SERVICE MODELS FOR AIRLINE REVENUE MANAGEMENT PROBLEMS

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ABSTRACT

SERVICE MODELS FOR AIRLINE REVENUE MANAGEMENT PROBLEMS

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In this thesis, the seat inventory control problem is studied for airlines from the perspective of a risk-averse decision maker. There are only a few studies in the revenue management literature that consider the risk factor. Most of the studies aim at finding the optimal seat allocations while maximizing the expected revenue and do not take the variability of the revenue and hence a risk measure into account. This study aims to decrease the variance of the revenue by increasing the capacity utilization called load factor in the revenue management literature. In addition to expected revenue, load factor is an important performance measure the state companies work with. For this purpose, two types of models with load factor formulations are proposed. This thesis is the first study in the revenue management literature for the airline industry that uses the load factor formulations in the mathematical models. It is an advantage to work with load factor formulations since the models with load factor formulations are much easier to formulate and solve as compared to other risk sensitive models in the literature. The results of the proposed models are evaluated by using simulation for a sample network under different scenarios. The models we propose allow us to control the variability of revenue by changing the used capacity of the aircraft. This is at the expense of a decrease in the revenue under some scenarios. The models we propose perform satisfactorily under all scenarios and they are strongly recommended to be used especially for the small-scale airline companies and state companies and for scheduling new flights even in large scale, well established airline companies.

Keywords: Revenue Management, Seat Inventory Control, Load Factor, Risk, Mathematical Models

ÖZ

HAVAYOLU GELİR YÖNETİMİ PROBLEMLERİ İÇİN SERVİS MODELLERİ

Eroğlu, Fatma Esra
Yüksek Lisans, Endüstri Mühendisliği Bölümü
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Bu tezde, havayolu ağları için koltuk stok kontrolü problemi risk almak istemeyen bir karar vericinin bakış açısıyla incelenmektedir. Havayolları için gelir yönetimi literatüründe risk faktörünü göz önüne alan sadece birkaç çalışma bulunmaktadır. Bu çalışmaların birçoğu beklenen gelir için en yüksek değeri bulmaya çalışırken, en uygun koltuk dağılımını bulmayı amaçlamaktadır ve gelirin değişkenliğini ve dolayısıyla bir risk ölçütünü dikkate almamaktadır. Bu çalışma gelir yönetimi literatüründe doluluk oranı olarak adlandırılan kapasite kullanımını arttırarak, gelirin değişkenliğini azaltmayı amaçlamaktadır. Beklenen gelire ek olarak doluluk oranı devlet kontrolündeki havayolu şirketlerinin kullandığı önemli bir peformans ölçütüdür. Bu amaçla, doluluk oranlarının modellendiği iki tip matematiksel model önerilmektedir. Bu tez, havayolları için gelir yönetimi literatüründeki matematiksel modellerde doluluk oranı formülasyonlarını kullanan ilk çalışmadır. Doluluk oranı formulasyonlarıyla çalışmak, bu modellerin literatürdeki diğer riske duyarlı çalışmalarla karşılaştırıldığında çok daha kolay modellenmesi ve çözülmesi açısından bir avantajdır. Önerilen modellerden elde edilen sonuçlar örnek bir havayolu ağı için, farklı senaryolar altında simülasyon kullanılarak değerlendirilmektedir. Bu simulasyon çalışmalarının sonuçlarına göre önerdiğimiz modeller uçağın kullanılan kapasitesini arttırarak gelirin değişkenliğini kontrol altında tutulmasına olanak sağlamaktadır. Öte yandan, bu durum bazı senaryolar altında gelirde bir azalmayı da beraberinde getirebilmektedir. Önerdiğimiz modeller bütün senaryolar altında tatmin edici bir performans göstermektedir ve özellikle küçük ölçekli havayolu şirketleri ve devlet kontrolündeki havayolu şirketleri ile yeni uçuş çizelgelemesi yapacak olan büyük ölçekli, oturmuş havayolu şirketleri için şiddetle tavsiye edilmektedir.

Anahtar Kelimeler: Gelir Yönetimi, Koltuk Stok Kontrolü, Doluluk Oranı, Risk, Matematiksel Modeller

To my family

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CHAPTER 1

INTRODUCTION

Revenue management (RM) is defined as a tool to maximize the revenue via demand management decisions. Revenue management is also called as Yield management (YM). There are many definitions for RM in the airline industry but the most common one is due to the American Airlines: "*Selling the right seats to the right people at the right time.*" According to Pak and Piersma (2002), RM is defined as "*the art of maximizing profit generated from a limited capacity of a product over a finite horizon by selling each product to the right customer at the right time for the right price.*"

Revenue management dates back to the deregulation of the airline industry in USA in 1970s. That is why revenue management is most successfully applied in the airline industry. To state it more clearly, the following typical characteristics of the airline industry, which enable a successful application of RM, are listed.

- The products are perishable. That is, the unsold seats at the departure time of the flight cannot be sold later.
- The main component of the airline operating costs is the fixed costs of the flight, such as fuel costs, airport costs and personnel costs. That is, the marginal cost of an extra passenger is very low when compared to the fixed costs. Therefore, the marginal cost of a passenger is assumed to be zero.
- Passengers have different characteristics. Therefore, revenue maximization is done by finding the right combination of the passengers.

Besides the airline industry, RM has a wide application area. The industries, where the demand decisions are critical, define the application area of the revenue management. Hotels, media and broadcasting, natural gas storage and transmission, car rental, retailing, air cargo, theaters, restaurants, sport events, electricity generation and transmission are the examples for the RM application areas.

As Pak and Piersma (2002) state in their study, "*RM is a powerful strategy in the markets, where the companies try to sell a fixed amount of product in a limited season and there are customers willing to pay different amounts of money for the products.*" Customer segmentation and price discrimination are mostly used strategies in those environments. Product differentiation is done by offering different prices for different customer segments and change the available mix of the prices during the selling period.

The scope of this thesis is the revenue management applications in the airline industry. The RM problem in the airline industry is defined as managing the flight capacities in a network. The objective of the airline RM problem is to maximize the revenue of the already scheduled flights. Flight scheduling is a structural decision for airline companies and is out of the scope of the revenue management.

The flights in a network can be in local or connecting traffic. In a *local traffic*, there is a direct flight from one node to another node. In a *connecting traffic*, however, there is a node called *the hub* of the network, which is an airport that an airline uses to transfer the passengers to their intended destination. The journey between an origin node and a destination node is called an itinerary. A product is defined as an origin-destination-fare combination and abbreviated as *ODF*. In single-leg flights (local traffic), there is only one origin destination pair and therefore the products are defined for the fare classes. However, the number of origin destination pairs increases in network (connecting) traffic. Therefore, network traffics is more complex.

Customer segments determine the *fare classes* in the airline industry. The type of customers and the condition of the tickets may differ from one customer segment to another. There are basically two customer types: *business traveler* and *leisure traveler*. The base of this customer segmentation is done according to the arrival time of the customers. Leisure travelers arrive earlier than business travelers, in general. Customer types can also be classified according to the location of the seats in the aircrafts, such as *first, business and economy class.*

The conditions of the tickets are also important in defining the customer segments. The fare is one of the factors to determine the condition of the tickets. The tickets are classified as *low price tickets (discounted fares)* and *high price tickets (full fare tickets)*. Low price tickets are generally offered to leisure travelers to attract them at the beginning of the booking horizon and to increase the used capacity of the flight. High fare tickets are generally offered to the business travelers. Another factor to determine the condition of the tickets is the options they have. Cancellation, refund, overnight stay and advance purchase are the examples of those options. The high fare tickets generally have some options such as cancellation and partial/full refund. *Cancellation* enables a customer to cancel his/her ticket and get a partial or full refund. Some passengers do not arrive at the time of departure without cancellation. This is called *no-show*.

The *load factor* is defined as the ratio of the seats filled on a flight to the total number of seats available and is an important performance measure for the airline companies. Offering discounted fares increases the load factor. On the other hand, offering full fare tickets increases the revenue earned per passenger, which is also a performance measure for the airline companies. As can be seen, RM aims at constructing a strategy to balance the conflicting objectives. In order to prevent empty departure of the aircrafts, overbooking is used. *Overbooking* means selling more tickets than capacity. Overbooking is used to cope with cancellations and no-shows.

There are two approaches for RM problems in the airline industry: *capacity allocation*, also called *seat inventory control* and *dynamic pricing*. Talluri and Van Ryzin (2005) define the former as the *Quantity based RM* and the latter as *Price Based RM*. In the seat inventory control approach, the decision maker is responsible for determining the capacity allocations. Based on the capacity allocations, the decision maker accepts or rejects the incoming ticket requests. The tickets are multi-type and differ in options, in terms of trade and/or in price. The changing parameter for the customer is the availability of the tickets during the booking period. That is, the tickets are opened for sale at the beginning of the booking period and, as the time to the flight departure gets shorter, the classes are closed. On the contrary to the seat inventory control approach, in dynamic pricing approach there is only one product and the changing parameter for the customer is the price of it. The price of the ticket changes throughout the booking period depending on the realized demand.

The application area of the aforementioned two approaches is directly related to the characteristics of the market. For a market like online retailing, where the price flexibility is high, dynamic pricing can be considered as an appropriate approach. Price felexibility refers to the ease of manipulation of the prices. That is, price flexible markets are able to change or update the prices of the products easily. For restaurants or that kind of markets, however, the price flexibility is low but the supply flexibility is high. Supply flexibility refers to the ease of manipulation of the availibility of the products. That is, the availibility of the products in restaurants or that kind of markets is easier to manipulate or update than the prices of the products. Therefore, seat inventory control is a more appropriate approach in such markets. Most of the airline companies prefer to work with the seat inventory control. They announce their prices over a given time interval and do not update them frequently. Moreover, the only variable that must be stored and announced is the status of the product, whether it is open or close. Therefore, seat inventory control is easy to implement and is mostly preferred by the airline companies. This thesis also focuses on seat inventory control.

There are two main headings under seat inventory control: *Single Leg Seat Inventory Control* and *Network Seat Inventory Control*. In the single-leg seat inventory control, there is only one origin destination pair isolated from the other flights in the network. This approach is far from being realistic since it optimizes the booking limits locally. In real life however, the airline companies aim to maximize the revenue for the whole network. In order to defeat this disadvantage of single-leg control, network seat inventory control deals with all of the legs in a network simultaneously. However, as the size of the network increases, the complexity of the network seat inventory control also increases, which is its main disadvantage.

It is expected that the decision maker implements the optimal seat allocations in a framework to decide on accepting or rejecting an arriving request for a product. The mentioned framework is drawn by the control polices. There are three control policies used for the RM problems in airline industry: *Partitioned Control Policy, Nested*

Control Policy and Bid Price Control Policy. In partitioned control policy, each fare class has a separate booking limit or protection level, which means a seat allocated to a fare class cannot be booked for another fare class. In nested booking limit control policy, the fare classes are ordered according to some criteria and the seats that are available for a low ranked fare class are also available for higher ranked ones. In bid price control policy, a request is accepted if the fare of the class exceeds the opportunity cost of selling the corresponding itinerary. The *opportunity cost* of an itinerary is defined as the expected loss in the revenue from using the capacity now rather than reserving it for future use. The opportunity cost is approximately calculated as the sum of the bid prices of the flight legs that the itinerary uses. A *bid price* is defined as the net value of an incremental seat on a particular flight leg in the airline network. Similarly, *shadow price* is defined as an increment in the revenue in case one more seat is allocated to a particular *ODF* when all other allocations remain unchanged. In bid price control policy, the class is open without any limit as long as the fare of the *ODF* exceeds the opportunity cost of selling the corresponding itinerary. This is the main difference between bid price control policy and the other control policies working with booking limits or protection levels.

The mathematical models developed for RM problems in airline industry are of two types: deterministic models and probabilistic models. *Deterministic models* assume that the demand of a particular *ODF* is equal to the expected value. *Probabilistic models*, on the other hand, include the probabilistic nature of the demand.

Most of the RM studies in the literature assume that the decision makers are risk-neutral. These studies mostly aim at maximizing the expected revenue. Therefore, the variability of revenue or any other risk factor such as competition in the market and the utility functions of the customers are not taken into account. However, it is quite important in the short term to include the risk factor especially for small-scale airline companies or for new flights. Small-scale airline companies are more vulnerable to risk as compared to large-scale, well established companies. They can change their routes according to the variations of demand whereas large scale, well established airline companies are not highly affected from demand variations. As Terciyanlı (2009) states in his study, the fares are relatively close and no spesific order of arrival is assumed in small-scale airline companies different than large-scale, well established airline companies. Therefore, small-scale companies try to increase the capacity utilization to increase the revenue earned. Large-scale companies, on the other hand, may achieve the same revenue as small-scale companies do by much lower fill rate with the help of the difference between fares. This situation also increases the risk for small-scale companies. Moreover, new flights can be more risky when compared to the existing ones even for the large scale airline companies. Isolating the risk factor from the framework of the problem results in the ignorance of the decision maker behavior. Without considering the behavior of the decision maker, the decision makers are treated as if they all have the same risk reaction pattern. As a result, the solution of the problem diverges from the reality.

The scope of this thesis is to handle a network seat inventory control problem by taking the risk factor into account. By incorporating the risk factor, the risk reaction pattern of the decision maker is taken into account in this thesis. The assumptions used in this study are as follows.

- Overbooking, cancellations and no-shows are not allowed.
- A shift between classes do not occur. That is, customers make their decisions for the fare class that they request.
- Batch booking is not allowed. That is, individual customers are assumed to arrive sequentially.

Two types of mathematical models are proposed in this thesis to find the optimal seat allocations by incorporating the load factor into the models. Although the load factor is an important performance measure considered to compare other alternative approaches, there is no study in the literature that directly uses the load factor formulations in the mathematical models. That is, this thesis is the first study in the RM literature for the airline industry that uses the load factor formulations in the mathematical models. The results of the numerical analysis in this thesis justify the observations of Terciyanlı (2009) regarding the relation between the load factor and the variance of the revenue. That is, the variance of the revenue and the load factor are negatively correlated. As Terciyanlı (2009) states in his study, "*decreasing risks causes an increase in the load factors*". In accordance with these observations due to Terciyanlı (2009), it is shown in this thesis that the variance of the revenue can be controlled by increasing the load factor. It is an advantage to work with load factor formulations to control the variance of the revenue since the models with load factor formulations are much easier to formulate and solve as compared to the models due to Çetiner (2007) and Terciyanlı (2009). Moreover, linearity is maintained in this thesis by load factor formulations. The maximization of the capacity utilization is an important performance measure that is used especially by the state airline companies under government regulation. Therefore, direct use of load factor for these airline companies is also appropriate. For airline RM problems, there exists a trade-off between the expected revenue and the capacity utilization of the aircraft, in general. Both the expected revenue and the capacity utilization are tried to be maximized. However, these objectives mostly conflict. In order to resolve this trade-off, both expected revenue and expected load factor are incorporated into mathematical models proposed in this thesis. That is, this thesis contibutes to the literaute about *MCDM* (Multi Criteria Decision Making). There are two criteria to be maximized: expected load factor and expected revenue. As a result of the numerical analysis, it is observed that there exists an efficient frontier for these objectives.

One type of the models we propose in this thesis aim at maximizing the expected revenue while working with service level constraints on the expected load factors. In these models, the service level is a predetermined threshold level. The other type of models we propose aim at maximizing a weighted average of the expected load factors of the network legs or maximizing the minimum expected load factor of the legs in the network while ensuring that the expected revenue is always above a predetermined threshold level. The variability of the revenue is taken into account in these two types of models by investigating the relation between load factor and risk aversion of the decision makers. The risk aversion is evaluated on the basis of the variation. Standard deviation and coefficient of variation of the revenue are two risk measures used in this thesis. The impact of a change in the load factor on these risk measures is investigated to take the variability of the revenue into account. The main advantage of the proposed models is the simplicity of their application since they are easy to formulate and solve. The models we propose do not have non-linear formulations. Moreover, no approximation is needed to solve the models we propose different than the models proposed by Terciyanlı (2009). We only use LP relaxations to solve the proposed models due to integrality constraints. The probabilistic nature of demand is taken into account in those proposed mathematical models.

The organization of the thesis is as follows: In Chapter 2, the related literature for RM problems is reviewed by introducing the notation that is used throughout the thesis. After mentioning the single-leg RM problems briefly, network seat inventory control models are presented in more detail. The risk sensitive models we propose are given in Chapter 3. In Chapter 4, we propose the bound models to determine the range of the threshold levels that are used in two types of models we propose in Chapter 3. The numerical ranges for the threshold levels in the proposed models are determined in Chapter 5 for a sample network due to de Boer (1999). Moreover, the numerical analysis and interpretations of our proposed risk sensitive models are also presented in this chapter. Chapter 6 is devoted to the simulation studies. In this chapter, the seat allocations and the bid prices obtained from the mathematical models are studied in simulation models for partitioned, nested and bid price controls. Moreover, the models we propose are compared to the existing models in the literature in Chapter 6. The thesis ends with the concluding remarks and suggestions for future research in Chapter 7.

CHAPTER 2

LITERATURE REVIEW

When the literature for the revenue management is reviewed from its point of origin in the airline industry, it is seen that the revenue management problems are handled basically in two ways: either by manipulating the capacity allocations of the airline flights, which is called *seat inventory control* or by manipulating the prices of the flight tickets, which is called *dynamic pricing*. Talluri and van Ryzin (2005) classify these two approaches as *Quantity based RM and Price based RM*, respectively.

As mentioned in Chapter 1, seat inventory control and dynamic pricing approaches differ in the following points of view.

- The aim of the decision maker is to determine the seat allocations in the seat inventory control and the changing parameter for customers is the availability of the products (tickets) during the booking period. On the other hand, the aim of the decision maker is to determine the price of the products (tickets) in dynamic pricing and the changing parameter for customers is the price.
- In seat inventory control, there are multiple types of tickets that differ in options. There is only one product in dynamic pricing.
- In seat inventory control, the decision of accepting or rejecting an incoming request is based on the seat allocations. In dynamic pricing, this decision is based on the price of the tickets, that changes throughout the booking period depending on the realized demand and the remaining time.
- The characteristics of the market determine the application area of these two approaches. As stated in Chapter 1, for markets having high price flexibility,

dynamic pricing is an appropriate approach. On the other hand, for markets having low price flexibility but high supply flexibility, seat inventory control is appropriate.

Although both of the seat inventory control and dynamic pricing approaches are used in the airline industry, seat inventory control is a more preferred approach. The following observations are due to Talluri and van Ryzin (2005). In the airline industry, most of the companies announce their price lists over a given time interval and do not update them frequently due to three reasons. One of them is the advertising concerns: announcing new prices to customers increases the operational costs. The second reason is the fact that the prices are not solely under the control of the company itself but are affected by the strategies of other companies. Therefore, it can be deducted that the prices are, in a way, determined by the market, especially for small scale companies. The last reason is the easy implementation of the approach since the only parameter to be stored is the availability information of the products. Since the scope of this thesis excludes price manipulation, our interest in this chapter is on the literature of seat inventory control problems.

There are two main research areas for seat inventory control problems in the literature: *Single-Leg Seat Inventory Control* and *Network Seat Inventory Control*. In single-leg seat inventory control problems, only a single flight leg is considered. In the network seat inventory control problems, a number of itineraries having connecting flights are considered aiming at the optimization for the whole network. Network seat inventory control problems gained popularity at the beginning of the 1990's by the changing environment of airline industry. Network traffics including connecting flights began to be used instead of single leg flights. Improvement in computational skills via computers started enabling the optimization of the network.

In this chapter, firstly the studies on single-leg seat inventory control problems are summarized in Section 2.1. The studies on the network seat inventory control problems are reviewed in Section 2.2. The studies that take the risk factor into account are presented in Section 2.3. Section 2.4 is devoted to the review of three control policies applied for network seat inventory control; namely, partitioned control, nested control and bid price control. The notation introduced in this chapter is used throughout the thesis.

2.1 Single-Leg Seat Inventory Control Problems

The concern of the single-leg inventory control problems is to allocate the seats of a single-leg flight to different customer types. In the literature, this problem is handled either with static or dynamic controls. For the static control case, the main assumption is the sequential arrival of the customers. For the dynamic control case, this assumption is relaxed. The assumptions for the static control case are listed by McGill and Talluri van Ryzin (1999) as follows:

1. single flight leg,
2. sequential arrival of different customer classes,
3. low-before-high arrival pattern (a low fare class customer books earlier than all of the passengers from higher fare classes),
4. statistically independent demands of the booking classes,
5. no cancellation, no-show and overbooking,
6. no batch booking.

The first study in the area of seat inventory control is due to Littlewood (1972). It is the first study that aims at maximizing the expected revenue by using mathematical formulations. In his study, all of the six assumptions of the static control case are considered for only two fare classes. In this study, it is suggested that a request of a lower fare class customer should be rejected in case the expected revenue of selling that seat to a higher fare class customer exceeds the lower fare.

The study of Littlewood (1972) inspired later studies in the literature. Mayer (1976) extends the study of Littlewood (1972) by updating the rule more than once throughout the booking period before the departure time via a simulation study. The assumption of low-before-high arrival pattern is relaxed in this study. Thereafter, Belobaba

(1987) extends the study of Littlewood (1972) for more than two fare classes. Moreover, he develops a heuristic called *Expected Marginal Seat Revenue (EMSR)* in order to determine the booking limits for each fare class. The booking limits obtained by using the *EMSR* heuristic may turn out to be much different than the optimal booking limits. However, Curry (1990), Wollmer (1992), Brumelle and McGill (1993) and Robinson (1995) observe that the revenue obtained by using the heuristic is quite close to the optimal revenue.

Curry (1990), Wollmer (1992), Brumelle and McGill (1993) also derive optimal booking limits for different fare classes by using the six assumptions for static control case. However, the demand distributions are different in these studies. For instance, Curry (1990) considers a continuous demand distribution and gives a recursive equation in order to determine the booking limits for each fare class. Wollmer (1992) uses real life discrete demand data while trying to determine the booking limits for fare classes. In the study of Brumella and McGill (1993), a booking limit control policy is developed that takes both continuous and discrete demand data into account.

Robinson (1995) relaxes the assumption of low-before-high arrival pattern for the multiple fare class problem. In his study, he assumes that all of the customers of a fare class book before any customers of another fare class. The main assumption of the static control is relaxed for the dynamic control: low-before-high arrival pattern. The first study on dynamic control is due to Lee and Hersh (1993). In this study, the batch booking assumption is also relaxed and it is allowed to book more than one seat for a request. Moreover, a discrete-time dynamic programming model is developed and the expected revenue is determined via a recursive function.

Lautenbacher and Stidham (1999) use a finite horizon Markov Decision Process to present the similarities and differences between static and dynamic control for single leg problems. In this study, cancellations, no-shows and overbooking are not allowed. Subramanian et al. (1999) extend the study of Lee and Hersh (1993) by incorporating cancellation, no-shows and overbooking. They show that the problem can be handled as a queuing system and the optimal booking policy is characterized by determining booking limits for the fare classes that are both time and state dependent. Their study reveals that the booking limits need not to be monotonic. Moreover, it is observed that it may be optimal to accept a low fare class rather than high one due to probability of cancellation.

Gosavi et al. (2002) consider a Semi Markov Decision Process and use the technique called *Reinforcement Learning*. In this study, random cancellation, overbooking and concurrent demand arrivals from different fare classes are allowed.

2.2 Network Seat Inventory Control Problems

As mentioned previously, the single-leg seat inventory control problems started to lose their popularity as the structure of the airline industry changes dramatically. With this structural change in the airline industry, it has become difficult to fly from one point to another directly. Transfer centers, which are called hubs, began to be used to transfer the passengers from one flight to another. The concern in the network inventory seat control problems is to allocate the seats of different flight legs in a network to different customer segments.

The mathematical models that are used for the network seat inventory control problems are classified into two groups: deterministic models and probabilistic models. The deterministic models assume that the demand of an *ODF* is the same as its expected value. In probabilistic models, on the other hand, probabilistic nature of the demand is taken into account.

In Section 2.2.1, the notation and assumptions used for the network seat inventory control problems are provided. The studies in the literature that are performed on network seat inventory control are reviewed in Section 2.2.2.

2.2.1 Notation

The legs between origins and destinations are defined as resources. m denotes the resources in a network. As defined in Chapter 1, a product which is called as *ODF*, refers to an origin-destination-fare combination and n denotes the number of products offered on those legs in the network. Demand of each *ODF* is assumed to be independent of the others. The passengers are not allowed to switch from one fare class to another fare class. The fares of the products are assumed to be known and they are constant throughout the reservation period. Recall that an itinerary is a trip from an origin to a destination.

The notation used in the network seat inventory control is as follows:

j : index for the products (ODFs) in the network, $j = 1, ..., n$,
l : index for the resources of the network, which are the flight legs, $l = 1, ..., m$,
S_l : set of ODFs that use flight leg l,
T_j : set of flight legs that are on the route of ODF j,
C_l : available capacity of flight leg l,
f_j : the fare of ODF j,
D_j : random variable for demand of ODF j in the booking period.

2.2.2 An Overview of the Studies

The first study for network seat inventory control is due to Buhr (1982) with two legs and one fare class. In this study, the notion of *expected marginal revenue of a seat* is defined. A network A-B-C with legs AB and BC is used as the sample network in this study.

Glover et al. (1982) performs the first study in the literature on a large network. Demand is assumed deterministic and integer programming is used to model a maximum profit network flow. The suggested integer programming is called Deterministic Mathematical Programming (DMP) and is given below. This model is a *"constrained knapsack"* problem.

$$DMP \quad : \quad \text{Maximize} \sum_{j=1}^{n} f_j x_j \tag{2.1}$$

subject to

$$\sum_{j \in S_l} x_j \leq C_l \quad \text{for } l = 1, ..., m, \tag{2.2}$$

$$x_j \leq E(D_j) \quad \text{for } j = 1, ..., n, \tag{2.3}$$

$$x_j \geq 0 \quad \text{and integer for } j = 1, ..., n. \tag{2.4}$$

The only decision variable of the model, x_j, represents the number of seats allocated to *ODF* j. Constraint (2.2) is the capacity constraint. The sum of the allocations for all *ODF*s on a particular leg is forced to be smaller than or equal to the capacity of that leg via constraint (2.2). Recall that S_l represents the set of *ODF*s using leg l. $E(D_j)$ in (2.3) is the expected demand of *ODF* j and it is required to be an upper bound for allocation of *ODF* j. The model maximizes the total revenue. *DMP* is a simple model and can be used to find the seat allocations for *ODF*s. However, it does not take the stochastic nature of the demand into account and that is the major drawback of it. That is why it is called a deterministic model. It is an interesting observation that *DMP* gives higher revenues when compared to the other probabilistic models in the literature. This phenomena is widely discussed in the literature. Almost all of the studies agree on the fact that deterministic as well as probabilistic models ignore nesting due to computational complexities. Nesting is incorporated by the simulation models. However this ignorance in the optimization models results in more severe results in probabilistic models. That is why *DMP* outperforms the probabilistic models. It is important to note that the demand forecasting quality affects the performance of the model.

The linear relaxation of this model is considered by Williamson (1992) and it is called Deterministic Linear Programming (*DLP*). The model is relaxed by allowing the decision variable, x_j, to take values between 0 and 1. The need for the relaxation stems from the integrality of the decision variable, x_j. Integrality causes computational burden and increases the solution time. However, a meaningful solution for a seat allocation would be an integer. At that point Williamson (1992) claims that *DLP* model provides integer solutions when integer expected demand values are used in the model since the upper and lower bounds on the decision variables are integer. However, de Boer (1999) disproves the claim of Williamson (1992) by a counterexample where the bounds on each variable are integer but the solution is not.

DMP and its relaxed model *DLP* are the only deterministic models in the literature. Other models in the literature on network seat inventory control take the stochastic nature of the demand into account and, therefore, they are called the "*Probabilistic Models*". The first probabilistic model introduced is called Probabilistic Mathematical Programming Model and its abbreviation is *PMP*. The model is also called *PNLP* (Probabilistic Non-Linear Programming) due to its non-linear characteristics. The model is given below.

$$PMP \quad : \quad \text{Maximize } E(\sum_{j=1}^{n} f_j \min\{x_j, D_j\}) \tag{2.5}$$

subject to

$$\sum_{j \in S_l} x_j \leq C_l \quad \text{for } l = 1, ..., m, \tag{2.6}$$

$$x_j \geq 0 \text{ and integer} \quad \text{for } j = 1, ..., n. \tag{2.7}$$

As in the *DMP* model, the decision variable x_j represents the number of seats allocated to $ODF\ j$. Actually, the definitions given for the *DMP* model is common for all of the models to be given in this chapter. The probabilistic nature of this model stems from the inclusion of the probability distribution of demand in the objective function by selecting the minimum of seats allocated to a particular $ODF\ j$ and the demand realized for that $ODF\ j$. In case the realized demand exceeds the number of seats allocated, all of the allocated seats are sold. In the opposite case, the number of seats sold is just equal to the realized demand.

Due to its non-linearity, *PMP* is hard to solve. There are efforts in the literature to relax the model. However, the mentioned relaxation is a remedy only for the integrality of the decision variable x_j but not for the non-linear characteristics of the objective function. As mentioned before, Williamson (1992) shows that deterministic model *DLP* outperforms the probabilistic models in terms of obtained revenue since both deterministic and probabilistic models ignore a nested environment. When this phenomenon is viewed in terms of *PMP*, the tendency of the probabilistic models can be better perceived. The probabilistic models allocate more seats to high fare class customers in order to obtain upward potential of high fare demand. This fact results in overprotection in terms of revenue. Since the uncertainty of demand is not incorporated into the deterministic models, they do not have such a tendency like allocating more seats to high fare classes and this fact becomes an advantage for them in a nested environment.

Wollmer (1986) proposes a linear probabilistic model for a multi-leg, multi-fare class problem. The model is based on the expected marginal revenues of the seats and is called *Expected Marginal Revenue (EMR)* model. The model is given below.

$$EMR \quad : \quad \text{Maximize} \sum_{j=1}^{n} \sum_{i=1}^{B_j} f_j P(D_j \geq i) x_j(i) \tag{2.8}$$

subject to

$$\sum_{j \in S_l} \sum_{i=1}^{B_j} x_j(i) \leq C_l \quad \text{for } l = 1, ..., m, \tag{2.9}$$

$$x_j(i) \in \{0, 1\} \quad \text{for } j = 1, ..., n, \text{ and } i = 1, ..., B_j. \tag{2.10}$$

Different from *DMP* and *PMP*, a binary decision variable, $x_j(i)$, is introduced in this model. The definition of the new decision variable is given as follows:

$$x_j(i) = \begin{cases} 1 & \text{if } i \text{ or more seats are allocated to } ODF \ j, \\ 0 & \text{otherwise.} \end{cases} \tag{2.11}$$

x_j can be expressed in terms of $x_j(i)$ as follows:

$$x_j = \sum_{i=1}^{B_j} x_j(i), \tag{2.12}$$

where B_j is the maximum number of seats that can be allocated to $ODF\ j$. There are alternative ways to determine the value of B_j. As Terciyanlı (2009) summarizes in his study, the capacities of the legs which are used by $ODF\ j$ are taken into consideration while determining the value of B_j. Mostly, the maximum capacity of the legs which are used by $ODF\ j$ is accepted as the value of B_j in case overbooking is not allowed. That is,

$$B_j = \max_{l \in T_j} \{C_l\}. \tag{2.13}$$

The *EMR* model maximizes the expected total marginal revenues of seats in the objective function. The only constraint is the capacity constraint. The model is hard to solve since it contains large number of binary variables. Williamson (1992) suggests to use the linear relaxation (LP) of the model and claims that the relaxation also provides integer solutions that are in accordance with the definition in (2.11). The following remark is useful in showing that the LP relaxation of the *EMR* model gives also integer results.

Remark 2.2.2.1 *(due to Williamson 1992) $P(D_j \geq i)$ is a monotonically decreasing function of i. This functional behavior ensures the following for an optimal solution* *of EMR and its LP relaxation:* $x_j(i+1)$ *cannot take a positive value unless* $x_j(i)$ *is equal to* 1. *In other words,* $x_j(i+1)$ *can take a positive value only if* $x_j(i)$ *is equal to* 1.

Although the integrality problem is solved with the relaxation, *EMR* is still hard to solve for large networks due to large number of variables.

de Boer (1999) proposes a stochastic model, *Stochastic Linear Programming (SLP)*, in which she uses the demand aggregation. In this sense, *SLP* is proposed as an approximation of *EMR*. In her study, de Boer (1999) partitions the demand into intervals rather than considering each value of demand separately. The formulation of the model is as follows:

$$SLP \quad : \quad \text{Maximize} \sum_{j=1}^{n} f_j x_j - \sum_{j=1}^{n} f_j \sum_{k=1}^{\kappa_j} P(D_j < d_j(k)) x_j(k) \tag{2.14}$$

subject to

$$\sum_{j \in S_l} x_j \leq C_l \quad \text{for } l = 1, ..., m, \tag{2.15}$$

$$x_j = \sum_{k=1}^{\kappa_j} x_j(k) \quad \text{for } j = 1, ..., n, \tag{2.16}$$

$$x_j(1) \leq d_j(1) \quad \text{for } j = 1, ..., n, \tag{2.17}$$

$$x_j(k) \leq d_j(k) - d_j(k-1) \quad \text{for } j = 1, ..., n, \text{ and } k = 2, ..., \kappa_j, \tag{2.18}$$

$$x_j(k) \geq 0 \quad \text{for } j = 1, ..., n, \text{ and } k = 1, ..., \kappa_j. \tag{2.19}$$

Recall that B_j in *EMR* is defined as the maximum capacity of the legs which are used by *ODF* j. Similarly, κ_j in *SLP* is the maximum number of demand groups. The decision variable $x_j(k)$ represents the number of seats allocated to the demand that falls in the interval $(d_j(k-1), d_j(k))$. The sum of $x_j(k)$s over k is equal to x_j used in *DLP* and *PMP*. The first term in the objective function represents the total revenue that would be generated if all of the allocated seats are sold. The second term is a correction factor for the uncertainty of the demand. de Boer et al. (2002) show that the linear relaxation of *EMR* is just a special case of *SLP*, where each demand interval is of unit size. For this special case, the objective function of *SLP* can be rewritten as follows:

$$\sum_{j=1}^{n} \sum_{k=1}^{\kappa_j} f_j P(D_j \geq d_j(k)) x_j(k) \tag{2.20}$$

by letting $(d_j(k+1) - d_j(k)) = 1$ and $d_j(1) = 1$ for all j and k. In this case, $\kappa_j = B_j$. *SLP* can be simplified by increasing the number of aggregation groups. However, the solution quality decreases in that case.

Until the study of Simpson (1989), booking limit controls are used in order to determine the allocation of seats. Simpson (1989) is the first one to develop the concept of bid price control for seat allocation decisions. Recall from Chapter 1 that bid price is defined as the net value for an incremental seat on a particular flight leg in the airline network. In the bid price control, a seat is sold if the fare of the class exceeds the opportunity cost of selling the corresponding itinerary. As mentioned in Chapter 1, the opportunity cost is approximately calculated as the sum of bid prices of the legs that the itinerary uses. Simpson (1989) and Williamson (1992) use deterministic linear programming models to obtain the bid prices. They suggest to use the dual prices of the capacity constraints as the bid prices of the network legs.

Williamson (1992) develops the concept of bid price control policy. Demand aggregation and simulation are the techniques that Williamson (1992) benefits from in her study. Her study reveals that the deterministic models perform better than the probabilistic models as mentioned before. In that sense, her study has inspired the following studies in the literature.

Talluri and van Ryzin (1998) also study on bid price control and derived the structure of the dynamic optimal control policy. In this study, they mostly analyze the theoretical basis of the control policy. One of the important result of this study is that the bid price control is not optimal in general when leg capacities and sales volumes are not large enough. It is revealed that the bid prices are asymptotically optimal for large networks. They propose a model which is called *Randomized Linear Programming (RLP)*. This model is the randomized version of *DLP*. In this model, a set of demand realizations are considered and the model is solved for every demand realization. It is claimed that *RLP* provides a slight improvement in revenue when compared to *DLP*. The formulation of the model is as follows:

$$
\begin{aligned}
RLP \quad : \quad & \text{Maximize } \sum_{j=1}^{n} f_j x_j && (2.21)\\
& \text{subject to} \\
& \sum_{j \in S_l} x_j \leq C_l \quad \text{for } l = 1, ..., m, && (2.22)\\
& 0 \leq x_j \leq d_j \quad \text{for } j = 1, ..., n. && (2.23)
\end{aligned}
$$

where d_j is the specific demand realization. Via *RLP*, Talluri and van Ryzin (1998) obtain approximate bid price of leg l by taking the average of dual variables of (2.22) for a specified set of demand realizations. The set of demand realization used is big enough to ensure the reliability of the results.

Another research area in the network seat inventory control is overbooking. As defined in Chapter 1, overbooking is allowing the total sales volume to be greater than the capacity of the flight. In spite of including a risk factor, overbooking increases the capacity utilization significantly due to presence of random cancellations and no-shows. The first study on overbooking is due to Beckmann (1958). In this study, a non-dynamic optimization model is proposed. Shlifer and Vardi (1975) propose an overbooking model, which is used for a single-leg flight with a single type of passenger. They develop this model also for a single-leg flight with two types of passengers and for two-leg flight. McGill and van Ryzin (1999) also perform related studies on overbooking. Biyalogorky et al. (1999) suggest to accept a high fare customer even if there is no remaining capacity and to cancel the ticket of a low fare class customer by paying a compensation. Ringbom and Shy (2002) propose the so called *adjustable curtain* strategy, which enables overbooking in favor of high fare customers. In this strategy, economy and business classes are adjusted before boarding. Karaesmen and van Ryzin (2004) propose a two-stage optimization in order to determine the overbooking levels. The requests are accepted in the first stage with the probabilistic knowledge of cancellations. In the second stage, the passengers who do not cancel their tickets are assigned to several inventory classes and the assignment penalties are minimized.

2.3 An Overview of the Risk-Sensitive Studies

All of the studies reviewed so far in this chapter aim at maximizing the revenue by determining the seat allocations and they do not take the risk factor into account. However, one may prefer to work with risk sensitive approaches according to the market conditions and this is mostly the case for real life situations. This thesis also has the consideration of the risk factor.

