MANAGING PRODUCTION AND LEAD TIME QUOTATION WITH MULTIPLE DEMAND CLASSES

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submitted by **ECE SAYIN** in partial fulfillment of the requirement for the degree of **Master of Science in Industrial Engineering Department, Middle East Technical University** by,

Prof. Dr. Canan Özgen
Dean, Graduate School of **Natural and Applied Sciences** ________________

Prof. Dr. Nur Evin Özdemirel ________________
Head of Department, **Industrial Engineering**

Asst. Prof. Dr. Seçil Savaşaneril ________________
Supervisor, **Industrial Engineering Dept., METU**

Examining Committee Members

Prof. Dr. Meral Azizoğlu
Industrial Engineering Dept., METU ________________

Prof. Dr. Sencer Yeralan
Agricultural and Biological Engineering Dept.,
University of Florida, USA ________________

Assoc. Prof. Dr. Canan Sepil
Industrial Engineering Dept., METU ________________

Asst. Prof. Dr. Seçil Savaşaneril
Industrial Engineering Dept., METU ________________

Asst. Prof. Dr. Ayşegül Toptal
Industrial Engineering Dept., Bilkent University ________________

Date: **17.09.2010**

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name : Ece SAYIN

Signature :

ABSTRACT

MANAGING PRODUCTION AND LEAD TIME QUOTATION WITH MULTIPLE DEMAND CLASSES

SAYIN, Ece

M.S., Department of Industrial Engineering

Supervisor : Asst. Prof. Dr. Seçil Savaşaneril

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In this study, we investigate several facets of a due-date quotation problem and develop a model considering jointly due-date quotation, order acceptance and base-stock decisions in a hybrid make-to-stock (MTS) / make-to-order (MTO) and multi-class system with lead time sensitive Poisson demand and exponentially distributed service times. We seek to maximize profit considering lateness penalties and holding costs in the model.

We consider three alternative due-date quotation policies each having different properties in terms of due-date flexibility as well as the utilization of state information. In order to evaluate the value of due-date flexibility as well as state information, the performances of the optimal policy and alternative policies are evaluated for various performance measures under different operating conditions. We also discuss the benefit of joint pooling of inventory and capacity under optimal policy and an accept-all policy.

Keywords: Due-Date Quotation, Multi-Class Systems,

ÖZ

ÇOKLU TALEP SINIFLARI ALTINDA ÜRETİM VE TESLİM TARİHİ BİLDİRME

SAYIN, Ece

Yüksek Lisans, Endüstri Mühendisliği Bölümü

Tez Yöneticisi : Yrd. Doç. Dr. Seçil Savaşaneril

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Bu çalışmada teslimat süresi atama probleminin farklı boyutları incelenmiştir. Çoklu talep sınıflarından oluşan ve teslimat süresi duyarlı Poisson tipi rassal talep ve üssel servis oranına sahip hibrit stoğa üretim ve siparişe üretim sisteminde dinamik teslimat süresi atama, sipariş kabulü ve güvenlik stoğu kararlarını dikkate alan bir model geliştirilmiştir. Modelde geç teslimat cezası ve stok maliyetleri altında karın eniyilenmesi amaçlanmıştır.

Teslimat süresi esnekliği ve sistem durumu bilgisinden faydalanma bakımından farklı özelliklere sahip alternatif teslimat süresi atama politikaları belirlenmiştir. Teslimat süresi esnekliği ve sistem durumu bilgisinin öneminin belirlenmesi için optimal politika ve alternatif politikaların performansları farklı işletme şartları altında çeşitli performans ölçütleri ile değerlendirilmiştir. Bunun yanında optimal politika ve tüm siparişler için anında teslimat vaad eden bir politika altında kapasite ve stok havuzlamasının getirileri ele alınmıştır.

Anahtar Kelimeler: Teslimat Süresi Atama, Çoklu Talep Sınıfları, Sipariş Kabulü

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CHAPTER 1

INTRODUCTION

Customer satisfaction is an important concept that nurtures competitive advantage in today's business environment with an increasing worldwide competition, hence it has become increasingly popular in both the manufacturing and the service sector. It increases customer loyalty and retention, therefore has a large impact on profitability. Accordingly, Davis et al. [10] states that "customer satisfaction is not the end objective, but rather an intermediate way station". Customer satisfaction, which may be attained via various factors such as low price, high quality, short delivery times, leads to customer loyalty, which in turn results in customer retention, and consequently increasing sales and finally providing higher profits. According to the study of Reicheld et al. [29], covering a wide array of industries, "a 5 percentage shift in customer retention results in 25-100% profit". Since most of the manufacturers and companies are aware of this significant impact of customer retention on profitability, they focus more on increasing customer satisfaction.

Meeting the customer demand within short and promised delivery times is essential for achieving customer satisfaction. Moreover in a competitive business environment, short delivery times are quite an effective tool for the companies to differentiate themselves from their competitors. Therefore, this motivates the manufacturers to shorten their response times with effective capacity planning and improve their production processes in order to be able to assign shorter due-dates, which will increase sales and reduce costs. Since capacity improvement can be done to a certain extent, the improvements in the production capacity can be supported with an effective due-date management.

For an effective due-date management, the manufacturers have to deal with the tradeoff between quoting small due-dates in order to increase customer satisfaction and achieving them with a limited production capacity. If the due-dates are set long, companies may lose customers because of the customers' limited delivery time tolerance, which may vary according to the industry and product. On the other hand, if the due-dates are unrealistically short to attain with the available production capacity, the customers again may have to wait a considerable amount of time for the delivery, and additionally, the due-date reliability of the company may deteriorate. Therefore an effective due-date quotation policy is an essential tool used in practice for production control which has a significant impact on improving lead times and customer satisfaction as addressed in Patil et al. [28].

In order to increase customer satisfaction, many firms may prefer to produce customized products for their customers, which in turn increase the production costs and delivery times as well. According to Lau [24] "Henry Ford's idea that "*customers can have any color as long as it's black*" is dead. The dilemma between standardization at low cost or customization at high cost is no longer there - customers want more product differentiation at the lowest possible price. The success of Japanese automobile and electronics manufacturers is attributed to their 'optimal' balance of product standardization and manufacturing flexibility". Moreover Keskinocak et al. [26] state that "The importance of lead time quotation becomes even more prevalent as many companies move from mass production to mass customization or from a make-to-stock (MTS) to a make-to-order (MTO) model to satisfy their customers′ unique needs." In such a business environment with changing customer needs and expectations, companies producing customized products can remain competitive by employing appropriate due-date management policies in order to shorten their lead times and increase their due-date reliabilities. However the effect of increasing demand in customized products does not only shorten lead time, but also propagates new concepts such as "delivery time differentiation". Especially companies having customers from different segments may encounter different delivery time sensitivities. We can see delivery time differentiation in various e-trade companies such as e-Bay, Amazon offering different delivery options to its customers each having a different price. Therefore, we can say that due-date quotation is an effective tool which may serve different purposes from capacity planning to product differentiation.

In this context, there are various due-date quotation policies in practice, such as "fixed due-date quotation", which is to promise constant due-dates to customers and has the risk of ignoring the changing level of congestion over time as well as not being able to respond to delivery time expectations of different customer segments. Therefore optimal due-date quotation policies are expected to perform better which seek to minimize various service level measures such as average tardiness, average queue length which are more deeply discussed in Keskinocak et al. [26].

In this study, we characterize a due-date quotation problem of a multi-class system with exponential production times and Poisson demand. We define two demand classes which differ in terms of revenues and due-date sensitivities. The high-priority class brings higher revenue and is more sensitive to due-dates quoted. Each customer arriving to the system is quoted a certain due-date, and may either accept or reject the due-date. The probability that the customer will accept the due-date quoted depends on the magnitude of the due-date and the class of the customer, hence we can evaluate the expected revenue to be gained for each due-date and class. Therefore, the assumption that the manufacturer knows the acceptance probabilities of each class provides the flexibility of determining the expected revenue to be gained as well as rejecting orders. When the manufacturer prefers to reject the order, he quotes the due-date which he knows that the customer will certainly reject. In order to shorten the response time, the manufacturer might prefer to keep an initial stock of items. Thus, we assume that the manufacturer operates in a hybrid MTS/MTO environment and decide on the optimal stocking level. According to the system defined above, we seek to maximize profits subject to holding cost incurred for every unassigned unit on-hand and lateness penalty incurred for every missed due-date.

In order to assess the performance of the optimal policy, we define three alternative policies which differ in terms of due-date flexibility and base-stock policy. Each alternative policy has a different character in terms of due-date flexibility and use different methods for the determination of due-dates and base-stock levels. Then we conduct numerical analysis comparing optimal model with predefined alternative policies by defining various performance measures. Throughout the analysis, we explore and compare the performances of optimal policy and alternative policies under different system parameters. Moreover we investigate the benefit obtained by joint pooling of inventory and capacity under optimal policy and one of the predefined alternative policies which is an accept-all policy. To the best of our knowledge, this is the first study in the literature considering due-date quotation, order acceptance, production and inventory decisions in a multi-class system.

The rest of the thesis is organized as follows. In Chapter 2, we review the related literature. Next we characterize the structure of the optimal policy for the due-date quotation problem and formulate alternative policies in Chapter 3. We present a linear programming model in order to solve the due-date quotation problem defined in Chapter 3, define various measures which we use for the comparison of the performances of the optimal policy and alternative policies, then explore the joint benefit of inventory and capacity pooling under optimal and accept-all policy in Chapter 4. Finally we present our main findings and point out some possible future research directions in Chapter 5.

CHAPTER 2

LITERATURE REVIEW

In our study, we consider several concepts related with inventory and production planning. In this chapter, we present the studies related with these concepts, most of which we take as a basis to our work. We grouped them under separate titles which are stock allocation, production scheduling, revenue management, inventory and capacity pooling and due-date quotation.

2.1. Stock Allocation

Stock allocation is one of the inventory policies where the stock is allocated for each demand class according to the defined threshold levels such that it is optimal to reject the incoming demand of a certain class if on-hand stock is below the threshold determined for that class. Therefore, stock allocation enables differential treatment of customer classes without using separate inventories and it is one of the problems which is addressed extensively in the literature. As in our study, we encounter queuing-based systems in the literature used heavily for inventory decisions. However, earlier studies such as Nahmias et al. [27] introduce the inventory rationing concept without taking into account the limited production capacity. In more recent studies, queuing-based systems begin to be used for stock allocation problems. Ha [18] is one of these studies which characterize a stock rationing problem and production policy of a multi-class MTS system with lost sales. He shows that the optimal model is a base-stock policy where the stock is allocated according to the threshold levels defined for each class. Ha [19] also considers a similar system, however this time instead of lost sales, backordering is allowed which necessitates to keep track of the number of customers in the backordering queue. He shows that the optimal policy is still the base-stock policy with stock rationing as in the lost sales case. De Véricourt et al. [11] extend this work to multiple demand classes.

Ha [20] extends his previous works Ha [18], [19] by investigating multi-class MTS system with Erlangian production times. In this study, he shows that the work storage level policy in which critical work storage levels are defined for each class yields the optimal solution for production and inventory rationing. Gayon et al. [15] also analyze a stock rationing problem for a similar MTS system with Poisson demand and Erlangian production times, however extends the work of Ha [20] by considering backordering.

One of the recent studies is Fadiloğlu et al. [14] which propose an optimum dynamic stock rationing policy utilizing from the information of outstanding order levels. Inventory is rationed dynamically according to the age information for all the outstanding orders.

2.2. Production Scheduling

As in our study, FCFS discipline is assumed for the incoming orders in most of the studies such as Ha [18], Hopp et al. [22] where an arriving customer is immediately processed in the order he arrives. In such a case, if the customer accepts the due-date quoted, the order begins to be processed and occupies the server from the beginning of the time that the order arrives to the system. Hence the manufacturer does not have the flexibility to postpone the production of the accepted customers and allocate the server to the high-priority class customer in the backordering queue. However, we can see this flexibility in several previous studies by means of a scheduling policy. Wein [37] is one of the earliest studies considering the joint problem of quoting due-dates and sequencing jobs. Similarly Duenyas [12] considers the joint problem of quoting due-dates and sequencing jobs in a MTO queue in a multi-class system, where the arrival of orders depends on the due-dates quoted to the customers. The flexibility of sequencing the orders other than FCFS discipline enables the manufacturer to dynamically decide on which order to serve the jobs in queue. Duenyas [12] and Duenyas and Hopp [13] investigate systems where the orders are processed according to FCFS discipline and where the orders can be sequenced according to a scheduling policy. Duenyas and Hopp [13] find that the optimal lead time quoting and optimal order sequencing policy is done according to the EDD rule for finite capacity case. Veatch et al. [35] also study a MTS multi-class system with both backordering and lost sales by developing and evaluating several scheduling policies.

Carr et al. [6] is the first to characterize both sequencing problem and admission control by considering customer classes that require different products and admission control which provides an option to accept or reject customers. Iravani et al. [23] also address a similar problem with Carr et al. [6], however assume backordering instead of lost sales. Keskinocak et al. [25] capture process scheduling by developing and comparing two models for both scheduling and lead time quotation problems, which are different in terms of being a single class and multi-class system. Finally Watanapa et al. (2005) [36] covers both EDD and FCFS sequencing disciplines in s similar system for the incoming orders.

2.3. Revenue Management

In our work, we use due-date quotation also as a revenue management tool in addition to inventory and capacity planning. Shorter due-dates increase the probability that the customer accepts the due-date quoted and places an order, therefore due-dates can change the expected revenue to be gained. In the literature, we see various studies related with pricing problem and revenue management, which generally consider that demand is highly sensitive to price and lead time. So et al. [32] is one of these studies investigating the pricing and delivery time sensitivities of customers and seeking to maximize the net profit by modeling demand as a function of price and lead time.

Similar to our work, Keskinocak et al. [25] considers that revenues obtained from the customers are sensitive to the lead time. Therefore, we see that revenues can be adjusted by setting an effective lead time policy considering customer classes with different lead time sensitivities. Charnsirisakskul et al. [9] also consider a system where the demand is highly sensitive to prices and lead times, and seek to find effective pricing, lead time quotation and scheduling policies. Watanapa et al. [36] and Aktaran et al. [1] are the other studies which take into account the price sensitivities of customers and explore effective pricing policies for maximizing profits.

Gupta et al. [17] and Caldentey et al. [5] study an order acceptance and capacity allocation problem under different contract terms related with price and lead time.

Finally, Hall et al. [21] compare three pricing policies, which do not have the same information level related to the system status, in order to investigate the significance of production capacity information on pricing.

2.4. Inventory and Capacity Pooling

We also explore the benefit of joint pooling of inventory and capacity under optimal base-stock policy with due-date quotation. In the literature, we encounter studies addressing capacity and inventory pooling jointly and/or separately. In De Véricourt et al. [11] benefit of inventory pooling is investigated in a multi-class system under stock allocation policies, and they show that under a system where stock allocation is not done optimally, pooled system may result with a loss compared to the system before pooling. The research also reveals that the benefit of pooling where optimal stock allocation policy is employed is not significant.

Iravani et al. [23] investigate the benefit of capacity pooling in order to evaluate new production and admission policies under low and high machine utilization.

Benjaafar et al. [4] also study pooling in production and inventory systems and shows that benefit obtained from inventory pooling is highly sensitive to various system parameters such as utilization, demand and process variability, control policy, service levels and the structure of production process.

2.5. Due-Date Quotation

There are various studies in the literature covering due-date quotation policies which take into account production capacity, customer due-date and price sensitivities as well as losses due to late deliveries. Wein [37] is one of the studies that address both due-date quotation and priority sequencing in a multiclass M/G/1 queuing system. He proposes several due-date policies which minimizes weighted average lead time subject to service level constraints such as the fraction of tardy jobs or the average job tardiness. He concludes that due-date setting improves the performance more than priority sequencing. Duenyas [12] extends this system by considering price and lead time sensitive customers and proposes a combined pricing and sequencing model for a multi-class system. Spearman et al. [33] characterize a due-date setting problem and show that minimizing the average “due-date lead time” (due-date minus arrival date) by setting fraction of tardy jobs as a service level constraint may lead to unethical policies and setting average tardiness as a service level constraint leads to a lead time policy quoting monotonically increasing due-dates with the congestion level. Hopp et al. [22] aim to quote the shortest possible lead time consistent under various service constraints such as fill rate and tardiness.

