COMPETITION AND COLLABORATION IN SERVICE PARTS MANAGEMENT SYSTEMS

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ABSTRACT

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Inventory management policies of two independent dealers in a service parts system with transshipment is studied in this thesis. Dealers can collaborate by pooling inventory or service. Revenue is shared in transshipment, can sometimes be contrary to profit maximization of one of the parties albeit sum of profits is increased. To assess the benefits of inventory pooling under equilibrium strategies, and the effect of competition on profits, a Markov Decision Process is formulated. A simpler variant of the optimal four-index threshold policy is used to characterize the production, service and transshipment related inventory decisions. A game theoretical approach as well as notions from policy iteration is taken to find the best response policy and equilibrium policies of the dealers. Numerical study is conducted to investigate the effect of cost, revenue and demand parameters, as well as dealer asymmetricities on benefit of pooling, service levels and transshipment flows. Analysis shows that commission schemes fairly allocating transshipment value to the players, high customer traffic intensities, and low transshipment costs are most suited environments for pooling. System centralization is beneficial when the inventory holding costs are high, transshipment costs are low, customer traffic intensities are high or the commission structure is distracting a party. Competition, within the experimental settings, dampens about 45% of the benefits of pooling.

Keywords: spare parts management, decentralized inventory pooling, centralized inventory pooling

ÖZ

YEDEK PARÇA YÖNETİM SİSTEMLERİNDE REKABET VE İŞBİRLİĞİ

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Bu tezde, bir yedek parça sistemindeki iki bağımsız, birbirleriyle stok ve hizmet alışverişi yapabilen bayilerin stok yönetim politikaları incelenmiştir. Bayiler bu alışveriş ile bir bakıma işbirliği yapar. Ancak bu işbirliği, gelir paylaşımını gerektirdiğinden, rekabetin öngördüğü bireysel kâr eniyilemesiyle çelişebilir. Denge politikalar üzerinden havuzlamanın ve rekabetin sistem kârlılığına etkisinin bulunması amacıyla bir Markov Karar Süreci formüle edilmiştir. Üretim, servis ve alışveriş kararlarını karakterize etmek için 4 parametreli bir stok yönetim politikasının basitleştirilmiş bir formu kullanılmıştır. Oyun teorisi ve sabit nokta iterasyonundan yararlanılarak en iyi tepki ve denge hesaplanmıştır. Sayısal analiz yoluyla, maliyet, gelir, talep parametrelerinin ve bayi asimetrisinin havuzlama getirisine, servis düzeyine ve bayiler arası parça akışlarına olan etkisi incelenmiştir. Havuzlama getirisini adil dağıtan komisyonların, yüksek talep trafiğinin ve düşük havuzlama maliyetlerinin havuzlama için en uygun ortamlar olduğu gözlenmiştir. Sistem merkezileşmesinin yüksek taşıma maliyeti, havuzlama maliyeti ve komisyonun bayilerden birinin havuzlamaya katılımını engelleyen miktarda olması durumunda faydalı olduğu görülmüştür. Deney uzayındaki örnekler üzerinde, rekabetten kaynaklanan kayıpların merkezi sistemde elde edilebilecek havuzlama kazancının yaklaşık %45 ine denk geldiği saptanmıştır.

Anahtar Kelimeler: yedek parça yönetimi, merkezi olmayan envanter havuzlaması, merkezi envanter havuzlaması

To all of those associated with Middle East Technical University

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TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

CHAPTER 1

INTRODUCTION

After-sales services, spare parts dealership associated to it, is a very profitable business area for companies involved in the production of expensive and complex repairable equipments. Mentioned in Kim et al. (2006), General Motor's $9B in after-sales revenues produced $2B in profits, which is a much higher rate of profit than its $150B in car sales generated over the same time period. After-sales services and parts contribute only 25 % of revenues across all manufacturing companies but are often responsible of 40 % - 50 % of profits.

Spare parts have considerable economical implication, since the costs associated to these items and prices of the items are very high. Furthermore, they have strategic importance, due to the indispensability of the items. For example, about 10% of the US military aircrafts are grounded, waiting for a failed repairable-item to be replaced at any point of time (Jung et al., 2003). The commercial aviation industry alone has as much as 44 billion dollars worth of spare parts in stock (Karsten et al., 2009).

Full utilization of this profit potential out of this strategic, high-value market is not so easy. Difficulties of operating a service parts vendor arise from the service level-inventory trade-off: parts under concern are high in value and customers demand prompt service to cover their economic losses as soon as possible. Replenishment times are much more variable than it was previously due to increasing globalization in supply. Spare parts dealers also have the challenge to meet customer demands with minimal backorders in order not to face loss of goodwill, ever gaining importance with the fast advent of global competition. In other words, they have to ensure enough part availability and hence provide high service levels. But having high service levels in a naïve fashion means capital wasted on unnecessary inventory. Moreover, the demand nature is usually very uncertain as demand is triggered through infrequent equipment failures from very large number of sources. Dealers thus face the hard task of managing their inventory effectively in an uncertain environment. The task of the dealer is to obtain high profits while keeping both goodwill losses and inventory costs low at the same time.

Dealer networks should be designed to meet customer demand for convenience, which is best at many small dealers. Network designers also have some concerns to reduce transportation/material handling costs, lead time and hence reduce the inventory pipeline and corresponding safety stocks, which merely requires the dealer network consist of few large locations to benefit from demand aggregation and economies of scale. As the demand is becoming more and more geographically dispersed with globalization, the pressure for a dispersed dealer network is felt more. This makes transportation costs a critical concern in effective dealer network design.

Interest in the subject of supply chain management (SCM) and SCM research has steadily increased since the 80's with the widespread use of IT in corporate operations: once practically not-available information -within the relevant time frame- became available and communicable, search for efficient policies/strategies with the help of so-called decision support systems, mathematical models/simulations, affordable. Both with the aid of widespread use of SCM tools and outputs of SCM research, companies started to see new feasible ways of collaborative relationships within and beyond their own organization.

Especially to cope with demand uncertainty, SCM scientists and experts started to propose lateral transshipments, institutionalized in the past by the airline industry[1] as a way to ensure spatial flexibility and lower levels of inventories along with sustained service levels (Satır et al., 2010). Lateral transshipment is defined as the

[1] Back in the 1960s there was a lot of fleet commonality between European airlines, which provided a foundation for two maintenance consortiums called KSSU and Atlas (Kilpi et al. [2004]).

redistribution of stock from retailers with stock on hand to retailers that cannot meet customer demands or to retailers that expect significant financial losses due to high supply risk (Tagaras, 1999). If retailers who are subject to long replenishment lead times from suppliers are located closer to each other or spend significant funds on construction and operation of storage facilities to prevent stock-out, a lateral transshipment policy can be used as an effective alternative to minimizing total cost. With the development of third party logistics operations, transshipment methods have improved in terms of speed and cost, and information systems in spare parts networks are enhanced. As a result, the cost of inventory and information sharing has been reduced and inventory pooling is becoming ever more attractive for after-sales service providers in spare parts networks (Satır et al., 2010).

To remind readers from the very beginning, it shall be noted that the words inventory pooling, lateral transshipment and transshipment will be used synonymously throughout the text.

There are quite many sound applications of lateral transshipments in practice. A classical example is the Saturn Corporation which re-constructed its service parts supply chain (Cohen et al., 2000). The key component in this process is pooling of the component inventories. Dealers that are in close proximity with each other are formed into groups such that if one of the dealers is out of stock, upon a demand arrival a part would be transshipped from another dealer in the group. In case of a lateral transshipment, a full reimbursement is made to the sending dealer. If the item does not exist in the group, dealer requests the part from Saturn. Pooling the inventory resulted in significant savings in the inventory holding costs while improving the service levels. Indeed, Saturn consistently ranks among the top ten brands of automobile manufacturers for supply-chain service, comparing favorably with luxury automobiles such as Lexus, Infiniti and Acura. Another example is ASML, an original equipment manufacturer in the semiconductor industry (Kranenburg and Van Houtum, 2009). Implemented since early 2005 between groups of component warehouses as in Saturn, they were able to achieve up to 50% reduction in their spare parts provisioning costs by efficient use of lateral transshipments, while keeping the service at the same level. With time, ASML obtained lower waiting times than it even anticipated, since actual performance further increased because of the lateral transshipment option being applied also within their upper echelon, so-called main local warehouses. A set of various other examples are reported by Narus and Anderson (1996). Volvo GM has contracted with Fed-Ex to provide its dealers with emergency transshipments: three warehouses were now obsolete and total inventory was reduced by about 15%, further benefits are accrued by demand aggregation. Japanese machine tool builder Okuma has built a shared information-technology system called Okumalink, keeping distributors informed about the location and availability of machine tools and parts in another and also central warehouses in Charlotte and Japan. At will, an Okuma distributor can contact other distributors or the central warehouses through Okumalink to negotiate transshipment of available parts in other distributors or can get guaranteed transshipments within a day.

Dealer inventory transshipment systems similar to the Okuma's have been implemented also at a number of overall equipment manufacturing/after-sales service companies, including Caterpillar, John Deere, General Motors, etc. (Zhao et al., 2006). There are reportedly similar systems in Turkey as well: TOFAŞ and Borusan Otomotiv are also known[2] to operate with an information system that enables all of its dealers throughout Turkey to see whether there are stocks on all other dealers as well as the central warehouse inventory: dealers are able to negotiate transshipment of parts.

Systems involving independent decentralized dealers, some discussed above, share a common characteristic of negotiation between dealers. Dealers may act with self-interest to deny transshipment requests at that specific time and at negotiated price intervals, if they believe that the given-away inventory would meet higher margin customer demand. They might not like to agree on receiving a part via transshipment if receiving an item has a very low profit margin for them. In order to encourage transshipment among the dealers, the manufacturer may provide monetary incentives

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[2] The information on TOFAŞ's system is gathered through an interview with Kavaklıdere authorized after-sales service coordinator. Borusan's system is learned through the Trabzon authorized service center manager.

to the transshipping dealer (Zhao et al., 2006). All in all, in the case of independent dealers, in other words independent companies, any individually rational company will only agree to pool their spare parts with other companies if doing so is anticipated to bring a positive net present value to it. This mere intricacy adds a layer of difficulty on an important portion of real-life applications of spare parts lateral transshipment: presence of competition might hinder some of the benefits of collaboration by availability of transshipment.

1.1 MOTIVATION OF THE STUDY

Examples above show evident attractiveness in the concept of inventory pooling and anticipated increasing number of practical applications of this concept in different types of spare parts networks. Common modes of decision making in spare parts dealer networks are centralized/collaborative decentralized or independent decentralized. Centralized/collaborative decentralized decision making structure means all transshipment decisions are made under central command by a central decision maker maximizing system-wide profits.

Independent decentralized decision making structure means all inventory management decisions as well as transshipment decisions are delegated to the dealers themselves that tend to maximize their sole profits. Therefore in this case, dealers have incentives to reject transshipment flows that would centrally be accepted. In other words, either of the parties can refuse transshipments that would be profitable from a central (i.e. sum of expected profits) perspective.

Hence spare parts suppliers, when strategically deciding on enabling transshipment between dealers and if transshipments are enabled, when deciding to delegate transshipment-enabled dealership sales/procurement operations to their independent dealers or deciding to make them centrally operated, they should correctly assess the value eroded by competition: to what extent would the benefit of pooling erode in the supply chain in the long run because of dealer’s self interest?

Independent dealers are also usually not equipped with sufficient information and/or decision support tools, hence can use naïve, evidently sub-optimal strategies such as meeting the transshipment requests all the time or meeting them only if the stock level is very high, placing a request only in case of a stock-out, sharing only partial information on stock levels, and so on. They may not be able[3] to share full information regarding their inventory levels even if they have incentive to truly report.

We ignore the imperfections caused by irrational behavior and deliberate/accidental loss of information and assume we have rational dealers with full, common information all the time. Yet there are questions also regarding the dealers themselves: What policies should rational independent firms choose under equilibrium if transshipment is enabled? How should they adapt to changes in cost/revenue parameters, given they are acting for sole self-benefit? Suppliers would like to be more informed on the extent of value erosion by competition and gather valuable insights on the value of centralization.

Both the dealer network designer and dealers are interested in the cost/revenue schemes under which either the benefits or the suggested extent of transshipment are considerable, both being indications to put weight on transshipment operations management and design. They are interested to know under which situations benefits of transshipment are so small that the net value can be eroded by the costs of implementing such schemes or the anticipated transshipment flows are so small that they can simply be regarded as a minor operation or a rare emergency. Thus, gathering insights on the benefit of pooling and long-run anticipated transshipment flows stands to be another interesting question.

Dealership networks might be homogenous or heterogeneous in terms of their demand and supply characteristics, inventory holding costs (e.g. differences in opportunity cost of space) and backordering costs (e.g. drastically different customer bases). This dimension should also be taken into consideration. If the dealership network is so heterogeneous, say a very large dealer with very small others, the

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[3] Mismatches between calculated and real physical stock levels prevail even in reputable businesses that are adherents to ERP: like a business in the architectural glass processing industry in Turkey that the author participated in consulting.

transshipment relationship might be exploiting, resulting to prevailing commensalist, even sometimes parasitic relationships among dealers: some dealers would have very large benefits from pooling, some will see a very small benefit even after carefully prepared pooling schemes to avoid loss. Non-optimal behavior might mean some dealers are merely subsidizing others (Karsten et al., 2009).

Motivated by the examples, questions and concerns above, this thesis focuses on the behavior of individual dealers in an even cost structure and symmetric uncertain demand/production timing environment. It looks upon the dynamics of the inventory sharing and rationing game between two dealers. Knowing that individual dealers' inventory management decisions are mainly influenced by the transfer payment scheme (i.e. commission) and the transshipment cost this thesis focuses on the following issues:

(i) How would the equilibrium policies be determined, and profitabilities, as well as service levels and transshipment flows be calculated? How would the benefits of pooling be assessed?

(ii) How would the equilibrium policies, profitabilities, transshipment flow volumes, and service performances of a two independent symmetric (in terms of demand/supply structure and internal costs) transshipment-enabled dealer spare parts inventory system be affected by different cost and demand/supply structures? How are the benefits from transshipment dispersed through homogenous dealers under various commissions?

(iii) What is the effect of asymmetricities in inventory holding costs, backordering costs and traffic intensities on transshipment benefits, equilibrium policies and transshipment flow volumes? How are the benefits from transshipment dispersed through heterogeneous dealers under various commissions?

(iv) What is the effect of competition on profits, and how is it affected by various parameters (i.e. inventory holding cost, backordering cost and demand rate) given the fact that inventory pooling causes both collaboration –*agents collectively increase their profitability*- and competition in an independent dealer context –*since some of the revenue is shared (and lost due to transshipment costs) if inventory is transshipped*-?

1.2 OUTLINE OF THE STUDY

There are detailed outlines in the beginning of each chapter and relevant sections for description of subsections. Without getting into technical detail, organization of the study is as follows:

In Chapter 2, literature on inventory pooling in spare and service parts, inventory rationing, competition in spare parts management systems, collaboration among competitors in terms of spare parts supply is presented and this study is positioned in the literature.

Chapter 3 (Model and Solution Approach), starts with clearly stating the mathematically modeled problem context and then fully describes the –numerical-solution approach to conduct a numerical study described in Chapter 4. It also discusses computational results concerning the performance of the heuristic to fasten up the numerical solution algorithm.

Chapter 4 (Computational Results), consists of four main sections (Sections 4.1, 4.2, 4.3, 4.4), listing detailed findings -extensive graphs and interpretations- on the research focus questions stated at the end of Section 1.1. Section 4.1 introduces the parameter combination base of cases, as well a the performance measures on which the numerical results are obtained. Section 4.2 contains the findings for the *symmetric competitive pooling* –identical competing dealers-. Section 4.3 yields the results for the *asymmetric competitive pooling* -different arrival rates/traffic intensities, inventory holding and backordering costs are allowed-. Section 4.4 yields the results for *centralized cooperative pooling*. System-wide value is now maximized instead of dealers trying to give the best response maximizing their own self-interest given other dealer’s inventory management policy. Effect of competition is assessed in this section.

General conclusions, managerial insights and future research directions are given in Chapter 5.

CHAPTER 2

LITERATURE REVIEW

In this chapter, the studies in the literature that are related with this study are summarized. The subjects of the papers, research questions, their differences with the previous works, models built and main results are explained and this study is positioned in the literature. Emphasis is placed on studies that consider game theory applications in spare parts inventory management as well as on studies that characterize optimal/near-optimal policies of dealers under similar contexts.

Under Sections 2.1, 2.2, 2.3 and 2.4, literature related to benefit of pooling is reviewed. Previous literature is categorized under those four headings, namely inventory pooling in spare parts inventory management (2.1), inventory rationing (2.2), collaboration among competitors in spare parts systems (2.3), and competition in spare parts management systems (2.4).

Section 2.4, where the main emphasis is put upon, probes into the line of research on transshipment & inventory management with independent dealers. Main results of relevant studies are listed and discussed. It also tries to verbally describe what an optimal/near-optimal policy looks like for a similar, two independent dealer context that is studied in this thesis. In Section 2.5, this study is positioned in the cited literature, finalizing this chapter.

2.1 INVENTORY POOLING IN SPARE PARTS INVENTORY MANAGEMENT

There are two crucial building blocks on the fundamentals of modern service parts management research area: namely the METRIC model (Sherbroke, 1968) and its mere modification MODMETRIC (Muckstadt, 1973).

METRIC is a mathematical model of a two-echelon supply system in which item demand is compound Poisson with a mean value estimated by a Bayesian –hence an approximate- procedure and replenishments are done one-for-one, where minimization of back-order levels is aimed under a budget constraint. Muckstadt (1973)'s modification allows for bill of materials to be accommodated within the METRIC management system. Repairable parts are supplied through the upper echelon without transshipment. METRIC and MODMETRIC, which are well-established, had found many applications in practice and had many modifications about its assumptions. A bibliography of related line of research is available in Muckstadt (2005) and Minner (2003a).

The first notable modification of METRIC model is by Lee (1987), where the author studies a multi-echelon system with identical retailers and transshipments for repairable items. If a retailer is out of stock, the demanded item is allowed to be sourced from another retailer within the same pooling group. Different priority rules for choosing the supplying retailer (random, retailer with maximum stock) are analyzed. Optimal stocking levels are determined subject to service level constraints (for the warehouse and retailers) on immediate and after transshipment fill rates.

Axsäter (1990) extends Lee (1987) by allowing non-identical retailers. In both studies, time fractions where demand is backordered, met from stock or met through emergency transshipment are evaluated and compared to values obtained through numerical experiments.

Erkip et al. (1990) have obtained general conditions under which inventory rebalancing (via transshipment) is required in a two-echelon system, allowing correlated demand. They find out that very large coefficients of variations for demand and supply are needed to justify transshipments in a two-echelon, identical depot, discrete time setting with deterministic lead times.

Alfredsson and Verrijdt (1999) present a two-echelon centralized model with emergency supply like Axsäter (1990), but now it is done either directly or via transshipments from other retailers. If parts are not available, direct shipments from the central warehouse, and if even this is not possible, direct shipments from the external supplier are allowed, as well as transshipments. Two main findings: emergency supply strategy always pays off and the performance is not sensitive to the lead time distribution.

Herer et al. (2006) consider a centralized supply chain, which consists of several retailers and one supplier. The retailers, who possibly differ in their cost and demand parameters, may achieve system-wide optimal profits through replenishment strategies and transshipments. They consider order-up-to policies. They demonstrate that the values of the order-up-to levels can be calculated using a sample-path-based optimization procedure. Given an order-up-to policy, they formulate a linear network flow framework to assess transshipment levels, i.e. transshipment quantities per demand realization in the period and inventory state. They try to assess optimal order-up-to policy parameters, as well as average costs of an experimental test-bed via numerical analysis where correlations in demands are also prevalent.

Kranenburg et al. (2009) model a real-life inventory control problem of ASML consisting of main and local warehouses. In this multi-item and multi-location system, where each local warehouse is clustered around a main warehouse, lateral transshipment is allowed from chosen main warehouses only. This type of pooling scheme is called as the partial-pooling situation. They show that partial-pooling captures almost all of the profits enabled by full possibility of transshipments and propose a heuristic algorithm to approximate base stock levels.

Implemented since early 2005 in between groups of (not all) component warehouses, ASML was able to reduce their spare parts related costs by up to 50% by the efficient use of lateral transshipments, while keeping the service at the same level. With time, ASML obtained lower waiting times than it even anticipated, since actual performance further increased because of the lateral transshipment option being applied also within their upper echelon, so-called main local warehouses.

The last piece of research is emphasized since it is shedding light on the optimal or near-optimal policy sets. In van Vijk et al. (2009), the authors consider a two stock point, single product centralized spare parts inventory system with Poisson demand arrivals and exponential repair times, fixed number of parts in circulation and infinite repair capacity at each stock point. Demand can either be satisfied from own stock, transshipped with some penalty or via an emergency procedure with some penalty (can also be thought as lost demand), there are no backorders. Aim is to minimize average system-wide costs over an infinite horizon. The problem is formulated as an MDP (Markov Decision Process). Through monotonicity, super-modularity and convexity properties of the cost function, the authors show that the optimal policy is threshold-type. The authors provide certain sufficient (but not necessary) conditions which further simplifies those policies. Their results follow:

(I) For a given stock level at the first stock point, demand at first stock point is open to be transshipped for sufficiently high levels of the second stock points' inventory, the second stock point serves itself below that level. Below a threshold level, emergency procedure is used.

(II) For a given stock level at the second stock point, demand at the first stock point is met from own stock if the stock is sufficiently high, else is transshipped. Below a threshold level, emergency procedure is used.

(III) Symmetric cost structure always implies a policy with a transshipment enabling threshold level. Complete pooling is optimal if the lateral transshipment penalty/emergency penalty is lower than a certain level.

(IV) Optimal lateral transshipment policy is either a hold-back/complete pooling policy at both locations or a complete pooling policy for at least one location (other location may be neither). If the hold-back condition does not hold at one location, the complete pooling condition is sure to hold at the other location.

A final result follows for a situation involving symmetric cost parameters but limited repair capacity: Range of cost parameters yielding benefit of pooling is narrower. Applicability of pooling is hand-in hand with availability of repair resources.

2.2 INVENTORY RATIONING

Transshipment makes the dealer face with multiple customer classes with different immediate revenue potentials, namely own customers, customers received via transshipment and customers forwarded via transshipment. The dealer hence faces the problem of proper inventory allocation among those multiple customer classes. This situation brings the inventory rationing concept. A good recent review concerning the literary taxonomy in inventory rationing area as a whole is Teunter and Haneveld (2008).

One the very first papers that studies the rationing problem is Topkis (1968). He analyzes problems associated with an inventory system in which demands for stock are of any *n* classes of varying importance. Procurement is only once made at the beginning, nevertheless the analysis is made by dividing the time between orders into large enough intervals, so that demand between each period are still independent. Aim is to minimize future expected costs. Two cases are considered: where backorders are allowed and where they are not, meaning that unsatisfied demand is lost. The author finds out that the optimal rationing policy can be expressed as a non-negative critical rationing level associated with each demand class such that one should satisfy as much demand of a given class as possible with existing stock as long as there is no unsatisfied demand of a higher class remaining and the stock level does not drop below the critical rationing level for that class. In each interval these critical levels are non-increasing functions of their associated demand class. Author concludes the study with analysis on some myopic policies and gives conditions under which the myopic policies are optimal for multi-period model.

A contemporary counterpart of the Topkis' (1968) study is Ha (1997a). He considers stock rationing problem of a manufacturer of single item in make-to-stock system with two demand classes. Demand is lost upon customer rejection. Optimal production policies (i.e. whether to continue production or stop production) and rationing policies (i.e. whether to serve customer or reject customer of first type) are investigated. Results show that rationing (threshold) levels for customer types are non-increasing with increasing penalties associated with lost sales. He shows that there exists a base stock level and both base stock and rationing policies are stationary. The policies can now be expressed in terms of switching curves for order satisfaction.

Ha (1997b) is a mere modification to Ha (1997a) with two types of products serving single type of customers each. Now, the policies can be expressed with added production switching curves (i.e. regions to produce product type one, two or do not produce at all) to the switching curves for order satisfaction.

Minner et al. (2003b) suggest a heuristic decision rule for a rationing model with multiple types and no backorders. Their decision rule also utilizes the remaining delivery times for outstanding orders of each type.

Rationing level in a transshipment context, defined as the level above which dealers satisfy arriving transshipment requests, may be zero or positive, i.e. the dealer may share all his inventory with another dealer at the same echelon level (i.e. rationing level is zero) or may spare some items for expected future own-customers (i.e. positive rationing levels). Pooling policies with positive rationing levels are called as partial pooling policy, as in Grahovac and Chakravarty (2001) or hold-back inventory policies, as in Çömez et al. (2007). Notice that, zero rationing level implies rationing is not actually made. Some studies do not allow for positive rationing levels and analyze the performance of full pooling policies vis-à-vis no pooling. On the other hand, in studies where positive rationing levels are allowed, strictly positive rationing levels may or may not be optimal depending on the cost and demand parameters.

Grahovac and Chakravarty (2001) analyze the benefit of sharing and lateral transshipment of low-demand expensive items under no-pooling, full-pooling and partial pooling contexts. A two-echelon, single-item system with transshipments is under consideration in order to show the benefit of inventory pooling under both centralized and decentralized settings. The upper echelon and the lower echelon share the backordering cost generated at the retail end. Inventory management and transshipment policies are not claimed to be optimal. The model allows asymmetric retailers facing different levels of demand. Lateral orders are allowed not only for the stock-out situation but also for arbitrarily chosen levels of net stock (so-called triggering level), a partial pooling situation. In a centralized setting, they find out that retailer stocking levels are at least equal to and distribution center (DC) stocking level is at most equal to those in without lateral transshipment.

Under decentralization and for lower emergency transshipment costs of retailers and higher backordering cost proportion imposed on the retailer, retailer tends to be more motivated for sharing and transshipment of inventory: the DC is just the opposite. Hence, free-riding by the distributor is prevalent in the decentralized setting, pointing to a moral hazard problem. Their numerical study indicates that, lateral transshipment policies applied to expensive low-demand items can result in a decrease of up to 20% for the transportation and inventory costs for both settings, but this does not necessarily imply a reduction in overall inventory levels. Indeed, in few cases, overall inventory level becomes larger with lateral transshipment.

Çömez et al. (2007) model a centralized system of two retailers with identical transshipment costs to analyze transshipment rationing decisions, where emergency transshipment acceptance/rejection decisions are made centrally and transshipment is done if inventory in the source is higher than the so-called hold-back level that is determined as a function of periods left until next replenishment period. Time is divided to sub-periods where demand can be at most one. Demand is a Binomial process. Rejected demand is lost. Study allows for non-negative replenishment and positive transshipment lead times. All costs (actually dispersed through time) associated with transshipment are incurred at the transshipment decision, which also ensures the independence of the hold-back levels from number of backorders or number of standing transshipment requests.

The system manager faces the clear trade-off between transshipment costs and backordering costs. Therefore, it is anticipated, and proven that the hold-back levels increase with time. The enabling result is that the marginal benefit of having an extra unit of inventory is a non-increasing function of time and is bounded and independent of initial stocking levels in a replenishment cycle. The marginal benefit of keeping inventory is also non-increasing with inventory and hence a hold-back level policy becomes optimal.

The optimal replenishment threshold parameters can be determined with search methods since the expected average cycle costs are positive.