Incorporating risk factors is a relatively new research area for revenue management although it is studied for many inventory control problems. There are only a few risk sensitive revenue management studies in the literature that incorporate the variability of the revenue, the utility functions of the customers or the competition in the market. Most of the risk sensitive revenue management studies are on dynamic pricing. The studies due to Feng and Xiao (1999), Lancester (2003), Weatherford (2004) and Chen et al. (2006), Barz and Waldman (2007) and Levin et al. (2008) are risk sensitive revenue management studies on dynamic pricing. Since the scope of this thesis excludes the price manipulation of the products, only the studies on risk sensitive seat inventory control are reviewed in this section.

One of the risk sensitive studies is due to Lancester (2003). He uses sensitivity analysis rather than a direct incorporation of risk aversion into the mathematical models. Weatherford (2004) and Chen et al. (2006) aim at maximizing the expected utility instead of expected revenue. Barz and Waldmann (2007) propose a Markov Decision Process for static and dynamic single leg revenue management problem by using the exponential utility functions. Levin et al. (2008) study optimal dynamic pricing of perishable goods and products.

There are only two studies in the literature on network seat inventory control that directly take the risk factor into account in the models. These studies have inspired the study in this thesis. One of those studies due to Çetiner (2007) and Çetiner and Avşar (2011) incorporates variance of the revenue into the models. The authors propose two *SLP* based models. The first model proposed, *EMVLP-1*, aims at maximizing the expected revenue while penalizing the variance of the revenue by a given factor. The model is given below.

$$
\begin{aligned}
EMVLP-1 \quad : \quad & \text{Maximize } \sum_{j=1}^{n}\sum_{k=1}^{\kappa_j} f_j P(D_j \geq k) x_j(k) \\
& -\theta \sum_{j=1}^{n}\sum_{k=1}^{\kappa_j} x_j(k) f_j^2 P(D_j \geq k) P(D_j < k) \qquad (2.24) \\
& \text{subject to} \\
& \sum_{j \in S_l}\sum_{k=1}^{\kappa_j} x_j(k) \leq C_l \quad \text{for } l = 1, ..., m, \qquad (2.25) \\
& x_j(1) \leq d_j(1) \quad \text{for } j = 1, ..., n, \qquad (2.26) \\
& x_j(k) \leq d_j(k) - d_j(k-1) \text{ for } j = 1, ..., n, \text{ and } k = 2, ..., \kappa_j \qquad (2.27) \\
& x_j(k) \geq 0 \quad \text{for } j = 1, ..., n, \text{ and } k = 1, ..., \kappa_j. \qquad (2.28)
\end{aligned}
$$

First term of the objective function is the expected marginal revenue. The second terms is the penalty that is applied on the variance of the revenue by the penalty factor θ. Note that the formulation for variance is an approximation and is given for the independent demand case. All of the constraints are the same as the one that is used in *SLP*. That is why we call this model an *SLP* based model. By changing the θ values, expected revenue and its variation can be controlled. However, it is not easy to set or determine the θ values. Therefore, the model is not practical to use for every day operational decisions.

The second model proposed by Çetiner (2007) and Çetiner and Avşar (2011), *EMVLP-2*, aims at maximizing the total expected revenue. The variance of the revenue is incorporated into the model using a constraint. The model is given below.

$$EMVLP-2 \quad : \quad \text{Maximize} \sum_{j=1}^{n}\sum_{k=1}^{\kappa_j} f_j P(D_j \geq k) x_j(k) \tag{2.29}$$

subject to

$$\sum_{j \in S_l}\sum_{k=1}^{\kappa_j} x_j(k) \leq C_l \quad \text{for } l = 1, ..., m, \tag{2.30}$$

$$x_j(1) \leq d_j(1) \quad \text{for } j = 1, ..., n, \tag{2.31}$$

$$x_j(k) \leq d_j(k) - d_j(k-1) \text{for } j = 1, ..., n, \text{ and } k = 2, ..., \kappa_j, \tag{2.32}$$

$$\sum_{j=1}^{n}\sum_{k=1}^{\kappa_j} x_j(k) f_j^2 P(D_j \geq k) P(D_j < k) \leq$$

$$\rho \sum_{j}\sum_{k} x_j(k) f_j P(D_j \geq k), \tag{2.33}$$

$$x_j(k) \geq 0 \text{ for } j = 1, ..., n, \text{ and } k = 1, ..., \kappa_j. \tag{2.34}$$

Different than *EMVLP-1*, the penalty factor with θ is dropped from the objective function in *EMVLP-2*. The constraint on the ratio of the expected value and the variance of the revenue is used. Çetiner and Avşar (2011) propose to use the two models together in a procedure. *EMVLP-1* is solved first and the ratio of the expected value and variance of the total revenue is found as the output of the model. Thereafter, *EMVLP-2* is solved by using the output of *EMVLP-1* as the ρ value on the right hand side of its additional constraint to find optimal seat allocations and bid prices. The results of both models are compared to our models proposed in this thesis in Chapter 6.

The second risk sensitive study in the literature on network seat inventory control is due to Terciyanlı (2009). Terciyanlı (2009) proposes a lexicographic optimization approach using two models, namely *PMP-RM-1* and *PMP-RM-2*. The abbreviation *PMP-RM* stands for *Probabilistic Mathematical Programming with Risk Measure*. Terciyanlı (2009) proposes to solve two models sequentially. *PMP-RM-1*, which minimizes the probability that the revenue is less than a threshold level, is solved first. The set of optimal solutions of *PMP-RM-1* is used as the set of feasible region of *PMP-RM-2* and *PMP-RM-2* is solved over this feasible region. The models are given below.

$$PMP-RM-1 \quad : \quad \text{Minimize} \sum_{d} p(\boldsymbol{d})v(\boldsymbol{d}) \tag{2.35}$$

subject to

$$\sum_{j \in S_l} \sum_{i=1}^{B_j} x_j(i) \leq C_l \quad \text{for } l = 1, ..., m, \tag{2.36}$$

$$Mv(\boldsymbol{d}) \geq - \sum_{(i,j) \ni d_j \geq i} f_j x_j(i) + L \quad \text{for all } \boldsymbol{d}, \tag{2.37}$$

$$M(1 - v(\boldsymbol{d})) \geq \sum_{(i,j) \ni d_j \geq i} f_j x_j(i) - L \quad \text{for all } \boldsymbol{d}, \tag{2.38}$$

$$x_j(i) \in \{0, 1\} \quad \text{for } j = 1, ..., n, \text{ and } i = 1, ..., B_j, \tag{2.39}$$

$$v(\boldsymbol{d}) \in \{0, 1\} \quad \text{for all } \boldsymbol{d}. \tag{2.40}$$

For a given seat allocation $\boldsymbol{x}$, $v(\boldsymbol{d})$ is the conditional probability that total revenue is less than L given that the demand is equal to $\boldsymbol{d} = (d_1, \cdots, d_n)$. $v(\boldsymbol{d})$ is defined as a binary decision variable since it is an indicator function. $p(\boldsymbol{d})$ is the probability that the demand vector is equal to $\boldsymbol{d}$. M is a big number. Via constraints (2.37) and (2.38), it is ensured that $v(\boldsymbol{d}) > 0$ when total revenue is less than L for a given demand $\boldsymbol{d}$.

$$PMP-RM-2|_v \quad : \quad \text{Maximize} \sum_{j=1}^{n} \sum_{i=1}^{B_j} f_j P(D_j \geq i) x_j(i) \tag{2.41}$$

subject to

$$\sum_{j \in S_l} \sum_{i=1}^{B_j} x_j(i) \leq C_l \quad \text{for } l = 1, ..., m, \tag{2.42}$$

$$Mv(\boldsymbol{d}) \geq - \sum_{(i,j) \ni d_j \geq i} f_j x_j(i) + L \quad \text{for all } \boldsymbol{d}, \tag{2.43}$$

$$M(1 - v(\boldsymbol{d})) \geq \sum_{(i,j) \ni d_j \geq i} f_j x_j(i) - L \quad \text{for all } \boldsymbol{d}, \tag{2.44}$$

$$x_j(i) \in \{0, 1\} \quad \text{for } j = 1, ..., n, \text{ and } i = 1, ..., B_j. \tag{2.45}$$

The set of optimal solutions of *PMP-RM-1* is used as the feasible region of *PMP-RM-2* and the expected revenue is maximized in this feasible region. The optimal $v(\boldsymbol{d})$ values obtained from *PMP-RM-1* are used as parameters in *PMP-RM-2*.

It must be noted here that there may exist an optimal allocation $\boldsymbol{x}^*$ for *PMP-RM-1* such that $x_j^*(i') = 0$ and $x_j^*(i'') = 1$ for some j although $i'' > i'$. This situation contradicts with Remark 2.2.2.1. However, Terciyanlı (2009) proves that there exists at least one optimal allocation for *PMP-RM-1* model and that is also a proper allocation in terms of the definition of $x_j(i)$.

Terciyanlı (2009) proposes an approximation for *PMP-RM* models since they contain high number of binary variables and therefore they are difficult to solve. Three approximations are used by Terciyanlı (2009). One of them is to use a given number of demand realizations instead of considering all $\boldsymbol{d}$. A set Ψ is defined as the set of sample demand realizations. $|\Psi|$ is the total number of demand realizations. Then, the objective function in *PMP-RM-1* becomes $\sum_{d \in \Psi} \frac{v(\boldsymbol{d})}{|\Psi|}$. The other approximation used is to relax the integrality constraints on $x_j(i)$ and $v(\boldsymbol{d})$ both in *PMP-RM-1* and *PMP-RM-2*. The last approximation is rounding positive $v(\boldsymbol{d})$ values that are less than 1 to 1 when *PMP-RM-1* is solved without integrality constraints. Terciyanlı (2009) considers the second and third approximations together and call the approximate model obtained as *PLP-RM*, which stands for *Probabilistic Linear Programming with Risk Measure*.

In order to decrease the computational time in solving the approximate *PLP-RM* models, the models are reformulated with aggregate demands as in *SLP* due to de Boer (1999). The resulting models are called *SLP-RM* which stands for *Stochastic Linear Programming with Risk Measure*.

Besides the lexicographic approach, Terciyanlı (2009) proposes another model *PMP-RC*, which stands for *Probabilistic Mathematical Programming with Risk Constraint*. The same risk measure, the probability that revenue is less than a threshold level L, is used in the constraint while the expected revenue is maximized. The same assumptions and approximations as the ones in the lexicographic approach are used in this model. The model is given below.

$$PMP-RC \quad : \quad \text{Maximize} \sum_{j=1}^{n} f_j x_j - \sum_{j=1}^{n} f_j \sum_{k=1}^{\kappa_j} P(D_j \leq d_j(k)) x_j(k) \tag{2.46}$$

subject to

$$\sum_{j \in S_l} \sum_{k=1}^{\kappa_j} x_j(k) \leq C_l \quad \text{for } l = 1, ..., m, \tag{2.47}$$

$$x_j = \sum_{k=1}^{\kappa_j} x_j(k) \quad \text{for } j = 1, ..., n, \tag{2.48}$$

$$x_j(1) \leq d_j(1), \tag{2.49}$$

$$x_j(k) \leq d_j(k) - d_j(k-1) \quad \text{for } k = 2, ..., \kappa_j, \tag{2.50}$$

$$\sum_{d \in \psi} \frac{v(\boldsymbol{d})}{|\psi|} < \rho, \tag{2.51}$$

$$Mv(\boldsymbol{d}) \geq - \sum_{(i,j) \ni d_j \geq i} f_j x_j(i) + L \quad \text{for } \boldsymbol{d} \in \Psi, \tag{2.52}$$

$$M(1 - v(\boldsymbol{d})) \geq \sum_{(i,j) \ni d_j \geq i} f_j x_j(i) - L \quad \text{for } \boldsymbol{d} \in \Psi, \tag{2.53}$$

$$x_j(k) \geq 0 \quad \text{for } j = 1, ..., n, \text{ and } k = 1, ..., \kappa_j, \tag{2.54}$$

$$x_j \geq 0 \quad \text{for } j = 1, ..., n, \tag{2.55}$$

$$v(\boldsymbol{d}) \in \{0, 1\} \quad \text{for } \boldsymbol{d} \in \Psi. \tag{2.56}$$

ρ is a predetermined constant number between 0 and 1. Terciyanlı (2009) argues that *PMP-RC* model is disadvantageous when compared to the *SLP-RM* models. The following observations are due to Terciyanlı (2009). For a detailed discussion, the reader is referred to Terciyanlı and Avşar (2011).

- Since *PMP-RC* is an integer programming formulation, it is harder to solve and requires more computational time when compared to the *SLP-RM* models.
- In the *SLP-RM* models, there is only one parameter, L, to be determined. However, in *PMP-RC* both L and ρ should be determined.
- Dual prices, which are used as the bid prices, cannot be obtained via *PMP-RC* due its integrality characteristics.

Terciyanlı (2009) also proposes a *Randomized Risk Sensitive Procedure*, whose abbreviation is *RRS Procedure*. The procedure is proposed not only for risk sensitive but also for risk taking decision makers. The procedure finds two bid prices for risk sensitive and risk taking decision makers. The level of risk sensitivity can be changed by adjusting the threshold level L. This study reveals the fact that risk sensitive decision makers tend to use lower bid prices as compared to risk taking decision makers. The results of the models proposed by Terciyanlı (2009) are compared to our models proposed in this thesis in Chapter 6.

2.4 Control Policies for Network Revenue Management Problems

In Section 2.1, 2.2 and 2.3, the studies on the revenue management are reviewed and the mathematical models for network seat inventory control problems are presented. The common aim of all those mathematical models presented is to obtain the optimal seat allocations, which are also called booking limits and/or optimal bid prices. This is called the optimization step of RM. It is expected that the decision maker implements the optimal seat allocation solution and bid prices in a framework to make acceptance-rejection decisions for an arriving request of a product (*ODF*). This step is called the control step. The aforementioned framework is drawn by the control policies that are studied in this section. These control policies are namely partitioned booking limit control, nested booking limit control and bid price control. As an alternative to the booking limits, one may work with the protection levels.

2.4.1 Partitioned Control

This control policy is the simplest and the most straightforward control policy since it directly uses the optimal seat allocations that are obtained from the mathematical models. As mentioned in the previous sections, the mathematical models do not take the nested environment into account. Therefore, they automatically provide optimal solutions for a partitioned environment. In partitioned booking limit control, the booking limits are used only for the corresponding *ODF*. The unsold capacity of an *ODF* cannot be sold to other *ODF*s even if they have higher fares. This is the major drawback of the partitioned control policy. The revenue gained via partitioned control policy is generally lower than the revenue obtained by nested and bid price control polices. Moreover, the load factor values obtained under the partitioned control is generally less than the load factor values under nested or bid price control policies. As Terciyanlı (2009) states in his study, one can overcome these drawbacks by updating the booking limits frequently. However, to update the booking limits one should re-optimize the system and re-forecast the future demand. Therefore, the implementation of this control policy is impractical for airline industry and it is rarely used despite its simplicity of implementation.

2.4.2 Nested Control

Recall that the major drawback of the partitioned control policy is not to allow selling the unused capacity of an *ODF* to other *ODF*s, even if they have higher fares. Nested booking limit control overcomes this drawback by ranking the fare classes. The main philosophy of this ranking is to sell the unsold capacity of an *ODF* with a low ranked fare to other *ODF*s with higher ranked fares. Therefore, the allocated seats of an *ODF* with the lowest ranked fare is open to sale for all other higher ranked fare classes. The booking limit of the *ODF* with the highest ranked fare class is equal to the capacity of the aircraft.

Although it sounds simple to implement, it is a handicap to determine the rule of ranking and this is the major difficulty encountered under this control policy. Williamson (1992) proposes three ranking methods: nesting by fare classes, nesting by fares and nesting by shadow prices. In the first, the ranking of the fare classes is done according the class types. A full fare class is ranked higher than a low fare class for an itinerary. That is, the allocated seats of a low fare class *ODF* can be sold to an *ODF* with full fare class. This ranking does not take the following fact into consideration: the revenue to be gained from a low fare class passenger with a long path can be higher than the revenue to be gained from a full fare class passenger with a short path. Therefore, the revenue to be gained from a low fare class passenger is sacrificed in order to gain probably less revenue from a full fare class passenger. Due to this drawback, this ranking generally gives poor results and is not preferred in the airline industry.

In the second ranking method, fare classes are ranked according to their fares. This method was first proposed by Boeing Commercial Airplane Company. The itinerary which has the highest fare is ranked in the first position. In contrast to the first method, the long path itineraries are ranked higher than the short path itineraries. However, this does not mean that the long path itineraries contribute more to the revenue. The fact that the long path itineraries may contribute less to the revenue is not taken into account and it is the major drawback of this method since the low yield itineraries with long paths can have access to an important amount of seats although they have small number of seat allocations in the optimization models. Therefore, this method also gives poor results for RM problems.

In the third method, ranking is performed according to the shadow prices. Recall from Chapter 1 that shadow price is an increment in the revenue in case one more seat is allocated to a particular *ODF* when all other allocations remain unchanged. The assumption under this method is that the itineraries with high shadow prices contribute to the revenue more than the itineraries with low shadow prices. Therefore, the itineraries with high shadow prices are ranked higher than the itineraries with low shadow prices. The determination of shadow prices for deterministic models such as *DLP* is quite simple. The dual prices of the demand constraints can be used as the shadow prices since there is a demand constraint associated with each *ODF*. However, it is not the case for probabilistic models due to the lack of demand constraints associated with each *ODF*. de Boer et al. (2002) propose that an estimate can be calculated for the shadow price of an *ODF* by using the dual prices of the capacity constraints in the probabilistic models. The sum of dual prices of the legs that are on the itinerary, which is used by an *ODF*, is accepted as the opportunity cost of that *ODF*. The opportunity cost obtained is then subtracted from the fare of the *ODF* and the net contribution of the *ODF* to the revenue is found.

The ranking methods proposed by Williamson (1992) are hard to implement. Therefore, heuristics are developed. de Boer et al. (2002) propose a heuristic method for nesting, which is also used in this thesis. The heuristic method works as follows.

- First of all, the capacities and number of booking requests that have been accepted so far are set to the initial values.

- A booking request arrives and is taken into consideration for acceptance.

- The number of available seats for sale are calculated for each leg on the itinerary that the *ODF j* uses. These seats are protected for fare classes that are ranked higher than the current *ODF j*.

- The number of all available seats are found considering all legs on *ODF j* by summing up the individual available seats on each leg. This seat number obtained is compared to the remaining capacity of the flight. If the result of the comparison is higher than zero, then the booking request is accepted and capacity is decreased for the legs of *ODF j* by one. Finally, the number of booking requests that are accepted is increased by one.

2.4.3 Bid Price Control

As an alternative to partitioned and nested control policies, Simpson (1989) and Williamson (1992) propose bid price control policy for RM problems. The main idea of this control policy is to set a threshold level and compare it to the fare of the demand request. If the fare of the demand request is higher than the threshold level, then the request is accepted. Otherwise, it is rejected and the fare class for this *ODF* is called closed. The threshold level is calculated by summing up the dual prices of the capacity constraints of the legs that the *ODF* uses. This threshold level definition is the same as the definition of opportunity cost given by de Boer et al. (2002). Optimality of such threshold levels results from the analysis of the dynamic programming formulations in RM for investigating the structure of the optimal policy.

Bid price control is easy to implement since there are only two necessary information to be kept: remaining capacity and class status. However, there is also a drawback of the bid price control since there is no limit for accepting the demand requests as long as its fare exceeds the bid price. Therefore, most of the capacity can be allocated to a fare class, which has small contribution to the revenue. Williamson (1992) shows that the results of partitioned booking limit control policy and bid price control policy are quite close when the bid prices are updated frequently. However, frequent update is generally impractical for airline industry since it is time consuming.

CHAPTER 3

THE PROPOSED RISK-SENSITIVE APPROACH

As mentioned in Chapter 2, there are only a few studies in the airline industry that take the risk factor into the account. Most of the studies aim at finding the optimal seat allocations while maximizing the expected revenue and do not take the variability of the revenue and hence a risk measure into account. Isolating the risk factor from the framework of the problem results in ignoring the decision maker behavior. Without considering the behavior of the decision maker in this respect, the decision makers are treated as if they all have the same risk reaction pattern and as if they are risk neutral. However, a decision maker may be risk-averse or risk-taking. As a result, the solution of the problem diverge from the reality.

Recall that there are some studies available on risk aversion in the literature. Barz and Waldmann (2007) propose a Markov Decision Process for static and dynamic single leg revenue management problem by using the exponential utility functions. Levin et al. (2008) study on optimal dynamic pricing of perishable goods and products, but their field of study is not the airline industry. Different than those studies, this thesis focuses on network structure rather than single leg applications and aims to determine the optimal seat allocations without manipulating the prices of the products.

The study in this thesis is inspired by two studies on risk aversion for airline network revenue management problems. One of these studies is due to Çetiner (2007) which is the first study on seat allocation problems in the airline industry that takes the variability of the revenue into account. The other study is due to Terciyanlı (2009) which restricts the probability that the revenue is less than a predetermined threshold level for airline network revenue management problems. The relation between the studies due to Çetiner (2007), Terciyanlı (2009) and the study in this thesis is explained in the subsequent paragraphs.

As mentioned in Chapter 1, expected revenue and capacity utilization are two conflicting criteria, which are aimed to be maximized simultaneously. However, there exists no such study in the literature that handles both objectives together. So, it is a trade-off whether to maximize the expected revenue or the capacity utilization of the aircraft. This thesis deals with multi criteria decision making in order to resolve this dilemma by handling expected revenue and expected load factor together. That is, this thesis contributes to the multi criteria decision making literature while taking the risk aversion into consideration.

In this chapter, two types of models are proposed to find the optimal seat allocations by incorporating the load factor into the risk neural models that maximize the expected revenue. Although the load factor is an important performance measure considered to compare alternative approaches, there is no existing study in the literature that direfctly uses the load factor formulations in the mathematical models. The study in this thesis is inspired from the following observations due to Terciyanlı (2009). Terciyanlı (2009) reveals in his study that the variability of the revenue decreases as load factor increases. That is, a negative correlation between the variability of revenue and the load factor is observed in his study. Therefore, Terciyanlı (2009) proposes the usage of load factor in the mathematical models in order to manage the risks. The usage of load factor in the formulations provides an important advantage for the decision makers since it is easier to formulate and solve as compared to the models due to Çetiner (2007) and Terciyanlı (2009). No approximation method is needed to solve the models we propose. We only use LP relaxation to overcome the integrality. Moreover, linearity is maintained by load factor formulations. Two types of models are proposed based on this premise. One of the models we propose aims at maximizing the expected revenue while working with service level constraints on the expected load factors of the network legs or a single service level constraint on a weighted average of the expected load factors of the legs. In this model, the service level is a predetermined threshold level. The other model we propose aims at maximizing a weighted average of the expected load factors of the network legs or the minimum expected load factor of the legs in the network while ensuring that the expected revenue is always above a predetermined threshold level.

These two types of models are considered for risk averse decision makers by investigating the relation between load factor and variability of the revenue. The impact of a change in the load factor on standard deviation and coefficient of variation of the revenue is investigated. Recall that the expected revenue and the expected load factor are two criteria in the proposed models. In these two types of models we propose, one criterion is aimed to be maximized while the other criterion is forced to stay above a threshold level. The proposed models are found promising because they yield satisfactory results in terms of variability of the revenue and are much easier to formulate and solve in terms of computational burden than the other risk sensitive models due to Çetiner (2007) and Terciyanlı (2009).

The load factor is formulated in Section 3.1. Section 3.2 is devoted to the proposed models.

3.1 Load Factor

In the airline terminology, load factor is defined as the ratio of seats sold on a flight to the total number of seats available. In this section, the load factor and its expected value are formulated.

Let the random variable $LF_l(x_1, ..., x_n)$ denote the load factor for leg l when the given seat allocation is $(x_1, ..., x_n)$. Here, we consider two formulations for load factor; $LF_{l(1)}$ and $LF_{l(2)}$ are expressed in terms of the random variables $Z_j(i)$ and Y_j, respectively. In Lemma 3.1, it is shown that the two formulations are equivalent.

Formulation 1.

- The load formulation $LF_{l(1)}$ in terms of Z_j is

  $$LF_{l(1)}(x_1, ..., x_n) = \frac{1}{C_l} \sum_{j \in S_l} Z_j, \tag{3.1}$$

  where $Z_j = \min\{D_j, x_j\}$. Recall that D_j is the demand for $ODF\ j$ and x_j is the number of seats allocated for $ODF\ j$ as in the existing models in the literature.

That is, Z_j denotes the number of tickets sold for $ODF\ j$. As a result, the ratio of the sum of Z_j over all js in S_l and C_l gives the load factor for leg l.

- For the load factor formulation $LF_{l(2)}$, we define the random variable $Y_j(i)$ as seen below.

$$Y_j(i) = \begin{cases} 1 & \text{if } D_j \geq i, \\ 0 & \text{otherwise.} \end{cases} \tag{3.2}$$

If the demand for $ODF\ j$ is greater than i and the i^{th} seat is allocated to $ODF\ j$, then the gain in load factor for that seat is equal to $1/C_l$. Otherwise, the gain is 0. Then, the total load factor for leg l can be obtained for the network by summing up the marginal load factors, $\frac{Y_j(i)}{C_l}$, of the seats.

That is,

$$LF_{l(2)}(\boldsymbol{x}) = \frac{1}{C_l} \sum_{j \in S_l} \sum_{i=1}^{B_j} Y_j(i) x_j(i), \tag{3.3}$$

where $\mathbf{x}$ is a given seat allocation such that $\mathbf{x} = (\boldsymbol{x}_1, ..., \boldsymbol{x}_n), \boldsymbol{x}_j = (x_j(1), ..., x_j(B_j))$ and $x_j(i)$ is defined in Chapter 2 as follows:

$$x_j(i) = \begin{cases} 1 & \text{if if } i \text{ or more seats are allocated to } ODF\ j, \\ 0 & \text{otherwise.} \end{cases} \tag{3.4}$$

□

Note that $LF_{l(1)}(x_1, ..., x_n)$ and $LF_{l(2)}(\mathbf{x})$ are two alternative formulations for the load factor. In Lemma 3.1, it is shown that $LF_{l(1)}$ is equal to $LF_{l(2)}$ for a given allocation.

Lemma 3.1: $LF_{l(1)}(x_1, ..., x_n) = LF_{l(2)}(\mathbf{x})$ for a given allocation $\mathbf{x}$ such that $|\mathbf{x_j}| = x_j$ for $j = 1, ..., n$.

Proof. By using the definition of $Y_j(i)$,

$$LF_{l(2)}(\mathbf{x}) = \frac{1}{C_l} \sum_{j \in S_l} (\sum_{i=1}^{D_j} 1 \cdot x_j(i) + \sum_{i=D_j+1}^{B_j} 0 \cdot x_j(i)) = \frac{1}{C_l} \sum_{j \in S_l} \sum_{i=1}^{D_j} x_j(i). \tag{3.5}$$

Recall that $\boldsymbol{x}_j = \sum_{i=1}^{B_j} x_j(i)$. Also, by definition, $x_j(i+1)$ cannot be 1 unless $x_j(i)$ is equal to 1. Then, $\sum_{i=1}^{D_j} x_j(i)$ in (3.5) can be rewritten as follows:

$$\sum_{i=1}^{D_j} x_j(i) = \begin{cases} D_j & \text{if } D_j \leq x_j, \\ x_j & \text{if } D_j > x_j. \end{cases} \tag{3.6}$$

That is, $\sum_{i=1}^{D_j} x_j(i) = \min\{D_j, x_j\}$ which is defined as Z_j and used for $LF_{l(1)}(x_1, ..., x_n)$. ■

Next, the expected load factor is formulated. Note that LF_l is a random variable that is formulated as a function of either Z_j or Y_j.

Formulation 2.

- Using the load factor formulation in (3.1), the expected load factor is given as follows:

$$\begin{aligned} E(LF_l(x_1, ..., x_n)) &= E(\frac{1}{C_l}\sum_{j\in S_l} Z_j) \\ &= \frac{1}{C_l}\sum_{j\in S_l} E(Z_j) \\ &= \frac{1}{C_l}\sum_{j\in S_l} E(\min(D_j, x_j)). \qquad (3.7) \end{aligned}$$

  This expected value function is a non-linear function of x_js. That is, when this formulation is used in a mathematical programming model to find seat allocations, x_js, one is to work with this non-linear function. On the other hand, the expected value obtained by working with the load factor formulation in (3.3) does not have this drawback as seen below.

- Using the load factor formulization in (3.3), the expected load factor is given as follows:

$$\begin{aligned} E(LF_l(\mathbf{x})) &= E(\frac{1}{C_l}\sum_{j\in S_l}\sum_{i=1}^{B_j} Y_j(i)x_j(i)) \\ &= \frac{1}{C_l}\sum_{j\in S_l}\sum_{i=1}^{B_j} E(Y_j(i))x_j(i) \\ &= \frac{1}{C_l}\sum_{j\in S_l}\sum_{i=1}^{B_j}[1\cdot Pr(D_j \geq i) + 0\cdot Pr(D_j < i)]x_j(i) \\ &= \frac{1}{C_l}\sum_{j\in S_l}\sum_{i=1}^{B_j} Pr(D_j \geq i)x_j(i). \qquad (3.8) \end{aligned}$$

□

Based on the equivalence result in Lemma 3.1, we drop superscripts (1) and (2) in the load factor notation to denote load factor of leg l for a given allocation. From now on, LF_l will be used. We use the expected load factor formulation in (3.8) in the proposed models given in Section 3.2.

3.2 Proposed Models

For the decision makers in the airline industry, there is sometimes a trade-off between reserving the tickets for the high income customers who generally arrive later in the booking horizon with the expectation of obtaining high income and selling the tickets more easily to the low fare customers who arrive earlier in the booking horizon in order to avoid having a low load factor. At first glance, one can consider two obvious strategies. The first one is to allocate more seats for high fare classes because they are more profitable. In this strategy, the revenue is aimed to be maximized by taking the risk of low load factor since most of the requests of lower fare class customers are rejected. However, the number of high fare class customers may not be enough to achieve a high total revenue level. In the second strategy, the decision maker accepts the requests for low fare class customers and reserves fewer tickets for future high fare class customers. In this strategy, the decision maker aims at maximizing the load factor and takes the risk of lower total revenue since most of the requests of lower fare class customers are accepted and fewer tickets are reserved for higher fare class customers. However, the high fare class customers may be lost by selling more tickets to low fare class customers and the number of low fare class customers may not be enough to achieve a high load factor level. This discussion reveals the relation between total revenue and load factor.

The probabilistic nature of the demand determines the best strategy to be used. As de Boer et al. (2002) mention, the revenue loss resulting from the decreased load factor of the flights could be larger than the increase in the revenue obtained from higher fare class customers. From this perspective, it can be seen that the effort of using the flight capacity as much as possible and increasing the revenue can be handled together. With that purpose, we propose two models and their variations: *RLF* and *LFR*. The acronym of *RLF* stands for *Revenue under Load Factor Constraint* and the acronym of *LFR* is used for the *Load Factor under Revenue Constraint*.

RLF aims at maximizing the expected revenue under the following constraint: the expected load factor of each leg is required to be greater than a predetermined threshold level. This model is called a service model where the service measure is the expected load factor and the service level is the threshold. A variation of *RLF* is *RLF-M*, which stands for *Load Factor Constraint-Modified*. *RLF-M* also aims at maximizing the expected revenue under the following constraint: a weighted average of the expected load factors of the network legs is required to be greater than a predetermined threshold level. *LFR* maximizes the weighted average of the expected load factors of the network legs while ensuring that the expected revenue is always above a predetermined threshold level. A variation of *LFR* is *MaxminLF*, which stands for *Maximize the Minimum Load Factor*. *MaxminLF* aims at maximizing the expected load factor of the leg which has the smallest expected load factor value. *RLF* and its variations are proposed as an alternative to *LFR* and its variations. The objectives and the distinctive constraints of the proposed models are summarized in Table 3.1.

Table 3.1: Proposed Models

Proposed Model	***Objective Function***	***Distinctive Constraint***
RLF	Max. $E(R)$	$E(LF_l) \geq SL_l$
RLF-M	Max. $E(R)$	weighted avg. $E(LF) \geq SL$
LFR	Max. weighted avg. $E(LF)$	$E(R) \geq RL$
MaxminLF	Max. min. $E(LF_l)$	$E(R) \geq RL$

With the proposed models, we expect to increase the revenue while not compromising the capacity utilization. The variance of the revenue is also expected to be kept at acceptable levels so that the risk-averse inclination of the decision maker is supported. In other words, the decreased variability is expected to be implied as a result of resolving the trade off between expected revenue and expected load factors of the legs. The performance of the models is evaluated under different booking control policies and a comparison with the models in the literature is given in Chapter 6.

The numerical studies in this thesis consist of two stages. In the first stage, we consider the optimization by solving the proposed mathematical models to obtain the optimal seat allocations and bid prices. The method of obtaining the bid prices of *RLF* and *RLF-M* is given in Section 3.2.2.

In the second stage of the numerical studies, we use the optimal seat allocations and bid prices obtained by solving the optimization models in a simulation model for implementation of different control polices; namely, partitioned booking limit control policy, nested booking limit control policy and bid price control policy.

The bound models to determine the range of the allowable threshold levels for the proposed models; namely, the service level (*SL*) for *RLF* and its variations and the revenue level (*RL*) for *LFR* and *MaxminLF* are proposed in Chapter 4.

The numerical threshold levels for the proposed models, namely; the service level (*SL*) for *RLF* and its variations and the revenue level (*RL*) for *LFR* and its variations are determined in Chapter 5 for a sample network. Moreover, the numerical analysis and the interpretations of the optimization results of the proposed models are presented.

The results of the simulation studies of the proposed models are reported in Chapter 6.

3.2.1 The Models with Constraints on Expected Load Factors

RLF is obtained by incorporating an additional set of constraints to the *EMR* model presented in Chapter 2. These additional constraints are for the expected load factors of the legs; the expected load factor of each leg l is required to be greater than a predetermined service level, SL_l. Depending on the preference of the decision maker or the structure of the airline network, different service levels can be used for the legs. Differentiating the service level requirements of the legs is more realistic when the real life situation is taken into account. One may appreciate that some legs are demanded more and it is easier for them to satisfy higher service levels when compared with the ones that have fewer customer intensity. However, to be able to differentiate the service level requirements of legs on a reasonable ground, more data about the leg characteristics are needed. The decision maker may identify such characteristics and set ranges for service levels accordingly. The following characteristics may be used in the determination of sets.

- Dominating customer profile of the leg (business or leisure traveler).
- Seasonality factors (for instance touristic destinations may attract more customers on specific seasons).
- Characteristics of the demand data.
- Competition in the market (for some origin-destination pairs, there may be harsh competition in the market so that it is not easy to satisfy a high service level for that leg).

The differentiation of the service level requirements of the legs by taking the above mentioned characteristics into account is not within the scope of this thesis and so it remains as a future work.

In our numerical analysis in Chapter 5, the case where the service levels of the legs are the same is investigated. The case where the service levels of the legs are different is also investigated and the impact of service level differentiation is analyzed via numerical outputs in Chapter 5.

$$RLF \quad : \quad \text{Maximize} \sum_{j=1}^{n} \sum_{i=1}^{B_j} f_j P(D_j \geq i) x_j(i) \tag{3.9}$$

subject to

$$\sum_{j \in S_l} \sum_{i=1}^{B_j} x_j(i) \leq C_l \quad \text{for } l = 1, ..., m, \tag{3.10}$$

$$\frac{1}{C_l} \sum_{j \in S_l} \sum_{i=1}^{B_j} P(D_j \geq i) x_j(i) \geq SL_l \quad \text{for } l = 1, ..., m, \tag{3.11}$$

$$x_j(i) \in \{0, 1\} \quad \text{for } j = 1, ..., n, \text{ and } i = 1, ..., B_j. \tag{3.12}$$

The only difference between *RLF* and *EMR* is constraint (3.11) in *RLF*. Due to the binary the decision variable $x_j(i)$, *RLF* is hard to solve. Therefore, LP relaxation of the model is considered by replacing (3.12) with $0 \leq x_j(i) \leq 1$ for $j = 1, ..., n$, and $i = 1, ..., B_j$. LP relaxation of *RLF* is used for numerical analysis throughout the rest of the thesis whenever the output of *RLF* model is required.

For *RLF*, it is worth to check the impact of the separate leg control on the network. Recall that, in *RLF*, there is an expected load factor constraint on each leg. What if the expected load factor of the whole network is required to be greater than or equal to a single, predetermined service level? Towards that end, the following modified version of *RLF* can be considered: the load factor measure considered for the whole network is a weighted average of the expected load factors of the legs.

$$RLF - M \quad : \quad \text{Maximize} \sum_{j=1}^{n} \sum_{i=1}^{B_j} f_j P(D_j \geq i) x_j(i) \tag{3.13}$$

subject to

$$\sum_{j \in S_l} \sum_{i=1}^{B_j} x_j(i) \leq C_l \quad \text{for } l = 1, ..., m, \tag{3.14}$$

$$\sum_{l=1}^{m} \frac{w_l}{C_l} \sum_{j \in S_l} \sum_{i=1}^{B_j} P(D_j \geq i) x_j(i) \geq SL, \tag{3.15}$$

$$x_j(i) \in \{0, 1\} \quad \text{for } j = 1, ..., n, \text{ and } i = 1, ..., B_j. \tag{3.16}$$

The parameter, w_l, stands for the weight of each leg. That is, $0 \leq w_l \leq 1$ and $\sum_{l=1}^{m} w_l = 1$. The weights for the legs should be determined according to the network characteristics. However, the determination of the weights may be difficult and cumbersome. The scope of this thesis excludes the determination of the weights and leaves it as a future work.

The weighted average of the expected load factors of the legs is used as the load factor measure in *RLF-M*. Different load factor measures can also be considered for the whole network, which remains as a future work.

As in the case of *RLF*, its modified version is also hard to solve due to the integrality constraints on the decision variables. Therefore, the relaxed version of *RLF-M* is solved for the numerical analysis in Chapter 6.

Remark 3.2.1. *RLF-M* with *SL* is a relaxation of *RLF* with SL_l=SL for all l. (3.11) in *RLF* with SL_l=SL can be written as $\frac{w_l}{C_l} \sum_{j \in S_l} \sum_{i=1}^{B_j} P(D_j \geq i) x_j(i) \geq w_l SL_l$. By summing up these inequalities over all l, we obtain (3.15) in *RLF-M*.

