LIFETIME CONDITION PREDICTION FOR BRIDGES

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ABSTRACT

LIFETIME CONDITION PREDICTION FOR BRIDGES

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Infrastructure systems are crucial facilities. They supply the necessary transportation, water and energy utilities for the public. However, while aging, these systems gradually deteriorate in time and approach the end of their lifespans. As a result, they require periodic maintenance and repair in order to function and be reliable throughout their lifetimes. Bridge infrastructure is an essential part of the transportation infrastructure. Bridge management systems (BMSs), used to monitor the condition and safety of the bridges in a bridge infrastructure, have evolved considerably in the past decades. The aim of BMSs is to use the resources in an optimal manner keeping the bridges out of risk of failure. The BMSs use the lifetime performance curves to predict the future condition of the bridge elements or bridges. The most widely implemented condition-based performance prediction and maintenance optimization model is the Markov Decision Process-based models (MDP). The importance of the Markov Decision Process-based model is that it defines the time-variant deterioration using the Markov Transition Probability Matrix and performs the lifetime cost optimization by finding the optimum maintenance policy. In this study, the Markov decision process-based model is examined and a computer program to find the optimal policy with discounted life-cycle cost is developed. The other performance prediction model investigated in this study is a probabilistic Bi-linear model which takes into account the uncertainties for the deterioration process and the application of maintenance actions by the use of random variables. As part of the study, in order to further analyze and develop the Bi-linear model, a Latin Hypercube Sampling-based (LHS) simulation program is also developed and integrated into the main computational algorithm which can produce condition, safety, and life-cycle cost profiles for bridge members with and without maintenance actions. Furthermore, a polynomial-based condition prediction is also examined as an alternative performance prediction model. This model is obtained from condition rating data by applying regression analysis. Regression-based performance curves are regenerated using the Latin Hypercube sampling method. Finally, the results from the Markov chain-based performance prediction are compared with Simulation-based Bi-linear prediction and the derivation of the transition probability matrix from simulated regression based condition profile is introduced as a newly developed approach. It has been observed that the results obtained from the Markov chain-based average condition rating profiles match well with those obtained from Simulation-based mean condition rating profiles. The result suggests that the Simulation-based condition prediction model may be considered as a potential model in future BMSs.

Keywords: lifetime condition prediction, Markov Decision Process, transition probability matrix, simulation methods, bridges.

ÖZ

KÖPRÜLERİN YAŞAM BOYU DURUM TAHMİNİ

Bayrak, Hakan
Doktora, Mühendislik Bilimleri Bölümü
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Altyapı sistemleri hayati öneme sahip tesislerdir. Bu sistemler kamu için gerekli ulaşım, su ve enerji hizmetlerini sağlarlar. Fakat bu sistemler zamanla yıpranır ve yaşam ömürlerinin sonuna yaklaşırlar. Sonuç olarak bu sistemler işlerlilikleri ve güvenilirlikleri için ömürleri boyunca düzenli aralıklarla bakım ve onarıma ihtiyaç duyarlar. Köprüler altyapı sistemleri içinde ulaşım altyapısının önemli bir bölümünü oluşturur. Son yıllarda, köprü altyapısında, köprülerin durumunu ve güvenliğini kontrol eden Köprü Yönetim Sistemleri (KYS) geliştirilmiştir. KYS'lerin amacı köprüleri çökme riskinden uzak tutarak, kaynakları en uygun şekilde kullanmaktır. KYS'ler köprülerin veya köprü elemanlarının gelecekteki durumlarını tahmin etmek için yaşamboyu performans eğrilerini kullanırlar. En yaygın kullanıma sahip olan duruma dayalı performans tahmini ve bakım optimizasyon modelleri Markov Karar Süreci'ne dayalı modellerdir. Optimum bakım politikasını bularak, yaşamboyu maliyet optimizasyonunu gerçekleştirmesi ve zamana bağlı yıpranmayı Markov geçiş olasılık matrisini kullanarak tanımlayabilmesi, Markov karar sürecine dayalı modelin özelliğidir. Bu çalışmada, Markov karar sürecine dayalı model incelendi ve iskontolu yaşamboyu maliyet hesabını kullanarak en uygun politikayı bulan bir bilgisayar programı geliştirildi. Bu çalışmada incelenen diğer bir performance tahmin modeli ise yıpranma süreci ve bakım uygulamaları için rasgele değişkenleri kullanarak birçok belirsizliği de hesaba katan olasılığa dayalı bi-linear modeldir. Çalışmanın bir bölümü olarak, Bi-linear modeli daha fazla analiz etmek ve geliştirmek için, Latin Hypercube örneklemeye dayalı bir simulasyon programı üretildi ve durum, güvenlik ve yaşamboyu maliyet profillerini bakım uygulamalarının uygulanma ve uygulanmama durumlarında üretebilen ana programa entegre edildi. Ayrıca, polinoma dayalı performans eğrisi alternatif bir performans tahmin modeli olarak incelendi. Bu model durum sıralama verisine regresyon analizi uygulayarak elde edildi. Regresyona dayalı performans eğrileri Latin Hypercube örnekleme metodu kullanılarak tekrar üretildi. Son olarak, Markov zincirine dayalı performans tahmin sonuçları simülasyona dayalı Bi-linear tahmin sonuçları ile kıyaslandı ve geçiş olasılığı matrisinin simüle edilen regrasyona dayalı durum profilinden elde edilmesi yeni geliştirilen bir yaklaşım olarak tanıtıldı. Markov zincirine dayalı ortalama durum sıralama profilinden elde edilen sonuçların simülasyona dayalı ortalama durum sıralama profilinden elde edilen sonuçlarla örtüştüğü gözlemlendi. Bu sonuç, simülasyona dayalı ortalama durum tahmini modelinin gelecekte KYS'lerde kullanılması muhtemel model olabileceğini gösterir.

Anahtar Kelimeler: yaşamboyu durum tahmini, Markov Karar Süreci, geçiş olasılık matrisi, simülasyon metodları, köprüler.

To my father and my mother

İsmail and Ayşe Bayrak

to my sister and brothers

Özlem, Haluk and Yusuf Haktan Bayrak

and

to my wife

Vildan Tokat Bayrak

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Infrastructure systems are crucial facilities for communities and countries. They supply the necessary transportation, water and energy utilities for the public. Due to increasing populations, the demand for these utilities is also increasing and consequently more facilities are being constructed to meet such needs. However, owning such large number of infrastructure systems presents new problems. Allocating funds and making decisions for maintenance and repair of these systems to ensure their survival and serviceability are two most important problems encountered during the lifespan of old facilities. For instance, lifetime of these systems ranges from approximately 30 and 100 years [1] which means that these systems require long term maintenance and repair in order to properly function and to be reliable.

An essential part of the transportation infrastructure is the bridge infrastructure. Bridge management systems (BMSs) used by developed countries to monitor the condition and safety of the bridges in a bridge infrastructure have evolved considerably in the past decades. Initially, a number of BMSs were developed such as Pontis [2], [3] and BRIDGIT [4] after unexpected failures of certain bridges have occurred such as the Silver Bridge in the U.S. Using bridge management systems, it is possible to establish maintenance and repair programs and to record conditions of bridges. Inspections can be performed to record conditions of bridges and a fixed time period may be established between these inspections. In addition to bridge replacement funding, the bridge management systems enable the allocation of resources for repair and maintenance of bridges. The aim of a BMS is to enable the maintenance repair or replacement of the bridges using life-cycle management techniques before their conditions fall below a critical level. In other words, the goal of design and management of highway bridges is to determine and implement the best lifetime design, maintenance and repair strategy that insures an adequate level of reliability at the lowest possible life-cycle cost. In developed countries, special attention is paid to preventive maintenance in order to prevent any problems in infrastructure facilities before they arise. Essential maintenance actions which are more expensive than others are either postponed or canceled by applying preventive maintenance actions which reduce the lifetime cost of a structural system [5]. Initially, these studies started with the use of alternative management model applications in order to efficiently use the resources allocated for maintenance planning. The aim of the BMSs which resulted from these applications was to establish the most effective maintenance planning for a network of bridges. Firstly, the U.S. lead the studies on BMSs. AASHTO LRFD Bridge Design Specifications was put into practice by the American Association of State Highway and Transformation Officials (AASHTO) in 1993. System reliability, aging and deterioration models were emphasized with more importance in this new bridge specification. Following such developments and recognizing the need of such systems for monitoring the health of their bridge infrastructure assets, numerous other countries have initiated the development of their own BMSs such as Finnra in Finland [6], Danbro+ in Denmark [7], APT in Italy [8], China [9], Japan [10], Sigpe in Mexico [11] and others.

Bridge Management Systems execute lifetime analysis for a stock of bridges. An important subject in life-cycle analysis of bridges is the deterioration prediction of bridges and their components. Deterioration prediction enables the determination of remaining service life of a bridge and planning of future maintenance activities.

There are various causes of performance deterioration of a structural system. In reinforced concrete bridges, deterioration is caused by corrosion and the main reason for corrosion in concrete is the cholorization (chloride diffusion into concrete, corrosion of steel reinforcement etc.). The other deterioration types in bridges are inadequate water insulation, inadequate design for thermal effects, excessive loading, vehicle collusions, inundations, the use of sea water in concrete mixture, damage resulted by periodic freezing and thawing, faulty expansion joints, faulty supports, cracking of reinforced concrete due to tension, alkali-silica reactions, and settlement and collapse in foundations. The causes of deterioration of performance may be grouped into three main categories. They include the aging (reduction of resistance and increase in loading), special actions (collusions by vehicles, earthquakes, pollution, etc.) and human errors (may arise at any stage in the lifetime of a structure) [12]. Existence of deterioration may have a major impact on the serviceability and load carrying capacity of bridges. For instance, small amount of local corrosion in prestressing steel cables of prestressed reinforced concrete beams may cause a sudden collapse in the structure.

Performance prediction of an infrastructure system is a difficult process due to existence of many uncertainties. Deterioration prediction models are produced to overcome this difficulty. In addition, some deterioration prediction models may treat the uncertainties as random variables. These random variables with known (or assumed) probability distributions are generated by simulation techniques and implemented within performance prediction models to predict the performance of a system. The generated random numbers for variables with known probability distributions are the main subject of the simulation process. Numerical simulation may be essential to solve the problems involving random variables with known (or assumed) probability distributions[13]. A sample obtained by simulation may present similar properties to a sample of experimental observations[13]. Results obtained by simulation may be presented statistically and applied to statistical methods[13]. Two most common simulation techniques are Monte Carlo Simulation and Latin Hypercube Sampling. In this study, Latin Hypercube Sampling method is programmed in Matlab environment to generate random variables. Using the Latin Hypercube method, it is possible to obtain a more reliable parameter space with fewer iterations. This improves the convergence rates and speed up execution. Therefore, the efficiency of Monte Carlo Simulations is improved using this superior technique. Furthermore, its embedded capability of handling multivariate distributions is advantageous in modeling studies. Comprehensive information on this subject is given in Chapter 4. Latin Hypercube method is a sampling technique and it is subjected to sampling errors. In other words, if sample size is not infinitely large, Latin Hypercube Sampling solutions are not exact.

1.2 Literature Review

In order to obtain the best maintenance and repair strategy, the lifetime performance prediction of an infrastructure system should be correctly predicted. Therefore, many studies have been performed to generate performance prediction models. These models may be divided into two main groups such as safety- and condition-based models. The safety-based models are based on continuous functions and consider the reliability index or rating factor as a performance indicator. For instance, a bi-linear model produced by Frangopol [14] is a continuous model. On the other hand, condition-based performance models are generally discrete models and studies the condition of a bridge members which is determined by visual inspection. For example, the Markov chain is a condition-based discrete model. Both methods maintain their validity because of their distinctive properties. In addition, a performance prediction model which contains bi-linear continuous functions was also developed.

Lifetime performance prediction for bridges can be performed using either a safety or a condition criteria. In a well designed BMS, both of these criteria should be implemented and monitored. In this thesis, safety prediction methods are described very briefly followed by analyses based on a Markovian process-based prediction technique and Polynomial-based prediction technique for condition prediction of bridge elements in time. As a part of the safety prediction, a very brief review of the structural reliability theory is presented as a background of performance prediction in Chapter 2.

Some researchers studied the safety-based continuous performance prediction models. A reliability-based structural maintenance methodology based on Monte Carlo Simulation was developed by Lin [15]. Lin optimized reliability-based inspection and rehabilitation strategy with minimum total expected cost for concrete girder bridges. Moreover, a set of lifetime repair strategies for infrastructure systems was optimized by Estes [16] with a system reliability approach using the first order reliability method.

In another study, Enright [17] suggested an approach, considering the time-variant system reliability for reinforced concrete highway girder bridges with time dependent resistance and loads. In that study, adaptive importance sampling and numerical integration were combined to predict performance levels of reinforced concrete bridges considering the environmental factors.

Kong and Frangopol [18] state that the assessment and prediction of structural deterioration is a difficult processes because of time dependent load and resistance parameters and applied maintenance actions . Therefore, some uncertainties should be introduced in a realistic lifetime analysis of infrastructure systems under multiple maintenance activities. In addition, Frangopol [14] mentioned that the reliability-based performance prediction models have to be implemented into bridge management systems to take into account many uncertainties during the lifetime of a bridge infrastructure system.

In another research study, performance prediction models are conducted with life-cycle cost procedure. In that study, maintenance and repair actions are applied to the structure throughout its lifetime and new maintenance strategies are presented.

Kong [12] proposed a method based on a modified decision tree to evaluate annual probability of rehabilitation. This proposed model was used to compute the present value of the expected annual and cumulative cost of rehabilitation action. Not only an individual bridge but also a group of bridges were examined with the modified decision tree method. Maintenance actions that were investigated in this method were time based strategies. The application time of the first and subsequent maintenance actions are described by probability mass functions in a relative time scale. Kong [12] used a deterioration model to evaluate bridge reliability profile and the related rehabilitation rate under no maintenance, preventive maintenance and essential maintenance actions. This is a probabilistic model with eight random variables, which include the deterioration rate, deterioration initiation time, initial reliability index. These random variables with known probability distribution functions were generated by using the Monte Carlo simulation technique. Furthermore, Kong conducted a sensitivity analysis for rehabilitation rate and parametric studies with discount rate and target reliability index. Several maintenance scenarios were composed by time controlled reliability profiles and safety controlled reliability profiles. In addition, an optimization algorithm was developed and optimum maintenance scenarios were investigated.

Studies about performance prediction of bridges indicate that deterioration is a nonlinear process. Although structures have similar characteristics based on design, construction and components, they may have different deterioration rates. For this reason, Petcherdchoo [1] investigated the bilinear and nonlinear deterioration functions and rehabilitation times of structures with and without preventive maintenance action strategy based on the Monte Carlo Simulation technique. Petcherdchoo generated condition index, reliability index and deterministic and random cost profiles under time-based maintenance strategies. The previously existing probabilistic model was developed and applied to a group of bridge components with a selected maintenance strategy. Time-based or preventive maintenance actions and performance-based or essential maintenance actions were defined and a combination of these actions were modeled and applied to a group of structures. In addition, Petcherdchoo stated that a minimum possible cumulative maintenance cost can not always be obtained by expected cumulative maintenance cost. Hence, percentiles of cumulative maintenance cost should be taken into account. Eventually, Petcherdchoo applied an optimization method for combined maintenance actions to obtain maintenance strategy to minimize total expected cumulative maintenance cost over a lifetime of an infrastructure system considering the minimum present value of expected cumulative cost and the percentile of cumulative maintenance cost.

Another researcher who studied the continuous performance prediction model is Neves [19, 5]. Neves investigated a model which integrates performance indicators based on visual inspection and structural assessment during the lifetime of a bridge. Condition, safety and cost profiles were generated by this probabilistic model which is defined as the bi-linear model. This model is a simplified performance-based method. It was proposed by Thoft-Christensen [20] and provides lifetime performance analysis with small computational effort using basic formulas. In this model, uncertainties were threated as random variables and generated by Latin Hypercube Sampling method. Neves examined a time dependent reliability model proposed by Kong and Frangopol [18] and obtained reliability index and cost profiles which exhibit interaction with each other. Furthermore, Neves generated several maintenance scenarios and selected optimum maintenance strategy considering the relationship between the cost and the effects of maintenance actions. In addition, nonlinear performance deterioration of a bridge infrastructure system under no maintenance and maintenance case was studied and applied to deteriorating reinforced concrete structures in the Netherlands. Furthermore, Neves conducted a multi-objective optimization procedure using Genetic Algorithm to select the best maintenance strategy. In this optimization procedure, the best situation for condition and safety index and minimum value of mean cumulative maintenance cost over lifetime were taken as the objective function.

Performance indicators are needed to describe time-dependent behavior of civil infrastructure systems such as bridges. Maintenance and management decisions may be made based on these performance indicators. Condition index, safety index, rating factor, and reliability index are the most commonly used performance indicators for bridges. Current BMSs use condition based deterioration prediction models in order to predict the lifetime deterioration of bridge elements. Condition based prediction generally relies on visual inspections of bridge elements.

The most widely implemented condition-based performance prediction and maintenance optimization models are the Markovian decision process-based models. This is a discrete performance prediction model. The laws of motion for a system in Markov process is described using a set of time independent transformation probabilities. In order to apply the Markov decision process to bridge cost analysis, a cost structure must be superimposed on the Markov process. The use of Markov processes to determine the optimal decision policy is the subject of Dynamic Programming. The solutions to such problems can be achieved by one of the three approaches: The Method of Successive Approximations, Policy Improvement Algorithm and Linear Programming [21]. It is not an easy task to obtain a solution using the Method of Successive Approximations within a finite number of iterations. However, a solution can be achieved if the method is slightly modified. Furthermore, the Policy Improvement Method is an alternative method based on iterations which also aims to obtain an optimal solution using finite iterations. In addition, dynamic programming problems may be stated as linear programming problem [22]. Markovian process models may include integer-programming techniques [23].The importance of the Markovian decision process-based model is that it may define the time-variant deterioration using different Markov transition probability matrices for different time periods and performs the lifetime cost optimization by finding the optimum maintenance policy. The

Simplex Method, developed by G. B. Dantzing [21], is an appropriate method to solve such linear optimization problems. In this thesis, a computer program is developed and implemented into Matlab which has embedded optimization toolbox commands using simplex algorithm to solve the linear programming problems. Markov process and its implementation in this thesis is explained thoroughly in Chapter 3.

Saito [24] developed a network level bridge management system to manage state owned bridges by evaluating the present and future needs of existing bridges and proposed a BMS composed of eight modules. The modules are: the data base, condition rating assistance, bridge safety evaluation, improvement activity identification, impact identification, project selection, activity recording and monitoring, and reporting modules. In Saito's study, classification factors were used to divide bridges into several subgroups according to identical properties to obtain more reliable results. Highway system (interstate, other state), traffic volume (low, medium and high), climatic region (northern, southern) and bridge types (concrete and steel) were used as the classification factors. Saito focused on consistency of condition ratings, bridge management costs and impacts, improvement of performance and need assessment models, and improvement of project selection models. The project selection module was divided into three sub-models, namely, life cycle costing, ranking and optimization. Saito divided management activities into three main groups. First, replacement indicating the replacement of the entire bridge structure including the approach slab. Second, rehabilitation which indicates major repair actions. Last, maintenance, which means minor repairs and preventive strategies. Some statistical analyses were performed to develop cost prediction models for bridge replacement, rehabilitation and maintenance. The data used in replacement cost analysis were obtained by only replaced state owned bridges between 1980 and 1985 in the State of Indiana in the U.S. A cost prediction model was generated by applying a non-linear regression approach and transformed linear regression analyses because of insufficiency of the use of linear regression approach for cost data. Saito considered deck reconstruction and deck replacement as two major rehabilitation activities. He states that maintenance activities should be applied periodically to avoid more expensive replacement and rehabilitation activities over the lifetime of the structures. Hand cleaning, bridge repair, flushing bridge, patching bridge decks and other bridge maintenance activities were used to determine the maintenance activity costs. Furthermore, Saito studied project level and network level life cycle cost using the Equivalent Uniform Annual Cost (EUAC) method. This method is suitable in order to evaluate the multiple maintenance actions with several analysis periods. In addition, a ranking method was used to set priorities on bridge rehabilitation and replacement projects. Moreover, analyses were conducted to determine the application time of bridge replacement and rehabilitation actions.

Jiang [25] studied to develop performance prediction models for bridge management systems. A dynamic optimization model was developed to select an optimal strategy. Jiang used a curve-based technique to predict the performance of bridges. The curve-based model was obtained by using regression analysis which was applied to Indiana bridge inventory data obtained by visual inspection. Genetic algorithms [26] or neural networks are suitable for maintenance optimization if curve-based techniques are used for deterioration prediction. Therefore, Jiang [25] generated transition probability matrices from regression curves by solving nonlinear programming optimization formula to implement dynamic optimization model. In this optimization model, the optimal strategies which maximize the benefit of the system was obtained during analysis period. Furthermore, this optimization model was subjected to budget constraints. Jiang [25], moreover, studied an alternative optimization method. In that method, a ranking model developed by Saito [24] and the dynamic optimization technique were combined.

Another study related to bridge management systems was conducted by Golabi *et al.* [27]. They studied a statewide pavement management system. The maximum benefit and minimum cost approaches were considered together and a mathematical model was developed. This mathematical model formulated the problem as a constrained Markov decision process and linear programming was used to determine the optimal policy. The long-term and the short-term model were studied to obtain maintenance policy which minimizes the expected long-term average cost and minimizes the total expected discounted cost during the first T years with short-term standards.

Madanat *et al.* [28] studied to estimate the transition probabilities from condition rating data which was based on discrete rating due to complexity of continuous condition indices. In addition, Madanat *et al.* [28] studied to develop incremental models. Similar to Jiang [25] and Saito [24], Madanat *et al.* [28] also used the bridge data from the Indiana State Bridge Inventory. Madanat introduced a new methodology based on the ordered probit technique. Madanat claimed that this new methodology gave a better estimation of the Markovian transition probabilities than those obtained using the commonly used methodology.

Another study on estimation of transition probability matrices was conducted by Ortiz-García *et al.* [29]. They generated six different data sets representing different condition rating distributions. Subsequently, three different methods for determination of transition probabilities from six different data sets was studied. The first method minimizes the sum of squared differences between the average condition obtained from the distribution of condition and each of data points. The second method uses the regression equations calculated from the data sets and aims to minimize the sum of squared differences between the condition rating value obtained from regression equations and the average condition value computed from the distribution of condition. Aim of the last method is to minimize the sum of the squared differences between the distributions from original data and distributions obtained from transition probabilities. Nonlinear optimization code was used for the solution by the three methods. Ortiz-García *et al.* [29] claimed that the three methods give a good estimation to the most original data, however, the third method yields better solutions for all data sets.

Another study on the Markov chains was conducted by Morcous [30]. This study examined the properties of Markov performance prediction model using field data. The field data was obtained from the Ministère des Transport du Québec (MTQ). This study focused on the bridge deck systems. In this study, two different condition rating systems are used, namely; material condition rating system and performance condition rating system. In addition, the transition probability matrices are generated by using the percentage prediction method for both of these condition rating systems. Examined properties are the effect of inspection period and state independence assumption on future condition prediction. As results of this study, it is concluded that the variation in inspection period presents an important effect on condition forecasting . Moreover, applied tests reveal that Markov chain model is a memoryless with a

95% level of confidence.

The Markov decision process is widely used maintenance and repair optimization algorithm in BMSs. Dynamic programming, on the other hand, is the name of the general technique to find an optimal maintenance and repair policy for a deteriorating system using transition probabilities. These transition probabilities are obtained from bridge condition data which is gathered from inspections. Smilowitz and Madanat [31] state that there may be measurement errors because of assumptions on inspection procedure. In their study [31], the assumptions which may cause the measurement errors are presented as error-free facility inspections and fixed inspection schedule. A methodology which is called the Latent Markov Decision Process (LMDP) is applied to take into account the presence of both assumptions in the selection of optimal maintenance and repair procedure by Madanat. The latent decision process does not assume the measurement of facility condition with no error. This is the major aspect which differs the latent Markov decision process from traditional Markov process. In the LMDP formulation, data information may be updated with subsequent inspections by using Bayes' law, the known transition and measurement probabilities. In Smilowitz and Madanat's study [31], the LMDP model is extended to include network-level problem by using randomized policies. The LMDP and traditional MDP produce normally nonrandomized policies which specify a single optimal policy. However, the extended methodology produces optimal probabilities for optimal policies for each state of the system. Smilowitz and Madanat [31] adapted the LMDP formulation for the finite and infinite horizon to optimize inspection procedure and maintenance and repair activities for a network-level with measurement and forecasting uncertainty. As a result of this study, it is observed that the expected costs increase as uncertainties increase for both planning horizon types.

Deterioration models are used to predict the future condition of deteriorating systems. In Markovian models, history of the condition states of infrastructure systems is not taken into account to predict the future conditions. This situation is a limitation on forecasting of bridge condition. In the study conducted by Robelin and Madanat [32], a history-dependent model of bridge deck deterioration is formulated as a Markov chain to overcome the limitation mentioned earlier. This developed method is an optimization approach for a bridge component maintenance and replacement. In their study, condition of the bridge component is represented by the reliability index. In addition, transition probabilities are estimated by using Monte Carlo simulation. Furthermore, the augmented state Markovian model is proposed by using a backward recursion algorithm. The proposed model includes new variables such as type of the latest action and time since the latest action. As a result of their study, if the proposed Markovian model is applied, only maintenance actions would be applied this way, and performance threshold for the component would not be reached with a lower budget during the planning horizon.

Another study related to Markovian deterioration model was conducted by Thompson and Johnson [33]. The purpose of that study was to develop a Markovian bridge deterioration model from historical data. It is stated that condition state data obtained from inspections may not be sufficient to predict future condition. More realistic condition prediction may be obtained by knowing actual maintenance records.

Lounis and Vanier conducted a study [34] that combines Markovian deterioration model with a multi-objective optimization procedure to obtain the optimal allocation of funds and to determine the optimum policy for maintenance, repair and replacement. In their study, a stochastic multi-objective optimization problem formulation is used for the bridge maintenance management problem. The maximization of bridge condition rating and reliability and minimization of maintenance costs may be the objectives of the problem. In addition, compromise programming and minimum Euclidean distance criterion and procedures were used to obtain the priority optimal ranking for deteriorated bridges. The minimum condition rating, minimum maintenance cost, and maximum average daily traffic were examined as three objectives in that study.

A study conducted by Scherer and Glagola [35] examined the Markovian deterioration model for bridge management system. In the study, it is stated that the Markov decision process is a powerful tool for representing deterioration model and for determining optimal policies to control a large-scale system. In their study, memoryless property of the Markovian chain is tested by an inference analysis using a chi-square statistic. It is seen that the past condition do not have an important effect on future condition.

Lifetime safety and condition prediction for bridges is also inherently related to the field of structural health monitoring. Time dependent change in performance can be observed through structural monitoring techniques. The latest development in this field is the use of sensors such as the sensor-based monitoring system for reinforced concrete and prestressed concrete structures [36]. This is a newly emerging field in performance evaluation of structures. However, its application is limited so far.

1.3 Research Objectives

The objective of this thesis is to develop an approach which combines the powerful ability of the Markovian maintenance optimization model with simulation-based transition probability generation. There are several performance prediction models such as bi-linear model, polynomial-based model, and Markov chain approach. Each model has its own distinctive properties. In this study, all of the existing models are investigated in order to achieve further development in this field. In current Bridge Management Systems (BMSs), Markov chain method is the preferred model since it gives the optimal strategy via dynamic programming. On the other hand, Markov chain approach has some limitations which is to be explained in the following chapters. Therefore, Markov chain approach and bi-linear model are investigated to combine powerful properties with an interactive relation as a new approach. Deterioration rate and transition probabilities are the same notions for deterioration prediction models. The mentioned new approach obtains this notion for both models via simulation-based deterioration model. Therefore, both models may be investigated in the dynamic programming to achieve optimal strategy for a bridge system during its lifetime.

In order to achieve the objective of this research, the task that needed to be completed are listed as follows:

1. Develop a simulation program in order to conduct probabilistic analyses including multiple random variables.
2. Examine existing performance prediction models which provide information about performance and deterioration condition of a deteriorating bridge or bridges

and to obtain performance prediction curves.

3. Determine the applicable maintenance and repair actions for bridge systems throughout their lifetime.

4. Investigate how the structural performance and life cycle cost of the structures may be affected by the applied maintenance and repair actions.

5. Study the theory of Markovian processes and chains and dynamic programming in general, derive the necessary formulations, develop an algorithm for the solution of the problem using linear programming and develop the necessary computer program.

6. Obtain an optimal solution for dynamic programming problem by using linear programming.

7. Produce regression-based performance curves using simulation in order to derive Markov transition probabilities.

8. Obtain optimal strategies using dynamic programming formulas and simulation-based continuous performance prediction models.

In this study, Latin Hypercube Sampling simulation technique has been implemented in Matlab computer environment to incorporate uncertainties in the problem by generating random variables. In addition, several other computer programs are developed and implemented for deterioration modeling, including; Bilinear, Markov process and Regression-based condition prediction models, in order to accomplish the objective of this study.

1.4 Organization of the Thesis

The thesis is divided into seven chapters.

Chapter 1 introduces, from general perspective, the performance prediction concept and deterioration modeling for deteriorating infrastructure systems. In addition, studies conducted on Bridge Management Systems (BMSs) to develop performance prediction methods are mentioned. Finally, objective and organization of the thesis are presented.

Chapter 2 presents subjects on condition, safety and combined performance prediction. Two general approaches for performance prediction, i.e condition and safety prediction, are briefly explained. Reliability-based prediction and rating factor-based prediction are discussed in safety-based prediction section. Probability of failure-based prediction and reliability index-based prediction constitute the reliability-based prediction methods. In addition, basic probability concepts, structural reliability and system reliability are defined. Condition prediction based on visual inspection is introduced. Finally, combination of condition and safety-based performance prediction is presented in combined performance prediction section.

Chapter 3 introduces the Markov process-based condition prediction. First, the chapter provides information about the dynamic programming problem in general. Then, linear programming is introduced as a solution technique for dynamic programming problem. A computer program, *Markov.m*, developed by using Simplex algorithm to obtain the optimal procedure for maintenance actions by dynamic decision process is introduced. Numerical examples are solved to reveal how the decision process works. Furthermore, complete formulation of the computational algorithm is derived for different number of states and number of action cases obtained. Finally, steady-state case and transition probabilities are explained and solutions are discussed.

Chapter 4 presents the simulation techniques in which Monte Carlo simulation technique and Latin Hypercube sampling method are described. In this Chapter, a simulation computer program, *latin_hs.m*, developed using Latin Hypercube sampling method to incorporate uncertainties in deterioration modeling is explained. Moreover, applicable maintenance actions for the bridges are investigated. In addition, a computer program, *lhs_csc.m*, developed to produce the bi-linear model performance prediction curves for condition, safety and cost profiles is also introduced. Maintenance actions are implemented into the *lhs_csc.m* program through an input file and performance curves under maintenance actions are obtained.

Chapter 5 presents the regression modeling used to predict the bridge performance over lifetime. Condition data for bridge components from Indiana Bridge Inventory

[25] is used to obtain performance prediction curves by using the regression model. Finally, obtained performance curves are regenerated by using simulation for the coefficients of regression-based performance curve formula. Effects of three different coefficient of variations values are observed for these coefficients with normal distribution. Probabilistic performance prediction curves are obtained from the bridge inventory data.

Chapter 6 explains further the studies performed in this thesis based on Markov chain approach for condition prediction of bridges. Determination of the future performance prediction of a structure with initial condition state vector and transition probability matrix is presented. Moreover, derivation of transition probability matrix from simulated condition profile is introduced as a new approach for estimation of transition probability matrix.

Chapter 7 summarizes the thesis by briefly presenting the studies performed. In addition, findings and developments are discussed. Finally, recommendations for future work are given.

CHAPTER 2

SAFETY AND CONDITION PREDICTION FOR BRIDGES

2.1 Introduction

Lifetime performance prediction for bridges can be performed using either a safety or a condition criterion. Reliability-based prediction and rating factor-based prediction are examples of safety-based performance prediction. Furthermore, condition criterion involves Markov decision process and polynomial based condition prediction. In a well designed BMS, both of these criteria should be implemented and monitored. However, currently, the BMSs do not have this capability. An overview of the safety and condition prediction methods is given in the following sections.

2.2 Safety Prediction

The assurance of system performance within the constraint of economy is one of the principal aims of engineering design. During the process of planning and design many decisions that are required are invariably made under conditions of uncertainty, and risk is often unavoidable. Therefore, the assurance of performance can seldom be perfect. Safety is a function of combinations of loads over the lifetime of the structure. Structural safety depends on the load carrying capacity of the structure and safety-based prediction models are produced by structural assessment. There are several procedures used for prediction of safety level of a bridge member or a bridge. The two main procedures used for safety prediction approaches are reliability-based prediction and rating factor-based prediction. These two safety prediction approaches are described in detail in sections 2.2.1 and 2.2.2. The purpose of the discussion presented in this section on safety prediction is to explain the meaning of the safety profile which will be discussed in Chapter 4. Safety index profiles are generated in Chapter 4 without using the formulations that will be presented in this section. This is because of the fact that the main focus of the thesis is the condition prediction for bridges. However, normally, value of a safety index must be calculated using the formulations that will be presented in this section. In other words, unlike the condition assessment which is generally performed visually, safety assessment normally requires structural engineering formulations or determination of quantified value of resistance degradation and load increase in a bridge member. The formulations presented herein form the background on how the safety index profiles presented in Chapter 4 would have been determined. Although the structural reliability analysis is outside the scope of this thesis, a brief overview of the subject is necessary before discussing the concept of safety index in future chapters.

2.2.1 Reliability-based Prediction

Prediction of element and system reliability are generally based on either the calculation of the reliability index or the probability of failure which are described in the following sections.

Reliability Index and Probability of Failure

The bridges are expected to be in service for a long time without adequate repair and maintenance. As bridges age, structural weakening due to heavy traffic and aggressive environmental factors such as impact of stream, climate, earthquake, additional dead loads and environmental pollution becomes more important since these factors lead to an increase in repair frequency and decrease in load carrying capacity.

Load and resistance have a time dependent effect on probability of failure throughout the service life of a structure as shown in Figure 2.1. As shown in this figure, expected resistance of a structure decreases in time because of environmental factors, whereas expected load increases in time.

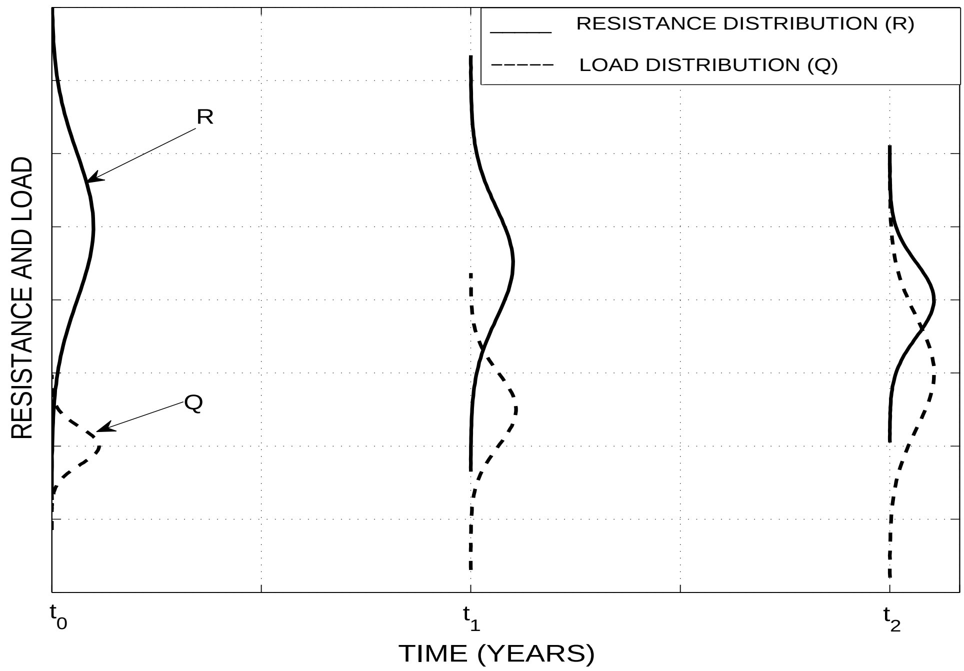


Figure 2.1: Load and resistance distribution throughout service life

Probabilistic measure of assurance of performance is defined as reliability. Nowadays, the use of moving load coefficient method and reliability analysis method for structural analysis of bridges are rapidly increasing in countries that have made progress in the subject of structural evaluation of bridges. In developed countries, in the area of structural assessment of bridges, structural safety criterion is the most important criterion among all other criteria taken into account which affects the determination of investment budgets for bridge maintenance and repair.

Reliability analysis methods are in the subject of the mechanics studies based on probability. Reliability can be formulated as the determination of the capacity of a system to meet certain requirements. Probabilistic nature of structural load and capacity can be modeled as follows.

$$R = \text{Supply Capacity (Resistance)}$$

$$Q = \text{Demand Requirement (Load)}$$

If it is assumed that R can be represented by the distribution of strength of a structural

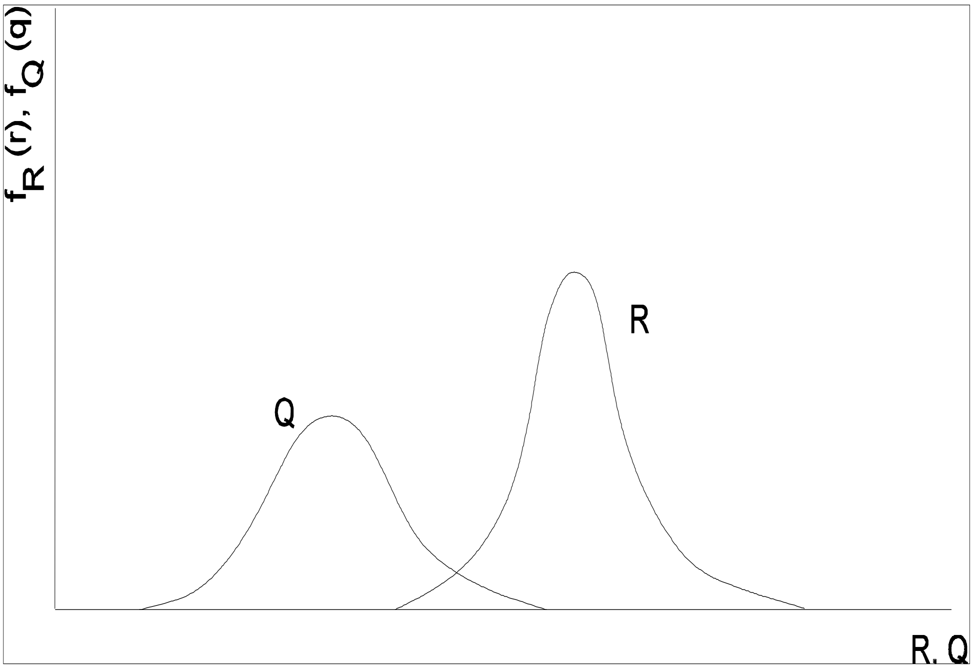


Figure 2.2: Probability density distributions of strength R and load Q

element and Q can be represented by the distribution of load as shown in Figure 2.2, reliability of that structural element P_s, (probability of safety or probability of having qualified level of structural performance) is defined by the area under the joint probability distribution function $f_{R,Q}(r,q)$. Insuring the event $(R > Q)$ throughout the lifetime of the structure is the objective of the reliability analysis. This is possible only if the probability $P(R > Q)$ is satisfied. Probability of occurrence of $R > Q$ is calculated by integration given in Eq. 2.1.

$$P_s = P(R > Q) = P(R - Q > 0) = \int\int_{R>Q} f_{R,Q}(r,q)drdq \qquad (2.1)$$

In Eq. 2.1, $f_{R,Q}(r,q)$ is the joint probability distribution function of R and Q. The term $(R - Q)$ defines another random variable which is defined as Safety Region (or Safety Margin) denoted by M, (i.e., $M = R - Q$). Mean of M and its standard deviation of M are denoted by μ_M and σ_M, respectively.

On the other hand, probability of failure can be defined as the probability of resistance being less than the load, which is formulated as shown in Eq. 2.2.

$$P_f = \int_{-\infty}^{\infty} \left[1 - F_q(x)\right] f_R(x) dx \tag{2.2}$$

If R and Q are independent random variables with normal distributions, probability of failure can be calculated as a function of the ratio $\frac{\mu_M}{\sigma_M}$ as given in Eq. 2.3.

$$P_f = P\left[R - Q \leq 0\right] = P\left[M \leq 0\right] = \Phi\left(-\frac{\mu_M}{\sigma_M}\right) \tag{2.3}$$

Φ function has the property shown in Eq. 2.4

$$\Phi\left(-\frac{\mu_M}{\sigma_M}\right) = 1 - \Phi\left(\frac{\mu_M}{\sigma_M}\right) \tag{2.4}$$

In the Eq. 2.3, Φ is the Laplace function (cumulative distribution function of standard normal variable). The ratio $\frac{\mu_M}{\sigma_M}$ is described as reliability index (or safety index) and shown by β. Therefore;

$$\beta = \left(\frac{\mu_M}{\sigma_M}\right) \tag{2.5}$$

Using the description of safety margin, for normally distributed random variables, the reliability index formula can be extended as:

$$\beta = \left(\frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}}\right) \tag{2.6}$$

The probability of occurrence of any event E in statistics, i.e *P(E)*, is between 0 and 1 as shown in Eq. 2.7.

$$0 \leq P(E) \leq 1 \tag{2.7}$$

If occurrence of an event is impossible, probability of occurrence of that event is 0. In addition, the probability of certain event is 1. These are fundamental axioms of probability. Safety and failure of a component or a system is mutually exclusive and collectively exhaustive because safety and failure of a component or system cannot happen at the same time; that is only one of them must occur in a sample space consisted of safety and failure. Therefore, probability of safety P_s in terms of the probability of failure P_f is defined as shown in Eq. 2.8.

$$P_s = 1 - P_f \tag{2.8}$$

Eq. 2.9 can be obtained by substituting Eq. 2.3 and 2.4 into Eq. 2.8.

$$P_s = \Phi(\beta) \tag{2.9}$$

The safety margin M also defines the so-called performance function $g(x)$. Performance functions depending on random variables can be generated to find the probability of failure of a system. The performance function $g(x)$ is defined as:

$$g(x) : \begin{cases} > 0 & \text{Safety} \\ = 0 & \text{Limit State} \\ < 0 & \text{Failure} \end{cases} \tag{2.10}$$

$g(X)$ describes the limit state of the system. The vector X contains the random variables. Solution of a limit state function yields the reliability index (or the probability of failure).

Structural systems are composed of structural members. In addition, reliability of structural systems may be different from the structural components that form these systems. In other words, capacity of the system are affected by the capacity and formation of the members. There are several types of systems, namely; series, parallel, and combination systems defined based on different combination of topologies and configuration of structural components. Furthermore, safety or failure of these systems are determined using different formulations. System reliability has a notable feature stated by Estes and Frangopol [37]. In their study, it was demonstrated that a component with the lowest reliability in a system may not always the one that is most needed to be repaired, because a component whose reliability index is below the target reliability level may not cause the reliability of the system to fall below the target reliability.

If components of the systems are connected in series, such systems are called series systems and the failure of these systems requires failures of any one of the components. In other words, the reliability or safety of the system requires that none of the components fail. Safety of a series system is defined as shown in Eq. 2.11,

$$\bigcap_{k=1}^{n} (g_k(x) > 0) \tag{2.11}$$

If components of the systems are connected in a parallel configuration, such systems are called parallel systems and the total failure of these systems requires failures of all components. In other words, the system remains safe if any one of the components survives. Failure of a parallel system is defined as shown in Eq. 2.12,

$$\bigcap_{k=1}^{n} (g_k(x) < 0) \tag{2.12}$$

Many structures in reality include a combination of series and parallel systems. Failure of a combined system is defined as shown in Eq. 2.13,

$$\bigcup_{l=1}^{m} \bigcap_{k=1}^{n} (g_k(x) < 0) \tag{2.13}$$

Illustration of Relationship between Reliability Index, Probability of Failure and Probability of Safety.

If resistance R and load Q are normally distributed and independent random variables, there exist a direct relationship among reliability index β, probability of failure P_f and probability of safety P_s as given by Eq. 2.14 and as shown in Table 2.1.

$$P_s = \Phi(\beta) = 1 - P_f \tag{2.14}$$

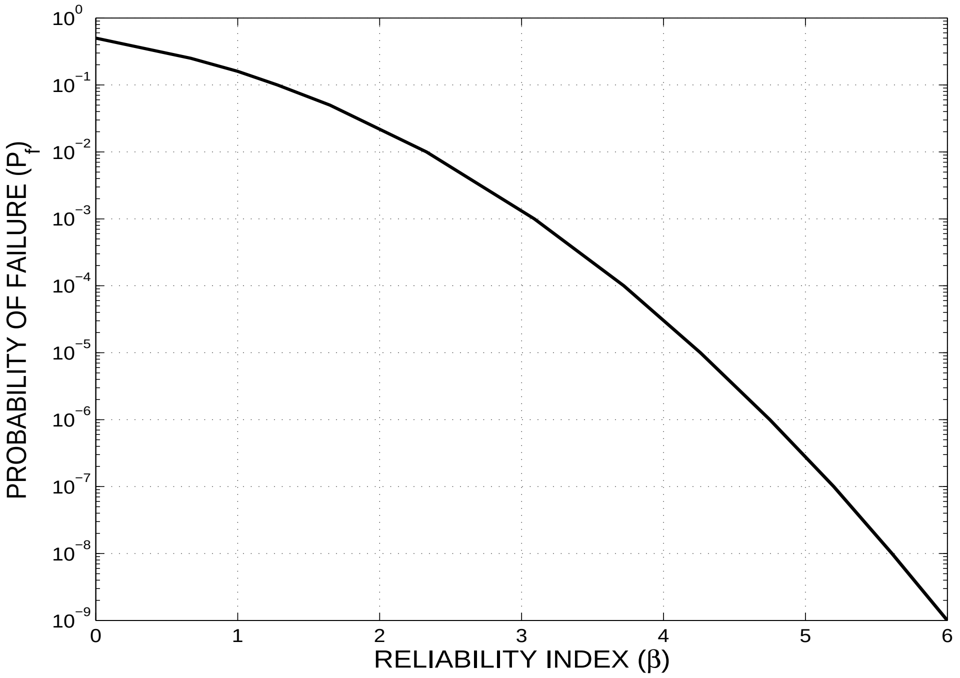


Figure 2.3: Relationship between probability of failure P_f and reliability index β

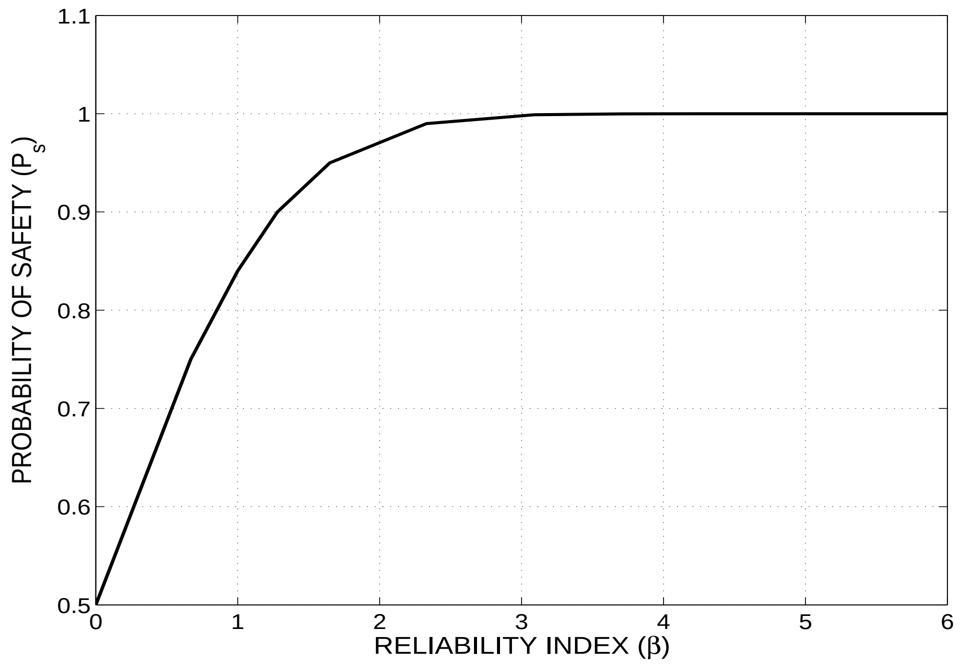


Figure 2.4: Relationship between probability of safety P_s and reliability index β

Table 2.1: Reliability indices and corresponding values of probability of failure and probability of safety

β	0	0.67	1.0	1.28	1.65	2.33	3.09	3.72	4.26	4.75	5.20	5.61	6.0
P_f	0.5	0.25	0.16	0.10	0.05	0.01	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-9}
P_s	0.5	0.75	0.84	0.90	0.95	0.99	0.999	0.9999	≈ 1	≈ 1	≈ 1	≈ 1	≈ 1

Figure 2.3 represents the relation between probability of failure and reliability index. In addition, relation between probability of safety and reliability index is shown in Figure 2.4. As can be seen in these figures, probability of failure decreases when reliability index increases. In other words, probability of failure and reliability index have inverse relation. In addition, probability of safety index and reliability index have direct relation. It may be stated that probability of safety index increases rapidly when reliability index increases.

Probability of Failure-based Safety Prediction

Probability of failure P_f can be used as a performance indicator to quantify the structural safety. Various researchers have studied the lifetime safety of bridges using probability of failure as the safety criterion.

Enright [17, 38, 39, 40, 41, 42] studied deterioration models of reinforced concrete bridges and investigated the reliability of reinforced concrete highway girder bridges under aggressive conditions using a time-variant series system reliability approach in which both load and resistance are time dependent. Enright used Monte Carlo Simulation technique with Adaptive Importance Sampling and Numerical Integration to determine the cumulative-time failure probability profiles. Enright determined nominal live load effect using AASHTO requirements (LRFD 1994), and described the live load by a Poisson point process.

Hong [43] extended the treatment of Mori and Ellingwood [44] and considered the uncertainties in the deterioration initiation time and in the degradation growth model. Hong has taken into account the correlation between the failures of structural elements in a structural system for the system reliability estimate. He used the nested reliability method which do not require the simulation to find $P_f(t)$, and used FORM

(First Order Reliability Method) to find reliability index modeling the structure as series and parallel systems. Hong used algorithms described in [45], [46] to find probability of failure for the system $P_{ij}(\tilde{x}, u)$ and presented an integrated methodology to evaluate the time-dependent reliability for deteriorating structures.

Probability of failure of a series system is determined from:

$$P_{ij}(x, u) = 1 - P\left[\bigcap_{k=1}^{n}\left(g_{k,ij}(x, s_k, u) > 0\right)\right] = 1 - \Phi(-\beta, \rho) \tag{2.15}$$

Similarly, the probability of failure of a parallel system can be determined from:

$$P_{ij}(x, u) = P\left[\bigcap_{k=1}^{n}\left(g_{k,ij}(x, s_k, u) < 0\right)\right] = \Phi(-\beta, \rho) \tag{2.16}$$

Reliability Index-based Safety Prediction

As an alternative criteria to probability of failure, reliability index has been more often used as a measure of safety for bridge elements and systems.

Estes and Frangopol [47] have studied to provide management decisions that will balance lifetime system reliability and expected life-cycle cost in an optimal manner and predicted remaining life reliability profiles for both bridge components and overall bridge system. Estes proposed a system reliability approach for optimizing the lifetime repair strategy for highway bridges and modeled the bridge as a series-parallel combination of failure modes, and developed limit-state equations for each of failure modes in terms of some random variables, and computed separately the reliability with respect to occurrence of each possible failure mode based on these limit equations using FORM. He transformed all random variables to uncorrelated standard normal variables and used an iterative search technique to compute the reliability index . Estes found the optimum lifetime repair strategy by examining all feasible combinations of developed options and considering the service life of the bridge.

Akgül and Frangopol [48, 49] conducted reliability analysis of bridge components using performance limit state functions defined in terms of standard code formulations in AASHTO (1996) specifications.

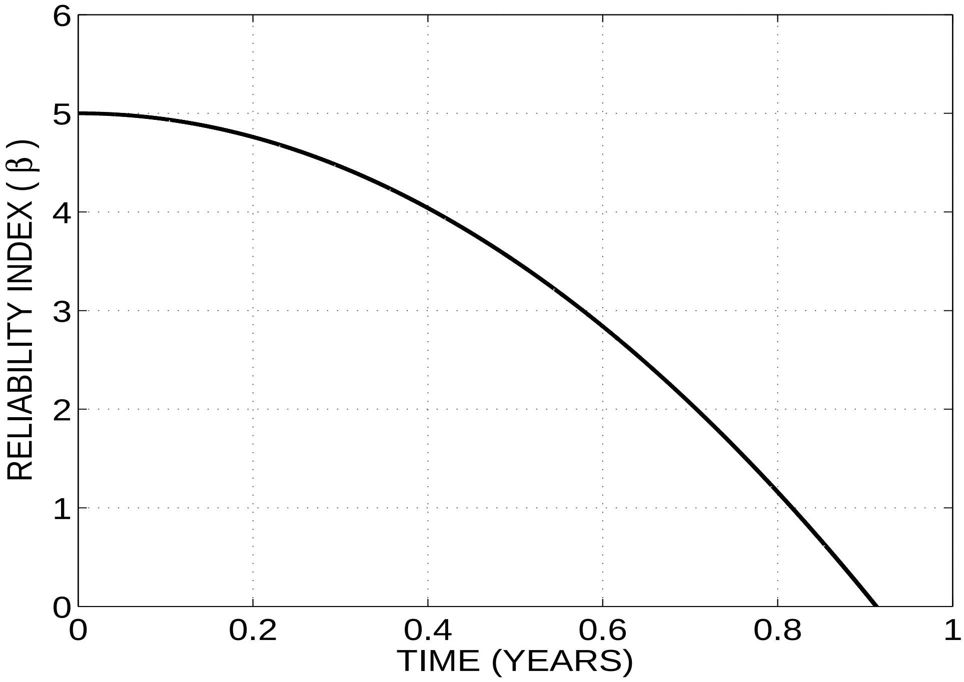


Figure 2.5: Reliability index profile

Reliability-index-based prediction of lifetime safety can be demonstrated by a time-variant reliability index profile (i.e. Safety Profile) as shown in Figure 2.5. In this approach, β is assumed to be a quadratic function of time, i.e., $\beta(t) = 5 - 6t^2$. The assumption is made to demonstrate the relationship between $\beta(t)$ and $P_f(t)$ in time, as shown in Figures 2.5 and Figure 2.6, respectively. Normally, $\beta(t)$ must be calculated using FORM, SORM or a similar reliability method.

For β values between 0 and 5, over a given time horizon of $t = 0$ to $t = 1$, the corresponding P_f values can be plotted as shown in Figure 2.6.

2.2.2 Rating Factor-based Prediction

Bridges are designed with respect to design vehicle loads. However, vehicle weights in traffic do not remain the same over the lifetime of bridges. Therefore, bridges are subjected to larger weights in time. In addition, bridges deteriorate and therefore,

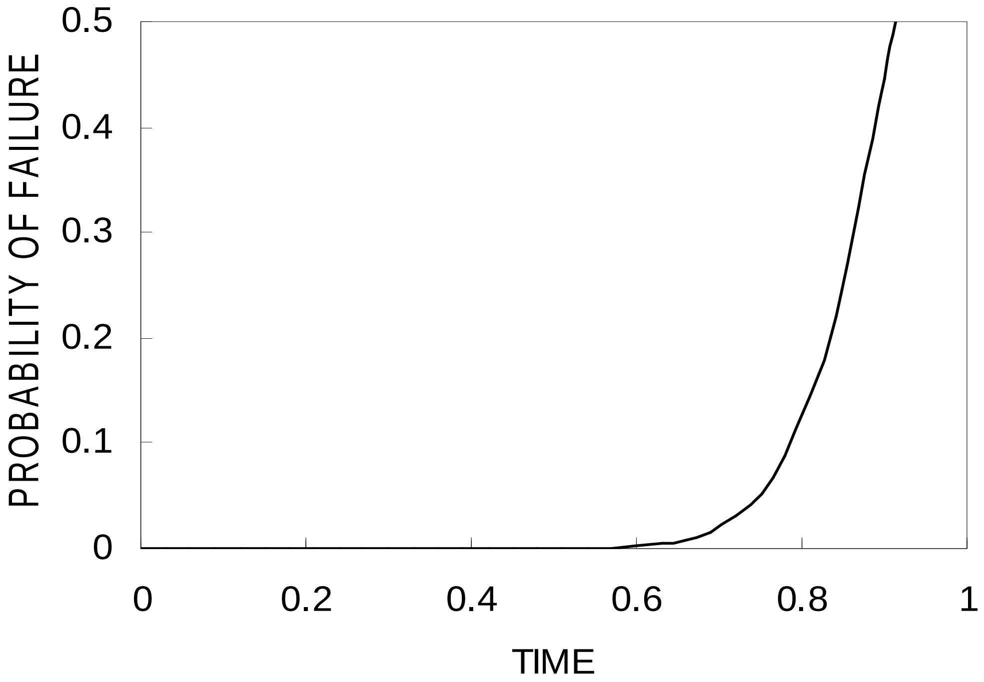


Figure 2.6: Probability of failure profile

their load carrying capacities also decrease in time. Hence, safety of older bridges should be examined to ensure safety of public. Rating is a score showing the safety level of a bridge structure at any age. In other words, the process of finding the safe live load capacity of a bridge is referred to as the rating. Load rating of a bridge member can be calculated using certain formulas, and if variations of resistance and load are known, time-variant rating value can be calculated for a bridge member or for a bridge. Such a performance prediction graph can be used as a Safety Index Profile.

In general, structure safety concept depends on the criteria which defines the relation between resistance R and load Q. The criteria is that resistance R should be greater than load Q as displayed in Eq. 2.17 [50].

$$R \geq Q_d + Q_l + \sum_i Q_i \tag{2.17}$$

where,

R is the resistance (or capacity) of the member

Q_d is the effect of dead load

Q_l is the effect of live load

Q_i is the effect of load i

Maximum allowable live load should be determined to evaluate the bridge rating factor. Rearranging Eq. 2.17 for Q_l yields

$$Q_l \leq R - \left(Q_d + \sum_i Q_i \right) \tag{2.18}$$

Q_l in Eq. 2.18 is called the maximum allowable live-load (or actual live load capacity). Rating Factor is described as the ratio of actual live load capacity to the required live load capacity of a bridge component and it is formulated as:

$$R.F = \frac{\text{(Actual Live Load Capacity)}}{\text{(Required Live Load Capacity)}} \tag{2.19}$$

As shown in Eq. 2.19, the rating factor has to be equal to or exceed unity to carry the rating vehicle safely. However, when the rating factor is less than unity, the bridge is subjected to overload.

Rating factor has different values for different bridge members. In other words, abutments, piers, columns, footing girder and slabs may have different actual live load capacities and required live load capacities. Therefore, structures may have several rating factors according to its components, but minimum of rating factor is considered as the rating of structure. In other words, component of a bridge with minimum rating factor defines the safety of the bridge.

There are three available rating factor methods for bridge elements including Allowable Stress rating (ASR), Load Factor Rating (LFR), and Load and Resistance Factor Rating (LRFR). There are two levels of rating which are classified by strength requirements [48]. First, inventory level rating can be defined as the safely carried load by the bridge for indefinite period. Second, operating level rating is described as the absolute maximum permissible load which may be safely carried by the bridge. The part of the actual live load capacity shown in Eq. 2.18, i.e; $(Q_d + \sum_i Q_i)$ can be denoted as:

$$\underbrace{\left(Q_d + \sum_i Q_i\right)}_{Q}$$

Then, Eq. 2.18 takes the following form.

$$Q_l \leq R - Q \tag{2.20}$$

Introducing the load and resistance factors, actual live load capacity can be written as:

$$\phi R - \gamma \beta_Q Q \tag{2.21}$$

where,

ϕ is the resistance (capacity) reduction factor,
R is the resistance (capacity) of the member,
γ is the load factor,
β_Q is the dead load coefficient,
Q is the dead load effect on member.

Similarly, required live load capacity is formulated as follows.

$$\gamma \beta_{(L+I)_n} L_{(L+I)} \tag{2.22}$$

where,

$\beta_{(L+I)_n}$ is the live load coefficient,
$L_{(L+I)}$ is the live load effect on member including the vehicle impact.

Substituting the actual live load capacity and required live load capacity into Eq. 2.19 yields

$$R.F = \frac{(\phi R - \gamma\beta_Q Q)}{(\gamma\beta_{(L+I)_n} L_{(L+I)})} \tag{2.23}$$

The dead load and live loads are the only loads considered in determining the rating factor because the probability of occurrence of other load types such as thermal, earthquake, hydraulic and wind load during the short live load loading is very small.

The load factors γ, $\beta_{(L+I)_n}$, and $L_{(L+I)}$ are defined in AASHTO (1996) Table 3.22.1A for each group of structure [48]. Load factor and load coefficient of member which work under flexure and tension are as follows: $\gamma = 1.3$, $\beta_Q = 1.0$, and $\beta_{(L+I)_n} = 1.67$, for operating rating level $\beta_{(L+I)_n} = 1$.

Consequently, basic rating factor formulas become:

For inventory rating,

$$R.F = \frac{(\phi R - 1.3Q)}{(2.17L_{(L+I)})} \tag{2.24}$$

For operating rating,

$$R.F = \frac{(\phi R - 1.3Q)}{(1.3L_{(L+I)})} \tag{2.25}$$

Once the rating factor is calculated for a given bridge member using the formulas given above, if the variation in time of R and Q can be predicted for the future, the time-variant rating values can e obtained for a bridge member.

However, determination of the time variation of resistance and load for a deteriorating structural member is not a straightforward procedure. It requires further modeling of both resistance degradation mechanisms in the member and increase in vehicular load levels in time. Modeling of resistance degradation mechanisms of reinforced concrete and steel are still be subject of considerable research in this field and are beyond the scope of this thesis. In this thesis, safety index profiles presented in Chapter 4 are not obtained considering R and Q formulations given in this chapter. Instead, simulation-based safety profiles are generated which are based on initial safety index distribution and deterioration rates, both of which are treated as random variables.

2.3 Condition Prediction

Safety-based performance prediction for bridges requires the use of formulas containing resistance and load requires the prediction of their lifetime variations. Condition-based prediction, on the other hand, is based on condition data of bridge members or bridges obtained through visual inspection techniques. Condition data is based on previously defined standards of damage categories (or classes) represented by numbers or letters, denoting little, medium or heavy damage levels. Damage categories may be as little as four or as many as ten or even more.

