OPTIMIZATION OF ENERGY HARVESTING WIRELESS COMMUNICATION SYSTEMS

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I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

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ABSTRACT

OPTIMIZATION OF ENERGY HARVESTING WIRELESS COMMUNICATION SYSTEMS

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In an energy harvesting communication system, energy is derived from outside sources and becomes partially available at different points in time. The constraints induced by this property on energy consumption plays an active role in the design of efficient communication systems. This thesis focuses on the optimal design of transmission and networking schemes for energy harvesting wireless communication systems. In particular, an energy harvesting transmitter broadcasting data to two receivers in an AWGN broadcast channel assuming that energy harvests and data arrivals occur at known instants is considered. In this system, optimal packet scheduling that achieves minimum delay is analyzed. An iterative algorithm, DuOpt, that achieves the same structural properties as the optimal schedule is proposed. DuOpt is proved to obtain the optimal solution when weaker user data is ready at the beginning. A dual problem is defined and shown to be strictly convex. Taking advantage of the dual problem, uniqueness of the solution of the main problem is proved. Finally, it is observed that DuOpt is almost two orders of magnitude faster than the SUMT (sequential unconstrained minimization technique) algorithm that solves the same problem.

Keywords: wireless communication, packet scheduling, energy harvesting, broadcast channel, optimization

ÖZ

ENERJİ HARMANLAYAN KABLOSUZ HABERLEŞME SİSTEMLERİN OPTİMİZASYONU

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Enerji harmanlayan haberleşme sistemlerinde, enerji dış ortamdaki kaynaklardan elde edilir ve belirli zamanlarda parça parça kullanıma hazır hale gelir. Bu özellikle birlikte gelen enerji harcama sınırlamaları verimli haberleşme sistemlerinin dizaynında önemli bir rol almaktadır. Bu tez enerji harmanlayan kablosuz haberleşme sistemlerinde en iyi iletim ve ağ iletişimi tasarımı üzerine odaklanmaktadır. Enerji harmanlama ve veri geliş zamanları önceden belli olan, toplanır beyaz Gauss gürültülü tüme gönderim kanalına yayın yapan, enerji harmanlayan bir verici ele alınmıştır. Bu sistemde, minimum iletim zamanını elde eden en iyi paket çizelgeleme incelenmiştir. Bu çizelgelemenin özelliklerini elde eden döngülü bir algoritma, DuOpt, önerilmiştir. Güçsüz kullanıcı verisinin en başta hazır olduğu durumda DuOptun en iyi çözüme ulaştığı gösterilmiştir. Eşlek problemin kesin surette dışbükey olduğu gösterilmiş ve eşlek problemden faydalanılarak ana problemin çözümünün tekliği ispat edilmiştir. Son olarak, DuOptun aynı problemi çözen SUMT (ardışık sınırlandırılmamış enküçültme yordamı) algoritmasına göre neredeyse iki mertebe daha hızlı olduğu gözlenmiştir.

Anahtar Kelimeler: kablosuz haberleşme, paket çizelgeleme, enerji harmanlama, tüme gonderim kanalı, eniyileme

To my family

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CHAPTER 1

INTRODUCTION

A recent study estimates that the energy consumption of ICT is nearly 4% of the worldwide energy consumption and this percentage is rising each year [1]. Energy efficiency of ICT plays an important role in the worldwide energy consumption and it has been an active research area since last two decades. With the emergence of battery limited wireless devices, i.e., cell phones, sensor nodes etc., energy efficiency become more and more important. These energy-limited devices can be used until the batteries run out of energy, then we need to recharge them. Recent improvements in energy harvesting techniques have made it feasible to recharge battery limited wireless devices by harvesting ambient energy from the environment. For example, many cell phone producers released solar energy powered cell phones so that users do not need to charge phones very frequently. Energy harvesting is even more critical for the sensor networks because in most cases recharging operation is very hard as it needs redeployment and in some cases it is even impossible, i.e., a sensor network deployed into an enemy territory.

Transmission delay is an important factor that determines the quality of service. Although energy efficiency is an important parameter to optimize, there is a fundamental trade-off between delay and energy [2, 3]. Energy minimization with respect to delay requirements has been studied widely [4, 5, 6, 7, 8, 9]. In this thesis we consider the delay minimization problem with respect to energy and data constraints. Specifically, we study the delay minimization problem in a two user broadcast channel with an energy harvesting transmitter. Energy harvests and data arrivals are assumed to occur arbitrarily but initially known instants and amounts. We have developed an efficient iterative algorithm , *DuOpt*, which is, under a special case, shown to achieve optimal schedule by locally optimizing consecutive epochs. We provide a detailed analysis on the local optimization problem. We have also solved the problem by a different approach using a dual problem in which energy is minimized given a transmission deadline. Then we compare two algorithms in terms of performance. In the following we present the outline of this thesis.

In Chapter 2, we provide background information on energy harvesting networks. We start by surveying recent literature on energy harvesting [10]. First energy harvesting models are presented and then information theoretic bounds on different channels are discussed. Since we assumed that data arrivals and energy harvest are known initially, we give a recent survey of offline studies in energy harvesting networks and state where our study lies in it. In Chapter 2, we also briefly discuss the information theoretic broadcast channel and provide some important properties on two user broadcast channel.

In Chapter 3, we begin with presenting the system model used throughout this thesis. Then, we restudy the problem stated in [11]. Providing some additional properties we prove that the solution of the problem is unique and algorithm proposed in [11] achieves the optimal solution [12, 13]. Afterwards, we extend the problem in [11] and provide some further properties to it [14, 15]. We propose an efficient iterative algorithm, *DuOpt*, to solve the extended problem. Under a special case we show that the solution of problem is unique and it can be obtained by *DuOpt* algorithm. We present numerical examples and discuss the results. Then, we study the dual of the extended problem. We prove that the dual problem is strictly convex and using this property we show that the solution of the extended problem is unique. We then present a variety of sequential unconstrained minimization technique [38] to solve the dual problem. Using the solution of dual problem, we propose an different algorithm to solve the extended problem. Finally, we compare the performance of this new algorithm and *DuOpt*.

In Chapter 4, we give final remarks and conclusions. We also briefly state the future work.

CHAPTER 2

BACKGROUND INFORMATION

2.1 Recent Developments in Energy Harvesting Networks

Energy efficiency has been considered as an important issue in the design of network architectures since the emergence in the last decade of sensor networks composed of small energy-limited autonomous units with wireless communication capability. Nodes of such networks are generally assumed to have limited energy supplies. When energy of a sensor node is depleted, it may not be replenished again, and commonly the node is assumed to be *dead*. Therefore, the *lifetime* of such a network is seriously dependent on the initial energies stored at nodes as well as the energy efficiency of network protocols. There is a vast body of literature [16] of transmission schemes and network protocols that strive to maximize network lifetime or related objective functions.

Recent developments in ambient energy harvesting technologies allow battery-limited devices to bear their own energy cost, so that network can sustain itself. In this case, as each energy harvest extends the network lifetime, the lifetime can in principle be unbounded. However, while the total energy available to the network may be unbounded, the amount harvested during each finite time interval may be erratic, which can make it challenging to maintain a continuous communication rate or quality, respecting delay constraints. Usually, energy is assumed to become available in time as harvested and the energy harvest process is a major factor determining the energy consumption schedules. This new set of constraints which introduce a new twist on communication as well as network problems have sparked active research effort in recent years. This section makes a short review of this very recent body of literature. We shall start by describing commonly used energy harvesting models and assumptions. Next, we shall review information theoretic studies considering the capacity of energy harvesting single- and multi-user channels, before turning our attention to *scheduling* problems. We will then review offline transmission scheduling formulations with different objectives, and the solutions of these problems.

2.1.1 Energy Harvesting Models

Various energy harvesting methods have been proposed in the literature [17, 18, 19, 20]. Solar and piezoelectric energy harvesting are among the most promising due to their high power densities. Different harvesting technologies lead to entirely different energy harvesting profiles. For example, in solar energy harvesting, the profile may follow a daily period where the peaks occur during midday, and the minima at night.

In conjunction with the harvesting profile, battery technology, in particular battery capacity and efficiency of energy storage circuitry are among significant parameters affecting the utilization of harvested energy. For example, when the battery capacity is small with respect to energy expected to be harvested during a peak harvest, some or perhaps most of the energy will be wasted. In this respect, regardless of the energy harvesting profile, bigger batteries mean better exploitation of energy harvests. In practice, battery size is limited. Since increasing battery size means increased node cost, it should be carefully optimized according to the needs of the system [21]. Inefficiencies in the storage circuitry also directly affect the utilization of harvested energy. With current battery technologies a portion of the harvested energy is lost due to leakage or other inefficiencies in the charging circuitry.

Although physically energy harvesting is typically continuous in time, in some works in the literature, it has been found convenient to model harvests that occur at discrete points in time, which corresponds to codeword or packet durations being much longer compared to the recharging durations.

2.1.2 Information Theoretic Bounds

Information-theoretic capacity of a point-to-point communication system with energy harvesting transmitter in AWGN channel has been considered in [22] and [23]. Assuming an energy harvesting transmitter with infinite capacity battery and a constant average energy harvest rate over large time scale, [22] and [23] show that by essentially introducing sufficient delay, the information-theoretic AWGN channel capacity of the communication system is equal to the capacity of a single transmitter under AWGN channel with an average power constraint, which is equal to the average energy harvest rate. Two different capacity achieving schemes, *save-and-transmit* and *best-effort-transmit*, are presented in [22]. The former scheme waits and stores energy initially for some time such that no energy scarcity will be experienced once transmission at the calculated average power, P, starts. On the other hand, the latter scheme acts more aggressively such that it transmits whenever enough energy is available in the battery to transmit a codeword at power P, and stays idle otherwise. The case where energy harvest rate varies with time is also considered and a capacity achieving power management scheme is proposed in [22].

While many studies focused solely on transmission energy, in practical systems other types of energy consumption, such as that occurs in the processor, sensor unit etc., can be significant. Information-theoretic analysis of an energy harvesting device with energy consuming components has been considered in [23] and the AWGN channel capacity of the system is found to be equal to the AWGN channel capacity of the ideal system with average energy harvest rate reduced by the average processor power consumption. A *randomized sleep policy* is found to be useful under processor energy consumption assumption.

As pointed out in subsection 2.1.1, batteries have limited capacity and not perfectly efficient in using that finite capacity. In [23], the capacity of an AWGN channel with an energy harvesting transmitter with an imperfect battery is shown to be equal to that of a system with average energy harvest rate P, where P is the maximum average transmission power that the battery can supply. The capacity of a fading Gaussian channel with energy harvests is investigated in [24], where the analysis starts with the following idealized assumptions: (1) energy is consumed solely by transmission, (2) battery capacity is infinite, (3) there is perfect CSI at the transmitter (CSIT), (4) there are no inefficiencies in storage. The harvesting process is modelled as stationary ergodic process, and the ergodic capacity is shown to be equal to that of the AWGN fading channel with average power constraint equal to the mean energy harvest rate. The capacity achieving scheme is instantaneous power allocated according to "water filling". Assumptions (2)-(4) are then relaxed and corresponding capacity expressions are derived. Assumption (4) is relaxed to include the following energy-harvesting architectures: HSU: Harvest-Store-Use (infinite/finite buffer), HUS: Harvest-Use-Store (infinite/finite buffer) and HU: Harvest-Use (no buffer). It is observed that HSU performs bad as compared to HUS when buffer size is small due to fact that in HSU, the harvested energy can be stored only up to the maximum buffer size and the rest is wasted. As the buffer size increases, the performance gap between the two architectures is shown to decrease, and they both approach the ideal. Moreover, as the inefficiency in storage increases, it is observed that HUS performs better than HSU without CSIT. But when there is no CSIT and energy store efficiency is very poor, the authors claim that the HU architecture is superior.

It is worth emphasizing that the above mentioned capacity results require coding over long blocks of data and harvests, and the delay required to approach these in practice may be excessive. The energy harvest profile plays an important part in determining how relevant the capacity results are in a given finite time horizon. The next subsection is devoted to scheduling algorithms that consider *delay* as an objective or constraint and we shall review *offline* approaches that model the harvest sequence as an arbitrary, but deterministic and known sequence.

2.1.3 Offline Problems

Definition of scheduling problems in energy harvesting networks are highly based on *energy harvesting profiles*. The profile of energy replenishment, and the relationship of this to the other problem parameters is a major factor determining the characteristics of optimal transmission policies for energy harvesting networks. In this subsection, we shall review studies that have assumed an energy harvest profile that is known in an *offline* manner.

In [25], the problem of minimizing transmission time of a given finite number of bits on a point-to-point energy harvesting link with AWGN noise is considered. Energy is assumed to be harvested in discrete units at arbitrary yet known points in time and infinite battery capacity is assumed. Some notable features of the unique optimal power/rate policy is found to be as follows

1. The transmission power is monotone non-decreasing in time, over the duration of the schedule.
2. The transmission power is held constant in between two packet arrival or energy harvest events.

3. Whenever transmission power increases, either the total harvested energy up to that instant has been consumed or all the number of packets arrived up to that instant has been transmitted.

An example of the optimal policy has been shown in Fig.2.1. An offline iterative algorithm that finds the optimal schedule has also been described in [25].

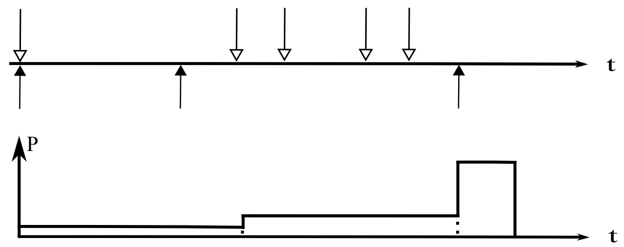

Figure 2.1: An optimal power schedule for a point-to-point energy harvesting AWGN link with infinite battery capacity. Top arrows show energy harvests and the bottom solid arrows indicate bit arrival times. Numerical values are omitted for simplicity [25].

A variation of this problem with finite battery capacity is considered in [26]. This practically motivated additional constraint complicates the problem and changes the structure of the solution, as it is not possible to replenish energy more than the battery capacity. The optimal transmission power profile is different from that described above, for example power may decrease in time. As expected, this decrease can only occur if the battery is fully depleted. Assuming no more amount of energy is available to be harvested than the battery size, it has also been shown that any schedule causing overflow of the battery is suboptimal. Assuming enough data available at the beginning, an algorithm which maximizes the throughput given a deadline is presented. Using the duality relationship between throughput maximization and transmission completion time minimization, a modified version of the algorithm which minimizes the transmission completion time is proposed.

Assuming that all the user data are ready at the beginning of transmission, the transmission completion time minimization problem has been extended to the energy harvesting broadcast channel in the concurrent works [13, 27]. Under a set of properties about the broadcast channel achievable rate region also satisfied by the AWGN broadcast channel, [13] has obtained the structure of the optimal policy, which is the unique optimal policy derived for the AWGN broadcast channel in [27]. It is shown that both the total transmit power as well as individual power levels allocated to each of the two users exhibit the same properties as in the single-user case. Again it is proven that the optimal schedule completes transmissions to both users at same time. In [27], the problem is shown to reduce to a problem of finding a cut-off power level for the strong user that splits power schedules of users for optimal power allocation and an iterative algorithm to attain optimal schedule has been proposed. The main objective in the algorithm is to reduce the problem to the single user problem as much as possible. In [13] the offline broadcast channel scheduling problem has been solved with a polynomial-time iterative algorithm, a slightly modified version of the *FlowRight* algorithm [28]. *FlowRight* gradually reaches the globally optimal schedule by computing a local optimization on two consecutive epochs at a time, and passes through all consecutive epoch pairs in each iteration. Each iteration strictly improves the schedule, and the algorithm can be stopped when the difference between iterations is sufficiently small.

The offline assumption about harvests (and data arrivals) facilitates the finding of optimal schedules by formulating the problem as an optimization problem. While the offline assumption may not be practical in some scenarios, where energy harvests or data arrive in an uncontrolled fashion, the offline approaches at least provide benchmarks and bounds to the best performance that can be achieved. Also, it may be possible to extend these to obtain online algorithms through the use of *look-ahead* buffers [5].

In general, arrivals of data and harvests may be modelled as stochastic processes. The analysis of such models is known to be less tractable, yet several different approaches have appeared to date [21, 29, 30, 31, 32, 33, 34, 35]. Online algorithms are out of the scope of this thesis; hence, they are not included in this chapter.

The contributions to the literature made by this thesis are as follows: In chapter 3 of [11], M. Akif Antepli has considered the transmission completion time minimization of a transmitter in AWGN broadcast channel assuming that all the packets to be transmitted are ready at the beginning of the schedule. Several important properties of the optimal schedule and an iterative algorithm, *FlowRight*, that solves the problem has also given in [11]. In this thesis, we first revisit the problem in [11] and providing some additional results about the optimal schedule and *FlowRight*, prove that the optimal schedule is unique. Uniqueness of the optimal schedule can be inferred from the idea in [27] that there is a cut-off level for the total transmit power, below which no transmission is made to the user with the smaller channel gain. However, the proof of uniqueness in this thesis is based on the structural properties of an optimal *two epoch* schedule; hence, it can be inferred that if every two consecutive epochs of a schedule is optimal, then the schedule is the unique global optimal schedule. As *FlowRight* optimize every consecutive epoch pair, with an analytical approach, we prove that the *FlowRight* returns the optimal schedule. After that, we extend the problem considering the case that packet arrivals occur during transmission, which is the main contribution of this thesis. The structural properties of the optimal schedule when data arrivals occur during transmission is studied in [14, 15]. The local optimizations that respect these properties are analyzed. Then, under a special case, no packet arrival occurs for the 2^{nd} user during transmission, the uniqueness of the optimal schedule is proved. And an iterative algorithm, *DuOpt*, is proposed and proved that it obtains the optimal schedule under specified special case. Then, we define a dual problem that minimize energy given a transmission deadline. The dual problem turns out to be strictly convex and has a unique solution. Using the dual problem we prove that main problem has also a unique solution. We describe another algorithm, which utilize sequential unconstrained minimization technique (SUMT), to solve the main problem and compare the performance of this algorithm with *DuOpt*. We observed that *DuOpt* runs nearly two orders of magnitude faster than the other algorithm.

In the next section, some important properties of the broadcast channel that characterize the problem are provided.

2.2 Broadcast Channel

Broadcasting is one of the earliest communication methods throughout the history. Considering the simplest example, talking is a way of broadcasting information if there are more than one listeners. In a formal definition, *broadcast channel* is a communication channel that distributes information from one sender to at least two receivers [36]. A two-user broadcast channel is illustrated in Figure 2.2.

Suppose that we have a two-user additive white Gaussian noise broadcast channel (AWGN BC). The received signals at the receivers are,

$$Y_1 = \sqrt{s_1}X + Z_1 \tag{2.1}$$

$$Y_2 = \sqrt{s_2}X + Z_2, \tag{2.2}$$

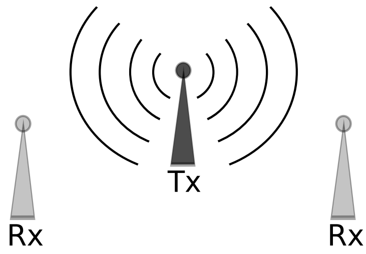

Figure 2.2: A broadcast channel with two receivers.

where X is the signal transmitted with average power constraint P, $\sqrt{s_1}$ and $\sqrt{s_2}$ are the channel gains, Z_1 and Z_2 are zero mean Gaussian noise with variance σ^2. Assuming $s_1 > s_2 > 0$, the receiver with the greater channel gain, $\sqrt{s_1}$, is defined as the *stronger* user where the receiver with the smaller channel gain, $\sqrt{s_2}$, is defined as the *weaker* user. The capacity region of this AWGN broadcast channel is defined by

$$r_1 \leq \frac{1}{2} log_2 \left(1 + \frac{s_1 \alpha P}{\sigma^2}\right) \tag{2.3}$$

$$r_2 \leq \frac{1}{2} log_2 \left(1 + \frac{s_2(1-\alpha)P}{s_2 \alpha P + \sigma^2}\right), \tag{2.4}$$

where α is the fraction of power reserved for the stronger user by the transmitter [36]. Transmitter first encodes the stronger user data with rate r_1 and power $P_1 = \alpha P$, and the weaker user data with rate r_2 and power $P_2 = (1-\alpha)P$, then combines encoded data and transmits it to the receivers. The stronger receiver initially decodes the weaker user data and subtracts it from the received signal Y_1. However, the weaker receiver does not decode the stronger user data due to its bad channel gain and treats it as noise. Figure 2.3 illustrates the capacity region of the AWGN broadcast channel.

Using capacity achieving codes, rate pair (r_1, r_2) can be selected from the boundary of the rate region, where inequalities (2.3) and (2.4) become equalities. Now, substituting (2.3) into (2.4) we can express the power level P in terms of rate pair as follows [11, 28]:

$$P = g(r_1, r_2) = \sigma^2 \left(\frac{(2^{2r_2} - 1)}{s_2} + \frac{(2^{2r_1} - 1)2^{2r_2}}{s_1} \right). \tag{2.5}$$

By rearranging (2.5), the maximum achievable rate of a user can also be expressed in terms of average power P and the rate of the other user.

$$r_1 = h_1(P, r_2) = \frac{1}{2}\log_2\left(\frac{s_1(s_2P + \sigma^2)}{s_2\sigma^2 2^{2r_2}} - \frac{s_1 - s_2}{s_2}\right) \quad (2.6)$$

$$r_2 = h_2(P, r_1) = \frac{1}{2}\log_2\left(\frac{\frac{s_2P}{\sigma^2} + 1}{\frac{s_2}{s_1}(2^{2r_1} - 1) + 1}\right). \quad (2.7)$$

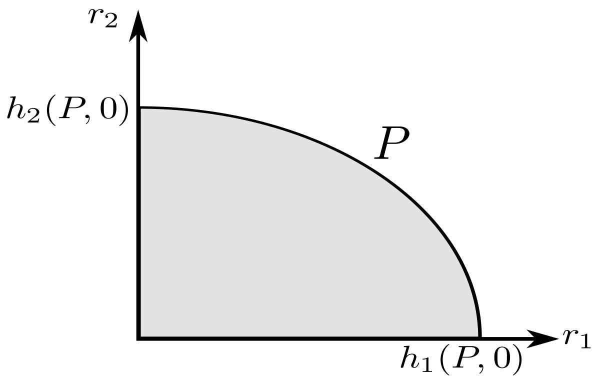

Figure 2.3: Capacity region of the AWGN BC

Proposition 1 outlines the properties satisfied by these rate functions at the boundary of the AWGN broadcast capacity region.

