each iteration the ratio

$$r_{in} = \begin{cases} \dfrac{\Delta W_{in}}{\Delta Z_{in}} & if\ \Delta Z_{in} > 0, \Delta W_{in} > 0 \\ \Delta W_{in}. \Delta Z_{in} & if\ \Delta Z_{in} \leq 0, \Delta W_{in} > 0 \\ \emptyset & if\ \ \Delta W_{in} \leq 0 \end{cases} \tag{54}$$

is calculated where

$$\Delta W_{in} = \begin{cases} \displaystyle\sum_{n \in N_{loc}} \left\{ \left(W_n(\underline{S}) - W_n^{target} \right)^+ - \left(W_n(\underline{S} + U_{in}) - W_n^{target} \right)^+ \right\} & n = 0 \\ \left(W_n(\underline{S}) - W_n^{target} \right)^+ - \left(W_n(\underline{S} + U_{in}) - W_n^{target} \right)^+ & n \in N_{loc} \end{cases} \tag{55}$$

$$\Delta Z_{in} = Z(\underline{S} + U_{in}) - Z(\underline{S}) \tag{56}$$

for all $i \in I$ and $n \in N$.

The total cost may increase or decrease at each iteration because the total inventory cost always increases but total transportation cost may decrease when the base stock level increases. Moreover; since increasing a base stock level will increase the fill rate of the corresponding OB, $E(W_{i0})$ will also increase. Then, ΔW_{in} can be smaller than or equal to 0. For that reason, r_{in} is defined as in (54).

$\Delta W_{i^*n^*}$, change in the waiting time value of SKU i^* at OB n^*, where $i^* \in I, n^* \in N$ are only dependent on the base stock level of $S_{i^*n^*}$. These values are not subject to change if a base stock level of another SKU increases. Computation time can be saved if only the results that change are updated. A formal description of the greedy procedure is given next.

Greedy Procedure for Model 3

Step 1: Set the initial solution $\underline{S} = 0$; calculate $W_n(0)$ for all warehouses

Step 2: For all combinations $i \in I$ and $n \in N$, calculate ΔW_{in} , ΔZ_{in} , and r_{in}.

Step 3: If $min\{\Delta Z_{in}\}$ is greater than 0, let i^* and n^* be defined as $r_{i^*n^*} = \max\ r_{in}$; otherwise let i^* and n^* be defined as $r_{i^*n^*} = \min\ r_{in}$. Set $\underline{S} = \underline{S} + U_{i^*n^*}$. If $W_n(\underline{S}) \leq W_n^{target}$ for all $n \in N_{loc}$, go to *Step 4*; otherwise go to *Step 2*.

Step 4: For all combinations $i \in I, n \in N$: Calculate ΔZ_{in}.

Step 5: Let i^* and n^* be defined as $\Delta Z_{i^*n^*} = \min\ \Delta Z_{i^*n^*}$. If $\Delta Z_{i^*n^*} < 0$, then set $\underline{S} = \underline{S} + U_{i^*n^*}$ and go *to Step 4*; otherwise end.

In some cases although the target waiting time values are met by every OB n, increasing the base stock level may decrease the total cost because this can decrease the lateral transshipment rates between OB-s, which in turn decreases total transportation costs. Thus, the greedy algorithm used in Model 3, checks the possible decrease in the total cost once the target waiting time values are met by every OB n.

3.4 Solution Procedure for Model 4

Since there is not any study in the literature which considers the system of Model 4, a new approximate evaluation method, which calculates the service levels and transportation cost for given base stock levels, is developed. The greedy algorithm of Model 3 is also used in Model 4 to minimize the total inventory and transportation cost with respect to a target waiting time constraint. The approximate evaluation method and the greedy algorithm are described below.

3.4.1 New Approximate Evaluation Method for Model 4

This method calculates the service levels and cost for given base stock levels.. There are two iterative solution procedures in this method. The first iteration calculates the $E(W_{i0})$ and β_{i0} iteratively with β_{in} of each OB n; and the second iteration calculates β_{in} iteratively with $\widehat{M}_{in}$ for every SKU i at every OB n where $i \in I, n \in N_{loc}$. The second iteration takes place within the first iteration as seen Figure 3.14.

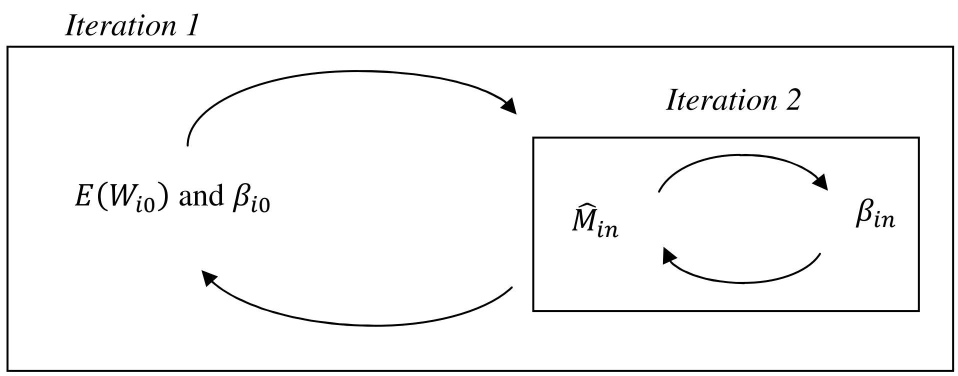

Figure 3.14: Iterations used in the approximate evaluation method of Model 4

Calculating $E(W_{i0})$ and β_{i0}:

When the inventory amount in RDC is greater than 0, because of the base stock policy used in the OB-s, $m_{i0} = \sum_{n \in N_{loc}} m_{in}$. However when the inventory amount in RDC is lower than or equal to 0; the probability that OB n requests a replenishment order from RDC is equal to the probability that there is at least one stock in any OB n where $n \in N_{loc}$. Because there are three possibilities that can happen when a demand comes to an OB n and the inventory amount in RDC is lower than or equal to 0:

1. If the OB has stock on hand, it will supply the demand by its stock and will give a replenishment order to RDC.
2. If the OB has not stock on hand, it will check the other OB-s because RDC does not have stock on hand. If any other OB has stock on hand, there will be a lateral transshipment to meet the demand, and the OB which sends the part for lateral transshipment will give a replenishment order to RDC.

3. If none of the OB-s has stock on-hand, there will be a direct shipment from CBT.

Let m'_{i0} denotes the demand rate coming to RDC for SKU i, when the inventory amount in RDC is lower than or equal to 0,

$$m'_{i0} = \left[1 - \prod_{n=1}^{|N_{loc}|}(1-\beta_{in})\right].m_{i0} \tag{57}$$

In (57), the events that there is positive stock of SKU i at OB n for all $n \in N_{loc}$ is assumed independent. Let the replenishment rate of RDC by CBT for each SKU i be

$$\mu_0 = 1/\,T_0 \tag{58}$$

and $\bar{S}_i$ be

$$\bar{S}_i = \sum_{n \in N_{loc}} S_{in} \qquad \forall \;\; i \in I \tag{59}$$

Because of the direct shipments from CBT to the OB-s, the inventory amount in RDC will be between $-\bar{S}_i$ and S_{i0}. There can be backorder in RDC, only when there is on-hand stock in at least one OB, thus $-\bar{S}_i$ is the minimum inventory amount or maximum number of backorders that can occur in RDC.

If all β_{in} are known and the demand coming to RDC for SKU i, when the inventory amount in RDC is lower than or equal to 0, is a Poisson process; then $X_{in}(t)$, the stock level for SKU i at RDC at time t, is a Continuous Time Markov Chain (CTMC) with a rate diagram given in Figure 3.15.

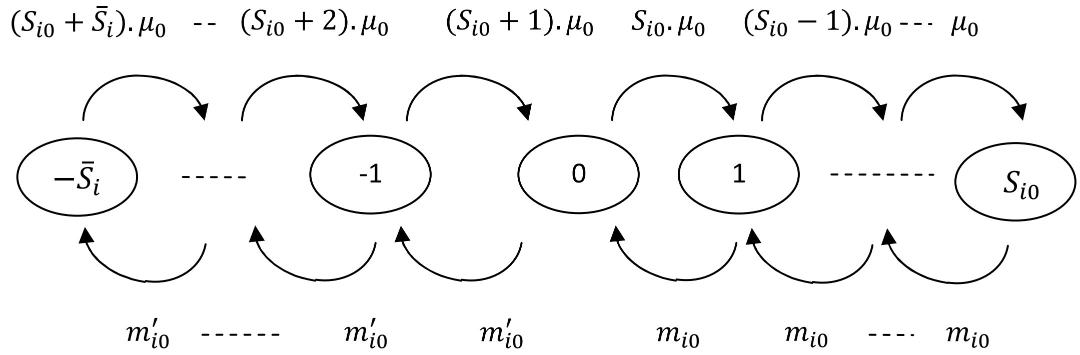

Figure 3.15: Rate diagram of the Markov process describing the stock level in RDC in Model 4

Let π_x denotes the steady state probabilities for the continuous time Markov chain shown in Figure 3.15 where $-\bar{S}_i \leq x \leq S_{i0}$ and x is integer. Then the steady state probabilities can be calculated as

$$\pi_x = \begin{cases} (S_{i0} - x + 1).\dfrac{\mu_0}{m'_{i0}}.\pi_{x-1} & -\bar{S}_i < x \leq 0 \\ (S_{i0} - x + 1).\dfrac{\mu_0}{m_{i0}}.\pi_{x-1} & 0 < x \leq S_{i0} \end{cases} \tag{60}$$

Then

$$\beta_{i0} = \sum_{x=1}^{S_{i0}} \pi_x \tag{61}$$

$$E(B_{i0}) = \sum_{x=-\bar{S}_i}^{-1} (-x).\pi_x \tag{62}$$

Let δ_{i0} be the average demand rate coming to RDC for each SKU i. Then,

$$\delta_{i0} = \left(\sum_{x=-\bar{S}_i+1}^{0} \pi_x\right).m'_{i0} + \left(\sum_{x=1}^{S_{i0}} \pi_x\right).\ m_{i0} \tag{63}$$

Note that (63) is an approximation because m'_{i0} is dependent to π_x. Then, by the Little's law,

$$E(W_{i0}) = E(B_{i0})/\delta_{i0} \tag{64}$$

Because of FCFS policy in CBT and Poisson demand, $E(W_{i0})$ is same for all OB-s.

Calculating $\boldsymbol{\beta}_{in}$ and the total demand rate:

Define

$\widehat{M}_{in}$: Total demand rate including the lateral demands coming for SKU i at OB n when there is positive stock in OB n

$\widetilde{M}_{ilk}$: Lateral demand rate coming from OB l to OB k for SKU i when there is positive stock in the OB k for $i \in I,\ \ n, k, l \in N_{loc},\ \ l \neq k$

$\sigma_n = \left(\sigma_1(n), \sigma_2(n), \dots, \sigma_{(|N_{loc}|-1)}(n)\right)$: The pre-specified order of the OB-s for asking lateral transshipment.

For each SKU i at OB n, the mean replenishment lead time is

$$LT_{in} = t_{reg} + E(W_{i0}) \tag{65}$$

At this point, it is assumed that the replenishment lead time for SKU i at OB n is exponentially distributed and its mean is LT_{in}. Then the total demand rates for SKU i at OB n and lateral demand rates for SKU i from OB n to the other OB-s respectively are

$$\widehat{M}_{in} = m_{in} + \sum_{x=1, x\neq n}^{|N_{loc}|} \widetilde{M}_{ixn} \tag{66}$$

$$\widetilde{M}_{in\sigma_1(n)} = (1-\beta_{i0}).(1-\beta_{in}).m_{in} \tag{67}$$

$$\widetilde{M}_{in\sigma_x(n)} = \left(1-\beta_{i\sigma_{x-1}(n)}\right).\widetilde{M}_{in\sigma_{x-1}(n)} \qquad 1 < x \leq |N_{loc}| - 1 \tag{68}$$

From the equations above, it is seen that $\widehat{M}_{in}$ is dependent on β_{in}. Moreover, if $\widehat{M}_{in}$ is known and since it is assumed that total demand rate coming for SKU i at OB n have a Poisson process, which also means that the overflow demands arrive according to a

Poisson process, then β_{in} can be calculated by Erlang Loss Probability for each SKU i at OB n.

$$\beta_{in} = 1 - L\left(S_{in}, \left(\widehat{M}_{in}.\ LT_{in}\right)\right) \tag{69}$$

Using the information above, the iterations of Model 4 are described below.

Iteration 2

Let ϵ =10^{-6}. For each SKU i:

Step 1: Assume no lateral transshipment exists between all of the OB-s. Then $\widehat{M}_{in} = m_{in}$; and calculate β_{in} for each OB n by using (23).

Step 2: Using β_{in}, calculate $\widehat{M}_{in}$ for one OB by using (66)-(68). Then calculate β_{in} for the same OB.

Step 3: Repeat *Step 2* for each OB n.

Step 4: Repeat *Step 2* and *Step 3* until $\widehat{M}_{in}$ does not change more than ϵ for each OB n where $n \in N_{loc}$.

The variables $\widehat{M}_{in}$and β_{in} converge in all cases considered in this study. The same convergence argument in the iterations of Model 2 is also valid here.

Iteration 1

Let ϵ =10^{-6}. For each SKU i:

Step 1: Assume $E(W_{i0}) = 0$ and $\beta_{i0} = 1$. Then, calculate β_{in} for every OB n by iteration 2 for $n \in N_{loc}$.

Step 2: Using β_{in}, calculate $E(W_{i0})$ and β_{i0}. Then calculate β_{in} for every OB n by iteration 2.

Step 3: Repeat *Step 2* until $E(W_{i0})$ does not change more than ϵ .

The variables $E(W_{i0})$, β_{i0} and β_{in} converge in all cases considered in this study. Figure 3.16, 3.17, and 3.18 show convergence of $E(W_{i0})$, β_{i0} and β_{in} for a setting.

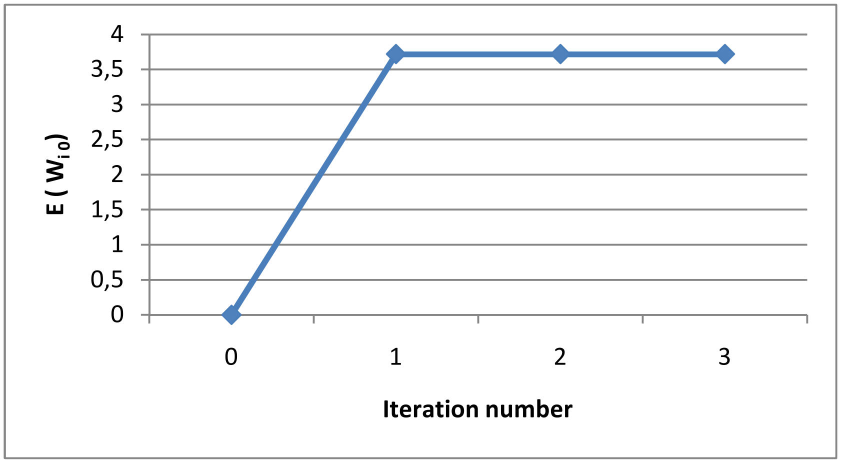

Figure 3.16: $E(W_{i0})$ at each iteration in instance 1 of Model 4

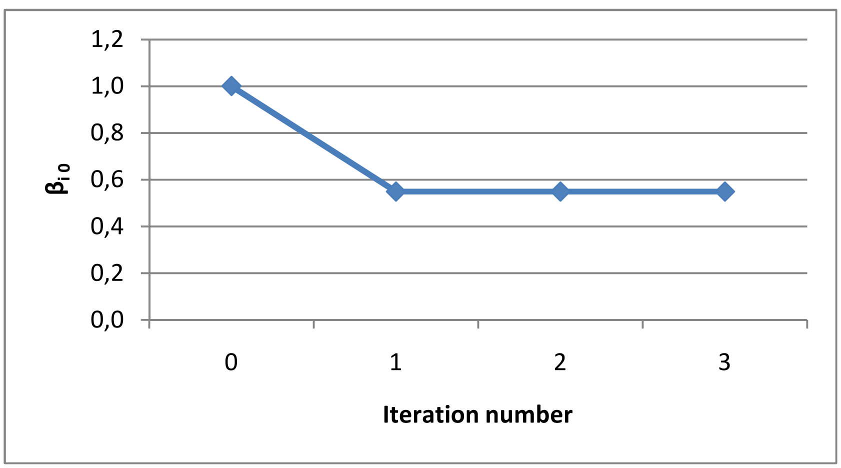

Figure 3.17: β_{i0} at each iteration in instance 1 of Model 4

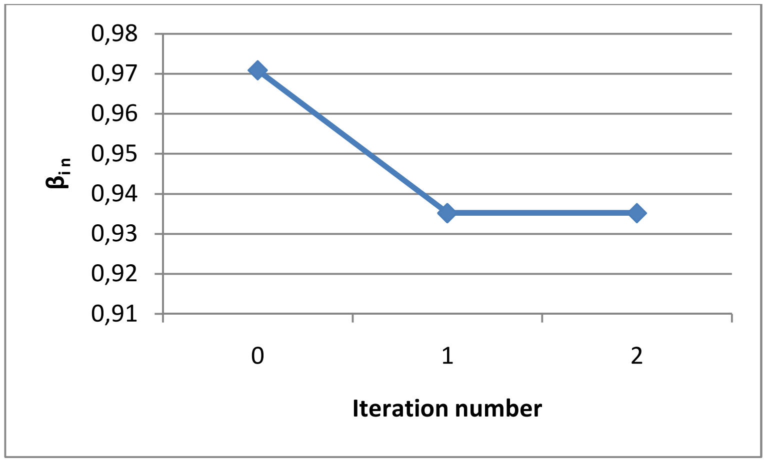

Figure 3.18: β_{in} at each iteration in instance 1 of Model 4

As it can be seen from the Figure 3.16 and 3.17, the initial values of $E(W_{i0})$ and β_{i0} are 0 and 1 respectively. In this case the corresponding β_{in} become the largest because the lower the expected delay in RDC, the lower the lead times of each OB; and the higher the β_{i0} value, the lower lateral demand coming to each OB. In the following iteration $E(W_{i0})$ and β_{i0} values becomes the largest and lowest respectively because the higher the fillrate values of the OB-s, the higher the number of replenishment orders from RDC and the higher the delay and number of backorders in RDC. Afterwards, β_{in} values become the lowest as seen in iteration number 1 in Figure 3.18, because the $E(W_{i0})$ and β_{i0} values are the largest and lowest respectively. Then; $E(W_{i0})$ and β_{i0} values becomes the second lowest and largest respectively; $E(W_{i0})$ becomes greater than 0 and β_{i0} becomes lower than 1 as seen in iteration number 2 in Figure 3.16 and 3.17. Then; β_{in} values become the second largest, because the current $E(W_{i0})$ and β_{i0} values are the second lowest and highest respectively as seen in iteration number 2 in Figure 3.18. Hence; at each odd numbered iteration, $E(W_{i0})$, β_{i0} and β_{in} will increase, decrease and decrease respectively, at each even numbered iteration, $E(W_{i0})$, β_{i0} and β_{in} will decrease, increase and increase respectively; and at each iteration, the difference between two consecutive $E(W_{i0})$, β_{i0} and β_{in} values will decrease. Thus sooner or later these values will converge.

Iteration 1 is robust with respect to the initial value of $E(W_{i0})$ and β_{i0}. For all possible initial values of $E(W_{i0})$ and β_{i0}, iteration 1 converges to the same values. However, the initial values of $E(W_{i0})$ and β_{i0} affect the number of iterations done, which in turn affects the computation time. When the initial value of β_{i0} is 1, number of iterations are minimized because in this situation, there will be no lateral demand in the system, then the number of iterations done in the first run of iteration 2 will be minimum. For the initial value of $E(W_{i0})$; generally the more the initial value is far from the exact value, the longer the computation time. However; in our cases, this increase was insignificant.

After convergence, θ_{in}, $\alpha_{i,l,k}$, and γ_{in} can be calculated as:

$$\theta_{in} = \beta_{i0}.(1 - \beta_{in}) \quad \forall\ i \in I, n \in N_{loc} \tag{70}$$

$$\alpha_{i,l,k} = \frac{\beta_{ik}.\widetilde{M}_{ilk}}{m_{il}} \quad \forall\ i \in I, l \in N_{loc}, k \in N_{loc}\ l \neq k \tag{71}$$

$$\gamma_{in} = \prod_{n=0}^{|N|}(1 - \beta_{in}) \quad \forall\ i \in I, n \in N \tag{72}$$

Note that for every SKU i, γ_{in} values for each OB n are equal to each other as it can be seen in (72), because direct shipments from CBT is done if there is not stock in all of the warehouses.

Consequently, the new approximate evaluation method used in Model 4 calculates the fillrates (β_{in}); the fraction of demand for SKU i met by direct shipment from RDC to OB n (θ_{in}); fraction of demand met by lateral transshipments ($\alpha_{i,l,k}$); and the fraction of demand for SKU i met by direct shipment from CBT to OB n (γ_{in}) for each SKU i at each OB n for $i \in I,\ n, l, k \in N_{loc},\ l \neq k$.

3.4.2 Numerical Experiments for the Approximate Evaluation Method of Model 4

In order to validate the approximate evaluation method used in Model 4, the model is simulated using ARENA Software. The results obtained from the simulation runs are considered as the exact values. The values obtained from the algorithm are compared with the exact values.

To test the approximation algorithm of Model 4, same instances used to test Model 2 are used. Pre-specified lateral demand orders are determined as: $\sigma_1 = (2,3,4)$, $\sigma_2 = (3,4,1)$, $\sigma_3 = (4,1,2)$, and $\sigma_4 = (1,2,3)$. Six different quantities of Model 4 are approximated and compared with the simulation. These are

$E(W_{i0})$: Expected delay in RDC SKU i

β_{i0}: Fillrate of SKU i at RDC

β_{in}: Fillrate of SKU i at OB n for $i \in I, n \in N_{loc}$

θ_{in}: Fraction of demand for SKU i met by direct shipment done from RDC to OB n

$\alpha_{i,l,k}$: Fraction of demands comes to OB l met by lateral transshipments from OB k for $i \in I,\ n, l, k \in N_{loc},\ l \neq k$

γ_{in}: Fraction of demand for SKU i met by direct shipment done from CBT to OB n

Table 3.12 shows the results of the symmetric instances of Model 4.

