

**SOME SET-THEORETICAL TRACES IN LEIBNIZ'S WORKS**

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SOME SET-THEORETICAL TRACES IN LEIBNIZ'S WORKS

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## **ABSTRACT**

### **SOME SET-THEORETICAL TRACES IN LEIBNIZ'S WORKS**

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The purpose of this dissertation is to search the primitives of Axiomatic Set Theory in Leibnizian Philosophy, nourishing, roughly, from Platonic idea of universal-particular distinction, Aristotelian syllogistic propositions of Organon-Categoria and Euclidean Methodology in Elements.

The main focus of the dissertation intends to examine the analyticity of Leibnizian Metaphysics and the analogies between the subject-predicate relation in The Philosophy of Leibniz and Axiomatic Method in general and Set Theory in particular.

In doing this, special emphasis will be ascribed to the notion of sets as to universality and/or nullness of a class, probable causes of paradoxes and in this context a critical analysis of Russell Paradox.

Key words: Analyticity, Axiomatic Method, Philosophy of Leibniz, Russell Paradox.

## ÖZ

### SET TEORİNİN LEİBNİZ'İN ÇALIŞMALARINDAKİ KÖKLERİ

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Bu tez çalışmasının amacı, esas itibariyle Platoncu tümel-tikel ayrımı, Organon-Categoria'daki Aristocu tasımsal önermeler ve Euclid Geometrisindeki Aksiyomatik yöntemden beslenen Aksiyomatik Set Teorinin Leibniz Felsefesindeki köklerini araştırmaktır.

Tezin yoğunlaştığı asıl alan Leibnizci Metafiziğin analitik özelliği ve Leibniz Felsefesinin temel prensibi olarak özne-yüklem ilişkisi ile genel olarak Aksiyomatik Yöntem özel olarak da Aksiyomatik Set Theory arasındaki karşılıklılıklar olacaktır.

Bunu yaparken, bir set/class'ın evrenselliği ve/veya boş küme olma özneliğine, paradoksların muhtemel nedenlerine ve Russell Paradoksunun eleştirel analizine özel bir önem verilecektir.

Anahtar Kelimeler: Analitiklik, Aksiyomatik Metod, Leibniz Felsefesi, Russell Paradoksu.

To my family

## ACKNOWLEDGMENTS

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## LIST OF ABBRIVIATIONS

### ABBREVIATIONS:

- G.            C.J.Gerhardt.            Berlin, 1875-90.
- G.M.         C.J.Gerhardt.            Halle, 1850-63.
- F. de C.     A.Foucher de Careil.     Paris, 1854.
- D.            *The Philosophical Works of Leibnitz*, with notes by George Martin  
Duncan. New Haven 1890.
- L.            *Leibniz: The Monadology and other philosophical writings*, translated,  
with introduction and notes, by Robert Latta. Oxford, 1898.
- N.E.         *New Essays Concerning Human Understanding* by Gottfried Wilhelm  
Leibnitz, together with an Appendix consisting of some of his shorter  
pieces, translated by Alfred Gideon Langey. New York and London,  
1869.

## CHAPTER 1

### INTRODUCTION

Sans les mathématiques on ne pénètre point au fond de la philosophie.  
Sans la philosophie on ne pénètre point au fond des mathématiques.  
Sans les deux on ne pénètre au fond de rien. — Leibniz

[Without mathematics we cannot penetrate deeply into philosophy.  
Without philosophy we cannot penetrate deeply into mathematics.  
Without both we cannot penetrate deeply into anything.]

**Tribute to Leibniz.**

#### 1.1. Philosophy Of Leibniz, A Quest For Reality:

Main motive of Philosophy, as it seems, has ever been to conceive the nature of Reality to the fullest extent possible and state it succinctly in a formal language in the strictest sense. This task necessarily applies to an inquiry from phenomena, that which is apparent, to the very first-stance which may mean the prime origin and to give it a body in language without hurting its integrity, power and beauty. So, there is a need for a formal language to convey what's in mind of analyticity to the proper appreciation of human understanding in common. And this, definitely, is a vital need, because, there is not always one-to-one correspondence between language of thought and language of ordinary discourse. This distinction, also subsumed as a programme as to Language-Reality Relation by Ludwig Wittgenstein, gave rise to the creation of formal languages, in that, Mathematics for Science and Logic for Philosophy. By the end of 19<sup>th</sup> and the beginning of 20<sup>th</sup> century, Mathematics was largely reduced into Set Theory by invaluable studies of Frege, Peano, Schröder, Peirce, Boole, Cantor and Whitehead-Russell, Zermelo&Fraenkel, W.V.Quine and many other first rate mathematicians and logicians respectively. And the two converged into Mathematical Logic, though philosophical works on the subject go back deep into medieval and ancient times up to Aquinas, Aristotle, Platon and Socrates. But, an exceptionally distinct figure among the very gratests like Aristotle, Aquinas, and Descartes, counts as mid-summit rising between Aristotle and Frege as Logician in eminence who offered Universal Characteristic, amounting to the invention of Calculus. That man was **Gottfried Wilhelm Leibniz**.

His methodology was deeply analogous with that of Axiomatic Method of 'Elements' of Euclid and Axiomatic Set Theory of 20<sup>th</sup> century and forth, as well as the natural laws that underlie a scientific theory. Because, all three consist in the least number of primitive<sup>1</sup> assumptions or laws from which all the original set of propositions or theorems can be deduced.

So, it would not be any proper to give him in an outline figured out of some principles, ascribed to him, which may sound somewhat vulgar to his intimate followers. Yet, if one is to give a simple principle with which Leibniz be entitled then *'to conceive thoroughly the very essence of the concept in question and the rationale behind, so as to remain true throughout every world in any nature, scientific, mathematical, logical, theological, possible, actual and so on, and examine the particular cases against the pervasive principle down to the details and up to the whole'* might identify his stance. Among the many, Principle of Sufficient Reason is what corresponds to that. The others, in his logic, are of the common principles of formal logic.

Given his resume up to 1686, Leibniz essentially was drawn up by a primary motivation which renders itself on two main pillars; the conception of world as multitude in unity and expression the whole idea through a mighty media of powerful grammar. This may seem an assumption no unusual given the philosophers' mind set before and after him. What's distinctive in him is a world of unusually coherent and logically interrelated units and a language of almost infallible consistency and mathematically provable formality. Even though it was endowed by Spinozistic Philosophy of absolute, attribute and modal forms of reality, Leibniz, as a mind of versatility, had captured the essence of entity in unity. The problem was, to him, deficiency of ordinary language in formally expressing of continuity in gradual steps of infinitesimally small changes. To him, language of ordinary discourse and language of ultimate reality were two different facts of two different worlds. He made another distinction, vitally momentous, to avoid inconsistencies and, perhaps,

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1. Note:By the word primitive what is meant is an immediacy to reality relating to being primary in contrast with premature.

contradictions in the final form of complete account of his description, namely ideal actual distinction.

So, he took his way to destiny from 1686 on, with his embroiled craftsmanship of possible world and mathematical logic. The former would lead him to a Monadic world view in metaphysics and the latter to Universal Characteristic and, further, Infinitesimal Calculus as the formal language of this conception. Leibniz was strictly axiomatic in construction of his system deducing it from a handful of axioms, to Russell, namely five. This would be examined and questioned in terms of consistency throughout a book by Bertrand Russell, *The Philosophy of Leibniz*(Routledge Book 1997).

## **1.2. Critical Philosophy of Leibniz with Russell:**

Key notion seems to be ‘unity’ in Leibniz’s system even if he had been in a desperate need of multiplicity to accord with that of actual world. He strikingly unifies those two in the conception of ‘multitude in unity’ and reaches infinitely versatile an element for his metaphysical system that fits any position from ultimately simple<sup>1</sup> to endlessly complex admitting every diversity in a perfect conformity and with no contradiction to the whole. This first build was so profound to the essence that the remaining hypothesis would necessarily follow in a genuine coherence with the notion of main support. Upcoming philosophical edifice was amazingly coherent not only in architecture but, perhaps more particularly, in notion. By this terse move forward, Leibniz, paved the ground for every element of his edifice to reflect the universe by definition.

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*1 Simplicity holds quantitatively not qualitatively, qualitatively, substance is infinitely complex*

Infinite divisibility of actuals in actual world which led Leibniz to actual infinity, would be replicated with infinite complexity in quality of substance in possible world. In principle, though not in cyclic routines, like sun's movement, seasonal changes etc., Leibniz oversees to remain in parallel with nature. What prevails behind the phenomena as a law in actual world underpins an unfolding inner power in the complex make up of simple substance in Monadic world. In this respect, aside from traditional meaning, Leibniz is a true practitioner of Philosophy of Nature. In essence Leibnizian definition of metaphysical world is merely an analogous of actual world and emerges through a simple integration operation in terms of Infinitesimal Calculus such that, he forms the least possible unit of the subject as a possible idea ( $dx$ ), an idea which is not self contradictory and a true representative of main body, namely individual substance, and then multiplies it with the extent of universe in question( $f(x)$ ). Hence  $\int f(x)dx$ , an idea also analogous to the relation between member, the least simple unit, and class, main body of the aggregate. This is what gives the logical ground to Leibnizian perception, namely multiplicity in unity, and makes his metaphysical world logically possible and mathematically provable. The idea is illustrated in "Distance=Speed.Time" relation:

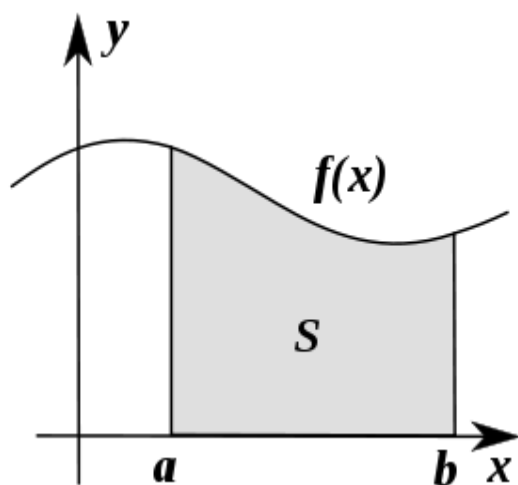
"The symbol of integration is  $\int$ , an elongated S (the S stands for "sum"). The definite integral is written as:

$$\int_a^b f(x) dx.$$

and is read "the integral from  $a$  to  $b$  of  $f$ -of- $x$  with respect to  $x$ ." The Leibniz notation  $dx$  is intended to suggest dividing the area under the curve into an infinite number of rectangles, so that their width  $\Delta x$  becomes the infinitesimally small  $dx$ "

Figure 1: The Integral as an Area.

Source: Courant, Richard *Differential and Integral Calculus*-New York Interscience Publishers 1988 CHAPTER II pp.77.





“If  $f(x)$  in the diagram on the left represents speed as it varies over time, the distance traveled (between the times represented by  $a$  and  $b$ ) is the area of the shaded region  $s$ . To approximate that area, an intuitive method would be **to divide up the distance between  $a$  and  $b$  into a number of equal segments**, the length of each segment represented by the symbol  $\Delta x$

**The sum of all such rectangles gives an approximation of the area between the axis and the curve**, which is an approximation of the total distance traveled. **A smaller value for  $\Delta x$  will give more rectangles and in most cases a better approximation**”<sup>1</sup>.

Therefore, no matter who says what; Leibniz has only two premisses which can be enfolded into one, instead of Mr. Russell’s five<sup>2</sup>, that lead him to the whole or almost to the whole of the necessary propositions of his Monadic world:

Every proposition has a subject and a predicate and,

Perception yields knowledge of an external world, i.e. of existents other than myself and my states.

The first premise involves time and space effects on states by memory capacity of soul and spirit monads, analytic-synthetic distinction is not central, though relevant secondarily, to the nature of metaphysical world in question and ego’s being substance to the extent of discussion seems tenable.

So there remains above two upon the unification of the other three to the first. But, Leibniz would not put the first premiss or axiom after all, as is given above; instead he would say ‘given a subject, **follows its all predicates** and vice versa that is, **given a predicate, follows its all subjects**’.

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<sup>1</sup> Richard Courant *Differential and Integral Calculus*-New York Interscience Publishers 1988 CHAPTER II pp.77 *The Integral as an Area*.

<sup>2</sup> Leibniz’s premisses according to Russell:

i. Every proposition has a subject and a predicate.

ii. A subject may have predicates which are qualities existing at various times. (Such a subject is called a substance.)

iii. True propositions not asserting existence at particular times are necessary and analytic, but such as assert existence at particular times are contingent and synthetic. The latter depend upon final causes.

iv. The Ego is a substance.

v. Perception yields knowledge of an external world. i.e. of existents other than myself and my states.

Bertrand Russell-*Philosophy of Leibniz* Routledge Book 1997 CHAPTER I Leibniz’s Premisses pp 4, para 4.

This last form would suffice to deduce all the rest of the propositions his metaphysical system necessitates. Because, it's complete in analyticity, that is, contained what's necessary for a priori deduction, from this axiom of manifestation his metaphysical edifice would potentially follow. This might reduce the number of premisses or axioms to one, namely that **every proposition has a subject and a predicate!** And perception holds a Leibnizian rather than an empiricist content, that is, being multitude in unity and converges into the major premiss in conformity with the main idea. Leibniz would agree with that!

### **1.3. Necessary Propositions and the Law of Contradiction:**

Now, axiomatic in propositions of metaphysics and mathematical in logic behind, Leibniz's assertions are twofold, involving two qualities concurrently with no discrimination in between. Less rethoric in metaphysics and more mathematics in logic to the maximum extent possible. Subject-predicate logic sees every proposition as an assertion connected with a binary connective. That should not be taken just as a statement vulgarly known to public and must be taken seriously. Such that, given every proposition is a binary relation and given that every proposition has a truth value then it has a truth value of affirmation or negation, that is, yes, true, **1**(one) or no, false, **0**(zero). Every truth value of every proposition asserts a fact about the world, **1** or **0**. Since the world is totality of facts not of things and since this also includes the world of possible ideas, that is, possible worlds one of which is Leibniz's Monadic world, the world continues to exist as the totality of affirming and negating series of combinational facts(This happens in the mind of percipient as a version of possible world). This is not only true for phenomenal facts of actual world directly visible or invisible but also true for notional and essential facts of logical and metaphysical worlds from universal measure down to the simple substance of monadic level. These series of binary encoding occur necessarily in simple substances by perception, appetite for action and registered in the notion of them. Soul and spirit monads apperceive these registrations on their own but bare monad depends upon the ultimate cause for that.

The truth value of every binary relation, that is every proposition, is determined by the notion of simple substance and environs. It's not impossible to imagine a spherical model of the universe at certain point of instant in **1** and **0** combination in lateral plan. Every simple substance would register only one binary code **1** or **0** at a certain instant of time and this would give a time series in vertical plan. Leibniz would substitute this record of registry with time series. So, any proposition asserting any of these relations is necessarily analytic because, all truths concerned are contained in the notions of subjects as predicates or attributes. Now, the world could have been profoundly different if the truth value of a single proposition had been different than it had been. This is the junction point of the route from which everything could continue adversely than it had been. And that's what makes the idea of possible worlds possible. It follows that all possible worlds are inherent in the notion of substance and any proposition as to that is necessarily analytic. The first build Leibniz seats at the bottom of his metaphysical world was not so large as a foundation stone, to the contrary, it was ultimately primitive, simple – indivisible, irreducable, indefinable – in quantity, and infinitely complex in quality. That would be an ideal material of nano character to an architecture. Because, the simpler in quantity the more resolutive in the construction of edifice. And it certainly would be an individual/simple substance, monad and its perceptual actions.

In fact, this was, most probably, an allusive admittance of oracular inscriptions appear on west terrace of Temple of Apollo at Delphi, 'Know Thyself' and 'Nothing in Excess' which perfectly match with Leibnizian principles of 'perception' and 'sufficient reason' respectively. This suggests the profound difference between Leibnizian notion of perception and that of Empiricists where in the former perception appears as 'comprehension of multitude in unity that one does experience within one's self'<sup>1</sup> while in the latter 'appreciation of phenomena as sense data or percept'. Virtually, how far seems the former analytic to that far is the latter synthetic.