Proposition 1 *Given $s_1 > s_2$, the functions h_1 and h_2, defined in* (2.6),(2.7) *on $\Re^+ \times \Re^+$ satisfy the following properties:*

1. *Nonnegativity: $h_1(P, r) \geq 0$, $h_2(P, r) \geq 0$.*
2. *Monotonicity: $h_1(P, r)$, $h_2(P, r)$ are both monotone decreasing in r, and monotone increasing in P.*
3. *Concavity: $h_1(P, r)$ and $h_2(P, r)$ are concave in P and r.*
4. *The rate of the user with the weaker channel satisfies the following: $\frac{\partial^2 h_2(P,r)}{\partial r \partial P} = 0$, $\frac{\partial^2 h_2(P,r)}{\partial P \partial r} = 0$.*

Proof of Proposition 1 is given in Appendix A.

This thesis concentrates on a broadcast channel that carries properties given in Proposition 1.

CHAPTER 3

OPTIMAL OFFLINE BROADCAST SCHEDULING WITH AN ENERGY HARVESTING TRANSMITTER

Over a decade ago, energy efficiency became an important property with the emergence of mobile battery limited wireless devices. In order to mitigate battery limitations, the offline problem of energy efficient packet scheduling [6, 7, 8, 9] is to find code rates to a set of packets that minimize the total energy consumption while transmitting all the packets before a predetermined deadline. Along with the recent developments in energy scavenging techniques, the problem is recast via addition of energy harvesting capability to the system where the objective is to minimize the transmission completion time of packets arrived within a time window [25]. The problem is extended to a multiuser model and considering that the packets to be transmitted are ready at the beginning of the schedule, the transmission duration in an AWGN broadcast channel is minimized [13, 27].

In this chapter, we first give a general system model. Then, in Section 3.2 we reformulate the problem in [11] and improve the performance of the algorithm given in [11] while providing a mathematical proof of convergence to optimal solution. In Section 3.3, we extend the problem by relaxing the assumption that all the data packets are available at the beginning of transmission.

3.1 System Model

Consider a broadcast channel with one transmitter and two receivers as described in Section 2.2. Arbitrary amounts of energy, $\{E_i < \infty, i = 1, 2, \ldots\}$, as well as data for each user $\{B_i^{(1)}, B_i^{(2)} < \infty, i = 1, 2, \ldots\}$ become available to the sender at arbitrary times t_i. A possible sequence of data and energy arrivals is illustrated in Fig. 3.1. $E(t)$ denotes the total amount of energy that has been *harvested* in $[0, t]$ (regardless of how much of it has been used.) Similarly, $B_1(t)$ and $B_2(t)$ denote the total number of bits destined to the first and second user respectively, that arrived to the sender in $[0, t]$. The interval between any two sequential arrival events (regardless of energy or data) will be called an *epoch*. The length of the i^{th} epoch is $\xi_i = t_{i+1} - t_i$.

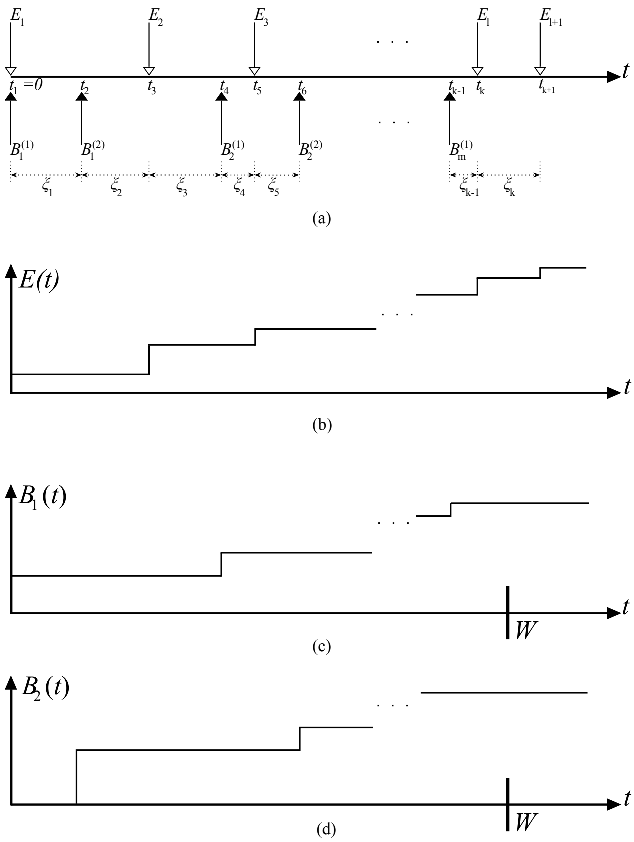

Figure 3.1: An example for energy harvest and data arrivals. In (a) a sequence of energy and data arrivals, in (b) the total harvested energy, in (c) and (d) the total data arrivals destined to first and second users respectively are shown.

In this offline problem, all the future arrival times and amounts of energy and bits are known by the sender at $t = 0$. It is also assumed that harvested energy and data are available for use instantaneously as they arrive, and code rate and transmission power decisions can be changed instantaneously. However, codeword block lengths will be chosen such that each codeword is sent completely within a single epoch (note that starting and ending times of epochs are known ahead of time), so that no arrival event occurs during a codeword. Consequently, the power and rate pair decision will be fixed throughout each codeword.

We are interested in minimizing the total transmission time for packets arriving by a certain time $W < \infty$, so W.L.O.G., set $B_i(t) = B_i(W)$ for $t \geq W$, $i = 1, 2$. A schedule, which is a sequence of power and rate allocations, is feasible if it sends $B_1(W) < \infty$ bits to the 1^{st} (*stronger*) user and $B_2(W) < \infty$ to the 2^{nd} (*weaker*) user (with a certain level of reliability[1]), without violating causality (at any time, using available energy and data by that time). We are interested in finding among all feasible schedules one with the smallest completion time, T^{opt}.

In the next section we examine the structural properties that need to be satisfied by any optimal schedule that completes the transmission by T^{opt}.

3.2 Broadcast Channel Packet Scheduling Revisited

In this section we reconsider the problem in [11] and assume that all the information bits destined to each user are ready at the beginning of transmission and no further data arrival occurs during transmission, i.e., $B_1(t) = B_1(0) = B_1$ and $B_2(t) = B_2(0) = B_2$ for $t > 0$. In Lemma 3.3.1 of [11] it has been shown that if there is a change in the total power level of a transmitter, then bringing the power level closer to each other gives a better schedule. In Corollary 1 of [11] it is stated that in between energy harvests changing the total power level of the transmitter is suboptimal. In Lemma 3.3.2 of [11] it has also been shown that it is suboptimal to change the rate assignments in between energy harvests. In Lemma 3.3.5 in [11] it has been shown that an optimal schedule, that minimizes the transmission duration, ends its transmission to both users at the same time if all the data for both users are available at the beginning. In Theorem 3.3.6 of [11] it has been shown that in an optimal schedule, powers assigned to epochs are monotonically nondecreasing, i.e., $P_1 \leq P_2 \leq ... \leq P_{n^{\text{opt}}}$, and the energy harvested in a constant power region is consumed in that region. Next, we claim in Lemma 3.2.1 that power assignment of the transmitter in an optimal schedule is unique.

Lemma 3.2.1 In an optimal schedule, the power assignment to epochs, $\mathbf{P}^{\text{opt}} = [P_1, P_2, ..., P_{n^{\text{opt}}}]$, is unique.

[1] The achievable rate regions will be implicitly assumed to correspond to a certain constant tolerable error probability respecting which it is possible to transmit a finite number of bits with a finite amount of energy per bit.

Proof. Suppose that there are two different optimal power allocation vectors, $\mathbf{P}^A$ and $\mathbf{P}^B$, where $P_k^A = P_k^B$ for $k = 1, 2, .., i-1$ and $P_i^A < P_i^B$. From Part-1 of the Theorem 3.3.6 in [11], power levels are monotonically nondecreasing in the optimal schedule. In this case, if P_k^A for $k \geq i$ stays constant, we have $\sum_{k=i+1}^{n} P_k^A \xi_k < \sum_{k=i+1}^{n} P_k^B \xi_k$, else $\exists\ j : \{P_i^A < P_{i+j}^A, 1 \leq j \leq n-i\}$ and we have $\sum_{k=i+1}^{j-1} P_k^A \xi_k < \sum_{k=i+1}^{j-1} P_k^B \xi_k$, both contradicting Part-2 of the Theorem 3.3.6 in [11]. ■

Considering the properties of the optimal schedule (See Lemmas 3.3.1 and 3.3.2 of [11]) the problem in [11] is defined by the epoch rates and powers in Problem 1. In order to bound the number of constraints in Problem 1, we shall assume that there is some $k^{\text{up}} < \infty$ such that there is at least one feasible schedule that ends within the first k^{up} epochs, i.e., at $t = T^{\text{up}} < \infty$. In other words, k^{up} is an upper bound for epochs to be considered. Similarly T^{up} is an upper bound on the transmission completion time.

Problem 1 Minimization of Transmission Time on an Energy Harvesting Broadcast Channel When Data is Available at the Beginning:

Minimize: $T = T(\{P_i, r_{2i}\}_{1 \leq i \leq k^{\text{up}}})$

Subject to:

$P_i \geq 0\,,\ 0 \leq r_{1i} \leq h_1(P_i, 0)\,,\ r_{2i} = h_2(P_i, r_{1i})\,,\ 1 \leq i \leq k^{\text{up}}\,,\ 0 \leq T \leq T^{\text{up}}$

$$\sum_{i=1}^{k} P_i \xi_i \leq E(t_k) \ \ \forall k \in \{1, .., k^* = \max\{i : \sum_{j=1}^{i} \xi_j \leq T\}\} \tag{3.1}$$

$$\sum_{i=1}^{k^*} P_i \xi_i + P_{(k^*+1)}(T - \sum_{i=1}^{k^*} \xi_i) = E(T) \tag{3.2}$$

$$\sum_{i=1}^{k^*} r_{1i} \xi_i + r_{1(k^*+1)}(T - \sum_{i=1}^{k^*} \xi_i) = B_1(T) \tag{3.3}$$

$$\sum_{i=1}^{k^*} r_{2i} \xi_i + r_{2(k^*+1)}(T - \sum_{i=1}^{k^*} \xi_i) = B_2(T) \tag{3.4}$$

The constraint set in (3.1) guarantee that the energy causality is respected. If inequality in (3.1) is met with equality at some t_k, then we will refer to this case by saying the *energy constraint is met (or active) at* t_k. The constraint in (3.2) ensures that all the energy harvested is consumed at the end of the schedule. Also, transmission of all the bits is guaranteed, B_1 and B_2 bits to stronger and weaker user respectively, by the constraints in (3.3) and (3.4).

Problem 1 is not a simple optimization problem since the parameter to be optimized, T, appear in the constraints. In order to solve this problem an iterative algorithm, *FlowRight* has been proposed in [11]. Before moving to the solution, in Theorem 3.2.3 we present a final observation on the structure of the optimal schedule using the general result presented in Lemma 3.2.2. These have originally been essentially shown in Lemma 4 and Corollary 1 of [27], through the observation therein that there is a cut-off level for the total power, below which the weaker user is assigned zero rate in the final optimal schedule. Thus the observations in Theorem 3.2.3 and Lemma 3.2.2, while not original to this thesis, will be stated and proved in the following form to preserve the flow of the thesis and be able to explicitly show (using Lemma 3.2.2) properties of the schedule returned by *FlowRight* in Theorem 3.2.4 which is not proven to be optimal until Theorem 3.2.5.

Lemma 3.2.2 Suppose the sender uses different rates for the stronger user in the intervals (τ_1, τ^*), (τ^*, τ_2), such that $\tau_1 < \tau^* < \tau_2$. Keeping power levels and the number of bits transmitted to the stronger user in (τ_1, τ_2) constant, a larger number of bits can be sent to the weaker user in (τ_1, τ_2) by bringing the rates of the stronger user closer to each other if feasible.

Proof. As illustrated in Fig.3.2, suppose that we have a schedule with two slots and total transmission duration $t = \tau_2 - \tau_1$. The slot lengths are βt and $(1 - \beta)t$. P_1 and P_2 denote the total power consumed in the first and second slot respectively. r_{ij} is the rate assigned to the i^{th} user at the j^{th} slot. Now consider the case $r_{11} < r_{12}$. Keeping the avg. rate of user 1, $\bar{r_1}$, constant, set r_{11} to r'_{11}, r_{12} to r'_{12} s.t. $r_{11} \leq r'_{11} \leq r'_{12} \leq r_{12}$ by transferring a certain amount of bits belonging to stronger user are transferred from the 2^{nd} slot to the 1^{st}. This is feasible unless r_{11} is already maximal for the given power level (i.e. $r_{21} = 0$).

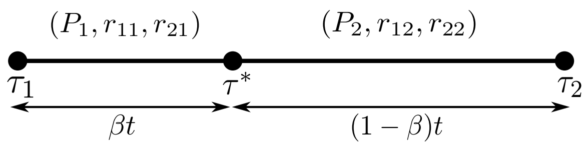

Figure 3.2: Illustration of the transmission scheme used in Lemma 3.2.2.

Average rate of the weaker user over the whole duration is increased from its original level, $\bar{r}_2$, to:

$$\begin{aligned}\bar{\bar{r}}_2 &= h_2(P_1, r'_{11})\beta + h_2(P_2, r'_{12})(1-\beta) \\ &> h_2(P_1, r_{11})\beta + h_2(P_2, r_{12})(1-\beta) = \bar{r}_2 \qquad (3.5)\end{aligned}$$

(3.5) follows from the fact that

$$\begin{aligned}& h_2(P_1, r'_{11})\beta + h_2(P_2, r'_{12})(1-\beta) \\ & - h_2(P_1, r_{11})\beta - h_2(P_2, r_{12})(1-\beta) \geq 0 \qquad (3.6)\end{aligned}$$

for all $\beta \neq \{0, 1\}$ (with equality achieved at $\beta = 0, 1$), unless $r_{21} = 0$, as proved in App. B. In the remaining case, $r_{11} > r_{12}$, set $r_{12} \leq r'_{12} \leq r'_{11} \leq r_{11}$, which is feasible unless $r_{22} = 0$, and strictly improves the average rate for the weaker user. ■

Theorem 3.2.3 In an optimal schedule,

1. [cf. Corollary 1 in [27]] the stronger user's rate is monotone nondecreasing, i.e., $r_{11} \leq r_{12} \leq ... \leq r_{1n^{\text{opt}}}$;

2. [cf. Corollary 1 in [27]] if $r_{1(i+1)} \neq r_{1i}$ for some $0 < i < n^{\text{opt}}$, then $r_{2i} = 0$, i.e., if the stronger user's rate changes at the start of the $(i+1)^{th}$ epoch, the weaker user's rate was zero during the i^{th} epoch;

3. [cf. Corollary 1 in [27]] the weaker user's rate is monotone nondecreasing, i.e., $r_{21} \leq r_{22} \leq ... \leq r_{2n^{\text{opt}}}$;

4. The vector of rate pairs, $\mathbf{R}^{\text{opt}} = [(r_{11}^{\text{opt}}, r_{21}^{\text{opt}}), ..., (r_{1n^{\text{opt}}}^{\text{opt}}, r_{2n^{\text{opt}}}^{\text{opt}})]$, is unique.

Proof. The first three parts essentially follow from Corollary 1 in [27] but are proved here for completeness.

1. Suppose the rate of the stronger user decreases at some point, i.e., $r_{1i} > r_{1(i+1)}$ for some i. From Part-1 of the Theorem 3.3.6 in [11] and Lemma 3.2.2, at least the same number of bits can be sent to each user (and more to at least one) in epochs $(i, i+1)$ by assigning the strong user the average rate $\bar{r}_1$.

2. Suppose that in an optimal schedule the weaker user's rate changes at the $(i+1)^{th}$ epoch and $r_{2i} \neq 0$. If the rate of the stronger user changes, it can only increase, i.e., $r_{1i} < r_{1(i+1)}$, by Part-1. From Lemma 3.2.2, the schedule could only be improved by bringing r_{1i} and $r_{1(i+1)}$ closer to each other using the energy available for the weaker user at the i^{th} epoch, if possible. Hence, the only reason why the rate of the stronger user can increase at the $(i+1)^{th}$ epoch is that there is no feasible energy available to equalize r_{1i} and $r_{1(i+1)}$, which contradicts $r_{2i} \neq 0$.

3. Suppose that in an optimal schedule $r_{2i} > r_{2(i+1)}$. From Part-2, $r_{1i} = r_{1(i+1)} = \bar{r_1}$ if $r_{2i} \neq 0$. By Part-1 of the Theorem 3.3.6 in [11] and 2^{nd} property of the rate region, $r_{2i} = h_2(P_i, \bar{r_1}) \leq h_2(P_{i+1}, \bar{r_1}) = r_{2(i+1)}$ which contradicts initial rate assumption.

4. Suppose that there are two distinct optimal rate-pair vectors, $\mathbf{R}^A$ and $\mathbf{R}^B$, where $(r^A_{1k}, r^A_{2k}) = (r^B_{1k}, r^B_{2k})$ for $k = 1, 2, .., i-1$ and $r^A_{1i} < r^B_{1i}$. Using Lemma 3.2.1, we have $r^A_{2i} = h_2(P_i, r^A_{1i}) > h_2(P_i, r^B_{1i}) \geq 0$. From Part-2, $r^A_{1j} = r^A_{1i} < r^B_{1i} \leq r^B_{1j}\ \forall\ j \in \{i+1, ..., n\}$. Hence, fewer bits will be transmitted by $\mathbf{R}^A$ than $\mathbf{R}^B$, which contradicts the optimality of $\mathbf{R}^A$. ■

From Lemma 3.2.1 and 3.2.3, we conclude that *the optimal schedule is unique* (henceforth abbreviated as OPT.) Next, we give the solution of Problem 1 by *FlowRight* algorithm which is be proved to obtain the unique optimal schedule.

3.2.1 Optimal scheduling with the FlowRight algorithm

FlowRight starts from a feasible initial schedule, and progresses iteratively. Each iteration strictly improves the schedule (decreases T), which ultimately converges to the unique optimal T^{opt}. Given an initial schedule *FlowRight* performs *local optimizations* on pairs of epochs sequentially, i.e., on epochs $(1,2)$, $(2,3)$, $(3,4)$, ... , until all epoch pairs are processed and it is called one iteration of the algorithm (Local optimizations on epoch pairs are studied in details in App. C). Afterwards, *FlowRight* starts from the first epoch pair and does another iteration over the schedule. After each iteration *FlowRight* obtains a schedule $S^n = \{r^n_{1i}, r^n_{2i}\}$ with transmission completion time T^n, where n is the iteration count from the beginning. It has been shown in Theorem 3.4.1 of [11] that each iteration of the *FlowRight* strictly improves the schedule decreasing the transmission completion time. *FlowRight* stops when an iteration cannot decrease the transmission completion time anymore, which happens when iteration count goes to infinity, and returns the schedule $\{r^{\infty}_{1i}, r^{\infty}_{2i}\}$ with transmission completion time $T^{\text{fr}} = T(\{r^{\infty}_{1i}, r^{\infty}_{2i}\})$ and energy consumption E^{∞}_{i} at each epoch $i \in \{1, 2, ..., n_{\infty}\}$. In Theorem 3.4.1 of [11] it has also been shown that the power of the transmitter in the schedule returned by *FlowRight* is monotonically nondecreasing which is one of the properties of the optimal schedule. Next, we will show that *FlowRight* also satisfies some other properties of the optimal schedule and we will prove that the schedule returned by *FlowRight* is the optimal schedule. In [11] the optimality of the schedule returned by *FlowRight* has been proved, yet in this thesis we will provide a mathematical proof of the optimality using Lemma 3.2.2.

Theorem 3.2.4 W*hen FlowRight stops,*

1. Energy consumed during any constant power allocation band equals the total energy harvested in that band.

2. The stronger user's rate is monotone nondecreasing, i.e., $r_{11} \leq r_{12} \leq ... \leq r_{1n}$,

3. If the stronger user's rate changes at the $(i+1)^{th}$ epoch, the weaker user's rate is zero at the i^{th} epoch,

4. The weaker user's rate is monotone nondecreasing, i.e., $r_{21} \leq r_{22} \leq ... \leq r_{2n}$.

Proof.

1. Suppose that $P_i = P_s \neq P_{s+1},\ s - m \leq i \leq s < n_{\infty}$ for some band of length $m < s$ such that $\sum_{i=s-m}^{s} P_i \xi_i < \sum_{i=s-m}^{s} E_i$. From Theorem 3.4.1 of [11] we know power is a monotone nondecreasing function, then we should have $P_s < P_{s+1}$. But we can transfer up to $\sum_{i=s-m}^{s} E_i - \sum_{i=s-m}^{s} P_i \xi_i$ units of energy from epoch $s+1$ to s, only improving the schedule (cf. Lemma 3.3.1 in [11]). This contradicts the assumption that *FlowRight* has stopped.

2. Suppose $\exists\ i \in \{1, .., n_{\infty} - 1\}\ s.t.\ r_{1i} > r_{1(i+1)}$. From Part-3 of Theorem 3.4.1 in [11] and Lemma 3.2.2, by assigning the average rate $\bar{r_1} = (r_{1i}\xi_i + r_{1(i+1)}\xi_{i+1})/(\xi_i + \xi_{i+1})$ to the stronger user in epochs, more bits can be transmitted to the weaker user which means further local improvement on these epochs is ensured. This contradicts the assumption that *FlowRight* stopped.

