INCENTIVE, SUBSIDY, PENALTY MECHANISMS AND POOLED, UNPOOLED ALLOCATION OF PRODUCTION CAPACITY IN SERVICE PARTS MANAGEMENT SYSTEMS

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ABSTRACT

INCENTIVE, SUBSIDY, PENALTY MECHANISMS AND POOLED, UNPOOLED ALLOCATION OF PRODUCTION CAPACITY IN SERVICE PARTS MANAGEMENT SYSTEMS

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In this thesis, two systems are analyzed in order to gain insight to the following issues: (i) Effect of incentive, subsidy and penalty designs on decentralized system, (ii) effect of using production facility as pooled capacity (pooled system) and dedicated capacity (unpooled system) on capacity utilization and system profit. Regarding the first issue, three models are defined; decentralized model, centralized model and decentralized model with incentive, subsidy, penalty designs. In all models, there are two dealers and one item is under consideration and lateral transshipments are allowed. Dealers operate with four inventory level decision (strategies) that consists of base stock level, rationing level, transshipment request level and customer rejection level. Under the decentralized system, a dealer sets its operating strategy according to the strategy of the other dealer and maximizes its own infinite horizon discounted expected profit. In the centralized system, a central authority (say manufacturer) exists, which considers the system-wide infinite horizon discounted expected profit, and makes all decisions. Under decentralized system with incentive, subsidy, penalty designs, manufacturer tries different designs on decentralized system namely revenue sharing, holding cost subsidy, request rejection penalty, transportation cost subsidy and commission subsidy in order to align decentralized system with centralized system. According to the results obtained, this alignment works best with nearly 40% revenue sharing percentage, low rejection penalty, high transportation cost subsidy under low transportation cost and commission subsidy under very low or very high commissions. Holding cost subsidy, on the other hand, is not a good strategy since it declines decentralized system profit. Considering the second issue, two systems are examined; pooled system and unpooled system. Both systems are centrally managed. In the pooled system, all capacity is dynamically allocated to either dealer considering maximization of system profit. In the unpooled system, capacity is shared among dealers and dealers are always allocated same percentage of the capacity. Infinite horizon average expected profit is maximized in both systems. The dealer having lower holding cost is allocated higher capacity in both pooled and unpooled system; however, exceptions exist in the unpooled system under low arrival rate. High-revenue dealer is always allocated higher capacity in both pooled and unpooled system. Arrival rate affects both systems such that total capacity utilization increases with increasing arrival rate. From the profit point of view, pooled system has great advantage under low demand rate in general.

Keywords: spare parts management, centralized system, decentralized system, incentive, subsidy, penalty, pooled production capacity, unpooled production capacity

ÖZ

YEDEK PARÇA YÖNETİM SİSTEMLERİNDE TEŞVİK, SÜBVANSİYON, CEZA MEKANİZMALARI VE ÜRETİM KAPASİTESİNİN HAVUZLANMIŞ YA DA PAYLAŞILMIŞ OLARAK TAHSİSİ

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Tez çalışması kapsamında, ana hatlarıyla iki konu incelenmiştir: (i) Teşvik, sübvansiyon, ceza uygulamalarının merkezi olmayan sistem üzerindeki etkisi, (ii) üretim tesisinin havuzlanmış ve paylaşılmış olarak tahsisinin kapasitenin bayilere tayinine ve toplam kara etkisi. İlk konu ile ilgili üç model tanımlanmıştır; merkezi olmayan model, merkezi model ve teşvik, sübvansiyon, ceza uygulamaları olan merkezi olmayan model. Tüm modellerde, iki bayii vardır ve tek bir ürün söz konusudur. Bayiler stratejilerini dört stok düzeyine göre belirler. Bu stok düzeyleri temel stok düzeyi, tayın verme düzeyi, parça aktarma isteği düzeyi ve müşteriyi reddetme düzeyidir. Merkezi olmayan sistemde, bir bayii diğer bayiinin stratejisine göre işletme stratejisini belirler ve sonsuz ufukta hesaplanan indirgenmiş karını en iyiler. Merkezi sistemde, merkezi bir otorite (üretici olduğu varsayılmıştır) bulunmaktadır. Merkezi otorite, sonsuz ufukta hesaplanan indirgenmiş toplam karı en iyiler ve tüm kararları alma yetkisine sahiptir. Merkezi olmayan sistemde, üretici farklı teşvik mekanizmalarını deneyerek merkezi olmayan sistemi merkezi sisteme yakınsatmaya çalışır. Bu mekanizmalar, gelir paylaşımı, stok tutma maliyetinin sübvanse edilmesi, parça aktarma isteğinin reddedilmesi cezası, ulaşım maliyetinin sübvanse edilmesidir. Merkezi olmayan sistemin merkezi sisteme yakınsaması;

yaklaşık %40 seviyesinde gelir paylaşımı, düşük reddetme cezası, küçük ulaşım maliyeti altında yüksek ulaşım sübvansiyonu ve çok düşük ve çok yüksek komisyon altında komisyon sübvansiyonu uygulanarak daha iyi gerçekleşmektedir. Merkezi olmayan sistemde stok tutma maliyetini sübvanse etmek toplam sistem karını düşürdüğünden iyi bir sonuç vermemiştir. İkinci konu göz önünde bulundurularak iki sistem incelenmiştir; üretim kapasitesinin havuzlanmış ve paylaşılmış olarak tahsisi. Her iki sistem de merkezi olarak yönetilir. Havuzlanmış tahsis sisteminde, tüm kapasite sadece bir bayiye tahsis edilebilir ve tahsis edilecek bayii seçimi toplam sistem karını en iyileyecek şekilde dinamik olarak güncellenir. Paylaşılmış tahsis sisteminde, kapasite bayilere paylaştırılmış durumdadır ve bayilere ayrılan kaynaklar zaman içinde güncellenmez, sabittir. Her iki sistemde de sonsuz ufukta hesaplanan ortalama kar en iyilenir. Elde edilen sonuçlara göre her iki sistemde de, stok tutma maliyeti düşük olan bayiye daha çok kapasite tahsis edildiği görülmüştür; ancak, paylaşılmış tahsis sisteminde –sayı olarak az da olsa- aykırı durumlar olduğu gözlemlenmiştir. Her iki sistemde de, geliri yüksek olan bayilere daha çok kapasite tahsis edildiği sonucuna ulaşılmıştır. Talep arttığında her iki sistemin de kapasite doluluklarının artış oluşmaktadır. Toplam sistem karı açısından bakıldığında, havuzlanmış tahsis sisteminin paylaşılmış tahsis sistemine karşı -özellikle talep az olduğunda- ciddi avantajlı olduğu söylenebilir.

Anahtar Kelimeler: yedek parça yönetimi, merkezi sistem, merkezi olmayan sistem, teşvik, sübvansiyon, ceza, üretim kapasitesinin havuzlanmış olarak tahsisi, üretim kapasitesi paylaşılmış kaynak olarak tahsisi

To my fiancée Bengi Kayışoğlu and to my family

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CHAPTER 1

INTRODUCTION

1.1 MOTIVATION OF THE STUDY

Spare parts industry is a special sector from several perspectives. First of all, this industry is critical from the sustainability point of view. To illustrate, think of an automobile requiring a vital spare part for the engine. This spare part is indispensable for the car; otherwise the engine would not start. Furthermore, think of a military helicopter that does not work due to lack of spare part that is special equipment. In such a situation, spare part inventory management can be strategic. Actually, spare part industry is a profitable sector. In automobile industry, a 22.2% profit margin exists in spare part sales (Kim et al. (2007)). So, marginalization is high compared to other sectors.

In today’s global and social economic conditions, manufacturers desire efficient production due to depleting resources, rivalry, constrained budget or some other managerial decision. Hence, the authority of a supply-chain system looks for ways to decrease system wide costs in order to gain more and compete with other organizations at every echelon of the supply chain.

In the basic model, suggested by Sherbrooke (1968), a one-way flow of items is considered. Lateral transshipment, which is transshipment of inventory between organizations in the same level of echelon, is not allowed. The objective is to minimize backorder costs given a limited budget. Lee (1987) has worked on multi-echelon systems. In case of stock out, a retailer is allowed to make a lateral transshipment request to another retailer in the same pooling group. The objective is to set optimum inventory levels.

Alfredsson and Verrijdt (1999) carried Lee's work one step further. They have worked on a centralized model where lateral transshipments directly from other retailers are allowed. In addition, stock of a retailer can be replenished by an external supplier in case of lack of parts. Realizing that the contribution of lateral transshipment is significant, Grahovac and Chakravarty (2001) have examined the advantages of pooling and lateral transshipment of demand in a single-item environment. Benefits are discussed under full pooling, no pooling and partial pooling in centralized management and decentralized management. Çömez et al. (2007) have worked on a centralized system where lateral transshipments are allowed. This time, lateral transshipment request acceptance levels are under consideration. In other words, rationing levels analyzed. Note that these levels are determined by central authority, not by retailers themselves.

In real life, supply chains may not always be managed by a central authority at the dealers' echelon (lower echelon) due to regulation or contracts. Instead, each player considers its own objective; so local optimal solutions are implemented. However, these solutions do not generally serve for the benefit of the whole supply chain. In that case, subsidizing, providing incentives or charging penalties could be a solution for bringing the system closer to optimal strategies. Then, decentralized system may align with the centralized system. This is the concern of the authority at lower echelon.

Another important issue in supply chain management is the right allocation of resources. Lee and Billington (1992) have mentioned the expanding view of supply chain as one of the opportunities of the manufacturer, which is located at the upper echelon. The main issue here is the understanding the lower echelon's, say dealer's, needs in order to set clearer targets and efficient operations. At first glance, efficient operations seem to serve only for the manufacturer. However, it also affects the stakeholders of the supply chain in terms of incurred costs. In other words, all players of the supply chain suffer from an inefficient operation of any stakeholder. Production resource allocation is one of the important issues in this manner where optimal allocation of resources has a major effect on production costs. Think of an environment that consists of dealers and a common production facility. Optimal allocation serves for both cost reduction and service level of the dealer. This is the other concern of the authority at production echelon (upper echelon).

1.2 OUTLINE OF THE STUDY

In this thesis, two main problems are discussed: (i) Effects of incentive, subsidy and penalty designs on profit under a decentralized environment and alignment of the decentralized system with the centralized system under those designs, (ii) Allocation of a common production resource to dealers with pooling and unpooling strategies under centralized system.

The supply chain consists of two echelons throughout the thesis; where lower echelon is the dealer echelon and upper echelon is the production echelon. The manufacturer manages the production environment; however, she has restricted rights on the dealer environment. The manufacturer can only regulate the costs and revenues at the lower echelon, and dealers decide on their own operating strategies accordingly. At the lower echelon, manufacturer wonders the optimal strategy of dealers if she is the decision maker (global optimal), then searches for different coordination mechanisms that diverts dealers to global optimal operating strategies. At the upper echelon, on the other hand, the manufacturer tries two different allocating strategies for the production facility. The manufacturer is now curious about the benefit of pooling the production capacity among dedicating certain percentage of the capacity to each dealer.

Incentive, subsidy and penalty design problem involves two-dealer, one manufacturer system where dealers are players. Two management views are considered; centralized system and decentralized system. The decentralized system is examined under no incentive setting and incentive, subsidy, penalty settings. In the centralized system, the manufacturer is assumed to be the authority and total system-wide costs are considered whereas dealers maximize their own profits in the decentralized system. In Chapter 3, decentralized system model, centralized system model and decentralized system with incentive, subsidy and penalty designs model have been described. In Chapter 4, results of computational analysis of the models discussed in Chapter 3 are interpreted.

Regarding the second research question, allocation of a common products resource problem has analyzed with the same stakeholders; two dealers and a manufacturer. In this model, pooled and unpooled strategies have been examined where capacity utilization of dealers, total idleness and system wide profit are the performance measures. The models are explained in Chapter 5 and findings upon computational analysis are discussed in Chapter 6.

In Chapter 7, general remarks and future research areas are discussed.

CHAPTER 2

LITERATURE SURVEY

METRIC model (Sherbrooke, 1968) and MODMETRIC model (Muckstadt, 1973) are the two core models in the research area of the spare parts management. In the METRIC model, a two-echelon supply chain is considered. Customer arrivals fit to compound Poisson distribution and item replenishments occur one at a time. The objective is to minimize backorder levels under a limited budget. The problem is solved using mathematical programming. In the Muckstadt version of METRIC model, bill of materials are accommodated within METRIC management system.

Zhao et al. (2005) have analyzed a two-dealer decentralized system with inventory sharing. Each dealer has a flexibility of sharing its inventory. Dealers operate with base stock level (S) and rationing level (K). When inventory level of a dealer is under S level, an order is placed and the item arrives τ time units later. When inventory level of a dealer is above K, dealer shares its inventory. When inventory level is under K, dealer does not share any inventory. Multiple demand classes are considered. That is, high-priority and low-priority demand classes exist such that transshipment request of another dealer is considered as low-priority demand whereas incoming customers directly to the dealer is considered as high-priority demand. The dealers incur holding cost per item per unit time, backorder cost per backordered unit and backorder cost per backordered unit per unit time. In addition to the inventory holding cost and backorder costs, commission payment and penalty cost exist in the system. The dealer that requests item makes a commission payment to the dealer that sends the item. This is also called cost of sharing. Penalty, on the other hand, is paid for each transshipment request rejected by the rejecting dealer. This penalty is considered as an incentive for sharing. The objective is to minimize the expected cost of each dealer's expected cost. Computational results and managerial insights are expressed as follows:

i. Increase in the incentive for sharing leads to decreases in the rationing level (K). Thus, dealer is willing to share more inventory.
ii. As cost of sharing increases, dealers prefer to increase their base stock levels (S) so that inventory sharing is used rarely compared to lower cost of sharing values. Dealer's reaction to an increase in the cost of sharing is not as responsive as increase in the penalty paid for each item rejected.
iii. From the individual dealer's cost point of view, cost increases with increasing penalty. When penalty is omitted from the decentralized system, the dealers are willing to share a specific amount of inventory, which results in a decrease in individual costs.
iv. The impact of increase in the incentive for sharing on system backorders is positive: Decentralized system's backorders decrease and align with centralized system's backorders.
v. When cost of sharing decreases due to subsidy provided by the manufacturer, system backorders increase.

As a result, last two observations lead to an important managerial insight to incentive and subsidy settings. Incentives for sharing is an effective way to direct dealers to share inventory whereas subsidizing dealers would lead to an increase in the system backorder levels, so a decrease in the service level.

Satır et al. (2009) have examined a two-center decentralized system. Dealers collaborate through inventory, service and information (inventory status) sharing. Customer arrival is distributed with Poisson distribution and time between arrivals of items from the manufacturer follows exponential distribution. In case of a customer arrival, center can either satisfy the demand or request a part from other center, which is called a lateral transshipment request. Upon the transshipment request, the requested center either accepts or rejects the request. At any time, a center can place production order to the manufacturing facility. Note that transshipment times are negligible since dealers are assumed to be in close proximity.

A center obtains revenue per sale. If the demand is satisfied via a lateral transshipment, then additionally, a commission payment is made and transportation cost is incurred by the requesting dealer. Dealers also incur holding cost per item per unit time and backorder cost per backorder per unit time.

A dealer (say Dealer 1) sets its operating strategy with respect to the other dealer's (say Dealer 2) strategy that operates under (S, K, Z) policy, where S is the base stock level, K is the rationing level and Z is the transshipment level. When the inventory level of Dealer 2 is below S, a production order is given. When the inventory level of Dealer 2 is above K, transshipment requests of Dealer 1 are accepted. If the inventory level of Dealer 2 is between K and Z, Dealer 2 rejects the transshipment requests of Dealer 1. When the inventory level of Dealer 2 drops below Z, it makes a transshipment request to Dealer 1.

The objective in the model is to find the optimal strategy of a dealer (say Dealer 1) given the operating strategy of the other dealer (say Dealer 2) since decentralized management is considered.

The system is modeled as an infinite horizon continuous time Markov decision process under expected discounted profit criterion. Four different environments are taken into account, namely no pooling, full pooling, dynamic pooling and static pooling. In no pooling, collaboration between dealers does not exist and neither of the dealers shares items or demand using lateral transshipments. In full pooling, dealers share their whole resources with each other non optimally. In other words, rationing level does not exist and every transshipment request is accepted by the dealers. In dynamic pooling, a dealer finds its optimal strategy with respect to the strategy of the other dealer. Dealer 1's strategy dynamically changes according to the inventory level of Dealer 2. In static pooling, inventory level information is not shared between dealers whereas dealers collaborate through lateral transshipments.

So, profit of Dealer 1 is maximized under a static operating strategy due to lack of information about the inventory status of Dealer 2.

The important results obtained are summarized as follows:

i. An increase in the arrival rate results in first an increase (for low values of arrival rate) then a decrease in the profit.
ii. Dynamic pooling environment is more beneficial for dealers than no pooling environment due to inventory sharing.

iii. Compared to no pooling, full pooling can be beneficial or unbeneficial under certain instances. Under low levels of base stock of Dealer 2, low levels of commission payment, full pooling is unbeneficial. As arrival rate to Dealer 1 increases, Dealer 1 uses own and Dealer 2's resources efficiently and full pooling is preferable to the no pooling environment.
iv. Comparing dynamic pooling and static pooling, for the 1684 results out of 1800 showed that benefit of dynamic policy is less than 1.5%.

Çömez et al. (2009) have analyzed multiple in-cycle transshipments with positive delivery times. There exist two dealers and a manufacturer. The supply chain is managed centrally by an inventory manager (IM). There are four types of cost; holding cost, backorder cost, replenishment cost and transshipment cost. The dealers and the manufacturer are not in close proximity. That is why, replenishment times and lateral transshipment times are also under consideration. The motivation behind lateral transshipment is to decrease inventory holding cost of requested dealer and to decrease backorder of requesting dealer because lateral transshipment time is less than replenishment time. Dealers are replenished by the manufacturer and IM obtains replenishment cost. Between two replenishments, a dealer can also request item from the other dealer if requesting dealer is out of stock. If the request is accepted, IM incurs the transportation cost, backorder cost of requesting dealer and holding cost of requested dealer during the transshipment time. If the request is rejected, IM incurs the backorder cost until the next replenishment. Note that more than one item can be requested at a time. At the end each period, Dealer 1 and

Dealer 2 incur holding cost per inventory. The objective of IM is to minimize long run average expected cost.

Rudi et al. (2005) have worked on a two location inventory model with transshipment and local decision making. In case of a stock out in one seller and stock surplus on the other seller, making lateral transshipment between sellers seems to be attractive. However, maximizing individual profit generally does not lead to an increase in the joint profits. This paper examines transshipment costs that lead sellers to set their operating strategies that serve for joint profit maximization.

Two retail firms exist in the system. The firms place purchase orders first; then demand occurs. Inventory level information of firms and demand realizations are shared in the system. A seller having surplus inventory can transship to the other firm whereas the other firm can only accept transshipped inventory in case of a stock out.

Each firm incurs a fixed cost of purchasing per item. In case of a sale, firm obtains revenue per unit sold. Firms incur penalty per each item backordered. Actually, summation of revenue and penalty is considered to be value of additional retail sales. Surplus inventory has a salvage value that is less than the purchasing cost. When a transshipment occurs from firm *i* to firm *j*, firm *i* receives transshipment payment per item from firm *j*. In addition, firm *i* incurs transportation cost per item to an outer transportation firm.

The system is analyzed under three environments: No transshipment, transshipment with central coordination and local decision making. Firstly, in no transshipment case, each seller maximizes its own profit since there is no interaction between sellers. The trade of is between expected marginal benefit and marginal cost. Note that, expected profit of each seller depends on the quantity purchased. The name of the model in the literature is The Newsvendor Problem, actually. Secondly, in transshipment with central coordination environment, central authority exists and the expected value of total profits for the two locations is considered. Lastly, each seller desires to maximize its own profits as in no transshipment environment. However, sellers share inventory through transshipment this time. So, expected profit of each seller depends on quantity purchase and also on transshipment price.

Assuming that transshipment prices are determined by the transshipping firm, then transshipment price is set to marginal value of additional retail sales at transshipped firm's location (sum of revenue and penalty). This is optimal solution for the transshipping firm; however the solution does not maximize the joint profits.

Oppositely, assuming that transshipment prices are determined by the transshipped firm, transshipment price is selected to be the sum of salvage cost and transportation cost. As in the previous case, joint profits are not still maximized.

As a third view, one of the sellers determines transshipment decision. Suppose seller 1 determines transshipment cost. Then, transshipping from seller 1 to seller 2 costs marginal value of additional sale (summation of revenue and penalty) whereas transshipping from seller 2 to seller 1 costs summation of salvage value of seller 2 and transportation cost of transshipping from seller 2 to seller 1. Thus, case 3 is a hybrid arrangement of cases 1 and 2. Consequently, dominated seller (seller 2 in this case), gains zero from transshipments. So, seller 2's operating strategy becomes independent of seller 1 and this solution is similar to the newsvendor solution.

Lastly discussed issue is the existence of a pair of transshipment prices which serve for aligning local decision making profit with centralized environment's profit. It is stated that pair of transshipment costs which diverts decentralized system to centralized system in terms of profit exist and this existence is proved in the paper.

Zhao (2008) has analyzed a system which consists of a supplier and oligopolistic retailers. The objective is to find a strategy for the supplier to align the ongoing system with optimal system.

Before the selling season, the supplier determines the wholesale price and buyback rates that differ for each retailer. Then, the retailers determine their order quantities and selling prices per item at the same time. Afterwards, products are transported to retailers and demand is realized. As the last step, profits are collected. The retailers are faced with demand that has originally three components: Price dependent demand, price independent demand and substituted demand from other retailers. Substituted demand from other retailers is also price independent demand. (The parameter illustrating other retailer's substitution of demand is named spill rate.) That is why, it is considered in the price independent demand from now on. Two perspectives are taken into account: Retailer perspective and supplier perspective.

The retailer determines selling price and safety stock regarding the demand. The retailer's objective is to maximize its profit (revenue - cost). The revenues are profit (selling price – wholesale price) of the total sales and revenue obtained due to price independent demand. The costs are total wholesale price paid to the supplier for the safety stock and cost incurred to the surplus inventory due to low demand (independent of price) realization compared to safety stock level; that is the total expected buy back paid to the surplus inventory.

It is important to note two extremes of the model; namely inventory competition and price competition. If the demand of a retailer depends only on its own selling price, this case is called inventory competition game. Price competition game, on the other hand, does not allow substitution of demand from a retailer to the other.

The supplier's problem is to set wholesale price and buy back rate that directs retailers to operate optimally. Note that wholesale price and buy back rate is contract parameters. A supplier could be interested in system optimality for several reasons:

i. Supplier may desire setting up a strategic partnership with the retailer in the long-term,
ii. Supplier may pay higher attention to the availability of its product in the markets; so optimum distribution of the product is considered,

So, supplier looks for a contract that maximizes system's total profit. Total profit consists of retailer's profit (selling price – whole sale price) obtained from price dependent demand, revenue obtained from price independent demand (given that price independent demand is under safety stock), and total production cost incurred for safety stocks.

Note that a competing retailer operates under low-selling price and high-safety stock strategy compared to the system optimum. When retailer increases its selling price, demand deviates to the other dealer. Likewise, when safety stock decreases, higher demand is spilled to the other dealer. A decentralized retailer, on the other hand, operates oppositely to the competing retailer; that is, under high-selling price and low-safety stock (if buyback rate is low) strategy compared to system optimum. When selling price is decreased, demand increases retailer gains less and supplier gains more. Regarding the insights and results obtained, to achieve system optimum, vertical conflict, which is also called double marginalization, and horizontal conflict (competition) should be eliminated in order to fully coordinate both competing and decentralized retailers.

If only price competition exists in the system, system coordination contract should be designed to set higher wholesale price. Remember that competing retailer sets lower selling price than the system optimum. Setting higher wholesale price leads retailer to set higher selling price and eliminate the harmful effect of competition. If only inventory competition exists, the coordination contract should force supplier (i) to set wholesale price equal to the marginal production cost and (ii) retailers to pay extra penalty for surplus inventory. By equating wholesale price with marginal production cost, double marginalization is eliminated. The motivation behind charging extra fee to retailers for surplus inventory is to decrease over stocks. In this contract, supplier has a single way revenue which is fees paid for surplus inventories.

Analyzing change in contract parameters (wholesale price and buy back rate) with respect to the price competition under linear demand function; as price competition toughens, both wholesale price and buy back rate increase. Intuitively, price competition leads retailer to decrease selling price. Thus, system's total profit decreases. To compensate this decrease, supplier increases wholesale price. As the wholesale price increases, buy back rate usually increases in order to neutralize the increase in wholesale price.

If both price competition and inventory competition exist in the system, then contract indicates payment from supplier to retailer for surplus inventories when price competition is tougher than the inventory competition. If inventory competition is tougher, on the other hand, contract suggests just the opposite: Retailer should pay fees for the surplus inventory to the supplier.

