USE OF AIR CHAMBERS AGAINST WATERHAMMER IN PENSTOCKS

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ABSTRACT

USE OF AIR CHAMBERS AGAINST WATERHAMMER IN PENSTOCKS

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All pipeline systems are susceptible to water hammer that can cripple critical infrastructure. One effective method to relieve excessive waterhammer pressures in pipelines is to use air chambers. This study aims to develop an empirical procedure for the quick analysis of penstock-turbine systems to determine dimensions and operating conditions of air-chambers that can effectively diminish the transient phenomena after sudden changes of flow rate in the system. A numerical study has been carried out by obtaining repeated solutions for variable system parameters using a commercial software. The relief brought by air chambers is found to approach to an asymptotic value for increasing chamber volumes. It is possible to determine the required chamber volume for a given discharge to limit the waterhammer pressures at a prescribed level in a given penstock-turbine system using the charts produced in the study.

Keywords: Waterhammer, Hydraulic Transients, Penstocks, Air Chamber

ÖZ

CEBRİ BORULARDA SU DARBESİNE KARŞI HAVA ODASI KULLANIMI

ADAL, Birand

Yüksek Lisans, İnşaat Mühendisliği Bölümü

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Tüm boru hatları kendi altyapısına zarar verebilen su darbesine maruz kalabilir. Hava odası kullanımı, boru hatlarında oluşabilecek aşırı su darbesi basıncına karşı kullanılabilecek etkili bir yöntemdir. Bu çalışmanın amacı, cebri boru-türbin sistemlerindeki ani debi değişikliklerinde oluşan su darbesinin sönümlenmesinde kullanılan hava odasının boyutlandırılması ve kullanım koşullarının belirlenmesi için bir hızlı analiz prosedürü geliştirmektir. Çalışma, ticari bir program kullanılarak değişken sistem parametreleri için tekrarlanan sayısal çözümler yapılarak gerçekleştirilmiştir. Hava odası hacmi arttıkça, hava odasının sağladığı rahatlamanın bir maximum değere asimtotik olarak yaklaştığı tespit edilmiştir. Bu çalışmada elde edilen çizelgelerden yararlanarak, cebri boru-türbin sistemlerindeki su darbesi basıncını belli bir seviyede tutmak için gereken hava odası hacminin sayısal çözüm yapılmadan saptanması mümkündür.

Anahtar Kelimeler: Su Darbesi, Zamanla Değişen Akım, Cebri Boru, Hava Odası

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A	Cross sectional area of the penstock, (m^2)
a	Sonic wave speed, (m/s)
D	Penstock diameter, (m)
D_T	Throttle diameter, (m)
e	Penstock thickness, (m)
E	Modulus of elasticity of pipe, (N/m^2)
g	Gravitational acceleration, (m/s^2)
$H=H_{max}$	Maximum total head in the system without air chamber, (m)
H_0	Static head of the system, (m)
H_a	Maximum total head in the system with air chamber, (m)
H_{ch}	Steady state head at air chamber, (m)
H_r	Head ratio
H_{ra}	Asymptotical head ratio
I_t	Turbine inertia, ($kg.m^2$)
K	Bulk modulus elasticity of the fluid, (N/m^2)
L	Penstock length, (m)
N	Rotational speed of the turbine, (rpm)
n_s	Turbine specific speed, (rad/s)
P	Pressure, (N/m^2)

P_{ini}	Preset pressure in the air chamber, (N/m^2)
P_T	Turbine power, (W,kW,MW,hp)
Q	Penstock discharge, (m^3/s)
T_c	Turbine closure time, (s)
T_f	Circumferential tensile force per unit length, (N/m)
t	Time, (s)
V_p	Penstock velocity, (m/s)
$\forall_{ch}$	Air chamber volume, (m^3)
γ	Unit weight of fluid, (N/m^3)
η	Turbine efficiency
ρ	Density of fluid, (kg/m^3)
σ_f	Maximum allowable tensile stress, (N/m^2)

CHAPTER 1

INTRODUCTION

1.1. General

Electricity demand has increased rapidly with increasing world population since 21th century. In Turkey, to supply increasing electricity demand as a result of rapid population growth, hundreds of power plants have been built since 1990s. In those days, to handle the energy deficit, Turkey has been forced to find quick solutions. So most of them are thermal power plants, thus they need fuel like coal and natural gas to generate electricity. This situation caused Turkey to import foreign supplies instead of using domestic, renewable and sustainable resources. However, in early 2000's, with increasing technical and practical knowledge, Turkey has espoused a renewable and sustainable resource with thousands of megawatts of domestic capacity; water. Hydropower has become a new trend among the state and the private sector. To encourage the private sector, Turkey has adopted new laws and regulations. Especially small hydropower has been considered the cleanest and the most economical method to produce energy, therefore the laws and regulations were mostly published for small hydropower. In March 2001, Electricity Market Law No. 4628 was published. Then in March 2003, Water Usage Right Agreement was legislated. With those implementations, investors started to get licences from Electricity Market Regulation Authority to build and operate hydropower plants. In May 2005, government guaranteed to buy electricity from these investors for duration of 10 years by the publication of Energy Law No. 5346.

All of these legislations attracted many investors to invest their money in the hydropower business. With increasing investment in hydropower, the proportion of hydropower in domestic energy output has significantly increased. Hundreds of small and medium scaled hydropower plants have been constructed. Thousands of engineers, technicians, operators, workers found employment. New techniques in hydropower have been developed such as pumped storage plants and run-off river plants. Hydropower has become an indispensible industry.

Building and operating a hydropower facility is not an easy work. It requires deep knowledge and care in order to be built and operated. Building the plant is the first challenge. The plant must be designed carefully to resist structural and hydraulic forces. If it is not designed safely, system failures resulting in structural damages may occur and it can cause casualties.

In hydraulic point of view, a HEPP, (Hydroelectric Power Plant) usually operates at steady state. That means, flow conditions do not change with time. In this state, only concern is hydrostatic and hydrodynamic forces. It is obvious that HEPP's must be designed to resist these forces. On the other hand, steady flow is not always possible. Sometimes flow pattern is changed for short time durations, either caused by malfunctioning component or human interference. This is called transient flow. Transient flow patterns and transient forces are complicated compared to steady flow. For this reason, transient forces are not easy to predict.

Waterhammer is a term used for pressure rises in a hydraulic system caused by transient events. Waterhammer may cause significant damage to the system if it is not designed accordingly. Especially penstocks, which are water conveyors from reservoir to the turbine, and turbines, are very prone to waterhammer. They may crack, deform and even collapse due to waterhammer. In the history, there are many huge accidents caused by transient forces and resulted in drastic damages and loss of human lives. The most significant one happened in 2009. Sanayo-Shushenskaya plant is one of the world's biggest HEPP located in Eastern Siberia 4000 km east of Moscow. Its installed capacity 6400 MW and it has an average annual production of 23.5 TWh. The accident occurred on 17 August 2009 at 08:13 local time. A loud bang heard from turbine 2, and then the rotor, which is 920 tones, shot out of its seat. As a result, the machinery hall and rooms below its level were flooded. According to media, the cause of the accident is transient forces aroused by excessive vibration at turbine 2. After the accident, seven people, including the plant's former head Nikholai Nevolko and his deputies were charged with safety breaches.

To avoid similar events, many methods to control waterhammer have been developed. Air chamber is one of these methods. It is a rather complex phenomenon which requires careful investigation.

1.2. Literature Survey

The studies on fluid transients have been carried out since 18th century. The following paragraph includes the historical background of fluid transients.

The study of hydraulic transients started with the investigation of the propagation of sound waves in air, the propagation of the sound waves in shallow water and the flow of blood in arteries. However, none of these problems could be solved until the development of the theories of elasticity and calculus and the solution of the partial differential equations.

First researchers interested in these topics in the history are Newton, Euler and Lagrange. Newton and Lagrange firstly defined the propagations of sound waves in air and the celerity of waves in a canal.

Then Euler developed a detailed theory of the propagation of the elastic waves and derived a partial differential equation for wave propagation (Chaudhry, 1987).

About 1808, Laplace figured out the reasons for the difference between the theoretical and practical values of the velocity of sound in air. In 1789, Monge developed a graphical method to integrate partial differential equations. He is the first person to introduce the term method of characteristics in the literature. Young, investigated flow in bloodstreams, friction and bend losses, and the propagation of pressure waves in pipes. In 1869, Riemann developed three dimensional equations of motion and simplified it to one dimension for sound waves. Weber studied flow of incompressible fluids in an elastic pipe and formed continuity and momentum equations which may be the milestones in the modern fluid mechanics. Michaud is the first person who made preliminary studies about waterhammer in closed conduits in 1878. Also he studied design of air chambers and safety valves (Chaudhry, 1987).

Waterhammer in HEPP's was firstly investigated by Frizell. He was working for Ogden Hydroelectrical Power Plant in Utah as an engineer and he conducted experiments on its long penstock. He developed expressions for the velocity of waterhammer waves and the pressure rise due to instantaneous reduction of the flow. He stated that the speed of waterhammer waves would be the same as the speed of sound in unconfined water if the modulus of elasticity of the pipe walls was infinite. He also stated the effects of branch lines, wave reflections and successive waves on speed regulation. However, Frizell's work has not been appreciated as much as that of his contemporaries, Joukowski and Allievi (Chaudhry, 1987).

In 1897, Joukowski conducted various experiments in Moscow's water supply pipes. Based on his studies he developed a formula for the wave velocity, containing elasticity of both water and pipe walls. He also discovered a relation between velocity drop and pressure rise by using two methods: the conservation of energy and the continuity equation. He discussed the propagation of the pressure wave along the pipeline and reflection of the wave from the open end. He examined the effects of surge tanks, air chambers and safety valves on waterhammer pressures. He also defined the closure times as rapid and gradual closure according to T<=2L/a formula. T is closure time, L is length of the pipe, and a is the speed of sound in that conduit. If T is less or equal to 2L/a value, closure is named rapid closure (Chaudhry, 1987).

In 1902, Allievi derived a dynamic equation and introduced two dimensionless parameters. He obtained an expression for the pressure rise at the valve and introduced charts for the pressure rise and drop caused by a uniform valve operation (Chaudhry, 1987).

Quite a few scholars studied transient flow and waterhammer in closed conduits. Many of them broadened their study with examining hydraulic transients in hydropower plants.

Shimada and Okushima (1984) developed a new numerical model and technique for waterhammer. They used a series of solution method and a Newton-Raphson method with new calculation steps. In this method, fewer calculations were required than previous methods.

Jimenez and Chaudhry (1987) studied the effects of pipe and wall elasticity and compressibility of water on waterhammer. They also investigated the stability of a single hydropower station unit. They derived a stability criterion and verified it by computer software.

Peicheng et al. (1989) performed tests on Linzhengqu Hydropower Plant to illustrate that pressure relieve valves and safety membranes can replace surge tanks in small hydropower stations. They proved that both are reliable and useful implementations.

Souza et al. (1999) conducted simulations on transient flow in hydropower plants by considering nonlinear model of the penstock and turbine. They developed a nonlinear simulation method and analyzed both the penstock and turbine by using their electrical equivalent circuit model.

Stephenson (2002) studied air chambers for waterhammer protection of pumping lines considering a pump trip case. He simplified sizing of air chamber with the monographs presented. He included air and water volumes in the monographs as well as inlet and connection diameters in order to optimize the chamber design. He also presented a guideline to make all these selections.

Elliot, Liou and Peterson (2006) investigated sizing and design of an air chamber with transient modeling results and field test comparisons. They studied on Wenatchee Regional Water system that was built in 1980 to supply all domestic water demands of the Greater Wenatchee, Washington service area. Their calculations showed that the existing air chamber was not sufficient enough to protect the system in case of a power failure. They used a numerical method for transients in order to resize the air chamber. Furthermore, after the new air chamber was installed and with the full service area operating, a series of pump failure tests were conducted. They found that the test results are consistent with the transient model they used and the air chamber size is appropriate.

1.3. Scope of the Study

In design and operation phase of a hydropower plant, waterhammer pressures usually result in tough situations. There is always a risk for confronting pressure rises at any time when the plant is in operation. Therefore, protective measures must be taken into account. The aim of the present study is to reveal an alternate solution to eliminate the consequences of waterhammer. Air chambers, at that point, may be useful if carefully designed. Unfortunately, guidelines for sizing and designing an air chamber for hydropower systems consisting of penstocks are very limited in the literature. The scope of this study is to present a practical guideline for designing air chambers to protect the system from waterhammer pressures. By using a commercial software which utilizes the method of characteristics to solve the non-linear partial differential equations of transient flow, a series of analyses will be conducted with various sets of input parameters for the system.

1.4. Organization of the Study

The study consists of six chapters which are organized as follows:

Chapter 1 presents the general information about the subject and brief history. It also includes literature survey and scope of the study.

Chapter 2 gives the basic information about waterhammer concept which is the main phenomenon for the air chamber requirement. It starts with general transient flow information. Then, waterhammer which is a special topic in transient flow is discussed. The equations of waterhammer are issued afterwards. The solution of waterhammer equations with the method of characteristics is explained. At the end of the chapter, the role of waterhammer in hydropower plants is defined. Briefly, causes and effects of waterhammer in hydropower plants are described. Moreover, the protection methods from harmful effects in hydropower plants are discussed with their governing equations.

Chapter 3 mainly focuses on air chamber. Definition of air chamber is issued more in detail. Then, advantages and disadvantages of air chamber compared to conventional surge tank are portrayed. At the end of the chapter, the governing equations of air chamber are clarified.

Chapter 4 describes the computer software that is used in this study, Bentley Hammer. Its functions and modeling procedures are described.

Chapter 5 is the main body of this study. Firstly, the computer model used in the study is defined with its components and parameters. Then, selection of the parameters that are chosen within specified rules and formulas are specified. Some of them are penstock diameter, penstock length, closure time, wave speed, penstock material, turbine speed, specific speed, turbine inertia, air chamber throttle diameter, and air chamber preset pressure. The related tables and charts obtained while selecting these parameters are listed herein. Thereafter, preliminary runs for a single chosen head and discharge values are illustrated and some preliminary results are listed. Also, the boundary conditions of the problem formed. At the end of the chapter, for various realistic and practical head and discharge values, main runs and their results are described with final charts and tables. Comments for the final results are stated.

Finally, in Chapter 7, conclusions and final remarks of the study are listed.

CHAPTER 2

WATERHAMMER CONCEPT

2.1. Transient Flow

In fluid mechanics, flow is identified with two different types with respect to its conditions: *steady* and *unsteady (transient) flow*. In steady flow, flow conditions like velocity, discharge and pressure do not change at a point with time. However in transient flow, the conditions at a point may change with time. From that definition, it can be said that steady flow is a special case of transient flow that the transient flow equations must satisfy.

In general, transient flow is classified in two types: *quasi-steady flow* and *true transient flow*. In quasi-steady flow, changes in flow parameters are gradual and short lasting. Lowering of a huge reservoir or drawdown of a huge water table may be examples of quasi-steady flow. On the other hand, in true transient flow, changes in flow parameters are rapid and significant. Oscillation of water in a surge tank and flow in a penstock after a valve operation may be examples of true transient flow.

2.2. Waterhammer

2.2.1. General Concepts

Waterhammer is a term used synonymously to describe unsteady flow of fluids in pipelines. Its difference from transient flow is that waterhammer is restricted to water. The name waterhammer comes from the sound of the water that is stopped suddenly in a pressurized pipeline is similar to hammering sound. That sound is nothing but the sound of the travelling pressure surge. The speed of this wave is analogous to speed of sound in the pipe. Some typical incidents that lead to waterhammer in pipe flow are as follows:

- Valve operations that results in a change in valve opening
- Vibrations of system elements like penstocks, turbines, pumps etc.
- Wave formations on the reservoirs and forebays
- Sudden water elevation changes of reservoirs and forebays
- Power failures or malfunctions in the system elements
- Emergency shutdowns of the systems elements
- Maintenance works in the system
- Emergency filling or emptying of the penstocks
- Human errors in operation

When these events occur in the system, kinetic energy of the water column transforms into elastic energy and hence waterhammer pressure. Both pipeline and fluid deform because of the waterhammer pressure. This pressure starts to travel through the pipeline and may harm the weak points in the pipeline. The amplitude of the pressure wave diminishes gradually due to the friction effect if transient conditions are vanished. Resonance may occur if the natural frequency of the pipeline coincides with the frequency of transient flow. The period of transient flow is the time interval at which transient flow conditions are repeated. The period of waves occurring on the reservoir surface can be given as an example for this concept. The number of transient cycles at a unit time is called as the frequency of transient flow. When resonance occurs in a hydraulic system, the amplitude of waterhammer pressure steadily increases with each cycle resulting in heavy damage or even failure of the system.

2.2.2. Derivation of Transient Flow Equations

To derive the transient flow equations, the unsteady momentum and continuity equations are applied to a control volume including a section of the pipe.

Firstly, the event of sudden stoppage of flow at a downstream valve is first identified, and then the continuity and momentum equations are assigned to an incremental change in valve setting. In Fig. 2.1a friction and minor losses are negligible. When the valve is closed, the fluid immediately adjacent to valve is brought from V_0 to rest by the force emerged due to the higher pressure developed at the frontal face of the valve. After the first layer is brought to rest, the same operation is applied to the next layer of fluid bringing it to rest. Consequently, a high pressure wave is emerged as traveling upstream with speed of sound named as *sonic wavespeed a*.

The momentum equation is applied to a control volume, Fig. 2.1b in which the wave is moving to the left with speed of $a- V_0$ caused by a small change in valve opening. The pressure change Δp at the valve is followed by a velocity change ΔV. The momentum equation for the x direction indicates that the resultant x component of force on the control volume is equal to the time rate of increase of x momentum within the control volume plus the net efflux of x momentum from the control volume (Wylie et al. 1993).

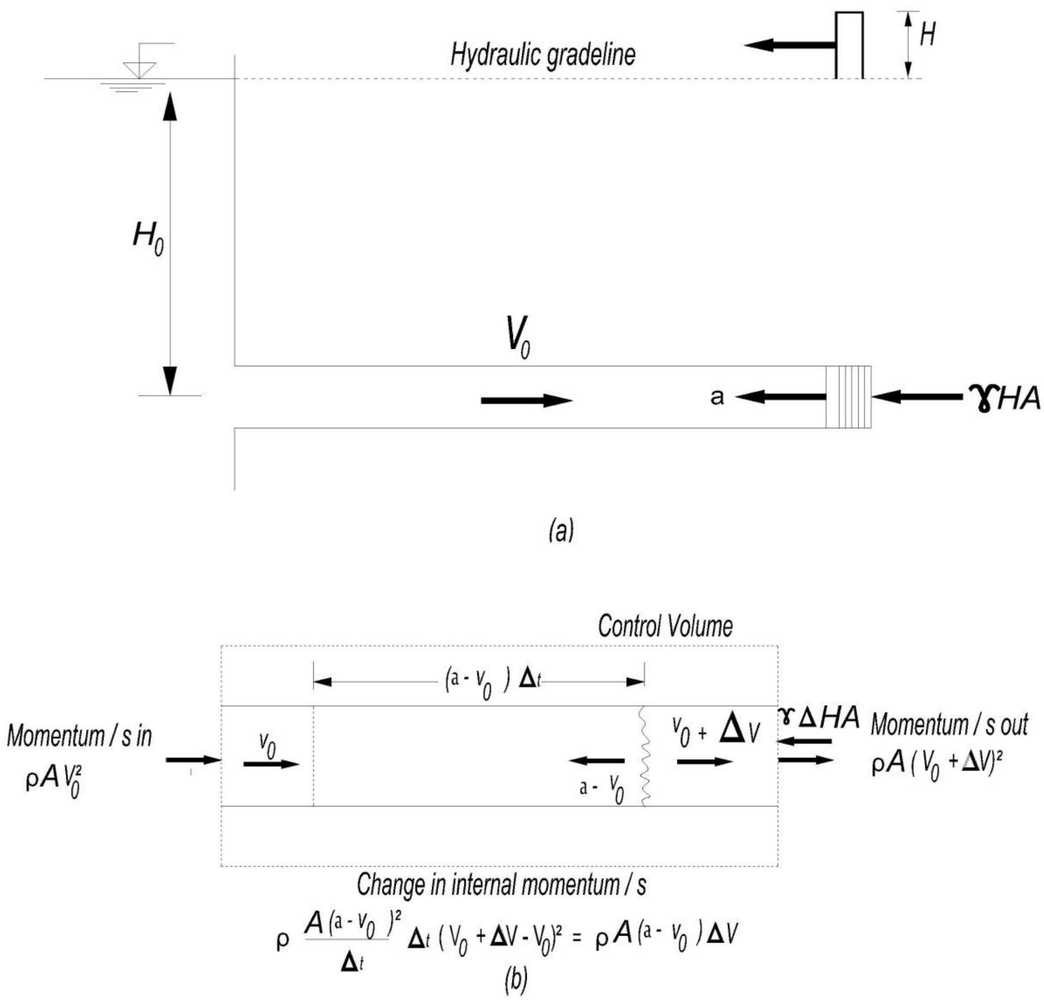

Figure 2.1 (a) Instantaneous stoppage of frictionless liquid in a horizontal pipe; (b) momentum equation applied to control volume

The time rate of increase of linear momentum is

$$\frac{A(a-V_0)\Delta t}{\Delta t}[(\rho+\Delta\rho)(V_0+\Delta V)-\rho V_0] \qquad (2.1a)$$

The momentum equation states

$$-\Delta pA = A(a-V_0)[(\rho+\Delta\rho)(V_0+\Delta V)-\rho V_0]+(\rho+\Delta\rho)A(V_0+\Delta V)^2-\rho A V_0^2 \qquad (2.1b)$$

where

ρ = mass density of fluid

$\Delta\rho$ = incremental density change

g = acceleration of gravity

γ = specific weight of the fluid = ρg

Δp = increment of pressure change

A = cross-sectional area of pipe

V_0 = initial velocity

ΔV = increment of flow velocity

a = unknown wavespeed

Conservation of mass in the control volume indicates that at any time the net mass influx equals the time rate of increase of mass inside the control volume. Because the same volume of fluid $A(a\text{-}V_0)\Delta t$ is having its density changed, the equation is

$$\rho A V_0 - (\rho + \Delta\rho)A(V_0 + \Delta V) = \frac{A(a - V_0)\Delta t[(\rho + \Delta\rho) - \rho]}{\Delta t} \qquad (2.1c)$$

When this equations is combined with the momentum equation, the following basic equation results

$$\Delta p = -\rho a \Delta V \qquad (2.1d)$$

Since $\Delta p = \rho g \Delta H$, in which ΔH is the *head change*,

$$\Delta H = -\frac{a\Delta V}{g} \qquad (2.1e)$$

If the flow is stopped entirely $\Delta V = -V_0$ and $\Delta H = aV_o/g$. Equations (2.1d) and (2.1e) also indicate that for an increase in velocity at the valve the head at the valve must decrease. If the valve is on the downstream end of a pipe and is closed by increments, the equations become

$$\sum \Delta p = -\rho a \sum \Delta V \tag{2.2a}$$

$$\sum \Delta H = -\frac{a}{g} \sum \Delta V \tag{2.2b}$$

which hold unless the pressure wave has not reached the upstream end of the pipe and returned as a reflected wave. For adjustments in an upstream gate, a similar derivation shows that $\Delta p = +\rho a \Delta V$ so

$$\sum \Delta p = +\rho a \Delta V \tag{2.3a}$$

$$\sum \Delta \mathrm{H} = +\frac{a}{g} \sum \Delta V \tag{2.3b}$$

characterize the change in flow resulting in change in pressure. It is the basic equation of waterhammer and always holds except in the presence of reflections. Equation (2.3b) is generally associated to Joukowski, however in the literature there are studies that Menabrea have made calculations with the equation (Chaudhry, 1987).

To find the pressure rise in the system, the magnitude of the *wavespeed 'a'* should be determined. Using continuity equation, together with equation (2.2), the numerical value of a can be computed. With reference to Fig. 2.2 (Wylie et al. 1993), if the valve at the downstream end of the pipe is instantaneously closed, the pipe may elongate in length Δs, depending on its supporting type. It can be assumed that the valve moves this distance in L/a seconds, or has the velocity $\Delta s a/L$. Hence the velocity of fluid at the gate has been changed by $\Delta V = \Delta s a/L - V_0$. The fluid mass entering the pipe during L/a seconds after valve closure is $\rho A V_0 L/a$, which is contained within the pipe by increasing its cross-sectional area, by filling additional volume caused by pipe extension Δs, and by squeezing the liquid due to its higher pressure

$$\rho A V_0 \frac{L}{a} = \rho L \Delta A + \rho A \Delta s + LA\Delta\rho \quad (2.4)$$

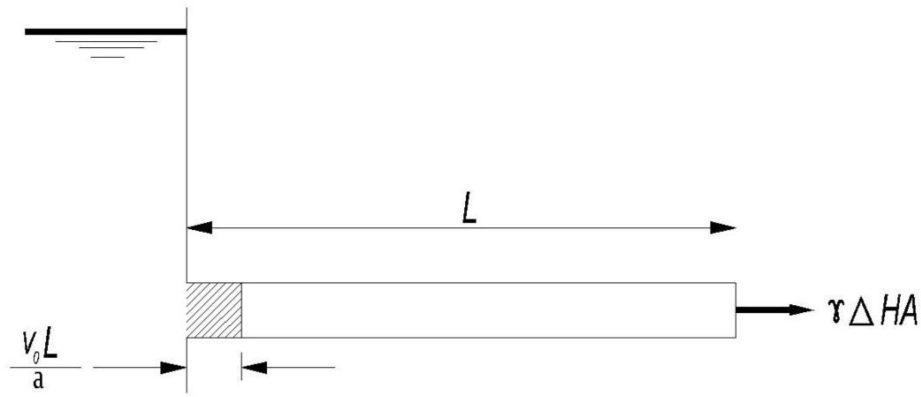

Figure 2.2 Continuity relations in pipe

To eliminate V_0, $\Delta V = \Delta s a/L - V_0$ is used and Eq. (2.4) simplifies to

$$-\frac{\Delta V}{a} = \frac{\Delta A}{A} + \frac{\Delta\rho}{\rho} \quad (2.4a)$$

By use of Eq. (2.1a) to eliminate ΔV,

$$a^2 = \frac{\Delta\rho/\rho}{\Delta A/A + \Delta\rho/\rho} \quad (2.5)$$

If the pipe is supported it cannot extend, so that $\Delta s=0$ and the same Eq. (2.5) is acquired, with or without expansion joints. By introducing in the *bulk modulus of elasticity K* of the fluid, defined by

$$K = \frac{\Delta p}{\Delta\rho/\rho} = -\frac{\Delta p}{\Delta\forall/\forall} \quad (2.6)$$

with $\Delta\forall/\forall$ the fractional volume change, Eq. (2.5) can be reorganized to obtain

$$a^2 = \frac{K/\rho}{1+(K/A)(\Delta A/\Delta p)} \tag{2.7}$$

For very thick-walled pipes $\Delta A/\Delta p$ is negligible, and $a \approx \sqrt{K/\rho}$ is the *acoustic speed* of a small disturbance in an infinite fluid. For very flexible pipe walls, the 1 in the denominator becomes small and negligible, so the equation becomes

$$a \approx \sqrt{\frac{A}{\rho}\frac{\Delta p}{\Delta A}} \tag{2.8}$$

The estimation of the *wavespeed* in a conduit requires the knowledge of the fluid bulk modulus of density, and the calculation of the conduit elasticity as defined by $\Delta A/\Delta pA$ in Eq. (2.7). A thin-walled pipeline is examined in Fig. 2.3 (Wylie et al. 1993) as an illustration. The change in pipe wall tensile stress, $\Delta\sigma$, is stated by

$$\Delta\sigma = \frac{\Delta T_f}{e} = \frac{\Delta pD}{2e} \tag{2.9}$$

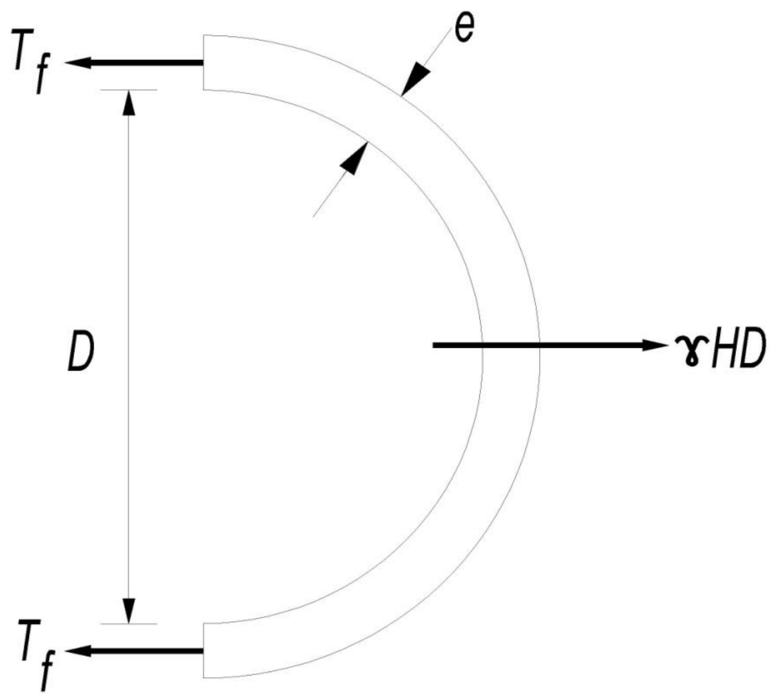

Figure 2.3 Forces on semicylinder of pipe
due to waterhammer

in which e is pipe wall thickness and T_f is the circumferential tensile force per unit length of pipe. The change in circumferential unit strain is acquired when Eq. (2.9) is divided by modulus of elasticity for the wall material, E. The radial extension is achieved by multiplying by the radius $D/2$, which, when multiplied by the perimeter πD, yields the change in cross-sectional area as a result of the pressure change: $\Delta A = \Delta p \pi D^3/(4eE)$. After dividing by A and Δp, the following equation is acquired

$$\frac{\Delta A}{A\Delta p} = \frac{D}{Ee} \tag{2.10}$$

which, when substituted into Eq. (2.7), states an equation that may be used for a thin-walled pipeline (Wylie et al. 1993).

$$a = \sqrt{\frac{K/\rho}{1+(K/E)(D/e)}} \tag{2.11}$$

2.2.3. Basic Differential Equations for Transient Flow

In this part, one-dimensional equations of motion and continuity are defined. Derivations of these equations are as follows (Wylie et al. 1993).

