DEVELOPING A FOUR-BAR MECHANISM SYNTHESIS PROGRAM IN CAD ENVIRONMENT

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ABSTRACT

DEVELOPING A FOUR BAR MECHANISM SYNTHESIS PROGRAM IN CAD ENVIRONMENT

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Flap, aileron, rudder, elevator, speed brake, stick, landing gear and similar movable systems used in aerospace industry have to operate according to the defined requirements and mechanisms used in those systems have to be synthesized in order to fulfill those requirements. Generally, without the use of synthesis tools, synthesis of mechanisms are done in CAD environment by trial-error and geometrical methods due to the complexity of analytical procedures. However, this approach is time consuming since it has to be repeated until the synthesized mechanism has suitable mechanism properties like transmission angle and connection points. Due to above reasons, a software developed for synthesis of mechanisms within the CAD environment can utilize all the graphical interfaces and provides convenience in mechanism design.

In this work, it is aimed to develop a four-bar mechanism synthesis tool which is compatible with CATIA V5 by considering the requirements of aerospace industry.

This tool performs function, path and motion synthesis and shows suitable mechanisms in CATIA according to input obtained from CATIA and mechanism properties.

Keywords: Mechanism Synthesis, Four-bar, Burmester Theory, CAD, CATIA

ÖZ

CAD ORTAMINDA DÖRT ÇUBUK MEKANİZMASI SENTEZLEME PROGRAMI GELİŞTİRİLMESİ

Erener, Kaan
Yüksek Lisans, Makine Mühendisliği Bölümü
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Havacılık sektöründe kullanılan flap, kanatçık, kuyruk ve irtifa dümeni, hız freni, levye, iniş takımları ve benzeri hareketli sistemlerin belli isterler doğrultusunda çalışabilmesi için kullanılacak olan mekanizmaların bu doğrultuda sentezlerinin gerçekleştirilmesi gerekmektedir. Bu mekanizmalar, sentez yazılımlarının kullanılmadığı durumlarda, analitik işlemlerin karmaşıklığı nedeniyle, genel olarak CAD programlarında deneme-yanılma ve geometrik metotlarla sentezlenmektedir. Ancak, bu sürecin oluşturulan mekanizmanın bağlantı noktaları ve bağlama açısı gibi mekanizma özellikleri uygun oluncaya kadar tekrarlanması zaman kaybına neden olmaktadır. Bu nedenlerden ötürü, mekanizma sentezini, CAD programlarının grafik arayüzlerini kullanarak gerçekleştiren bir yazılım, mekanizma tasarımlarında kolaylık sağlayacaktır.

Bu çalışmada, havacılık sektörünün gereksinimleri ön planda tutularak CATIA V5 programı ile birlikte çalışabilen, dört çubuk mekanizma sentezleme programının geliştirilmesi amaçlanmıştır. Bu program, CATIA'dan aldığı girdilerle, konum, yörünge ve fonksiyon sentezlerini gerçekleştirebilen ve istenilen mekanizma özelliklerine göre uygun mekanizma alternatiflerini yine CATIA'da çıktı olarak gösterebilme özelliklerine sahiptir.

Anahtar Kelimeler: Mekanizma Sentezi, Dört Çubuk, Burmester Teorisi, CAD, CATIA

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CHAPTER 1

INTRODUCTION

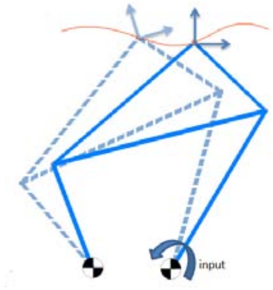
1.1 GENERAL

Kinematic synthesis of mechanism is one of the essential steps of the machine design. According to the duty of machine, several types of mechanisms can be synthesized and largely number of different configurations can be found. After the type of mechanism is determined, the dimensional synthesis has to be performed. Prescribed position synthesis is the most common method for dimensional synthesis and this is the basis of this thesis subject.

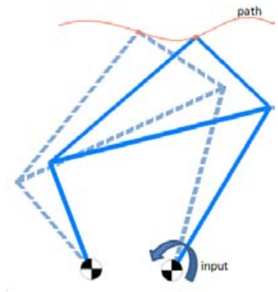
The prescribed position synthesis is commonly divided into three parts. These are namely, motion generation, path generation and function generation. Motion generation deals with rotation and translation of a body while it passes from several positions. Path generation deals only translation of a point and function generation is about correlation of input and output motion.

For all these tasks, two curves are obtained which satisfies the prescribed positions namely center and circle point locus. These loci show the fixed and moving pivots of the suitable mechanism respectively.

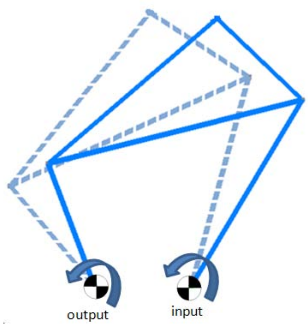
In mechanism synthesis, if the design conditions are suitable, there are an infinite number of solutions and it is engineer’s ability to judge and select suitable mechanism type and configuration. Even though various analytical methods have been developed for synthesis of mechanisms, it still depends on trial-error and repetitive tasks which causes loss of time and money.



Motion Generation



Path Generation



Function Generation

Figure 1 Illustrations for motion, path and function generation

Today, there is a need to reduce cost and save time for early stages of design namely for preliminary design. Therefore, a program which synthesizes mechanism according to user inputs and does trial-error for numerous requirements and conditions will be advantageous especially for aerospace industries which have lots of requirements and conditions.

The aim of this thesis is to create visual and interactive computer software package which works with CATIA V5 in fully parametric form and to apply this software package in aircraft mechanisms design. The ability of program is planned to cover motion synthesis, function generation and path synthesis of planar four-bar mechanisms. Moreover, in order to satisfy possible requirement; velocity, acceleration and transmission angle analysis shall be included in the program.

Since Visual Basic macros can be run under CATIA V5, the software is written in Visual Basic environment, with graphical user interface for ease of usage.

1.2 LITERATURE SURVEY

The history of machine design goes back to 300 B.C. However, with the studies of Ampere who excluded forces from kinematic analysis and studies of Reuleaux in which mechanisms are classified (type synthesis) and their symbolic representations are identified, "Kinematics" developed into a separate discipline in the 19^{th} century. Gruebler worked on number synthesis and developed criteria for the mobility of a mechanism and in the end of 19^{th} century, Burmester contributed on dimensional synthesis by introducing the precision position concept. He used geometrical methods and his work is known as Burmester Theory. [9, 21, 26]

After Burmester Theory, valuable contributions were done in synthesis of mechanism by geometrical methods in the beginning of 20^{th} century. Hartenberg [18], in his textbook, considered both techniques namely analytical and graphical approaches and used kinematic inversion and center point method as a geometrical method for function and motion generation synthesis of mechanism with three and four accuracy points. He defined the geometrical method as quick and staying close to the physical problem but tedious for repetitive tasks. Moreover, Hall [16] used geometrical method for synthesis of mechanism up to five specified position. In addition to Hartenberg and Hall, Harrisberger [17] worked on geometrical methods and used overlay method for synthesis of function generation, center point method for synthesis of motion generation, catalog, center point and inflection circle methods for synthesis for path generation.

After 50 years later from Burmester Theory, Ferdinand Freudenstein who is known as “father of modern kinematics” developed an analytical approach to finitely separated prescribed position concept [14]. With the help of this approach, he introduced digital computation in the kinematic synthesis which reduced the importance of trial-error and geometrical approaches [9]. Erdman and Sandor introduced the dyadic approach and used it in path, function and motion generation synthesis [10].

Kramer and Sandor [20] developed a selective precision synthesis method (SPS). In this method, unlike precision point approach the prescribed points are satisfied with a specified accuracy. This method can be used where exact accuracy is not needed or attainable due to the manufacturing tolerance and joint clearances. A few years later, Schaefer and Kramer [29] extended this approach to include the synthesis of mechanisms whose tracer points satisfy velocity as well as position specifications which is applicable the path, function and motion generation problems [9]. Bagci and Lee [2] developed optimum synthesis of plane mechanism by linear superposition technique for four-bar mechanism with six and eight unknown dimensions. Dimensions of the optimum mechanism are determined by minimizing the error in the loop-closure equations.

Tesar [33] and Eschenbach [32] worked on infinitesimally separated position synthesis and introduced multiply separated position (MSP) synthesis which is the combination of finitely and infinitesimally separated positions. He used PP notation for infinitesimally and P-P for finitely prescribed position. Therefore he obtains three combinations for three MSP (PPP, PP-P, P-P-P), five combinations for four MPS (PPPP, PPP-P, PP-PP, PP-P-P, P-P-P-P).

With the trend of using analytical approach at synthesis of mechanism, some problems arise. Due to the mathematical modeling, there may be branch and order problem in synthesized mechanisms. Previously, the points on the center and circle points curve are selected arbitrarily and check whether it has defect or not. Filemon [15] studied on this problem and set the basis of the theory. The aim of his computer program was to show the solutions only which fulfill the conditions. Waldron [35] introduced an efficient method for elimination of defects. In his search, he showed that, by selecting the driven crank first, it is possible to determine all possible driving cranks which satisfy the Grashof inequality. Moreover, he [34] extended his theory for infinitesimally separated positions and grouped the linkages by means of their complete or restricted rotations about their joint [26, 30].

Beloiu and Gupta [3], showed that in his works, the studies done by Filemon and Waldron fails for finding defects in some cases. Filemon and Waldron's works eliminates the branch defects if the design positions are belong to different modes and the input link is fully rotatable, however, Beloiu and Gupta prove that if the input link is partially rotatable the branch defects cannot be eliminated. They introduced a new approach by combining the previous studies to overcome this problem. The hyperbolic and elliptic boundaries are determined due to the selected output link and input link is determined in the boundaries. About defects of mechanisms, Chen and Fu [6] have published a new method to determine regions of the center point curve by using the Grashof inequality, which give the driving cranks of double crank or crank-rocker mechanism when the driven link is selected.

The usage of all these approaches becomes very efficient after the developments in the computers due to the nonlinear equations and high number of calculations. In this respect there have been some studies which use mathematical and programming tools and combine the theories mentioned above in computer environment and offer a complete solution.

Martin, Russell and Sodhi [23] presented an algorithm for motion generation in MathCAD for selecting planar four-bar from Burmester curve solutions. The algorithm works in this way; after the Burmester curve solution is given as an input, firstly it calculates all the link lengths of every mechanism solution so, the user can specify the interested region of curve. Secondly, mechanisms are selected which have feasible transmission angles. Thirdly, mechanisms are eliminated which are not desired type according to the Grashof classifications and finally minimum perimeter solution is selected among the other mechanism solutions. Bourrelle and Chen [4] studied on a program with graphical user interface which solves five position synthesis Burmester problem for RR and RP dyads by MATLAB with considering the mobility of the mechanisms.

Kinzel, Schmiedeler and Pennock [19] studied a new approach called “geometric constraint programming” (GCP) which enables to use sketching mode of CAD programs in order to synthesis mechanisms. This new approach uses geometric constraints and constructions rather than non-linear equations like most of commercial synthesis software. The study based on the motion generation for five finitely separated positions, path generation for nine finitely separated precision points and function generation for four finitely separated positions. The working principle of GCP is can be understood by motion generation for five separated positions problem. In this problem, five different four-bar mechanisms are drawn separately for every specified separated position. Then, every dimension of linkages of every four-bar is set to equal and corresponding center points are constraint to be coincident. Moreover, since GCP works on sketch module of CAD program, it is highly parametric that user can visualize every change on parameters.

Talekar and DePauw [24] has developed a function generation synthesize for planar four bar for three, four and five points and spatial four bar (RSSR) for three points on Msc. ADAMS software by using kinematic inversion. It has ability to draw Burmester curve in order to give ability to user to select mechanism according to suitable one.

Moreover, Polat [26] developed a computer program called “MECSYN” to determine center and circle points for three and four multiply separated positions by using Dyadic approach. With the same approach, Sezen [30] built “Quad-Link” in Delphi 4 environment for synthesis planar mechanisms which has graphical user interface unlike “MECSYN”. In addition to these, Demir [7] created “CADSYN” with same approach. However, the main difference is, “CADSYN” is integrated in AutoCAD. The other difference is, “CADSYN” is capable of taking into account the approximate position inputs. Therefore, it provides flexibility and ease of usage to designer. Furthermore, there are also various kinematic synthesizing programs are developed as a commercial products. Some of them are as follows;

WATT is a product of a Company of Heron in cooperation with the University of Twente. It is a user-friendly synthesis program which can create four-bar, slider crank, five bar, six-bar (Watt1, Watt2, Stephenson1, Stephenson2) and eight-bar mechanism. The program has capability for path and motion generation. After the problem type is specified, according to the user parameters like rotation, length of links, area of pivot points and etc. program synthesizes the mechanism and analyses it. The main advantage of the program is, the user can define several positions and program finds the most suitable mechanism with minimum error [37].

Lincages, which stands for Linkage INteractive Computer Analysis and Graphically Enhanced Synthesis, is developed by the University of Minesota. The software has ability to synthesize and analyze four and six bar mechanisms. The features of program are motion, path, and function synthesizing which for 3 or 4 positions; creating Burmester curves; and doing analysis of created mechanisms [22].

Sphinx is an interactive graphics based software package for designing spherical 4R mechanisms. It has capability to find center and circle point curves for four position synthesis [25].

ANALYTIX/CAM enables user to create a cam profile due to motion or geometry of follower for existing cam according to selected parameters like curve type, acceleration, velocity and dwell period. Moreover, ANALYTIX/CAM can make force analysis of created cam pairs also [1].

Synthetica is robotic system design software developed by Virtual Reality and Mechanism Lab, University of Maryland Baltimore County. It is used to synthesize, analyze and simulate spatial linkages. The main features of the program are; Dimensional synthesis of spatial mechanism, kinematic analysis for serial and parallel linkages and Trajectory planning [31].

CHAPTER 2

THEORY AND FORMULATION

2.1 GENERAL

Theory and formulation used in this study for synthesis of mechanism is based on the works of Erdman and Sandor [11, 12]. Multiply separated positions synthesis has been developed by Tesar [32]. Polat [26], derived the necessary equations using the dyadic approach for multiply separated positions. Demir [7], applied this formulation in Autocad environment. He used Visual Basic as the programming language. The theory will be summarized in the following sections.

The motion of a body can be defined independently and uniquely as follows;

Let point A is on the moving body with the coordinates $A(X, Y)$, $A(x, y)$ and position vectors $\vec{Z}(t), \vec{z}(t)$ relative to fixed and moving frame respectively. (Figure 2) Therefore, if the position of moving plane is defined by the time dependent quantities $\vec{a}(t)$ and $\emptyset(t)$, the coordinates of point A and position can be expressed in fixed frame as follows;

Let $\vec{a} = a + ib$, then;

$$X = a + xcos(\emptyset) - ysin(\emptyset) \quad (2.1)$$

$$Y = b + xsin(\emptyset) + ycos(\emptyset) \quad (2.2)$$

$$\vec{Z} = \vec{a} + \vec{z}e^{-i\phi} \quad (2.3)$$

In the analysis of mechanisms, the geometry of motion is not related with the angular velocity of links.

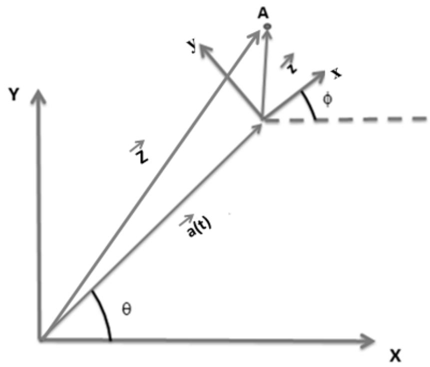


Figure 2 Motion of a moving plane respect to fixed frame

Therefore, without loss of generality, the assumption $\frac{d\phi}{dt} \neq 0$ and $\frac{d\phi}{dt} = 1 \; or \; \phi = t$ can be made.

If the derivatives of equations of (2.1), (2.2) and (2.3) are taken with respect to ϕ;

Let $\acute{K} = \frac{dK}{d\phi}$, $a_n = \frac{d^n a}{d\phi^n}$ and $b_n = \frac{d^n b}{d\phi^n}$ $(n = 1,2,3..)$

$$\grave{X} = a_1 - xsin(\phi) - ycos(\phi) \tag{2.4}$$

$$\grave{Y} = b_1 + xcos(\phi) - ysin(\phi) \tag{2.5}$$

$$\grave{Z} = \overrightarrow{a_1} + i\vec{z}e^{i\emptyset} \tag{2.6}$$

are obtained.

If a point P called as instantaneous center or pole which exists in every plane motion where the angular velocity of moving plane is zero is searched, equations (2.4), (2.5) and (2.6) become;

$$\grave{X}_p = a_1 - x_p sin(\phi) - y_p cos(\phi) = 0 \quad (2.7)$$

$$\grave{Y}_p = b_1 + x_p cos(\phi) - y_p sin(\phi) = 0 \quad (2.8)$$

$$\grave{Z}_p = \overrightarrow{a_1} + i\vec{z}_p e^{i\emptyset} = 0 \quad (2.9)$$

Using Cramer's rule, x_p, y_p and $\vec{z}_p$ found as;

$$x_p = a_1 sin(\emptyset) - b_1 cos(\emptyset) \quad (2.10)$$

$$y_p = b_1 sin(\emptyset) + a_1 cos(\emptyset) \quad (2.11)$$

$$\vec{z}_p = i\vec{a}_1 e^{-i\emptyset} \quad (2.12)$$

Therefore, point P can be expressed in fixed frame as follows;

$$X_p = a - b_1 \quad (2.13)$$

$$Y_p = b + a_1 \quad (2.14)$$

$$\vec{Z}_p = \vec{a} + i\vec{a}_1 \quad (2.15)$$

The differentials of the instantaneous center on the fixed and moving frames are;

$$ds = \sqrt{dx^2 + dy^2}$$

$$(ds)^2 = (\frac{dX_p}{d\emptyset})^2 + (\frac{dY_p}{d\emptyset})^2 = (a_1 + b_2)^2 + (b_1 + a_2)^2 \quad (2.16)$$

$$(ds)^2 = (\frac{dx_p}{d\emptyset})^2 + (\frac{dy_p}{d\emptyset})^2 = (a_1 + b_2)^2 + (b_1 + a_2)^2 \quad (2.17)$$

respectively.

Equations (2.16) and (2.17) imply that the rates of change of the pole are same which means that, the curve attached to moving frame which is defined as $\vec{z}_p$, is called the moving centrode and the curve attached to the fixed frame which is defined as $\vec{Z}_p$, is called the fixed centrode.

The method defined above is called as canonical representation of plane motion. The logic of this method is to define the motion of a moving plane as pure rolling of the moving centrode curve which is at the moving frame on the fixed centrode which is at the fixed frame. (Figure 3)

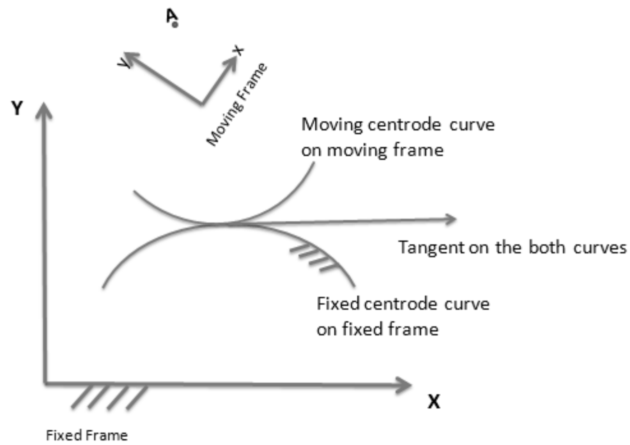


Figure 3 Fixed and moving centrode curves

2.2 MULTIPLY SEPARATED POSITION AND DYADIC APROACH

Multiply Separated Position Synthesis is one of the methods of dimensional synthesis of mechanisms. The other major methods are Infinitesimal Position Synthesis and Optimization Synthesis. Since these methods are not used in the software package, they are beyond the scope of this thesis.

Multiply Separated Position Synthesis deals whether the mechanism passes from desired prescribed positions or not and the displacements between two prescribed positions can be both finite and infinitesimal. In order to see all possible combinations of finitely and infinitesimally separated positions, the basic notations used are shown in Table 1 which is introduced by Tesar [33].

Table 1 Representation for finitely and infinitesimally separated positions

P-P	Two finitely separated positions
PP	Two infinitesimally separated positions

Therefore, P-PP is used for three multiply separated positions and the infinitesimally separated position belongs to the second finitely separated position.