3. Suppose that the stronger user's rate changes at the $(i+1)^{th}$ epoch and $r_{2i} \neq 0$. The stronger user's rate can only increase, i.e., $r_{1i} < r_{1(i+1)}$, by Part-2. From Lemma 3.2.2, by bringing r_{1i} and $r_{1(i+1)}$ closer to each other using the energy available for the weaker user at the i^{th} epoch, overall transmission duration is decreased which contradicts the assumption that *FlowRight* stopped.

4. Suppose that $r_{2i} > r_{2(i+1)}$. From Part-3, $r_{1i} = r_{1(i+1)}$ since $r_{2i} \neq 0$. From Part-2, the power is monotone increasing. Since $h_2(P, r)$ is monotone increasing in P by the properties of rate region, $r_{2(i+1)} = h_2(P_{i+1}, r_{1i}) \geq h_2(P_i, r_{1i}) = r_{2i}$, which contradicts $r_{2i} > r_{2(i+1)}$. ■

Theorem 3.2.5 The schedule returned by FlowRight is optimal, i.e., $T(\{r_{1i}^{\infty}, r_{2i}^{\infty}\}) = T^{\text{opt}}$.

Proof. Suppose that *FlowRight* stops and returns a schedule $\{r_{1i}^{\infty}, r_{2i}^{\infty}\} \triangleq S^{\text{fr}}$, with completion time $T(\{r_{1i}^{\infty}, r_{2i}^{\infty}\}) \triangleq T^{\text{fr}}$.

As a matter of fact, T^{fr} can not be smaller than T^{opt}. Suppose $T^{\text{fr}} > T^{\text{opt}}$. Consider the case that T^{opt} is in the m^{th} epoch and T^{fr} is in the n^{th} epoch with $n \geq m$.

There must be a schedule $\{r_{1i}^{\text{opt}}, r_{2i}^{\text{opt}}\} \triangleq S^{\text{opt}}$ that achieves T^{opt}. Suppose that S^{opt} and S^{fr} are equal up to epoch s, which is the *first* time they differ either in terms of power level or rates, or both. Let us denote the power allocated in epoch s in S^{opt} as P_s^{opt} and in S^{fr} as P_s^{fr}. Consider the following.

1. $P_s^{\text{fr}} > P_s^{\text{opt}}$: From Part-2 of Theorem 3.3.6 in [11], all the harvested energies are consumed within any constant power band of S^{opt}. Then, starting from epoch s when S^{opt} consumes all the energy at the end of that constant power region, S^{fr} would have consumed more energy than S^{opt} by Part-3 of Theorem 3.4.1 in [11], contradicting the fact that *FlowRight* always respects energy causality.

2. $P_s^{\text{fr}} < P_s^{\text{opt}}$: Suppose that P_s^{fr} increases to P_{s+m}^{fr} at some further epoch $s+m$ before T^{opt}. From Lemma 3.3.1 in [11], by bringing power levels $P_{s+m-1}^{\text{fr}} = P_s^{\text{fr}}$ and P_{s+m}^{fr} of the epoch pair $(s+m-1, s+m)$ closer to each other (this never violates energy causality), further local improvement on these epochs is ensured which contradicts the fact that *FlowRight* has stopped.

Hence, S^{fr} cannot have higher power level than S^{opt} until T^{opt}. Moreover, if power level of S^{fr} becomes lower than that of S^{opt}, then it should stay constant until T^{opt}. These results are shown in the general case in Fig. 3.3.

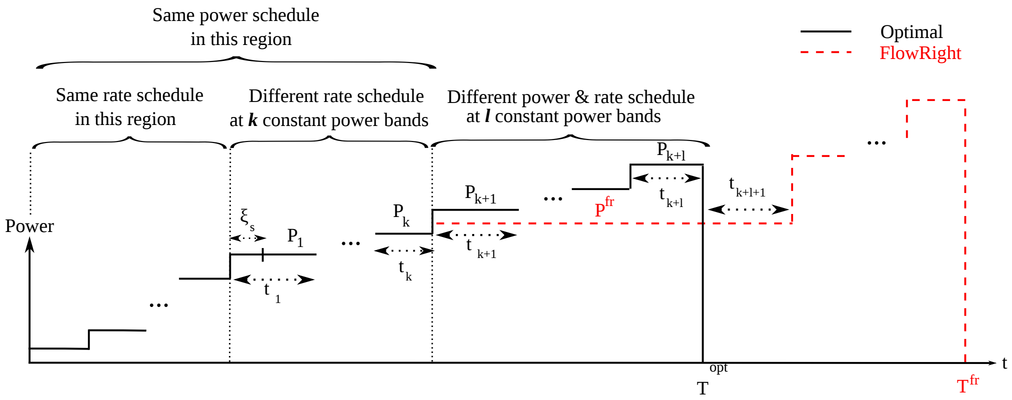

Figure 3.3: Illustration of the general case (in the proof of Theorem 3.2.5) that S^{fr} and S^{opt} differ in power at k constant power bands and differs in both power and rate at l constant power bands.

Now, suppose that the *first* change occurs when $P_s^{\text{fr}} = P_s^{\text{opt}}$ and the rate pairs, $\{r_{1s}^{\text{opt}}, r_{2s}^{\text{opt}}\}$ and $\{r_{1s}^{\text{fr}}, r_{2s}^{\text{fr}}\}$, differ from each other in the general case. Consider the following.

1. $r_{1s}^{\text{opt}} < r_{1s}^{\text{fr}}$: Since $r_{2s}^{\text{opt}} = h_2(P_s^{\text{opt}}, r_{1s}^{\text{opt}}) > h_2(P_s^{\text{fr}}, r_{1s}^{\text{fr}}) \geq 0$, rate of stronger user in the S^{opt} should stay constant after epoch s by Theorem 3.2.3. Since $T^{\text{fr}} \geq T^{\text{opt}}$ and $r_{1s}^{\text{fr}} \leq r_{1u}^{\text{fr}}$ $\forall u \in \{s+1, ..., n\}$, S^{fr} should have transmitted more bits to stronger user than S^{opt}, which contradicts the fact that *FlowRight* always respects *bit feasibility*, i.e., S^{fr} transmits exactly the same number of bits to each user as S^{opt} by the time T^{fr}.

2. $r_{1s}^{\text{opt}} > r_{1s}^{\text{fr}}$: We have $r_{2s}^{\text{fr}} = h_2(P_s^{\text{fr}}, r_{1s}^{\text{fr}}) > h_2(P_s^{\text{opt}}, r_{1s}^{\text{opt}}) \geq 0$; therefore, $r_{1u}^{\text{fr}} = r_{1s}^{\text{fr}}$ $\forall u \in \{s+1, ..., m\}$ by Theorem 3.2.4. Then, bit feasibility requires $r_1^{\text{fr}}\left(\sum_{i=1}^{k+l+1} t_i\right) \leq \sum_{i=1}^{k+l} t_i r_{1(i)}^{\text{opt}}$, where $r_{1(i)}^{\text{opt}} \geq r_{1s}^{\text{opt}}$ is the rate of stronger user for the i^{th} constant power band whereas t_i is the duration of that band. Rearranging the terms we have $r_1^{\text{fr}} \leq \sum_{i=1}^{k+l} \gamma_i r_{1(i)}^{\text{opt}}$, where $\gamma_i = t_i / \sum_{i=1}^{k+l+1} t_i \in (0,1)$, $\forall i \in \{1, 2, .., k+l\}$. Moreover, from Part-1 of Theorem 3.2.4 we have $P^{\text{fr}}\left(\sum_{i=k+1}^{k+l+1} t_i\right) \geq \sum_{i=k+1}^{k+l} t_i P_i$, where P_i is the power of the i^{th} constant power band (see Fig.3.3). Rearranging the terms we have $P^{\text{fr}} \geq \beta \sum_{i=k+1}^{k+l} (\alpha_i/\beta) P_i$ where $\alpha_i = t_i / \sum_{i=k+1}^{k+l+1} t_i \in (0,1)$, $\forall i \in \{1, 2, .., k+l\}$ and $\beta = \sum_{i=k+1}^{k+l} t_i / \sum_{i=k+1}^{k+l+1} t_i \in (0,1)$. Now, let $\tilde{b}_2^{\text{fr}}$ and $\tilde{b}_2^{\text{opt}}$ be the number of bits transmitted to the 2^{nd} user from epoch s till $T^{\text{opt}} + t_{k+l+1}$ by

S^{fr} and till T^{opt} by S^{opt}, respectively. Let $C = (\sum_{i=k+1}^{k+l+1} t_i)$. Then, we have $\tilde{b}_2^{\text{fr}} - \tilde{b}_2^{\text{opt}} > 0$ from (3.13). Eq (3.7), Eq (3.8) and Eq (3.13) follows from the 2^{nd} property of the rate region while Eq (3.9) and Eq (3.10) follows from the 3^{rd}. Hence, S^{fr} transmits more bits to the 2^{nd} user than S^{opt}, contradicting the fact that *FlowRight* always respects bit feasibility.

$$
\begin{aligned}
\tilde{b}_2^{\text{fr}} - \tilde{b}_2^{\text{opt}} &= \sum_{i=1}^{k} t_i h_2(P_i, r_1^{\text{fr}}) + C h_2(P^{\text{fr}}, r_1^{\text{fr}}) - \sum_{i=1}^{k+l} t_i h_2(P_i, r_{1(i)}^{\text{opt}}) \\
&= C\{\sum_{i=1}^{k} \alpha_i h_2(P_i, r_1^{\text{fr}}) + h_2(P^{\text{fr}}, r_1^{\text{fr}}) - \sum_{i=1}^{k} \alpha_i h_2(P_i, r_{1(i)}^{\text{opt}})\} \\
&= C\{\sum_{i=1}^{k} \alpha_i (\underbrace{h_2(P_i, r_1^{\text{fr}}) - h_2(P_i, r_{1(i)}^{\text{opt}})}_{>0}) + h_2(P^{\text{fr}}, r_1^{\text{fr}}) - \sum_{i=k+1}^{k+l} \alpha_i h_2(P_i, r_{1(i)}^{\text{opt}})\} \\
&> C\{h_2(P^{\text{fr}}, r_1^{\text{fr}}) - \sum_{i=k+1}^{k+l} \alpha_i h_2(P_i, r_{1(i)}^{\text{opt}})\} \qquad (3.7) \\
&\geq C\{h_2(\beta \sum_{i=k+1}^{k+l} (\alpha_i/\beta) P_i, r_1^{\text{fr}}) - \sum_{i=k+1}^{k+l} \alpha_i h_2(P_i, r_{1(i)}^{\text{opt}})\} \qquad (3.8) \\
&> C\{\beta h_2(\sum_{i=k+1}^{k+l} (\alpha_i/\beta) P_i, r_1^{\text{fr}}) - \sum_{i=k+1}^{k+l} \alpha_i h_2(P_i, r_{1(i)}^{\text{opt}})\} \qquad (3.9) \\
&> C\{\beta \sum_{i=k+1}^{k+l} (\alpha_i/\beta) h_2(P_i, r_1^{\text{fr}}) - \sum_{i=k+1}^{k+l} \alpha_i h_2(P_i, r_{1(i)}^{\text{opt}})\} \qquad (3.10) \\
&= C\{\sum_{i=k+1}^{k+l} \alpha_i (\underbrace{h_2(P_i, r_1^{\text{fr}}) - h_2(P_i, r_{1(i)}^{\text{opt}})}_{>0})\} \\
&> \ 0 \qquad (3.11)
\end{aligned}
$$

Then, power allocation and rate pairs of S^{opt} and S^{fr} cannot differ, so $S^{\text{fr}} = S^{\text{opt}}$ and $T^{\text{fr}} = T^{\text{opt}}$. ∎

In the next section, we will relax the assumption that all the bits are available at the beginning of the schedule. In other words, arbitrary amount of data destined to each user arrive at arbitrary points in time as well as the transmitter gets replenished with arbitrary amounts of energy at arbitrary points in time.

3.3 Broadcast Channel Packet Scheduling Extended

Consider the system model in Section 3.1. In the previous section, it is assumed that all the data bits are available at the beginning of the schedule. However, a more realistic system model would consider data arrivals during transmission. In this section, we extend Problem 1 to Problem 2 by considering the case that the data arrivals occur during transmission.

Problem 2 Minimization of Transmission Time on an Energy Harvesting Broadcast Channel When Data Arriving at Arbitrary Points:

$$\textit{Minimize: } T = T(\{(P_i, r_{1i})\}_{1\le i\le k^{\mathrm{up}}})$$

$$\begin{aligned}
& P_i \ge 0\,,\ 0 \le r_{1i} \le h_1(P_i, 0)\,,\ r_{2i} = h_2(P_i, r_{1i})\,,\ 1 \le i \le k^{\mathrm{up}}\,,\ 0 \le T \le T^{\mathrm{up}} \\
& \forall k \in \{1, .., k^* = \max\{i : \sum_{j=1}^{i} \xi_j \le T\}\} \\
& \sum_{i=1}^{k} P_i \xi_i \le E(t_k) \\
& \sum_{i=1}^{k} r_{1i}\xi_i \le B_1(t_k)\ ,\quad \sum_{i=1}^{k} r_{2i}\xi_i \le B_2(t_k) \\
& \sum_{i=1}^{k^*} P_i\xi_i + P_{(k^*+1)}(T - \sum_{i=1}^{k^*} \xi_i) = E(T) \\
& \sum_{i=1}^{k^*} r_{1i}\xi_i + r_{1(k^*+1)}(T - \sum_{i=1}^{k^*} \xi_i) = B_1(T) \\
& \sum_{i=1}^{k^*} r_{2i}\xi_i + r_{2(k^*+1)}(T - \sum_{i=1}^{k^*} \xi_i) = B_2(T)
\end{aligned} \tag{3.12}$$

In addition to the constraints in Problem 1, the set of constraints (3.12) is added to Problem 2. Constraint set (3.12) ensure that no information is transmitted to the users before it becomes available at the sender. Similar to the energy causality constraint, the equality case of inequalities in (3.12) for some t_k will be referred as the constraint is met (or active) at t_k.

The structural properties of an optimal schedule with arbitrary data arrivals are studied in [15]. It has been shown that in an optimal schedule with arbitrary data arrivals, transmit power of a sender is monotonically nondecreasing and can only rise if at least one of the specific conditions hold (cf. Lemma 2 and Lemma 3 in [15]). Also, it has been shown in Lemma 5 of [15] that in an optimal schedule, all the harvested energies during transmission should be consumed until the end of the schedule. In the following, we study the structure of optimal schedule if weaker user data is ready at the beginning of schedule, i.e., $B_2(t) = B_2(0)\ \forall t \in [0, W]$. We shall abbreviate this condition as follows:

Definition 3.3.1 Weaker User Full Buffer Condition (WUFBC) *is said to be satisfied whenever all of the data of the weaker user is available at the beginning of transmission. That is,* $B_2(t) = B_2(0)\ \forall t \in [0, W]$.

In Lemma 3.3.2 we will show that stronger user's rate is monotonically nondecreasing under WUFBC and may increase only certain conditions satisfied. Then, in Theorem 3.3.3 we will prove that optimal schedule is unique under WUFBC.

Lemma 3.3.2 *Consider two consecutive epochs i and* $i+1$ *of a given schedule, ending at* t_{i+1} *and* t_{i+2} *by definition, and suppose WUFBC holds for the problem instance. The following is necessary for the rate and power allocation to these two epochs of the given schedule to be locally optimal: The stronger user's rate is constant throughout* $[t_i, t_{i+1})$, *and* $[t_{i+1}, t_{i+2})$. *Furthermore, the rate may jump up at* $t = t_{i+1}$ *(staying constant otherwise) if at least one of the below is true:*

1. *There is data arrival to the stronger user at* $t = t_{i+1}$ *and all the data that arrived before* $t = t_{i+1}$ *has been transmitted by* t_{i+1}.
2. *An energy harvest occurs at* $t = t_{i+1}$ *and all of the power has been used for the stronger user during epoch i.*

Proof. Suppose that $r_{1i} \leq r_{1(i+1)}$. One can find a better schedule by bringing the rates of the stronger user closer by Lemma 3.2.2. Therefore, the stronger user's rate cannot decrease. However, the stronger user's rate may increase because it may be against to either bit or energy causality to transfer some stronger user bits from epoch $i+1$ to i. Firstly, it is against *bit causality* to transfer some stronger user bits from epoch $i+1$ to epoch i, if the first condition holds. Secondly, if the second condition is satisfied we cannot bring stronger users rates closer to each other as it would violate *energy causality*. ∎

We investigate the unique solution of Problem 2 in the next section.

Theorem 3.3.3 There is a unique optimum schedule under WUFBC, i.e., a unique power-rate allocation achieving T^{opt}.

Proof. Suppose that there are two distinct optimal schedules, S^A and S^B, which have equal power and rate assignments until t_s and differ for the first time at epoch s. Consider that the corresponding power allocation vectors, $\mathbf{P}^A$ and $\mathbf{P}^B$, also differ at epoch s such that $P_i^A = P_i^B, \forall i \in \{1, 2, .., s-1\}$ and $P_s^A < P_s^B$. First, assume that $\mathbf{P}^A$ remains constant after epoch s, i.e., $P_i^A = P_s^A, \forall i > s$. By definition, both schedules end at T^{opt}. The total energy consumption of S^A would be less than that of S^B by T^{opt}, i.e., $\left(\sum_{i=1}^{k^*} P_i^A \xi_i + (T^{\text{opt}} - t_{k^*+1}) P_{k^*+1}^A\right) < \left(\sum_{i=1}^{k^*} P_i^B \xi_i + (T^{\text{opt}} - t_{k^*+1}) P_{k^*+1}^B\right)$, which contradicts Lemma 5 of [15]. Hence, total transmit power of S^A cannot remain constant after t_s. Since total transmit power is nondecreasing (See Lemma 2 of [15]), it should increase after epoch s and before the end of transmission, i.e., $P_u^A < P_{u+1}^A$, $\exists u \in \{s, s+1, ..., k^*\}$. Since there are no data arrivals for the weaker user, the increase in total transmit power is either due to energy constraint being met or due to *all* the packets arrived by the time t_{u+1} having been transmitted (cf. conditions (a) or (c) in Lemma 3 of [15]). As $\sum_{i=s}^{u} P_i^A \xi_i < \sum_{i=s}^{u} P_i^B \xi_i$, S^A has not consumed all the available energy at the end of epoch u. Hence, S^A must have transmitted all the bits arrived until t_{u+1}, which means that S^A has transmitted at least the same number of bits to both users while consuming less energy than S^B between t_0 and t_{u+1}, which contradicts the optimality of S^B. Therefore, if there are two distinct optimal schedules, S^A and S^B, their power allocation vectors cannot be different, i.e., $\mathbf{P}^A = \mathbf{P}^B$.

Now, consider two rate pair vectors, $\mathbf{R}^A$ and $\mathbf{R}^B$, where $(r_{1i}^A, r_{2i}^A) = (r_{1i}^B, r_{2i}^B)$, $\forall i \in \{1, 2, .., s-1\}$ and $r_{1s}^A < r_{1s}^B$. Let the rate of the stronger user in S^A, $\{r_{1j}^A\}$ stay constant after t_{s+1}. By Lemma 3.3.2 rate of the stronger user cannot decrease, hence the rate of the stronger user in S^B would be larger than that of S^A after epoch s, i.e., $r_{1(j+1)}^A = r_{1s}^A < r_{1s}^B \le r_{1j}^B$, $\forall j \in \{s, s+1, ..., k-1\}$. Since both schedules end transmission at the same time, S^A transmits fewer bits to the stronger user than S^B does, which contradicts the fact that optimal schedule transmits all the packet arrivals by the end of transmission. Therefore, the rate of the stronger user in S^A cannot stay constant after epoch s. Now suppose that rate of the stronger user in S^A increases at the end of epoch u, i.e., $r_{1u}^A < r_{1(u+1)}^A$, $\exists u \in \{s, s+1, ..., k^*\}$. This increase cannot be due to (1) in Lemma 3.3.2 because S^B has transmitted more bits to the stronger user by

t_{u+1}, i.e., $\sum_{i=1}^{u} r_{1i}^{A}\xi_i < \sum_{i=1}^{u} r_{1i}^{B}\xi_i$. Moreover, this increase cannot be due to (2) in Lemma 3.3.2 since rate of the weaker user in S^A is greater than zero in epoch u, i.e., $r_{2u}^{A} = h_2(P_u, r_{1u}^{A}) > h_2(P_u, r_{1u}^{B}) \geq 0$. Hence rate of stronger user in S^A cannot increase after epoch s. Finally, rate of the stronger user in S^A cannot *decrease* (See Lemma 3.3.2) as this would also contradict optimality. Hence, there cannot be two optimal schedules with different rate pair vectors.

As both the power allocation vector and the rate pair vector of an optimal schedule are unique, we conclude that the optimal schedule is *unique* under WUFBC. ■

Next, we give the solution of Problem 2 with the *DuOpt* algorithm which is proved to obtain the unique optimal schedule iteratively under WUFBC.

3.3.1 Optimal Scheduling with the DuOpt Algorithm

The Problem 1 which is a special case of Problem 2, where both users' data is available at the beginning, was shown to be solved by the *FlowRight* algorithm [28]. Along similar lines, we develop an algorithm that we call *DuOpt* for solving Problem 2 in its general form. As a matter of fact, *DuOpt* simply reduces to *FlowRight* when the given problem instance has all the data arriving at $t = 0$. *DuOpt* starts with any feasible schedule and reduces the transmission completion time iteratively. Let the number of epochs and the transmission completion time of the initial schedule be k^{up} and T^{up} respectively. In each iteration, *DuOpt* sequentially updates rates and powers of two consecutive epochs at a time, i.e., epochs $(1, 2), (2, 3), ...,$ until all epochs are updated. Then, starting from the first epoch pair, *DuOpt* continues with the next iteration. *DuOpt* stops after N iterations such that $N = \min\{n : T^{n-1} - T^{n} \leq \epsilon, i = 1, ..., k^n, j = 1, 2\}$, where $T^n \leq T^{\mathrm{up}}$ is the transmission completion time, $k^n \leq k^{\mathrm{up}}$ is the number of epochs used at the end of n^{th} iteration and ϵ is a predefined threshold.