Regarding computational analysis, as buy back rate increases, retailers hold higher levels of inventory. So, buy back rate should be decreased under high spill rate environment. Under supplier's optimal contract, increase in the spill rate decreases buyback rate. Defining supplier's efficiency as ratio of the absolute difference between supplier's profit under her optimal contract and under coordination contract to the supplier's profit under her optimal contract, this efficiency decreases as price competition gets tougher.

Briefly, the following results can be obtained from the analysis of Zhao (2008):

i. When both vertical and horizontal conflicts exist in the system, coordination contract leads to higher wholesale price than production cost.
ii. Total buy back cost due to surplus inventory can be incurred either by supplier or by retailer depending on the relative effect of the inventory competition or price competition.
iii. Buyback rate increases with increasing price competition.
iv. Buyback rate decreases and drops below zero with increasing inventory competition.
v. Under linear demand, higher buy back rate leads retailer to increase selling price and the level of safety stocks.
vi. Under optimal contract, as price competition gets fiercer, wholesale price and buy back rate increase.
vii. With computational analysis, increase in the proportion of demand spill leads to increase in the wholesale price and decrease in the buyback rate.
viii. Regarding system's optimal contract and supplier's optimal contract, both contract parameters diverge from each other as competition is quitted.

Rudi et al (2005) has majored multi-period inventory control environment in which retailers compete on the product availability. Generally when a retailer is out of stock, demand is either backordered or directed to the rival retailer. In this paper, there are two retailers in the system. If a retailer is out of stock, customer is allowed to switch to the rival retail. In a multi-period environment, customer has four options in case of a stock in retail 1:

i. Customer can be rejected; so, demand is lost.
ii. Demand can be backordered by original retail,
iii. Demand can be backordered by rival retail,
iv. Customer explores its own way of supplying the product.

The environment is analyzed in order to answer the following question: How do customers' backordering attitudes affect the inventory levels and profits of retailers? During the analysis of the system, four different backordering strategies of customers are examined. Each version of the scenarios is modeled as a stochastic, multi-period game. It is proved that static base stock strategy is a Nash equilibrium of the multi-period game.

Gao et al. (2005) have analyzed coordination mechanisms among supply chain. Coordination issues include price discount, volume discount, integer-ratio policy and power-of-two policy. Note that price discount and volume discount strategies are related with order quantity whereas integer-ratio policy and power-of-two policy are concerned with the frequency of order. In addition to these coordination strategies, cooperative advertising, cooperative information sharing and VMI application are commonly applied issues in supply chain coordination.

Price discount has a wide range of use to implement supply chain inventory coordination. Two types of price discounts exist; quantity discount and volume discount. Under quantity discount setting, vendor announces a lower price in case of order quantity being higher than a minimum threshold level. In this case, retailers tend to order bigger batch sizes. So, vendor can take advantage of economies of scale. The attention is on determining optimal price discount so that system wide profits are improved.

Volume discount, on the other hand, reflects the demand dependence price. Demand is assumed to be met by the retailers and annual demand is taken into account while setting the price.

Integer ratio policy indicates that coinciding replenishment interval of retailers and the supplier decreases system wide costs since replenishment orders of different suppliers can be joined and advantage of economies of scale arises. Power-of-two policy, on the other hand, is a modified case of integer ratio policy. In a power-of-two policy, the supplier still takes advantage of coinciding the intervals of its own replenishment and the retailer-replenishments. However, there is a base time period and the replenishments of retailers can only be done in time periods which is the multiplication of base time period with an integer power of two.

Cooperative advertising is an advertising strategy where retailer triggers the advertisement and manufacturer pays portion of the advertisement costs. It is an effective strategy that strengthens the brand’s potential.

Cooperative information sharing is one of the most important issues in coordination of the supply chain. This sharing strongly influences many decision including scheduling, inventory and logistics planning, safety stock level of both raw materials and finished products and so on.

VMI stands for vendor-managed inventory. In gross markets, shelf employees of the many suppliers, who control the inventory level on the shelf and makes an replenishment order if necessary, can be detected. This is the basic explanation of vendor-managed inventory control: The vendor on the upper echelon of the system controls its inventory that is in the lower echelon. VMI has advantages on elimination of bullwhip effect among supply chain. In addition, VMI supports smooth production and lower-cost logistics plan of the vendor.

Fugate et al. (2006) worked on an interdisciplinary study. Actually, several contracts are implemented in many supply chains for the sake of global optimality rather than local optimal solutions. In this environment, perception of the system users is focused. In other words, perception of managers that are responsible for certain supply chain management issues are taken into account. Those managers are interviewed with respect to the academically suggested sampling ways and results are analyzed using qualitative research procedures suggested by Strauss and Corbin (1998).

Queuing control problem is analyzed by Çil et al. (2008). Admission control and pricing are applied to the queuing system in order to keep them under control. Optimal policies under different system parameter settings are examined. Main motivation under queuing control problem is the curiosity for the effect of queue size on the reward function.

Lee and Billington (1993) have analyzed common mistakes in the supply chain systems. According to Lee and Billington (1993), manufacturers generally pay attention to the quality of raw materials and finished products; however, they do not put emphasis on the logistics and holding costs. Common mistakes are lack of performance measures, incorrect definition of customer service, lack of accurate delivery information in terms of quantity and time, use of poor information technologies that does not allow data security and reliability, disregarding effect of uncertainties, use of primitive planning process, negative discrimination against internal customers, considering economic issues rather than service level during transportation, lack of coordination among supply chain in multiple item orders, lack of correct valuation to inventory holding costs, organizational barriers, concentrating on local optimal solutions rather than global optimal solutions, unconsidering supply chain point of view and concentrating on operational decisions and assuming manufacturer or retailer as the end of the supply chain.

Anupindi et al. (2001) have focused on a system with N retailers. The retailers observe stochastic demands. They can hold inventory at different centers. In other words, inventory ownership and location of inventories are separated in the system. An allowable portion of the backordered demand can be satisfied by the surplus inventory of the retailers. The issue discussed is the correct inventory levels and correct allocation of them. The study also concentrates on setting correct costs and revenues coherent with incentives provided to several independent retailers. In such an environment, the allocation strategy that aligns decentralized system with centralized system is developed.

Aktaran and Ayhan (2009) have examined a queuing system in which price is dependent on customer arrivals. Objective is to maximize long run average reward. The system is modeled as Poisson arrivals, exponential service times, restricted number of parallel servers and restricted number of customers in the system. Upon analysis, the following results are obtained:

i. Optimal price setting decreases with increasing facility capacity; that is, decrease in the time between services or increase in the number of parallel servers.
ii. Optimal price setting increases with increasing demand or increasing time between services.

This study extends the two dealer system with *S, K, Z* levels (Satır et al (2009)) to one manufacturer and two dealers system with *S, K, Z, T* levels where manufacturer searches for more efficient supply chain both at the lower echelon, by giving incentives, subsidies or obtaining penalties, and at the upper echelon, by comparing pooled capacity allocation and unpooled capacity allocation. Actually, coordination mechanisms with dealers having *S, K, Z, T* levels are not previously studied in the literature. Furthermore, benefit of pooling in the production facility in two-echelon supply chain system is also focused in this thesis work.

CHAPTER 3

DESCRIPTION OF THE DECENTRALIZED MODEL, CENTRALIZED MODEL AND DECENTRALIZED MODEL WITH COORDINATION MECHANISMS

In this chapter, three models are described:

- Decentralized model,
- Centralized model,
- Decentralized model with incentive, subsidy, penalty designs.

In all models, two dealers and one manufacturer exist. Dealers obtain revenue per item sold and incur inventory holding cost per item per unit time, backorder (customer waiting) cost per item per unit time and loss-of-goodwill cost per customer rejected. The manufacturer obtains revenue per item sold (by the dealers) and incurs lateness cost for each waiting customer per unit time as a loss-of-goodwill. In the decentralized model, the dealers are the decision makers and they determine their own operating strategies. Profit of the manufacturer is calculated according to the dealers' decisions. In the centralized model, profits of the dealers and the manufacturer are taken into account while determining the operating strategy of the dealers. To align the decentralized model with the centralized model, manufacturer could design coordination mechanisms. In the decentralized model with incentive, subsidy, penalty designs, such coordination mechanisms are examined. Under those mechanisms, profitability of the manufacturer and the dealers are analyzed. In the decentralized model with incentive, subsidy, penalty designs, revenue and cost components of the dealers and the manufacturer depend on the coordination mechanism that in use. To illustrate, if the manufacturer subsidizes a portion of the transportation cost, this portion is additional revenue for the dealer whereas it is additional cost for the manufacturer. Note that, manufacturer revenue per sale is assumed as 0 (zero) at the beginning.

Note that decentralized and centralized systems are adopted from Usta (2010).

3.1 DECENTRALIZED MODEL

In this model, there are two independent dealers. Note that one item is under consideration. Each dealer has its own objective; to maximize own profit. Dealers can hold inventory or form a queue of customers that are waiting for the item. In other words, backorder is allowed. The inventory levels of the dealers reflect system state. Let *i* be the inventory level of Dealer 1 and *j* be the inventory level of Dealer 2. System state (*i*,*j*) changes when customer arrives or item arrives. (Note that negative values of *i* or *j* indicate backorders.)

Number of customer arrivals fits to Poisson distribution with rate λ_1 for Dealer 1 and λ_2 for Dealer 2. Customers arriving to Dealer 1 and Dealer 2 are selected from different populations. One customer arrives at a time, bulk arrivals are not allowed. Item replenishment, on the other hand, is done independently; each dealer has a dedicated production capacity. Two production lines can be thought to be allocated for different dealers. Time between arrivals of items is exponentially distributed with rate μ_1 and μ_2 for Dealer 1 and Dealer 2, respectively. Item replenishments also occur one at a time.

In case of a customer arrival, Dealer 1 can either (i) satisfy the demand using own resources, (ii) satisfy demand using Dealer 2's resources or (iii) reject the customer. Dealer 1 chooses its action according to system state and policy of Dealer 2. Dealer

2's policy is (S_2, K_2, Z_2, T_2). S_2 is the base stock level; that is, when j reaches S_2, item replenishment to Dealer 2 stops. K_2 is the rationing level. When $K_2<j\leq S_2$, transshipment request of Dealer 1 is accepted. Note that if inventory does not exist physically at Dealer 2 ($j<0$) when $K_2<j\leq S_2$, then Dealer 2 also backorders transshipment request and cannot satisfy the demand immediately. When $T_2<j\leq K_2$, Dealer 2 rejects the transshipment requests of Dealer 1. If request is rejected, demand is satisfied by Dealer 1 instead of rejecting the customer. Z_2 is the transshipment request level. When $Z_2 < j \leq S_2$, Dealer 2 satisfies demand using its own resources. When $T_2<j\leq Z_2$, Dealer 2 makes lateral transshipment request to Dealer 1. Dealer 1 either (i) accepts or (ii) rejects the request. If Dealer 1 accepts, demand is satisfied by Dealer 1 or corresponding customer joins to the queue of Dealer 1. If Dealer 1 rejects the request, demand is either satisfied by Dealer 2 or customer joins to the queue of Dealer 2. T_2 is the customer rejection level. When $j=T_2$, Dealer 2 rejects customers; in other words, demand is lost. Other than the demand satisfaction and incoming request decisions, Dealer 1 makes decision on production. Dealer 1 either (i) gives production order or (ii) stops production.

Based on recently discussed decisions, Dealer 1 has actions, (a,b,c), where:

- a Є A = {satisfy with own resources (SOR), satisfy with other dealer's resources (SDR), reject the customer (REJ)}
- b Є B = {accept transshipment request (AR), reject transshipment request (RR)}
- c Є C = {produce (P), do not produce (DP)}.

When Dealer 1 satisfies demand using its own sources, it obtains revenue (R) per sale. If Dealer 1 (Dealer 2) satisfies demand using Dealer 2's (Dealer 1's) resources, Dealer 1 (Dealer 2) pays commission (r) to Dealer 2 (Dealer 1) per item laterally transshipped. In addition, Dealer 1 (Dealer 2) incurs transportation cost (tr) to an outer transportation company. Thus, if Dealer 1 (Dealer 2) satisfies demand using Dealer 2's (Dealer 1's) resources, Dealer 1 obtains R-r-tr per sale. Cost of replenishment is so small compared to inventory holding and loss of goodwill cost that it is negligible. Dealers also incur holding cost (*h*) per item per unit time, backorder cost (*l*) per item per unit time and loss-of-goodwill (*w*) cost per customer rejected.

Problem of Dealer 1 is examined through infinite horizon continuous time Markov chain. Dealer 1's objective is to maximize infinite horizon discounted expected profit. Dealer 1's optimality equation is expressed as follows:

$$v(i,j) = \frac{-hi^{+} + li^{-}}{\alpha+\beta} + \frac{1}{\alpha+\beta}\left[\lambda_1\phi_1 v(i,j) + \lambda_2\phi_2 v(i,j) + \mu_1\phi_3 v(i,j) + \mu_2\phi_4 v(i,j)\right] \qquad Eq.\ 3.1.1$$

where *v(i,j)* is the infinite horizon expected discounted profit of the dealer starting from state *(i,j)*, i^{+} is max{*i,0*}, i^{-} is min{*i,0*}, β is $\lambda_1 + \lambda_2 + \mu_1 + \mu_2$, α is the discount rate and ϕ_i operators are expressed as follows:

$$\phi_1 v(i,j) = \begin{cases} max\{v(i-1,j) + R, v(i,j-1) + R - r - tr, v(i,j) - w\}, & K_2 < j \le S_2 \\ max\{v(i-1,j) + R, v(i,j) - w\} & , otherwise \end{cases}$$

$Eq.\ 3.1.2$

$$\phi_2 v(i,j) = \begin{cases} v(i,j) & , j = T_2 \\ v(i,j-1) & , j > Z_2 \\ max\{v(i-1,j) + r, v(i,j-1)\}, & otherwise \end{cases} \qquad Eq.\ 3.1.3$$

$$\phi_3 v(i,j) = max\{v(i,j), v(i+1,j)\} \qquad Eq. 3.1.4$$

$$\phi_4 v(i,j) = \begin{cases} v(i,j+1), j < S_2 \\ v(i,j) \quad\quad ,j = S_2 \end{cases} \qquad Eq. 3.1.5$$

Operator $\phi_1 v(i,j)$ is the reflection of customer arrival to Dealer 1. If the inventory level of Dealer 2 is within the transshipment request acceptance range, Dealer 1 chooses one of the two choices; accept customer-satisfy demand using own sources (SOR), accept customer-satisfy demand using Dealer 2's sources (SDR) or reject customer (REJ), which maximizes its profit (condition 1). If Dealer 2 is not willing to accept a transshipment request, Dealer 1 chooses one of the two choices; accept customer- satisfy demand using own sources (SOR) or reject customer (REJ), which maximizes its profit (condition 2).

Operator $\phi_2 v(i,j)$ is associated with customer arrival to Dealer 2. If the inventory level of Dealer 2 is at its customer rejection level (T_2), the customer is rejected (condition 1). If the inventory level of Dealer 2 is above Z_2, it accepts the customer and satisfies demand using its own sources (condition 2). If the inventory level of Dealer 2 is less than or equal to Z_2, Dealer 1 chooses one the two choices; accept the request (AR) or reject the request (RR), which maximizes Dealer 1's profit (condition 3).

Operator $\phi_3 v(i,j)$ is the production order decision of Dealer 1. Upon customer or item arrival, Dealer 1 chooses one of the two choices; give production order (P) or stop production (DP), which maximizes its profit.

Operator $\phi_4 v(i,j)$ reflects production order of Dealer 2. If inventory level of Dealer 2 is at its base stock level (S_2), it stops production (condition 1). If inventory level of Dealer 2 to is below S_2, it places production order (condition 2).

In order to reach to the equilibrium of the problem, firstly one-dealer problem is focused (say Dealer 1). Dynamic policy of Dealer 1 is determined assuming that operating policy of Dealer 2 is exogenous and converted to static policy. Then, Dealer 2's static policy is obtained considering the static operating policy of Dealer 1 as exogenous and so on. After sufficient number of iterations, the equilibrium policies for Dealer 1 and Dealer 2 are obtained, where both policies are static. The equilibrium policies are obtained with the same heuristic method developed in Usta (2010). Item flow schema and revenue-cost flow schema under static policies are expressed in Figure 3.1.1 and 3.1.2 respectively.

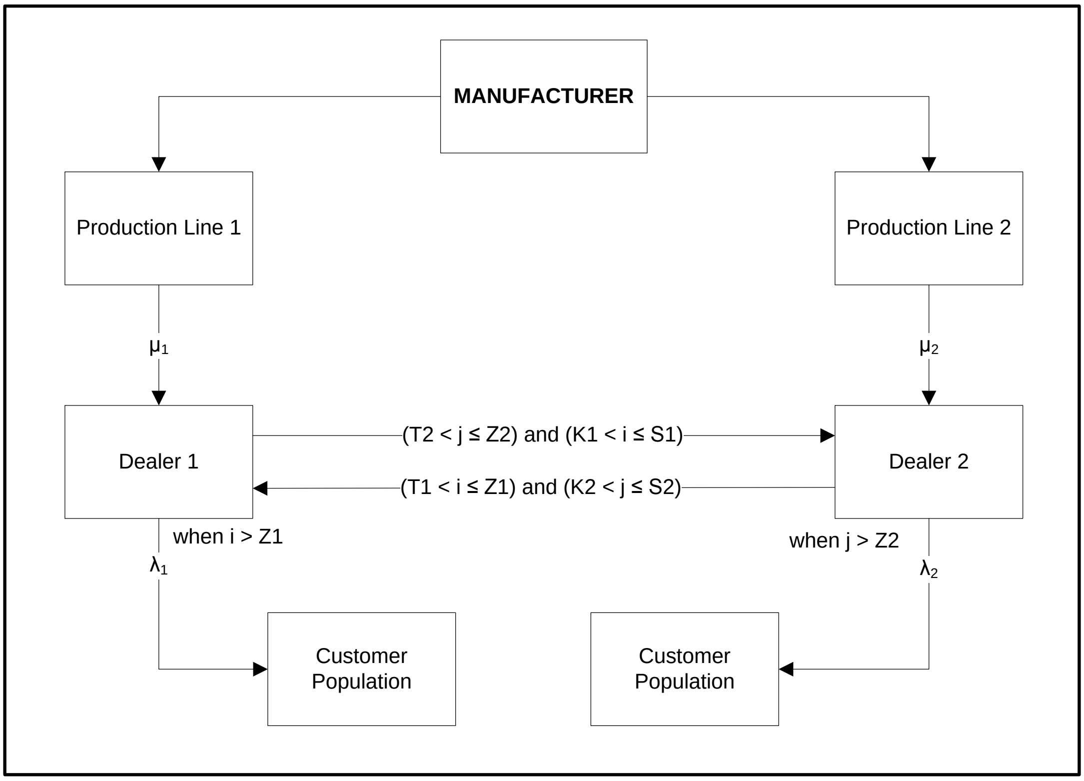

Figure 3.1.1 Item flow under static operating strategies in decentralized system

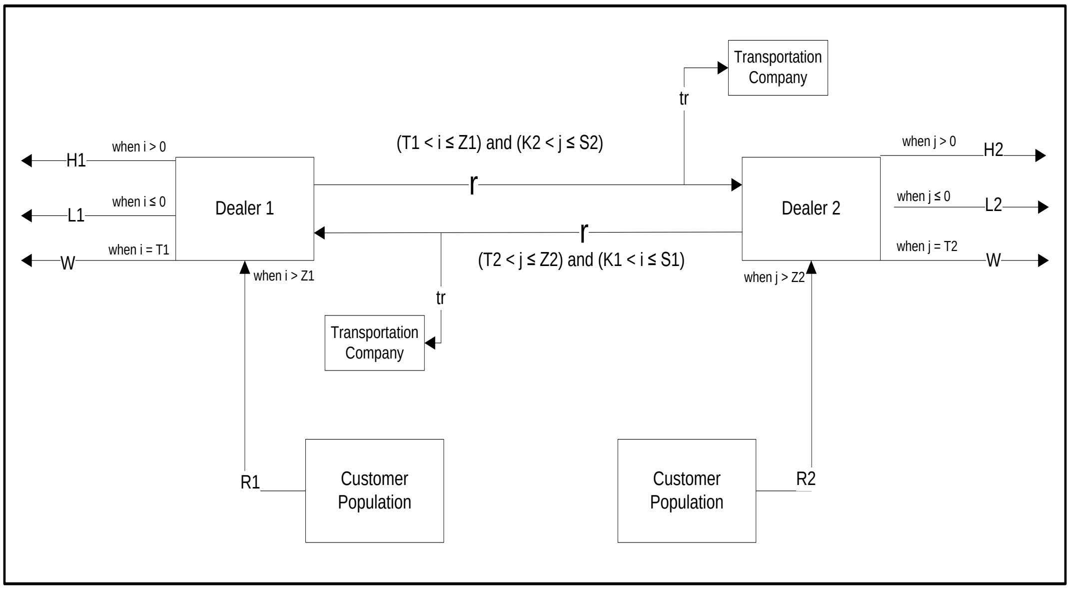

Figure 3.1.2 Revenues obtained and costs incurred under static operating strategies in decentralized system

3.2 CENTRALIZED MODEL

Under centralized system, there are two dealers and a central authority who is the manufacturer. One item is under consideration. The objective is to maximize infinite horizon discounted expected system wide profit of two dealers and the manufacturer. Backorder is still allowed under the centralized system. The inventory levels of the dealers reflect the system state. Let *i* be the inventory level of Dealer 1 and *j* be the inventory level of Dealer 2.

Under the centralized system, customer arrival rates, time between item replenishments, state definition, action space, revenues and costs are similar except the commission payment (*r*) and the backorder cost paid by the manufacturer (l_m). Commission is paid by a dealer obtained by the other dealer. From system wide profit point of view, this payment is irrelevant since both payer and receiver are within the system. Backorder cost of the manufacturer, on the other hand, is denoted by l_m and defined as the backorder cost incurred per item per unit time. One may wonder why manufacturer pays backorder cost although she does not obtain any revenues. Actually, revenue of the manufacturer is embedded in the revenue obtained by the dealer. In other words, dealer pays an amount to the manufacturer for each item sold. Since this payment is within the system, like the commission payment, it is not separately considered in the centralized system.

Centralized system problem is examined through infinite horizon continuous time Markov chain. Manufacturer's objective is to maximize infinite horizon discounted expected system wide profit. The centralized system's optimality equation is expressed as follows:

$$v(i,j) = \frac{l_m(i^- + j^-)}{\alpha+\beta} + \frac{-hi^+ + li^-}{\alpha+\beta} + \frac{-hj^+ + lj^-}{\alpha+\beta} + \frac{1}{\alpha+\beta}[\lambda_1\phi_1 v(i,j) + \lambda_2\phi_2 v(i,j) + \mu_1\phi_3 v(i,j) + \mu_2\phi_4 v(i,j)] \qquad Eq. 3.2.1$$

where $v(i,j)$ is the infinite horizon expected discounted profit of the system starting from state (i,j), i^+ is max{$i,0$}, i^- is min{$i,0$}, j^+ is max{$j,0$}, j^- is min{$j,0$}, β *is* $\lambda_1+\lambda_2+\mu_1+\mu_2$, α is the discount rate and ϕ_i operators are expressed as follows:

$$\phi_1 v(i,j) = max\{v(i-1,j) + R, v(i,j-1) + R - tr, v(i,j) - w\} \qquad Eq. 3.2.2$$

$$\phi_2 v(i,j) = max\{v(i-1,j) + R - tr, v(i,j-1) + R, v(i,j) - w\} \qquad Eq. 3.2.3$$

$$\phi_3 v(i,j) = max\{v(i,j), v(i+1,j)\} \qquad Eq. 3.2.4$$

$$\phi_4 v(i,j) = max\{v(i,j), v(i,j+1)\} \qquad Eq. 3.2.5$$

Operator $\phi_1 v(i,j)$ is the reflection of customer arrival to Dealer 1. Central authority chooses one of the three choices; accept customer-satisfy demand using Dealer 1's sources (SOR), accept customer-satisfy demand using Dealer 2's sources (SDR) or reject customer (REJ), which maximizes system-wide profit.

Operator $\phi_2 v(i,j)$ is the reflection of customer arrival to Dealer 2. Central authority chooses one of the three choices; accept customer-satisfy demand using Dealer 1's sources (SDR), accept customer-satisfy demand using Dealer 2's resources (SOR) or reject customer (REJ), which maximizes system-wide profit.

Operator $\phi_3 v(i,j)$ is the production order decision of Dealer 1. Upon customer or item arrival to Dealer 1, central authority chooses one of the two choices; give production order for Dealer 1 (P) or stop production for Dealer 1 (DP), which maximizes system-wide profit.