As expected, the hold back levels increase with transshipment cost, demand to own retailer and decrease with holding costs. However, an interesting result is that the hold-back level is insensitive to the demand probability of the other retailer. This result is actually a corollary of proven result of non-decreasing hold-back levels with time: a rejected request today is rejected thereafter, so the benefit of having inventory can be decoupled from demands at other retailer. If a transshipment request is accepted at a given time and inventory level, it is accepted at a later time at the same inventory level, since the benefit of transshipment at an inventory level is non-decreasing with time.

Relationship between hold back levels and backordering costs is a complex one, there can be situations where hold back levels are increasing or decreasing with backordering cost depending on cost parameters.

An interesting result is that the order-up-to levels or replenishment quantities are not interacting with transshipment policies, a counter-intuitive but claimed in the study to be a well reported result in the literature.

Numerical study over 21 problem instances to survey the main parameter effects has the following observations:

(I) As demand increases, fewer transshipments occur as proven theoretically.

(II) There is numerical evidence that complete pooling is more likely to occur under symmetric demand

(III) There is numerical evidence that increased holding costs motivates retailers to share more inventory

(IV) There is numerical evidence that sufficiently large backordering costs leads to complete pooling

(V) There is numerical evidence that transshipment costs deteriorates benefit of transshipment

An average cost improvement of 5.4% over no pooling and 2% over complete pooling is observed, where the highest improvement over no pooling is about 17%.

Hold-back levels under positive lead times could not be analytically expressed in the paper. A heuristic is developed in this case (simply add the lead time expected total demand per each retailer to the optimal target quantities with zero replenishment lead time) and compared to a lower bound cost (one with rebalancing of replenishment quantities) and the gap found to be less than 2% in 95% and 1.5% in 80% of the cases.

2.3 COLLABORATION AMONG COMPETITORS IN SPARE PARTS SYSTEMS

There is a body of literature considering collaborative decentralized spare parts inventory management formed as coalitions among competitors, utilizing notions from collaborative game theory in their contexts. Moncrief et al. (2005) give a good summary of real-life practical cases, as well as a compact review on this line of research. Another very recent and comprehensive review is Paterson et al. (2009).

Wong et al. (2007) study a spare parts inventory system with lateral transshipments, where parts can be repaired and delayed lateral transshipments are possible. Using a game-theoretic approach, the authors show that there are cost allocation policies for decentralized setting which are acceptable for all participants. Four cost allocation policies are proposed, namely to:

(I) Account downtime and inventory holding costs accrued at each company to itself, and lateral transportation cost is always paid by the receiving company

(II) Account the inventory holding cost and lateral transportation cost based on the demand rate of each company and account the downtime cost accrued at each company to itself.

(III) Allocate the total cost based on the demand rate of each company

(IV) Shapley value principle.

The authors apply these four cost allocation policies to a numerical example of a three-company pooling problem, and all four policies above give cost allocations that are in the core of the game. They do not prove non-emptiness of the core. They don't show that allocation policies will always have a core for any input parameters. There are also no explanations for the choice of these allocation policies.

Authors give an example about how false information causes the companies to become worse-off to show the importance of building mutual trust between the cooperating companies.

In Karsten et al. (2009), effect of inventory pooling in a decentralized setting with arbitrary number of independent players on an infinite time horizon is analyzed. Authors prove non-emptiness of the cost sharing core for generic spare parts inventory pooling games, where the problem environment is restricted to no transshipment cost and full pooling to ensure analytical tractability. There is a fixed number of repairable parts (i.e. is not altered even if a coalition is formed), equal to each other in each facility. A failure leads to a demand, which is a Poisson process and repairs are iid with a certain mean. There is always ample repair capacity and parts are perfectly repairable. If there are no parts available, an emergency transshipment occurs (no backorders allowed). They note Wong et al. (2007) as the most similar problem formulation. However, there is finite repair capacity, transshipments are for a fee and partial pooling is enabled. They prove that the core of the game is non-empty if and only if the game is balanced, meaning that if any coalition of more than one member can be feasibly formed, it implies a grand coalition. Following results are reported:

(I) No transfer payments between players is in the core of symmetric base stock level, demand, repair and emergency cost setting (allowing for different inventory holding costs)

(II) Having asymmetric emergency costs does not alter the result in (I)

(III) Having either asymmetric base stock levels or demand rates do not alter the result in (I), however one can construct a case with an empty core where two or more of the following are different: emergency costs, demand rates and base stock levels are asymmetric.

2.4 COMPETITION IN SPARE PARTS MANAGEMENT SYSTEMS

In all cases above independent dealers have incentives to deny transshipment flows that would centrally (or jointly-decided) be done so: they may forfeit transshipments. Hence, a standpoint from non-cooperative game theory is another viable approach. Having an independent dealer setting, this thesis also takes this stance.

Cachon and Netessine (2004) and Leng and Parlar (2005) are good reviews that include non-cooperative game theoretic applications in supply chain management as a whole that also considers the line of literature concerning lateral transshipments.

Rudi et al. (2001) consider a decentralized two-location, single-echelon, single-period, single-product, news-vendor model where the lateral transshipment price is negotiable. The model is claimed to be representative of the real-life problem of independently operating Bosch automotive parts dealers of Norway. Unique Nash equilibrium is shown to exist for the decentralized model and the behavior of optimal policy (i.e. order quantities) with respect to transshipment price is characterized. Obtaining, analytically characterizing and comparing optimal centralized and decentralized solutions, authors show that transshipment prices can always be uniquely adjusted to achieve the centralized solution even under correlated demand.

Zhao et al. (2005) is one of the relevant studies to the thesis setting. It considers a two independent dealer (symmetric as well as asymmetric) decentralized model, dealers are assumed to make replenishment from a one-for-one replenishing manufacturer with constant lead times, where demand is a Poisson process. Dealers tend to minimize their long term expected cost rate consisting of backorder, inventory holding and transshipment costs minus transshipment incentives and subsidies. Without claiming optimality, authors conjecture that ordering and rationing decisions are carried through threshold base-stock and rationing policies that are "static", i.e. policy levels independent of other dealer's inventory level. The followings issues are analyzed: (I) The order policy of dealers under full or partial-pooling (i.e. zero or positive rationing level) conditions, (ii) the effect of dealers' decisions on each other's profitability and the service level and (iii) manufacturer's impact on dealer's strategies, as well as system-wide profits via incentives and subsidies. Steady-state probabilities of dealers for inventory levels are used to calculate expected cost functions. Three different strategy sets are investigated, which are full sharing (threshold-rationing level is zero and base-stock level is the sole decision variable), fixed sharing (for a given arbitrary threshold-rationing level, base-stock level is the sole strategy parameter) and inventory rationing (for a given base-stock level, threshold-rationing level is the sole strategy parameter). Cost function cannot be shown to be always supermodular on inventory levels: violating the sufficient condition for the existence of pure-strategy Nash equilibria. However, this does not mean that there is no equilibrium solution for the games: Nash equilibria are checked using an extensive numerical study through a test-bed of about a thousand instances, no equilibrium is observed in a very little portion of cases.

Main findings are as follows: Dealers respond to higher incentives by decreasing their threshold-rationing levels rather than increasing their base-stock levels, manufacturer subsidies increase backorders (meaning worsens customer service level which is against manufacturer's interests, where it is just the reverse for incentives), inventory sharing in decentralized system for very expensive items increases backorders (for other items, backorders are decreased, as it is always the case for centralized system).

Zhao et al. (2006) is an optimal operating policy characterization study. Demand is a Poisson process. Production to each dealer is dedicated and one-for-one. There are no production lead times. A requesting (i.e. sending a request to another dealer for lateral transsshipment) decision variable is considered: a probability variable, shown to be depending on the policy indices of both dealers. All transshipment requests are subject to acceptance from the party who receives the request. Inventory process of a dealer is hence modeled as a continuous time Markov chain (actually, as a uniformized discrete time Markov decision problem).

Equilibrium order policy of dealers under full or partial-pooling (i.e. zero or positive rationing level), effect of dealers' decisions on each other's profitability and the service level and manufacturer's impact on dealer's strategies, as well as system-wide profits via incentives and subsidies are analyzed. Steady-state probabilities of dealers for inventory levels are used to calculate expected cost functions.

A three-index policy consisting of base-stock (S), rationing (K) and requesting levels (Z) with $S \geq K \geq Z$ is proven to be optimal among all policies that do not depend on other dealer's inventory level (e.g. static or stationary). Sufficient conditions for the existence of a pure-strategy Nash equilibrium could not be shown.

Each dealer wants to minimize a long-run average individual cost function. A competitive game theoretic approach is used for finding a equilibrium policy set, where the objective function is based on steady state probabilities, proven to exist. Best response mapping is used to obtain equilibrium set of policies. An exhaustive search algorithm is run to find best response threshold policy parameters.

An extensive numerical study comprising about a thousand scenarios (used parameters not explicitly shown) is used for two dealers case and four different non-optimal policy types are investigated for all scenarios, including optimal rationing and requesting policy under centralized network. Therefore, the authors could characterize benefits of optimal pooling and centralization.

The key findings are as follows: With increasing transshipment cost, dealers stock more and share less, where high-demand dealers are more sensitive to the increase.

An increased transshipment incentive makes dealers to decrease both stocking and rationing levels. The dealers are sensitive even the incentive is very small, which is meaningful in practice. Including a requesting threshold policy index to the base-stock and rationing policy in a decentralized network makes costs, stocks and backorders less.

Finally, settings with infinitely many dealers are analyzed. When there is a large number of dealers, the effects of the actions of one dealer on others is negligible, so other dealers' actions are considered as exogenous constants. Hence, dynamic programming is used for the case with infinitum of dealers for optimal policy determination. They find out that an increased transshipment incentive makes dealers to increase base stock-levels and decrease rationing levels.

Zhao et al. (2008) is another study that aims to characterize the optimal operating policies in a centralized dealer network. A centralized network with two dealers which are linked to dedicated production facilities is assumed. An expected long-term discounted (not average) system cost is minimized. All other system settings are the same as in Zhao et al. (2006), except the proxy probability parameter for transshipment acceptance is omitted. Only other minor difference is that lateral transshipments are allowed both after production completion and after demand realization in this study. Structure of the optimal policy is proven again to be S≥K≥Z type, namely order-up-to level (S), production transshipment (K) and demand-filling transshipment (Z). The authors show that aforementioned policy levels dynamically change depending on the inventory level at the other dealer, with the increasing inventory level of one dealer, other dealer's order-up-to level is non-increasing, while other control variables increase. Optimal values of control variables are found (i.e. the optimal policy is defined) by an exhaustive search algorithm; a newsvendor heuristic is proposed for finding good values of control variables.

Two competing dealers that maximize their individual profits and collaborate through lateral transshipments are studied in Çömez et al. (2009). In a decentralized, two location, unit-by-unit transshipment and demand setting, the expected revenue maximizing retailers first decide on their initial stocking level and then, until the end of sales horizon (at the end of which all remaining units are salvaged with a salvage value), use this inventory but are allowed to transship if they are stocked-out. These independent retailers can accept/reject transshipment requests.

The replenishment period is divided into sub periods where the demand can be meaningfully at most one.

When demand arrives, it chooses its retailer (can be with different retail prices, where the gap should be less than transportation cost to avoid arbitrage) with a certain probability, and is satisfied if there is an item on that retailer. If the retailer is stocked-out, it requests transshipment from the other retailer. Accepted, demand is again satisfied, the retailer pays the freight and a transshipment commission (at least the salvage value, at most its sales revenue minus freight) to the other retailer. Rejected, the customer may go to the other retailer with some probability (called overflow probability) or is lost forever. Therefore the decision problem retailer faces when a transshipment request arrives reflects the trade-off between the benefit of having one extra inventory and the ability to directly or indirectly (e.g. via overflown customer) getting revenues versus the revenues imposed by transshipment.

The study also includes a discussion on optimal transshipment prices relying on retailer powers dependent on demand scheme or exogenous bargaining powers, but assumes those prices fixed or exogenously assigned through the rest of the study.

Authors show that if the overflow probability is equal to one, then the retailers reject all transshipment requests. Thus if geographical proximity and intensity of competition can be represented with high overflow probability, the benefit of pooling becomes limited; an analogous result is that the bargaining solution for transshipment prices yields no utility for the requester side.

The study proves monotonicity (with respect to time and inventory level) and boundedness (both above and below) of the marginal benefit (independent of stocking levels) with respect to inventory and remaining periods. Since the benefit by transshipment is assumed as exogenous, the transshipment acceptance/rejection decision can be represented via a single inventory hold back parameter above which transshipment requests are accepted. This level is decreasing as time goes by, as the benefit of having an extra inventory diminishes.

For sufficiently low overflows, a complete pooling strategy over the entire horizon can be optimal, further it might be optimal not to pool any inventory as the overflow probability approaches one. Further the hold-back level cannot decrease by more than one in each period because maximum demand in each period is bounded by one.

Another important proof states the quasi-concavity of profit functions with respect to initial stocking levels when fractional initial stocking levels are embedded into the profit function with a linear interpolation scheme. Therefore, pure-strategy Nash equilibria always exist when fractional values of the initial inventory are allowed.

It is proven that hold-back levels increase with demand and demand overflow probability, sales price and salvage value. As expected, hold-back levels decrease with the transshipment price charged by that retailer. Keeping the market size (i.e. sum of probabilities) constant, the hold-back levels are shown to still increase at the retailer where the demand probability increases.

A numerical study of 21 experimental settings to observe main effects is reported. 3000 randomly generated problem sets are also solved to show confidence bounds on results. Authors state that a low demand retailer coupled with a high demand retailer, retailer with limited salvage value or high ordering costs or low transportation costs or low overflows from other retailer are observed to have the highest relative benefit from transshipment.

Taking expected total sales and lost sales as proxy to the performance of the manufacturer where retailers order their parts from, the total sales of the manufacturer is numerically observed to be higher (lost sales lower) if the salvage prices are higher, transportation cost is higher or the purchase cost is lower, at the expense of manufacturer profits. It is also shown that, if overflow probability is relatively small, expected lost sales under complete pooling are lesser than that with no pooling.

A study contributing Zhao's line of studies is Satır et al. (2010). The study has similar setting to Zhao et al. (2006), namely a single-echelon, single-product, dedicated production, one-for-one production and demand, lateral transshipment. Demand is a Poisson process and production times are exponentially distributed. In line with Zhao et al. (2006), customers are exchangeable and hence backorders can be transferred in addition to on-hand, physical inventory.

Unlike Zhao et al. (2006), each dealer has full and common information on costs, profit functions and inventory levels. A discounted profit function instead of a cost function is used: incentives and subsidies for transshipment are replaced by commission payment (which the consigner receives and the consignee pays) and transshipments are assumed to take place without any cost.

This study proves the optimality of a threshold S, K, Z policy parameters that depend on other dealer's inventory (i.e. dynamic) for the centralized case and as a best response for the decentralized case. It then establishes a numerical analysis of benefit of inventory/backorder pooling under optimal operating policies where S, K, Z is dynamic and static. Full and no pooling strategies are also assessed.

Decentralized setting is merely best response assessments to an exogenous dealer, however: not the long-run expected rational behavior, i.e. equilibrium strategy/Nash equilibrium.

Under this setting, dealers are found to form a conflict of interest by choosing two opposite extremes of the commission payments (i.e. 0 and retail price) for themselves.

Out of 1,684 instances (out of 1,800) the benefit of dynamic policy over static policy is found out to be less than 1.5%, and the benefit obtained under dynamic policy is bounded above by 4%. Hence, a static 3-index policy captures most of the benefits obtainable via transshipment.

It is reported in Satır et al. (2010) that decentralized and centralized systems have opposite trends for pooling threshold control variables with inventory level. As the inventory level of the other dealer decreases, authors find numerical evidence that the dealer under consideration shares more of his inventory under centralized system in order to achieve a better system-wide profit, while under competition, the dealer shares less to prevent profit deterioration via giving away inventory that might possibly serve own customers. Although existence of the optimal threshold policies can be analytically proven as a best response and in the centralized setting, monotonicity properties, unlike Zhao et al. (2008) and Çömez et al. (2009) does not necessarily hold in the decentralized setting. Monotonicity holds in the centralized setting. A few non-monotonic instances are found to occur during instances run through numerical study, one is shown in the study.

Other main results are reported as follows: An inappropriately designed pooling system, such as full-pooling, can be worse than no-pooling system. Under certain parameter values, full-pooling profit is observed to be less than half of the non-pooling profit, examples are shown. Diminishing marginal returns on profit over customer arrival rate of dealer under consideration is observed, whereas for customer arrival rate of the other dealer, effect on the profit of the dealer under concern depends on the commission.

2.5 POSITIONING OF THE STUDY IN CITED LITERATURE

Four of the studies in the literature discussed up to this point are closer to this study than the others in terms of its modeling aspects, namely Zhao et al. (2006), Zhao et al. (2008), Çömez et al. (2009) and Satır et al. (2010). These studies, as well as this thesis analyze an inventory management model and consider a cost/profit function optimization scheme under a system of two-locations with continuous inventory review, single echelon, single-item, multi-period, multiple types of customer, endogenous lateral transshipments, centralized/decentralized decision making, exponentially distributed inter-demand time and production time under S,K,Z type threshold-level-type control policies with full information (except Zhao et al. [2006]) about both dealers' inventory.

To refer to relevant Supply Chain Management taxonomies, Kennedy et al. (2002) is a good example along with Teunter and Haneveld (2008). Paterson et al. (2009) was already mentioned earlier.

The demand structure is like Satır (2010), unlike Çömez et al. (2009) which consider demand overflows: there are no demand overflows in this thesis. Like Satır et al. (2010), production times and demand time intervals are exponentially distributed and inventory is continuously reviewed, whereas Çömez et al. (2009) consider a periodic review model.

Policy parameters in Satır (2010), Çömez et al. (2009) and Zhao et al. (2008) are adopted for the model in this thesis except that there is an additional rejection parameter T with positive customer rejection cost[4]. Çömez et al. (2009) does not consider a non-zero transshipment request level, Z.

In line with Zhao et al. (2006) and Satır et al. (2010), customers are exchangeable and hence backorders can be transferred in addition to on-hand, physical inventory.

Unlike the works cited in this section, there is also a positive cost of transshipment is incurred by the consignee of transshipment in this study. This is paid even under centralized authority as a transactional cost.

Also, a heuristic is developed to find best response strategies; the extent of exhaustive search as in Zhao et al. (2006) and Zhao et al. (2008) is hence conjectured to be avoided.

This thesis, like Çömez et al. (2009) and Zhao et al. (2006), focuses its attention on the long-term, equilibrium behavior of individual dealers. Emphasis of this thesis is more numerical: it includes extensive numerical experimentation and observations. This numerical emphasis is felt in all studies, but is stronger in Zhao et al. (2006) and Satır et al. (2010). Zhao et al. (2006) and Satır et al. (2010) have parts to prove the

[4] Existence and monotonicity of the base stock level, in the optimal policy, if can be proven, automatically and trivially guarantees that of customer rejection under non-trivial cost parameters like negative inventory holding costs.

optimality of policy parameters for some, if not all, of their settings, unlike this thesis.

This thesis looks upon and tries to infer from the dynamics of the inventory sharing and rationing game between two dealers by choosing a symmetric cost/demand environment as its base case and focuses on numerical studies to study the impact of main demand/cost parameters, as well as decentralization on dealer profitability, system profitability, policies, inventory/backorder levels and customer service levels. Benefit of pooling is assessed and studied like in all four studies. Impact of asymmetricities, cited in Çömez et al. (2009) and Satır et al. (2010) are also studied. Like Zhao et al. (2006) and Satır et al. (2010), impact of decentralization on policies and profits is assessed and studied. It should be noted that Satır et al. (2010) does not assess equilibrium behavior whereas this thesis does.

All in all, this thesis can be considered as an extensive numerical application of the sum of all findings and a mere, extensive numerical complement of all of those four aforementioned studies with a strong, but previously less touched emphasis on characterizing decentralized equilibrium behavior, benefits of pooling and impact of decentralization.

However, there stand the differences from the individual works cited in this section and our motivation to conduct this research. Zhao et al. (2008) study a centralized model with information sharing whereas we study a decentralized model with information sharing albeit policies are simpler. Satır et al. (2010) study a decentralized model with information sharing, but strategic interaction between the dealers is not under consideration. Zhao et al. (2006) consider a decentralized system with strategically interacting dealers but there is no information sharing, information on other dealer is approximate. Çömez et al. (2009) is a periodic review model that does not decide on Z and T levels. In their model, the base-stock level is determined only once and for a finite number of periods. We do consider the effect of limited production capacity on the performance measures through traffic intensity.

This thesis tries to look upon interactions between cost/demand parameters rather than only on main effects. It also proposes a viable heuristic to find best response strategies in a more efficient manner like in Zhao et al. (2008).

Table 2.1 tries to clarify the comparison between assessments of this thesis and those closely related pieces of literature. A dynamic policy means that threshold policies depend on the inventory level of the other dealer.

Table 2.1 Comparison of the study with closely related literature

Study	**S,K,Z,(T) Policies considered**	**What kind of pooling strategies are considered?**	**Is benefit of pooling assessed?**	**Authority types assessed**
Çömez et al. (2009)	Dynamic	Optimal, complete and non pooling	Analyzed under optimal policies	Decentralized equilibrium, no arguments for benefit of centralization
Zhao et al. (2006)	Static	Four different settings: S,K,Z policy not claimed to be optimal	Not analyzed	Decentralized equilibrium, Centralized
Zhao et al. (2008)	Dynamic and static	Optimal dynamic, static and no pooling also some heuristic policies	Analyzed under optimal policies	Centralized
Satır et al. (2010)	Dynamic and static	Optimal dynamic, static and no pooling	Analyzed under optimal policies	Decentralized best response and centralized
MS Thesis	Static	Static and no pooling. S,K,Z, T policy not claimed to be optimal	Analyzed under optimal policies derived from Satır et al. (2010)	Decentralized equilibrium and centralized

CHAPTER 3

MODEL AND SOLUTION APPROACH

This chapter is comprised of four sections. In 3.1 the problem context is introduced. Models under decentralized and centralized systems are described.

Section 3.2 describes the solution approach for the decentralized setting to compute the performance measures of interest. A heuristic aimed to fasten up the numerical solution algorithm is embedded as a stage of the algorithm.

In Section 3.3, a centralized setting is formulated to assess the impacts of decentralization.

This chapter finalizes with section 3.4 where the solution approach is discussed and computational results concerning the performance of the algorithm stage described in sections 3.2 and 3.3 are presented.

3.1 PROBLEM CONTEXT

A single echelon, single product, two-dealer inventory management system is modeled to address the research questions in Section 1.1.

Single-item demand to Dealer *i* follows Poisson distribution with a *demand rate*, λ_i. Single-item production times (i.e. production is capacitated and one at a time) have exponential distribution with a dedicated production line for each dealer, Dealer *i* with production rate μ_i. Therefore, the inventory levels constitute a Markov process.

Customers arrive to either dealer, requesting a single part. This is called the *customer demand* event. The dealer can either *accept* this request, meaning that the demand is satisfied from dealer's own stock (or backordered), *or* can deny service, issue a *Denial of Service (DoS)*, i.e. reject the customer and make the demand lost *or* can *place a transshipment request* to the other dealer and meet the demand from the other dealer provided that *other dealer is also willing to transship* (if there is no physical stock at the other dealer and this dealer is still willing to transship, this demand is backordered at the dealer requesting the transshipment[5]).

A dealer is subject to transshipment requests from the other dealer. This is called as the *transshipment request* event. The request can *either* be *accept*ed, meaning that the demand of the other dealer is satisfied from own stock (or backordered) *or* can be *reject*ed, meaning that the dealer is not willing to fulfill other dealer's request.

Dedicated production lines to each dealer feed them with single-item replenishments. At any time, the dealer can either make a *new product request (production on)*, continuing the replenishment process *or* can *stop production (production off)*, making the replenishment process idle.

Dealers are subject to[6] revenues generated by each satisfied demand (*R* per unit), commissions paid by the requesting dealer to the other dealer if a transshipment request is fulfilled (*r* per unit) and transshipment costs paid by the requesting dealer if the request is fulfilled (*tr* per unit). If a transshipment request is fulfilled, fulfilling dealer gets r, requesting dealer gets *R-r-tr*. Otherwise, no immediate revenues/costs are incurred.

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[5] Hence, it is implicitly assumed that the customer is informed whether his part is satisfied via transshipment or not in case of inventory deficiency in the other dealer and the customer appreciates this situation as a service deficiency in the requested dealer.

[6] Relevancy of a production cost: although it cannot be said that production cost does not have an impact on control variables, we may assume that production cost is embedded in R and is incurred at the time of sale.

Dealers are also subject to inventory holding costs (c_h per unit per unit time), backordering costs (c_l per unit per unit time) and goodwill costs associated to rejected (DoS) customers (py per unit).

Independent dealers want to maximize their expected discounted infinite horizon individual profits. A centralized authority would rather like to maximize the system-wide profit. Since commissions are within-system payments, it is irrelevant for the centralized authority.

A dealer knows the net inventory level of itself and the other dealer. All demand and cost parameters, as well as profit functions are also common knowledge. Assessment of the value of working with policies which is a function of the real-time inventory information is left untouched in this piece of research.

3.2 SOLUTION APPROACH: DECENTRALIZED PROBLEM

In this thesis, the benefit of pooling for independent dealers as well as the effect of competitive behavior of the dealers on profits is aimed to be assessed. It is assumed that the dealers are having strategic interactions, hence operating under equilibrium policies and thus equilibrium policies and equilibrium profits are claimed to be determined. Performance measures such as service levels or part flow rates between the dealers are determined under the equilibrium policies.

First the problem of one dealer given an exogenous other dealer is analyzed. Then it is assumed that dealers get engaged in a game where in each turn a dealer gives his best response to the other dealer's decision. Iterating between dealers' best-responses, equilibrium set of policies for the dealers are obtained. If the iterations yield a singleton set of policies, that policy set is pure-strategy Nash equilibrium. There might be multiple pure strategy Nash equilibria or no pure strategy Nash equilibrium (a mixed strategy Nash equilibrium instead).

Rest of the argument for the mathematical models is made in terms of Dealer 1 and problem of Dealer 1 against an exogenous Dealer 2. This is stated without any loss of generality because Dealer 2's problem against an exogenous Dealer 1 will be shown to be easily obtained by swapping the relevant dealer indices.

The independent two dealer system can be represented as a Markov chain with inventory levels at the dealers, Dealer 1 and 2, *(i, j)*, -*i* for Dealer 1 (D1) and *j* for 2- as its state variables. Exponential distributed interarrival/production times enable the state transitions to possess the Markov property.

The problem of one dealer given an exogenous dealer is to *maximize* the discounted profits for infinite horizon. Under the modeling properties and event descriptions listed in Section 3.1, a Markov Decision Process (MDP) is formed to model the problem. Actions are defined as possible decisions on each event:

Customer Arrival: accept, Denial of Service (DoS), send transshipment request
Transshipment Request: accept, reject
Production: New product request (production on), stop production (production off)

Dealer 2 (D2) is assumed to have an exogenous fixed (S_2, K_2, Z_2, T_2) policy (where $S_2 \geq K_2 \geq Z_2 \geq T_2$) and acts as explained in Figure 3.1:

	D2's Inventory/Queue Level, j				
EVENT	j≥S2	S2>j>K2	K2≥j>Z2	Z2≥j>T2	j≤T2
production	off	on			
transshipment req.	accept		deny		
customer demand	accept			req. first	DoS

Figure 3.1 Policy of D2 for the decentralized model

In the figure, "Req. First" means, a transshipment request is made. If D1 rejects this request, the demand is satisfied by its own resources.