2.2.3.1. Equation of Motion

The equation of motion (momentum equation) is derived for fluid flow through a conical tube, a cylindrical tube or prismatic section which is illustrated in Fig. 2.4 (Wylie et al. 1993). The tube is filled with fluid with mass density ρ. It is assumed that an average cross-sectional pressure and velocity is equal to the *centerline pressure* $P(x,t)$ and *average cross-sectional velocity* $V(x,t)$ respectively. Pressure is converted to the hydraulic grade line $H(x,t)$, called *piezometric head*, or in short, *head*.The *volumetric discharge* $Q(x,t)$ is the product of the velocity and the pipe area and it is used as the dependent variable, along with either p or H. Distance x and time t are the independent variables.

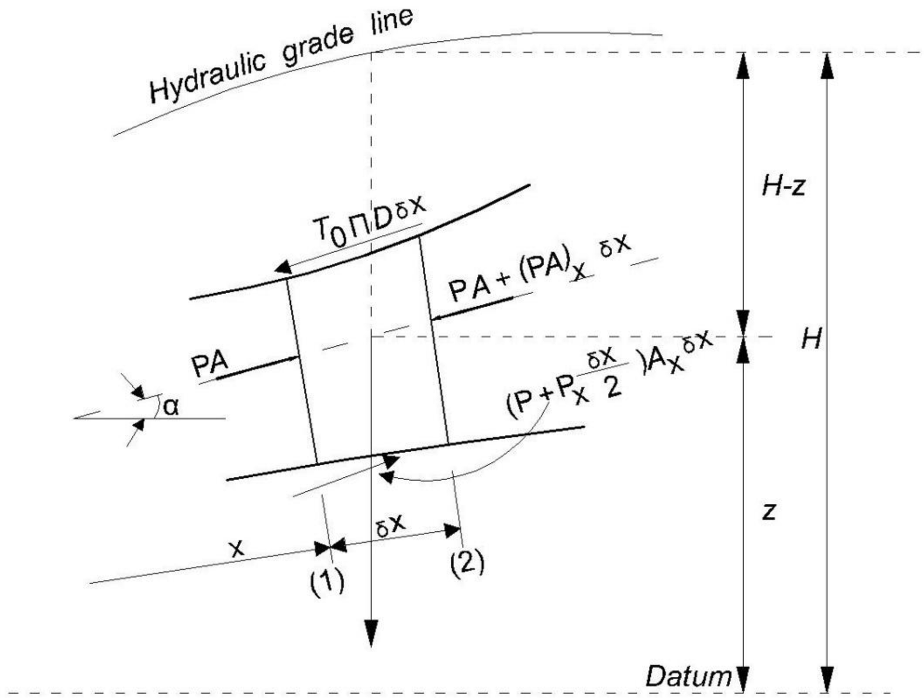

Figure 2.4 Free-body diagram for application of equation of motion

With reference to the figure, summation of all forces exerted on the control volume (CV) is equal to the summation of time rate of change of momentum in the CV and momentum flux through the control surface (CS) 1 and 2. Summation of surface and body forces in *x* direction is

$$\sum F_x = PA - \left[PA + \frac{\partial}{\partial x}(PA)\delta x\right] - \tau_w \pi D \delta x - \rho g A sin\theta \delta x \tag{2.12}$$

The time rate of change of linear momentum in the CV is

$$\frac{\partial}{\partial t}\int_{CV} \rho V d\forall = \frac{\partial}{\partial t}(\rho V A)\delta x \tag{2.13}$$

The linear momentum flux through the CS is

$$\frac{\partial}{\partial x}(\rho V^2 A)\delta x \tag{2.14}$$

Combining all terms the equation of motion becomes

$$-\left(A\frac{\partial P}{\partial x}+\tau_w\pi D+\rho gAsin\theta\right)\delta x=\frac{\partial}{\partial t}(\rho VA)\delta x+\frac{\partial}{\partial x}(\rho V^2A)\delta x \tag{2.15}$$

Dividing Eq. (2.15) by $\delta x\rho A$ and then rearranging, the final form of the *equation of the motion* is obtained as

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}+\frac{1}{\rho}\frac{\partial P}{\partial x}+gsin\theta+\frac{4\tau_w}{\rho D}=0 \tag{2.16}$$

2.2.3.2. Continuity Equation

Continuity equation is derived using the law of conservation of mass principle. Control volume shown in Fig.2.5 (Wylie et al. 1993) is used to derive the continuity equation. The fluid inside is single phase liquid and compressible. Conduit walls are elastic. Thus, due to pressure changes CV may stretch. The flow is assumed one-dimensional and pressure is uniform over the CS.

Continuity equation indicates that the time rate of change of mass inside the CV is equal to the net mass flux across the entire CS sections.

$$\frac{\partial}{\partial t}\int_{CV}\rho d\forall+\int_{CS}\rho(\vec{V}.\vec{n})dA=0 \tag{2.17}$$

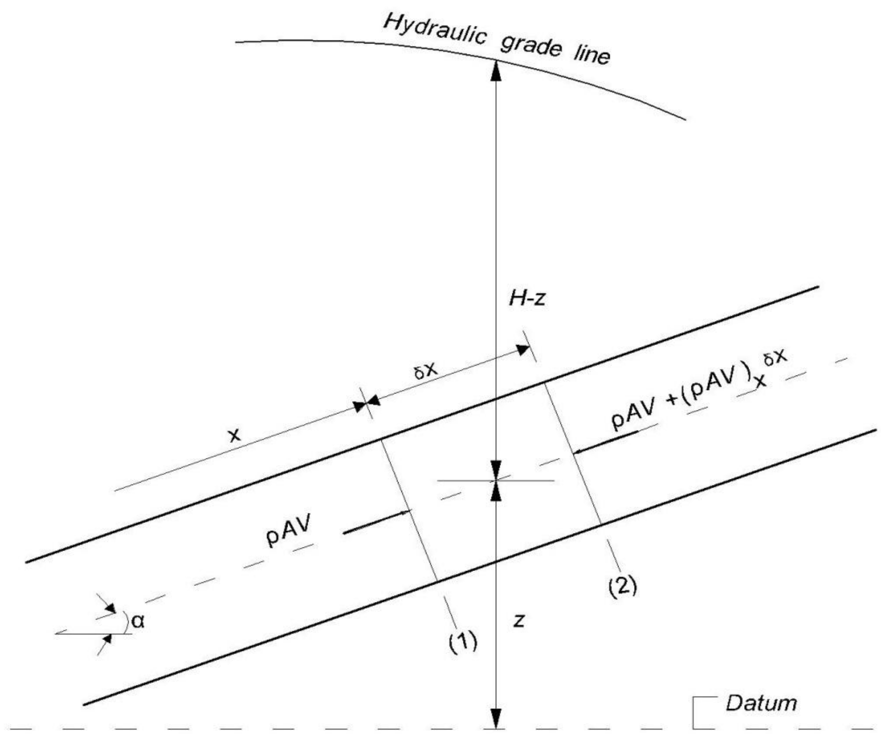

Figure 2.5 Control volume for continuity equation

Assuming that the *ρ* is constant over the control surface

$$\frac{\partial}{\partial t}(\rho A)\delta x + \left[\rho AV + \frac{\partial}{\partial x}(\rho AV)\delta x\right] - \rho AV = 0 \qquad (2.18)$$

After simplifications, Eq. (2.18) becomes

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho AV) = 0 \qquad (2.19)$$

With differentiation of Eq. (2.19) by parts, and then substitution of bulk modulus of elasticity *(K)* and the modulus of elasticity of the pipe into this equation gives

$$\left(\frac{1}{K} + \frac{D}{Ee}\right)\left(\frac{\partial P}{\partial t} + V\frac{\partial P}{\partial x}\right) + \frac{\partial V}{\partial x} = 0 \qquad (2.20)$$

Subsituting Eq. (2.11) into Eq. (2.20) and simplifying the resulting equation gives the general form of *continuity equation*.

$$\frac{\partial P}{\partial t} + V\frac{\partial P}{\partial x} + \rho a^2 \frac{\partial V}{\partial x} = 0 \tag{2.21}$$

2.2.4. Solutions of Basic Differential Equations for Transient Flow with Method of Characteristics

The governing equations (Eqs. (2.16) (2.21)) are non-linear partial differential equations. It is inconvenient to solve these equations numerically in this form. Therefore, these two equations are needed to be transformed into appropriate forms. By implementing *method of characteristics*, they can be transformed into four ordinary differential equations, which can be integrated to get finite difference equations. Resulting equations can be utilized for obtaining numerical solutions. The derivation of method of characteristics is as follows (Wylie et al. 1993)

The, continuity and momentum equations can be designated as L_1 and L_2.

$$L_1 = \frac{\partial P}{\partial t} + V\frac{\partial P}{\partial x} + \rho a^2 \frac{\partial V}{\partial x} \tag{2.22}$$

$$L_2 = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + \frac{1}{\rho}\frac{\partial P}{\partial x} + F = 0 \tag{2.23}$$

where $$F = gsin\theta + \frac{4\tau_w}{\rho D} \tag{2.24}$$

Linear combination of Eq. (2.22) and Eq. (2.23) can be considered as

$$L = L_1 + \lambda L_2 = 0 \tag{2.24a}$$

By writing the full forms of L_1 and L_2 in combination

$$\left[\frac{\partial P}{\partial t} + \left(V + \frac{\lambda}{\rho}\right)\frac{\partial P}{\partial x}\right] + \lambda\left[\frac{\partial V}{\partial t} + \left(V + \frac{\rho a^2}{\lambda}\right)\frac{\partial V}{\partial x}\right] + \lambda F = 0 \tag{2.25}$$

From calculus

$$\frac{\partial P}{\partial t} + \left(V + \frac{\lambda}{\rho}\right)\frac{\partial P}{\partial x} = \frac{dP}{dt} \tag{2.26}$$

if $$V + \frac{\lambda}{\rho} = \frac{dx}{dt}$$

$$\frac{\partial V}{\partial t} + \left(V + \frac{\rho a^2}{\lambda}\right)\frac{\partial V}{\partial x} = \frac{dV}{dt} \tag{2.27}$$

if $$V + \frac{\rho a^2}{\lambda} = \frac{dx}{dt}$$

So Eq. (2.25) comes in a simpler form

$$\frac{dP}{dt} + \lambda\frac{dV}{dt} + \lambda F = 0 \tag{2.28}$$

The description of the unknown multiplier can be done by using the constraints in Eq. (2.26) and Eq. (2.27)

$$\lambda = \pm\rho a \tag{2.29}$$

By substituting the values of the λ into the constraints in Eq. (2.26) and Eq. (2.27) and ignoring the flow velocity compared with acoustic speed

$$\frac{dx}{dt} \cong \pm a \tag{2.30}$$

This equation represents the change in position of the surge wave related to the change in time. When two λ values are inserted into Eq. (2.28), two sets of equations, which are called *characteristic equations* C^+ *and* C^- appear.

$$\frac{1}{\rho}\frac{dP}{dt} + a\frac{dV}{dt} + aF = 0 \tag{2.31}$$

if $$\frac{dx}{dt} = +a$$

$$\frac{1}{\rho}\frac{dP}{dt} - a\frac{dV}{dt} - aF = 0 \tag{2.32}$$

if $$\frac{dx}{dt} = -a$$

Eqs. (2.31) and (2.32) are valid as long as their constraints are satisfied. They provide elimination of one independent variable x and enable the conversion of non-linear partial differential equations of transient flow into ordinary differential equations. Only problem is that they are valid only along their straight lines which are defined by their constraints, whereas Eqs. (2.16) and (2.21) are valid everywhere on the *x-t* plane.

Space time *(x-t)* plane is shown in Fig.2.6 and Fig.2.7 (Wylie et al. 1993). Fig.2.6 illustrates the general view of characteristic lines in *x-t* plane while Fig.2.7 shows the grid system used in solving single-pipe problems.

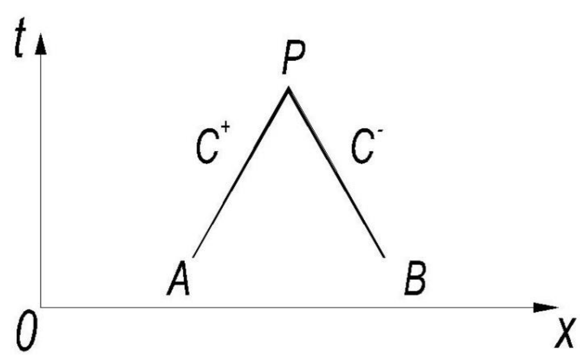

Figure 2.6 Characteristic lines in the x-y plane

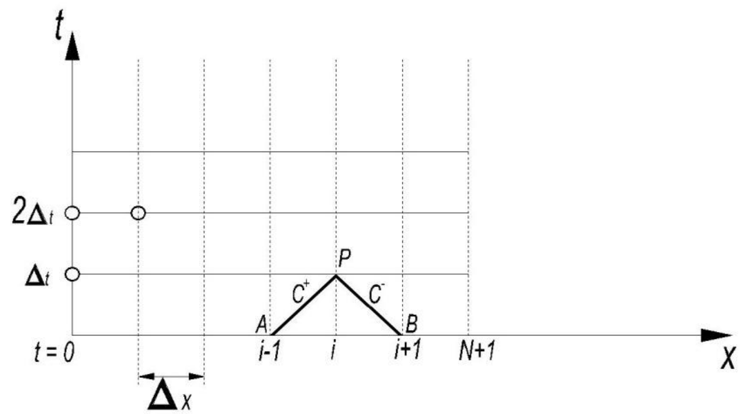

Figure 2.7 x-t grid for solving single-pipe problems

C^+ is the line having *+1/a* slope whereas C^- is the line having *-1/a* slope. They represent the followed path of transient disturbance. The *x-t* grid can be formed by dividing the pipe into *N* reaches virtually. At each time step, characteristic equations need to be solved for *N+1* nodes. According to Courant condition, time step can be calculated as $\Delta t = \Delta x/a$.

If dependent variables *V* and *H* are known at nodes *A* and *B*, C^+ and C^- equations can be integrated along lines *AP* and *BP*. Then, two equations with two unknowns *V* and *H* are obtained for node *P*. By solving these equations simultaneously, unknowns can be attained for node *P*. It should be noted that, node *P* is one time step ahead of node *A* and *B*. Its meaning is with known *V* and *H* parameters for a time step, *V* and *H* parameters for subsequent time step can be calculated.

Darcy-Weishbach definition of shear stress for steady flow can be applied in transient flow in order to simplify the case.

$$\tau_w = \frac{\rho f V|V|}{8} \tag{2.33}$$

therefore,

$$F = gsin\theta + f\frac{V|V|}{2D} \quad (2.34)$$

The relation between pressure and head is, $P = \rho g(H - sin\theta)$. By multiplying C^+ equation by $a\frac{dt}{g} = \frac{dx}{g}$ and by substituting velocity term with discharge term *(Q=V.A)* the equation can be integrated along C^+ characteristic line.

$$\int_{H_A}^{H_P} dH + \frac{a}{gA}\int_{Q_P}^{Q_A} dQ + \frac{f}{2gDA^2}\int_{X_A}^{X_P} Q|Q|dx = 0 \quad (2.35)$$

Similar integration can be carried out along C^- characteristic line, and following equations are acquired.

$$H_P - H_A + \frac{a}{gA}(Q_P - Q_A) + \frac{f\Delta x}{2gDA^2}Q_A|Q_A| = 0 \quad (2.36)$$

$$H_P - H_A - \frac{a}{gA}(Q_P - Q_B) + \frac{f\Delta x}{2gDA^2}Q_B|Q_B| = 0 \quad (2.37)$$

Both equations are algebraic relations that define the transient flow in single-pipe flow.

Subsituting

$$B = \frac{a}{gA} \qquad \text{and} \qquad R = \frac{f\Delta x}{2gDA^2}$$

C^+ and C^- equations become

$$C^+: H_P = H_A - B(Q_P - Q_A) - RQ_A|Q_A| \quad (2.38)$$

$$C^-: H_P = H_B + B(Q_P - Q_B) - RQ_B|Q_B| \quad (2.39)$$

In general form

$$C^+: H_{Pi} = C_P - BQ_{P_i} \quad \text{and} \quad C_P = H_{i-1} + BQ_{i-1} - RQ_{i-1}|Q_{i-1}| \quad (2.40)$$

$$C^-: H_{Pi} = C_M + BQ_{P_i} \quad \text{and} \quad C_M = H_{i+1} - BQ_{i+1} + RQ_{i+1}|Q_{i+1}| \quad (2.41)$$

2.3. Waterhammer in Hydropower Plants

Waterhammer can cause considerable problems in hydropower plants such as deformation of pipelines, turbine failures or system failures; therefore, in a hydropower plant design, waterhammer calculations should be done carefully. However, unpredictable events may happen and system may be exposed to undesirable waterhammer pressures. In order to avoid harmful effects of waterhammer, system should be designed for the worst scenario that may happen. Yet in this case, design costs may become excessive. A detailed optimization analysis should be performed to find optimal design for waterhammer.

Waterhammer in hydropower plants mainly occur due to

- Load rejection
- Load acceptance
- Load variation

The turbine of a hydropower plant is connected with a generator that converts the mechanical energy of turbine into electricity. Electricity is transferred to a distribution grid afterwards. Any malfunction in the transmission lines or sudden drop of the power demand causes *load rejection* in the system. If load rejection occurs, power produced by generator cannot be transferred to the grid and rotational speeds of the turbine and the generator starts to increase. In order to avoid the turbine and the generator suffering from excessive speed, wicket gates of the turbine or needle valves should start to close. That operation should continue until the system frequency falls to normal operating frequency. Valves should respond to that situation as quickly as possible. However, with faster closure of valves, the waterhammer pressure in the system will be more significant. If power demand falls to zero, the event is called *instant load rejection*.

When the turbine and the generator gets connected to the grid or starts operating, *load acceptance* occurs. In this case, wicket gates or needle valves are opened to speed up the system and to meet the power demand. Waterhammer pressure occurs in the system after this process. Unlike in load rejection case, formation of low pressure occurs in the system in load acceptance case. Although low pressure formation is less severe than high pressure formation in penstocks, it must be analyzed carefully in order to avoid vapor formation in penstocks.

The characteristic of the electrical load is that the demand is not fixed, it varies with time. With changing load demand, flow pattern changes accordingly. Thus, valves adjust themselves to their new position continuously. This event is called *load variation*. Waterhammer pressure rises and drops occur as a result of load variation but it is generally not critical as load acceptance and load rejection as load variations are not sharp compared to other cases.

2.4. Protection Methods

Waterhammer may cause significant pressure rises in a system. To avoid system failures penstocks should be designed with high safety. However, penstocks designed with high safety may not be economical. Instead of designing uneconomical penstocks, other protection methods can be implemented. Common protection methods from waterhammer pressure are

- Surge tank
- Air chamber
- Valves
- Flywheel
- Safety Membrane

2.4.1. Surge Tank

A *surge tank* is a kind of reservoir that is connected to penstock. It allows water inflow and outflow to absorb the transient discharges and pressures. During a transient event, it reflects the pressure wave and stores excess water from the system and supplies water to the system. The oscillation inside the surge tank is maximum at the beginning of a transient event, and then it dampens gradually due to the friction inside the surge tank and penstock. Generally surge tank is constructed as near as possible to the turbine, so that it reduces the length of the penstock considerably. The types of the surge tank are; simple, orifice, differential, one-way, closed tank and tank with galleries. Types of surge tanks are shown in Fig. 2.8 (Chaudhry, 1987).

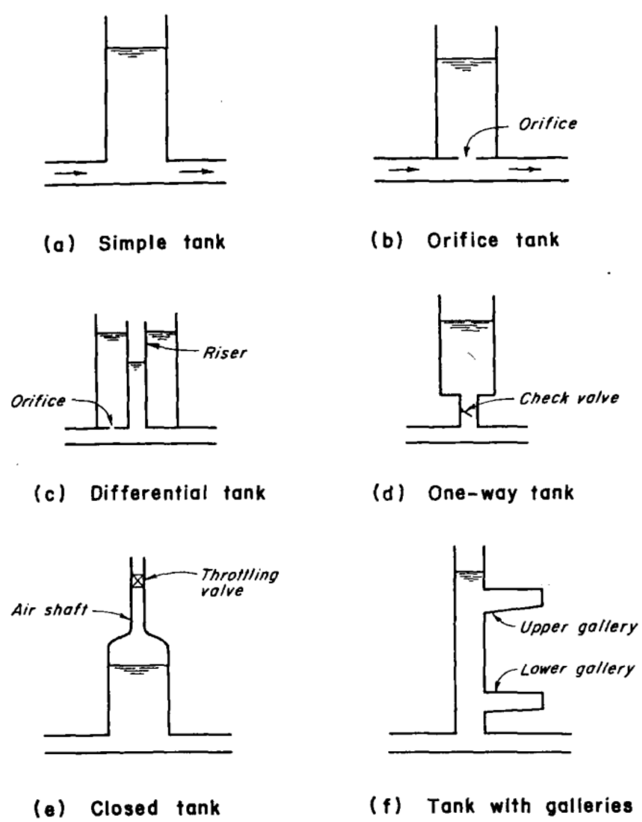

Figure 2.8 Types of surge tanks

2.4.2. Air Chamber

Air chamber is a vessel filled with gas. It acts like a surge tank. The difference of air chamber from surge tank is that air chamber is not open to atmosphere. It is generally pre-pressurized by a compressor to give more relaxation to the system. Detailed information for air chamber is given in the next chapter.

2.4.3. Valves

Valves are used to discharge water if the pressure in the pipe exceeded a pre-defined limit value or entraining air into the pipe in order to avoid the pressure dropping to the liquid vapor pressure. Types of valves used to control transients are safety, pressure-relief, pressure-regulating, air-inlet and check valves. Types of valves are shown in Fig. 2.9 (Chaudhry, 1987).

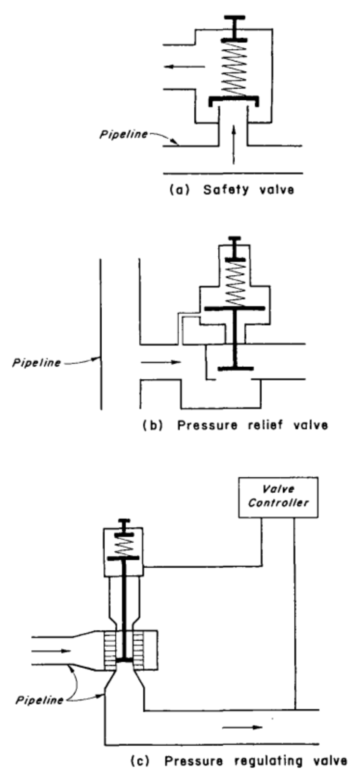

Figure 2.9 Types of valves

2.4.4. Flywheel

Flywheel is a heavy disc fixed on the rotated parts of the system. Its function is to increase the polar moment of inertia of the rotating parts. In an emergency case, flywheel slowly reduces the turbine speed and increases the time to reach its runaway speed. Also it reduces the time to stop the units. However, the disadvantage of flywheel is that the increase in the moment of inertia of the units may retard the start up of the unit.

2.4.5. Safety Membrane

Safety membrane is another protection method which is not very common in practice. It is made of a material that is more fragile than the pipe material. In steel pipes, aluminum is used for membrane. Membrane is placed near to the turbines. When the pressure in the system increases due to waterhammer, weak membrane ruptures and water discharges outside from that orifice. It may not be practical but it is cheaper than other protection methods.

CHAPTER 3

AIR CHAMBER

3.1. Definition

An *air chamber* (Fig. 3.1) is a vessel having compressed air at its top and having liquid in its lower part (Chaudhry, 1987).

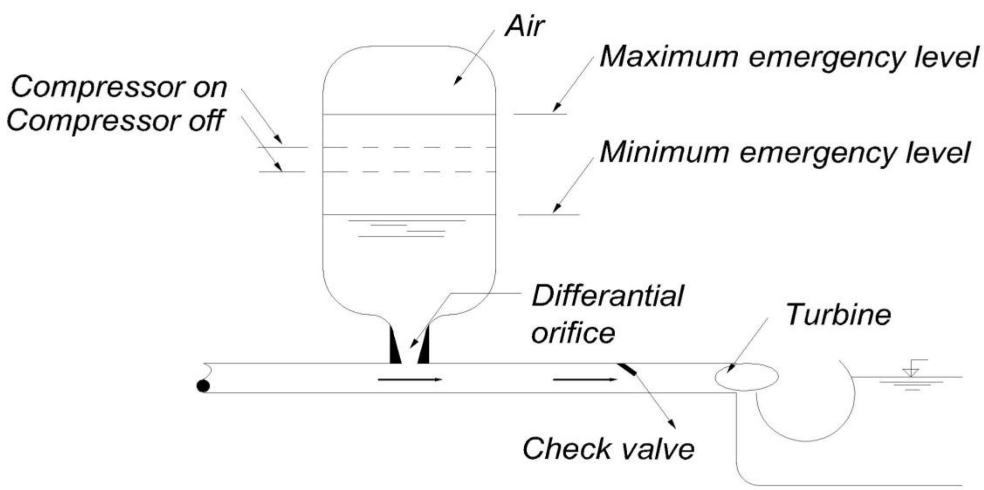

Figure 3.1 Air chamber

An air chamber is composed of a cylindrical or spherical body and a differential orifice (throttle). Body of an air chamber is generally made of steel as it is capable of resisting tensile forces. Differential orifice is the inlet of the air chamber which is connected to penstock. It is shaped such that it provides more head loss for inflow than outflow. The reason is to prevent very low minimum pressures in the pipeline by letting free outflow from the orifice and restricting the inflow into the chamber. Usually, the ratio of 2.5:1 is used for inflow/outflow head loss proportion.

Before operating an air chamber, some pre-set pressure should be given to the system. With this practice, air volume in the chamber is increased while the chamber is operating at steady state. This also means, the chamber volume is increased artificially. In order to give preset pressure to the chamber, strong compressors should be used. Also compressors can be used for maintaining the system pressure in case of an emergency case such as gas leakage from the chamber.

There are two kinds of air chambers, air chambers without bladder and air chambers with bladder. Bladder is an expandable and flexible tool that is used to keep gas and fluid in an air chamber. With this application, dissolution of gas into the water is prevented; therefore, pressure loss in the chamber is avoided. Before starting operation, air chambers without bladder are filled with water and then given a pre-set pressure by inflating air whereas in air chambers with bladder the bladder is filled with air to get some pre-set pressure and then water is introduced to the chamber. In Fig. 3.2 (HAMMER V8i Help), filling procedure of a bladder type air chamber is shown.

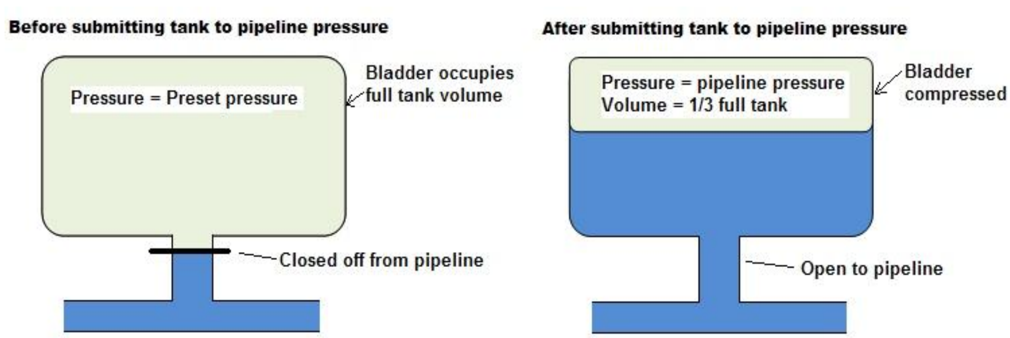

Figure 3.2 Bladder type air chambers

In operation, liquid level becomes stabilized to a fixed elevation determined by static head, chamber volume and pre-set pressure. When positive surge caused by any transient event is introduced to the system, liquid is discharged from pipe into the chamber. Liquid level starts to rise and liquid starts to compress the air. After compressing the air to a maximum extent, liquid starts to discharge out of the chamber to the pipe resulting in liquid level drop in the chamber, and then the same cycle repeats itself. Because of friction and entrance losses in the system, the amplitude of liquid level oscillation dampens. After the effects of waterhammer passes, the liquid level becomes fixed at its steady state level.

3.2. Advantages and Disadvantages of Air Chamber Compared to Conventional Surge Tank

Air chamber is an efficient way of obtaining relaxation in hydraulic systems. It has many advantages over conventional surge tank. Mainly:

- The volume of an air chamber required for keeping the system pressure within the prescribed limits is smaller than an equivalent surge tank.
- Generally, foundation costs of air chamber are less than surge tank and foundation of air chamber is more resistant to wind and earthquake loads since an air chamber can be installed with its axis parallel to the ground slope.
- In some cases, providing the surge tank near the turbine is not practical due to excessive height. Air chamber can be installed near the turbine and penstock length can be designed shorter.

- It is easier and cheaper to heat the liquid in an air chamber than in a surge tank because of smaller size and proximity to the turbine to prevent freezing of water in cold climates.

The main disadvantage of an air chamber is the necessity to provide air compressors and auxiliary equipment, which require constant maintenance (Chaudhry, 1987).

3.3. Equations of Air Chamber

Air chamber can be modeled using characteristic equations (Chaudhry, 1987). Control volume is illustrated in Fig. 3.3.