In Multiply Separated Positions Synthesis, the mechanisms have to satisfy a finite number of constraint equations at the prescribed points. In order to obtain those constraint equations dyadic approach is used which introduced by Erdman [22].

In dyadic approach, the planar linkages are thought of as combinations of vector pairs called dyads. For example, the four-bar mechanism shown in Figure 4 can be perceived as two dyads. These dyads have to be solved separately and then have to be combined in order to form a whole mechanism.

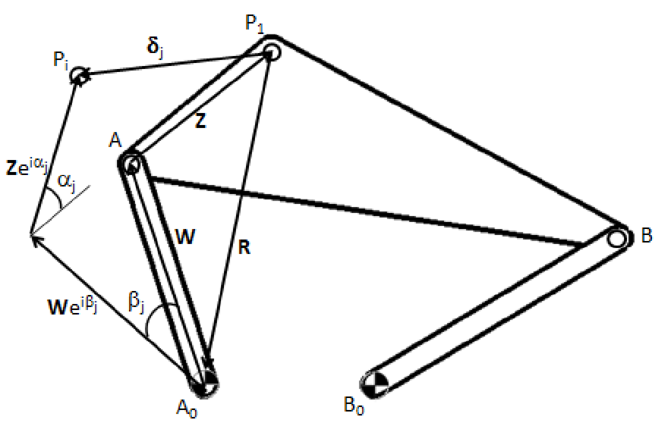


Figure 4 Dyadic representation of four-bar mechanism

In Figure 4, one dyad of the four-bar mechanism is shown. If one takes the P_1 (first prescribed position) as a center of the fixed and moving frame then $\vec{R}$ shows the center point and $\vec{R} + \vec{W}$ shows the circle point for the half of the four-bar mechanism. Moreover, notations used to define for every prescribed position are listed in Table 2.

Table 2 Notations for prescribed position

β_j	The rotation of crank from 1st position to jth position
P_j	The coordinates of point P at the jth position
δ_j	The position vector of point P at the jth position
α_j	The rotation of moving plane from 1st position to jth position

Therefore, if the loop defined by A_0-A_1-P_1-P_j-A_j-A_0 is considered, the following loop closure equation is obtained.

$$\vec{W} + \vec{Z} + \vec{\delta}_j - \vec{Z}e^{i\alpha_j} - \vec{W}e^{i\beta_j} = 0 \quad j = 1,2,3\ldots \tag{2.18}$$

$$\vec{W}\left(e^{i\beta_j} - 1\right) + \vec{Z}\left(e^{i\alpha_j} - 1\right) = \vec{\delta}_j \tag{2.19}$$

Since $\vec{R} = -\vec{Z} - \vec{W}$, the equation can be written by using fixed pivot vector.

$$\vec{R}\left(1 - e^{i\beta_j}\right) + \vec{Z}\left(e^{i\alpha_j} - e^{i\beta_j}\right) = \vec{\delta}_j \tag{2.20}$$

Equation (2.20) is used for finitely separated positions. However, for infinitesimally separated positions, derivative of equation (2.20) has to be taken with respect to the given angular displacement value. For motion generation synthesis, the loop closure equation for infinitesimally separated positions is as follows;

$$\vec{R}\frac{d}{d\alpha}(1-e^{i\beta})\Big|_{\beta=\beta_j}+\vec{Z}\frac{d}{d\alpha}(e^{i\alpha}-e^{i\beta})\Big|_{\substack{\alpha=\alpha_j\\ \beta=\beta_j}}=\frac{d}{d\alpha}\vec{\delta}_j(\alpha)\Big|_{\alpha=\alpha_j} \qquad (2.21)$$

For situations where the finitely and infinitesimally separated positions are combined following equation is used.

$$\vec{R}\frac{d^k}{d\alpha^k}(1-e^{i\beta})\Big|_{\beta=\beta_j}+\vec{Z}\frac{d^k}{d\alpha^k}(e^{i\alpha}-e^{i\beta})\Big|_{\substack{\alpha=\alpha_j\\ \beta=\beta_j}}=\frac{d^k}{d\alpha^k}\vec{\delta}_j(\alpha)\Big|_{\alpha=\alpha_j} \qquad (2.22)$$

where;

j: index of the finitely separated position

k: index of the infinitesimally separated position corresponding to a finitely separated position

l: index of the total separated position

More compact form of equation (2.22) can be written as;

$$\vec{R}(\sigma_l-b_l)+\vec{Z}(a_l-b_l)=\vec{\delta}_l \qquad (2.23)$$

$$where \quad a_l=\frac{d^k}{d\alpha^k}e^{i\alpha}\bigg|_{\alpha=\sigma_l\alpha_j}\ ;\ b_l=\frac{d^k}{d\alpha^k}e^{i\alpha}\bigg|_{\beta=\sigma_l\beta_j}\ ;\ \delta_l=\frac{d^k}{d\alpha^k}\vec{\delta}\bigg|_{\vec{\delta}=\sigma_l\vec{\delta}_j}$$

$$\sigma_l k=0\ \ (k=0,\sigma_l=1\ or\ k\neq 0,\ \sigma_l=0)$$

In Multiply Separated Position Synthesis, there are mainly three different tasks for mechanism synthesis. They are namely; motion, path and function generation. According to those tasks, the unknowns and prescribed positions can be changed.

2.3 SYNTHESIS TASKS

2.3.1 Synthesis for Motion Generation

The position of a moving plane can be defined by $\vec{\delta}_j$, α_j finitely and $\frac{d^k\vec{\delta}}{d\alpha^k}$ infinitesimally separated positions. ($\frac{d\vec{\delta}}{dt} = \frac{d\vec{\delta}}{d\alpha}\frac{d\alpha}{dt}$ and $\frac{d\alpha}{dt} = 1$) Therefore for different combinations of finitely and infinitesimally multiply separated positions are summarized in Table 3.

Table 3 The specified parameters for motion generation

	CASE	Specified Parameters
THREE POSITION SYNTHESIS	*P-P-P*	$\overrightarrow{\delta_2},\ \alpha_2, \overrightarrow{\delta_3},\ \alpha_3$
	PP-P	$\overrightarrow{\delta_2},\ \alpha_{2,}\ \left.\frac{d\vec{\delta}}{d\alpha}\right\|_{\alpha=0}$
	PPP	$\left.\frac{d\vec{\delta}}{d\alpha}\right\|_{\alpha=0} \left.\frac{d^2\vec{\delta}}{d^2\alpha}\right\|_{\alpha=0}$
FOUR POSITION SYNTHESIS	*P-P-P-P*	$\overrightarrow{\delta_2},\ \alpha_2, \overrightarrow{\delta_3},\ \alpha_3, \overrightarrow{\delta_4},\ \alpha_4$
	PP-P-P	$\overrightarrow{\delta_2},\ \alpha_2, \overrightarrow{\delta_3},\ \alpha_3, \left.\frac{d\vec{\delta}}{d\alpha}\right\|_{\alpha=0}$
	PPP-P	$\overrightarrow{\delta_2},\ \alpha_2, \left.\frac{d\vec{\delta}}{d\alpha}\right\|_{\alpha=0} \left.\frac{d^2\vec{\delta}}{d^2\alpha}\right\|_{\alpha=0}$
	PP-PP	$\overrightarrow{\delta_2},\ \alpha_2, \left.\frac{d\vec{\delta}}{d\alpha}\right\|_{\alpha=0}, \left.\frac{d\vec{\delta}}{d\alpha}\right\|_{\alpha=\alpha_2}$
	PPPP	$\left.\frac{d\vec{\delta}}{d\alpha}\right\|_{\alpha=0}, \left.\frac{d^2\vec{\delta}}{d^2\alpha}\right\|_{\alpha=0}, \left.\frac{d^3\vec{\delta}}{d^3\alpha}\right\|_{\alpha=0}$

2.3.1.1 Three Multiply Separated Positions

If equation (2.23) is expanded for three multiply separated positions the following equations are obtained. The values for α_i, b_i and σ_i can be found in Table 4 for different finitely and infinitesimally separated positions.

$$\vec{R}(\sigma_2 - b_2) + \vec{Z}(a_2 - b_2) = \delta_2 \tag{2.24}$$

$$\vec{R}(\sigma_3 - b_3) + \vec{Z}(a_3 - b_3) = \delta_3 \tag{2.25}$$

Set of equations (2.24) and (2.25) has twelve scalar variables and the six of them are unknown ($\vec{R}, \overline{Z}$, b_2, b_3). Therefore, in order to determine the fixed and moving pivots; one has to select b_2 and b_3 arbitrarily. If equations (2.24) and (2.25) are solved for $\vec{R}$ and $\vec{Z}$, the coordinates of center and circle points are found respectively in a compact form.

$$\vec{R} = \frac{(\alpha_3 - b_3)\delta_2 - (\alpha_2 - b_2)\delta_3}{(\alpha_3 - b_3)\sigma_2 + (-\alpha_2 + b_2)\sigma_3 + \alpha_2 b_3 - \alpha_3 b_2} \tag{2.26}$$

$$\vec{Z} = \frac{(b_3 - \sigma_3)\delta_2 - (b_2 - \sigma_2)\delta_3}{(\alpha_3 - b_3)\sigma_2 + (-\alpha_2 + b_2)\sigma_3 + \alpha_2 b_3 - \alpha_3 b_2} \tag{2.27}$$

Instead of selecting β_2 and β_3 freely, one can choose $\vec{R}$ or $\vec{Z}$ freely which result a unique circle that shows the coordinates of moving or fix centrode namely center or circle points.

Table 4 Explicit terms used in the dyad loop equation for three positions

CASE	σ_2	σ_3	α_2	α_3	b_2	b_3	δ_2	δ_3	Selected Parameters
P-P-P	1	1	$e^{i\alpha_2}$	$e^{i\alpha_3}$	e^{ib_2}	e^{ib_3}	$\overrightarrow{\delta_2}$	$\overrightarrow{\delta_3}$	$\beta_2\ \beta_3$
PP-P	0	1	i	$e^{i\alpha_2}$	$i\dot{\beta}$	e^{ib_2}	$\frac{d\vec{\delta}}{d\alpha}$	$\overrightarrow{\delta_2}$	$\dot{\beta}\ \beta_3$
PPP	0	0	i	-1	$i\dot{\beta}$	$-\dot{\beta}^2 + i\ddot{\beta}$	$\frac{d\vec{\delta}}{d\alpha}$	$\frac{d^2\vec{\delta}}{d\alpha^2}$	$\dot{\beta}\ \ddot{\beta}$

In order to specify the locations of center points $\vec{R}$ has to be specified. Therefore, the dyad loop closure equation can be rewritten as;

$$\vec{Z} + \overrightarrow{W} = -\vec{R} \tag{2.28}$$

$$a_2\vec{Z} + b_2\overrightarrow{W} = \delta_2 - \sigma_2\vec{R} \tag{2.29}$$

$$a_3\vec{Z} + b_3\overrightarrow{W} = \delta_3 - \sigma_3\vec{R} \tag{2.30}$$

This set of equation can be solved for $\overrightarrow{W}$ and $\vec{Z}$ if the determinant of the coefficient matrix is zero.

$$\begin{vmatrix} 1 & 1 & -\vec{R} \\ b_2 & a_2 & \delta_2 - \sigma_2\vec{R} \\ b_3 & a_3 & \delta_3 - \sigma_3\vec{R} \end{vmatrix} = 0$$

by expanding the matrix about the first column;

$$D_1 + D_2 b_2 + D_3 b_3 = 0 \tag{2.31}$$

where;

$D_1 = a_2(\delta_3 - \sigma_3\vec{R}) - a_3(\delta_2 - \sigma_2\vec{R})$

$D_2 = -a_3\vec{R} - \delta_3 + \sigma_3\vec{R}$

$D_3 = \delta_2 - \sigma_2\vec{R} + a_2\vec{R}$

In order to specify the locations of center points $\vec{Z}$ has to be specified. If the same procedure is applied;

$$D_1 + D_2 b_2 + D_3 b_3 = 0 \qquad (2.32)$$

where;

$D_1 = \sigma_2(\delta_3 - a_3\vec{Z}) - \sigma_3(\delta_2 - a_2\vec{R})$

$D_2 = -\sigma_3\vec{Z} - \delta_3 + a_3\vec{Z}$

$D_3 = \delta_2 - a_2\vec{Z} + \sigma_2\vec{Z}$

Different $\vec{R}$ or $\vec{Z}$ can be found for different b_2 and b_3 by solving equations (2.31) and (2.32).

2.3.1.2 Four Multiply Separated Positions

In four multiply separated positions synthesis, the number of equation increases and by selecting one variable arbitrarily the set of equation can be solved. Therefore, by varying this parameter, the center and circle points can be found which forms two curves called as Burmester curves.

As an example, the solution procedure of finitely separated position (P-P-P-P) synthesis will be given. By equation (2.23) the following set of equation is obtained.

$$\left(1-e^{i\beta_2}\right)\vec{R}+\left(e^{i\alpha_2}-e^{i\beta_2}\right)\vec{Z}=\delta_2 \tag{2.33}$$

$$\left(1-e^{i\beta_3}\right)\vec{R}+\left(e^{i\alpha_3}-e^{i\beta_3}\right)\vec{Z}=\delta_3 \tag{2.34}$$

$$\left(1-e^{i\beta_4}\right)\vec{R}+\left(e^{i\alpha_4}-e^{i\beta_4}\right)\vec{Z}=\delta_4 \tag{2.35}$$

In order to solve the set of equation, β_2, β_3 or β_4 is selected arbitrarily and every selection will yield the same curves. The determinant of the augmented matrix of the set of equation for selection of β_2 arbitrarily will be;

$$\begin{vmatrix} 1-e^{i\beta_2} & e^{i\alpha_2}-e^{i\beta_2} & \overrightarrow{\delta_2} \\ 1-e^{i\beta_3} & e^{i\alpha_3}-e^{i\beta_3} & \overrightarrow{\delta_3} \\ 1-e^{i\beta_4} & e^{i\alpha_4}-e^{i\beta_4} & \overrightarrow{\delta_4} \end{vmatrix}=0$$

$$D_1+D_2e^{i\beta_3}+D_3e^{i\beta_4}=0 \tag{2.36}$$

where;

$$D_1=\left(1-e^{i\beta_2}\right)\left(\overrightarrow{\delta_4}e^{i\alpha_3}-\overrightarrow{\delta_3}e^{i\alpha_4}\right)+\overrightarrow{\delta_2}\left(e^{i\alpha_4}-e^{i\alpha_3}\right)+\overrightarrow{\delta_3}\left(e^{i\alpha_2}-e^{i\beta_2}\right)$$
$$+\overrightarrow{\delta_4}(e^{i\beta_2}-e^{i\alpha_2})$$

$$D_2=\overrightarrow{\delta_2}\left(1-e^{i\alpha_4}\right)-\overrightarrow{\delta_4}(1-e^{i\alpha_2})$$

$$D_3=\overrightarrow{\delta_2}\left(e^{i\alpha_3}-1\right)-\overrightarrow{\delta_3}(e^{i\alpha_2}-1)$$

2.3.2 Synthesis for Path Generation

In path generation task, the points on the coupler path are correlated with the input link positions. The synthesis procedure of path generation is same as motion generation. However, at this time, crank rotations (β_j) are known, but, coupler rotations (α_j) are not known. Therefore, the motion generation calculation routines can be used if β_j values are entered as if they were α_j. As a result, the first dyad can be constructed. Since the coupler rotations are found, the crank rotations in the second dyad are found by the same routines of motion generation synthesis.

2.3.3 Synthesis for Function Generation

In function generation task, the prescribed rotations of the input link are correlated with the output link. Since the rotations of input links are given, the dyad containing the input link is known. The other dyad which contains the output link can be found by the path generation synthesis procedure.

The dyad loop closure equation for output link for finite displacements is:

$$\overrightarrow{\delta_j} = -\overrightarrow{R_B}(e^{i\psi_j} - 1) \tag{2.37}$$

For the infinitesimal displacements, the derivative of equation (2.37) has to be taken with respect to β.

Table 5 Coefficients of the dyad loop equations for MSP motion generation

CASE		
P-P-P-P	DYAD LOOP EQUATIONS	$\left(1 - e^{i\beta_2}\right)\vec{R} + \left(e^{i\alpha_2} - e^{i\beta_2}\right)\vec{Z} = \overrightarrow{\delta_2}$ $\left(1 - e^{i\beta_3}\right)\vec{R} + \left(e^{i\alpha_3} - e^{i\beta_3}\right)\vec{Z} = \overrightarrow{\delta_3}$ $\left(1 - e^{i\beta_4}\right)\vec{R} + \left(e^{i\alpha_4} - e^{i\beta_4}\right)\vec{Z} = \overrightarrow{\delta_4}$

Table 5 (Cont'd)

	SOLUTION	$\boldsymbol{D_1} + \boldsymbol{D_2}e^{i\beta_3} + \boldsymbol{D_3}e^{i\beta_4} = 0 \quad \beta_2\ is\ varied$ $\boldsymbol{D_1} = \left(1 - e^{i\beta_2}\right)\left(\overrightarrow{\delta_4}e^{i\alpha_2} - \overrightarrow{\delta_3}e^{i\alpha_4}\right) + \overrightarrow{\delta_2}\left(e^{i\alpha_4} - e^{i\alpha_3}\right) + \overrightarrow{\delta_3}\left(e^{i\alpha_2} - e^{i\beta_2}\right) + \overrightarrow{\delta_4}\left(e^{i\beta_2} - e^{i\alpha_2}\right)$ $\boldsymbol{D_2} = \overrightarrow{\delta_2}\left(1 - e^{i\alpha_4}\right) - \overrightarrow{\delta_4}\left(1 - e^{i\alpha_2}\right)$ $\boldsymbol{D_3} = \overrightarrow{\delta_2}\left(e^{i\alpha_3} - 1\right) - \overrightarrow{\delta_3}\left(e^{i\alpha_2} - 1\right)$
PP-P-P	DYAD LOOP EQUATIONS	$-i\dot{\beta}\vec{R} + \left(i - i\dot{\beta}\right)\vec{Z} = \dot{\delta}$ $\left(1 - e^{i\beta_2}\right)\vec{R} + \left(e^{i\alpha_2} - e^{i\beta_2}\right)\vec{Z} = \overrightarrow{\delta_2}$ $\left(1 - e^{i\beta_3}\right)\vec{R} + \left(e^{i\alpha_3} - e^{i\beta_3}\right)\vec{Z} = \overrightarrow{\delta_3}$
	SOLUTION	$\boldsymbol{D_1} + \boldsymbol{D_2}i\dot{\beta} + \boldsymbol{D_3}e^{i\beta_3} = 0 \quad \beta_2\ is\ varied$ $\boldsymbol{D_1} = \boldsymbol{A_1}\boldsymbol{A_4} - \boldsymbol{A_2}\boldsymbol{A_3}$ $\boldsymbol{D_2} = \boldsymbol{A_4}\overrightarrow{\delta_2} - \boldsymbol{A_3} + \boldsymbol{A_3}e^{i\alpha_2}$ $\boldsymbol{D_3} = \boldsymbol{A_2}\overrightarrow{\delta_2} - \boldsymbol{A_1} + \boldsymbol{A_1}e^{i\alpha_2}$ $\boldsymbol{A_1} = e^{i\alpha_2}\dot{\delta} - e^{i\beta_2}\dot{\delta} - i\overrightarrow{\delta_2}$ $\boldsymbol{A_2} = -i + ie^{i\beta_2}$ $\boldsymbol{A_3} = e^{i\alpha_3}\overrightarrow{\delta_2} - e^{i\alpha_2}\overrightarrow{\delta_3} + e^{i\beta_2}\overrightarrow{\delta_3}$ $\boldsymbol{A_4} = e^{i\alpha_3} - e^{i\alpha_2} + e^{i\beta_2} - e^{i\beta_2}e^{i\alpha_3}$
PP-PP	DYAD LOOP EQUATIONS	$-i\dot{\beta}\vec{R} + \left(i - i\dot{\beta}\right)\vec{Z} = \dot{\delta}$ $\left(1 - e^{i\beta_2}\right)\vec{R} + \left(e^{i\alpha_2} - e^{i\beta_2}\right)\vec{Z} = \overrightarrow{\delta_2}$ $-i\dot{\beta}_2\vec{R} + \left(i - i\dot{\beta}_2\right)\vec{Z} = \dot{\delta}_2$
	SOLUTION	$\boldsymbol{D_1} + \boldsymbol{D_2}i\dot{\beta} + \boldsymbol{D_3}\dot{\beta}_2 = 0 \quad \beta_2\ is\ varied$ $\boldsymbol{D_1} = -i\dot{\delta}_2\left(1 - e^{i\beta_2}\right) + ie^{i\alpha_2}\dot{\delta}\left(1 - e^{i\beta_2}\right)$ $\boldsymbol{D_2} = -\dot{\delta}_2\left(e^{i\alpha_2} - e^{i\beta_2}\right) + ie^{i\alpha_2}\overrightarrow{\delta_2} + \dot{\delta}_2\left(1 - e^{i\beta_2}\right)$ $\boldsymbol{D_3} = -\dot{\delta}_2e^{i\beta_2} - e^{i\beta_2}\dot{\delta}\left(1 - e^{i\beta_2}\right) + e^{i\beta_2}\left(e^{i\alpha_2} - e^{i\beta_2}\right)$
PPP-P	DYAD LOOP EQUATIONS	$-i\dot{\beta}\vec{R} + \left(i - i\dot{\beta}\right)\vec{Z} = \dot{\delta}$ $\left(\dot{\beta}^2 - i\ddot{\beta}\right)\vec{R} + \left(-1 + \dot{\beta}^2 - i\ddot{\beta}\right)\vec{Z} = \ddot{\delta}$ $\left(1 - e^{i\beta_2}\right)\vec{R} + \left(e^{i\alpha_2} - e^{i\beta_2}\right)\vec{Z} = \overrightarrow{\delta_2}$