□

The following remark is given to show why it is ensured that there exist solutions obtained by the proposed *RLF* and *RLF-M* models and their relaxations that are in accordance with the definition of the decision variable $x_j(i)$ in (3.4).

Remark 3.2.2. For each of *RLF*, *RLF-M* and their relaxations, there exists an optimal solution $\mathbf{x}$ such that $x_j(i+1)$ takes a positive value only if $x_j(i)$=1 for all i and j.

This observation results from the behaviour of the objective function in the aforementioned models: $f_j P(D_j \geq i)$ decreases in i as for every j (recall Remark 2.2.2.1 given for *EMR* with the same objective function).

□

3.2.2 Determination of Bid Prices for *RLF* and *RLF-M*

Recall from Chapter 1 and Chapter 2 that bid price is introduced as the threshold level to make the acceptance/rejection decision for a ticket request. It is the expected value of an incremental seat. In other words, it is the opportunity cost of accepting the ticket request. Bid price is compared with the fare of the ticket requested, e.g., f_j, and the acceptance/rejection decision is made accordingly.

Simpson (1989) and Williamson (1992) propose to sum up the dual prices of the capacity constraints of the legs used by product j in order to obtain the bid price of product j. For *RLF* and *RLF-M* due to the existence of the service level constraint(s), it is not possible to work with only the dual prices of the capacity constraints. In this case, the duals of the capacity constraints are adjusted (corrected) by considering also the impact of the service level on the expected revenue in the objective function as explained below in Remark 3.2.3. Note that the service level constraint is also a function of the capacities, C_l, of the legs. The impact of a unit change of C_l on the optimal expected revenue is determined not only by the dual prices of capacity constraints but also the dual prices of the service level constraint(s).

Remark 3.2.3.

- Consider the LP relaxation of *RLF*. Rewrite the service level constraint in (3.11) as follows: $\frac{1}{SL_l}\sum_{j\in S_l}\sum_{i=1}^{B_j} P(D_j \geq i)x_j(i) \geq C_l$. Let y_l and v_l be the dual variables of the capacity and service level constraints, respectively. Let $u_j(i)$ be the dual variable of the constraint $x_j(i) \leq 1$.

  Then, the objective function of the dual is

  Minimize $\sum_{l=1}^{m} C_l(y_l + v_l) + \sum_{j=1}^{n}\sum_{i=1}^{B_j} u_j(i)$.

  Consider the objective function of the dual problem for the LP relaxation of *EMR*. The only difference between this objective and the one in given above is the additional term $\sum_{l=1}^{m} C_l v_l$ in the objective function above due to the service level constraints. That is, the change in the optimal expected revenue that results from a unit increase in C_l is given by $y_l + v_l$ in the case of *RLF* unlike only y_l in the case of *EMR*. Note that $y_l \geq 0$ and $v_l \leq 0$. That is, v_l is a correction term for including the service level constraint of leg l in the proposed *RLF* model.

- An alternative way to calculating the bid prices of *RLF* is as follows. Consider the LP relaxation of *RLF* by keeping the service level constraint as in (3.11). Define y_j and $u_j(i)$ as in the previous item above. Let $\bar{v}_l$ be the dual variable of the service level constraint for leg l. Then, the objective function of the dual problem is

  Minimize $\sum_{l=1}^{m}(C_l y_l + SL_l\bar{v}_l) + \sum_{j=1}^{n}\sum_{i=1}^{B_j} u_j(i)$.

  Rearranging the $\sum_{l=1}^{m} SL_l\bar{v}_l$, we obtain

  Minimize $\sum_{l=1}^{m} C_l(y_l + \frac{SL_l\bar{v}_l}{C_l}) + \sum_{j=1}^{n}\sum_{i=1}^{B_j} u_j(i)$,

  where the correction term for the dual price y_l is $\frac{SL_l\bar{v}_l}{C_l}$. That is, the bid price of *RLF* is $y_l + \frac{SL_l\bar{v}_l}{C_l}$.

  Note that $y_l \geq 0$ and $v_l \leq 0$.

- The bid prices of *RLF-M* are given proceeding as in the second item above. Consider the LP relaxation of *RLF-M*. Let y_l and $\bar{v}$ be the dual variables of the capacity constraints and the service level constraints, respectively. Let $u_j(i)$ be the dual variable of the constraint $x_j(i) \leq 1$. Then, the objective function of the dual problem is

Minimize $\sum_{l=1}^{m}(C_l y_l) + SL\bar{v} + \sum_{j=1}^{n}\sum_{i=1}^{B_j} u_j(i)$.

Rewriting the second term as

$SL\bar{v} = SL\bar{v}\frac{\sum_{l=1}^{m}\frac{C_l}{C_l}}{m}$

$= \sum_{l=1}^{m} C_l(\frac{SL\bar{v}}{mC_l})$,

the correction term for the dual price y_l is obtained as $\frac{SL\bar{v}}{mC_l}$ for leg l. That is, the bid price of *RLF-M* is $y_l + \frac{SL\bar{v}}{mC_l}$.

Note that $y_l \geq 0$ and $v_l \leq 0$.

□

3.2.3 The Models with a Constraint on Expected Revenue

LFR is also an *EMR*-based model, where a weighted average of the expected load factors of the network legs is maximized while ensuring that the expected revenue is always above a predetermined threshold level, *RL*.

$$LFR \;:\; \text{Maximize} \sum_{l=1}^{m}\frac{w_l}{C_l}\sum_{j\in S_l}\sum_{i=1}^{B_j} P(D_j \geq i)x_j(i) \tag{3.17}$$

subject to

$$\sum_{j\in S_l}\sum_{i=1}^{B_j} x_j(i) \leq C_l \quad \text{for } l = 1, ..., m, \tag{3.18}$$

$$\sum_{j=1}^{n}\sum_{i=1}^{B_j} f_j P(D_j \geq i)x_j(i) \geq RL, \tag{3.19}$$

$$x_j(i) \in \{0, 1\} \quad \text{for } j = 1, ..., n, \text{ and } i = 1, ..., B_j. \tag{3.20}$$

As in the case of *RLF*, the *LFR* model also contains integrality constraints that need to be relaxed for numerical analysis. The LP relaxation of *LFR* is used for numerical analysis throughout the rest of the thesis whenever the output of *LFR* model is required.

In *LFR*, a weighted average of the expected load factors of the network legs is used in the objective function. However, different load factor measures can also be used in the objective. For example, we propose the model *MaxminLF*. As an alternative to the weighted average of the expected load factors of the network legs, the expected load factor of the leg which has the smallest expected load factor value is maximized in *MaxminLF*. This is a max(min) type of problem. The objective of *MaxminLF* is formulated as

$$\max_{\mathbf{x}} \ \min_{l}\left(\frac{1}{C_l}\sum_{j\in S_l}\sum_{i=1}^{B_j} P(D_j \geq i)x_j(i)\right). \tag{3.21}$$

The *MaxminLF* model is given below.

$$MaxminLF \quad : \quad \text{Maximize } z \tag{3.22}$$

subject to

$$\frac{1}{C_l}\sum_{j\in S_l}\sum_{i=1}^{B_j} P(D_j \geq i)x_j(i) \geq z \quad \text{for } l = 1, ..., m, \tag{3.23}$$

$$\sum_{j\in S_l}\sum_{i=1}^{B_j} x_j(i) \leq C_l \quad \text{for } l = 1, ..., m, \tag{3.24}$$

$$\sum_{j=1}^{n}\sum_{i=1}^{B_j} f_j P(D_j \geq i)x_j(i) \geq RL, \tag{3.25}$$

$$x_j(i) \in \{0, 1\} \quad \text{for } j = 1, ..., n, \text{ and } i = 1, ..., B_j, \tag{3.26}$$

$$z \geq 0. \tag{3.27}$$

In this model, the expected load factor of each leg is forced to be greater than or equal to the decision variable z and then z is maximized. Different from *LFR*, it is not necessary to determine a weight for each leg, which is an advantage of *MaxminLF*.

As in the case of *LFR*, its modified version is also hard to solve due to the integrality constraints. Therefore, the relaxed version of *MaxminLF* is solved for numerical analysis presented in Chapter 6.

The following remark is analogous to Remark 3.2.2; it is ensured that there exist solutions obtained by the proposed *LFR* and *MaxminLF* models and their relaxations that are in accordance with the definition of $x_j(i)$ in (3.4). Note that the remark gives a sketch of the proof.

Remark 3.2.4. For each of *LFR*, *MaxminLF* and their LP relaxations, there exists an optimal solution $\mathbf{x}$ such that $x_j(i + 1)$ takes a positive value only if $x_j(i)$=1 for all i and j. a) For *LFR*, this observation results from the rearrangement of the objective function: $P(D_j \geq i)\sum_{l\in T_j}\frac{w_l}{C_l}$ decreases in i for every j. b) For *MaxminLF*, the result follows from the analysis of $E(LF_l)$ for a $\max_{\boldsymbol{x}} \min_l$ problem: $P(D_j \geq i)$ in $E(LF_l)$ decreases in i for all $j \in S_l$ and for all l.

CHAPTER 4

DETERMINATION OF THE THRESHOLD BOUNDS

Recall that the service level of each leg in *RLF*, the network service level in *RLF-M*, the revenue level in *LFR* and *MaxminLF* are the threshold levels considered in this thesis. The bound models for determining the ranges of numerical threshold values for a given network are presented in this chapter. The range excludes inappropriate assignments to the threshold levels by the decision maker and hence prevents infeasibility. Also, by working with different threshold levels over the allowable range, we can see the impact of a change in the threshold level on the system performance measures like expected revenue and expected load factor. A range is defined by a lower bound and an upper bound. An upper bound is the value for the threshold level, above which the proposed models *RLF*, *RLF-M*, *LFR* and *MaxminLF* from Chapter 3 are infeasible. On the other hand, a lower bound is the value for the threshold level, where the solution of the proposed models in Chapter 3 starts to diverge from the results of the unconstrained models in terms of SL_l, SL and *RL*.

The bound models to be used for determining the range of the service levels for expected load factors are presented in Sections 4.1 and 4.2 for *RLF* and *RLF-M*, respectively. The bound models for determining the range of the threshold level for expected revenue are presented in Sections 4.3 and 4.4 for *LFR* and *MaxminLF*, respectively. Some of the bound models we propose give exact bounds for the threshold levels whereas some of them serve the purpose of finding approximate bounds for the threshold levels. In Chapter 5, the range of the service levels for the expected load factors and the range of the threshold level for an expected revenue are determined numerically by using the proposed bound models for a sample network. To avoid integrality constraints, LP relaxations of the proposed bound models are solved for numerical analysis. Also, the numerical results of the proposed models in Chapter 3 are analyzed as a function of the threshold levels in Chapter 5 for a sample network. In Chapter 6, the resulting seat allocations and the bid prices obtained by the proposed models are used in the simulation studies for different control policies, and the proposed models and the models in the literature are compared.

The bound models proposed in this section to determine the ranges for the threshold levels to be used in the proposed models (*RLF*, *RLF-M*, *LFR* and *MaxminLF*) are summarized in Table 4.1 and 4.2.

Table 4.1: Bound Models for *RLF* and *RLF-M*.

Proposed Model	***Lower Bound of Service Level***	***Upper Bound of Service Level***
RLF with $SL_l \neq SL$	*EMR*, ***MinmaxSL***	*MaxminLF-M*
RLF with $SL_l = SL$ for all l	*EMR* with equal $E(LF_l)$	*MaxELF*
RLF-M	*EMR* with *WLF*	*MaxWLF*

Table 4.2: Bound Models for *LFR* and *MaxminLF*.

Proposed Model	***Lower Bound of Revenue Level***	***Upper Bound of Revenue Level***
LFR	*LFR-M*	*EMR*
MaxminLF	*MinRL*	*EMR*

The following remark is to ensure that there exist solutions obtained by the bound models that satisfy (3.4) for $x_j(i)$s.

Remark 4.1. For each of the bound models and their relaxations, there exists an optimal solution **x** such that $x_j(i+1)$ takes a positive value only if $x_j(i)$=1 for all i and j. For *EMR*, *EMR* with equal E(LF_l) and *EMR* with *WLF*, the result follows Remark 3.2.2. For *MaxWLF* and *LFR-M*, the result follows Remark 3.2.4 (a). For *MaxminLF-M*, *MaxELF* and *MinRL*, the result follows Remark 3.2.4 (b). For *MinmaxSL*, the result is similar to Remark 3.2.4 (b).

4.1 Service Levels for *RLF*

Recall that *RLF* aims at maximizing the expected revenue under a service level constraint on the expected load factor of each leg. In this section, we propose bound models to determine the range of the service levels that are to be considered for numerical tests. Two cases considered in the following subsections are the following: (1) $SL_l = SL$ for all l and (2) allowing SL_l to take different values for different legs. LSL and USL denote the lower bound and the upper bound of the service level, SL, respectively, used in (1). LSL_l and USL_l denote the lower bounds and upper bounds of the service levels, SL_l, respectively, used in (2).

4.1.1 *RLF* with Unequal SL_l

The lower bounds on the service levels to be used in *RLF* with $SL_l \neq SL$ are determined by the *EMR* model. The bound model we propose to determine the upper bound to be used in *RLF* with $SL_l \neq SL$ is similat to the *MaxminLF*model.

In order to determine the lower bounds on the service levels to be used in *RLF*, firstly the results of the *EMR* model are investigated. Recall that *RLF* is obtained by incorporating an additional constraint to *EMR* to keep the expected load factor of each leg to be greater than a specified service level. That is why the expected load factors corresponding to the optimal allocation obtained by *EMR* can be considered as the lowest expected load factor values to be used in *RLF* as the service levels. In order to evaluate the expected load factor value obtained by *EMR* for each leg, the following model is used by incorporating (4.3) to the original *EMR* model. Note that this model is not different than *EMR*. This model allows us to evaluate $E(LF_l)$ for each leg for the optimal seat allocation obtained by *EMR* by using the decision variable ELF_l. The values of the decision variables ELF_l obtained from this model can be used in *RLF* as the lower bounds, LSL_l, of the service levels.

$$\text{Maximize } \sum_{j=1}^{n} \sum_{i=1}^{B_j} f_j P(D_j \geq i) x_j(i) \tag{4.1}$$

subject to

$$\sum_{j \in S_l} \sum_{i=1}^{B_j} x_j(i) \leq C_l \quad \text{for } l = 1, ..., m, \tag{4.2}$$

$$ELF_l = \frac{1}{C_l} \sum_{j \in S_l} \sum_{i=1}^{B_j} P(D_j \geq i) x_j(i) \quad \text{for } l = 1, ..., m, \tag{4.3}$$

$$x_j(i) \in \{0, 1\} \quad \text{for } j = 1, ..., n, \text{ and } i = 1, ..., B_j, \tag{4.4}$$

$$ELF_l \geq 0 \quad \text{for } l = 1, ..., m. \tag{4.5}$$

As an alternative to the use of *EMR*, one can think of using the following model to find the lower bounds for service levels in *RLF*. The model is called *MinmaxSL*, which stands for *Minimize the Maximum Service Level*.

$$MinmaxSL \; : \; \text{Minimize } z \tag{4.6}$$

subject to

$$ELF_l \leq z \quad \text{for } l = 1, ..., m, \tag{4.7}$$

$$\sum_{j=1}^{n} \sum_{i=1}^{B_j} f_j P(D_j \geq i) x_j(i) = r, \tag{4.8}$$

$$\sum_{j \in S_l} \sum_{i=1}^{B_j} x_j(i) \leq C_l \quad \text{for } l = 1, ..., m, \tag{4.9}$$

$$ELF_l = \frac{1}{C_l} \sum_{j \in S_l} \sum_{i=1}^{B_j} P(D_j \geq i) x_j(i) \quad \text{for } l = 1, ..., m, \tag{4.10}$$

$$x_j(i) \in \{0, 1\} \quad \text{for } j = 1, ..., n, \text{ and } i = 1, ..., B_j, \tag{4.11}$$

$$ELF_l \geq 0 \quad \text{for } l = 1, ..., m, \tag{4.12}$$

$$z \geq 0. \tag{4.13}$$

This model is a min-max type of model. Expected revenue is forced to be equal to r, which is the optimal expected revenue value obtained by *EMR*. This model aims at minimizing the maximum expected load factor of the network while equating the expected revenue to the value obtained by *EMR*. The values of the decision variable ELF_l obtained from this model can also be used in *RLF* as the lower bounds, LSL_l, for the service level as long as the solution obtained satisfies (3.4).

To sum up, for decision makers who intend to use *RLF*, it is suggested to solve *EMR* to obtain lower bounds of the service levels that can be used in *RLF*.

In order to determine the upper bounds on the service levels to be used in *RLF* with unequal SL_l, the bound model we propose is comparable with the *MaxminLF* model given in Section 3.2.3. The proposed bound model is given below, which is called *MaxminLF-M*, where *M* stands for *Modified*. We do not need the revenue constraint (3.25) of *MaxminLF* to find upper bounds for the service levels. Therefore, this constraint is eliminated. In order to evaluate the expected load factor value, $E(LF_l)$, for each leg for the optimal solution of this bound model, the additional constraint set (4.17) is introduced. The optimal value of the decision variable ELF_l can be used in *RLF* as the upper bound, USL_l, of the service level for leg l.

$$MaxminLF-M \quad : \quad \text{Maximize } z \tag{4.14}$$

subject to

$$ELF_l \geq z \quad \text{for } l = 1, ..., m, \tag{4.15}$$

$$\sum_{j \in S_l} \sum_{i=1}^{B_j} x_j(i) \leq C_l \quad \text{for } l = 1, ..., m, \tag{4.16}$$

$$ELF_l = \frac{1}{C_l} \sum_{j \in S_l} \sum_{i=1}^{B_j} P(D_j \geq i) x_j(i) \quad \text{for } l = 1, ..., m, \tag{4.17}$$

$$x_j(i) \in \{0, 1\} \quad \text{for } j = 1, ..., n, \text{ and } i = 1, ..., B_j, \tag{4.18}$$

$$ELF_l \geq 0 \quad \text{for } l = 1, ..., m, \tag{4.19}$$

$$z \geq 0. \tag{4.20}$$

4.1.2 *RLF* with Equal SL_l

In order to determine the lower bound on the service level to be used in *RLF* with $SL_l = SL$ for all l, again the results of *EMR* are investigated for the case of equal $E(LF_l)$ for all l. In this section, the following model is used by incorporating constraint (4.23) to the original *EMR* model. This model allows us to evaluate the expected load factor value by forcing the legs in the network to take equal SL_l values. Due to integrality constraints, the LP relaxation of the model is solved for numerical analysis. The value of the decision variable *ELF* obtained from this model can be used in *RLF* as a lower bound, *LSL*, for the service level.

$$\text{Maximize } \sum_{j=1}^{n} \sum_{i=1}^{B_j} f_j P(D_j \geq i) x_j(i) \tag{4.21}$$

subject to

$$\sum_{j \in S_l} \sum_{i=1}^{B_j} x_j(i) \leq C_l \quad \text{for } l = 1, ..., m, \tag{4.22}$$

$$ELF = \frac{1}{C_l} \sum_{j \in S_l} \sum_{i=1}^{B_j} P(D_j \geq i) x_j(i) \quad \text{for } l = 1, ..., m, \tag{4.23}$$

$$x_j(i) \in \{0, 1\} \quad \text{for } j = 1, ..., n, \text{ and } i = 1, ..., B_j, \tag{4.24}$$

$$ELF \geq 0. \tag{4.25}$$

In order to determine the upper bound on the service level to be used in *RLF* with $SL_l = SL$ for all l, the proposed bound model is *MaxELF*, which stands for *Maximum Expected Load Factor*. Due to integrality constraints, the LP relaxation of the model is solved for numerical analysis.

$$MaxELF \; : \; \text{Maximize } ELF \tag{4.26}$$

subject to

$$\sum_{j \in S_l} \sum_{i=1}^{B_j} x_j(i) \leq C_l \quad \text{for } l = 1, ..., m, \tag{4.27}$$

$$ELF = \frac{1}{C_l} \sum_{j \in S_l} \sum_{i=1}^{B_j} P(D_j \geq i) x_j(i) \quad \text{for } l = 1, ..., m,, \tag{4.28}$$

$$x_j(i) \in \{0, 1\} \quad \text{for } j = 1, ..., n, \text{ and } i = 1, ..., B_j, \tag{4.29}$$

$$ELF \geq 0. \tag{4.30}$$

The optimal value of *ELF* obtained by solving *MaxELF* can be used as an upper bound for the service level, *USL*.

4.2 Service Level for *RLF-M*

Recall that *RLF-M* is proposed as a variation of *RLF* in Chapter 3. In this section, we propose bound models for *RLF-M* in order to determine the lower bound, *LSL*, and the upper bound, *USL*, for the service level *SL*.

In order to determine the lower bound on the service level to be used in *RLF-M*, the following model is used by incorporating constraint (4.33) to the original *EMR* model. Note that this model is not different than *EMR*. This model allows us to evaluate the weighted average of the expected load factor values for the optimal seat allocation obtained by *EMR* by using the decision variable *WLF*. The value of *WLF* obtained from this model can be used in *RLF-M* as a lower bound, *LSL*, for the service level.

$$\text{Maximize} \sum_{j=1}^{n} \sum_{i=1}^{B_j} f_j P(D_j \geq i) x_j(i) \tag{4.31}$$

subject to

$$\sum_{j \in S_l} \sum_{i=1}^{B_j} x_j(i) \leq C_l \quad \text{for } l = 1, ..., m, \tag{4.32}$$

$$WLF = \sum_{l=1}^{m} \frac{w_l}{C_l} \sum_{j \in S_l} \sum_{i=1}^{B_j} P(D_j \geq i) x_j(i), \tag{4.33}$$

$$x_j(i) \in \{0, 1\} \quad \text{for } j = 1, ..., n, \text{ and } i = 1, ..., B_j, \tag{4.34}$$

$$WLF \geq 0. \tag{4.35}$$

In order to determine the upper bound on the service level to be used in *RLF-M*, the proposed bound model is *MaxWLF*, which stands for *Maximum Weighted Load Factor*. Note that *MaxWLF* is a variation of *LFR* that is obtained by replacing (3.19) in *LFR* with (4.38) to evaluate the maximum weighted avarege.

$$MaxWLF \ : \ \text{Maximize } WLF \tag{4.36}$$

subject to

$$\sum_{j \in S_l} \sum_{i=1}^{B_j} x_j(i) \leq C_l \quad \text{for } l = 1, ..., m, \tag{4.37}$$

$$WLF = \sum_{l=1}^{m} \frac{w_l}{C_l} \sum_{j \in S_l} \sum_{i=1}^{B_j} P(D_j \geq i) x_j(i), \tag{4.38}$$

$$x_j(i) \in \{0, 1\} \quad \text{for } j = 1, ..., n, \text{ and } i = 1, ..., B_j, \tag{4.39}$$

$$WLF \geq 0. \tag{4.40}$$

The optimal value of *WLF* obtained by solving *MaxWLF* can be used as an upper bound, *USL*, for the service level.

4.3 Revenue Level for *LFR*

Recall that *LFR* aims at maximizing a weighted average of the expected load factors of the network legs while ensuring that the expected revenue is always above a specified threshold level. In this section, we propose bound models to determine the range of the revenue level that is to be considered for numerical tests.

In order to determine the lower bound on the revenue level to be specified in *LFR*, the *LFR-M* model is proposed. *LFR-M* is obtained by removing the revenue constraint (3.19) from *LFR*, in which the revenue is forced to take values greater than or equal to a threshold level, *RL*. Instead, the additional constraint (4.43) is needed to determine the smallest *RL* value, *LRL*, that can be used in *LFR*.

$$LFR-M \quad : \quad \text{Maximize} \sum_{l=1}^{m} \frac{w_l}{C_l} \sum_{j \in S_l} \sum_{i=1}^{B_j} P(D_j \geq i) x_j(i) \tag{4.41}$$

subject to

$$\sum_{j \in S_l} \sum_{i=1}^{B_j} x_j(i) \leq C_l \quad \text{for } l = 1, ..., m, \tag{4.42}$$

$$LRL = \sum_{j=1}^{n} \sum_{i=1}^{B_j} f_j P(D_j \geq i) x_j(i), \tag{4.43}$$

$$x_j(i) \in \{0, 1\} \quad \text{for } j = 1, ..., n \text{ and } i = 1, ..., B_j, \tag{4.44}$$

$$LRL \geq 0. \tag{4.45}$$

The optimal *LRL* value that *LFR-M* gives can be used as a lower bound for the specified revenue level to be used in *LFR*.

The upper bound on the revenue level to be specified in *LFR* is the maximum expected revenue that is obtained by solving *EMR*.

4.4 Revenue Level for *MaxminLF*

Recall that *MaxminLF* is proposed as an alternative to *LFR* in Chapter 3. In this section, we propose bound models in order to determine the lower bound, *LRL* and the upper bound, *URL* for the revenue level, *RL*, to be used in *MaxminLF*.

In order to determine the lower bound on the revenue level to be specified in *MaxminLF*, the following model is proposed. The model is called *MinRL*, which stands for *Minimum Revenue Level*. Note that *MinRL* is a variation of *MaxminLF* that is obtained by replacing (3.5) in *MaxminLF* with (4.49) to evaluate the revenue level.

$$MinRL \quad : \quad \text{Maximize } z \tag{4.46}$$

subject to

$$\frac{1}{C_l} \sum_{j \in S_l} \sum_{i=1}^{B_j} P(D_j \geq i) x_j(i) \geq z \quad \text{for } l = 1, ..., m, \tag{4.47}$$

$$\sum_{j \in S_l} \sum_{i=1}^{B_j} x_j(i) \leq C_l \quad \text{for } l = 1, ..., m, \tag{4.48}$$

$$LRL = \sum_{j=1}^{n} \sum_{i=1}^{B_j} f_j P(D_j \geq i) x_j(i), \tag{4.49}$$

$$x_j(i) \in \{0, 1\} \quad \text{for } j = 1, ..., n \text{ and } i = 1, ..., B_j, \tag{4.50}$$

$$z \geq 0 \tag{4.51}$$

$$LRL \geq 0. \tag{4.52}$$

The *LRL* value that *MinRL* yields can be used as a lower bound for the specified revenue level in *MaxminLF*.

The upper bound on the revenue level to be specified in *MaxminLF* is the maximum expected revenue that is obtained by solving *EMR*.

CHAPTER 5

NUMERICAL ANALYSIS OF THE PROPOSED MODELS

In Chapter 4, the bound models are presented to determine the range of the threshold levels used in *RLF*, *RLF-M*, *LFR* and *MaxminLF*. In this chapter, the ranges for the threshold levels are determined numerically by using the proposed bound models for a sample network that is commonly used in the RM literaure. The network data is provided in Appendix A. Moreover, the optimization results of the proposed models are investigated to reveal the relation between the service levels and performance measures.

For the models we propose in Chapter 3 and for the bound models proposed in Chapter 4, the LP relaxations of these models are coded in MATLAB for a relaxation of the integrality constraints. The MATLAB codes of the proposed models in Chapter 3 and the bound models in Chapter 4 are provided in Appendix B.

Recall that the parameter, w_l, in *RLF-M* and in *LFR* in Chapter 3, stands for the weight of each leg. As stated in Chapter 3, different weights can be assigned to each leg according to the characteristics of them. In this chapter, the weight of each leg is assumed to be the same and equal to $1/m$. Determination of the weights for different legs and the use of them remain as a future work.

For the optimization and the simulation studies, the data given by de Boer (1999) for a sample three-leg airline network is used. The network consists of 4 nodes (A, B, C, D) and these nodes are connected with 3 identical legs (AB, BC and CD), each having a capacity of 200 seats. There are 6 itineraries on those three legs (AB, AC, AD, BC, BD, and CD). Each itinerary consists of 3 fare classes; that is, we have 18 origin-destination-fare (*ODF*) combinations in total. The reservation period is assumed to be 150 days. The sample network, which is a directed graph, is illustrated in Figure 5.1. As in the study of de Boer (1999), five different scenarios are tested. First one is

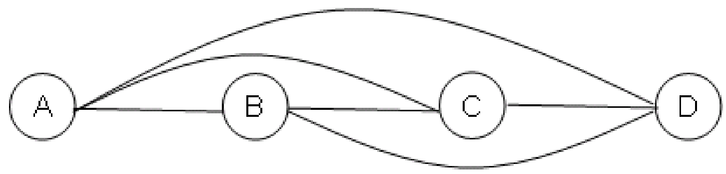

Figure 5.1: Sample Network with three legs

called the base problem and is studied in this chapter. The remaining four scenarios are studied in Chapter 6. The network data of all scenarios are provided in Appendix A.

The ranges of the threshold levels in *RLF*, *RLF-M*, *LFR* and *MaxminLF* and the related numerical analysis of the performance measures over these ranges are in Sections 5.1, 5.2, 5.3 and 5.4, respectively. In Section 5.5, the range for the service levels and the range for the threshold revenue level are presented under five different scenarios, which are "base problem", "increased variance of low fare demand", "smaller differences between fares", "realistic variations and close fares" and "low before high arrival pattern". The ranges found under different scenarios are used in the simulation studies in Chapter 6.

5.1 *RLF* Model

The solutions of the bound models proposed in Section 4.1.1 for the case of unequal SL_l are summarized in Table 5.1 below.

Table 5.1: Bounds for *RLF* with unequal SL_l

Lower Bound Model	LSL_{AB}	LSL_{BC}	LSL_{CD}
EMR	0.850427	0.849087	0.897118
MinmaxSL	0.850427	0.849087	0.897118
Upper Bound Model	USL_{AB}	USL_{BC}	USL_{CD}
MaxminLF-M	0.965798	0.965798	0.965798

The solutions of the bound models proposed in Section 4.1.2 for the case $SL_l = SL$ for all l are summarized in Table 5.2 below.

The upper bound in Table 5.2 is the same as the upper bounds of the legs in Table 5.1. This is an expected situation since all of the legs in the sample network has the same upper bound value for *SL* in Table 5.1. On the other hand, the lower bound in Table 5.2 is between the minimum and maximum lower bound values in Table 5.1. This is also an expected situation since all of the legs are forced to take equal SL_l values in *EMR* to obtain the values in Table 5.2.

In order to numerically justify the solutions obtained by the bound models above, the following model is solved by incrementally increasing the $SL_l = SL$ values. An increment of 0.0025 is used for *SL* in order to find the feasible range of *SL* numerically. This model is not different than the LP relaxation of *RLF* presented in Chapter 3 and allows us to evaluate $E(LF_l)$ for each leg for the optimal seat allocation obtained by *RLF* by using the decision variable ELF_l. The results obtained are summarized in Table 5.3 for the case $SL_l = SL$ for all l.

$$\text{Maximize } \sum_{j=1}^{n} \sum_{i=1}^{B_j} f_j P(D_j \geq i) x_j(i) \tag{5.1}$$

subject to

$$\sum_{j \in S_l} \sum_{i=1}^{B_j} x_j(i) \leq C_l \quad \text{for } l = 1, ..., m, \tag{5.2}$$

$$ELF_l \geq SL_l \quad \text{for } l = 1, ..., m, \tag{5.3}$$

$$ELF_l = \frac{1}{C_l} \sum_{j \in S_l} \sum_{i=1}^{B_j} P(D_j \geq i) x_j(i) \quad \text{for } l = 1, ..., m, \tag{5.4}$$

$$0 \leq x_j(i) \leq 1 \quad \text{for } j = 1, ..., n, \text{ and } i = 1, ..., B_j, \tag{5.5}$$

$$ELF_l \geq 0 \quad \text{for } l = 1, ..., m. \tag{5.6}$$

Table 5.2: Bounds for *RLF* with $SL_l = SL$ for all l

Lower Bound Model	$LSL(LSL_{AB}=LSL_{BC}=LSL_{CD})$
EMR with equal $E(LF_l)$	0.857145
Upper Bound Model	$USL(USL_{AB}=USL_{BC}=USL_{CD})$
MaxELF	0.965798

Table 5.3: Results of the *RLF* with $SL_l = SL$ for the Base Problem

SL_l=SL	E(R)	$E(LF_{AB})$	$E(LF_{BC})$	$E(LF_{CD})$
0.8000	71765.7848	0.8504	0.8491	0.8971
0.8025	71765.7848	0.8504	0.8491	0.8971
0.8050	71765.7848	0.8504	0.8491	0.8971
0.8075	71765.7848	0.8504	0.8491	0.8971
0.8100	71765.7848	0.8504	0.8491	0.8971
0.8125	71765.7848	0.8504	0.8491	0.8971
0.8150	71765.7848	0.8504	0.8491	0.8971
0.8175	71765.7848	0.8504	0.8491	0.8971
0.8200	71765.7848	0.8504	0.8491	0.8971
0.8225	71765.7848	0.8504	0.8491	0.8971
0.8250	71765.7848	0.8504	0.8491	0.8971
0.8275	71765.7848	0.8504	0.8491	0.8971
0.8300	71765.7848	0.8504	0.8491	0.8971
0.8325	71765.7848	0.8504	0.8491	0.8971
0.8350	71765.7848	0.8504	0.8491	0.8971
0.8375	71765.7848	0.8504	0.8491	0.8971
0.8400	71765.7848	0.8504	0.8491	0.8971
0.8425	71765.7848	0.8504	0.8491	0.8971
0.8450	71765.7848	0.8504	0.8491	0.8971
0.8475	71765.7848	0.8504	0.8491	0.8971
0.8491	**71765.7848**	**0.8504**	**0.8491**	**0.8971**
0.8500	**71765.2508**	**0.8513**	**0.8500**	**0.8971**
0.8525	71763.7755	0.8529	0.8525	0.8972
0.8550	71759.5472	0.8550	0.8550	0.8974
0.8575	71748.3285	0.8575	0.8575	0.8998
0.8600	71728.1412	0.8600	0.8600	0.9005
0.8625	71705.4402	0.8625	0.8625	0.9005
0.8650	71682.0248	0.8650	0.8650	0.9005
0.8655	71677.3079	0.8675	0.8675	0.9010
0.8675	71657.4593	0.8700	0.8700	0.9026
0.8700	71628.8953	0.8725	0.8725	0.9026
0.8725	71597.1128	0.8750	0.8750	0.9026
0.8750	71564.8700	0.8775	0.8775	0.9034
0.8775	71530.1841	0.8800	0.8800	0.9034
0.8800	71489.3009	0.8825	0.8825	0.9034

Table 5.3: Results of the *RLF* with $SL_l = SL$ for the Base Problem

SL$_l$=SL	***E(R)***	***E(LF$_{AB}$)***	***E(LF$_{BC}$)***	***E(LF$_{CD}$)***
0.8825	71446.3567	0.8850	0.8850	0.9038
0.8850	71401.4511	0.8875	0.8875	0.9054
0.8875	71355.1372	0.8900	0.8900	0.9065
0.8900	71306.1975	0.8925	0.8925	0.9069
0.8925	71253.6549	0.8950	0.8950	0.9073
0.8950	71198.3994	0.8975	0.8975	0.9073
0.8975	71141.6877	0.8975	0.8975	0.9073
0.9000	71080.9484	0.9000	0.9000	0.9065
0.9025	71013.4477	0.9025	0.9025	0.9068
0.9050	70943.0053	0.9050	0.9050	0.9059
0.9075	70869.2429	0.9075	0.9075	0.9075
0.9100	70792.2400	0.9100	0.9100	0.9100
0.9125	70709.3328	0.9125	0.9125	0.9125
0.9150	70615.9511	0.9150	0.9150	0.9150
0.9175	70521.8629	0.9175	0.9175	0.9175
0.9200	70422.7178	0.9200	0.9200	0.9200
0.9225	70311.8048	0.9225	0.9225	0.9225
0.9250	70194.9531	0.9250	0.9250	0.9250
0.9275	70074.0724	0.9275	0.9275	0.9275
0.9300	69949.6318	0.9300	0.9300	0.9300
0.9325	69819.8156	0.9325	0.9325	0.9325
0.9350	69669.7699	0.9350	0.9350	0.9350
0.9375	69509.5550	0.9375	0.9375	0.9375
0.9400	69345.1705	0.9400	0.9400	0.9400
0.9425	69162.9732	0.9425	0.9425	0.9425
0.9450	68958.6206	0.9450	0.9450	0.9450
0.9475	68737.6633	0.9475	0.9475	0.9475
0.9500	68497.2873	0.9500	0.9500	0.9500
0.9525	68221.5936	0.9525	0.9525	0.9525
0.9550	67904.5550	0.9550	0.9550	0.9550
0.9575	67536.7614	0.9575	0.9575	0.9575
0.9600	67090.9719	0.9600	0.9600	0.9600
0.9625	66522.6315	0.9625	0.9625	0.9625
0.9650	**65618.8187**	**0.9650**	**0.9650**	**0.9650**
0.9658	**Infeasible**			

According to Table 5.3, the upper bound, USL, is between 0.9650 and 0.9658. In fact, the largest *SL* value for which there exists a feasible solution is 0.965798. Recall that both USL_l in case of $SL_l \neq SL$ in Table 5.1 and *USL* in case of $SL_l = SL$ in Table 5.2 are found as 0.965798.

According to Table 5.3, the lower bound, *LSL*, is between 0.8491 and 0.8500. The $E(LF_l)$ values in Table 5.3 corresponding to *SL*=0.8491 are totally in accordance with the lower bounds in Table 5.1. On the other hand, the lower bound, *LSL*, in Table 5.2 is 0.857145. The difference between 0.857145 and the interval (0.8491-0.8500) seems to result from keeping $E(LF_l)$=*ELF* for all l in (4.23) used for Table 5.2.

When the concern is to analyze the impact of the service levels on the expected revenue and the expected load factors of the network, we can benefit from the results in Table 5.3 which are obtained for the case $SL_l = SL$ for all l. The influence of the service level on expected load factors of each leg of the network and on the expected revenue is given in Figure 5.2 and Figure 5.3, respectively.

- For service levels less than 0.8491, the expected revenue is constant at 71765.7848 and the vector of the expected load factor values of the network are constant at $[0.8504, 0.8491, 0.8971]$. These are in accordance with the definition of lower bound of *SL*.