Operator $\phi_4 v(i,j)$ is the production order decision of Dealer 2. Upon customer or item arrival to Dealer 2, central authority chooses one of the two choices; give production order for Dealer 2 (P) or stop production for Dealer 2 (DP), which maximizes system-wide profit.

To solve centralized system problem and obtain static policy for each dealer, the heuristic developed by Usta (2010) is used. First the dynamic policy found through the optimality equations is converted into a static policy which is considered to be the candidate policy. Then a local search is conducted using the candidate policy as a starting node. Note that steepest ascent method is used during the search.

3.3 DECENTRALIZED MODEL WITH INCENTIVE-SUBSIDY-PENALTY DESIGNS

In this section, a decentralized system with incentive, subsidy or penalty designs is analyzed. Manufacturer provides incentives, subsidizes dealers or obtains penalty payments in order to align decentralized system with centralized system. Note that dealers still seek for their best strategies to maximize their own profits. The difference from the pure decentralized system is that, dealers make their decisions considering previously defined cost and revenues (Section 3.1) and also incentive, subsidy payments from the manufacturer to dealers and penalty payments from the dealers to the manufacturer.

Dealer roles and objectives, manufacturer role, customer arrival rates, time between arrivals, state definition and action space are all similar to the decentralized system (Section 3.1). Revenue and cost definitions, on the other hand, differ from the pure decentralized system.

Comparing centralized and decentralized system, initial observation is the higher rate of item flow between dealers under centralized system. Regarding inventory levels, S and T values coincide under centralized system. K and Z values of centralized and decentralized system, on the other hand, can be quite different. As demand rate increases or commission decreases, K and Z increase under decentralized system. In centralized system, K and Z values coincide and lie between -1 and 1. Regarding the observations, five coordination mechanisms are under consideration:

1 Revenue Sharing: The parameter associated with this incentive is R_{shr}, meaning revenue sharing incentive percentage ($0 \leq R_{shr} \leq 1$). In this setting, R is shared such that manufacturer receives ($R_{shr} R$) whereas dealer receives the remaining part, $(1- R_{shr})R$ per sale to customer.

2 Holding Cost Subsidy: The parameter associated with this incentive is h_{subs}, meaning holding cost subsidy percentage ($0 \leq h_{subs} \leq 1$). In this setting, h is shared such that manufacturer subsidizes (h_{subs} h) whereas dealer incurs the remaining part, $(1- h_{inc})h$ per item per unit time.

3 Request Rejection Penalty: The parameter associated with this incentive is ltr_{pen}, meaning amount of penalty that is incurred by rejecting dealer when lateral transshipment request is rejected (RR). Remember that there is no cost in case of request rejection under pure decentralized system. With this

penalty, if Dealer 1 rejects a lateral transshipment request of Dealer 2, Dealer 1 pays ltr_{pen} amount of penalty per item to the manufacturer.

4 Transportation Cost Subsidy: The parameter associated with this incentive is tr_{subs}, meaning transportation cost subsidy percentage $(0 \leq tr_{subs} \leq 1)$. In this setting, the transportation cost (*tr*) is shared such that the manufacturer subsidizes ($tr_{subs}\, tr$) whereas dealer that requests an item incurs the remaining part, $(1- tr_{subs})tr$ per item. With this subsidy setting, if Dealer 1 (Dealer 2) satisfies a demand using Dealer 2's (Dealer 1's) source, Dealer 1 receives $R-r-(1-tr_{subs})tr$.

5 Commission Subsidy: The parameter associated with this subsidy is $r_{x,y}$. In this setting, manufacturer is sure that the commission paid by a dealer does not exceed *y* and commission received by a dealer is not less than *x*. Suppose that commission is 5 and $r_{7,3}$ setting is in use. When Dealer 1 (Dealer 2) satisfies demand using Dealer 2's (Dealer 1's) sources, it pays 5 but manufacturer re-pays 2 back to Dealer 1 (Dealer 2) since *y* is 3. Manufacturer also pays 2 to Dealer 2 (Dealer 1) so that it receives 7 as commission payment because *x* is 7.

Regarding these settings, Dealer 1's optimality equation is expressed as follows:

$$v(i,j) = \frac{(1-h_{subs})hi^{+} - li^{-}}{\alpha+\beta} + \frac{1}{\alpha+\beta}\left[\lambda_1\phi_1 v(i,j) + \lambda_2\phi_2 v(i,j) + \mu_1\phi_3 v(i,j) + \mu_2\phi_4 v(i,j)\right] \qquad Eq. 3.3.1$$

where $v(i,j)$ is the infinite horizon expected discounted profit of the dealer starting from state (i,j), i^+ is max$\{i,0\}$, i^- is min$\{i,0\}$, β is $\lambda_1 + \lambda_2 + \mu_1 + \mu_2$, α is the discount rate and ϕ_i operators are expressed as follows:

$$\phi_1 v(i,j) = \begin{cases} max\{v(i-1,j) + R(1-R_{shr}), v(i,j-1) + R(1-R_{shr}) - min(r,y) - tr(1-tr_{inc}), v(i,j) - w\} & ,K_2 < j \leq S_2 \\ max\{v(i-1,j) + R(1-R_{shr}), v(i,j) - w\} & ,otherwise \end{cases}$$

$Eq.$ 3.3.2

$$\phi_2 v(i,j) \begin{cases} v(i,j) & ,j = T_2 \\ v(i,j-1) & ,j > Z_2 \\ max\{v(i-1,j) + max(r,x), v(i,j-1) - ltr_{pen}\}, & otherwise \end{cases}$$

$Eq.$ 3.3.3

$$\phi_3 v(i,j) = max\{v(i,j), v(i+1,j)\} \qquad Eq. 3.3.4$$

$$\phi_4 v(i,j) \begin{cases} v(i,j+1), & j < S_2 \\ v(i,j) & ,j = S_2 \end{cases} \qquad Eq. 3.3.5$$

Meaning of ϕ_i operators and solution approach are similar to pure decentralized system. For detailed explanation, please refer to Section 3.1.

Considering incentive, subsidy and penalty settings, dealer’s profit, manufacturer’s profit and system wide profit are written as follows:

Dealers' Profit = $R(1 - R_{shr})$(total discounted number of demands satisfied using own resources) + $[R(1 - R_{shr}) - tr\ (1 - tr_{subs}) - min\{r,y\}]$(total discounted number of demands satisfied using other dealer's resource) + $max\{r,x\}$ (total discounted number of accepted lateral transshipment request made by other dealer) - ltr_{trn}(total discounted number of rejected requests made by other dealer) – l(total discounted number of waiting customers of the dealer) – $h(1 - h_{subs})$(total discounted inventory level of the dealer) – w(total discounted number of rejected customers by the dealer)

Manufacturer's Profit = -(l_m)(total discounted number of waiting customers in the system) + $R\ R_{shr}$(total discounted number of total sales) – $tr\ tr_{subs}$(total discounted number of laterally transshipped items between dealers) – $min\{0,r\text{-}y\}$(total discounted number of laterally transshipped items between dealers) – $min\{0,x\text{-}r\}$ (total discounted number of laterally transshipped items between dealers) + ltr_{pen}(total discounted number of rejected lateral transshipment requests) – $h\ h_{subs}$(total discounted inventory level of the system) - w(total discounted number of rejected customers)

System Wide Profit = R (total discounted number of total sales) – tr (total discounted number of laterally transshipped items between dealers) – (l+ l_m) (total discounted number of waiting customers in the system) – h(total discounted inventory level of the system) – w(total discounted number of rejected customers).

Note that, system wide profit is summation of dealers' profits and manufacturer's profit. Also note that only one incentive, subsidy or penalty setting is applied at a time.

CHAPTER 4

COMPUTATIONAL ANALYSIS OF CENTRALIZED/DECENTRALIZED SYSTEM INCLUDING INCENTIVE, SUBSIDY AND PENALTY DESIGNS

In this chapter, the issue discussed is the incentive, subsidy and penalty designs under decentralized system in order to determine the appropriate designs that can align the decentralized system with the centralized system. Infinite horizon discounted expected profit criterion is regarded while determining optimal operating policies. To obtain static operating policies of dealers, heuristic developed by Usta (2010) is used.

Performance measures are profit (infinite horizon discounted expected profit) and service level (infinite horizon discounted number of waiting customers). In the centralized system, following parameter sets are used:

- Revenue per sale for the dealers (R_1 = R_2) = 10,
- Revenue per sale for the manufacturer = 0
- Holding Cost (h_1 = h_2) = 0.5, 1, 2,
- Backorder Cost (l_1 = l_2) = 3,
- Loss of good will cost (w) = 5,
- Transportation Cost (tr) = 2, 6,

- Customer arrival rate to any dealer ($\lambda_1 = \lambda_2$) = 0.3, 0.6, 0.75, 0.9, 0.99,
- Item arrival rate to any dealer ($\mu_1 = \mu_2$) = 1,
- Discount Rate (α) = 0.05.

In the decentralized system with coordination mechanisms, in addition to the parameter sets of centralized system, the following sets are used:

- Commission Payment (r) = 1, 3, 6, 9,
- Revenue Sharing Percentage (R_{shr}) = 0, 0.2, 0.4, 0.6, 0.8,
- Holding Cost Subsidy Percentage (h_{subs}) = 0, 0.2, 0.4, 0.6, 0.8,
- Request Rejection Penalty (ltr_{pen}) = 0, 2, 4, 6, 8, 10,
- Transportation Cost Subsidy Percentage (tr_{subs}) = 0, 0.2, 0.4, 0.6, 0.8,
- Commission Incentive Setting (r_{inc}) = $r_{6,4}$, $r_{7,3}$.

In this section, six environments are considered: Centralized system versus decentralized system and centralized system versus decentralized system with five coordination mechanisms. It is assumed that only one incentive, subsidy or penalty can be applied at the same time.

4.1 CENTRALIZED SYSTEM VERSUS DECENTRALIZED SYSTEM WITH NO INCENTIVE

In the decentralized system, there are two independent dealers. Manufacturer is not a player. Any dealer determines its policy with respect to the other dealer's policy (Note that, inventory status information is shared between Dealer 1 and Dealer 2 and between dealers and manufacturer). Then, other dealer determines its policy in the same way. At the end of this iterative game, dealers set their policies. Actually, dealers' objectives are to maximize their own profits under decentralized system.

In centralized system, on the other hand, central authority takes into account system-wide profit rather than letting dealers to maximize their own profits.

In order to compare decentralized system with centralized system, the following indicator is used:

$$Relative\ Gap\ (RG) = \frac{\Pi_{cent} - \Pi_{dec}}{\Pi_{dec}} \qquad Eq. 4.1.1$$

where Π_{cent} is the profit under centralized system, and Π_{dec} is the profit under decentralized system. Both Π_{cent} and Π_{dec} are obtained upon determining the operating strategies of dealers in centralized and decentralized system, respectively using the heuristic of Usta (2010). The relative gap is quite high under high holding cost (See Figure 4.1.1).

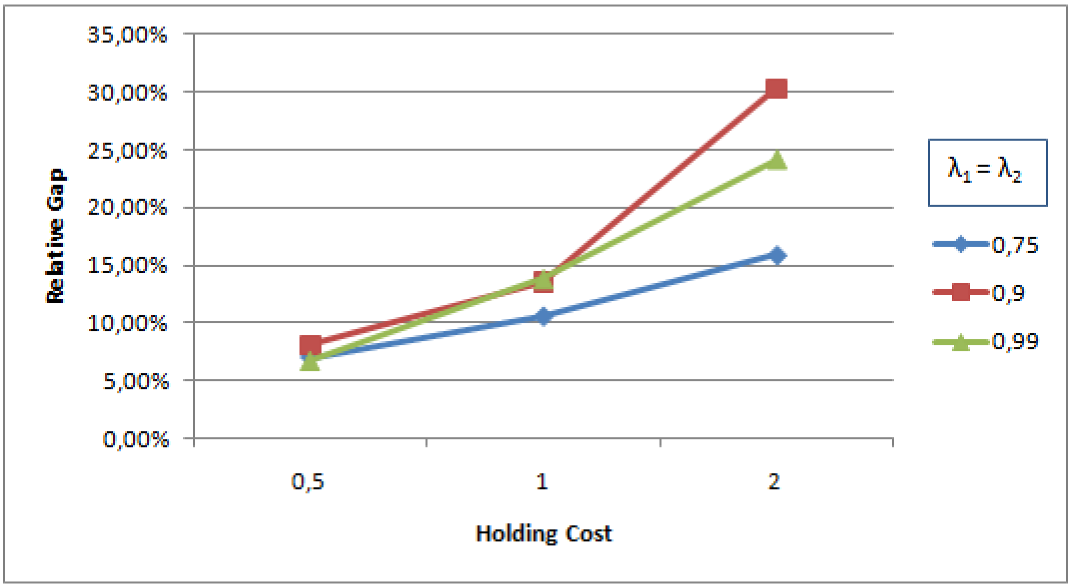

Figure 4.1.1 Relative gap (*R_1= R_2=10, r=1, l_1= l_2=3, w=5, tr=2*)

Regarding analysis, the incentive, subsidy or penalty settings are expected to make *K* and *Z* levels closer to each other without changing the base stock (*S*) and customer rejection (*T*) levels. In this case, higher number of customers is satisfied and this leads to an increase in the manufacturer's profit. Request rejection penalty, transportation cost subsidy and commission subsidy serve for this purpose.

These settings are also expected to narrow the gap between centralized and decentralized profit.

The incentive, subsidy and penalty settings are expressed as follows:

1 Revenue Sharing: Revenue *(R)* is shared with the manufacturer.

2 Holding Cost Subsidy: Particular percentage of the holding cost is subsidized by the manufacturer.

3 Request Rejection Penalty: In case of a lateral transshipment request rejection, rejecting dealer pays penalty to the manufacturer.

4 Transportation Cost Subsidy: In case of transshipment between dealers, particular percentage of the transportation cost is subsidized by the manufacturer.

5 Commission Subsidy: The parameter associated with this incentive is $r_{x,y}$. In this setting, manufacturer makes sure that the commission paid by a dealer does not exceed y and commission received by a dealer is not less than x.

The designs are discussed in detail in Sections 4.2, 4.3, 4.4, 4.5, 4.6, respectively.

4.2 EFFECT OF REVENUE SHARING

In this design, revenue per sale gained by a dealer is shared with the manufacturer with parameter R_{shr}. To illustrate, R_{shr} = 40% means dealer receives 60% of the revenue whereas manufacturer receives 40% of it. As revenue sharing percentage decreases; that is, if revenue of the dealer increases, higher number of demands could be satisfied and number of waiting and rejected customers could decrease. As a result, total number of sales increases, and decentralized system's profit could align with the centralized system's profit.

Revenue sharing effect on profit is expressed in Figure 4.2.1. At first glance, manufacturer's profit increases with increasing revenue sharing percentage whereas that of dealers' decreases. Note that similar behavior hold under different demand rates. The result is very logical: If the bigger part of the revenue is shared, manufacturer gains more and dealers gain less.

Decentralized system profit is maximized when revenue sharing percentage is around 40%. Actually, as revenue sharing percentage increases, dealer gains nearly 0 for each item sold. This situation could be profitable for the manufacturer; however, this is the worst performance if system profit is taken into account.

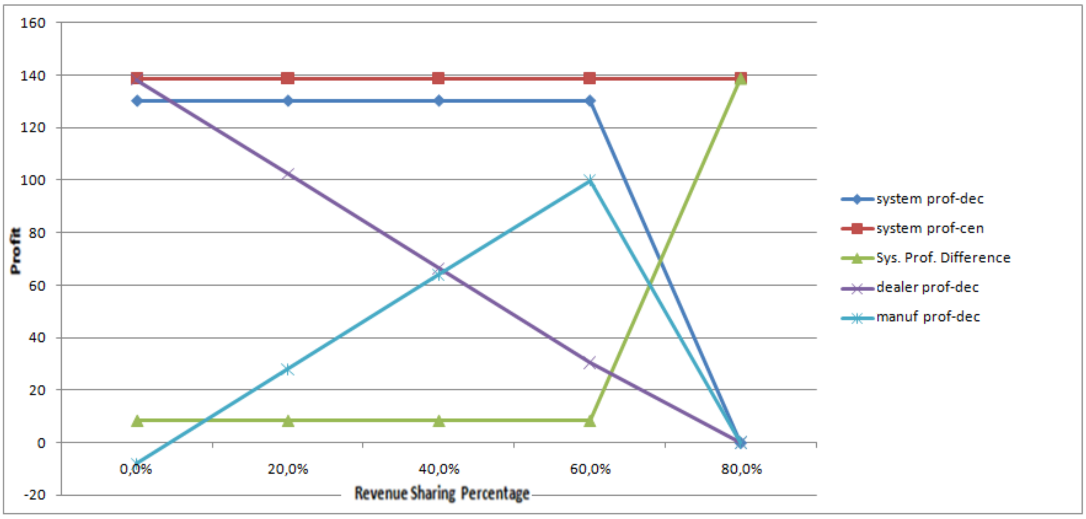

Figure 4.2.1 Profit ($h_1 = h_2 = 1$, $r = 1$, $\lambda_1 = \lambda_2 = 0.45$, $tr = 2$)

As revenue sharing percentage increases, customer rejection level (T) increases and approaches to the request level (Z) (Figure 4.2.2). Actually, the two levels coincide when the commission payment is high. The increase in T results in 1% - 2% decrease in the number of units sold and 7% - 8% decrease in number of customers in the queue. Intuitively, as revenue sharing percentage increases; that is, revenue of the dealer decreases, satisfying demand using other dealer's resource is not as profitable as using own resources due to commission payment and transportation cost. That is why dealer increases its T level and narrows the gap between Z and T with increasing revenue sharing. Note that under high revenue sharing percentage (R_{shr}=80%), dealers cannot make profit. Hence, they stop to operate and system profit becomes zero.

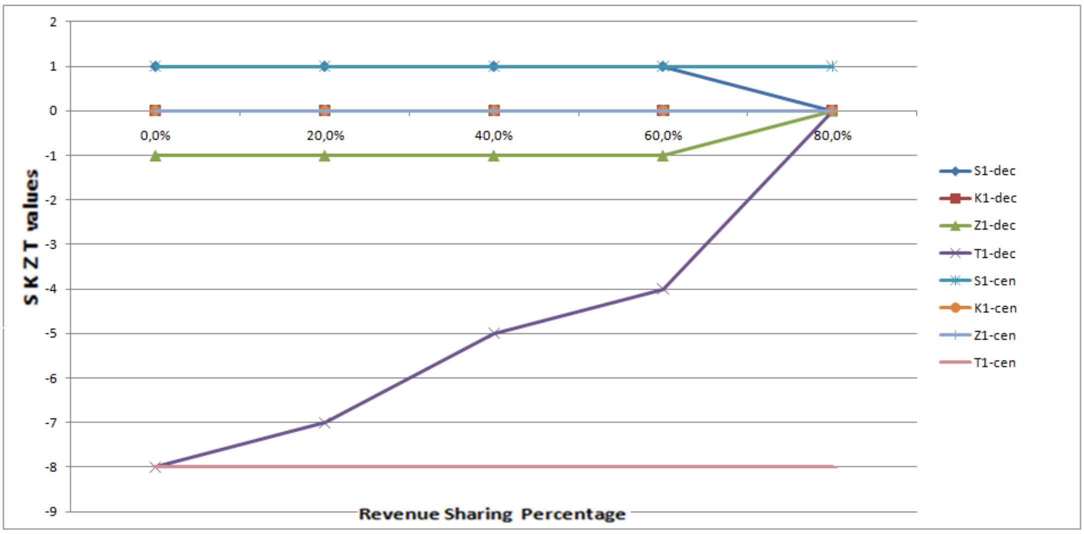

Figure 4.2.2 *SKZT* decisions ($h_1 = h_2 = 1$, $r = 1$, $\lambda_1 = \lambda_2 = 0.45$, $tr = 2$)

4.3 EFFECT OF HOLDING COST SUBSIDY

In this design, portion of holding cost of the dealer is subsidized by the manufacturer with parameter h_{subs}. h_{subs} = 40% means dealer incurs 60% of the holding cost whereas manufacturer subsidizes 40% of it. As holding cost subsidy percentage increases; that is, holding cost of the dealer decreases, dealer holds higher amount of inventory and this leads to increase in the number of sales. From the manufacturer's point of view, it incurs specific percentage of holding cost.

The effect of holding cost subsidy on the system profit and *S-K-Z-T* are shown in Figure 4.3.1 and Figure 4.3.2 respectively.

As holding cost subsidy percentage increases, Z value is not affected. S, K, T values, on the other hand, diverges from the centralized values. In addition, gap between K

and *Z* widens under decentralized system. These observations are clearer under high demand rates and high commission payment.

Increase in holding cost percentage leads to increase in base stock level (*S*) under decentralized system. As *S* increases, inventory level increases. This leads to increase in number of sales and in inventory level.

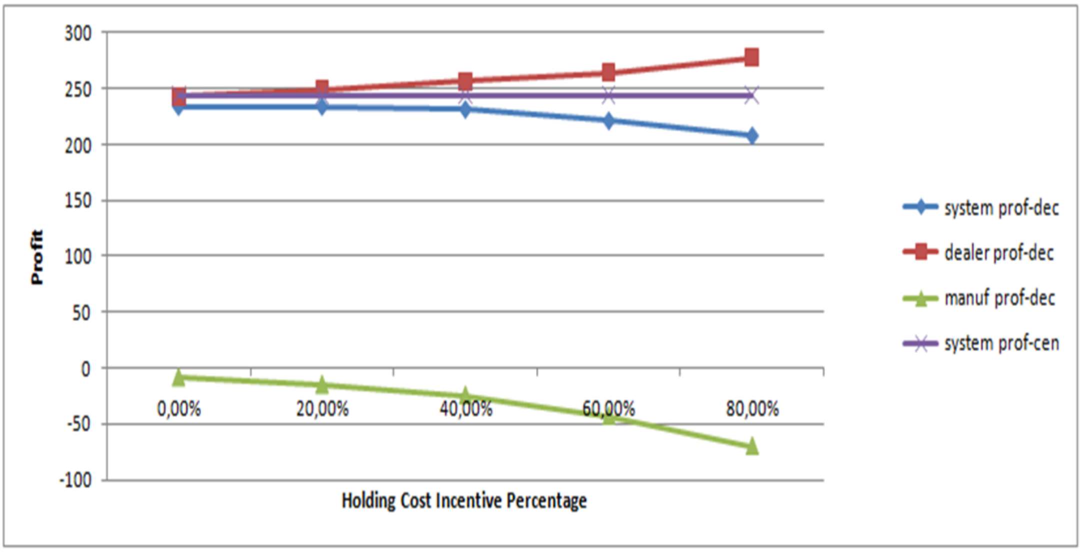

Figure 4.3.1 Profit (h_1= h_2= *0.5*, *r* = *6*, λ_1=λ_2= *0.75*, *tr* = *2*)

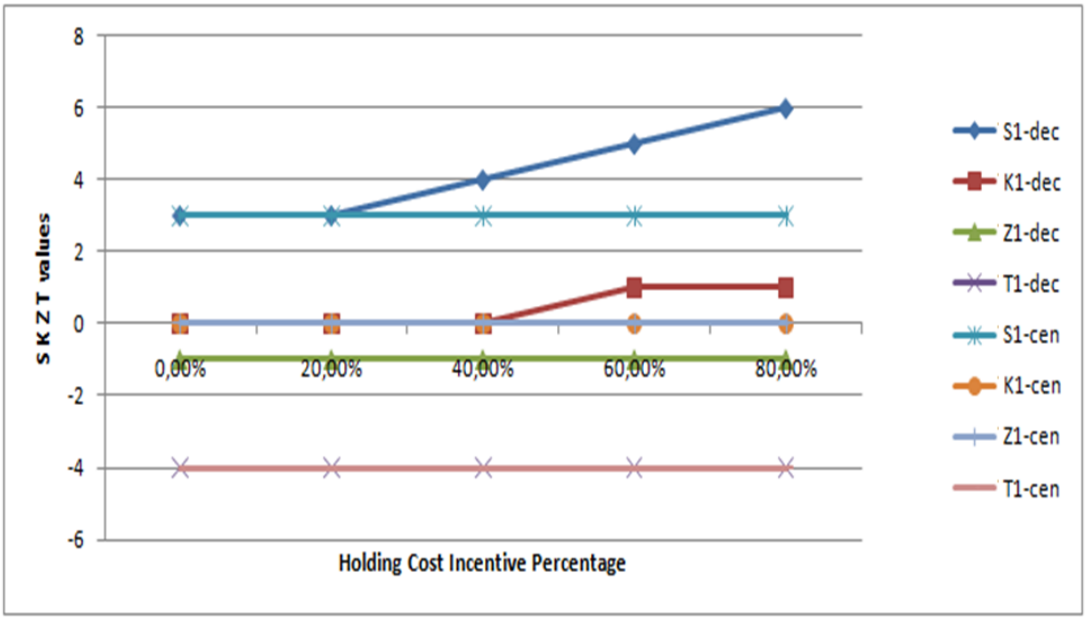

Figure 4.3.2 SKZT decisions (h_1= h_2= *0,5*, *r* = *6*, λ_1=λ_2= *0.75*, *tr* = *2*)

As gap between *K* and *Z* widens, dealers accept less number of lateral transshipment requests. So, dealer is willing to satisfy its own demand instead of the other dealer's request. The result is the decrease in item flow between dealers.