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1. Nicholas Rescher, *G.W.Leibniz's Monadology* ROUTLEDGE London 2002 CHAPTER 3 *The Thematic Structure of the Monadology*, pp.31 para B-V (16)

A deeper inquiry into the origin of perceptual experience that ‘reflects a comprehension of multiplicity in unity what we ourselves can and do experience within ourselves’<sup>1</sup> may prove to be adopted from Iranian sufi Mansur Al-Hallaj’s mystic assertion that "Ana al-haqq" ("I am the Truth"—i.e., God)’ which essentially appeals to ‘unification with God-Tawhid’. So, Leibniz obliges to read him in quite a different manner<sup>2</sup>. To that far, Leibnizian perception is ‘inexplicable by mechanical reasons and ever-changing internally complex make up of monad’s qualitative states. It is subject to an inner drive of developmental change known as appetition. Perception in Leibniz, proceeds entirely from the monad’s **own** internal complexity through an inherent “**programming**” that makes every substance into a fully independent source of its **own** actions’<sup>3</sup>. Two consequences follow from the last statement; in the first place, a monad, as an individual substance, is not dependent upon the ultimate cause at all. Because, hereafter, it’s on its own sufficient reason which makes it an independent source of its own choices. And this independence in reasoning endows it the self sufficiency of an individual identity. Now, it’s fully on its own and responsible for its every action, in all respects, good or bad, right or wrong, true or false. Taking into consideration the final cause in the last analysis, the efficient cause is throughly in effect. So, no excuse is admissable on account of actions or passions. And secondly; Leibniz proposes a distributed mode of processing structure which even today is so contemporary in networking architecture of modern information systems instead of main frame topology based on central processing units. The distributed processing systems which stand for sufficient reason, proved flexibly viable as in Microsoft and Internet examples, while the main frame systems which correspond to ultimate reason dependency, has become extinct like dinosaurs of paleontological times as in IBM instance, once a giant. Now,

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1. Nicholas Rescher, *G.W.Leibniz’s Monadology* ROUTLEDGE London 2002 CHAPTER 3 *The Thematic Structure of the Monadology*, pp.31 para B-V (16).

2. Note: However; Leibniz finds somewhat subjective, the Cartesian assertion as to existence, “*cogito ergo sum*”, havily emphasizes the same interconnection between individual substance ‘I’ and eternal substance ‘God’. Granted that thought/mind is a substance in Cartesian terminology; Descartes, burdens his existence on eternal substance’s being necessarily existent by his all known assertion that “*I think therefore I am*”. In this proposition main emphasis is, to our understanding, not on ‘I’, as is widely believed, but on ‘think’ as an agent of substance that necessarily exists. Since Cartesians take the existence of God for granted, they need no other evidence to assure that an individual substance, that is, ‘man’ exists.

3 .Nicholas Rescher, *G.W.Leibniz’s Monadology* ROUTLEDGE London 2002 CHAPTER 3 *The Thematic Structure of the Monadology*, pp.31 para B-V (16).

given that every perception is on its own sufficient reason; a multiplicity, not only in complexity but also in quality immediately follows. That's Leibniz!

'Now, monadic change involves in the qualities of substances being ever-present. It comes from within through unfolding of an inner law of development and is driven by appetition but different from it holding a cause and effect relation to it. Following a law of continuity forms one causal series and roots in the inner complexity of substances'<sup>1</sup>. So far so good in his metaphysics yet, a doubt of inconsistency slightly stirs in one's mind. This is impervious analyticity of necessity in all acts of substance. Because, the necessity is ultimate and indefinable, that which is somehow necessary can never be known to third parties!. Leibniz maintains that change in qualitative states, which is necessarily necessary, is perceived by allied of substance and registered in the notion of them. Now, the question is if those registrations are knowable by any substance other than Absolute! Apparently unlikely! Because, due to the ultimate isomorphism in the notion of substances and change in qualitative state's being the mode of existence would make it impossible to distinguish one state from the others. Speaking more expressly, in perception of change in qualitative states, attributes, there seems a problematic area! Because, every perception is to involve a sufficient reason. An act of reasoning presupposes a distinction between the states in question. But, given that the reasoning substance is also subject to the same change as qualitative states in parallel, the question 'how can it be possible to perceive a necessary change in quality, attribute, and distinguish one from the others' remains open! Because, in this case, the subject of the reason, which is substance, and object of the reason, which is also the same substance, turns out to be one and the same thing. Unlike the other propositions of Leibniz as to the logical notion of his metaphysics, this time, that seems logically not possible. Because, it presupposes to judge the one and the same quality through the one and the same quality as incapable of referring to a third party, by definition, because change in qualitative states, attribute, is necessary and assumed to be impervious. To apply to the difference in clarity of perception does not seem to provide a way out. Because, the problem is of

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*1* Nicholas Rescher, *G.W. Leibniz's Monadology* ROUTLEDGE London 2002 CHAPTER 3 *The Thematic Structure of the Monadology*, pp.31 para B.

self reference nature and pervades throughout the domain of necessity. To this account, the proposition that ‘the substance changes necessarily in quality’ stands meaningless and truth or falsity of it remains unknowable *per se*. That suggests an analyticity of Kantian sort which leaves a room for deity.

On the other hand what’s perceived of the change of qualitative states is merely predicable state of changes, not the essential notion of substance behind, attribute. It can only be apperceived, as it seems. The essence of substance, subject of existence, by all means, remains intact. What’s known to percipient is, again, just predicates by their phenomenal indications. So, predicatives, presuppose attributives. In other words, the change in qualitative states is presupposed by change in quantitative states. Most probably the former depends on action and the latter causation to proceed. This brings about a typical reality-appearance distinction case as traditionally understood or conceiving the analytic truth through actual, contingent, truth. This point of view seems to provide a great convenience against indeterminacy of necessary truths from actual stand point.

Now, how can it be possible to reconcile the two canonical definition of perception? The internally complex make up of a monad’s qualitative states which is ever-changing and the multiplicity in unity! Given multiplicity in unity emerges through the ever-changing internally complex make up of a monad’s qualitative states and vice versa the two definitions squarely overlap. This certainly constitutes the ground **subject-predicate relation** emerges through. The multiplicity in its unity, as the collection of predicates or registries of changes in qualitative states, is wholly contained in internally and infinitely complex make up of the substance’s ever-changing qualitative states. That means that every predicate gives a particular instance of subject-predicate relation over an individual substance and its possible predicates and implies that Leibniz’s metaphysical propositions, at least for the most part, are deduced from **subject-predicate logic**.

Perception registers the changes in qualitative state in an inherent orderliness which means ‘what comes later emerges through a natural explanation from what has gone before’<sup>1</sup> Thus, a linear one dimensional registry programming is offered for the perceptual changes. It can validly be said that this simply forms a bit stream of binary encoding emerging from truth functionality of **subject-predicate relation**. Thus, in case the relation is affirmed the code is **1**-true, otherwise, that is, in denial of relation the code is **0**-false. That’s the idea which underlies **the world’s being an infinite line code software**, in lateral, vertical and spherical plan, back and forth in time. The registry of states, says Leibniz, makes one causal series in continuity of time and can be substituted for it. What’s particularly noteworthy in his fabrication of metaphysical edifice is that; whatever the matter of discussion is, perception, appetite, action or self sufficiency, once one has captured the subject from a genuine end, one can proceed towards the successive notions by a truly natural explanation, as it were, through unfolding of a progression law of mathematical series or time series almost with no gap in between, hence, following law of continuity. Thus, one with comprehensive appreciation of Leibnizian principles of metaphysics can travel through from the immediate end to the distant end without having to confront with a break-through, that is, to reap the envelope of universe defined or to come over a dead end virtually by pointing out a consistency and completeness. That presents an amazing applied demonstration of the principle of sufficient reason and the law of contradiction in his philosophy. That’s the full analyticity of Leibnizian type! Not only in propositions but also in architecture. This is what gives the logical ground to Leibnizian perception namely multiplicity in unity and makes his metaphysical world logically possible!

#### **1.4. Contingent Propositions and the Law of Sufficient Reason:**

Any fact consists in a sufficient reason, that is, there is nothing in the world which does not depend on a sufficient reason for its being so and not otherwise. By the word sufficient, Leibniz means not only a scalable proportion in amount but also

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<sup>1</sup> *Nicholas Rescher, G.W.Leibniz’s Monadology ROUTLEDGE London 2002 CHAPTER 3 The Thematic Structure of the Monadology, pp.31 para B-V (22).*

a necessity for existence. What is meant by the attribute sufficient, is explicitly to denote adequacy but implicitly to conform with necessity as well. So, the principle of sufficient reason is, in fact, for the principle of necessary and sufficient reason for being existent or true in actual or possible worlds.

Principle of sufficient reason determines two vital processes; perfection in being and perfection in action. These processes are fully analogous to the process of optimization in engineering. While perfection in being optimizes the use of source in bringing something into existence, perfection in action secures the efficiency in transition from preceeding to successive state in conformity between the sufficient reason of simple substance and the necessary reason of ultimate cause. This process happens without any gap or leap in continuum, between thought and things, nature and mind harmoniously and in full agreement such that no discernible redundancy or surplus or waste of source is left. Regardless of any concept of magnitude or measure, this process keeps working detailing down to the infinitesimals and simplifying and unifying up to the eternal truth. Every state and every infinitely small bit of differentiation of this process is conceivable and mathematically provable in Leibniz.

Perfection in being, shapes the individuality of simple substance while perfection in action, keeps in touch with the final ends of necessary substance. Now, such a system features every property of a Digital Network of a modern Information Systems. Yet, it is not so simple as a digital network in complexity. Because, it contains a multi-layered and multi-chanelled architecture both in physical and metaphysical realms for the best possibles and just possibles as well as compossibles and impossibles in the past, present and future. It's admissable to say that, Leibniz theorizes a natural philosophy provided that essences and ultimate causes of the laws behind the phenomena are acknowledged<sup>1</sup>.

In this domain, also, he distinguishes contingent from necessary. In parallel with this idea, Leibniz reconstructs his metaphysical world using the true atoms (monads) of

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<sup>1</sup> *Litteral principles are simplified and transposed from the book G.W.Leibniz's Monadology by Nicholas Rescher.Routledge LONDON 2002.*



universe through a series of analogies to the laws of physical world. Today, the solution of a problem by the sole power of mind as such is known “Mathematical Modelling and Simulation”.

By saying that ‘for every truth whatever there is a sufficient reason for its being so rather than otherwise’ he gives the Principle of Sufficient Reason a prime place among the other principles of truth, supposing it to unify them into one on itself. Thus, the Law of Contradiction, for instance, provides a sufficient reason for necessary and eternal truths, in his system, by being possible or not contradictory. In other words, being possible or not contradictory is a sufficient reason for a necessary truth. God’s decree might also be considered to conform to this custom, but seemingly, nor can it flee from accounting for his acts on the basis of Principle of Sufficient Reason efficiently and teleologically. And Principle of Perfection appears as the sufficient reason at every instance of contingent truths. So, Principle of Sufficient Reason gives a common root to necessary and contingent truths over itself and unifies them into one at the ultimate ground. Therefore, the sufficient reason for any truths can be found by a process of analysis. In eternal truth’s case this analysis is finite and terminates in logically self evident, indefinable, irreducible or identical propositions. These are to necessary logical truths what axioms or postulates are to mathematical truths. The point that the root proposition that appears in the finite analysis of the necessary truths is ultimately simple in quantity but infinitely complex in quality is vitally important to always be borne in mind!<sup>1</sup>

The process of analysis, on the other hand, to find a sufficient reason for truths of fact is infinite and of unending complexity. The distinctive character of this analysis is that the process is infinite in quantity, due to infinite divisibility of actuals, but simple in quality namely an instance of universal hypothetical proposition

“ $\forall_x (F_x \rightarrow G_x)$ ”.

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<sup>1</sup> *Littoral principles are simplified and transposed from the book G.W.Leibniz’s Monadology by Nicholas Rescher. Routledge LONDON 2002.*

## CHAPTER 2

### 2.1. The Power of Analyticity That Suggests Universal Characteristic:

Now, since the topical field of interest of this essay is a through examination of subject-predicate relation and an inquiry into what analogy could be established between subject predicate relation and class member relation, consecutive asserts as to what the notion of the relations appears to be and what outcomes will follow from to occur:

As a passionate Universal Characteristicist he was in pursuit of statements that prove true in all possible worlds. He would find this essence in analyticity, perfectly matching with the notion of folding multiplicity in unity and unfolding complexity out of simplicity. Given an analytic proposition is true by the meaning of its elements, that is, by definition; the denial of such a proposition is impossible, hence, an express contradiction. So, his proposition would be analytically true, thus, not contradictory. Such a proposition would virtually be identical to a possible idea in its exact meaning! Thus a possible idea would be that which is not self contradictory and necessarily true! Now, a thing which necessarily exists would perfectly match that definition and constitute the essential build of his subject. This would give him the ever existent essence of things. Substance! Still one question as to the notion of his subject remains suspending. Plurality! Leibniz responds this thirst of yeast with the notion of individual substance, a metaphysical nano character comprising the genetic codes of all that will happen ever after! Leibniz's prudent zoom into the reality ends with the framing of individual substance! It would serve as the universally quantified variable in deduction of his edifice. He would employ it, as it were, the differential element of an integration process and place it as the navel piece of his big puzzle or the keystone of his havenly domed edifice to complete the puzzle and make the big picture visible. It can safely be asserted that Leibniz, at first,

conceived his edifice in its full integrity and after that gave its principles of constitution in *Monadology* and the logical grounds for those principles in *Discourse on Metaphysics*. This may seem an early conclusion but, he heavily underlines that the world, in the final analysis, is analytic and proceeds on the principles of continuum in actual and family resemblance in ideal domain. Mathematics is the interface in between with its infinitesimality!

Pure mathematics is analytic but applied mathematics also interfaces to syntheticity of actual world. Hence, Wittgensteinian assertion that ‘whole mathematics being a tautology’ holds and seems to base on this analyticity. In this connection **circularity seems inevitable, in fact, it appears even necessary to express one term in the notion of the other**. Otherwise successive conveyance of reality or meaning would be impossible!

Typical examples of what’s been said above would be, among others, mathematical entities of transcendental or irrational numbers. To our understanding they are immediate result of lack of analyticity in their nature, forcing two essentially different entity to interact. Hence,  $\pi$  is an unnatural outcome of line through curve attempt. This is no different from the self contradictory idea of round square! So, reconciliation or peaceful unity between two notions, line and curve, is impossible and can not result in a natural or rational number. Thus, it’s a vacuous or vicious attempt to keep talking on the transcendentality of the number  $\pi$ . It’s the mere consequent of incommensurability.

## **2.2. Ideal-Actual vs. Necessary-Contingent Distinction:**

Every proposition, whatever, analytic or synthetic, is ultimately reducible to subject-predicate form. In other words, every true proposition of subject-predicate form is analytic, that is, the predicate is contained in the subject, provided existence is not claimed to be the predicate. Existence as predicate is attributed only to God. Thus, existential propositions, except God’s case, are synthetic. Actual existence is, metaphysically, not *per se*, rather, is contingent and fragmentary. So, it does not

assert any necessity and denial of the case leads no contradiction. In analytic propositions, even in such pure tautologies as “*A is A*” subject is always complex.

The subject of such a proposition may be ultimately simple in quantity but will be infinitely complex in quality. Predicates can be abstract and/or concrete. Necessary propositions have necessary and synthetic propositions have contingent predicates and vice versa Contingent predicates are concrete or perceptible forms of states and express the momentary states of a substance.

All the concrete predicates are necessarily connected to subject but not to one another. The series of predicates attributed to a subject determines the notion of subject, that is, different predicates would amount to different subjects.

Ideal-actual distinction is vital to avoid any ambiguity between necessary and contingent truth. Supposing line of numbers, actual stands at zero point of continuum line. Reason experiences a two way journey starting from actuals; one upwards abstracts other downwards true atoms of actuals. The former would proceed by association to abstraction the latter would proceed by division to the ultimately simple. The former would immediately be abstract and analytic right after the first order terms and be simply a work of mind from the second order terms on. The first order terms, as Leibniz sees, are of semi-mental objects. From this point on only reason/thought or in Rationalist terms substance would be at work. So, you get to ultimate ground of substance. The journey to the center of the actuals would, virtually, result in the capture of simple substance if the analysis by division be completed but this seems only in the capacity of God. Nevertheless, let's suppose that by virtue of mathematical series operation one succeeded to reach the end of the series. Substance would be what one captured in the end because it is supposed that by division the aggregation was eliminated and substance was reached in the end. “Necessary and contingent truths differ as rational numbers and surds. The resolution of the latter proceeds to infinity” (N.E. 309; G. v.268). In eternal truths we proceed from essences and predicates to determine their relations and this holds a finite process; while in contingent truths we proceed from an undetermined existence like Ego and enquire into its predicates. Since the number of predicates goes to infinity,

theoretically, the collection of predicates cannot decisively give the nature of its subject.

So, the idea is that two eagles of Zeus going upward and downward would meet at the navel of the world that is substance. In either cases, the truth would be ultimately simple in quantity and infinitely complex in quality. The eternal truths are all hypothetical and do not assert the existence of their subjects (N.E.p515; G.v.428).

The truth of hypothetical propositions lies in the connection of ideas (N.E.p516; G.v.429).