Watanapa et al. [36] develop a simplified pattern search algorithm which optimize price and due-dates with multiple customer classes for MTO systems more efficiently. In the model, customer classes has different due-date sensitivities, willingness to pay and quality level requirements.

Previous studies related with due-date quotation in the literature always assume linear customer delay costs. However Ata et al. [3] study dynamic due-date quotation under a MTO system where the customer delay costs are nonlinear (investigate both delay costs with a convex and concave structure). Another work related with due-date quotation is Alfrei [2] which combines due-date quoting and process scheduling decisions in computer-based simulation and represents that service level as a function of the due-date quoted to the customers. Shabtay [31] also study a joint due-date setting and process scheduling problem by minimizing earliness, tardiness, and other costs related with due-date assignment, delivery and inventory.

There are also studies considering both due-date quotation and order acceptance. Some of those are Keskinocak et al. [25], Chatterjee et al. [7], Iravani et al. [23], Charnsirisakskul et al. [8], [9], Zorzini et al. [38]. Similar to our work, order acceptance flexibility is provided in most of these studies via acceptable due-date levels determined for each customer class such that the customers may be rejected by quoting a due-date which is greater than the latest acceptable due-date for that customer.

One of the most recent works in the literature regarding lead time and inventory decisions is Savaşaneril et al. [30], capturing both inventory decisions and due-date quotation as well as providing the flexibility of order acceptance by using due-dates. Our work extends Savasaneril et al. [30] by considering a multi-class system and evaluating the benefit obtained from joint pooling of inventory and capacity where the optimal due-date quotation and inventory policy is employed. Therefore our study strives to contribute to the literature by jointly considering due-date quotation, order acceptance, production and inventory decisions in a multi-class system.

CHAPTER 3

THE DUE-DATE QUOTATION MODEL

We consider a hybrid of MTO and MTS production system with one server producing one type of item for two different customer classes with different revenues, possibly with different arrival rates and due-date sensitivities. The production time is exponential with a mean $1/\mu$ and one item is produced at a time. Demand arrives according to a Poisson process with rate λ_c for class c, $c \in C=\{1,2\}$, and customers can place an order one at a time. Class 1 is regarded as high-priority demand class since it brings higher revenue. Incoming demand is served according to the FCFS discipline.

The system state is denoted by i, where the positive values of i correspond to the number of customers in the system, and negative values of i correspond to the inventory level. The state space is defined as $I=\{-\infty,\ldots,-1,0,1,\ldots\infty\}$.

There are two types of actions to be taken by the manufacturer; which are the decision of production and the quotation of due-dates for each class. For production decision, the manufacturer may choose to produce or stop production at any time. However due-date quotation is done only when the customer arrives to the system. Hence the frequency of production action and due-date quotation are different. We denote the production action with the binary variable a, where a=1 is to produce, and a=0 is not to produce. As the second type of action, the manufacturer can respond to any incoming demand with a due-date range of $D_c = [0,d_{max,c}]$ for each demand class. There are two different d_{max} values for each class denoted by $d_{max,c}$ and the action is denoted as $d_c \in D_c$ which is the due-date quoted to customer class c upon arrival. The manufacturer can accept or reject the incoming demand by quoting a due-date in D_c depending on the state of the system. Additionally the manufacturer can quote a due-date between zero and $d_{max,c}$ which the customer accepts with a probability less than or equal to 1. Depending on the system status, there can be a delay in the delivery and total waiting time of the customer in the system may be greater than the due-date quoted.

According to the quoted due-date d, customer in class c places an order with probability $f_c(d)$, where $f_c(d)$ is a decreasing function of d. We assume that in case of zero due-date quotation, the customer accepts order with probability 1. There is also a maximum value of due-date $d_{max,c}$ which the customer class c rejects to wait that much time and $f_c(d)$ becomes zero. Hence quoting due-date of $d_{max,c}$ can be regarded as the action of rejection. A due-date quote d affects the decision of customer classes differently, as their acceptance probability functions differ according to their due-date sensitivities. One may be inclined to think that class 1 customer which belongs to the higher priority class is more due-date sensitive compared to class 2 customer, hence we set class 1 acceptance probability function $f_1(d)$ always less than or equal to acceptance probability function of class 2 $f_2(d)$ for all d.

Each customer in class c brings a revenue of Rev_c (where $Rev_1 \geq Rev_2$) if he accepts the due-date quoted and places an order. For unmet due-dates, a lateness cost, l, is incurred per unit time per item regardless of the demand class. Another cost item is holding cost, h, which is incurred for any item per unit time waiting in the inventory. Unit holding cost h is incurred for each item and negative state at which the inventory level is greater than zero.

3.1. The Optimal Model

Considering the operating conditions and the actions to be taken by the manufacturer, we aim to maximize the profit under average gain criterion. Therefore we formulate the problem as a Markov decision process under the average gain criterion, and the optimality equation is expressed as follows:

$$g + v(i) = \max_{d_c}\left[\sum_{c=1}^{2} \frac{\lambda_c}{(\sum_{c=1}^{2}\lambda_c) + \mu}.f_c(d_c).(Rev_c - l.L_i(d_c) + v(i+1)) + \sum_{c=1}^{2} \frac{\lambda_c}{(\sum_{c=1}^{2}\lambda_c) + \mu}.\big(1 - f_c(d_c)\big).v(i)\right] + \frac{hi^-}{(\sum_{c=1}^{2}\lambda_c) + \mu} + \frac{\mu}{(\sum_{c=1}^{2}\lambda_c) + \mu}\max_a v(i-a), \qquad \forall i \in I,\ i^- = \min\{i, 0\} \qquad (1)$$

where g is the optimal average profit per transition time and v(i) is the relative value of starting in state i under the optimal policy. The profit obtained from an arriving customer who accepts the due-date quoted is expressed with the term $\big(Rev_c. - l.L_i(d_c)\big)$. This expression is multiplied with $f_c(d_c)$ which is the acceptance probability of class c customer calculated according to due-date d_c.

We define a reward function $r_i(d_1, d_2, a)$ which is the net profit obtained until the next transition when due-date of d_1 and d_2 is quoted to class 1 and 2 customers respectively and a production action of a is taken in state i. It consists of three items which are the revenue, lateness cost and holding cost. There is no production cost incurred in case of any production decision a=1 or a=0 is given. The reward function is given in the following:

$$r_i(d_1, d_2, a) = \sum_{c=1}^{2}\left(\frac{\lambda_c.f_c(d_c)}{(\sum_{c=1}^{2}\lambda_c) + \mu_c}.\big(Rev_c - l.L_i(d_c)\big)\right) + \frac{1}{(\sum_{c=1}^{2}\lambda_c) + \mu}.h.i^- \quad (2)$$

The expected actual waiting time $L_i(d)$ corresponds to the expected time that an arriving customer waits after the quoted due-date. This delay depends on the value of due-date quoted and the state of the system at the time of the due-date decision. Since the arriving customer has to wait for his service time plus the service times of the customers in the queue minus the due-date quoted, and the service times of each customer class are exponentially distributed with the same mean, the distribution of waiting time can be defined as an Erlang - (k+1) distribution where k is the number of customers in the system upon arrival given that the system is out-of-stock.

$$L_i(d) = \begin{cases} \int_d^{\infty} (t-d)E_{i+1}dt & \text{if } i \geq 0 \\ 0, & \text{otherwise,} \end{cases} \tag{3}$$

where $\quad E_j(t) = \frac{\mu(\mu t)^{j-1}}{(j-1)!} e^{-\mu t}$

After the substitution of $E_j(t)$ to $L_i(d)$, the new expression obtained for $L_i(d)$ is below, the detail of which can be found in APPENDIX A.

$$L_i(d) = e^{-\mu d}\left(\frac{(i+1)}{\mu} + \sum_{k=0}^{i} \frac{d^{i+1-k}.k.\mu^{i-k}}{(i+1-k)!}\right) \tag{4}$$

In the later analysis expected actual waiting time will be expressed as $L_i(d_c)$ which is the expected actual waiting time of an arriving customer when the quoted due-date is d and given the state is i.

Properties of the Optimal Solution

1. At positive states where the inventory level is zero and there are customers waiting in the system, production action taken by the optimal model is always a=1 "to produce" since not producing in the existence of customers will incur additional lateness cost. Therefore the production decision can be restricted to only "to produce" in positive states in order to reduce the computation time.

2. When the inventory level is greater than zero (at negative states), the model quotes only zero to class 1 customer so as to guarantee that the customer will certainly accept the due-date. When the inventory level is greater than zero, demand of class 1 customers should be met immediately as in the single-class system. Note also that the model will quote only zero or d_{max} values but not the other due-dates between 0 and $d_{max,2}$ for class 2 customers in negative states. In positive states, where there is no inventory, there is a tradeoff between quoting a lower or higher due-date. In case of lower due-dates, the model increases the acceptance probability of the due-date quoted and this increases the expected revenue to be gained; however, this may increase the actual waiting time of the customer as well (the difference between the waiting time of the customer and the due-date quoted) by incurring higher lateness cost. Similarly, higher due-dates decrease the probability of

gaining the revenue but this time lateness cost decreases. Because of this tradeoff, due-dates to be quoted can take some intermediate values other than zero or d_{max} in the states with zero inventory level. However this situation is not encountered in negative states. Since the incoming orders are processed under FCFS sequencing, the only tradeoff related with due-date quotation of class 2 customers will be to keep the stock for future class 1 customers by quoting d_{max} due-date and rejecting the demand or immediately meeting the demand of class 2 customers by quoting zero due-date. Hence it is not meaningful to quote due-date values between 0 and d_{max} to class 2 customers for negative states.

3.2. Alternative Policies

In order to understand the relative benefit of using the optimal model, we define three alternative policies with different performances in terms of both model results and computational time under the same operating conditions. In positive states where inventory level is zero, the production action is taken as “to produce” (a=1). Instead of individual determination of production decision for each state, only base-stock level is determined by certain methods i.e. newsvendor model, exhaustive-search. For each alternative policy, a base-stock policy is assumed, which implies producing up to a certain level and stopping production for inventory levels greater than the base-stock level. Hence for each state greater than state -s, “to produce” action is given as a stationary base-stock policy. Each policy may perform well under certain operating conditions and may sometimes yield very close results to the optimal policy. In each alternative policy, expression (4) is used for expected actual waiting time $L_i(d)$.

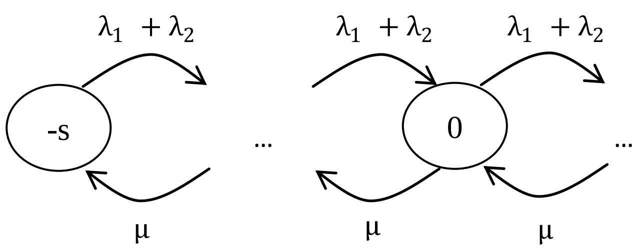
Flow diagram of each policy as well as steady-state probabilities and average profit expressions of each alternative policy are explained in the following subsections.

3.2.1. Policy 1

We define Policy 1 as an accept-all policy where no rejection of customer is possible and only zero due-date can be quoted at every state. In practice, firms may tend to quote unrealistically short due-dates and never reject an arriving customer in order to avoid loss of goodwill. However, contrary to expectations, this behavior may decrease the customer satisfaction, since the production facility may not be able to attain these unrealistic due-dates. Therefore an accept-all policy in which the demand is promised to be met immediately regardless of the system status, generally shows poor performance compared to other policies where different due-dates other than zero can be quoted to the customer. In practice, there can be penalties incurred when the manufacturer cannot meet the demand on the promised due-date. In the system we define, the rejection of a customer can be more profitable in situations where the lateness cost incurred due to a late delivery begins to outweigh the revenue gained from that customer (i.e. when the system is overcrowded). Therefore more due-date options enable the manufacturer to be more flexible in due-date quotation under different system conditions and expected to decrease the losses occurring from late deliveries. Given that due-date quoted is zero at every state, expected lateness cost function becomes only a function of the state of the system. By substituting d with zero in (4), we obtain the expression below;

$$L_i = \frac{(i+1)}{\mu} \tag{5}$$

which is the expected actual waiting time of an arriving customer to state i under Policy 1. Basically the expression can also be interpreted as the sum of expected service times of all the customers in the system including the arriving customer. The flow diagram of the system for Policy 1 and the expression of steady-state probabilities and average profit per unit time are given in the following:



Let π_i be the long-run fraction of decision epochs at state i, and $\rho = \frac{\lambda_1+\lambda_2}{\mu}$

$$\pi_{-s} + \rho\pi_{-s} + \cdots = 1$$

$$\pi_{-s} = \frac{1}{\sum_{n=0}^{\infty} \rho^n}, \qquad \text{note that} \quad \sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \qquad \text{for } 0 \leq x < 1$$

We obtain,

$$\pi_{-s} = 1 - \rho$$

$$\pi_i = \rho^{i+s}.\pi_{-s}$$

Note that under Policy 1, it must hold that $\rho<1$

Given the steady-state probabilities, next we calculate the optimal base-stock level. Since there is no opportunity to change the acceptance probability of the customers in Policy 1, revenue will no longer be a decision parameter for Policy 1 in the determination of the base-stock level. In such a case, newsvendor model is appropriate for base-stock calculation, which considers only lateness and holding costs. Cumulative distribution function F(s) is the steady-state distribution, which is the sum of steady-state probabilities from state –s (where inventory level is s) to state zero (where inventory level is zero). The understocking cost corresponds to the unit lateness cost, l, and overstocking cost corresponds to the unit holding cost, h.

