CASH FLOW ANALYSIS OF CONSTRUCTION PROJECTS USING FUZZY SET THEORY

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ABSTRACT

CASH FLOW ANALYSIS OF CONSTRUCTION PROJECTS USING FUZZY SET THEORY

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Construction industry is a one of the most risky sectors due to high level of uncertainties included in the nature of the construction projects. Although there are many reasons, the deficiency of cash is one of the main factors threatening the success of the construction projects and causing business failures. Therefore, an appropriate cash planning technique is necessary for adequate cost control and efficient cash management while considering the risks and uncertainties of the construction projects.

The main objective of this thesis is to develop a realistic, reliable and cost-schedule integrated cash flow modeling technique by using fuzzy set theory for including the uncertainties in project cost and schedule resulting from complex and ambiguous nature of construction works. The linguistic expressions are used for utilizing from human judgment and approximate reasoning ability of users for reflecting their experience into the model to create cash flow scenarios. The uncertain cost and duration estimates gathered from experts are inserted in the model as fuzzy numbers. The model provides the user different net cash flow scenarios with fuzzy formats that are beneficial for foreseeing possible cost and schedule threats to the project during the tender stage. The model is generated in Microsoft Excel 2007 using Visual Basic for applications and the model is applied to a case example.

Keywords: Cash Flow, Cost Estimate, Fuzzy Sets

ÖZ

İNŞAAT PROJELERİ NAKİT AKIŞI ANALİZLERİNİN BULANIK KÜMELER YÖNTEMİYLE YAPILMASI

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İnşaat sektörü, içerdiği üst düzey belirsizlikler nedeniyle en riskli sektörlerden birisidir. Çok sayıda farklı nedeni olmasına rağmen, inşaat projelerinin başarısını tehdit eden ve inşaat şirketlerin iflasına neden olan en önemli etkenlerden bir tanesi nakit yetersizliğidir. Bundan dolayı, projenin risklerini ve belirsizliklerini dikkate alarak maliyet kontrolü ve nakit yönetimini etkili biçimde yapabilecek bir nakit planlama tekniğine ihtiyaç vardır.

Bu tezin amacı, bulanık kümeler yöntemini kullanarak, inşaat işlerinin ve inşaat sektörünün doğasından kaynaklanıp projenin maliyetinde ve iş programında değişimlere neden olan karmaşık ve muğlâk yapılı belirsizlikleri dikkate alan, gerçekçi, güvenilir, maliyet-zaman bütünleşmesini sağlayacak bir nakit akışı modelini geliştirmektir. Sözel ifadeler kullanarak, insani karar verme ve yaklaşık akıl yürütme yeteneklerinden yararlanılıp kullanıcıların tecrübelerinin modele yansıtılması ve nakit akışı senaryolarının üretilmesi hedeflenmiştir. Uzmanlardan alınan maliyet ve zaman bilgileri tahminleri bulanık sayılar olarak modele girilmiştir. Modelin çalıştırılmasıyla, kullanıcının ihale hazırlığı aşamasında projedeki olası maliyet ve zaman risklerini önceden görebilmesini sağlayacak farklı nakit akışı senaryolarının üretilmesi sağlanmıştır. Model Microsoft Excel 2007 programında Visual Basic uygulamalarıyla hazırlanmış ve modelin uygulanabilirliği bir örnekle gösterilmiştir.

Anahtar Kelimeler: Nakit Akışı, Maliyet Tahmini, Bulanık Kümeler

To My Beloved Family

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LIST OF ABBREVIATIONS

FCFM	Fuzzy Cash Flow Modeling
BOQ	Bill of Quantities
MCS	Monte Carlo Simulation
PERT	Program Evaluation and Review Technique
VBA	Visual Basic for Applications
MP	Most Promising Value
PU	Predicate Unit

CHAPTER 1

INTRODUCTION

1.1. Motivation

Business failure of the construction companies is the most important result of the fragile structure of the construction sector. Although there are various reasons of business failure, according to many construction management researches like Peer and Rosental (1982), Pate Cornell et al. (1990), Singh and Lakanathan (1992), Kaka and Price (1993), Boussabaine and Kaka (1998), the main reasons of the bankruptcy of the construction companies is the inefficient control and management of cash. Therefore, controlling and regulating the movement of the cash is necessary for the success of the construction projects.

Cash flow is one of the major tools required for controlling the cash movement of the company by determining the cash in and cash out in the project and demonstrating the possible results clearly. Due to importance of the cash flow in construction sector; many studies have been made by researches for developing a reasonable cash flow model for the construction projects. The researches have experienced many ways of generating a reliable cash flow model such as mathematical techniques, curve fitting equations and soft computing models that would help the financial management of the construction companies, projects and determination of the bidding cost during the tender stage. In spite of the high number of studies about the cash flow, there is no consensus for the reliability and applicability of the existing techniques for obtaining an efficient cash flow. The reasons are listed as follow:

- Most of the existing techniques are only based on the mathematical equation trying to predict suitable cash in and cash out curve but the generated curves are not able to fit different type of construction projects and give dependable results appropriate for different projects (Kaka and Price, 1991).

- Due to the time limitation, the studies do not examine the project in details but seek a basic approach for describing percentage of the total project cost by total project duration so the real reasons of the cash flow problem can not be clearly observed.

- Most of the studies do not consider the uncertainties and risks included in construction projects so that the cash flow results can not comply with cost and time variations of the projects.

- Most of the models are developed for giving only deterministic results with point estimations but these models do not provide range predictions resulting from the uncertainties.

- The cash flow models including the risks of the projects do not consider the non - probabilistic, ambiguous nature of the uncertainties of the construction projects.

1.2. Objectives

In order to overcome the common problems of the existing models, an alternative cash flow model is proposed in this study by using fuzzy set theory. The objectives of this study are listed as follows:

- To obtain a reliable cash flow model considering project risks such as cost overruns and schedule delays.

- Different than other methods, to propose a new cash flow methodology by using fuzzy set theory with an integrated cost - schedule system and give possibilistic results with range estimations.

- To make realistic cash flow analysis by using the expert judgment and human approximate reasoning processes rather than historic data of the past project.

- To create different possible cash flow scenarios during the tender stage.

- To get a computerized, user friendly cash flow model enabling cost - schedule integration and accelerating the cash flow processes.

1.3. Scope

The scope of this study is limited to the development of a cost - schedule integrated cash flow model by using fuzzy set theory and demonstration of applicability of the generated model with an example.

1.4. Organization of Thesis

This study is organized as in the following:

- In chapter 2, the background information about the concepts of cash, cash flow and cash flow management are explained briefly. Besides, cash flow studies made throughout the construction management studies are examined in details and the inefficiency of the past studies are discussed.

- In chapter 3, the concept of fuzzy set theory, fuzzy logic and fuzzy numbers are explained with examples. Also, the applications of fuzzy sets into the construction management studies are reviewed briefly.

- In chapter 4, the methodology of the Fuzzy Cash Flow Modeling (FCFM) is presented, the applications of the processes are illustrated with an example and the results of the analysis are discussed.

- In chapter 5, the conclusion of thesis is presented by the benefits and limitations of the study and some recommendations are offered for future studies.

CHAPTER 2

CASH FLOW IN CONSTRUCTION MANAGEMENT

In this chapter, the fundamental concept of cash and cash flow are explained and the cash flow management techniques are introduced.

2.1. Cash

In main economy dictionaries, the word cash is generally described as "Literally notes and coin; but cash is mostly used as a synonym for money in general" (Black, 1997).

In construction industry, cash is the main engine of the companies for making new investments, starting up new projects and let the projects going on track. It has operational functions in business transaction to get essential resources for providing necessary goods and services used in construction industry. Cash shortage is one of the most dangerous problems that may appear while projects are in progress. If the contractor does not have any plan for covering the amount of cash shortage, the works will stop due the insufficient source of money for compensating the indirect and direct expense of the project such as labor cost, material cost, equipment and overhead costs. Even when the contractors decide to continue to project by lending money from suppliers, they will have to lend money usually with a high interest rate by increasing project cost. Therefore, it is very risky to get loan before identifying the working capital requirements of the project and without having a reasonable cash plan. The consequences of these unplanned loan and extra costs will not only threaten the completion and profitability of the project but will cause the great loss of money and bankruptcy of the company. According to Kaka and Price (1991), Kenley (2003); the inadequacy or the absence of the cash is the main reason of construction companies going bankruptcy rather than lack of profit in the projects. Likewise, Singh and Lakanathan (1992) stated that cash is the most important resource for supporting the day to day activities of the ongoing projects so that absence of this resource will cause the failure of the company. Therefore, the construction companies should control and anticipate the financial situation of the projects and its effects on to the company in terms of cash during the tendering stage and while the projects are in progress.

2.2. Cash Flow

Cash Flow is one of the most common cash forecasting and cost control technique has been widely used by most of the construction companies for a long time. In economy, cash flow is described as “The pattern over time of a firm’s actual receipts and payments in money as opposed to credit” (Black, 1997) or “The flow of money payments to or from a firm” (Bannock et al. 1988). Basically, cash flow defines the expenses and revenues of the single project or whole company per time and reflects their present and future situations by demonstrating net cash conditions. Cash flow is a financial model necessary to count the demand for money to meet the project cost and the pattern of income it will generate (Smith, 2008). Therefore, the usage of cash flow technique is beneficial for both the projects in tender stage and while the projects are in progress since the contractors want to know in all stages of the project that if their predicted cash flow is sufficient for covering the possible financial deficit of the project.

Cash flow is very important for construction projects as:

- A cash flow chart visualizes the net amount of money that will be required during the project as a function of time and gives an alert before the project/company will be in trouble. Therefore, cash flow chart will give chance for displaying the financial risk of the project.

- It enables tracking both cost and revenue of the project through time.

- Cost and time are the two major items for the success of a construction project. Therefore, cash flow analysis is important for visualizing of cost - time integration of the project.

- A cash flow chart summarizes and gives a snapshot of the whole picture of the financial situation of the project, which is easy to understand by users such as project managers, contractors, clients and financial suppliers.

- It is required for describing financial situation of the whole company.

- It provides cash management strategies in order to plan, monitor and control the cash shortage or surplus.

- Cash flow is a useful tool for capital budgeting practices in decision - making process during making new investments (CIB, 2000).

- It is a good cost planning technique helps in taking bid/no bid decisions of the company during tendering stage of the project (Kirkham, 2007). Besides, cash flow will assist the contractors in the selection of contracts that will not cause serious cash problems due to the lack of sufficient financial resources (Kaka and Price, 1991).

- It will be useful in pretender stage for making good estimation and determine the contingency, mark-up percentage of the bid cost.

- It develops a cash conscious culture in the company by promoting allocation, usage and control of resources effectively (CIB, 2000).

2.2.1. Cash Management

Cash management is basically required for planning, monitoring and controlling the cash flow of the project and taking necessary actions to the anticipated cash flow problems for completing the project on time within the budget.

According to CIB (2000), an efficient cash management should:

- Reduce the financial risk of the project, volatility of the company's cash flow and maintain its position by providing enough liquidity.

- Control the expense of the project and consider the possible rate of increase in inflation and its pressure onto the project expenses.

- Optimize cash collection and improve cash capacity to make the project more profitable.

- Plan the company's total credit capacity with banks to supply the foreseeable funding needs.

- Find necessary funds with lowest possible cost.

- Maintain and improve the company's credit control and its credit worthiness to protect against a credit compress from suppliers, banks or from other creditor.

The financial management strategy and the cash flow are the two interrelated items of the project effecting and determining each other. Since cash flow is the plan of predicting the future cash requirement of the project, all attitudes about the prospect of the project should be taken into account while developing cash flow. For instance, for the same project, the final cash flow curve will change considerably if the contractor planning to apply front - loading strategy. Besides, if cash shortage is foreseen by the cash flow analysis of the project, the company should prepare financial management strategies in order to cover the cash deficit and complete the project. Therefore, it is important to determine possible strategies while making cash flow analysis. In spite of the discussions about the morality of using them, there are some tactics generally applied by the contractor in order to improve the cash deficiency of the project stated by Marc (2009) as below:

- Front-Loading: Front-loading is mostly used in unit price type of contracts. In tendering stage, the contractors enhance the cash flow conditions without changing the tender price by increasing the work items going to be constructed at early stages and reducing the those going to be held on at the end in order to balance the cost of the original tender price.

- Back-Loading: When the contractors foreseen cash problems due to inflation, they try to postpone the items to be constructed at the expense of the earlier ones.

Besides, there are some policies should be taken to enhance cash flow of the project and reduce project expense for funding the project in case of cash shortage. Atallah (2006) suggests some techniques for maximizing, accelerating cash inflow and controlling cash outflow:

- To negotiate with the client for getting fair and logical payment terms and retention amount so that the cash requirement of the project will not threaten the project success.

- To submit the first invoice as soon as possible and get the cost of mobilization (site office setup, supervision, temporary facilities), bonding and insurance cost.

- To introduce the completed works to the client as soon as possible for making checks and strictly following up the deserved receivables.

- To practice prudent contract and change order management for improving the chances of getting paid.

- To accelerate the schedule for improving the cash inflow and decreasing the overall indirect cost of the project.

- To retain at least the same amount of money from the subcontractors in progress payments.

If the company could not take the necessary actions contractually for improving cash flow, lending strategies should be developed for meeting the financial needs of the project. As discussed before, due to the risky nature of the construction industry, high rates of business failure and bankruptcy occurred in the construction sector and many banks are unwilling to lend money to the contractors unless they are reliable (Atallah, 2006). Besides, even if the company is found eligible by the financial supplier, the lenders will loan with high rate of interest at time of cash shortage since the late interference on to project may not reduce the financial risk (Halphin and Woodhead, 1998).

2.3. Cash Flow Studies in Literature

Since making cash flow is crucial and inevitable for taking healthy decisions, making good estimations and having efficient financial control in construction industry, researches developed cash flow techniques for making more accurate and reliable cash flows. Most of the cash flow models have been developed for the aim of assisting the client in decision making processes while making new investments and helping the contractor during the tender stage.

2.3.1. Mathematical and Statistical Models

Due to the inefficient time and cost resources during the bidding stage, most of the researches suggested mathematical or statistical models based on historical data of the previous projects. The overall cash flow of the project is estimated by plotting curves demonstrating the cumulative project cost percentage in terms of the cumulative project duration percentage.

Wray (1965) made the first study emphasizing importance of describing cumulative cost in terms of cumulative duration of the project. Wray (1965) suggested that both the clients and the contractors should have a curve showing the monthly cumulative value of project in order to make compression with budgeted plan and enable efficient cost control (cited from Kaka and Price, 1993).

Nazem (1968) presented a methodology based on data provided from the completed projects. Nazem developed a standard reference used to anticipate the future capital requirement of the project. Nazem concluded that an ideal reference curve could be gathered by taking the average values of the previously completed similar projects and the cash balance curve could be obtained indirectly from the cash - inflow and cash outflow curves. Although some construction firms used this model as a cash flow prediction tool, Nazem's model was not commonly preferred due to the difficulty in deriving an ideal, average curve.

In contrast to Nazem, Jepson (1969) declared that S-curves were not reliable for making estimation and controlling the performance of the project since the actual values would be different that the estimations and Jepson (1969) offered to use 'Generating and Component Curves' for making individually net cash flow of the project.

The aim of explaining the mathematical relationship between project cost and project duration was explored by Bromilow (1969). Bromilow presented a formula describing time as an equation of cost such that $T = KC^B$ where K is a variable showing efficiency, C is the total value of the project, B is factor of sensitivity and T is actual duration of the construction. However, Bromilow's suggestion did not consider the possible delays in progress payments.

Cash flow analysis began to be more important in the construction management studies since the beginning of 1970s when the construction sector suffered from the increasing interest rates and its considerable negative effects to the ongoing construction projects (Kenley and Wilson, 1989). Hardy (1970) made analyses by using the data of 25 projects. In the study, Hardy (1970) applied systematic delays to the inflow and outflow curves. Finally, Hardy (1970) concluded that there was no similarity between the shapes of the curves even all the projects selected from the same category. It is difficult to discuss the reliability of Hardy's model and make comparison with the actual data due to the insufficient information about the payments of the project.

O'Keefe (1971) analyzed more than one project to estimate the possible financial requirements and determine the factors effecting cash deficiency and presented that profitability of the project is one of the important factors effecting financial disorder. Different than S-curves, Cleaver (1971) suggested a cash flow model estimating the financial requirement of the project based on the information coming from balance sheets but this model was not widely preferred or used for making cash flow analysis.

Mackay (1971) used a computer program for analyzing the profiles of different projects. Mackay (1971) searched that whether the selected shape of the value curved used for cash flow estimating would affect the results of the cash flow model. Firstly, Mackay classified the input data into cost categories and entered the project based data (such as value of the contract, expected profit and estimated retention amounts) into the program by applying systematic delays. Breaking the curve into straight lines, Mackay analyzed the sensitivity of cash flow with different shape of value curves. Finally, Mackay concluded that the shape of the selected curve does not have any considerable effect to the results of the cash flow. Trimble (1972) also investigated the effects of the shape of the selected curve onto the net cash flow and Trimble (1972) judged the same result as Mackay did.

Bromilow and Henderson (1974) developed an ideal standard S-Curve by making regression analyses where the historical data of 4 projects with different characteristics were used for curve fitting. However, having too many constants decreased the flexible usage of the model. Zoiner (1974) generated cost commitment curves by using data of the projects with different progress rates and preparing work schedule in details but Zoiner did not take into account the possible errors occurred while making schedule. Therefore, the result of the Zoiner's model may not be accepted as reliable.

Specific studies were made to develop a reliable cash flow model for clients. Kennedy et al. (1970) prepared scheduled based cash flow model for efficient cost control purpose. Peterman (1972) generated a computerized cash flow model using bar charts to deliver value curves of a single contract. Balkau (1975) developed a value curve for estimating total cash flow of the project by performing certain delays in payment time and later, this model was improved and used by Bromilow and Davies (1978).

Ashley and Teicholz (1977) proposed a model for estimating future cash flow of the project by providing a standard curve that would be used instead of detailed cost and schedule calculations. Ashley and Teicholz (1977) classified the cost items into main cost groups such as labor cost, material, equipment cost and entered the input by performing certain delay intervals. Ashley and Teicholz (1977) concluded that making a cash flow without having a financial strategy would be the main reason of the failure of cash flow prediction models and Ashley and Teicholz suggested strategies about improvement of financial disorders. Peterman (1973), Reinschmidt and Frank (1976), and Bericevsky (1978) also studied for generate cash flow models making accurate estimates.

Kerr (1973), Bromilow and Henderson (1974), Balkau (1975), Peer (1982), Tucker and Rahilly (1982), Drake (1978), Gates and Scarpa (1979), Singh and Woon (1984), Miskawi (1989), Khosrowshahi (1991), Skitmore (1992) used mathematical models (such as linear or polynomial regression, biquadratic equations) in the tender or planning stage of the project that based on historical data for developing standard value curves by fitting them into the collected data.

Hudson (1978) also studied a mathematical model by utilizing the data of some hospital projects in order to generate an ideal curve. Different than other mathematical/statistical models, Hudson used less constant while developing S - curve. Finally, Hudson confessed the difficulty of explaining the results of the historical data and estimating the future cash requirements of the projects with simple mathematical models. Keller and Ashrafi (1984) emphasized the importance of considering sophisticated features of the projects while making cash flows.

In some studies, the researches focused on speeding up delivery of the results and enhanced their mathematical models by using computer programs. McCaffer (1979) generated computerized value curve models that would be used as an alternative to the complex and time consuming schedules based on network analyses. McCaffer applied certain time delays both into S - curves describing cash inflow and cash outflow so that a more realistic net cash flow results could be obtained. Similarly, Khung (1982) generated a computer program for delivering value curves faster. Besides, Allsop (1980) constituted a library of S - curves linked to a computer program by which the user could select value curves of the similar projects to get the cost and time prediction and make cash flows analysis.

The cash flow models mentioned above are nomothetic type of studies since the researchers presented general rules and principles for defining all type of construction cost estimations and developing cash flows without bearing in mind the unique nature of the construction projects. Oliver (1984) made cash flow models with four projects and tested the accuracy of the existing models. According to the results, Oliver (1984) judged the unique nature of construction projects. Oliver stated that the historical data are not reliable for making accurate cash flow. Besides, generating models by only including time and cost factors will not give realistic results. There are more factors have to be considered for meeting the quality, cost and time related requirements of the projects such as the political and economical factors, managerial systems and actions of the project team, the relationship between the labors etc (Ireland, 1983).

Berney and Howers (1983) were the first researches attempted to create a unique curve for a single project with a general equation. This attempt became a touchstone in cash flow studies and effected following researches such as Kenley and Wilson (1986), Tucker (1986) and Kaka and Price (1991) who considered each construction projects individually. Kenley and Wilson (1986, 1989) examined the most common problems of the cash flow models, explained the reasons of the inaccurate results of previous studies, suggested comparison technique to reflect the unique characteristic of each project and named their model type as ideographic.

Kaka and Price (1991) suggested a new model based on cost commitment curves rather than value curves for developing cash flow in tender stage. In previous models, Kaka and Price obtained an ideal standard value for describing the net cash flow profile of the project. The cash in curve was calculated by the applying certain retention percentage and time lags into the value of the contract and cash out curve was delivered by taking the definite percentage of the cash in value and applying lagging periods. The authors proposed that a more reliable ideal curve would be obtained by using cost commitment curves instead of value curves. The net cash flow profile was obtained by the deducting the cash out values from cash in ones.

The model was tested with five different projects. The logit transformation technique was used while measuring the accuracy o the model. According to the results, the systematic errors between the actual and estimated ones were found small. Although this model seemed reliable at first glance, making tests with a set having only 5 projects in the same type of construction made the accuracy of the model questionable. Besides, important risk factors such as complexity and unique nature of each project were not considered.

Soft computing techniques also used for developing a more reliable cash flow estimation model. For instance, Lowe et al. (1993) used expert systems for modeling cash flow. Besides, Boussabaine and Kaka (1998) suggested a non-linear technique that neural network were used while developing the cash flow model. According to Boussabaine and Kaka, the previous models had many disadvantages such as it is difficult to determine the correlation between the variables effecting cash flow. Also, the models developed by regression technique are not able to learn and find general solutions by using inadequate and unreliable historical data and since the factors effecting the shape of the S - curve is not clear, the relation between the input and output data is complex and uncontrollable. Therefore, the authors proposed a model based on neural networks which is good at adapting non-linear data format. Although the model aims to fill the gaps of the previous models, it can not meet the target since the model is rely on the quality of the historical data as previous mathematical models did. Besides, the model does not consider the retention amount made in progress payments, lagging time applied to the cost outflow and inflow of the project that will change in different projects.

The reliability of S - curve based cash flow models have been discussed in more specific studies. Evans and Kaka (1998) searched the possibility of delivering accurate standard cash flow curves if the historical data of a specific type of projects are clustered and analyzed. The authors examined the data of 20 food retail building projects and applied logit model for getting an accurate average S-curve. Then, Evans and Kaka divided the project cluster into more specific groups according to time and cost of the projects and again applied the logit transformation model. The test results showed that a value curve should not be used for developing a reliable standard cash flow curve even if they are examined in a specific type of projects.

2.3.2. Cash Flow Models with Cost Categories

Dividing cost outflow into categories is another method has been commonly used by many researches while making cash flow models since 1970s. As developing cash flows in details is considered as time consuming, many researches generated models which were easy to build; however, as discussed above, this time the accuracy of the rapid models did not satisfy the users. Therefore, the researches created new models which were again easy to build but included more information than previous models. The main idea behind the classification of cost categories is to inspect the total project cost in details and to generate cost flow curve for each category rather than a single curve for the whole expense.

Fondahl and Bacarreza (1972) made the first study about developing cash flows with different cost classes. They declared that the cash flow models should use different cost flow curves for different type of resources. As mentioned before, Ashley and Teicholz (1977) made a cash flow model by grouping the expense of the project under the label of labor, material and equipment costs by assigning each of them a specified percentage and describing them in terms of the percentage of the total project cost. Finally, Park et al. (2005) presented this technique in a more realistic way and developed a cash flow estimation model. Different than previous models, the authors considered time lags and established the model under this principle. In the model, the total cost of the project was divided into labor, material, equipment, subcontractor, indirect cost and different time lags was applied to each category so that cash flow anticipation was made by generating different cost flow curves.

In this study, although a realistic model was explored to be developed by providing varying S - curves, the proposed model did not consider the reliability problems inherited from the mathematical models. Besides, although the model claimed to meet the requirement of determining the financial statements of the project by progress, examining cost categories will be too general for inspection of the problems occurred during the project and taking the necessary precautions properly. Additionally, the model did not consider the uncertainty of the construction projects and was not flexible enough for covering the risky nature of the project. Therefore, in order to overcome these cash flow problems, a more reliable and integrated approach is required.

2.3.3. Integrated Cash Flow Models

As stated and declared many times by construction engineering and management researches, cost and schedule are two important items commonly used for determining the success of the project by enabling efficient control process in which initial schedule and budget can be compared with the progressed ones. Hence, the unexpected problems causing the failure of the project can be foreseen with details. The main idea behind the integration is to reflect the interrelations and effects of the cost and schedule into the project's monitoring and controlling mechanisms and provide to take necessary actions to the evaluated problems. In contrast to the models developed by mathematical and statistical models, the cost - schedule integration technique is used in activity base level. Therefore, the user should have enough information while constructing an integrated cash flow model. The integrated models enable the users to have a well organized management system presenting efficient cost and schedule control. Besides, integrated models provide an accurate cash flow analysis since the estimations are made with detail information. The main drawback of the integrated systems is they cannot be efficiently used when the information about the project is not sufficient or when there is no enough time for preparing such models in details.

Cost - schedule integration has been widely used in construction management studies for making cash flow models. Although, the models generated by this technique require too much time and effort in the absence of a high speed computerized systems, the practitioners realized the importance and enhanced the integration models. Reinschmidt and Frank (1976) used a probabilistic simulation method that integrate cost and schedule items by giving varying cost and durations into the activities. Sears (1981) stated that the schedule of the project should be considered while developing cash flow models that formulated by project duration. In this study, the resource and cost items were tried to be assigned the related activities with computer software but the lag between time of using the resource and payment made for it is ignored. Besides, Bennet and Ormerdo (1984) used bar charts for integrating cost and schedule integration and considered the uncertainty occurred in the projects duration due to the unexpected weather conditions which also cause cost variations in the project initial cash flow. Assigning range estimates to the cost and duration items while generating stochastic predictions was also applied by other researches like Isidore et al. (2001).

Teicholz (1987), Mawdesley et al. (1989), Harris and McCaffer (1989), Booth et al. (1991), Carr (1993), Abudayyeh and Rasdorf (1993) also used computerized cost - schedule integration models. In these models, the researches included much more detail than previous mathematical models for getting a more accurate estimate. For instance, the bill of quantities of the project, the expense of each resource and the duration of each activity were determined in details by dividing the project elements into cost codes and activity codes.