- Beyond the service level 0.8491 on, the expected revenue shows a decreasing concave behaviour. This situation is not surprising since the model has tendency to accept the low fare class requests in order to increase the capacity utilization and to reserve fewer tickets for future high fare class requests. This is in accordance with the numerical seat allocations in Table C.1 in Appendix C.

- The *SL* value at which the expected load factor is equal to the service level differs among three legs. For leg *AB*, the *SL* value where the load factor constraint becomes binding is 0.8975. For leg *BC* it is 0.8975 and for leg *CD* it is 0.9075.

- It is observed that, for $SL \geq 0.9075$, $E(LF_l) = E(LF)$ for all l. That is, for service level greater than or equal to 0.9075, the expected load factor value of all legs in the sample network take the same value.

- Beyond the upper bound value of the service level, which is determined as 0.965798, it is not possible to find an allocation that satisfies the service level constraints.

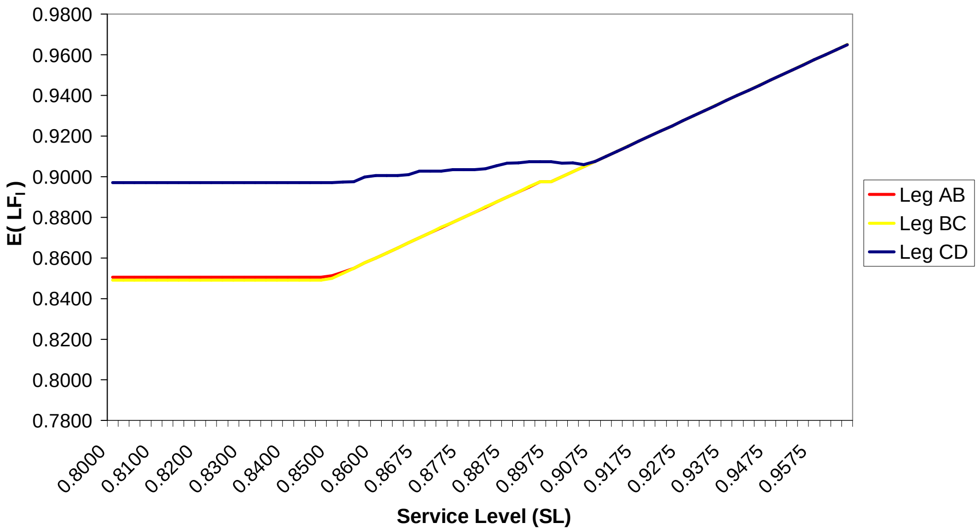

Figure 5.2: The Relation between *SL* and $E(LF_l)$ ($SL_l = SL$ for all l)

The relation between the service levels and the bid prices is also worth investigating. Recall from Chapter 1 and Chapter 2 that bid price is defined as the net value for an incremental seat on a particular flight leg in the airline network. The method proposed in Section 3.2.2 is used throughout this thesis to find the bid prices of *RLF* and *RLF-M*. In order to present the relationship between service levels and bid prices, the bid prices of the network legs are provided in Table C.2 in Appendix C.

When the relation between the service levels and bid prices are of concern, it can be said that bid prices generally show almost a constant pattern for low service levels. However, for $SL > 0.94$ bid prices of leg AB and CD decrease and a decreasing pattern is observed thereafter. As can be seen in Table C.2 in Appendix C, there is a tremendous difference between the bid prices of leg CD at service levels 0.80 and 0.9600. On the other hand, for $SL > 0.94$ bid prices of leg BC increase and a

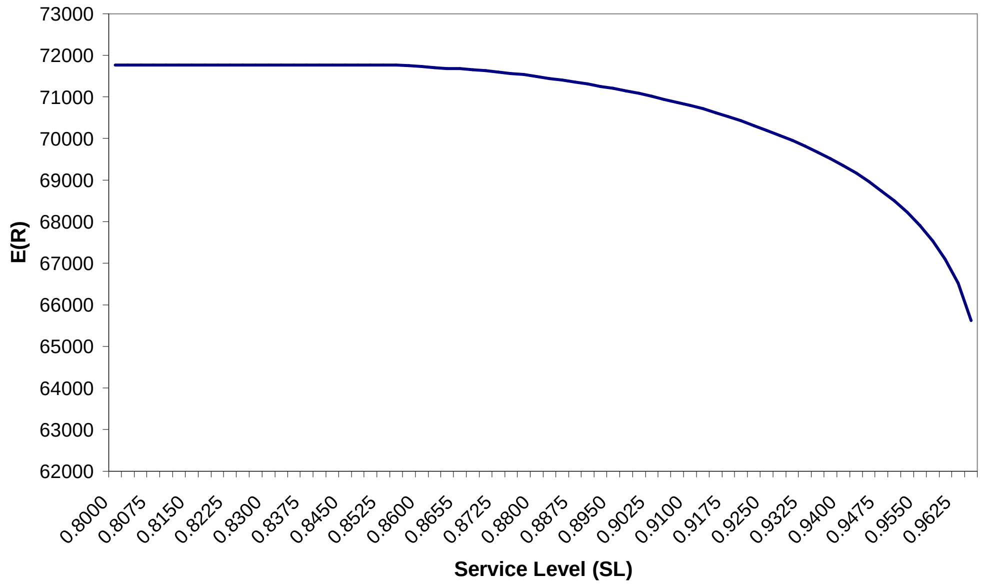

Figure 5.3: The Relation between *SL* and *E(R)* ($SL_l = SL$ for all l)

slightly increasing pattern is observed thereafter. The difference in the behavior of bid prices of leg BC and legs AB and CD results from the leg characteristics that differ in the input parameters. It is also worth to note that negative bid prices are obtained for $SL > 0.96$. It makes sense since the contribution of an additional seat to the expected revenue decreases as the service level increases and an additional seat starts to contibute negatively to the expected revenue for high service levels. The relation between the service levels and the bid price of each leg is seen in Figure 5.4. π_l denotes the bid price of leg l (optimal value of the dual variable corresponding to the capacity constraint of leg l).

So far, the legs of the network are assumed completely identical. That is, their service level requirements are set the same. However, it is also worth to see the results when the legs are forced to satisfy different service levels. At this point, it is important to decide on the service levels of the legs. When the network data of the base problem provided by de Boer is investigated, it can be seen that the demands for leg AB and leg CD have the same expectation and variation whereas leg BC has a smaller expectation and variance. From this point of view, the same service levels can be used for legs AB and CD. In our case, the service level of leg BC takes a value smaller than that of the other two legs. The case, where leg BC has larger service level values than leg AB

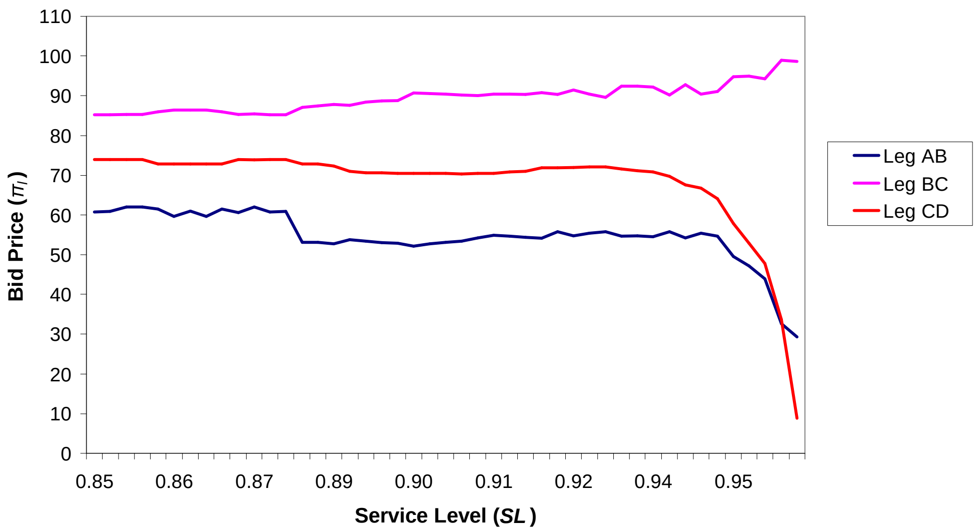

Figure 5.4: The Relation between *SL* and Π_l (*RLF* with $SL_l = SL$ for all l)

and CD may also be studied in future studies. Let ***SL*** denote the service level vector for $(SL_{AB}, SL_{BC}, SL_{CD})$.

In order to see the results for different service level combinations, *RLF* is solved for several cases and the results in Table 5.4 are obtained. The service levels of leg l are allowed to take values in the interval $[LSL_l, USL_l]$ given in Table 5.1. A number of possible combinations are considered in Table 5.4.

As a result of a limited number of numerical observations, it is seen that the service level combinations given in Table 5.4 do not result in a remarkable difference in the expected revenue when compared to the case of equal service levels for the legs. Simulation study in Chapter 6 to compare the use of different service levels and the use of equal service levels for all legs justifies this observation. Note that, for every feasible vector of $(E(LF_{AB}), E(LF_{BC}), E(LF_{CD}))$ that can be obtained, *SL* value can be chosen as in the model allowing $SL = SL_l$ for all l.

Table 5.4: Results of the *RLF* with unequal SL_l for Base Problem

SL				*E(LF$_l$)*			π_l		
SL$_{AB}$	***SL$_{BC}$***	***SL$_{CD}$***	***E(R)***	***l:AB***	***l:BC***	***l:CD***	***l:AB***	***l:BC***	***l:CD***
0.9000	0.8500	0.9000	71211.5502	0.9000	0.8723	0.9000	52.1376	85.2410	70.5002

Table 5.4: Results of the *RLF* with unequal SL_l for Base Problem

SL				***E(LF$_l$)***			π_l		
SL$_{AB}$	***SL$_{BC}$***	***SL$_{CD}$***	***E(R)***	***l:AB***	***l:BC***	***l:CD***	***l:AB***	***l:BC***	***l:CD***
0.9025	0.8525	0.9025	71162.3938	0.9025	0.8746	0.9025	52.7852	85.1944	70.5002
0.9050	0.8550	0.9050	71109.1113	0.9050	0.8767	0.9050	53.1078	85.3303	70.5002
0.9075	0.8575	0.9075	71051.0415	0.9075	0.8769	0.9075	53.3623	85.3303	70.3985
0.9100	0.8600	0.9100	70990.0477	0.9100	0.8790	0.9100	54.2581	85.9152	70.5169
0.9125	0.8625	0.9125	70922.6056	0.9125	0.8819	0.9125	54.8821	86.4684	70.5196
0.9150	0.8650	0.9150	70851.6259	0.9150	0.8829	0.9150	54.5927	86.4684	70.8791
0.9175	0.8675	0.9175	70772.7054	0.9175	0.8847	0.9175	54.3604	86.4684	70.9507
0.9200	0.8700	0.9200	70685.9643	0.9200	0.8869	0.9200	54.1219	85.9152	71.8578
0.9225	0.8725	0.9225	70595.7033	0.9225	0.8921	0.9225	55.6883	85.3303	71.8353
0.9250	0.8750	0.9250	70496.3082	0.9250	0.8943	0.9250	54.7305	85.4235	71.9606
0.9275	0.8775	0.9275	70388.5306	0.9275	0.8945	0.9275	55.3125	85.2410	72.0913
0.9300	0.8800	0.9300	70266.7842	0.9300	0.8986	0.9300	55.7670	85.1944	72.1086
0.9325	0.8825	0.9325	70138.8545	0.9325	0.9005	0.9325	54.6557	87.1087	71.5449
0.9350	0.8850	0.9350	70004.0251	0.9350	0.9034	0.9350	54.7848	87.4121	71.0585
0.9375	0.8875	0.9375	69862.0720	0.9375	0.9082	0.9375	54.4668	87.7412	70.8887
0.9400	0.8900	0.9400	69709.3049	0.9400	0.9084	0.9400	55.6867	87.5296	69.7613
0.9425	0.8925	0.9425	69543.4195	0.9425	0.9109	0.9425	54.2560	88.4231	67.6197
0.9450	0.8950	0.9450	69365.0826	0.9450	0.9132	0.9450	55.3497	88.6115	66.7830
0.9475	0.8975	0.9475	69175.8283	0.9475	0.9149	0.9475	54.6645	88.7572	64.1281
0.9500	0.9000	0.9500	68959.3408	0.9500	0.9147	0.9500	49.5234	90.6408	57.9156
0.9525	0.9025	0.9525	68721.7651	0.9525	0.9179	0.9525	47.1795	90.5574	52.8763
0.9550	0.9050	0.9550	68454.2488	0.9550	0.9194	0.9550	43.9127	90.3974	47.7848
0.9575	0.9075	0.9575	68133.9484	0.9575	0.9203	0.9575	32.6141	90.2009	33.6434
0.9600	0.9100	0.9600	67757.6073	0.9600	0.9226	0.9600	29.2893	89.9993	8.8503

5.2 *RLF-M* Model

The solutions of the bound models proposed in Section 4.2 are summarized in Table 5.5 below.

Table 5.5: Bounds for *RLF-M*

Lower Bound Models	***LWL***
EMR with WLF	0.865544
Upper Bound Model	***UWL***
MaxWLF	0.968887

In order to numerically justify the solutions obtained by the bound models above, *RLF-M* is solved by incrementally increasing the SL values in *RLF-M*. An increment of 0.0025 is used for *SL* in order to find the feasible range of *SL* numerically. We consider the case of $w_l = 1/m$ for all l. The results obtained are summarized in Table 5.6.

Table 5.6: Results of *RLF-M* for the Base Problem

			π_l		
SL	***E(R)***	***Weighted Avg. E(LF)***	***l:AB***	***l:BC***	***l:CD***
0.8500	71765.7848	0.8655	61.9916	85.3303	73.9393
0.8525	71765.7848	0.8655	59.5551	86.4263	72.8012
0.8550	71765.7848	0.8655	59.5551	86.4263	72.8012
0.8575	71765.7848	0.8655	60.9468	86.4684	72.8012
0.8600	71765.7848	0.8655	61.4999	85.9152	72.8012
0.8625	71765.7848	0.8655	61.4999	85.9152	72.8012
0.8650	**71765.7848**	**0.8655**	**61.2042**	**84.7771**	**73.9393**
0.8675	**71763.9402**	**0.8675**	**60.1076**	**85.1858**	**74.0600**
0.8700	71759.9691	0.8700	60.7182	84.6738	74.0558
0.8725	71752.4767	0.8725	58.6325	84.5106	74.5480
0.8750	71730.7988	0.8750	54.9234	83.0870	74.7611
0.8775	71703.3604	0.8775	53.0910	82.4067	73.1214
0.8800	71669.6967	0.8800	53.0363	82.3503	74.2481
0.8825	71633.1581	0.8825	53.5468	82.7554	73.1511
0.8850	71591.5427	0.8850	53.3205	83.1868	73.2537
0.8875	71548.4624	0.8875	53.1490	83.3928	73.2709
0.8900	71501.3535	0.8900	52.6500	84.4604	73.5551
0.8925	71452.2292	0.8925	52.5686	85.4262	73.9971
0.8950	71396.3691	0.8950	52.5697	85.2644	73.9167
0.8975	71336.9566	0.8975	54.1705	84.6110	74.3862
0.9000	71272.7016	0.9000	54.0635	84.5039	74.2791
0.9025	71206.3344	0.9025	54.2134	84.8140	74.4999
0.9050	71135.9302	0.9050	53.4917	85.6384	74.5510
0.9075	71059.3800	0.9075	53.7741	85.8226	74.7596
0.9100	70978.1377	0.9100	55.1038	85.0764	74.7448
0.9125	70891.4625	0.9125	54.1182	86.5192	74.8396
0.9150	70797.3889	0.9150	53.9961	86.8270	74.5277

Table 5.6: Results of *RLF-M* for the Base Problem

			π_l		
SL	***E(R)***	***Weighted Avg. E(LF)***	***l:AB***	***l:BC***	***l:CD***
0.9175	70698.7620	0.9175	55.8890	85.1830	74.5477
0.9200	70593.7112	0.9200	54.2597	87.2540	74.2446
0.9225	70481.5958	0.9225	54.4034	87.8556	74.0771
0.9250	70359.7190	0.9250	53.8481	88.5602	72.6089
0.9275	70230.0255	0.9275	53.6305	89.3919	70.3252
0.9300	70091.9663	0.9300	53.8056	89.5351	69.9619
0.9325	69944.2624	0.9325	55.5338	88.0290	69.5796
0.9350	69786.1641	0.9350	54.4581	89.9510	65.8893
0.9375	69617.2864	0.9375	54.5953	89.9386	65.4975
0.9400	69436.7142	0.9400	53.2253	91.7929	63.2667
0.9425	69234.2399	0.9425	53.0637	92.3728	62.2613
0.9450	69017.8570	0.9450	52.3417	94.3333	56.9601
0.9475	68779.0327	0.9475	52.7355	93.7720	53.5220
0.9500	68528.5597	0.9500	53.8519	91.2258	51.1273
0.9525	68235.2605	0.9525	49.8280	94.8871	44.8122
0.9550	67911.8030	0.9550	49.3326	94.5912	43.7175
0.9575	67543.2017	0.9575	38.7249	95.4733	30.2146
0.9600	67112.9871	0.9600	35.4424	92.5866	26.8744
0.9625	66570.3516	0.9625	20.1812	86.7167	2.0330
0.9650	65867.9759	0.9650	-9.4266	86.5032	-30.2386
0.9675	**64772.2840**	**0.9675**	**-82.1376**	**53.6075**	**-133.8659**
0.9700	**Infeasible**				

- For $SL < 0.8675$, the *RLF-M* model gives exactly the same results as the *EMR* model in terms of expected revenue and the weighted average expected load factor of the network, which is in accordance with the definition of lower bound of *SL*.

- For service levels equal to or greater than 0.8675, the model diverges from the *EMR* model. That is, the lower bound for the service level, *LWL*, is between 0.8650 and 0.8675 in Table 5.6. As presented in Table 5.5, the proposed bound model, *EMR*, gives 0.86558844 as *LWL*. That is, the numerical analysis justifies the bound model proposed in Section 4.2.

- The highest *SL* value for which *RLF-M* is feasible is 0.968887. Beyond this value, the model becomes infeasible. The upper bound for service level, *UWL*, is between 0.9675 and 0.9700 in Table 5.6. As presented in Table 5.5, the proposed bound model, *MaxWLF*, gives 0.968887 as *UWL*. That is, the numerical analysis justifies the bound model proposed in Section 4.2.

- Note that the load factor constraint becomes binding at the divergence point, 0.8675. For the original *RLF* model, the divergence point was 0.849087 and the service level where all of the load factor constraints become binding was 0.9075.

5.3 *LFR* Model

The solutions of the bound models proposed in Section 4.3 are summarized in Table 5.7 below.

Table 5.7: Bounds for *LFR*

Lower Bound Model	***LRL***
LFR-M	62948.3292
Upper Bound Model	***URL***
EMR	71765.7848

Note that, *LFR-M* is solved for the base problem with $w_l = 1/m$ for all legs in the network.

In order to numerically justify the solutions obtained by the proposed bound models, the optimization results of *LFR* are checked for the base problem. An increment of 250 is used for revenue level in order to find the feasible range of revenue level, *RL* numerically. w_l values are assumed as $1/m$ for all legs in the network. The results obtained are summarized in Table 5.8.

Table 5.8: Results of *LFR* for the Base Problem

RL	***E(R)***	***Weighted Avg. E(LF)***		***RL***	***E(R)***	***Weighted Avg. E(LF)***
59000	62948.3292	0.968887		65500	65500.0000	0.965987
59250	62948.3292	0.968887		65750	65750.0000	0.965324
59500	62948.3292	0.968887		66000	66000.0000	0.964599
59750	62948.3292	0.968887		66250	66250.0000	0.963746
60000	62948.3292	0.968887		66500	66500.0000	0.962783
60250	62948.3292	0.968887		66750	66750.0000	0.961751
60500	62948.3292	0.968887		67000	67000.0000	0.960560
60750	62948.3292	0.968887		67250	67250.0000	0.959220
61000	62948.3292	0.968887		67500	67500.0000	0.957762
61250	62948.3292	0.968887		67750	67750.0000	0.956157
61500	62948.3292	0.968887		68000	68000.0000	0.954325
61750	62948.3292	0.968887		68250	68250.0000	0.952385
62000	62948.3292	0.968887		68500	68500.0000	0.950280
62250	62948.3292	0.968887		68750	68750.0000	0.947797
62500	62948.3292	0.968887		69000	69000.0000	0.945193
62750	**62948.3292**	**0.968887**		69250	69250.0000	0.942308
63000	**63000.0000**	**0.968878**		69750	69750.0000	0.935543
63250	63250.0000	0.968830		70000	70000.0000	0.931575
63500	63500.0000	0.968722		70250	70250.0000	0.927124
63750	63750.0000	0.968575		70500	70500.0000	0.922112
64000	64000.0000	0.968384		70750	70750.0000	0.916227
64250	64250.0000	0.968158		71000	71000.0000	0.909350
64500	64500.0000	0.967893		71250	71250.0000	0.900868
64750	64750.0000	0.967536		71500	71500.0000	0.890070
65000	65000.0000	0.967107		**71750**	**71750.0000**	**0.872851**
65250	65250.0000	0.966594		**72000**	**Infeasible**	
65500	65500.0000	0.965987				

According to Table 5.8, it can be seen that the lower bound for *RL*, is between 62750 and 63000. That is, the numerical analysis justifies the lower bound model proposed in Section 4.3 as seen in Table 5.7: $62948.3292 \in (62750, 63000)$. According to the results presented in Table 5.8, the upper bound for *RL*, is between, 71750 and 72000. In fact, the highest *RL* value for which there exists a feasible solution of *LFR* is 71765.7848 as seen in Table 5.7. That is, the numerical analysis justifies the upper bound model proposed in Section 4.3.

When the concern is to analyze the impact of the threshold revenue level on the expected revenue and the weighted average of the expected load factors of the network, we can benefit from the results presented in Table 5.7, which is obtained by assuming $w_l = 1/m$ for all legs in the network.

- For *RL* below 62948.3292, the constraint on the expected revenue and the weighted average expected load factor are constant at 62948.3292 and 0.9689, respectively. This is in accordance with the definition of the lower bound of *RL*.
- The weighted average of the expected load factors show a decreasing concave behaviour as *RL* increases. The reason of this pattern in the weighted average of the expected load factors is the tendency of the model to reserve more tickets for the future high fare class requests in order to be able to keep the revenue above the threshold revenue level. This is in accordance with the numerical seat allocations in Appendix C.
- As the *EMR* model gives the expected revenue level as 71765.7848, we would expect that *LFR* model is infeasible beyond the revenue level 71765.7848. Consistent with that expectation, when *RL* in the model is forced to be equal to or greater than 71765.7848, the problem becomes infeasible. The highest *RL* for which an optimal solution exists is 71765.7848.

The relation between the threshold revenue level, *RL*, and expected revenue is given in Figure 5.5. The relation between the threshold revenue level, *RL*, and the weighted average of the expected load factors of the network is given Figure 5.6.

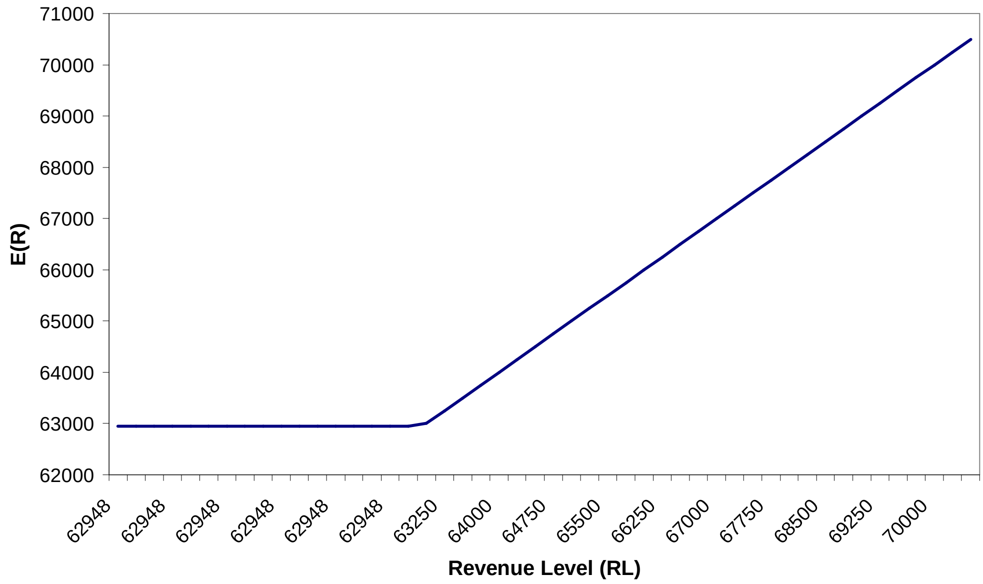

Figure 5.5: The Relation between *RL* and *E(R)* (*LFR* with $w_l = 1/m$ for all l)

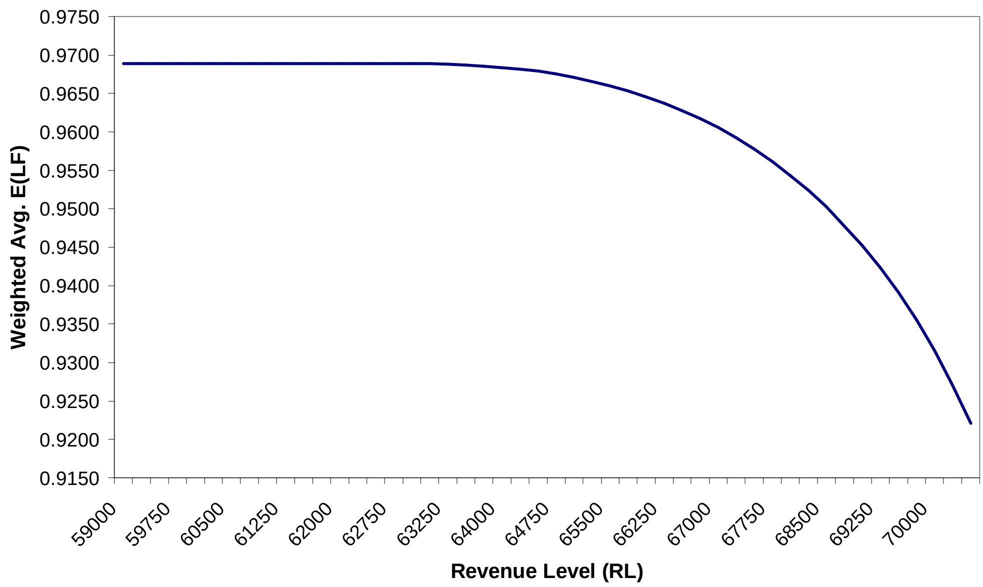

Figure 5.6: The Relation between *RL* and $\sum_l w_l E(LF_l)$ (*LFR* with $w_l = 1/m$ for all l)

5.4 *MaxminLF* Model

The solutions of the bound models proposed in Section 4.4 are summarized in Table 5.9 below.

Table 5.9: Bounds for *MaxminLF*

Lower Bound Model	***LRL***
MinRL	62481.3044
Upper Bound Model	***URL***
EMR	71765.7848

In order to numerically justify the solutions obtained by the proposed bound models, the optimization results of *MaxminLF-R* are checked for the base problem. An increment of 250 is used for revenue level in order to find the feasible range of revenue level, *RL* numerically. The results obtained are summarized in Table 5.10.

Table 5.10: Results of *MaxminLF* for Base Problem

RL	***E(R)***	$\min_l$ ***E(LF$_l$)***	***E(LF$_{AB}$)***	***E(LF$_{BC}$)***	***E(LF$_{CD}$)***
62000	63672.6378	0.965798	0.965798	0.965798	0.965798
62250	64471.4773	0.965798	0.965798	0.965798	0.965798
62500	64498.8030	0.965798	0.965798	0.965798	0.965798
62750	64428.4046	0.965798	0.965798	0.965798	0.965798
63000	63000.0000	0.965798	0.965798	0.965798	0.965798
63250	64523.5430	0.965798	0.965798	0.965798	0.965798
63500	64435.2149	0.965798	0.965798	0.965798	0.965798
63750	64565.0876	0.965798	0.965798	0.965798	0.965798
64000	64343.0930	0.965798	0.965798	0.965798	0.965798
64250	64250.0000	0.965798	0.965798	0.965798	0.965798
64500	64500.0000	0.965798	0.965798	0.965798	0.965798
64750	**64750.0000**	**0.965798**	**0.965798**	**0.965798**	**0.965798**
65000	**65000.0000**	**0.965753**	**0.965753**	**0.965753**	**0.965753**
65250	65250.0000	0.965554	0.965554	0.965554	0.965554
65500	65500.0000	0.965203	0.965203	0.965203	0.965203
65750	65750.0000	0.964728	0.964728	0.964728	0.964728
66000	66000.0000	0.964125	0.964125	0.964125	0.964125
66250	66250.0000	0.963397	0.963397	0.963397	0.963397
66500	66500.0000	0.962585	0.962585	0.962585	0.962585
66750	66750.0000	0.961581	0.961581	0.961581	0.961581
67000	67000.0000	0.960453	0.960453	0.960453	0.960453
67250	67250.0000	0.959193	0.959193	0.959193	0.959193
67500	67500.0000	0.957735	0.957735	0.957735	0.957735
67750	67750.0000	0.956073	0.956073	0.956073	0.956073
68000	68000.0000	0.954283	0.954283	0.954283	0.954283
68250	68250.0000	0.952262	0.952262	0.952262	0.952262
68500	68500.0000	0.949974	0.949974	0.949974	0.949974
68750	68750.0000	0.947365	0.947365	0.947365	0.947365
69000	69000.0000	0.944514	0.944514	0.944514	0.944514
69250	69250.0000	0.941358	0.941358	0.941358	0.941358
69500	69500.0000	0.937646	0.937646	0.937646	0.937646
69750	69750.0000	0.933687	0.933687	0.933687	0.933687
70000	70000.0000	0.929000	0.929000	0.929000	0.929000
70250	70250.0000	0.923835	0.923835	0.923835	0.923835
70500	70500.0000	0.918070	0.918070	0.918070	0.918070
70750	70750.0000	0.911287	0.911287	0.911287	0.911287

Table 5.10: Results of *MaxminLF* for Base Problem

RL	***E(R)***	$\min_l$ ***E(LF$_l$)***	***E(LF$_{AB}$)***	***E(LF$_{BC}$)***	***E(LF$_{CD}$)***
71000	71000.0000	0.902987	0.902987	0.902987	0.906500
71250	71250.000	0.892667	0.892667	0.892667	0.907040
71500	71500.000	0.879346	0.879346	0.879346	0.903425
71750	71750.000	0.857177	0.857177	0.857177	0.899453
72000	**Infeasible**				

- *MaxminLF* is not feasible beyond the revenue level 72000. The highest *RL* for which *MaxminLF* gives a feasible solution is 71765.7848. Therefore, the numerical analysis justifies the proposed bound model as seen in Table 5.9.
- Up to $RL = 64750$, there is a zig-zag pattern in E(R) values, which we can not explain. However, for $RL \geq 64250$, E(R) increases as RL increases, as expected. That is, *MaxminLF* reaches the stability for $RL \geq 64250$.
- According to Table 5.10, the expected load factor values of leg AB, BC and CD is equal for $RL \leq 70750$, while the revenue earned changes as RL changes. The $E(LF_l)$ values in Table 5.10 corresponding to $E(LF_l) = 0.965798$ are totally in accordance with the $E(LF_l)$ values of the proposed lower bound model, *MinRL*. Based on this observation, it can be said that there are alternative optimal solutions for *MaxminLF*.
- The constraint on *RL* in *MaxminLF* becomes binding at a revenue level of 64250 whereas the same constraint in *LFR* becomes binding in terms of revenue at its lower bound 63000.

5.5 Threshold Levels under Different Scenarios

The upper and lower bounds for threshold levels used in *RLF* with $SL_l = SL$, *RLF-M*, *LFR* and *MaxminLF* under different scenarios are summarized in Table 5.11. The upper and lower bounds for the service level used in *RLF* with unequal SL_l under different scenarios are summarized in Table 5.12.

Table 5.11: Bounds on Threshold Levels under Different Scenarios

	RLF with SL_l=SL		***LFR***	
Scenerio	***LSL***	***USL***	***LRL***	***URL***
Base Problem	0.857145	0.965798	62948.3292	71765.7848
Increased Variance of Low Fare Demand	0.846324	0.942825	63491.6418	70679.1388
Smaller Differences Between Fares	0.901853	0.965798	56706.7123	60547.7284
Realistic Variations and Close Fares	0.924922	0.965653	62168.2883	65035.9662
Low Before High Arrival Pattern	0.857145	0.965798	62948.3292	71765.7848
	RLF-M		***MaxminLF***	
Scenerio	***LWL***	***UWL***	***LRL***	***URL***
Base Problem	0.865544	0.968887	62481.3044	71765.7848
Increased Variance of Low Fare Demand	0.841761	0.944037	62811.1964	70679.1388
Smaller Differences Between Fares	0.893495	0.968887	56139.9793	60547.7284
Realistic Variations and Close Fares	0.916565	0.951410	61937.0757	65035.9662
Low Before High Arrival Pattern	0.865544	0.968887	62481,3044	71765.7848

Table 5.12: Bounds on Threshold Levels of *RLF* with $SL_l = SL$ under Different Scenarios

	RLF with SL_l=SL		
Scenario	***LSL_{AB}***	***LSL_{BC}***	***LSL_{CD}***
Base Problem	0,850427	0,849087	0,897118
Increased Variance of Low Fare Demand	0,821081	0,825079	0,879122
Smaller Differences Between Fares	0,874064	0,888714	0,917708
Realistic Variations and Close Fares	0,916759	0,928640	0,944348
Low Before High Arrival Pattern	0,850427	0,849087	0,897118
	RLF with SL_l=SL		
Scenario	***USL_{AB}***	***USL_{BC}***	***USL_{CD}***
Base Problem	0,965798	0,965798	0,965798
Increased Variance of Low Fare Demand	0,942825	0,942825	0,942825
Smaller Differences Between Fares	0,965798	0,965798	0,965798
Realistic Variations and Close Fares	0,965653	0,965653	0,965653
Low Before High Arrival Pattern	0,965798	0,965798	0,965798

CHAPTER 6

SIMULATION STUDIES

This chapter includes the simulation studies performed to evaluate the performance of the proposed models under different booking control polices as compared to the existing models in the literature. The results of the optimization models provide just an approximation for the performances of the models. Recall from Chapter 2, nesting environment is not incorporated into the optimization models. In order defeat this, control polices are incorporated in simulation studies and the performances of the models under these control polices are evaluated. By simulation studies, not only nesting control but also bid price control and even partitioned control are tested in a more realistic environment with the help of the statistical distributions. In the simulation studies, the airline network data given by de Boer (1999) is used as in Chapter 5. As mentioned in Chapter 5, the network consists of 3 legs, which are AB, BC and CD. 4 nodes of the network are connected via these legs and there are 6 itineraries among these nodes as represented in Figure 5.1. It is assumed that the flight legs are identical and each has a flight capacity of 200. There are 3 fare classes for each itinerary and therefore 18 origin-destination-fare combinations (*ODF*) in total. The booking period is accepted as 150 days.

In general, the arrival process of the booking requests for RM problems is modeled with Poisson Processes. The arrival rate of the Poisson Process is a random variable and it is fitted with the Gamma Distribution. However, the low fare class customers tend to arrive earlier than high fare class customers. Therefore, the arrival rate for RM problems is not constant throughout the booking period and a Non-Homogeneous Poisson Process (NHPP) is used in order to introduce this situation into simulation models. The random arrival rate of NHPP is defined as the product of Beta and Gamma distributed random variables. Beta density functions for different fare classes that are used in this thesis are given in Figure A.1 in Appendix A. For the derivations of the required distribution functions for simulation models, the reader is referred to an overview of Terciyanlı (2009). The simulation of NHPP is not straightforward as in the case of Poisson Process with constant rate. For the simulation techniques to generate arrivals from NHPP, the reader is referred to the study of Law and Kelton (2000).

Bayesian update is a frequently used method in RM problems to update the demand distribution at certain points in time during the booking period. Bayesian update is generally used where the historical data is not sufficient to estimate the distributions exactly. For an overview of the adaptation of the Bayes' formula to RM problems, the reader is referred to Terciyanlı (2009). Bayesian update is not considered within the scope of this thesis. The update of the bid prices is leaved as future research.

As mentioned in Chapter 5, 5 scenarios considered by de Boer (1999) are studied in this thesis. The first scenario is the base problem on which we perform the model interpretations in Chapter 5. The second one is the case with increased variance of low-fare demand and, in the third case; the differences between fares are lowered. In the fourth scenario, the realistic coefficient of variations of demand and relatively close fares are used and the order of the arrivals is not specified. The last case is with low-before-high arrival pattern. The chracteristics of these scenarios and the motivation behind them is explained in the related subsections. The parameters of the scenarios are provided in Appendix A.

The input for the simulation studies is the optimal seat allocations and the bid prices, which are obtained by solving the optimization models. In this chapter *DLP*, *EMR*, *SLP*, *EMVLP*, *SLP-RM*, *PMP-RC*, *RLP* models and *RRS* Procedure are compared to our proposed models *RLF* with $SL_l = SL$ for all l, *RLF-M*, *LFR* and *MaxminLF*. For *RLF-M* and *LFR*, we use $w_l = 1/m$ for all l. The fact that the models we propose are solved much more easily than the other risk sensitive models in the literature is an advantage. It can be said that the proposed models lead to reasonable results as compared to other models and give nearly close results to *EMVLP*, *SLP-RM* and

PMP-RC.

Recall that, in Section 5.5, the intervals for threshold levels to be used in *RLF* with $SL_l = SL$, *RLF-M*, *LFR* and *MaxminLF* are provided in Table 5.11 for different scenarios by solving the bound models proposed for them. The simulation intervals of the proposed models in this chapter are in accordance with Table 5.11.

As Terciyanlı (2009) suggests, the replication number used in the simulation model for partitioned and nested controls is taken as 10000.