Hereafter, we will briefly outline the local optimizations over epoch pairs. In Theorem 3.3.4, it will be shown that local optimizations can only improve the schedule. We will also prove that under WUFBC, successive iterations strictly improves the schedule unless it is optimal.

Local Optimizations

Let E_i^n denote the energy used during the i^{th} epoch and b_{ji}^n denote the number of bits transmitted to the j^{th} user during epoch i at the end of n^{th} iteration. Suppose that *DuOpt* is at the n^{th} iteration and running a local optimization over epoch pair $(i, i+1)$. The values of b_{jz}^n and E_z^n, $\forall z \in \{1, 2, ..., i-1\}$ have already been found by previous local optimizations. At the end of this optimization, E_i^n and b_{ji}^n will be determined; E_{i+1}^{n-1}, E_{i+2}^{n-1} and $b_{j(i+1)}^{n-1}$ will be reset to new values that conserve total energy consumption and data transmission in these epochs. The goal of the local optimization is surely to minimize the total transmission completion time of all the packet arrivals. Hence, it is logical to minimize the transmission time in the local optimization problem, which results in a gap[2] if transmission ends before the end of $(i+1)^{th}$ epoch. This gap is used in the next local optimization to further reduce the transmission time via transferring bits or energy between epochs $(i+1)$ and $(i+2)$; hence, a new gap occurs at the end of the next local optimization. This new gap propagates to the end of the transmission resulting a reduction in the total transmission completion time [13]. However, in some cases an epoch long gap occurs and this gap is useless for the next local optimization, i.e., energy or data transfer between epochs in the next local optimization is impossible because of constraints. In that case, it is better to just spread the data out till the end of the second epoch in the local problem and minimize the energy consumption so that the excess energy can be used to further reduce the transmission time in the next local optimization. This leads to two different local optimization functions: *time minimization* and *energy minimization*. These functions both support the global objective in different ways. Time minimization aims to find the minimum amount of time, $T_{(i,i+1)}^n$, to transmit $b_{j(i,i+1)}^n = b_{ji}^{n-1} + b_{j(i+1)}^{n-1}$ bits to each user using the energy available in epoch pair $(i, i+1)$, i.e., $E_{(i,i+1)}^{n-1} = E_i^{n-1} + E_{i+1}^{n-1}$. On the other hand, energy minimization aims to find the minimum energy, $E_{(i,i+1)}^n$, to transmit $b_{j(i,i+1)}^n$ bits to each user in two epoch durations, i.e., $\xi_i + \xi_{i+1}$, and excess energy, $E_i^{n-1} + E_{i+1}^{n-1} - E_{(i,i+1)}^n$, is transferred to the $(i+2)^{th}$ epoch in order to conserve energy. Both of the optimizations respect energy and bit causalities, i.e., $E_i^n \leq E(t_i) - \sum_{s=1}^{i-1} E_s^n$ and $b_{ji}^n \leq B_j(t_i) - \sum_{s=1}^{i-1} b_{js}^n$, $j \in \{1, 2\}$. For details of the local optimization, see App. C.

Suppose that all the feasible packets have been transmitted until the end of the i^{th} epoch and there are still packets to arrive after t_i. Then, further minimization of transmission com-

[2] A gap is a time period with zero power allocation.

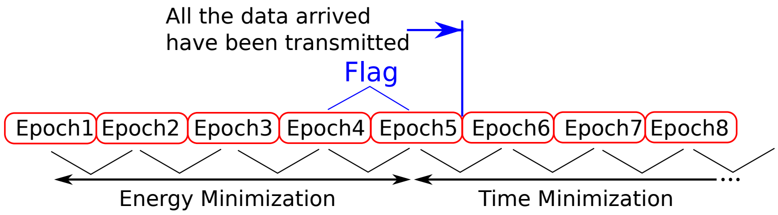

Figure 3.4: Illustration of the *Flag* and local optimizations, where all the feasible bits have been transmitted until the end of 5^{th} epoch; hence, a *Flag* is set to 4^{th} epoch pair, i.e., $(4, 5)$. Energy minimization is performed upto epoch pair with the *Flag* and time minimization is performed for the rest.

pletion time of sequential epochs before t_i will be suboptimal. On the other hand, we can minimize the energy consumption until t_i and use the excess energy to minimize the transmission completion time. Therefore, utilization of energy minimization for local optimizations in Problem 2 is very crucial. If it is guaranteed that current schedule uses at least the same amount of energy as optimal schedule until t_i, *DuOpt* uses the energy minimization function up to i^{th} epoch pair and the time minimization function for the rest. In order to determine when to switch from energy minimization to time minimization, a *Flag* is placed at i^{th} epoch pair. Initially, the *Flag* is set to zero and *DuOpt* starts with performing time minimization on epoch pairs. During n^{th} iteration, if all the feasible bits are transmitted by the i^{th} epoch for $\exists i \in \{1, 2, ..., k^{n-1}\}$, then the *Flag* is set to i ($Flag < i$). In the following iterations, energy minimization function is used up to i^{th} epoch pair. Fig. 3.4 illustrates the *Flag* usage and the pseudo-code in Algorithm 1 outlines the *DuOpt* algorithm. We have also observed that utilization of the energy minimization function after *Flag* is also useful when an epoch long gap occurs.

Theorem 3.3.4 Following statements hold:

1. Successive iterations of DuOpt can only improve the schedule.

2. DuOpt stops and returns a schedule with $\{r_{1i}^{\infty}, r_{2i}^{\infty}\}$.

Proof.

1. Suppose that *DuOpt* is running its n^{th} iteration. After the local optimization on i^{th}

Algorithm 1 DuOpt Algorithm

Initialize();

n ← 0, Flag ← 0, $T^0 \leftarrow T^{up}$

repeat

  n++

  for $i = 1$ to $(k^n - 1)$ **do**

    $e^n_{i,max} \leftarrow E(t_i) - \sum_{m=1}^{i-1} e^n_m$

    $b^n_{1i,max} \leftarrow B_1(t_i) - \sum_{m=1}^{i-1} b^n_{1m}$

    $b^n_{2i,max} \leftarrow B_2(t_i) - \sum_{m=1}^{i-1} b^n_{2m}$

    $b^n_1 \leftarrow b^{n-1}_{1i} + b^{n-1}_{1(i+1)}$

    $b^n_2 \leftarrow b^{n-1}_{2i} + b^{n-1}_{2(i+1)}$

    if $i \leq$ Flag **then**

      $[b^n_{1i}, b^{n-1}_{1(i+1)}, b^n_{2i}, b^{n-1}_{2(i+1)}, E^n_i, E^{n-1}_{i+1}, E^{n-1}_{i+2}] =$

        `Minimize_Energy(`$E^{n-1}_i, E^{n-1}_{i+1}, E^{n-1}_{i+2}, b^n_1, b^n_2, e^n_{i,max}, b^n_{1i,max}, b^n_{2i,max}$`)`

    else

      $[b^n_{1i}, b^{n-1}_{1(i+1)}, b^n_{2i}, b^{n-1}_{2(i+1)}, E^n_i, E^{n-1}_{i+1}] =$

        `Minimize_Time(`$E^{n-1}_i, E^{n-1}_{i+1}, b^n_1, b^n_2, e^n_{i,max}, b^n_{1i,max}, b^n_{2i,max}$`)`

    end if

    if $b^n_{1i,max} == b^n_{1i}$ && $b^n_{2i,max} == b^n_{2i}$ && Flag $< i$ && $i < k^n - 1$ **then**

      Flag = i

    end if

  end for

  `Calculate_T(`&T^n`)` {Calculate current transmission completion time.}

until $T^n == T^{n-1}$

epoch pair, we obtain $\{(r^n_{1i}, r^n_{2i}), E^n_i\}$ and reset the values of $\{(r^{n-1}_{1(i+1)}, r^{n-1}_{2(i+1)}), E^{n-1}_{i+1}, E^{n-1}_{i+2}\}$. If the *Flag* is not placed before i^{th} epoch pair, i.e., $Flag \geq i$, then the aim of the local optimization will be energy minimization. Following the local optimization on i^{th} epoch pair, the excess energy will be transferred to E^{n-1}_{i+2}. In the next local optimization this excess energy is either further transferred or is used to reduce the transmit time. On the other hand, if $Flag < i$, then the aim of the local optimization on i^{th} epoch pair will be time minimization. After the local optimization the transmission completion time of the bits in epochs $(i, i+1)$ will either be equal to or before the end of the epoch $(i+1)$. That is, a gap may occur within i^{th} epoch pair. In the next local optimization, this gap would propagate to the $(i+2)^{th}$ epoch [13]. During the n^{th} iteration of *DuOpt*, if a gap occurs or excess energy is transferred during local optimizations, then the gap (or the excess energy respectively) will propagate to the last epoch pair resulting in an ultimate reduction the transmission completion time at the end of the iteration, i.e., $T(r^n_{1i}, r^n_{2i}) < T(r^{n-1}_{1i}, r^{n-1}_{2i})$. If neither excess energy nor a gap occurs during local optimizations, then transmission completion time cannot be decreased and *DuOpt* will stop by definition. Both local optimizations are in favor of the next local optimization. Therefore, if in either one of the local optimizations a gap occurs or excess energy is transferred then it would propagate till the last epoch pair and finally the transmission completion time would decrease at the end of n^{th} iteration, i.e., $T(r^n_{1i}, r^n_{2i}) < T(r^{n-1}_{1i}, r^{n-1}_{2i})$. If neither excess energy nor gap occurs during local optimizations, then transmission completion time would not be decreased and *DuOpt* would stop. ■

2. In Part-1 we have shown that transmission completion time, $T(r^n_{1i}, r^n_{2i})$, is strictly decreasing in each iteration; meanwhile it is bounded below by T^{OPT}. Therefore, the iterations of *DuOpt* stop and return a schedule $\{r^\infty_{1i}, r^\infty_{2i}\}$. ■

Theorem 3.3.5 If WUFBC is guaranteed, the schedule returned by *DuOpt* is optimal, i.e., $T(\{r^\infty_{1i}, r^\infty_{2i}\}) = T^{\mathrm{opt}}$.

Proof. Suppose that *DuOpt* stopped and returned a schedule $\{r^\infty_{1i}, r^\infty_{2i}\} \triangleq S^{\mathrm{Du}}$, with completion time $T(\{r^\infty_{1i}, r^\infty_{2i}\}) \triangleq T^{\mathrm{Du}}$. Let S^{opt} be the unique optimal schedule with transmission completion time T^{opt}. We will now prove that $S^{\mathrm{Du}} = S^{\mathrm{opt}}$. Let us suppose $S^{\mathrm{Du}} \neq S^{\mathrm{opt}}$, then these schedules have to differ in either the power allocation or rate allocation (or both). First, suppose $P^{\mathrm{Du}}_i = P^{\mathrm{opt}}_i, i \in \{1, 2, ..., s-1\}$ and $P^{\mathrm{Du}}_s \neq P^{\mathrm{opt}}_s$ for some s. We will show that this case is impossible. There are two possible cases for epoch s: (i) $P_s^{\mathrm{Du}} > P_s^{\mathrm{opt}}$, (ii) $P_s^{\mathrm{Du}} < P_s^{\mathrm{opt}}$. Let us begin with the first case.

(i) We assumed $P_s^{\mathrm{Du}} > P_s^{\mathrm{opt}}$. If P^{opt} stays constant after epoch s till the end of transmission, this would mean that S^{Du} consumes more energy than S^{opt} until T^{opt}, which would contradict the fact that optimal schedule consumes all the harvested energy till the end of transmission. Therefore the power of the optimal schedule must increase at the end of epoch $(s+m)$ for some $m \geq 0$ before the end of transmission. As S^{Du} has been able to use more energy than the optimal schedule until t_{s+m+1}, the optimal schedule cannot have run into an energy constraint at t_{s+m+1}, hence the rise in the power can only be due to a data constraint at t_{s+m+1}, i.e., all the bits arrived have been transmitted by the optimal schedule until t_{s+m+1}. In order to contradict the assumption that $P_s^{\mathrm{Du}} > P_s^{\mathrm{opt}}$, we will now analyze the rate assignments for both schedules. First let us focus on the case that both schedules use exactly the same rates for the stronger user up to t_{s+m+1}, i.e., $r_{1i}^{\mathrm{Du}} = r_{1i}^{\mathrm{opt}}$, $\forall i \in \{s, ..., s+m\}$. As we have shown above, S^{opt} should have transmitted all the bits available until t_{s+m+1}. However, if we compare the weaker user bits transmitted by both schedules until t_{s+m+1}, we observe that S^{Du} transmits more bits to the weaker user than S^{opt} does, because $\sum_{i=1}^{s+m}(r_{2i}^{\mathrm{Du}} - r_{2i}^{\mathrm{opt}})\xi_i = \sum_{i=s}^{s+m}(r_{2i}^{\mathrm{Du}} - r_{2i}^{\mathrm{opt}})\xi_i = \sum_{i=s}^{s+m}(h_2(P_i^{\mathrm{Du}}, r_{1i}^{\mathrm{Du}}) - h_2(P_i^{\mathrm{opt}}, r_{1i}^{\mathrm{opt}}))\xi_i > 0$. On the other hand, *DuOpt* respects *bit causality*, i.e., *DuOpt* does not transmit bits that have not arrived yet, so we reach contradiction. That is, rates cannot stay constant up to t_{s+m+1}, i.e., there is some $k \in \{1, 2, ..., s+m-1\}$ such that $r_{1i}^{\mathrm{Du}} = r_{1i}^{\mathrm{opt}}$ for $i < k$ and $r_{1k}^{\mathrm{Du}} \neq r_{1k}^{\mathrm{opt}}$. But we shall now show that this is not possible. First consider the case that $r_{1k}^{\mathrm{Du}} < r_{1k}^{\mathrm{opt}}$. From Lemma 3.3.2, the stronger user's rate cannot decrease under WUFBC. If $r_{1i}^{\mathrm{Du}} = r_{1k}^{\mathrm{Du}}, i \in \{k, ..., s+m\}$, then S^{Du} transmits more bits to the weaker user than S^{opt} does by t_{s+m+1}, i.e., $\sum_{i=1}^{s+m}(r_{2i}^{\mathrm{Du}} - r_{2i}^{\mathrm{opt}})\xi_i = \sum_{i=k+1}^{s+m}(r_{2i}^{\mathrm{Du}} - r_{2i}^{\mathrm{opt}})\xi_i = \sum_{i=k+1}^{s+m}(h_2(P_i^{\mathrm{Du}}, r_{1i}^{\mathrm{Du}}) - h_2(P_i^{\mathrm{opt}}, r_{1i}^{\mathrm{opt}}))\xi_i > 0$. However, at the end of $(s+m)^{th}$ epoch, S^{Du} cannot send more bits to weaker user because S^{opt} should have transmitted all the weaker user bits. Therefore, r_1^{Du} should increase before t_{s+m+1}, i.e., at the end of epoch $k+n$, where $0 < n < (s+m-k)$. We have $r_{1(k+n)}^{\mathrm{Du}} < r_{1(k+n+1)}^{\mathrm{Du}}$, hence either one of the two conditions in Lemma 3.3.2 must hold. Since $\sum_{i=1}^{k+n}(r_{1i}^{\mathrm{opt}} - r_{1i}^{\mathrm{Du}})\xi_i > 0$, until t_{k+n+1}, S^{opt} has transmitted more bits to the stronger user than S^{Du} does; therefore, all the stronger user's bits arrived have not been transmitted by S^{Du} at the end of epoch $(k+n)$. Also, $r_{2(k+n)}^{\mathrm{Du}} = h_2(P_{k+n}^{\mathrm{Du}}, r_{1(k+n)}^{\mathrm{Du}}) > h_2(P_{k+n}^{\mathrm{opt}}, r_{1(k+n)}^{\mathrm{opt}}) \geq 0$. Hence neither of the two conditions in Lemma 3.3.2 holds and stronger user's rate cannot increase at t_{k+n+1}, which implies $r_{1k}^{\text{Du}} \not< r_{1k}^{\text{opt}}$. Thus, we are left with the case $r_{1k}^{\text{Du}} > r_{1k}^{\text{opt}}$. If $r_{1i}^{\text{opt}} = r_{1k}^{\text{opt}}, i \in \{k, ..., s+m\}$, then $\sum_{i=1}^{s+m}(r_{1i}^{\text{Du}} - r_{1i}^{\text{opt}})\xi_i > 0$, which contradicts the fact that *DuOpt* respects bit feasibility. Hence, stronger user's rate in S^{opt} cannot remain constant after epoch k. Then we should have $r_{1i}^{\text{opt}} = r_{1k}^{\text{opt}}, i \in \{k, ..., k+n\}$ and $r_{1(k+n)}^{\text{opt}} < r_{1(k+n+1)}^{\text{opt}}$. Since there is an increase in the stronger user rate, at least one of the conditions in Lemma 3.3.2 should hold at t_{k+n+1}. However, we have $\sum_{i=1}^{k+n}(P_i^{\text{Du}} - P_i^{\text{opt}})\xi_i > 0$ and $\sum_{i=1}^{k+n}(r_{1i}^{\text{Du}} - r_{1i}^{\text{opt}})\xi_i > 0$, which tells us that neither one of the conditions in Lemma 3.3.2 holds, which implies that this final case is also not possible. Hence, we conclude that the case $P_s^{\text{Du}} > P_s^{\text{opt}}$ is not possible.

(ii) Now consider the case $P_s^{\text{Du}} < P_s^{\text{opt}}$. We will prove that this case is also not possible by following a similar method to the one in case (i). First, suppose that the power of S^{Du} increases after s^{th} epoch. This increase cannot be due to an energy constraint, since S^{opt} consumes more energy than S^{Du} does until the increase in power. Hence, it should be due to data constraint and under WUFBC both user data constraints should be active. That is, S^{Du} transmits all the feasible data until the increase in power. This implies that while consuming less energy, S^{Du} transmits at least the same number of bits than S^{opt} does, which contradicts the optimality of S^{opt}. Thus, power of S^{Du} cannot increase after epoch s. Also, it cannot decrease in time, otherwise a local optimization results in either a gap or excess energy that propagates till the end of the schedule and transmission duration decreases. Therefore, we power of S^{Du} should stay constant after epoch s until T^{opt}. Now, we will analyze the rate assignments for both schedules. Let the transmission of S^{opt} end in epoch $(s+m)$ for $m \geq 0$ and suppose that $r_{1i}^{\text{Du}} = r_{1i}^{\text{opt}}\ \ \forall i < k$, $0 < k < (s+m)$. At the k^{th} epoch there are three possible cases: $r_{1k}^{\text{Du}} > r_{1k}^{\text{opt}}$, $r_{1k}^{\text{Du}} < r_{1k}^{\text{opt}}$ and $r_{1k}^{\text{Du}} = r_{1k}^{\text{opt}}$. We will first consider the case $r_{1k}^{\text{Du}} > r_{1k}^{\text{opt}}$ and prove that this is not possible. Let $r_{1k}^{\text{Du}} > r_{1k}^{\text{opt}}$ and consider the rate of the stronger user in S^{opt} after k^{th} epoch. It cannot stay constant until T^{opt}, because it contradicts the fact that S^{opt} transmits all the feasible bits before T^{opt}, i.e., $\sum_{i=1}^{s+m}(r_{1i}^{\text{Du}} - r_{1i}^{\text{opt}})\xi_i > 0$. Since the stronger user's rate in S^{opt} cannot decrease by Lemma 3.3.2, it should increase at the end of epoch $(k+n)$ for $0 \leq n < (s+m-k)$, i.e., $r_{1(k+n)}^{\text{opt}} < r_{1(k+n+1)}^{\text{opt}}$. However, we have $\sum_{i=1}^{k+n}(r_{1i}^{\text{Du}} - r_{1i}^{\text{opt}})\xi_i > 0$ and $r_{2(k+n)}^{\text{opt}} = h_2(P_{k+n}^{\text{opt}}, r_{1(k+n)}^{\text{opt}}) > h_2(P_{k+n}^{\text{Du}}, r_{1(k+n)}^{\text{Du}}) \geq 0$ which implies that none of the conditions in Lemma 3.3.2 holds and the stronger user's rate in S^{Du} cannot increase after epoch k. Hence, we conclude that $r_{1k}^{\text{Du}} \not> r_{1k}^{\text{opt}}$. Now we consider the case $r_{1k}^{\mathrm{Du}} < r_{1k}^{\mathrm{opt}}$. Suppose that the stronger user's rate in S^{opt} increase at epoch $(k+n)$ for $0 \le n < (s+m-k)$. This increase in stronger user's rate requires that at least one of the conditions in Lemma 3.3.2 should hold. However, we have $\sum_{i=1}^{k+n}(r_{1i}^{\mathrm{Du}} - r_{1i}^{\mathrm{opt}})\xi_i > 0$ and $r_{2(k+n)}^{\mathrm{opt}} = h_2(P_{k+n}^{\mathrm{opt}}, r_{1(k+n)}^{\mathrm{opt}}) > h_2(P_{k+n}^{\mathrm{Du}}, r_{1(k+n)}^{\mathrm{Du}}) \ge 0$, so the stronger user's rate in S^{Du} cannot increase, i.e., $r_{1k}^{\mathrm{Du}} \not> r_{1k}^{\mathrm{opt}}$. Since the stronger user's rate in S^{opt} cannot decrease by Lemma 3.3.2, it should stay constant until T^{opt}.

Thus far we have shown that if S^{Du} is different than S^{opt}, then S^{Du} cannot have higher power level than S^{opt} until T^{opt}. Moreover, if power level of S^{Du} becomes lower than that of S^{opt}, then it should stay constant until T^{opt} and if the stronger user's rate of S^{Du} becomes lower than that of S^{opt}, then it should stay constant until T^{opt}. These results are shown in the general case in Fig. 3.5.