### **2.3. From Substance to Individual Substance with Leibniz:**

Philosophy of substance, as is known, inherited to Rene' Descartes and his followers from ancient Greeks, dominates The Rationalist Philosophy and '*it's that which only needs God's concurrence for its existence in Descartes, while in Spinoza, self caused or that which is in itself and conceived through itself. Descartes was on dualistic view while Spinoza was unifying extension and thought into attributes of the one and the same substance. In both views, the substance was a notion dependent upon the purely logical notion of subject and predicate. The **attributes of a substance** were the **predicates of subject** and existence of predicates depended on their subject but not conversely*'<sup>11</sup>. Leibniz, noticing that his predecessors depended their notion of substance on two principles, namely independence of existence and logical notion of subject-predicate relation, favoured the more profound logical relation of subject-predicate as the ground for his notion of substance with one exception that while Cartesian definition of substance admitting extension as substance Leibniz saw it just as a repetition and not substance but substances.

In the first place, when all that he had said about substance considered together and interrelatedly one fact proves discernible at the intersection of those arguments; the Leibniz's substance is an animate subject. It may not be breeding, growing, dying as ordinary living things do, but, it is capable of every mental activity like, perception, appetite, activity, reason and making choices for the good, better or

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<sup>1</sup> Bertrand Russell-Philosophy of Leibniz Routledge Book 1997 CHAPTER IV Conception of Substance pp.40.para.16

best. When those attributes are put in an outline it gives a picture that substance in Leibniz is present in mental acts rather than bodily acts of an animal things. Or it's a mind always favoring mental acts to animal desires on the principle of sufficient reason ultimately related to necessary reason. His notion of substance would behave like an animate thing with mind and body. But with one exception, not as a living thing with one mind on one body, as in his predecessors, causing inconsistencies among the elements of the metaphysical system and in their relations, instead that of many minds and many bodies! A notion of this sort would respond also to the plurality of actual world as well as the problem of evil which is not the subject of this essay. By the notion of individual substance he would individualize and numerically minimize his substance down to the least bit of existence with sufficient reason and its entailed estates and from this bottom build up his edifice. By the capacity of perception he would enable his system to be comprised of infinitely many individual simple substance of atomic level to act like one being acting in its extensional and mental integrity. That would be multitude in unity! The notion of individual substance with the capacity of perception would save the notion of substance from appearing in a bulky massive aggregation and make it into, as it were, a transparent extension of Reality. That would perfectly fit to his denial of infinity in numbers for, to him, there is only one number instead of many or infinitely many and it's the number 'one'-1 the rest is the fractions of it in a particular ratio of denomination. Thus, individual simple substances, monads, not perfectly alike in indiscernible identity, stand for numerator and the universe stands for denominator in the continuum of ultimate proportion, hence, monads are for multiplicity and universe for unity in the idea of multiplicity in unity. And mathematically, universe stands for the final whole 'one'-1 and monads for individual whole from their point of views, perspectives. In the 'multitude in unity' as the total sum of his entire philosophy, as it seems, one can find its sufficient and necessary reasons in conformity, hence, sufficient reason behind multiplicity and necessary reason behind unity!

Along with subject-predicate logic Leibniz saw another constituent in the notion of substance; persistence through change. It suggests a subject which preserves its identity while altering its quality. By the discussion of the element of persistence a deeper inquiry into the nature of substance brings itself about as the notion of subject and predicate applied what exists in time. This gives the ground to the idea of individual substance as being only subjects and not predicates. Internal experience, to say that I am the same person as I was, is not sufficient for Leibniz; an a priori reason is needed.

A subject, predicable of many predicates and not predicated to any other subject is called an **individual substance**<sup>1</sup>. This is the true atom of Leibniz's metaphysical world and almost in the same relations to its environs as has the **variable** in the set abstract in set theory  $\{x:\dots x\dots\}=\emptyset$ . More expressly, the correspondence lies between the function of individual substance in Leibniz's Metaphysics and the function of a variable particularly in Set Theory and generally in mathematics. Thus, individual substance(monad), ultimately simple in quantity but infinitely complex in quality, yields diversity in phenomena by repetition, similarly a variable in a set abstract can turn out to be any particular in phenomena by instantiation. Under initial conditions of set domain, since there are no sets the number of sets that exists is  $\emptyset$ , hence  $\{x:\dots x\dots\}=\emptyset$ . Although we will define a putative set 'null' as an axiom later in CHAPTER 3, 3.3. Empty Class and Natural Numbers; we do not have the concept of null class yet as  $\{x: x \neq x\} = \emptyset$ . The individual substance is the one who says *I*. It involves a reference to existence and time. As is said above, it's a possible existent and expressed by its predicates of different times. Such predicates would be contingent or concrete in nature and not follow, laterally, from any other, rather the last one would follow from the preceding one by activity. The sequential change of qualitative states, perception, is driven by appetite and governed by sufficient reason! To say that I am the person that I was, one requires an a priori reason rather

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<sup>1</sup> Similar to Aristotle's definition: "that which is neither predicable of a subject nor present in a subject is called substance e.g. individual man or horse.." Richard McKeon-The Basic Works of ARISTOTLE; CATEGORIA pp. 9 para 5.

than a mere internal experience. This reason is that one's present and past attributes all belong to one and the same substance which is *I*. Saying that all my states are involved in the notion of me is just asserting that the predicate is in the subject! That would make an individual the universe folded in him. Therefore, every individual substance, soul, is a world apart to Leibniz and as a subject has eternally all the predicates, the states, that time will bring it.

He would find this in the fact that one's present and past attributes belong to one as the one and same subject. Thus, substance is that which only be subject, not predicate and persists through change as the subject of change. It is attributed many predicates at many different times and once a predicate is attributed to a subject it's eternally the predicate of it and can be found by an infinite analysis. Because 'the notion of predicate, according to Leibniz, is always contained in the notion of the subject. Thus, to say that all my states are involved in the notion of me is merely to say that the predicate is in the subject'<sup>1</sup>. So, every individual substance is a distinct world and independent of everything else except God. And its development by change in time is the consequent of its notion and can not depend upon any other substance. Such a subject is mere a possibility. It may or may not exist. But if it exists, all its states follow from its own notion and not from any action from without.

#### **2.4. Activity vs. Causality:**

Activity vs. causality takes quite a significant place in Leibniz's notion of substance. It's a regulated tendency from within and metaphysically necessary to substance through which phenomena emerge in an orderly manner. Hence, activity appears as an element or quality in every state of a substance by which that state is not permanent, but tends to pass into the next state. It may be due to distinguish activity from causation at this point of discussion. Causation appears as a relation between two phenomena by which one is succeeded by the other whereas activity is a quality of one phenomenon by which it tends to cause another on the basis of sufficient reason. In this sense, it seems more profound than causation. Activity

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<sup>1</sup> *Bertrand Russell-Philosophy of Leibniz Routledge Book 1997 CHAPTER IV Conception of Substance pp.42.para.17.*



happens to be an actual quality of a substance, an attribute, that corresponds to the relation of causality. A closer look would reveal that causality applies to a cause rather external while activity to a reason, namely sufficient and internal. Activity in a substance, as it were, is a life energy which causes the substance to continue to be. 'Given a substance is the permanent subject of changing attributes it follows that activity in the above sense is essential to substance and thus metaphysically necessary'<sup>1</sup>. In activity two successive states hold a particular relation to one another; the preceding state being activity or desire and succeeding state desired. This distinction enables one to interpret their order of succession as a result of their own natures. This inner drive of developmental change is called appetite. Thus, the process of activity is twofold, one is in preceding state and the other in succeeding. The former is called desire and the latter desired. Very similar to a love affair provided that it's divine. So, this takes one to a mystic conclusion that '*whatever can one see diverse is on love in the universe.*' The point is that the object of activity or desire should be non-existent but should be regarded as capable of becoming existent. This is what appeals to eternity in perpetuity of activity as a relation **from intension to extension**. So, no relation is purely external it roots in subject and predicate reciprocally. 'We can say that a body acts when there is spontaneity in its change, and suffers when it is pushed or impeded by another-G.V. 196(N.E.219).' Owing to its perfection on more distinct perception, an active substance acts upon a passive substance that suffers due to its confused perception. The former is activity and the latter is passivity. Activity results in action which implies some joy and passivity results in passion which implies some pain.

Activity which involves a reference to the time is the principle behind the change to the state of substance resulting in the predicates that make the subject which is a substance. It is essential to every substance. So, the notion of an individual substance is so complete that it suffices to distinguish its subject or to comprehend and deduce all its predicates. Hence, from this identity of indiscernibility follows, namely that, no two substances can perfectly be alike!

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<sup>1</sup> Bertrand Russell-Philosophy of Leibniz Routledge Book 1997 CHAPTER IV Conception of Substance pp.42,para.17.

Now, although whatever it may happen to a substance is contained in the notion of it that does not necessarily mean that how a subject will act in the future can be inferred from any general proposition about it. No future actions of a substance follows necessarily from its general notions or essence or whatever. The only means for this purpose is the principle of sufficient reason that one has. The principle with its defined form and function is substituted for the law of causation. It gives a connection between events at different times. What is further than causality is that ‘it gives an account of why and not merely that certain sequences occur’<sup>1</sup>. To emphasize the role of sufficient reason Leibniz says that if there were only one substance; since everything would remain in the same state, there would be no need to make any choice, on the principle of sufficient reason, and the substance in question could not drive its developmental change and cease to exist. So, it takes two to tango. In free souls, that is, the ones with memory or reason, the sufficient reason underlies the choices made for the good proportional to the clarity of their perceptions. In bare monads sufficient reason is provided for their actions in God’s perception. Anyway, in no case, is the connection between two states in itself necessary. If it were necessary, the principle would not be the sufficient reason but, principle of contradiction. Because, the sufficient reason in necessary propositions is the principle of contradiction.

Those predicates registered in the change of monad’s qualitative states, as can evidently be seen, form one causal series such that, ‘all singular things are subject to succession nor is there anything but the law itself. Succession have the property that given the first term and the law of progression the remaining terms follow in order. The only difference is that the order is temporal in successions but in numbers it is that of logical priority’<sup>2</sup>. The law of progression also gives the connection between preceding and succeeding states of the same substance changing through time. This defines simply an **adaptive algorithm in modern sense** on the basis of sufficient reason to constitute a persistent law to provide the state of persistence through time. If this is the case, Russell says that the law of progression or persistent law could

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<sup>1</sup> .Bertrand Russell-Philosophy of Leibniz Routledge Book 1997 CHAPTER IV Conception of Substance pp.47.para.19 line 3.

<sup>2</sup> Bertrand Russell-Philosophy of Leibniz Routledge Book 1997 CHAPTER IV Conception of Substance pp.47.para.20.

have been safely substituted for the appeal to subject-predicate concluding that a given existent at a given moment might have been caused not by the whole preceding state of the universe but one definite existent in the preceding moment-keeping in mind that any definite existent mirrors the whole universe in its unity. And the fact that there are many causal series and not one only would have been substituted for the denial of the interaction of substances and from this, persistence and independence of substance would have necessarily followed. But, in this case, a substance would amount to the sum of its states which Leibniz denies. For he says that those states inhere in a substance thus they cannot exist without it. ‘The substance is not an idea or a predicate, or collection of predicates it is the substratum in which predicates inhere(cf.N.E.pp225-6; G.v.201-3; esp.para 2).’<sup>1</sup>.

Apparently, the ultimate origin of the motivation that fuels, seemingly, the unending activity of substances lies behind the state they were residing in until “everything started up”. Substances seem to have been in a state that can be described as “simplicity in unity” as opposed to multiplicity in unity at the beginning of everything. At that state, it can be inferred that they had been condensed into the least possible quantity so as to have the maximum possible quality. They were, as it were, in the germinal form of universe which reflected a state as the simplest possible in quantity and the most complex possible in quality. This state could be said to be the “essence of existence”. The major attribute of this state was unification. Existence was in a unified form as the substance! Everything was in an absolute happiness and grace as is a baby in its mother’s uterus. To this account, the activity as a love is driven by the desire to come back to the initial state of unified era. A love, divine or earthly, appears to be the desire for unification missing the initial conditions. So, everything is in a quest of being one with its mate to reverse the actual multiplicity into possible unity backwards the oldest total unity. This aspiration runs as the energizer of activity process as a whole.

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<sup>1</sup> .Bertrand Russell-Philosophy of Leibniz Routledge Book 1997 CHAPTER IV Conception of Substance pp.49.para.21

## CHAPTER 3

### 3.1. Emergence of Empty Class Through Substance:

We, thus, come to the point where it all begins; how the notion of empty set emerges through substance! Leibniz's assertion that substance is more than the sum of its predicates or subject to the change which results in, in the last analysis, the states of qualitative changes, attributes, or predicates leaves the substance wholly destitute of meaning. And this is where we want to reach in the world of meanings. Because, that may form the ultimate origin of everything. Meaninglessness! Acknowledge a subject incapable of being named.-It is the ultimate truth, identical to it's identity, indefinable, irreducible to any simpler thing other than itself. It is predicable of everyting, that is, every predicate can define itself over it but it can not be defined by any predicate. Since a meaning is conveyed by a predicate, it remains fully destitute of meaning. Meaninglessness and emptiness turns out to be two logically equivalent concepts of two equivalent worlds mirroring one another; conceptual world of Mathematics and philosophical world of Metaphysics which goes in paralell with the former in Leibniz's mind. This state might appear as a radically different and unusual thing, as it were, emptiness full of emptiness, appealing to the space-time field dimension as an emptiness big with possibles. Thus, 'It seems essential that the object of activity or desire should be **non-existent** but should be regarded as **capable of becoming existent**'<sup>1</sup>. This notion perfectly fits to the idea of class, that is, **empty class**. It is the very first class with no members and in the domain of initial conditions where nothing is member or class of anything, it can be a member of no class for there is no class to be member of, since it is indefinable, irreducible, unprovable and identical to itself it can putatively be said to be Set $\emptyset$  (i.e.  $\emptyset$  is a set) as an axiom. It enables one to generate the class of natural numbers from it through a law of progression or persistence. Hence if Leibniz were here, he

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<sup>1</sup> Bertrand Russell-Philosophy of Leibniz Routledge Book 1997 CHAPTER IV Conception of Substance pp.50.para.21

would say that; there is only one class it's the idea of class which is empty class. And the classes are universals and works of mind, that is, they are mental things and logically possible, just as a substance is the logical subject of change. Hence **substance is what to the ultimate origin of things so is empty class to the class of natural numbers.**

Before proceeding any further, it would be worthy of underlining a point, the interrelation between connectives as membership ( $\in$ ), identity ( $=$ ) and inclusion ( $\subseteq$ ). And, ambiguous predicate 'is', owing to its multi-use meaning in ordinary language, intersects with the sense of ( $=$ ), ( $\in$ ) and ( $\subseteq$ ) in different uses.

Now; 'is' is identical with ( $=$ ), this is for sure. In membership relation to a class ( $\in$ ) is identical with 'is'<sup>1</sup> in terms of the same predicate shared by the class and its members both. And when self inclusion of a class, given that every class is included by it self, is concerned, virtually, the binary connective ( $\subseteq$ ) merges into 'is'. In the meantime, the notation  $\in$  is short for  $\xi\sigma\tau\iota$  due to Peano<sup>2</sup>. Furthermore; in tautologies like  $A$  is  $A$  or  $A=A$  virtually, there seems no difference between  $A$  is  $A$  or  $A=A$  and, for the above said reason,  $A\subseteq A$  seems to be added to the former two at that level of primacy. So it is seen that the ambiguity of 'is' causes a multi-use interpretation. To illustrate this let us consider the following examples:

- (i) "is" of identity: e.g. Mark Twain = Samuel Clemens.

- (ii) "is" of predication which appears in two forms:

- (ii.1) e.g. "Socrates is a human being." Here, "is", is for " $\in$ " to set the membership relation between the individual 'Socrates' and the class 'human being', hence, 'Socrates  $\in$  H (where "H" stands for the class of all human beings.).

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<sup>1</sup> W.V. Quine - *Mathematical Logic* Cambridge Harvard University Press 1961 Chapter III Terms, Class and Member pp.119 para.22.

<sup>2</sup> W.V. Quine - *Mathematical Logic* Cambridge Harvard University Press 1961 Chapter III Terms, Class and Member pp.119 Footnote 1.