There are other approaches for condition prediction of bridges other than Markov process-based models. A thorough background on such studies was presented in Chapter 1. The other approaches include bi-linear or polynomial-based condition predictions and regression-based prediction methods. Another approach is the use of simulation techniques in combination with the approaches listed above. As an example, Neves [51, 5] analyzed the variations in time of probabilistic performance indicators of existing bridges according to condition, safety and cost under maintenance strategies. Neves and Frangopol [19] have proposed a model which helps prediction of uncertainties in the application times of maintenance actions, the effects of maintenance actions and the deterioration process of existing structure on the performance indicator and life-cycle cost of structure. Uncertainties in Neves' model was generated by Latin Hypercube Sampling. Neves used Genetic Algorithms to optimize the time of application of maintenance actions.

In this thesis, separate computer programs are developed for solving the lifetime condition prediction for bridges with the ultimate objective of combining the results of simulation techniques and hence the randomness of the problem with the lifetime optimum policy determination capability of Markov process-based on discrete time intervals.

Notations in Chapter 2

R	:	Supply Capacity (Resistance)
Q	:	Demand Requirement (Load)
P_s	:	Probability of safety
P_f	:	Probability of failure
$f_{R,Q}(r,q)$	:	Joint probability distribution function of R and Q
M	:	Safety region (Safety Margin)
μ_M	:	Mean of M
σ_M	:	Standard deviation of M
Φ	:	Laplace function (cumulative distribution function of standard normal variable)
β	:	Reliability index
μ_R	:	Mean of R
σ_R	:	Standard deviation of R
μ_Q	:	Mean of Q
σ_Q	:	Standard deviation of Q
$P(E)$	:	Probability of occurrence of an event
$g(x)$	:	Limit state function (Performance function)
Q_d	:	Effect of dead load
Q_l	:	Effect of live load
Q_i	:	Effect of load i
$R.F$	:	Rating factor
ϕ	:	Resistance (capacity) reduction factor
γ	:	Load factor
β_Q	:	Dead load coefficient
$\beta_{(L+I)_n}$	:	Live load coefficient
$L_{(L+I)}$	:	Live load effect on member including the vehicle impact

CHAPTER 3

MARKOV PROCESS BASED CONDITION PREDICTION

A few of the currently used maintenance management systems for bridges use the visual inspection-based discrete condition states and Discounted Dynamic Programming methods. In Dynamic Programming, the state of a deteriorating system change with respect to time. The prior states have more important effect on the transition from one state to another. However, the process of change can be modeled with the Markov process if the transition probability depends on the current state. Markov process is currently used in a few bridge management systems. However, the research on such systems is still continuing to improve the existing methods by incorporating new performance measures such as structural reliability. The objective of the optimization of bridge maintenance through bridge lifetime is to determine how to use the existing resources in order to keep the bridges at acceptable reliability levels while having the lowest lifetime cost. Because of discrete nature of its formulation, Markov process-based condition deterioration model match well with the fact that condition data on deteriorating structural elements is also collected at discrete time intervals. Morcous [52] used Markov-chain models which are based on two assumptions for predicting the future condition of bridge components, systems, and networks. The first assumption is constant inspection period, and the second is state independence. Markov chains are a special case of the Markov processes, and are used as performance prediction models for bridge components by defining discrete condition states and accumulating the probability of transition from one condition state to another over multiple discrete time intervals. Morcous developed transition probability matrices for different elements of the deck system, and used Bayes' rule to adjust for the variation in the inspection period. Transition probabilities are obtained either from accumulated condition data or by using an expert judgment.

The laws of motion for a system in Markov process is described using a set of time independent transformation probabilities. P^t_{ija} defines the probability that the system may be at level j at the beginning of the next time interval t when the system is at level i now and action a is chosen without any consideration about the past condition.

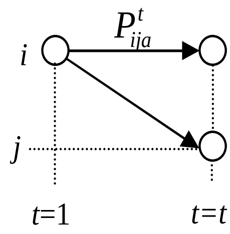


Figure 3.1: States, Time and Transition Probability in Markov Process

In order to apply the Markov decision process to bridge reward analysis, a reward structure must be superimposed on the Markov process. If action a is chosen when the system is at level i, let the associated reward be denoted by r_{ja}. The use of Markov processes to determine optimal decision policy is the subject of Dynamic Programming. Solution of such problems can be achieved by one of the three approaches; the Method of Successive Approximations, Policy Improvement Algorithm and Linear Programming.

It is not an easy task to obtain a solution using method of successive approximations with a finite number of iterations. However, a solution can be achieved if the method is slightly modified. Furthermore, the policy improvement method is an alternative method based on iterations which also aims to obtain an optimal solution using finite iterations.

Let us consider an arbitrary policy $R \in C_D$. A policy refers to a set of optimal actions in each state maximizing the total reward. In Dynamic Programming, the Discounted long-term Discounted Life-cycle cost, i.e, $v_i^\alpha(R)$ must satisfy the following Equation [22].

$$v_i^\alpha(R) = \sum_{t=1}^{\infty} \alpha^{t-1} \sum_j \sum_a P_R(X_t = j, Y_t = a | X_1 = i) \cdot r_{ja} \tag{3.1}$$

where,

$v_i^\alpha(R)$: Discounted long-term life-cycle cost under policy R at state i,
R : Selected policy,
α : Discount factor,
r_{ja} : Reward earned at state j when action a is chosen,
P_R : Probability that the system will be at state j at the beginning of the next time interval if action a is chosen when the system is currently at state i.

The objective is to find R which maximizes $v_i^\alpha(R)$. In other words, the aim is to find a series of optimal actions (decisions) which will maximize the lifetime reward over a certain time horizon. The Linear Programming formulation of the problem is stated as follows [22].

$$\begin{aligned} &\text{Minimize} && \sum_j \beta_j v_j \\ &\text{subject to} && v_i \leq r_{ja} + \alpha \sum_j P_{ija}^t v_j \\ & && \beta_j > 0, \quad j \in A, \quad \sum_j \beta_j = 1 \end{aligned} \tag{3.2}$$

The problem may be solved as a maximization problem after transforming it to a Dual Linear Problem. The Simplex Method is an appropriate method to solve such linear optimization problems. At this stage of study, simplex method routine has been implemented in a main program and the optimization results of example problems have been verified. According to the obtained results, if the program could be improved to solve the linear optimization problems faultlessly, the main program could be extended to a more general program to solve Markov process problems with linear optimization. After that, time dependent and discounted life-cycle cost optimization problems which rely on Markov decision process could be solved. At a later stage, it was intended to apply the developed sequential decision cost optimization program to optimum bridge maintenance and repair decision making.

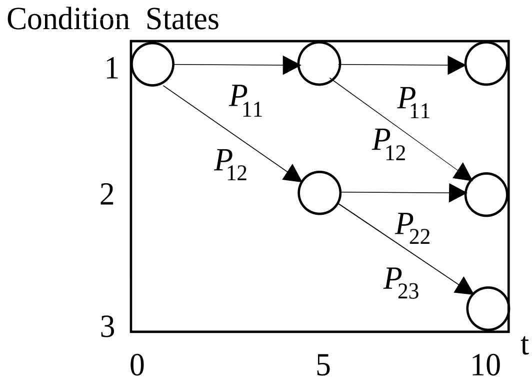


Figure 3.2: Markovian Deterioration Model

3.1 Dynamic Programming Solution

In order to apply the Markov decision process to a system cost analysis, a cost structure must be superimposed on the Markov process. Let $q_{ij}(a)$ define the probability of the system being at state j at the the next time interval when the system is at state i if action a is chosen. If action a is chosen when system is at level i, the associated cost can be denoted by w_{ia}.

Let us consider an arbitrary policy $R \in C_D$ where C_D denotes the subclass of C_S consisting of the deterministic policies, and C_S denotes the class of all Markovian policies which are time variant. In this case, the discounted life-cycle cost $\Psi_R(i, \alpha)$ satisfies the following equation [21].

$$\Psi_R(i, \alpha) = E_R \sum_{t=1}^{\infty} \alpha^t \cdot W_t \tag{3.3}$$

where,

$W_t = w_{ia}$ if $Y_t = i$, $A_t = a$, $a \in K$, $i \in I$

Therefore,

$$\Psi_R(i, \alpha) = \sum_{t=0}^{\infty} \alpha^t \sum_i \sum_a q_{ij}^R(a) \cdot w_{ja} \tag{3.4}$$

E_R : Expected value,

$\Psi_R(i,\alpha)$: Discounted life-cycle cost,

R : Selected policy,

α : Discount factor,

w_{ia} : Cost incurred when system is in state i and action a is taken,

i : State i,

a : Action a,

$q_{ij}(a)$: The probability of the system being in state j at the next instant the system is observed when the system is in state i now and action a is taken regardless of its history (referred to as the transition probability).

K_i : Number of actions possible when the system is at state i.

I : State space (Space of possible states)

The objective is to find the policy R which minimizes $\Psi_R(i,\alpha)$. In other words, the aim is to find a series of optimal actions (decisions) which will minimize the life time cost over a certain time horizon. Linear Programming can be used to find optimum policy R. Linear programming is a useful approach to derive finite algorithms for a number of Markovian control problems. Denoting the discounted life cycle cost $\Psi_R(i,\alpha)$ as v_i, the Linear Programming formulation of the problem is stated as follows.

$$\begin{aligned} \text{Maximize} \quad & \sum_j \beta_j v_j \\ \text{subject to} \quad & v_i \leq w_{ia} + \alpha \sum_j q_{ij}(a) v_j \\ & \beta_j > 0, \quad j \in A, \quad \sum_j \beta_j = 1 \end{aligned} \tag{3.5}$$

The problem may be solved as a minimization problem after transforming it to a Dual Linear Problem. Again, the Simplex Method is the appropriate method to solve the linear optimization problems. In this study, a program is developed to solve the primal and dual optimization problems stated above.

3.2 Linear Programming

Dynamic Programming is used to implement Markov processes to determine optimal decision policy for a system that changes states in time. Discounted life-cycle cost problem is also a type of Dynamic Programming. In order to find the discounted life-cycle cost and optimal policy for a dynamic system, Dynamic Programming problem must be solved. Linear Programming Problem is one of the solution methods for Dynamic Programming problem. Linear Programming problem for the cost-based formulation (i.e cost minimization) can be solved using the following procedure if w_{ia} and $q_{ij}(a)$ are known.

Primal Problem:

$$\begin{aligned}
&\text{Maximize} && \sum_j \beta_j v_j \\
&\text{subject to} && v_i \leq w_{ia} + \alpha \sum_j q_{ij}(a) v_j, \quad a \in K_i, \quad i \in I \\
&\text{where} && \beta_j > 0, \quad j \in I, \quad \text{and} \quad \sum_j \beta_j = 1 \quad \text{are given numbers.}
\end{aligned} \tag{3.6}$$

Dual problem for the above primal problem is:

$$\begin{aligned}
&\text{Minimize} && \sum_i \sum_a x_{ia} w_{ia} \\
&\text{subject to} && x_{ia} \geq 0, \quad a \in K_i, \quad i \in I \\
& && \sum_i \sum_a x_{ia}(\delta_{ij} - \alpha q_{ij}(a)) = \beta_j, \quad j \in I
\end{aligned} \tag{3.7}$$

where;

$\delta_{ij} = 0$ if $i \neq j$ and $\delta_{ij} = 1$ if $i = j$ (Kronecker delta)
i, j : States of the system.

According to the Expected Average Cost Criterion, $\sum_i \sum_a x_{ia} = 1$ is added as a new constraint.

The formulation presented above is implemented in a computer program (both in FORTRAN and Matlab environments) in order to solve numerical examples. In the following section, the problem is solved for combining different number of states and different number of actions. First, a two-state and two-action case is solved, then a three-state and two-action case is considered.

After the numerical examples, formulation of the computational algorithm is explained. Derivation of the coefficient matrix for the two-state and two-action problem is presented followed by the derivation of the coefficient matrix for the dual form of the same problem.

After the derivation of the coefficient matrices are presented, a flowchart of the algorithm used for the Markov Process is introduced. Then steady state transition probabilities are explained using a two-state transition model and a three-state transition model. Finally, an example of bridge element condition state transition model is presented.

3.3 Computational Examples

3.3.1 A Two - State, Two - Action Case

In this problem, the dynamic system is periodically observed in time and at any given time, the system can only be at one of the two states: $i = 0$, $i = 1$, i.e; $I = \{0, 1\}$, and there are two possible actions at each state $a = 1$, $a = 2$, i.e; $K_i = 2$, [21]. The cost matrix $[w]$ and transition probability matrix $[q]$ are as follows.

$$\begin{Bmatrix} w_{01} & w_{02} \\ w_{11} & w_{12} \end{Bmatrix} = \begin{Bmatrix} 1 & 0 \\ 2 & 2 \end{Bmatrix}$$

$$\begin{Bmatrix} (q_{00}(1), q_{00}(2)) & (q_{01}(1), q_{01}(2)) \\ (q_{10}(1), q_{10}(2)) & (q_{11}(1), q_{11}(2)) \end{Bmatrix} = \begin{Bmatrix} (\frac{1}{2}, \frac{1}{4}) & (\frac{1}{2}, \frac{3}{4}) \\ (\frac{2}{3}, \frac{1}{3}) & (\frac{1}{3}, \frac{2}{3}) \end{Bmatrix}$$

Fig. 3.3 shows the two possible states and the actions which can be taken at each state for a two state, two action case. Transition probabilities of two state, two action case are given in Fig. 3.4 through Fig. 3.7.

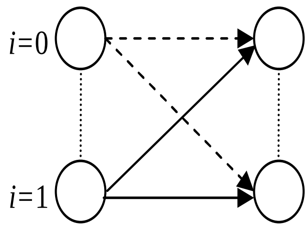


Figure 3.3: Possible states and action paths for each state for a two state , two action case

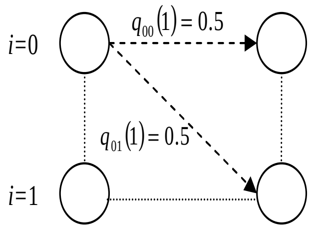


Figure 3.4: Transition probabilities when action 1 is taken at state 0

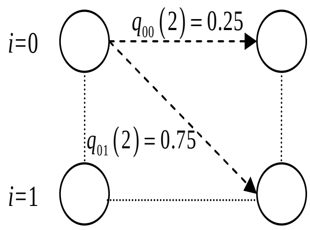


Figure 3.5: Transition probabilities when action 2 is taken at state 0

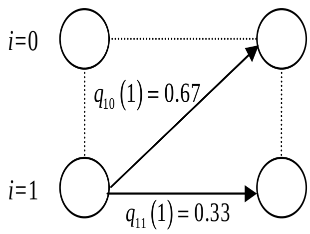


Figure 3.6: Transition probabilities when action 1 is taken at state 1

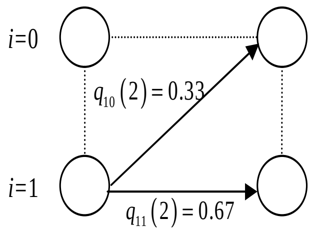


Figure 3.7: Transition probabilities when action 2 is taken at state 1

Transition probability matrix can be divided into two parts considering the number of actions. $q(1)$ and $q(2)$ as follows.

$$q(1) = \begin{Bmatrix} 0.5 & 0.5 \\ 0.67 & 0.33 \end{Bmatrix}$$

$$q(2) = \begin{Bmatrix} 0.25 & 0.75 \\ 0.33 & 0.67 \end{Bmatrix}$$

The problem can be converted to a linear programming problem using the formulations explained earlier. Primal Linear programming problem is formulated as follows.

Letting $\beta_0 = \beta_1 = \frac{1}{2}$ and $\alpha = \frac{1}{2}$ formulation for objective function becomes.

$$\text{Maximize} \quad \sum_j \beta_j v_j = \beta_0 v_0 + \beta_1 v_1$$

Substituting the values of β_i's

$$\text{Maximize} \quad \tfrac{1}{2} v_0 + \tfrac{1}{2} v_1$$

Constraints are expanded as,

$$v_i \leq w_{ia} + \alpha \sum_j q_{ij}(a) v_j$$

$$v_0 \leq w_{01} + \alpha[q_{00}(1)v_0 + q_{01}(1)v_1]$$

$$v_0 \leq w_{02} + \alpha[q_{00}(2)v_0 + q_{01}(2)v_1]$$

$$v_1 \leq w_{11} + \alpha[q_{10}(1)v_0 + q_{11}(1)v_1]$$

$$v_1 \leq w_{12} + \alpha[q_{10}(2)v_0 + q_{11}(2)v_1]$$

Substituting the values of w_{ia}, $q_{ij}(a)$ and α, we obtain,

$$\frac{3}{4}v_0 - \frac{1}{4}v_1 \leq 1$$

$$\frac{7}{8}v_0 - \frac{3}{8}v_1 \leq 0$$

$$-\frac{1}{3}v_0 + \frac{5}{6}v_1 \leq 2$$

$$-\frac{1}{6}v_0 + \frac{2}{3}v_1 \leq 2$$

Therefore, the primal linear programming problem becomes,

$$\text{Maximize} \qquad \frac{1}{2}v_0 + \frac{1}{2}v_1$$

$$\text{subject to} \qquad \frac{3}{4}v_0 - \frac{1}{4}v_1 \leq 1$$

$$\frac{7}{8}v_0 - \frac{3}{8}v_1 \leq 0$$

$$-\frac{1}{3}v_0 + \frac{5}{6}v_1 \leq 2$$

$$-\frac{1}{6}v_0 + \frac{2}{3}v_1 \leq 2$$

The objective function of dual linear programming problem becomes

$$\text{Minimize} \qquad \sum_i \sum_a x_{ia}w_{ia} = x_{01}w_{01} + x_{02}w_{02} + x_{11}w_{11} + x_{12}w_{12}$$

Substituting the values of w_{ia}, we obtain

$$\text{Minimize} \qquad x_{01} + 0x_{02} + 2x_{11} + 2x_{12}$$

The constraints are generated as follows.

$$\sum_i \sum_a x_{ia}(\delta_{ij} - \alpha q_{ij}(a)) = \beta_j$$

for $j = 0$

$$x_{01}(\delta_{00} - \alpha q_{00}(1)) + x_{02}(\delta_{00} - \alpha q_{00}(2)) + x_{11}(\delta_{10} - \alpha q_{10}(1)) + x_{12}(\delta_{10} - \alpha q_{10}(2)) = \beta_0$$

for $j = 1$

$$x_{01}(\delta_{01} - \alpha q_{01}(1)) + x_{02}(\delta_{01} - \alpha q_{01}(2)) + x_{11}(\delta_{11} - \alpha q_{11}(1)) + x_{12}(\delta_{11} - \alpha q_{11}(2)) = \beta_1$$

Substituting the values of δ_{ij}, $q_{ij}(a)$ and α, we obtain,

$$\frac{3}{4}x_{01} + \frac{7}{8}x_{02} - \frac{1}{3}x_{11} - \frac{1}{6}x_{12} = \frac{1}{2}$$

$$-\frac{1}{4}x_{01} - \frac{3}{8}x_{02} + \frac{5}{6}x_{11} + \frac{2}{3}x_{12} = \frac{1}{2}$$

Thus, the dual linear programming problem takes the following form

$$\text{Minimize} \qquad x_{01} + 0x_{02} + 2x_{11} + 2x_{12}$$

$$\text{subject to} \qquad \frac{3}{4}x_{01} + \frac{7}{8}x_{02} - \frac{1}{3}x_{11} - \frac{1}{6}x_{12} = \frac{1}{2}$$

$$-\frac{1}{4}x_{01} - \frac{3}{8}x_{02} + \frac{5}{6}x_{11} + \frac{2}{3}x_{12} = \frac{1}{2}$$

The rearranged form of the problem can be stated as follows. The primal problem is a maximization problem. The solution of this problem gives the values of v_0, v_1 which maximizes the objective function. The values of the variables give the minimum cost.

$$\text{Maximize} \qquad \frac{1}{2}v_0 + \frac{1}{2}v_1$$

$$\text{subject to} \qquad \frac{3}{4}v_0 - \frac{1}{4}v_1 \leq 1$$

$$\frac{7}{8}v_0 - \frac{3}{8}v_1 \leq 0$$

$$-\frac{1}{3}v_0 + \frac{5}{6}v_1 \leq 2$$

$$-\frac{1}{6}v_0 + \frac{2}{3}v_1 \leq 2$$

Since the program implemented solves only the minimization problems, in oder to perform this maximization, the primal problem must be converted to a minimization problem. Firstly, objective function is multiplied by -1 and then inequalities in constraints are converted to equalities adding or subtracting new variables which are referred to as the slack variables. Each constraint has only one slack variable and all slack variables are different from each other. The variables are shown in the objective function with 0 coefficient, which means that the slack variables have no effect on the objective function.

$$\text{Minimize} \qquad -\frac{1}{2}v_0 - \frac{1}{2}v_1 + 0v_2 + 0v_3 + 0v_4 + 0v_5$$

$$\text{subject to} \qquad \frac{3}{4}v_0 - \frac{1}{4}v_1 + v_2 = 1$$

$$\frac{7}{8}v_0 - \frac{3}{8}v_1 + v_3 = 0$$

$$-\frac{1}{3}v_0 + \frac{5}{6}v_1 + v_4 = 2$$

$$-\frac{1}{6}v_0 + \frac{2}{3}v_1 + v_5 = 2$$

Simplex Method is used to solve the problem. The solution is $v_0 = 1.2414$, $v_1 = 2.8966$. v_0 denotes the minimum discounted life-cycle cost if optimal policy R is taken when the system is at state 0, and v_1 denotes the minimum discounted life-cycle cost if optimal policy R is taken when the system is at state 1.

Dual linear programming problem was formulated as

$$\text{Minimize} \qquad x_{01} + 0x_{02} + 2x_{11} + 2x_{12}$$

$$\text{subject to} \qquad \frac{3}{4}x_{01} + \frac{7}{8}x_{02} - \frac{1}{3}x_{11} - \frac{1}{6}x_{12} = \frac{1}{2}$$

$$-\frac{1}{4}x_{01} - \frac{3}{8}x_{02} + \frac{5}{6}x_{11} + \frac{2}{3}x_{12} = \frac{1}{2}$$

Solution vector is

$$\tilde{x}_{ia} = \begin{Bmatrix} x_{01} \\ x_{02} \\ x_{11} \\ x_{12} \end{Bmatrix}.$$

For which the following values are obtained

$$\tilde{x}_{ia} = \begin{Bmatrix} 0 \\ 0.9655 \\ 1.0345 \\ 0 \end{Bmatrix}.$$

The values of x_{01} and x_{12} are zero and x_{02} and x_{11} have nonzero values. This result is interpreted according to D_{ia} values. Let $D_{ia} = D_a\{H_{t-1}, Y_t\}$ for $R \in C_s$. $D_a\{H_{t-1}, Y_t\}$ denotes the probability of taking action a at time t using a random mechanism. In this random mechanism formula, H_{t-1} shows the history of the system up to time $t-1$ and Y_t shows the state of the system at time t. Therefore, in order to find the optimal policy, D_{ia} values must be obtained. D_{ia}'s formula is

$$D_{ia} = \frac{x_{ia}}{\sum_a x_{ia}} \tag{3.8}$$

and D_{ia} values can be obtained as follows.

$i = 0, a = 1$

$$D_{01} = \frac{x_{01}}{\sum_a x_{0a}} = \frac{x_{01}}{x_{01} + x_{02}} = \frac{0}{0 + 0.9655} = 0$$

$i = 0, a = 2$

$$D_{02} = \frac{x_{02}}{\sum_a x_{0a}} = \frac{x_{02}}{x_{01} + x_{02}} = \frac{0.9655}{0 + 0.9655} = 1$$

$i = 1, a = 1$

$$D_{11} = \frac{x_{11}}{\sum_a x_{1a}} = \frac{x_{11}}{x_{11} + x_{12}} = \frac{1.0345}{1.0345 + 0} = 1$$

$i = 1, a = 2$

$$D_{12} = \frac{x_{12}}{\sum_a x_{1a}} = \frac{x_{12}}{x_{11} + x_{12}} = \frac{0}{1.0345 + 0} = 0$$

Nonzero values of D_{ia} correspond to $(i = 0, a = 2)$, $(i = 1, a = 1)$. This means that action 2 should be taken at state 0, and action 1 should be taken at state 1 in order to achieve the optimal policy.

3.3.2 A Three - State, Two - Action Case

In this problem, the dynamic system is periodically observed in time and at any given time, the system can only be at one of the three states: $i = 0$, $i = 1$, and $i = 2$, i.e; $I = \{0, 1, 2\}$, and there are two possible actions at each state $a = 1$, $a = 2$, i.e; $K_i = 2$ [21]. The cost $[w]$ and transition probability matrices $[q]$ are as follows.

$$\begin{Bmatrix} w_{01} & w_{02} \\ w_{11} & w_{12} \\ w_{21} & w_{22} \end{Bmatrix} = \begin{Bmatrix} 1 & 0 \\ 2 & 1 \\ 1 & 2 \end{Bmatrix}$$

$$\begin{Bmatrix} (q_{00}(1), q_{00}(2)) & (q_{01}(1), q_{01}(2)) & (q_{02}(1), q_{02}(2)) \\ (q_{10}(1), q_{10}(2)) & (q_{11}(1), q_{11}(2)) & (q_{12}(1), q_{12}(2)) \\ (q_{20}(1), q_{20}(2)) & (q_{21}(1), q_{21}(2)) & (q_{22}(1), q_{22}(2)) \end{Bmatrix} = \begin{Bmatrix} (\frac{1}{2}, \frac{1}{3}) & (\frac{1}{4}, \frac{1}{3}) & (\frac{1}{4}, \frac{1}{3}) \\ (0, \frac{1}{3}) & (1, \frac{2}{3}) & (0, 0) \\ (\frac{2}{3}, 0) & (0, \frac{1}{3}) & (\frac{1}{3}, \frac{2}{3}) \end{Bmatrix}$$

Fig. 3.8 shows three possible states and the actions that can be taken at each state for a three state, two action case. Transition probabilities of three state, two action case are given in Fig. 3.9 through Fig. 3.14.

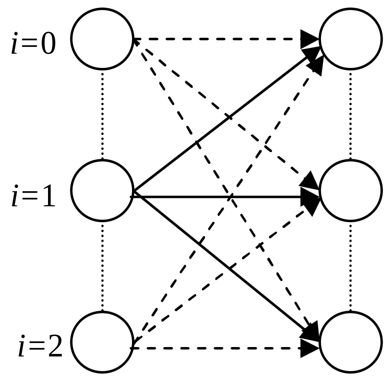


Figure 3.8: Possible states and actions that can be taken at each state for a three state, two action case

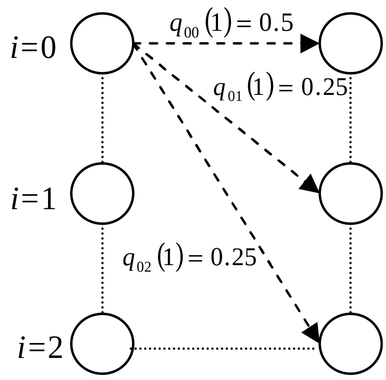


Figure 3.9: Transition probabilities when action 1 is taken at state 0

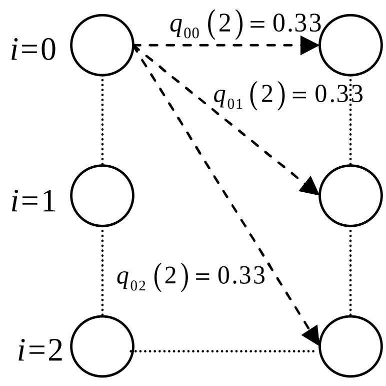


Figure 3.10: Transition probabilities when action 2 is taken at state 0

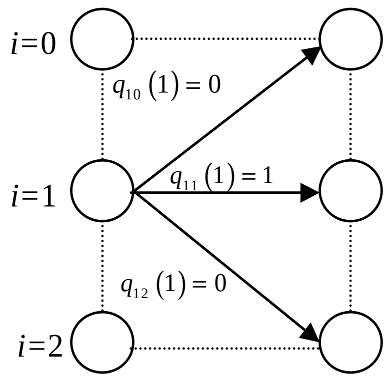


Figure 3.11: Transition probabilities when action 1 is taken at state 1

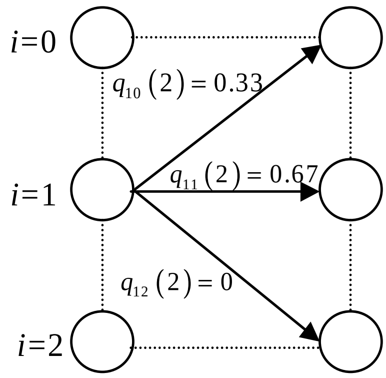


Figure 3.12: Transition probabilities when action 2 is taken at state 1

The problem can be converted to a linear programming problem using the formulations explained earlier. Primal Linear programming problem is formulated as follows.

Letting $\beta_0 = \beta_1 = \beta_2 = \frac{1}{3}$ and $\alpha = \frac{1}{2}$, formulation for objective function becomes

$$\text{Maximize} \quad \sum_j \beta_j v_j = \beta_0 v_0 + \beta_1 v_1 + \beta_2 v_2$$

Substituting the values of β_i's, we obtain

$$\text{Maximize} \quad \tfrac{1}{3}v_0 + \tfrac{1}{3}v_1 + \tfrac{1}{3}v_2$$

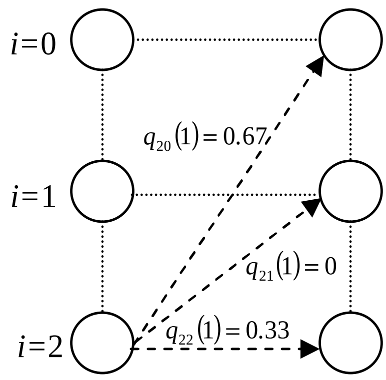


Figure 3.13: Transition probabilities when action 1 is taken at state 2

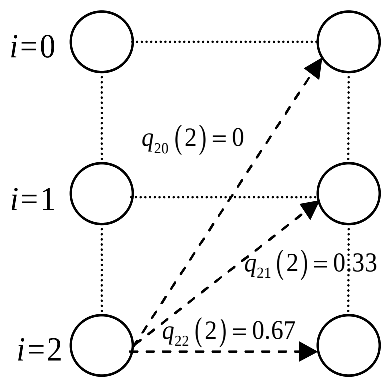


Figure 3.14: Transition probabilities when action 2 is taken at state 2

Constraints are expanded as,

$$v_i \leq w_{ia} + \alpha \sum_j q_{ij}(a) v_j$$

$$v_0 \leq w_{01} + \alpha[q_{00}(1)v_0 + q_{01}(1)v_1 + q_{02}(1)v_2]$$

$$v_0 \leq w_{02} + \alpha[q_{00}(2)v_0 + q_{01}(2)v_1 + q_{02}(2)v_2]$$

$$v_1 \leq w_{11} + \alpha[q_{10}(1)v_0 + q_{11}(1)v_1 + q_{12}(1)v_2]$$

$$v_1 \leq w_{12} + \alpha[q_{10}(2)v_0 + q_{11}(2)v_1 + q_{12}(2)v_2]$$

$$v_2 \leq w_{21} + \alpha[q_{20}(1)v_0 + q_{21}(1)v_1 + q_{22}(1)v_2]$$

$$v_2 \leq w_{22} + \alpha[q_{20}(2)v_0 + q_{21}(2)v_1 + q_{22}(2)v_2]$$

Substituting the values of w_{ia}, $q_{ij}(a)$ and α, we obtain,

$$\frac{3}{4}v_0 - \frac{1}{8}v_1 - \frac{1}{8}v_2 \leq 1$$

$$\frac{5}{6}v_0 - \frac{1}{6}v_1 - \frac{1}{6}v_2 \leq 0$$

$$0v_0 + \frac{1}{2}v_1 + 0v_2 \leq 2$$

$$-\frac{1}{6}v_0 + \frac{2}{3}v_1 + 0v_2 \leq 1$$

$$-\frac{1}{3}v_0 + 0v_1 + \frac{5}{6}v_2 \leq 1$$

$$0v_0 - \frac{1}{6}v_1 + \frac{2}{3}v_2 \leq 2$$

Therefore, the primal linear programming problem becomes

$$\text{Maximize} \quad \frac{1}{3}v_0 + \frac{1}{3}v_1 + \frac{1}{3}v_2$$

$$\frac{3}{4}v_0 - \frac{1}{8}v_1 - \frac{1}{8}v_2 \leq 1$$

$$\frac{5}{6}v_0 - \frac{1}{6}v_1 - \frac{1}{6}v_2 \leq 0$$

$$0v_0 + \frac{1}{2}v_1 + 0v_2 \leq 2$$

$$-\frac{1}{6}v_0 + \frac{2}{3}v_1 + 0v_2 \leq 1$$

$$-\frac{1}{3}v_0 + 0v_1 + \frac{5}{6}v_2 \leq 1$$

$$0v_0 - \frac{1}{6}v_1 + \frac{2}{3}v_2 \leq 2$$

The objective function of the dual linear programming problem becomes

$$\text{Minimize} \quad \sum_i \sum_a x_{ia}w_{ia} = x_{01}w_{01} + x_{02}w_{02} + x_{11}w_{11} + x_{12}w_{12} + x_{21}w_{21} + x_{22}w_{22}$$

Substituting the values of w_{ia}, we obtain,

$$\text{Minimize} \qquad x_{01} + 0x_{02} + 2x_{11} + x_{12} + x_{21} + 2x_{22}$$

The constraints are generated as follows.

$$\sum_i \sum_a x_{ia}(\delta_{ij} - \alpha q_{ij}(a)) = \beta_j$$

for $j = 0$

$x_{01}(\delta_{00} - \alpha q_{00}(1)) + x_{02}(\delta_{00} - \alpha q_{00}(2)) + x_{11}(\delta_{10} - \alpha q_{10}(1)) + x_{12}(\delta_{10} - \alpha q_{10}(2)) + x_{21}(\delta_{20} - \alpha q_{20}(1)) + x_{22}(\delta_{20} - \alpha q_{20}(2)) = \beta_0$

for $j = 1$

$x_{01}(\delta_{01} - \alpha q_{01}(1)) + x_{02}(\delta_{01} - \alpha q_{01}(2)) + x_{11}(\delta_{11} - \alpha q_{11}(1)) + x_{12}(\delta_{11} - \alpha q_{11}(2)) + x_{21}(\delta_{21} - \alpha q_{21}(1)) + x_{22}(\delta_{21} - \alpha q_{21}(2)) = \beta_1$

for $j = 2$

$x_{01}(\delta_{02} - \alpha q_{02}(1)) + x_{02}(\delta_{02} - \alpha q_{02}(2)) + x_{11}(\delta_{12} - \alpha q_{12}(1)) + x_{12}(\delta_{12} - \alpha q_{12}(2)) + x_{21}(\delta_{22} - \alpha q_{22}(1)) + x_{22}(\delta_{22} - \alpha q_{22}(2)) = \beta_2$

Substituting the values of δ_{ij}, $q_{ij}(a)$ and α, we obtain,

$$\frac{3}{4}x_{01} + \frac{5}{6}x_{02} + 0x_{11} - \frac{1}{6}x_{12} - \frac{1}{3}x_{21} + 0x_{22} = \frac{1}{3}$$

$$-\frac{1}{8}x_{01} - \frac{1}{6}x_{02} + \frac{1}{2}x_{11} + \frac{2}{3}x_{12} + 0x_{21} - \frac{1}{6}x_{22} = \frac{1}{3}$$

$$-\frac{1}{8}x_{01} - \frac{1}{6}x_{02} + 0x_{11} + 0x_{12} + \frac{5}{6}x_{21} + \frac{2}{3}x_{02} = \frac{1}{3}$$

Thus, the dual linear programming problem takes the following form.

$$\text{Minimize} \qquad x_{01} + 0x_{02} + 2x_{11} + x_{12} + x_{21} + 2x_{22}$$

$$\text{subject to} \quad \frac{3}{4}x_{01} + \frac{5}{6}x_{02} + 0x_{11} - \frac{1}{6}x_{12} - \frac{1}{3}x_{21} + 0x_{22} = \frac{1}{3}$$

$$-\frac{1}{8}x_{01} - \frac{1}{6}x_{02} + \frac{1}{2}x_{11} + \frac{2}{3}x_{12} + 0x_{21} - \frac{1}{6}x_{22} = \frac{1}{3}$$

$$-\frac{1}{8}x_{01} - \frac{1}{6}x_{02} + 0x_{11} + 0x_{12} + \frac{5}{6}x_{21} + \frac{2}{3}x_{02} = \frac{1}{3}$$

The solution of primal problem is $v_0 = 0.6207$, $v_1 = 1.6552$, $v_2 = 1.4483$. v_0 denotes the minimum discounted life-cycle cost if optimal policy R is taken when the system is at state 0, v_1 denotes the minimum discounted life-cycle cost if optimal policy R is taken when the system is at state 1, and v_2 denotes the minimum discounted life-cycle cost if optimal policy R is taken when the system is at state 2.

Solution of dual problem is summarized as follows.

$$\tilde{x}_{ia} = \begin{Bmatrix} 0 \\ 0.7586 \\ 0 \\ 0.6897 \\ 0.5517 \\ 0 \end{Bmatrix}.$$

The values of x_{01} and x_{11} and x_{22} are zero and x_{02}, x_{12} and x_{21} have nonzero values. Using D_{ia} values, the optimal policy can be determined as follows.

$i = 0, a = 1$

$$D_{01} = \frac{x_{01}}{\sum_a x_{0a}} = \frac{x_{01}}{x_{01} + x_{02}} = \frac{0}{0 + 0.7586} = 0$$

$i = 0, a = 2$

$$D_{02} = \frac{x_{02}}{\sum_a x_{0a}} = \frac{x_{02}}{x_{01} + x_{02}} = \frac{0.7586}{0 + 0.7586} = 1$$

$i = 1, a = 1$

$$D_{11} = \frac{x_{11}}{\sum_a x_{1a}} = \frac{x_{11}}{x_{11} + x_{12}} = \frac{0}{0 + 0.6897} = 0$$

$i = 1, a = 2$

$$D_{12} = \frac{x_{12}}{\sum_a x_{1a}} = \frac{x_{12}}{x_{11} + x_{12}} = \frac{0.6897}{0 + 0.6897} = 1$$

$i = 2, a = 1$

$$D_{21} = \frac{x_{21}}{\sum_a x_{2a}} = \frac{x_{21}}{x_{21} + x_{22}} = \frac{0.5517}{0.5517 + 0} = 1$$

$i = 2, a = 2$

$$D_{22} = \frac{x_{22}}{\sum_a x_{2a}} = \frac{x_{22}}{x_{21} + x_{22}} = \frac{0}{0.5517 + 0} = 0$$

Results correspond to $(i = 0, a = 2)$, $(i = 1, a = 2)$ and $(i = 2, a = 1)$. This means that action 2 should be taken at state 0, and action 2 should be taken at state 1 and action 1 should be taken at state 2 in order to achieve the optimal policy.

3.4 Formulation of the Computational Algorithm

3.4.1 Derivation of the Coefficient Matrix A for the Two-State Two-Action Problem

Constraints of linear programming problem compose a linear system of equations as shown below.

$$[\mathrm{A}]_{m\times n}\{v\}_{n\times 1} \leq \{w\}_{m\times 1} \tag{3.9}$$

Matrix A is referred to as the coefficient matrix and it is derived using the transformation formula which transforms discounted life-cycle cost problem from dynamic programming to linear programming.

Transformation formula is given as shown below.

$$v_i \leq w_{ia} + \alpha \sum_j q_{ij}(a) v_j$$

The elements of the coefficient matrix depend on cost, transition probability and discounted rate as shown below.

$$a_{mn} = f\left\{w_{ia}, \alpha, q_{ij}(a)\right\} \tag{3.10}$$

Explicit form of matrix A can be written as,

$$\mathrm{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{bmatrix}$$

As an example problem, let us assume that the cost matrix and the transition probability matrix are given as below.

$$w_{ia} = \begin{Bmatrix} w_{01} & w_{02} \\ w_{11} & w_{12} \end{Bmatrix} = \begin{Bmatrix} 1 & 0 \\ 2 & 2 \end{Bmatrix}$$

$$q_{ia} = \begin{Bmatrix} (q_{00}(1), q_{00}(2)) & (q_{01}(1), q_{01}(2)) \\ (q_{10}(1), q_{10}(2)) & (q_{11}(1), q_{11}(2)) \end{Bmatrix} = \begin{Bmatrix} (\frac{1}{2}, \frac{1}{4}) & (\frac{1}{2}, \frac{3}{4}) \\ (\frac{2}{3}, \frac{1}{3}) & (\frac{1}{3}, \frac{2}{3}) \end{Bmatrix}$$

Based on the given data, elements of matrix A may be formulated in terms of w_{ia}, $q_{ij}(a)$ and α. First row of the coefficient matrix implies that the system is at state 0 and action 1 is taken, i.e;

$$i = 0, \quad a = 1 \quad \text{and} \quad \alpha = \frac{1}{2}$$

$$v_0 \leq w_{01} + \alpha[q_{00}(1)v_0 + q_{01}(1)v_1]$$

$$v_0 \leq w_{01} + \alpha q_{00}(1)v_0 + \alpha q_{01}(1)v_1$$

$$v_0 - \alpha q_{00}(1)v_0 - \alpha q_{01}(1)v_1 \leq w_{01}$$

$$[1 - \alpha q_{00}(1)]v_0 - \alpha q_{01}(1)v_1 \leq w_{01}$$

$$[1 - \alpha q_{00}(1)]v_0 + [-\alpha q_{01}(1)]v_1 \leq w_{01}$$

Letting $a_{11} = [1 - \alpha q_{00}(1)]$, $a_{12} = [-\alpha q_{01}(1)]$, we obtain

$$a_{11}v_0 + a_{12}v_1 \leq w_{01}$$

Second row of the coefficient matrix corresponds to the system being at state 0 and action 2 is taken.

$$i = 0, \quad a = 2 \quad \text{and} \quad \alpha = \frac{1}{2}$$

$$v_0 \leq w_{02} + \alpha[q_{00}(2)v_0 + q_{01}(2)v_1]$$

$$v_0 \leq w_{02} + \alpha q_{00}(2)v_0 + \alpha q_{01}(2)v_1$$

$$v_0 - \alpha q_{00}(2)v_0 - \alpha q_{01}(2)v_1 \leq w_{02}$$

$$[1 - \alpha q_{00}(2)]v_0 - \alpha q_{01}(2)v_1 \leq w_{02}$$

$$[1 - \alpha q_{00}(2)]v_0 + [-\alpha q_{01}(2)]v_1 \leq w_{02}$$

Letting $a_{21} = [1 - \alpha q_{00}(2)]$, $a_{22} = [-\alpha q_{01}(2)]$, we obtain

$$a_{21}v_0 + a_{22}v_1 \leq w_{02}$$

Third row of the coefficient matrix corresponds to the system being at state 1 and action 1 is taken.

$$i = 1, \quad a = 1 \quad \text{and} \quad \alpha = \frac{1}{2}$$

$$v_1 \leq w_{11} + \alpha[q_{10}(1)v_0 + q_{11}(1)v_1]$$

$$v_1 \leq w_{11} + \alpha q_{10}(1)v_0 + \alpha q_{11}(1)v_1$$

$$v_1 - \alpha q_{10}(1)v_0 - \alpha q_{11}(1)v_1 \leq w_{11}$$

$$[-\alpha q_{10}(1)]v_0 + [1 - \alpha q_{11}(1)]v_1 \leq w_{11}$$

Letting $a_{31} = [-\alpha q_{10}(1)]$, $a_{32} = [1 - \alpha q_{11}(1)]$, we obtain

$$a_{31}v_0 + a_{32}v_1 \leq w_{11}$$

Fourth row of the coefficient matrix corresponds to the system being at state 1 and action 2 is taken.

$$i = 1, \quad a = 2 \quad \text{and} \quad \alpha = \frac{1}{2}$$

$$v_1 \leq w_{12} + \alpha[q_{10}(2)v_0 + q_{11}(2)v_1]$$

$$v_1 \leq w_{12} + \alpha q_{10}(2)v_0 + \alpha q_{11}(2)v_1$$

$$v_1 - \alpha q_{10}(2)v_0 - \alpha q_{11}(2)v_1 \leq w_{12}$$

$$[-\alpha q_{10}(2)]v_0 + [1 - \alpha q_{11}(2)]v_1 \leq w_{12}$$

Letting $a_{41} = [-\alpha q_{10}(2)]$, $a_{42} = [1 - \alpha q_{11}(2)]$, we obtain

$$a_{41}v_0 + a_{42}v_1 \leq w_{12}$$

At this stage, all elements of matrix A and vector w are determined. Therefore, linear system of equations for the constraints can be written explicitly as,

$$[\mathrm{A}] = \begin{bmatrix} [1 - \alpha q_{00}(1)] & [-\alpha q_{01}(1)] \\ [1 - \alpha q_{00}(2)] & [-\alpha q_{01}(2)] \\ [-\alpha q_{10}(1)] & [1 - \alpha q_{11}(1)] \\ [-\alpha q_{10}(2)] & [1 - \alpha q_{11}(2)] \end{bmatrix} \quad \text{and} \quad \{w\} = \begin{Bmatrix} w_{01} \\ w_{02} \\ w_{11} \\ w_{12} \end{Bmatrix}$$

Substituting [A] and $\{w\}$ into the system of equations, we obtain,

$$\begin{bmatrix} [1-\alpha q_{00}(1)] & [-\alpha q_{01}(1)] & 1 & 0 & 0 & 0 \\ [1-\alpha q_{00}(2)] & [-\alpha q_{01}(2)] & 0 & 1 & 0 & 0 \\ [-\alpha q_{10}(1)] & [1-\alpha q_{11}(1)] & 0 & 0 & 1 & 0 \\ [-\alpha q_{10}(2)] & [1-\alpha q_{11}(2)] & 0 & 0 & 0 & 1 \end{bmatrix} . \begin{Bmatrix} v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \\ 2 \\ 2 \end{Bmatrix}$$

3.4.2 Derivation of the Coefficient Matrix B for the Dual Form of the Two - State Two - Action Problem

Constraints of dual linear programming problem compose a linear system of equations as shown below.

$$[\mathrm{B}]_{m\times n}\{x\}_{n\times 1} \leq \{\beta\}_{m\times 1} \tag{3.11}$$

Matrix B is referred to as the coefficient matrix and it is derived using the transformation formula which transforms discounted life-cycle cost problem from dynamic programming to dual linear programming.

Transformation formula is given as shown below.

$$\sum_i \sum_a x_{ia}(\delta_{ij} - \alpha q_{ij}(a)) = \beta_j \tag{3.12}$$

The elements of the coefficient matrix depend on transition probability, β_j and discounted rate as shown below.

$$b_{mn} = f(q_{ij}(a), \beta_j, \alpha) \tag{3.13}$$

Explicit form of matrix B can be written as,

$$\mathrm{B} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \end{bmatrix} \tag{3.14}$$

Transformation formula has Kronecker Delta (δ), β, and α. Properties and values of these variables are shown below.

$$\sum_j \beta_j = 1 \quad \text{and} \quad \alpha = \frac{1}{2}$$

$$\delta_{ij} = 0 \quad \text{if} \quad i \neq j \quad \text{and} \quad \delta_{ij} = 1 \quad \text{if} \quad i = j$$

where β_j are any set of arbitrary numbers that when summed, add to one.

There are summations over i and a indices. Thus, the formulation of coefficient matrix of constraint of dual linear programming problem can be generated for j values.

for $j = 0$

$$\sum_i \sum_a x_{ia}(\delta_{i0} - \alpha q_{i0}(a)) = \beta_0$$

Substituting i and a values and performing summation over i and a, we obtain

$$x_{01}(\delta_{00} - \alpha q_{00}(1)) + x_{02}(\delta_{00} - \alpha q_{00}(2)) + x_{11}(\delta_{10} - \alpha q_{10}(1)) + x_{12}(\delta_{10} - \alpha q_{10}(2)) = \beta_0$$

Letting $b_{11} = [\delta_{00} - \alpha q_{00}(1)]$, $b_{12} = [\delta_{00} - \alpha q_{00}(2)]$, $b_{13} = [\delta_{10} - \alpha q_{10}(1)]$ and $b_{14} = [\delta_{10} - \alpha q_{10}(2)]$, we obtain

$$b_{11}x_{01} + b_{12}x_{02} + b_{13}x_{11} + b_{14}x_{12} = \beta_0$$

for $j = 1$

$$\sum_i \sum_a x_{ia}(\delta_{i1} - \alpha q_{i1}(a)) = \beta_1$$

Substituting i and a values and performing summation over i and a, we obtain

$$x_{01}(\delta_{01} - \alpha q_{01}(1)) + x_{02}(\delta_{01} - \alpha q_{01}(2)) + x_{11}(\delta_{11} - \alpha q_{11}(1)) + x_{12}(\delta_{11} - \alpha q_{11}(2)) = \beta_1$$

Letting $b_{21} = [\delta_{01} - \alpha q_{01}(1)]$, $b_{22} = [\delta_{01} - \alpha q_{01}(2)]$, $b_{23} = [\delta_{11} - \alpha q_{11}(1)]$ and $b_{24} = [\delta_{11} - \alpha q_{11}(2)]$, we obtain

$$b_{21}x_{01} + b_{22}x_{02} + b_{23}x_{11} + b_{24}x_{12} = \beta_1$$

Coefficient matrix of dual linear programming problem for this problem may be written explicitly as:

$$\mathrm{B} = \begin{bmatrix} \delta_{00} - \alpha q_{00}(1) & \delta_{00} - \alpha q_{00}(2) & \delta_{10} - \alpha q_{10}(1) & \delta_{10} - \alpha q_{10}(2) \\ \delta_{01} - \alpha q_{01}(1) & \delta_{01} - \alpha q_{01}(2) & \delta_{11} - \alpha q_{11}(1) & \delta_{11} - \alpha q_{11}(2) \end{bmatrix}$$

Substituting [B] and β's into system of equations, we obtain,

$$\begin{bmatrix} [\delta_{00} - \alpha q_{00}(1)] & [\delta_{00} - \alpha q_{00}(2)] & [\delta_{10} - \alpha q_{10}(1)] & [\delta_{10} - \alpha q_{10}(2)] \\ [\delta_{01} - \alpha q_{01}(1)] & [\delta_{01} - \alpha q_{01}(2)] & [\delta_{11} - \alpha q_{11}(1)] & [\delta_{11} - \alpha q_{11}(2)] \end{bmatrix} \cdot \begin{Bmatrix} x_{01} \\ x_{02} \\ x_{11} \\ x_{12} \end{Bmatrix} = \begin{Bmatrix} \beta_0 \\ \beta_1 \end{Bmatrix}$$

Figure 3.15 shows the algorithm developed for programming to find an optimal policy for a dynamic system using dynamic programming Markov Process reduced to linear programming. The aim of the program is to find an optimal policy which minimizes the expected discounted cost. First, dynamic programming problem is reduced to a linear programming problem. The form of the linear programming problem is referred to as primal problem which includes the minimum discounted cost variable, action cost and transition probability matrix. Second, in order to find feasible solution variables, the primal problem is converted to dual linear problem. Finally, solution of the dual problem yields the optimal policy.

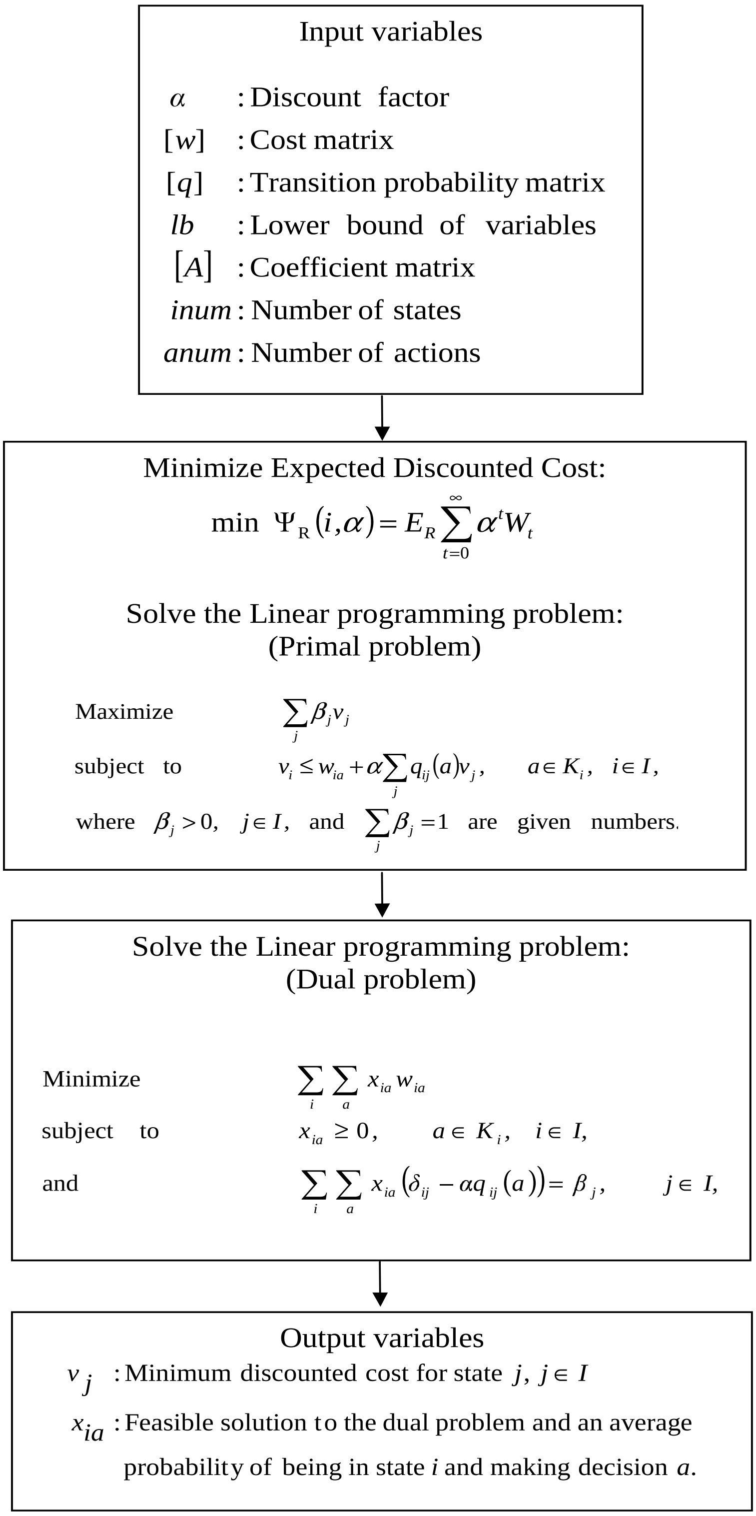


Figure 3.15: Flowchart Algorithm of Markov Process

3.5 Steady - State Probabilities

Condition of a dynamic system that changes its state is defined by a set of states. The system moves from one state to another in time. Moving to another state is called the state transition. Transition of the states can be in terms of time or space change. After a certain number of transitions, the change in values of the elements of the transition matrix diminishes. This state of the matrix is referred to as the steady-state transition matrix. The following examples are generated in order to demonstrate this process.

3.5.1 A Two State Transition Model

The transition of two states is modeled as a homogeneous Markov chain with the following transition probability matrix [13].

$$\begin{bmatrix} 0.8 & 0.2 \\ 0.5 & 0.5 \end{bmatrix}$$

(a) If it is dry today, the probability that it will be dry 2 days from now may be computed by first determining P(2).

$$\begin{aligned} P(2) &= P(0)P^2 \\ &= [1 \quad 0] \begin{bmatrix} 0.8 & 0.2 \\ 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 0.8 & 0.2 \\ 0.5 & 0.5 \end{bmatrix} \\ &= [0.8 \quad 0.2] \begin{bmatrix} 0.8 & 0.2 \\ 0.5 & 0.5 \end{bmatrix} \\ &= [0.74 \quad 0.26] \end{aligned}$$

Hence, it is 74% probable that the day after tomorrow will be dry. Now, let us advance this procedure in time in order to find the steady state condition state.

$$
\begin{aligned}
P(3) &= P(0)P^3 \\
&= [1 \quad 0]\begin{bmatrix} 0.8 & 0.2 \\ 0.5 & 0.5 \end{bmatrix}^3 \\
&= [0.722 \quad 0.278]
\end{aligned}
$$

$$
\begin{aligned}
P(4) &= P(0)P^4 \\
&= [1 \quad 0]\begin{bmatrix} 0.8 & 0.2 \\ 0.5 & 0.5 \end{bmatrix}^4 \\
&= [0.7166 \quad 0.2834]
\end{aligned}
$$

$$
\begin{aligned}
P(5) &= P(0)P^5 \\
&= [1 \quad 0]\begin{bmatrix} 0.8 & 0.2 \\ 0.5 & 0.5 \end{bmatrix}^5 \\
&= [0.7150 \quad 0.2850]
\end{aligned}
$$

$$
\begin{aligned}
P(6) &= P(0)P^6 \\
&= [1 \quad 0]\begin{bmatrix} 0.8 & 0.2 \\ 0.5 & 0.5 \end{bmatrix}^6 \\
&= [0.7145 \quad 0.2855]
\end{aligned}
$$

$$
\begin{aligned}
P(7) &= P(0)P^7 \\
&= [1 \quad 0]\begin{bmatrix} 0.8 & 0.2 \\ 0.5 & 0.5 \end{bmatrix}^7 \\
&= [0.7143 \quad 0.2857]
\end{aligned}
$$

$$P(8) = P(0)P^8$$
$$= [1 \quad 0]\begin{bmatrix} 0.8 & 0.2 \\ 0.5 & 0.5 \end{bmatrix}^8$$
$$= [0.7143 \quad 0.2857]$$

Therefore, steady state condition state is found at $P(8)$.

3.5.2 A Three State Transition Model

Let us assumed that the following transition probability matrix is given for a dynamic system having three states.

$$P = \begin{bmatrix} 0.4 & 0.5 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.1 & 0.7 & 0.2 \end{bmatrix}$$

and the initial state probabilities are given as:

$$P = [0.1 \quad 0.1 \quad 0.8]$$

After one transition, the state probabilities become:

$$P(1) = [0.1 \quad 0.1 \quad 0.8]\begin{bmatrix} 0.4 & 0.5 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.1 & 0.7 & 0.2 \end{bmatrix}$$
$$= [0.15 \quad 0.64 \quad 0.21]$$

After second and the following transitions, the state probabilities are calculated as follows:

$$P(2) = [0.1 \quad 0.1 \quad 0.8]\begin{bmatrix} 0.4 & 0.5 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.1 & 0.7 & 0.2 \end{bmatrix}^2$$

$$= [0.273 \quad 0.414 \quad 0.313]$$

$$P(3) = [0.1 \quad 0.1 \quad 0.8]\begin{bmatrix} 0.4 & 0.5 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.1 & 0.7 & 0.2 \end{bmatrix}^3$$

$$= [0.2647 \quad 0.4798 \quad 0.2555]$$

$$P(4) = [0.1 \quad 0.1 \quad 0.8]\begin{bmatrix} 0.4 & 0.5 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.1 & 0.7 & 0.2 \end{bmatrix}^4$$

$$= [0.2754 \quad 0.4551 \quad 0.2695]$$

$$P(5) = [0.1 \quad 0.1 \quad 0.8]\begin{bmatrix} 0.4 & 0.5 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.1 & 0.7 & 0.2 \end{bmatrix}^5$$

$$= [0.2736 \quad 0.4629 \quad 0.2635]$$

$$P(6) = [0.1 \quad 0.1 \quad 0.8]\begin{bmatrix} 0.4 & 0.5 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.1 & 0.7 & 0.2 \end{bmatrix}^6$$

$$= [0.2747 \quad 0.4601 \quad 0.2652]$$

$$P(7) = [0.1 \quad 0.1 \quad 0.8]\begin{bmatrix} 0.4 & 0.5 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.1 & 0.7 & 0.2 \end{bmatrix}^7$$

$$= [0.2744 \quad 0.4601 \quad 0.2646]$$

$$P(8) = [0.1 \quad 0.1 \quad 0.8]\begin{bmatrix} 0.4 & 0.5 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.1 & 0.7 & 0.2 \end{bmatrix}^8$$

$$= [0.2745 \quad 0.4607 \quad 0.2648]$$

$$P(9) = [0.1 \quad 0.1 \quad 0.8]\begin{bmatrix} 0.4 & 0.5 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.1 & 0.7 & 0.2 \end{bmatrix}^9$$

$$= [0.2745 \quad 0.4608 \quad 0.2647]$$

$$P(10) = [0.1 \quad 0.1 \quad 0.8]\begin{bmatrix} 0.4 & 0.5 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.1 & 0.7 & 0.2 \end{bmatrix}^{10}$$

$$= [0.2745 \quad 0.4608 \quad 0.2647]$$

Therefore, the steady state condition state is achieved at the tenth transition.