As holding cost subsidy percentage increases, number of customers in the queue, number of lost sales decrease and number of sales increases. It is so clear that dealers' profit increase. Also, system profit could be expected to increase. However, effect of *S*, *K*, *T* values and decrease in inventory sharing lead to decrease in the system profit (Figure 4.3.1). Thus, giving holding cost subsidy is not a good strategy in terms of system-wide profit.

4.4 EFFECT OF REQUEST REJECTION PENALTY

In this design, dealer pays "ltr_{pen}" amount of penalty to the manufacturer for each transshipment request it rejects. If dealer pays penalty for each request rejection, inventory pooling between dealers increases and decentralized system may align with centralized system. In addition, manufacturer profit may increase with increasing penalty.

The effect of rejection penalty on the system profit and *S*-*K*-*Z*-*T* are shown in Figure 4.4.1 and Figure 4.4.2 respectively.

Request rejection penalty does not have effect on the base stock level (*S*). However, it has major effect on rationing level (*K*): As penalty increases, *K* decreases and coincides with *Z*. Centralized *K* and *Z* values stay above with respect to the decentralized values under high rejection penalty. This is a rational decision since dealer does not prefer a rejecting-dealer role under high penalty. To do this, rationing level (*K*) is decreased so that higher number of transshipment requests is accepted. Penalty has minor effect on customer rejection level (*T*): *T* slightly increases under high penalty. The effect of the penalty is similar under different demand rates.

The rejection penalty does not have a major effect on units sold and lost sales. Instead, it has an effect on inventory transfer between dealers and number of customers in queue. Low values of penalty (2 and 4) increase inventory flow between dealers and decrease the number of customers in queue. So, customer satisfaction increases and system aligns with the centralized environment.

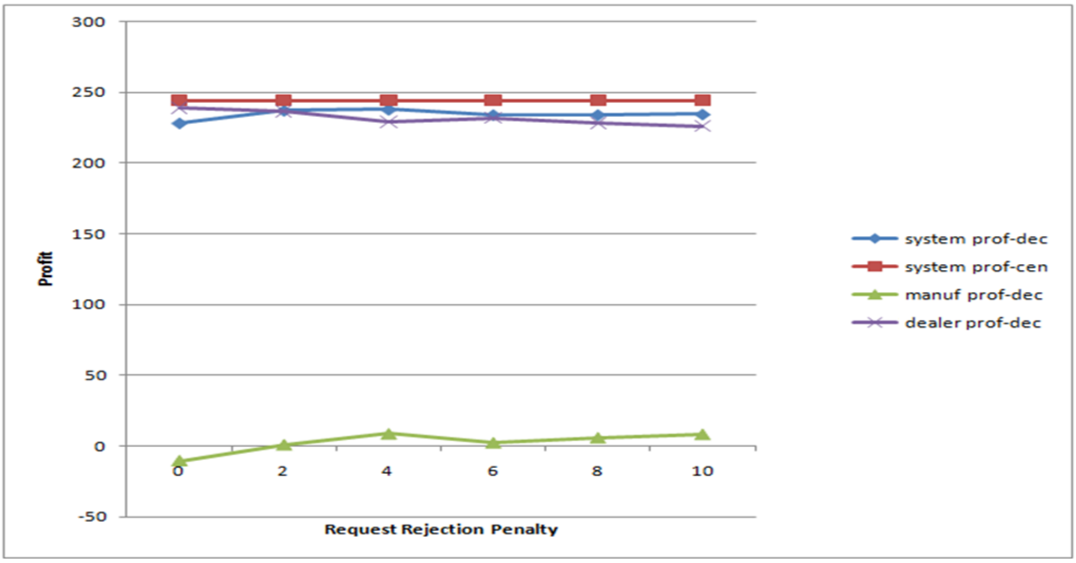

Figure 4.4.1 **Profit (h_1= h_2= 0.5, r = 1, λ_1=λ_2= 0.75, tr = 2)**

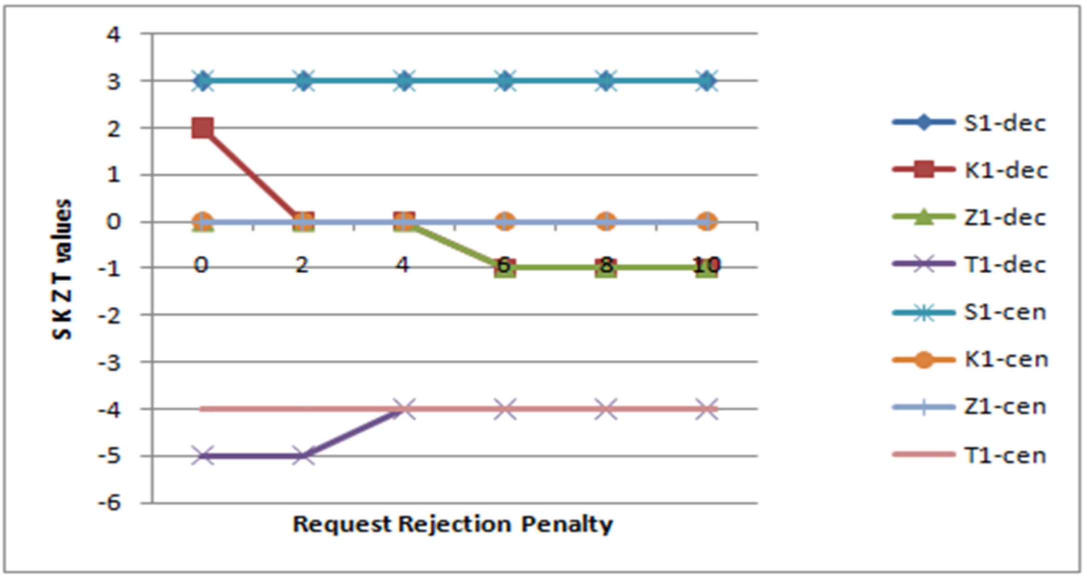

Figure 4.4.2 SKZT decisions (h_1= h_2= 0.5, r = 1, λ_1=λ_2= 0.75, tr = 2)

As penalty gets higher (> 4), dealers prefer to share less inventory and number of customers in the queue increases. System diverges from centralized system. Intuitively, the initial reaction of dealers is to decreases *K* values in order to share more inventory and to decrease rejection penalty costs. Under high rejection penalty, dealers decrease *Z* levels. By that way, probability of request rejection is decreased. Low *Z* values also serve for decrease in the lateral transshipment requests because dealer having low *Z* makes fewer requests. That is why under very high rejection penalty, inventory flow between dealers decreases. Note that under any parameter setting, inventory flow under centralized system is always higher than inventory flow under decentralized system.

From the profit point of view, when rejection penalty is low (2 and 4), system profit in decentralized environment approaches to centralized system's profit. This result is closely related to the inventory flow between dealers. As rejection penalty increases, system diverges from the centralized system. The cause of this behavior is the decrease in the *K-Z* values and decrease in inventory flow between dealers: Since increase in the rejection penalty leads to divergence form centralized system in terms of *S-K-Z-T* values, it also affects profit and triggers divergence from centralized environment.

4.5 EFFECT OF TRANSPORTATION COST SUBSIDY

In this design, portion of transportation cost is subsidized by the manufacturer with parameter tr_{subs}. tr_{subs} = 40% means dealer incurs 60% of the transportation cost whereas manufacturer subsidizes 40% of it. As transportation cost subsidy percentage increases; that is, transportation cost of the dealer decreases, inventory sharing is expected to increase. As a result, decentralized system may align with centralized system.

The effect of transportation cost subsidy on the system profit under low ($tr=2$) and high ($tr=6$) transportation cost is shown in Figure 4.5.1 and Figure 4.5.2 respectively.

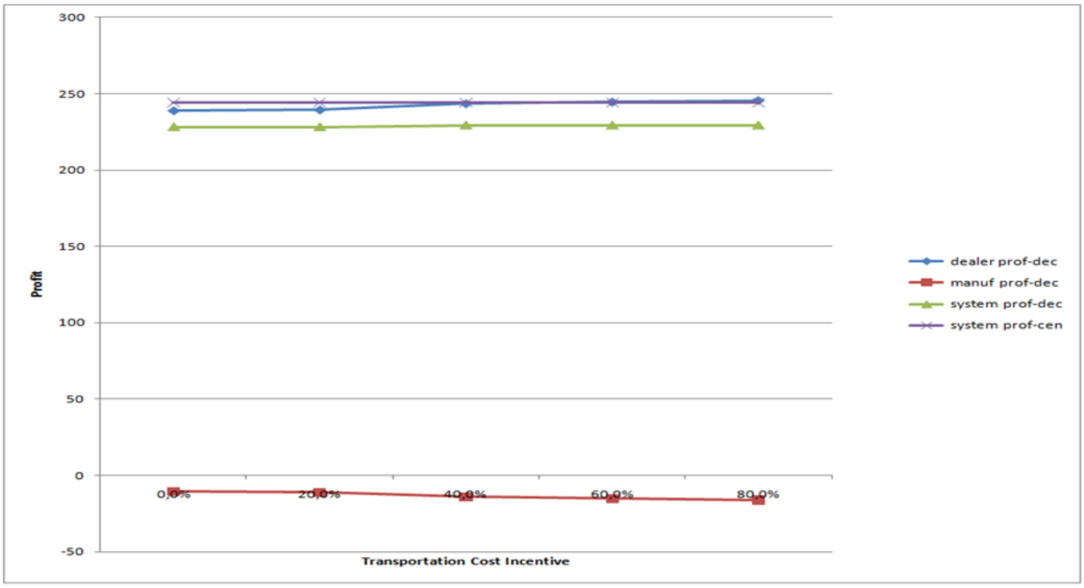

Figure 4.5.1 Profit (h_1= h_2= 0.5, *r* = 1, λ_1=λ_2 = 0.75, *tr* = 2)

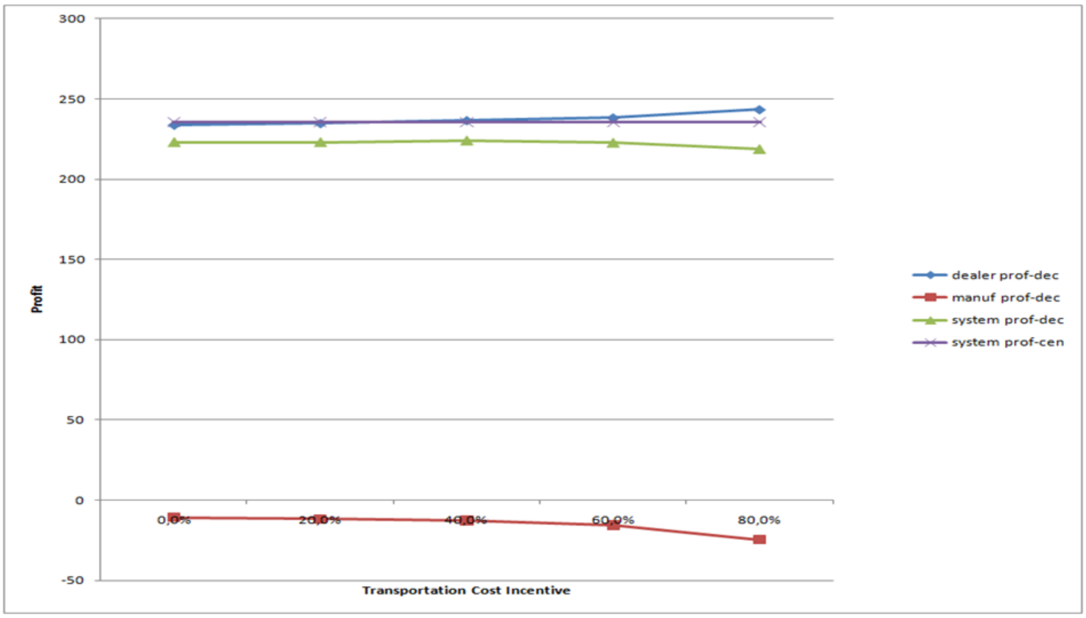

Figure 4.5.2 Profit (h_1= h_2= 0.5, *r* = 1, λ_1=λ_2 = 0.75, *tr* = 6)

Under high demand rate ($\lambda_1 = \lambda_2 > 0.60$) and high transportation cost, transportation cost subsidy is more effective: As it increases, inventory flow between dealers increases. This increase is as high as 107.6% (comparing incentive percentages 0.0% and 80.0%) when transportation cost is 6 and as low as 20.4% when it is 2. Comparing with centralized system, decentralized system first aligns with than diverges in terms of inventory flow under high transportation cost (tr = 6). When transportation cost is low (tr = 2), inventory flow increases and gets closer to centralized system with increasing transportation cost incentive.

Under high demand rate (λ_1=λ_2>0.60), total sales and number of waiting customers increase with increasing incentive. Under low demand rate (λ_1=λ_2≤0.60), on the other hand, number of waiting customers decreases and aligns with centralized environment. Number of total sales is not majorly affected by the incentive.

From the profit point of view, when transportation cost is low (tr = 2), decentralized system profit gets closer to the centralized system profit as transportation cost subsidy increases (Figure 4.5.1). When transportation cost is high, on the other hand, decentralized system profit initially aligns with then diverges from the centralized system (Figure 4.5.2). Under low transportation cost dealer decreases its base stock level (S) with increasing transportation cost subsidy since inventory sharing increases. As a result, inventory holding cost decreases and decentralized system aligns with centralized system. When transportation cost is high, similar behavior is observed when transportation cost subsidy percentage is less than 80%. When this subsidy is high (≥ 80%), dealers prefer to increase Z levels so that more lateral transshipment requests can be made. From the requested dealer's point of view, increasing Z leads to higher number of rejected transshipment requests. This behavior decreases inventory flow between dealers. As a result, decentralized system diverges from centralized system.

4.6 EFFECT OF COMMISSION SUBSIDY

In this design commission payer and commission receiver are subsidized by the manufacturer such that commission paid by a dealer does not exceed 'y' and commission received by a dealer is not less than 'x'. The design parameter is $r_{x,y}$. When commission is subsidized, inventory flow between dealers can increase and decentralized system may align with centralized system.

The effect of commission subsidy on the system profit under low ($r=1$) and high ($r=9$) commission is shown in Figure 4.6.1 and Figure 4.6.2 respectively.

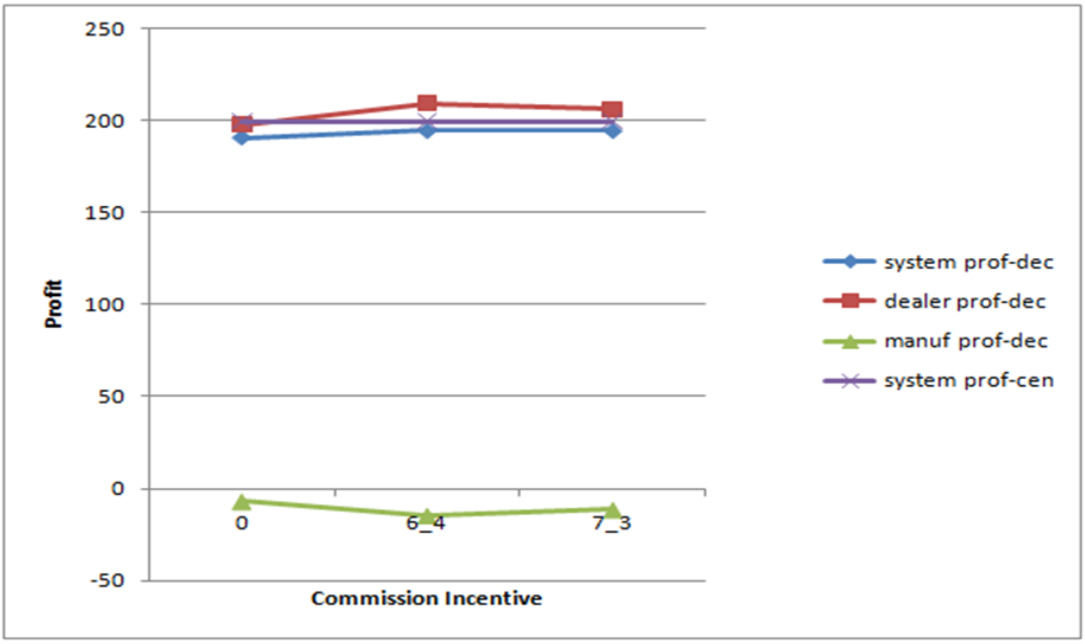

Figure 4.6.1 Profit ($h_1 = h_2 = 0.5$, $r = 1$, $\lambda_1 = \lambda_2 = 0.6$, $tr = 2$)

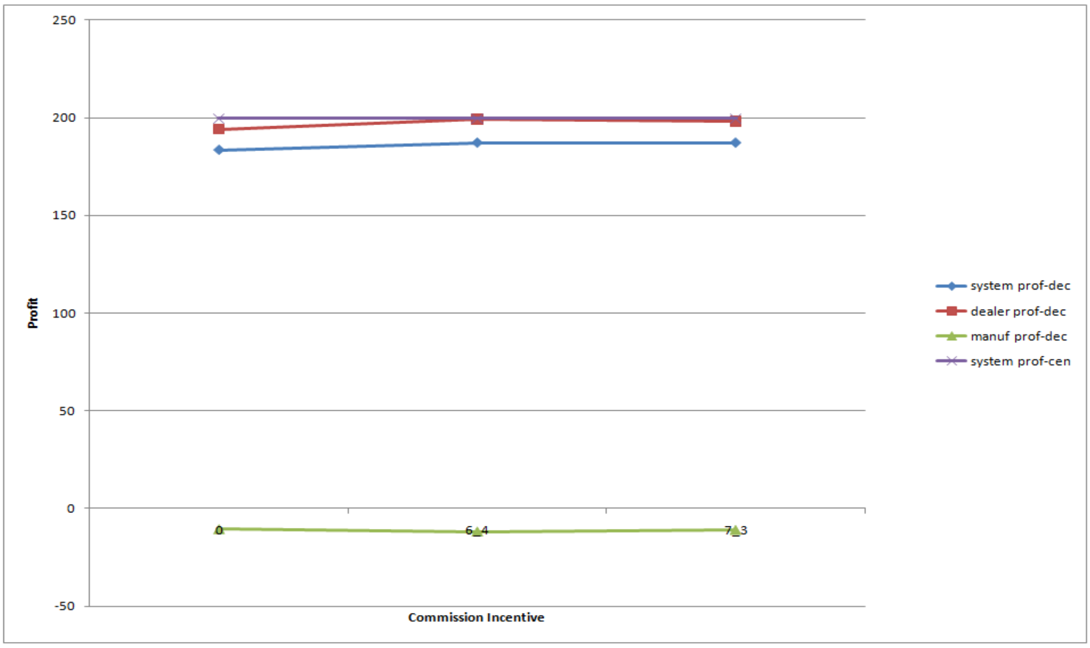

Figure 4.6.2 Profit ($h_1 = h_2 = 0.5$, $r = 9$, $\lambda_1 = \lambda_2 = 0.6$, $tr = 2$)

Commission subsidy has effect on K, Z and T values. Under low commission (r = 1), K decreases with both commission subsidy settings. Z and T is not affected. Under high commission (r=9), Z increases and T decreases with both subsidy settings. Under middle values of commission (r=3, 6), incentive does not influence S,K,Z,T values. Intuitively, when commission is low, the requested dealer starts to gain higher revenue from commission payment of the other dealer. So, it wants to satisfy more lateral transshipment request and decreases rationing level (K). When commission payment is high, requesting dealer's commission payment is decreased; so, it wants to place lateral transshipment requests more frequently. Consequently, Z value is increased. Under middle values of commission payment (r=3,6), the incentive payment to the dealers is not as profitable as the payment under low and high commissions. So, decentralized system is not affected.

Inventory flow between dealers and number of waiting customers, on the other hand, increases and decreases respectively with commission subsidy under low (r=1) and high (r=9) commissions.

From the profit point of view, decentralized system aligns with centralized system with the help of commission subsidy under low and high commissions. For the middle values of commission, subsidy does not affect decentralized system. This behavior is closely related with the commission subsidy effect on rationing level (K) and request level (Z). When commission is low (r=1) or high (r=9), commission subsidy serves for alignment to the centralized system. This also leads system profit to get closer to centralized system's profit. After subsidy is applied, the gap between decentralized and centralized system's profit is narrower when commission is low because it triggers inventory sharing whereas high commission decreases. When commission is high, decentralized profit gets closer to centralized system's profit but the gap between the centralized profit is wider.

4.7 SUMMARY

To sum up, three systems, namely decentralized system, centralized system and decentralized system with incentive, subsidy, penalty designs are analyzed in order to see the effect of coordination mechanisms on the decentralized system. Under the decentralized system, dealers are independent and want to maximize their own profits. In centralized system, on the other hand, manufacturer, which is assumed to be the system authority, considers the system wide profit and maximizes it. In decentralized system with incentive, subsidy, penalty designs, which is the modified system of pure decentralized system, manufacturer tries five different coordination mechanisms. Firstly, revenue sharing leads to decrease in service level, increase in number of waiting customers and number of rejected customers. System profit under decentralized environment is maximum when revenue sharing percentage is around 40%. Secondly, holding cost subsidy decreases number of waiting customers and lost demand under decentralized environment; however, system profit decreases due to decrease in the number of lateral transshipments. Thirdly, lower values of rejection penalty leads to higher number of laterally transshipped items. Increase in the number of lateral transshipments results in increase in the profit under decentralized environment. Fourthly, increase in the transportation cost subsidy aligns the decentralized system with the centralized system due to increase in the number of lateral transshipments. This alignment is best under low arrival rates. Lastly, commission subsidy effect is observable under extreme values of the commission payment. When commission is 1 or 9, commission subsidy increases number of lateral transshipments and decreases backordered demands.

CHAPTER 5

DESCRIPTION OF THE MODELS FOR A SYSTEM WITH UNPOOLED CAPACITY AND POOLED CAPACITY

Upon examining different coordination mechanisms at the lower echelon of the supply chain system, the manufacturer is now interested in the allocation strategy at the upper echelon: The manufacturer wonders the advantage of pooling the production capacity over separately dedicating it to dealers.

In this chapter, system is always operated under a central authority (manufacturer). In this centralized environment, the decisions are given to maximize system wide profits. Note that all cost parameters are known by the manufacturer. Two systems are compared:

i. System with unpooled capacity,
ii. System with pooled capacity.

In the unpooled system, it is assumed that total capacity is shared among the dealers. In the pooled system, on the other hand, total capacity is allocated to single dealer at unit time.

5.1 SYSTEM WITH UNPOOLED CAPACITY

In this system, there are two dealers and one manufacturer. System is operated under the central authority. Note that one item is under consideration and inventory sharing does not exist. The objective is to maximize infinite horizon average expected system profit. Backorder is allowed.

Total capacity is shared by the dealers in the unpooled system. To illustrate, when capacity allocated to Dealer 2 is idle, Dealer 1 cannot use Dealer 2's idle capacity. Instead, Dealer 1 uses only the capacity that is allocated to itself. In this system, dealers have individual profit functions. Manufacturer firstly decides on the time between item arrivals of each dealer. Secondly, the manufacturer determines optimal operating policies of each dealer using their individual profit functions where time between arrivals of items is given. The policy that satisfies maximum total system profit (Dealer 1's profit + Dealer 2's profit) by deciding on the share of the production capacity to dealers is the optimal operating policy. Since dealers have individual profit functions, system state is one dimensional: m is the number of customers in the queue of Dealer 1 and reflects the Dealer 1's state and n is the number of customers in queue of Dealer 2 and reflects Dealer 2's state. (Note that negative values of m or n indicate inventory on hand.)

Number of customer arrivals fits to Poisson distribution with rate λ_1 for Dealer 1 and λ_2 for Dealer 2. Customers arriving to Dealer 1 and Dealer 2 are selected from different populations. One customer arrives at a time, bulk arrivals are not allowed. Item replenishment, on the other hand, is done independently and each dealer has a dedicated production capacity due to the unpooled system. Time between arrivals of items is exponentially distributed with rate μ_T and this capacity is shared to Dealer 1 and Dealer 2 such that time between arrivals is μ_1 for Dealer 1 and μ_2 for Dealer 2. The manufacturer makes sure that the whole capacity is shared ($\mu_T = \mu_1 + \mu_2$) and item replenishments occur one at a time. It is assumed that upon sharing the capacity to the dealers, time between item replenishments is still exponentially distributed. Item flow diagram is given in Figure 5.1.1.

