INTEGRATED RISK MANAGEMENT APPLICATIONS IN OFFSHORE WIND FARM CONSTRUCTION

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ABSTRACT

INTEGRATED RISK MANAGEMENT APPLICATIONS IN OFFSHORE WIND FARM CONSTRUCTION

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This thesis is motivated by a construction company whose main business operation is foundation installations of offshore wind farm projects. Steel is the main input for most of the foundations. This construction company is confronted with price risk when involved in the procurement part of the foundations. The company receives a fixed payment for the project while paying variable raw material costs which depend on the steel spot price on the procurement date. The construction company should use financial markets to eliminate the price risk. However, in most wind farm projects, the steel requirement of a foundation is not known in advance, i.e., there is also a quantity risk. Therefore, it is not possible to completely eliminate the associated risk. In this thesis we analyze the hedging decisions of a value maximizing construction company confronted with both price and quantity risks under the presence of capital market imperfections.

Key words: Futures markets, risk management, construction industry

ÖZ

OFFSHORE RÜZGAR ENERJİSİ İNŞAATINDA ENTEGRE RİSK YÖNETİMİ UYGULAMALARI

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Ortak Tez Yöneticisi: Yrd. Doç. Dr. Fehmi Tanrısever

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Bu tez, temel operasyonu offshore rüzgar çiftliklerinin temel inşaatı olan bir inşaat şirketinden etkilenerek oluşturulmuştur. Çoğu türbin temeli için çelik ana ham maddedir. Bu inşaat şirketi temellerin tedariğinden sorumlu olduğunda çeliğin fiyat riskine maruz kalmaktadır. Şirket, proje için sabit bir ödeme alırken, tedarik zamanındaki çelik fiyatına bağlı olan değişken ham madde maliyeti ödemektedir. İnşaat şirketi, fiyat riskini yok etmek için finansal marketleri kullanmalıdır. Ama, çoğu rüzgar enerjisi projesinde, bir temelde kullanılacak çelik miktarı önceden bilinmemektedir, yani aynı zamanda talep riski vardır. Bu yüzden ilişkili riski tamamen yok etmek mümkün değildir. Bu tezde, biz talep ve fiyat riskiyle karşı karşıya olan, firma değerini maksimize etmeye çalışan bir inşaat şirketinin riskten korunma kararlarını inceliyoruz.

Anahtar kelimeler : Vadeli işlemler piyasası, risk yönetimi, inşaat sektörü

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$\tilde{s}_t$ **:** *Unit price of steel at time t*

P: Unit contract price fixed at time 1. (The unit price that will be paid by the developer to construction company when steel is delivered.)

$(\tilde{s}_t + \lambda)$ *: Unit price paid by construction company to steel manufacturer if steel delivery is at time t*

δ *: Profit margin*

$\tilde{\xi}$ *: Steel quantity requirement*

$F_{t,k}$ *: Futures price of steel long at time t for time k*

y_t *: The cash reserve of the construction company at the beginning of time t*

η_t *: The cash outflow of the construction company at time t*

$x_{t,k}$ *: The amount of steel futures long at time t for time k*

$\phi_F^Q(.)$ *: Risk neutral probability density function of* $F_{2,3}$

$\phi_s^Q(.)$ *: Risk neutral probability density function of* S_3

$\phi_\xi(.)$ *: Probability density function of* ξ

$\Phi_F(.)$ *: Cumulative density function of* $F_{2,3}$

CHAPTER 1

INTRODUCTION AND MOTIVATION

Renewable energy has started to gain more importance in the recent years with increasing concerns about environmental issues and security of energy supply. One of the most popular renewable energy sources is the offshore wind energy. Offshore wind technology started to be used in Europe in 1991 with the first application in Denmark. According to EWEA statistics, the capacity of offshore wind farms in Europe reached to 2946 MW by the end of 2010, which produce 11.5 TWh of electricity. There are 3 main parts of an offshore wind turbine which are foundation, turbine tower and turbine. A figure of a wind turbine is included in Appendix A.

This thesis is motivated by a construction company that is involved in offshore wind energy projects and exposed to fluctuations in steel price. Before starting to focus on the risk exposure of the company, it is beneficial to have a look at all parties involved in offshore wind energy projects to see the big picture. Important parties are wind farm developers, government, construction companies, turbine manufacturers, steel manufacturers and steel mills. These parties and their relations are shown in Figure 1.1.

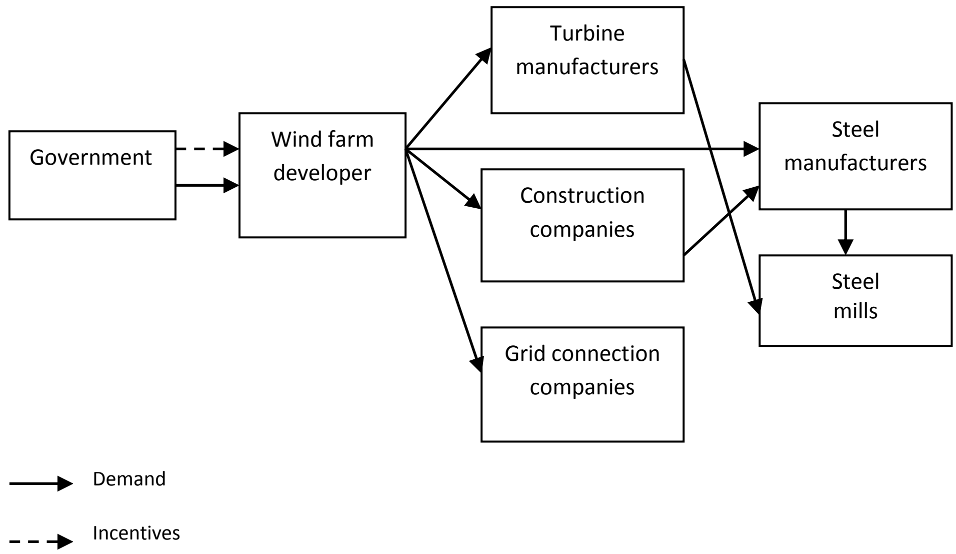

Figure 1.1: Parties involved in offshore wind energy projects

Wind farm developer is the party that arranges the construction of a wind farm on land or offshore site in order to efficiently exploit the wind energy. There are two types of wind farm developers; utility companies and private companies. Utility companies are government related and subject to tender regulations for energy sector. These utility companies should announce in EU newspapers that they will open a tender process for the specified operations and call the interested companies to perform those operations. They should select their subcontractors according to the already defined rules. Utility companies operate the wind farms after the construction is completed. However, private companies are not subject to tender regulations and they can select their subcontractors freely. Private companies may select to operate the wind farms or may sell the constructed wind farm via Power Purchasing Agreement (PPA).

Government is the party which initiates the offshore wind energy demand. Governments try to promote the renewable energy projects via incentives, since the cost of constructing an offshore wind farm is higher than constructing a conventional power plant/electricity price. A cost comparison of conventional power plant and wind technology is included in Appendix B. The government wants to be sure that the wind farm will be operational in the targeted period in the cheapest way. Therefore, the wind farm developer that is economically and technically eligible and demanding for the least incentive obtains the construction right of the wind farm. Wind farm developer should get the required permits from energy agencies and ministries to be able to get the construction right of the wind farm. Turbine manufacturers are the companies that manufacture the turbines and turbine towers. Grid connection companies are responsible for connecting offshore wind farms to grids by laying cables under water. Steel mills manufacture steel plates and steel manufacturers use those steel plates to manufacture the foundations.

Construction companies are mainly involved in the design and installation of foundations. However, these companies may be involved in the procurement and manufacturing of the foundation in some projects as well. When a construction company is responsible from the procurement, the construction company becomes the party that contacts with the steel manufacturer. Construction company bargains the price with the steel manufacturer and undertakes some of the risks related to procurement. A contract that involves Engineering (design of foundations), Procurement (procurement and manufacturing of foundations), and Construction (installation of foundations) is called as an EPC contract.

A wind farm developer may prefer an EPC contract or may demand separate project tasks from different subcontractors (multi-contractual case). For example, a developer may agree with a construction company to make the design of foundations, may agree with another construction company to make the installations, and may directly agree with steel manufacturer for the procurement in multi-contractual case. “Indeed, to reduce the costs of the construction process, the interfaces between the various projects steps, from manufacturing to commissioning, should be kept as smooth as possible. Each interface within this chain of project steps is associated with a number of expenses, e.g, documentation hand-off, inspections, insurance, damage assessment and clarification etc. “ POWR (2010).

According to the European offshore wind industry key trends and statistics 2010 report, 65% of offshore wind farm’s foundations are monopiles, 23% are gravity foundations and 8 % are jacket type. A figure of these foundation types is included in Figure 1.2. From these foundation types; only the gravity based structures are mainly made up of concrete, steel is also started to be used for gravity based foundations. Therefore, steel is the main input for most foundations.

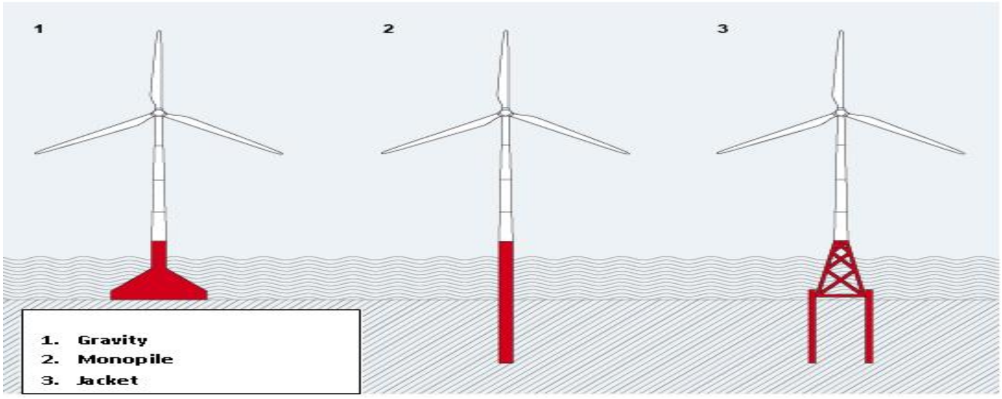

Figure 1.2: Foundation types

In this thesis, we will focus on a construction company that is involved in the procurement of foundations. The parties in our focus system and material-demand flow between them are shown in Figure 1.3.

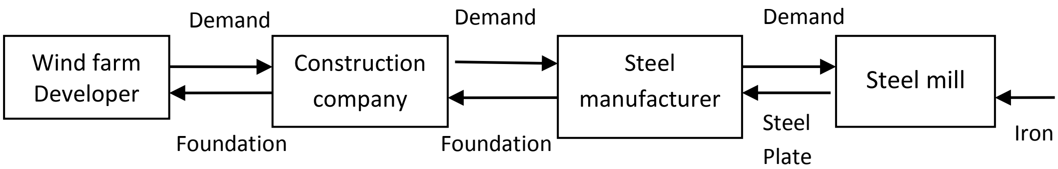

Figure 1.3: Focus system

Steel mill produces steel plates processing iron and steel manufacturer produces foundations based on the foundation design using these steel plates. As it is seen from Figure 1.3, the construction company procures foundations from steel manufacturers and delivers the foundations to the project area/wind farm developer. So, the construction company is an intermediary company between developer and steel manufacturer. When the construction company is responsible from either design or installation and is also involved in procurement, it is easier to coordinate the project schedule and inspect the foundations. Therefore, developers prefer that construction companies handle the procurement. Then, there is no need for the developer to contact with steel manufacturer and to negotiate project costs, risks, project schedule.

The developer demands from the construction company a fixed price contract for the procurement of steel foundations. Developers want to fix the procurement costs for two reasons. The first reason is the easiness in borrowing from banks when costs are certain. The second reason is that the incentive granted by the government to the developer becomes certain before the project costs are realized. Therefore, fluctuations in project costs make the profit of the developer uncertain. When the developer asks for a fixed price contract, the construction company also asks to the subcontractors a fixed price contract not to take any price risk. Steel plates are produced by steel mill and the fluctuations in steel prices are important for the steel mill. Therefore, steel mill is the party that decides to offer a fixed price contract or not. When there is less than six months between contract sign and steel plates delivery from the steel mill, steel mill accepts to fix the price, however, if there is more than six months, steel mill does not want to be exposed to steel price fluctuations and therefore does not accept to give a fixed price. Instead, steel mill prefers to be paid according to the spot steel prices at plates delivery date. Steel manufacturer does not want to take price risk and charges the cost of buying steel plates from steel mill plus a profit margin to the construction company. However, construction company is paid a fixed price by the developer. Therefore, a price risk arises for the construction company. Moreover, in general, final design of the foundations is not completed at the contract sign which means the exact steel requirement for the foundations is not known. Generally, in real life projects steel requirement in final designs can change in $\pm$ 25 % compared to the requirement in draft design. Therefore, construction company is exposed to both price and quantity risks and should consider using the financial markets to decrease the risk exposure.

The history of financial markets that trade steel is not long. Steel contracts started to be traded over-the-counter (OTC) in 2004. Some of the exchange markets started to trade steel futures after 2007. We have found 6 exchange markets that launched steel futures. These exchange markets are trading steel contracts that differ in steel types, delivery dates and quantities. The list of exchange markets and a summary of the traded steel contracts are included in Appendix C.

Steel price is volatile similar to the other metal prices. Therefore, handling fluctuations in steel prices is really important for the parties that will buy or sell steel. The volatility of steel prices is illustrated in Figure 1.4 for three different steel types; hot rolled USA, cold rolled USA and standard plate USA. The figures are obtained with three year steel price data and taken from metal prices.

Figure 1.4: Steel prices

In this research, we analyze the hedging decisions of a construction company which is exposed to price and quantity risks. The construction company is paid a fixed price by the developer, however, is exposed to variable costs. The construction company prefers to use financial markets to decrease the exposure to fluctuations in input prices; however, since quantity is not known at the contract sign, it is not possible to eliminate all the associated risk. Moreover, the construction company will be exposed to financial distress cost if the company does not have enough cash. We use a two-period framework to model the hedging decisions of the company. We consider three real life scenarios with different assumptions about the accumulation of margin calls. Under these scenarios we analyze the optimal hedging decision of the construction company in the futures market.

The rest of the thesis is organized as follows. In Chapter 2, we give a brief introduction to the financial derivatives and their working mechanism. In Chapter 3, the related literature on financial hedging is summarized. In Chapter 4, our mathematical model is introduced and the hedging behavior of the construction company under three business cases is discussed. In Chapter 5, an extension of the mathematical model is included. In Chapter 6, numerical analysis for the optimal hedging behavior of the company is included. Finally, a summary of the thesis is given in Chapter 7.

CHAPTER 2

INTRODUCTION TO FINANCIAL DERIVATIVES

Derivative is a financial instrument whose payoffs depend on or derives from the values of other more basic underlying variables. Underlying variables can be commodities (gold, oil, gas, steel, wheat, etc.), financial assets (stocks, bonds, etc.), another derivative (e.g. options on futures), index, interest rate, temperature at a given day, and wind at a specific location.

Three kinds of agents use financial derivatives with different objectives: hedgers, speculators and arbitrageurs. Hedgers want to reduce the uncertainty about future price movements they will face via using financial instruments. For example, a flour miller who is already committed to sell flour at a fixed price in the future takes a long position in wheat futures. Speculators want to make money by betting on the movements of the market. For example, Fortis Bank takes a long position in oil futures anticipating that oil prices will increase. Arbitrageurs want to make riskless profits by participating in two or more markets and using the price differences in these markets.

Some commonly traded financial derivatives include futures contracts, forward contracts, options and swaps. Explanations for only forward contracts and futures contracts are included since they are similar and futures contracts are used in the hedging decisions of the firm and in our mathematical model.

2.1 Forward contracts

A forward contract is defined as "an agreement to buy or sell an asset at a certain future time for a certain price" (Hull, 2006). Forward contracts are traded on the over- the-counter (OTC) market which is a telephone and computer linked network of dealers. In OTC markets, the contracts are not standardized and the dealers can agree to buy/sell any asset in any quantity/quality, which is a great advantage to the dealers. However, one disadvantage of forward contracts is any party can cancel the contract when they want; default risk. There are two parties involved in a forward contract. The party who agrees to buy the asset takes a long position and the party who agrees to sell the asset takes a short position. When we denote the price of the asset at time t as S_t and the agreed priced at time 0 with a delivery at time t as $F_{0,t}$ (forward price), then the payoff to the party that takes long position is ($S_t - F_{0,t}$) and the payoff to the party that takes short position is ($F_{0,t} - S_t$). This means that if the asset price at the delivery date is higher than forward price, the party that has a long position will have a positive payoff and the party that has a short position will have a negative payoff. However since forward contract is a zero-sum game, the payoff to the short position party and the payoff to the long position party will cancel each other.