Navon (1995) determined main problems of the cost - time integration models and proposed a model that enabling cost and resource compatibility while making integration. In this study, Navon developed a computerized integrated model in project level which was easy to use and did not spent too much time while loading input data. According to Navon, the main handicap of the integration models was the integration of cost and schedule data since the schedule is constructed in activity base but the cost items are defined and classified in physical items of the project. To overcome this problem, first of all different type of relationship between the activity and cost items were defined as one to one (one cost items for one activity), one to many (more than one cost items into one activity), many to one (one cost items for more than one activity) and many to many (one to many and many to one relations are combined). The cash flow forecasting system was developed by using the BOQ , including total price and quantity information of the project; schedule, including duration and starting date of the project and estimate, describing the assignment of the resource items into with their cost. Each cost, BOQ and schedule items were defined by a code system and the integration was made by matching of the cost and activity codes. Besides, with the help of the predefined resource data base and estimation list, the model commits to assign each resource to the related activity of the project automatically. After that, the project specific time lags are applied to the inflow and outflow of the items of the cash flow, the subcontractors and overhead costs are determined and the retention amount of the progress payment is assigned and finally the program was run. Navon presented another study in 1996. In this time, Navon introduced a company level cash flow model which computing the whole cash requirement of the company by dividing the projects that the company have into two that the projects with limited data and the projects with the detailed data. After finding the cash flow of each project individually based on the logic of the study in 1995 and adjusting their costs according to the inflation rate, the model calculated the all cash requirement of the company expected to be beneficial in giving alert about the cash flow requirement of the company.

The studies of Navon show how a cost - time integration technique is applied and accurate cash flow results can be delivered by using computer programs without having too much human involvement. However, the models that the Navon proposed does no really consider that effect of possible problems into the cash flow of the project due to the uncertainties arising from the nature of the construction projects like unexpected project cost, weather conditions, labor, equipment, material expenses etc. Besides, in the study about the cost - time integration model and accuracy of the cash flows, Chen et al. (2005) criticized model of the Navon about applying the same amount of time lag into different subcontractors although in real life, the contractors generally adjust their payment strategy according to the performance and credibility of the subcontractor.

The cash flow models introduced up to this point have certain drawbacks. For instance, as it is declared by many researches before, due to its easy and rapid use, although the mathematically/statistically developed cash flow models are required in tender stage, they are not reliable for making estimation and determining the financial requirements of the projects. The reasons of the failure of traditional methods are stemmed from the uniqueness of each project having different characteristics so that each construction project should be examined individually. Besides, the traditional models have not considered the varying nature of the construction projects due to potential risks inherited in the nature of the construction industry and only focused on fitting the historical data of previous projects into the their modeling curve. Therefore, the proposed models become totally deterministic and they are not flexible enough for meeting the changing nature of construction projects. Khosrowshahi and Kaka (2007) states that the strong dependency of models into the polynomial equation and historical data confines the applicability of the model in different cases, enforces the user to develop new mathematical equation in every project and decrease the reliability of generated models due to the generalization made while presenting each model. Moreover, the traditional models did not consider and appreciate the opinion of experts while making cash flow models that is very important for understanding the nature of the project. Additionally, the most of the models mentioned before did consider the risk factors causing uncertainty in the cash flow estimation so that the proposed models are not realistic enough for meeting the changing nature of the construction conditions.
Therefore, alternative models including the cost and schedule variations of the project are required for obtaining a more reliable cash flow model.

2.3.4. Cash Flow Models with Uncertainty

The construction activities maintain both risk and uncertainty. Risk is the state of uncertainty can be placed between the certainty and uncertainty. Some authors define risk and uncertainty as different phenomena. According to Flanagan and Norman (2000), risk can be defined with quantitative expression and takes place in a calculus of probabilities, however, in uncertainty there are no suitable historical data for existing situation since it is the first time, when such kind of event occurs. However, as many writers like Loosemore et al. (2006) did, in this study, the risk and uncertainty is used for expressing the same meaning such that risk is mentioned as synonymous of uncertainty.

The construction works include too many activities from different disciplines and construction industry is subjected to more risk (or uncertainty) than other industries. Although similar activities take place, each project shall be considered individually due the changing conditions of the projects like the location, contract type, cost, quality, time situation etc. Additionally, there are lots of uncontrollable external factors affecting the fate of the projects such as the economic situation of the country where the construction takes place, fore-majeure events or the weather conditions, threatening the construction performance. Besides, the interrelations between the activities make the present situation more complex and it is very difficult to give exact prediction about the possible outcomes of the project. In short, uncertainty and risk are two important phenomena take place in the nature of the construction work. Although risk and uncertainty are unavoidable in construction projects, their negative, devastating impacts will be limited by identifying the reasons behind, making further analyses and giving though responses as it is normally done in risk management procedure.

The risk that not considered during the planning stage may cause the failure of the project since the improper decisions made in the construction industry are mainly arising from the illusion of certainty and knowledge. Therefore, the risky nature of the construction projects should be considered while making estimation for the future outcomes. The process from the design stage to the end of construction, there are many risk factors that a construction project will possibly have and in the same project each party (owner, contractor, consultant, subcontractor etc.) contributing the completion of the project will have different types and degrees of risks. There are many studies in literature concentrate on the identification and classification of risk factors. In the studies reflecting risk from the contractor’s point of view, the risk can be separated into main categories as the financial, economical, political, cultural, legal and market risk and the detail risk management strategies will be developed during bidding stage according to the importance and impact of the each risk factor.

The main aim of cash flow modeling is to warn the practitioners in case of possible financial problems may happen in the future. The models without including uncertainty will not meet the objective of cash flow analysis. Therefore, in construction management studies, the researches realized the importance of risk factors and included them in cash flow models for reflecting the uncertainty. There are several studies examining the factors affecting the cash flow of the project and determining the reasons of uncertainty in cash flow models. Lowe (1987) declared that pricing, valuation, contractual, programming and economic factors are the main items responsible for the changes in cash flow forecast. Moreover, Odeyinka and Lowe (2001) stated that the design and specification changes occurred different than original project and the changes in the schedule are the most critical factors effecting cash flow prediction. Additionally, Smith (2008) claimed that the contract type, the characteristic of the payments have to be done according to contract, the delay between the incurring cost and paying the bill, the delays in project schedule and the speed and extent of reimbursement of variations and construction claims are the factors that the cash flow of the project with relatively short duration is sensitive to.

Uncertainty factor is considered in different type of cash flow models like mathematical, integrated and soft computing models. In these studies, the researchers generally identified the main factors behind the uncertainty and then presented a cash flow model for prediction. In previous mathematical models, the uncertainty exerted in the models by defining different payment lags into the cost inflow and outflow curves that also gives deterministic results so that these models does not really consider the risk in cash flows. After the study of Kenley and Wilson (1986), the investigators realized the variability of cost curves and developed more reliable models.

Kaka (1996) made a comprehensive study, examining the factors affecting the cash flow accuracy and tried to generate more flexible cost curves used in cash flow prediction. After determining the cash inflow and outflow curves with the concerned mark-up and retention amounts, the stochastic cash flow is obtained by making subtraction. Kaka's model is a good sample for application of uncertainty in mathematical models but the proposed model is also complex and may not give satisfactory estimations due to generalizations made while obtaining the cash inflows and outflows.

Boussabaine and Elhag (1999) applied fuzzy technique for development of cash flow. Boussabaine and Elhag used fuzzy technique for providing an alternative suggestion to the cash flow problems resulting from the ambiguity of the construction projects and trying to help in decision making process for choosing the appropriate cash flow alternative. The data of 30 projects were used and divided into nine completion periods. In the method, the imprecision was handled by using different weighted degree of beliefs with different alpha-cuts. However, the model could not utilize from possibility theory and reflect the human decision procedure properly.

In stochastic modeling, since there is not enough information in the bidding stage or in the beginning of the project, the researches used a technique for making reliable estimations with limited data and proposed simulation techniques. Monte Carlo Simulation (MCS) is a powerful technique generally used in construction management studies. In cost - time integration based models, MCS is generally used for developing cash flow projection of the project. The general process is performed by the following steps: First, the project is divided into activities. Then, the network relationship between the activities is developed. After that, the uncertain cost and scheduling items are assigned to the related activities by selecting the suitable probability distribution. For instance, triangular distribution is generally preferred by users and optimistic (lowest), most likely and pessimistic (highest) values are assigned accordingly. The possible correlations between the items are entered to the model. Then, the plots of N number of cumulative frequency histogram are obtained for the project cost and schedule results are delivered. Finally, the results are interpreted by user/experts (Flanagan and Forman, 2000).

Bennett and Ormerod (1984) performed an example of MCS technique for cash flow analysis. Bennett and Ormerod developed a computer program for developing a simulation based model including external factors effecting cash flow model. Bennett and Ormerod used direct cost, indirect cost, weather data, resources, resource constraints, bar chart schedule as input data and assigned probability distribution to the each activity to generate cost and cash flow curve based on stochastic cost and durations. Finally, the cash flow curves with confidence interval were delivered.

Despite its advantages, there are certain drawbacks of using MCS cost and schedule estimation. In order to get reliable results, large number of iterations should be made. The time required and spent in developing risk analysis model and making analysis is one of the disadvantages of MCS technique the practitioners complain about. Besides, there will be correlations between the parameters used in the analysis such that each correlated item should be entered the program manually. If the user does not enter the correlations properly, the results of the analysis will not be reliable and will mislead the decision makers. Additional, the probability distribution selected for each probabilistic cost and duration item is important since the result is sensitive to the selected input distribution (Ferson, 2002). Besides, the probabilistic approach may not be appropriate for all construction projects since the uncertainties met in the construction projects are not really appropriate for the axiomatic fundamentals of the probability theory (Behrens and Choobineh, 1989).

CHAPTER 3

FUZZY SET THEORY

In this chapter, the fuzzy sets, fuzzy logic theory and applications are explained briefly.

3.1. Fuzzy Theory and Fuzzy Logic

Intelligence could be measured by the adaptation talent of the living creatures having ability to survive. Likewise, an intelligent prediction model trying to anticipate future outcomes of the project should adapt itself into uncertain, changeable conditions of the real life in order to give outstanding anticipations (Ayyub and Klir, 2006). The first rule of the adaptation lies on the accepting the deficiency realistically and realizing the nature of the varying conditions threatening the success of the project.

As discussed in previous chapter, almost every activity, event, action happening in the world surrounding us contains uncertainty. When the source of the uncertainty is questioned, certain reasons will be mentioned as the source of uncertainty like lack of knowledge, illusion of knowledge; ignorance and complexity. As there are many reasons of uncertainty, the nature of them are also differs. In history, the uncertainty has generally been defined by using probability theory which gives mathematical explanation of an uncertain event due to the randomness. According to Ross (1995), “A random process is the one where the outcomes of any particular realization of the process are strictly a matter of chance; a prediction of a sequence of an event is not possible.” It means that randomness is related to occurrence of an event by chance and the results can be estimated by using probability theory. However, there are also uncertain events cannot be treated with probability theory due to the ambiguity in its nature. The uncertainty existing from the situations related to the human perception and judgment are not related to randomness and they could not be expressed by general mathematical theory. For instance, the linguistic expression used in describing a person or situation like “tall person”, “old people”, “good weather “, “bad conditions”, “beautiful woman”, “talented labor”, “slow car” or the actions of defining the weather, choosing clothes, preferring a car have nonrandom type of uncertainties and they cannot be clarified by occurrence or tests. Since the decision of complex daily issues are generally related to human decisions and the general probabilistic theories are not satisfactorily explain the uncertainty resulting from human subjectivity, a new powerful tool was proposed by Lutfi Asker Zadeh (1965) called Fuzzy Set Theory. Fuzzy set theory is a mathematical theory which is used for modeling the imprecise, ambiguous, vagueness nature of complex systems when there is not enough of information about the problem. The idea behind the fuzzy sets is related to fuzzy logic. In classical logic, the world is defined by binary extremes such as zero or one, black or white, good or bad, true or false, big or small, short or tall, guilty not guilty etc. However, in fuzzy logic, there are also gray areas where the answer of the question is ambiguous and could not be classified in a polarized cluster as it is in real world. For instance, in Figure 3.1, it is required to make a model describing the weight of students in a class, according to binary logic. According to graph, the students heavier than 80 kg are defined as fat with one membership value and lighter than 80 are called slim with zero membership value. It means that there is no difference between the 80 kg and 120 kg in terms of binary logic since both of them are labeled as fat whereas 79 kg is defined as slim although it is near the border. Therefore the classical logic is not enough for proper modeling of such questions. However, in Figure 3.2, the state of being fat is graduated from zero to one by using fuzzy logic so that the meaning of being near could be used in the modeling.

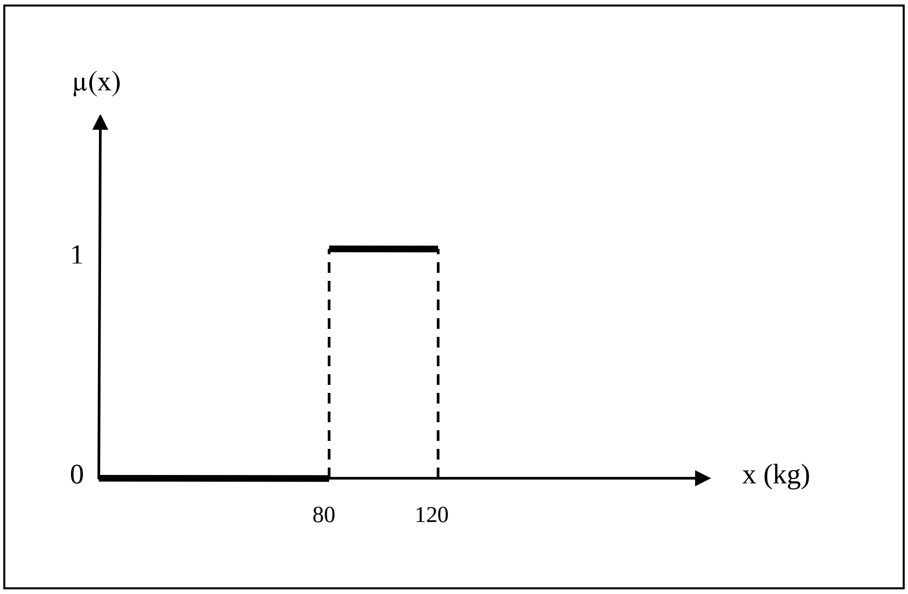

Figure 3.1: Defining Fat Students with Classical Logic

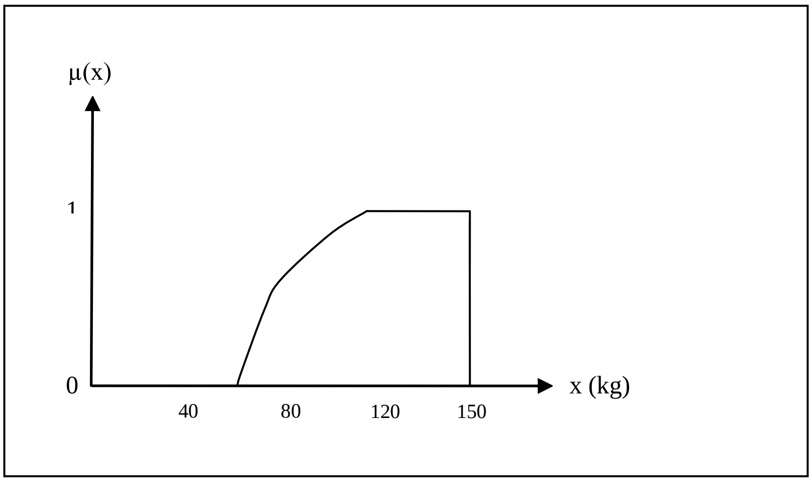

Figure 3.2: Defining Fat Students with Fuzzy Logic

Different than computers' binary world, the human beings generally understand and make judgment about the imprecise situation with approximate reasoning which gives a proper relation between the input and output of a complex system. The power of the technique is about the well reflection of human intuition into the certain models with mathematical expressions. However, there are also limits of using fuzzy theory. It is proposed to use fuzzy logic when there is lack of information about the too complex problem and the required precision is not high. In Figure 3.3, the relation between the complexity of the system and required precision of the model is described (Ross, 1995). It can be seen that mathematical equations are good enough for the systems having little complexity and model - free methods as neural networks are sufficient methods in decreasing uncertainty with their learning capacity and they will be used in defining more complex systems. Whereas, fuzzy systems will be preferred for the case when there is no enough precise data about the too complex systems. Therefore, it is important to determine if it is useful to use fuzzy systems.

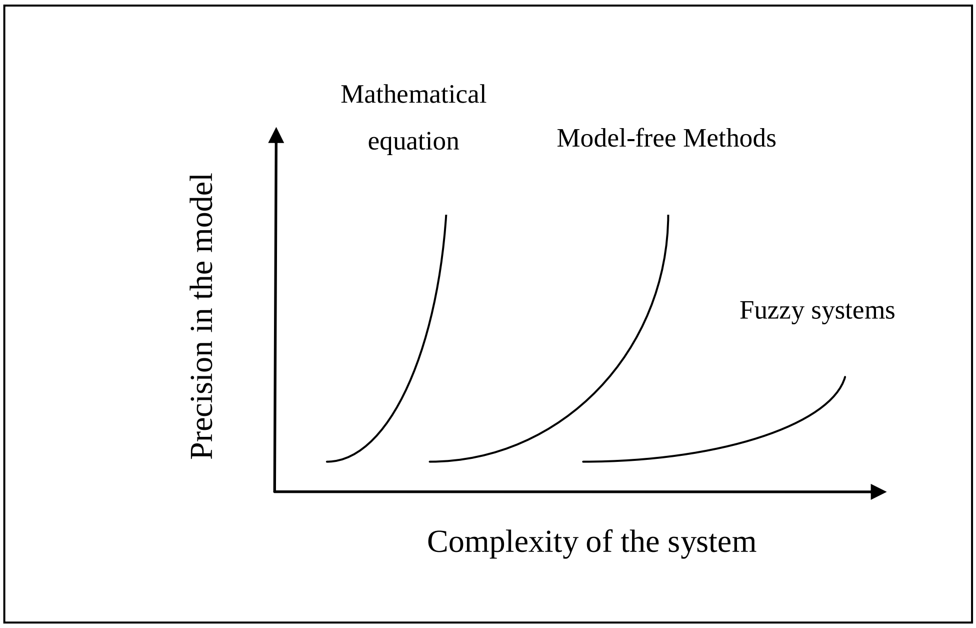

Figure 3.3: Complexity of System vs. Precision of the System

Fuzzy sets are the mathematical explanation of fuzzy logic. In order to understand fuzzy sets, it is better to define the classical set theory with its applications.

3.1.1. Classical Sets

Set will be defined as the mathematical abstraction of the universe that the objects in space are collected. The elements of a set are labeled and classified according to the boundaries including them and the classical sets are the ones having certain prescribed limits that there is no ambiguity about the boundary lines. That's why in classical set theory, an object is either an element of a set or not. It means that if an element is not a member of a set, it is not used while making calculations. The notation of element x belonging to a crisp set A is shown as $x \in A$ and outside the set is shown as $x \notin A$.

Membership Function:

The classical sets are also named as crisp (well-defined) sets. The function showing the element of x is either member of set A or not called membership function. In a crisp set A, the elements are defined with the membership function μ_A that the membership value is either 0 or 1:

$$\mu_A(x) = 1 \text{ for } x \in A$$

$$\mu_A(x) = 0 \text{ for } x \notin A$$

Therefore the null sets, "$\varnothing$", are defined as for $\forall\ x \in U$, $\mu_A(x) = 0$ where U describes universe.

Although there are many operations, the main operations made with crisp sets are:

Union:

The union of two crisp sets, A and B is indicated as $A \cup B$ and it shows the elements in the universe are either belongs to A, B or both of them.

$$A \cup B = \{x | x \in A \text{ or } x \in B\}$$

Intersection:

The intersection of two sets, A and B is shown as $A \cap B$ and it shows the elements in the universe are belonging to both A and B.

$$A \cap B = \{x|x \in A \text{ and } x \in B\}$$

Complement:

The complement of a crisp set A designates the elements in the universe do not participate in crisp set A.

$$\bar{A} = \{x|x \notin A \text{ and } x \in X\}$$

Difference:

The difference of set A with respect to set B is indicated the elements belongs to A but does not belong to B.

$$A \mid B = \{x|x \in A \text{ and } x \notin B\}$$

Although there are many properties, the most important ones showing similarities with fuzzy sets are as follows:

Commutativity: $A \cup B = B \cup A$

$A \cap B = B \cap A$

Associativity: $A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cap C) = (A \cap B) \cap C$

Distributivity: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Idempotency: $A \cup A = A$

$A \cap A = A$

Identity: $A \cup \varnothing = A$, $A \cap \varnothing = \varnothing$

$A \cap X = A$, $A \cup X = X$

3.1.2. Fuzzy Sets

Fuzzy set theory can be defined as the formulation of uncertainty and obtained by the widening the binary logic of the classical sets into multivalent logic and partial membership concept (Baykal and Beyan, 2004).

There are many differences between the fuzzy set theory and the crisp sets. In crisp sets, an element either belongs to set or not, therefore the conversion from membership to non-membership is certain and clear. However, in fuzzy sets, the boundaries of the sets are indefinite such that the elements are both member of a set and not. Therefore, different than binary logic of classical sets, the membership concept of the fuzzy sets gradually changes. It means that the elements in a fuzzy set have varying degree of belonging graduating from full membership to non-membership. The well-known apple case is a good example for explaining the difference of fuzzy sets from crisp sets. In a set of apple, all apples are full member of sets. But when one of them is bitten, the question arises if bitten apple is still fully belonging to apple set or not. Besides, what is the boundary of an eaten apple, full membership or non-membership? The crisp sets theory could not answer those questions properly and fuzzy sets are beneficial and preferred for understanding, defining and describing such kind of cases where the boundaries become vagueness and ambiguous.

3.1.3. Membership Function

In a fuzzy set *A* (written in italic format) of the universe U, the elements are defined with the membership function μ_A and the membership value varies from 0 to 1.

$$A = \{(\mu_A(x)/x, x \in A, \mu_A(x) \in [0, 1]\}$$

where the $\mu_A(x)$ is the membership value of element x in fuzzy set *A*.

A fuzzy set is called normal or normalized when at least one item has full membership. The membership grade equals to 1 and in a set where max $\mu_A(x) < 1$ is called subnormal or non-normalized fuzzy set. The Figure 3.4 and 3.5 show the normalized and non - normalized fuzzy sets.

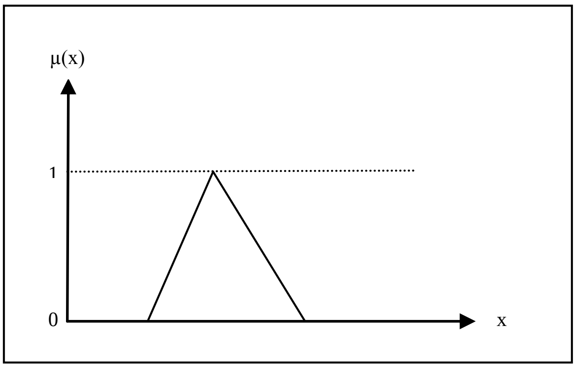

Figure 3.4: Normalized Fuzzy Sets

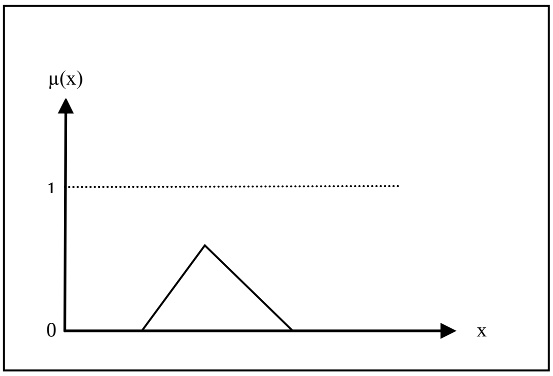

Figure 3.5: Non - Normalized Fuzzy Sets

Besides, fuzzy sets are labeled as convex when the membership function values are strictly "monolithically increasing, monolithically decreasing or monolithically increasing then monolithically decreasing with increasing values for elements in universe" (Ross, 1995). The Figure 3.6 shows the normalized non - convex fuzzy set and the Figure 3.7 indicates the non - normalized, non - convex fuzzy set.

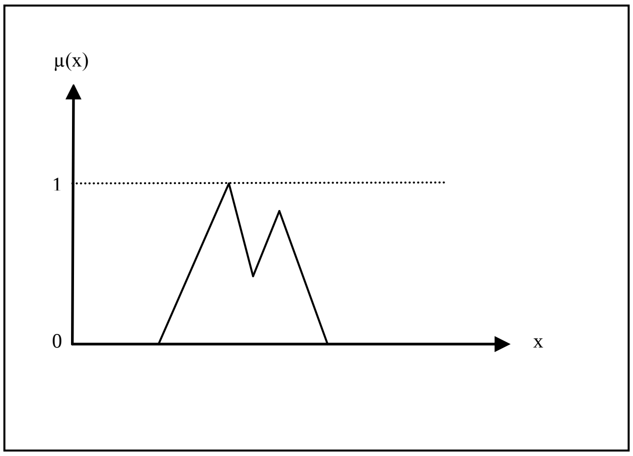

Figure 3.6: The Normalized Non - Convex Fuzzy Sets

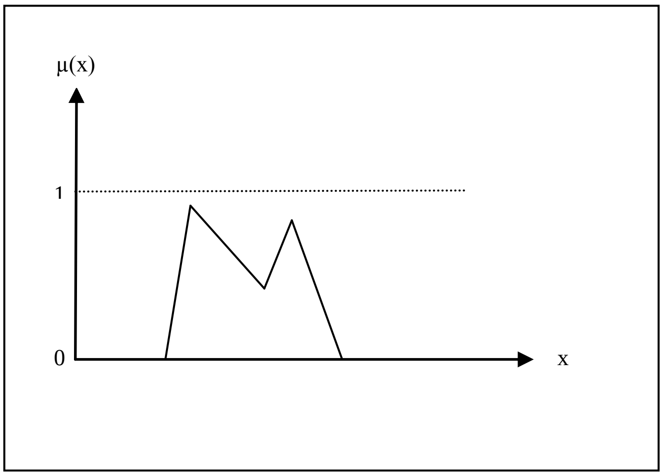

Figure 3.7: Non - Normalized Non - Convex Fuzzy Sets

For fuzzy sets $A = \{(\mu_A(x)/x,\ x \in A,\ \mu_A(x) \in [0,1]\}$ and $B = \{(\mu_B(x)/x,\ x \in B,\ \mu_B(x) \in [0,1]\}$, U is universe, the main operations made with fuzzy sets are:

Union:

The union of two fuzzy sets, A and B is indicated as $A \vee B$ where

$$\mu_{A \cup B}(x) = \mu_A(x) \vee \mu_B(x) = \max(\mu_A(x), \mu_B(x)),\ x \in U \quad \textbf{(3.1.1)}$$

Intersection:

The intersection of two fuzzy sets, A and B is shown as $A \wedge B$ where,

$\mu_{A \cap B}(x) = \mu_A(x) \wedge \mu_B(x) = \min(\mu_A(x), \mu_B(x)), x \in U$ **(3.1.2)**

Complement:

The complement of a set A designates the elements in the universe do not participate in fuzzy set A where

$\mu_{\bar{A}}(x) = 1 - \mu_A(x)$ **(3.1.3)**

Difference:

The difference of fuzzy sets A with respect to set B is indicated the elements belongs to A but do not belong to B.