Table 5 (Cont'd)

	SOLUTION	$\boldsymbol{D_1} + \boldsymbol{D_2} i\dot{\beta} + \boldsymbol{D_3}(-\dot{\beta}^2 + i\ddot{\beta}) = 0 \quad \beta_2\ is\ varied$ $\boldsymbol{D_1} = i\ddot{\delta}\left(1 - e^{i\beta_2}\right) + \dot{\delta}\left(1 - e^{i\beta_2}\right)$ $\boldsymbol{D_2} = \overrightarrow{\delta_2} + \ddot{\delta}\left(e^{i\alpha_2} - 1\right)$ $\boldsymbol{D_2} = i\overrightarrow{\delta_2} + \dot{\delta}\left(1 - e^{i\alpha_2}\right)$
PPPP	DYAD LOOP EQUATIONS	$-i\dot{\beta}\vec{R} + \left(i - i\dot{\beta}\right)\vec{Z} = \dot{\delta}$ $\left(\dot{\beta}^2 - i\ddot{\beta}\right)\vec{R} + \left(-1 + \dot{\beta}^2 - i\ddot{\beta}\right)\vec{Z} = \ddot{\delta}$ $\left(3\dot{\beta}\ddot{\beta} - i\dddot{\beta} + i\dot{\beta}^3\right)\vec{R} + \left(-i + 3\dot{\beta}\ddot{\beta} - i\dddot{\beta} + +i\dot{\beta}^3\right)\vec{Z}$ $= \dddot{\delta}$
	SOLUTION	$\boldsymbol{D_1} + \boldsymbol{D_2}\ddot{\beta} + \boldsymbol{D_3}\dddot{\beta} = 0 \quad \beta_2\ is\ varied$ $\boldsymbol{D_1} = (-i\dddot{\delta} - \ddot{\delta} + i\left(\dot{\delta} + \dddot{\delta}\right)\dot{\beta} + (\ddot{\delta} - i\dot{\delta})\dot{\beta}^2)\dot{\beta}$ $\boldsymbol{D_2} = \dot{\delta} + \dddot{\delta} + \dot{\beta}(-3\dot{\delta} - 3i\ddot{\delta})$ $\boldsymbol{D_3} = -\ddot{\delta} + i\dot{\delta}$

2.4 DEFECTS IN MECHANISM SYNTHESIS

When the points on the Burmester curve are used to construct mechanism they may not fulfill the prescribed geometrical conditions, although it satisfies the conditions theoretically. This occurs due to branch and circuit defects, order problem and Grashof condition.

In branch defect, the mechanism has to be disassembled from a joint and then assembled again in order to obtain prescribed positions. Circuit defect occurs when the prescribed positions are not in the same disjointed range of motion of input link. Order problem occurs since orders of prescribed positions cannot be considered in synthesis procedures. Therefore, the mechanism selected from Burmester curve, may not pass in desired order. Moreover, the obtained mechanism may be double rocker, double crank or crank-rocker mechanism. Therefore, according to the desired type of mechanism, the usable points of Burmester curve have to be defined.

In this work, these problems are eliminated by analysis of mechanism alternatives obtained from Burmester curves. In SynCAT, after one crank is selected, all points on the Burmester curve are searched for Grashof condition due to desired types of mechanism. With the determined points, every mechanism is constructed in its first prescribed position and analyses are done for several positions up to last prescribed positions by small increments.

Therefore, mobility of mechanism between prescribed position, existence of prescribed position in the same branch and order can be checked.

2.5 SPECIAL CASES

The general synthesis procedures mentioned in section 2.3 fails in some cases and for these cases center and circle point curves degenerates into a line, a circle and a line or every point on the coupler plane. In the below, these cases are examined by the method mentioned in the study of Polat [26].

2.5.1 Case P-P-P

The center and circle point curves for Case P-P-P can be found by Table 4. The $\vec{Z}$ and $\vec{R}$ are as follows;

$$\vec{R} = \frac{\left(e^{i\alpha_3} - e^{i\beta_3}\right)\overrightarrow{\delta_2} - \left(e^{i\alpha_2} - e^{i\beta_2}\right)\overrightarrow{\delta_3}}{e^{i\alpha_3} - e^{i\beta_3} - e^{i\alpha_2} + e^{i\beta_2} + e^{i\alpha_2}e^{i\beta_3} - e^{i\alpha_3}e^{i\beta_2}} \tag{2.38}$$

$$-\vec{Z} = \frac{-\left(e^{i\beta_3} - 1\right)\overrightarrow{\delta_2} + \left(e^{i\beta_2} - 1\right)\overrightarrow{\delta_3}}{e^{i\alpha_3} - e^{i\beta_3} - e^{i\alpha_2} + e^{i\beta_2} + e^{i\alpha_2}e^{i\beta_3} - e^{i\alpha_3}e^{i\beta_2}} \tag{2.39}$$

If the prescribed all positions are parallel (i.e. $\alpha_{2=}\alpha_3 = 0$), $\vec{Z}$ and $\vec{R}$ becomes as follows;

$$\vec{R} = \frac{\left(1 - e^{i\beta_3}\right)\overrightarrow{\delta_2} - \left(1 - e^{i\beta_2}\right)\overrightarrow{\delta_3}}{0} \tag{2.40}$$

$$-\vec{Z} = \frac{-\left(e^{i\beta_3} - 1\right)\overrightarrow{\delta_2} + \left(e^{i\beta_2} - 1\right)\overrightarrow{\delta_3}}{0} \tag{2.41}$$

The equations above have a solution if the numerators are zero.

$$-\left(e^{i\beta_3} - 1\right)\overrightarrow{\delta_2} + \left(e^{i\beta_2} - 1\right)\overrightarrow{\delta_3} = 0 \tag{2.42}$$

In the equation, if $\frac{\overrightarrow{\delta_3}}{\overrightarrow{\delta_2}}$ is pure real then the crank rotations (i.e. $\beta_2 = \beta_3 = 0$) have to be zero and if $\frac{\overrightarrow{\delta_3}}{\overrightarrow{\delta_2}}$ is not pure real, the crank rotations are fixed and can be obtained from above equation. In both cases, the every point of the moving plane can be selected as a circle point. The other configurations for special cases are shown in table below.

Table 6 Special cases for P-P-P

Condition		Circle Point	Center Point
$\alpha_2 = \alpha_3 = 0$	$\frac{\overrightarrow{\delta_3}}{\overrightarrow{\delta_2}}$ is real	At infinity	Every point
	$\frac{\overrightarrow{\delta_3}}{\overrightarrow{\delta_2}}$ is not real	Every point	Every point
$\alpha_2 = 0, \alpha_3 \neq 0$		Line	Line
$\alpha_2 \neq 0, \alpha_3 = 0$		Line	Line
$\alpha_2 = \alpha_3 \neq 0$		Line	Line

2.5.2 Case P-P-P-P

Special cases handled in P-P-P case can be applied to P-P-P-P case. Locus for center and circle point for different types of conditions can be seen in the following table.

Table 7 Special cases for P-P-P-P

Condition		Circle Point Locus	Center Point Locus
$\alpha_2 = \alpha_3 = 0$ $\alpha_4 \neq 0$	$\frac{\overrightarrow{\delta_3}}{\overrightarrow{\delta_2}} = \mu$	At infinity	Line
	$\frac{\overrightarrow{\delta_3}}{\overrightarrow{\delta_2}} \neq \mu$	Circle	Circle
$\alpha_2 = \alpha_4 = 0$ $\alpha_3 \neq 0$	$\frac{\overrightarrow{\delta_4}}{\overrightarrow{\delta_2}} = \mu$	At infinity	Line
	$\frac{\overrightarrow{\delta_4}}{\overrightarrow{\delta_2}} \neq \mu$	Circle	Circle
$\alpha_3 = \alpha_4 = 0$ $\alpha_2 \neq 0$	$\frac{\overrightarrow{\delta_4}}{\overrightarrow{\delta_3}} = \mu$	At infinity	Line
	$\frac{\overrightarrow{\delta_4}}{\overrightarrow{\delta_3}} \neq \mu$	Circle	Circle
$\alpha_2 = \alpha_3 = \alpha_4 \neq 0$	$\frac{\overrightarrow{\delta_4}}{\overrightarrow{\delta_2}} = \mu, \frac{\overrightarrow{\delta_3}}{\overrightarrow{\delta_2}} = \upsilon$	At infinity	Line
	$\frac{\overrightarrow{\delta_4}}{\overrightarrow{\delta_2}} = \mu, \frac{\overrightarrow{\delta_3}}{\overrightarrow{\delta_2}} = \upsilon$	Circle	Circle
$\alpha_3 = \alpha_4$ $\alpha_2 = 0$	$\frac{\overrightarrow{\delta_3} - \overrightarrow{\delta_4}}{\overrightarrow{\delta_2}} = \mu$	Line	Two Line
	$\frac{\overrightarrow{\delta_3} - \overrightarrow{\delta_4}}{\overrightarrow{\delta_2}} \neq \mu$	General Procedure	General Procedure
$\alpha_2 = \alpha_4$ $\alpha_3 = 0$	$\frac{\overrightarrow{\delta_2} - \overrightarrow{\delta_4}}{\overrightarrow{\delta_3}} = \mu$	Line	Two Lines
	$\frac{\overrightarrow{\delta_2} - \overrightarrow{\delta_4}}{\overrightarrow{\delta_3}} \neq \mu$	General Procedure	General Procedure
$\alpha_2 = \alpha_3$ $\alpha_4 = 0$	$\frac{\overrightarrow{\delta_2} - \overrightarrow{\delta_3}}{\overrightarrow{\delta_4}} = \mu$	Line	Two Lines
	$\frac{\overrightarrow{\delta_2} - \overrightarrow{\delta_3}}{\overrightarrow{\delta_4}} \neq \mu$	General Procedure	General Procedure

2.5.3 Case PP-P-P

Special case in PP-P-P occurs when the coupler rotations are same. The following table shows the locus equations for different types of conditions.

Table 8 Special cases for PP-P-P

Condition		Circle Point Locus	Center Point Locus
$\alpha_2 = 0$ $\alpha_3 = 0$	$\frac{\overrightarrow{\delta_3}}{\overrightarrow{\delta_2}}$ is real	At infinity	$-\vec{Z} = -i\overrightarrow{\delta_2} - \frac{\left(e^{i\beta_2} - 1\right)}{i\beta}\overrightarrow{\delta_3}$
	$\frac{\overrightarrow{\delta_3}}{\overrightarrow{\delta_2}}$ is not real	Line $\vec{R} = \frac{\left(1 - e^{i\beta_2}\right)\overrightarrow{\delta_2} - \left(i - i\dot{\beta}\right)\overrightarrow{\delta_3}}{i(e^{i\beta_2} - 1)}$	Line $-\vec{Z} = \frac{-\left(e^{i\beta_2} - 1\right)\overrightarrow{\delta_2} - i\dot{\beta}\overrightarrow{\delta_3}}{i(e^{i\beta_2} - 1)}$

2.5.4 Case PPPP

The equation of synthesis of PPPP generation and the coefficients are as follows;

$$\boldsymbol{D_1} + \boldsymbol{D_2}\ddot{\beta} + \boldsymbol{D_3}\dddot{\beta} = 0$$

$$\boldsymbol{D_1} = (-i\dddot{\delta} - \ddot{\delta} + i\left(\dot{\delta} + \dddot{\delta}\right)\dot{\beta} + (\ddot{\delta} - i\dot{\delta})\dot{\beta}^2)\dot{\beta} \quad (2.43)$$

$$\boldsymbol{D_2} = \dot{\delta} + \dddot{\delta} + \dot{\beta}(-3\dot{\delta} - 3i\ddot{\delta})$$

$$\boldsymbol{D_3} = -\ddot{\delta} + i\dot{\delta}$$

The general solution procedure fails when $\dot{\delta}$, $\dddot{\delta}$ are real and $\ddot{\delta}$ is imaginary or vice versa. In such a cases $\boldsymbol{D_1}, \boldsymbol{D_3}$ are real and $\boldsymbol{D_2}$ is imaginary or vice versa respectively. In these cases, the equations can be satisfied only if $\boldsymbol{D_2} = 0$ or $\ddot{\beta} = 0$.

If $\boldsymbol{D_2} = 0$ is solved for $\dot{\beta}$, then $\dot{\beta}$ is;

$$\dot{\beta} = \frac{\dot{\delta} + \dddot{\delta}}{3(\dot{\delta} + i\ddot{\delta})} \quad (2.44)$$

Then by fixing $\dot{\beta}$ and varying $\ddot{\beta}$, the center points $(\vec{Z})$ and circle points $(\overrightarrow{R})$ which results circles can be found as follows;

$$-\vec{Z} = \frac{\left(\dot{\beta}^2 - i\,\ddot{\beta}\right)\dot{\delta} + i\dot{\beta}\ddot{\delta}}{i\dot{\beta} + i(-\dot{\beta}^2 + i\,\ddot{\beta})} \tag{2.45}$$

$$\vec{R} = \frac{\left(-1 + \dot{\beta}^2 - i\dot{\beta}^2\right)\dot{\delta} + (i - i\dot{\beta})\ddot{\delta}}{i\dot{\beta} + i(-\dot{\beta}^2 + i\,\ddot{\beta})} \tag{2.46}$$

The other solution is found when $\ddot{\beta} = 0$. In this case $\dot{\beta}$ is varied and $\vec{Z}$ and $\vec{R}$ which results straight line becomes as follows;

$$-\vec{Z} = \frac{\dot{\beta}^2\dot{\delta} + i\dot{\beta}\ddot{\delta}}{i\dot{\beta} - i\dot{\beta}^2} \tag{2.47}$$

$$\vec{R} = \frac{\left(-1 + \dot{\beta}^2\right)\dot{\delta} - (i - i\dot{\beta})\ddot{\delta}}{i\dot{\beta} - i\dot{\beta}^2} \tag{2.48}$$

2.6 SynCAT

2.6.1 Usage

In this thesis, a computer program called SynCAT is created for the synthesis of four-bar mechanism. It is capable of synthesizing for four multiply separated positions and analyzing the synthesized mechanisms.

The major importance of SynCAT is, it is written in Visual Basic which commands works under CATIA V5. Therefore, it works with CATIA V5 CAD program fully compatible which means designer synthesize mechanism in its working environment without an extra effort. The designer need not to change its design criteria like working plane or input points due to the CATIAs global coordinate. Moreover, during creation of SynCAT, the needs of Aerospace Industry are taken into account. Since space allocation is very limited in such industries, selection of interested regions can be used in SynCAT. Also in order to increase the possibilities, SynCAT is capable of modifying the input variables.

SynCAT consists three different synthesis types namely path, function and motion generation. For every synthesis type there are five different variations for four multiply separated position synthesis which are P-P-P-P, PP-P-P, PP-PP, PPP-P and PPPP. In order to show usage of SynCAT, function generation for PP-P-P will be shown as an example.

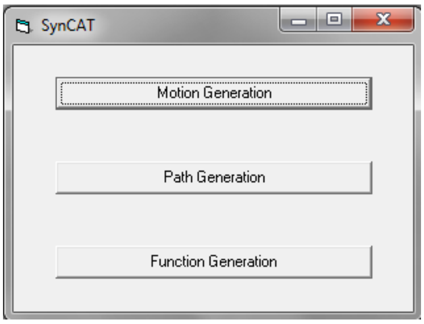


Figure 5 SynCAT synthesis generation types

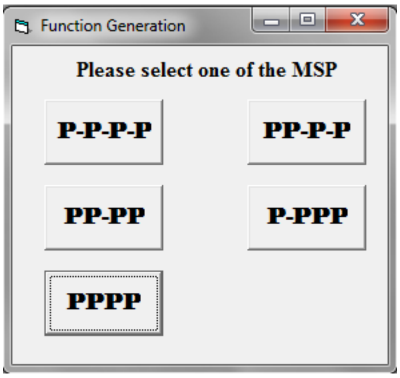


Figure 6 SynCAT function generation MSP types

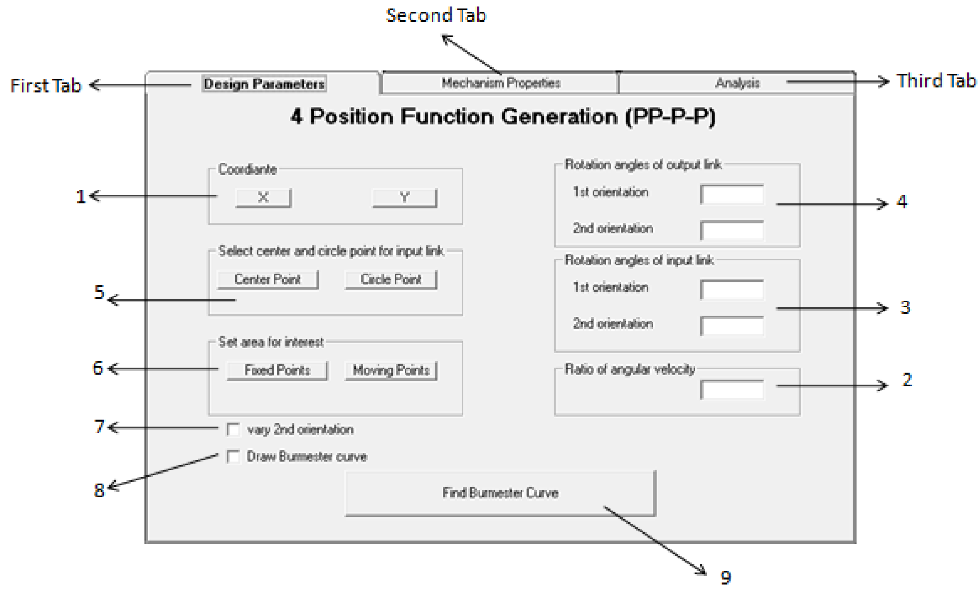


Figure 7 SynCAT design parameter entrance screen

As seen in the above figures, in the first tab, the design input are entered to SynCAT, in the second tab, the mechanism properties are defined and in the third tab, analysis of selected mechanism can be performed.

Firstly designer has to define its coordinate system (1) in order to be consistent with input variables. This property gives chance user to work in any orientated and translated plane with respect to global axis system. Secondly, the finitely and infinitesimally prescribed positions (2, 3, 4) are entered to SynCAT. After that, input link and interested regions for moving and fixed centrode are specified (5, 6). If the designer clicks the "vary 2nd orientation" option (7), the second rotation angle of input link is varied in a region in order to increase possibilities. Moreover, if the "Draw Burmester curve" option (8) is clicked and "Draw Burmester Curve" button (9) is pressed, the all points belongs to fixed and moving centrode in the interested region are shown in CATIA screen. If just button is pressed, the centrode are found but not shown in CATIA screen.

After SynCAT found all possibilities, it asks for mechanism properties. In the second tab, user can enter the mechanism type (10), possible link length ratio (11) and minimum transmission angle in the mechanism between prescribed positions (12) and by pressing “Search for suitable mechanism” the all possible output cranks are shown in CATIA screen. Moreover, in Third Tab, selected suitable mechanisms can be analyzed for its kinematic properties.

The all output reflected to CATIA can be seen in figure below.