We calculate the minimum base-stock level holding the equation below;

$\min_s \left(F(s^*) = (1-\rho)\sum_{i=0}^{s}\rho^i \geq \frac{l}{l+h}\right),\ \rho < 1.$ Equivalently,

$$s^* = \min\left\{s:\ 1-\rho^{s+1} \geq \frac{l}{l+h}\right\}$$

$$= \min\left\{s:\ s \geq \log_\rho\left(\frac{h}{l+h}\right) - 1\right\}$$

$$= \left\lceil \log_\rho\left(\frac{h}{l+h}\right) - 1\right\rceil \tag{6}$$

By using the base-stock level s and given production and due-date actions for each state, average profit per unit time obtained under Policy 1 denoted by g_1 is calculated by using the expression below:

$$g_1 = (\lambda_1 Rev_1 + \lambda_2 Rev_2)\sum_{i=-s}^{\infty}\pi_i - l\sum_{i=1}^{\infty} i.\pi_i - h\sum_{i=-s}^{0}(-i)\pi_i\ , \tag{7}$$

where $\pi_i = \rho^{i+s}.(1-\rho)$ (8)

$$g_1 = (\lambda_1 Rev_1 + \lambda_2 Rev_2) - l.\rho^s(1-\rho)\sum_{i=1}^{\infty} i.\rho^i + h\sum_{i=1}^{s+1}(i-s-1).\rho^{i-1}(1-\rho)$$

$$= (\lambda_1 Rev_1 + \lambda_2 Rev_2) - \frac{l.\rho^{s+1}}{(1-\rho)} + h.(1-\rho)\left(\sum_{i=1}^{s+1} i.\rho^{i-1} - (s+1)\sum_{i=1}^{s+1}.\rho^{i-1}\right)$$

$$= (\lambda_1 Rev_1 + \lambda_2 Rev_2) - \frac{l.\rho^{s+1}}{(1-\rho)}$$
$$+ h\left(\frac{\left((s+1)\rho^{s+2} - (s+2)\rho^{s+1} + 1\right)}{1-\rho} + (s+1)(\rho^{s+1} - 1)\right)$$

Note that,

$\sum_{i=-s}^{\infty} \pi_i = 1$,

$$\sum_{i=1}^{s+1} \rho^{i-1} = \frac{1-\rho^{s+1}}{1-\rho},$$

$$\sum_{i=1}^{s+1} i.\rho^{i-1} = \frac{1}{(1-\rho)^2}\left((s+1)\rho^{s+2} - (s+2)\rho^{s+1} + 1\right),$$

$$\sum_{i=1}^{\infty} i.\rho^i = \frac{\rho}{(1-\rho)^2} .$$

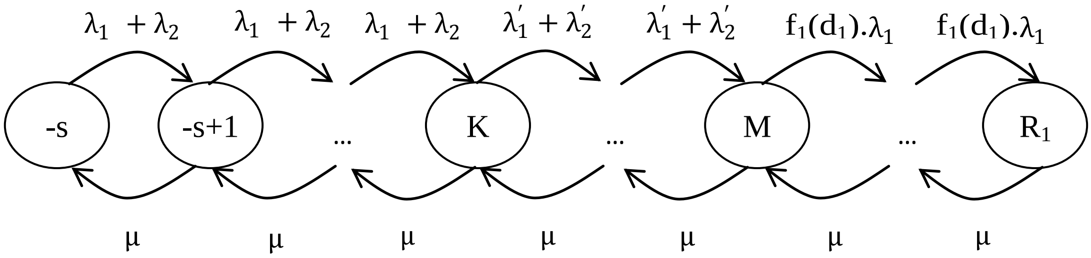
3.2.2. Policy 2

As already mentioned in Policy 1, there are several shortcomings of fixed due-date policies. For accept-all policy, we may observe these shortcomings when the demand is high. However under a fixed due-date policy which has relatively higher fixed due-dates, we may also lose customers when demand is low as a result of disappointed customers due to high due-dates. On the other hand fixed due-date policy is a common policy used since it does not use state information which may be costly to keep track of. However this property of fixed due-date policies may lead to poor performance in certain conditions depending on the lack of state information. In our study, we had already analyzed fixed due-date quotation in Policy 1 in order to understand the benefit of using the state information. However fixed due-date quotation policy may be extended such that fixed due-dates can be determined separately for each predefined condition or period. Considering that, we developed Policy 2 which uses state information in only certain intervals hence can be regarded as a semi-state dependent policy. This property of Policy 2 can be also be interpreted as the lack of flexibility in due-date quotation compared to optimal policy. Therefore by comparing the benefit of Policy 2 and optimal policy, we also would like to assess the benefit of "dynamic" due-date quotation.

In Policy 2, we assume that all arriving class 1 customers are quoted zero due-date if inventory level is greater than zero (similar to the optimal policy), and quoted a constant due-date if there is no stock. Different than Policy 1, the manufacturer also has the option of quoting different due-dates from the range of D_1 and D_2 which are the due-date sets also used in the optimal policy. However fixed due-dates should be quoted for each state between the states zero and the rejection points of the classes, R_c, where at states greater than or equal to the rejection point, the customer will be quoted d_{max} due-date, in other words will be rejected. Similarly for class 2 customers, in case of the rejection point of class 2 customers, R_2, being greater than or equal to zero, the same d_2 value within the range $d_2 \epsilon D_2$ should be set for each state between states zero and R_2. This reduces the flexibility of due-date quotation in Policy 2 and makes the optimal model more favorable than Policy 2.

In Policy 2, due-date actions, d_c, the rejection points, R_c (the policy begins to quote $d_{max,c}$ due-date to class c customer in states greater than R_c) and base-stock level, s are determined optimally through exhaustive-search method. The action space for due-date quotation is the same with the optimal case, however the usage of due-dates are restricted in certain state intervals. For class 1 customers, the policy is free to quote $d_1 \epsilon D_1$.

For class 1 customers, rejection point is assumed to be greater than or equal to zero since we know that demand of higher-priority class should be met when there is stock on-hand, as already been discussed in the previous section. Rejection point of class 1 customers is also expected to be greater than rejection point of class 2 customers, since the rejection of class 2 customers will be less costly when there is insufficient capacity to meet the incoming demand. However there are two possible cases for the rejection point of class 2 customers. The model may begin to reject class 2 customers although inventory level is greater than zero. Another case would be that class 2 customers may be rejected after inventory is depleted. Average profit per unit time denoted by g_2 is maximized through exhaustive-search method. The flow diagram of the system for Policy 2 and expression of steady-state probabilities and average unit profit are given in the following:



Note that the rejection point of class 2, R_2, may be positive or negative. Therefore we define two conditional parameters K and M denoting 0 and R_2 respectively for positive R_2 value; R_2 and 0 respectively for negative R_2 value.

$$K = \begin{cases} 0, & R_2 > 0 \\ R_2, & R_2 \leq 0 \end{cases}$$

$$M = \begin{cases} R_2, & R_2 > 0 \\ 0, & R_2 \leq 0 \end{cases}$$

Note that $K, M \leq R_2$, $K \leq 0$, $M \geq 0$ and $K.M = 0$ and holds

$$\lambda_1' = \begin{cases} f_1(d_1).\lambda_1, & R_2 > 0 \quad \forall i \\ \lambda_1, & R_2 \leq 0 \quad \forall i \end{cases}$$

$$\lambda_2' = \begin{cases} f_2(d_2).\lambda_2, & R_2 > 0 \quad \forall i \\ 0, & R_2 \leq 0 \quad \forall i \end{cases}$$

Let,

$$\rho = \frac{\lambda_1 + \lambda_2}{\mu}$$

$$\rho' = \frac{\lambda_1' + \lambda_2'}{\mu}$$

$$\rho'' = \frac{f_1(d_1).\lambda_1}{\mu}$$

$$\pi_i = \begin{cases} \rho^{s+i}.\,\pi_{-s}, & [-s, K] \\ \rho'^{\,i-K}.\,\rho^{s+K}.\,\pi_{-s}, & (K, M] \\ \rho''^{\,i-M}.\,\rho'^{\,M-K}.\,\rho^{s}.\,\pi_{-s}, & (M, R_1] \end{cases}$$

$$\pi_{-s}(1+\rho+\rho^2+\cdots+\rho^{s+K}) + \rho^{s+K}\pi_{-s}\left(\left(\rho'+\rho'^2+\cdots+\rho'^{M-K}\right)+\rho'^{M-K}\left(\rho''+\rho''^2+\cdots+\rho''^{R_1-M}\right)\right)=1$$

For $\rho \neq 1, \rho' \neq 1, \rho'' \neq 1$

$$\pi_{-s}\left(\frac{\rho^{s+K+1}-1}{\rho-1}+\rho^{s+K}\left(\rho'\left(\frac{\rho'^{M-K}-1}{\rho'-1}\right)+\rho'^{M-K}\rho''\left(\frac{\rho''^{R_1-M}-1}{\rho''-1}\right)\right)\right)=1$$

$$\pi_{-s} = \frac{(\rho-1)(\rho'-1)(\rho''-1)}{(\rho^{s+K+1}-1)(\rho'-1)(\rho''-1)+\rho^{s+K}\rho'\left(\rho'^{M-K}-1\right)(\rho-1)(\rho''-1)+\rho^{s+K}\rho'^{M-K}\rho''\left(\rho''^{R_1-M}-1\right)(\rho-1)(\rho'-1)} \tag{9}$$

If any of ρ, ρ', ρ'' or the combinations of these parameters are equal to 1, then the expression (9) should be updated as in APPENDIX B.

For a given s, K, M, R_1, d_1, d_2, let π_i be the corresponding steady state probability as derived in (9). Then the average profit under Policy 2 is expressed as:

$$g_2 = \max_{R_1,K,M,d_c,s} \begin{cases} \left(\begin{array}{l} \sum_{i=-s}^{K}\left(\sum_{c=1}^{2}\lambda_c.\left(Rev_c - l.\,L_i(0)\right)+h.\,i^-\right).\,\pi_i \\ +\sum_{i=K+1}^{M}\sum_{c=1}^{2} f_c(d_c).\,\lambda_c.\left(Rev_c - l.\,L_i(d_c)\right).\,\pi_i \\ +\sum_{i=M+1}^{R_1}\left(f_1(d_1).\,\lambda_1.\left(Rev_1 - l.\,L_i(d_1)\right)\right).\,\pi_i \end{array} \right), & \text{if } K=0, M\neq 0 \\ \left(\begin{array}{l} \sum_{i=-s}^{K}\left(\sum_{c=1}^{2}\lambda_c.\left(Rev_c - l.\,L_i(0)\right)+h.\,i^-\right).\,\pi_i \\ +\sum_{i=K+1}^{M}\left(\lambda_1.\left(Rev_1 - l.\,L_i(0)\right)+h.\,i^-\right).\,\pi_i \\ +\sum_{i=M+1}^{R_1} f_1(d_1).\,\lambda_1.\left(Rev_1 - l.\,L_i(d_1)\right).\,\pi_i \end{array} \right), & \text{if } K\neq 0, M=0 \end{cases} \tag{10}$$

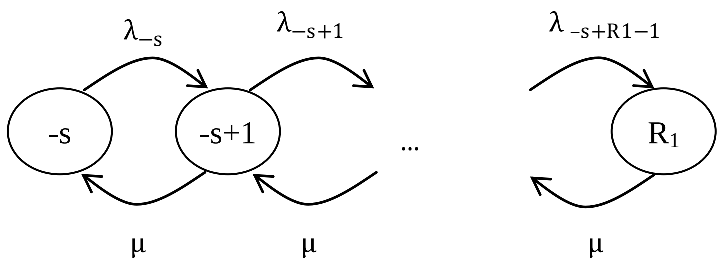
3.2.3. Policy 3

Policy 3 is a myopic policy where in each state due-dates are determined so as to maximize the expected profit of that individual state. This optimization is done for each class and state independently. At every state, the expected reward excluding holding cost is maximized through line search of due-dates for each class separately as in the following:

$$\max_{d_{c,i}}\left(f_c\left(d_{c,i}\right).\left(Rev_c - l_c L_i\left(d_{c,i}\right)\right)\right), \qquad \forall c \in C = \{1,2\}, \forall i \in I \tag{11}$$

Holding cost and arrival rate components are excluded since they have no effect on the due-dates under Policy 3. After the determination of due-dates, the average gain for a given s is calculated and maximized through line-search of base-stock level s. The rejection point of class c is denoted as R_c (where the policy begins to quote $d_{max,c}$ due-date to class c customer in states greater than R_c). As in Policy 2, it is assumed that R_1 is greater than R_2.

The flow diagram of the system for Policy 3 and expression of steady-state probabilities are given below:



$$\lambda_i = \sum_{c=1}^{2} f_c(d_c, i).\lambda_c$$

$$\pi_i = \frac{\pi_{-s}}{\mu}\prod_{n=-s}^{i-1}\lambda_n$$

OR

$$\pi_i = \frac{\lambda_{-s}\lambda_{-s+1}\ldots\lambda_{i-1}}{\mu^{i+s}}\pi_{-s}$$

$$\pi_{-s} = \frac{1}{1 + \frac{\lambda_{-s}}{\mu} + \frac{\lambda_{-s}\lambda_{-s+1}}{\mu^2} + \cdots + \frac{\lambda_{-s}\lambda_{-s+1} \dots \lambda_{R_1-1}}{\mu^{R_1+s}}}$$

OR

$$\pi_{-s} = \left(1 + \sum_{n=-s}^{R_1-1} \prod_{i=-s}^{n} \frac{\lambda_i}{\mu}\right)^{-1} \tag{12}$$

Let $\mathbf{d_c}$ ={ $d_{c,-s}$,…,$d_{c,\infty}$} (as found in expression (11)). For a given s and $\mathbf{d_c}$, let π_i be the corresponding steady-state probabilities. The average profit under Policy 3 can be expressed as:

$$g_3 = \max_s \sum_{i=-s}^{\infty} \left(\sum_{c=1}^{2} \left(\lambda_c . f_c\left(d_{c,i}\right) . \left(\mathrm{Rev}_c - l . L_i\left(d_{c,i}\right) \right) \right) + h . i^- \right) \pi_i \tag{13}$$

CHAPTER 4

COMPUTATIONAL ANALYSIS

In this chapter we seek to analyze the performance of the optimal policy and three alternative policies mentioned in the previous chapter. We hereby defined and computed several performance measures and explored the results under different operating conditions. We observed that certain system parameters have discernable effect on average profit and several performance measures. Furthermore, we investigate the benefit of inventory and capacity pooling by running the optimal model under different conditions and compare the average profit of pooled and individual system in order to measure the benefit of keeping a common stock and having a common server for two demand classes.

4.1. The Optimal Model

For the solution of the optimization model defined in Chapter 3, we use linear programming method. We use GAMS V22.5 for the solution of the model. The details of the linear model are given in the following:

$$\text{Maximize} \sum_{i \epsilon I} \sum_{d_1 \epsilon D_1} \sum_{d_2 \epsilon D_2} \sum_{a \epsilon A(i)} r_i(d_1, d_2, a)\, \pi_i(d_1, d_2, a)$$

subject to,

$$\sum_{d_1 \epsilon D_1} \sum_{d_2 \epsilon D_2} \sum_{a \epsilon A} \pi_j(d_1, d_2, a) - \sum_{i \epsilon I} \sum_{d_1 \epsilon D_1} \sum_{d_2 \epsilon D_2} \sum_{a \epsilon A(i)} p_{ij}(d_1, d_2, a)\, \pi_i(d_1, d_2, a) = 0\,, \forall j \epsilon I$$

$$\sum_{i \epsilon I} \sum_{d_1 \epsilon D_1} \sum_{d_2 \epsilon D_2} \sum_{a \epsilon A} \pi_i(d_1, d_2, a) = 1, \quad \pi_i(d_1, d_2, a) \geq 0 \qquad \forall i, d_1, d_2, a \qquad (14)$$

The objective of the linear program is to maximize the average profit per transition time subject to the balance equations which represent the probability that the system is in state j in steady-state is equal to flow rate into j (i.e. flow rate out of all states into j). Moreover last constraint guarantees that steady-state probabilities defined for each state must sum up to 1. The parameters and decision variables of the model are as follows:

Parameters:

i: state index, $i \in I$ (# of customers waiting in queue: i^+ ; inventory level: i^-),

$i = [-\infty,\infty]$

λ_c: arrival rate of customer class $c \in C=\{1,2\}$

μ: service rate

$\gamma = \mu + \sum_{c=1}^{2} \lambda_c$, To find profit per unit time, we simply multiply the objective function with γ.

$r_i(d_1, d_2, a)$: expected reward to be gained, given that the state is i, when due-dates d_1 and d_2 are quoted to customer 1 and 2 until the next transition respectively and action a is chosen

$p_{ij}(d_1, d_2, a)$: probability that next state will be j, given that the state is i, when due-dates $d_{1,i}$ and $d_{2,i}$ are quoted to customer 1 and 2 respectively and action a is chosen

$$p_{ij}(d_1, d_2, a = 1) = \begin{cases} \dfrac{\sum_{c=1}^{C} f_c(d_c).\lambda_c}{\gamma}, & j = i + 1 \\ \dfrac{\sum_{c=1}^{C} \left(1 - f_c(d_c)\right).\lambda_c}{\gamma}, & j = i \\ \dfrac{\mu}{\gamma}, & j = i - 1 \end{cases}$$

$$p_{ij}(d_1, d_2, a = 0) = \begin{cases} \dfrac{\sum_{c=1}^{C} \lambda_c}{\gamma} f_c(d_c), & j = i + 1 \\ \dfrac{\sum_{c=1}^{C} \lambda_c}{\gamma} \left(1 - f_c(d_c)\right) + \dfrac{\mu}{\gamma}, & j = i \end{cases}$$

Decision Variables

$\pi_i(d_1, d_2, a) =$ long-run fraction of decision epochs at which the system is in state i and actions $d_{1,i}$, $d_{2,i}$ and a are chosen

a= 1, to produce; 0, not to produce, $a \in A = \{0,1\}$

d_c: due-date quoted to customer c, $d_c \in D_c = \{0,1,2,3,4,5\}$

The expression to be used for the calculation of each performance measure is exhibited in detail in the following.

