

ANALYSIS AND CLASSIFICATION OF SPELLING PARADIGM EEG DATA
AND AN ATTEMPT FOR OPTIMIZATION OF CHANNELS USED

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

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IN PARTIAL FULLFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
ELECTRICAL AND ELECTRONICS ENGINEERING

DECEMBER 2010

Approval of the thesis:

**ANALYSIS AND CLASSIFICATION OF SPELLING PARADIGM EEG
DATA AND AN ATTEMPT FOR OPTIMIZATION OF CHANNELS USED**

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ABSTRACT

ANALYSIS AND CLASSIFICATION OF SPELLING PARADIGM EEG DATA AND AN ATTEMPT FOR OPTIMIZATION OF CHANNELS USED

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December 2010, 98 pages

Brain Computer Interfaces (BCIs) are systems developed in order to control devices by using only brain signals. In BCI systems, different mental activities to be performed by the users are associated with different actions on the device to be controlled. Spelling Paradigm is a BCI application which aims to construct the words by finding letters using P300 signals recorded via channel electrodes attached to the diverse points of the scalp. Reducing the letter detection error rates and increasing the speed of letter detection are crucial for Spelling Paradigm. By this way, disabled people can express their needs more easily using this application.

In this thesis, two different methods, Support Vector Machine (SVM) and AdaBoost, are used for classification in the analysis. Classification and Regression Trees is used as the weak classifier of the AdaBoost. Time-frequency domain characteristics of P300 evoked potentials are analyzed in addition to time domain characteristics. Wigner-Ville Distribution is used for transforming time domain

signals into time-frequency domain. It is observed that classification results are better in time domain. Furthermore, optimum subset of channels that models P300 signals with minimum error rate is searched. A method that uses both SVM and AdaBoost is proposed to select channels. 12 channels are selected in time domain with this method. Also, effect of dimension reduction is analyzed using Principal Component Analysis (PCA) and AdaBoost methods.

Keywords : Brain Computer Interface (BCI), Spelling Paradigm, Wigner-Ville Distribution, Principal Component Analysis (PCA), Support Vector Machine (SVM), AdaBoost.

ÖZ

HECELEME PARADİGMASI EEG VERİSİNİN ANALİZİ VE SINIFLANDIRILMASI VE EN UYGUN KANALLARIN KULLANILMASI ÜZERİNE BİR ÇALIŞMA

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Aralık 2010, 98 sayfa

Beyin Bilgisayar Arayüzleri (BBA) sadece beyin sinyalleri kullanılarak cihazların kontrol edilmesi için geliştirilen sistemlerdir. BBA sistemlerinde kontrol edilecek cihazlardaki değişik eylemler için kullanıcı tarafından farklı zihinsel aktiviteler gerçekleştirilmektedir. Heceleme Paradigması, kafa derisinin çeşitli noktalarına tutturulmuş kanal elektrotlarıyla kaydedilen P300 sinyaller kullanılarak bulunan harfleri bir araya getirerek kelimeleri oluşturmayı amaçlayan bir BBA uygulamasıdır. Harf tanınmasındaki hata oranlarının azaltılması ve harf tanıma hızının artırılması Heceleme Paradigması açısından çok önemlidir. Bu şekilde engellilerin bu uygulamayı kullanarak isteklerini daha kolay bir biçimde ifade etmeleri mümkündür.

Bu tezde analizlerde sınıflandırma yöntemi olarak iki farklı metot, Destek Vektör Makinesi (DVM) ve AdaBoost, kullanılmıştır. AdaBoost zayıf sınıflandırıcısı olarak Sınıflandırma ve Regresyon Ağaçları kullanılmıştır. Zaman alanına ek olarak zaman-frekans alanında da uyarıyla tetiklenen P300 geriliminin karakteristiği analiz edilmiştir. Zaman alanındaki sinyallerin zaman-frekans alanına çevrilmesi için Wigner-Ville Dağılımı kullanılmıştır. Zaman alanında sınıflandırma sonuçlarının daha iyi çıktığı gözlemlenmiştir. Bundan başka, P300 sinyallerini en az hata oranı ile modelleyen en uygun kanal alt kümesi araştırılmıştır. Hem DVM hem de AdaBoost kullanan bir kanal seçme metodu önerilmiştir. Bu metot ile zaman alanında 12 kanal seçimi yapılmıştır. Ayrıca, Ana Bileşen Analizi (ABA) ve AdaBoost yöntemleri ile boyut azaltmanın etkileri incelenmiştir.

Anahtar Kelimeler : Beyin Bilgisayar Arayüzü (BBA), Heceleme Paradigması, Wigner-Ville Dağılımı, Ana Bileşen Analizi (ABA), Destek Vektör Makinesi (DVM), AdaBoost.

To My Family

ACKNOWLEDGEMENTS

I would like to thank to my supervisor Prof. Dr. Uğur Halıcı for her guidance, supervision, encouragement and patience throughout the thesis work.

I would like to thank to my friends and my colleagues for their encouragement and support. I also thank to ASELSAN Inc. the facilities they provided throughout the thesis work.

I would like to thank to TÜBİTAK for the scholarship throughout the thesis work.

Last, but not least, I am grateful to my family for supporting me in my entire life.

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CHAPTER 1

INTRODUCTION

Human being performs all voluntary movements by controlling related organs with the brain. Diseases may cause partial or full loss of abilities of the patients. In diseases such as Amyotrophic Lateral Sclerosis (ALS), all voluntary movements of the patient can be disabled and only cognitive abilities of the brain remain unaffected. So, these patients can interact with outside world only using their brain signals. Brain Computer Interface (BCI) is an area of research that tries to increase the interaction of such patients with outside world by processing activities of the brain and controlling external devices. For example, a cursor was moved in a two-dimensional maze by human using signals recorded via Electroencephalography (EEG) in 1977 [1]. With the improvements in technology, studies on BCI are increased and research groups are founded over past decades. Researchers try the increase the applications of BCI and increase the usability of the systems developed ([6]-[11]).

There are two types of BCI's according to their input types. These input types are endogenous and exogenous electrophysiological activities. In endogenous BCI's, the subject creates control signals by imagination without any external stimulus. One of main methods to create these signals is imagination of muscle movement ([2], [3]) such as imagining right and left hand movements. Another method is thinking abstract feelings ([4], [5]) such as stress and relaxation. In exogenous BCI's, the subject creates control signals evoked by an external stimulus. For instance, the method developed in [17] is allowing subjects to communicate letters

and words using Event-Related Potentials (ERP's). Endogenous BCI's require excessive training while learning creation of control signals. However, once it is learned, subject can create signals on its own. On the other hand, exogenous BCI's does not require too much training but it needs an external stimulus such as a display to create control signals.

A laboratory environment is needed to gather data and work on BCI. Fortunately, in recent years, BCI research groups started to organize competitions and provide data sets to the competitors ([12]-[15]). These competitions aim to improve the BCI development process by introducing challenging problems and creating environment to compare different methods. Competition datasets are still available and can be accessed through the webpage given in [12]. So, new approaches can be tried on the same data set and results can be compared with previous studies.

In this thesis, Spelling Paradigm, a specific exogenous BCI application, is studied. Spelling Paradigm was first introduced by Donchin et al. [17] in 1988. In 2000, a new version of Spelling Paradigm with increased data transformation rate was introduced by Donchin et al. [18]. Spelling Paradigm is an application for disabled people to express their thoughts by constructing words from detected letters. Spelling Paradigm problems exist in BCI competitions [14] and [15]. Data set is also available on [20] and [22]. In this thesis, time domain and time-frequency domain responses of P300 evoked potentials of Spelling Paradigm data set are analyzed. Time domain signals are transformed into time-frequency domain using Wigner-Ville Distribution (WVD). EEG signals are recorded using several electrodes attached to the scalp. Optimum electrode subset among the recorded ones is searched. Working on small data sets increases the speed of the process and real time use of the system. Effect of feature reduction methods Principal Component Analysis (PCA) and AdaBoost on feature vectors is analyzed. Support Vector Machine (SVM) and AdaBoost are used for classification throughout the process.

Organization of the thesis is given as:

In Chapter 2, introductory information about Spelling Paradigm is given and previous studies on this problem are analyzed.

In Chapter 3, background information about filtering, normalization, feature transformation, dimension reduction and classification methods are given.

In Chapter 4, details of implemented methods are given.

In Chapter 5, experimental results about feature transformation methods, optimum channel selection results, effect of dimension reduction and classification methods are given.

In Chapter 6, results are discussed and thesis is concluded.

CHAPTER 2

SPELLING PARADIGM

Spelling Paradigm is a BCI application which aims to detect the words by finding letters from brain activities of the person. It is introduced by Farewell and Donchin [17]. In this chapter, first, Spelling Paradigm experimental setup and data set is described in detail. Then, previous studies on this topic are summarized. Finally, implemented methodologies are discussed.

2.1 Spelling Paradigm Definition

The goal of the spelling paradigm is to give an interface to construct words by using only the electrical activity of the brain. 6 by 6 matrix which is composed of ASCII characters is displayed on a screen (Figure 2.1). Subject is requested to focus on a character in the matrix. Then, rows and columns of the matrix are successively and randomly intensified. There are total 12 rows and columns and only 2 of them contain the focused character. From now on, rows and column intensifications that contain desired character will be named as *target intensifications* and rows and column intensifications that do not contain desired character will be named as *non-target intensifications*.

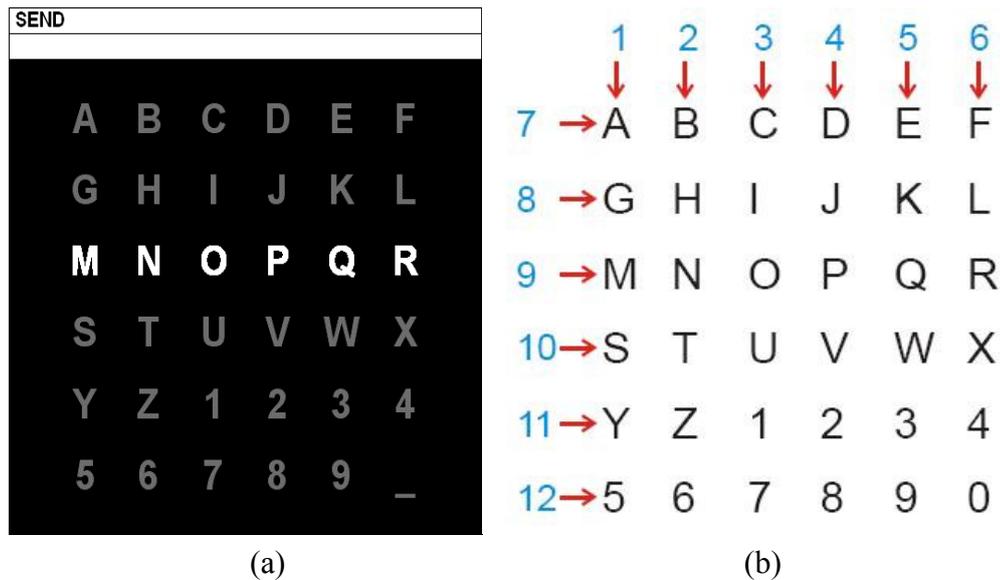


Figure 2.1 Spelling Paradigm character matrix that is displayed to user is given in (a) and row and column number assignments of the matrix is given in (b), [20]

Response of the brain against target and non-target intensifications is different and their responses are similar to the P300 evoked potentials reported by Farewell and Donchin [17]. The P300 evoked potential is the reaction of the subject to the stimulus. Its occurrence is not related with the physical attributes of a stimulus. It is a positive deflection in voltage with latency (delay between stimulus and response) of roughly 300 to 600 milliseconds. In spelling paradigm, P300 evoked potentials are elicited using the Oddball Paradigm [23]. The oddball paradigm is a technique used in evoked potential research in which trains of stimuli that are usually auditory or visual are used to assess the neural reactions to unpredictable but recognizable events. In this case, low-probability target intensifications are inter-mixed with high-probability non-target (or "standard") intensifications. The subject's reaction to the target intensifications elicits the P300 evoked potentials (Figure 2.2).

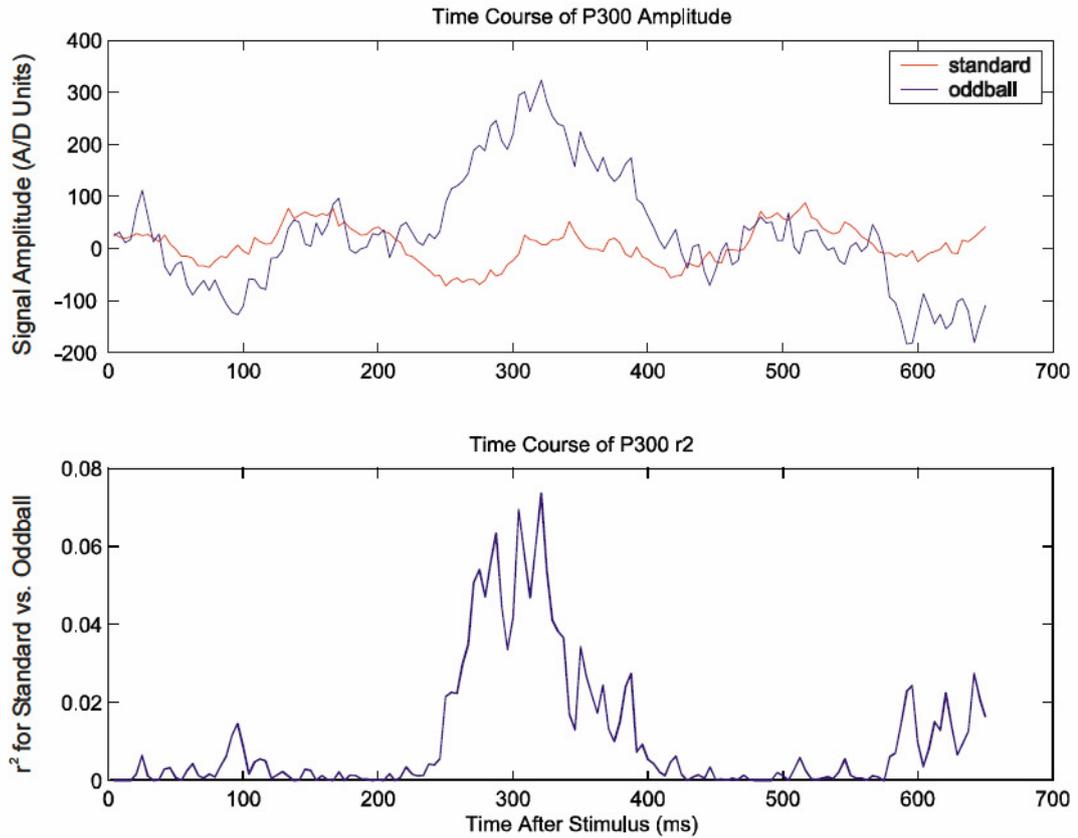


Figure 2.2 P300 evoked potentials, standard is the response of the non-target intensification and oddball is the response of the target intensification, [20]

To get more accurate results while predicting the character, row/column intensifications are repeated. For instance, in [20] and [22], row/column intensifications are repeated 15 times for each character. Since row/column intensification consists of 12 intensifications, there are total 180 intensifications for each character. A single intensification is completed in 175 ms, so character intensification lasts 31.5 seconds. So, there is a trade off between time and accuracy. It is important to use less number of repetitions and still predict the character accurately.

2.2 Spelling Paradigm Solution Approach

Spelling paradigm is separated into three main steps. These steps are extracting samples, creating features and classifying features.

First step is extracting relevant information from recorded EEG signals. EEG signals consist of multiple channel data which are recorded using electrodes placed on different points of the skull. Using less number of channels reduces the computational effort. However, relevant information must be kept by selecting proper set of channels. Part of the EEG signal that contains relevant information must be extracted. P300 evoked potential characteristics are taken into account while determining the duration of the signal after the stimulus. Then, extracted signals can also be filtered to suppress noise.

Second step is extracting features from the preprocessed signals which are the output of the first step. Time domain signals can be transformed into other domains and transformed channels are concatenated to create features. Then, features can be normalized and size of the features can be reduced.

Final step is the classification of the features. First, each intensification is classified separately and then character decision is done using the repetitions.

2.3 Previous Studies on Spelling Paradigm

Studies on spelling paradigm are encouraged by competitions [20] and [22]. Data set is provided by the organizers of results are evaluated. By this way, effect of different algorithms can tested on same data and results can be compared.

7 competitors attended to the competition [20] and 5 of them managed to classify all the words correctly using all 15 repetitions. Other than success rate, using less number of repetitions is also an evaluation criterion of the performance. All words are correctly classified using only 5 repetitions by Kaper [24].

10 competitors attended to the competition [22], and Rakotomamonjy [30] won the competition by achieving %96.5 accuracy with 15 repetitions and %73.5 accuracy with 5 repetitions.

Studies are continued after the completion of the competition. For instance, Erdoğan successfully implemented Wiener filter in his thesis study and gave the results in [29].

Previous solution attempts on Spelling Paradigm are evaluated in terms of their methods on three main steps, preprocessing signals, extracting features and classifying features.

2.3.1 Preporcessing Signals

In preprocessing step, a channel subset is decided and a signal portion is extracted from these channel sets after intensification. Filtering and downsampling are other methods applied at this step.

Kaper [24], used 10 channels in his study, used a bandpass filter with cutoff frequencies (0.5 – 30) and extracted signals of interval 0-600 ms after stimulus. Erdoğan [29] used same channel subset and the same time interval after stimulus with [24]. But he applied the adaptive Wiener filter instead of standard filters. Rakotomamonjy [30] did not use fixed channels. Instead, channels are selected automatically among the whole channel set with elimination. So, an adaptive channel selection method is applied. Then signals are filtered with (0.1 – 10 Hz) bandpass filter and downsampled to 20 Hz. 0-667 ms signals are extracted after stimulus. Bostanov [25] used all channels, a 10 Hz lowpass filter and a logarithmic downscaling. One second interval after stimulus is extracted. Xu [26] used all channels, 2-8 Hz bandpass filter and 0-650 ms interval. Hoffmann [32] used the same 10 channels given in [24]. 0-9 Hz filter, 0-600 ms time interval after stimulus and downsampling is used. Yang [31] used 55 channels selected with F-score, 0.1 – 20 Hz bandpas filter, 0-667 ms time interval after stimulus and downsampled

signals to 20 Hz. Rivet [33] used 1-20 Hz bandpass filter with 1 second interval after stimulus.

2.3.2 Feature Extraction

Preprocessed signals are transformed into feature spaces and used channels are concatenated to create feature vectors. Dimension of the feature vectors are also reduced in this step.

Kaper [24], Erdoğan [29], Rakotomamonjy [30], Yang [31], Hoffmann [32], concatenated preprocessed signals without transforming or reducing its size. Bostanov [25] transformed signals using Continuous Wavelet Transform and Student's two-sample t statistics. Xu [26] used Principal Component Analysis (PCA) and Independent Component Analysis (ICA) for both transforming signals into feature space and reducing dimension. Rivet [33] used xDAWN algorithm, a spatial filter, to transform signals.

2.3.3 Classification

Feature vectors are classified with learning methods at final step.

Kaper [24], Erdoğan [29], Rakotomamonjy [30], Yang [31], used SVM in their studies. Hoffmann [32] used gradient boosting, Bostanov [25] used Linear Discriminant Analysis (LDA), and Rivet [33] used Bayesian LDA for classification.

2.4 Conclusion

Although the classification methods in most of the previous studies are similar to each other, performances of the systems are different. So, pre-processing and feature extraction steps distinguish the performance of the systems. Filtering, is generally applied to enhance the performance of the system. Downsampling is also preferred after filtering. Channel selection varies significantly between methods. Methods can use all channels and then reduces data size at feature extraction or

select a subset of channels and create already reduced number of features at the end of preprocessing step. Time domain is generally preferred for P300 analysis.

CHAPTER 3

BACKGROUND

3.1 Wigner-Ville Distribution

Wigner-Ville Distribution (WVD) transforms time signals into time-frequency domain. WVD of a signal $s(t)$ is given in equation (3-1).

$$W(t, f) = \int_{-\infty}^{+\infty} s(t + \tau/2) s^*(t - \tau/2) e^{-j2\pi f\tau} d\tau \quad (3-1)$$

WVD satisfies the marginal conditions given in equations (3-2) and (3-3) for the signal $s(t)$, defined for t between 0 and t_1 and f between 0 and f_1 .

$$\int_0^{t_1} W(t, f) dt = ESD(f) \quad (3-2)$$

$$\int_0^{f_1} W(t, f) df = |s(t)|^2 \quad (3-3)$$

where $ESD(f)$ is the energy spectral density of the signal (the intensity of energy per unit frequency), and $|s(t)|^2$ is the signal power at time t .

For a time-series $x(n)$, the expression of the discrete-time Wigner-Ville distribution, $W(n,f)$ is:

$$W(n, f) = 2 \sum_{k=-\infty}^{\infty} h_N^2(k) x(n+k) x^*(n-k) e^{-j4\pi f k} \quad (3-4)$$

where $h_N(k)$ is a data-window, which performs a frequency smoothing.

3.2 Principal Component Analysis

Principal Component Analysis (PCA) is an orthogonal linear transformation method that transforms the data to a new basis according to variance of the data. The axes of the new coordinate system (principal components) are ordered with decreasing variance. That is, the greatest variance direction is the first axis (first principal component) and smallest variance direction is the last axis. PCA is commonly used for reducing dimensionality of the data set. Possibly correlated variables are also eliminated while projecting data to the lower dimensional space.

PCA Example:

In Figure 3.1, data is sampled from noisy $y = x$ line is plotted. Principal components for the data set are also plotted.

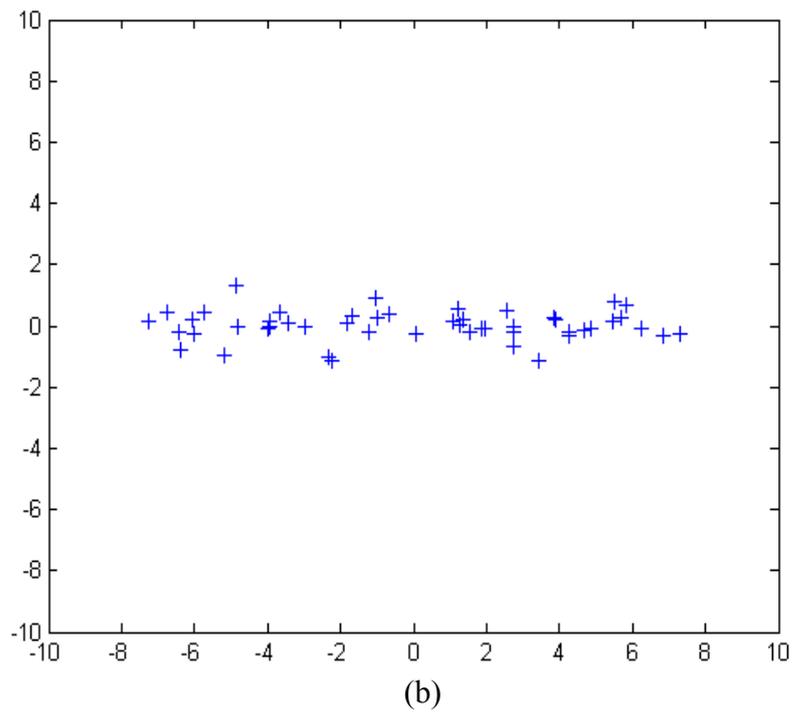
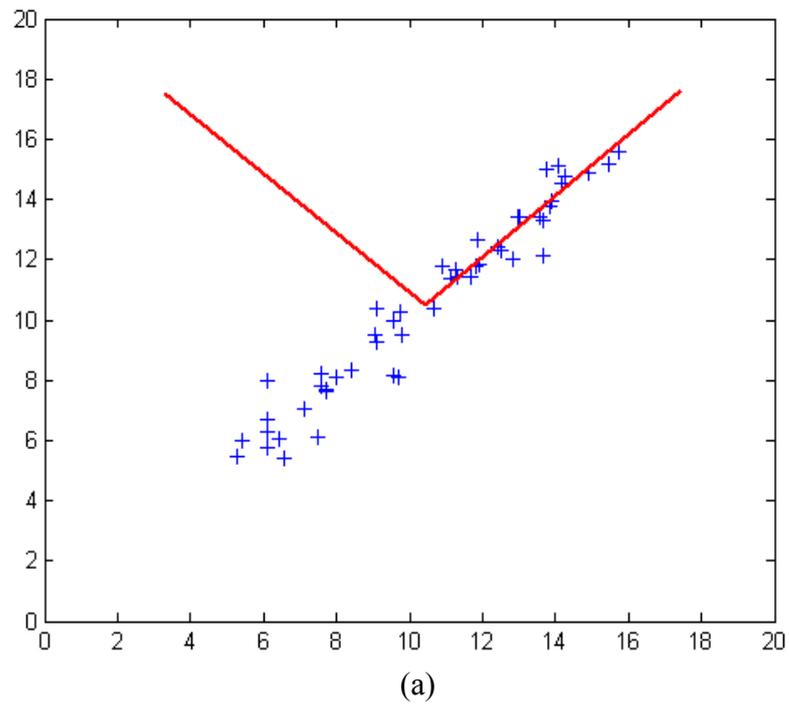


Figure 3.1 Principal components (drawn with red lines) for data set that sampled from noisy $y = x$ line is given in (a). PCA transformation for data is given in (b).