Table 3.12: Comparison of the approximations with the exact values of the symmetric instances of Model 4

				$E(W_{i0})$		β_{i0}		β_{in}		θ_{in}		γ_{in}		$\alpha_{i,n,\sigma_1(n)}$		$\alpha_{i,n,\sigma_2(n)}$		$\alpha_{i,n,\sigma_3(n)}$	
Inst.	m_{in}	S_{i0}	S_{in}	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.
1	0,01	1	1	3,73	3,72	0,55	0,55	0,93	0,94	0,00	0,04	0,00	0,00	0,06	0,03	0,01	0,00	0,00	0,00
2		2	1	0,69	0,67	0,88	0,88	0,96	0,96	0,02	0,03	0,00	0,00	0,02	0,00	0,00	0,00	0,00	0,00
3	0,04	1	1	8,76	8,83	0,11	0,10	0,58	0,56	0,00	0,04	0,10	0,03	0,19	0,22	0,09	0,10	0,04	0,04
4		2	1	4,67	4,75	0,33	0,31	0,71	0,72	0,02	0,09	0,05	0,00	0,14	0,14	0,06	0,04	0,02	0,01
5		1	2	9,31	9,31	0,09	0,09	0,90	0,91	0,00	0,01	0,00	0,00	0,08	0,07	0,01	0,01	0,00	0,00
6		2	2	5,00	4,99	0,31	0,31	0,95	0,96	0,00	0,01	0,00	0,00	0,04	0,03	0,01	0,00	0,00	0,00
7	0,08	1	1	10,33	10,05	0,03	0,03	0,30	0,27	0,00	0,02	0,34	0,28	0,17	0,19	0,11	0,14	0,07	0,10
8		2	1	7,12	7,06	0,10	0,08	0,40	0,36	0,01	0,05	0,25	0,15	0,18	0,21	0,11	0,13	0,06	0,09
9		2	2	8,70	8,81	0,05	0,05	0,73	0,74	0,00	0,01	0,04	0,00	0,15	0,18	0,06	0,05	0,02	0,01
10		2	3	8,92	8,92	0,05	0,05	0,92	0,94	0,00	0,00	0,00	0,00	0,06	0,06	0,01	0,00	0,00	0,00
11		3	1	4,81	4,95	0,22	0,18	0,50	0,47	0,02	0,10	0,17	0,07	0,17	0,20	0,09	0,11	0,05	0,06
12		6	1	1,10	1,15	0,70	0,66	0,72	0,73	0,11	0,18	0,04	0,00	0,07	0,07	0,04	0,02	0,02	0,00
13	0,16	1	1	10,98	10,40	0,01	0,01	0,14	0,12	0,00	0,01	0,61	0,58	0,10	0,11	0,08	0,09	0,07	0,08
14		2	1	8,29	7,72	0,02	0,03	0,17	0,16	0,00	0,02	0,54	0,49	0,12	0,13	0,09	0,11	0,07	0,09
15		2	2	10,65	10,51	0,00	0,00	0,35	0,30	0,00	0,00	0,28	0,23	0,18	0,21	0,12	0,15	0,07	0,10
16		3	1	6,36	5,97	0,06	0,06	0,22	0,20	0,01	0,04	0,47	0,40	0,13	0,15	0,10	0,12	0,07	0,10
17		3	2	8,95	8,91	0,01	0,01	0,41	0,36	0,00	0,01	0,23	0,17	0,19	0,23	0,11	0,15	0,07	0,09
18		6	1	2,92	3,05	0,28	0,20	0,37	0,32	0,06	0,13	0,28	0,17	0,14	0,18	0,09	0,12	0,06	0,08
19		6	2	4,88	5,24	0,12	0,10	0,61	0,58	0,00	0,04	0,10	0,03	0,17	0,22	0,08	0,09	0,04	0,04
20		6	3	5,66	5,79	0,09	0,08	0,81	0,84	0,00	0,01	0,02	0,00	0,12	0,12	0,04	0,02	0,01	0,00

Figure 3.19, 3.20, and 3.21 show the β_{in}, θ_{in}, and γ_{in} of the approximate method and of the simulation for the symmetric instances. Note that the instances of the exact results are ranked from smallest to largest. Figure 3.20 and 3.21 show the θ_{in} and γ_{in} calculated by the (70) and (72) with the exact β_{i0} and β_{in} values.

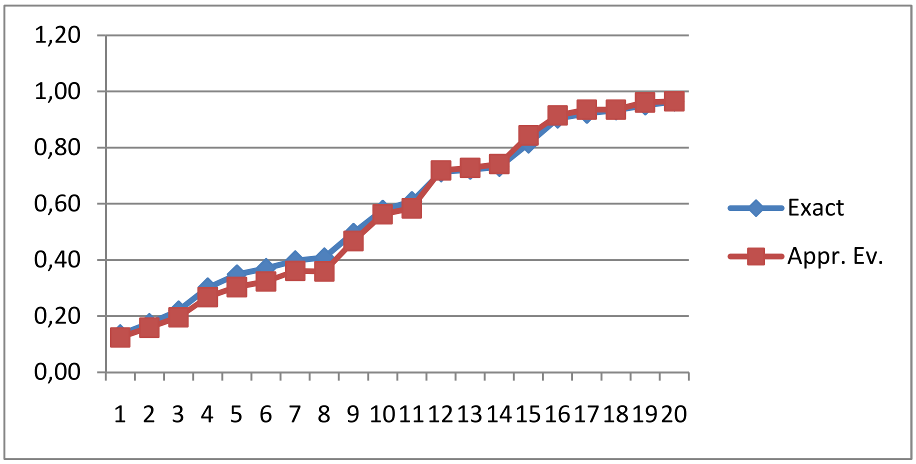

Figure 3.19: β_{in} in the symmetric instances in Model 4

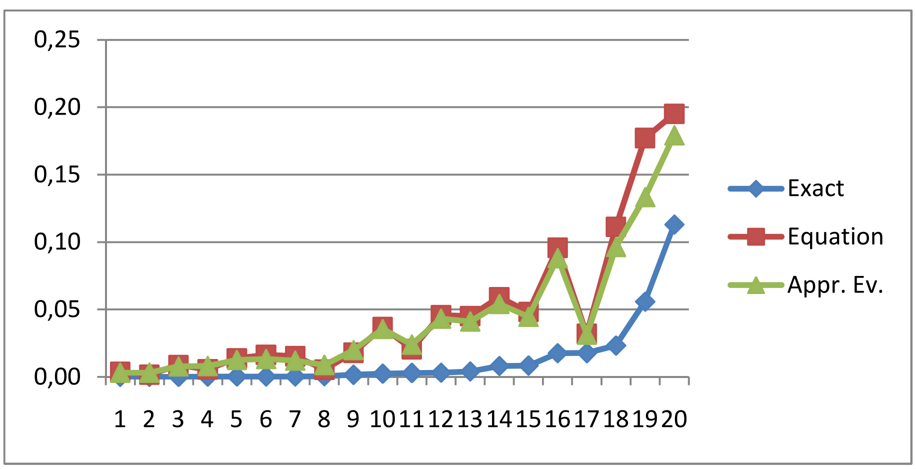

Figure 3.20: θ_{in} in the symmetric instances in Model 4

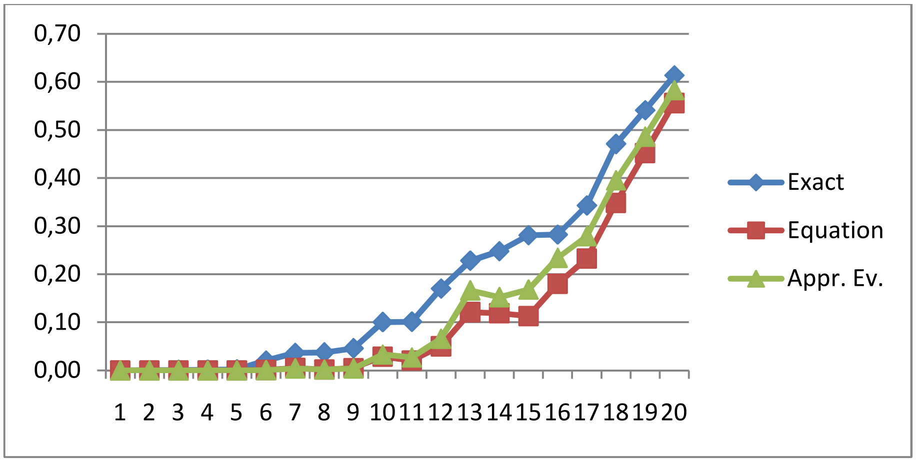

Figure 3.21: γ_{in} in the symmetric instances in Model 4

According to these results, it can be said that the new approximate evaluation method estimates the β_{in} values quite accurately. The average absolute percentage deviation for β_{in} is 5.5%. However, the new approximate evaluation method overestimates θ_{in} and underestimates γ_{in} in every instance. In (70) and (72), the events that there is positive stock for SKU i at RDC and OB n are assumed independent although these events are dependent. To see that the deviation is not due to approximation, θ_{in} and γ_{in} calculated by (70) and (72) with the exact β_{i0} and β_{in} are compared with the exact values. Figure 3.20 and 3.21 show these results in the series named 'Equation'. As it can be seen from these figures, (70) overestimates θ_{in} and (72) underestimates γ_{in} in every symmetric instance. Exact θ_{in} is equal to the probability that there is positive stock for SKU i at RDC given that OB n is out of stock for SKU i. However in (70), θ_{in} is calculated by just multiplying the probability that there is positive stock for SKU i at RDC with the probability that OB n is out of stock for SKU i. Intuitively, it is seen that when there is positive stock in RDC, there is more likely positive stock in an OB either, because OB-s are replenished by the RDC. Thus, when there is positive stock in RDC, it is less likely that an OB is out of stock. However, (70) does not consider this situation and as a result it overestimates the θ_{in}. Underestimation of θ_{in} by (70) can be explained in a similar way.

Table 3.13, 3.14 and 3.15 show the results of the asymmetric instances of Model 4. Parameters settings used in the asymmetric instances can be seen in Table 3.2.

Table 3.13: Comparison of the approximations with the exact values of the asymmetric instances of Model 4

	$E(W_{i0})$		β_{i0}		β_{i1}		β_{i2}		β_{i3}		β_{i4}		θ_{i1}		θ_{i2}		θ_{i3}	
Inst.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.
1	7,07	7,12	0,23	0,22	0,79	0,82	0,79	0,81	0,75	0,75	0,69	0,68	0,00	0,04	0,00	0,04	0,00	0,06
2	2,80	2,77	0,56	0,56	0,90	0,93	0,88	0,89	0,85	0,85	0,81	0,80	0,01	0,04	0,02	0,06	0,02	0,08
3	2,82	2,81	0,56	0,56	0,94	0,94	0,90	0,89	0,98	0,99	0,97	0,98	0,01	0,03	0,02	0,06	0,00	0,01
4	10,63	10,21	0,02	0,02	0,24	0,21	0,25	0,22	0,23	0,21	0,21	0,19	0,00	0,02	0,00	0,01	0,00	0,02
5	7,63	7,38	0,06	0,06	0,32	0,28	0,33	0,29	0,31	0,28	0,28	0,25	0,00	0,04	0,01	0,04	0,01	0,04
6	8,93	8,93	0,03	0,03	0,40	0,33	0,36	0,32	0,53	0,50	0,50	0,47	0,00	0,02	0,00	0,02	0,00	0,02
7	6,68	6,86	0,09	0,08	0,49	0,44	0,45	0,41	0,62	0,61	0,59	0,57	0,01	0,04	0,01	0,05	0,00	0,03
8	10,90	10,36	0,01	0,01	0,16	0,14	0,16	0,15	0,16	0,14	0,15	0,14	0,00	0,01	0,00	0,01	0,00	0,01
9	8,15	7,65	0,03	0,03	0,21	0,19	0,21	0,19	0,21	0,19	0,18	0,17	0,00	0,03	0,00	0,03	0,00	0,03
10	9,59	9,35	0,01	0,01	0,24	0,20	0,23	0,19	0,36	0,33	0,36	0,33	0,00	0,01	0,00	0,01	0,00	0,01
11	8,62	8,69	0,02	0,02	0,50	0,45	0,51	0,47	0,48	0,44	0,43	0,39	0,00	0,01	0,00	0,01	0,00	0,01
12	9,16	9,32	0,01	0,01	0,59	0,55	0,56	0,53	0,66	0,65	0,62	0,60	0,00	0,01	0,00	0,01	0,00	0,00
13	4,72	4,77	0,16	0,16	0,91	0,96	0,91	0,94	0,88	0,90	0,83	0,84	0,00	0,01	0,00	0,01	0,00	0,02
14	4,77	4,78	0,16	0,16	0,94	0,97	0,92	0,94	0,95	0,97	0,92	0,94	0,00	0,00	0,00	0,01	0,00	0,01
15	4,57	4,72	0,17	0,16	0,59	0,58	0,74	0,76	0,84	0,87	0,89	0,93	0,02	0,07	0,00	0,04	0,00	0,02
16	6,20	6,21	0,31	0,30	0,85	0,86	0,84	0,84	0,82	0,83	0,81	0,81	0,00	0,04	0,00	0,05	0,00	0,05
17	6,27	6,26	0,30	0,30	0,99	0,99	0,98	0,99	0,98	0,99	0,98	0,98	0,00	0,00	0,00	0,00	0,00	0,00
18	2,06	2,03	0,66	0,66	0,94	0,94	0,92	0,92	0,90	0,91	0,89	0,89	0,01	0,04	0,01	0,05	0,02	0,06
19	7,76	7,47	0,06	0,05	0,35	0,32	0,33	0,30	0,31	0,28	0,29	0,26	0,00	0,04	0,00	0,04	0,01	0,04
20	12,16	12,17	0,00	0,00	0,56	0,52	0,54	0,51	0,52	0,49	0,50	0,47	0,00	0,00	0,00	0,00	0,00	0,00
21	9,63	9,74	0,02	0,02	0,65	0,64	0,64	0,63	0,62	0,60	0,59	0,57	0,00	0,01	0,00	0,01	0,00	0,01
22	8,45	7,83	0,02	0,03	0,21	0,19	0,19	0,17	0,17	0,16	0,16	0,15	0,00	0,02	0,00	0,02	0,00	0,02
23	12,68	12,52	0,00	0,00	0,33	0,28	0,31	0,27	0,30	0,26	0,28	0,25	0,00	0,00	0,00	0,00	0,00	0,00
24	10,74	10,59	0,00	0,00	0,39	0,34	0,37	0,33	0,35	0,31	0,34	0,29	0,00	0,00	0,00	0,00	0,00	0,00

Table 3.14: Comparison of the approximations with the exact values of the asymmetric instances of Model 4

	θ_{i4}		$\alpha_{i,1,2}$		$\alpha_{i,1,3}$		$\alpha_{i,1,4}$		$\alpha_{i,2,1}$		$\alpha_{i,2,3}$		$\alpha_{i,2,4}$		$\alpha_{i,3,1}$		$\alpha_{i,3,2}$	
Inst.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.
1	0,00	0,07	0,13	0,11	0,04	0,02	0,01	0,00	0,02	0,01	0,12	0,11	0,04	0,03	0,06	0,05	0,02	0,01
2	0,03	0,11	0,07	0,03	0,02	0,00	0,00	0,00	0,01	0,00	0,07	0,04	0,02	0,01	0,03	0,01	0,01	0,00
3	0,00	0,01	0,04	0,02	0,01	0,00	0,00	0,00	0,00	0,00	0,08	0,05	0,00	0,00	0,00	0,00	0,00	0,00
4	0,00	0,02	0,15	0,17	0,10	0,13	0,07	0,09	0,08	0,10	0,14	0,16	0,10	0,12	0,11	0,13	0,08	0,11
5	0,01	0,04	0,16	0,20	0,10	0,13	0,06	0,09	0,07	0,10	0,16	0,19	0,10	0,12	0,12	0,15	0,08	0,11
6	0,00	0,02	0,16	0,20	0,17	0,22	0,08	0,10	0,04	0,06	0,28	0,33	0,12	0,15	0,06	0,09	0,04	0,05
7	0,00	0,03	0,16	0,21	0,15	0,19	0,06	0,07	0,03	0,04	0,28	0,33	0,10	0,12	0,05	0,07	0,03	0,04
8	0,00	0,01	0,12	0,12	0,09	0,10	0,07	0,08	0,07	0,09	0,11	0,12	0,09	0,10	0,09	0,11	0,07	0,09
9	0,00	0,03	0,13	0,15	0,10	0,12	0,07	0,09	0,07	0,10	0,13	0,15	0,09	0,11	0,10	0,12	0,08	0,10
10	0,00	0,01	0,13	0,15	0,18	0,21	0,10	0,14	0,05	0,07	0,24	0,26	0,14	0,17	0,07	0,09	0,05	0,07
11	0,00	0,01	0,19	0,25	0,10	0,13	0,05	0,06	0,06	0,08	0,18	0,23	0,09	0,11	0,12	0,15	0,06	0,09
12	0,00	0,00	0,17	0,24	0,11	0,13	0,04	0,04	0,03	0,04	0,24	0,30	0,08	0,10	0,06	0,08	0,03	0,03
13	0,00	0,03	0,07	0,04	0,01	0,00	0,00	0,00	0,01	0,00	0,06	0,04	0,01	0,00	0,03	0,01	0,01	0,00
14	0,00	0,01	0,04	0,02	0,01	0,00	0,00	0,00	0,00	0,00	0,06	0,05	0,01	0,00	0,01	0,00	0,00	0,00
15	0,00	0,01	0,24	0,27	0,09	0,07	0,03	0,01	0,00	0,00	0,17	0,18	0,05	0,03	0,01	0,00	0,01	0,00
16	0,01	0,06	0,10	0,08	0,03	0,01	0,01	0,00	0,01	0,00	0,11	0,09	0,03	0,02	0,04	0,02	0,01	0,00
17	0,00	0,01	0,01	0,01	0,00	0,00	0,00	0,00	0,00	0,00	0,02	0,01	0,00	0,00	0,00	0,00	0,00	0,00
18	0,03	0,07	0,04	0,02	0,01	0,00	0,00	0,00	0,00	0,00	0,05	0,02	0,01	0,00	0,01	0,00	0,00	0,00
19	0,01	0,04	0,16	0,19	0,10	0,13	0,06	0,08	0,08	0,11	0,16	0,19	0,10	0,12	0,12	0,16	0,08	0,10
20	0,00	0,00	0,19	0,24	0,09	0,11	0,04	0,06	0,05	0,07	0,19	0,24	0,09	0,12	0,11	0,14	0,05	0,07
21	0,00	0,01	0,17	0,22	0,07	0,08	0,03	0,03	0,04	0,04	0,17	0,22	0,07	0,08	0,09	0,11	0,04	0,04
22	0,00	0,02	0,12	0,14	0,09	0,11	0,06	0,08	0,08	0,11	0,12	0,13	0,08	0,10	0,11	0,13	0,08	0,10
23	0,00	0,00	0,17	0,20	0,11	0,14	0,06	0,09	0,08	0,12	0,17	0,19	0,11	0,13	0,13	0,16	0,08	0,11
24	0,00	0,00	0,18	0,22	0,10	0,14	0,06	0,09	0,08	0,11	0,18	0,21	0,11	0,13	0,13	0,17	0,08	0,11

Table 3.15: Comparison of the approximations with the exact values of the asymmetric instances of Model 4

	$\alpha_{i,3,4}$		$\alpha_{i,4,1}$		$\alpha_{i,4,2}$		$\alpha_{i,4,3}$		γ_{in}	
Inst.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.
1	0,14	0,13	0,19	0,20	0,06	0,04	0,02	0,01	0,03	0,00
2	0,08	0,05	0,11	0,08	0,03	0,01	0,01	0,00	0,01	0,00
3	0,02	0,01	0,02	0,01	0,00	0,00	0,00	0,00	0,00	0,00
4	0,13	0,15	0,16	0,17	0,11	0,14	0,08	0,10	0,44	0,39
5	0,15	0,17	0,18	0,20	0,12	0,15	0,07	0,10	0,34	0,26
6	0,18	0,23	0,13	0,17	0,08	0,11	0,09	0,12	0,19	0,12
7	0,16	0,20	0,13	0,18	0,07	0,09	0,07	0,08	0,13	0,05
8	0,11	0,12	0,12	0,12	0,09	0,11	0,07	0,09	0,57	0,53
9	0,12	0,14	0,13	0,15	0,10	0,12	0,11	0,10	0,48	0,43
10	0,18	0,21	0,11	0,13	0,08	0,10	0,11	0,14	0,34	0,29
11	0,17	0,21	0,22	0,27	0,12	0,15	0,06	0,08	0,17	0,10
12	0,16	0,21	0,16	0,22	0,08	0,09	0,05	0,05	0,09	0,03
13	0,08	0,07	0,13	0,13	0,03	0,01	0,01	0,00	0,01	0,00
14	0,04	0,03	0,06	0,05	0,01	0,00	0,00	0,00	0,00	0,00
15	0,12	0,10	0,03	0,03	0,03	0,02	0,02	0,00	0,02	0,00
16	0,12	0,10	0,13	0,11	0,03	0,02	0,01	0,00	0,01	0,00
17	0,02	0,01	0,02	0,01	0,00	0,00	0,00	0,00	0,00	0,00
18	0,06	0,03	0,07	0,04	0,01	0,00	0,00	0,00	0,00	0,00
19	0,16	0,18	0,19	0,22	0,11	0,14	0,07	0,09	0,33	0,24
20	0,19	0,24	0,22	0,28	0,11	0,13	0,05	0,06	0,12	0,06
21	0,18	0,22	0,21	0,27	0,09	0,09	0,04	0,03	0,08	0,02
22	0,11	0,12	0,14	0,15	0,10	0,12	0,07	0,09	0,53	0,47
23	0,17	0,18	0,19	0,21	0,12	0,15	0,08	0,10	0,33	0,29
24	0,18	0,20	0,21	0,24	0,12	0,15	0,07	0,10	0,26	0,21

According to these results, it can be said that the new approximate evaluation method estimates the β_{in} values quite accurately. The average absolute percentage deviation for β_{in} is 6,2%. Like in the symmetric cases, the new approximate evaluation method overestimates the θ_{in} and underestimates the γ_{in} in every instance. The reason is the same with that in the symmetric cases. θ_{in} and γ_{in} calculated by (70) and (72) with the exact β_{i0} and β_{in} are compared with the exact values and it is seen that (70) overestimates θ_{in} and (72) underestimates γ_{in} in every asymmetric instance.

The computation time of the new approximate evaluation method is quite short. Average computation time of one instance for the new approximate evaluation method is smaller than 7 milliseconds in a Intel Dual Core 3GHz computer.

3.4.2.1 Cost and Service Level Validation

Following parameters are used in the cost and service level validation of Model 4:

$$t_n^{RDC} = 0{,}3\ days,\ t_{n,k}^{lat} = 0{,}3\ days,\ t_n^{CBT} = 2{,}1\ days,\ p_i = €\,5.000$$

$$h = 20\%,\ C_n^{RDC} = €\,125,\quad C_{n,k}^{lat} = €\,125,\ \ C_n^{CBT} = €\,375 \qquad \forall\, n, l \in N_{loc}, l \neq n$$

Table 3.16 shows the total yearly cost and aggregate waiting time values for all OB-s for the simulation and the new approximate evaluation method for the symmetric instances.

Table 3.16: Total yearly cost and aggregate waiting time values for all OB-s for the symmetric instances of Model 4

	Total yearly cost values (€)		**W_{in} (days)**	
Instance	Exact	Appr.	Exact	Appr.
1	5.126	5.118	0,02	0,02
2	6.067	6.065	0,01	0,01
3	9.550	8.672	0,31	0,19
4	8.769	8.116	0,17	0,09
5	9.733	9.622	0,03	0,03
6	10..371	10.285	0,02	0,01
7	25.214	23.841	0,83	0,72
8	22.026	19.771	0,63	0,47
9	14.983	13.896	0,15	0,09
10	15.195	14.944	0,03	0,02
11	19.317	16.697	0,46	0,28
12	15.162	14.023	0,15	0,08
13	66.049	64.598	1,36	1,31
14	61.663	58.933	1,22	1,13
15	45.517	44.004	0,70	0,63
16	57.277	53.575	1,08	0,95
17	41.590	39.422	0,59	0,49
18	44..785	39.580	0,69	0,51
19	31.343	27.727	0,30	0,17
20	24.601	22.603	0,09	0,05

Average absolute percentage deviation from the exact results in the total yearly cost value for the new approximate evaluation method is 5,87%. The new approximate evaluation method underestimates the total cost and waiting time values, because it overestimates of θ_{in} and underestimates of γ_{in} and it is more costly and time consuming to make direct shipment from CBT.

