CHANCE CONSTRAINED OPTIMIZATION OF BOOSTER DISINFECTION IN WATER DISTRIBUTION NETWORKS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

EZGİ KÖKER

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
CIVIL ENGINEERING

SEPTEMBER 2011

Approval of the thesis:

CHANCE CONSTRAINED OPTIMIZATION OF BOOSTER DISINFECTION IN WATER DISTRIBUTION NETWORKS

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ABSTRACT

CHANCE CONSTRAINED OPTIMIZATION OF BOOSTER DISINFECTION IN WATER DISTRIBUTION NETWORKS

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September 2011, 109 pages

Quality of municipal water is sustained by addition of disinfectant, generally chlorine, to the water distribution network. Because of health problems, chlorine concentration in the network is limited between maximum and minimum limits. Cancerogenic disinfectant by-products start to occur at high concentrations so it is desired to have minimum amount of chlorine without violating the limit. In addition to the health issues, minimum injection amount is favorable concerning cost. Hence, an optimization model is necessary which covers all of these considerations. However, there are uncertain factors as chlorine is reactive and decays both over time and space. Thus, probabilistic approach is necessary to obtain reliable and realistic results from the model. In this study, a linear programming model is developed for the chance constrained optimization of the water distribution network. The objective is to obtain minimum amount of injection mass subjected to maintaining more uniformly distributed chlorine concentrations within the limits while considering the randomness of chlorine concentration by probability distributions. Network hydraulics and chlorine concentration computations are done by the network simulation software, EPANET.

Keywords: Water Distribution Network, Linear Optimization, Chance Constraint, Chlorine Concentration, EPANET

ÖZ

SU DAĞITIM ŞEBEKELERİNDE EK KLORLAMA MİKTARININ OLASILIK SINIRLAMALI OPTİMİZASYONU

Köker, Ezgi

Yüksek Lisans, İnşaat Mühendisliği Bölümü

Tez Yöneticisi: Doç. Dr. Ayşe Burcu Altan Sakarya

Eylül 2011, 109 sayfa

İçme suyunun dezenfektasyonu su dağıtım şebekesine klor enjekte edilerek sağlanır. Şebekedeki klor miktarı, sağlık sorunlarına yol açması göz önünde bulundurularak belli bir minimum ve maksimum değer arasında sınırlandırılmıştır. Klor konsantrasyonu arttığında kanserojen etkiye sahip yan ürünler oluşmaya başlar. Bu yüzden şebekede en istenen durum, minimum sınırı ihlal etmeden en az düzeyde klor miktarına sahip olmaktır. İstenen bu durum aynı zamanda maliyet açısından da avantajlıdır. Böylece, bütün ihtiyaçları karşılayacak bir optimizasyon modeli gerekmektedir. Klor, tepkisel bir madde olduğundan zamana ve mesafeye göre azalmaktadır. Bu durum değişken faktörlerin ortaya çıkmasına sebebiyet verir. Modelden güvenilir ve gerçekçi sonuçlar alabilmek için değişken faktörlerden kaynaklanan olasılık hesaplarının formülizasyonun içine katılması gerekir. Bu çalışmada, su dağıtım şebekesinin olasılık sınırlamalı optimizasyonu için lineer bir programlama modeli oluşturulmuştur. Amaç, sisteme uygulanan toplam klor miktarını en aza indirmektir. Optimizasyondaki kısıtlama klor miktarını, klor konsantrasyonunun rastgele davranışını dağılımlarla hesaba katarak daha düzgün bir dağılıma sahip şekilde gerekli değerler arasında tutmaktır. Sistem hidrolik değerleri ve klor konsantrasyonu hesaplamaları su şebekesi modelleme ve çözüm yazılımı olan EPANET ile elde edilmiştir.

Anahtar Kelimeler: Su Dağıtım Şebekesi, Lineer Optimizasyon, Olasılık Sınırlaması, Klor Konsantrasyonu, EPANET

To my dear family,

ACKNOWLEDGEMENTS

Just like all the other theses's writing, this thesis was a difficult progress too. In this process, besides all those analysers, numbers, results and writing, I had lots of memories that made me smile. I know that whether good or bad every memory will fade away as the time passes. However, I must make those moments which are more valuable than the results unforgettable. All those precious people deserve to be remembered a lifetime.