$A \backslash B = A \cap B(x) = \min(\mu_A(x), 1 - \mu_B(x))$ **(3.1.4)**

For instance, for A and B are both fuzzy sets as $A = \{(x_1, 0.5), (x_2, 0.4), (x_3, 0.7)\}$ $B = \{(x_1, 0.7), (x_2, 0.1), (x_3, 0.4)\}$ and universe, $U = (x_1, x_2, x_3)$ the main operations are as follows:

$A \cup B = \{(x_1, 0.7), (x_2, 0.4), (x_3, 0.7)\}$

$A \cap B = \{(x_1, 0.5), (x_2, 0.1), (x_3, 0.4)\}$

$\bar{A} = \{(x_1, 0.5), (x_2, 0.6), (x_3, 0.3)\}$

$\bar{B} = \{(x_1, 0.3), (x_2, 0.9), (x_3, 0.6)\}$

$A \backslash B = \{(x_1, 0.3), (x_2, 0.4), (x_3, 0.6)\}$

Properties of Fuzzy Sets:

Commutativity: $A \cup B = B \cup A$ **(3.1.5)**

$A \cap B = B \cap A$

Associativity: $A \cup (B \cup C) = (A \cup B) \cup C$ **(3.1.6)**

$A \cap (B \cap C) = (A \cap B) \cap C$

Distributivity: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ **(3.1.7)**

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Idempotency: $A \cup A = A$ **(3.1.8)**

$A \cap A = A$

Identity: $A \cup \varnothing = A$ **(3.1.9)**

$A \cap X = A$

$A \cap \varnothing = \varnothing$

$A \cup X = X$

As it is seen, the main properties of fuzzy sets and crisp sets are very similar.

3.2. Fuzzy Numbers

Fuzzy numbers are defined in the universe R as a convex, normalized fuzzy set. As fuzzy sets, fuzzy numbers are also used for describing complex situations and modeling imprecise quantities such as about 6 or below 10 (Pedrycz and Gomide, 1998). Fuzzy numbers are used in practical application of fuzzy sets due to its ease of presentation. Dubois and Prada (1979, 1980), Dijkman and van Haeringen and De Lange (1983), Kaufmann and Gupta (1988) made considerable contributions to fuzzy numbering concept by enabling the usage of fuzzy set theory in mathematical forms and applications.

Actually the roots of fuzzy numbers rely on interval analysis and interval arithmetic and α-cut is one of the basic computing methods used in arithmetic operations with fuzzy numbers (Moore, 1996). Although the basic mathematical calculations are mainly related to interval arithmetic, fuzzy numbers differ in the graduation of degree of membership assigned to the number. For instance, the interval number A = [-10, 10] represents an uncertain number "x" located in the interval [-10, 10] (See Figure 3.8). In one-level interval arithmetic, the number "x" will take any value in that interval and there is no value in the interval that being more plausible than others. In the Figure 3.8, all values in the interval have the same level grade. However for A is a fuzzy number and $A = \{(\mu_A(x)/x,\ x \in A,\ \mu_A(x) \in [0, 1]\}$, the membership values of x varies from 0 to 1. Figure 3.9 describes fuzzy number A in the condition of around zero between the numbers -10 to 10. It is clear that the membership degree $\mu_A(x)$ is high for the numbers close to 0 and becomes to decrease while the numbers becoming distant to 0.

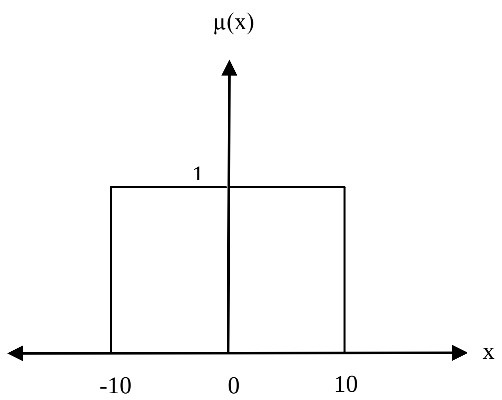

Figure 3.8: Interval Number

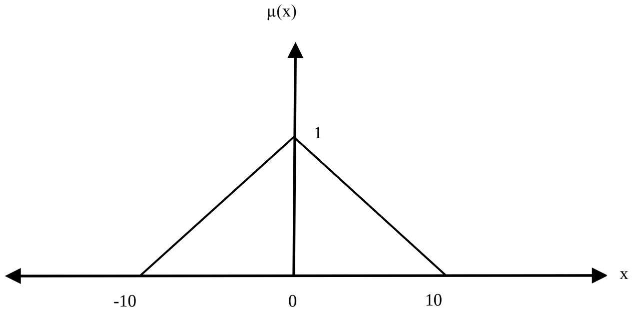

Figure 3.9: Fuzzy Number

α - cuts:

α - cut is the specific representation interval arithmetic for graduating membership degree of fuzzy sets. For a fuzzy set A, $A_\alpha = \{x \mid \mu_A(x) \geq \alpha\}$; $\alpha \in [0, 1]$ and for $x \in A$, if degree of membership increases from 0 to 1, the confidence that x belonging to A also increases.

Fuzzy number A can be defined for an interval $A = [a_1, a_2]$ with membership range $F_A(x) \in [0,1]$ and there is only one a_M value that having maximum degree of membership equal to 1, the function can be defined as:

$$\alpha = F_A(x) = \begin{cases} F^l_A(x) \text{ for } a_1 \leq x \leq a_M \\ F^r_A(x) \text{ for } a_M \leq x \leq a_2 \end{cases} \qquad \textbf{(3.1.10)}$$

$F^l_A(x)$ represents the left range of F_A and $F^r_A(x)$ shows the right range and for $x = a_M$, $F^l_A(a_M) = F^r_A(a_M)$.

By using α - cuts, the fuzzy number A can be denoted by:

$A_\alpha = [a_1^{(\alpha)}, a_2^{(\alpha)}]_\alpha$, $\alpha \in [0, 1]$

Besides, the number will be denoted as left and right as follows:

$$\alpha = \begin{cases} F^l_A\,(a_1^{(\alpha)}) \text{ for } a_1 \le x \le a_M \\ F^r_A\,(a_2^{(\alpha)}) \text{ for } a_M \le x \le a_2 \end{cases} \qquad \textbf{(3.1.11)}$$

The graphical representation of α-cuts are demonstrated in Figure 3.10 (Bojadziev and Bojadziev, 1995).

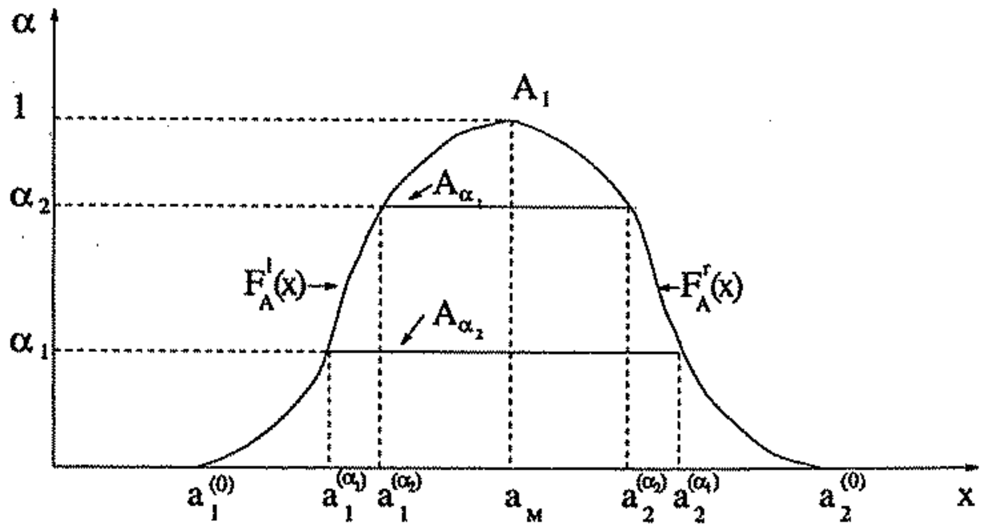

Figure 3.10: Fuzzy Number A and α - level intervals

For example, the α - cut values of Figure 3.9 can be calculated as follows:

$$\text{The membership function } F_A(x) = \alpha = \begin{cases} x * 1/10 + 1 & \text{for } -10 \le x \le 0 \\ (-x) * 1/10 + 1 & \text{for } 0 \le x \le 10 \\ \text{Otherwise} & 0 \end{cases} \qquad \textbf{(3.1.12)}$$

The left and right side of the function can be arranged by describing x in terms of α.

$$x^l = a_1^{(\alpha)} = 10\alpha - 10 \text{ and } x^r = a_2^{(\alpha)} = -10\alpha + 10$$

$A_0 = [a_1^{(0)}, a_2^{(0)}]_0 = [-10, 10]$

$A_{0.1} = [a_1^{(0.1)}, a_2^{(0.1)}]_{0.1} = [-9, 9]$

$A_{0.2} = [a_1^{(0.2)}, a_2^{(0.2)}]_{0.2} = [-8, 8]$

$A_{0.3} = [a_1^{(0.3)}, a_2^{(0.3)}]_{0.3} = [-7, 7]$

$A_{0.4} = [a_1^{(0.4)}, a_2^{(0.4)}]_{0.4} = [-6, 6]$

$A_{0.5} = [a_1^{(0.5)}, a_2^{(0.5)}]_{0.5} = [-5, 5]$

$A_{0.6} = [a_1^{(0.6)}, a_2^{(0.6)}]_{0.6} = [-4, 4]$

$A_{0.7} = [a_1^{(0.7)}, a_2^{(0.7)}]_{0.7} = [-3, 3]$

$A_{0.8} = [a_1^{(0.8)}, a_2^{(0.8)}]_{0.8} = [-2, 2]$

$A_{0.9} = [a_1^{(0.9)}, a_2^{(0.9)}]_{0.9} = [-1, 1]$

$A_{1.0} = [a_1^{(1.0)}, a_2^{(1.0)}]_{1.0} = [0, 0]$

3.2.1. Shape of Fuzzy Number

A fuzzy number can be defined with various shapes. Bell- shaped, trapezoidal and triangular fuzzy numbers are the most popular ones used in engineering applications.

3.2.1.1. Bell - Shaped Fuzzy Number

There are two types of bell-shaped fuzzy numbers.

a) Fuzzy Normal Distribution:

It is obtained by arranging typical Gauss distribution function such that

$$f(x) = \frac{1}{\sigma * \sqrt{2\pi}} * e^{\frac{-0.5*(x-\mu)^2}{\sigma^2}} \qquad \text{for } -\infty < x < \infty$$

In which $\sigma = \frac{1}{2\pi}$ so that

$$\alpha = \mathbf{F_A(x) = e}^{-\pi*(x-\mu)^2} \quad \text{for } x \in (-\infty, \infty) \text{ and } \alpha \in [0, 1] \qquad \textbf{(3.1.13)}$$

b) Piecewise-quadratic fuzzy numbers:

It is the number that combines three quadratic functions. The membership function of the fuzzy number is stated below and the shape of it presented in Figure 3.11 (Bojadziev and Bojadziev, 1995).

$A = [a_1, a_2]$

$$\alpha = F_A(x) = \begin{cases} \dfrac{(x - a_1)^2}{2 * (p - \beta - a_1)^2} & \text{for} \quad a_1 \leq x \leq p-\beta \\ \dfrac{-1}{2\beta^2} * (x - p)^2 + 1 & \text{for} \quad p-\beta \leq x \leq p+\beta \\ \dfrac{(x - a_2)^2}{2 * (p - \beta - a_2)^2} & \text{for} \quad p+\beta \leq x \leq a_2 \\ 0 & \text{otherwise} \end{cases} \qquad \textbf{(3.1.14)}$$

where $p = ½ * (a_1+a_2)$ and for $\beta \in (0, a_2 - p)$, 2β shows the bandwidth

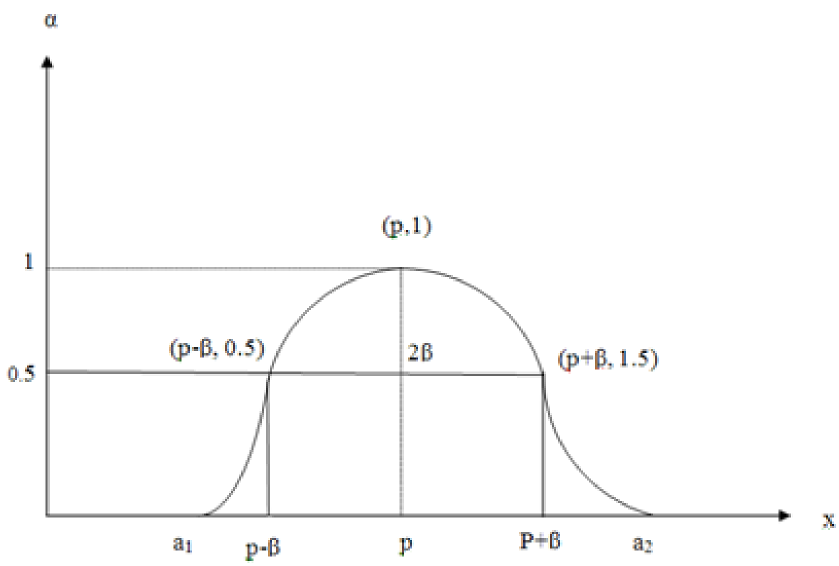

Figure 3.11: Piecewise - Quadratic Fuzzy Number

3.2.1.2. Trapezoidal Fuzzy Number

Trapezoidal fuzzy numbers are commonly used since the flat part of these numbers is long for describing the members in the interval having full membership. The membership function of the fuzzy number is stated below and the shape of it presented in Figure 3.12.

$A = [a_1, a_2]$

$$\alpha = F_A(x) = \begin{cases} \dfrac{x - a_1}{a_1^{(1)} - a_1} & \text{for} \quad a_1 \le x \le a_1^{(1)}, \\ 1 & \text{for} \quad a_1^{(1)} \le x \le a_2^{(1)}, \\ \dfrac{x - a_2}{a_2^{(1)} - a_2} & \text{for} \quad a_2^{(1)} \le x \le a_2, \\ 0 & \text{otherwise} \end{cases} \qquad \textbf{(3.1.15)}$$

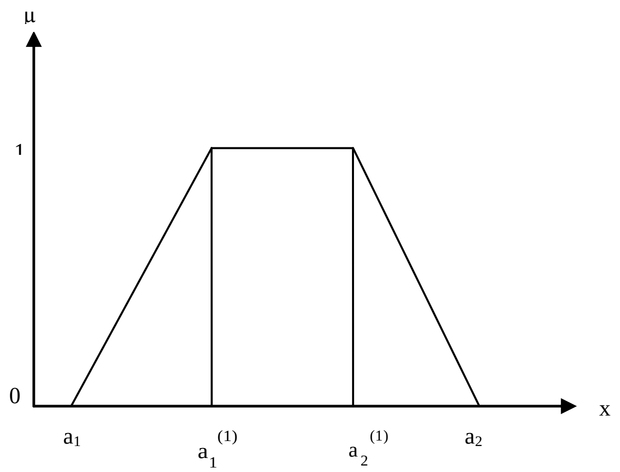

Figure 3.12: Trapezoidal Fuzzy Number

3.2.1.3. Triangular Fuzzy Number

Triangular fuzzy numbers are special case of trapezoidal fuzzy numbers where there is only one number having full membership degree. Triangular fuzzy numbers are the most preferred ones used in engineering and science applications due its ease of use.

The membership function of the fuzzy number is stated below and the shape of it presented in Figure 3.13 (Bojadziev and Bojadziev, 1995).
$A = [a_1, a_2]$ and in this case the peak point is $(a_M, 1)$ where $a_1^{(1)} = a_2^{(1)} = a_M$ different than trapezoidal fuzzy numbers.

$$\alpha = F_A(x) = \begin{cases} \frac{x-a_1}{a_M-a_1} & \text{for } a_1 \le x \le a_M \\ \frac{x-a_2}{a_M-a_2} & \text{for } a_M \le x \le a_2 \\ 0 & \text{otherwise} \end{cases} \quad \textbf{(3.1.16)}$$

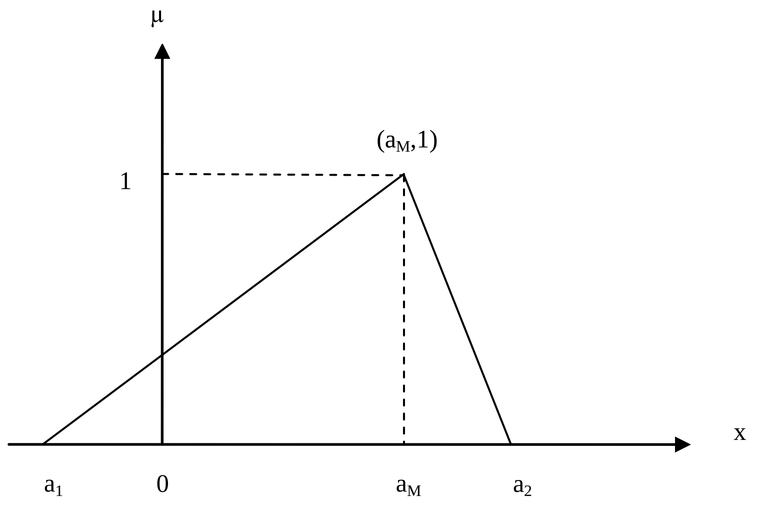

Figure 3.13: Triangular Fuzzy Number

In case of symmetrical dispersion of the numbers around the peak point (a_M, 1), the membership functions becomes as it is stated below and shape of it presented in Figure 3.14.

$$\alpha = F_A(x) = \begin{cases} \dfrac{2*(x - a_1)}{a_2 - a_1} & \text{for} \quad a_1 \le x \le \dfrac{a_1 + a_2}{2} \\ \dfrac{2*(x - a_2)}{a_1 - a_2} & \text{for} \quad \dfrac{a_1 + a_2}{2} \le x \le a_2 \\ 0 & \text{otherwise} \end{cases} \quad \textbf{(3.1.17)}$$

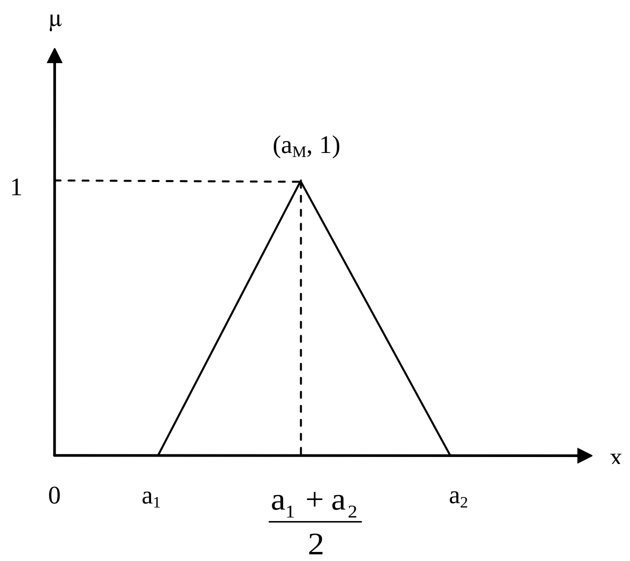

Figure 3.14: Central Triangular Fuzzy Number

3.2.2. Arithmetic Operations with Fuzzy Numbers

As mentioned before, fuzzy numbers are the generalized version of interval numbers so that the basic mathematical applications are computed similar to interval arithmetic calculations.

For two fuzzy numbers $A_\alpha = [a_1^\alpha, a_2^\alpha]_\alpha$, $B_\alpha = [b_1^\alpha, b_2^\alpha]$ and $\alpha \in [0, 1]$; the basic arithmetic operations made with fuzzy numbers are:

Summation: $A_\alpha + B_\alpha = [a_1^\alpha + b_1^\alpha, a_2^\alpha + b_2^\alpha]$ **(3.2.1)**

Subtraction: $A_\alpha - B_\alpha = [a_1^\alpha - b_2^\alpha, a_2^\alpha - b_1^\alpha]$ **(3.2.2)**

Multiplication: $A_\alpha * B_\alpha = [\min (a_1^\alpha * b_1^\alpha, a_1^\alpha * b_2^\alpha, a_2^\alpha * b_1^\alpha, a_2^\alpha * b_2^\alpha), \max (a_1^\alpha * b_1^\alpha, a_1^\alpha * b_2^\alpha, a_2^\alpha * b_1^\alpha, a_2^\alpha * b_2^\alpha)]$ **(3.2.3)**

Division: $A_\alpha/B_\alpha = \{[a_1^\alpha, a_2^\alpha]_{\alpha}:[b_1^\alpha, b_2^\alpha]\} = \{[a_1^\alpha, a_2^\alpha]_\alpha * [\frac{1}{b_2(\alpha)}, \frac{1}{b_1(\alpha)}], 0 \notin [b_1, b_2]$ **(3.2.4)**

Example: For two triangular fuzzy numbers A and B, the arithmetic applications and the Figures 3.15-3.18 are demonstrated below.

$$F_A(x) = \begin{cases} \frac{x}{3} & \text{for } 0 \le x \le 3 \\ 2 - \frac{x}{3} & \text{for } 3 \le x \le 6 \\ 0 & \text{otherwise} \end{cases}$$

$$F_B(x) = \begin{cases} \frac{x-1}{3} & \text{for } 1 \le x \le 4 \\ \frac{7-x}{3} & \text{for } 4 \le x \le 7 \\ 0 & \text{otherwise} \end{cases}$$

$A_\alpha + B_\alpha = [1, 13]$

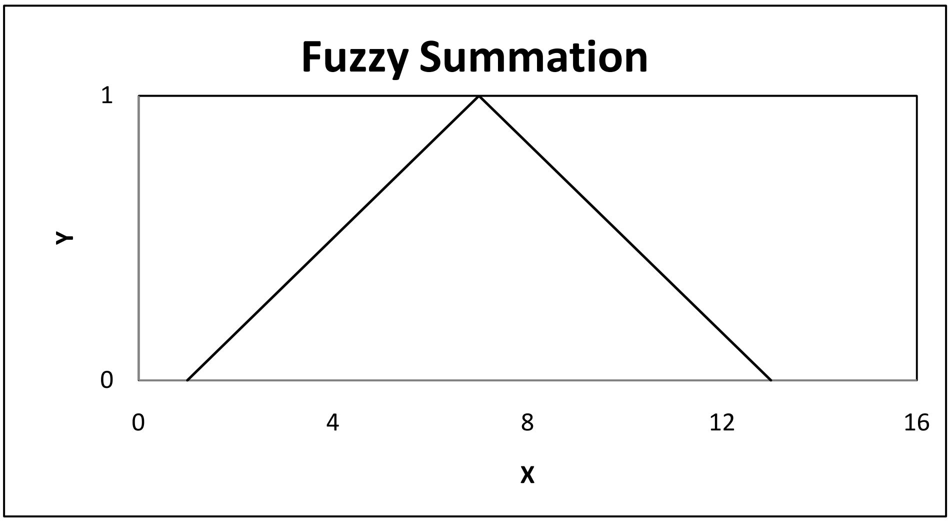

Figure 3.15: Summation of Two Fuzzy Numbers

$A_\alpha - B_\alpha = [-7, 5]$

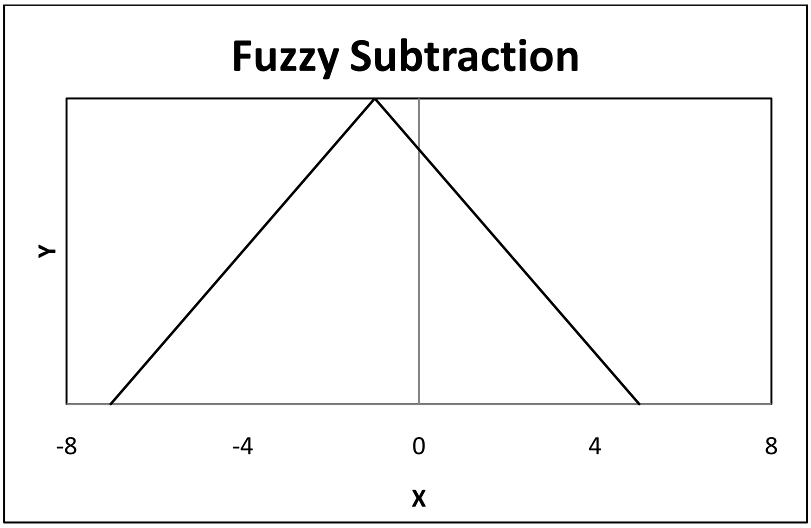

Figure 3.16: Subtraction of Two Fuzzy Numbers

$A_\alpha * B_\alpha = [0, 42]$

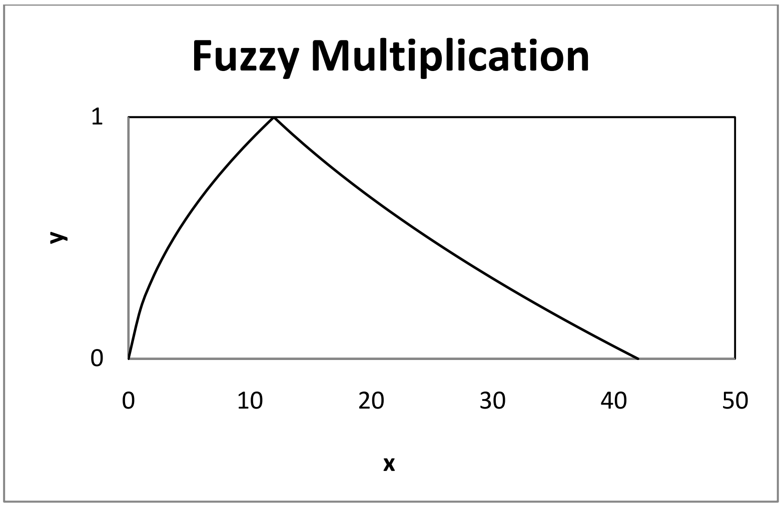

Figure 3.17: Multiplication of Two Fuzzy Numbers

$A_\alpha / B_\alpha = [0, 6]$

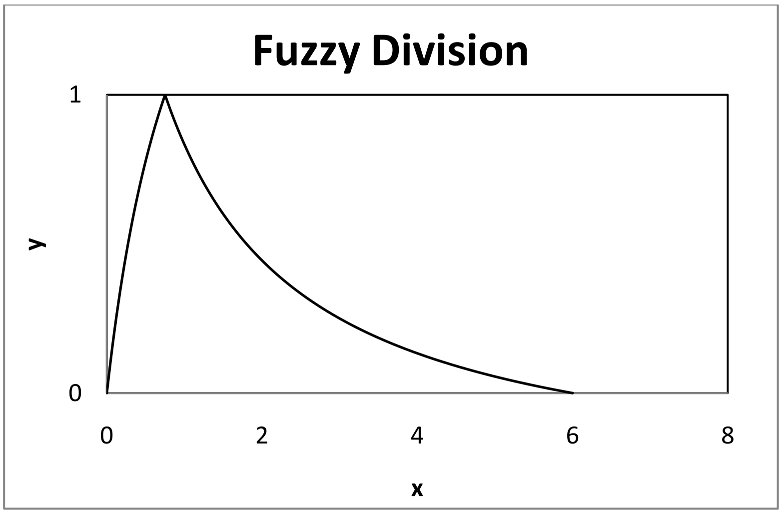

Figure 3.18: Division of Two Fuzzy Numbers

As can be seen from Figure 3.17 and 3.18, the multiplication and division of two fuzzy numbers will not result in a triangular shape.