Table 3.17 shows the total yearly cost and aggregate waiting time values for each OB for the simulation and the new approximate evaluation method for the asymmetric instances.

Table 3.17: Total yearly cost and aggregate waiting time values for each OB for the asymmetric instances of Model 4

	Total Yearly Cost (€)		W_{i1} **(days)**		W_{i2} **(days)**		W_{i3} **(days)**		W_{i4} **(days)**	
Inst.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.	Exact	Appr.
1	6.428	6.213	0,11	0,06	0,11	0,06	0,12	0,08	0,14	0,10
2	6.777	6.697	0,05	0,02	0,05	0,03	0,06	0,05	0,07	0,06
3	8.206	8.180	0,02	0,02	0,03	0,03	0,01	0,00	0,01	0,01
4	35.016	33.582	1,01	0,93	1,01	0,93	1,02	0,93	1,02	0,94
5	31.300	28.813	0,82	0,68	0,82	0,68	0,83	0,68	0,84	0,69
6	24.600	22.603	0,52	0,41	0,53	0,42	0,48	0,36	0,49	0,37
7	21.825	19.392	0,39	0,26	0,40	0,27	0,35	0,21	0,36	0,22
8	55.510	54.078	1,27	1,21	1,27	1,21	1,27	1,21	1,28	1,22
9	51.138	48.605	1,10	1,01	1,10	1,01	1,11	1,01	1,12	1,01
10	42.981	41.164	0,84	0,76	0,85	0,76	0,81	0,72	0,81	0,72
11	32.964	30.525	0,45	0,34	0,45	0,33	0,46	0,34	0,47	0,36
12	27.309	24.910	0,28	0,19	0,29	0,19	0,26	0,16	0,27	0,17
13	21.721	20.692	0,04	0,01	0,04	0,02	0,05	0,03	0,07	0,05
14	21.791	21.155	0,02	0,01	0,03	0,02	0,02	0,01	0,03	0,02
15	22.242	20.459	0,16	0,13	0,12	0,07	0,09	0,04	0,07	0,02
16	5.705	5.606	0,06	0,04	0,07	0,05	0,07	0,05	0,08	0,06
17	9.064	9.048	0,00	0,00	0,00	0,00	0,01	0,00	0,01	0,01
18	6.343	6.311	0,02	0,02	0,03	0,02	0,03	0,03	0,04	0,03
19	30.376	27.780	0,78	0,64	0,79	0,64	0,80	0,65	0,80	0,66
20	21.933	20.429	0,35	0,26	0,35	0,26	0,36	0,26	0,37	0,27
21	19.684	17.927	0,24	0,15	0,25	0,15	0,25	0,16	0,26	0,17
22	60.654	57.715	1,19	1,09	1,19	1,09	1,20	1,10	1,20	1,10
23	48.259	47.344	0,79	0,74	0,79	0,74	0,80	0,74	0,80	0,75
24	44.052	42.418	0,66	0,58	0,67	0,59	0,67	0,59	0,68	0,60

Average absolute percentage deviation in the total yearly cost value for the new approximate evaluation method with the exact results is 4,87%. Again, it is seen that the new approximate evaluation method underestimates the total cost and waiting time values because the new method overestimates θ_{in} and underestimates γ_{in}.

The greedy algorithm used in Model 4 is also used in Model 3.

CHAPTER 4

RESULTS AND SENSITIVITY ANALYSIS

In this chapter, the results of the experiments are given. Note that the real service levels and total cost values observed in Nedtrain are not compared with the results of the models. This has two main reasons: Firstly, according to a recent study; employees did not implement the decisions of the Xelus in 80% of the time. Thus, the real cost and service level values are not the results of Xelus. Secondly, even if the control policy proposed by Xelus is simulated, there is not enough information about the parameters and the policies used by Xelus. Thus the designed models are compared with each other. In this chapter, firstly, preparation of the test beds is explained. Then, the main results of each model are given and analyzed. Afterwards a short summary of the results and sensitivity analysis is given. Lastly, additional experiments which analyze the aggregation of planned and unplanned demands is given.

4.1 Preparation of the Test Beds

Several test beds are formed to evaluate the models. Due to repairable classes and sensitivity analysis with respect to different parameters, the models were run 268 times in total.

Initially, test beds are formed for the critical repairables which have only unplanned demand because approximately 71% of all critical repairables have only unplanned demand. For the critical repairables which have both planned and unplanned demand and form the 25% of the all critical repairables, different test beds are formed. Before the test beds are determined, repairables are classified with respect to two main criteria:

1. **Price of the repairables**: Four different price ranges are determined as seen in Table 4.1.

Table 4.1: Determined price ranges for the repairables

Price Range	Name of the range
(€ 0, € 100]	A
(€ 100, € 1.000]	B
(€ 1.000, € 10.000]	C
(€ 10.000, + ∞)	D

2. **Number of OB-s that demand comes for the repairables**: Among the critical repairables which have only unplanned demand, 72 % of them have demand which comes to only one OB, thus they are stocked at one OB only. For this type of critical repairables, only Model 1 and 3 are examined. However for the remaining 28% of them, which have demand coming to more than one OB, all of the models are examined.

With respect to these two criteria, 7 different test beds are formed. The first three is solved using Model 1-2-3-4 and the last four is solved using Model 1-3. In addition to these test beds, additional 12 different test beds are formed from the critical repairables which have both planned and unplanned demand. Thus in total 19 different test beds are formed as seen in Table 4.2.

The target waiting time value is determined as 2 hours for the test beds 1-2-3 and 4 hours for the test beds 4-5-6-7 (see Appendix F for more detailed information).

Table 4.2: Test Beds

Demand Type		Test Bed Number	Price Range	Sample Size	Models run
Only Unplanned Demand	Demand comes to more than one OB	1	A	30	1-2-3-4
		2	B	30	1-2-3-4
		3	C	30	1-2-3-4
	Demand comes to just one OB	4	A	30	1-3
		5	B	30	1-3
		6	C	30	1-3
		7	D	10	1-3
Both Planned and Unplanned demand	Unplanned Demand	8	A	30	1-2-3-4
		9	B	30	1-2-3-4
		10	C	30	1-2-3-4
		11	D	8	1-2-3-4
	Planned Demand	12	A	30	1-2-3-4
		13	B	30	1-2-3-4
		14	C	30	1-2-3-4
		15	D	8	1-2-3-4
	Aggregated Demand	16	A	30	1-2-3-4
		17	B	30	1-2-3-4
		18	C	30	1-2-3-4
		19	D	8	1-2-3-4

All of the test beds have sample size of 30, except test beds 7-11-15-19. The reason for the smaller sample size of these test beds is that they contain repairables which are in the price range ‘D’ (their price is higher than € 10.000), and total number of this type of items are less than 30. Then all of the repairables which are in this price level are chosen for these test beds. There is not any test bed in the price range D for the ‘demands coming to more than one OB since there is not any repairable which fits to the criteria of this test bed.

4.2 Results

In this part, the results of each model for the test beds 1 to 7 are analyzed. Note that in Model 3, the new approximate evaluation method is used instead of the method of Muckstadt and Thomas with the greedy algorithm, because it gives more accurate results.

4.2.1 Results of Test Beds 1-2-3

These test beds contain critical repairables which have both only unplanned demand and demand coming to more than one OB. The results can be seen in Table 4.3.

Table 4.3: Total costs of the models for test beds 1-2-3 (€ / year)

Test Bed	Price Range	Model 1	Model 2	Model 3	Model 4
1	A	1.320	2.494	2.056	1.963
2	B	8.079	10.360	10.113	9.304
3	C	46.377	46.141	41.150	38.359

In price ranges A and B, Model 1 gives the lowest cost. Thus it is not profitable to implement lateral or direct shipments in these price ranges. This is because the average cost of one lateral transshipment between two OB-s is €140, the average cost of one direct shipment from RDC to an OB is € 125; and average cost of one direct shipment from CBT to an OB is € 375. However, keeping one additional repairable in stock in price range A is lower than € 20 and in price range B is between € 20 and € 200. Since the models do not prefer lateral or direct shipments, this means that the increase in the service level in case of lateral or direct shipments does not compensate the additional cost which they bring.

In price range C, Model 4 gives the lowest cost. Total cost given by Model 4 is 17% lower than Model 1. In this price range, keeping one additional repairable in stock is between € 200 and € 2000. However, the average cost of one lateral transshipment between two OB-s is €140, cost of direct shipment from RDC to an OB is € 125 and the cost of one direct shipment from CBT to an OB is € 375 on the average. This result shows that making direct and lateral transshipments compensate the transportation cost which they bring in this price range.

Hence, in the price ranges A and B, lateral or direct shipments are not useful; but in the price range C, implementing these supply options decreases the total cost. Another interesting fact is seen when the results of Model 2 and 3 are examined. In the price range C, either lateral or direct shipments decrease the total cost but using them simultaneously decreases the total cost the most.

In the validation of the models, it is seen that Model 3 and 4 underestimate the total yearly cost by 3,5% and 5,87% on the average respectively. In the price range C, where Model 3 and 4 are profitable than Model 1, it is seen that these models give 11% and 17,3% lower total yearly cost than Model 1 respectively. Thus, the fact that these models underestimate the total cost is probably insignificant in the price range C.

It is also seen in the validation process that Model 2, 3 and 4 underestimate the waiting time values. Table 4.4 shows the aggregate mean waiting time for all OB-s for each model in the test beds 1-2-3.

Table 4.4: Aggregate mean waiting time (hrs) for all OB-s for each model in the test beds 1-2-3.

Price Range	Model 1	Model 2	Model 3	Model 4	Target
A	1,95	1,01	0,02	0,02	2
B	1,96	1,24	0,34	0,16	2
C	1,86	1,64	1,77	1,14	2

According to the Table 4.4, the aggregate mean waiting times of the OB-s in Model 2, 3, and 4 are much lower than the target waiting time value, 2 hours. Thus, the fact that these models underestimate the waiting time values may be insignificant.

Below in Table 4.5, the sum of the base stock levels for all items in the RDC and OB-s at each model of test beds 1-2-3 can be seen:

Table 4.5: Total base stock levels of the test beds 1-2-3 in RDC and OB-s

Price	Model 1			Model 2			Model 3			Model 4		
R.	RDC	OB-s	Total	RDC	OB-s	Total	RDC	OB-s	Total	RDC	OB-s	Total
A	57	66	123	119	101	220	58	114	172	43	118	161
B	51	72	123	76	67	143	48	74	122	27	82	109
C	46	62	108	52	51	103	31	47	78	23	47	70

Model 4 keeps the lowest stock in the price range B and C. The reason is that implementing lateral and direct shipments at the same time uses the inventory more efficiently. However, Model 3 and 4 keep more stock then Model 1 in the price range A; Model 2 keeps more stock than Model 1 in the price range A and B. It may be expected that the total stock kept by Model 2, 3 and 4 should be lower than that kept by Model 1, because faster supply options, lateral or direct shipments, are used in these models. In Model 2, the reason of high stock levels is that the greedy algorithm used in this model decreases the high lateral transshipment costs by increasing the stock; because the higher the stock levels, the lower the lateral transshipment rates

In Model 3 and 4, the reason of high stock levels in the price range A is the greedy algorithm. As it is mentioned before, after the greedy algorithm meets the target waiting time constraint, it checks whether adding new items to stock decreases the total cost of the system; if does, the algorithm increases the stock. In the price range A, Model 3 and 4 increase the stock in order to decrease the high lateral or direct shipment costs after the target waiting time constraint is met. For instance, if the greedy algorithm stops when it meets the target waiting time constraint in price range A, then the total stock kept in RDC and OB-s become 47 and 37 respectively for Model 3, and 37 and 33 respectively for Model 4 which are lower than the stock levels kept in the RDC and OB-s by Model 1, which is 57 and 66 respectively. However, in this case the total cost of the Model 3 and 4 becomes € 14.063 and € 19.545 respectively, which is much higher than the total costs given if the greedy algorithm does not stop although the target waiting time constraint is met (total cost for Model 3 and 4 in this situation are € 2.056 and € 1.963 respectively).

An interesting result can be seen in Table 4.6, which shows the percentage of stocks kept in the OB-s and RDC for each of the models in the test beds 1-2-3.

Table 4.6: Percentage of stock kept in the network of Nedtrain in the test beds 1-2-3

Price R.	**Model 1**			**Model 2**			**Model 3**			**Model 4**		
	RDC	OB-s	Total	RDC	OB-s	Total	RDC	OB	Total	RDC	OB-s	Total
A	46%	54%	100%	54%	46%	100%	34%	66%	100%	27%	73%	100%
B	41%	59%	100%	53%	47%	100%	39%	61%	100%	25%	75%	100%
C	43%	57%	100%	50%	50%	100%	40%	60%	100%	33%	67%	100%

From Table 4.6, it is seen that when lateral transshipments are implemented, the percentage of the stocks kept in OB-s is the lowest. This is because the lateral transshipments pool the stock of all OB's; they use total stock more efficiently. However; in Model 3, where direct shipments from RDC and CBT to OB-s implemented, percentage of the stock kept in RDC becomes lower than Model 1. This is because the direct shipments decrease the lead time from CBT to OB-s and RDC to OB-s which decreases the necessity of the buffer stock kept in the RDC. Another interesting result is seen in Model 4 where lateral and direct shipments are implemented at the same time. In this case; although percentage of the stock kept in RDC is expected between corresponding values of Model 2 and Model 3, it is the lowest in Model 4.

One of the assumptions done for Model 3 and 4 is that CBT can always make direct shipment to OB-s, independent of the demand rate which is supplied by the direct shipment from CBT. However, this assumption may not be realistic because the higher the fraction of the demands supplied by direct shipment from CBT, the more difficult for CBT to make emergent repairs and send them to OB-s. Below, in Table 4.7; percentage of demand supplied by different options can be seen for Model 3 and 4 in the test beds 1-2-3. Percentage of demands supplied by the stock of OB directly is denoted by β, supplied by direct shipment from RDC is denoted by θ, supplied by lateral transshipment from other OB-s is denoted by α, and supplied by direct shipment from CBT is denoted by γ.

Table 4.7: Percentage of demands supplied by different options in Model 3 and 4 in the test beds 1-2-3

Price Range	Model 3				Model 4				
	β	**θ**	**γ**	**Total**	**β**	**θ**	**α**	**γ**	**Total**
A	99,69%	0,27%	0,04%	100%	99,53%	0,30%	0,16%	0,00%	100%
B	94,61%	4,66%	0,73%	100%	94,31%	2,90%	2,67%	0,12%	100%
C	85,26%	8,98%	5,76%	100%	81,58%	9,59%	6,77%	2,07%	100%

Percentage of demands supplied by direct shipment from CBT has its maximum value at price range C, which is 5,76% and 2,07% respectively for Model 3 and 4. The reason is that direct shipment is preferred to decrease stock levels in this price range, but in the price range A and B, Model 3 and 4 keeps high stock to decrease the lateral or direct shipments, since they are costly compared to inventory costs. Another interesting fact is that Model 4 makes direct shipments from CBT to OB-s less than Model 3 in each price range. The reason of this situation is lateral transshipments, which increases the efficiency of using the total stock in RDC and OB-s.

4.2.2 Results of Test Beds 4-5-6-7

These test beds contain critical repairables which have both only unplanned demand and demand coming to only one OB. For these test beds, the results can be seen in Table 4.8.

Table 4.8: Total costs of the models for test beds 4-5-6-7 (€ / year)

Test Bed	Price Range	Model 1	Model 3
4	**A**	531	782
5	**B**	3.263	3.806
6	**C**	25.561	19.741
7	**D**	52.304	49.915

Model 1 gives lower cost than Model 3 in the price ranges A and B. Thus it is not profitable to implement direct shipments in these price ranges. However, in the price ranges C and D; Model 3 gives lower cost than Model 1. In the price range C, total cost given by Model 3 is 22.8 % lower than Model 1 and in the price range D, this difference is 4.6 %. This means that increase in the service level in case of direct shipments, compensates the additional cost which they bring in the price range C and D. So it is rational to implement direct shipments in these price ranges.

In the validation of the models, it is seen that Model 3 underestimates the total yearly cost 3,5% on the average. In the price range C and D, where implementing Model 3 is profitable than Model 1, it is seen that this model gives 4,5% and 22,8% lower total yearly cost than Model 1 respectively. Thus, the fact that Model 3 underestimates the total cost is probably insignificant in the price range C and D.

It is also seen during validation that Model 3 underestimates the waiting time. Table 4.9 shows the aggregate mean waiting time for all OB-s for each model in the test beds 4-5-6-7.

Table 4.9: Aggregate mean waiting time for all OB-s for each model in the test beds 4-5-6-7 (hrs).

Price Range	Model 1	Model 3	Target
A	3,45	0,10	4
B	3,23	0,42	4
C	2,90	2,82	4
D	1,18	1,09	4

Aggregate mean waiting time values of the OB-s in Model 3 is much lower than the target waiting time, which is 4 hours. Thus, the fact that this model underestimates the waiting time values may be insignificant.

In Table 4.10, summation of the base stock levels for all items in the RDC and OB-s in the test beds 4-5-6-7 can be seen:

Table 4.10: Total base stock levels of the test beds 4-5-6-7 in RDC and OB-s

	Model 1			**Model 3**		
Price Range	**RDC**	**OB-s**	**Total**	**RDC**	**OB-s**	**Total**
A	12	32	44	5	50	55
B	14	36	50	10	39	49
C	18	32	50	11	25	36
D	3	10	13	3	9	12

Model 3 keeps more stock than Model 1 in the price range A; because it decreases the high direct shipment costs by increasing the stock. However in the price range C, Model 3 keeps much lower stock than Model 1, because direct shipments decrease the necessity to inventory.

Table 4.11 shows the percentage of stocks kept in the OB-s and RDC for each of the models in the test beds 4-5-6-7.

Table 4.11: Percentage of stock kept in the network of Nedtrain in the test beds 4-5-6-7

Price Range	Model 1			Model 3		
	RDC (%)	OB-s (%)	Total (%)	RDC (%)	OB-s (%)	Total (%)
A	27%	73%	100%	9%	91%	100%
B	28%	72%	100%	20%	80%	100%
C	36%	64%	100%	31%	69%	100%
D	27%	73%	100%	9%	91%	100%

Again; when direct shipments from RDC and CBT to OB-s implemented, percentage of the stock kept in RDC becomes the lowest in general; because direct shipments decrease the lead time from CBT and RDC to OB-s, which decreases the necessity of the buffer stock, kept in the RDC.

In Table 4.12; the percentage of demands supplied by the stock of OB directly (β), supplied by direct shipment from RDC (θ), and supplied by direct shipment from CBT (γ) in Model 3 in the test beds 4-5-6-7 can be seen:

Table 4.12: Percentage of demands supplied by OB-s, RDC, and CBT in Model 3 in the test beds 4-5-6-7

Price Range	β	θ	γ	Total
A	99,54%	0,07%	0,39%	100%
B	97,45%	0,99%	1,56%	100%
C	85,99%	3,11%	10,90%	100%
D	93,97%	1,79%	4,23%	100%

Percentage of demands supplied by direct shipment from CBT is higher at price ranges C and D. The reason is that direct shipment is preferred to decrease stock levels in these price ranges; but in the price range A and B, Model 3 keeps high stock to decrease the direct shipments, since they are costly compared to inventory costs.

4.3 Summary of the Results and the Sensitivity Analysis

The models which give the lowest costs at each price range can be seen in Table 4.13:

Table 4.13: The best models of each test bed in the normal case

Demands come to:	Test Bed Number	Price Range	Models run	The Best Model
More than one OB	1	A	1-2-3-4	Model 1
	2	B	1-2-3-4	Model 1
	3	C	1-2-3-4	Model 4
Only one OB	4	A	1-3	Model 1
	5	B	1-3	Model 1
	6	C	1-3	Model 3
	7	D	1-3	Model 3

From the results of the test beds 1-2-3; it is seen that Model 1 is the best option for the repairables within the price range A and B, and Model 4 is the best option for the repairables within the price range C. This result shows that implementing direct and lateral transshipments becomes rational when the inventory holding costs of the repairables are high. From the results of the test beds 4-5-6-7; it is seen that Model 1 is the best option for the repairables within the price range A and B, and Model 3 is the best option for the repairables within the price range C and D. Again, it is seen that implementing direct shipments in the high price ranges decreases the total cost.

In the sensitivity analysis, six different parameters are analyzed. The following points are observed:

1. **Lateral and direct shipment costs**: When these costs decrease (increase), Model 2, 3 and 4 become more attractive (unattractive). However, in the price range A, Model 1 gives the best results at every cost setting.

2. **Target waiting time values**: When the target waiting time decreases, performance of Model 2, 3 and 4 increases in general. Benefit of increasing the

stocks decreases when the target waiting time values decrease; lateral or direct shipments become more preferable in this situation.

3. **Replenishment lead time of RDC by CBT**: When the lead time between CBT and RDC decreases, total cost of the models decrease. However, the shorter the lead time, the lower the improvement in the performance of Model 3 and 4 with respect to the other models, because the relative importance of the direct shipments decreases in this situation.

4. **Transportation time between RDC and OB-s**: When the transportation time between RDC and OB-s decreases, decrease in the total cost of Model 2, 3 and 4 is lower than the one of Model 1. The reason of this situation is that the relative importance of the lateral and direct shipments decreases in this situation.

5. **Uplift factor**: The higher the uplift factor[8], the higher the total cost of Model 3 and 4; and the higher the direct shipment rates, the higher the effect of uplift factor. However, in general Model 3 and 4 are not affected by the uplift factor value significantly in our experiments.

6. **Demand rates**: Model 2, 3 and 4 are more stable to the change in the demand rates in terms of keeping the waiting time value below target, because these models can use lateral or direct shipments to fulfill the extra demand, but naturally this process will increase the total cost.

Implementing lateral or direct shipments has some advantages and disadvantages. The first advantage of these supply options occurs when the target waiting time decreases. The lower the target waiting time value, the better the performance of Model 2, 3 and 4. Since Nedtrain gives great importance to increase its service level, a better performance in high service levels is important. The second advantage occurs when the demand changes. Model 2, 3 and 4 are more stable than Model 1when the demand changes.

[8] Uplift factor is a parameter used to calculate the costs of direct shipments from CBT to OB-s.

When the real demand is higher than the estimated demand[9], the increase in the expected waiting time value in Model 2, 3 and 4 is lower than the one in Model 1. Since Nedtrain always gives great importance to achieve high service levels, stability with respect to service level is important. Lastly, Model 3 and 4 are not significantly affected by the changes in uplift factor. This result is desired because generally it is difficult to estimate the emergent repair costs done in CBT. Thus, even if the cost parameters are not estimated well, the results of Model 3 and 4 are still robust.