2.2 Futures contracts

Similar to the forward contracts, a future contract is an agreement to buy or sell an asset at a certain future time for a certain price. Futures contracts are traded on an exchange such as Chicago Mercantile Exchange (CME). Standardized contracts are traded on exchange markets, which mean only certain type of an asset in defined quality and quantity and with a specific maturity can be traded. By daily marking to market, exchange markets eliminate the default risk.

How hedging using futures contracts eliminates all price risk and fix the price is shown via an example. Suppose that a steel manufacturer needs to buy 100 tons of steel at t=1 and the quoted t=1 futures price of steel is 900 $/ton. The spot price at t=1 is not known in advance. Two possible cases are considered. In the first one, the spot price at t=1 is 950$/ton, i.e., higher than futures price. The cash flows in this case are shown in Table 2.1. In the second one, the spot price at t=1 is 850$/ton, i.e., lower than futures price. The cash flows in this case are shown in Table 2.2.

Table 2.1: Cash flows when spot price is larger than futures price

Strategies	t=0 Cash flow	t=1 Cash flow
Long a futures contract for 100 ton of steel at t=0 for t=1	0	(950-900)*100 = $ 5000
Pay for steel delivery at t=1	0	-950*100 = $-95000
TOTAL	**0**	**$ -90000**

Table 2.2: Cash flows when spot price is smaller than futures price

Strategies	t=0 Cash flow	t=1 Cash flow
Long a futures contract for 100 ton of steel at t=0 for t=1	0	(850-900)*100 = - $ 5000
Pay for steel delivery at t=1	0	-850*100 = $-85000
TOTAL	**0**	**$ -90000**

As it is seen from the tables, whether the spot price at t=1 is high or low, hedging fixes the purchase price to futures price.

Generally forward contracts are held until maturity and then physically settled, i.e., physical delivery is realized. However, majority of futures contracts are not held until maturity. Futures contracts are generally closed out before delivery by entering into the opposite type of trade from the original one (Hull, 2006).

As mentioned before, futures contracts are marked to market daily to prevent the default risk. Marking to market functions as follows; when the contract is entered into, some amount of money is deposited which is known as initial margin. Some fraction of initial margin is known as maintenance margin. At the end of each trading day, margin account of the dealer is updated to reflect the gain/loss which is resulted from price movements. If the margin account of the dealer is below the maintenance margin, then dealer receives a margin call to deposit some more money to top up the margin account to the initial margin level. So, the futures contracts are cash settled every day. Marking to market is illustrated with an example.

Suppose that an investor enters into a long steel futures position at 7th of July for 1st of September with a futures price of 100 $/oz ($F_{7July,1September}$ =100 $/oz). Contract size is 100 oz. Assume the initial margin per contract as $4000 and maintenance margin as $3000. How marking to market functions and the margin account of the firm is updated is shown in Table 2.3.

Table 2.3: Operations of margins

Day	**Futures Price ($)**	**Daily Gain (Loss) ($)**	**Cumulative Gain (Loss) ($)**	**Margin Account Balance ($)**	**Margin call ($)**
	100			4000	
8-July	97	(300)	(300)	3700	0
9-July	98	100	(200)	3800	0
10-July	88	(1000)	(1200)	2800	1200
11-July	92	400	(800)	4400	0

The first column shows the futures price of a steel contract between 7-11 July. The second column shows the daily gain/loss from the futures position. On 8th of July, the futures price is 97 $/oz ($F_{8July,1September}$=97 $/oz), which means the futures price has decreased by $ 3. Since the contract size is 100 oz, daily loss is 3*100=$300. Every day, margin account of the dealer is updated. If margin account of the dealer decreases below maintanence margin, the dealer receives a margin call. For example, the dealer receives a margin call of $ 1200 on 10-July, since margin account is $2800 on that day. By depositing margin call, the dealer should increase margin account to initial margin level. In the end of 11th July, the cumulative loss of the dealer from the futures position is (100-92)*100=$ 800.

Eliminating all price risk, i.e., perfect hedging is not usually possible in real life. There are three reasons of imperfect hedging. The first reason is the possible mismatch between the expiration date of the futures and the actual selling date of the asset. For example, the available steel futures contracts may be monthly and the payments to a steel mill may be bi-weekly. The second reason is the possible difference between the asset whose price to be hedged and the asset underlying the futures contract. The third reason is the uncertainty on the exact date when the asset will be bought or sold. Under these conditions, the spot price of the asset, and the futures price, may not converge on the expiration date of the future. The amount by which the two quantities differ measures the value of the basis risk. In our motivating example, we assume that there is no basis risk. In this thesis, we assume that there are available futures contracts with a maturity matching with steel delivery. Moreover, we assume that the steel traded on exchange markets are same with the steel used for foundations. Finally, we assume that the steel delivery date is known when contract is signed and there is not any delay in the project.

CHAPTER 3

LITERATURE REVIEW

Firms can reduce the volatility of their future cash flows via hedging using financial instruments such as futures, forwards, options etc. The literature on hedging practices of firms is immense. There are mainly two types of objectives used in these articles, namely maximization of expected utility and maximization of firm value.

The literature using expected utility framework focuses on risk averse firms who use financial markets to reduce the variability of their cash flows. Although expected utility framework is a useful basis for hedging operations of individual proprietorships and closely held corporations, it is not applicable to widely held corporations. This is because of establishing a common utility function for all owners, stockholders and bondholders is not easy.

Value maximization means the maximization of total discounted value of expected cash flows of a company. In our analysis, we will use a value maximization framework since our motivating company is a widely held corporation. Since the seminal work of Modigliani and Miller (1958), under perfect and complete markets, it is well known that financial hedging does not add any value to the firm. Smith and Stulz (1985) show that for hedging to add value to a value maximizing firm, there should be market imperfections. Under market imperfections, there are different cost rates for different cash flow streams. Therefore, decreasing the volatility of cash flows is important in reducing the expected costs for a firm. These market imperfections are taxes, costs of financial distress, bankruptcy, and managerial risk aversion.

When firms are exposed to a convex tax function, hedging the volatility of cash flows may decrease the expected tax costs (Smith and Stulz (1985)). Financial distress is defined as a low cash-flow state of the firm in which it incurs deadweight losses without being insolvent. “There are three important sources of deadweight losses from financial distress. First, a financially distressed firm may lose customers, valuable suppliers and key employees. Secondly, a financially distressed firm is more likely to violate its debt covenants or miss coupon/principal payments without being insolvent. These violations impose deadweight losses in the form of financial penalties, accelerated debt-repayment, operational inflexibility and managerial time and resources spent on negotiations with the lenders. Finally, a financially distressed firm may have to forego positive NPV projects due to costly external financing” (Purnanandam, 2008). Hedging lowers the probability of facing bankruptcy by reducing the volatility of cash flows, which decreases the expected costs of bankruptcy.

In this thesis, we will use value maximization objective under financial distress cost. The construction companies first rely on their own cash balance and cash inflows to fulfill the cash outflows as operating expenses. If the internally generated cash is not enough to satisfy the cash outflows, then the foundation installers face financial distress cost and use costly external funds.

There is a set of papers that consider financial and operational decisions of the firms together as Archibald et al. (2002) and Babich and Sobel (2004). Archibald et al. (2002) consider a start-up firm that aims to maximize survival. The firm will survive if the capital plus earnings can meet the cash outflow in each period. They investigate the optimal inventory management framework of the firm under survival maximization. Babich and Sobel (2004) consider a startup firm that wants to avoid bankruptcy. The firm aims to maximize the expected value of initial public offerings (IPO) with integrating production and capacity increase decisions of the firm with the loan size decision.

In our motivating example, we consider that the company is confronted with both price and quantity risks. The literature which has analyzed the hedging decisions of the firms under price and quantity risk generally uses a utility maximization framework. Rolfo (1980) analyzes the optimal hedge ratio of cocoa producers under price and quantity risk. In this paper the individual preferences are modeled via logarithmic and quadratic utility function where risk parameter is less than 0.0001. They find that unlike traditional hedging where the optimal hedge ratio is unity, under price and quantity risk, optimal hedge ratio is well below unity. Moreover, they find that when the correlation between price and quantity is low, optimal hedge ratios are higher. Similarly Losq (1982) investigates the optimal hedge in forward market using a concave utility function when confronted with both price and quantity risks. He shows that the optimal hedge is less than expected output when price and quantity are independent. Ho (1984) analyzes the hedging behavior of a farmer that is exposed to both price and quantity risks in a continuous time framework. Hedging decision of the farmer is modeled using a exponential utility function. He shows that it is not possible to eliminate all risks with the use of futures contracts. In general, the hedge ratio is less than unity and falls with the longer the time to harvest. Xing and Pietola (2005) figure out optimal hedge ratios of a wheat producer under quantity and price risk using a mean-variance and expected utility framework. They suggest that the correlation between price and quantity is important in the determination of the optimal hedge ratio. Their results suggest that price and quantity are negatively correlated which creates a natural hedge. They show that when there is natural hedge, the optimal hedge is always less than the expected quantity. Moreover, they suggest that hedging effectiveness decreases as quantity uncertainty increases.

Although there is a large literature dealing with both price and quantity under utility maximization, there is not much literature that uses value maximization framework. Two exceptions are Maes (2011) and Tanrisever and Gutierrez (2011). Moreover, Goel and Tanrisever (2011), Goel and Gutierrez (2009) and Goel and Gutierrez (2011) use value maximization for different purposes. Goel and Tanrisever (2011) consider a firm that procures, processes and distributes a commodity. The firm can procure the commodity from forward/options markets or from spot markets. The transportation cost paid for spot market is higher than forward/options market. They model the tradeoff between procurement in spot market and forward/options market in a multi-period framework. Goel and Gutierrez (2009) model how the change in price of futures and spot price can be used on the inventory management decisions of a firm. Similarly, Goel and Gutierrez (2011) investigate the additional value created in procurement via the use of commodity markets.

Maes (2011) uses value maximization framework to analyze the hedging decisions of a wheat miller that is confronted with price, quantity and blend risks. He uses forward contracts as financial instrument and incorporates logistical frictions, which means the differences in logistical costs depending on how much in advance the delivery of a contract is arranged. He shows that increase in the uncertainty leads to a lower optimal hedge ratio.

He compares the optimal results found with a complete hedge and concludes that optimal solution gives a higher expected profit and higher variation in profits than a complete hedge. Tanrisever and Gutierrez (2011) examine the operating and financial hedging decisions of a value maximizing flour miller under financial distress cost. They show that hedging adds value to the firm and increases the production level. In MTO (make-to-order) situation, where there is only price risk, all risk can be eliminated via hedging when hedging quantity equals to the production amount. However, in MTS (make-to-stock) situation, where both price and quantity risks are present, it is not possible to eliminate all the associated risk.

Different from the discussed articles, we also want to capture the effect of margin calls on the hedging decisions of companies. However, there is not any article that deals with margin risk management except Tanrisever and Levij (2011). They consider the hedging decision of a value maximizing electricity trading firm that is confronted with margin risks (price risk). They model the tradeoff between acquiring put options to decrease the margin risk and the cost of acquiring put options in a two period framework. Different from the discussed articles, we analyze the hedging behavior of a construction company that is confronted with both price and quantity risk by incorporating margin calls into our model. We use a value maximization framework under financial distress cost. To the best of our knowledge, this is the first time this topic is studied in literature.

CHAPTER 4

MATHEMATICAL MODEL

In this thesis, we use maximization of firm value as the objective which is defined as the total discounted value of expected cash flows. We use a two period framework to model the hedging decisions of the company. The important times/events in an offshore wind farm construction project are shown in Figure 4.1.

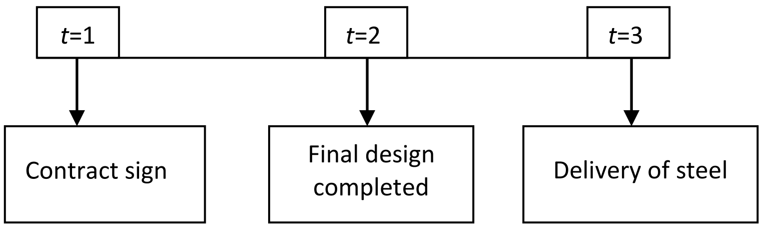

Figure 4.1: Important milestones over the event timeline

At t=1, contract is signed between the developer and the construction company. The developer wants to fix the project cost at t=1. Therefore, the developer and foundation installer agrees on a fixed price per quantity at t=1. This fixed unit price is denoted as p and paid to the construction company when steel is delivered at t=3. Simultaneously, the construction company signs contract with steel manufacturer and steel manufacturer with the steel mill at t=1. In our motivating example, there is more than 6 months between t=1 and t=3, thus steel mill does not accept to fix the price. Therefore, construction company is exposed to fluctuations in steel price. This is a realistic case since in large wind farm projects generally there is 1 to 2 years between *t*=1 and *t*=3. Generally, final design of the foundations is completed after contract sign, which means the exact steel quantity requirement is not known at *t*=1. At *t*=2, when final design is completed, exact steel requirement is observed. In our models, we consider different cases. If steel quantity requirement is deterministic in a model, it is denoted as ξ, if it is stochastic , it is denoted as $\tilde{\xi}$ .Probability density function of steel quantity requirement is denoted as $\phi_{\xi}(.)$ when it is stochastic. At *t*=3*,* steel plates are delivered from steel mill to steel manufacturer. Since the steel mill does not fix the steel price, the construction company is exposed to the steel price at *t*=3. The spot price of steel at time *t* is denoted as s_t . The payment by the construction company to the steel manufacturer is done when steel is delivered to steel manufacturer and is denoted as $(s_t+\lambda)$ where λ denotes the profit margin of steel manufacturer.

The construction company will use the financial markets to decrease the risk exposure. In general, the amount of long futures contracts bought at time *k* for time *t* is denoted as $x_{t,k}$. At *t*=1, construction company will long futures contracts with a delivery at *t*=3, which is denoted as $x_{1,3}$. The construction company may prefer to update his hedging quantity at *t*=2 when exact steel quantity is known. Then, the new hedging quantity of the company is denoted as $x_{2,3}$. The futures price of steel at time *t* for time *k* is denoted as $F_{t,k}$. The construction company has a cash reserve y_t and cash outflow η_t at time *t*. If the cash reserve of the company is negative at any time, then the company will face financial distress cost and will use costly external financing. Financial distress premium will be denoted with a constant rate denoted as *r*. We have assumed that there is no basis risk. In addition to the defined notation above, risk neutral probability density function of futures price at *t*=2, $F_{2,3}$, is denoted as $\phi_F^Q(.)$ and risk neutral probability density function of S_3 is denoted as $\phi_S^Q(.)$. A detailed list of notation is included in the beginning of the thesis.

In real life gain/loss from the futures position is realized daily. For ease of modeling in a two period framework, we make an assumption about the accumulation of gain/loss from futures position. This assumption is illustrated with Table 4.1.

Table 4.1: Margins

Day	Futures Price ($)	Daily Gain (Loss) ($)	Cumulative Gain (Loss) ($)
t=1	100		
-	97	(300)	(300)
-	98	100	(200)
-	88	(1000)	(1200)
t=2	92	400	**(800)**
-	95	300	(500)
t=3	94	(100)	**(600)**

For example, on the first date \$ 300 loss, on the second day \$ 100 gain is realized from the futures position. In our models, we assume that gain/loss is realized at the end of a period, which is at *t*=2 and *t*=3. When it is stated that there is not margin call between periods in a model that will be explained later, it means that the company's margin is updated only at *t*=3. For example, the company only realizes a loss of \$ 600 at *t*=3; which is the sum of daily gain/loss between *t*=1 and *t*=3. When it is stated that there is margin call between periods, it means that the company's margin is updated both at *t*=2 and at *t*=3. For example, the company loses \$ 800 at *t*=2; which is the sum of daily gain/loss between *t*=1 and *t*=2 and gains \$ 200 at *t*=3; which is the sum of daily gain/loss between *t*=2 and *t*=3.