Note: The basic arithmetic operations made by a fuzzy number and a crisp number has the same logic of the equations mentioned above such that the corresponding numbers of the fuzzy interval are summed , subtract, multiply and divide by the crisp number

Example: For the triangular fuzzy number A and crisp number B, the arithmetic applications are stated below.

$$F_A(x) = \begin{cases} \frac{x}{3} & \text{for } 0 \leq x \leq 3 \\ 2-\frac{x}{3} & \text{for } 3 \leq x \leq 6, \quad F_B(x) = 2 \\ 0 & \text{otherwise} \end{cases}$$

$A_\alpha + B_\alpha = [2, 8]$

$A_\alpha - B_\alpha = [-2, 4]$

$A_\alpha * B_\alpha = [0, 12]$

$A_\alpha / B_\alpha = [0, 3]$

3.3. Fuzzy Linguistic Variables

Linguistic variables are generally used for describing the situation based on many observations. According to Zadeh (1975), linguistic variables "serve as a means of approximate characterization of phenomena that are too ill-defined or too complex or both to permit a description in sharp terms". For instance, "warm", "hot", "cold" are common expressions declaring the state of weather conditions. Although, these expressions could not accurately define the exact conditions such as defining the temperature with 30˚C, the terms give an idea about the situation of weather when it is impossible to measure the temperature. Besides, it will be useful to use linguistic terms into numerical forms for including the information related to human expertise. Therefore, fuzzy numbers are used for translating the appropriate linguistic terms into the numbers for using them calculation of the models. In Figure 3.19, it can be seen that fuzzy numbers are used for translating the linguistic variables for defining the temperature such as “very cold”, “cold”, “warm”, “hot”, and “very hot”.

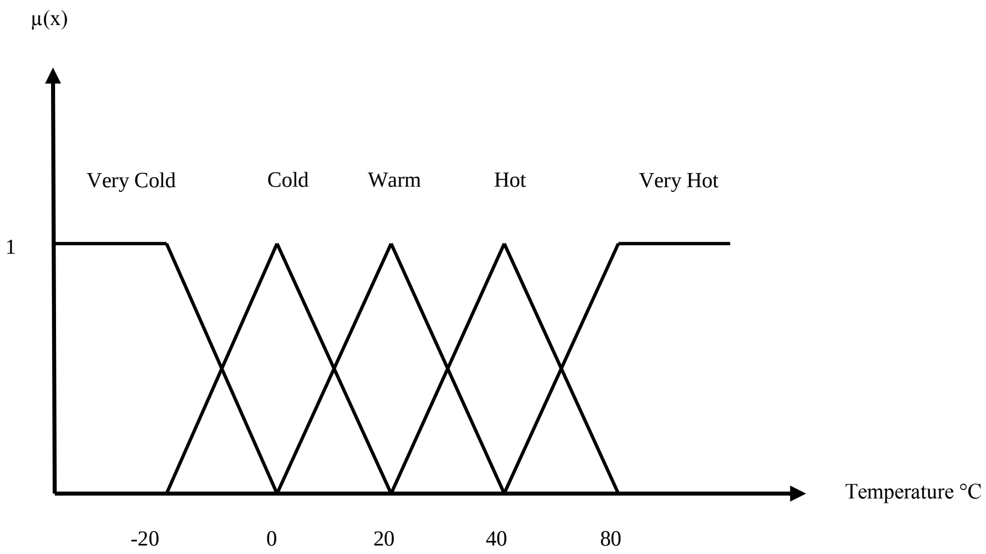

Figure 3.19: Terms of the Linguistic Variable “Weather”

3.4. Defuzzification

As mentioned, fuzzy numbers are useful for describing and modeling the complex situations including uncertainty. However, many scientific and engineering applications are based on binary logic and the tools such as computers that are used for making decisions give deterministic results. Therefore, for efficient usage of fuzzy numbers in real world problem solving, there is need of defuzzifying the fuzzy numbers into crisp numbers. In other words, defuzzification is the conversion of fuzzy numbers into the precise ones so that rather than interval numbers, a unique value, successfully representing the set, will be obtained and used in making estimates. There are several methods of defuzzification operations. The most commonly used ones are stated as follows:

1. Maximum Membership Principle:

   It is also called as height method that the value having the maximum membership degree is used as the defuzzifyed value of the fuzzy number. The method is limited since only one number used for representing the whole range. In Figure 3.20, the value a* has the maximum degree of membership and is the defuzzifyed value of the fuzzy number (Ross, 1995).

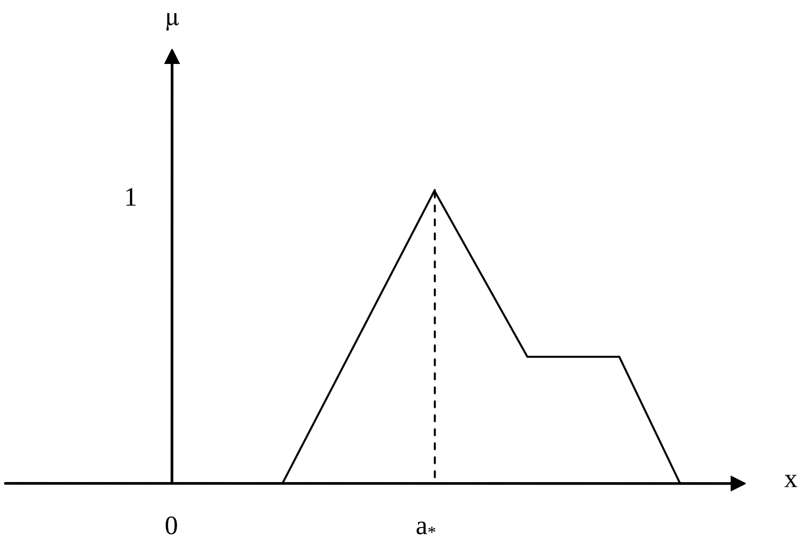

Figure 3.20 : Maximum Membership Defuzzification Method

2. Mean-Maximum Method:

   This method is the special usage of Maximum Membership Principle where there are more than one number having full membership. In Figure 3.21, the defuzzifyed value c* is calculated by taking the average of the boundaries of the maximum plateau so that

$$c^* = \frac{a+b}{2} \tag{3.4.1}$$

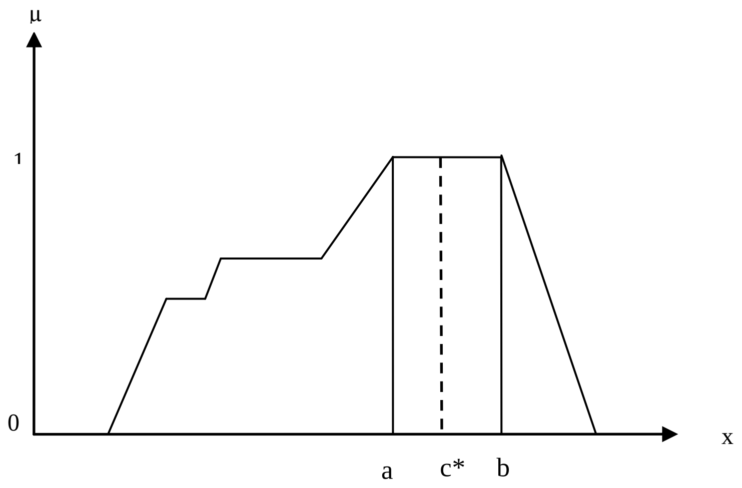

Figure 3.21: Mean - Maximum Defuzzification Method

3. Height Defuzzification Method:

It is the generalized method of mean-maximum method used in case of having more than one plateaus with different membership degree. In Figure 3.22, the defuzzifyed value z_h is calculated by the formula below:

$$z_h = \frac{p * \frac{(a_1 + a_2)}{2} + q * \frac{(b_1 + b_2)}{2}}{p + q} = w_1 * \frac{(a_1 + a_2)}{2} + w_2 * \frac{(b_1 + b_2)}{2} \quad \textbf{(3.4.2)}$$

where w_1 and w_2 are the weighted average of the midpoints of the plateaus and

$w_1 = p / (p+q)$, $w_2 = q / (p+q)$

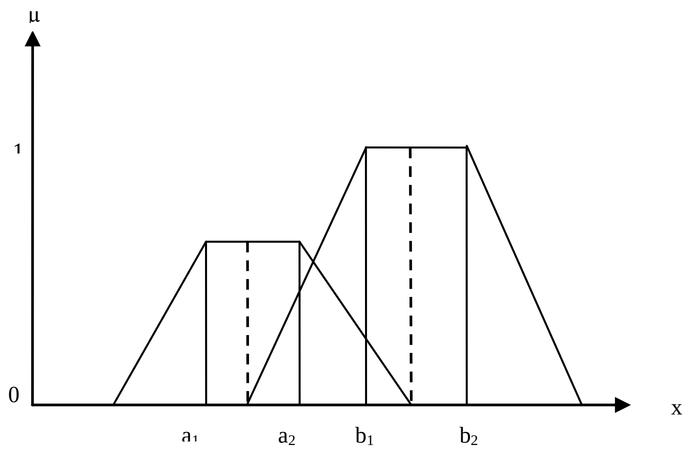

Figure 3.22: Height Defuzzification Method

4. Center of Area Method:

Center of area method is commonly used since it gives an adequate representation of fuzzy numbers. The logic is similar to geometrical computation of a centroid of a curve. The interval of the fuzzy number [a_1, a_n] is subdivided into n equal subintervals and the crisp number is calculated by using membership degree of each point for taking the weighted average of whole number. In Figure 3.23, the defuzzifyed value z_c is calculated by using equation below:

$$z_c = \frac{\sum_{k=1}^{n-1} z_k * \mu_X(z_k)}{\sum_{k=1}^{n-1} \mu_X(z_k)} \qquad \textbf{(3.4.3)}$$

where z_k is any value in the interval [a_1, a_n] and $\mu_x(z_k)$ is the membership degree of each value.

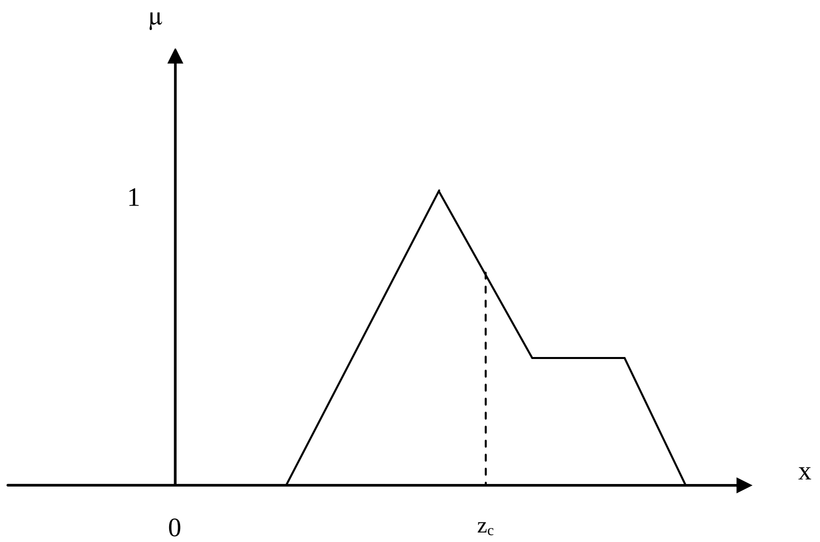

Figure 3.23: Center of Area Defuzzification Method

3.5. Applications of Fuzzy Set Theory in Construction Management Studies

The studies related to fuzzy set theory has begun since the publication of the seminal notes of Prof. Zadeh in 1965. Although the genesis of the fundamentals is quite new, fuzzy set theory and fuzzy logic has been widely used in many mathematical, scientific and engineering applications for about 45 years. Fuzzy sets were first introduced in America but the theoretical and practical applications were intensively made in far - east due to the similarity in philosophy denying dual logic. Especially the researches made in Japan increased the popularity of fuzzy logic. Up to know, fuzzy logic has been used in different scientific disciplines. For instance, due to theoretical background of fuzzy logic, many studies were generated on modern mathematic such as fuzzy topology, fuzzy measure, fuzzy integral, fuzzy factor space theory etc (Lin and Pang, 1994). Likewise, the scientist have used fuzzy logic for production of new devices and numerous technologic commercial products made with fuzzy logic begun to appear in sales markets such as washing machines, cameras, medical diagnosis, computers, braking systems, vacuum cleaners. Besides, fuzzy controlling systems have been used for upgrading existing machines, controlling automatic driving systems, subway systems, helicopters etc.

As mentioned before, construction actions are unique and complex. It is difficult to make a reliable test of a prototype as it is made in other disciplines (Klir and Yuan, 1995). There are many parameters effecting success of the construction works. In case the lack of existing reliable data, it is more difficult to solve the existing problems and make appropriate decisions. Therefore, fuzzy sets and fuzzy logic are one the most suitable techniques construction management for modeling uncertain parameters (like climate, labor, equipment, activities depending on time) and also for making decisions (Malek, 2000).

There are many studies conducted with fuzzy logic for overcoming the ill - defined, imprecise, uncertain, ambiguous nature of the construction works. In the study of Chan et al. (2009), the application of fuzzy techniques in construction management studies are extensively overviewed. In this study, the authors divide the fuzzy research fields into two parts as fuzzy set/logic and hybrid fuzzy techniques. Fuzzy sets and logic are the pure application of the fuzzy theory in which the complexity and vagueness of the system is avoided with only fuzzy techniques. Whereas the hybrid systems are used in combining appropriate soft computing techniques (like neural networks, genetic algorithms, evaluation theory, chaos theory) related to nature of problems with fuzzy set theory. Besides, Chan et al. (2009) clustered the fuzzy applications in four main groups as decision making, performance, evaluation and modeling. Although there are many studies for each field, only the important ones are mentioned in below.

In construction management, decision - making is a challenging issue in case of lack of enough data and information to make reliable judgment. When the decision makers face with such complex situations with uncertainty, they usually try approximate reasoning based on human knowledge and experiences (Malek, 2000). The researches applied fuzzy decision making systems for benefiting from human decision system. Fayek (1998) studied a competitive bidding strategy based on fuzzy sets for determining the tender margin. Boussabine and Elhag (1999) proposed to use fuzzy techniques for making cash flows that is compulsory tool for taking financial decisions in construction projects. Boussabine and Elhag (1999) inspected the past performance of similar projects and used statistical techniques for forming the membership functions of the fuzzy sets and tried to determine cost inflow, cost outflow and project progress curves. Lam and Runeson (1999) suggested a financial decision tool based on fuzzy applications to help contractors to take investment decisions by minimizing the use of resources. Mohammed and McCowan (2001) used possibility theory for ranking the project for making new investments. Wang and Liang (2004) generated a multiple fuzzy goal programming in order to help decision makers for solving decision making problems. Wang and Liang (2004) use Zimmerman's linear membership function (1978) for modeling the real word project management decisions with intervals to minimize the total project cost, project duration and crash cost. Lin and Chen (2004) proposed a fuzzy linguistic approach for modeling the uncertain things with regarding the subjectivity of the experts to get a proper the bid/ no-bid decision process. Singh and Tong (2005) suggested the owners to using fuzzy decision framework in contractor selection.

Fuzzy application techniques are also commonly used for determining and improving the construction project performance. Chua and Kog (2001) used hybrid neurofuzzy technique for providing the efficient allocation of resources and obtaining satisfactory project budget and schedule. Leu et al. (2001) generated a cost - time trade off model based on hybridization of genetic algorithms and fuzzy sets. Zheng and Ng (2005) also utilized the combination of fuzzy sets and genetic algorithms for making cost - time optimization model. The Zheng and Ng (2005) stated that the duration and cost items of a construction project dynamically change due to many uncertain variables such as productivity, weather conditions and availability of resource etc. Therefore, Zheng and Ng (2005) used fuzzy techniques in understanding the behavior of the experts to get realistic inputs into system and applied genetic algorithms to enhance time-cost relations. Li et al. (2006) predicted the status of a construction project with regarding the possible cost overruns and schedule delays. The model enabled the users to reflect the possible risk of projects and graded them with fuzzy logic. Eshtehardian et al. (2008) also studied a hybrid model for time - cost optimization problem where different levels of risk could be defined by the users with α - cut approach. Both the cost and duration of the activities were entered into model as fuzzy numbers and genetic algorithms used for suggesting solutions to the fuzzy multi - objective time cost model.

Fuzzy scheduling is another important field in which fuzzy techniques have been implemented. Due to the uncertainties resulting from complexity of the construction works, variable productivity rates and unpredictable events, the activity durations in a project will vary considerably. According to Bonnal et al. (2004) fuzzy set theory is appropriate in using project scheduling with uncertainty since it is realistic and fit the nature of the construction works. The basic idea behind the usage of fuzzy set theory in construction project scheduling is to determination of the uncertain activity duration by reliable experts. Chanas and Kamburowski (1981), Ayyup and Haldar (1984), Lootsma (1989) applied fuzzy variables into PERT by assigning ranging activity durations gathered from experts verbally. McCahon (1993) generated project network analysis fuzzy PERT and used degree of critically for finding the activities on the critical path. Dobois and Prada (1988), Geidel (1989), Hapke and Slovinski (1993), Nasution (1994), Wu et al. (1994), Galvagnon (2000), Castro-Locouture (2009) are the other researches studied the fuzzy scheduling concept.

Lorteraprog and Moselhi (1996) made a comprehensive study for the application fuzzy network scheduling different than Fuzzy PERT. Lorteraprog and Moselhi only studied in theoretical basis by developing some assumptions for the backward pass and critical path calculations. The authors compared the results of the scheduling with the ones calculated by using Monte Carlo simulation technique and presented the superiority of fuzzy scheduling model. Oliveros and Fayek (2005) enhanced this study for fuzzy schedule updating and activity delay analysis. It should be noted that although there are numerous studies about fuzzy scheduling, it could not be used as efficient as deterministic scheduling methods due to the theoretical difficulties leading from finding critical paths and making backward pass calculations.

Fuzzy set theory is also used in evaluation and modeling purposes especially for making risk analysis. Paek et al. (1993) carried out fuzzy numbers for introducing risk pricing methodology and analyzing and pricing construction project risks to help contractors in making decision about bid price. Tah and Carr (2000) made qualitative risk assessment model by using common language that includes cause effect diagrams for describing the relationship between risk factors and consequences. Knight and Fayek (2002) used fuzzy logic in determining the relationship between the characteristic of the project and risk of the project and proposed a model for estimating cost overrun. Choi at all (2004) suggested a risk assessment methodology for modeling underground construction projects by considering both probability theory and human judgment. Dikmen et al. (2006) generated a fuzzy risk rating for predicting cost overruns in international projects. Shaheen et al. (2007) applied cost range estimation with fuzzy numbers gathered from experts and compared the results with the one generated by Monte Carlo simulation. The authors finally stated that fuzzy set approach can be used as an alternative to Monte Carlo simulation in predicting cost of the project. Li et al. (2007) proposed fuzzy approach for prequalifying the contractors. Bendana et a.l (2008) proposed a fuzzy contractor selection technique based on fuzzy control technique with computerized application for clients.

CHAPTER 4

FUZZY CASH FLOW MODELING

Up to this point, the basic logic behind cash flow analysis is explained and the positive and negative parts of the existing cash flow studies are discussed in details. Although there have been studies about fuzzy cash flow and capital budgeting in economy and industrial engineering such as Çetin and Kahraman (1999), Kahraman et al. (2006), a complete integrated fuzzy cash flow methodology has not been developed yet in construction management literature. In this chapter, a new cash flow technique based on fuzzy set theory, Fuzzy Cash Flow Modeling (FCFM), is introduced. The methodology and process of the model are explained in detail and the model is applied to a case study.

4.1. General Overview

This study aims to provide a new cash flow model based on fuzzy logic for enabling the users coping with uncertainties while preparing a reliable cash flow projection of the construction projects. Also, it is intended to warn the practitioners about the cost and schedule threats of the project before the commencement date. Besides, the purpose of this study is to let the practitioners make a financial plan for cash management of the project including necessary precautions against the possible risks. The model is actually generated for the help the contractors in lump-sum project due to the high risk that contractor undertakes.

FCFM is based on the possibility theory for reflecting the power of human knowledge and approximate reasoning in making estimation when there is no reliable data for giving dependable predictions. It intends to generate an alternative way to the models utilizing from statistics for dealing with uncertainty such as simulations. The model aims to examine the project in activity level, use linguistic terms for modeling cash flow inputs and provide a user friendly cash flow projection using Microsoft Excel 2007 with VBA (Visual Basic for applications).

4.2. Research Methodology

The flow chart of the FCFM is illustrated in Figure 4.1. The methodology of the system is composed of four steps:

1. Entering of basic input by construction experts,
2. Fuzzification of the input information,
3. Computation fuzzy expense and calculation of project schedule based on defuzzifyed activity durations,
4. Obtaining different cash flow scenarios by using fuzzy net cash flow output.

4.2.1. Input Data

When there is insufficient data about the project and high level of uncertainty resulting from ambiguity, it is unreliable to use statistical data of the past projects for dealing with the risk of the projects. In such cases, according to the Page (2000), it is better to use expert opinion and prediction to make a proper estimation. FCFM is generated to overcome the uncertainty problem by relying on approximate reasoning talents of human judgment. The first step of the model is entering the basic input information about the project by the experts into the main menu form of the program (See Figure 4.2).

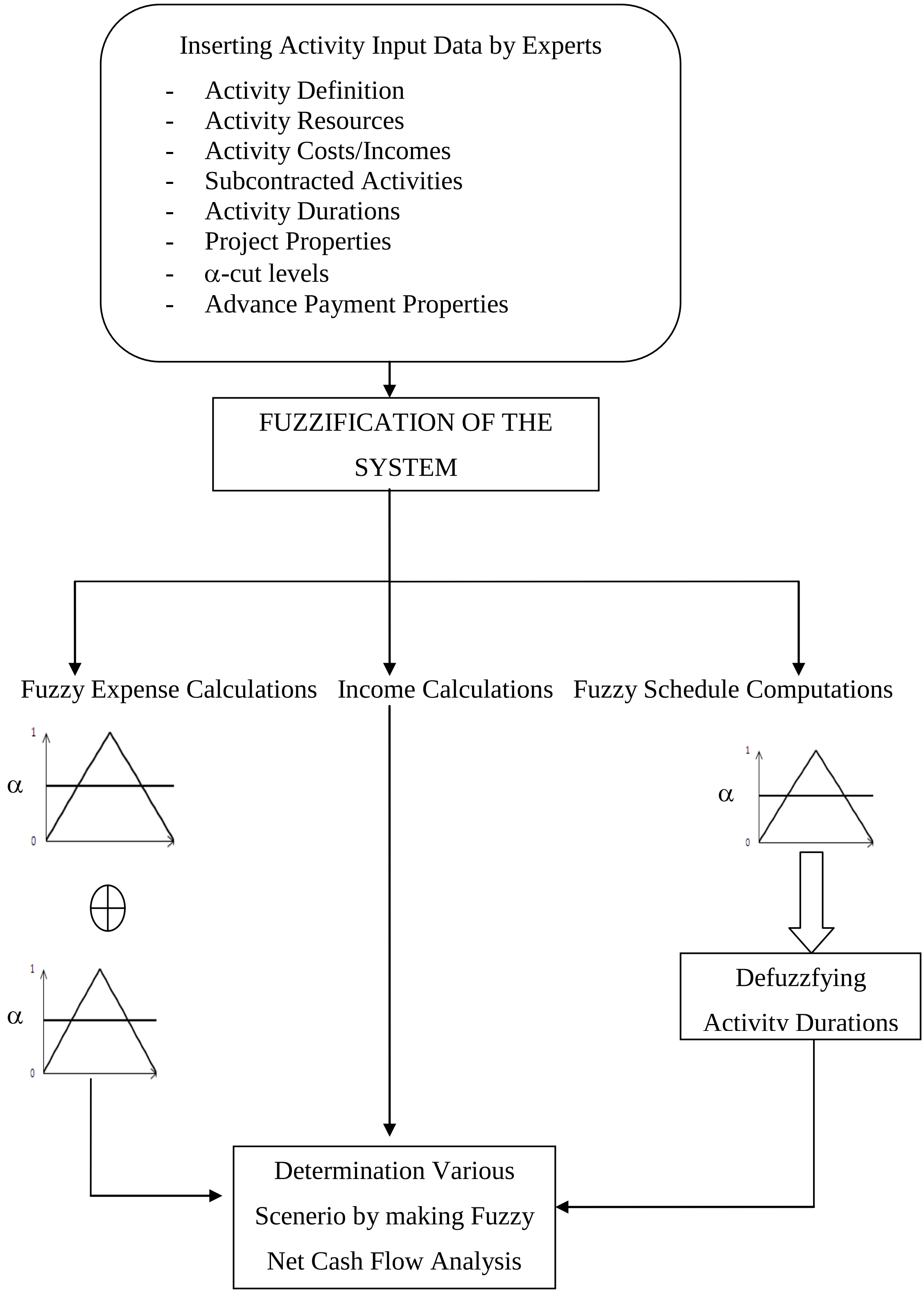

Figure 4.1: FCFM Flow Chart

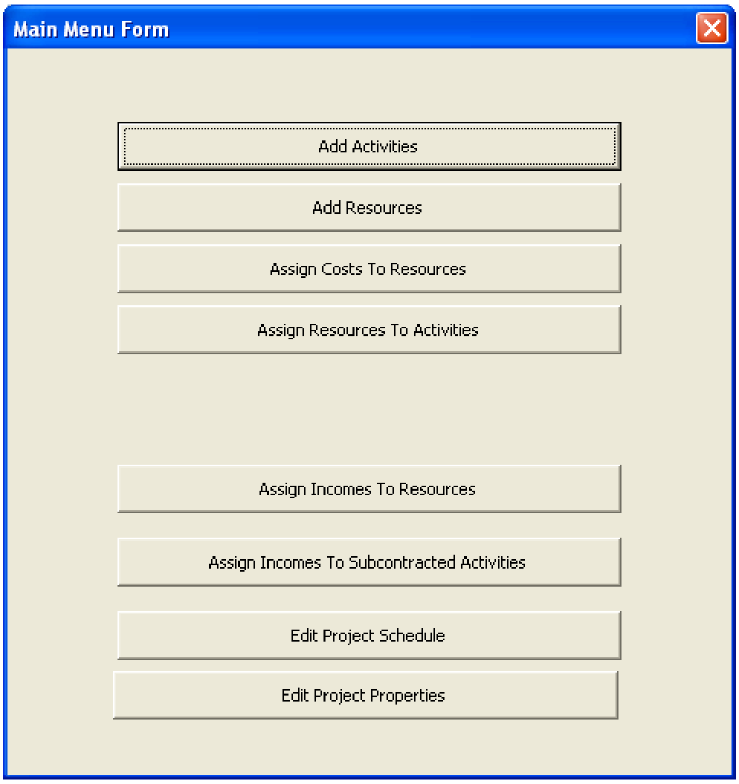

Figure 4.2: Main Menu Form

First of all, all the activities related to project are presented to the model by defining the activities with Activity ID and explanations (See Figure 4.3). The form is generated to insert 20 activities as a prototype.