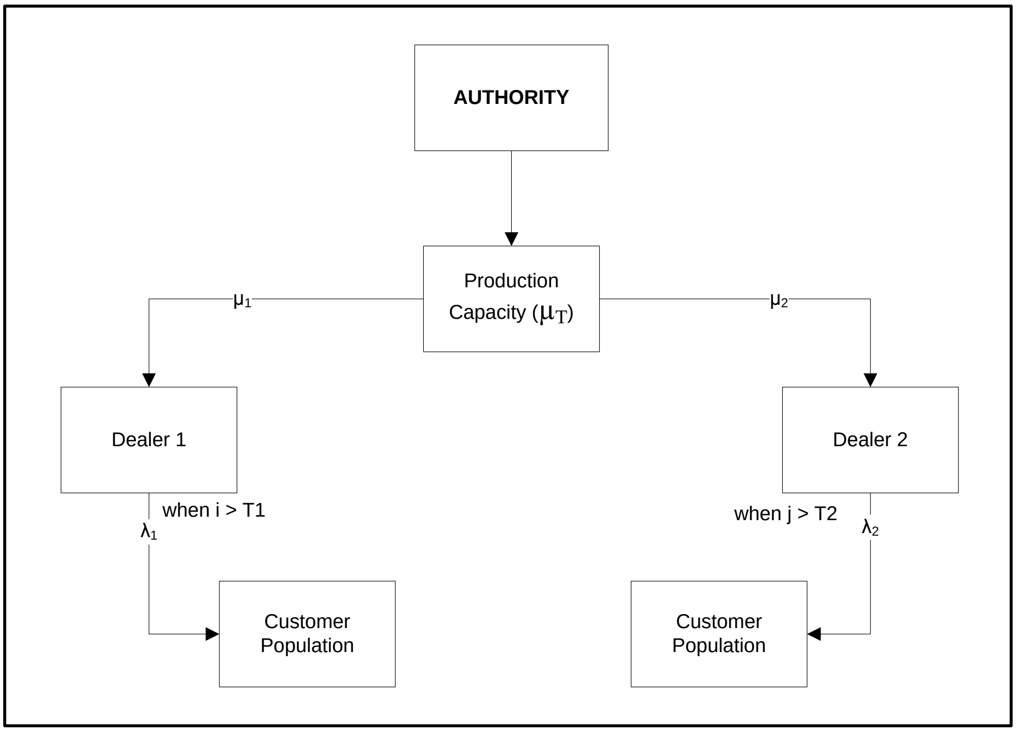

Figure 5.1.1 Item flow diagram in unpooled system

In case of a customer arrival, Dealer 1 can either (i) satisfy demand using own resources or (ii) reject the customer. Other than customer arrivals, Dealer 1 makes decision on item replenishment. Dealer 1 either (i) gives production order or (ii) does not order a new item.

Dealer 2, on the other hand, is identical in terms of actions in case of a customer arrival and decisions about production order.

When dealer *i* satisfies a demand, it obtains revenue (R_i) per sale. Cost of replenishment is so small compared to inventory holding that it is negligible. Dealer *i* also incurs holding cost (h_i) per item per unit time and backorder cost (l_i) per item per unit time.

Problem of central authority is examined through infinite horizon continuous time Markov chain. Central authority's objective is to maximize infinite horizon average expected system wide profit. Under these criterions, optimality equation is expressed as follows:

$$v_1(m) + g_1 = \frac{h_1 m^{-} - l_1 m^{+}}{\lambda_1 + \mu_1} + \frac{1}{\lambda_1 + \mu_1}[\lambda_1 \phi_1 v(m) + \mu_1 \phi_2 v(m)] \qquad Eq. 5.1.1$$

$$v_2(n) + g_2 = \frac{h_2 n^{-} - l_2 n^{+}}{\lambda_2 + \mu_2} + \frac{1}{\lambda_2 + \mu_2}[\lambda_2 \phi_3 v(n) + \mu_2 \phi_4 v(n)] \qquad Eq. 5.1.2$$

subject to $\mu_1 + \mu_2 = \mu_T$ $\quad Eq. 5.1.3$

where $v_1(m)$ is the benefit of starting in state m for dealer 1, g_1 is the long-run expected average profit of Dealer 1 per transition, $v_2(n)$ is the benefit of starting in state n for Dealer 2, g_2 is the long-run expected average profit of Dealer 2 per transition, m^+ is max{$m,0$}, m^- is min{$m,0$}, n^+ is max{$n,0$}, n^- is min{$n,0$} and ϕ_i operators are expressed as follows:

$$\phi_1 v(m) = max\{v(m+1) + R, v(m)\} \qquad Eq. 5.1.4$$

$$\phi_2 v(m) = max\{v(m), v(m-1)\} \qquad Eq. 5.1.5$$

$$\phi_3 v(n) = max\{v(n+1) + R, v(n)\} \qquad Eq. 5.1.6$$

$$\phi_4 v(n) = max\{v(n), v(n-1)\} \qquad Eq. 5.1.7$$

Operator $\phi_1 v(m)$ and $\phi_3 v(n)$ are the reflection of customer arrivals to Dealer 1 and Dealer 2, respectively. Customer can arrive at either Dealer 1 or Dealer 2. Authority chooses one of the two choices; satisfy demand (SD) or reject customer (RC), which maximizes system wide profit.

Operator $\phi_2 v(m)$ and $\phi_4 v(n)$, on the other hand, reflect production decision of Dealer 1 and Dealer 2, respectively. Note that the decision is given by the authority, not dealers. Upon customer or item arrival to any dealer, authority chooses one of the two choices; give production order (P) or stop production (DP) for the corresponding dealer, which maximizes system wide profit.

To obtain the optimal solution for a single dealer, steady state probabilities are used. Note that state of the one-dealer system is defined as the number of customers in the queue, as stated recently. State space of dealer i is $\{-S_i, -S_i+1, -S_i+2, \ldots, T_i-2, T_i-1, T_i\}$ where S_i is the base stock level of dealer i and T_i is the customer rejection level of dealer i. Being x_i current state of dealer i, system jumps to x_i-1 with rate μ_i, x_i+1 with rate λ_i. Rate diagram of dealer i is expressed in Figure 5.1.2.

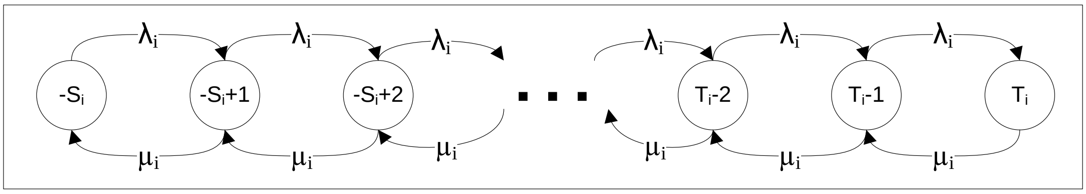

Figure 5.1.2 Rate diagram of the unpooled system

Given S_i and T_i, the steady state probabilities are as follows:

$$\Pi_{(-S_i)} = \begin{cases} \dfrac{1-\rho_i}{1-{\rho_i}^{S_i+T_i+1}} & ,\rho_i \neq 1 \\ \dfrac{1}{S_i+T_i+1} & ,\rho = 1 \end{cases} \qquad Eq. 5.1.8$$

$\Pi_{X_i} = ({\rho_i}^{X_i+S_i})\Pi_{(-S_i)}$ $\quad ,x_i \in \{-S_i, -S_i+1, -S_i+2, \ldots, T_i-2, T_i-1, T_i\}$

$Eq. 5.1.9$

where $\rho_i = \dfrac{\lambda_i}{\mu_i}$.

Let x_i represents the state of dealer i. Reward function of state x_i is expressed as below:

$Reward(x_i) =$

$$\begin{cases} \left(\frac{1}{\lambda_i+\mu_i}\, h_i\, {x_i}^{-}\right) - \left(\frac{1}{\lambda_i+\mu_i}\, l_i\, {x_i}^{+}\right) & ,x_i = T_i \\ \left(\frac{1}{\lambda_i+\mu_i}\, h_i\, {x_i}^{-}\right) - \left(\frac{1}{\lambda_i+\mu_i}\, l_i\, {x_i}^{+}\right) + \left(\frac{\lambda_i}{\lambda_i+\mu_i}\, R_i\right), & -S_i \leq x_i < T_i \end{cases}$$

$Eq. 5.1.10$

where ${X_i}^{+}$ is max $\{X_i, 0\}$ and ${X_i}^{-}$ is min $\{X_i, 0\}$.

Let G_i be the dealer i's infinite horizon average expected profit. It is expressed as follows:

$$G_i = \sum_i Reward_{X_i} \pi_{X_i}$$

$$= \frac{\lambda_i}{\lambda_i + \mu_i} R_i \sum_{X_i=-S_i}^{T_i-1} \Pi_{X_i} + \frac{1}{\lambda_i + \mu_i} h_i \sum_{X_i=-S_i}^{0} X_i \, \Pi_{X_i} + \frac{1}{\lambda_i + \mu_i} l_i \sum_{X_i=1}^{T_i} (-X_i) \, \Pi_{X_i}$$

$$= \left(\frac{\lambda_i}{\lambda_i + \mu_i} R_i\right)$$
$$+ \left(\frac{1}{\lambda_i + \mu_i}\right) \left(\frac{1-\rho_i}{(1-{\rho_i}^{S_i+T_i+1})}\right) \Bigg[-\lambda_i R_i {\rho_i}^{S_i+T_i}$$
$$+ \frac{h_i(-S_i - S_i\rho_i + \rho_i - {\rho_i}^{S_i+1}) + l_i {\rho_i}^{S_i}(T_i {\rho_i}^{T_i+1} - T_i {\rho_i}^{T_i+2} - \rho_i + {\rho_i}^{T_i+1})}{(1-\rho_i)^2} \Bigg]$$

$Eq.$ 5.1.11

where $\rho_i = \dfrac{\lambda_i}{\mu_i}$ and $\rho_i \neq 1$. (Please see Appendix A for the details of the derivation.)

When $\rho_i = 1$, closed form profit function is expressed below:

$$\boldsymbol{G_i} = \left(\frac{\lambda_i}{\lambda_i + \mu_i}\right)\left(\frac{S_i + T_i}{S_i + T_i + 1}\right) R_i - \left(\frac{1}{\lambda_i + \mu_i}\right)\left(\frac{S_i(S_i+1)}{2(S_i + T_i + 1)}\right) h_i$$
$$- \left(\frac{1}{\lambda_i + \mu_i}\right)\left(\frac{T_i(T_i+1)}{2(S_i + T_i + 1)}\right) l_i \qquad Eq. 5.1.12$$

For a given μ_i, the problem of determining S_i and T_i levels of dealer i is relatively easier problem. However, jointly determining *Si, Ti* and μ_i is more challenging.

Figure 5.1.3 expresses the profit curves of a single dealer under different *S, T* and μ. Note that *S, T* and μ values are arbitrarily chosen. Regarding the curves, each of which corresponds to a specific *S*-*T* pair, best solution lies on different *S*-*T* curves as μ changes. The profit function under optimal solution is expressed in Figure 5.1.4. In figure 5.1.4, it can easily be detected that optimal profit curve of a dealer is constructed by different *S*-*T* pair curves. Thus, optimal curve is not concave. Due to jumps on the optimal profit curve, closed form of the profit function cannot be obtained and system is solved through a search algorithm.

The search algorithm assigns different combinations of μ_1 and μ_2 to Dealer 1 and Dealer 2, satisfying "$\mu_1+\mu_2=\mu_T$". When dealer *i* is assigned μ_i, the optimal operating policy (S_i and T_i values) is determined accordingly for that dealer. Afterwards, profit of both dealers is summed and G_1+G_2 is obtained. μ_1 and μ_2 pair that provides highest profit is the optimal solution for the unpooled system. (See Appendix A for search algorithm flow)

After the solution procedure, optimal μ_1 and μ_2 values are obtained. They are the capacity allocated to Dealer 1 and Dealer 2, respectively. Capacity usage of dealer *i*, on the other hand, is a different measure. When dealer *i* is at its base stock level (S_i), production stops and idleness occurs. Thus, dealer *i* does not utilize the capacity that is allocated to it.

Idleness percentage of dealer *i* is denoted with $Idle_i$ and percentage of time the production facility is busy for dealer *i* is denoted with CU_i. $Idle_i$ and CU_i are calculated as follows:

$$Idle_i = \Pi_{(-S_i)} \frac{\mu_i}{\mu_T} \qquad Eq. 5.1.13$$

$$CU_i = \left(1 - \Pi_{(-S_i)}\right) \frac{\mu_i}{\mu_T} \qquad Eq. 5.1.14$$

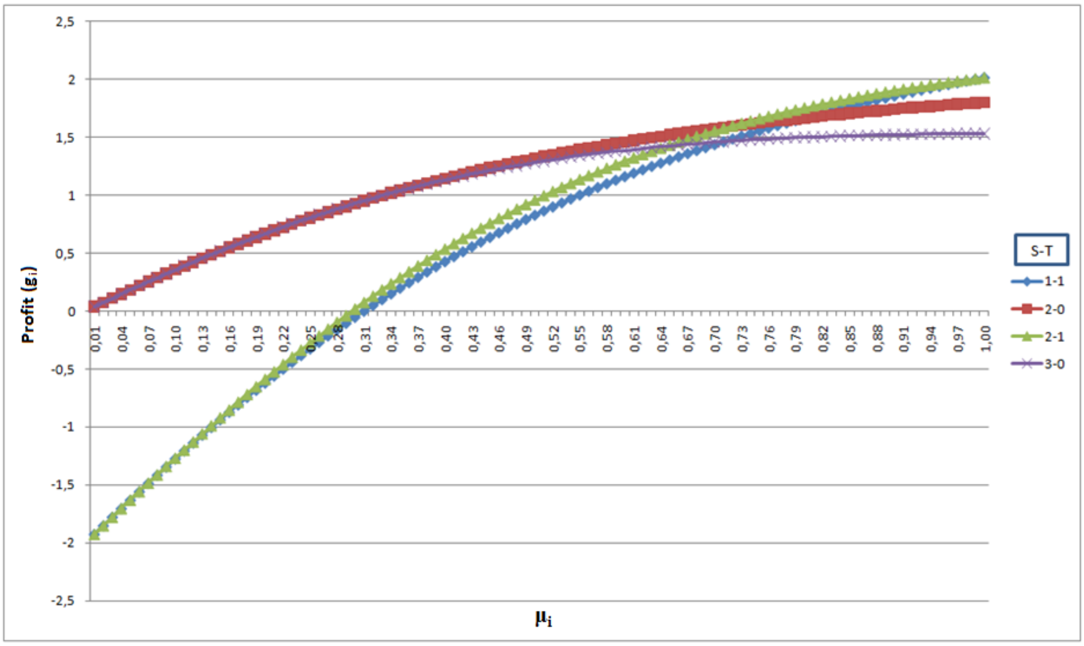

Figure 5.1.3 ***S*** **and** ***T_i*****,** ***μ_i*** **vs. Profit (*g_i*) (*h_1=1, R_1=5, l_1=2, λ_1=0.8*)**

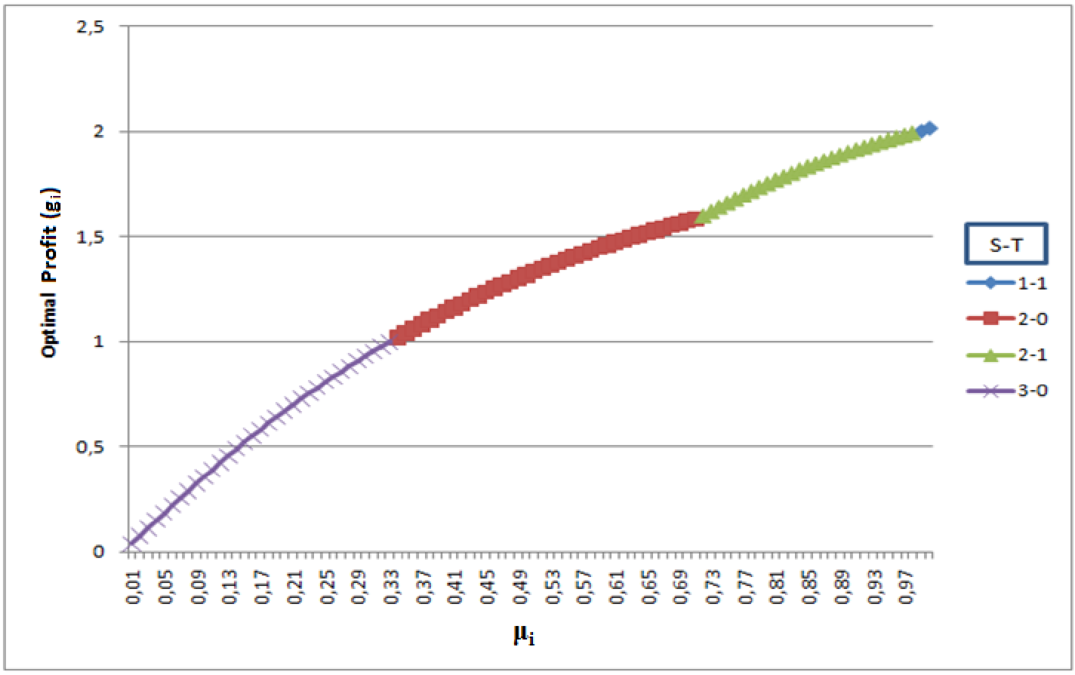

Figure 5.1.4 ***μ_i*** **vs. Profit (*g_i*) under optimal strategy (*h_1=1, R_1=5, l_1=2, λ_1=0.8*)**

5.2 SYSTEM WITH POOLED CAPACITY

In this system, two dealers and one manufacturer exist. System is operated under the central authority (manufacturer). Note that one item is under consideration and inventory sharing does not exist. Objective is to maximize infinite horizon average expected system profit. Backorder is allowed.

Different than the unpooled system, whole capacity is allocated to a single dealer in a unit time or production facility is kept idle. In other words, capacity is not separately dedicated to the dealers.

Number of customer arrivals fits to Poisson distribution with rate λ_1 for Dealer 1 and λ_2 for Dealer 2. Customers arriving to Dealer 1 and Dealer 2 are selected from different populations. One customer arrives at a time, bulk arrivals are not allowed. Item replenishment, on the other hand, is done independently and each dealer uses the whole capacity during different time intervals. In other words, production facility is allocated to a single dealer within a unit time. Time between arrivals of item is exponentially distributed with rate μ_T to dealer *i*. Item replenishments occur one at a time. Item flow diagram is expressed in Figure 5.2.1.

In case of a customer arrival, dealer *i* can either (i) satisfy demand using own resources or (ii) reject the customer. Production facility, on the other hand, is either (i) allocated to Dealer 1 (one unit is produced and sent to Dealer 1), (ii) allocated to Dealer 2 (one unit is produced and sent to Dealer 2) or (iii) kept idle. Note that manufacturer is the decision maker.

Based on recently discussed decisions, manufacturer has three actions, (a,b,c), where:

- a ∈ A = {stop production (SP), produce for Dealer 1 (P1), produce for Dealer 2 (P2)}
- b ∈ B = {satisfy demand of Dealer 1 (SD1), reject customer of Dealer 1(RC1)}
- c ∈ C = { satisfy demand of Dealer 2 (SD2), reject customer of Dealer 2(RC2)}.

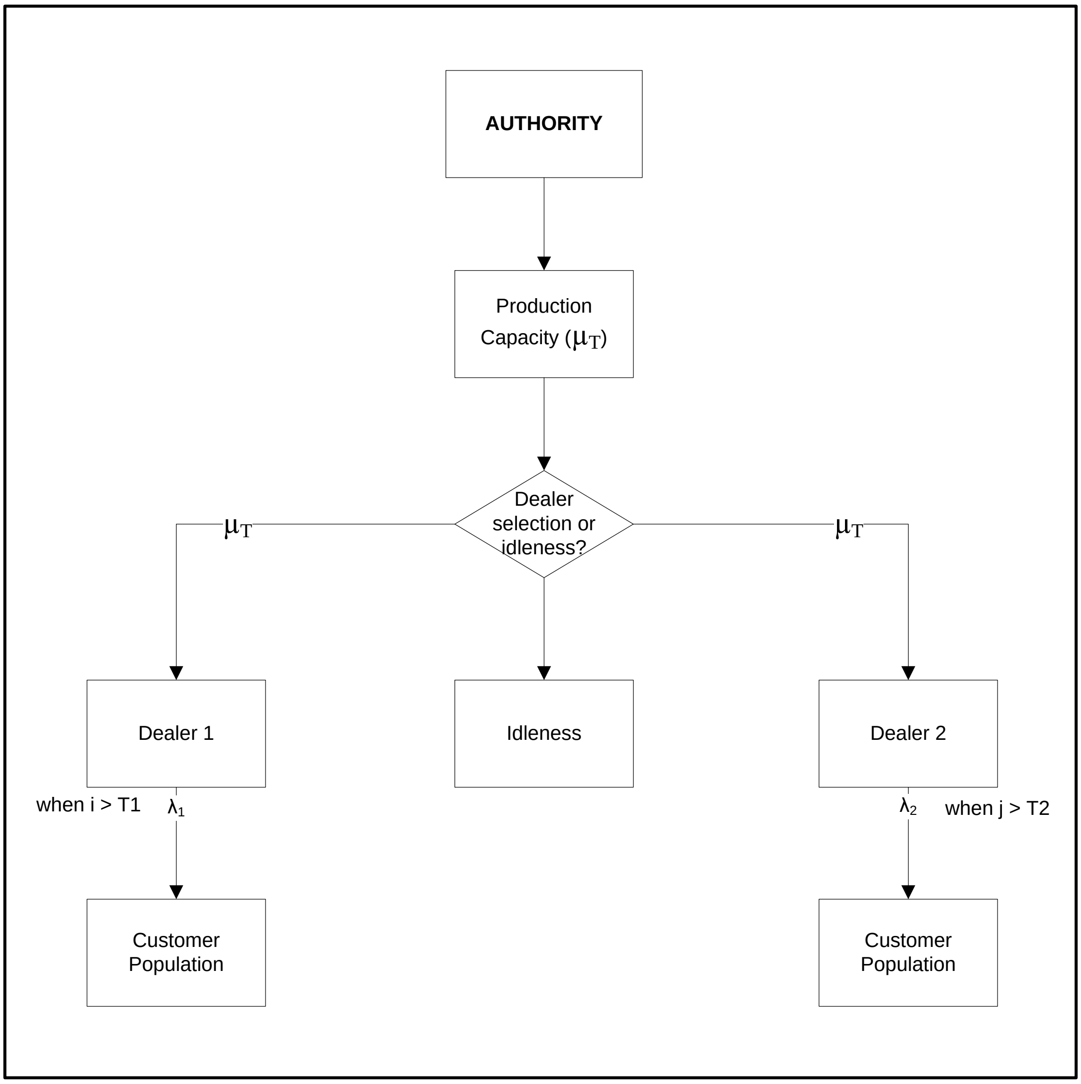

Figure 5.2.1 Item flow diagram in pooled system

When dealer *i* satisfies demand, it obtains revenue (R_i) per sale. Cost of replenishment is so small compared to inventory holding that it is negligible. Dealer *i* also incurs holding cost (h_i) per item per unit time and backorder cost (l_i) per item per unit time.

Two-dealer system can be modeled as a Markov chain, with (*m*,*n*) being its state variable, where *m* is the inventory level of Dealer 1 and *n* is the inventory level of Dealer 2. Aim of the manufacturer is to maximize infinite horizon average system-wide profit. To solve the system, the following linear model is used:

$$\text{Min} \sum_{(m,n)} \sum_{(a,b,c)} X_{(m,n),(a,b,c)} C_{(m,n),(a,b,c)}$$

s. to

$$\sum_{(a,b,c)} X_{(p,r),(a,b,c)} - \sum_{(m,n)} \sum_{(a,b,c)} X_{(m,n),(a,b,c)} P_{(m,n),(p,r),(a,b,c)} = 0 \quad , \forall p, r \in Z$$

$$\sum_{(m,n)} \sum_{(a,b,c)} X_{(m,n),(a,b,c)} = 1$$

$$X_{(m,n),(a,b,c)} \geq 0 \ , \forall m, n, a, b, c$$

where m is the inventory level of Dealer 1 at current state, n is the inventory level of Dealer 2 at current state, p is the inventory level of Dealer 1 at succeeding state, r is the inventory level of Dealer 2 at succeeding state, $X_{(m,n),(a,b,c)}$ (decision variable) is steady state probabilities at state (m,n) under action (a,b,c), $C_{(m,n),(a,b,c)}$ (parameter) is the reward function of state (m,n) under action (a,b,c) and $P_{(m,,n),(p,r),(a,b,c)}$ (parameter) is the transition probabilities from state (m,n) to state (p,r) under action (a,b,c).