-(ii.2) e.g. “Human beings are mammals.” Here, “are” (as “is” in plural) is for “ $\subseteq$ ” to set the inclusion relationship between ‘human beings’ and ‘mammals’, hence,  $H \subseteq M$  (where “H” stands for the class of all human beings and M for the class of all mammals)

Having this in mind, ‘ $\forall x(x \in \{x: \dots x \dots\} \leftrightarrow \text{Set}x \wedge \dots x \dots)$ ’ stands at the ultimate origin as the law of progression agreeing with the above connectives. Hence if the membership relation is true then the class,  $\{x: \dots x \dots\}$ , is set around the predicate common to the member  $x$ , and  $\text{Set}x$  which means **1**-true in binary encoding, If otherwise the said connectives stands in denial form and means **0**. Again, a monadic change in qualitative states at every infinitely small instant, corresponds to an emerging world composed of facts determined by the combination of binary encodings **1** and **0**. Once again, this appeals to **a world of infinite line of software encoded in 1 and 0** which emerges at every instant of monadic change in qualitative states as a possible world in continuity.

Analytic propositions are hypothetical, that is, in ‘if, then’ form. When one considers that necessary truths are in hypothetical form and when the antecedent is ‘ $\emptyset$ ’(false) then the implication itself is rightly ‘1’(true), regardless of the truth value of consequent, as in “ $0 \rightarrow 1$ ”. This is for a very reason that predicative logic, hence Mathematical Logic and on this basis Axiomatic Set Theory is a formal system of binary, 2 based, propositions; calculations are in 2 based mathematics. And in calculation of sub-sets of an arbitrary set, the 2 based mathematical formula  $2^n$ , where  $n$  is the number of elements(members) of the set, is used. Given that the class is the null class which has no members at all, then 1 sub-set still exists as a null class, that is,  $\emptyset, 2^0=1$ , which means; the number of sub-sets of a null class is 1. This, most probably, is because the null class is included in every class, hence,  $\forall x(\emptyset \subseteq x)^1$ . It is assumable that this is the fact which gives a ground for being true of conclusion of an

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<sup>1</sup> Lemmon, E.J., (1968). *Introduction to Set Theory. Chapter I Sets Relations and Functions pp.31, T47.*

implication with an antecedent '0'(false). Because, even if it's a proposition with no term, it still has a predicate, emptiness, as in the case of, again, null class. Although there is no existent, as a member to a set, there is still a set,  $\emptyset$  included in  $\emptyset$ . So, in hypothetical propositions it's eligible to prove true even the antecedent is false, zero.

This point may be constituting a very essence of the concept of sets. Because, stripping off the members, the remaining is merely the idea of set which means sethood, the *form* of setness in Platonic realm of ideas. Given that this simplification is granted, then there will remain no difference between null class and universe class in terms of *form* except the number or existence of members. In this case, the other classes in between empty( $\emptyset$ ) and universal (U) classes reduces to the particular instances of universal class identical to null class in *form*. Hence, classes denote, essentially; **none**-null class, **all**-universal class and **many**-arbitrary classes.

If I would be asked about what the most primitive, in immediate primacy, thing is; my answer would definitely be 'empty class'. Empty class, as it seems, is more proper to the idea of class rather than 'null class' or universe class. Because, null class, on the ground that 'nullness' refers to non-existence, therefore existence, of particulars, that is, finitude, whereas 'empty class' involves no particularity. It's this notion that reduces it to being a set as opposes to properness. **'Emptiness' is all about being a priori and in the realm of pure reason, for it's the idea of class, hence, universal.**

I dare to say that, 'null class' is the existential instantiation of 'empty class' whereas 'empty class' is the Universal Generalization of nullness. Since they are unusually bold assertions, to put them in formulae may require to develop a new vocabulary and a corpus of symbols in a totally different concept. Taking this into account, this task is left to another research attempt.

As was said above, in the realm of ideas, there is only one class it's the idea of class which is truly universal, and null class and universal class, as traditionally understood, act as mathematical operators between the entities of **domain**, **range** and **field** ontology of relations. That may be the very identification of Axiomatic Method, namely, that the most elegant and beautiful in idea and maximum complexity,

richness and diversity in derivation. That's how Leibniz considers the acts of God. 'God simultaneously maximizes the richness of the world, and minimizes the complexity of the laws which determine this world' he says. That's what the word 'universe' implicitly signifies with its etymological decomposition, namely that, 'uni', to denote, 'of unit or that which unified and reduced into **one**' whereas, 'verse', to denote, a word or a poetic phrase intensively rich and deep in content which gives rise to hardly expressible lyric exaltations. The lyrical content of the word "verse" contains the sense of harmony which Leibniz greatly emphasizes, in its notion. Thus, "universe" may mean to say "the **whole** expressed **in one** word" An interim conclusion might be that there is only one set, it's the idea of set  $\{ \}$  or set abstraction  $\{x:\dots x.\}$  and from this, all the other combinations of sets set forth as all the numbers are deduced from only one single concept, that is, empty set  $\{ \}$ . Null set is also derived from the set abstract formally in the absence of any members but, empty class, as the idea of class, precedes the set abstract and the latter is formalized from the former.

### **3.2. Empty Class And Vacuous Truth:**

At the ultimate origin of things there is only emptiness. Emptiness is not a vacuum nor is a non-existence. It's abstractness of infinite possibility not yet actualized. It's universally quantified a variable which can be employed in a set abstract. It appears as the abstract idea in Berkeley, though he does not defend rather attacks to it, qualia in Russell, and individual substance-monad in Leibniz, though different in form, yet equivalent in meaning. It's the subject of predication and the object of attribution but not predicated of anything. There appears a notion; the notion of class, that is, any collection of things. But, at the ultimate origin there is only emptiness, no other objects, hence, no class is formed with any objects. Therefore, the idea of class remains as the sole entity in emptiness. Now that there is a putative object in emptiness, though abstract and mental, the idea of class  $\{ \}$ . It is



denoted with  $\emptyset$  and named empty or null class, though we take them differently; that is, null class as the particular instance of empty class. Still, there is no class with members. The only difference from the very beginning of the process is that there was only emptiness then, now we introduced an idea of class to it. Having the idea of class in mind, we define a class with no members. We have to, because, there is no other objects. If we had had some other objects we would have assigned them to this idea of class as members. But none! So, we have a class with no members. Empty class is unique with its having no members, as defined on page 30. The notion of class, other than empty class is defined by its members. Because, they are non-empty classes. But, empty class is also defined by its members. It's empty, because, it has no members. In this form it's infinitely versatile, that is, by defining a membership condition, provided non-circularity or non-contradictory, infinitely many classes can be formed. Class members can be concrete or abstract. In this process of set formation, empty class is the concept of all sets. In Platonic sense, it's the form of set, hence, a universal. **That is the very reason of every class has a subset of empty set or empty set is included in every class as a subset.** The other sets with concrete or abstract members are the only particular instances of empty set.

In a Leibnizian discourse, there is not any sufficient reason to form two indiscernibly identical sets to proceed from this ground it may be reasonable to multiply this empty set by the process of repetition. Thus, in this domain, since there is nothing other than emptiness to make a difference everything is identical to everything, including two supposed empty classes. Thus, every assertion of identity is true and every assertion of membership or sethood is false. Since Axiom of Extensionality and Axiom of Classification are both conditional, under these conditions they prove to be true. This is called **vacuous truth**. 'Any statement that begins "for every element of  $\emptyset$ " is not making any substantive claim; it is a **vacuous truth**. This is often paraphrased as "everything is true of the elements of the empty set." For instance, zero is the identity element for

addition and one is the identity element for multiplication. In mathematics, an **empty product**, or **nullary product**, is the result of multiplying no factors. It is equal to the multiplicative identity<sup>1</sup>.

Two often-seen instances are  $a^0=1$  (any number raised to the zeroth power is one) and  $0!=1$  (the factorial of zero is one).

### 3.3. Empty Class and Natural Numbers:

Let's consider the domain where it enables us to define the very first axioms of Set Theory. Our domain will contain just one object,  $\emptyset$ , as was said above it is a putative  $\text{Set}\emptyset$ . It is a set, the idea of set. Under this circumstance;

We've been given identity '= $\text{'}$ .

We have a membership connective ' $\in$ ' and

Logical signs ' $\rightarrow$ ', ' $\neg$ ' and ' $\forall$ '.

We know that nothing is member or class of anything. Because, there is only empty class in the domain and nothing at all. That means that;  $x \in y$  is always false and  $\emptyset$  can not be shown to be a set if it's not axiomatically asserted as a set, for there is nothing for it to be a member of. Hence, there are no sets on their own thus, we have:  $z \in x$ ,  $z \in y$ ,  $x \in y$ , and  $\neg \text{Set}x$  are always false, i.e, have the truth-value 0, and  $x = \emptyset$ ,  $y = \emptyset$ ,  $z = \emptyset$

For this interpretation we have:

Axiom of Extensionality<sup>2</sup>:  $\forall_x \forall_y (\forall_z (z \in x \equiv z \in y) \leftrightarrow x = y) \equiv 0 \leftrightarrow 0: 1$  -True

Thus,  $[(0 \equiv 0) \rightarrow \emptyset = \emptyset; 0 \equiv 0$  is True (1) and  $\emptyset = \emptyset$  is True-1]. Hence;  $1 \rightarrow 1: \equiv 1$

Axiom of Classification:  $\exists_y \forall_x (x \in y \leftrightarrow \text{Set}x \wedge \dots x \dots) \equiv 0 \rightarrow (1 \wedge \dots x \dots): 1$

<sup>1</sup> PlanetMath.org-. <http://planetmath.org/encyclopedia/EmptyProduct.html>

<sup>2</sup> E.J.Lemmon-Introduction to Axiomatic Set Theory Routledge/Dover Publications 1969 Chapter Two Sets, Relations and Functions pp.47-48 para 1-7.

So, following the same strategy to proceed from emptiness to the idea of class we proceed from the idea of class to ‘class of idea of class’ which is one level higher in rank of terms ( $n+1$ ). The former is of first order whereas the latter is of second order term type. And you get  $\{\{\}\}=\{\emptyset\}$  which is a unit class, if we use the natural numbers to model sets, you have;

1.  $\{\}=\emptyset=0$

2.  $\{\{\}\}=\{\emptyset\}=1$

3.  $\{\{\{\}\}\}=\{\emptyset,\{\emptyset\}\}=\{\emptyset,1\}=2$  this also gives the pair set of  $\{\emptyset,1\}$ .

These three numbers derived from the notion of set suffice to validate Leibniz’s assertions as to the logic he held behind the complete picture of his Metaphysics. 2 based-binary mathematics! He says that only 0 and 1 suffice to express the whole mathematics, in fact, to him there is not any number other than 1. All the rest is either repetitions or fractions of it. To him, there is nothing in the world which is not subsumable in numbers and only 1 and 0 suffice for this. Informally, it’s possible to show that entire class of natural numbers can be reduced into 1 in infinitely many parenthesis,  $\{\{\{\{\emptyset\}\}\}\}... N$  ary.

$$4=\{0,1,2,3\}=\{\emptyset, \{\emptyset\}, \{\emptyset,\{\emptyset\}\}, \{\emptyset, \{\emptyset\}, \{\emptyset,\{\emptyset\}\}\}=\{0\cup\{1\}\cup\{0\cup\{1\}\}\cup\{0\cup\{1\}\cup\{0\cup\{1\}\}\}$$

Number 4 is built from 0 and 1 in binary system, and this can go to infinity following law of progression, that is, **the sum of preceding numbers by union operation**. In the centre of natural number parenthesis  $\emptyset$  is seen at first sight but this is illusive because in the central position the **unit class of null**, that is, **1** inheres.

A unit class is a class whose sole member is itself and denoted with  $\{\emptyset\}=1$ . Unit class is complete with its having its only member as a subset. Now that we have the unit class of numbers which is 1. Thus we could proceed from this and construct the entire class of natural numbers N. From now on Leibniz, in paralell with the process in construction of natural numbers could build up his Metaphysical edifice. And so

did he. Individual substance would be the unique unit class of substance and the successive substances be repetitions.

When the question is taken from an unusually different point of view, that is, free from ordinary way of thinking a possible idea may prove itself consistent. Thus, Leibniz contents that ultimate reality renders the power of an omnipotent mind which seems to take two courses which are in agreement in process, individual or particular mind in one hand and universal or eternal mind in the other. The former would depend on Sufficient Reason and the latter Necessary Reason. Sufficient reason would cause and govern the micro-cosmos on contingent truths and necessary reason would do so for macro-cosmos on necessary truths. However the universe is said to be full of monads and every monad is a world apart, that is, micro-cosmos, micro-cosmos is attributed vertically to man and macro-cosmos is referred to universe or, in some interpretations, God. Different may they seem but originate from the same intellectual root and be substituted for one another in due cases. So, the “idea” as the corollary of the “mind” recognized above, is applied as that which stands after all has gone! Leibniz who builds his metaphysics modelling the nature follows the footprints of Set Theory sticking to the basic principles of his philosophy.

Leibniz constructs his metaphysical system in five steps; primitive entelechy, primary matter or primitive passive power, the monad made up of these two, mass or secondary matter or organic body, an animal of corporeal substance around the dominating monad. In conceptual world it goes as emptiness-empty class, unit class of emptiness and N ary repetition of unit class through a mathematical law of progression.

The idea/mind; primitive entelechy.	The idea of class.
Primary matter-material basis.	Emptiness-conceptual basis.
Monad-Individual substance.	Empty class-Unit class.
Mass-secondary matter or phenomena by repetition.	2,...N: Repetition by the law of progression.
The animal around the dominating monad.	Mathematical modelling of a thing around a basic mathematical function.

The correspondence or analogy between the two course is obvious. Progression in conceptual world of Mathematics proceeds from idea of class, emptiness to empty class and unit class of empty class, that is, 1 and following 2 and up to the class of Natural Numbers and so on. On the other hand, in metaphysical world of existence emergence of things follows a course from primitive entelechy or idea-mind to primary matter or primitive passive power and monad-individual substance and by repetition to phenomena and so on. Emptiness's being at the ultimate origin of conceptual world must not be confusing. It's just a concept which may mean that which is not named yet or a concept to which no predicate has been attributed yet. It would have been, for instance, named as only "that" as is used by some philosophers. 'Emptiness' has just an intuitive advantage *per se* and the point is in thought anything is abstract and analytic, governed by sufficient reason on the basis of well foundedness! What you call it does not matter but just you call it and successive consistency matter. That's naming which designates a meaning to the object called and from now on this discussion goes to another domain of argument. So, it may be well to stop it here.

At the ultimate origin of existence there is "multitude in unity", for mind, matter, reason, meaning, activity and probability. Now, we have seen that individual substance is supposed to be the **least unit of existence** ultimately simple in quantity and infinitely complex in quality by Leibniz. This metaphysical entity functions as a differential element of mathematical integration operation to proceed from the least to the whole, thus, universe. Leibniz affirms this analogy, in our view, when he is saying that extension is repetition. There is only a unique individual substance the remaining is repetition though they are still individuals. This is the reason for Leibniz to believe in actual infinity; repetition appears as a multiplication sort of operation. And he says that 'instead of infinite number we ought to say that there are more than any number can express (G.II.304).'

### 3.4. Critical Philosophy of Leibniz with Russell as to Identity of Indiscernibles and the Law of Continuity<sup>1</sup>:

A quick overview of Leibniz's analyticity in his logical principles and well founded propositions tacitly suggests that the notion of identity of indiscernibles is immediate. But the foundations of the argument is ambiguous to Russell's view. Conflict arises from the inconsistency between premisses, principles and notions that apply to substance, and subsequent requirements of the principle of identity of indiscernibles. Reconciliation of subject predicate form and the difference between necessary and contingent truths with the principle of identity of indiscernibles is disputed and does not seem to be evaded without some damage to Leibniz. Russell is quite consistent and credible in his comments on the points of contact of the principle to the main course of Leibniz's metaphysics. With him critical points of the impact of the principle upon overall system will be examined to see the outcomes more clearly.

Now, the principle of The Identity of Indiscernibles and The Law of Continuity is given as 'all created substances form a series, in which every possible position intermediate between the first and last terms filled once and only once'<sup>2</sup>. The first part with 'is filled once' forms the Law of Continuity and the second part with 'is filled only once' makes the Identity of Indiscernibles. In fact, the meaning of the Identity of Indiscernibles is quite clear and asserted in the New Essays of Leibniz, thus, "that there are not in nature two indiscernible real absolute beings" (D.259; G. vii. 393) or again "no two substances are completely similar or differ solo numero"(G.vii. 433).<sup>3</sup> It applies to substances only. Leibniz admits particular attributes, to be indiscernible.

Discussion goes on over three arguments:

- Whether diversity is just of content,
- Is it just numerical or material, or

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<sup>1</sup> . Bertrand Russell-Philosophy of Leibniz Routledge Book 1997 CHAPTER V The Identity of Indiscernible and The Law of Continuity pp.54.para.23.

<sup>2</sup>.Bertrand Russell-Philosophy of Leibniz Routledge Book 1997 CHAPTER V The Identity of Indiscernible and The Law of Continuity pp.54.para.23.