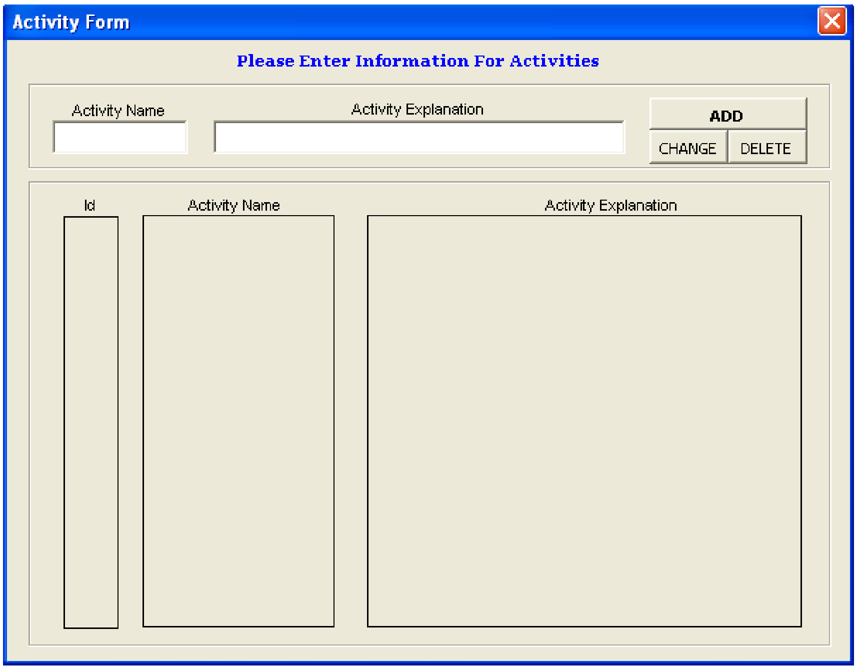

Figure 4.3: Activity Form

After the first step, the resources of the project are defined with the resource ID and resource type as Labor-Material and Equipment (See Figure 4.4). Then, the costs of the resources determined by the experts are assigned as expenses of the project (See Figure 4.5). The resources are defined such that the user enters the cost of resources for the completion of the one unit of the total quantity of the activity that the resource is assigned. For example, if the user inserts lean concrete cost as an expense, he should define to the model the cost of casting 1 m^3 lean concrete. Due to the variety of reasons such as inflation, political instability of the country, monetary strategies of the government, material shortage, inability of finding qualified labor with low cost etc., variability of resource cost is one of the main risks that the contractors meet during the project. Akpan and Igwe (2001) state that increases in the material price and labor price are the major factors leading to cost overruns in a project. Therefore, the labor and material prices should be evaluated carefully.

A stated before, statistical index may be inaccurate for determining the price of the resources at tendering stage (Fitzgerald and Akintoye, 1995). Hence, using the view and prediction of a specialist who is well – experienced and aware of the possible risks of the project will be more beneficial while making cash flow anticipation. For that reason, the model includes the possible cost overruns of the resources by using fuzzy logic that enable to assign cost resources with range estimations the experts with graduating the range from zero to one. The aim of using fuzzy logic is utilizing from human intuition and thinking. Humans mind begins to think with using language and the experts use linguistic expressions during making approximate reasoning. Therefore, linguistic labels are preferred to reflect the risk of the cost of resources such as Low - Medium - High. As it is shown in Figure 4.4, the model allows using linguistic expression for assigning the cost of resources with fuzzy terms and develops different scenarios by assigning three numbers for each linguistic term such as Low Cost - Medium Cost - High Cost.

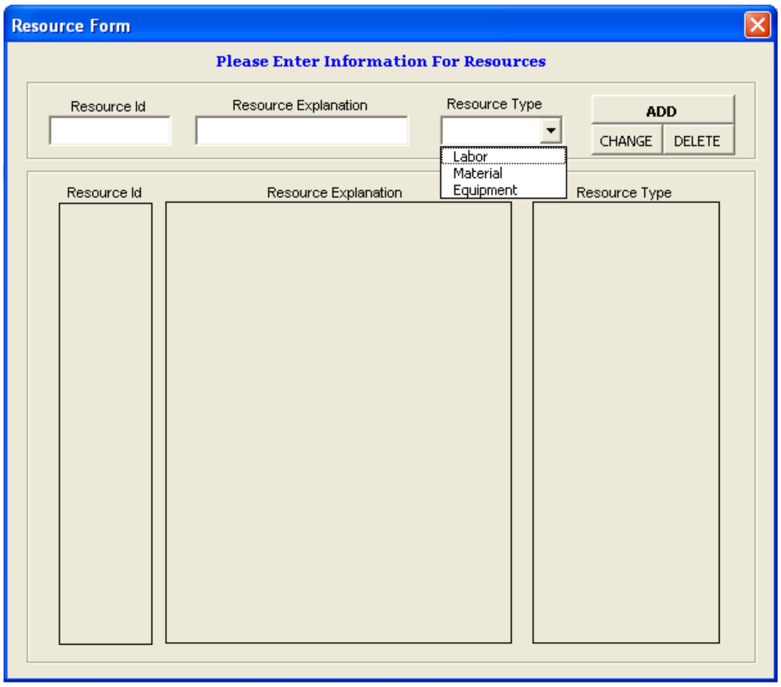

Figure 4.4: Resource Form

As it was done by some of the previous cash flow studies such as the model of Navon (1995), a computerized model that is developed to prepare a reliable cash flow. Hence, the model is created for studying the project in activity level and the whole sources of input are expected to be received from experts. Obtaining whole data from experts may be overburden to the estimator, may cause time consumption and may prevent the flexible usage of the model. Therefore, it is expected from experts to insert only the most promising value and predicate units. Most promising value is the one that having largest possibility for the cost of the resources. It means that the expert will assign the value as most promising to be. Besides, predicate unit is used for dispersing the deviation of the cost symmetrically and for determining the borders of the range of estimation by finding smallest possible and largest possible values (Chiu, 1992). If the expert knows the cost of the resource with less uncertainty, predicate unit is equal to zero and the value certainly known is inserted to model as a crisp number. However, if the expert is more suspicious about the future cost of that resource, predicate unit will be large to reveal the high risk of the cost overrun.

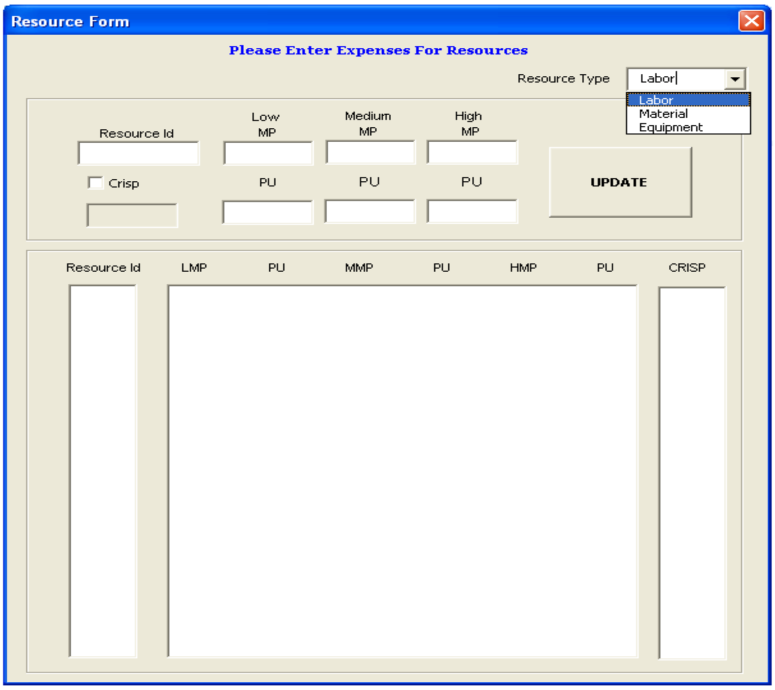

Figure 4.5: Assigning Cost to Resources

When the cost of the resources is determined, the user will assign the resources (labor, material, equipment) into the related activities (See Figure 4.6). During resource assignment process, the user determines the total quantity of resources amount that is going to be consumed while performing the related activity. Since inserting every input as an uncertain variable will cause the overestimations about the total cost of the project by enlarging the extreme limits of cost flow, BOQ of the activities is assumed to be unchanged during the project and the activity quantities will be expected to be as a crisp number. The system also allows some activities to be subcontracted. Since the expense of the subcontracted items are determined by a contract, the cost of the activities planned to be done by subcontractor will be put in as crisp values.

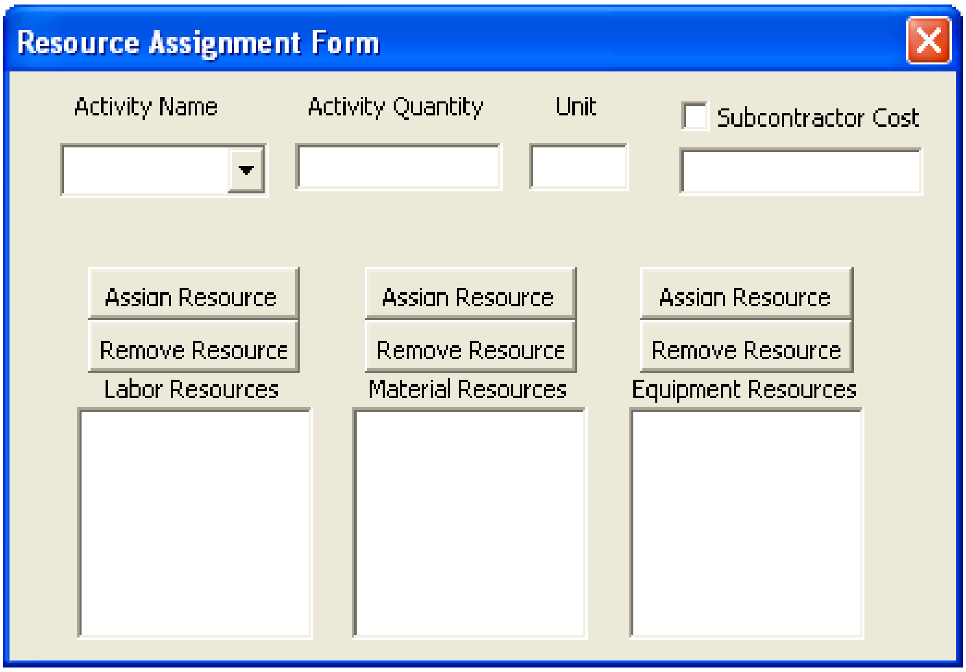

Figure 4.6: Resource Assignment Form

As it is aimed to find a net cash flow of the project, the expected income of the contractor will also be put in to the model. The money that incurred by the owner is definite so that the income value of the project is calculated by assigning deterministic prices to the predefined resources and subcontracted activities with an expected profit percentage (See Figure 4.7 and 4.8).

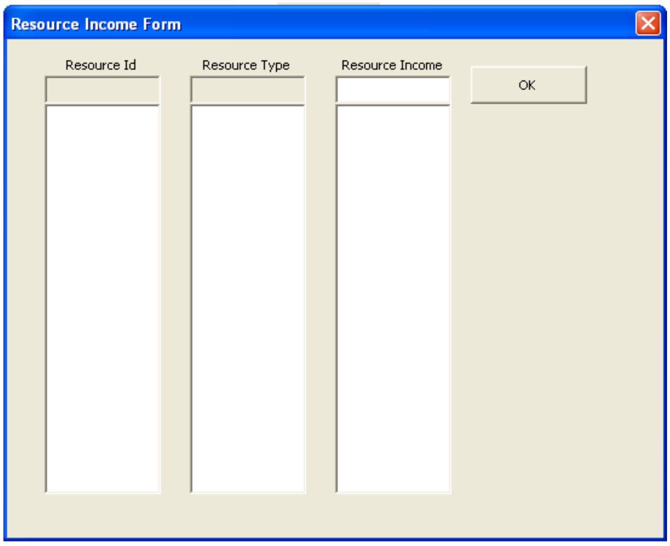

Figure 4.7: Resource Income Form

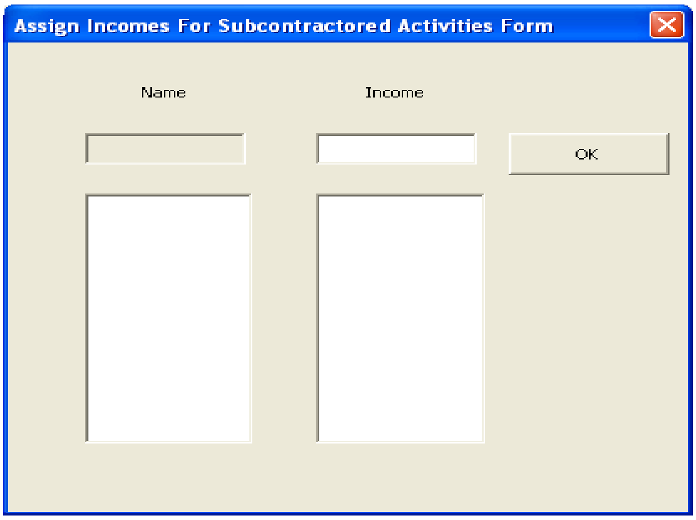

Figure 4.8: Income Form for Subcontracted Activities

Activity durations will also show variations due to the changes in productivity rate of labors, adverse weather conditions, insufficient material capacity etc. Therefore, the duration of the project is also uncertain. There are various ways of determining activity durations like the dividing the total quantity by productivity rate. In this study, the activity durations are determined by directly use of estimate of the experts. The experts define the activity durations with range estimations by determining the most promising value and predicate unit of the activity for developing different scenarios by assigning three numbers for each linguistic term such as Pessimistic Durations - Normal Duration - Optimistic Duration are entered in schedule input form. After that, the user enters the logical relation between the activities of the project and enters a lag time if it is required (See Figure 4.9).

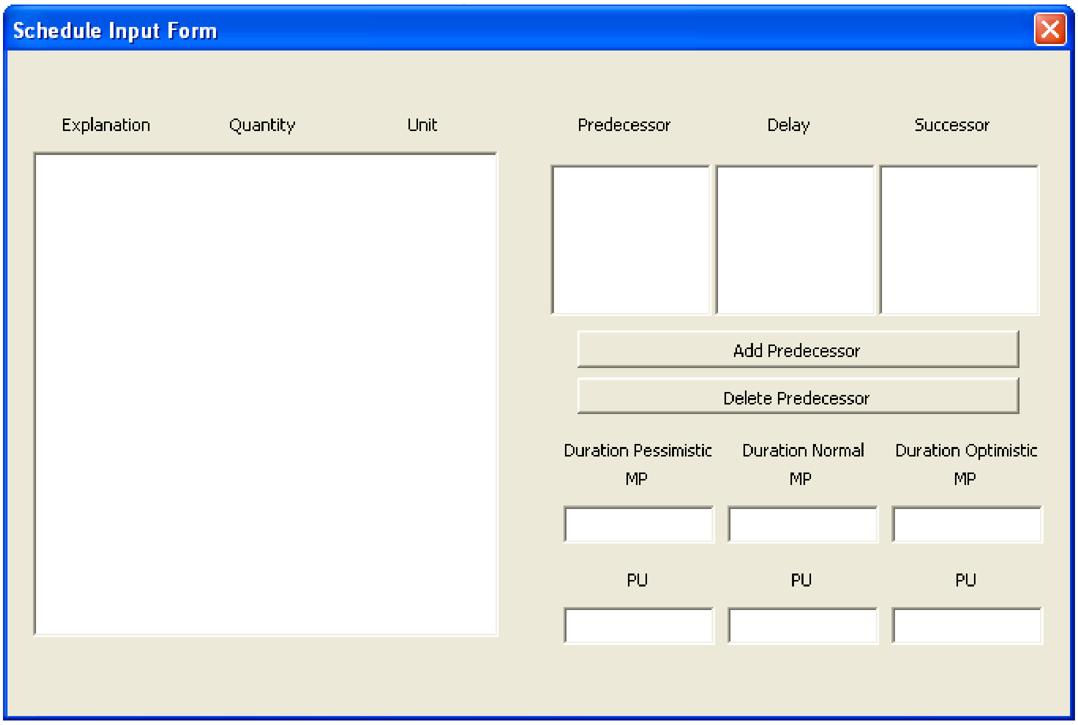

Figure 4.9: Schedule Input Form

The general project properties such as indirect cost per day, starting date of the project, advance payment percentage, advance payment deduction percentage and of cost and schedule α (alpha) - cut values are entered into system by project properties form (See Figure 4.10).

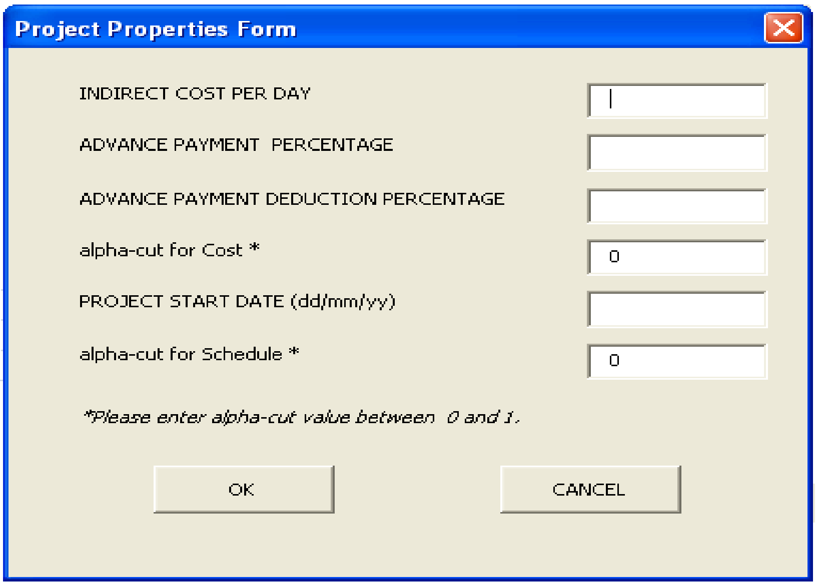

Figure 4.10: Project Properties

α - cut values, are used to represent fuzzy sets into crisp sets and give the opportunity to the users to put in his/her risk attitude into the model by adjusting the α - cut value from zero to one. The determination of the α - cut value is important since it directly effects the results generated by the model. Zero α - level means that the user determines the cost and schedule of the project with a wider range due to the high risk of variation. Raising the α - level shows the reduction of the risk for the variation of cost and schedule of the project. When the α - level is equal to one, the user is totally certain about the cost and schedule estimations since there is no risk for range estimations and the cash flow results will be crisp values.

4.2.2. Fuzzification of the System

In this step, the cost and schedule inputs entered to the model by linguistic expression with range intervals are fuzzified. The fuzzified inputs are used to form fuzzy numbers that are normalized and convex fuzzy sets used in arithmetic calculations of fuzzy set theory. Fuzzy membership functions are assigned for each fuzzy number. Selection of the shape of the fuzzy membership functions is a challenging issue. The expert will prefer to use any shape for defining a fuzzy number that is believed to be suitable for the estimation of the inputs as it is mentioned in chapter 3. According to Klir et al. (1997), fuzzy set applications are not very sensitive to the shape of the fuzzy number. For this reason, all fuzzy numbers are selected as triangular shaped due to the simplicity of getting input data for constructing fuzzy number by describing most promising value (MP) with predicate units (PU) as it is demonstrated in Figure 4.11 and the typical fuzzy numbers established for different cost scenarios with different linguistic labels are illustrated in Figure 4.12.

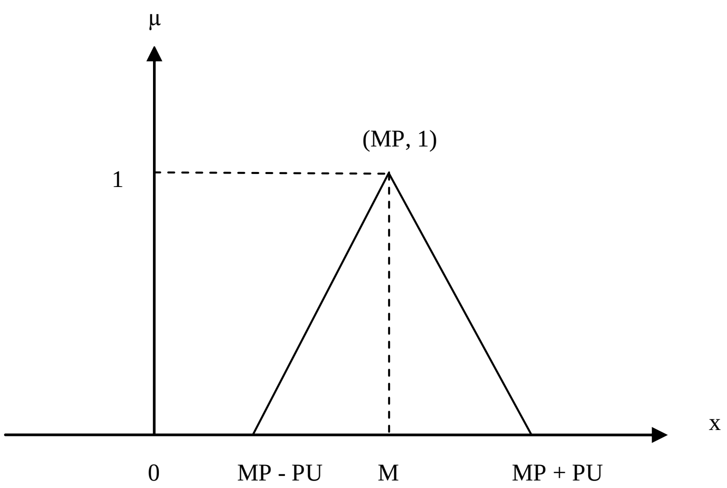

Figure 4.11: Fuzzy Numbers with MP and PU

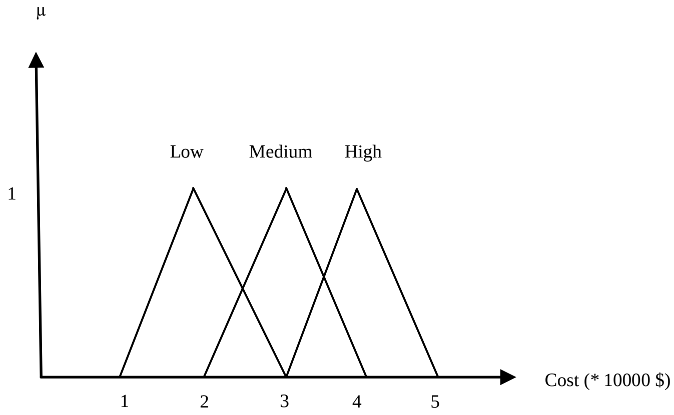

Figure 4.12: Typical Demonstration of Fuzzy Numbers with Linguistic Labels

4.2.3. Expense and Income Calculations

After the determination of the fuzzy numbers, the arithmetic operations are performed by the certain α - cut level with predefined intervals. The cost of one resource is calculated by the multiplication of total quantity of the resource planned to be consumed in certain activity with fuzzy or crisp price. Then, the resources assigned to the same activity such as cost of rebar material and wages of rebar labor are summed and assigned as the total expense of the corresponding activity. Similarly, the income value planned to be received from the owner is calculated by the multiplication of income values with total quantity of resources going to be consumed at that activity.

4.2.4. Schedule

Making a proper schedule is compulsory for the success of the any project management application.

In cash flow modeling applications, scheduling is an important tool for dispersing the expense and incomes of the project over the project duration. In this study, the project schedule is made in Microsoft Excel 2007 by making forward pass calculations.

As previously mentioned, due to the possible variations of the activity duration, it is decided to use the durations of the activities with range estimations and the users insert the activity durations with most promising values and predicate units. The uncertainty of schedule could affect the cash flow of the project since any delay in the project time will raise the indirect costs of the project. Therefore, a possibilistic schedule is established for reflecting the uncertainty into the model. When the previous studies about the fuzzy scheduling are inspected, the problem about the backward pass calculations and determination of critical path are observed as mentioned chapter 3. Besides, since the cash flow models shows the breakdown of the net cash into the time, there should be a border for limiting the time such as one week, 15 days, one month etc. If all scheduling dates are described with range intervals, the cash flow will not be distributed to certain time periods so that the users could not benefit from the cash flow projection for developing an appropriate strategy. To overcome these problems, in this model, the duration inputs obtained as range intervals and exposed to α-cut leveling, are scheduled by only making forward pass calculations to present the uncertainty of the project duration. Lorterapong and Moselhi's (1996) forward pass rules, as stated below, are used since both the most promising values and deviations of the project durations are considered while making forward pass comparisons.

"For $A = (a_1, b_1, c_1)$ and $B = (a_2, b_2, c_2)$ are two triangular fuzzy numbers;

$\text{Max}(A, B) = [\vee(a_1, a_2), \vee(b_1, b_2), \vee(c_1, c_2)]$

$\text{Min}(A, B) = [\vee(a_1, a_2), \vee(b_1, b_2), \vee(c_1, c_2)]$

$$FES_x = \max_{p \in P} (FEF_p)$$

$FEF_x = FES_X \oplus FD_X$

$T_{proj} = FEF_e$

where FES_x = fuzzy early start time of activity x,

P = a predecessor activity, P = set of predecessors, FEF = fuzzy early finish time

FD = fuzzy activity duration, T_{proj} = fuzzy project duration, e = last activity in the project".

In cost-schedule integration, the activity durations of each scenario (Pessimistic – Normal - Optimistic) defuzzifyed with center of area method (by the equations **3.4.3**). Then the defuzzifyed durations are rounded for obtaining the crisp scheduling dates while generating the cash flow. As a result, the uncertainty of the project durations will affect the total project cost with presenting different indirect cost and an effective cash flow projection is obtained with the certain time periods.

4.2.5. Net Cash Flow Computations

After the scheduling of the project, the cost and income of the activities are distributed to the project months in the interface sheets of the program and the lagging time of the payments, advance payment and advance payment deductions are applied to the model. In the model, the lag time of the progress payments paid to contractor by the owner is one month and the lag time of the payments of the subcontracted activities incurred by the contractor to the subcontractors is also one month. The advance payment is assumed to be made in the beginning of the project and advance payment deductions are assumed to start in the first progress payments of the project. No retention amount is deducted from both the progress payments between the owner – contractor and contractor – subcontractor. The net cash flow is calculated by the subtracting the cost the performed activities from incurred incomes with regarding to advance payment deductions. Finally, the net cash flow is demonstrated in cash flow diagrams.