3.5.3 A Bridge Element Condition State Transition Model

An example of transition probabilities under optimal actions for a bridge element's condition states is given as follows [53]:

$$P = \begin{bmatrix} 0.95 & 0.05 & 0 & 0 \\ 0.47 & 0.49 & 0.04 & 0 \\ 0.18 & 0.48 & 0.31 & 0.03 \\ 0.39 & 0.39 & 0.16 & 0.06 \end{bmatrix}$$

Based on these transition probabilities, the process of how the steady state condition state is achieved will be demonstrated. Let us assume that initial condition state for the system is:

$$P = [0 \quad 0 \quad 1 \quad 0]$$

This means that the condition of the system is at state 3. This is an assumption based on regular standard inspection period for bridge evaluation and that the system changes state at every 2 year period. Now, steady state condition state can be found the same following procedure that was followed in the previous examples.

$$P(1) = [0 \quad 0 \quad 1 \quad 0]\begin{bmatrix} 0.95 & 0.05 & 0 & 0 \\ 0.47 & 0.49 & 0.04 & 0 \\ 0.18 & 0.48 & 0.31 & 0.03 \\ 0.39 & 0.39 & 0.16 & 0.06 \end{bmatrix}$$

$$= [0.18 \quad 0.48 \quad 0.31 \quad 0.03]$$

$$P(2) = [0 \quad 0 \quad 1 \quad 0]\begin{bmatrix} 0.95 & 0.05 & 0 & 0 \\ 0.47 & 0.49 & 0.04 & 0 \\ 0.18 & 0.48 & 0.31 & 0.03 \\ 0.39 & 0.39 & 0.16 & 0.06 \end{bmatrix}^2$$

$$= [0.4641 \quad 0.4047 \quad 0.1201 \quad 0.0111]$$

$$P(3) = [0 \quad 0 \quad 1 \quad 0]\begin{bmatrix} 0.95 & 0.05 & 0 & 0 \\ 0.47 & 0.49 & 0.04 & 0 \\ 0.18 & 0.48 & 0.31 & 0.03 \\ 0.39 & 0.39 & 0.16 & 0.06 \end{bmatrix}^3$$

$$= [0.6571 \quad 0.2835 \quad 0.0552 \quad 0.0043]$$

$$P(4) = [0 \quad 0 \quad 1 \quad 0]\begin{bmatrix} 0.95 & 0.05 & 0 & 0 \\ 0.47 & 0.49 & 0.04 & 0 \\ 0.18 & 0.48 & 0.31 & 0.03 \\ 0.39 & 0.39 & 0.16 & 0.06 \end{bmatrix}^4$$

$$= [0.7690 \quad 0.1999 \quad 0.0291 \quad 0.0019]$$

$$P(5) = [0 \quad 0 \quad 1 \quad 0]\begin{bmatrix} 0.95 & 0.05 & 0 & 0 \\ 0.47 & 0.49 & 0.04 & 0 \\ 0.18 & 0.48 & 0.31 & 0.03 \\ 0.39 & 0.39 & 0.16 & 0.06 \end{bmatrix}^5$$

$$= [0.8305 \quad 0.1511 \quad 0.0173 \quad 0.0010]$$

$$P(6) = [0 \quad 0 \quad 1 \quad 0]\begin{bmatrix} 0.95 & 0.05 & 0 & 0 \\ 0.47 & 0.49 & 0.04 & 0 \\ 0.18 & 0.48 & 0.31 & 0.03 \\ 0.39 & 0.39 & 0.16 & 0.06 \end{bmatrix}^6$$

$$= [0.8636 \quad 0.1243 \quad 0.0116 \quad 0.0006]$$

$$P(7) = [0 \quad 0 \quad 1 \quad 0]\begin{bmatrix} 0.95 & 0.05 & 0 & 0 \\ 0.47 & 0.49 & 0.04 & 0 \\ 0.18 & 0.48 & 0.31 & 0.03 \\ 0.39 & 0.39 & 0.16 & 0.06 \end{bmatrix}^7$$

$$= [0.8811 \quad 0.1099 \quad 0.0087 \quad 0.0004]$$

$$P(8) = [0 \quad 0 \quad 1 \quad 0]\begin{bmatrix} 0.95 & 0.05 & 0 & 0 \\ 0.47 & 0.49 & 0.04 & 0 \\ 0.18 & 0.48 & 0.31 & 0.03 \\ 0.39 & 0.39 & 0.16 & 0.06 \end{bmatrix}^8$$

$$= [0.8904 \quad 0.1022 \quad 0.0071 \quad 0.0003]$$

$$P(9) = [0 \quad 0 \quad 1 \quad 0]\begin{bmatrix} 0.95 & 0.05 & 0 & 0 \\ 0.47 & 0.49 & 0.04 & 0 \\ 0.18 & 0.48 & 0.31 & 0.03 \\ 0.39 & 0.39 & 0.16 & 0.06 \end{bmatrix}^9$$

$$= [0.8953 \quad 0.0981 \quad 0.0063 \quad 0.0002]$$

$$P(10) = [0 \quad 0 \quad 1 \quad 0]\begin{bmatrix} 0.95 & 0.05 & 0 & 0 \\ 0.47 & 0.49 & 0.04 & 0 \\ 0.18 & 0.48 & 0.31 & 0.03 \\ 0.39 & 0.39 & 0.16 & 0.06 \end{bmatrix}^{10}$$

$$= [0.8979 \quad 0.0960 \quad 0.0059 \quad 0.0002]$$

$$P(11) = [0 \quad 0 \quad 1 \quad 0]\begin{bmatrix} 0.95 & 0.05 & 0 & 0 \\ 0.47 & 0.49 & 0.04 & 0 \\ 0.18 & 0.48 & 0.31 & 0.03 \\ 0.39 & 0.39 & 0.16 & 0.06 \end{bmatrix}^{11}$$

$$= [0.8992 \quad 0.0949 \quad 0.0057 \quad 0.0002]$$

$$P(12) = [0 \quad 0 \quad 1 \quad 0]\begin{bmatrix} 0.95 & 0.05 & 0 & 0 \\ 0.47 & 0.49 & 0.04 & 0 \\ 0.18 & 0.48 & 0.31 & 0.03 \\ 0.39 & 0.39 & 0.16 & 0.06 \end{bmatrix}^{12}$$

$$= [0.9000 \quad 0.0943 \quad 0.0056 \quad 0.0002]$$

$$P(13) = [0 \quad 0 \quad 1 \quad 0]\begin{bmatrix} 0.95 & 0.05 & 0 & 0 \\ 0.47 & 0.49 & 0.04 & 0 \\ 0.18 & 0.48 & 0.31 & 0.03 \\ 0.39 & 0.39 & 0.16 & 0.06 \end{bmatrix}^{13}$$

$$= [0.9003 \quad 0.0939 \quad 0.0055 \quad 0.0002]$$

$$P(14) = [0 \quad 0 \quad 1 \quad 0]\begin{bmatrix} 0.95 & 0.05 & 0 & 0 \\ 0.47 & 0.49 & 0.04 & 0 \\ 0.18 & 0.48 & 0.31 & 0.03 \\ 0.39 & 0.39 & 0.16 & 0.06 \end{bmatrix}^{14}$$

$$= [0.9005 \quad 0.0938 \quad 0.0055 \quad 0.0002]$$

$$P(15) = [0 \quad 0 \quad 1 \quad 0]\begin{bmatrix} 0.95 & 0.05 & 0 & 0 \\ 0.47 & 0.49 & 0.04 & 0 \\ 0.18 & 0.48 & 0.31 & 0.03 \\ 0.39 & 0.39 & 0.16 & 0.06 \end{bmatrix}^{15}$$

$$= [0.9007 \quad 0.0937 \quad 0.0055 \quad 0.0002]$$

$$P(16) = [0 \quad 0 \quad 1 \quad 0]\begin{bmatrix} 0.95 & 0.05 & 0 & 0 \\ 0.47 & 0.49 & 0.04 & 0 \\ 0.18 & 0.48 & 0.31 & 0.03 \\ 0.39 & 0.39 & 0.16 & 0.06 \end{bmatrix}^{16}$$

$$= [0.9006 \quad 0.0936 \quad 0.0055 \quad 0.0002]$$

$$P(17) = [0 \quad 0 \quad 1 \quad 0]\begin{bmatrix} 0.95 & 0.05 & 0 & 0 \\ 0.47 & 0.49 & 0.04 & 0 \\ 0.18 & 0.48 & 0.31 & 0.03 \\ 0.39 & 0.39 & 0.16 & 0.06 \end{bmatrix}^{17}$$

$$= [0.9007 \quad 0.0936 \quad 0.0055 \quad 0.0002]$$

Therefore, by comparing $P(16)$ and $P(17)$, it can be observed that the steady state condition is achieved at 34 years, since each multiplication is equal to 2 years.

3.6 Applications for Finding the Optimal Policy

Table 3.1 gives detailed information for the application of Markov Decision Process used by the California Department of Transportation (Caltrans) [2]. The information presented is for Element 107 which is the Painted Steel Open Girder. Similarly, Table 3.2 presents the costs of actions defined in Table 3.1. Caltrans uses the Pontis Bridge Management System to find the optimal solution for bridge element maintenance, repair and rehabilitation (MR&R) by using the linear programming technique.

Markov.m program, written in this study, is used to find the optimal policy for the same problem. The Transition Probability Matrix and the Cost Matrix denoted as $q_{ij}(a)$ and w_{ia} presented below each table in matrix form are implemented into the developed computer program.

Table 3.1: Transition probabilities of Do Nothing and other maintenance actions for the application example.

States	Actions	Transition Probabilities (%)				
		1	2	3	4	5
1	0 - Do Nothing	93.81	6.19	0	0	0
	1 - Surface Clean	100	0	0	0	0
2	0 - Do Nothing	0	88.88	11.12	0	0
	1 - Surface Clean	1	99	0	0	0
	2 - Surface Clean & Repaint	96	4	0	0	0
3	0 - Do Nothing	0	0	87.12	12.88	0
	1 - Spot Blast, Clean & Repaint	88	12	0	0	0
4	0 - Do Nothing	0	0	0	88.88	11.12
	1 - Spot Blast, Clean & Repaint	61	14	5	20	0
	2 - Replace Paint System	97	3	0	0	0
5	0 - Do Nothing	0	0	0	0	90.55
	1 - Major Rehabilitation	30	9	1	20	40
	2 - Replace Unit	100	0	0	0	0

Table 3.2: Costs of Do Nothing and Other Maintenance Actions for the application example.

States	Actions	Cost ($)
1	0 - Do nothing	0
	1 - Surface clean	62.34
2	0 - Do nothing	0
	1 - Surface clean	80.84
	2 - Surface clean & repaint	225.26
3	0 - Do nothing	0
	1 - Spot blast, clean & repaint	328.48
4	0 - Do nothing	0
	1 - Spot blast, clean & repaint	455.90
	2 - Replace paint system	396.32
5	0 - Do nothing	0
	1 - Major rehabilitation	1279.52
	2 - Replace unit	2394.82

$$q_{ij}(a) = \begin{bmatrix} (93.81, 100, 0) & (6.19, 0, 0) & (0, 0, 0) & (0, 0, 0) & (0, 0, 0) \\ (0, 1, 96) & (88.88, 99, 4) & (11.12, 0, 0) & (0, 0, 0) & (0, 0, 0) \\ (0, 88, 0) & (0, 12, 0) & (87.12, 0, 0) & (12.88, 0, 0) & (0, 0, 0) \\ (0, 61, 97) & (0, 14, 3) & (0, 5, 0) & (88.88, 20, 0) & (11.12, 0, 0) \\ (0, 30, 100) & (0, 9, 0) & (0, 1, 0) & (0, 20, 0) & (90.55, 40, 0) \end{bmatrix}$$

As shown in Table 3.1, the Element 107 has 5 different condition state and there are several feasible actions for each state. For instance, State 1 and State 3 have 2 different MR&R actions. However, the State 2, 4, and 5 have 3 different MR&R actions. The Do-Nothing action is listed in all condition states as a feasible action. There are 7 unique actions considering all of the condition states. In addition, Table 3.1 lists the transition probabilities of being in State j at the next instant, when action a is taken at the present state i.

Maintenance, Repair, Rehabilitation and Replacement costs for Element 107 is given in Table 3.2. As shown in Table 3.2, Do-Nothing action has zero cost. It should be noted that the same maintenance action may have different cost values in different states. For example, both Surface Clean and Spot Blast, Clean & Repaint actions are feasible in more than one state having different maintenance costs in different condition states.

$$w_{ia} = \begin{bmatrix} 0 & 62.34 & 0 \\ 0 & 80.84 & 225.26 \\ 0 & 328.48 & 0 \\ 0 & 455.90 & 396.32 \\ 0 & 1279.52 & 2394.82 \end{bmatrix}$$

For the above problem, solution of the linear programming problem is obtained as follows.

$$\tilde{x}_{ia} = \begin{Bmatrix} 0.4688 \\ 0 \\ 0.2690 \\ 0 \\ 0 \\ 0.2323 \\ 0 \\ 0 \\ 0 \\ 0.0299 \\ 0 \\ 0 \\ 0 \end{Bmatrix}.$$

The values of x_{12}, x_{22}, x_{23}, x_{32}, x_{41}, x_{42}, x_{51}, x_{52}, and x_{53} are zero and x_{11}, x_{21}, x_{31} and x_{43} have nonzero values. Calculating the D_{ia} values, the optimal policy can be defined as follows.

$i = 1, a = 1$

$$D_{11} = \frac{x_{11}}{\sum_a x_{1a}} = \frac{x_{11}}{x_{11} + x_{12}} = \frac{0.4688}{0.4688 + 0} = 1$$

$i = 1, a = 2$

$$D_{12} = \frac{x_{12}}{\sum_a x_{1a}} = \frac{x_{12}}{x_{11} + x_{12}} = \frac{0}{0.4688 + 0} = 0$$

$i = 2, a = 1$

$$D_{21} = \frac{x_{21}}{\sum_a x_{2a}} = \frac{x_{21}}{x_{21} + x_{22} + x_{23}} = \frac{0.2690}{0.2690 + 0 + 0} = 1$$

$i = 2, a = 2$

$$D_{22} = \frac{x_{22}}{\sum_a x_{2a}} = \frac{x_{22}}{x_{21} + x_{22} + x_{23}} = \frac{0}{0.2690 + 0 + 0} = 0$$

$i = 2, a = 3$

$$D_{23} = \frac{x_{23}}{\sum_a x_{2a}} = \frac{x_{23}}{x_{21} + x_{22} + x_{23}} = \frac{0}{0.2690 + 0 + 0} = 0$$

$i = 3, a = 1$

$$D_{31} = \frac{x_{31}}{\sum_a x_{3a}} = \frac{x_{31}}{x_{31} + x_{32}} = \frac{0.2323}{0.2323 + 0} = 1$$

$i = 3, a = 2$

$$D_{32} = \frac{x_{32}}{\sum_a x_{3a}} = \frac{x_{32}}{x_{31} + x_{32}} = \frac{0}{0.2323 + 0} = 0$$

$i = 4, a = 1$

$$D_{41} = \frac{x_{41}}{\sum_a x_{4a}} = \frac{x_{41}}{x_{41} + x_{42} + x_{43}} = \frac{0}{0 + 0 + 0.0299} = 0$$

$i = 4, a = 2$

$$D_{42} = \frac{x_{42}}{\sum_a x_{4a}} = \frac{x_{42}}{x_{41} + x_{42} + x_{43}} = \frac{0}{0 + 0 + 0.0299} = 0$$

$i = 4, a = 3$

$$D_{43} = \frac{x_{43}}{\sum_a x_{4a}} = \frac{x_{43}}{x_{41} + x_{42} + x_{43}} = \frac{0.0299}{0 + 0 + 0.0299} = 1$$

$i = 5, a = 1$

$$D_{51} = \frac{x_{51}}{\sum_a x_{5a}} = \frac{x_{51}}{x_{51} + x_{52} + x_{53}} = \frac{0}{0 + 0 + 0} = \text{NaN}$$

$i = 5, a = 2$

$$D_{52} = \frac{x_{52}}{\sum_a x_{5a}} = \frac{x_{52}}{x_{51} + x_{52} + x_{53}} = \frac{0}{0 + 0 + 0} = \text{NaN}$$

$i = 5, a = 3$

$$D_{53} = \frac{x_{53}}{\sum_a x_{5a}} = \frac{x_{53}}{x_{51} + x_{52} + x_{53}} = \frac{0}{0 + 0 + 0} = \text{NaN}$$

Results correspond to $(i = 1, a = 1)$, $(i = 2, a = 1)$, $(i = 3, a = 1)$ and $(i = 4, a = 3)$ which means that action 1 should be taken at state 1, action 1 should be taken at state 2, action 1 should be taken at state 3, and action 3 should be taken at state 4 in order to achieve the optimal policy. If the optimal actions are followed for a sufficiently long term period of time, the element will not reach the state 5.

According to the optimal policy, the transition probability matrix takes the form below.

$$q_{ij}(a) = \begin{bmatrix} 93.81 & 6.19 & 0 & 0 & 0 \\ 0 & 88.88 & 11.12 & 0 & 0 \\ 0 & 0 & 87.12 & 12.88 & 0 \\ 97 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 90.55 \end{bmatrix}$$

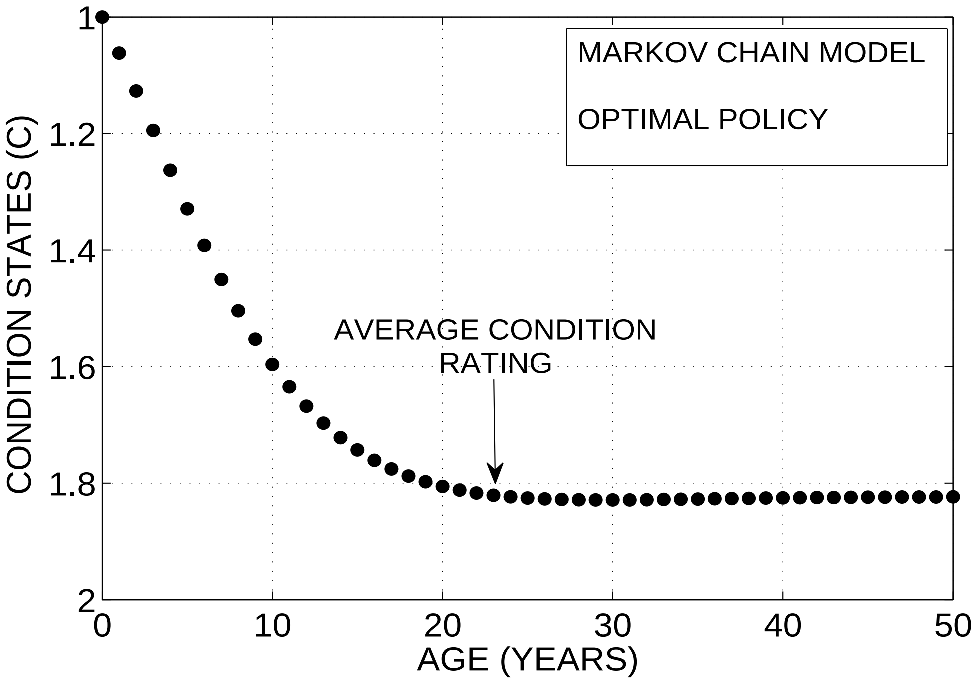


Figure 3.16: Performance data obtained from optimal policy

Fig. 3.16 showing the variation in time of the average condition rating may be obtained by following the optimal policy. As shown this graph, the steady state condition state is achieved at nearly 30 years.

As another example, the transition probability is given in Table 3.3. In this example, the transition probabilities are selected different than that of the former example.

The deterioration rate of this element is higher than the former one. The costs of the actions are the same as the former element costs. In this example, effect of the transition probabilities on selecting optimal policy is investigated. In order to implement the transition probabilities into the developed *Markov.m* computer program,

Table 3.3: Transition probabilities of Do Nothing and MR&R actions.

States	Actions	Transition Probabilities (%)				
		1	2	3	4	5
1	0 - Do nothing	70	20	10	0	0
	1 - Surface clean	80	20	0	0	0
2	0 - Do nothing	0	70	15	10	5
	1 - Surface clean	30	50	20	0	0
	2 - Surface clean & repaint	70	15	15	0	0
3	0 - Do nothing	0	0	70	10	10
	1 - Spot blast, clean & repaint	15	30	30	15	10
4	0 - Do nothing	0	0	0	75	30
	1 - Spot blast, clean & repaint	5	15	30	35	15
	2 - Replace paint system	20	25	40	10	5
5	0 - Do nothing	0	0	0	0	70
	1 - Major rehabilitation	10	25	30	25	5
	2 - Replace unit	100	0	0	0	0

$q_{ij}(a)$ matrix is composed as shown below.

$$q_{ij}(a) = \begin{bmatrix} (70,80,0) & (20,20,0) & (10,0,0) & (0,0,0) & (0,0,0) \\ (0,30,70) & (70,50,15) & (15,20,15) & (10,0,0) & (5,0,0) \\ (0,15,0) & (0,30,0) & (70,30,0) & (20,15,0) & (10,10,0) \\ (0,5,20) & (0,15,25) & (0,30,40) & (70,35,10) & (30,15,5) \\ (0,10,100) & (0,25,0) & (0,30,0) & (0,25,0) & (70,5,0) \end{bmatrix}$$

Based on the given transition probability and cost matrices, the solution to the linear programming problem is obtained as follows.

$$\tilde{x}_{ia} = \left\{ \begin{array}{l} 0 \\ 0.7471 \\ 0 \\ 0 \\ 0.1943 \\ 0 \\ 0.0460 \\ 0 \\ 0 \\ 0.0077 \\ 0 \\ 0 \\ 0.0050 \end{array} \right\}.$$

The x_{12}, x_{23}, x_{32}, x_{43} and x_{53} have nonzero values, so optimal policy can be achieved by applying action 2 when in State 1, action 3 in State 2, action 2 in State 3, action 3 in State 4, and action 3 when in State 5.

The transition probability matrix of the optimal policy is as shown below and Fig. 3.17 is obtained by following the optimal policy.

$$q_{ij}(a) = \begin{bmatrix} 80 & 20 & 0 & 0 & 0 \\ 70 & 15 & 15 & 0 & 0 \\ 15 & 30 & 30 & 15 & 10 \\ 20 & 25 & 40 & 10 & 5 \\ 100 & 0 & 0 & 0 & 0 \end{bmatrix}$$

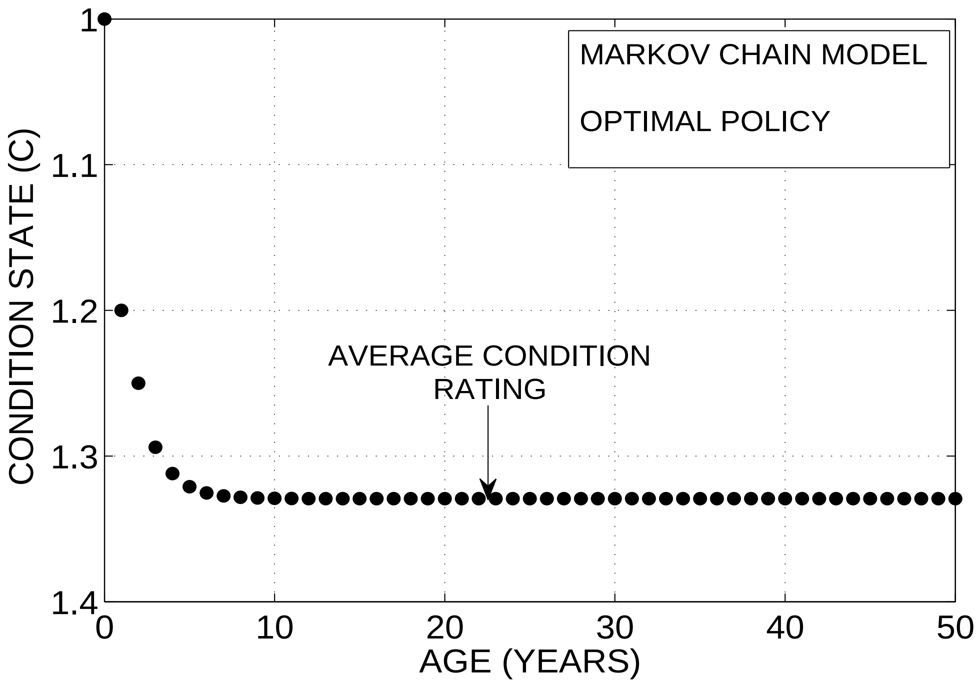


Figure 3.17: Condition State Performance curve obtained from optimal policy.

3.7 Summary

In this chapter, Markov process-based condition prediction is investigated. The condition prediction method is used for the Dynamic Systems. Dynamic system problems can be solved by dynamic programming technique. Policy improvement, successive approximation, and linear programming are mentioned as solution methods for Dynamic Programming problems and linear programming technique is investigated to obtain a solution technique for Markov process-based condition prediction.

Dynamic programming problem can be reduced to linear programming which is referred to as primal problem. In order to obtain optimal solution, primal problem is converted to dual problem. A two-state two-action and three-state two action cases are investigated.

Furthermore, the steady state procedure obtained by following the optimal policy for a sufficiently long period is investigated. Finally, the developed computer program is used to find optimal policy for a structure with different deterioration model and same cost matrix.

Notations in Chapter 3

a	:	A maintenance action
i and j	:	Condition states (Condition level)
P^t_{ija}	:	Probability that the system may be at level j at the beginning of the next time interval t when the system is at level i now and action a is chosen without any consideration about the past condition
$v^\alpha_i(R)$	:	Discounted long-term life-cycle cost under policy R at state i
R	:	Selected policy
α	:	Discount factor
r_{ja}	:	Reward earned at state j when action a is chosen
$q_{ij}(a)$	:	The probability of the system being in state j at the next instant the system is observed when the system is in state i now and action a is taken regardless of its history (referred to as the transition probability)
E_R	:	Expected policy R
β_j	:	Probability of occurrence of a system in state j
C_S	:	Class of all Markovian policies
C_D	:	Subclass of C_S
$\Psi_R(i, \alpha)$	:	Expected long-term discounted cost under policy R
w_{ia}	:	Cost incurred when system is in state i and action a is taken
K_i	:	Number of actions possible when the system is at state i
I	:	State space (Space of possible states)
H_{t-1}	:	History of a system up to time $t-1$
Y_t	:	State of a system at time t
$D_a H_{t-1}, Y_t$	:	Probability of taking action a at time t using a random mechanism
$\tilde{x}_{ia}$	:	Solution vector of a dual linear problem

CHAPTER 4

BI-LINEAR AND POLYNOMIAL BASED CONDITION PREDICTION AND EFFECT OF MAINTENANCE ACTIONS

4.1 Simulation

Simulation is an artificially generated state of a real physical process. The act of simulating a system generally requires representing essential key characteristics or behavior of a selected system.

Simulation is used in many events, including the modeling of systems in nature in order to gain insight into their functioning. Simulation is also used in such fields as performance optimization, safety engineering, and testing. Simulation can be used to illustrate the eventual real effects of alternative conditions and courses of action.

Simulation is an important feature in engineering systems that involve many processes. The term simulation may be used in engineering with two different meanings. First, term simulation may refer to a computer simulation of the behavior of an engineering system. This means that the real world system is re-created using computer modeling and its real life behavior is imitated. For example in civil engineering, simulation allows scientists to observe the behavior of a building under earthquake effect prior to the occurrence of such an event. Second, the term simulation may refer to a numerical simulation technique. This meaning of simulation is the subject of this thesis. There are different types of simulation techniques, two of which are mentioned here; the Monte Carlo Simulation Method and the Latin Hypercube Sampling Method.

4.1.1 Monte Carlo Simulation

Monte Carlo Simulation method is a computational algorithm that relies on repeated random sampling. Monte Carlo methods are often used to simulate physical and mathematical systems. They are most suited for computer computation because of their reliance on fast repeated computation and random number generation. Monte Carlo methods are used when it is infeasible or impossible to compute an exact result using a deterministic algorithm. If systems have large number of coupled random variables, it is especially suitable to use the Monte Carlo simulation method.

4.1.2 Latin Hypercube Sampling

The Latin Hypercube sampling was developed to generate a distribution of a rational collection of parameter values from a multidimensional distribution. This sampling method was first introduced by McKay *et al.*[54]. Simply stated, Latin Hypercube sampling is a constrained Monte Carlo sampling scheme. It samples the entire domain more systematically while Monte Carlo simulation method typically picks points randomly within the domain. Therefore, the efficiency of Monte Carlo simulations can be improved using Latin Hypercube sampling.

Latin Hypercube sampling works in the following manner. The range of each variable is divided into n nonoverlapping intervals on the basis of equal probability. The method selects n different values from each of k variables $X_1, X_2, ..., X_k$. Each variable has a probability density function and Latin hypercube algorithm selects one value from each interval randomly taking the corresponding probability density function into account. The n values obtained for the random variable X_1 are paired in equally likely combinations with n values of X_2 and these n pairs are combined in a random manner with n values of X_3 to form n triplets, and so on, until n k-tuplets are formed. These n k-tuplets are the same as the n k-dimensional input vectors which form the Latin Hypercube sample.

Using the Latin hypercube method, the whole parameter space can be obtained more reliable with fewer iterations. This can help improve convergence rates and speed up execution [54].

Monte Carlo Simulation method and Latin Hypercube Sampling method use several techniques to generate random numbers from a given probability distribution. Some of the techniques are the inverse transform method, composition approach, convolution method, and acceptance-rejection technique. In this study, inverse transform method is used. Inverse transform is a method of generating sample numbers randomly from any probability distribution whose cumulative distribution function $F(x)$ (CDF) is given. Inverse transform method can generate a number x from a random variable with the probability density function (PDF) $P(x)$ by first finding $F(x)$ from $P(x)$ and then inverting it by solving $p = F(x)$ for x, which gives $x = F^{-1}(p)$. Then, a uniform random number $0 < p < 1$ is generated and $x = F^{-1}(p)$ is computed. Although this method is generally applicable, there may be computational difficulties obtaining cumulative distribution functions for some probability distributions.

Matlab, which is used for simulation in this study, contains a Toolbox application to generate random numbers from Latin Hypercube sampling. Matlab Toolbox functions that can generate numbers using Latin Hypercube sampling are named as *lhsdesign* and *lhsnorm*. The *lhsdesign* function generates a Latin Hypercube sample X containing n values for each of p variables. For each column, the n values are randomly distributed with one from each interval (0,1/n), (1/n,2/n), ..., (1-1/n,1), and they are randomly permuted. The *lhsnorm* function, on the other hand, generates a Latin Hypercube sample X of size n from the multivariate normal distribution with mean vector μ and covariance matrix σ. X is similar to a random sample from the multivariate normal distribution, but the marginal distribution of each column is adjusted so that its sample marginal distribution is close to its theoretical normal distribution.

Since Matlab is a programmable tool, in this study, its programming capabilities are combined with its embedded simulation functions. Furthermore, the Latin Hypercube sampling has been improved so that it can be used with a different probability distribution type other than the ones existing in Matlab. The random variables used in this study have triangular distributions but *lhsdesign* and *lhsnorm* functions cannot generate random numbers from triangular distribution. Thus, a Matlab function *M*-file, named as *latin_hs_tri.m* is developed and desired random numbers are generated based on triangular distribution. Inverse cumulative distribution function (CDF) used to generate random numbers from any distribution type is derived from this distribution. Cumulative distribution function of a triangular distribution and its inverse cumulative distribution function are given as Eqs. 4.1 and 4.2, respectively.

$$F_x = \begin{cases} \left(\frac{(\text{mode-min})}{(\text{max-min})}\right), & x = \text{mode} \\ \left(\frac{(x - \text{min})^2}{(\text{mode-min})\cdot(\text{max-min})}\right), & x < \text{mode} \\ \left(1 - \frac{(\text{max}- x)^2}{(\text{max-mode})\cdot(\text{max-min})}\right), & x > \text{mode} \end{cases} \tag{4.1}$$

$$G_p = \begin{cases} \text{mode}, & p = \text{mode} \\ \left(\text{min} + \sqrt{p \cdot (\text{max-min}) \cdot (\text{mode-min})}\right), & p < \text{mode} \\ \left(\text{max} - \sqrt{(1 - p) \cdot (\text{max-min}) \cdot (\text{max-mode})}\right), & p < \text{mode} \end{cases} \tag{4.2}$$

Table 4.1: Parameters in the cumulative and inverse cumulative distribution functions of triangular distribution.

Parameter	Description
min	Minimum value of the triangular distribution
mode	Mode value of the triangular distribution
max	Maximum value of the triangular distribution
F_x	Cumulative distribution function of triangular distribution
G_p	Inverse cumulative distribution function of triangular distribution
x	Any selected number in the distribution
p	probability of cumulative distribution function

Eq. 4.2 is derived from Eq. 4.1, transforming cumulative distribution function into the inverse function. The parameters introduced in equation 4.1 and 4.2 are described in Table 4.1.

Simulation results obtained from *latin_hs_tri.m* function are presented in Figures 4.2, 4.3, 4.4, 4.5, 4.6, 4.7 and 4.8. These figures show how an increase in sample size increases the convergence of simulation results to the theoretical triangular distribution. Triangular nature of the simulated distribution is not visible in Fig. 4.4 with 100 samples. However, it is clearly visible in Fig. 4.7 with 1000 samples. As shown in Fig. 4.4 through Fig. 4.8, a triangularly distributed random variable, whose minimum, mode and maximum values are known, is simulated with 10, 100, 1000, 10000 and 50000 sample sizes. Minimum, mode and maximum values of the random variable are 10, 20 and 30, respectively. As illustrated, if the sample size is small, simulation results are not accurate. When the number of sample size is greatly increased (e.g, 10000 or 50000 simulations), an exact distribution fitting the triangular distribution is achieved.

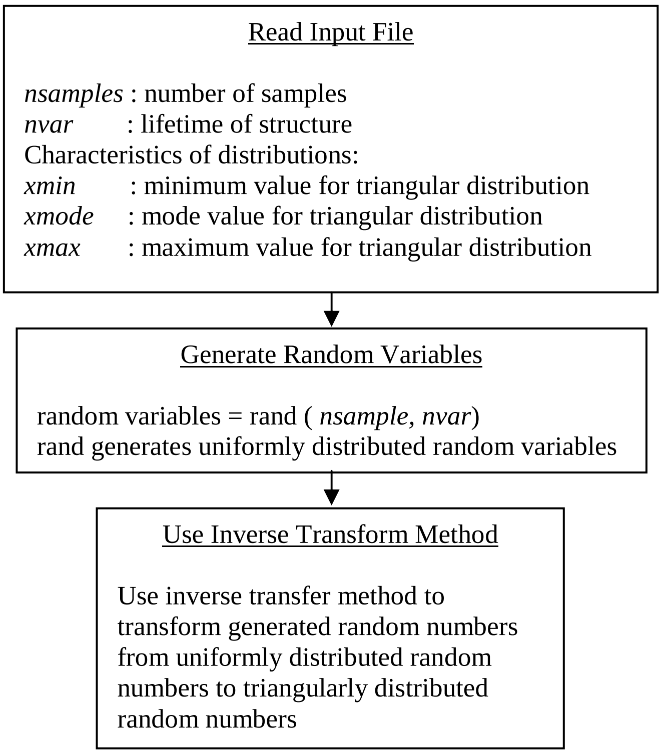


Figure 4.1: Flowchart of Latin Hypercube Sampling simulation program.

It can also be observed that the interval length of bar chart graph has an effect on the display of the simulation results. Although Fig. 4.2 and Fig. 4.3 have the same sample size, these figures do not look the same. This can be seen more clearly from differences between Figure 4.4 and Figure 4.5. As shown in Fig. 4.4, there is a disharmony among bar heights deviating substantially from a triangular distribution. In Fig. 4.5, however, the bars converge to the triangular distribution shape. Therefore, it should be realized that bar charts with larger interval lengths can represent the desired distribution with smaller sample sizes.

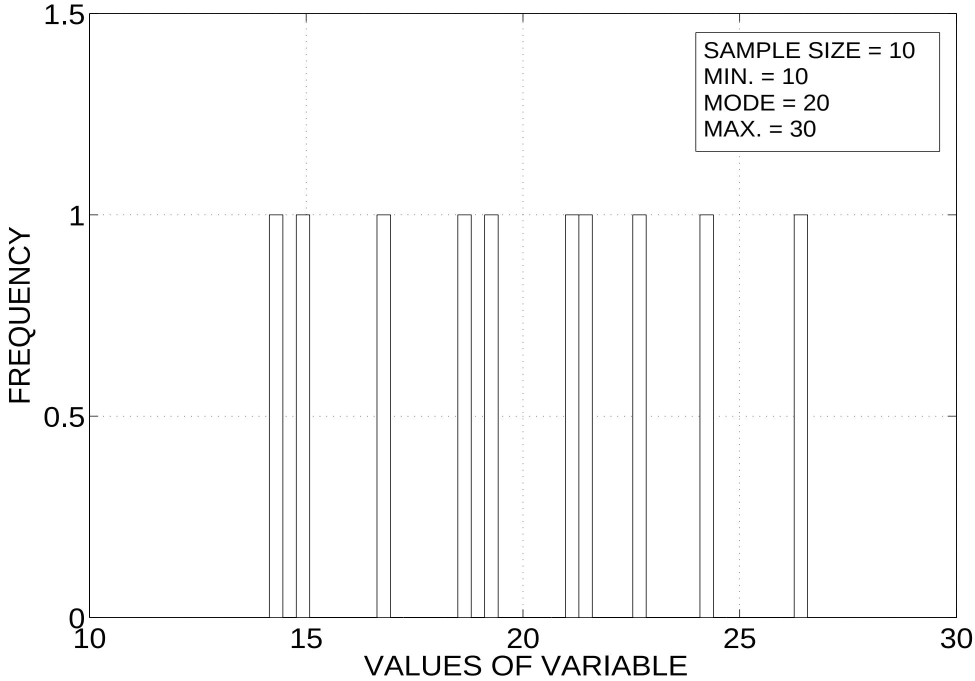


Figure 4.2: Latin Hypercube Sampling with a sample size of 10 and 60 intervals

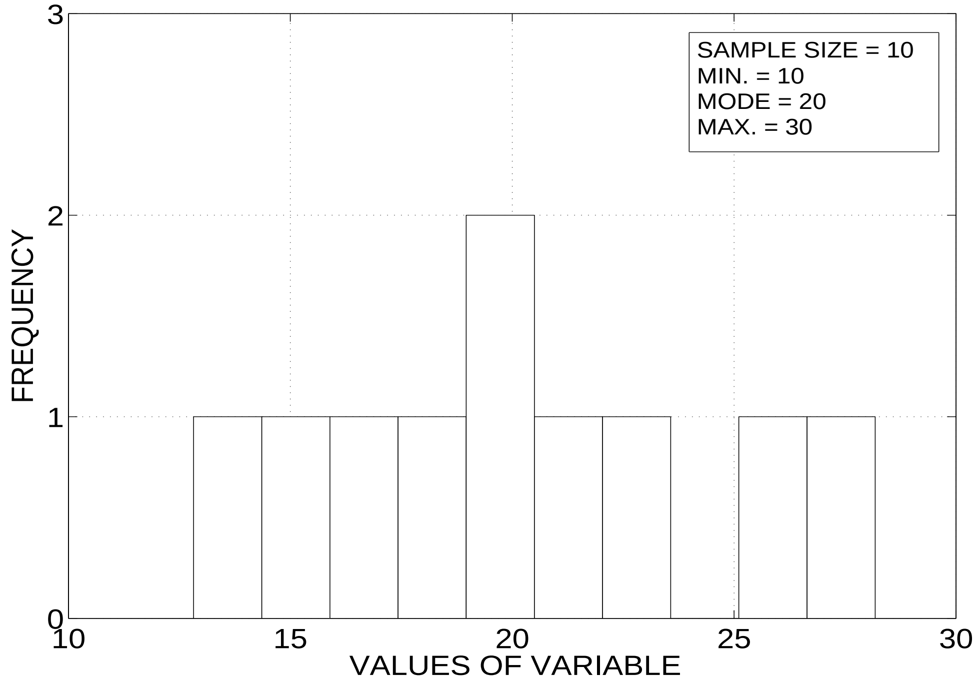


Figure 4.3: Latin Hypercube Sampling with a sample size of 10 and 13 intervals

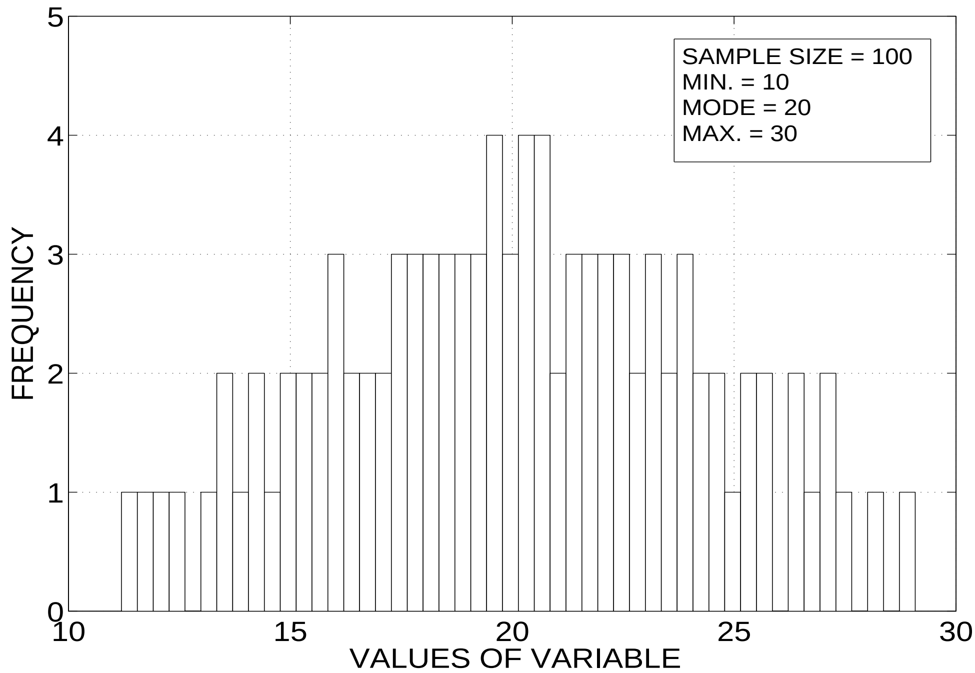


Figure 4.4: Latin Hypercube Sampling with a sample size of 100 and 60 intervals

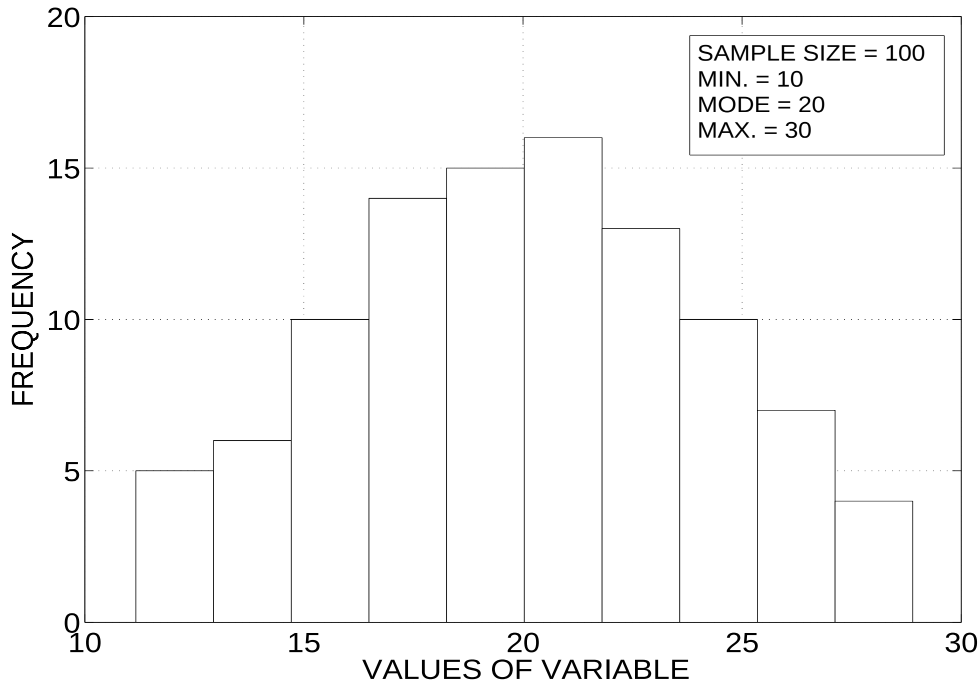


Figure 4.5: Latin Hypercube Sampling with a sample size of 100 and 11 intervals

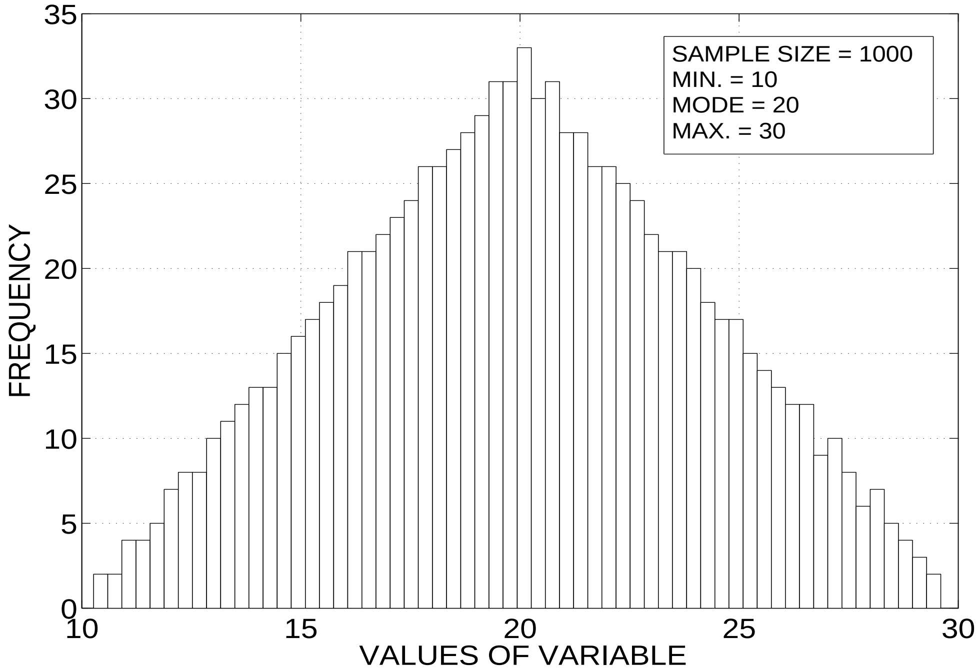


Figure 4.6: Latin Hypercube Sampling with a sample size of 1000

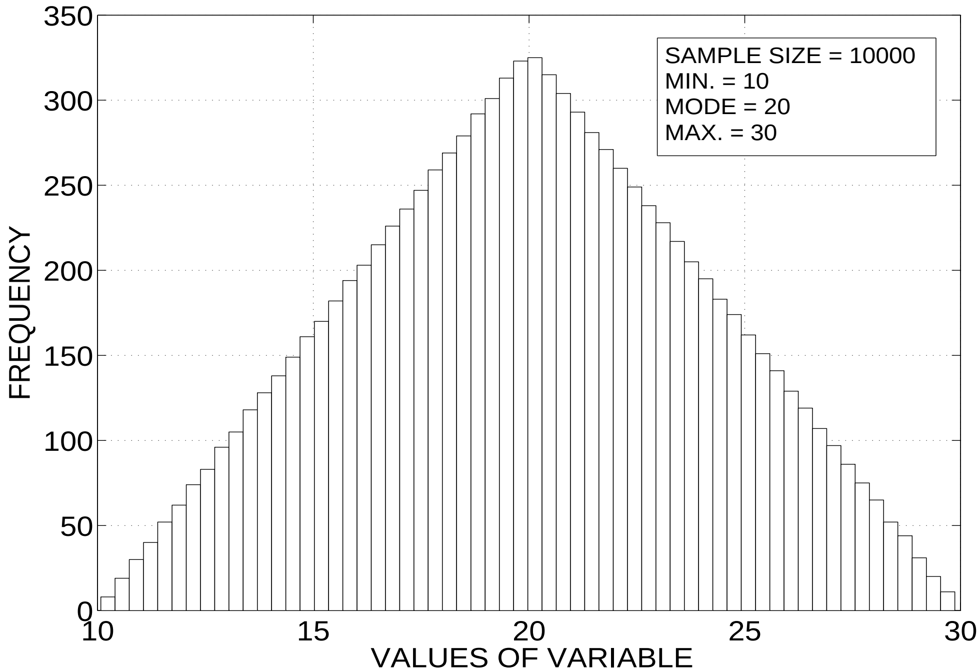


Figure 4.7: Latin Hypercube Sampling with 10000 Sample Size

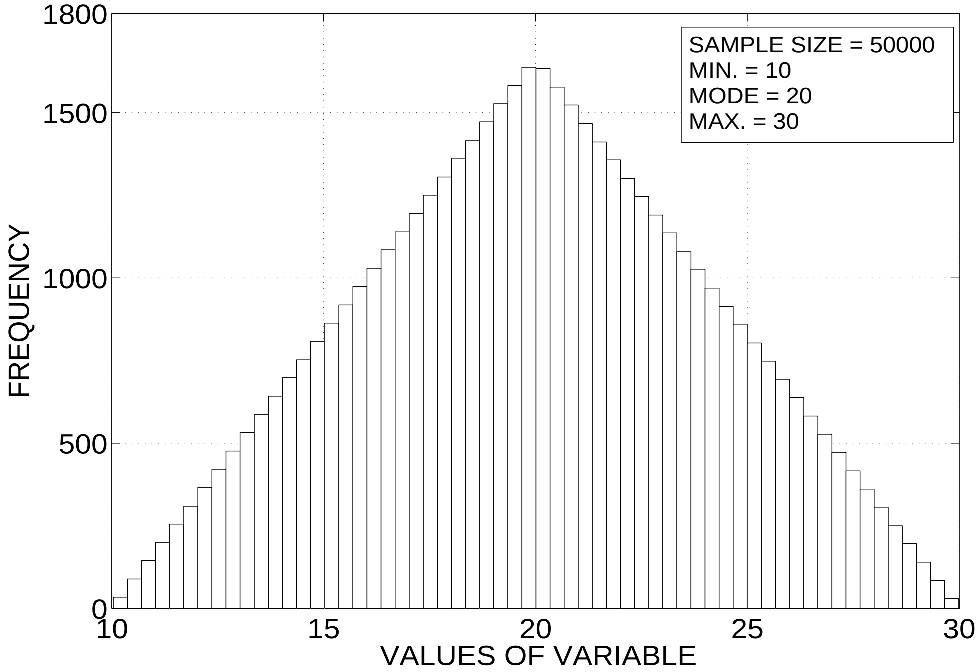


Figure 4.8: Latin Hypercube Sampling with a sample size of 50000

4.2 Maintenance Actions for Bridges

Maintenance is any activity applied to a structural system other than the new construction. The objective of all maintenance actions is to keep the structural system in good condition throughout its lifetime. Cleaning, painting as well as repairs and replacements of components are examples of maintenance actions for the bridges. The main actions of bridge maintenance can be classified as cleaning, sealing, painting, coating, reseting, repairing, replacement, modification, rehabilitation and emergency maintenance actions. Specific maintenance actions that can be performed under these action categories may be grouped as follows [55].

1. Cleaning Actions:
    - Wash
    - Zone wash
    - Sweeping

- Flushing
- Removal of incompressible material
- Removal of vegetation
- Removal of material in channels
- Unclog cleanouts
- Clean Debris / drift
- Clean Graffiti

2. Sealing, Painting, Coating actions:

    - Spot clean
    - Partial or complete application of fluid sealers
    - Painting
    - Coating or Applying preservatives
    - Chemical treatments
    - Surface preparation

3. Reset actions:

    - Re-positioning
    - Lubrication
    - Tightening (of bolts and rods)
    - Other minor corrective actions
    - Consumables
    - Caulking
    - Resetting Gates or signals
    - Resetting Mechanical equipments
    - Resetting Electrical equipments

4. Repair actions return elements to better condition, or even to as-built condition

    - Patching

- Re-attach or Re-anchor
- Straightening
- Jacking or Aligning
- Reinforcing
- Dredging or Grading

5. Replacement actions :

    - Individual replacement
    - Section replacement
    - Complete replacement
    - Span replacement

6. Modification actions :

    - Modify Geometry
    - Modify Protection
    - Modify Vulnerability
    - Modify Strengthen capacity
    - Modify Function
    - Modify Assembly

7. Emergency maintenance actions are taken in response to sudden acute problems that must correct restore or continue traffic operations

    - Posting the bridge
    - Shoring the bridge
    - Closure, full
    - Closure, partial
    - Detour
    - Temporary bridge placement

Based on policies of developed countries that have well designed BMSs, maintenance actions may be defined more appropriately. In general, cleaning and minor repairing are always classified as maintenance. Also, repairs or replacements of components are usually categorized as maintenance actions. Moreover, some improvements obtained in small scaled projects might be considered as maintenance activity. However, improvement in large scaled project, bridge replacement and bridge reconstruction are never classified as maintenance.

Maintenance actions mentioned below and Table 4.2 and Table 4.3 are taken from Bridge Rehabilitation [56]. Existing bridge infrastructure needs maintenance and repair actions because aging leads to deterioration of bridge elements. In very general terms, according to [56], the following maintenance actions can be undertaken when a bridge is deteriorated.

- Repair
- Replacement
- Rehabilitation
- Strengthening
- Modernization

Repair, restores the defects on the structure and it generally deals with local damages of structural members than whole structures.

Replacement actions substitute or change the bridge members and equipment elements that need to be fixed. Equipment elements are expansion joints, bearings and barriers, deck elements, bracing elements are some structural members.

Rehabilitation actions restore the bridge structure. Rehabilitation is applied on the whole bridge structure.

The fourth action; i.e, strengthening, increases the load carrying capacity by adding new members or material.

Some actions upgrade the facilities, e.g., new traffic flow arrangement, new signs, new barriers. These actions are grouped under the Modernization.

Moreover, there exist an action type, named as retrofitting. Retrofit is applied to existing structure and is a strengthening procedure. In light of the newly gained experiences, retrofitting may be applied if it is found that initial design is not sufficient.

These five maintenance actions have many specific sub-actions for concrete and steel bridges. These actions and the material or structural members to which they can be applied are shown in Table 4.2 and Table 4.3 [56].

Table 4.2: General classification of standard repair techniques and materials applied to concrete bridge superstructures.

Type of work	**Material or structural member to be repaired**
Removal of deteriorated concrete	Concrete, All structural members
Corrosion removal	Reinforcing steel Strands Anchorages Steel bearings or joints Balustrades or other steel elements
Surface cleaning	Concrete Steel
Crack repair	Concrete
Bounding the repair materials	Concrete
Patching	Concrete
Replacement or addition of the reinforcement	Reinforcing steel
Reinforcement protection	Reinforcing steel
Applying the repair materials	Concrete
Surface coating and sealing	Concrete
Repair of collusion damage in structural members	Mostly reinforced or prestressed concrete beams or box girders

Table 4.3: General classification of standard repair techniques and materials applied to steel bridge superstructures

Type of work	**Structural members or their joints to be repaired**
Corrosion removal and surface cleaning	Any
Repair of deformed elements	Any
Removal of structural elements or some parts of them as well as removal of structural joints or some of their parts, e.g., welds or rivets with defects, cracked gusset plates, etc.	Any, if necessary
Strengthening of structural elements with reduced cross-sections by corrosion or the elements with other defects (e.g., fatigue cracks) or the elements weakened by plastic deformation	Any, if necessary
Strengthening of the the structure after its repairing	Mostly main structural elements, e.g., the girders Relatively seldom other elements, e.g., floor-beams
Installation of the new elements after removal of the existing ones	Any, if necessary and technically justified and possible
Anti-corrosion protection	Any steel elements and their joints

According to PONTIS Technical Manual [57], there are five condition (damage) states for a steel open girder that is painted. Also, there are some feasible actions for each condition state. Five condition states and suitable maintenance and repair actions that can be applied in each condition state are listed below.

1. There is no evidence of active corrosion and the paint system is sound and functioning as intended to protect the metal surface.

   - Do nothing or
   - Surface clean unit

2. There is little or no active corrosion.The paint system may be chalking, peeling, curling, or showing other early evidence of paint system distress but there is no exposure of metal.

   - Do nothing or
   - Surface clean unit or
   - Surface clean and restore top coat of unit

3. Surface or fracled rust has formed or is forming. The paint system is no longer effective. There may be exposed metal but there is no active corrosion which is causing loss of section

   - Do nothing or
   - Spot blast, clean and paint unit

4. The paint system has failed. Surface pitting may be present but any section loss due to active corrosion does not yet warrant structural analysis of either the element or the bridge.

   - Do nothing or
   - Spot blast, clean and paint unit or
   - Replace paint system on unit

5. Corrosion has caused section loose and is sufficient to warrant structural analysis to ascertain the impact on the ultimate strength and/or serviceability of either the element or the bridge.

- Do nothing or
- Major replacement unit or
- Replace unit

According to Neves and Frangopol [51], there are four different condition levels for structures. In the light of the inspection, it is decided which maintenance actions are to be applied to the structure according to its condition level. Also, a safety index is included in their study.

In [51], maintenance strategy is composed of three maintenance action types. These are: No maintenance, Preventive maintenance and Essential maintenance actions. Furthermore, depending on the time of application of the maintenance action, the maintenance actions are classified as follows.

1. Time-based maintenance actions
   - Silane treatment
   - Replacement of expansion joints
2. Performance-based maintenance actions
   - Minor concrete repairs
   - Do nothing and rebuild
3. Time-and Performance-based maintenance actions
   - Cathodic protection

Preventive maintenance actions are usually time-based maintenance actions because they are applied independently of the performance of the structure.

If performance of the structure is at or below an acceptable threshold level, essential maintenance actions are applied. Therefore, essential maintenance actions are usually performance-based maintenance actions.

Maintenance and Repair actions taken from [51], [55], [56] are grouped in Table 4.4.

Table 4.4: A comparison and matching of Maintenance Actions

Bridge Rehabilitation	**National Database System for Maintenance Actions on Highway Bridges**	**Neves and Frangopol**
Removal of deteriorated concrete	Replace	Minor concrete repair
Corrosion removal, Surface coating and sealing	Coat / Paint	Cathodic protection
Surface cleaning	Clean / Clear	Silane treatment
Crack repair, Patching, Bonding the repair material, Replacement or addition of the reinforcement, Applying the repair materials	Repair	Replacement of expansion joints
Reinforcement protection	Modify	Rebuild
Repair of collusion damage in structural members	Emergency	Rebuild

4.3 Bi-Linear Performance Model

The lifetime deterioration of infrastructure systems may be predicted using condition profiles. The profiles can be obtained using different models, such as Markovian Process, Polynomial functions and Regression Models. The prediction model described in this section combines the condition and safety assessment in order to determine the condition and safety profiles considering maintenance actions. The model is developed by Neves and Frangopol [19]. In this study, the model is re-created by developing a computer program. The objective of the program development is to verify the developed simulation program by comparing the results from Neves [19] and then to further develop the model taking into account the regression modeling and Markov Process. The performance prediction model has many variables similar to other prediction models. Some of the variables are initial condition index, deterioration rate, and deterioration initiation time. If maintenance or repair actions are applied to the system, new variables are introduced to predict the performance profiles of the systems, too. Fig. 4.9 shows the performance indicator versus time graph when two maintenance actions are applied throughout the lifetime of the system [19]. Variables shown in Fig. 4.9 are described in Table 4.5. All random variables are assumed to have triangular distribution [19].

A computer program is developed as part of this thesis in order to have a simulation program in hand to be used for simulation-based bridge performance prediction. As an initial study, the developed program is first used to verify some of the results reported in [19].

The developed program named as *lhs_csc.m* obtains the condition, safety, and cost profiles by using the equations given between Equation 4.3 and Equation 4.12. This developed computer program generates the random variables using Latin Hypercube sampling method with *latin_hs_tri.m*. Flowchart of the *lhs_csc.m* is presented in Figure 4.10. First, this program reads the random variables from an input file. All random variables related with condition, safety and cost profiles are described in this input file. The described random variables in the input file are presented in Table 4.5. Second, the program performs simulation for each random variable according to their characteristic parameters. Third, time loop starts from $t = 0$ as the construction year and maintenance type is decided to be applied according to the random variables. The calculation part is included in the time loop. In addition, analysis is conducted for every year. The values of performance profiles and maintenance cost are calculated for each year throughout the lifetime. Maintenance action is applied if application of any action is necessary at a certain instant. The application time is stored and effects of maintenance actions are implemented into the profile and then new profile values are calculated as a next step in the calculation. Thereafter, it is determined whether the time loop comes to the end of the time horizon. If time loop continuous, calculations are conducted for the next year. Otherwise, simulation number and number of simulation variables are controlled whether the program comes to the end. If simulation number is still smaller than the number of simulations, program goes to the start of the simulation loop and then generates new values for the random variables. Otherwise, the *lhs_csc.m* is accomplished. Finally, condition, safety, and cost profiles are composed of the average values of condition, safety and cost variables obtained from simulation based computer program.