Reward function is expressed as follows:

$$Reward_{(m,n)(a,b,c)}= \begin{cases} \left(\frac{lm^-}{\beta}\right) + \left(\frac{ln^-}{\beta}\right) - \left(\frac{hm^+}{\beta}\right) - \left(\frac{hn^+}{\beta}\right) + \left(\frac{\lambda_1}{\beta}R_1\right) + \left(\frac{\lambda_2}{\beta}R_2\right), & b = SD1, c = SD2 \\ \left(\frac{lm^-}{\beta}\right) + \left(\frac{ln^-}{\beta}\right) - \left(\frac{hm^+}{\beta}\right) - \left(\frac{hn^+}{\beta}\right) + \left(\frac{\lambda_2}{\beta}R_2\right) & , b = RC1, c = SD2 \\ \left(\frac{lm^-}{\beta}\right) + \left(\frac{ln^-}{\beta}\right) - \left(\frac{hm^+}{\beta}\right) - \left(\frac{hn^+}{\beta}\right) + \left(\frac{\lambda_1}{\beta}R_1\right) & , b = SD1, c = RC2 \\ \left(\frac{lm^-}{\beta}\right) + \left(\frac{ln^-}{\beta}\right) - \left(\frac{hm^+}{\beta}\right) - \left(\frac{hn^+}{\beta}\right) & , b = RC1, c = RC2 \end{cases}$$

$Eq. 5.2.1$

where m^+ is max{$m,0$}, m^- is min{$m,0$}, n^+ is max{$n,0$}, n^- is min{$n,0$} and $\beta=\lambda_1+\lambda_2+\mu_T$.

Transition probabilities are expressed as follows. Note that state space has physical lower and upper bounds. This can be thought to be physical inventory capacity of the system. From calculation point of view, transition probabilities cannot be calculated from -∞ to +∞. That is why probabilities are calculated from a lower bound (LB) to upper bound (UB). Due to physical constraint, each probability is written one under the other.

***P*((*p*,*r*) | (*m*,*n*), (*a*,*b*,*c*)) =** $Eq. 5.2.2$

1. $P_{(m,n),(m+1,n)} = \frac{\mu_T}{\beta}(IF\ a = 2),\ m < UB$

2. $P_{(m,n),(m,n+1)} = \frac{\mu_T}{\beta}(IF\ a = 3),\ n < UB$
3. $P_{(m,n),(m-1,n)} = \frac{\lambda_1}{\beta}(IF\ b = 1),\ m > LB$

4. $P_{(m,n),(m,n-1)} = \frac{\lambda_2}{\beta}(IF\ c = 1),\ n > LB$

5. $P_{(m,n),(m,n)} = \frac{\mu_T}{\beta}(IF\ a = 1) + \frac{\lambda_1}{\beta}(IF\ b = 2) + \frac{\lambda_2}{\beta}(IF\ c = 2),\ LB < m < UB\ and\ LB < n < UB$

6. $P_{(m,n),(m,n)} = \frac{\mu_T}{\beta}(IF\ a = 1) + \frac{\lambda_1}{\beta} + \frac{\lambda_2}{\beta}(IF\ c = 2),\ m = LB\ and\ LB < n < UB$

7. $P_{(m,n),(m,n)} = \frac{\mu_T}{\beta}(IF\ a \neq 3) + \frac{\lambda_1}{\beta}(IF\ b = 2) + \frac{\lambda_2}{\beta}(IF\ c = 2),\ m = UB\ and\ LB < n < UB$

8. $P_{(m,n),(m,n)} = \frac{\mu_T}{\beta}(IF\ a = 1) + \frac{\lambda_1}{\beta}(IF\ b = 2) + \frac{\lambda_2}{\beta},\ LB < m < UB\ and\ n = LB$

9. $P_{(m,n),(m,n)} = \frac{\mu_T}{\beta}(IF\ a \neq 2) + \frac{\lambda_1}{\beta}(IF\ b = 2) + \frac{\lambda_2}{\beta}(IF\ c = 2),\ LB < m < UB\ and\ n = UB$

10. $P_{(m,n),(m,n)} = \frac{\mu_T}{\beta}(IF\ a = 1) + \frac{\lambda_1}{\beta} + \frac{\lambda_2}{\beta},\ m = LB\ and\ n = LB$

11. $P_{(m,n),(m,n)} = \frac{\mu_T}{\beta}(IF\ a \neq 2) + \frac{\lambda_1}{\beta} + \frac{\lambda_2}{\beta}(IF\ c = 2),\ m = LB\ and\ n = UB$

12. $P_{(m,n),(m,n)} = \frac{\mu_T}{\beta}(IF\ a \neq 3) + \frac{\lambda_1}{\beta}(IF\ b = 2) + \frac{\lambda_2}{\beta},\ m = UB\ and\ n = LB$

13. $P_{(m,n),(m,n)} = \frac{\mu_T}{\beta} + \frac{\lambda_1}{\beta}(IF\ b = 2) + \frac{\lambda_2}{\beta}(IF\ c = 2),\ m = UB\ and\ n = UB$

14. $P_{(m,n),(m+1,n)} = 0, m = UB$

15. $P_{(m,n),(m-1,n)} = 0, m = LB$

16. $P_{(m,n),(m,n+1)} = 0, n = UB$

17. $P_{(m,n),(m,n-1)} = 0, n = LB$

where

$\beta = \lambda_1 + \lambda_2 + \mu_T$,

"IF X=k" $= \begin{cases} 1, \mathrm{X} = \mathrm{k} \\ 0, \mathrm{X} \neq \mathrm{k} \end{cases}$ where X is variable and k is integer. $Eq. 5.2.3$

Let idleness percentage of the system is denoted with $Idle_T$ and percentage of time the production facility is busy for Dealer 1 and Dealer 2 is denoted with CU_1 and CU_2 respectively. $Idle_T$, CU_1 and CU_2 are calculated as follows:

$$Idle_T = \sum_{m,n,b,c} X_{i,j,(a=SP),b,c} \qquad Eq. 5.2.4$$

$$CU_1 = \sum_{m,n,b,c} X_{m,n,(a=P1),b,c} \qquad Eq. 5.2.5$$

$$CU_2 = \sum_{m,n,b,c} X_{m,n,(a=P2),b,c} \qquad Eq. 5.2.6$$

Note that action set ‘a’ consists of three sub-actions; namely, stop production (SP), produce for Dealer 1 (P1) and produce for Dealer 2 (P2). Concentrating on action ‘a’, system idleness is calculated by taking summation of all steady state probabilities given that ‘a=SP’ since production facility does not operate under that condition. Likewise, capacity utilization of Dealer 1 and Dealer 2 is calculated by taking summation of all steady state probabilities given that ‘a=P1’ and ‘a=P2’, respectively.

GAMS (version 22.5) is used as a programming language to solve the LP model.

CHAPTER 6

COMPUTATIONAL ANALYSIS OF CAPACITY ALLOCATION OF DEALERS

In this chapter, effect of (i) holding cost, (ii) revenue, (iii) arrival rate on capacity utilization and (iv) system profit under the unpooled system and the pooled system are discussed. Infinite horizon average expected profit criterion is used while determining the optimal policies. Performance measures are capacity utilized by each dealer, total idleness and total profit.

In the unpooled and the pooled systems, manufacturer is the authority. The difference is that in the unpooled system, capacity allocation percentage is static, not dynamic. In other words, specific percentage of the total capacity is dedicated to the dealer regardless of how frequently dealer uses its allocated capacity. In the pooled system, on the other hand, whole capacity is dedicated to one dealer between two events. The dealer that is allocated whole capacity is chosen at the beginning of each state. By analogy, pooled system resembles centralized system whereas unpooled system resembles decentralized system.

6.1 EFFECT OF HOLDING COST ON CAPACITY UTILIZATION

In this subsection, the effect of holding cost on the capacity utilization is analyzed by incurring Dealer 1 different holding costs while fixing other dealer's holding cost.

The parameter set used is as follows:

- Revenue of Dealer 1 (R_1) = 5,
- Revenue of Dealer 2 (R_2) = 5,
- Holding Cost of Dealer 1 (H_1) = 0.3, 0.6, 1.0, 1.5, 2.0, 5.0,
- Holding Cost of Dealer 2 (H_2) = 1.0,
- Backorder Cost of Dealer 1 (l_1) = 2,
- Backorder Cost of Dealer 2 (l_2) = 2,
- Customer Arrival Rate to Any Dealer ($\lambda_1 = \lambda_2$) = 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0,
- Total Capacity of the Manufacturer (μ_T) = 1.

Under the pooled system, the effect holding cost is significant on the capacity utilization. Dealer having lower holding cost possesses higher rate of capacity utilization compared to dealer having higher holding cost (Figure 6.1.1). This is due to base stock level: When $h_1 < h_2$, Dealer 1's base stock level is higher than that of Dealer 2 and consequently Dealer 1's capacity utilization is higher and replenishes its stock more frequently. However, as holding cost of Dealer 1 increases ($h_2 < h_1$), Dealer 1 decreases its base stock level under Dealer 2's base stock level (Figure 6.1.2). This time Dealer 2's capacity utilization is higher and replenishes its own inventory more frequently. Thus, Dealer 2's capacity utilization becomes higher than Dealer 1's utilization, relatively.

As holding cost of Dealer 1 increases under the pooled system, its capacity allocation and capacity utilization decreases. This is due to decrease in the base stock level (S_1). Decrease in the Dealer 1's allocation of capacity results in increase in Dealer 2's allocation of capacity. Increase in the capacity allocation means higher rate of inventory replenishment to Dealer 2. Since S_2 remains the same with increasing h_1, Dealer 2 reaches its base stock level more frequently. So, Dealer 2's utilization of capacity decreases. Consequently, both Dealer 1's and Dealer 2's capacity utilization decreases and idleness increases with increasing holding cost of Dealer 1.

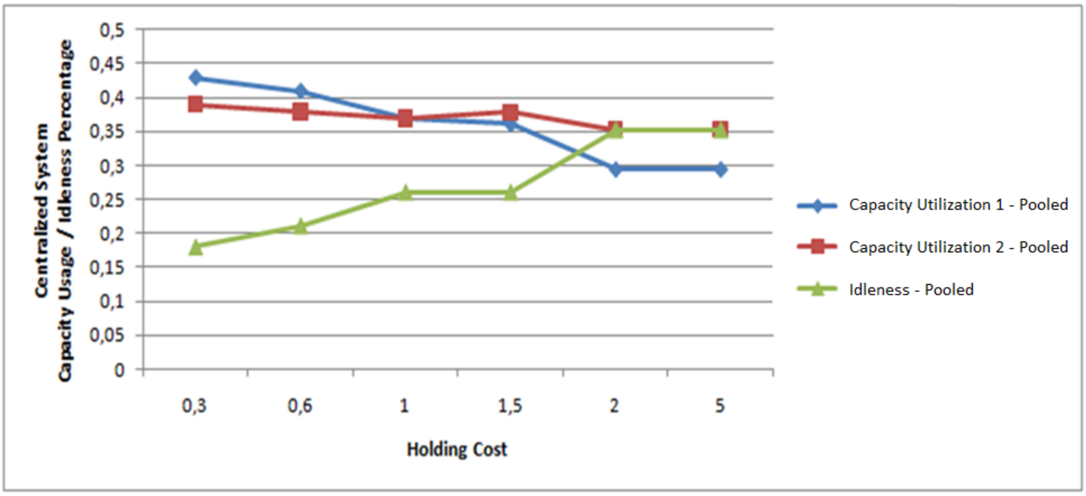

Figure 6.1.1 Capacity utilization under pooled system (*R_1=R_2=5, H_2=1, l_1=l_2=2, λ_1=λ_2=0.5*)

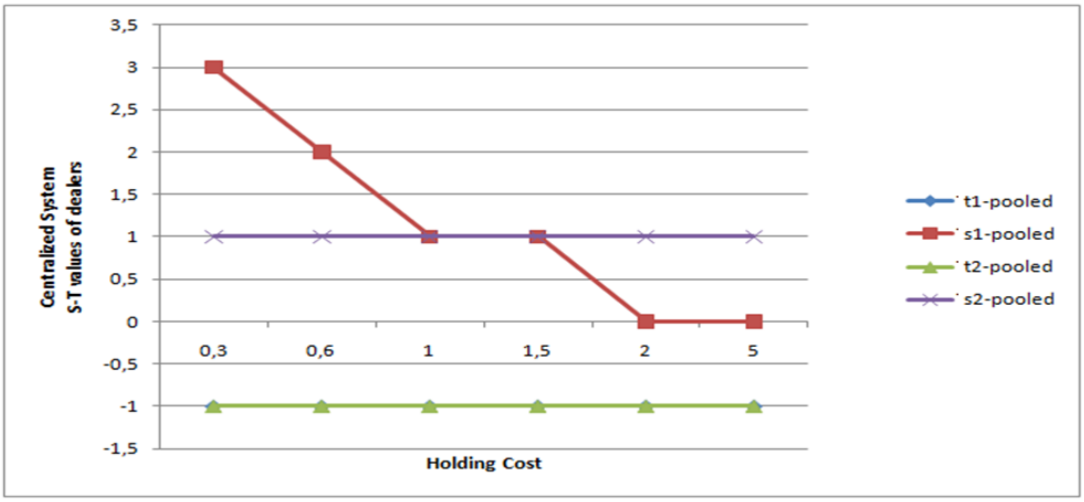

Figure 6.1.2 *S-T* decisions under the pooled system (*R_1=R_2=5, H_2=1, l_1=l_2=2, λ_1=λ_2=0.5*)

Taking into account both holding cost and customer arrival rate change under pooled system, total capacity usage increases and idleness decreases with increasing customer arrivals. Dealer, incurring lower holding cost, is always allocated higher capacity and high-capacity-allocated dealer's utilization of capacity is higher due to high base stock level. However, holding cost does not have much effect on capacity utilization under low arrival rate.

In the unpooled system, under low arrival rate, one of the dealers prefers not to keep stock for $h_1 \geq 1$. Thus, this dealer is indifferent to the holding cost if $h_1 \geq 1$ (Figure 6.1.3). This is due to cost structure of the optimality equation (Figure 6.1.4). Under low arrival rate and high holding cost, central authority prefers allocating whole capacity to either dealer. Intuitively, dealer allocated low capacity (low μ_i) cannot make profit under low demand rate. In order to make profit, dealer should be allocated high capacity (high μ_i).

Under low demand rate and low holding cost (H_1= 0.3, 0.6), Dealer 1 holds inventory whereas Dealer 2 does not keep inventory. In this situation, item arrival to Dealer 1 leads to an increase in the inventory holding cost since inventory level of Dealer 1 increases. Item arrival to Dealer 2, on the other hand, results in decrease in the backorder cost. Since system's cost structure serves for allocating capacity to both dealers under low demand rate and low holding cost, capacity utilization percentage of the backordering dealer (Dealer 2) is higher.

On the other hand, when demand rate is high under the unpooled system, capacity utilization percentage of the dealer having lower holding cost is higher (Figure 6.1.5). In this situation, both dealers prefer to hold inventory due to high demand rates. As holding cost of Dealer 1 increases, base stock level (S_1) decreases while rejection level (T_1) remains the same. Thus, Dealer 1's capacity utilization is lower where Dealer 2's is higher. Due to higher capacity utilization, Dealer 2 replenishes items more frequently. As a result, Dealer 2 also decreases its base stock level (S_2) with increasing h_1.

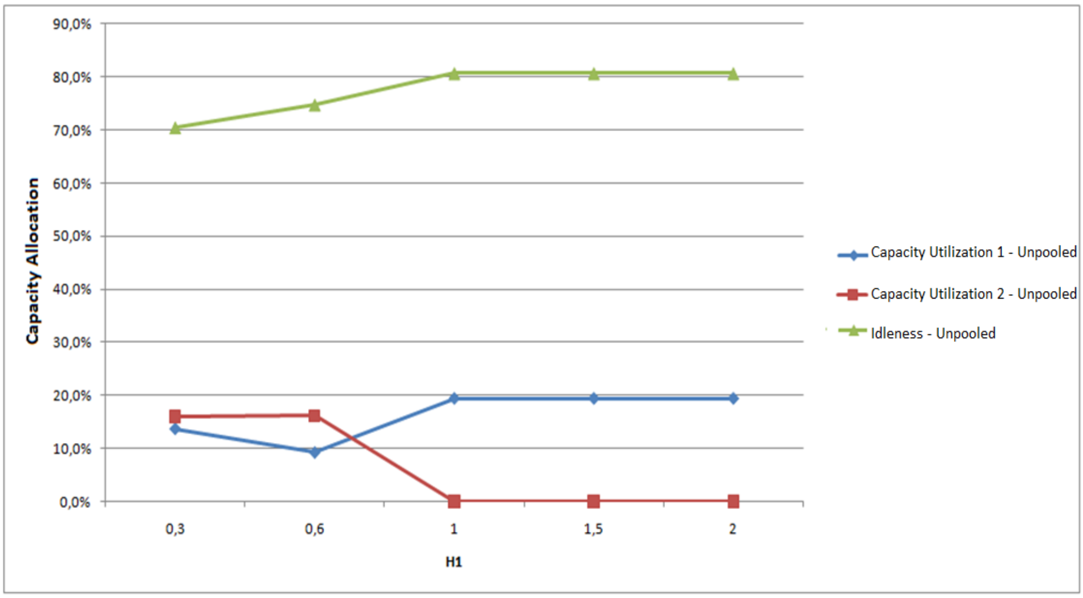

Figure 6.1.3 Capacity utilization under the unpooled system (*R_1=R_2=5, H_2=1, l_1=l_2=2, $\lambda_1 = \lambda_2 = 0.2$*)

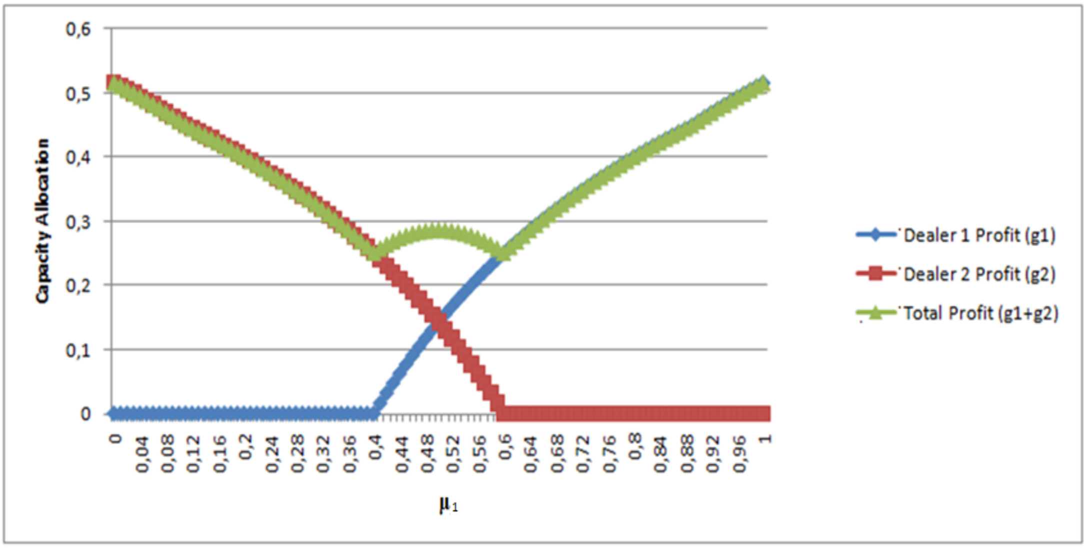

Figure 6.1.4 **Cost structure of the unpooled system (*R_1=R_2=5, H_1= H_2=1, l_1=l_2=2, $\lambda_1 = \lambda_2 = 0.2$*). (This structure is also valid for *H_1* > 1.)**

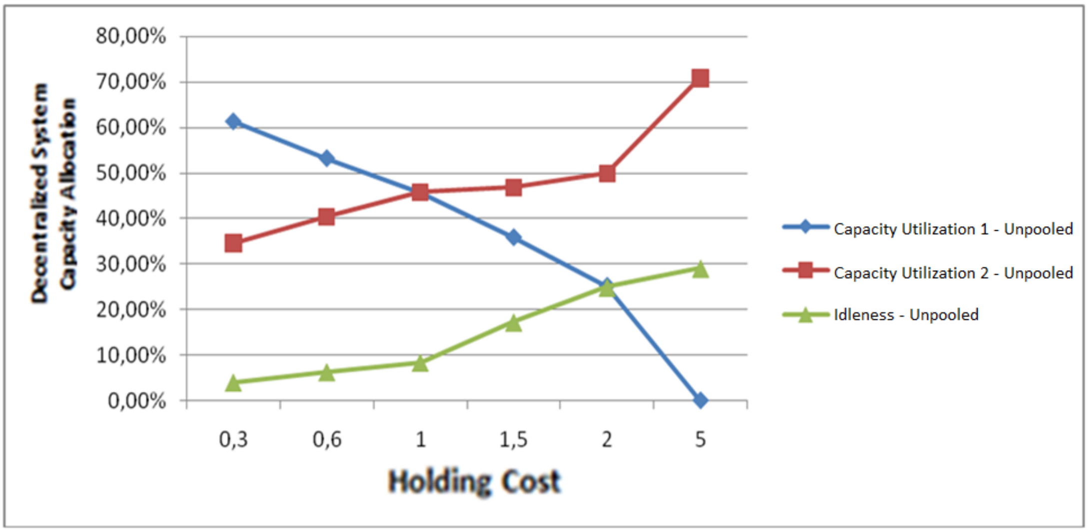

Figure 6.1.5 Capacity allocation under the unpooled system (*R_1=R_2=5, H_2=1, l_1=l_2=2, λ_1=λ_2=0.9*)

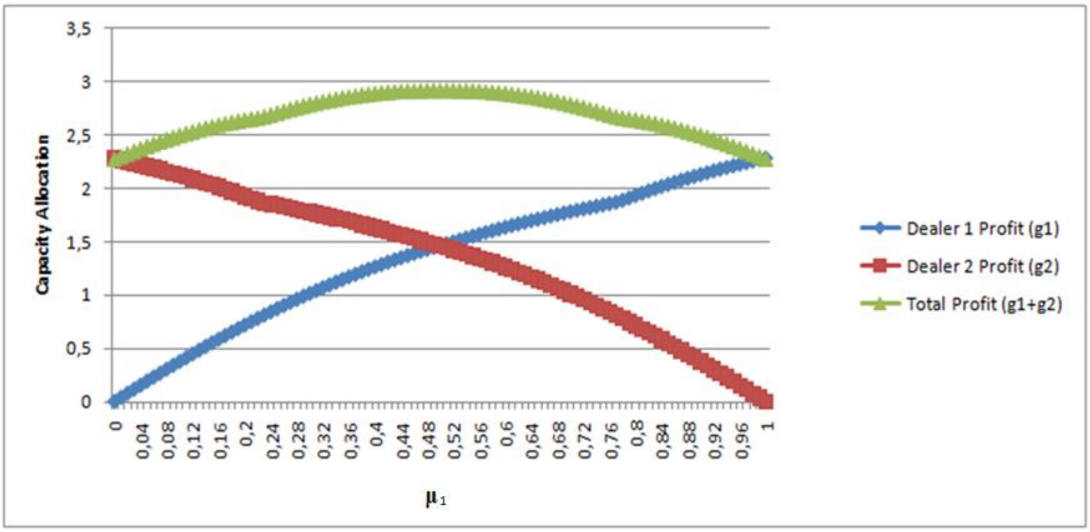

Figure 6.1.6 **Cost structure of the unpooled system (*R_1=R_2=5, H_1= H_2=1, l_1=l_2=2, $\lambda_1 = \lambda_2 = 0.9$*)**

In the unpooled system, under low demand rate and high h_1, whole capacity allocation to either Dealer 1 or Dealer 2 is indifferent from each other since dealers do not keep inventory. Under low demand and low h_1, capacity utilization of the dealer having higher holding cost (Dealer 2) is higher since Dealer 2 does not keep inventory and frequent item replenishments leads to an increase in the backorder cost of Dealer 2 where Dealer 1 incurs extra holding cost with more frequent replenishments. Oppositely, in the unpooled system under high demand rate and in the pooled system, capacity utilization of the dealer having lower holding cost is higher since both dealers keep inventory and base stock level (S_i) of the dealer having low holding cost is higher than the dealer having higher holding cost.

6.2 EFFECT OF REVENUE ON CAPACITY UTILIZATION

In this subsection, the effect of revenue on capacity utilization and total profit is analyzed by incurring Dealer 1 different revenues while fixing other dealer's revenue.

The parameter set used is as follows:

- Revenue of Dealer 1 (R_1) = 1.0, 1.5, 2.0, 3.0, 5.0, 10.0, 15.0,
- Revenue of Dealer 2 (R_2) = 5.0,
- Holding Cost of Dealer 1 (H_1) = 1.0,
- Holding Cost of Dealer 2 (H_2) = 1.0,
- Backorder Cost of Dealer 1 (l_1) = 2,
- Backorder Cost of Dealer 2 (l_2) = 2,
- Customer arrival rate to any dealer ($\lambda_1 = \lambda_2$) = 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0,
- Total Capacity of the Manufacturer (μ_T) = 1.