This model gives the users more than one net cash flow choice that will be used for making decisions and taking actions for the success of the project. According to the input data entered to the program, the model introduces 9 different scenarios stated in Table 4.1. Finally, the user is expected to select the most appropriate choice suitable for the establishing strategy for tendering and cash management plans of the project.

Table 4.1 : The Scenario Matrix of Net Cash Flow

SCHEDULE

COST	L-P	L-N	L-O
	M-P	M-N	M-O
	H-P	H-N	H-O

where L: Low Cost, M: Medium Cost, H: High Cost

P: Pessimistic Schedule, N: Normal Schedule, O: Optimistic Schedule

The application of the model and process are discussed with a case study as follow:

4.3. Analysis of Test Problem

The operational functions of the Fuzzy Cash Flow Modeling (FCFM) are introduced with an illustrated project as an example. The case study is a warehouse project to be constructed in Ankara. The model is applied with help of the experienced engineers of the tendering department of a Turkish construction company generally dealing national and international infrastructure, transportation, building construction, superstructure, industrial and environment type of construction projects. The model was introduced to engineers by giving information about the fuzzy set theory and the process of the model. Then, the description of activities and the quantity of works going to be constructed are determined; the cost and schedule estimations of the engineers are taken as input for the model.

The basic information and assumptions about the project are listed below:

- The lump-sum types of contracts are made between the owner and contractor, contractor and subcontractor.

- The project is assumed to start on 01.01.2010.

- The logical relationship between the activities is only finish to start. The network diagram of the schedule is demonstrated in Figure 4.13.

- Three predecessors and successors could be assigned to each activity.

- The model enables to assign four labor, material and equipment resources to each activity.

- Some of the activities are assumed to be subcontracted. (The ones marked by * in Table 4.2).

- The construction site works all days of the week and no holiday is defined for stopping the work due to the short duration of the project.

- The activities having many subactivities like electrical works -mechanical works are grouped and the duration is given to the whole group.

- The advance payment is assumed to be paid to the contractor at the beginning of the project and the advance payment percentage is 10 % of the total price of the contract.

- The advance payment borrowed to the contractor in the beginning of the project is going to be deducted from the progress payments at each month and the amount of deduction rate is also 10 %.

- No extra retention amount is applied for the protection of owner against contractor and contractor against subcontractor.

- The interm payments are incurred to the contractor one month after the completion of each work.

- The progress payment of the subcontractors is going to be made one month after the completion of each work.

- The income values of the resources and the subcontracted items are inserted as crisp numbers.

- No resource is assigned to the activities planned to be subcontracted to another party.

- No equipment is inserted as resource since the equipment based activities like excavation are subcontracted

- The labor expense are inserted the model as crisp values.

- All of the costs are inserted to the model in terms of dollar value.

- The indirect cost of the project including the wages of the engineers, labors working on the site for the contractor, cost of water, accommodation, electricity etc. is assumed as 200 $/day.

- At the end of the process, 9 scenarios are aimed to be obtained. The crew size and the cost for them are assumed to be fixed so that the users do not consider making time - cost optimization while making different estimates. It is assumed that the differences of the cost and schedule data are resulting from the quality of management, weather conditions, productivity rate, inflation rate etc.

- Different α - cut levels are applied to both cost and schedule calculations of the project for measuring the effect of different α - cuts to the cash flow analysis.

The tabular form of the inputs and graphical form of the outputs are demonstrated as follows:

Table 4.2: Activity Inputs

Activity	**Activity**	**Activity Explanation**
1	A	Site Preparation*
2	B	Excavation*
3	C	Formworks of Foundation
4	D	Rebar of Foundation
5	E	Pouring Foundation Concrete
6	F	Structural Steel Erection
7	G	Masonry Works
8	H	Insulation
9	I	Leveling
10	J	Plastering
11	K	Floor Covering
12	L	Paint Interior
13	M	Paint Exterior
14	N	Doors &Windows*
15	P	Mechanical Works*
16	Q	Electrical Works*
17	R	Pouring Concrete for Protection
18	S	Thermal Moisture

“*” shows the subcontracted activities

Table 4.3: Resource Input

Resource ID	Resource Name	Resource Type	Resource Type
		Labor	Material
F1	Formwork Material		F1
R1	Rebar Material		R1
C1	Concrete Material		C1
S1	Structural Steel Material		S1
F2	Formwork Labor	F2	
R2	Rebar Labor	R2	
C2	Concrete Labor	C2	
L1	Leveling Material		L1
L2	Leveling Labor	L2	
Pl1	Plastering Material		Pl1
Pl2	Plastering Labor	Pl2	
P1	Interior Painting Material		P1
P2	Painting Labor	P2	
S2	Structural Steel Labor	S2	
Mas1	Masonry Material		Mas1
Mas2	Masonry Labor	Mas2	
Fl1	Floor Covering Material		Fl1
Fl2	Floor Covering Labor	Fl2	
Ins1	Foundation Insulation Material		Ins1
Ins2	Foundation Insulation Labor	Ins2	
C3	Lean Concrete Material		C3
TM1	Insulation Material of WC		TM1
P3	Exterior Painting Material		P3
TM2	Insulation Labor of WC	TM2	

Table 4.4: Resource Expense Input

Resource ID	Resource Name	Resource Type Labor	Resource Type Material	Labor Cost CRISP ($)	Material Cost Low ($)		Material Cost Med.		Material Cost High	
					MP ($)	PU ($)	MP ($)	PU ($)	MP ($)	PU ($)
F1	Formwork Material		F1		5	0,8	7	1,5	9	2
R1	Rebar Material		R1		400	25	500	50	600	75
C1	Concrete Material		C1		55	5	60	4	75	5
S1	Structural Steel Material		S1		525	20	575	60	800	50
F2	Formwork Labor	F2		6,75						
R2	Rebar Labor	R2		165						
C2	Concrete Labor	C2		1,2						
L1	Leveling Material		L1		1	0,25	1,35	0,2	1,5	0,15
L2	Leveling Labor	L2		2,02						
Pl1	Plastering Material		Pl1		5	0,5	8,2	1	13	3
Pl2	Plastering Labor	Pl2		9,88						
P1	Interior Painting Material		P1		1	0,1	1,24	0,1	2,5	0,25
P2	Painting Labor	P2		1,86						
S2	Structural Steel Labor	S2		480						
Mas1	Masonry Material		Mas1		70	7	82	9	90	8
Mas2	Masonry Labor	Mas2		36						
Fl1	Floor Covering Material		Fl1		5	1	9,2	1,2	12	2
Fl2	Floor Covering Labor	Fl2		5,65						
Ins1	Foundation Insulation		Ins1		10	1,5	12,5	2	14	2
Ins2	Foundation Insulation	Ins2		1,96						
C3	Lean Concrete Material		C3		47	5	51,13	5,5	60	5
TM1	Insulation Material of WC		TM1		7	2,5	9	2	12	2,5
P3	Exterior Painting Material		P3		2,5	0,5	3,76	0,4	5	0,5
TM2	Insulation Labor of WC	TM2		7,35						

Table 4.5: Resource Income Input

Resource ID	Resource Name	Resource Type			Labor	Material
		Labor	Material	Equipment	Crisp ($)	Crisp ($)
F1	Formwork Material		F1			8,05
R1	Rebar Material		R1			575
C1	Concrete Material		C1			69
S1	Structural Steel Material		S1			661,25
F2	Formwork Labor	F2			7,77	
R2	Rebar Labor	R2			189,75	
C2	Concrete Labor	C2			1,38	
L1	Leveling Material		L1			1,56
L2	Leveling Labor	L2			2,32	
Pl1	Plastering Material		Pl1			9,43
Pl2	Plastering Labor	Pl2			11,36	
P1	Interior Painting Material		P1			1,42
P2	Painting Labor	P2			2,14	
S2	Structural Steel Labor	S2			552	
Mas1	Masonry Material		Mas1			94,3
Mas2	Masonry Labor	Mas2			41,4	
Fl1	Floor Covering Material		Fl1			10,58
Fl2	Floor Covering Labor	Fl2			6,5	
Ins1	Foundation Insulation Material		Ins1			14,38
Ins2	Foundation Insulation Labor	Ins2			2,26	
C3	Lean Concrete Material		C3			58,79
TM1	Insulation Material of WC		TM1			10,35
P3	Exterior Painting Material		P3			4,33
TM2	Insulation Labor of WC	TM2			8,69	

Table 4.6: Resource Assignment and Subcontracted Costs - Incomes

Activity Name	Total Quantity of Work	Unit	Labor	Material	Subcontracted Cost ($)	Subcontracted Income ($)
A					25000	28750
B					25000	28750
C	392	m^2	F2	F1		
D	34,00	ton	R2	R1		
E	324,00	m^3	C2	C1		
F	44,5	ton	S2	S1		
G	30,00	m^3	Mas2	Mas1		
H	680	m^2	Ins2	Ins1		
I	650,00	m^2	L2	L1		
J	465,00	m^2	Pl2	Pl1		5000
K	650,00	m^2	Fl2	Fl1		
L	465,00	m^2	P2	P1		
M	330,00	m^2	P2	P3		
N					20000	23000
P					18000	20700
Q					12000	13800
R	680,00	m^2	C2	C3		
S	40	m^2	TM2	TM1		

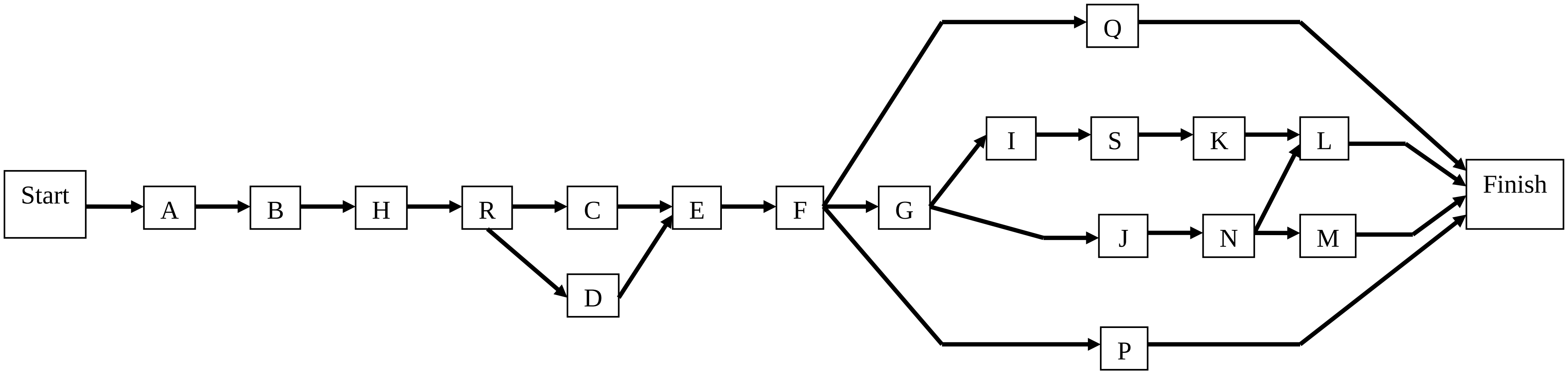

Figure 4.13: Activities on Node Diagram of the Warehouse Project

4.4. Discussion of Results

The user enters all necessary input data for obtaining 9 different cash flow scenarios. The results of the Optimistic Schedule – Low Cost scenario are demonstrated here and the rest of the results are presented in the tables A. 1 – A. 16 and the net cash flow graphs of the zero alpha cuts are shown in Figures B1 – B8 (See Appendix A and B). The Figures show the variability of the project net cash flow among project duration. Since the cost and schedule inputs are gathered as range estimation in triangular fuzzy shape, most of the graphs show the results as triangular fuzzy number. The graphs enable the user to observe the net cash flow profiles of the project with different possibilities. For instance, the Figure 4.14 shows the optimistic schedule – low cost case of the model. It is observed from the graph that the net cash flow of the project will be negative in the first and second month of construction and the ranges changes from -5.000 $ to -30.000 $. Whereas, the positive cash flow continues for the rest of the project duration while ranging from 17000 $ to 36.000$. Although the contractor takes advance payment in the beginning of the project, the net cash flow of first two months is negative. There are many activities going to be performed in the first two months so that the most of the project expenses accumulate and pass the project incomes in these months. It means that for the beginning of the project, the contractor should prepare a cash management plan for compensating the gap of the negative cash. Besides, in the same graph, it is observed that the cash flow of the last month has only one value. That’s why the shape of the last month is a deterministic straight line rather than a fuzzy triangle.

When the results of the all 9 scenarios are examined, it is observed that the total project cost is ranging from 292.137,49 $ to 382.143,34 $ and the net cash profile differs from the 16000 $ to -74000 $ where the total income of the project does not change. The total duration of the project is 105 days in optimistic schedule, 144 days in normal schedule and 182 days in pessimistic schedule calculated by the defuzzifyed project durations. These large differences between the project durations of the different schedule scenarios cause variations in expense of the project by effecting the total indirect cost.

From Tables A. 1 – A. 16, it can be observed that applying different α - cut values changes the project expense and net cash flow profile by adjusting the range of the estimate. For instance, when the results of the Optimistic Schedule - Low Cost scenario is examined, it can be clearly seen that the project expense and net cash flow data get closer to the most promising values of the results as the α - cut values increases from 0 to 1 (See Table 4.7). It means that the users preferring using the small α - cut values want to foreseen low risk while preparing the tender and the users choose high α-cut values get high risk while evaluating the project cash flow since results only depend on the most promising value. Similarly, the fuzzy project dates are also effected by different α - cuts. For instance, in Table 4.8, it is observed that the pessimistic project finish date of the scenario "c" is 03.05.2010 when α - cut level is zero and 16.04.2010 when α - cut level is one. However, since all the estimates are symmetrically distributed while assigning durations rounded up in case of rational numbers, the crisp project durations are not changed. The experts will examine the different scenarios and choose the best suitable case so that they will prepare different cash flow plans for pretending negative cash flow situation. Also, the expert will decide to bid or not to bid with the help of the net cash flow profiles determined by the model and adjust the contingency amount according to the risks foreseen by the possibilistic cash flow data.

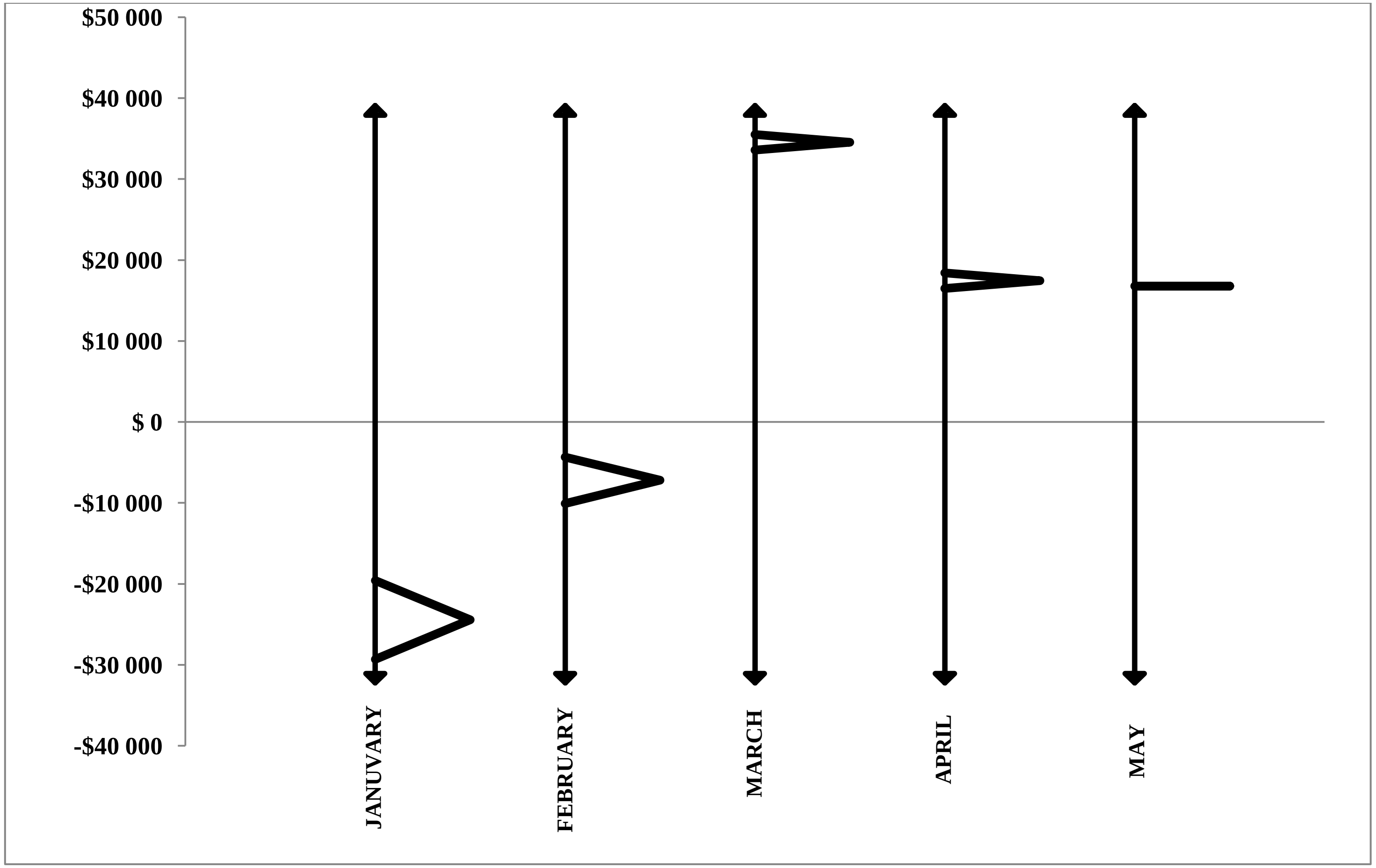

Figure 4.14: Optimistic Schedule Low Cost

Table 4.7: Optimistic Schedule Low Cost – Net Cash Flow

Optimistic Schedule High Cost	**Total Project Income ($)**	**Fuzzy Total Project Expense ($)**			**Fuzzy Net Cash Flow ($)**			**Net Cash Flow ($)**
α - cuts	Crisp	a	b	c	a	b	c	Crisp
0	308.140,94	292.137,49	301.797,59	311.457,69	-3.316,76	6.343,34	16.003,44	6.343,34
0,1	308.140,94	293.103,50	301.797,59	310.491,68	-2.350,75	6.343,34	15.037,43	6.343,34
0,2	308.140,94	294.069,51	301.797,59	309.525,67	-1.384,74	6.343,34	14.071,42	6.343,34
0,3	308.140,94	295.035,52	301.797,59	308.559,66	-418,73	6.343,34	13.105,41	6.343,34
0,4	308.140,94	296.001,53	301.797,59	307.593,65	547,28	6.343,34	12.139,40	6.343,34
0,5	308.140,94	296.967,54	301.797,59	306.627,64	1.513,29	6.343,34	11.173,39	6.343,34
0,6	308.140,94	297.933,55	301.797,59	305.661,63	2.479,30	6.343,34	10.207,38	6.343,34
0,7	308.140,94	298.899,56	301.797,59	304.695,62	3.445,31	6.343,34	9.241,37	6.343,34
0,8	308.140,94	299.865,57	301.797,59	303.729,61	4.411,32	6.343,34	8.275,36	6.343,34
0,9	308.140,94	300.831,58	301.797,59	302.763,60	5.377,33	6.343,34	7.309,35	6.343,34
1	308.140,94	301.797,59	301.797,59	301.797,59	6.343,34	6.343,34	6.343,34	6.343,34

Table 4.8: Optimistic Schedule Low Cost – Schedule Results

Optimistic Schedule Low Cost	Project Start Date	Project Finish Date	Project Duration	Fuzzy Project Finish Date		
α - cuts	Crisp	Crisp	Crisp	a	b	c
0	01.01.2010	16.04.2010	105	30.03.2010	16.04.2010	03.05.2010
0,1	01.01.2010	16.04.2010	105	07.04.2010	16.04.2010	02.05.2010
0,2	01.01.2010	16.04.2010	105	07.04.2010	16.04.2010	02.05.2010
0,3	01.01.2010	16.04.2010	105	07.04.2010	16.04.2010	02.05.2010
0,4	01.01.2010	16.04.2010	105	09.04.2010	16.04.2010	30.04.2010
0,5	01.01.2010	16.04.2010	105	09.04.2010	16.04.2010	26.04.2010
0,6	01.01.2010	16.04.2010	105	12.04.2010	16.04.2010	26.04.2010
0,7	01.01.2010	16.04.2010	105	14.04.2010	16.04.2010	24.04.2010
0,8	01.01.2010	16.04.2010	105	14.04.2010	16.04.2010	24.04.2010
0,9	01.01.2010	16.04.2010	105	14.04.2010	16.04.2010	24.04.2010
1	01.01.2010	16.04.2010	105	14.04.2010	16.04.2010	16.04.2010

CHAPTER 5

CONCLUSION

This study aims to present a realistic cash flow model by using fuzzy set theory called Fuzzy Cash Flow Modeling (FCFM). Fuzzy set theory is mostly preferred in decision making processes for coping with uncertainties of an event resulting from the complexity and deficiency of the appropriate statistical information. Since construction projects are unique and complex, the historical data will not be always suitable for generating a reliable cash flow model. Therefore, in this study, it is decided to utilize from the experience of the practitioners, human ability of thinking and approximate reasoning by using fuzzy set theory with the help of linguistic labels while developing a cash flow model and to it is aimed to obtain possibilistic range estimation rather than a single deterministic one.

FCFM relies on the range estimations of the experts. All of the related input data (cost of the resource, duration of the activities, general information about the project) are inserted by the users as triangular fuzzy number by defining the related most possible and dispersion values for reflecting the possible cost and schedule uncertainty of the project. The input data is described by the users with linguistic expressions for creating scenarios and grading the inputs while making data entering. If it is required by the user, the model allows creating 9 different scenarios based on the matching of 3 different cost (Low – Medium - High Cost) and schedule (Pessimistic – Normal - Optimistic Schedule) situation and changing the range of the estimates according to risk approach of the experts about the project by changing the α - cut level. The model is applied to a case project and the results are demonstrated in both tabular form and graphical view.

There are certain advantages of using the proposed cash flow model. First, FCFM is a user-friendly model for making cash flow analysis and it is developed with a well - known computer program. Therefore, it will be easily used by construction management practitioners for financial management of the project. Second, the results of the cash flow analysis demonstrates overall cash situation of the project over project duration. The users could realize the requirement of cash flow with graduated possibilities and take necessary actions for preventing the negative cash flow and developing necessary cash management strategies for the completion of the project in success. Third, since the projects are examined in details, the users have chance to establish the problem in activity level and make appropriate point solutions for improving the cash flow of the whole project. Also, the users could designate a more realistic bid price by the created different cash flow scenarios after realizing the possible cost and schedule risks of the project or generate bidding strategies like applying front-loading, back loading etc. Hence, examining different risk scenarios may help the users in bid/no bid decision making process. Furthermore, this study reveals that with fuzzy set theory, cash flow model can be achieved for overcoming the problem of the risk in construction projects and develop realistic cash management strategies. Therefore, FCFM will be a good alternative of the probabilistic simulation models for dealing with uncertainties of the construction projects resulting from complexity and ambiguity.

In spite of its advantages, the model has certain limitations. First of all, since all inputs are obtained from the experts, the reliability of the model depends on the accuracy and quality of the estimates. It means that the model generated by different experts will give different results. Moreover, while establishing fuzzy numbers, the membership functions are assumed to be linear but different membership functions could be used in different cases for better explanation of the expert opinion. Likewise, different defuzzification methods, time lags for interm payments and subcontractor progress payments can be preferred by different users. Besides, for the practical usage of the model, only three linguistic variables were used for expressing the expert judgment such as low – medium - high. The number of the linguistic terms may change and more linguistic variables can be used such as very low - slightly low - moderately high etc. Similarly, for preventing the time consumption while gathering input data from experts, all the fuzzy numbers are constructed by the symmetrical distribution of the predicate units into the left and right span of the most promising value but it is possible to disperse the predicate unit for obtaining unsymmetrical fuzzy numbers.

In future studies, it is recommended to generate the model by increasing the number of the activities and resources so that the model will be used for generating the cash flow of the larger projects. Also, a decision support tool can be made for helping the user during the selection of the appropriate scenario related to nature of the project.