4.2. Performance Measures

For a given policy p, let $\pi_{i,p}$ denote the probability that the chain is in state i in steady-state under policy p, let $d_{1,i,p}$ and $d_{2,i,p}$ denote the due-date quoted to class 1 and class 2 customers in state i respectively under policy p and let $a_{i,p}$ denote the production action chosen in state i under policy p. Additionally $i_{max,c}$, corresponds to the first state at which class c customers are rejected and we define $i_{max}=\max(i_{max,1}, i_{max,2})$. Finally, –s corresponds to the state at which production stops.

By using the steady-state probabilities found with policy p, the following performance measures are defined for each policy:

Effective Arrival Rate of Class c Customers

$$\lambda_{eff,c} = \lambda_c \sum_{i=-s}^{i_{max,c}} f_c(d_{c,i,p})\pi_{i,p}$$

Effective Arrival Rate

$$\lambda_{eff} = \lambda_{eff,1} + \lambda_{eff,2}$$

Expected Rejection Rate for Class c Customers

$$\mathrm{Rej}_c = \lambda_c \pi_{i_{max,c},p}$$

Expected Rejection Rate

$$\mathrm{Rej} = \sum_{c=1}^{C} \mathrm{Rej}_c$$

Expected Waiting Time for Class c Orders

It is the expected time a class c customer order waits in the production queue from the arrival of the customer to the delivery of the item.

$$W_c = \sum_{i=-s}^{i_{max,c}} E_i[\text{wait}] \frac{\lambda_c f_c(d_{c,i,p}) \pi_{i,p}}{\lambda_{eff,c}}$$

where $E_i[\text{wait}] = \frac{i+1}{\mu}$ if $i \geq 0$, $E_i[\text{wait}]=0$ otherwise

Expected Waiting Time for Orders

It is the expected time a customer order waits in the production queue from the arrival of the customer to the delivery of the item.

$$W = \frac{\sum_{c=1}^{C} \lambda_{eff,c} W_c}{\lambda_{eff}}$$

Expected Actual Waiting Time for Class c Customers

It is the expected additional time that class c customers wait after the quoted due-date.

$$AW_c = \sum_{i=-s}^{i_{max,c}} E_i[\text{wait}] \frac{\lambda_c f_c(d_{c,i,p}) \pi_{i,p}}{\lambda_{eff,c}}$$

where $E_i[\text{wait}] = L_i(d_{c,p,i})$ if $i \geq 0$, $E_i[\text{wait}]=0$ otherwise

Expected Actual Waiting Time of an Arriving Customer

It is the expected additional time that customers wait after the quoted due-date.

$$AW = \frac{\sum_{c=1}^{C} \lambda_{eff,c} AW_c}{\lambda_{eff}}$$

Expected Number of Outstanding Orders

It is the total number of orders waiting in the production queue who accept the due-date quoted and wait until the item is produced.

$$L = \sum_{i=0}^{i_{max}} i.\, \pi_{i,p}$$

Expected Number of Outstanding Class c Orders

It is the total number of class c customers waiting in the production queue who accept the due-date quoted and wait until the item is produced.

$L_c = \lambda_{eff,c}.W_c$

The sum of expected outstanding class 1 and 2 orders yields the expected number of customers in the system:

$$\sum_{c=1}^{2} L_c = \sum_{i=0}^{i_{max}} i.\pi_{i,p}$$

4.3. Computational Analysis for Optimal Policy

In this section, we seek to analyze the response of the optimal policy to different operating parameters. We investigate the behavior of the optimal policy under different revenues of each class, Rev_1 and Rev_2, unit holding cost h, traffic intensity ρ (which is equal to $(\lambda_1+\lambda_2)/\mu$) and arrival rates ratio λ_1/λ_2.

We include five and six different due-dates in the due-date set of class 1 and 2 respectively, in which $d_{max,1}$ is equal to 4 days and $d_{max,2}$ is equal to 5 days where $d_1 \epsilon D_1 = \{0,1,2,3,4\}$ and $d_2 \epsilon$ $D_2=\{0,1,2,3,4,5\}$. The corresponding acceptance probabilities of each day in the predefined due-date ranges are exhibited for each class in Figure 1. As already been mentioned in the previous chapter, class 1 customers are more due-date sensitive compared to class 2 customers.

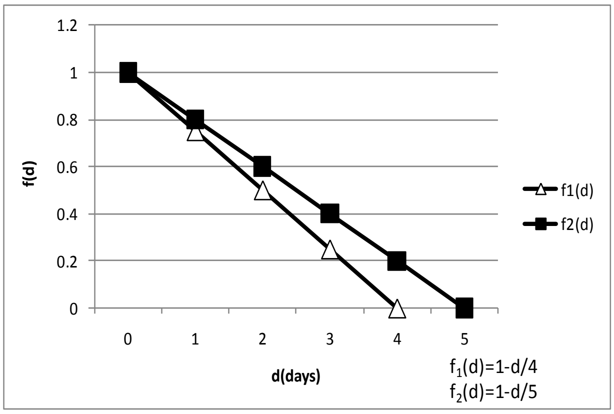


Figure 1 - Acceptance Probability Functions of Class 1 and 2

Throughout the analysis, we keep service rate as μ=1 and take unit lateness cost, l, the same for each class. For the comparison of model results under different policies, we use average profit per unit time. Hence we multiply the average profit per transition time under optimal policy with $\gamma = \mu + \sum_{c=1}^{2} \lambda_c$ in order to obtain the average profit per unit time. In the remainder parts of the thesis, we use the statement "average unit profit" for average profit per unit time.

It is intuitive that other parameters kept constant, average unit profit increases with increasing total revenue of each class. Similarly, for increasing unit holding and lateness costs, average unit profit decreases. However the negative effect of holding cost continues up to the point where the model begins not to keep base-stock due to high holding cost. It is also observed that for higher class 1 revenues, earlier due-dates are quoted and the rejection point for class 1 moves forward, because more base-stock is kept in order to benefit from higher revenues. For class 2 customers, the number of rejections increases and longer due-dates are quoted with increasing revenue of class 1.

4.3.1. Effect of System Parameters on the Performance of the Optimal Policy

4.3.1.1. Effect of Traffic Intensity (ρ)

It is seen that average unit profit is increasing with increasing traffic intensity. However, the effect of this increase on average unit profit diminishes with increasing ρ and disappears when it reaches to a certain value (see Figure 2). As traffic intensity ρ increases, the model tends to keep more base-stock and bears more holding cost. Moreover longer due-dates are quoted in higher traffic intensities which decrease the expected revenue and lateness cost when there are customers waiting in the system. After a certain point, the model begins to reject all the incoming demand due to limited capacity and average unit profit begins not to change. This diminishing return can be interpreted such that the decrease in lateness cost is dominated by the decrease in expected revenue and increase in the holding cost in higher traffic intensities, and the aggregate effect of these three system parameters turn out to be a decrease in the marginal gain of one more customer.

It is also seen from the graph that average unit profit increases with increasing arrival rate ratio λ_1/λ_2. It is straightforward that under the same traffic intensity, increasing the arrival rate of the class with higher revenue will increase the profit.

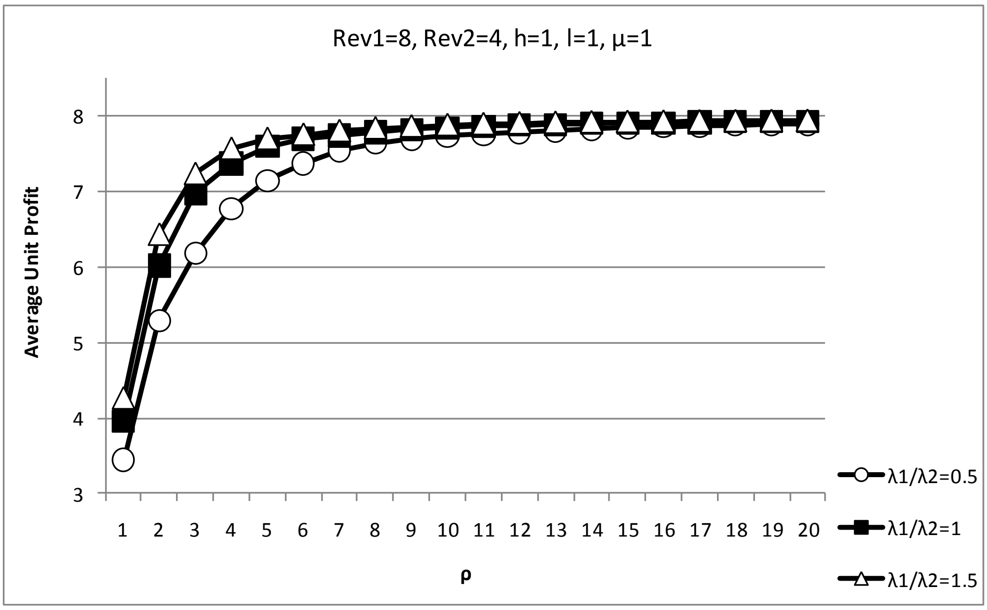


Figure 2 - Average Unit Profit vs. Traffic Intensity

4.3.1.2. Effect of Revenues (Rev_1, Rev_2)

We also investigated the response of average unit profit to the increasing revenue of a certain class by keeping the total revenue of the classes constant. As the revenue of class 1 increases, average unit profit behaves differently for different arrival rate pairs due to the tradeoff between the high revenue and high arrival rate of the classes. With the increasing revenue of class 1, the model quotes shorter due-dates to class 1 customers which increases the effective arrival rate of class 1, and after a certain value of class 1 revenue, the effective arrival rate of class 1, $\lambda_{eff,1}$ converges to λ_1 as the model begins to accept all class 1 customers by quoting zero due-date (see Figure 3a). Similarly with increasing revenue of class 1, longer lead times are quoted to class 2 customers and number of rejected class 2 customer increases. Therefore, the effective arrival rate of class 2, $\lambda_{eff,2}$ decreases with increasing class 1 revenue. After a certain value, all class 2 demands are rejected and the effective arrival rate of class 2 customers converges to zero as seen in Figure 3b.

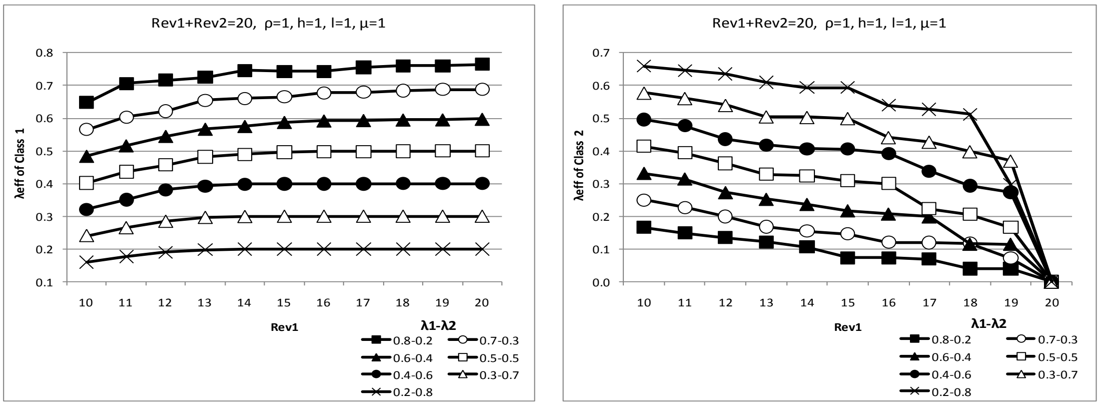


Figure 3 - Effective Arrival Rate of Class 1- 2 Customers vs. Class 1 Revenue

a) λ_{eff} of Class 1 vs. Revenue 1 b) λ_{eff} of Class 2 vs. Revenue 1

After investigating the response of effective arrival rates to different class 1 revenues, we can better explain the effect of class 1 revenue on the average unit profit. When class 1 arrival rate is dominant, increasing class 1 revenue affects average unit profit positively as expected. However, for the dominant class 2 arrival rate case, the average unit profit decreases up to a certain value with increasing class 1 revenue and after that point, begins to increase again and thus does not show monotone behavior (see Figure 4). The breaking point is generally realized at the level where the non-dominant class begins to outweigh the other class with its effective arrival rate. After this point, the graph follows a similar pattern with the previous instances having dominant class 1 arrival rates. We can conclude that increasing revenue of the class with dominating effective arrival rate, increases average unit profit, however increasing revenue of the class with non-dominating effective arrival rate decreases average unit profit.

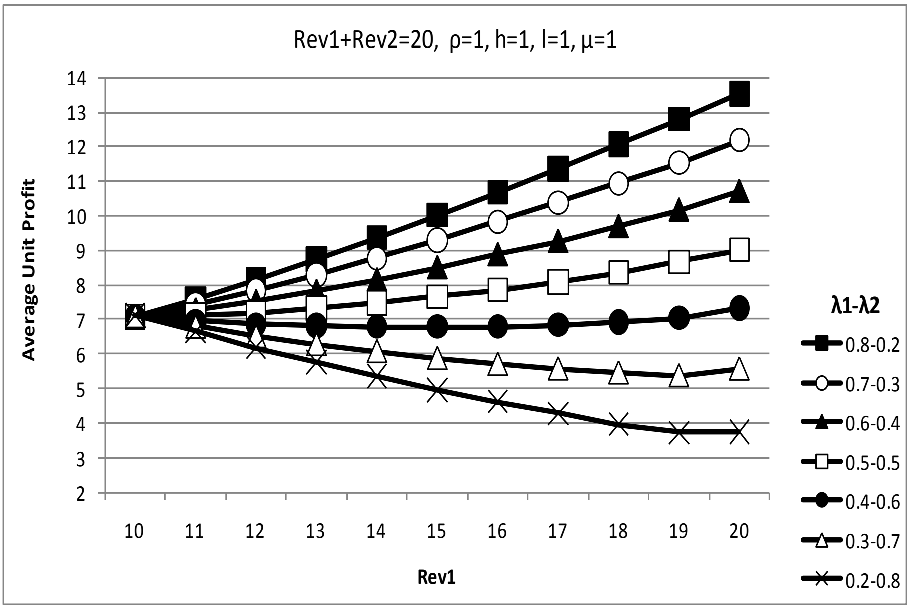


Figure 4 - Average Unit Profit vs. Revenue of Class 1

4.3.1.3. Effect of Arrival Rates (λ_1, λ_2)

Similar to profitability, customer satisfaction is also an important issue which in turn can result in again higher profits. Therefore, manufacturers may sometimes set service level targets such as expected waiting time of customers, etc. in order to protect the customer satisfaction. Especially under high traffic intensities (as a result of high arrival rates), it becomes harder to achieve these targets. Hence, profitability may be sacrificed when necessary by limiting the arrivals of customers to the system, i.e. by rejecting the customers, in order to decrease the traffic intensity for the sake of achieving service level target. Therefore, we next analyze the effect of arrival rates on service levels. In order to analyze the effect of arrival rates to the service level more deeply, we investigate the effect of arrival rates to the expected actual waiting time of class 1 and expected number of outstanding class 1 orders.