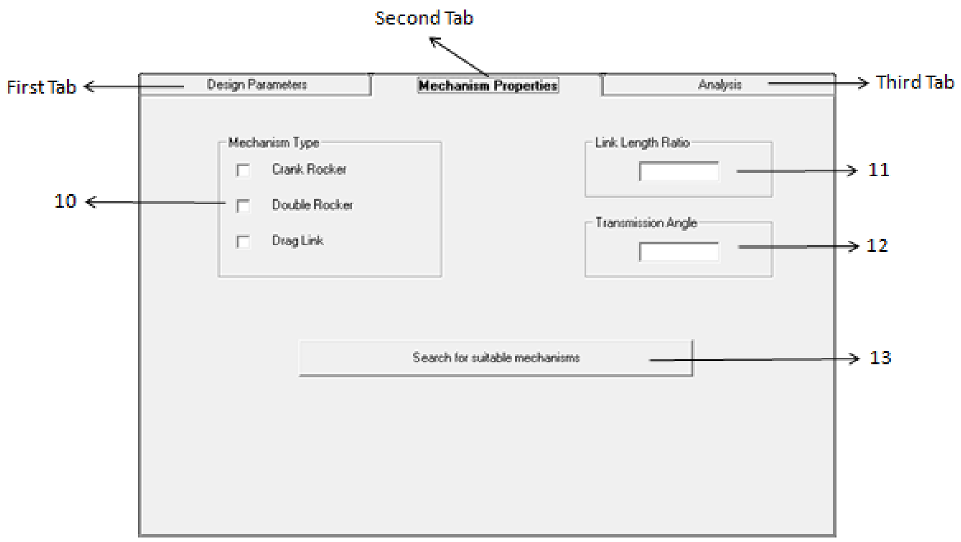


Figure 8 SynCAT mechanism properties entrance screen

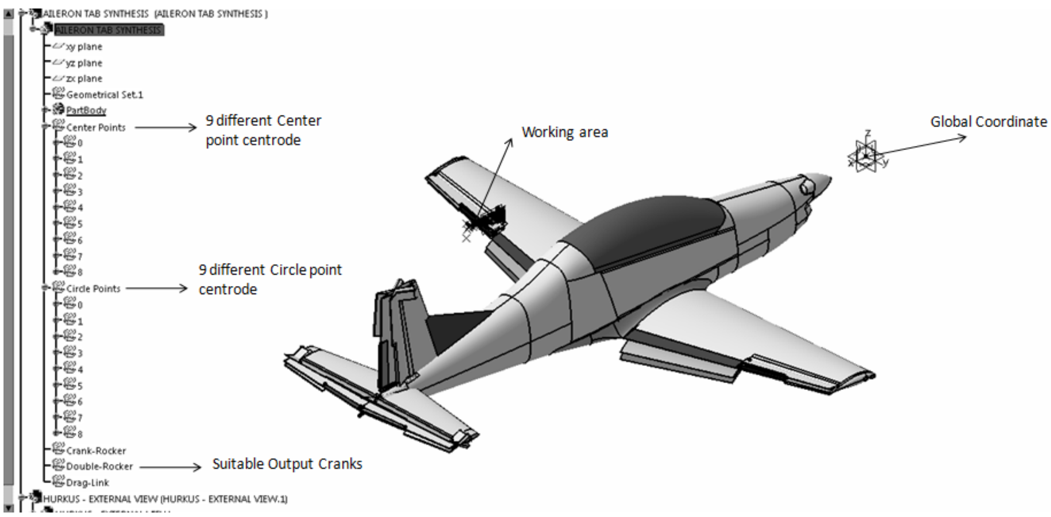


Figure 9 View of SynCAT output in CATIA screen

2.6.2 Interface between CATIA and SynCAT

Communication between CATIA and SynCAT starts by selecting points for desired positions and region of interests. In this step, coordinates of points (X, Y, Z) with respect to CATIAs global coordinates are stored in SynCAT by visual basic code given in Appendix B.

Nevertheless, in order to use the stored points in Burmester theory, they have to be transformed into a new coordinate system (x, y, z) such that all points lie on the –xy plane. After transformation, Burmester curves can be found by easily by Burmester theory. However, points at Burmester curves have to be transformed into CATIAs global coordinate before printing them onto CATIA.

Moreover, printing points at Burmester curve and suitable cranks onto CATIA are done by visual basic code given in Appendix C and D.

CHAPTER 3

MECHANISM SYNTHESIS PROBLEMS AND SOLUTIONS

3.1 NOSE LANDING GEAR DOOR MECHANISM

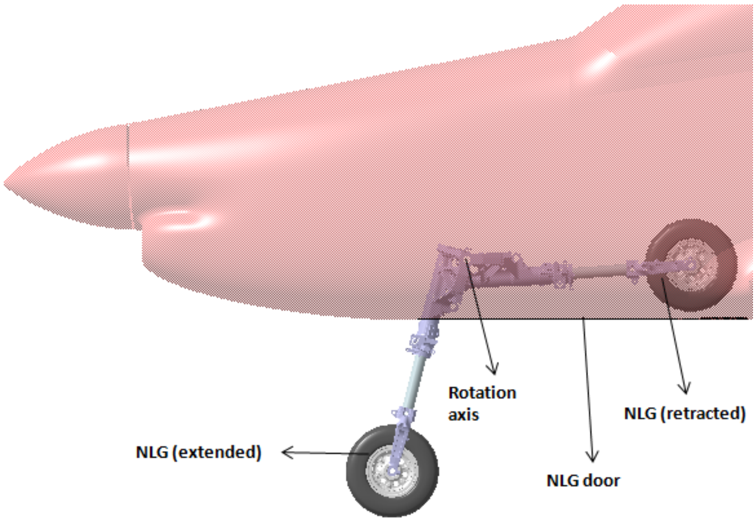


Figure 10 General representation for NLG

3.1.1 Problem Definition

Nose Landing Gear (NLG) is the landing gear of an aircraft which is placed at the front. It is driven by linear hydraulic actuator. As seen in the Figure 10, NLG rotates about its rotation axis and the door is expected to be open while it is rotating.

Some aircrafts use separate actuators to drive only doors which increases cost and weight. Therefore, in this synthesis problem, there is a need to build a mechanism which is driven by NLG itself to open NLG doors.

Since the door rotation axis and NLG rotation axis is not parallel, the mechanism as a whole cannot be considered as a planar mechanism. Therefore, a crank which reciprocates by the motion of NLG can open and close the door which leads to usage of two four-bar mechanisms. In the first step, the 3-D four-bar will be constructed roughly by geometrical and trial-error synthesis method and then results will be used for the second four-bar mechanism as an input.

The NLG rotates 108° about its rotation axis. The door is expected to be closed at the end of motion. Moreover, the door has to be open quickly in order to prevent clash between door and NLG. The problem can be thought as a function generation for four position synthesis. Two positions will be used to satisfy retracted and extracted positions and the other two positions will be used to open doors more rapidly.

3.1.2 Problem Solution (P-P-P-P Alternative – 1)

If two positions for rapid movement are taken as finitely, then the following data set can be used for program as input.

Table 9 Crank rotation correlations for SynCAT for alternative-1

	NLG Rotation (input)	Crank (output)
1st	0°	0°
2nd	-5°	~9°
3rd	-10°	~18°
4th	-108°	0°

In order to find center and circle points for output crank, firstly, one has to define position for circle point for input crank and then secondly, select areas of interests for center and circle points in software as seen in the Figure 11.

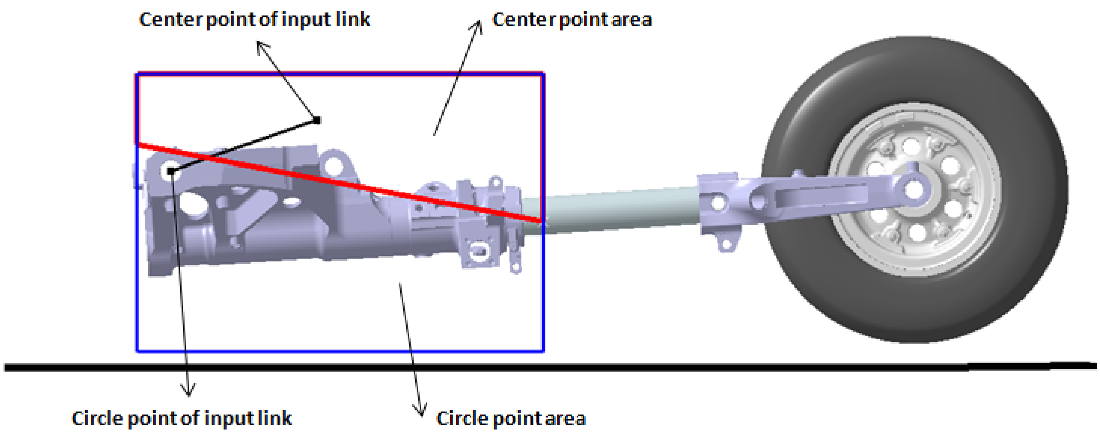


Figure 11 Suitable regions for circle and center points

Therefore, after giving the inputs from CATIA to SynCAT and selecting the "vary 2nd orientation" nine different Burmester curves can be found by varying corresponding rotation angle of NLG by small amounts.

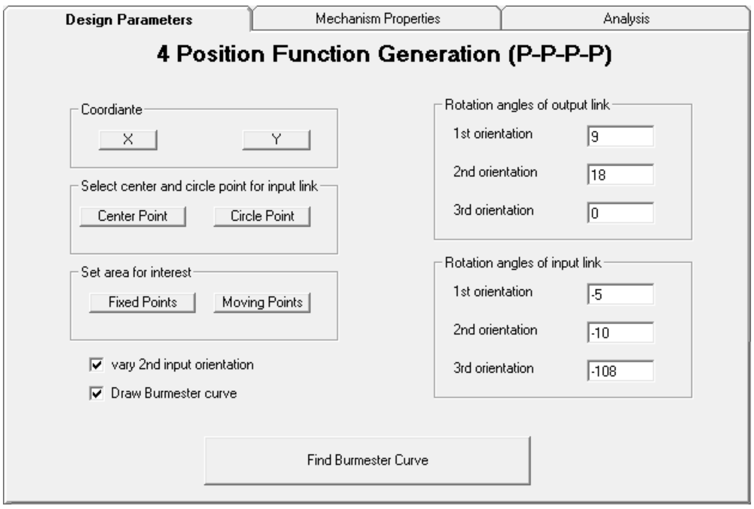


Figure 12 Input screen for P-P-P-P function generation

After Burmester Curves are found and all the sets of Burmester curves are drawn onto CATIA, suitable output cranks are drawn for every set of curve onto CATIA according to the desired mechanism type. It is decided that transmission angle should be greater than 20° and ratio of link lengths should not be greater than 5. Moreover, all kinds of mechanism types are suitable for such an application.

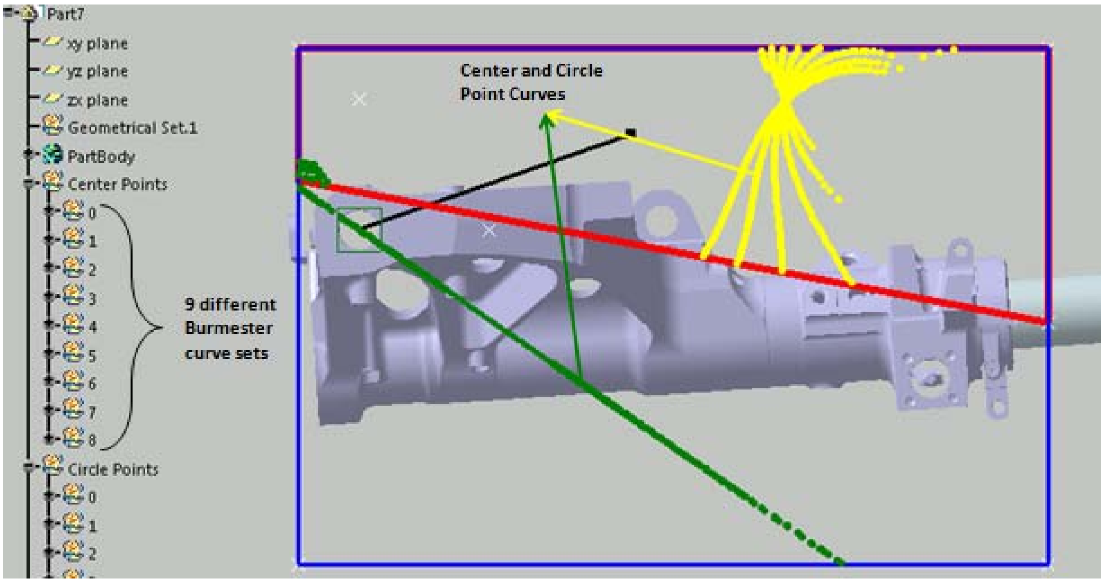


Figure 13 Burmester curve alternatives for NLG door mechanism

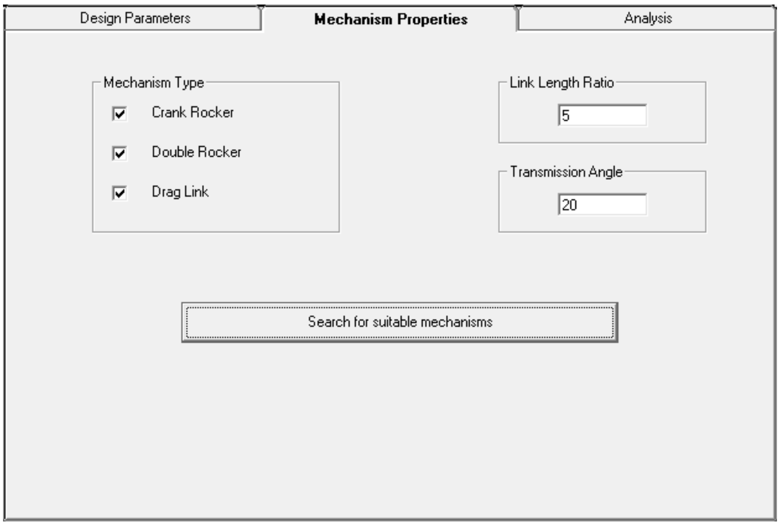


Figure 14 Design criteria entrance screen

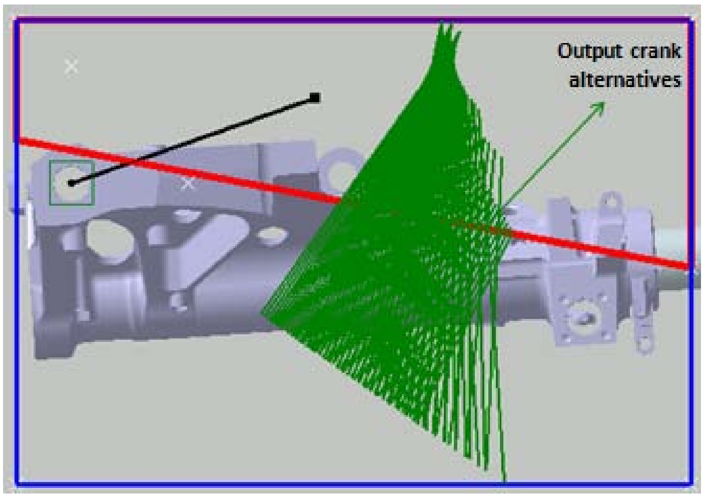


Figure 15 Result of suitable output links

After the all possible solutions are found, the constructed mechanism by most suitable points can be seen in the Figure 16.

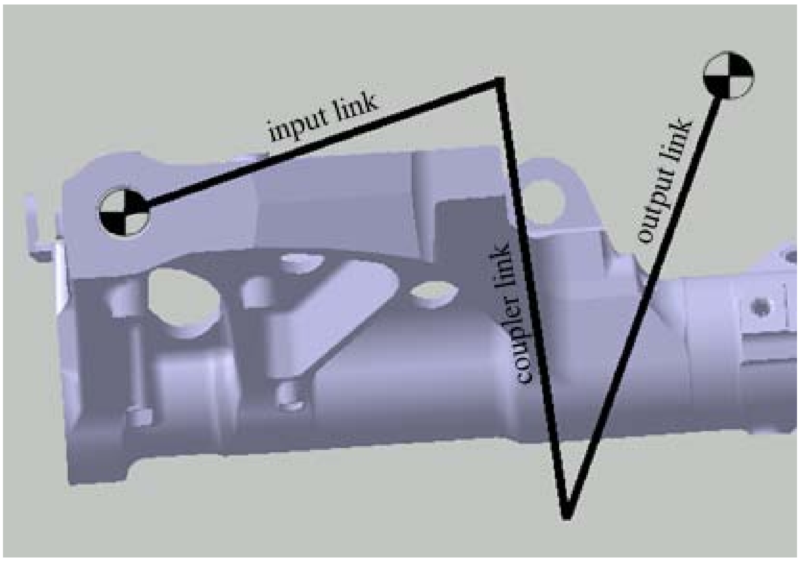


Figure 16 Selected output link and constructed mechanism

Kinematic analysis of constructed mechanism can be performed in SynCAT for coupler link rotation (q13), output link rotation (q14) and transmission angle as shown in Figure 17.

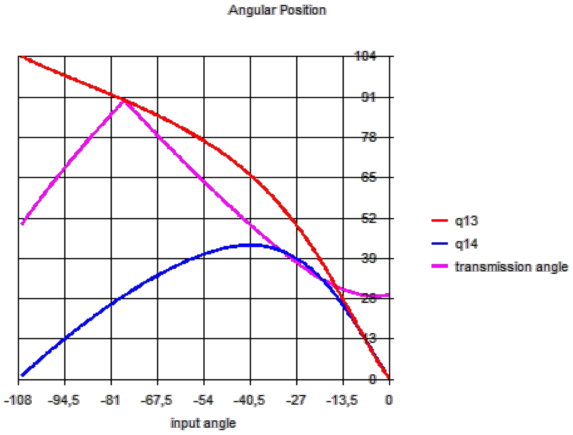


Figure 17 Kinematic analysis of selected mechanism

Demonstration of whole mechanism can be seen in below table.

Table 10 Motion demonstration of obtained mechanism

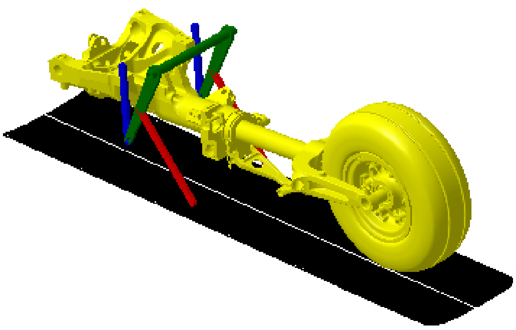
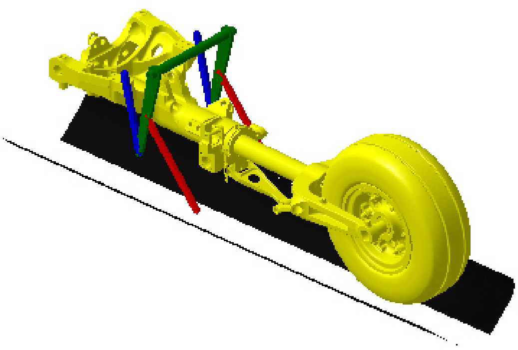
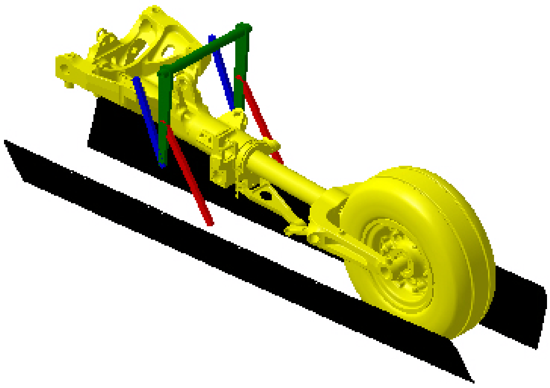
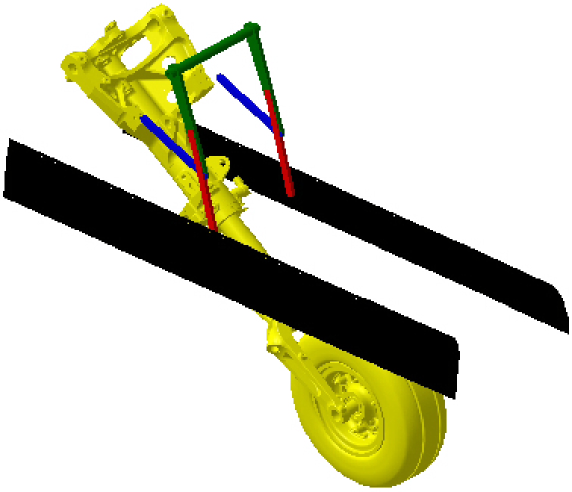
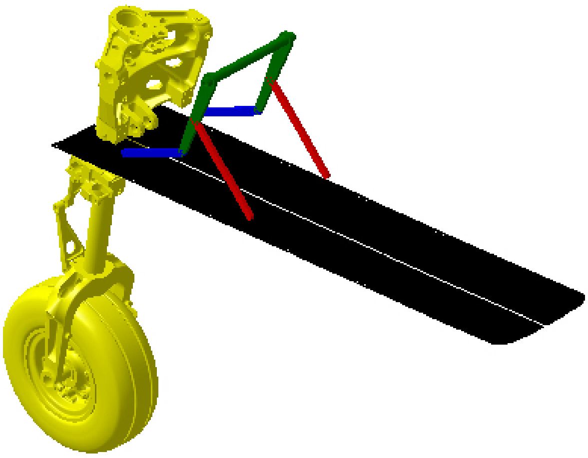
NLG Rotation		Output Angle	Door Rotation
0		0	0
-5		9	37

Table 10 (Cont'd)

-10		18	64
-30		40	100
-108		0	0

3.1.3 Problem Solution (PP-P-P Alternative – 2)

The example studied in section 3.1.1 can be treated in a different way also. Since the first and last positions are mandatory and the mid two positions are to prevent clash between NLG and door, one of the mid finitely defined position can be replaced by infinitesimally defined position which specifies the rotation angles of input and output links. Therefore, the problem can be treated as PP-P-P for four position function generation.