For Dealer 1 (D1), the corresponding optimality equation is (3.1)

$$v^*(i,j) = \frac{\left[-c_h i^+ - c_l i^-\right]}{\alpha+\beta} + \left(\frac{\beta}{\alpha+\beta}\right)\left[\frac{\lambda_1}{\beta}\Phi_1 v^*(i,j) + \frac{\lambda_2}{\beta}\Phi_2 v^*(i,j) + \frac{\mu_1}{\beta}\Phi_3 v^*(i,j) + \frac{\mu_2}{\beta}\Phi_4 v^*(i,j)\right] \tag{3.1}$$

where

i^+ and i^- denotes *max {0, i}* and *max {0,-i}* respectively,

β is the sum of rates on the dealer system (i.e. $\beta=\lambda_1+\lambda_2+\mu_1+\mu_2$), used for uniformization of the MDP (see Lippman [1975]: Uniformization leads a uniform transition rate and the infinite horizon continuous time decision process is converted into a discrete time decision process),

α is the continuous discount rate accounting for time value of money,

λ_i is the customer *demand rate* on dealer *i*,

μ_i is the *production rate* on dealer *i*, and each Φ-operator (3.2)-(3.5) is tied to each event explained below.

Φ_1 accounts for actions associated with customer arrivals to D1:

$$\Phi_1(i,j) = \begin{cases} \max\{v^*(i-1,j)+R, v^*(i,j-1)+(R-r-tr), v^*(i,j)-py\} & K_2 < j \le S_2 \\ \max\{v^*(i-1,j)+R, v^*(i,j)-py\} & o/w \end{cases} \tag{3.2}$$

D1 can accept, ask from D2 or DoS if the other dealer is willing to accept the transshipment request, otherwise the D1 can accept or DoS.

Φ_2 accounts for actions associated with customer arrivals to D2:

$$\Phi_2(i,j) = \begin{cases} v^*(i,j) & T_2 \ge j \\ v^*(i,j-1) & Z_2 < j \\ \max\{v^*(i-1,j)+r, v^*(i,j-1)\} & o/w \end{cases} \tag{3.3}$$

D1 can accept or reject transshipment requests from D2 if the other dealer is willing to make transshipments.

Φ_3 accounts for actions associated with production in D1:

$$\Phi_3(i,j) = \max\{v^*(i,j), v^*(i+1,j)\} \tag{3.4}$$

D1 can either stop production or initiate a new replenishment.

Φ_4 accounts for actions associated with production in D2:

$$\Phi_4(i,j)=\begin{cases} v^*(i+1,j) & S_2>j \\ v^*(i,j) & o/w \end{cases} \tag{3.5}$$

Let $S_1(j)$, $K_1(j)$, $Z_1(j)$, $T_1(j)$ define action thresholds as in Figure 3.1 for D1 depending on the inventory level j in D2. It could not be shown (see Satır et al. [2010]) that the D1's best response has a $S_1(j) \geq K_1(j) \geq Z_1(j) \geq T_1(j)$ type structure under general settings, but it is known through Satır et al. (2010) that it will hold if *v* is concave in *i* and sub-modular in *i,j*. It was shown in Satır et al. (2010) that sub-modularity fails to hold for a number of settings. Nevertheless, we work with the space of this type of responses.

These level-type structures, if exist, are indeed defined with indifference equations (3.6) between event-related actions, given the threshold nature of the policies:

$$\begin{aligned}
&S_1(j)=\min\{i \mid v(i+1,j)-v(i,j)\leq 0\} \\
&K_1(j)=\begin{cases} \min\{i-1 \mid v(i,j-1)-v(i-1,j)\leq r\} & T_2<j\leq Z_2 \\ S_1(j) & o/w \end{cases} \\
&Z_1(j)=\begin{cases} \max\{i \mid v(i-1,j)-v(i,j-1)\leq -(r+tr) \wedge v(i,j)-v(i,j-1)\leq R+py-r-tr\} & S_2\geq j>K_2 \\ T_1(j) & o/w \end{cases} \\
&T_1(j)=\begin{cases} \max\{i \mid v(i,j)-v(i-1,j)\geq R+py \wedge v(i,j)-v(i,j-1)\geq R+py-r-tr\} & S_2\geq j>K_2 \\ \max\{i \mid v(i,j)-v(i-1,j)\geq R+py & o/w \end{cases}
\end{aligned} \tag{3.6}$$

In almost every cost structure, policy levels that monotonically descend with j are observed for a given S_2, K_2, Z_2, T_2-policy, as sketched in Figure 3.2. Note for instance that no transshipment occurs in the highlighted region (i.e. $Z_1(j) < i \leq K_1(j)$ and $Z_2 < j \leq K_2$) of the state space.

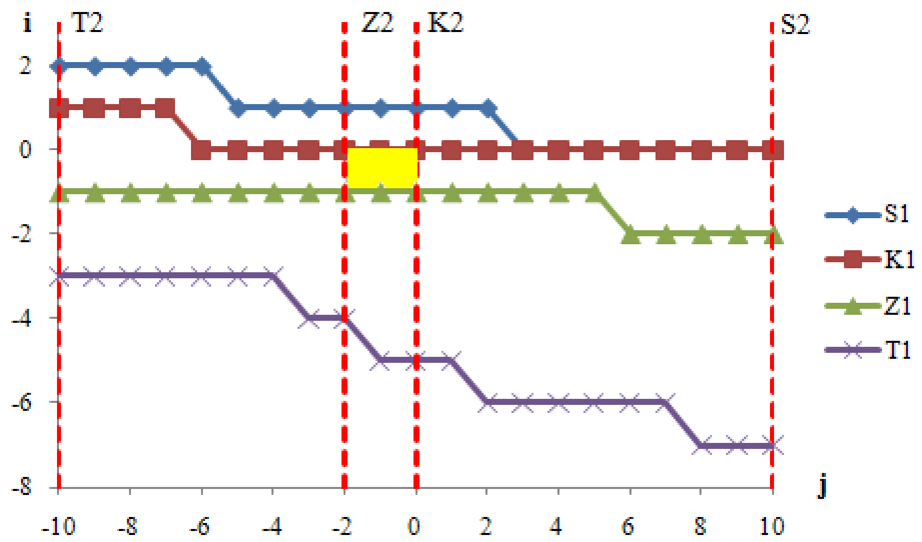

Figure 3.2 A possible best response policy of D1 vis-à-vis policy of D2 in Figure 3.1

For few instances there is slightly non-monotonic behavior, i.e. trendlines in the figure may not be always descending, depending on properties of the cost structure.

This thesis will work with inventory levels independent of the inventory level of the other dealer, i.e. under the set of *static policies.* This will eliminate the requirement for real-time inventory information to be directly incorporated in choosing every action. Even if such information would be available, reaching equilibrium would be hard since every action changes the current state and each action is dependent on the state.

In Satır et al. (2010), a test-bed of 1,684 instances (out of 1,800) the benefit of dynamic policy over static policy (static policies are even not claimed to be the best static policies) is found out to be less than 1.5%, and the benefit obtained under dynamic policy is bounded above by 4%. Hence, a static 3-index policy was found to capture most of the benefits obtainable via transshipment. Therefore we conjecture that there will be a minor loss from optimality in our setting too, noted deviations from the model in Satır et al. (2010), since model formulations are similar, albeit some differences like existence of T-level and transshipment cost.

3.2.1 HEURISTIC FOR DETERMINING A STATIC POLICY: DECENTRALIZED PROBLEM

When determining a static best-response policy for D1 (and then iteratively for D2), the following approach is taken. First a candidate static policy is constructed out of the dynamic policy obtained through the optimality equations. Then a local search, based on the steepest ascent method, is performed around that candidate static policy. Finally, a static policy for D2 is determined. Since D1's best response facing an exogenous static policy for D2 is stated as a static policy, it is clear D2's best response for the stated static best response of D1 can be easily obtained by swapping the relevant indices.

First, it will be described how a candidate static policy is determined through a so-called Policy Iteration phase. Consider a dynamic policy for D1 as represented in Figure 3.2. Given a dynamic policy, the parameter values at highest possible inventory levels of D2 are selected[7] for the relevant policy level. Let us define $x \Rightarrow y$ as x is inferred from the $x(y)$ function at $j=y$. Hence, $S_1 \Rightarrow S_2$, $K_1 \Rightarrow Z_2$ –highest level to send a transshipment request-, $Z_1 \Rightarrow S_2$ –highest level to receive a transshipment request- and $T_1 \Rightarrow K_2$ –highest level for being not eligible to receive a transshipment-.

The best response dynamic policy of a dealer could be solved through a large scale MIP formulation, but it is a computationally very costly way. An iterative method, policy iteration, starting from an initial candidate, can be sought to reduce the computational burden.

This method enjoys the fact that a S_1, K_1, Z_1, T_1, S_2, K_2, Z_2, T_2 policy yields v(i, j) values that can be evaluated through a series of simultaneous equations (3.7) and these $v(i,j)$ values are expected to imply a $S_1(j)$-$K_1(j)$-$Z_1(j)$-$T_1(j)$ dynamic policies defined as in Section 3.2 that has a better objective.

From this dynamic policy, it was discussed that a static policy is obtained by grabbing the policy levels at highest possible inventory levels of D2 relevant for the

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[7] This merely arbitrary choice is fortified by the property stated in the computational analysis: the initial mass is placed at (i, j)=(S_1, S_2), i.e., the highest possible levels of inventory.

decision. This policy can be different than the S_1, K_1, Z_1, T_1 levels previously evaluated. This policy is used as a better candidate for the best response and so on.

If the sole, implied dynamic policy would be evaluated in each turn, it would be exact policy iteration and it is known to be improving. However, the number of parameters to be tuned each iteration will be very huge. An LP formulation and solution per iteration can be viable under this context, but it is computationally costly and will very likely offset the benefits of the policy iteration phase in terms of computer time. We still anticipate that there will be significant gains by iterating through the transformed static policies. All in all, iterations are terminated if it is non-improving.

Since these iterations are inspired by the method of policy iteration, this phase of the heuristic is called the Policy Iteration phase. We expect a very good candidate for the best response to be yielded at the termination of the phase.

Since actions are fixed if S_1, K_1, Z_1, T_1, S_2, K_2, Z_2, T_2 is given, the one-step transition matrix, P can be easily constructed. Further, since the actions are fixed, the expected immediate profit in each stage are also fixed and can be given as the C vector. The ratio $\rho = \beta / \alpha + \beta$ is the one-uniformized step discount factor.

C can be explicitly written as follows:

$$C(i, j) = \frac{-c_h i^+ - c_l i^-}{\alpha + \beta} + \left(\frac{\beta}{\alpha + \beta}\right)\left[\frac{\lambda_1}{\beta} R\right] \; if\ i > T_1\ and\ Z_2 < j \le K_2\ \ or\ if\ i > Z_1\ and\ K_2 < j \le S_2$$

$$or\ if\ i > T_1\ and\ j = T_2\ or\ if\ T_1 < i \le K_1\ and\ T_2 < j \le Z_2$$

$$C(i, j) = \frac{-c_h i^+ - c_l i^-}{\alpha + \beta} + \left(\frac{\beta}{\alpha + \beta}\right)\left[\frac{\lambda_1}{\beta}(R - r - tr)\right] \; if\ T_1 < i \le Z_1\ and\ K_2 < j \le S_2$$

$$C(i, j) = \frac{-c_h i^+ - c_l i^-}{\alpha + \beta} + \left(\frac{\beta}{\alpha + \beta}\right)\left[\frac{\lambda_1}{\beta} R + \frac{\lambda_2}{\beta} r\right] \; if\ K_1 < i \le S_1\ and\ T_2 < j \le Z_2$$

$$C(i, j) = \frac{-c_h i^+ - c_l i^-}{\alpha + \beta} - \left(\frac{\beta}{\alpha + \beta}\right)\left[\frac{\lambda_1}{\beta} py\right] \; if\ i = T_1$$

Also, P can be explicitly written as follows:

$$P((i,j),(i+1,j)) = \begin{cases} \dfrac{\mu_1}{\beta} & if\ i < S_1 \\ 0 & o/w \end{cases}$$

$$P((i,j),(i,j+1)) = \begin{cases} \dfrac{\mu_2}{\beta} & if\ j < S_1 \\ 0 & o/w \end{cases}$$

$$P((i,j),(i-1,j)) = \frac{\lambda_1}{\beta} \quad if\ i > T_1\ and\ Z_2 < j \le K_2\ or\ if\ i > Z_1\ and\ K_2 < j \le S_2$$
$$or\ if\ i > T_1\ and\ j = T_2\ or\ if\ T_1 < i \le K_1\ and\ T_2 < j \le Z_2$$

$$P((i,j),(i-1,j)) = \frac{\lambda_1}{\beta} + \frac{\lambda_2}{\beta} \quad if\ K_1 < i \le S_1\ and\ T_2 < j \le Z_2$$
$$P((i,j),(i-1,j)) = 0 \quad o/w$$
$$P((i,j),(i,j-1)) = \frac{\lambda_2}{\beta} \quad if\ j > T_2\ and\ Z_1 < i \le K_1\ or\ if\ j > Z_2\ and\ K_1 < i \le S_1$$
$$or\ if\ j > T_2\ and\ i = T_1\ or\ if\ T_2 < j \le K_2\ and\ T_1 < i \le Z_1$$

$$P((i,j),(i,j-1)) = \frac{\lambda_1}{\beta} + \frac{\lambda_2}{\beta} \quad if\ K_2 < j \le S_2\ and\ T_1 < i \le Z_1$$
$$P((i,j),(i,j-1)) = 0 \quad o/w$$

$$P((i,j),(i,j)) = 1 - P((i,j),(i+1,j))$$
$$-P((i,j),(i,j+1)) - P((i,j),(i-1,j)) - P((i,j),(i,j-1))$$

$$P((i,j),(k,l)) = 0 \quad o/w$$

Hence, the expected profit as well as the value functions corresponding to the static policy defined above is obtained from equation set (3.7) as follows:

$$C + \rho PV = V$$
$$\pi = \delta V \tag{3.7}$$

Where V is the vector of the value functions v*(i, j), δ is the initial probability distribution among states and π denotes the expected discounted infinite horizon individual profit. The process is assumed to start in state *(S_1, S_2)*. S and T levels for both dealers yield the natural truncation points of the state space (i.e. it is impossible to go beyond T levels and go above S levels); hence the state space is always finite. Note further that $(I - \rho P)$ matrix is invertible, therefore $\pi = \delta V = \delta(I - \rho P)^{-1} C$

By the Policy Iteration Phase, we anticipate a reduction of the number of iterations for the next phase, the steepest ascent neighborhood search algorithm in the space of static policies, which guarantees a local optimal solution. Once the best response is found, players are swapped to find the best response of D2 to new policies of D1 and so on. It is implicitly conjectured that local search sufficiently ensures the best response to the opponent's choice of a static policy.

The solution procedure to find the equilibrium (i.e. best responses of two dealers in terms of profit) S_1, K_1, Z_1, T_1, S_2, K_2, Z_2, T_2 policy levels for two independent dealers has two phases to find the best response and a player interchange (i.e. best response mapping algorithm) for this two person non-constant sum game.

- Policy Iteration Phase
  - Starts with any fixed S_1, K_1, Z_1, T_1, S_2, K_2, Z_2, T_2 policy.
  - Actions are fixed for every state, *v(i, j)* can be solved through a linear system of equations
  - *v(i, j)*'s imply *$S_1(j)$-$K_1(j)$-$Z_1(j)$-$T_1(j)$* dynamic policy parameters. These are found.
  - Transform the dynamic policy to a static S_1,K_1,Z_1,T_1 policy through highest j values[8]
  - Continue until the phase does not improve or converges to the same policy

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[8] Since S and T can go beyond the evaluation boundaries for the following iterations, the state space for D1 is kept 5 (arbitrarily chosen) below and above than current levels in each iteration of the policy iteration phase.

- Steepest Ascent Phase
    - Search the whole neighborhood of the policy (+1, 0, -1 combinations for all policy parameters) where $S_1 \geq K_1 \geq Z_1 \geq T_1$ is satisfied.
    - Go to the policy with most improvements. Stop when no such direction is available
- Best response $S_1 \geq K_1 \geq Z_1 \geq T_1$ found given S_2, K_2, Z_2, and T_2. Switch dealers. Proceed until best response mapping stops at a fixed point or loops at a set of equilibrium policies.

The solution algorithm is coded in MATLAB environment and run under single core of an Intel® Core 2 Duo™ 2.5GhZ CPU. *Appendix A* can be visited to see the workspace definition, a detailed pseudocode and the modules. Both pseudo-codes and MATLAB codes guided with the pseudo-code are available for each module.

3.3 SOLUTION APPROACH: CENTRALIZED PROBLEM

The aim is now to *maximize* the expected discounted infinite-horizon system-wide profit. Necessary modifications to the arguments for the decentralized problem do hence follow. Please refer to the "Solution Approach: Decentralized Problem", section 3.2 for the relevant arguments.

The state space representation and the events are still the same as in the decentralized problem. So are the relevant costs except commissions. Commissions are now irrelevant since the dealer system is centrally operated.

The aim of the *centralized MDP* is to *maximize* the expected total discounted profit under infinite horizon. The corresponding optimality equation is (3.8), as follows:

$$v^*(i,j) = \frac{-c_h(i^+ + j^+) - c_l(i^- + j^-)}{\alpha + \beta} + \left(\frac{\beta}{\alpha+\beta}\right)[\frac{\lambda_1}{\beta}\Phi_1 v^*(i,j) + \frac{\lambda_2}{\beta}\Phi_2 v^*(i,j) + \frac{\mu_1}{\beta}\Phi_3 v^*(i,j) + \frac{\mu_2}{\beta}\Phi_4 v^*(i,j)] \qquad (3.8)$$

Where j^+ and j^- denotes *max {0, j}* and *max {0,-j}* respectively.

The most important change arises within the definition of the Φ-operators (3.9)-(3.12) and the definition of policy management levels since the revenue function is changed.

Φ_1 accounts for actions associated with customer arrivals to D1

$$\Phi_1(i,j) = \max\{v^*(i-1,j)+R, v^*(i,j-1)+(R-tr), v^*(i,j)-py\} \tag{3.9}$$

D1 can accept, transship from D2 or DoS an arriving customer.
Φ_2 accounts for actions associated with customer arrivals to D2

$$\Phi_2(i,j) = \max\{v^*(i-1,j)+(R-tr), v^*(i,j-1)+R, v^*(i,j)-py\} \tag{3.10}$$

D2 can transship from D1, accept or make DoS an arriving customer.
Φ_3 accounts for actions associated with production in D1

$$\Phi_3(i,j) = \max\{v^*(i,j), v^*(i+1,j)\} \tag{3.11}$$

Φ_4 accounts for actions associated with production in D2

$$\Phi_4(i,j) = \max\{v^*(i,j), v^*(i,j+1)\} \tag{3.12}$$

With supporting arguments stated in Satır et al. (2010) as well as Zhao et al. (2008) proving the existence of the threshold policy parameters and the fact that *three index policy* under non-trivial (i.e. not considering trivial cases like commissions exceeding revenues, negative penalty cost, negative backorder and inventory holding costs) cost and revenue structures exist, dealers are supposed to have a $S_1(j) \geq K_1(j) \geq Z_1(j) \geq T_1(j)$ and $S_2(i) \geq K_2(i) \geq Z_2(i) \geq T_2(i)$ structure. Dealers are supposed to act in line with Figure 3.3, Table 3.1 and Table 3.2 if such a policy occurs.

Note that A, DoS and T correspond to accept, Denial of Service (DoS) and send transshipment request actions in case of the customer demand event, respectively. AC and RJ correspond to accept and reject actions in case of a transshipment request event, in that order. Finally, N and S corresponds to new product request and stop production actions, respectively.

Table 3.1 Regions of dealers for each action, regions as in Figure 3.3 and Table 3.2

Dealer	Action	Region	Dealer	Action	Region
D1	A	1,2,3,4,5,6,7,8 and along T2 line	D2	A	2,3,4,5,6,7,8,9 and along T1 line
	DoS	Along T1 line		DoS	Along T2 line
	T	9		T	1
	AC	1,2,3		AC	3,6,9
	RJ	4,5,6,7,8,9 and along T1 and T2 lines		RJ	1,2,4,5,7,8 and along T1 and T2 lines
	N	Except S1 line		N	Except S2 line
	S	Along S1 line		S	Along S2 line

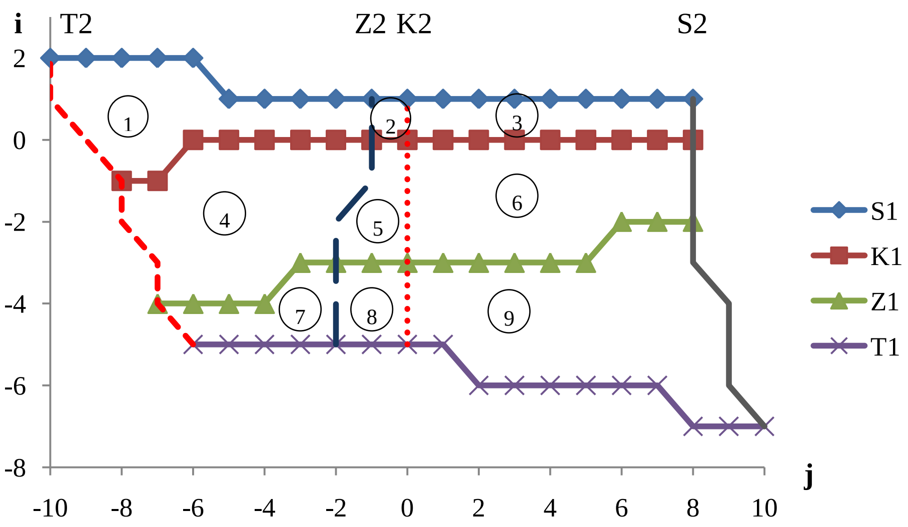

Figure 3.3 Policy regions of dealers for the centralized model

Table 3.2 Region descriptions in inventory levels of both dealers

Region	Description	Region	Description
1	K1<i≤S1, T2<j≤Z2	6	Z1<i≤K1,K2<j≤S2
2	K1<i≤S1, Z2<j≤K2	7	T1<i≤Z1,T2<j≤Z2
3	K1<i≤S1, K2<j≤S2	8	T1<i≤Z1,Z2<j≤K2
4	Z1<i≤K1,T2<j≤Z2	9	T1<i≤Z1,K2<j≤S2
5	Z1<i≤K1,Z2<j≤K2	Along T1-line	i=T1
		Along T2-line	j=T2
		Along S1-line	i=S1
		Along S2-line	j=S2

The $S_1(j),K_1(j),Z_1(j),T_1(j),S_2(i),K_2(i),Z_2(i),T_2(i)$ levels, (3.13)-(3.14) are defined explicitly as follows, given the threshold nature of the policies:

$$\begin{aligned}
S_1(j) &= \min\{i \mid v(i+1, j) - v(i, j) \le 0\} \\
K_1(j) &= \min\{i-1 \mid v(i, j-1) - v(i-1, j) \le -tr \wedge v(i, j) - v(i-1, j) \le R + py - tr\} \\
Z_1(j) &= \max\{i \mid v(i-1, j) - v(i, j-1) \le -tr \wedge v(i, j) - v(i, j-1) \le R + py - tr\} \\
T_1(j) &= \max\{i \mid v(i, j) - v(i-1, j) \ge R + py \wedge v(i, j) - v(i, j-1) \ge R + py - tr\}
\end{aligned} \tag{3.13}$$

And similarly,

$$\begin{aligned}
S_2(i) &= \min\{j \mid v(i, j+1) - v(i, j) \le 0\} \\
K_2(i) &= \min\{j-1 \mid v(i-1, j) - v(i, j-1) \le -tr \wedge v(i, j) - v(i, j-1) \le R + py - tr\} \\
Z_2(i) &= \max\{j \mid v(i, j-1) - v(i-1, j) \le -tr \wedge v(i, j) - v(i-1, j) \le R + py - tr\} \\
T_2(i) &= \max\{j \mid v(i, j) - v(i, j-1) \ge R + py \wedge v(i, j) - v(i-1, j) \ge R + py - tr\}
\end{aligned} \tag{3.14}$$

3.3.1 HEURISTIC FOR DETERMINING A STATIC POLICY: CENTRALIZED PROBLEM

When determining the static best policies for dealers, the following approach is taken. First a candidate static policy for both dealers is constructed out of the dynamic policy obtained through the optimality equations. Then a local search, based on the steepest ascent method, is performed around that candidate static policy.

First, it will be described how a candidate static policy is determined through a so-called Policy Iteration phase. A static policy from the policies as in Figure 3.3 is obtained by grabbing the policy levels at highest possible inventory levels of D1 and

D2 relevant for the decision, like in Section 3.2.1. Defined x ➔ y as x is inferred from the x(y) function at j=y, S_1➔ S_2, K_1➔ Z_2 –highest level to send a transshipment request-, Z_1 ➔ S_2 –highest level to receive a transshipment request- and T_1 ➔ K_2 –highest level for being not eligible to receive a transshipment-.

Similarly, S_2➔ S_1, K_2➔ Z_1, Z_2 ➔ S_1, T_2 ➔ K_1.

The best dynamic policies of dealers could be solved through a large scale MIP formulation, but it is a computationally very costly way. An iterative method, policy iteration, starting from an initial candidate, can be sought to reduce the computational burden.

This method enjoys the fact that a S_1, K_1, Z_1, T_1, S_2, K_2, Z_2, T_2 policy yields v(i, j) values that can be evaluated through a series of simultaneous equations (3.7) and these v(i,j) values are expected to imply $S_1(j)$-$K_1(j)$-$Z_1(j)$-$T_1(j)$-$S_2(i)$-$K_2(i)$-$Z_2(i)$-$T_2(i)$ dynamic policy defined as in Section 3.3 that has a better objective.

As a S_1, K_1, Z_1, T_1, S_2, K_2, Z_2, T_2 policy is evaluated, v(i, j) values can be obtained through equations (3.7), these imply a $S_1(j)$-$K_1(j)$-$Z_1(j)$-$T_1(j)$-$S_2(i)$-$K_2(i)$-$Z_2(i)$-$T_2(i)$ dynamic policy. From this, it is discussed that a static policy can be obtained by grabbing the policy levels at highest possible inventory levels of D1 and D2 relevant for the decision. This policy can be different than the S_1, K_1, Z_1, T_1, S_2, K_2, Z_2, T_2 levels evaluated. Evaluating this policy will be usually improving.

If the sole, implied dynamic policy would be evaluated in each turn, it would be exact policy iteration and it is known to be improving. However, the number of parameters to be tuned each iteration will be very huge. An LP formulation and solution per iteration can be viable under this context, but it is computationally costly and will very likely offset the benefits of the policy iteration phase in terms of computer time. We still anticipate that there will be significant gains by iterating through the transformed static policies. All in all, iterations are terminated if it is non-improving. However we still anticipate that there will be significant gains by iterating through the transformed static policies.

Since these iterations are inspired by the method of policy iteration, this phase of the heuristic is called the Policy Iteration phase. We expect a very good candidate for the best policy set to be yielded at the termination of the phase.

Since actions are fixed if S_1, K_1, Z_1, T_1, S_2, K_2, Z_2, T_2 is given, the one-step transition matrix, P can be easily constructed. Further, since the actions are fixed, the expected immediate profits in each stage are also fixed and can be given as the C vector.