<sup>3</sup> Bertrand Russell-Philosophy of Leibniz Routledge Book 1997 CHAPTER V The Identity of Indiscernible and The Law of Continuity pp.54.para.23.

- Does it follow from the diversity of both content and numerical.

‘Diversity of content proper is the difference between one content and another of the same subject. Material or numerical diversity is the difference between one subject or one substance and another. Leibniz’s doctrine is that two things which are materially diverse, i.e. two different substances, always differ also as to their predicates’.<sup>1</sup>

Diversity certainly would come from the difference in predicates. Because material diversity could also be reduced into predicates. Leibniz says that if two things are materially diverse that entails the diversity of predicates so the doctrine is that any two substances differ as to their predicates. Now, the question is that if the principle is necessary or contingent in this form. The answer presupposes a knowledge of substance and calls for the definition of substance.

The definition of substance was given in due detail in the foregoing paragraph. What is strictly relevant to this discussion in that definition is the substance’s being the subject of change. And difficulty in comprehension of the nature of substance gets tougher at this point. Because, as, again, is stated above, Leibniz emphasizes namely two attributes of the substance, being defined by its predicates and yet being not the collection or sum of them. To our understanding this is the point that makes the discussion so controversial in Russell’s view. To keep substance beyond phenomena Leibniz seems obliged to maintain that substance is not an idea or a predicate or a collection of predicate but rather the subject of change. Because, that would give him what persists through the time. Russell tends to propose to substitute the law of progression for substance and subject predicate form. But, it’s immediately clear that one needs a material basis to house all those predicates, activity, law of progression and infinitely complex make up of substance to reflect the universe within. It may be well to remind that this is just an illustration to make an easier consideration of it otherwise any false idea of substance with parts or divisibility or whatever not attributable to a substance is excluded. Having conceived that Leibniz maintains a definition with visible and invisible features Russell urges

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<sup>1</sup> . Bertrand Russell-*Philosophy of Leibniz* Routledge Book 1997 CHAPTER V *The Identity of Indiscernible and The Law of Continuity* pp.55.para.23

him to give a full definition of his substance without excluding any property of it. And says that if you hold that a substance is defined by its predicates then you must admit the identity of predicates with their subject, that is, substance or if you say that substance is not the sum of its predicates and so do you, then what is the true and comprehensive definition of substance emerging through your premisses, axioms or propositions confounding with your metaphysics.

Those questions are truly legitimate and well founded from Russell's point of view. But, there may be an idea of different perspective. They can be both taken with their context and be tried to understand in their mind set. Given Russell is an early logical positivist he is assumed to take any subject from factual point of view. Whereas Leibniz as an intellectual rationalist would see the question rather introspectively and from an individual point of view. However they seem to state what they have in mind in apparently similar concepts of closer terminology they might not saying virtually the same thing when seemingly so doing. This would be the position we assume to take in this disputed discussion.

So, let's remember the major attributes of Leibniz's substance deduced from its definition. Individuality, emerging through subject-predicate form, thus, being subject but not predicate; defined by its predicates but not being the sum of its predicates; being subject of change and persistence through time. When the discussion is put on this frame then the positions of two great men become more explicit. Thus, in our view Russell is right synthetically and Leibniz is true analytically. That is, from contingent truth's point of view Russell seems infallible but from necessary truth's perspective Leibniz seems consistent. As to our opinion, analytic truth appears to be the language of eternity and would prove capable of explaining the contingent in the end. Not just for a reason to believe we would favor Leibniz's position, it's also for it's being quite tenable. Thus, when Leibniz is giving the substance as that which is defined by its predicates but not being the sum of them, which seems the most problematic assertion of all, he is also giving what Russell seeks to find, that is, phenomena in the form of predicates. But, to our



understanding it cannot consist of more than 1/7 of the whole reality as a ratio followed by the nature. Yet it's a true representative of what's behind. So, one can analyze what has been let by the substance as predicative phenomena and be acquainted with the nature of the substance without being content with the facts deduced from definitions. That's what science does in the realm of physical aggregate. Russell does not seem right when he says that if the subject is wholly defined by its predicates no room remains for substance in its definition, or if not there must be something other than predicates involved in determining the substance.<sup>1</sup> Because, substance is still there behind the phenomena as the subject of change. A substance is wholly defined when all its predicates are numerated. That may appear true but the numeration process is not finite. Because, the substance, qualitatively, is in a perpetual change resulting in successive states yielding predicates and proceeding to infinity. This unending and activity driven process inheres necessarily in the substance as the subject of change. Every individual substance is unique on the basis of sufficient reason which furnishes it with individuality and self sufficiency, thus uniqueness! Given what all he had said Leibniz implies that validity of inference on a substance applies only to present and past states of it, and future is still to be fully at its decree! That involves probability and presupposes the stochastic reality of Quantum Mechanics. So, all one can validly say about a substance applies to past and immediate present. Future remains intact and beyond, consisted in the substance as the subject of change! Thus, this sort of supposition of substance by Leibniz seems to hold substance more profoundly, appealing past, present and future states of it.

On the other hand Russell should have known that given a substance is ultimately simple it would be indefinable, indivisible, irreducible and identical a notion and only a complex term could be given a full definition not a simple substance. Thus, as Russell rightously concludes substance does not fit into words and seems indefinable. One can only show it, if any. And perhaps that moment would be the

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<sup>1</sup> *Bertrand Russell-Philosophy of Leibniz Routledge Book 1997 CHAPTER V The Identity of Indiscernible and The Law of Continuity pp.58.para.24*

instant when Wittgenstein was urged to say “*Whereof one cannot speak, thereof one must be silent*”-Tractatus. Most probably this would be the best possible definition of substance, namely, that which can be given no definition, hence, meaninglessness, thus, emptiness, therefore, as was explained in detail in foregoing paragraphs, the ground of **empty class**.

Which is more, Leibniz would counter with the idea that if predicates were capable of defining the complete substance or if the substance could be given in a definition by predicates then the entire actions of an individual substance, past, present and future, could be inferred from the general propositions about it. This would contradict the Principle of Sufficient Reason and prove impossible. As opposed to Russell’s idea that predicates inhere in the substance in the same sense as letters inhere in the alphabet, it may seem a true analogy or metaphor at first sight but unlikely because the number of letters in any alphabets is finite whereas fusion capacity of a substance is infinite. That underlines the profound difference between the two! That’s why a substance cannot be reduced into its predicates in Leibniz’s sense. Russell should have perceived that the activity in a substance is an infinitely complex concurrent process which, in some sense, may be named as the process of being, its divinely orderly and optimal, that is, it’s a perfect event which makes the subject a complete being, that is, self sufficient in any respect to sustain its being. So, the only primacy seems the existence of substance the rest is the life processes of substance as an animate thing.

When we come to its relevance to the Identity of Indiscernibles, it’s due to predicates, the phenomena charged with identity or diversity of subjects. Thus, a subject is defined by its predicates as well as differed. It can validly be said that two indiscernibly identical things seem intuitively impossible and not rational. It can also be given such a reason that a fashion magazine editor would never, except mistakenly, place the same design picture in the magazine twice. It would be contrary to the main course of action which may be summarized as intensity of reader’s interest, efficient use of content composition and page capacity of the issue or whatever. It seems also true of nature on sufficient reason. Another reference can

also be given as to time space concurrency of two objects. In that, Leibniz means not only difference of place but difference as to the predicates to which place must be reduced. Bearing in mind that “every extrinsic denomination-i.e. every relation- has an intrinsic foundation, i.e. a corresponding predicate”(G. II. 240) he says that, besides the spatio temporal difference there must be an internal principle of distinction, for places and times are distinguished by means of things not vice versa. Given that every spatio temporal coordinate can be reduced into predicates and vice versa., and that every possible position is filled once and only once then it can safely be asserted that there would be no two subjects indiscernibly identical in the universe. Leibniz says that two indiscernibly identical objects are not even conceivable and if they were, nature would conceive it and not bring one of them into being. This principle proves true down to the quantal domain of sub atomic particles.

Russell finds difficult to employ the principle of Identity of Indiscernibles for it appears in various forms as to the domain of propositions. Sometimes it occurs as contingent and as metaphysically necessary at other times. This requires a careful interpretation to evade inconsistency. For instance Leibniz finds the principle possible in abstract terms whereas in the order of things it seems inconsistent he says.

Hence, in his view, to suppose two drops of water perfectly alike is possible provided that its being contrary to the divine wisdom be admitted. Russell says that the confusion is between eternal truths about the contingent which involves necessary propositions about the natures of substance and contingent truth about the existence of substance. He maintains that truths neither about possible nor actual worlds cannot be contingent. But he does not expressly assert that they are all necessary. So, his judgement is still ambiguous. But, from this one can hold that ‘contingency consists in phenomena the principle behind remains necessary’ may follow. The Identity of Indiscernibles is usually formulated as follows: if, for every property  $F$ , object  $x$  has  $F$  if and only if object  $y$  has  $F$ , then  $x$  is identical to  $y$ . Or in the notation of symbolic logic:

$$\forall F(Fx \leftrightarrow Fy) \rightarrow x=y.$$

Now, **the Law of Continuity**<sup>1</sup> holding a prominent place in Leibniz's expositions and related with Identity of Indiscernibles asserts that every place in the series is filled whereas the latter asserts that no place is filled twice over. The latter is logically prior to former and metaphysically necessary whereas the former is contingent. Leibniz gives three kinds of continuity;

- Spatio temporal continuity,
- Continuity of cases,
- Continuity of actual existents or of forms.

Spatio temporal continuity is twofold. It involves continuity of space and time and that of existents in space and time. That sort of continuity involves a gradual change in spatio temporal coordinates. Continuity of cases is properly a mathematical principle and asserts that when the difference of two successive cases diminishes without limit the difference in their results also diminishes without limit. Continuity of forms asserts that nature makes no leaps and every change is gradual. Therefore, Identity of Indiscernibles is what continuum rests upon.

When Russell comments on continuity as Leibniz puts it, he seems not perceived the profound effect of it on the existence. He says, "In Mathematics,...it's still in constant use. But in philosophy it seems of no very great moment"<sup>2</sup> and "Why Leibniz held that substances form a continuous series it is difficult to say"<sup>3</sup>.

Continuum seems to be analyticity and what makes an idea or a metaphysical system possible. In this sense, it is of the greatest moment. W.V.Quine also recognizes it momentous by his appeal to continuum between synthetic and analytic propositions. But, in progressive discussion of the subject he seems acquainted with the profound effect of continuum on the Leibniz's whole deduction of his system. Thus, he says that 'we must agree therefore, that Leibniz's view as to infinity(tacitly continuity) are by no means so simple or so naïve as is often supposed'.

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<sup>1</sup> *Bertrand Russell-Philosophy of Leibniz Routledge Book 1997 CHAPTER V The Identity of Indiscernible and The Law of Continuity pp.63.para.27.*

<sup>2</sup> *Bertrand Russell-Philosophy of Leibniz Routledge Book 1997 CHAPTER V The Identity of Indiscernible and The Law of Continuity pp.64.para.27(2).*

<sup>3</sup> *Bertrand Russell-Philosophy of Leibniz Routledge Book 1997 CHAPTER V The Identity of Indiscernible and The Law of Continuity pp.65.para.28.*



Leibniz admits only space, time and numbers-not the class of natural numbers but only the number one, to be continuous and these are purely ideal in Leibniz's sense. He presupposes the true infinity lies in continuum. Because, the true infinity is also ideal, that is, conceptual not real. According to him, 'there are two sorts of indivisibles, namely simple ideas and single substances'<sup>1</sup>. The confusion of ideal and the actual causes the labyrinth of continuum. And in ideals, numbers, space and time, the whole is prior to the part. Whereas in actuals, the simple/part is prior to the aggregate. There is a distinction between intensive and extensive quantities. 'Extensive quantities presupposes all the constituents whose sum they are; intensive quantities on the contrary, do not in any way presuppose the existence of smaller quantities of the same kind. The spatial continuum is the assemblage of all possible distances'.<sup>2</sup> Difference of spatial position at the same time means at least two different substances but difference of temporal positions means only two different predicates of the same substance. Thus 'the time order consists of relations between predicates, but the space order holds between substances'<sup>3</sup>. 'Composed of' and 'results from' is different in consequent such that, though both introduce plurality in the former whole is prior to part and in the latter the part is prior to the whole.

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<sup>1</sup> Bertrand Russell-*Philosophy of Leibniz* Routledge Book 1997 CHAPTER IX *The Labyrinth of Continuum* pp.111.para.59.

<sup>2</sup> Bertrand Russell-*Philosophy of Leibniz* Routledge Book 1997 CHAPTER IX *The Labyrinth of Continuum* pp.114.para.61.

<sup>3</sup> Bertrand Russell-*Philosophy of Leibniz* Routledge Book 1997 CHAPTER X *The Theory of Space and Time* pp.128.para.73.



**Figure2: Medallion commemorating Leibniz's discovery of binary arithmetic:**

Source: G J Chaitin Home Page <http://www.cs.auckland.ac.nz/~chaitin/>

On January 2nd, 1697, Leibniz wrote a letter to Rudolf August, Duke of Braunschweig-Wolfenbüttel, in which he detailed the design of a commemorative coin or medallion which he suggested could be minted in silver. The design he described posited an analogy between “the creation of all from nothing through the omnipotence of God” and the fact that “all numbers (could) be created from zeros and one’s”.

### 3.5. Possibility and Compossibility<sup>1</sup>:

In Leibniz's system possible is wider than actual. An idea or thing is possible when it's not self-contradictory. Two or more things are compossible when they belong to one and the same possible world. All possible worlds have their own laws analogous to those of actual world. Things which cannot conform to the same set of general laws are not compossible.

There are infinite number of possible worlds free from self contradiction. When several notions of possible existents form part of the notion of one the same possible world they are compossible for in this case they may all exist.

Compossibility of predicates in parallel causal series may cause some confusion and must be posited carefully. Predicates of two different causal series do not interact between themselves. But, every series of predicates is necessarily connected to their subjects. Thus, **given a subject each predicate follows.**

In this connection Leibniz gives several kinds of necessity;

-Metaphysical or geometrical<sup>2</sup> necessity of which the opposite is self-contradictory.

-Hypothetical necessity.

-Moral necessity.

It is deemed sufficient to give head lines for the purpose of this essay.

### 3.6. A Critical Analysis of Russell Paradox:

In his all objections to the propositions either of Set Theory or Metaphysics Russell seems to pay less attention to what makes a relation a law or if Ockham's razor is in effect in his explanations of the matter. Rather, he seems to enlarge the number of constraints the author of the material felt himself bound with. This caused some 'minor in appearance but major in essence' distractions from the main course

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<sup>1</sup> Bertrand Russell-Philosophy of Leibniz Routledge Book 1997 CHAPTER V The Identity of Indiscernible and The Law of Continuity pp.66.para.29.

<sup>2</sup> All geometrical proposition are deemed necessary on the assumption that geometry consists of universal elements of the realms of the forms.



of argument. Sometimes it amounted to a drift on the ground of debate and most of the time, two philosophers did not seem talking the same thing.

A more positive and contributing attitude could have been adopted by comprehension and assimilation, without any prejudice, if possible, of the true position and insight of the 'author' and with this in mind criticising the builds from bottom up to top down. In this case, the result would be highly appreciable and more respectable a contribution in every respect. His position, as it seems, has always been explanatory not resolatory. Hence, in his *Principia Mathematica* what appears is just a picturesque animation of individuals, species, generas etc. which results in Theoy of Types, an infinitely complicated and somewhat obscure account of class member relations. His Theory of Description is also tending to expand in prose so as the main core of the subject, as a word of one name, to fade out, in the end. Consequently you are given a tool that serves you to climb the roof but you are forced to take many times as much longer a way to go just like in the case of a pulley, not an efficient engineering. It offers no gain in the end. Leibniz would disagree with that. Because, if there is no condensation to simplicity in theory and expansion to complexity in application then the theory is vacuous and there is no law at all. Because, then it comes to the statement of the same proposition in another notation namely symbolic logic or mathematics. This does not mean a law stepping forward nor is a theory. From this, it can validly be said that description in Leibniz and Russell is disputed in form and meaning such that, it, in Leibniz is analysis into indefinable simple ideas whereas in Russell an explanation in finitely many words. In other words description in Leibniz is in **verse** but in Russell in **prose**.

As it seems, Russell generally shows a strong criticism but a weak analyticity in his comments. Apparently, he stricts himself into abstract concrete distinction and the relations in between and commenting within this boundaries. His analyticity is restricted to this domain and a pervasive analyticity is hard to find throughout his comments on Philosophy of Leibniz.