In this analysis, arrival rate of class 2, λ_2, is kept zero in order to simulate a single-class system. Since the marginal cost of accepting customers increases due to the increasing lateness cost which is a function of both due-dates and the state of the system, the net benefit is expected to decrease. In order to prevent this decrease, the model begins to quote longer due-dates to class 1 and finally rejects the customers when λ_1 is increasing. However with increasing λ_1, the system spends more time in states with higher number of customers, which increases the expected actual waiting time of class 1. Hence with increasing λ_1, expected actual waiting time of class 1 may either increase or decrease according to the net effect of λ_1 and longer due-dates quoted to class 1. Moreover the impact of base-stock level increase is also relatively strong compared to the change in due-dates and λ_1. The impact of the base-stock increase can be observed as a sharp decrease in the expected actual waiting time of class 1 under high λ_1 and high revenues, as seen in Figure 5. Hence the expected actual waiting time of class 1 does not display a monotonous structure under increasing λ_1 due to the effect of changes in base-stock level and due-dates. However we observe from Figure 5 that when other parameters are kept constant, expected actual waiting time increases with increasing revenues.

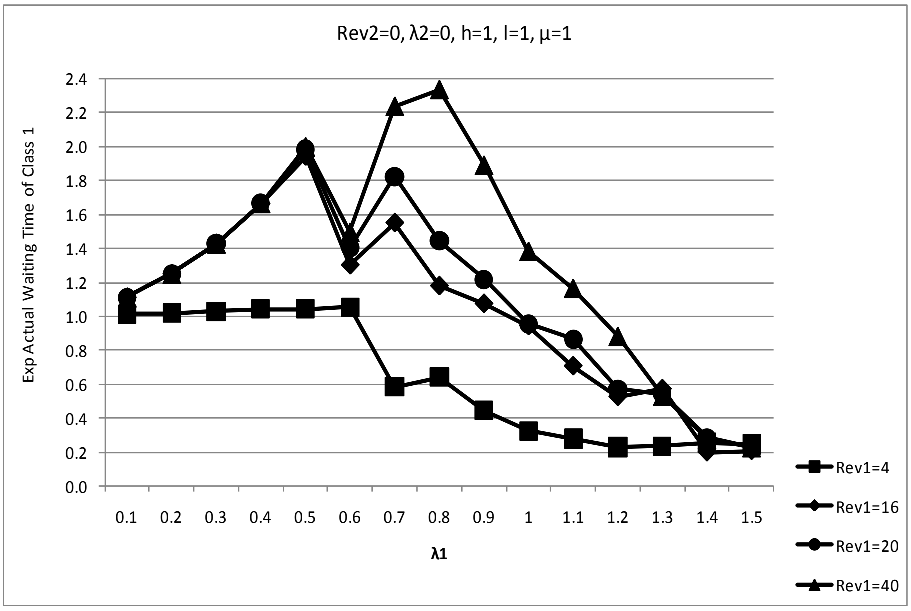


Figure 5 - Expected Actual Waiting Time of Class 1 vs. Arrival Rate of Class 1

Considering again the same single-class system above and similar to the expected actual waiting time of class 1, we see that expected number of outstanding class 1 orders also does not display a monotonous structure under increasing λ_1. Again the impact of base-stock increase due to the increase in traffic intensity can be clearly observed under high λ_1. However the model does not keep base-stock under lower λ_1, hence the pattern of expected number of outstanding class 1 orders under increasing λ_1 is mainly determined by only the due-date policy for lower λ_1 values.

In practice, as traffic intensity in a queuing system increases, the service level deteriorates, since the waiting time and the number of the customers increases. However as we can see from Figure 5 and Figure 6 that when base-stock level and due-dates are decided effectively, the service level does not necessarily deteriorate, although the shop-floor would have to deal with an increased level of congestion.

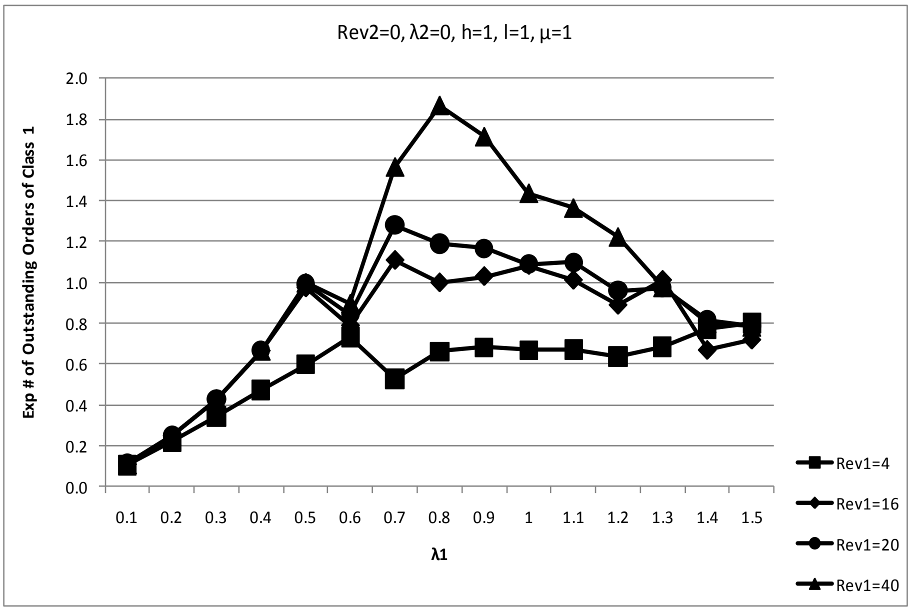


Figure 6 - Expected Number of Outstanding Class 1 Orders vs. Arrival Rate of Class 1

Next we analyze the change in service level with respect to demand mix λ_1/λ_2 under a given traffic intensity. We observe that with increasing arrival rates ratio λ_1/λ_2, expected actual waiting time of class 1 may either increase or decrease. On the other hand with increasing $\lambda_1/\lambda_{2,}$ the model may quote longer due-dates to class 1 and shorter due-dates to class 2 with the effect of increasing λ_1, or may quote shorter due-dates to class 1 and longer due-dates to class 2 with the effect of decreasing λ_2; and the opposite effect of increasing λ_1 and decreasing λ_2, makes it hard to predict the net effect of these two factors on class 1 and class 2 due-dates. However we see that due-dates of both classes are generally increasing with increasing λ_1/λ_2 which means that the effect of increasing λ_1 is more dominant on class 2 due-dates. Another significant result is that due-dates and base-stock levels are more robust under increasing λ_1/λ_2, in the sense that there are smaller increases in the due-dates quoted compared to the single-class case and nearly no change in the base-stock level under increasing λ_1/λ_2. Accordingly, the effect of λ_1 (on increasing the probability that there are more customers in the system) is generally more dominant than the effect of due-dates under most of the instances. Hence, we can conclude that increasing λ_1/λ_2, which also increases λ_1, is expected to increase the expected actual waiting time of class 1 in most of the instances exhibited in Figure 7.

We observe that λ_1/λ_2 does not have a significant effect on the expected actual waiting time under low revenues. However for higher revenues, base-stock levels and due-dates are more sensitive to λ_1/λ_2, which also makes expected actual waiting time of class 1 sensitive to λ_1/λ_2. Hence we can observe more fluctuations in the expected actual waiting time of class 1 under high revenues as seen in Figure 7.

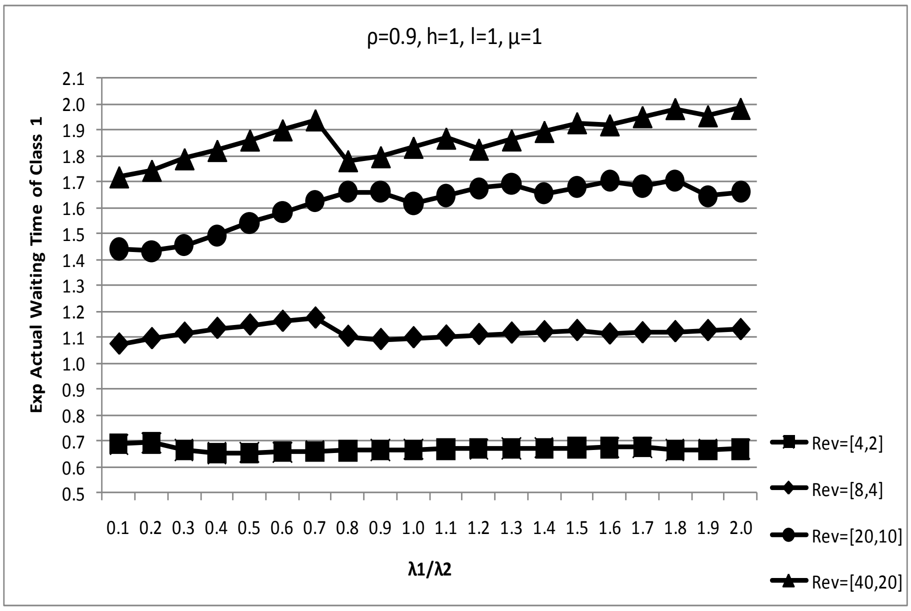


Figure 7 - Expected Actual Waiting Time vs. Arrival Rates Ratio

We also analyzed the effect of arrival rates ratio λ_1/λ_2 on the expected number of outstanding class 1 orders under constant traffic intensity. We see that the effect of increase in λ_1 due to the increase of λ_1/λ_2, is more dominant than the effect of longer due-dates quoted. Hence the number of outstanding orders of class 1 increases with increasing λ_1/λ_2. Differently from single-class system, we see that increasing λ_1/λ_2 does not decrease the expected number of outstanding orders of class 1 including the instances where longer due-dates are quoted. Again we do not see any impact of base-stock level on the expected number of outstanding orders of class 1, since no change is observed for any λ_1/λ_2 value.

Under higher revenues, shorter due-dates are quoted in order not to lose customer and more customers are accepted to the system, hence the number of outstanding class 1 orders increase with increasing revenues.

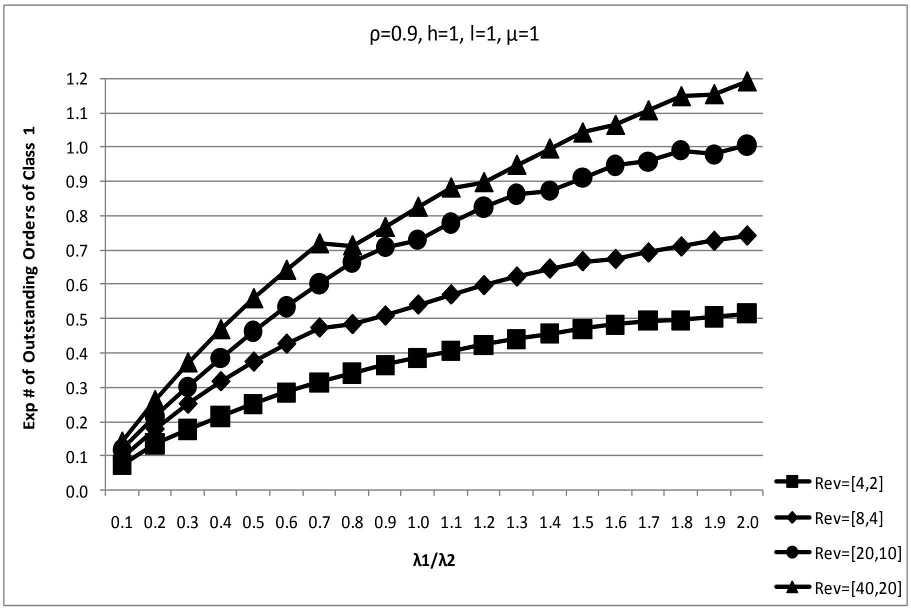


Figure 8 - Expected Number of Outstanding Class 1 Orders vs Arrival Rates Ratio

4.4. Comparison of the Optimum Policy and Alternative Policies

In this section, we investigate the behavior of the alternative policies relative to the optimal policy under several instances with different system parameters. For the comparison, we use the parameters unit holding cost, traffic intensity ρ and arrival rate ratio λ_1/λ_2. As we already explained the alternative policies in Chapter 3, we prefer to summarize the certain properties of these alternative policies again in order to emphasize the expected responses of these policies under different operating conditions.

Policy 1 is an accept-all policy where no rejection of customer demand takes place. Therefore the weakness of this policy is that there is only one option for due-date quotation which is to quote zero due-date to arriving customers regardless of the status of the system. Since no rejection is allowed, the model may sometimes even yield negative average unit profit (losses). This shortcoming is expected to be noticed when the flexibility of due-date quotation and customer rejection is important, i.e. under considerably high arrival rates.

In Policy 2, base stock level, rejection points of both classes and constant due-dates are determined optimally (through exhaustive-search). Since customer rejection is allowed for both types of customer classes, Policy 2 is expected to yield better results than Policy 1 especially when the arrival rates are high. Additionally, base-stock level, rejection points of both classes and constant due-dates are determined by exhaustive-search method. Differently than Policy 1, a customer can be rejected by quoting d_{max} due-date. Also at the states between zero and rejection points of each class, different due-dates from the range used in the optimal policy can also be quoted to the customers, which makes Policy 2 superior to Policy 1. However, the shortcoming of Policy 2 is again the lack of flexibility in the due-date quotation compared to the optimal policy. In Policy 2, when there is stock, class 1 is accepted by quoting zero due-date and class 2 is quoted a fixed due-date within the predefined range D_2; when there is no stock, either a fixed due-date is quoted or rejected. Hence the due-dates quoted can only be changed at certain states.

Policy 3 is a myopic policy where the due-dates of both classes are determined for each state independently. After the determination of due-dates, base-stock level is determined by line-search method. The shortcoming of this method is that every state is independently optimized by maximizing the difference of revenue and lateness cost by ignoring the interaction between the states. Also the due-date quoted for any class has no effect on the due-date selection of the other class and due-dates are determined regardless of the status of the system. Therefore, the policy never rejects any customer when there is stock on-hand. In negative states, there is no lateness cost which creates a tradeoff between quoting shorter or longer due-dates together with the holding cost, hence Policy 3 simply accepts the customer demand by quoting zero due-date in negative states in order to gain the revenue without any additional cost.

Analyses show that in terms of average unit profit, neither the performance of Policy 2 nor that of Policy 3 dominates the other. However, it is expected that the performance of both policies should be greater than or equal to Policy 1. Policy 1 is not flexible enough to reject a demand when the total expected lateness and holding cost of the customer overweighs the revenue brought by the customer. Therefore Policy 1 may sometimes quote unrealistic due-dates to the customers which may lead to unethical situations.

In order to measure the relative loss of using alternative policy instead of the optimal policy, we define the relative profit loss for each alternative policy. Let O and P_p be the average unit profit obtained by using optimum model and alternative policy p respectively. The % profit loss of policy p denoted by ΔP_p is calculated as follows:

$$\Delta P_p = \frac{O - P_p}{O} \text{ x } 100$$

ΔP_p indicates the performance of the policy; the policy with lower ΔP_p yields closer results to the optimal model.

4.4.1. Policy 1: Value of Controlled Arrivals and Customer Rejection

Keeping the total traffic intensity of the system constant, it is seen that the relative profit loss of Policy 1 is decreasing with increasing arrival rate ratio λ_1/λ_2. This is due to the relative flexibility of the optimum policy in due-date quotation compared to Policy 1. The shortcoming of Policy 1 comes out when class 2 customers have considerably high arrival rate, since rejection becomes a more frequently used tool as the existence of class 2 customers become more prominent. Therefore, Policy 1 yields closer results to the optimal model when class 1 demand is relatively higher. Similarly, the results worsen when the relative arrival rate of class 2 customers increases.

The effect of arrival rate ratio λ_1/λ_2 mentioned above decreases with increasing revenues as expectedly. With increasing revenues, optimal policy begins to quote shorter due-dates in order to increase the expected revenue by ignoring more stock-outs, and Policy 1 yields closer results to the optimal policy (see Figure 9).