C can be explicitly written as follows:

$C(i,j) = C_1(i,j) + C_2(i,j)$, where

$$C_1(i,j) = \frac{-c_h i^+ - c_l i^-}{\alpha+\beta} + \left(\frac{\beta}{\alpha+\beta}\right)\left[\frac{\lambda_1}{\beta}R\right] \ \ if\ i > T_1\ and\ Z_2 < j \le K_2\ \ or\ if\ i > Z_1\ and\ K_2 < j \le S_2$$
$$or\ if\ i > T_1\ and\ j = T_2\ or\ if\ T_1 < i \le K_1\ and\ T_2 < j \le Z_2$$

$$C_1(i,j) = \frac{-c_h i^+ - c_l i^-}{\alpha+\beta} + \left(\frac{\beta}{\alpha+\beta}\right)\left[\frac{\lambda_1}{\beta}(R-r-tr)\right] \ (T_1 < i \le Z_1, K_2 < j \le S_2)$$

$$C_1(i,j) = \frac{-c_h i^+ - c_l i^-}{\alpha+\beta} + \left(\frac{\beta}{\alpha+\beta}\right)\left[\frac{\lambda_1}{\beta}R + \frac{\lambda_2}{\beta}r\right] \ (K_1 < i \le S_1, T_2 < j \le Z_2)$$

$$C_1(i,j) = \frac{-c_h i^+ - c_l i^-}{\alpha+\beta} - \left(\frac{\beta}{\alpha+\beta}\right)\left[\frac{\lambda_1}{\beta}py\right] \ if\ i = T_1$$

and

$$C_2(i,j) = \frac{-c_h j^+ - c_l j^-}{\alpha+\beta} + \left(\frac{\beta}{\alpha+\beta}\right)\left[\frac{\lambda_2}{\beta}R\right] \ \ if\ j > T_2\ and\ Z_1 < i \le K_1\ \ or\ if\ j > Z_2\ and\ K_1 < i \le S_1$$
$$or\ if\ j > T_2\ and\ i = T_1\ or\ if\ T_2 < j \le K_2\ and\ T_1 < i \le Z_1$$

$$C_2(i,j) = \frac{-c_h j^+ - c_l j^-}{\alpha+\beta} + \left(\frac{\beta}{\alpha+\beta}\right)\left[\frac{\lambda_2}{\beta}(R-r-tr)\right] \ if\ T_2 < j \le Z_2\ and\ K_1 < i \le S_1$$

$$C_2(i,j) = \frac{-c_h j^+ - c_l j^-}{\alpha+\beta} + \left(\frac{\beta}{\alpha+\beta}\right)\left[\frac{\lambda_2}{\beta}R + \frac{\lambda_1}{\beta}r\right] \ if\ K_2 < j \le S_2\ and\ T_1 < i \le Z_1$$

$$C_2(i,j) = \frac{-c_h j^+ - c_l j^-}{\alpha+\beta} - \left(\frac{\beta}{\alpha+\beta}\right)\left[\frac{\lambda_2}{\beta} py\right] \quad if \;\; j = T_2$$

P is exactly the same as in pg. 40.

Hence, the set of equations, equation set (3.7) follows. The initial mass is again placed at (i, j) equal to (S_1, S_2). Since $(I-\rho P)$ matrix is invertible, the system of equations (3.7) is guaranteed to have a unique solution for a finite state space system. S and T levels for both dealers yield the natural truncation points of the state space (i.e. it is impossible to go beyond T levels and go above S levels); hence the state space is always finite.

By the Policy Iteration Phase, we anticipate a reduction of the number of iterations for the next phase, the steepest ascent neighborhood search algorithm in the space of static policies, which guarantees a local optimal solution. It is implicitly conjectured that local search sufficiently ensures the best response to the opponent's choice of a static policy.

Policy iteration phase is now done for the whole 8 parameters, and then the implied static policy (as discussed in pg. 38) is found, another step is done and so on until the same static policy is implied or the objective does not improve. Then, the best static policy with the highest objective is estimated by steepest ascent neighborhood search, which is guaranteed to give a local optimal solution.

Policy Iteration Phase

- Starts with any S_1, K_1, Z_1, T_1, S_2, K_2, Z_2, T_2 policy.
- Actions are fixed for every state, *v(i, j)* can be solved through a linear system of equations
- *v(i, j)*'s imply $S_1(j)$-$K_1(j)$-$Z_1(j)$-$T_1(j)$-$S_2(i)$-$K_2(i)$-$Z_2(i)$-$T_2(i)$ dynamic policy parameters. These are found.
- Transform new S_1,K_1,Z_1,T_1,S_2,K_2,Z_2,T_2 values from highest i and j values[9]

  - Continue until the phase does not improve or converges to the same policy
- Steepest Ascent Phase
  - Search the whole neighborhood of the policy (+1,0,-1 combinations for all policy parameters $S_1,K_1,Z_1,T_1,S_2,K_2,Z_2,T_2$) where $S_1 \geq K_1 \geq Z_1 \geq T_1$ and $S_2 \geq K_2 \geq Z_2 \geq T_2$ is satisfied.
  - Go to the policy set with most improvements. Stop when no such direction is available

Therefore, it is implicitly conjectured that local search sufficiently ensures the optimal centralized solution.

The solution algorithm is coded in MATLAB environment and run under single core of an Intel® Core 2 Duo™ 2.5GhZ CPU.

3.4 PERFORMANCE OF THE POLICY ITERATION PHASE

S(.),K(.),Z(.),T(.) policies imposed by v(i,j) are dynamic, i.e. policy levels of one dealer is a function of the inventory level of the other dealer. The Policy Iteration phase was not exact because a static policy was arbitrarily obtained from these dynamic policies.

Given the range of S(.),K(.),Z(.),T(.) functions, this arbitrarily obtained static policy is only one of such alternatives. Chances of choosing a good static policy out of a dynamic policy by such an arbitrary choice deteriorates as the ranges of policies are enlarged, e.g. when holding cost decreases or when customer traffic increases so that the S and T ranges are enlarged.

An approach that would iterate with dynamic policies could be sought, but it would be computationally burdensome. The number of parameters to be tuned each iteration will be very huge. An LP formulation and solution per iteration is viable

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[9] Since S and T can go beyond the evaluation boundaries for the following iterations, the state space for both dealers are kept 5 below and above than current levels in each iteration of the policy iteration phase.

under this context, but it is computationally costly and will very likely offset the benefits of the policy iteration phase in terms of computer time. Likewise, the best response dynamic policies could be solved through a large scale MIP formulation, but it is much more costly.

Therefore, as the quality of policy iteration deteriorates, steepest ascent phase is anticipated to yield greater improvements to the solution obtained from the policy iteration phase.

As performance of the policy iteration phase for the numerical experiments stated in Chapter 4 will be discussed, please refer to Section 4.1 for the definition of cases as well as parameter combinations.

Policy iteration phase in the symmetric competitive case greatly reduces the required number of evaluations[10], this phase finishes at up to 7 (avg. 3.03) iterations. In 336/750 (44%) of the Symmetric Competitive case experiments, steepest ascent did not change the solution (indicated by the fact that steepest ascent phase only checking the neighbors, total number of evaluations less than or equal to total number of neighbors)

Steepest Ascent Phase improves the solutions by 3.58% in average, in the range of 0% - 59%. Independent of other parameters, when holding cost is greater than 0.1 per unit per unit time, the average improvement reduces to 0.10% and the range shrinks to 0% - 5.04%. Likewise, if traffic intensity is less than 0.6, the average is 0.02% and the range is 0% - 0.83%.

For policy iteration phase in asymmetric competitive pooling case, results are even better in terms of average performance. Cases with significant improvements in the steepest ascent phase (>80%) occurs at high traffic flows to both dealers and higher holding and backorders costs of D1 vs. D2. Results get better with lower demand flows to the system and very interestingly, lower holding and especially lower

[10] Recall that for each steepest ascent neighborhood search, up to 3^4=81 evaluations can be required.

backordering costs of D1 with respect to D2 irrespective of the traffic intensity at D2. Improvement is more drastic if the traffic intensity at D2 is high.

Policy iteration phase in asymmetric case finishes at up to 8 (avg. 4.61) iterations. In 3573/6250 (57%) of the experiments, steepest ascent did not change the solution. Steepest Ascent Phase improves the solutions by 0.57% in average, in the range of 0% - 83%. Table 3.3 summarizes the average and range of improvement of the Steepest Ascent phase under different cost/traffic parameters. Refer to Section 4.1 for parameter combinations.

Table 3.3 Improvement of Steepest Ascent Phase for asymmetric competitive pooling case under different cost/traffic parameters

	Traffic Intensity of D2	
	0.3	0.9
All such cases	0.27% (0-66%)	0.87% (0-83%)
Traffic intensity of D1< 0.6	0.17% (0-66%)	0.26% (0-40%)
Holding cost of D1< 1 per unit per unit time	0.08% (0-22%)	0.25% (0-25%)
Backordering cost of D1< 2 per unit per unit time	0.02% (0-1.1%)	0.04%(0-2.9%)

The improvement of the steepest ascent phase in the symmetric centralized pooling case is higher than in other cases, but it has the best performance in the range. This is believed to be associated with the merely naïve solution procedure modification for the policy iteration phase. It is observed that greater holding costs and lower traffic intensities improved the results.

Policy iteration phase in centralized case finished at up to 8 (avg. 2.91) iterations. In 6/210 (3%) of the experiments, steepest ascent did not change the solution. Steepest Ascent Phase improves the solutions by 8.38% in average, in the range of 0% -

34.2%. When holding cost is greater than 0.1 per unit per unit time, the average improvement reduces to 7.95%, range is still the same. Likewise, if traffic intensity is less than 0.6, the average is 3.34% and the range squeezes to 0% - 15.7%.

It can be said that a powerful method to find the game equilibriums in the case of independent dealers is introduced.

The main cause of further improvement by local search was observed to be the process of finding good levels of static policy levels from a dynamic best response. A base-stock approximation to S and T levels may prove useful. Another challenge is to find a better performance method for the centralized system, the merely naïve solution procedure modification did not work as intended.

CHAPTER 4

COMPUTATIONAL RESULTS

This chapter outlines the experimental base and listing findings to the research focus questions stated. It consists of four sections.

In Section 4.1, the numerical study setting, through which the computational results are obtained, is described: cases and parameter base of each case as well as the performance measures are introduced.

In Section 4.2 the results for the *symmetric competitive pooling* –benchmark- case are presented. By “symmetric”, it is meant that all demand parameters, inventory holding costs and backordering costs are the same across the system. Interpretations are given on the effect of holding cost, transshipment cost and traffic intensity, as well as commissions on equilibrium policies, profits, transshipment flow volumes, and service performances of a two independent symmetric (in terms of demand/supply structure and internal costs) dealer spare parts system under the possibility of inventory pooling.

In Section 4.3 the results for the *asymmetric competitive pooling* are presented. Allowing for different arrival rates/traffic intensities, inventory holding and backordering costs between two dealers, performance measures are examined and interpretations provided, notably on the impact of asymmetry in environment parameters on transshipment benefits, equilibrium policies and transshipment flow volumes. This section also aims to answer how the benefits from transshipment are dispersed through heterogeneous dealers under various commissions.

Section 4.4 yields the results for *centralized cooperative pooling*. The aim is to quantify the negative effect of competition and to observe how centralized policies differ from decentralized policies. The system-wide value is now maximized instead of dealers trying to give the best response maximizing their own self-interest given other dealer's inventory management policy. Under symmetry of cost and demand parameters, numerical results are presented to show the effect of demand and cost parameters on the value of centralized inventory management in a symmetric setting as well as on the centrally achievable benefits of pooling. If a manufacturer aims to introduce incentive mechanisms to motivate the independent dealers to collaborate, he would benefit from this analysis.

4.1 NUMERICAL SETTING

This section has two subsections, 4.1.1 and 4.1.2. In 4.1.1 the three cases that are dealt in the study is introduced. Parameter base of each case is introduced, as well as the factors analyzed in each case. In 4.1.2, performance measures are expressed.

4.1.1 CASES CONSIDERED

Symmetric Competitive Pooling is the benchmark case. It is defined as two independent dealers with identical economic environments: same backorder/holding/penalty costs, same customer traffic intensity and production rate.

Factors Analyzed: Traffic intensity (synonymous to demand rate since production rate is unity), transshipment cost, commission, holding cost attributed to both dealers are the factors analyzed. Table 4.1 states the numerical combinations.

Table 4.1 Factor combinations of symmetric competitive pooling study

Demand Rate	0.3, 0.45, 0.6, 0.75, 0.9, 0.99 per unit time
Prod. Rate	1.0 per unit time
Commission	1, 3, 4.5, 6, 9 per unit
Trans. Cost	1,2,4,6,8 per unit
Sales Rev.	10 per unit
DoS Penalty	5 per unit
Inv. Hold Cost	0.1,0.5,1,2,4 per unit per unit time
Backordering cost	2 per unit per unit time
TOTAL	**750 Combinations**

Asymmetric Competitive Pooling is system with not-necessarily-identical economic environments: backorder/holding costs and customer traffic intensities now differ across the dealers.

Transshipment cost, commission payment, holding cost of D1, backordering cost of D1 and customer traffic intensity (production rate is the same for both D1 and D2) are the factors analyzed. Table 4.2 states the numerical combinations.

Table 4.2 Factor combinations of asymmetric competitive pooling study

Demand Rate of D1	0.3, 0.45, 0.6, 0.75, 0.9 per unit time
Demand Rate of D2	0.3,0.9 per unit time
Prod. Rate for both dealers	1.0 per unit time
Commission for both dealers	1, 3, 4.5, 6, 9 per unit
Trans. Cost for both dealers	1,2,4,6,8 per unit
Sales Rev. for both dealers	10 per unit
DoS Penalty for both dealers	5 per unit
Inv. Holding Cost for D1	0.1,0.5,1,2,4 per unit per unit time
Inv. Holding Cost for D2	1 per unit per unit time
Backordering cost for D1	0.2,1,2,4,8 per unit per unit time
Backordering cost for D2	2 per unit per unit time
TOTAL	**6250 Combinations**

Note that traffic intensities to D1 are either lower than or equal to the traffic intensity to the second dealer, or higher than or equal to that. Hence, it is convenient to recall half of the combinations as “low traffic intensity at D2” and the other half as “high traffic intensity at D2” case and treat them separately.

Symmetric Centralized Pooling is the case with two dealers that are centrally operated with identical economic environments.

Factors analyzed are transshipment cost, traffic intensity and holding cost. Note that commission, an in-system transfer payment, is irrelevant to this system (i.e. the system-wide revenue is R-tr if a customer demand is met through transshipment) are the factors analyzed. Table 4.3 states the numerical combinations.

Table 4.3 Factor combinations of symmetric centralized pooling

Demand Rate	0.3, 0.45, 0.6, 0.75, 0.9, 0.99 per unit time
Prod. Rate	1.0 per unit time
Commission	Irrelevant
Trans. Cost	1,2,4,6,8 per unit
Sales Rev.	10 per unit
DoS Penalty	5 per unit
Inv. Holding Cost	0.1,0.5,1,2,4,8,12 per unit per unit time
Backordering cost	2 per unit
TOTAL	**210 Combinations**

α, the continuous discount rate accounting for time value of money, is kept at 0.05 for all experimental settings.

4.1.2 PERFORMANCE MEASURES

The benefit of pooling for the D1 (i.e. percentage added to D1 profits under pooling equilibrium vis-à-vis without pooling) and the effect of competition on profits.

The *(relative) benefit of pooling* for D1 and D2 follows (4.1):

$$\frac{\pi_{pooling} - \pi_{nopooling}}{\pi_{nopooling}} \times 100\% \qquad (4.1)$$

Where $\pi_{pooling}$ is the profits of D1 and D2 under the centralized/decentralized model and $\pi_{nopooling}$, the no-pooling profits are computed by restricting the policies to be S, T type and fixing K=S and Z=T and using the same algorithm under same traffic intensity and cost/revenue parameters.

The *(relative) benefit of centralization* for the 1st (2nd) dealer is defined as in (4.2)

$$\frac{\pi_{centralized} - \pi'_{pooling}}{\pi'_{pooling}} \times 100\% \qquad (4.2)$$

Where $\pi'_{pooling}$ is the arithmetic average profits of D1(2) via the decentralized model under different commissions and $\pi_{centralized}$ is computed under the same cost/revenue structure (except commission, which is irrelevant) under the centralized model.

Note that the benefit of pooling[11] and the benefit of centralization is always non-negative in our modeling context, since pooling can be forfeited at will and centralization does transshipments that will increase system-wide profits but unilaterally decrease one of the dealer's profits.

Although the model seeks to find the best responses of each dealer solely in terms of their profits in the decentralized case and best system-wide profit for the centralized case, the total discounted expected inventory levels, the total discounted expected backorders, total discounted (transshipment) flow rates between dealers and the total discounted expected DoS rate are also assessed to fully monitor the impact of centralization and inventory pooling on supply chain performance.

To calculate the remaining performance measures, total discounted expected time fractions in each state is obtained through equation set (4.3).

$$X = \delta + \rho P X \tag{4.3}$$

X is proportional to the total discounted expected time fraction up to a multiplicative constant, which depends only on α and β. The initial mass is placed at (i, j) = (S_1, S_2). Please, refer to Chapter 3 for the definitions of δ, ρ, β and P.

Given X, the abovementioned performance measures can be mathematically defined.

The total discounted *expected inventory level* for D1 is simply as in (4.4):

$$\frac{1}{\alpha+\beta}\sum_{j=T2}^{S2}\sum_{i=T1}^{S1} i^{+} X(i,j) \tag{4.4}$$

Where $i^{+} = max\{0, i\}$

[11] Benefit of pooling is non-negative since a dealer always has the option to unilaterally waive pooling in a competitive setting by setting K_1=S_1 and Z_1=T_1.

The total discounted *expected backorders* for D1 is as in (4.5):

$$\frac{1}{\alpha+\beta}\sum_{j=T2}^{S2}\sum_{i=T1}^{S1} i^{-} X(i,j) \tag{4.5}$$

Where $i^{-}=max\{0,-i\}$

The total discounted expected *flow rate from D1 to D2* is as in (4.6):

$$\frac{1}{\alpha+\beta}\sum_{j=T2+1}^{Z2}\sum_{i=K1+1}^{S1} \lambda_2 X(i,j) \tag{4.6}$$

The total discounted expected *flow rate from D2 to D1* is as in (4.7):

$$\frac{1}{\alpha+\beta}\sum_{j=K2+1}^{S2}\sum_{i=T1+1}^{Z1} \lambda_1 X(i,j) \tag{4.7}$$

The total discounted expected *DoS rate for D1* is as in (4.8):

$$\frac{1}{\alpha+\beta}\sum_{j=T2}^{S2} \lambda_1 X(T_1,j) \tag{4.8}$$

Note that the $\sum_{a}^{b}.$ operand is null if $a>b$.

4.2 SYMMETRIC COMPETITIVE POOLING RESULTS

In this section, In Section 4.2 the results for the *symmetric competitive pooling* – benchmark- case are presented. By "symmetric", it is meant that all demand parameters, inventory holding costs and backordering costs are the same across the system.

This section consists of 5 subsections. In subsection 4.2.1, some introductory results are stated on the equilibrium structure and the distribution of relative benefit of pooling. In the following three subsections, interpretations are given on the effect of transshipment cost (subsection 4.2.2), inventory holding cost (subsection 4.2.3), commission (subsection 4.2.4) on equilibrium policies, profitabilities, transshipment flow volumes, and service performances of a two independent symmetric (in terms of demand/supply structure and internal costs) dealer spare parts system under the possibility of inventory pooling. Subsection 4.2.5 summarizes the results obtained so far and discusses the effect of traffic intensity on the system performance.

4.2.1 INTRODUCTORY RESULTS

This section has some notes on the equilibrium structures and the distribution of the benefit of pooling.

The equilibrium structure is found to be mostly single, pure-strategy equilibrium. 599 out of 750 combinations (or 79.9% of cases) indicated a single, symmetric, pure-strategy equilibrium.

141 of the remainder cases or 18.8% of all cases indicated multiple equilibria: a multitude of pure strategy equilibrium policies for the game. Due to symmetry, if (a, b), $a \neq b$ is an equilibrium policy, (b, a) is also an equilibrium policy on these equilibrium policy sets. We believe those equilibriums are observed since applied policies are restricted to be invariant of the state.

Final 10 combinations, or 1.3% of all cases indicated a multitude of equilibrium policies for the game resulting from the best response mapping algorithm, no pure strategy equilibrium. There are more than couple of (a, b) - (b, a), $a \neq b$ elements in these equilibrium policy sets.

The relative benefit of pooling is highly right-skewed: ranged between 0 to 16% (there are cases with no improvement of pooling, since the system in equilibrium does not involve inventory pooling), and has a mean of 3.14%. It will be clearly seen after subsequent sections that competition is dampening the relative benefit of pooling.

4.2.2 EFFECT OF TRANSSHIPMENT COST ON PERFORMANCE MEASURES

Relative benefit decreases with transshipment cost, a mere market friction. Increased transshipment cost acts as a barrier for requesting transshipments for both dealers, therefore decreasing the advent of transshipments, as well as directly decreasing the profit gains by pooling. The effect is more intensely felt with increasing traffic intensity, where the transshipment flows get higher due to higher customer demand and higher demand variance (since demand is a Poisson process). Both the benefit of pooling and the decrease of it is minimal at the lowest traffic intensity and maximal at the highest. Figure 4.2.1 follows.

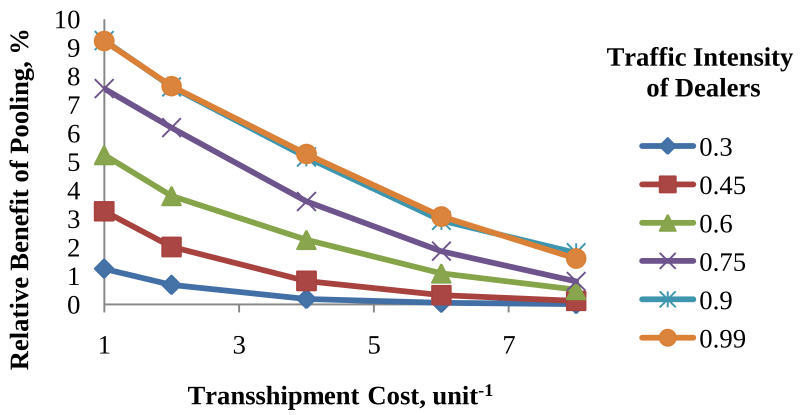

Figure 4.2.1 Average relative benefit of pooling versus transshipment cost averaged over all of the remaining parameters

This behavior (along with the behavior with traffic intensity) repeats itself also for the asymmetric competitive pooling case (Figure 4.3.8) for the benefit of pooling for D1 with low arrivals to the D2 (the trend with D1 traffic intensity is reversed when there are high arrivals to the D2) and the cooperative pooling case (Figure 4.4.1).

Notice that what is dealt is a symmetric, bilateral phenomenon that has a cleared market at equilibrium. That fact is best embodied in the merely counterintuitive clause, “if it is not requested, it cannot be granted”. When transshipment is requested, the granting interval (i.e. S-K) also gets narrower: The dealer cannot profitably unilaterally widen up his sending interval because it will cause decrease in his revenues due to unnecessarily increased profit margin losses (e.g. receiving the commission instead of the full payment) and cause imbalance between supply and demand. This is at best characterized with the Figure 4.2.2, average S, K, Z, T levels under traffic intensity of 0.9. Both S-K and Z-T margins decrease with increased transshipment costs.

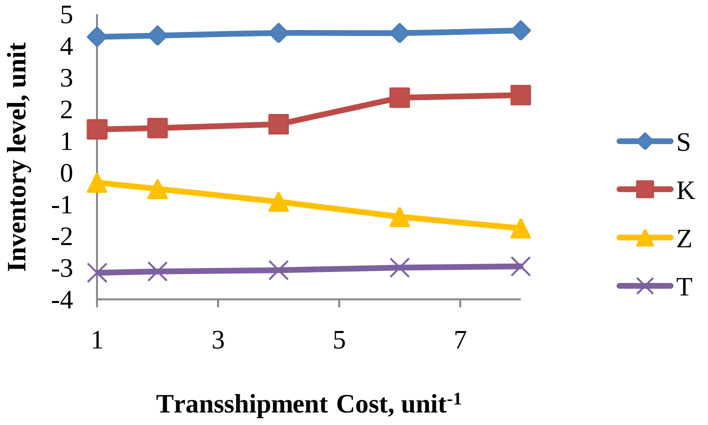

Figure 4.2.2 Average S, K, Z, T levels under 0.9 traffic intensity vs. transshipment cost over all relevant experimental runs

When trends of performance measures with transshipment cost are investigated, averaging over all of the other factors, the following are observed: Inventory slightly increases, profit slightly decreases, flow between dealers decays, benefit of pooling decays, backorders slightly increases and DoS increases with transshipment cost.

From this point on, it is felt necessary that numerical results should be more thoroughly treated to infer more through parameter interactions.

For a low inventory holding cost (c_h: 0.1 per unit per unit time), low commission (r: 1 per unit) environment, the S,K,Z,T trend looks like in that of Figure 4.2.3 for a low traffic intensity and high traffic intensity combination.

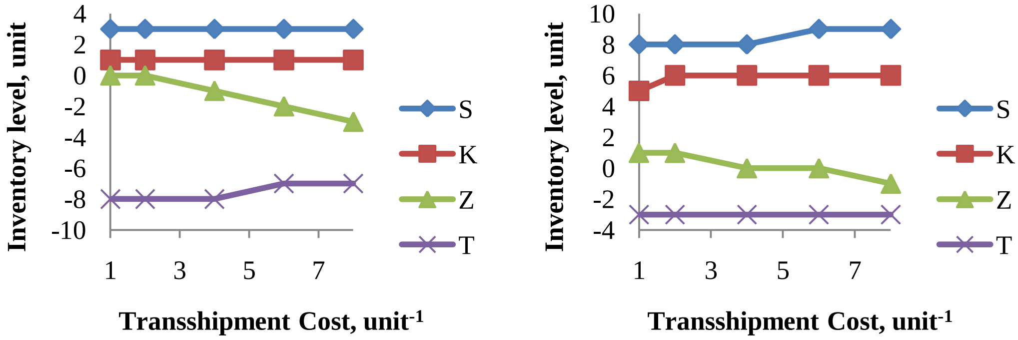

Figure 4.2.3 S, K, Z, T levels under 0.45 (left) and 0.9 (right) traffic intensity vs. transshipment cost over experimental runs having inventory holding cost of 0.1 per unit per unit time and commission 1 per unit

The increase in transportation costs decreases the propensity towards requesting from the other party, but does not change its inclination towards offering. Hence, the Z-T (requesting) region narrows down and the S-K (accepting) region enlarges. Since the system is expected to spend more time around S under low traffic intensities and around T under high traffic intensities, T is more responsive at low traffic intensities and S is at high. Increasing transshipment costs signify increased base stock levels because of increased variability of demand due to lack of pooling. Hence, lack of pooling results in less effective usage of capacity.

It is to be noted that high holding cost combined with small commission tends to eliminate pooling unless there is enough traffic intensity (e.g. ~0.9) to justify pooling. At high holding costs and small commissions, dealers tend to approach a zero base stock, sole-source policy: small commissions do not provide enough incentive to exchange backorders. Policy trends on 0.9 traffic intensity look like that of Figure 4.2.3 (left); the low traffic pattern for a small holding cost and small commission.