Despite the above advantages, implementing lateral or direct shipments has also some disadvantages. The first one occurs when the transportation time decreases. The shorter the transportation time, the lower the improvement in the performance of Model 2, 3 and 4. Another disadvantage occurs in Model 3 and 4 when the lead time between CBT and RDC decreases. The shorter the lead time, the lower the improvement in the performance of Model 3 and 4. More detailed information about the sensitivity analysis can be found in Appendix F.

4.4 Planned & Unplanned Demand Aggregation

Nedtrain distinguishes between planned and unplanned demand. 25% of all critical repairables have both planned and unplanned demand. In the initial studies, it is seen that the aggregation of these two demand types decreases the coefficient of variation which increases predictability. Thus, the effect of aggregation of the planned and unplanned demand is analyzed in this part. For this analysis, test beds 8-19 are formed. Test beds 8-11 contain unplanned demands, test beds 12-15 contain planned demands, and test beds 16-19 contain aggregated planned and unplanned demands for each price range.

Table 4.14 - 4.17 show the total costs of unplanned demand and planned demand, sum of the costs of unplanned and planned demand, total cost of the aggregation of these demand types and percentage decrease in the total cost when the two demand types are aggregated. The 'Total' rows in the tables show the sum of the costs of each price range.

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[9] Estimated demand is the one used as input to the models

Table 4.14: Total yearly costs of unplanned, planned and aggregation of these demands for Model 1 (€ / year)

Price Range	Unplanned	Planned	Unplanned + Planned	Aggregated	% Decrease
A	1.200	837	2.036	1.563	23,3%
B	7.906	7.584	15.491	11.698	24,5%
C	46.710	24.819	71.529	53.038	25,9%
D	52.996	47.142	100.138	66.442	33,6%
Total	108.812	80.382	189.194	132.740	29,8%

Table 4.15: Total yearly costs of unplanned, planned and aggregation of these demands for Model 2 (€ / year)

Price Range	Unplanned	Planned	Unplanned + Planned	Aggregated	% Decrease
A	2.344	1.527	3.871	3.234	16,5%
B	10.472	9.518	19.990	15.845	20,7%
C	49.860	24.294	74.154	55.530	25,1%
D	49.885	47.212	97.098	59.693	38,5%
Total	112.562	82.552	195.113	134.301	31,2%

Table 4.16: Total yearly costs of unplanned, planned and aggregation of these demands for Model 3 (€ / year)

Price Range	Unplanned	Planned	Unplanned + Planned	Aggregated	% Decrease
A	1.998	1.637	3.635	2.946	18.9%
B	10.657	10.275	20.931	17.015	18.7%
C	44.925	21.729	66.654	54.719	17.9%
D	38.599	29.920	68.519	48.456	29.3%
Total	96.177	63.561	159.739	123.136	22.9%

Table 4.17: Total yearly costs of unplanned, planned and aggregation of these demands for Model 4 (€ / year)

Price Range	Unplanned	Planned	Unplanned + Planned	Aggregated	% Decrease
A	1.912	1.594	3.505	2.833	19,2%
B	10.044	10.081	20.125	16.373	18,6%
C	46.212	21.235	67.446	52.766	21,8%
D	38.639	29.920	68.559	48.181	29,7%
Total	96.806	62.829	159.635	120.152	24,7%

In each of the Models total costs decreases when the planned and unplanned demand aggregated, because aggregation increases the predictability which decreases the total cost. The percentage decrease in the total costs for the models, when the planned and unplanned demands are aggregated, are 29,8%, 31,2%, 22,9%, and 24,7% respectively. Moreover, in the price range A and B, Model 1 gives the lowest costs; in the price range C and D, Model 3 and 4 gives the lowest costs. These results support the former results given by the test beds 1 to 7 in the previous analysis.

CHAPTER 5

IMPLEMENTATION ASPECTS

In this chapter, information about the implementation of the designed models is given. Since Nedtrain uses Xelus Software in the control of repairables, integration should be done between Xelus and the designed models.

In the design of the models, two significant assumptions are done. The first one assumes an exponential and independent lead time for the repair process in CBT. However, CBT makes its repairs with respect to a min-max policy currently. Thus in case of implementation of the designed Models, CBT should be aware of this assumption. Secondly, it is assumed that one-for-one replenishment policy is used in the OB-s and RDC. However; Nedtrain uses batch sizes for the repairables. Thus Nedtrain should be aware of this situation.

In the implementation phase, a prototype control mechanism tool is designed for each model in Excel Software. These tools use the demand rates, transportation and lead-time parameters, and cost parameters as inputs; and give the total cost values, service levels, and base stock levels as the outputs with respect to a target service level. The prototype tools are written in the visual basic language in Excel Software; however, any other programming language can also be used.

In order to calculate the demand rates, moving average forecasting method is used considering the demand of the last three years. However, Nedtrain can use any other appropriate forecasting method in the calculation of the demand rates. Since demand rates are input of the control mechanism tools, Nedtrain should update this parameter at predetermined time intervals (For instance; every one year, two year etc.). The control mechanism tools should be run at the beginning of each period and the base stock levels in the OB-s and RDC should be arranged with respect to the results given by the models. If the number of SKU-s that Nedtrain have is higher than the results that the control mechanism tool give, than the excess SKU-s can be distributed to the warehouses considering demand rates or any other factors which are important to Nedtrain. However, if the number of SKU-s that Nedtrain has, is lower than the results that the control mechanism tool give, than the difference should be purchased.

The designed control mechanism tools are computationally efficient. They spend approximately four seconds for the computation of the described test beds. This means that the designed algorithms can be used for data which have larger sample size easily. Moreover, the designed tools can be redesigned even faster by the software experts.

5.1 Stakeholders

In this part the direct and indirect stakeholders are described when the designed models are implemented. Direct stakeholders are LLC, CBT and purchasing department; and the indirect stakeholders are OB-s.

- **LLC**: This department is responsible for the control of repairables and Xelus. They will be dealing with the integration and implementation of the designed models in Nedtrain. Thus they are the main stakeholder in this case.
- **CBT**: This department is responsible for the repair process of the repairables. If the designed models are implemented, the repair policy of CBT will change, thus they will be affected directly, which makes them a direct stakeholder.
- **Purchasing Department**: Implementing the designed models will affect the purchasing decisions, which makes this department as a direct stakeholder.
- **OB**-s: Implementing designed models will affect the stock levels and the stock replenishment policy in the OB-s. Since these stock points do not have any control in the determination of base stock levels they can be considered as indirect stakeholders.

CHAPTER 6

CONCLUSION AND RECOMMENDATIONS

This chapter contains final conclusions about the master thesis project and the recommendations for future research.

6.1 Conclusions

In this master thesis project, main focus is given to the control of repairables in Nedtrain. Initially, demand structure of the repairables is examined and it is found that the demands are generally low and they fit Poisson distribution the best. Then, the price values of the repairables are examined and it is found that the most of the repairables are more expensive than € 100. With respect to these findings, it is decided to use stochastic inventory models which use Poisson distributed demand. Afterwards, Model 1, which uses METRIC, a well known multi echelon technique for repairable item control is designed, because it perfectly fits with the demand distribution of the repairables and echelon structure of Nedtrain. Then three additional models are suggested to see the effects of lateral and direct shipments. Model 2 considers the lateral transshipments between the OB-s, Model 3 considers the direct shipments from RDC to OB-s and CBT to OB-s, and Model 4 considers both the lateral and direct shipments. These are optimization models which minimize the total inventory and transportation costs with respect to a target waiting time constraint.

In Model 2, the method developed by Kranenburg and van Houtum [17] is integrated to the network of Nedtrain; in Model 3 and 4, new solution methods are developed. Simulation is used to validate the solution procedures of Model 2, 3 and 4. After the simulation, it is seen that solution procedures designed for Model 2, 3 and 4 work efficiently. Then three different greedy algorithms are developed for these solution procedures to determine the optimal base stock levels. Afterwards, the models are solved with different test beds and sensitivity analysis is done with respect to several parameters. The results show that implementing lateral or direct shipments can decrease the total cost values for the expensive repairables. Meanwhile, the designed models are run for the planned and unplanned demand, and aggregation of them separately; and it is found that aggregating the planned and unplanned demand decreases the total cost. Lastly, some suggestions are given for the implementation aspects of the results.

During the analysis, the main goal was to find answers to the research questions that can be acceptable and implementable by Nedtrain. These questions and their brief answers are given below.

How can the repairables be classified?

In the initial analysis; firstly, demand structure of the repairables are examined. It is found that the demand of the repairables has similar characteristics with respect to demand distribution, thus a classification with respect to demand is not necessary. Secondly, the prices of the repairables are examined and it is seen that repairables have a very wide price range. Then four different classes are determined so that that the different control mechanisms may be good for different price ranges; and this expectation is proved to be true when the models were run. According to the results; for the cheap repairables implementing only regular shipments is the best; whereas for the expensive repairables, implementing lateral and direct shipments simultaneously is the best.

What is the optimal control policy for each class of the repairables?

Several test beds are formed and run by each of the designed models. According to the results, Model 1 is the best option for the repairables which are cheaper than € 1.000; Model 4 is the best option for the repairables which are more expensive than € 1.000 and have demand coming to more than one OB; and Model 3 is the best option for the repairables which are more expensive than € 1.000 and have demand coming to only one OB. This means that direct or lateral transshipments are beneficial supply option for the expensive repairables. Lastly, sensitivity analysis proved that these results can change when the parameter values change. Thus, any change in the value of a parameter can change the optimal control policy, which should be taken into account.

How can the results of these models be implemented?

For each model, a prototype control mechanism tool is designed with the visual basic language in Excel Software. However; since Nedtrain uses Xelus Software in the control of repairables, integration should be done between Xelus and the control mechanism tools. Moreover, the repair policy of CBT should be reconsidered such that it will be suitable for the exponential repair lead-time assumption, and one for one replenishment policy should be implemented in the network of Nedtrain.

In summary, all of the research questions are answered in this master thesis. The objective is to propose Nedtrain efficient control mechanisms for repairables, and give some clues about possible future research areas.

6.2 Recommendations for Future Research

In this part, suggestions for the future research projects for Nedtrain are given.

As mentioned before, Nedtrain uses Xelus in the control of repairables as well as main parts and consumables. However; Xelus is like black box, because there is not much information about how it works. Thus examining the working structure of Xelus can be a possible future research topic for Nedtrain. By this research, a reliable comparison can be done with the proposed models and Xelus.

CBT uses a min-max policy to supply RDC. A possible future research topic can be analyzing the min-max policy that CBT uses. Remember that in this master thesis project, an exponential lead time is assumed for the repair process in CBT. The reason of this assumption is to make the network system of Nedtrain simpler so that one control mechanism can be used for repairables in the whole network of Nedtrain (OB-s, RDC and CBT). However; currently, there are two control mechanisms used for the repairables. These are the control mechanism for the OB-s and RDC, and the control mechanism that CBT uses. In my opinion in the situation of Nedtrain, optimizing two different control mechanisms at the same time is more difficult than optimizing only one control mechanism; because even if each of the control mechanisms optimized separately; since they are dependent, the overall solution may not be the optimal one. Moreover; as far as I know, there is not any research in the literature which considers a min-max system for the supply of a two echelon network. So, there is not any scientific proof for the min-max policy. Thus, more research should be done to analyze this policy.

The necessity of RDC can be reanalyzed. Since CBT and RDC are in the same city, it may not be logical to keep a central stock in the same location of CBT. Supplying the OB-s directly by the CBT may be economically more beneficial. Moreover, this process will make the echelon structure of Nedtrain simpler, which will make the system more controllable. There are also control mechanism tools in the literature which can deal with this kind of systems efficiently (For example, see Kranenburg and van Houtum [17]).

The designed models can be implemented for the main parts. Since main parts are the most expensive parts in the network of Nedtrain and they are repairable either, implementing lateral or direct shipments can be advantageous for them.

Another possible research area can be analyzing the Grave's Approximate evaluation method (Graves [11]) over repairables. In the literature, it is shown that this method gives better results than METRIC, thus this method can be beneficial to Nedtrain in the control of repairables.

In this study, direct shipments from CBT and RDC to OB-s are implemented at the same time. A possible research area is to analyze the direct shipments from CBT to OB-s and RDC to OB-s separately.

In the lateral transshipment case, full pooling, in which every local warehouse shares its stock with the others, is considered. In the literature, there is also a partial pooling, in which only some of the local warehouses share their stock for lateral transshipment. Naturally, the coordination efforts between the local warehouses in partial pooling will be less than the full pooling. Moreover, Kranenburg and van Houtum [17] proved that partial pooling is sufficient to obtain the full pooling benefits. Thus partial pooling can be considered in Model 2 and 4 in the future research. Note that there is not any study in the literature which considers partial pooling in a two echelon system.

In the lateral transshipment case, a local warehouse always sends repairables by lateral transshipments independent of its stock level. A possible research area is to analyze the situation where lateral transshipment is allowed until a pre-determined inventory level. In this case; if the inventory level is below a certain point, then the local warehouse will not share its stock for lateral transshipments. There is not any study in the literature which considers this case in a multi-item, multi-echelon system.

In Model 4, it is assumed that when a local warehouse is out of stock, if first tries to make a direct shipment from RDC, then it tries to make a lateral transshipment from the other local warehouses. Another possible research area can be analyzing the situation when a local warehouse first tries to make a lateral transshipment, then a direct shipment. Alfredsson and Verrijdt [2] considers this situation.

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APPENDIX A

TRANSPORTATION TIMES AND COSTS

Replenishment time of each OB by RDC, t_{reg}, is 2 days; and repair lead time of a repairable in CBT, T_0, is 20 days.

The locations of the RDC, CBT and the OB-s are in Table 7.1:

Table 7.1: Locations of the RDC, CBT and the OB-s

	Location		**Location**
CBT	Tilburgh	**OB4**	Rotterdam
RDC	Tilburgh	**OB5**	Amsterdam
OB1	Zwolle	**OB6**	Maastricht
OB2	Onnen	**OB7**	Watergraafsmeer
OB3	Leidschendam		

Distance matrix between these locations can be seen in Table 7.2:

Table 7.2: Distances between OB-s and RDC (km)

km	**OB1**	**OB2**	**OB3**	**OB4**	**OB5**	**OB6**	**OB7**
RDC	154	251	106	86	116	126	114
OB1	-	99	151	148	119	236	107
OB2	-	-	235	244	194	332	182
OB3	-	-	-	23	53	231	56
OB4	-	-	-	-	74	209	77
OB5	-	-	-	-	-	228	5,5
OB6	-	-	-	-	-	-	210

Since CBT is in the same city with RDC, distances between CBT and OB-s are equal to the ones between RDC and OB-s.

Total transportation time, which is denoted by t_l^k where $k, l \in N$, between RDC and OB-s are calculated by the formula:

$$t_l^k = \frac{d_l^k}{90} + 2 \quad \forall\, k, l \in N\,, i \in I$$

- Average travelling speed of the vehicle is assumed as 90 $^{km}/_{hour}$, which is the lowest travelling speed in a highway in the Netherlands.
- d_l^k denotes the distance between $\forall\, k, l \in N$
- The additional 2 hours time is the summation of order processing time (1hour), and loading/unloading time for the repairable (1hour)

Total lateral or direct shipment time between RDC and OB-s are in Table 7.3:

Table 7.3: Lateral or direct shipment time between RDC and OB-s (hours)

hour	**OB1**	**OB2**	**OB3**	**OB4**	**OB5**	**OB6**	**OB7**
RDC	3,71	4,79	3,18	2,96	3,29	3,40	3,27
OB1	-	3,10	3,68	3,64	3,32	4,62	3,19
OB2		-	4,61	4,71	4,16	5,69	4,02
OB3			-	2,26	2,59	4,57	2,62
OB4				-	2,82	4,32	2,86
OB5					-	4,53	2,06
OB6						-	4,33

Note that, since CBT is in the same city with RDC, direct shipment time between CBT and OB-s should be equal to the ones between RDC and OB-s. However, since there is the delivery of broken repairable from the OB to CBT and the emergent repair time of the broken repairable in CBT, total direct shipment of a repairable from CBT to OB n is:

$$t_n^{CBT} = 2.\left({}^{t_l^k}/_{12}\ hours \right) + 1.5\ days \quad \forall\, k, l \in N\,, i \in I$$

Emergent repair time of the broken repairable in CBT is assumed 1.5 days and one day is assumed 12 hours. Direct shipment time between CBT and OB-s are in Table 7.4:

Table 7.4: Direct shipment time between CBT and OB-s (days)

	OB1	**OB2**	**OB3**	**OB4**	**OB5**	**OB6**	**OB7**
days	2,12	2,30	2,03	1,99	2,05	2,07	2,04

Transportation cost per repairable for a lateral transshipment between the OB-s and a direct shipment between the RDC and the OB-s is denoted by C_l^k where $k, l \in N$ and calculated by the formula:

$$C_l^k = (c_{usage} + c_{fuel}).d_l^k + \frac{d_l^k}{Average\ Speed}.c_{driver} \quad \forall\, k, l \in N\,, i \in I$$

- $c_{usage} = 0.30\ {€}/_{km}$
- $c_{fuel} = 0.06\ {€}/_{km}$
- $c_{driver} = 50\ {€}/_{hr}$
- Average speed of the truck = 90 ${km}/_{hour}$

Cost of lateral or direct shipments between RDC and OB-s can be seen in Table 7.5:

Table 7.5: Cost of lateral or direct shipments between RDC and OB-s (€)

km	**OB1**	**OB2**	**OB3**	**OB4**	**OB5**	**OB6**	**OB7**
RDC	141,00	229,80	97,05	78,74	106,20	115,36	104,37
OB1	-	90,64	138,25	135,50	108,95	216,07	97,96
OB2		-	215,16	223,40	177,62	303,96	166,63
OB3			-	21,06	48,52	211,49	51,27
OB4				-	67,75	191,35	70,50
OB5					-	208,75	5,04
OB6						-	192,27

An uplift factor, which is equal to 3, is used to calculate the direct shipment cost between CBT and the OB-s. Hence;

$$C_n^{CBT} = C_n^{RDC}.\lambda \quad \forall\, i \in I$$

In the sensitivity analyses, two additional transportation cost situation is considered: High cost situation and low cost situation. In the calculation of the high cost situation, costs of the employees who will participate in the lateral or direct shipment ordering process (€ 200) and the costs of the employees who will make the loading & unloading of the repairables to/from the trucks (€ 100) are added to the cost values mentioned in the normal case. However, in the low cost case, the cost values are determined as 25% of the normal cost values. Table 7.6 and 7.7 shows the lateral or direct shipment costs between RDC and OB-s in the high and low cost situation.

Table 7.6: Cost of lateral and direct shipments between RDC and OB-s in the high cost case (€)

km	**OB1**	**OB2**	**OB3**	**OB4**	**OB5**	**OB6**	**OB7**
RDC	441,00	529,80	397,05	378,74	406,20	415,36	404,37
OB1	-	390,64	438,25	435,50	408,95	516,07	397,96
OB2		-	515,16	523,40	477,62	603,96	466,63
OB3			-	321,06	348,52	511,49	351,27
OB4				-	367,75	491,35	370,50
OB5					-	508,75	305,04
OB6						-	492,27

Table 7.7: Cost of lateral and direct shipments between RDC and OB-s in the low cost case (€)

km	**OB1**	**OB2**	**OB3**	**OB4**	**OB5**	**OB6**	**OB7**
RDC	35,25	57,45	24,26	19,68	26,55	28,84	26,09
OB1	-	22,66	34,56	33,88	27,24	54,02	24,49
OB2		-	53,79	55,85	44,40	75,99	41,66
OB3			-	5,26	12,13	52,87	12,82
OB4				-	16,94	47,84	17,62
OB5					-	52,19	1,26
OB6						-	48,07

APPENDIX B

PRE-SPECIFIED ORDER FOR LATERAL TRANSHIPMENTS

In Model 2 and 4, the pre-specified order of the OB-s for asking lateral transshipment is determined with respect to the distances between OB-s. This means that when an OB is out of stock, it first checks the OB which is closest to it, then checks the second closest one… However, this order can be rearranged with respect to any other criteria. Table 7.8 shows the σ_n vector for each OB n of Model 2 and 4.

Table 7.8: σ_n vector for each OB n

OB	$\sigma_n(1)$	$\sigma_n(2)$	$\sigma_n(3)$	$\sigma_n(4)$	$\sigma_n(5)$	$\sigma_n(6)$
1	2	7	5	4	3	6
2	1	7	5	3	4	6
3	4	5	7	1	6	2
4	3	5	7	1	6	2
5	7	3	4	1	2	6
6	4	7	5	3	1	2
7	5	3	4	1	2	6

APPENDIX C

A SIMPLE EXAMPLE FOR THE EQUATIONS (18), (19), AND (20) OF CHAPTER 3

In this part a simple example is given about the equations (18), (19), and (20) of chapter 4. Let's consider a network with three local warehouses and let $\sigma_1 = (2,3)$, $\sigma_2 = (1,3)$ and $\sigma_3 = (2,1)$. Then for an SKU i:

$$\widetilde{M}_{i12} = (1-\beta_{i1}).m_{i1} \qquad \widetilde{M}_{i13} = (1-\beta_{i2})(1-\beta_{i1}).m_{i1}$$

$$\widetilde{M}_{i21} = (1-\beta_{i2}).m_{i2} \qquad \widetilde{M}_{i23} = (1-\beta_{i1})(1-\beta_{i2}).m_{i2}$$

$$\widetilde{M}_{i32} = (1-\beta_{i3}).m_{i3} \qquad \widetilde{M}_{i31} = (1-\beta_{i2})(1-\beta_{i3}).m_{i3}$$

and;

$$\widehat{M}_{i1} = m_{i1} + \widetilde{M}_{i21} + \widetilde{M}_{i31}$$

$$\widehat{M}_{i2} = m_{i2} + \widetilde{M}_{i12} + \widetilde{M}_{i32}$$

$$\widehat{M}_{i3} = m_{i3} + \widetilde{M}_{i13} + \widetilde{M}_{i23}$$

APPENDIX D

EXACT CALCULATION OF FILLRATES AND AVERAGE BACKORDER WAITING TIME

D.1 Exact Calculation of Fillrates

In this part, calculation of β_{in} from the Markov process of Model 2 will be shown; when $\widehat{M}_{in}$ and λ_{in} values are known. Figure 7.1, below, is the same of Figure 3.1.