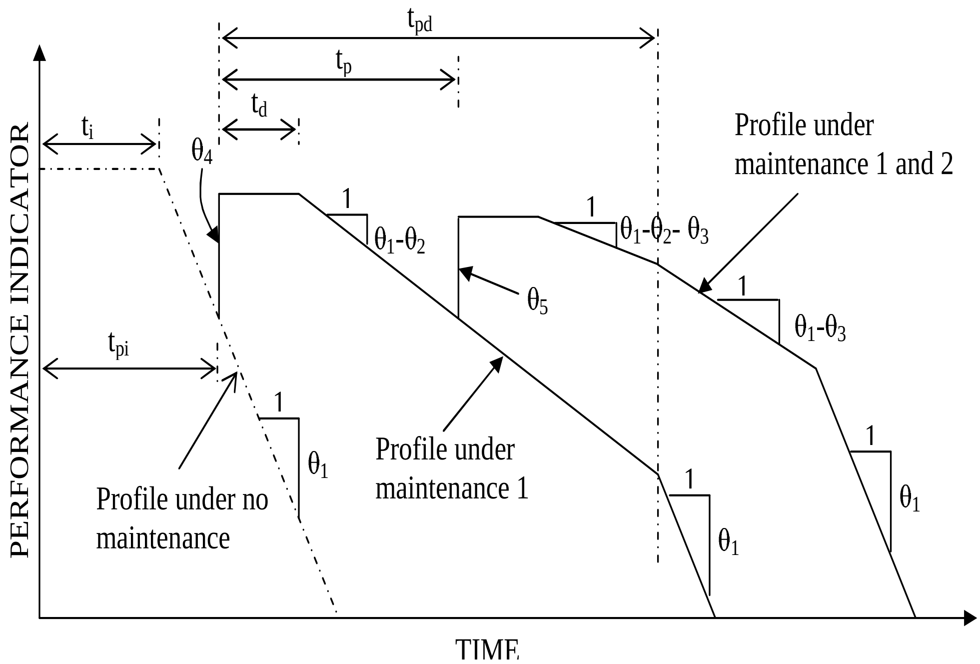


Figure 4.9: Superposition of the effects of two maintenance actions in a performance prediction model

Table 4.5: Description of Random Variables Used in Figure 4.9

Random Variable	**Description**
θ_1	Deterioration rate of condition index
θ_2	Change in deterioration rate due to first maintenance action
θ_3	Change in deterioration rate due to second maintenance action
θ_4	Increase in performance indicator due to first maintenance action
θ_5	Increase in performance indicator due to second maintenance action
t_i	Time of initiation of deterioration of performance indicator
t_{pi}	Time of first application of maintenance action
t_p	Time of subsequent application of maintenance action
t_d	Time during which the deterioration effect on performance indicator is suppressed
t_{pd}	Time during which the deterioration effect on performance indicator is suppressed or reduced

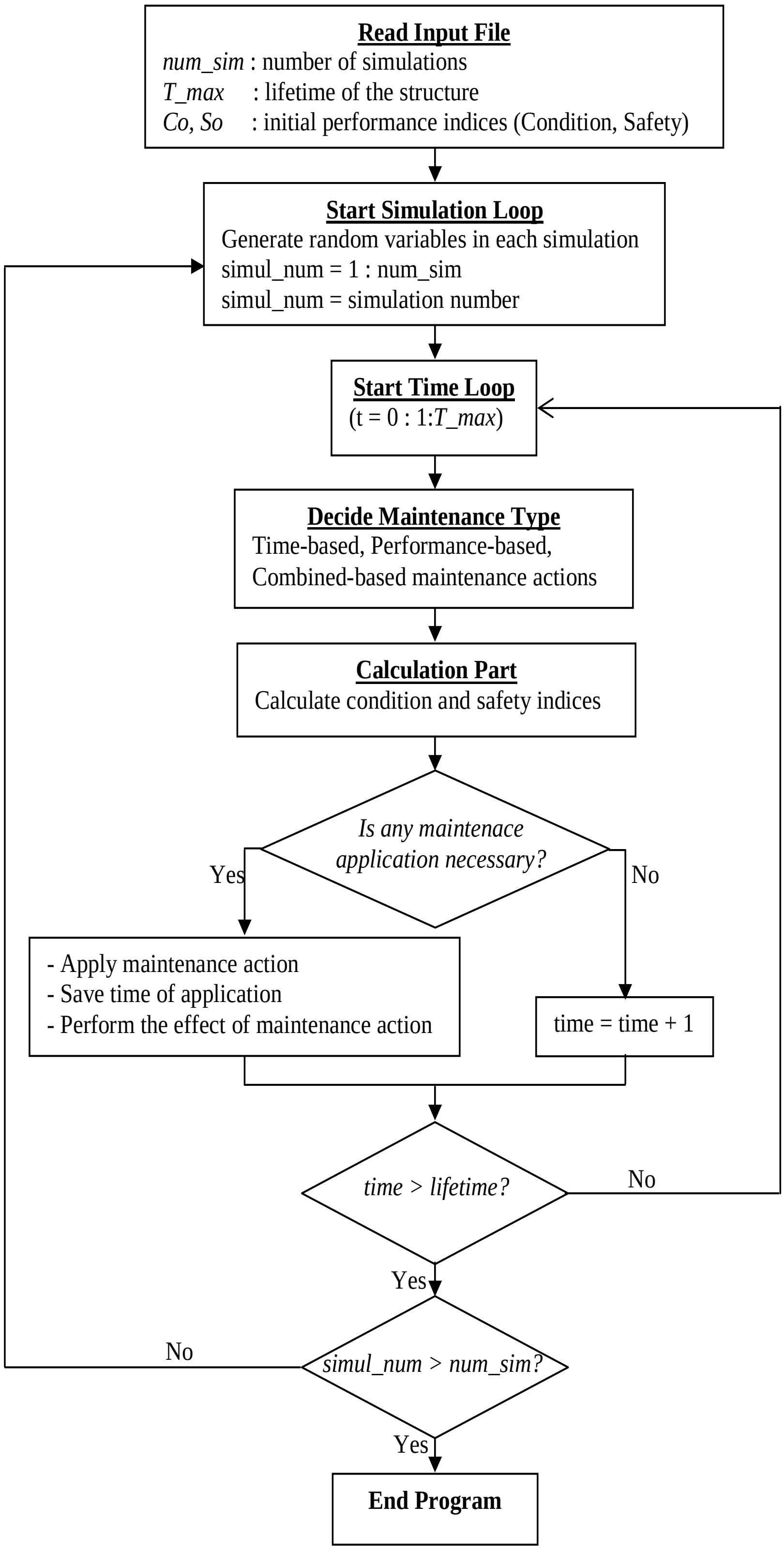


Figure 4.10: Flowchart of *lhs_csc.m* program

4.3.1 Condition Profiles

Condition index is a kind of performance indicator. It is obtained by visual inspection. In addition, condition index is an important indicator to see the bridge deterioration without any mechanical test. Moreover, Bilinear model presents condition prediction formula for infrastructure system throughout the lifetime.

Equation from 4.3 to 4.6 are used to compose the condition profiles under no maintenance case. Figures from 4.11 to 4.19 are obtained from formulas used for no maintenance case which are presented below.

$$\epsilon_T = \theta \cdot t_{det} \tag{4.3}$$

$$t_{zero} = \min(t_i; T) - T + 1 \geq 0 \tag{4.4}$$

$$t_{zero} + t_{det} = 1 \text{ year} \tag{4.5}$$

$$C_T = \begin{cases} C_0, & \text{if } T = 0 \\ C_{T-1} - \epsilon_T, & \text{if } T \geq 0 \end{cases} \tag{4.6}$$

Definitions of the random variables used in the above formulations are given in Table 4.6.

The condition profiles under the maintenance action is obtained by formulas given in Eqs 4.7 through 4.12.

$$\epsilon_T = (\theta_1 - \theta_2) \cdot t_{reduced} + \theta_1 \cdot t_{no} \tag{4.7}$$

$$t_{zero} = \min(t_d; \tau) - \max(0; \tau - 1) \geq 0 \tag{4.8}$$

Table 4.6: Random variables used in condition index formulation under No Maintenance Case

Random Variable	Description
ϵ_T	Deterioration rate of condition index under no maintenance during the specified one year time interval
t_{zero}	Fractions of the year during which there is no deterioration of condition under no maintenance
t_{det}	Fractions of the year during which there is deterioration of condition under no maintenance
t_i	The initiation time of deterioration of condition
T	Time
C_0	Initial condition index
C_T	Condition index at time T

$$t_{effect} = \min(t_{pd}; \tau) - \max(0; \tau - 1) \geq 0 \tag{4.9}$$

$$t_{reduced} = \min(1; t_{effect} - t_{zero}) \tag{4.10}$$

$$t_{zero} + t_{reduced} + t_{no} = 1 \text{ year} \tag{4.11}$$

$$C_T = \begin{cases} C_{T-1} - \epsilon_T, & \text{if } \tau \geq 1 \, year \\ C_{T-1} - \epsilon_T + \theta_4, & \text{if } 0 \leq \tau < 1 \, year \end{cases} \tag{4.12}$$

The variables in Eqs. 4.7 through 4.12 are defined in Table 4.7.

Condition profiles can be obtained based on given different values of the random variables including initial condition index C_0, deterioration initiation time T_i, and deterioration rate θ_1. The formulas presented above are used to develop the simulation-based performance prediction program. Flowchart of computer program's algorithm is presented in Fig. 4.10. The condition profiles and values of descriptors of the three random variables are presented Figs. 4.11 through 4.20. Ten different profiles are obtained using different values for the random variables. Nine of these figures are

Table 4.7: Random variables used in condition index formulation under Maintenance Case

Random Variable	Description
ϵ_T	Equivalent rate of condition deterioration during one year interval
t_{no}	Fraction of the year during which there is no effect of the maintenance on condition
t_{zero}	Fraction of the year during which there is no deterioration of condition due to the effect of maintenance action
t_{effect}	Fraction of the year during which maintenance action reduces or suppresses the deterioration of the condition
$t_{reduced}$	Fraction of the year during which the deterioration rate of condition is reduced due to the maintenance action
τ	The time elapsed since the maintenance is applied

obtained for under no maintenance case and are grouped into three subgroups based on the variables values. For every group, the values are changed for only one variable and other two variables are kept constant. These Figures show the effects of the random variables on the condition index profile. All used variables to achieve these figures have triangular distribution.

The graphs contain the descriptor values the random variables which define the shape of the condition index profile. The random variables, initial condition index (C_0), deterioration initiation time (T_i), deterioration rate (θ_1), and the number of simulations are displayed in all the figures. The number of simulation used is 100000. In all graphs, the probability density functions (PDFs) of the condition index distribution are computed using Latin Hypercube Sampling and are displayed at discrete points in time. Mean value and standard deviation profiles of the condition index are shown separately in the graphs.

The first three graphs, Fig. 4.11, Fig. 4.12 and Fig. 4.13, show the effect of deterioration rate θ_1 on condition index C. Initial condition index C_0 and deterioration initiation time T_i are kept at constant values, and the value of deterioration rate is changed. Minimum, mode, and maximum values of initial condition index C_0 are de-

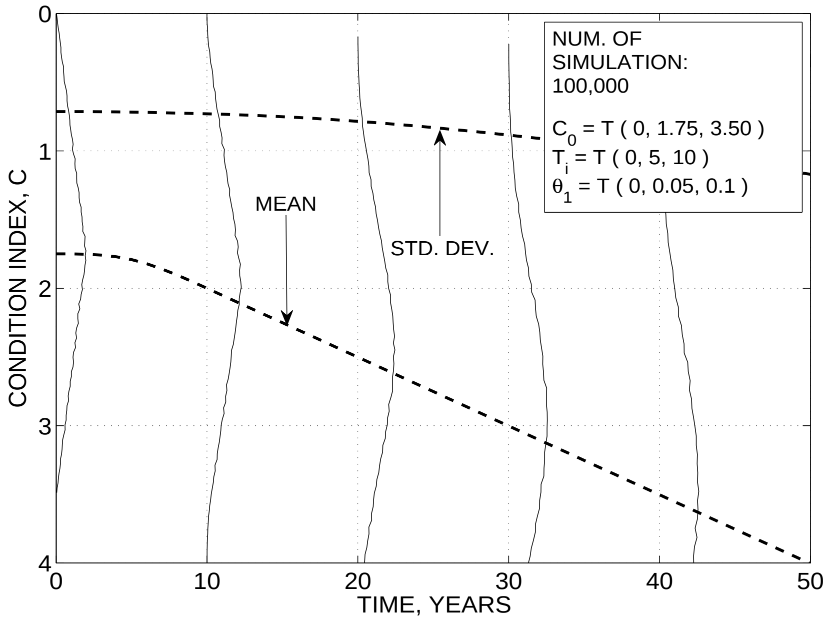


Figure 4.11: Effect of Deterioration Rate on Condition Index Profile under No Maintenance Case for θ_1 = T (0, 0.05, 0.1)

fined as 0, 1.75 and 3.50, respectively and 0, 5 and 10 years are used as the minimum, mode and maximum values of deterioration time initiation T_i.

For Fig. 4.11, distribution values of deterioration rate (θ_1) are used as 0, 0.05, and 0.1. As shown in the figure, value of the mean initial condition index is around 1.75 and during the first 5 year, no deterioration is observed. Afterwards, mean condition index increases gradually (i.e, downward), and violates the condition threshold (C_{target} = 3.0) by the end of 30 years if a condition threshold value is assumed to be 3. Finally, the mean condition index reaches 4 at 50 years. Variation of the standard deviation of the condition index throughout the analysis remains almost at the same value. The standard deviation is observed as 0.7 at the beginning time and reaches 1.2 at the end of 50 years.

Fig. 4.13 is the last figure of the first group of three graphs. For this profile, the minimum, mode and maximum values of deterioration rate (θ_1) are 0, 0.15, 0.3, respectively. Initial condition index C_0 and deterioration initiation time T_i have the same values as the first two analyses. As shown in Fig. 4.13, deterioration rate values

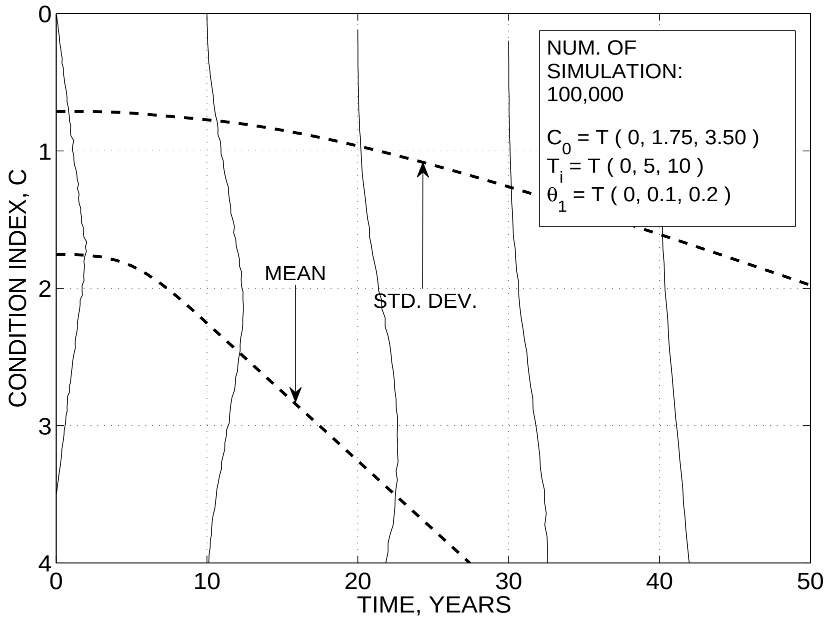


Figure 4.12: Effect of Deterioration Rate on Condition Index Profile Under No Maintenance Case for θ_1 = T (0, 0.1, 0.2)

for this analysis are higher than the other two analyses. Therefore, the higher value of the deterioration rate leads the condition index to reach $C_{target} = 3.0$ very early and causes a larger variation of standard deviation. Mean condition index reaches threshold value by the end of 13 years and standard deviation of the condition index 3 at the end of 50 years.

Consequently, it can be observed that deterioration rate (θ_1) has an important effect on the condition index profile. When deterioration rate changes, the standard deviation and the mean of condition index present significant variations. If deterioration rate increases, mean and standard deviation of condition index increase, too.

Condition index of a bridge component may not be exactly known when the bridge is constructed. However, initial condition index distribution should be known to examine the condition profiles. For this reason, the initial condition index is a random variable. Figs. 4.14, 4.15 and 4.16 illustrate the effect of initial condition index on the condition profile. All random variables except initial condition index have the same distributions values for these three profiles. Minimum, mode, and maximum values

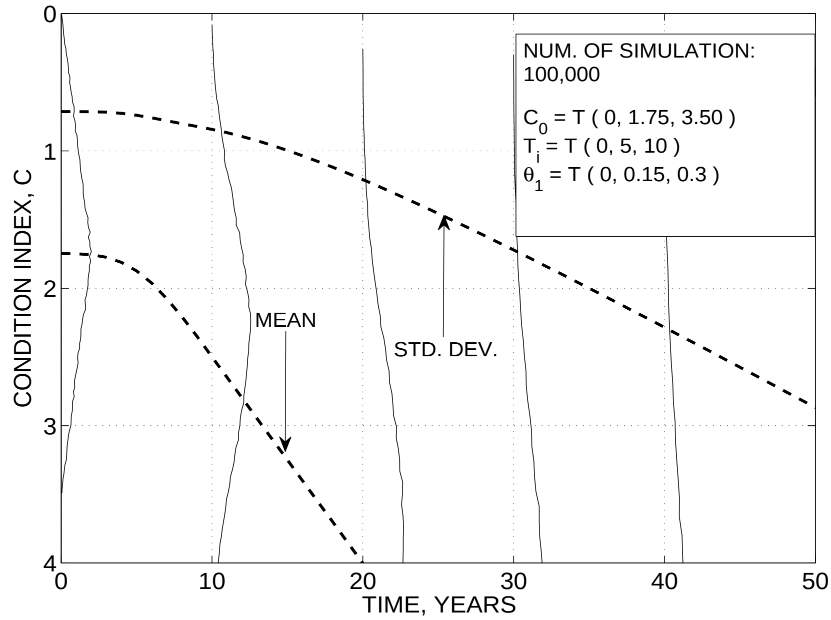


Figure 4.13: Effect of Deterioration Rate on Condition Index Profile Under No Maintenance Case for θ_1 = T (0, 0.15, 0.3)

of deterioration rate θ_1 are 0, 0.08 and 0.16, respectively and 0, 5 and 10 years are the minimum, mode and maximum values of deterioration time initiation T_i.

As shown from the Fig. 4.14, 0, 0.5, and 1 are the minimum, mode and maximum values of initial condition index for the first profile. For this profile, mean condition index is higher than condition threshold $C_{target} = 3.0$ until 37 years. At the end of 48 years, mean of the condition index reaches 4. On the other hand, standard deviation of the condition index is 0.2 at the beginning of analysis time and 1.5 at the end of the lifetime.

Figure 4.15 shows the effect of initial condition index on condition profile when minimum, mode, and maximum values of initial condition index are 0, 1 and 2, respectively. Mean of the condition index crosses the condition threshold $C_{target} = 3.0$ after 30 years because of having larger value of initial condition index than that in Fig. 4.14. Changes in standard deviation of the condition index due to variation of initial condition index is very small. Standard deviation of the condition index is 0.3 at the beginning and it reaches 1.5 at the end of the time horizon.

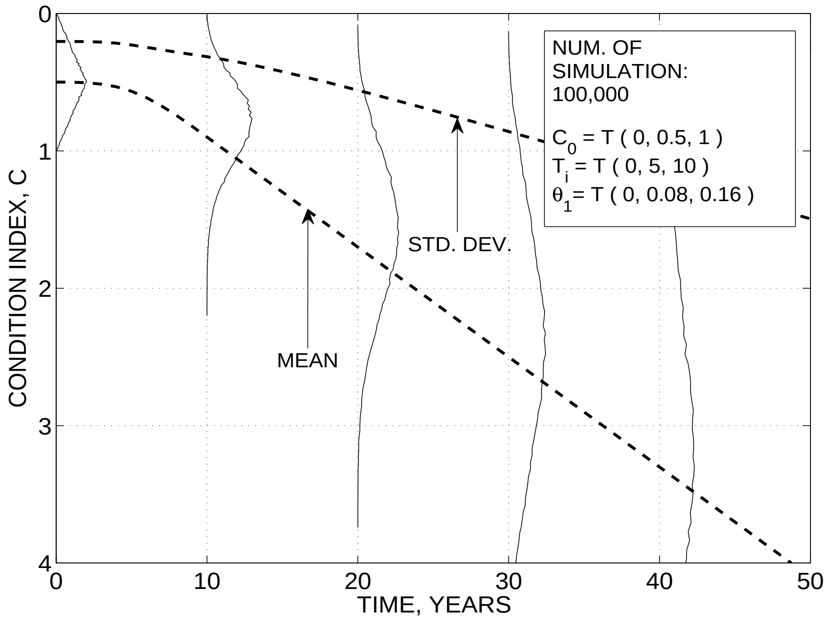


Figure 4.14: Effect of Initial Condition Index on Condition Index Profile under No Maintenance Case for C_0 = T (0, 0.5, 1)

If triangular distribution of initial condition index is $C_0 = T(0, 1.5, 3)$ as shown in Fig. 4.16, mean condition index crosses the threshold very early as shown in the Fig. 4.16. Condition index reaches 3 at the end of the 24 years. Standard deviation behaves in the same manner as in Figure 4.14 and Figure 4.15.

As stated in [19], it is verified that the initial condition index C_0 has an important effect on the condition index profile. The higher initial condition index values cause condition index to cross the threshold sooner. Value of the initial condition index has no remarkable effect on the standard deviation of the condition index.

Onset of deterioration of in a bridge member may not be known exactly. Therefore, deterioration initiation time is modeled as random variable when predicting the lifetime of a deteriorating system more accurately. The next three figures of the condition profile, i.e, Figs. 4.17, Fig. 4.18 and Fig. 4.19, show the effect of deterioration initiation time T_i on the condition profile. Hence, all variables except deterioration initiation time have constant values for the three figures. Minimum, mode, and maximum values of deterioration rate θ_1 are 0, 0.08 and 0.16, respectively. Besides, 0,

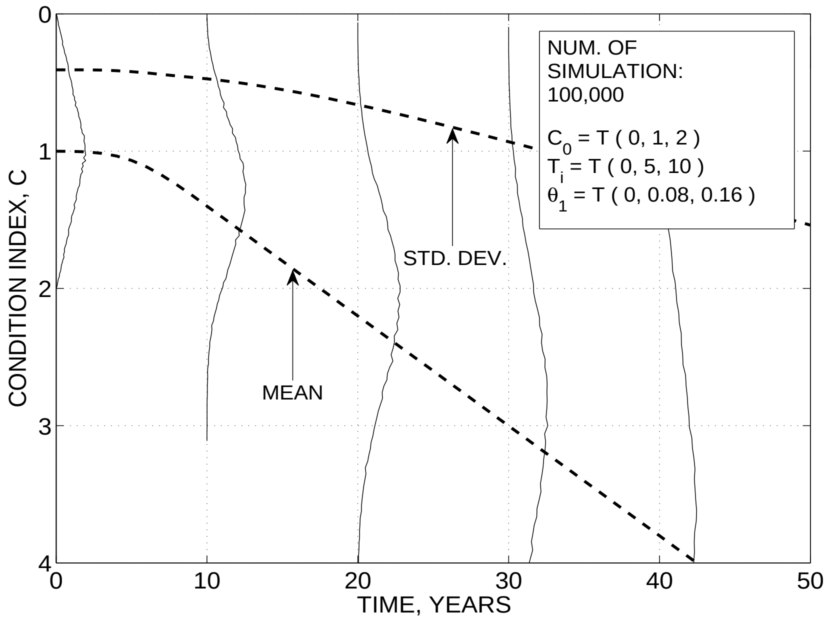


Figure 4.15: Effect of Initial Condition Index on Condition Index Profile under No Maintenance Case for C_0 = T (0, 1, 2)

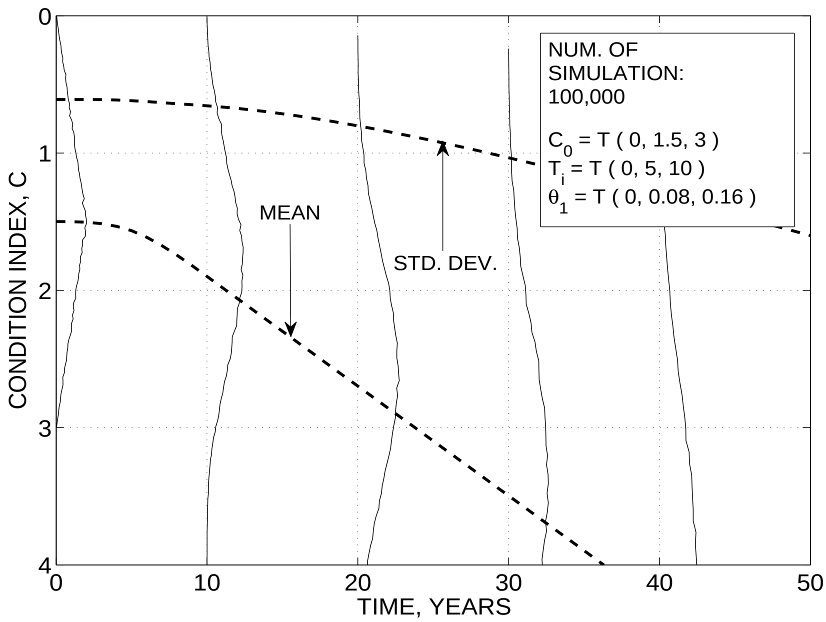


Figure 4.16: Effect of Initial Condition Index on Condition Index Profile Under No Maintenance for C_0 = T (0, 1.5, 3)

1.75 and 3.50 are the minimum, mode and maximum values of initial condition index C_0.

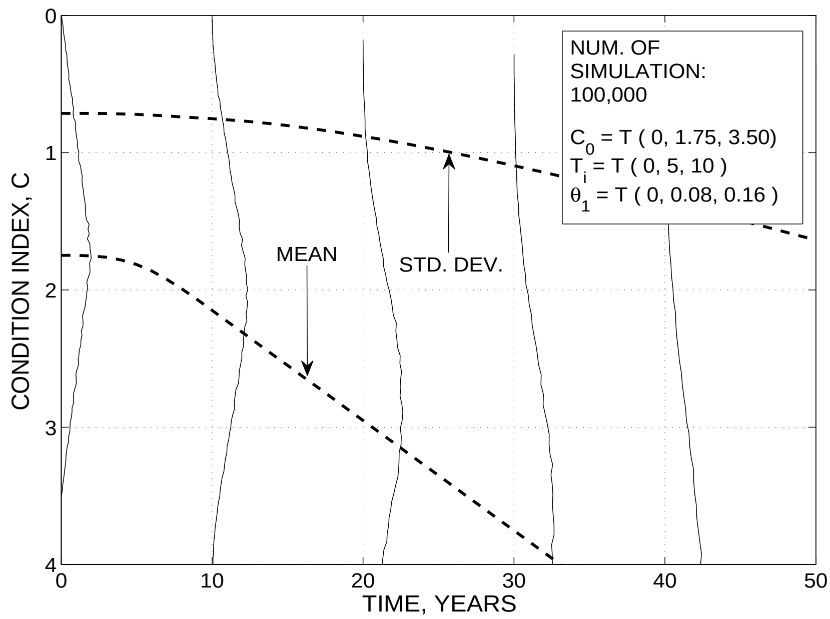


Figure 4.17: Effect of Deterioration Initiation Time on Condition Index Profile under No Maintenance Case for T_i = T (0, 5, 10)

In Fig. 4.17, the system starts deteriorating approximately 5 years later after construction and then condition index continues to increase (downward) linearly. Mean condition index reaches the threshold level at the end of 20 years and standard deviation while 0.8 at the beginning of the analysis, it reaches 1.7 at the end of 50 years.

Fig. 4.18 is generated with new distribution values of deterioration initiation time. i.e; T(10, 15, 20). This increment causes the deterioration to start later and the lifetime to be longer.Deterioration starts approximately between 10 and 20 years after the construction. Consequently, condition index reaches the condition threshold at 30 years.

Fig. 4.19 is the last profile showing the effect of deterioration initiation time on condition index. In this figure, characteristic values of deterioration initiation time distribution are 20, 25 and 30. As shown in the Fig. 4.19, deterioration starts between

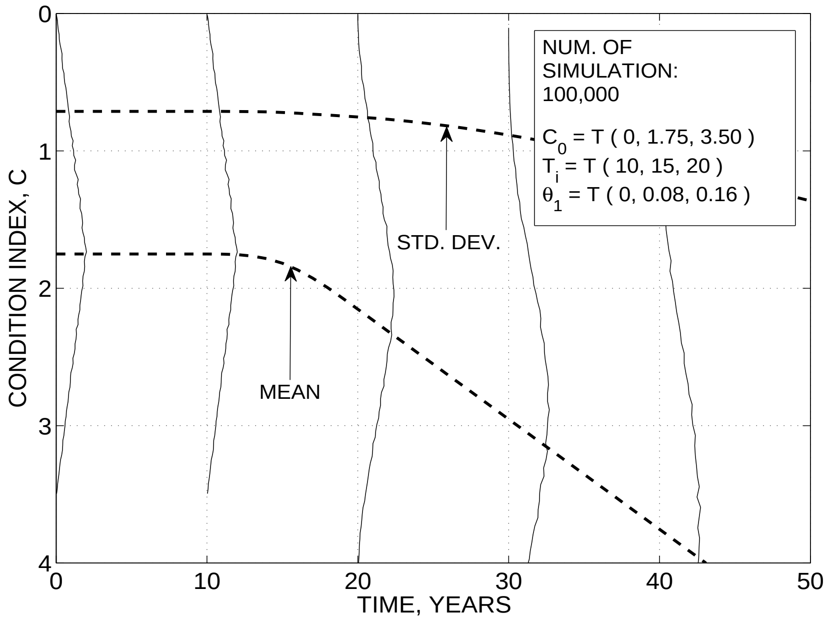


Figure 4.18: Effect of Deterioration Initiation Time on Condition Index Profile under No Maintenance Case for T_i = T (10, 15, 20)

20 and 30 years and this causes lifetime to be longer than the profiles in Fig. 4.17 and Fig. 4.18. Mean condition index reaches the threshold level at 40 years and the variation on standard deviation becomes smaller.

Deterioration initiation time has a significant effect on condition index profile and it effects the lifetime of the bridge. The later the deterioration onset starts, the longer the lifetime of the bridge becomes. In addition, there is a inverse proportion between deterioration initiation time and standard deviation of condition index.

As the next step, maintenance scenarios are applied for a deteriorating bridge using the deterioration model developed by Neves and Frangopol [19]. No maintenance case, silane treatment, replacement of expansion joints, minor concrete repairs, do nothing and rebuild, and cathodic protection actions are applied as maintenance and repair actions. Performance (condition) profiles are obtained for each maintenance scenario. The data is obtained from the model presented by Neves and Frangopol [19]. Condition profiles presented between Figure 4.20 and Figure 4.25 are obtained from the random variables defined in the Table 4.8. All random variables for the

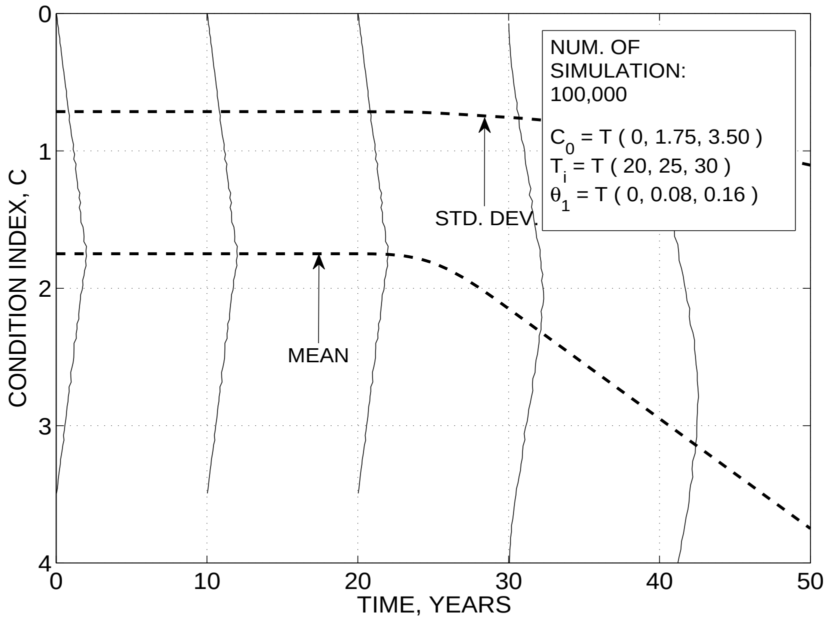


Figure 4.19: Effect of Deterioration Initiation Time on Condition Index Profile under No Maintenance Case. for T_i = T (20, 25, 30)

maintenance scenario are generated using the Latin Hypercube sampling method and the number of simulations is selected as 1000.

Fig. 4.20 is condition profile under no maintenance case. As shown in this figure, the bridge is assumed to be subjected to deterioration since $t = 0$. Performance curve gradually decreases over years with condition index value increasing. Performance curve in 4.20 is linear because the selected maintenance type is no maintenance case. For no maintenance case, condition index increases very rapidly and reaches its critic level between 15 and 20 years.

The profiles in Fig. 4.21, 4.22, 4.23, 4.24, and 4.25 are obtained using data defined in Table 4.8.

The profiles in Figure 4.21 is obtained when silane treatment maintenance activity is applied to the bridge infrastructure. Silane treatment is a preventive maintenance activity and only leads to reduction of deterioration rate. Hence, this action only extends the lifetime of the bridge. Silane treatment has no improvement effect on the

Table 4.8: Data for Condition Index with Maintenance

	Silane Treatment	Replace Expansion Joints	Cathotic Protection	Minor Concrete Repair	Do nothing and Rebuild
t_{pi}	T(0, 7.5, 15)	T(0, 20, 40)	when $C = 2$	when $C = 3$	when $S = 0.91$
t_p	T(0, 12.5, 15)	T(20, 30, 40)	T(7.5, 10, 12.5)	when $C = 3$	when $S = 0.91$
θ_4	0	0	0	T(2.0, 2.5, 3)	To 0
t_d	0	0	12.5	0	T(10, 15, 30)
$(\theta_1 - \theta_2)$	T(0.0, 0.01, 0.03)	T(0.0, 0.04, 0.08)	0	0	0
t_{pd}	T(7.5, 10, 12.5)	T(10, 15, 30)	12.5	0	T(10, 15, 30)

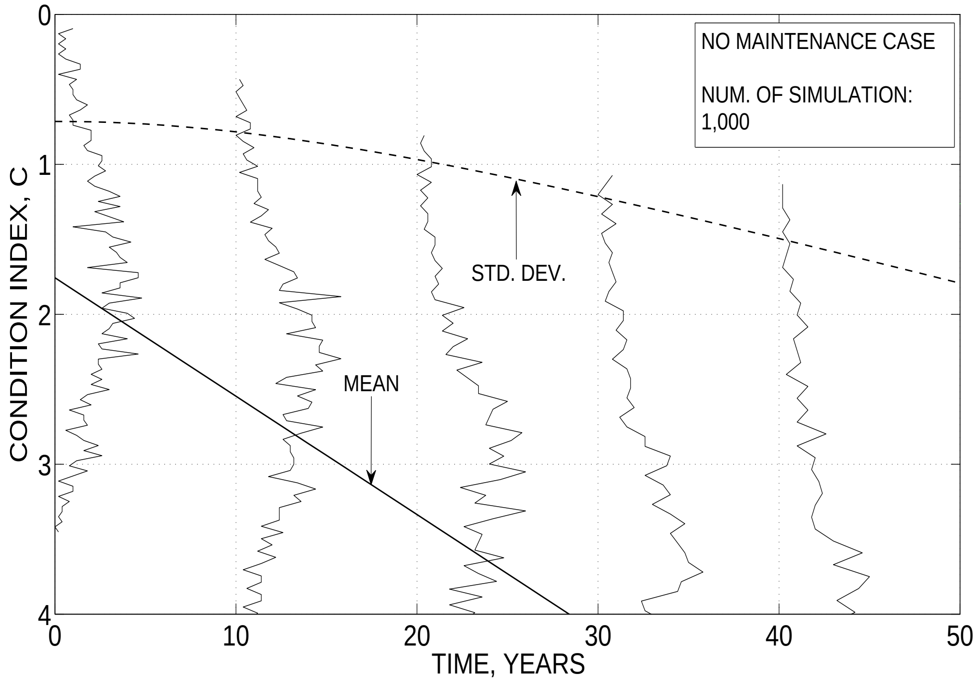


Figure 4.20: Condition Profile under No Maintenance Case.

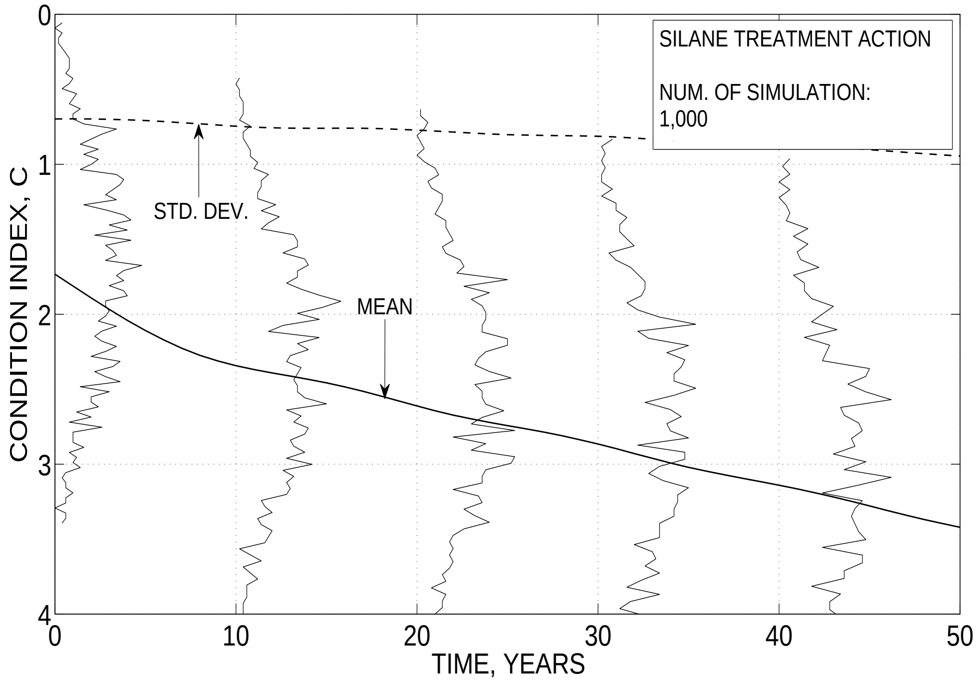


Figure 4.21: Condition Profile under Silane Treatment.

condition profile. As shown in Figure 4.21, no improvement of performance curve is achieved by silane treatment but condition index reaches at critical level between 30 and 40 years. Therefore, the lifetime of the structure extends if silane treatment is applied as maintenance action.

The profile in Fig. 4.22 is obtained by replacement of expansion joints which is a preventive maintenance actions and a time-based maintenance strategy. Therefore, this action has first and subsequent maintenance application times which are random variables. The only effect of this maintenance activity is the reduction of deterioration rate of the condition profile (performance curve). Replacement of expansion joints has no improvement effect on the condition profile.

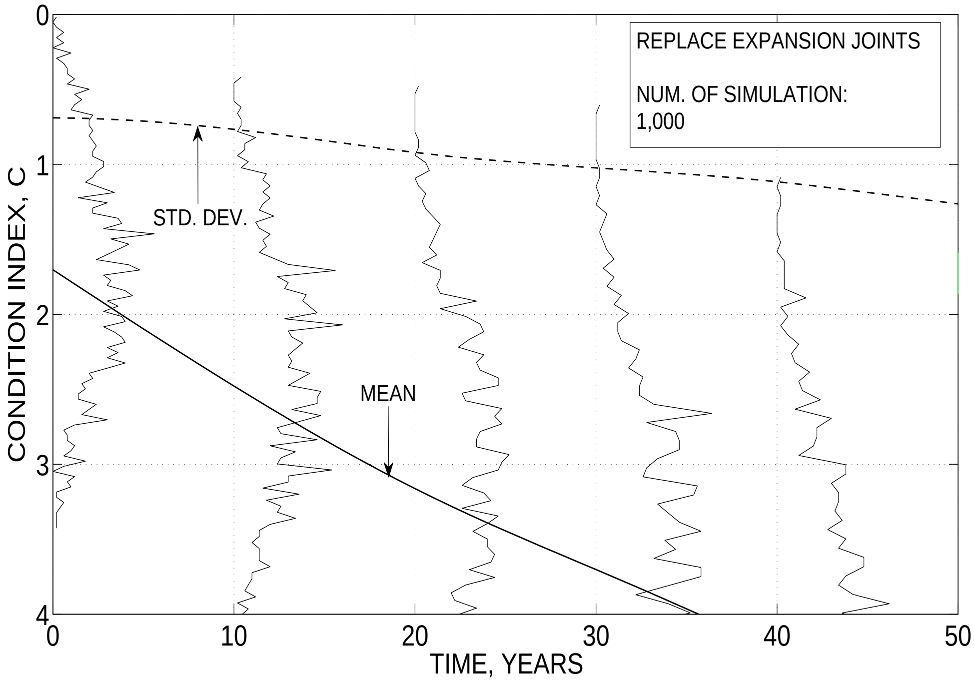


Figure 4.22: Condition Profile when Replacement of Expansion Joints action is applied.

The profile in Fig. 4.23 is obtained when minor concrete repair is applied to the bridge. Minor concrete repair is an essential maintenance action. First and subsequent maintenance actions are applied when condition index reaches the threshold level. Minor concrete repair has an important improvement effect on the condition profile. Minor concrete repair prevents the bridge condition from reaching the critical level throughout its lifetime.

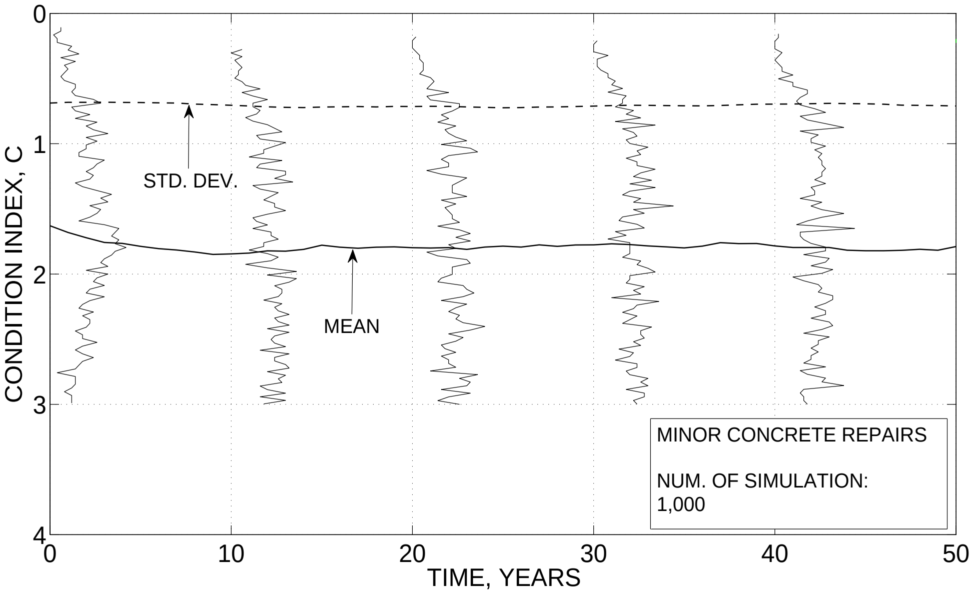


Figure 4.23: Condition Profile under Minor Concrete Repair action.

Do nothing and rebuild is an essential maintenance action and it is dependent on the safety index profile. Do nothing and rebuild applied when safety index reaches the threshold level. Neves and Frangopol [19], based on statistical data, assumed the threshold level for the safety index is 0.91. In other words, Neves and Frangopol [19] assumed the existence of a coupling between the condition and safety indices. As shown in Figure 4.24, condition (performance) of bridge gradually gets worse until the maintenance action is applied.

The profile in Fig. 4.25 is obtained by applying the cathodic protection maintenance to the structure. Cathodic protection is a time and performance-based maintenance action. First cathodic protection action is applied when condition index reaches the threshold level but subsequent actions are applied at subsequent application times. In [19], threshold level for the condition index for cathodic protection is selected as 2. Cathodic protection only delays the deterioration in time and it has no improvement effect on the condition index.

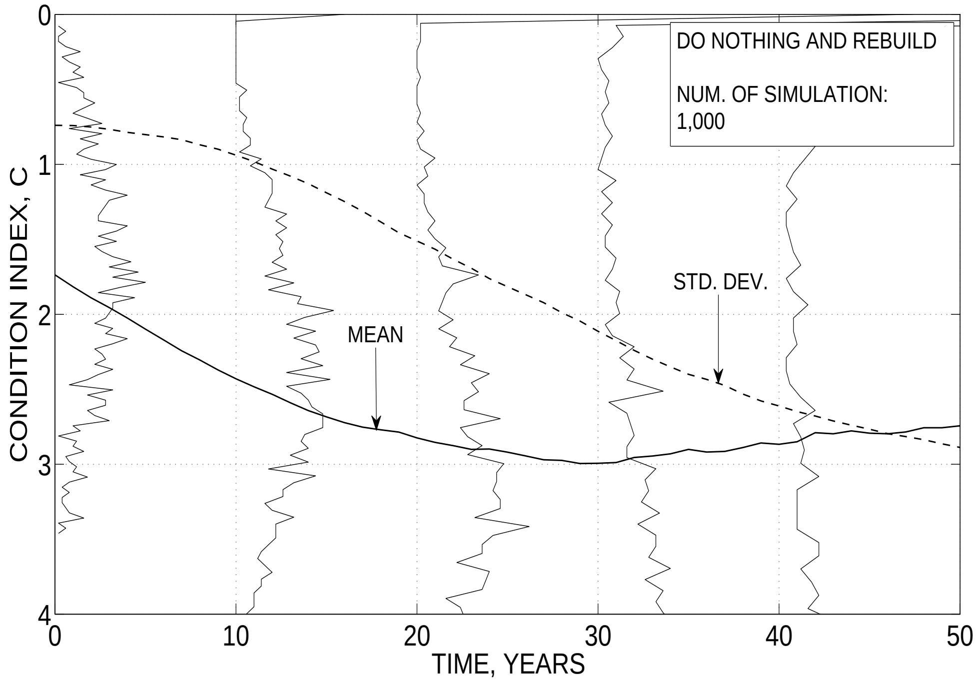


Figure 4.24: Condition Profile under Do Nothing and Rebuild action.

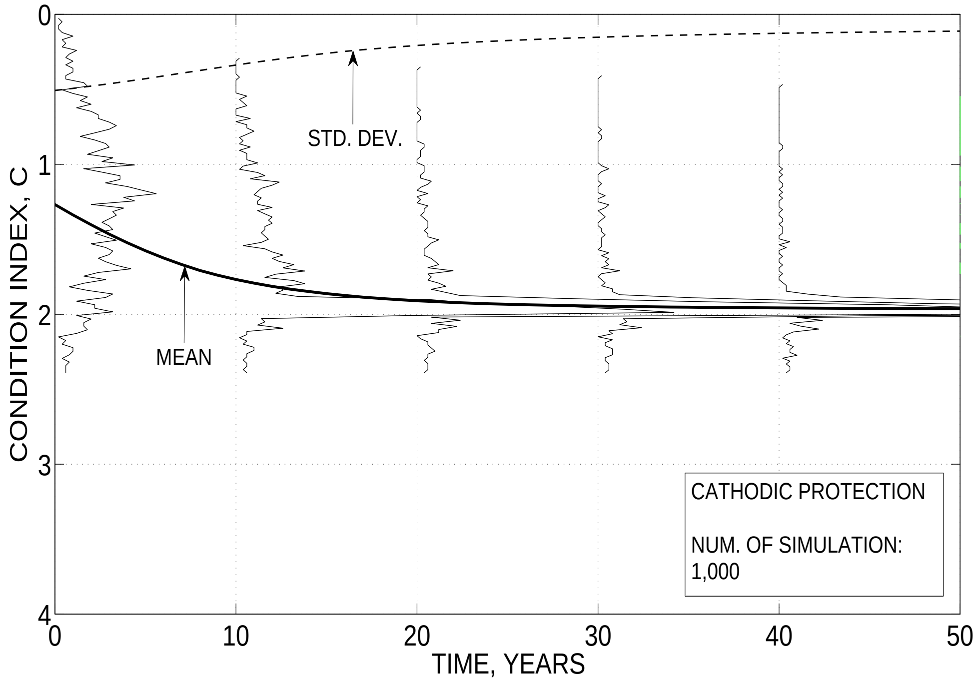


Figure 4.25: Condition Profile when Cathodic Protection action is applied.

4.3.2 Safety Profile

In well designed BMS, maintenance and repair strategies are decided according to performance curves of the structure. Another performance profile other than the condition profile is the safety profile. This performance profile is determined based on structural assessment formulas which were explained in Chapter 2. Structure's safety level is impaired when safety index is small. In the study [19], the threshold level for the safety index is selected as 0.91. Structure whose safety index reaches threshold level will be out of service and must be subjected to an essential maintenance action. Safety profile is obtained using the similar formulas of the condition profile. Safety profiles presented between Figure 4.26 and Figure 4.31 are obtained using the random variables defined in the Table 4.9.

Using the developed maintenance simulation program, the safety profile under no maintenance case is obtained as shown in Fig.4.26. The bridge is subjected to deterioration since the beginning. Therefore, performance curve gradually decreases over the years. Safety index of the structure reaches the threshold level at nearly 40 years if no action is taken. Performance curve is linear because maintenance type selected is no maintenance case.

Figures 4.27 through Figure 4.31 are obtained using the random variables values given in Table 4.9.

The Safety Profile in Fig. 4.27 is obtained when silane treatment action is applied to the bridge. Silane treatment is a time-based maintenance action so first and subsequent maintenance actions are applied at specified times which are random variables. Silane treatment leads to a reduction of deterioration rate only. Hence, this action extends the lifetime of the bridge. In addition, silane treatment has no improvement effect on safety profile. Therefore, no improvement on Safety Profile is achieved by silane treatment.

Replacement of expansion joints is a preventive maintenance action and it is a time-based maintenance strategy. The action has first and subsequent maintenance application times which are random variables. Since the only effect of this maintenance

Table 4.9: Data for Safety Index with Maintenance

	Silane Treatment	Replace Expansion Joints	Cathotic Protection	Minor Concrete Repair	Do nothing and Rebuild
t_{pi}	T(0, 7.5, 15)	T(0, 20, 40)	$C = 2$	$C = 3$	$S = 0.91$
t_p	T(10, 12.5, 15)	T(20, 30, 40)	T(7.5, 10, 12.5)	$C = 3$	$S = 0.91$
θ_4	0	0	0	0	T(1.0, 1.25, 1.50)
t_d	0	0	12.5	while $C < 1$	while $C < 1$
$(\theta_1 - \theta_2)$	T(0.0, 0.007, 0.018)	T(0.0, 0.007, 0.018)	0	0	0
t_{pd}	T(7.5, 10, 12.5)	T(10, 15, 30)	12.5	while $C < 1$	while $C < 1$

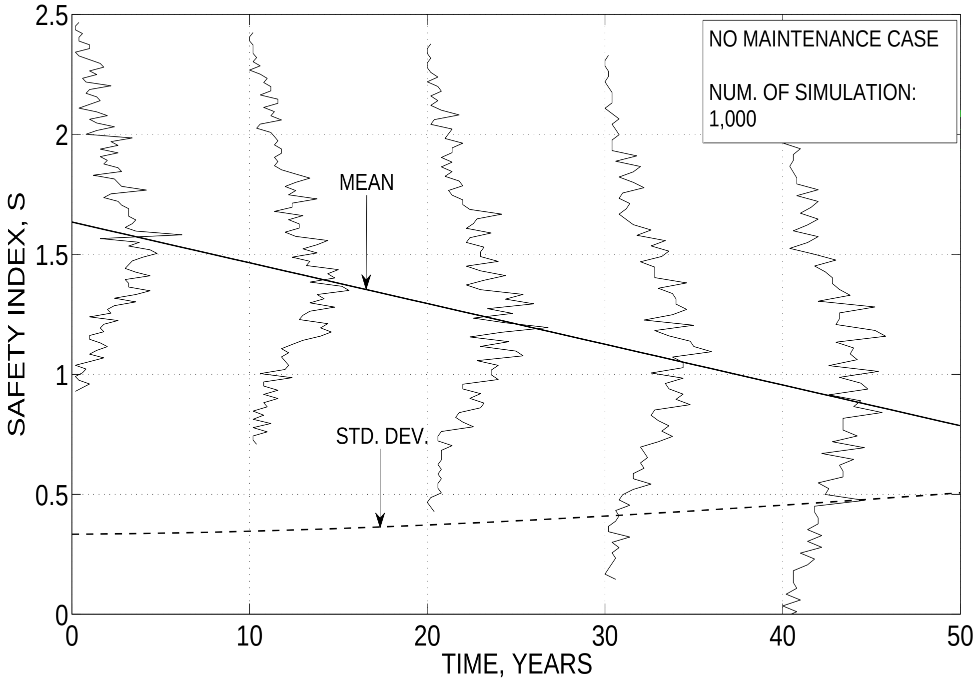


Figure 4.26: Safety Profile under No Maintenance Case

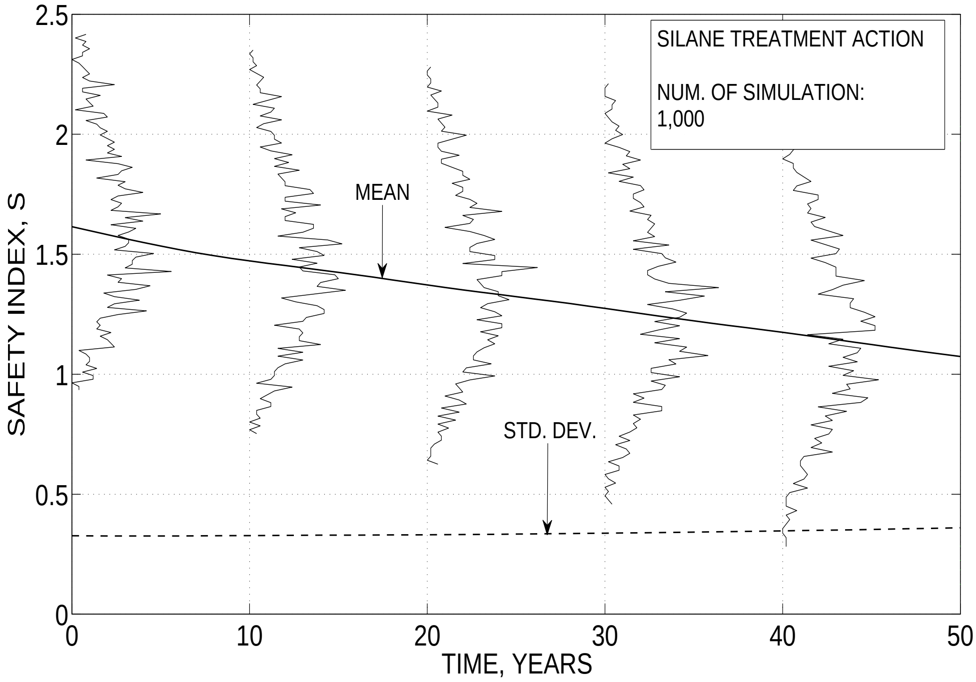


Figure 4.27: Safety Profile under Silane Treatment action.

activity is the reduction of deterioration rate of condition profile only, this maintenance activity extends lifetime of bridge infrastructure, however as shown in Fig. 4.28, it has no improvement effect on the Safety Profile.

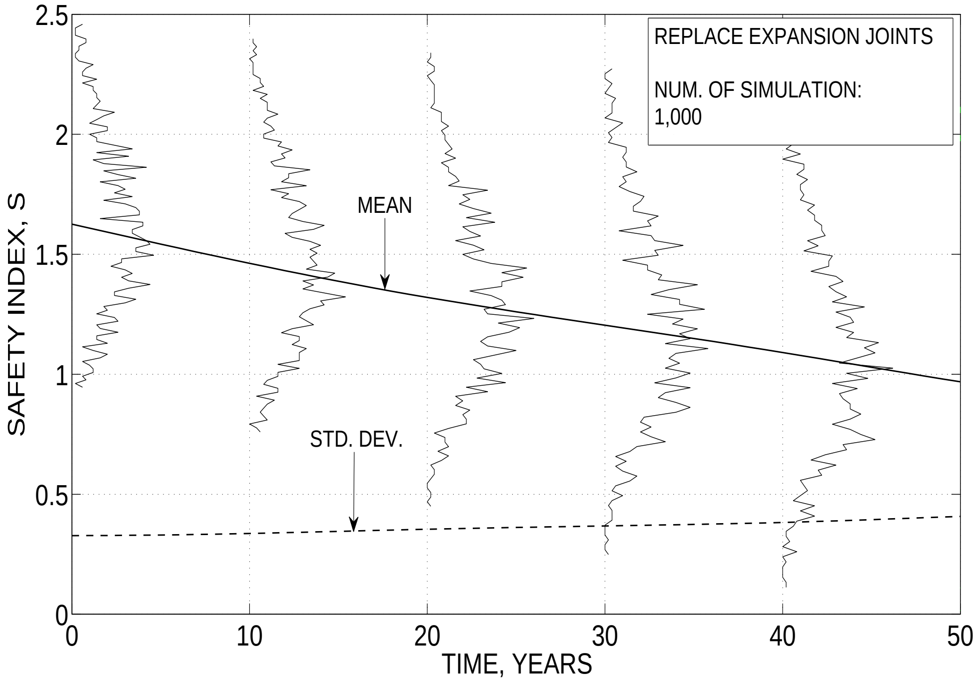


Figure 4.28: Safety Profile when Replacement of Expansion Joints action is applied.

The Safety index profile shown in Fig. 4.29 is obtained when minor concrete repair is applied the bridge. Minor concrete repair is an essential and performance-based maintenance action. First and subsequent maintenance actions are applied when condition index reaches the threshold level. While minor concrete repair has an important improvement effect on condition profile, as shown in Fig. 4.29, it has no improvement effect on the safety profile.

Do nothing and rebuild is an essential and performance-based maintenance action and applied when safety index reaches the threshold level. As shown in Fig. 4.30, safety index of the bridge is gradually decreasing until the maintenance action is applied. Safety index gains substantial improvement immediately after the application of rebuild action.

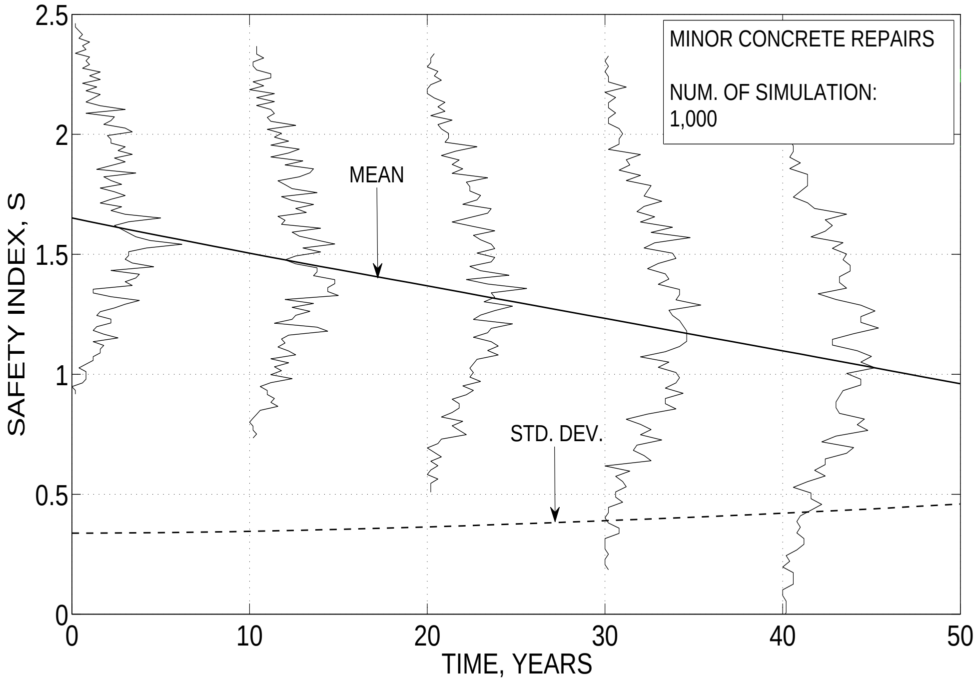


Figure 4.29: Safety Profile Under Minor Concrete Repair Action

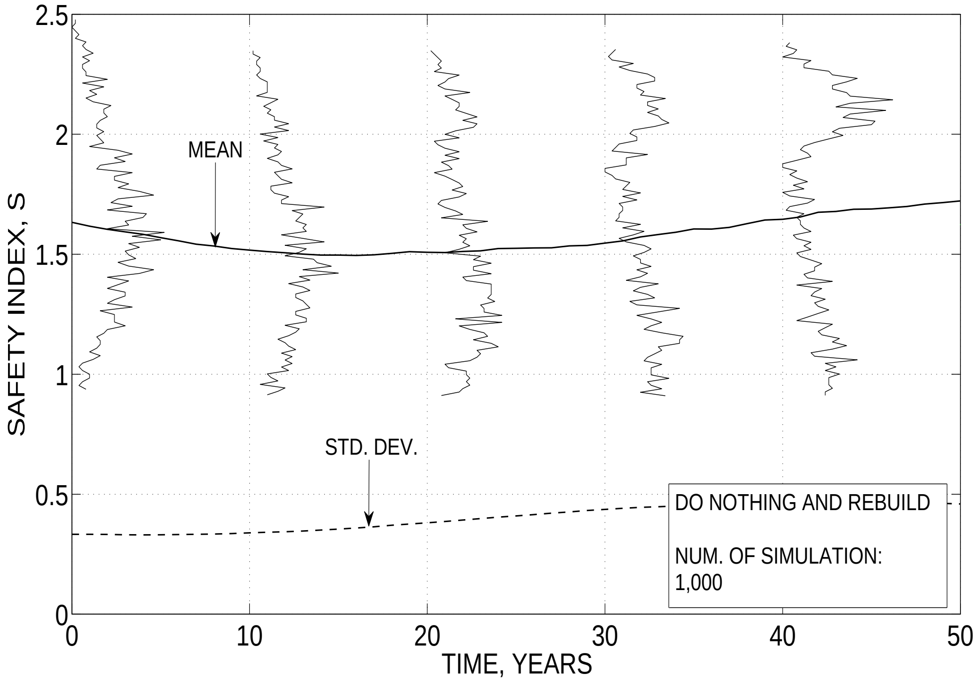


Figure 4.30: Safety Profile Under Do Nothing and Rebuild Action

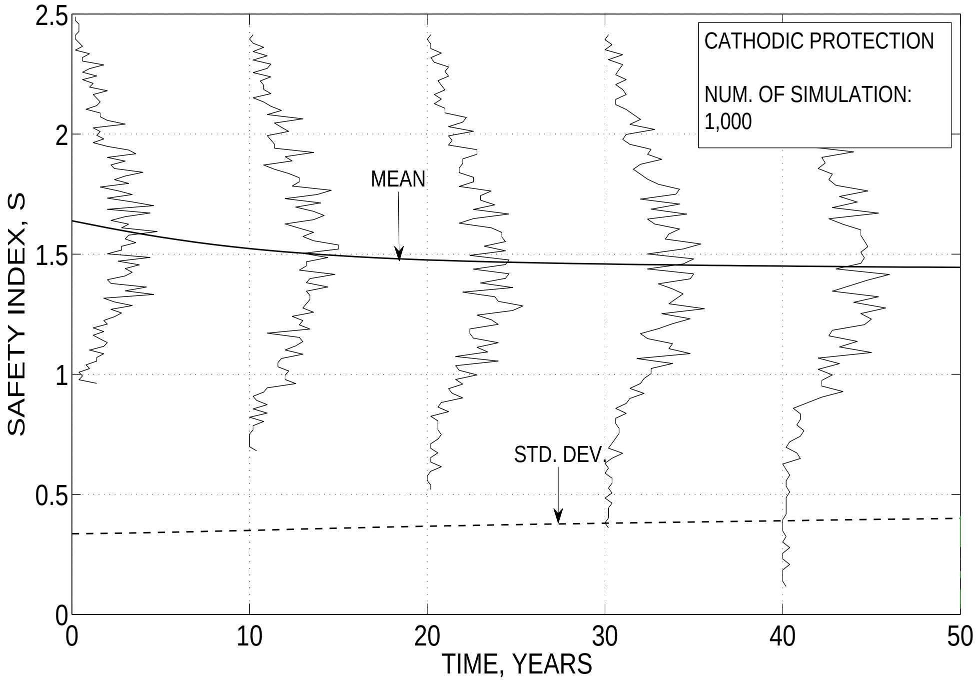


Figure 4.31: Safety Profile when Cathodic Protection action is applied

If cathodic protection is applied to the structure the safety profile is as shown in Fig. 4.31. Cathodic protection is a time and performance-based maintenance action. First cathodic protection action is applied when condition index reaches the threshold level but subsequent actions are applied according to subsequent application times. Threshold level of condition index for cathodic protection is selected as 2. Cathodic protection only delays the deterioration in time, it has no improvement effect on safety index. Although there is no improvement on safety index, cathodic protection prevents further deterioration of the structure. Therefore, service life of structure automatically extends.