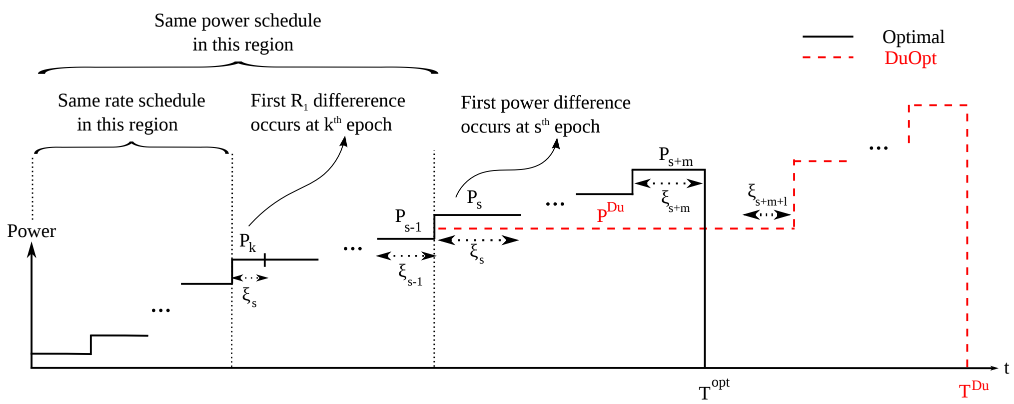

Figure 3.5: Illustration of the general case (in the proof of Theorem 3.3.5) that at the first change between OPT and the schedule returned by *DuOpt*, power level of OPT is greater than that of the schedule returned by *DuOpt*.

Now, let $\tilde{b}_2^{\mathrm{Du}}$ and $\tilde{b}_2^{\mathrm{opt}}$ be the number of bits transmitted to the weaker user till t_{s+m+l} by S^{Du} and till T^{opt} by S^{opt}, respectively. Then, we have

$$
\begin{aligned}
\tilde{b}_2^{\mathrm{Du}} - \tilde{b}_2^{\mathrm{opt}} &= \sum_{i=1}^{s+m+l} \xi_i h_2(P_i^{\mathrm{Du}}, r_1^{\mathrm{Du}}) - \sum_{i=1}^{s+m+l} \xi_i h_2(P_i^{\mathrm{Opt}}, r_1^{\mathrm{Opt}}) \\
&= \sum_{i=k}^{s-1} \xi_i \underbrace{(h_2(P_i^{\mathrm{Du}}, r_1^{\mathrm{Du}}) - h_2(P_i^{\mathrm{Opt}}, r_1^{\mathrm{Opt}}))}_{>0} + \left(\sum_{i=s}^{s+m+l} \xi_i\right) h_2(P_s^{\mathrm{Du}}, r_{1s}^{\mathrm{Du}}) - \sum_{i=s}^{s+m} \xi_i h_2(P_i^{\mathrm{Opt}}, r_{1i}^{\mathrm{opt}}) \\
&> \left(\sum_{i=s}^{s+m+l} \xi_i\right) h_2(P_s^{\mathrm{Du}}, r_{1s}^{\mathrm{Du}}) - \sum_{i=s}^{s+m} \xi_i h_2(P_i^{\mathrm{Opt}}, r_{1i}^{\mathrm{opt}}) \\
&\geq \left(\sum_{i=s}^{s+m+l} \xi_i\right) h_2(\sum_{i=s}^{s+m} \frac{\xi_i}{\sum_{i=s}^{s+m+l} \xi_i} P_i^{\mathrm{Opt}}, r_{1s}^{\mathrm{Du}}) - \sum_{i=s}^{s+m} \xi_i h_2(P_i^{\mathrm{Opt}}, r_{1i}^{\mathrm{opt}}) \\
&> \left(\sum_{i=s}^{s+m+l} \xi_i\right)\left(\sum_{i=s}^{s+m} \frac{\xi_i}{\sum_{i=s}^{s+m+l} \xi_i} h_2(P_i^{\mathrm{Opt}}, r_{1s}^{\mathrm{Du}})\right) - \sum_{i=s}^{s+m} \xi_i h_2(P_i^{\mathrm{Opt}}, r_{1i}^{\mathrm{opt}}) \\
&= \sum_{i=s}^{s+m} \xi_i h_2(P_i^{\mathrm{Opt}}, r_{1s}^{\mathrm{Du}}) - \sum_{i=s}^{s+m} \xi_i h_2(P_i^{\mathrm{Opt}}, r_{1i}^{\mathrm{opt}}) \\
&= \sum_{i=s}^{s+m} \xi_i \underbrace{(h_2(P_i^{\mathrm{Opt}}, r_{1s}^{\mathrm{Du}}) - h_2(P_i^{\mathrm{Opt}}, r_{1i}^{\mathrm{opt}})}_{>0} \\
&> 0 \qquad (3.13)
\end{aligned}
$$

From (3.13), S^{Du} transmits more bits to the weaker user than S^{opt} does, then this final case also cannot happen. Therefore, we conclude that the schedule returned by *DuOpt* cannot be different than the unique optimal schedule, i.e., $S^{\mathrm{Du}} = S^{\mathrm{opt}}$. ■

3.3.2 Numerical Examples

In this section we consider a two-user AWGN BC with $B = 1KHz$ bandwidth and noise spectral density of $N_0 = 10^{-12}$ Watts/Hz. Path loss factors on the links of stronger and weaker user are assumed to be $s_1 = 70dB$ and $s_2 = 75dB$, respectively. Choosing from the boundary of the capacity region of the BC, rate of each user is calculated as follows:

$$
\begin{aligned}
R_1 &= B \cdot log_2\left(1 + \frac{s_1 P_1}{BN_0}\right) \\
R_2 &= B \cdot log_2\left(1 + \frac{s_2 P_2}{s_2 P_1 + BN_0}\right)
\end{aligned}
$$

Let us first consider a problem with all of the weaker user data ready at the beginning. As depicted in Fig.3.6, [3, 10, 4, 7, 13, 3, 5, 8, 6, 12] joules of energy is harvested at [0, 3, 5, 8, 9, 10, 11, 13, 15, 17] seconds. 25 Kbits data for the weaker user arrives at the beginning of the schedule whereas data arrivals occur for the stronger user at [0, 2, 4, 8, 10] seconds with amounts [8, 25, 12, 20, 15] Kbits. Under these circumstances *DuOpt* algorithm is run and the final schedule is calculated, as drawn in Fig. 3.6.

Decrease of transmission duration after each iteration is used as the stopping criterion. *DuOpt* stops whenever the change in the transmission duration is within $\epsilon\xi_{av}$ where $\epsilon = 10^{-14}$ and $\xi_{av} = \frac{1}{k^*}\sum_{i=1}^{k^*}\xi_i$ is the average epoch duration. With this criterion, *DuOpt* runs for 61 iterations for a duration of 0.316 seconds on a computer with a 3GHz Core2Duo processor.

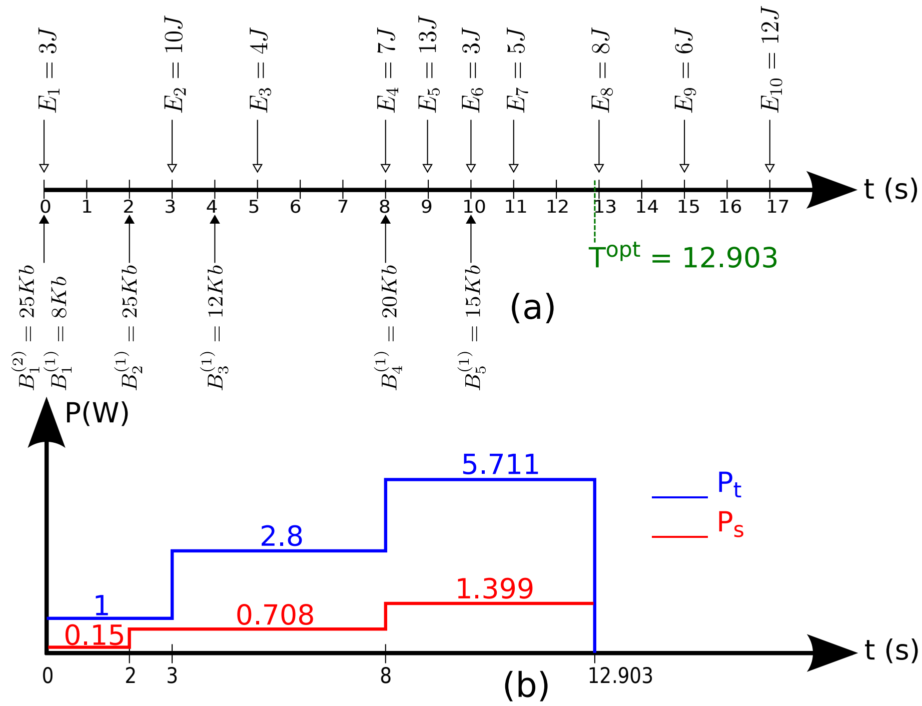

Figure 3.6: (a) An illustration of energy harvest and bit arrival sequences with the weaker user data is ready at the beginning of schedule. E_i is the i^{th} energy arrival and $B_i^{(j)}$ is the i^{th} data arrival to j^{th} user. (b) Final schedule calculated by *DuOpt*, where P_s is the power used for transmission to stronger user and P_t is the total transmit power of the transmitter.

As shown in 3.6, the transmission duration of the final schedule returned by *DuOpt* is $T^{Du} = 12.9027$ seconds. The power allocation vector is calculated as

$\mathbf{P} = \{(P_{1i}, P_{2i})\} = [(0.15\ ,\ 0.85)\ ,\ (0.708\ ,\ 0.292)\ ,\ (0.708\ ,\ 2.092)\ ,\ (1.399\ ,\ 4.312)]$ watts with durations [2 , 1 , 5 , 4.903] seconds. This power allocation vector at the transmitter corresponds to SNR levels of [11.7609 , 18.5026 , 18.5026 , 21.4593] dB at the stronger user and

effective SNR (or SINR) levels of [6.7025 , -4.0441 , 4.5125 , 4.7902] dB at the weaker user. Corresponding rate allocation vector of this schedule is can be calculated as:
$\mathbf{r} = \{(r_{1i}, r_{2i})\}$ = [(4.000 , 2.506) , (6.166 , 0.480) , (6.166 , 1.937) , (7.138 , 2.005)] Kbps.

The power, hence the rate, of the stronger user in this example is an increasing function of time. On the other hand power (or rate) function of the weaker user in this example decreases at $t = 2s$ as shown in Fig. 3.6. Only the stronger user data constraint is active at $t = 2s$; hence, the total transmission power does not change. At $t = 3s$ an energy constraint is met with equality and total transmission power increases. Both stronger user data and energy constraints are active at $t = 8s$, hence, we observe an increase in both stronger user and the total power.

Next we consider an example with both user data arrive during transmission. As shown in Fig.3.7, [11, 8, 6, 1, 6, 4, 4, 13, 6] joules of energy is harvested at [0, 2, 5, 6, 9, 12, 13, 14, 16] seconds. Stronger user data arrivals occur at [0, 1, 2, 3, 6] seconds with amounts [11, 15, 19, 15, 15] Kbits totaling 75 Kbits of data. A total of 26 Kbits weaker user data arrives at [0, 3, 6, 7, 8] seconds with amounts [5, 3, 6, 3, 9] Kbits. With these data arrivals and energy harvests, *DuOpt* algorithm is run and less than one second *DuOpt* stops with 330 iterations.

As shown in Fig. 3.7, the final schedule returned has a transmission completion time of T^{Du} = 12.4906 seconds. The power allocation vector of this schedule is
$\mathbf{P} = \{(P_{1i}, P_{2i})\}$ = [(1.023 , 1.603) , (0.404 , 2.304) , (0.404 , 7.749)] watts with durations [6, 6, 0.4906] seconds. This power allocation vector at the transmitter corresponds to SNR levels of [20.0988 , 16.0595 , 16.0595] dB at the stronger user and effective SNR (or SINR) levels of [1.8180 , 7.2374 , 12.5055] dB at the weaker user. Corresponding rate allocation vector of this schedule is can be calculated as:
$\mathbf{r} = \{(r_{1i}, r_{2i})\}$ = [(6.6907 , 1.3333) , (5.3702 , 2.6539) , (5.3702 , 4.2331)] Kbps.

Contrary to the first example, in this example the power of the stronger user is not an increasing function of time. When we let packet arrivals for both users, power function for both users do not have monotonicity anymore and they can both increase and decrease in time. Another important observation in this example is that, at $t = 6s$ the weaker user data constraint is

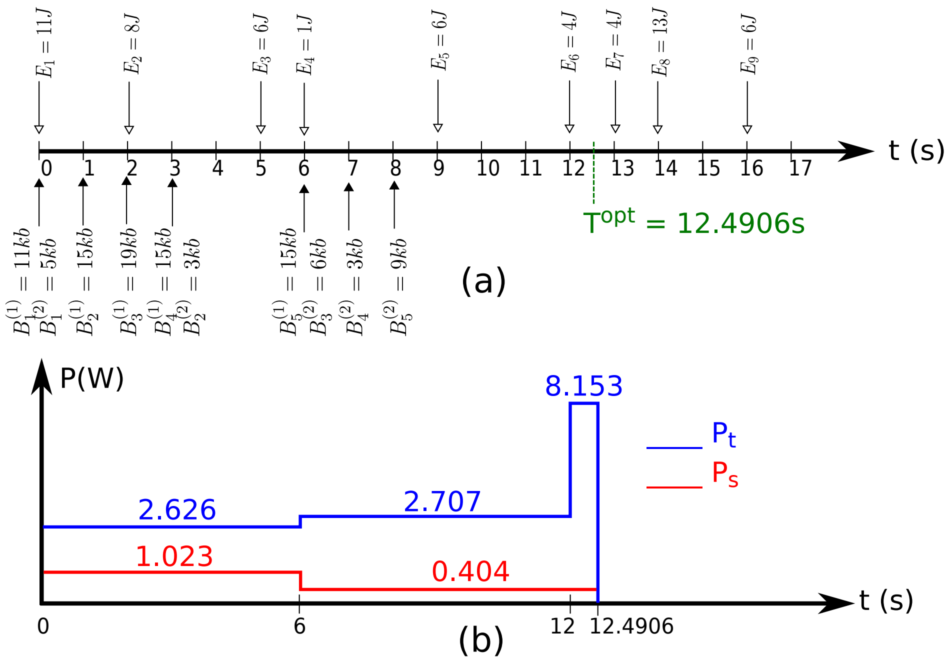

Figure 3.7: (a) An illustration of energy harvest and bit arrival sequences for the second example where E_i is the i^{th} energy arrival and $B_i^{(j)}$ is the i^{th} data arrival to j^{th} user. (b) Final schedule calculated by *DuOpt*, where P_s is the power used for transmission to stronger user and P_t is the total transmit power of the transmitter.

active hence the weaker user power increases while the stronger user power decreases. We know from 3.2.2 that bringing rates of stronger user we can have a better schedule. Also, from Lemma 3.3.1 of [11] bringing the power levels closer to each other improves the schedule. However, bringing rates of stronger user closer to each other in this example will increase the difference in the total power level. Therefore we have a trade-off here. And the optimum point in this problem is to find the schedule that preserve its sum of rates. For example, at $t = 6s$, the rates of the stronger and the weaker user changes but the sum of the weaker and the stronger user rates remain constant at 8.024 Kbps.

3.4 Dual Problem & Solution

In the previous section, we have shown in Theorem 3.3.3 that solution of Problem 2 is unique under WUFBC. It is very difficult to prove the uniqueness of the problem in the general case with the same method. To circumvent this difficulty, we shall analyze a dual problem. Problem 3 presents a dual of Problem 2 that minimizes transmitter energy consumption for a given transmission duration T with both energy and data constraints. We note that Problem 3 is not the Lagrangian dual of Problem 2. However, we will show in Lemma 3.4.2 that their solutions are identical. Consequently, we refer Problem 3 as the dual problem.

Problem 3 Energy Consumption Minimization of an Energy Harvesting Transmitter with Data and Energy Arriving at Arbitrary Points on a Broadcast Channel:

Minimize: $E = \sum_{i=1}^{k^*} g(r_{1i}, r_{2i})\xi_i + g(r_{1(k^*+1)}, r_{2(k^*+1)})(T - \sum_{i=1}^{k^*} \xi_i)\ ,\ k^* = \max\{i : \sum_{j=1}^{i} \xi_j \leq T\}$

subject to: $\forall k \in \{1, .., k^*\}$

$$r_{1k} \geq 0\ ,\ r_{1(k^*+1)} \geq 0\ ,\ r_{2k} \geq 0\ ,\ r_{2(k^*+1)} \geq 0 \tag{3.14}$$

$$\sum_{i=1}^{k} g(r_{1i}, r_{2i})\xi_i \leq E(t_k) \tag{3.15}$$

$$\sum_{i=1}^{k} r_{1i}\xi_i \leq B_1(t_k)\ ,\ \sum_{i=1}^{k} r_{2i}\xi_i \leq B_2(t_k) \tag{3.16}$$

$$\sum_{i=1}^{k^*} r_{1i}\xi_i + r_{1(k^*+1)}(T - \sum_{i=1}^{k^*} \xi_i) = B_1(T)\ ,\ \sum_{i=1}^{k^*} r_{2i}\xi_i + r_{2(k^*+1)}(T - \sum_{i=1}^{k^*} \xi_i) = B_2(T) \tag{3.17}$$

Non-negativity constraints in (3.14) ensure that rates do not take negative values. Set of constraints (3.15) guarantee that no energy is consumed before it is harvested. We note that there is no energy constraint for the last epoch, so that there will always be a solution to the problem. If the energy constraint for the last epoch had been included, the problem could have had no solution because it may be impossible to find a schedule that can transmit all the data with the given data and energy consumption constraints. The set of constraints (3.16) guarantee that no data is transmitted before it arrives and constraints in (3.17) assure that all the data arrivals are transmitted by T.

In the following lemma we will state that the solution of Problem 3 is strictly convex. By using Lemma 3.4.1, we will state in Corollary 1 that the solution of Problem 3 is unique. Then, we will declare in Lemma 3.4.2 that solutions of Problems 2 and 3 are identical. And using Lemma 3.4.2 and Corollary 1, we will state in Corollary 2 that Problem 2 also has a unique solution. Finally, we will state in Lemma 3.4.3 that the solution of Problem 3 is a strictly decreasing function of total transmission duration.

Lemma 3.4.1 Problem 3 is strictly convex.

Proof. $g(\mathbf{r}_1, \mathbf{r}_2)$ is a strictly convex function of rates (See Appendix A of [11]). A non-negative weighted sum of strictly convex functions is also a strictly convex function [37]. Since the objective function E in Problem 3 is the weighted sum of $g(r_{1i}, r_{2i})$ where $i \in \{1, 2, ..., k^* + 1\}$, E should also be a strictly convex function. The set of constraints (3.15) is composed of strictly convex functions by the same reasoning. And the rest of the constraints are linear functions. Since objective function is strictly convex and constraints are either strictly convex or linear, Problem 3 is strictly convex.

Let us prove this lemma with a different approach. Suppose that we have two different schedules, $S^A = (\mathbf{r}_1^A, \mathbf{r}_2^A)$ and $S^B = (\mathbf{r}_1^B, \mathbf{r}_2^B)$ with energy consumptions E^A and E^B respectively, that satisfy all the constraints in Problem 3. Let $S^\star = \theta(\mathbf{r}_1^A, \mathbf{r}_2^A) + (1-\theta)(\mathbf{r}_1^B, \mathbf{r}_2^B)$ be any linear combination of S^A and S^B where $0 \le \theta \le 1$. In order to prove the strict convexity of problem we have to show that $S^\star$ satisfies all constraints of Problem 3 and consumes less energy than $\theta E^A + (1-\theta)E^B$. The energy consumption of $S^\star$, $E^\star$, is

$$
\begin{aligned}
E^\star &= \sum_{i=1}^{k^*} g(r_{1i}^\star, r_{2i}^\star)\xi_i + g(r_{1(k^*+1)}^\star, r_{2(k^*+1)}^\star)(T - \sum_{i=1}^{k^*}\xi_i) \\
&= \sum_{i=1}^{k^*} g(\theta r_{1i}^A + (1-\theta)r_{1i}^B\,,\, \theta r_{2i}^A + (1-\theta)r_{2i}^B)\xi_i \\
&\quad + g(\theta r_{1(k^*+1)}^A + (1-\theta)r_{1(k^*+1)}^B\,,\, \theta r_{2(k^*+1)}^A + (1-\theta)r_{2(k^*+1)}^B)(T - \sum_{i=1}^{k^*}\xi_i) \\
&\le \left(\sum_{i=1}^{k^*} \theta g(r_{1i}^A, r_{2i}^A)\xi_i + \theta g(r_{1(k^*+1)}^A, r_{2(k^*+1)}^A)(T - \sum_{i=1}^{k^*}\xi_i)\right) \\
&\quad + \left(\sum_{i=1}^{k^*}(1-\theta) g(r_{1i}^B, r_{2i}^B)\xi_i + (1-\theta) g(r_{1(k^*+1)}^B, r_{2(k^*+1)}^B)(T - \sum_{i=1}^{k^*}\xi_i)\right) \qquad (3.18) \\
&= \theta E^A + (1-\theta)E^B,
\end{aligned}
$$

where (3.18) follows from the strict convexity of $g(r_1, r_2)$ and becomes equality only if $S^A = S^B$ or $\theta \in \{0, 1\}$. Hence, we have $E^\star < \theta E^A + (1-\theta)E^B$. Now, let us check whether $S^\star$ satisfies constraints or not. The energy consumption of $S^\star$ at t_k, $k \in \{1, 2, ...k^*\}$ is as follows:

$$\begin{aligned}
\sum_{i=1}^{k} g(r_{1i}^{\star}, r_{2i}^{\star})\xi_i &= \sum_{i=1}^{k} g(\theta r_{1i}^{A} + (1-\theta) r_{1i}^{B}, \theta r_{2i}^{A} + (1-\theta) r_{2i}^{B})\xi_i \\
&\leq \sum_{i=1}^{k} \theta g(r_{1i}^{A}, r_{2i}^{A})\xi_i + \sum_{i=1}^{k} (1-\theta) g(r_{1i}^{B}, r_{2i}^{B})\xi_i \qquad (3.19) \\
&\leq \theta E(t_k) + (1-\theta) E(t_k) \\
&= E(t_k). \qquad (3.20)
\end{aligned}$$

(3.20) indicates that $S^{\star}$ satisfies energy constraints, i.e., (3.15). Transmitted data for the j^{th} user by $S^{\star}$ at t_k is as follows:

$$\min\left\{\sum_{i=1}^{k} r_{ji}^{A}\xi_i, \sum_{i=1}^{k} r_{ji}^{B}\xi_i\right\} \leq \sum_{i=1}^{k} (\theta r_{ji}^{A} + (1-\theta) r_{ji}^{B})\xi_i \leq \max\left\{\sum_{i=1}^{k} r_{ji}^{A}\xi_i, \sum_{i=1}^{k} r_{ji}^{B}\xi_i\right\} \qquad (3.21)$$

and at T, we have

$$\begin{aligned}
&\sum_{i=1}^{k^*} (\theta r_{ji}^{A} + (1-\theta) r_{ji}^{B})\xi_i + (\theta r_{j(k^*+1)}^{A} + (1-\theta) r_{j(k^*+1)}^{B})(T - \sum_{i=1}^{k^*} \xi_i) \\
=& \left(\theta \sum_{i=1}^{k^*} r_{ji}^{A}\xi_i + \theta r_{j(k^*+1)}^{A}(T - \sum_{i=1}^{k^*} \xi_i)\right) + \left((1-\theta) \sum_{i=1}^{k^*} r_{ji}^{B}\xi_i + (1-\theta) r_{j(k*+1)}^{B}(T - \sum_{i=1}^{k^*} \xi_i)\right) \\
=& \theta B_j(T) + (1-\theta) B_j(T) \\
=& B_j(T). \qquad (3.22)
\end{aligned}$$

(3.21) and (3.22) imply that $S^{\star}$ satisfies (3.16) and (3.17). Since rates of S^A and S^B are both non-negative, rates of $S^{\star}$ should also be non-negative and satisfy (3.14).