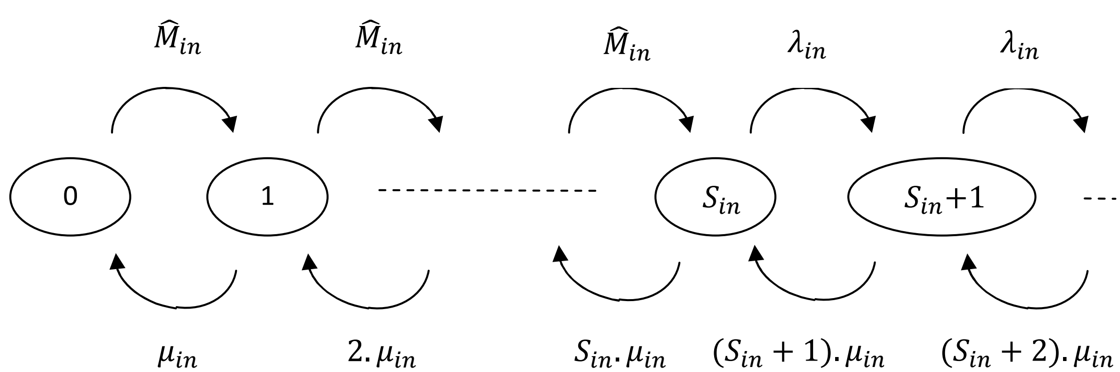

Figure 7.1: Rate diagram of the Markov process describing the number of SKU-s in replenishment in Model 2

Let π_x denotes the steady state probabilities for the continuous time Markov chain shown in Figure 7.1, where $0 \leq x \leq +\infty$ and x is integer. Then,

$$\beta_{in} = \sum_{x=0}^{S_{in}-1} \pi_x \tag{8.1}$$

where,

$$\pi_x = \begin{cases} \frac{1}{x!} \cdot \left(\frac{\widehat{M}_{in}}{\mu_{in}}\right)^x \cdot \pi_0 & \forall\ 0 < x \le S_{in} \\ \frac{1}{x!} \cdot \frac{\widehat{M}_{in}{}^{(S_{in})} \cdot \lambda_{in}{}^{(x-S_{in})}}{\mu_{in}{}^x} \cdot \pi_0 & \forall\ S_{in} < x < +\infty \end{cases} \tag{8.2}$$

Let $\rho_1 = {\widehat{M}_{in}}/{\mu_{in}}$ and $\rho_2 = {\lambda_{in}}/{\mu_{in}}$. Then,

$$\pi_0 \cdot \left\{ \sum_{x=0}^{S_{in}} \left(\frac{1}{x!} \cdot\ \rho_1{}^x \right) + \sum_{x=S_{in}+1}^{+\infty} \left(\frac{1}{x!} \cdot\ \rho_1{}^{S_{in}} \cdot\ \rho_2{}^{x-S_{in}} \right) \right\} = 1 \tag{8.3}$$

Let $A = \left({\rho_1}/{\rho_2} \right)^{S_{in}}$ .Then;

$$\pi_0 \cdot \left\{ \sum_{x=0}^{S_{in}} \left(\frac{1}{x!} \cdot\ \rho_1{}^x \right) + \sum_{x=S_{in}+1}^{+\infty} \left(\frac{1}{x!} \cdot A \cdot\ \rho_2{}^x \right) \right\} = 1 \tag{8.4}$$

$$\pi_0 \cdot \left\{ \sum_{x=0}^{S_{in}} \left(\frac{1}{x!} \cdot\ \rho_1{}^x \right) + A \cdot \left(e^{\rho_2} - \sum_{x=0}^{S_{in}} \frac{\rho_2{}^x}{x!} \right) \right\} = 1 \tag{8.5}$$

π_0 can be calculated exactly by (8.5), and β_{in} can be calculated exactly by (8.1) and (8.2). However, due to the numerical problems encountered, β_{in} is calculated approximately. In this case, π_x values are calculated until $\pi_{x+1} \le 10^{-6} \cdot \pi_0$; then β_{in} is calculated by (8.1).

D.2 Exact Calculation of Average Backorder Waiting Time

In this part, calculation of $\mathrm{BW_{in}}$ of SKU i at OB n from the Markov process of Model 2 will be shown. Let,

- π_x denote the steady state probabilities for the continuous time Markov chain shown in Figure 3.1 or 7.1; where $0 \le x \le +\infty$ and x is integer
- AW_{in} denote the average waiting time of an order for SKU i at OB n
- $E(B_{in})$ denote the expected backorder rate for SKU i at OB n

- δ_{in} denote the average demand rate for SKU i at OB n.

Then by Little's Law:

$$AW_{in} = \frac{E(B_{in})}{\delta_{in}} \tag{8.6}$$

where,

$$E(B_{in}) = \sum_{x=S_{in}+1}^{\infty} (x - S_{in}).\pi_x \tag{8.7}$$

$E(B_{in})$ can be calculated exactly as below.

$$E(B_{in}) = \sum_{x=S_{in}+1}^{\infty} (x - S_{in}).\frac{1}{x!}.\ \rho_1^{S_{in}}.\ \rho_2^{x-S_{in}}.\pi_0 = \sum_{x=S_{in}+1}^{\infty} (x - S_{in}).\frac{1}{x!}.A.\ \rho_2^{x}.\pi_0$$

$$E(B_{in}) = A.\pi_0.\left(\sum_{x=S_{in}+1}^{\infty} x.\frac{1}{x!}.\ \rho_2^{x}. - S_{in}.\sum_{x=S_{in}+1}^{\infty} \frac{1}{x!}.\ \rho_2^{x}\right)$$

$$E(B_{in}) = A.\pi_0.\left[p_2.e^{p_2} - \sum_{x=0}^{S_{in}} x.\frac{\rho_2^{x}}{x!} - S_{in}.\left(e^{p_2} - \sum_{x=0}^{S_{in}} \frac{\rho_2^{x}}{x!}\right)\right]$$

where,

$$\delta_{in} = \left(\sum_{x=0}^{S_{in}-1} \pi_x\right).\widehat{M}_{in} + \left(\sum_{x=S_{in}}^{+\infty} \pi_x\right).\lambda_{in} \tag{8.8}$$

Note that equation 9.5 is an approximate formula because $\widehat{M}_{in}$ and λ_{in} are dependent to π_x. Then, δ_{in} can be calculated as below.

$$\delta_{in} = \pi_0.\left[\left(\sum_{x=0}^{S_{in}-1} \frac{\rho_1^{x}}{x!}\right).\widehat{M}_{in} + \left(\frac{\rho_1^{S_{in}}}{S_{in}!} + \sum_{x=S_{in}+1}^{\infty} \frac{1}{x!}.A.\ \rho_2^{x}\right).\lambda_{in}\right]$$

$$\delta_{in} = \pi_0.\left\{\left(\sum_{x=0}^{S_{in}-1} \frac{{\rho_1}^x}{x!}\right).\widehat{M}_{in} + \left[\frac{{\rho_1}^{S_{in}}}{S_{in}!} + A.\left(e^{p_2} - \sum_{x=0}^{S_{in}} \frac{{\rho_2}^x}{x!}\right)\right].\lambda_{in}\right\}$$

However, due to the numerical problems encountered, $E(B_{in})$ and δ_{in} are calculated approximately. In this case, π_x values are calculated until $\pi_{x+1} \leq 10^{-6}.\pi_0$; then $E(B_{in})$ and δ_{in} are calculated by using equation (8.7) and (8.8) respectively.

Then, BW_{in}, which is average backorder waiting time for SKU i at OB n can be calculated by (8.10).

$$AW_{in} = \left(\sum_{x=0}^{S_{in}-1} \pi_x\right).0 + \left(\sum_{x=S_{in}}^{+\infty} \pi_x\right).BW_{in} \qquad (8.9)$$

$$BW_{in} = \frac{AW_{in}}{\sum_{x=S_{in}}^{+\infty} \pi_x} = \frac{AW_{in}}{1-\beta_{in}} \qquad (8.10)$$

Equation (8.9) and (8.10) use the fact that the orders, which come when there is stock on hand, will not wait. Thus, BW_{in} can be calculated by dividing AW_{in} to the probability that there is not any on-hand stock.

APPENDIX E

DETAILED SENSITIVITY ANALYSIS

The purpose of making sensitivity analyses is to understand the effect of input parameters to each model. The parameters that are analyzed are:

1. Lateral and direct shipment costs
2. Target waiting time values
3. Replenishment lead time of RDC by CBT
4. Transportation time between RDC and OB-s
5. Uplift factor (used in Model 3 and 4 to calculate the cost of direct shipment from CBT to OB-s)
6. Demand rates

Test beds from 1 to 7 are used in these analyses.

E.1 Lateral and Direct Shipment Costs

In the normal case, lateral transshipment costs between the OB-s and the direct shipment costs from RDC to OB-s are calculated by considering the truck renting cost, fuel cost, and the wage rate of the driver; and the direct shipment cost from CBT to OB-s is calculated by multiplying the direct shipment costs from RDC to OB-s by an uplift factor.

To understand the behavior of Model 2, 3 and 4 with respect to different transshipment costs, two additional cost scenarios are considered (see Appendix A). In the high cost scenario, costs of the employees who will participate in the lateral or direct shipment ordering process and the costs of the employees who will make the loading & unloading of the repairables to/from the trucks are added to the cost values mentioned in the normal case. In reality, these new cost factors are not expected to occur because the order processing and loading & unloading of the repairables are a part of the daily jobs of the related employees. However, considering these additional cost factors will represent the worst case cost scenario for the lateral and direct shipments in which almost all possible cost factors will be considered.

In the low cost scenario, the cost values are determined as 25% of the normal cost values. The reason of considering this situation is to understand the behavior of the related models when the lateral and direct shipment costs are much lower than the normal case.

Figure 7.2, 7.3 and 7.4 show the total yearly cost values for each of the models and price range with respect to different cost situations for the test beds 1-2-3 (Note that Model 1 is independent of the lateral and direct shipment costs).

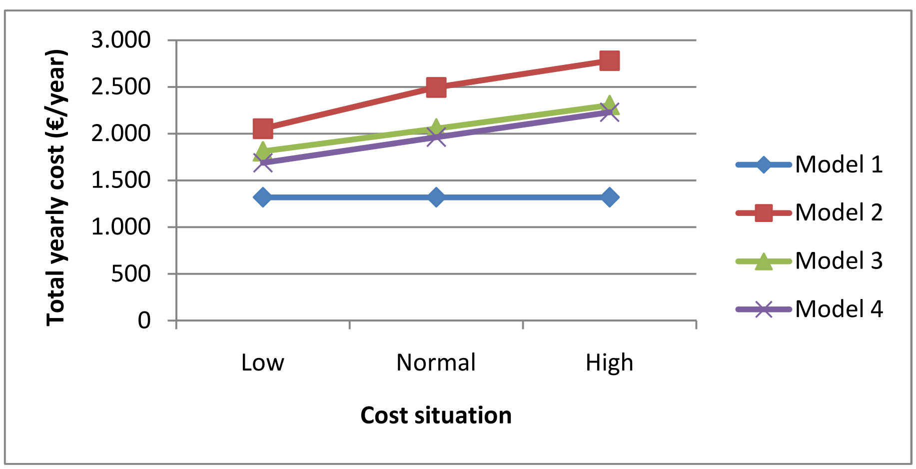

Figure 7.2: Cost values with respect to different cost situations for price range A

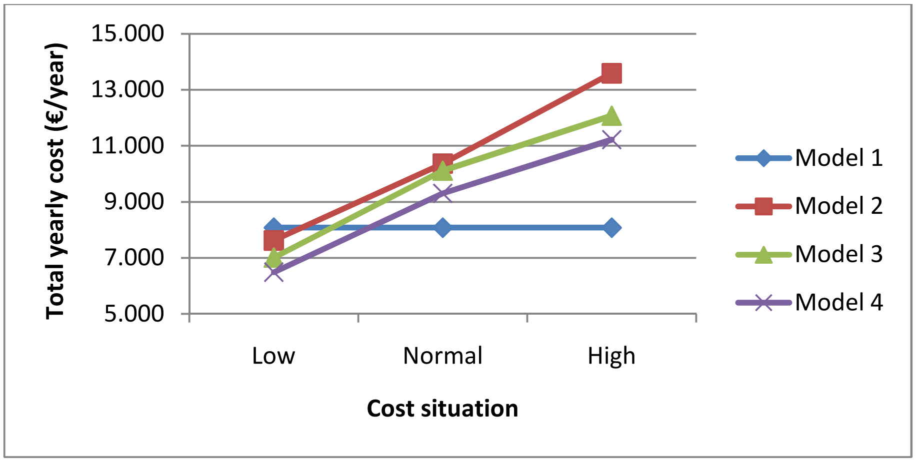

Figure 7.3: Cost values with respect to different cost situations for price range B

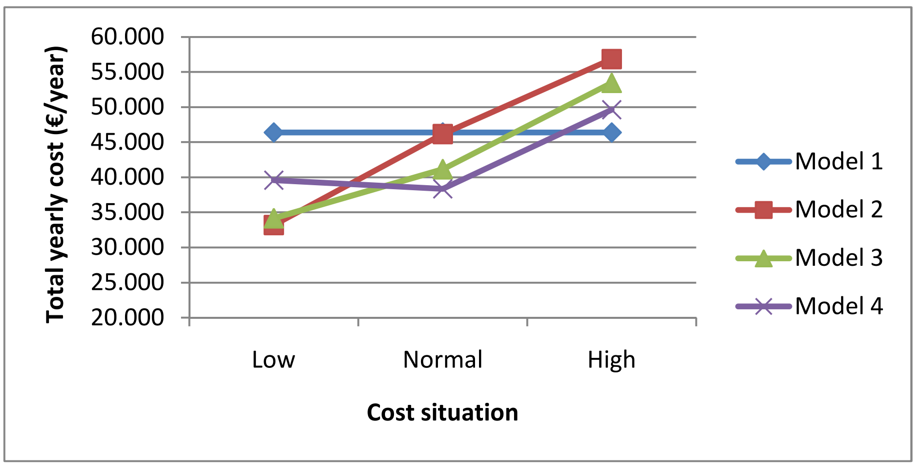

Figure 7.4: Cost values with respect to different cost situations for price range C

According to the results, when the lateral or direct shipment costs decrease (increase), total cost values of Model 2, 3 and 4 decrease (increase). There is only one exception in Model 4 in the price range C. In this scenario, when the lateral or direct shipment costs decrease, total cost value increases. The reason of this situation is the greedy algorithm used in Model 4. This algorithm finds only feasible solutions; it does not find the optimal or an extreme point. Thus the greedy algorithm does not need to find a lower total cost value, when the cost parameters decrease. This situation is also valid for the greedy algorithms of the other models.

In the high cost situation, Model 1 gives the lowest total cost values in each price range. Thus it is not logical to implement lateral or direct shipments in the high cost situations. However, in the low cost situation, lateral or direct shipments become more attractive.

Figure 7.5, 7.6, 7.7 and 7.8 show the total yearly cost values for each of the models and price range with respect to different cost situations for the test beds 4-5-6-7.

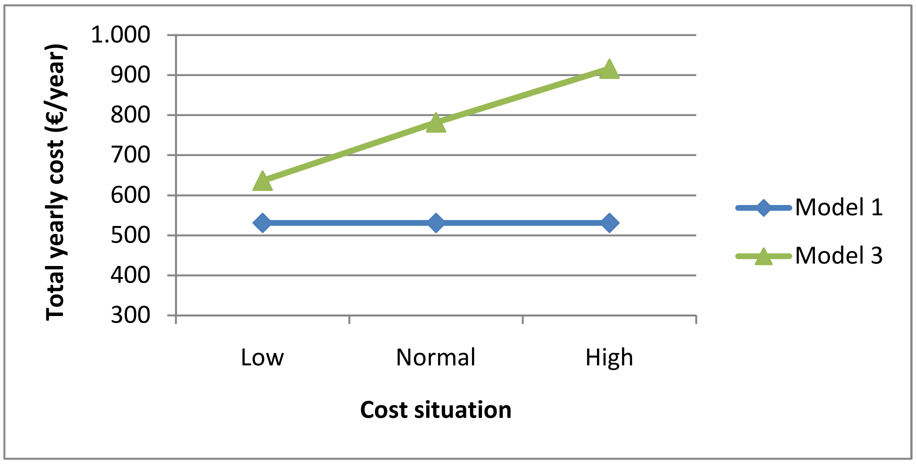

Figure 7.5: Cost values with respect to different cost situations for price range A

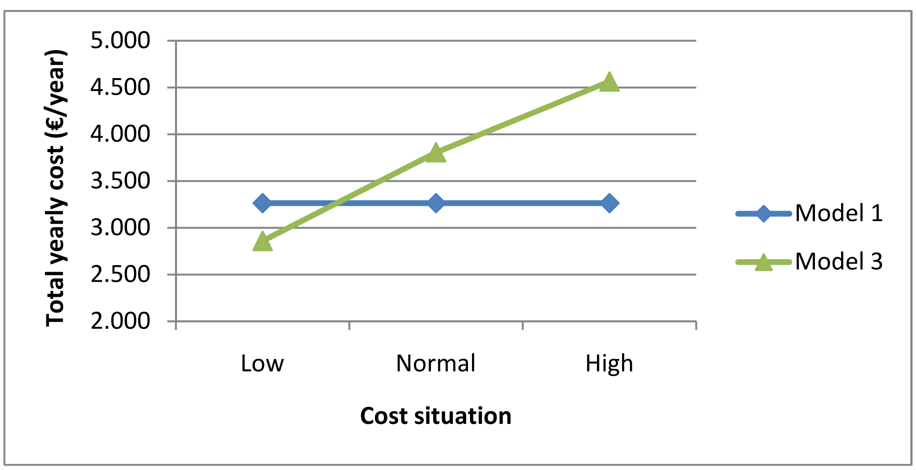

Figure 7.6: Cost values with respect to different cost situations for price range B

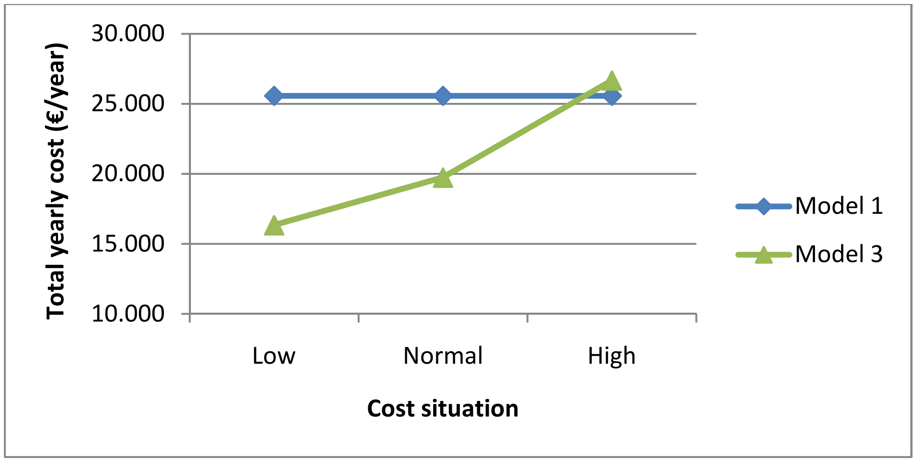

Figure 7.7: Cost values with respect to different cost situations for price range C

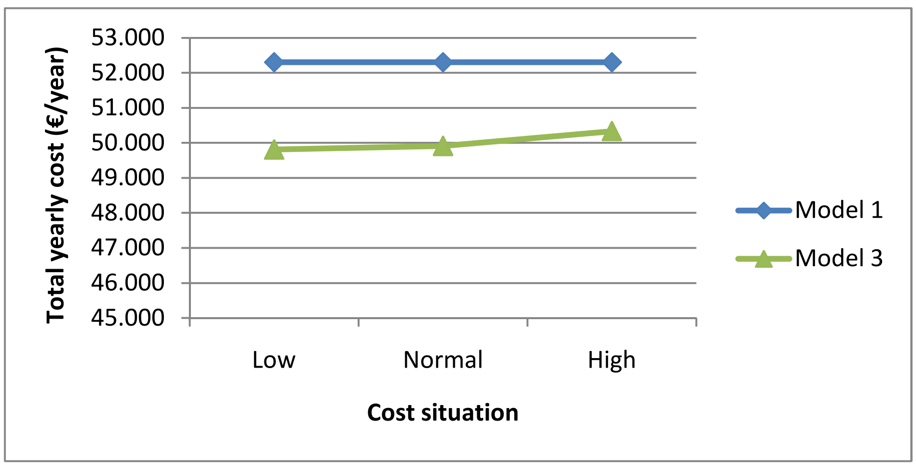

Figure 7.8: Cost values with respect to different cost situations for price range D

As mentioned before; in the normal case, Model 3 gives the lowest cost values in the price ranges C and D. In the high cost case, Model 3 gives lower cost values than Model 1 only in the price range D. This means that when the prices of the repairables are within range D, Model 3 is a better alternative than Model 1 in each of the cost cases. Another interesting fact is that Model 3 is not affected significantly by the cost of direct shipments in the price range D, because the total yearly cost values of Model 3 are close to each other in each case. The reason of this situation is that the inventory costs in this price range make the direct shipment costs less important.

In the low cost situation; Model 1 gives the lowest cost values only in the price range A. In the other price ranges Model 3 is a better alternative. In the price range C, in the low cost situation, the percentage cost difference between Model 3 and 1 is -36.1% which is a very significant value.

In conclusion; Model 2, 3 and 4 becomes more (less) attractive when the cost of lateral or direct shipment costs decrease (increase). However, using Model 1 is rational for the price range A in all cost cases.

E.2 Target Waiting Time Values

In the test beds 1-2-3, for the normal case, target waiting time is determined as 2 hours, because this value provides 93% average fill rate for the OB-s in Model 1, which is the target fill rate used by Nedtrain for each OB. Other than this, 1 and 3 hours target waiting time values are also used in the sensitivity analysis which give 96% and 90% average fill rate for the OB-s in Model 1 respectively as it can be seen in Table 7.9. Hence, the behavior of each model can be examined in different target waiting time situations.

Table 7.9: Fill rate values of Model 1 with respect to different target waiting time values for test beds 1-2-3

Test Bed	**Price Range**	**1 hour**	**2 hours**	**3 hours**
1	A	0,96	0,93	0,91
2	B	0,96	0,93	0,91
3	C	0,96	0,93	0,90

In the test beds 4-5-6-7, for the normal case, target waiting time is determined as 4 hours, because this value provides 93% average fill rate for the OB-s in Model 1. Other than this value, 2 and 8 hours target waiting time values are used in the sensitivity analysis which give 96% and 90% average fill rate for the OB-s in Model 1 as it can be seen in Table 7.10).

Table 7.10: Fill rate values of Model 1 with respect to different target waiting time values for test beds 4-5-6-7

Test Bed	Price Range	2 hours	4 hours	8 hours
4	A	0,95	0,94	0,91
5	B	0,97	0,94	0,92
6	C	0,97	0,95	0,90
7	D	0,97	0,97	0,97

Figure 7.9, 7.10 and 7.11 show the total yearly cost values for each model and price range with respect to different target waiting time values for the test beds 1-2-3.