4.3.3 Cost Profiles

Cost profiles are obtained by using Life-Cycle Cost Analysis (LCCA). This analysis is used to find out the action cost throughout the bridge lifetime. Fig. 4.32 presents effects of the maintenance action on the bridge condition and cost profiles. As shown in Fig. 4.32[58], costs depends on application time and amount of the improvement.

The triangular shapes shown in Fig. 4.32 represents the cost of the routine actions.

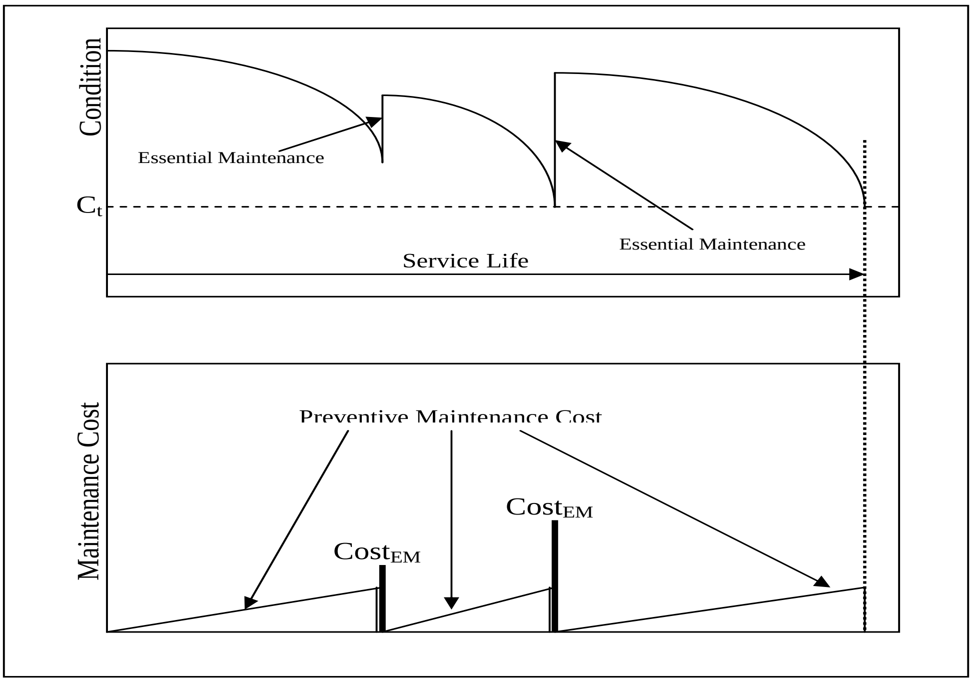


Figure 4.32: Interaction between maintenance costs and actions

Cost Profiles are obtained by inserting life-cycle cost analysis procedure into the *lhs_csc.m* program. Cost values are stored at the application time. The cost values are called as the annual maintenance cost. In addition, summation of these cost values gives the cumulative maintenance costs. Both of these formulas are presented in Equation 4.13 and Equation 4.14.

$$\text{Annual Maintenance Cost}(t) = C(t) \tag{4.13}$$

$$\text{Cumulative Maintenance Cost}(t) = \sum_{t=0}^{T} C(t) \tag{4.14}$$

Annual and cumulative maintenance costs can be transformed to the equivalent cost of the application time in order to have a chance for making comparison at a reference point in time. Discount rate is used to obtain the present value of the maintenance cost. Discounted cost formulas are given in Eq. 4.15 and Eq. 4.16.

$$\text{P.V of Annual Maintenance Cost}(t) = \frac{C(t)}{(1+\nu)^t} \tag{4.15}$$

$$\text{P.V of Cumulative Maintenance Cost}(t) = \sum_{t=0}^{T} \frac{C(t)}{(1+\nu)^t} \tag{4.16}$$

In order to calculate these maintenance cost with *lhs_csc.m*, two new formulas are required as shown.

$$E(\text{P.V of Annual Maintenance Cost}(t)) = \frac{\sum_{sim=1}^{N} \frac{C(t)}{(1+\nu)^t}}{N} \tag{4.17}$$

$$E(\text{P.V of Cumulative Maintenance Cost}(t)) = \frac{\sum_{sim=1}^{N} \sum_{t=0}^{T} \frac{C(t)}{(1+\nu)^t}}{N} \tag{4.18}$$

where,

$C(t)$ is the cost value at time t,
T is the lifetime of the bridge,
ν is the discount rate,
N is the total number of the simulations.

Cost profiles for applied maintenance actions are calculated and plotted using the values in Table 4.10 [19]. Fig. 4.33 through 4.36 show the present value of expected cumulative cost over the time horizon for four different maintenance actions. Lifetime maintenance cost profile for silane treatment is shown in Fig. 4.33 with discount rates 0 % and 6 %. As mentioned before, silane treatment action is a preventive maintenance action. Hence, application procedure of this action is based on time. As show in Fig. 4.33, silane maintenance strategy is a recursive precedure applied at definite time intervals. If only silane treatment is applied over lifetime of structure, present value of the expected cumulative cost is approximately 150 k£ with discount rate 0 %.

Present value of expected cumulative cost of replacement of expansion joints action is presented in Fig. 4.34. This action is also a time-based strategy. Probability distributions of first and subsequent maintenance actions were given in the previous section. Based on these distributions, replacement of expansion joints occurs one or two times in a 50 year lifetime. As shown in Fig. 4.34, present value of expected cumulative cost of replacement of expansion joints is approximately 27.5 k£ over the lifetime period if only this strategy is applied on the structure.

Table 4.10: Cost distribution for maintenance actions

Maintenance Actions	**Maintenance Cost (k£)**
Silane Treatment	T(0.3, 39, 77)
Cathodic Protection	T(19, 2604, 5189)
Minor Concrete Repair	T(16, 3605, 14437)
Rebuild	T(247, 7410, 28898)
Replace Expansion Joints	T(0.7, 19, 39)

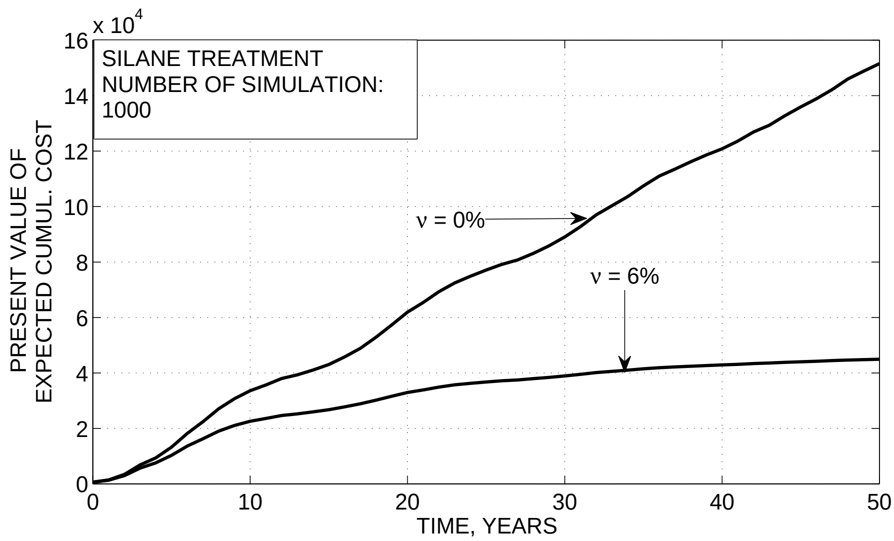


Figure 4.33: Present value of expected maintenance cost under time-based silane treatment for discount rates 0% and 6%.

The other maintenance action considered for cost profile generation is minor concrete repair. This maintenance action is a performance-based (essential) maintenance action. Minor concrete repair should be applied when the condition performance index

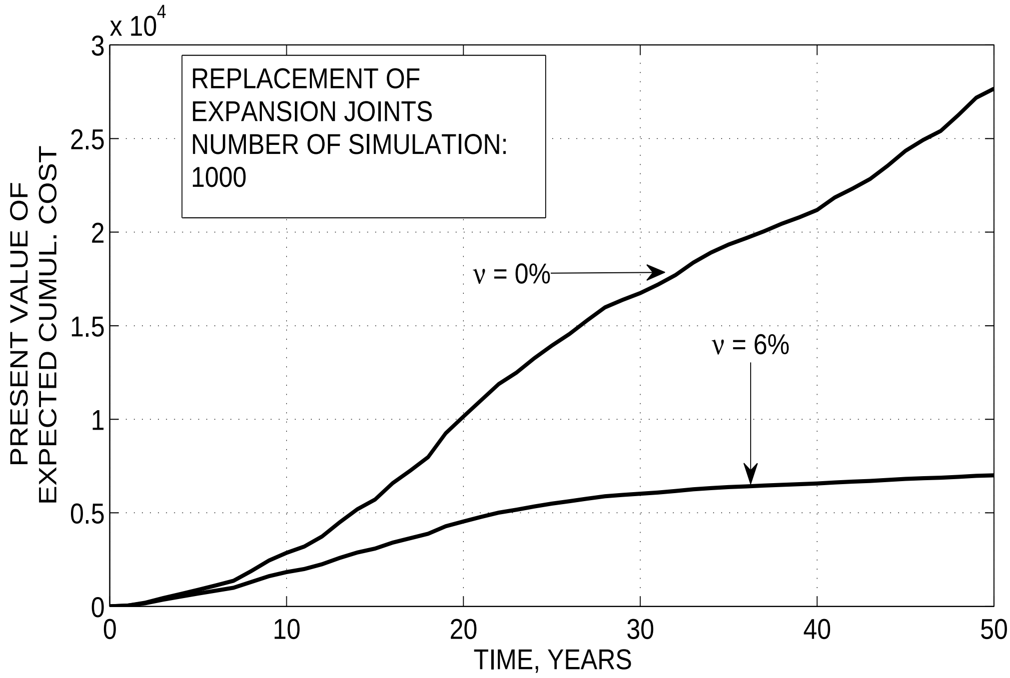


Figure 4.34: Present value of expected maintenance cost under time-based replacement of expansion joints for discount rate 0% and 6%

reaches its target value (i.e., 3). Therefore, there is relationship between deterioration rate and the number of applied minor concrete repair actions during the lifetime of the structure. This maintenance strategy has an important effect on performance indices of the structure. Minor concrete repair causes improvement in condition index and delay in safety index. If minor concrete repair is selected as maintenance strategy for lifetime maintenance analysis, cost profile for that action is obtained as shown in Fig. 4.35. For this case, present value of expected cumulative cost reaches approximately 10000 k£ at the end of 50 years. This cost is higher than that of silane treatment and replacement of expansion joints.

Rebuild maintenance strategy is a performance-based (essential) maintenance action type. It is refered to as the do-nothing and rebuild maintenance action. In this action procedure, any maintenance activity is not applied to the bridge until the safety index reaches the safety threshold level. The rebuild action is applied to the bridge when the safety index reaches the safety threshold level. Fig. 4.36 shows present value profile of the expected cumulative cost of the do-nothing and rebuild action. Approximate

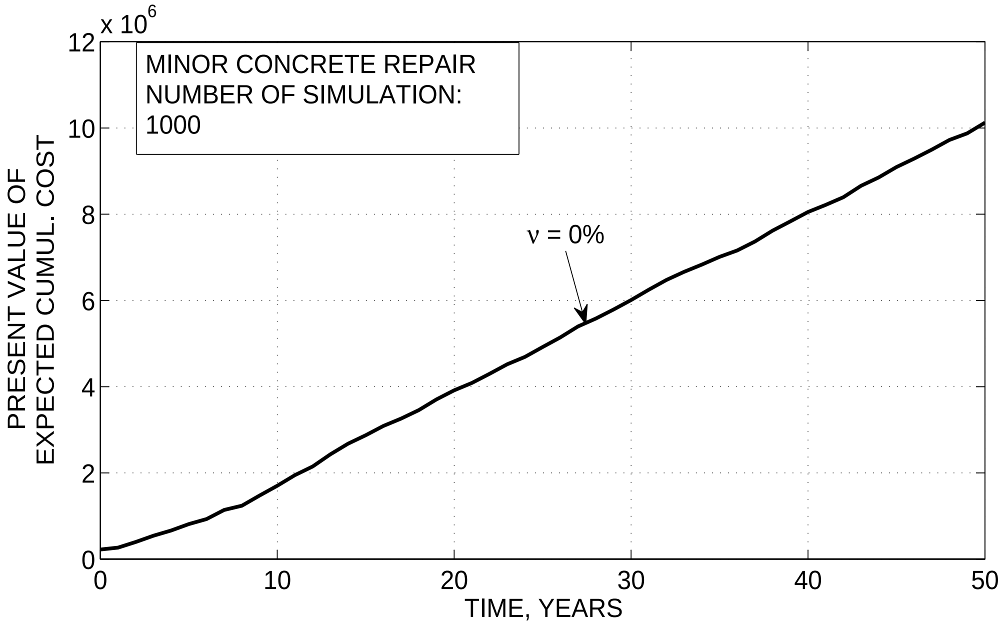


Figure 4.35: Present value of expected maintenance cost under performance-based minor concrete repair.

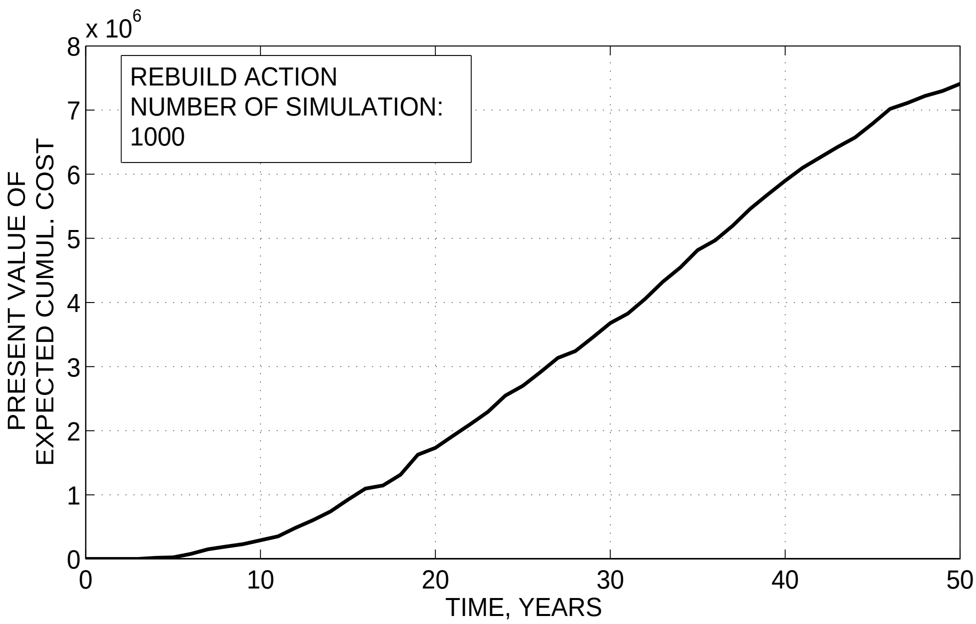


Figure 4.36: Present value of expected maintenance cost under performance-based rebuild action.

value of the expected cumulative cost for this maintenance action at the end of the lifetime is 7300 k£.

4.4 Polynomial-Based Condition Prediction

An example of a Condition Index is the condition rating. The two terms are generally used interchangeably. They both represent a range of numerical or alphanumerical values representing different levels of deterioration of bridge components or bridges. Condition ratings of bridge components or bridges at different ages can be predicted using different methods. In addition to simulation-based probabilistic method described in the previous section, another method is the so called regression model-based method. There are several regression models used in statistical studies. Linear regression, piecewise linear regression and polynomial regression are the common types of regression models. In this study, polynomial regression model is used because it has more advantages than the other regression models when condition deterioration of bridges is considered. Polynomial regression model is more realistic than linear regression model and easier to use than piecewise linear regression model for bridge condition prediction.

Regression models have important applications in bridge condition analysis. The first application is to predict the average condition ratings of bridge components at different ages. Second, regression models is used to find the deterioration rates at different ages. Third, they can be used to determine the improvement benefit gained by rehabilitation.

An example of polynomial regression equation for deck condition rating, the following equation is proposed by Jiang [59].

$$C_{deck} = 9 - 0.3498T + 0.0104T^2 - 0.0001T^3 \tag{4.19}$$

In Jiang's formula, condition rating ranges between 0 and 9. 9 represents the best condition while 0 represents the failed condition. The value comes from NBI (National Bridge Inventory) condition rating classification in U.S. However, initial condition index value can be set equal to any index value based on any condition evaluation rating scale. Condition rating profile for bridge decks over a 65 year lifetime period based on Eq. 4.19 is shown Fig. 4.37. In this polynomial regression model, time T(years) is the independent variable and condition rating changes at different ages.

Let us represent the coefficient in Eq. 4.19 by β's as shown in Fig. 4.37. Deck polynomial regression model gives rationally different condition rating profiles when β's are changed. The effect of β's on condition rating profile will be presented at the end of this section.

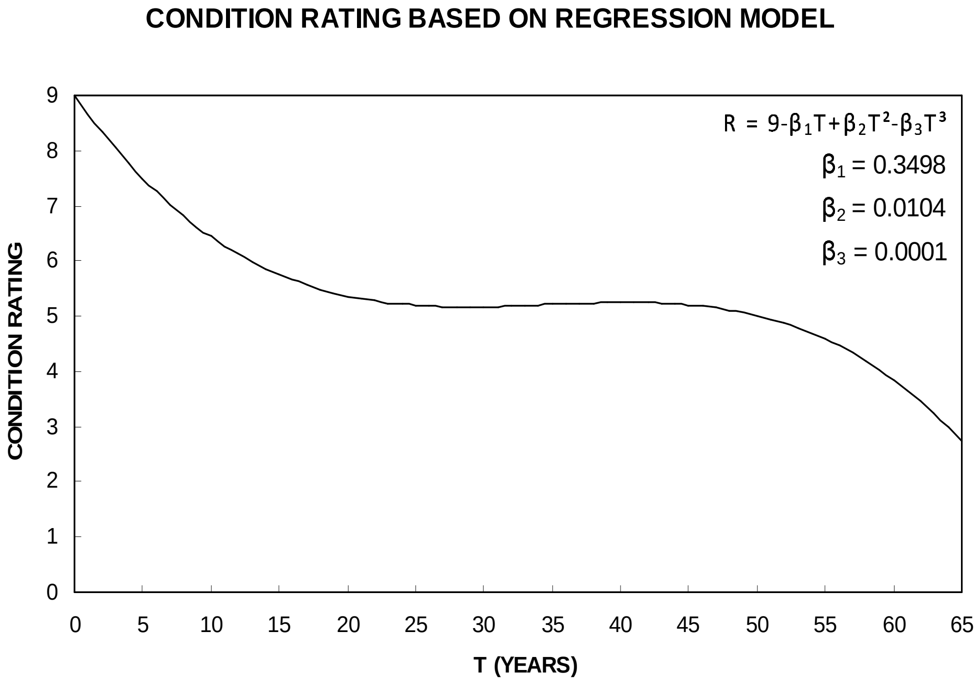


Figure 4.37: Polynomial regression of deck condition

Fig. 4.38 shows the predicted and actual condition ratings for the decks of 40 bridges presented by Jiang [59]. Statistical bridge data provided by Jiang is reproduced here. As shown the regression model for bridge decks approximate the actual condition ratings with some discrepancy.

Fig. 4.39 shows the number of bridges by age condition ratings of which were plotted in Fig. 4.38. Mean age of the 40 bridges is 39.3 years and the standard deviation of the bridges age is 19.8 years as indicated in the figure. Most of the bridges, (17 bridges), between 15 and 30 years old.

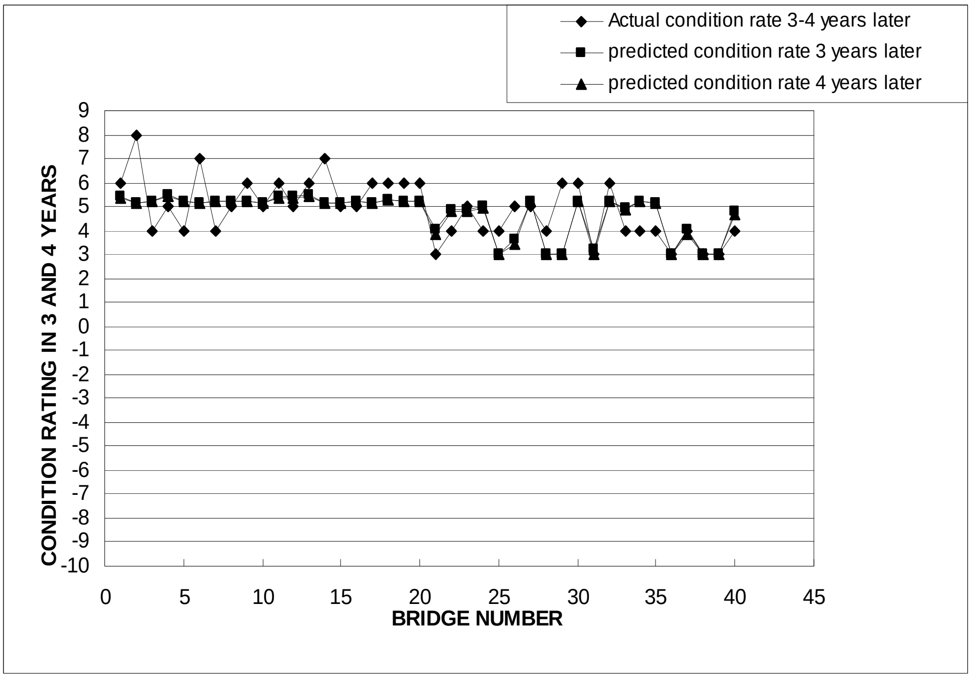


Figure 4.38: Actual and predicted condition rate of selected 40 bridges.

Fig. 4.40 shows the individual ages of the 40 bridges. The youngest age in the bridge stock is 15 years old and there are two bridges at that age. Age of the oldest bridge is 84.

In Fig. 4.41 through 4.49, effect of the coefficients (β's) in Eq. 4.19 on the condition rating profiles are investigated and displayed.

In Fig. 4.41, condition rating profile is examined based on regression model for bridge deck. Every variables except β_1 are kept constant, and effect of β_1 on condition rating profile of bridge deck is observed. The values of β_1 vary between 0.1 and 0.6. As shown in Fig. 4.41, some of these results of the polynomial regression model for bridge deck are irrational because condition rating curve must decrease gradually but the mentioned values (β_1=0.1, 0.2, and 0.3) increase the condition rating curve after 20 years. This leads to conceptual error for the deterioration model. Therefore, condition rating curves leading to irrational results are removed from Fig. 4.41, and as a result, Fig. 4.42 is obtained. As shown in Fig. 4.42, the smallest acceptable value for β_1 a value between 0.3498 and 0.4 the condition rating profile of bridge deck when the other β values are kept constant. Therefore, increasing β_1 value causes

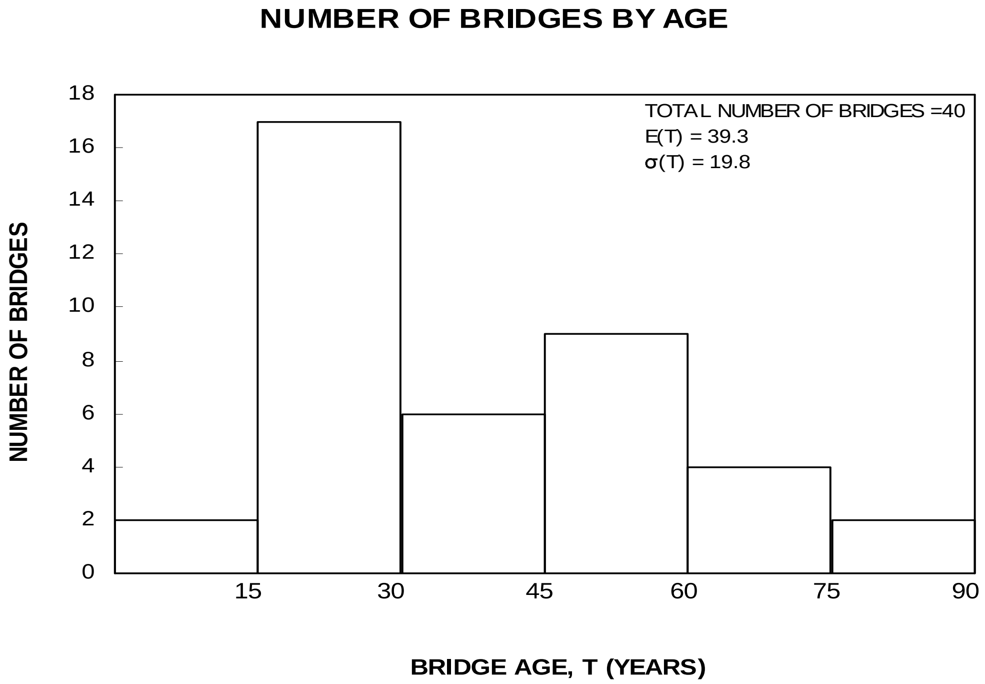


Figure 4.39: Number of bridges by age for which condition rating analysis is performed.

a decrease in service life of the bridge deck. When value of β_1 is increased to 0.4, 0.5, and 0.6, condition rating decreases radically and reaches 3 after 44 years, 17 years and 12 years, respectively.

In Fig. 4.43, condition rating profiles are examined based on the values of β_2. Every variables except β_2 are kept constant, and the effect of β_2 on condition rating of bridge deck is observed. The values of β_2 vary between 0.0098 and 0.011. Acceptable value is for β_2 is 0.0102 based on a visual inspection of the profiles in this graph. As shown in Fig. 4.43, if the value of β_2 is increased, irrational results arise from regression model for bridge deck because the condition rating profile starts increasing (for $\beta_2 \geq 0.0104$) after approximately 30 years.

Figure 4.44 is obtained by removing the regression curves using values of β_2 larger than 0.0102 from Fig. 4.43. As shown in Figure 4.43, the largest acceptable value for β_2 is 0.0102 and when value of β_2 is decreased keeping other variables constant, deterioration of the bridge condition accelerates more rapidly. Therefore, decreasing

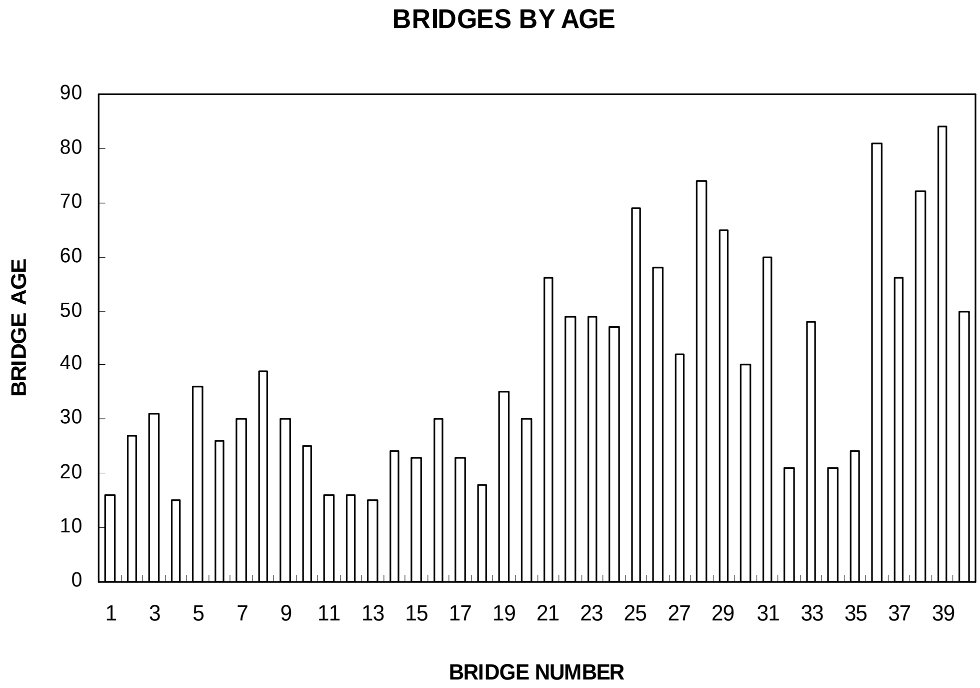


Figure 4.40: Individual ages of 40 bridges.

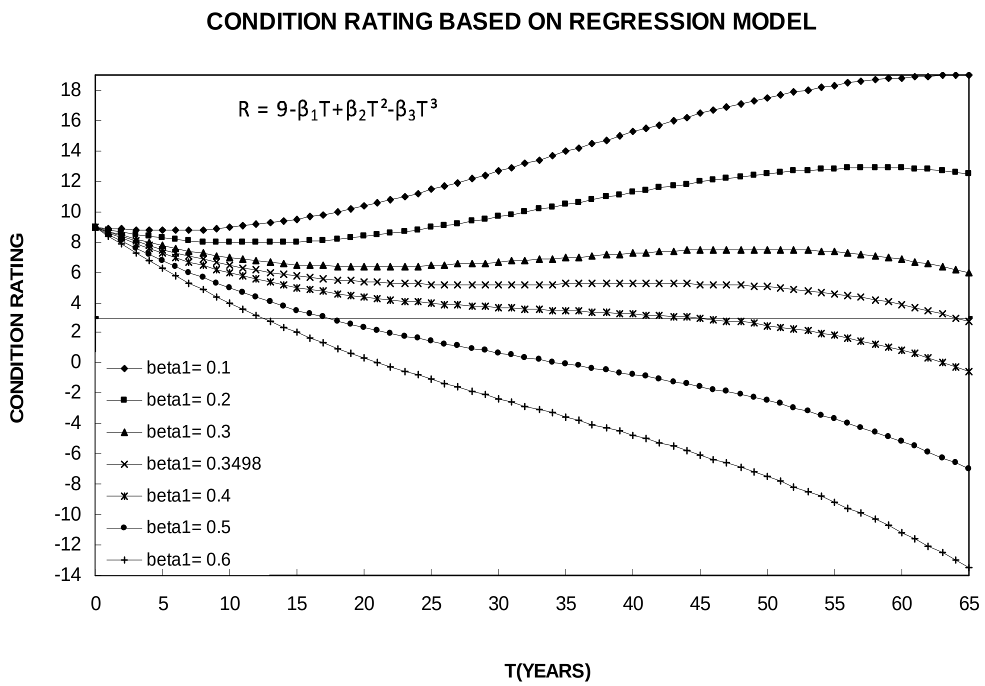


Figure 4.41: Polynomial regression-based condition rating profile for bridge deck when only β_1 ranges between 0.1 and 0.6

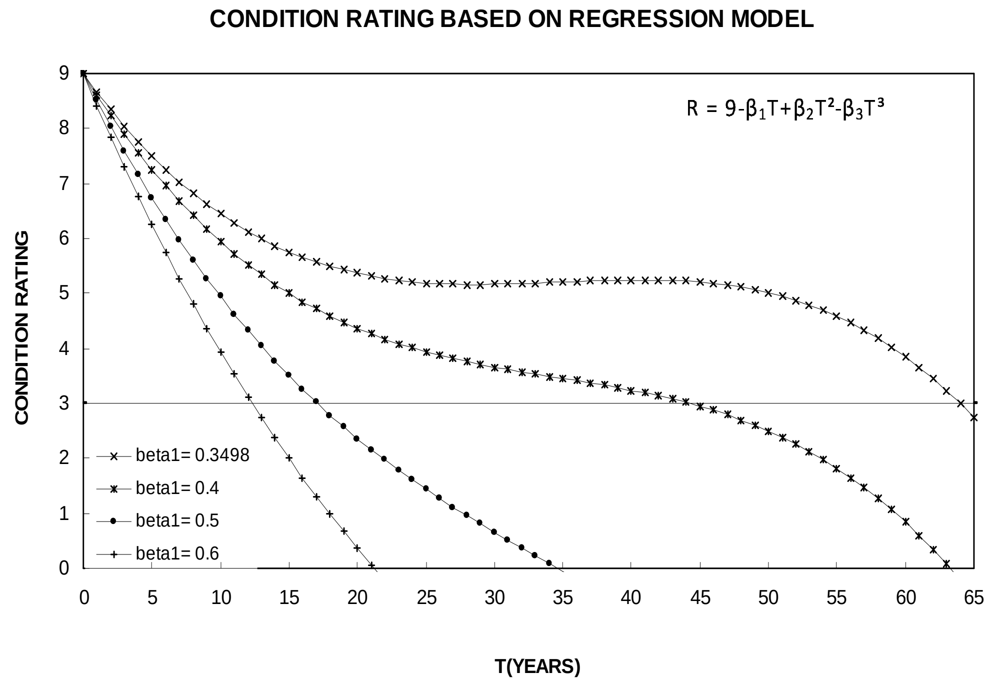


Figure 4.42: Polynomial regression-based condition rating profile for bridge deck when only β_1 ranges between 0.3498 and 0.6

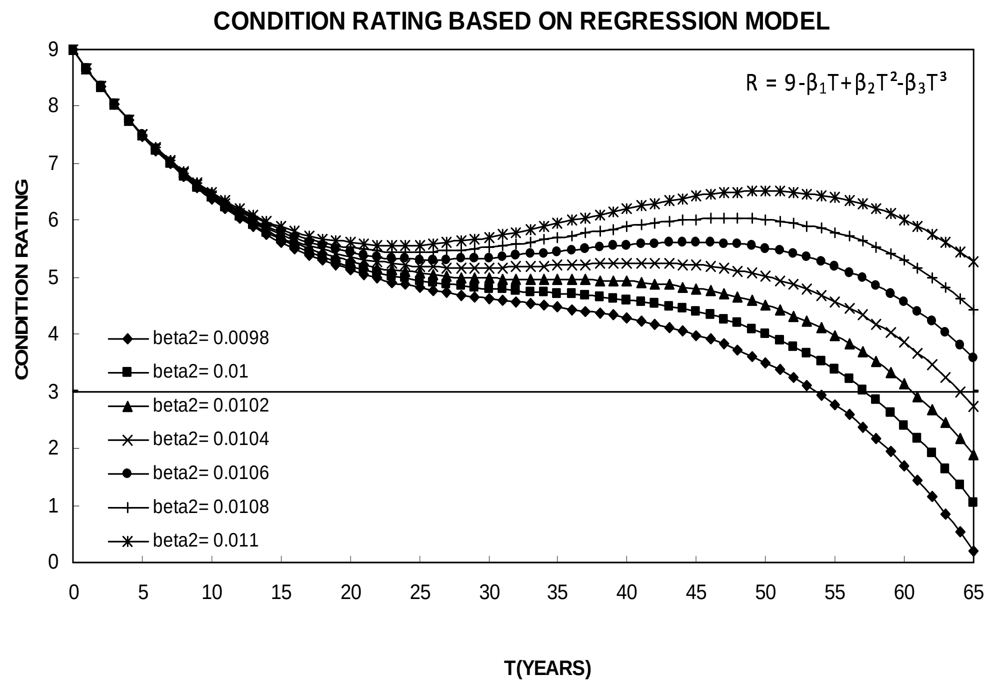


Figure 4.43: Polynomial regression-based condition rating profile for bridge deck when only β_2 ranges between 0.0098 and 0.011

of value of β_2 leads to a reduction of the service life of the bridge deck based on the regression model for bridge deck element. For example, when the 0.0098 is used as the value of β_2, condition rating of deck reaches 3 at the end of 54 years.

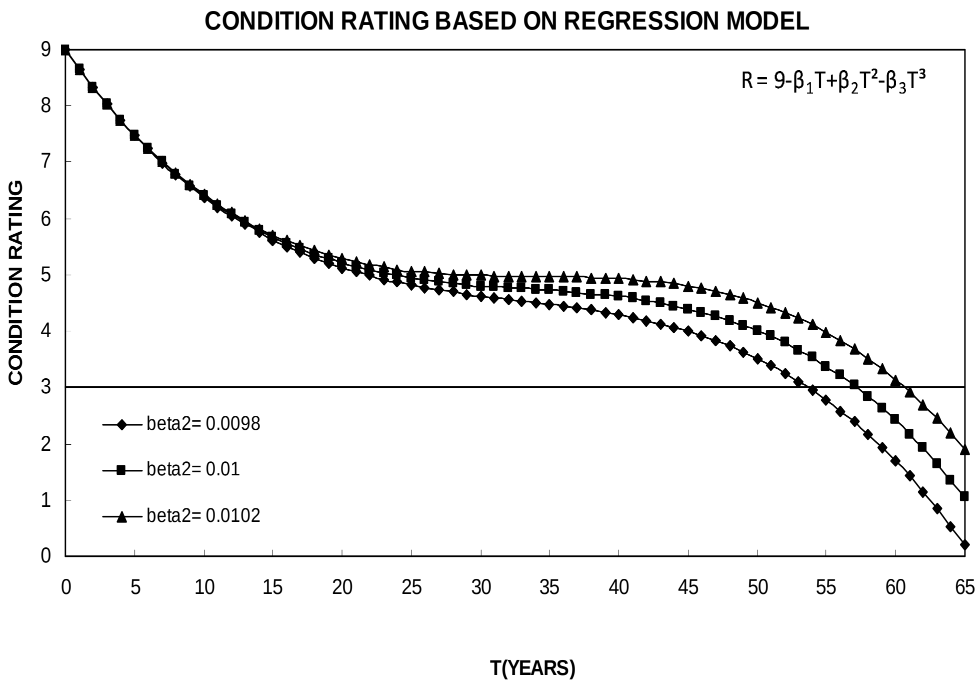


Figure 4.44: Polynomial regression-based condition rating profile for bridge deck when only β_2 ranges between 0.0098 and 0.0104

Fig. 4.45 shows the effect of the coefficient β_3 on the condition rating curve of the bridge deck based on polynomial regression model. In order to Fig. 4.45, every coefficient except β_3 are kept constant, and effect of β_3 on condition rating of bridge deck is observed. The value of β_3 is varied between 0.00008 and 0.00014. The smallest acceptable value for β_3 is between 0.0001 and 0.00011. As shown in Fig. 4.45, if value of β_3 is decreased, irrational results arise from regression model for bridge deck and the condition rating increases after 30 years.

Figure 4.46 is obtained by removing the irrational regression curves from Fig. 4.45. When value of β_3 is taken as 0.0001, an acceptable regression curve is obtained for 65 years service life. The larger values of β_3 (larger than 0.0001) causes deterioration of the bridge condition to accelerate more rapidly. Therefore, increasing the value of β_3 leads to reduction of the service life of the bridge deck based on regression model

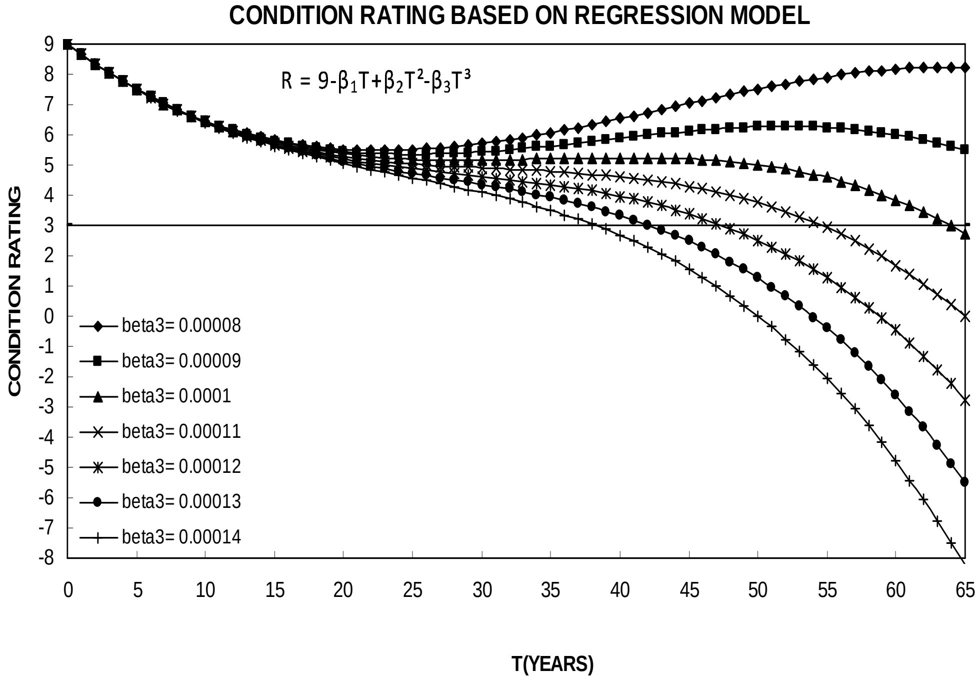


Figure 4.45: Polynomial regression-based condition rating profile for bridge deck when only β_3 ranges between 0.00008 and 0.00014

for bridge deck element. For example, if the 0.00014 is used for β_3, condition rating of deck reaches 3 at the end of 37 years. However, service life of bridge deck extends to 65 years when value of β_3 is taken as 0.0001.

Each of Figures 4.47, 4.48 and 4.49 consists of two regression curves. The two regression curves are obtained using the same regression model of bridge deck and have the same values for β_1 except for β_2 and β_3.

The value of coefficient β_1 is 0.4 for each regression curve in Fig. 4.47. The values of β_2 and β_3, on the other hand, are assigned the values shown in Eq. 4.19. In other words β_2 and β_3 are assumed independent of β_1. For the other regression curve, values of β_2 and β_3 are determined depending on β_1. To achieve this, β_2 and β_3 are divided by β_1 in order to find a ratio between β_1 and the other coefficients. In Eq. 4.19, ratio between β_1 and β_2, and β_1 and β_3 are 0,029731 and 0,000286, respectively. If β_1 is changed, the value of β_2 and β_3, depending on the value of β_1, are found using these ratio constant.

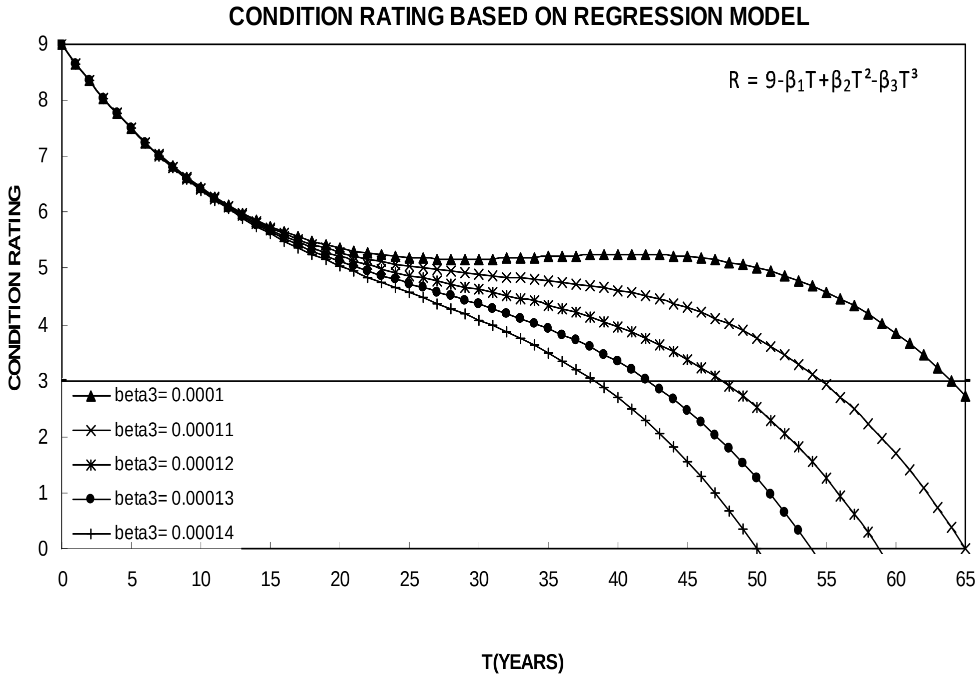


Figure 4.46: Polynomial regression-based condition rating profile for bridge deck when only β_3 ranges between 0.0001 and 0.00014

As shown in Fig. 4.47, two polynomial curves are obtained. β_1 is equal to 0.4 for both regression curves. However, values of β_2 and β_3 are different for each curve. All coefficients are independent for one of the polynomial curves. In the other curve, β_2 and β_3 values change with respect to ratio depending on β_1. β_2 and β_3 values in independent curve are 0.0104 and 0.0001, respectively. On the other hand, β_2 and β_3 values in dependent curve are 0.01189 and 0.000114, respectively. This application gives good regression model because regression curve obtained based on the coefficient ratios leads bridge deck member to have longer service life than ones in which β_2 and β_3 are kept as constant even if the values of β_1 is varied. Regression curve obtained from application mentioned above is rational. For example, service life of the regression curve whose coefficients are related to each other is approximately 62 years. However, the other regression curve whose coefficients are not related to each other has 44 years of service life.

In Fig. 4.48, both of the regression models of deck member has $\beta_1 = 0.5\,\beta_1$. The regression curve for which the coefficients β_2 and β_3 depend on β_1 gives a more rational

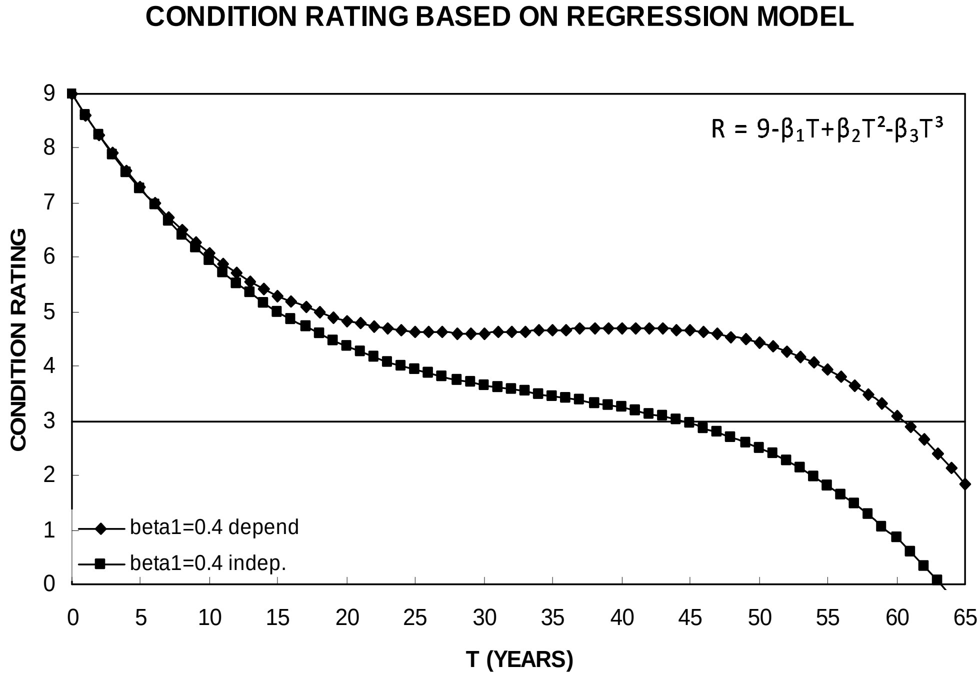


Figure 4.47: Comparison of polynomial regression-based profiles of deck condition based on $\beta_1 = 0.4$ while $\beta2$ andβ_3 are changed as being independent or dependent of the value of β_1.

result than the other curve. The first regression curve starts from condition rating 9 and decreases sharply to rating 5 in approximately 12 years, then remains nearly at rating 3.5, starting from 20 years old for a duration of approximately 27 years. The regression curve reaches the condition rating 3 when the bridge deck member is approximately 54 years old. The other regression curve obtained by changing β_1 value only submit little service life profile. When the bridge deck member is approximately 18 years old, condition rating reaches the rating 3.

The two regression curves shown in Fig. 4.49 has $\beta_1 = 0.6$. Both curves reaches the condition rating 3 earlier than the curves shown in Fig. 4.47 and Fig. 4.48. The polynomial curve with independent coefficient values reaches the condition rating 3 approximately at 12 years. However, the other polynomial curve reaches the condition rating 3 at 18 years. It can be noted that β_1 has an important effect on condition rating. The larger values of β_1 leads bridge condition rating to faster deterioration.

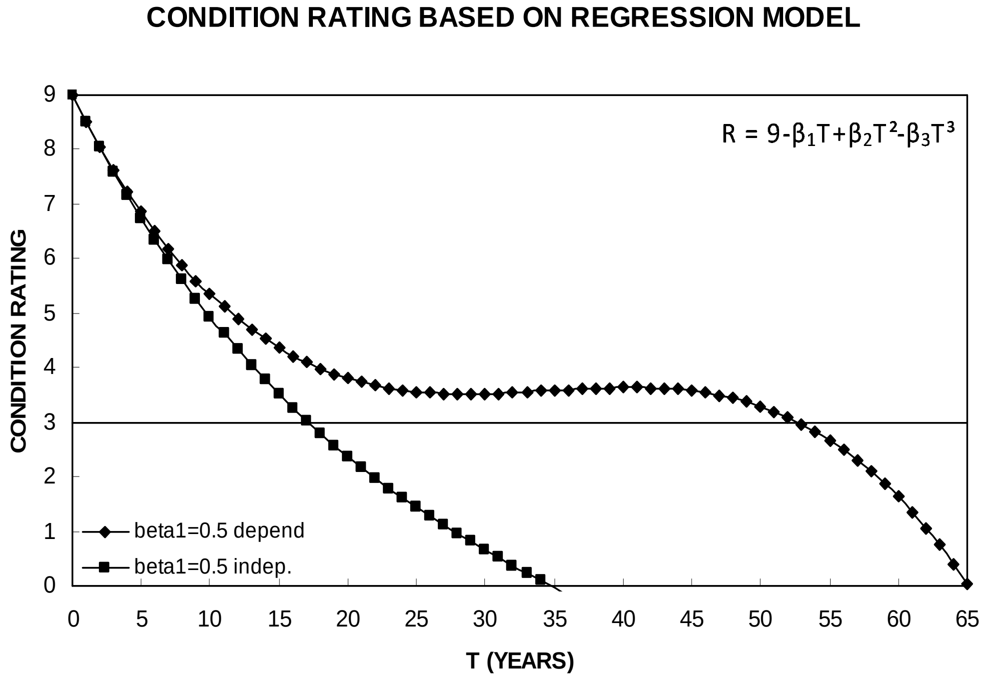


Figure 4.48: Comparison of polynomial regression-based profiles of deck condition based on $\beta_1 = 0.5$ while $\beta 2$ andβ_3 are changed as being independent or dependent of the value of β_1.

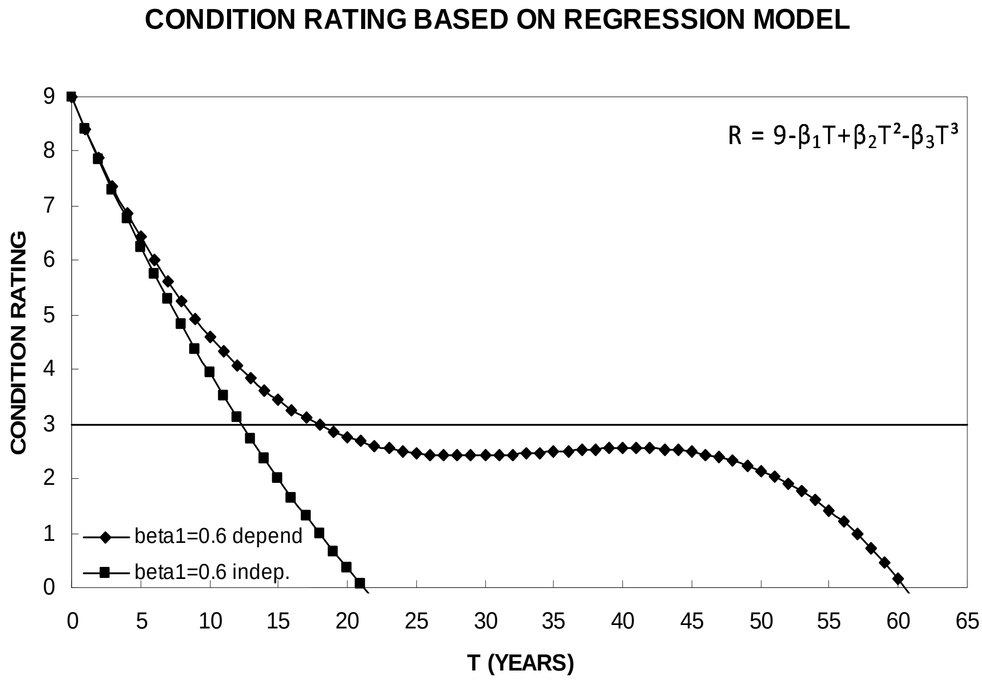


Figure 4.49: Comparison of polynomial regression-based profiles of deck condition based on $\beta_1 = 0.6$ while $\beta 2$ andβ_3 are changed as being independent or dependent of the value of β_1.

4.5 Summary

In this chapter, a probabilistic Bi-linear model is investigated. The model contains numerous uncertainties defined as random variables. In order to generate random variables and investigate Bi-linear model, a Latin Hypercube sampling-based computer program is developed in Matlab environment. The developed simulation program generates values for random variables. The simulation program is integrated in a main algorithm to produce the condition, safety, and cost profiles. Deterioration rate, initial condition index, and deterioration initiation time are three substantial random variables which determine the condition and safety profiles under no maintenance case. Various condition profiles are obtained for the low, moderate and high values of these three random variables. In addition, condition, safety, and cost profiles are re-generated under the five different maintenance actions and investigated effect of the maintenance actions. Furthermore, maintenance and repair actions are investigated for different bridge types and several components of bridges. Finally, polynomial regression-based prediction curve is investigated. Effects of the changes of polynomial equation on the performance curve are examined.

Notations in Chapter 4

$F(x)$	:	Cumulative distribution function of variable x
$P(x)$	:	Probability density function of variable x
G	:	Inverse cumulative distribution function
x	:	Any selected number in the distribution
p	:	Probability of cumulative distribution function
θ_1	:	Deterioration rate of condition index
θ_2	:	Change in deterioration rate due to first maintenance action
θ_3	:	Change in deterioration rate due to second maintenance action
θ_4	:	Increase in performance indicator due to first maintenance action
θ_5	:	Increase in performance indicator due to second maintenance action
t_i	:	Time of initiation of deterioration of performance indicator
t_{pi}	:	Time of first application of maintenance action
t_p	:	Time of subsequent application of maintenance action
t_d	:	Time during which the deterioration effect on performance indicator is suppressed
t_{pd}	:	Time during which the deterioration effect on performance indicator is suppressed or reduced
ϵ_T	:	Deterioration rate of condition index under no maintenance during the specified one year time interval
t_{zero}	:	Fractions of the year during which there is no deterioration of condition under no maintenance
t_{det}	:	Fractions of the year during which there is deterioration of condition under no maintenance
T	:	Time
C_0	:	Initial condition index
C_T	:	Condition index at time T
t_{no}	:	Fraction of the year during which there is no effect of the maintenance on condition
t_{effect}	:	Fraction of the year during which maintenance action reduces or suppresses the deterioration of the condition
$t_{reduced}$	:	Fraction of the year during which the deterioration rate of condition is reduced due to the maintenance action
τ	:	The time elapsed since the maintenance is applied
$C(t)$	:	Annual maintenance cost at time t
ν	:	Discount rate
N	:	Total number of the simulations
C_{deck}	:	Condition rating for deck
β_i	:	Coefficients of the polynomial regression equation

CHAPTER 5

CONDITION PREDICTION COMBINING BOTH SIMULATION AND REGRESSION TECHNIQUES

5.1 Introduction

Bridge condition profiles and bridge condition rating data are important and essential measures that assist the decision making process regarding bridge management, rehabilitation and repair actions. Through a bridge inspection, bridge condition rating should be predicted as accurately as possible in order to make a accurate selections for maintenance, repair and rehabilitation actions over the bridge lifetime. For this reason, bridge performance prediction is an essential component for Bridge Management Systems (BMSs). Bridge performance prediction models are produced by several bridge performance prediction methods using condition or safety rating data. These methods were discussed and studied in Chapter 4 and include a simulation-based bi-linear deterioration model [19], a regression analysis model [25, 60] and the Markov decision process model [25]. Simulation-based and Regression-based condition prediction models will be combined in this chapter in order to develop a new condition prediction model incorporating the powerfull features of each method..

5.2 Regression Models for Bridge Performance Prediction

Regression analysis is a method used to determine relationship between dependent and independent variables and to generate new data sets from these variables. Regression analysis method is used to obtain bridge performance prediction curves. These performance prediction curves assist bridge administrations to predict bridge condition and remaining service life [60]. Based on the prediction curves, applications of required maintenance actions can be planned. Bridge condition prediction curves are developed based on the inspection data collected over time. Bridge condition rating data depend on many different parameters. Some of these parameters are bridge age, bridge types (concrete, steel and timber bridge), highway conditions, traffic volume and climatic conditions. In addition, bridge condition rating data differ according to substructure, superstructure and deck which are considered as major bridge components [61]. Therefore, several performance prediction models can be produced by regression analysis method for different parameters and for bridge components of a bridge.There are different regression analysis models. As an introduction to this subject, an example regression-based condition prediction model developed by Jiang was introduced in Sect. 4.4 of Chapter 4. The coefficients of the regression model were analyzed in detail. A more general discussion of the regression models for bridge performance prediction is presented in the following sections.

5.2.1 Linear Regression Model

Linear regression is an approach to generate a linear formulation of the relationship between random variables composing a data set. One of the variables is considered as the independent variable, while the rest of the variables are dependent variable in the data set.

A commonly used method for fitting a regression line to an observed data is the least square method. The best fitting line for the data set is calculated by minimizing the sum of the squares of the vertical differences between actual data and estimated regression line.

Linear Regression Model with One Independent Variable. Form of a linear equation with one independent variable is shown in Eq. 5.1.

$$Y_i = \alpha_0 + \alpha_1 X_{i1} + \epsilon_i \tag{5.1}$$

where

Y_i is the dependent variable,
X_{i1} is the independent variable,
α_1 is the slope of the linear line,
α_0 is the intercept of linear equation and is also interpreted as value of Y when $X = 0$,
ϵ_i is the error term which has standard normal distribution.

Linear Regression Model with Multiple Independent Variable. Linear regression with multiple variables is used when the dependent variable is expressed by two or more random variables. In BMSs, linear regression with one or two random variables may be used as a simple form of performance prediction model. Each additional variable term leads to a new coefficient to be estimated. As a result, if number of variables increases, the complexity of the regression increases. Form of a linear regression equation with two independent variable is given as;

$$Y_i = \alpha_0 + \alpha_1 X_{i1} + \alpha_2 X_{i2} + \epsilon_i \tag{5.2}$$

where

X_{i1} is the first independent variable,
X_{i2} is the second independent variable,
α_1, α_2 are the coefficient of the first and second independent variables respectively.

There are several factors which affect the bridge condition in time. In BMSs, however, more than two factors are not generally used in regression models because of computational complexity. Therefore, if linear regression model is used in a BMSs, two independent variables are generally used to represent the bridge condition rating. Bridge age is the first independent variable generally used, and the other variable is the average daily traffic (ADT).

Fig. 5.1 show the result of a linear regression analysis for a data set consisted of one independent variable. Values of dependent and independent variables are listed in the figure. Linear regression obtained when the least square fitting method is applied to this data set.

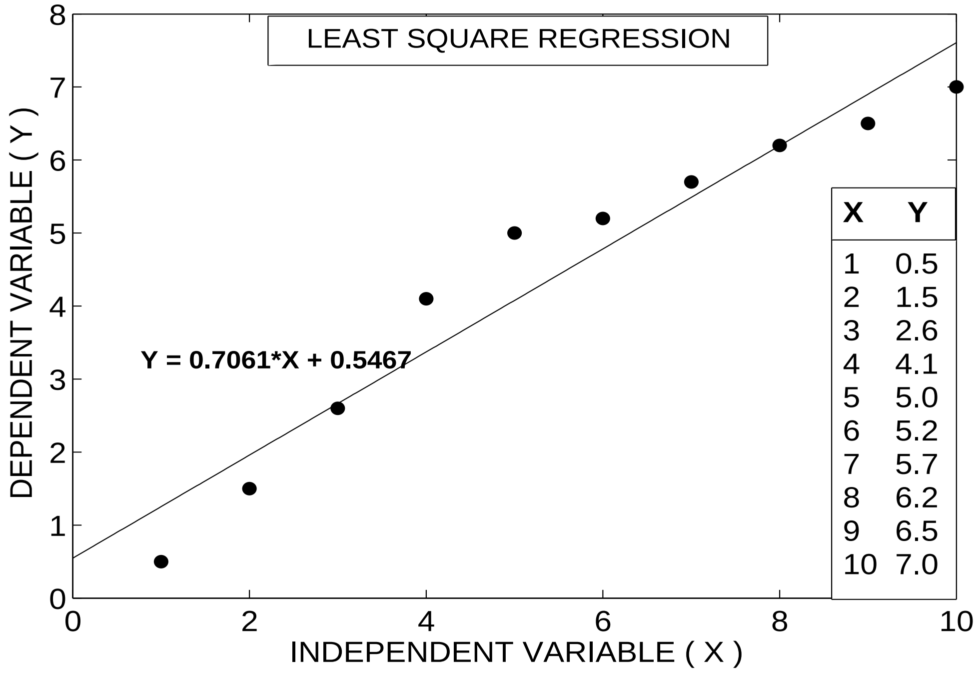


Figure 5.1: Linear Regression for a Data Set having one independent variable

5.2.2 Piecewise Linear Regression Model

Another linear regression model is the piecewise linear regression model. Piecewise linear regression model includes different slopes at different intervals. For data that displays a nonlinear relationship between the variables, linear regression may not be suitable. In this case, piecewise linear regression can be used to obtain a better fit for nonlinear data.

Figure 5.2 presents application of piecewise linear regression for a data set taken from Figure 5.1. As shown in Figure 5.2, linear regression has two-pieces linear equation with different slopes and piecewise linear regression gives better approximation than linear regression for that sample data.