Table 11 Crank rotation correlations for SynCAT for alternative-2

	NLG Rotation (input)	Crank (output)
1st	0°	0°
2nd	$\frac{d\psi}{d\beta} = -0{,}6$ (input/output)	
3rd	-10°	~16°
4th	-108°	0°

If the same input link is selected as the example in section 3.1.1 and the above inputs are entered to the software, the suitable output cranks can be found.

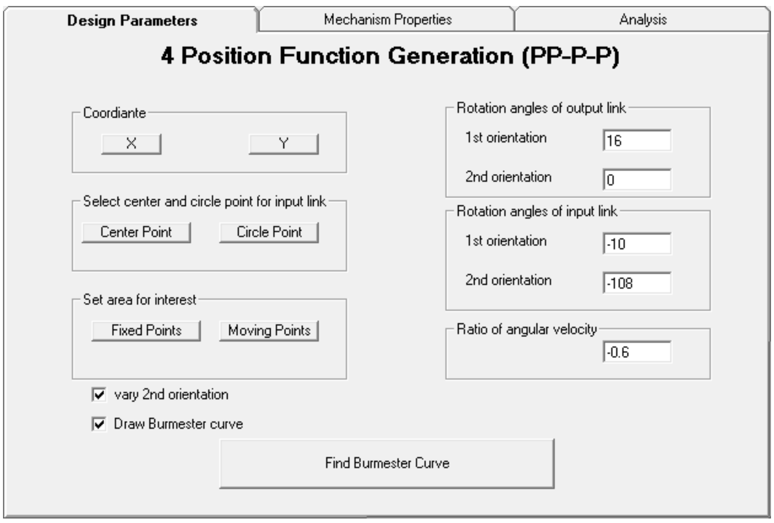


Figure 18 Input screen for PP-P-P function generation

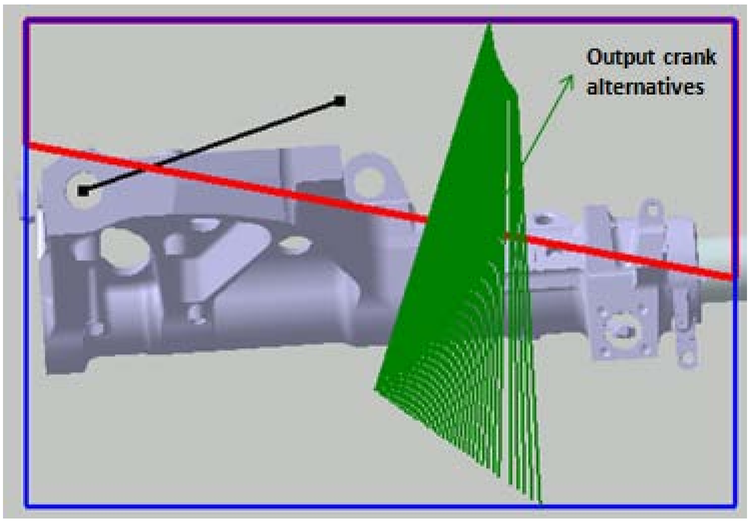


Figure 19 Result of suitable output links

If one of the possible output links is selected, mechanism which satisfies desired conditions can be constructed. Therefore, kinematic analysis of constructed mechanism can be performed in SynCAT for angular velocity ratio of output link to input link (w14), coupler link to input link (w13) and rotational angles of output link (q14), coupler link (q13) as shown in figures below.

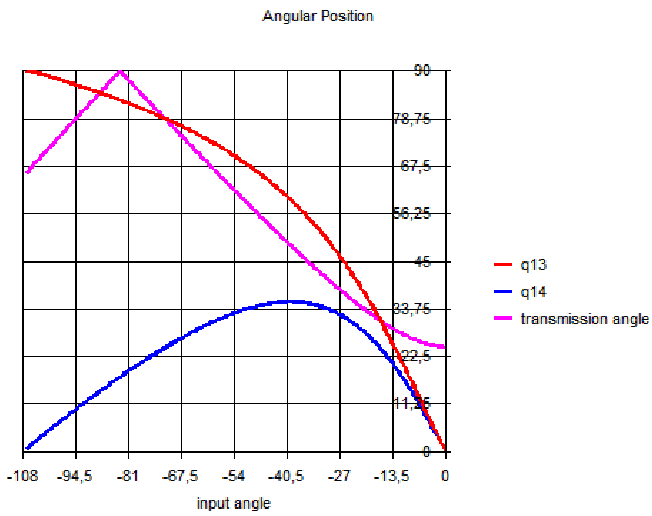


Figure 20 Angular position relation of selected mechanism

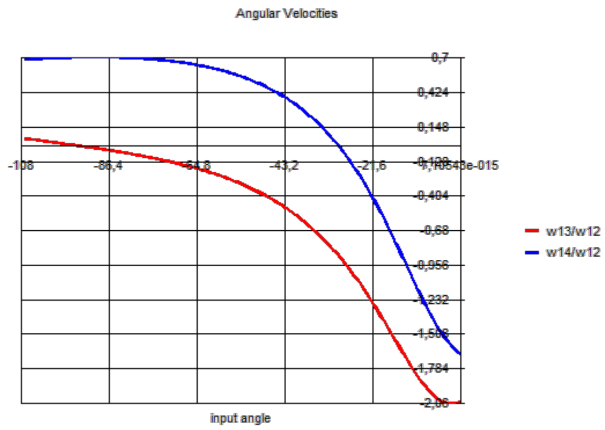


Figure 21 Angular velocity relation of selected mechanism

Demonstration of motion of constructed mechanism can be seen in the figure below.

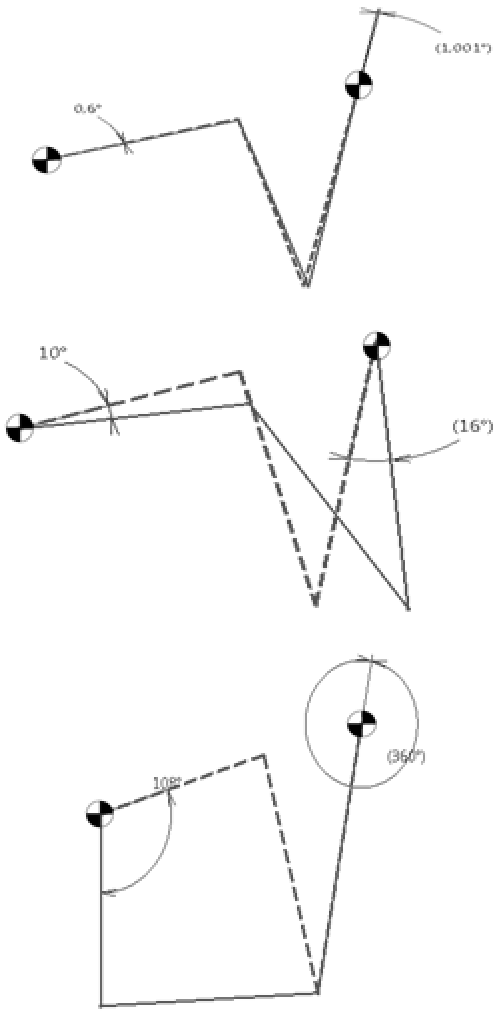


Figure 22 Kinematic analysis of obtained mechanism

3.1.4 Problem Solution (PPP-P Alternative – 3)

The same example can be treated as PPP-P in a third way. The mandatory position is entered as finitely separated position and the rate of first order of rotation angles of input and output cranks is given as in order not to have a clash. Finally, the rate of second order of rotation angles of input and output cranks is given as zero in order to increase the effect of first order. Therefore following table is used for solution of the problem;

Table 12 Crank rotation correlations for SynCAT for alternative-3

	NLG Rotation (input)	Crank (output)
1^{st}	0°	0°
2^{nd}	$\frac{d\psi}{d\beta} = -0.6$ (input/output)	
3^{rd}	$\frac{d^2\psi}{d\beta^2} = 0$ (input/output)	
4^{th}	-108°	0°

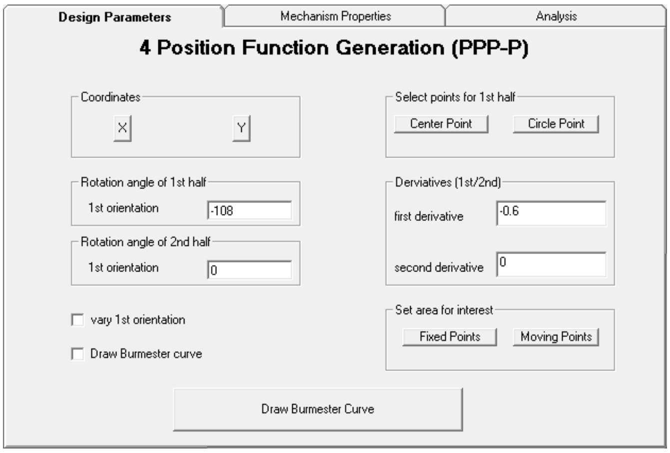


Figure 23 Input screen for PPP-P function generation

If the similar procedure applied at above examples, the possible links can be seen in below;

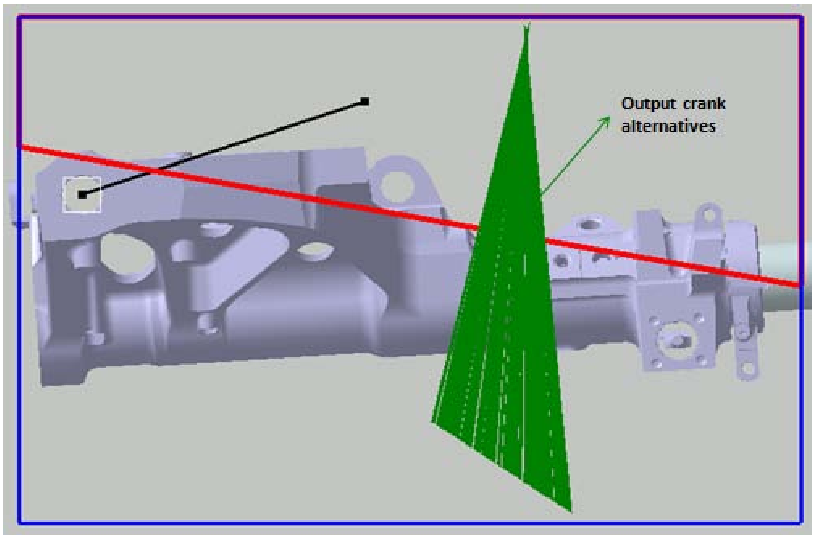


Figure 24 Result of suitable output links

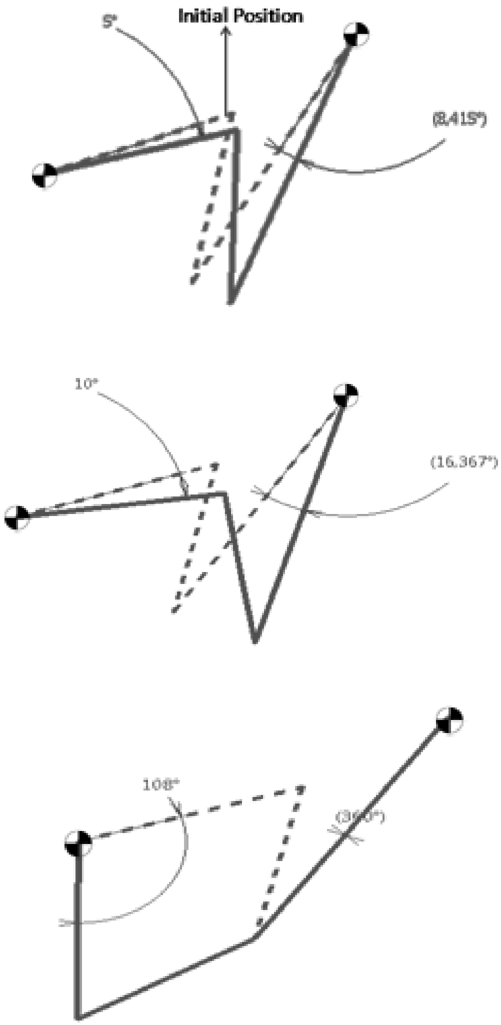


Figure 25 Kinematic analysis of obtained mechanism

If one of the possible output links is selected, the angular velocity, rotational relations between input and output cranks and demonstration of motion can be seen in the Figure 25. As it can be seen, the ratio between output and finitely prescribed positions are satisfied.

3.2 FLAP CONTROL SURFACE MECHANISM

3.2.1 Problem Definition

Flap Control Surfaces are the high-lift devices of an aircraft. The aircraft requires extra lift force at low speeds like take-off or landing conditions and for these situations different flap positions are required.

The flap positions are determined according to the type and mission of an aircraft and for this example, the flaps of HURKUS are concerned. The required positions of flaps are determined as follows;

Table 13 Desired flap rotations

Name of Position	Flap angle
UP	0°
Take-off (TO)	-20°
Landing (LD)	-35°

Since the positions are not about an axis, the problem cannot be thought as a function generation. The problem can be thought as a P-P-P motion generation synthesis, however, in order to draw a Burmester curve and see the possibilities easier, P-P-P-P motion generation synthesis will be used for this example by adding an extra flexible prescribed point. Therefore, the prescribed positions of flap control surface that will be used in the software are as follows.

Table 14 Flap rotations input for SynCAT

Name of Position	Flap angle
UP	0°
Take-off (TO)	-20°
Flexible Position (FP)	~-27.5°
Landing (LD)	-35°

3.2.2 Problem Solution

The flap mechanism will be driven by a four-bar mechanism. Therefore, the fixed pivots shall be close to spar in order to make proper fittings. In addition to that, the moving pivots which will be connected to flap control surface should be selected so that they will not be too far from aircraft external surface. If they will be selected away from the external surfaces, the aircraft will be subjected to extra drag force.

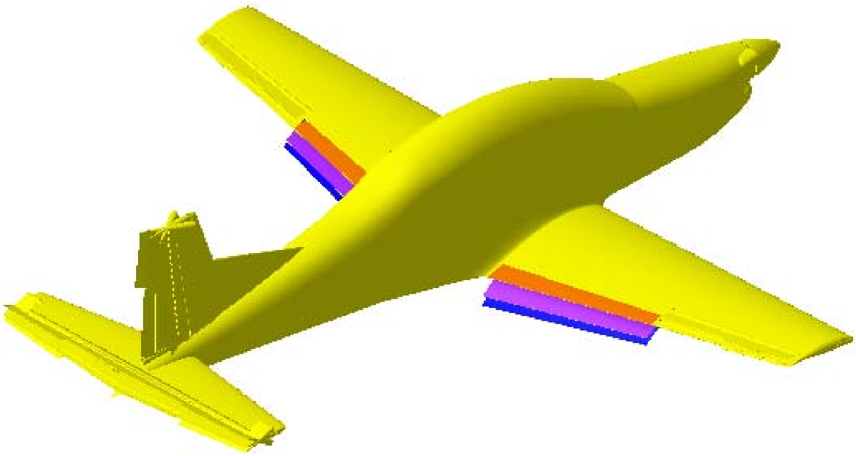


Figure 26 General view of flap control surface

Moreover, in order to examine the problem as P-P-P-P, the third position will be considered approximately between second and fourth position as mentioned above, so, the problem schematic becomes as follows;

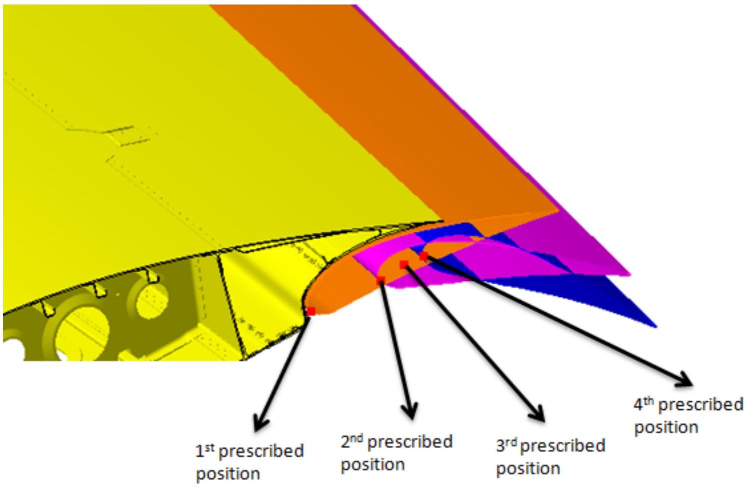


Figure 27 Problem schematic

The red points show the leading edge of the flap at corresponding positions. Then, designer selects the points from CATIA in the SynCAT and enters the corresponding flap angles. Moreover, in order to increase the possible mechanism, one can select "vary 2nd orientation". By clicking this option, the software will be able to find different Burmester curve by changing the orientation of flap control surface at the third position.

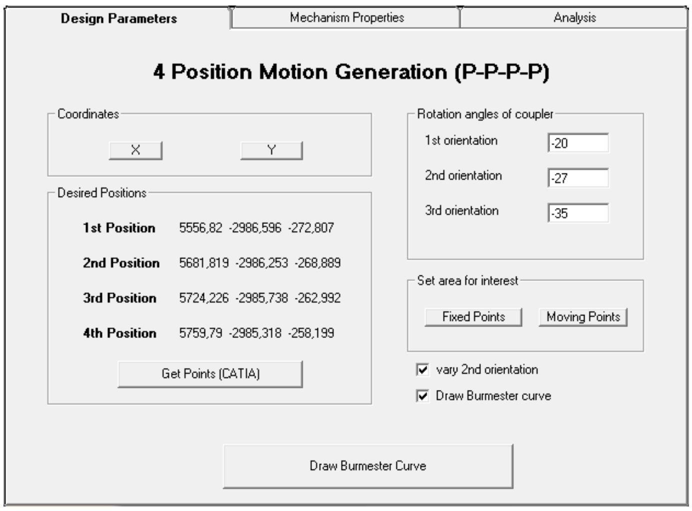


Figure 28 Input screen for P-P-P-P motion generation

After the positions are determined, the areas for fixed and moving pivots have to be determined in order to eliminate unwanted mechanism options.

