INVENTORY MANAGEMENT THROUGH VENDOR MANAGED INVENTORY IN A SUPPLY CHAIN WITH STOCHASTIC DEMAND

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ABSTRACT

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Vendor Managed Inventory (VMI) is a business practice in which vendors monitor their customers' inventories, and decide when and how much inventory should be replenished. VMI has attracted a lot of attention due to its benefits. In this study, we analyze the benefits of VMI in a supply chain consisting of a single retailer and a single capacitated supplier under stochastic demand. We propose a VMI setting and compare the vendor managed system with the traditional system to quantify the benefits of VMI. In our proposed VMI system, the retailer shares the inventory level information with the supplier, which is not available in traditional system; and the supplier is responsible to keep the retailer's inventory level between the specified minimum and maximum values, called (z,Z) levels, set by a contract. We examine the benefits of such a VMI system for each member and for the overall chain; and analyze the effects of system parameters on these benefits. The performance of VMI in coordinating the overall chain is examined under different system parameters.

Keywords: Vendor Managed Inventory, Supply Chain Coordination

ÖZ

BİR TEDARİK ZİNCİRİ İÇİN STOKASTİK TALEP ALTINDA TEDARİKÇİ KONTROLÜ İLE ENVANTER YÖNETİMİ

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Tedarikçi Kontrollü Envanter Yönetimi (VMI), tedarikçilerin müşterilerinin envanterini kontrol ettiği, ne zaman ve ne miktarda yeniden doldurulacağına karar verdiği bir iş yöntemidir. VMI, sağladığı faydalar nedeniyle oldukça dikkat çekmektedir. Bu çalışmada, VMI uygulamasının stokastik talep altında bir perakendeci ve bir tedarikçiden oluşan tedarik zincirindeki faydaları incelenmiştir. Bir VMI uygulaması tasarlanmış ve tedarikçi kontrollü sistemle geleneksel sistem kıyaslanarak VMI uygulamasının faydaları niceliksel olarak gösterilmiştir. Tasarladığımız VMI uygulamasında perakendeci, geleneksel sistemde geçerli olmayan bir şekilde stok seviyesine dair bilgileri tedarikçisiyle paylaşmakla; tedarikçi ise perakendecinin stok seviyesini aralarındaki sözleşmece belirlenen ve (z,Z) seviyesi olarak adlandırılan minimum ve maksimum seviyeler arasında tutmakla yükümlüdür. Bu şekilde tasarlanan bir VMI sisteminin faydaları tedarik zincirinin her bir üyesi ve zincirin bütünü için incelenmiş, sistem parametrelerinin bu faydalara etkileri analiz edilmiştir. VMI uygulamasının tedarik zincirini bütünüyle koordine edebilme performansı farklı parametreler altında incelenmiştir.

Anahtar Kelimeler: Tedarikçi Kontrollü Envanter Yönetimi, Tedarik Zinciri Koordinasyonu

Rüzgar'a

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CHAPTER 1

INTRODUCTION

A company which is involved in supply chain operations should perform the processes of sourcing of raw materials and components, producing and dispatching the finished products to customers. The main concern of each company is to minimize the operational cost and to maximize profit.

Although being a part of the supply chain, a firm still focuses on optimizing its own costs or profits, because the decisions concerning production and replenishment are made separately and independently by the members of that chain. However, a supply chain implies the interaction of its members, even when they have different operational goals. Technological advancements and the realization that each company is a part of one or more supply chains that dictate its performance and effectiveness have resulted to the adoption of cooperative strategies through information sharing and contracting.

“In fact, the performance of a supply chain depends not only on how well each member manages its operational processes, but also on how well the members coordinate their decisions” (Achabal 2000). For this reason, coordinating the decisions is significant in a supply chain, even each member of the chain has different own operational goals.

Vendor Managed Inventory (VMI) is a popular and widely discussed partnership between the members of the supply chain which was popularized in the late 1980’s by Wal-Mart and Procter & Gamble and resulted in significant benefits.

After this successful application, many other firms, such as Campbell Soup Company, Barilla and Intel, have also implemented VMI in their supply chains.

VMI is an inter-organizational relationship that the retailer shares the information of end-item demand and the inventory level information with the supplier and the supplier uses this information for better management of the retailer's inventory. "Although it depends on the form of the agreement, the benefits of VMI generally include improved service level and reduced supply chain costs, reduced customer-demand uncertainty, reduced stock-outs and stock-out frequency, and reduced bullwhip effect" (Waller, Johnson and Davis 1999).

In this study, we model a supply chain which is composed of a single retailer (he) and a single capacitated supplier (she). We define a traditional system under which the retailer manages his own inventory, and then introduce a system with VMI agreement. We analyze the benefits of VMI for the retailer, for the supplier and for the total chain. The retailer faces stochastic customer demand at each period and in the traditional setting, he periodically places orders to the supplier. In the traditional setting, the supplier is assumed to know the end-demand distribution and the inventory policy of the retailer with its parameter values. She infers the probability distribution of the retailer's demand by using this information. The information of the retailer's inventory level is not available to the supplier in the traditional system, although she has full information in the VMI system. Availability of information in traditional and VMI systems is similar to partial and full information models defined and analyzed in Gavirneni, Kapuscinski and Tayur (1999).

In a VMI system, if the supplier's control were unlimited, then without some form of penalty costs at the supplier, it would be optimal for the supplier to dispatch large volumes of inventory to the retailer. In real life systems, however, the supplier is limited in her replenishments and in other operational decisions under VMI by an agreement or contract between the participant firms (Fry, Kapuscinski and Olsen 2001). The implementation and details of VMI contracts differ from company to company and may specify various parameters to control the replenishments of the supplier, such as customer service level, allocated storage or shelf space (Fry, Kapuscinski and Olsen (2001) provides the examples of VMI agreements used in practice). In our study, under the VMI setting, the supplier is expected (or forced) to keep the retailer's inventory level between the specified two values, which are denoted as (z,Z) levels. These specified levels should satisfy that each player in the chain is never worse off under the VMI setting when compared to the traditional case. In the new system, the fixed ordering cost of the retailer (transportation cost) is also shared between the retailer and the supplier in different ratios, although it is fully incurred by the retailer in traditional setting.

We aim to examine the benefits of VMI for the retailer, for the supplier and for the overall chain in this study and we analyze how these benefits under VMI setting change with system parameters.

Depending on the form of the agreement between the supplier and the retailer, the supply chain under VMI may be very close to a centralized system, that is, it may operate on conditions which are very close to optimal. One of our aims is to measure the performance of such an agreement to coordinate the chain by comparing its benefit on total supply chain by a centralized system.

The remainder of the study is organized as follows. A literature review related to the VMI systems is provided in Chapter 2. In Chapter 3, we present the supply chain environments and the models for the traditional, VMI and the centralized settings. We present our computational study results in Chapter 4. The benefits of VMI for the supplier, for the retailer and for the overall supply chain under different system parameters are analyzed in this chapter. In Chapter 5, we conclude the study by stating a summary of our work and by discussing on the further study issues.

CHAPTER 2

LITERATURE REVIEW

Vendor Managed Inventory (VMI) systems have been studied from many aspects. A considerable amount of researches in this literature addresses the benefits of information sharing in a supply chain. Benefit of VMI as a supply chain coordinator is studied by many researchers. VMI contractual agreements are also studied in the related literature.

Co-operative policies and information sharing are one of the focuses in VMI research. There exist considerable amount of articles studying these issues. Gavirneni, Kapuscinski and Tayur (1999) analyze the value of information in a supply chain with single retailer and a single capacitated supplier. The authors study the partial and the full information sharing and compare these to a traditional case of no information. As a partial information sharing, the benefit of knowing he retailer's inventory policy parameters, (*s,S*) policy, is examined. In complete information sharing case, they examine the benefits of knowing the retailer's inventory level. They found that the information sharing is most beneficial when the capacity is not restrictive. Cachon and Fisher (2000) find only limited benefits to information sharing. They analyze a supply chain which consists of multiple retailers and a single supplier. The retailers order an integer number of batches and the supplier uses a batch priority allocation process among these retailers. They conclude that the extra benefit of demand information is limited for the supplier, since it is most valuable just before an order is placed by the retailer. Aviv and Federgruen (1998) analyze VMI from the perspective of information sharing. They find that information sharing with VMI always provides more benefits than pure information sharing.

The incentive issues in VMI contractual agreements are also studied in VMI literature. Fry, Kapuscinski and Olsen (2001) model a specific (z,Z)-type of VMI agreement in a supply chain environment with a single manufacturer and a single retailer with stochastic demand and compare the VMI system with a traditional case under full information sharing. In this type of contract, the retailer sets the inventory levels, z and Z, that represent the lowest and highest inventory levels, respectively. The manufacturer should keep the retailer's inventory level between these specified levels. The manufacturer follows a fixed production schedule, but can replenish the retailer in any period and she incurs a penalty cost if she cannot keep the retailer's inventory level between the specified levels. They make a comparison between the VMI system and the traditional system with information sharing and find that VMI can perform significantly better than the traditional case in many settings but can perform worse in others. Their numerical analyses also show that when the outsourcing cost and the demand variation is high, VMI performs close to a centralized system.

VMI provides flexibility in delivery and in operational decisions. Several studies focus on these benefits provided by the VMI agreements in a single supplier-multiple retailer supply chain environments. "The flexibility may enable a supplier to combine routes from multiple origins and delay stock assignments, consolidate shipments to two or more customers, or postpone a decision on the quantity destined for each of them" (Gumus 2006). Aviv and Federgruen (1998) study the effects of information sharing under periodic inventory review in a supply chain with single supplier and multiple retailer. They assume a VMI agreement that leads to a fully centralized planning model where the supplier minimizes the system-wide total cost of inventory holding and distribution. They construct the optimal policies for the supplier and the retailers under VMI with information sharing and under information sharing alone. They conclude that VMI provides more benefits than information sharing alone. Cheung and Lee (2002) also study the flexibility of delivery to multiple retailers offered by VMI in terms of shipment coordination and stock balancing. Similarly, Çetinkaya and Lee (2000) analyze the synchronization of inventory and dispatch decisions under VMI. They assume that under VMI, the supplier can hold orders until a suitable shipment time, at which orders can be economically consolidated. They derive the optimal policy under VMI and conclude that shipment consolidation is beneficial if the outsourcing and holding costs are low. Waller, Johnson and Davis (1999) studies the impacts of VMI under different levels of demand variability. They demonstrate that a reduction in inventory is achieved in VMI and this is the result of frequent reviews and shorter intervals between replenishments. Kleywegt, Nori and Savelsbergh (2002) also study a single supplier-multiple retailer supply chain environment in order to determine the optimal distribution policy of a single product.

Supply chain coordination is another stream of research. A supply chain may be very close to a centralized system depending on the performance of the VMI agreement between the supplier and the retailer. The effects of VMI agreements to the coordination of the supply chain is studied in the literature. Cachon (2001) studies VMI in a supply chain with a single supplier-multiple retailers. He analyzes various strategies with the aim of channel coordination. He employs game theory to find the equilibrium for each member of the supply chain. Cachon (2001) remarks that "VMI alone does not guarantee an optimal supply-chain solution; both the vendor and retailers must also agree to make fixed transfer payments to participate in the VMI contract, and then be willing to share the benefits." Bernstein, Chen and Federgruen (2006) study a partially centralized VMI model. For the supply chain a replenishment strategy is determined. They find that when the demand rate is constant and a single retailer retains the decision rights on pricing, the channel coordination can be achieved. In their supply chain environment, the supplier incurs holding costs of the retailer. Therefore, their system considers consignment inventory with VMI. Dong and Xu (2002) analyze the different impacts of a VMI program on a supply channel in terms of total chain cost and supplier's cost. The purchasing price is set by the retailers in the contract; however the selling quantity is determined by the supplier in their model. They find that total chain cost is reduced by VMI, but the profit of the supplier could decrease under some certain conditions. They argue that VMI is an effective strategy that can realize many of the benefits obtainable only in a fully integrated supply channel. Narayanan and Raman (1997) and Fry, Kapuscinski and Olsen (2001) also analyze VMI agreements under stochastic demand and discuss centralization. One aim of our study is to analyze the benefits of the VMI agreement on the total supply chain and to measure the performance of VMI in coordinating the supply chain by comparing the supply chain profits under VMI setting with the centralized system.

The VMI agreement may also specify a consignment stock, whereby the retailer will not be invoiced until he sells the product to end customers. Several research employ the consignment stock with VMI. Valentini and Zavanella (2003) discuss the use of consignment stock by a manufacturer who manages the inventory of her customer based on (s,S) policy. Their analysis and computational studies show that consignment stock outperforms the usual inventory models. In a very recent study, Zavanella and Zanoni (2009) examine a consignment stock system in a single supplier-multiple retailers supply chain environment with discrete demand. They offer an analytical model to analyze the benefits of consignment stock and to obtain the optimal replenishment decisions both for the supplier and the retailers. They show that according to the structure of the chain, the joint management of the inventory gives modest or relevant rise to economic benefits. In our study, the retailer is invoiced just after the replenishment. Therefore, our supply chain environment does not represent a consignment system.

We study a periodic review inventory control problem in a supply chain with a single retailer and a single supplier under stochastic demand. We propose a VMI agreement and compare the VMI setting with the traditional case to quantify the benefits of VMI. In our proposed VMI agreement, the retailer shares the inventory level information with the supplier, which is not available in traditional system, and the supplier is responsible to keep the retailer's inventory level between the minimum and maximum levels set by the contract. We analyze the benefits of such an agreement for each member and also for the supply chain. Furthermore, we aim to measure the performance of the VMI agreement in coordinating the overall chain by comparing it with the centralized system.

Our work is one of the few studies that consider a specific type of VMI agreement and differs from the existing studies in the following aspects. (i) We analyze the benefits VMI from the perspective of each member and the supply chain. (ii) We utilize fixed cost sharing as the motivation for the retailer to join in a VMI partnership. (iii) Apart from the benefits over the traditional system, we analyze the performance of the vendor managed system in coordinating the supply chain. (iv) Finally, we analyze how the system parameters affect the benefits of VMI system.

CHAPTER 3

MODEL

We consider a periodic review inventory control problem in a supply chain which consists of a single retailer (he) and a single capacitated supplier (she). Our aim is to model a particular type of Vendor Managed Inventory (VMI) agreement between the supplier and the retailer, and to investigate the benefits provided by such an agreement.

First, a setting is constructed for the traditional way of managing inventory in the supply chain. In the traditional retailer-managed inventory case (will be named as "traditional setting"), the decision of order quantity and time is made by the retailer. The retailer faces identically and independently distributed positive integer demands ("the customer demand") in any period which is and places orders with the supplier according to the (s,S) policy (Figure 3.1). That is, the retailer incurs a fixed ordering cost at each time he places an order to the supplier. The supplier, on the other side, should decide on the production quantity at each period. If an order is placed by the retailer, the supplier should fully satisfy it without any lead time. If her on-hand inventory is not sufficient, the supplier satisfies the retailer's demand by outsourcing. The supplier has the knowledge of the customer demand distribution that the retailer faces and also she is aware of the inventory policy used by the retailer. These are the only information available to the supplier in traditional case.

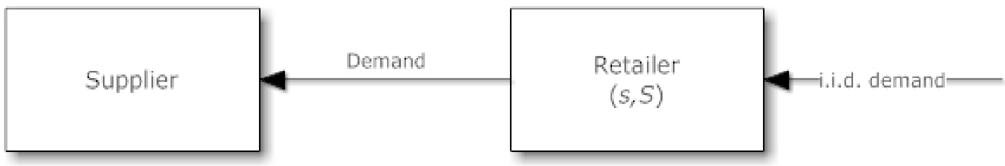

Figure 3. 1: Traditional Setting

The system is then modeled as a specific type of VMI agreement. In this case, the supplier receives periodic information about the customer demands. That is, in addition to the knowledge of the customer demand distribution and the inventory policy being followed by the retailer, the supplier also knows the inventory level of the retailer in VMI setting (Figure 3.2). According to the applied VMI agreement, the replenishment decision is made by the supplier, and the retailer's inventory level is kept between a specified minimum and a maximum value, *z* and *Z* values respectively, which are set in the contract.

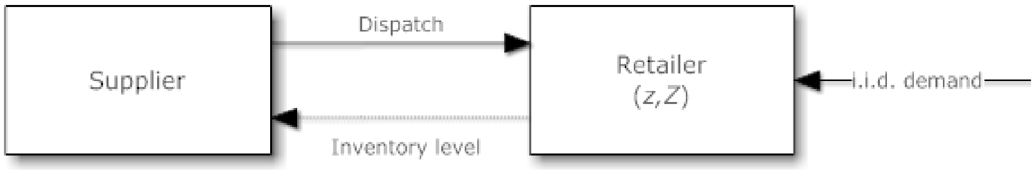

Figure 3. 2: VMI Setting

Finally, a centralized setting is defined (Figure 3.3), where the production and the replenishment related decisions are made by a central decision maker. In the centralized setting, the aim of the central decision maker is to maximize the total supply chain profit by coordinating the decisions of production and replenishment. The performance of the VMI is determined by measuring how the supply chain profit under VMI is close to the chain profit in the centralized setting.