All of the constraints are satisfied by $S^{\star}$ and the cost function of $S^{\star}$, $E^{\star}$, is always less than respective linear combination of the cost functions of S^A and S^B. Hence, we conclude that Problem 3 is strictly convex. ■

Corollary 1 The solution of Problem 3 is unique.

Proof. Proof is due to convexity. Since Problem 3 is strictly convex, solution of it should be unique [37]. ■

Let the transmission completion time of the solution of Problem 2 be T^{opt} and consider that transmission deadline of Problem 3 is taken as T^{opt}. In Lemma 3.4.2, we claim that the solution of Problem 3, which has a transmission deadline T^{opt}, consumes exactly the same amount of energy, E^{opt}, as the solution of Problem 2 consumes. Then, for the same harvest and data arrivals, optimal schedule resulting from solution of Problem 3 for a given T^{opt} is also a solution of Problem 2. We propose in Lemma 3.4.3 that the energy consumption of the solution of Problem 3 strictly decreases as the transmission deadline increases. Lemmas 3.4.2 and 3.4.3 allow us to solve Problem 2 by solving Problem 3 for varying transmission deadlines.

Lemma 3.4.2 Suppose that the solution of Problem 2 has a transmission completion time of T^{opt} and consider Problem 3 with transmission deadline T^{opt}. Any solution of Problem 2 is also a solution to Problem 3 and vice versa.

Proof. Since Problem 2 is probably non-convex, we will prove this lemma by contradiction. Consider that S^{opt} is the optimal solution of Problem 2 with transmission completion time T^{opt}. Suppose that we have a dual problem in which transmission deadline is selected as T^{opt} and the optimal solution to this problem is $\hat{S}^{\text{opt}}$. Let us assume that $\hat{S}^{\text{opt}}$ is not equal to S^{opt} and the energy consumption of $\hat{S}^{\text{opt}}$ is different than that of S^{opt}. If $\hat{S}^{\text{opt}}$ consumes less energy than S^{opt}, then we can always find a schedule that has a transmission duration less than T^{opt}. As energy is strictly decreasing function of time for one epoch (See App. C.1), we can increase the energy consumption in the last epoch of $\hat{S}^{\text{opt}}$ while ending the transmission before T^{opt}, which would contradict the optimality of S^{opt}. On the other hand, if $\hat{S}^{\text{opt}}$ consumes more energy than S^{opt}, then presence of S^{opt} contradicts the optimality of $\hat{S}^{\text{opt}}$. Therefore, S^{opt} and $\hat{S}^{\text{opt}}$ should consume exactly same amount of energy. As the solutions of Problem 2 and 3 has the same transmission completion time and same energy consumption, the solution of Problem 2 is also a solution to Problem 3 and vice versa. ■

Corollary 2 The solution of Problem 2 is unique.

Proof. By Lemma 3.4.2 the solutions of Problem 2 and 3 should consume exactly same amount of energy, which implies that every solution of Problem 2 should also be a solution to Problem 3. On the other hand, the solution of Problem 2 is unique by Corollary 1. Hence, the solution of Problem 2 should also be unique. ■

In the following lemma we present a final observation that guide us to construct another algorithm that achieve the optimal solution of Problem 2.

Lemma 3.4.3 The total energy consumption of an optimal schedule is a strictly decreasing function of total transmission time.

Proof. Suppose that we have a unique optimal solution $S^{\text{opt}} = (\mathbf{r}_1, \mathbf{r}_2)$ to Problem 3. Let us modify the problem by increasing the duration of last epoch by ϵ. In order to find a schedule that satisfy all the constraints of this modified problem, let us increase duration of last epoch in S^{opt} by ϵ. In App. C.1 we have shown that the energy consumption is strictly decreasing function of transmit duration for one epoch. Energy consumption of this new schedule, which is strictly less than that of S^{opt}, becomes an upper bound for the solution of the new problem. Hence we conclude that the total energy consumption of an optimal schedule strictly decreases as the total transmission time increases. ■

3.4.1 Solution of the Dual Problem with SUMT Algorithm

Gradient descent and Newton's method are a commonly used techniques for solving linear equality constrained convex problems. Problem 3 is strictly convex, but has both linear equality and convex inequality constraints. Interior-point methods are utilized for solving inequality constrained convex problems [37]. Specifically, in this thesis, we use the *sequential unconstrained minimization technique (SUMT)* which has been proposed by Fiacco and McCormick [38]. Today, SUMT is also called as the *Barrier Method* [37].

SUMT algorithm converts a constrained convex problem into an unconstrained problem by adding penalty terms into objective function. For example, we transform Problem 3 into Problem 4 by adding penalty terms that increase the objective function if constraints are not met.

Problem 4 Unconstrained Convex Optimization Problem for Energy Consumption Minimization of an Energy Harvesting Transmitter with Data and Energy Arriving at Arbitrary Times on a Broadcast Channel:

$$\textit{Minimize: } F(\boldsymbol{r}) = \sum_{i=1}^{k^*} g(r_{1i}, r_{2i})\xi_i + g(r_{1(k^*+1)}, r_{2(k^*+1)})(T - \sum_{i=1}^{k^*} \xi_i) + \mu P(\boldsymbol{r}),$$

where $k^* = \max\{i : \sum_{j=1}^{i} \xi_j \leq T\}$ and

$$P(\mathbf{r}) = \sum_{i=1}^{k^*+1} (\max(0, -r_{1i}))^2 + \sum_{i=1}^{k^*+1} (\max(0, -r_{2i}))^2 \tag{3.23}$$

$$+ \sum_{k=1}^{k^*} \left(\max(0, \sum_{i=1}^{k} g(r_{1i}, r_{2i})\xi_i - E(t_k)) \right)^2 \tag{3.24}$$

$$+ \sum_{k=1}^{k^*} \left(\max(0, \sum_{i=1}^{k} r_{1i}\xi_i - B_1(t_k)) \right)^2 + \sum_{k=1}^{k^*} \left(\max(0, \sum_{i=1}^{k} r_{2i}\xi_i - B_2(t_k)) \right)^2 \tag{3.25}$$

$$+ \left(\sum_{k=1}^{k^*} r_{1i}\xi_i + r_{1(k^*+1)}(T - \sum_{i=1}^{k^*} \xi_i) - B_1(T) \right)^2 \tag{3.26}$$

$$+ \left(\sum_{k=1}^{K} r_{2i}\xi_i + r_{2(k^*+1)}(T - \sum_{i=1}^{k^*} \xi_i) - B_2(T). \right)^2 \tag{3.27}$$

Penalty terms (3.23), (3.24) and (3.25) are due to constraints (3.14), (3.15) and (3.16) respectively. Penalties (3.26) and (3.27) are due to (3.17). If all the constraints are met, then the penalty will be equal to zero. The penalty terms actually force the solution to satisfy all the constraints. In order to control the effect of the penalty terms, a penalty parameter μ is defined. The larger the parameter μ, the bigger the effect of the penalty terms. Selecting a sufficiently large μ, we can reach the optimal solution with an arbitrarily small error. However, choosing a very large μ might also have some computational disadvantages and choosing a very small μ might also lead to a slow convergence or early termination [39]. Hence, an arbitrary small penalty parameter, i.e., $\mu = 1$, is selected at the beginning of algorithm and doubled after each iteration of the SUMT algorithm until a predefined maximum value, i.e., $\mu_{max} = 10^{10}$, is reached.

SUMT algorithm starts with an initial rate vector: $\mathbf{r}^0 = [r_{11}^0, r_{21}^0, ..., r_{1(k^*+1)}^0, r_{2(k^*+1)}^0]^T$. We know that if no constraints are present, the optimal schedule will transmit to both users at constant rates. With this idea in mind (without considering the constraints (3.15) and (3.16) for simplicity) we aim to transmit $B_1(T)$ and $B_2(T)$ at constant rates to the stronger and the weaker user respectively. Initial rate vector is obtained by

$$r_{1i}^0 = \frac{B_1(T)}{T} \quad , \; r_{2i}^0 = \frac{B_2(T)}{T} \quad \forall i \in \{1, 2, .., k^* + 1\}.$$

After the initialization step, we update the rate vector in each iteration by an inner method and then increase the penalty parameter. The SUMT algorithm terminates if either μ reaches maximum value or $P(\mathbf{r})$ is smaller than a predefine threshold ϵ_S. Algorithm 2 summarize the SUMT algorithm.

Algorithm 2 SUMT Algorithm

```
1: r_1i ← B_1(T)/T , r_2i ← B_2(T)/T , μ ← 1
2: repeat
3:     r ← InnerMethod(r)
4:     μ ← 2μ
5: until μ ≥ μ_max  ∨  P(r) ≤ ε_S
```

Generalized Newton's method is used as the inner method of the SUMT algorithm. Newton's method iteratively decrease the objective function $F(\mathbf{r})$ by utilizing first and second order partial derivatives of $F(\mathbf{r})$. Given an initial rate vector from SUMT algorithm, Newton's method revise the rate vector iteratively. At the n^{th} iteration, rate vector $\mathbf{r}^n$ is updated as follows

$$\mathbf{r}^{n+1} = \mathbf{r}^n - [\mathbf{H}F(\mathbf{r}^n)]^{-1}\nabla F(\mathbf{r}^n),$$

where $[\mathbf{H}F(\mathbf{r}^n)]^{-1} = [\nabla^2 F(\mathbf{r}^n)]^{-1}$ is inverse of the Hessian matrix, $\nabla F(\mathbf{r}^n)$ is the gradient and $\mathbf{r}^{n+1}$ is the updated rate vector after n^{th} iteration. The update term $\Delta\mathbf{r}^n = -[\mathbf{H}F(\mathbf{r}^n)]^{-1}\nabla F(\mathbf{r}^n)$ is called *Newton step* and $\lambda(\mathbf{r}^n) = \left(\nabla F(\mathbf{r}^n)^T[\mathbf{H}F(\mathbf{r}^n)]^{-1}\nabla F(\mathbf{r}^n)\right)^{\frac{1}{2}}$ is named as *Newton decrement*. $\lambda(\mathbf{r})$ is found to be useful as a stopping criterion [37]. The aim of this algorithm is to decrease the objective function in each iteration; however, we observed that fixed point implementation of this algorithm may induce an increase in the objective function after an iteration. Hence, we choose two stopping criteria. We finalize the algorithm if either $\lambda(\mathbf{r})$ is smaller than a predefined threshold ϵ_N or the objective function increases. Algorithm 3 outlines Newton's method.

So far we have seen that an iterative method, SUMT algorithm, can be used to solve Problem 3 that minimizes the energy consumption given a transmission deadline. In the next subsection,

Algorithm 3 Newton's Method

repeat
  $F_{Previous} \leftarrow F(\mathbf{r})$
  $\Delta\mathbf{r} = -[\mathbf{H}F(\mathbf{r})]^{-1}\nabla F(\mathbf{r})$
  $\lambda(\mathbf{r}) = \left(\nabla F(\mathbf{r})^T[\mathbf{H}F(\mathbf{r})]^{-1}\nabla F(\mathbf{r})\right)^{\frac{1}{2}}$
  $\mathbf{r} \leftarrow \mathbf{r} + \Delta\mathbf{r}$
until $F_{Previous} \leq F(\mathbf{r}) \quad \vee \quad \lambda(\mathbf{r}) \leq \epsilon_N$

we present a new algorithm , which use SUMT algorithm, to solve Problem 2.

3.4.2 Solution of Problem 2 with SUMT Algorithm

Given a transmission deadline, we can find the minimum energy consumption by solving Problem 3 using SUMT algorithm. And Lemma 3.4.3 states that energy consumption of an optimal schedule strictly decreases as the transmission completion time increases. So, using the SUMT algorithm, we first find lower and upper bounds on the transmission completion time of Problem 2. Then, we iteratively narrow down the distance between the bounds and finally obtain the solution of Problem 2 within some error, i.e., $\epsilon > 0$.

Let us define the last epoch with data arrival as

$$N = \underset{k\in\{1,2,\ldots k^{up}\}}{\operatorname{argmin}} \left(B_1(t_{k^{up}}) = B_1(t_k) \quad \vee \quad B_2(t_{k^{up}}) = B_2(t_k)\right).$$

As we cannot transmit a data before it arrives, we choose the last data arrival time t_N as a (loose) lower bound on the transmission completion time. After having a lower bound, we start searching for an upper bound. Using SUMT algorithm, we first calculate the minimum energy consumption when we select the deadline as the end of the last data arriving epoch, i.e., t_{N+1}. If the energy consumption is smaller than the energy harvested till t_{N+1}, then t_{N+1} becomes an upper bound on the transmission duration. Otherwise, it is not feasible to transmit all the data with the given constraints and t_{N+1} becomes a lower bound. We continue this search with the next epoch until we find an upper bound. We should note that as the number of epochs increases, the total harvested energy increases and the total energy consumption decreases (see Lemma 3.4.3); hence we will eventually find an upper bound. After we find a lower and an upper bound, we proceed the search for the minimum transmission duration by using the bisection method. Note that we can use the bisection method because minimum energy consumption is an decreasing function of the transmission completion time (see Lemma 3.4.3). At each iteration we set the transmission completion time to the middle point in between the upper and lower bounds and then check whether this point is feasible in terms of total energy consumption. If the calculated minimum energy consumption is less than the total harvested energy, we use the current transmission duration to update the upper bound; otherwise it is used to update the lower bound. By this method the the time difference between the lower and the upper bound halved at each iteration. We stop the algorithm when the possible range of the minimum transmission duration is smaller than a predefined threshold ϵ. Algorithm 4 outlines the method to solve Problem 2.

Algorithm 4 Time Minimization with SUMT Algorithm

$N \leftarrow \underset{k \in \{1,2,\ldots k^{up}\}}{\operatorname{argmin}} \; (B_1(t_{k^{up}}) = B_1(t_k) \quad \vee \quad B_2(t_{k^{up}}) = B_2(t_k))$
repeat
  $N \leftarrow N + 1$
  $E_{min} \leftarrow$ SUMT_Algorithm(t_N)
until $E_{min} \leq E(t_N)$
$T^{min} \leftarrow t_{N-1}$, $T^{max} \leftarrow t_N$
repeat
  $T \leftarrow (T^{min} + T^{max})/2$
  $E_{min} \leftarrow$ SUMT_Algorithm(T)
  if $E_{min} < E(T)$ **then**
    $T^{max} \leftarrow T$
  else
    $T^{min} \leftarrow T$
  end if
until $T^{max} - T^{min} \leq \epsilon$
$T_{min} \leftarrow T$

3.5 Comparison of DuOpt and SUMT Algorithms

In order to solve Problem 2, we have proposed two different algorithms, *DuOpt* and time minimization with SUMT. The main difference in between these two algorithms is the parameter update procedure. *DuOpt* algorithm acts like a block coordinate descent algorithm and optimize only a portion of parameters (rates and powers in consecutive epochs) at a time.

Suppose that we have a K epoch problem. In every step of *DuOpt* algorithm, two consecutive epochs are optimized. Each iteration of *DuOpt* has roughly K steps, thus complexity of an iteration is linear with the number of epochs. It has been observed that iteration count indeed increases linearly with the number of epochs. On the other hand, SUMT algorithm optimizes all the parameters at the same time. In each iteration a $2K \times 2K$ Hessian matrix is constructed and inverse of this matrix is found. We have simulated DuOpt and SUMT algorithms with different number of epochs and observed that simulation time for both algorithms increase quadratically as the number of epochs increase. However, the termination time of *DuOpt* algorithm is almost two orders of magnitude shorter than the SUMT algorithm, as the number of epochs is varied (see Fig. 3.8). In the simulation, energy harvest and data arrival amounts are chosen as zero with a probability of 0.25 and otherwise chosen from Pareto distribution with probability 0.75. Probability density function of Pareto distribution is as follows.

$$f(x) = \frac{\alpha}{b}\left(\frac{b}{x}\right)^{(\alpha+1)} \quad , \ x \geq b.$$

We should also compare the memory requirements of algorithms. In each step of *DuOpt* algorithm only two consecutive epochs are considered; therefore, memory requirement of an iteration is constant across varying number of epochs. But, SUMT algorithm constructs a $2K \times 2K$ Hessian matrix in each iteration and the size of this matrix increases quadratically with the increasing epoch numbers.

The only drawback of *DuOpt* algorithm seems to be the termination process. In SUMT algorithm we always know the ultimate distance of the optimal schedule and halve this distance at each iteration. However, transmission time reduction of *DuOpt* algorithm can be erratic and may induce an early termination. We have observed that controlling total power levels before termination is very useful in *DuOpt* algorithm. If the total power level decreases greater than a predefined threshold, i.e., an ϵ of average power level, then we should not let the algorithm terminate.

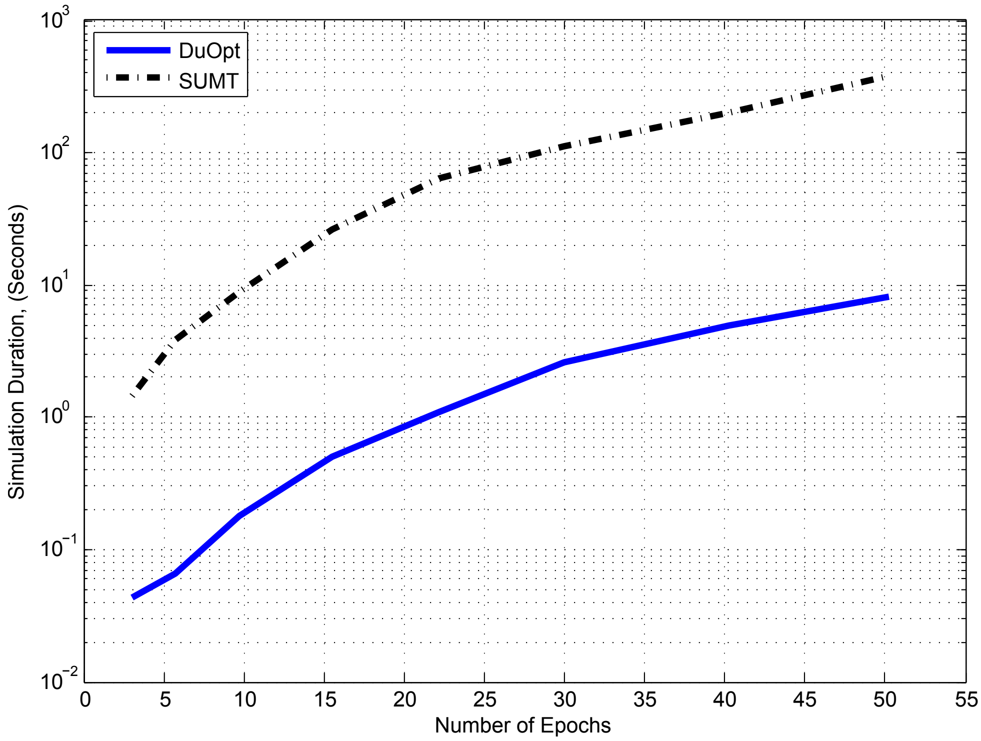

Figure 3.8: Average computation time (log scale) versus number of epochs for the optimal solutions of Problem 2 with DuOpt and SUMT Algorithms: Solid blue line is the average computation time of the optimal schedule with *DuOpt* and black dashed line represents the computation time of SUMT algorithm. A two user AWGN BC with $1KHz$ bandwidth and noise spectral density of $N_0 = 10^{-12}$ Watts/Hz is considered. Path loss factors on the links of stronger and weaker user are assumed to be $s_1 = 70dB$ and $s_2 = 75dB$, respectively. There is a probability of 0.25 that no energy harvest occurs at the start of an epoch. With probability 0.75 energy harvest amounts (in Watts) are chosen from a Pareto distribution with parameters $b = 2$ and $\alpha = 2$. Similarly, there is a probability of 0.25 that no data arrival occurs, otherwise data arrival amounts (in Kbits) are selected from a Pareto distribution with parameters $b = 4$, $\alpha = 2$ and then rounded to the nearest integer.