We use 4 performance measures in order to evaluate the results of the optimization models in the simulation model: sample mean (*SM*) and sample standard deviation (*SSD*) for revenue, sample coefficient of variation (*SCV*) for revenue and load factor (*LF*). These measures are calculated using the simulation results. The sample coefficient of variation is defined as follows:

$$SCV = \frac{SSD}{SM}. \tag{6.1}$$

The organization of the simulation studies in this chapter is scenario-based. In Section 6.1, the base problem is considered under partitioned, nested and bid price control policies. It is observed that the performances of partitioned and nested controls are comparable in different respects. In addition, partitioned and nested controls are consistent whereas bid price control exhibits inconsistencies in the expected trend of the performance measures. Also nested control is preferred in practice to partitioned control. Based on these observations, only nested control is evaluated for the other problem scenarios. In this thesis, we use the nesting heuristic that is proposed by de Boer et al. (2002). The case with increased variance of low-fare demand is studied in Section 6.2. In Section 6.3, the third case in which the differences between fares are lowered is investigated. The fourth scenario, where the realistic coefficient of variations of demand and relatively close fares are used and the order of the arrivals is not specified, is given in Section 6.4. Section 6.5 is devoted to the last case with low-before-high arrival pattern. The concluding remarks about the simulation studies are given in Section 6.6.

6.1 Base Problem

According to de Boer (1999), the base problem is specified with the following characteristics. The fare classes inherit huge differences among themselves. That is, the low fare and high fare classes diverge significantly in terms of their parameters. The low fare class customers are usually the leisure travelers and they tend to arrive earlier than the high fare class customers do. However, there is not a strict rule about this situation. There may be cases where a low fare class customer arrives later than a high fare class customer. The long-haul flights are cheaper than the single-leg flights.

6.1.1 Partitioned Control Policy

The simulation results under partitioned control are given in Table 6.1. Note that, the parameter, *RL*, is used in the optimization models *LFR* and *MaxminLF* whereas, the sample mean, *SM*, is obtained from the simulation model. Therefore, *RL* and *SM* values may not coincide. In accordance with this, it is observed that *RL* values are mostly greater than or equal to *SM* values for *LFR* and *MaxminLF*.

Table 6.1: Simulation Results under Partitioned Control for the Base Problem

Model	Parameter	*SM*	*SSD*	*SCV*	*LF*
DLP	-	70735.10	5581.89	0.0789	0.8638
EMR	-	**71916.23**	6243.67	0.0868	0.8669
SLP	-	71744.89	5922.72	0.0826	0.8796
EMVLP	$\theta = 0.001$	71709.15	5616.34	0.0783	0.8847
	$\theta = 0.005$	67442.93	2950.30	0.0437	0.9527
SLP-RM	L=60000	70830.87	4798.45	0.0677	0.9052
	L=65000	70268.85	4299.09	0.0612	0.9203
	L=70000	71229.74	5044.26	0.0708	0.8966
	L=75000	71544.67	5660.58	0.0791	0.8780
	L=80000	71772.31	6124.84	0.0853	0.8662
PMP-RC	L=70000, $\rho = 0.28$	71899.31	6205.83	0.0863	0.8683
	L=70000, $\rho = 0.29$	71461.74	5486.52	0.0768	0.8864
	L=70000, $\rho = 0.30$	71791.26	5966.37	0.0831	0.8750
	L=70000, $\rho = 0.31$	71830.97	6309.32	0.0878	0.8645

Table 6.1: Simulation Results under Partitioned Control for the Base Problem

Model	**Parameter**	***SM***	***SSD***	***SCV***	***LF***
RLF with $SL_l = SL$ for all l	*SL*=0.86	71862.81	5962.27	0.0830	0.8756
	SL=0.87	71765.42	5796.44	0.0808	0.8817
	SL=0.88	71596.28	5555.78	0.0776	0.8893
	SL=0.89	71484.05	5327.96	0.0745	0.8980
	SL=0.90	71130.30	5097.01	0.0717	0.9037
	SL=0.91	70834.03	4861.06	0.0686	0.9101
	SL=0.92	70378.28	4501.83	0.0640	0.9208
	SL=0.93	69949.03	4216.93	0.0603	0.9314
	SL=0.94	69168.97	3788.52	0.0548	0.9400
	SL=0.95	68240.22	3378.41	0.0495	0.9475
	SL=0.96	66891.29	2824.70	0.0422	0.9601
LFR	*RL*=63000	62742.02	**1785.81**	**0.0285**	**0.9672**
	RL=64000	63712.43	1949.42	0.0306	0.9667
	RL=65000	64782.22	2185.34	0.0337	0.9653
	RL=66000	65733.78	2461.71	0.0374	0.9628
	RL=67000	66749.88	2798.81	0.0419	0.9588
	RL=68000	67822.02	3171.96	0.0468	0.9529
	RL=69000	68889.49	3601.72	0.0523	0.9447
	RL=70000	70015.13	4208.64	0.0601	0.9309
	RL=71000	71064.54	4986.30	0.0702	0.9104
RLF-M	*SL*=0.87	71910.69	6136.20	0.0853	0.8705
	SL=0.88	71797.86	5790.78	0.0807	0.8818
	SL=0.89	71620.12	5550.37	0.0775	0.8913
	SL=0.90	71370.99	5259.00	0.0737	0.9010
	SL=0.91	71043.47	4980.67	0.0701	0.9108
	SL=0.92	70617.57	4591.84	0.0650	0.9208
	SL=0.93	70015.13	4208.64	0.0601	0.9309
	SL=0.94	69346.81	3843.23	0.0554	0.9396
	SL=0.95	68345.44	3383.29	0.0495	0.9492
	SL=0.96	66887.69	2831.52	0.0423	0.9584
MaxminLF	*RL*=63000	63034.64	1888.84	0.0300	0.9655
	RL=64000	64289.75	2067.10	0.0322	0.9646
	RL=65000	64830.98	2232.53	0.0344	0.9640
	RL=66000	65802.96	2441.08	0.0371	0.9619

Table 6.1: Simulation Results under Partitioned Control for the Base Problem

Model	Parameter	*SM*	*SSD*	*SCV*	*LF*
	RL=67000	66701.67	2765.14	0.0415	0.9592
	RL=68000	67784.77	3194.22	0.0471	0.9527
	RL=69000	68851.15	3642.78	0.0529	0.9436
	RL=70000	69948.90	4263.56	0.0610	0.9284
	RL=71000	71061.16	5056.41	0.0712	0.9046

- As can be seen form Table 6.1, the highest *SM* value is obtained from the *EMR* model. The second highest *SM* value is obtained via *RLF-M* at $SL = 0.87$. As expected, *RLF-M* yields higher *SM* values than *RLF* (recall Remark 3.2.1). However, up to the service level 0.92, the *RLF* model gives close results to the risk sensitive models *EMVLP*, *SLP-RM* and *PMP-RC* in terms of *SM*. When the revenue level is at its upper bound, 71000, *LFR* outperforms *DLP*, *EMVLP* at $\theta = 0.005$ and *SLP-RM* at $L = 60000$ and $L = 65000$. The *SM* values obtained from *MaxminLF* are slightly higher than those of *LFR*. It is observed that the *SM* values obtained from our proposed risk sensitive models are quite close to those of existing risk sensitive models. It is also observed that *RLF* and *RLF-M* yield higher *SM* values as compared to *LFR* and *MaxminLF*.

- *LFR* yields the lowest *SSD* and *SCV* values among all other models in Table 6.1 when $RL = 63000$ at the cost of a considerable decrease in *SM* as compared to the highest *SM* obtained by *EMR*. Also for higher *RL* values, *LFR* outperforms all existing risk sensitive models in terms of variation of the revenue. *RLF* also performs satisfactorily in terms of variation as compared to other existing models. Especially, when $SL = 0.96$, *RLF* outperforms other existing models in the literature in terms of *SSD* and *SCV* values. It is observed that *LFR* is more effective in terms of risk sensitivity as compared to *RLF*. *RLF-M* and *MaxminLF* gives higher variation values than *RLF* and *LFR*, respectively. The highest variation is obtained via *PMP-RC*, when $L = 70000$, $\rho = 0.31$.

- *LFR* yields the highest *LF* value among all other models in Table 6.1 when

$RL = 63000$. *RLF* also performs more satisfactorily than the existing models in terms of load factor. Especially, at $SL = 0.96$, *RLF* yields higher *LF* value than all of the existing models. The maximum load factor value that can be obtained from other risk sensitive models, *EMVLP*, *SLP-RM* and *PMP-RC* is 0.9527. *RLF-M* and *MaxminLF* yield higher *LF* values than most of the existing models as *RLF* and *LFR* do. The poorest model in terms of load factor is *DLP*.

Depending on the above observations, *LFR* is found quite effective in terms of risk sensitivity as compared to other models including the risk sensitive ones. In addition, *RLF* seems to perform satisfactorily when compared to the existing models. It is especially promising that *RLF* gives better than or as satisfactory results as the risk sensitive models *EMVLP*, *SLP-RM* and *PMP-RC*. It is observed that *RLF* yields lower variation and higher load factor values than *RLF-M*. Similarly, *LFR* yields lower variation and higher load factor values than *MaxminLF*.

Under partitioned policy, the lowest *SSD* and *SCV* values and the highest *LF* value can be obtained by *LFR*, in case the sample mean of the revenue is sacrificed. However, also for higher threshold levels, variation and load factor values of *LFR* are close to and even mostly better than those of the existing risk sensitive models. The performance of *RLF* follows the peformance of *LFR* in terms of those performance measures. To sum up, all proposed models can be used as risk sensitive models that give reasonable revenue levels while keeping the variation at tolerable levels.

SM, *SCV* and *LF* values obtained by *RLF*, *RLF-M*, *LFR* and *MaxminLF* are presented in Figures 6.1, 6.2, 6.3, respectively. According to these figures it can be said that the models with a constraint on expected revenue (*LFR* and *MaxminLF*) may be preferable to those models with constraints on expected load factors (*RLF* and *RLF-M*) in terms of risk sensitivity since the former models provide lower variation but higher load factor values as compared to latter models. Moreover, it is observed that the same type of models exhibit a very similar behavior in terms of all performance measures.

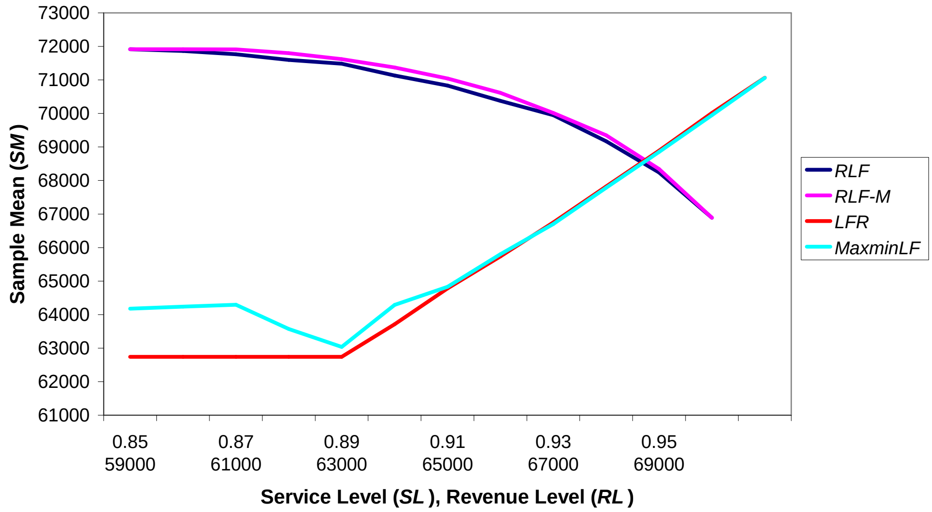

Figure 6.1: *SM* Values of Proposed Models under Partitioned Control

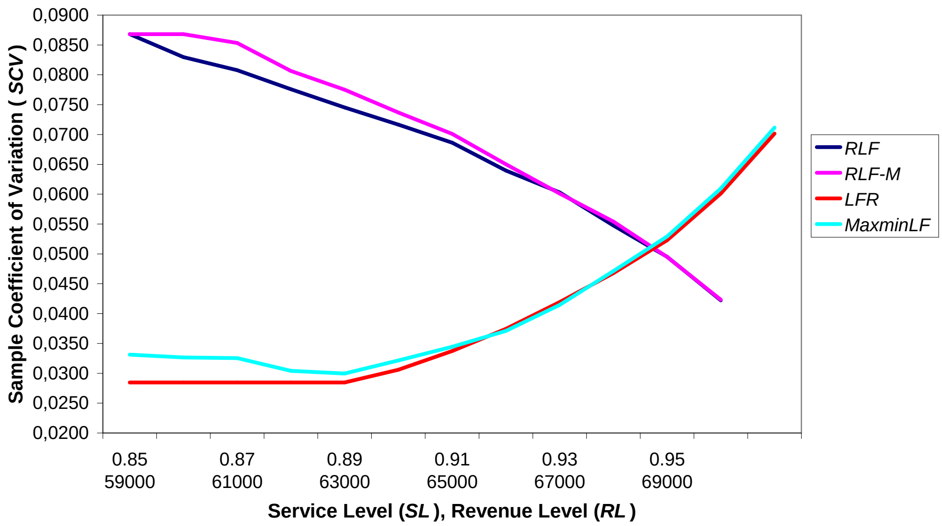

Figure 6.2: *SCV* Values of Proposed Models under Partitioned Control

6.1.2 Nested Control Policy

The simulation results for the nested booking policy are given in Table 6.2.

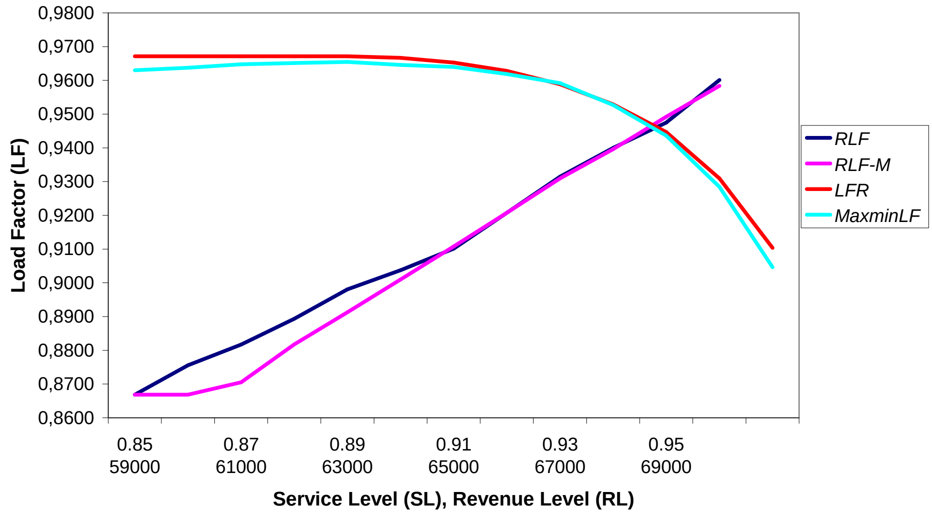

Figure 6.3: *LF* Values of Proposed Models under Partitioned Control

Table 6.2: Simulation Results under Nested Control for the Base Problem

Model	**Parameter**	***SM***	***SSD***	***SCV***	***LF***
DLP	-	**75857.76**	6791.64	0.0895	0.8964
EMR	-	74440.36	6965.04	0.0936	0.8799
SLP	-	74305.64	6507.56	0.0876	0.8924
EMVLP	$\theta = 0.001$	74683.00	6234.07	0.0835	0.9013
	$\theta = 0.005$	70575.22	3241.07	0.0459	0.9651
SLP-RM	L=60000	74896.23	5671.38	0.0757	0.9197
	L=65000	73757.56	4790.14	0.0649	0.9368
	L=70000	74900.44	5784.49	0.0772	0.9147
	L=75000	74841.38	6354.80	0.0849	0.8999
	L=80000	74503.56	6884.54	0.0924	0.8811
PMP-RC	L=70000, $\rho = 0.28$	74476.11	6895.56	0.0926	0.8815
	L=70000, $\rho = 0.29$	75391.19	6181.95	0.0820	0.9081
	L=70000, $\rho = 0.30$	74597.24	6670.12	0.0894	0.8890
	L=70000, $\rho = 0.31$	74273.41	7091.90	0.0955	0.8752
RLF with $SL_l = SL$ for all l	SL=0.86	74606.57	6643.54	0.0890	0.8897
	SL=0.87	74452.72	6437.31	0.0865	0.8950
	SL=0.88	74294.31	6152.78	0.0828	0.9014
	SL=0.89	74200.36	5877.27	0.0792	0.9089

Table 6.2: Simulation Results under Nested Control for the Base Problem

Model	**Parameter**	***SM***	***SSD***	***SCV***	***LF***
	SL=0.90	73752.97	5528.66	0.0750	0.9149
	SL=0.91	73442.41	5265.69	0.0717	0.9205
	SL=0.92	72965.35	4856.38	0.0666	0.9300
	SL=0.93	72380.52	4502.25	0.0622	0.9384
	SL=0.94	71727.05	4027.01	0.0561	0.9481
	SL=0.95	70974.14	3611.47	0.0509	0.9566
	SL=0.96	69771.81	3086.18	0.0442	0.9662
LFR	*RL*=63000	66759.56	**2378.37**	**0.0356**	**0.9794**
	RL=64000	67309.29	2416.94	0.0359	0.9772
	RL=65000	68121.30	2570.88	0.0377	0.9746
	RL=66000	68800.96	2760.44	0.0401	0.9714
	RL=67000	69622.02	3039.21	0.0437	0.9662
	RL=68000	70585.24	3420.18	0.0485	0.9599
	RL=69000	71660.40	3865.76	0.0539	0.9520
	RL=70000	72807.43	4590.77	0.0631	0.9379
	RL=71000	74051.76	5526.43	0.0746	0.9185
RLF-M	*SL*=0.87	74554.40	6830.99	0.0916	0.8843
	SL=0.88	74494.01	6440.26	0.0865	0.8950
	SL=0.89	74446.40	6131.53	0.0824	0.9038
	SL=0.90	74366.74	5811.00	0.0781	0.9125
	SL=0.91	73960.50	5499.54	0.0744	0.9190
	SL=0.92	73590.02	5043.76	0.0685	0.9292
	SL=0.93	72807.43	4590.77	0.0631	0.9379
	SL=0.94	72048.68	4147.41	0.0576	0.9471
	SL=0.95	71004.05	3629.13	0.0511	0.9564
	SL=0.96	69684.42	3071.09	0.0441	0.9657
MaxminLF	*RL*=63000	67354.51	2512.86	0.0373	0.9763
	RL=64000	68287.11	2538.66	0.0372	0.9755
	RL=65000	68510.92	2622.73	0.0383	0.9743
	RL=66000	68925.16	2787.03	0.0404	0.9716
	RL=67000	69615.56	3023.40	0.0434	0.9667
	RL=68000	70423.45	3399.30	0.0483	0.9597
	RL=69000	71394.54	3873.27	0.0543	0.9504
	RL=70000	72425.71	4539.91	0.0627	0.9374

Table 6.2: Simulation Results under Nested Control for the Base Problem

Model	Parameter	*SM*	*SSD*	*SCV*	*LF*
	RL=71000	73643.61	5491.33	0.0746	0.9156

- The *SM* value of all models increases under nested control as compared to partitioned control. The highest *SM* value is obtained by *DLP*. This is in accordance with the observation due to Williamson (1992), which states that *DLP* outperforms the probabilistic models in terms of obtained revenue since both deterministic and probabilistic models ignore a nested environment. Up to $SL = 0.92$, *RLF* and *RLF-M* give close *SM* values to existing risk sensitive models. For those *SL* values, the variation and load factor values of *RLF* and *RLF-M* are also close to those of the existing models. *LFR* yields lower *SM* values as compared to other risk sensitive models even at its highest *SM* value when $RL = 71000$. Under nested control, *RLF* yields higher *SM* values than *LFR* as in the case of partitioned control. *RLF-M* and *MaxminLF* give close results to *RLF* and *LFR*, respectively.

- Under nested control, the *SSD* and *SCV* values of all models increase as compared to partitioned control. The lowest *SSD* and *SCV* values are obtained by *LFR* when $RL = 63000$. However, also for higher *RL* requirements, *LFR* gives close or better *SSD* and *SCV* values as compared to existing risk sensitive models. *RLF* also yields satisfactory variation values especially for $SL > 0.91$.

- Under nested control, the *LF* values of all models increase as compared to partitioned control. The highest *LF* value is obtained by *LFR* when $RL = 63000$. For higher *RL* requirements, the load factor values obtained from *LFR* are still satisfactory as compared to other models. *RLF* also yields satisfactory *LF* values at reasonable *SM* values as compared to other existing models.

According to the simulation results, it can be said that for risk-averse decision makers *LFR* and *MaxminLF* are preferable to *RLF* and its variation *RLF-M* since they yield lower variation and higher load factor values. Note that *RLF* and *RLF-M* yield higher *SM* values than *LFR* and *MaxminLF*, respectively.

When we compare our proposed models to the existing models in the literature, it can be said that all of the four proposed models give satisfactory results in terms of almost all performance measures. Moreover, the fact that the models we propose are solved much easier than the other risk sensitive models in the literature is an advantage. The CPU times of the models we propose and those of existing models are given in Table 6.7. In summary, it can be said that the proposed models do not lead to an unacceptable result when compared to other models and give nearly close results to that of *EMVLP*, *SLP-RM* and *PMP-RC*.

SM, *SCV* and *LF* values obtained by *RLF*, *RLF-M*, *LFR* and *MaxminLF* are presented in Figures 6.4, 6.5, 6.6, respectively.

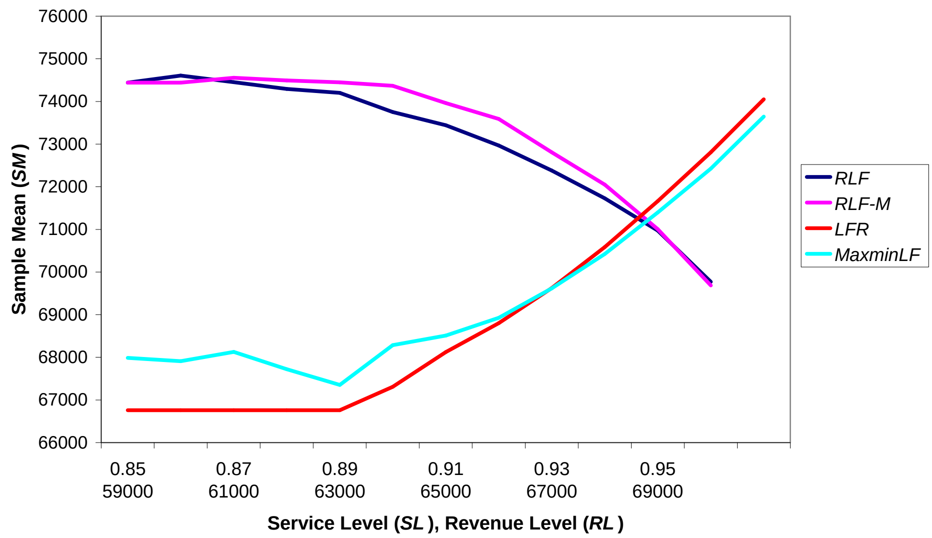

Figure 6.4: *SM* Values of Proposed Models under Nested Control (Base Problem)

6.1.3 Bid Price Control Policy

Different than partitioned and nested controls, the input for the bid price control to be evaluated in the simulation model is the bid prices obtained by solving the optimization models. Note that bid price control can not be considered for *LFR* and

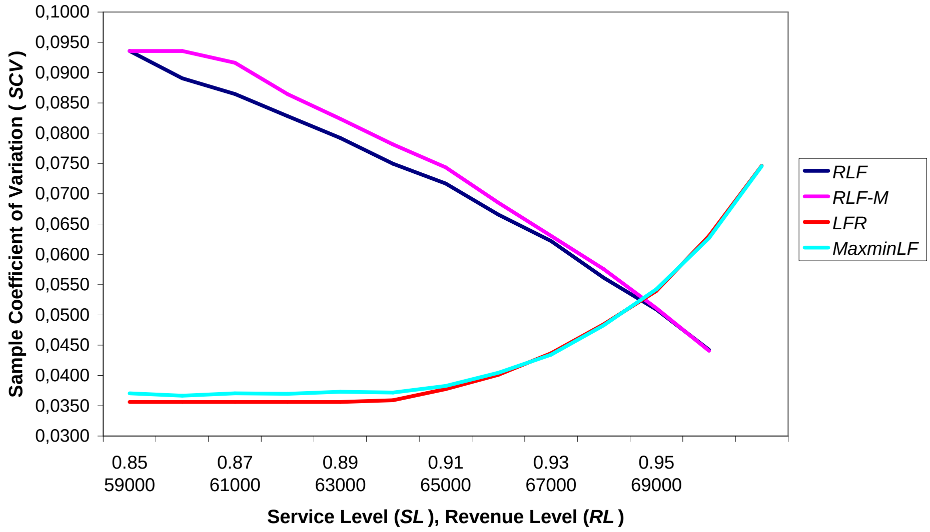

Figure 6.5: *SCV* Values of Proposed Models under Nested Control (Base Problem)

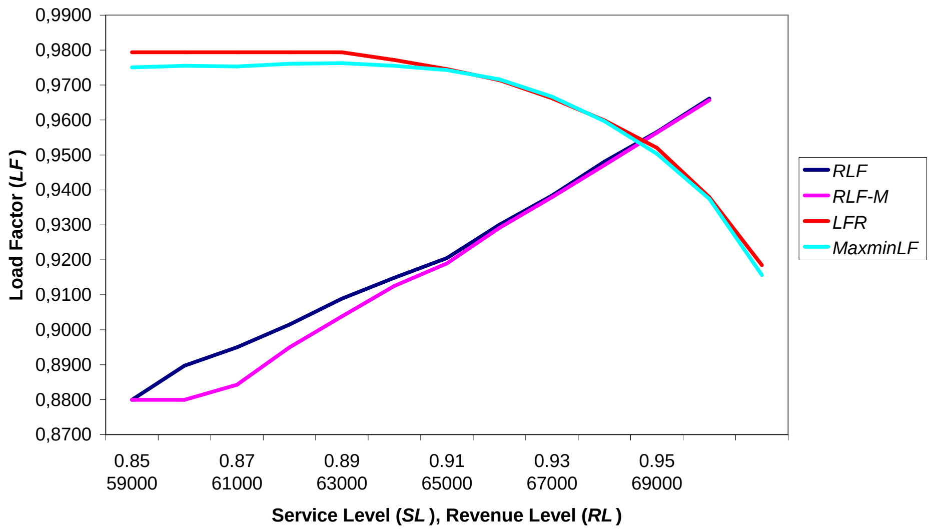

Figure 6.6: *LF* Values of Proposed Models under Nested Control (Base Problem)

MaxminLF because the objective in these models is not revenue maximization. That is, bid prices (dual variables) in *LFR* and *MaxminLF* are not comparable with the fares of the products.

In this section, the *RLF* and *RLF-M* models are compared with *DLP*, *EMR*, *SLP*, *SLP-RM*, *RLP* models and *RRS* Procedure. *RLP* and *RRS* Procedure are due to Terciyanlı

(2009). The simulation results for the bid price control are given in Table 6.3.

Table 6.3: Simulation Results under Bid-Price Control for the Base Problem

Model	**Parameter**	***SM***	***SSD***	***SCV***	***LF***
DLP	-	72878.75	4523.30	0.0630	0.9551
EMR	-	72878.75	4523.30	0.0630	0.9551
SLP	-	72878.75	4523.30	0.0630	0.9551
EMVLP	$\theta = 0.001$	66896.50	3789.70	0.0550	0.9797
	$\theta = 0.005$	59843.30	**3000.24**	**0.0525**	**0.9942**
SLP-RM	*L*=60000	58060.85	12671.35	0.2182	0.5200
	L=65000	73846.35	7018.79	0.0955	0.8641
	L=70000	59843.85	**3125.44**	**0.0526**	**0.9942**
	L=75000	72118.35	8182.59	0.1132	0.8246
	L=80000	73846.41	7018.29	0.0952	0.8641
RLP	-	**75968.29**	5868.81	0.0772	0.9289
RRS	$L < 75000$	72878.75	4523.30	0.0630	0.9551
	$L > 75000$	**75968.21**	5868.81	0.0772	0.9291
	$L < 80000$	72878.75	4523.30	0.0630	0.9551
	$L > 80000$	**75968.21**	5868.81	0.0772	0.9291
	$L < 85000$	**75968.20**	5868.81	0.0772	0.9291
	$L > 85000$	**75968.20**	5868.81	0.0772	0.9291
	$L < 90000$	**75968.20**	5868.81	0.0772	0.9291
	$L > 90000$	71942.30	8272.50	0.1159	0.8269
RLF with $SL_l = SL$ for all l	*SL*=0.86	72878.75	4523.30	0.0630	0.9551
	SL=0.87	72878.71	4523.30	0.0630	0.9551
	SL=0.88	72878.71	4523.30	0.0630	0.9551
	SL=0.89	72878.71	4523.30	0.0630	0.9551
	SL=0.90	**75968.21**	5868.81	0.0772	0.9291
	SL=0.91	**75968.21**	5868.81	0.0772	0.9291
	SL=0.92	**75968.21**	5868.81	0.0772	0.9291
	SL=0.93	**75968.21**	5868.81	0.0772	0.9291
	SL=0.94	72878.75	4523.30	0.0630	0.9551
	SL=0.95	59843.85	**3125.44**	**0.0526**	**0.9942**
	SL=0.96	59843.85	**3125.44**	**0.0526**	**0.9942**
RLF-M	*SL*=0.87	**72878.75**	4523.30	0.0630	0.9551
	SL=0.88	72878.75	4523.30	0.0630	0.9551

Table 6.3: Simulation Results under Bid-Price Control for the Base Problem

Model	**Parameter**	***SM***	***SSD***	***SCV***	***LF***
	SL=0.89	72878.75	4523.30	0.0630	0.9551
	SL=0.90	72878.75	4523.30	0.0630	0.9551
	SL=0.91	72878.75	4523.30	0.0630	0.9551
	SL=0.92	75968.21	5868.81	0.0772	0.9291
	SL=0.93	72878.75	4523.30	0.0630	0.9551
	SL=0.94	72878.75	4523.30	0.0630	0.9551
	SL=0.95	66486.35	3723.25	0.0571	0.9842
	SL=0.96	59843.85	**3125.44**	**0.0526**	**0.9942**

Different from partitioned and nested controls, a number of alternative models yield the highest sample mean, the lowest variation and the highest load factor values. *RLF* is one of those models that yield the highest sample mean. Contrary to the partitioned and nested controls, existing risk-sensitive models *EMVLP* and *SLP-RM* yield the lowest variation and the highest load factor values as *RLF* does. That is, *EMVLP* and *SLP-RM* improve their performance in terms of risk sensitivity under bid price control. *EMVLP*, *SLP-RM*, *RLF* and *RLF-M* reach to their utmost load factor value under bid price control for some instances. It is observed that there is not a significant decrase in *SM* values for *RLF* and *RLF-M* as *SL* increases. Different than partitioned and nested controls, there is not a continuous trend in the performance measures obtained by bid price control for *RLF* and *RLF-M*. It is observed that for all performance measures, the values obtained by bid price control for *RLF* and *RLF-M* display a step-wise behaviour.

According to the simulation results, it can be said that *RLF* and *RLF-M* perform satisfactorily under bid price control when compared to other models in the literature.

6.1.4 Comparison of the Control Policies

- When the partitioned, nested and bid price controls are evaluated, it is observed that the nested control gives higher sample mean than partitioned control for all models. Bid price control yields higher sample means than the partitioned control policy for *DLP*, *EMR* and *SLP*. However, for *EMVLP*, partitioned control performs better than bid price control in terms of sample mean. For *SLP-RM*, bid price control policy gives higher sample mean than the partitioned control at some revenue levels. Bid price control does not a have continious trend in sample mean. It displays a step-wise behaviour. For high service levels, bid price control policy displays a sharply decreasing trend in sample mean values for *RLF* and *RLF-M* since the opportunity cost of the products increase as service level increases.

- For all models, nested booking limit control policy yields higher sample standard deviation and sample coefficient of variation than partitioned control. Bid price control policy yields lower sample standard deviation and sample coefficient of variation values for *DLP*, *EMR* and *SLP* as compared to partitioned and nested booking control limit policies. Such a deduction is not valid for *RLF* and *RLF-M* since the values of the performance measures obtained by bid price control exhibit an inconsistent behaviour.

- Bid price control policy yields the highest values for the load factor for all models.

- The behavior of *RLF-M* is quite similar to that of *RLF* under different control policies. Similarly, *MaxminLF* exhibits a very close behavior to *LFR* under different control policies.

The comparison of partitioned, nested and bid price controls for *SM*, *SSD*, *SCV* and *LF* values of *RLF* is presented in Figures 6.7, 6.8, 6.9, 6.10, respectively. A similar comparison for *LFR* is presented in Figures 6.11, 6.12, 6.13, 6.14, respectively.

6.2 Increased Variance of Low Fare Demand

In the base problem considered in Chapter 5 and in Section 6.1, the demand variance of high fare class (class 1) is relatively higher than the demand variances of two other low fare classes (class 2 and 3). de Boer (1999) considers increases in the demand

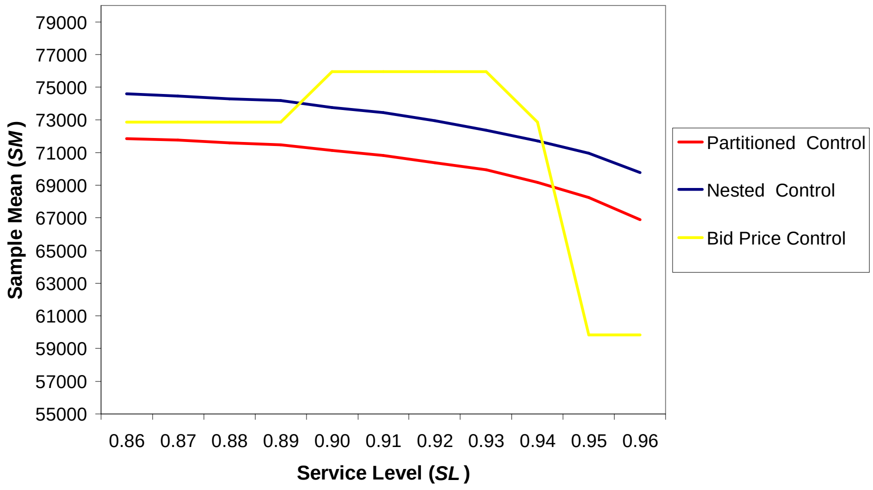

Figure 6.7: *SL* versus *SM* for alternative control polices (*RLF*)

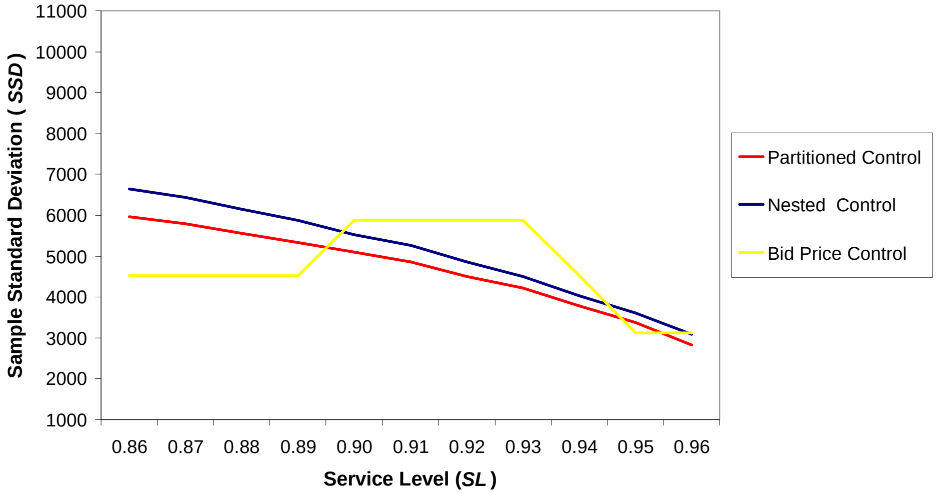

Figure 6.8: *SL* versus *SSD* for alternative control polices (*RLF*)

variance of low fare classes according to the studies of Belobaba (1987). The data for the "increased variance of low fare demand" case is given in Appendix A. The seat allocations obtained from *RLF*, *RLF-M*, *LFR* and *MaxminLF* to be used in the simulation model for nested control are provided in Appendix C.

Recall that, in Section 5.5, the intervals for threshold levels to be used in *RLF* with

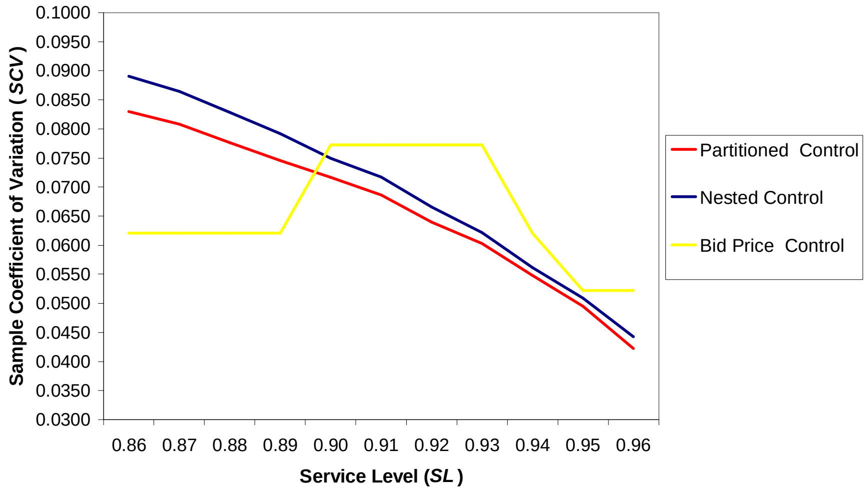

Figure 6.9: *SL* versus *SCV* alternative control polices (*RLF*)

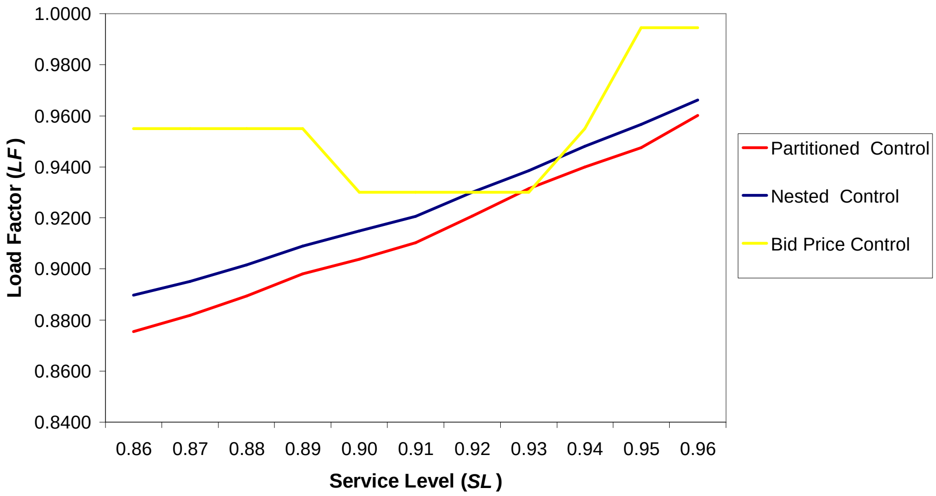

Figure 6.10: *SL* versus *LF* for alternative control polices (*RLF*)

$SL_l = SL$, *RLF-M*, *LFR* and *MaxminLF* are provided in Table 5.11 for different scenarios by solving the bound models proposed for them. The simulation intervals of the proposed models in this section are in accordance with Table 5.11.