Table 3.1 Principal components' eigenvectors and eigenvalues for the data set given in Figure 3.1

	Principal Component 1	Principal Component 2
Eigenvector	[0.7002 0.7139]	[-0.7139 0.7002]
Eigenvalue	18.2577	0.2394

Assumptions and Limitations of PCA:

- PCA is based on mean vector and covariance matrix of the data. If distribution of the data can not be characterized by these attributes, PCA is not useful.
- Principal components with larger variance contain more information about data set.
- Principal components are selected as orthogonal to reduce the computational complexity. So, it only rotates the coordinate system.

3.3 Normalization

Normalization is a standardization of collected data in order to recover errors of the repeated measurements. Variety of normalization methods exist, which are using the properties of each sample or statistical properties of the whole data set. Collected samples are assumed to be in vector form.

Scaling magnitude, which is called as *simple normalization*, is one of the most common and easy normalization methods. In simple normalization, properties of a single vector are sufficient to apply normalization. Minimum and maximum magnitude values of the vector are used while normalizing the vector. Using these properties, vector is scaled to the predefined range.

For a vector x , where maximum value of x is $\max(x)$ and minimum value of x is $\min(x)$, formula that maps x into the region $[\min(\bar{x}), \max(\bar{x})]$ is given in (3-5).

$$\bar{x} = \frac{\max(\bar{x}) - \min(\bar{x})}{\max(x) - \min(x)} \times (x - \min(x)) + \min(\bar{x}) \quad (3-5)$$

Scaling mean and standard deviation of the signals, which is called *Gaussian normalization*, is another normalization method. Mean and standard deviation are statistical properties of data set. So, whole data set is required to calculate these properties. Mean is normalized to zero and standard deviation is normalized to unity at Gaussian normalization.

Gaussian normalization formula is given in equation (3-6) for the vector x , where $\bar{\mu}_x$ and $\bar{\sigma}_x$ are mean and standard deviation respectively.

$$\bar{x} = \frac{x - \bar{\mu}_x}{\bar{\sigma}_x} \quad (3-6)$$

3.4 Kernel Methods

Kernel Methods (KMs) map the data into a higher dimensional space. This space is called feature space and each coordinate of the feature space corresponds to a feature of the data. Non-linear models in the input space can be solved by linear models in the feature space. Transformation from the input space to the feature space is done by the help of kernel functions. Kernel functions enable to operate in the feature space without even computing the coordinates of the data in that space. Instead, inner products between the images of all pairs of data are computed in the feature space, which is generally less complex than the explicit computation of the coordinates. Various methods such as SVM, Fisher's Linear Discriminant Analysis (LDA) and Principal Component Analysis (PCA) are capable of operating with kernels.

A kernel function K is non-negative, real valued and integrable. Kernel function satisfies the following the requirements:

- $$\int_{-\infty}^{+\infty} K(u)du = 1 \tag{3-7}$$

- $$K(-u) = K(u) \text{ for all values of } u \tag{3-8}$$

Examples of kernel functions:

Linear Kernel:

$$K(x, y) = x.y \tag{3-9}$$

Polynomial Kernel:

Control parameter is polynomial degree d .

$$K(x, y) = (x.y + 1)^d \tag{3-10}$$

Radial Basis Function:

$$K(x, y) = \exp\left(\frac{-|x - y|^2}{2\sigma^2}\right) \tag{3-11}$$

Exponential Kernel:

$$K(x, y) = \exp\left(\frac{-|x - y|}{2\sigma^2}\right) \tag{3-12}$$

3.5 Support Vector Machine

Support Vector Machine (SVM), which is introduced by Vapnik in 1963, is a supervised learning method, which analyzes the input-output relation of the training data to predict the outputs for the new input data. When the outputs are discrete,

prediction is called classification of the input data. SVM is a linear classifier, which uses hyperplane to separate data into classes. SVM is originally applied to two class classification problems. There are various solutions to separate linearly separable two classes given in Figure 3.2. SVM is aimed to find the maximum margin hyperplane while separating data into two classes (see Figure 3.3).

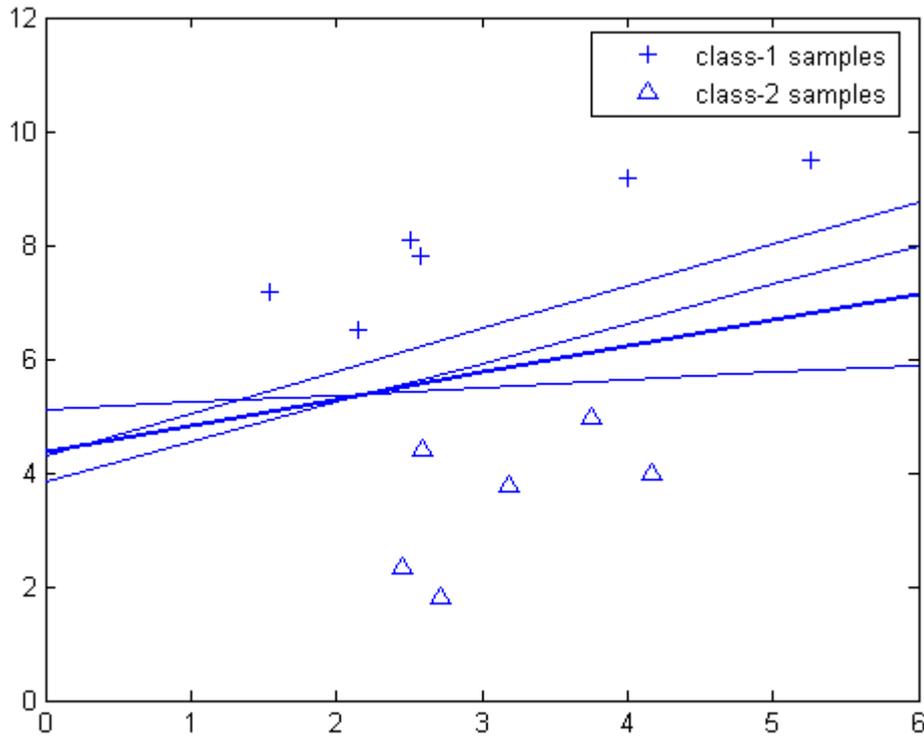


Figure 3.2 Lines (1 dimensional hyperplane) that are separating two classes in two dimensional space

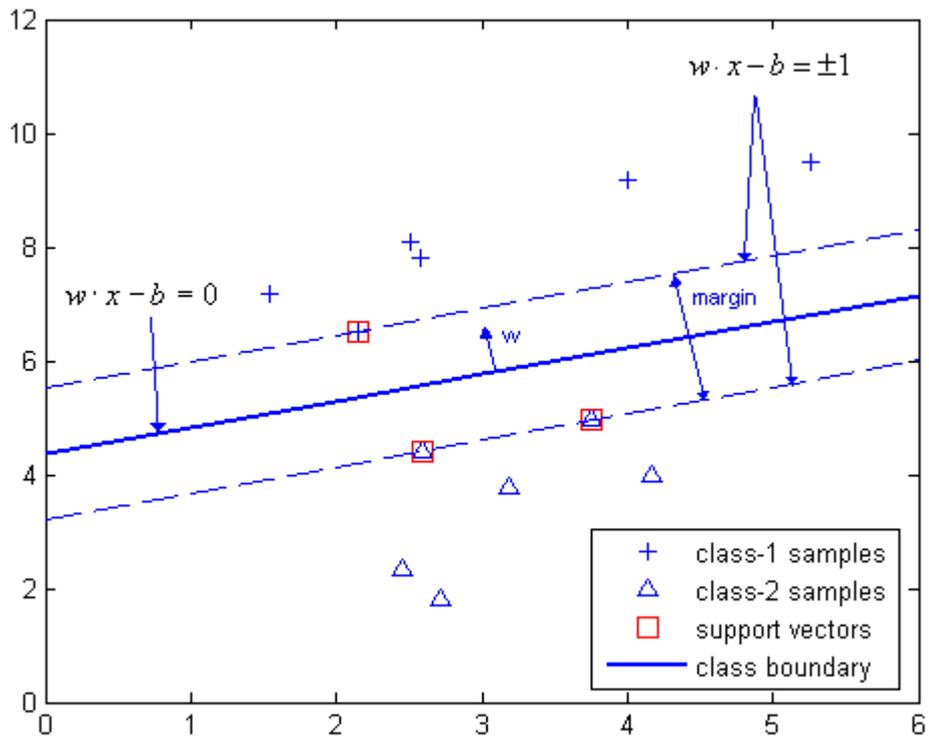


Figure 3.3 SVM solution for maximum-margin line (1 dimensional hyperplane) separating two classes for the data set given in Figure 3.2

Suppose that data set consists of p -dimensional n samples x belonging to linearly separable two classes c . The aim is to find the $p-1$ dimensional hyperplane which separates the classes with maximum margin. Labels $c = \{1, -1\}$ will be used for two classes. Mathematical form of a hyperplane is

$$w \cdot x - b = 0, \tag{3-13}$$

where w is the normal vector and b is the bias vector. Hyperplanes passing through the margins of the classes can be chosen as

$$w \cdot x - b = \pm 1 \tag{3-14}$$

Since samples belonging to two classes have to be the outside of the hyperplanes defining the margin,

$$w \cdot x_i - b \geq 1, \quad (3-15)$$

where x_i belongs to class labeled with $\{1\}$ and

$$w \cdot x_i - b \leq -1, \quad (3-16)$$

where x_i belongs to class labeled with $\{-1\}$.

$$c_i(w \cdot x_i - b) \geq 1, \text{ for all } 1 \leq i \leq n \quad (3-17)$$

The aim is to maximize the margin between two classes. The distance between two hyperplanes given in (3-14) is $\frac{2}{\|w\|}$. So, minimizing $\|w\|$, the norm of w , will maximize the distance between two classes. Using the quadratic programming $\frac{1}{2}\|w\|^2$ will be minimized while satisfying (3-17).

The optimization problem can be constructed with the Lagrange multipliers technique:

$$L(w, b, \Lambda) = \frac{1}{2}\|w\|^2 - \sum_{i=1}^n \lambda_i [c_i(w \cdot x_i + b) - 1], \quad (3-18)$$

where $\Lambda = \{\lambda_1, \dots, \lambda_n\}$ is the vector of non-negative Lagrange multipliers corresponds to the constraints in (3-17). The Lagrangian given in (3-18) is minimized with respect to w and b and maximized with respect to $\Lambda \geq 0$.

The solution of the problem can be expressed as a linear combination of training vectors:

$$w = \sum_{i=1}^n \lambda_i c_i x_i \quad (3-19)$$

So, maximum margin hyperplane is found by solving the optimization problem given in the Lagrangian dual form:

$$\begin{aligned} \text{Maximize } L(\Lambda) &= \sum_{i=1}^n \lambda_i - \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \lambda_i \lambda_j c_i c_j x_i \cdot x_j \\ \text{subject to } \sum_{i=1}^n \lambda_i c_i &= 0 \\ \lambda_i &\geq 0 \text{ for } i = 1, \dots, n \end{aligned} \quad (3-20)$$

The training vectors that have coefficients satisfying $\lambda > 0$ are called support vectors. Support vectors satisfy the equation $c_i(w \cdot x_i - b) = 1$. So, b can be calculated using any support vector.

Decision function becomes:

$$f(x) = \text{sign} \left(\sum_{i=1}^n \lambda_i c_i (x \cdot x_i) + b \right) \quad (3-21)$$

Soft Margin Hyperplane

Train vectors can not be separated by a hyperplane due to mislabeled data in Figure 3.4. SVM is modified [27] to handle linearly inseparable classes by introducing *Soft Margin* method.

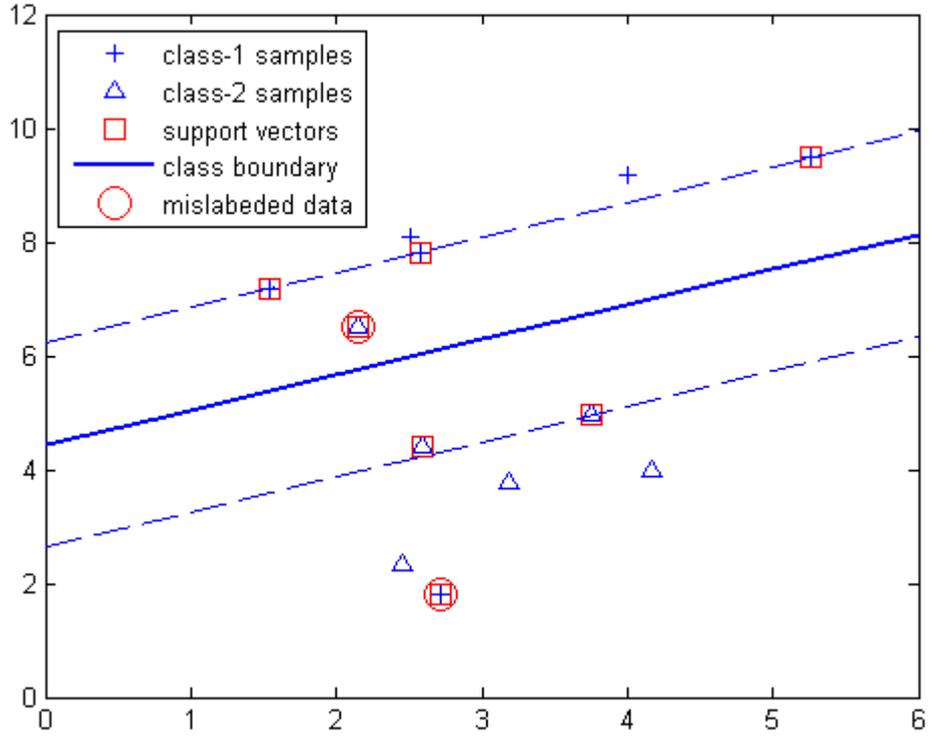


Figure 3.4 SVM solution for soft margin line (1 dimensional hyperplane) for $C=100$

Soft Margin method introduces slack variables ξ_i that measure the amount of violation of the constraints. A penalty coefficient is added to the optimization problem with regularization parameter C :

$$\begin{aligned} & \text{Minimize } \frac{1}{2} \|w\|^2 + C \sum_{i=1}^n \xi_i \\ & \text{subject to } c_i (w \cdot x_i - b) \geq 1 - \xi_i, \quad \text{for } i = 1, \dots, n \\ & \quad \quad \quad \xi_i \geq 0, \quad \text{for } i = 1, \dots, n \end{aligned} \quad (3-22)$$

The modified optimization problem can be constructed with the Lagrange multipliers technique:

$$L(w, b, \Lambda, \Xi, \Gamma) = \frac{1}{2} \|w\|^2 - \sum_{i=1}^n \lambda_i [c_i (w \cdot x_i + b) - 1 + \xi_i] - \sum_{i=1}^n \gamma_i \xi_i + C \sum_{i=1}^n \xi_i, \quad (3-23)$$

where $\Lambda = \{\lambda_1, \dots, \lambda_n\}$ and $\Gamma = \{\gamma_1, \dots, \gamma_n\}$ are the vector of non-negative Lagrange multipliers corresponds to the constraints in (3-22). The Lagrangian given in (3-23) is minimized with respect to w , E and b and maximized with respect to $\Lambda \geq 0$ and $\Gamma \geq 0$.

Maximum margin hyperplane is found by solving the optimization problem given in the Lagrangian dual form:

$$\begin{aligned} \text{Maximize } L(\Lambda) &= \sum_{i=1}^n \lambda_i - \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \lambda_i \lambda_j c_i c_j x_i \cdot x_j \\ \text{subject to } \sum_{i=1}^n \lambda_i c_i &= 0 \\ C \geq \lambda_i &\geq 0 \text{ for } i = 1, \dots, n \end{aligned} \quad (3-24)$$

Decision function becomes:

$$f(x) = \text{sign} \left(\sum_{i=1}^n \lambda_i c_i (x \cdot x_i) + b \right) \quad (3-25)$$

where bias b is calculated from the equation:

$$\lambda_i [c_i (w \cdot x_i - b) - 1 + \xi_i] = 0 \text{ for } i = 1, \dots, n \quad (3-26)$$

There are constraints in the equation (3-26) coming from (3-22) that limits the value of the λ_i to $0 < \lambda_i < C$. $\lambda_i > 0$ is valid for only the support vectors. $\lambda_i < C$ is valid for only correctly classified vectors. So, only correctly classified support vectors can be used for calculation of bias b . Note that there can be multiple solutions for b . Common approach is to solve all equations and then take the averages of the results:

$$b = \frac{1}{N_{SV}} \sum_{i=1}^{N_{SV}} w \cdot x_i - c_i, \quad (3-27)$$

where x_i are correctly classified support vectors and N_{SV} is the number of x_i .

Non-Linear Decision Surfaces

As it is stated in KMs, SVM are also used for classifying non-linear data set by transforming it into higher dimensional feature space where data can become linearly separable. In (3-25), decision function depends on dot product of vectors which defines hyperplanes for decision boundaries. If the dot product is changed with a kernel function, non-linear decision surfaces are obtained for decision boundaries. This is called *kernel trick*.

Decision function becomes:

$$f(x) = \text{sign}\left(\sum_{i=1}^n \lambda_i c_i K(x \cdot x_i) + b\right) \quad (3-28)$$

3.6 AdaBoost

AdaBoost is an adaptive boosting algorithm which is introduced in 1995 by Freund and Schapire [28]. AdaBoost constructs a strong classifier as a linear combination of weak classifiers.

Suppose that train set consists of m elements $(x_1, y_1), \dots, (x_m, y_m); x_i \in X, y_i \in \{-1, +1\}$.

Then, pseudo code of the AdaBoost algorithm is:

Initialize

$$D_1(i) = 1/m \quad (3-29)$$

For $t = 1, \dots, T$

- Train weak classifier using the distribution D_t
- Get weak hypothesis $h_t : X \rightarrow \{-1, +1\}$ with error

$$\varepsilon_t = \Pr_{i \sim D_t} [h_t(x_i) \neq y_i] = \sum_{i: h_t(x_i) \neq y_i} D_t(i) \quad (3-30)$$

- Choose

$$\alpha_t = \frac{1}{2} \ln \left(\frac{1 - \varepsilon_t}{\varepsilon_t} \right) \quad (3-31)$$

- Update:

$$\begin{aligned} D_{t+1}(i) &= \frac{D_t(i)}{Z_t} \times \begin{cases} e^{-\alpha_t} & \text{if } h_t(x_i) = y_i \\ e^{\alpha_t} & \text{if } h_t(x_i) \neq y_i \end{cases} \\ &= \frac{D_t(i) \exp(-\alpha_t y_i h_t(x_i))}{Z_t} \end{aligned} \quad (3-32)$$

where Z_t is a normalization factor (chosen so that D_{t+1} will be a distribution).

- Output the final hypothesis:

$$H(x) = \sum_{t=1}^T \alpha_t h_t(x) \quad (3-33)$$

In (3-31), α_t is inversely proportional with ε_t . That is, having a larger α_t coefficient means that the corresponding weak hypothesis classifies more accurately. The distribution $D_t(i)$ is updated such that weight of the correctly classified samples are decreased and weight of the misclassified samples are increased in the next cycle. By this way, algorithm focuses on the misclassified samples in next cycles.

3.6.1 Classification and Regression Trees

Classification and Regression Trees (CART) is used as the weak classifier of the AdaBoost. Decision tree consists of nodes and leaves. Tree is tracked from root to leaves to find out the class of the corresponding input.

Suppose that x is an n dimensional input, where x_i is the value of the i 'th dimension of x , and y is the binary classification output which is an element of $\{-1, +1\}$. Then, leaf of the tree contains the predicted class label y and node of the tree contains the relational condition which compares x_i with a threshold value. Decision tree is

constructed at training phase by choosing the dimension variable i and threshold value of the node and class label of the leaf. A sample CART is given in Figure 3.5.

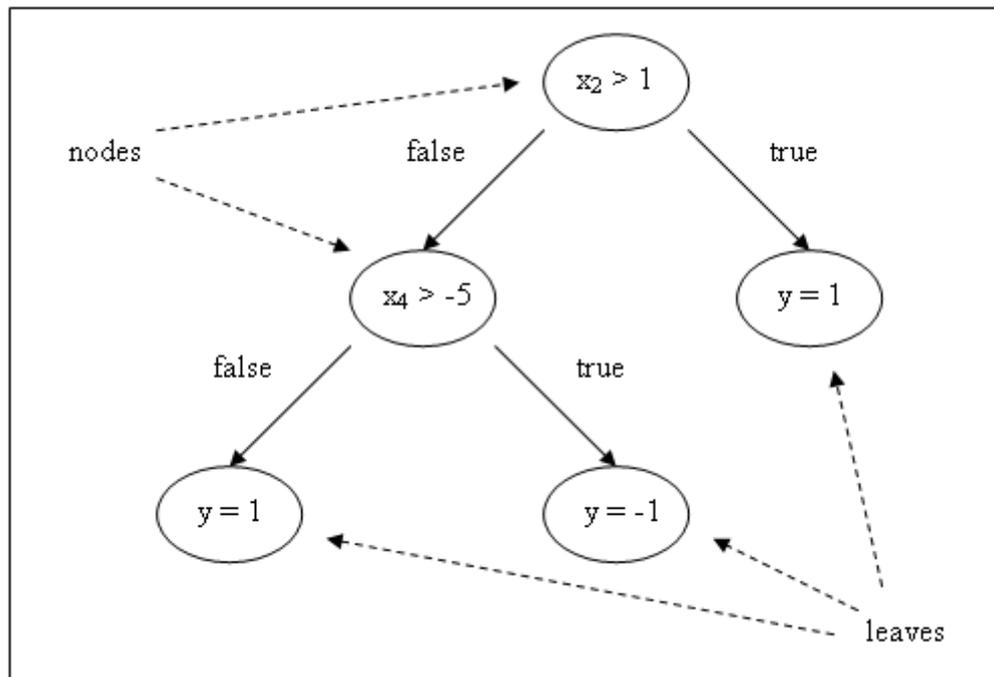


Figure 3.5 Classification and Regression Tree example, x_i is the i 'th dimension value of x and y is the class label

3.7 Cross Validation

In predictive models, if the validation set used for testing the parameters of the model is selected from the training set, model parameters may become quite dependant on the training data. Thus, model's prediction performance on unseen data may not be as satisfactory as if it is on the train data. This is called overfitting problem. To overcome this problem, cross-validation is used. Cross-validation is a method to evaluate the accuracy of the predictive model. Train set is divided into two subsets. One subset is used for training and the other subset is used for validating the model. It prevents overfitting problem since training and validation data sets are constructed from different subsets of the training set.

In k-fold cross-validation, training data is divided into k partitions and k-1 partitions are used for training and 1 partition is used for testing the model. This cross-validation process is repeated k times such that at each round, another partition is used for testing the model. While creating partitions, data can be also stratified to have a better representation of the whole data. Stratification ensures that samples of each class are equally distributed into each fold. That is, each fold is composed of equally sized members of the whole classes.

CHAPTER 4

IMPLEMENTATION

EEG signals are processed in three steps. First, relevant part of the signal is extracted from data. Then, signals are transformed into feature space and dimension of the feature vector is reduced. Finally, classification is performed using the extracted features. Spelling Paradigm data set provided in [20] is used to test the proposed methods.

4.1 Preprocessing EEG Signals

The whole activity of the brain is recorded by the electrodes while collecting data during the intensifications. However, only the part related with the stimulus is needed for processing. To remove irrelevant information, collected data is preprocessed. Preprocessing consists of three steps. First one is selecting signal portion related with stimulus and second one is filtering the selected portion of the signal to suppress noise and third one is downsampling data.

4.1.1 Extracting Data

EEG signals for 64 channels, row/column number (or stimulus code), intensification's start time are available in data set provided in [20]. Besides, character labels for train data set are also given. Since competition is over, test labels of the data set are also available. To increase the flexibility of the analysis, channel subset, signal duration, and number of repetitions are made configurable in

the system. By this way, different setups are easily established. Stimulus code is used for separating rows and columns in next steps. Number of repetitions used is also adjustable to analyze the prediction results for less number of repetitions.

For each target intensification, there are 5 non-target intensifications in train data set. Number of target and non-target intensifications need to be equalized to obtain a well-balanced train data set. For each target row and column, a non-target row and column is selected randomly. A sample for a single target and non-target intensifications are given in Figure 4.1.