5.2.3 Polynomial Regression Model

It is common to use the polynomial regression model when linear regression can not represent the relationship between dependent and independent variables for a data

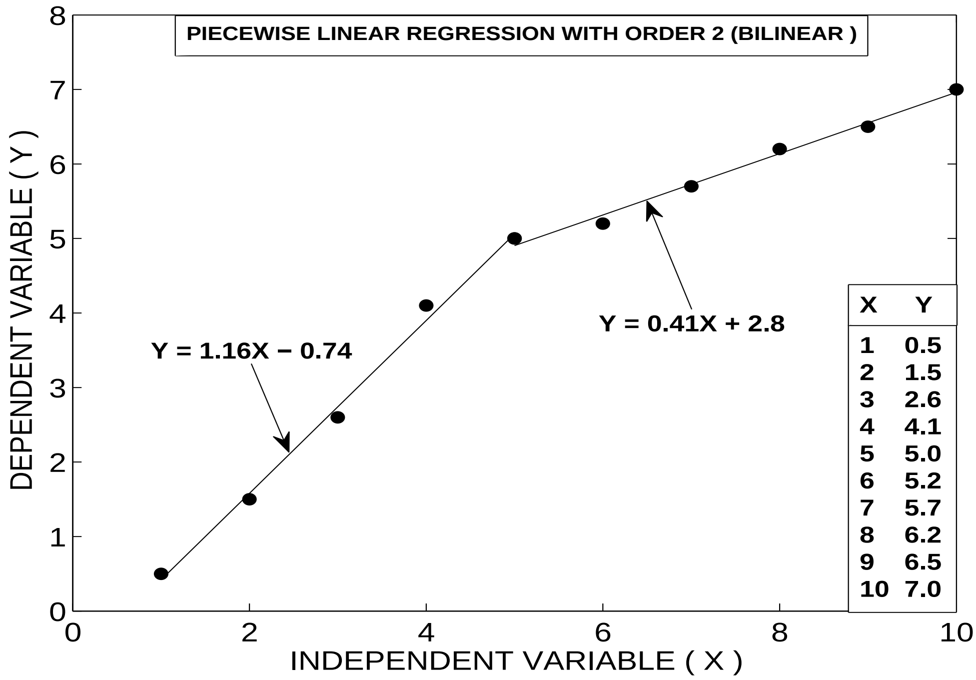


Figure 5.2: Piecewise Linear Regression with Order Two (Bilinear) for Complete Data Set

set. Polynomial regression equation may be a second order, a third order or a higher order equation. For example, a third order polynomial regression equation with one independent variable is presented as follows.

$$Y_i = \alpha_0 + \alpha_1 X_i + \alpha_2 X_i^2 + \alpha_3 X_i^3 + \epsilon_i \tag{5.3}$$

The bridge condition rating data presented as data points in Figs. 5.3 through 5.14. The bridge group data is from interstate and otherstate bridges for different age located in the state of Indiana in the U.S, and represent the bridge condition ratings obtained by visual inspections [25]. Furthermore, the data is classified by bridge component types, representing the condition ratings of decks, substructures, and superstructures of these bridges based on the bridge age.

The bridge condition rating data can be used to generate performance prediction models by applying polynomial regression analysis. As a result of polynomial regression, polynomial-based performance prediction equations are obtained. The coefficients of these regression equations are presented in Table 5.1 through Table 5.4.

Table 5.1: The Coefficients of Polynomial-based Performance Prediction Equation for Interstate Highway Concrete Bridge Condition Rating Data Used

	C_0	α_1	α_2	α_3	R^2
Deck	8.1911	-.2013	0.0070	$-1.2727e^{-4}$	0.6053
Substructure	8.0962	-0.2047	0.0081	$-1.4960e^{-4}$	0.4816
Superstructure	8.0074	-0.1773	0.0044	$-5.4061e^{-5}$	0.5474

In regression analysis, R^2 term is the coefficient of determination and it shows the prediction performance of the regression equation for the statistical data.

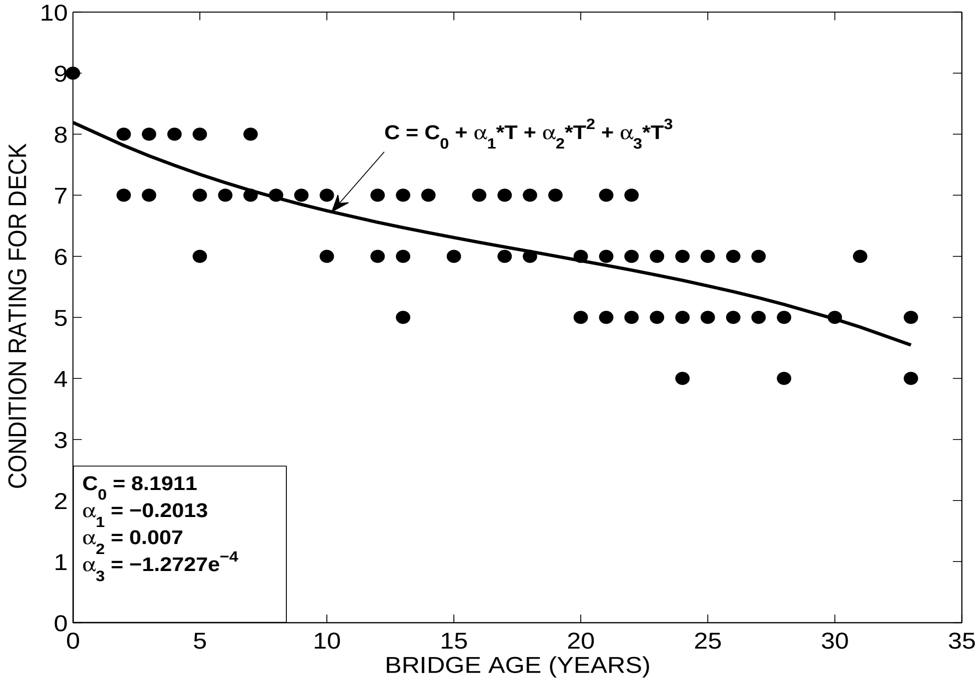


Figure 5.3: Condition Ratings and Polynomial Regression Curve for Concrete Bridge Decks of Interstate Highway Bridges.

The bridge condition ratings and polynomial-based regression curves and equations for concrete bridge components are shown in Figures 5.3 through 5.5. There are concrete bridges with different ages. The ages of the bridges for this data group range between 0 and 33. In addition, the condition rating data set of the concrete bridges are composed of deck condition rating data for 54 bridges, substructure data for 58 bridges, and superstructure condition data for 55 bridges. Subsequently, all data is analyzed by polynomial regression method. As a result, the coefficients of

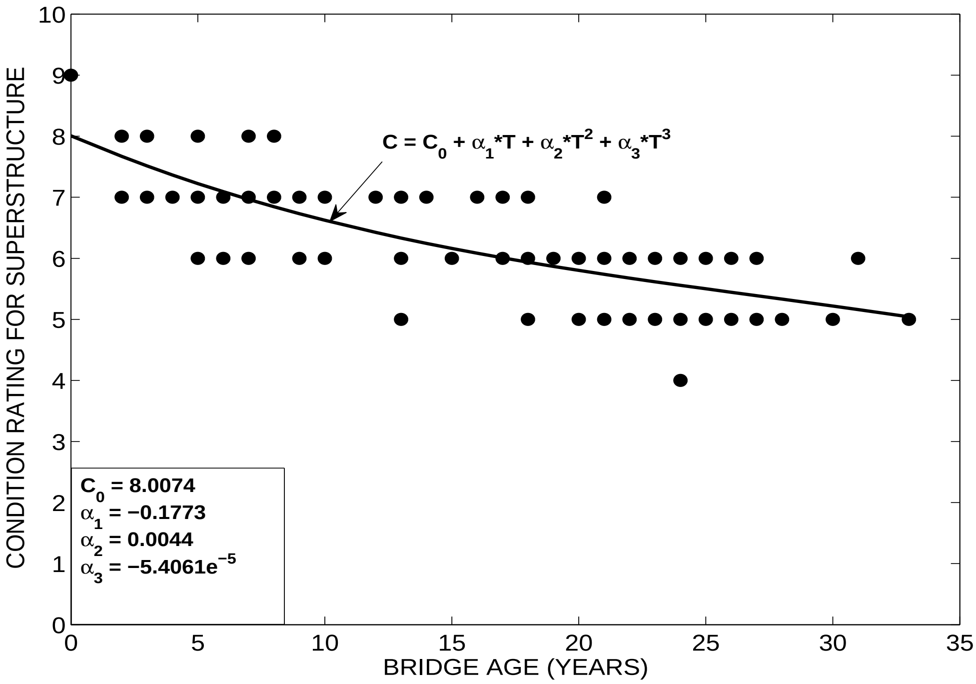


Figure 5.4: Condition Ratings and Polynomial Regression Curve for Concrete Bridge Superstructures of Interstate Highway Bridges.

polynomial-based regression equations are obtained and presented in Table 5.1. Furthermore, R^2 values for deck, substructure, and superstructure of concrete bridges on interstate highways are shown in the last column of Table 5.1.

Another data set is the condition ratings of steel bridge components presented from Fig. 5.6 through Fig. 5.8. The ages of bridges in this data group range between 0 and 39. Total number of steel bridges on interstate highways in this data set are 169. 57 bridges out of a total of 169 bridges represent the condition ratings of the decks as shown in Fig. 5.6 and 56 bridges represent the superstructure data as shown in Fig. 5.7. Finally, 56 bridges are shown in Fig. 5.8 representing substructure condition ratings. All of the condition rating data is examined by using the polynomial regression method. Consequently, polynomial-based regression equations are obtained and the coefficients of these equations are presented in Table 5.2 for steel bridges. In addition, R^2 values for deck, substructure, and superstructure of steel bridges on interstate highways are shown in the last column of Table 5.2.

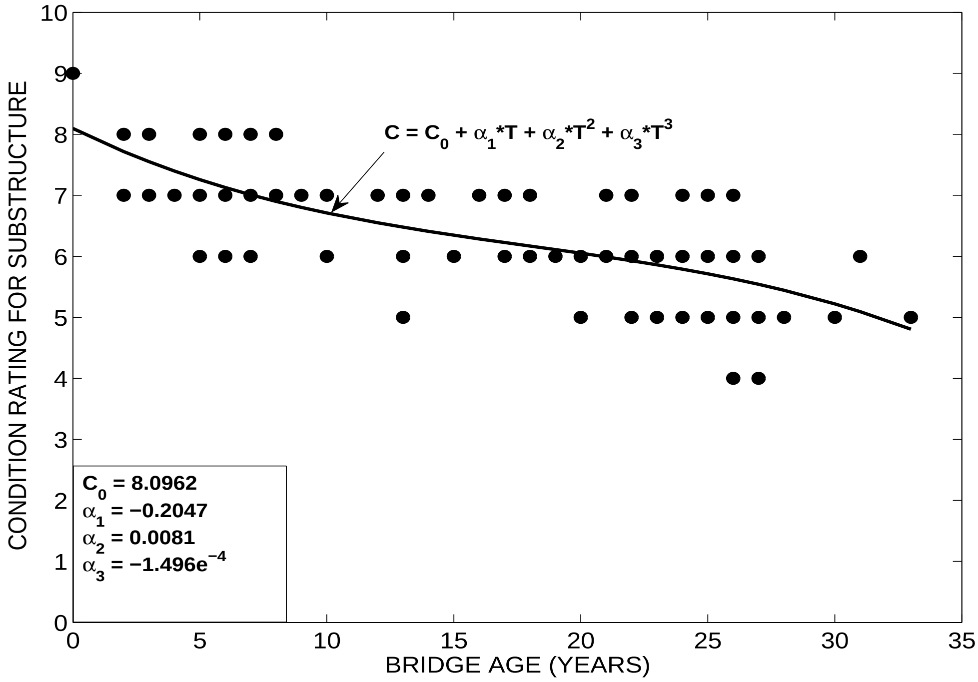


Figure 5.5: Condition Ratings and Polynomial Regression Curve for Concrete Bridge Substructures of Interstate Highway Bridges.

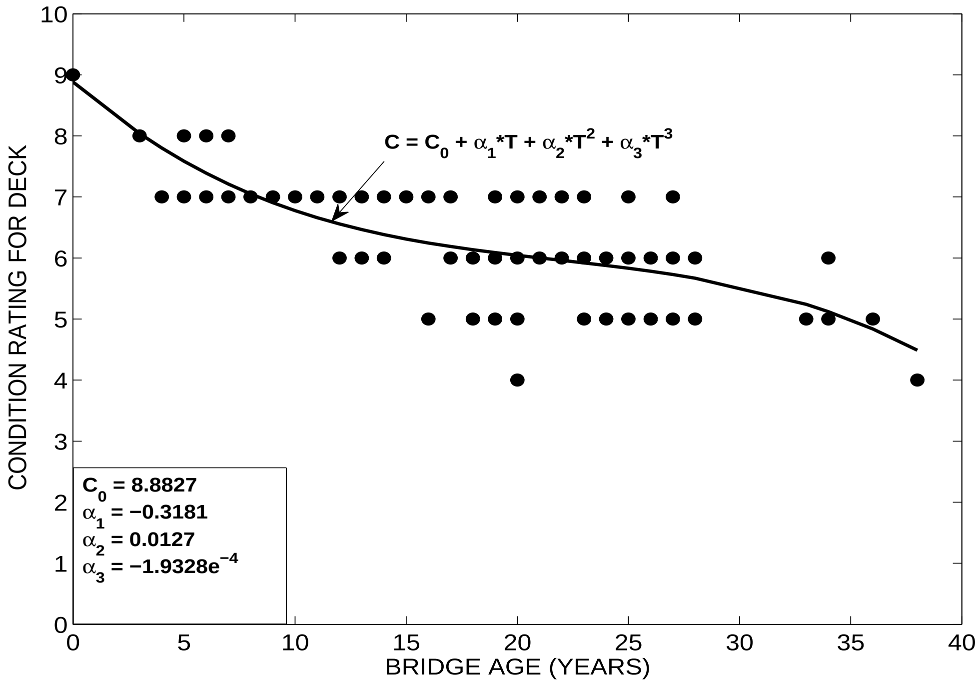


Figure 5.6: Condition Ratings and Polynomial Regression Curve for Steel Bridge Decks of Interstate Highway Bridges.

Table 5.2: The Coefficients of Polynomial-based Performance Prediction Equation for Interstate Highway Steel Bridge Condition Data used.

	C_0	α_1	α_2	α_3	R^2
Deck	8.8827	-.3181	0.0127	$-1.9328e^{-4}$	0.5496
Substructure	8.6403	-0.2415	0.009	$-1.4589e^{-4}$	0.5680
Superstructure	8.6471	-0.2522	0.0103	$-1.7273e^{-4}$	0.5333

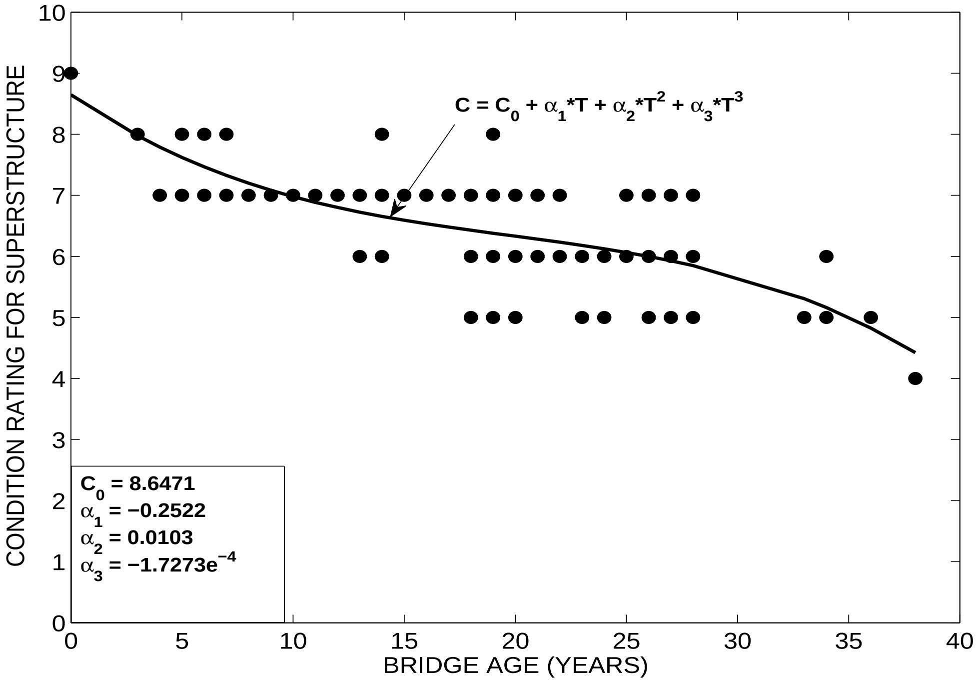


Figure 5.7: Condition Ratings and Polynomial Regression Curve for Steel Bridge Superstructures of Interstate Highway Bridges.

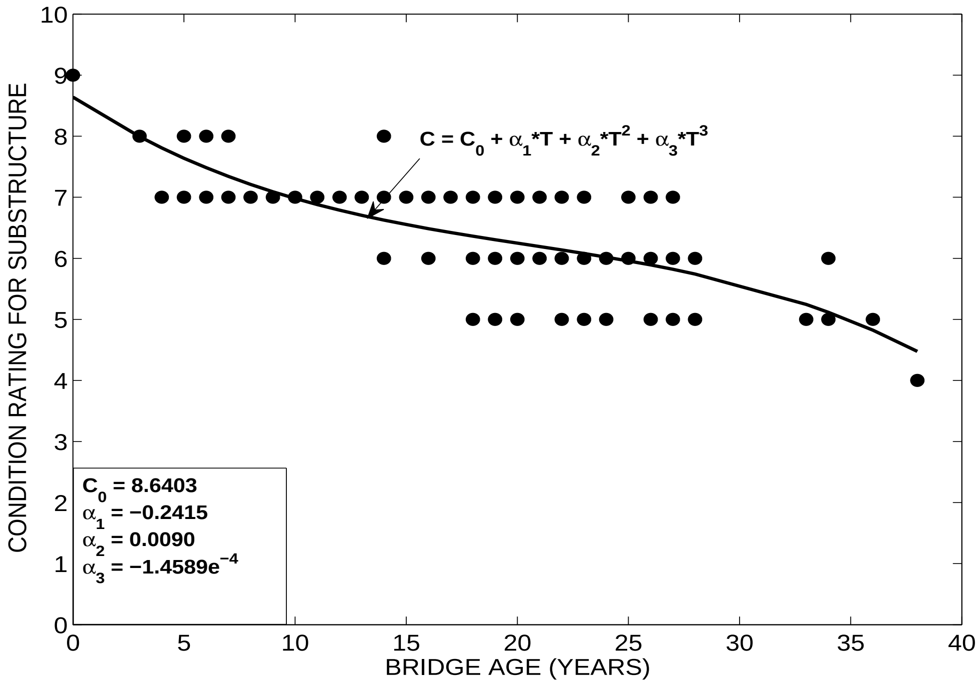


Figure 5.8: Condition Ratings and Polynomial Regression Curve for Steel Bridge Substructures of Interstate Highway Bridges.

The condition rating data of concrete bridge components on highways other than interstate highways (i.e, State and Local highways) which will be referred to as Otherstate highways in the State of Indiana are shown in Figures 5.9 through Figure 5.11. The age range of the otherstate highways are greater than that of Interstate highways. As shown in Fig. 5.13, the ages of concrete bridges on Otherstate highways range between 0 and 60 years old. Moreover, the sample size of this data group is larger than total number of Interstate highways. Total number of bridges for the condition rating data for concrete bridge components on Otherstate highways are 278.

Polynomial regression method is applied to this data and polynomial-based regression equations are obtained. The coefficients of regression equations and the coefficient of determination values (R^2) are presented in Table 5.3.

The steel bridge components on Otherstate highways is the last group of condition rating data set. These condition rating data sets are presented in Figures 5.12 through 5.14. These data sets represent the maximum number of bridges as compared to the previous data sets. The number of steel bridges on Otherstate highways is 384. The

Table 5.3: The Coefficients of Polynomial-based Performance Prediction Equation for Otherstate Highway Concrete Bridges Condition Rating Data used.

	C_0	α_1	α_2	α_3	R^2
Deck	8.3647	-.2707	0.0068	$-6.5485e^{-5}$	0.5397
Substructure	8.1872	-0.2648	0.0079	$-8.4097e^{-5}$	0.4609
Superstructure	8.5785	-0.3299	0.0101	$-1.0518e^{-4}$	0.5115

bridge age for these bridges is older than the bridge age of data sets mentioned earlier. The ages of the bridges range from 0 to 63 for steel bridges on Otherstate highways. All of the data is analyzed by polynomial regression method, and polynomial-based regression equations are obtained for deck, substructure, and superstructure, separately. These coefficients of equations and coefficient of determinations for these bridge components are presented in Table 5.4.

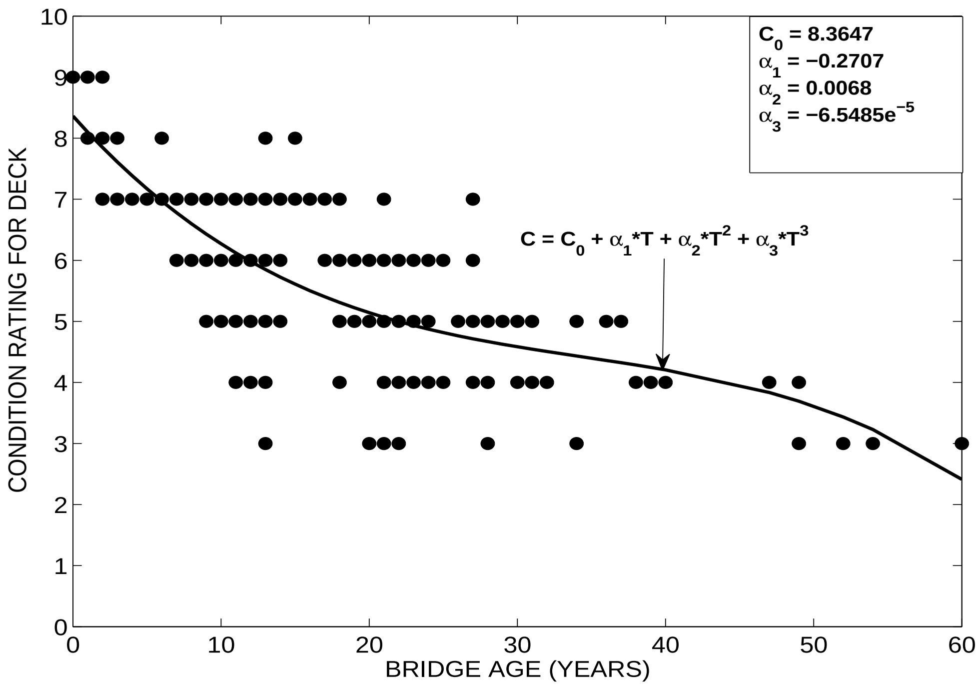


Figure 5.9: Condition Ratings and Polynomial Regression Curve for Concrete Bridge Decks of Otherstate Highway Bridges.

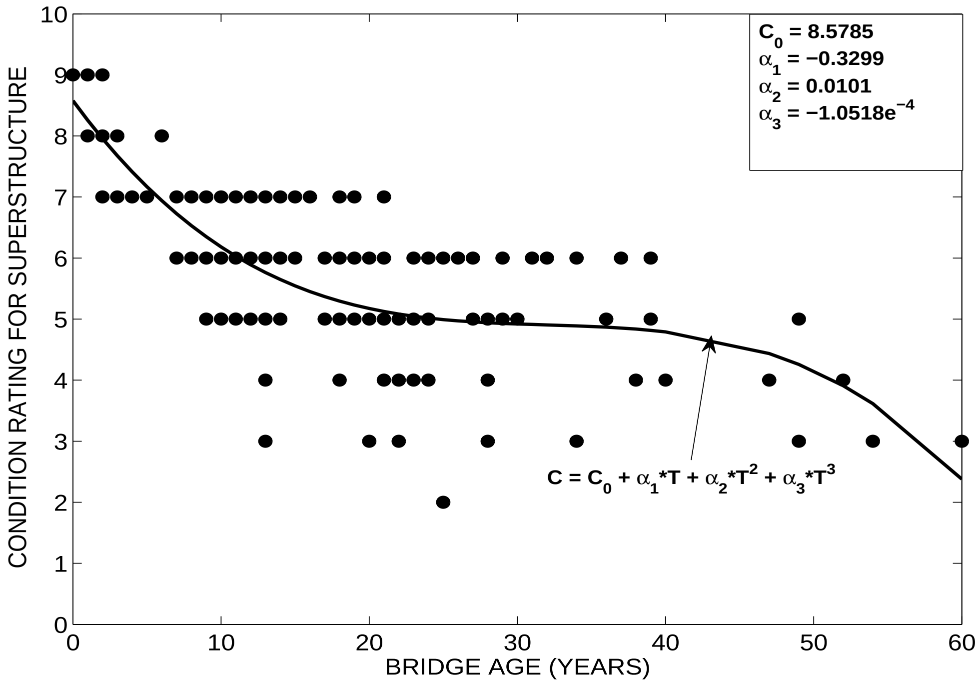


Figure 5.10: Condition Ratings and Polynomial Regression Curve for Concrete Bridge Superstructures on Otherstate Highway Bridges.

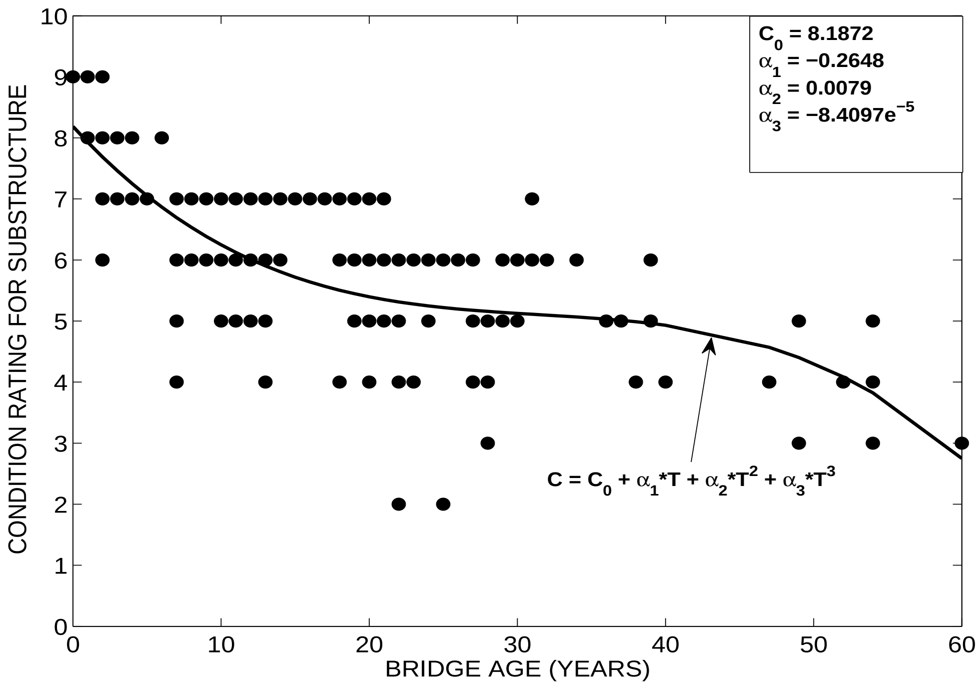


Figure 5.11: Condition Ratings and Polynomial Regression Curve for Concrete Bridge Substructures on Otherstate Highway Bridges.

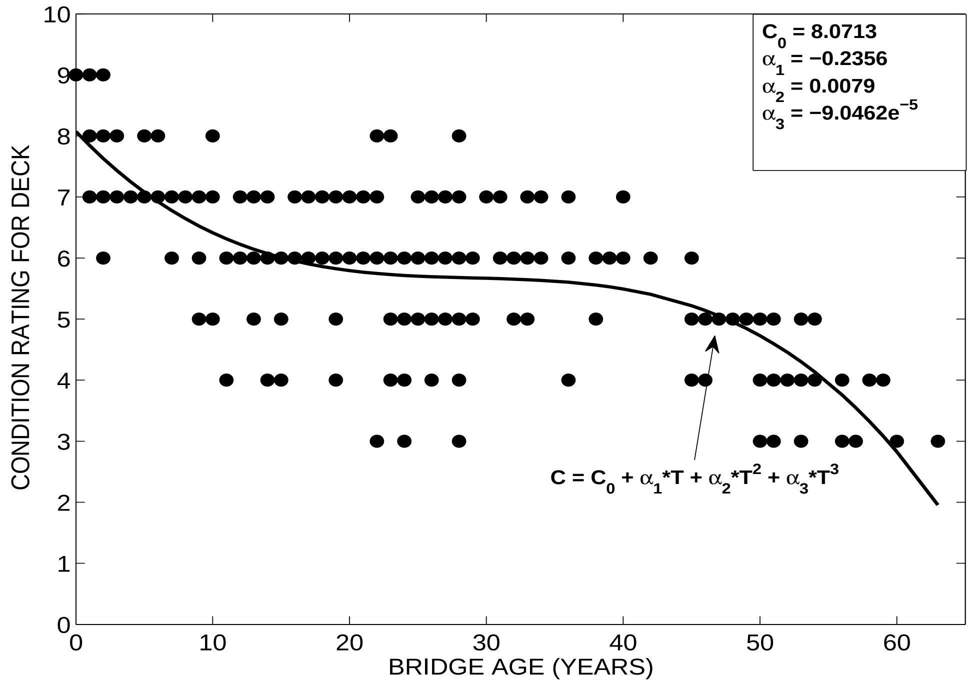


Figure 5.12: Condition Ratings and Polynomial Regression Curve for Steel Bridge Decks on Otherstate Highway Bridges.

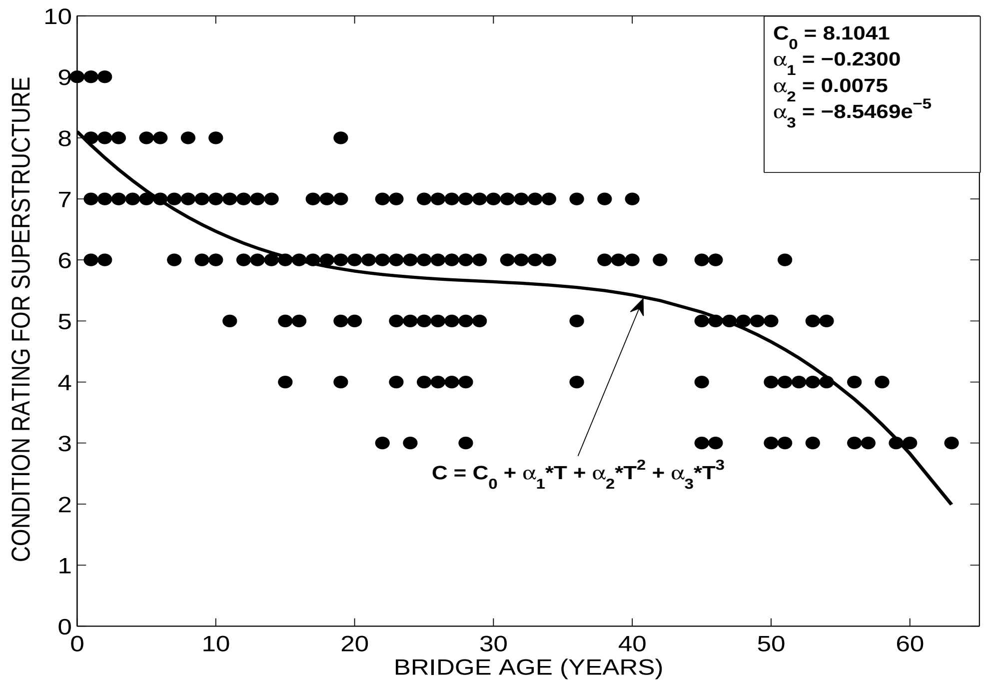


Figure 5.13: Condition Ratings and Polynomial Regression Curve for Steel Bridge Superstructures on Otherstate Highway Bridge

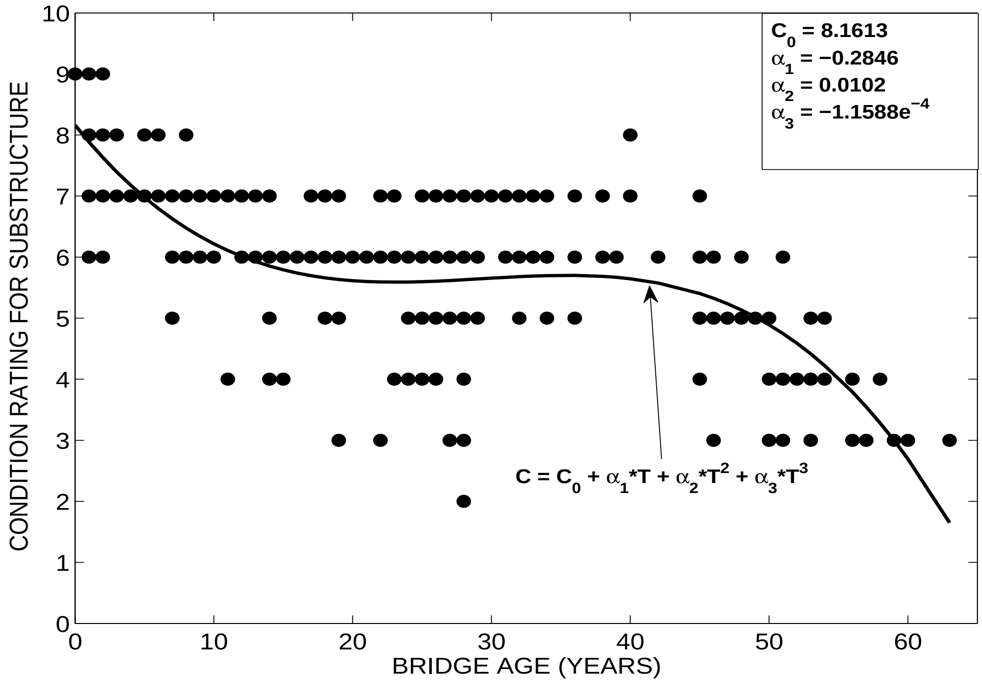


Figure 5.14: Condition Ratings and Polynomial Regression Curve for Steel Bridge Substructures on Otherstate Highway Bridge

Table 5.4: The Coefficients of Polynomial-based Performance Prediction Equation for Otherstate Highway Steel Bridges Condition Ratings Data used

	C_0	α_1	α_2	α_3	R^2
Deck	8.0713	-.2356	0.0079	$-9.0462e^{-5}$	0.4693
Substructure	8.1613	-0.2846	0.0102	$-1.1588e^{-4}$	0.4371
Superstructure	8.1041	-0.23	0.0075	$-8.5469e^{-5}$	0.4966

5.3 Simulation-Based Condition Prediction

The regression-based performance prediction is one of the performance prediction models. This performance prediction model may be obtained by applying the regression analysis to a real condition rating data set for bridges. In such a data set, condition rating is the dependent variable whereas bridge age is the independent variable. In addition, the fitting accuracy of the obtained regression equations for condition rating data sets may be checked by examining the coefficient of determination R^2. Once obtained, regression-based performance curve model can be directly used to predict the condition of an infrastructure group at any future time.

In this section, a new concept of condition prediction combining both simulation and regression techniques will be introduced. The coefficients of the polynomial-based performance curves obtained by regression analysis are now treated as random variables and their distributions are generated using Latin Hypercube simulation technique with different coefficient of variation (COV) values, and a new mean condition rating profile is computed based on the random coefficient parameters. Simulations with different COV values lead to many condition rating values with different values to store in the same year. As a result, a simulation-based regression curve can be plotted by using the mean condition rating based on all condition rating data in the same year. In addition, as shown in Figures 5.15 through 5.50, probability density distributions of these condition rating data at every ten year intervals are plotted to show the distribution (or dispersion) of the condition rating at each point in time (i.e, every 5 years) over the structure's lifetime.

All simulations to obtain the simulation-based performance prediction curves generated using the Latin Hypercube simulation technique. The coefficients of the polynomial-based regression equations, i.e, C_0, α_1, α_2, and α_3 are simulated. The probability distributions of these coefficients are assumed as Normal distribution. In addition, number of simulations used is 1000 in order to obtain reliable sample space to represent all coefficients. Furthermore, the deterministic values of these coefficients are taken as expected values. Standard deviation of these parameters are not known. Therefore, in these simulations, therefore, the standard deviation values for the coefficients are obtained based on three different assumed values of the coefficient of variation COV.

For example, 0.05, 0.10, and 0.15 are used as COV values for each of the regression equation parameters of the condition rating data for all bridge component types.

In this study, simulation process is applied to the regression equations for all bridge component types. There are 12 different bridge component types used in this study (Based on deck, superstructure, substructure, concrete, steel, interstate, otherstate criteria). 12 different regression equation curves are subjected to simulation process with 3 different COV values. Thus, this simulation process yields 36 different simulated performance prediction curves which are presented in Figures 5.15 through 5.50.

In Figures 5.15 through 5.17, regression-based condition prediction curves and probability density distributions of condition rating data generated by using simulation for the deck components of the concrete bridges on Interstate highways are presented. As shown these figures, simulation analysis is conducted for 40 years of lifetime. In addition, the condition rating value is approximately 8.2 at the beginning of the analysis and it reaches 3 at the end of the analysis time. Figures 5.15 through 5.17 are plotted for three different values of the coefficient of variation for the regression equation coefficients such as 0.05, 0.10, and 0.15. For instance, Fig. 5.15 is plotted by using the value of COV as 0.05 for the regression parameters. As shown in probability density plots of in figure, dispersion of the condition rating data is very small at a given year. Furthermore, the variation of the standard deviation plotted by the dotted line in the same figure is very small throughout the analysis period. The graph in Fig. 5.16 is plotted by using the COV of 0.10. In this figure, there is a big difference between the maximum and minimum values of simulated condition rating values (i.e, samples) in the same year. In addition, standard deviation for condition rating increases toward the end of the lifetime. Fig. 5.17 is the third figure for this bridge component type and represents the simulated condition prediction curve using the COV of 0.15. As shown in the figure, there is a significant variation (very large dispersion) in the condition ratings at a given year. Therefore, standard deviation in this figure indicates the big dispersion by substantially increasing at the end of the simulation analysis period.

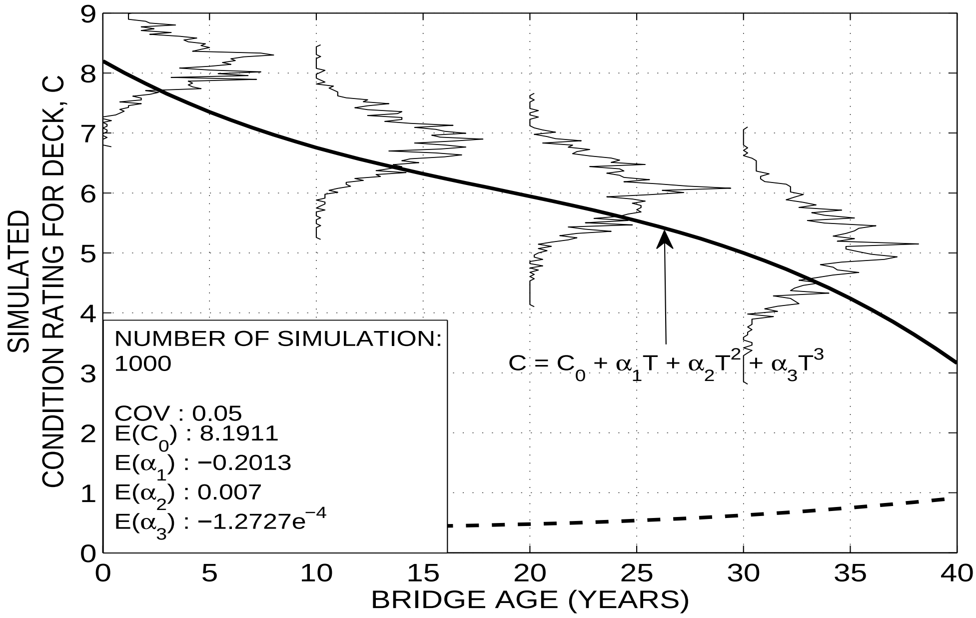


Figure 5.15: Simulation of Polynomial Regression Equation with 5 % COV to predict the Condition Rating of Concrete Bridge Decks on Interstate Highways

The next three figures, i.e, Fig. 5.18 through Fig. 5.20 show the simulated regression-based condition prediction curves for the substructure components of the concrete bridges on Interstate highways. Similar to the deck component, simulation analysis for the substructure components is conducted for the 40 year period. As shown in these figures, the condition rating value is approximately 8.1 at the beginning of the analysis and it reaches about 3.3 at the end of the analysis period. Figures 5.18 through 5.20 are plotted for three different values of the coefficient of variation for the regression equation coefficient, i.e; 0.05, 0.10, and 0.15. As shown in Fig. 5.18, there is a small difference between the maximum and minimum values of the samples (i.e values of the probability density distribution plotted along vertical axes) of condition rating data for the same year. Furthermore, the change in standard deviation in the same figure is very small throughout the analysis period. Fig. 5.19 is plotted using a COV of 0.10. In this figure, dispersion of the condition rating data is larger at time periods. In addition, standard deviation for condition rating is also increased. Fig. 5.20 is plotted using a COV of 0.15. As shown in the figure, there is a substantial variation in simulated values of the condition ratings at displayed time periods. Standard deviation displays a very large dispersion at the end of the simulation analysis time.

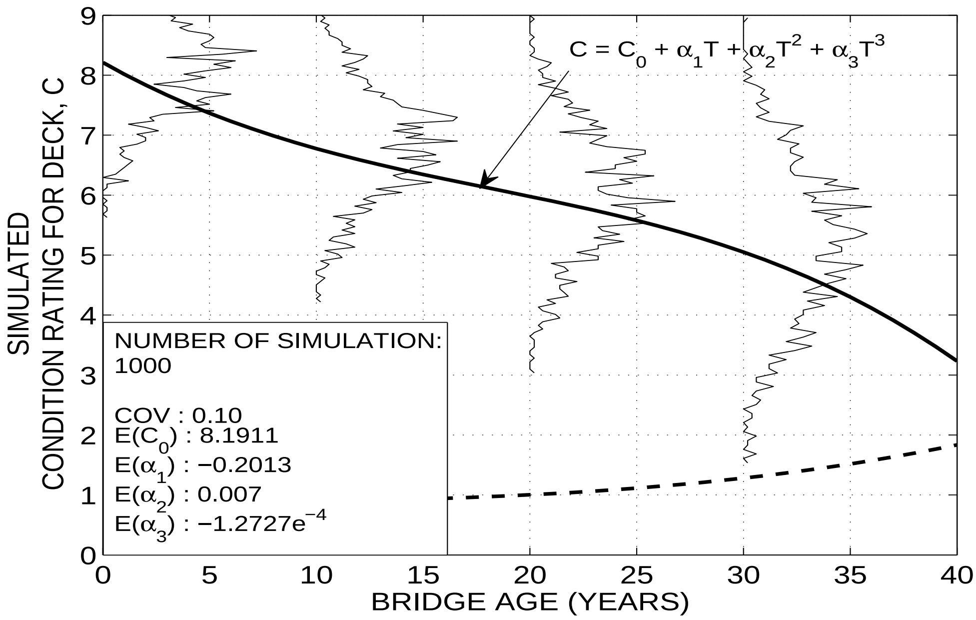


Figure 5.16: Simulation of Polynomial Regression Equation with 10 % COV to predict the Condition Rating of Concrete Bridge Decks on Interstate Highways

Similarly, Figures 5.21 through 5.23 are generated for the superstructure components of concrete bridges. The condition rating value is approximately 8 at the beginning of the analysis and reaches 4.5 at the end of the analysis time. Similar to the previous analyses, three different values of the coefficient of variation used for the regression equation coefficients are 0.05, 0.10, and 0.15. The profile in Fig. 5.21 is based on a COV of 0.05 for the regression parameters. There is a little dispersion of the condition rating in time. Standard deviation plotted by the dotted line is very small throughout the analysis time and reaches 0.8 at the end of 40 years. The profile in Fig. 5.22, is based on COV of 0.10. Standard deviation for condition rating reaches 1.2 at the end of the 40 years. Finally, Fig. 5.23 shows the simulated condition prediction curve based on a COV of 0.15. A very large dispersion of the condition ratings is visible. Standard deviation also displays a very large dispersion by reaching 2 at the end of the simulation analysis time.

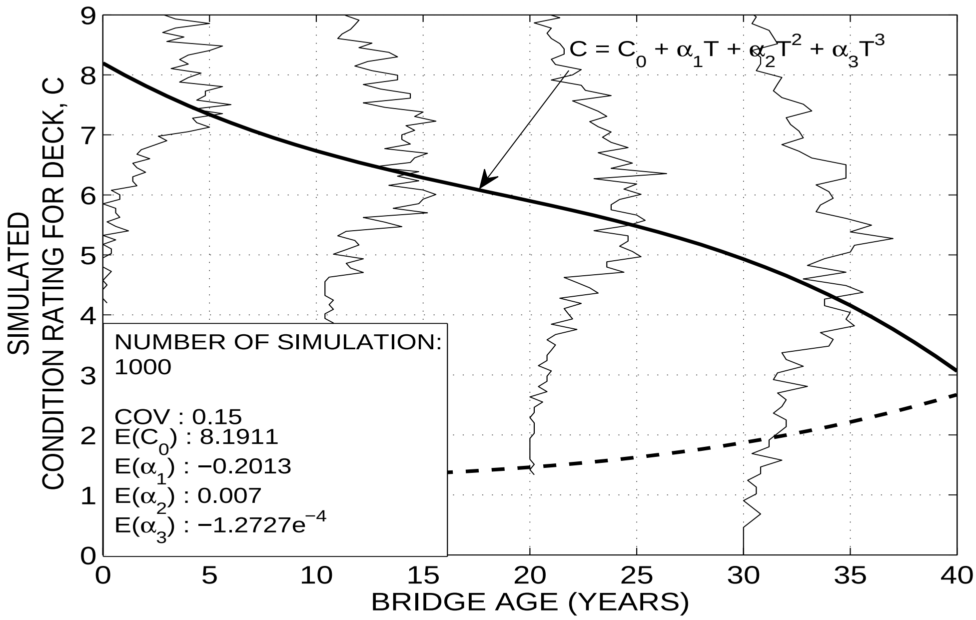


Figure 5.17: Simulation of Polynomial Regression Equation with 15 % COV to predict the Condition Rating of Concrete Bridge Decks on Interstate Highways.

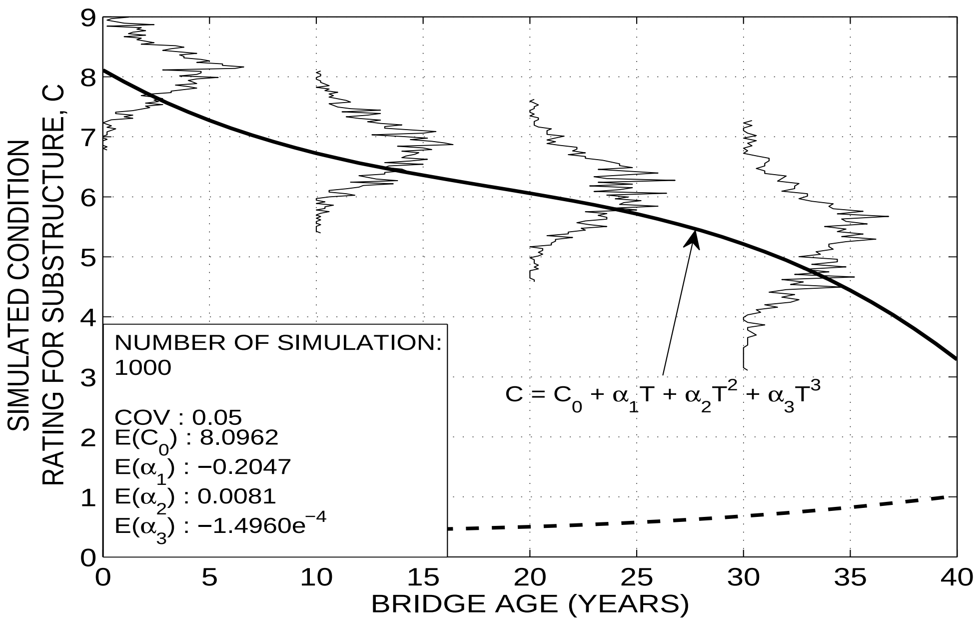


Figure 5.18: Simulation of Polynomial Regression Equation with 5 % COV to predict the Condition Rating of Concrete Bridge Substructures on Interstate Highways

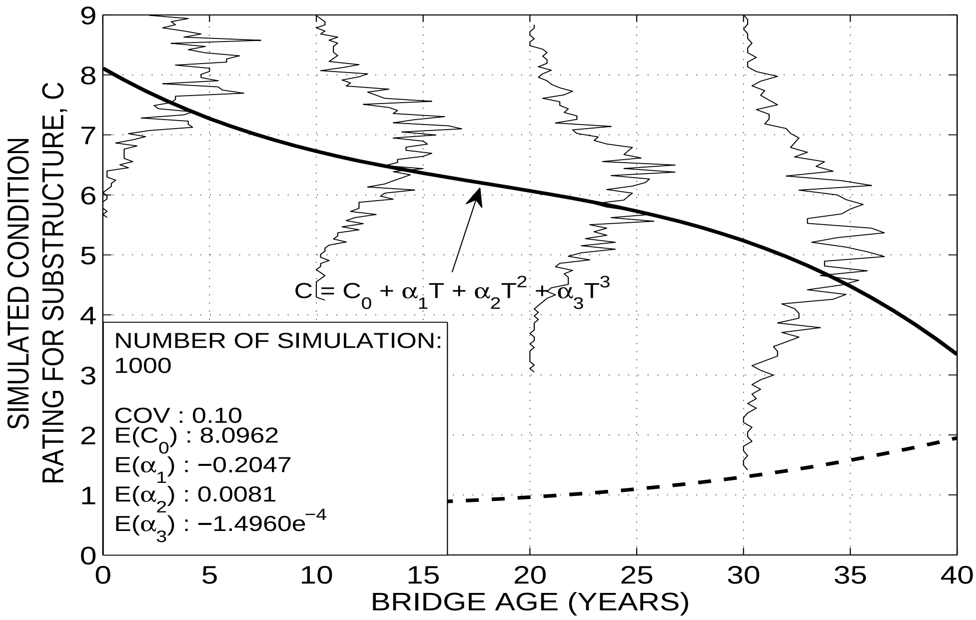


Figure 5.19: Simulation of Polynomial Regression Equation with 10 % COV to predict the Condition Rating of Concrete Bridge Substructures on Interstate Highways

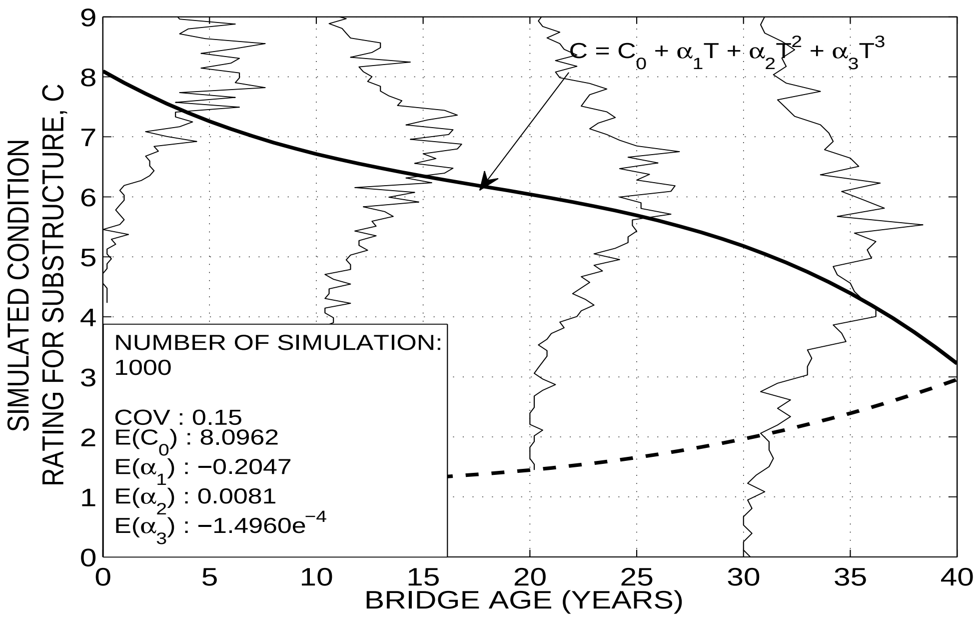


Figure 5.20: Simulation of Polynomial Regression Equation with 15 % COV to predict the Condition Rating of Concrete Bridge Substructures on Interstate Highways

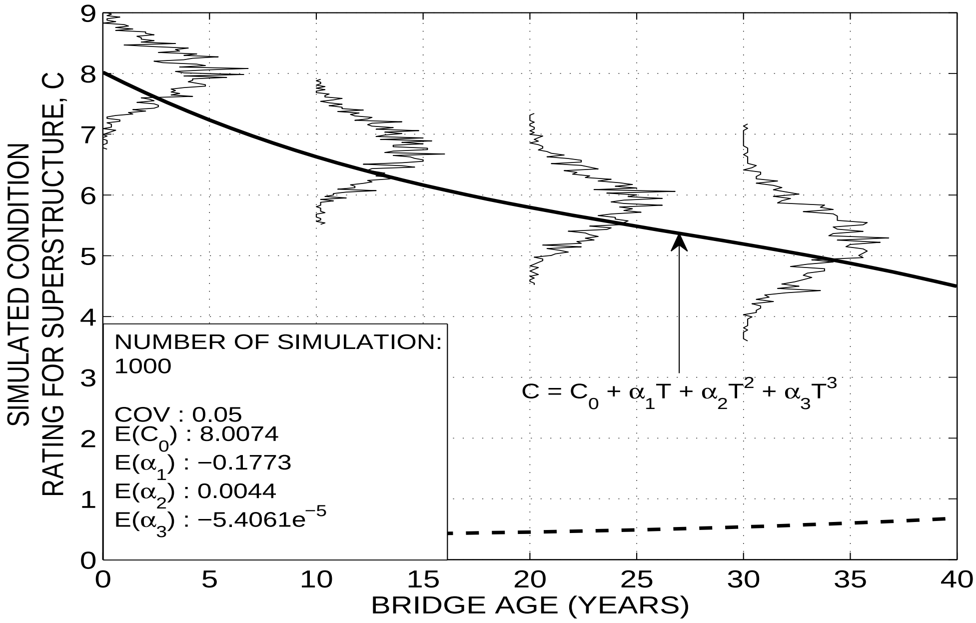


Figure 5.21: Simulation of Polynomial Regression Equation with 5 % COV to predict the Condition Rating of Concrete Bridge Superstructures on Interstate Highways

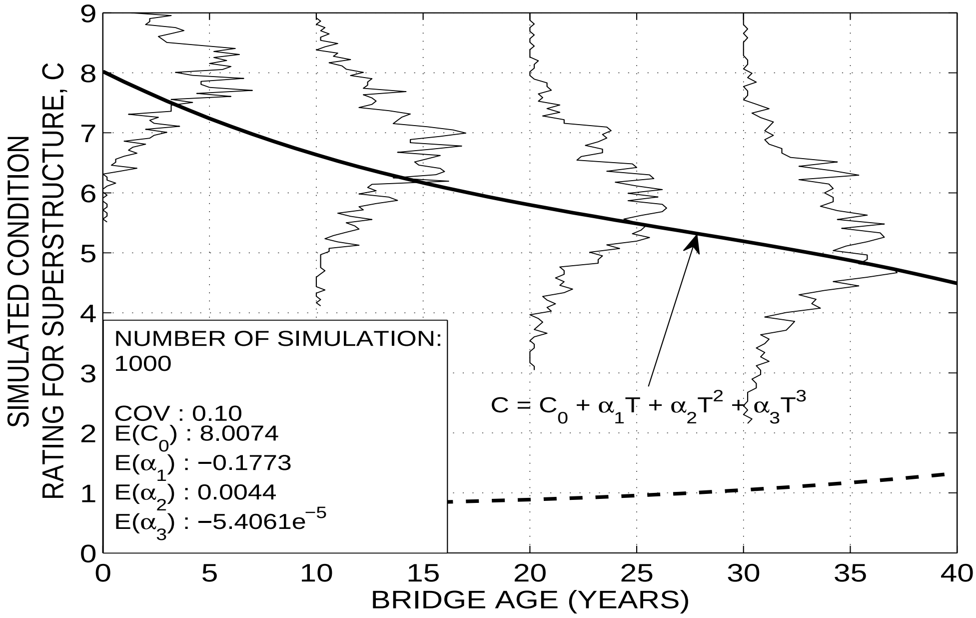


Figure 5.22: Simulation of Polynomial Regression Equation with 10 % COV to Predict the Condition Rating of Concrete Bridge Superstructures on Interstate Highways

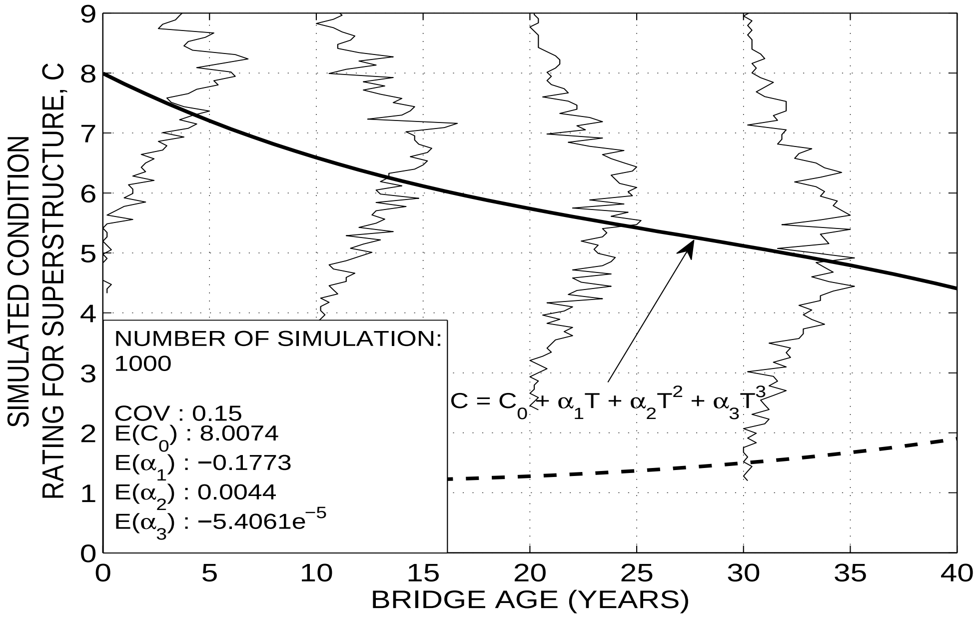


Figure 5.23: Simulation of Polynomial Regression Equation with 15 % COV to Predict the Condition Rating of Concrete Bridge Superstructures on Interstate Highways

In Figures 5.24 through 5.26, regression-based condition prediction curves and probability density distributions of condition rating data generated by using simulation for the deck components of the steel bridges on Interstate highways are presented. The profiles show that the condition rating value is approximately 8.9 at the beginning of the analysis and reaches 4.2 at the end of 40 years. The profile in Fig. 5.24 is plotted using a COV as 0.05 for the regression parameters. Variation of the standard deviation is very small throughout the analysis time and reaches 1.5 at the end of 40 years. The profile in Fig. 5.25 is plotted based on a COV of 0.10. In this graph, there are large differences between the maximum and minimum simulated sample values of condition rating data. Standard deviation for condition rating reaches 2.8 at the end of 40 years. Finally, the profile in Fig. 5.26 is based on a COV of 0.15. Standard deviation indicates a large dispersion and reaches 4.3 at 40 year period.

The profiles in Fig. 5.27 through 5.29 are for the substructure components of steel bridges on Interstate highways. The condition rating value is approximately 8.6 at the beginning and reaches 4 at the end of 40 years. In Fig. 5.27, standard deviation reaches approximately 1 at the end of 40 years. The profile in Fig. 5.28 is based on a COV of 0.10. Standard deviation for condition rating reaches about 2.1 at the end of the analysis period. Finally, Fig. 5.29 is for a COV of 0.15. Standard deviation in this case reaches 3.2 at the end of 40 years.

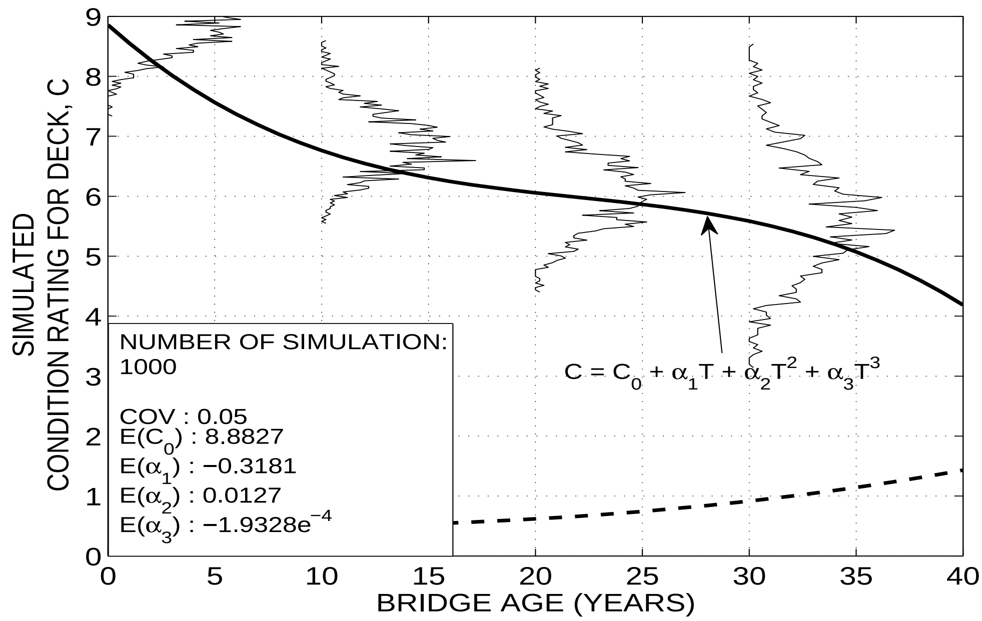


Figure 5.24: Simulation of Polynomial Regression Equation with 5 % COV to predict the Condition Rating of Steel Bridge Decks on Interstate Highways

The profiles in Figures 5.30 through 5.32 are for the superstructure components of the steel bridges on Interstate highways. The condition rating value is approximately 8.7 at the beginning and reaches approximately 4 at the end of 40 years. The profile in Fig. 5.30 is based on a COV of 0.05. Standard deviation plotted by the dotted line reaches around 1.3 at the end of 40 years. Similarly, the profile in Fig. 5.31 is based on a COV of 0.10. Standard deviation for condition rating reaches 2.5 at the end of the analysis period. Finally, Fig. 5.32 is for a COV of 0.15. As shown, there is substantial in the condition rating value in time. Standard deviation indicates this very large dispersion by reaching 3.6 at the end of the simulation analysis. A standard deviation of 3.6 for a mean condition rating of 4 corresponds to a COV of 90 % which represents an extremely large dispersion of mean value of condition rating at the end of 40 years. i.e, 4 is not a reliable estimate of condition rating of the structure at 40 years. It represents 4 ∓ 3.6 which corresponds to a condition rating value at the end of 40 years of anywhere between 0.4 and 7.6.