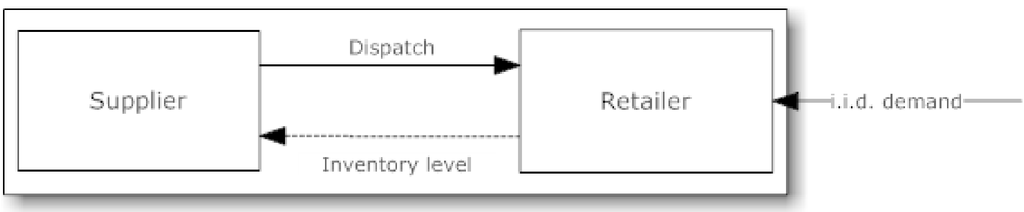

Figure 3. 3: Centralized Setting

Table 3.1 summarizes the properties of the traditional, VMI and the centralized settings.

Table 3. 1: Problem Environment For The Traditional, VMI and Centralized Settings

	Traditional	**VMI**	**Centralized**
Customer demand distribution	*Available to the supplier*	*Available to the supplier*	*Available to the supplier*
Inventory level of the retailer	*Not available to the supplier*	*Available to the supplier*	*Available to the supplier*
Inventory policy of the retailer	*(s,S)*	*(z,Z)*	-
Replenishment / Order decision is made by	*Retailer*	*Supplier*	*Supplier (Central Decision Maker)*
Fixed Cost of Ordering (K_r)	*Paid by the retailer*	*Shared*	*Indifferent*
Lead time of transshipment	*Zero*	*Zero*	*Zero*
Excess customer demand	*Lost*	*Lost*	*Lost*

In all three settings, the common relevant costs for the supplier are per-unit holding cost, the unit cost of outsourcing, production variable and fixed costs. For the retailer the cost components are composed of per-unit holding cost, unit purchasing cost and a fixed ordering cost. Different from the traditional setting, the fixed ordering cost of the retailer is shared between the supplier and the retailer in the VMI setting, while it is fully incurred by the retailer in the traditional case. The fixed ordering cost of the retailer can be considered as the transportation cost and although the retailer does not place any order in the VMI setting, the fixed cost is incurred at each occurrence of replenishment. The details of the traditional, VMI and the centralized environment of the supply chain and the profit maximization models are described in sections 3.1, 3.2 and 3.3.

3.1 TRADITIONAL SETTING

In traditional way of managing inventory, there is no agreement or contract between the supplier and the retailer. That is, there is no shared information about the inventory level of the retailer. The retailer manages his own inventory, decides on the replenishment time and quantity by himself. On the other hand, the supplier should decide on the optimal production quantity in order to satisfy the retailer's demand while maximizing her own profit. The sequence of events in traditional setting is as follows: (1) First, the supplier decides how much to produce for the period. (2) Customer demand occurs at the retailer and he satisfies the demand either fully or partially (unsatisfied demand is lost). After satisfying the demand, he reviews his inventory level and if required, places an order with the supplier. (3) The supplier ships the demanded quantity to the retailer with a negligible lead time at the end of the period. If her on-hand inventory is not sufficient to fully satisfy the retailer's demand, then she does outsourcing. (4) Costs are incurred for the period.

The problem environments both for the retailer and the supplier in traditional setting are evaluated separately and discussed in detail in sections 3.1.1 and 3.1.2. Although the notation is described within the text when necessary, a full list of the notation is provided in Appendix A.

3.1.1 The Model For The Retailer

In traditional setting the retailer manages his own inventory and focuses on maximizing his profit by optimizing the ordering decisions.

The retailer faces stochastic demand and periodically places order to the supplier. At the beginning of each period, the retailer checks his inventory level (I_r) and decides on the order quantity (q). When he places an order, the supplier makes the replenishment with a negligible lead time, and the retailer gets the exact quantity of order each time. That is, the supplier fully satisfies the retailer's demand and backorders are not allowed. After the replenishment, the retailer faces an integer demand (customer demand). If the customer demand exceeds the retailer's on-hand inventory, then it is assumed that the excess demand is lost. Since the ordered quantity is fully dispatched to the retailer within the same period and the excess demand of the customer is not backlogged, the inventory level of the retailer at the beginning of the next period (I_r') is independent of the previous order decisions and previous customer demands. The system represents a Markov chain with these properties and the profit maximization problem of the retailer is modeled as a Markov Decision Process (MDP).

The inventory level of the retailer at the beginning of each period, I_r, represents the state of the system. At each state, the retailer decides on the order quantity, q, and places order to the supplier, which represents the action. So, the state space E and the action set A is composed of I_r and q values respectively, where $I_r \geq 0$ and $q \geq 0$.

If the system is in state I_r and the retailer places q units of order, then the retailer's inventory level after the replenishment increases to $I_r + q$ units. When D units of customer demand occurs, then the retailer starts to the next period with I_r' units of item with a probability of $P(I_r' \mid I_r,q)$. The new state of the system, I_r', will be equal to $max\ (I_r + q - D, 0)$. The transition probability, $P(I_r' \mid I_r,q)$, is expressed as;

$$P(I_r' \mid I_r,q) = \begin{cases} P(I_r + q - I_r') & \text{when } I_r' > 0; \\ \sum_{D \geq I_r+q} P(D) & \text{when } I_r' = 0; \end{cases}$$

where $P(D)$ is the probability that the customer demand is equal to D units ($D>0$).

The cost components for the retailer at each period are composed of per-unit holding cost, unit purchasing cost and a fixed ordering cost. In order to express the cost and revenue functions, the following notations are used:

$PC_{(I_r,q)}$: *purchasing cost* (*includes fixed and variable cost*) of the retailer when the system is in state $I_r \in E$ and decision $q \in A$ is made.

$HC_{(I_r,q)}$: expected *inventory holding cost* of the retailer if the system is in state $I_r \in E$ and decision $q \in A$ is made.

$R_{(I_r,q)}$: expected *revenue* of the retailer if the system is in state $I_r \in E$ and decision $q \in A$ is made.

$r_{(I_r,q)}$: expected *profit* of the retailer if system is in state $I_r \in E$ and decision $q \in A$ is made

p_r : *unit selling price* of the retailer (retail price)

w : *unit selling price* of the supplier (wholesale price)

h_r : *unit holding cost/period* for the retailer

K_r : *fixed ordering cost* of the retailer

The retailer incurs a fixed ordering cost at each time he places an order to the supplier. So, the purchasing cost of the retailer is defined as:

$$PC_{(Ir,q)} = \begin{cases} w.\,q + K_r & \text{for} \quad q > 0 \\ 0 & \text{for} \quad q = 0 \end{cases} \tag{1}$$

After satisfying the customer demand, the retailer incurs holding cost for unsold items. That is, if the retailer faces D units of demand, he will incur holding cost for $I_r + q - D$ units of item if $I_r + q \geq D$. Expected holding cost of the retailer is provided in Eq. (2)

$$HC_{(I_r,q)} = \sum_D h_r \,.\, max(I_r + q - D, 0) \,.\, P(D) \qquad \forall\, I_r \in E \text{ and } q \in A \tag{2}$$

If the customers demand, D, in a period exceeds the retailer's on-hand inventory, $I_r + q$, then the excess demand is lost. The retailer's expected revenue function is then expressed as:

$$R_{(I_r,q)} = \sum_D p_r \, . \, min(I_r + q, D) \, . \, P(D) \qquad \forall \, I_r \in E \text{ and } q \in A \qquad (3)$$

If the system is in state I_r and q units of items are ordered, the expected profit of the retailer for the period is simply the difference between the revenue and the total cost, that is:

$$r_{(I_r,q)} = R_{(I_r,q)} - (PC_{(I_r,q)} + HC_{(I_r,q)}) \qquad (4)$$

Let $X_{(I_r,q)}$ be the steady- state probability that the system is in state I_r and decision q is made, then the LP formulation that maximizes the expected long-run profit of the retailer is written as;

$$\textit{Maximize} \qquad \sum_{I_r \epsilon E} \sum_{q \epsilon A} r_{(I_r,q)} . X_{(I_r,q)} \qquad (5)$$

s. t.

$$\sum_{q \epsilon A} X_{(I_r',q)} - \sum_{I_r \epsilon E} \sum_{q \epsilon A} X_{(I_r,q)} . P(I_r' \mid I_r, q) = 0 \qquad \forall \, I_r' \in E \qquad (6)$$

$$\sum_{I_r \epsilon E} \sum_{q \epsilon A} X_{(I_r,q)} = 1 \qquad (7)$$

$$X_{(I_r,q)} \geq 0 \qquad \forall \, I_r \in E \text{ and } q \in A \qquad (8)$$

The objective function given in Eq. (5) maximizes the retailer's long-run expected profit. Constraint (6) preserves the flow balance. Constraints (7) and (8) ensure that sum of the steady-state probabilities is exactly 1 and the steady state probabilities are non-negative, respectively.

In traditional setting, the optimal ordering policy of the retailer is obtained by the given MDP model. According to the modeled environment in traditional setting, the retailer uses (s,S) policy in managing his inventory. That is, if the level of inventory drops below the reorder point s, then the retailer places an order to bring his inventory level to S.

3.1.2 The Model For The Supplier

In traditional way of managing inventory, there is no agreement or contract between the supplier and the retailer. That is, there is no shared information about the inventory level of the retailer. The retailer manages his own inventory, decides on the replenishment time and quantity by himself. The supplier, on the other hand, should decide on the optimal production quantity in order to satisfy the retailer’s demand while maximizing her own profit. The sequence of events in traditional setting is as follows: (1) First, the supplier decides how much to produce for the period. (2) Customer demand occurs at the retailer and he satisfies the demand either fully or partially (unsatisfied demand is lost). After satisfying the demand, if his ending inventory level is below s, he places an order with the supplier to bring his inventory level to S. (3) The supplier ships the demanded quantity to the retailer with a negligible lead time at the end of the period. If her on-hand inventory is not sufficient, the supplier satisfies the retailer’s demand by outsourcing. (4) Costs are incurred for the period.

Although the retailer does not share the inventory level information with the supplier in traditional case, it is assumed that the supplier knows the distribution function of the customer demand, the fact that the retailer places orders according to an (s,S) policy, and the specific parameters s and S.

Having the knowledge of the customer demand distribution and the retailer’s (s,S) values enable the supplier to infer the distribution function for orders placed by the retailer, conditioning on the elapsed time since the last order of the retailer (or the last replenishment). At the beginning of each period, the supplier knows the number of periods, t, that have elapsed since the last order was placed. We assume that the customer demand is at least one unit per period, therefore, the states are $t = 1, 2, \ldots, \Delta$ (where $\Delta = S\text{-}s$). Future demands at the supplier are independent of past demands. Because the retailer obtains all of the ordered quantity in a period, he brings his inventory level to S (after ordering) independent of the supplier's inventory level. Since the supplier knows that the retailer follows an (s,S) policy and since she knows the value of $\Delta = S\text{-}s$, she can determine the probability p_t that an order will be generated at the end of the period and the cumulative distribution function, $\Phi_t(x)$, of the order size x, given that an order is placed. Let $P(D)$ be the probability that the customer demand is equal to D units. As in Gavirneni (1999), p_t and $\Phi_t(x)$ are expressed as;

$$p_t \quad = P(d_1 + d_2 + \ldots + d_t \geq \Delta \,|\, d_1 + d_2 + \ldots + d_{t-1} < \Delta) \tag{9}$$

$$\Phi_t(x) = P(d_1 + d_2 + \ldots + d_t \leq x \,|\, d_1 + d_2 + \ldots + d_{t-1} < \Delta \textit{ and } d_1 + d_2 + \ldots + d_t \geq \Delta) \tag{10}$$

where d_i represents the customer demand in period i.

Let $P(t,Q)$ be the probability that the retailer's order size is Q when t periods elapsed since the last order. The following algorithm is used to generate the probability, $P(t,Q)$.

The algorithm for generating P(t,Q)

Let $P(D)$ be the probability that the customer demand is equal to D units ($D > 0$),

J denotes the set of all possible discrete demand values D,

and d_i represents the customer demand at period i.

Step 0: Set $t = 1$;

Step 1: Generate $d_t \in$ J;

Step 2: If $\sum_{i=1}^{t} d_i < \Delta$; then set $t = t + 1$; go to Step 1.

If $\sum_{i=1}^{t} d_i \geq \Delta$; then set $P'(t,Q) = P'(t,Q) + \prod_{i=1}^{t} P(d_i)$

where $Q = \min \{\sum_{i=1}^{t} d_i, S\}$; go to Step 0.

Once all possible combinations of d_i series providing $d_1 + d_2 + \ldots + d_{t'} \geq \Delta$ are evaluated, the probability of the retailer's order size, conditioning on the elapsed time since the last order, is obtained as;

$$P(t,Q) = \frac{P'(t,Q)}{\sum_{t' \geq t} \sum_{Q} P'(t',Q)}$$

(A numerical example of the algorithm is provided in Appendix B)

At the beginning of each period, the supplier checks her inventory level (I_s), time elapsed since the last order of the retailer (t), and decides on the production quantity for that period. The supplier's inventory level at the beginning of the next period only depends on her current inventory level, the quantity of production and the retailer's demand at the current period. The profit maximization problem of the supplier is modeled as a Markov Decision Process (MDP). The state of the system is characterized by two parameters, inventory level of the supplier, I_s, and time elapsed since the last replenishment, t, and represented by (I_s,t). We denote the state space by E which consists of (I_s,t) pairs where $I_s \geq 0$ and $1 \leq t \leq \Delta$.

The supplier should decide on the quantity of production at each state. So, action set A consists of the production quantity a ($a \in A$). Production quantity is limited with the capacity of the supplier (that is, $a \leq C$). So action set A can be expressed as $A = \{0, 1, 2, 3, \ldots, C\}$. When the supplier decides to produce a units at state (I_s,t), the system will go to a new state (I_s',t') with the probability $P(I_s',t' \mid I_s,t,a)$.

If the retailer's demand is zero, then t' will be $t+1$ in the new state and the inventory level of the supplier I_s will increase by the produced quantity ($I_s' = I_s + a$). However, if the demand is greater than zero, then t' will be equal to 1 in the new state. Supplier should fully satisfy the retailer's demand. While doing this, supplier will use the on-hand inventory (I_s+a) first. If it is not enough to satisfy the all demand, then she will provide the remaining by outsourcing. So, the inventory level of the supplier at the beginning of the next period, I_s', will be zero if the retailer's demand is greater than or equal to the on-hand inventory of the supplier. Otherwise, I_s' will be greater than zero and will be equal to $I_s + a - Q$. As a result, the state transition probability, $P(I_s',t' \mid I_s,t,a)$, can be expressed as;

$$P(I_s',t' \mid I_s,t,a) \begin{cases} P(t,0) & \text{when } I_s' = I_s + a \text{ and } t' = t + 1; \\ P(t,Q) & \text{when } I_s' = I_s + a - Q > 0 \text{ and } t' = 1; \\ \sum_{Q \geq Is+a} P(t,Q) & \text{when } I_s' = 0 \text{ and } I_s + a > 0 \text{ and } t' = 1; \\ 0 & \text{otherwise.} \end{cases}$$

The model for traditional setting that maximizes the supplier's expected long-run profit is written as;

$$\textit{Maximize} \qquad \sum_{(I_s,t)\epsilon E} \sum_{a \epsilon A} r_{(I_s,t)a}\, X_{(I_s,t)a} \tag{11}$$

s. t.

$$\sum_{a\epsilon A} X_{(I_s{'},t{'})a} - \sum_{(I_s,t)\epsilon E} \sum_{a\epsilon A} X_{(I_s,t)a} P(I_s{'},t' \mid I_s,t,a) = 0 \qquad \forall\ (I_s',t') \in E \tag{12}$$

$$\sum_{(I_s,t)\epsilon E} \sum_{a\epsilon A} X_{(I_s,t)a} = 1 \tag{13}$$

$$X_{(I_s,t)a} \geq 0 \qquad \forall\ (I_s,t) \in E \text{ and } a \in A \tag{14}$$

where

$X_{(I_s,t)a}$ is the steady state probability that the system is in state (I_s,t) and decision a is made, and $r_{(I_s,t)a}$ is the expected profit gained if the system is in state (I_s,t) and decision a is made.

The objective function given in (11) maximizes the long run expected average profit of the supplier. Constraint (12) preserves the flow balance. Constraint (13) ensures that sum of the steady-state probabilities is exactly 1 and Constraint (14) ensures that steady state probabilities should be non-negative.