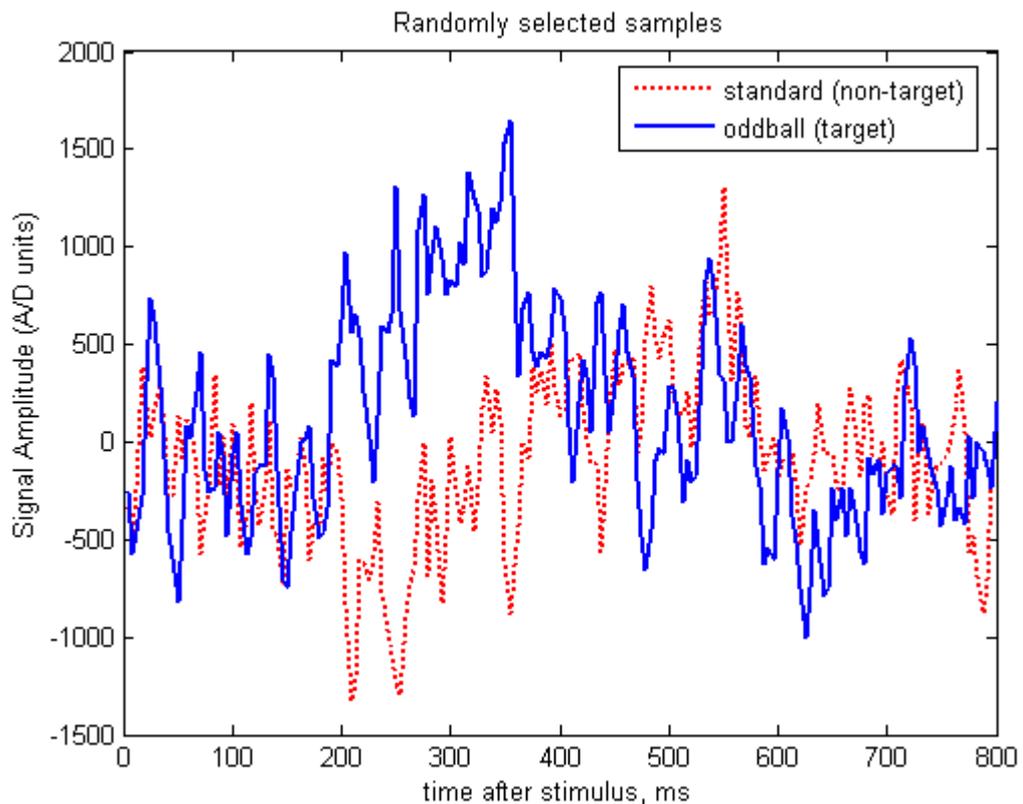


Figure 4.1 Randomly selected target and non-target intensifications extracted 0-800 ms after stimulus

Target and non-target intensifications for train and test data set is extracted for 800 ms duration after the stimulus and averaged plots are given in Figure 4.2, Figure

4.3, Figure 4.4, and Figure 4.5 for channels 1-16, 17-32, 33-48 and 49-64 respectively.

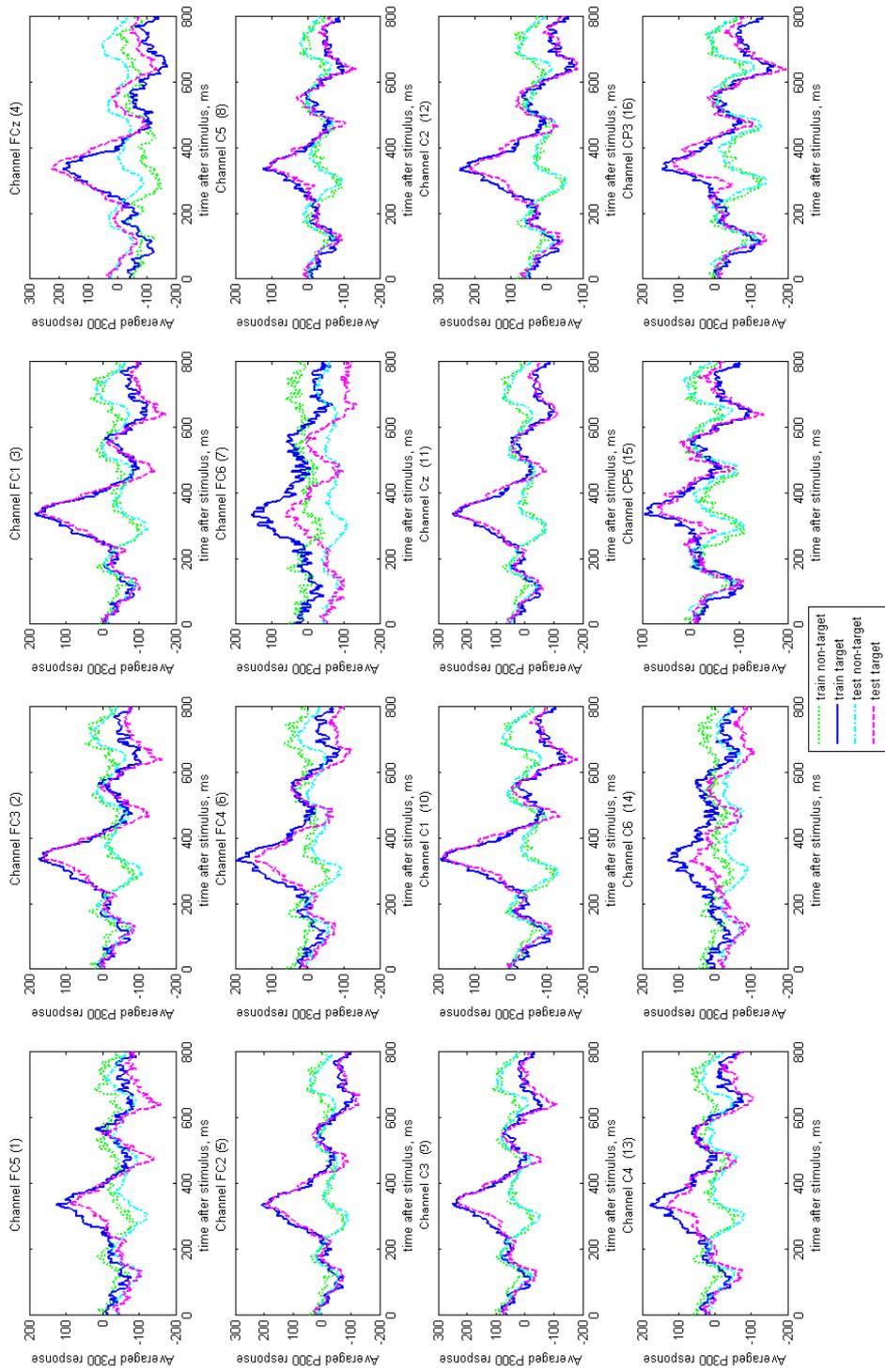


Figure 4.2 Averaged target and non-target intensifications for train and test data set extracted 0-800 ms after stimulus, channels 1-16

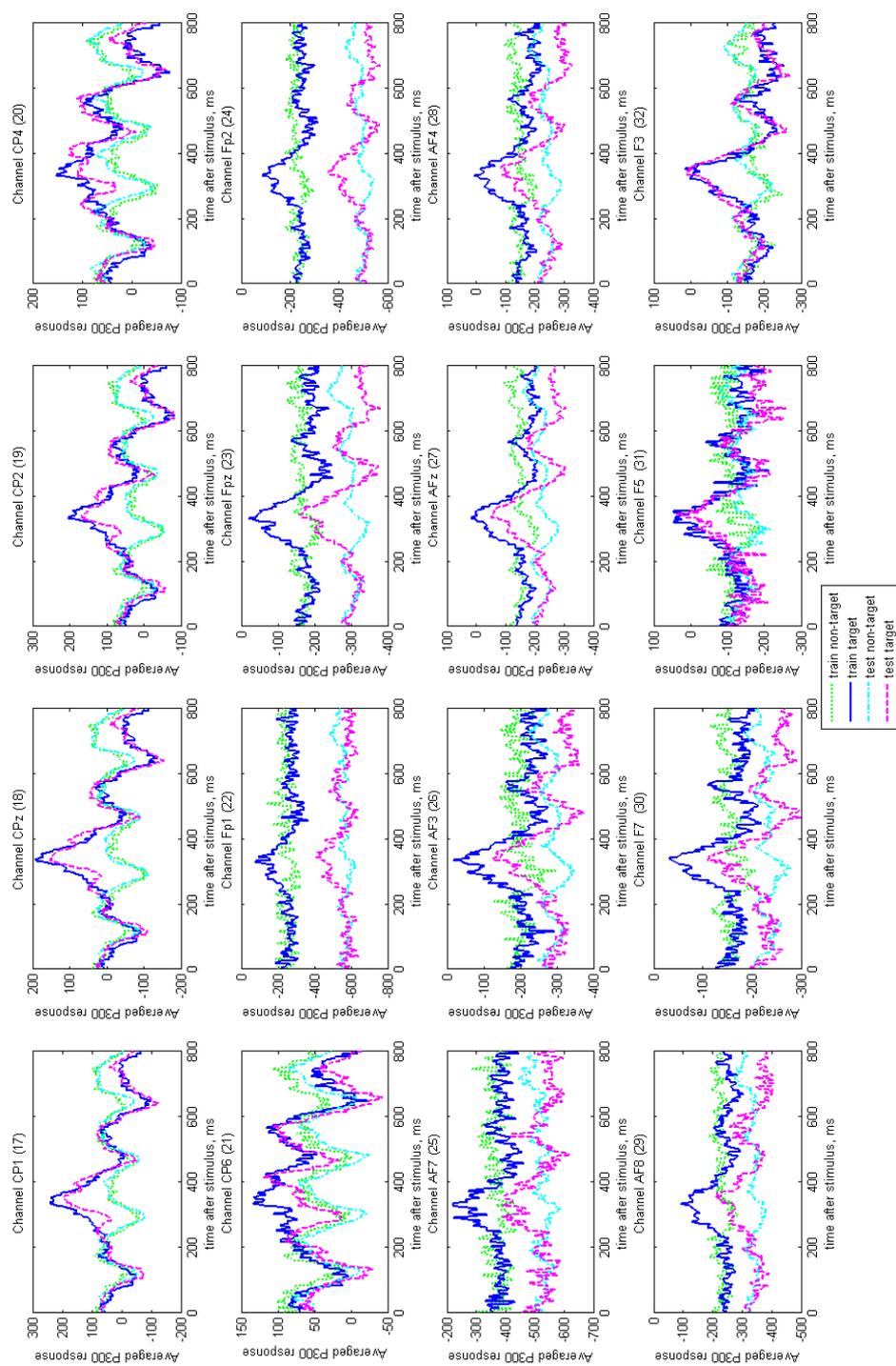


Figure 4.3 Averaged target and non-target intensifications for train and test data set extracted 0-800 ms after stimulus, channels 17-32

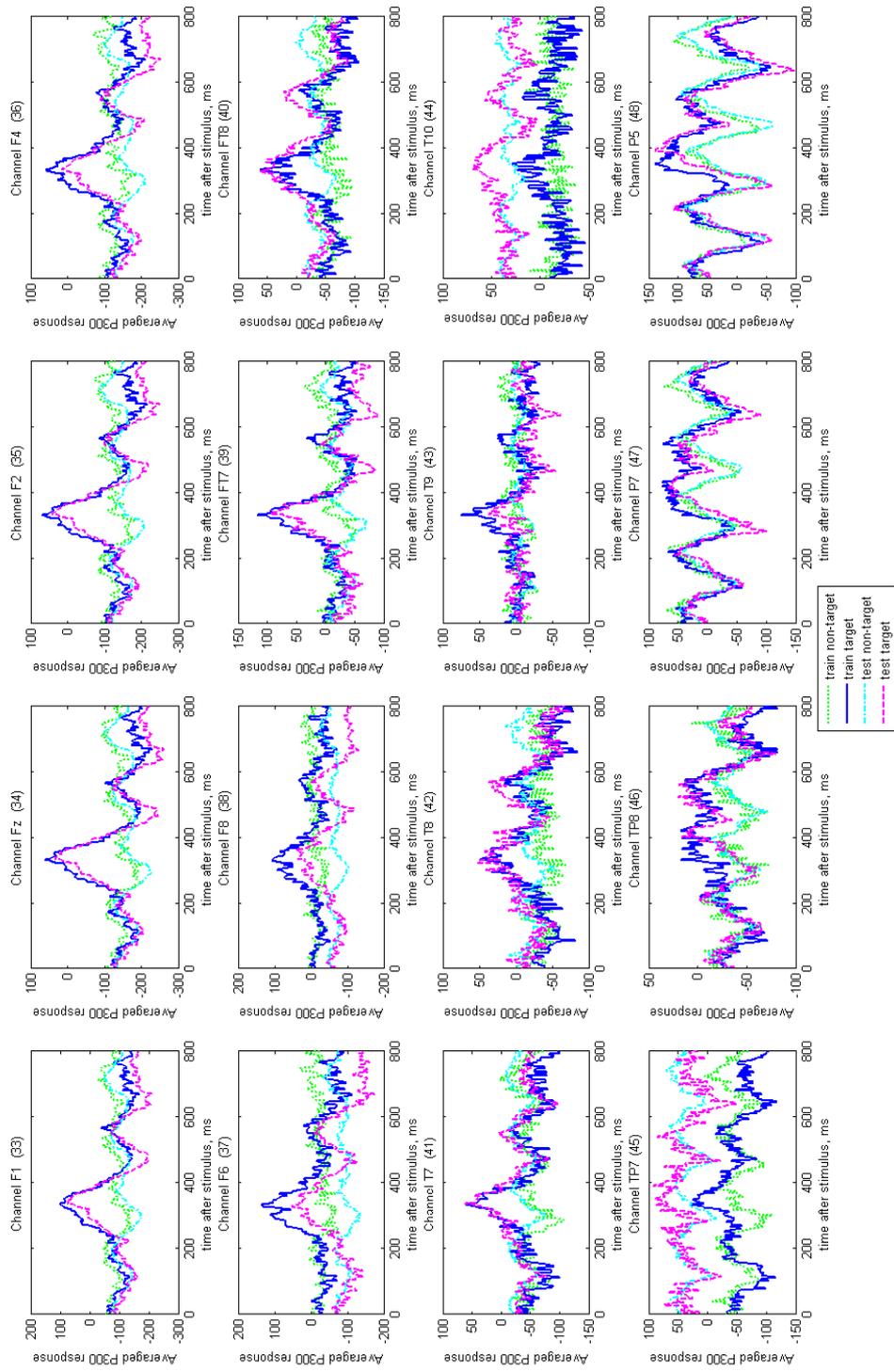


Figure 4.4 Averaged target and non-target intensifications for train and test data set extracted 0-800 ms after stimulus, channels 33-48

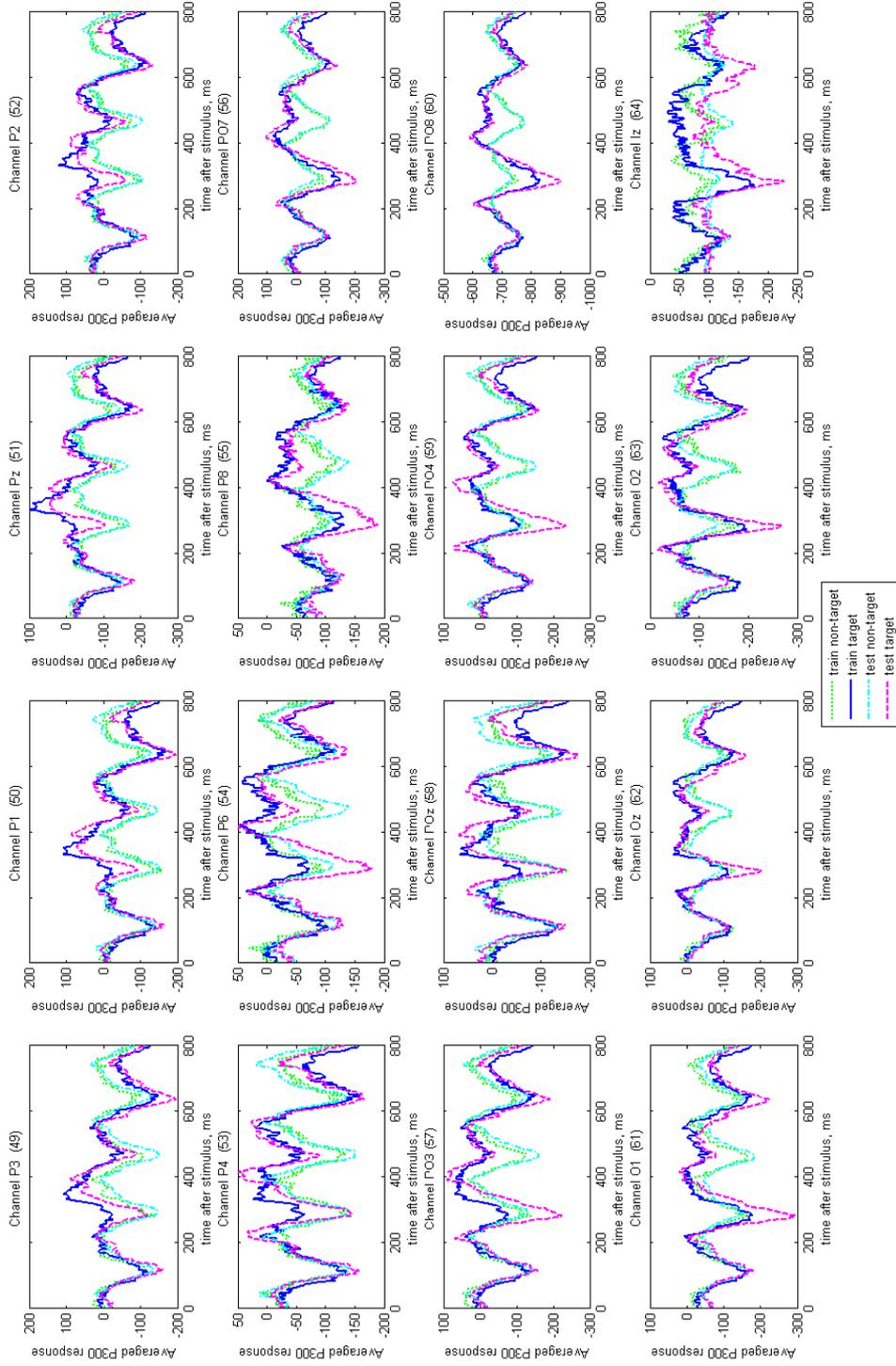


Figure 4.5 Averaged target and non-target intensifications for train and test data set extracted 0-800 ms after stimulus, channels 49-64

P300 evoked potentials are observed approximately 300 ms after the stimulus. It is seen from 4 figures, Figure 4.2, Figure 4.3, Figure 4.4, and Figure 4.5, that extracting signals [100-600] ms after stimulus keeps all of the information related with P300 evoked potentials.

4.1.2 Filtering

When 4 figures, Figure 4.2, Figure 4.3, Figure 4.4, and Figure 4.5, are investigated, it is seen that P300 evoked potentials contain the information in low frequency components. Since signals are sampled at 240 Hz, high frequency components are filtered to reduce the effect of noise. Characteristics of 8th order Chebyshev Type-I bandpass filter with cut-off frequencies 0.1-10 Hz is given in Figure 4.6. This filter was used by Rakotomamonjy et al. [30].

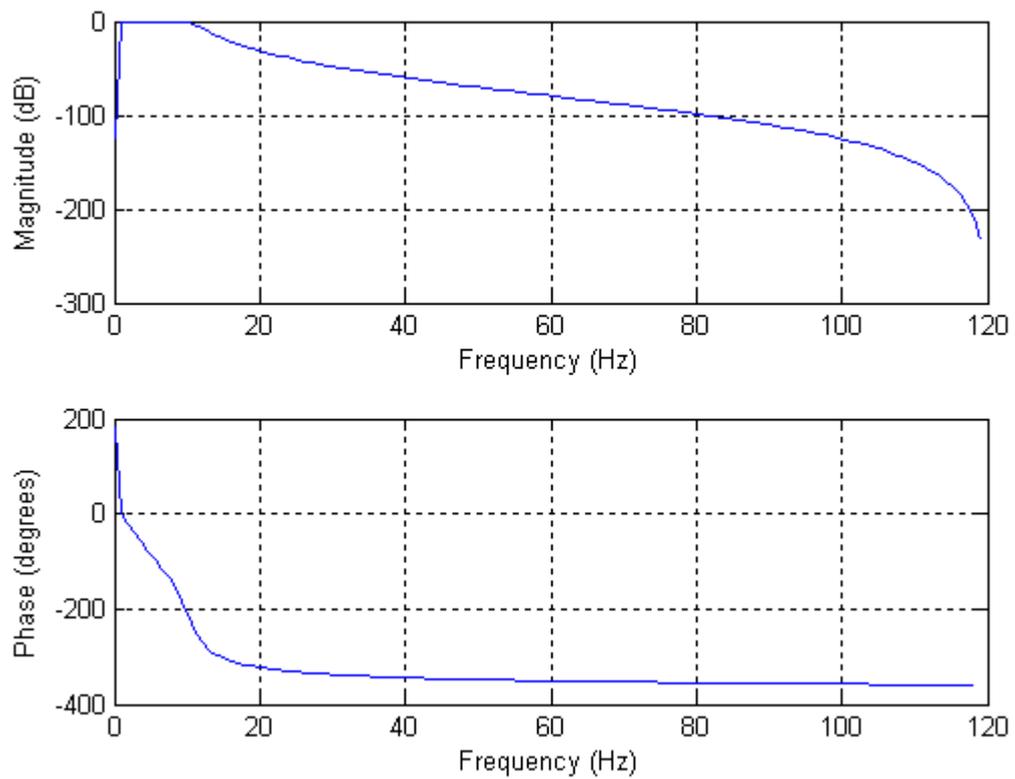


Figure 4.6 8th order bandpass Chebyshev Type I filter with cut-off frequencies 0.1 Hz and 10 Hz

Effect of the filtering with Chebyshev Type-I filter given in Figure 4.6 on the target and non-target signals of Figure 4.1 are given in Figure 4.7 and Figure 4.8 respectively.