Dear me in fourty years, when you first started master's degree you selected an advisor instead of selecting a topic. The subject was not important, only she must have thought it. Well, you were right. In the past two years you have seen that you can handle anything with an advisor that trusts you and cares about you more than yourself. Do not forget how peaceful was it to have an advisor who works with full effort for your aim, reacts mildly even though she is worried about your working speed, explains over and over again until it eases your worries, listens and embraces you. Maybe you won't become a teacher as good as Burcu Altan Sakarya but continually walk the path that she lead.

Remember one night your father told you that "the only being that loves you unconditionally and infinitely is your family". That was really true. Love always saved you when you were in stress. The fruit plates that your mother brought to freshen you, the kisses your father dropped on your head at midnights, the chit chats and laughs with Ekin, dancing all together at evenings and discussions at dinner table were your doping. You realized that nothing can give the serenity of a family who looks upon your face with proud and excitement even if they did not understand your thesis presentation.

In this two years, you saw that love is not a function of distance one more time. Emre Ölçeroğlu's trustful hug and the support of his calming words took away the anxieties of how will the doctorate be and what the future will bring.

Then you noticed that kilometers are not an obstacle to sleep early and wake up early. You took every step of your life with Yaprak Onat and for the rare steps you did not took together, you have spent hours on the phone without getting bored. The shining sun of your life has made you complete this thesis with german discipline while she was kilometers away. Then you once again understood that both space and time are relative when the subject comes to caring more than yourself.

It was your "Life" that you walked for hours and shared everything. You poured the troubles into pastas with Gülseda Karakuş who called you everyday to check whether you are fine just like she did in all of the troubled times of your life. Remember the happiness of a cake plate that appears above your head after finishing a long and intense thesis period.

Life has given you a new chance during this two years. When words were not enough to show that she understands, Nazlı Aslıcan Yılmaz sheltered in "lüp lüp makintoş". Every night she fitted the ocean distance into a single room and tried to keep your hopes alive for your aims.

It was preparatory class when you met him but maybe your best sharings happened in these two years. Alper Erişen is the one that hugs you tightly when you are crying without questioning. He became the "Zihni Sinir" of your house, partner of sunflower seeds and a hard time friend.

Do not forget Burcu Şimşir who has an unbelievably big and pure heart. Even if you did not talk for long, everything went on from where you left when you came together with her and she spend all the efford that she got for making someone happy although she struggles with life.

Always remember the ones who are responsible from your smiling everyday. You added lots of laughs to lunches, happy hours and long roads with Tuğçe Yıldırım, Meriç Apaydın, Alper Önen, Gizem Okyay, Ürün Bakar. They made everything to make life easier for you and make you social again during the thesis writing.

Do not forget Halit Şahiner, who tried to find solutions for your departure more than he did for himself and tried to make you happy by appreciating the work you have done every day. Ali Baykara who sat and scanned the papers with you even if he was not related with the subject and Deniz Şen who gave you peace with her smile and love.

Can Gökgöl is an exceptional person. Keep in mind how he was generous while showing his love, how hard times are easier with him and how you like sharing with him, telling him and listening from him.

Remember Harun Serin who made you feel like the very first thesis writer and gave you motivation during thesis period. Alper Mutlu with whom you take enthusiasm for writing more sentences by playing with protos and how both of them embrace you as sister.

You understood that being MSc. is not obtained by completing your masters degree but with the great respect of the ones who loves you; Belin Karagöz, Murat Ayhan, Emir Alimoğlu, Orhun Günel. Do not forget how much they tried for your happiness.

Besides all of your academic work, these smilings were what you got. Please do not ever underestimate or forget them. Present you believes that life is composed of smilings and at any way it should be owned and made others have it.

TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

LIST OF SYMBOLS AND ABBREVIATIONS

DBP	: **D**isinfectant **B**y-**P**roducts
GA	: **G**enetic **A**lgorithm
LP	: **L**inear **P**rograming
PDF	: **P**robability **D**istribution **F**unction
CDF	: **C**umulative **D**istribution **F**unction
UL	: **U**pper **L**imit
LL	: **L**ower **L**imit
BL	: **B**oth of the **L**imits
LPS	: **L**iters **P**er **S**econd
WHO	: **W**orld **H**ealth **O**rganization
SDWA	: **S**afe **D**rinking **W**ater **A**ct
N	: **N**ormal Distribution
LN	: **L**og-**N**ormal Distribution
ir	: Monitoring node
n_b	: Number of booster disinfection stations
n_s	: Number of source locations
n_t	: Number of scheduling time intervals of booster location
Δt	: Booster mass injection time interval
u_i^j	: Mass injection rate at booster node i and time period j
tr	: Monitoring time
$\beta_{ir,tr}^{i,j}$	: Response coefficient at monitoring node ir at time tr
$C_{ir,tr}$	: Concentration value at monitoring node ir at time tr
Y	: Objective function notation
$\overline{C}$	: Upper limit of the concentration