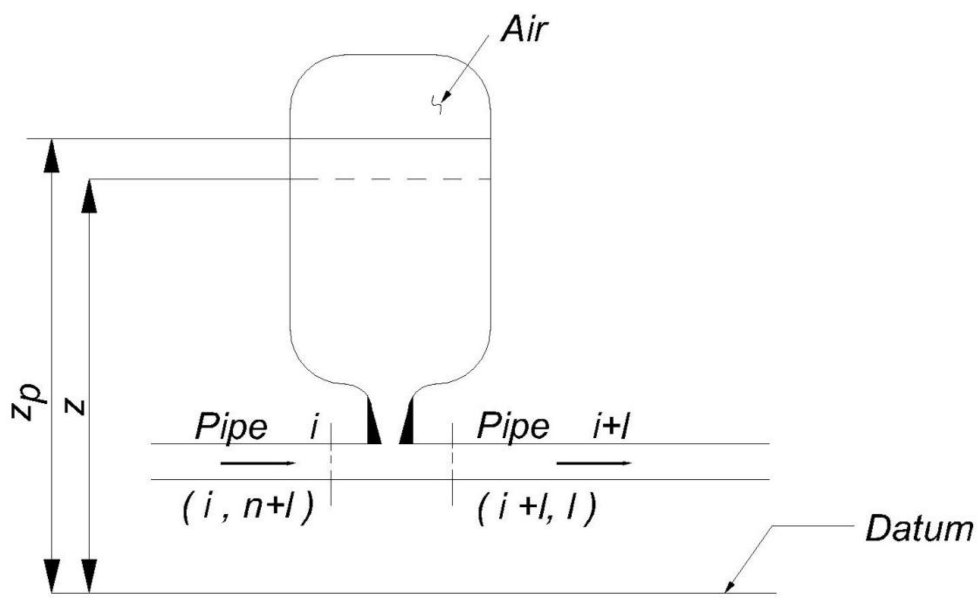

Figure 3.3 Notation for air chamber

C^+ equation for section *(i,n+1)*

$$Q_{P_{i,n+1}} = \frac{C_P - H_{P_{i,n+1}}}{B} \tag{3.1}$$

C^- equation for section *(i+1,1)*

$$Q_{P_{i+1,1}} = \frac{H_{P_{i+1,1}} - C_M}{B} \tag{3.2}$$

The continuity equation at the junction

$$Q_{P_{i,n+1}} = Q_{P_{i+1,1}} + Q_{P_{orf}} \tag{3.3}$$

where $Q_{P_{orf}}$ is the discharge through the orifice either in positive or negative direction. If minor losses are ignored at the orifice

$$H_{P_{i,n+1}} = H_{P_{i+1,1}} \tag{3.4}$$

Head loss through the orifice is

$$h_{P_{orf}} = C_{orf} Q_{P_{orf}} \left| Q_{P_{orf}} \right| \tag{3.5}$$

Assuming the gas in the chamber is an ideal gas; its behavior can be expressed with polytropic relation:

$$H^*_{P_{air}} \forall^m_{P_{air}} = C \tag{3.6}$$

where $H^*_{P_{air}}\ and\ \forall_{P_{air}}$ are the absolute head and volume of the entrapped air at the end of the time step respectively. *C* is a constant whose value is determined from the initial (steady state) conditions. The values of m are equal to *1.0* and *1.4* for an *isothermal* and for *adiabatic* expansion or contraction of air. Terms *isothermal* and *adiabatic* refer to the gas property of tendency for permitting heat exchange. Following equations can be written for the air inside the chamber.

$$H^*_{P_{air}} = H_{P_{i,n+1}} + H_b - z_P - h_{P_{orf}} \tag{3.7}$$

$$\forall_{P_{air}} = \forall_{air} - A_c(z_P - z) \tag{3.8}$$

$$z_P = z + \frac{1}{2}(Q_{orf} + Q_{P_{orf}})\frac{\Delta t}{A_c} \tag{3.9}$$

where H_b is barometric pressure head; z and z_p are the elevations of the liquid surface in the chamber at the beginning and at the end of the time step Δt respectively; $\forall_{air}$ is the volume of the air in the chamber at the beginning of the time step; Q_{orf} is the discharge through the orifice at the beginning of the time step, and A_c is the *cross*-sectional area of the chamber. The solution of these nine equations from Eq. (3.1) to Eq. (3.9) yields the head and discharge values at the junction.

CHAPTER 4

BENTLEY HAMMER

4.1. Overview and Functions

Solving transient flow equations by hand calculation is not possible since they are non-linear partial differential equations. They require numerical methods to be solved. There are many numerical methods that can be used for solving transient flow equations that are completely different from each other; but they have one common ground. They are time consuming and non-practical. For this reason, numerous computer programs are developed to be a practical method for modeling and solving transient flow problems.

HAMMER, developed by Bentley, is a commercial software that solves hydraulic problems, either steady or unsteady. It helps designers to analyze hydraulic systems like pipe networks, pumping systems, power plants, etc. It has many functions to allow the designer making an appropriate model for analysis. The main aim of the program is to compute hydraulic transients along a pipeline and offer protective measures for reducing the transient effects. Uses of HAMMER are:

- Developing cost-effective surge control strategies.
- Preventing costly infrastructure damage.
- Reducing, operation and maintenance costs.
- Eliminating costly over-design.
- Modeling any surge protection device.
- Minimizing wear and tear on pipes.
- Simulating any transient condition.

- Ensuring the longevity of water systems
- Preparing for power failures and minimizing service interruptions.
- Accurately determining transient forces.
- Preventing catastrophic failures.

HAMMER V8i is used in this thesis work to model and analyze a typical air chamber problem. All of the numerical results are based on solutions by HAMMER.

4.2. Modeling in Bentley HAMMER

4.2.1. Input Parameters

To model a hydraulic system in HAMMER, several input parameters are needed to be entered into the program. These input parameters enable the program to make the program steady-state analysis of the model. These parameters are:

- Elevations of reservoirs, pipe nodes, turbines, pumps and other elements in the model.
- Static head of reservoirs.
- Dimensions, material types and characteristics of pipe elements to compute acoustic wave speed which is another input parameter.
- Liquid properties such as Young's modulus and vapor pressure.
- Turbine characteristics such as inertia, rotational speed, specific speed and efficiency.
- Pump characteristics such as pump curve, inertia and efficiency.
- Specifications for node requirements such as discharge requirement if present.
- Operational patterns of valves such as head loss and discharge characteristics.

- Dimensions and characteristics of surge protection devices.

4.2.2. Steps for Modeling

Modeling in Bentley HAMMER requires several steps to be followed.

- Firstly, model layout should be drawn by using the user friendly interface of the HAMMER. Also a model can be created via importing the drawing of the model from other software such as EPANET and WaterCAD. To make more realistic viewed models, pictures and figures can be imported as background layers.
- Defining the input parameters and properties of the system is the second step. In this step, required parameters for steady and transient state flow simulation should be entered. Also water quality can be defined in this step if water quality analysis is desired. This step is very important because a little mistake can lead into unrealistic and erroneous results which may confuse or misguide the designer. For this reason, if results are unreasonable for designer point of view, this step should be repeated.
- Computation is the next step. Firstly, a steady state analysis is conducted by Hammer. In this step, steady state parameters of the system like discharge, head loss across the system and hydraulic grade line of the system are computed. Then, transient analysis is executed. In this step, transient pressure variations and discharge variations are computed.
- Viewing the results is the final step. Results can be viewed either in tabular or graphical forms. There is an animation tool available for viewing transient results. Variations in head, pressure, discharge, and vapor volume at a point in time and transient head and pressure envelopes can be animated using this tool. Also

miscellaneous information can be gathered using transient report tool.

4.2.3. Interface and Tools of HAMMER

4.2.3.1. Interface

Hammer has a user friendly interface that enables users to save time while creating models and conducting analyses. Figure 4.1 shows the main window of HAMMER. In the main window, there are many tools for utilizing the program. These are listed in this section.

- File tab contains commands for opening, closing and saving projects.
- Edit tab involves undo, redo and select commands.
- Analysis tab includes scenario, calculation options, compute and transient result viewer tools.
- Components tab contains tools for listing whole models pump definitions, valve characteristics, unit demands and head loss curves.
- View tab involves background layers, flex tables, graphs, profiles and contours. Also pan and zoom options are available in this tab.
- Tools tab includes useful auxiliary applications such as demand control center and wave speed calculator.
- Report tab contains reports that can be gathered after computing.
- Help tab is a comprehensive tool that includes a tutorial of the program.

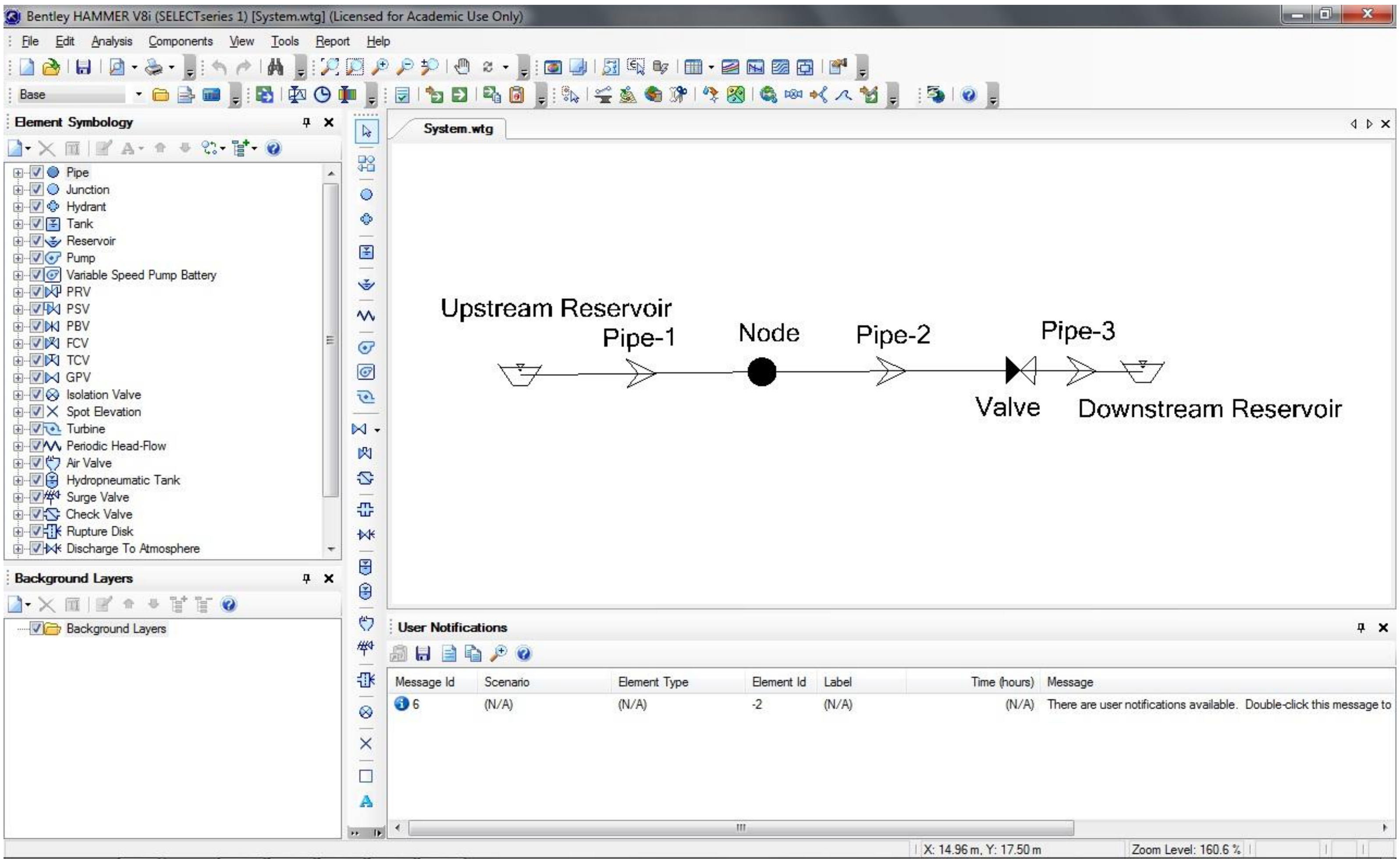

Figure 4.1 Main Window of HAMMER

4.2.3.2. Tools for Drawing the Model Layout

HAMMER includes comprehensive database for drawing and defining the desired model. The model can be drawn schematic or scaled. A layout toolbar is present on the main interface that elements of model can be selected by simply clicking on it. This toolbar is shown in Figure 4.1 as the vertical line at the left side of drawing plane. Elements included in HAMMER are listed herein.

- *Pipe* is the main element of hydraulic model. It is the element that conveys the fluid from a reservoir to a reservoir or a turbine or valve. In HAMMER pipes should be connected to another element or a node. After a pipe element is placed in the model, physical properties like length, diameter, material type and wave speed should be identified. Wave speed can be calculated using wave speed calculator tool.
- *Reservoir* is used to represent a free surface in the hydraulic model. This free surface can be a dam reservoir, a forebay or a tailwater surface. After placed, its elevation should be determined. It may be entered as fixed or variable.
- *Junction* is the element that connects two pipe segments. Pipe segments that are connected may be identical or different. Number of junctions can be increased for getting more results from the model. Elevation of the junction should be entered after placed.
- *Turbine* is the element that represents reaction turbines such as Francis turbine. Impulse turbines are modeled with a valve or discharge to atmosphere element. Turbine element is identified with its elevation, inlet diameter, inertia, rotational speed and specific speed. Its head flow relationship should be defined in order to adjust the discharge of the system. Also for transient analysis its gate opening pattern should be assigned. Four

operating cases can be analyzed in HAMMER; namely, instant load rejection, load rejection, load acceptance and load variation.

- *Pump* element similar to turbine element defines hydro pumps. It requires elevation, pump curve, gate opening pattern to be defined.
- *Valve* element in HAMMER is much in detail. Various kinds of valves are modeled such as PRV (pressure reducing valve), PSV (pressure sustaining valve, PBV (pressure breaking valve), FCV (flow control valve), TCV (throttle control valve) and GPV (general purpose valve). GPV element can be used to model common valves. To define a valve, its elevation and flow head loss curve should be assigned.
- *Surge tank* element is a beneficial tool in HAMMER used to simulate surge tanks in hydraulic systems. Simple and differential surge tank types can be modeled. Also an overflow spillway can be inserted and the amount of water spilled from the spillway can be investigated after the analysis. Its height, body diameter, orifice diameter, maximum and minimum elevations and head loss coefficient should be identified after placed.
- *Hydropneumatic tank* element is the tool for modeling air chambers. It is also detailed like other elements in layout toolbar. Its type, whether with bladder or not, should be selected. Also physical properties such as chamber volume, elevation, preset gas pressure, throttle (tank inlet) diameter, and ratio of inflow to outflow losses should be determined. It solves the equations of air chamber on two different bases: constant area approximation and gas law model. Gas law model is used in this study. It requires gas law exponent to be assigned.

In addition to these, following elements are also available for use in layout toolbar: check valve, orifice between pipes, hydrant, air valve, surge valve, rupture disk and isolation valve.

4.2.3.3. Tools for Computation and Viewing Results

Analysis tab is the main tool for computation. Before starting the analysis, *calculation options* for steady state and transient solver should be revised. In steady state calculation options, hydraulic properties such as liquid kinematic viscosity and liquid specific gravity can be redefined. Also friction method can be selected between Hazen-Williams, Darcy-Weisbach and Manning formulas. In transient calculation options, time step interval, run duration time, transient friction method and vapor pressure of water can be redefined.

Once calculation options are arranged, initial conditions need to be calculated. For this purpose, *compute initial conditions* tool in the analysis tab can be used. After this step, transient calculations can be executed by using *compute* tool in the analysis tab.

Viewing results of steady state and transient analysis is an easy process in HAMMER. In analysis tab, *calculation summary* tool provides steady state calculation results such as discharge through pipes while *transient calculation summary* tool provides transient calculation results in tabular form. Transient pressure, head, velocity and discharge results can be viewed at desired nodes. These results can be illustrated in graphs and changes in these variables can be animated with respect to time using the *transient results viewer* tool. Moreover, *reports* that contain transient results can be obtained using *reports tab*. These reports can be saved in Microsoft Word format if needed.

Figure 4.2 is a screenshot covering calculation summary and transient calculation summary tool. Figure 4.3 shows transient results viewer tool.

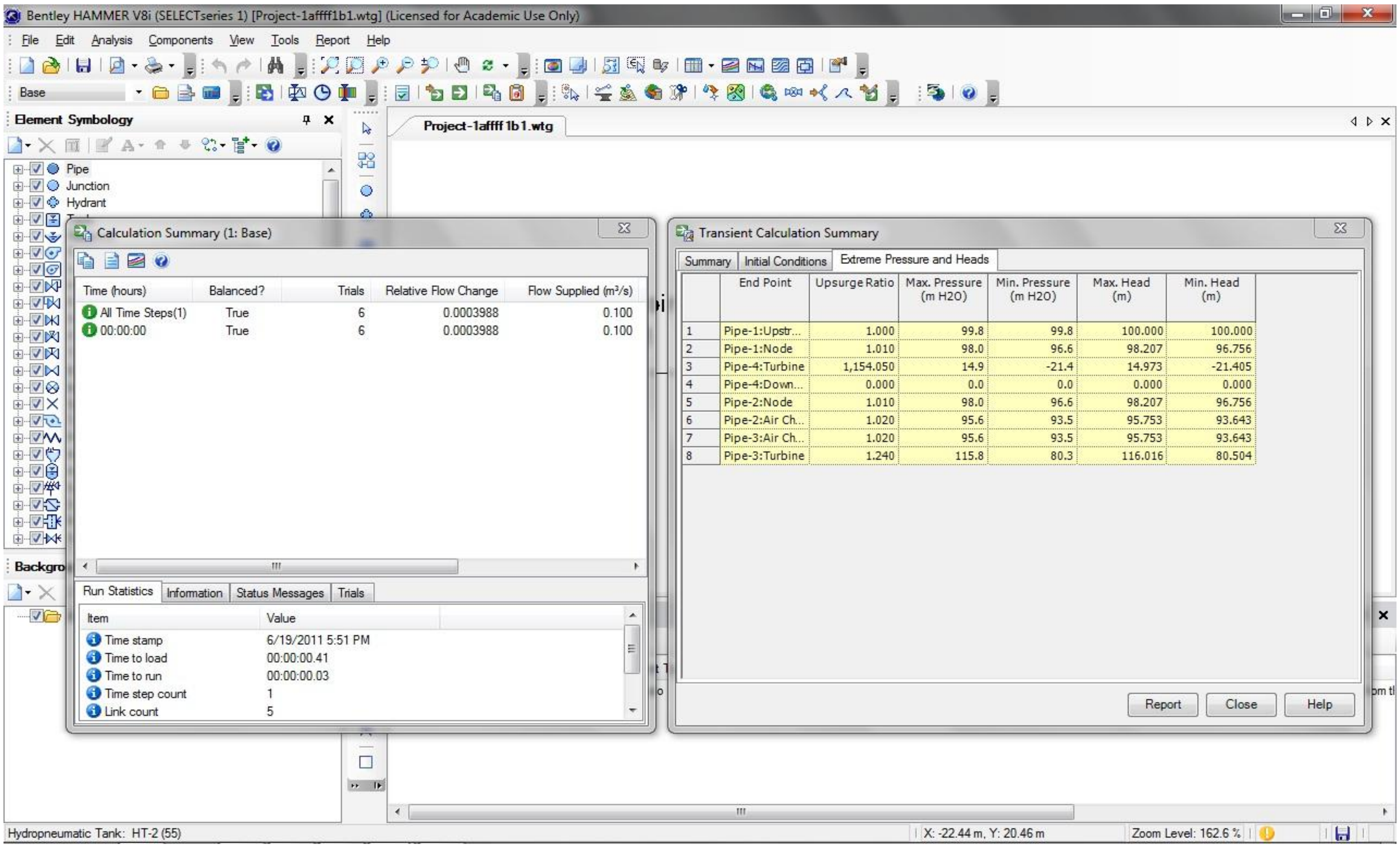

Figure 4.2 Calculation Summary and Transient Calculation Summary Tool

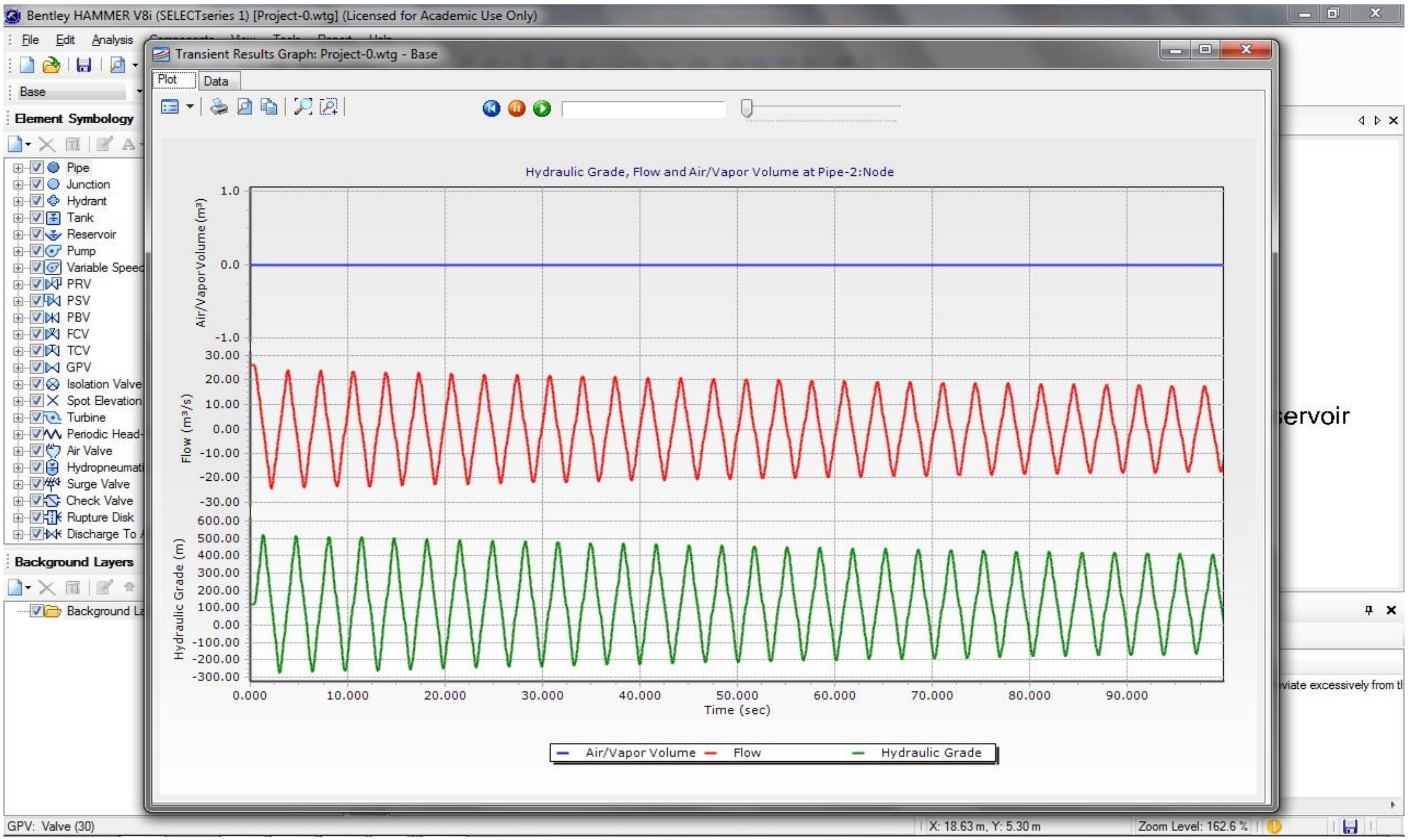

Figure 4.3 Transient Results Viewer Tool

CHAPTER 5

ANALYSIS OF AIR CHAMBER MODEL

5.1. Model

The aim of this study is to investigate the correlation between the volume of an air chamber and the relaxation it provides to the hydraulic system. To achieve this objective a typical model system should be defined first. In this study, two typical models are identified. First model consists of a constant elevation reservoir, a single penstock, a Francis turbine and a downstream reservoir. Second model includes an air chamber in addition to the same elements. The reason to identify two different models is to investigate the effect of air chamber to the system by comparing the results from the model with an air chamber and the model without an air chamber. Models are designed as basic hydropower systems to simplify the problem. After defining the models, basic parameters of the models are selected.

Figures 5.1 and 5.2 illustrate the models defined for this problem. It should be noted that the drawings are not scaled, they are only schematic.

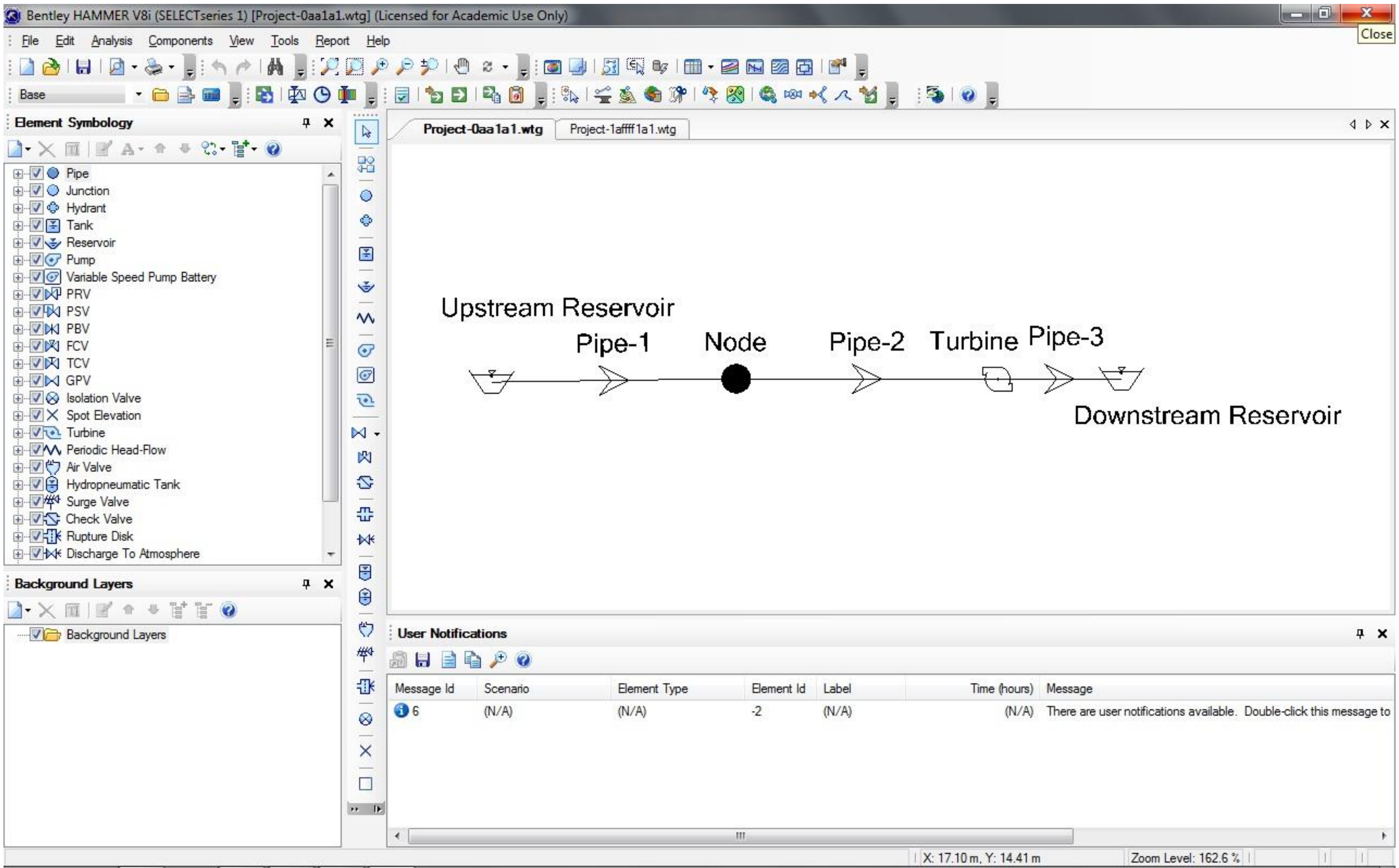

Figure 5.1 Model without an air chamber

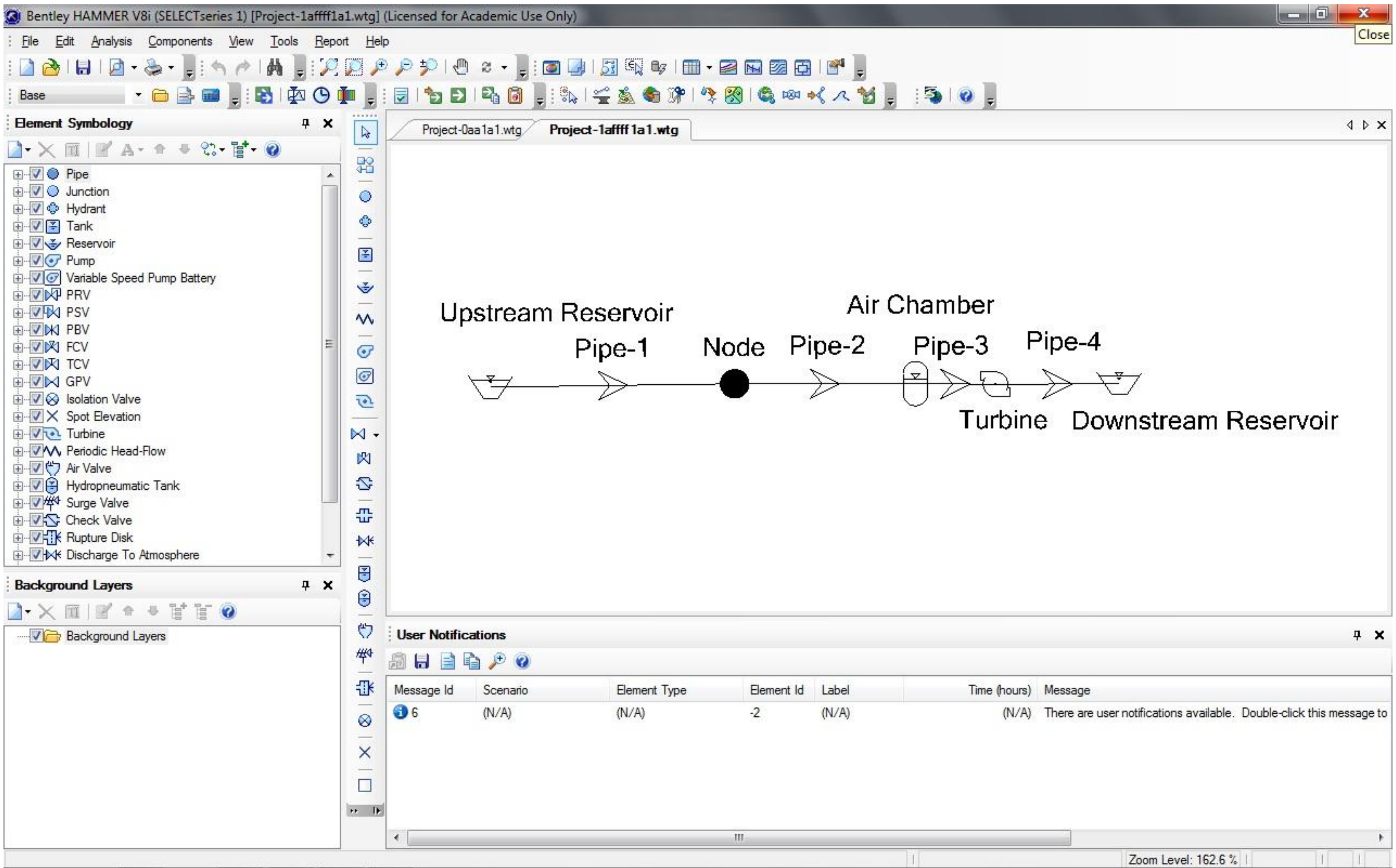

Figure 5.2 Model with an air chamber

5.2. Selection of the Parameters of the Model

Selection of the parameters of the models is very important as obtaining a useful data set is possible only by defining a set of consistent parameters. Some parameters should be assigned as basic parameters to reduce the number of unknowns. They are either constant or based on a specified formula. Basic parameters specified for the models are: static head *(H_0),* penstock discharge *(Q),* flow velocity *(V_p),* penstock diameter *(D),* turbine closure time *(T_c),* penstock length *(L),* penstock material, penstock thickness, sonic wave speed *(a),* turbine rotational speed *(N),* turbine specific speed *(n_s),* turbine inertia *(I_t),* turbine efficiency *(η),* gas law exponent and air chamber ratio of losses *(inflow/outflow)*. From these parameters, static head and penstock discharge are independent variables. They are selected within a specified range in conformity with common practice. Unknown parameters that need to be calculated are air chamber throttle diameter and preset gas pressure.