In this thesis, we observe and compare the hedging decisions of the construction company under three real life business cases. In the first case, we investigate the hedging decision of the company under deterministic quantity with no margin calls between the periods. In the second case, we investigate the hedging decision of the company under deterministic quantity and with margin calls between the periods. In the third case, we investigate the hedging decision of the company under stochastic quantity and with margin calls between the periods.

4.1 Deterministic Quantity with no Margin Call

In this case, we assume that there is no quantity uncertainty at *t*=2. There is only steel price risk at *t*=3. The construction company longs steel futures contracts at *t*=1 maturing at *t*=3, in amount of $x_{1,3}$ units and with a futures price of $F_{1,3}$, to decrease the exposure to fluctuations in steel prices. We have assumed that the firm's margin account is updated only at *t*=3. That is like in a forward contract, the firm does not receive margin calls as the steel price fluctuates. The firm will face financial distress cost (FDC) if the firm has a negative net cash balance at *t*=3. Our aim is to find the optimal amount of futures contracts to be longed at *t*=1, i.e., $x_{1,3}$ value that maximizes the firm value. The events are summarized in the following table.

Table 4.2: Summary of events for deterministic quantity with no margin call case

***t*= 1**	***t*=3**
Observe: S_1 *and* $F_{1,3}$ **Action:** Long $x_{1,3}$ units of futures contract	**Observe**: S_3 **Cash Flows:** Receive revenues from developer : $p\xi$ Cost paid to steel manuf.: $(s_3+\lambda)\xi$ Cash from futures position: $(s_3-F_{1,3})x_{1,3}$ Pay FDC premium if y_3 is negative : $r\left[y_3\right]^{-}$

Then, the problem can be formulated as follows:

$$V_3(y_1) = \max_{x_{1,3}} E_{\tilde{s}_3}^{Q}[(p-\tilde{s}_3-\lambda)\xi + (\tilde{s}_3 - F_{1,3})x_{1,3} - r\left[y_3\right]^{-}]$$
$$s.t.\;\; y_3 = y_1 + (p-\tilde{s}_3-\lambda)\xi + (\tilde{s}_3 - F_{1,3})x_{1,3} - \eta_2 - \eta_3. \qquad (1)$$

$V_3(y_1)$ denotes the value of the company at *t*=3 with the initial value of cash reserve y_1 and the decision of the optimal futures to be long. The first term in the value function is the profit/loss of the construction company from the procurement operation. If the steel price at *t*=3 (s_3) is high, then the profit is low; if steel price at *t*=3 is low, then the profit is high. The second term is the gain/loss from the futures position. If the steel price increases above the futures price ($s_3 > F_{1,3}$), the company gains from futures contracts and vice versa. The last term is the financial distress cost. If the net cash of the firm at *t*=3 is negative, i.e., $y_3 < 0$; then the company pays a premium proportional to the deficit. The construction company aims to maximize the firm value by deciding on the optimal $x_{1,3}$ value. We use a risk neutral measure when taking the expectations. Under a risk neutral measure, a derivative's price is the discounted expected value of the future payoff. Therefore, price of a futures contract maturing at time *k* equals to the risk neutral expected value of spot price at time *k*, i.e. $F_{t,k} = E^Q(s_k)$. The constraint shows the net cash of the firm at *t*=3, which equals the net cash reserve at *t*=1 plus profit from procurement plus gain/loss from futures position and minus the cash outflow at *t*=3. The known parameters at *t*=1 are $y_1, F_{1,3}, \eta_3, \eta_2, p, \lambda, \xi$, which are denoted as set *K.*

Since $E^Q(\tilde{s}_3) = F_{1,3}$; the problem can be shown to reduce to a financial distress cost (FDC) minimization as the follows:

$$\min_{x_{1,3}} J(x_{1,3} \mid K) = \min_{x_{1,3}} \begin{cases} \int\limits_{s_3=0}^{a} r(-y_1 - (p - \tilde{s}_3 - \lambda)\xi - (\tilde{s}_3 - F_{1,3})x_{1,3} + \eta_3 + \eta_2)\phi_s^Q(s)ds \text{ if } \mathrm{x}_{1,3} \geq \xi \\ \int\limits_{s_3=a}^{\infty} r(-y_1 - (p - \tilde{s}_3 - \lambda)\xi - (\tilde{s}_3 - F_{1,3})x_{1,3} + \eta_3 + \eta_2)\phi_s^Q(s)ds \text{ if } \mathrm{x}_{1,3} \leq \xi \end{cases}$$

where;

$$a = \frac{F_{1,3}x_{1,3} - (p - \lambda)\xi + \eta_3 + \eta_2 - y_1}{(x_{1,3} - \xi)}. \tag{2}$$

J(.|.) denotes the FDC term at *t*=3. The Equation (2) means that if the steel price at *t*=3 (s_3) is smaller than *a* and hedging quantity is larger than steel quantity requirement, then the firm will face FDC. Similarly, when the steel price at *t*=3 is larger than *a* and hedging quantity is smaller than steel quantity requirement, then the firm will face FDC.

Theorem 1: *The optimization problem in* (2) *is convex and the optimal futures position at t = 1 is given by:* $x_{1,3} = \xi$

When there is no quantity uncertainty; it is possible to hedge perfectly and eliminate all price risk. Hence, a complete hedge which removes all the price risk in the cash flows is optimal. The proofs of all theorems are included in Appendix D.

4.2 Deterministic Quantity with Margin Call

In this business case, we assume that there is no quantity uncertainty at *t*=2. For ease of exposition, we assume that the margin calls are received at the end of each period, i.e., at *t*=2 and at *t*=3. Then, there is steel price risk at *t*=2 because of futures position and at *t*=3 because of raw material cost variability and futures position. The construction company will long steel futures contracts at *t*=1 with maturing at *t*=3 in $x_{1,3}$ units with a futures price of $F_{1,3}$, to decrease the exposure to fluctuations in steel price. The firm updates the hedging decision at *t*=2 when the gain/loss from futures position is realized. The firm closes the futures position of $x_{1,3}$ amounts at *t*=2 and buys new futures contracts maturing at *t*=3 with a futures price of $F_{2,3}$. The new hedging quantity of the firm is denoted as $x_{2,3}$. The firm will face FDC if the firm has a negative net cash balance at *t*=2and at *t*=3. Our aim is to find the optimal $x_{1,3}$ and $x_{2,3}$ values that maximize the firm value. The scenario is summarized in Table 4.3.

Table 4.3: Summary of events for deterministic quantity with margin call case

***t*=1**	***t*=2**	***t*=3**
Observe: S_1 *and* $F_{1,3}$ **Action:** Long $x_{1,3}$ units of futures contracts	**Observe**: $F_{2,3}$ **Actions:** Close contracts longed at *t*=1 Long $x_{2,3}$ units of contracts **Cash Flows:** Cash from futures position: $(F_{2,3}-F_{1,3})x_{1,3}$ Pay FDC premium if y_2 is negative : $r\left[y_2\right]^-$	**Observe**: S_3 **Cash Flows:** Revenues from developer : $p\xi$ Cost paid to steel manuf.: $(s_3+\lambda)\xi$ Cash from futures position: $(s_3-F_{2,3})x_{2,3}$ Pay FDC premium if y_3 is negative : $r\left[y_3\right]^-$

Then, the problem can be formulated as follows:

$$
\begin{aligned}
&V_2(y_1) = \max_{x_{1,3}} E^Q_{\tilde{F}_{2,3}}[(\tilde{F}_{2,3} - F_{1,3})x_{1,3} - r\left[y_2\right]^- + \beta E^Q_{\tilde{\xi}} V_3(y_2)] \\
&\qquad s.t.\ \ y_2 = y_1 + (\tilde{F}_{2,3} - F_{1,3})x_{1,3} - \eta_2 \\
&\ where\ \beta = 1/(1+r_f) \text{ and} \\
&V_3(y_2) = \max_{x_{2,3}} E^Q_{\tilde{s}_3}[(\tilde{s}_3 - F_{2,3})x_{2,3} + (p - \tilde{s}_3 - \lambda)\xi - r\left[y_3\right]^-] \\
&\qquad s.t.\ \ y_3 = y_2 + (\tilde{s}_3 - F_{2,3})x_{2,3} + (p - \tilde{s}_3 - \lambda)\xi - \eta_3\ .
\end{aligned} \tag{3}
$$

$V_2(y_1)$ denotes the value of the firm at t=2 with the initial value of y_1 and the decision of the optimal futures to be long at t=1. The first term is the gain/loss from futures position. The second term is the FDC premium which will be realized if the net cash reserve of the firm is negative at t=2. The last term is the discounted value of the cash flows at t=3. The constraint shows the net cash of the firm at t=2, which equals the net cash reserve at t=1 plus gain/loss from futures position and minus the cash outflow at t=2.

$V_3(y_2)$ denotes the value of the firm at t=3 with the initial value of y_2 and the decision of the optimal futures to be long at t=2. The first term is the gain/loss from futures position. The second term is the profit/loss of the construction company from the procurement operation. The last term is the FDC premium which will be realized if the net cash reserve of the firm is negative at t=3. The constraint shows the net cash of the firm at t=3. We aim to maximize the firm value by deciding on the optimal $x_{1,3}$ and $x_{2,3}$ values.

Lemma 1: *The solution to the second period is given by* $x_{2,3} = \xi$ *.*

This result follows from Theorem 1. Since there is no quantity risk at t=2, the firm will prefer a perfect hedge which removes all price risk.

Considering Lemma 1 and $E^Q(F_{2,3}) = F_{1,3}$, the two-period problem in (3) reduces to the following financial distress cost minimization problem.

$$\min J(x_{1,3} \mid K) = \begin{cases} \beta r \int\limits_{F_{2,3}=0}^{b} (-y_1 - (F_{2,3} - F_{1,3})x_{1,3} + \eta_2 - (p - F_{2,3} - \lambda)\xi + \eta_3)\phi_F^Q(F)dF \\ +r \int\limits_{F_{2,3}=0}^{c} (-y_1 - (F_{2,3} - F_{1,3})x_{1,3} + \eta_2)\phi_F^Q(F)dF \qquad \text{if } \mathrm{x}_{1,3} \geq \xi \\ \beta r \int\limits_{F_{2,3}=b}^{\infty} (-y_1 - (F_{2,3} - F_{1,3})x_{1,3} + \eta_2 - (p - F_{2,3} - \lambda)\xi + \eta_3)\phi_F^Q(F)dF \\ +r \int\limits_{F_{2,3}=0}^{c} (-y_1 - (F_{2,3} - F_{1,3})x_{1,3} + \eta_2)\phi_F^Q(F)dF \qquad \text{if } \mathrm{x}_{1,3} \leq \xi \end{cases}$$

$$where\ b = \frac{F_{1,3}x_{1,3} - (P - \lambda)\xi + \eta_3 + \eta_2 - y_1}{(x_{1,3} - \xi)},\ c = \frac{F_{1,3}x_{1,3} + \eta_2 - y_1}{x_{1,3}}. \qquad (4)$$

The equation means that when the optimal first period position, i.e., $x_{1,3}$ is larger than steel quantity requirement and $F_{2,3}$ is smaller than *b,* the firm will face FDC at *t*=3. If $F_{2,3}$ is smaller than *c,* the firm will face FDC at *t=2*. Similarly, when the optimal first period position, i.e., $x_{1,3}$ is smaller than steel quantity requirement and $F_{2,3}$ is larger than *b,* the firm will face FDC at *t*=3. If $F_{2,3}$ is smaller than *c,* the firm will face FDC at *t=2*.

Theorem 2: *The optimization problem in (4) is convex and the optimal hedging decision of the firm at t=1, x_{13}*, is described by:*

$$-\beta \int\limits_{F_{2,3}=0}^{b} (F_{1,3} - F_{2,3})\phi_F^Q(F)dF + \int\limits_{F_{2,3}=0}^{c} (F_{1,3} - F_{2,3})\phi_F^Q(F)dF = 0$$

$$where\ b = \frac{F_{1,3}x_{1,3}^* - (P - \lambda)\xi + \eta_3 + \eta_2 - y_1}{(x_{1,3}^* - \xi)},\ c = \frac{F_{1,3}x_{1,3}^* + \eta_2 - y_1}{x_{1,3}^*}. \qquad (5)$$

Further, $x_{1,3}^* \leq \xi$, i.e. ,the optimal first period futures position is to underhedge.

The only difference between this business case and the previous one is that the firm receives margin calls at *t*=2 in this case. Therefore, there is an extra price risk at *t*=2. If the steel price decrease at *t*=2, then the firm will lose money from the futures position and may face FDC. It is shown in Theorem 1 that when the firm does not receive any margin calls until *t*=3, it is optimal to hedge as steel quantity requirement. However, when the firm receives the accumulated margin updates between *t*=1 and *t*=2 at *t*=2, then the firm prefers to underhedge at *t*=1 to decrease the exposure to the margin calls at *t*=2.

4.3 Stochastic Quantity with Margin Call

In this business case, quantity uncertainty is resolved at t = 2. Similar to the previous case, we assume that the margin calls are received at the end of each period, i.e., at *t=2* and *t=3.* Therefore, the construction company is confronted with both price and quantity risks. The company will long steel futures contracts at *t=1* maturing at *t=3,* which is denoted as $x_{1,3}$, to decrease the exposure to fluctuations in steel price under stochastic demand. The firm may prefer to update its futures position after observing the demand at *t=2*. The firm closes the futures position of $x_{1,3}$ amounts at t=2 and buys new futures contracts maturing at t=3 with a futures price of $F_{2,3}$. The firm's new hedging position at t=2 is denoted as $x_{2,3}$. As in previous cases, the firm faces FDC if the firm has a negative net cash balance at t=2 and at t=3. Our aim is to find the optimal $x_{1,3}$ and $x_{2,3}$ values that maximize the firm value. The events are summarized in Table 4.4.

Table 4.4: Summary of events for stochastic quantity with margin call case

***t*=1**	***t*=2**	***t*=3**
Observe: S_1 *and* $F_{1,3}$ **Action:** Long $x_{1,3}$ units of futures contracts	**Observe**: $F_{2,3}$ and ξ **Action:** Close contracts longed at t=1 Long $x_{2,3}$ units of contracts **Cash Flows:** Cash from futures position: $(F_{2,3} - F_{1,3})x_{1,3}$ FDC premium if y_2 is negative: $r[y_2]^-$	**Observe**: S_3 **Cash flows:** Revenues from developer : $p\xi$ Paid to steel manuf.: $(s_3 + \lambda)\xi$ Cash from futures position: $(s_3 - F_{2,3})x_{2,3}$ FDC premium if y_3 is negative : $r[y_3]^-$

The two-stage problem can be written as follows:

$$V_2(y_1)=\max_{x_{1,3}} E^Q_{\tilde{F}_{2,3}}[(\tilde{F}_{2,3}-F_{1,3})x_{1,3}-r\left[y_2\right]^- +\beta E^Q_{\tilde{\xi}}V_3(y_2,\tilde{\xi})]$$

$$s.t.\ \ y_2=y_1+(\tilde{F}_{2,3}-F_{1,3})x_{1,3}-\eta_2$$

$$where\ \beta=1/(1+r_f)\text{ and}$$

$$V_3(y_2,\xi)=\max_{x_{2,3}} E^Q_{\tilde{s}_3}[(\tilde{s}_3-F_{2,3})x_{2,3}+(p-\tilde{s}_3-\lambda)\xi-r\left[y_3\right]^-]$$

$$s.t.\ \ y_3=y_2+(\tilde{s}_3-F_{2,3})x_{2,3}+(p-\tilde{s}_3-\lambda)\xi-\eta_3\ . \tag{6}$$

The explanation of the model is similar to deterministic quantity with margin call case. The only difference is that there is quantity uncertainty in this case. The known parameters at *t*=1 are; $y_1,\eta_2,\eta_3,p,\lambda,F_{1,3}$ which are denoted as set *L*.

Lemma 2: *The solution to the second period is given by* $x_{2,3}=\xi$ *.*

This result follows from Theorem 1. Since there is no quantity risk at *t=2*, the firm will prefer a perfect hedge which removes all price risk.