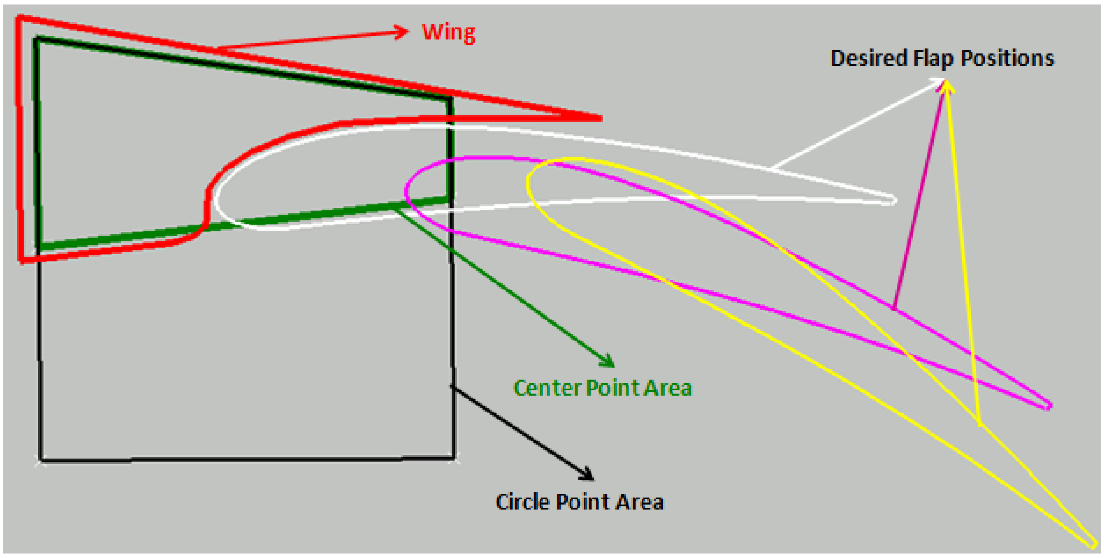


Figure 29 Cross-sectional view of flap control surface and interested regions

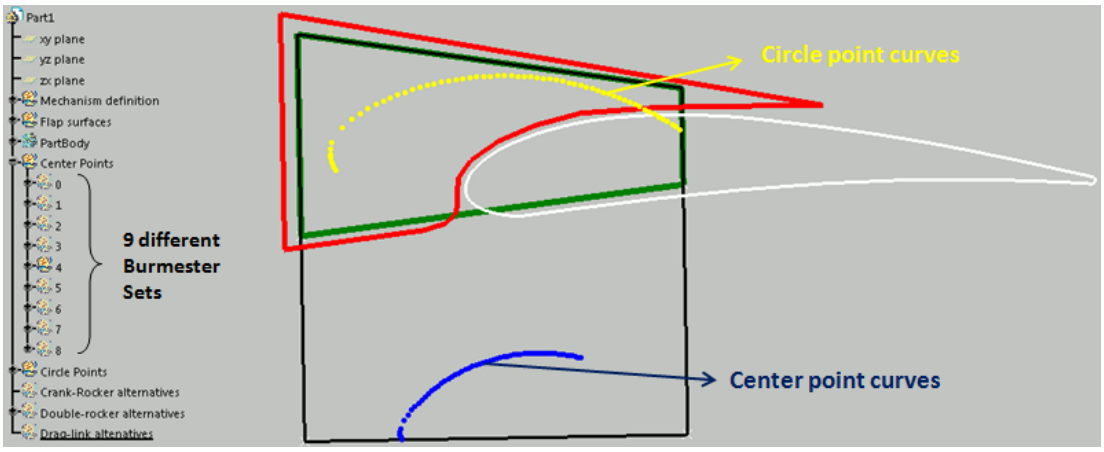


Figure 30 Suitable points for moving and fixed points

The area enclosed by green lines is for center points in order to connect them properly to structure and the area enclosed by the black lines is for circle points. The area for circle points is not selected bigger in order not to increase drag as mentioned above. Therefore, the eight different Burmester curve is obtained according to the given inputs as shown in the figure above.

After the Burmester curves are drawn onto CATIA, the designer decide which curve is most suitable for him and then select the center point for one branch and seek the other suitable branch by entering the design parameters.

The type of mechanism can be crank-rocker, double rocker or drag-link. In addition to that, the transmission angle shall not be less than 30 degrees and the link length ratios shall not be larger than five. According to those design parameters, the designer can select a circle point from 4th set and find the possible mechanisms.

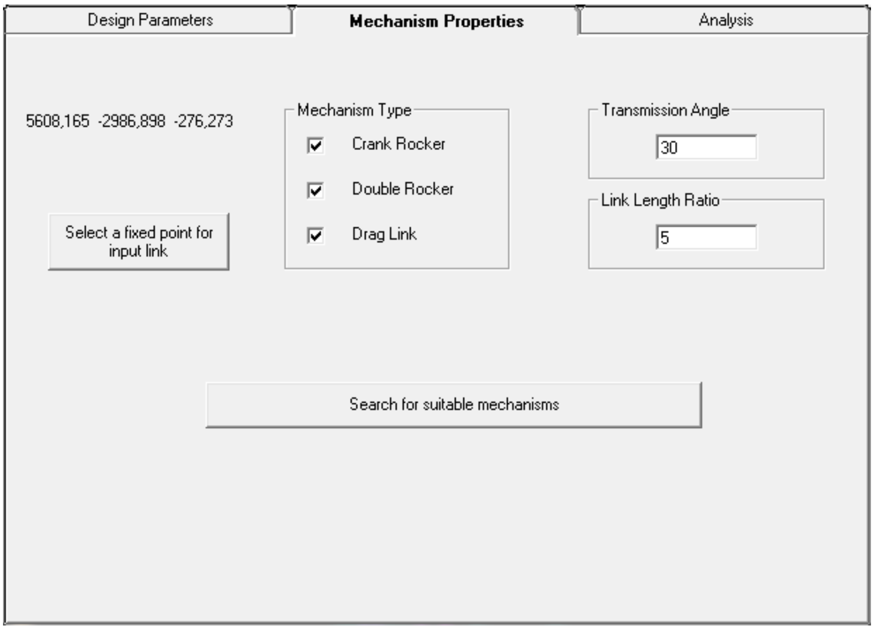


Figure 31 Design criteria entrance screen

As a result, all possible alternatives are shown in CATIA screen as shown in figure below. Therefore, the user can build a mechanism by selecting one of possible output-link which will have a transmission angle greater than 30 degrees during motion.

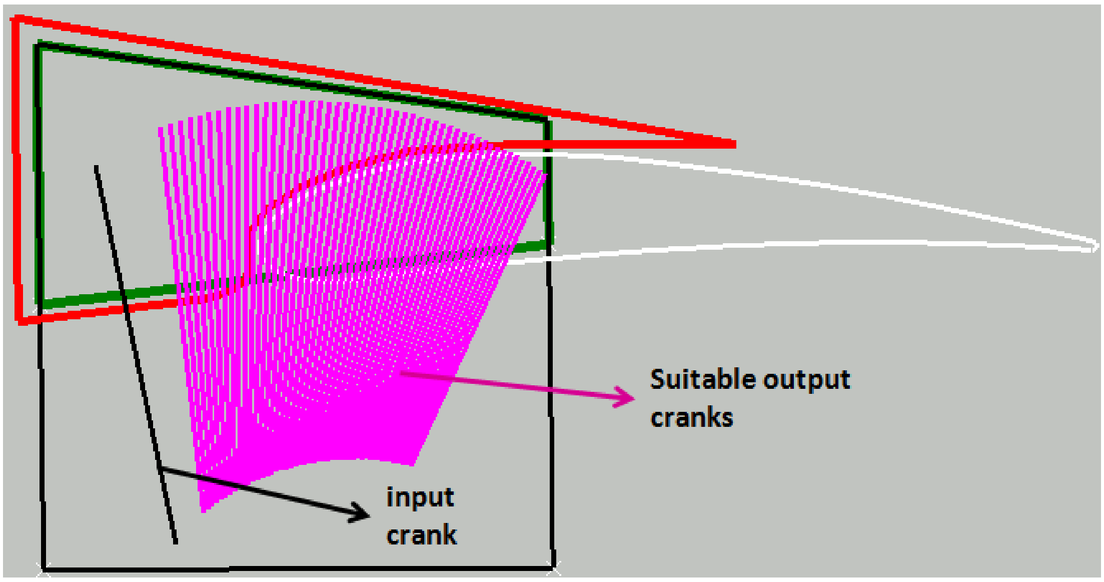


Figure 32 Output link alternatives for selected input link

After building a mechanism as an example, kinematic analysis of constructed mechanism can be performed in SynCAT for output link rotation (q14) and coupler link rotation (q13) as shown in figure below.

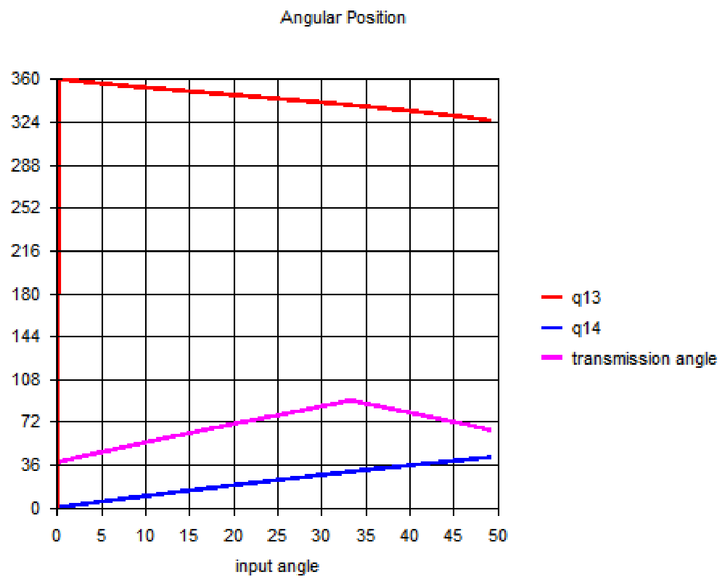


Figure 33 Kinematic analysis of selected mechanism

Demonstration of motion of constructed mechanism can be seen in the figure below.

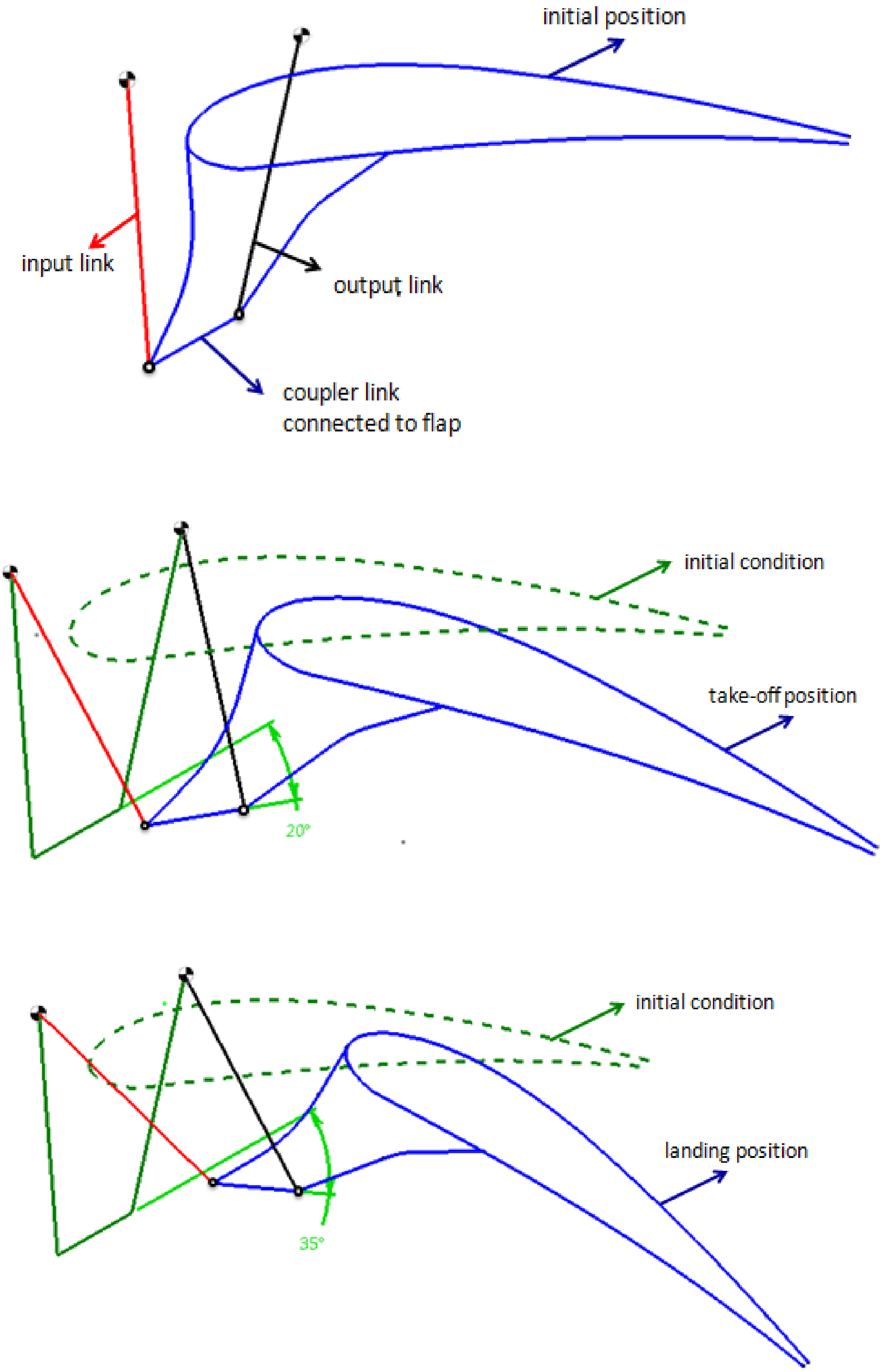


Figure 34 Motion demonstration of obtained mechanism

3.3 PRIMARY CONTROL SURFACE MECHANISMS

3.3.1 Problem Definition

Primary control surfaces are aileron, elevator and rudder which are mainly used to give a roll, pitch and yaw motion to an aircraft respectively. These control surfaces are controlled by the pilot in the cockpit and the motion is generally given by control stick or pedals. According to the aircraft performance and pilot comfort, the motion of control surfaces and motion of pedal or stick are determined.

In general, stick or pedal motion is considered as input and the control surface motion considered as output. Therefore, these mechanisms can be thought as function generation. As a result, the construction of them can be done by using the software. However, since the places of control surfaces are far away from the cockpit, the stick or pedal are connected to control surfaces by many linkages or cables for some aircrafts which are not fly-by-wire. This software also can be used for such designs for some portion or for whole of that. In this example, the usage of this software in such designs will be demonstrated for elevator mechanisms. The orientations of control surfaces and stick are given below for illustration.

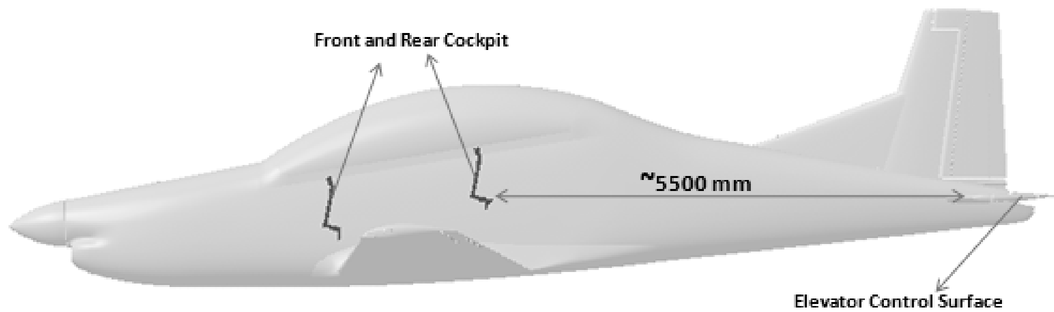


Figure 35 General scheme for elevator control system

The elevator control surface has to rotate 30° degree upward (CW) and 20° downward (CCW) for stick rotation 18° to back (CCW) and 14° to forward (CW).

As seen in the figure, the stick and the elevator are distanced from each other and they have to be connected with several rods and cables. With the help of push-pull rods and suitable bell cranks, the desired function correlation will be obtained.

In this design, two four bar mechanism will be constructed and then they will be connected by cables. The first one will be correlated with the stick motion and the other one will be correlated with elevator control surface such that the output links of both mechanisms will rotate same amount, but in different directions in order to obtain 30° CW rotation of elevator at 18° CCW rotation of stick. In addition to that, the amount of rotations of cranks will be increased gradually to obtain elevator control surface rotation. Therefore, the following tables show the orientation of cranks.

Table 15 First mechanism design input

	Stick Rotation	1^{st} Mechanism output link
Lowermost position	14	-20
Neutral	0	0
Approximate position	-11	+15
Uppermost position	-18	+24

Table 16 Second mechanism design input

	Elevator Rotation	2^{nd} Mechanism output link
Lowermost position	-20	-20
Neutral	0	0
Approximate position	+17	+15
Uppermost position	+30	+24

As a result, the mechanisms will be considered at their lowermost position for full motion. So, the following tables will be used for synthesis problem.

Table 17 Summary of the desired motion of mechanism

	Stick Rotation	1st Mechanism output link	Elevator Rotation	2nd Mechanism output link
1st orientation	-14	20	20	20
2nd orientation	-25	35	37	35
3rd orientation	-32	44	50	44

3.3.2 Problem Solution

In the first mechanism, the input link is stick and the suitable area for circle points is determined according to the allowable space in the fuselage and center points area is drawn near to floor in order to make proper fitting.

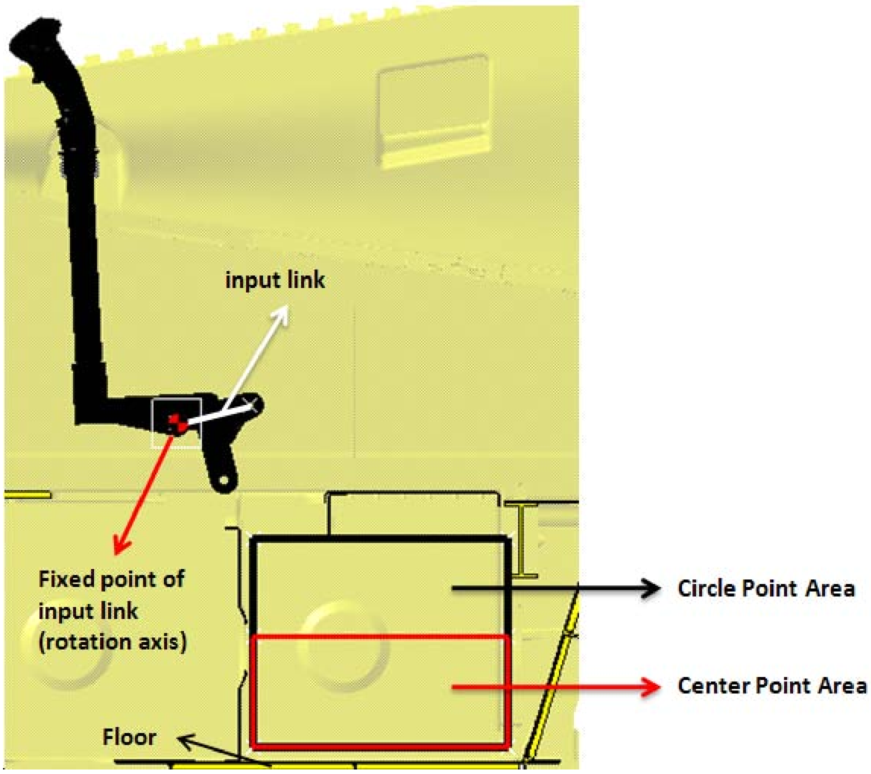


Figure 36 Suitable regions for circle and center points

After the rotations of cranks shown in Table 17 and the available areas for circle and center points entered in the software, designer selects mechanism type and transmission angle and link length ratio in order to eliminate unwanted results.

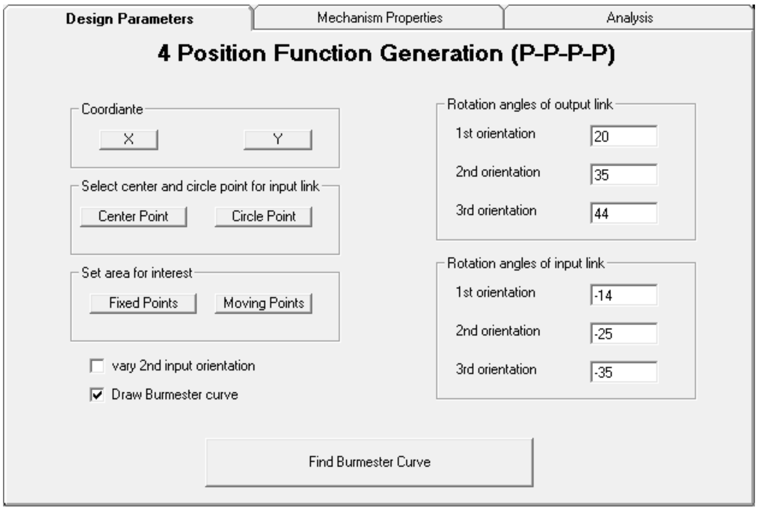


Figure 37 Input screen for P-P-P-P function generation

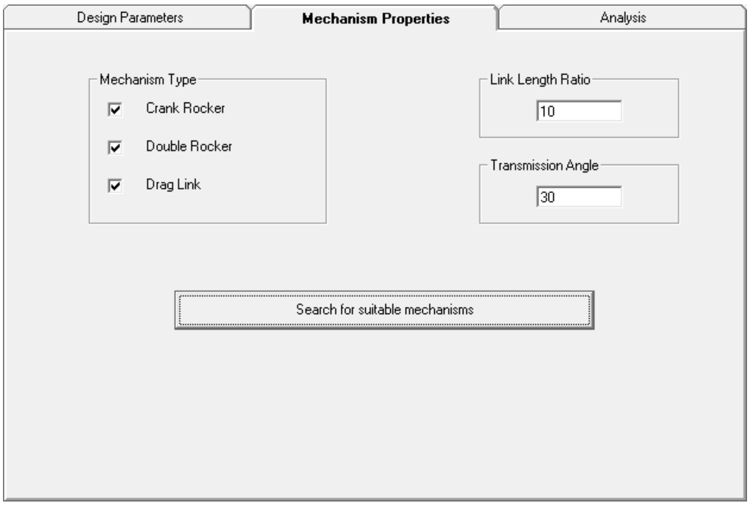


Figure 38 Design criteria entrance screen

After the software is run, the results can be seen in the CATIA screen and with the most appropriate crank the first four-bar mechanism can be constructed.