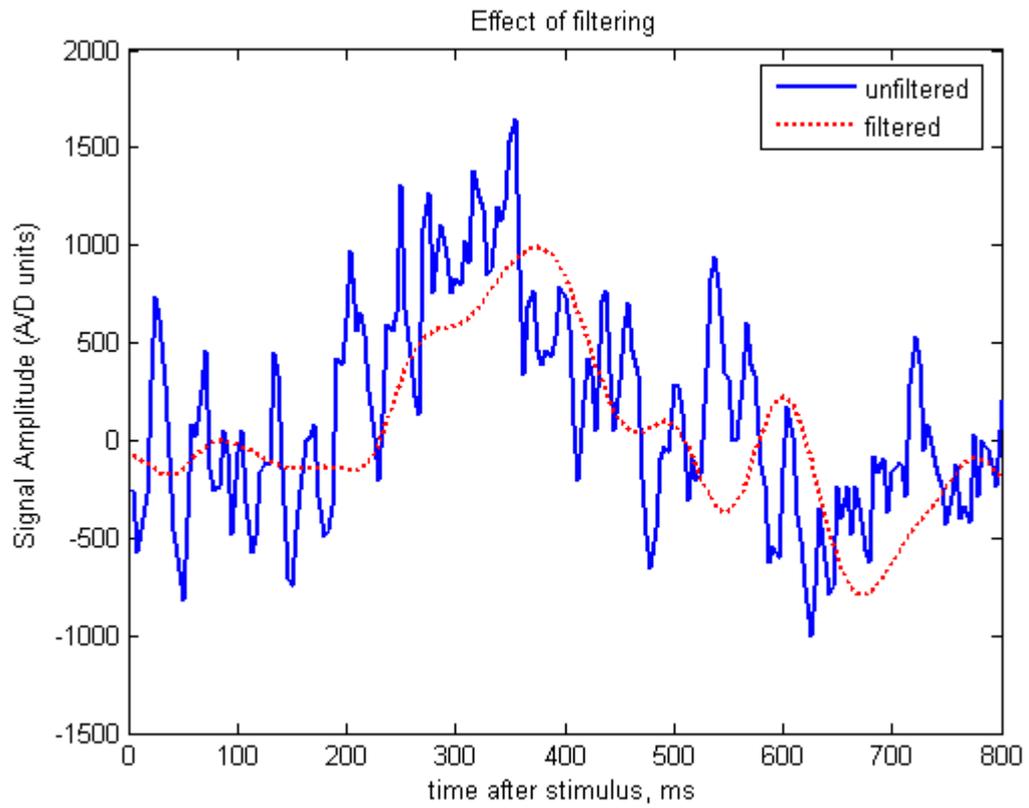


Figure 4.7 Effect of filtering with 8th order 0.1 – 10 Hz bandpass Chebyshev Type-I filter on a target intensification

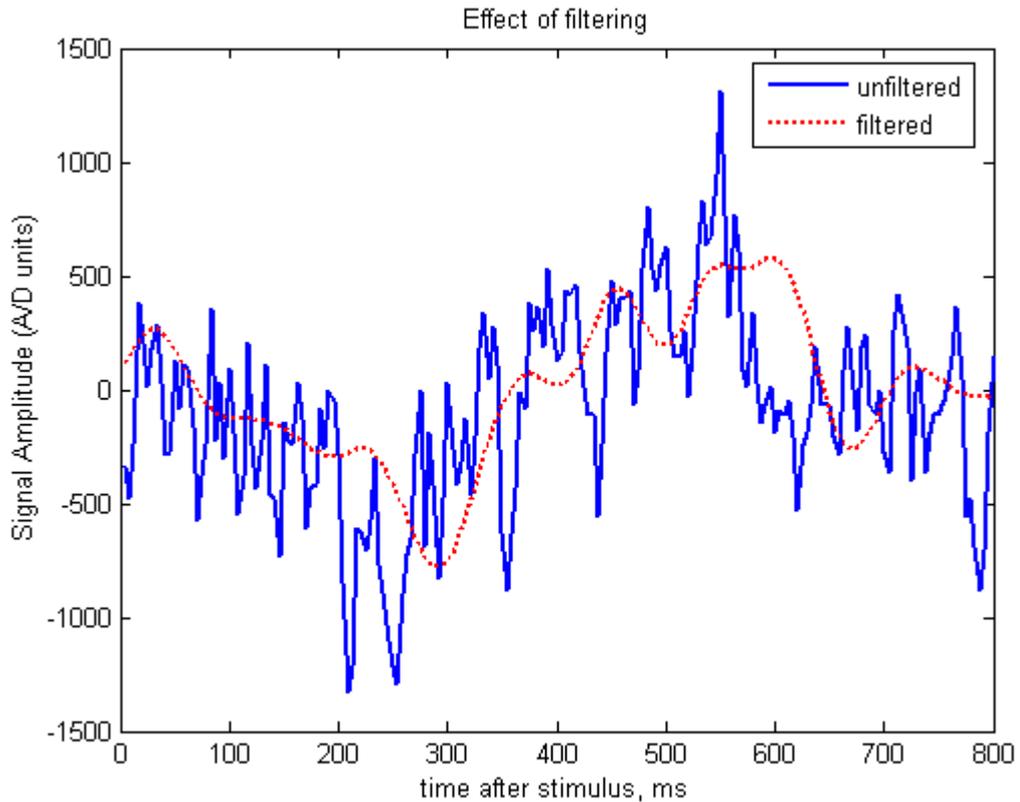


Figure 4.8 Effect of filtering with 8th order 0.1 – 10 Hz bandpass Chebyshev Type-I filter on a non-target intensification

4.1.3 Downsampling

Downsampling is the last method that is applied in the preprocessing step. Signals are generally sampled at frequencies higher than the interest. Filtering suppresses frequency components that are out of interest. However, it does not change the sampling rate. Since signals sampled at high frequency rates are filtered, sampling rate of the signal can be reduced by representing a group of samples with a single sample. By this way, size of the signal is also reduced and thus process speed of the next steps is increased. Downsampling is done by averaging samples with non-overlapping windows. That is, signal is partitioned into pieces and each piece is represented with the average of samples in it. Length of each piece is determined by the window length parameter. New sampling frequency becomes original sampling frequency divided by the window length. Window length is adjustable and if it is

selected to be 1, downsampling is skipped. When the window length is greater than 1, high frequency noise components are also suppressed with averaging consecutive samples in addition to dimension reduction.

4.2 Feature Extraction

Feature vectors are extracted after relevant part of the recorded brain signals are extracted from data. While extracting feature vectors, each channel is transformed into the feature space and then all channels are concatenated. Time and time-frequency domains are used as feature spaces in this study. Feature vectors are normalized with simple or Gaussian normalization methods to minimize the effect of time varying offset and amplitude of the signal. That is, dependency on sampling time and subject is reduced by normalizing features. Samples are extracted from time interval [100-600] ms after stimulus unless otherwise it is stated.

4.2.1 Time Domain Features

No transformation is done in time domain. Channel outputs are directly concatenated to obtain feature vectors. Average feature vectors extracted without filtering and normalization for a single channel are given in Figure 4.9.

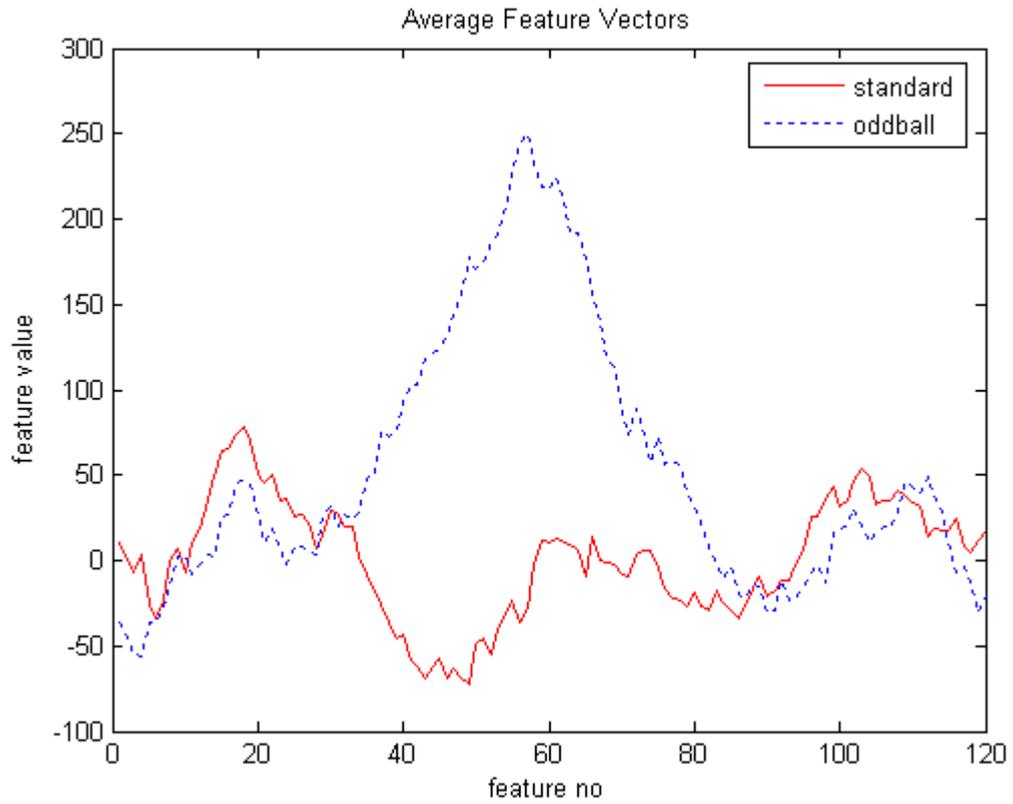


Figure 4.9 Average time domain feature vectors of target (oddball) and non-target (standard) classes for channel 11

Simple normalization and Gaussian normalization methods are applied to channel 11 data and results are given in Figure 4.10 and Figure 4.11 respectively.

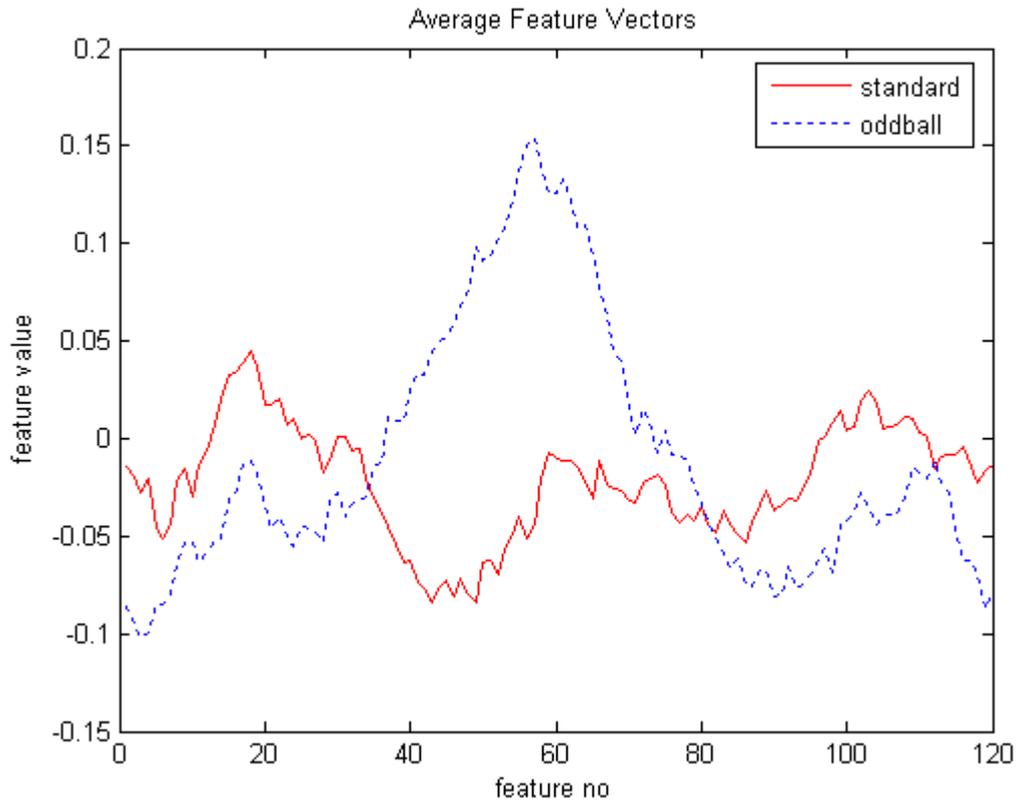


Figure 4.10 Average time domain simple normalized (mapped to [-1, 1]) feature vectors of target (oddball) and non-target (standard) classes for channel 11

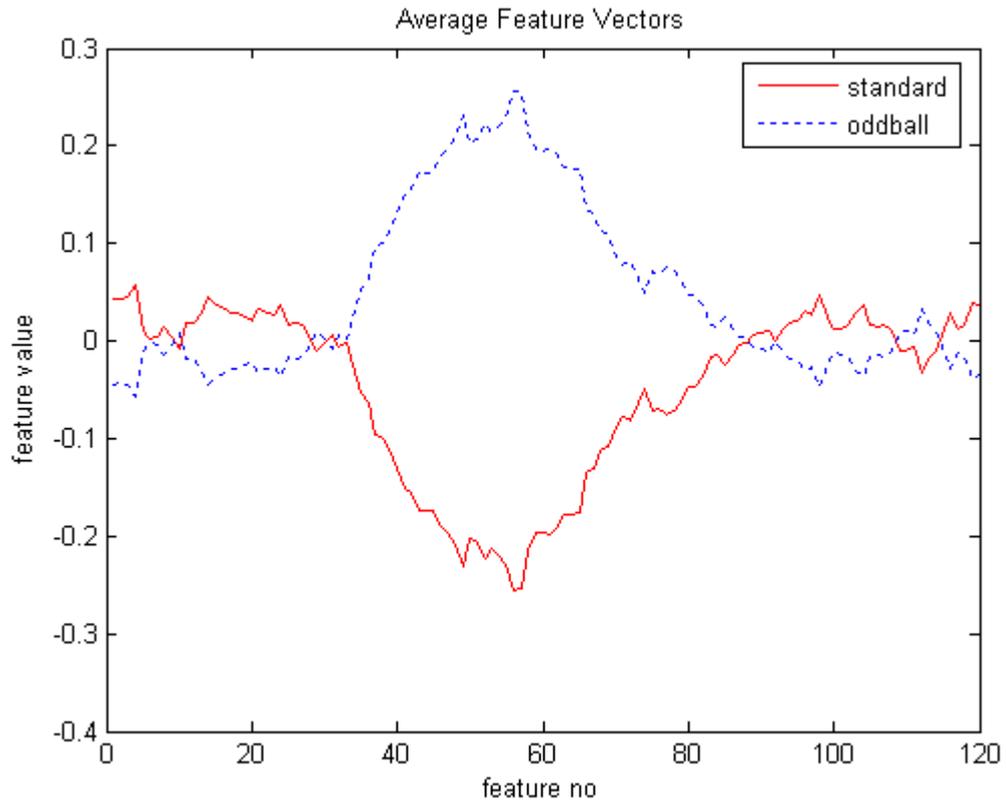


Figure 4.11 Average time domain Gaussian normalized feature vectors of target (oddball) and non-target (standard) classes for channel 11

4.2.2 Time-Frequency Domain Features

Wigner-Ville Distribution (WVD) is used to create time-frequency features of the system. Before starting the analysis in time-frequency domain, energy of the signal is investigated. Because, integral of WVD over frequency gives energy distribution over time and keeping the information that distinguishes target and non-target classes while transforming signal into energy domain is the key point for creating time-frequency features successfully.

4.2.2.1 Energy Analysis

Energy of a Discrete-Time Signal is:

$$E = \sum_{n=-\infty}^{\infty} |x_n|^2 \quad (4-1)$$

As it is seen from (4-1), square operation is involved in energy calculation. Square operation causes loss of sign information and thus the signal and its absolute value becomes equivalent to each other. Effect of absolute value operation on channel 11 is given in Figure 4.12. The average feature vectors given in Figure 4.12 are distorted compared to the ones given in Figure 4.9.

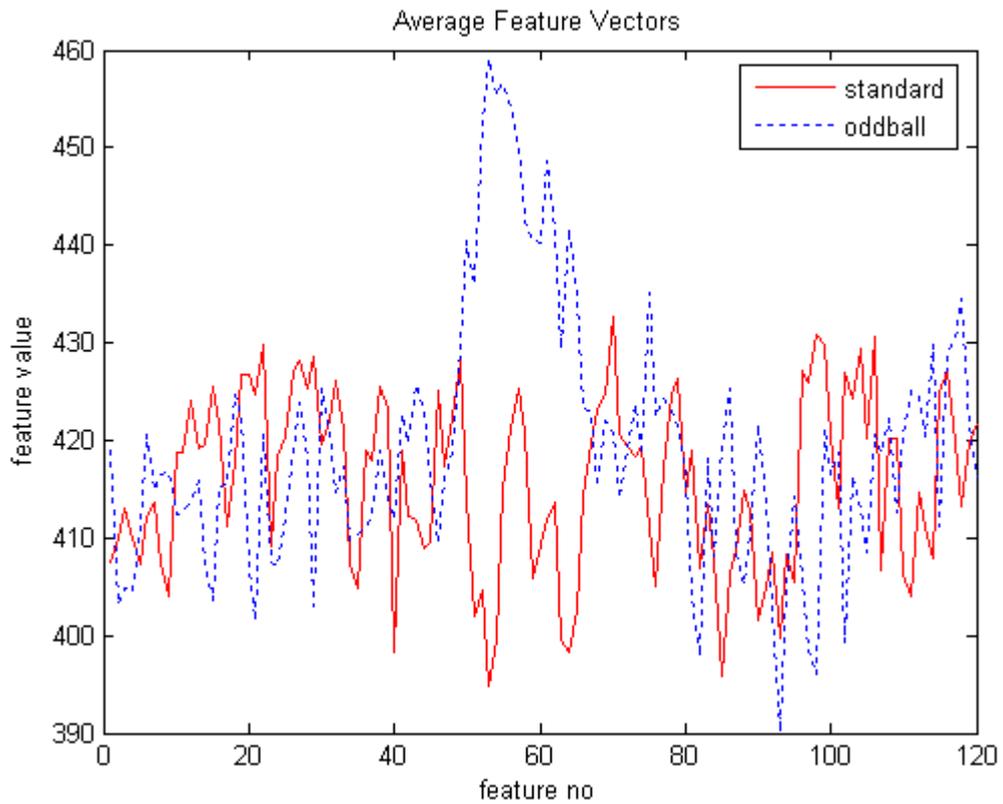


Figure 4.12 Average time domain feature vectors of target (oddball) and non-target (standard) classes obtained by the absolute value of the signals for channel 11

Figure 4.13 shows the normalized energy distribution over time. Normalization of the energy is done by dividing energy distribution to total energy. That is, each signal has total energy of 1 after normalization. Distortion of the absolute value operation is also seen in Figure 4.13.

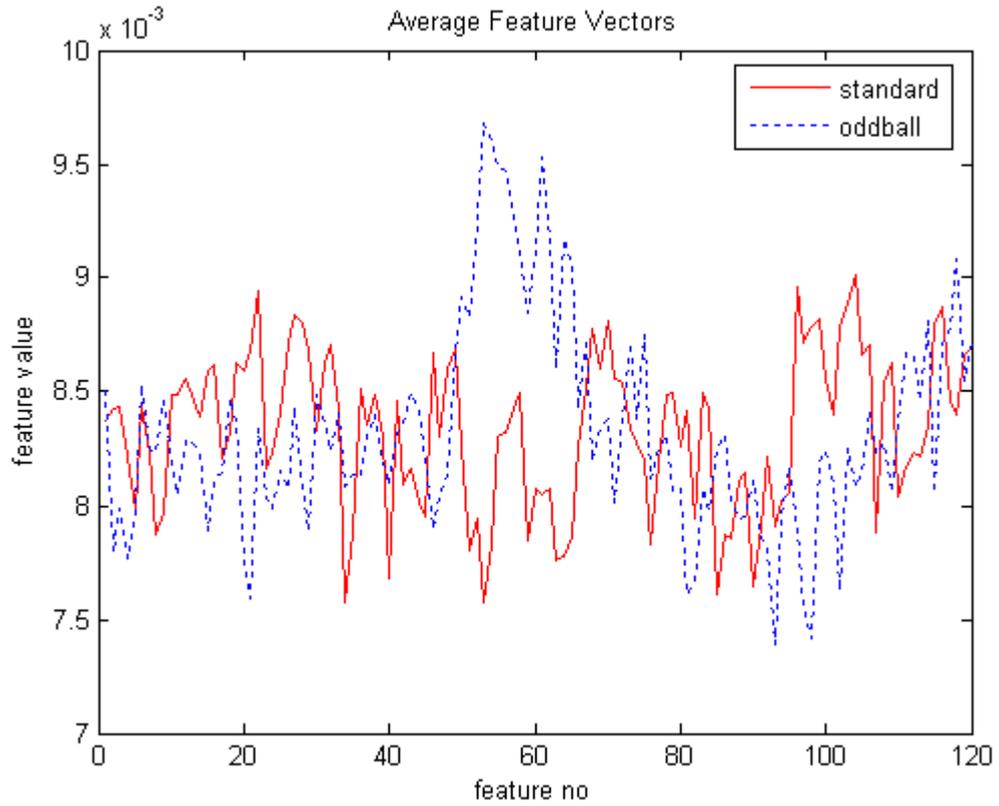


Figure 4.13 Average time domain feature vectors of target (oddball) and non-target (standard) classes obtained by the normalized energy distribution of the signals for channel 11

To get rid of the distortion due to absolute value operation, offset is applied to set minimum value of the signal to 0. After the offset trick, normalized energy distribution is given in Figure 4.14. Distortion due to absolute value operation is recovered and average feature vector characteristics obtained in this way are similar to the ones given in Figure 4.9.

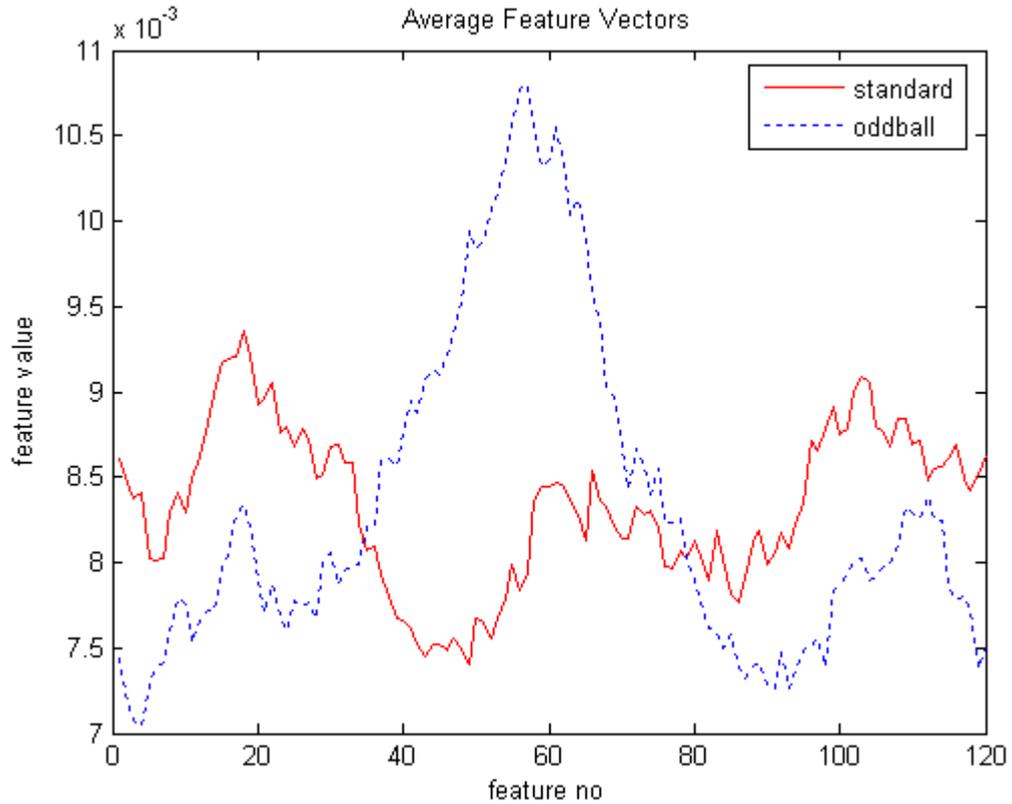


Figure 4.14 Average time domain feature vectors of target (oddball) and non-target (standard) classes obtained by the normalized energy distribution of the signals with offset trick for channel 11

4.2.2.2 Wigner-Ville Distribution

Integral of WVD over time and frequency gives total energy of the signal. WVD is also normalized to make total energy of signal equal to 1. In Figure 4.15, all frequency components of the WVD are summed to get energy distribution over time. Offset trick is applied to signals before calculating WVD.

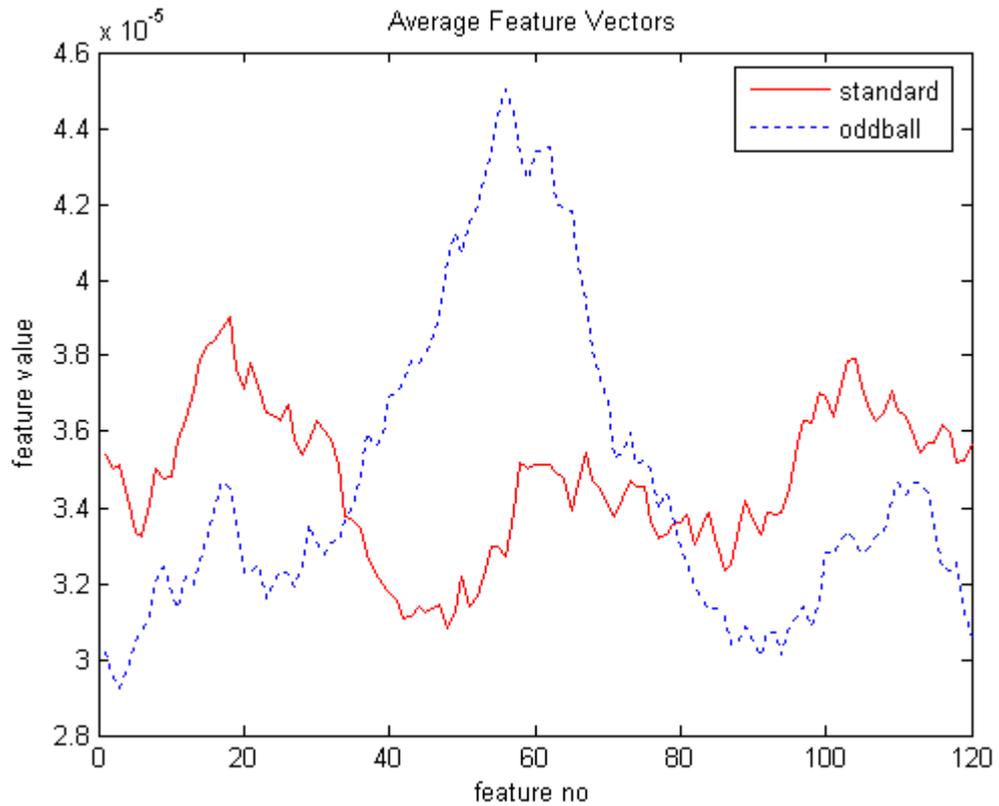


Figure 4.15 Average time domain feature vectors of target (oddball) and non-target (standard) classes obtained by the sum of all frequency components in WVD for channel 11.