Increasing low-fare demand variance results in an increase in deviations from the mean for low-fare demands. Therefore, we expect that there is a decrease in the

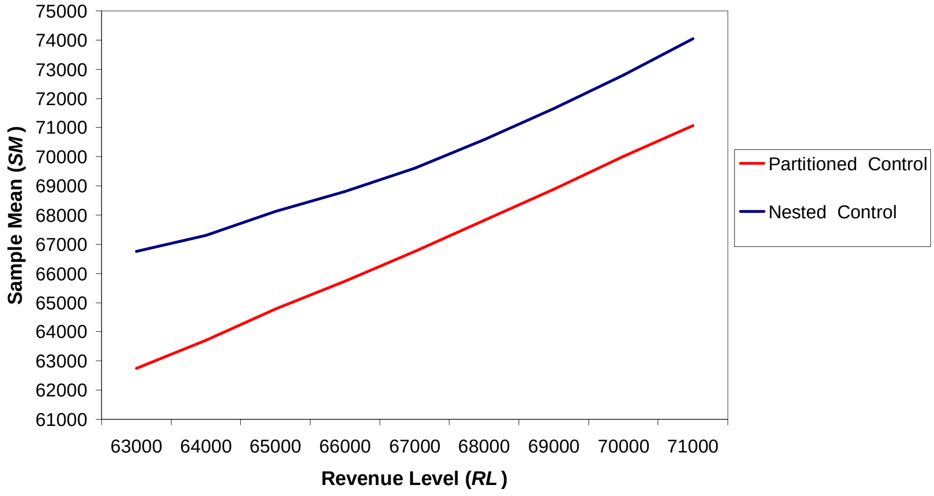

Figure 6.11: *RL* versus *SM* for alternative control polices (*LFR*)

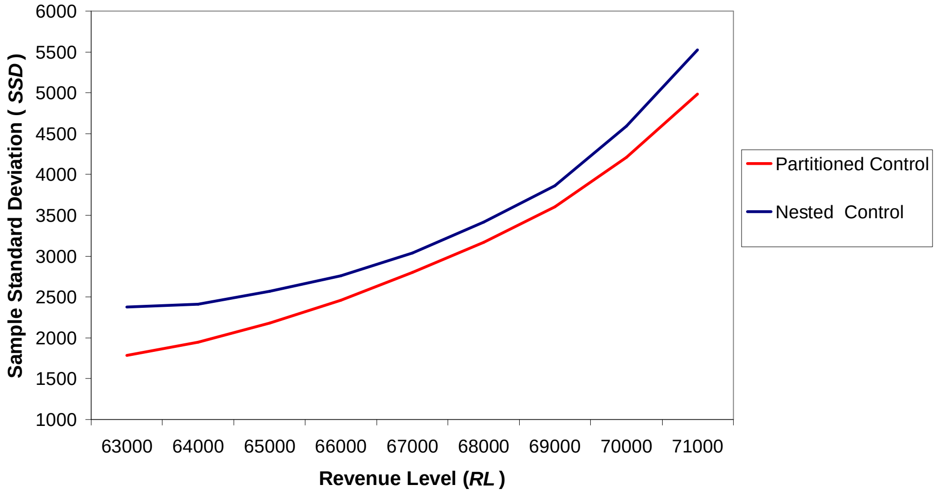

Figure 6.12: *RL* versus *SSD* for alternative control polices (*LFR*)

number of seats allocated for low-fare classes. As it is expected, there is a slight decrease in seat allocations for low-fare classes. The decrease in low-fare class seat allocations results in slight decreases in revenue and load factors for most of the models. The simulation results under nested control are provided in Table 6.4.

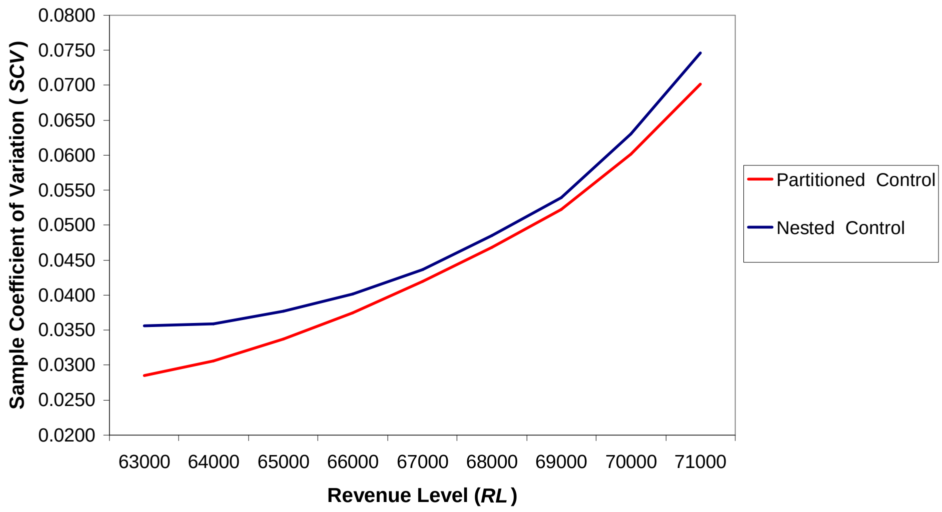

Figure 6.13: *RL* versus *SCV* for alternative control polices (*LFR*)

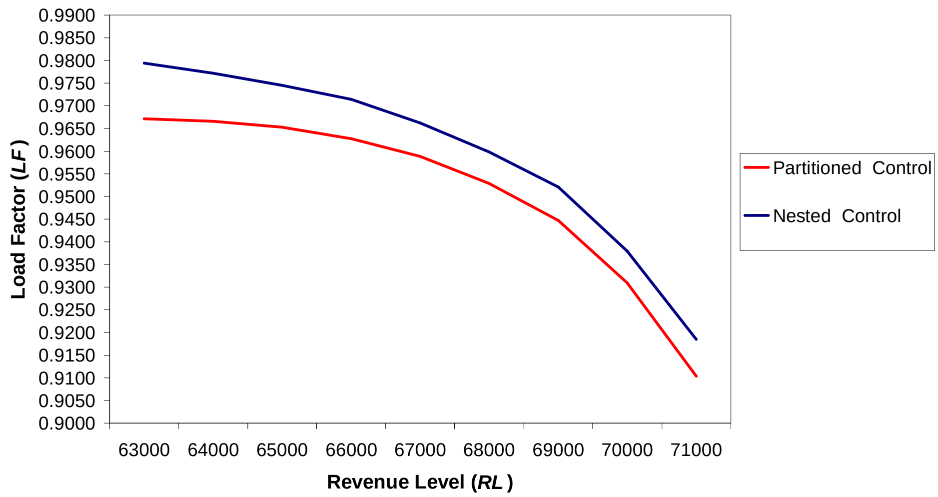

Figure 6.14: *RL* versus *LF* for alternative control polices (*LFR*)

Table 6.4: Simulation Results under Nested Control for “Increased Variance of Low Fare Demand” Case

Model	Parameter	*SM*	*SSD*	*SCV*	*LF*
DLP	-	**76002.79**	6836.85	0.0900	0.8975
EMR	-	74380.88	7257.46	0.0976	0.8730

Table 6.4: Simulation Results under Nested Control for "Increased Variance of Low Fare Demand" Case

Model	Parameter	*SM*	*SSD*	*SCV*	*LF*
SLP	-	74250.58	6689.30	0.0901	0.8879
EMVLP	θ = 0.001	74212.16	6189.47	0.0834	0.9036
	θ = 0.005	70357.69	3311.78	0.0471	0.9634
SLP-RM	*L*=60000	73331.47	5279.22	0.0720	0.9230
	L=65000	74099.97	5257.27	0.0709	0.9274
	L=70000	74225.44	5667.37	0.0764	0.9161
	L=75000	74501.94	6462.15	0.0867	0.8964
	L=80000	74596.30	7263.08	0.0974	0.8755
RLF with $SL_l = SL$	*SL*=0.85	73987.73	6639.93	0.0897	0.8899
	SL=0.86	73882.44	6421.13	0.0869	0.8960
	SL=0.87	73604.36	6145.68	0.0835	0.9029
	SL=0.88	73325.62	5916.14	0.0807	0.9076
	SL=0.89	73072.14	5618.67	0.0769	0.9151
	SL=0.90	72652.86	5272.76	0.0726	0.9227
	SL=0.91	72313.48	4919.06	0.0680	0.9302
	SL=0.92	71755.78	4497.68	0.0627	0.9392
	SL=0.93	71071.63	3950.83	0.0556	0.9498
	SL=0.94	69562.20	3118.99	0.0448	0.9648
LFR	*RL*=64000	68155.82	**2572.29**	**0.0377**	**0.9733**
	RL=65000	68881.16	2775.95	0.0403	0.9689
	RL=66000	69367.00	3034.47	0.0437	0.9650
	RL=67000	70279.34	3416.14	0.0486	0.9589
	RL=68000	71093.92	3932.35	0.0553	0.9505
	RL=69000	72008.09	4641.53	0.0645	0.9377
	RL=70000	73199.73	5671.21	0.0775	0.9151
RLF-M	*SL*=0.85	74319.87	7032.83	0.0946	0.8795
	SL=0.86	74163.08	6488.51	0.0875	0.8952
	SL=0.87	73885.73	6136.53	0.0831	0.9046
	SL=0.88	73493.74	5872.42	0.0799	0.9114
	SL=0.89	73493.74	5872.42	0.0799	0.9114
	SL=0.90	73082.63	5427.17	0.0743	0.9212
	SL=0.91	72418.65	4998.14	0.0690	0.9308
	SL=0.92	71906.64	4525.48	0.0629	0.9396

Table 6.4: Simulation Results under Nested Control for "Increased Variance of Low Fare Demand" Case

Model	**Parameter**	***SM***	***SSD***	***SCV***	***LF***
	SL=0.93	71093.92	3932.35	0.0553	0.9505
	SL=0.94	69698.21	3130.10	0.0449	0.9641
MaxminLF	*RL*=63000	68493.57	2684.24	0.0392	0.9724
	RL=64000	68493.57	2684.24	0.0392	0.9724
	RL=65000	68908.07	2805.37	0.0407	0.9693
	RL=66000	69562.20	3118.99	0.0448	0.9648
	RL=67000	70291.58	3463.12	0.0493	0.9583
	RL=68000	71071.63	3950.83	0.0556	0.9498
	RL=69000	71921.21	4629.06	0.0644	0.9362
	RL=70000	73121.15	5722.90	0.0783	0.9129

The following observations are made based on the comparison with the base problem.

- The *SM* values in Table 6.4 are smaller than the *SM* values in Table 6.2 for all of the models except *LFR* and *MaxminLF*. In addition, *DLP* and *SLP-RM* at $L = 65$ and $L = 80$ show an increase in *SM* values as compared to the base problem. The highest *SM* value is obtained by *DLP* as in the case of base problem.

- The *LF* values decrease for almost all of the existing models as compared to the base problem. Only *DLP*, *EMVLP* and *SLP-RM* show an increase in *LF* values at particular instances. The same decrease in the *LF* values is observed also for *LFR* and *MaxminLF* at all *RL* values. On the contrary, *LF* values for *RLF* and *RLF-M* show an increase at all *SL* values as compared to base problem.

- The *SSD* values increase for *DLP*, *SLP*, *EMR* and for all *RL* values of the proposed models *LFR* and *MaxminLF* as compared to the base problem. On the contrary, *EMVLP* and *SLP-RM* show a slight decrease in *SSD* for some instances. The proposed models *RLF* and *RLF-M* have also a decrease in *SSD* values for all *SL* values. *SCV* values increase for almost all models except *RLF* and *RLF-M*.

In case the variance of low fare classes increases, the performances of the proposed models *RLF*, *RLF-M* and *LFR*, *MaxminLF* change in opposite directions. It can be seen that the performance of *RLF* and its variation *RLF-M* improves in terms of risk sensitivity when the variance of low fare classes increases as compared to the base problem. Although the performance of *LFR* and *MaxminLF* is worse in this sense as compared to base problem, they still provide the lowest variance and highest *LF* values among all other models at *RL* = 63000.

SCV and *LF* values obtained by *RLF*, *RLF-M*, *LFR* and *MaxminLF* are presented in Figures 6.15 and 6.16, respectively.

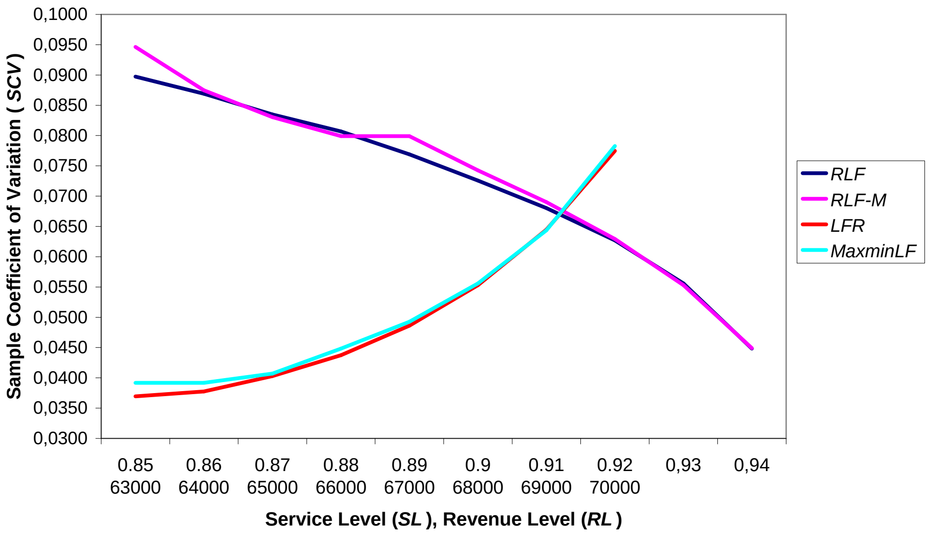

Figure 6.15: *SCV* Values of Proposed Models under Nested Control (Increased Variance of Low Fare Demand)

6.3 Smaller Differences between Fares

de Boer (1999) considers decreasing the difference between the fare classes and investigates the impact of this change on the performances of the models in the literature. de Boer (1999) states that decreasing differences between fare classes improves

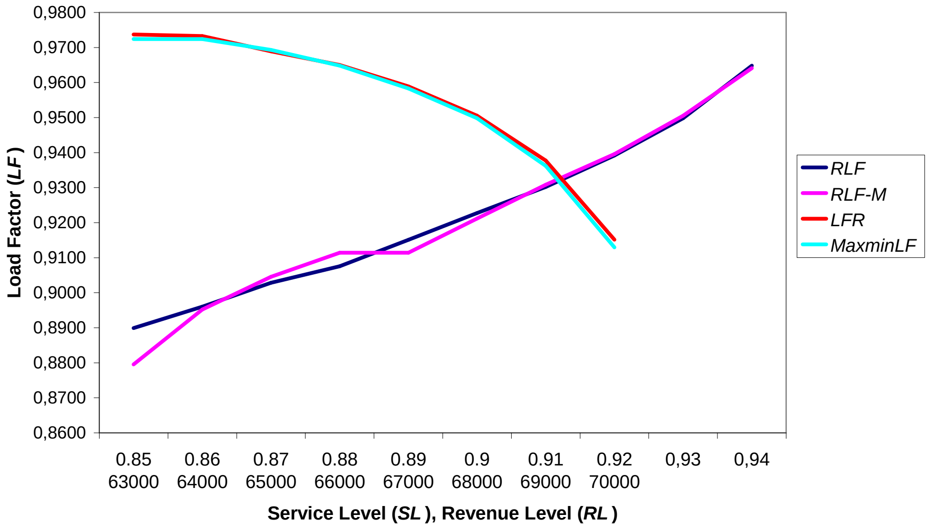

Figure 6.16: *LF* Values of Proposed Models under Nested Control (Increased Variance of Low Fare Demand)

the results of the *EMR*-based stochastic models as compared to the deterministic *DLP* model. The data for the "smaller differences between fares" case is given in Appendix A.

The decrease in difference between fare classes is obtained by decreasing the high fares. This situation affects the seat allocations of high fare classes in a negative way since it is now less attractive to allocate a seat for high fare classes with lower fares. The seat allocations obtained by *RLF* with $SL_l = SL$, *RLF-M*, *LFR* and *MaxminLF* to be used in the simulation model for nested booking limit control policy are provided in Appendix C.

Decreasing the difference between the fare classes results in a decrease both in the expected revenue and in the standard deviation of revenue. That is, more seats are allocated to the low fare classes that have small variations in demand when compared to the high fare classes.

The simulation intervals of the proposed models in this section are in accordance with Table 5.11. That is, the service levels, the corresponding seat allocations **of which** are used in the simulation are chosen from the interval in Table 5.11. The simulation results under nested control are provided in Table 6.5.

Table 6.5: Simulation Results under Nested Control for “Smaller Differences between Fares” Case

Model	***Parameter***	***SM***	***SSD***	***SCV***	***LF***
DLP	-	**75596.06**	6698.37	0.0886	0.8993
EMR	-	74084.68	5740.71	0.0775	0.9169
SLP	-	73085.45	4920.56	0.0673	0.9366
EMVLP	$\theta = 0.001$	73191.81	4880.64	0.0667	0.9369
	$\theta = 0.005$	69622.04	3086.56	0.0443	0.9704
SLP-RM	L=60000	73116.35	4296.74	0.0588	0.9500
	L=65000	74571.03	5536.01	0.0742	0.9231
	L=70000	73384.76	5985.59	0.0816	0.9097
	L=75000	73085.45	4920.56	0.0673	0.9366
	L=80000	73085.45	4920.56	0.0673	0.9366
RLF with $SL_l = SL$	$SL = 0.91$	73016.15	4869.91	0.0667	0.9367
	$SL = 0.92$	72589.56	4690.57	0.0646	0.9400
	$SL = 0.93$	71972.61	4338.55	0.0603	0.9465
	$SL = 0.94$	71179.26	3986.76	0.0560	0.9532
	$SL = 0.95$	70256.24	3598.67	0.0512	0.9603
	$SL = 0.96$	69122.97	3087.31	0.0447	0.9691
LFR	$RL = 57000$	66329.71	**2394.41**	**0.0361**	**0.9796**
	$RL = 57500$	67052.31	2471.64	0.0369	0.9779
	$RL = 58000$	67599.18	2619.97	0.0388	0.9758
	$RL = 58500$	68268.67	2822.63	0.0413	0.9730
	$RL = 59000$	69204.05	3125.23	0.0452	0.9681
	$RL = 59500$	70282.76	3526.99	0.0502	0.9618
	$RL = 60000$	71538.23	4080.39	0.0570	0.9524
	$RL = 60500$	73334.48	5168.58	0.0705	0.9298
RLF-M	$SL = 0.90$	73673.60	5519.94	0.0749	0.9212
	$SL = 0.91$	73344.27	5136.70	0.0700	0.9307
	$SL = 0.92$	72796.03	4781.22	0.0657	0.9385
	$SL = 0.93$	72358.77	4452.09	0.0615	0.9453
	$SL = 0.94$	71573.00	4082.54	0.0570	0.9525
	$SL = 0.95$	70506.70	3563.68	0.0505	0.9614
	$SL = 0.96$	68849.74	3044.20	0.0442	0.9691
MaxminLF	$RL = 56500$	67626.74	2585.23	0.0382	0.9777
	$RL = 57000$	67766.29	2583.86	0.0381	0.9780
	$RL = 57500$	68024.94	2631.50	0.0387	0.9774

Table 6.5: Simulation Results under Nested Control for "Smaller Differences between Fares" Case

Model	***Parameter***	***SM***	***SSD***	***SCV***	***LF***
	RL = 58000	68055.21	2696.66	0.0396	0.9755
	RL = 58500	68612.29	2904.55	0.0423	0.9722
	RL = 59000	69329.12	3180.86	0.0459	0.9678
	RL = 59500	70187.45	3499.74	0.0499	0.9622
	RL = 60000	71604.07	4141.62	0.0578	0.9506
	RL = 60500	73453.34	5294.98	0.0721	0.9268

- For all of the models in the literature and the proposed models, *SM* values decrease as compared to the base problem. In addition, the "smaller differences between fares" case provides lower *SM* values than the "increased variance of low fare demand" case for almost all models except the proposed models *RLF* and *RLF-M*. As in the case of the previous scenarios, *DLP* yields the highest *SM* value in accordance with the observation due to Williamson (1992).
- The *SSD* and *SCV* values decrease for all of the models as compared to the base problem. The *SSD* and *SCV* values obtained under this case are also lower than the "increased variance of low fare demand" case except for the proposed models *RLF* and *RLF-M* up to $SL = 0.92$.
- The load factor values increase for all of the models as compared to the base problem.

According to the above observations, it can be stated that the performances of all of the models improve in terms of risk sensitivity in case the difference between the fares is lowered. Although the *SM* values decrease for all models, a decrease in variance and an increase in *LF* values are obtained for all models.

Under nested control, the highest *LF* values and the lowest *SSD* and *SCV* values are obtained by *LFR* at $RL = 57000$. Also for higher *RL* values, *LFR* gives satisfactory results in terms of risk sensitivity. The revenue obtained from *LFR*, however, is less than but quite close to the revenue values of other models in the literature. The performance of *MaxminLF* follows the performance of *LFR* in terms of effectiveness in risk sensitivity. In addition, *RLF* gives results close to those of other models for all performance measures. The gap between the lowest *SM* value of *RLF* and the *SM* value of other models is not high. Therefore, *RLF* is also preferable by taking its ease of use into consideration. The performance of *RLF-M* follows the performance of *RLF*.

SCV and *LF* values obtained by *RLF*, *RLF-M*, *LFR* and *MaxminLF* are presented in Figures 6.17 and 6.18, respectively.

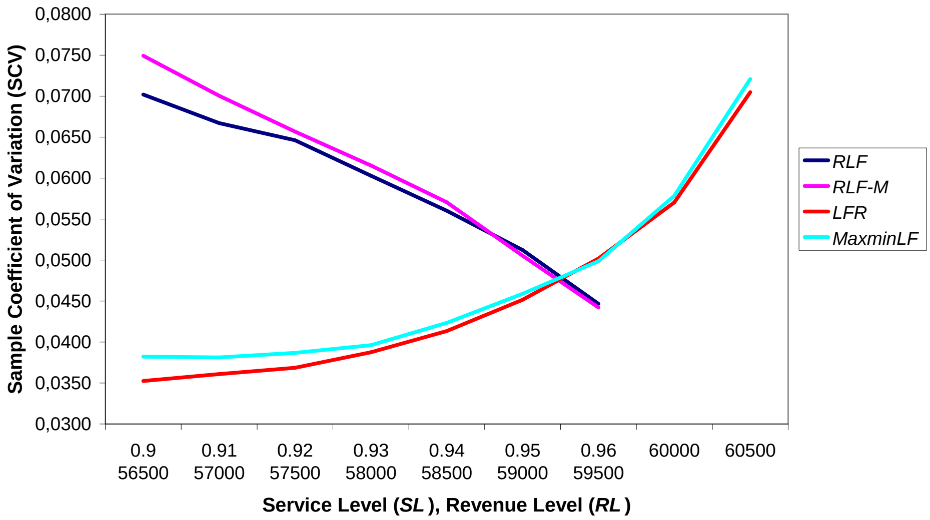

Figure 6.17: *SCV* Values of Proposed Models under Nested Control (Smaller Differences between Fares)

6.4 Realistic Variations and Close Fares

In this section, the case of realistic coefficients of variation of demand and relatively close fares is considered. Moreover, no specific order of arrivals is assumed for this case different than the base problem. According to the study of de Boer (1999), no specific order of arrivals favors the occurrence of nesting. Recall that the decrease of difference between fare classes is studied in Section 6.3. In addition to this decrease between fare classes, the coefficient of variation of demand is set almost equal to

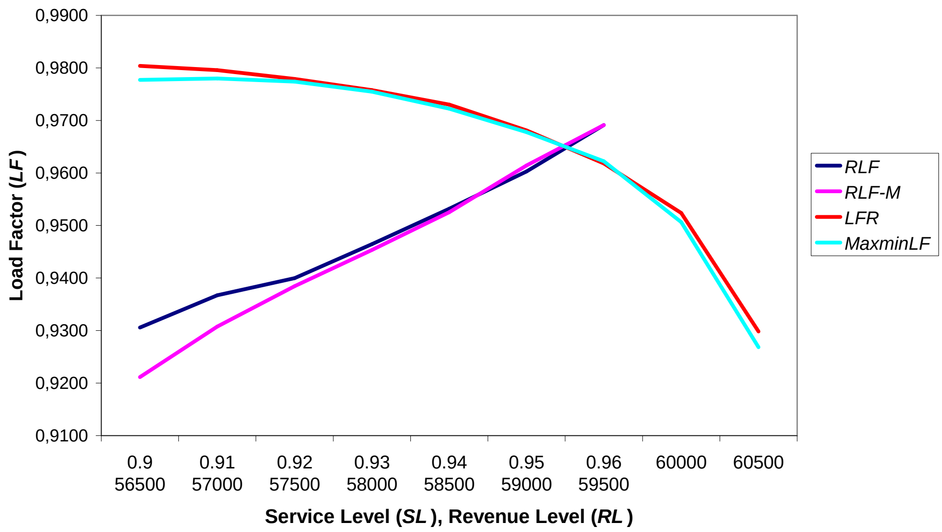

Figure 6.18: *LF* Values of Proposed Models under Nested Control (Smaller Differences between Fares)

0.33 for all fare classes, which is stated as the realistic case by Belobaba (1987). As Terciyanlı (2009) states in his study, this case is best suited to small-scaled airlines which have relatively close fares and no specific order of arrival. The data for the "realistic variations and close fares" case is given in Appendix A.

The seat allocations obtained by *RLF* with $SL_l = SL$, *RLF-M*, *LFR* and *MaxminLF* to be used in the simulation model for nested control are provided in Appendix C. The simulation intervals of the proposed models in this section are in accordance with Table 5.11. The simulation results under nested control are provided in Table 6.6.

Table 6.6: Simulation Results under Nested Control for "Realistic Variations and Close Fares" Case

Model	***Parameter***	***SM***	***SSD***	***SCV***	***LF***
DLP	-	**75343.68**	7034.09	0.0934	0.8803
EMR	-	74248.70	7143.41	0.0962	0.8663
SLP	-	73447.83	6353.36	0.0865	0.8845
EMVLP	$\theta = 0.001$	73935.70	6592.00	0.0892	0.8806
	$\theta = 0.005$	72988.79	5485.27	0.0752	0.9076
SLP-RM	*L*=60000	73447.83	6353.36	0.0865	0.8845
	L=65000	74164.57	6270.87	0.0846	0.8943
	L=70000	73450.65	6522.32	0.0888	0.8790

Table 6.6: Simulation Results under Nested Control for "Realistic Variations and Close Fares" Case

Model	***Parameter***	***SM***	***SSD***	***SCV***	***LF***
	L=75000	74324.70	7496.49	0.1009	0.8552
	L=80000	73447.83	6353.36	0.0865	0.8845
RLF with $SL_l = SL$	*SL*=0.9250	73546.19	5810.37	0.0790	0.9102
	SL=0.93	73323.10	5576.89	0.0761	0.9169
	SL=0.9350	73069.17	5474.81	0.0749	0.9191
	SL=0.94	72698.99	5361.55	0.0737	0.9222
	SL=0.9450	72360.28	5135.74	0.0710	0.9276
	SL=0.95	72146.83	4987.59	0.0691	0.9313
	SL=0.9550	71568.15	4739.83	0.0662	0.9367
	SL=0.96	71208.63	4514.91	0.0634	0.9421
LFR	*RL*=62500	69223.05	**3495.69**	**0.0505**	**0.9594**
	RL=63000	69618.54	3744.31	0.0538	0.9558
	RL=63500	70128.55	4029.94	0.0575	0.9501
	RL=64000	70761.42	4363.99	0.0617	0.9439
	RL=64500	71651.95	4785.42	0.0668	0.9354
	RL=65000	73630.12	5676.47	0.0771	0.9150
RLF-M	*SL*=0.92	73682.98	5907.07	0.0802	0.9080
	SL=0.9250	73682.98	5907.07	0.0802	0.9080
	SL=0.93	73682.98	5907.07	0.0802	0.9080
	SL=0.9350	73658.38	5692.55	0.0773	0.9145
	SL=0.94	73251.93	5459.93	0.0745	0.9196
	SL=0.9450	72695.02	5258.76	0.0723	0.9246
	SL=0.95	72264.83	4957.56	0.0686	0.9315
MaxminLF	*RL*=62000	70209.31	3829.56	0.0545	0.9543
	RL=62500	70197.56	3885.31	0.0553	0.9538
	RL=63000	70398.38	3881.66	0.0551	0.9542
	RL=63500	70463.40	4098.20	0.0582	0.9500
	RL=64000	70929.82	4384.34	0.0618	0.9445
	RL=64500	71790.05	4901.82	0.0683	0.9329
	RL=65000	73546.19	5810.37	0.0790	0.9102

- For all of the existing models, *SM* values decrease when compared to the base problem and the case of increased variance of low fare demand. On the other hand, *SM* values of the proposed models *RLF*, *RLF-M*, *LFR* and *MaxminLF* increase in the scenario under consideration. That is, this scenario contributes to the performance of the proposed models in terms of *SM* values. Note that this scenario provides higher *SM* values as compared to the case of smaller differences between fares. Similar to all of the scenarios studied so far, the highest *SM* value is obtained by *DLP* in accordance with the observation due to Williamson (1992).

- *SSD* and *SCV* values increase for almost all of the models when compared to the base problem. *LFR* outperforms all existing models in terms of variation. Even its highest variation value at $RL = 65000$ is smaller than those values of other existing models except *EMVLP* at $\theta = 0.005$. The performance of *MaxminLF* follows the performance of *LFR* in terms of variation. As in the case of the previous scenarios, *RLF* and *RLF-M* are also effective in deceasing variation of the revenue. Their results are mostly close to and for some instances better than those of the existing risk sensitive models.

- The *LF* values decrease for all models including the proposed models as compared to the base problem.

In case the difference between the fare classes decreases, the coefficient of variation is set to a realistic value and no specific arrival order is specified, the proposed models perform successfully. When compared to other risk sensitive models in the literature, it can be said that the solution quality improves for the proposed models more in terms of all performance measures. That is, especially for small-sized airline companies, the proposed models can be applied successfully.

SCV and *LF* values obtained by *RLF*, *RLF-M*, *LFR* and *MaxminLF* are presented in Figures 6.19 and 6.20, respectively.

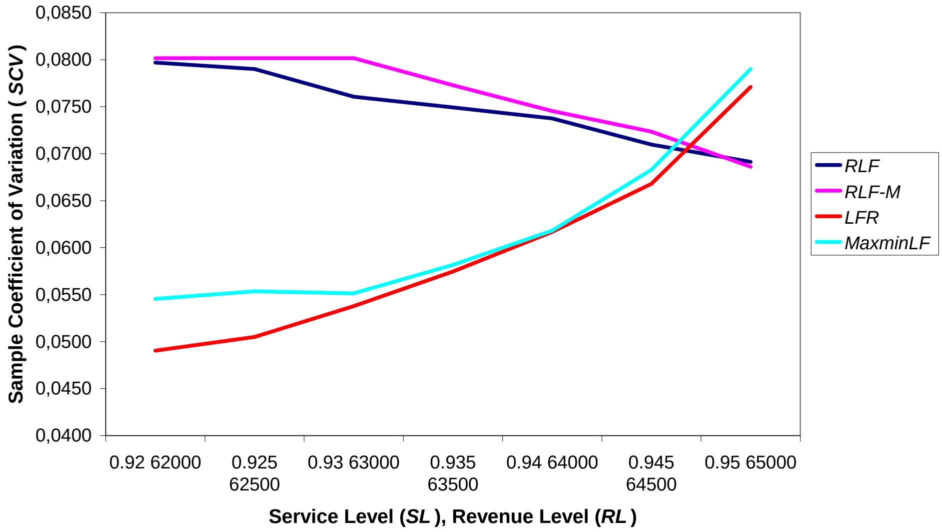

Figure 6.19: *SCV* Values of Proposed Models under Nested Control (Realistic Variations and Close Fares)

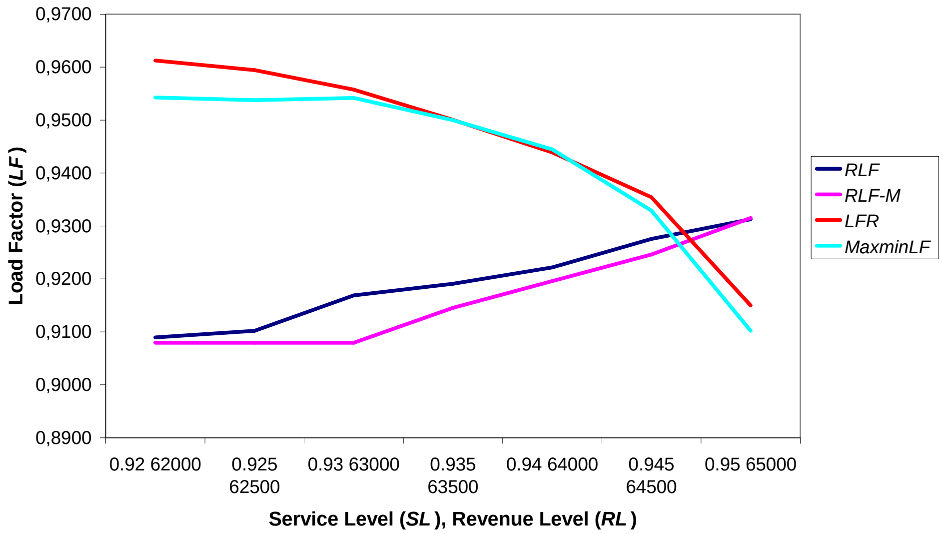

Figure 6.20: *LF* Values of Proposed Models under Nested Control (Realistic Variations and Close Fares)

6.5 Low before High Arrival Pattern

In this section, the impact of a change in the arrival pattern on the performance measures is investigated. It is assumed in the base problem that the low-fare class customers arrive earlier than the high fare class customers. However, an important amount of low fare class customers arrive later than the first customer of a high fare class in real life. Therefore, the assumption of the base problem on the arrival pattern of the customers is relaxed and the impact of this relaxation on the performance measures is investigated in this section. For this relaxation, the Beta distribution parameters of the base problem are changed as in Appendix A. The data for the arrival rates in the ”low before high arrival pattern” case is also given in Appendix A.

Recall from Table 5.11 that the range of *SL* for *RLF* and *RLF-M* and the range of *RL* for *LFR* and *MaxminLF* are the same as the ranges obtained for the base problem. The reason of this is the fact that the input data of the “low before high arrival pattern” case is the same as the input data of the base problem except the arrival rates. Since the arrival pattern is not considered in the mathematical models, the seat allocations and the bid prices obtained from the mathematical models are same for the base problem and the scenario considered in this section.

The simulation results for the “low before high arrival pattern” are almost the same as the results of the base problem in Table 6.2 for nested control because a seat allocated to a low fare class customer can be used by a higher fare class customer in the nested control. Therefore, the deductions in Section 6.1 are also valid in the “low before high arrival pattern” case. By observing this situation, it can be concluded that a change in the arrival pattern of the fare classes do not affect the performance measures of the models significantly as long as nested booking limit control policy is used.

6.6 Concluding Remarks

The numerical experiments in this chapter show that the performances of the mathematical models depend on the scenario considered. The concluding remarks about the scenarios and the performances of the mathematical models are summarized below.

- The base problem and the “low before high arrival pattern” case give very similar results for all of the models under nested control. This situation results from the fact that the input data of the two scenarios are the same except the arrival rates of the fare classes.

- The highest sample mean values are obtained under the base problem for all of the models.
- The deterministic model, *DLP* gives the highest revenue value in base problem. However, variation value takes the smallest value and load factor takes the highest value in the "realistic variations and close fares" case.
- The deterministic model, *DLP*, gives the highest sample mean values for all of the scenarios except the base problem. This is in accordance with the observation due to Williamson (1992), which states that *DLP* outperforms the probabilistic models in terms of obtained revenue since both deterministic and probabilistic models ignore a nested environment.
- For the "realistic variations and close fares" case, the proposed models are quite successful. That is, for small-scaled airline companies which show the characteristics of the "realistic variations and close fares" case, the proposed models are promising.
- In accordance with the simulation results, *RLF* is slightly more effective than *RLF-M* in terms of risk sensitivity since it yields lower variation and higher load factor values than *RLF-M*. Similarly, *LFR* is slightly more effective than *MaxminLF* in terms of risk sensitivity because it gives lower revenue variation and higher load factor values than *MaxminLF*. On the other hand, *RLF-M* and *MaxminLF* give slightly higher revenue than *RLF* and *LFR*, respectively.
- *LFR* outperforms *RLF* in terms of variation and load factor for all of the scenarios.
- The models we propose are formulated and solved much more easily than other risk sensitive models in the literature. The avarage CPU times of the models we propose and those of the models proposed in the literature are given in Table 6.7. It should be noted that *EMR* and other bound models we propose in Chapter 4 are used in order to obtain the bounds of the threshold levels. Therefore, the time elapsed for solving the bound models should also be taken into account. However, as long as the bounds do not change, the bound models are used just once. The values in Table 6.7 excludes the solving time of the bound models.