Expected profit of the supplier in any state (I_s,t) for decision a, $r_{(I_s,t)a}$, is simply the difference between the expected revenue and the expected total cost. Total cost of the supplier includes outsourcing cost, inventory holding cost and production cost. In order to define the revenue and cost equations we use the following additional notations.

$OC_{(I_s,t)a}$: expected *outsourcing cost* incurred if the system is in state (I_s,t) and decision a is made.

$HC_{(I_s,t)a}$: expected *inventory holding cost* incurred if the system is in state (I_s,t) and decision a is made.

$PC_{(I_s,t)a}$: *production cost* incurred if the system is in state (I_s,t) and decision a is made.

$R_{(I_s,t)a}$: expected revenue of the supplier if the system is in state (I_s,t) and decision a is made.

w : unit selling price of the supplier (wholesale price)

c_s : unit production cost of the supplier

h_s : unit holding cost/period for the supplier

o_s : unit outsourcing cost

K_s : setup cost for the supplier (production fixed cost)

If the system is in state (I_s,t) and decision a is made, then total inventory of the supplier for that period will be equal to $I_s + a$. If the retailer's demand (Q) exceeds the supplier's total inventory, i.e., if $Q > I_s + a$, then the supplier will outsource $Q - (I_s + a)$ units of item. On the other hand, if the demand is less than the supplier's total inventory ($Q < I_s + a$), then the supplier will incur holding cost for $(I_s + a) - Q$ units of item. So, for a given state (I_s,t) and decision a, expected outsourcing and holding costs are given in Eq. (15) and (16) respectively;

$$OC_{(Is,t)a} = \sum_D o_s \,.\, \max(Q - I_s - a, 0) \,.\, P(t, Q) \tag{15}$$

$$HC_{(Is,t)a} = \sum_D h_s \,.\, \max(I_s + a - Q, 0) \,.\, P(t, Q) \tag{16}$$

Production cost of the supplier is defined as a function of the produced quantity, a. At any state, if the supplier decides to produce a units ($a > 0$), then she will charge a fixed setup cost, K_s, and unit production cost c_s for each produced unit of item. That is;

$$PC_{(Is,t)a} = \begin{cases} c_s \,.\, a + K_s & \text{for} \quad a > 0 \\ 0 & \text{for} \quad a = 0 \end{cases} \tag{17}$$

The revenue of the supplier is independent from the state of the system and the decision of production quantity, because she should satisfy the retailer's demand fully in any case and she gets w dollars per unit of item dispatched to the retailer. When the retailer demands Q units of item, then the revenue of the supplier will be equal to (w . Q). Then the expected revenue of the supplier is as in Eq. (8).

$$R_{(Is,t)a} = \sum_Q w \,.\, Q \,.\, P(t, Q) \tag{18}$$

Using the revenue function defined in Eq. (8) and the cost components defined in Eq. (5), (6) and (7), we obtain the expected profit of the supplier. When the system is in state (I_s,t) and decision a is made, the expected profit, $r_{(Is,t)a}$, is the difference between the expected revenue and total cost.

$$r_{(Is,t)a} = R_{(Is,t)a} - (OC_{(Is,t)a} + HC_{(Is,t)a} + PC_{(Is,t)a}) \qquad (19)$$

3.2 VMI SETTING

In traditional setting, it is assumed that the end-item demand probabilities and the retailer's inventory policy with its parameter values ((s,S) policy with s and S values) are the only information known by the supplier. In VMI setting, the inventory level of the retailer is also available to the supplier and the management of the retailer's inventory is left to the supplier. According to the VMI agreement, the supplier follows a (z,Z) type policy in managing the retailer's inventory. That is, the retailer sets a minimum inventory level, z, and a maximum inventory level, Z, which represents the lowest and highest inventory levels, respectively. The supplier decides not only to the quantity of production but also the quantity of shipment to the retailer. She should keep the inventory level of the retailer between the specified (z,Z) values. The sequence of events relating to the replenishment decision under VMI is as follows: (0) (z,Z) levels are specified by a contract. (1) The supplier examines the retailer's inventory level and decides how much to produce and how much to ship to the retailer. If the retailer's inventory level is below z, then the supplier should make a replenishment to carry the inventory level between z and Z. (2) The material is shipped to the retailer with negligible lead time. (3) Demand occurs at the retailer. (4) Holding costs are incurred.

In traditional case, the fixed order cost of the retailer (can be named as transportation cost) was charged to the retailer. In VMI setting, however, this cost will be shared between the participants in different ratios.

Since the supplier knows the retailer's inventory level in VMI model, the state definition is changed and this time it represents the entire system. At the beginning of each period, the supplier checks her inventory level, I_s, and also controls the inventory level of the retailer I_r, in order to decide on the production quantity *a*, and the quantity of shipment *b*, for the period. The state of the system is now characterized by the inventory levels of the supplier and the retailer at the beginning of the period and represented by (I_s,I_r). The state space E consists of all possible (I_s,I_r) pairs where $I_s \geq 0$ and $0 \leq I_r \leq Z$.

The supplier decides not only the quantity of production but also how much to ship to the retailer. That is, in VMI model, actions (decisions) are represented by two parameters, *a* and *b*, where *a* denotes the quantity of production as in the traditional model, and *b* denotes the quantity of shipment. So, the action set A consists of *a* and *b* pairs, $(a,b) \in A$, where $a \leq C$ and $z \leq I_r + b \leq Z$.

Suppose that the system is in state (I_s,I_r) and the supplier decides to produce *a* units and dispatch *b* units of item. After the replenishment the retailer's inventory level will be $I_r + b$. If D units of demand occurs at the retailer (end-item demand is equal to D), then the inventory level of the retailer at the beginning of the next period will be, $I_r' = max\,(I_r + b - D, 0)$. The supplier's inventory level after replenishment will be, $I_s' = max\,(I_s + a - b, 0)$. If the supplier decides to produce *a* and dispatch *b* units of item in state (I_s,I_r), then the system will go to a new state (I_s',I_r') with the probability $P(I_s',I_r' \mid I_s,I_r,a,b)$. This transition probability is expressed as;

$$P(I_s',I_r' \mid I_s,I_r,a,b) = \begin{cases} P(I_r + b - I_r') & \text{when } I_s' = max\,(I_s + a - b, 0) \text{ and } I_r' > 0; \\ \sum_{D \geq Ir+b} P(D) & \text{when } I_s' = max\,(I_s + a - b, 0) \text{ and } I_r' = 0; \\ 0 & \text{otherwise} \end{cases}$$

where $P(D)$ is the probability that the customer demand is equal to D units ($D>0$), as it was defined in traditional model.

Let $X_{(Is,Ir)(a,b)}$ be the steady-state probability that the system is in state (I_s,I_r) and decision (a,b) is made. Then, the LP formulation for VMI setting that maximizes the supplier's expected long-run profit is written as;

$$\textit{Maximize} \qquad \sum_{(I_s,I_r)\epsilon E} \sum_{(a,b)\epsilon A} r_{(I_s,I_r)(a,b)} . X_{(I_s,I_r)(a,b)} \tag{20}$$

s. t.

$$\sum_{(a,b)\epsilon A} X_{(I_s{'},I_r{'})(a,b)} - \sum_{(I_s,I_r)\epsilon E} \sum_{(a,b)\epsilon A} X_{(I_s,I_r)(a,b)} . P(I_s{'}, I_r{'} \mid I_s, I_r, a, b) = 0$$

$$\forall\ (I_s{'},I_r{'}) \in E \tag{21}$$

$$\sum_{(I_s,I_r)\epsilon E} \sum_{(a,b)\epsilon A} X_{(I_s,I_r)(a,b)} = 1 \tag{22}$$

$$X_{(I_s,I_r)(a,b)} \geq 0 \qquad \forall (I_s,I_r) \in E \text{ and } (a,b) \in A \tag{23}$$

where $r_{(I_s,I_r)(a,b)}$denotes the expected profit gained if system is in state (I_s,I_r) and decision (a,b) is made.

The objective function given in Eq. (20) maximizes the long run expected average profit of the supplier for a given (z,Z) band. Constraint (21) preserves the flow balance. Constraints (22) and (23) ensure that sum of the steady-state probabilities is exactly 1 and the steady state probabilities are non-negative, respectively.

In order to generate the equation for the profit of the supplier when the system is in state (I_s,I_r) and decision (a,b) is made ($r_{(Is,Ir)(a,b)}$), we should first specify the revenue and cost equations. In VMI case, there is an additional cost component for the supplier when compared with the traditional model, which is dispatching cost (or transportation cost).

Additional notation for VMI setting is provided below:

$DC_{(I_s,I_r)(a,b)}$: *dispatching cost (transportation cost)* incurred if the system is in state (I_s,I_r) and decision (a,b) is made.

$OC_{(I_s,I_r)(a,b)}$: expected *outsourcing cost* incurred if the system is in state (I_s,I_r) and decision (a,b) is made.

$HC_{(I_s,I_r)(a,b)}$: expected *inventory holding cost* incurred if the system is in state (I_s,I_r) and decision (a,b) is made.

$PC_{(I_s,I_r)(a,b)}$: *production cost* incurred if the system is in state (I_s,I_r) and decision (a,b) is made.

$R_{(I_s,I_r)(a,b)}$: expected revenue of the supplier if the system is in state (I_s,I_r) and decision (a,b) is made.

When the supplier decides to produce a units of item in state (I_s,I_r), then her inventory level will be $I_s + a$ just before the shipment. If the supplier dispatches b units of item to the retailer, then she will incur holding cost for $I_s + a - b$ units of item if $b < I_s + a$. However, if the dispatching quantity b is greater than $I_s + a$, then outsourcing cost will be incurred for $b - (I_s + a)$ units of item. So, holding and outsourcing cost equations will be as follows:

$$OC_{(I_s,I_r)(a,b)} = o_s \,.\, max\,(b - I_s - a,\, 0) \tag{24}$$

$$HC_{(I_s,I_r)(a,b)} = h_s \,.\, max\,(I_s + a - b,\, 0) \tag{25}$$

Each time the supplier makes a replenishment, a specified fraction (denoted as α_s) of the retailer's fixed order cost will be charged to the supplier as dispatching cost.

$$DC_{(I_s,I_r)(a,b)} = \begin{cases} \alpha_s \,.\, K_r & \text{for} \quad b > 0 \\ 0 & \text{for} \quad b = 0 \end{cases} \tag{26}$$

Production cost expression in VMI model is the same with the expression in traditional setting. At any state, if the supplier decides to produce a units ($a > 0$), then she will charge a fixed setup cost, K_s, and unit production cost c_s for each produced unit of item. That is;

$$PC_{(Is,Ir)(a,b)} = \begin{cases} c_s \,.\, a + K_s & \text{for} \quad a > 0 \\ 0 & \text{for} \quad a = 0 \end{cases} \tag{27}$$

If the supplier dispatches b units of item in any state, then the revenue of the supplier is simply the multiplication of wholesale price, w, by dispatch quantity b. That is,

$$R_{(I_s,I_r)(a,b)} = w \,.\, b \tag{28}$$

Profit function of the supplier can now be expressed by using the revenue function in Eq. (28) and cost components described in Eqns. (24), (25), (26) and (27).

$$r_{(Is,Ir)(a,b)} = R_{(I_s,I_r)(a,b)} - \left(OC_{(I_s,I_r)(a,b)} + HC_{(I_s,I_r)(a,b)} + PC_{(I_s,I_r)(a,b)} + DC_{(I_s,I_r)(a,b)} \right) \tag{29}$$

In VMI setting, the expected profit of the retailer is also calculated within the model. The notation that is related with the cost, revenue and profit of the retailer in

VMI setting is given below. In order to specify 'the retailer', superscript 'R' is used in representation.

$DC^R{}_{(I_s,I_r)(a,b)}$: purchasing cost *(includes fixed and variable cost)* of the retailer when the system is in state (I_s,I_r) and decision (a,b) is made.

$HC^R{}_{(I_s,I_r)(a,b)}$: expected *inventory holding cost* of the retailer if the system is in state (I_s,I_r) and decision (a,b) is made.

$R^R{}_{(I_s,I_r)(a,b)}$: expected *revenue* of the retailer if the system is in state (I_s,I_r) and decision (a,b) is made.

$r^R{}_{(I_s,I_r)(a,b)}$: expected *profit* of the retailer if system is in state (I_s,I_r) and decision (a,b) is made.

p_r : unit selling price of the retailer (end-item price)

h_r : unit holding cost/period for the retailer

When the system is in state (I_s,I_r) and the supplier decides to dispatch b units of item to the supplier, the retailer's inventory level will be $I_r + b$ before the customer demand occurs. If the retailer's inventory level after replenishment ($I_r + b$) is enough to fully satisfy the demand, i.e., if $I_r + b \geq D$, then the retailer's revenue will be ($p_r \cdot D$) and he will incur holding cost for the remaining $I_r + b - D$ units of inventory, after satisfying the demand. On the other hand, if the demand is greater than on-hand inventory, excess demand will be lost as in the traditional case. The holding cost and the revenue functions are provided in Eqns. (30) and (31), respectively.

$$HC^R{}_{(I_s,I_r)(a,b)} = \sum_j h_r \,.\, max(I_r + b - j, 0) \,.\, P(j) \qquad \forall\, (I_s,I_r) \in E\,, (a,b) \in A \quad (30)$$

$$R^R{}_{(I_s,I_r)(a,b)} \quad = \sum_j \; p_r \;.\; min(I_r + b, j) \;.\; P(j) \qquad \forall \, (I_s,I_r) \in E, (a,b) \in A \quad (31)$$

One other cost component for the retailer is purchasing cost, which contains ordering fixed cost and variable cost. Each time the supplier makes a replenishment, the retailer will pay w for each unit. Ordering fixed cost (transportation cost) is shared between the supplier and the retailer in the VMI setting. So, the retailer will pay the specified fraction (will be denoted as α_r) of fixed cost. (α_r = 1 - α_s). Purchasing cost function of the retailer is given in Eq. (32).

$$DC^R{}_{(I_s,I_r)(a,b)} = \begin{cases} (1\text{-}\alpha_s) \,.\, K_r + w \,.\, b & \text{for} \quad b > 0 \\ 0 & \text{for} \quad b = 0 \end{cases} \qquad (32)$$

Expected profit of the retailer in state (I_s,I_r) when decision (a,b) is made, can now be expressed as;

$$r^R{}_{(I_s,I_r)(a,b)} \quad = \; R^R{}_{(I_s,I_r)(a,b)} - (HC^R{}_{(I_s,I_r)(a,b)} + DC^R{}_{(I_s,I_r)(a,b)}) \qquad (33)$$

We defined $X_{(I_s,I_r)(a,b)}$ as the steady-state probability that the system is in state (I_s,I_r) and decision (a,b) is made. So, the expected total long-run profit of the retailer in VMI setting is obtained by the following summation:

$$\sum_{(I_s,I_r)\epsilon S} \sum_{(a,b)\epsilon A} (r^R{}_{(I_s,I_r)(a,b)} . X_{(I_s,I_r)(a,b)}). \qquad (34)$$

3.3 CENTRALIZED SETTING

In the centralized setting, the supplier and the retailer act as a single firm for a unique objective. The decisions, concerning production and replenishment, are assumed to be made with the focus of maximizing the total supply chain profit. The supplier, which has full information about the retailer and the chain, can be considered as the central decision maker. She aims to maximize the total chain profit by managing the inventory in the supply chain.

The sequence of events in a period relating to the replenishment decision is the same with the VMI setting and as follows: (1) The supplier, as a central decision maker, reviews her own and the retailer's inventory level, and then decides on the production and dispatch quantity. (2) The material is shipped to the retailer with negligible lead time. (3) Demand occurs at the retailer. (4) Costs are incurred.

The state of the system represents the entire supply chain in the centralized setting and defined as the inventory levels of the supplier and the retailer at the beginning of each period, (I_s,I_r). The definition of the state is the same with the VMI setting. However, in centralized case, the retailer's inventory level is not bounded by an agreement. So, the state space E consists of all (I_s,I_r) pairs where $I_s \geq 0$ and $I_r \geq 0$.

The decisions of production and replenishment quantities are made by the supplier after checking the inventory levels at the beginning of each period. The action set A is composed of the production and dispatch quantities, a and b respectively, where $0 \leq a \leq C$ and $b \geq 0$. After the action is taken, customer demand occurs at the retailer and the system goes to a new state, (I_s',I_r'), with the probability $P(I_s',I_r' \mid I_s,I_r,a,b)$, which is expressed in VMI setting.