4.2.2.3 Creating Time-Frequency Domain Features

WVD gives time-frequency analysis of the signals. These signals are converted into features by dividing WVD into sections and taking averages of each section to create features. Frequency band is divided according to types of the brain waves.

Table 4.1 Brain waves

Type	Frequency (Hz)	Activity
Delta: Δ	0.5 – 3.5	Prepare to Motion
Theta: Θ	3.5 – 8	Memory
Alpha (μ): α (μ)	8 – 13	Sensor Idling
Beta: β	13 – 22	Motor Idling
Gamma: γ	22 - 40	Property Fusion

Mean value of each band is calculated to reduce frequency band into 5 components. Using these 5 components as features causes 5 times increase in feature vector size. To overcome this problem, time is also divided into uniform sections. For a time section of length 25 ms, feature vector size reduces 6 times. Averages of time-frequency domain feature vectors are given in Figure 4.16 for channel 11.

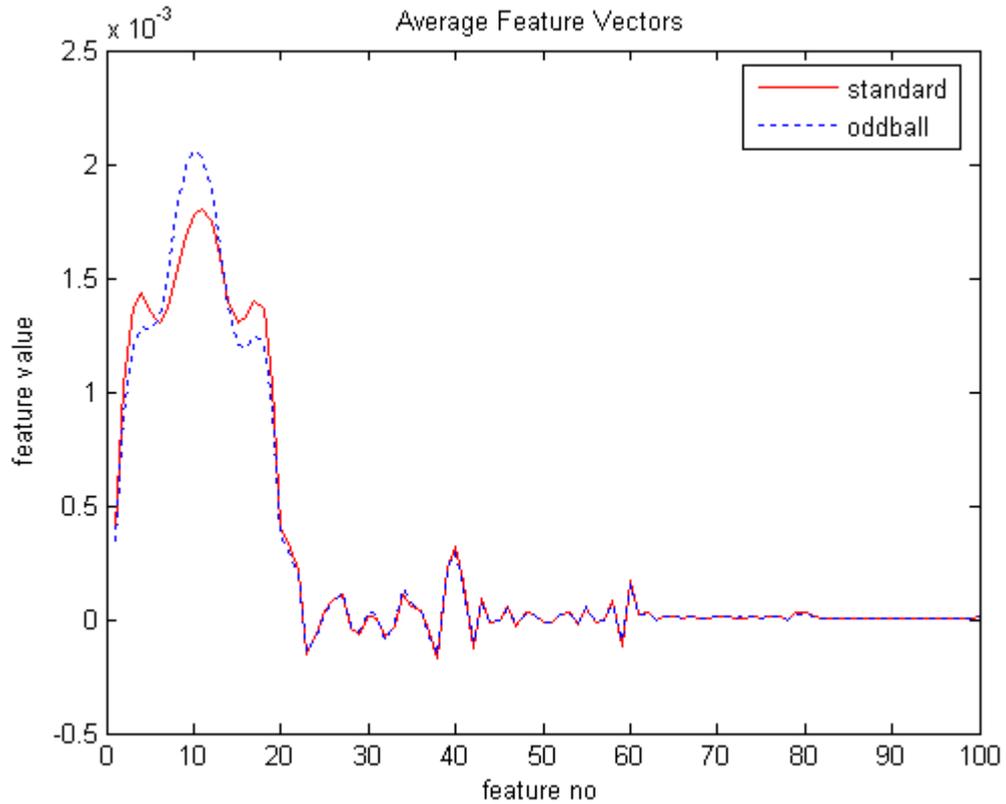


Figure 4.16 Average time-frequency domain feature vectors of target (oddball) and non-target (standard) classes obtained from the WVD for channel 11.

4.3 Dimension Reduction

Reducing the size of the data is important to reduce memory usage and processing time. However, valuable information must be kept while reducing the size of the data. With a proper dimension reduction, even the classification performance can be increased by getting rid of irrelevant information.

If same information can be obtained from different channels due to the correlation of data, then redundancy can be removed by dimension reduction methods. Correlation of data sampled from different channels at the same instant is measured to see that if dimension reduction is needed when multiple channel data is used. Measurement is done by concatenating extracted signals and keeping the chronological sequence.

Correlation coefficients of channel 11 (Cz) with all 64 channels are given in Figure 4.17. Correlation coefficients are greater than 0.8 for neighbor channels (3, 5, 6, 9, 10, 12, 13, 17, 18, and 19) and decreasing with the distance between two channels.

Calculation of correlation coefficients is given in APPENDIX A.

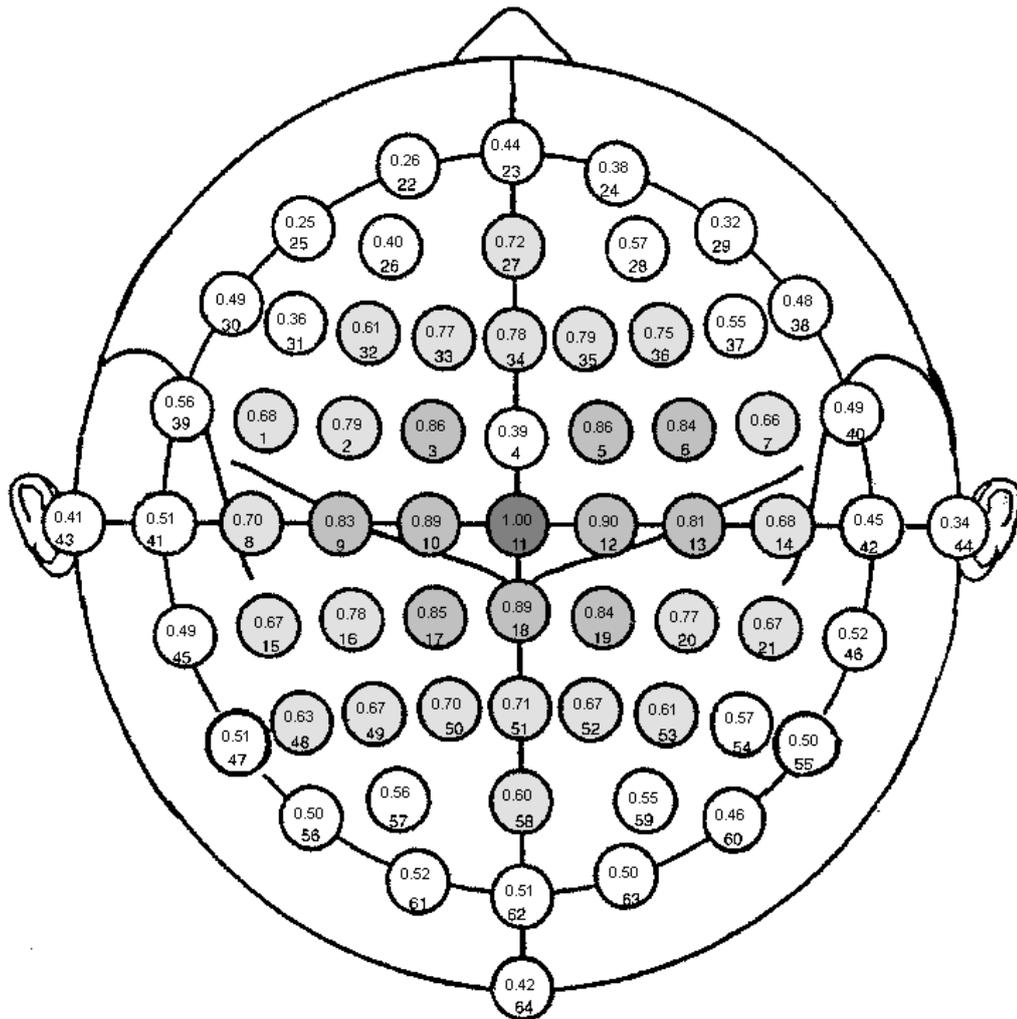


Figure 4.17 Correlation coefficients between channel 11 and other 64 channels

Correlation matrix for channels 3, 4, 5, 10, 11, 12, 17, 18, 19 (placed near to the center of the skull) are given in Table 4.2. Table 4.2 also shows that channels that are closer to each other have higher correlation.

Table 4.2 Correlation Coefficient Matrix for the subset of channels 3, 4, 5, 10, 11, 12, 17, 18, 19

Channel Number	3	4	5	10	11	12	17	18	19
3	1	0.4	0.9	0.92	0.86	0.88	0.83	0.85	0.79
4	0.4	1	0.39	0.37	0.39	0.38	0.33	0.34	0.32
5	0.9	0.39	1	0.88	0.86	0.93	0.81	0.86	0.84
10	0.92	0.37	0.88	1	0.89	0.91	0.94	0.94	0.87
11	0.86	0.39	0.86	0.89	1	0.9	0.85	0.89	0.84
12	0.88	0.38	0.93	0.91	0.9	1	0.89	0.95	0.94
17	0.83	0.33	0.81	0.94	0.85	0.89	1	0.96	0.9
18	0.85	0.34	0.86	0.94	0.89	0.95	0.96	1	0.96
19	0.79	0.32	0.84	0.87	0.84	0.94	0.9	0.96	1

Correlation coefficient analysis shows that there is a big correlation among some channels and feature vector size can be reduced by eliminating redundant information. For this purpose, Principal Component Analysis (PCA) and AdaBoost are examined in this study as explained below.

4.3.1 Dimension Reduction Using PCA

Dimension reduction with PCA is done by first transforming features into another space, in which new features are ordered in decreasing variance, and then eliminating features with small variance. PCA does not consider differences between classes in data set while reducing dimension. Effect of dimension reduction using PCA is analyzed for data given channels given in Table 4.2. Filtering and normalization is not applied to data set. Average feature vectors for target and non-target intensifications are given in Figure 4.18.

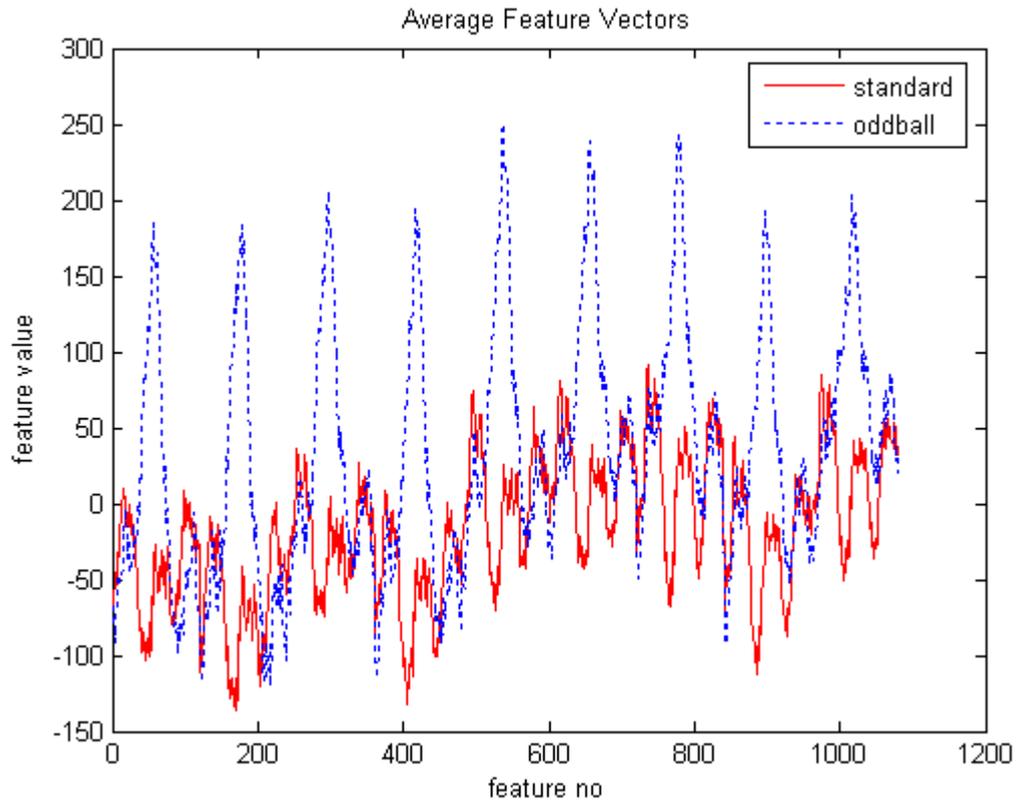


Figure 4.18 Average feature vectors of target (oddball) and non-target (standard) classes for channels 3, 4, 5, 10, 11, 12, 17, 18, 19

All feature vectors are transformed using PCA and averages of transformed target and non-target feature vectors are given in Figure 4.19. Dimension reduction is not done after transformation. Because of that, feature vector size is equivalent with non-transformed feature vectors given in Figure 4.18.

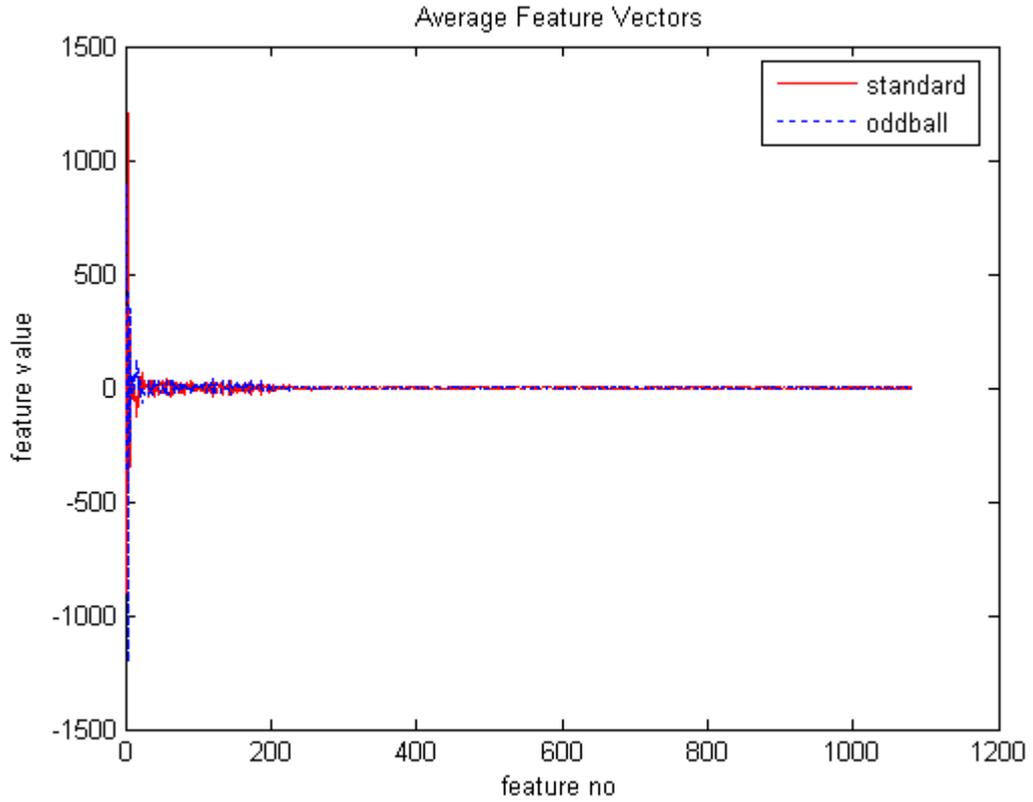


Figure 4.19 Average PCA transformed feature vectors of target (oddball) and non-target (standard) classes for channels 3, 4, 5, 10, 11, 12, 17, 18, 19

Information kept while reducing the dimensionality in PCA is calculated with:

$$r = \frac{\sum_{i=1}^p \lambda_i}{\sum_{i=1}^n \lambda_i} \times 100, \quad (4-2)$$

where λ_i is the eigenvalue of the i 'th feature, n is the number of features and p is the reduced feature number. In Figure 4.20, change of r with p is shown for the feature vectors analyzed in Figure 4.19.

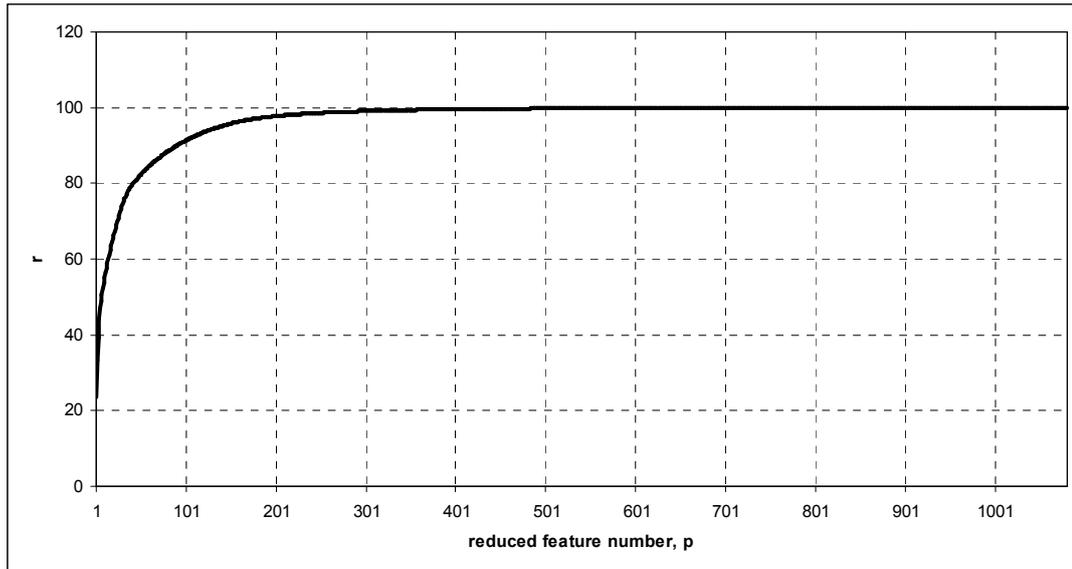


Figure 4.20 Percentage of information kept (r) vs. number of feature components included (p) for channels 3, 4, 5, 10, 11, 12, 17, 18, 19

In Figure 4.20, dimension of the feature vector to keep 90% of the information is found to be the first 91 features of PCA transformed feature vectors. In Figure 4.21, average target and non-target PCA transformed feature vectors with 91 components are given.

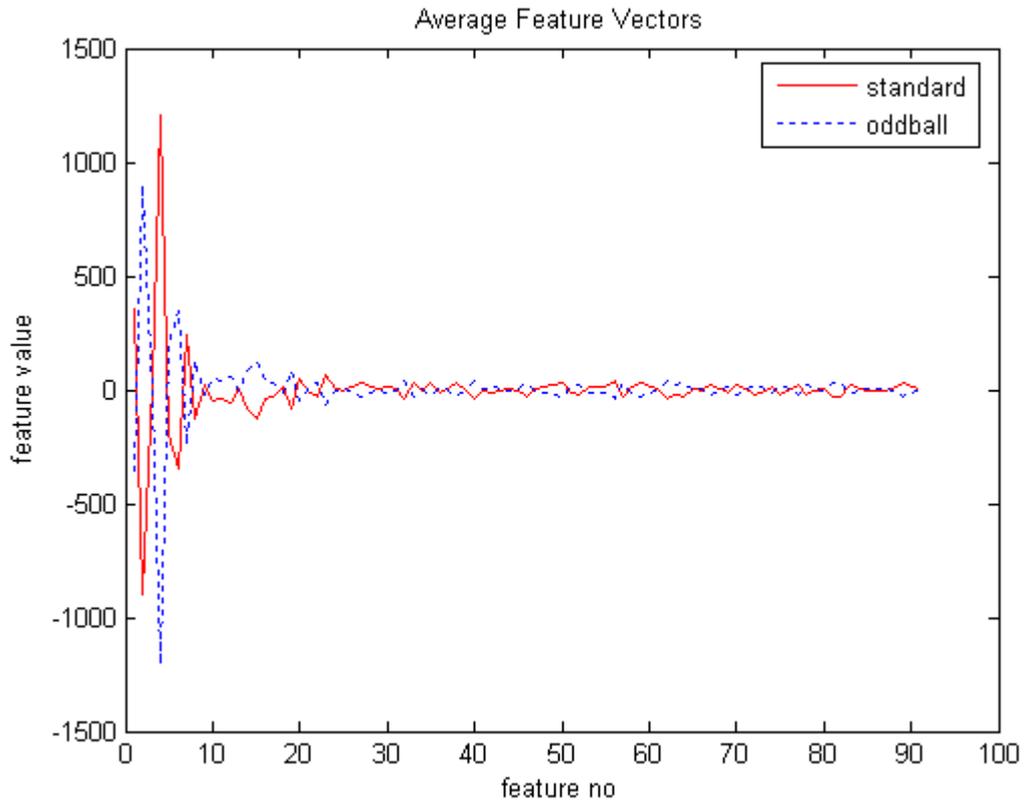


Figure 4.21 Average PCA transformed and reduced feature vectors of target (oddball) and non-target (standard) classes for channels 3, 4, 5, 10, 11, 12, 17, 18, 19

4.3.2 Dimension Reduction Using AdaBoost

AdaBoost selects features using the discriminative properties of the target and non-target classes. AdaBoost does not transform features during selection. It only determines a subset of features using the information provided in training data. That is, dimension reduction via AdaBoost is performed by eliminating unselected features from feature vectors. Because of that, calculation overhead does not exist after system is trained. In Figure 4.22, first 30 features of channel 11 selected by AdaBoost with CART as the weak classifier are marked on average target and non-target classes. Selected features are sorted according to their selection order in Figure 4.23.

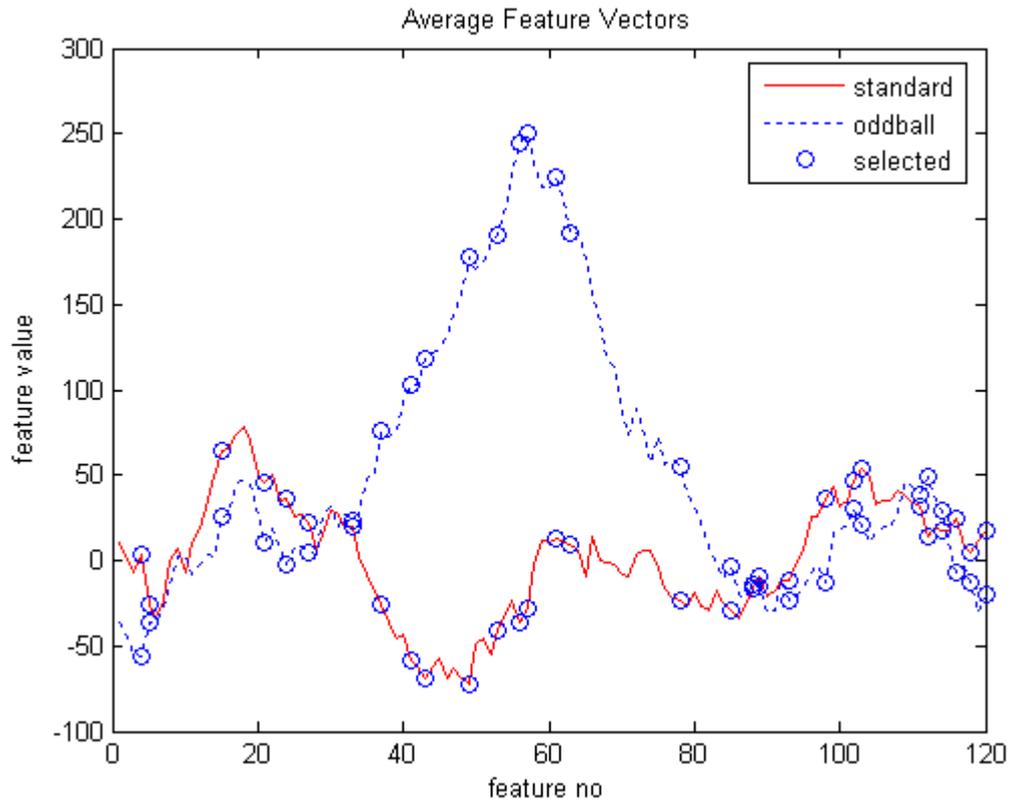


Figure 4.22 30 features selected by AdaBoost from channel 11; selected features are marked with 'o' on both averages of target (oddball) and non-target (standard) classes

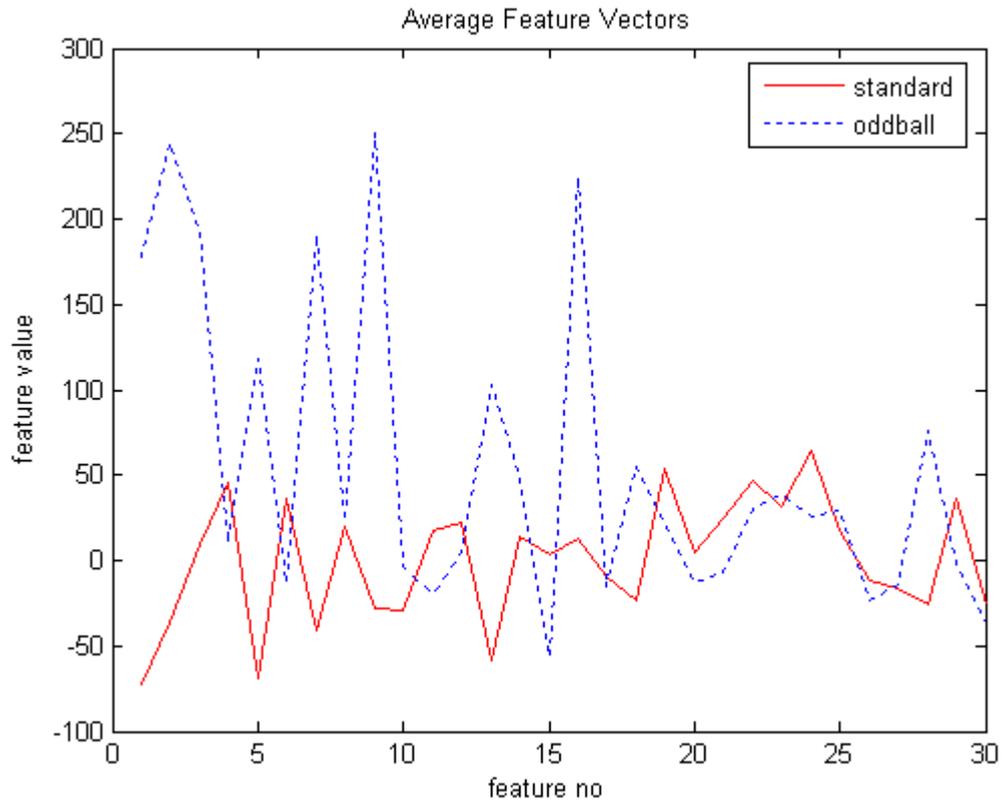


Figure 4.23 Averages of target (oddball) and non-target (standard) classes for the first 30 features selected by AdaBoost; features are given in selection order and selected from channel 11

4.4 Classification

Classification is done in two steps. In first step, binary classification is done to decide if the feature vectors belong to target or non-target classes. In second step, decision fusion is done to select a target character using binary classification results.