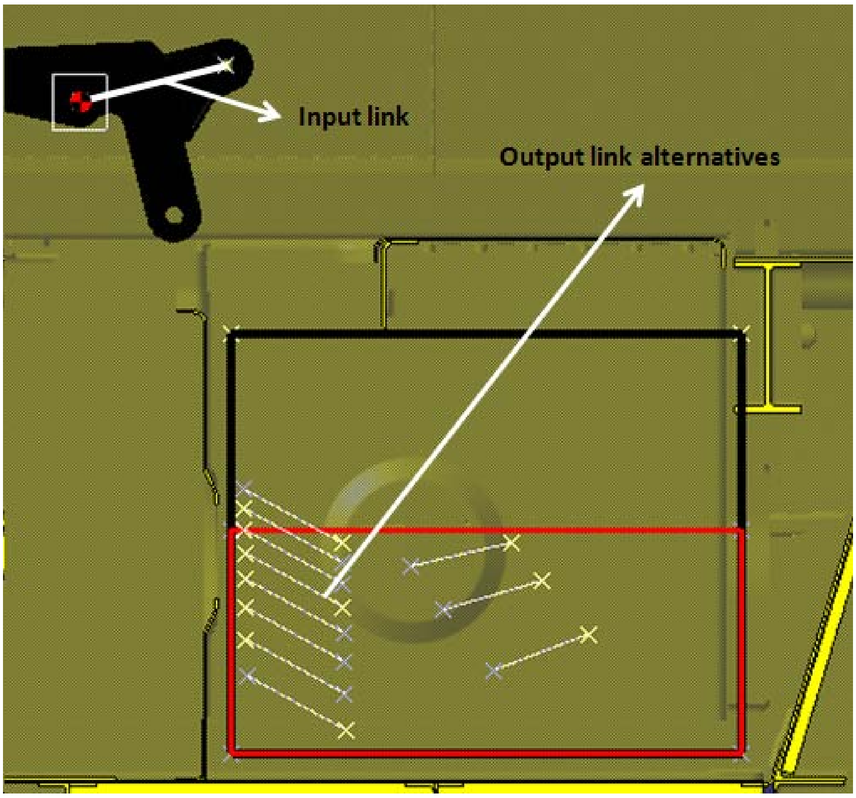


Figure 39 Suitable output links according to the selected input link

The second four-bar mechanism will be constructed due to the output link of the first mechanism. With the same procedure applied in the first mechanism, the second four-bar can be constructed as shown figures shown below.

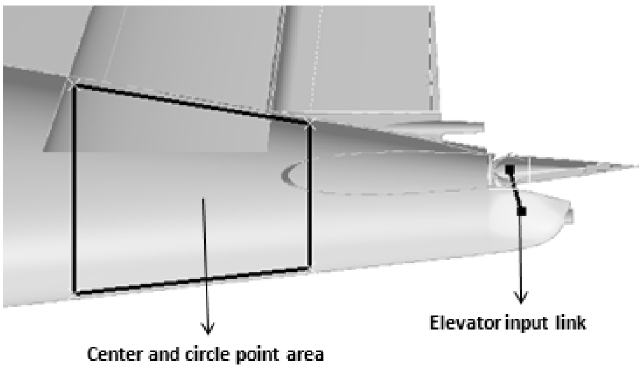


Figure 40 Suitable region for center and circle points

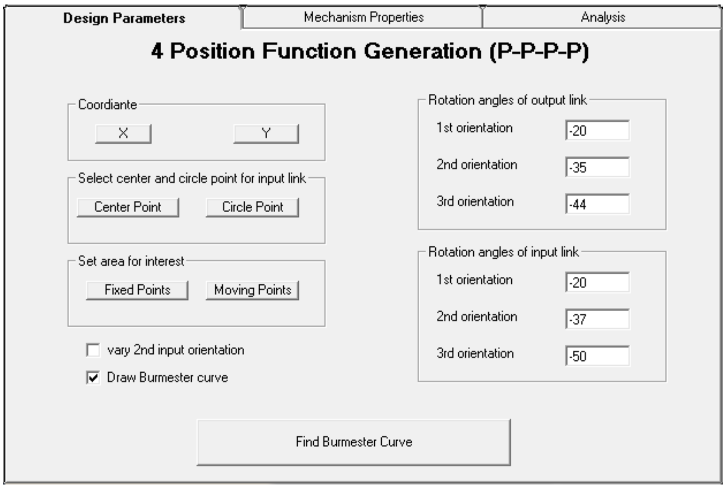


Figure 41 Input screen for P-P-P-P function generation

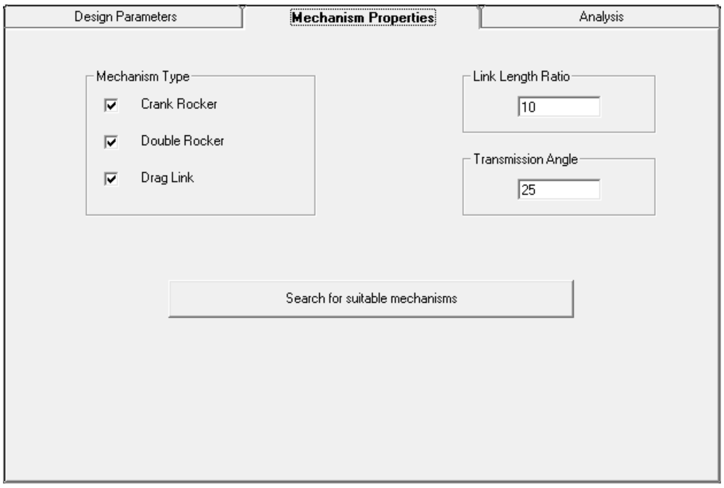


Figure 42 Design criteria entrance screen

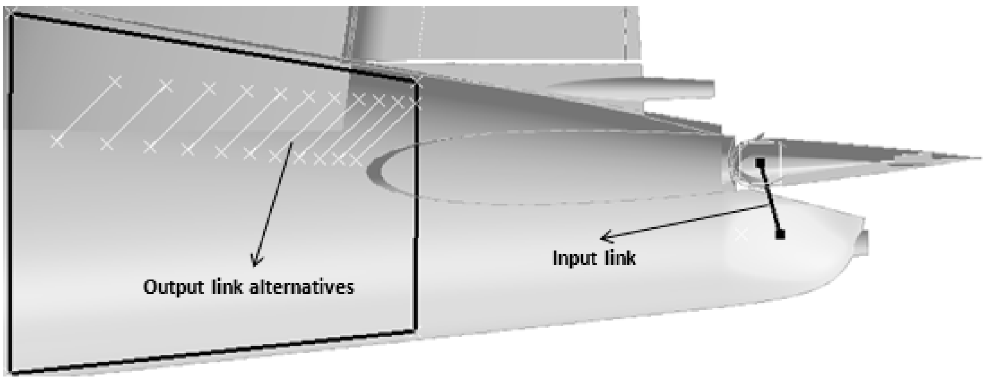


Figure 43 Suitable cranks for obtained link

As in the first mechanism, with the most appropriate crank, the second four-bar can be constructed. After that, the first and second mechanism can be connected to each other by cables, so, that, the synthesis of whole system accomplished.

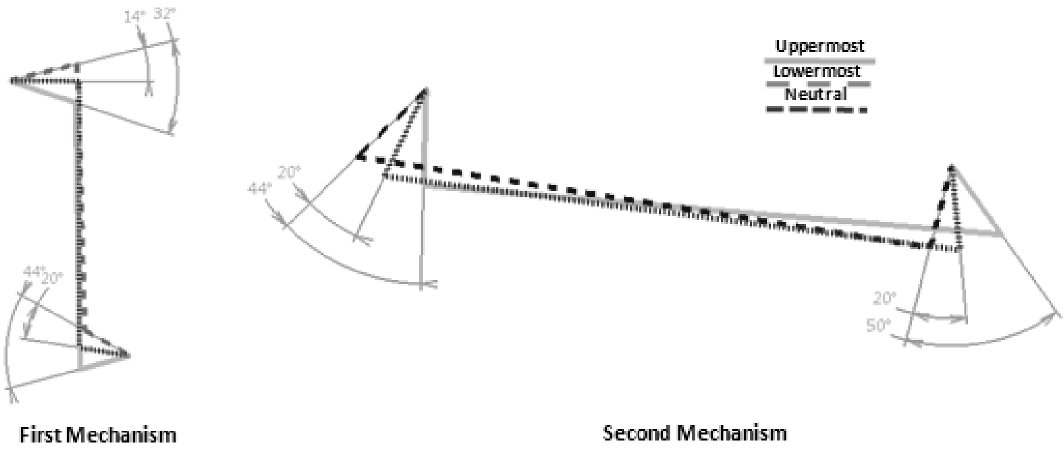


Figure 44 Demonstration of motion of mechanism

3.4 ACCESS DOOR MECHANISM

3.4.1 Problem Definition

In the air vehicles, there are some removable panels on the outer skin for maintainability and inspection requirements of parts and equipments. With these panels, the parts in the air vehicle can be visually checked and/or replaced easily. These panels can be completely removal or simply hinged to structure or designed with some properties for some special purposes. These special designed access panels are generally used for maintainability of large and important equipments like motor of an aircraft and flir of a helicopter.

In this example, a special access panel for a helicopter will be studied. In this design the access panel is desired to be opened 90° degree and has to maintain its open position for disturbances on the access panel for maintainability process. This can be achieved by a four-bar mechanism with its one of the dead center positions. In such a case, since the system is in a dead-center position, mechanism is not movable by the inputs given from access panel. The access panel can only be closed by the BB_0 link.

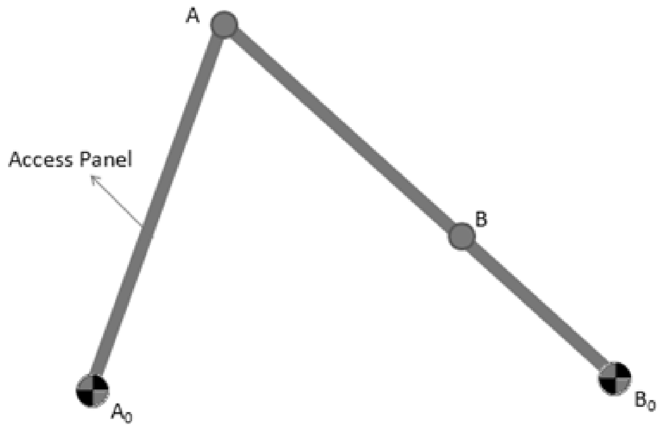


Figure 45 Dead-center configuration for four-bar

This position occurs where the angular velocity of the output link changes sign namely where the angular velocity of a crank is zero. For this problem, there are two finitely prescribed positions for opened and closed cases. In addition to that, there is one infinitesimally prescribed position which shows the sign change for opened case. If one arbitrary finitely prescribed position is selected as an extra, the problem can be solved for PP-P-P function generation.

3.4.2 Problem Solution

In this problem, since the rotation of access door is defined, one needs to specify a center point for access door. After that, suitable regions for center and circle points for other cranks have to be determined.

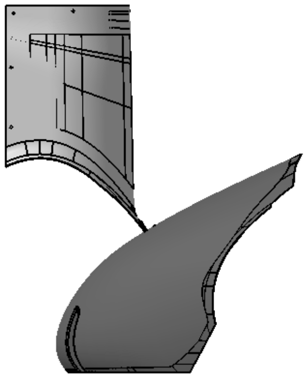


Figure 46 Open configuration for access panel

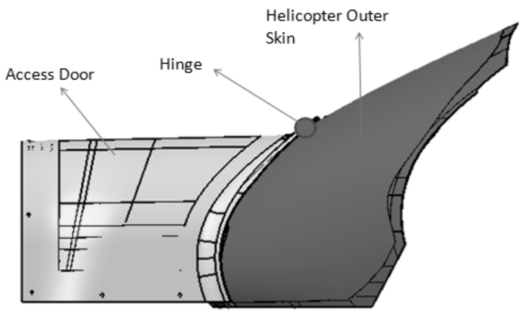


Figure 47 Closed configuration for access panel

Center point region is selected so that, they will be near to outer skin of helicopter in order to make a proper fitting (shown in blue in below picture) and they will be inside the helicopter when the access door is closed. Circle point region is also selected as they will remain inside the helicopter.

After determining the suitable regions for circle and center points, the prescribed positions are entered into the software. After a few trials, the arbitrary prescribed position is determined for most suitable outputs. The following tables summarize the prescribed positions of the desired mechanism.

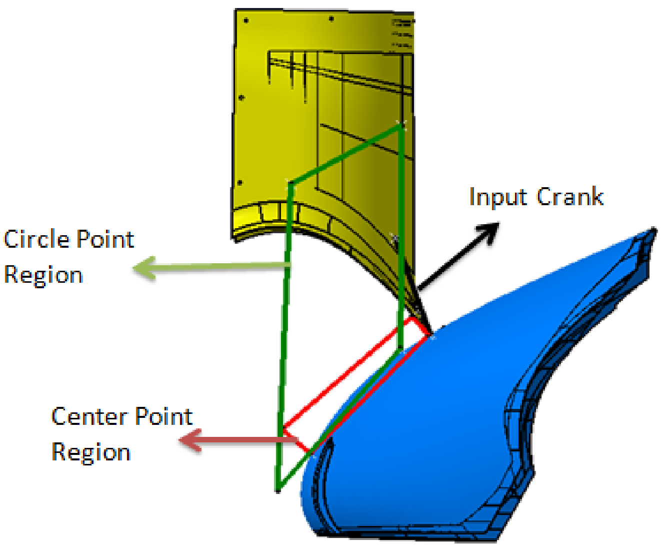


Figure 48 Suitable regions for center and circle points

Table 18 Input for SynCAT

	Access Panel	Crank
First Prescribed Position (Opened Case)	0°	0°
Second Prescribed Position	$\left.\frac{d\varphi}{d\beta}\right\|_{\beta=0} = 0$ (input/output)	
Third Prescribed Position (Arbitrary)	45°	70°
Fourth Prescribed Position (Closed Case)	90°	110°

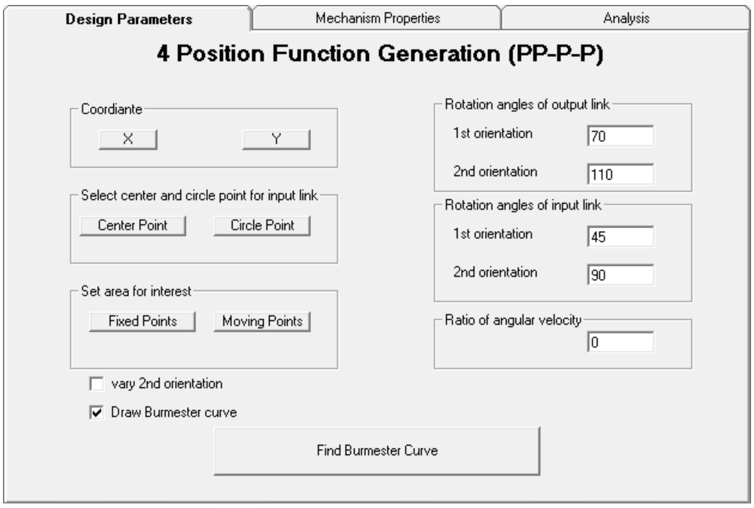


Figure 49 Input screen for PP-P-P function generation

As a result, by clicking "draw Burmester curve" options the center and circle points can be obtained. Designer can select most suitable points and build a four-bar mechanism which accomplishes the desired task.

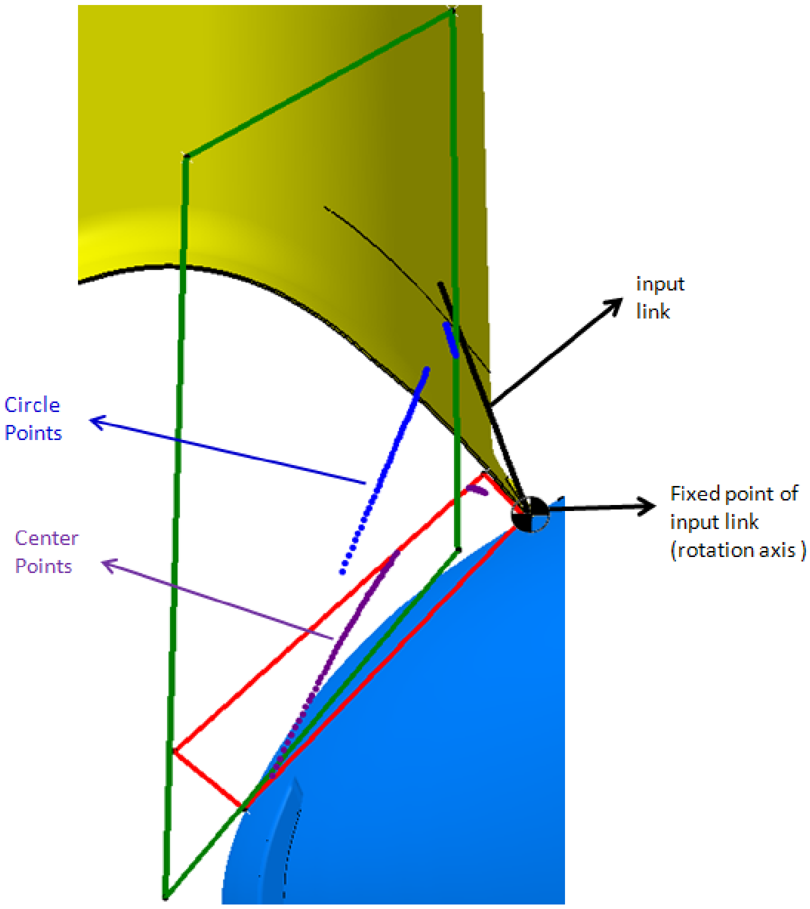


Figure 50 Suitable cranks for selected link

After the center and circle points are selected the mechanism can be constructed which achieves all desired prescribed position as seen in the below figure.

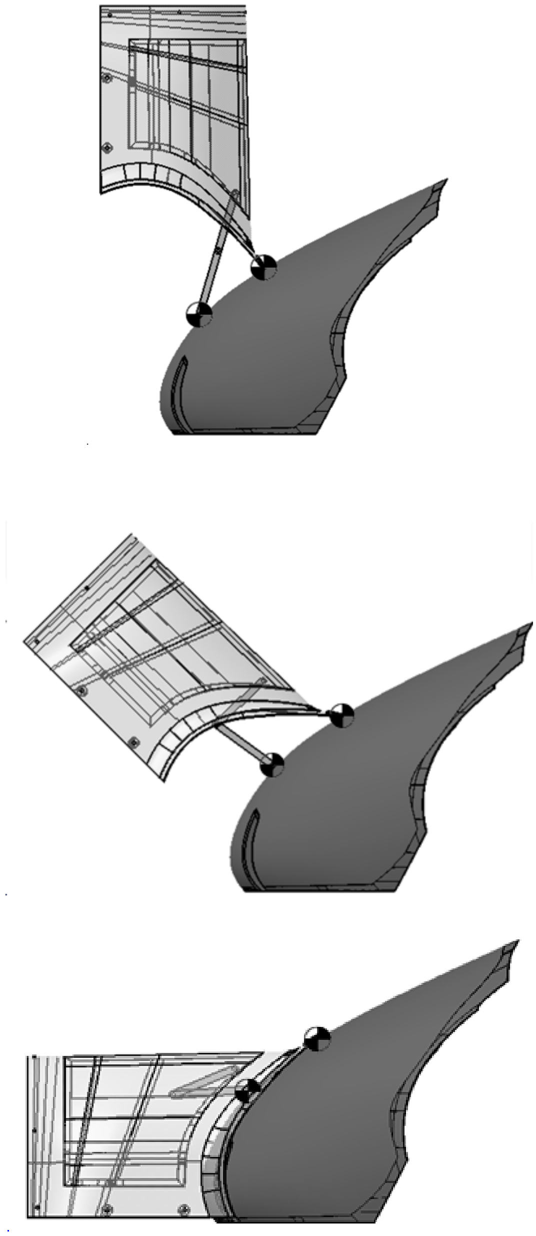


Figure 51 Kinematic analysis of obtained mechanism

3.5 MISCELLANEOUS TYPES OF MECHANISMS

3.5.1 Approximation of a straight line

In some applications like conveyers and cutting machines, the working part is desired to move along a line for a period of time. To illustrate such an example, a cutting machine will be used. In a cutting a machine, a circular knife cuts object with moving back and forth by just rolling without slipping.