The selection of these parameters is explained in following subtopics.

5.2.1. Selection of Static Head and Penstock Discharge

Static head *(H_0)* and penstock discharge *(Q)* are variable parameters of the model. Since transient flow behavior is directly related to head and discharge, range and number of selected head and discharge values increases the inclusiveness of results and provides detailed description of the hydraulic behavior.

Turbine used in the model is Francis turbine; therefore, head values must be selected from its application range. According to the Francis turbine manual published by Voith Hydro, standard Francis turbines operate between 10 meters and 350 meters head. With some modifications, custom made Francis turbines can operate at higher heads. Maximum head that a Francis turbine can operate efficiently is approximately 900~1000 meters. To get accurate results, head values should be chosen from this range. But, due to some computational discrepancies which will be explained in later sections, head values are chosen between 10 meters and 500 meters in this study.

Penstock discharge of the model should also be selected from a logical range. It should cover a wide range to be comprehensive. But, similar to head parameter, range of discharge values is restricted to avoid computational errors of HAMMER software. Penstock discharge values are selected between 0.1 and 20 cubic meters per second in final computations.

5.2.2. Selection of Turbine Closure Time, Penstock Length, Penstock Diameter, Penstock Material, Penstock Thickness and Sonic Wave Speed

Turbine closure time (T_c) is a critical parameter of the model. It directly affects the amplitude of waterhammer pressure in the system. As closure time decreases, the closure becomes more rapid and effects of waterhammer becomes more severe. In the literature, turbine closure is named rapid closure if $T_c<2L/a$. L is the penstock length and a is the sonic wave speed in this relationship. Since rapid closure is more critical in hydraulic systems, rapid closure is investigated in this study. A constant 1 second closure time is selected for both models.

Joukowsky equation (Eq.(2.1b)) indicates the waterhammer rise in the system for rapid closure case. This equation is independent of penstock length *(L)*. Since rapid closure is investigated, penstock length becomes an insignificant parameter. In the model, penstock length is chosen in such a way that it satisfies the rapid closure condition in every data set.

Penstock diameter is computed according to Sarkaria formula (Yıldız, 1992). It is an empirical equation that is used to find the optimum penstock diameter for different systems. Parameters in the equation are turbine power in horsepower and static head of the system in meters.

$$D = 0.634\left(\frac{{P_T}^{1/2}}{{H_0}^{3/4}}\right)^{0.86} \tag{5.1}$$

In this equation, P_T is the turbine power in horsepower and H_0 is the static head of the system in meters.

Penstock material is chosen as steel due to the widespread usage of steel in hydraulic systems. The friction factor, modulus of elasticity and other material parameters are chosen for steel by default.

Penstock thickness is a parameter which affects *sonic wave speed* of the system. Wave speed is a variable that is related to material and liquid properties, penstock thickness, penstock supporting and penstock diameter. It can vary between 600-1200 m/s. For the sake of simplicity, sonic wave speed is chosen as constant 800 m/s for every data set. It can be selected as a constant because it is a reasonable value and wave speed hasn't got a crucial effect on the results. Penstock thickness becomes a trivial parameter after sonic wave speed is determined.

5.2.3. Selection of Turbine Rotational Speed, Specific Speed, Inertia and Efficiency

Selection of turbine parameters such as rotational speed, specific speed, inertia and efficiency is simplified with empirical relationships and assumptions. According to USBR following empirical formula can be used for determination of preliminary n_s value (Pekçağlayan, 2010).

$$n_s = \frac{2000}{\sqrt{H}} \tag{5.2}$$

In this equation, n_s is the specific speed and H net head at the turbine in meters. After determining n_s, turbine rotational speed can be computed. Empirical equation for rotational speed is (Pekçağlayan, 2010):

$$N = \frac{n_s {H_0}^{1.25}}{\sqrt{P_T}} \tag{5.3}$$

HAMMER allows users to select from three predefined ns values: 115, 170, 230. It offers a formula to compute n_s. After computing the new n_s value, user should select the closest predefined value to his n_s. The formula offered by HAMMER is:

$$n_s = \frac{N\sqrt{Q}}{H^{3/4}} \tag{5.4}$$

In this formula, N is the rotational speed of the turbine in rpm, Q is the discharge in cubic meters per second and H is the net head in meters.

Inertia of the turbine is a mechanical characteristic. An empirical and approximate formula stated by turbine manufacturers is used for inertia parameter.

$$I_t = 3943\left[\frac{P_T}{N^{1.5}}\right]^{1.25} \tag{5.5}$$

Here, P_T is the turbine power in horsepower units. N is the rotational speed in rpm. I_t is the moment of inertia of the turbine in kg.m^2.

Turbine efficiency is selected as 0.9 (90%). It is constant for every data set.

5.2.4. Selection of Throttle Diameter

Throttle is the inlet pipe of the air chamber which is connected to penstock. Its diameter is an unknown parameter. For a sufficiently large air chamber, the parameters that influence optimum throttle diameter are head and discharge; therefore, the relationship of throttle diameter with these parameters should be tested individually. In other words, other parameters should be kept constant while the inspected parameter is being changed. Then, relationship with that parameter and resulting transient head at any specified point can be observed. At the end, throttle diameter should be selected as a fraction of penstock diameter.

Static head of the system may affect throttle diameter selection. In order to analyze the correlation between throttle diameter and static head at reservoir, following parameters are entered to the Model 2 (model with an air chamber).

Table 5-1 Input parameters for head-throttle diameter analysis

H_0(m)	Q(m^3/s)	D(m)	$\forall_{ch}$(m^3)	P_{ini}/γ(m)
20	26	3.90	4000	16
120	26	2.65	4000	96
300	26	2.18	4000	240

In these data sets, $\forall_{ch}$ is the air chamber volume in cubic meters. P_{ini} is the preset air pressure in the air chamber before the chamber is in operation. It is an unknown parameter whose optimum value will be determined in later sections. For the time being, it is assumed that the preset pressure is equal to the static head multiplied by 0.8. It should be noted that other parameters such as discharge and chamber volume are identical for three data sets. Difference between diameters is caused by the definition of diameter which is a head and discharge dependent relationship. Similarly, difference between preset pressures is caused by different static head values. These three data sets are simulated by HAMMER to achieve transient results.

Results of data sets are as follows:

Table 5-2 Result of first data set (H_0=20m)

D_T(m)	H_{max}(m)	H_{max}/H_0	D_T/D
0.1	211.63	10.582	0.026
0.4	179.92	8.996	0.103
0.7	133.11	6.556	0.179
1.0	88.52	4.426	0.256
1.3	57.94	2.897	0.333
1.6	40.62	2.031	0.410
1.9	31.51	1.576	0.487
2.2	26.72	1.336	0.564
2.5	24.11	1.206	0.641
2.8	23.93	1.197	0.718
3.1	24.07	1.204	0.795
3.4	24.48	1.224	0.872
3.7	24.62	1.231	0.949

Table 5-3 Result of second data set (H_0=120m)

D_T(m)	H_{max}(m)	H_{max}/H_0	D_T/D
0.1	490.52	4.088	0.038
0.3	448.70	3.739	0.113
0.5	378.00	3.150	0.188
0.7	300.79	2.507	0.264
0.9	236.20	1.968	0.339
1.1	191.37	1.595	0.414
1.3	163.51	1.363	0.490
1.5	146.92	1.224	0.565
1.7	137.06	1.142	0.640
1.9	131.06	1.092	0.716
2.1	128.77	1.073	0.791
2.3	130.62	1.089	0.866
2.5	132.33	1.103	0.942

Table 5-4 Result of third data set (H_0=300m)

D_T(m)	H_{max}(m)	H_{max}/H_0	D_T/D
0.10	824.18	2.747	0.046
0.25	775.58	2.585	0.115
0.40	696.31	2.321	0.183
0.55	604.18	2.014	0.252
0.70	519.48	1.732	0.321
0.85	450.44	1.501	0.380
1.00	400.29	1.334	0.459
1.15	366.34	1.221	0.528
1.30	344.17	1.147	0.596
1.45	329.75	1.099	0.665
1.60	322.02	1.073	0.734
1.75	315.76	1.053	0.803
1.90	315.25	1.051	0.872
2.05	314.89	1.050	0.940
2.15	314.50	1.048	0.986

In Tables 5-2, 5-3 and 5-4 optimum throttle diameters corresponding for minimum H_{max}/H_0 values are marked with grey color. D_T/D ratio is a dimensionless term which can represent D_T. From the results, it is clear that D_T/D ratio lies within 0.7-0.9 range with changing head parameter.

The other parameter that may affect throttle diameter is penstock discharge. To investigate its relationship, data sets including different discharge values are selected.

Table 5-5 Input parameters for discharge-throttle diameter analysis

H_0(m)	Q(m^3/s)	D(m)	$\forall_{ch}$(m^3)	P_{ini}/γ(m)
120	0.5	0.49	4000	96
120	26	2.65	4000	96
120	150	5.64	4000	96

These data sets are also simulated by Hammer and following results are obtained. It should be noted that result of the second data set is identical with the second data set of head-throttle diameter analysis (Table 5-3), hence it is not duplicated.

Table 5-6 Result of first data set ($Q=0.5m^3/s$)

D_T(m)	H_{max}(m)	H_{max}/H_0	D_T/D
0.10	233.17	1.943	0.204
0.13	195.66	1.631	0.265
0.16	167.46	1.396	0.327
0.19	148.79	1.240	0.388
0.22	137.18	1.143	0.449
0.25	130.05	1.084	0.510
0.28	125.64	1.047	0.571
0.31	122.85	1.024	0.633
0.34	121.04	1.009	0.694
0.37	120.16	1.001	0.755
0.40	120.19	1.002	0.816
0.43	120.19	1.002	0.878
0.46	120.18	1.002	0.939

Table 5-7 Result of third data set ($Q=150m^3/s$)

D_T(m)	H_{max}(m)	H_{max}/H_0	D_T/D
0.10	599.18	4.993	0.012
0.50	567.54	4.729	0.089
0.90	500.71	4.173	0.150
1.30	415.48	3.462	0.230
1.70	331.50	2.762	0.301
2.10	262.66	2.189	0.372
2.50	213.33	1.778	0.443
2.90	187.00	1.558	0.514
3.30	173.70	1.447	0.585
3.70	151.01	1.258	0.656
4.10	142.09	1.184	0.727
4.50	157.86	1.315	0.798
4.90	134.64	1.122	0.869
5.30	155.58	1.296	0.940

In tables 5-3, 5-6 and 5-7 the optimum throttle diameters corresponding for minimum H_{max}/H_0 values are illustrated in grey color. The results indicate that with varying discharge values, again the throttle diameter ratio lies in the 0.7-0.9 range.

Head and discharge are the two main parameters of the model. Other parameters are either constant or dependent to head and discharge. Thus, an approximate throttle diameter can be assigned for the model considering its relationship with head and discharge. By taking the results of throttle diameter analysis into consideration, throttle diameter can be 0.7-0.9 times penstock diameter. For simplicity, throttle diameter is selected as 0.8 times penstock diameter. This value is used in all later calculations.

5.2.5. Selection of Preset Pressure

Preset pressure of the air chamber is the initial pressure that is obtained by pumping air into the chamber before air chamber is connected to the penstock. After air chamber is connected to the penstock, chamber is filled with water to some extent, until the pressure inside the air chamber become equal to the pressure in the penstock. Air mass contained in the chamber can be increased by increasing the preset pressure. Thus, this procedure artificially increases the effective chamber volume and relaxation of the transient pressures. For this reason preset pressure is beneficial for the system and it should be increased as much as possible. On the other hand, there is a drawback of excess preset pressure. It is obvious that water level in the chamber will decrease if preset pressure is increased. In operation, when a transient event occurs, water level in the chamber will oscillate. When water level is decreasing, there is a risk that water in the chamber totally drains and air enters into the penstock. This is an undesirable event that may harm the turbine and its components. In order to avoid this event, preset pressure should not exceed a limit value. This is called *maximum limit* of preset pressure. Maximum limit value varies with changing head and discharge of the system. Volume of the chamber does not affect the event since the preset pressure artificially acts as extra volume of the chamber.

These facts indicate that preset pressure should have an optimum value. To determine this value, various cases are simulated by HAMMER. Results are illustrated in table 5-8 and figure 5.3.

Table 5-8 Results of preset pressure analysis

H_0(m)	Q(m^3/s)	D(m)	D_T(m)	P_{ini}/γ(m)	$\%P_{ini}/H_0$
30	1	0.88	0.70	29	96.67
30	10	2.37	1.90	28	93.33
30	30	3.80	3.04	28	93.33
30	50	4.74	3.79	27	90.00
30	100	6.38	5.11	27	90.00
30	150	7.60	6.08	26	86.67
30	200	8.60	6.88	23	76.67
50	1	0.79	0.63	49	98.00
50	10	2.12	1.70	48	96.00
50	30	3.41	2.73	46	92.00
50	50	4.24	3.40	46	92.00
50	100	5.72	4.57	43	86.00
50	150	6.81	5.45	40	80.00
50	200	7.70	6.16	36	72.00
100	1	0.68	0.54	96	96.00
100	10	1.83	1.46	96	96.00
100	30	2.94	2.35	95	95.00
100	50	3.66	2.93	92	92.00
100	100	4.93	3.94	85	85.00
100	150	5.87	4.69	77	77.00
100	200	6.64	5.31	65	65.00
150	1	0.62	0.50	141	94.00
150	10	1.68	1.34	145	96.67
150	30	2.69	2.15	144	96.00
150	50	3.35	2.68	137	91.33
150	100	4.52	3.61	123	82.00
150	150	5.38	4.30	104	69.33
150	200	6.08	4.87	86	57.33
200	1	0.59	0.47	184	92.00
200	10	1.58	1.26	191	95.50
200	30	2.53	2.02	185	92.50
200	50	3.15	2.52	179	89.50
200	100	4.24	3.40	157	78.50
200	150	5.05	4.04	133	66.50
200	200	5.72	4.57	110	55.00

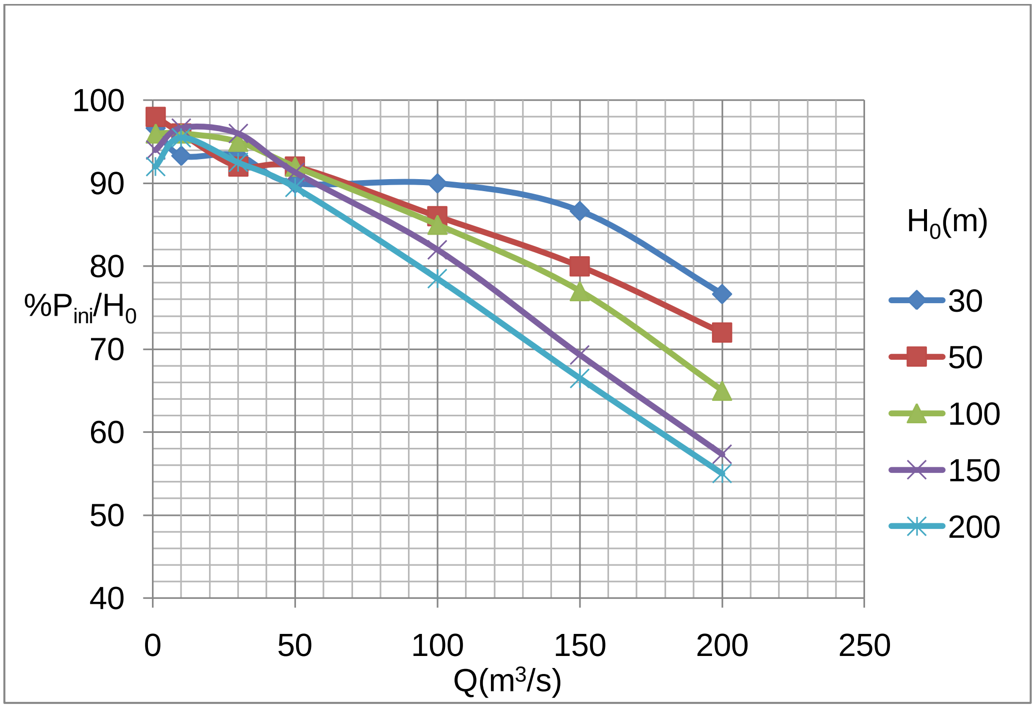

Figure 5.3 Graphical illustration of preset pressure analysis

In table 5-8, head, discharge, penstock diameter and throttle diameter parameters are listed respectively. P_{ini} column represents the maximum limit that preset pressure can take. Dividing that value with static head of the system and multiplying with 100, limit value is expressed with the percentage of the static head.

In Fig. 5.3, this percentage is drawn against penstock discharge individually for each static head value. It is clear that head percentage decreases with increasing penstock discharge. The main reason behind this event is increasing amplitude of water level oscillation inside the air chamber. In other words, the risk of total drainage of water increases with increasing discharge. Static head of the system also influences the preset pressure value. With increasing head, percentage slightly decreases.

Considering the section between 0 and 100 m^3/s which is covered in final computations, a constant value of 80% of static head can be adequate and safe. Thus, it is selected for later computations.

5.3. Preliminary Analysis of Air Chamber Model

Parameters that will be used in the analysis of air chamber are determined so far. At this section of the study, the behavior of the air chamber is analyzed. In the first analysis, static head of the system is held constant. Penstock discharge and volume of the chamber are variable parameters. Transient heads at the chamber are calculated by HAMMER for models with and without air chamber. For the model without air chamber, transient head at the turbine can be utilized since the air chamber is very close to turbine in the model with air chamber. After transient heads are computed, their proportion gives the relaxation rate of the system that air chamber provides. For each volume of the air chamber, the relationship of the relaxation rate with increasing discharge values is determined afterwards.

In the second analysis, the same procedure is applied for the fixed penstock discharge and variable static head values. Also H_{max} is renamed as H in this analysis and later analyses.

5.3.1. Analysis for Fixed Static Head

Static head is fixed as 200 meters in this analysis. Penstock discharge takes values between 0.1-75 cubic meters per second. Air chamber volume is between 1000-10000 cubic meters. For these input parameters series of analyses is conducted by HAMMER. Results are illustrated in tables (Table 5-9 – 5-18) below.

H_{ch} is the normal operation head at the air chamber junction. $\forall_{ch}$ is the chamber volume. H_a is the transient head at the air chamber junction and H is the transient head at that location without an air chamber.

The graphical illustration of the results is provided in Fig. 5.4 in logarithmic plot.

Table 5-9 Analysis for Q=0.1 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_a/H
200	174.29	0.1	160	1000	178.7	414.6	0.4310
200	174.29	0.1	160	2000	177.7	414.6	0.4285
200	174.29	0.1	160	3000	177.3	414.6	0.4276
200	174.29	0.1	160	4000	177.3	414.6	0.4276
200	174.29	0.1	160	5000	177.3	414.6	0.4276
200	174.29	0.1	160	6000	177.3	414.6	0.4276
200	174.29	0.1	160	7000	177.3	414.6	0.4276
200	174.29	0.1	160	8000	177.3	414.6	0.4276
200	174.29	0.1	160	9000	177.3	414.6	0.4276
200	174.29	0.1	160	10000	177.3	414.6	0.4276

Table 5-10 Analysis for Q=0.2 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_a/H
200	175.28	0.2	160	1000	182.2	450.9	0.4040
200	175.28	0.2	160	2000	180.1	450.9	0.3994
200	175.28	0.2	160	3000	179.4	450.9	0.3979
200	175.28	0.2	160	4000	179.1	450.9	0.3971
200	175.28	0.2	160	5000	178.9	450.9	0.3968
200	175.28	0.2	160	6000	178.9	450.9	0.3968
200	175.28	0.2	160	7000	178.9	450.9	0.3968
200	175.28	0.2	160	8000	178.9	450.9	0.3968
200	175.28	0.2	160	9000	178.9	450.9	0.3968
200	175.28	0.2	160	10000	178.9	450.9	0.3968

Table 5-11 Analysis for Q=0.5 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_a/H
200	181.209	0.5	160	1000	193.467	479.695	0.4033
200	181.209	0.5	160	2000	189.025	479.695	0.3941
200	181.209	0.5	160	3000	187.365	479.695	0.3906
200	181.209	0.5	160	4000	186.526	479.695	0.3888
200	181.209	0.5	160	5000	186.032	479.695	0.3878
200	181.209	0.5	160	6000	185.606	479.695	0.3869
200	181.209	0.5	160	7000	185.358	479.695	0.3864
200	181.209	0.5	160	8000	185.262	479.695	0.3862
200	181.209	0.5	160	9000	185.262	479.695	0.3862
200	181.209	0.5	160	10000	185.261	479.695	0.3862

Table 5-12 Analysis for Q=1 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_a/H
200	185.165	1	160	1000	202.684	501.814	0.4039
200	185.165	1	160	2000	197.043	501.814	0.3927
200	185.165	1	160	3000	194.275	501.814	0.3871
200	185.165	1	160	4000	192.92	501.814	0.3844
200	185.165	1	160	5000	191.84	501.814	0.3823
200	185.165	1	160	6000	191.296	501.814	0.3812
200	185.165	1	160	7000	190.833	501.814	0.3803
200	185.165	1	160	8000	190.562	501.814	0.3797
200	185.165	1	160	9000	190.345	501.814	0.3793
200	185.165	1	160	10000	190.246	501.814	0.3791

Table 5-13 Analysis for Q=2 m^3/s

$H_0(m)$	$H_{ch}(m)$	$Q(m^3/s)$	$P_{ini}/\gamma(m)$	$\forall_{ch}(m^3)$	$H_a(m)$	$H(m)$	H_a/H
200	187.143	2	160	1000	207.562	535.758	0.3874
200	187.143	2	160	2000	203.588	535.758	0.3800
200	187.143	2	160	3000	200.238	535.758	0.3737
200	187.143	2	160	4000	198.293	535.758	0.3701
200	187.143	2	160	5000	196.557	535.758	0.3669
200	187.143	2	160	6000	195.666	535.758	0.3652
200	187.143	2	160	7000	195.086	535.758	0.3641
200	187.143	2	160	8000	194.639	535.758	0.3633
200	187.143	2	160	9000	194.212	535.758	0.3625
200	187.143	2	160	10000	193.694	535.758	0.3615

Table 5-14 Analysis for Q=5 m^3/s

$H_0(m)$	$H_{ch}(m)$	$Q(m^3/s)$	$P_{ini}/\gamma(m)$	$\forall_{ch}(m^3)$	$H_a(m)$	$H(m)$	H_a/H
200	189.912	5	160	1000	219.182	576.907	0.3799
200	189.912	5	160	2000	210.766	576.907	0.3653
200	189.912	5	160	3000	207.417	576.907	0.3595
200	189.912	5	160	4000	205.564	576.907	0.3563
200	189.912	5	160	5000	203.598	576.907	0.3529
200	189.912	5	160	6000	202.621	576.907	0.3512
200	189.912	5	160	7000	200.654	576.907	0.3478
200	189.912	5	160	8000	199.728	576.907	0.3462
200	189.912	5	160	9000	199.479	576.907	0.3458
200	189.912	5	160	10000	198.128	576.907	0.3434

Table 5-15 Analysis for Q=10 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_a/H
200	191.409	10	160	1000	234.093	614.259	0.3811
200	191.409	10	160	2000	219.09	614.259	0.3567
200	191.409	10	160	3000	212.644	614.259	0.3462
200	191.409	10	160	4000	209.113	614.259	0.3404
200	191.409	10	160	5000	207.206	614.259	0.3373
200	191.409	10	160	6000	205.477	614.259	0.3345
200	191.409	10	160	7000	203.111	614.259	0.3307
200	191.409	10	160	8000	200.939	614.259	0.3271
200	191.409	10	160	9000	200.921	614.259	0.3271
200	191.409	10	160	10000	200.907	614.259	0.3271

Table 5-16 Analysis for Q=30 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_a/H
200	193.275	30	160	1000	258.642	687.31	0.3763
200	193.275	30	160	2000	226.543	687.31	0.3296
200	193.275	30	160	3000	214.729	687.31	0.3124
200	193.275	30	160	4000	214.149	687.31	0.3116
200	193.275	30	160	5000	210.685	687.31	0.3065
200	193.275	30	160	6000	211.022	687.31	0.3070
200	193.275	30	160	7000	209.802	687.31	0.3053
200	193.275	30	160	8000	209.522	687.31	0.3048
200	193.275	30	160	9000	209.216	687.31	0.3044
200	193.275	30	160	10000	208.346	687.31	0.3031

Table 5-17 Analysis for Q=50 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_a/H
200	194.066	50	160	1000	259.718	720.927	0.3603
200	194.066	50	160	2000	223.842	720.927	0.3105
200	194.066	50	160	3000	211.839	720.927	0.2938
200	194.066	50	160	4000	212.937	720.927	0.2954
200	194.066	50	160	5000	214.814	720.927	0.2980
200	194.066	50	160	6000	214.807	720.927	0.2980
200	194.066	50	160	7000	214.303	720.927	0.2973
200	194.066	50	160	8000	209.132	720.927	0.2901
200	194.066	50	160	9000	209.058	720.927	0.2900
200	194.066	50	160	10000	208.942	720.927	0.2898

Table 5-18 Analysis for Q=75 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_a/H
200	194.56	75	160	1000	254.12	752.644	0.3376
200	194.56	75	160	2000	227.462	752.644	0.3022
200	194.56	75	160	3000	224.313	752.644	0.2980
200	194.56	75	160	4000	234.705	752.644	0.3118
200	194.56	75	160	5000	218.824	752.644	0.2907
200	194.56	75	160	6000	212.092	752.644	0.2818
200	194.56	75	160	7000	217.254	752.644	0.2887
200	194.56	75	160	8000	218.292	752.644	0.2900
200	194.56	75	160	9000	211.572	752.644	0.2811
200	194.56	75	160	10000	228.976	752.644	0.3042

Fig. 5.4 indicates that relaxation of transient pressure increases with increasing air chamber volume which is an expected result. For small discharges chamber volume does not affect the relaxation considerably and with increasing discharges chamber volume becomes more significant. After 40-50 m^3/s discharge, transient phenomena becomes more complicated and numerical error that HAMMER introduce is more frequent. Therefore, it is not probable to achieve accurate results for high discharge region.

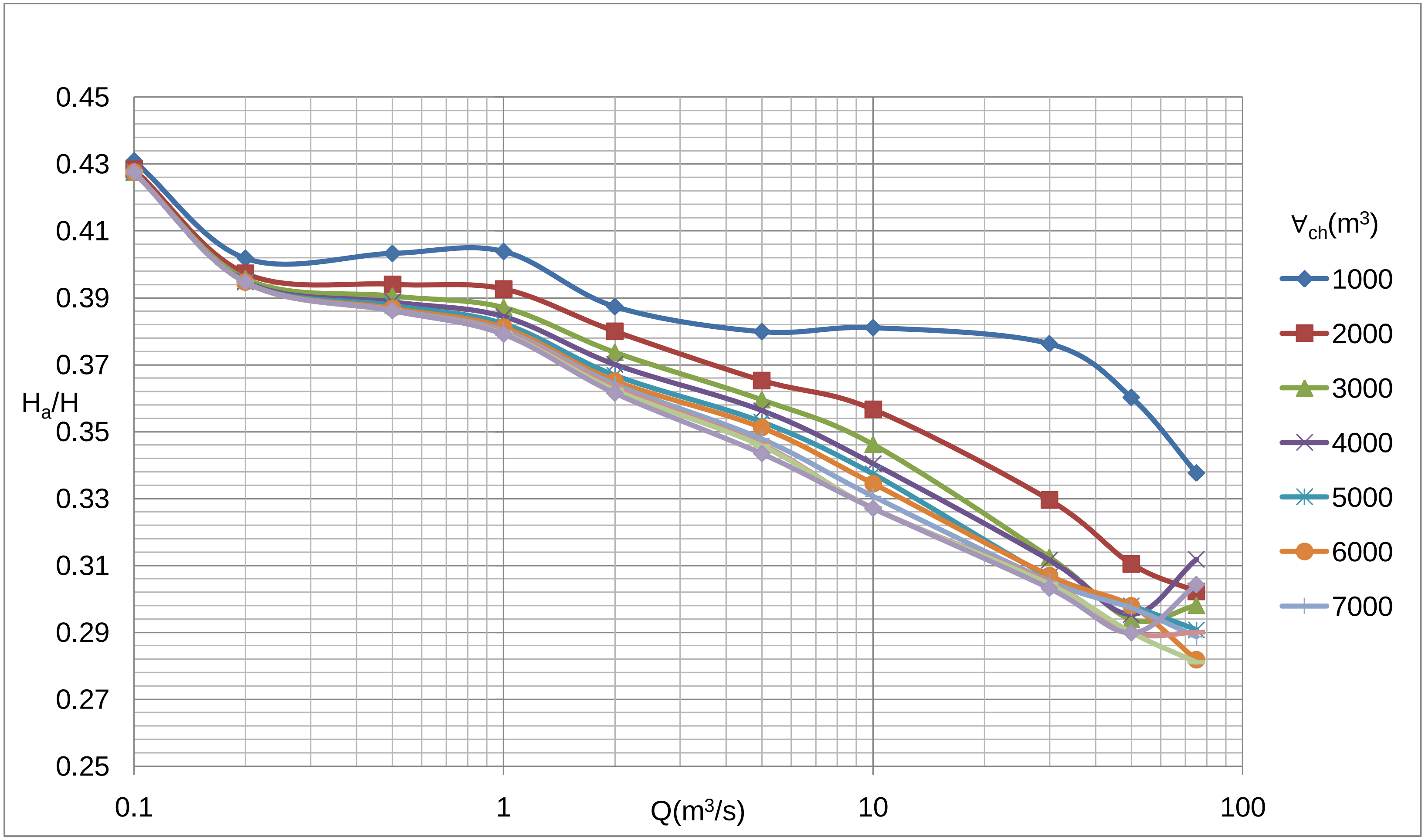

Figure 5.4 Graphical illustration of preliminary analysis for fixed head

5.3.2. Analysis for Fixed Penstock Discharge

Penstock discharge is not the only parameter that affects the relaxation rate. Static head of the system is the other parameter affecting the relaxation. In order to investigate the impacts of static head to relaxation rate, different data sets are arranged. In these sets, discharge has a constant value of 30 cubic meters per second while static head takes values between 20 and 500 meters. Following tables (Table 5-19 -5-26) are obtained after series of analyses conducted via HAMMER.