The LP formulation for centralized setting that maximizes the expected long-run profit of the supply chain is written as;

$$\textit{Maximize} \qquad \sum_{(I_s,I_r)\epsilon E}\sum_{(a,b)\epsilon A} r^{SC}{}_{(I_s,I_r)(a,b)} . X_{(I_s,I_r)(a,b)} \tag{35}$$

s. t.

$$\sum_{(a,b)\epsilon A} X_{(I_s',I_r')(a,b)} - \sum_{(I_s,I_r)\epsilon E}\sum_{(a,b)\epsilon A} X_{(I_s,I_r)(a,b)} . P(I_s', I_r' \mid I_s, I_r, a, b) = 0$$

$$\forall\ (I_s',I_r') \in E \tag{36}$$

$$\sum_{(I_s,I_r)\epsilon E}\sum_{(a,b)\epsilon A} X_{(I_s,I_r)(a,b)} = 1 \tag{37}$$

$$X_{(I_s,I_r)(a,b)} \geq 0 \qquad \forall\ (I_s,I_r) \in E \text{ and } (a,b) \in A \tag{38}$$

where

$X_{(Is,Ir)(a,b)}$ is the steady-state probability that the system is in state (I_s,I_r) and decision (a,b) is made;

and $r^{SC}{}_{(I_s,I_r)(a,b)}$denotes the expected profit of the supply chain if the system is in state (I_s,I_r) and decision (a,b) is made.

All cost components of the retailer and the supplier in the centralized setting are the same with those defined and expressed in the VMI setting. The expected profit of the supply chain in state (I_s,I_r) when decision (a,b) is made, $r^{SC}{}_{(I_s,I_r)(a,b)}$, is the sum of the profit of each member. That is;

$$r^{SC}{}_{(I_s,I_r)(a,b)} = r_{(I_s,I_r)(a,b)} + r^{R}{}_{(I_s,I_r)(a,b)} \tag{39}$$

where $r_{(I_s,I_r)(a,b)}$ and $r^{R}{}_{(I_s,I_r)(a,b)}$ are the expected profits of the supplier and the retailer, respectively, and expressed in Eqns. (19) and (23).

CHAPTER 4

COMPUTATIONAL STUDY

In this section, a computational study is performed in order to investigate the benefits of VMI and to examine the performance of the agreement in coordinating the supply chain. The benefit of VMI on the profits of the retailer, the supplier and the total supply chain are evaluated separately and the effects of the system parameters to the benefit of VMI on profits are investigated. In addition to the benefits of the VMI over the traditional setting, the performance of VMI in coordinating the supply chain is examined.

Evaluation of VMI is performed according to changing values of four main system parameters. These are the retail price, wholesale price, the production capacity of the supplier and the coefficient of customer demand variation (mean customer demand is 5.5 with different variances). In addition to these parameters, three different order cost sharing scenarios are constructed. The parameters and their values are listed in Table 4.1.

Table 4. 1: Parameters and Their Values Used in Computational Study

Parameter	Values
Selling price (retail price) of the end-item– p_r	10 - 15 – 20
Selling price (wholesale price) of the supplier– w	5 – 8
Production capacity (units/period) of the supplier (C)	5 - 10 – 15 - 20
Coefficient of variation of customer demand (cv_d)	0.122 – 0.260 - 0.522 – 0.672 - 0.802
Order cost ratio of the supplier (α_s)	0.25 – 0.50 – 0.75

The other parameters (cost components) are kept fixed. These parameters and their fixed values are given in Table 4.2.

Table 4. 2: Cost Parameters and Fixed Values Used in Computational Study

Cost Component	Cost
Holding cost of the retailer (h_r)	1.2
Holding cost of the supplier (h_s)	0.5
Order cost of the retailer (K_r)	10
Production setup cost of the supplier (K_s)	20
Unit production cost (p)	1
Unit outsource cost (o_s)	11

4.1 Benefit of VMI on Supplier's Profit:

In analyzing the benefits of VMI on supplier's profit, we assume that the supplier is the powerful player in the chain, and the values of contract parameters, z and Z, are set by her. However, in order to encourage the retailer to participate in such a VMI agreement, she has to provide that the retailer must be better than the traditional setting in terms of profit with the decided values of (z,Z) levels. Among all possible levels that satisfy the retailer's criteria, the supplier selects the one that maximizes her profit.

In the computational study, for each parameter set, the profits of the retailer and the supplier are calculated under all (z,Z) levels ($0 \leq z \leq Z$) with a complete enumeration, and the (z,Z) level satisfying the mentioned criteria is obtained.

For each set of parameter, the profit of the supplier under VMI is compared to the profit in traditional setting, and the benefit of VMI over traditional setting is analyzed under different values of system parameters. For each parameter set, the profit value of the supplier under the traditional and the VMI setting is provided in Appendix C.

4.1.1 Effect of Demand Variation:

In the VMI setting, the replenishment decision is made by the supplier and this provides her flexibility in terms of the dispatch quantity. She can replenish the retailer with any quantity as soon as she guarantees the inventory level of the retailer after replenishment is kept between the specified (z,Z) levels. The specified values of these levels, z and Z, determine the flexibility of the supplier in VMI setting. As the difference between these levels (Z - z) increase, the flexibility increases.

In contrast to the traditional setting, the supplier does not face a demand by the retailer in VMI setting that should be fully satisfied. Because of the gained flexibility in dispatch quantity, the supplier does not use the expensive outsource option in VMI. In addition, since the supplier has full information about the retailer and flexible in replenishment quantity, she operates on just-in-time basis; that is, as soon as she make a production, she dispatches all produced items to the retailer and does not carry inventory. If the specified (z,Z) levels in the VMI agreement provide enough flexibility to the supplier to keep the retailer's inventory levels between the specified values without outsourcing and holding inventory; in the long-run, the expected revenue and the profit of the supplier are not affected by the coefficient of customer demand variation as long as the mean demand is the same.

Figure 4.1 monitors the effect of customer demand variation to the supplier's profit in traditional and VMI setting (C=15; p_r = 15; α_s = 0.25 for this example). It is seen that, the expected profit of the supplier in VMI setting does not change with the coefficient of variation. The (z,Z) levels, set by the supplier at each cv_d value in this example, are the optimal levels for her and these levels are also acceptable by the retailer in VMI setting. For increasing values of cv_d in this example, the (z,Z) levels, set by the supplier, are (7,21), (8,22), (10,24), (10,24) and (10,24), respectively. At each pair, the difference between the maximum and the minimum level (Z-z) is 14. So, when the retailer's inventory level drops below z, the supplier makes a production with full capacity (a = 15) and dispatches all produced items to the retailer. With this policy, the supplier does not incur any holding and outsourcing cost. In addition, she minimizes the unit production cost by allocating the fixed cost to the full capacity and reaches the optimal profit. Under such a situation, the expected profit of the supplier is not affected by different values of coefficient of variation, since the mean demand is the same at each value.

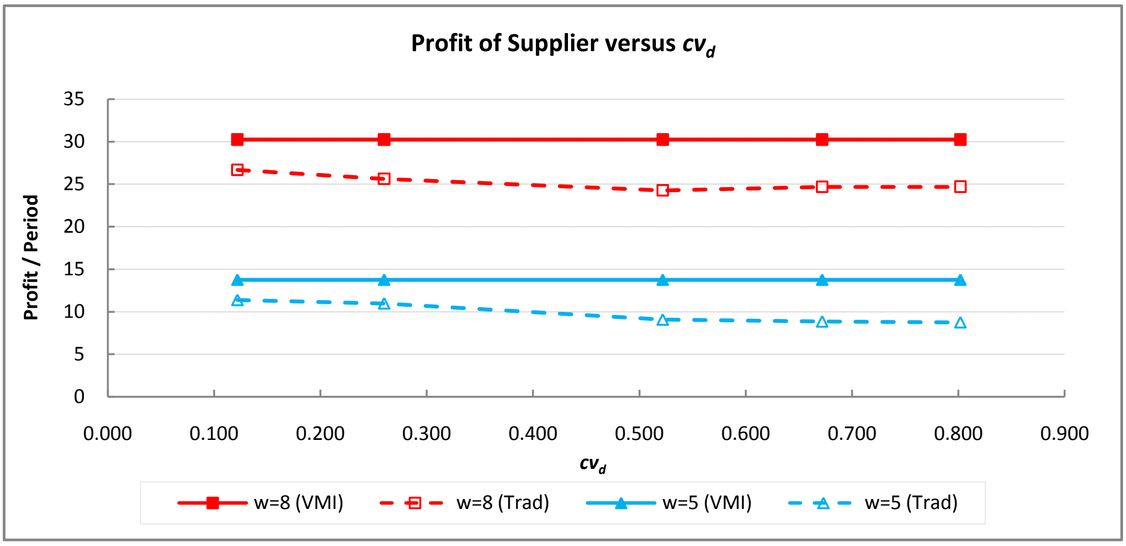

Figure 4. 1: Profit of The Supplier vs Coefficient of Variation (cv_d) Under Traditional and VMI Setting (C = 15; p_r = 15; α_s = 0.25)

In traditional case, however, the supplier periodically faces demand from the retailer, and she infers the distribution of this demand by using customer demand distribution and the (s,S) values as expressed in Eqns. (9) and (10). Changing values of coefficient of customer demand variation and (s,S) parameters yields different demand distributions, inferred by the supplier. Therefore, the expected profit of the supplier in traditional setting is affected by the coefficient of customer demand variation.

In Figure 4.1, when the wholesale price is low (w = 5), the expected profit of the supplier decreases with increasing values of the coefficient of customer demand variation in traditional setting. As the demand variation increases, the difference between the order-up-to level (S) and the reorder point (s) is decreased by the retailer to become more sensitive to the variations on demand, and the retailer keeps more safety stock. Decreased (S-s) provides the supplier to better infer the distribution of the retailer orders. However, as the difference (S-s) decreases, the retailer's order frequency increases in traditional setting. That is, he is replenished by the supplier more frequently with smaller quantities. Due to the decreased dispatch quantity, the supplier incurs higher inventory holding cost. Although the revenue of the supplier is also increased with the increasing values of the reorder point (s) at high variance case, increasing holding cost is the most important factor in decreasing profits. For the higher value of wholesale price (w=8), the profit is maximum when the coefficient of demand variation is low. With increasing values of cv_d, the profit initially decreases and reaches minimum at moderate values of cv_d, then increases slightly. For w=8, at higher values of cv_d, the increase in the revenue is slightly greater than the increase in holding cost, which results in increasing profit.

The effect of customer demand variation to the benefit of VMI is displayed in Figure 4.2. The benefit of VMI increases with increasing demand variation when the wholesale price is low. For a higher wholesale price, VMI is most beneficial at moderate values of variation and provide 25% increase on profit.

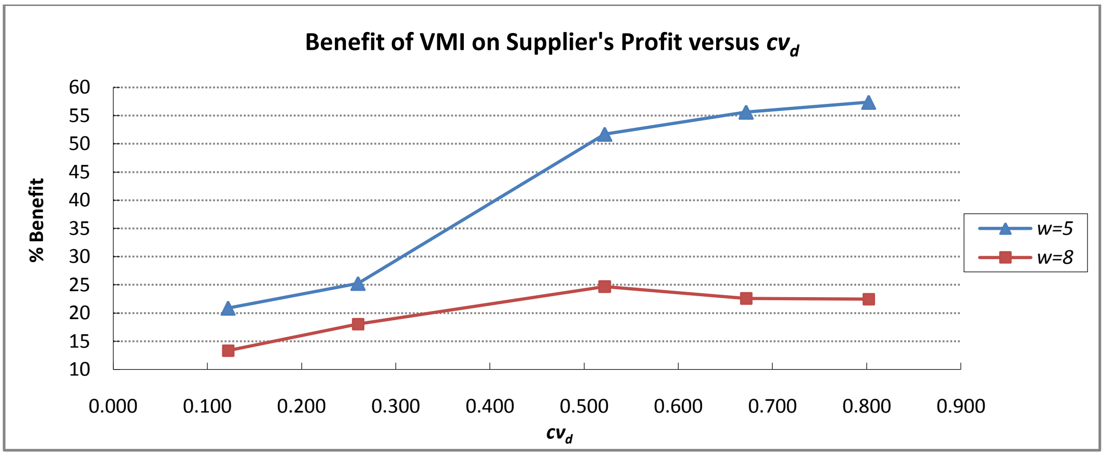

Figure 4. 2: Benefit of VMI on Supplier's Profit vs Coefficient of Variation (C = 15; p_r = 15; α_s = 0.25)

4.1.2 Effect of Capacity

Figure 4.3 displays the effect of the capacity to the benefit of VMI on the supplier's profit. For every value of capacity and each coefficient of variation of customer demand, since they show similar patterns, the percentage benefits are averaged over all the retail and wholesale prices in the figure.

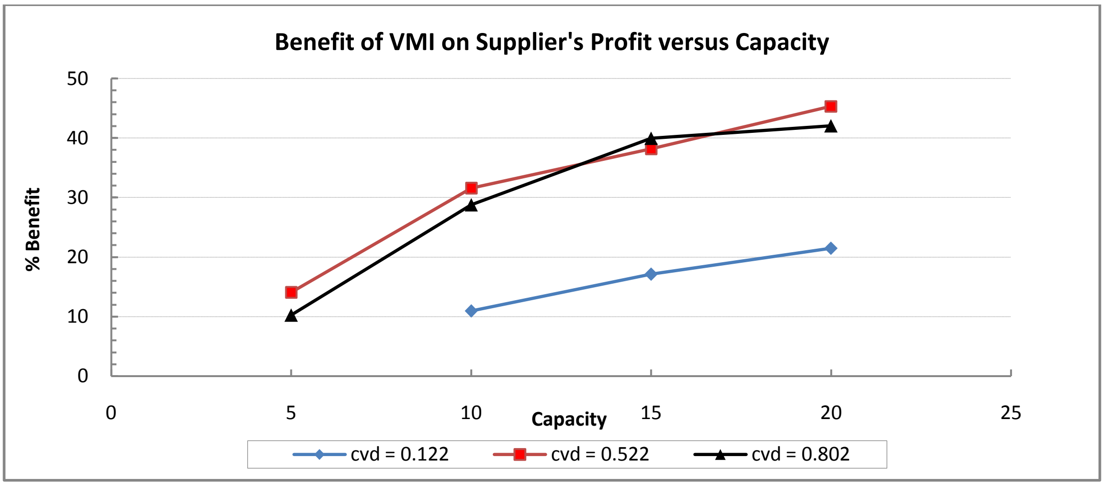

Figure 4. 3: Benefits of VMI on Supplier's Profit vs Capacity (α_s = 0.25)

It is seen in the figure that the benefit of VMI increases with capacity. In low coefficient of variation and capacity (i.e., cv_d = 0.122 and C = 5), a common (z,Z) level that makes the VMI beneficial both for the supplier and the retailer does not exist. For other values of variance, the benefit of VMI is minimal when the capacity is 5. The (z,Z) policy in the VMI setting provides flexibility to the supplier in dispatch quantity and this flexibility increases with the difference (Z-z). At each replenishment, the supplier tries to increase the retailer's inventory level as close as possible to the upper limit Z. This flexibility cannot be utilized fully when the capacity is low. Since the customer demand mean is 5.5, the supplier does not have much choice other than to produce in every period to meet the demand in low capacities. As the capacity increases, the supplier use the higher portion of the provided flexibility in dispatch quantity. Therefore, the benefit of VMI increases with capacity.

4.1.3 Effect of Retail and Wholesale Price

As it was stated before, in the VMI setting, the replenishment decision is made according to the inventory level of the retailer and the specified (z,Z) levels. The supplier decides on the dispatch quantity at each period and replenishes the retailer. The wholesale price is the only factor that specifies her profit in such a situation. The retail price has no effect on supplier's profit in the VMI setting. However, in traditional case, it plays a role in determining the retailer's inventory policy parameters s and S, thus it has an indirect effect on the supplier's profit since she infers the retailer's demand distribution function by using these parameters. Increasing retail price means higher profit for the retailer. As the profit of the item increases, the retailer keeps more safety stock to minimize lost sales. Therefore, the reorder point (s) increases with higher values of the retail price.

Figure 4.4 presents the profit of the supplier in traditional and VMI setting for each value of retail price (for this example, C = 15 and cv_d = 0.522). It is seen that the profit of the supplier increases with the retail price in traditional case, and does not change in VMI. The figure is plotted for two different values of wholesale price and as it is expected, higher wholesale price yields higher profit in any setting.