4.4.1 Binary Classification

In binary classification, each feature vector is processed independently and labeled as either target or non-target classes. SVM and AdaBoost binary classification methods are used to classify features. 5-fold CV is used in both methods to prevent overfitting problem.

4.4.1.1 Support Vector Machine

SVM is trained using all of the train feature vectors. Soft Margin SVM is used to introduce a penalty or regularization parameter (C) for misclassified samples. This parameter is used to tune the system to increase classification performance. Also, Gaussian Radial Basis Function (RBF) is used as kernel. Gaussian RBF brings an extra control parameter σ to SVM which provides more suitable boundary selection. 5-fold cross validation is used to tune SVM control parameters (C, σ) and prevent overfitting problem. Soft margin outputs of cross validation results are summed and label of the feature vector is decided. For every label, a label rate showing the strength of the guess is also provided using the summation of cross validation soft margin results.

4.4.1.2 AdaBoost

AdaBoost learns data using the whole train data set with cross validation. AdaBoost maximum iteration number is the only parameter that needs to be adjusted according to the data set. AdaBoost mislabeled target and non-target feature vectors rate is saturated after maximum iteration number becomes sufficiently large. So, once maximum iteration number is properly selected, AdaBoost can adapt to data set without requiring any other adjustments. CART is used as the weak classifier of the AdaBoost. In Figure 4.24, binary classification error of AdaBoost is given up to maximum iteration number 200. So, maximum iteration of 100 is sufficiently large to reach saturation point. Binary classification outputs of cross validation results are summed and label of the feature vector is decided. For every label, a label rate showing the strength of the guess is also provided using the summation of cross validation binary classification results.

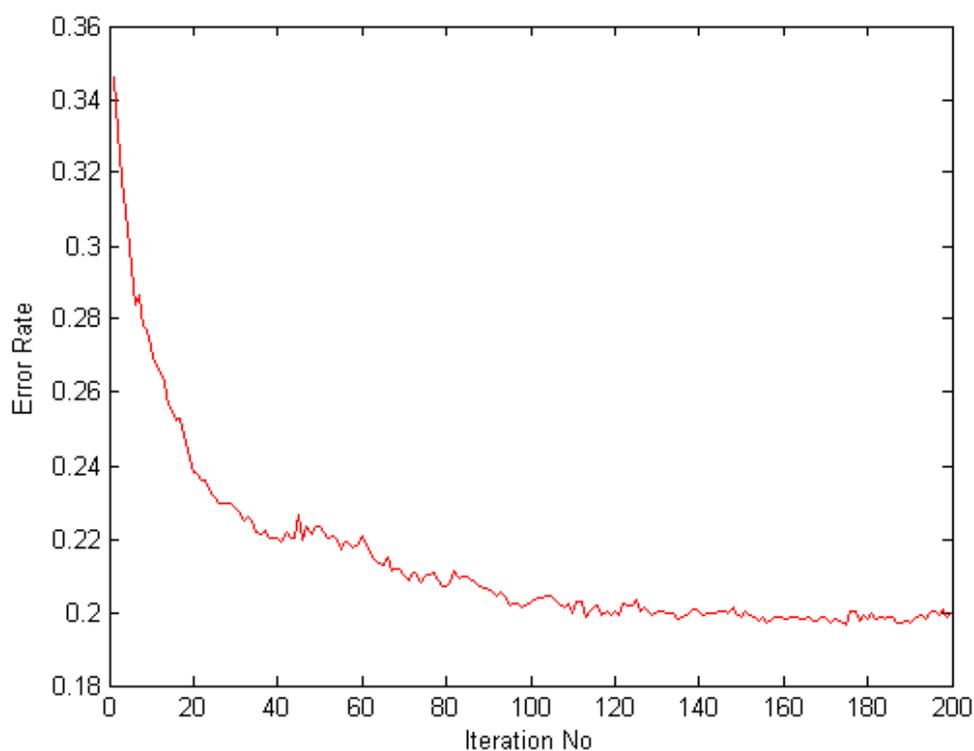


Figure 4.24 AdaBoost Iteration number vs. error rate graphics for channels 9,11,13,34,49,51,53,56,60,62

4.4.2 Decision Fusion

Target character is selected among 36 characters using binary classification results with a voting based decision fusion. Binary classification gives target/non-target classification results for 6 rows and 6 columns but only one of the rows and columns include the target character. To select a row and a column, repetitions of the target character are used. There can be up to 15 repetitions for each character. The possibility of being target row/column is found using the label rate of each binary classification. Then, repetition results are combined by summing this value for each row and column and the row and the column with the maximum possibility are selected as target row and column. Then, interception of the row and the column gives the target character. Note that it is important to classify target character with less number of repetitions to increase the usability of the system.

CHAPTER 5

EXPERIMENTAL RESULTS

Designed system is composed of three main steps called as preprocessing data, creating feature vectors and classification. Data is extracted using relevant channels and filtered at preprocessing step. Then, data is transformed into feature space, normalized, and its dimensionality is reduced to obtain feature vectors in next step. At final step, binary classification of the feature vectors are done and target character is predicted. Overview of the designed system is given in Figure 5.1.

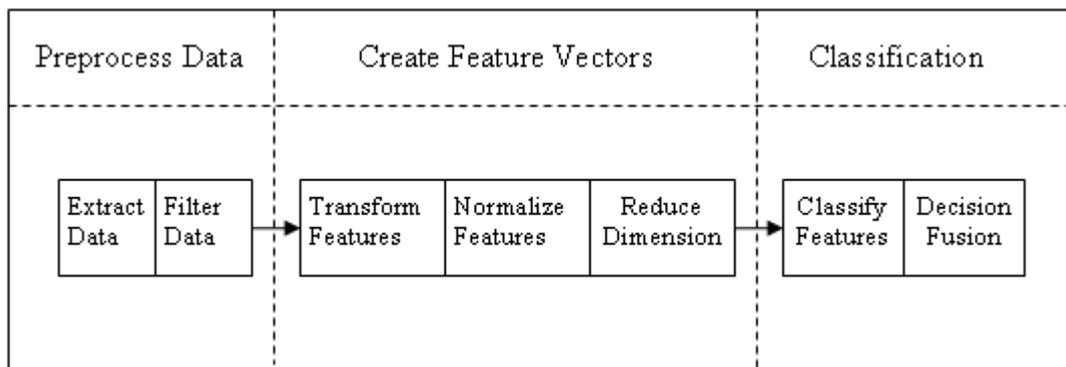


Figure 5.1 Overview of the designed system

First, filtering and normalization methods that best fit on transformation methods are selected. Then, optimum channels are selected using binary classification methods for each transformation method. Classification performances of transformation methods are evaluated and channel subset, transformation method

and binary classification method that gives maximum classification performance is determined. Then, effect of dimension reduction methods on feature vectors is analyzed. Finally, character prediction results are analyzed with the designed method.

First of all, performance measurement criteria are required while training and testing the system. Because of that, this chapter starts with explanation of evaluation criteria used while measuring performance of the methods and then continues with the outputs of the experimental results.

At training step, 5-fold Cross Validation (CV) results are used as success criteria. At each fold of CV, while 80% of data is used for training, the remaining 20% of data is used only for testing purpose to prevent overfitting. This is repeated 5 times, keeping a different 20% portion of data for validation. Success criterion is obtained by averaging classification results of these validation sets. For example, average misclassification rate of the sample CV result given in Table 5.1 is 16.79%. This means that %83.21 of test set labels are classified correctly. Aim is to maximize classification rate or minimize misclassification/error rate of the averaged CV results.

Table 5.1 Sample Cross Validation misclassification rates for 5-fold cross validation

CV fold-1	CV fold-2	CV fold-3	CV fold-4	CV fold-5
16.87%	16.27%	17.66%	14.29%	18.85%

Each CV result is calculated by first counting True Positive (TP), False Negative (FN), True Negative (TN), and False Positive (FP) samples, where meanings of TP, FN, TN, and FP are given as:

True Positive (TP): Number of correctly labeled target feature vectors

False Negative (FN): Number of mislabeled target feature vectors

True Negative (TN): Number of correctly labeled non-target feature vectors

False Positive (FP): Number of mislabeled non-target feature vectors

Then, *sensitivity* (True Positive Rate) given in (5-1) and *specificity* (True Negative Rate) given in (5-2) are calculated.

$$TruePositiveRate(TPR) = \frac{TP}{TP + FN} \quad (5-1)$$

$$TrueNegativeRate(TNR) = \frac{TN}{TN + FP} \quad (5-2)$$

Finally TPR and TNR results are averaged (see (5-3)) to obtain success of the classification.

$$SuccessRate(SR) = \frac{TPR + TNR}{2} \quad (5-3)$$

Equivalently, error rate can be also obtained by using the success rate of the classification (see (5-4)).

$$ErrorRate(ER) = 1 - SR \quad (5-4)$$

In train set, number of non-target samples is reduced by random selection to have equal number of target samples (TP + FN) and non-target samples (TN + FP).

Overall classification performance is also evaluated using (5-4) on a separate test set that is not used in CV. Since character detection is done on test set, number of non-target samples is not reduced by random selection. Because of that, there are 5 times more non-target samples than target samples in test set. Weight of target and non-target samples on success criteria is equalized by using (5-3).

Another performance criterion, which is named as *Quality Rate* (5-5), is maximized at channel selection.

$$QualityRate(QR) = \frac{TP}{TP + FP + FN} \quad (5-5)$$

Equivalently, *Quality Error Rate* (5-6) can also be minimized instead of maximizing (5-5).

$$QualityErrorRate(QER) = 1 - QR = \frac{FP + FN}{TP + FP + FN} \quad (5-6)$$

As it is stated before, there are 5 non-target samples for each target sample in test set. Because of that, importance of target feature classification is increased by omitting TN while evaluating the performance of the classification at channel selection. Equation (5-5) is also used by Rakotomamonjy [30] while selecting channels in his study. 5-fold cross-validation is also used at channel selection to prevent overfitting.

After individual prediction of target and non-target samples, target character prediction is done. Character recognition performance is evaluated by dividing correctly classified target characters to total character numbers.

Note that all performance evaluation formulas, (5-1), (5-2), (5-3), (5-4), (5-5) and (5-6), take values between 0 and 1.

5.1 Analyzing Effects of Filtering and Normalization Methods on Time and Time-Frequency Domain Signals

There are two types of filtering that are applied to data set. First one is applying a band-pass filter without changing the sampling rate and second one is down-sampling data by averaging consecutive samples. Applying band-pass filter is optional and can be skipped. However, down-sampling, which is applied at the end of filtering step of Preprocess, is not optional. Data is sampled at 240 Hz. However, maximum frequency which is required for Time-Frequency Domain features is 40 Hz. In order to reduce the dimension of the data and get rid of high frequency signal components, data is down-sampled to 40 Hz by averaging the observations with

non-overlapping windows. That is, at the end of filtering, each 6 sample is averaged and represented as a single value. So, data is both filtered and its size is reduced 6 times by down-sampling.

The word “filtering” represents applying band-pass filter without changing sampling rate. Since down-sampling is always implemented, it is excluded from “filtering” definition.

First, effects of filtering and normalization on Time and Time-Frequency signals are analyzed to find out the best combination in terms of classification. Also, optimum control parameters (C, σ) for SVM are found for each combination. These decisions are done using reference channels used by Kaper et al. [24] and Erdoğan et al. [29] and all repetitions. Overview of applied method is given in Figure 5.2.

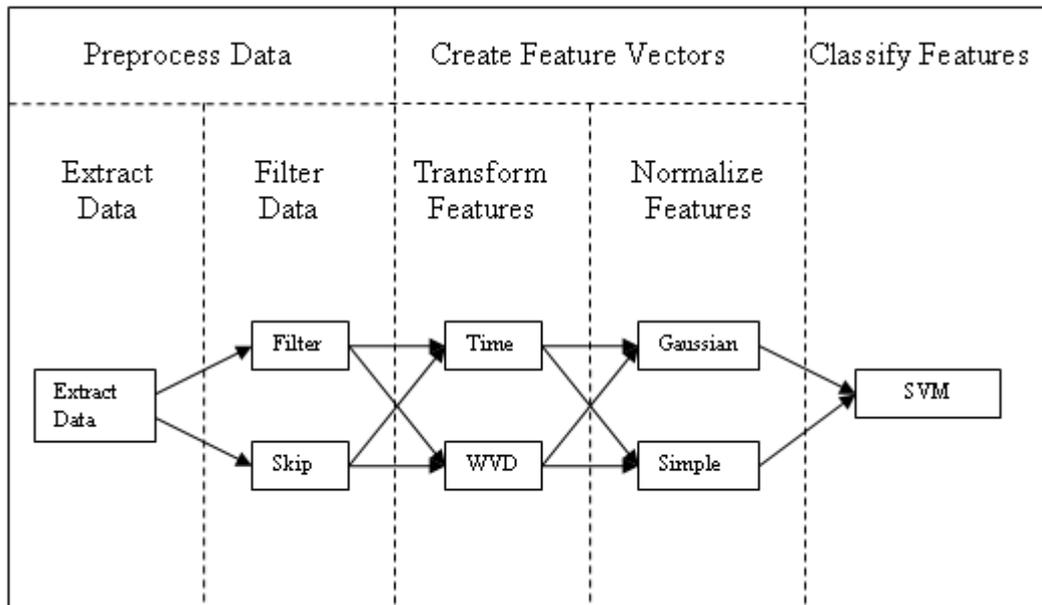


Figure 5.2 Schematic of filtering and normalization analysis

Basic settings given in Table 5.2 are applied to all channels while evaluating the methods. Configurable settings are summarized in Table 5.3. Results of the analysis are given in Table 5.4. As it is seen from the results, best performance for time

domain signals are obtained when filtering is applied and Gaussian normalization is selected. For time-frequency domain signals, highest score is reached when filtering is not applied and Simple normalization is selected. Note that time-frequency domain features are created using the frequencies up to 40 Hz. So, applying a band-pass filter with 10 Hz cut-off frequency should also be avoided.

Table 5.2 Basic settings that are used at evaluation of filtering and normalization on time and time-frequency domain signals

Sampling Period	100-600 ms
Selected Channels	9,11,13,34,49,51,53,56,60,62
Repetition Number	15
Feature Size Reduction	None
Classification	SVM

Table 5.3 Configurable settings that are used at evaluation of filtering and normalization on time and time-frequency domain signals

Filter Type	None / Chebyshev Type-I (0.1 – 10 Hz, 8 th order)
Feature Type	Time Domain / Time Frequency Domain
Normalization	Gaussian / Simple
SVM Control Parameters	C, σ

Table 5.4 Success rates of filtering and normalization options on time and time-frequency domain signals

Domain	Filtering	Normalization	C	σ	Success
Time	Disabled	Gaussian	110	55	%82.62
Time	Enabled	Gaussian	110	55	%83.21
Time	Disabled	Simple	120	25	%82.66
Time	Enabled	Simple	120	25	%82.9
Time-Freq.	Disabled	Gaussian	110	140	%77.86
Time-Freq.	Enabled	Gaussian	110	140	%77.78
Time-Freq.	Disabled	Simple	110	40	%80.24
Time-Freq.	Enabled	Simple	110	40	%79.01

Kaper [24] and Erdoğan [29] did not use down-sampling in their studies. So, effect of down-sampling is also analyzed using the maximum success rate configuration given in Table 5.4. In this configuration, transformation domain is Time, filtering is enabled and Gaussian normalization is applied. SVM parameters are optimized independently for each configuration and results are given in Table 5.5. Note that down-sampling actually slightly improved success rate.

Table 5.5 Effect of down-sampling on performance

Down-sampling	Feature Size	C	σ	Success
Skipped	1200	110	140	%83.02
Applied	200	110	55	%83.21

5.2 Selecting Channels

Optimum channel subset is searched for time and time-frequency domain signals. Channels are selected according to their classification performances. First, the

channel with best classification performance is selected. Then, rest of the channels is considered and the one giving the best classification performance when concatenated to previously selected channel is selected. Channels are sorted by concatenating all remaining channels one by one with previously selected ones until 32 out of 64 channels are sorted. Both SVM and AdaBoost methods are used to find out classification performance at channel selection. AdaBoost results are obtained with the weak classifier CART. Overview of the channel selection methods are given in Figure 5.3.

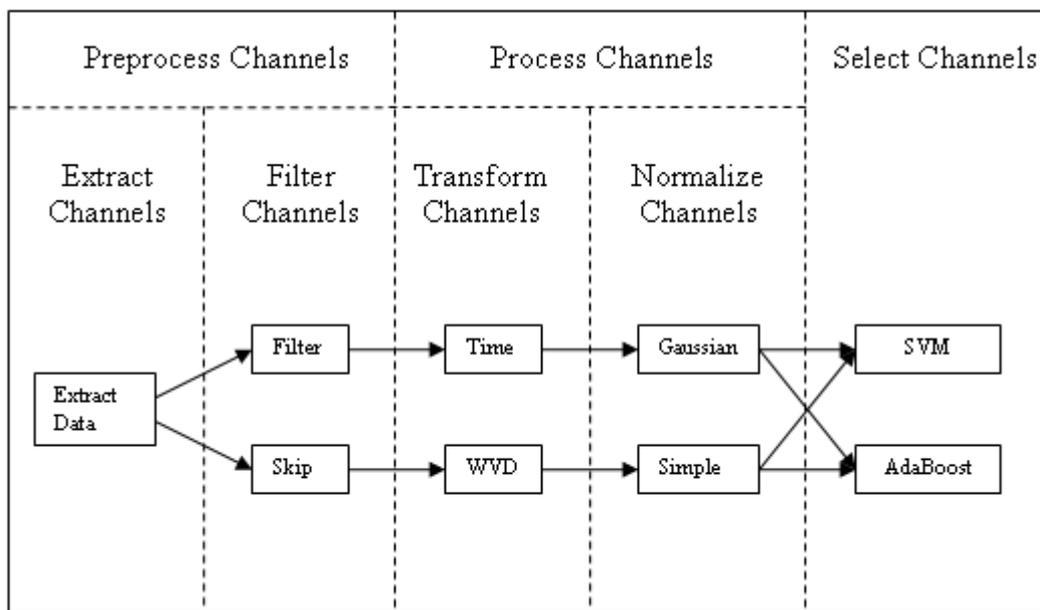


Figure 5.3 Schematic of channel selection methods

AdaBoost classification parameters do not depend on data set. However, as it is seen from section 5.1, SVM parameters highly depend on data set. Parameters obtained at section 5.1 are used for SVM classification. Error is plotted according to the equation given in (5-6) for each number of selected channels and optimum number of channels to be used is selected accordingly in the following sections.

Channel selection completion time is also given for each configuration. PC used for channel selection has dual core 3.00 GHz CPU and 4 GB RAM.

5.2.1 Time Domain and AdaBoost Channel Selection

AdaBoost channel selection order for Time Domain signals is given in Table 5.6. Completion of 32 channel selection takes 124 hours or equivalently 5.17 days. First 10 channels are shown in Figure 5.4.

Table 5.6 Selected channels for Time Domain and AdaBoost configuration

Order	1	2	3	4	5	6	7	8
Channel ID	9	56	51	60	6	17	28	18
Order	9	10	11	12	13	14	15	16
Channel ID	62	46	55	61	14	44	49	38
Order	17	18	19	20	21	22	23	24
Channel ID	7	52	21	58	27	1	20	3
Order	25	26	27	28	29	30	31	32
Channel ID	2	29	40	47	50	35	30	41

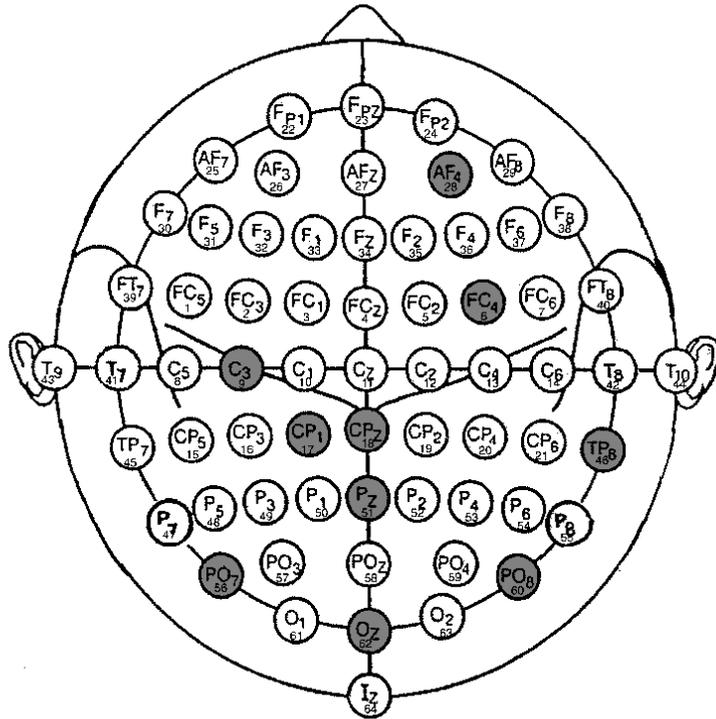


Figure 5.4 Selected 10 channels for Time Domain and AdaBoost configuration

Figure 5.5 shows that using first 12 channels is sufficient and adding more channels actually does not improve the result. QER is 0.33 for first 10 channels and 0.323 for first 12 channels.

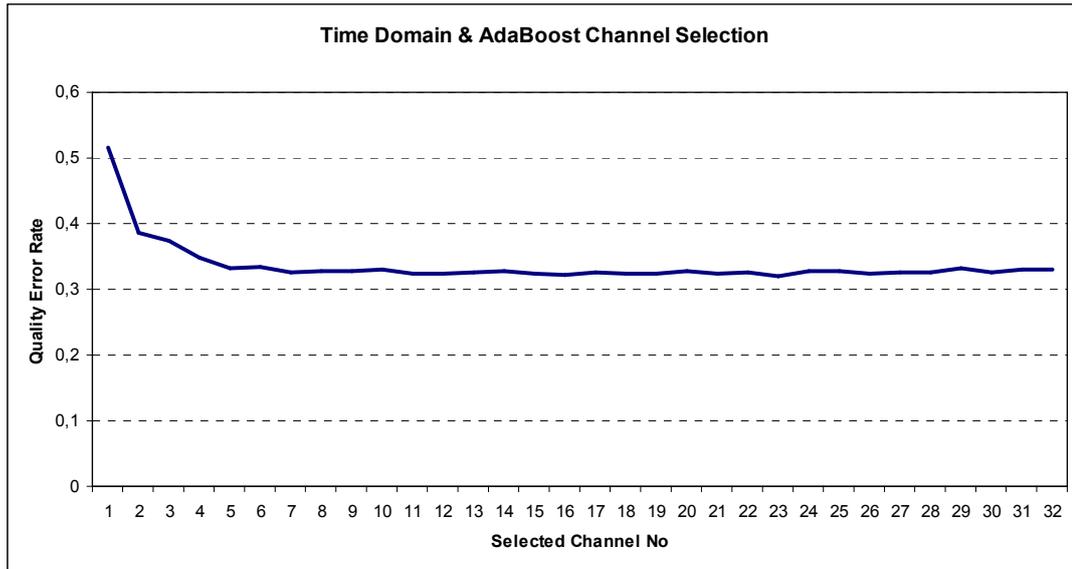


Figure 5.5 Quality Error Rate vs. number of selected channels for Time Domain and AdaBoost configuration

5.2.2 Time Domain and SVM Channel Selection

SVM channel selection order for Time Domain signals is given in Table 5.7. Completion of 32 channel selection takes 12 hours. First 10 channels are shown in Figure 5.6.