Since the rotation of knife is important and to approximate a straight line motion, PPPP motion generation synthesis case is selected. If the radius of knife is r and the rotation angle is α in radians, the knife travels on the cutting table is α.r.

$$\vec{\delta} = -\alpha r$$

The first prescribed position is taken as the current position of the knife and the other prescribed positions are as follows;

First Prescribed Position

$\delta_1 = 0$

Second Prescribed Position

$$\delta_2 = \left.\frac{d\vec{\delta}}{d\alpha}\right|_{\alpha=0} \qquad \delta_2 = -r$$

Third Prescribed Position

$$\delta_3 = \left.\frac{d^2\vec{\delta}}{d\alpha^2}\right|_{\alpha=0} \qquad \delta_3 = 0$$

Fourth Prescribed Position

$$\delta_4 = \left.\frac{d^3\vec{\delta}}{d\alpha^3}\right|_{\alpha=0} \qquad \delta_4 = 0$$

According to these parameters, obtained circle and center points can be seen in Figure 53.

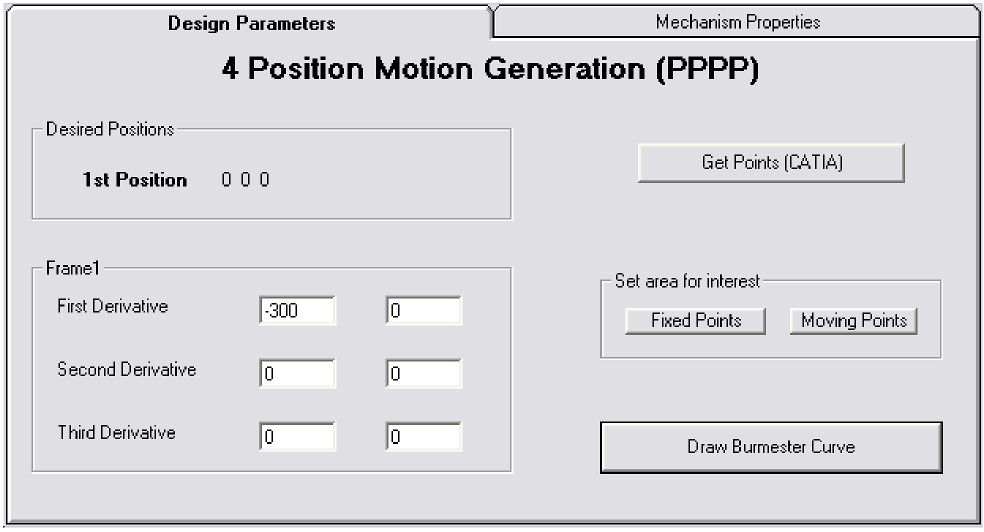


Figure 52 Input screen for PPPP motion generation

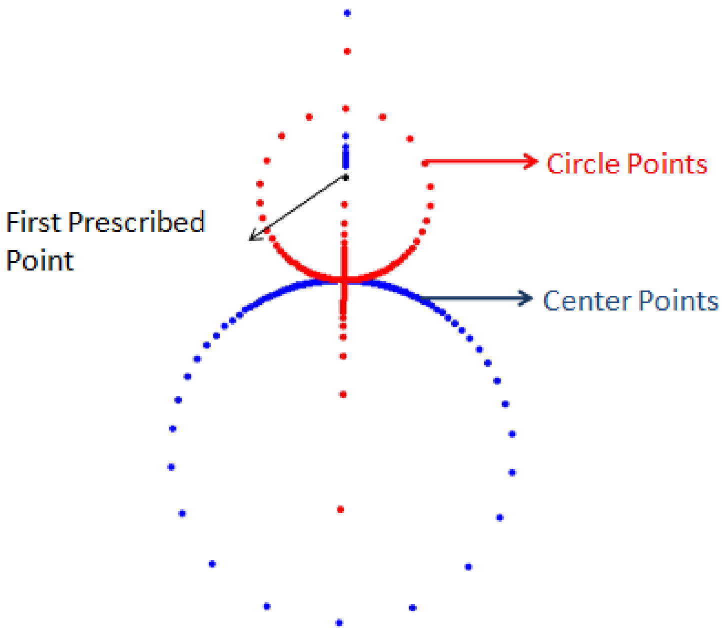


Figure 53 Center and Circle points for desired mechanism

Therefore, according to the mechanism properties and selected center point for one branch, the other cranks are obtained. The obtained mechanism can be seen in the figure below.

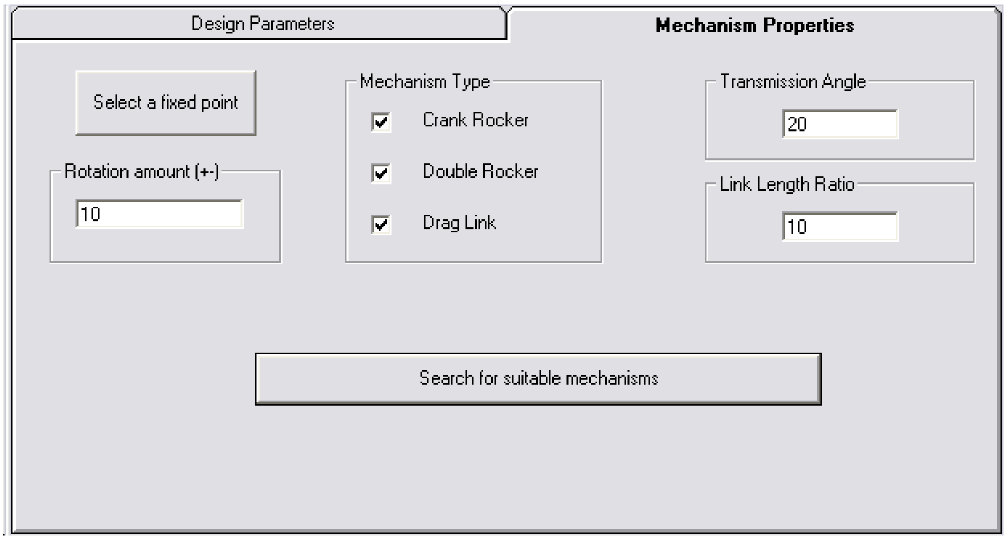


Figure 54 Mechanism properties entrance screen

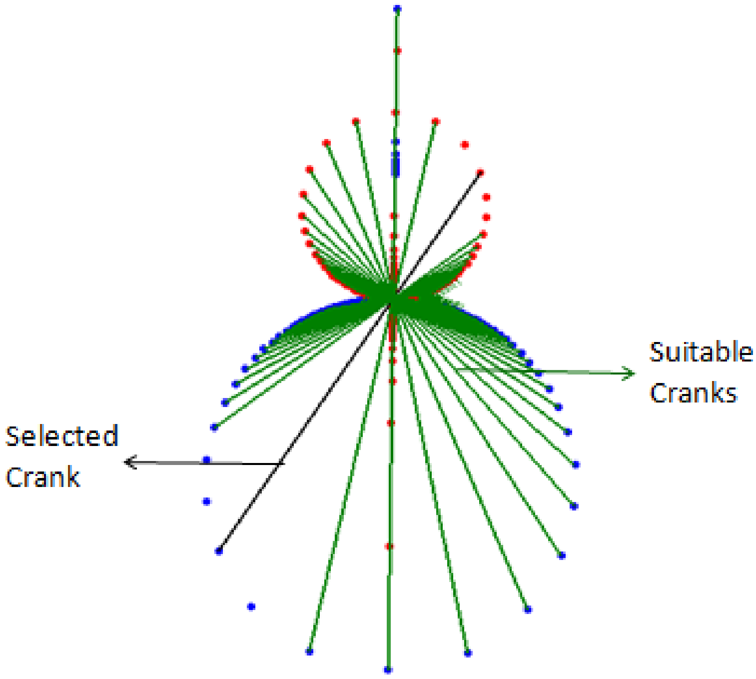


Figure 55 Suitable cranks according to the selected fixed point

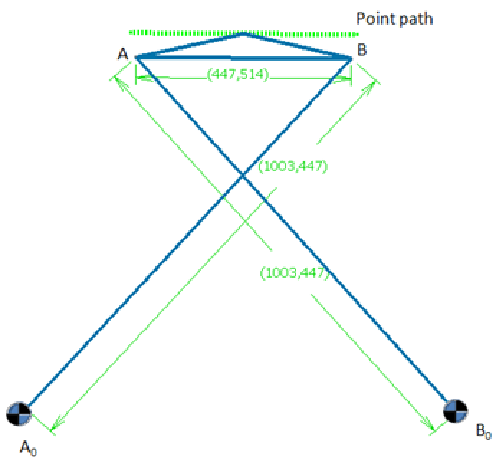


Figure 56 Constructed mechanism and straight line path

3.5.2 Correlation of crank angles

In some applications, a constant ratio between input and output motions may be desired like in epicyclic gear trains. In order to obtain such a mechanism, the functional relationship between the arm and the planet gear of the gear train will be studied and to be more precise, the relationship between input and output approximated to the third. Therefore, the synthesis problem can be treated as PPPP function generation.

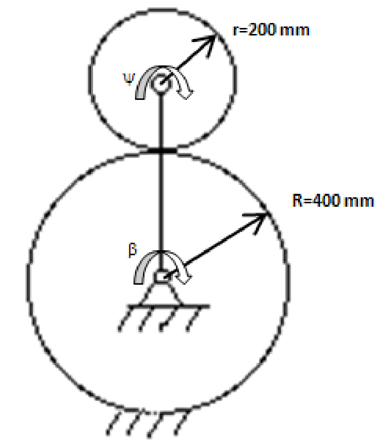


Figure 57 Epicyclic gear train

As it can be seen from above figure, with pure rolling condition, the linear relation relationship between links can be taken as $\psi = \frac{R+r}{r}\beta$.

Therefore the prescribed positions are as follows;

<u>First Prescribed Position</u>

$\delta_1 = 0$

<u>Second Prescribed Position</u>

$$\delta_2 = \frac{d\psi}{d\beta} \qquad \delta_2 = \frac{R+r}{r} \qquad \delta_2 = 3$$

<u>Third Prescribed Position</u>

$$\delta_3 = \frac{d^2\vec{\psi}}{d\beta^2} \qquad \delta_3 = 0$$

<u>Fourth Prescribed Position</u>

$$\delta_4 = \frac{d^3\psi}{d\beta^3} \qquad \delta_4 = 0$$

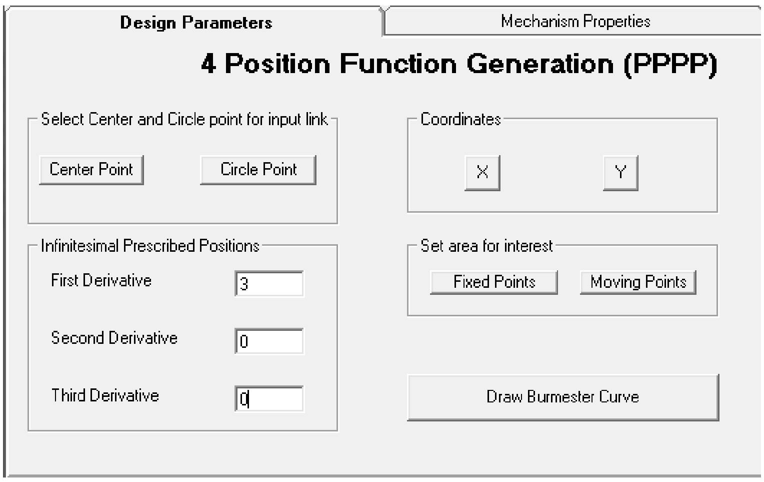


Figure 58 Input entrance screen for PPPP function generation

According to these parameters, obtained circle and center points can be seen in the below figure.

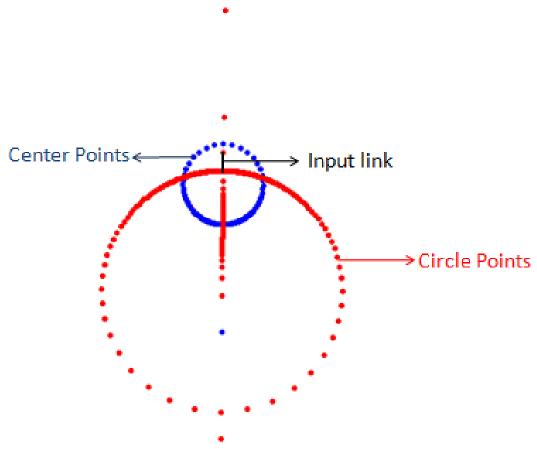


Figure 59 Center and Circle points for desired mechanism

In this case, the center and circle point curves degenerate into a line and a circle which is a special case for PPPP function generation

If the center and circle points are selected for the output crank from CATIA screen, the following mechanism is obtained and the ratio between two cranks can be satisfied as desired up to 20° degree input crank rotation with a 0.5 degree error. As it can be seen from the figure below, for 20° degree input crank rotation, the output link rotates 60,577° degree.

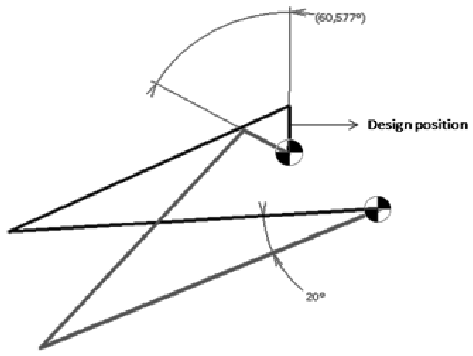


Figure 60 Constructed mechanism and correlation between rotation angles

CHAPTER 4

DISCUSSION AND CONCLUSION

4.1 SUMMARY AND DISCUSSION

In this study, visual and interactive computer software package which works with CATIA V5 in fully parametric form is created for solving aerospace mechanism synthesis problems. The created software is named as SynCAT and SynCAT is capable of synthesizing planar four-bar mechanisms for four multiply separated positions for motion, path and function generation synthesis types.

SynCAT is written in Visual Basic (VB) with graphical user interface since commands of CATIA V5 is available in VB. Thanks to compatibility with CATIA and VB, SynCAT performs synthesis of mechanism in CATIA in fully parametric form. Therefore, designer can use geometrical element in CATIA and get output in the same design environment.

SynCAT can solve path generation with prescribed timing, function generation and motion generation problems. Depending on the problem to be solved, different inputs must be provided to the program. Therefore, after design parameters are given as the input, SynCAT is capable of drawing Burmester curve exactly for design parameters or Burmester curves by varying one of the design parameter.

Furthermore, SynCAT has been tested for several aerospace applications and six of them explained in this thesis in details. During testing these examples in SynCAT, no problems are encountered and synthesis of these mechanisms is performed.

4.2 FUTURE WORKS

In dyadic approach, Burmester curves are found by varying one of the independent parameters with a certain increment. For example, in motion generation problem, there are three different independent variables namely the crank angle in one of the design positions: β_2, β_3, β_4 and SynCAT determines the resulting Burmester curves by varying β_2 from 0 to 360 with a fixed increment. The resulting points found on the circle and center point curves may be too close or too far apart. This problem can be eliminated by changing the increment and/or the independent parameter. In such a method the software may check the distances between two consecutive points and increase or decrease the increment if the distances are not in the specified range or change the independent parameter to another crank angle such as β_3. Therefore, with this improvement, points that can't be found with fixed increment can be determined.

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APPENDIX A

FLOWCHART OF SynCAT

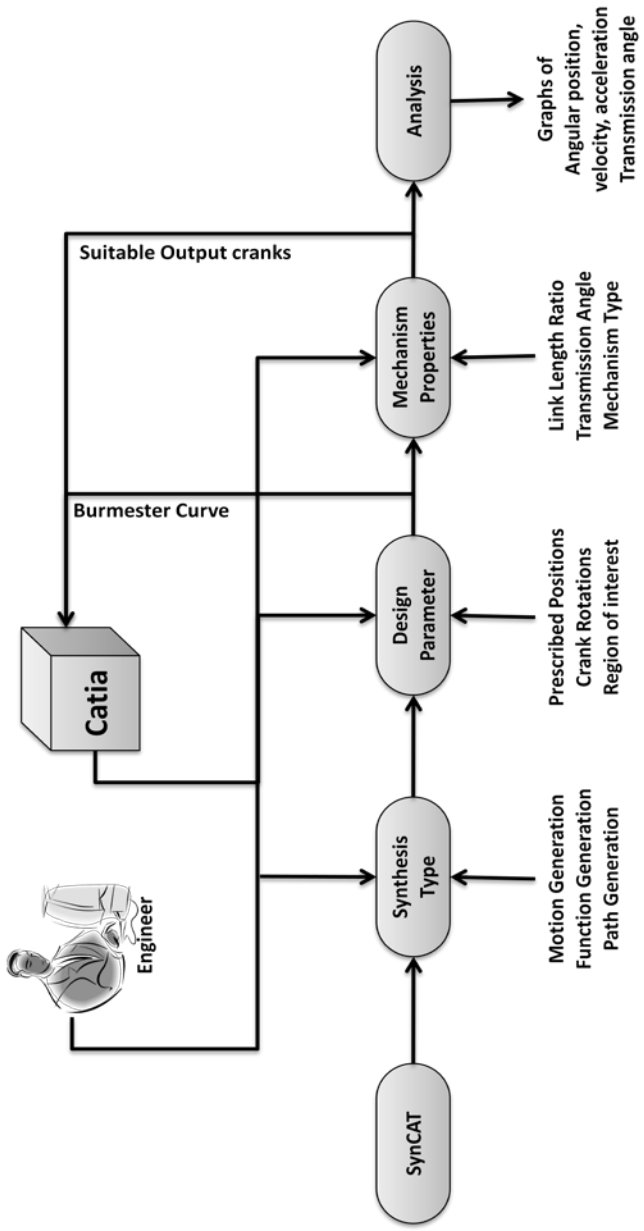

Figure 61 Flowchart of SynCAT

APPENDIX B

OBTAINING COORDINATES OF A POINT FROM CATIA

```
Sub CATgetcoord(point_name$, p1$, p2$, p3$)

Set CATIA = GetObject(, "CATIA.Application")

Set PartDocument = CATIA.ActiveDocument

Set Selection = PartDocument.Selection

Dim Point As Variant

MsgBox "Select a Point for" & point_name$

Set Point = Selection.Item(1).Value

ReDim koord(2)

Point.GetCoordinates koord

p1 = koord(0) ‘X coord

p2 = koord(1) ‘Y coord

p3 = koord(2) ‘Z coord

End Sub
```

APPENDIX C

PRINTING POINT ONTO CATIA

```
Set CATIA = GetObject(, "CATIA.Application")

Set partDocument1 = CATIA.ActiveDocument

Set Part1 = partDocument1.Part

Set hybridBodies1 = Part1.HybridBodies

Set hybridBody1 = hybridBodies1.Add()

hybridBody1.Name = "Center Points" ‘naming of the geometrical set

Set hybridShapeFactory1 = Part1.HybridShapeFactory

Set Point_1 = hybridShapeFactory1.AddNewPointCoord(X,Y,Z)

hybridBody1.AppendHybridShape Point_1

Part1.InWorkObject = Point_1
```

APPENDIX D

PRINTING LINE ONTO CATIA

```
Set CATIA = GetObject(, "CATIA.Application")
Set partDocument1 = CATIA.ActiveDocument
Set Part1 = partDocument1.Part
Set hybridBodies1 = Part1.HybridBodies
Set hybridBody1 = hybridBodies1.Add()
hybridBody1.Name = "Crank-Rocker alternatives" ‘naming geometrical set
Set hybridShapeFactory1 = Part1.HybridShapeFactory
Set Point_B0 = hybridShapeFactory1.AddNewPointCoord(X1,Y1,Z1)
Set Point_B = hybridShapeFactory1.AddNewPointCoord(X2,Y2,Z2)
Set reference1 = Part1.CreateReferenceFromObject(Point_B0)
Set reference2 = Part1.CreateReferenceFromObject(Point_B)
Set Line_1 = hybridShapeFactory1.AddNewLinePtPt(reference1, reference2)
hybridBody1.AppendHybridShape Line_1
Line_1.Name = "output crank"
Part1.InWorkObject = Line_1
 Part1.Update
```