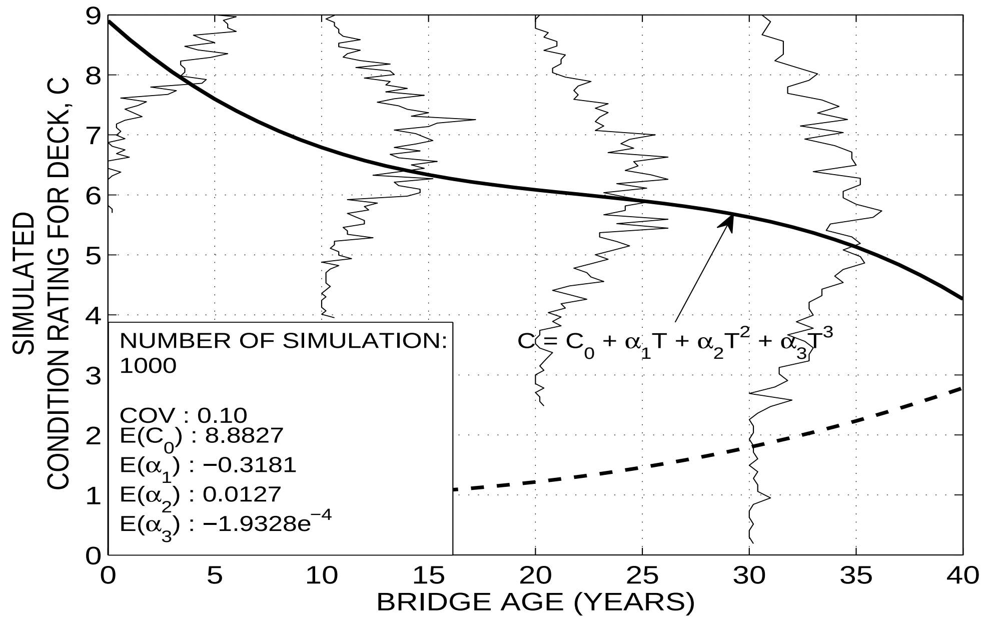


Figure 5.25: Simulation of Polynomial Regression Equation with 10 % COV to Predict the Condition Rating of Steel Bridge Decks on Interstate Highways

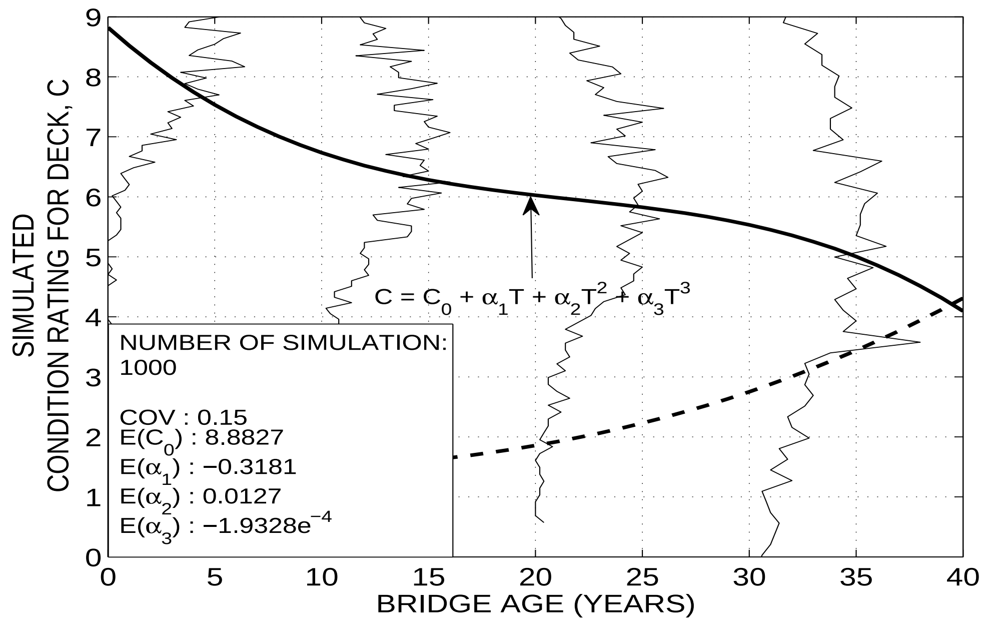


Figure 5.26: Simulation of Polynomial Regression Equation with 15 % COV to Predict the Condition Rating of Steel Bridge Decks on Interstate Highways

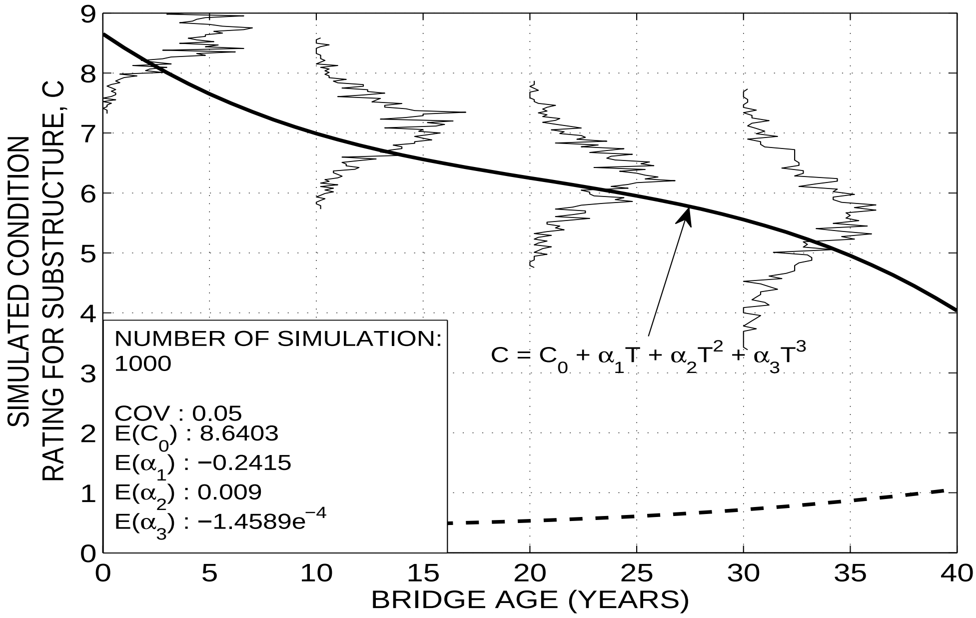


Figure 5.27: Simulation of Polynomial Regression Equation with 5 % COV to Predict the Condition Rating of Steel Bridge Substructures on Interstate Highways

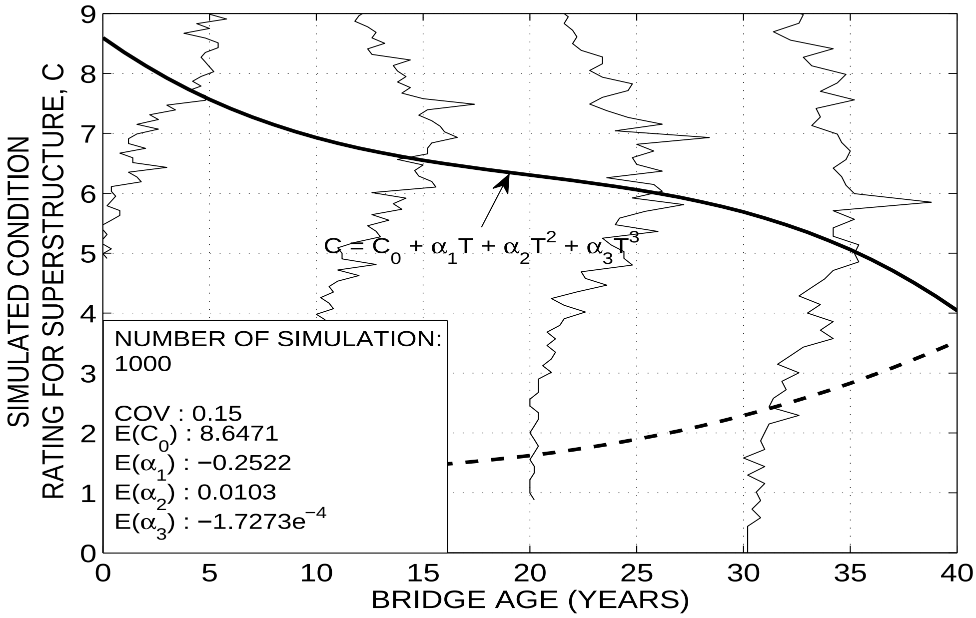


Figure 5.32: Simulation of Polynomial Regression Equation with 15 % COV to predict the Condition Rating of Steel Bridge Superstructures on Interstate Highways

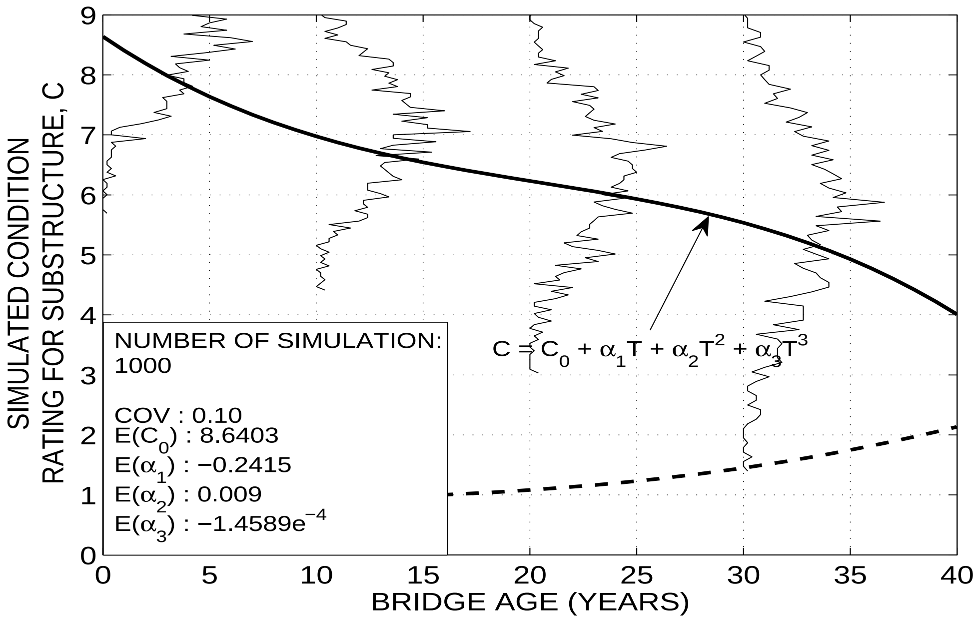


Figure 5.28: Simulation of Polynomial Regression Equation with 10 % COV to predict the Condition Rating of Steel Bridge Substructures on Interstate Highways

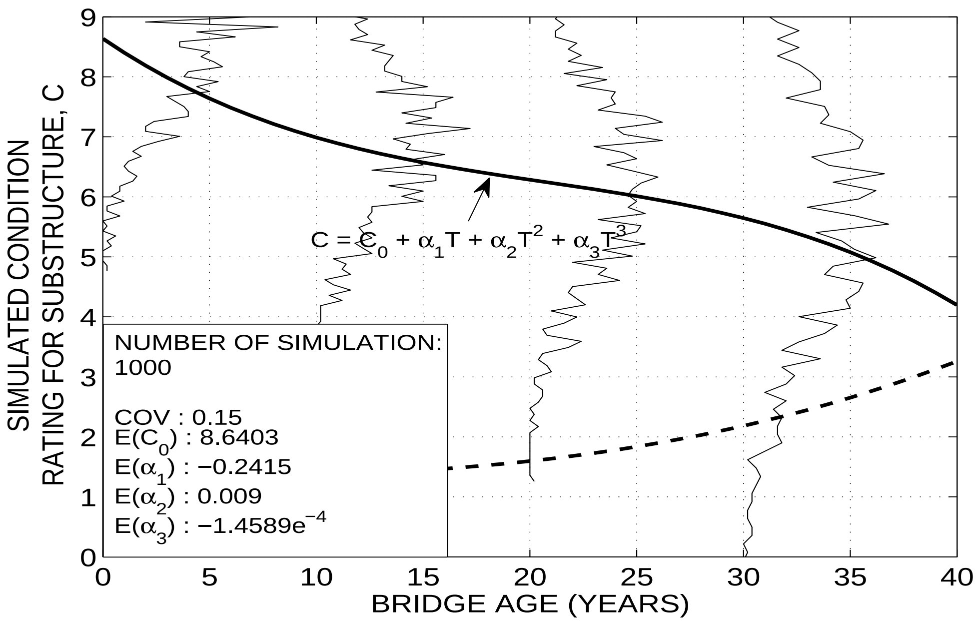


Figure 5.29: Simulation of Polynomial Regression Equation with 15 % COV to predict the Condition Rating of Steel Bridge Substructures on Interstate Highways

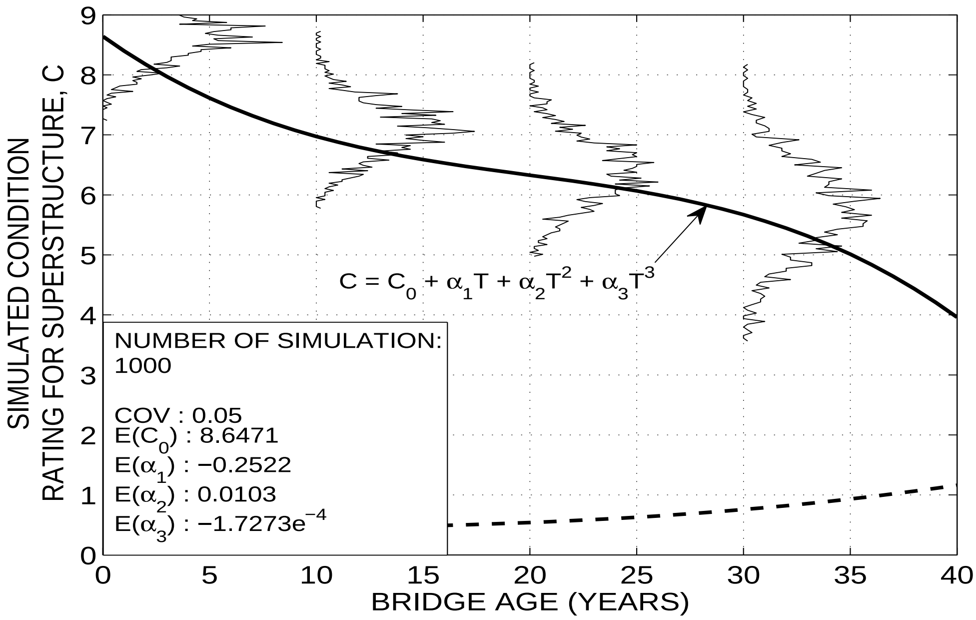


Figure 5.30: Simulation of Polynomial Regression Equation with 5 % COV to predict the Condition Rating of Steel Bridge Superstructures on Interstate Highways

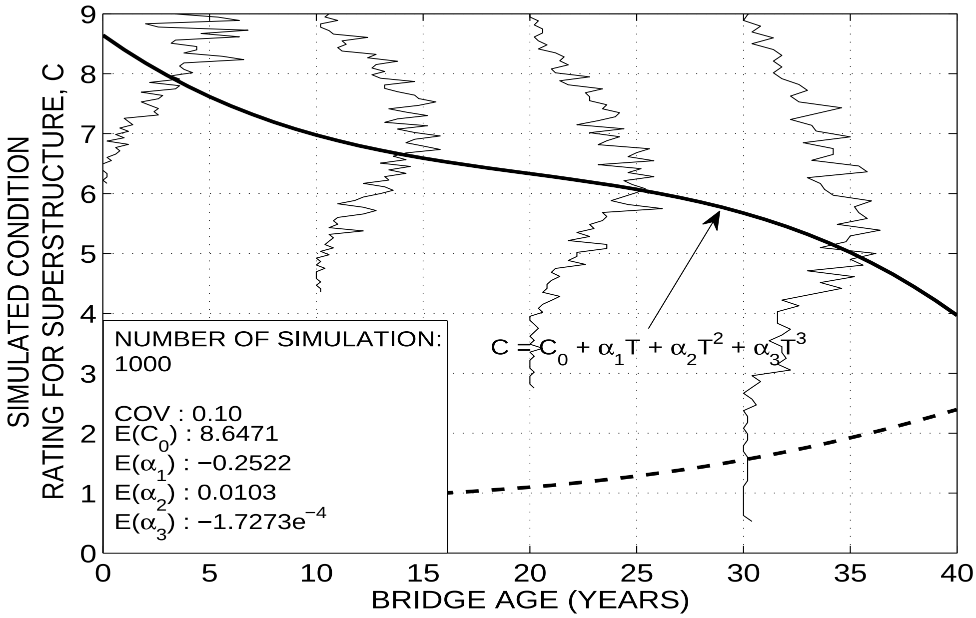


Figure 5.31: Simulation of Polynomial Regression Equation with 10 % COV to predict the Condition Rating of Steel Bridge Superstructures on Interstate Highways

The profiles in Figures 5.33 through 5.35 are for the deck components of the concrete bridges on Otherstate highways. The lifetime period of statistical data for deck is 60 years. As shown in these graphs, initial condition rating value is approximately 8.4 and its final value reaches around 2.5. Again three different values of the coefficient of variation are used for the regression equation coefficients consisting of 0.05, 0.10, and 0.15. The profile in Fig. 5.33 is based on a COV of 0.05 for the regression parameters. As shown in the graph, variation of dispersion of the condition rating data in time is small. However it is larger than that of the previous cases. Standard deviation has a small value throughout the period and reaches 1.8 at 40 years. The profile in Fig. 5.34 is based on a COV of 0.10. In this graph, there is substantial differences between the maximum and minimum values simulated sample values of the condition rating data. Standard deviation for condition rating reaches 3.3 at the end of 40 years, which is larger than the mean value (2.5). This means an high uncertainty for condition rating value at 40 years. Finally, Fig. 5.35 is based on a COV of 0.15. Standard deviation in this figure indicates an extremely large dispersion by reaching approximately 5 at the end of the analysis period.

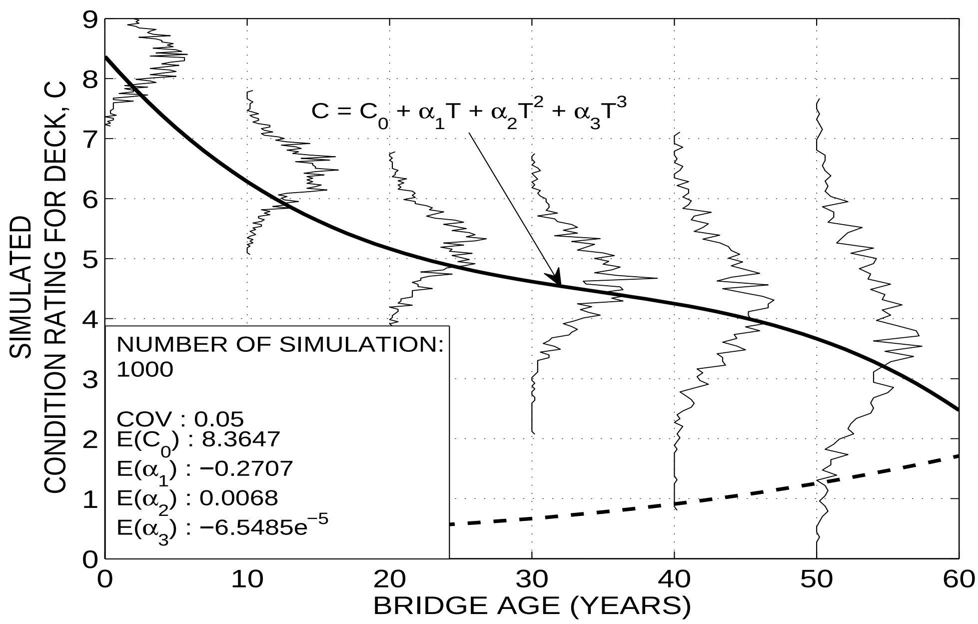


Figure 5.33: Simulation of Polynomial Regression Equation with 5 % COV to predict the Condition Rating of Concrete Bridge Decks on Otherstate Highways

Simulation-based regression condition prediction curves for the substructure components of the concrete bridges on Interstate highways are presented in Fig. 5.36 through 5.38. The lifetime of statistical data for substructure is 60 years. The initial condition rating is approximately 8.2 at the beginning and the condition rating value reaches 2.5 at the end of 60 years. The coefficient of variation used for the regression equation coefficient are 0.05, 0.10, and 0.15. Fig. 5.36 is based on a COV of 0.05. As shown in garph, dispersion of the condition rating data is very small at the beginning. However, dispersion of the condition rating data gets larger toward the end of the lifetime period. Standard deviation for substructure component reaches 2 at the end of the analysis. The profile in Fig. 5.37, is based on a COV of 0.10. As shown in figure, standard deviation reaches 3.7 at the end of 60 years. Fig. 5.38 is based on a COV of 0.15. As shown in figure, standard deviation reaches 5.7 at the end of 60 year. Standard deviation gets larger than the mean condition rating at the end of the lifetime period.

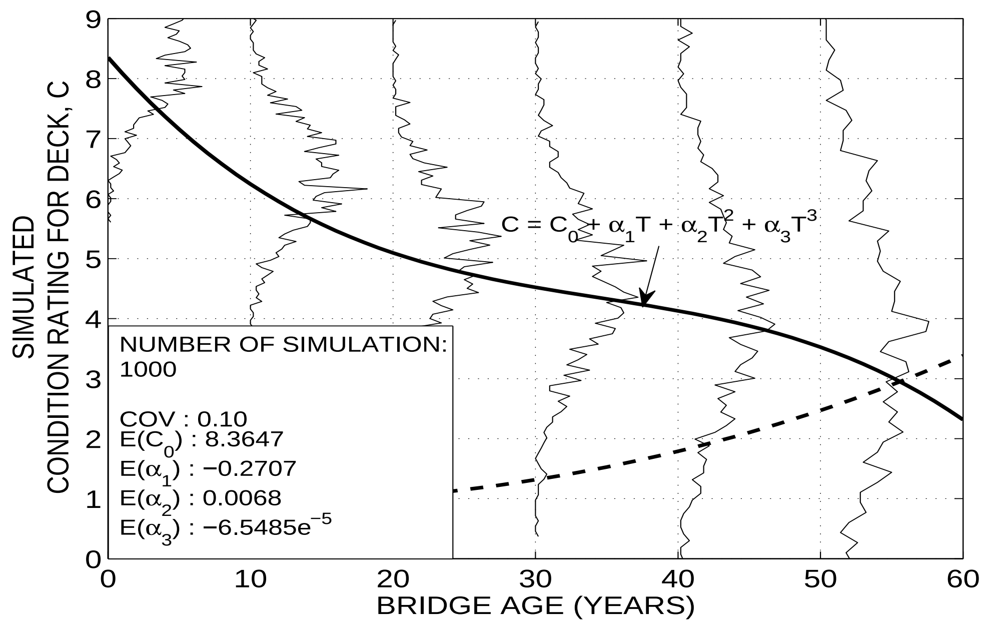


Figure 5.34: Simulation of Polynomial Regression Equation with 10 % COV to predict the Condition Rating of Concrete Bridge Decks on Otherstate Highways

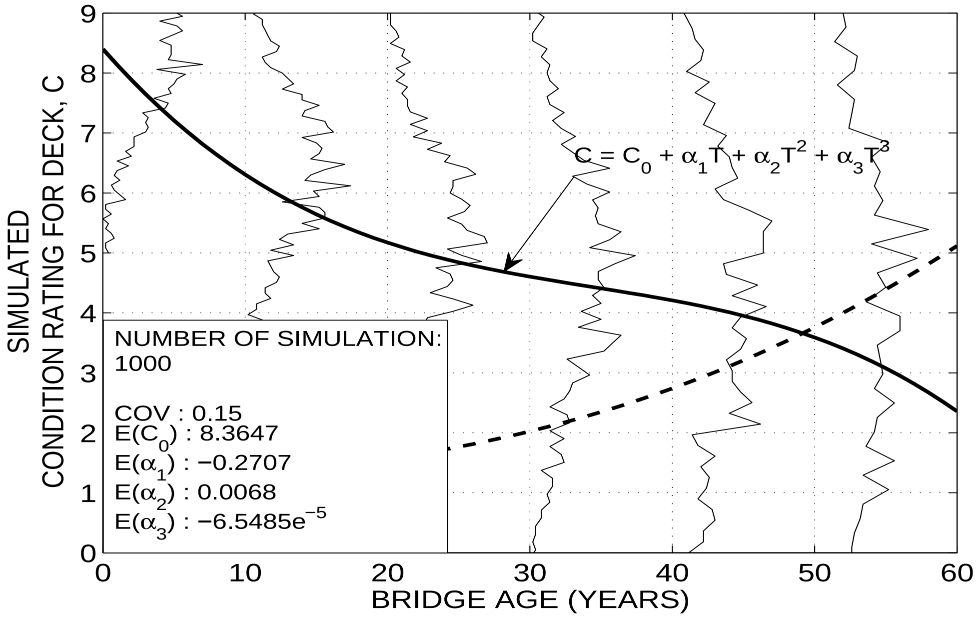


Figure 5.35: Simulation of Polynomial Regression Equation with 15 % COV to predict the Condition Rating of Concrete Bridge Decks on Otherstate Highways

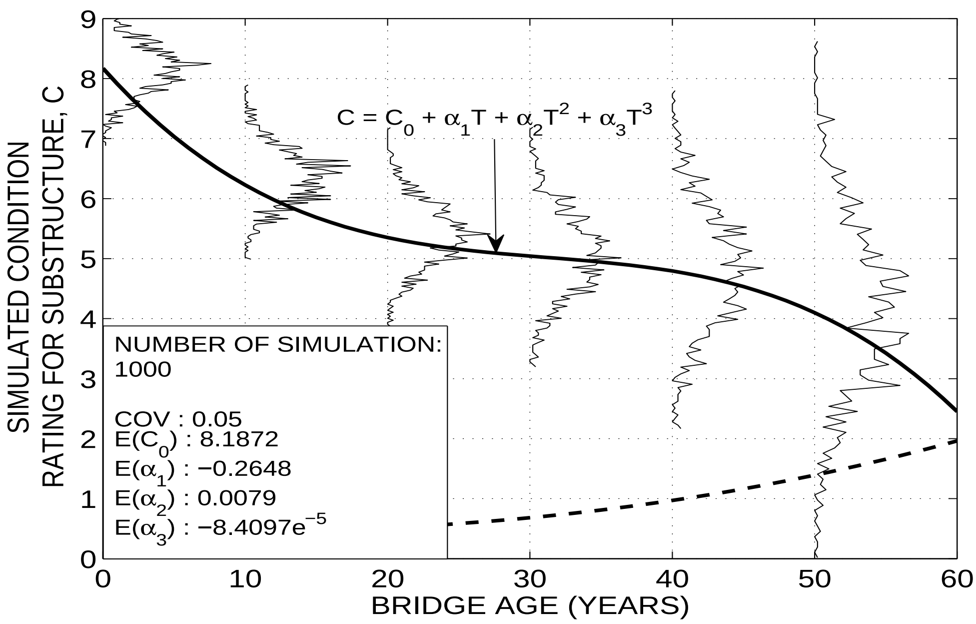


Figure 5.36: Simulation of Polynomial Regression Equation with 5 % COV to predict the Condition Rating of Concrete Bridge Substructures on Otherstate Highways

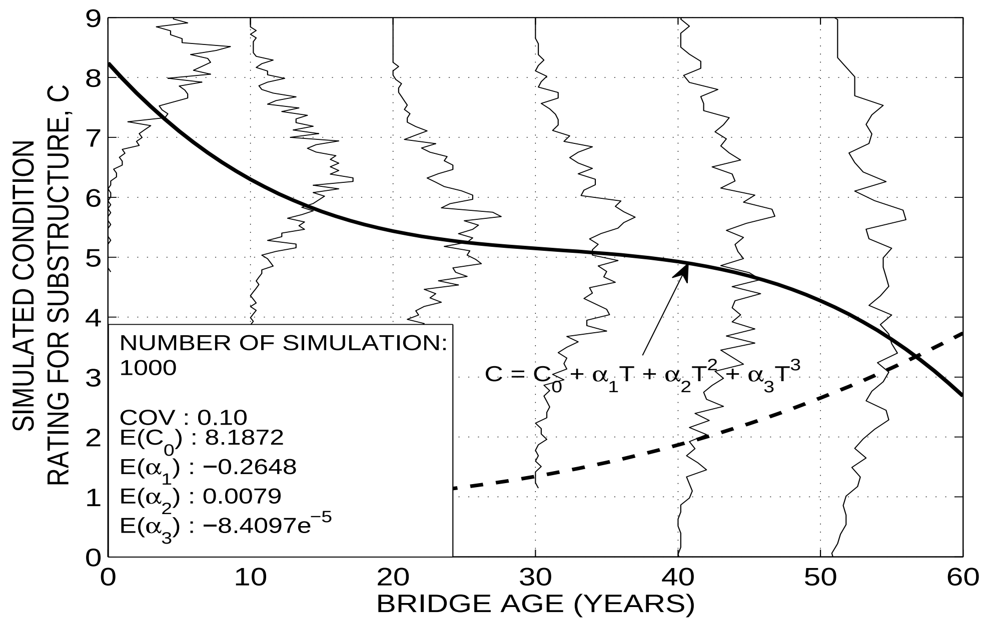


Figure 5.37: Simulation of Polynomial Regression Equation with 10 % COV to predict the Condition Rating of Concrete Bridge Substructures on Otherstate Highways

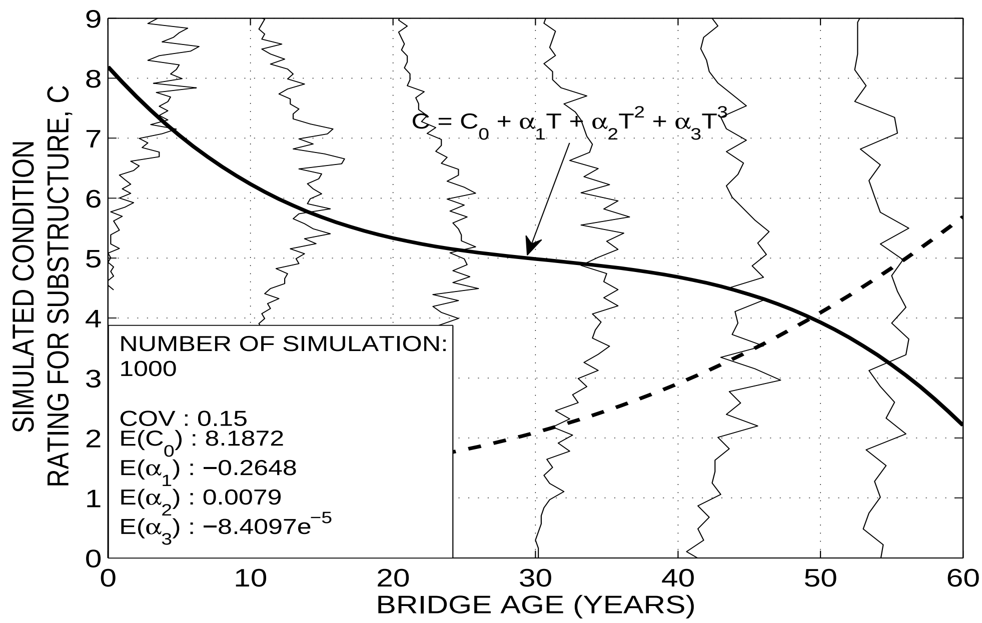


Figure 5.38: Simulation of Polynomial Regression Equation with 15 % COV to predict the Condition Rating of Concrete Bridge Substructures on Otherstate Highways

The profiles in Figures 5.39 through 5.41 are for the superstructure components of the concrete bridges on Otherstate highways. The mean condition rating value is approximately 8.6 at the beginning and reaches 2.5 at the end of the analysis time. Similar to previous graphs, the figures are obtaine using the coefficient of variation of 0.05, 0.10, and 0.15. The profile in Fig. 5.39 is based on a COV of 0.05. Standard deviation gets an important value at the end of the lifetime period and reaches 2.5 at 60 years. Fig. 5.40, is based on COV of 0.10. Standard deviation for condition rating reaches approximately 4.7 at the end of the analysis. Fig. 5.41 is based on a COV of 0.15. As shown in the figure, there is substantial difference between the maximum and minimum values simulated sample values of condition rating data. Therefore, standard deviation reaches 7.3 at the end of 60 years which cause high uncertainty for condition rating value at the end of the lifetime period.

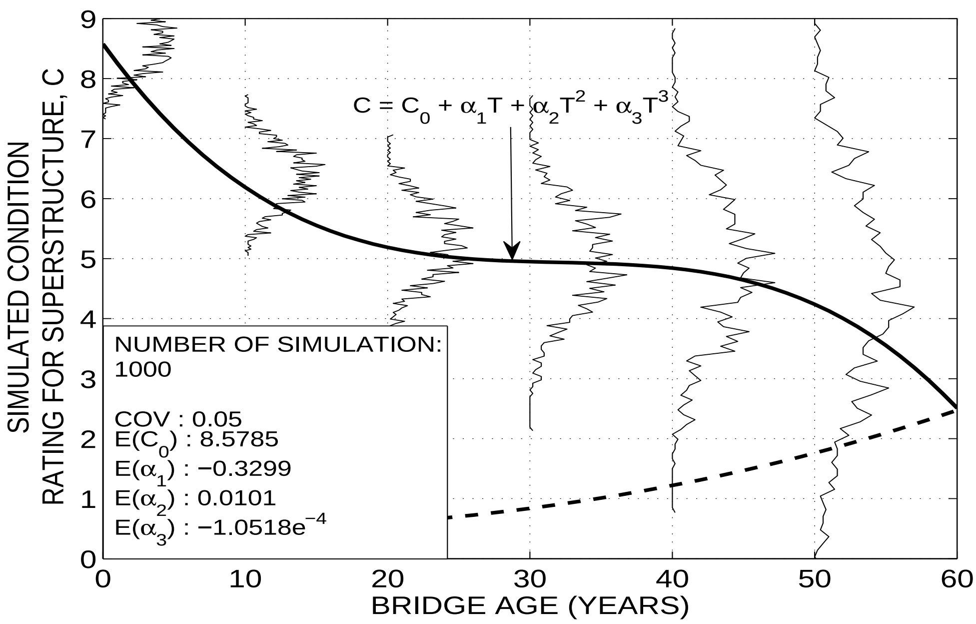


Figure 5.39: Simulation of Polynomial Regression Equation with 5 % COV to predict the Condition Rating of Concrete Bridge Superstructures on Otherstate Highways

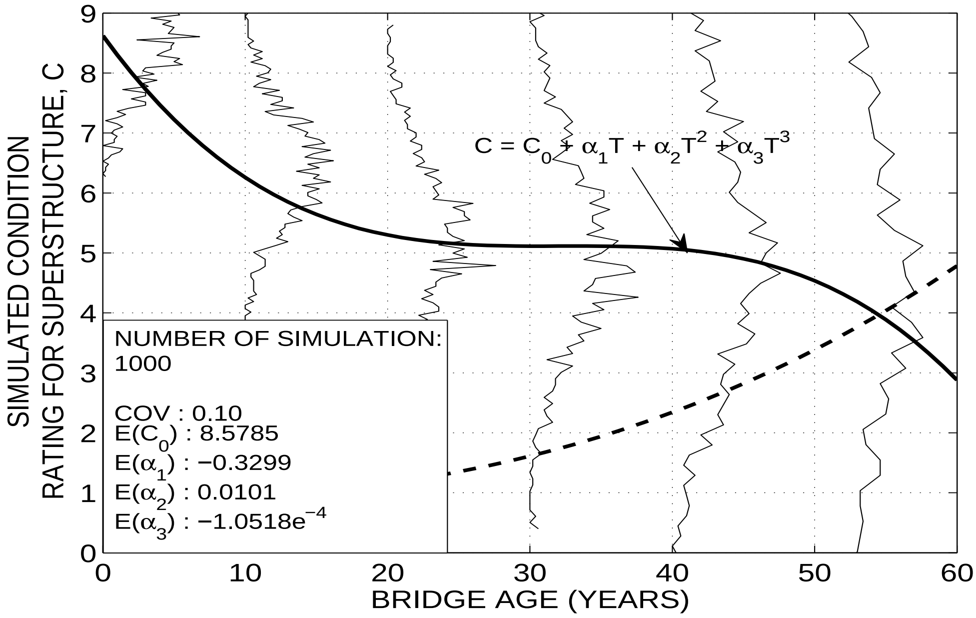


Figure 5.40: Simulation of Polynomial Regression Equation with 10 % COV to predict the Condition Rating of Concrete Bridge Superstructures on Otherstate Highways

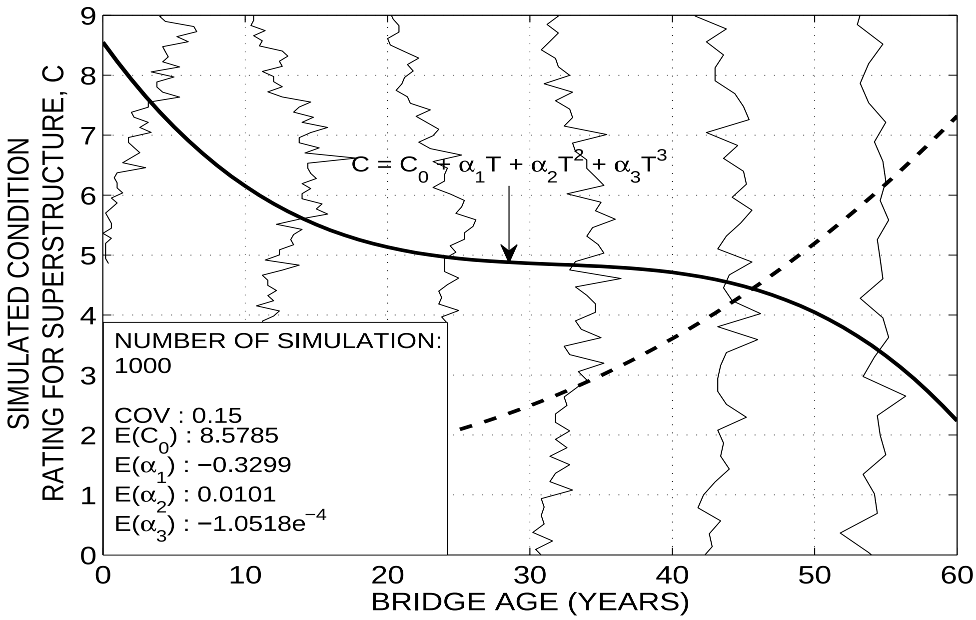


Figure 5.41: Simulation of Polynomial Regression Equation with 15 % COV to predict the Condition Rating of Concrete Bridge Superstructures on Otherstate Highways

The profiles in Fig. 5.42 through 5.44 are for the deck components of the steel bridges on Otherstate highways. The lifetime of available statistical data for steel bridges on Otherstate is 70 years. The condition rating value is approximately 8.1 at the beginning and reaches about 0 at the end of lifetime period. Three different values of the coefficient of variation are used for the regression equation coefficient consisting of 0.05, 0.10, and 0.15. Fig. 5.42 is based on a COV of 0.05. As shown in figure, dispersion of the condition rating data is very small at the beginning and increases gradually toward the end of 70 years. Standard deviation reaches 2.7 at the 70 years. Standard deviation gets larger than the mean value at the end of lifetime period. The profile in Fig. 5.43, is based on a COV of 0.10. Standard deviation reaches approximately 5.3 at the end of 70 years which causes an substantially high uncertainty. Fig. 5.44 is based on a COV of 0.15. Standard deviation displays too much dispersion at the end of 70 years. Standard deviation reaches approximately 7.8 at 70 years.

The profiles in Figures 5.45 through 5.47 are for the substructure components of the steel bridges on Otherstate highways. For this component, available statistical data is for 70 years. The condition rating value is approximately 8.2 at the beginning and reaches 0 at the end of the lifetime period. The profile in Fig. 5.45 is based on a COV of 0.05. Standard deviation has also remarkable value throughout the analysis and reaches 3.4 at the 70 years. Fig. 5.46, is based on a COV of 0.10. As shown in figure, standard deviation for condition rating reaches about 7 at the end of the analysis. Fig. 5.47 is produced for the COV of 0.15. Standard deviation reaches approximately 10 at the end of the simulation analysis. It means that the generated condition rating data with COV of 0.15 for the substructure components of the steel bridges on Otherstate highways shows extremely high uncertainty and it is very difficult to make a reliable estimate of condition rating of structure after 40 years.

Figures 5.48 through 5.50 are obtained for the superstructure components of the steel bridges on Otherstate highways. The condition rating value is approximately 8.1 at the beginning and it reaches 0 at the end of 70 years. The profile graphs are obtained based on a COV of 0.05, 0.10, and 0.15. Fig. 5.48 is based on a COV of 0.05. Standard deviation is small at the beginning but reaches 2.5 at 70 years. The profile in Fig. 5.49 is based on a COV of 0.10. Standard deviation reaches 5 at the end of the analysis. Finally, Fig. 5.50 is obtained by using a COV of 0.15. Standard deviation reaches approximately 7.8 at the end of 70 years.

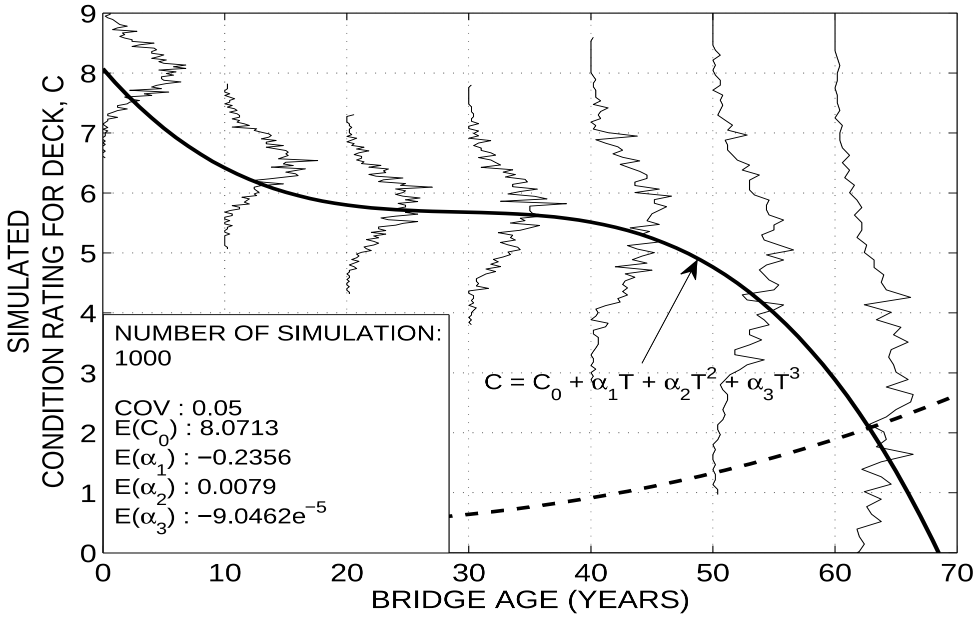


Figure 5.42: Simulation of Polynomial Regression Equation with 5 % COV to predict the Condition Rating of Steel Bridge Decks on Otherstate Highways

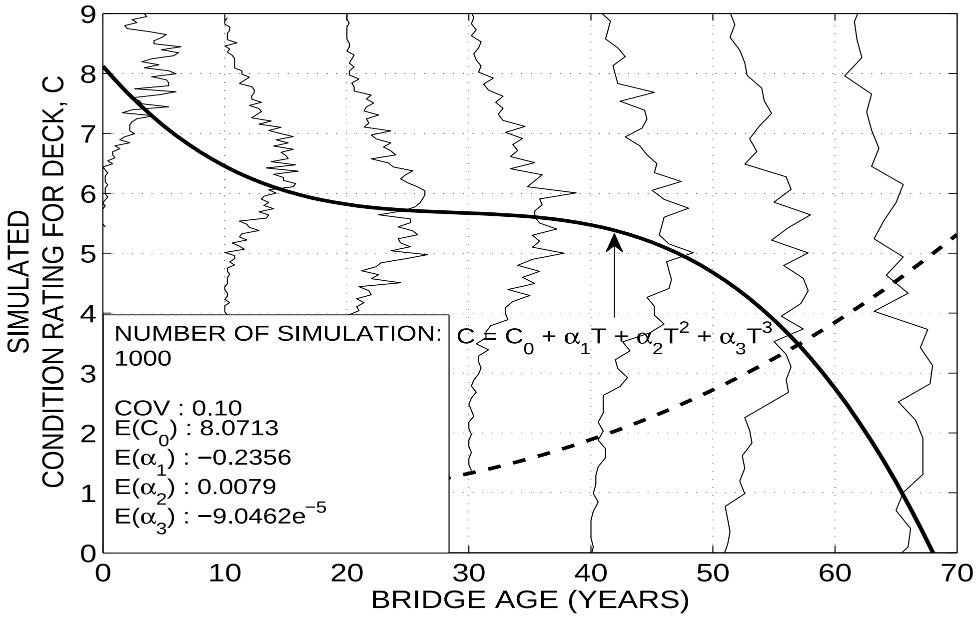


Figure 5.43: Simulation of Polynomial Regression Equation with 10 % COV to predict the Condition Rating of Steel Bridge Decks on Otherstate Highways

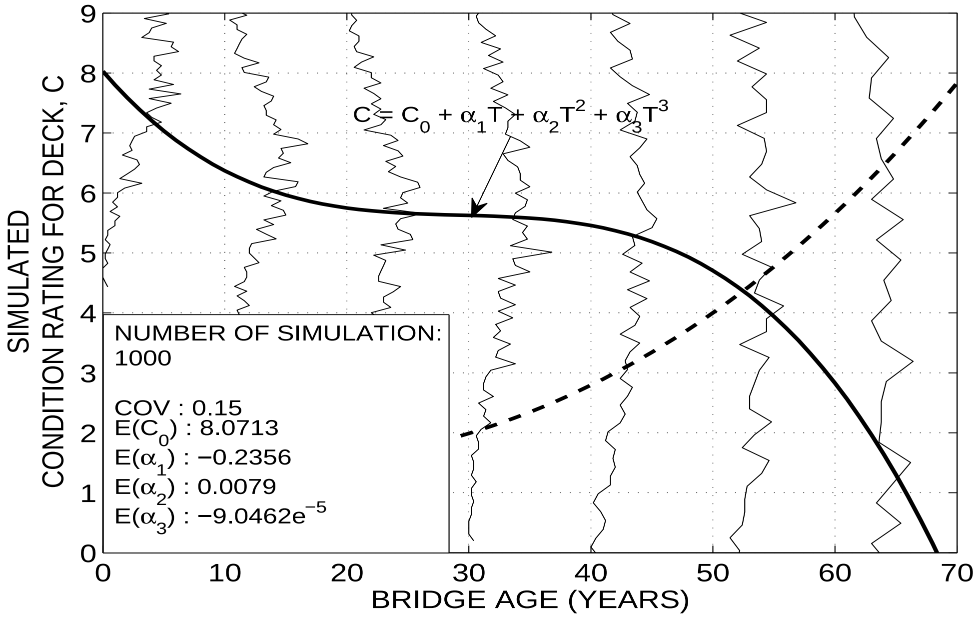


Figure 5.44: Simulation of Polynomial Regression Equation with 15 % COV to predict the Condition Rating of Steel Bridge Decks on Otherstate Highways

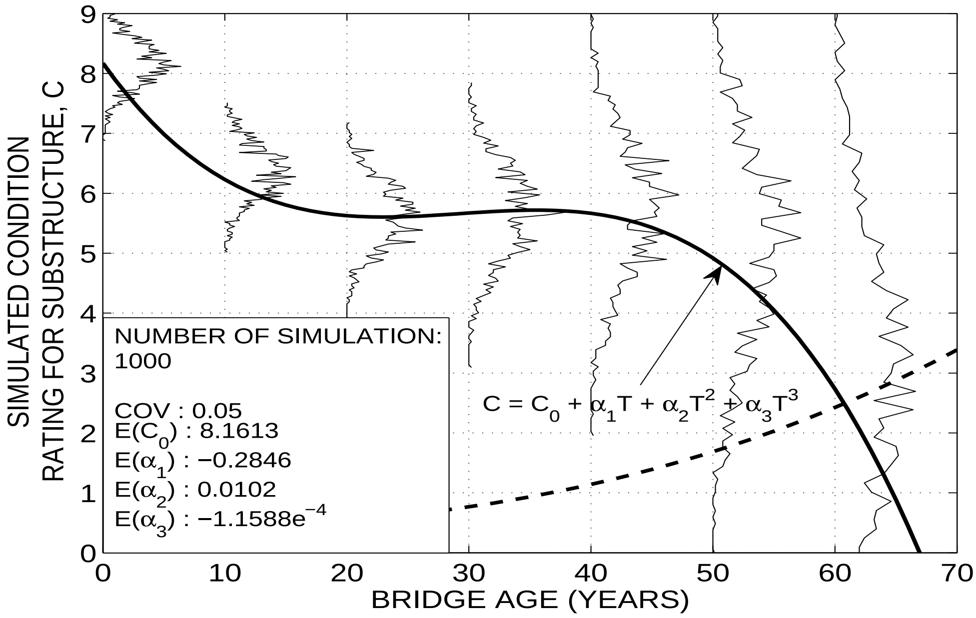


Figure 5.45: Simulation of Polynomial Regression Equation with 5 % COV to predict the Condition Rating of Steel Bridge Substructures on Otherstate Highways

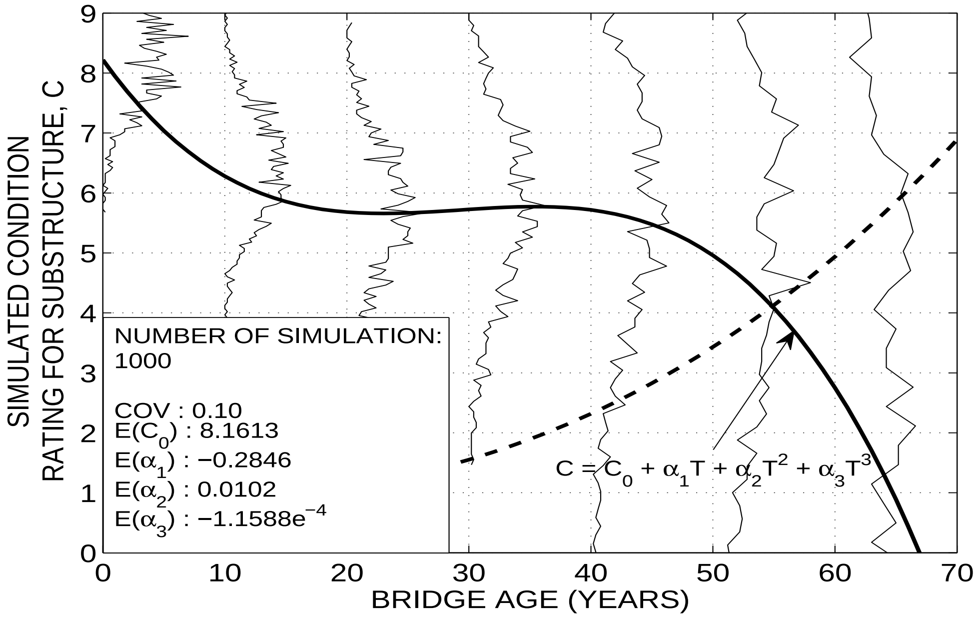


Figure 5.46: Simulation of Polynomial Regression Equation with 10 % COV to Predict the Condition Rating of Steel Bridge Substructures on Otherstate Highways

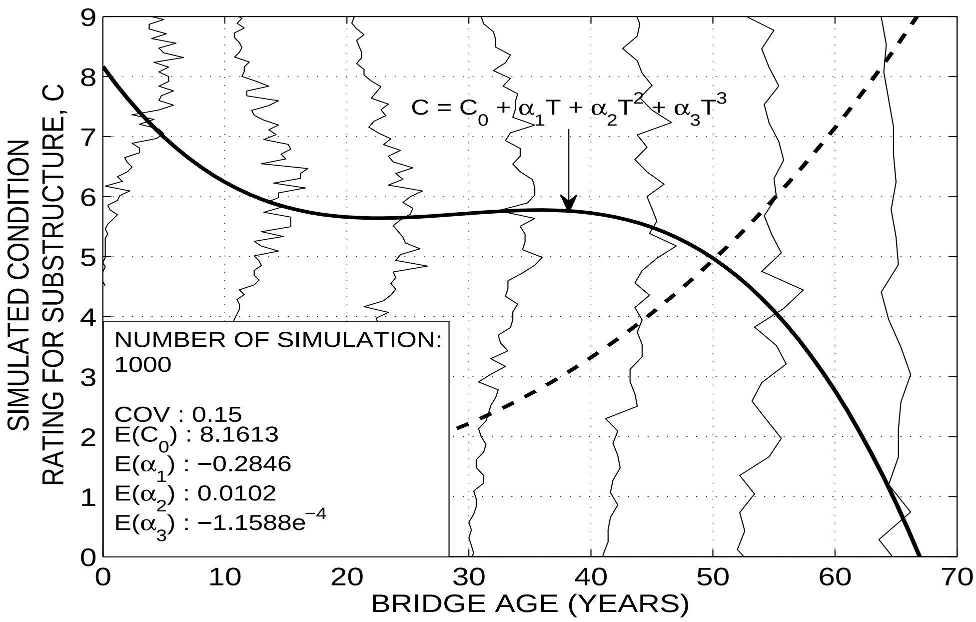


Figure 5.47: Simulation of Polynomial Regression Equation with 15 % COV to predict the Condition Rating of Steel Bridge Substructures on Otherstate Highways

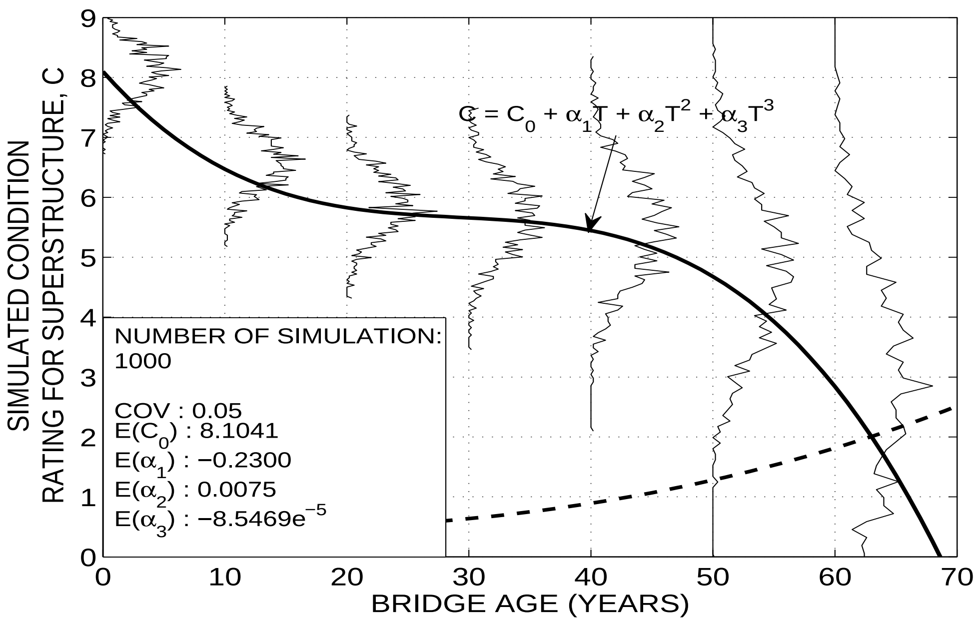


Figure 5.48: Simulation of Polynomial Regression Equation with 5 % COV to predict the Condition Rating of Steel Bridge Superstructures on Otherstate Highways

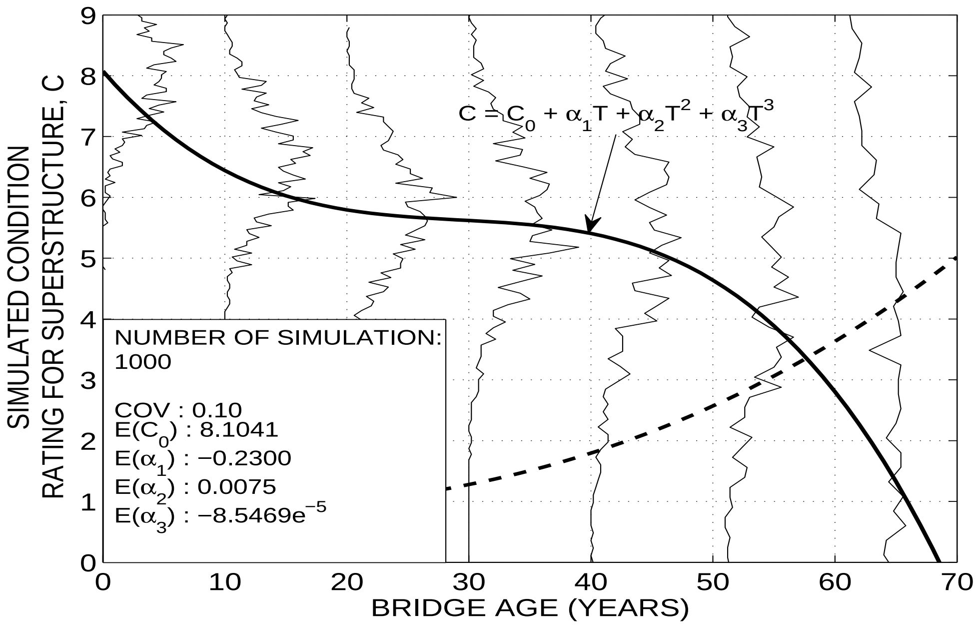


Figure 5.49: Simulation of Polynomial Regression Equation with 10 % COV to predict the Condition Rating of Steel Bridge Superstructures on Otherstate Highways

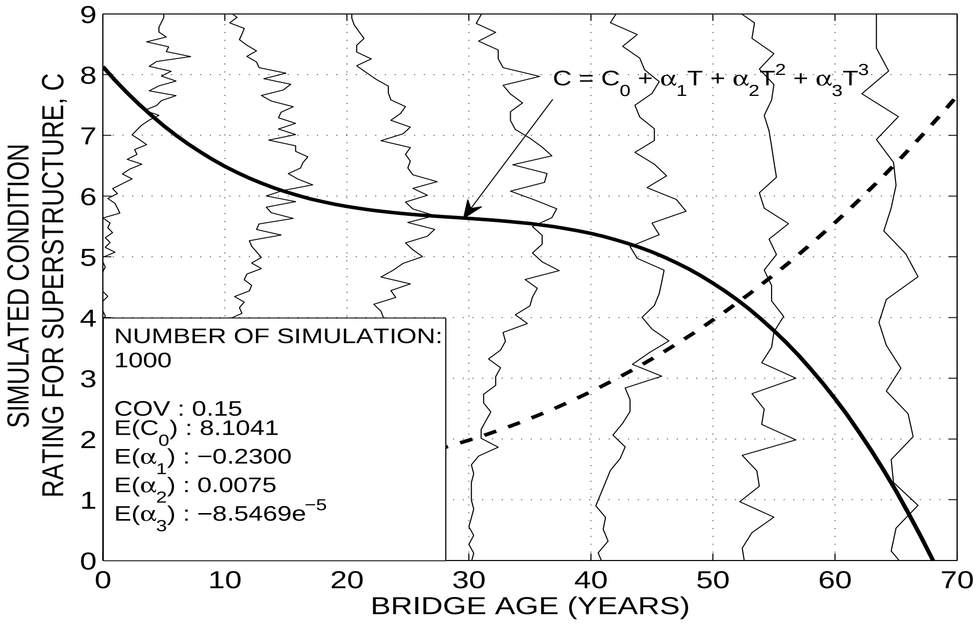


Figure 5.50: Simulation of Polynomial Regression Equation with 15 % COV to predict the Condition Rating of Steel Bridge Superstructures on Otherstate Highways

5.4 Summary

In this chapter, regression-based condition prediction models investigated as an alternative prediction model. Linear and piecewise linear regression models are mentioned. However, an important part of this chapter is devoted to polynomial regression-based performance prediction curves. The polynomial regression equations are obtained for deck, superstructure, and substructure components of bridges in the existing BMS. The coefficients of the polynomial regression equations are treated as random variables with normal distribution. The values of these coefficients are used as mean value to generate polynomial regression curves using Latin Hypercube Sampling technique. Therefore, a large number of condition rating data for each year can be obtained.

Notations in Chapter 5

Y_i	:	Dependent random variable
X_{i1}	:	The first independent random variable
X_{i2}	:	The second independent random variable
α_0	:	Intercept of linear equation
α_1	:	Slope of linear line
ϵ_i	:	Error term which has standard normal distribution
C_0	:	Initial condition rating
R^2	:	Coefficient of determination
COV	:	Coefficient of variation
$E(C_0)$	:	Expected value of C_0

CHAPTER 6

DERIVATION OF MARKOV TRANSITION PROBABILITY MATRIX FROM SIMULATED CONDITION PROFILE

6.1 Introduction

As mentioned in previous chapters, there are several approaches to predict the performance of infrastructure systems. Markov chain approach, regression-based model and bi-linear model are the three different approaches studied in this thesis, that can be used for forecasting the conditions of structures. Markov chain approach is the predominantly used stochastic approach in BMSs. This approach uses the transition probabilities to predict the future condition of the structure. Transition probabilities represent the probabilities of the state transitions from one state to another. Transition probabilities are represented in matrix form which i referred to as the Transition Probability Matrix (TPM) denoted by P. If the initial condition state and transition probability matrix of a structural member are known, its future condition state can be obtained by multiplying the initial state vector with the transition probability matrix.

The form of the transition probability matrix is given in Figure 6.1. Transition probability matrix is obtained from inspection data of bridge components. In order to obtain the transition probability matrix and build the deterioration prediction model for structural elements, Markov chain model needs only two successive cycles of inspections [33]. A transition probability matrix represents "do-nothing" case for a structure that is not subjected to any maintenance and repair actions. This transition probability matrix is referred to as the deterioration model for the structure. Both the rows and columns of the matrix represent the number of possible states the structure may be at any given time. Therefore, the transition probability matrix is a square matrix with number of rows and colums ($m = n$).

$$P_{m,n} = \begin{bmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,n} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m,1} & P_{m,2} & \cdots & p_{m,n} \end{bmatrix}$$

Figure 6.1: Transition Probability Matrix.

There are different methods in literature on derivation of the transition probability matrix. The percentage prediction method [25], regression-based non-linear programming optimization [25], and ordered probit and random effects model [62] are some of the derivation methods of the transition probability matrix.