Table 5-19 Analysis for H_0=20 m

$Q(m^3/s)$	$H_{ch}(m)$	$H_0(m)$	$P_{ini}/\gamma(m)$	$\forall_{ch}(m^3)$	$H_a(m)$	$H(m)$	H_a/H
30	19.786	20	16	1000	32.592	198.815	0.1639
30	19.786	20	16	2000	27.162	198.815	0.1366
30	19.786	20	16	3000	25.265	198.815	0.1271
30	19.786	20	16	4000	24.441	198.815	0.1229
30	19.786	20	16	5000	23.937	198.815	0.1204
30	19.786	20	16	6000	23.435	198.815	0.1179
30	19.786	20	16	7000	23.212	198.815	0.1168
30	19.786	20	16	8000	22.824	198.815	0.1148
30	19.786	20	16	9000	22.441	198.815	0.1129
30	19.786	20	16	10000	22.283	198.815	0.1121

Table 5-20 Analysis for H_0=50 m

$Q(m^3/s)$	$H_{ch}(m)$	$H_0(m)$	$P_{ini}/\gamma(m)$	$\forall_{ch}(m^3)$	$H_a(m)$	$H(m)$	H_a/H
30	49.416	50	40	1000	68.434	316.137	0.2165
30	49.416	50	40	2000	61.593	316.137	0.1948
30	49.416	50	40	3000	57.875	316.137	0.1831
30	49.416	50	40	4000	56.302	316.137	0.1781
30	49.416	50	40	5000	55.027	316.137	0.1741
30	49.416	50	40	6000	54.776	316.137	0.1733
30	49.416	50	40	7000	54.327	316.137	0.1718
30	49.416	50	40	8000	53.817	316.137	0.1702
30	49.416	50	40	9000	53.673	316.137	0.1698
30	49.416	50	40	10000	53.547	316.137	0.1694

Table 5-21 Analysis for H_0=100 m

$Q(m^3/s)$	$H_{ch}(m)$	$H_0(m)$	$P_{ini}/\gamma(m)$	$\forall_{ch}(m^3)$	$H_a(m)$	$H(m)$	H_a/H
30	98.455	100	80	1000	133.438	460.136	0.2900
30	98.455	100	80	2000	121.038	460.136	0.2630
30	98.455	100	80	3000	113.866	460.136	0.2475
30	98.455	100	80	4000	110.942	460.136	0.2411
30	98.455	100	80	5000	109.001	460.136	0.2369
30	98.455	100	80	6000	107.245	460.136	0.2331
30	98.455	100	80	7000	106.871	460.136	0.2323
30	98.455	100	80	8000	106.861	460.136	0.2322
30	98.455	100	80	9000	107.348	460.136	0.2333
30	98.455	100	80	10000	106.771	460.136	0.2320

Table 5-22 Analysis for H_0=150 m

$Q(m^3/s)$	$H_{ch}(m)$	$H_0(m)$	$P_{ini}/\gamma(m)$	$\forall_{ch}(m^3)$	$H_a(m)$	$H(m)$	H_a/H
30	146.354	150	120	1000	199.287	515.167	0.3868
30	146.354	150	120	2000	170.904	515.167	0.3317
30	146.354	150	120	3000	164.231	515.167	0.3188
30	146.354	150	120	4000	163.51	515.167	0.3174
30	146.354	150	120	5000	159.752	515.167	0.3101
30	146.354	150	120	6000	158.926	515.167	0.3085
30	146.354	150	120	7000	157.478	515.167	0.3057
30	146.354	150	120	8000	158.425	515.167	0.3075
30	146.354	150	120	9000	157.961	515.167	0.3066
30	146.354	150	120	10000	156.77	515.167	0.3043

Table 5-23 Analysis for H_0=200 m

$Q(m^3/s)$	$H_{ch}(m)$	$H_0(m)$	$P_{ini}/\gamma(m)$	$\forall_{ch}(m^3)$	$H_a(m)$	$H(m)$	H_a/H
30	193.275	200	160	1000	258.642	687.31	0.3763
30	193.275	200	160	2000	226.543	687.31	0.3296
30	193.275	200	160	3000	214.729	687.31	0.3124
30	193.275	200	160	4000	214.149	687.31	0.3116
30	193.275	200	160	5000	210.685	687.31	0.3065
30	193.275	200	160	6000	211.022	687.31	0.3070
30	193.275	200	160	7000	209.802	687.31	0.3053
30	193.275	200	160	8000	209.522	687.31	0.3048
30	193.275	200	160	9000	209.216	687.31	0.3044
30	193.275	200	160	10000	208.346	687.31	0.3031

Table 5-24 Analysis for H_0=300 m

$Q(m^3/s)$	$H_{ch}(m)$	$H_0(m)$	$P_{ini}/\gamma(m)$	$\forall_{ch}(m^3)$	$H_a(m)$	$H(m)$	H_a/H
30	284.514	300	240	1000	383.589	874.52	0.4386
30	284.514	300	240	2000	322.336	874.52	0.3686
30	284.514	300	240	3000	315.706	874.52	0.3610
30	284.514	300	240	4000	314.316	874.52	0.3594
30	284.514	300	240	5000	312.623	874.52	0.3575
30	284.514	300	240	6000	311.251	874.52	0.3559
30	284.514	300	240	7000	303.164	874.52	0.3467
30	284.514	300	240	8000	306.735	874.52	0.3507
30	284.514	300	240	9000	306.94	874.52	0.3510
30	284.514	300	240	10000	306.873	874.52	0.3509

Table 5-25 Analysis for H_0=400 m

$Q(m^3/s)$	$H_{ch}(m)$	$H_0(m)$	$P_{ini}/\gamma(m)$	$\forall_{ch}(m^3)$	$H_a(m)$	$H(m)$	H_a/H
30	371.557	400	320	1000	467.618	1050.711	0.4450
30	371.557	400	320	2000	413.805	1050.711	0.3938
30	371.557	400	320	3000	407.629	1050.711	0.3880
30	371.557	400	320	4000	403.735	1050.711	0.3842
30	371.557	400	320	5000	405.798	1050.711	0.3862
30	371.557	400	320	6000	406.209	1050.711	0.3866
30	371.557	400	320	7000	405.573	1050.711	0.3860
30	371.557	400	320	8000	406.915	1050.711	0.3873
30	371.557	400	320	9000	400.24	1050.711	0.3809
30	371.557	400	320	10000	398.136	1050.711	0.3789

Table 5-26 Analysis for H_0=500 m

$Q(m^3/s)$	$H_{ch}(m)$	$H_0(m)$	$P_{ini}/\gamma(m)$	$\forall_{ch}(m^3)$	$H_a(m)$	$H(m)$	H_a/H
30	454.7	500	400	1000	550.824	1214.916	0.4534
30	454.7	500	400	2000	508.429	1214.916	0.4185
30	454.7	500	400	3000	504.422	1214.916	0.4152
30	454.7	500	400	4000	490.703	1214.916	0.4039
30	454.7	500	400	5000	500.347	1214.916	0.4118
30	454.7	500	400	6000	493.457	1214.916	0.4062
30	454.7	500	400	7000	498.432	1214.916	0.4103
30	454.7	500	400	8000	491.077	1214.916	0.4042
30	454.7	500	400	9000	486.961	1214.916	0.4008
30	454.7	500	400	10000	498.216	1214.916	0.4101

Figure 5.5 states that the relaxation of the system decreases with increasing static head. Hence, the air chamber works more efficiently in small hydropower plants. Volume curves in the graph are nearly parallel. That means, chamber volume can be expressed as a function of discharge only, whereas relaxation rate depends on both head and discharge. In order to obtain the exact relationship, more analyses should be conducted for various head and discharge values.

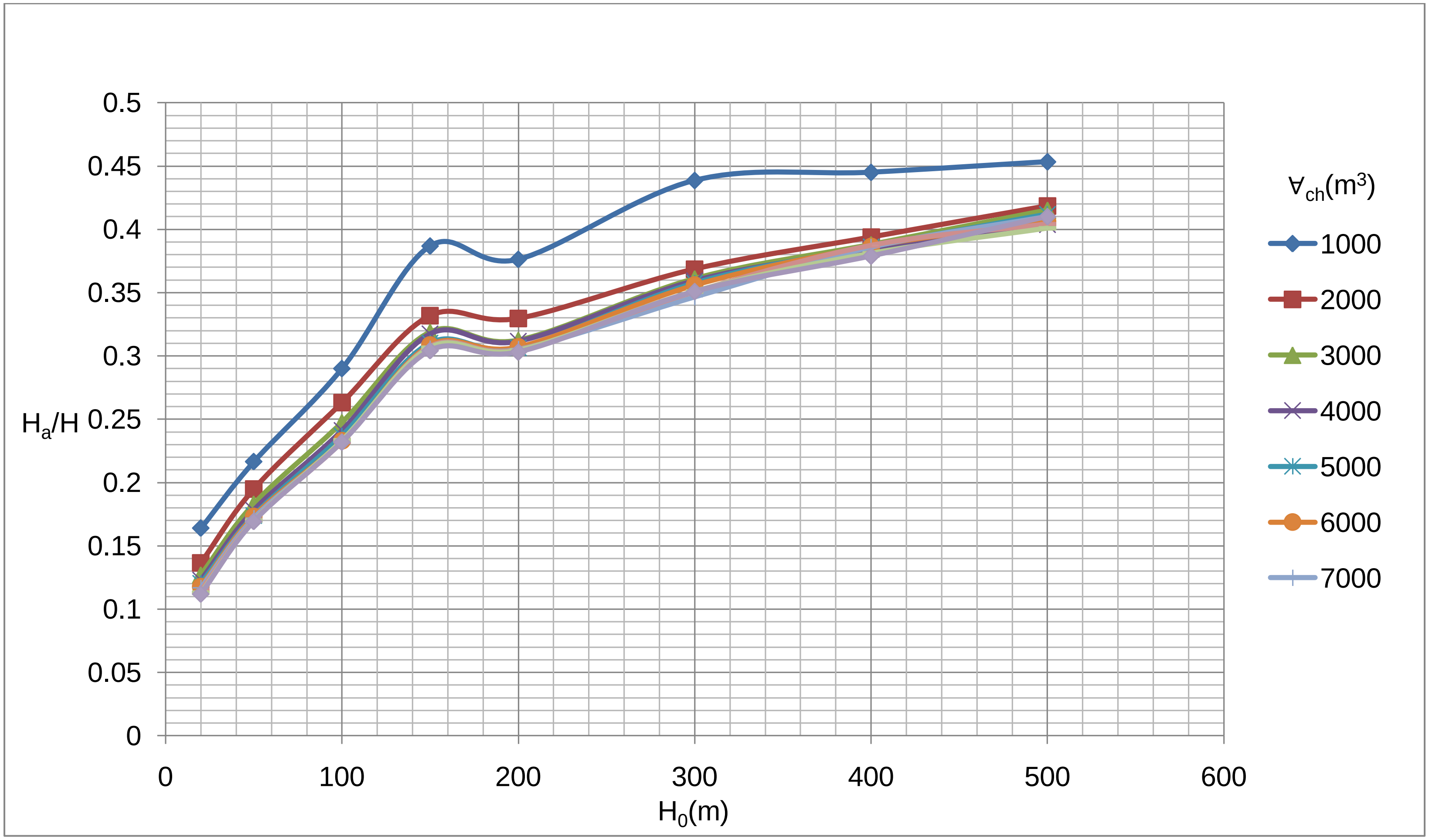

Figure 5.5 Graphical illustration of preliminary analysis for fixed discharge

5.4. Formulation of Air Chamber Model

The preliminary analysis gave some idea about air chambers behavior against different values of head and discharge. However, the exact relationship hasn't been revealed yet. The main objective of this study is to investigate the hydraulic response of the systems with an air chamber. A consequent objective is to provide the information to enable people to make practical air chamber designs for desired relaxation rates for their systems. By using charts provided in this study, one should be able to choose the volume of the air chamber which offers desired relaxation for the system. To achieve this purpose, more detailed and systematic analysis is required.

In the preliminary analysis, the relationship of air chamber volume and system relaxation was observed. According to observations, it is definite that an increase in chamber volume increases relaxation. However, the rate of increase of relaxation decreases with increasing chamber volume. That means, the relaxation has an asymptotic value. H_a/H ratio will be renamed as H_r *(the head ratio)* obtained for a certain chamber volume. Another abbreviation, H_{ra}, should be identified for the ratio of corresponding heads to indicate the maximum relaxation with sufficiently large chamber volume, being the asymptotic value of H_r. Value of asymptotic head ratio is dependent on the head and discharge of the system. It should be analyzed for both parameters individually. After long trials with numerical experiments, it is found that the asymptotic value of the relaxation can best be represented by H_0/V_p ratio. V_p is the velocity of flow in the penstock and the ratio gives a time scale of the unsteady phenomena taking place in the penstock.

Execution of the final analysis requires sets of head and discharge values. The head values chosen for the analysis are 10, 20, 50, 100, 200 and 500 meters. The discharge values are 0.2, 0.5, 1, 2, 5, 10, 20 cubic meters per second. The air chamber volumes selected for the analysis are 10, 20, 50, 100, 200, 500, 1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000, 9000 and 10000 cubic meters. These data sets are analyzed by HAMMER and the resulting tables are listed at the appendix section. These sets are put in a graphical form according to their H_0/V_p value in Fig. 5.6 where the head ratio is plotted against the chamber volume.

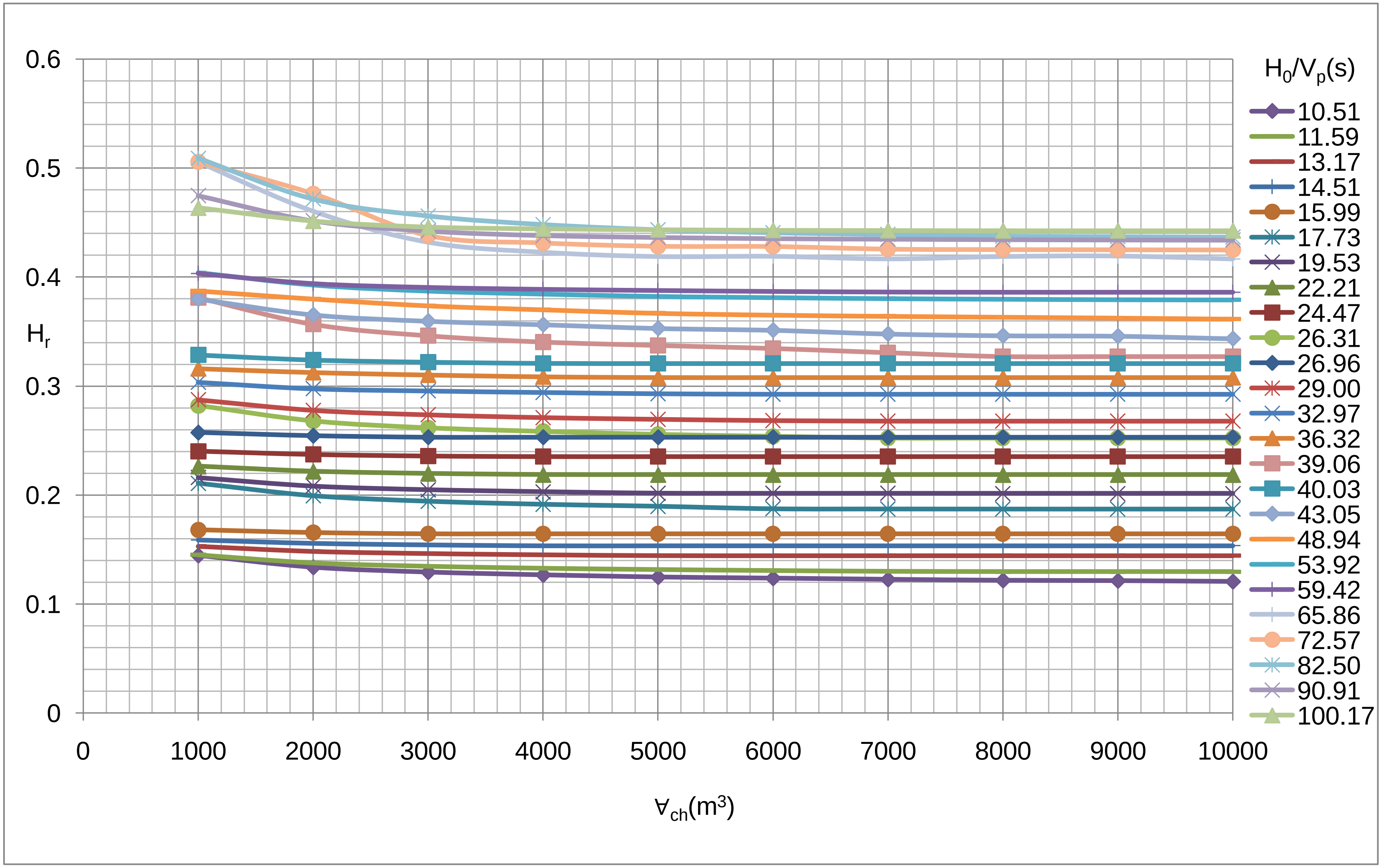

Figure 5.6 Graphical illustration of H_r vs $\forall_{ch}$ for different H_0/V_p values

From Fig. 5.6 remarkable implications are obtained. First of all, asymptotical values of H_r can be clearly seen. They are also sequential for their own H_0/V_p value. That means, H_{ra} values can be plotted against H_0/V_p values valid for every system with any static head and discharge. This brings a major convenience. Asymptotical head relaxation rate of a unique system having sufficiently large air chamber can be easily calculated. Data in Figure 5.7 is fitted to a second order polynomial given by:

$$H_{ra} = 4 \times 10^{-7} \left(\frac{H_0}{V_P}\right)^3 - 0.0001 \left(\frac{H_0}{V_P}\right)^2 + 0.0125 \left(\frac{H_0}{V_P}\right) \qquad (5.6)$$

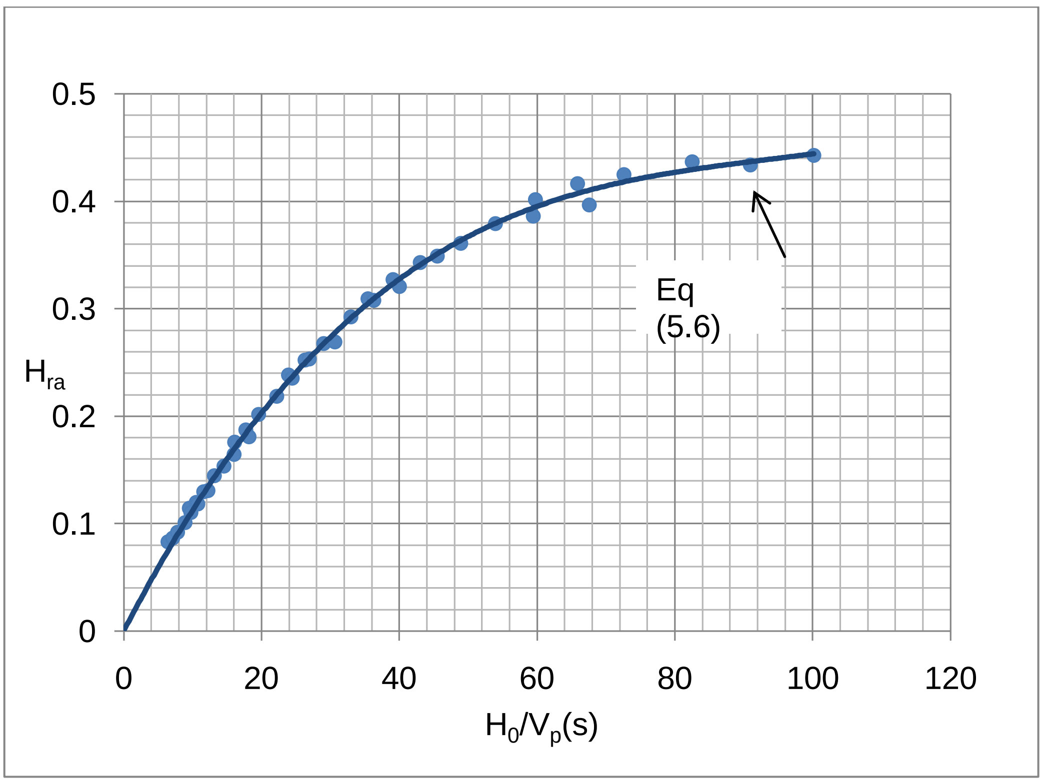

Figure 5.7 Graphical illustration of H_{ra} versus H_0/V_p

Fig. 5.7 provides the maximum values of relaxation only for sufficiently large chamber volume. It is clearly seen that air chamber performs better at low H_0/V_p values. Increasing H_0/V_p or time scale of the motion reduces the effectiveness of the chamber volume.

To make a decision on the sufficient chamber volume, the definition of H_{ra}/H_r is introduced. With increasing chamber volume, H_{ra}/H_r value approaches to 1, which means system is approaching its maximum relaxation ratio. Unfortunately, H_{ra}/H_r value is dependent on chamber volume, head and discharge which render H_{ra}/H_r values inconvenient to be illustrated in a single graph. Hence, they are plotted against volume for various discharges in the same plot for a given head.

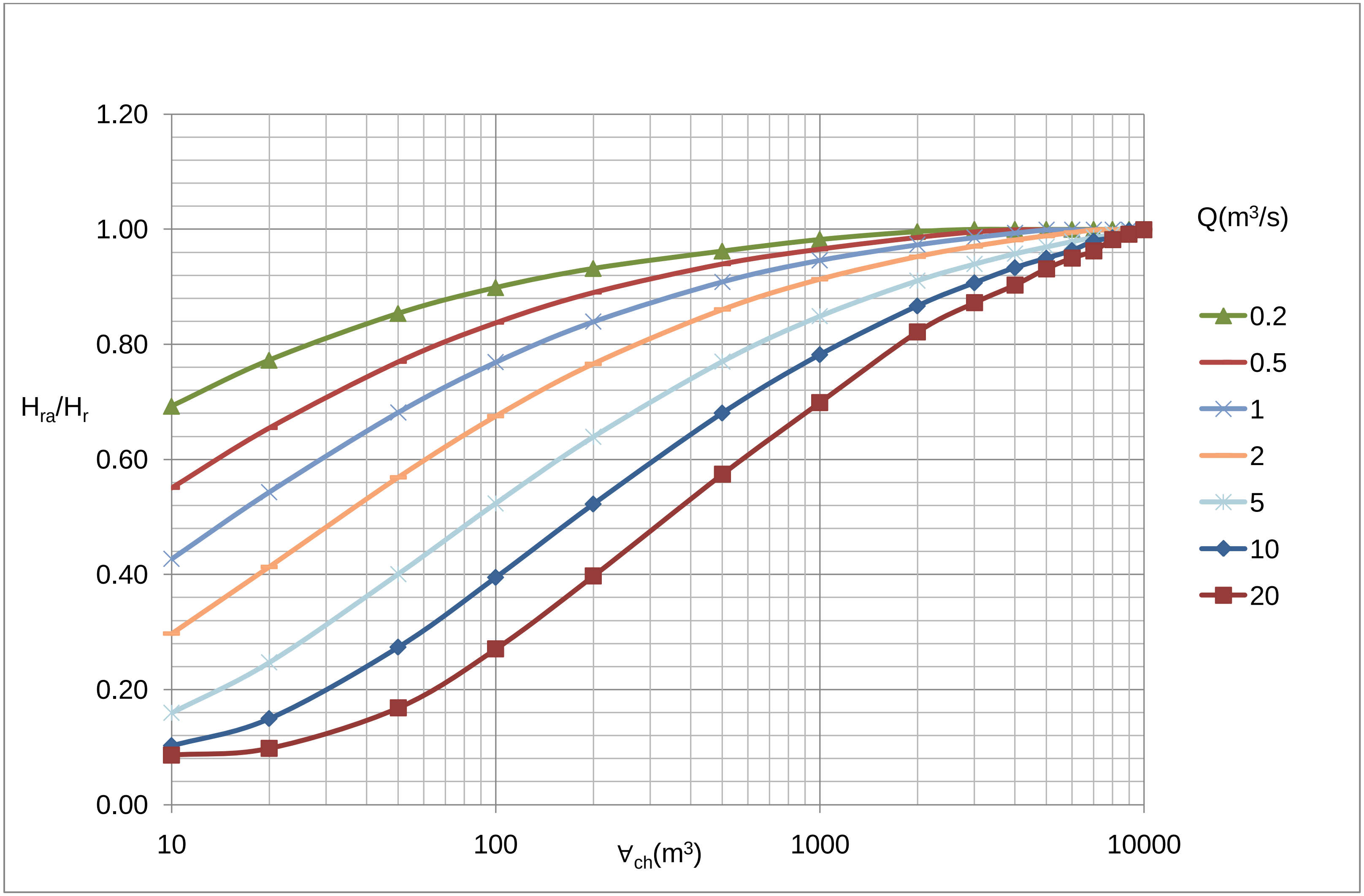

Figure 5.8 Graphical illustration of H_{ra}/H_r versus $\forall_{ch}$ for H_0=10 m

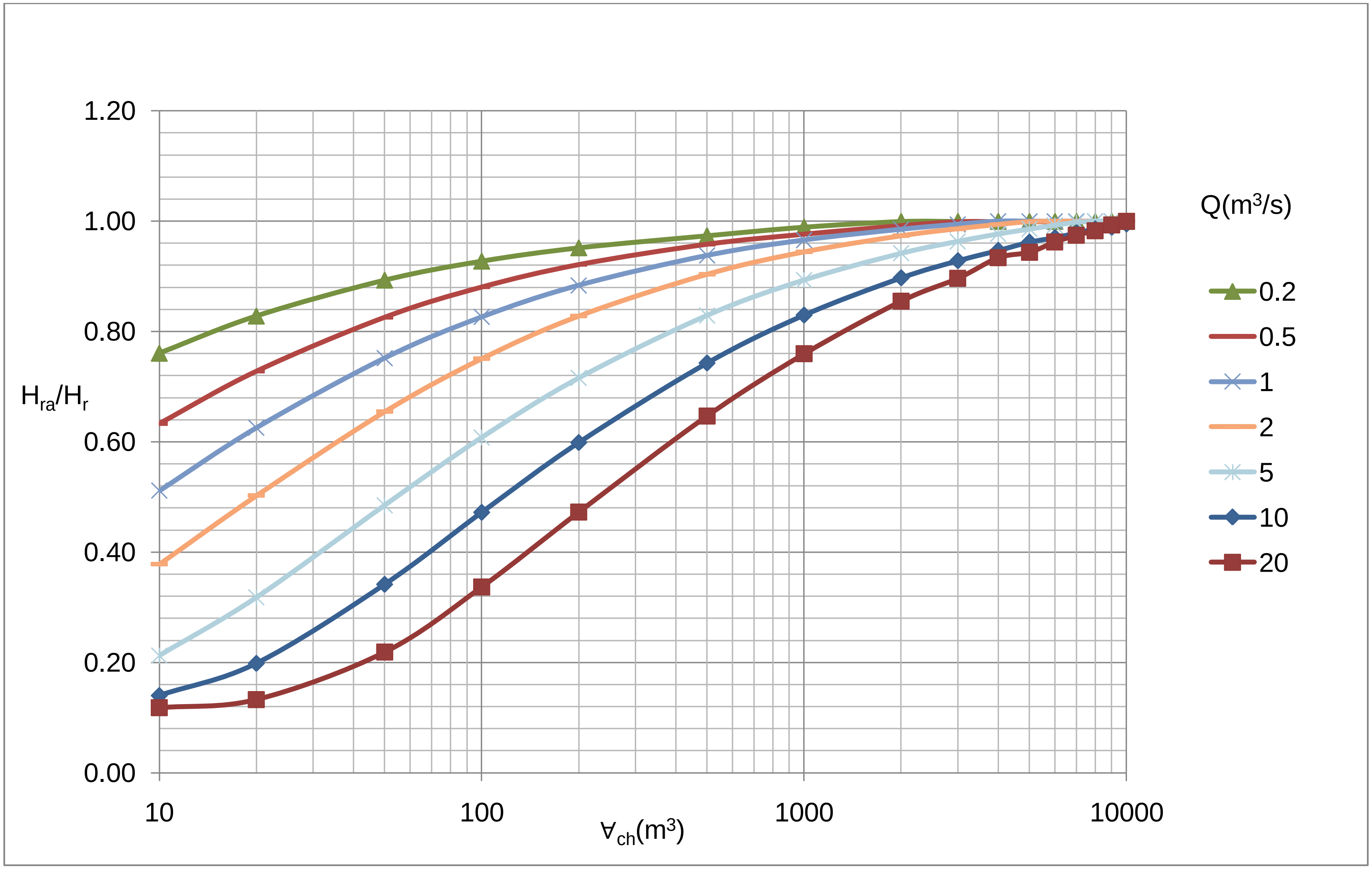

Figure 5.9 Graphical illustration of H_{ra}/H_r versus $\forall_{ch}$ for H_0=20 m