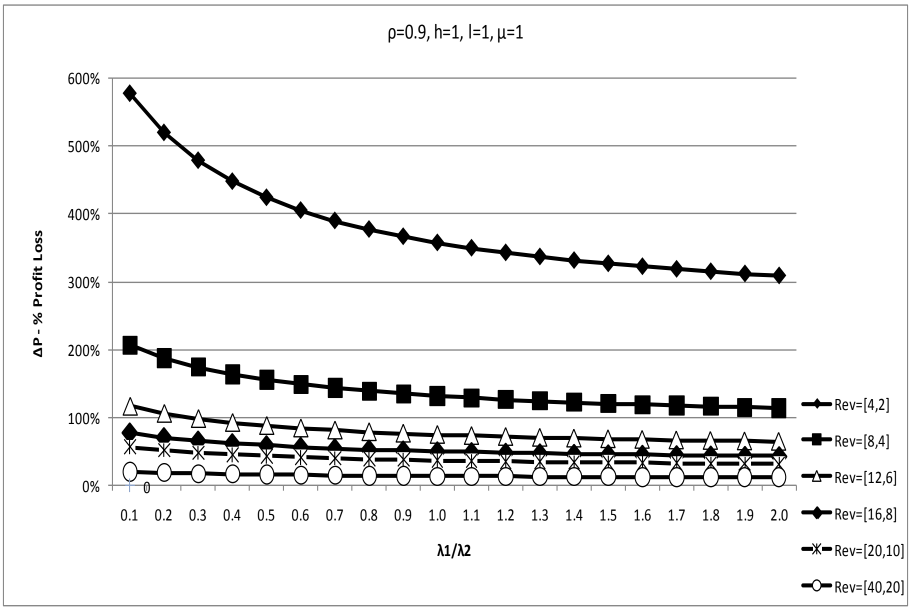


Figure 9 - Relative Profit Loss of Policy 1 vs. Arrival Rates Ratio

When the arrival rate of class 2 is kept constant, Policy 1 gives better results under smaller λ_1 values. Note that keeping the arrival rate of a certain class constant, while increasing the arrival rate of the other class, will also increase the traffic intensity ρ of the system. The increase in the traffic intensity of the system necessitates the quotation of longer due-dates and even the rejection of incoming class 2 demand. In such a case, Policy 1 will incur relatively high lateness cost due to the accept-all property. However for smaller traffic intensity values, optimal policy is expected to quote similar due-date policies with Policy 1, hence Policy 1 will yield closer results to the optimal in smaller λ_1 values.

The effect of arrival rate ratio λ_1 decreases with increasing revenues. With increasing revenues, the performance of Policy 1 becomes closer to the optimal policy, since the optimal policy tends to accept more customers by quoting shorter due-dates and the flexibility advantage of optimal policy in due-date quotation lose importance in high revenues (see Figure 10).

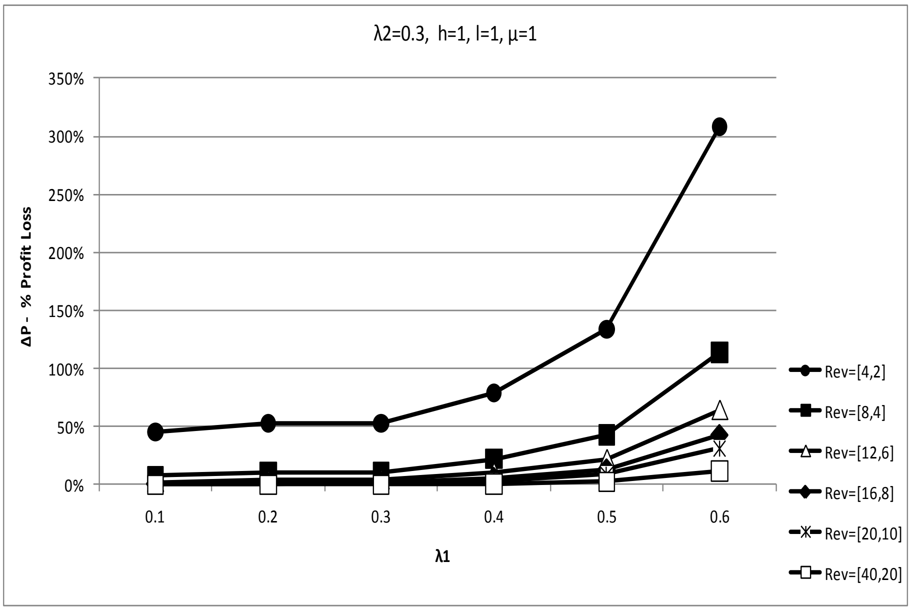


Figure 10 - Relative Profit Loss under Policy 1 vs. Arrival Rate of Class 1

4.4.2. Policy 2: Value of Information on Number of Waiting Customers

Since the rejection points of class 1 and 2 customers as well as base-stock level and fixed due-dates are determined by exhaustive-search in Policy 2, the structure of relative profit loss displays a non-monotonous structure according to different parameters such as revenue, holding cost, traffic intensity ρ and arrival rates ratio λ_1/λ_2. Base-stock levels of Policy 2, determined through exhaustive-search method for different instances are very similar to that of the optimal policy. We can conclude that the lack of flexibility in due-date quotation does not worsen the performance of Policy 2 very much, since optimal base-stock level is found in most of the instances through exhaustive-search. Hence we observe that the performance of Policy 2 is very close to the optimal results. During the analysis, we use due-date sets consisting of five and six elements for class 1 and 2 respectively, which may not be regarded as rich. If we further limit the due-date range with only two elements consisting of zero and d_{max}, two policies are expected to yield the same results, since the limitation of fixed due-date quotation for certain states would be pointless. On the contrary, we expect that the difference between the performances of these two policies would be more divergent in case of a due-date range with more precision, since the flexibility of due-date quotation becomes more valuable in such case.

Both the optimal policy and Policy 2 has an increasing pattern with increasing traffic intensity ρ as expected. However, due to the nature of Policy 2 as explained above, the difference of optimal policy and Policy 2 results does not exhibit a monotonous structure. As seen in Figure 11, the increase in the average unit profit of Policy 2 has a diminishing rate, since both in optimal policy and Policy 2, the model tends to keep more base-stock in order not to lose class 1 customers. Therefore the cost of gaining class 1 revenue increases with increasing class 1 demand, resulting in diminishing returns in both policies. At the instances with lower revenues, diminishing returns and the convergence of the return to a constant value is expected to be more observable.

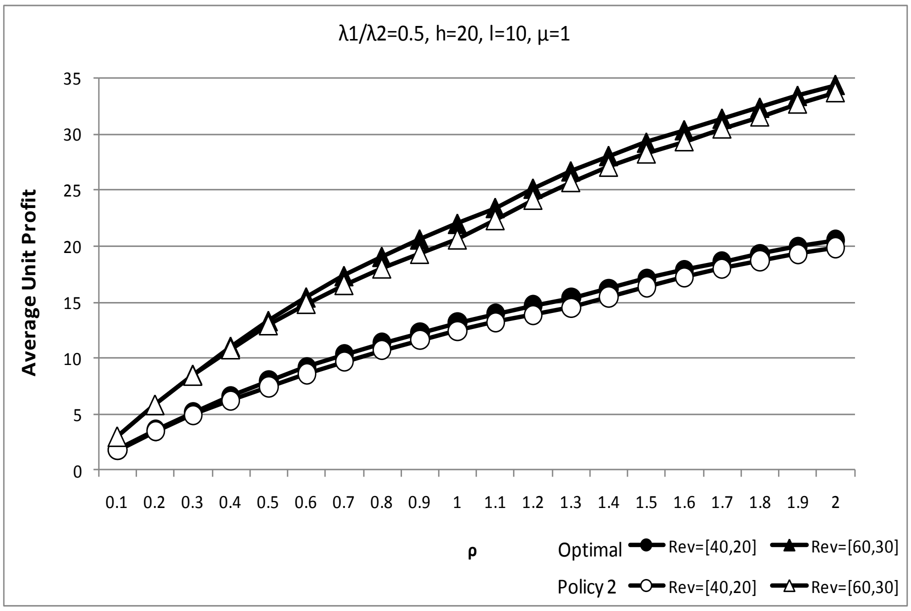


Figure 11 - Average Unit Profit of Optimal Policy and Policy 2 vs. Traffic Intensity

Profit loss of Policy 2 ΔP_2 under increasing traffic intensity ρ, also has a non-monotonous structure (see Figure 12). However, it is observed that in very small and very large ρ values ΔP_2 decreases. When there is not sufficient capacity to meet demand, the number of rejections increases, and the flexibility of due-date quoting becomes irrelevant since it becomes sufficient to be able to reject the demand. On the other hand, when there is excessive capacity (with small ρ) to meet demand, most of the demand is generally met immediately by quoting zero due-date and similarly the flexibility of due-date quoting loses its importance. Hence the performance of Policy 2 becomes closer to that of optimal policy when ρ is very small or very large (see Figure 12). Moreover we see that the profit loss converges to zero at low and high traffic intensities. Since, in low traffic intensities, both the optimal model and Policy 2 accept few customers to the system which is resulting with a less profit. Similarly, when traffic intensity is high, both optimal policy and Policy 2 tends to keep more base-stock and spend more time in negative states and the number of customer rejections increases. Hence the profit of both optimal policy and Policy 2 converge to zero.

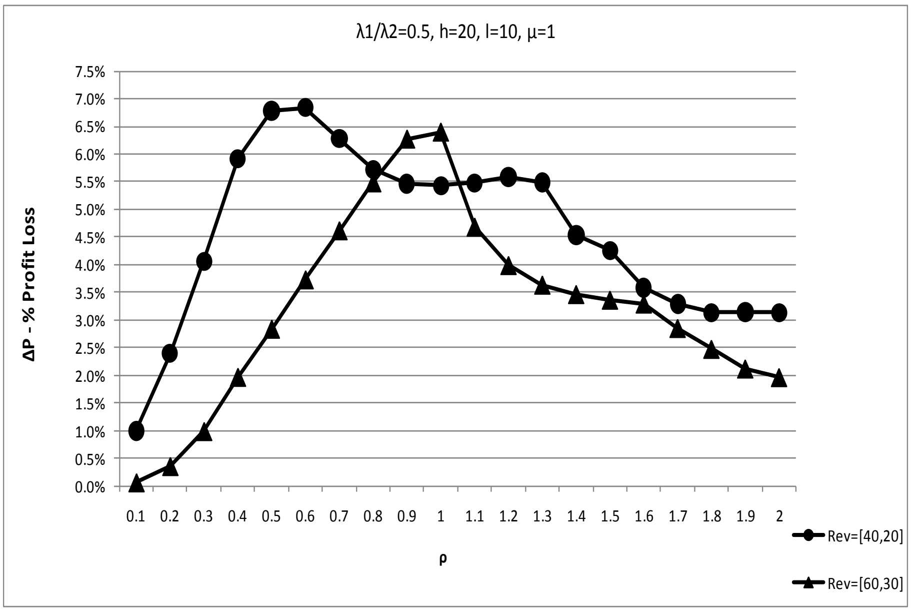


Figure 12 - Relative Profit Loss of Policy 2 vs. Traffic Intensity

When we analyze the performance of Policy 2 with respect to a change in unit holding cost, we observe that Policy 2 results diverge from the optimal results as unit holding cost increases (see Figure 13). The computational results show that, base-stock level of Policy 2 is very close to the optimal policy when the unit holding cost is small. It can be explained such that optimal policy and Policy 2 both tend to meet demand from inventory by keeping more base-stock when the unit holding cost is small. However for instances with higher unit holding cost, optimal policy and Policy 2 both begin to keep fewer items in stock and quoting due-dates optimally and flexibly becomes more important. Since Policy 2 is not as flexible as optimal policy in quoting due-dates, the performance difference of optimal policy and Policy 2 becomes clearer in system conditions with higher unit holding cost.

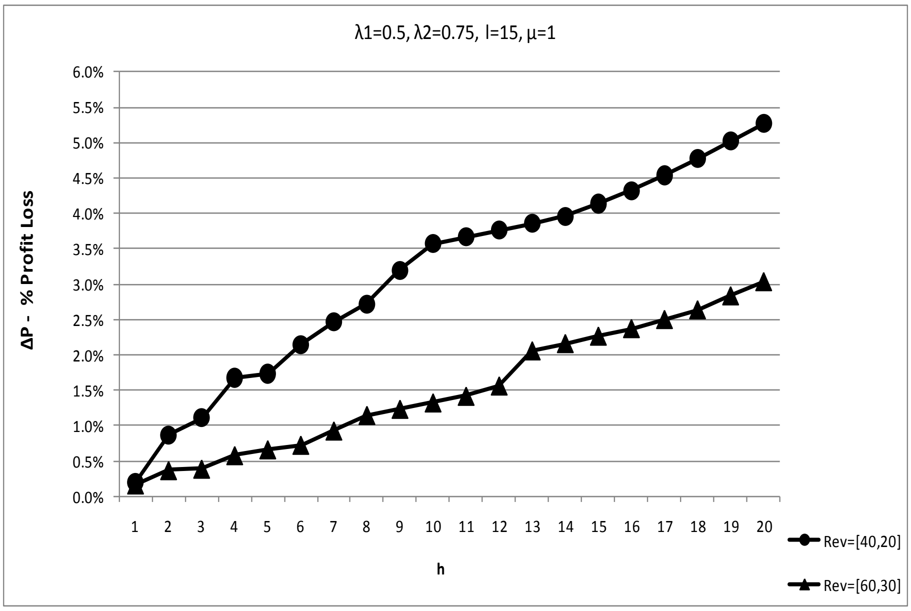


Figure 13 - Relative Profit Loss of Policy 2 vs. Holding Cost

4.4.3. Policy 3: Value of a Far-Sighted Policy

Relative loss ΔP % of Policy 3 is expected to increase with increasing traffic intensity. Under small traffic intensity ρ, most of the customers are quoted shorter due-dates in each policy since there is relatively low risk of stock-out in a system with small traffic intensity ρ. Hence optimal policy and Policy 3 are expected to yield nearly the same results in such a case. However for larger ρ values, the performance of Policy 3 is expected to diverge from the optimal result. This increasing difference between the optimal policy and Policy 3 can be explained by the due-date determination method of Policy 3. In Policy 3, due-date for each class is determined by maximizing the function below in every state;

$$\max_{d_{c,i}} \left(f_c(d_{c,i}).\left(Rev_c - l_c L_i(d_{c,i})\right)\right), \qquad \forall c \in C = \{1,2\}, \forall i \in I$$

The function is composed of revenue and expected lateness cost when due-date $d_{c,i}$ is quoted to class c customer. This maximization is done independently for each class, so there is no relation between the due-dates quoted to different classes of customers as a result of this approach. Also note that the arrival rate of each class does not have any effect on the maximizer of the immediate return function. This implies that the due-date quotes under Policy 3 are ignorant to the traffic intensity in the system. Thus, for the instances with large ρ, Policy 3 is expected to quote shorter due-dates relative to optimal policy. In some instances optimal policy rejects most of the arriving customers, whereas Policy 3 continues to quote due-dates other than d_{max}. In this situation, the number of customers in the system is expected to be greater in Policy 3 compared to the optimal policy. Hence, for a given base-stock level, the gap between the results of optimal policy and Policy 3 widens with increasing traffic intensity and total lateness cost under Policy 3 is likely to be greater than that value under the optimal policy.

We further observe that the increase in relative profit loss ΔP_3 % exhibits a decreasing behavior and converges to a constant value as traffic intensity increases (see Figure 14). Due to the insufficient capacity (in sufficiently large states), the number of rejections increases in both optimal policy and Policy 3 (see Figure 15). Due to the increase in customer rejections, the difference in the results of optimal policy and Policy 3 stays constant after sufficiently large ρ.