Considering Lemma 2 and $E^Q(F_{2,3})=F_{1,3}$, the two-period problem in (6) reduces to the following financial distress cost minimization problem.

$$\min J(x_{1,3}\,|\,L)=\min E^Q\begin{pmatrix}\beta r\left[y_1+(\tilde{F}_{2,3}-F_{1,3})x_{1,3}-\eta_2+(p-\tilde{F}_{2,3}-\lambda)\tilde{\xi}-\eta_3\right]^- \\ +r\left[y_1+(\tilde{F}_{2,3}-F_{1,3})x_{1,3}-\eta_2\right]^-\end{pmatrix}. \tag{7}$$

Theorem 3: *The optimization problem in (7) is convex and the optimal hedging decision of the firm at t=1 , x_{13}*, is described by:*

$$\beta\int_{\xi=0}^{x^*_{1,3}}\int_{F_{2,3}=0}^{b}(F_{1,3}-\tilde{F}_{2,3})\phi^Q_F(F)dF\phi_\xi(\xi)d\xi+\beta\int_{\xi=x^*_{1,3}}^{\infty}\int_{F_{2,3}=b}^{\infty}(F_{1,3}-\tilde{F}_{2,3})\phi^Q_F(F)dF\phi_\xi d\xi$$

$$+\int_{F_{2,3}=0}^{c}(F_{1,3}-\tilde{F}_{2,3})\phi^Q_F(F)dF=0$$

$$where\ b=\frac{F_{1,3}x^*_{1,3}-(P-\lambda)\tilde{\xi}+\eta_3+\eta_2-y_1}{(x^*_{1,3}-\tilde{\xi})},\ c=\frac{F_{1,3}x^*_{1,3}+\eta_2-y_1}{x^*_{1,3}}\ . \tag{8}$$

The equation in Theorem 3 provides the necessary and sufficient optimality condition. The first and third integrals in Equation (8) always take positive values and the second integral always takes negative values. The discussion about the behavior of integrals is included in Appendix D. Next we explore the optimal solution as a function of the key problem parameters including:

1) $\Delta = (y_1 - \eta_2)$: the net cash reserve of the firm at *t*=2.

2) η_3 : Cash outflow of the firm at *t*=3.

3) $\bar{\pi} = (P - F_{1,3} - \lambda)\bar{\xi}$: Expected profit of the firm which equals to the multiplication of expected profit margin and expected demand.

Theorem 4: *If $\Delta = 0$; then the firm prefers not to take any futures position at t=1; i.e. , $x^*_{1,3} = 0$. However, if $\Delta \neq 0$; then the firm prefers to long futures contracts at t=1, i.e., $x^*_{1,3} > 0$.*

If the firm has zero net cash reserve at *t*=2 ($\Delta = 0$) and longs futures contracts at *t*=1, and if the steel prices decrease at *t*=2, then the firm will be exposed to financial distress at *t*=2. Therefore, to avoid financial distress at *t*=2, the firm prefers not to take any futures position at *t*=1, i.e., $x^*_{1,3} = 0$. However, increase in net cash reserve at *t*=2, i.e., increase in Δ; gives a flexibility to the firm. By increasing the hedging position above 0, the firm can decrease the steel price risk at *t*=3. Similarly, when the firm has a negative net cash reserve at *t*=2, i.e., $\Delta < 0$; the firm knows that whether the firm takes any futures position or not , the firm will face a FDC at *t*=2. If the firm takes a futures position at *t*=1and steel prices increase, the firm will decrease the FDC exposure at *t*=2. Moreover, by longing futures contracts at *t*=1, the firm can decrease the price risk at *t*=3. Therefore, when $\Delta < 0$, the firm prefers to long futures contracts, i.e., $x^*_{1,3} > 0$.

Theorem 5: *When the net cash reserve of the firm at t=2 is approaching to minus/positive infinity, i.e., $\Delta \to \pm\infty$; the hedging becomes a value-neutral proposition.*

Suppose that the firm has a lot of net cash reserve at *t*=2. If the steel prices decrease at *t*=2, then taking any futures position will not cause any FDC risk at *t*=2 or *t*=3. Therefore, the firm is indifferent about the hedging position. Similarly, suppose that the firm has a large negative cash reserve at *t*=2. Even if the steel prices increase at *t*=2, taking any futures position will not eliminate the FDC risk at *t*=2. In these extreme cases, hedging has no effect on the firm value.

Theorem 6: *The firm prefers to hedge as expected steel quantity requirement, i.e.,* $x^*_{1,3} = \bar{\xi}$ *, if* $\Delta \geq F_{1,3}\bar{\xi}$ *and* $\Delta + \bar{\pi} = \eta_3$.

The first condition means that the firm has enough cash to cover the margin calls of contracts in expected steel quantity requirement size under worst $F_{2,3}$ scenario. The second condition means that the expected cash reserve and cash outflow of the firm at t=3 are equal. If the firm has enough cash to cover the margin calls at t=2 of the hedged amount, then there will not be any expected risk of FDC at t=3 if the hedged amount equals to expected quantity. Therefore, the firm chooses to hedge about expected quantity to decrease the FDC risk at t=3 without causing any risk at t=2.

Theorem 7: *When* $F_{2,3}$ and ξ *have symmetric distributions, if* $\Delta \geq F_{1,3}\bar{\xi}$ *and* $\Delta + \bar{\pi} < \eta_3$*, then the firm prefers to overhedge, i.e.,* $x^*_{1,3} \geq \bar{\xi}$ *.*

The first condition means that the firm has enough cash to cover the FDC of hedged contracts under worst case $F_{2,3}$ scenario. The second condition means that the expected cash reserve of the firm at $t=3$ is less than cash outflow at t=3. When the firm has enough cash to cover margin calls of hedged amount, and there is a certain expected risk of FDC at t=3; which means the net cash outflow is higher than cash reserve; then the firm prefers to overhedge. By overhedging the firm wants to eliminate the FDC risk at t=3. If steel prices increase at t=3, then the firm will profit from the futures position, so by overhedging the firm can eliminate the cash deficit.

Theorem 8: *When* $F_{2,3}$ and ξ *have symmetric distributions, if* $(0 < \Delta < F_{1,3}\bar{\xi}$ *and* $\Delta + \bar{\pi} = \eta_3)$ *or* $\Delta + \bar{\pi} > \eta_3$ *, the firm prefers to underhedge.*

In the first case, the first condition means that the firm has a positive net cash reserve at t=2, however this cash reserve is not enough to cover the FDC of contracts in expected steel quantity requirement size under worst $F_{2,3}$ scenario. Therefore, the company can face FDC at t=2 if the steel price decreases. The second condition means that the expected cash reserve and cash outflow of the firm at t=3 are equal. Under these conditions, the firm knows that the firm does not have any cash surplus at t=3. The firm may choose to hedge as expected quantity to eliminate the FDC risk at t=3 under expected conditions. However, when the steel prices decrease as in worst price scenario, this will cause a FDC risk at t=2 since the firm has not enough cash, so the firm prefers to hedge less than expected quantity.

In the second case, the condition means that the expected cash reserve of the firm at *t*=3 is larger than cash outflow at *t*=3. The firm knows that there will be a cash surplus at *t*=3 under expected conditions. If the firm hedges less than expected quantity and steel prices increase so much, the firm can compensate the losses by the positive cash surplus. Therefore, to decrease the FDC risk at *t*=2, the firm prefers to underhedge.

Theorem 9: *When* $F_{2,3}$ and ξ *have symmetric distributions, if* $0 < \Delta < F_{1,3}\bar{\xi}$ *and* $\Delta + \bar{\pi} < \eta_3$ *or* $\Delta < 0$ *and* $\Delta + \bar{\pi} \leq \eta_3$*, the firm may prefer to underhedge or overhedge.*

In the first case , under these conditions, the firm knows that under expected conditions, they will have a cash deficit at *t*=3, since cash outflow is higher than expected cash reserve. However, the firm also does not have enough cash to cover the margin updates under worst price scenario at *t*=2 when the firm hedges as expected quantity. Under these conditions, the firm may prefer to overhedge or underhedge depending on the tradeoff of FDC at *t*=2 and FDC at *t*=3. If the cash deficit at *t*=3 is not so large, the firm may prefer to overhedge to decrease the FDC risk at t=*3*. However, if the cash deficit is large at *t*=3, the firm prefers to underhedge.

In the second case, the first condition means that the firm has a negative net cash reserve at *t*=2. The second condition means that the expected cash reserve of the firm at *t*=3 is less than cash outflow at *t*=3. Similar to the first case, the firm will have a cash deficit at *t*=3. Moreover, the firm has a negative net cash reserve at *t*=2. The firm may prefer to overhedge or underhedge depending on how large the cash deficit and the negative net cash reserve are.

Theorem 10: *When* $F_{2,3}$ *and* ξ *have symmetric distributions and* $\Delta > 0$ *and* $\eta_3 = 0$*; as the profit margin increases, the optimal first period position; i.e.,* $x^*_{1,3}$ *decreases.*

As the profitability increases, price risk at *t*=3 decreases. Since the price risk at *t*=3 decreases, the firm prefers to hedge less. The summary of the theorems for stochastic quantity with margin call case when $F_{2,3}$ and ξ have symmetric distributions is illustrated in

Table 4.5. This table makes the theorems and the relations of parameters clearer. For example, we can easily see the effect of an increase in cash outflow at *t*=3.

Table 4.5 : Summary of theorems

		η_3		
		$\eta_3 < \Delta + \bar{\pi}$	$\eta_3 = \Delta + \bar{\pi}$	$\eta_3 > \Delta + \bar{\pi}$
Δ	$\Delta < 0$	Underhedge	Underhedge	Under/Overhedge
	$\Delta = 0$	0	0	0
	$0 < \Delta < F_{1,3}\bar{\xi}$	Underhedge	Underhedge	Under/Overhedge
	$\Delta \geq F_{1,3}\bar{\xi}$	Underhedge	Fullhedge	Overhedge

The optimal results for three business cases are summarized in Table 4.6. Case 1 corresponds to "Deterministic quantity without margin call" case, Case 2 corresponds to "Deterministic quantity with margin call" case, Case 3 corresponds to "Stochastic quantity with margin call" case. The optimal second period futures position for all business cases is same, which is a full hedge since the settings for the second period decision is same. The difference of Case 1 and Case 2 is the extra price risk at *t*=2. The difference of case 3 from case 2 is the quantity uncertainty. Although in deterministic case, underhedging is always optimal, in stochastic case, the company can prefer to overhedge, fullhedge or underhedge depending on expected profit, net cash reserve at *t*=2 and cash outflow at *t*=3.

Table 4.6: Summary of results for three business cases

	Optimal first period futures position	**Optimal second period futures position**
Case 1	Fullhedge	Fullhedge
Case 2	Underhedge	Fullhedge
Case 3	Overhedge/Fullhedge/Underhedge	Fullhedge

CHAPTER 5

EXTENSIONS

In this part, we extend our research with a new scenario. In this scenario, we change the timeline of events. Quantity uncertainty is resolved at *t=3* instead of *t=2*.

5.1 Stochastic Quantity with no Margin Call

In a two period model, suppose that there is demand (quantity) uncertainty that will be resolved at *t*=3. Moreover, there is a price risk at *t*=3 because of uncertainty in raw material costs. The firm longs futures contracts at *t*=1 with a delivery at *t*=3, which is denoted as $x_{1,3}$, for protection against price and quantity risk. The margin updates of the firm between *t*=1 and *t*=3 are accumulated at *t*=3. The events in this scenario are summarized in Table 5.1.

Table 5.1: Summary of events for stochastic quantity with no margin call case

***t*=1**	***t*=3**
Observe: $F_{1,3}$ **Action:** Long $x_{1,3}$ units of futures contracts	**Observe**: s_3 and ξ **Cash flows:** Revenue from developer : $p\xi$ Cost paid to steel manuf.: $(s_3+\lambda)\xi$ Cash flows from futures position: $(s_3-F_{1,3})x_{1,3}$ FDC premium if y_3 is negative : $r\left[y_3\right]^-$

The problem can be formulated as follows:

$$V_3(y_1,\tilde{\xi}) = \max_{x_{1,3}} E^Q_{\tilde{s}_3}[(p-\tilde{s}_3-\lambda)\tilde{\xi}+(\tilde{s}_3-F_{1,3})x_{1,3}-r\left[y_3\right]^-]$$
$$s.t.\ \ y_3 = y_1-\eta_2+(p-\tilde{s}_3-\lambda)\tilde{\xi}+(\tilde{s}_3-F_{1,3})x_{1,3}-\eta_3\ . \tag{9}$$

The explanation of the model is similar to the previous cases.

Since $E^Q(\tilde{s}_3)=F_{1,3}$; the problem reduces to a financial distress cost minimization problem.

$$\min J(x_{1,3}\,|\,.) = \min E^Q\left(r\left[y_1+(p-\tilde{s}_3-\lambda)\tilde{\xi}+(\tilde{s}_3-F_{1,3})x_{1,3}-\eta_3\right]^-\right)$$

$$\min J(x_{1,3}\,|\,.) = \min\begin{pmatrix} r\int\limits_{\xi=0}^{x_{1,3}}\int\limits_{s_3=0}^{b}(-y_1-(p-\tilde{s}_3-\lambda)\tilde{\xi}-(\tilde{s}_3-F_{1,3})x_{1,3}+\eta_3)\phi_s^Q(s)ds\phi\xi d\xi \\ +r\int\limits_{\xi=x_{1,3}}^{\infty}\int\limits_{s_3=b}^{\infty}(-y_1-(p-\tilde{s}_3-\lambda)\tilde{\xi}-(\tilde{s}_3-F_{1,3})x_{1,3}+\eta_3)\phi_s^Q(s)ds\phi\xi d\xi \end{pmatrix}$$

where;

$$b=\frac{F_{1,3}x_{1,3}-(p-\lambda)\tilde{\xi}+\eta_3+\eta_2-y_1}{(x_{1,3}-\tilde{\xi})}\ . \tag{10}$$

Equation (10) means that when realized quantity is less than hedging quantity and steel price at *t*=3(s_3) is less than *b*, and then the firm faces FDC. Similarly, when steel quantity is higher than hedging quantity and steel price at *t*=3 is larger than *b*, then the firm faces FDC.

Theorem 11: *The optimization problem in (10) is convex and the optimal hedging decision of the firm, x_{13}*, is given by:*

$$\frac{\partial J(x_{1,3}\,|\,L)}{\partial x_{1,3}} = \int\limits_{\xi=0}^{x_{1,3}^*}\int\limits_{s_3=0}^{b}(F_{1,3}-\tilde{s}_3)\phi_s^Q(s)ds\phi_\xi(\xi)d\xi+\int\limits_{\xi=x_{1,3}^*}^{\infty}\int\limits_{s_3=b}^{\infty}(F_{1,3}-\tilde{s}_3)\phi_s^Q(s)ds\phi_\xi d\xi=0$$

where;

$$b=\frac{F_{1,3}x^*_{1,3}-(p-\lambda)\tilde{\xi}+\eta_3+\eta_2-y_1}{(x^*_{1,3}-\tilde{\xi})}. \tag{11}$$

Theorem 12: *If* $\Delta+\overline{\pi}=\eta_3$ *; then the optimal first period futures position equals to expected steel quantity, i.e.,* $x_{1,3}^{*}=\overline{\xi}$ *.*

When the firm knows that under expected price and quantity conditions, expected cash reserve at *t*=3 will equal to the cash outflows, then the firm can eliminate all risk under expected conditions with a full hedge.

Theorem 13: *When* $F_{2,3}$ and ξ *have symmetric distributions, if* $\Delta+\overline{\pi}>\eta_3$ *then underhedging, i.e.,* $x_{1,3}\leq\xi$ *is optimal. If* $\Delta+\overline{\pi}<\eta_3$*, overhedging ,i.e. ,* $x_{1,3}\geq\xi$ *is optimal.*

If the expected cash reserve of the firm at *t*=3 is larger than the cash outflow, then the firm knows that there will be a cash surplus at *t*=3 under expected conditions, so prefers to underhedge. If the realized steel quantity is less than hedging quantity and steel price increased above futures price, then the firm can compensate the loss with the cash surplus. When, there is a certain cash deficit risk at *t*=3 under expected conditions, then the firm prefers to overhedge. By overhedging, the firm tries to eliminate FDC risk at *t*=3 if steel price increases.