High holding costs combined with high commissions create a case in between: trends on both high and low traffic intensities look like that of Figure 4.2.3 (left); the low traffic pattern for a small holding cost and small commission. High holding costs hinder inventory holding and the available inventory to share but high commissions tend to yield incentives toward sharing under this specific context.

4.2.3 EFFECT OF COMMISSION ON PERFORMANCE MEASURES

Each dealer's profit is observed to be maximized at middle values of the commission payment. Middle values are more preferable than extremes. Refer to Figure 4.2.4.

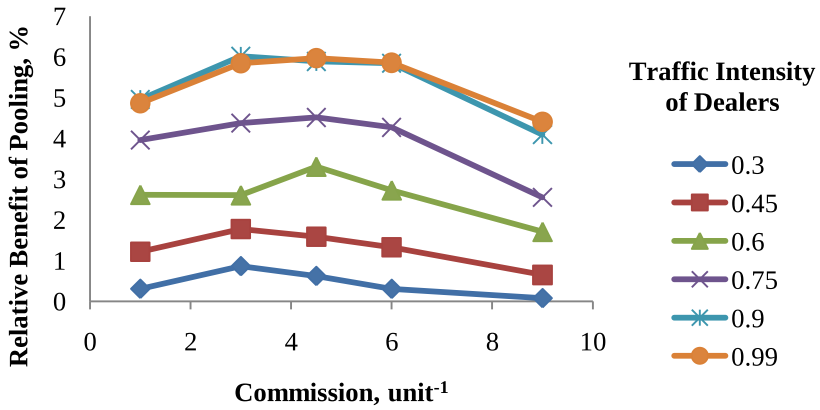

Figure 4.2.4 Average benefit of pooling vs. commission under different traffic intensities on D1

When the other dealer would not respond to the best response of the dealer (that can well occur if a tiny player is exploiting a very large player or there are many numerous close players) profit and the transshipment benefit is maximized at either extremes of the commission since either the revenues from shipping or the revenues from receiving is higher: a finding from Satır (2010). Hence, the optimizing dealer is expected to choose among one of these rates and best response is reached when the commission is at one of the extremes.

Therefore, under presence of gaming, opposite of what would be observed when the best response policy of a dealer is under consideration.

There is a much more interesting trend with the traffic intensity on the asymmetric competitive pooling case (for those who can foresee, it looks like a twisted rainbow twisting around middle levels of commission).

When the dealers are symmetric, the outflow equals inflow for each dealer and hence commission does not affect the dealer profits directly. However, it may affect the policy and indirectly change the profits with change in policy. Suppose for a given commission, the optimal policy is symmetric, e.g. X-X: As commission increases, transshipment cost gains prominence. The transshipment cost acts as a transaction cost type market imperfection and both parties should optimize their lateral transshipment benefits against the transaction cost. Hence, dealers are observed to request less and give more, i.e. increase the S-K margin and decrease the Z-T margin, as can be seen on Figure 4.2.5.

As commission decreases, dealers are observed to decrease the S-K margin and increase the Z-T margin. Since the tendency to give or offer less will be reflected on the overall flows between dealers which should equalize in symmetric equilibrium, middle levels of commission encourage transshipment, which provides a fair incentive for transshipment in the case of symmetric dealers. A trend similar to that of Figure 4.2.4 is observed for the DoS, inventory & queue decrease vis-à-vis non-pooling situation.

Albeit the changes in policy parameters are characterized more generally as a decrease in K and Z levels (therefore widening of the S-K [accepting] gap and narrowing of Z-T [requesting] gap) with commission, a low holding cost/high traffic intensity situation may further trigger an increase in S and a high holding cost/low traffic intensity situation may further trigger an increase in T level.

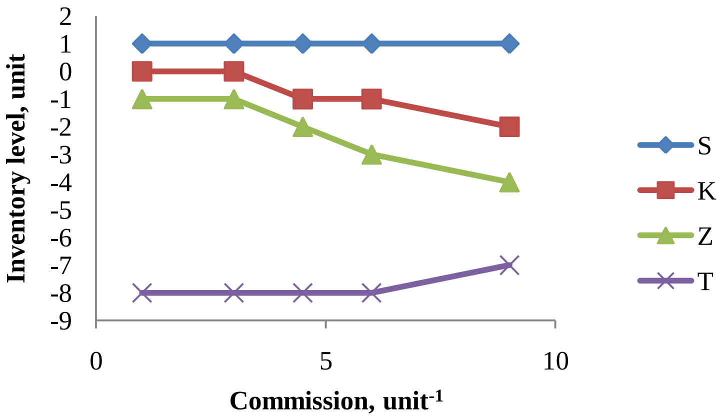

Figure 4.2.5 Policy parameters versus commission for traffic intensity 0.45, inventory holding cost 1 per unit per unit time and transshipment cost 2 per unit

As expected, a similar trend like Figure 4.2.4 is observed with profit and flow between dealers. Inventory, Queue and DoS rate: on average, decreases and then increases.

The behavior on extreme commissions is then inquired: identifying which parameter combinations of inventory holding, transshipment costs and traffic intensities enable pooling or cause it to merely diminish.

We have a theoretical result under restrictive conditions for the no commission, $r=0$ extreme: If the best response $S_1(j)$ in equilibrium is a constant function and $K_1(j)$ in equilibrium is a constant function, definition of S and K, along with the symmetry property in equilibrium establishes the fact that S=K for both dealers when the commission is zero. The proof is in Appendix B and it holds regardless of the holding, transshipment costs and traffic intensities.

However, a similar conjecture for T & Z levels cannot be said for the commission equal to retail price, $r=R$ end. Here, the presence of the rejection penalty might enable parties to pool even if the immediate margin (also reduced by the transshipment cost) of a transshipped good is negative. Numerical evidence shows that in almost all cases, Z=T as the commission gets sufficiently large.

Flow between dealers is an indicator of pooling behavior. The relative benefit of pooling is observed to have trends in line with the flows for the oncoming cases in this subsection.

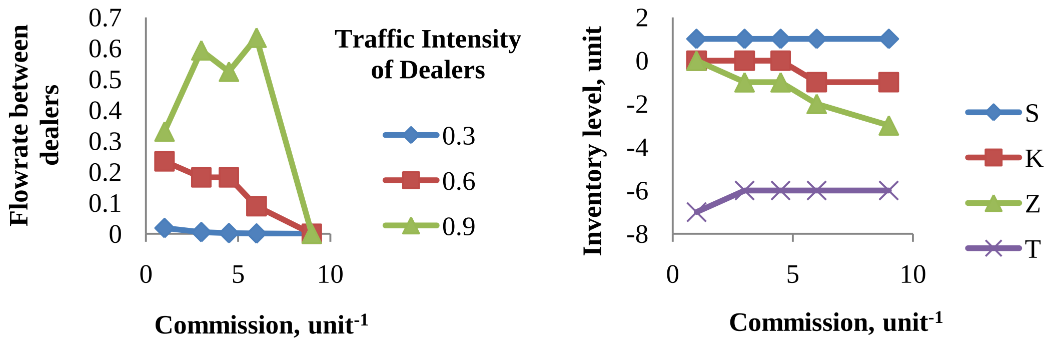

Figure 4.2.6 Flow rate between dealers versus commission for various traffic intensities (left) and policy parameters at traffic intensity 0.6 (right) under inventory holding cost of 0.5 per unit per unit time and transshipment cost of 8 per unit

In Figure 4.2.6 it is observed that under low traffic intensities there is a steady decrease with the commission payment. When traffic intensity is low, so is the need for sharing parts. As commission payment increases, the request for parts decrease and a decrease in pooling is observed. The flow rate between dealers is especially low when traffic intensity is low and transshipment cost is high.

Note the relative robustness of S and K parameters at the lower extreme of commission, Z and T parameters at the higher end. The Z-T (requesting) margin is tweaked first, which is more effective (since low commissions provide incentives over requesting), then the S-K (granting) margin.

At high traffic intensities, tendency to give away describes the pooling behavior in symmetric environment. Low inventory holding cost at very low commissions provides a reluctance to give away inventory due to high anticipated customer flow.

As commission payment increases, dealers are willing to share resources, give away inventory. However, under very high commission, the willingness starts to decrease due to diminishing margins of transshipment requests: firms are reluctant to request transshipments; therefore equilibrium behavior has lesser flow-rate. Thus, the flow rate is expected to increase, and then decrease with commission.

It is also observable from Figure 4.2.6 (right) that policy parameters become more irresponsive to commission at the lower traffic intensity extreme (0.3). This can be attributed to the fact that the transshipment flows are already small.

When the inventory holding cost is at a high extreme, transshipment flows with commission are now steady decreasing for high traffic intensities and peaking around middle levels for middle traffic intensities, as in Figure 4.2.7. In a system with high inventory holding costs and high traffic intensities, inventory/service would be given away even at low commissions. A mere trivial fact is that high holding costs and high traffic intensities with low commissions do foster the tendency of parties to request for transshipments. Therefore, high traffic intensity environments prefer low commissions. Lower traffic intensity environments now prefer higher commissions since lower commissions do not enable enough incentives to receive costly inventory when the anticipated customer flow is small. Due to high transshipment costs, no significant transshipment is observable if traffic intensity gets too low.

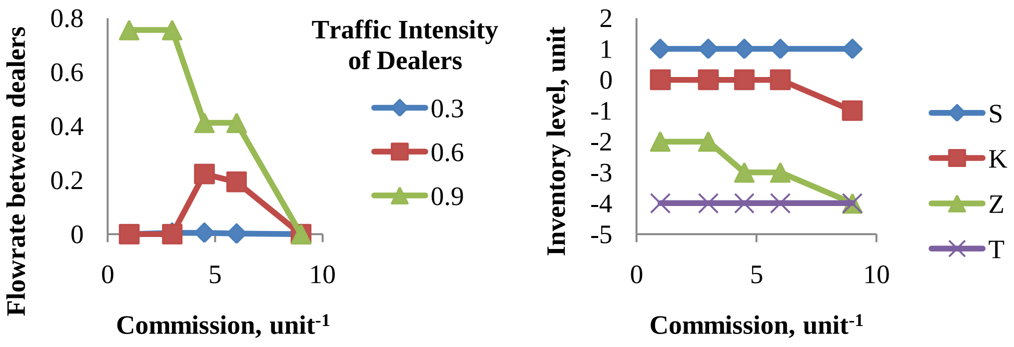

Figure 4.2.7 Flow rate between dealers versus commission Figure for various traffic intensities (left) and policy parameters at traffic intensity 0.9 (right) under inventory holding cost of 4 per unit per unit time and transshipment cost of 8 per unit

Note the relative robustness of S and K parameters at the lower extreme of commission, Z and T parameters at the higher end. The Z-T (requesting) margin is tweaked first, which is more effective (since low commissions provide incentives over requesting), then the S-K (granting) margin.

Trends as in Figure 4.2.6 (for low inventory holding cost) and Figure 4.2.7 (for high inventory holding cost) is less clearly observed under a low transshipment cost, but still in its place, as can be observed in Figure 4.2.8 (right) and Figure 4.2.8 (left) respectively. Note that the transshipment flows are larger.

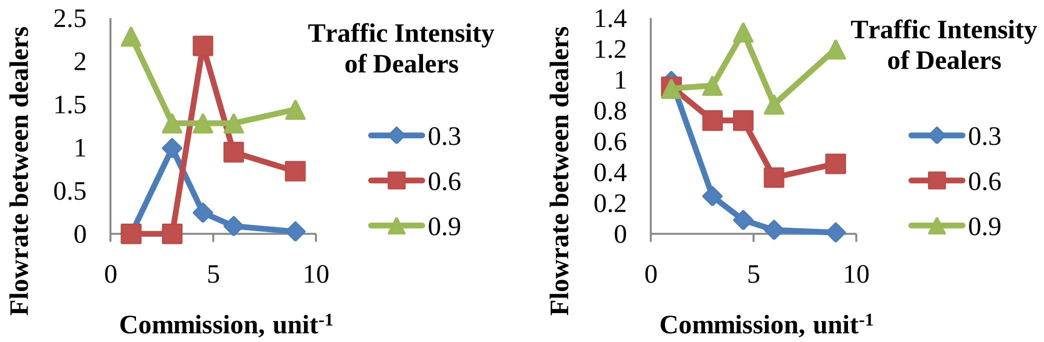

Figure 4.2.8 Flow rate between dealers versus commission for various traffic intensities and transshipment cost of 1 per unit under (left) inventory holding cost of 4 per unit per unit time (right) inventory holding cost of 0.5 per unit per unit time

When the effect of commission payment on control parameters is analyzed, it is observed that different transshipment cost values affect the behavior. Note that, as the transshipment cost is increased, the willingness to give would not be affected. However, the willingness to request –high with lower commissions simply due to high margins- makes a counter-balance; transshipments naturally decrease with increasing commission due to reduced margins. Therefore, the Z-T gap more abruptly narrows down as commission payment increases, compared to the trend in Figure 4.2.6 (right) –high transshipment costs-. In other words, an increase in transportation cost is more likely to exhibit an increased responsiveness on the Z-T gap, as can be observed in Figure 4.2.9.

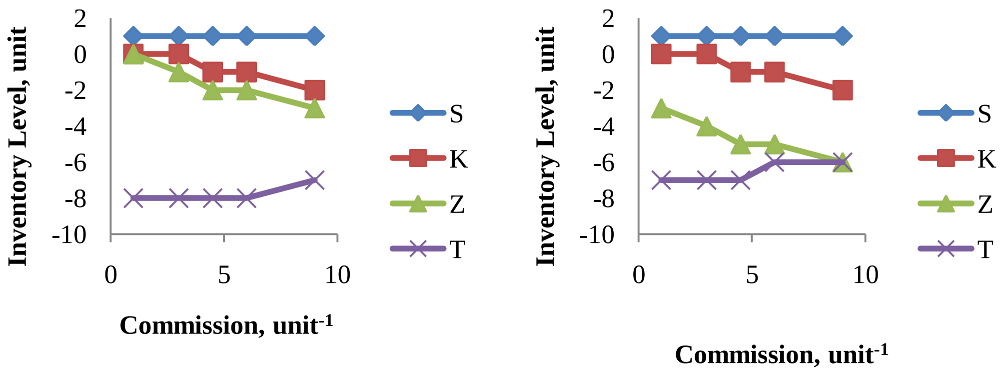

Figure 4.2.9 Policy parameters vs. commission under (left) transshipment cost of 1 per unit and (right) transshipment cost of 8 per unit, inventory holding cost of 1 per unit per unit time and traffic intensity 0.45

4.2.4 EFFECT OF INVENTORY HOLDING COST ON PERFORMANCE MEASURES

As the transshipment cost increases, equilibrium tends to no pooling. When all other factors are averaged out, benefit of pooling, which is more or less aligned with expected flows between the dealers, show an interesting characteristic with holding cost. Figure 4.2.10 follows.

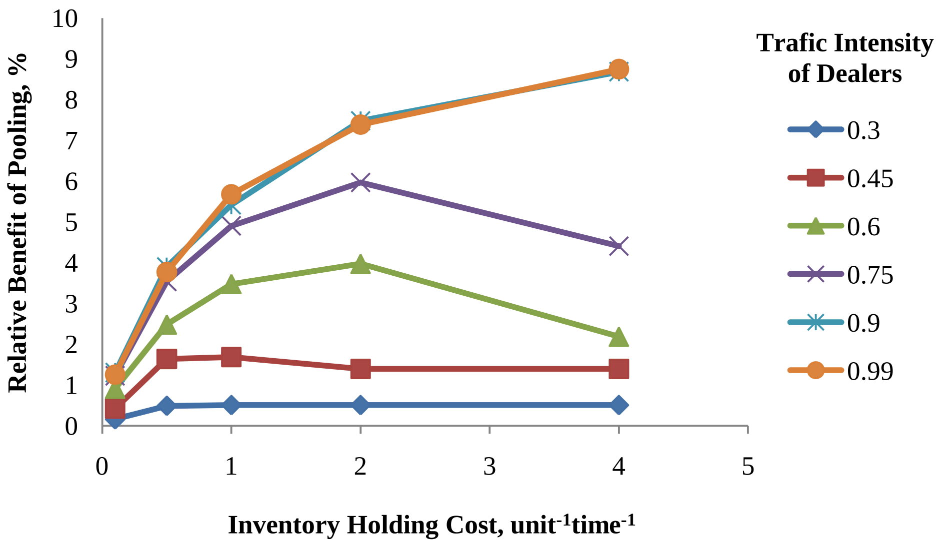

Figure 4.2.10 Average relative benefit of pooling versus inventory holding cost

Benefit of pooling increases with increasing holding cost at low holding costs due to more balanced inventory. Benefit of pooling gets to a peak and then the benefit slowly diminishes with increasing holding cost (diminishing effect slower with increased traffic intensity due to increased benefits of backorder sharing), since increased holding cost means firms are holding less inventory, and hence the effective capability of exchanging on hand inventory becomes limited.

With increasing holding cost, gains of sharing on hand inventory decreases, whereas gains by sharing backorders/service gains prominence.

The benefit eventually reaches to a limit where only queues are pooled (since there are no more base stocks), situation invariant of the holding cost. It is no longer wise to allocate more physical inventory or backorders just for the sake of pooling.

When trends are further investigated under parameter combinations, there will be combinations where pooling is forfeited with holding cost. However, we can safely say that under almost all cases concerned, flow rate between dealers follows Figure 4.2.10.

As expected, inventory decays to zero, profit decays asymptotically to a limit, average number of customers waiting in the system and rejected/DoS customers increases asymptotically to a limit with increased holding cost.

It will be seen on consequent sections that this behavior (along with the behavior with traffic intensity) repeats itself also for the cooperative pooling case (see Figure 4.4.2)

To gain a more comprehensive insight, the effect of inventory holding cost on the benefit of pooling is analyzed with respect to changes in commission and transshipment cost.

The trend of policy parameters with inventory holding cost under low transshipment costs are generally characterized (except extremely low commissions, which disables transshipments) as in Figure 4.2.11: a rapid decrease in S level, followed by a slight decrease in K, Z and T levels to facilitate working under lower inventory. The S, hence K & Z levels are more responsive to the holding cost when the traffic intensity is higher.

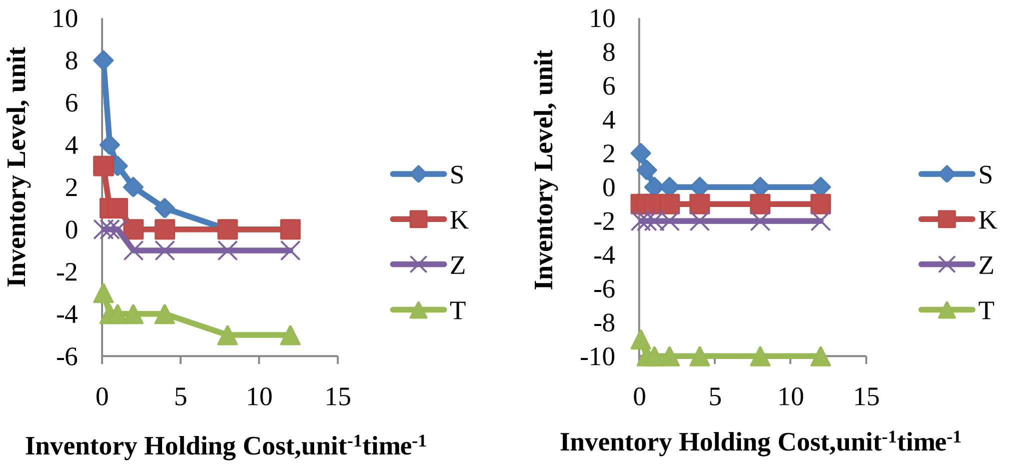

Figure 4.2.11 Policy parameters vs. inventory holding cost under commission of 4.5 per unit and transshipment cost of 1 per unit with (left) traffic intensity 0.9 and (right) with traffic intensity 0.3

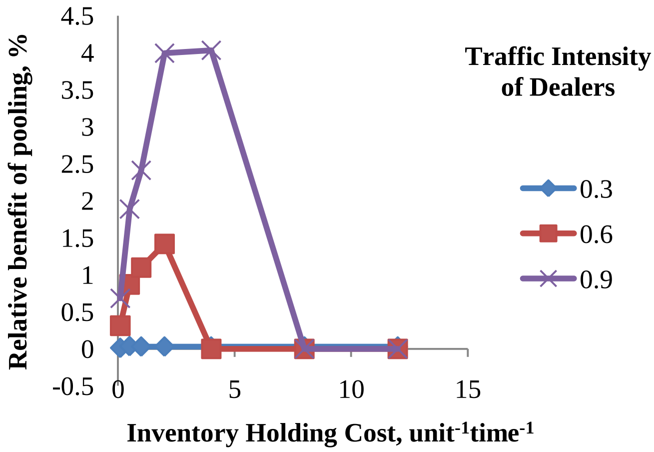

Figure 4.2.12 Relative benefit of pooling versus holding cost Figure for low commission of 3 per unit and high transshipment cost of 8 per unit

A cost environment as in Figure 4.2.12, low commission is a disincentive for dealers to accept transshipment requests and high transshipment cost is a disincentive to request transshipments. Note that the benefits from transshipment at this commission and transshipment cost are significantly lower than the averages over commission and transshipment cost (i.e. Figure 4.2.10) in this case.

However, transshipments could still be done at low enough inventory holding cost levels to benefit from decreased variability in demand, especially when traffic intensity is high.

Benefit of pooling becomes zero at high enough inventory costs under low traffic intensity (already low in average) almost regardless of commission if transshipment cost is high, this can be clearly seen through Figures 4.2.12, 4.2.13 and 4.2.14.

Another case where no pooling is observed with high inventory holding costs are those with all-low commission, low transshipment cost and low traffic intensities (see: Figure 4.2.15). A very rapid decay of S-K gap, then a rapid decrease in Z, and an increase in T is observed to induce lower queue levels in an environment where there is no transshipment. Since the commission is low, willingness to give is also low. This case is also observed to occur at the other extreme of commission. At medium commissions however, there is still inventory sharing at high inventory holding costs (see: Figure 4.2.14).

Benefit of pooling becomes zero at high traffic intensities and high enough inventory costs if the transshipment cost is high or commission is low (even if transshipment cost is low), refer to Figure 4.2.11. In this case, it is clearly unwise to accept or conduct transshipment requests; the opportunity cost of keeping the inventory for dealer’s own is high. Demand for transshipments is also very low. High enough commissions still justify backorder/service sharing due to high customer demand and hence the benefit of pooling keeps being positive even at very high inventory costs. Refer to Figures 4.2.11, 4.2.12, 4.2.13 as well as 4.2.15.

A final case is under high commission and high transshipment cost regardless of traffic intensity. There is clearly no incentive for placing a transshipment request and hence there is no benefit of pooling in the equilibrium; although the retailers would like to accept a transshipment offer because of the high commission.

When averaged over all other factors, it was observed that the benefit of pooling was increasing, then decreasing and staying constant for low traffic intensities and decreasingly increasing for large traffic intensities: refer to Figure 4.2.10. However, this tendency can be altered with the interaction of high traffic intensity, high transshipment cost and high inventory holding costs. Observed in Figure 4.2.13 as well as Figure 4.2.14, benefit of pooling is observed to be increasing, then decreasing then staying constant for large traffic intensities and decreasingly increasing for small traffic intensities.

A credible explanation can be that a mere "opportunity cost of inventory" is in its place: when the inventory holding cost is high and inventory is demanded due to high customer flow, reserving/expecting inventory for/from transshipment loses credibility due to the total transportation costs to be paid to the environment, especially intensified when the commission scheme is also not favorable either for receiving transshipments or accepting those. Note the trends in Figure 4.2.14 are milder than that in Figure 4.2.13 and the y-axis scales are considerably larger.

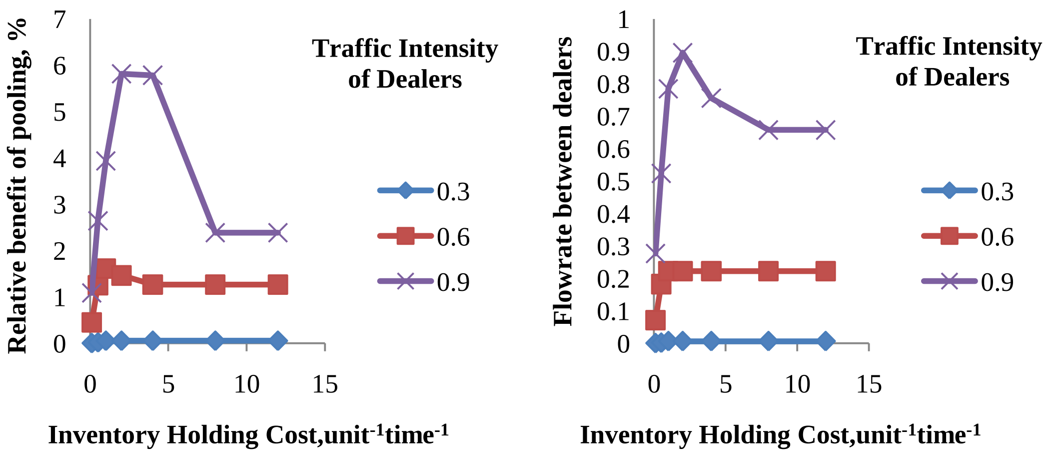

Figure 4.2.13 Relative benefit of pooling (left) and discounted average transshipment flows between dealers (right) with respect to holding cost at transshipment cost of 6 per unit and commission of 6 per unit

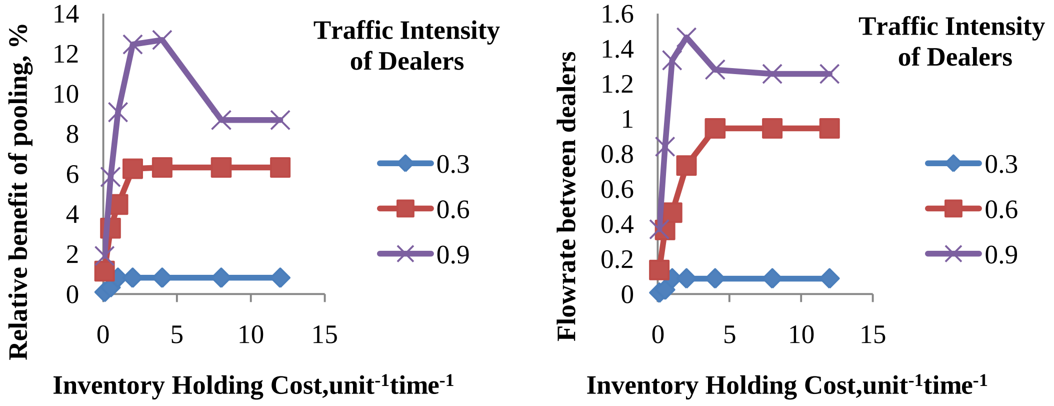

Figure 4.2.14 Relative benefit of pooling (left) and discounted average transshipment flows between dealers (right) with respect to holding cost at transshipment cost of 2 per unit and commission of 6 per unit

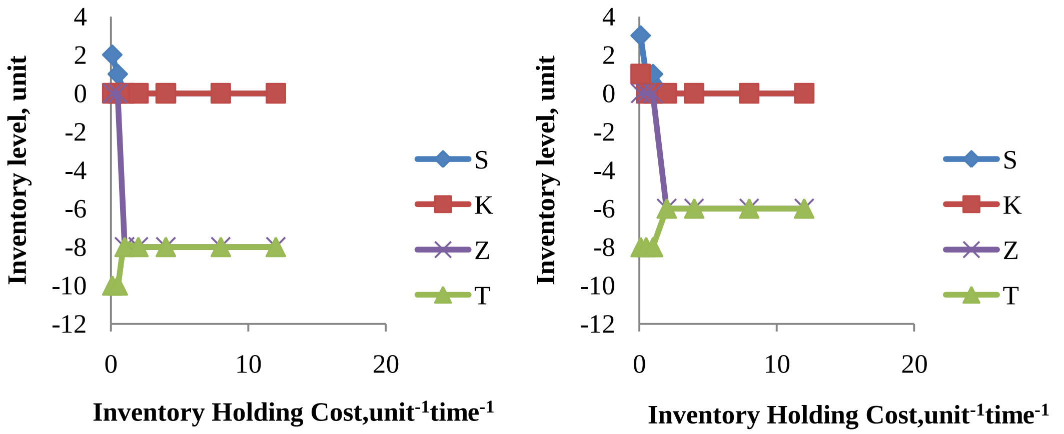

Figure 4.2.15 S, K, Z, T levels vs. inventory holding cost over experimental runs having traffic intensity (left) 0.3 and (right) traffic intensity 0.45, transshipment cost 1 per unit, and commission 1 per unit

Another set of interesting results stems from the analysis of the service level: one would expect that the service level (number of waiting customers in the system) improves under pooling. However, maximization of profit not necessarily always improve the service level, decreases the amount of backorders and inventory in the system. Actually what happens may sometimes be increased inventory and decreased backorder levels if holding cost is low and vice versa if holding costs are high with still a strictly positive benefit of pooling. Observe Figures 4.2.16, 4.2.17 and 4.2.18.