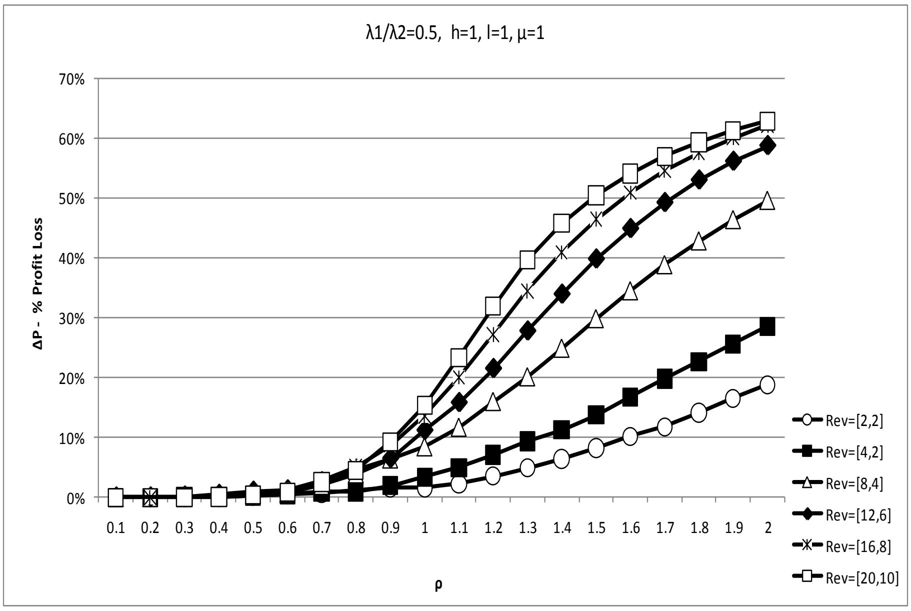


Figure 14 - Relative Profit Loss of Policy 3 vs. Traffic Intensity

For the case we analyzed, the rejections are only observed for class 2 customers; however at conditions where the capacity is much more limited and is not sufficient to meet class 1 demand, class 1 rejections also tend to increase.

It is also observed that base-stock level of Policy 3 is likely to be greater than optimal, since in Policy 3, more customers are accepted by quoting shorter due-dates compared to the optimal policy. Therefore the number of customers in the system is expected to be higher in Policy 3. And the policy tends to keep more base-stock in order to compensate the high lateness cost resulting from these short due-dates quoted.

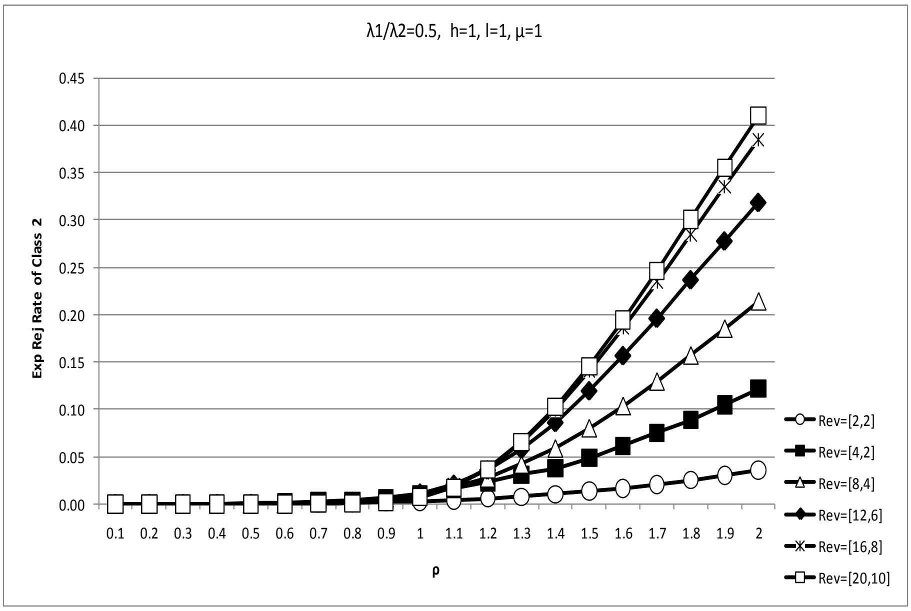


Figure 15 - Expected Class 2 Rejection Rate of Policy 3 vs. Traffic Intensity

Expected actual waiting time of class 1, which is the time between the delivery of the item and the due-date quoted, is expected to increase with increasing traffic intensity for Policy 3. The increase is higher for instances with higher revenues, since the policy has a tendency to quote shorter due-dates in order not to lose customer which brings relatively high revenues. Consequently the additional time the customers wait after the due-date, is expected to increase with increasing lateness cost due to the shorter due-dates quoted. This increase is diminishing with increasing number of rejections and converges to a constant value when the system becomes overcrowded with increasing ρ and the policy accepts no more customers.

Actual waiting time of class 2 customers is also expected to exhibit the same behavior with actual waiting time of class 1 customers when traffic intensity is increasing. Note that the opposite trend in service level was observed under the optimal policy.

Similarly, the expected number of outstanding of class 1 orders increases with increasing traffic intensity ρ of the system. In Policy 3, more customers accumulate in the system due to the different behavior of optimal policy and Policy 3 in due-date quotation for high traffic intensity cases.

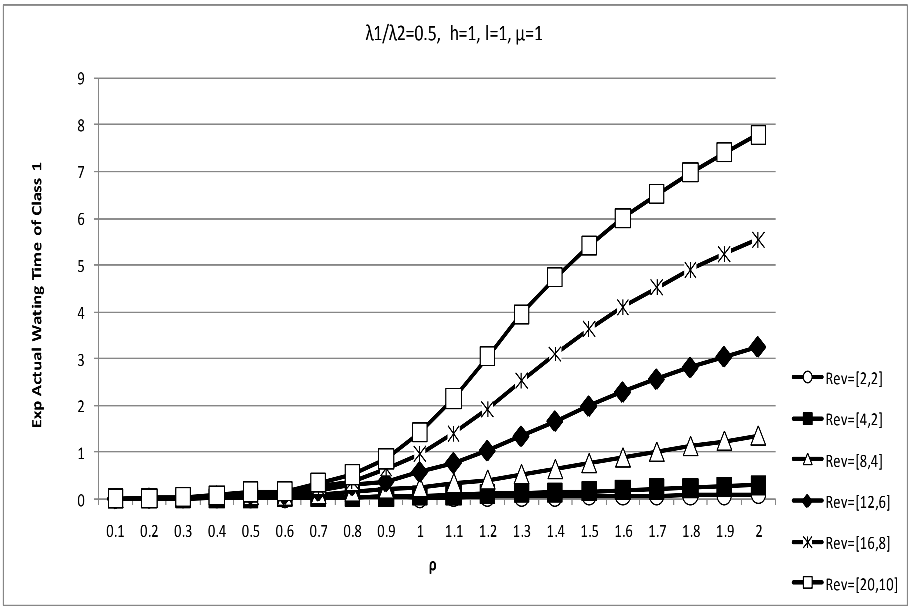


Figure 16 - Expected Actual Waiting Time of Class 1 vs. Traffic Intensity

4.5. The Benefit of Inventory and Capacity Pooling

In this section we seek to illustrate the benefit of joint pooling of inventory and capacity in a system with limited production capacity and facing two customer classes. Basically, in a system with both inventory and capacity pooling, a common stock and a server are used in order to meet the incoming orders of different demand classes (see Figure 17). In order to explore the benefit of pooling, we define an alternative system with two servers and inventories, each of which is allocated to a different customer class, and compare this alternative system with the original system where the inventory and capacity is pooled. It is straightforward to think that, both inventory and capacity pooling, jointly or separately, would increase the total system profit.

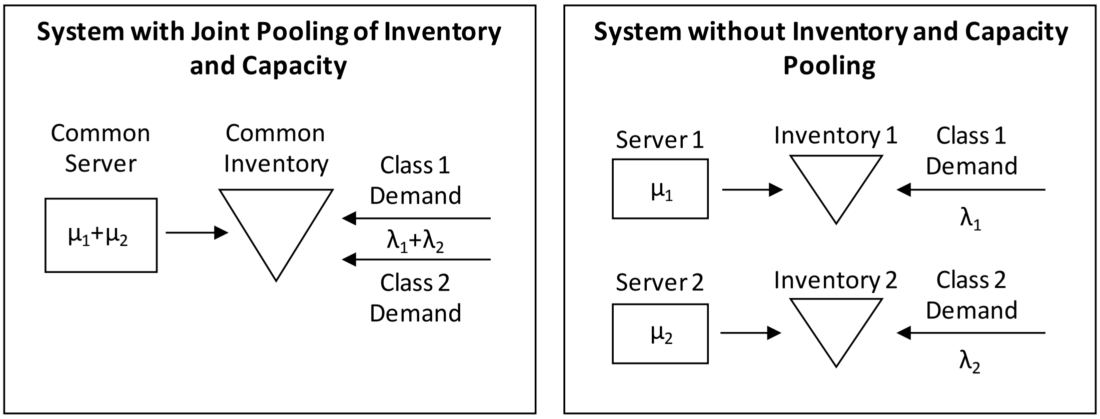


Figure 17- Joint Pooling of Inventory and Capacity

According to the previous studies conducted, inventory pooling results with a reduction in the inventory costs through statistical economies of scale. When the system with one common server and two separate stocks for each demand class and the system with one common server and inventory are compared in terms of costs incurred, it is expected that inventory pooling decreases the amount of base-stock to be kept, thereby decreasing the total holding cost, by protecting the same service level. Similarly, when a system with two separate servers for both demand classes having service rates μ_1 and μ_2 and a system with a common server having a service rate of $\mu_1+\mu_2$ are compared, it is expected that in the latter system with capacity pooling the expected lead time decreases, since the utilization rate of the server increases when the capacity is pooled. In both inventory and capacity pooling cases, the total system cost is expected to decrease. Therefore it is reasonable to think that the joint effect of inventory and capacity pooling is also expected to decrease the total system cost. In order to verify it, we analyze the benefit of inventory and capacity pooling under different operating conditions with different revenue, arrival rate and unit holding cost values for the optimal policy and Policy 1 we defined in Chapter 3.

4.5.1. Benefit Obtained from Joint Pooling of Inventory and Capacity in Optimal Policy

In order to simulate the joint pooling of inventory and capacity under optimal policy, we ran the optimum model for 1080 instances consisting of different combinations of 15 revenue pairs and 18 arrival rate pairs for class 1 and 2, and 4 unit holding cost parameters exhibited in Table 1. The other parameters are kept constant for each run.

Table 1 - System Parameters of 1080 Instances Used in the Optimal Policy

Revenue 1	Revenue 2	Arrival Rate of Class 1 (λ1)	Arrival Rate of Class 2 (λ2)	Unit Holding Cost (h)
2	2	0.125	0.25	0.5
4	4	0.25	0.5	1
6	6	0.375	0.75	1.5
8	8	0.5	1	2
10	10	0.625	1.25	
4	2	0.75	1.5	
8	4	0.1	0.4	
12	6	0.2	0.3	
16	8	0.3	0.2	
20	10	0.4	0.1	
8	2	0.2	0.8	
16	4	0.4	0.6	
24	6	0.6	0.4	
32	8	0.8	0.2	
40	10	0.4	1.6	
		0.8	1.2	
		1.2	0.8	
		1.6	0.4	

For the simulation of a system where both inventory and servers are pooled for two demand classes, we ran two different models for each class with the parameters in the abovementioned instances in order to simulate a system with two separate supply chains receiving a demand of only one class and having a service rate of μ=1. In the first run, we keep the corresponding arrival rate of class 1 for the same instance while setting the arrival rate of class 2 to zero and service rate μ to 1. Similarly in the second run, we keep the corresponding arrival rate of class 2 for the same instance while setting the arrival rate of class 1 to zero and service rate μ to 1. Then, we sum up the results of these two runs, in order to obtain the optimum average unit profit for a system without inventory and capacity pooling. For the case with joint pooling of inventory and capacity, we ran the model for a system receiving both class 1 and 2 demand with a service rate of μ=2 (which is equal to the sum of class 1 and 2 service rates in the former case without inventory and capacity pooling).

For the calculation of the % benefit obtained from inventory and capacity pooling when optimal policy is employed, we define % benefit loss ΔB_O as follows:

$$\Delta B_O = \frac{g_{p,o} - g_{s,o}}{g_{s,o}} \text{ x } 100$$

Where $g_{p,o}$ is the average unit profit obtained with joint pooling of inventory and capacity under optimal policy and $g_{s,o}$ is the average unit profit obtained for the system with separate inventory and capacity for different demand classes under optimal policy.

The results support the expected profit advantage of the system with joint pooling of inventory and capacity (see Table 2).

Table 2 - Benefit of Joint Pooling of Inventory and Capacity for Optimal Policy

% Benefit Interval	Number of Instances
[-1,0]	29
[0,5]	490
(5,10]	281
(10,20]	200
(20,30]	45
(30,40]	21
(40,50]	14
Max % Benefit	46.47%

4.5.2. Benefit Obtained from Joint Pooling of Inventory and Capacity in Policy 1

We investigated the joint effect of inventory and capacity pooling on Policy 1. Similar to the analysis made in the previous subsection, we ran the model for both cases with and without pooling for the system where Policy 1 is employed. In order to measure the benefit obtained, we define ΔB_1 as the % benefit obtained from the joint pooling of inventory and capacity where Policy 1 (accept-all policy) is employed.

$$\Delta B_1 = \frac{g_{p,1} - g_{s,1}}{g_{s,1}} \text{ x } 100$$

Where $g_{p,1}$ is the average unit profit obtained with joint pooling of inventory and capacity and $g_{s,1}$ is the average unit profit obtained for the system with separate inventory and capacity for different demand classes.

In order to simulate the joint pooling of inventory and capacity pooling case in Policy 1, we ran the optimum model for 54 instances consisting of different combinations of 6 revenue pairs and 9 arrival rate pairs for class 1 and 2 exhibited in Table 3. The other parameters are kept constant for each run.

Table 3 - System Parameters of 54 Instances Used in the Optimal Policy

Revenue 1	Revenue 2	Arrival Rate of Class 1 (λ_1)	Arrival Rate of Class 2 (λ_2)
4	2	0.033	0.067
8	4	0.067	0.133
12	6	0.1	0.2
16	8	0.133	0.267
20	10	0.167	0.333
40	20	0.2	0.4
		0.233	0.467
		0.267	0.533
		0.3	0.6

During the analysis, we keep the arrival rate ratio λ_1/λ_2 constant in order to display the pure effect of the traffic intensity ρ on the benefit obtained from inventory and capacity pooling. According to the results, it is seen that the benefit obtained from joint pooling of inventory and capacity increases with increasing ρ. This can be explained with the requirement of additional capacity and/or inventory when the system is exposed to higher traffic intensity. In such a case, the benefit of inventory and capacity pooling becomes more distinctive.

4.5.3. Comparison of Optimal Policy and Policy 1 in Terms of Benefit Obtained from Joint Pooling of Inventory and Capacity

In order to compare the impact of inventory and capacity pooling on different policies, we ran the cases with and without pooling for both optimal policy and Policy 1 by using the same parameter set exhibited in Table 3. The results show that benefit of pooling is higher in Policy 1 (see Table 4). This can be explained with the relatively high lateness and holding cost of Policy 1 compared to the optimal policy.

Table 4 - % Benefit Difference of Optimal Policy and Policy 1 for the Joint Pooling of Inventory and Capacity

% Benefit Interval	Number of Instances in Optimal Policy	Number of Instances in Policy 1
[0,5]	53	12
(5,10]	1	19
(10,20]	0	11
(20,30]	0	6
(30,45]	0	3
(45,60]	0	3
Max % Benefit	5.64%	57.55%

When we compare the benefit obtained in optimal policy and Policy 1, we see that the benefit obtained for two policies become closer when traffic intensity is small, and this is distinctive especially in the instances with higher revenues. Both in optimal policy and Policy 1, increasing traffic intensity also increases the benefit obtained from inventory and capacity pooling.