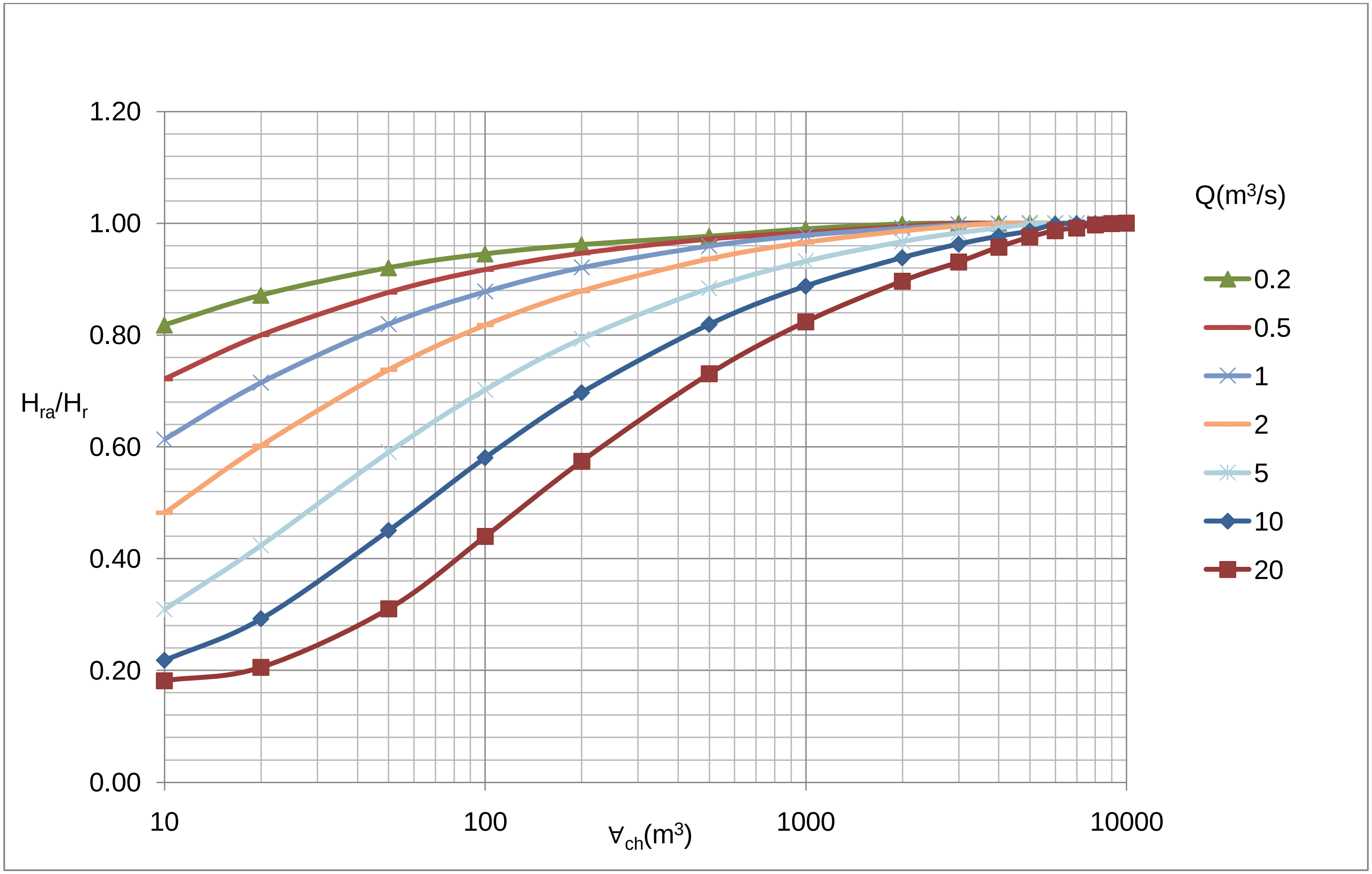

Figure 5.10 Graphical illustration of H_{ra}/H_r versus $\forall_{ch}$ for H_0=50 m

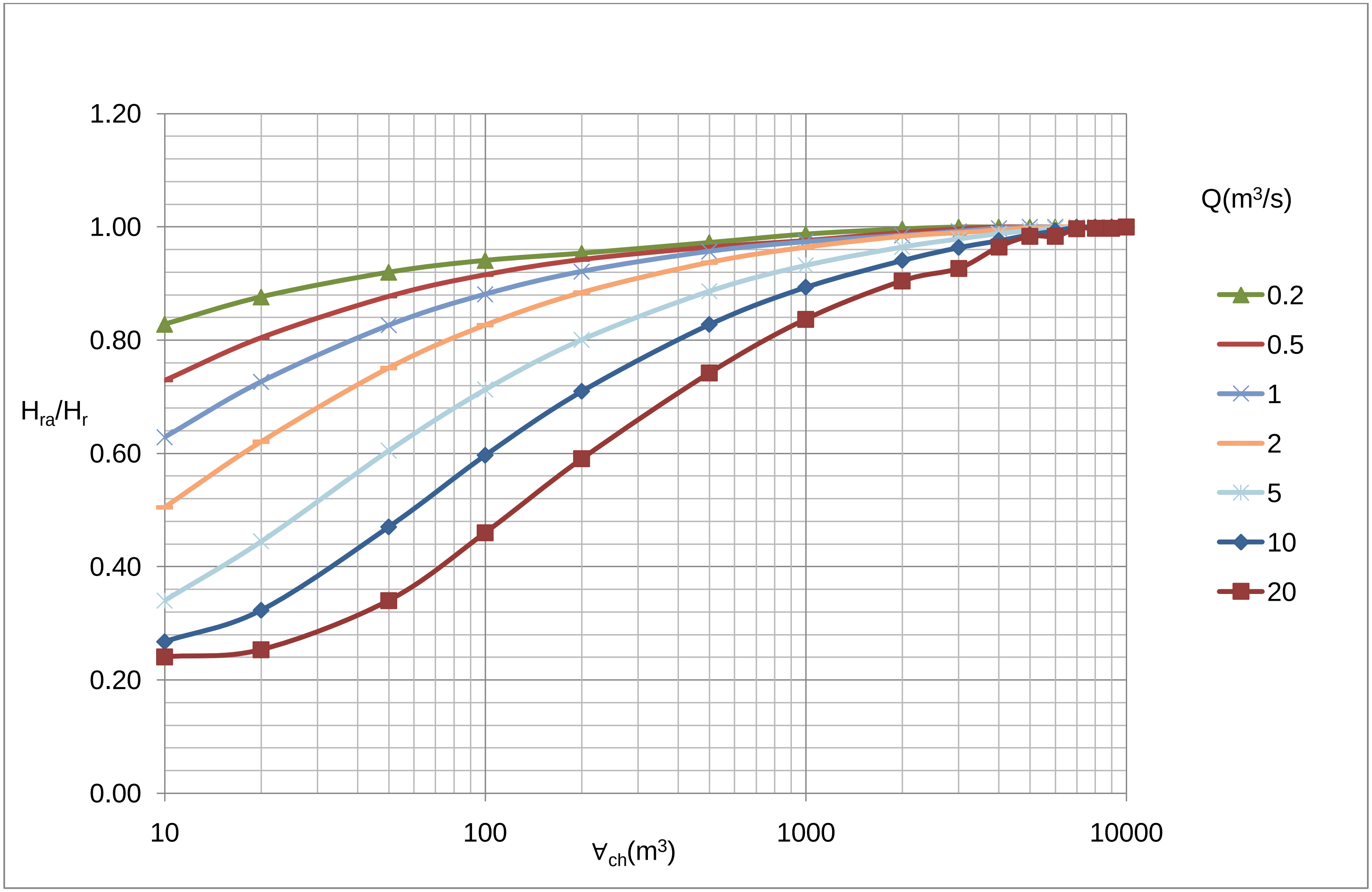

Figure 5.11 Graphical illustration of H_{ra}/H_r versus $\forall_{ch}$ for H_0=100 m

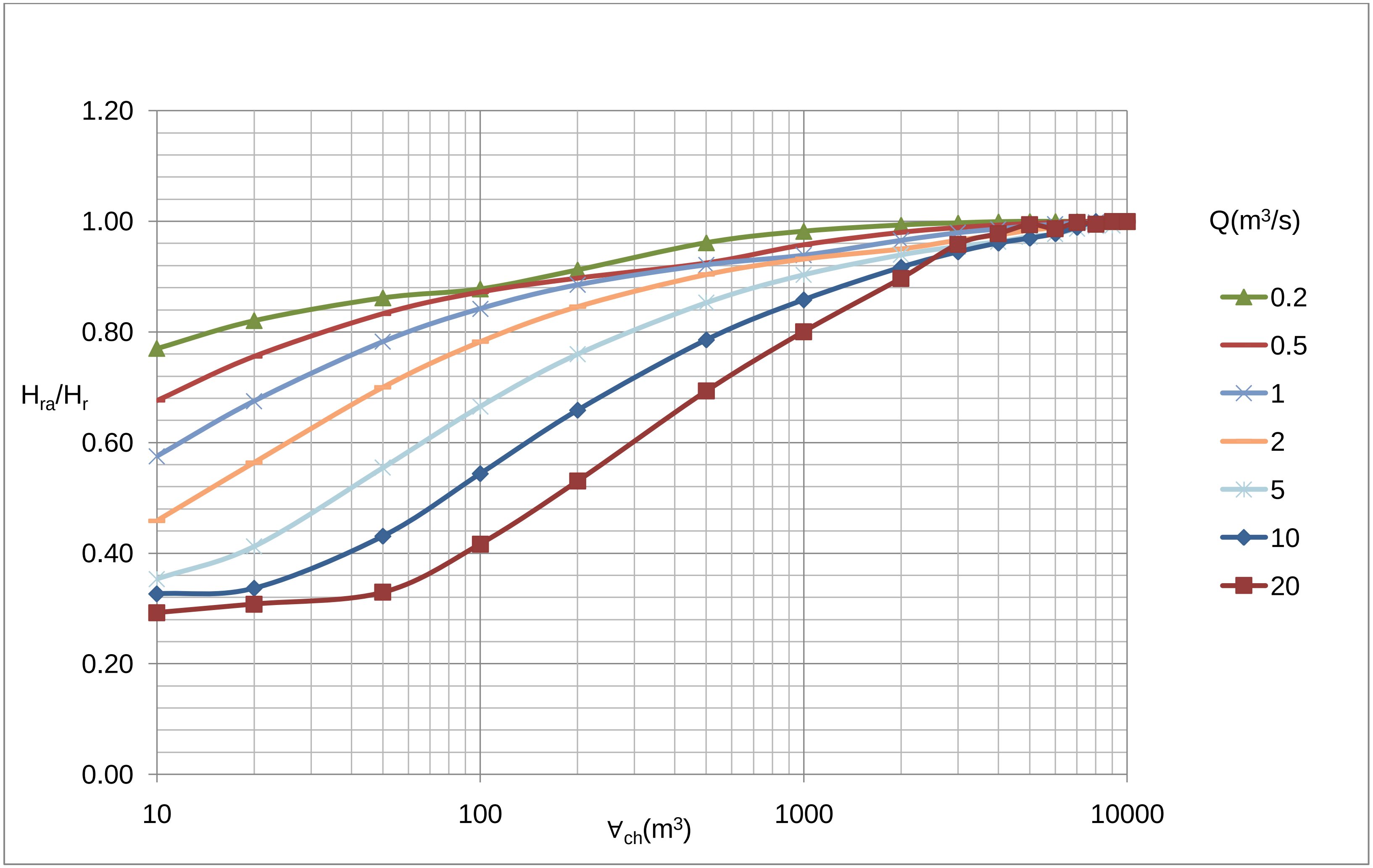

Figure 5.12 Graphical illustration of H_{ra}/H_r versus $\forall_{ch}$ for H_0=200 m

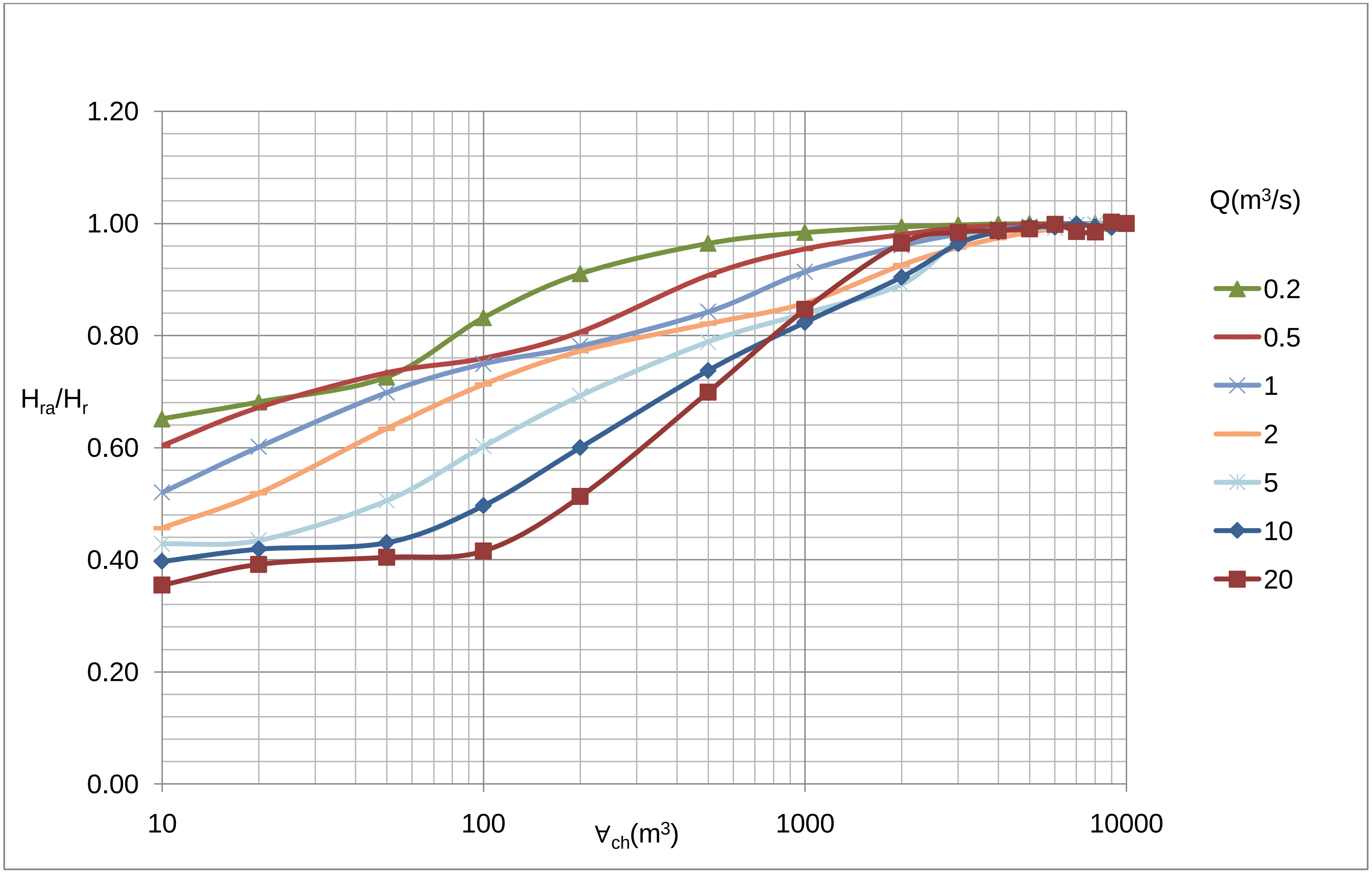

Figure 5.13 Graphical illustration of H_{ra}/H_r versus $\forall_{ch}$ for H_0=500 m

From the evaluation of Figures 5.8 to 5.13 it is seen that it is relatively easy to reach high relaxation rates at smaller chamber volumes for small discharges. The same is true for low heads. Therefore, air chambers are more efficient in low head and low discharge systems, or in general for small hydropower plants.

Maximum relaxation for different system parameters varies between 0.05 and 0.5. It means that air chamber may damp the possible waterhammer pressures in the system by 5% ~ 50% when a rapid valve closure occurs.

The reason of inefficiency of air chamber in high head and discharge systems is the solidification of air inside the chamber. At high pressures air is squeezed much and behaves like an incompressible material, thus relaxation given by the chamber decreases.

HAMMER gives more accurate results for low head and discharge systems for air chamber analysis. This is mainly due to numerical errors caused by method of characteristics. When head and discharge values are bigger, numerical errors become bigger.

One can use the charts produced in this study to determine the required air chamber volume for a certain hydropower system. With known H_0, Q and D parameters for the system, the time scale, H_0/V_p should be calculated first. Maximum possible relaxation rate, H_{ra}, corresponding to H_0/V_p can be obtained from Fig. 5.7. Then, one can choose the chamber volume required for a certain relaxation rate, H_r, from Figs 5.8~5.13. Final value of the chamber volume can best be obtained by a cost optimization of the whole system. Cost of air chamber together with waterhammer affected portion of the penstock must be included in the analysis to design the most economical system. This requires an optimization study for the penstock material thickness and air chamber volume.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

Rapid increase in demand for electricity resulted in rapid construction of big number of small hydropower plants. Smaller hydropower plants are becoming increasingly feasible due to rise in electricity prices. As smaller hydro become feasible, air chambers may be more frequently involved to provide the most economical solution against large waterhammer pressures. Air chamber, unlike other protective methods, is rather complicated and thus it should be investigated more rigorously. This study is conducted to analyze hydraulic response of air chambers and provide some technical material for quick prediction of system performance for an assumed air chamber volume. Typical systems with Francis turbines are considered. Based on the work conducted these conclusions can be stated:

- The throttle diameter between the penstock and the air chamber can be selected as 80% of the penstock diameter.
- Providing preset pressure to the air chamber improves the performance significantly, reducing the need for larger chamber volumes. The preset pressure value can be selected as 80% of the static head in the system.
- Charts provided in Figs. 5.7~5.13 can be used to predict possible performance of a certain volume of air chamber in a hydropower system which then allows a cost optimization study for the most economical design.

- An increase in the volume of air chamber provides additional relaxation until the maximum is reached asymptotically.
- Usage of air chamber provides reduction of waterhammer pressures for hydraulic systems, therefore; pipe thicknesses used for those systems would be thinner.
- Air chamber is a strong alternative to conventional protective measures if the required chamber volume can be provided at low cost.
- Theoretically, air chamber can be used for all scales of hydropower plants. However, it is more efficient for small hydropower plants. Also, it can be used for protection of distribution systems, plumbing systems and other kinds of pressured flow systems.
- HAMMER, which is the commercial software used for the transient analysis of air chamber system is a useful tool for transient flow simulation. Modeling with a user-friendly interface and its analysis is simple and time saving. However, there are limitations on head and discharge to keep numerical errors negligible.

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APPENDIX A

RESULTING TABLES

In the appendix section resulting tables from final analysis of air chamber are listed.

Table A-1 Analysis results for H_0=10 m, Q=0.2 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
10	9.66	0.2	8	10	14.301	75.835	0.1886	0.1306	0.6925	0.816	12.248
10	9.66	0.2	8	20	12.821	75.835	0.1691	0.1306	0.7725	0.816	12.248
10	9.66	0.2	8	50	11.601	75.835	0.1530	0.1306	0.8537	0.816	12.248
10	9.66	0.2	8	100	11.021	75.835	0.1453	0.1306	0.8987	0.816	12.248
10	9.66	0.2	8	200	10.628	75.835	0.1401	0.1306	0.9319	0.816	12.248
10	9.66	0.2	8	500	10.294	75.835	0.1357	0.1306	0.9621	0.816	12.248
10	9.66	0.2	8	1000	10.083	75.835	0.1330	0.1306	0.9823	0.816	12.248
10	9.66	0.2	8	2000	9.944	75.835	0.1311	0.1306	0.9960	0.816	12.248
10	9.66	0.2	8	3000	9.906	75.835	0.1306	0.1306	0.9998	0.816	12.248
10	9.66	0.2	8	4000	9.906	75.835	0.1306	0.1306	0.9998	0.816	12.248
10	9.66	0.2	8	5000	9.906	75.835	0.1306	0.1306	0.9998	0.816	12.248
10	9.66	0.2	8	6000	9.906	75.835	0.1306	0.1306	0.9998	0.816	12.248
10	9.66	0.2	8	7000	9.906	75.835	0.1306	0.1306	0.9998	0.816	12.248
10	9.66	0.2	8	8000	9.906	75.835	0.1306	0.1306	0.9998	0.816	12.248
10	9.66	0.2	8	9000	9.906	75.835	0.1306	0.1306	0.9998	0.816	12.248
10	9.66	0.2	8	10000	9.906	75.835	0.1306	0.1306	0.9998	0.816	12.248

Table A-2 Analysis results for H_0=10 m, Q=0.5 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
10	9.73	0.5	8	10	18.254	85.14	0.2144	0.1181	0.5508	0.928	10.773
10	9.73	0.5	8	20	15.352	85.14	0.1803	0.1181	0.6550	0.928	10.773
10	9.73	0.5	8	50	13.061	85.14	0.1534	0.1181	0.7699	0.928	10.773
10	9.73	0.5	8	100	12.003	85.14	0.1410	0.1181	0.8377	0.928	10.773
10	9.73	0.5	8	200	11.294	85.14	0.1327	0.1181	0.8903	0.928	10.773
10	9.73	0.5	8	500	10.697	85.14	0.1256	0.1181	0.9400	0.928	10.773
10	9.73	0.5	8	1000	10.412	85.14	0.1223	0.1181	0.9657	0.928	10.773
10	9.73	0.5	8	2000	10.196	85.14	0.1198	0.1181	0.9862	0.928	10.773
10	9.73	0.5	8	3000	10.099	85.14	0.1186	0.1181	0.9956	0.928	10.773
10	9.73	0.5	8	4000	10.062	85.14	0.1182	0.1181	0.9993	0.928	10.773
10	9.73	0.5	8	5000	10.062	85.14	0.1182	0.1181	0.9993	0.928	10.773
10	9.73	0.5	8	6000	10.062	85.14	0.1182	0.1181	0.9993	0.928	10.773
10	9.73	0.5	8	7000	10.062	85.14	0.1182	0.1181	0.9993	0.928	10.773
10	9.73	0.5	8	8000	10.062	85.14	0.1182	0.1181	0.9993	0.928	10.773
10	9.73	0.5	8	9000	10.062	85.14	0.1182	0.1181	0.9993	0.928	10.773
10	9.73	0.5	8	10000	10.062	85.14	0.1182	0.1181	0.9993	0.928	10.773

Table A-3 Analysis results for H_0=10 m, Q=1 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
10	9.77	1	8	10	23.797	92.163	0.2582	0.1103	0.4272	1.023	9.777
10	9.77	1	8	20	18.710	92.163	0.2030	0.1103	0.5433	1.023	9.777
10	9.77	1	8	50	14.911	92.163	0.1618	0.1103	0.6818	1.023	9.777
10	9.77	1	8	100	13.220	92.163	0.1434	0.1103	0.7690	1.023	9.777
10	9.77	1	8	200	12.110	92.163	0.1314	0.1103	0.8394	1.023	9.777
10	9.77	1	8	500	11.186	92.163	0.1214	0.1103	0.9088	1.023	9.777
10	9.77	1	8	1000	10.744	92.163	0.1166	0.1103	0.9462	1.023	9.777
10	9.77	1	8	2000	10.446	92.163	0.1133	0.1103	0.9732	1.023	9.777
10	9.77	1	8	3000	10.313	92.163	0.1119	0.1103	0.9857	1.023	9.777
10	9.77	1	8	4000	10.231	92.163	0.1110	0.1103	0.9936	1.023	9.777
10	9.77	1	8	5000	10.176	92.163	0.1104	0.1103	0.9990	1.023	9.777
10	9.77	1	8	6000	10.175	92.163	0.1104	0.1103	0.9991	1.023	9.777
10	9.77	1	8	7000	10.175	92.163	0.1104	0.1103	0.9991	1.023	9.777
10	9.77	1	8	8000	10.174	92.163	0.1104	0.1103	0.9992	1.023	9.777
10	9.77	1	8	9000	10.174	92.163	0.1104	0.1103	0.9992	1.023	9.777
10	9.77	1	8	10000	10.174	92.163	0.1104	0.1103	0.9992	1.023	9.777

Table A-4 Analysis results for H_0=10 m, Q=2 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
10	9.80	2	8	10	34.589	101.69	0.3402	0.1011	0.2972	1.127	8.873
10	9.80	2	8	20	24.869	101.69	0.2446	0.1011	0.4134	1.127	8.873
10	9.80	2	8	50	18.072	101.69	0.1777	0.1011	0.5689	1.127	8.873
10	9.80	2	8	100	15.225	101.69	0.1497	0.1011	0.6752	1.127	8.873
10	9.80	2	8	200	13.417	101.69	0.1319	0.1011	0.7662	1.127	8.873
10	9.80	2	8	500	11.949	101.69	0.1175	0.1011	0.8604	1.127	8.873
10	9.80	2	8	1000	11.258	101.69	0.1107	0.1011	0.9132	1.127	8.873
10	9.80	2	8	2000	10.793	101.69	0.1061	0.1011	0.9525	1.127	8.873
10	9.80	2	8	3000	10.593	101.69	0.1042	0.1011	0.9705	1.127	8.873
10	9.80	2	8	4000	10.476	101.69	0.1030	0.1011	0.9813	1.127	8.873
10	9.80	2	8	5000	10.399	101.69	0.1023	0.1011	0.9886	1.127	8.873
10	9.80	2	8	6000	10.34	101.69	0.1017	0.1011	0.9942	1.127	8.873
10	9.80	2	8	7000	10.294	101.69	0.1012	0.1011	0.9987	1.127	8.873
10	9.80	2	8	8000	10.288	101.69	0.1012	0.1011	0.9993	1.127	8.873
10	9.80	2	8	9000	10.288	101.69	0.1012	0.1011	0.9993	1.127	8.873
10	9.80	2	8	10000	10.288	101.69	0.1012	0.1011	0.9993	1.127	8.873

Table A-5 Analysis results for H_0=10 m, Q=5 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
10	9.84	5	8	10	65.947	114.021	0.5784	0.0922	0.1594	1.281	7.804
10	9.84	5	8	20	42.557	114.021	0.3732	0.0922	0.2470	1.281	7.804
10	9.84	5	8	50	26.225	114.021	0.2300	0.0922	0.4009	1.281	7.804
10	9.84	5	8	100	20.077	114.021	0.1761	0.0922	0.5236	1.281	7.804
10	9.84	5	8	200	16.446	114.021	0.1442	0.0922	0.6392	1.281	7.804
10	9.84	5	8	500	13.65	114.021	0.1197	0.0922	0.7702	1.281	7.804
10	9.84	5	8	1000	12.384	114.021	0.1086	0.0922	0.8489	1.281	7.804
10	9.84	5	8	2000	11.544	114.021	0.1012	0.0922	0.9107	1.281	7.804
10	9.84	5	8	3000	11.19	114.021	0.0981	0.0922	0.9395	1.281	7.804
10	9.84	5	8	4000	10.98	114.021	0.0963	0.0922	0.9574	1.281	7.804
10	9.84	5	8	5000	10.846	114.021	0.0951	0.0922	0.9693	1.281	7.804
10	9.84	5	8	6000	10.742	114.021	0.0942	0.0922	0.9787	1.281	7.804
10	9.84	5	8	7000	10.665	114.021	0.0935	0.0922	0.9857	1.281	7.804
10	9.84	5	8	8000	10.608	114.021	0.0930	0.0922	0.9910	1.281	7.804
10	9.84	5	8	9000	10.553	114.021	0.0926	0.0922	0.9962	1.281	7.804
10	9.84	5	8	10000	10.516	114.021	0.0922	0.0922	0.9997	1.281	7.804

Table A-6 Analysis results for H_0=10 m, Q=10m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
10	9.87	10	8	10	105.589	125.118	0.8439	0.0865	0.1025	1.412	7.083
10	9.87	10	8	20	72.341	125.118	0.5782	0.0865	0.1496	1.412	7.083
10	9.87	10	8	50	39.537	125.118	0.3160	0.0865	0.2737	1.412	7.083
10	9.87	10	8	100	27.403	125.118	0.2190	0.0865	0.3949	1.412	7.083
10	9.87	10	8	200	20.719	125.118	0.1656	0.0865	0.5224	1.412	7.083
10	9.87	10	8	500	15.903	125.118	0.1271	0.0865	0.6805	1.412	7.083
10	9.87	10	8	1000	13.841	125.118	0.1106	0.0865	0.7819	1.412	7.083
10	9.87	10	8	2000	12.488	125.118	0.0998	0.0865	0.8666	1.412	7.083
10	9.87	10	8	3000	11.934	125.118	0.0954	0.0865	0.9069	1.412	7.083
10	9.87	10	8	4000	11.599	125.118	0.0927	0.0865	0.9331	1.412	7.083
10	9.87	10	8	5000	11.397	125.118	0.0911	0.0865	0.9496	1.412	7.083
10	9.87	10	8	6000	11.239	125.118	0.0898	0.0865	0.9630	1.412	7.083
10	9.87	10	8	7000	11.052	125.118	0.0883	0.0865	0.9793	1.412	7.083
10	9.87	10	8	8000	10.985	125.118	0.0878	0.0865	0.9852	1.412	7.083
10	9.87	10	8	9000	10.849	125.118	0.0867	0.0865	0.9976	1.412	7.083
10	9.87	10	8	10000	10.823	125.118	0.0865	0.0865	1.0000	1.412	7.083

Table A-7 Analysis results for H_0=10 m, Q=20 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
10	9.89	20	8	10	130.980	136.238	0.9614	0.0831	0.0864	1.556	6.428
10	9.89	20	8	20	115.484	136.238	0.8477	0.0831	0.0980	1.556	6.428
10	9.89	20	8	50	67.316	136.238	0.4941	0.0831	0.1682	1.556	6.428
10	9.89	20	8	100	41.862	136.238	0.3073	0.0831	0.2704	1.556	6.428
10	9.89	20	8	200	28.496	136.238	0.2092	0.0831	0.3973	1.556	6.428
10	9.89	20	8	500	19.722	136.238	0.1448	0.0831	0.5740	1.556	6.428
10	9.89	20	8	1000	16.203	136.238	0.1189	0.0831	0.6987	1.556	6.428
10	9.89	20	8	2000	13.784	136.238	0.1012	0.0831	0.8213	1.556	6.428
10	9.89	20	8	3000	12.981	136.238	0.0953	0.0831	0.8721	1.556	6.428
10	9.89	20	8	4000	12.537	136.238	0.0920	0.0831	0.9030	1.556	6.428
10	9.89	20	8	5000	12.155	136.238	0.0892	0.0831	0.9314	1.556	6.428
10	9.89	20	8	6000	11.919	136.238	0.0875	0.0831	0.9499	1.556	6.428
10	9.89	20	8	7000	11.766	136.238	0.0864	0.0831	0.9622	1.556	6.428
10	9.89	20	8	8000	11.525	136.238	0.0846	0.0831	0.9823	1.556	6.428
10	9.89	20	8	9000	11.421	136.238	0.0838	0.0831	0.9913	1.556	6.428
10	9.89	20	8	10000	11.327	136.238	0.0831	0.0831	0.9995	1.556	6.428

Table A-8 Analysis results for H_0=20 m, Q=0.2 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
20	19.27	0.2	16	10	25.955	109.330	0.2374	0.1806	0.7607	1.100	18.182
20	19.27	0.2	16	20	23.844	109.330	0.2181	0.1806	0.8281	1.100	18.182
20	19.27	0.2	16	50	22.109	109.330	0.2022	0.1806	0.8931	1.100	18.182
20	19.27	0.2	16	100	21.289	109.330	0.1947	0.1806	0.9275	1.100	18.182
20	19.27	0.2	16	200	20.747	109.330	0.1898	0.1806	0.9517	1.100	18.182
20	19.27	0.2	16	500	20.281	109.330	0.1855	0.1806	0.9736	1.100	18.182
20	19.27	0.2	16	1000	19.961	109.330	0.1826	0.1806	0.9892	1.100	18.182
20	19.27	0.2	16	2000	19.757	109.330	0.1807	0.1806	0.9994	1.100	18.182
20	19.27	0.2	16	3000	19.756	109.330	0.1807	0.1806	0.9994	1.100	18.182
20	19.27	0.2	16	4000	19.756	109.330	0.1807	0.1806	0.9994	1.100	18.182
20	19.27	0.2	16	5000	19.756	109.330	0.1807	0.1806	0.9994	1.100	18.182
20	19.27	0.2	16	6000	19.756	109.330	0.1807	0.1806	0.9994	1.100	18.182
20	19.27	0.2	16	7000	19.756	109.330	0.1807	0.1806	0.9994	1.100	18.182
20	19.27	0.2	16	8000	19.756	109.330	0.1807	0.1806	0.9994	1.100	18.182
20	19.27	0.2	16	9000	19.755	109.330	0.1807	0.1806	0.9995	1.100	18.182
20	19.27	0.2	16	10000	19.755	109.330	0.1807	0.1806	0.9995	1.100	18.182