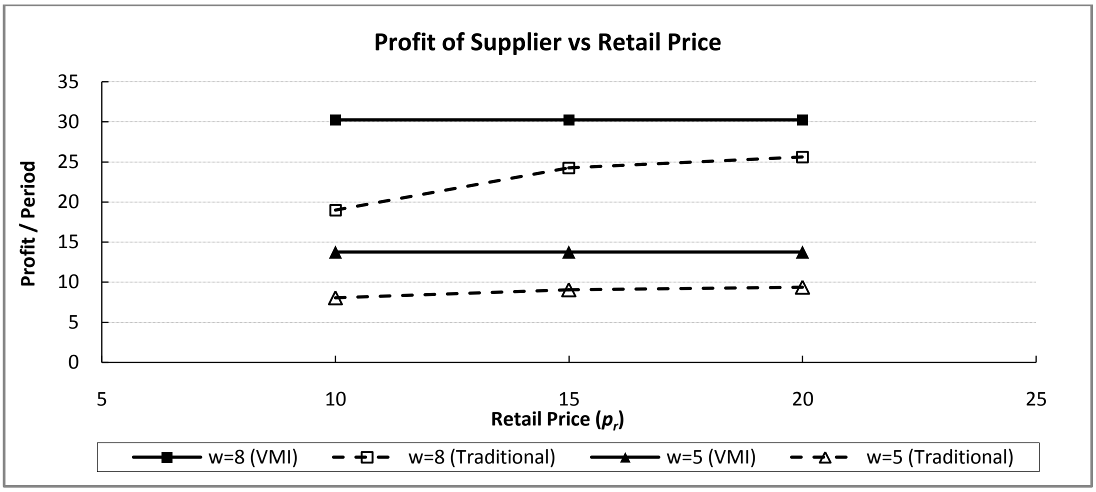

Figure 4. 4: Profit of The Supplier vs Retail Price (p_r) Under Traditional and VMI Settings (C = 15; cv_d = 0.522; α_s = 0.25)

In traditional setting, for increasing values of the retail price, the values of (s,S) parameters are (0,10), (4,10) and (6,10), respectively, when the wholesale price is 8; and (3,10), (5,10) and (6,10) when the wholesale price is 5, for the example in Figure 4.4. As expected, since he wants to minimize lost sales, the reorder point of the retailer (s) increases with the retail price while the order-up-to level (S) remains the same. Due to the decreased (S-s), the supplier faces rather frequent orders and the order size of the retailer increases in average, which yields higher profit at the supplier. As a result, as the retail price increases, the expected profit of the supplier also increases in traditional setting.

For the example shown in Figure 4.4, the profits are calculated under the moderate value of coefficient of customer demand variation (cv_d = 0.522). However, the profit values at different values of cv_d displays the same pattern. The percentage benefit of VMI on profit of the supplier are averaged over low, moderate and high values of cv_d for each value of retail and wholesale price, and displayed in Figure 4.5. Although it remains the same in VMI setting; since the supplier's profit increases with retail price in traditional setting, the percentage benefit of VMI on the supplier's profit decreases with retail price. It is observed in Figure 4.5 that the VMI agreement is most beneficial in low values of retail and wholesale price. For different values of retail and wholesale price, the benefit of VMI on profit varies from 16% to 52% in average.

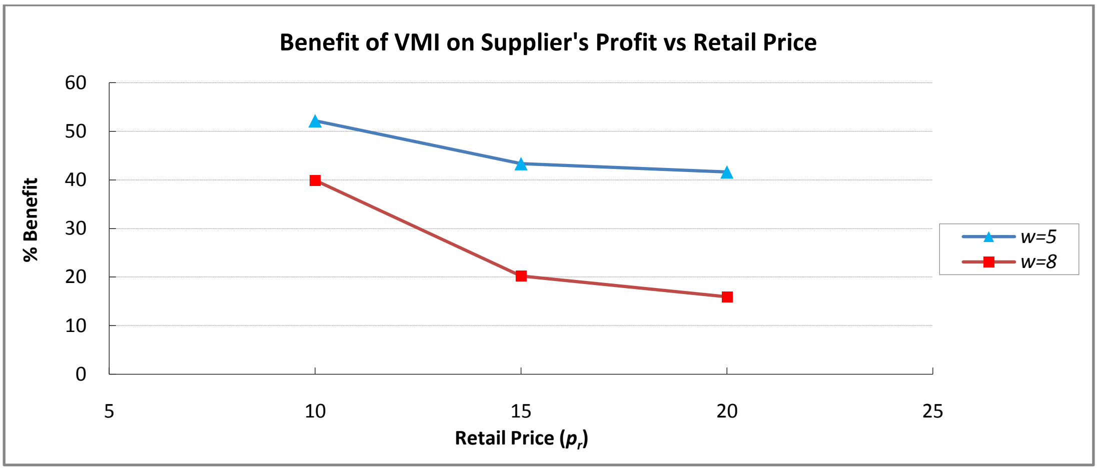

Figure 4. 5: Benefit of VMI on Supplier's Profit vs Retail Price (C = 15; α_s = 0.25)

4.2 Benefit of VMI on Retailer's Profit

In this section, the benefit of VMI on the retailer's profit over traditional setting is analyzed. The analysis are performed with the assumption that the (z,Z) levels are set by the retailer in the VMI agreement. According to the agreement, the profit of the supplier under VMI should not be less than her profit in traditional setting. For each parameter set, the profit value of the retailer under the traditional and the VMI setting is provided in Appendix D.

4.2.1 Effect of Demand Variation

Figure 4.6 provides the profit of the retailer in traditional and VMI setting according to the coefficient of variation. Both in traditional and VMI setting, the retailer gets the maximum profit in low variance case. When the coefficient of variation is increased to moderate values, the profit value decreases and reaches the minimum among all variance values. Although we set the retail price to 15 for this example, the same pattern is observed at different retail price values. The changing pattern of the profits with respect to the demand variation are similar under both traditional and the VMI setting.

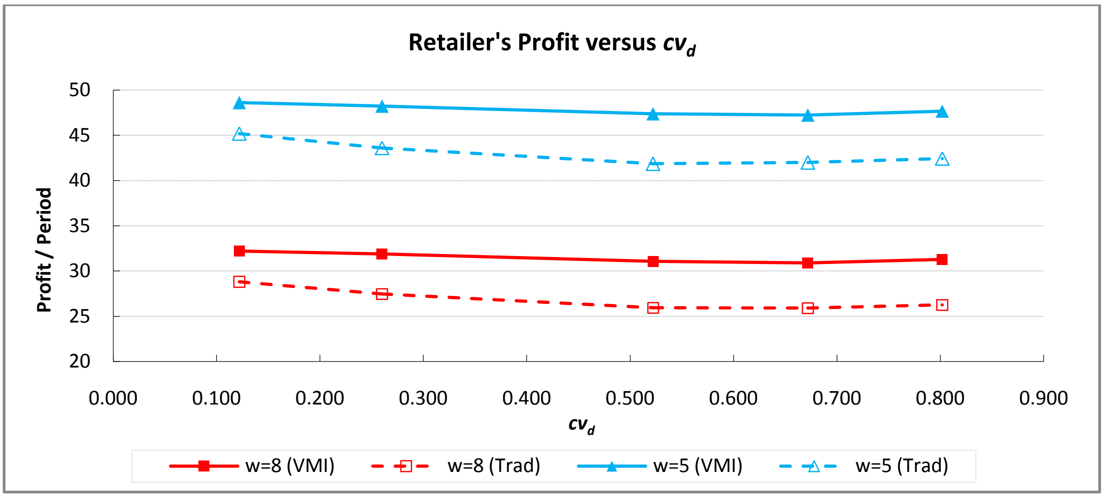

Figure 4. 6: Profit of The Retailer vs Coefficient of Variation (cv_d) Under Traditional and VMI Settings (C = 15; p_r = 15; α_s = 0.25)

The effect of customer demand variation to the benefit of VMI on the retailer's profit is displayed in Figure 4.7. For each value of wholesale price, VMI provides the maximum benefit at moderate values of coefficient of customer demand variation, while the benefit is small at low variance case.

When the coefficient of variation increases from a low value to a moderate one (from 0.122 to 0.522 in the example), the percent benefit of VMI increases sharply and it is almost doubled for each value of wholesale price. Increasing the coefficient of variation from a moderate to a high value slightly decreases the benefit of VMI, which is less than 1%.

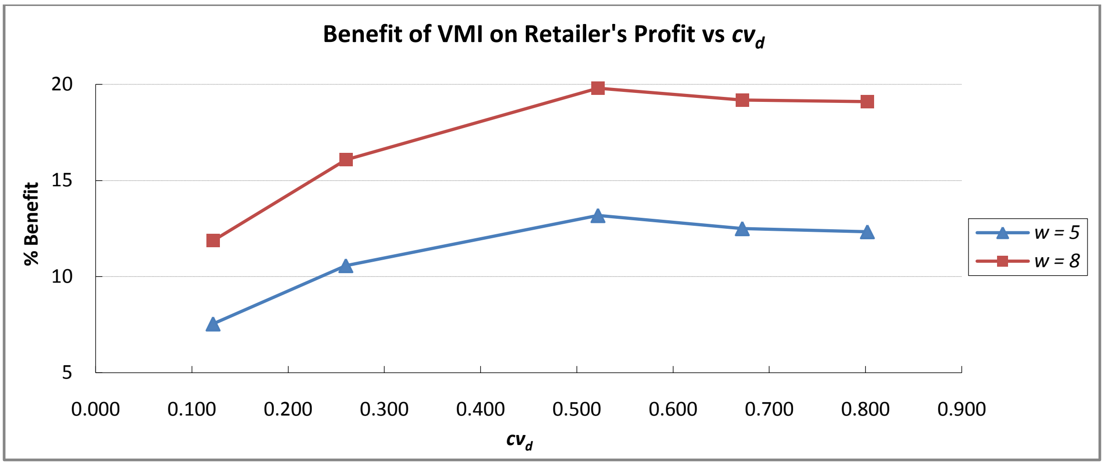

Figure 4. 7: Benefit of VMI on Retailer's Profit vs Coefficient of Variation (C = 15; α_s = 0.25)

4.2.2 Effect of Retail and Wholesale Price

In order to analyze the effect of the retail and wholesale price to the benefit of VMI on retailer's profit, we provide a sample in Figure 4.8, which displays the profit values of the retailer for each value of retail and wholesale price under traditional and VMI settings. Naturally, increasing retail price or decreasing wholesale price provides higher profits to the retailer under both settings. However, the critical point here is that, the differences on profit between two settings do not change significantly with increasing retail price.

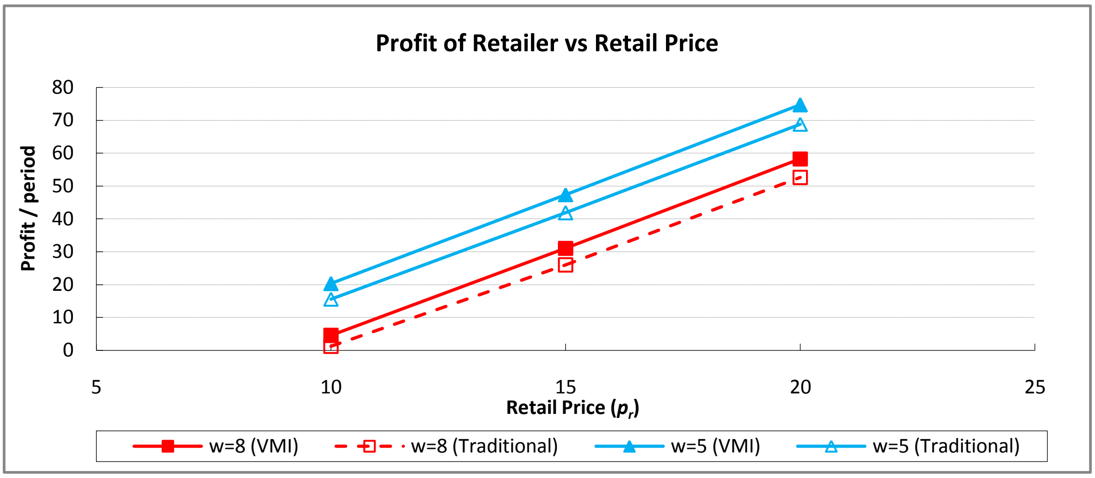

Figure 4. 8: Profit of The Retailer vs Retail Price (p_r) Under Traditional and VMI Settings (C = 15; α_s = 0.25; cv_d = 0.522)

In traditional setting, when the retail price of an item increases, or the wholesale price decreases, it becomes more profitable and the retailer keeps more safety stock to capture the greater portion of the lost sales. This is also valid for the VMI setting. As the retail price increases, the retailer sets the minimum inventory level, z, to a higher value in the VMI agreement. For the example given in Figure 4.8, the (z,Z) levels set by the retailer are (6,17), (7,18) and (8,19) respectively, for the increasing values of the retail price.

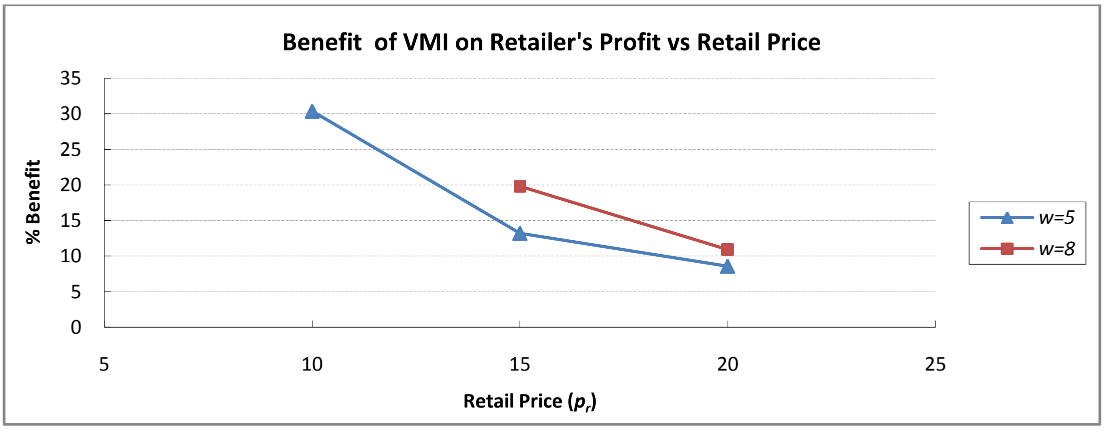

Figure 4. 9: Benefit of VMI on Retailer's Profit vs Retail Price (C = 15; α_s = 0.25)

Figure 4.9 presents the percentage benefit of VMI on retailer's profit for each value of the retail and wholesale price. VMI is most beneficial at lower retail and higher wholesale prices. In other words, as the unit profit of the item increases, VMI provides less benefit to the retailer.

4.3 The Performance of VMI and Benefit on Supply Chain Profit

In this part, the benefit of VMI on total supply chain profit is analyzed under different parameter sets. In the VMI agreement, it is assumed that the specified values of (*z,Z*) levels makes the total supply chain profit maximum while making both parties better off than traditional setting. For each parameter set, the profit values of the total supply chain under the traditional and the VMI setting are provided in Appendix E.

Our aim in this part is not only to analyze the benefit of VMI over traditional setting, but also to examine its performance in coordinating the supply chain. That is, the supply chain profit under VMI is compared with the profits under traditional and the centralized case.

In a centralized system, that is, if both the supplier and the retailer act as a single firm, the supply chain profit reaches its maximum due to the best coordination of production and shipment. Therefore, a supply chain operating in traditional case has a potential of additional profit that can be gained by better coordination. Our aim in this section is to investigate the ability of VMI to capture this potential at each parameter set. So, the performance of VMI (% Potential Benefit Captured by VMI) is measured as;

$$\frac{\text{VMI Profit} - \text{Traditional Profit}}{\text{Centralized Profit} - \text{Traditional Profit}} \times 100\%$$

4.3.1 Effect of Demand Variation

Figure 4.10 displays the supply chain profits in traditional, VMI and centralized setting for each value of coefficient of customer demand variation. The figure represents the supply chain profits for the setting C = 15, α_s = 0.25, p_r = 15 and w = 8. The supply chain profit in VMI setting is very close to the centralized profit at each coefficient variation of demand value, and decreases with increasing variance. In traditional case, the supply chain profit reaches the maximum value in low variation, and the minimum in moderate values of variance.

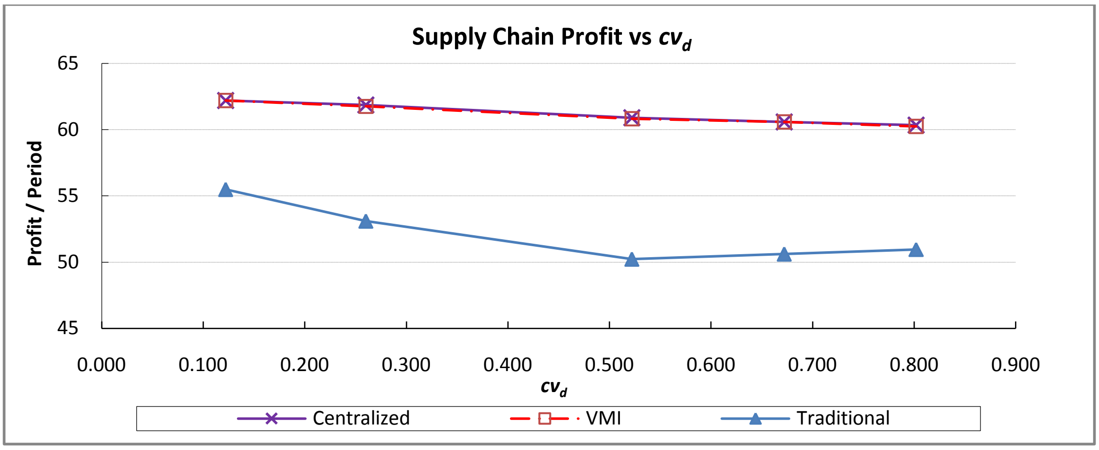

Figure 4. 10: Profit of The Supply Chain vs Coefficient of Variation (cv_d) (C = 15; p_r = 15; w = 8; α_s = 0.25)

It is seen in the figure that the VMI setting has benefits for the supply chain at each demand variance and operates very close to the optimal (i.e., centralized setting). Figure 4.11 and Figure 4.12 represent the percent benefits of VMI over traditional setting and the percent potential benefits captured by VMI for each value of coefficient of customer demand variation.