Table 6.7: CPU Times of the Models

Model	CPU Time
RLF	2.64
RLF-M	2.48
LFR	2.61
MaxminLF	2.68
SLP-RM	63.13
PMP-RC	34.48
EMVLP	2.36
RRS	1100
DLP	0.22
EMR	1.86

- In summary, it can be said that the proposed models do not lead to an unacceptable result when compared to other models and give nearly close results to *EMVLP*, *SLP-RM* and *PMP-RC*.

In order to visualize the relation among the scenarios, the performances of *RLF* and *LFR* under different scenarios are given in the subsequent figures. As stated in Section 6.1, the behavior of *RLF-M* is similar to that of *RLF* and the performance of *MaxminLF* is similar to that of *LFR* under different alternative scenarios.

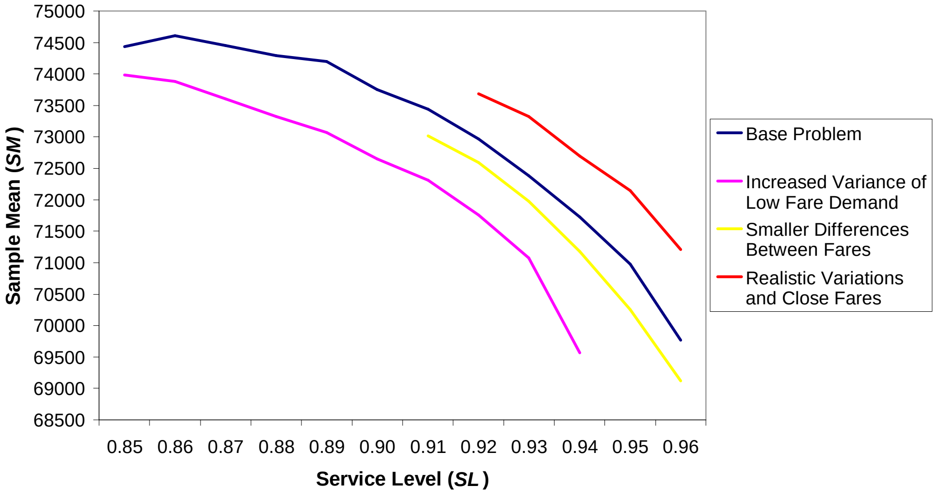

Figure 6.21: *SL* versus *SM* for Alternative Scenarios under Nested Control (*RLF*)

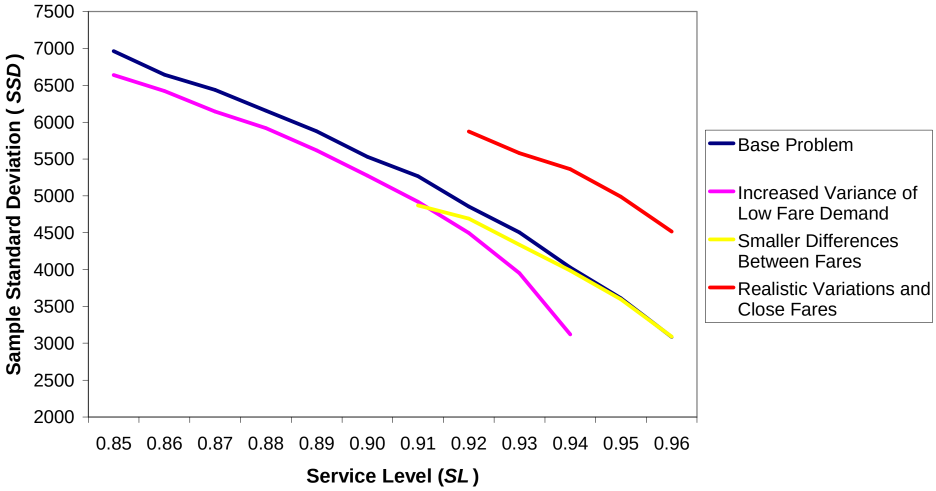

Figure 6.22: *SL* versus *SSD* for Alternative Scenarios under Nested Control (*RLF*)

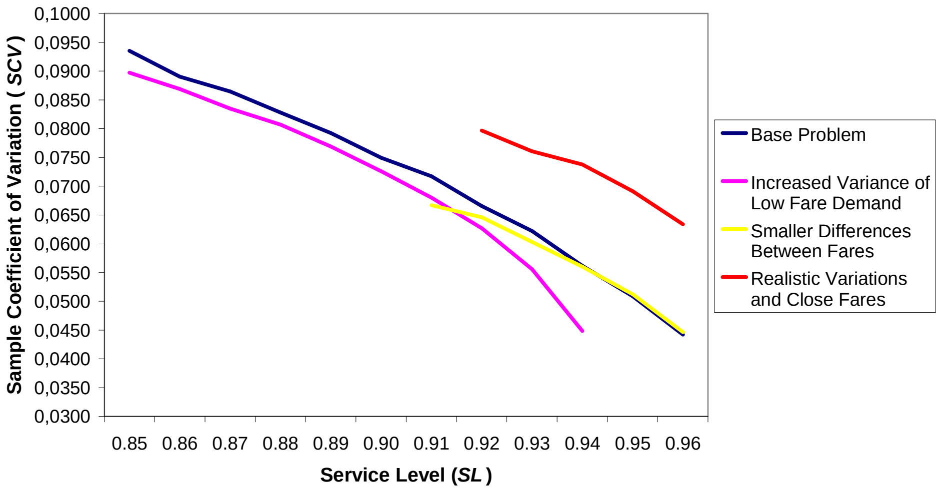

Figure 6.23: *SL* versus *SCV* for Alternative Scenarios under Nested Control (*RLF*)

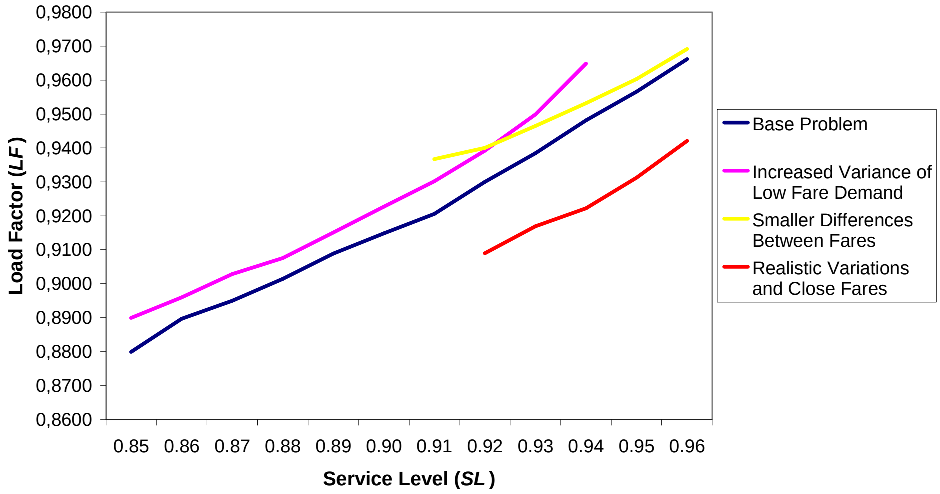

Figure 6.24: *SL* versus *LF* for Alternative Scenarios under Nested Control (*RLF*)

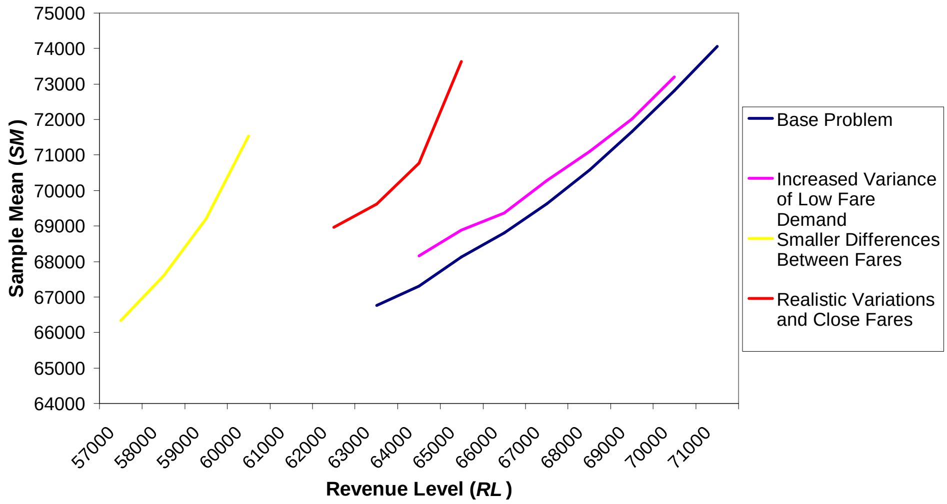

Figure 6.25: *RL* versus *SM* for Alternative Scenarios under Nested Control (*LFR*)

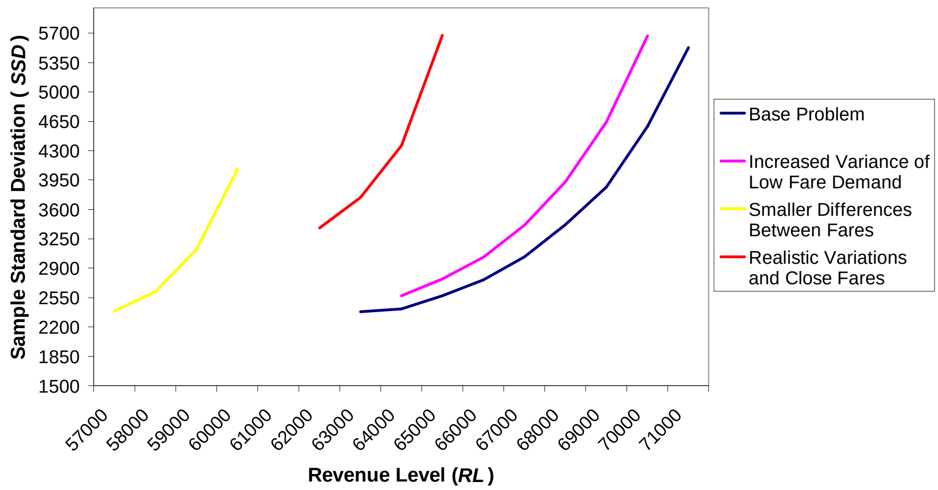

Figure 6.26: *RL* versus *SSD* for Alternative Scenarios under Nested Control (*LFR*)

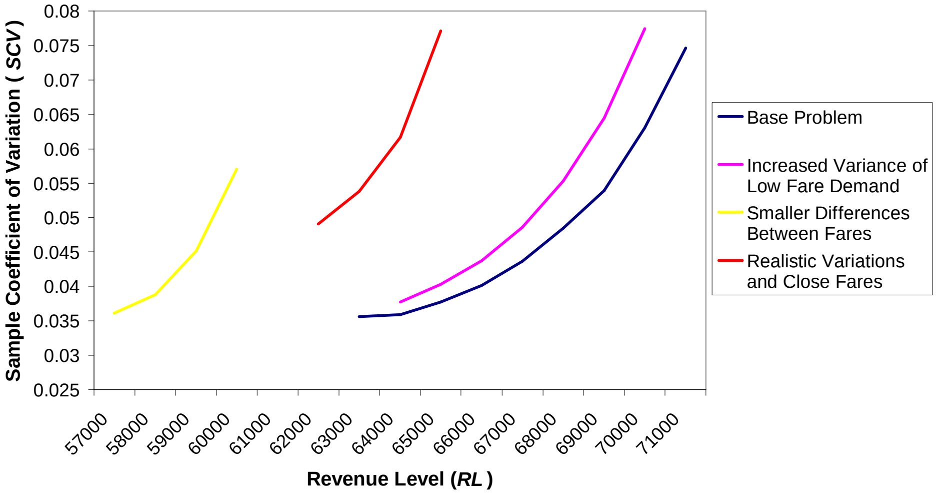

Figure 6.27: *RL* versus *SCV* for Alternative Scenarios under Nested Control (*LFR*)

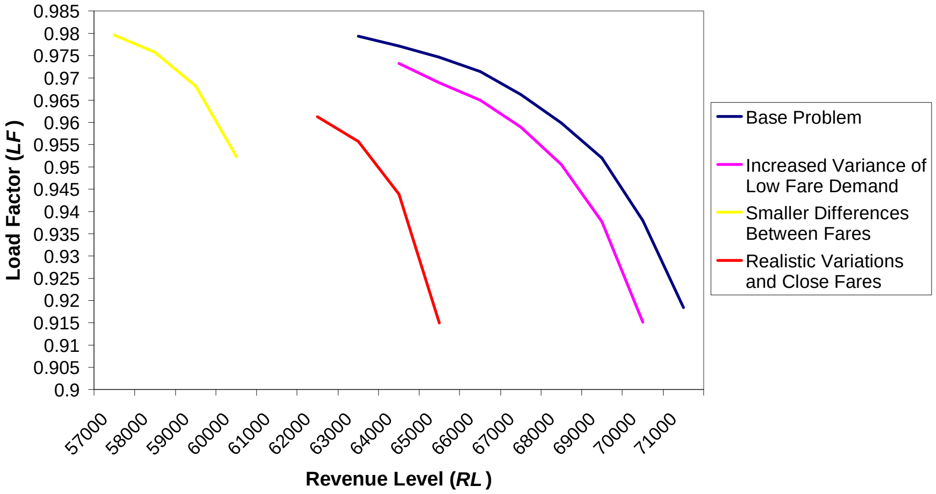

Figure 6.28: *RL* versus *LF* for Alternative Scenarios under Nested Control (*LFR*)

CHAPTER 7

CONCLUSION

In this study, the network seat inventory problem is studied for the airline industry from the perspective of a risk sensitive decision maker. The aim of the study is to consider a number of objectives of the decision maker such as increasing the revenue earned, increasing the flight capacity utilization and decreasing the risk factor or the variability. These are sought by proposing mathematical models. By incorporating both expected revenue and expected load factor into mathematical formulations, we handle two conflicting objectives together in this study. By incorporating a load factor constraint into the risk-neutral models in the literature, we control the revenue variation while maximizing the expected revenue in one type of the models we propose (*RLF* and *RLF-M*). Moreover, in the second type of models we propose, we aim to control the variance of the revenue by maximizing the load factor under a constraint for a revenue target level (*LFR* and *MaxminLF*). This way, we perform multi criteria decision making and also take the behavior of a risk-averse decision maker into account. In this sense, this thesis contributes to the risk sensitive studies in the literature on revenue management for network seat inventory problems as well as to the *MCDM* literature. Overbooking, cancellations and no-shows are not allowed in this study.

This thesis is the first study that uses the load factor directly in the mathematical formulations. In this study, it is aimed to decrease the variance of the revenue by increasing the capacity utilization. To increase the capacity utilization, load factor is forced to be increased in the models we propose in this thesis. The main advantage of working with load factor formulations to decrease the variance of the revenue is the ease of modeling and solving the models with load factor formulations as compared to the models due to Çetiner (2007) and Terciyanlı (2009).

Two types of mathematical models are proposed in this thesis to find the optimal seat allocations by incorporating the load factor into the models. Although the load factor is an important and widely used performance measure considered to compare alternative approaches, there is no existing study in the literature that uses the load factor formulations directly in the mathematical models. That is, this thesis is the first study in the RM literature for the airline industry that uses the load factor formulations in the mathematical models. Recall that there are only a few studies on the network seat inventory problems for the airline industry that take the risk factor into account.

One type of the models we propose in this thesis aims at maximizing the expected revenue while working with service level constraints on the expected load factors (*RLF* and *RLF-M*). In these models, the service level is a predetermined threshold level. The other type of models we propose aim at maximizing a weighted average of the expected load factors of the network legs (*LFR*) or maximizing the minimum expected load factor of the legs in the network (*MaxminLF*) while ensuring that the expected revenue is always above a predetermined threshold level. The variability of the revenue is taken into account in these two types of models by investigating the relation between load factor and risk aversion of the decision makers. The risk aversion is evaluated on the basis of the variability of the revenue that can be tolerated by the decision maker. Standard deviation and coefficient of variation of the revenue are the two risk measures used in this thesis. The impact of a change in the load factor on these risk measures is numerically investigated. The main advantage of the proposed models is the simplicity of their application since they are easy to formulate and solve as compared to other risk-sensitive models in the literature since they maintain linearity and do not need approximation methods.

The booking limits and the bid prices obtained from the proposed models are used in the simulation studies. The performance of our models are compared with the performances of the other models in the literature for a sample network under different scenarios. The models we propose decrease the variability of revenue and so they increase the used capacity of the aircraft. However, an acceptable decrease in the revenue occurs as well under some scenarios. Despite this fact, there are also instances, where our models give higher revenues than existing risk sensitive models under different scenarios for specific threshold values. Our models perform well and satisfactorily under all scenarios. Especially for the case where the coefficients of variation are set realistically, the difference between the fare classes is close and no specific arrival pattern of the fare classes is determined, our models perform well. Therefore, the models we propose are strongly recommended to be used for the small-scale airline companies, which have these chracretistiscs. Moreover, the models we propose are strongly recommended also for state companies and for scheduling new flights in large scale, well established airline companies. The reason of this recommendation is the high risk factor in those companies and operations. Small-scale airline companies are more vulnerable against risk as compared to large-scale ones. Competition in the airline market and the variation of demand increases their risk. Similarly, it is risky for large-scale airline companies to schedule a new flight due to chracteristics of the flight leg and competition in the market. Besides, risk is an important indication of performance especially for state companies.

The following remains as a future work.

- Recall that the parameter, w_l, in *RLF-M* and in *LFR* in Chapter 3, stands for the weight of each leg. In this study, the weight of each leg is assumed to be the same and equal to $1/m$ for numerical analysis. However, different weights can be assigned to the legs according to the characteristics of them. The weights for the legs should be determined according to the network characteristics. Determination of the weights for different legs and the use of them is a research area for future studies.

- The weighted average of the expected load factors of the legs is used as the load factor measure in *RLF-M*. However, different load factor measures can also be considered for the whole network in the future studies.

- The need of the investigation of the differentiation in the service level requirements of the legs are tried to be simply illustrated in this study. However a thorough analysis is needed to analyze the impact of such a differentiation. Investigating the network characteristics and differentiating the service level requirements of the legs according to those network characteristics is another future research direction.

- The update of the bid prices is not considered within the scope of this thesis. A further analysis is needed to investigate the impact of the update mechanism on the performance measures.

- Recall that the sample network of de Boer (1999) is used both for the optimization and simulation studies in this thesis. However, a larger network with a different structure can be used in the further studies. Evaluating the performances of the models we propose for such a network is a future research direction.

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APPENDIX A

SIMULATION DATA

A.1 Base Problem

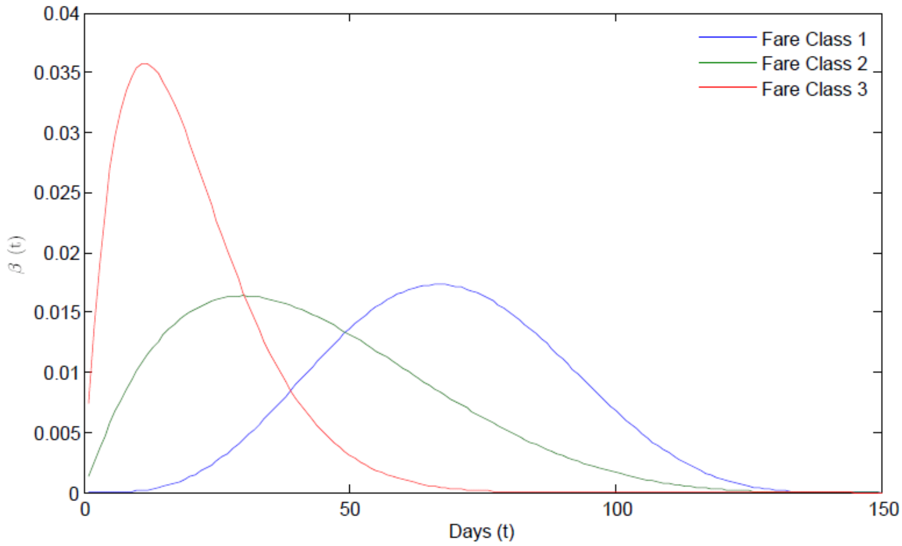

Figure A.1: Beta Density Functions for three Fare Classes

Table A.1: Fare Settings for the Base Problem

OD Number	Origin-Destination	Fare Class 3	Fare Class 2	Fare Class 1
1	A-B	75	125	250
2	A-C	130	370	400
3	A-D	200	320	460
4	B-C	100	150	330
5	B-D	160	200	420
6	C-D	80	110	235

Table A.2: Demand Settings for the Base Problem

	Fare Class 3				Fare Class 2				Fare Class 1			
Itinerary	p_j	δ	E_j	SD_j	p_j	δ	E_j	SD_j	p_j	δ	E_j	SD_j
AB	80	1.6	50	9.01	80	2	40	7.75	3	0.1	30	18.17
AC	80	2	40	7.75	50	2	25	6.12	2	0.1	20	14.83
AD	60	2	30	6.71	72	3	24	5.66	2	0.1	20	14.83
BC	60	2	30	6.71	40	2	20	5.48	2	0.1	20	14.83
BD	60	2	30	6.71	60	3	20	5.16	6	0.3	20	9.31
CD	80	1.6	50	9.01	80	2	40	7.75	6	0.2	30	13.42

Table A.3: Request Arrival Setting for the Base Problem

	Fare Class 3		Fare Class 2		Fare Class 1	
Itinerary	α	β	α	β	α	β
1-6	5	6	2	5	2	13

A.2 Increased Variance Of Low Fare Demand

Table A.4: Demand Settings for the "Increased Variance of Low Fare Demand" Case

	Fare Class 3				Fare Class 2				Fare Class 1			
Itinerary	p_j	δ	E_j	SD_j	p_j	δ	E_j	SD_j	p_j	δ	E_j	SD_j
AB	20	0.4	50	13.23	20	0.5	40	10.95	3	0.1	30	18.17
AC	20	0.5	40	10.95	5	0.2	25	12.25	2	0.1	20	14.83
AD	15	0.5	30	9.49	18	0.75	24	7.48	2	0.1	20	14.83
BC	15	0.5	30	9.49	10	0.5	20	7.75	2	0.1	20	14.83
BD	15	0.5	30	9.49	15	0.75	20	6.83	6	0.3	20	9.31
CD	20	04	50	13.23	20	0.5	40	10.95	6	0.2	30	13.42

A.3 Smaller Differences Between Fares

Table A.5: Fare Settings for the “Smaller Differences Between Fares” Case

OD Number	Origin-Destination	Fare Class 3	Fare Class 2	Fare Class 1
1	A-B	75	125	175
2	A-C	130	170	220
3	A-D	200	320	440
4	B-C	100	150	210
5	B-D	160	200	250
6	C-D	80	110	160

A.4 Realistic Variations and Close Fares

Table A.6: Fare Settings for the “Smaller Differences Between Fares” Case

OD Number	Origin-Destination	Fare Class 3	Fare Class 2	Fare Class 1
1	A-B	75	125	175
2	A-C	130	170	220
3	A-D	200	320	460
4	B-C	100	150	210
5	B-D	180	210	250
6	C-D	80	110	160

Table A.7: Demand Settings for Realistic Variations and Close Fares

	Fare Class 3				Fare Class 2				Fare Class 1			
Itinerary	p_j	δ	E_j	SD_j	p_j	δ	E_j	SD_j	p_j	δ	E_j	SD_j
AB	80	1.6	50	9.01	80	2	40	7.75	3	0.1	30	18.17

Table A.7: Demand Settings for Realistic Variations and Close Fares

	Fare Class 3				**Fare Class 2**				**Fare Class 1**			
Itinerary	p_j	δ	E_j	SD_j	p_j	δ	E_j	SD_j	p_j	δ	E_j	SD_j
AC	80	2	40	7.75	50	2	25	6.12	2	0.1	20	14.83
AD	60	2	30	6.71	72	3	24	5.66	2	0.1	20	14.83
BC	60	2	30	6.71	40	2	20	5.48	2	0.1	20	14.83
BD	60	2	30	6.71	60	3	20	5.16	6	0.3	20	9.31
CD	80	1.6	50	9.01	80	2	40	7.75	6	0.2	30	13.42

Table A.8: Request Arrival Setting for Realistic Variations and Close Fares

	Fare Class 3		**Fare Class 2**		**Fare Class 1**	
Itinerary	α	β	α	β	α	β
1-6	2	2	2	2	2	2

A.5 Low Before High Arrival Pattern

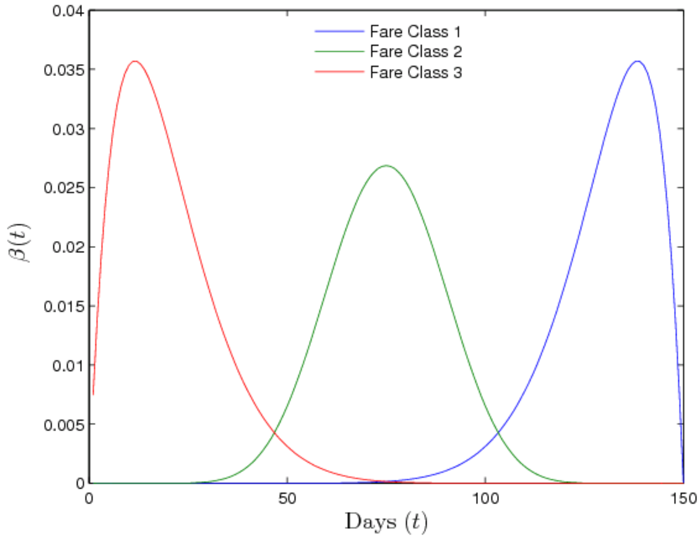

Figure A.2: Low before High Arrival Rates

Table A.9: Request Arrival Setting for Low before High Arrival Pattern

	Fare Class 3		**Fare Class 2**		**Fare Class 1**	
Itinerary	α	β	α	β	α	β
1-6	13	2	13	13	2	13

APPENDIX B

MATLAB PROGRAMMING CODES

The MATLAB codes of the proposed models are given in this section. For the codes of the models in the literature, the reader is referred to the studies due to Çetiner (2007) and Terciyanlı (2009). Moreover, the simulation codes for the partitioned control, nested control and bid price control are given in the study due to Terciyanlı (2009).

The definitions of the MATLAB.m files are given below.

Mainpart.m: The main part for optimization models that are used to run all of the mathematical models.

Demandpart.m: This .m file is used to calculate the probabilities of demands for all predetermined integer values.

Input.m: Initial data for legs, seats and demand are provided in this file.

RLF.m: File for *RLF* Model

RLF_M.m: File for *RLF-M* Model

LFR.m: File for *LFR* Model

MaxminLF.m: File for *MaxminLF* Model

EMR_model_with_load_factor_constraint.m: File for first the model to find the lower bound for the service level used in *RLF* with $SL_l \neq SL$.

MinmaxSL.m: File for second the model to find the lower bound for the service level used in *RLF* with $SL_l \neq SL$.

MaxminLF_M.m: File for the model to find the upper bound for the service level used in *RLF* with $SL_l \neq SL$.

EMR_model_with_equal_E(LF$_l$).m: File for the EMR model with equal $E(LF_l)$ to find the lower bound for the service level used in *RLF* with $SL_l = SL$.

MaxELF.m: File for the *MaxELF* model to find the upper bound for the service level used in *RLF* with $SL_l = SL$.

EMR_model_with_WLF.m: File for *EMR* model with *WLF* that is used to find the lower bound for the service level used in *RLF-M*.

MaxWLF.m: File for *MaxWLF* model that is used to find the upper bound for the service level used in *RLF-M*.

LFR_M.m: File for *LFR-M* model that is used to find the lower bound for the revenue level used in *LFR*.

MinRL.m: File for *MinRL* model that is used to find the lower bound for the revenue level used in *MaxminLF*.

B.1 Mainpart.m

% This .m file is the main part for optimization models and used for test purposes. % By setting test and den values and changing parameters of the models, performances of the models are tested. % By setting ”a” values and increasing them incrementally, the service level for RLF and the revenue level for *LFR* are changed for our models to see the effect of a change in the service level or in the revenue level on the performance measures.

tic; % Used for calculating CPU time

a=170; % Set the service level for *RLF* or the revenue level for *LFR*

for test=1:1

for den=1:10 a=a+0.5; %Increase the service level for *RLF* or the revenue level for

LFR incrementally.

Input % Go to Input.m file and get the input data Demandpart % Go to Demand.m file and solve the problem revden(den,:)=rev; % Get expected revenue values bidden(den,:)=bid; % Get bid prices seatden(den,:)=seat; % Get seat allocations loadfden(den,:)=loadf; % Get the weighted average expected load factor values

loadflegden(den,:)=loadfleg; % Get the expected load factor value of each leg in the network aden(den,:)=a; %Get the matrix for service level or revenue level maxminden(den,:)=maxmin; % Get the maximized minimum expected load factor value of the network

end

revtest(:,:,test)=revden; % Write expected revenue values to a 3-d matrix

bidtest(:,:,test)=bidden; % Write bid prices to a 3-d matrix

seattest(:,:,test)=seatden; % Write seat allocations to a 3-d matrix

end

toc; % Used for calculating CPU time

clear test den % clear variables test and den

B.2 Demandpart.m

% This .m file is used for calculating probabilities of demands for all predetermined integer values.

%*****************Calculate Pr(Dj¿=i) and Pr(Dj¡i)***********************

pdfno=(0:C-1);

for i=1:D

ProbDist(:,i)=1-nbincdf(pdfno,DDODF(i,1),(DDODF(i,2)+beT(1,i))/(DDODF(i,2)+1));

```
end
ProbDist=1-ProbDist;
for i=1:K
ProbDistcdf(i,:)=ProbDist(i*C/K,:); % Group demands
end
ProbDistrcdf=1-ProbDistcdf;
%***********************Calculate Pr(Dj=i)**************************
pdfno=(0:C); for i=1:D
ProbDistpdf(:,i)=nbinpdf(pdfno,DDODF(i,1),DDODF(i,2)/(DDODF(i,2)+1));
ProbDistpdf(C,i)=1-nbincdf(C-1,DDODF(i,1),DDODF(i,2)/(DDODF(i,2)+1));
end
clear pdfno i ProbDist
RLF % Go to mathematical model
```

B.3 Input.m

```
% This .m file is used for describing initial values for legs, seats and demands.
%*********NETWORK STRUCTURE DATA************
C=200; % Upper bound of x variables CAPACITY=[200 200 200]; % Capacity of the flights D=18; % Number of ODFs NL=3; % Number of legs NF=3; % Number of fare classes K=200; % Number of demand segments Leg=[1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0;0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0;... 0 0 0 0 0 0 1 1 1 0 0 0 1 1 1 1 1 1]; % ODFs in legs
% **************Base Problem*******************
```

FF=[75 125 250 130 170 400 200 320 460 100 150 330 160 200 420 80 110 235]; % Price of tickets DDODF=[80 1.6;80 2;3 0.1;80 2;50 2;2 0.1;60 2;72 3;2 0.1;60 2;... 40 2;2 0.1;60 2;60 3;6 0.3;80 1.6;80 2;6 0.2]; % Demand parameters Inbeta=[5 6;2 5;2 13]; % Beta distribution parameters

%*******Increased Low-Fare Demand Variance******

% FF=[75 125 250 130 170 400 200 320 460 100 150 330 160 200 420 80 110 235]; % Price of tickets % DDODF=[20 0.4;20 0.5;3 0.1;20 0.5;5 0.2;2 0.1;15 0.5;18 0.75;2 0.1;15 0.5;... % 10 0.5;2 0.1;15 0.5;15 0.75;6 0.3;20 0.4;20 0.5;6 0.2]; % Demand parameters % Inbeta=[13 2;13 13;2 13]; % Beta distribution parameters

%

% FF=[75 125 175 130 170 220 200 320 440 100 150 210 160 200 250 80 110 160]; % Price of tickets % DDODF=[80 1.6;80 2;3 0.1;80 2;50 2;2 0.1;60 2;72 3;2 0.1;60 2;... % 40 2;2 0.1;60 2;60 3;6 0.3;80 1.6;80 2;6 0.2]; % Demand parameters % Inbeta=[13 2;13 13;2 13]; % Beta distribution parameters

%

% FF=[75 125 175 130 170 220 230 340 460 100 150 210 180 210 250 80 110 160]; % Price of tickets % DDODF=[20 0.4;20 0.5;30 1;20 0.5;5 0.2;20 1;15 0.5;18 0.75;20 1;15 0.5;... % 10 0.5;20 1;15 0.5;15 0.75;60 3;20 0.4;20 0.5;60 2]; % Demand parameters % Inbeta=[2 2;2 2;2 2]; % Beta distribution parameters

% CAPACITY=[220 220 220]; % Capacity of the flights % C=220; % K=220;

%

% FF=[75 125 250 130 170 400 200 320 460 100 150 330 160 200 420 80 110 235]; % Price of tickets % DDODF=[80 1.6;80 2;3 0.1;80 2;50 2;2 0.1;60 2;72 3;2 0.1;60 2;... % 40 2;2 0.1;60 2;60 3;6 0.3;80 1.6;80 2;6 0.2]; % Demand parameters % Inbeta=[13 2;13 13;2 13]; % Beta distribution parameters

for i=1:D Expdem(1,i)=DDODF(i,1)/DDODF(i,2); % Calculate expected demand end clear i;

B.4 RLF.m

% Our Proposed Model RLF % This model aims at maximizing the expected revenue under the following constraint: % the expected load factor of each leg is required to be greater than a predetermined threshold level.

% VARIABLES

xodf=sdpvar(K,D,'full');

% CONSTRAINTS

for i=1:NL b(i,1)=CAPACITY(1,i); % RHS of the capacity constraint end clear i;

lfbound=[a;a;a]; % RHS of the load factor constraint

cap=sum(xodf*Leg')'; % Capacity constraint

lfrisk=sum((ProbDistrcdf.*xodf)*Leg')'; % Load factor constraint

o=sum(CAPACITY);

% Constraint Set

F=set($cap \leq b$)+set($xodf \leq C/K$)+set($0 \leq xodf$)+set($lfrisk/200 \geq lfbound$);

% OBJECTİVE FUNCTION

obj=-sum(xodf*FF')+sum((ProbDistcdf.*xodf)*FF');

% SOLVE MODEL

solvesdp(F,obj,sdpsettings('solver','glpk'));

% OUTPUT

rev=-double(obj); % Expected revenue

bid=dual(F(1))-((dual(F(4))*a)/200); % Bid price

seat=round(sum(double(xodf))); % Seat allocation loadf=double(sum(sum((ProbDistrcdf.*xodf)*Leg')')/sum(CAPACITY)); % Avarage expected load factor value of the network

loadfleg=double(lfrisk)/200; % Expected load factor of each leg in the network

clear F b beT cap obj;

B.5 RLF-M.m

% The Variation of RLF: RLF-M % This model also aims at maximizing the expected revenue under the following constraint: % the weighted average expected load factor of the whole network is required to be greater than a predetermined threshold level.

% VARIABLES

xodf=sdpvar(K,D,'full');

% CONSTRAINTS

for i=1:NL b(i,1)=CAPACITY(1,i); % RHS of the capacity constraint end clear i;

lfbound=[a]; % RHS of the load factor constraint

cap=sum(xodf*Leg')'; % Capacity constraint

lfrisk=sum(sum((ProbDistrcdf.*xodf)*Leg')')/sum(CAPACITY); % Load factor constraint

o=sum(CAPACITY);

% Constraint Set

F=set($cap \leq b$)+set($xodf \leq C/K$)+set($0 \leq xodf$)+set($lfrisk \geq lfbound$);

% OBJECTİVE FUNCTION

obj=-sum(xodf*FF')+sum((ProbDistcdf.*xodf)*FF');

% SOLVE MODEL

solvesdp(F,obj,sdpsettings('solver','glpk'));

% OUTPUT

rev=-double(obj); % Expected revenue

bid=dual(F(1))-((dual(F(4))*a)/600); % Bid price

seat=round(sum(double(xodf))); % Seat allocation

loadf=double(lfrisk); % Weighted average expected load factor value of the network

clear F b beT cap obj;

B.6 LFR.m

% Our Proposed Model LFR % This model maximizes the weighted average of the expected load factor of the network while ensuring that the expected revenue is always above a predetermined threshold level.

% VARIABLES

xodf=sdpvar(K,D,'full');

% CONSTRAINTS

for i=1:NL

b(i,1)=CAPACITY(1,i); % RHS of the capacity constraint

end

clear i;

cap=sum(xodf*Leg')'; % Capacity constraint

lfrisk=sum((ProbDistrcdf.*xodf)*Leg')'; % Load factor constraint

rb=[a]; % RHS of the revenue constraint

% Constraint Set

F=set($cap \leq b$)+set($xodf \leq C/K$)+set($0 \leq xodf$)+set($sum((ProbDistrcdf.*xodf)*FF') \geq rb$);

% OBJECTİVE FUNCTION

obj=-sum(lfrisk)/sum(CAPACITY);

% SOLVE MODEL

solvesdp(F,obj,sdpsettings('solver','glpk'));

% OUTPUT

rev=double(sum((ProbDistrcdf.*xodf)*FF')); % Expected revenue

bid=dual(F(1)); % Bid price

seat=round(sum(double(xodf))); % Seat allocation

loadf=-double(obj); % Weighted average expected load factor value of the network

clear F b beT cap obj;

B.7 MaxminLF.m

% The Variation of LFR: MaxminLF % This model aims at maximizing the expected load factor of the leg, which has the smallest expected load factor value.