$\underline{C}$	: Lower limit of the concentration
U	: Booster mass injection matrix
B	: Response coefficient matrix
C	: Nodal concentration matrix
T_c	: Sampling time horizon
$f(x)$	: Probability distribution function of a normal variable
$f_X(x)$	: Probability distribution function of a log- normal variable
μ	: Mean of the normal distribution
λ	: Mean of the log-normal distribution
σ	: Standard deviation of the normal distribution
ξ	: Standard deviation of the log-normal distribution
X	: Random variable
Z	: Standardized variate of a random variable
$\phi(z)$	: Probability distribution function of a standardized variable
$\Phi(z)$	: Cumulative distribution function of a standardized variable
α_U	: Specified reliability of upper chlorine concentration limit
α_L	: Specified reliability of lower chlorine concentration limit
$\mu_{\overline{C}}$	: Mean of the normally distributed upper limit
$\mu_{\underline{C}}$	: Mean of the normally distributed lower limit
$\lambda_{\overline{C}}$	: Mean of the log-normally distributed upper limit
$\lambda_{\underline{C}}$	: Mean of the log-normally distributed lower limit
$\sigma_{\overline{C}}$	: Standard deviation of the normally distributed upper limit
$\sigma_{\underline{C}}$	: Standard deviation of the normally distributed lower limit
$\xi_{\overline{C}}$	: Standard deviation of the log-normally distributed upper limit
$\xi_{\underline{C}}$	: Standard deviation of the log-normally distributed lower limit
$Z_{\overline{C}}$	: Standardized variate of upper limit
$Z_{\underline{C}}$	: Standardized variate of lower limit
k_b	: Bulk coefficient
k_w	: Wall decay coefficient

CHAPTER 1

INTRODUCTION

1.1 Statement of the Problem

Water quality of the municipal water is generally provided by addition of the disinfectants. In order to supply sufficient quality, disinfectant concentration, i.e. chlorine, is limited between a maximum and a minimum limit. The reason of setting a maximum limit is that after that concentration, taste and odor problems start to occur. In addition to these problems, formations of the disinfectant by-products (DBPs) start which can lead to serious health problems. Likewise, minimum limit is set by considering the biological regrowth. Hence, having lowest amount of chlorine concentrations without violating the minimum limit is the most favorable condition.

Amount of disinfectant mass supplied to the system should be adjusted carefully in order to supply water to the far ends of the water distribution networks with sufficient quality. High amount of disinfectant injection in order to cover the problems in the far ends, may result in excessive concentration values at the nodes near the booster stations. Moreover, it is not easy to calculate the total disinfectant demand as chlorine is decaying over time and space. Thus, it is necessary to adjust an appropriate scheduling of the booster disinfection facility in order to overcome this problem.

An optimization model is needed to arrange the scheduling of the booster disinfection facility. While modeling, it should be considered that there are many uncontrollable factors that will result in significant difference between the analysis results and the real case as nothing is as certain as it is modeled. Main uncertainty in the booster disinfection is the chlorine decaying. So, in order to get more realistic results, probability concept should be introduced to the optimization procedure.

Consequently, an optimization model which will cover uncertainties in the network should be developed in order to minimize the total mass injected to the system while maintaining the residual concentrations within maximum and minimum limits.

1.2 Literature Survey

To develop a well supported study on chance constrained optimization of booster disinfection, previous works related with the optimization of the water distribution network and uncertainty concept for water distribution network are reviewed and summarized in the following sections.

1.2.1 Optimization of the Water Distribution Network

Studies related with the optimization of the water distribution network are based on 1970s and they are mostly depending on the physical characteristics of the hydraulic components. For example, Deb and Sarkar (1971), applied the optimization procedure to a network consisting of pumps, elevated reservoir and pipes with known nodal pressures and water consumptions. Aim in the study is to minimize the cost by taking pipe sizes as decision variable.