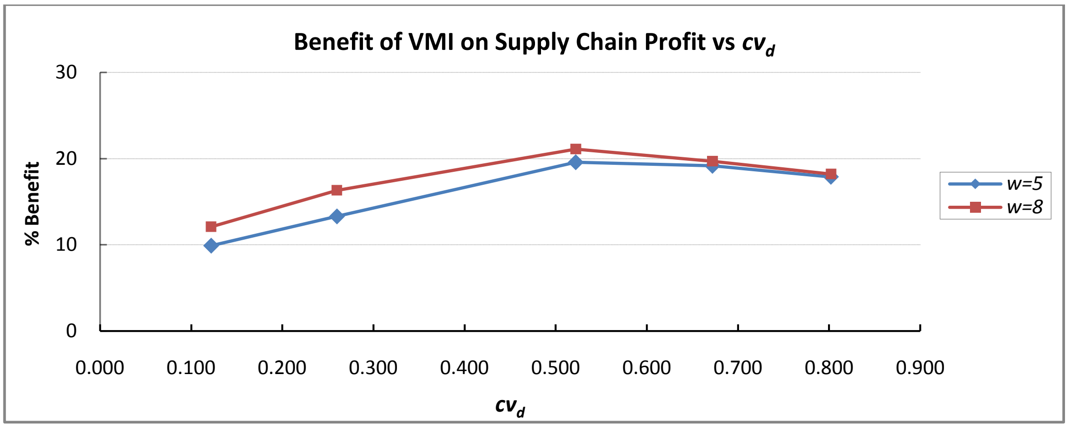

Figure 4. 11: Benefit of VMI on Supply Chain Profit vs Coefficient of Variation (C = 15; p_r = 15; w = 8; α_s = 0.25)

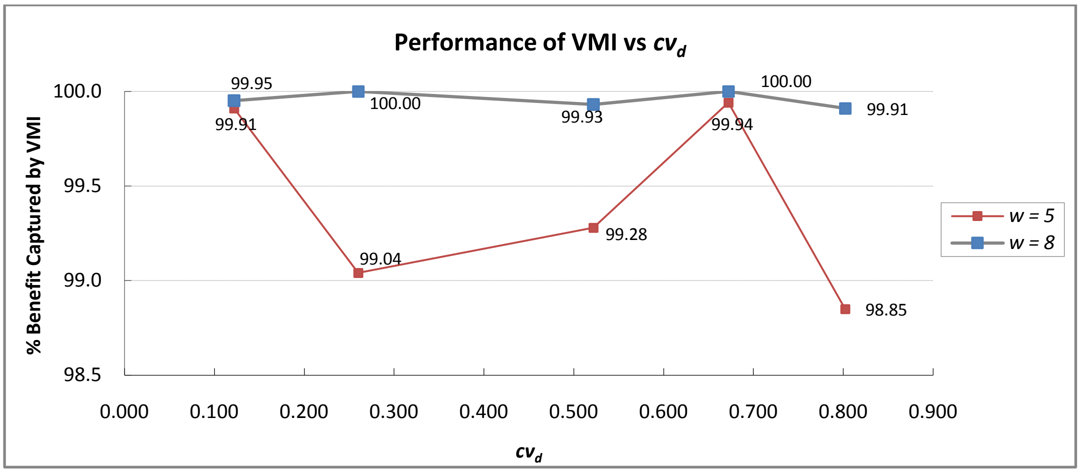

Figure 4. 12: Performance of VMI vs Coefficient of Variation (C = 15; p_r = 15; α_s = 0.25)

For the supply chain, the VMI agreement provides the higher benefit at moderate values of variance. For the example, VMI provides 20% increase in supply chain profit at moderate value of cv_d and high value of wholesale price. The performance of VMI displays a random change with increasing values of variation. At each value of coefficient of variation, VMI captures the more than 98% of the potential benefit.

4.3.2 Effect of Capacity

Figure 4.13 displays the benefit of VMI on supply chain profit at each value of capacity. As the capacity of the supplier increases, VMI provides greater benefits for the supply chain over traditional setting. When the customer demand variance is low, there is almost no benefit of VMI in low capacity. As it is discussed in section 4.1.2, since mean demand is 5.5, the supplier should fully produce each period to satisfy the mean demand under low capacity case (C = 5). Although the VMI agreement provides flexibility in production and replenishment decisions, the supplier cannot use this due to the tight capacity. Therefore, the benefit of VMI on the profit of supply chain increases with increasing values of capacity.

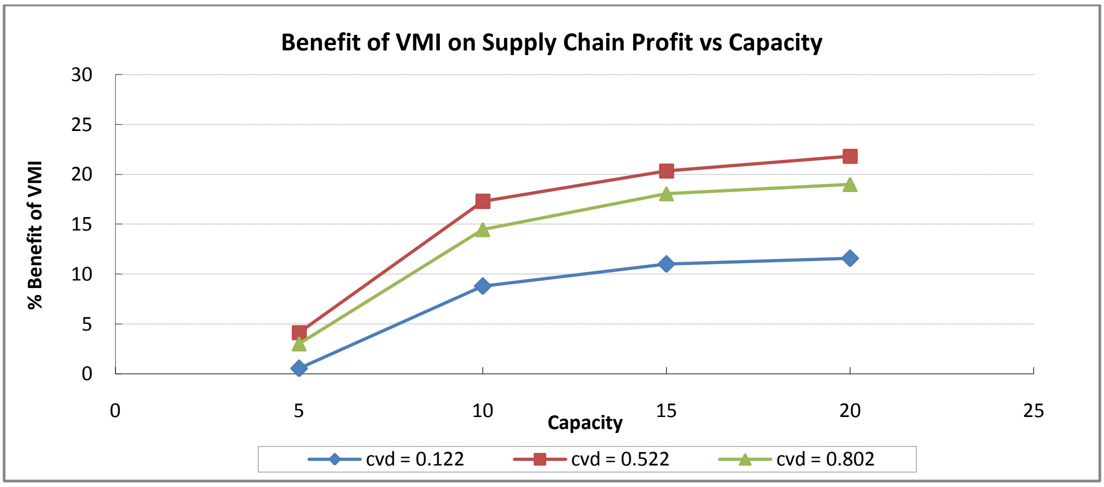

Figure 4. 13: Benefit of VMI on Supply Chain Profit vs Capacity (p_r = 15; α_s = 0.25)

The performance of VMI at each value of capacity is displayed in Figure 4.14 For every value of capacity and three different demand distribution (low, medium and high variation), the percentage potential benefits captured by VMI are averaged over all the retail and wholesale prices. When the capacity is 5, VMI captures 38.8% of potential benefit in average. As the capacity increases, the performance of VMI increases significantly and gets closer to the centralized case. When the capacity is increased to 10, the VMI setting reaches its maximum performance value, 99.97% in average. Although it slightly decreases with increasing capacity, VMI captures more than 99% of the potential benefit for higher values of capacity.

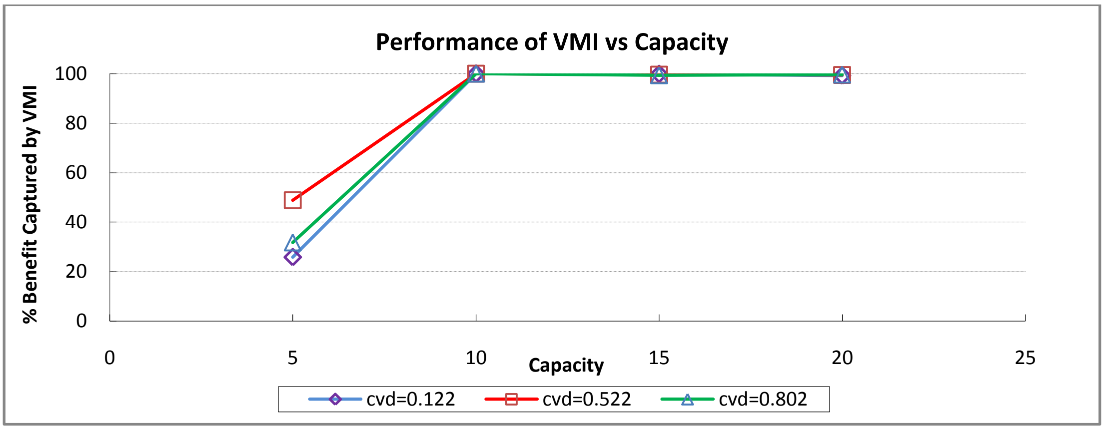

Figure 4. 14: Performance of VMI vs Capacity (α_s = 0.25)

4.3.3 Effect of Order Cost Sharing

In VMI setting, the fixed order cost of the retailer (K_r) is shared between the retailer and supplier in different ratios, while it is fully incurred by the retailer in traditional setting. Three ratio-scenarios are constructed for sharing the fixed order cost of the retailer. In these scenarios 25%, 50% and 75% of the fixed cost (K_r) is incurred by the supplier (i.e., $\alpha_s = 0.25$; $\alpha_s = 0.50$; $\alpha_s = 0.75$), while the remaining is charged to the retailer. For every value of K_r sharing ratio, α_s, and each capacity value, the percent benefit of VMI on supply chain is averaged over all retail and wholesale prices. The results are displayed in Figure 4.15. As the supplier incurs the higher portions of the order cost the benefit of VMI on supply chain increases. However, these increases are negligible for the capacity values of 10 and 15. If the capacity is 5, the benefit of VMI increases from 6.57% to 10.63% by increasing the supplier's ratio of order cost payment from 0.25 to 0.75.

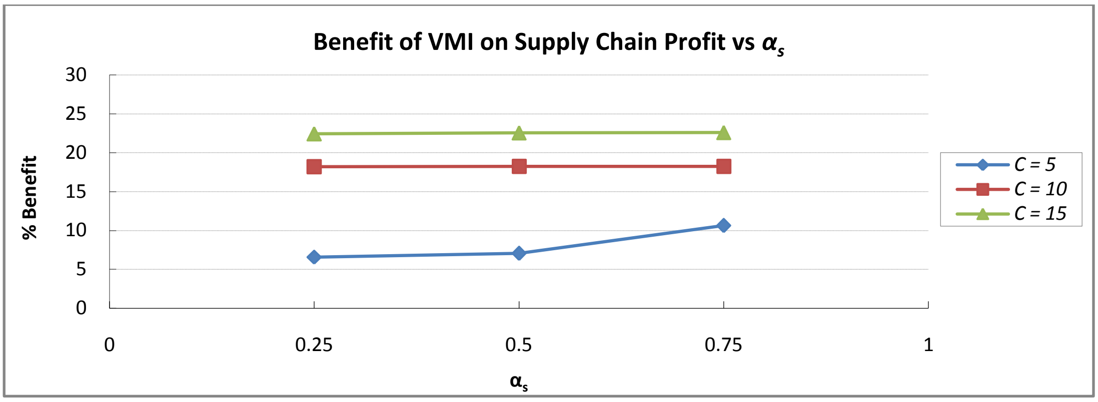

Figure 4. 15: Benefit of VMI on Supply Chain Profit vs Order Cost Sharing Ratio (α_s)

The performance of the VMI setting under different K_r sharing ratios are evaluated and represented in Figure 4.16. In higher capacities, the performance of VMI in coordinating the supply chain is greater than 99% and it increases with α_S. If the capacity is 5, the performance of VMI increases significantly with α_S. That is, if the supplier's share of order cost is increased from 25% to 75%, the potential benefit captured by VMI increases from 38.8% to 89.39%. As a result; under high capacity, α_S does not have a coordinating effect, while under tight capacity the higher the portion is shared by the supplier, the more coordinated the supply chain.

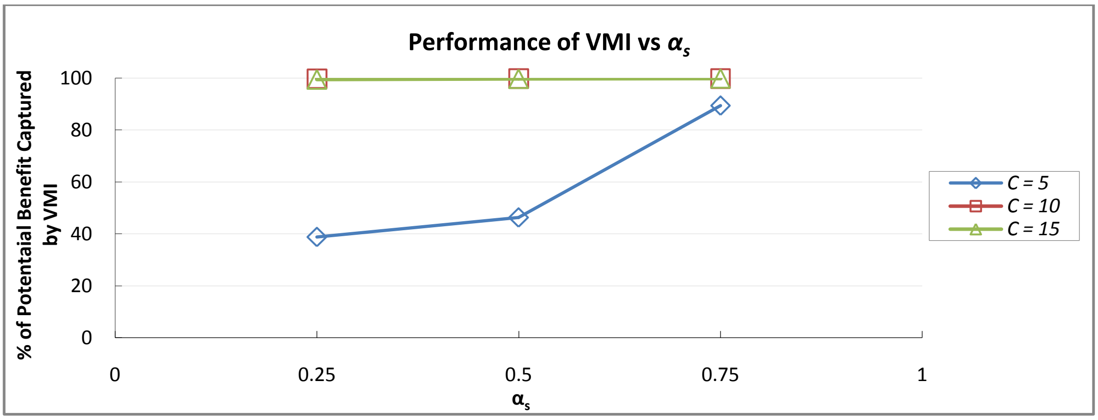

Figure 4. 16: Performance of VMI vs Order Cost Sharing Ratio (α_s)

4.3.4 Effect of Retail and Wholesale Price

The benefit of VMI on supply chain profit for each value of retail price is provided in Figure 4.17. For this sample, we averaged the percent benefits over all values of coefficient of variation (the pattern is the same for each distinct value of coefficient of variation). The benefit of VMI on supply chain profit decreases with increasing retail price. VMI provides 40.3% of benefit in average when the retail price is 10. The benefit decreases to 10.55% in average, at the retail price value of 20. In addition, increasing values of retail price diminishes the effect of the wholesale price.

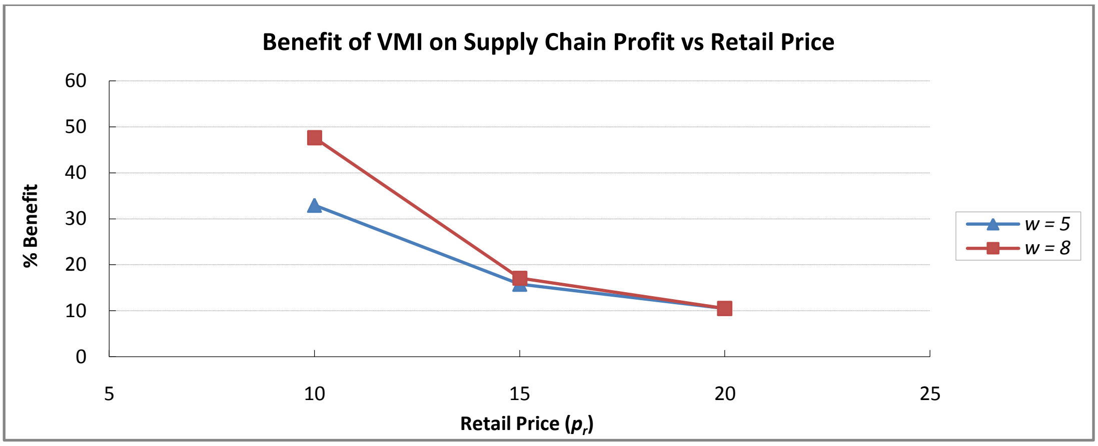

Figure 4. 17: Benefit of VMI on Supply Chain Profit vs Retail Price (C = 15; α_s = 0.25)

Figure 4.18 displays the performance of VMI for each value of retail price. Increasing values of the retail price in the supply chain makes the VMI better coordinate the system. When the retail price is 20 and the wholesale price is 5, VMI captures almost all of the potential benefit.