As to subject-predicate relation, Russell, argues that the treatment of subject and predicate is itself relational as in the propositions, 'this is red' and 'red is a colour' assigning 'red' as object in the first and subject in the second and exemplifying the 'use v.s mention' of subject or predicate. Mr. Russell always follows the same route to let his opponents into a contradiction, hence, paradox, such that, he first, conceives the world described and the relations between the objects of discussion in this world and then adjuncts the said world to the class of objects supposing it an arbitrary member of the world- a class in Set Theory- concerned and in the last analysis, because the world, that is, class does not exist in the same habitat as its constituent objects, he has his opponent faced with a fierce contradiction, paradox.

Now, in his criticism of Leibniz we see the same attacking strategy. But this time it is not as successful as it was with Frege. Because, Frege did not foresee vulnerability of his naïve class definition  $\{x:Fx\}$  to paradoxes and faced with an example of the fatal attack of this type, unsatisfiable membership condition  $x \notin x$ . He had to exclude this type of impurities from his system to hold it consistent. Now, the same maneuver is in use for the betweenness relation abstracted from greater than and less than relations of two magnitudes, lengths of L and M lines. Briefly, Mr. Russell says that, however Leibniz maintains that a mere subject-predicate logic suffices to express all propositions, subject-predicate logic is not always applicable to all abstract relations whose subject and predicate are not definite as in the ratio or proportion of M and N. This, initially may seem a formative detail in general which does not hurt the whole philosophy and the logic behind, nevertheless it may require to look over the matter from a new perspective much as a normative objection attempt. Within the scope of this thesis it may suffice to underline the fact that any such deficiency of dead end may lead to an unexpected paradox and prove the system to be inconsistent.

Mr. Russell's stress test under paradoxically adverse, if not alien, conditions uses a similar token just as in the case of Russell Paradox, that is, penetrates a subversive hence self contradictory agent into the herd of objects which peacefully set membership relations to the class in question and undermines the footings of the

whole idea essentially aimed for the good and the happiness of the community. But Russell shows that Reality can occur in a very diverse manner. Now, as is usual, he formulates an idea essentially of the notion of second order relations and inquires for its consistency in the first order relations. In other words, Russell takes a proposition from species-genera relation and wants one to put it in individual-species relation to see if it acts consistently and harmoniously. Most of the time he shows that his addressees are confused the ideal with the actual or he so posits themselves deliberately to show that their systems allow him to formulate such formulae to which the system is not immune. That's, as Leibniz pointed out, the confusion of ideals with actuals and an express contradiction. So, however Russell seems to multiply the folds of knot in his Theory of Description he definitely shows the way to a more consistent, robust and non-contradictory systems in his Theory of Types. Leibniz, certainly, tries to unfold the problem to simple in his descriptions following the rule that 'ultimately simple in idea and infinitely complex in phenomena'. However he left no Theory of Sets but if he had had one, owing to his strong analyticity, most possibly, it would be consistent and free of paradoxes.

“To say that nothing fulfills a given condition, *in class-identity relation* is to say that everything fulfills the denial of that condition”<sup>1</sup> says W.V. Quine in his *Mathematical Logic*. On the other hand, to say that there are objects satisfying a given condition, is, to deny that there are none! Quantificationally, that amounts to;  $\forall x \sim (x \neq x)$  for the above first case, and given,  $F =$  a is black and  $G =$  a is swan, then;  $\sim \forall x \sim (Fx \wedge Gx)$  for the second. Thus, nothing fulfills  $\sim (x = x)$  condition and everything fulfills the denial of that condition,  $\sim (x \neq x)$ , i.e.,  $(x = x)$ . The first gives null class,  $\emptyset = \{x : x \neq x\}$ , and the second, universal class  $U = \{x : x = x\}$ . Consider then the following;

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<sup>1</sup> W.V. Quine-*Mathematical Logic*; Cambridge-Harvard University Press; 1961, pp.101, para 19, *Existential Quantification*.

$$(1) U = \{x:x=x\}$$

$$(2) \emptyset = \{x:x \neq x\}$$

$$(3) \forall x \sim(x \neq x)$$

Thus, if any class membership condition, even indirectly, refers to any of the above three (the first two being definitions and the last a theorem) then it can not be counted as legitimate because all three are implicitly giving false **reference to existence**. Because, as Kant said ‘exist’ is not a predicate. And existence has no distinctive power among contingents and because it is only attributable to God.

But, Russellian class condition refers to objects not capable of membership relation to its own, that is, regardless of any predicative properties, everything. There is only one property common to all enough to subsume the whole universe. Existence! Existence, *per se*, on his own. Thus, under universal quantification, denial of it implies null class, and recognition signifies universe. And it is not predicable to contingents. Phenomena are of predicative and of contingents, whereas, existence is of primary substance and constitutes an existential basis for phenomena when attributed. So, phenomena is eligible to set a membership condition to a class from the aggregate of entities, not existence directly or indirectly. For Russellian class condition gives an indirect reference to existence, it’s not eligible to form a class. Because, for Kant ‘existence is not a predicate’! It’s not the largeness that causes the contradiction and hence paradox in Russell class, it’s the controversial nature of it that conflicts with the entire texture of conceivability.

In the above mentioned statements of identity regardless of their being a class membership condition in a class abstract or a hypothesis in theorem, the necessary truth is that nothing is false to its nature,  $x=x$ . But in the case of membership relation of an entity to a class, the necessary truth is that a class can never set a membership relation to its own. Because, a class is categorically different in nature from its members, if such an assertion is claimed to be true then, the fact that a thing is assumed to be so and otherwise at the same time is implied. And this is contrary to

the fact that a thing is never false to its nature i.e.  $a=a$ . So the Russellian class condition  $\{x: x \notin x\}$  is self contradictory. But, this is not due to Russell. It is the weakness of system(s) which allow(s) him to set such statements through them. Recognizing the paradoxical threat, Russell sets the Theory of Types to ban this contradictory agent from his system.

Now, every proposition is ultimately reducible to one which attributes a predicate to a subject. From this, every predicate, except existence, is ascribable as a membership condition follows. All predicates, except existence, are contained in the notion of the subject. Therefore, their assertion of the subject is analytic whereas the assertion of existence is contingent, except of God. Consequently, it can be asserted that, existence is not a predicate for actuals at all. Now, if it's not a predicate at all it can not legitimately given as a membership condition to a class. So, the fatal condition  $x \notin x$  in the class abstract that gives notorious Russell Paradox is not a valid condition. I think, Leibniz would admit that.

Having reached the foregoing resolution he would proceed to time-space relational identity and denial of divisibility of ideals including infinity in numbers. The conceptual details of the discussion shall be given in progressive sections.

Nevertheless, having this in mind, Bertrand Russell, in his attempts of this sense, deserves much credit, since he spent every effort, resulting in the Typed Set Theory, to avoid paradoxes . The trick is that Russell banished setting membership relations from the other types as a constituting principle of his own theory. Therefore, it was not possible to deliver the same attack to his system by the same token. Whereas the other set theories, except ZF Set Theory with the Axiom of Choice and Quine's NFU (New Foundations with Urelements), were wide open to the attacks of that nature. Thus, Russell's strenuous works, not to mention, is highly respectable and, in the final analysis, aims to broaden the domain of consistency in the development of Axiomatic Set Theory.

### 3.7. Subject-Predicate Logic As The Foundations Of Universal Characteristic:

Since a meaning comes from a predicate that identifies its own subject; a meaningless substance is that which has no predicate. If one states this in subject-predicate logic grammar following the rule ‘given a predicate find its all subjects’ one finds just the emptiness, because, if no predicate then no subject, hence, the result is emptiness, that is, empty class. Recollecting that, by definition of a priori truth, Leibniz means the decomposition of a term down to its identicals/indivisibles/irreducibles/indefinables, that is, its self evident root. Given self evident primitivity, meaninglessness necessarily follows. In any case, even it is in ultimate primitivity, its abstractness as being the name of the object and not the object itself holds involving ‘use versus mention’ distinction.

This introduces the very essence of the origin of things in metaphysical world, corresponding to a universally quantified variable or a class abstract with no membership condition in conceptual world of Set Theory, hence, Mathematics. The first would lead Leibniz to his metaphysical edifice of monadic world and the second to Universal Characteristic or Infinitesimal Calculus. This signifies the etymological roots of the word universe. Hence, ‘uni’ and ‘verse’ which may hold the creation of multitude from the unity and inreturn, explanation of complexity in simplicity.

So, that which is before anything exists is emptiness, empty class, that is where it all begins. Just like the notion of substance, emptiness can not be defined without a relation to an existent. Thus, the ultimate origin of things is the empty-null class, though, as is pointed out above, there is a quantificational difference between null and empty class notions. This may also be the very reason of the truth value of conditional connective in propositional logic that, where antecedent is 0-False then the conclusion is always 1-True, regardless of the truth value of consequent, that is,  $0 \rightarrow p \equiv 1$ .

The class of natural numbers in mathematics forms a causal series initiating with empty class- $\emptyset$ . In causal series given the first term and law of causation the remaining terms follow.

Now it's time to give Peano construction of natural numbers.

$$0 = \{ \}$$

$$1 = \{0\} = \{ \{ \} \}$$

$$2 = \{0,1\} = \{0, \{0\}\} = \{ \{ \}, \{ \{ \} \} \}$$

$$3 = \{0,1,2\} = \{0, \{0\}, \{0, \{0\}\}\} = \{ \{ \}, \{ \{ \} \}, \{ \{ \}, \{ \{ \} \} \} \}$$

Empty class,  $\emptyset$ , which stands for the number 0-zero has no members at all. But, 1 has a member, though its member is also an empty class. But, the point is that the class of the empty class which gives the number 1 is not the object null, 'it' is the name of the object null that counts as an object, in other words it is not the use of (first order) null, it is the mention of (second order) null. In this respect, empty class remains in the realm of necessary truths as the only class, that is, the idea of class, and universe class forms a mere particular instance of empty class,  $\{ \}$  with one possible world already actualized.

In saying "a simple substance is ultimately simple in quantity and infinitely complex in quality" Leibniz, in some sense, is giving a definition of Mathematical Limit Operation, minimizing the quantity down to  $\emptyset$  and maximizing the quality up to U. These two properties are unified in the notion of simple substance, that is, monad. In this sense, the identity of empty class to universe class is made into assertable. Hence,  $\emptyset=U$ , which represents simplicity in quantity and complexity in quality of a monad.

Some say that the analysis of necessary truths comes to an end with primitive notions whereas the analysis of contingents goes to infinity. These two are not comparable and analysis of contingents is going nowhere. This is a mere confusion of universal with particular or necessary with contingent. Because, analysis and synthesis of all that happens are completed in and by the necessary reason once for all on the basis of sufficient reason the rest is just the variety or diversity of complexity in/of particulars. This being so must not be that surprising because that's what axiomatic method is all about, ultimately simple/primitive in the idea and infinitely complex/diverse in the world. And the Law of Sufficient Reason suffices to uncover

the principle followed or conformed. The diversity which seems to go to infinity is a mere particular instances of the necessary truth of the principle of sufficient reason. Before passing through; a first order-simple-term, particular or universal, in class abstract form, proves to be impossible except a paradoxical field, domain- range-relation, is introduced due to incommensurability as in Russell Paradox. Hence, { } denotes a second order term, mention of nullness, in species-individual relation whereas, null as member of this,{{ }}, is a third order term in species-genera relation.

In essence, a membership relation between particular and universal or individual and class, by definition, proves impossible, hence, individuals are ignored not only for simplicity and elegance but also for commensurability of relations between the terms of class theory. So, in class theory, anything is a class and we are discussing classes and only classes in the **field**.

The strict correlation and immediate analogy between properties and activities of simple substances or monads in the process of producing phenomena and the emergence of natural numbers from empty class can evidently and naturally be seen from the following mappings;

Simple substance or monad is the ultimate unit of existence in nature.	Empty class is the ultimate unit of Natural Numbers in Mathematics.
It has no parts,	Empty class has no parts,
It must exist,	It must exist as a putative set/class at the origin of natural numbers, hence, Set $\emptyset$
Lacks extension,	Lacks extension,
Figure,	Figure,
Divisibility.	Divisibility.
Individual substance.	Unit class of empty class.
An individual substance is that which can only be the subject but not the predicate of anything.	Proper class, namely, that which can only be the class of things but not the member of anything.

Every monad and its repetition through one casual, time, series gives an independent class of natural numbers.



Given that an individual substance is that which can only be the subject but not the predicate of anything in monadic metaphysics then, it strongly correlates with the definition of proper class, namely, that which can only be the class of things but not the member of anything in set/class domain.

An individual substance in Leibnizian metaphysics also correlates with the definition of unit class in set/class domain having in mind that a proper class can also be seen as the unit class of its species or genera. Thus, the unit class of empty class, which is '1',  $\{\{\}\} = \{\emptyset\} = 1$ , is the class of all number '1's as the isomorphic class of all '1's.

So, Leibniz's metaphysics seems to correlate with set theory in the notion of subject-predicate logic. Thus, an individual substance is a complete subject on the ground that every predicate of it is included in it, as is the fact that a set is said to be complete if and only if every member of it is included in it. The universe is full of substances. Every simple substance/monad mirrors the universe from its own perspective, that is, from its own point of view. A monad's point of view forms a mathematical point in space. And, every simple substance has a different perception proportional to its distinctive power and every individual substance is complete on the ground that every predicate of it is included in it. In this sense every natural number is complete since every unit number of it which totals up the number itself is also included in itself as in the example of unit class. As is known, a unit class is that whose only member is itself. Given that every class is included in itself<sup>1</sup>, then; a unit class is complete and on this ground, a proper class is complete.  $U$  is proper  $\text{Prop } U$ , thus, complete, hence;

$$\forall x(x \in U \leftrightarrow \text{Set } x) \text{ and } \forall x(x \subseteq U)^2$$

Inclusion operation proceeds from inclusion in itself to inclusion in upper aggregations i.e. 1 is included in 1 first and then 2 and 3 successively... This correlates with causal series of predicates and time. Thus, they are included in future

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<sup>1</sup> Lemmon, E.J., (1968). *Introduction to Set Theory* pp.30. Chapter I The General Theory of Classes T44:  $\forall x(x \subseteq x)$

<sup>2</sup> Lemmon, E.J., (1968). *Introduction to Set Theory* pp.30. Chapter I The General Theory of Classes T36 and T48

occurrences of phenomena. Taking the ultimate origin of things as ‘zero’ point, ‘now’ proves older than ‘then’. This signifies a consistency with the expansion of universe.

To make ideal actual distinction; substances do not interact and every substance is a world apart. Set operation is ideal, that is, mental, and can only be seen as the progression law of the activities of substances, hence, they are conceptual not actual and actual world pervades contingently according to the Law of Physics, causal laws, which are explicable mathematically by set operations.

Since a further analysis would directly involve the field of Number Theory, it may suffice to stop the discussion at this point.

The whole idea is that all truths can be deduced from a small number of simple truths, by an analysis of the concepts, terms, ideas or numbers, they contain, and that from these in turn, all ideas can be reduced, by decomposition, to a small number of primitive, undefinable identical ideas. This is a genuine definition of Axiomatic Method applicable both necessary and contingent truths such that, the sufficient reason for necessary truths can be found by a finite analysis that ends in a self-evident, that is, identical propositions. But to provide sufficient reason for contingent truths by such an analysis is infinite and of unending complexity. For necessary truths; this is a three level categorization of truth, simplifying and purifying the cogency of reality, namely that; primitive, undefinable and irreducible ideas what Leibniz calls identicals would stand at the utmost bottom of the conceptual world and be named as first order terms, the following layers would be derivable from the preceding notions as second, third and successive orders as the combinations of the fundamental notions. In this way, one would construct the alphabet of human thoughts, and all derivative notions would be no more than combinations of the fundamental notions, just as the words and sentences of speech are no more than combinations, indefinitely varied, of the 26, finite number of, letters of the alphabet.

Application of that idea to logic led Leibniz to develop, the new branch of mathematics known today as “*combinatorics*, deeply Axiomatic in concept and of *two-based* mathematics in operation.”

To Leibniz the root of the logic of invention was based on a fundamental problem such that: "Given a subject, find all its possible predicates, *system analysis*; given a predicate, find all its possible subjects, *progressive system synthesis*." In other words, find all the *true* propositions which contains a concept, as a subject or as a predicate (for two reasons; either for hypotheticality in form or for true concepts with correct connectives). As a proposition is a combination of two terms, a subject and a predicate, the problem turns out to be combinational. Here Leibniz was inspired by the example of Ramon Lull, who in order to find all the possible propositions among nine terms taken two by two is 36, or  $(9 \times 8) / (1 \times 2)$ , from the formula;  $C = \frac{9!}{(9-2)!2!}$  uses a nonagonal geometry in a circle<sup>1</sup>.