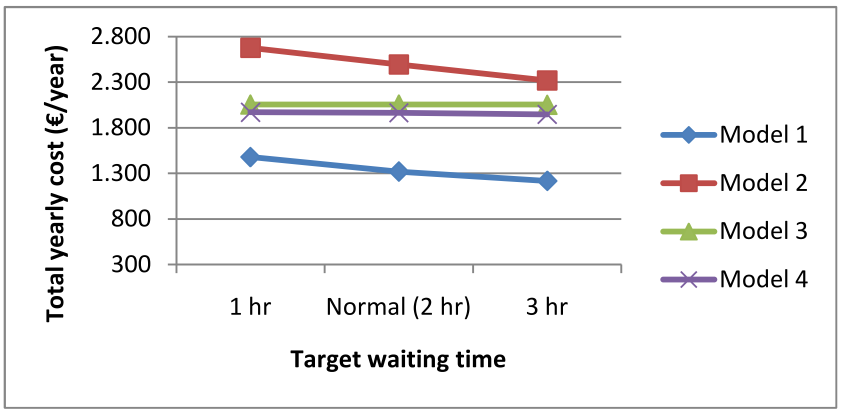

Figure 7.9: Cost values with respect to different target values for price range A

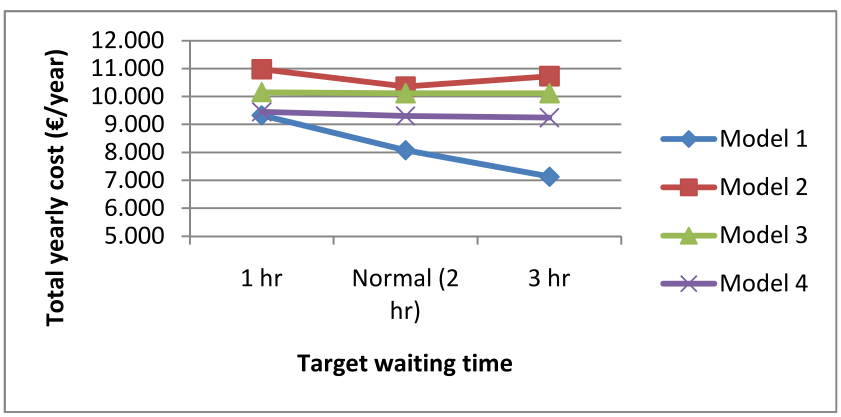

Figure 7.10: Cost values with respect to different target values for price range B

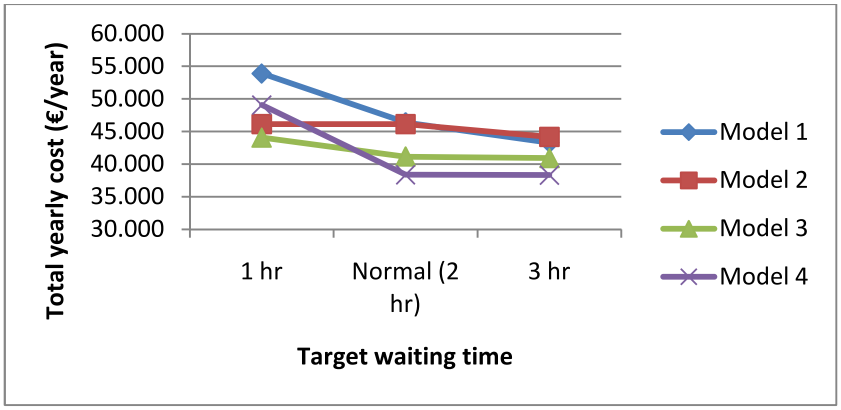

Figure 7.11: Cost values with respect to different target values for price range C

In Model 1 and 4, when the target waiting time decreases (increases), total cost generally increases (decreases).

In Model 2, in the price range B, total yearly cost increases when the target waiting time value is increased to 3 hours in price range B. This result seems irrational, however the reason of the high cost level in 3 hours target waiting time is that the greedy algorithm used in Model 2 does not stop when the target waiting time constraint is met, it continues to add items to stock because increasing the stock decreases the high lateral transshipment costs (When the target is met, total yearly cost value is € 12.325, and when the greedy algorithm stops total cost value becomes € 10.728). Thus 3 hours target waiting time is not the only factor which determines total yearly cost in the price range B for Model 2. Same situation exists in price range C either, however this time the reason is the fact that the greedy algorithm finds only feasible points; it does not need to find the optimal or an extreme point. Thus the greedy algorithm does not need to find a higher total cost value, when the target waiting time value decreases.

In Model 3; in the price range C, when the target waiting time decreases (increases), total cost increases (decreases). However, in the price range A, total yearly cost does not change when the target waiting time decreases; and in the price range B, total yearly cost increases, when the target waiting time increases. Since greedy algorithm used in Model 3 does not stop when the target waiting time constraint is met, it continues to add items to stock because increasing the stock decreases the high direct shipment costs. In the price range A, when the 1 hour target waiting time constraint is met total yearly cost value is € 8.689, and when the greedy algorithm stops total cost value becomes € 2.056; and in the price range B, when the 3 hours target is met total yearly cost value is € 19.714, and when the greedy algorithm stops total cost value becomes € 10.116. Note that when the target is met total cost values are very high, because at that time the models use direct shipments a lot.

When the target waiting time decreases, the performance of Model 2, 3 and 4 increases with respect to Model 1. For instance, in price range A and B; when the target waiting time values decrease; the cost difference between Model 2-3 and Model 1 decreases. In the price range C, when the target is 1 hour, Model 2-3-4 gives much lower cost values relative to Model 1.

Figure 7.12, 7.13, 7.14 and 7.15 show the total yearly cost values for each model and price range with respect to different target waiting time values for the test beds 4-5-6-7.

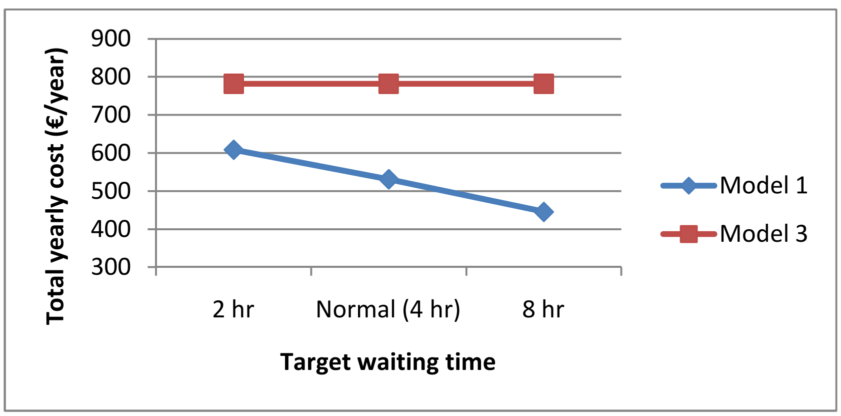

Figure 7.12: Cost values with respect to different target values for price range A

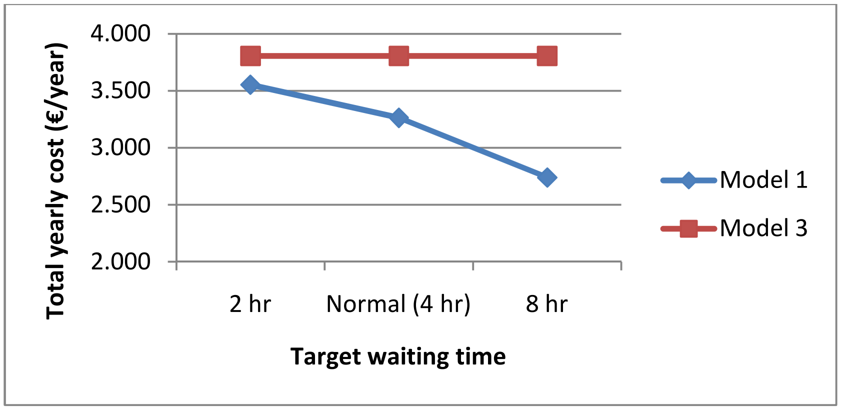

Figure 7.13: Cost values with respect to different target values for price range B

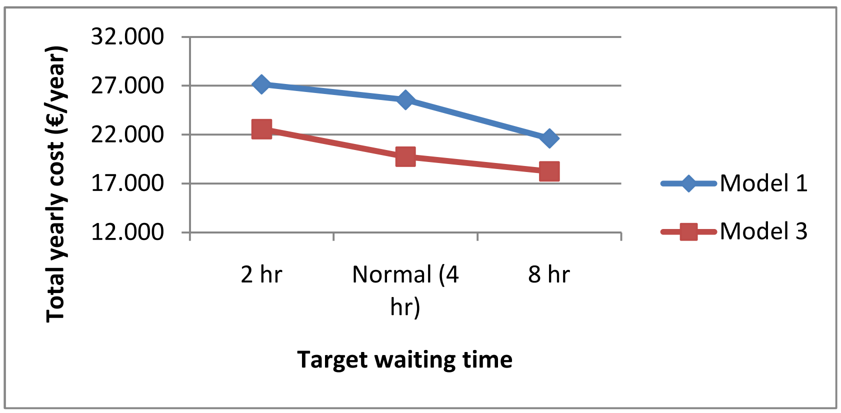

Figure 7.14: Cost values with respect to different target values for price range C

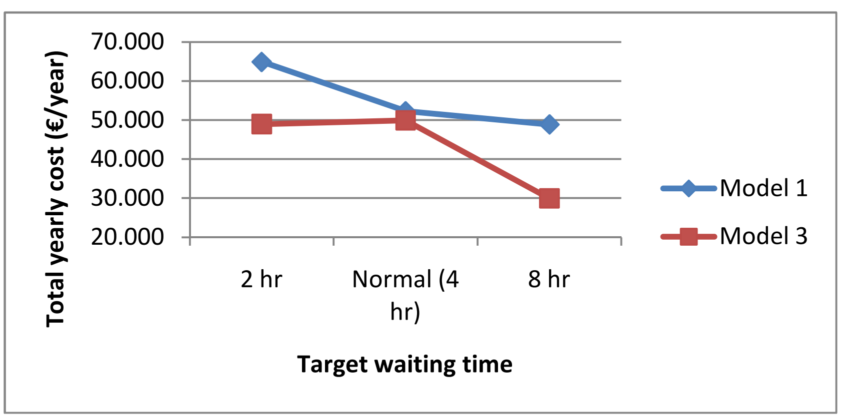

Figure 7.15: Cost values with respect to different target values for price range D

In Model 1; when the target waiting time decreases (increases), total cost increases (decreases); because the stock levels increase when the target decreases.

In Model 3; when the target waiting time decreases (increases), total cost increases (decreases) only in the price range C. In the price range D, when the target waiting time is 2 hours, total cost value decreases in this case. The reason of this situation is the greedy algorithm. This algorithm finds only feasible solutions; it does not find the optimal or an extreme point. Thus the greedy algorithm does not need to find a higher total cost value, when the target waiting time value decreases. In the price range A and B, total cost value is not affected by the target waiting time value in Model 3. Since greedy algorithm used in Model 3 does not stop when the target waiting time constraint is met, it continues to add items to stock because increasing the stock decreases the high direct shipment costs.

In Table 7.11; total cost values for Model 3 in the price ranges A and B in the test beds 4-5, when the target waiting time is met and the greedy algorithm stops, can be seen.

Table 7.11: Total yearly cost values of Model 3 when the target is met and greedy algorithm stops for the test beds 4-5

Price Range	**When target is met**			**When Greed A. stops**		
	Normal (4 hr)	**2 hr**	**8 hr**	**Normal (4 hr)**	**2 hr**	**8 hr**
A	4.587	2.353	7.193	782	782	782
B	6.139	4.659	9.971	3.806	3.806	3.806

As it can be seen from Table 7.11; greedy algorithm continues to add items to stock although the target is met, because increasing stock decreases the total cost in these price ranges. Thus target waiting time is not the only factor which determines total yearly cost in Model 3, in the price ranges A and B.

In conclusion; for the test beds from 1 to 7, when the target waiting time value decreases, the relative performance of Model 2, 3, and 4 generally increases with respect to Model 1. One last remark can be said about Model 2, 3 and 4. These models are not affected significantly by the target waiting time values in the price ranges where they are dominated by Model 1. The reason of this fact is that these models make overstocking to decrease the high lateral and direct shipment costs in these price ranges, although they already meet the target waiting time constraint.

E.3 Replenishment Lead Time of RDC by CBT

Mean replenishment lead time of RDC by CBT is 20 days. In the sensitivity analysis, two additional mean replenishment lead time values are used: 15 and 10 days. The reason of choosing these values is to understand the behavior of the total inventory and transportation costs, if Nedtrain decreases this lead time.

Figure 7.16, 7.17 and 7.18 show the total yearly cost values of each model for each price range with respect to different lead time values for the test beds 1-2-3.

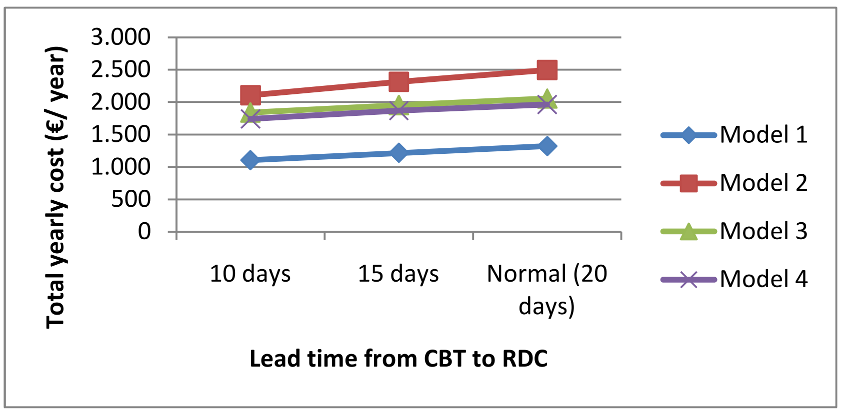

Figure 7.16: Total cost values of the models with respect to different lead time values for price range A

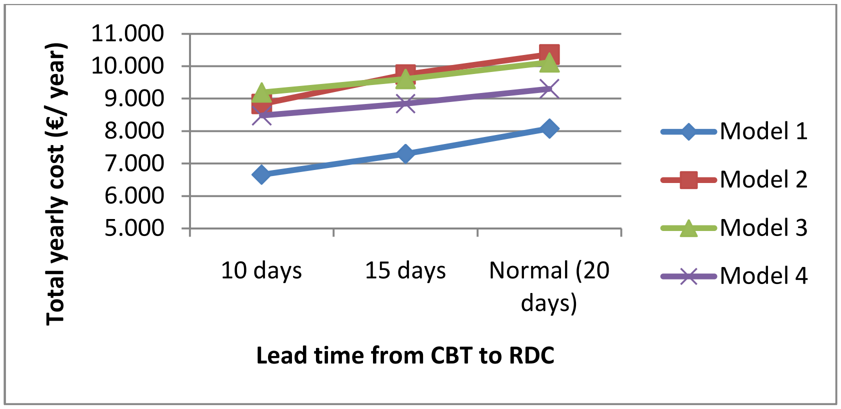

Figure 7.17: Total cost values of the models with respect to different lead time values for price range B

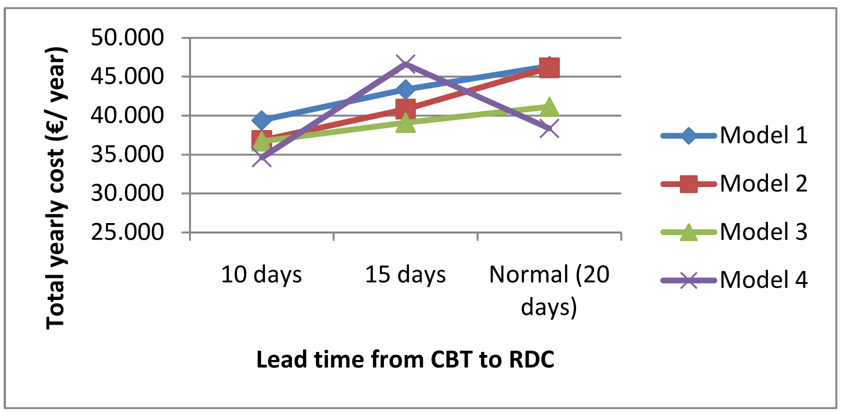

Figure 7.18: Total cost values of the models with respect to different lead time values for price range C

In each of the models; when the lead time decreases, total cost decreases. There is only one exception occurs in Model 4, in the price range C. In this situation, when the lead time decreases to 15 days, total cost value increases. The reason of this situation is the greedy algorithm. This algorithm finds only feasible solutions; it does not find the optimal or an extreme point. Thus the greedy algorithm does not need to find a lower total cost value, when the lead time between CBT and RDC decreases.

In Table 7.12, percentage difference between the 15 and 10 days lead time cases and normal lead time case for each of the models in each of the price ranges in the test beds 1-2-3 can be seen.

Table 7.12: Percentage difference between the 15 and 10 days lead time cases and normal lead time case for the test beds 1-2-3

Price Range	Model 1		Model 2		Model 3		Model 4	
	10 days	**15 days**	**10 days**	**15 days**	**10 days**	**15 days**	**10 days**	**15 days**
A	-16,5%	-8,0%	-15,6%	-7,2%	-10,4%	-5,0%	-11,3%	-4,9%
B	-17,6%	-9,7%	-15,4%	-6,7%	-9,2%	-5,0%	-8,9%	-4,9%
C	-15,0%	-6,5%	-22,1%	-11,6%	-10,8%	-5,0%	-9,7%	21,5%

From Table 7.12, it can be concluded that Model 3 and 4 is affected the least by the change in lead time parameter. This result is logical because; direct shipments are used as another option for the lead time between CBT and RDC, thus they decrease the effect of this long lead time. This means that the shorter the lead time between CBT and RDC, the lower the improvement in the performance of Model 3 and 4 with respect to the other models.

Figure 7.19, 7.20, 7.21 and 7.22 show the total yearly cost values of Model 1 and 3 for each price range with respect to different lead time values for the test beds 4-5-6-7.

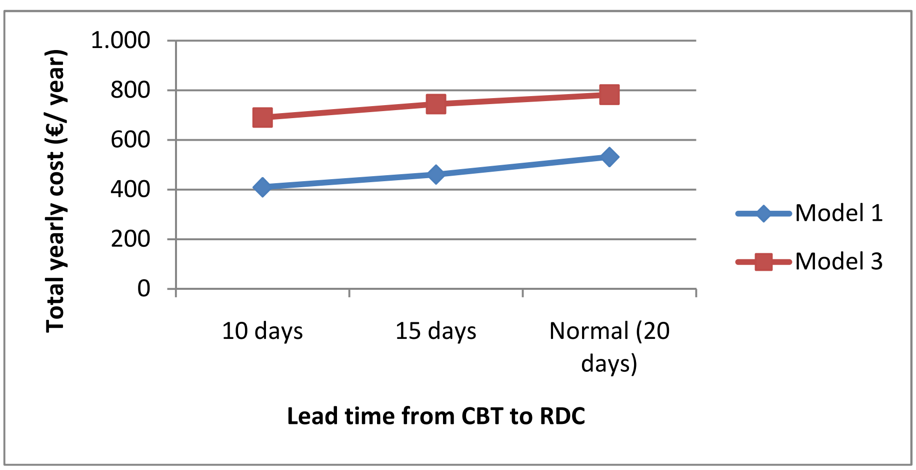

Figure 7.19: Total cost values of the models with respect to different lead time values for price range A

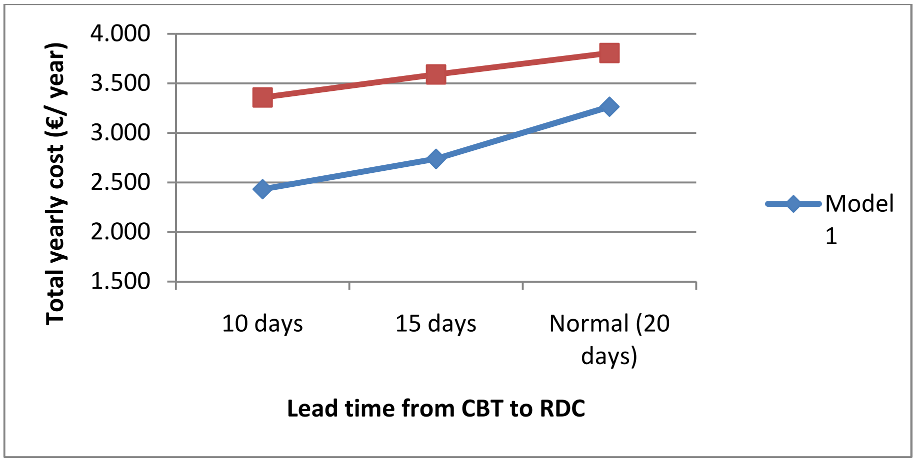

Figure 7.20: Total cost values of the models with respect to different lead time values for price range B

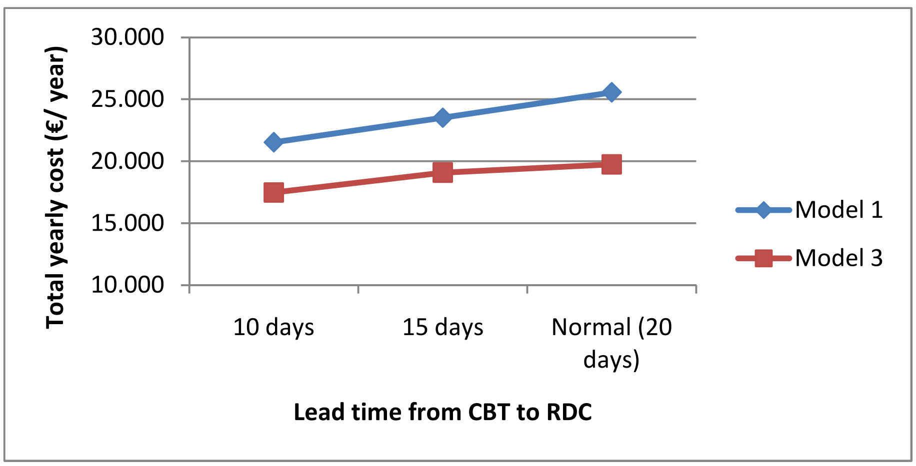

Figure 7.21: Total cost values of the models with respect to different lead time values for price range C

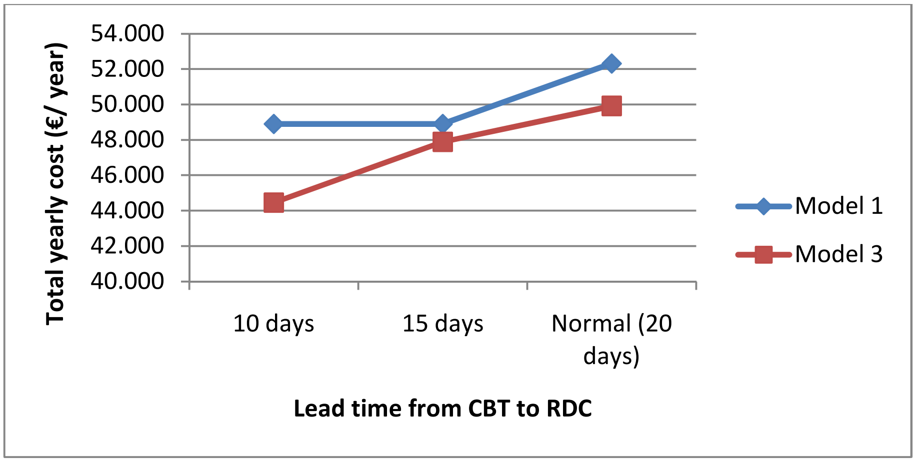

Figure 7.22: Total cost values of the models with respect to different lead time values for price range D

In each of the models; when the lead time decreases, total cost decreases except for Model 1 in the price range D. When the lead time from CBT to RDC is decreased to 15 or 10 days; in the price range A and B, Model 1 gives the lowest cost values. However, in the price range C and D, Model 3 gives the best results.