However, all of the cases considered yet have significant decreases in the DoS rate; i.e. an increase in service availability/acceptance.

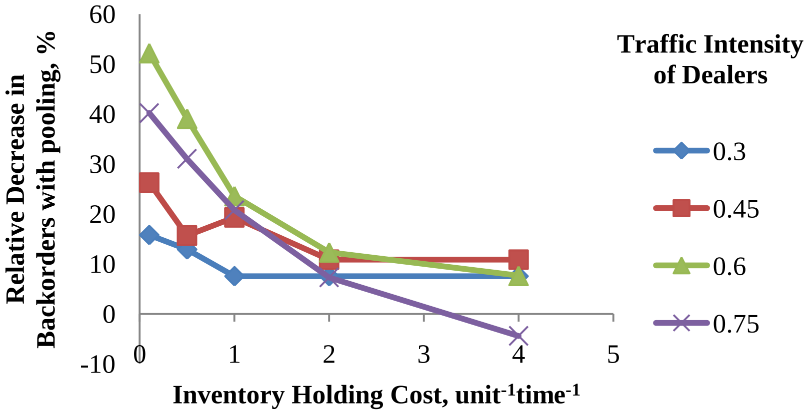

Figure 4.2.16 Relative decrease in service level vs. inventory holding cost at commission 4.5 per unit and transshipment cost 2 per unit

As can be seen in Figure 4.2.16, although in many cases pooling decreases backorders, under high holding cost values pooling may result in an increase in backorders.

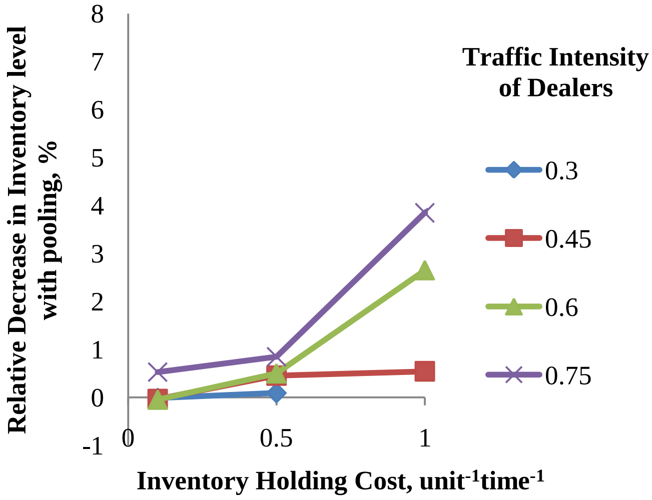

Figure 4.2.17 Relative decrease in inventory level vs. inventory holding cost at commission 4.5 per unit and transshipment cost 2 per unit

Note that the lines are not complete. This is because there is no base stock held without pooling at some cases and hence the figure is undefined. As can be seen, there are figures slightly below zero at low inventory cost levels, indicating increase in inventory levels under pooling.

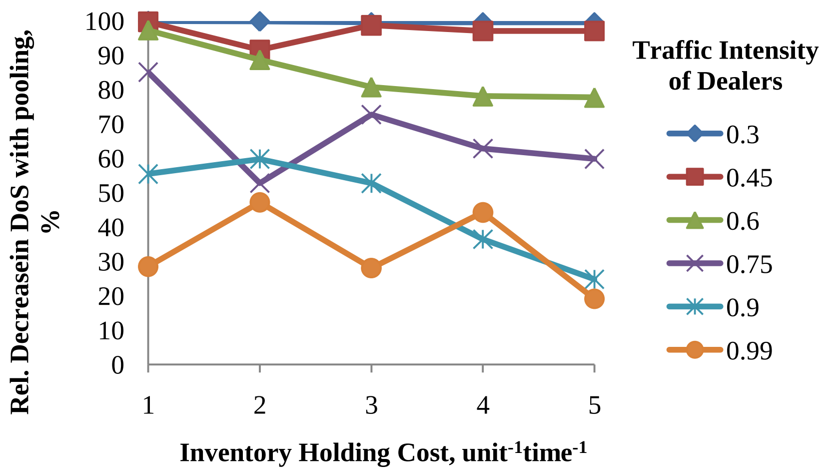

Figure 4.2.18 Relative benefit of pooling vs. DoS rate at commission of 4.5 per unit and transshipment cost of 2 per unit

There is a drastic recovery from rejected customers and hence increased service capability.

4.2.5 SUMMARY OF RESULTS SO FAR

The benefit of pooling in the symmetric case varies between 0%-16%, where higher benefits (≥10%) are obtained under high traffic intensities (≥0.6), low transshipment cost (≤2) and medium inventory holding costs (1,2 and 4 per unit per unit time). It is shown that other performance measures such as expected discounted number of waiting customers or the expected discounted inventory levels might be higher with pooling depending on the weight of the profit function to backorders and holding costs. Numbers of waiting customers can be also higher with higher traffic intensities and higher inventory holding costs. However, numerical evidence shows that the service levels, i.e. number of customers which are rejected service are always lower.

The section will conclude by discussing an observation on the effect of traffic intensity on the pooling benefit with low inventory holding cost and low transshipment cost. It is credible to state high traffic intensities as a disincentive to give away inventory as well as service for transshipment requests, therefore a net decrease of benefit of pooling is observed with increasing traffic intensities on very low commission levels even if it is profitable to ask for transshipments and there is increasing demand for it. An increase of pooling benefits can be observed with high commissions. Even if the tendency to give away inventory is diminishing with traffic intensity, high enough commission keeps the door open for transshipment as the demand for transshipment requests also increase even if it is not profitable to do so. Figure 4.2.19 follows.

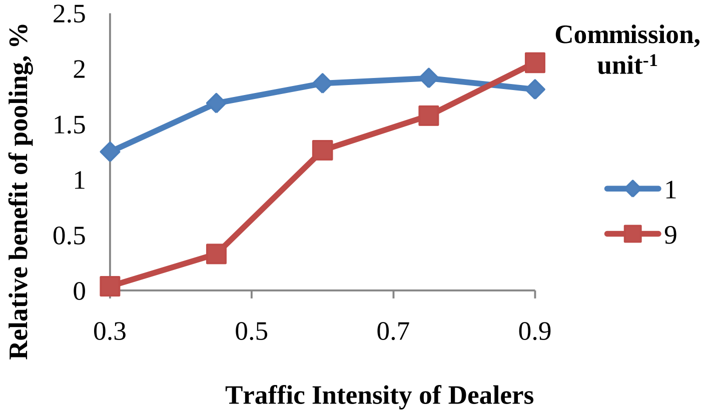

Figure 4.2.19 Benefit of pooling versus traffic intensity under different commission schemes and traffic intensities and with transshipment cost 1 per unit and inventory holding cost 0.1 per unit per unit time

4.3 ASYMMETRIC COMPETITIVE POOLING

The main aspect of asymmetric competitive pooling is the fact that more can be gained through pooling. The average benefit of pooling is higher: (8.66% for D1, 5.11% for D2). The range is much wider: (0%-557% for D1, 0%-101% for D2). The benefit of pooling for D1 is less than 4% in 56% of the cases (3519/6250), between 4% and 20% in 34% of the cases (2128/6250) , 20%-75% in 8% of the cases (499/6250) and extremely large for the remainder 2% (104/6250).

Although some revenue opportunities are absorbed by competition, it will be seen in this section that pooling in asymmetric cases benefits both rational players to be better off by exchanging their best capabilities. The benefits are especially high since there is now a significant net flow of goods. For example, the dealer with low customer arrivals would tend to send parts to receive customers from the dealer with high customer arrivals or the dealer with high inventory costs tends to send parts to receive customers from the dealer with low inventory costs under similar traffic intensities, increasing the benefit of pooling under favorable commissions depending on the anticipated net flow of transshipment. A detailed analysis now follows for the benefit of pooling having identified those drivers.

As mentioned above, benefits of pooling are impacted by the difference in traffic intensities. That is, when D2 faces customer arrivals at rate 0.9, it is most beneficial for D1 to pool when it faces a traffic intensity of 0.3, the smallest rate tested. It is just the vice versa when D2 faces 0.3 rate arrivals; most benefits to D1 are at D1 facing 0.9 rate arrivals. Of cases which have more than 20% benefit of pooling for D1, 27% (134/603) are where D1 has a traffic intensity at the lower end (0.3) and D2 has at higher end (0.9), 12% (74/603) is for the case reversed, D1 has 0.9 and D2 has 0.3 traffic intensity: whereas only 5% (34/603) are 0.3-0.3 and 7% (43/603) are 0.9-0.9.

When the lower extreme end of the benefit distribution (≤2%) is analyzed, 73% (477/650) of all 0.3-0.3 cases and 24% of all (157/650) 0.9-0.9 cases are observed to be at this region. It can be safely said that low-low and high-high traffic intensity combinations, in other words symmetry, tend to lower benefits of pooling in general.

At the higher extreme of the benefit distribution however, (Benefit of pooling to D1 >75%) the 0.3-0.3 traffic intensity combination has 17% (18/104) weight, outperforming the 0.9-0.3 and 0.3-0.9 cases which have 13% (13/104) each.

Thus, difference in holding and backordering costs do have a significant impact on the benefit of pooling. A large 58% (582/1000) of the cases where the inventory holding cost and backordering cost of D1 is strictly smaller than that of D2 (holding cost 0.1 or 0.5 per unit per unit time and backordering cost 0.2 or 1 per unit per unit time) has lower than 2% benefits for D1. Only 33 of those cases have larger than

20% benefits, none is having larger than 75% benefits. On the other hand, on cases where the costs of D1 are strictly larger (holding cost 2 or 4 and backordering cost 4 per unit or 8 per unit): 171/1000, or only a small 17% of cases has lower than 2% benefits, 421 of the cases has larger than 20% benefit. All of the 104 <75% benefit cases are the cases where costs of D1 are strictly larger.

Looking at the very extreme, the 556% benefit case for D1 arises at the largest holding and backordering cost, smallest traffic intensities (i.e. 0.3-0.3) and transshipment cost (1 per unit). In this case, it can be anticipated that there is a net flow from D2 to D1; the net stock and backorders of D1 are stored in D2. D1 would be expected to benefit from this situation under low commission and to have relatively deteriorated gains under high commission, which is the case: D1 has 556% benefits under commission of 1 per unit and 146% under commission of 9 per unit. D2 would be expected to benefit from this situation under high commission and deteriorated gains under low commission: 16% under commission of 9 per unit and 2% under commission of 1 per unit.

Figure 4.3.1 below shows the effect of commission payment on the benefit of pooling to D2 under D2 traffic intensity of 0.9. D1 has lower traffic intensities; D2 tends to request and is willing to receive from D1. Under low commission, the benefit obtained by D2 is the highest when D1 has the lowest traffic intensity.

As commission increases, D2 will be less likely to place a transshipment request since it does not get good deals while receiving and hence it tends more towards sending. D2 will start to get more requests and be more willing to send units or D1 will be more willing to share backorders when the traffic intensity to D1 increases. Hence, under high commission, the benefit second dealer will have is the highest when D1 has the highest traffic intensity.

Among all traffic intensities, the benefit of pooling to D2 is observed to be unimodal in commission payment. D1 will be more willing to share when there is enough incentive for it to send, i.e. under high commissions. However, when the commission is too high, D2, facing high arrivals, will not be tending to receive, and hence the benefit degrades from this maximum.

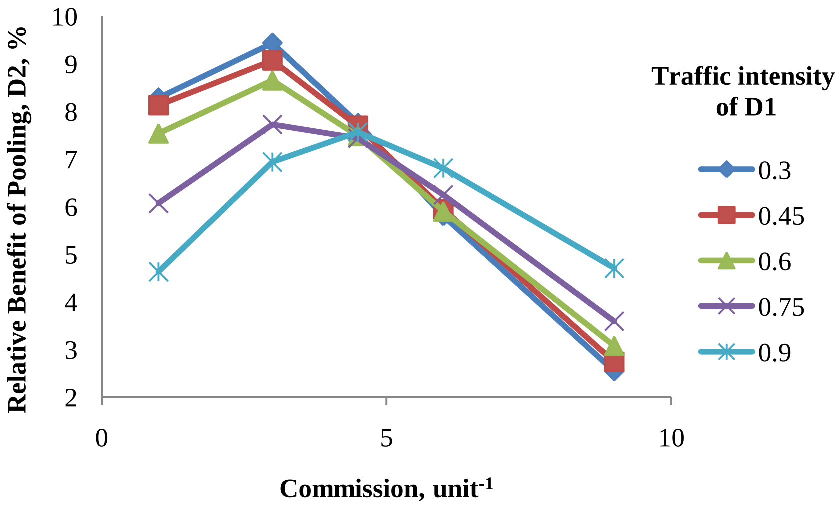

Figure 4.3.1 Average benefit of pooling for D2 (at traffic intensity 0.9) versus lateral transshipment revenue.

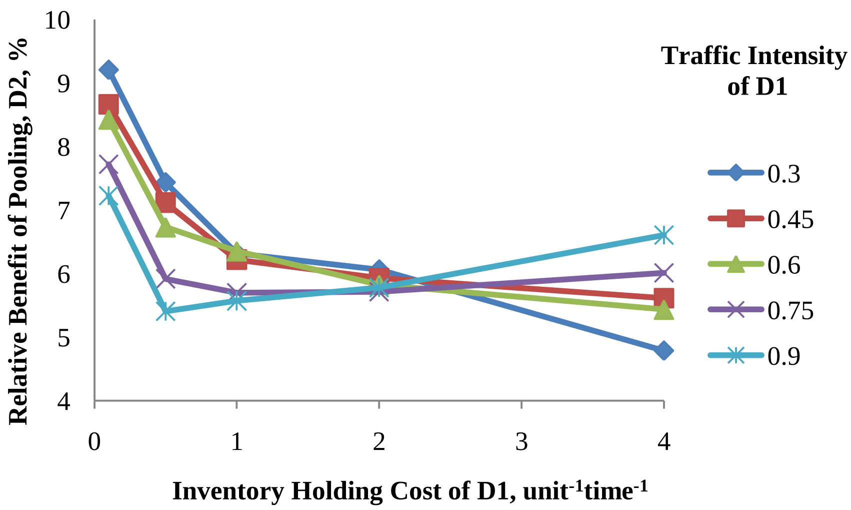

Figure 4.3.2 Average benefit of pooling for D2 (at traffic intensity 0.9) versus inventory holding cost of D1.

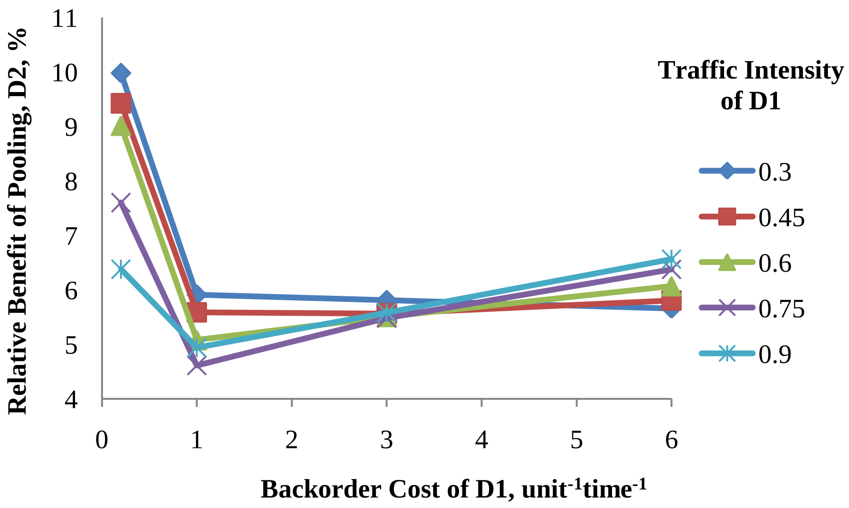

Figure 4.3.3 Benefit of pooling for D2 (at traffic intensity 0.9) versus backordering cost.

Notice Figures 4.3.2 and 4.3.3. Under low inventory holding costs and backordering costs of D1, D2 is more likely to receive from D1 as it has lower holding costs and hence can offer inventory. The chances of receiving inventory increase with decreased traffic intensity of D1. Under high holding/backordering costs of D1 however, pooling is most beneficial under high traffic intensity of D2. High holding costs mean that D2 is not so likely to receive as D2 now faces a lower inventory holding/backordering cost. Hence, D2 enjoys from transshipment when the traffic intensity to D1 increases.

Among all traffic intensities, the graph, Figure 4.3.2, retains its convex-looking shape and attains a minimum when the unit holding cost of D1 and D2 are close to each other. D1 will be more willing to share when there is enough incentive for him to send, i.e. a sufficiently high inventory holding cost difference. When the costs in D1 is too high, D2 will be tending to send and D1 tending to receive, and hence the benefit increases from this minimum.

General trends of performance measures versus transshipment cost are similar to that in the symmetric competitive pooling case, except the fact that it has now variations due to asymmetry of traffic intensity and inventory holding/backordering costs between dealers.

General trends of performance measures versus inventory backordering cost are also similar to that in the symmetric competitive pooling case. As expected, queue decays to zero, Profit decays asymptotically to a limit, inventory and DoS customer rate increases asymptotically to a limit with increased backordering cost.

Some interesting results arise when the asymmetry of the inventory holding cost and backordering cost are analyzed.

Figure 4.3.4 shows the effect of holding cost difference on the flows from D1 to D2 and D2 to D1 when the customer traffic intensities and backordering costs at D1 are equal. Customer traffic intensities are parameters. Commission is 4.5 per transshipment.

Figure 4.3.5 shows the effect of backorder cost difference on the flows from D1 to D2 and D2 to D1 when the customer traffic intensities and inventory holding costs are equal. Customer traffic intensities are parameters. Commission is again 4.5 per transshipment.

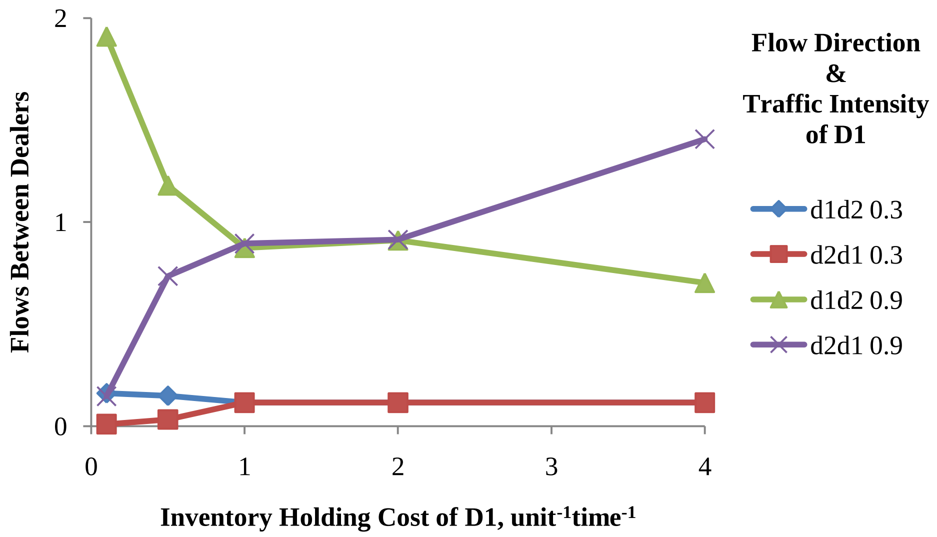

Figure 4.3.4 Flows between dealers versus holding cost of D1 when backordering cost is 2 per unit per unit time

Notice that flows between dealers are equal to each other when inventory holding costs of the dealers are equal, both at 1 per unit per unit time.

Transshipment flow difference between dealers is higher at high traffic intensities. Cheaper cost environment facing D2 feeds D1 with parts: D1 is urged to keep lesser inventory and correspondingly requests more. This difference is low at low traffic intensities: risk of backordering is not significant at low traffic intensities.

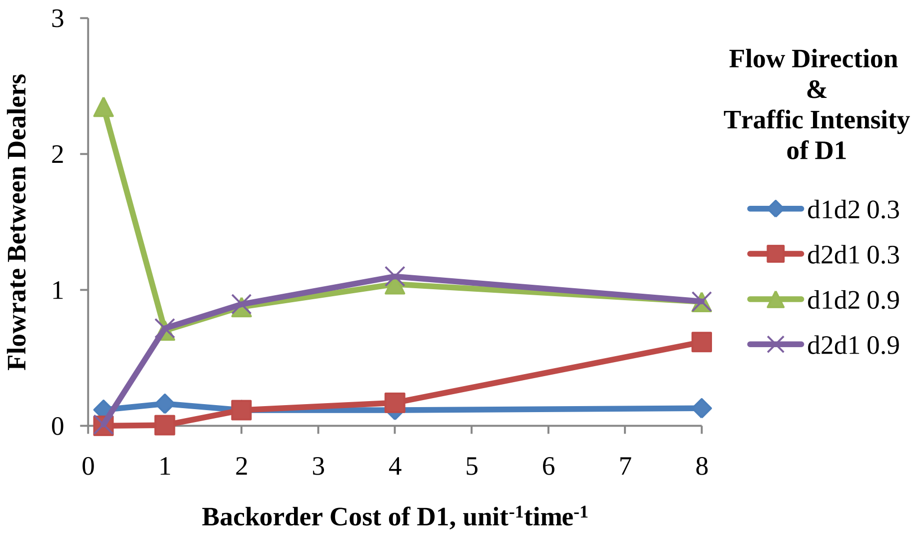

Figure 4.3.5 Flows between dealers versus backordering cost of D1 when holding cost is 1 per unit per unit time

Likewise, notice that flows between dealers are equal to each other when backordering costs of the dealers are equal, both at 2 per unit per unit time. There is a higher net flow of demand from D1 to D2 at low customer traffic intensities, D1 needs to share more as the risk of backordering becomes more prominent and having an additional unit of stock is still costly since demand is low. At high traffic intensities, D1 keeps more inventories and does not increase its requests.

Next, the benefit of pooling for D1 is analyzed. Figure 4.3.6 depicts the trends of benefit of pooling of D1 with respect to changes in commission payment. When the holding cost and backordering cost are high vis-à-vis D2, the benefit of pooling for D1 is anticipated to decrease with the commission. On these cases, D2 is “reserving” stock for D1 especially when the traffic intensity to the second dealer is low; requesting is more beneficial at lower commissions. On the other hand, when these costs are low at D1, then D1 is “reserving” stock and service for D2. In this case, especially when the traffic intensity to the second dealer is high, high commission values are preferred by D1: Benefit increases as commission increases, giving away is more beneficial at higher commissions.

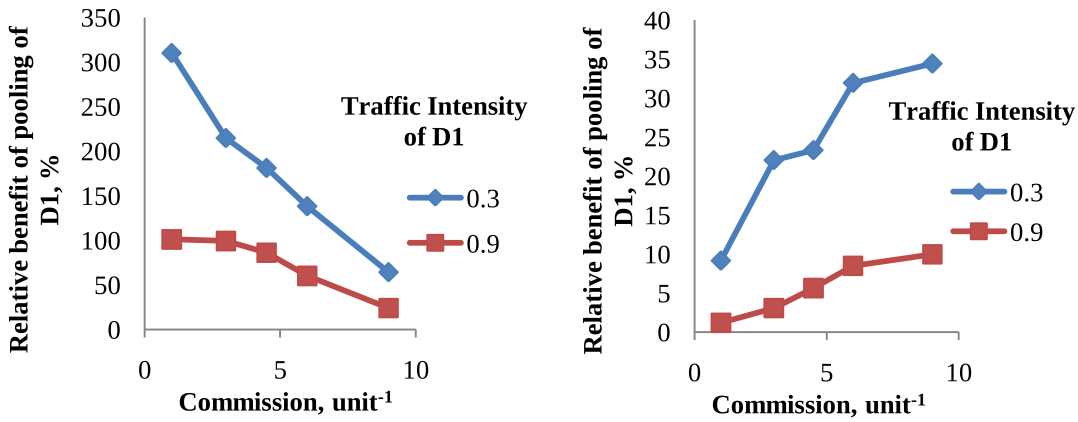

Figure 4.3.6 Benefit of pooling for D1 versus commission under (left) holding cost of 4 per unit per unit time and backordering cost of 8 per unit for D1 and traffic intensity of 0.3 to D2 (right) holding cost of 0.1 per unit per unit time and backordering cost of 0.2 per unit per unit time for D1 and traffic intensity of 0.9 to D2. Transshipment cost is held at 1 per unit.

The effect of traffic intensity at D1 on the benefit of pooling for D1 and flow rates when D2 has high and low traffic intensity will be discussed.

A low traffic intensity to D1 (0.3), along with a high traffic intensity at D2 (0.9) coupled with high commission (9 per unit) results in high benefit of pooling for D1: on 45/125, or 36% of such cases, benefit of pooling is higher than 20% Furthermore, high traffic intensity to D1 (0.9), along with low traffic intensity to D2 (0.3) coupled with low commission (1 per unit) triggers the benefit of pooling: for 19 out of 125, or 15% of such experiments, benefit of pooling is above 20%. Note that only 10% of all experiments lie above 20% benefit.

Figure 4.3.7 (left) shows the behavior of benefit of pooling for D1 with respect to traffic intensity at D1 when traffic intensity is 0.9 at D2. Under low traffic intensities at D1, transshipment requests will be accepted much more in frequency by D1, value of holding additional inventory at D2 (serving its own customer vis-à-vis providing transshipment) is low due to high customer demand there, as well as availability at D1. The flow-gap diminishes with the increased traffic intensity at D1, as the number of transshipment requests placed by D1 increases (inflow for D1, d2d1 flow) and satisfied transshipment requests by D1 (outflow for D1, d1d2 flow) decreases with traffic intensity on D1. The result is a significant decrease in outflow (d1d2 flow) and decrease in the benefit of pooling for the first dealer.