Having analyzed every concept by defining it, that is, by reducing it to a combination of simpler <sup>2</sup> concepts, a certain number of absolutely simple concepts, primitive enough to be irreducible and undefinable are obtained. As is said above these will be the first order terms and the second order terms will set a second class obtained by combining in pairs those of the first; then in a third class the terms of the third, obtained by combining in triples the terms of the first, and so on. Each term, derived from simple terms, will be referred by a numeral which is the (symbolic) product of the corresponding marks or numerals. That numeral will constitute the correct definition of the term thus formed. For simplicity in the notation as well as in the definitions, each term following the second order terms can be formed variously so that sum of the simplers would amount to the class number of the term at hand. Terms of an order higher than the first order would be represented by a rational number such that, numerator would be the number of the term/denominator would be

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<sup>1</sup> A definition of a concept, as particularly understood by Leibniz, is an analysis by reducing it to a combination of simpler concepts, that is, identifying all its basic components which enter into its composition. Given that a set is an aggregate of things under a common property; this concept of definition is in an exact agreement with the description of set, namely that, a set is determined by its members, that is, constituents.

<sup>2</sup> The idea about Combinatorics is simplified and generalized from the Texts and Translations by Donald Rutherford-G.W.Leibniz: *The Logic of Leibniz*; Chapter 3 *The Universal Language*.  
<http://philosophyfaculty.ucsd.edu/faculty/rutherford/Leibniz/ch3.htm>

the number of the order. Hence 1/2 would designate the first term of the second class, 2/3 the second term of the third class, and so on.

As can be well understood, the same complex term will be expressible in several ways, according to how one combines the simple terms that enter into it as factors. Identity of the complex term, thus expressed, could be verified by reducing it to simple terms and thereby to recover the primitive definition of the complex term in question. This operation is analogous to the decomposition of numbers into prime factors, which proves that all correct reasoning is a correct calculation. After all, one can find all the (logically) possible predicates of a given subject. They are all the factors or divisors of the given term. Because, they express each and every one of the characteristics or qualities of its **intension** and define it. One can therefore attribute to a given subject, first, each of its prime factors, and then each of the combinations formed from these.

One thus discovers all the possible predicates of a given subject, from the simplest up to the subject itself which constitutes the subject of the proposition whose predicate is the product of all its simple terms and can be considered its proper definition, that is, an identity. Avoiding any technicality of second importance, to say that the number of possible predicates can be calculated by a formula:  $2^k - 1$  in brief, where  $k$  is the number of simple terms (prime factors) that enter into the definition (or formula) of a given term. To be more illustrative let's give a numerical example;

Let's suppose that our subject is some whose identity is 30030 in numeral.

We would obtain  $\{2, 3, 5, 7, 11, 13\}$  if we decomposed it into its prime factors.

So, the class of first order terms:  $\{2, 3, 5, 7, 11, 13\}$  according to formula:  $2^k - 1$  we would find 63.

According to the formula  $C = \frac{n!}{(n-r)!r!}$  where  $n=6$  (number of prime factors which

gives the **class number** of the subject), one can find all combinations of first order terms combining them 1 by 1, 2 by 2, 3 by 3 and so on as the value of  $r$ . Thus;

$r_1=1$	$C_1=6$	number of first order terms	irreducibly simple terms
$r_2=2$	$C_2=15$	number of second order terms	complex terms
$r_3=3$	$C_3=20$	number of third order terms	complex terms
$r_4=4$	$C_4=15$	number of fourth order terms	complex terms
$r_5=5$	$C_5=6$	number of fifth order terms	complex terms
$r_6=6$	$C_6=1$	number of sixth order terms	complex terms <sup>1</sup>

So;  $C = \sum_{i=0}^{i=6} C_i = 63$  hence;  $2^k - 1 = 2^6 - 1 = 64 - 1 = 63$ .

In other words; the number of possible predicates of the subject 30030, combining first order terms 1 by 1, 2 by 2, 3 by 3 and so on, would be 63. That detail may suffice for the purpose of this dissertation. The inverse of the preceding problem: given a term, find all its possible subjects could be calculated by the formula;  $2^{n-k}$  where  $k$  is the number of prime factors of the given combination and  $n$  the total number of simple terms.

In this context, the number of particular predicates of a given term is defined as that which can be attributed to the term in a particular proposition. A particular proposition is derivable from a universal proposition by subalternation, that is, predicate from predicate or by conversion, predicate from subject. Thus, the number of particular predicates of a given term is equal to the sum of its universal predicates and universal subjects which means to say that the number of particular subjects is equal to that of particular predicates. As a result, every particular affirmative subject of a given term is also a particular affirmative predicate of it.

Leibniz also considers negative propositions from a point of view analogous to that of previous paragraph and additionally, searches for the number of arguments (syllogisms) but since further details of these problems may be far beyond of this essay it will suffice just to touch with in name.<sup>2</sup>

In sum, Universal Characteristic would not be a kind of disguised arithmetic whose use would require constant mental calculation, instead it would be a genuine

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<sup>1</sup> The idea about Combinatorics at this page, except the specific example, is simplified and generalized from the Texts and Translations by Donald Rutherford-G.W.Leibniz: *The Logic of Leibniz; Chapter 3 The Universal Language*.

<sup>2</sup> The idea about Combinatorics at this page is simplified and generalized from the Texts and Translations by Donald Rutherford-G.W.Leibniz: *The Logic of Leibniz; Chapter 3 The Universal Language*.  
<http://philosophyfaculty.ucsd.edu/faculty/rutherford/Leibniz/ch3.htm>

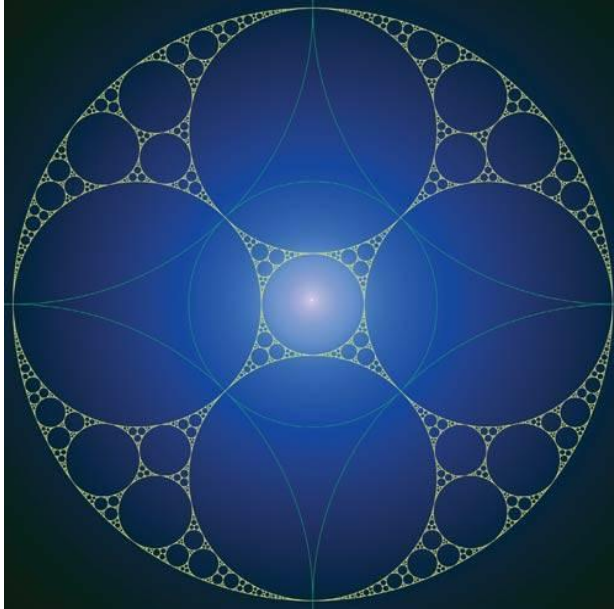
spoken and written language, with sentences as in ordinary languages formed from nouns, verbs, and particles in simplified and unified. It would be completely different from a logical calculus employing mainly logical algebra. All projective properties of figures common to arithmetic and geometry would definitely be excluded from the content<sup>1</sup>.

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<sup>1</sup> *The attempt could not reach an integrity because of the massive works it required and volume of the corpus it was likely to amount, but a strong Mathematical Logic thereby resulted in in addition to his Calculus, hence, the conceptual spin offs of pathfinder ideas thereby served greatly to the intellectual works of next generation.*

### 3.8. Apollonius' Circles Argument for the Expressive Power of Set Theory:

Many variations on the Apollonian gasket construction are possible. In this beautiful example, each pore is occupied by three inscribed circles rather than by one.



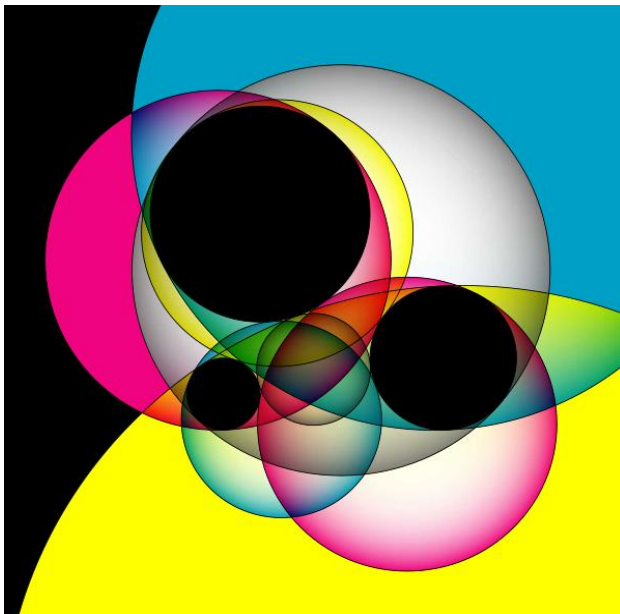
Light blue arcs represent five “curved mirrors.” Reflections in these curved mirrors—known technically as circle inversions—create a kaleidoscopic effect. Every circle in the gasket is generated by repeated inversions of the first six circles through these curved mirrors.

Image courtesy of Jos Leys.

Figure 3: Depicts a symmetrical Apollonian gasket, also called the Leibniz packing, after its inventor Gottfried Wilhelm Leibniz in the 17<sup>th</sup> century.

Source: <http://www.americanscientist.org/issues/pub/a-tisket-a-tasket-an-apollonian-gasket/4>

Apollonius' problem is a four circle problem. In that, each is expected to be mutually tangent to the other three in every solution. And this is figured in eight possible combinations in Figure 4. Circles of Apollonius



What is interesting and relevant to the Set Theory is that, combinations of solution are in binary form and exhaustively agreeing with the combinations of three first order terms, or predicates or prime factors or members of a complex proposition analogous to a given three member class subset combinations.

In Figure 4; the three given circles are in black but, the solutions are in colour. In Leibnizian vocabulary; given three circles, find all mutually tangent circles.

Figure 4: Circles of Apollonius.

Source: <http://demonstrations.wolfram.com/ProblemsOnCirclesIIIApolloniussProblem/>

Interestingly Apollonius' circle problem sets a good example for convenience of Axiomatic Set Theory to Geometry as well as to Mathematics universally. The brief demonstration is as follows.

Apollonius circle is a class with three elements, the three given circles.

Since membership is a binary relation, two based, subset of the class is obtained by  $2^n$  where  $n$  is the number of elements. The combination formula  $C = \frac{n!}{(n-r)!r!}$  is

eligible for this calculation. And solution number is 8.

$C_1=3$             one in and two out, three times for three given circles.

$C_2=3$             two in and one out, three times for three given circles.

$C_3=1$             all three in.

$C_4=1$             all three out.

*Totally*            $C = \sum_{i=1}^{i=3} C = 2^n = 8.$

That all combinations are in one to one correspondence to membership and proper inclusion relation of three element class to its own members and subsets can obviously be seen by a close examination. Thus;

All three externally tangent, out, to the fourth, in this case, the circle class, the fourth, has no member, hence, corresponds to Null Class contained as a subset.

All three internally tangent, in, to the fourth, in this case, the class circle, the fourth, has all members, hence, corresponds to Universe Class or Self Class.

One in and two out corresponds to 3 one-member subsets of the circle class.

One out and two in corresponds to 3 two-member subsets of the circle class.

This example verifies the triplet categorization of classes given above, in general, that is, null class-none, universe class-all, and arbitrary class-many, in a full accord with Aristotelian Syllogistic Propositions for No, All and Some.

As to universe class and self class equity; metaphysically, every individual substance is a universe on his own, but set theoretically it's the universe analogous to that of invalidating model. Hence, the instantiating universe is the domain of the



membership relation from class to members resulting in an infinite array of universes and verifying Bernard Russell's Theory of Types each of which exhausts a type. It's perfectly finite and particular to the case it sets forth in.

Quine admits the multiplicity of universe class to avoid paradoxical results and inconsistency on the basis of Russell's Typed Set Theory. But, since it's somewhat obscure and tiresome to prove or to deduce a theorem through the Typed Theory, for simplicity, Quine strips off the subscripts representing the number of type they belong to and generalize the formula then solves the problem or proves the theorem once and for all at the end he reassigns the subscripts to variables of the solution and obtains the original formula of the solution. This is the very equivalent an operation to substitution of variables in Mathematics especially in integration algebra. So it would not be much mistaken to say that Quine's New Foundations-NF, is merely a generalized form of Russell's Typed Set Theory or Typed Set Theory is the aggregation of particular instances of NF. However, historically, since Russell precedes Quine, the former would be prior to the latter.

In other words the notion of universe may be stochastic and relational and every distinct set of relations may set a new notion of universe which manifests itself on the ground of emerging relations in quality and quantity provided that it's the best possible. However, the notion of possibility occurs here is particularly restricted to precision of class-member relationship and does not subsume the idea of possible worlds. That would be in a strong accord with the idea, Leibniz maintains, that every individual substance is a world apart and there is no measure of magnitude between infinitesimals and finite whole is prior to the parts as regards number.

Hence, the class of predicate equates in size and diversity to the class of phenomena. Now, is existence just phenomenal or otherwise? Well, it seems to be not phenomenal but, to give a full account of whether it's so or otherwise is far beyond the purposes of this essay! But, what all thinkers are agreed upon is that existence is not predicable of contingents. It's only admissible for the Absolute. The reason why may take a long route to conclusion, for the present, in reasoning, above mentioned prohibition will be kept in mind and judged accordingly.

Leibniz always believed in an infallible, as it were, ‘language of thought’ as the key to an everlasting solution to the problems of subjectivity in reasoning. That would be his magnum opus providing true happiness based on the science and for that reason he had to have of the nature of God and the soul.

“Whence it is manifest that if we could find **characters or signs appropriate for expressing all our thoughts** as definitely and as exactly as arithmetic expresses numbers or geometric analysis expresses lines, we could in all subjects in so far as they are amenable to reasoning accomplish what is done in Arithmetic and Geometry”<sup>1</sup>.

Now, three assertions are identified from the above mentioned propositions analogous to that of Axiomatic Set Theory; such that,

Objects in the membership domain, set relation merely to the class to which they are members.

Members do not set relations to one another.

Identity of two classes is determined by their members.

So, in outline, subject predicate relation is closely analogous to member class relation.

General but still contingent laws hold only in the full sequence of states in one possible world being already actual and not in other possible worlds. Thus, generality on its own, does not suffice to prove necessity though they have an a priori proof by means of final causes.

There are two kinds of existent, to which the law of sufficient reason apply, possible and actual, either are contingent. The motivation of sufficient reason in possibles being desire or appetite whereas in actuals desire for the good, a notion more concrete. The sufficient reason in possible contingents is metaphysically necessary and it necessitates its decrees whereas that in actual contingents is contingent and applies merely to contingents by inclination as the pursuit of the best.

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<sup>1</sup> Roger Bishop Johns Website-The Method of Mathematics by Gottfried Wilhelm Leibniz Preface to General Science.  
[http://texts.rbjones.com/rbjpub/philos/classics/leibniz/meth\\_math.htm](http://texts.rbjones.com/rbjpub/philos/classics/leibniz/meth_math.htm)

Leibniz rightfully believed that it's the confusion of ideals with actuals which caused the labyrinth of continuum. So, the distinction between the two will strictly be borne in mind to avoid any inconsistency in judgements.

Predicates nominate the classes and act as the name of classes, the membership condition to a class<sup>1</sup>, therefore, can be given as membership conditions to classes. Classes are multitude in unity as the aggregate of particulars under a common predicate. In fact a class does not aggregate its members to anywhere, its just a distinct property common to all its members and it's the mind which sets this relation in and around particulars on its own. Therefore, classes/sets are in mind and not out in the world. Membership relation is from objects to the class. It's a vertical relation, so to speak, not lateral, that is, members or objects do not set membership relations to one another, if they do, then they are not the members of the class in question at all, they are just subsets. This principle is also in a perfect accord with the principle that substances do not interact with one another. In the perceptible world. There is a universal circle of movement from synthetic to analytic and vice versa.

The mind is flooded by an intensive flux of facts. Repetition of accidents, that is, frequency of occurrence, plays a significant role in the formation of the notions in the mind. The efficacy of the number of repetition is admitted by Leibniz, though as a statistics which measures the powers of things, this is also true with the formation of notions in the mind. As the number of occurrence increases, the effect is penetrated deeper into the nature of mind. After a certain number, perception is handed over to psycho-motor and stimulus is perceived unconsciously. Thereafter comes the level of intuition and the knowledge is internalized. The intuitive knowledge by acquaintance could be described as identification of necessary in contingent. The journey of the fact, as it were, continues up to sufficient and, perhaps, to necessary reason and finally unifies with it. Summing it up; unification of man with the knowledge of

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<sup>1</sup> W.V. Quine, says 'in propositions like "Paris is a city= $Paris \in city$ " and "Socrates is wise= $Socrates \in wise$ "; predicates 'city' and 'wise', serving as membership conditions, can be treated as the names of the classes 'city' and 'wise' in W.V. Quine- *Mathematical Logic*; Cambridge-Harvard University Press; 1961, pp.119, Class and Member para 3.

world seems to follow a gradual sequence such that perception-empirical, knowledge-phenomenal, repetition, common sense, repetition, intuition, repetition, acquaintance-by common nature in between, perception-monic, bare, soul, spirit. That would be a metaphysical and, perhaps, a transcendental journey having no reference to time, space or matter whose only stops were merely stages of Reality. As one can evidently see, this is analogous to the change of Monads from bare to soul and spirit rate respectively.