In Table 7.13, percentage difference between the 15 and 10 days lead time cases and normal lead time case for each of the models in each of the price ranges in the test beds 4-5-6-7 can be seen:

Table 7.13: Percentage difference between the 15 and 10 days lead time cases and normal lead time case for the test beds 4-5-6-7

Price Range	Model 1		Model 3	
	10 days	15 days	10 days	15 days
A	-23,0%	-13,3%	-11,7%	-4,9%
B	-25,5%	-16,2%	-11,8%	-5,7%
C	-15,7%	-8,0%	-11,5%	-3,3%
D	-6,5%	-6,5%	-10,9%	-4,1%

From Table 7.13, again it can be seen that Model 3 is affected less than Model 1 by the change in lead time parameter. This means that the shorter the lead time between CBT and RDC, the lower the improvement in the performance of Model 3 with respect to the Model 1.

In conclusion; when the lead time between CBT and RDC decreases, total cost values of the models generally decrease. However, the shorter the lead time, the lower the improvement in the performance of Model 3 and 4 with respect to the other models.

E.4 Transportation Time between RDC and OB-s

Transportation time between the RDC and OB-s is assumed 2 days. In the sensitivity analysis, one additional transportation time value is used which 1 day. The reason of choosing this value is to understand the behavior of the total inventory and transportation costs, if Nedtrain decreases the transportation time between the RDC and OB-s.

Figure 7.23, 7.24 and 7.25 show the total yearly cost values of each model for each price range with respect to different transportation time values for the test beds 1-2-3.

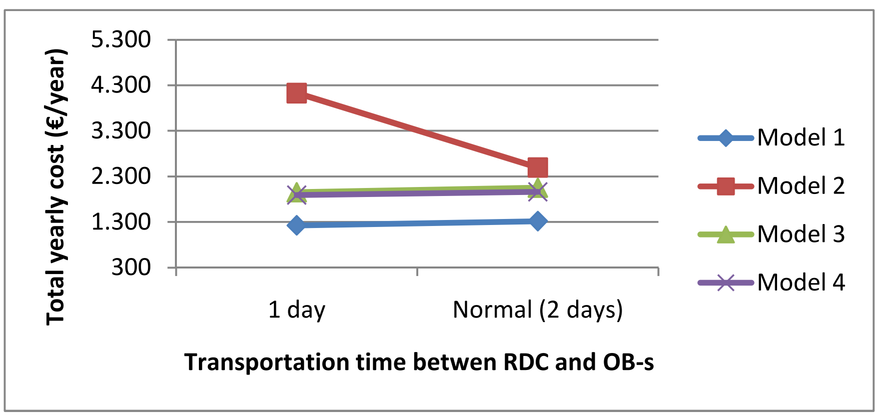

Figure 7.23: Cost values of the models with respect to different transportation time values for price range A

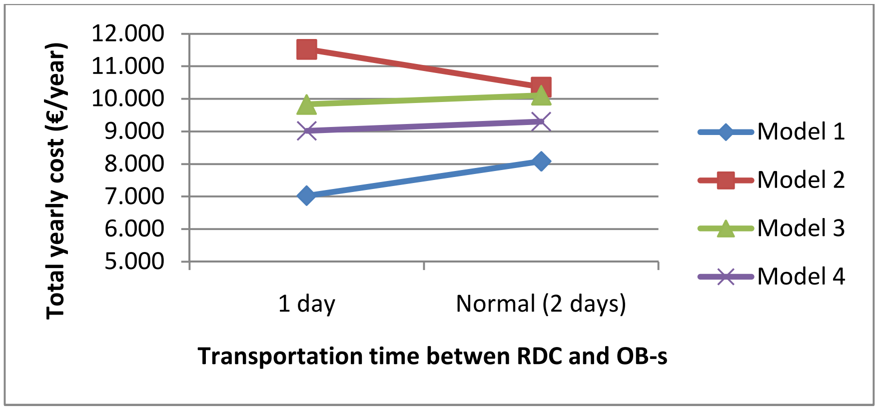

Figure 7.24: Cost values of the models with respect to different transportation time values for price range B

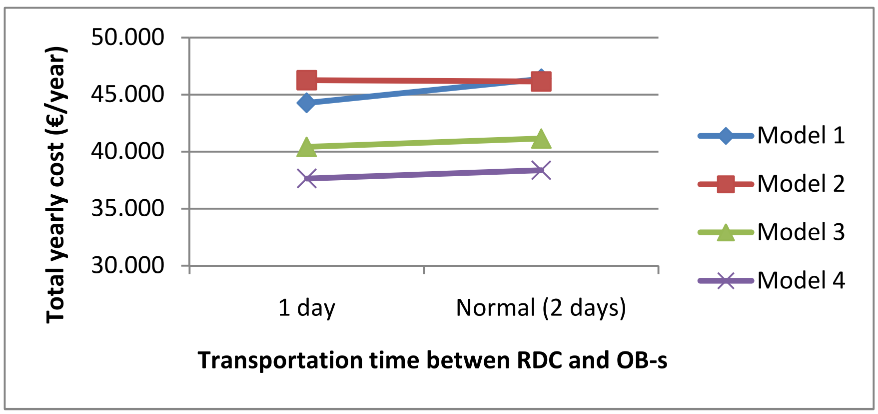

Figure 7.25: Cost values of the models with respect to different transportation time values for price range C

When the transportation time decreases from 2 days to 1 day, total cost decreases in each of the models except Model 2. Total cost increased, when the transportation time decreases in Model 2 because of the greedy algorithm. This algorithm finds only feasible solutions; it does not find the optimal or an extreme point. Thus the greedy algorithm does not need to find a lower total cost value, when the transportation time between the RDC and OB-s decreases. In Table 7.14, percentage difference between the 1 day transportation time case and normal case for each of the models in each of the price ranges in the test beds 1-2-3 can be seen.

Table 7.14: Percentage difference between 1 day transportation time case and normal case in the test beds 1-2-3

Price R.	Model 1	Model 2	Model 3	Model 4
A	-7,1%	65,5%	-4,9%	-3,7%
B	-13,0%	11,2%	-2,8%	-3,1%
C	-4,6%	0,2%	-1,8%	-1,9%

When the transportation time decreases, the improvement in Model 1 is larger than the other Models; because, when the transportation time decreases, the necessity to lateral or direct transshipments decrease. Figure 7.26, 7.27, 7.28 and 7.29 shows the total yearly cost values of each model for each price range with respect to different transportation time values for the test beds 4-5-6-7.

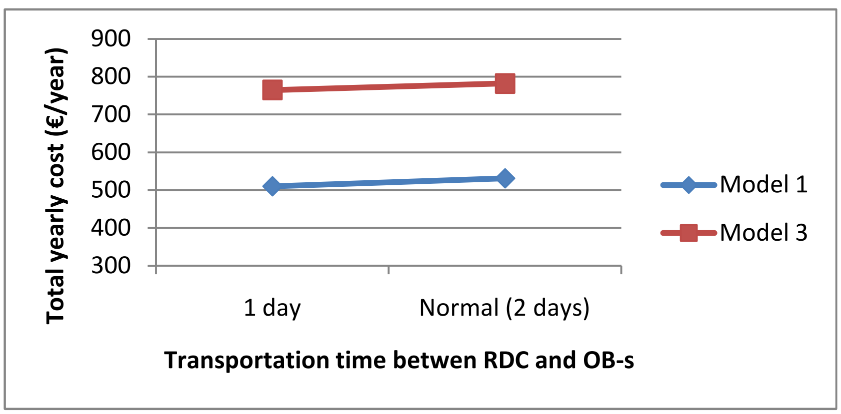

Figure 7.26: Cost values of the models with respect to different transportation time values for price range A

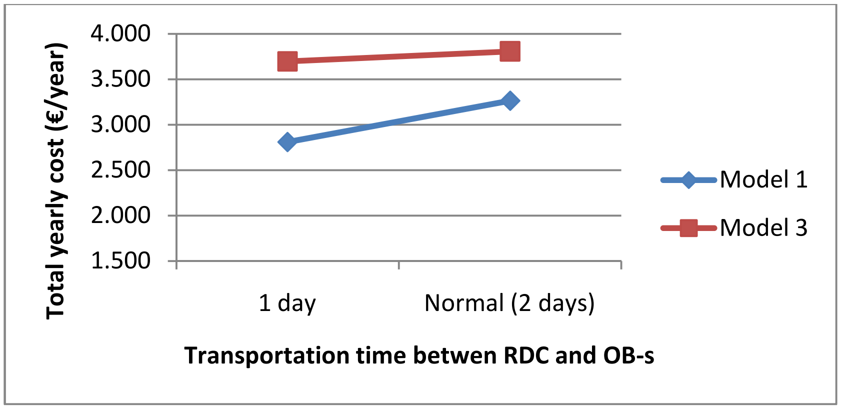

Figure 7.27: Cost values of the models with respect to different transportation time values for price range B

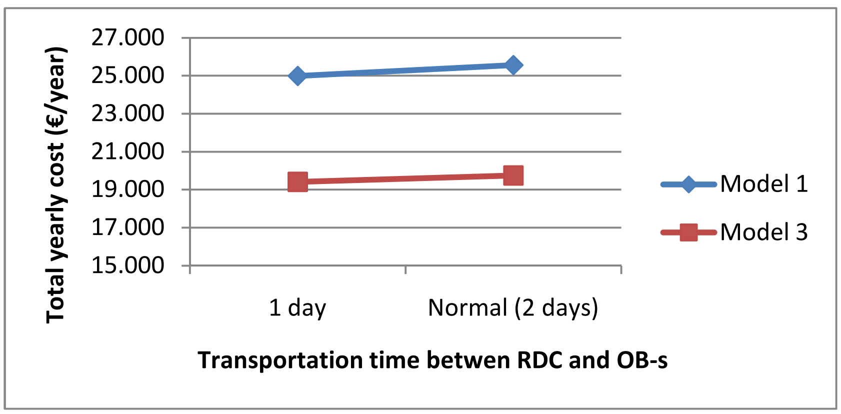

Figure 7.28: Cost values of the models with respect to different transportation time values for price range C

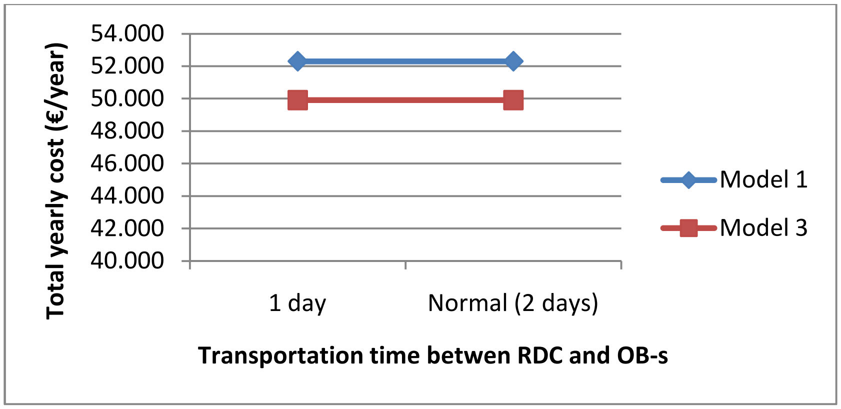

Figure 7.29: Cost values of the models with respect to different transportation time values for price range D

Again when the transportation time decreases from 2 days to 1 day, total cost decreases. The only exception occurs in the price range D for Model 1.

In Table 7.15, percentage difference between the 1 day transportation time case and normal case for each of the model in each of the price ranges in the test beds 4-5-6-7 can be seen.

Table 7.15: Percentage difference between the 1 day transportation time case and normal case in the test beds 4-5-6-7

Price R.	Model 1	Model 3
A	-4,0%	-2,2%
B	-13,9%	-2,9%
C	-2,2%	-1,7%
D	0,0%	0,0%

When the transportation time decreases, the improvement in Model 1 is larger than Model 3; because, when the transportation time decreases, the necessity to direct shipments decrease.

In conclusion; when the transportation time between RDC and OB-s decreases, performance of Model 2, 3 and 4 with respect to Model 1 decreases. However, Model 3 and 4 are still a better alternative than Model 1 in the price ranges C and D in each of the transportation time cases.

E.5 Uplift Factor

This parameter is used in Model 3 and 4 to calculate the cost of direct shipments from CBT to OB-s. In a direct shipment from CBT to an OB, there is the delivery of broken part from OB to CBT, emergent repair of the broken part in CBT, and the delivery of ready-for-use part from CBT to the OB. Because of these three processes, uplift factor is considered as '3'. This means that one direct shipment from CBT to an OB is three times the cost of fast delivery of a part between the CBT and the corresponding OB. In this case, it is assumed that the emergent repair of the broken part has the same cost value with the delivery of a part between the CBT and the corresponding OB.

In the sensitivity analysis, two additional uplift factor values are considered: 4 and 2. The first case reflects the situation in which the cost of the emergent repair of the broken part is twice the cost value of the delivery of a part between the CBT and an OB; and the second case reflects the situation in which the emergent repair of the broken part does not have a cost at all. Thus, the effect of different cost values of the emergent repair of the broken part in CBT will be considered in this part.

Figure 7.30, 7.31 and 7.32 show the total yearly cost values of each model for each price range with respect to different uplift factor values for the test beds 1-2-3 (Note that Model 1 and 2 are independent of the uplift factor values).

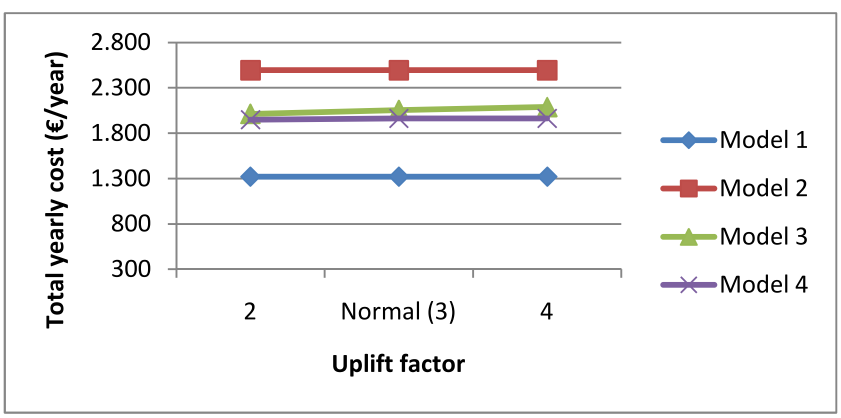

Figure 7.30: Cost values of the models with respect to different uplift factor values for the price range A

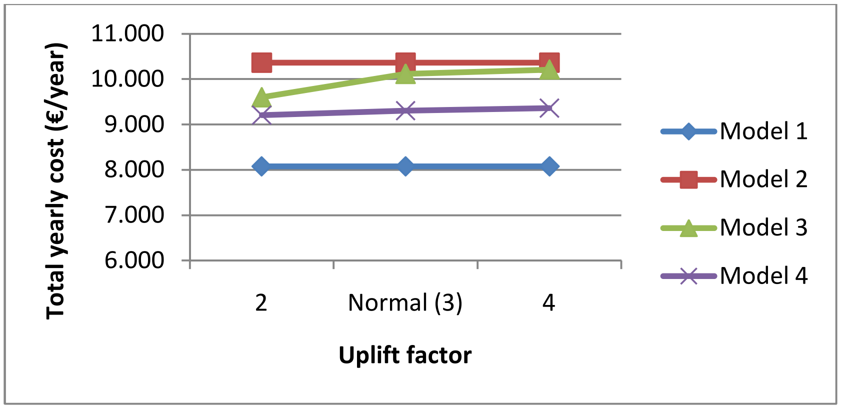

Figure 7.31: Cost values of the models with respect to different uplift factor values for the price range B

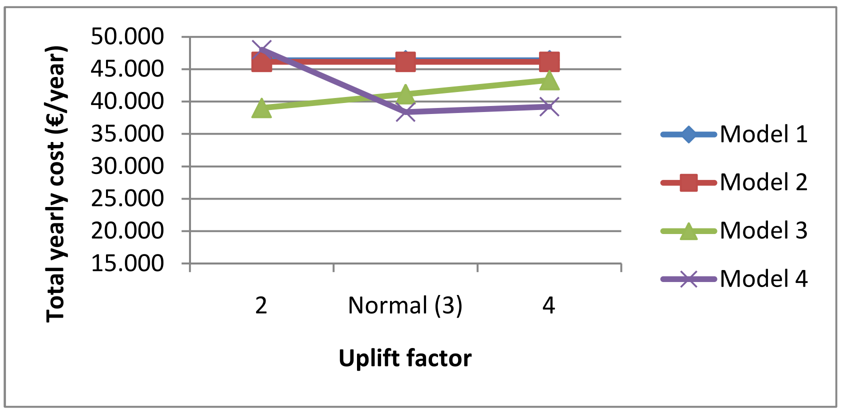

Figure 7.32: Cost values of the models with respect to different uplift factor values for the price range C

The higher the uplift factor, the higher the cost value of Model 3 and 4. The only exception occurs in the price range C for Model 4. In this situation, total cost of the Model 4 increases, although the uplift factor decreases. The reason of this situation is the greedy algorithm. This algorithm finds only feasible solutions; it does not find the optimal or an extreme point. Thus the greedy algorithm does not need to find a lower total cost value, when the uplift factor decreases.

In Table 7.16, percentage difference between the uplift factor 2 - 4 and normal case (3) for Model 3 and 4 in each of the price ranges in the test beds 1-2-3 can be seen:

Table 7.16: Percentage difference between the uplift factor 2 - 4 and normal case (3) in the test beds 1-2-3

Price Range	Model 3		Model 4	
	Uplift f. = 2	Uplift f. = 4	Uplift f. = 2	Uplift f. = 4
A	-2,1%	1,6%	-0,7%	0,0%
B	-5,1%	1,0%	-1,1%	0,6%
C	-5,1%	5,2%	25,1%	2,2%

It is seen that the change in the uplift factor does not affect the total cost values much. This means that Model 3 and 4 is not affected by the uplift factor value significantly. Note that the maximum percentage difference occurs at price range C, in which direct shipments used the most. This means that the higher the direct shipment rates, the higher the effect of uplift factor.

Figure 7.33, 7.34, 7.35 and 7.36 show the total yearly cost values of Model 1 and 3 for each price range with respect to different uplift factor values for the test beds 4-5-6-7.

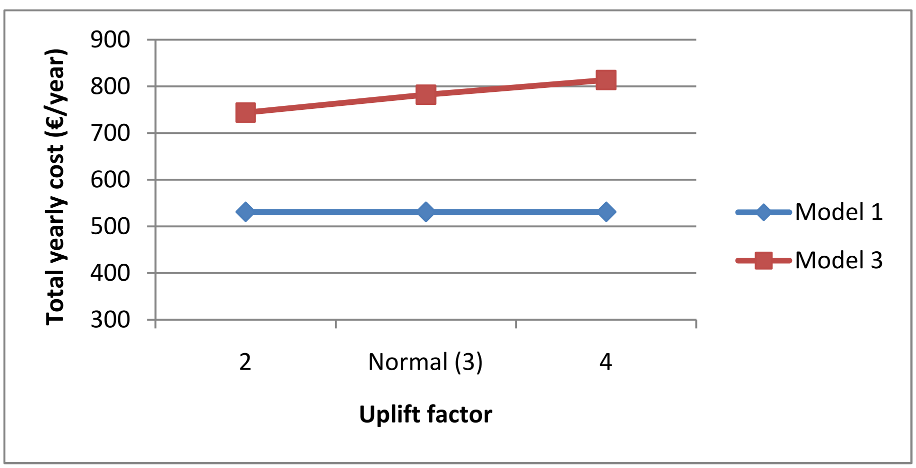

Figure 7.33: Cost values of the models with respect to different uplift factor values for the price range A

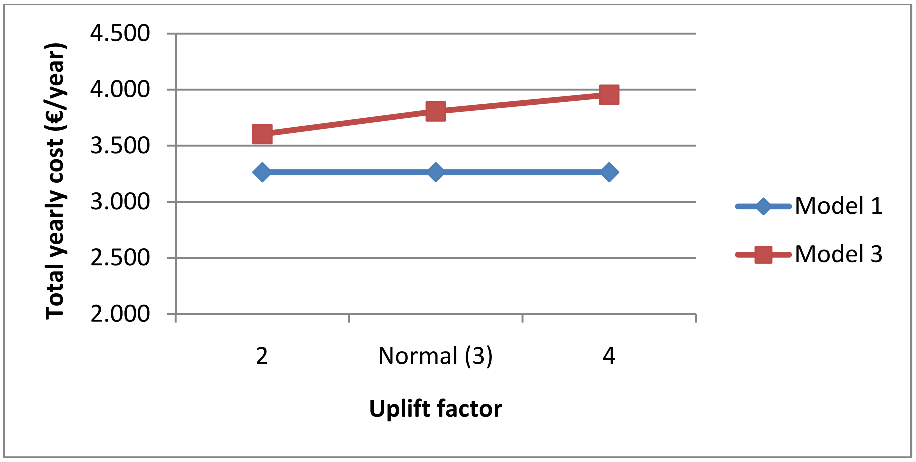

Figure 7.34: Cost values of the models with respect to different uplift factor values for the price range B

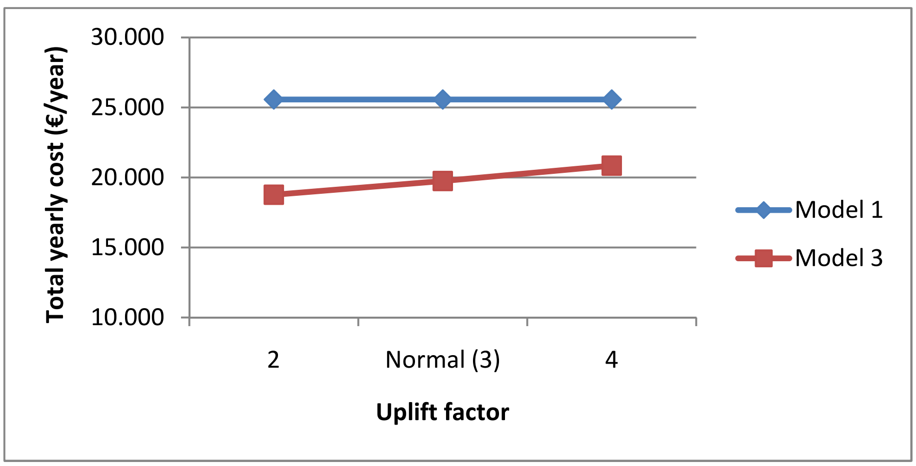

Figure 7.35: Cost values of the models with respect to different uplift factor values for the price range C

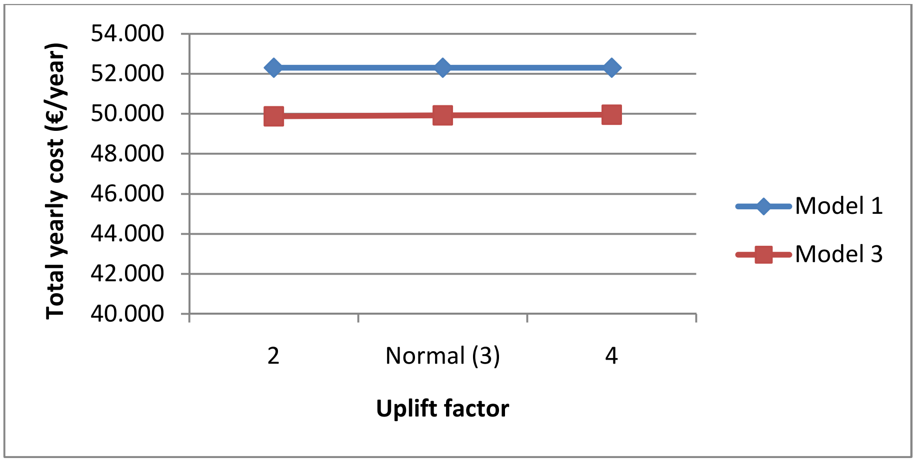

Figure 7.36: Cost values of the models with respect to different uplift factor values for the price range D

Again it is seen that the higher the uplift factor, the higher the cost value of Model 3 and the sequence of the models with respect to cost is not affected by the uplift factor values. In price range A and B, Model 1 gives the best result; and in the price range C and D Model 3 gives the best results in each of the cases.