Both the pooled and the unpooled system's behavior with respect to revenue change resemble. At first glance, it can easily be interpreted that dealer having higher revenue always utilizes higher portion of the capacity (Figures 6.2.1 and 6.2.2). The reason is an increase in R_i results in higher base stock to be kept and higher number of customers to be accepted. This is in line with the effect of lower holding cost on capacity utilization. Under low arrival rate, dealer having low revenue does not utilize any capacity (when R_1 = 1.0, 1.5 or 2.0). As arrival rate increases, the capacity utilization of the low-revenue dealer increases.

From the capacity usage point of view, it decreases as revenue increases in both pooled and unpooled system. This is due to increase in the base stock level: As base stock level increases, dealer's capacity utilization increases. However, there are exceptions to decreasing idleness with increasing revenue: Especially when revenue is high (R_1 > 5), the high-revenue dealer utilizes higher portion of the capacity in the unpooled system. The reason is the high asymmetry (in contrast to the pooled system) in utilization of capacity under high revenues in the unpooled system (Figure 6.2.2).

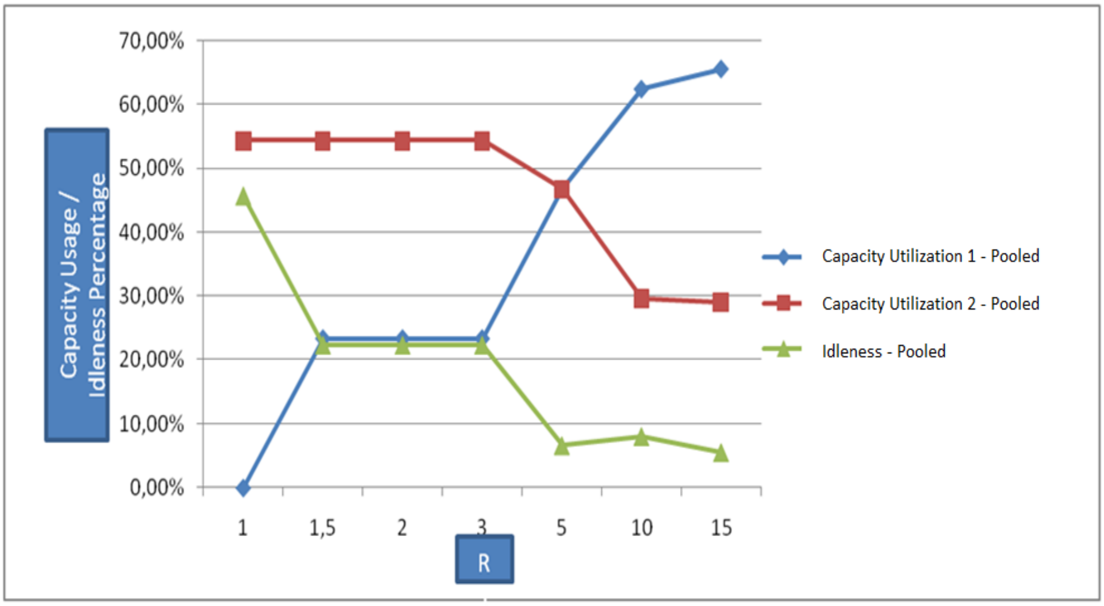

Figure 6.2.1 Capacity utilization in pooled system (*R_2=5, H_1=H_2=1, l_1=l_2=2, λ_1=λ_2=0.7*)

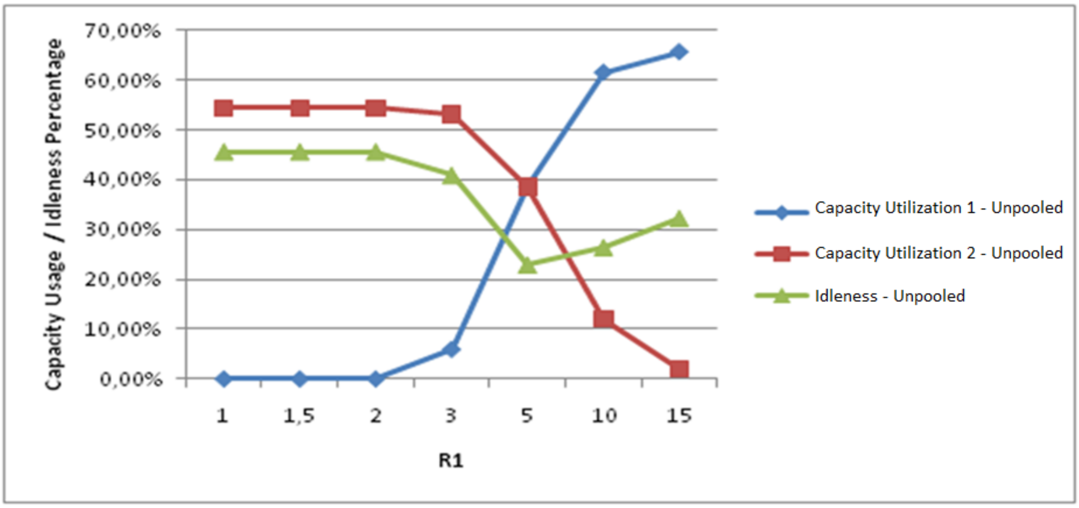

Figure 6.2.2 Capacity utilization in unpooled system (R_2=5, H_1=H_2=1, l_1=l_2=2, λ_1=λ_2=0.7)

6.3 EFFECT OF ARRIVAL RATE ON CAPACITY UTILIZATION

In this subsection, the effect of arrival rate on capacity utilization is analyzed by assigning Dealer 1 and Dealer 2 different arrival rates. Note that arrival rates of the dealers are equal to each other throughout the analysis.

The parameter set used is as follows:

- Revenue of Dealer 1 (R_1) = 5,
- Revenue of Dealer 2 (R_2) = 5,
- Holding Cost of Dealer 1 (H_1) = 0.3, 0.6, 1.0, 1.5, 2.0, 5.0,
- Holding Cost of Dealer 2 (H_2) = 1.0,
- Backorder Cost of Dealer 1 (l_1) = 2,
- Backorder Cost of Dealer 2 (l_2) = 2,
- Customer arrival rate to any dealer (λ_1= λ_2) = 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0,
- Total Capacity of the Manufacturer (μ_T) = 1.

Under the pooled system, increase in the arrival rate leads to decrease in the system idleness (Figure 6.3.1). As arrival rate increases, both dealer's capacity utilization increases (so idleness decreases) (Figure 6.3.1) since dealers replenish their inventory more frequently. In addition, dealer having lower holding cost increases the base stock level. This leads to higher capacity to be utilized by the dealer having lower holding cost than the dealer having higher holding cost with increasing arrival rate.

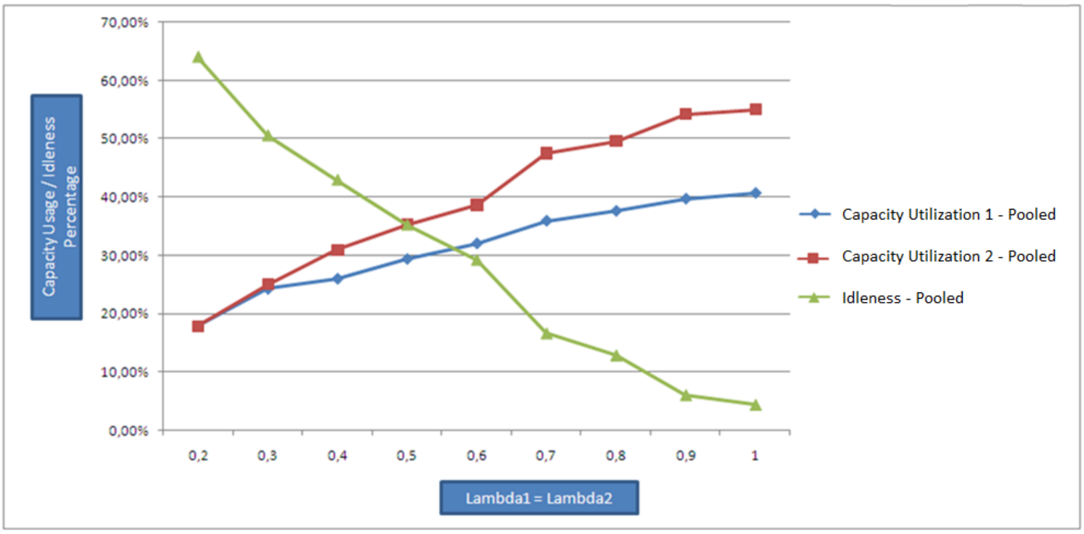

Figure 6.3.1 Capacity utilization in the pooled system (*R_1=R_2=5, H_1=5, H_2=1, l_1=l_2=2*)

In the unpooled system, on the other hand, under low demand rate ($\lambda_1 = \lambda_2 \leq 0.4$) and low h_1, Dealer 1 utilizes higher portion of the capacity (Figure 6.3.2). When $\lambda_1 = \lambda_2 \leq 0.4$, Dealer 1 does not keep inventory whereas Dealer 2 holds inventory and does not allow backordering. In this situation, item replenishment to Dealer 1 decreases total backorder cost, item replenishment to Dealer 2 increases total inventory holding cost. So, capacity utilization of Dealer 1 is high under low demand rate and low h_1.

Under high arrival rate ($\lambda_1 = \lambda_2 > 0.4$) in the unpooled system, dealer having lower holding cost utilized higher portion of the capacity (Figure 6.3.2) as opposite to the low arrival rate case. This time, both dealers hold inventory but dealer having lower holding cost has higher capacity utilization since base stock level of that dealer is higher.

When demand rate increases from 0.8 to 0.9 in Figure 6.3.2, Dealer 2 increases its rejection level (T_2) due to high backorder cost and does not change its base stock level (S_2) in unpooled system. Since gap between S_2 and T_2 gets narrower, Dealer 2 utilizes lower portion capacity specific to that situation (Figure 6.3.2).

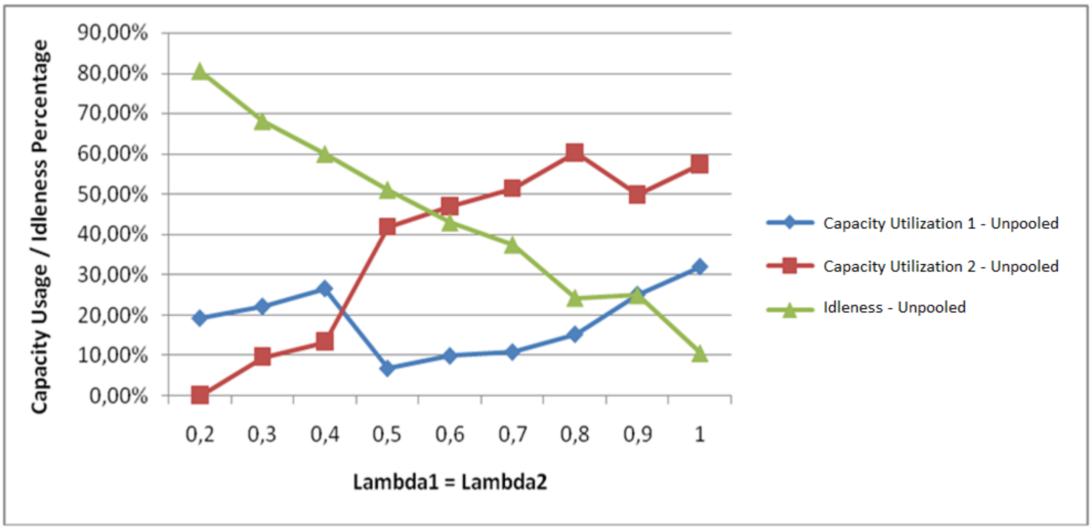

Figure 6.3.2 Capacity utilization under decentralized system (R_1=R_2=5, H_1=2, H_2=1, l_1=l_2=2)

Comparing the pooled and the unpooled system under low demand rate, capacity utilization of the dealer having higher holding cost is higher in the unpooled system since dealer having higher inventory cost does not keep inventory. Under high demand rate, capacity utilization of the dealer having lower holding cost is higher in the unpooled system. In the pooled system, capacity utilization of the dealer having lower holding cost is higher regardless of the arrival rate. Idleness, on the other hand, decreases in both systems as demand rate increases due to more frequent item replenishments.

6.4 BENEFIT OF POOLING

Pooling benefit is the relative gap between the unpooled system profit and the pooled system profit. It is calculated as follows:

$$Pooling\ Benefit\ (PB) = \frac{\Pi_{pooled} - \Pi_{unpooled}}{\Pi_{unpooled}} \qquad Eq. 6.4.1$$

where Π_{pooled} is the pooled system profit and $\Pi_{unpooled}$ is the unpooled system profit.

The pooling benefit is analyzed under three effects:

i. Holding Cost,
ii. Revenue,
iii. Arrival Rate.

Firstly, effect of holding cost is examined. As holding cost of Dealer 1 increases, pooling benefit increases regardless of the arrival rate (Figure 6.4.1). Due efficient use of capacity in pooled system, dealers do not hold high level of base stock in pooled system as they hold in the unpooled system. Thus, higher holding cost illustrates base stock level difference between the pooled and the unpooled system more clearly.

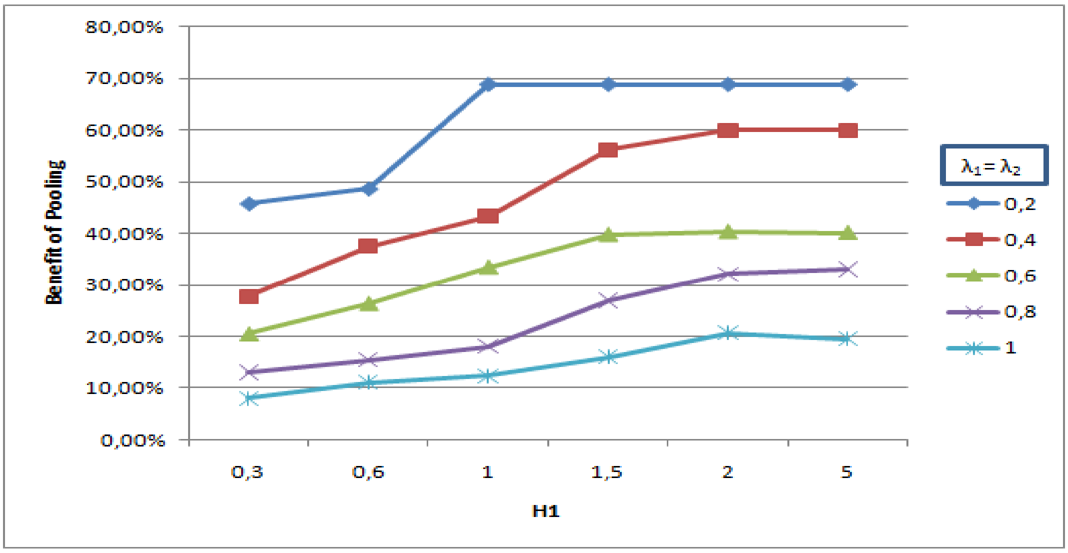

Figure 6.4.1 Relative gap with respect to holding cost (*R_1=R_2=5, H_2=1, l_1=l_2=2*)

Secondly, effect of revenue is examined. There is no pooling benefit under low revenue of Dealer 1 (Figure 6.4.2). As recently discussed in section 6.2, dealer having low revenue is not allocated any capacity in the unpooled system. So, whole capacity is allocated to one dealer and the unpooled system becomes a pooled system. As a result, the unpooled system aligns with pooled system and no pooling benefit exists under low revenue of Dealer 1. For middle and high values of revenue (R_1 > *2*), relative gap exists; so, benefit of pooling is observable. The relative gap first increases than decreases for high values of revenue ($R_1 \geq 5$). Note that pooling benefit is very high when dealers are identical ($R_1 = 5$) regardless of the demand rate. This is due to asymmetric capacity utilization in the unpooled system although dealers are identical. In the pooled system, however, each dealer utilizes same amount of capacity when they are identical. Thus, relative gap is the highest when dealers are identical.

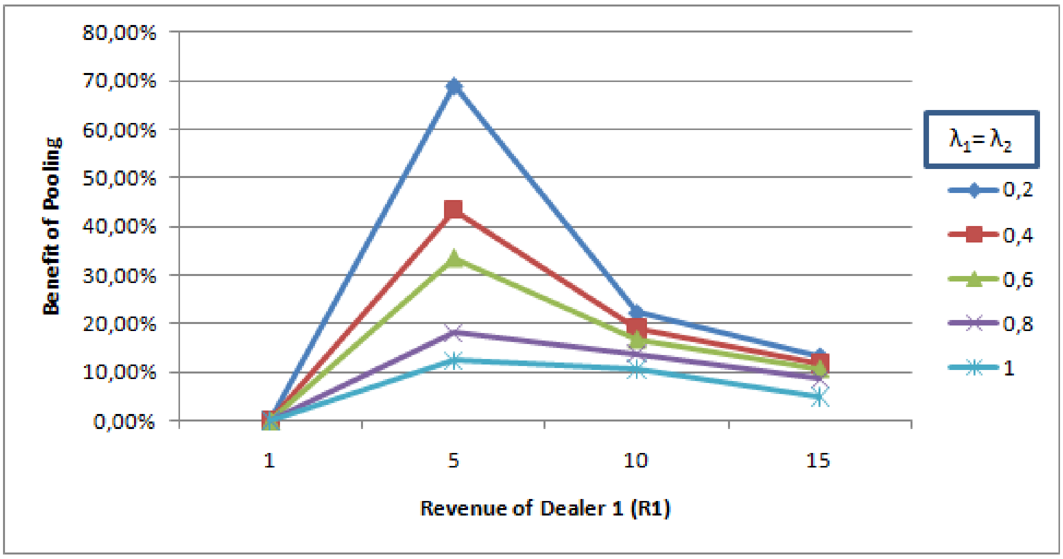

Figure 6.4.2 Relative gap with respect to revenue (R_2=5, H_1=H_2=1, l_1=l_2=2)

Thirdly, effect of demand rate is discussed. Under low revenue and low demand rate, pooling benefit does not exist since both systems allocate capacity to one dealer (Figure 6.4.3 – R_1 = 1.0, 1.5, 2.0). As demand gets higher under low revenue, other dealer is also allocated capacity under decentralized system. This leads to divergence from centralized system since decentralized system looses the advantage of pooling and pooling benefit increases (Figure 6.4.3 – R_1 = 1.5, 2.0). Under high revenue, relative gap decreases with increasing demand rate (Figure 6.4.3 - R_1 = 5.0, 10.0). So, the unpooled system aligns with the pooled system as demand rate increases under high R_1. In other words, benefit of pooling is higher under high revenue and low demand rates.

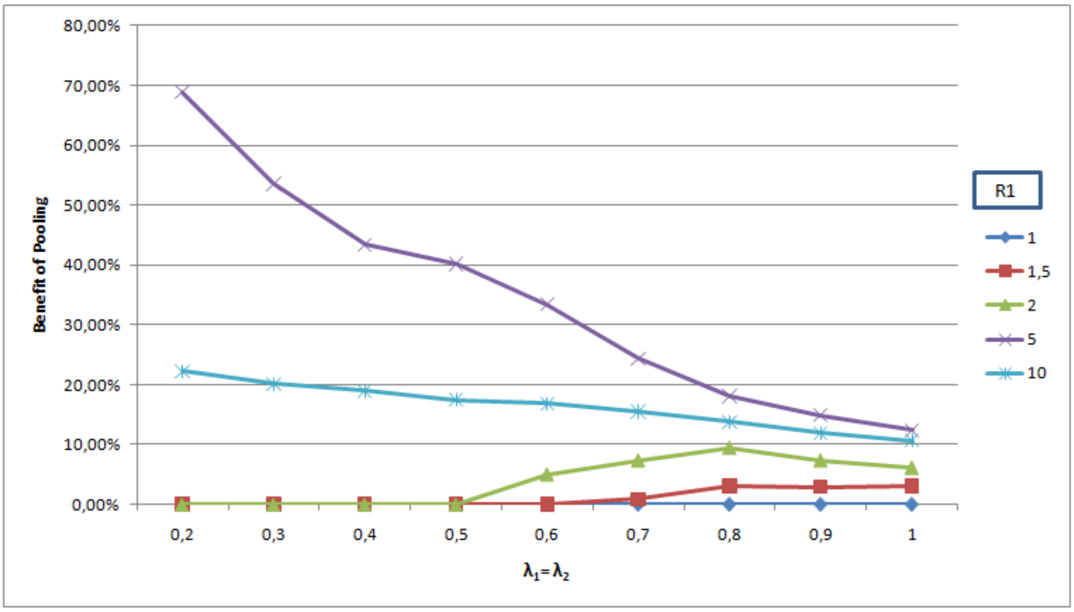

Figure 6.4.3 Relative gap with respect to arrival rate (*R_2=5, H_1=H_2=1, l_1=l_2=2*)

To sum up, pooling is beneficial under:

- High holding cost,
- Symmetric revenue,
- Low arrival rate.

To illustrate, when $\lambda_1 = \lambda_2$=0.2 and H_1>1, pooling benefit is nearly 70% whereas it is nearly 50% when $\lambda_1 = \lambda_2$=0.2 and H_1=0.6. Likewise when $\lambda_1 = \lambda_2$=0.2 and revenues are asymmetric (*R_1=10, R_2=5*), pooling benefit is nearly 20% whereas it is nearly 70% when revenues are symmetric (*R_1=5, R_2=5*). So, it can be concluded that asymmetry in arrival rate or holding cost does not necessarily align the unpooled system with pooled system but asymmetry in revenues does so.

CHAPTER 7

CONCLUSION AND FUTURE WORK

In this thesis two questions are under consideration: (i) “Can a decentralized system be aligned with a centralized system with incentive, subsidy or penalty mechanisms?” and (ii) “How does production capacity allocation strategies, namely pooled capacity allocation and unpooled capacity allocation, affect the percentage of capacity utilization of each dealer, total idleness and system wide profit?”.

Regarding the first research question, six cross comparisons are carried out among different incentive, subsidy or penalty designs. Firstly, comparing the centralized system with the pure (no incentive setting) decentralized setting, S and T values of the centralized and the decentralized system coincides whereas K and Z values can be quite different. Increasing demand or decreasing commission result in increase in K and Z values under decentralized demand. In centralized system, K and Z values coincide.

Under high holding cost, percentage of the profit gap between decentralized system and centralized system is high regardless of demand rate. These observations result in two consequences: (i) Incentive, subsidy or penalty designs can be expected to narrow gap between K and Z in decentralized system. In addition, such designs are also expected align decentralized system profits, especially under high holding costs, to centralized system wide profits.

Focusing on revenue sharing, service level increases, number of waiting and rejected customers decrease with decreasing revenue sharing. When revenue sharing percentage is around 40%, the decentralized system profit is maximized. Operating strategy of a dealer changes with respect to revenue sharing percentage such that *T* and *Z* values coincide with increasing revenue sharing percentage under high commission payment. The reason for such a response is the loss profitability in case of lateral transshipment under high commission and low revenue.

Holding cost subsidy is considered as another support given by the manufacturer. As holding cost subsidy percentage increases, base stock level (*S*) of the dealer increases, *K* and *Z* values diverge from each other resulting in less lateral transshipments between dealers and under the decentralized system. Number of waiting customers and number of lost sales, on the other hand, decreases with holding cost subsidy; however, system wide profit of the decentralized system decreases with holding cost subsidy because advantage of higher service level is neutralized by the disadvantage due to decrease in the lateral transshipments.

Request rejection penalty, on the other hand, has great effect on rationing level whereas other operating parameters (*S*, *T*) do not seem to be affected by the penalty under decentralized system. Rationing level (*K*) and transshipment level (*Z)* decrease with increasing rejection penalty. This is intuitive since dealer desires to reject less number of lateral transshipment requests. Number of waiting customers and lost demands are not affected by the penalty whereas item flow between dealers first increase than decrease with increasing penalty. So, only low values of rejection penalty leads the decentralized system to align with the centralized system.

Transportation cost subsidy is designed for the sake of higher item flow between dealers. When transportation cost is low, increasing transportation cost subsidy increases item flow between dealers. So, the decentralized system aligns with centralized system. Under high transportation cost, on the other hand, system wide profit of the decentralized system gets closer to that of centralized system when demand rate is low ($\lambda_1 = \lambda_2 \leq 0.60$).

Lastly, commission subsidy is applied to the decentralized system with 6-4 and 7-3 setting (Please refer to Chapter 3 for notation definition). On the middle values of commission payments (r=3, 6), commission subsidy is not effective. However, under low and high values of commission (r=1, 9), item flow between dealers increase and number of waiting customers decreases with the commission subsidy. Thus, the decentralized system aligns with the centralized system under low and high commissions using commission subsidy.