CHAPTER 4

CONCLUSIONS

In this thesis, we studied the AWGN broadcast channel transmission completion time minimization problem of an energy harvesting transmitter with two-users. It has been shown that the solution of the problem is unique. In order to obtain the unique optimal schedule, transmitter should aim to have a nondecreasing transmit power level. In fact, transmitting at a constant power level to both users is the best strategy if all the data and energy is present at the beginning of transmission. However, energy constraints due to energy harvesting procedure may hinder this operation. If energy harvests at the early part of schedule are insufficient, then transmitter starts from a lower sustainable power level and increase it as more energy is harvested from the environment. On the other hand, if energy harvests diminish as time passes, then transmitter will save the harvested energy from the earlier parts of the schedule for later use and achieve a constant power level.

Data arrivals also affect the transmission strategy. For example under WUFBC the rate of the stronger user is nondecreasing in an optimal schedule. In the general case, this statement is not correct and weak user data arrivals may induce a decrease in the rate of the stronger user in an optimal schedule. In fact, the stronger user's rate decrease so that the sum of rates do not decrease. If we analyze the solution of two epoch problem we will observe that in all possible solutions the sum of rates is nondecreasing.

We have showed that under WUFBC, an efficient iterative algorithm, *DuOpt* solves the delay minimization problem. *DuOpt* optimize two consecutive epochs at a time and decrease the transmission completion time in each iteration. With a slightly different approach, we also developed a second algorithm that solves the problem. Second algorithm converge to the optimal schedule by solving the dual problem with SUMT algorithm in each iteration. The difference between these two algorithms underlie in their approach to the solution. *DuOpt* optimize selected variables at a time, whereas the second algorithm optimize all the variables at a time. *DuOpt* does more iterations, yet it an iteration takes much less time than an iteration of the second algorithm. Hence, *DuOpt* achieves the optimal schedule faster. Another issue is the memory requirements of algorithms. *DuOpt* needs nearly no additional memory since it optimize two epochs at a time. However, the memory requirement of second algorithm increase quadratically since for a K epoch problem $2K \times 2K$ Hessian matrix is constructed in each iteration.

There are several future directions from this work. We have assumed that the battery has infinite capacity. Extending the problem with a finite capacity battery would be interesting. In single user problem it has been shown that with finite battery, transmission power is no longer non-decreasing and power may decrease whenever the battery is fully charged [26]. We expect a similar result for the broadcast channel. In fact, if power does not decrease, some of the harvested energy would be wasted due to battery overflow.

In this thesis we have assumed that all the harvested energy is consumed for transmission. However, in practice transmitter would also use its energy for other purposes, i.e., processor energy consumption. An extension of the problem with a more realistic model, which considers a constant processor power consumption whenever transmission occurs, is also interesting and may help us in development of practical algorithms.

In this thesis, we have considered the AWGN broadcast channel. Another future work is to reconsider the problem in a fading broadcast channel. In a fading channel the channel gains vary in time; hence, the stronger and the weaker users may interchange during transmission. And a further extension to the problem, which considers no channel state information at the transmitter side, makes the transmitter unaware of the stronger/weaker user at the moment.

We have considered an offline problem in this thesis. In many practical scenarios, times and amounts of energy harvests as well as data arrivals are not known in advance, which makes the online approach to the formulation of problem more relevant. Online approaches range from dynamic programming to machine learning algorithms. Our results give lower bound on transmission completion time (or total energy consumption) to online problems. Finally, online algorithms that utilize the results of this study is thought to achieve better performance.

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APPENDIX A

PROOF OF PROPOSITION 1

First and second order derivatives of $h_1(P,r)$ and $h_2(P,r)$ for the AWGN broadcast channel can easily be found from (2.6) and (2.7) as follows

$$\frac{\partial h_1(P,r)}{\partial P} = \frac{1}{2}(\log_2 e)\frac{s_1 s_2}{s_1 s_2 P + s_1\sigma^2 - (s_1 - s_2)\sigma^2 2^{2r}} \geq 0 \tag{A.1}$$

$$\frac{\partial h_1(P,r)}{\partial r} = -\frac{s_1 s_2 P + s_1\sigma^2}{s_1 s_2 P + s_1\sigma^2 - (s_1 - s_2)\sigma^2 2^{2r}} \leq 0 \tag{A.2}$$

$$\frac{\partial^2 h_1(P,r)}{\partial P^2} = -\frac{1}{2}(\log_2 e)\frac{(s_1 s_2)^2}{(s_1 s_2 P + s_1\sigma^2 - (s_1 - s_2)\sigma^2 2^{2r})^2} \leq 0 \tag{A.3}$$

$$\frac{\partial^2 h_1(P,r)}{\partial r^2} = -\frac{(2\ln 2)(s_1 - s_2)(s_1 s_2 P + s_1\sigma^2)\sigma^2 2^{2r}}{(s_1 s_2 P + s_1\sigma^2 - (s_1 - s_2)\sigma^2 2^{2r})^2} \leq 0 \tag{A.4}$$

$$\frac{\partial^2 h_1(P,r)}{\partial P \partial r} = -\frac{s_1 s_2 (s_1 - s_2)\sigma^2 2^{2r}}{(s_1 s_2 P + s_1\sigma^2 - (s_1 - s_2)\sigma^2 2^{2r})^2} \leq 0$$

$$\frac{\partial^2 h_1(P,r)}{\partial r \partial P} = \frac{(s_1 - s_2)(s_1 - s_2)\sigma^2 2^{2r}}{(s_1 s_2 P + s_1\sigma^2 - (s_1 - s_2)\sigma^2 2^{2r})^2} \geq 0$$

$$\frac{\partial h_2(P,r)}{\partial P} = \frac{1}{2}(\log_2 e)\frac{s_2}{s_2 P + \sigma^2} \geq 0 \tag{A.5}$$

$$\frac{\partial h_2(P,r)}{\partial r} = -\frac{2^{2r_1}}{2^{2r_1} + \frac{s_1 - s_2}{s_2}} \leq 0 \tag{A.6}$$

$$\frac{\partial^2 h_2(P,r)}{\partial P^2} = -\frac{1}{2}(\log_2 e)\frac{{s_2}^2}{(s_2 P + \sigma^2)^2} \leq 0 \tag{A.7}$$

$$\frac{\partial^2 h_2(P,r)}{\partial r^2} = -\frac{(2\ln 2)2^{2r_1}\frac{s_1 - s_2}{s_2}}{(2^{2r_1} + \frac{s_1 - s_2}{s_2})^2} \leq 0 \tag{A.8}$$

$$\frac{\partial^2 h_2(P,r)}{\partial P \partial r} = \frac{\partial^2 h_2(P,r)}{\partial r \partial P} = 0 \tag{A.9}$$

Rate functions $h_1(P,r)$ and $h_2(P,r)$ are nonnegative by definition. $h_1(P,r)$ and $h_2(P,r)$ are monotone increasing with power by (A.1) and (A.5) and monotone decreasing with rate by (A.2) and (A.6) respectively. $h_1(P,r)$ and $h_2(P,r)$ are concave in power by (A.3) and (A.7) as well as in rate by (A.4) and (A.8) respectively. (A.9) proves the last property of Proposition 1.

APPENDIX B

PROOF OF LEMMA 3.2.2

Substituting $r'_{11} = r_{11} + (1-\beta)\Delta r$ and $r'_{12} = r_{12} - \beta\Delta r$ in to (3.6), we have the following.

$$f(\beta) = h_2(P_1, r_{11} + (1-\beta)\Delta r)\beta + h_2(P_2, r_{11} - \beta\Delta r)(1-\beta) - h_2(P_1, r_{11})\beta - h_2(P_2, r_{12})(1-\beta).$$

The 2^{nd} order derivative of f with respect to β is the following

$$\begin{aligned}\frac{\partial^2 f}{\partial \beta^2} &= 2\{\underbrace{h_{2_y}(P_1, r_{11} + (1-\beta)\Delta r)(-\Delta r) - h_{2_y}(P_2, r_{12} - \beta\Delta r)(-\Delta r)}_{\leq 0}\} \\ &\quad +\{\underbrace{\beta(\Delta r)^2 h_{2_{yy}}(P_1, r_{11} + (1-\beta)\Delta r)}_{\leq 0} + \underbrace{(1-\beta)(\Delta r)^2 h_{2_{yy}}(P_2, r_{12} - \beta\Delta r)}_{\leq 0}\} \leq 0 \qquad \text{(B.1)}\end{aligned}$$

According to the properties of the rate region (1)-(4), (B.1) always holds. Hence f is concave in β. ■

APPENDIX C

TWO EPOCH OPTIMIZATIONS

In this appendix, we study the optimization of a schedule with just two epochs. Since both *FlowRight* and *DuOpt* algorithms rely on the optimization of two consecutive epochs, the results given in appendix provide a basis for solutions of Problem 1 and Problem 2.

Suppose that we have two epochs as shown in Fig. C.1, where B_{ij} is the data arrival for the i^{th} user, E_j is the energy harvest at the beginning of j^{th} epoch for $i, j \in \{1, 2\}$. T_1 and T_2 are the length of the first epoch and second epoch respectively.

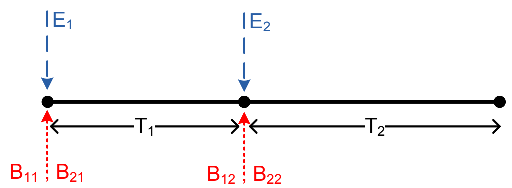

Figure C.1: Illustration of local optimization problem.

Let P_{ij} and r_{ij} be the power and rate assigned to the i^{th} user during j^{th} epoch. All schedules should respect three causality constraints given below.

$$E_1 \geq (P_{11} + P_{12}) \cdot T_1 \tag{C.1}$$

$$B_{11} \geq r_{11} \cdot T_1 \tag{C.2}$$

$$B_{21} \geq r_{21} \cdot T_1, \tag{C.3}$$

where (C.1) is called the energy constraint, (C.2) and (C.3) are referred as the *stronger* and *weaker* user data constraints respectively.

Given these energy harvest and data arrivals (hence the constraints), we have derived two different problems:

Problem 5 Minimization of Total Energy Consumption in Local Optimization:

$$\textbf{Minimize :}\ E = \sum_{i=1}^{2} T_i P_i$$

$$\textbf{Subject To:}\ 0 \leq r_{1i} \leq h_1(P_i, 0)\ ,\ 0 \leq P_i\ ,\ r_{2i} = h_2(P_i, r_{1i})\ ,\ i \in \{1, 2\}$$

$$T_1 P_1 \leq E_1\ ,\ T_1 r_{11} \leq B_{11}\ ,\ T_1 r_{21} \leq B_{21}$$

$$T_1 r_{11} + T_2 r_{12} = B_{11} + B_{12}\ ,\ T_1 r_{21} + T_2 r_{22} = B_{21} + B_{22}$$

Problem 6 Minimization of Transmission Time in Local Optimization:

$$\textbf{Minimize :}\ T(\{r_{1i}, r_{2i}\})\ ,\ i \in \{1, 2\}$$

$$\textbf{Subject To:}\ 0 \leq r_{1i} \leq h_1(P_i, 0)\ ,\ 0 \leq P_i\ ,\ r_{2i} = h_2(P_i, r_{1i})\ ,\ i \in \{1, 2\}$$

$$min(T, T_1)(P_1) \leq E_1$$

$$min(T, T_1) r_{11} \leq B_{11}\ ,\ min(T, T_1) r_{21} \leq B_{21}$$

$$min(T, T_1) r_{11} + (T - T_1) r_{12} = B_{11} + B_{12}$$

$$min(T, T_1) r_{21} + (T - T_1) r_{22} = B_{21} + B_{22}$$

Problem 5 and Problem 6 are dual in the sense that solution to one of them provides a basis for the solution to other one. In this section, we will provide methods to obtain solutions of Problem 5 and 6.

The total transmit power and the rate of the stronger user should be as close as possible for the optimal schedule (see 3.2.2 and Lemma 3.3.1 of [11]). If no constraint oppose, the optimal schedule would use the same transmit power (and rates) at the first and second epoch. However, the structure of the solutions to the local optimizations changes if either one of the constraints satisfied with equality. Since there are 3 different constraints, after optimization one will encounter with one of the $2^3 = 8$ local optimization results. We have studied all 8 cases and came up with solutions to each case for both energy minimization and time minimization methods. In each case, the solution to the problem can be calculated analytically for energy minimization and it can be found iteratively for time minimization. Before starting a local optimization, if we have known which constraints would be active, then the result would be obtained solving just that case. However, we do not know which event would occur before optimization, hence we find the results for each case and then pick the best one[1] that respects energy and data causalities.

Before moving to the solutions, let us define b_{ij} as the data sent to i^{th} user in j^{th} epoch in any schedule. Both delay optimal and energy optimal schedules operate at the boundary of AWGN BC. We will find the data sent to each user by the following equations:

$$\begin{aligned} b_{11} &= \frac{T_1}{2}\log_2(1+\frac{s_1P_{11}}{\sigma^2}) && \text{(C.4)}\\ b_{21} &= \frac{T_1}{2}\log_2(1+\frac{s_2P_{21}}{s_2P_{11}+\sigma^2}) && \text{(C.5)}\\ b_{12} &= \frac{T_2}{2}\log_2(1+\frac{s_1P_{12}}{\sigma^2}) && \text{(C.6)}\\ b_{22} &= \frac{T_2}{2}\log_2(1+\frac{s_2P_{22}}{s_2P_{12}+\sigma^2}), && \text{(C.7)} \end{aligned}$$

where P_{ij} is the power reserved for the i^{th} user in the j^{th} epoch by the transmitter and s_j is the channel gain for the j^{th} epoch and σ^2 is the variance of the AWGN noise.

Next we will analyze all 8 cases and give algorithms that obtain optimum results for both energy and time minimization.

C.1 Local optimization when no constraint is active

The first case to be considered is the one that has no active constraints. Since no constraint is active in this problem, the total transmit power and the rate of the stronger user should be unchanging through two epochs (c.f. 3.2.2 and Lemma 3.3.1 of [11]). The solution to this problem is illustrated in Figure C.2. Since transmission parameters (power and rates) do not change in between epochs, it is reasonable to treat this problem as one epoch problem with transmission duration $(T_1 + T_2)$. The minimum energy to transmit $B_1 = B_{11} + B_{12}$ and $B_2 = B_{21} + B_{22}$ bits in $T = (T_1 + T_2)$ can be obtained by the following equation:

$$E(T) = T\sigma^2\left(\frac{2^{\frac{2B_2}{T}}-1}{s_2}+\frac{(2^{\frac{2B_1}{T}}-1)2^{\frac{2B_2}{T}}}{s_1}\right). \tag{C.8}$$

[1] Best result is the one that consumes minimum energy for the *energy minimization* method and one that has the minimum transmit time for the *time minimization* method.

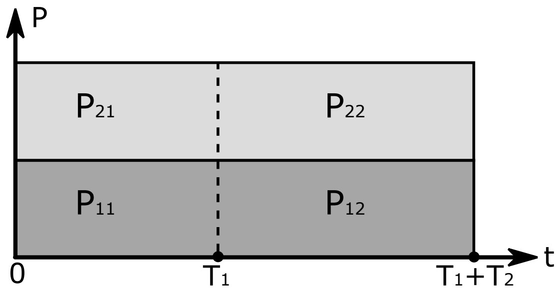

Figure C.2: Optimal power allocation for the stronger and weaker user when no constraint is active.

Taking first and second order partial derivative of E_{min} with respect to T we have

$$\begin{aligned}
\frac{\partial E}{\partial T} &= \sigma^2\left(\frac{2^{\frac{2B_2}{T}}-1}{s_2}+\frac{(2^{\frac{2B_1}{T}}-1)2^{\frac{2B_2}{T}}}{s_1}\right) \\
&\quad -\frac{\sigma^2\ln 2}{T}\left(\frac{2^{\frac{2B_2}{T}}2B_2(s_1-s_2)}{s_1 s_2}+\frac{2^{\frac{2(B_1+B_2)}{T}}2(B_1+B_2)}{s_1}\right) \\
\frac{\partial^2 E}{\partial T^2} &= \frac{\sigma(\ln 2)^2}{T^3}\left(\frac{(s_1-s_2)4B_2^2 2^{\frac{2B_2}{T}}}{s_1 s_2}+\frac{2^{\frac{2(B_1+B_2)}{T}}(2(B_1+B_2))^2}{s_1}\right) \\
&> 0 \qquad \text{(C.9)}
\end{aligned}$$

From (C.9), $E(T)$ is a strictly convex function. As T goes to infinity, E is as follows

$$\begin{aligned}
\lim_{T\to\infty} E(T) &= \lim_{T\to\infty}\sigma^2\ln 2\left(\frac{2^{\frac{2B_2}{T}}2B_2(s_1-s_2)}{s_1 s_2}+\frac{2^{\frac{2(B_1+B_2)}{T}}2(B_1+B_2)}{s_1}\right) \\
&= \sigma^2\ln 2\left(\frac{2B_1}{s_1}+\frac{2B_2}{s_2}\right) \\
&< \infty. \qquad \text{(C.10)}
\end{aligned}$$

(C.9) and (C.10) together proves that E is a strictly decreasing convex function of T.

Given $E = E_1 + E_2$, B_1 , and B_2 , minimum transmission time for all the data, T_{min}, can be obtained by solving a nonlinear equation. (C.8) is a strictly decreasing convex function of transmission time T, which implies that for a given E, there is always a unique T value that can be found iteratively by using bisection method (see Algorithm 5). In the bisection method, the objective is to find the single root of the equation $E(T) - (E_1 + E_2) = 0$. Assuming an upper bound for the transmission time $T_{upper} > T_{min}$ and taking lower bound as 0, bisection method starts from initial domain interval $[T^{min} = 0\ ,\ T^{max} = T_{upper}]$ and narrows down this interval by bisecting it in each iteration. Algorithm converges to the unique solution as $E(T)$

is a continuous function of T and can be terminated within a certain arbitrarily small tolerance $\epsilon > 0$ in a practical implementation.

Algorithm 5 Local Time Minimization Algorithm for the Case: No Constraint is Active

$T^{max} \leftarrow T_{upper}$, $T^{min} \leftarrow 0$
repeat
  $\hat{T} \leftarrow (T^{max} + T^{min})/2$
  $\hat{E} \leftarrow \hat{T}\sigma^2\left(\frac{2^{\frac{2B_2}{\hat{T}}}-1}{s_2} + \frac{(2^{\frac{2B_1}{\hat{T}}}-1)2^{\frac{2B_2}{\hat{T}}}}{s_1}\right)$
  if $\hat{E} > E$ **then**
    $T^{min} \leftarrow \hat{T}$
  else
    $T^{max} \leftarrow \hat{T}$
  end if
until $\hat{E} == E$
$T_{min} \leftarrow \hat{T}$

C.2 Local optimization when only stronger user data constraint is active

Suppose $P_1 = P_{11} + P_{21}$ is the total power used by the transmitter at the first epoch whereas $P_2 = P_{12} + P_{22}$ is the total power used at the latter epoch. If only stronger user data constraint is active, then total power of the local optimal solution should stay constant (See Lemma 3 of [15]), i.e., $P_1 = P_2$. Also, we should have $P_{11} \leq P_{12}$, otherwise stronger user data constraint would not be active. Fig. C.3 illustrates the optimal power allocation for this case.

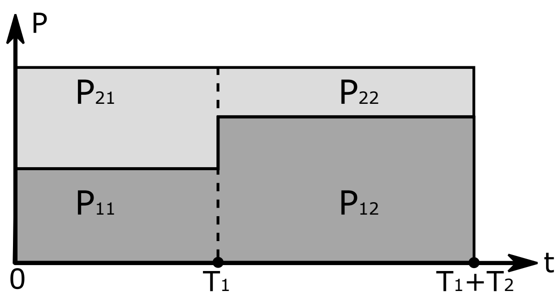

Figure C.3: Optimal power allocation for the stronger and weaker user when only stronger user data constraint is active.