CHAPTER 6

COMPUTATIONAL RESULTS

In this section, we aim to analyze the hedging behavior of the company that we could not capture analytically. Moreover, we want to illustrate how the hedging decisions change when price distribution changes. Therefore, numerical analysis is performed with a normally distributed quantity and normally and lognormally distributed price distribution. We have investigated how the optimal first period futures position ($x_{1,3}^{*}$) changes as the parameters listed changes.

- Δ : Net cash reserve at *t*=2.
- η_3 : Cash outflow at *t*=3.
- δ : Profit margin

6.1 Stochastic Quantity with Margin Call Case

6.1.1 Symmetric Price Distribution

In this part, numerical analysis is performed with a normally distributed price and quantity. We have used C++ for the numerical analysis. 100,000 random variables are generated for price and quantity with the following distribution parameters, $F_{2,3} \sim N\ (10, 2^2)$ and $\xi \sim N\ (100, 25^2)$. According to the generated $F_{2,3}$ and ξ values, $F_{2,3} = 9.99507$ and $\bar{\xi} = 99.9761$ is realized. We have assumed that $\beta = 0.95$ and r = 0.1

a) $\Delta > 0$ and $\eta_3 = 0$

In this part, we consider the case where the company has a positive net cash reserve at *t=2* and does not have any cash outflow at *t=3*. Optimal first period futures position ($x_{1,3}$) for the changing Δ values are given in Figure 6.1.

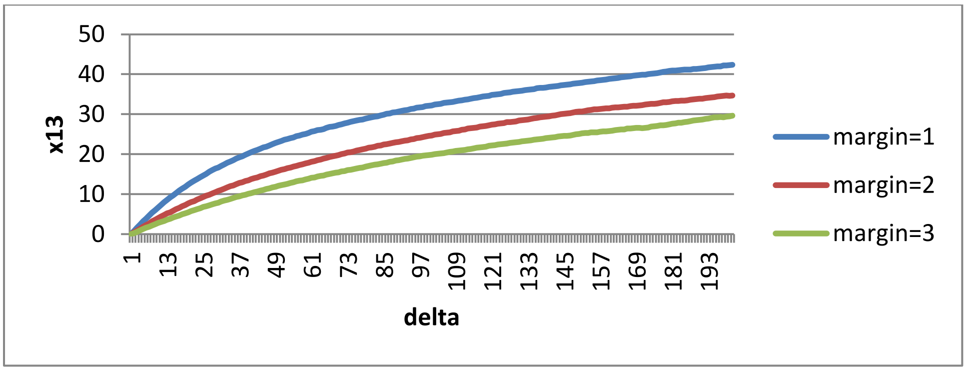

Figure 6.1: $x_{1,3}$ versus Δ

As it is mentioned in Theorem 5, when the net cash reserve at *t*=2 (Δ) is so large or so small, hedging becomes a value-neutral proposition. Therefore, we consider moderate levels of Δ in the analysis. As it is seen in the figure, as Δ increases, the optimal first period position, i.e. ,$x_{1,3}$ increases. Margin shows the profit margin of the company. margin=1 equals to an approximate profit of 100. For the considered range of Δ and profit margins, underhedging is optimal. This figure is also a visualization for the second case of Theorem 8. If $\eta_3 = 0$ and $\Delta > 0$, then $\Delta + \bar{\pi} > \eta_3$ so, underhedging is optimal. As it is stated in Theorem 10, we see that as the profit margin increases optimal first period position decreases.

b) $\Delta > 0$ and $\eta_3 > 0$

In this part, we perform the numerical analysis for the case where the company has a positive cash reserve at *t*=2 and a positive cash outflow at *t*=3. We investigate three scenarios considering different margin levels. In Figure 6.2, a smaller value of delta , in Figure 6.3 a moderate level of delta and in Figure 6.4 a larger level of delta is considered. In the generated price data, minimum $F_{2,3}$ is realized as 1.65925. Therefore, delta that can cover the FDC of futures contracts under worst price scenario equals to 833.

$$\Delta^* = (F_{1,3} - F_{2,3_{\min}})\bar{\xi}$$
$$\Delta^* = (9.99507 - 1.65925)99.9761 \cong 833.$$

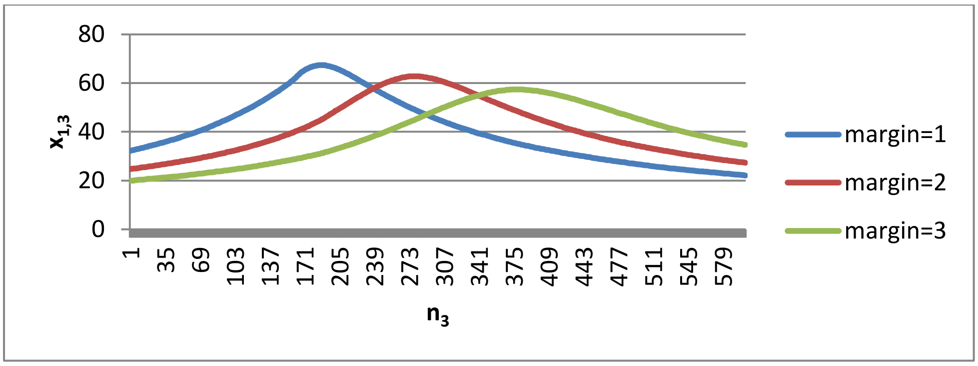

Figure 6. 2: $x_{1,3}$ versus n_3 when Δ =100

In Figure 6.2, always underhedging is optimal. The hedging patterns in Figure 6.2 and 6.3 are similar. For a fixed net cash reserve at *t*=2 (Δ), as η_3 increases, $x^*_{1,3}$ first increases but then starts to decrease. As η_3 increases, the price risk at *t=3* increases, so the firm prefers to increase the optimal first period futures position. However, as $x^*_{1,3}$ increases, the price risk at *t=2* also increases, so after a while $x^*_{1,3}$ starts to decrease. In Figure 6.2, the maximum $x^*_{1,3}$ is realized when $\eta_3 = 184$ and margin=1 and the maximum $x^*_{1,3}$ is realized when $\eta_3 = 272$ and margin=2. When margin increases, the price risk at *t=3* decreases so, the firm can tolerate higher cash outflows at *t=3*. One of the differences in Figure 6.2 and 6.3 is that maximum $x^*_{1,3}$ value that can be realized when delta=500 is higher than delta=100 case, since the increase in delta increases the flexibility of the company, thus the company can hedge in a higher amount.

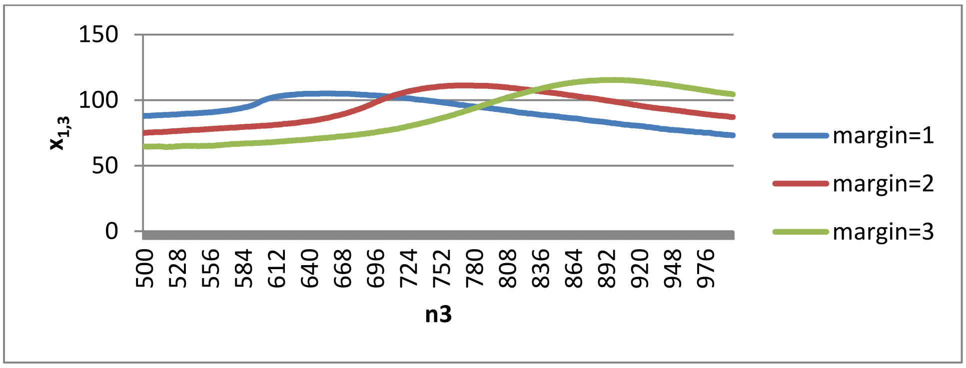

Figure 6.3: $x_{1,3}$ versus n_3 when Δ =500

As it is seen from the Figure 6.3, underhedging and overhedging are both possible. This case is a visualization for Theorem 8 and 9. When the cash outflow at *t=3*, i.e., η_3 is slightly higher than the sum of expected profit and net cash reserve, i.e., $\eta_3 > \Delta + \bar{\pi}$ overhedging is optimal, however for the larger values of η_3, underhedging is optimal. As it is stated in Theorem 8, when $\Delta + \bar{\pi} = \eta_3$, underhedging is optimal since Δ is not large enough.

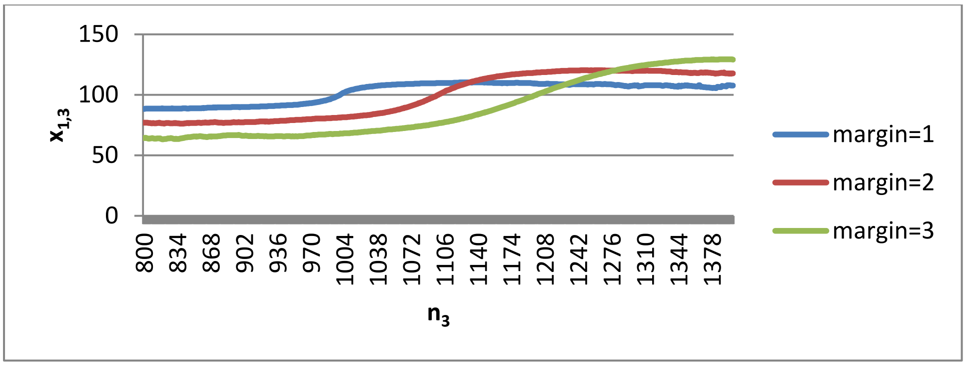

Figure 6. 4: $x_{1,3}$ versus n_3 when Δ =900

In Figure 6.4, numerical analysis for a higher level of delta which equals to 900 is shown. When $\Delta = 900 > 833$, the firm knows that under worst price scenario, the firm will not face FDC at *t=2* if hedges as expected steel quantity. As η_3 increases, $x_{1,3}^*$ first increases. After a threshold value, $x_{1,3}^*$ becomes nearly steady. This case is a visualization for Theorem 7. When $\eta_3 > \Delta + \bar{\pi}$ and delta is large enough, overhedging is optimal.

c) $\Delta < 0$ and $\eta_3 > 0$

In this part, we consider that the firm has a negative cash reserve and a cash outflow at *t*=3. The results are shown in Figures 6.5 and 6.6.

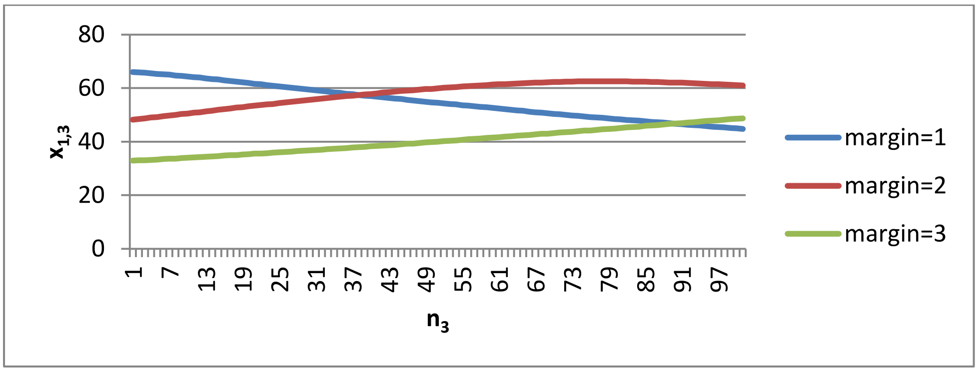

Figure 6.5: $x_{1,3}$ versus n_3 when Δ = -100

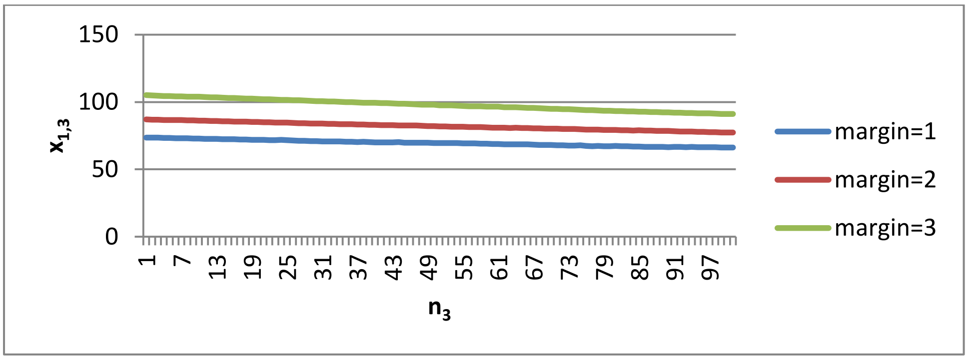

Figure 6.6: $x_{1,3}$ versus n_3 when Δ = -450

As stated in the theorems, hedging quantity depends on the level of delta and cash reserve cash outflow relation at *t*=3. If delta is not so negative as in Figure 6.6, the firm prefers to underhedg since the FDC cost faced at *t*=2 is small and the firm does not want to increase this risk via increasing hedging position. If steel price increases a little at *t*=2, the firm may avoid FDC. However, when delta is as negative as in Figure 6.7, the firm may prefer to overhedge.

6.1.2 Asymmetrical Price Distribution

In this case we try to capture how the hedging behavior changes when price does not have a symmetric distribution and standard deviation of price distribution changes. We have generated 100,000 rvs for $F_{2,3}$ with the following parameters, $F_{2,3} \sim LogN(10, 2^2)$ and $F_{2,3} \sim LogN(10, 4^2)$.

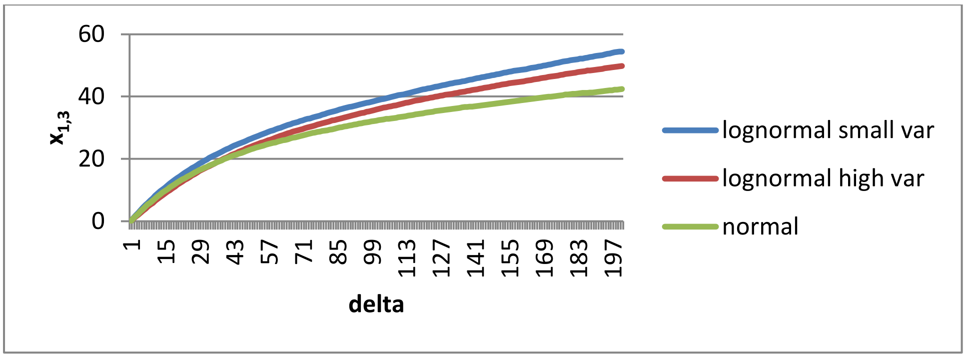

Figure 6.7: $x_{1,3}$ versus Δ for three distributions when margin=1

We compare optimal $x_{1,3}$ under three different distributions when $\Delta > 0$ and $\eta_3 = 0$. The results are shown in Figure 6.7. "Normal" means that $F_{2,3}$ has a normal distribution with mean 10 and standard deviation 2; "lognormal small var." means that $F_{2,3}$ has a lognormal distribution with mean 10 and standard deviation 2; "lognormal high var." means that $F_{2,3}$ has a lognormal distribution with mean 10 and standard deviation 4. We can conclude that under the same mean and standard deviation, if the price turns out to be distributed lognormally, optimal hedging quantity increases since under lognormal distribution the steel price is more probable to have higher values so the risk at *t*=2 decreases. Therefore the company prefers to hedge more. Another conclusion is that, under the same mean and distribution if variance increases, optimal hedging quantity decreases.

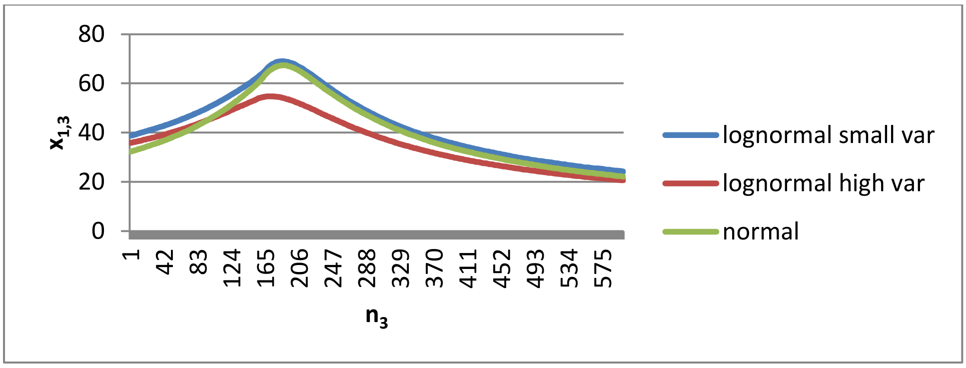

Figure 6.8: $x_{1,3}$ versus n_3 for three distributions when margin=1

A different scenario is considered in Figure 6.8. Similar to the previous one, under the same distribution and mean, when variance increases hedging quantity decreases. When we compare normal distribution and lognormal distribution that have same mean and variance, we see that there is not much difference in hedging quantities.