Table 5.7 Selected channels for Time Domain and SVM configuration

Order	1	2	3	4	5	6	7	8
Channel ID	10	60	51	56	3	64	18	54
Order	9	10	11	12	13	14	15	16
Channel ID	59	63	45	49	58	61	55	42
Order	17	18	19	20	21	22	23	24
Channel ID	4	6	9	53	17	1	19	40
Order	25	26	27	28	29	30	31	32
Channel ID	12	26	34	16	46	21	48	32

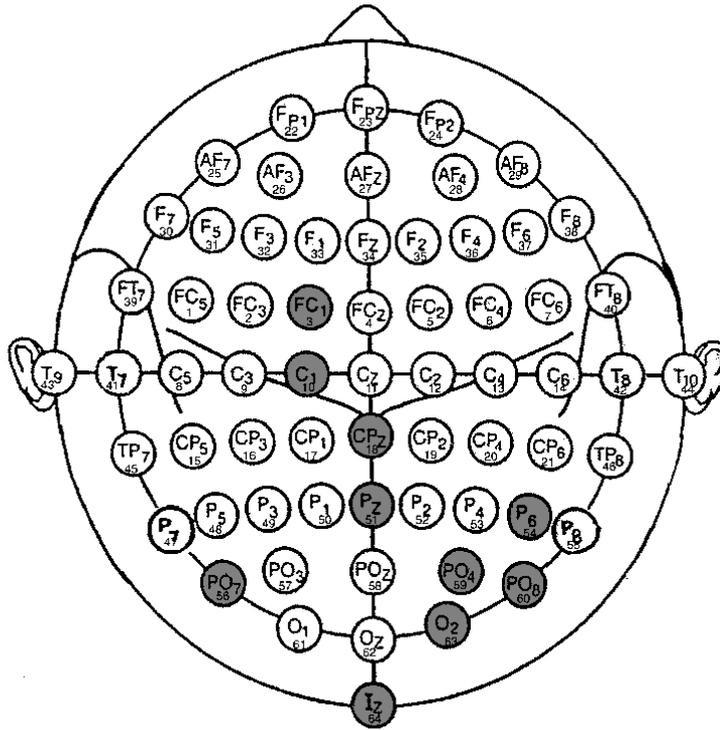


Figure 5.6 Selected 10 channels for Time Domain and SVM configuration

Figure 5.7 shows that using first 22 channels is sufficient. QER is 0.273 for first 10 channels and 0.226 for first 22 channels.

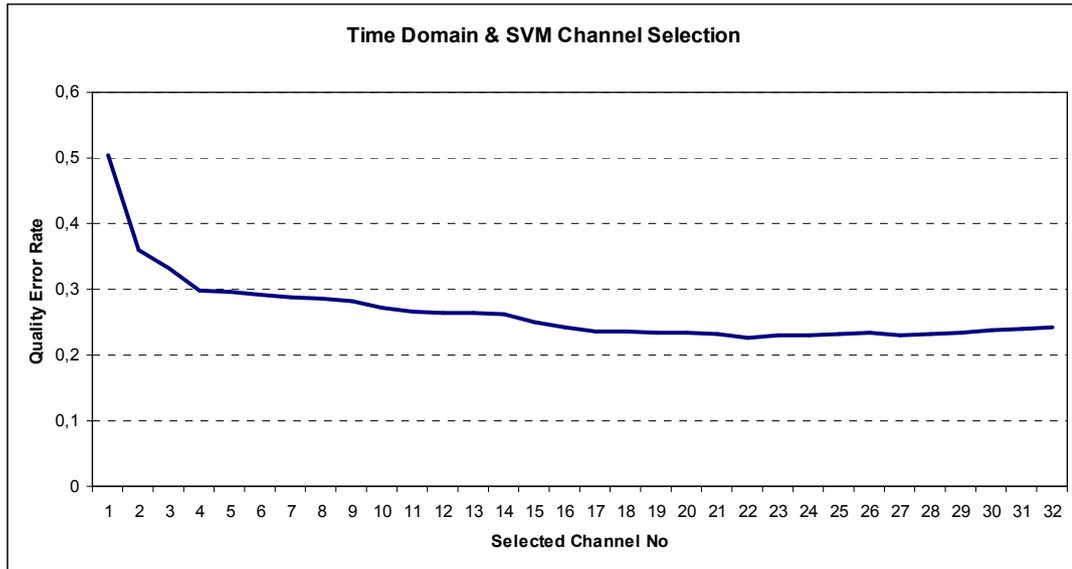


Figure 5.7 Quality Error Rate vs. number of selected channels for Time Domain and SVM configuration

5.2.3 Time-Frequency Domain and SVM Channel Selection

SVM channel selection order for Time-Frequency Domain signals is given in Table 5.8. Completion of 32 channel selection takes 52 hours. First 10 channels are shown in Figure 5.6.

Table 5.8 Selected channels for Time Domain and SVM configuration

Order	1	2	3	4	5	6	7	8
Channel ID	11	60	18	52	56	41	62	58
Order	9	10	11	12	13	14	15	16
Channel ID	48	54	40	59	8	42	64	9
Order	17	18	19	20	21	22	23	24
Channel ID	55	61	25	17	10	51	63	1
Order	25	26	27	28	29	30	31	32
Channel ID	16	46	2	3	57	7	44	13

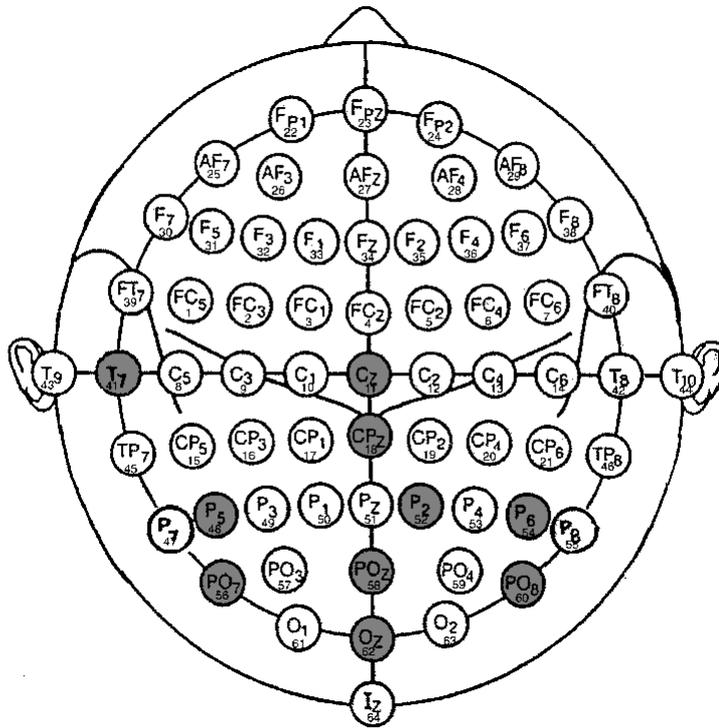


Figure 5.8 Selected 10 channels for Time-Frequency Domain and SVM configuration

Figure 5.7 shows that using first 21 channels is sufficient. QER is 0.31 for first 10 channels and 0.285 for first 21 channels.

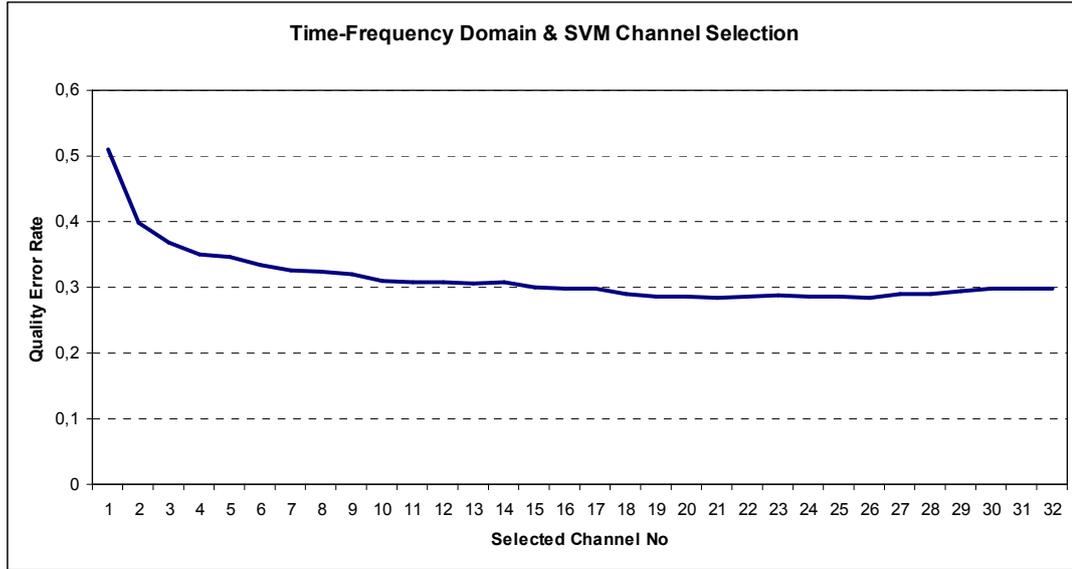


Figure 5.9 Quality Error Rate vs. number of selected channels for Time-Frequency Domain and SVM configuration

5.2.4 Time-Frequency Domain and AdaBoost Channel Selection

Time-Frequency Domain and AdaBoost channel selection configuration is skipped due to the expected completion of 32 channels selection time. Length of Time-Frequency Domain features are 5 times more than Time Domain features. AdaBoost channel selection linearly increases with data length. So, expected completion time of Time-Frequency Domain and AdaBoost channel selection is $124 \times 5 = 620$ hours, or equivalently 25.83 days.

Effects of skipping Time-Frequency Domain and AdaBoost channel selection is analyzed in 5.2.5.

5.2.5 Evaluation of Channel Selection Methods

Classification performances of 10 channels used by Kaper et al. [24] and Erdoğ an et al. [29] are used as reference while evaluating channel selection methods. Classification results for 10 channels and optimum number of channels are compared with reference results. Performances are evaluated in terms of both

averaged CV results on train set and classification results on test set. Error rates are calculated according to (5-4). Results are given in Table 5.9.

Table 5.9 Classification results of channel selection methods

Channel Selection Method	No of Channels	Domain	Classification	Train Error	Test Error
Reference [24], [29]	10	Time	SVM	0.1679	0.1317
SVM & Time	10	Time	SVM	0.1571	0.1214
SVM & Time	22	Time	SVM	0.127	0.1618
Reference [24], [29]	10	Time-Freq.	SVM	0.1976	0.1692
SVM & Time-Freq.	10	Time-Freq.	SVM	0.1948	0.163
SVM & Time-Freq.	21	Time-Freq.	SVM	0.1798	0.1798
Reference [24], [29]	10	Time	AdaBoost	0.2064	0.1852
AdaBoost & Time	10	Time	AdaBoost	0.2035	0.1865
AdaBoost & Time	12	Time	AdaBoost	0.1941	0.1839

When 10 channel classification results are analyzed, it is seen that performance is improved at SVM and Time channel selection method, slightly improved at SVM and Time-Frequency channel selection method and not changed at AdaBoost and Time channel selection method with respect to reference channels. When optimum channel number classification results are analyzed, it is seen that train errors at all methods are decreased. However, test error is only decreased at AdaBoost and Time channel selection method. Test error is increased at the methods that use SVM. This shows that there is an overfitting problem for SVM that can not be prevented even with cross-validation. The reason for this may be due to SVM parameters (C, σ) that are optimized for 10 channels. When channel length is increased, SVM may not model the data set with the given parameters. On the other hand, AdaBoost has no parameter to be adjusted for data set and so less vulnerable to overfitting problem. To evaluate channel selection methods more accurately, train and test errors vs.

number of selected channels are plotted. Results are given in Figure 5.10, Figure 5.11 and Figure 5.12.

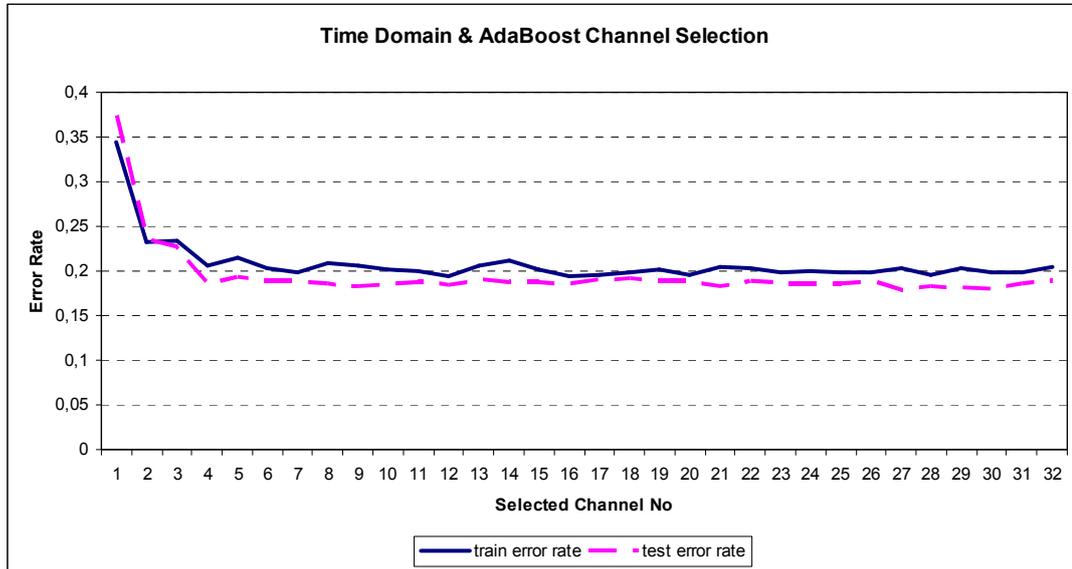


Figure 5.10 Train and test error rates vs. number of channels for Time Domain and AdaBoost channel selection

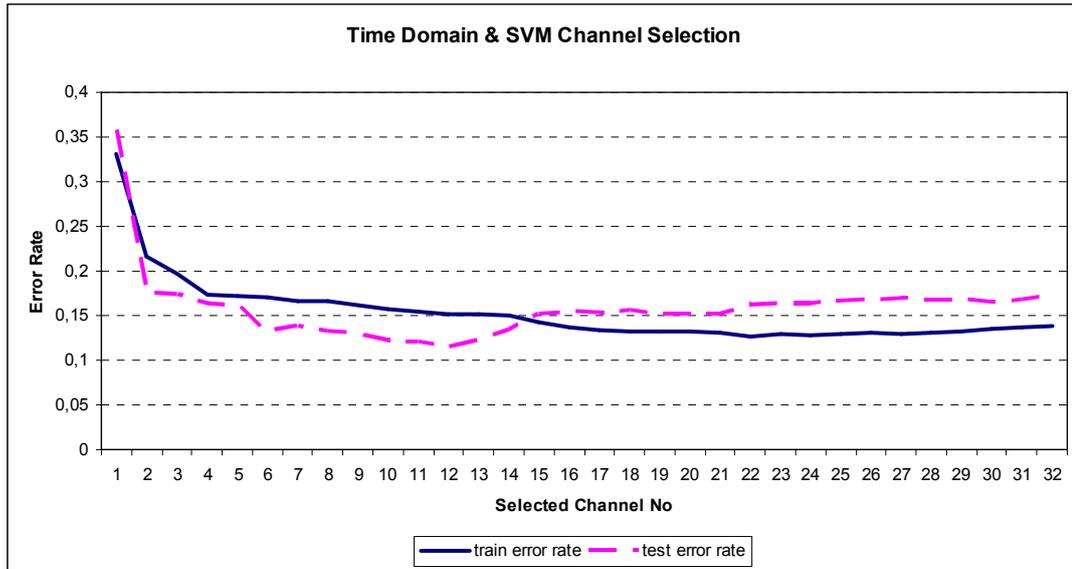


Figure 5.11 Train and test error rates vs. number of channels for Time Domain and SVM channel selection

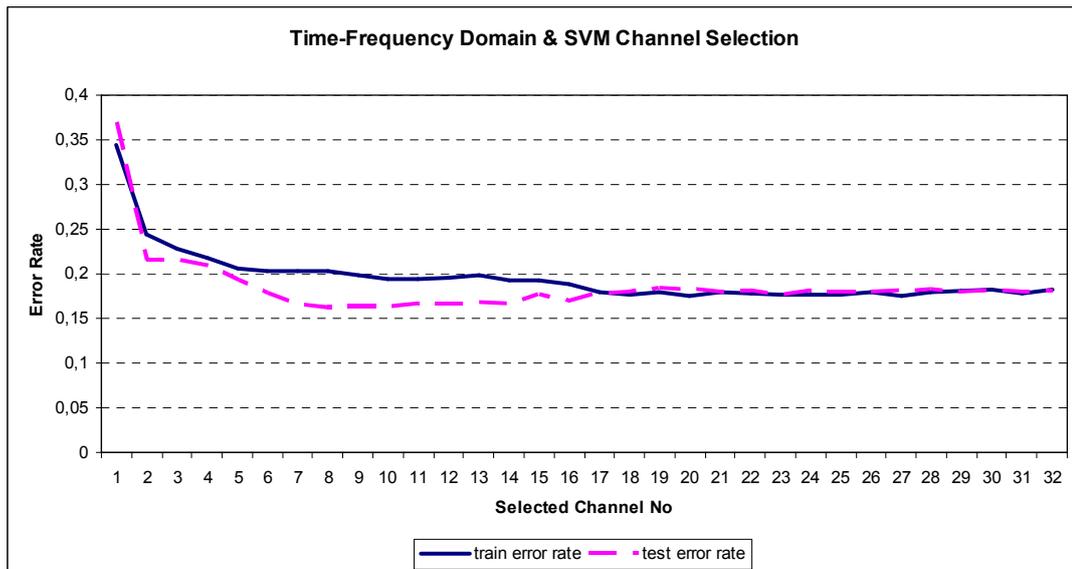


Figure 5.12 Train and test error rates vs. number of channels for Time-Frequency Domain and SVM channel selection

In Figure 5.10, AdaBoost and Time channel selection method train and test error behaviors are similar. On the other hand, in Figure 5.11 and Figure 5.12, train and test error behaviors are different. Train error bottom point is around 20 channels and

test error bottom point is around 10 channels. Test error behavior is as expected since SVM parameters (C , σ) are already optimized for 10 channels. However, there is an overfitting problem in train set. Because of that, SVM can not be used for optimizing number of channels. AdaBoost is better while determining number of channels to be used, but its classification performance is not as good as SVM. SVM classification performance is measured with channels selected with AdaBoost to decide whether it is sufficient to use only AdaBoost channel selection method and results are given in Table 5.10.

Table 5.10 Comparison of AdaBoost and SVM Classification performances

Channel Selection Method	No of Channels	Domain	Classification	Train Error	Test Error
Reference [24], [29]	10	Time	SVM	0.1679	0.1317
AdaBoost & Time	10	Time	AdaBoost	0.2035	0.1865
AdaBoost & Time	10	Time	SVM	0.1782	0.1404
SVM & Time	10	Time	SVM	0.1571	0.1214

SVM classification performance is better than AdaBoost even with channel set selected with AdaBoost. But, AdaBoost and Time channel selection performance is worse than reference channels. So, channels selected using AdaBoost are not suitable for SVM.

Considering all these, we propose the 3 step approach for channel selection:

- 1 Deciding number of channels to be used via AdaBoost
- 2 Optimizing SVM parameters using channels selected by AdaBoost
- 3 Selecting channels with optimized SVM parameters.

12 channels are decided to be optimum at AdaBoost and Time channel selection. SVM channel selection is already optimized for 10 channels, so results can be

directly used without reselecting channels. SVM and Time channel selection results for 12 channels are given in Table 5.11. SVM classification results are improved by using 12 channels.

Table 5.11 Comparison of 10 and 12 channels selection performance with the proposed 3 step approach

Channel Selection Method	No of Channels	Domain	Classification	Train Error	Test Error
Reference [24], [29]	10	Time	SVM	0.1679	0.1317
SVM & Time	10	Time	SVM	0.1571	0.1214
SVM & Time	12	Time	SVM	0.1512	0.1143

Another important result of Figure 5.11 and Figure 5.12 is the similarity in both train and test error characteristics. Main difference is the values of the error rates. Classification performance of Time-Frequency Domain signals is worse than Time Domain. Time-Frequency Domain brings extra computational cost without improving the performance. That is why, it is not reasonable to use this domain and optimum channel number search is not necessary for this domain. Note that Time-Frequency Domain and AdaBoost channel selection method is needed only to determine the number of channels to be used, and it is skipped due to the reasons explained above.

So, 12 channels (see Figure 5.13) selected by Time Domain and SVM channel selection method is used, since it gives the best performance. Channels placed on center and rear part of the skull are selected in this configuration.

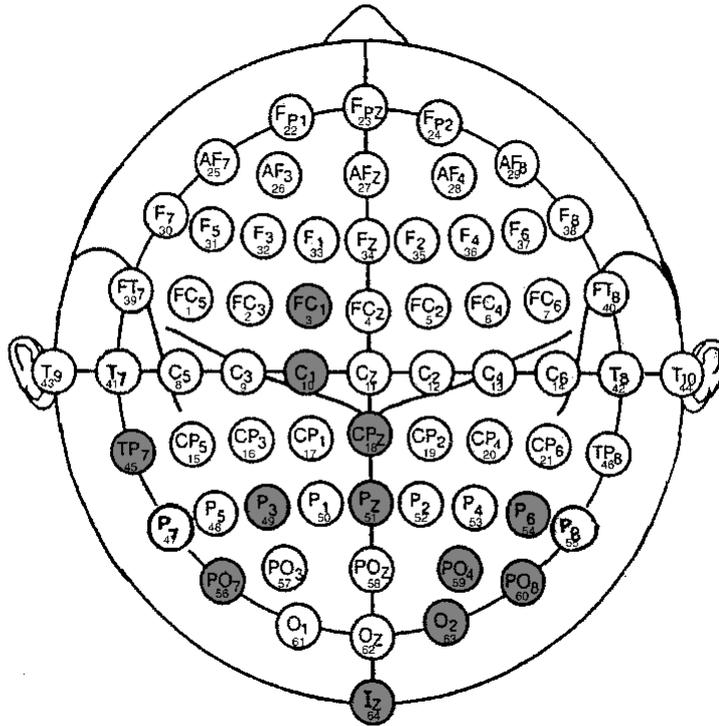


Figure 5.13 12 channels selected by Time Domain and SVM channel selection method

Configuration parameters that are determined up to now are summarized in Table 5.12.

Table 5.12 Fixed configuration parameters after channel selection

Sampling Period	100-600 ms, 40 Hz down-sampled
Selected Channels	3, 10, 18, 45, 49, 51, 54, 56, 59, 60, 63, 64
Filter Type	Chebyshev Type-I (0.1 – 10 Hz, 8 th order)
Feature Type	Time Domain
Normalization	Gaussian
Classification	SVM (C=110,σ=55)

5.3 Reducing Dimension of the Features

Feature dimension is already reduced 6 times at down-sampling. In this section, the effect of further reducing dimension of features is analyzed. PCA and AdaBoost feature selection methods are used at analysis. CART is used as the weak classifier of the AdaBoost. Overview of the analysis method, which uses the fixed configuration parameters given in Table 5.12, is given in Figure 5.14.