When the traffic intensity to D2 is low (see: Figure 4.3.7 [right]) and that in D1 is high, transshipment requests will be sent much more in frequency by D1, value of holding additional inventory at D1 (serving its own customer vis-à-vis providing transshipment) is high due to high customer demand and already available inventory at D2. The flow-gap diminishes with the decreasing traffic intensity at D1, as the number of transshipment requests placed by D1 decreases (inflow for D1, d2d1 flow). The result is a significant decrease in inflow (d2d1 flow) and corresponding decrease in the benefit of pooling for the first dealer. Hence, benefit of pooling decreases with decreasing traffic intensity at D1.

The increase in relative benefit of pooling is felt more abrupt because the outflow from D1 is insignificant anyway.

There are more flows in the 0.9-0.9 case than the 0.3-0.3 case, as can be anticipated through the arguments placed under the benchmark –symmetric- case analysis.

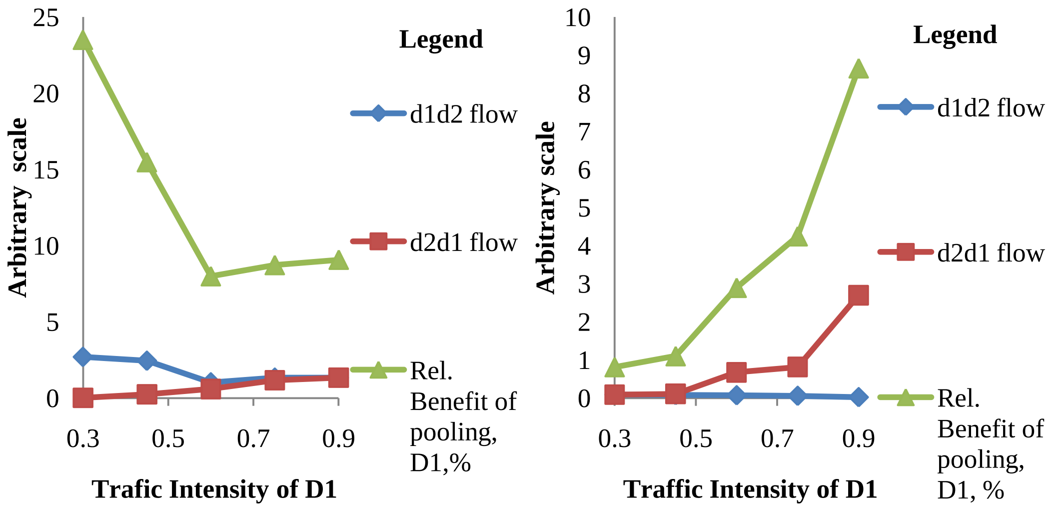

Figure 4.3.7 Benefit of pooling, inflow and outflow of D1 versus traffic intensity at D1 under (left) traffic intensity of 0.9 at D2 (right) traffic intensity of 0.3 at D2. Transshipment cost is 2 per unit, inventory holding costs and backordering costs are equal, the commission is at 6 per unit.

4.4 SYMMETRIC CENTRALIZED POOLING

Benefit of pooling is higher in the case of centralized pooling, (compared to competitive pooling: 4.53% vs. 2.54%). Range also increases slightly: (0-18.1% vs. 0-16.0% in competition). It can be roughly said that decentralization/competition, on average, dampens benefit of pooling about to its half, about 45% to that without gaming between dealers.

Effect of competition on benefit of pooling/Benefit of centralization average is 1.94%, ranges up to 13.8%. This figure can also be deduced and hence verified from the symmetric centralized benefit of pooling and symmetric competitive benefit of pooling: Average benefit of centralization is 1.94%. Average benefit of pooling in symmetric competitive pooling is 2.54%. Effects added up, symmetric centralized benefit of pooling mean comes to be 4.53%, exactly the observed figure.

The trends in the centralized case look similar to that of symmetric competitive pooling, but are more robust to increases in transshipment cost and the holding cost. To re-characterize these trends, Figures 4.4.1 and 4.4.2 follow. Yet still, relevant comments on these Figures can be found under Figures 4.2.1 and 4.2.3 respectively.

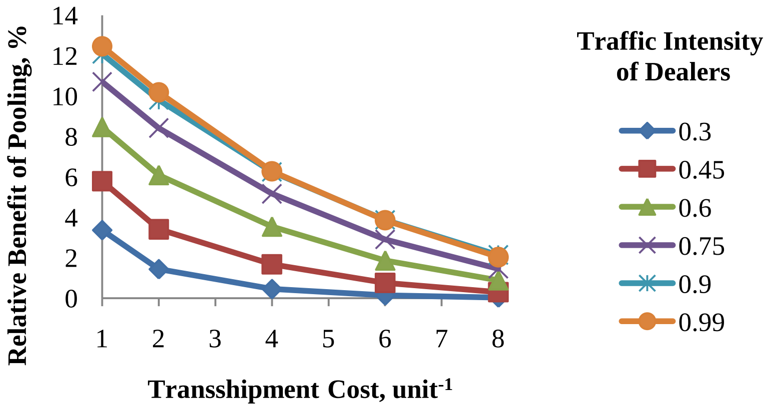

Figure 4.4.1 Average relative benefit of pooling versus transshipment cost

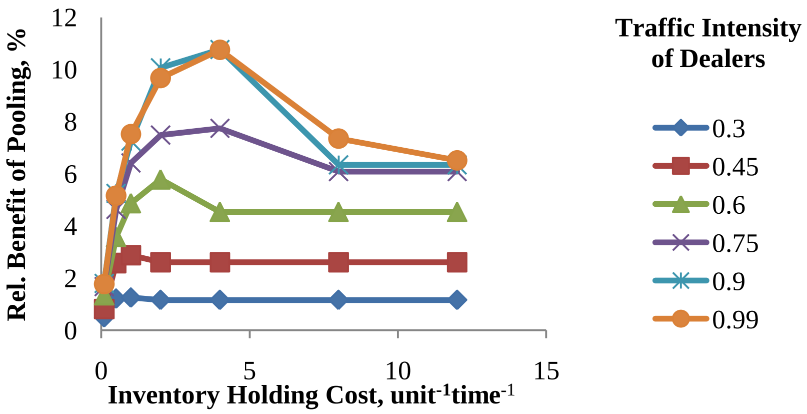

Figure 4.4.2 Average relative benefit of pooling versus inventory holding cost

Effect of competition on relative benefit is more pronounced, as expected, at low transshipment cost, high inventory holding cost and large traffic intensities, where commissions trigger conflicts of interest and wash-off attainable profit tuples without competition.

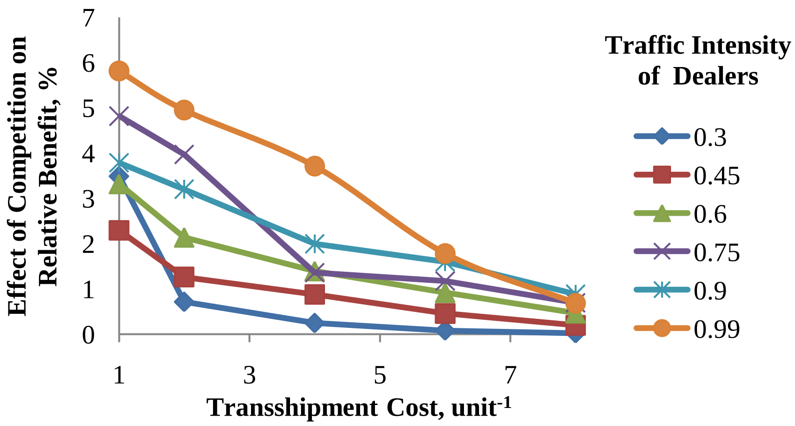

Figure 4.4.3 Average effect of competition on relative benefit versus transshipment cost

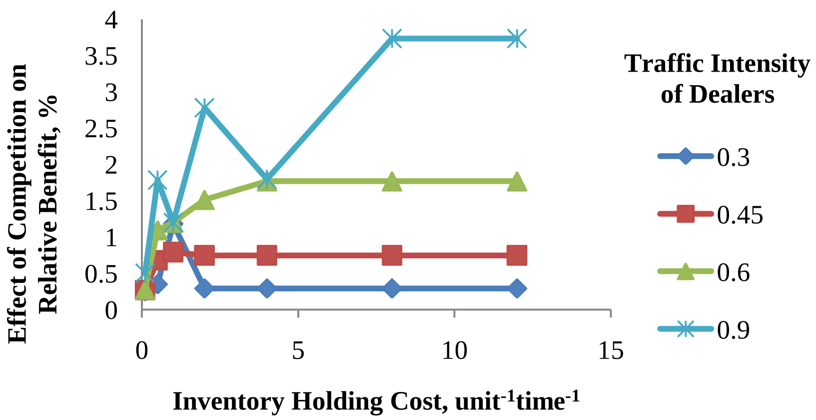

Figure 4.4.4 Average effect of competition on relative benefit versus inventory holding cost

The policies under cooperative pooling generally shows a larger interval of pooling and a narrower interval for the non-interacting (i.e. K-Z) interval (Compare with Figure 4.2.2).

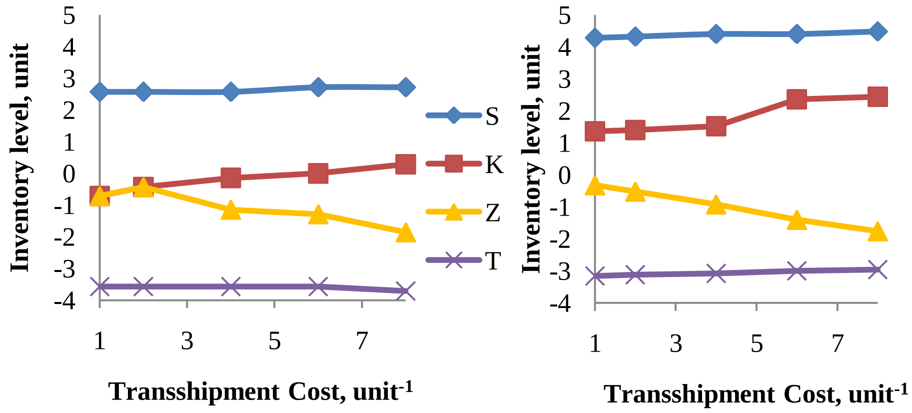

Figure 4.4.5 (left). Average S, K, Z, T levels under 0.9 traffic intensity vs. transshipment cost

Compare with Figure 4.2.2, provided right of Figure 4.4.5 to the same scale.
As a consequence, competition is observed in all cases to decrease the transshipment flows between dealers. Centralized authority enables transshipments which would be otherwise not be done under decentralization: not requested or not accepted due to the fact that doing so is anticipated to decrease profits of the deciding dealer. Therefore, K and Z levels are anticipated to be more robust to cost parameters.

Competition is also observed to generally increase the DoS; customer rejections are usually lesser under centralized authority.

There is a mere 15.4% (116/750) of cases where the DoS is decreased by competition, more than half (59/116) of the cases having high traffic intensities (≥0.9), and about three quarters (86/116) having low inventory holding costs (≤1). In these cases, either inventory is raised to account for the decreased flow and benefit from low inventory holding cost.

As can be seen on Figure 4.4.6 and Figure 4.4.7, increase in DoS and decrease in flows is more drastic when the commission is at either extreme and traffic intensity is low. Increase in DoS with competition is slightly more if commission is at the higher (rather than lower) extreme, where requesting becomes less profitable.

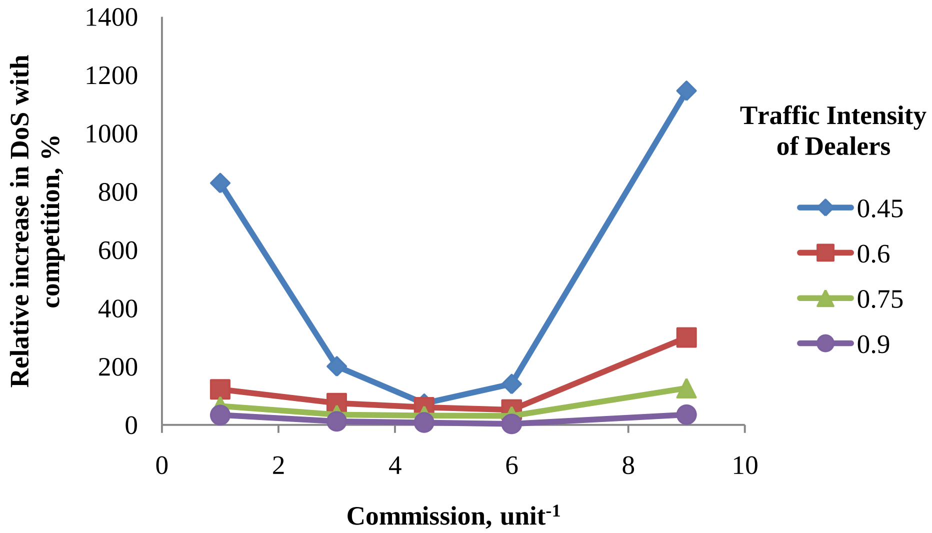

Figure 4.4.6 Average increase in DoS with competition

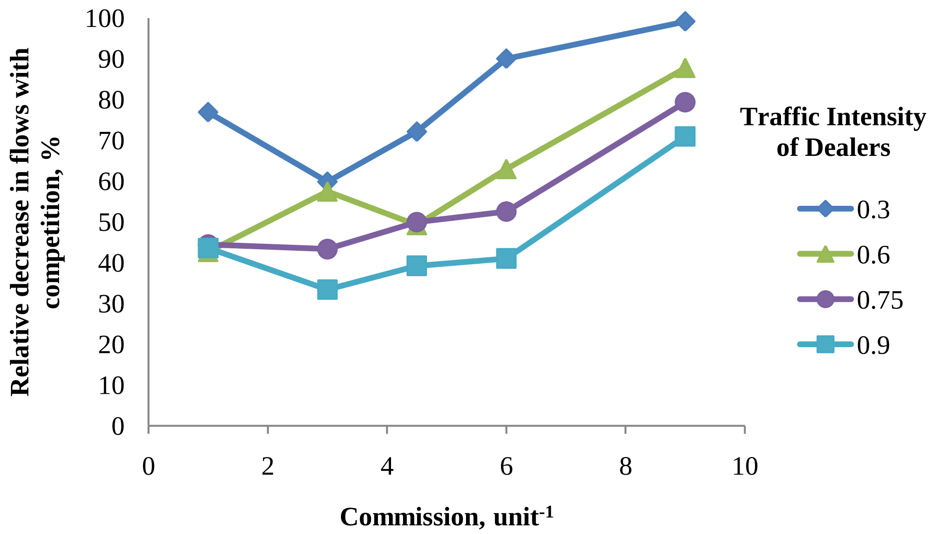

Figure 4.4.7 Average decrease in flows with competition

Another set of results stems from the impact of competition on the service level: one would expect that the service level (number of waiting customers in the system) improves with competition. Actually what happens may sometimes be increased inventory and decreased backorder levels if holding cost is low and vice versa (i.e., decreased inventory and increased backorders) if holding costs are high with still a negative impact of decentralization on profitability as well as flows between dealers.

In 8.1% (61/750) cases, a decrease in inventory with decentralization is observed. Almost half of the cases (31/61) of them are having high inventory holding cost (>1). In all of these 61 cases, service levels are increased with decentralization. Likewise, 5.0% or (38/750) cases are observed to have a decrease in service level with decentralization. Almost two thirds (24/38) of them are having low inventory holding cost (≤1). In all of these 38 cases, inventory levels are increased with decentralization. Observe Figures 4.4.8 and 4.4.9. In Figure 4.4.8, lines are incomplete due to cases with zero inventory without competition.

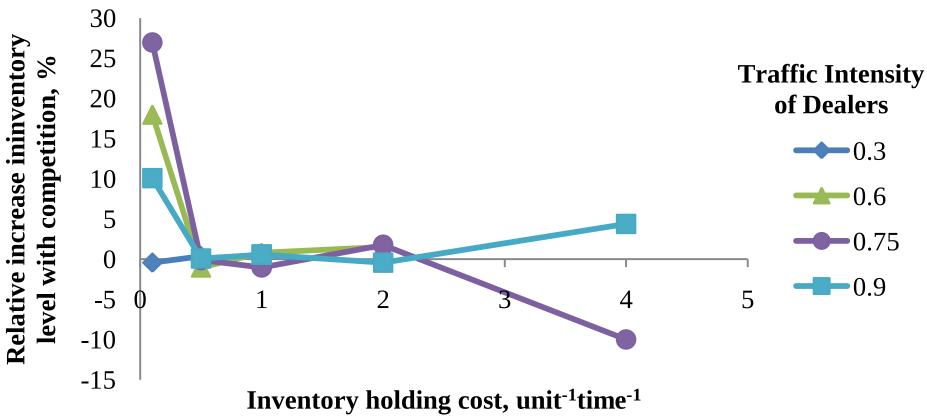

Figure 4.4.8 Relative increase in inventory vs. inventory holding cost at commission 3 per unit and transshipment cost 2 per unit

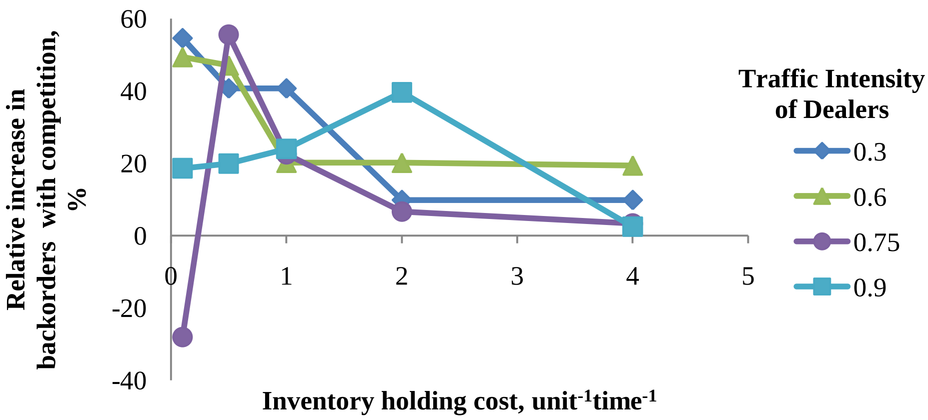

Figure 4.4.9 Relative decrease in service level vs. inventory holding cost at commission 3 per unit and transshipment cost 2 per unit

CHAPTER 5

CONCLUSIONS AND FUTURE WORK

This research attempts to investigate the possible effects of competition and collaboration on an inventory pooling system. A large set of experimental cost/traffic factor combinations are run in a two-dealer, single-echelon, capacitated production and stochastic (Markov) customer/production arrival processes. Both a competitive equilibrium and a system-wide profit maximizing solution (centralized case) are considered. Therefore, gains of centralization and pooling as well as service levels and customer rejection levels could be observed under different cost environments.

The benefit of pooling in the symmetric case, within the span of the experimental factor combinations regarded, varies between 0%-17% (average 2.54%), where higher benefits (≥10%) are obtained under high traffic intensities (≥0.6), low transshipment cost (≤2) and medium inventory holding costs (1,2 and 4 per unit per unit time).

It can be safely said that proper commission schemes that allocate the transshipment value to the players fairly (e.g. at medium levels of commissions if the cost structure is symmetric), high customer traffic intensities, and low transshipment costs are most suited environments for pooling. Benefit of pooling is observed to rise with inventory holding cost, but then decline, under some conditions totally wipe out, if it is too much.

Competition, on the average and within the span of the experimental factor combinations regarded, dampens about 45% of the benefits associated with pooling.

If the vendor system would be operated centrally, a 1.94% on the average would be the gain, ranging up to 13.8%.

Further, centralization is observed to increase the extent of pooling on all cases. Centralized authority enables transshipments which would be otherwise not be done under decentralization: not requested or not accepted due to the fact that doing so is anticipated to decrease profits of the deciding dealer. Therefore, K and Z levels are anticipated to be more robust to cost parameters.

Deterioration of the profits under competition is higher under higher unit inventory holding costs, lower transshipment costs, higher customer traffic intensities and the commission structure is distracting either the receiver or sender such that eventually -in competitive equilibrium-, both sides become reluctant to share.

Although some revenue opportunities are absorbed by competition, pooling with vendors facing different –asymmetric- cost/traffic environments benefit both rational players to be better off by exchanging their best capabilities. The average benefit of pooling is higher. Up to a mere 557% increase in profits by pooling is achieved for D1 (101% for D2). The benefits are especially high since there is a significant net flow of goods. For example, the dealer with low customer arrivals would tend to send parts to receive customers from the dealer with high customer arrivals or the dealer with high inventory costs tends to send parts to receive customers from the dealer with low inventory costs under similar traffic intensities, increasing the benefit of pooling under commissions depending on the anticipated net flow of transshipment.

One would expect that the presence of pooling should increase service levels along with profitability. This intuition also claims that inventory/service should decrease vis-à-vis a no pooling situation. Profit maximization does not necessarily meet all of these three trends simultaneously. Actually what it does may sometimes be to increase inventory and decrease backorders if holding cost is low and vice versa (i.e., decreased inventory and increased backorders) if holding costs are high and provide still a non-negative benefit of pooling. Note that benefit of pooling is always non-negative since the dealers always have the option to forfeit pooling even in a competitive environment.

A powerful heuristic method to quickly find the set of equilibrium policies in the competitive setting is outlined. So called a “policy iteration phase”, it drastically reduces the number of evaluations to find the equilibrium policy set. The main cause of further improvement by local search was observed to be the process of finding good levels of static policy levels from a dynamic best response. A base-stock approximation to S and T levels may prove useful. Another challenge is to find a better performance transformation method for the centralized system, the merely naïve solution procedure modification did not work as intended.

Since the effects of cost/revenue/demand are observed to be interrelated to each other with seemingly high degrees of interaction, use of Experimental Design or Response Surface Methodologies to better express the effect of parameters might prove useful, despite its discussed fallacies. One way or another, a statistically reinforced picture clearly highlighting all statistically significant main effects, as well as interactions can be depicted via these methods.

The value of information and common information on inventory levels and value of information and common information on cost and product/traffic intensity parameters are left untouched. Further, the outlying policies and performance trends might be different, these should also be investigated.

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APPENDIX A

PSEUDO-CODE AND MATLAB CODES FOR THE SYMMETRIC COMPETITIVE CASE SOLUTION ALGORITHM

The **MATLAB code** has a modular structure using the same workspace (hence the variable pool), having the following modules:

- **main.m** contains the experimental setting, includes the policy iteration stage, steepest ascent phase and the best response mapping algorithm. It calls all other modules.
- **evaluation.m** builds the one-step probability matrix P and computes the value function V and profit π through the equation system $C+\rho PV=V$ and $\pi=\delta V$
- **xeval.m** has the same structure with evaluation.m except that X, which is the vector measure proportional to the total discounted expected time fraction is computed via $X=\delta+\rho PX$.

The **MATLAB workspace** contains the following *input parameters*:

- *alpha*, discount factor
- br, the revenue (per unit)
- *beta*, uniformization constant (per unit time) = l1+l2+m1+m2
- *envseth*, contains the set of holding costs for experimentation (per unit per unit time)
- *envsetl*, contains the set of traffic intensities for experimentation (per unit time)
- *envsettr*, contains the set of transshipment costs for experimentation (per unit)

- *envsetsr*, contains the set of commissions for experimentation (per unit)
- *h*, the inventory holding cost (per unit per unit time)
- *iterlim* is an upper limit for the total iterations in the best response mapping algorithm
- *l*, the backordering cost (per unit per unit time)
- *l1*, traffic intensity to D1 (per unit time)
- *l2*, traffic intensity to D2 (per unit time)
- *m1*, production rate at dedicated line to D1 (per unit time)
- *m2*, production rate at dedicated line to D2 (per unit time)
- *py*, the negative of the penalty cost (per unit)
- *rho*, modified discounting factor = beta/(beta+alpha)
- *sr*, the commission (per unit)
- *tr*, the transshipment cost (per unit)

The **workspace** contains the following *indices*:

- *a,b,c* holds the actions deducted in each state (i, j) given S1,K1,Z1,T1,S2,K2,Z2,T2.
    - *a* is the production completion event decision. a=1 means "stop production" and a=2 means "new product request"
    - *b* is the receive transshipment request event decision. b=1 means "reject" and b=2 means "accept"
    - *c* is the customer arrival event decision. c=1 means "DoS", c=2 means "accept", c=3 means "send transshipment request"
- *cycleindex* holds the *iteration* number where the best response mapping loops, hence the set of equilibrium policies are found. If the set of equilibrium policies are singleton, i.e. cycleindex=iteration-1, a single pure-strategy equilibrium is output
- *expiter* counts the output line to be printed for the experiment log *expout*
- *counter* counts the experimental runs conducted so far
- *h_indx, l_indx,sr_indx,tr_indx* denotes the indices to be used in the current experiment

- *i,j* and *d,e (alias)* indices hold the inventory levels of each dealer
- *ion* index is an index for printed lines
- *iteration* counts the number of best response calculations done in each experimental setting
- *k and m (alias)* holds the single-dimensionalized version of *(i, j)* and *(d, e)* tuples
- *locsearchiter* counts the iterations to exit the steepest ascent algorithm
- *policyiter* counts the iterations to exit the policy iteration phase

The **workspace** contains the following *variables*:

- *best* holds the best objective function in policy iteration
- *bestimp* has the best improvement (i.e. steepest ascent) amount for local search
- *cumpro* is used for construction of *pro*. For each state, it holds the total probability to the neighboring states, hence the remainder from 1 becomes the own-stage probability
- *d1tod2, d2tod1* has the expected discounted infinite horizon flow amounts between dealers computed for all equilibrium policies
- *del*, initial probability distribution vector (always unit mass at S1,S2)
- *impliedbest* and *best* has the best objective function out of the policy iteration search
- *improvement* has the difference between the local search best solution and the output from policy iteration= best-impliedbest
- *inventory* has the expected discounted infinite horizon inventory measure for the *mode* player
- *lostsales* has the expected discounted infinite horizon number of rejections measure for the *mode* player
- *log* holds the best response mapping in each iteration. The column headings are as in the following order:
    - *1-S1best,2-K1best,3-Z1best,4-T1best,5-S2,6-K27-,Z2,8-T2,9-mode,10-objold*

- *mode* , having value of either 1 or 2, holds the player index of whose best response is currently being obtained
- *obj* is the profit of the *mode* player given S1,K1,Z1,T1,S2,K2,Z2,T2
- *objold* has the profit of the *mode* player of the previous iteration
- *NOS* contains the total number of states in the current evaluation
- *S2,K2,Z2,T2* denote the policy parameters of the other dealer
- *S1,K1,Z1,T1* denote the current policy parameter set to be evaluated
- *S1old,K1old,Z1old,T1old* denote the previous policy parameter set
- *S1local,K1local,Z1local,T1local* denote the initiation point for the local search (i.e. output from the policy iteration phase)
- *S1search,K1search,Z1search,T1search* denote the result from the local search, i.e. the local optimal best response solution
- *S1best, K1best,Z1best,T1best* denote the policy parameter combination with the highest value of objective function found so far
- *swap* is the variable alloted to swap players
- *pro* is the one step probability matrix in *k,m*-index form
- *prof* is the immediate profit matrix in *k*-index form
- *v* has the value function vector in *k*-index form
- *waitcust* has the expected discounted infinite horizon backorder measure for the *mode* player
- *x* has the vector measure proportional to the total discounted expected time fraction in k-index form

The workspace contains the following *flags*:

- *cycleflag* flags whether the best response mapping has looped
- *flag* flags whether there is an improving direction (flag=1) or not (flag=0)
- *out* denotes the status of the experiment, out=1 means that a single equilibrium policy is found, out=2 means that a multitude of equilibrium policies are found, out=3 means that the *iterlim* is exceeded.