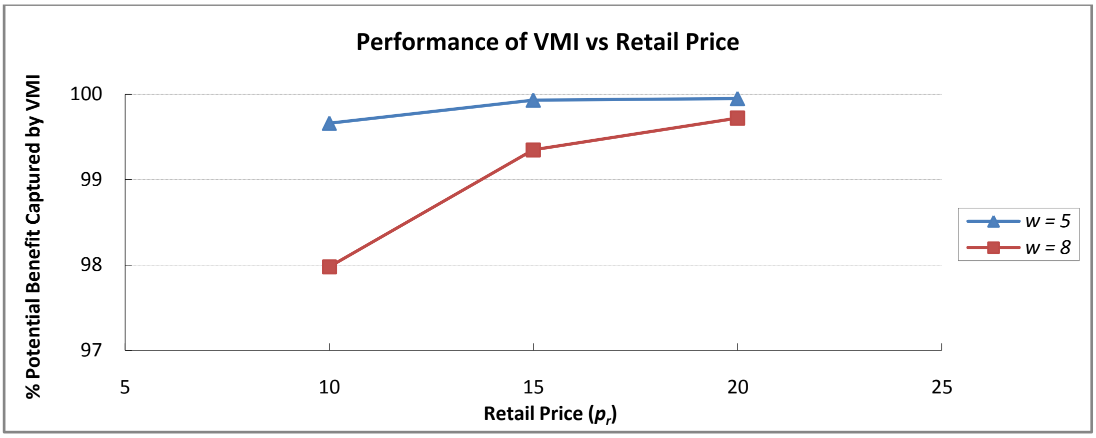

Figure 4. 18: Performance of VMI vs Retail Price (C = 15; α_s = 0.25)

CHAPTER 5

CONCLUSION

In this study, we analyze a VMI system for a supply chain consisting of a single capacitated supplier and a single retailer. According to the proposed VMI agreement, the supplier is informed periodically about the inventory level of the retailer, and she manages the retailer's inventory by deciding on the timing and the quantity of the replenishments on behalf of him. The motivation of the retailer for participating in such a relationship is provided by transferring some portion of his fixed ordering cost to the supplier. The replenishments in the VMI setting are forced to be done according to the (z,Z) policy. That is, the minimum and the maximum inventory levels, z and Z respectively, are specified in a contract and the supplier should keep the inventory level of the retailer between these levels. We conduct a computational study and compare the VMI system with the traditional setting, under which the supplier has limited (partial) information and the replenishment decision is made by the retailer himself according to (s,S) policy. The analyses of the benefits provided by VMI are done from the perspective of the supplier, the retailer and the overall supply chain. The effects of the capacity, variation of customer demand, retail price, wholesale price and the order cost sharing ratio to these benefits are analyzed.

VMI system provides significant benefits in many settings, but not for all. The (z,Z) type of inventory management in the VMI system provides flexibility to the supplier on the quantity of production and dispatch. However, the utilization of this flexibility is mainly dependent on the production capacity of the supplier; therefore the capacity plays an important role on the benefit provided by VMI. When the capacity of the supplier is low (close to mean demand), the VMI system does not provide benefit to the supplier, that is, at any (z,Z) level, the supplier cannot get better than traditional setting, although the retailer can. On the other hand, increasing production capacity yields significant benefits both for the supplier and the retailer.

We analyze the effects of customer demand variability on the benefits. In the VMI system, as soon as the supplier can utilize the flexibility on dispatch quantity, her profit does not affected by the customer demand variation in the long-run. VMI provides the higher benefit for the supplier at moderate or high variation cases, depending on the retailer's policy parameters in the traditional setting. The VMI system, however, is most beneficial at moderate values of demand variance when we consider the profit of the retailer and the total supply chain. Under the VMI setting, the system performs very close to the centralized setting at any value of the demand variance according to our data set.

Future research can extend the analysis here in many directions. In our supply chain environment, as soon as the material is shipped to the retailer, the ownership of the inventory is also transferred to the retailer. A further extension may consider the consignment stock case under the VMI setting. Another extension could be to aim a specified service level under the VMI setting.

Finally, one other extension of the study may consider a supply chain environment with a single supplier who joins in a VMI partnership with two or more retailers. For such a case, the model we construct for VMI setting can be easily modified by defining the state with more parameters. However, this case is not simple to conduct a numerical analysis due to the increased complexity of the model.

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APPENDIX A

List of Notation

I_r	: Inventory level of the retailer
I_s	: Inventory level of the supplier
q	: Order quantity of the retailer in the traditional setting
a	: Production quantity of the supplier.
b	: Quantity of item dispatched to the retailer
p_r	: Unit selling price of the retailer (retail price)
w	: Unit selling price of the supplier (wholesale price)
h_r	: Unit holding cost/period for the retailer
h_s	: Unit holding cost/period of the supplier
c_s	: Unit production cost of the supplier
o_s	: Unit outsourcing cost of the supplier
K_r	: Fixed ordering cost of the retailer
K_s	: Setup cost of the supplier (production fixed cost)
t	: Elapsed time since the last order of the retailer.
d_i	: Customer demand quantity in period i
α_s	: The ratio of the fixed ordering cost incurred by the supplier in the VMI setting
$P(D)$	: The probability that the customer demand is equal to D units ($D>0$)
$P(t,Q)$	: Probability that the order size of the retailer is Q units when t periods elapsed since the last order.
$PC_{(I_r,q)}$	: Purchasing cost (includes fixed and variable cost) of the retailer in the traditional setting, when the system is in state $I_r \in E$ and decision $q \in A$ is made

$HC_{(I_r,q)}$: Expected inventory holding cost of the retailer in the traditional setting, when the system is in state $I_r \in E$ and decision $q \in A$ is made.

$R_{(I_r,q)}$: Expected revenue of the retailer in the traditional setting, when the system is in state $I_r \in E$ and decision $q \in A$ is made.

$r_{(I_r,q)}$: Expected profit of the retailer in the traditional setting, when the system is in state $I_r \in E$ and decision $q \in A$ is made

$X_{(I_r,q)}$: Steady-state probability that the system is in state I_r and decision q is made by the retailer in the traditional setting.

$X_{(I_s,t)a}$: Steady state probability that the system is in state (I_s,t) and decision a is made by the supplier in the traditional setting

$r_{(I_s,t)a}$: Expected profit of the supplier in the traditional setting when the system is in state (I_s,t) and decision a is made

$OC_{(I_s,t)a}$: Expected outsourcing cost of the supplier in the traditional setting when the system is in state (I_s,t) and decision a is made

$HC_{(I_s,t)a}$: Expected inventory holding cost of the supplier in the traditional setting when the system is in state (I_s,t) and decision a is made.

$PC_{(I_s,t)a}$: Production cost of the supplier in the traditional setting when the system is in state (I_s,t) and decision a is made.

$R_{(I_s,t)a}$: Expected revenue of the supplier if the system is in state (I_s,t) and decision a is made.

$X_{(I_s,I_r)(a,b)}$: Steady-state probability that the system is in state (I_s,I_r) and decision (a,b) is made in the VMI setting

$r_{(I_s,I_r)(a,b)}$: Expected profit of the supplier in the VMI setting when the system is in state (I_s,I_r) and decision (a,b) is made

$DC_{(I_s,I_r)(a,b)}$: Dispatching cost of the supplier in the VMI setting when the system is in state (I_s,I_r) and decision (a,b) is made.

$OC_{(I_s,I_r)(a,b)}$: Expected outsourcing cost of the supplier in the VMI setting when the system in state (I_s,I_r) and decision (a,b) is made.

$HC_{(I_s,I_r)(a,b)}$: Expected inventory holding cost of the supplier in the VMI setting when the system is in state (I_s,I_r) and decision (a,b) is made.

$PC_{(I_s,I_r)(a,b)}$: Production cost of the supplier in the VMI setting when the system is in state (I_s,I_r) and decision (a,b) is made.

$R_{(I_s,I_r)(a,b)}$: Expected revenue of the supplier in the VMI setting when the system is in state (I_s,I_r) and decision (a,b) is made.

$DC^R{}_{(I_s,I_r)(a,b)}$: Purchasing cost (includes fixed and variable cost) of the retailer in the VMI setting when the system is in state (I_s,I_r) and decision (a,b) is made.

$HC^R{}_{(I_s,I_r)(a,b)}$: Expected inventory holding cost of the retailer in the VMI setting when the system is in state (I_s,I_r) and decision (a,b) is made.

$R^R{}_{(I_s,I_r)(a,b)}$: Expected revenue of the retailer in the VMI setting when the system is in state (I_s,I_r) and decision (a,b) is made.

$r^R{}_{(I_s,I_r)(a,b)}$: Expected profit of the retailer in the VMI setting when the system is in state (I_s,I_r) and decision (a,b) is made.

$r^{SC}{}_{(I_s,I_r)(a,b)}$: Expected profit of the supply chain in the centralized setting when the system is in state (I_s,I_r) and decision (a,b) is made.

APPENDIX B

The Algorithm For Generating *P(t,Q)* – A Numerical Example

Let $(s,S) = (1,4)$, that is $\Delta = 3$;

and the customer demand distribution is defined as $P(D = 1) = P(D = 2) = 0.5$.

Step 0: $t = 1$

Step 1: $d_1 = 1$

Step 2: $\sum_{i=1}^{t} d_i = 1 < \Delta$; set $t = 2$;

Step 1: $d_2 = 1$

Step 2: $\sum_{i=1}^{t} d_i = 2 < \Delta$; set $t = 3$;

Step 1: $d_3 = 1$

Step 2: $\sum_{i=1}^{t} d_i = 3 \geq \Delta$; $Q = min(\sum_{i=1}^{t} d_i, S) = 3$;

set $P'(3,3) = P'(3,3) + \prod_{i=1}^{3} P(d_i) = 0.125$

$(d_1, d_2, d_3) = (1, 1, 1)$

Step 0: $t = 1$

Step 1: $d_1 = 1$

Step 2: $\sum_{i=1}^{t} d_i = 1 < \Delta$; set $t = 2$;

Step 1: $d_2 = 1$

Step 2: $\sum_{i=1}^{t} d_i = 2 < \Delta$; set $t = 3$;

Step 1: $d_3 = 2$

Step 2: $\sum_{i=1}^{t} d_i = 4 \geq \Delta$; $Q = min(\sum_{i=1}^{t} d_i, S) = 4$;

set $P'(3,4) = P'(3,4) + \prod_{i=1}^{3} P(d_i) = 0.125$

$(d_1, d_2, d_3) = (1, 1, 2)$

Step 0: t = 1

Step 1: $d_1 = 1$

Step 2: $\sum_{i=1}^{t} d_i = 1 < \Delta$; set $t = 2$;

Step 1: $d_2 = 2$

Step 2: $\sum_{i=1}^{t} d_i = 3 \geq \Delta$; $Q = min(\sum_{i=1}^{t} d_i, S) = 3$;

set $P'(2,3) = P'(2,3) + \prod_{i=1}^{2} P(d_i) = 0.25$

(d_1, d_2) = (1, 2)

Step 0: $t = 1$

Step 1: $d_1 = 2$

Step 2: $\sum_{i=1}^{t} d_i = 2 < \Delta$; set $t = 2$;

Step 1: $d_2 = 1$

Step 2: $\sum_{i=1}^{t} d_i = 3 \geq \Delta$; $Q = min(\sum_{i=1}^{t} d_i, S) = 3$;

set $P'(2,3) = P'(2,3) + \prod_{i=1}^{2} P(d_i) = 0.25 + 0.25 = 0.50$

(d_1, d_2) = (2, 1)

Step 0: $t = 1$

Step 1: $d_1 = 2$

Step 2: $\sum_{i=1}^{t} d_i = 2 < \Delta$; set $t = 2$;

Step 1: $d_2 = 2$

Step 2: $\sum_{i=1}^{t} d_i = 4 \geq \Delta$; $Q = min(\sum_{i=1}^{t} d_i, S) = 4$;

set $P'(2,4) = P'(2,4) + \prod_{i=1}^{2} P(d_i) = 0.25$

(d_1, d_2) = (2, 2)

All possible combinations of d_i series, providing $d_1 + d_2 + \ldots + d_t \geq \Delta$, are evaluated and the unconditional probability distribution, $P'(t,Q)$, is obtained. That is;

$P'(2,3)=0.5$; $P'(2,4)=0.25$; $P'(3,3)=0.125$; $P'(3,4)=0.125$.

The probability of the order placed by the retailer, conditioning on the elapsed time, *P(t,Q)*, is calculated by using the following equation;

$$P(t,Q) = \frac{P'(t,Q)}{\sum_{t' \geq t} \sum_{Q} P'(t',Q)}$$

and the results are summarized in the following table.

		Q				
		0	**1**	**2**	**3**	**4**
t	**1**	1	0	0	0	0
	2	0.25	0	0	0.5	0.25
	3	0	0	0	0.5	0.5

APPENDIX C

Computational Study – The Supplier's Profit in Traditional and VMI Setting

Table C. 1: Profit of The Supplier Under The Traditional and VMI Settings For Each Parameter Set (α_s = 0.25)

Parameters			Profit of The Supplier								Benefit (%)			
			Traditional Setting				VMI Setting							
			Capacity				Capacity				Capacity			
Retail Price (p_r)	Wholesale Price (w)	Coef. of Var. (cv_d)	5	10	15	20	5	10	15	20	5	10	15	20
10	5	0.122	-3.26	8.63	11.14		*	9.63	13.75	15.76	-	11.65	23.38	
10	8	0.122	13.56	24.78	25.60		*	26.13	30.25	32.31	-	5.44	18.15	
15	5	0.122	-4.14	8.55	11.38	12.46	*	9.63	13.75	15.76	-	12.68	20.87	26.85
15	8	0.122	12.66	23.93	26.68	27.82	*	26.13	30.25	32.31	-	9.21	13.39	16.14
20	5	0.122	-4.14	8.55	11.38		*	9.63	13.75	15.76	-	12.68	20.87	
20	8	0.122	12.29	24.97	27.80		*	26.13	30.25	32.31	-	4.64	8.81	
15	5	0.260			10.98			9.63	13.75	15.76			25.28	
15	8	0.260			25.62			26.13	30.25	32.31			18.08	
10	5	0.522	-4.99	6.41	8.07		-3.71	9.63	13.75	15.76	25.62	50.19	70.41	
10	8	0.522	9.53	18.33	19.00		10.98	26.13	30.25	32.31	15.26	42.58	59.25	
15	5	0.522	-5.44	6.68	9.06	9.74	-4.57	9.63	13.75	15.76	15.92	44.25	51.72	61.84
15	8	0.522	10.37	21.98	24.26	25.08	11.63	26.13	30.25	32.31	12.19	18.88	24.68	28.85
20	5	0.522	-5.55	6.91	9.38		-5.26	9.63	13.75	15.76	5.24	39.40	46.56	
20	8	0.522	10.69	23.15	25.62		11.31	26.13	30.25	32.31	5.81	12.89	18.07	
15	5	0.672			8.84			9.63	13.75	15.76			55.60	
15	8	0.672			24.68			26.13	30.25	32.31			22.57	
10	5	0.802	-8.24	6.58	8.46		-6.88	9.63	13.75	15.76	16.50	46.37	62.63	
10	8	0.802	9.38	18.48	21.24		9.93	26.13	30.25	32.31	5.92	41.37	42.41	
15	5	0.802	-8.19	6.75	8.74	9.90	-7.41	9.63	13.75	15.76	9.56	42.73	57.38	59.48
15	8	0.802	7.90	22.76	24.69	25.92	8.76	26.13	30.25	32.31	10.96	14.80	22.51	24.64
20	5	0.802	-8.19	6.75	8.74		-7.26	9.63	13.75	15.76	11.39	42.73	57.38	
20	8	0.802	8.11	23.05	25.04		9.24	26.13	30.25	32.31	13.98	13.38	20.82	

(*) The supplier's profit is not better off than the traditional setting at any (*z*,*Z*) level.

Table C. 2: Profit of The Supplier Under The Traditional and VMI Settings For Each Parameter Set (α_s = 0.50)

Parameters			Profit of The Supplier						Benefit (%)		
			Traditional Setting			VMI Setting					
			Capacity			Capacity			Capacity		
Retail Price (p_r)	Wholesale Price (w)	Coef. of Var. (cv_d)	5	10	15	5	10	15	5	10	15
10	5	0.122	-3.26	8.63	11.14	*	*	12.83	-	-	15.13
10	8	0.122	13.56	24.78	25.60	*	*	29.33	-	-	14.55
15	5	0.122	-4.14	8.55	11.38	*	*	12.83	-	-	12.78
15	8	0.122	12.66	23.93	26.68	*	24.75	29.33	-	3.44	9.94
20	5	0.122	-4.14	8.55	11.38	*	*	12.83	-	-	12.78
20	8	0.122	12.29	24.97	27.80	*	*	29.33	-	-	5.50
10	5	0.522	-4.99	6.41	8.07	-4.22	8.25	12.83	15.40	28.67	59.00
10	8	0.522	9.53	18.33	19.00	9.92	24.75	29.33	4.14	35.05	54.41
15	5	0.522	-5.44	6.68	9.06	*	8.25	12.83	-	23.58	41.56
15	8	0.522	10.37	21.98	24.26	*	24.75	29.33	-	12.60	20.88
20	5	0.522	-5.55	6.91	9.38	*	8.25	12.83	-	19.43	36.75
20	8	0.522	10.69	23.15	25.62	*	24.75	29.33	-	6.93	14.48
10	5	0.802	-8.24	6.58	8.46	-5.70	8.25	12.83	30.83	25.40	51.74
10	8	0.802	9.38	18.48	21.24	9.53	24.75	29.33	1.65	33.91	38.08
15	5	0.802	-8.19	6.75	8.74	-7.71	8.25	12.83	5.90	22.28	46.85
15	8	0.802	7.90	22.76	24.69	8.26	24.75	29.33	4.62	8.74	18.78
20	5	0.802	-8.19	6.75	8.74	*	8.25	12.83	-	22.28	46.85
20	8	0.802	8.11	23.05	25.04	*	24.75	29.33	-	7.39	17.15

(*) The supplier's profit is not better off than the traditional setting at any (z,Z) level.