Table A-9 Analysis results for H_0=20 m, Q=0.5 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
20	19.42	0.5	16	10	31.585	121.798	0.2593	0.1642	0.6332	1.251	15.993
20	19.42	0.5	16	20	27.467	121.798	0.2255	0.1642	0.7281	1.251	15.993
20	19.42	0.5	16	50	24.212	121.798	0.1988	0.1642	0.8260	1.251	15.993
20	19.42	0.5	16	100	22.706	121.798	0.1864	0.1642	0.8808	1.251	15.993
20	19.42	0.5	16	200	21.703	121.798	0.1782	0.1642	0.9215	1.251	15.993
20	19.42	0.5	16	500	20.866	121.798	0.1713	0.1642	0.9585	1.251	15.993
20	19.42	0.5	16	1000	20.477	121.798	0.1681	0.1642	0.9767	1.251	15.993
20	19.42	0.5	16	2000	20.158	121.798	0.1655	0.1642	0.9921	1.251	15.993
20	19.42	0.5	16	3000	20.014	121.798	0.1643	0.1642	0.9993	1.251	15.993
20	19.42	0.5	16	4000	20.012	121.798	0.1643	0.1642	0.9994	1.251	15.993
20	19.42	0.5	16	5000	20.011	121.798	0.1643	0.1642	0.9994	1.251	15.993
20	19.42	0.5	16	6000	20.011	121.798	0.1643	0.1642	0.9994	1.251	15.993
20	19.42	0.5	16	7000	20.011	121.798	0.1643	0.1642	0.9994	1.251	15.993
20	19.42	0.5	16	8000	20.011	121.798	0.1643	0.1642	0.9994	1.251	15.993
20	19.42	0.5	16	9000	20.011	121.798	0.1643	0.1642	0.9994	1.251	15.993
20	19.42	0.5	16	10000	20.010	121.798	0.1643	0.1642	0.9995	1.251	15.993

Table A-10 Analysis results for H_0=20 m, Q=1 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
20	19.51	1	16	10	39.552	131.775	0.3001	0.1535	0.5114	1.378	14.514
20	19.51	1	16	20	32.334	131.775	0.2454	0.1535	0.6256	1.378	14.514
20	19.51	1	16	50	26.901	131.775	0.2041	0.1535	0.7519	1.378	14.514
20	19.51	1	16	100	24.478	131.775	0.1858	0.1535	0.8264	1.378	14.514
20	19.51	1	16	200	22.889	131.775	0.1737	0.1535	0.8837	1.378	14.514
20	19.51	1	16	500	21.571	131.775	0.1637	0.1535	0.9377	1.378	14.514
20	19.51	1	16	1000	20.946	131.775	0.1590	0.1535	0.9657	1.378	14.514
20	19.51	1	16	2000	20.533	131.775	0.1558	0.1535	0.9851	1.378	14.514
20	19.51	1	16	3000	20.344	131.775	0.1544	0.1535	0.9943	1.378	14.514
20	19.51	1	16	4000	20.233	131.775	0.1535	0.1535	0.9997	1.378	14.514
20	19.51	1	16	5000	20.232	131.775	0.1535	0.1535	0.9998	1.378	14.514
20	19.51	1	16	6000	20.232	131.775	0.1535	0.1535	0.9998	1.378	14.514
20	19.51	1	16	7000	20.231	131.775	0.1535	0.1535	0.9998	1.378	14.514
20	19.51	1	16	8000	20.231	131.775	0.1535	0.1535	0.9998	1.378	14.514
20	19.51	1	16	9000	20.231	131.775	0.1535	0.1535	0.9998	1.378	14.514
20	19.51	1	16	10000	20.231	131.775	0.1535	0.1535	0.9998	1.378	14.514

Table A-11 Analysis results for H_0=20 m, Q=2 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
20	19.60	2	16	10	54.067	141.645	0.3817	0.1444	0.3783	1.518	13.172
20	19.60	2	16	20	40.701	141.645	0.2873	0.1444	0.5025	1.518	13.172
20	19.60	2	16	50	31.244	141.645	0.2206	0.1444	0.6546	1.518	13.172
20	19.60	2	16	100	27.249	141.645	0.1924	0.1444	0.7506	1.518	13.172
20	19.60	2	16	200	24.705	141.645	0.1744	0.1444	0.8279	1.518	13.172
20	19.60	2	16	500	22.636	141.645	0.1598	0.1444	0.9036	1.518	13.172
20	19.60	2	16	1000	21.663	141.645	0.1529	0.1444	0.9442	1.518	13.172
20	19.60	2	16	2000	21.014	141.645	0.1484	0.1444	0.9733	1.518	13.172
20	19.60	2	16	3000	20.739	141.645	0.1464	0.1444	0.9862	1.518	13.172
20	19.60	2	16	4000	20.579	141.645	0.1453	0.1444	0.9939	1.518	13.172
20	19.60	2	16	5000	20.474	141.645	0.1445	0.1444	0.9990	1.518	13.172
20	19.60	2	16	6000	20.46	141.645	0.1444	0.1444	0.9997	1.518	13.172
20	19.60	2	16	7000	20.459	141.645	0.1444	0.1444	0.9997	1.518	13.172
20	19.60	2	16	8000	20.459	141.645	0.1444	0.1444	0.9997	1.518	13.172
20	19.60	2	16	9000	20.458	141.645	0.1444	0.1444	0.9998	1.518	13.172
20	19.60	2	16	10000	20.458	141.645	0.1444	0.1444	0.9998	1.518	13.172

Table A-12 Analysis results for H_0=20 m, Q=5 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
20	19.67	5	16	10	97.784	160.315	0.6099	0.1297	0.2126	1.726	11.586
20	19.67	5	16	20	65.385	160.315	0.4079	0.1297	0.3180	1.726	11.586
20	19.67	5	16	50	42.862	160.315	0.2674	0.1297	0.4851	1.726	11.586
20	19.67	5	16	100	34.203	160.315	0.2133	0.1297	0.6079	1.726	11.586
20	19.67	5	16	200	29.051	160.315	0.1812	0.1297	0.7157	1.726	11.586
20	19.67	5	16	500	25.076	160.315	0.1564	0.1297	0.8292	1.726	11.586
20	19.67	5	16	1000	23.275	160.315	0.1452	0.1297	0.8934	1.726	11.586
20	19.67	5	16	2000	22.081	160.315	0.1377	0.1297	0.9417	1.726	11.586
20	19.67	5	16	3000	21.584	160.315	0.1346	0.1297	0.9633	1.726	11.586
20	19.67	5	16	4000	21.288	160.315	0.1328	0.1297	0.9767	1.726	11.586
20	19.67	5	16	5000	21.096	160.315	0.1316	0.1297	0.9856	1.726	11.586
20	19.67	5	16	6000	20.947	160.315	0.1307	0.1297	0.9926	1.726	11.586
20	19.67	5	16	7000	20.835	160.315	0.1300	0.1297	0.9980	1.726	11.586
20	19.67	5	16	8000	20.796	160.315	0.1297	0.1297	0.9998	1.726	11.586
20	19.67	5	16	9000	20.796	160.315	0.1297	0.1297	0.9998	1.726	11.586
20	19.67	5	16	10000	20.794	160.315	0.1297	0.1297	0.9999	1.726	11.586

Table A-13 Analysis results for H_0=20 m, Q=10 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
20	19.72	10	16	10	149.84	175.1	0.8557	0.1200	0.1402	1.902	10.514
20	19.72	10	16	20	105.833	175.1	0.6044	0.1200	0.1985	1.902	10.514
20	19.72	10	16	50	61.48	175.1	0.3511	0.1200	0.3418	1.902	10.514
20	19.72	10	16	100	44.498	175.1	0.2541	0.1200	0.4722	1.902	10.514
20	19.72	10	16	200	35.093	175.1	0.2004	0.1200	0.5988	1.902	10.514
20	19.72	10	16	500	28.287	175.1	0.1615	0.1200	0.7428	1.902	10.514
20	19.72	10	16	1000	25.323	175.1	0.1446	0.1200	0.8298	1.902	10.514
20	19.72	10	16	2000	23.418	175.1	0.1337	0.1200	0.8973	1.902	10.514
20	19.72	10	16	3000	22.635	175.1	0.1293	0.1200	0.9283	1.902	10.514
20	19.72	10	16	4000	22.188	175.1	0.1267	0.1200	0.9470	1.902	10.514
20	19.72	10	16	5000	21.84	175.1	0.1247	0.1200	0.9621	1.902	10.514
20	19.72	10	16	6000	21.657	175.1	0.1237	0.1200	0.9702	1.902	10.514
20	19.72	10	16	7000	21.467	175.1	0.1226	0.1200	0.9788	1.902	10.514
20	19.72	10	16	8000	21.312	175.1	0.1217	0.1200	0.9859	1.902	10.514
20	19.72	10	16	9000	21.249	175.1	0.1214	0.1200	0.9888	1.902	10.514
20	19.72	10	16	10000	21.123	175.1	0.1206	0.1200	0.9947	1.902	10.514

Table A-14 Analysis results for H_0=20 m, Q=20 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
20	19.76	20	16	10	184.008	190.298	0.9669	0.1143	0.1182	2.096	9.542
20	19.76	20	16	20	163.491	190.298	0.8591	0.1143	0.1330	2.096	9.542
20	19.76	20	16	50	99.317	190.298	0.5219	0.1143	0.2190	2.096	9.542
20	19.76	20	16	100	64.537	190.298	0.3391	0.1143	0.3370	2.096	9.542
20	19.76	20	16	200	46.023	190.298	0.2418	0.1143	0.4726	2.096	9.542
20	19.76	20	16	500	33.624	190.298	0.1767	0.1143	0.6469	2.096	9.542
20	19.76	20	16	1000	28.642	190.298	0.1505	0.1143	0.7594	2.096	9.542
20	19.76	20	16	2000	25.444	190.298	0.1337	0.1143	0.8549	2.096	9.542
20	19.76	20	16	3000	24.266	190.298	0.1275	0.1143	0.8964	2.096	9.542
20	19.76	20	16	4000	23.297	190.298	0.1224	0.1143	0.9336	2.096	9.542
20	19.76	20	16	5000	23.048	190.298	0.1211	0.1143	0.9437	2.096	9.542
20	19.76	20	16	6000	22.608	190.298	0.1188	0.1143	0.9621	2.096	9.542
20	19.76	20	16	7000	22.316	190.298	0.1173	0.1143	0.9747	2.096	9.542
20	19.76	20	16	8000	22.130	190.298	0.1163	0.1143	0.9829	2.096	9.542
20	19.76	20	16	9000	21.898	190.298	0.1151	0.1143	0.9933	2.096	9.542
20	19.76	20	16	10000	21.763	190.298	0.1144	0.1143	0.9995	2.096	9.542

Table A-15 Analysis results for H_0=50 m, Q=0.2 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
50	48.06	0.2	40	10	59.972	182.375	0.3288	0.2690	0.8180	1.631	30.653
50	48.06	0.2	40	20	56.315	182.375	0.3088	0.2690	0.8712	1.631	30.653
50	48.06	0.2	40	50	53.310	182.375	0.2923	0.2690	0.9203	1.631	30.653
50	48.06	0.2	40	100	51.915	182.375	0.2847	0.2690	0.9450	1.631	30.653
50	48.06	0.2	40	200	51.027	182.375	0.2798	0.2690	0.9614	1.631	30.653
50	48.06	0.2	40	500	50.244	182.375	0.2755	0.2690	0.9764	1.631	30.653
50	48.06	0.2	40	1000	49.574	182.375	0.2718	0.2690	0.9896	1.631	30.653
50	48.06	0.2	40	2000	49.132	182.375	0.2694	0.2690	0.9985	1.631	30.653
50	48.06	0.2	40	3000	49.063	182.375	0.2690	0.2690	0.9999	1.631	30.653
50	48.06	0.2	40	4000	49.063	182.375	0.2690	0.2690	0.9999	1.631	30.653
50	48.06	0.2	40	5000	49.062	182.375	0.2690	0.2690	0.9999	1.631	30.653
50	48.06	0.2	40	6000	49.062	182.375	0.2690	0.2690	0.9999	1.631	30.653
50	48.06	0.2	40	7000	49.062	182.375	0.2690	0.2690	0.9999	1.631	30.653
50	48.06	0.2	40	8000	49.062	182.375	0.2690	0.2690	0.9999	1.631	30.653
50	48.06	0.2	40	9000	49.062	182.375	0.2690	0.2690	0.9999	1.631	30.653
50	48.06	0.2	40	10000	49.061	182.375	0.2690	0.2690	1.0000	1.631	30.653

Table A-16 Analysis results for H_0=50 m, Q=0.5 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
50	48.52	0.5	40	10	68.961	196.669	0.3506	0.2530	0.7215	1.854	26.962
50	48.52	0.5	40	20	62.199	196.669	0.3163	0.2530	0.8000	1.854	26.962
50	48.52	0.5	40	50	56.778	196.669	0.2887	0.2530	0.8763	1.854	26.962
50	48.52	0.5	40	100	54.252	196.669	0.2759	0.2530	0.9172	1.854	26.962
50	48.52	0.5	40	200	52.577	196.669	0.2673	0.2530	0.9464	1.854	26.962
50	48.52	0.5	40	500	51.206	196.669	0.2604	0.2530	0.9717	1.854	26.962
50	48.52	0.5	40	1000	50.612	196.669	0.2573	0.2530	0.9831	1.854	26.962
50	48.52	0.5	40	2000	50.046	196.669	0.2545	0.2530	0.9942	1.854	26.962
50	48.52	0.5	40	3000	49.778	196.669	0.2531	0.2530	0.9996	1.854	26.962
50	48.52	0.5	40	4000	49.772	196.669	0.2531	0.2530	0.9997	1.854	26.962
50	48.52	0.5	40	5000	49.771	196.669	0.2531	0.2530	0.9997	1.854	26.962
50	48.52	0.5	40	6000	49.771	196.669	0.2531	0.2530	0.9997	1.854	26.962
50	48.52	0.5	40	7000	49.770	196.669	0.2531	0.2530	0.9997	1.854	26.962
50	48.52	0.5	40	8000	49.770	196.669	0.2531	0.2530	0.9997	1.854	26.962
50	48.52	0.5	40	9000	49.770	196.669	0.2531	0.2530	0.9997	1.854	26.962
50	48.52	0.5	40	10000	49.770	196.669	0.2531	0.2530	0.9997	1.854	26.962

Table A-17 Analysis results for H_0=50 m, Q=1 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
50	48.72	1	40	10	81.906	213.626	0.3834	0.2352	0.6134	2.043	24.469
50	48.72	1	40	20	70.290	213.626	0.3290	0.2352	0.7148	2.043	24.469
50	48.72	1	40	50	61.309	213.626	0.2870	0.2352	0.8195	2.043	24.469
50	48.72	1	40	100	57.254	213.626	0.2680	0.2352	0.8776	2.043	24.469
50	48.72	1	40	200	54.574	213.626	0.2555	0.2352	0.9207	2.043	24.469
50	48.72	1	40	500	52.361	213.626	0.2451	0.2352	0.9596	2.043	24.469
50	48.72	1	40	1000	51.333	213.626	0.2403	0.2352	0.9788	2.043	24.469
50	48.72	1	40	2000	50.683	213.626	0.2373	0.2352	0.9914	2.043	24.469
50	48.72	1	40	3000	50.374	213.626	0.2358	0.2352	0.9974	2.043	24.469
50	48.72	1	40	4000	50.259	213.626	0.2353	0.2352	0.9997	2.043	24.469
50	48.72	1	40	5000	50.257	213.626	0.2353	0.2352	0.9998	2.043	24.469
50	48.72	1	40	6000	50.256	213.626	0.2353	0.2352	0.9998	2.043	24.469
50	48.72	1	40	7000	50.255	213.626	0.2352	0.2352	0.9998	2.043	24.469
50	48.72	1	40	8000	50.255	213.626	0.2352	0.2352	0.9998	2.043	24.469
50	48.72	1	40	9000	50.254	213.626	0.2352	0.2352	0.9998	2.043	24.469
50	48.72	1	40	10000	50.254	213.626	0.2352	0.2352	0.9998	2.043	24.469

Table A-18 Analysis results for H_0=50 m, Q=2 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
50	48.89	2	40	10	105.321	231.989	0.4540	0.2187	0.4817	2.252	22.206
50	48.89	2	40	20	84.260	231.989	0.3632	0.2187	0.6021	2.252	22.206
50	48.89	2	40	50	68.758	231.989	0.2964	0.2187	0.7379	2.252	22.206
50	48.89	2	40	100	62.056	231.989	0.2675	0.2187	0.8176	2.252	22.206
50	48.89	2	40	200	57.735	231.989	0.2489	0.2187	0.8788	2.252	22.206
50	48.89	2	40	500	54.201	231.989	0.2336	0.2187	0.9361	2.252	22.206
50	48.89	2	40	1000	52.549	231.989	0.2265	0.2187	0.9655	2.252	22.206
50	48.89	2	40	2000	51.460	231.989	0.2218	0.2187	0.9859	2.252	22.206
50	48.89	2	40	3000	50.994	231.989	0.2198	0.2187	0.9949	2.252	22.206
50	48.89	2	40	4000	50.759	231.989	0.2188	0.2187	0.9995	2.252	22.206
50	48.89	2	40	5000	50.751	231.989	0.2188	0.2187	0.9997	2.252	22.206
50	48.89	2	40	6000	50.749	231.989	0.2188	0.2187	0.9997	2.252	22.206
50	48.89	2	40	7000	50.748	231.989	0.2188	0.2187	0.9998	2.252	22.206
50	48.89	2	40	8000	50.746	231.989	0.2187	0.2187	0.9998	2.252	22.206
50	48.89	2	40	9000	50.746	231.989	0.2187	0.2187	0.9998	2.252	22.206
50	48.89	2	40	10000	50.745	231.989	0.2187	0.2187	0.9998	2.252	22.206

Table A-19 Analysis results for H_0=50 m, Q=5 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
50	49.13	5	40	10	166.596	255.780	0.6513	0.2014	0.3092	2.560	19.532
50	49.13	5	40	20	121.543	255.780	0.4752	0.2014	0.4238	2.560	19.532
50	49.13	5	40	50	87.212	255.780	0.3410	0.2014	0.5907	2.560	19.532
50	49.13	5	40	100	73.371	255.780	0.2869	0.2014	0.7021	2.560	19.532
50	49.13	5	40	200	64.921	255.780	0.2538	0.2014	0.7935	2.560	19.532
50	49.13	5	40	500	58.286	255.780	0.2279	0.2014	0.8838	2.560	19.532
50	49.13	5	40	1000	55.261	255.780	0.2160	0.2014	0.9322	2.560	19.532
50	49.13	5	40	2000	53.251	255.780	0.2082	0.2014	0.9674	2.560	19.532
50	49.13	5	40	3000	52.402	255.780	0.2049	0.2014	0.9831	2.560	19.532
50	49.13	5	40	4000	51.936	255.780	0.2030	0.2014	0.9919	2.560	19.532
50	49.13	5	40	5000	51.581	255.780	0.2017	0.2014	0.9987	2.560	19.532
50	49.13	5	40	6000	51.547	255.780	0.2015	0.2014	0.9994	2.560	19.532
50	49.13	5	40	7000	51.544	255.780	0.2015	0.2014	0.9994	2.560	19.532
50	49.13	5	40	8000	51.540	255.780	0.2015	0.2014	0.9995	2.560	19.532
50	49.13	5	40	9000	51.538	255.780	0.2015	0.2014	0.9995	2.560	19.532
50	49.13	5	40	10000	51.538	255.780	0.2015	0.2014	0.9995	2.560	19.532

Table A-20 Analysis results for H_0=50 m, Q=10 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
50	49.24	10	40	10	239.253	278.625	0.8587	0.1871	0.2179	2.821	17.726
50	49.24	10	40	20	178.408	278.625	0.6403	0.1871	0.2922	2.821	17.726
50	49.24	10	40	50	115.791	278.625	0.4156	0.1871	0.4502	2.821	17.726
50	49.24	10	40	100	89.807	278.625	0.3223	0.1871	0.5805	2.821	17.726
50	49.24	10	40	200	74.801	278.625	0.2685	0.1871	0.6969	2.821	17.726
50	49.24	10	40	500	63.644	278.625	0.2284	0.1871	0.8191	2.821	17.726
50	49.24	10	40	1000	58.729	278.625	0.2108	0.1871	0.8876	2.821	17.726
50	49.24	10	40	2000	55.551	278.625	0.1994	0.1871	0.9384	2.821	17.726
50	49.24	10	40	3000	54.151	278.625	0.1944	0.1871	0.9627	2.821	17.726
50	49.24	10	40	4000	53.372	278.625	0.1916	0.1871	0.9767	2.821	17.726
50	49.24	10	40	5000	52.862	278.625	0.1897	0.1871	0.9862	2.821	17.726
50	49.24	10	40	6000	52.224	278.625	0.1874	0.1871	0.9982	2.821	17.726
50	49.24	10	40	7000	52.174	278.625	0.1873	0.1871	0.9992	2.821	17.726
50	49.24	10	40	8000	52.157	278.625	0.1872	0.1871	0.9995	2.821	17.726
50	49.24	10	40	9000	52.153	278.625	0.1872	0.1871	0.9996	2.821	17.726
50	49.24	10	40	10000	52.149	278.625	0.1872	0.1871	0.9996	2.821	17.726

Table A-21 Analysis results for H_0=50 m, Q=20 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
50	49.36	20	40	10	291.398	301.427	0.9667	0.1757	0.1817	3.108	16.087
50	49.36	20	40	20	257.726	301.427	0.8550	0.1757	0.2055	3.108	16.087
50	49.36	20	40	50	170.775	301.427	0.5666	0.1757	0.3101	3.108	16.087
50	49.36	20	40	100	120.497	301.427	0.3998	0.1757	0.4395	3.108	16.087
50	49.36	20	40	200	92.270	301.427	0.3061	0.1757	0.5740	3.108	16.087
50	49.36	20	40	500	72.504	301.427	0.2405	0.1757	0.7305	3.108	16.087
50	49.36	20	40	1000	64.305	301.427	0.2133	0.1757	0.8236	3.108	16.087
50	49.36	20	40	2000	59.091	301.427	0.1960	0.1757	0.8963	3.108	16.087
50	49.36	20	40	3000	56.918	301.427	0.1888	0.1757	0.9305	3.108	16.087
50	49.36	20	40	4000	55.327	301.427	0.1836	0.1757	0.9572	3.108	16.087
50	49.36	20	40	5000	54.319	301.427	0.1802	0.1757	0.9750	3.108	16.087
50	49.36	20	40	6000	53.675	301.427	0.1781	0.1757	0.9867	3.108	16.087
50	49.36	20	40	7000	53.407	301.427	0.1772	0.1757	0.9916	3.108	16.087
50	49.36	20	40	8000	53.113	301.427	0.1762	0.1757	0.9971	3.108	16.087
50	49.36	20	40	9000	52.975	301.427	0.1757	0.1757	0.9997	3.108	16.087
50	49.36	20	40	10000	52.962	301.427	0.1757	0.1757	1.0000	3.108	16.087

Table A-22 Analysis results for H_0=100 m, Q=0.2 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
100	94.81	0.2	80	10	116.774	277.232	0.4212	0.3488	0.8281	2.198	45.505
100	94.81	0.2	80	20	110.340	277.232	0.3980	0.3488	0.8764	2.198	45.505
100	94.81	0.2	80	50	105.116	277.232	0.3792	0.3488	0.9199	2.198	45.505
100	94.81	0.2	80	100	102.784	277.232	0.3708	0.3488	0.9408	2.198	45.505
100	94.81	0.2	80	200	101.417	277.232	0.3658	0.3488	0.9535	2.198	45.505
100	94.81	0.2	80	500	99.452	277.232	0.3587	0.3488	0.9723	2.198	45.505
100	94.81	0.2	80	1000	97.950	277.232	0.3533	0.3488	0.9872	2.198	45.505
100	94.81	0.2	80	2000	97.052	277.232	0.3501	0.3488	0.9964	2.198	45.505
100	94.81	0.2	80	3000	96.731	277.232	0.3489	0.3488	0.9997	2.198	45.505
100	94.81	0.2	80	4000	96.720	277.232	0.3489	0.3488	0.9998	2.198	45.505
100	94.81	0.2	80	5000	96.719	277.232	0.3489	0.3488	0.9998	2.198	45.505
100	94.81	0.2	80	6000	96.719	277.232	0.3489	0.3488	0.9998	2.198	45.505
100	94.81	0.2	80	7000	96.718	277.232	0.3489	0.3488	0.9998	2.198	45.505
100	94.81	0.2	80	8000	96.718	277.232	0.3489	0.3488	0.9998	2.198	45.505
100	94.81	0.2	80	9000	96.718	277.232	0.3489	0.3488	0.9998	2.198	45.505
100	94.81	0.2	80	10000	96.718	277.232	0.3489	0.3488	0.9998	2.198	45.505

Table A-23 Analysis results for H_0=100 m, Q=0.5 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
100	95.75	0.5	80	10	134.499	305.937	0.4396	0.3206	0.7293	2.498	40.026
100	95.75	0.5	80	20	121.906	305.937	0.3985	0.3206	0.8046	2.498	40.026
100	95.75	0.5	80	50	111.791	305.937	0.3654	0.3206	0.8774	2.498	40.026
100	95.75	0.5	80	100	107.136	305.937	0.3502	0.3206	0.9155	2.498	40.026
100	95.75	0.5	80	200	104.100	305.937	0.3403	0.3206	0.9422	2.498	40.026
100	95.75	0.5	80	500	101.749	305.937	0.3326	0.3206	0.9640	2.498	40.026
100	95.75	0.5	80	1000	100.525	305.937	0.3286	0.3206	0.9757	2.498	40.026
100	95.75	0.5	80	2000	99.085	305.937	0.3239	0.3206	0.9899	2.498	40.026
100	95.75	0.5	80	3000	98.477	305.937	0.3219	0.3206	0.9960	2.498	40.026
100	95.75	0.5	80	4000	98.148	305.937	0.3208	0.3206	0.9993	2.498	40.026
100	95.75	0.5	80	5000	98.096	305.937	0.3206	0.3206	0.9999	2.498	40.026
100	95.75	0.5	80	6000	98.095	305.937	0.3206	0.3206	0.9999	2.498	40.026
100	95.75	0.5	80	7000	98.094	305.937	0.3206	0.3206	0.9999	2.498	40.026
100	95.75	0.5	80	8000	98.094	305.937	0.3206	0.3206	0.9999	2.498	40.026
100	95.75	0.5	80	9000	98.093	305.937	0.3206	0.3206	0.9999	2.498	40.026
100	95.75	0.5	80	10000	98.093	305.937	0.3206	0.3206	0.9999	2.498	40.026

Table A-24 Analysis results for H_0=100 m, Q=1 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
100	96.58	1	80	10	158.028	322.833	0.4895	0.3078	0.6288	2.753	36.324
100	96.58	1	80	20	136.732	322.833	0.4235	0.3078	0.7267	2.753	36.324
100	96.58	1	80	50	120.247	322.833	0.3725	0.3078	0.8264	2.753	36.324
100	96.58	1	80	100	112.769	322.833	0.3493	0.3078	0.8812	2.753	36.324
100	96.58	1	80	200	107.855	322.833	0.3341	0.3078	0.9213	2.753	36.324
100	96.58	1	80	500	103.850	322.833	0.3217	0.3078	0.9568	2.753	36.324
100	96.58	1	80	1000	102.052	322.833	0.3161	0.3078	0.9737	2.753	36.324
100	96.58	1	80	2000	100.897	322.833	0.3125	0.3078	0.9848	2.753	36.324
100	96.58	1	80	3000	100.156	322.833	0.3102	0.3078	0.9921	2.753	36.324
100	96.58	1	80	4000	99.618	322.833	0.3086	0.3078	0.9975	2.753	36.324
100	96.58	1	80	5000	99.399	322.833	0.3079	0.3078	0.9997	2.753	36.324
100	96.58	1	80	6000	99.397	322.833	0.3079	0.3078	0.9997	2.753	36.324
100	96.58	1	80	7000	99.395	322.833	0.3079	0.3078	0.9997	2.753	36.324
100	96.58	1	80	8000	99.395	322.833	0.3079	0.3078	0.9997	2.753	36.324
100	96.58	1	80	9000	99.393	322.833	0.3079	0.3078	0.9997	2.753	36.324
100	96.58	1	80	10000	99.393	322.833	0.3079	0.3078	0.9997	2.753	36.324

Table A-25 Analysis results for H_0=100 m, Q=2 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
100	97.16	2	80	10	198.996	343.627	0.5791	0.2925	0.5051	3.034	32.965
100	97.16	2	80	20	161.933	343.627	0.4712	0.2925	0.6207	3.034	32.965
100	97.16	2	80	50	133.830	343.627	0.3895	0.2925	0.7510	3.034	32.965
100	97.16	2	80	100	121.597	343.627	0.3539	0.2925	0.8266	3.034	32.965
100	97.16	2	80	200	113.692	343.627	0.3309	0.2925	0.8841	3.034	32.965
100	97.16	2	80	500	107.254	343.627	0.3121	0.2925	0.9371	3.034	32.965
100	97.16	2	80	1000	104.266	343.627	0.3034	0.2925	0.9640	3.034	32.965
100	97.16	2	80	2000	102.233	343.627	0.2975	0.2925	0.9832	3.034	32.965
100	97.16	2	80	3000	101.561	343.627	0.2956	0.2925	0.9897	3.034	32.965
100	97.16	2	80	4000	101.085	343.627	0.2942	0.2925	0.9943	3.034	32.965
100	97.16	2	80	5000	100.701	343.627	0.2931	0.2925	0.9981	3.034	32.965
100	97.16	2	80	6000	100.538	343.627	0.2926	0.2925	0.9997	3.034	32.965
100	97.16	2	80	7000	100.535	343.627	0.2926	0.2925	0.9998	3.034	32.965
100	97.16	2	80	8000	100.533	343.627	0.2926	0.2925	0.9998	3.034	32.965
100	97.16	2	80	9000	100.531	343.627	0.2926	0.2925	0.9998	3.034	32.965
100	97.16	2	80	10000	100.530	343.627	0.2926	0.2925	0.9998	3.034	32.965