In Table 7.17, percentage difference between the uplift factor 2 - 4 and normal case (3) for Model 3 in each of the price ranges in the test beds 4-5-6-7 can be seen.

Table 7.17: Percentage difference between the uplift factor 2 - 4 and normal case (3) in the test beds 4-5-6-7

Price Range	Uplift Factor	
	Uplift f. = 2	**Uplift f. = 4**
A	-4,9%	4,1%
B	-5,3%	3,9%
C	-5,0%	5,6%
D	-0,1%	0,1%

Again it is seen that the change in the uplift factor does not affect the total cost values much. This means that Model 3 is not affected by the uplift factor value significantly. Note that the maximum percentage difference occurs at price range C, in which direct shipments used the most. This means that the higher the direct shipment rates, the higher the effect of uplift factor.

In conclusion, it is seen that the higher the uplift factor, the higher the cost value of Model 3 and 4; and the higher the direct shipment rates, the higher the effect of uplift factor. However, in general Model 3 and 4 is not affected by the uplift factor values significantly.

E.6 Demand Rates

In the demand estimations, moving average forecasting method with the demand data of the last three years is used. However; in real life, it is almost impossible to estimate the demand rates with 100% accuracy. Because of this fact, three additional demand rate values are used in this part: 80%, 120%, and 150% of the estimated demand rates. The reason for choosing these values is to examine the waiting time and total cost values when the real demand rates are higher or lower than the estimated demand rates.

Unlike the sensitivity analysis done with the previous parameters, optimization is not used in this part. The stock levels found by the initial demand rates are used as input for the additional demand rate values, and the behavior of the waiting time and total cost values are examined.

Figure 7.37 to 7.42 show the change in the aggregate mean waiting time values for all SKU i at all OB n and the change in total cost values for each model for each price range with respect to different demand rates for the test beds 1-2-3.

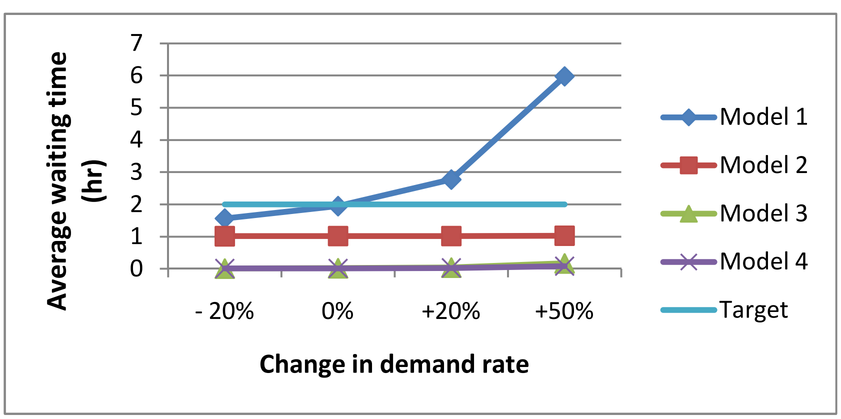

Figure 7.37: Change in aggregate mean waiting time value with respect to different demand rates in price range A

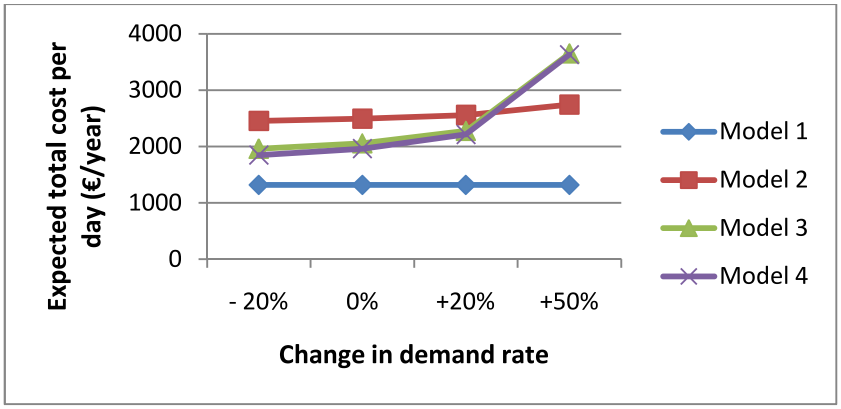

Figure 7.38: Change in total cost value with respect to different demand rates in price range A

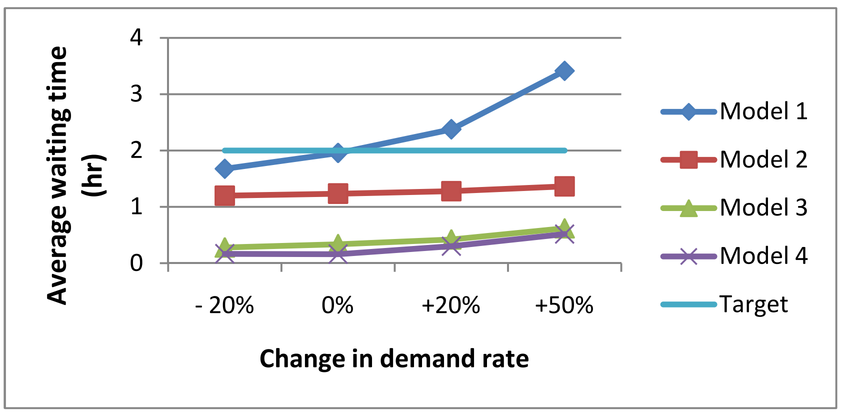

Figure 7.39: Change in aggregate mean waiting time value with respect to different demand rates in price range B

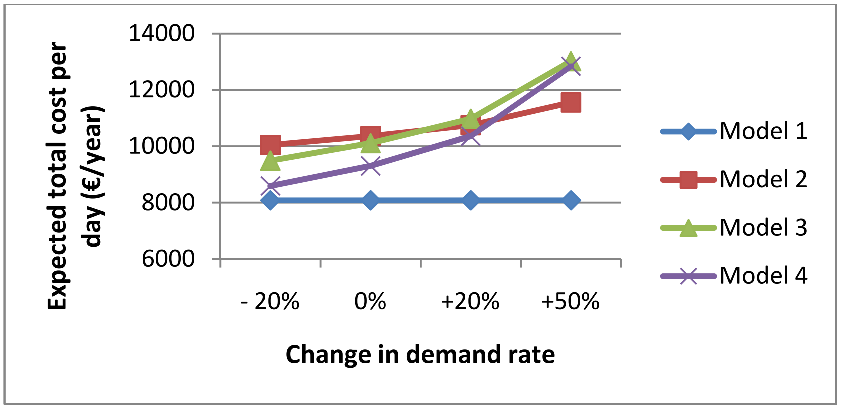

Figure 7.40: Change in total cost with respect to different demand rates in price range B

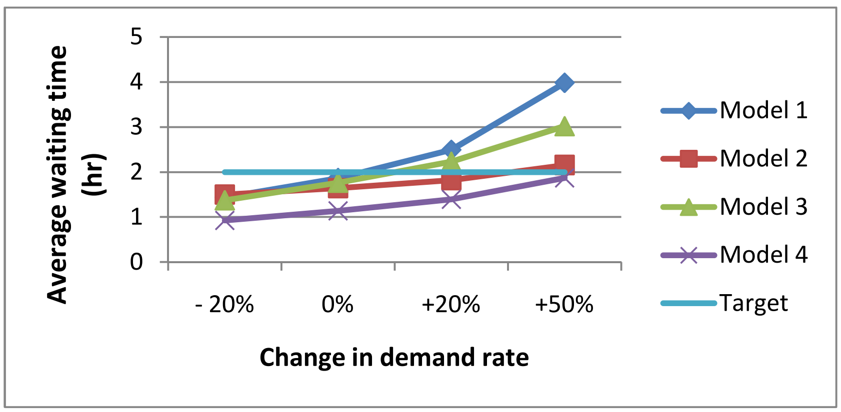

Figure 7.41: Change in aggregate mean waiting time value with respect to different demand rates in price range C

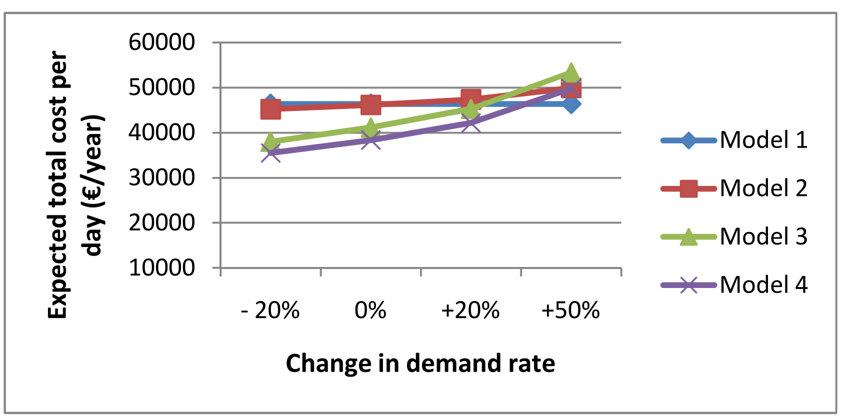

Figure 7.42: Change in total cost with respect to different demand rates in price range C

From the Figures 7.37 to 7.42, it is seen that change in the aggregate mean waiting time value is lower in Model 2, 3 and 4 than Model 1. Thus it can be concluded that the Model 2, 3 and 4 are more stable to the demand changes than Model 1 in terms of the waiting time value. The reason of this situation is that Model 2, 3 and 4 increases (decreases) the lateral or direct shipments when the demand rate increases (decreases), so lateral or direct shipments are the key factors in the stability of Model 2, 3 and 4 when the demand rate changes.

In the price range A and B; when the demand rate increases 20%, aggregate mean waiting time value in Model 1, which gives the lowest cost values in these price ranges, exceeds the target waiting time value. However; in the price range C, even when the demand rate increases 50%, aggregate mean waiting time value in Model 4, which gives the lowest cost values in this price range, does not exceed the target waiting time value.

Since Model 2, 3 and 4 increase or decrease lateral or direct shipments, when the demand rate changes; total cost values of these models change at the same time (Note that when demand rate changes, that total cost values of Model 1 does not change; because the only cost factor in this model is the inventory costs.).

Table 7.18 shows the percentage change in the total cost values in Model 2, 3 and 4 for each of the demand rates with respect to the normal demand case for test beds 1-2-3.

Table 7.18: Percentage change in the total cost values in Model 2, 3 and 4 for each of the demand rates with respect to the normal demand case in the test beds 1-2-3

Price Range	**Model 2**			**Model 3**			**Model 4**		
	-20%	**20%**	**50%**	**-20%**	**20%**	**50%**	**-20%**	**20%**	**50%**
A	-1,7%	2,6%	10%	-4,6%	10,8%	77,6%	-5,8%	12,7%	84,8%
B	-3%	3,8%	11,5%	-6,2%	8,6%	28,7%	-7,7%	11,3%	38,0%
C	-2,1%	2,7%	8,1%	-7,6%	10,0%	29,7%	-7,5%	9,8%	29,8%

From Table 7.18, it is seen that the greatest cost increase for Model 3 and 4 occurs at price range A. This is rational because normally Model 3 and 4 make overstocking to prevent direct shipments in this price range; and when the demand rate increases, in order to supply the extra demands, Model 3 and 4 use more lateral or direct. Since these supply options are costly in this price range with respect to inventory costs, increase in the total cost becomes high in this situation. However; in the price range C, Model 3 almost meets the target waiting time constraint when the demand increases 20% with a 10% increase in total cost and Model 4 meets the target waiting time constraint when the demand increases 20% with a 9.8% increase in total cost.

Figure 7.43 to 7.50 shows the change in the aggregate mean waiting time values for all SKU i at all OB n and the change in total cost values for Model 1 and 3 for each price range with respect to different demand rates for the test beds 4-5-6-7:

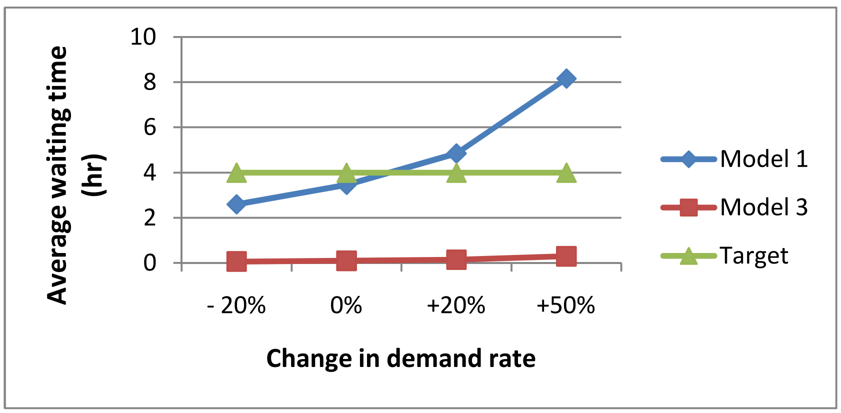

Figure 7.43: Change in aggregate mean waiting time value with respect to different demand rates in price range A

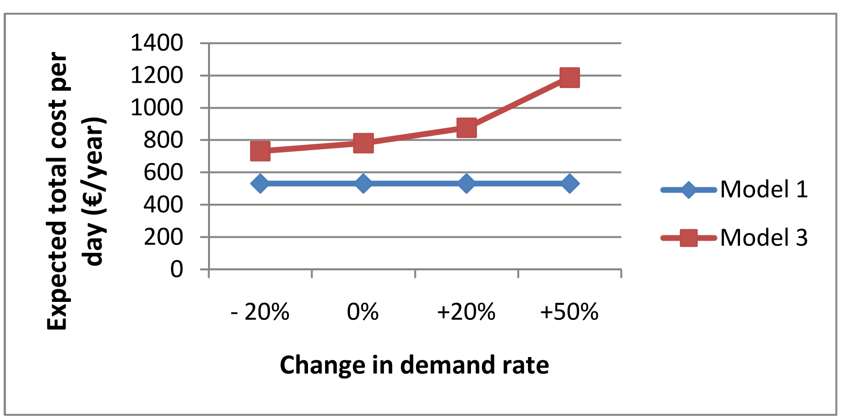

Figure 7.44: Change in total cost value with respect to different demand rates in price range A

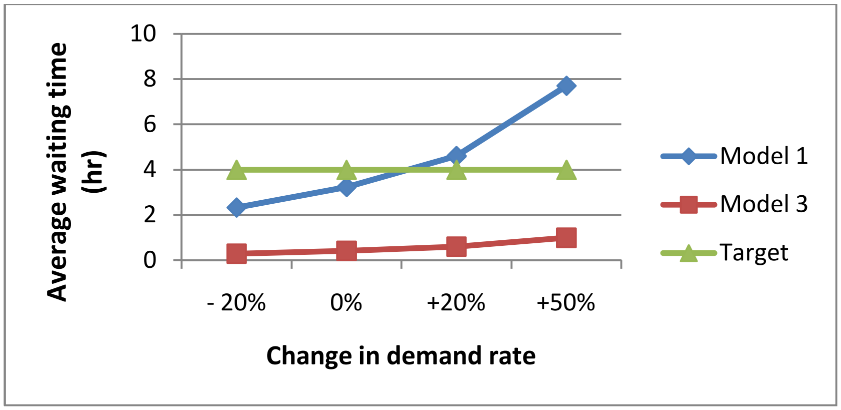

Figure 7.45: Change in aggregate mean waiting time value with respect to different demand rates in price range B

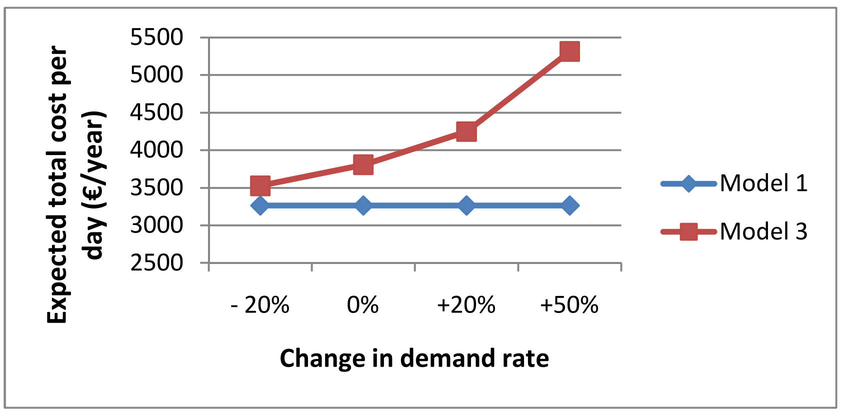

Figure 7.46: Change in total cost value with respect to different demand rates in price range B

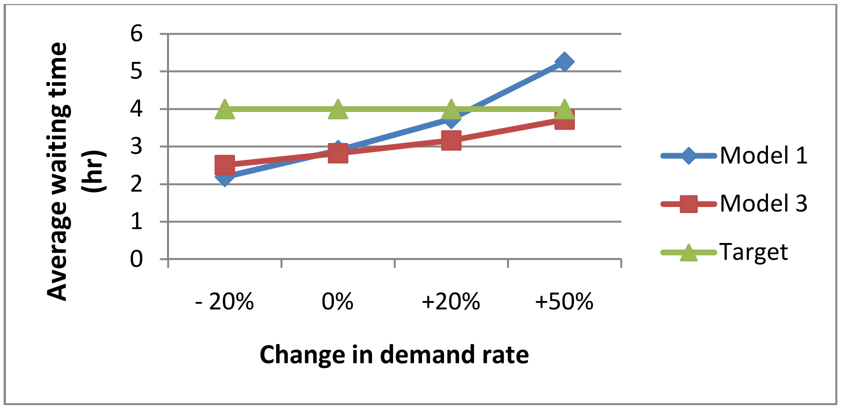

Figure 7.47: Change in aggregate mean waiting time value with respect to different demand rates in price range C

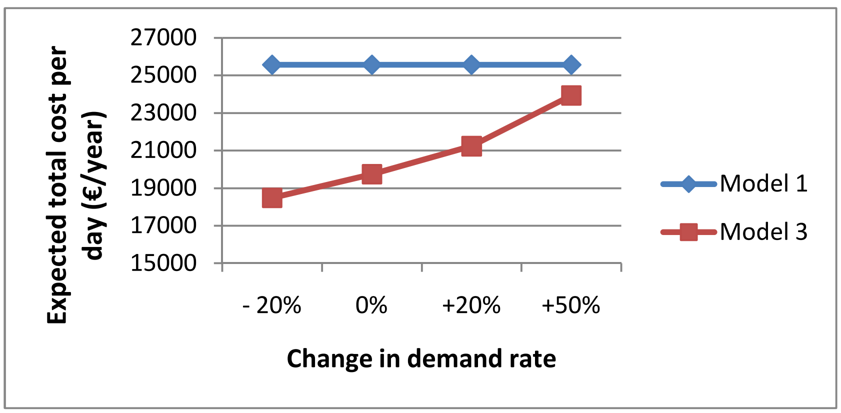

Figure 7.48: Change in total cost value with respect to different demand rates in price range C

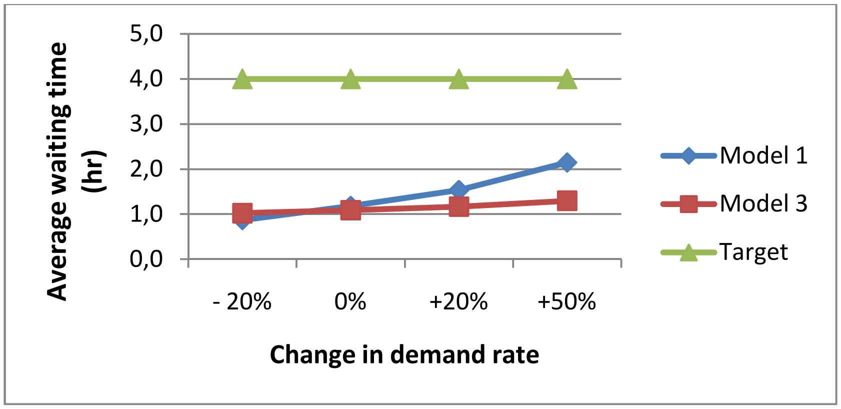

Figure 7.49: Change in aggregate mean waiting time value with respect to different demand rates in price range D

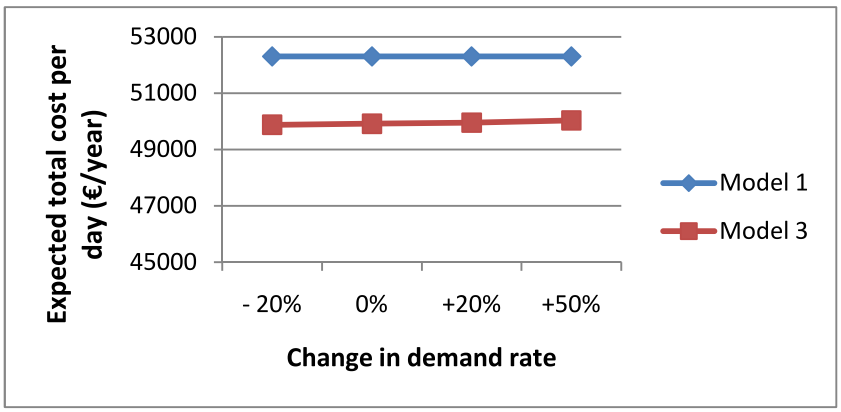

Figure 7.50: Change in total cost value with respect to different demand rates in price range D

From the Figures 7.43 to 7.50, it is seen that the change in the aggregate mean waiting time value is lower in Model 3 than Model 1. Thus it can be concluded that the Model 3 is more stable to the demand changes than Model 1 in terms of the waiting time value. The reason of this situation is that Model 3 increases (decreases) the direct shipments when the demand rate increases (decreases).

In the price range A and B; when the demand rate increases 20%, aggregate mean waiting time value in Model 1, which gives the lowest cost values in these price ranges, exceeds the target waiting time value. However; in the price range C and D; even when the demand rate increases 50%, aggregate mean waiting time value in Model 3, which gives the lowest cost values in this price range, does not exceed the target waiting time value. Moreover total cost values of Model 3 are still lower than the one of Model 1 in these cases. This means that when the demand rate increases even 50%, Model 3 keeps the waiting time value below the target and still gives the lowest cost value in the price ranges C and D.

In conclusion, Model 2, 3 and 4 are more stable to the change in the demand rates in terms of the aggregate mean waiting time value and the reason of this situation is that these models can use lateral or direct shipments to fulfill the extra demand; but naturally this process will increase the cost values, because lateral or direct shipment rates will increase.