Table C. 3: Profit of The Supplier Under The Traditional and VMI Settings For Each Parameter Set (α_s = 0.75)

Parameters			Profit of The Supplier						Benefit (%)		
			Traditional Setting			VMI Setting					
			Capacity			Capacity			Capacity		
Retail Price (p_r)	Wholesale Price (w)	Coef. of Var. (cv_d)	5	10	15	5	10	15	5	10	15
10	5	0.122	-3.26	8.63	11.14	*	*	11.92	-	-	6.96
10	8	0.122	13.56	24.78	25.60	*	*	28.42	-	-	11.00
15	5	0.122	-4.14	8.55	11.38	*	*	11.92	-	-	4.78
15	8	0.122	12.66	23.93	26.68	*	*	28.42	-	-	6.53
20	5	0.122	-4.14	8.55	11.38	*	*	11.92	-	-	4.78
20	8	0.122	12.29	24.97	27.80	*	*	28.42	-	-	2.23
10	5	0.522	-4.99	6.41	8.07	*	6.88	11.92	-	7.30	47.73
10	8	0.522	9.53	18.33	19.00	*	23.38	28.42	-	27.57	49.62
15	5	0.522	-5.44	6.68	9.06	*	6.88	11.92	-	3.06	31.52
15	8	0.522	10.37	21.98	24.26	*	23.38	28.42	-	6.37	17.13
20	5	0.522	-5.55	6.91	9.38	*	*	11.92	-	-	27.05
20	8	0.522	10.69	23.15	25.62	*	23.38	28.42	-	1.01	10.92
10	5	0.802	-8.24	6.58	8.46	-5.72	6.88	11.92	30.58	4.58	40.98
10	8	0.802	9.38	18.48	21.24	*	23.38	28.42	-	26.49	33.80
15	5	0.802	-8.19	6.75	8.74	*	6.88	11.92	-	1.97	36.43
15	8	0.802	7.90	22.76	24.69	*	23.38	28.42	-	2.72	15.10
20	5	0.802	-8.19	6.75	8.74	*	6.88	11.92	-	1.97	36.43
20	8	0.802	8.11	23.05	25.04	*	23.38	28.42	-	1.44	13.51

(*) The supplier's profit is not better off than the traditional setting at any (z,Z) level.

APPENDIX D

Computational Study – The Retailer's Profit Under The Traditional and The VMI Setting

Table D. 1: Profit of The Retailer Under The Traditional and VMI Settings For Each Parameter Set (α_s = 0.25)

Parameters			Profit of The Retailer					Benefit (%)			
			Traditional Setting	VMI Setting							
				Capacity				Capacity			
Retail Price (p_r)	Wholesale Price (w)	Coef. of Var. (cv_d)		5	10	15	20	5	10	15	20
10	5	0.122	18.13	*	21.22	21.36		-	17.04	17.82	
10	8	0.122	2.22	*	4.94	5.04		-	122.52	127.03	
15	5	0.122	45.22	*	48.46	48.63	48.87	-	7.16	7.54	8.07
15	8	0.122	28.81	*	32.08	32.23	32.50	-	11.35	11.87	12.81
20	5	0.122	72.60	*	75.93	76.11		-	4.59	4.83	
20	8	0.122	56.17	*	59.45	59.62		-	5.84	6.14	
15	5	0.260	43.62			48.24				10.59	
15	8	0.260	27.48			31.90				16.08	
10	5	0.522	15.61	16.33	19.98	20.35		4.61	27.99	30.37	
10	8	0.522	1.22	1.64	3.97	4.56		34.43	225.41	273.77	
15	5	0.522	41.86	43.15	47.01	47.38	47.02	3.08	12.30	13.19	12.33
15	8	0.522	25.96	27.38	30.72	31.10	31.15	5.47	18.34	19.80	19.99
20	5	0.522	68.80	70.38	74.33	74.70		2.30	8.04	8.58	
20	8	0.522	52.56	54.16	57.94	58.29		3.04	10.24	10.90	
15	5	0.672	42.00			47.25				12.50	
15	8	0.672	25.93			30.91				19.21	
10	5	0.802	15.58	16.51	20.03	20.38		5.97	28.56	30.81	
10	8	0.802	0.28	0.41	4.07	4.30		46.43	1353.57	1435.71	
15	5	0.802	42.45	43.36	47.29	47.69	47.41	2.14	11.40	12.34	11.68
15	8	0.802	26.27	27.08	30.87	31.29	31.29	3.08	17.51	19.11	19.11
20	5	0.802	69.62	70.86	74.79	75.04		1.78	7.43	7.79	
20	8	0.802	53.32	54.36	58.29	58.63		1.95	9.32	9.96	

(*) There not exist a common (z,Z) level that makes both players better off than the traditional setting.

Table D. 2: Profit of The Retailer Under The Traditional and VMI Settings For Each Parameter Set (α_s = 0.50)

Parameters			Profit of The Retailer				Benefit (%)		
			Traditional Setting	VMI Setting					
				Capacity			Capacity		
Retail Price (p_r)	Wholesale Price (w)	Coef. of Var. (cv_d)		5	10	15	5	10	15
10	5	0.122	18.13	*	*	22.30	-	-	23.00
10	8	0.122	2.22	*	*	6.13	-	-	176.13
15	5	0.122	45.22	*	*	49.58	-	-	9.64
15	8	0.122	28.81	*	33.48	33.17	-	16.21	15.13
20	5	0.122	72.60	*	*	77.06	-	-	6.14
20	8	0.122	56.17	*	*	60.57	-	-	7.83
10	5	0.522	15.61	16.96	21.32	21.32	8.65	36.58	36.58
10	8	0.522	1.22	3.34	5.61	5.52	173.77	359.84	352.46
15	5	0.522	41.86	*	48.38	48.38	-	15.58	15.58
15	8	0.522	25.96	*	32.10	32.10	-	23.65	23.65
20	5	0.522	68.80	*	75.70	75.70	-	10.03	10.03
20	8	0.522	52.56	*	59.31	59.31	-	12.84	12.84
10	5	0.802	15.58	17.52	21.53	21.69	12.45	38.19	39.22
10	8	0.802	0.28	1.34	5.56	5.62	378.57	1885.71	1907.14
15	5	0.802	42.45	43.01	48.71	48.89	1.32	14.75	15.17
15	8	0.802	26.27	26.66	32.38	32.55	1.48	23.26	23.91
20	5	0.802	69.62	*	76.21	76.30	-	9.47	9.59
20	8	0.802	53.32	*	59.71	59.85	-	11.98	12.25

(*) There not exist a common (*z,Z*) level that makes both players better off than the traditional setting.

Table D. 3: Profit of The Retailer Under The Traditional and VMI Settings For Each Parameter Set (α_s = 0.75)

Parameters			Profit of The Retailer				Benefit (%)		
			Traditional Setting	VMI Setting					
				Capacity			Capacity		
Retail Price (p_r)	Wholesale Price (w)	Coef. of Var. (cv_d)		5	10	15	5	10	15
10	5	0.122	18.13	*	*	23.24	-	-	28.19
10	8	0.122	2.22	*	*	7.02	-	-	216.22
15	5	0.122	45.22	*	*	50.53	-	-	11.74
15	8	0.122	28.81	*	*	34.12	-	-	18.43
20	5	0.122	72.60	*	*	78.01	-	-	7.45
20	8	0.122	56.17	*	*	61.52	-	-	9.52
10	5	0.522	15.61	*	22.65	22.48	-	45.10	44.01
10	8	0.522	1.22	*	7.08	6.89	-	480.33	464.75
15	5	0.522	41.86	*	49.74	49.47	-	18.82	18.18
15	8	0.522	25.96	*	33.53	33.26	-	29.16	28.12
20	5	0.522	68.80	*	*	76.70	-	-	11.48
20	8	0.522	52.56	*	60.67	60.40	-	15.43	14.92
10	5	0.802	15.58	18.53	22.43	21.63	18.93	43.97	38.83
10	8	0.802	0.28	*	6.92	6.93	-	2371.43	2375.00
15	5	0.802	42.45	*	49.36	48.70	-	16.28	14.72
15	8	0.802	26.27	*	33.34	32.69	-	26.91	24.44
20	5	0.802	69.62	*	76.66	76.09	-	10.11	9.29
20	8	0.802	53.32	*	60.46	59.70	-	13.39	11.97

(*) The retailer's or the supplier's profit is worse than the traditional setting.

APPENDIX E

Computational Study – Total Supply Chain Profit Under The Traditional and The VMI Settings

Table E. 1: Supply Chain Profit Under The Traditional and VMI Settings For Each Parameter Set (α_s = 0.25)

Parameters			Total Profit of The Supply Chain								Benefit (%)			
			Traditional Setting				VMI Setting							
			Capacity				Capacity				Capacity			
Retail Price (p_r)	Wholesale Price (w)	Coef. of Var. (cv_d)	5	10	15	20	5	10	15	20	5	10	15	20
10	5	0.122	14.88	26.76	29.27		14.95	30.53	34.85		0.50	14.11	19.05	
10	8	0.122	15.78	27.00	27.82		14.95	30.46	34.72		0.00	12.80	24.78	
15	5	0.122	41.08	53.77	56.60	57.68	41.49	57.94	62.20	63.81	0.99	7.76	9.90	10.62
15	8	0.122	41.47	52.74	55.49	56.63	41.50	57.93	62.20	63.73	0.08	9.85	12.10	12.54
20	5	0.122	68.46	81.15	83.98		68.77	85.41	89.68		0.45	5.25	6.79	
20	8	0.122	68.46	81.14	83.97		68.77	85.41	89.68		0.46	5.26	6.80	
15	5	0.260			54.60				61.86				13.30	
15	8	0.260			53.10				61.78				16.34	
10	5	0.522	10.62	22.02	23.68		12.52	29.39	33.73		17.87	33.46	42.45	
10	8	0.522	10.75	19.55	20.22		12.87	29.26	33.49		19.77	49.69	65.67	
15	5	0.522	36.43	48.54	50.92	51.60	37.89	56.57	60.90	62.55	4.02	16.55	19.59	21.23
15	8	0.522	36.33	47.94	50.22	51.04	37.88	56.58	60.83	62.49	4.28	18.02	21.12	22.45
20	5	0.522	63.25	75.71	78.18		64.96	83.89	88.27		2.71	10.81	12.90	
20	8	0.522	63.25	75.71	78.18		64.96	83.90	88.20		2.71	10.82	12.82	
15	5	0.672			50.84				60.59				19.17	
15	8	0.672			50.61				60.58				19.70	
10	5	0.802	7.34	22.16	24.04		10.24	29.03	33.01		39.51	31.01	37.34	
10	8	0.802	9.66	18.76	21.52		11.07	28.93	32.81		14.66	54.19	52.46	
15	5	0.802	34.26	49.20	51.19	52.35	35.24	56.22	60.35	62.24	2.87	14.28	17.90	18.89
15	8	0.802	34.17	49.03	50.96	52.19	35.24	56.22	60.25	62.17	3.15	14.66	18.23	19.12
20	5	0.802	61.43	76.37	78.36		62.74	83.58	87.75		2.14	9.45	11.99	
20	8	0.802	61.43	76.37	78.36		62.74	83.58	87.75		2.14	9.45	11.99	

Table E. 2: Supply Chain Profit Under The Traditional and VMI Settings For Each Parameter Set (α_s = 0.50)

Parameters			Total Profit of The Supply Chain						Benefit (%)		
			Traditional Setting			VMI Setting					
			Capacity			Capacity			Capacity		
Retail Price (p_r)	Wholesale Price (w)	Coef. of Var. (cv_d)	5	10	15	5	10	15	5	10	15
10	5	0.122	14.88	26.76	29.27	14.95	30.54	34.90	0.50	14.15	19.22
10	8	0.122	15.78	27.00	27.82	14.95	30.46	34.72	0.00	12.80	24.78
15	5	0.122	41.08	53.77	56.60	41.49	57.93	62.21	0.99	7.74	9.92
15	8	0.122	41.47	52.74	55.49	41.50	57.94	62.21	0.08	9.87	12.11
20	5	0.122	68.46	81.15	83.98	68.77	85.41	89.69	0.45	5.25	6.80
20	8	0.122	68.46	81.14	83.97	68.77	85.41	89.69	0.46	5.26	6.81
10	5	0.522	10.62	22.02	23.68	12.51	29.39	33.73	17.77	33.46	42.45
10	8	0.522	10.75	19.55	20.22	12.88	29.39	33.65	19.86	50.36	66.46
15	5	0.522	36.43	48.54	50.92	38.33	56.58	60.91	5.23	16.57	19.61
15	8	0.522	36.33	47.94	50.22	38.13	56.57	60.91	4.97	18.00	21.28
20	5	0.522	63.25	75.71	78.18	65.33	83.90	88.27	3.29	10.82	12.90
20	8	0.522	63.25	75.71	78.18	65.32	83.90	88.27	3.27	10.82	12.90
10	5	0.802	7.34	22.16	24.04	10.12	29.04	33.01	37.87	31.05	37.34
10	8	0.802	9.66	18.76	21.52	11.08	28.93	32.97	14.76	54.19	53.20
15	5	0.802	34.26	49.20	51.19	36.00	56.22	60.36	5.09	14.28	17.92
15	8	0.802	34.17	49.03	50.96	36.00	56.22	60.30	5.37	14.66	18.32
20	5	0.802	61.43	76.37	78.36	63.50	83.57	87.75	3.37	9.43	11.99
20	8	0.802	61.43	76.37	78.36	63.50	83.57	87.75	3.37	9.43	11.99

Table E. 3: Supply Chain Profit Under The Traditional and VMI Settings For Each Parameter Set (α_s = 0.75)

Parameters			Total Profit of The Supply Chain						Benefit (%)		
			Traditional Setting			VMI Setting					
			Capacity			Capacity			Capacity		
Retail Price (p_r)	Wholesale Price (w)	Coef. of Var. (cv_d)	5	10	15	5	10	15	5	10	15
10	5	0.122	14.88	26.76	29.27	15.83	30.53	34.90	6.42	14.11	19.22
10	8	0.122	15.78	27.00	27.82	15.84	30.46	34.85	0.39	12.80	25.25
15	5	0.122	41.08	53.77	56.60	41.94	57.94	62.20	2.09	7.76	9.90
15	8	0.122	41.47	52.74	55.49	41.93	57.93	62.20	1.11	9.85	12.10
20	5	0.122	68.46	81.15	83.98	69.39	85.41	89.69	1.36	5.25	6.80
20	8	0.122	68.46	81.14	83.97	69.38	85.41	89.69	1.35	5.26	6.81
10	5	0.522	10.62	22.02	23.68	13.10	29.40	33.73	23.33	33.50	42.45
10	8	0.522	10.75	19.55	20.22	13.88	29.40	33.65	29.16	50.41	66.46
15	5	0.522	36.43	48.54	50.92	39.06	56.57	60.91	7.23	16.55	19.61
15	8	0.522	36.33	47.94	50.22	39.06	56.57	60.91	7.53	18.00	21.28
20	5	0.522	63.25	75.71	78.18	66.22	83.90	88.28	4.70	10.82	12.92
20	8	0.522	63.25	75.71	78.18	66.23	83.89	88.28	4.71	10.81	12.92
10	5	0.802	7.34	22.16	24.04	10.97	29.03	33.01	49.46	31.01	37.34
10	8	0.802	9.66	18.76	21.52	12.34	28.95	32.97	27.81	54.29	53.20
15	5	0.802	34.26	49.20	51.19	36.82	56.21	60.36	7.48	14.25	17.92
15	8	0.802	34.17	49.03	50.96	36.82	56.22	60.31	7.77	14.66	18.34
20	5	0.802	61.43	76.37	78.36	64.32	83.58	87.75	4.71	9.45	11.99
20	8	0.802	61.43	76.37	78.36	64.32	83.58	87.75	4.71	9.45	11.99