% VARIABLES

xodf=sdpvar(K,D,'full');

z=sdpvar(1);

% CONSTRAINTS

for i=1:NL

b(i,1)=CAPACITY(1,i); % RHS of the capacity constraint

end

clear i;

cap=sum(xodf*Leg')'; % Capacity constraint

lfrisk=sum((ProbDistrcdf.*xodf)*Leg')'; % Load factor constraint

loadfleg=double(lfrisk)/200; % Expected load factor of each leg

rb=a; % RHS of the revenue constraint

% Constraint Set

F=set($cap \leq b$)+set($xodf \leq C/K$)+set($0 \leq xodf$)+set($sum((ProbDistrcdf. * xodf) * FF') \geq rb$)

+set($lfrisk/200 \geq z$);

% OBJECTİVE FUNCTION

obj=-z;

% SOLVE MODEL

solvesdp(F,obj,sdpsettings('solver','glpk'));

% OUTPUT

rev=double(sum((ProbDistrcdf.*xodf)*FF')); % Expected revenue

bid=dual(F(1)); % Bid price

seat=round(sum(double(xodf))); % Seat allocation

maxmin=-double(obj); % Objective function

loadf=double(sum(sum((ProbDistrcdf.*xodf)*Leg')')/sum(CAPACITY)); % Average expected load factor of network

loadfleg=double(lfrisk)/200; %Expected load factor of each leg in the network

clear F b beT cap obj;

B.8 EMR_Model_with_Load_Factor_Constraint.m

% EMR model with load factor constraint (LB-1 for RLF with $SL_l \neq SL$)

% An additional load factor constraint is added to the original EMR model to find the lower bound for the service level used in RLF with $SL_l \neq SL$.

% VARIABLES

xodf=sdpvar(K,D,'full');

% CONSTRAINTS

for i=1:NL

b(i,1)=CAPACITY(1,i); % RHS of the capacity constraint

end

clear i;

cap=sum(xodf*Leg')'; % Capacity constraint

% Constraint Set

F=set($cap \leq b$)+set($xodf \leq C/$)+set($0 \leq xodf$);

% OBJECTİVE FUNCTION

obj=-sum(xodf*FF')+sum((ProbDistcdf.*xodf)*FF');

% SOLVE MODEL

solvesdp(F,obj,sdpsettings('solver','glpk'));

% OUTPUT

rev=-double(obj); % Expected revenue

bid=dual(F(1)); % Bid price

seat=sum(double(xodf)); % Seat allocation loadf=double(sum(sum((ProbDistrcdf.*xodf)*Leg')')/sum(CAPACITY)); % Average expected load factor value of the network

lfrisk=sum((ProbDistrcdf.*xodf)*Leg')'; % Load factor constraint

loadfleg=double(lfrisk)/200; % Expected load factor of value each leg in the network

clear F b beT cap obj xodf;

B.9 MinmaxSL.m

% MinmaxSL Model (LB-2 for RLF with $SL_l \neq SL$) % This model is a min-max type of model. Expected revenue is forced to be equal to 71765.7848, % which is the optimal expected revenue value obtained by the EMR model. % The maximum expected load factor value of the network is tried to be minimized in order to find a lower bound for the service level used in % RLF with $SL_l \neq SL$.

% VARIABLES

xodf=sdpvar(K,D,'full');

SL=sdpvar(1);

% CONSTRAINTS

for i=1:NL

b(i,1)=CAPACITY(1,i); % RHS of the capacity constraint

end

clear i;

cap=sum(xodf*Leg')'; % Capacity constraint

lfrisk=sum((ProbDistrcdf.*xodf)*Leg')'; % Load factor constraint

o=sum(CAPACITY);

rev=sum((ProbDistrcdf.*xodf)*FF'); % Expected revenue

% Constraint Set

F=set($cap \leq b$)+set($xodf \leq C/K$)+set($0 \leq xodf$)+
set($lfrisk \leq SL$)+set($0 \leq SL$)+set(71765.7848 == rev);

% OBJECTİVE FUNCTION

obj=SL;

% SOLVE MODEL

solvesdp(F,obj,sdpsettings('solver','glpk'));

% OUTPUT

rev=double(rev); % Expected revenue

bid=dual(F(1)); % Bid price

seat=round(sum(double(xodf))); % Seat allocation

maxmin=obj; % Objective function

loadf=double(sum(sum((ProbDistrcdf.*xodf)*Leg')')/sum(CAPACITY)); % Avarage expected load factor of the network

loadfleg=double(lfrisk)/200; % Expected load factor of each leg in the network

clear F b beT cap obj;

B.10 MaxminLF-M.m

% MaxminLF-M model (UB for RLF with $SL_l \neq SL$) % This model is used to find the upper bound for the service level used in RLF with $SL_l \neq SL$.

% VARIABLES

xodf=sdpvar(K,D,'full');

z=sdpvar(1);

```
% CONSTRAINTS
for i=1:NL
b(i,1)=CAPACITY(1,i); % RHS of the capacity constraint
end
clear i;
cap=sum(xodf*Leg')'; % Capacity constraint
lfrisk=sum((ProbDistrcdf.*xodf)*Leg')'; % Load factor constraint
% Constraint Set
F=set(cap <= b)+set(xodf <= C/K)+set(0 <= xodf)+set(lfrisk/200 >= z);
% OBJECTİVE FUNCTION
obj=-z;
% SOLVE MODEL
solvesdp(F,obj,sdpsettings('solver','glpk'));
% OUTPUT
rev=double(sum((ProbDistrcdf.*xodf)*FF')); % Expected revenue
bid=dual(F(1)); % Bid price
seat=round(sum(double(xodf))); % Seat allocation
maxmin=-double(obj); % objective function
loadf=double(sum(sum((ProbDistrcdf.*xodf)*Leg')')/sum(CAPACITY)); %total Expected Load factor of network
loadfleg=double(lfrisk)/200; % Expected load factor of each leg
clear F b beT cap obj;
```

B.11 EMR_with_equal_E(LF_l).m

% EMR model with equal E(LFl) (LB for RLF with SLl=SL) % An additional load factor constraint is added to the original EMR model to find a lower bound for the service level used in RLF with SLl=SL.

% VARIABLES

xodf=sdpvar(K,D,'full');

% CONSTRAINTS

for i=1:NL

b(i,1)=CAPACITY(1,i); % RHS of the capacity constraint

end

clear i;

cap=sum(xodf*Leg')'; % Capacity constraint

lfrisk=sum((ProbDistrcdf.*xodf)*Leg')';

loadfleg=double(lfrisk)/200;

% Constraint Set

F=set($cap \leq b$)+set($xodf \leq C/K$)+set($0 \leq xodf$)+set(lfrisk(1,1)==
lfrisk(2,1))+set(lfrisk(2,1)==lfrisk(3,1));

% OBJECTİVE FUNCTION

obj=-sum(xodf*FF')+sum((ProbDistcdf.*xodf)*FF');

% SOLVE MODEL

solvesdp(F,obj,sdpsettings('solver','glpk'));

% OUTPUT

rev=-double(obj); % Expected revenue

bid=dual(F(1)); % Bid price

seat=sum(double(xodf)); % Seat allocation

loadf=double(sum(sum((ProbDistrcdf.*xodf)*Leg')')/sum(CAPACITY)); % Average expected load factor value of the network

lfrisk=sum((ProbDistrcdf.*xodf)*Leg')'; % Load factor constraint

loadfleg=double(lfrisk)/200; % Expected load factor of value each leg in the network

clear F b beT cap obj xodf;

B.12 MaxELF.m

% MaxELF Model (UB for RLF with SLl=SL) % This model is used to find the upper bound for the service level used in RLF with SLl=SL.

% VARIABLES

xodf=sdpvar(K,D,'full');

ELF=sdpvar(1);

% CONSTRAINTS

for i=1:NL

b(i,1)=CAPACITY(1,i); % RHS of the capacity constraint

end

clear i;

cap=sum(xodf*Leg')'; % Capacity constraint

lfrisk=sum((ProbDistrcdf.*xodf)*Leg')'; % Load factor constraint

% Constraint Set

F=set($cap \leq b$)+set($xodf \leq C/K$)+set($0 \leq xodf$)+set($lfrisk/200 \geq ELF$)+set(lfrisk(1,1)

```
==lfrisk(2,1))+set(lfrisk(2,1)==lfrisk(3,1));

% OBJECTİVE FUNCTION

obj=-ELF;

% SOLVE MODEL

solvesdp(F,obj,sdpsettings('solver','glpk'));

% OUTPUT

rev=double(sum((ProbDistrcdf.*xodf)*FF')); % Expected revenue

bid=dual(F(1)); % Bid price

seat=round(sum(double(xodf))); % Seat allocation

maxmin=-double(obj); % objective function

loadf=double(sum(sum((ProbDistrcdf.*xodf)*Leg')')/sum(CAPACITY)); %total Expected Load factor of network

loadfleg=double(lfrisk)/200; % Expected load factor of each leg

clear F b beT cap obj;
```

B.13 EMR_with_WLF.m

```
% EMR model with load factor constraint (LB for RLF-M ) % This model is used to find a lower bound for the service level used in RLF-M.

% VARIABLES

xodf=sdpvar(K,D,'full');

% CONSTRAINTS

for i=1:NL

b(i,1)=CAPACITY(1,i); % RHS of the capacity constraint
```

end

clear i;

cap=sum(xodf*Leg')'; % Capacity constraint

% Constraint Set

F=set($cap \leq b$)+set($xodf \leq C/K$)+set($0 \leq xodf$);

% OBJECTİVE FUNCTION

obj=-sum(xodf*FF')+sum((ProbDistcdf.*xodf)*FF');

% SOLVE MODEL

solvesdp(F,obj,sdpsettings('solver','glpk'));

% OUTPUT

rev=-double(obj); % Expected revenue

bid=dual(F(1)); % Bid price

seat=sum(double(xodf)); % Seat allocation

lfrisk=sum(sum((ProbDistrcdf.*xodf)*Leg')')/sum(CAPACITY); % Load factor constraint

loadf=double(lfrisk);

clear F b beT cap obj xodf;

B.14 MaxWLF.m

% MaxWLF Model (UB for RLF-M) % This model is used to find the upper bound for the service level used in RLF-M.

% VARIABLES

xodf=sdpvar(K,D,'full');

WLF=sdpvar(1);

% CONSTRAINTS

for i=1:NL

b(i,1)=CAPACITY(1,i); % RHS of the capacity constraint

end

clear i;

cap=sum(xodf*Leg')'; % Capacity constraint

lfrisk=sum(sum((ProbDistrcdf.*xodf)*Leg')')/sum(CAPACITY); % Load factor constraint

% Constraint Set

F=set($cap \leq b$)+set($xodf \leq C/K$)+set($0 \leq xodf$)+set($lfrisk/200 \geq WLF$);

% OBJECTİVE FUNCTION

obj=-WLF;

% SOLVE MODEL

solvesdp(F,obj,sdpsettings('solver','glpk'));

% OUTPUT

rev=double(sum((ProbDistrcdf.*xodf)*FF')); % Expected revenue

bid=dual(F(1)); % Bid price

seat=round(sum(double(xodf))); % Seat allocation

maxmin=-double(obj); % objective function

loadf=double(lfrisk); %total Expected Load factor of network

% son=double(z);

clear F b beT cap obj;

B.15 LFR-M.m

% LFR-M Model (LB for LFR) % This model is used to find the lower bound for the revenue level used in LFR.

% VARIABLES

xodf=sdpvar(K,D,'full');

% CONSTRAINTS

for i=1:NL

b(i,1)=CAPACITY(1,i); % RHS of the capacity constraint

end clear i;

cap=sum(xodf*Leg')'; % Capacity constraint

lfrisk=sum((ProbDistrcdf.*xodf)*Leg')'; % Load factor constraint

% Constraint Set

F=set($cap \leq b$)+set($xodf \leq C/K$)+set($0 \leq xodf$);

% OBJECTİVE FUNCTION

obj=-sum(lfrisk)/sum(CAPACITY);

% SOLVE MODEL

solvesdp(F,obj,sdpsettings('solver','glpk'));

% OUTPUT

rev=double(sum((ProbDistrcdf.*xodf)*FF')); % Expected revenue

bid=dual(F(1)); % Bid price

seat=round(sum(double(xodf))); % Seat allocation

loadf=-double(obj); % Average expected load factor of the network

clear F b beT cap obj;

B.16 MinRL.m

% MinRL Model (LB for MaxminLF) % This model is used to find a lower bound for the revenue level used in MaxminLF.

% VARIABLES

xodf=sdpvar(K,D,'full');

z=sdpvar(1);

% CONSTRAINTS

for i=1:NL

b(i,1)=CAPACITY(1,i); % RHS of the capacity constraint

end

clear i;

cap=sum(xodf*Leg')'; % Capacity constraint

lfrisk=sum((ProbDistrcdf.*xodf)*Leg')'; % Load factor constraint

% Constraint Set

F=set($cap \leq b$)+set($xodf \leq C/K$)+set($0 \leq xodf$)+set($lfrisk/200 \geq z$);

% OBJECTİVE FUNCTION

obj=-z;

% SOLVE MODEL

```
solvesdp(F,obj,sdpsettings('solver','glpk'));

% OUTPUT

rev=double(sum((ProbDistrcdf.*xodf)*FF')); % Expected revenue

bid=dual(F(1)); % Bid price

seat=round(sum(double(xodf))); % Seat allocation

loadf=-double(obj); % objective function= avarage Expected Load factor of total system

clear F b beT cap obj;
```

APPENDIX C

OPTIMIZATION RESULTS OF THE MATHEMATICAL MODELS FOR DIFFERENT SCENARIOS

C.1 BASE PROBLEM

C.1.1 *RLF* with $SL_l=SL$ for all l

Table C.1: Optimal Seat Allocations of *RLF* with $SL_l = SL$ for Base Problem

ODF		*RLF* with SL_l=*SL*										
Itinerary	**Class**	**0.86**	**0.87**	**0.88**	**0.89**	**0.90**	**0.91**	**0.92**	**0.93**	**0.94**	**0.95**	**0.96**
AB	**3**	44	44	43	43	42	42	41	41	40	40	41
	2	40	39	39	38	38	37	37	36	36	35	34
	1	38	37	35	34	33	32	30	29	27	25	22
AC	**3**	1	4	8	12	17	20	20	20	21	24	27
	2	19	19	19	19	18	18	18	17	17	17	17
	1	20	20	19	18	17	16	15	14	13	12	10
AD	**3**	0	0	0	0	0	1	6	12	16	18	21
	2	21	21	21	21	20	20	20	19	19	19	19
	1	17	16	16	15	15	14	13	12	11	10	9
BC	**3**	23	23	22	22	22	21	21	21	21	20	19
	2	19	18	18	18	17	17	16	16	16	15	14
	1	26	25	23	23	21	20	19	19	17	16	14
BD	**3**	17	17	18	18	18	18	18	18	18	19	22
	2	16	16	15	15	15	15	15	15	14	14	14
	1	21	21	21	20	20	20	19	18	17	16	15
CD	**3**	37	38	38	39	40	40	39	38	39	40	42
	2	36	36	36	37	37	37	36	35	35	35	34

Table C.1: Optimal Seat Allocations of *RLF* with $SL_l = SL$ for Base Problem

ODF		*RLF* with SL_l=*SL*										
Itinerary	**Class**	**0.86**	**0.87**	**0.88**	**0.89**	**0.90**	**0.91**	**0.92**	**0.93**	**0.94**	**0.95**	**0.96**
	1	35	35	35	35	35	35	34	33	31	28	24

C.1.2 Bid Prices of *RLF* with SL_l=*SL* for all *l*

Table C.2: Bid Prices of *RLF* for Base Problem

	Bid Price for Leg *l*				Bid Price for Leg *l*		
SL_l=*SL*	*l:AB*	*l:BC*	*l:CD*	SL_l=*SL*	*l:AB*	*l:BC*	*l: CD*
0.8000	59.5551	86.4263	72.8012	0.8850	53.0854	87.4121	72.8012
0.8025	60.9468	86.4684	72.8012	0.8875	52.7277	87.7412	72.3745
0.8050	60.9468	86.4684	72.8012	0.8900	53.6974	87.5296	70.9289
0.8075	60.9468	86.4684	72.8012	0.8925	53.3539	88.4231	70.5536
0.8100	60.9468	86.4684	72.8012	0.8950	53.0170	88.6115	70.5536
0.8125	59.5551	86.4263	72.8012	0.8975	52.8551	88.7572	70.5002
0.8150	59.5551	85.9152	72.8012	0.9025	52.7852	90.5574	70.5002
0.8200	61.9916	84.7771	73.9393	0.9050	53.1078	90.3974	70.5002
0.8225	59.5551	86.4684	72.8012	0.9075	53.3623	90.2009	70.3985
0.8250	61.9916	85.4235	73.9393	0.9125	54.8821	90.4120	70.5196
0.8300	61.9916	85.3303	73.9393	0.9150	54.5927	90.4691	70.8791
0.8325	61.4999	85.9152	72.8012	0.9175	54.3604	90.7458	71.8578
0.8375	60.9468	86.4684	72.8012	0.9225	55.6883	90.3272	71.8353
0.8400	59.5551	85.9152	72.8012	0.9275	55.3125	90.3730	72.0913
0.8450	60.6511	85.3303	73.9393	0.9300	55.7670	89.5176	72.1086
0.8475	61.9916	85.4235	73.8461	0.9325	54.6557	92.4282	71.5449
0.8500	60.7658	85.2410	73.9393	0.9350	54.7848	92.4625	71.0585
0.8525	60.8055	85.1944	73.9393	0.9375	54.4668	92.1152	70.8887
0.8550	61.9916	85.3303	73.9393	0.9400	55.6867	90.1914	69.7613
0.8575	61.9916	85.3303	73.9393	0.9425	54.2560	92.8249	67.6197
0.8600	61.4999	85.9152	72.8012	0.9450	55.3497	90.4304	66.7830
0.8625	59.5551	86.4684	72.8012	0.9475	54.6645	91.0428	64.1281

Table C.2: Bid Prices of *RLF* for Base Problem

	Bid Price for Leg *l*				**Bid Price for Leg *l***		
SL_l=SL	***l:AB***	***l:BC***	***l:CD***	***SL_l=SL***	***l:AB***	***l:BC***	***l: CD***
0.8650	60.9468	86.4684	72.8012	0.9500	49.5234	94.7283	57.9156
0.8675	59.5551	86.4684	72.8012	0.9525	47.1795	94.8899	52.8763
0.8700	60.6511	85.3303	73.9393	0.9575	32.6141	98.8483	33.6434
0.8750	61.9916	85.4235	73.8461	0.9600	29.2893	98.6319	8.8503
0.8775	60.7658	85.2410	73.9393	0.9625	15.3393	102.9723	-21.1226
0.8800	60.8055	85.1944	73.9393	0.9650	-4.0411	102.6050	-211.0228
0.8825	53.1495	87.1087	72.8012	0.9675	0.0000	0.0000	0.0000

C.1.3 RLF with unequal SL_l

Table C.3: Optimal Seat Allocations of *RLF* with unequal SL_l for Base Problem

ODF		**RLF with unequal SL_l**						
Itinerary	**Class**	**0.90-0.85-0.90**	**0.91-0.86-0.91**	**0.92-0.87-0.92**	**0.93-0.88-0.93**	**0.94-0.89-0.94**	**0.95-0.90-0.95**	**0.96-0.91-0.96**
AB	**3**	42	42	42	41	41	40	40
	2	37	37	36	36	35	34	34
	1	31	30	28	27	25	23	21
AC	**3**	9	8	9	13	17	23	27
	2	19	18	18	18	18	17	17
	1	18	17	17	15	15	14	12
AD	**3**	7	13	16	17	18	19	21
	2	21	20	20	20	19	19	19
	1	16	15	14	13	12	11	9
BC	**3**	22	22	20	17	12	6	1
	2	18	18	18	18	17	17	17
	1	26	26	26	25	24	24	24
BD	**3**	7	7	7	10	15	18	22
	2	16	15	15	15	14	14	15
	1	21	21	20	19	19	18	16
CD	**3**	40	39	39	40	40	40	41
	2	37	36	36	35	34	34	33
	1	35	34	33	31	29	27	24

C.1.4 *RLF-M*

Table C.4: Optimal Seat Allocations of $RLF - M$ for Base Problem

ODF		***RLF-M***									
Itinerary	**Class**	**0.87**	**0.88**	**0.89**	**0.90**	**0.91**	**0.92**	**0.93**	**0.94**	**0.95**	**0.96**
AB	**3**	42	44	43	43	42	42	41	41	40	40
	2	40	40	39	39	38	38	37	36	35	34
	1	40	38	37	35	34	32	30	28	26	23
AC	**3**	0	2	2	2	4	7	11	16	22	27
	2	19	19	19	19	18	18	18	17	17	17
	1	21	20	19	19	18	16	15	14	12	10
AD	**3**	0	0	4	8	12	14	17	18	19	21
	2	21	21	21	20	20	20	19	19	19	19
	1	17	16	16	15	14	13	12	11	10	9
BC	**3**	23	23	23	23	22	22	21	21	20	20
	2	19	18	18	18	17	17	17	16	15	14
	1	27	25	24	23	22	21	19	18	16	13
BD	**3**	16	19	19	19	19	19	19	19	20	21
	2	16	16	15	15	15	15	14	14	14	14
	1	21	21	20	19	19	18	18	17	16	15
CD	**3**	38	38	38	38	37	37	38	40	41	42
	2	36	36	35	35	34	34	34	34	34	34
	1	35	33	32	31	30	30	29	28	27	25

C.1.5 Bid Prices of *RLF-M*

Table C.5: Bid Prices of *RLF-M* for Base Problem

	π_l		
SL	***l: AB***	***l: BC***	***l: CD***
0.87	60.7182	84.6738	74.0558
0.88	53.0363	82.3503	74.2481
0.89	52.6500	84.4604	73.5551
0.90	54.0635	84.5039	74.2791
0.91	55.1038	85.0764	74.7448

Table C.5: Bid Prices of *RLF-M* for Base Problem

	π_l		
SL	***l: AB***	***l: BC***	***l: CD***
0.92	54.2597	87.2540	74.2446
0.93	53.8056	89.5351	69.9619
0.94	53.2253	91.7929	63.2667
0.95	53.8519	91.2258	51.1273
0.96	35.4424	92.5866	26.8744

C.1.6 *LFR*

Table C.6: Optimal Seat Allocations of *LFR* for Base Problem

ODF		***LFR***								
Itinerary	**Class**	**63000**	**64000**	**65000**	**66000**	**67000**	**68000**	**69000**	**70000**	**71000**
AB	**3**	43	42	41	41	40	40	40	41	42
	2	34	34	34	34	34	35	36	37	38
	1	15	17	19	20	22	24	27	30	34
AC	**3**	32	31	30	29	28	24	18	11	3
	2	19	18	18	18	17	17	17	18	19
	1	7	7	8	9	10	11	13	15	18
AD	**3**	24	24	23	22	21	20	19	17	12
	2	19	19	19	19	19	19	19	19	20
	1	7	8	8	8	9	10	11	12	14
BC	**3**	22	22	20	20	19	19	20	21	22
	2	14	14	14	14	14	15	16	17	17
	1	5	7	9	11	13	15	17	19	22
BD	**3**	24	23	23	22	21	20	20	19	19
	2	15	15	15	14	14	14	14	14	15
	1	12	12	13	14	15	16	16	18	19
CD	**3**	44	43	43	43	42	41	40	38	37
	2	35	35	34	34	34	34	34	34	34
	1	20	21	22	24	25	26	27	29	30

C.1.7 *MaxminLF*

Table C.7: Optimal Seat Allocations of *MaxminLF* for Base Problem

ODF		*MaxminLF*								
Itinerary	**Class**	**63000**	**64000**	**65000**	**66000**	**67000**	**68000**	**69000**	**70000**	**71000**
AB	**3**	35	38	41	41	41	40	40	41	42
	2	34	35	34	34	34	35	35	36	38
	1	17	18	19	20	22	24	26	30	33
AC	**3**	33	30	29	28	27	25	22	20	18
	2	20	18	17	17	17	17	17	17	18
	1	8	9	9	9	10	11	13	14	17
AD	**3**	25	25	24	23	21	20	17	11	0
	2	19	19	19	19	19	18	19	19	20
	1	6	8	8	8	9	10	11	12	14
BC	**3**	16	15	17	17	19	19	20	21	22
	2	15	15	13	14	14	15	15	16	17
	1	4	9	11	12	13	15	16	19	21
BD	**3**	24	24	24	23	22	20	19	18	18
	2	16	16	16	15	14	14	14	15	15
	1	12	12	13	14	15	16	17	18	20
CD	**3**	43	43	43	42	42	41	39	38	40
	2	34	34	34	34	34	34	34	36	37
	1	19	19	19	22	24	27	30	33	36

C.2 INCREASED VARIANCE OF LOW FARE DEMAND

C.2.1 *RLF* with SL_l=SL for all l

Table C.8: Optimal Seat Allocations of *RLF* with $SL_l = SL$ for "Increased Variance of Low Fare Demand"

ODF		*RLF* with SL_l=SL for all l									
Itinerary	**Class**	**0.85**	**0.86**	**0.87**	**0.88**	**0.89**	**0.90**	**0.91**	**0.92**	**0.93**	**0.94**
AB	**3**	40	40	39	39	38	38	38	38	39	41
	2	39	38	38	37	37	36	35	35	34	33
	1	38	37	36	35	33	33	31	29	26	21
AC	**3**	7	9	12	15	17	20	22	24	26	30
	2	14	14	13	13	13	12	12	12	13	14
	1	21	20	19	18	18	16	15	14	13	11
AD	**3**	4	5	7	8	10	12	14	17	19	22
	2	20	20	20	19	19	19	19	18	18	18
	1	17	17	16	16	15	14	14	13	12	10
BC	**3**	21	21	21	20	20	20	20	19	19	18
	2	18	17	17	17	16	16	15	15	14	13
	1	25	25	24	23	22	21	20	19	17	14
BD	**3**	16	16	16	17	16	16	16	17	18	21
	2	15	15	14	14	14	14	14	14	14	14
	1	22	21	21	21	20	20	19	18	17	15
CD	**3**	35	35	35	35	35	35	35	36	38	41
	2	35	35	35	35	35	35	35	34	34	33
	1	36	36	36	36	36	35	34	33	30	26

C.2.2 *RLF-M*

Table C.9: Optimal Seat Allocations of *RLF-M* for "Increased Variance of Low Fare Demand"

ODF		***RLF-M***									
Itinerary	**Class**	**0.85**	**0.86**	**0.87**	**0.88**	**0.89**	**0.90**	**0.91**	**0.92**	**0.93**	**0.94**
AB	**3**	41	40	39	39	39	38	38	38	39	40
	2	40	39	38	38	38	37	36	35	34	34
	1	40	38	36	35	35	34	32	30	27	23
AC	**3**	1	2	6	9	9	13	19	22	25	29
	2	15	14	14	13	13	13	12	12	13	14
	1	22	21	20	19	19	18	16	15	13	11
AD	**3**	2	10	11	13	13	14	15	17	19	21
	2	21	20	20	19	19	19	19	18	18	18
	1	18	16	16	15	15	14	13	13	12	10
BC	**3**	22	21	21	20	20	20	19	19	19	20
	2	18	18	17	17	17	16	15	15	14	14
	1	27	25	24	24	24	22	21	19	17	13
BD	**3**	17	17	17	17	17	18	18	18	19	21
	2	15	15	14	14	14	14	14	14	14	14
	1	22	21	20	20	20	19	19	18	17	15
CD	**3**	35	34	35	36	36	36	37	38	39	41
	2	35	34	34	34	34	34	34	34	34	34
	1	35	33	33	32	32	32	31	30	28	26

C.2.3 *LFR*

Table C.10: Optimal Seat Allocations of *LFR* for "Increased Variance of Low Fare Demand"

ODF		***LFR***						
Itinerary	**Class**	**64000**	**65000**	**66000**	**67000**	**68000**	**69000**	**70000**
AB	**3**	42	41	40	39	39	38	38
	2	33	33	33	34	34	35	37
	1	19	20	23	25	27	31	35
AC	**3**	31	30	29	28	25	21	12
	2	15	15	14	13	13	12	13
	1	9	10	11	12	13	15	18
AD	**3**	23	22	22	20	19	17	13
	2	19	19	18	18	18	18	19
	1	9	10	10	11	12	13	15
BC	**3**	21	20	20	19	19	19	20
	2	13	13	13	14	14	15	16
	1	9	10	12	14	17	20	23
BD	**3**	23	22	22	21	19	18	17
	2	15	15	14	14	14	14	14
	1	13	14	15	16	17	18	20
CD	**3**	42	41	41	40	39	38	36
	2	34	33	33	34	34	34	34
	1	22	24	25	26	28	30	32

C.2.4 *MaxminLF*

Table C.11: Optimal Seat Allocations of *MaxminLF* for "Increased Variance of Low Fare Demand"

ODF		***MaxminLF***							
Itinerary	**Class**	**63000**	**64000**	**65000**	**66000**	**67000**	**68000**	**69000**	**70000**
AB	**3**	41	41	41	41	40	39	38	38
	2	33	33	33	33	34	34	35	37
	1	17	17	18	21	23	26	30	34
AC	**3**	33	33	31	30	28	26	23	16
	2	16	16	15	14	14	13	12	13
	1	9	9	10	11	12	13	15	18
AD	**3**	23	23	23	22	20	19	16	10
	2	19	19	19	18	18	18	18	19
	1	9	9	10	10	11	12	13	15
BC	**3**	16	16	18	18	19	19	19	20
	2	14	14	12	13	13	14	15	16
	1	10	10	12	14	15	17	19	23
BD	**3**	22	22	22	21	20	18	17	16
	2	15	15	14	14	14	14	14	14
	1	14	14	14	15	16	17	18	20
CD	**3**	42	42	41	41	40	38	36	35
	2	33	33	33	33	33	34	34	35
	1	23	23	24	26	28	30	34	36

C.3 SMALLER DIFFERENCES BETWEEN FARES

C.3.1 *RLF* with SL_l=SL for all l

Table C.12: Optimal Seat Allocations of *RLF* with $SL_l = SL$ for "Smaller Differences between Fares"

ODF		***RLF* with SL_l=SL**					
Itinerary	**Class**	**0.91**	**0.92**	**0.93**	**0.94**	**0.95**	**0.96**
AB	**3**	44	43	42	42	41	41
	2	39	38	38	37	36	35
	1	30	29	27	26	24	21
AC	**3**	13	16	20	21	23	26
	2	20	19	19	18	18	18
	1	12	11	10	9	8	8
AD	**3**	5	7	8	13	17	21
	2	21	21	21	20	20	19
	1	16	16	15	14	13	11
BC	**3**	23	23	22	22	21	19
	2	19	19	18	18	17	16
	1	20	19	18	17	16	14
BD	**3**	19	17	17	18	18	21
	2	16	16	16	15	15	15
	1	16	16	15	15	14	13
CD	**3**	40	40	40	39	40	41
	2	37	37	37	37	35	35
	1	30	30	30	29	27	24

C.3.2 *RLF-M*

Table C.13: Optimal Seat Allocations of *RLF-M* for “Smaller Differences between Fares”

ODF		***RLF-M***						
Itinerary	**Class**	**0.90**	**0.91**	**0.92**	**0.93**	**0.94**	**0.95**	**0.96**
AB	3	44	44	43	42	41	41	40
	2	40	40	39	39	38	37	36
	1	35	33	32	30	28	26	23
AC	3	8	7	10	10	13	19	26
	2	20	20	19	19	19	18	18
	1	13	13	12	11	10	9	7
AD	3	0	5	8	14	17	18	20
	2	22	21	21	20	20	20	19
	1	18	17	16	15	14	12	11
BC	3	25	25	24	24	23	22	21
	2	19	19	19	18	18	17	16
	1	21	20	19	18	17	15	12
BD	3	21	21	21	20	20	21	22
	2	16	16	16	16	15	15	15
	1	17	16	15	15	14	14	13
CD	3	39	39	39	38	39	40	41
	2	37	36	36	35	35	35	35
	1	30	29	28	27	26	25	24

C.3.3 *LFR*

Table C.14: Optimal Seat Allocations of *LFR* for "Smaller Differences between Fares"

ODF		*LFR*							
Itinerary	**Class**	**57000**	**57500**	**58000**	**58500**	**59000**	**59500**	**60000**	**60500**
AB	3	42	42	41	41	40	41	41	43
	2	34	34	35	35	36	37	38	40
	1	16	18	19	21	23	26	28	34
AC	3	32	31	30	28	25	20	14	7
	2	19	18	18	18	18	18	19	20
	1	6	7	7	7	8	8	10	13
AD	3	24	23	22	21	20	18	16	5
	2	19	19	19	19	19	20	20	21
	1	8	8	9	10	11	12	14	17
BC	3	22	22	21	21	21	22	23	25
	2	14	14	15	16	16	17	18	19
	1	6	8	9	11	13	15	17	20
BD	3	24	23	23	22	21	21	20	21
	2	15	15	15	15	15	15	15	16
	1	11	12	12	12	13	14	14	16
CD	3	43	43	42	42	41	40	39	39
	2	35	35	35	35	35	35	35	36
	1	21	22	23	24	25	25	27	29

C.3.4 *MaxminLF*

Table C.15: Optimal Seat Allocations of *MaxminLF* for "Smaller Differences between Fares"

ODF		*MaxminLF*								
Itinerary	**Class**	**56500**	**57000**	**57500**	**58000**	**58500**	**59000**	**59500**	**60000**	**60500**
AB	**3**	38	38	39	42	41	41	41	42	44
	2	36	36	35	35	35	35	36	37	40
	1	19	18	19	19	21	22	24	27	33
AC	**3**	31	30	30	28	27	26	23	20	11
	2	18	18	18	18	18	18	18	19	20
	1	7	8	9	7	7	8	8	10	12
AD	**3**	24	25	24	23	22	20	18	11	1
	2	19	19	19	19	19	19	20	20	22
	1	8	8	8	9	10	11	12	14	17
BC	**3**	14	15	15	17	18	19	21	22	24
	2	16	16	16	15	16	16	17	18	19
	1	9	9	9	12	13	15	16	18	21
BD	**3**	25	24	25	24	22	21	18	18	20
	2	16	16	16	15	15	15	15	15	16
	1	12	12	12	13	13	13	14	15	17
CD	**3**	43	43	43	43	42	41	40	40	40
	2	34	34	34	34	35	35	35	37	37
	1	19	19	19	20	22	25	27	30	30

C.4 REALISTIC VARIATIONS AND CLOSE FARES

C.4.1 *RLF* with SL_l=SL for all l

Table C.16: Optimal Seat Allocations of RLF with $SL_l = SL$ for ”Realistic Variations and Close Fares”

ODF		*RLF* with SL_l=SL							
Itinerary	**Class**	**0.9250**	**0.9300**	**0.9350**	**0.9400**	**0.9450**	**0.9500**	**0.9550**	**0.9600**
AB	**3**	41	40	40	39	39	38	38	38
	2	38	38	37	37	35	35	34	34
	1	31	31	30	30	29	29	28	27
AC	**3**	0	3	5	9	12	15	18	21
	2	14	14	14	13	13	12	12	12
	1	17	17	17	16	16	16	15	15
AD	**3**	18	18	18	17	17	17	17	18
	2	21	20	20	20	20	20	19	18
	1	20	19	19	19	19	18	18	17
BC	**3**	20	20	19	18	18	17	17	16
	2	18	18	17	17	17	16	16	15
	1	21	21	21	21	20	20	19	19
BD	**3**	18	18	18	18	17	17	17	18
	2	15	14	14	14	14	14	14	14
	1	18	18	18	18	17	17	17	17
CD	**3**	27	29	29	30	32	32	34	37
	2	33	34	34	34	34	35	34	33
	1	30	30	30	30	30	30	29	28

C.4.2 *RLF-M*

Table C.17: Optimal Seat Allocations of *RLF-M* for "Realistic Variations and Close Fares"

ODF		*RLF-M*						
Itinerary	**Class**	**0.9200**	**0.9250**	**0.9300**	**0.9350**	**0.9400**	**0.9450**	**0.9500**
AB	3	39	39	39	40	39	39	38
	2	40	40	40	40	38	37	37
	1	33	33	33	32	32	31	30
AC	3	0	0	0	0	3	7	11
	2	14	14	14	14	14	13	13
	1	17	17	17	17	17	16	16
AD	3	16	16	16	19	18	18	18
	2	21	21	21	20	20	20	19
	1	20	20	20	19	19	19	18
BC	3	21	21	21	21	21	20	20
	2	18	17	18	18	17	17	16
	1	21	21	21	21	20	20	19
BD	3	19	19	19	19	19	19	19
	2	15	15	15	15	15	14	14
	1	18	17	18	18	17	17	17
CD	3	27	27	27	29	30	32	33
	2	34	34	34	33	33	32	33
	1	30	30	30	29	29	29	29

C.4.3 *LFR*

Table C.18: Optimal Seat Allocations of *LFR* for "Realistic Variations and Close Fares"

ODF		***LFR***					
Itinerary	**Class**	**62500**	**63000**	**63500**	**64000**	**64500**	**65000**
AB	**3**	38	38	37	36	38	41
	2	31	32	32	34	36	39
	1	24	25	26	28	29	32
AC	**3**	28	27	25	22	15	0
	2	13	12	12	11	12	14
	1	14	14	14	15	15	17
AD	**3**	21	20	20	19	18	18
	2	17	17	18	18	19	20
	1	14	15	16	17	18	19
BC	**3**	19	18	17	18	19	21
	2	11	12	13	14	15	18
	1	13	15	16	17	19	21
BD	**3**	21	21	20	19	19	19
	2	14	14	13	14	14	15
	1	15	15	16	16	17	18
CD	**3**	40	40	38	37	35	29
	2	32	32	32	32	32	33
	1	26	26	27	28	28	29

C.4.4 *MaxminLF*

Table C.19: Optimal Seat Allocations of *MaxminLF* for "Realistic Variations and Close Fares"

ODF		***MaxminLF***						
Itinerary	**Class**	**62000**	**62500**	**63000**	**63500**	**64000**	**64500**	**65000**
AB	3	31	33	33	37	37	38	41
	2	33	33	34	33	33	35	38
	1	25	26	25	26	27	29	31
AC	3	28	27	26	24	22	17	0
	2	13	11	12	12	12	12	14
	1	15	15	15	14	15	15	17
AD	3	22	22	22	21	19	17	18
	2	18	18	18	18	18	19	21
	1	15	15	15	16	17	18	20
BC	3	10	11	12	14	15	17	20
	2	13	13	14	14	15	16	18
	1	14	15	15	17	18	20	21
BD	3	22	22	22	21	19	18	18
	2	14	14	14	14	14	14	15
	1	16	16	16	16	16	17	18
CD	3	39	39	39	38	37	33	27
	2	30	30	30	31	33	35	33
	1	24	24	24	25	27	30	30