6.2 Deterministic Quantity with Margin Call Case

6.2.1 Symmetric Price Distribution

In this part, numerical analysis is performed with a normally distributed price. To have comparable results, we have used the same distribution with the previous case, which is $F_{2,3} \sim N\ (10, 2^2)$.

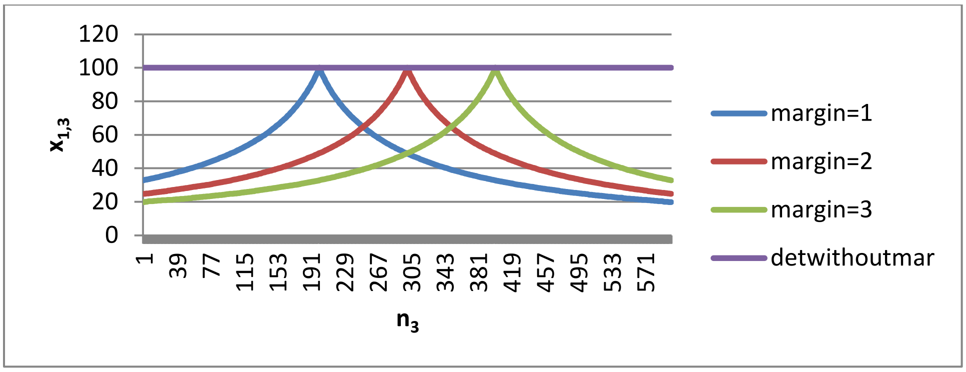

Figure 6.9: $x_{1,3}$ versus n_3 when Δ =100

Hedging behavior when Δ=100 is shown in Figure 6.9 for different profit margins. It is stated in Theorem 1 that when quantity is deterministic and there is no margin call between the periods, then hedging as expected demand is optimal. This is also included in the figure as the purple line to highlight the effect of margin call risk on hedging. As it is stated in Theorem 2, when there is margin call risk between periods under deterministic demand, underhedging is optimal. This result can be seen in Figure 6.9. Maximum hedging quantity, which equals to steel quantity requirement, is realized when $\Delta + \bar{\pi} = \eta_3$.

6.2.2 Asymmetrical Price Distribution

In this section, we want to investigate hedging decisions of the company when price has a lognormal distribution; we use the same distribution as in the previous case which is; $F_{2,3} \sim LogN(10, 2^2)$ and $F_{2,3} \sim LogN(10, 4^2)$

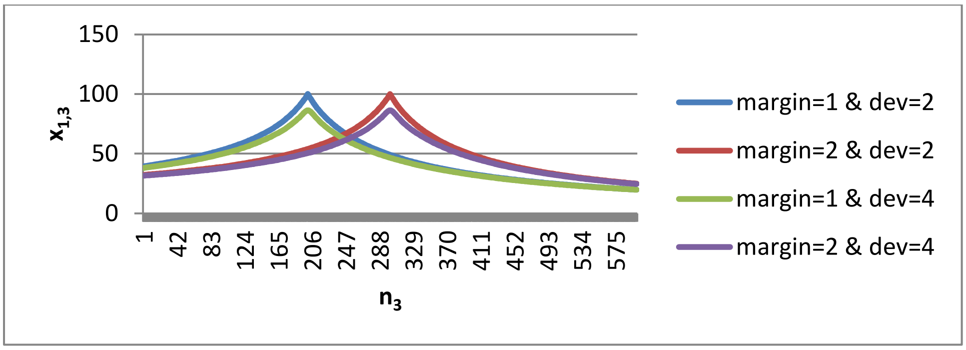

Figure 6.10: $x_{1,3}$ versus n_3 when Δ =100 for lognormal distribution

The results are shown in Figure 6.10. We see that hedging patterns are same even if standard deviation of the price changes. As η_3 increases, $x^*_{1,3}$ first increases, reaches maximum when $\Delta + \bar{\pi} = \eta_3$ and then decreases. Another conclusion is that hedging quantity decreases, if standard deviation of price increases. If we compare the results in Figures 6.9 and 6.10, we see that under the same mean and standard deviation if price turns out to be lognormally distributed, optimal hedging quantity does not differ so much.

CHAPTER 7

CONCLUSION AND FUTURE RESEARCH

7.1 Conclusion

In this thesis, we consider a construction company that is confronted with both price and quantity risks. Different from the related literature, we investigate the hedging decisions of the value maximizing company by incorporating margin calls and financial distress cost in a two period framework. We consider three business cases. In the first case, we observe the hedging behavior of the company under deterministic demand without any margin call and we see that a complete hedge is optimal. In the second case, we incorporate margin call risk between periods under deterministic demand and see that the firm prefers to underhedge. In the last case, we consider stochastic demand with margin call risk between periods. We conclude that optimal hedging quantity depends on the expected profits, net cash reserve and cash outflow. The company may prefer to overhedge or underhedge depending on the values of these parameters. We extend our mathematical model with varying timeline of events. By computational results, we investigate the hedging behavior in a more detailed way. We see that under same distribution and mean, if the standard deviation of the price increases, the firm becomes more conservative and hedges less.

7.2 Future Research

In this thesis, we consider the hedging behavior of the construction company in a two period framework. We assume that margin calls can be received only at the end of these two periods. However, in real life margin calls are received every day. As a future research, hedging behavior of the company can be investigated within a multi-period framework (more than two). Moreover, we provide a research on the optimal financial decisions of the company under only price and quantity risks. However, in real life construction industry has other risks. For example, the projects are subject to delays. Indeed, a construction company should update the hedging position as the steel delivery date changes. Moreover, we assume that there is not any basis risk. However, in real life there may be basis risk. For example, the steel that is used in foundations may not be the same with the steel that is traded on exchange markets. As a future research, investigating the hedging behaviors under different combinations of these risks would be interesting.

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APPENDIX A

PARTS OF A TURBINE

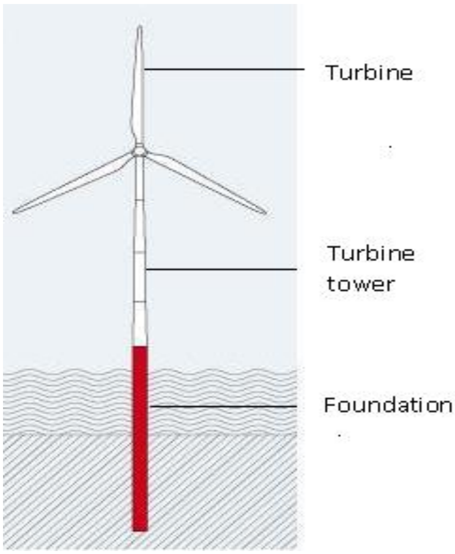


Figure A.1: Turbine

There are mainly three parts of a wind turbine, which are turbine, turbine tower and foundation. These parts are shown in Figure A.1.

APPENDIX B

COST COMPARISON OF DIFFERENT ELECTRICITY GENERATION TECHNOLOGIES

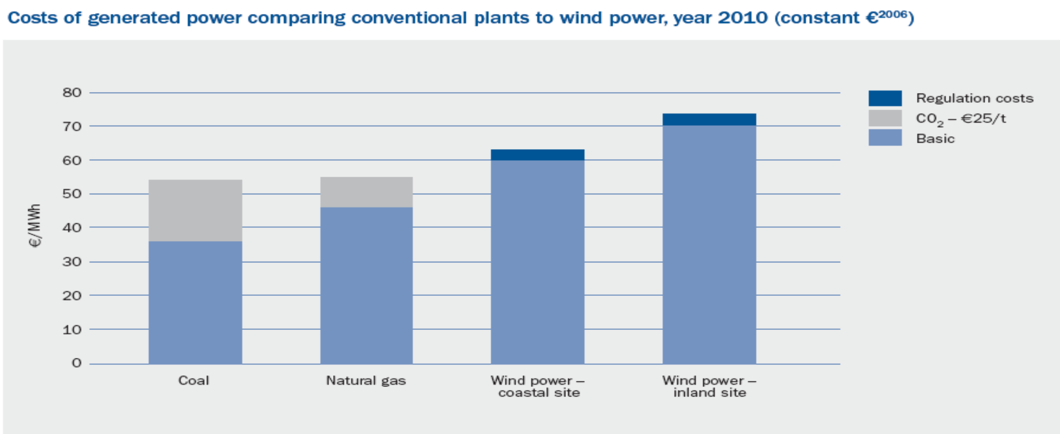


Figure B.1: Costs of different technologies

This figure is taken from a report written by European Wind Energy Association, The Economics of Wind Energy. EWEA (2009). The figure shows that according to 2010 prices, cost of electricity generation via offshore windfarm is about % 35 higher than cost of electricity generation via conventional plants. The cost of conventional electricity generation depends on four factors, which are fuel prices, O&M costs, cost of CO_2 emissions and capital costs. Crude oil price is taken as $59/barrel, coal price as €1.6/GJ and natural gas price as €6.05/GJ in 2010 (constant terms). Since power plants that use coal and natural gas are not clean energy, they should pay CO_2 price that is determined in ETS markets. The CO_2 price is around 25 €/t. Regulation costs which is indicated in the figure means the balancing costs of wind power.

APPENDIX C

EXCHANGE MARKETS

The exchange markets that have launched steel futures contracts are listed as;

- ***London Metal Exchange (LME):***LME is trading steel billet futures actively. The available contracts are daily from spot to 3 months, weekly from 3 months to 6 months and, monthly from 7 months to 15 months.
- ***New York Mercantile Exchange (NYMEX):*** NYMEX launched Hot rolled US steel coil futures. The available contracts are monthly with at most 24 months forward. NYMEX is the only exchange market that has financially settled contracts; not physically settled.
- ***Dubai Gold and Commodities Exchange (DGCX):*** DGCX launched steel rebar futures contracts. The available contracts are monthly with at most 4 months forward. However, the traded volumes for these contracts are 0 which makes DGCX steel market illiquid.
- ***Shangai Futures Exchange (SFE):*** SFE is trading steel wire rod and rebar futures contracts actively. The available contracts are monthly with at most 12 months forward.
- ***Multicommodity exchange of India (MCX)*****:** MCX is trading mild steel ingot/billets futures contracts actively. The available contracts are monthly with at most 3 months forward.
- ***National Commodity and Derivatives Exchange (NCDEX):*** NCDEX is trading mild steel ingot futures actively. The available contracts are monthly with at most 5 months forward.

From these exchange markets the contracts of LME, SFE and NYMEX can be considered since contracts with maturity date longer than 6 months can be found from these exchange markets.

APPENDIX D

PROOFS

D.1) Proof of Theorem 1:

By using the Leibniz rule, first derivative of the function in (2) is given as;

$$\frac{\partial J(x_{1,3} \mid K)}{x_{1,3}} = \begin{cases} r \int\limits_{s_3=0}^{a} (F_{1,3} - s_3)\phi_s^Q(s)ds & \text{if } \mathrm{x}_{1,3} \geq \xi \\ r \int\limits_{s_3=a}^{\infty} (F_{1,3} - s_3)\phi_s^Q(s)ds & \text{if } \mathrm{x}_{1,3} \leq \xi \end{cases}$$

The second derivative of the function in (2) is;

$$\frac{\partial^2 J(x_{1,3} \mid K)}{\partial x_{1,3}^2} = \begin{cases} r\,\dfrac{\left[(p-\lambda-F_{1,3})\xi + y_1 - \eta_2 - \eta_3\right]^2}{(x_{1,3}-\xi)^3}\phi_s^Q(\mathrm{s'}) & \text{if } x_{1,3} \geq \xi \\ r\,\dfrac{-\left[(p-\lambda-F_{1,3})\xi + y_1 - \eta_2 - \eta_3\right]^2}{(x_{1,3}-\xi)^3}\phi_s^Q(\mathrm{s'}) & \text{if } x_{1,3} \leq \xi \end{cases}$$

where,

$\phi_s^Q(\mathrm{s'})$ means replacing s_3 in density function with *a*

Since second derivative is positive for both cases, $\mathrm{J}(\mathrm{X}_{1,3} | \mathrm{K})$ is convex, and the optimality condition is;

$$\frac{\partial J(x_{1,3} \mid K)}{x_{1,3}} = \begin{cases} \int\limits_{s_3=0}^{a} (F_{1,3} - s_3)\phi_s^Q(s)ds = 0 & \text{if } \mathrm{x}_{1,3} \geq \xi \\ \int\limits_{s_3=a}^{\infty} (F_{1,3} - s_3)\phi_s^Q(s)ds = 0 & \text{if } \mathrm{x}_{1,3} \leq \xi \end{cases}$$

Since, $\frac{\partial J(x_{1,3} \mid K)}{x_{1,3}}|_{x_{1,3}=\xi} = 0$, the optimal hedging decision is $x_{1,3} = \xi$.

D.2) Proof of Lemma 1:

Proof is similar to the proof of Theorem 1.

D.3) Proof of Theorem 2:

By using the Leibniz rule, the first derivative of the function in (4) is given as;

$$\frac{\partial J(x_{1,3} \mid K)}{\partial x_{1,3}} = \begin{cases} \beta r \int\limits_{F_{2,3}=0}^{b} (F_{1,3} - F_{2,3}) \phi_F^Q(F) dF + r \int\limits_{F_{2,3}=0}^{c} (F_{1,3} - F_{2,3}) \phi_F^Q(F) dF & \text{if } \mathrm{x}_{1,3} \geq \xi \\ \beta r \int\limits_{F_{2,3}=b}^{\infty} (F_{1,3} - F_{2,3}) \phi_F^Q(F) dF + r \int\limits_{F_{2,3}=0}^{c} (F_{1,3} - F_{2,3}) \phi_F^Q(F) dF & \text{if } \mathrm{x}_{1,3} \leq \xi \end{cases}$$

The second derivative is;

$$\frac{\partial^2 J(x_{1,3} \mid K)}{\partial x_{1,3}^2} = \begin{cases} \dfrac{r\left[(p - \lambda - F_{1,3})\xi + y_1 - \eta_2 - \eta_3\right]^2}{(x_{1,3} - \xi)^3} \phi_F^Q(\mathrm{F'}) & \text{if } x_{1,3} \geq \xi \\ \dfrac{-r\left[(p - \lambda - F_{1,3})\xi + y_1 - \eta_2 - \eta_3\right]^2}{(x_{1,3} - \xi)^3} \phi_F^Q(\mathrm{F'}) & \text{if } x_{1,3} \leq \xi \end{cases}$$

where,

$\phi_F^Q(\mathrm{F'})$ means replacing $F_{2,3}$ in density function with *b.*

Since second derivative is positive for both cases, $\mathrm{J}(\mathrm{X}_{1,3}|\mathrm{K})$ is convex, and the optimality condition is;

$$\frac{\partial J(x_{1,3} \mid K)}{x_{1,3}} = \begin{cases} \beta \int\limits_{F_{2,3}=0}^{b} (F_{1,3} - F_{2,3}) \phi_F^Q(F) dF + \int\limits_{F_{2,3}=0}^{c} (F_{1,3} - F_{2,3}) \phi_F^Q(F) dF = 0 & \text{if } \mathrm{x}_{1,3} \geq \xi \\ \beta \int\limits_{F_{2,3}=b}^{\infty} (F_{1,3} - F_{2,3}) \phi_F^Q(F) dF + \int\limits_{F_{2,3}=0}^{c} (F_{1,3} - F_{2,3}) \phi_F^Q(F) dF = 0 & \text{if } \mathrm{x}_{1,3} \leq \xi \end{cases}$$

When $x_{1,3} > \xi$, both integrals take positive values, so the optimal $x_{1,3} \leq \xi$, which means the firm prefers underhedging.