It would be an unending motion with no displacement, that is, a relative motion. It mirrors the eternal circle of steam, cloud, rain, river, pond, sea, ocean and steam again. Facts of the physical world would follow a route as a posteriori, a priori, sufficient reason and necessary reason. This would cause a new world accordingly. So, it would be agreeable to say that the world is in an infinite/eternal recreation and transformation, not only physically, perhaps, rather, essentially. In upward stream the facts are less Real and more phenomenal, and in downward, more Real and less phenomenal. I believe, Leibniz would agree with that.

A soul/spirit has no beginning or end in time<sup>1</sup>.

It's perfectly still relative to its own.

If no motion then no time and place.

Contingents are in a motion relative to theirs and necessary.

The motion of contingents is a chain of states folded in individually and universally.

Phenomena is properly included in Reality but not conversely. So, the language of Reality a priori enables the community of Grace, spirits, to see all that happens in perfect clarity but, language of phenomena (object language) still has a long way to go to perfection. That's why Leibniz saw Universal Characteristic as a divine gift to man.

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<sup>1</sup> *Platon's proof conclusion of the immortality of soul.*

I dare to say that, if Leibniz had attempt to construct an Axiomatic Set Theory, he would have avoided any paradoxes. Because, he would give the solution of major problem, distinct and clear conception of the whole system-a priori, the first priority, before any syntactical or notational concern. Granting their vital importance in regard to formality and consistency of the vocabulary theyare likely to employ, namely logical and mathematical symbols, Leibniz believed in two fold, if not two based, compatibility, from contingent to necessary, that is, from sufficient reason to necessary reason and from necessary to contingent. Every contingent would act on a sufficient reason of its own in agreement with the necessary reason. Whereas, necessary would act merely on his own without having to comply with the sufficient reason of contingents, because, act of necessary reason would necessarily include both necessary and sufficient in consistency. Hence he would say that consistency would consist essentially in necessary reason rather than sufficient, since the former is seated at the deepest and more primitive than the latter, thus the former is analogous to first-order terms and the latter to the second order therefore; contingent truths with sufficient reason is derivable from the necessary truths with necessary reason.

He would also use his Universal Characteristic as the language of the Leibnizian Axiomatic Set Theory, perfectly corresponding to its refrents and yet maintaining its derivation power from its own notion.

If Leibniz had attempted to construct an Axiomatic Set Theory he certainly would have restricted axioms to the least number. The notion of axioms would have been ultimately simple, distinct and acquainted with intuition yet proved strictly mathematical and formal in deduction. Components of the system would have a notion common in essence but diverse in identity. Class formation would be based on attribution to essence and predication of subject and class operation would be based on numerical identity of none, all and many holding harmonious unity in time, space and number.

Class relations would be set consistently allowing no contradiction. For they all would be possible ideas, not self contradictory. And the system would, most probably, be a succinct verse of multitude in unity. By this he would mean to say ultimately simple, in number and verse, axioms and infinitely complex, in number and verse, theorems. The system would, as it were, breath to inhale the multitude into unity and exhale the multitude out of unity. All would emerge from one and merge into one. That would be the only Reality Leibniz would wish to substantiate, a multitude in unity. That's the analyticity of Leibnizian type!

Now, what is it to perfectly fit to this outline? A universally quantified variable and a whole or unity. He would instantiate that universal quantifier for any particular case to amount one in the end. So, Leibniz would enfold the very codes of his universe in the name of the universe itself, as **uni** and **verse**. Where uni for 1 and verse for 0. 0, zero, stands, in our view, for empty class which is the only class as the idea of class and universally quantifiable variable of class. So it can be instantiated for any sort of class from without. There is a strong insight as to the relation of 1 to joint denial '↓' and 0 to alternative denial '↑' as binary connectives. It suffices, for the present, to allude that similarity, if not identity, in between.

Since the genesis of his entire idea is deeply analogous to the metaphysical conception of the universe and mirroring it from the identity of indiscernibles to possible and compossible worlds and since they were acting and existing in a perfect harmony and consistency, agreeing with this Grand Harmony, he would not allow any paradoxes in his Axiomatic Set Theory. Because, paradoxes arise from the controversial nature of the statements or adversary of grounds.

Leibniz would unfold the multi-layer knots of indeterminacy in the nature of light, rendering particle and wave nature concurrently, and incomputability of position and speed of a particle at a time, as the one aware of probabilistic approaches, hence, quantiles. Because, he believed that infinitesimals, that is, being infinitely small, would not mean to say that they are absolutely small as a measure of magnitude, it's just a ratio between two, not an independent, per se, property of reality. Having this in mind, he would enter into a world apart from this hybrid transition point and conceive it to unfold the ration behind.

## CHAPTER 4

### Conclusions:

Major motive of Leibniz was to compile an opus magnum to stand at the center of his full corpus as 'The Language of Thought'. He believed that human mind was to operate in accord with The Principle of Sufficient Reason as micro cosmos enfolding the universe in. He absolutely believed that 'reasoning is calculation'. Given that mathematics is infallible then in case humanity has a tool of thinking of mathematical precision any dispute or problem of any nature could be solved easily, clearly and irreversibly. For this serve his solution was Universal Characteristic, uncompleted yet resulted in, as a spin off, Differential and Integral Calculus, striking a middle course combining Algebra and Geometry in between.

Deducing genes of binary algebra long before those being titled the founder of this sharp and high resolution tool of computational judgement paved the road to invention of divine machine of computer.

He was a mind of a priori thinking, analytic to the supreme rate, a mathematician, a logician, a philosopher of possible worlds, an enabler of impossibles, simplifier and unifier of Platon, Aristotle, Euclid and Descartes, merger of Egypt, Greece and China, east and west, the oldest and the newest.

Now, let's summarize how a simple substance gains individuality and develops all the necessary components of a metaphysical edifice from nothing in Leibniz's view and, owing to Leibnizian analyticity, follow all the journey it experiences across the perceptual space-time environs.

In this process governing rule will be twofold principle of Sufficient Reason of which the other fold is The Law of Contradiction when necessary truths are concerned, since the sufficient reason for necessary truths is being not self contradictory which makes them each a possible idea that underlies a possible world.

And key notion to this process will be 'multitude in unity', that is, perception distinctness of which will determine the perfection of its subject i.e. monads.

Comprehension of multiplicity in unity occurs as a form of existence such that 'what's not truly **one** being is also not truly a **being**' which refers to being one in individuality and still integral to the universe, mirroring the whole universe in itself from part to the whole and vice versa. Perception does not have to be conscious, that is, apperceptive because it comes from within a monad through a somehow intrinsic acquaintance based on the common nature substances hold that fill the whole universe.

Key element to this construction mission will be simple substance, monad, the ultimate unit of existence which gains individuality by possession of reason as a spirit and being always a subject but not predicate. Monads are quantitatively simple but qualitatively complex. They differ in their qualities and have appetite, an inner drive of developmental change which makes them active. Activity holds conformity with efficient cause for contingent truths following the principle of sufficient reason and final cause for necessary truths. Action comes up with the desire of desired and attributed to the monads which have a more perfect perception, that is, souls and further spirits. Perfection is determined by the distinctness of perception. A more perfect monad, spirit, acts on its own perception which makes it active. A spirit with more distinct perception acts on his own and this gives it some joy. A bare monad with confused perception suffers and this give it some pain. So, activity is the tendency to pass the next state by the perception of good, if it be free, on its own, if not on God's decree. Activity appears to be a response to environs by a simple substance some in action and some in passion. In action, a spirit plays the game to its own rules yet accountable to eternal-necessary truths, in passion, a monad with confused perception is dependent the actions from without and complies with them. In this respect, activity is necessary for the existence of a substance.

And the logic behind will be in subject- predicate form, deemed sufficient to express the whole world by Leibniz, namely that, every true proposition has a subject and a predicate and the predicate is somehow contained within the subject. If it is contained explicitly the proposition is analytic; if only implicitly, it is synthetic. The



last statement is known as Predicate Containment Principle (PCP). This is the logic Leibniz builds his entire philosophy upon.

In Leibniz's world simple substance is in a perpetual change of qualitative states. But no change is made by leaps to the contrary every change is in infinitesimally small steps and orderly manner, that is, every event can naturally be explained by the preceding one. This results in the continuum which means every possible position intermediate between the first and last terms filled once and only once and results in identity of indiscernibles. Changes in qualitative states give the predicates and every predicate mirrors internal and external factors that cause it to emerge. To this account, a substance is defined by its predicates but is more than that as the subject of change. Emergence of predicates follow a progression law and form a causal series which can be substituted for time sequence. Hence, the time order consists of relations between predicates but, the space order holds between substances. No relation is fully external, every extrinsic denomination, has an intrinsic foundation i.e. predicates. There is no absolute space and time notion in Leibniz, instead, both are relational and convertible to one another. Space is an assemblage of possible relations of distance.

In this connection matter is a phenomenon resulting from the aggregates of real unities or monads, extension is repetition and extended is plural. The unity of phenomena is realized in the mind of percipient by confused perception.

It can be said that one can travel through every point of Leibniz's metaphysical world following the fundamental concepts, principles and logic of his philosophy owing to its strong analyticity. Thus, every proposition asserted throughout his philosophical works is logically possible and mathematically provable. His method is strongly axiomatic and correlated with mathematics. Believing in the power of mathematical truths he said that the world is subsumable in the series of zeros-0 and ones-1, (hence an **infinite line code software**). In his philosophy, his model was the nature and his method was mathematics. Correlation between his philosophy and set theoretical progression of natural numbers is highly noteworthy.

Thus;

Essence of existence: Substance.	Emptiness.
Simple substance/monad	Empty class.
Individual substance.	1: Unit class of empty class.
Phenomena: Repetitive substances.	2,...N: Repetition by the law of progression.

If he had stated the notion of class, Leibniz would say, on the basis of multiplicity in unity, that there is only one class it's the idea of class, hence empty class,  $\{ \}$ . The rest is the particular instances of the idea of class. Given that the essence of existence is multiplicity in unity then, this also proves true in set theory, hence;  $\{\{ \}\} = \{\emptyset\} = 1$ , which perfectly fits to the simplicity v.s complexity principle, namely that, ultimately simple in the idea-emptiness and infinitely complex in diversity, The Class of Natural Numbers which has correspondences in Leibniz's metaphysics as monad and phenomena-whole world.

The correlation between Leibniz's metaphysics and Set Theory can explicitly be seen from the followings:

An individual substance is a complete subject on the ground that every predicate of it is included in it, as is the fact that a set is said to be complete if and only if every member of it is included in it.

On this ground, it would not be any false to assert that the world is the aggregate of real unities, that is, monads, that appears as phenomena which is mapped into the Class of Natural Numbers in agreement with the Principle of Sufficient Reason, the Law of Contradiction, the Law of Continuity, monadic perception and the other Principles of Leibnizian Metaphysics on the basis of subject-predicate logic.

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## APPENDIX A

### **Basic Definitions of Mathematical Logic as the Servant of Set Theory<sup>1</sup>:**

Set/Class Theory is axiomatic in method and mathematical in language, in other words, follows from axioms in the grammar of Mathematical Logic. In modern sense Axiomatic Set Theory is ascribed to Georg Philipp Cantor as the founder while Mathematical Logic in binary form to George Boole, though traces date as far back as Leibniz. So, Mathematical Logic, though not a theory rather a new science, consists of the major component of Set Theory as that which provides the terms in necessary formality to the Theory. The point, by this allusion, is namely that, Mathematical Logic is indispensably necessary to entertain with Set Theory, axiomatic or otherwise, formal. For, The Set Theory is on its own without necessitating to be axiomatic . But, this would restrict it to being a particular branch of Mathematics and not the whole account of Mathematics through axiomatization and in the vocabulary of Mathematical Logic. Hence as is known well, The Set Theory arises from the study of Reals, a branch of Mathematics known as Analysis. The second and, perhaps, the most obligatory reason for being axiomatic is to hold the contradictory or circulatory elements out to avoid particularly paradoxical results and generally inconsistency in Set Theoretic Systems. ZFC, Zermelo-Fraenkel System with the Axiom of Choice is widely admitted to achieve this goal, i.e. consistency.

Now, from Mathematical Logic to Axiomatic Set Theory basic definition may be as follows in brief:

Logical vocabulary will be introduced in the ontology of binary connectives, namely that; “ is, not, and, or, unless, if, then, neither, nor, some, all, members, etc.” These logical connectives enable us to judge the truth value of any statement

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<sup>1</sup> *W.V.Quine Mathematical Logic, Cambridge-Harvard University Press 1961 Chapter I STATEMENTS, Chapter II QUANTIFICATION, Chapter III TERMS*

independently from their rhetorical content. Further, it's possible to reduce them into minimum in number, down to one as joint denial '↓'.

A formula without free variables is a statement.

A term without free variables is a name.

All the names needed for pure logic and mathematics are provided by abstraction. Numerals, '0', '1', variables essentially pronouns  $x$ ,  $y$ ,  $z$ , and abstracts with/without free variables are all names of definite classes. All names are abstracts but not conversely and all variables are name matrices but not conversely. As primitive notation the only terms needed are variables, pronouns.

A matrice is the general form of a statement in which variables occur in place of names.

All variables are name matrices but not conversely.

A formula comprises statements and statement matrices so terms comprise names and name matrices.

Quantifiers, universal or existential, govern the variables bound to them in a statement.

A closure is a statement whose all variables are bound to quantifiers.

A quantifier, signifies, practically, that;

- a variable is bound or not to a quantifier, bondage or freedom.

- if the declaration of the statement applies to bound variables in a universal or particular domain, highly momentous in instantiation.

-which variable is governed by which quantifier in the statement.

Since the language is formal the elements of statements are to be described. Thus, 'the object  $x$  such that ... $x$ ...', is a description.

That, from simple to complex, formula, statement, theorem proceeding is followed.

That statements have quantifiers but matrices not, that is, statements have bound but matrices free variables.

A matrice is neither true nor false, hence, has no truth value.

Thus, only statements are true or false, have truth value.

If a statement is concluded by a sequence of statements each of which is either a ponential, derivable from antecedents by Modus Ponens, or axiom of quantification of earlier statements of the sequence, then it's a Theorem.

For consecutive quantifiers of the same sort the order does not count but for the quantifiers of different type can make a difference such that,  $\exists x \forall y (x=y)$  is false whereas  $\forall y \exists x (x=y)$  is true.

An atomic formula in set theory is such that, one set is connected to other by the binary connective  $\in$ . Hence,  $(x \in y)$  is an atomic formula and henceforward will be spoken of logical formula when occurred with ' $\downarrow$ ', that is, all other logical connectives.

The logical formulae contain no names neither concrete nor abstract.

In case a name has to necessarily occur in a logical formulae, then a description of the name which is a special sort of abstract can perfectly be used instead.

Abstraction, namely the class abstraction, is a short for the class definition. Thus, to say 'the class of all entities  $x$  such that  $x$  is such and such' it suffices to give a class abstract;  $\{x: \dots x \dots\}$ .

Variables and abstracts are collectively to be called as terms.

There are three sorts of terms; variables which are essentially pronouns, abstracts with and without free variables.

Abstracts without free variables are names of definite classes. According to standard convention;  $\{x: \dots x \dots\}$  is called a class abstraction.  $\forall x (x \in x)$ ,  $\forall x (x \notin x)$ ,  $\forall x (x=x)$ , (or  $U$ ),  $\forall x (x \neq x)$ , (or  $\emptyset$ ), since all variables appear in the formula are bound to universal quantifier to make the formula a statement. They are all called as the names of definite classes accordingly.



Abstracts with free variables will be classed as name matrices, as in this formula  $\forall x (y \in x)$  Since the variable  $y$  is free it can be quantified in any way and the formula may gain a new meaning accordingly. So it's not a name but a name matrix.

Formulae comprise statements and statement matrices whereas terms comprise names and name matrices

*Last Note:*

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<sup>i</sup> - *The deed is primary to the word*

- *Leibniz was one of the supreme intellect of all time! Bertrand Russell.*

- *Whereof one cannot speak, thereof one must be silent.-Ludwig Wittgenstein.-*

- *Russell referes De Morgan's Law to Leibniz and Quine to William of Ockham in his Mathematical Logic 1961 pp.61.*