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APPENDIX A

TABLES

Table A.1: Optimistic Schedule Medium Cost – Net Cash Flow

Optimistic Schedule Medium Cost	**Total Project Income($)**	**Fuzzy Total Project Expense ($)**			**Fuzzy Net Cash Flow ($)**			**Net Cash Flow ($)**
α - cuts	Crisp	a	b	c	a	b	c	Crisp
0	308.140,94	306.490,39	319.747,89	333.005,39	-24.864,46	-11.606,96	1.650,54	-11.606,96
0,1	308.140,94	307.816,14	319.747,89	331.679,64	-23.538,71	-11.606,96	324,79	-11.606,96
0,2	308.140,94	309.141,89	319.747,89	330.353,89	-22.212,96	-11.606,96	-1.000,96	-11.606,96
0,3	308.140,94	310.467,64	319.747,89	329.028,14	-20.887,21	-11.606,96	-2.326,71	-11.606,96
0,4	308.140,94	311.793,39	319.747,89	327.702,39	-19.561,46	-11.606,96	-3.652,46	-11.606,96
0,5	308.140,94	313.119,14	319.747,89	326.376,64	-18.235,71	-11.606,96	-4.978,21	-11.606,96
0,6	308.140,94	314.444,89	319.747,89	325.050,89	-16.909,96	-11.606,96	-6.303,96	-11.606,96
0,7	308.140,94	315.770,64	319.747,89	323.725,14	-15.584,21	-11.606,96	-7.629,71	-11.606,96
0,8	308.140,94	317.096,39	319.747,89	322.399,39	-14.258,46	-11.606,96	-8.955,46	-11.606,96
0,9	308.140,94	318.422,14	319.747,89	321.073,64	-12.932,71	-11.606,96	-10.281,21	-11.606,96
1	308.140,94	319.747,89	319.747,89	319.747,89	-11.606,96	-11.606,96	-11.606,96	-11.606,96

Table A.2: Optimistic Schedule Medium Cost – Schedule Results

Optimistic Schedule Medium Cost	Project Start Date	Project Finish Date	Project Duration	Fuzzy Project Finish Date		
α - cuts	Crisp	Crisp	Crisp	a	b	c
0	01.01.2010	16.04.2010	105	30.03.2010	16.04.2010	03.05.2010
0,1	01.01.2010	16.04.2010	105	07.04.2010	16.04.2010	02.05.2010
0,2	01.01.2010	16.04.2010	105	07.04.2010	16.04.2010	02.05.2010
0,3	01.01.2010	16.04.2010	105	07.04.2010	16.04.2010	02.05.2010
0,4	01.01.2010	16.04.2010	105	09.04.2010	16.04.2010	30.04.2010
0,5	01.01.2010	16.04.2010	105	09.04.2010	16.04.2010	26.04.2010
0,6	01.01.2010	16.04.2010	105	12.04.2010	16.04.2010	26.04.2010
0,7	01.01.2010	16.04.2010	105	14.04.2010	16.04.2010	24.04.2010
0,8	01.01.2010	16.04.2010	105	14.04.2010	16.04.2010	24.04.2010
0,9	01.01.2010	16.04.2010	105	14.04.2010	16.04.2010	24.04.2010
1	01.01.2010	16.04.2010	105	14.04.2010	16.04.2010	16.04.2010

Table A.3: Optimistic Schedule High Cost – Net Cash Flow

Optimistic Schedule High Cost	**Total Project Income ($)**	**Fuzzy Total Project Expense ($)**			**Fuzzy Net Cash Flow ($)**			**Net Cash Flow ($)**
α - cuts	Crisp	a	b	c	a	b	c	Crisp
0	308.140,94	336.007,84	351.360,59	366.743,34	-58.602,41	-43.219,66	-27.866,91	-43.224,65
0,1	308.140,94	337.543,12	351.360,59	365.205,07	-57.064,13	-43.219,66	-29.402,18	-43.224,15
0,2	308.140,94	339.078,39	351.360,59	363.666,79	-55.525,86	-43.219,66	-30.937,46	-43.223,65
0,3	308.140,94	340.613,67	351.360,59	362.128,52	-53.987,58	-43.219,66	-32.472,73	-43.223,15
0,4	308.140,94	342.148,94	351.360,59	360.590,24	-52.449,31	-43.219,66	-34.008,01	-43.222,65
0,5	308.140,94	343.684,22	351.360,59	359.051,97	-50.911,03	-43.219,66	-35.543,28	-43.222,15
0,6	308.140,94	345.219,49	351.360,59	357.513,69	-49.372,76	-43.219,66	-37.078,56	-43.219,66
0,7	308.140,94	346.754,77	351.360,59	355.975,42	-47.834,48	-43.219,66	-38.613,83	-43.219,66
0,8	308.140,94	348.290,04	351.360,59	354.437,14	-46.296,21	-43.219,66	-40.149,11	-43.219,66
0,9	308.140,94	349.825,32	351.360,59	352.898,87	-44.757,93	-43.219,66	-41.684,38	-43.219,66
1	308.140,94	351.360,59	351.360,59	351.360,59	-43.219,66	-43.219,66	-43.219,66	-43.219,66

Table A.4: Optimistic Schedule High Cost – Schedule Results

Optimistic Schedule High Cost	Project Start Date	Project Finish Date	Project Duration	Fuzzy Project Finish Date		
α - cuts	Crisp	Crisp	Crisp	a	b	c
0	01.01.2010	16.04.2010	105	30.03.2010	16.04.2010	03.05.2010
0,1	01.01.2010	16.04.2010	105	07.04.2010	16.04.2010	02.05.2010
0,2	01.01.2010	16.04.2010	105	07.04.2010	16.04.2010	02.05.2010
0,3	01.01.2010	16.04.2010	105	07.04.2010	16.04.2010	02.05.2010
0,4	01.01.2010	16.04.2010	105	09.04.2010	16.04.2010	30.04.2010
0,5	01.01.2010	16.04.2010	105	09.04.2010	16.04.2010	26.04.2010
0,6	01.01.2010	16.04.2010	105	12.04.2010	16.04.2010	26.04.2010
0,7	01.01.2010	16.04.2010	105	14.04.2010	16.04.2010	24.04.2010
0,8	01.01.2010	16.04.2010	105	14.04.2010	16.04.2010	24.04.2010
0,9	01.01.2010	16.04.2010	105	14.04.2010	16.04.2010	24.04.2010
1	01.01.2010	16.04.2010	105	14.04.2010	16.04.2010	16.04.2010

Table A.5: Normal Schedule Low Cost – Net Cash Flow

Normal Schedule Low Cost	**Total Project Income ($)**	**Fuzzy Total Project Expense ($)**			**Fuzzy Net Cash Flow ($)**			**Net Cash Flow ($)**
α - cuts	Crisp	a	b	c	a	b	c	Crisp
0	308.140,94	299.937,49	309.597,59	319.257,69	-11.116,76	-1.456,66	8.203,44	-1.456,66
0,1	308.140,94	300.903,50	309.597,59	318.291,68	-10.150,75	-1.456,66	7.237,43	-1.456,66
0,2	308.140,94	301.869,51	309.597,59	317.325,67	-9.184,74	-1.456,66	6.271,42	-1.456,66
0,3	308.140,94	302.835,52	309.597,59	316.359,66	-8.218,73	-1.456,66	5.305,41	-1.456,66
0,4	308.140,94	303.801,53	309.597,59	315.393,65	-7.252,72	-1.456,66	4.339,40	-1.456,66
0,5	308.140,94	304.767,54	309.597,59	314.427,64	-6.286,71	-1.456,66	3.373,39	-1.456,66
0,6	308.140,94	305.733,55	309.597,59	313.461,63	-5.320,70	-1.456,66	2.407,38	-1.456,66
0,7	308.140,94	306.699,56	309.597,59	312.495,62	-4.354,69	-1.456,66	1.441,37	-1.456,66
0,8	308.140,94	307.665,57	309.597,59	311.529,61	-3.388,68	-1.456,66	475,36	-1.456,66
0,9	308.140,94	308.631,58	309.597,59	310.563,60	-2.422,67	-1.456,66	-490,65	-1.456,66
1	308.140,94	309.597,59	309.597,59	309.597,59	-1.456,66	-1.456,66	-1.456,66	-1.456,66

Table A.6: Normal Schedule Low Cost – Schedule Results

Normal Schedule Low Cost	Project Start Date	Project Finish Date	Project Duration	Fuzzy Project Finish Date		
α - cuts	Crisp	Crisp	Crisp	a	b	c
0	01.01.2010	25.05.2010	144	27.04.2010	25.05.2010	22.06.2010
0,1	01.01.2010	25.05.2010	144	06.05.2010	25.05.2010	22.06.2010
0,2	01.01.2010	25.05.2010	144	06.05.2010	25.05.2010	21.06.2010
0,3	01.01.2010	25.05.2010	144	09.05.2010	25.05.2010	19.06.2010
0,4	01.01.2010	25.05.2010	144	12.05.2010	25.05.2010	15.06.2010
0,5	01.01.2010	25.05.2010	144	13.05.2010	25.05.2010	12.06.2010
0,6	01.01.2010	25.05.2010	144	18.05.2010	25.05.2010	09.06.2010
0,7	01.01.2010	25.05.2010	144	22.05.2010	25.05.2010	06.06.2010
0,8	01.01.2010	25.05.2010	144	24.05.2010	25.05.2010	03.06.2010
0,9	01.01.2010	25.05.2010	144	25.05.2010	25.05.2010	03.06.2010
1	01.01.2010	25.05.2010	144	25.05.2010	25.05.2010	25.05.2010

Table A.7: Normal Schedule Medium Cost – Net Cash Flow

Normal Schedule Medium Cost	**Total Project Income ($)**	**Fuzzy Total Project Expense ($)**			**Fuzzy Net Cash Flow ($)**			**Net Cash Flow ($)**
α - cuts	Crisp	a	b	c	a	b	c	Crisp
0	308.140,94	314.290,39	327.547,89	340.805,39	-32.664,46	-19.406,96	-6.149,46	-19.406,96
0,1	308.140,94	315.616,14	327.547,89	339.479,64	-31.338,71	-19.406,96	-7.475,21	-19.406,96
0,2	308.140,94	316.941,89	327.547,89	338.153,89	-30.012,96	-19.406,96	-8.800,96	-19.406,96
0,3	308.140,94	318.267,64	327.547,89	336.828,14	-28.687,21	-19.406,96	-10.126,71	-19.406,96
0,4	308.140,94	319.593,39	327.547,89	335.502,39	-27.361,46	-19.406,96	-11.452,46	-19.406,96
0,5	308.140,94	320.919,14	327.547,89	334.176,64	-26.035,71	-19.406,96	-12.778,21	-19.406,96
0,6	308.140,94	322.244,89	327.547,89	332.850,89	-24.709,96	-19.406,96	-14.103,96	-19.406,96
0,7	308.140,94	323.570,64	327.547,89	331.525,14	-23.384,21	-19.406,96	-15.429,71	-19.406,96
0,8	308.140,94	324.896,39	327.547,89	330.199,39	-22.058,46	-19.406,96	-16.755,46	-19.406,96
0,9	308.140,94	326.222,14	327.547,89	328.873,64	-20.732,71	-19.406,96	-18.081,21	-19.406,96
1	308.140,94	327.547,89	327.547,89	327.547,89	-19.406,96	-19.406,96	-19.406,96	-19.406,96

Table A.8: Normal Schedule Medium Cost – Schedule Results

Normal Schedule Medium Cost	Project Start Date	Project Finish Date	Project Duration	Fuzzy Project Finish Date		
α - cuts	Crisp	Crisp	Crisp	a	b	c
0	01.01.2010	25.05.2010	144	27.04.2010	25.05.2010	22.06.2010
0,1	01.01.2010	25.05.2010	144	06.05.2010	25.05.2010	22.06.2010
0,2	01.01.2010	25.05.2010	144	06.05.2010	25.05.2010	21.06.2010
0,3	01.01.2010	25.05.2010	144	09.05.2010	25.05.2010	19.06.2010
0,4	01.01.2010	25.05.2010	144	12.05.2010	25.05.2010	15.06.2010
0,5	01.01.2010	25.05.2010	144	13.05.2010	25.05.2010	12.06.2010
0,6	01.01.2010	25.05.2010	144	18.05.2010	25.05.2010	09.06.2010
0,7	01.01.2010	25.05.2010	144	22.05.2010	25.05.2010	06.06.2010
0,8	01.01.2010	25.05.2010	144	24.05.2010	25.05.2010	03.06.2010
0,9	01.01.2010	25.05.2010	144	25.05.2010	25.05.2010	03.06.2010
1	01.01.2010	25.05.2010	144	25.05.2010	25.05.2010	25.05.2010

Table A.9: Normal Schedule High Cost – Net Cash Flow

Normal Schedule High Cost	**Total Project Income ($)**	**Fuzzy Total Project Expense ($)**			**Fuzzy Net Cash Flow ($)**			**Net Cash Flow ($)**
α - cuts	Crisp	a	b	c	a	b	c	Crisp
0	308.140,94	343.807,84	359.160,59	374.543,34	-66.402,41	-51.019,66	-35.666,91	-51.024,65
0,1	308.140,94	345.343,12	359.160,59	373.005,07	-64.864,13	-51.019,66	-37.202,18	-51.024,15
0,2	308.140,94	346.878,39	359.160,59	371.466,79	-63.325,86	-51.019,66	-38.737,46	-51.023,65
0,3	308.140,94	348.413,67	359.160,59	369.928,52	-61.787,58	-51.019,66	-40.272,73	-51.023,15
0,4	308.140,94	349.948,94	359.160,59	368.390,24	-60.249,31	-51.019,66	-41.808,01	-51.022,65
0,5	308.140,94	351.484,22	359.160,59	366.851,97	-58.711,03	-51.019,66	-43.343,28	-51.022,15
0,6	308.140,94	353.019,49	359.160,59	365.313,69	-57.172,76	-51.019,66	-44.878,56	-51.021,65
0,7	308.140,94	354.554,77	359.160,59	363.775,42	-55.634,48	-51.019,66	-46.413,83	-51.021,15
0,8	308.140,94	356.090,04	359.160,59	362.237,14	-54.096,21	-51.019,66	-47.949,11	-51.019,66
0,9	308.140,94	357.625,32	359.160,59	360.698,87	-52.557,93	-51.019,66	-49.484,38	-51.019,66
1	308.140,94	359.160,59	359.160,59	359.160,59	-51.019,66	-51.019,66	-51.019,66	-51.019,66

Table A.10: Normal Schedule High Cost – Schedule Results

Normal Schedule High Cost	**Project Start Date**	**Project Finish Date**	**Project Duration**	**Fuzzy Project Finish Date**		
α - cuts	Crisp	Crisp	Crisp	a	b	c
0	01.01.2010	25.05.2010	144	27.04.2010	25.05.2010	22.06.2010
0,1	01.01.2010	25.05.2010	144	06.05.2010	25.05.2010	22.06.2010
0,2	01.01.2010	25.05.2010	144	06.05.2010	25.05.2010	21.06.2010
0,3	01.01.2010	25.05.2010	144	09.05.2010	25.05.2010	19.06.2010
0,4	01.01.2010	25.05.2010	144	12.05.2010	25.05.2010	15.06.2010
0,5	01.01.2010	25.05.2010	144	13.05.2010	25.05.2010	12.06.2010
0,6	01.01.2010	25.05.2010	144	18.05.2010	25.05.2010	09.06.2010
0,7	01.01.2010	25.05.2010	144	22.05.2010	25.05.2010	06.06.2010
0,8	01.01.2010	25.05.2010	144	24.05.2010	25.05.2010	03.06.2010
0,9	01.01.2010	25.05.2010	144	25.05.2010	25.05.2010	03.06.2010
1	01.01.2010	25.05.2010	144	25.05.2010	25.05.2010	25.05.2010

Table A.11: Pessimistic Schedule Low Cost – Net Cash Flow

Pessimistic Schedule Low Cost	**Total Project Income ($)**	**Fuzzy Total Project Expense ($)**			**Fuzzy Net Cash Flow ($)**			**Net Cash Flow ($)**
α - cuts	Crisp	a	b	c	a	b	c	Crisp
0	308.140,94	307.537,49	317.197,59	326.857,69	-18.716,76	-9.056,66	603,44	-9.056,66
0,1	308.140,94	308.503,50	317.197,59	325.891,68	-17.750,75	-9.056,66	-362,57	-9.056,66
0,2	308.140,94	309.669,51	317.397,59	325.125,67	-16.984,74	-9.256,66	-1.528,58	-9.256,66
0,3	308.140,94	310.435,52	317.197,59	323.959,66	-15.818,73	-9.056,66	-2.294,59	-9.056,66
0,4	308.140,94	311.401,53	317.197,59	322.993,65	-14.852,72	-9.056,66	-3.260,60	-9.056,66
0,5	308.140,94	312.367,54	317.197,59	322.027,64	-13.886,71	-9.056,66	-4.226,61	-9.056,66
0,6	308.140,94	313.333,55	317.197,59	321.061,63	-12.920,70	-9.056,66	-5.192,62	-9.056,66
0,7	308.140,94	314.299,56	317.197,59	320.095,62	-11.954,69	-9.056,66	-6.158,63	-9.056,66
0,8	308.140,94	315.265,57	317.197,59	319.129,61	-10.988,68	-9.056,66	-7.124,64	-9.056,66
0,9	308.140,94	316.231,58	317.197,59	318.163,60	-10.022,67	-9.056,66	-8.090,65	-9.056,66
1	308.140,94	317.197,59	317.197,59	317.197,59	-9.056,66	-9.056,66	-9.056,66	-9.056,66

Table A.12: Pessimistic Schedule Low Cost – Schedule Results

Pessimistic Schedule Low Cost	**Project Start Date**	**Project Finish Date**	**Project Duration**	**Fuzzy Project Finish Date**		
α - cuts	Crisp	Crisp	Crisp	a	b	c
0	01.01.2010	02.07.2010	182	25.05.2010	02.07.2010	09.08.2010
0,1	01.01.2010	02.07.2010	182	05.06.2010	02.07.2010	09.08.2010
0,2	01.01.2010	03.07.2010	183	05.06.2010	02.07.2010	07.08.2010
0,3	01.01.2010	02.07.2010	182	11.06.2010	02.07.2010	03.08.2010
0,4	01.01.2010	02.07.2010	182	14.06.2010	02.07.2010	28.07.2010
0,5	01.01.2010	02.07.2010	182	17.06.2010	02.07.2010	28.07.2010
0,6	01.01.2010	02.07.2010	182	20.06.2010	02.07.2010	22.07.2010
0,7	01.01.2010	02.07.2010	182	26.06.2010	02.07.2010	19.07.2010
0,8	01.01.2010	02.07.2010	182	29.06.2010	02.07.2010	13.07.2010
0,9	01.01.2010	02.07.2010	182	02.07.2010	02.07.2010	13.07.2010
1	01.01.2010	02.07.2010	182	02.07.2010	02.07.2010	02.07.2010

Table A.13: Pessimistic Schedule Medium Cost – Net Cash Flow

Pessimistic Schedule Medium Cost	**Total Project Income ($)**	**Fuzzy Total Project Expense ($)**			**Fuzzy Net Cash Flow ($)**			**Net Cash Flow ($)**
α - cuts	Crisp	a	b	c	a	b	c	Crisp
0	308.140,94	321.890,39	335.147,89	348.405,39	-40.264,46	-27.006,96	-13.749,46	-27.006,96
0,1	308.140,94	323.216,14	335.147,89	347.079,64	-38.938,71	-27.006,96	-15.075,21	-27.006,96
0,2	308.140,94	324.741,89	335.347,89	345.953,89	-37.812,96	-27.206,96	-16.600,96	-27.206,96
0,3	308.140,94	325.867,64	335.147,89	344.428,14	-36.287,21	-27.006,96	-17.726,71	-27.006,96
0,4	308.140,94	327.193,39	335.147,89	343.102,39	-34.961,46	-27.006,96	-19.052,46	-27.006,96
0,5	308.140,94	328.519,14	335.147,89	341.776,64	-33.635,71	-27.006,96	-20.378,21	-27.006,96
0,6	308.140,94	329.844,89	335.147,89	340.450,89	-32.309,96	-27.006,96	-21.703,96	-27.006,96
0,7	308.140,94	331.170,64	335.147,89	339.125,14	-30.984,21	-27.006,96	-23.029,71	-27.006,96
0,8	308.140,94	332.496,39	335.147,89	337.799,39	-29.658,46	-27.006,96	-24.355,46	-27.006,96
0,9	308.140,94	333.822,14	335.147,89	336.473,64	-28.332,71	-27.006,96	-25.681,21	-27.006,96
1	308.140,94	335.147,89	335.147,89	335.147,89	-27.006,96	-27.006,96	-27.006,96	-27.006,96

Table A.14: Pessimistic Schedule Medium Cost – Schedule Results

Pessimistic Schedule Medium Cost	**Project Start Date**	**Project Finish Date**	**Project Duration**	**Fuzzy Project Finish Date**		
α - cuts	Crisp	Crisp	Crisp	a	b	c
0	01.01.2010	02.07.2010	182	25.05.2010	02.07.2010	09.08.2010
0,1	01.01.2010	02.07.2010	182	05.06.2010	02.07.2010	09.08.2010
0,2	01.01.2010	03.07.2010	183	05.06.2010	02.07.2010	07.08.2010
0,3	01.01.2010	02.07.2010	182	11.06.2010	02.07.2010	03.08.2010
0,4	01.01.2010	02.07.2010	182	14.06.2010	02.07.2010	28.07.2010
0,5	01.01.2010	02.07.2010	182	17.06.2010	02.07.2010	28.07.2010
0,6	01.01.2010	02.07.2010	182	20.06.2010	02.07.2010	22.07.2010
0,7	01.01.2010	02.07.2010	182	26.06.2010	02.07.2010	19.07.2010
0,8	01.01.2010	02.07.2010	182	29.06.2010	02.07.2010	13.07.2010
0,9	01.01.2010	02.07.2010	182	02.07.2010	02.07.2010	13.07.2010
1	01.01.2010	02.07.2010	182	02.07.2010	02.07.2010	02.07.2010

Table A.15: Pessimistic Schedule High Cost – Net Cash Flow

Pessimistic Schedule High Cost	**Total Project Income ($)**	**Fuzzy Total Project Expense ($)**			**Fuzzy Net Cash Flow ($)**			**Net Cash Flow ($)**
α - cuts	Crisp	a	b	c	a	b	c	Crisp
0	308.140,94	351.407,84	366.760,59	382.143,34	-74.002,41	-58.619,66	-43.266,91	-58.624,65
0,1	308.140,94	352.943,12	366.760,59	380.605,07	-72.464,13	-58.619,66	-44.802,18	-58.624,15
0,2	308.140,94	354.678,39	366.960,59	379.266,79	-71.125,86	-58.819,66	-46.537,46	-58.823,65
0,3	308.140,94	356.013,67	366.760,59	377.528,52	-69.387,58	-58.619,66	-47.872,73	-58.623,15
0,4	308.140,94	357.548,94	366.760,59	375.990,24	-67.849,31	-58.619,66	-49.408,01	-58.622,65
0,5	308.140,94	359.084,22	366.760,59	374.451,97	-66.311,03	-58.619,66	-50.943,28	-58.622,15
0,6	308.140,94	360.619,49	366.760,59	372.913,69	-64.772,76	-58.619,66	-52.478,56	-58.621,65
0,7	308.140,94	362.154,77	366.760,59	371.375,42	-63.234,48	-58.619,66	-54.013,83	-58.621,15
0,8	308.140,94	363.690,04	366.760,59	369.837,14	-61.696,21	-58.619,66	-55.549,11	-58.619,66
0,9	308.140,94	365.225,32	366.760,59	368.298,87	-60.157,93	-58.619,66	-57.084,38	-58.619,66
1	308.140,94	366.760,59	366.760,59	366.760,59	-58.619,66	-58.619,66	-58.619,66	-58.619,66

Table A.16: Pessimistic Schedule High Cost – Schedule Results

Pessimistic Schedule High Cost	**Project Start Date**	**Project Finish Date**	**Project Duration**	**Fuzzy Project Finish Date**		
α - cuts	Crisp	Crisp	Crisp	a	b	c
0	01.01.2010	02.07.2010	182	25.05.2010	02.07.2010	09.08.2010
0,1	01.01.2010	02.07.2010	182	05.06.2010	02.07.2010	09.08.2010
0,2	01.01.2010	03.07.2010	183	05.06.2010	02.07.2010	07.08.2010
0,3	01.01.2010	02.07.2010	182	11.06.2010	02.07.2010	03.08.2010
0,4	01.01.2010	02.07.2010	182	14.06.2010	02.07.2010	28.07.2010
0,5	01.01.2010	02.07.2010	182	17.06.2010	02.07.2010	28.07.2010
0,6	01.01.2010	02.07.2010	182	20.06.2010	02.07.2010	22.07.2010
0,7	01.01.2010	02.07.2010	182	26.06.2010	02.07.2010	19.07.2010
0,8	01.01.2010	02.07.2010	182	29.06.2010	02.07.2010	13.07.2010
0,9	01.01.2010	02.07.2010	182	02.07.2010	02.07.2010	13.07.2010
1	01.01.2010	02.07.2010	182	02.07.2010	02.07.2010	02.07.2010