D.4) Proof of Theorem 3:

$$\min J(x_{1,3}\,|\,L)=\min\left(\begin{array}{l}\beta r\int\limits_{\xi=0}^{X_{1,3}}\int\limits_{F_{2,3}=0}^{b}\left(-y_1-(\tilde{F}_{2,3}-F_{1,3})x_{1,3}+\eta_2-(p-\tilde{F}_{2,3}-\lambda)\tilde{\xi}+\eta_3\right)\phi_F^Q(F)dF\phi_\xi(\xi)d\xi \\ +\beta r\int\limits_{\xi=X_{1,3}}^{\infty}\int\limits_{F_{2,3}=b}^{\infty}\left(-y_1-(\tilde{F}_{2,3}-F_{1,3})x_{1,3}+\eta_2-(p-\tilde{F}_{2,3}-\lambda)\tilde{\xi}+\eta_3\right)\phi_F^Q(F)dF\phi_\xi(\xi)d\xi \\ +r\int\limits_{F_{2,3}=0}^{c}(-y_1-(\tilde{F}_{2,3}-F_{1,3})x_{1,3}+\eta_2)\phi_F^Q(F)dF\end{array}\right)$$

where,

$$b=\frac{F_{1,3}x_{1,3}-(P-\lambda)\tilde{\xi}+\eta_3+\eta_2-y_1}{(x_{1,3}-\tilde{\xi})}\quad c=\frac{F_{1,3}x_{1,3}+\eta_2-y_1}{x_{1,3}}$$

By using the Leibniz rule, first derivative of the function is given as;

$$\frac{\partial J(x_{1,3}\,|\,L)}{\partial x_{1,3}}=\beta r\int\limits_{\xi=0}^{x_{1,3}}\int\limits_{F_{2,3}=0}^{b}(F_{1,3}-\tilde{F}_{2,3})\phi_F^Q(F)dF\phi_\xi(\xi)d\xi$$

$$+\beta r\int\limits_{\xi=x_{1,3}}^{\infty}\int\limits_{F_{2,3}=b}^{\infty}(F_{1,3}-\tilde{F}_{2,3})\phi_F^Q(F)dF\phi_\xi d\xi+r\int\limits_{F_{2,3}=0}^{c}(F_{1,3}-\tilde{F}_{2,3})\phi_F^Q(F)dF$$

The second derivative is;

$$\frac{\partial^2 J(x_{1,3}\,|\,L)}{\partial x_{1,3}{}^2}=\int\limits_{\xi=0}^{x_{1,3}}\frac{\left((P-\lambda-F_{1,3})\xi-\eta_3-\eta_2+y_1\right)^2}{(x_{1,3}-\xi)^3}\phi(F')\phi(\xi)d\xi$$

$$+\int\limits_{\xi=x_{1,3}}^{\infty}\frac{-\left((P-\lambda-F_{1,3})\xi-\eta_3-\eta_2+y_1\right)^2}{(x_{1,3}-\xi)^3}\phi(F')\phi(\xi)d\xi+\frac{(\eta_2-y_1)^2}{x_{1,3}{}^3}\phi(F'')\geq 0$$

where $\phi(F')$ means replacing $F_{2,3}$ in density function with $x_{1,3}$. $\phi(F'')$ means replacing $F_{2,3}$ in density function with *c*.

Since all terms in the second derivative equation are positive, second derivative is positive, so $\mathrm{J}(\mathrm{X}_{1,3}|\mathrm{L})$ is convex, and the optimality condition is;

$$\frac{\partial J(x_{1,3}\,|\,L)}{\partial x_{1,3}}=\beta\int\limits_{\xi=0}^{x_{1,3}}\int\limits_{F_{2,3}=0}^{b}(F_{1,3}-\tilde{F}_{2,3})\phi_F^Q(F)dF\phi_\xi(\xi)d\xi$$

$$+\beta\int\limits_{\xi=x_{1,3}}^{\infty}\int\limits_{F_{2,3}=b}^{\infty}(F_{1,3}-\tilde{F}_{2,3})\phi_F^Q(F)dF\phi_\xi d\xi+\int\limits_{F_{2,3}=0}^{c}(F_{1,3}-\tilde{F}_{2,3})\phi_F^Q(F)dF=0$$

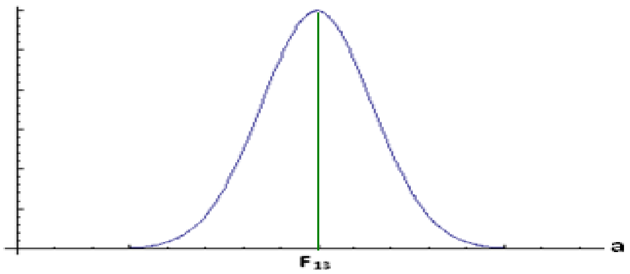
Similarly, the optimality condition can be expressed in the following way.

$$\frac{\partial J(x_{1,3}\,|\,L)}{\partial x_{1,3}} = \beta \int_{\xi=0}^{x_{1,3}} \int_{F_{2,3}=0}^{b} (F_{1,3} - \tilde{F}_{2,3})\phi_F^Q(F)dF\phi_\xi(\xi)d\xi$$

$$-\beta \int_{\xi=x_{1,3}}^{\infty} \int_{F_{2,3}=0}^{b} (F_{1,3} - \tilde{F}_{2,3})\phi_F^Q(F)dF\phi_\xi d\xi + \int_{F_{2,3}=0}^{c} (F_{1,3} - \tilde{F}_{2,3})\phi_F^Q(F)dF = 0$$

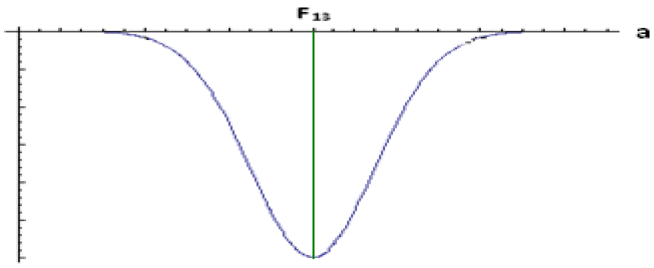
D.5) Behavior of the integrals in equation (8)

In Figure F.1 and F.2, representative integrals for a normally distributed *$F_{2,3}$* are given.



$$\int_{F_{2,3}=0}^{a} (F_{1,3} - \tilde{F}_{2,3})\phi_F(F)dF$$

Figure D.1 : Representative integral 1



$$\int_{F_{2,3}=a}^{\infty} (F_{1,3} - \tilde{F}_{2,3})\phi_F(F)dF$$

Figure D.2: Representative integral 2

The first figure is a representative for the first and third integrals in the general FOC equation. The second figure is a representative for the second integral in the general FOC equation. As it is seen from the figures, the integral in the first figure always takes positive values and reaches its largest value when *$a=F_{1,3}$*. The integral in the second figure always takes negative values and reaches its smallest value when *$a = F_{1,3}$*.

D.6) Proof of Theorem 4:

When $\Delta = 0$ and $(P-\lambda) = F_{1,3} + \delta$ where δ is a positive number showing the profit margin,

$c = F_{1,3}$ and $b = F_{1,3} + \dfrac{\eta_3 - \delta\xi}{(x_{1,3} - \xi)}$

Then, the general FOC equation becomes;

$$\frac{\partial J(x_{1,3} \mid L)}{\partial x_{1,3}} = \beta \int_{\xi=0}^{x_{1,3}} \int_{F_{2,3}=0}^{F_{1,3}+\frac{\eta_3-\delta\xi}{(x_{1,3}-\xi)}} (F_{1,3} - \tilde{F}_{2,3})\phi_F^Q(F)dF\phi_\xi(\xi)d\xi +$$

$$\beta \int_{\xi=x_{1,3}}^{\infty} \int_{F_{2,3}=F_{1,3}+\frac{\eta_3-\delta\xi}{(x_{1,3}-\xi)}}^{\infty} (F_{1,3} - \tilde{F}_{2,3})\phi_F^Q(F)dF\phi_\xi(\xi)d\xi + \int_{F_{2,3}=0}^{F_{1,3}} (F_{1,3} - \tilde{F}_{2,3})\phi_F^Q(F)dF$$

$If\ x_{1,3} = 0;$

$$\frac{\partial J(x_{1,3} \mid L)}{\partial x_{1,3}} = -\beta \int_{\xi=0}^{\infty} \int_{F_{2,3}=0}^{F_{1,3}+\delta-\frac{\eta_3}{\xi}} (F_{1,3} - \tilde{F}_{2,3})\phi_F^Q(F)dF\phi_\xi(\xi)d\xi + \int_{F_{2,3}=0}^{F_{1,3}} (F_{1,3} - \tilde{F}_{2,3})\phi_F^Q(F)dF > 0$$

Since the function is convex, increase in $x_{1,3}$ will increase the total FDC, so $x_{1,3}^* = 0$ when $\Delta = 0$.

When $\Delta \neq 0$,

$b = F_{1,3} - \dfrac{\delta\xi + \Delta - \eta_3}{(x_{1,3} - \xi)}$ and $c = F_{1,3} - \dfrac{\Delta}{x_{1,3}}$

When $x_{1,3} = 0$, the general FOC equation becomes,

$$\frac{\partial J(x_{1,3} \mid .)}{x_{1,3}} = \int_{\xi=0}^{\infty} \int_{F_{2,3}=b}^{\infty} (F_{1,3} - F_{2,3})\phi_F^Q(F)dF\phi(\xi)d\xi < 0$$

Since the function is convex, $x_{1,3}^* > 0$.

D.7) Proof of Theorem 5:

In the general FOC equation in (8),

As $\Delta \to \infty$; $b \to -\infty$ and $c \to -\infty$

As $\Delta \to -\infty$; $b \to \infty$ and $c \to \infty$

In either case; the general FOC equation equals to 0; without depending on the value of $x_{1,3}$; so the firm is indifferent about the hedging quantity.

D.8) Proof of Theorem 6:

If $\Delta + \delta * \overline{\xi} = \eta_3$, then

$$b = \frac{F_{1,3}x_{1,3} - (F_{1,3} + \delta)\xi + \delta * \overline{\xi}}{(x_{1,3} - \xi)} \text{ and } c = F_{1,3} - \frac{\Delta}{x_{1,3}}$$

If $x_{1,3} = \overline{\xi}$; then $b = F_{1,3} + \delta$

So;

$$\frac{\partial J(x_{1,3}\,|\,.)}{x_{1,3}} = \beta \int_{\xi=0}^{x_{1,3}} \int_{F_{2,3}=0}^{F_{1,3}+\delta} (F_{1,3} - F_{2,3})\phi_F^Q(F)dF\phi_\xi(\xi)d\xi$$

$$-\beta \int_{\xi=x_{1,3}}^{\infty} \int_{F_{2,3}=0}^{F_{1,3}+\delta} (F_{1,3} - F_{2,3})\phi_F^Q(F)dF\phi_\xi(\xi)d\xi + \int_{F_{2,3}=0}^{F_{1,3}-\frac{\Delta}{\overline{\xi}}} (F_{1,3} - F_{2,3})\phi_F^Q(F)dF$$

If $\Phi_F(F_{1,3} - \frac{\Delta}{\overline{\xi}}) = 0$; equally $\Delta \geq \mathrm{F}_{1,3}\overline{\xi}$; which means the firm has enough cash

$$\frac{\partial J(x_{1,3}\,|\,.)}{x_{1,3}}\Big|_{x_{1,3}=\overline{\xi}} = 0 \text{ so, the optimal } x_{1,3}^* = \overline{\xi}$$

D.9) Proof of Theorem 7:

It is easier to prove that theorem by discretizing the integrals in equation (8).

A double integral can be simplified to a single integral as follows if x is normally distributed and when x can only take integer values;

$$\int_{x=a}^{x=b} \int_{y=0}^{h(x)} f(x,y)dydx = \sum_{x=a}^{x=b} \left[P(x = x_i) \int_{y=0}^{h(x)} f(x,y)dy \right]$$

where $P(x = x_i)$ is the probability of *x* taking x_i value

$$\int_{x=a}^{x=b} \int_{y=0}^{h(x)} f(x,y)dxdy = P(x=a) \int_{y=0}^{h(x=a)} f(x,y)dxdy + P(x=a+1) \int_{y=0}^{h(x=a+1)} f(x,y)dxdy +$$

$$... + P(x=b) \int_{y=0}^{h(x=b)} f(x,y)dxdy$$

This discretizing approach is a realistic approach since quantity can take only integer values.

Using that discretizing approach, the theorem can be proved as follows:

$$\text{If } \Delta+\delta\overline{\xi}<\eta_3 \text{ and } \Delta+\delta\overline{\xi}=\eta_3^*;$$
$$\text{then}, \eta_3=\eta_3^*+\psi \text{ where } \psi>0$$
$$b=\frac{F_{1,3}x_{1,3}-(F_{1,3}+\delta)\xi+(\eta_3^*-\Delta)+\psi}{(x_{1,3}-\xi)}=F_{1,3}+\frac{-\delta\xi+\delta\overline{\xi}+\psi}{(x_{1,3}-\xi)}$$

$$\text{If } x_{1,3}=\overline{\xi};$$
$$b=F_{1,3}+\delta+\frac{\psi}{(\overline{\xi}-\xi)} \text{ and } c=F_{1,3}-\frac{\Delta}{\overline{\xi}}$$

$$\frac{\partial J(x_{1,3}\,|.)}{x_{1,3}}\Big|_{x_{1,3}=\overline{\xi}}=\int_{\xi=0}^{x_{1,3}}\int_{F_{2,3}=0}^{F_{1,3}+\delta+\frac{\psi}{(\overline{\xi}-\xi)}}(F_{1,3}-F_{2,3})\phi_F^Q(F)dF\phi_\xi(\xi)d\xi$$
$$-\int_{\xi=x_{1,3}}^{\infty}\int_{F_{2,3}=0}^{F_{1,3}+\delta+\frac{\psi}{(\overline{\xi}-\xi)}}(F_{1,3}-F_{2,3})\phi_F^Q(F)dF\phi_\xi(\xi)d\xi+\beta\int_{F_{2,3}=0}^{F_{1,3}-\frac{\Delta}{\overline{\xi}}}(F_{1,3}-F_{2,3})\phi_F^Q(F)dF$$

For ease of notation, we make the following abbreviation;

$$\int_{F_{2,3}=0}^{F_{1,3}+\delta+\frac{\psi}{(\overline{\xi}-\xi)}}(F_{1,3}-F_{2,3})\phi_F^Q(F)dF=\int_{F_{2,3}=0}^{F_{1,3}+\delta+\frac{\psi}{(\overline{\xi}-\xi)}}..$$

$$\frac{\partial J(x_{1,3}\,|.)}{x_{1,3}}\Big|_{x_{1,3}=\overline{\xi}}=\beta\left\{P(\xi=\xi_{\min})\int_{F_{2,3}=0}^{F_{1,3}+\delta+\frac{\psi}{(\overline{\xi}-\xi_{\min})}}..+......+P(\xi=\overline{\xi}-2\alpha)\int_{F_{2,3}=0}^{F_{1,3}+\delta+\frac{\psi}{2\alpha}}..+P(\xi=\overline{\xi}-\alpha)\int_{F_{2,3}=0}^{F_{1,3}+\delta+\frac{\psi}{\alpha}}..\right\}$$
$$-\beta\left\{P(\xi=\overline{\xi}+\alpha)\int_{F_{2,3}=0}^{F_{1,3}+\delta-\frac{\psi}{\alpha}}..+P(\xi=\overline{\xi}+2\alpha)\int_{F_{2,3}=0}^{F_{1,3}+\delta-\frac{\psi}{2\alpha}}..+.......+P(\xi=\xi_{\max})\int_{F_{2,3}=0}^{F_{1,3}+\delta-\frac{\psi}{(\xi_{\max}-\overline{\xi})}}..\right\}+\int_{F_{2,3}=0}^{F_{1,3}-\frac{\Delta}{\overline{\xi}}}..$$

$$\text{Since } \Phi_F(F_{1,3}+\delta+\frac{\psi}{(\overline{\xi}-\xi_{\min})})\geq\Phi_F(F_{1,3}+\delta-\frac{\psi}{(\xi_{\max}-\overline{\xi})});..........;\Phi_F(F_{1,3}+\delta+\frac{\psi}{\alpha})\geq\Phi_F(F_{1,3}+\delta-\frac{\psi}{\alpha})$$

1st integral -2nd integral) $\leq$ 0.