- *redundant* flags whether the local search was redundant: it did not improve (redundant=1) or improve (redundant=0) the best response guess by the policy iteration phase

- *trunc* flags whether the policy iteration phase was truncated because a non-improving solution is found

The **workspace** has the following output:

- Each line of *expout* has the experiment summary. The columns are arranged as follows:
  - *1-br,2-sr,3-h,4-l,5-py,6-tr,7-l1,8-l2,9-m1,10-m2,11-policyiter,12-locsearchiter,13-redundant,14-trunc,15-S1,16-K1,17-Z1,18-T1,19-S2,20-K2,21-Z2,22-T2,23-obj for D1,24- obj for D2 (25-waitcust for D1 (26-waitcust for D2 (27-lostsales for D1 (28-lostsales for D2, 29-d1tod2, 30-d1tod2 where d1 is D2 and d2 is D1, 31-d2tod1, 32-d2tod1 where d1 is D2 and d2 is D1, 33-inventory for D1, 34-inventory for D2, 35-improvement*
  - Having blank entries in the columns 1-14 implies that the policy combination is one of the equilibrium policy tuple for the nearest above parameter combination.

A.1 PSEUDO-CODES

A.1.1 PSEUDO-CODE FOR MAIN.M

S0. Initialize the workspace

S1. Initialize all variables

S2. Grab *h,l,tr,sr* values of the current experiment via *h_indx*, *l_indx*,*sr_indx*,*tr_indx* values on *envseth*,*envsetl*,*envsettr*,*envsetsr* respectively.

S3. Compute *rho* and *beta*

-Policy Iteration Phase

S4. Initialize *pro,prof,v,x,del,S1old,K1old,Z1old,T1old,best*

S5. Call evaluation.m

S6. If there is an improvement, replace S1old,K1best,Z1best,T1best with S1,K1,Z1,T1. Else, trunc=1, go to S11.

S7. Update *policyiter* and *S1old,K1old,Z1old,T1old*

S8. Compute implied policy parameters and update S1,K1,Z1,T1 via indifference equations inferred at D2 inventory level S2 for S1, Z2 for K1, S2 for Z1 and K2 for T1

S9. Ensure that S1≥K1≥Z1≥T1

S10. If a fixed point is found, go to S11, else revert to S5.

S11. Record the solution out of implied policy iteration under *S1local,K1local,Z1local,T1local* and *impliedbest* by calling evaluation.m

-Steepest Ascent Phase

S12. Initialize S1old,K1old,Z1old,T1old, objold and flag=1

S13. Update *locsearchiter*. Evaluate the profit of D1 in one policy neighbor with S1≥K1≥Z1≥T1. Call evaluation.m. Calculate imp=obj-objold

S14. Update *S1best,K1best,Z1best,T1best* and *bestimp* if imp>bestimp. Else g oto S15.

S15. Go to S13 until all neighbors are exhausted.

S16. If there are no improving directions, go to S17 . Else, S1old=S1best,K1old=K1best, Z1old=Z1best, T1old=T1best and return to S13.

S17. Record the best response into *S1search*, *K1search*,*Z1search* and *T1search* and compute the *improvement* of local search to the objective function. Local search objective is stored at *objold*.

S18. Determine if the local search phase did not change the best response guess

S19. Update *iteration* and the best response log depending on *mode*

-Best response mapping

S20. Change *mode*, swap players

S21. Check if the best response *log* is looping. cycleflag=1 if such a loop is found.

S22.Return to S4 until either a set of equilibrium policies are found (i.e. cycleflag=1), or *iterlim* is exceeded by *iteration*

-Output

S23.Determine *out*. out=1 if iteration-cycleindex=1, out=2 if iteration-cycleindex>1 but iteration<iterlim, out=3 if iteration-iterlim=0.

S24. Swap *l1,l2,m1,m2* if *mode*=2

S25. If out=1 or out=2, write columns 1-14 and 35 of *expout*. Initialize *ion=cycleindex*. Else (i.e. out=3), proceed to S35.

S26. Take *S1,K1,Z1,T1,S2,K2,Z2,T2* from respective columns of the *ion* line of *log*

S27. Write columns 15-22 of *expout*

S28.Call xeval.m

S29. Calculate *waitcust, lostsales, d1tod2,d2tod1* and *inventory*

S30.Write *obj, waitcust, lostsales, d1tod2, d2tod1* and *inventory* to relevant columns of *expout* giving D1 measures

S31. Swap players, call xeval.m

S32. Calculate *waitcust, lostsales, d1tod2,d2tod1* and *inventory*

S33. Write *obj, waitcust, lostsales, d1tod2, d2tod1* and *inventory* to relevant columns of *expout* giving D2 measures

S34.Update ion=ion+1. Return to S25 until all equilibrium policy tuples S1,K1,Z1,T1,S2,K2,Z2,T2 are exhausted (i.e. ion=iteration-1)

-Terminal

S35.Update *counter, expiter, h_indx, l_indx,sr_indx,tr_indx.*

S36.Return to S1 until all experimental settings are exhausted.

A.1.2 PSEUDO-CODE FOR EVALUATION.M AND XEVAL.M

S1. Convert the state k onto i,j components

S2. Compute *del* value for the current state

S3. Determine actions *a,b,c* given *i,j*

S4. Determination of the immediate expected profits for the state (i,j) given actions

S5.Initialize (1-own transition probability), cumpro=0

S6. Convert the state m onto d,e components

S7. Compute transition probabilities, deduct from own transition probability

S8. Return to S6 until all states *m* are exhausted (i.e. m=NOS)

S9. Compute own transition probability

S10. Return to S1 until all states *k* are exhausted (i.e. k=NOS)

S11. Compute *v* and *obj* via the equation system prof+rho*pro*v=v and obj=del*v

S12. *(for xeval.m)* Compute *x* via the equation system x=del+rho*pro*x

A.2 MATLAB CODES

A.2.1 MATLAB CODE FOR MAIN.M

```
% S0. Initialize the workspace
envsetsr=[1 3 4.5 6 9];
envseth=[0.1 0.5 1 2 4];
envsettr=[1 2 4 6 8];
envsetl=[0.3 0.45 0.6 0.75 0.9 0.99];
expiter=1;
expout(1,1)=0;
clear expout;
counter=0;
for sr_indx=1:length(envsetsr)
    for h_indx=1:length(envseth)
        for tr_indx=1:length(envsettr)
            for l_indx=1:length(envsetl)
% S1. Initialize all variables
waitcust=0;
lostsales=0;
d1tod2=0;
d2tod1=0;
inventory=0;
policyiter=0;
locsearchiter=0;
redundant=0;
trunc=0;
mode=1;
iteration=1;
iterlim=30;
log(1,1)=0;
clear log;
cycleindex=1;
br=10;
l=2;
py=-5;
m1=1.0;
m2=1.0;
alpha=0.05;
```

```
%S1a. initial policy guess for each experiment
S1=2;
K1=0;
Z1=-2;
T1=-4;
S2=2;
K2=0;
Z2=-2;
T2=-4;
%S1b. initialize best response mapping
%tracking
log(iteration,1)=S1;
log(iteration,2)=K1;
log(iteration,3)=Z1;
log(iteration,4)=T1;
log(iteration,5)=S2;
log(iteration,6)=K2;
log(iteration,7)=Z2;
log(iteration,8)=T2;
cycleflag=0;
%S2. Grab h,l,tr,sr values of the current experiment
% via h_indx, l_indx,sr_indx,tr_indx values on envseth,
% envsetl,envsettr,envsetsr respectively.
sr=envsetsr(sr_indx);
h=envseth(h_indx);
tr=envsettr(tr_indx);
l1=envsetl(l_indx);
l2=envsetl(l_indx);
% S3. Compute rho and beta
beta=l1+l2+m1+m2;
rho=beta/(beta+alpha);
while ((iteration<=iterlim) && (cycleflag==0))
%S4. Initialize pro,prof,v,x,del,S1old,
%K1old,Z1old,T1old,best
clear pro;
clear prof;
clear v;
clear x;
clear del;
S1old=9999;
K1old=9999;
Z1old=9999;
T1old=9999;
best=-9999;
%implied policy loop
while (~((S1==S1old)&&(K1==K1old)&&(Z1==Z1old)&&(T1==T1old)))
```

```
% S5. Call evaluation.m
NOS=31*(S2-T2+1);
evaluation;
%S6. If there is an improvement, replace S1old,K1best,Z1best,
% T1best with S1,K1,Z1,T1. Else, trunc=1 go to S11.
if (obj>best)
   best=obj;
   S1best=S1;
   K1best=K1;
   Z1best=Z1;
   T1best=T1;
else
   trunc=1;
   break;
end;
%S7. Update policyiter and S1old,K1old,Z1old,T1old
S1old=S1;
K1old=K1;
Z1old=Z1;
T1old=T1;
if (iteration==1)
   policyiter=policyiter+1;
end;
% S8. Compute implied policy parameters and update S1,K1,Z1,T1
% via indifference equations inferred at D2 inventory level
% S2 for S1, Z2 for K1, S2 for Z1 and K2 for T1
% Determination of implied policies
% Determination of the implied policy: S1
for k = NOS-30:NOS-1
if ((v(k+1)-v(k)) <= 0)
S1 = -15+mod((k-1),31);
break;
end;
end;
% Determination of the implied policy: K1
if ((Z2-T2)>0)
for k = (Z2-T2+1)*31-29:(Z2-T2+1)*31
if (v(k-31)-v(k-1) <= sr)
K1 = -15+mod((k-2),31);
break;
end;
end;
end;
if ((Z2-T2)==0)
for k = (Z2-T2+2)*31-29:(Z2-T2+2)*31
if (v(k-31)-v(k-1) <= sr)
K1 = -15+mod((k-2),31);
```

```
break;
end;
end;
end;
% Determination of the implied policy: Z1
for k = NOS:-1:NOS-29
if (((v(k-1)-v(k-31)) <= -(sr+tr)) && (v(k)-v(k-31) <= (br-py-sr-tr)))
Z1 = -15+mod((k-1),31);
break;
end;
end;
% Determination of the implied policy: T1
for k = (K2-T2+1)*31-1:-1:(K2-T2+1)*31-29
if ( v(k)-v(k-1) >= (br-py))
T1 = -15+mod((k-1),31);
break;
end;
end;
% S9. Ensure that S1?K1?Z1?T1
if (K1>S1)
   K1=S1;
end;
if ((Z1>K1)||(Z1>S1))
   Z1=K1;
end;
if ((T1>Z1)||(T1>K1)||(T1>S1))
   T1=Z1;
end;
%S10. If a fixed point is found, go to S11. Else,
%revert to S5
end;
%S11. Record the solution out of implied policy
%iteration under S1local,K1local,Z1local,T1local and
%impliedbest by calling evaluation.m
if (iteration==1)
S1local=S1best;
K1local=K1best;
Z1local=Z1best;
T1local=T1best;
S1=S1best;
K1=K1best;
Z1=Z1best;
T1=T1best;
NOS=31*(S2-T2+1);
evaluation;
impliedbest=obj;
end;
```

```
%S12. Initialize S1old,K1old,Z1old,T1old,
% objold and flag=1
S1old=S1best;
K1old=K1best;
Z1old=Z1best;
T1old=T1best;
objold=best;
flag=1;
while (flag==1)
bestimp=0;
for S1=S1old-1:S1old+1
   for K1=K1old-1:K1old+1
      for Z1=Z1old-1:Z1old+1
         for T1=T1old-1:T1old+1
         if ((S1>=K1)&&(K1>=Z1)&&(Z1>=T1))
% S13. Update locsearchiter.
% Evaluate the profit of D1 in one policy neighbor.
% with S1?K1?Z1?T1. Call evaluation.m. Calculate
% imp=obj-objold
            NOS=(S1-T1+3)*(S2-T2+3);
            if (iteration==1)
            locsearchiter=locsearchiter+1;
            end;
            clear pro;
            clear prof;
            clear v;
            clear x;
            clear del;
            evaluation;
            imp=obj-objold;
%S14. Update best improvement if imp>bestimp. Else
%go to S15.
            if (imp>bestimp)
               bestimp=imp;
               S1best=S1;
               K1best=K1;
               Z1best=Z1;
               T1best=T1;
            end;
% S15. Go to S13 until all neighbors are exhausted.
         end;
         end;
      end;
   end;
end;
```

```
%S16. If there are no improving directions, go to S17. Else,
%S1old=S1best,K1old=K1best, Z1old=Z1best, T1old=T1best
%and return to S13.
if (bestimp<=0)
   flag=0;
else
   objold=objold+bestimp;
   S1old=S1best;
   K1old=K1best;
   Z1old=Z1best;
   T1old=T1best;
end;
end;
%S17. Record the best response into S1search, K1search,Z1search and
%T1search and compute the improvement of local search to the objective
%function. Local search objective is stored at objold.
if (iteration==1)
   S1search=S1best;
   K1search=K1best;
   Z1search=Z1best;
   T1search=T1best;
   improvement=objold-impliedbest;
end;
%S18. Determine if the local search phase did not change
%the best response guess
if
((S1search==S1local)&&(K1search==K1local)&&(Z1search==Z1local)&&(T1searc
h==T1local))
redundant=1;
end;
%S19. Update iteration and the best response log
%depending on mode
iteration=iteration+1;
log(iteration,9)=mode;
log(iteration,10)=objold;
if (mode==1)
log(iteration,1)=S1best;
log(iteration,2)=K1best;
log(iteration,3)=Z1best;
log(iteration,4)=T1best;
log(iteration,5)=S2;
log(iteration,6)=K2;
log(iteration,7)=Z2;
log(iteration,8)=T2;
end;
if (mode==2)
log(iteration,1)=S2;
```

```
log(iteration,2)=K2;
log(iteration,3)=Z2;
log(iteration,4)=T2;
log(iteration,5)=S1best;
log(iteration,6)=K1best;
log(iteration,7)=Z1best;
log(iteration,8)=T1best;
end;
%S20. Change mode,  swap players
   mode=(mode==1)*2+(mode==2)*1;
   %swap l1 and l2
   swap=l2;
   l2=l1;
   l1=swap;
   %swap m1 and m2
   swap=m2;
   m2=m1;
   m1=swap;
   %swap S,K,Z,T
   swap=S2;
   S2=S1best;
   S1=swap;
   swap=K2;
   K2=K1best;
   K1=swap;
   swap=Z2;
   Z2=Z1best;
   Z1=swap;
   swap=T2;
   T2=T1best;
   T1=swap;
%S21. Check if the best response log is looping. cycleflag=1 if such a loop
%is found.
if ((iteration>2))
for i=1:(iteration-1)
   if (isequal (log(i,1:8),log(iteration,1:8)))
   cycleflag=1;
   cycleindex=i;
   break;
   end;
end;
end;
% S22.Return to S4 until either a set of equilibrium policies are found
%(i.e. cycleflag=1), or iterlim is exceeded by iteration
end;
%S23.Determine out. out=1 if iteration-cycleindex=1, out=2 if
%iteration-cycleindex>1 but iteration<iterlim, out=3 if
```

```
%iteration-iterlim=0.
if ((iteration-cycleindex)==1)
    out=1;
end;
if (((iteration-iterlim)<0) && ((iteration-cycleindex)>1))
    out=2;
end;
if ((iteration-iterlim)==0)
    out=3;
end;
%S24. Swap l1,l2,m1,m2 if mode=2
if (mode==2)
    %swap l1 and l2
    swap=l2;
    l2=l1;
    l1=swap;
    %swap m1 and m2
    swap=m2;
    m2=m1;
    m1=swap;
end;

%S25. If out=1 or out=2, write columns 1-14 and 35 of expout. Initialize
%ion=cycleindex. Else (i.e. out=3), proceed to S35.
if ((out==1)||(out==2))
expout(expiter,1)=br;
expout(expiter,2)=sr;
expout(expiter,3)=h;
expout(expiter,4)=l;
expout(expiter,5)=py;
expout(expiter,6)=tr;
expout(expiter,7)=l1;
expout(expiter,8)=l2;
expout(expiter,9)=m1;
expout(expiter,10)=m2;
expout(expiter,11)=policyiter;
expout(expiter,12)=locsearchiter;
expout(expiter,13)=redundant;
expout(expiter,14)=trunc;
expout(expiter,35)=improvement;
    for ion=cycleindex:(iteration-1)
%S26. Take S1,K1,Z1,T1,S2,K2,Z2,T2 from respective columns of the ion
%line of log
S1=log(ion,1);
K1=log(ion,2);
Z1=log(ion,3);
T1=log(ion,4);
```

```
S2=log(ion,5);
K2=log(ion,6);
Z2=log(ion,7);
T2=log(ion,8);
% S27. Write columns 15-22 of expout
expout(expiter,15)=S1;
expout(expiter,16)=K1;
expout(expiter,17)=Z1;
expout(expiter,18)=T1;
expout(expiter,19)=S2;
expout(expiter,20)=K2;
expout(expiter,21)=Z2;
expout(expiter,22)=T2;
% S28.Call xeval.m
NOS=31*(S2-T2+1);
   clear pro;
   clear prof;
   clear v;
   clear del;
xeval;
% S29. Calculate waitcust, lostsales, d1tod2,d2tod1 and inventory
for k=1:NOS
i=-15+mod((k-1),31);
j=T2+floor((k-1)/31);
      if ((i>=T1) && (i<0))
         waitcust=waitcust-i/(alpha+beta)*x(k);
      end;
      if ((i>=T1) && (i<=S1) && (i>0))
         inventory=inventory+i/(alpha+beta)*x(k);
      end;
      if (i==T1)
         lostsales=lostsales+l1/(alpha+beta)*x(k);
      end;
      if ((i<=S1)&&(i>K1)&&(j<=Z2)&&(j>T2))
         d1tod2=d1tod2+l2/(alpha+beta)*x(k);
      end;
      if ((j<=S2)&&(j>K2)&&(i<=Z1)&&(i>T1))
      d2tod1=d2tod1+l1/(alpha+beta)*x(k);
      end;
end;
% S30.Write obj, waitcust, lostsales, d1tod2, d2tod1 and inventory to
% relevant columns of expout giving D1 measures
expout(expiter,23)=obj;
expout(expiter,25)=waitcust;
expout(expiter,27)=lostsales;
expout(expiter,29)=d1tod2;
expout(expiter,31)=d2tod1;
```

```
expout(expiter,33)=inventory;
% S31. Swap players, call xeval.m
   %swap l1 and l2
   swap=l2;
   l2=l1;
   l1=swap;
   %swap m1 and m2
   swap=m2;
   m2=m1;
   m1=swap;
   %swap S,K,Z,T
   swap=S2;
   S2=S1;
   S1=swap;
   swap=K2;
   K2=K1;
   K1=swap;
   swap=Z2;
   Z2=Z1;
   Z1=swap;
   swap=T2;
   T2=T1;
   T1=swap;
waitcust=0;
lostsales=0;
d1tod2=0;
d2tod1=0;
inventory=0;
NOS=31*(S2-T2+1);
   clear pro;
   clear prof;
   clear v;
   clear x;
   clear del;
xeval;
% S32. Calculate waitcust, lostsales, d1tod2,d2tod1 and inventory
for k=1:NOS
      i=-15+mod((k-1),31);
      j=T2+floor((k-1)/31);
      if ((i>=T1) && (i<0))
         waitcust=waitcust-i/(alpha+beta)*x(k);
      end;
      if ((i>=T1) && (i<=S1) && (i>0))
         inventory=inventory+i/(alpha+beta)*x(k);
      end;
      if (i==T1)
         lostsales=lostsales+l1/(alpha+beta)*x(k);
```

```
        end;
        if ((i<=S1)&&(i>K1)&&(j<=Z2)&&(j>T2))
            d1tod2=d1tod2+l2/(alpha+beta)*x(k);
        end;
        if ((j<=S2)&&(j>K2)&&(i<=Z1)&&(i>T1))
        d2tod1=d2tod1+l1/(alpha+beta)*x(k);
        end;
end;
% S33. Write obj, waitcust, lostsales, d1tod2, d2tod1 and inventory to
% relevant columns of expout giving D2 measures
expout(expiter,24)=obj;
expout(expiter,26)=waitcust;
expout(expiter,28)=lostsales;
expout(expiter,30)=d1tod2;
expout(expiter,32)=d2tod1;
expout(expiter,34)=inventory;
expiter=expiter+1;
% S34.Update ion=ion+1. Return to S25 until all equilibrium policy
% tuples S1,K1,Z1,T1,S2,K2,Z2,T2 are exhausted  (i.e. ion=iteration-1)
end;
end;
counter=counter+1;
counter
            end;
        end;
    end;
end;
% S35.Update counter, expiter, h_indx, l_indx,sr_indx,tr_indx.
%S36.Return to S1 until all experimental settings are exhausted
return;
```

A.2.2 MATLAB CODE FOR EVALUATION.M AND XEVAL.M

```
for k=1:NOS
% S1. Convert the state k onto i,j components
% Evaluation of (i,j) tuple for each loop index k
    i=-15+mod((k-1),31);
    j=T2+floor((k-1)/31);
% S2. Compute del value for the current state
% Determination of del(k), initial state probability
    if ((j==S2) && (i==S1))
        del(k)=1;
    else
        del(k)=0;
    end;
```

```
% S3. Determine actions a,b,c given i,j
% Determination of the action set (a,b,c) for each (i,j)
% Action a
   if (i>=S1)
      a=1;
   else
      a=2;
   end;
% Action b
   if(i>K1)
      b=2;
   else
      b=1;
   end;
% Action c
   if(i>Z1)
     c=2;
   end;
   if ((i<=Z1) && (i>T1))
   c=3;
   end;
   if (i<=T1)
   c=1;
   end;
% S4. Determination of the immediate expected profits for the state (i,j)
% given actions
if ( (i<0) && (j<=K2) && (j>Z2))
   prof(k) = (i*l)/(alpha+beta) +  (beta/(alpha+beta))*((3-c)*(c-1)*l1*br/beta + (2-
c)*(3-c)/2*py*l1/beta + (c-1)*(c-2)/2*l1*br/beta) ;
      else if ( (i>=0) && (j<=K2) && (j>Z2))
      prof(k) = (-i*h)/(alpha+beta) +  (beta/(alpha+beta))*((3-c)*(c-1)*l1*br/beta +
(2-c)*(3-c)/2*py*l1/beta + (c-1)*(c-2)/2*l1*br/beta) ;
         else if ( (i<0) && (j>K2) && (j<=S2))
         prof(k) = (i*l)/(alpha+beta) +  (beta/(alpha+beta))*((3-c)*(c-1)*l1*br/beta +
(c-1)*(c-2)/2*l1*(br-sr-tr)/beta + (2-c)*(3-c)/2*l1*py/beta) ;
            else if ( (i>=0) && (j>K2) && (j<=S2))
            prof(k) = (-i*h)/(alpha+beta) +  (beta/(alpha+beta))*((3-c)*(c-
1)*l1*br/beta + (c-1)*(c-2)/2*l1*(br-sr-tr)/beta + (2-c)*(3-c)/2*l1*py/beta) ;
               else if ( (i<0) && (j<=Z2))
               prof(k) = (i*l)/(alpha+beta) +  (beta/(alpha+beta))*((3-c)*(c-
1)*l1*br/beta + (2-c)*(3-c)/2*py*l1/beta + (j>T2)*(b-1)*l2*sr/beta + (c-1)*(c-
2)/2*l1*br/beta) ;
                  else if ( (i>=0) && (j<=Z2))
                  prof(k) = (-i*h)/(alpha+beta) +  (beta/(alpha+beta))*((3-c)*(c-
1)*l1*br/beta + (2-c)*(3-c)/2*py*l1/beta + (j>T2)*(b-1)*l2*sr/beta + (c-1)*(c-
2)/2*l1*br/beta) ;
                  end;
```

```
                    end;
                end;
            end;
        end;
     end;
% S5.Initialize (1-own transition probability), cumpro=0
cumpro=0;
%Determination of the transition probabilities for the state (i,j) to (d,e)
for m=1:NOS
 % S6. Convert the state m onto d,e components
    d=-15+mod((m-1),31);
    e=T2+floor((m-1)/31);
%S7. Compute transition probabilities, deduct from own transition
%probability
    if ( (d==i) && (e==j+1) && (j<S2))
                pro(k,m) = (m2)/beta;
                cumpro = cumpro + pro(k,m);

            else if ( (d==i+1) && (e==j) )
            pro(k,m) = (m1*(a-1))/beta;
            cumpro = cumpro + pro(k,m) ;
            else if ( (d==i-1) && (e==j) && (j>Z2) && (j<=K2))
            pro(k,m) = ((3-c)*(c-1)+(c-1)*(c-2)/2)*l1/beta;
            cumpro = cumpro + pro(k,m) ;
            else if ( (d==i) && (e==j-1) && (j>Z2) && (j<=K2))
            pro(k,m) = (l2)/beta;
            cumpro = cumpro + pro(k,m) ;
            else if ( (d==i-1) && (e==j) && (j>K2) && (j<=S2))
            pro(k,m) = (l1*(3-c)*(c-1))/beta;
            cumpro = cumpro + pro(k,m) ;
            else if ( (d==i) && (e==j-1) && (j>K2) && (j<=S2))
            pro(k,m) = (l1*(c-1)*(c-2)/2 + l2)/beta;
            cumpro = cumpro + pro(k,m) ;
            else if ( (d==i-1) && (e==j) && (j<=Z2))
            pro(k,m) = (l1*((3-c)*(c-1))+l1*((c-1)*(c-2)/2) + l2*(b-1)*(j>T2))/beta;
            cumpro = cumpro + pro(k,m) ;
            else if ( (d==i) && (e==j-1) && (j<=Z2))
            pro(k,m) = (l2*(2-b)*(j>T2))/beta;
            cumpro = cumpro + pro(k,m) ;
                else pro(k,m)=0;
                end;
                end;
                end;
                end;
                end;
                end;
                end;
```

```
      end;
%S8. Return to S6 until all states m are exhausted
end;
%S9. Compute own transition probability
         pro(k,k)= 1-cumpro;
%S10. Return to S1 until all states k are exhausted
end;
%S11. Compute v and obj via the equation system prof+rho*pro*v=v and
%obj=del*v
v=(eye(NOS)-rho*pro)\prof';
obj=del*v;
%xeval.m has the additional S12, i.e. the line x=del*inv(eye(NOS)-rho*pro);
return;
```

APPENDIX B

PROOF OF NO POOLING IF THERE IS NO COMMISSION

Suppose $S_1(j)=S2=S \; \forall j$ be the level where $v(S+1,j) \leq v(S,j)$ and $v(i+1,j) \geq v(i,j)$ if $i<S$. Similarly, $v(i,S+1) \leq v(i,S)$ if $j<S$. The definition of K at r=0 requires $v(i+1,j-1) \leq v(i,j)$.

Suppose that $K_1(j)=K2=K<S \; \forall j$, but then $v(i+1,j-1) \geq v(i+1,j) \geq v(i,j)$ is obtained which is a contradiction to the above statement. Therefore, K=S should hold for both dealers, thus there is no possibility of pooling.