Regarding the second research question, which related with the pooled capacity and the unpooled capacity, effect of holding cost, revenue and arrival rate on capacity utilization and system profits are discussed.

Holding cost effect is stable regardless of arrival rate in pooled environment: Dealer having lower holding cost always utilizes higher portion of the capacity. In the unpooled system, dealer having higher holding cost utilizes higher portion of the capacity under low arrival rate. This result is surprising. The cause is that dealer having higher holding cost always backorders; so, higher capacity utilization of backordering dealer is logical. In the unpooled system, under high arrival rate, dealer having lower holding cost utilizes higher portion of the capacity as in the pooled system.

Revenue change has same effect on both pooled and unpooled system: Dealer having higher revenue utilizes higher portion of the capacity regardless of the arrival rate.

Eliminating revenue and holding cost effects and only concentrating on system response to the arrival rate change, total capacity utilization increases and idleness decreases with increasing arrival rate under the pooled system. System response in the unpooled system generally in line with the pooled system; however, exceptions exists.

From the profit point of view, relative gap between the pooled and the unpooled system increases with increasing holding cost. Considering revenue effect, on the other hand, relative gap decreases with increasing revenue of Dealer 1. Taking into account arrival rate change, relative gap decreases with increasing arrival rate under high revenue of Dealer 1. Under low revenue of Dealer 1, relative gap increases with increasing arrival rate.

In this thesis, first system consists of two dealers in close proximity and a manufacturer. Dealers operate with four inventory level strategies (*S, K, Z, T*). Since dealers are in close proximity, transshipment times are negligible. In addition, number of customers in the system is not restricted. A further research can be conducted taking into account transshipment times and maximum allowable customers in the system. In addition, analyzing asymmetric dealers can be another research area regarding coordination mechanisms.

Second system, on the other hand, concentrates of efficient allocation of the production capacity. Set-up times or loss due to maintenance operations in the production facility are left untouched. Further research can be conducted considering those losses. Actually, extending the models to multiple-item environment can be another research area. In addition, not allowing backorders (sales can still be lost), which is a special case of the discussed models, can also be analyzed.

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APPENDIX A

KEY POINTS OF POOLED AND UNPOOLED SYSTEMS SOLUTIONS

Unpooled System – Closed Form Profit Derivation (ρ≠1)

Note that $\rho=\lambda/\mu$

$$\boldsymbol{G} = \frac{\lambda}{\lambda+\mu} R \sum_{i=-S}^{T-1} \Pi_i + \frac{1}{\lambda+\mu} h \sum_{i=-S}^{0} i\,\Pi_i + \frac{1}{\lambda+\mu} l \sum_{i=1}^{T} (-i)\,\Pi_i$$

$$= \frac{\lambda}{\lambda+\mu} R\left(\Pi_{(-S)} + \rho\Pi_{(-S)} + \rho^2\Pi_{(-S)} + \cdots + \rho^{S+T-1}\Pi_{(-S)}\right) \qquad Eq\ A.1$$

$$+ \frac{1}{\lambda+\mu} h\left(-S\Pi_{(-S)} + (-S+1)\rho\Pi_{(-S)} + (-S+2)\rho^2\Pi_{(-S)} + \cdots + (-1)\rho^{S-1}\Pi_{(-S)}\right) \quad Eq\ A.2$$

$$+ \frac{1}{\lambda+\mu} l\left((-1)\rho^{S+1}\Pi_{(-S)} + (-2)\rho^{S+2}\Pi_{(-S)} + \cdots + (-T)\rho^{S+T}\Pi_{(-S)}\right) \quad Eq\ A.3$$

Solution of Equation A.1

$$\frac{\lambda}{\lambda+\mu}R\left(\Pi_{(-S)}+\rho\Pi_{(-S)}+\rho^2\Pi_{(-S)}+\cdots+\rho^{S+T-1}\Pi_{(-S)}\right)=\frac{\lambda}{\lambda+\mu}R\left(1-\Pi_{(T)}\right)$$

$$=\boxed{\frac{\lambda}{\lambda+\mu}R\left(1-\rho^{S+T}\Pi_{(-S)}\right)}$$

Solution of Equation A.2

$$\frac{1}{\lambda+\mu}h\left(-S\Pi_{(-S)}+(-S+1)\rho\Pi_{(-S)}+(-S+2)\rho^2\Pi_{(-S)}+\cdots+(-1)\rho^{S-1}\Pi_{(-S)}\right)$$

$$=\frac{1}{\lambda+\mu}h\Pi_{(-S)}(-S+(-S+1)\rho+(-S+2)\rho^2+\cdots+(-1)\rho^{S-1})$$

Sub calculation

Suppose:

$$m=(-S+(-S+1)\rho+(-S+2)\rho^2+\cdots+(-1)\rho^{S-1})$$

Then:

$$\rho m=(-S\rho+(-S+1)\rho^2+(-S+2)\rho^3+\cdots+(-1)\rho^{S})$$

$$m-\rho m=-S+\rho+\rho^2+\cdots+\rho^S$$

$$m(1-\rho)=-S-1+(1+\rho+\rho^2+\cdots+\rho^S)$$

$$m(1-\rho)=-S-1+\frac{1-\rho^{S+1}}{1-\rho}$$

$$\boxed{m=\frac{-S+S\rho+\rho-\rho^{S+1}}{(1-\rho)^2}}$$

$$= \frac{1}{\lambda + \mu} h\Pi_{(-S)} \left(\frac{-S + S\rho + \rho - \rho^{S+1}}{(1-\rho)^2} \right)$$

Solution of Equation A.3

$$\frac{1}{\lambda + \mu} l\left((-1)\rho^{S+1}\Pi_{(-S)} + (-2)\rho^{S+2}\Pi_{(-S)} + \cdots + (-T)\rho^{S+T}\Pi_{(-S)}\right)$$

$$= \frac{-1}{\lambda + \mu} l\rho^{S}\Pi_{(-S)}(\rho + 2\rho^2 + 3\rho^3 + \cdots + T\rho^T)$$

Sub calculation

Suppose:

$$r = \rho + 2\rho^2 + 3\rho^3 + \cdots + T\rho^T$$

Then:

$$\rho r = \rho^2 + 2\rho^3 + 3\rho^4 + \cdots + T\rho^{T+1}$$

$$r - \rho r = -T\rho^T + \rho + \rho^2 + \cdots + \rho^T$$

$$r(1-\rho) = -T\rho^T - 1 + (1 + \rho + \rho^2 + \cdots + \rho^T)$$

$$r(1-\rho) = -T\rho^T - 1 + \frac{1 - \rho^{T+1}}{1-\rho}$$

$$r = \frac{-T\rho^{T+1} + T\rho^{T+2} + \rho - \rho^{T+1}}{(1-\rho)^2}$$

$$\boxed{= \frac{-1}{\lambda + \mu} l\rho^S \Pi_{(-S)} \left(\frac{-T\rho^{T+1} + T\rho^{T+2} + \rho - \rho^{T+1}}{(1-\rho)^2}\right)}$$

By substituting Equations A.1, A.2 and A.3 in the G, substituting $\Pi_{(-S)}$ with $\left(\frac{1-\rho}{1-\rho^{S+T+1}}\right)$ and re-arranging the terms, the final equation is obtained:

$$\boldsymbol{G} = \left(\frac{\lambda}{\lambda + \mu} R\right) + \left(\frac{1}{\lambda + \mu}\right) \left(\frac{1-\rho}{(1-\rho^{S+T+1})}\right) \left[-\lambda R \rho^{S+T} + \frac{h(-S - S\rho + \rho - \rho^{S+1}) + l\rho^S (T\rho^{T+1} - T\rho^{T+2} - \rho + \rho^{T+1})}{(1-\rho)^2}\right]$$

Unpooled System – Closed Form Profit Derivation (ρ=1)

If ρ=1, than λ=μ by definition.

When ρ=1, all steady state probabilities are equal to each other. Then:

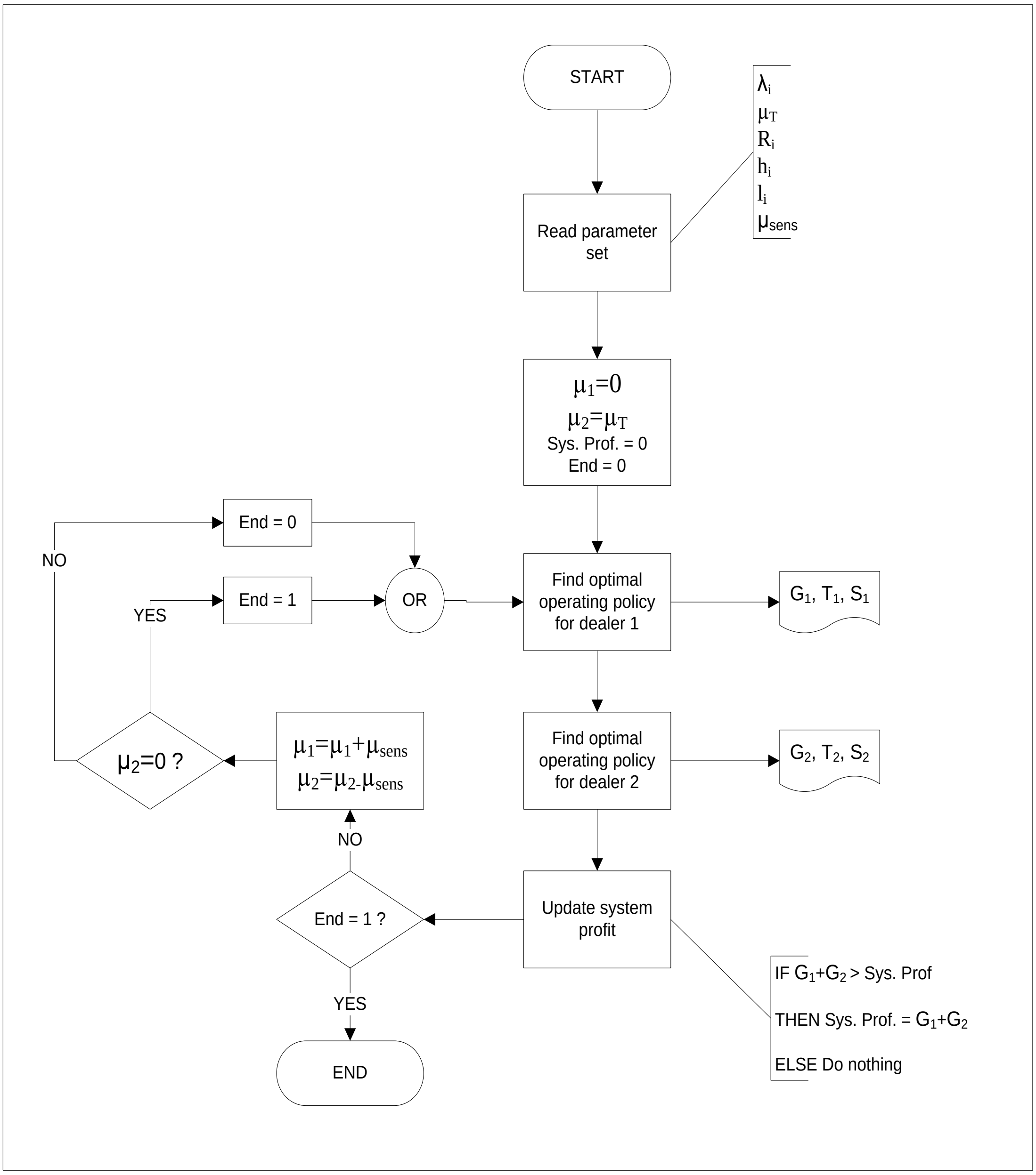
$$\Pi_i = \frac{1}{S+T+1} \quad \forall i$$

$$\boldsymbol{G} = \frac{\lambda}{\lambda + \mu} R \sum_{i=-S}^{T-1} \Pi_i + \frac{1}{\lambda + \mu} h \sum_{i=-S}^{0} i\, \Pi_i + \frac{1}{\lambda + \mu} l \sum_{i=1}^{T} (-i)\, \Pi_i$$

$$= \frac{\lambda}{\lambda + \mu} R(S+T)\Pi_i - \frac{1}{\lambda + \mu} h\Pi_i (S + S - 1 + S - 2 + \cdots + 0) - \frac{1}{\lambda + \mu} l\Pi_i (1 + 2 + \cdots + T)$$

$$= \left[\frac{\lambda}{\lambda+\mu} R \frac{(S+T)}{S+T+1}\right] - \left[\frac{1}{\lambda+\mu} h \frac{S(S+1)}{2(S+T+1)}\right] - \left[\frac{1}{\lambda+\mu} l \frac{T(T+1)}{2(S+T+1)}\right]$$

Search Algorithm Flow



Pooled System – Parameter Preparation Source Code (C++)

```
// Para.cpp : Defines the entry point for the console application.

//

//

#include "stdafx.h"

//#include <stdlib.h>


int _tmain(int argc, _TCHAR* argv[])

{

/*      const double S2 = 10.0;            // order upto level of D2

        const double K2 = 0.0;             // treshold of D2 for D1 customer

        const double Z2 = -5.0;            // ltr treshold of D2

   const double T2 = -10.0;           // cust acceptance treshold of D2

*/      const double r1 = 5;          // revenue1

        const double r2 = 5;          // revenue2

        const double h1 = 0.3;        // holding cost of dealer1

        const double h2 = 1.0;        // holding cost of dealer2

   const double l = 2.0;        // lateness cost

//   const double w = 3.0     ;      // backlog cost

//   const double tr = 1.7    ;      // transportation cost

        const double lambda1 = 0.3;        // lambda 1

        const double lambda2 = 0.3;   // lambda 2
```

```
   const double m =  1.0;                  // pooled mu
//         const double m2 = 1.0;                       // mu 2
//         const double alpha = 0.05;      // discount rate
           const int low = -10;
           const int high = 10;       // order-up-to level
           int identical = 0;   //identical dealer indicator
//         const int high = 10;


           double beta = lambda1+lambda2+m;
//         double tao = 1/beta;


           long double prof;
           long double pro;
           long double cumpro;


//         const int temp = 1;


if ((lambda1 == lambda2) && (l == l) && (h1 == h2) && (r1 == r2))
  {
  identical = 1;


  }
```

```
ofstream fout ("prof.inc", ios_base::out);

fout.precision(16);


// set a(act) 1 --> do not produce, 2 --> produce for Dealer 1, 3 --> produce for Dealer 2

//set b(act) 1 --> Dealer 1 accepts customers, 2 --> Dealer 1 rejects customers

//set c(act) 1 --> Dealer 2 accepts customers, 2 --> Dealer 2 rejects customers


// parameter prof(i,j,a,b,c) one step profit;

fout<<"*parameter profit(i,j,a,b,c)"<<"\n";


for (int k=low; k<(high+1); k++)

{

for (int n=low; n<(high+1); n++)

{

for (int a=1;a<4;a++)

{

for (int b=1;b<3;b++)

{

for (int c=1;c<3;c++)

{
```

```
			prof = (k*l)*(k<0) + (n*l)*(n<0) + (-k*h1)*(k>=0) + (-n*h2)*(n>=0)
+ (lambda1*r1)*(b==1) + (lambda2*r2)*(c==1) ;
			fout<<"\""
			<<k<<"\".\""
			<<n<<"\".\""
			<<a<<"\".\""
			<<b<<"\".\""
			<<c<<"\" "
			<<prof<<" "
			<<"\n";


		}
		}
		}
		}
		}


//fout.open("debug\\ip_discP_4_0_-5_1_2_1_2_0.6_0.6_1_1_-65_13.inc",
ios_base::out);
fout.close();


fout.open("probability.inc", ios_base::out);
```

```
fout.precision(16);


//parameter prob(i,j,m,n,a,b,c) one step transition probabilities from (i j) to (m n);
fout<<"*parameter prob(i,j,d,e,a,b,c)"<<"\n";
        for (int i=low; i<(high+1); i++)
        {
        for (int j=low; j<(high+1); j++)
        {
        for (int a=1;a<4;a++)
        {
        for (int b=1;b<3;b++)
        {
        for (int c=1;c<3;c++)
        {
        cumpro=0;
        for (int d=low; d<(high+1); d++)
        {
        for (int e=low; e<(high+1); e++)
        {


                if ( (d==i+1) && (e==j) && (i < high)) //1
                {
```

```
        pro = (m/beta)*(a==2);
        cumpro = cumpro + pro;
        if (pro > 0)
        {
        fout<<"\""
        <<i<<"\".\""

        <<j<<"\".\""
        <<d<<"\".\""

        <<e<<"\".\""
        <<a<<"\".\""
        <<b<<"\".\""
        <<c<<"\" "
        <<pro<<" "
        <<"\n";

        }
}

else if ( (d==i) && (e==j+1) && (j<high)) //2
{
        pro = (m/beta)*(a==3);
```

```
            cumpro = cumpro + pro ;
            if (pro > 0)
            {
            fout<<"\""
            <<i<<"\".\""

            <<j<<"\".\""
            <<d<<"\".\""

            <<e<<"\".\""
            <<a<<"\".\""
            <<b<<"\".\""
            <<c<<"\" "
            <<pro<<" "
            <<"\n";
            }
}

else if ( (d==i-1) && (e==j) && (i>low)) //3
{
            pro = (lambda1/beta)*(b == 1);
            cumpro = cumpro + pro ;
```

```
                if (pro > 0)
                {
                fout<<"\""
                <<i<<"\".\""

                <<j<<"\".\""
                <<d<<"\".\""

                <<e<<"\".\""
                <<a<<"\".\""
                <<b<<"\".\""
                <<c<<"\" "
                <<pro<<" "
                <<"\n";
                }
        }
else if ( (d==i) && (e==j-1) && (j>low)) //4
        {
                pro = (lambda2/beta)*(c == 1);
                cumpro = cumpro + pro ;
                if (pro > 0)
                {
```

```
fout<<"\""
<<i<<"\".\""

<<j<<"\".\""
<<d<<"\".\""

<<e<<"\".\""
<<a<<"\".\""
<<b<<"\".\""
<<c<<"\" "
<<pro<<" "
<<"\n";
}
}

else if ( (d==i) && (e==j) && (i>low) && (i<high) && (j>low) &&
(j<high) ) //5
{
pro = (m/beta)*(a==1) + (lambda1/beta)*(b==2) +
(lambda2/beta)*(c==2) ;
cumpro = cumpro + pro ;
if (pro > 0)
{
```

```
fout<<"\""
<<i<<"\".\""

<<j<<"\".\""
<<d<<"\".\""

<<e<<"\".\""
<<a<<"\".\""
<<b<<"\".\""
<<c<<"\" "
<<pro<<" "
<<"\n";
}
}
else if ( (d==i) && (e==j) && (i==low) && (j>low) && (j<high) )
//6
{
pro = (m/beta)*(a==1) + (lambda1/beta) +
(lambda2/beta)*(c==2) ;
cumpro = cumpro + pro ;
if (pro > 0)
{
fout<<"\""
```

```
            <<i<<"\".\""

            <<j<<"\".\""

            <<d<<"\".\""

            <<e<<"\".\""

            <<a<<"\".\""

            <<b<<"\".\""

            <<c<<"\" "

            <<pro<<" "

            <<"\n";

            }

        }

        else if ( (d==i) && (e==j) && (i==high) && (j>low) && (j<high) )
//7

        {

            pro = (m/beta)*(a!=3) + (lambda1/beta)*(b==2) +
(lambda2/beta)*(c==2) ;

            cumpro = cumpro + pro ;

            if (pro > 0)

            {

            fout<<"\""
```

```
<<i<<"\".\""

<<j<<"\".\""
<<d<<"\".\""

<<e<<"\".\""
<<a<<"\".\""
<<b<<"\".\""
<<c<<"\" "
<<pro<<" "
<<"\n";
}
}
else if ( (d==i) && (e==j) &&  (i>low) && (i<high) && (j==low) )
//8
{
pro = (m/beta)*(a==1) + (lambda1/beta)*(b==2) +
(lambda2/beta) ;
cumpro = cumpro + pro ;
if (pro > 0)
{
fout<<"\""
```

```
<<i<<"\".\""

<<j<<"\".\""
<<d<<"\".\""

<<e<<"\".\""
<<a<<"\".\""
<<b<<"\".\""
<<c<<"\" "
<<pro<<" "
<<"\n";
}
}
else if ( (d==i) && (e==j) &&  (i>low) && (i<high) && (j==high) )
//9
{
pro = (m/beta)*(a!=2) + (lambda1/beta)*(b==2) +
(lambda2/beta)*(c==2) ;
cumpro = cumpro + pro ;
if (pro > 0)
{
fout<<"\""
```

```
            <<i<<"\".\""


            <<j<<"\".\""

            <<d<<"\".\""


            <<e<<"\".\""

            <<a<<"\".\""

            <<b<<"\".\""

            <<c<<"\" "

            <<pro<<" "

            <<"\n";

            }

}

else if ( (d==i) && (e==j) &&  (i==low) && (j==low) ) //10

{

            pro = (m/beta)*(a==1) + (lambda1/beta) + (lambda2/beta) ;

            cumpro = cumpro + pro ;

            if (pro > 0)

            {

            fout<<"\""

            <<i<<"\".\""


            <<j<<"\".\""
```

```
<<d<<"\".\""

<<e<<"\".\""
<<a<<"\".\""
<<b<<"\".\""
<<c<<"\" "
<<pro<<" "
<<"\n";
}
}

else if ( (d==i) && (e==j) &&  (i==low) && (j==high) ) //11
{
pro = (m/beta)*(a!=2) + (lambda1/beta) +
(lambda2/beta)*(c==2) ;
cumpro = cumpro + pro ;
if (pro > 0)
{
fout<<"\""
<<i<<"\".\""

<<j<<"\".\""
```

```
                    <<d<<"\".\""


                    <<e<<"\".\""

                    <<a<<"\".\""

                    <<b<<"\".\""

                    <<c<<"\" "

                    <<pro<<" "

                    <<"\n";

                    }

            }


            else if ( (d==i) && (e==j) &&  (i==high) && (j==low) ) //12

            {

                    pro = (m/beta)*(a!=3) + (lambda1/beta)*(b==2) +
(lambda2/beta) ;

                    cumpro = cumpro + pro ;

                    if (pro > 0)

                    {

                    fout<<"\""

                    <<i<<"\".\""


                    <<j<<"\".\""
```

```
<<d<<"\".\""

<<e<<"\".\""
<<a<<"\".\""
<<b<<"\".\""
<<c<<"\" "
<<pro<<" "
<<"\n";
}
}
else if ( (d==i) && (e==j) &&  (i==high) && (j==high) ) //13
{
pro = (m/beta) + (lambda1/beta)*(b==2) +
(lambda2/beta)*(c==2) ;
cumpro = cumpro + pro ;
if (pro > 0)
{
fout<<"\""
<<i<<"\".\""

<<j<<"\".\""
<<d<<"\".\""
```

```
                    <<e<<"\".\""
                    <<a<<"\".\""
                    <<b<<"\".\""
                    <<c<<"\" "
                    <<pro<<" "
                    <<"\n";
                    }
                }


        }
}




                }
                        }
                    }
                }
        }

fout.close();
```

```
//lambda1 file output
fout.open("lambda1.inc", ios_base::out);
fout.precision(16);
fout<<"*lambda1"<<"\n";
fout<<lambda1<<"\n";
fout.close();

//lambda2 file output
fout.open("lambda2.inc", ios_base::out);
fout.precision(16);
fout<<"*lambda2"<<"\n";
fout<<lambda2<<"\n";
fout.close();

//l1 file output
fout.open("l1.inc", ios_base::out);
fout.precision(16);
fout<<"*l1"<<"\n";
fout<<l<<"\n";
fout.close();
```

```
//l2 file output
fout.open("l2.inc", ios_base::out);
fout.precision(16);
fout<<"*l2"<<"\n";
fout<<l<<"\n";
fout.close();

//h1 file output
fout.open("h1.inc", ios_base::out);
fout.precision(16);
fout<<"*h1"<<"\n";
fout<<h1<<"\n";
fout.close();

//h2 file output
fout.open("h2.inc", ios_base::out);
fout.precision(16);
fout<<"*h2"<<"\n";
fout<<h2<<"\n";
fout.close();
//r1 file output
fout.open("r1.inc", ios_base::out);
```

```
fout.precision(16);

fout<<"*r1"<<"\n";

fout<<r1<<"\n";

fout.close();


//r2 file output

fout.open("r2.inc", ios_base::out);

fout.precision(16);

fout<<"*r2"<<"\n";

fout<<r2<<"\n";

fout.close();


//identical output

fout.open("identical.inc", ios_base::out);

fout.precision(16);

fout<<"*identical"<<"\n";

fout<<identical<<"\n";

fout.close();

//system("gams.exe pooled_capacity_model_26032011");


        return 0;

}
```