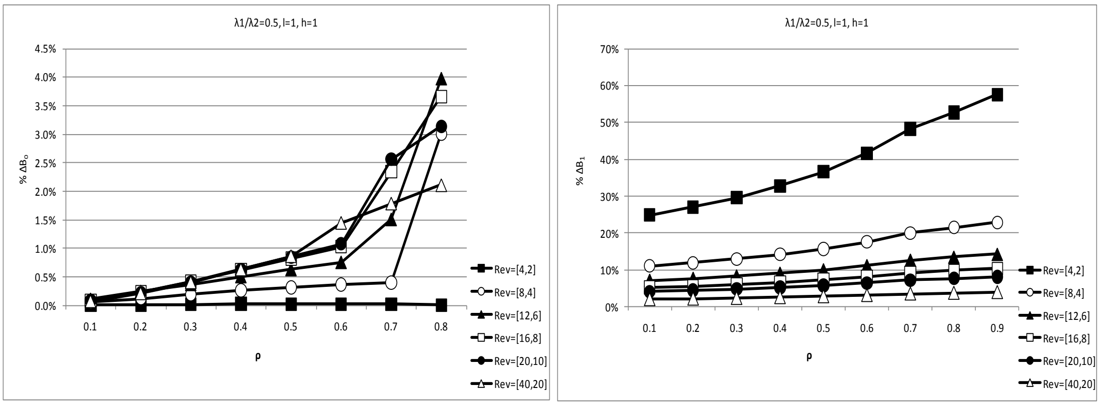


Figure 18 - % Benefit Obtained from Inventory and Capacity Pooling in Optimal Policy and Policy 1 vs. Traffic Intensity

Optimal policy differs from Policy 1, in that, it uses the capacity effectively by balancing the arrival rates to the system with the net profit obtained under a certain state.

Figure 18 shows that the benefit of pooling in the presence of effective quotation is significantly lower compared to the benefit of pooling under Policy 1. The reason is though effective due-date quotation; the optimal policy is using the capacity effectively and balances the lateness cost with holding cost. However, Policy 1 does not have the tool to balance lateness cost with holding cost and, therefore capacity pooling and inventory pooling significantly helps reducing the cost. On the other hand, we note that there might be cases where inventory pooling actually deteriorates the profit under Policy 1. When traffic intensity is not high, and two classes are too wide apart (in terms of revenues and arrival rates) pooling under Policy 1 may deteriorate the profit, since by pooling, Policy 1 loses the ability to effectively ration the stock. In other words, when the two classes are managed separately, then each class uses its own stock. For instance two manufacturer may prefer to operate close to a MTS system for high revenue customers, while operate with a MTO for low revenue customers.

When the stock is pooled, then both customers must be met through a hybrid MTS/MTO system, which may disrupt the balance of the system and deteriorate the profits. De Véricourt et al. [11] makes a similar observation and states that if in an inventory pooling environment stock allocation (rationing) is not done properly, then profits may deteriorate when moving into inventory pooling, especially if products significantly differs in terms of lateness cost.

Note that inventory and capacity pooling may also have a similar (but less pronounced) impact under optimal due-date policy, since we assume that in the inventory and capacity pooled system, FCFS policy is adopted. The only mechanism that alleviates the negative impact of FCFS is the rejection of class 2 customers possibly when there is stock. But this action limits the flexibility moving from the separate system setting to the pooled system setting, which may result in loss of profit. We observe such cases in the 29 instances with negative % benefit ratios (see Table 1) where pooling deteriorates the profit. We expect that these cases are more observable when holding cost is high, so that the benefit of optimum stock rationing is less effective. Under s=0, the only benefit of pooling is due to capacity pooling. When $Rev_1 >> Rev_2$ if $\lambda_1 < \lambda_2$, then under pooling capacity is not rationed properly, while the separate system can do the rationing in the right way. Therefore, under pooling, class 2 abuses the capacity that could be allocated to class 1. To avoid the abuse, the system quotes longer due-dates to class 2 and ends up with loss of revenue from class 2.

We see that there is a strong relation between the benefit obtained from pooling and the revenues and arrival rates of the two classes. In a pooled system, intuitively we prefer to ration the major portion of the capacity and inventory to the high-priority demand class in case of equal arrival rates, since it brings higher revenue and net profit, and it is more important to meet the demand of this class. And one of the options may be to allocate the total capacity and/or inventory according to the revenue ratios of each class. However arrival rate is also another important factor for the effective capacity and stock allocation, since we may prefer to allocate less capacity and/or stock to the high-priority class when its arrival rate is relatively very low ($Rev_1 >> Rev_2$ and $\lambda_1 < \lambda_2$). Therefore, the value of the demand (i.e. revenue) and its arriving rate for the allocation of capacity and inventory are taken into account in the optimal capacity allocation. Accordingly, in an environment where optimal policy is employed, we see that the % benefit is minimized in a certain λ_1/λ_2 ratio (i.e. which may be a ratio close to the revenue ratios of the classes) and reaches its maximum value when λ_1/λ_2 is either very small or very large, where the system becomes closer to a single-class system ($\lambda_1 >> \lambda_2$ or $\lambda_1 << \lambda_2$). Since, differently from the pooled system where the manufacturer is free to allocate the total service rate to any of the class, in the separate case there are two servers with equal fixed service rates. Therefore, it is reasonable that the revenue and arrival rate pairs under which the optimum policy allocates equal capacity for two classes is expected to minimize the % benefit of the pooled system, i.e. when the values of class 1 and 2 demand are inversely proportional to the arrival rates such that $Rev_1 > Rev_2$ and $\lambda_1 < \lambda_2$ or vice versa. If these ratios are not proportional to each other such that $Rev_1 >> Rev_2$ and $\lambda_1 > \lambda_2$, then an effective capacity allocation is expected to bring a considerable amount of benefit compared to the former system. Therefore we expect that the % benefit of joint pooling of inventory and capacity under increasing λ_1/λ_2 will be convex.

CHAPTER 5

CONCLUSION

In this study, we characterize a due-date quotation problem of a multi-class system with exponential production times and Poisson arrivals. We define the problem as a Markov decision process and include both due-date quotation and production decisions in our model. For the solution of the due-date quotation problem, we use a linear programming formulation.

Next we define three alternative policies in order to assess the performance of the optimal policy and to measure the value of due-date quotation flexibility and quoting reliable due dates based on the current workload in the system. We consider Policy 1 as an accept-all policy which does not have order acceptance flexibility or any due-date option greater than zero. Policy 2 is more flexible in terms of due-date quotation however does not use state information efficiently for the determination of due-dates by applying fixed due-date quotation policy between certain states. Policy 3 can be as flexible as the optimal policy in due-date quotation however it is a myopic policy which completely ignores the long-run effect of the due-dates on the traffic intensity of the system. In order to assess the performance of each policy more deeply, we define various performance measures and assess the optimal policy and alternative policies under different operating conditions. According to the computational analysis, we can summarize our main findings as follows:

- The dominance of high-priority class deteriorates the performance of the accept-all policy. Moreover the accept-all policy perform close to the optimal policy under low level of congestion.

- We observe that Policy 2 perform very close to the optimal policy, hence we can conclude that quoting due-dates under limited state information may also yield remarkably close results to the optimal model.

- Policy 2 and Policy 3 possess different shortcomings as mentioned above, therefore neither Policy 2 nor Policy 3 dominates each other in terms of performances. However each of these policies may perform superior than the other one under different system parameters.

- We see that service level performance does not necessarily deteriorate under optimal policy when the level of congestion increases.

- Policy 3 which is a myopic policy diverges from the optimal policy in terms of profit when the congestion level increases.

- Joint pooling of inventory and capacity can significantly increase the profit when Policy 1 employed which either does not make stock or capacity allocation. However under optimal policy, the benefit obtained from pooling is not as significant.

- Joint pooling of inventory and capacity may result with a loss compared to the separate system under any policy including the optimal one when an effective capacity allocation policy is not employed.

According to our observations we can conclude that our study can further be extended with an order scheduling policy which is another essential component of due-date management. In our study, optimal policy can make capacity planning to a certain extent, by only rejecting the customers. However a scheduling policy can be employed by keeping track of the length of backordering queue, as a result more important orders can be first accepted although they arrive to the system later than the other class. Another extension would be that the model we study can further be relaxed by assuming general arrivals instead of Poisson arrivals.

In order to use as a benchmark, we define Policy 2 which uses an exhaustive-search method having a poor runtime performance. In order to decrease the computation time, various search algorithms may be developed for Policy 2, resulting with less computational burden and again very close performance to the optimal policy.

One of the other potential future works related with the optimal policy may include nonlinear lateness penalties in order to reflect different lead time sensitivities and preferences of customers which are nonlinearly correlated with the length of due-dates.

Finally the benefit of capacity and inventory pooling may be investigated separately again under the optimal policy and an accept-all policy. This time, a state variable must be defined for each class in order to evaluate the benefit of capacity pooling.

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APPENDIX A

COMPUTATION OF EXPECTED ACTUAL WAITING TIME

$$L_i(d) = \begin{cases} \int_d^{\infty} (t-d)E_{i+1}(t)dt\,, & \text{if } i \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Where $E_j(t) = \frac{\mu(\mu t)^{j-1}}{(j-1)!}e^{-\mu t}$ which is the pdf of Erlang Distribution. Hence;

$$L_i(d) = \int_d^{\infty} (t-d)\frac{\mu(\mu t)^i}{i!}e^{-\mu t}dt$$

$$L_i(d) = \frac{\mu^{i+1}}{i!}\int_d^{\infty} \left(t^{i+1}e^{-\mu t} - t^i e^{-\mu t}d\right)dt$$

<u>Integration by Parts</u>

$u = t^i \qquad du = it^{(i-1)}dt$

$v = -\frac{e^{-at}}{a} \qquad dv = e^{-at}dt$

$$\int u.dv = uv - \int v.du$$

$$\int t^i e^{-at}dt = -\frac{e^{-at}}{a}t^i - \int -\frac{e^{-at}}{a}it^{(i-1)}$$

$$= -\frac{e^{-at}}{a}t^{i} + \frac{i}{a}\int e^{-at}t^{(i-1)}$$

$$= -\frac{e^{-at}}{a}t^{i} + \frac{i}{a}\left(-\frac{e^{-at}}{a}t^{(i-1)}\frac{i-1}{a}\left(-\frac{e^{-at}}{a}t^{(i-2)} + \frac{i-2}{a}\left(\dots\frac{1}{a}\left(-\frac{e^{-at}}{a}t^{0}\right)\right)\right)\right)$$

$$= -\frac{e^{-at}}{a}\left(t^{i} + \frac{i}{a}t^{(i-1)} + \frac{i(i-1)}{a^{2}}t^{(i-2)} + \frac{i(i-1)(i-2)}{a^{3}}t^{(i-3)} + \cdots + \frac{i!}{a^{i}}t^{0}\right)$$

$$\int t^{i}e^{-at}dt = -\frac{e^{-at}}{a}\left(\sum_{k=0}^{i}\frac{t^{i-k}i!}{a^{k}(i-k)!}\right)$$

$$L_{i}(d) = \frac{\mu^{i+1}}{i!}\left(\frac{e^{-\mu d}}{\mu}\left(\sum_{k=0}^{i+1}\frac{d^{i+1-k}(i+1)!}{\mu^{k}(i+1-k)!} - d\sum_{k=0}^{i}\frac{d^{i-k}\,i!}{\mu^{k}(i-k)!}\right)\right)$$

$$L_{i}(d) = \mu^{i}e^{-\mu d}\left(\sum_{k=0}^{i+1}\frac{d^{i+1-k}(i+1)}{\mu^{k}(i+1-k)!} - \sum_{k=0}^{i}\frac{d^{i+1-k}}{\mu^{k}(i-k)!}\right)$$

$$L_{i}(d) = e^{-\mu d}\left(\frac{(i+1)}{\mu} + \sum_{k=0}^{i}\frac{d^{i+1-k}.\,k.\,\mu^{i-k}}{(i+1-k)!}\right)$$

APPENDIX B

STEADY-STATE PROBABILITY EXPRESSIONS OF POLICY 2

In case of different combinations of $\rho, \rho^{'}$ and $\rho^{''}$ being 1, the expression for steady-state probability of state i = –s in Policy 2 will be as follows;

$$\pi_{-s}(1+\rho+\rho^2+\cdots+\rho^{s+K})+\rho^{s+K}\pi_{-s}\left(\left(\rho'+\rho'^2+\cdots+\rho'^{M-K}\right)+\rho'^{M-K}\left(\rho''+\rho''^2+\cdots+\rho''^{R_1-M}\right)\right)=1$$

For $\rho = 1, \rho^{'} = 1$ and $\rho^{''} = 1$;

$$\pi_{-s}\left(s+K+1+\left(M-K+(R_1-M)\right)\right)=1$$

$$\pi_{-s}=\frac{1}{s+R_1+1}$$

For $\rho = 1, \rho^{'} = 1$ and $\rho^{''} \neq 1$;

$$\pi_{-s}\left(s+K+1+\left(M-K+\rho''\left(\frac{\rho''^{R_1-M}-1}{\rho''-1}\right)\right)\right)=1$$

$$\pi_{-s}=\frac{(\rho''-1)}{(s+M+1)(\rho''-1)+\rho''\left(\rho''^{R_1-M}-1\right)}$$

For $\rho = 1, \rho^{'} \neq 1$ and $\rho^{''} = 1$;

$$\pi_{-s}\left(s+K+1+\left(\rho'\left(\frac{\rho'^{M-K}-1}{\rho'-1}\right)+\rho'^{M-K}(R_1-M)\right)\right)=1$$

$$\pi_{-s}=\frac{(\rho'-1)}{(s+K+1)(\rho'-1)+\rho'\left(\rho'^{M-K}-1\right)+\rho'^{M-K}(R1-M)(\rho'-1)}$$

For $\rho \neq 1, \rho' = 1$ and $\rho'' = 1$;

$$\pi_{-s}\left(\frac{\rho^{s+K+1}-1}{\rho-1}+\rho^{s+K}\big(M-K+(R_1-M)\big)\right)=1$$

$$\pi_{-s}=\frac{(\rho-1)}{(\rho^{s+K+1}-1)+\rho^{s+K}(R_1-K)(\rho-1)}$$

For $\rho = 1,\ \rho' \neq 1$ and $\rho'' \neq 1$;

$$\pi_{-s}\left(s+K+1+\left(\rho'\left(\frac{\rho'^{M-K}-1}{\rho'-1}\right)+\rho'^{M-K}\rho''\left(\frac{\rho''^{R_1-M}-1}{\rho''-1}\right)\right)\right)=1$$

$$\pi_{-s}=\frac{(\rho'-1)(\rho''-1)}{(s+K+1)(\rho'-1)(\rho''-1)+\rho'\left(\rho'^{M-K}-1\right)(\rho''-1)+\rho'^{M-K}\rho''\left(\rho''^{R_1-M}-1\right)(\rho'-1)}$$

For $\rho \neq 1, \rho' = 1$ and $\rho'' \neq 1$;

$$\pi_{-s}\left(\frac{\rho^{s+K+1}-1}{\rho-1}+\rho^{s+K}\left(M-K+\rho''\left(\frac{\rho''^{R_1-M}-1}{\rho''-1}\right)\right)\right)=1$$

$$\pi_{-s}=\frac{(\rho-1)(\rho'_1-1)}{(\rho^{s+K+1}-1)(\rho''-1)+\rho^{s+K}\,(M-K)\,(\rho-1)(\rho''-1)+\rho^{s+K}\rho''\left(\rho''^{R_1-M}-1\right)(\rho-1)}$$

For $\rho \neq 1, \rho' \neq 1$ and $\rho'' = 1$;

$$\pi_{-s}\left(\frac{\rho^{s+K+1}-1}{\rho-1}+\rho^{s+K}\left(\rho'\left(\frac{\rho'^{M-K}-1}{\rho'-1}\right)+\rho'^{M-K}(R_1-M)\right)\right)=1$$

$$\pi_{-s}=\frac{(\rho-1)(\rho'-1)}{(\rho^{s+K+1}-1)(\rho'-1)+\rho^{s+K}\rho'\left(\rho'^{M-K}-1\right)(\rho-1)+\rho^{s+K}\rho'^{M-K}(R_1-M)(\rho-1)(\rho'-1)}$$