Table A-26 Analysis results for H_0=100 m, Q=5 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
100	97.65	5	80	10	299.456	380.555	0.7869	0.2678	0.3403	3.449	28.996
100	97.65	5	80	20	229.278	380.555	0.6025	0.2678	0.4445	3.449	28.996
100	97.65	5	80	50	168.511	380.555	0.4428	0.2678	0.6048	3.449	28.996
100	97.65	5	80	100	143.016	380.555	0.3758	0.2678	0.7126	3.449	28.996
100	97.65	5	80	200	127.304	380.555	0.3345	0.2678	0.8005	3.449	28.996
100	97.65	5	80	500	115.007	380.555	0.3022	0.2678	0.8861	3.449	28.996
100	97.65	5	80	1000	109.370	380.555	0.2874	0.2678	0.9318	3.449	28.996
100	97.65	5	80	2000	105.709	380.555	0.2778	0.2678	0.9641	3.449	28.996
100	97.65	5	80	3000	104.149	380.555	0.2737	0.2678	0.9785	3.449	28.996
100	97.65	5	80	4000	103.179	380.555	0.2711	0.2678	0.9877	3.449	28.996
100	97.65	5	80	5000	102.581	380.555	0.2696	0.2678	0.9935	3.449	28.996
100	97.65	5	80	6000	102.147	380.555	0.2684	0.2678	0.9977	3.449	28.996
100	97.65	5	80	7000	101.947	380.555	0.2679	0.2678	0.9997	3.449	28.996
100	97.65	5	80	8000	101.942	380.555	0.2679	0.2678	0.9997	3.449	28.996
100	97.65	5	80	9000	101.937	380.555	0.2679	0.2678	0.9998	3.449	28.996
100	97.65	5	80	10000	101.935	380.555	0.2679	0.2678	0.9998	3.449	28.996

Table A-27 Analysis results for H_0=100 m, Q=10 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
100	98.00	10	80	10	385.732	409.649	0.9416	0.2521	0.2677	3.800	26.315
100	98.00	10	80	20	319.482	409.649	0.7799	0.2521	0.3232	3.800	26.315
100	98.00	10	80	50	219.579	409.649	0.5360	0.2521	0.4703	3.800	26.315
100	98.00	10	80	100	173.046	409.649	0.4224	0.2521	0.5968	3.800	26.315
100	98.00	10	80	200	145.520	409.649	0.3552	0.2521	0.7097	3.800	26.315
100	98.00	10	80	500	124.786	409.649	0.3046	0.2521	0.8276	3.800	26.315
100	98.00	10	80	1000	115.587	409.649	0.2822	0.2521	0.8935	3.800	26.315
100	98.00	10	80	2000	109.777	409.649	0.2680	0.2521	0.9407	3.800	26.315
100	98.00	10	80	3000	107.185	409.649	0.2617	0.2521	0.9635	3.800	26.315
100	98.00	10	80	4000	105.830	409.649	0.2583	0.2521	0.9758	3.800	26.315
100	98.00	10	80	5000	104.712	409.649	0.2556	0.2521	0.9863	3.800	26.315
100	98.00	10	80	6000	103.991	409.649	0.2539	0.2521	0.9931	3.800	26.315
100	98.00	10	80	7000	103.329	409.649	0.2522	0.2521	0.9995	3.800	26.315
100	98.00	10	80	8000	103.317	409.649	0.2522	0.2521	0.9996	3.800	26.315
100	98.00	10	80	9000	103.313	409.649	0.2522	0.2521	0.9996	3.800	26.315
100	98.00	10	80	10000	103.303	409.649	0.2522	0.2521	0.9997	3.800	26.315

Table A-28 Analysis results for H_0=100 m, Q=20 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
100	98.32	20	80	10	435.045	438.911	0.9912	0.2386	0.2407	4.187	23.881
100	98.32	20	80	20	413.191	438.911	0.9414	0.2386	0.2535	4.187	23.881
100	98.32	20	80	50	307.724	438.911	0.7011	0.2386	0.3403	4.187	23.881
100	98.32	20	80	100	227.627	438.911	0.5186	0.2386	0.4601	4.187	23.881
100	98.32	20	80	200	177.390	438.911	0.4042	0.2386	0.5904	4.187	23.881
100	98.32	20	80	500	141.184	438.911	0.3217	0.2386	0.7418	4.187	23.881
100	98.32	20	80	1000	125.132	438.911	0.2851	0.2386	0.8369	4.187	23.881
100	98.32	20	80	2000	115.772	438.911	0.2638	0.2386	0.9046	4.187	23.881
100	98.32	20	80	3000	113.016	438.911	0.2575	0.2386	0.9266	4.187	23.881
100	98.32	20	80	4000	108.562	438.911	0.2473	0.2386	0.9646	4.187	23.881
100	98.32	20	80	5000	106.529	438.911	0.2427	0.2386	0.9831	4.187	23.881
100	98.32	20	80	6000	106.526	438.911	0.2427	0.2386	0.9831	4.187	23.881
100	98.32	20	80	7000	105.054	438.911	0.2394	0.2386	0.9969	4.187	23.881
100	98.32	20	80	8000	105.007	438.911	0.2392	0.2386	0.9973	4.187	23.881
100	98.32	20	80	9000	104.940	438.911	0.2391	0.2386	0.9979	4.187	23.881
100	98.32	20	80	10000	104.727	438.911	0.2386	0.2386	1.0000	4.187	23.881

Table A-29 Analysis results for H_0=200 m, Q=0.2 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
200	175.28	0.2	160	10	232.273	450.913	0.5151	0.3968	0.7703	2.961	67.553
200	175.28	0.2	160	20	218.028	450.913	0.4835	0.3968	0.8206	2.961	67.553
200	175.28	0.2	160	50	207.624	450.913	0.4605	0.3968	0.8618	2.961	67.553
200	175.28	0.2	160	100	203.804	450.913	0.4520	0.3968	0.8779	2.961	67.553
200	175.28	0.2	160	200	196.110	450.913	0.4349	0.3968	0.9124	2.961	67.553
200	175.28	0.2	160	500	186.095	450.913	0.4127	0.3968	0.9615	2.961	67.553
200	175.28	0.2	160	1000	182.174	450.913	0.4040	0.3968	0.9822	2.961	67.553
200	175.28	0.2	160	2000	180.117	450.913	0.3994	0.3968	0.9934	2.961	67.553
200	175.28	0.2	160	3000	179.420	450.913	0.3979	0.3968	0.9972	2.961	67.553
200	175.28	0.2	160	4000	179.050	450.913	0.3971	0.3968	0.9993	2.961	67.553
200	175.28	0.2	160	5000	178.940	450.913	0.3968	0.3968	0.9999	2.961	67.553
200	175.28	0.2	160	6000	178.940	450.913	0.3968	0.3968	0.9999	2.961	67.553
200	175.28	0.2	160	7000	178.939	450.913	0.3968	0.3968	0.9999	2.961	67.553
200	175.28	0.2	160	8000	178.939	450.913	0.3968	0.3968	0.9999	2.961	67.553
200	175.28	0.2	160	9000	178.939	450.913	0.3968	0.3968	0.9999	2.961	67.553
200	175.28	0.2	160	10000	178.938	450.913	0.3968	0.3968	0.9999	2.961	67.553

Table A-30 Analysis results for H_0=200 m, Q=0.5 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
200	181.21	0.5	160	10	274.061	479.695	0.5713	0.3862	0.6760	3.366	59.420
200	181.21	0.5	160	20	245.066	479.695	0.5109	0.3862	0.7560	3.366	59.420
200	181.21	0.5	160	50	222.372	479.695	0.4636	0.3862	0.8331	3.366	59.420
200	181.21	0.5	160	100	212.398	479.695	0.4428	0.3862	0.8722	3.366	59.420
200	181.21	0.5	160	200	206.453	479.695	0.4304	0.3862	0.8973	3.366	59.420
200	181.21	0.5	160	500	200.342	479.695	0.4176	0.3862	0.9247	3.366	59.420
200	181.21	0.5	160	1000	193.467	479.695	0.4033	0.3862	0.9576	3.366	59.420
200	181.21	0.5	160	2000	189.025	479.695	0.3941	0.3862	0.9801	3.366	59.420
200	181.21	0.5	160	3000	187.365	479.695	0.3906	0.3862	0.9888	3.366	59.420
200	181.21	0.5	160	4000	186.526	479.695	0.3888	0.3862	0.9932	3.366	59.420
200	181.21	0.5	160	5000	186.032	479.695	0.3878	0.3862	0.9958	3.366	59.420
200	181.21	0.5	160	6000	185.606	479.695	0.3869	0.3862	0.9981	3.366	59.420
200	181.21	0.5	160	7000	185.358	479.695	0.3864	0.3862	0.9995	3.366	59.420
200	181.21	0.5	160	8000	185.262	479.695	0.3862	0.3862	1.0000	3.366	59.420
200	181.21	0.5	160	9000	185.262	479.695	0.3862	0.3862	1.0000	3.366	59.420
200	181.21	0.5	160	10000	185.261	479.695	0.3862	0.3862	1.0000	3.366	59.420

Table A-31 Analysis results for H_0=200 m, Q=1 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
200	185.17	1	160	10	330.789	501.814	0.6592	0.3791	0.5751	3.709	53.925
200	185.17	1	160	20	281.836	501.814	0.5616	0.3791	0.6750	3.709	53.925
200	185.17	1	160	50	243.165	501.814	0.4846	0.3791	0.7823	3.709	53.925
200	185.17	1	160	100	225.931	501.814	0.4502	0.3791	0.8420	3.709	53.925
200	185.17	1	160	200	214.930	501.814	0.4283	0.3791	0.8851	3.709	53.925
200	185.17	1	160	500	206.508	501.814	0.4115	0.3791	0.9212	3.709	53.925
200	185.17	1	160	1000	202.684	501.814	0.4039	0.3791	0.9386	3.709	53.925
200	185.17	1	160	2000	197.043	501.814	0.3927	0.3791	0.9655	3.709	53.925
200	185.17	1	160	3000	194.275	501.814	0.3871	0.3791	0.9792	3.709	53.925
200	185.17	1	160	4000	192.920	501.814	0.3844	0.3791	0.9861	3.709	53.925
200	185.17	1	160	5000	191.840	501.814	0.3823	0.3791	0.9916	3.709	53.925
200	185.17	1	160	6000	191.296	501.814	0.3812	0.3791	0.9945	3.709	53.925
200	185.17	1	160	7000	190.833	501.814	0.3803	0.3791	0.9969	3.709	53.925
200	185.17	1	160	8000	190.562	501.814	0.3797	0.3791	0.9983	3.709	53.925
200	185.17	1	160	9000	190.345	501.814	0.3793	0.3791	0.9994	3.709	53.925
200	185.17	1	160	10000	190.246	501.814	0.3791	0.3791	1.0000	3.709	53.925

Table A-32 Analysis results for H_0=200 m, Q=2 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
200	187.14	2	160	10	422.035	535.758	0.7877	0.3611	0.4584	4.087	48.938
200	187.14	2	160	20	343.095	535.758	0.6404	0.3611	0.5639	4.087	48.938
200	187.14	2	160	50	276.408	535.758	0.5159	0.3611	0.6999	4.087	48.938
200	187.14	2	160	100	247.285	535.758	0.4616	0.3611	0.7823	4.087	48.938
200	187.14	2	160	200	228.741	535.758	0.4269	0.3611	0.8458	4.087	48.938
200	187.14	2	160	500	214.024	535.758	0.3995	0.3611	0.9039	4.087	48.938
200	187.14	2	160	1000	207.562	535.758	0.3874	0.3611	0.9321	4.087	48.938
200	187.14	2	160	2000	203.588	535.758	0.3800	0.3611	0.9503	4.087	48.938
200	187.14	2	160	3000	200.238	535.758	0.3737	0.3611	0.9662	4.087	48.938
200	187.14	2	160	4000	198.293	535.758	0.3701	0.3611	0.9756	4.087	48.938
200	187.14	2	160	5000	196.557	535.758	0.3669	0.3611	0.9843	4.087	48.938
200	187.14	2	160	6000	195.666	535.758	0.3652	0.3611	0.9887	4.087	48.938
200	187.14	2	160	7000	195.086	535.758	0.3641	0.3611	0.9917	4.087	48.938
200	187.14	2	160	8000	194.639	535.758	0.3633	0.3611	0.9940	4.087	48.938
200	187.14	2	160	9000	194.212	535.758	0.3625	0.3611	0.9961	4.087	48.938
200	187.14	2	160	10000	193.694	535.758	0.3615	0.3611	0.9988	4.087	48.938

Table A-33 Analysis results for H_0=200 m, Q=5 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
200	189.91	5	160	10	560.229	576.907	0.9711	0.3432	0.3534	4.646	43.046
200	189.91	5	160	20	480.244	576.907	0.8324	0.3432	0.4123	4.646	43.046
200	189.91	5	160	50	357.053	576.907	0.6189	0.3432	0.5545	4.646	43.046
200	189.91	5	160	100	297.612	576.907	0.5159	0.3432	0.6653	4.646	43.046
200	189.91	5	160	200	260.578	576.907	0.4517	0.3432	0.7598	4.646	43.046
200	189.91	5	160	500	232.163	576.907	0.4024	0.3432	0.8528	4.646	43.046
200	189.91	5	160	1000	219.182	576.907	0.3799	0.3432	0.9033	4.646	43.046
200	189.91	5	160	2000	210.766	576.907	0.3653	0.3432	0.9394	4.646	43.046
200	189.91	5	160	3000	207.417	576.907	0.3595	0.3432	0.9546	4.646	43.046
200	189.91	5	160	4000	205.564	576.907	0.3563	0.3432	0.9632	4.646	43.046
200	189.91	5	160	5000	203.598	576.907	0.3529	0.3432	0.9725	4.646	43.046
200	189.91	5	160	6000	202.621	576.907	0.3512	0.3432	0.9772	4.646	43.046
200	189.91	5	160	7000	200.654	576.907	0.3478	0.3432	0.9867	4.646	43.046
200	189.91	5	160	8000	199.728	576.907	0.3462	0.3432	0.9913	4.646	43.046
200	189.91	5	160	9000	199.479	576.907	0.3458	0.3432	0.9926	4.646	43.046
200	189.91	5	160	10000	198.128	576.907	0.3434	0.3432	0.9993	4.646	43.046

Table A-34 Analysis results for H_0=200 m, Q=10 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
200	191.41	10	160	10	615.067	614.259	1.0013	0.3270	0.3266	5.120	39.065
200	191.41	10	160	20	596.441	614.259	0.9710	0.3270	0.3368	5.120	39.065
200	191.41	10	160	50	466.233	614.259	0.7590	0.3270	0.4308	5.120	39.065
200	191.41	10	160	100	369.303	614.259	0.6012	0.3270	0.5439	5.120	39.065
200	191.41	10	160	200	304.987	614.259	0.4965	0.3270	0.6586	5.120	39.065
200	191.41	10	160	500	255.646	614.259	0.4162	0.3270	0.7857	5.120	39.065
200	191.41	10	160	1000	234.093	614.259	0.3811	0.3270	0.8580	5.120	39.065
200	191.41	10	160	2000	219.090	614.259	0.3567	0.3270	0.9168	5.120	39.065
200	191.41	10	160	3000	212.644	614.259	0.3462	0.3270	0.9446	5.120	39.065
200	191.41	10	160	4000	209.113	614.259	0.3404	0.3270	0.9605	5.120	39.065
200	191.41	10	160	5000	207.206	614.259	0.3373	0.3270	0.9694	5.120	39.065
200	191.41	10	160	6000	205.477	614.259	0.3345	0.3270	0.9775	5.120	39.065
200	191.41	10	160	7000	203.111	614.259	0.3307	0.3270	0.9889	5.120	39.065
200	191.41	10	160	8000	200.939	614.259	0.3271	0.3270	0.9996	5.120	39.065
200	191.41	10	160	9000	200.921	614.259	0.3271	0.3270	0.9997	5.120	39.065
200	191.41	10	160	10000	200.907	614.259	0.3271	0.3270	0.9998	5.120	39.065

Table A-35 Analysis results for H_0=200 m, Q=20 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
200	192.58	20	160	10	697.126	659.047	1.0578	0.3095	0.2926	5.641	35.452
200	192.58	20	160	20	662.189	659.047	1.0048	0.3095	0.3080	5.641	35.452
200	192.58	20	160	50	619.609	659.047	0.9402	0.3095	0.3292	5.641	35.452
200	192.58	20	160	100	489.594	659.047	0.7429	0.3095	0.4166	5.641	35.452
200	192.58	20	160	200	384.630	659.047	0.5836	0.3095	0.5303	5.641	35.452
200	192.58	20	160	500	294.325	659.047	0.4466	0.3095	0.6930	5.641	35.452
200	192.58	20	160	1000	254.856	659.047	0.3867	0.3095	0.8004	5.641	35.452
200	192.58	20	160	2000	227.495	659.047	0.3452	0.3095	0.8966	5.641	35.452
200	192.58	20	160	3000	212.802	659.047	0.3229	0.3095	0.9585	5.641	35.452
200	192.58	20	160	4000	208.670	659.047	0.3166	0.3095	0.9775	5.641	35.452
200	192.58	20	160	5000	205.273	659.047	0.3115	0.3095	0.9937	5.641	35.452
200	192.58	20	160	6000	206.726	659.047	0.3137	0.3095	0.9867	5.641	35.452
200	192.58	20	160	7000	204.425	659.047	0.3102	0.3095	0.9978	5.641	35.452
200	192.58	20	160	8000	205.041	659.047	0.3111	0.3095	0.9948	5.641	35.452
200	192.58	20	160	9000	204.044	659.047	0.3096	0.3095	0.9997	5.641	35.452
200	192.58	20	160	10000	204.021	659.047	0.3096	0.3095	0.9998	5.641	35.452

Table A-36 Analysis results for H_0=500 m, Q=0.2 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
500	350.76	0.2	400	10	549.321	842.150	0.6523	0.4249	0.6514	4.390	113.885
500	350.76	0.2	400	20	525.121	842.150	0.6235	0.4249	0.6814	4.390	113.885
500	350.76	0.2	400	50	492.849	842.150	0.5852	0.4249	0.7260	4.390	113.885
500	350.76	0.2	400	100	430.091	842.150	0.5107	0.4249	0.8320	4.390	113.885
500	350.76	0.2	400	200	393.046	842.150	0.4667	0.4249	0.9104	4.390	113.885
500	350.76	0.2	400	500	370.976	842.150	0.4405	0.4249	0.9646	4.390	113.885
500	350.76	0.2	400	1000	363.645	842.150	0.4318	0.4249	0.9840	4.390	113.885
500	350.76	0.2	400	2000	359.986	842.150	0.4275	0.4249	0.9940	4.390	113.885
500	350.76	0.2	400	3000	358.676	842.150	0.4259	0.4249	0.9976	4.390	113.885
500	350.76	0.2	400	4000	358.093	842.150	0.4252	0.4249	0.9993	4.390	113.885
500	350.76	0.2	400	5000	357.875	842.150	0.4250	0.4249	0.9999	4.390	113.885
500	350.76	0.2	400	6000	357.874	842.150	0.4250	0.4249	0.9999	4.390	113.885
500	350.76	0.2	400	7000	357.873	842.150	0.4250	0.4249	0.9999	4.390	113.885
500	350.76	0.2	400	8000	357.872	842.150	0.4250	0.4249	0.9999	4.390	113.885
500	350.76	0.2	400	9000	357.872	842.150	0.4250	0.4249	0.9999	4.390	113.885
500	350.76	0.2	400	10000	357.871	842.150	0.4249	0.4249	0.9999	4.390	113.885

Table A-37 Analysis results for H_0=500 m, Q=0.5 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
500	385.61	0.5	400	10	652.681	889.822	0.7335	0.4426	0.6034	4.991	100.174
500	385.61	0.5	400	20	586.398	889.822	0.6590	0.4426	0.6716	4.991	100.174
500	385.61	0.5	400	50	536.863	889.822	0.6033	0.4426	0.7336	4.991	100.174
500	385.61	0.5	400	100	518.575	889.822	0.5828	0.4426	0.7595	4.991	100.174
500	385.61	0.5	400	200	488.413	889.822	0.5489	0.4426	0.8064	4.991	100.174
500	385.61	0.5	400	500	434.008	889.822	0.4877	0.4426	0.9074	4.991	100.174
500	385.61	0.5	400	1000	412.506	889.822	0.4636	0.4426	0.9547	4.991	100.174
500	385.61	0.5	400	2000	401.742	889.822	0.4515	0.4426	0.9803	4.991	100.174
500	385.61	0.5	400	3000	396.865	889.822	0.4460	0.4426	0.9924	4.991	100.174
500	385.61	0.5	400	4000	395.430	889.822	0.4444	0.4426	0.9960	4.991	100.174
500	385.61	0.5	400	5000	394.730	889.822	0.4436	0.4426	0.9977	4.991	100.174
500	385.61	0.5	400	6000	394.278	889.822	0.4431	0.4426	0.9989	4.991	100.174
500	385.61	0.5	400	7000	394.077	889.822	0.4429	0.4426	0.9994	4.991	100.174
500	385.61	0.5	400	8000	393.917	889.822	0.4427	0.4426	0.9998	4.991	100.174
500	385.61	0.5	400	9000	393.916	889.822	0.4427	0.4426	0.9998	4.991	100.174
500	385.61	0.5	400	10000	393.914	889.822	0.4427	0.4426	0.9998	4.991	100.174

Table A-38 Analysis results for H_0=500 m, Q=1 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
500	397.55	1	400	10	785.615	942.078	0.8339	0.4338	0.5202	5.500	90.910
500	397.55	1	400	20	679.422	942.078	0.7212	0.4338	0.6015	5.500	90.910
500	397.55	1	400	50	585.079	942.078	0.6211	0.4338	0.6985	5.500	90.910
500	397.55	1	400	100	545.110	942.078	0.5786	0.4338	0.7497	5.500	90.910
500	397.55	1	400	200	522.849	942.078	0.5550	0.4338	0.7816	5.500	90.910
500	397.55	1	400	500	485.140	942.078	0.5150	0.4338	0.8424	5.500	90.910
500	397.55	1	400	1000	447.292	942.078	0.4748	0.4338	0.9137	5.500	90.910
500	397.55	1	400	2000	425.409	942.078	0.4516	0.4338	0.9607	5.500	90.910
500	397.55	1	400	3000	416.625	942.078	0.4422	0.4338	0.9809	5.500	90.910
500	397.55	1	400	4000	412.741	942.078	0.4381	0.4338	0.9901	5.500	90.910
500	397.55	1	400	5000	411.104	942.078	0.4364	0.4338	0.9941	5.500	90.910
500	397.55	1	400	6000	410.066	942.078	0.4353	0.4338	0.9966	5.500	90.910
500	397.55	1	400	7000	409.613	942.078	0.4348	0.4338	0.9977	5.500	90.910
500	397.55	1	400	8000	409.194	942.078	0.4344	0.4338	0.9987	5.500	90.910
500	397.55	1	400	9000	408.929	942.078	0.4341	0.4338	0.9994	5.500	90.910
500	397.55	1	400	10000	408.764	942.078	0.4339	0.4338	0.9998	5.500	90.910

Table A-39 Analysis results for H_0=500 m, Q=2 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
500	415.47	2	400	10	941.678	984.480	0.9565	0.4366	0.4564	6.060	82.503
500	415.47	2	400	20	829.041	984.480	0.8421	0.4366	0.5185	6.060	82.503
500	415.47	2	400	50	678.169	984.480	0.6889	0.4366	0.6338	6.060	82.503
500	415.47	2	400	100	603.120	984.480	0.6126	0.4366	0.7127	6.060	82.503
500	415.47	2	400	200	556.596	984.480	0.5654	0.4366	0.7722	6.060	82.503
500	415.47	2	400	500	523.538	984.480	0.5318	0.4366	0.8210	6.060	82.503
500	415.47	2	400	1000	501.014	984.480	0.5089	0.4366	0.8579	6.060	82.503
500	415.47	2	400	2000	464.259	984.480	0.4716	0.4366	0.9258	6.060	82.503
500	415.47	2	400	3000	449.030	984.480	0.4561	0.4366	0.9572	6.060	82.503
500	415.47	2	400	4000	441.299	984.480	0.4483	0.4366	0.9740	6.060	82.503
500	415.47	2	400	5000	436.641	984.480	0.4435	0.4366	0.9844	6.060	82.503
500	415.47	2	400	6000	433.927	984.480	0.4408	0.4366	0.9905	6.060	82.503
500	415.47	2	400	7000	432.319	984.480	0.4391	0.4366	0.9942	6.060	82.503
500	415.47	2	400	8000	430.794	984.480	0.4376	0.4366	0.9977	6.060	82.503
500	415.47	2	400	9000	430.336	984.480	0.4371	0.4366	0.9988	6.060	82.503
500	415.47	2	400	10000	429.954	984.480	0.4367	0.4366	0.9997	6.060	82.503

Table A-40 Analysis results for H_0=500 m, Q=5 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}(m^3)$	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
500	430.41	5	400	10	1,048.08	1,058	0.9909	0.4247	0.4286	6.89	72.57
500	430.41	5	400	20	1,032.97	1,058	0.9767	0.4247	0.4348	6.89	72.57
500	430.41	5	400	50	888.29	1,058	0.8399	0.4247	0.5057	6.89	72.57
500	430.41	5	400	100	745.76	1,058	0.7051	0.4247	0.6023	6.89	72.57
500	430.41	5	400	200	648.45	1,058	0.6131	0.4247	0.6927	6.89	72.57
500	430.41	5	400	500	569.43	1,058	0.5384	0.4247	0.7888	6.89	72.57
500	430.41	5	400	1000	535.01	1,058	0.5058	0.4247	0.8396	6.89	72.57
500	430.41	5	400	2000	503.95	1,058	0.4765	0.4247	0.8913	6.89	72.57
500	430.41	5	400	3000	463.44	1,058	0.4382	0.4247	0.9692	6.89	72.57
500	430.41	5	400	4000	456.24	1,058	0.4314	0.4247	0.9845	6.89	72.57
500	430.41	5	400	5000	453.04	1,058	0.4283	0.4247	0.9915	6.89	72.57
500	430.41	5	400	6000	452.79	1,058	0.4281	0.4247	0.9920	6.89	72.57
500	430.41	5	400	7000	450.27	1,058	0.4257	0.4247	0.9976	6.89	72.57
500	430.41	5	400	8000	449.82	1,058	0.4253	0.4247	0.9986	6.89	72.57
500	430.41	5	400	9000	449.66	1,058	0.4251	0.4247	0.9989	6.89	72.57
500	430.41	5	400	10000	449.42	1,058	0.4249	0.4247	0.9995	6.89	72.57

Table A-41 Analysis results for H_0=500 m, Q=10 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
500	442.36	10	400	10	1,162.67	1,109	1.0482	0.4164	0.3973	7.59	65.86
500	442.36	10	400	20	1,101.72	1,109	0.9932	0.4164	0.4192	7.59	65.86
500	442.36	10	400	50	1,072.44	1,109	0.9668	0.4164	0.4307	7.59	65.86
500	442.36	10	400	100	929.90	1,109	0.8383	0.4164	0.4967	7.59	65.86
500	442.36	10	400	200	769.46	1,109	0.6937	0.4164	0.6003	7.59	65.86
500	442.36	10	400	500	626.13	1,109	0.5645	0.4164	0.7377	7.59	65.86
500	442.36	10	400	1000	560.87	1,109	0.5056	0.4164	0.8235	7.59	65.86
500	442.36	10	400	2000	510.78	1,109	0.4605	0.4164	0.9043	7.59	65.86
500	442.36	10	400	3000	479.21	1,109	0.4320	0.4164	0.9639	7.59	65.86
500	442.36	10	400	4000	468.96	1,109	0.4228	0.4164	0.9849	7.59	65.86
500	442.36	10	400	5000	464.62	1,109	0.4189	0.4164	0.9941	7.59	65.86
500	442.36	10	400	6000	464.86	1,109	0.4191	0.4164	0.9936	7.59	65.86
500	442.36	10	400	7000	462.23	1,109	0.4167	0.4164	0.9993	7.59	65.86
500	442.36	10	400	8000	464.65	1,109	0.4189	0.4164	0.9941	7.59	65.86
500	442.36	10	400	9000	465.13	1,109	0.4193	0.4164	0.9930	7.59	65.86
500	442.36	10	400	10000	462.14	1,109	0.4166	0.4164	0.9995	7.59	65.86

Table A-42 Analysis results for H_0=500 m, Q=20 m^3/s

H_0(m)	H_{ch}(m)	Q(m^3/s)	P_{ini}/γ(m)	$\forall_{ch}$(m^3)	H_a(m)	H(m)	H_r	H_{ra}	H_{ra}/H_r	V_p(m/s)	H_0/V_p
500	449.13	20	400	10	1,336.49	1,181	1.1312	0.4015	0.3549	8.37	59.77
500	449.13	20	400	20	1,209.78	1,181	1.0240	0.4015	0.3921	8.37	59.77
500	449.13	20	400	50	1,172.53	1,181	0.9925	0.4015	0.4045	8.37	59.77
500	449.13	20	400	100	1,140.56	1,181	0.9654	0.4015	0.4159	8.37	59.77
500	449.13	20	400	200	924.87	1,181	0.7828	0.4015	0.5129	8.37	59.77
500	449.13	20	400	500	678.32	1,181	0.5742	0.4015	0.6993	8.37	59.77
500	449.13	20	400	1000	560.35	1,181	0.4743	0.4015	0.8465	8.37	59.77
500	449.13	20	400	2000	491.60	1,181	0.4161	0.4015	0.9649	8.37	59.77
500	449.13	20	400	3000	481.92	1,181	0.4079	0.4015	0.9843	8.37	59.77
500	449.13	20	400	4000	480.44	1,181	0.4067	0.4015	0.9873	8.37	59.77
500	449.13	20	400	5000	478.93	1,181	0.4054	0.4015	0.9904	8.37	59.77
500	449.13	20	400	6000	474.96	1,181	0.4020	0.4015	0.9987	8.37	59.77
500	449.13	20	400	7000	481.07	1,181	0.4072	0.4015	0.9860	8.37	59.77
500	449.13	20	400	8000	481.67	1,181	0.4077	0.4015	0.9848	8.37	59.77
500	449.13	20	400	9000	473.36	1,181	0.4007	0.4015	1.0021	8.37	59.77
500	449.13	20	400	10000	474.45	1,181	0.4016	0.4015	0.9998	8.37	59.77