Moreover, when $\Phi_F(F_{1,3}-\frac{\Delta}{x_{1,3}^*})=0$; i.e.; $\Delta>F_{1,3}\overline{\xi}$ which means the firm has enough cash. Then,

$$\frac{\partial J(x_{1,3}\,|.)}{x_{1,3}}|_{x_{1,3}=\bar{\xi}} \leq 0,\ \text{so } \mathrm{x}_{1,3}^{*} \geq \bar{\xi}$$

In this approach, we assume that minimum price is 0 and maximum price is infinity. Moreover, we assume that this assumption does not violate the symmetricity of price.

D.10) Proof of Theorem 8:

Proof of first case:

The proof is similar to the proof of Theorem 6.

$$If\ \ \mathrm{x}_{1,3} = \bar{\xi};$$

$$b = F_{1,3} + \delta \quad \mathrm{c}{=}F_{1,3} - \frac{\Delta}{\bar{\xi}}$$

$$So;$$

$$\frac{\partial J(x_{1,3}\,|.)}{x_{1,3}} = \beta \int_{\xi=0}^{x_{1,3}} \int_{F_{2,3}=0}^{F_{1,3}+\delta} (F_{1,3} - F_{2,3}) \phi_F^Q(F) dF \phi_\xi(\xi) d\xi$$

$$-\beta \int_{\xi=x_{1,3}}^{\infty} \int_{F_{2,3}=0}^{F_{1,3}+\delta} (F_{1,3} - F_{2,3}) \phi_F^Q(F) dF \phi_\xi(\xi) d\xi + \int_{F_{2,3}=0}^{F_{1,3}-\frac{\Delta}{\bar{\xi}}} (F_{1,3} - F_{2,3}) \phi_F^Q(F) dF$$

If $\Phi_F(F_{1,3} - \frac{\Delta}{\bar{\xi}}) \neq 0;$ and equally $\Delta < F_{1,3}\bar{\xi}$, which means the firm has not enough cash

$$\frac{\partial J(x_{1,3}\,|.)}{x_{1,3}}|_{x_{1,3}=\bar{\xi}} > 0 \ \ \text{so, the optimal } x_{1,3}^{*} < \bar{\xi}$$

Proof of second case:

$$If\ \ \mathrm{x}_{1,3} = \bar{\xi};$$

$$b = F_{1,3} + \delta - \frac{\psi}{(\bar{\xi} - \xi)} \ \ \mathrm{c}{=}\mathrm{F}_{1,3} - \frac{\Delta}{\bar{\xi}}$$

$$\frac{\partial J(x_{1,3}\,|.)}{x_{1,3}}|_{x_{1,3}=\bar{\xi}} = \int_{\xi=0}^{x_{1,3}} \int_{F_{2,3}=0}^{F_{1,3}+\delta-\frac{\psi}{(\bar{\xi}-\xi)}} (F_{1,3} - F_{2,3}) \phi_F^Q(F) dF \phi_\xi(\xi) d\xi$$

$$- \int_{\xi=x_{1,3}}^{\infty} \int_{F_{2,3}=0}^{F_{1,3}+\delta-\frac{\psi}{(\bar{\xi}-\xi)}} (F_{1,3} - F_{2,3}) \phi_F^Q(F) dF \phi_\xi(\xi) d\xi + \beta \int_{F_{2,3}=0}^{F_{1,3}-\frac{\Delta}{\bar{\xi}}} (F_{1,3} - F_{2,3}) \phi_F^Q(F) dF$$

By using the discretizing approach,

$$
\begin{aligned}
&If\ \ \Delta + \delta\bar{\xi} = \eta_3^*; \\
&\Delta + \delta\bar{\xi} > \eta_3\ ,\ \eta_3 = \eta_3^* - \psi \\
&b = \frac{F_{1,3}x_{1,3} - (F_{1,3}+\delta)\xi + (\eta_3^* - \Delta) - \psi}{(x_{1,3}-\xi)} = F_{1,3} + \frac{-\delta\xi + \delta\bar{\xi} - \psi}{(x_{1,3}-\xi)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial J(x_{1,3}\,|.)}{x_{1,3}}\Big|_{x_{1,3}=\bar{\xi}} = \beta\left\{ P(\xi=\xi_{\min}) \int_{F_{2,3}=0}^{F_{1,3}+\delta-\frac{\psi}{(\bar{\xi}-\xi_{\min})}} .. + ...+ P(\xi=\bar{\xi}-2\alpha) \int_{F_{2,3}=0}^{F_{1,3}+\delta-\frac{\psi}{2\alpha}} .. + P(\xi=\bar{\xi}-\alpha) \int_{F_{2,3}=0}^{F_{1,3}+\delta-\frac{\psi}{\alpha}} .. \right\} \\
-\beta\left\{ P(\xi=\bar{\xi}+\alpha) \int_{F_{2,3}=0}^{F_{1,3}+\delta+\frac{\psi}{\alpha}} .. + P(\xi=\bar{\xi}+2\alpha) \int_{F_{2,3}=0}^{F_{1,3}+\delta+\frac{\psi}{2\alpha}} .. + ... + P(\xi=\xi_{\max}) \int_{F_{2,3}=0}^{F_{1,3}+\delta+\frac{\psi}{(\xi_{\max}-\bar{\xi})}} .. \right\} + \int_{F_{2,3}=0}^{F_{1,3}-\frac{\Delta}{\bar{\xi}}} ..
\end{aligned}
$$

$$
\begin{aligned}
&\text{Since } \Phi_F(F_{1,3} + \delta + \frac{\psi}{(\xi_{\max}-\bar{\xi})}) \ge \Phi_F(F_{1,3} + \delta - \frac{\psi}{(\bar{\xi}-\xi_{\min})})\ ;...... \\
&.....;\ \Phi_F(F_{1,3} + \delta + \frac{\psi}{\alpha}) \ge \Phi_F(F_{1,3} + \delta - \frac{\psi}{\alpha})
\end{aligned}
$$

(1st integral-2nd integral) $\geq$ 0. Since 3rd integral is nonnegative,

$$
\frac{\partial J(x_{1,3}\,|.)}{x_{1,3}}\Big|_{x_{1,3}=\bar{\xi}} \ge 0,\ \text{so } x_{1,3}^* \le \bar{\xi}
$$

D.11) Proof of Theorem 9:

The proof is similar to the proof of Theorem 7.

$$
\begin{aligned}
\frac{\partial J(x_{1,3}\,|.)}{x_{1,3}}\Big|_{x_{1,3}=\bar{\xi}} = \int_{\xi=0}^{x_{1,3}} \int_{F_{2,3}=0}^{F_{1,3}+\delta+\frac{\psi}{(\bar{\xi}-\xi)}} (F_{1,3}-F_{2,3})\phi_F^Q(F)dF\phi_\xi(\xi)d\xi \\
-\int_{\xi=x_{1,3}}^{\infty} \int_{F_{2,3}=0}^{F_{1,3}+\delta+\frac{\psi}{(\bar{\xi}-\xi)}} (F_{1,3}-F_{2,3})\phi_F^Q(F)dF\phi_\xi(\xi)d\xi + \beta \int_{F_{2,3}=0}^{F_{1,3}-\frac{\Delta}{\bar{\xi}}} (F_{1,3}-F_{2,3})\phi_F^Q(F)dF
\end{aligned}
$$

$$\frac{\partial J(x_{1,3}|.)}{x_{1,3}}|_{x_{1,3}=\bar{\xi}}=\beta\left\{P(\xi=\xi_{\min})\int_{F_{2,3}=0}^{F_{1,3}+\delta+\frac{\psi}{(\bar{\xi}-\xi_{\min})}}..+......+P(\xi=\bar{\xi}-2\alpha)\int_{F_{2,3}=0}^{F_{1,3}+\delta+\frac{\psi}{2\alpha}}..+P(\xi=\bar{\xi}-\alpha)\int_{F_{2,3}=0}^{F_{1,3}+\delta+\frac{\psi}{\alpha}}..\right\}$$

$$-\beta\left\{P(\xi=\bar{\xi}+\alpha)\int_{F_{2,3}=0}^{F_{1,3}+\delta-\frac{\psi}{\alpha}}..+P(\xi=\bar{\xi}+2\alpha)\int_{F_{2,3}=0}^{F_{1,3}+\delta-\frac{\psi}{2\alpha}}..+.......+P(\xi=\xi_{\max})\int_{F_{2,3}=0}^{F_{1,3}+\delta-\frac{\psi}{(\xi_{\max}-\bar{\xi})}}..\right\}+\int_{F_{2,3}=0}^{F_{1,3}-\frac{\Delta}{\bar{\xi}}}..$$

Since $\Phi_F(F_{1,3}+\delta+\frac{\psi}{(\bar{\xi}-\xi_{\min})})\geq\Phi_F(F_{1,3}+\delta-\frac{\psi}{(\xi_{\max}-\bar{\xi})})$;..........; $\Phi_F(F_{1,3}+\delta+\frac{\psi}{\alpha})\geq\Phi_F(F_{1,3}+\delta-\frac{\psi}{\alpha})$

(1^{st}integral-2^{nd}integral) ≤ 0

When $0<\Delta<F_{1,3}\bar{\xi}$ or $\Delta<0$, 3^{rd} integral has a positive value so;

$\frac{\partial J(x_{1,3}|.)}{x_{1,3}}|_{x_{1,3}=\bar{\xi}}$ may be positive or negative, so $x^*_{1,3}\geq\bar{\xi}$ or $x^*_{1,3}\leq\bar{\xi}$

D.12) Proof of Theorem 10:

$$\frac{\partial\left(\frac{J(x_{1,3}|.)}{x_{1,3}}\right)}{\partial\delta}=-\int_{\xi=0}^{x_{1,3}}\frac{\xi(\xi\delta+\Delta)}{(x_{1,3}-\xi)^2}\phi(F')\phi(\xi)d\xi+\int_{\xi=x_{1,3}}^{\infty}\frac{\xi(\xi\delta+\Delta)}{(x_{1,3}-\xi)^2}\phi(F')\phi(\xi)d\xi$$

where $\phi(F')$ means replacing $F_{2,3}$ with b in density function

The derivative of the equation in (8) wrt to δ is positive since $x_{1,3}<\bar{\xi}$. So, as δ increases, the equation in (8) increases, which means the optimal $x_{1,3}$ should decrease.

D.13) Proof of Theorem 11:

By using the Leibniz rule, first derivative of the function is given as;

$$\frac{\partial J(x_{1,3}|L)}{\partial x_{1,3}}=r\int_{\xi=0}^{x_{1,3}}\int_{s_3=0}^{b}(F_{1,3}-\tilde{s}_3)\phi_s^Q(s)ds\phi_\xi(\xi)d\xi+r\int_{\xi=x_{1,3}}^{\infty}\int_{s_3=b}^{\infty}(F_{1,3}-\tilde{s}_3)\phi_s^Q(s)ds\phi_\xi d\xi$$

The second derivative is;

$$\frac{\partial^2 J(x_{1,3}|L)}{\partial x_{1,3}{}^2}=r\int_{\xi=0}^{x_{1,3}}\frac{\left((P-\lambda-F_{1,3})\xi-\eta_3-\eta_2+y_1\right)^2}{(x_{1,3}-\xi)^3}\phi(F')\phi(\xi)d\xi$$

$$+r\int_{\xi=x_{1,3}}^{\infty}\frac{-\left((P-\lambda-F_{1,3})\xi-\eta_3-\eta_2+y_1\right)^2}{(x_{1,3}-\xi)^3}\phi(F')\phi(\xi)d\xi\geq 0$$

where $\phi(F')$ means replacing s_3 in density function with $x_{1,3}$, $\phi(F'')$ means replacing s_3 in density function with *c.*

Since all terms in the second derivative equation are positive, second derivative is positive, so $\mathrm{J}(\mathrm{X}_{1,3}|\mathrm{L})$ is convex, and the optimality condition is;

$$\frac{\partial J(x_{1,3}\,|\,L)}{\partial x_{1,3}} = \int_{\xi=0}^{x_{1,3}}\int_{s_3=0}^{b}(F_{1,3}-\tilde{s}_3)\phi_s^Q(s)ds\phi_\xi(\xi)d\xi + \int_{\xi=x_{1,3}}^{\infty}\int_{s_3=b}^{\infty}(F_{1,3}-\tilde{s}_3)\phi_s^Q(s)ds\phi_\xi d\xi = 0$$

The optimality condition can be expressed also as;

$$\frac{\partial J(x_{1,3}\,|\,L)}{\partial x_{1,3}} = \int_{\xi=0}^{x_{1,3}}\int_{s_3=0}^{b}(F_{1,3}-\tilde{s}_3)\phi_s^Q(s)ds\phi_\xi(\xi)d\xi - \int_{\xi=x_{1,3}}^{\infty}\int_{s_3=0}^{b}(F_{1,3}-\tilde{s}_3)\phi_s^Q(s)ds\phi_\xi d\xi = 0$$

D.14) Proof of Theorem 12:

If $\eta_3 = \Delta + \bar{\xi}\delta$, then $b = F_{1,3} + \dfrac{-\delta\xi + \bar{\xi}\delta}{(x_{1,3}-\xi)}$

If $x_{1,3} = \bar{\xi}$,

$$\frac{\partial J(x_{1,3}\,|\,L)}{\partial x_{1,3}}\Big|_{x_{1,3}=\bar{\xi}} = \int_{\xi=0}^{x_{1,3}}\int_{s_3=0}^{F_{1,3}+\delta}(F_{1,3}-s_3)\phi_s^Q(s)ds\phi(\xi)d\xi - \int_{\xi=x_{1,3}}^{\infty}\int_{s_3=0}^{F_{1,3}+\delta}(F_{1,3}-s_3)\phi_s^Q(s)ds\phi(\xi)d\xi = 0$$

So, $x_{1,3}{}^* = \bar{\xi}$.

D.15) Proof of Theorem 13:

If $\eta_3 = \Delta + \bar{\xi}\delta + \psi$, then $b = \dfrac{-\delta\xi + \bar{\xi}\delta + \psi}{(x_{1,3}-\xi)}$

If $x_{1,3} = \bar{\xi}$,

$$b = F_{1,3} + \delta + \frac{\psi}{(\bar{\xi}-\xi)}$$

By using the discretizing approach,

$$\frac{\partial J(x_{1,3}\,|\,L)}{x_{1,3}}\Big|_{x_{1,3}=\bar{\xi}} = \beta\left\{P(\xi=\xi_{\min})\int_{F_{2,3}=0}^{F_{1,3}+\delta+\frac{\psi}{(\bar{\xi}-\xi_{\min})}}..+......+P(\xi=\bar{\xi}-2\alpha)\int_{F_{2,3}=0}^{F_{1,3}+\delta+\frac{\psi}{2\alpha}}..+P(\xi=\bar{\xi}-\alpha)\int_{F_{2,3}=0}^{F_{1,3}+\delta+\frac{\psi}{\alpha}}..\right\}$$

$$-\beta\left\{P(\xi=\bar{\xi}+\alpha)\int_{F_{2,3}=0}^{F_{1,3}+\delta-\frac{\psi}{\alpha}}..+P(\xi=\bar{\xi}+2\alpha)\int_{F_{2,3}=0}^{F_{1,3}+\delta-\frac{\psi}{2\alpha}}..+.......+P(\xi=\xi_{\max})\int_{F_{2,3}=0}^{F_{1,3}+\delta-\frac{\psi}{(\xi_{\max}-\bar{\xi})}}..\right\}+\int_{F_{2,3}=0}^{F_{1,3}-\frac{\Delta}{\bar{\xi}}}..$$

Since $\Phi_F(F_{1,3}+\delta+\frac{\psi}{(\bar{\xi}-\xi_{\min})}) \geq \Phi_F(F_{1,3}+\delta-\frac{\psi}{(\xi_{\max}-\bar{\xi})})$;..........; $\Phi_F(F_{1,3}+\delta+\frac{\psi}{\alpha}) \geq \Phi_F(F_{1,3}+\delta-\frac{\psi}{\alpha})$

(1^{st}integral-2^{nd}integral) ≤ 0

So, $\frac{\partial J(x_{1,3}\,|\,L)}{x_{1,3}}\Big|_{x_{1,3}=\bar{\xi}} \leq 0$, so $x^*_{1,3} \geq \bar{\xi}$

Proof of the other case is similar.