Optimum schedule that minimize energy can be found as follows

$$P_{11} = \frac{\sigma^2}{s_1}(2^{\frac{2B_{11}}{T_1}} - 1)\ ,\ P_{12} = \frac{\sigma^2}{s_1}(2^{\frac{2B_{12}}{T_2}} - 1)\ \text{and}\ P_{22} = P_{11} + P_{21} - P_{12} \quad (C.11)$$

$$B_2 = B_{21} + B_{22} = \frac{T_1}{2}\log_2(1 + \frac{s_2P_{21}}{s_2P_{11} + \sigma^2}) + \frac{T_2}{2}\log_2(1 + \frac{s_2P_{22}}{s_2P_{12} + \sigma^2}). \quad (C.12)$$

From (C.11) and (C.12) we derive

$$P_{21} = -P_{11} + \frac{1}{s_2}\left[\left(2^{2B_2}(s_2P_{11} + \sigma^2)^{T_1}(s_2P_{12} + \sigma^2)^{T_2}\right)^{\frac{1}{T_1+T_2}} - \sigma^2\right]. \quad (C.13)$$

Then, total energy consumed in two epochs is calculated by

$$E_{min} = (P_{11} + P_{21})(T_1 + T_2). \quad (C.14)$$

Substituting (C.11) into (C.12) and arranging terms we have

$$B_2 = \frac{T_1}{2}\log_2\left(\frac{\frac{E_1+E_2}{T_1+T_2} + \frac{\sigma^2}{s_2}}{\frac{\sigma^2}{s_1}(2^{\frac{2B_{11}}{T_1}} - 1) + \frac{\sigma^2}{s_2}}\right) + \frac{T_2}{2}\log_2\left(\frac{\frac{E_1+E_2}{T_1+T_2} + \frac{\sigma^2}{s_2}}{\frac{\sigma^2}{s_1}(2^{\frac{2B_{12}}{T_2}} - 1) + \frac{\sigma^2}{s_2}}\right) \quad (C.15)$$

First and second order derivatives of B_2 with respect to T_2 are as follows:

$$\begin{aligned}\frac{\partial B_2}{\partial T_2} &= \frac{1}{2}\log_2\left(\frac{E_1+E_2}{T_1+T_2} + \frac{\sigma^2}{s_2}\right) - \frac{1}{2\ln(2)\left(1 + \frac{\sigma^2}{s_2}\frac{T_1+T_2}{E_1+E_2}\right)} \\ &\quad -\frac{1}{2}\log_2\left(\frac{\sigma^2}{s_1}2^{\frac{2B_{12}}{T_2}} + \sigma^2\frac{s_1 - s_2}{s_1s_2}\right) + \frac{B_{12}}{T_2}\frac{1}{1 + \frac{s_1-s_2}{s_2}2^{-\frac{2B_{12}}{T_2}}} \qquad (C.16)\\ \frac{\partial^2 B_2}{\partial T_2^2} &= -\frac{1}{2\ln(2)(T_1+T_2)\left(1 + \frac{\sigma^2}{s_2}\frac{T_1+T_2}{E_1+E_2}\right)^2} - \frac{2\ln(2)B_{12}^2\frac{s_1-s_2}{s_2}2^{-\frac{2B_{12}}{T_2}}}{T_2^3\left(1 + \frac{s_1-s_2}{s_2}2^{-\frac{2B_{12}}{T_2}}\right)^2} \qquad (C.17)\\ &< 0\end{aligned}$$

As shown in (C.17), second derivative of B_2 is always negative for $s_1 > s_2$, which implies that B_2 is a strictly concave function of T_2. As T_2 goes to infinity, B_2 is as follows,

$$\begin{aligned}\lim_{T_2\to\infty} B_2 &= -\frac{T_1}{2}\log_2\left(\frac{s_2}{s_1}(2^{\frac{2B_{11}}{T_1}} - 1) + 1\right) - \frac{s_2}{s_1}B_{12} + \frac{(E_1+E_2)s_2}{2\ln(2)\sigma^2} \qquad (C.18)\\ &> -\infty\end{aligned}$$

By (C.18), B_2 goes to a finite number as T_2 goes to infinity, which implies that B_2 is a strictly increasing concave function of T_2. Since B_2 given in (C.15) is a strictly increasing concave function of T_2, for a given T_2, there exist a unique B_2 value. In order to find the minimum time to transmit, we iteratively find the transmission duration in the second epoch, $\hat{T}_2$, that sends exactly B_2 bits to weaker user by using bisection method. Minimum time to transmit $B_1 = B_{11} + B_{12}$ and B_2 bits to users in these two epochs is found to be equal to $T_{min} = T_1 + \hat{T}_2$. Algorithm 6 presents a pseudo-code of time minimization algorithm for the case that only stronger user data causality event occurs.

Algorithm 6 Local Time Minimization Algorithm for the Case: Only Stronger User Data Constraint is Active

$T^{max} \leftarrow T_{upper}$, $T^{min} \leftarrow 0$
repeat
  $\hat{T} \leftarrow (T^{max} + T^{min})/2$
  $\hat{B}_2 \leftarrow \frac{T_1}{2}\log_2\left(\frac{\frac{E_1+E_2}{T_1+\hat{T}}+\frac{\sigma^2}{s_2}}{\frac{\sigma^2}{s_1}(2^{\frac{2B_{11}}{T_1}}-1)+\frac{\sigma^2}{s_2}}\right) + \frac{\hat{T}}{2}\log_2\left(\frac{\frac{E_1+E_2}{T_1+\hat{T}}+\frac{\sigma^2}{s_2}}{\frac{\sigma^2}{s_1}(2^{\frac{2B_{12}}{\hat{T}}}-1)+\frac{\sigma^2}{s_2}}\right)$
  if $\hat{B}_2 < B_2$ **then**
    $T^{min} \leftarrow \hat{T}$
  else
    $T^{max} \leftarrow \hat{T}$
  end if
until $\hat{B}_2 == B_2$
$T_{min} \leftarrow \hat{T} + T_1$

C.3 Local optimization when only weaker user data constraint is active

If only weaker user data constraint is active, then total power of the local optimal solution may increase (See Lemma 3 of [15]), i.e., $P_1 \leq P_2$. Also, we should have $P_{21} \leq P_{22}$, otherwise weaker user data constraint would not be active. Fig. C.4 illustrates the optimal power allocation for this case.

Optimum schedule that minimize energy can be found as follows

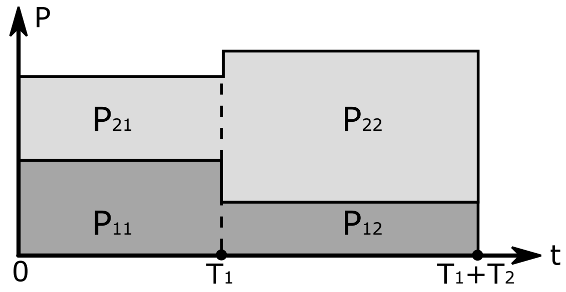

Figure C.4: Optimal power allocation for the stronger and weaker user when only weaker user data constraint is active.

From (C.5) and (C.7) we have

$$P_{21} = (2^{\frac{2b_{21}}{T_1}} - 1)(P_{11} + \frac{\sigma^2}{s_2}) \tag{C.19}$$

$$P_{22} = (2^{\frac{2b_{22}}{T_2}} - 1)(P_{12} + \frac{\sigma^2}{s_2}). \tag{C.20}$$

And from (C.4) and (C.6) we have

$$P_{12} = \frac{\sigma^2}{s_1}\left(2^{\frac{2B_1}{T_2}}\left(1 + \frac{s_1 P_{11}}{\sigma^2}\right)^{-\frac{T_1}{T_2}} - 1\right), \tag{C.21}$$

where $B_1 = b_{11} + b_{12}$. The total energy consumed in two epochs can be calculated as follows

$$E = (P_{11} + P_{21})T_1 + (P_{12} + P_{22})T_2. \tag{C.22}$$

By using (C.21), (C.19), (C.20) and (C.22) we can write E in terms of P_{11} as follows

$$E = \frac{\sigma^2}{s_2}\left(T_1(2^{\frac{2b_{21}}{T_1}} - 1) + T_2(2^{\frac{2b_{22}}{T_2}} - 1)\right) - T_2\frac{\sigma^2}{s_1}2^{\frac{2b_{22}}{T_2}} + T_1 2^{\frac{2b_{21}}{T_1}} P_{11} + T_2\frac{\sigma^2}{s_1}2^{\frac{2(B_1+b_{22})}{T_2}}\left(1 + \frac{s_1 P_{11}}{\sigma^2}\right)^{-\frac{T_1}{T_2}} \tag{C.23}$$

The first and second order derivatives of E with respect to P_{11} are as follows

$$\frac{\partial E}{\partial P_{11}} = T_1\left(2^{\frac{2b_{21}}{T_1}} - 2^{\frac{2(B_1+b_{22})}{T_2}}\left(1 + \frac{s_1 P_{11}}{\sigma^2}\right)^{-\frac{T_1+T_2}{T_2}}\right) \tag{C.24}$$

$$\frac{\partial^2 E}{\partial P_{11}^2} = \frac{T_1(T_1 + T_2)}{T_2}\left(1 + \frac{s_1 P_{11}}{\sigma^2}\right)^{-\frac{T_1+2T_2}{T_2}} 2^{\frac{2(B_1+b_{22})}{T_2}} > 0. \tag{C.25}$$

Since (C.25) is always positive, E is strictly convex in P_{11}. Therefore, the minimum value of E occurs when $\frac{\partial E}{\partial P_{11}} = 0$. By equating (C.24) to 0, we obtain P_{11} that minimize total energy as follows

$$P_{11} = \frac{\sigma^2}{s_1}\left(2^{\frac{2(T_1(B_1+b_{22})-T_2 b_{21})}{T_1(T_1+T_2)}} - 1\right). \tag{C.26}$$

From a different perspective, in order to find the local optimal schedule, one has to transmit

$$b_{11} = \frac{T_1}{T_1+T_2}(B_1+b_{22}) - \frac{T_2}{T_1+T_2}b_{21} \tag{C.27}$$

bits to stronger user in the first epoch. Rearranging the terms in (C.27) we have

$$\frac{b_{11}+b_{21}}{T_1} = \frac{b_{12}+b_{22}}{T_2},$$

which means that *sum of the rates of stronger and weaker user in the first should be equal to sum of the rates of stronger and weaker user in the second epoch.*

Substituting (C.26) into (C.23), we obtain minimum transmission energy as follows

$$E_{min} = \frac{\sigma^2(s_1-s_2)}{s_1 s_2}\left(T_1 2^{\frac{2b_{21}}{T_1}} + T_2 2^{\frac{2b_{22}}{T_2}}\right) - \frac{\sigma^2}{s_2}(T_1+T_2) + \frac{\sigma^2}{s_1}(T_1+T_2)2^{\frac{2(B_1+b_{21}+b_{22})}{T_1+T_2}}. \tag{C.28}$$

In order to find the minimum energy consumption, we first set $b_{21} = B_{21}$ and $b_{22} = B_{22}$ as weaker user data constraint is active. Then, we obtain b_{11} from (C.27) and get $b_{12} = B_1 - b_{11}$. Finally, we obtain E_{min} by (C.28).

Before moving to time minimization method, let us first examine (C.28) by taking first and second order derivatives of with respect to T_2.

$$\begin{aligned}
\frac{\partial E_{min}}{\partial T_2} &= \frac{\sigma^2}{s_2}\left[\left(1 - 2\ln 2\frac{B_{22}}{T_2}\right)\frac{(s_1-s_2)}{s_1}2^{\frac{2B_{22}}{T_2}} - 1\right] + \frac{\sigma^2}{s_1}\left(1 - 2\ln 2\frac{B_1+B_2}{T_1+T_2}\right)2^{\frac{2(B_1+B_2)}{T_1+T_2}} \\
\frac{\partial^2 E_{min}}{\partial T_2^2} &= \frac{4(\ln 2)^2 B_{22}^2\sigma^2(s_1-s_2)}{T_2^3 s_1 s_2}2^{\frac{2B_{22}}{T_2}} + \frac{4(\ln 2)^2(B_1+B_2)^2\sigma^2}{(T_1+T_2)^3 s_1}2^{\frac{2(B_1+B_2)}{T_1+T_2}} > 0
\end{aligned} \tag{C.29}$$

As shown in (C.29), second derivative of E_{min} is always positive for $s_1 > s_2$, which implies that E_{min} is a strictly convex function of T_2. As T_2 goes to infinity, E_{min} is as follows,

$$\begin{aligned}
\lim_{T_2\to\infty} E_{min} &= \left(\left(T_1 2^{\frac{2B_{21}}{T_1}} - 1\right) + 2\ln 2B_{22}\right)\frac{\sigma^2(s_1-s_2)}{s_1 s_2} + 2(B_1+B_2)\frac{\sigma^2}{s_1} \\
&< \infty
\end{aligned} \tag{C.30}$$

Since E_{min} is a strictly convex function of T_2 and goes to a finite number as T_2 goes to infinity, E_{min} in (C.28) is a strictly decreasing convex function of T_2 and there is a unique E_{min} for each value of T_2. In order to find the minimum transmission duration, we first set $b_{21} = B_{21}$ and $b_{22} = B_{22}$. Then, from (C.28) we iteratively search, by using bisection method, the transmission duration in the second epoch, $\hat{T}$, that induce an exact energy consumption of $E_{min} = E_1 + E_2$. Minimum time to transmit B_1 and B_2 bits to users in these two epochs is found to be equal to $T_{min} = T_1 + \hat{T}$. Algorithm 7 presents a pseudo-code of time minimization algorithm when only weaker user data constraint is active.

Algorithm 7 Local Time Minimization Algorithm for the Case: Only Weaker User Data Constraint is Active

$T^{max} \leftarrow T_{upper}$, $T^{min} \leftarrow 0$
repeat
    $\hat{T} \leftarrow (T^{max} + T^{min})/2$
    $\hat{E} \leftarrow \frac{\sigma^2(s_1 - s_2)}{s_1 s_2}\left(T_1 2^{\frac{2B_{21}}{T_1}} + \hat{T} 2^{\frac{2B_{22}}{\hat{T}}}\right) + \frac{\sigma^2}{s_1}(T_1 + \hat{T}) 2^{\frac{2(B_1+B_2)}{T_1+\hat{T}}}$
    if $\hat{E} > E_1 + E_2$ **then**
        $T^{min} \leftarrow \hat{T}$
    else
        $T^{max} \leftarrow \hat{T}$
    end if
until $\hat{E} == E_1 + E_2$
$T_{min} \leftarrow \hat{T} + T_1$

C.4 Local optimization when only energy constraint is active

In this section, we will analyze the solution of local optimization when only energy constraint is active. Theorem 3.2.3 tells us that if there are only energy causality constraints, then stronger user rate is non-decreasing and may only increase if weaker user rate in the first epoch is zero. Thus, we will encounter with two sub-cases:

1. The rate (hence the power) of stronger user remains constant during transmission, i.e., $r_{11} = r_{12}$.

2. The rate of the stronger user increases at the end of the first epoch and the weaker user rate in the first epoch is zero, i.e., $r_{11} < r_{12}$, $r_{21} = 0$.

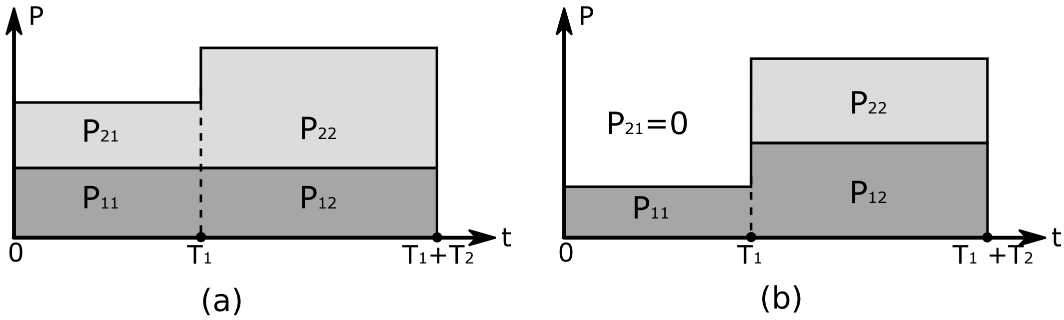

Figure C.5: Optimal power allocation for the stronger and weaker user when energy constraint is active. There are two sub-cases. (a): Rate of the stronger user is constant during transmission. (b): Rate of the stronger user increases at the end of the first epoch and the weaker user rate in the first epoch is zero.

These two sub-cases are illustrated in Fig. C.5. In the following, we provide the method to find the optimum schedule that minimize energy.

First we have to check whether the rate of the stronger user changes or not.

$$\frac{\sigma^2}{s_1}(2^{2\frac{B_{11}+B_{12}}{T_1+T_2}} - 1) \leq \frac{E_1}{T_1} = P_1, \tag{C.31}$$

If inequality in (C.31) holds, then we should have $r_{11} = r_{12}$ and we can obtain the minimum energy consumption as follows:

$$\begin{aligned}
P_{11} = P_{12} &= \frac{\sigma^2}{s_1}(2^{2\frac{B_{11}+B_{12}}{T_1+T_2}} - 1) \\
P_{21} &= \frac{E_1}{T_1} - P_{11} \\
b_{21} &= \frac{T_1}{2}\log_2\left(1 + \frac{s_2 P_{21}}{s_2 P_{11} + \sigma^2}\right) \\
b_{22} &= B_{12} + B_{22} - b_{12} \\
P_{22} &= \frac{s_2 P_{12} + \sigma^2}{s_2}(2^{\frac{b_{22}}{T_2}} - 1) \\
E_{min} &= T_1(P_{11} + P_{21}) + T_2(P_{12} + P_{22})
\end{aligned}$$

If the inequality in (C.31) does not hold, then rate of the stronger user cannot stay constant and we should have $r_{11} < r_{12}$ and $r_{21} = 0$. We analytically obtain the minimum energy consumption in this case as follows:

$$
\begin{aligned}
P_{11} &= P_1 = \frac{E_1}{T_1} \\
b_{11} &= \frac{T_1}{2}\log_2\Big(1+\frac{s_1 P_{11}}{\sigma^2}\Big) \\
b_{12} &= B_{11}+B_{12}-b_{11} \\
P_{12} &= \frac{\sigma^2}{s_1}(2^{\frac{2b_{12}}{T_1}}-1) \\
b_{22} &= B_{21}+B_{22} \\
P_{22} &= \frac{s_2 P_{12}+\sigma^2}{s_2}(2^{\frac{b_{22}}{T_2}}-1) \\
E_{min} &= T_1 P_{11} + T_2(P_{12}+P_{22})
\end{aligned}
$$

With a similar method used in section C.2, it is easy to prove that the number of bits transmitted to the weaker user is an increasing concave function of the transmission duration in the second epoch for both sub-cases. The minimum total transmission duration is again obtained by using bisection method. Algorithm 8 presents a pseudo-code for the time minimization method when only energy constraint is active.

C.5 Local optimization when more than one constraint is active

Selecting the rates from the boundary of the AWGN BC region, transmission in an epoch can be described by the parameters $\{P, r_1, r_2\}$. Since any of these parameters can be calculated from the others, i.e., $r_1 = h_1(P, r_2)$, $r_2 = h_2(P, r_1)$ and $P = g(r_1, r_2)$, transmission schedule in one epoch can be uniquely identified by any two of these parameters. If the solution of two epoch energy (or time) minimization problem has at least two active constraints, then the power and rates of the first epoch in can be calculated from active constraints and problem simplifies to finding parameters in one epoch.

Suppose that both energy and stronger user data constraints are active in the solution of Problem 5 (or Problem 6). Then, given the duration of the first epoch, T_1, total power and the stronger user's rate in the first epoch can be calculated by simply dividing the constraints with T_1, i.e., $P_1 = E_1/T_1$ and $r_{11} = B_{11}/T_1$. And the weaker user rate can also be calculated by $r_2 = h_2(P_1, r_1)$. Having identified the all the parameters in the first epoch, problem simplifies

Algorithm 8 Local Time Minimization Algorithm for the Case: Only Energy Constraint is Active

$B_1 \leftarrow B_{11} + B_{12}$, $B_2 \leftarrow B_{21} + B_{22}$
$P_{11} \leftarrow \frac{E_1}{T_1}$, $P_{12} \leftarrow \frac{E_1}{T_1}$
$\hat{T} \leftarrow 2B_1 / \left(log_2(1 + \frac{s_1 E_1}{\sigma^2 T_1}) \right) - T_1$
$P_{22} \leftarrow \frac{E_2}{\hat{T}} - P_{11}$
$\hat{B}_2 \leftarrow \frac{\hat{T}}{2} \log_2 \left(1 + \frac{s_2 P_{22}}{s_2 P_{12} + \sigma^2} \right)$
if $\hat{B}_2 < B_2$ **then**
    {We should have a constant rate for the stronger user during transmission.}
    $T^{max} \leftarrow T_{upper}, T^{min} \leftarrow \hat{T}$
    repeat
        $\hat{T} \leftarrow (T^{max} + T^{min})/2$
        $P_{11} \leftarrow \frac{\sigma^2}{s_1} \left(2^{\frac{2B_1}{T_1 + \hat{T}}} - 1 \right)$, $P_{12} \leftarrow P_{11}$
        $P_{21} \leftarrow (\frac{E_1}{T_1} - P_{11})$, $P_{22} \leftarrow (\frac{E_2}{\hat{T}} - P_{12})$
        $\hat{B}_2 \leftarrow \frac{T_1}{2} \log_2 \left(1 + \frac{s_2 P_{21}}{s_2 P_{11} + \sigma^2} \right) + \frac{\hat{T}}{2} \log_2 \left(1 + \frac{s_2 P_{22}}{s_2 P_{12} + \sigma^2} \right)$
        if $\hat{B}_2 < B_2$ **then**
            $T^{min} \leftarrow \hat{T}$
        else
            $T^{max} \leftarrow \hat{T}$
        end if
    until $\hat{B}_2 = B_2$
else
    {Whole energy in the first epoch should be used for the stronger user and rate (power) of the stronger user should increase at the end of the first epoch.}
    $T^{max} \leftarrow \hat{T}, T^{min} \leftarrow 0$, $b_{11} \leftarrow \frac{T_1}{2} \log_2(1 + \frac{s_1 E_1}{\sigma^2 T_1})$, $b_{12} \leftarrow B_1 - b_{11}$
    repeat
        $\hat{T} \leftarrow (T^{max} + T^{min})/2$
        $P_{12} \leftarrow \frac{\sigma^2}{s_1} \left(2^{\frac{2b_{12}}{\hat{T}}} - 1 \right)$, $P_{22} \leftarrow (\frac{E_2}{\hat{T}} - P_{12})$
        $\hat{B}_2 \leftarrow \frac{\hat{T}}{2} \log_2 \left(1 + \frac{s_2 P_{22}}{s_2 P_{12} + \sigma^2} \right)$
        if $\hat{B}_2 < B_2$ **then**
            $T^{min} \leftarrow \hat{T}$
        else
            $T^{max} \leftarrow \hat{T}$
        end if
    until $\hat{B}_2 = B_2$
end if
$T_{min} \leftarrow \hat{T} + T_1$

to finding the parameters in the second epoch.

In order to minimize transmission energy, we can simply calculate rates in the second epoch by dividing the remaining bits to the duration of second epoch, T_2, i.e., $r_{12} = B_{12}/T_2$ and $r_{22} = (B_{21} + B22 - r_{21}T_1)/T_2$. Then we calculate the transmission power of the second epoch from rates, i.e., $P_2 = g(r_{12}, r_{22})$. Finally, by using (C.8) we can find E_{min} that minimize energy consumption.

Delay minimization problem, Problem 6, also simplifies to minimization of transmission duration of bits $b_{12} = B_{12}$ and $b_{22} = (B_{21} + B22 - r_{21}T_1)$ in minimum time by using $E = E_1 + E_2 - P_1T_1$ units of energy. Solution of this problem is similar to the one in section C.1 and can be solved iteratively by Algorithm 5.

Solution method of other cases with two or more active constraints follow the same methodology given in the previous paragraphs, hence omitted here.