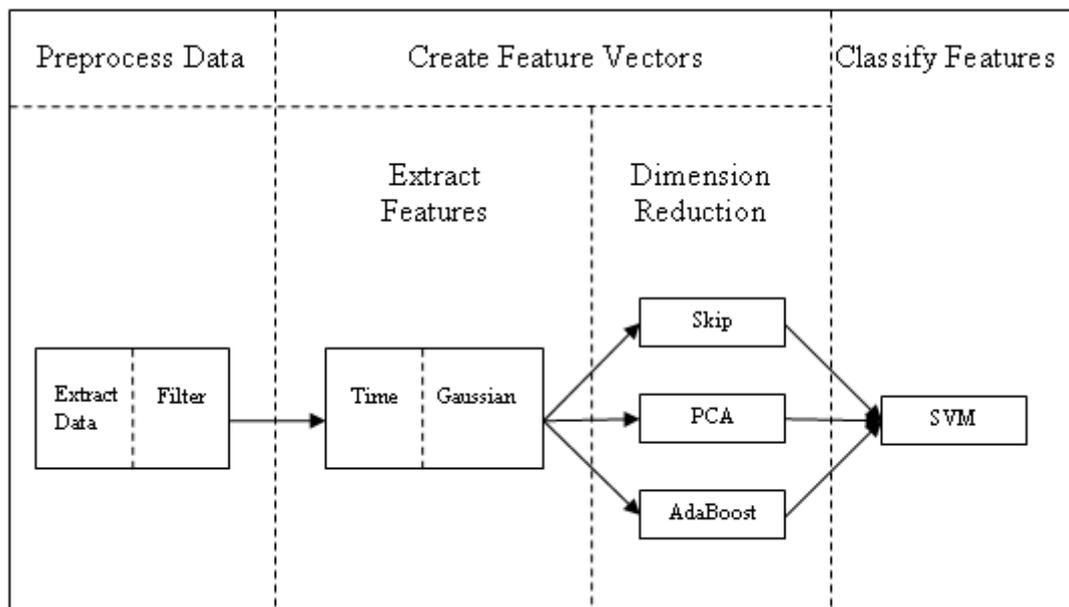


Figure 5.14 Schematic of dimensionality reduction methods

PCA does not reduce the number features used as input to the system when it is used for dimension reduction. PCA reduces the dimension through a transformation at an intermediate step. That is, PCA must be applied to both train and test set feature vectors. Also, PCA does not consider class labels in data set while reducing dimension. On the other hand, AdaBoost is a feature selection method. So, the number of features used as input to the system can be reduced by AdaBoost. That is, once subset of features are determined at training, train and test set features are easily created by getting rid of unselected features without any calculation effort.

Besides, AdaBoost selects features by taking into account discriminative properties of classes.

5.3.1 Dimensionality Reduction Using PCA

PCA eigenvalue ratio which is calculated according to Equation (4-2) is given in Figure 5.15. First 100 and 160 features contain 99.5% and %99.97 of the data respectively. So, there are correlated features in feature vectors.

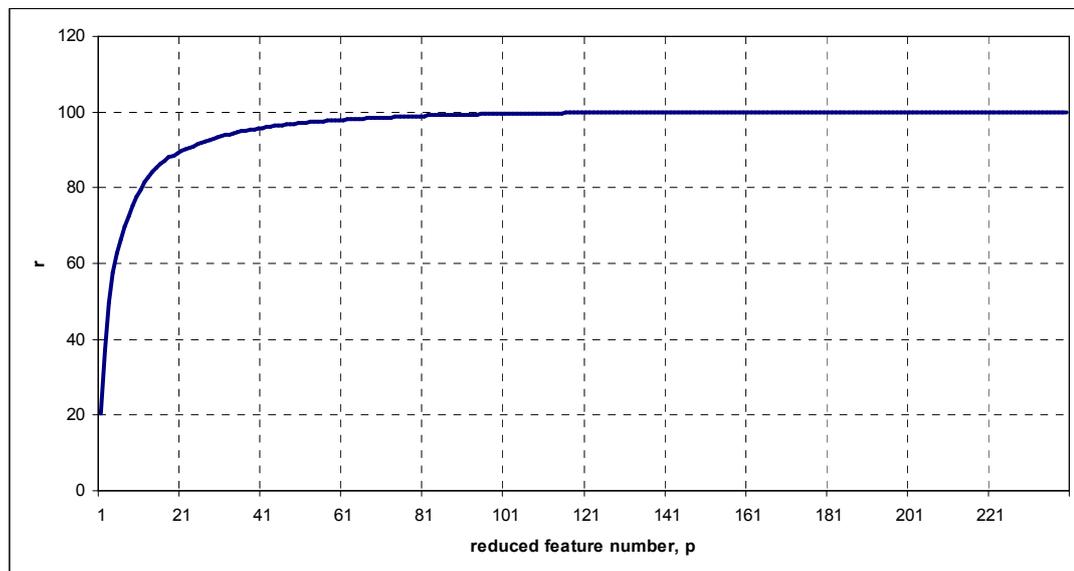


Figure 5.15 Percentage of information kept (r) vs. number of feature components included (p) for selected 12 channels

In Figure 5.16, train and test error rates vs. number of selected features are given. As expected, eigenvalue ratio and error characteristics are similar. First 160 PCA features contain all of the information and give same classification results with using all features.

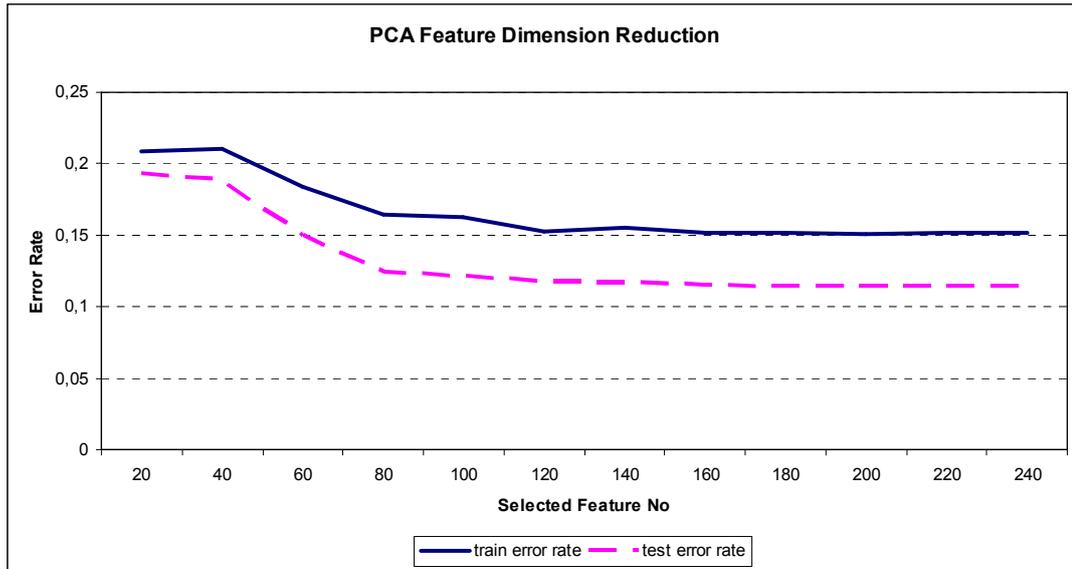


Figure 5.16 PCA dimension reduction results

5.3.2 Dimensionality Reduction Using AdaBoost

AdaBoost dimension reduction results are given in Figure 5.17. Error rates are first decreasing and then saturating with increasing number of features. Minimum error rate is observed when all features are selected. So, reducing the feature number increases error rate.

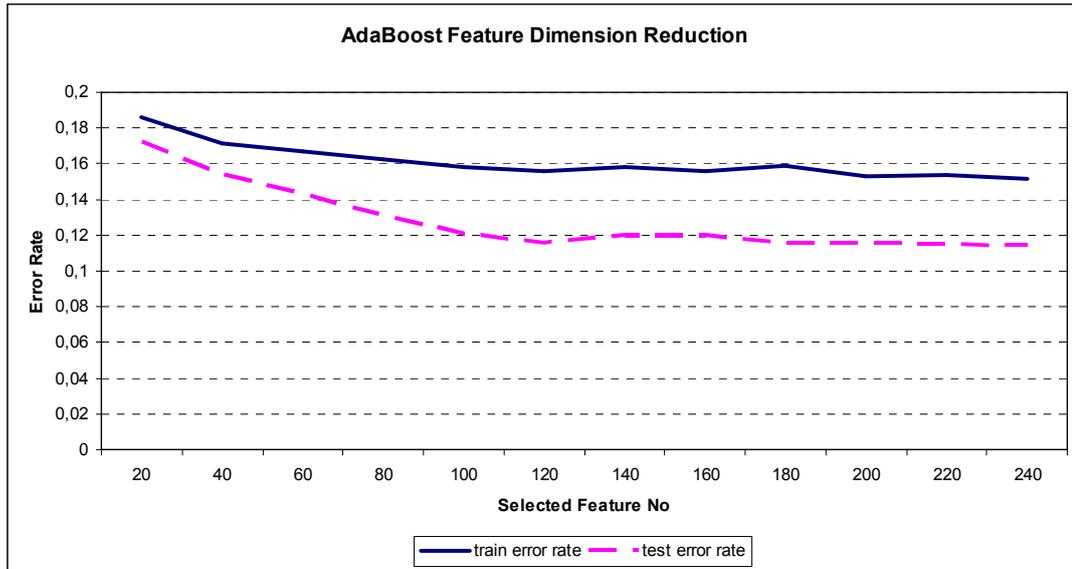


Figure 5.17 AdaBoost dimension reduction results

5.3.3 Evaluation of Dimensionality Reduction

When error rates given in Figure 5.16 and Figure 5.17 are compared, it is seen that AdaBoost feature selection results are better for less than 120 features and PCA feature selection results are better for more than 120 features. PCA assumes that features with larger variances contain more important information and reduces data with this principle. Note that PCA is a transformation method and it successfully eliminates correlated features. Since first 100 PCA features already contains 99.5% of the data variance, eliminating rest of the features removes correlation. On the other hand, AdaBoost selects features using discriminative properties of classes. Since features with smaller variance can also contain valuable information while separating classes, AdaBoost is more powerful at smaller number of features. However, AdaBoost gains less information with increasing number features because it does not transform features.

AdaBoost always loses some information because error rates are increased with AdaBoost dimensionality reduction. However, PCA reduces feature size by 80 without losing performance by getting rid of correlated variables. In order not to

increase error rates, PCA is selected as dimensionality reduction method with feature size 160.

If one needs to further reduce the dimension of the features, it can combine powerful sides of both methods by first applying PCA and getting rid of correlated data and then applying AdaBoost to further reduce data size.

5.4 Classifying Features and Deciding Focused Characters

Superiority of SVM over AdaBoost is already shown during channel selection. Because of that, only SVM is used while classifying features. Target (or focused) characters are selected based on voting according to soft margin outputs of SVM.

There are total 42 characters in train set and 31 characters in test set provided in [20]. Trained SVM is both tested on 42 character train set and 31 character test set. Performance is measured by dividing correctly classified target characters to total character numbers. Results are given for both reference channels used by Kaper [24] and 12 channels selected by SVM in time domain. Performance of target character detection is measured for repetition numbers from 1 to all. Repetitions are selected in chronological order. In Figure 5.18, results of train set and reference channels are given. In Figure 5.19, results of train set and 12 channels selected by SVM are given. In Figure 5.20, results of test set and reference channels are given. In Figure 5.21, results of test set and 12 channels selected by SVM are given.

When Figure 5.18 and Figure 5.19 are compared, it is seen that selected channels give better performance results than reference channels. Note that a character is always misclassified by both channel sets. Character ‘G’ is always found instead of ‘H’ in second “HAT” word of train set. This character can be dismissed from train set. Erdoğan [29] already constructed train set with 39 characters and did not use this misclassified character.

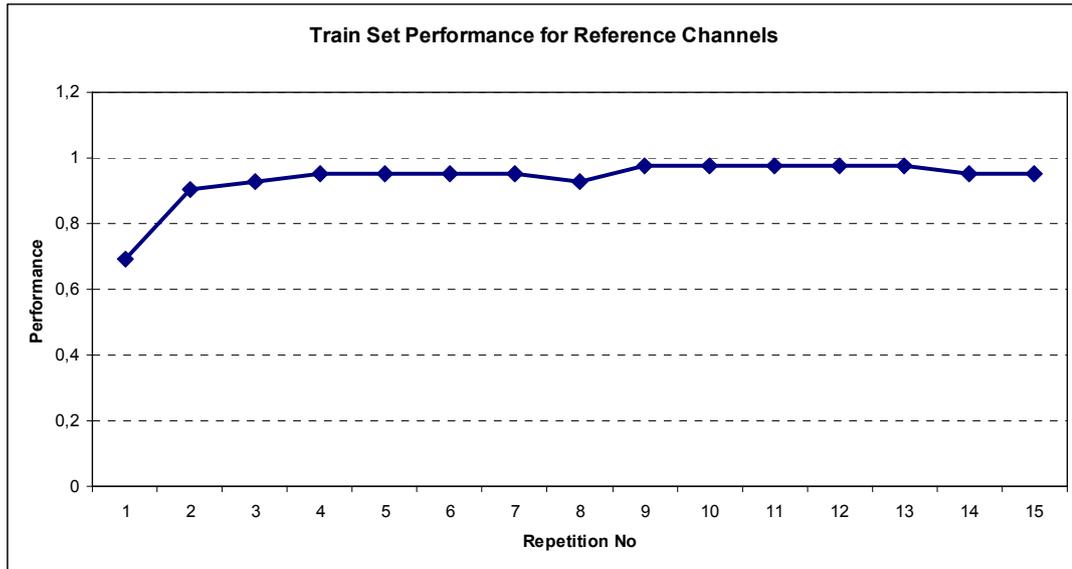


Figure 5.18 Performance vs. repetition number results for reference channels on 42 character train set



Figure 5.19 Performance vs. repetition number results for selected 12 channels on 42 character train set

Results given in Figure 5.20 and Figure 5.21 also show that performance of selected channels is better than reference channels. In Figure 5.20, a character is misclassified for repetitions 7, 8 and 10. Kaper [24] and Erdoğan [29] reported

%100 performance for these repetitions. There are numerous factors, such as randomly selected train set and soft margin outputs of SVM, that may affect the performance results. However, only variable is channel subset in the experiment and thus comparison of channel sets are done under same conditions.

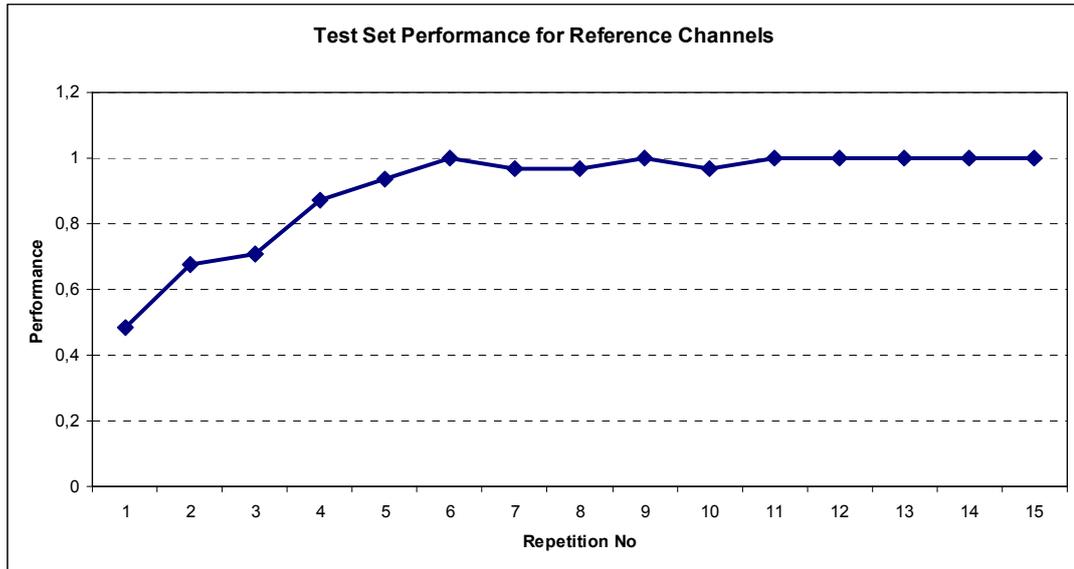


Figure 5.20 Performance vs. repetition number results for reference channels on 31 character test set

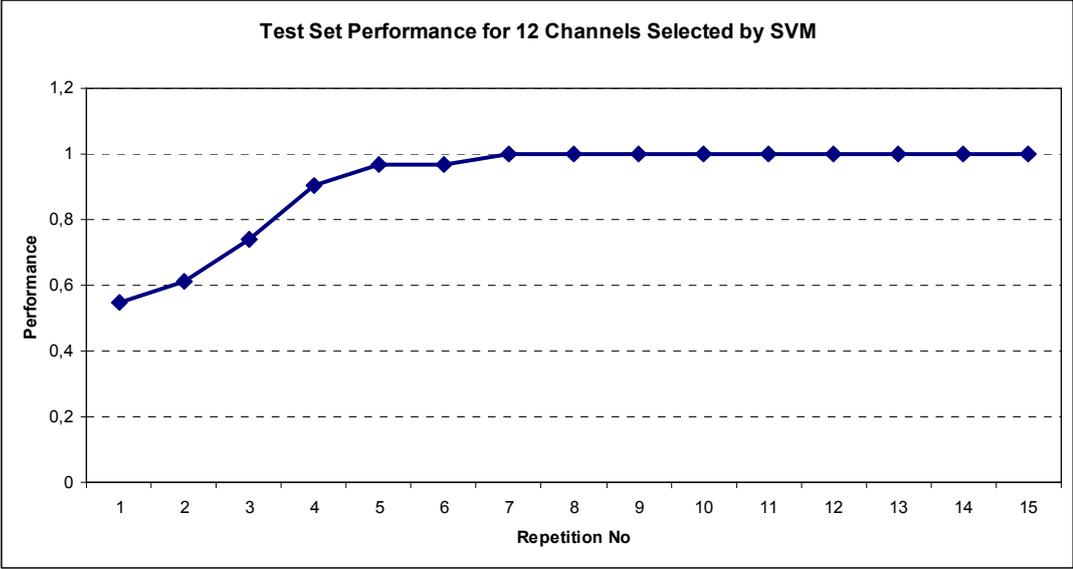


Figure 5.21 Performance vs. repetition number results for selected 12 channels on 31 character test set

CHAPTER 6

CONCLUSIONS

In this thesis, Spelling Paradigm, a specific implementation of BCI, is studied. P300 signals are collected from different parts of the brain. Using all collected data slows down the system and increases the noise level because P300 response does not exist at same intensity at all regions of the brain. Channel subset that contains relevant data with minimum noise is searched. AdaBoost with the weak classifier CART and SVM classification methods are used in this study. Besides, P300 responses are analyzed in time and time-frequency domains. Best subset of channels is analyzed for the combinations of classification and transformation domain methods. Since SVM parameters (C , σ) depend on data set, it needs optimization before starting the analysis. So, number of channels to be used and a channel subset is required before starting the analysis. Using a reference subset of channels can not solve this problem, because although SVM gives successful results around the region that its parameters are optimized, it loses its learning ability at other regions and can not model data set properly. On the other hand, AdaBoost does not require parameter optimization for each subset of data and suitable to detect optimum subset of channels. However, AdaBoost with the weak classifier CART classification performance is worse than the optimized SVM and so it is wise to select SVM as primary classification method of the system. Success of SVM classification with channels selected by AdaBoost is analyzed and it is seen that SVM is more successful if channels are also selected by SVM. So, AdaBoost is better while deciding number of channels to be used and SVM is better at classification. Strong sides of both methods are combined while selecting channels by deciding number of

channels to be used via AdaBoost channel selection method and optimizing SVM with the channel subset selected with AdaBoost and reselecting channels with optimized SVM parameters.

P300 signals are analyzed in time and time-frequency domains. Filtering and normalization methods that best suits for these domains are analyzed. It is decided to use time domain with filtering and Gaussian normalization and time-frequency domain without filtering and simple normalization. Since signals already exist in time domain, no transformation is required for time domain analysis. To transform time domain signals into time-frequency domain, WVD is used. Frequency band of WVD output is divided according to the types of brain waves namely delta, theta, alpha, beta, and gamma. Averages of WVD for each brain wave type are calculated and results are concatenated to create feature vectors. Since there are five brain wave types, feature vector size is five times of the one obtained from time domain. After channel selection analysis is done, it is observed that time domain is both cheaper in terms of computational effort and better in terms of classification. So, time domain is preferred as the primary transformation domain.

In time domain, AdaBoost selects 12 as the optimum number of channels to be used. 12 channels selected by SVM are 3, 10, 18, 45, 49, 51, 54, 56, 59, 60, 63, and 64. Classification results are compared with 10 reference channels used by Kaper [24], and Erdoğan [29]. Train and test errors are found to be 0.1679 and 0.1317 respectively for reference channels and 0.1512 and 0.1143 respectively for 12 selected channels. So, channel selection method improved the classification performance of the system.

Dimension is tried to be reduced without decreasing the success of the implementation. 100 – 600 ms period after intensification is used for each channel. Since signals are sampled at 240 Hz, there are total 120 features for each channel. Total feature size becomes 1440 for 12 channels. This increases training time and also may cause memory errors with the increasing number of samples. It is shown that downsampling signals to 40 Hz do not cause information loss at all. Also, PCA and AdaBoost with the weak classifier CART dimension reduction methods are

analyzed. PCA successfully eliminates correlated data. However, it is not as successful as AdaBoost if feature size is further reduced. This due to the fact that AdaBoost takes into account discriminative properties of classes, but PCA does not consider this information. AdaBoost dimension reduction always increases error rates in comparison to full set and not preferred. PCA reduces feature size up to 160 without any information loss. So, PCA is selected as dimension reduction method. If further dimension reduction is needed, then information loss should be minimized by first getting rid of correlated features by PCA and then applying AdaBoost to the feature set already reduced by PCA.

As it is stated before, SVM is used for labeling test set feature vectors due to its superior performance over AdaBoost with the weak classifier CART. Margin outputs of SVM are used while selecting target rows and columns. Then target character is decided combining target rows and columns. Effect of reducing repetition numbers on performance of system is also analyzed and it is seen that 12 channels selected by SVM in time domain gives also better results.

Note that since brain signals up to 40 Hz are required for time-frequency analysis, downsampling frequency is selected to be 40 Hz in this study. Results already show that time domain features are preferred against time-frequency domain signals. So, downsampling frequency can be further reduced and information loss can be analyzed.

In this study, CART is used as the weak classifier of the AdaBoost. Different weak classifiers can be analyzed to improve the classification and dimension reduction performance of the AdaBoost. If AdaBoost with another weak classifier gives better classification results than SVM, then it can be used at classification step in addition to the decision of the number of channels to be used. That is, the necessity to use SVM can be eliminated by improving the classification performance of the AdaBoost.

In this study, rows and columns are processed together. Effects of separating rows and columns while processing signals can be analyzed.

In this study, standard filter is used at preprocessing step. Adaptive filters such as Wiener filter can be implemented.

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APPENDIX A

MATHEMATICAL BACKGROUND

Variance

Definition:

$$\text{Var}(X) = E(X - E(X))^2 = E(x^2) - (E(X))^2 \quad (\text{A-1})$$

Empirical:

$$\text{Var}(X) = \sigma^2 = \frac{1}{N} \sum_{i=1}^N (x_i - \mu_x)^2, \quad (\text{A-2})$$

where μ_x is the mean of X .

Covariance

Definition:

$$\text{cov}(X, Y) = E[(X - E(X))(Y - E(Y))] = E(X.Y) - E(X).E(Y) \quad (\text{A-3})$$

Empirical:

$$\text{cov}(X, Y) = \frac{1}{N} \sum_{i=1}^N (x_i - \mu_x)(y_i - \mu_y) \quad (\text{A-4})$$

Covariance is measured between two dimensions.

Properties:

- $\text{cov}(X, X) = \text{Var}(X)$
- $\text{cov}(X, Y) = \text{cov}(Y, X)$
- If X and Y are uncorrelated, $\text{cov}(X, Y) = 0$.

That is, X and Y are independent.

- If X and Y are correlated, $\text{cov}(X, Y) > 0$.

That is, X and Y both increase and decrease together.

- If X and Y are anti-correlated, $\text{cov}(X, Y) < 0$.

That is, if X is increasing, then Y is decreasing and vice versa.

Correlation

$$\text{cor}(X, Y) = \frac{\text{cov}(X, Y)}{\sigma_x \sigma_y} \quad (\text{A-5})$$

$$-1 \leq \text{cor}(X, Y) \leq 1 \quad (\text{A-6})$$

Covariance Matrix

$$C^{n \times n} = (c_{i,j}, \quad c_{i,j} = \text{cov}(\text{Dim}_i, \text{Dim}_j)), \quad (\text{A-7})$$

where $C^{n \times n}$ matrix has n rows and n columns and Dim_x is the x'th dimension.

3 dimensional data set example where x, y, z denotes a dimension:

$$C^{3 \times 3} = \begin{pmatrix} \text{cov}(x, x) & \text{cov}(x, y) & \text{cov}(x, z) \\ \text{cov}(y, x) & \text{cov}(y, y) & \text{cov}(y, z) \\ \text{cov}(z, x) & \text{cov}(z, y) & \text{cov}(z, z) \end{pmatrix} \quad (\text{A-8})$$

Eigenvector and Eigenvalue

Given a linear transformation A , a non-zero vector x is defined to be an eigenvector of the transformation if it satisfies the eigenvalue equation;

$$Ax = \lambda x \tag{A-9}$$

for some scalar λ . In this situation, the scalar λ is called an eigenvalue of A corresponding to the eigenvector x .