APPENDIX B

FIGURES

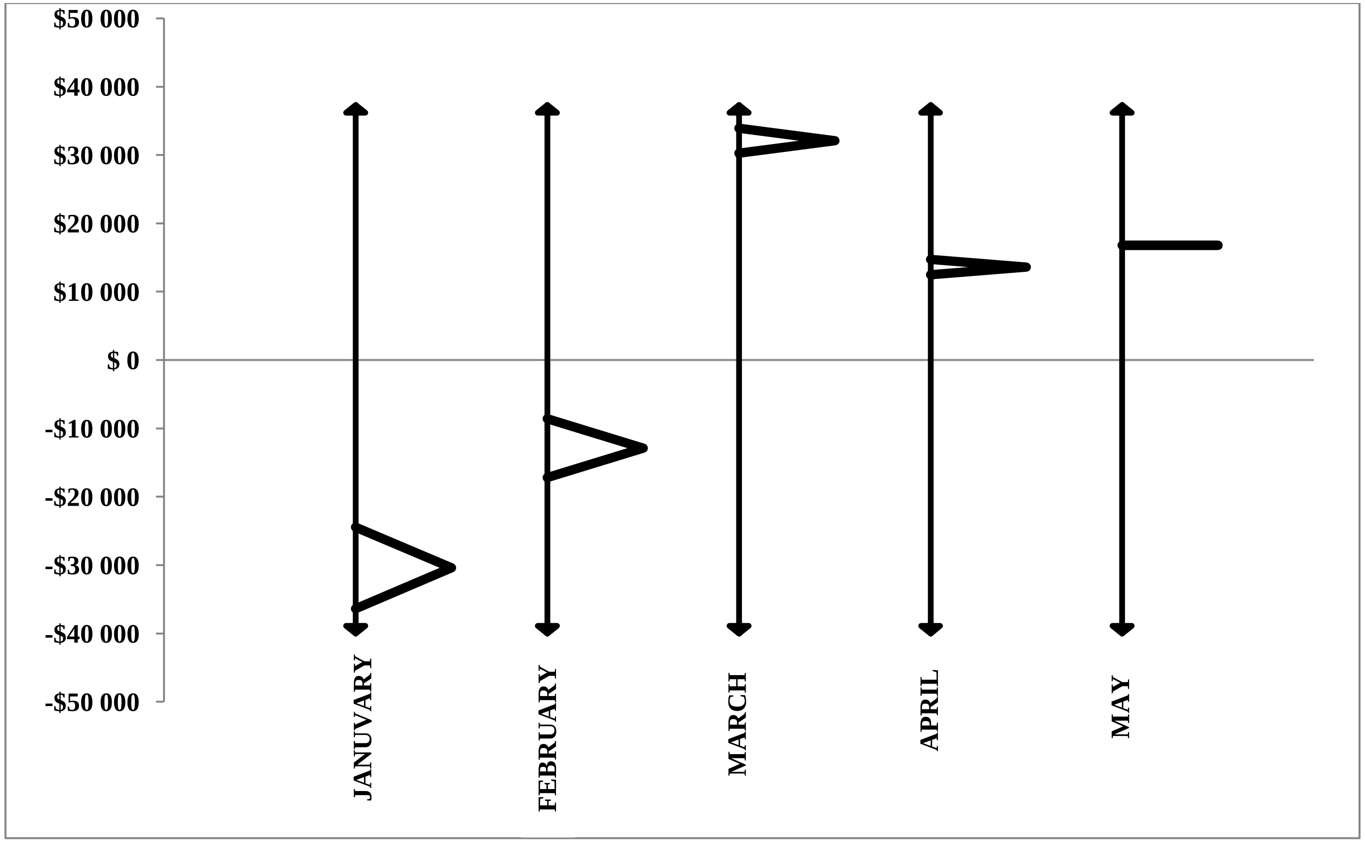

Figure B.1: Optimistic Schedule Medium Cost

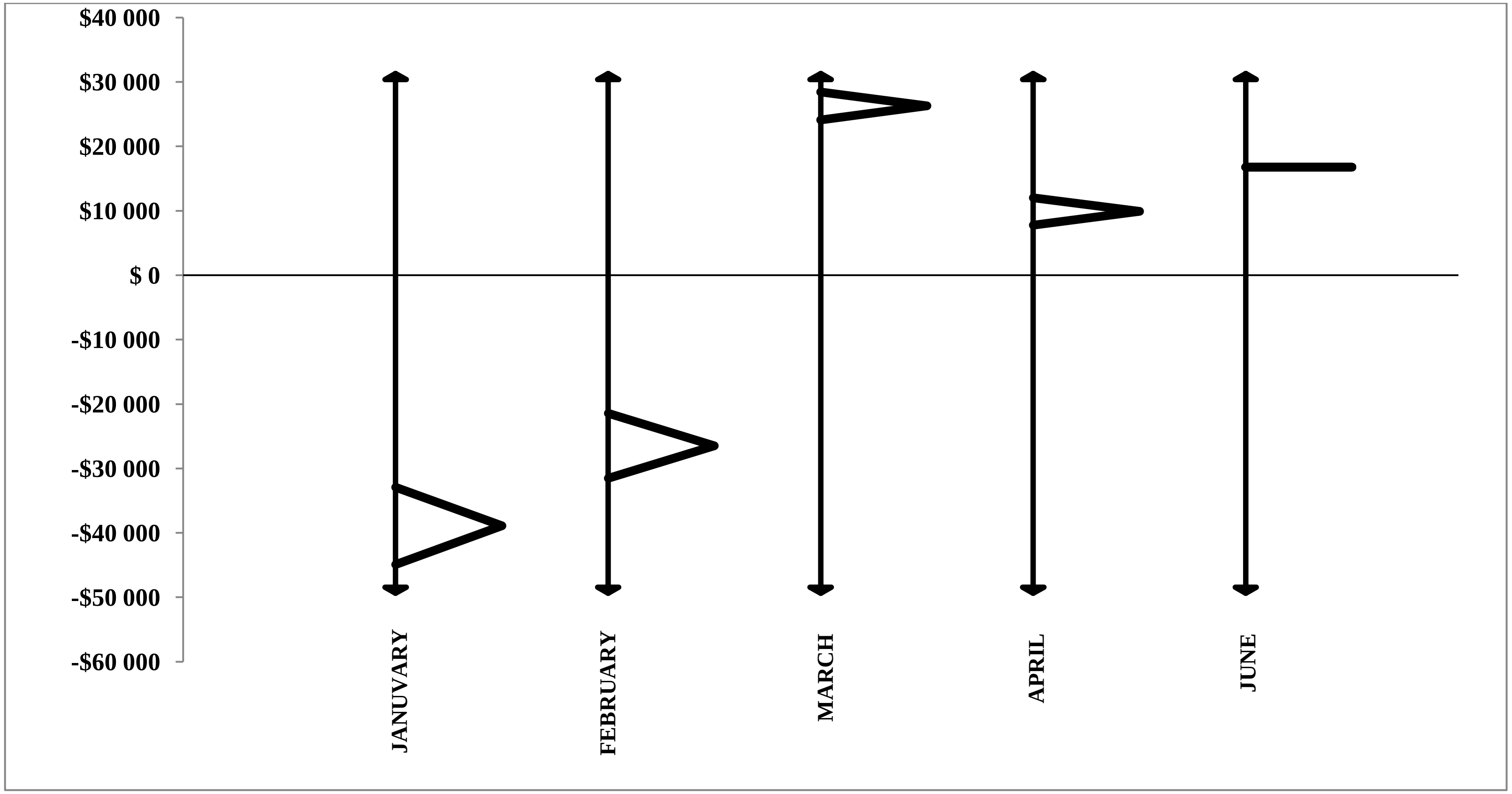

Figure B.2: Optimistic Schedule High Cost

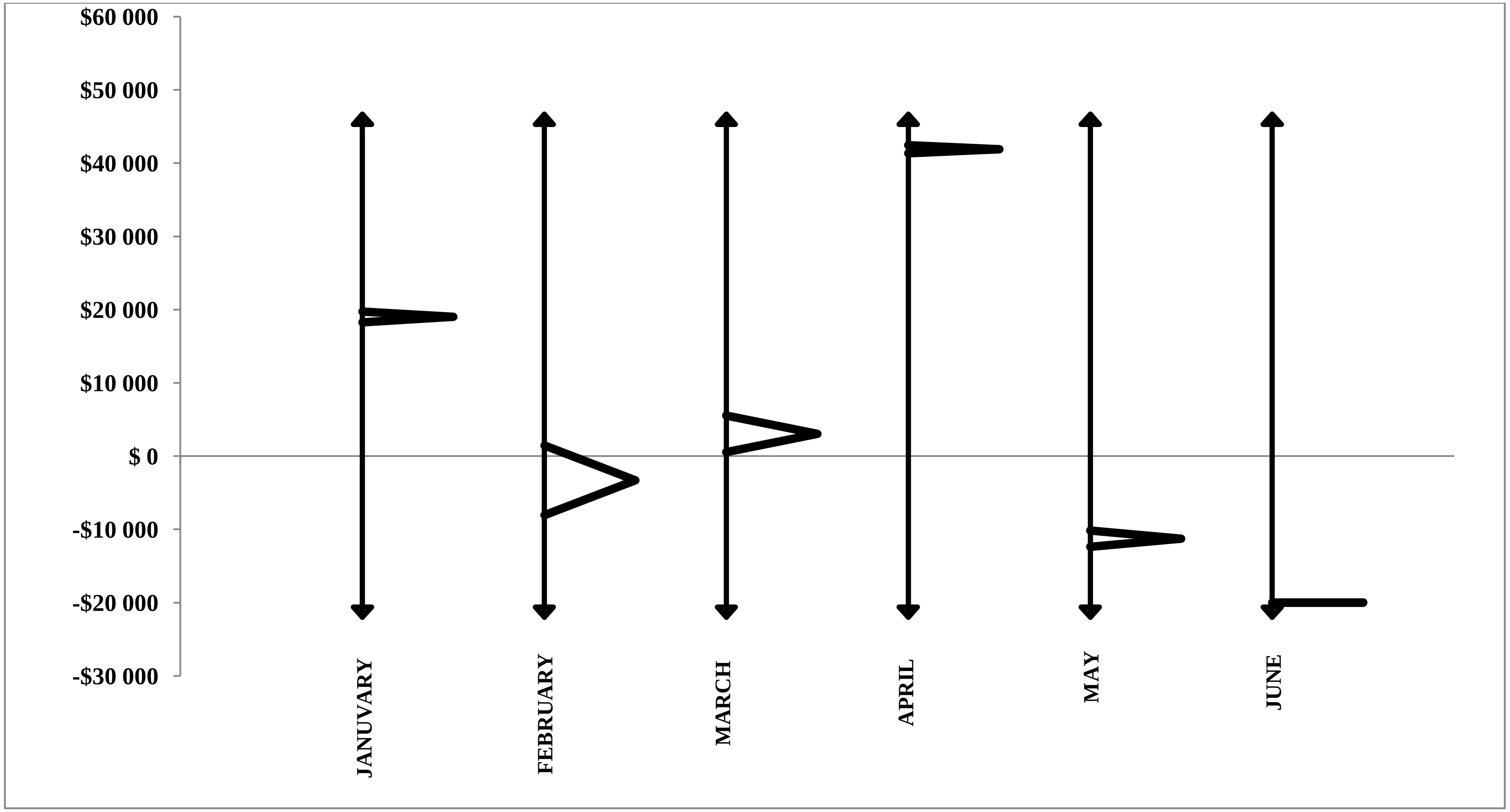

Figure B.3: Normal Schedule Low Cost

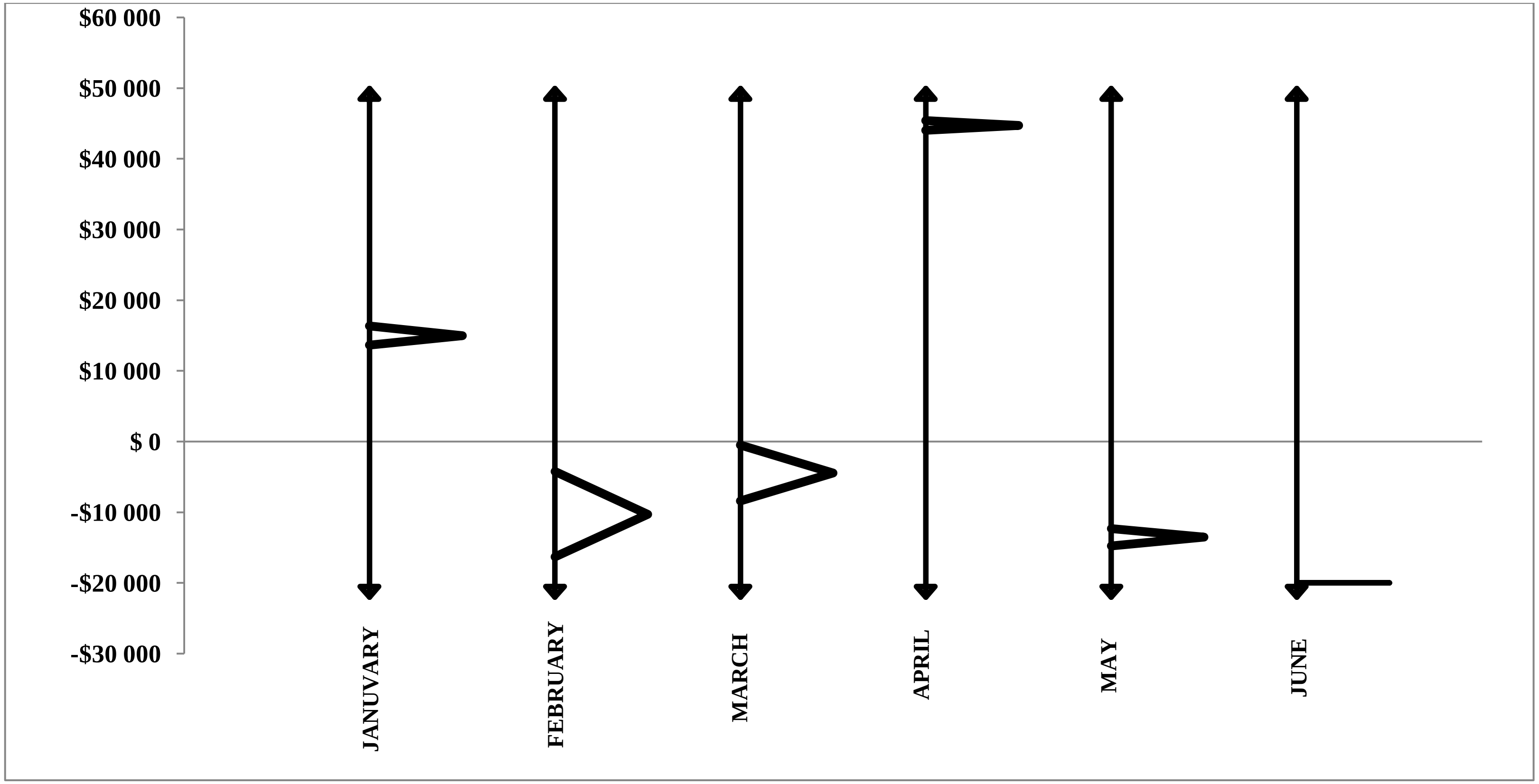

Figure B.4: Normal Schedule Medium Cost

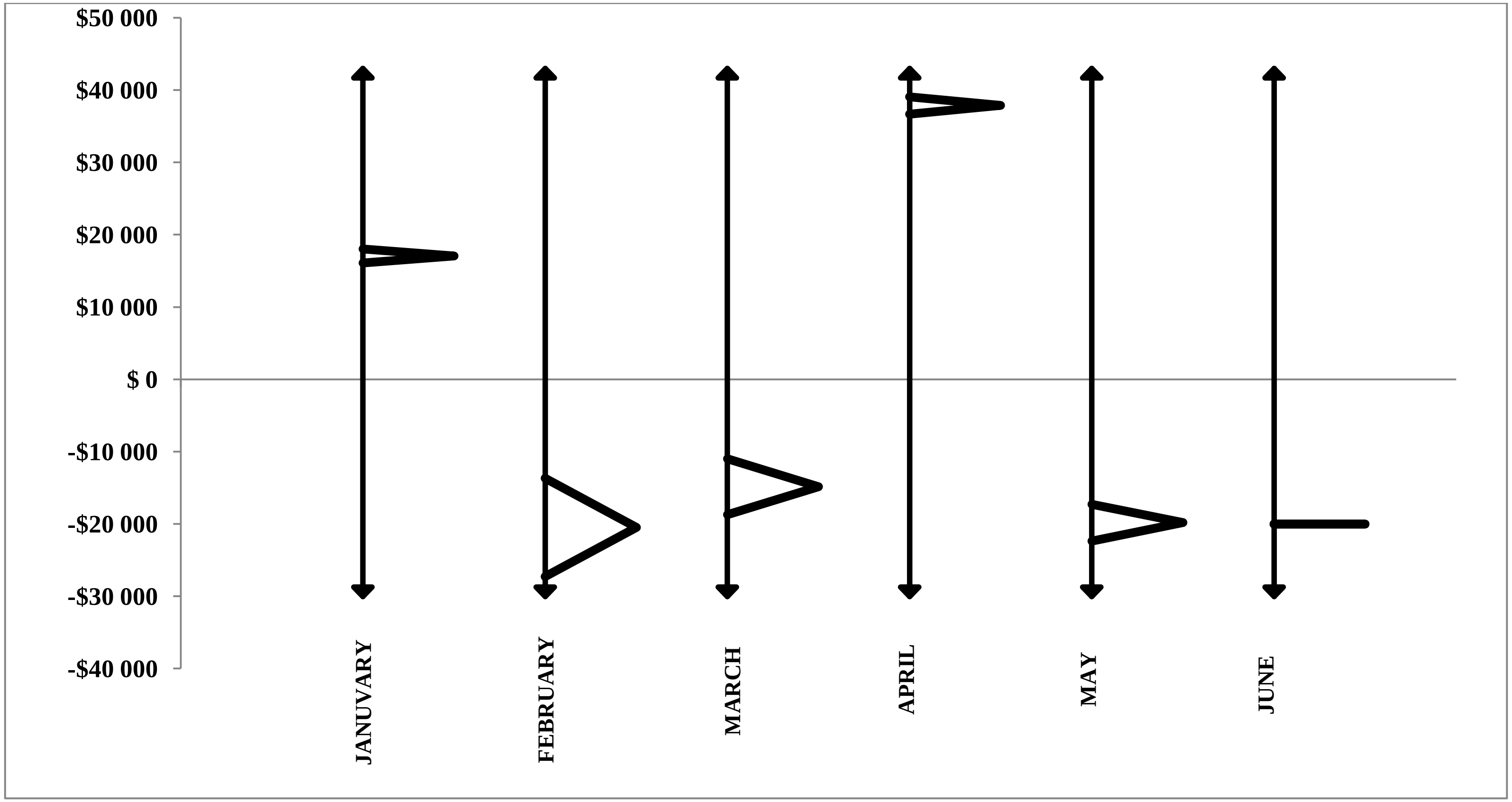

Figure B.5: Normal Schedule High Cost

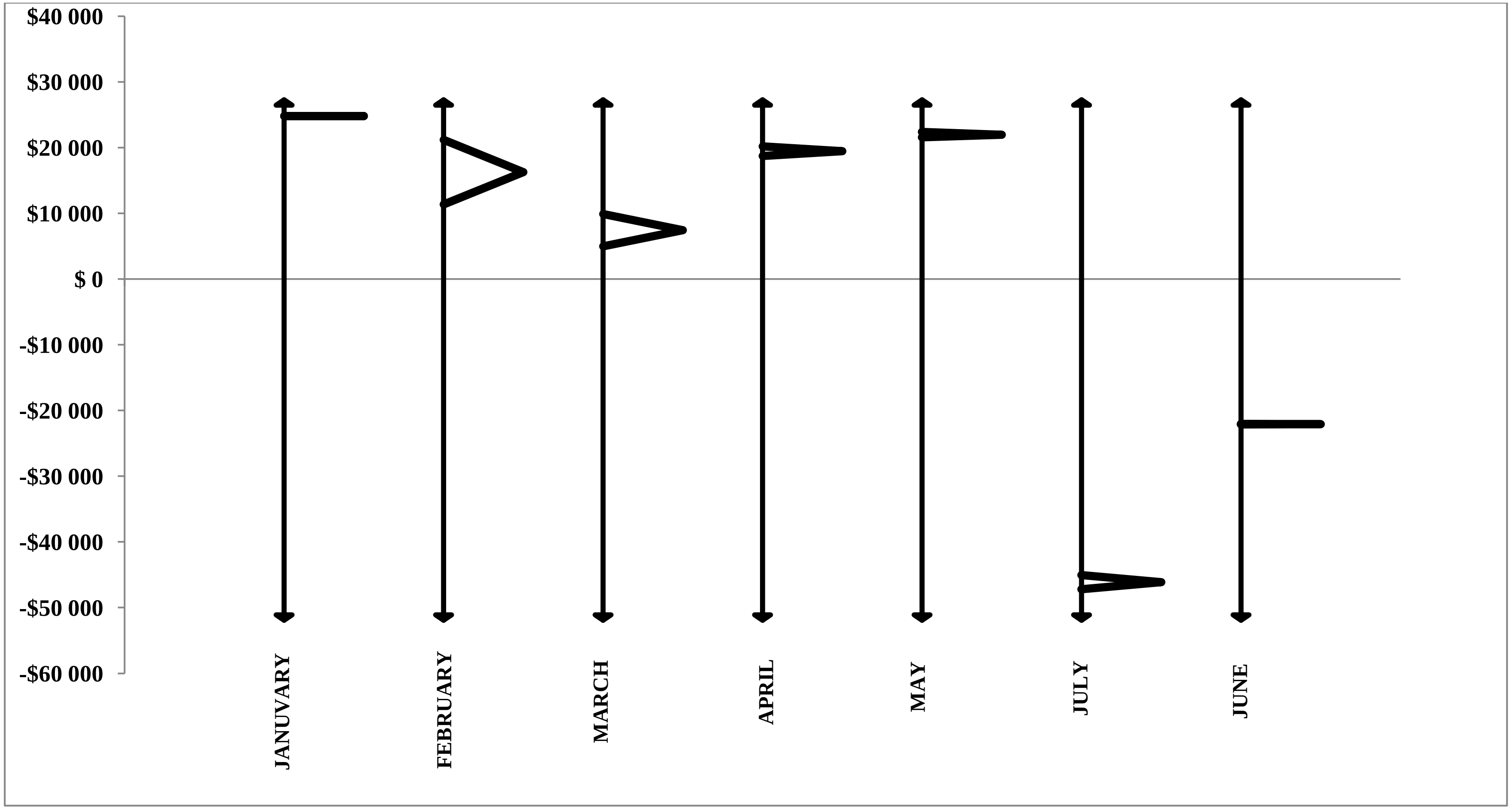

Figure B.6: Pessimistic Schedule Low Cost

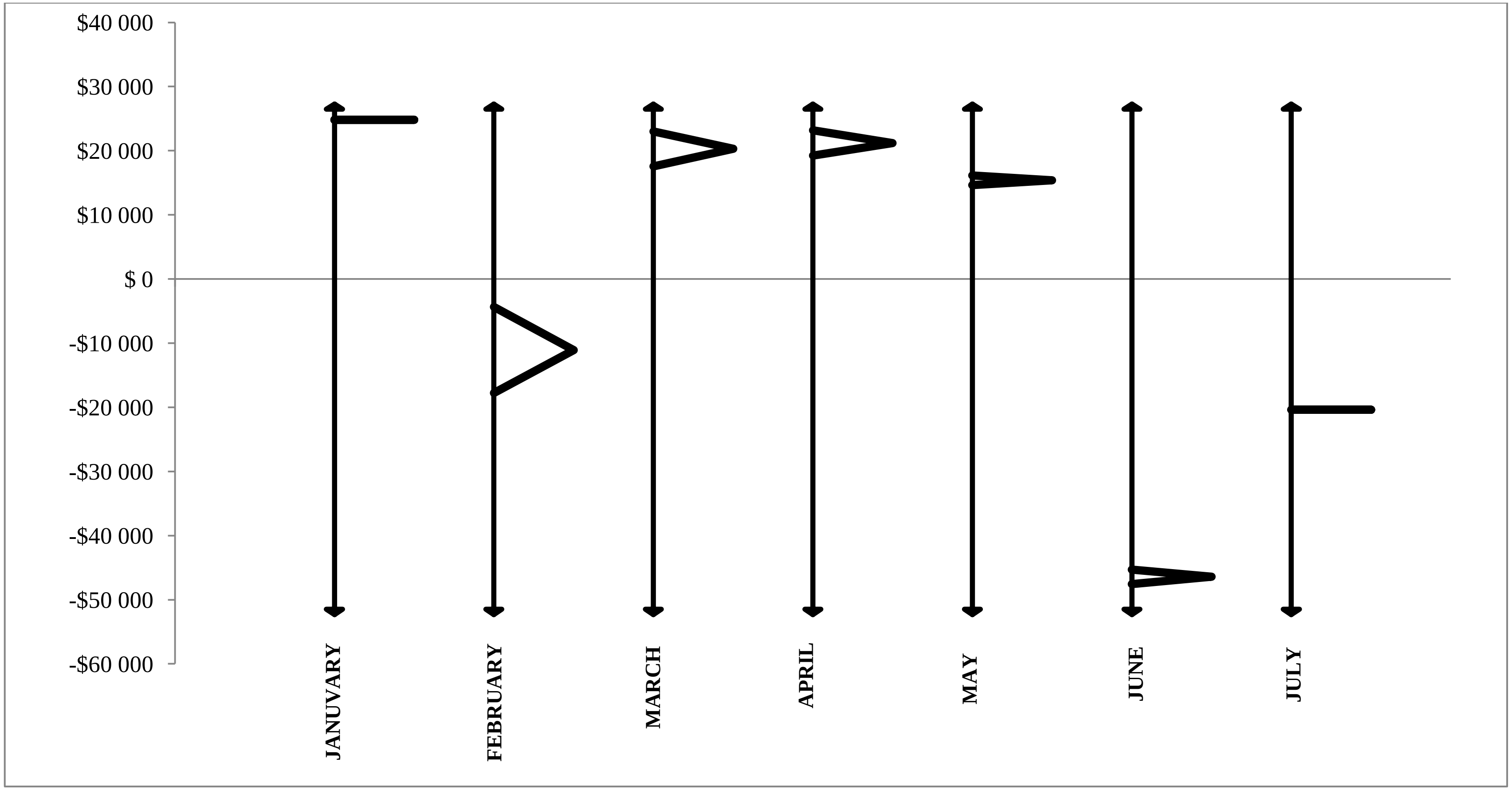

Figure B.7: Pessimistic Schedule Medium Cost

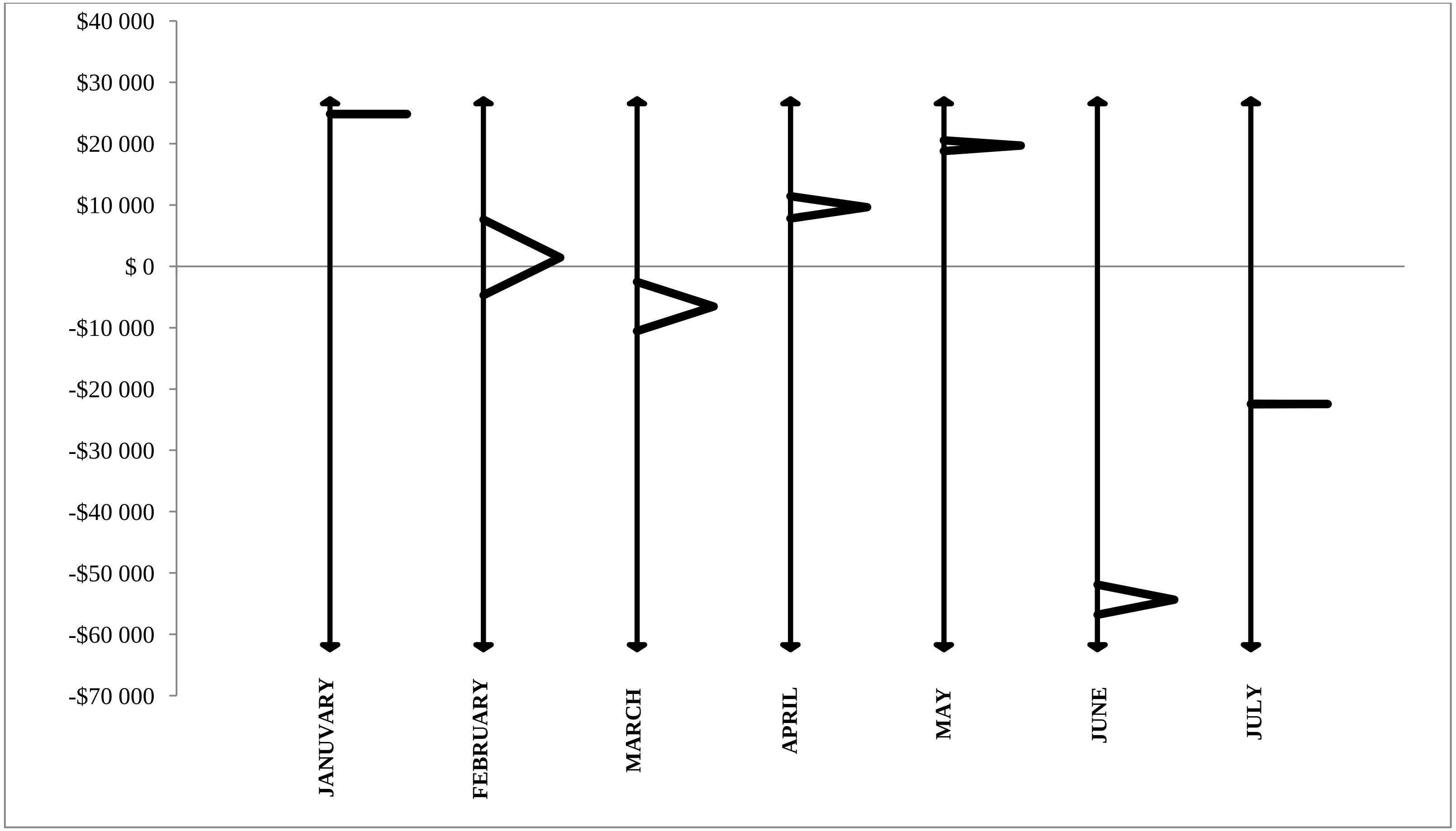

Figure B.8: Pessimistic Schedule High Cost