In the percentage prediction method, the formula presented in Eq. 6.1 is used. This formula gives the transition probabilities for the states of the structural systems. The variables in Eq. 6.1 defined in Eq. 6.2.

$$\hat{p}_{i,j} = \frac{n_{i,j}}{n_i} \tag{6.1}$$

where,

$\hat{p}_{i,j}$ is the estimated transition probability of the system between state i and state j.

$$n_i = \sum_j n_{i,j} \tag{6.2}$$

where,

$n_{i,j}$ is the number of bridges or bridge elements passing from state i to state j during the observation (or given) time period.

n_i is the total number of bridges or bridge elements in state i before the transition.

Another method used for deriving the transition probability matrix is the regression-based method by solving a nonlinear optimization formulation. The nonlinear optimization method minimizes the sum of absolute differences between condition ratings obtained from regression curve and from the Markov-chain model using the formulation presented below [25].

$$\begin{aligned} \text{Minimize} \quad & \sum_{t=1}^{N} |Y(t) - E(t, P)| \\ \text{subject to} \quad & 0 \leq p(i) \leq 1 \\ & \sum_{i=1}^{n} p(i) = 1 \end{aligned} \tag{6.3}$$

where,

t is the operation time,

N is the total number of transition years,

$Y(t)$ is the value of condition rating obtained from regression equation at time t,

$E(t, P)$ is the value of condition rating estimated by Markov chain model at time t,

$p(i)$ is the probability that a structure will remain in the same state during the transition period.

Eq. 6.3 can be solved by the Quasi-Newton method [25]. Solution of the equation yields $p(i)$'s which form the transition probability matrix. There are several factors that affect the matrix dimension and values of transition probabilities. They include the number of possible states and the transition period. The more the number of possible states, are the bigger the transition probability matrix becomes. Moreover, if the transition period is relatively small, transition probabilities to different states will also be small values. In other words, probabilities of transition to the same states will be large. Computation with a large matrix arises certain difficulties. Therefore, certain assumptions are made in order to keep the computations simple. For example, the transition to different states are limited if the transition period is small. In this case, it can be assumed that the condition of a structure may change only one state in a small transition period. In this case, the transition probability matrix takes the form as shown in Fig. 6.2 which is referred to as the Restricted Form of the TPM.

$$P = \begin{bmatrix} p(1,1) & p(1,2) & 0 & 0 & 0 & 0 & 0 \\ 0 & p(2,2) & P(2,3) & 0 & 0 & 0 & 0 \\ 0 & 0 & P(3,3) & P(3,4) & 0 & 0 & 0 \\ 0 & 0 & 0 & p(4,4) & p(4,5) & 0 & 0 \\ 0 & 0 & 0 & 0 & p(5,5) & p(5,6) & 0 \\ 0 & 0 & 0 & 0 & 0 & p(6,6) & p(6,7) \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Figure 6.2: Transition Probability Matrix in Restricted Form

For a bridge element, the transition period may be one, two or a five year periods. Based on the transition period, Markov chain gives the condition of the structure at the end of the transition period. As shown in Fig. 6.2, each row has only two transition probabilities in the form of $p(i, j)$, where i represents the condition state at present or at the beginning of the transition period, and, j is the condition state at future or at the end of the transition period. There is a relationship between i and j given by $j = i + 1$. Since each row includes only two probabilities, condition of the structure either remains in the same state or only drops to one worse state during the transition period. Since there are only two possibilities between state i and state j, the sum of the probabilities in each row is equal to 1, which is stated as:

$$p(i,i) + p(i, j) = 1 \tag{6.4}$$

Moreover, there is one additional property of the transition probability matrix in restricted form. That is, the probabilities are null for $i>j$. This means that a structure's condition can get only worse without any rehabilitation or repair action as the structure ages.

The transition probability matrix shown in Fig. 6.2 is assembled based on the condition rating scale of the National Bridge Inventory (NBI) system in the United States [25]. As shown in Table 6.1, condition ratings of the bridges in NBI [63] are categorized by a rating system which ranges from 0 to 9. In this rating system, condition rating 9 represents the newly constructed system, on the contrary, the worst situation for the bridge is represented by 0. However, a constraint can be introduced below

Table 6.1: Condition states and definitions of structural elements used in National Bridge Inventory (NBI) in the U.S

NBI Rating	Description	Repair Action
9	Excellent condition	None
8	very good condition	None
7	Good condition	Minor maintenance
6	Satisfactory condition	Major maintenance
5	Fair condition	Minor repair
4	Poor condition	Major repair
3	Serious condition	Rehabilitate
2	Critical condition	Replace
1	Imminent failure condition	Close bridge and evacuate
0	Failed condition	Beyond corrective action

which the bridges may not be permitted to fall. For instance, in BMS of the State of Indiana, the condition rating of a bridge is not permitted to drop under rating 3 by applying repair and replacement actions. Based on this constraint, the transition probability matrix shown in Fig. 6.2 is assembled using the BMS criteria of the State of Indiana, and hence uses 7 condition states. In addition, $p(7)$ in Fig. 6.2 is 1 because the lowest acceptable rating level is 3. In BMS of State of Indiana, the NBI Condition Rating Scale is converted to Condition States as shown in Table 6.2 [25].

Table 6.2: Relationship between Condition Ratings and Condition States used in BMS of the State of Indiana

NBI Bridge Rating Scale									
9	8	7	6	5	4	3	2	1	0
1	2	3	4	5	6	7	8	9	10
Condition States Used by IBMS									

Pontis uses element-based condition state classification instead of a bridge-based classification. The condition states and their definitions used for the Bare Concrete Bridge Deck Element in Pontis, the predominant BMS in U.S., are shown in Table 6.3. Conditions of elements for bridges are categorized by visual inspection for their discrete states. Pontis uses 5 different condition states to define the visual condition of a bridge element. The small number of condition states produce a small transition probability matrix and results in computational simplicity. As shown in Table 6.3, condition state 1 represents the newly built or perfect condition of an element. If an element, however, is in the worst condition, its condition state is represented by rating 5.

Table 6.3: Condition States and definitions for Bare Concrete Bridge Deck Element in Pontis

Pontis Condition Rating	Description	Feasible Repair Action
1	The surface of deck has no repaired areas and there is no spalls/delamination in the deck surface	Add a protective system
2	Repair area and/or spalls/ delaminations exist in the deck surface. The combined distress areas is 2 % or less of the deck area	Repair spalled/ delaminated area Add a protective system
3	Repaired area and/or spalls/ delaminations exist in the deck surface. The combined distress areas is 10 % or less of the deck area	Repair spalled/ delaminated area Add a protective system
4	Repaired area and/or spalls/ delaminations exist in the deck surface. The combined distress areas is more than 10 % but less than 20 % of the deck area	Repair spalled/ delaminated area Add a protective system
5	Repaired area and/or spalls/ delaminations exist in the deck surface. The combined distress areas is more than 25 % of the deck area	Repair spalled/ delaminated area Add a protective system

6.2 Performance Prediction using Markov Chain Approach

As mentioned earlier, Markov process is a stochastic process for predicting the future states of a dynamic system if its future state depends only on its present value. In other words, the past state of the system has no effect on its future state. Because of this characteristic, Markov process is said to have memoryless property, which is stated as follows.

$$P(X_{t+1} = i_{t+1}|X_t = i_t, X_{t-1} = i_{t-1}, \ldots X_0 = i_0) = P(X_{t+1} = i_{t+1}|X_t = i_t) \qquad (6.5)$$

where, t is the present time.

In addition, the Markov process is called a Markov chain when the parameter set is discrete. In order to find the future condition state of a structural element, it is necessary to know its initial condition state vector and its transition probability matrix. Initial condition state vector represents the condition rating value of a structural element in terms of a probability distribution. Dimension of this vector is defined by the number of possible states. Initial condition state vector can be written as follows.

$$I.C_{(1\times n)} = \begin{bmatrix} p(1) & p(2) & p(3) & \cdots & p(n) \end{bmatrix} \qquad (6.6)$$

where,

$I.C$ is the initial condition state vector, and n is the total number of condition states.

The sum of the probabilities in this row vector is equal to 1. Formulation to predict the future condition rating vector can be written as follows.

$$C.D(t)_{(1\times n)} = I.C_{(1\times n)} \cdot P^{\,t}_{(n\times n)} \qquad (6.7)$$

where, $C.D(t)$ is the condition rating distribution of a structural element at time t.

As shown in Eq. 6.7, future condition prediction process is conducted by multiplying the initial condition state vector (I.C) with the transition probability matrix. The future time of the condition vector to be predicted depends on the initial condition vector, the transition period, and the power of the transition probability matrix. This relationship in an explicit form is written as follows.

$$
\begin{aligned}
C.D(0) &= I.C \cdot P^{\,0} \\
C.D(1) &= I.C \cdot P^{\,1} \\
C.D(2) &= I.C \cdot P^{\,2} \\
C.D(3) &= I.C \cdot P^{\,3} \\
&\vdots \\
C.D(t) &= I.C \cdot P^{\,t}
\end{aligned} \tag{6.8}
$$

As shown in Eq. 6.8, condition state vector at $t = 0$ is equal to the initial condition state vector itself.

In order to obtain the average condition rating $C(t)$ of a structure at time t, $C.D(t)_{(1\times n)}$ should be multiplied by the condition rating vector $R_{(n\times 1)}$. Condition rating vector R is a column vector which ranges between 1 and 5 for Pontis BMS and between 0 and 9 for NBI. The formula for the average condition rating, therefore, is as follows.

$$
C(t) = C.D(t)_{(1\times n)} \cdot R_{(n\times 1)} \tag{6.9}
$$

6.2.1 An Example for the Application of the Markov Chain Approach

An example is presented in this section to explain the performance prediction using Markov chain approach. In this example, transition probability matrix is taken from actual bridge element data which belongs to Bridge Element 107, painted steel open girders, used by California Department of Transportation in Pontis [2]. The condition states and their descriptions are represented in Table 6.4. The transition probability matrix, P, shown in Eq. 6.10 is obtained for the "Do Nothing" action case which also forms deterioration model for Bridge Element 107. In P matrix, the sum of the 5^{th} row is not equal to 1 and it has a nonzero value of 90.55 %. It means that the probability of failure is 9.45 % for this element if no maintenance action is applied at the last condition state.

Table 6.4: Condition states and definitions of the Bridge Element 107, (Painted open steel girders) in Pontis.

State	Name	Description
1	No corrosion	No evidence of active corrosion; paint system sound and functioning as intended.
2	Paint distress	Little or no active corrosion. Surface or freckled rust has formed or is forming.
3	Rust formation	Surface or freckled rust is prevalent. There may be exposed metal but no active corrosion.
4	Active corrosion	Corrosion present but any section loss resulting from active corrosion does not yet warrant structural analysis.
5	Section loss	Corrosion has caused section loss sufficient to warrant structural analysis to ascertain the effect of the damage.

$$P = \begin{bmatrix} 93.81 & 6.19 & 0 & 0 & 0 \\ 0 & 88.88 & 11.12 & 0 & 0 \\ 0 & 0 & 87.12 & 12.88 & 0 \\ 0 & 0 & 0 & 88.88 & 11.12 \\ 0 & 0 & 0 & 0 & 90.55 \end{bmatrix} \tag{6.10}$$

According to the Markov chain approach, the condition prediction for the newly built Bridge Element 107 is performed as follows. Since the element 107 is newly built, initial condition state vector takes the form shown below.

$$I.C = [1 \quad 0 \quad 0 \quad 0 \quad 0]$$

In addition, the condition rating vector may be represented as follows.

$$R' = [1 \quad 2 \quad 3 \quad 4 \quad 5]$$

The Condition Rating Distribution C.Dt and the Condition Rating Ct at time t may be found by using Eqs. 6.7 and 6.9, respectively.

$$\begin{aligned} C.D(0) &= [1 \quad 0 \quad 0 \quad 0 \quad 0] \cdot [P]^0 \\ &= [1 \quad 0 \quad 0 \quad 0 \quad 0] \end{aligned}$$

$$C(0) = [1 \quad 0 \quad 0 \quad 0 \quad 0] \cdot \{R\}$$
$$= 1$$

$$C.D(1) = [1 \quad 0 \quad 0 \quad 0 \quad 0] \cdot [P]^1$$
$$= [93.8 \quad 6.2 \quad 0 \quad 0 \quad 0]$$

$$C(1) = [93.8 \quad 6.2 \quad 0 \quad 0 \quad 0] \cdot \{R\}$$
$$= 1.0619$$

$$C.D(2) = [1 \quad 0 \quad 0 \quad 0 \quad 0] \cdot [P]^2$$
$$= [88 \quad 11.31 \quad 0.7 \quad 0 \quad 0]$$

$$C(2) = [88 \quad 11.3 \quad 0.7 \quad 0 \quad 0] \cdot \{R\}$$
$$= 1.1269$$

$$C.D(3) = [1 \quad 0 \quad 0 \quad 0 \quad 0] \cdot [P]^3$$
$$= [82.5 \quad 15.50 \quad 1.9 \quad 0.1 \quad 0]$$

$$C(3) = [82.5 \quad 15.50 \quad 1.9 \quad 0.1 \quad 0] \cdot \{R\}$$
$$= 1.1948$$

$$C.D(4) = [1 \quad 0 \quad 0 \quad 0 \quad 0] \cdot [P]^4$$
$$= [77.45 \quad 18.88 \quad 3.34 \quad 0.32 \quad 0.01]$$

$$C(4) = [77.45 \quad 18.88 \quad 3.34 \quad 0.32 \quad 0.01] \cdot \{R\}$$
$$= 1.2656$$

$$C.D(5) = [1 \quad 0 \quad 0 \quad 0 \quad 0] \cdot [P]^5$$
$$= [72.65 \quad 21.58 \quad 5.02 \quad 0.71 \quad 0.04]$$

$$C(5) = [72.65 \quad 21.58 \quad 5.02 \quad 0.71 \quad 0.04] \cdot \{R\}$$
$$= 1.3392$$

The condition rating distribution and the average condition rating may be computed as described above for any point in time t. Condition distributions obtained above are plotted in Fig. 6.3 as bar chart distributions along the vertical axes. The graph shows how the condition distribution of the bridge element changes form in time.

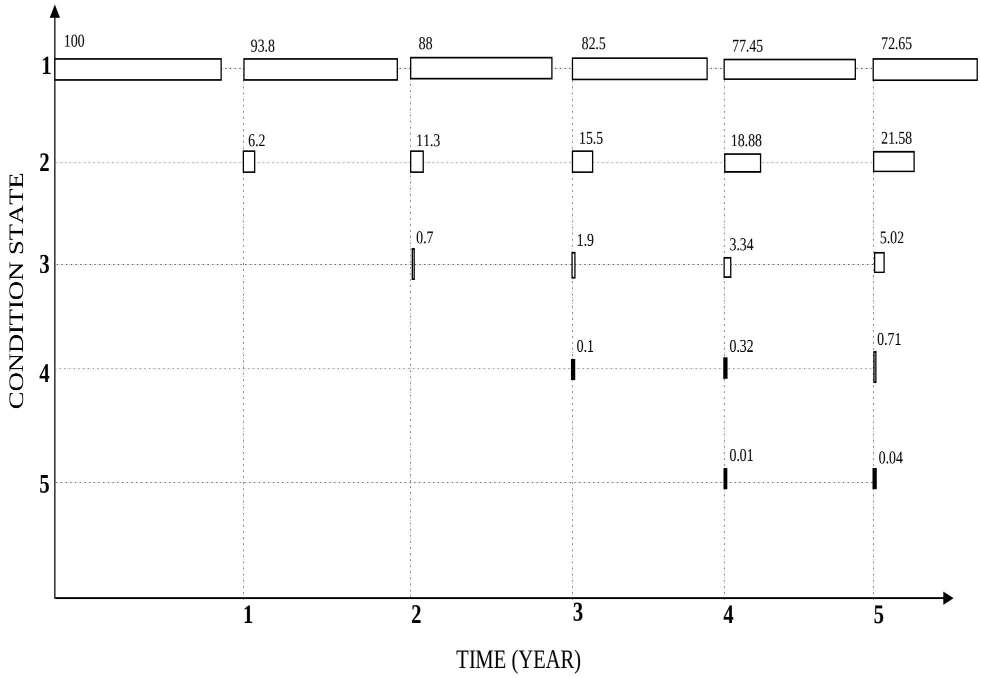


Figure 6.3: Transition Probabilities Under Do Nothing Case

The average condition rating values for Bridge Element 107 can be computed by Markov Chain approach throughout its lifetime. The computations can be performed using the transition probability matrix of the element. The matrix can be subjected a slight modification so that the value at the last row becomes 100 %. In other words, the probability of failure for the element is diminished. Then, the deterioration model or the transition probability matrix takes the following form.

$$P = \begin{bmatrix} 93.81 & 6.19 & 0 & 0 & 0 \\ 0 & 88.88 & 11.12 & 0 & 0 \\ 0 & 0 & 87.12 & 12.88 & 0 \\ 0 & 0 & 0 & 88.88 & 11.12 \\ 0 & 0 & 0 & 0 & 100 \end{bmatrix} \qquad (6.11)$$

The average condition rating values calculated for the element are listed in Table 6.5 for a 50 year time period (considered as the lifetime). Fig. 6.4 shows the time average condition ratings or the variation of the deterioration model obtained by Markov chain throughout the lifetime.

Table 6.5: Markov chain approach-based condition prediction.

Age(t)	0	5	10	15	20	25	30	35	40	45	50
C(t)	1	1.34	1.74	2.18	2.63	3.05	3.43	3.76	4.03	4.25	4.43

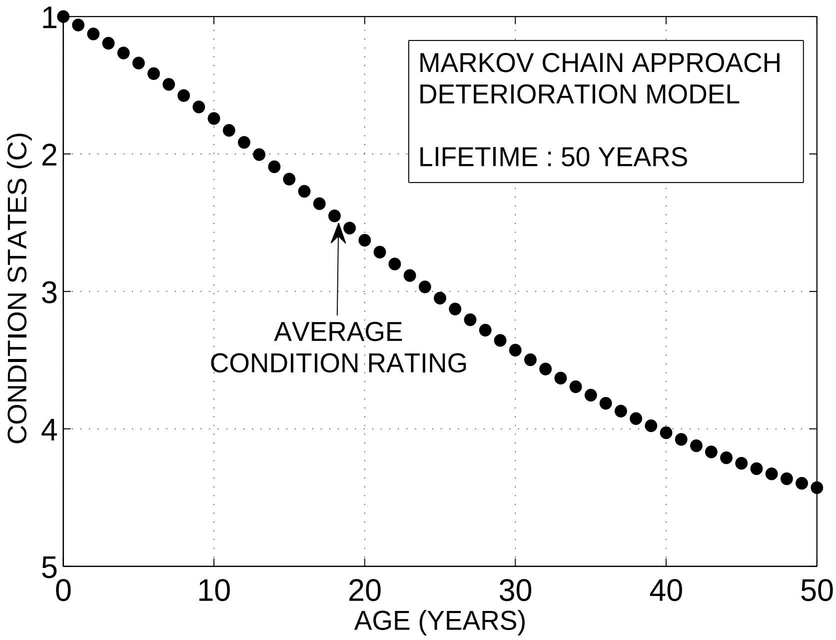


Figure 6.4: Markov chain approach-based performance prediction.

6.3 Derivation of Transition Probability Matrix

In the previous section, the methods and formulations that can be used to obtain transition probability matrix for a bridge element were described. In this section, a new approach developed in this thesis that can be used to generate the transition probability matrix from condition rating data by using simulation will be explained.

As mentioned earlier, one of the methods that can be used to obtain the transition probability matrix is the percentage prediction method. However, in order to use this method, the sample space must contain large number of data. Otherwise, this method can not give an effective result. This is the important limitation which restricts the use of the percentage prediction method. This limitation may be overcome by applying simulation which produces enough data to use this practical method.

The regression curve obtained from condition rating data achieved by visual inspection can be regenerated by Latin Hypercube simulation technique. Therefore, simulation produces enough condition rating data to apply the percentage prediction method. During the application process of this method, the procedure may be subjected to certain difficulties arising from the probabilistic nature of the simulation. In that case, certain assumptions which will be mentioned later can be made to overcome these difficulties.

Simulated condition profile for the condition rating data of concrete bridge substructures on the Otherstate highway in the State of Indiana in U.S is used to explain the method of derivation of the Markov transition probability matrix.

The formulations used to obtain the transition probability matrix are presented as follows.

$$p_{i,i} = \frac{n_{i,i}^{t+1}}{n_i^t} \tag{6.12}$$

$$trans_{(i,j)} = n_i^t - n_i^{t+1} \tag{6.13}$$

$$n_{j,j}^{t+1} = n_j^{t+1} - trans_{(i,j)} \tag{6.14}$$

$$j = i + 1 \tag{6.15}$$

where,

$p_{i,i}$ is the probability that the element which is currently in condition rating i will stay in the same condition rating during the transition period.

$n_{i,i}^{t+1}$ is the number of condition ratings data which stay in the same condition rating during the transition period.

n_i^t is the total number of condition ratings i at the beginning of the transition period.

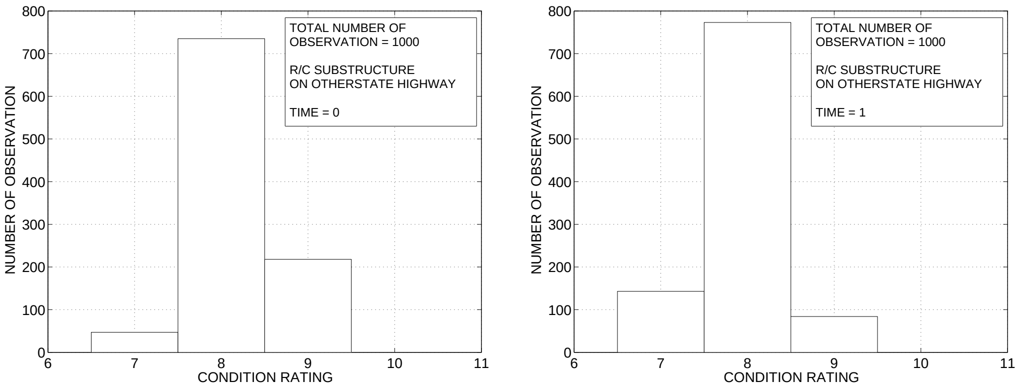
$trans_{(i,j)}$ is the number of condition ratings which pass to a worse condition during the transition period.

n_i^{t+1} is the total number of condition ratings i at the end of the transition period.

The transition probability matrix can be derived from the simulated regression-based performance curve if the Eqs. 6.12 through 6.15 are applied to the distribution of the condition rating data for each year.

Table 6.6: Distribution of the condition rating data obtained from simulation at time $t = 0$ and $t = 1$

Rating	**State**	**t = 0**	**t = 1**
9	1	218	84
8	2	735	773
7	3	47	143



(a) At the beginning of the first inspection period (b) At the end of the first inspection period

Figure 6.5: Distribution of simulated condition rating data

The distributions of condition rating data at time $t = 0$ and $t = 1$ are presented in

Figure 6.5(a) and 6.5(b). The number of observation values in these graph are listed in Table 6.6. One step of the procedure of the percentage prediction method can be applied as follows.

$$p_{1,1} = \frac{84}{218}$$
$$= 0.39$$

$$trans_{(1,2)} = 218 - 84$$
$$= 134$$

$$n^1_{2,2} = 773 - 134$$
$$= 639$$

$$p_{2,2} = \frac{639}{735}$$
$$= 0.87$$

$$trans_{(2,3)} = 735 - 639$$
$$= 96$$

$$n^1_{3,3} = 143 - 96$$
$$= 47$$

$$p_{3,3} = \frac{47}{47}$$
$$= 1$$

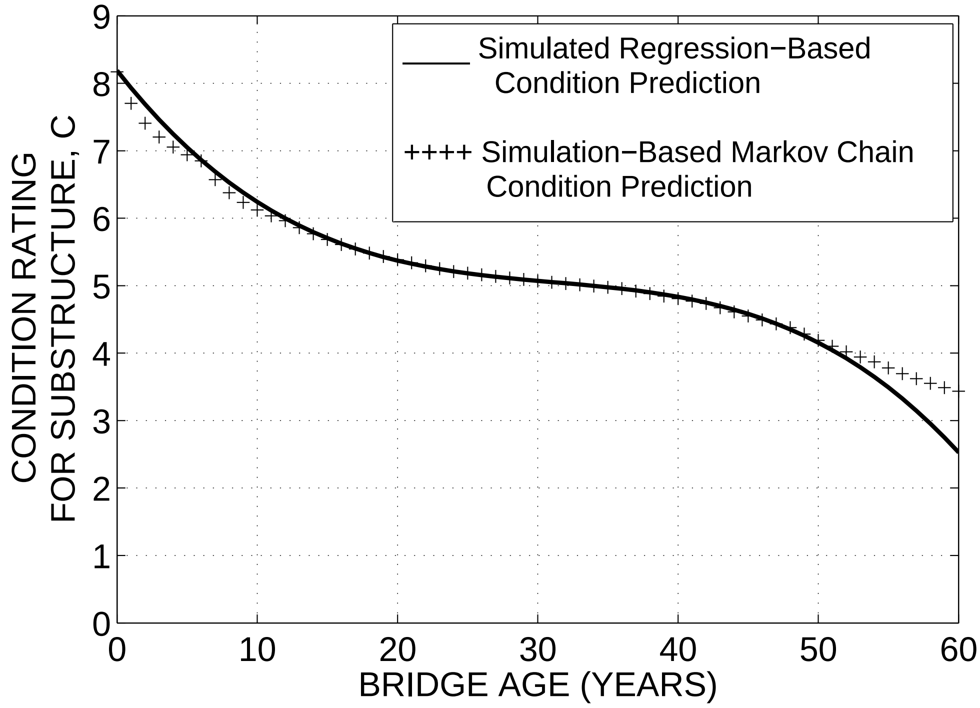


Figure 6.6: Simulated regression-based Condition prediction curve vs. Markov chain-based Condition prediction

A structure has different deterioration rates in the lifetime period. Therefore, lifetime of a structural element may be divided into several time periods based on these deterioration rates. It is why the deterioration is predicted well if the lifetime of the structure is divided many time periods. For each time period, different transition probability matrices should be computed. For instance, lifetime of the concrete substructure components on Otherstate highway is divided into ten periods [25]. Therefore, ten different transition probability matrices should be computed for the lifetime of those components.

Moreover, each time period consists of several years. Presented method for derivation of the transition probability matrix should be applied for each year as transition period step in each time period. In that case, there may be computed different values of the same $p_{i,i}$'s for each year. However, there must be one $p_{i,i}$ value for each time period. Therefore, the arithmetic mean of same transition states of $p_{i,i}$'s gives the value of $p_{i,i}$ for each time period.

There may not be any condition rating data in a time period. In that case, assumptions must be made. For instance, newer structural elements may not have condition state ratings larger than 4. For this case, $p_{5,5}$ and $p_{6,6}$ can be taken as 1, and then $p_{5,6}$ and $p_{6,7}$ are forced to be 0. In addition, older structural elements may not have condition state ratings less than 3. For this case, $p_{1,2}$, $p_{2,3}$ and $p_{3,4}$ can be taken as 1, and then $p_{1,1}$, $p_{2,2}$ and $p_{3,3}$ are forced to be 0.

It is assumed that the condition rating can drop at most one state in one year period to decrease the computational work. Moreover, sum of each row in transition probability matrix is equal to 1. Therefore, transition probability of ($p_{i,i+1}$) can be computed as follows.

$$\begin{aligned} p_{1,2} &= 1 - 0.39 \\ &= 0.61 \end{aligned}$$

$$\begin{aligned} p_{2,3} &= 1 - 0.87 \\ &= 0.13 \end{aligned}$$

$$\begin{aligned} p_{3,4} &= 1 - 1 \\ &= 0 \end{aligned}$$

A transition probability matrix with seven elements convenient for IBMS form can be seen in Table 6.7. As can be seen in Figure 6.6, Markov chain-based condition prediction curve is obtained by using this transition probability matrix. In addition, both simulated regression-based performance curve and Markov chain-based condition prediction curve are presented together in Figure 6.6. As can be seen in Figure 6.6, these two prediction curves are very similar. However, Markov chain-based condition prediction curve displays a certain deviation beginning from the approximately 50 year. Condition rating of the Markov chain-based condition prediction curve does not drop to 3 in the lifetime of the structure because of $p_{7,7} = 1$.

Table 6.7: Transition probabilities for concrete substructure component on otherstate highway in Indiana for 7 different states

Age	$p_{1,1}$	$p_{2,2}$	$p_{3,3}$	$p_{4,4}$	$p_{5,5}$	$p_{6,6}$	$p_{7,7}$
0-6	0.35	0.62	0.93	1	1	1	1
7-12	0	0.33	.70	0.96	1	1	1
13-18	0	0	0.75	0.90	0.99	1	1
19-24	0	0	0.92	0.93	0.97	0.99	1
25-30	0	0	1	0.98	0.97	0.99	1
31-36	0	0	1	1	0.97	0.98	1
37-42	0	0	1	0.97	0.96	0.95	1
43-48	0	0	0.97	0.95	0.92	0.92	1
49-54	0	0	0.94	0.88	0.87	0.86	1
55-60	0	0	0.90	0.85	0.82	0.80	1

The transition probability matrix should have 10 elements to obtain good match curve as the Markov chain-based condition prediction for the simulated regression-based performance curve. Therefore, it is allowed that the condition rating value can drop under the value of 3. In that case, the transition probability matrix takes the form presented in Table 6.8. If this transition probability matrix is used, the Markov chain -based condition prediction presented in Figure 6.8 can be obtained. As shown in Figure 6.8, presented method for the derivation of transition probability matrix from simulated regression-based performance prediction.

Table 6.8: Transition probabilities for concrete substructure component on otherstate highway in Indiana for 10 different states

Age	$p_{1,1}$	$p_{2,2}$	$p_{3,3}$	$p_{4,4}$	$p_{5,5}$	$p_{6,6}$	$p_{7,7}$	$p_{8,8}$	$p_{9,9}$	$p_{10,10}$
0-6	0.35	0.62	0.93	1	1	1	1	1	1	1
7-12	0	0.33	.70	0.96	1	1	1	1	1	1
13-18	0	0	0.75	0.90	0.99	1	1	1	1	1
19-24	0	0	0.92	0.93	0.97	0.99	1	1	1	1
25-30	0	0	1	0.98	0.97	0.99	1	1	1	1
31-36	0	0	1	1	0.97	0.98	0.96	1	1	1
37-42	0	0	1	0.97	0.96	0.95	0.96	0.99	1	1
43-48	0	0	0.97	0.95	0.92	0.92	0.93	0.90	0.98	1
49-54	0	0	0.94	0.88	0.87	0.86	0.87	0.87	0.85	1
55-60	0	0	0.90	0.85	0.82	0.80	0.79	0.81	0.79	1

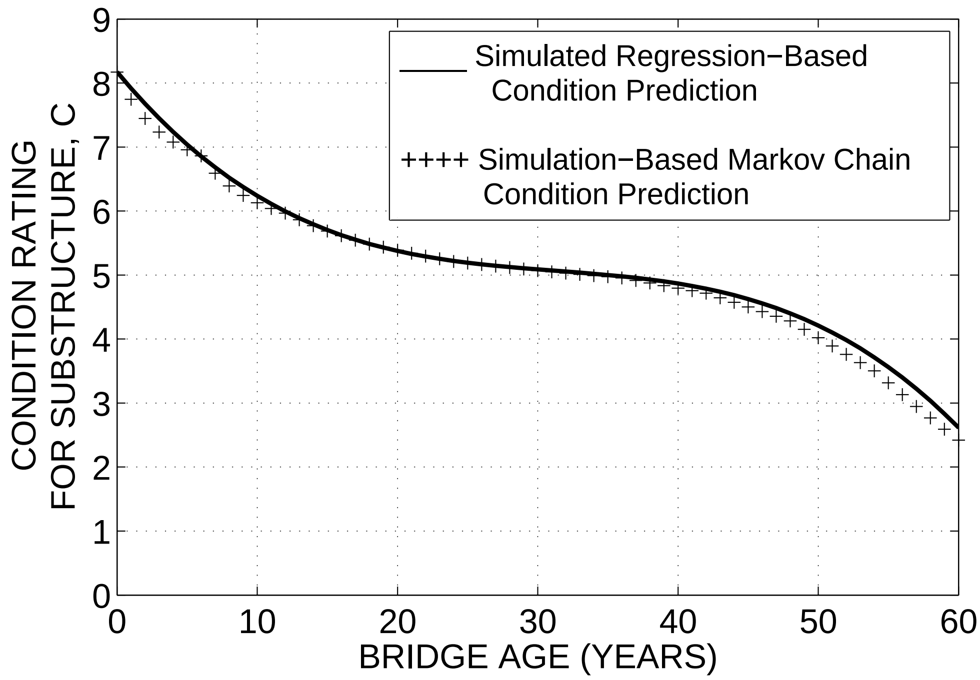


Figure 6.7: Simulated regression-based performance prediction curve vs Markov chain performance prediction for the whole condition rating scale

6.3.1 Derivation of Maintenance Profiles of Bilinear Model Using Markov Chain Approach

As mentioned earlier, the bi-linear model is an important performance prediction model which gives the decision makers the ability to observe both safety and condition prediction for a bridge. Furthermore, the model is a simulation based probabilistic model, and the effects of maintenance actions can be implemented into this prediction model. However, the bi-linear model needs to be further developed to find the optimum maintenance policy for the structure. This necessity may be overcome by applying an optimization procedure to the bi-linear model, and the Markov process can be used to find the optimum policy.

Markov chain approach predicts the lifetime condition of a structural element by using transition probabilities. In addition, Markov process finds the optimal policy by using dynamic programming formulations. In order to use Markov Decision Process, transition probabilities can be obtained from condition profiles of the bi-linear model. The transition probabilities are derived for every action procedure by using the Simulation-based Performance Prediction Method.

The simulation based percentage prediction method obtains the transition probability matrix by calculating the transitions of condition data between the condition states for each year. However, this process may require substantial computational effort. In addition, deterioration of the structure may not be the same throughout the lifetime. Therefore, lifetime of the structure is divided into several intervals and transition probability matrix is calculated for each interval separately. The transition probabilities of each interval can be found by taking the average of transition of each year in the analyzed subgroup.

Bi-linear model defines the condition of the structure with 4 different condition indices. However, more than 4 condition indices are taken into account in order to find the transition probabilities from the simulation based percentage prediction method. This is why the simulated condition data shows very large dispersion towards the end of the time horizon. In other words, condition indices are distributed to many states when the analysis time increases. Therefore, transition probabilities derived from profiles of bilinear model may be presented in an 8×8 matrix.

The Bi-linear condition profiles obtained for maintenance strategies and the Markov chain prediction curves obtained using the transition probabilities derived from the bilinear condition profiles are presented in Figures 6.8 through 6.13. The transition probabilities of the maintenance strategies are presented in Tables 6.9 through 6.14.

Fig. 6.8 presents both the Bi-linear condition profile and Markov chain approach-based condition profile under No Maintenance case. Due to No Maintenance case, the profiles are linear. In Fig. 6.8, the solid line is the bi-linear model and the circular markers represent the Markov chain condition profile obtained by simulation-based transition probability matrix. As shown in this figure, simulation-based Markov chain approach matches the bi-linear model well. The transition probability matrix presented in Table 6.9 is obtained by dividing the lifetime of the element into 5 equal subgroups. The transition probability values are the average values of the transitions of each group.

Fig. 6.9 shows the Bi-linear and Markov chain condition profiles for the silane treat-

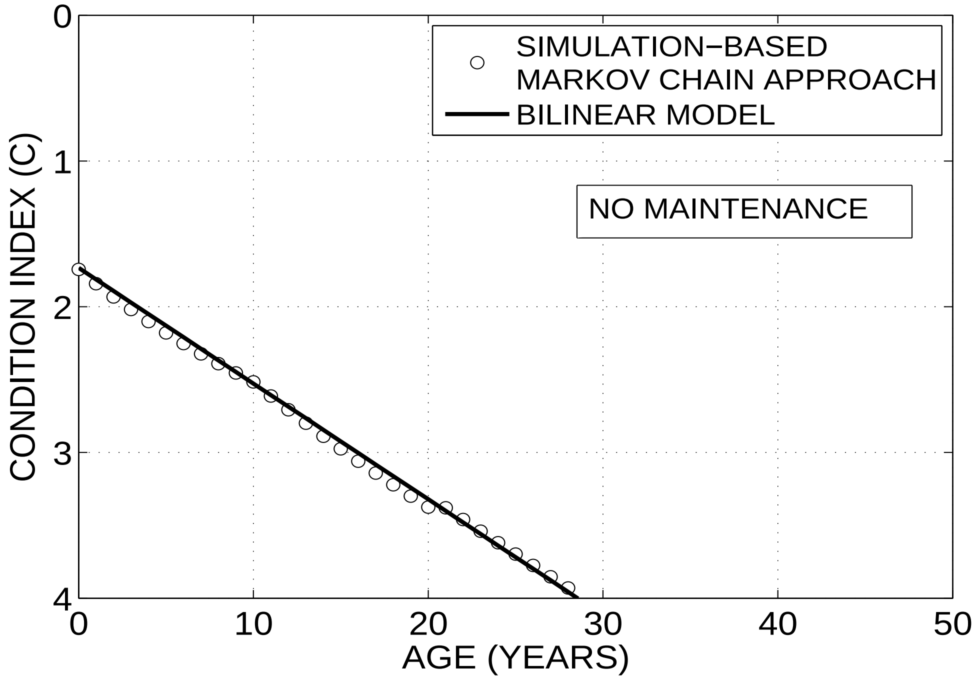


Figure 6.8: Simulation-based Markov chain performance prediction vs Bilinear model prediction for no maintenance

ment action. The silane treatment is a time-based maintenance action. In other words, time of the first and subsequent maintenance applications are specified by probability distributions defined in terms of time as the random variable. In order to obtain the transition probabilities from the bi-linear model, time horizon is also divided into many intervals according to maintenance application times. As shown in Fig. 6.9, simulation-based Markov chain condition profile gives a good approximation for the bi-linear model.

Table 6.9: Transition probabilities obtained from Bilinear model for no maintenance case

Age	$p_{0,0}$	$p_{1,1}$	$p_{2,2}$	$p_{3,3}$	$p_{4,4}$	$p_{5,5}$	$p_{6,6}$	$p_{7,7}$
0-10	0.716	0.878	0.917	0.966	0.997	1	1	1
11-20	1	0.858	.905	0.914	0.95	0.979	1	1
21-30	1	0.934	0.917	0.92	0.917	0.933	1	1
31-40	1	0.954	0.947	0.931	0.919	0.915	0.927	1
41-50	0	0.981	0.951	0.943	0.92	0.913	0.903	1

Figure 6.10 is generated for Replacement of Expansion Joints maintenance action.

The simulation based transition probabilities are presented in Table 6.11. As shown in Table 6.11, the transition probabilities are calculated for three different lifetime intervals.

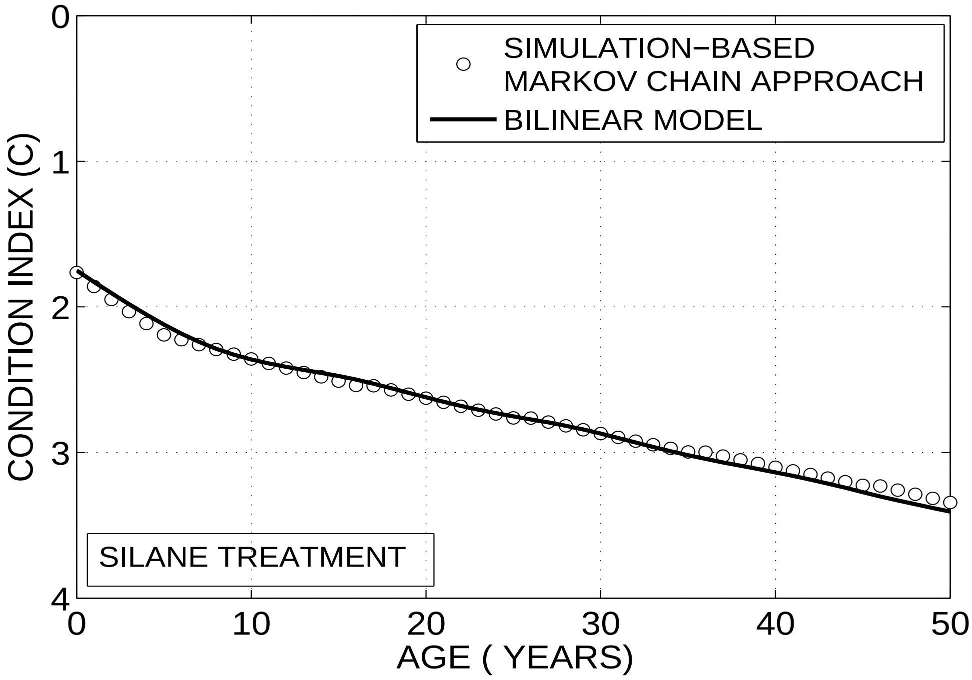


Figure 6.9: Simulation-based Markov chain performance prediction vs Bilinear model prediction for Silane Treatment maintenance action

Table 6.10: Transition probabilities obtained from Bi-linear model for Silane Treatment maintenance action.

Age	$p_{0,0}$	$p_{1,1}$	$p_{2,2}$	$p_{3,3}$	$p_{4,4}$	$p_{5,5}$	$p_{6,6}$	$p_{7,7}$
0-5	0.57	0.9	0.94	0.88	1	1	1	1
6-16	1	0.945	.965	0.978	0.99	1	1	1
17-25	0	0.94	0.967	0.98	0.985	1	1	1
26-35	0	0.937	0.97	0.975	0.984	0.996	1	1
36-45	0	0.932	0.972	0.972	0.984	0.981	1	1
46-50	0	0.933	0.963	0.97	0.978	0.983	0.988	1

Fig. 6.11 is obtained for the Cathodic Protection maintenance action. In order to calculate the transition probabilities for cathodic protection, the lifetime of the structure is divided into 5 intervals. As shown in Figure 6.11, the condition profile remains unchanged between 20th and 50th year of the lifetime under the cathodic protection

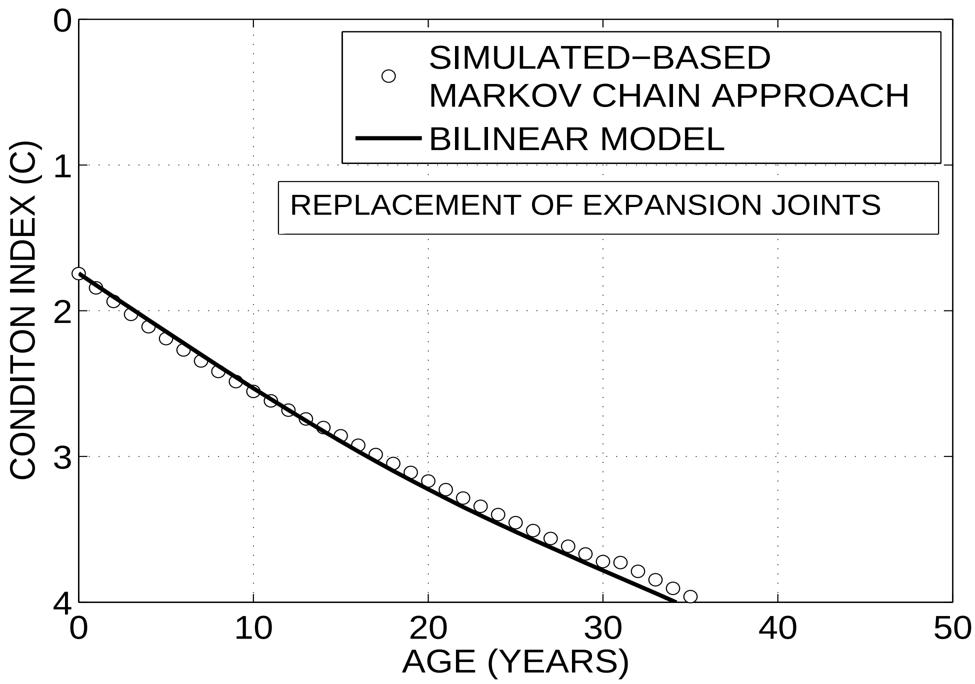


Figure 6.10: Simulation-based Markov chain performance prediction vs. Bi-linear model prediction for Replacement of Expansion Joints

action. Therefore, the lifetime of the structure is not divided any interval after the 20th year. The transition probabilities for the Cathodic Protection action are represented in Table 6.12. Four condition indices are sufficient to represent the Cathodic Protection profile of the bi-linear model.

Table 6.11: Transition probabilities obtained from Bi-linear model for Replacement of Expansion Joints

Age	$p_{0,0}$	$p_{1,1}$	$p_{2,2}$	$p_{3,3}$	$p_{4,4}$	$p_{5,5}$	$p_{6,6}$	$p_{7,7}$
0-15	0.71	0.88	0.918	0.95	0.984	0.994	1	1
16-30	0	0.838	.929	0.943	0.952	0.96	0.97	1
31-50	0	0.929	0.935	0.928	0.946	0.949	0.962	1

The bi-linear model profile and simulation based Markov chain profile under Minor Concrete Repair action are presented in Fig. 6.12. As shown in this graph, there are improvements for the structure. The transition probability matrix for Minor Concrete Repair is presented in Table 6.13. However, this table presents only one age group. The transition probability matrix for essential maintenance actions can only be presented in separate Tables for every age group because of improvements in condition index. The age group presented in Table 6.13 includes the transition probabilities between 11 and 15 years.

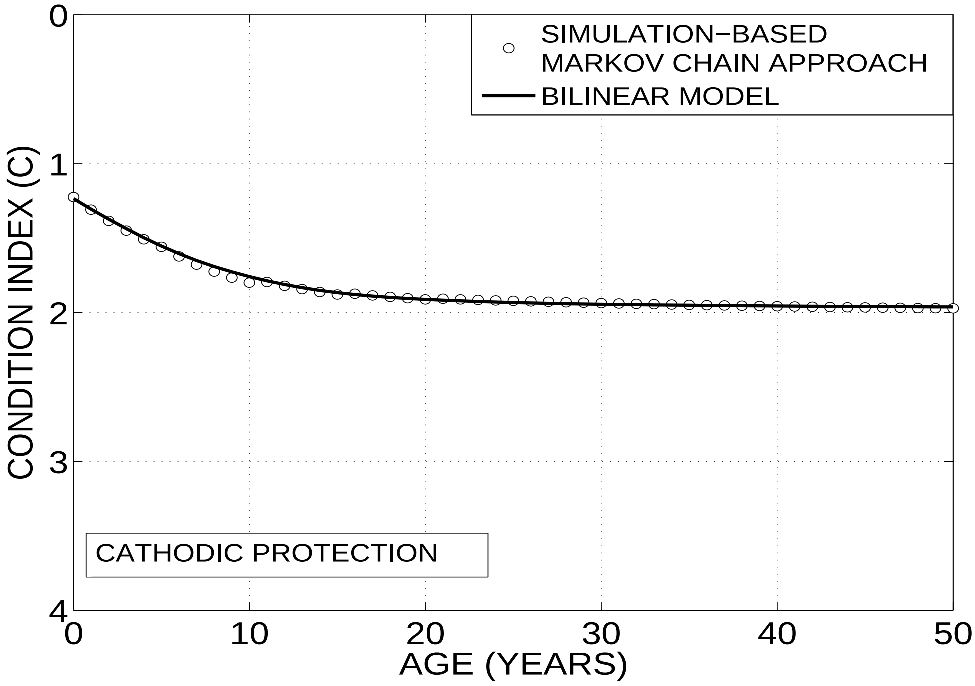


Figure 6.11: Simulation-based Markov chain performance prediction vs Bi-linear model prediction for the Cathodic Protection action.

Table 6.12: Transition probabilities obtained from Bi-linear model for the Cathodic Protection action.

Age	$p_{0,0}$	$p_{1,1}$	$p_{2,2}$	$p_{3,3}$
0-5	0.71	0.90	1	1
6-10	0.856	0.848	1	1
11-15	0.96	0.866	1	1
16-20	1	0.90	1	1
21-50	0.968	0.954	1	1

The Do Nothing and Rebuild action profile for both the bi-linear model and Markov chain approach is presented in Fig. 6.13. This maintenance procedure enables the condition of the structure to reach the threshold condition index level and then applies the rebuild action. The transition probabilities are represented by 8×8 matrix. Simulation based transition probabilities obtained by the bi-linear model is presented

Table 6.14. The do nothing and rebuild is an essential maintenance action. Therefore, the condition profile shows some improvement at the application times of the maintenance action. The transition probabilities between 30 and 40 years are presented in Table 6.14.

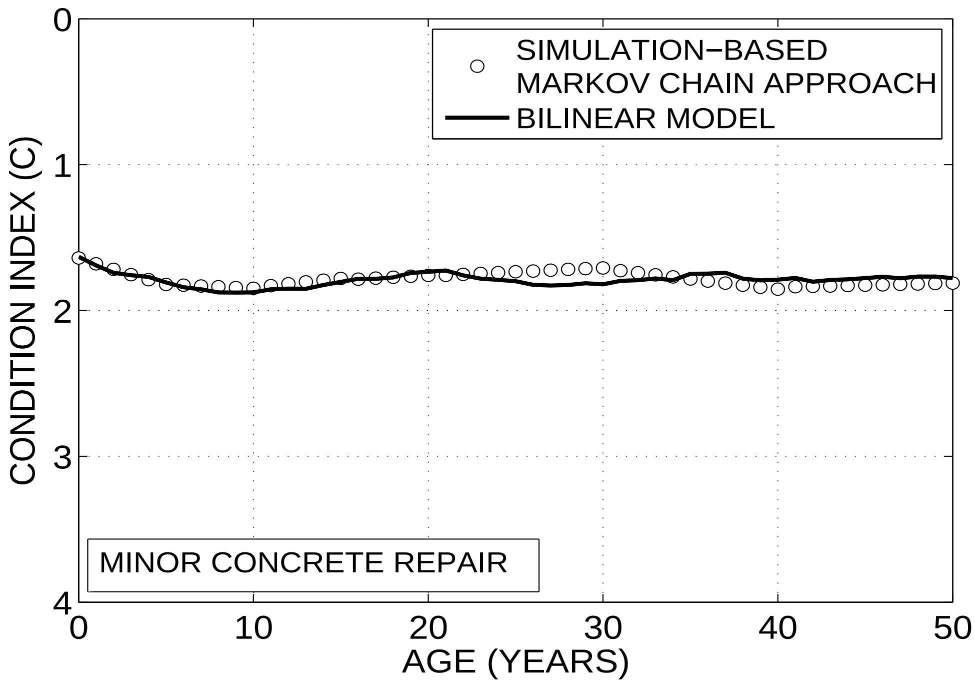


Figure 6.12: Simulation-based Markov chain performance prediction vs Bi-linear model prediction for Minor Concrete Repair action

Table 6.13: Transition probabilities obtained from Bilinear model for Minor Concrete Repair action

$$\begin{bmatrix} p(0,0) & p(0,1) & 0 & 0 \\ p(1,0) & p(1,1) & 0 & 0 \\ 0 & p(2,1) & P(2,2) & P(2,3) \\ 0 & 0 & p(3,2) & p(3,3) \end{bmatrix} = \begin{bmatrix} 0.957 & 0.043 & 0 & 0 \\ 0.008 & 0.992 & 0 & 0 \\ 0 & 0.026 & 0.97 & 0.004 \\ 0 & 0 & 0.011 & 0.989 \end{bmatrix}$$

Six different condition profiles obtained from both the Bi-linear and Markov chain approaches are presented together in Figures 6.8 through 6.13. In addition, the transition probability matrices used in Markov chain approach are presented in the Tables 6.9 through 6.14.

Table 6.14: Transition probabilities obtained from Bi-linear model for Do Nothing and Rebuild action.

$$\begin{bmatrix} p(0,0) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p(1,0) & p(1,1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & p(2,1) & P(2,2) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & p(3,2) & p(3,3) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & p(4,3) & p(4,4) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & p(5,4) & p(5,5) & p(5,6) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & p(6,6) & p(6,7) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & p(7,7) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.363 & 0.637 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.444 & 0.556 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.162 & 0.838 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.04 & 0.93 & 0.03 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.005 & 0.943 & 0.052 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.951 & 0.052 \\ 0 & 0 & 0) & 0 & 0 & 0 & 0 & 0.049 \end{bmatrix}$$

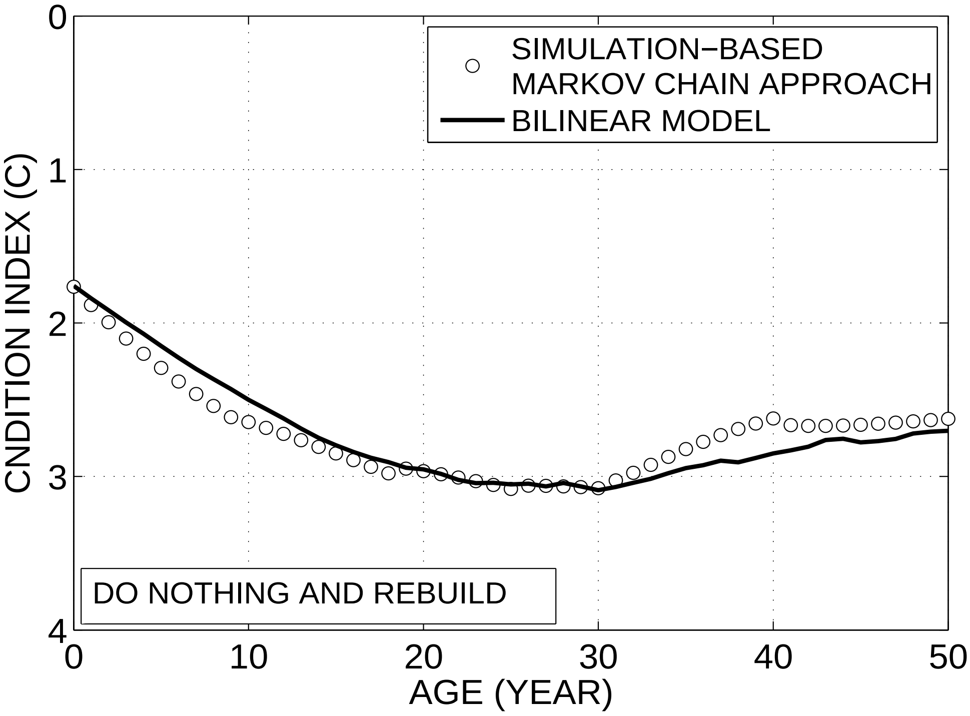


Figure 6.13: Simulation-based Markov chain performance prediction vs Bilinear model prediction for do nothing and rebuild

6.4 Summary

In this chapter, derivation of transition probabilities are studied. Two different methods for derivation of transition probability matrix are mentioned such as the percentage prediction method and the nonlinear optimization method. The percentage prediction method is a straight forward method, however, it is not reliable if the data on number of ratings is not sufficient. This limitation can be overcome by using the simulation-based results of condition profiles.Transition probabilities are derived from simulated regression-based condition curve and Bi-linear condition profiles for five different maintenance case and no maintenance. Finally, simulated regression-based condition curve and Bi-linear profiles can also be obtained from the Markov chain approach condition prediction method with transition probabilities computed from simulated condition rating data.

Notations in Chapter 6

P	:	Transition probability matrix
$\hat{p}_{i,j}$	:	Estimated transition probability of the system between state i and state j
$n_{i,j}$	:	Number of bridges or bridges elements passing from state i to state j during the observation (or given) time period
n_i	:	Total number of bridges or bridge elements in state i before the transition
$Y(t)$	:	Value of condition rating obtained from regression equation at time t
$E(t, P)$	:	Value of condition rating estimated by Markov chain model at time t
$p(i)$	:	Probability that a structure will remain in the same state during the transition period
$I.C$	:	Initial condition state vector
$C.D(t)$	:	Condition rating distribution of a structural element at time t
R	:	Condition rating vector
$C(t)$	:	Average condition rating
$p_{i,i}$	:	Probability that the element which is currently in condition rating i will stay in the same condition rating during the transition period
$n_{i,i}^{t+1}$	:	Number of condition rating data which stay in the same condition rating during the transition period
n_i^t	:	Total number of condition ratings i at the beginning of the transition period
$trans_{(i,j)}$	:	Number of condition ratings which pass to a worse condition during the transition period
n_i^{t+1}	:	Total number of condition ratings i at the end of transition period

CHAPTER 7

SUMMARY AND CONCLUSION

Bridge networks are one of the most important infrastructure systems. All constructed structures deteriorate throughout their lifetimes due to various environmental factors and loading conditions. If maintenance, repair, rehabilitation and replacement actions are not applied to deficient bridges at required times with adequate funds, irreversible problems may arise such as inadequate funds for further improvements, sudden accumulation of repair and rehabilitation needs, substantial economical losses in terms of infrastructure assets and ultimately endangering the safety of general public. Therefore, bridge conditions should be inspected periodically and specific actions should be applied to improve their performance when necessary. These requirements create the need for Bridge Management Systems (BMSs). BMSs are designed to manage maintenance, repair, rehabilitation and replacement actions for bridge networks and to keep bridges away from risk of failure by using the necessary resources in an optimal manner. In order to achieve these objectives, BMSs may employ one of the performance prediction and optimization models. This study presents a new model for the derivation of the transition probability matrix from a simulated regression based condition profiled for lifetime performance prediction for bridges through condition evaluation.

In this thesis, first, a brief overview of Bridge Management Systems is presented, followed by the investigations of BMSs and performance prediction models. Safety and condition prediction are the two performance prediction techniques for bridges. Safety performance prediction is described and then reliability index and rating factor are presented as two performance indicators. Rating factor-based prediction is presented and basic principles and rating formulas are also given.

As the next step, Markov process-based condition prediction is investigated. Markov process is a category of the dynamic programming problem. Dynamic programming problem should be solved in order to find the optimal decision policy from a Markov process. The problem can be reduced to a linear programming problem which becomes one of the solution methods for dynamic programming. A computer program is developed to obtain an optimal policy for a dynamic system that has multiple maintenance actions in multiple condition states. Both expected average and discounted cost problems are solved by using the developed computer program. The developed program solves the Markov process problem to obtain the optimal policy with minimum life-cycle cost. It is noted that Markov process generally choses the do-nothing and rebuild action types as part of the optimal policy to obtain the minimum cost solution for a structural element with a low deterioration rate. However, it is also observed that the process choses the maintenance actions with significant improvement effects for the bridge elements at each condition state for a structural element with a high deterioration rate. The process follows such a policy in an attempt to keep the bridge at higher condition states with minimum cost during its lifetime. As part of the studies, steady-state probabilities are also calculated by solving the expected average cost problem for the long term optimal policies.

Furthermore, maintenance action types applicable for different bridge components are investigated and these actions are categorized based on their effects on the structure.

A part of the study is devoted to an existing probabilistic Bi-linear performance prediction model. The model provides an analysis of life-cycle performance prediction for bridges under no maintenance case and different maintenance cases. The Bi-linear model is also able to define numerous uncertainties existing in bridge performance prediction. The uncertainties are related to the bridge itself and also to the maintenance actions. The uncertainties are defined as random variables. In order to study and further develop the Bi-linear model, and to generate values for random variables, a Latin Hypercube sampling-based computer program is developed using Matlab. The program generates values for random variables having triangular distributions. The program is integrated into the another specially developed computer program which is used to generate condition, safety and life-cycle cost profiles for a bridge and to verify the results presented by the model's developers. Various condition profiles are obtained for different distribution values of deterioration rate, initial condition index, and deterioration initiation time to present the effect of certain random variables on condition performance of the bridge. Using the developed computational algorithm, condition, safety and cost profiles obtained under the five different maintenance actions were re-created. It was also noted that different performance profiles were obtained under different type of maintenance actions. For instance, preventive maintenance actions prevented the condition profile to reach the target level for a while, however, they had no improvement effect on the condition profiles. In addition, it was observed that preventive maintenance actions had no remarkable effect on safety profiles. Whereas, essential maintenance actions were applied to structures whose performance close to target level and these actions indicate sudden and significant improvement on condition and safety profiles. Moreover, it was illustrated that life-cycle cost of preventive maintenance actions was smaller than one of the essential maintenance actions.

As an alternative performance prediction model and for the purpose of comparing the results of different models, a condition rating data-based polynomial regression prediction model is also developed. Effects of the changes of coefficients of the polynomial equation on the performance curve are examined. Ranges of the values of the coefficients which yield the rational performance curves are determined. As an introduction to this subject, a short introduction on regression-based models used for bridge performance prediction is presented. Then, linear and piecewise linear regression models are described. Polynomial regression-based performance prediction curves are obtained from the data for deck, superstructure and substructure components of the bridges in the existing BMS. The developed regression- based performance curve models can be directly used to predict the condition of a bridge group at any future time. The regression-based condition prediction curves are generated using the Latin Hypercube Sampling technique. Therefore, a large number of condition rating data for each year can be obtained.

A large part of this study is devoted to the derivation of Markov transition probability matrix from a simulated condition profile. The developed programs and knowledge gained from studying different performance prediction techniques formed the foundation for the development of this new approach. The transition probability matrix determines the deterioration model of a bridge element. For this reason, it is an important instrument to predict the lifetime performance of a structure. Furthermore, in this study, several different approaches for derivation of transition probability matrix are described such as the percentage prediction method and the nonlinear optimization method. The percentage prediction is a relatively straight forward method, however, it is not reliable if the data on number of ratings is not sufficient. It has been found in this study that this limitation can be overcome by using the simulation-based results of condition profiles. Consequently, it has been observed that the performance prediction curves obtained using the Bi-linear prediction method can also be obtained from the Markov chain-based condition prediction method with transition probabilities computed from simulated condition rating data.

As a conclusion, in this study, the Markov decision process-based model is examined and a computer program to find the optimal policy with discounted life-cycle cost is developed. The other performance prediction model investigated in this study is a probabilistic Bi-linear model which takes into account the uncertainties for the deterioration process and the application of maintenance actions by the use of random variables. As part of the study, in order to further analyze and develop the Bi-linear model, a Latin Hypercube Sampling-based (LHS) simulation program is also developed and integrated into the main computational algorithm which can produce condition, safety, and life-cycle cost profiles for bridge members with and without maintenance actions. Furthermore, a polynomial-based condition prediction is also examined as an alternative performance prediction model. This model is obtained from condition rating data by applying regression analysis. Regression-based performance curves are regenerated using the Latin Hypercube sampling method. Finally, the results from the Markov chain-based performance prediction are compared with Simulation-based Bi-linear prediction and the derivation of the transition probability matrix from simulated regression based condition profile is introduced as a newly developed approach. It has been observed that the results obtained from the Markov chain-based average condition rating profiles match well with those obtained from Simulation-based mean condition rating profiles. The result suggests that the Simulation-based condition prediction model may be considered as a potential model in future BMSs.

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