Then, in 1980s scope of the optimization studies started to cover operational characteristics. Zessler and Shamir (1989) conducted a study to find the optimal scheduling of pump operation of a water supply system with an iterative dynamic programming method.

In the similar times, optimization of water quality in municipal water distribution systems was also studied. Mark et al. (1987) developed a nonlinear optimization problem with the purpose of minimizing the cost. Analyses are done with a simulation model and it is significant that the formation of simulation models for water distribution networks enabled long period and complex analyses for network hydraulics and water quality.

Later in 1990s, water quality and operational characteristics of a water distribution network started to be combined. Boccelli et al. (1998), worked with the optimal scheduling of booster disinfection facilities in the water distribution networks. Aim in this study was to minimize the total mass injected to the network while maintaining the chlorine concentrations at the consumer nodes between a maximum and a minimum limit. Using a linear programming formulation, it was proven that optimal scheduling of booster station reduces the general chlorine concentration in the network and number of the booster stations with their locations effect the optimal schedule.

In 2000s Tryby et al. (2002), enhance their work with Bocelli et al. (1998) by introducing operation types of booster disinfection facilities to the analyses. Also, minimization of the total number of booster stations was the additional aim of the study. Similar results were obtained from the mixed integer linear programing problem.

After the studies of Tryby and Bocelli (2002), Munavalli and Kumar (2003) formulated the problem in a nonlinear way in order to cover set-point sources and non-first-order reactions. Aim is formulated in this work as obtaining lowest difference between the nodal chlorine concentrations and the minimum concentration amount for all consumer nodes and for all time intervals. Using a genetic algorithm (GA) approach for the solution, it was proven that GA is useful for calculating optimal schedule of water quality sources.

Propato and Uber (2004) introduced a linear least-squares formulation to the problem with the purpose of optimal disinfectant scheduling that minimizes the variation in the system residual space-time distribution. Different type of booster facilities for both linear least squares and linear programming method were compared in the study and it was concluded that booster disinfection effects the reduction in disinfectant residual variations while minimizing the total injected mass.

Lansey et al. (2007) improved the optimal schedule of the booster disinfection formulation by taking initial conditions into consideration. By taking initial concentrations equal to final concentrations, long water quality simulations are avoided.

1.2.2 Uncertainty Concept for Water Distribution Network

In 1980s, uncertainties started to play role in the optimization studies. Lansey et al. (1989), formulated a chance constrained optimization problem as nonlinear programing model for the minimization of the cost in a water distribution network. Uncertainties of necessary amount of demands, pressure heads and pipe roughness coefficients were taken into consideration and it was the accepted fact that they were independent from each other. Problem was solved with a generalized reduced gradient method and it was concluded that considering uncertainties have important effect on the optimal network design.

Then, Goulter and Bouchart (1990) introduced a least-cost optimization model for looped water distribution networks which covered the probability of pipe failure and probability of actual demand exceeding the design value. By combining these probabilities, a single reliability measure was obtained as probability of no node failure.

In the same year, Bao and Mays (1990) conducted a study on nodal and system hydraulic reliabilities of water distribution systems. Using Monte Carlo simulation, impact of uncertainties of water demands, pressure heads and pipe roughness’s on nodal and system reliability examined. In addition to that, different probability distributions were tried to find the sensitivity of reliability.

In the study of Jacobs (1991), a mixed integer chance constrained optimization model was formed for structural design. Although its subject is not related with the water distribution networks, the study contains explanation of obtaining the deterministic equivalent of chance constraint using established cumulative distribution function of the variable.

Cullinane et al. (1992) extended the work of Lansey et al. (1989) with integrating a reliability based procedure. In this study, reliability of water distribution systems based on hydraulic availability represented and it is combined with nonlinear optimization procedure for component sizing.

Datta and Dhiman (1996) formed a two parted mathematical model of ground water quality monitoring network. First one was the ground water pollution transport simulation model and the second one was the chance constrained optimization model. Uncertainties were resulting from the prediction of pollutant movement. Response matrix approach and chance constraint formulations with using cumulative distribution functions were examined in the study.

Later, El-Gamel and Harrell (2003) developed a chance constraint optimization model with GA based search procedure in order to minimize the costs of water use and canal cleaning while maintaining stability of gates, maintaining adequate water levels in canals and preventing flood and water shortages. Uncertain parameters in the study were crop distributions and water demands in the network.

Das (2007) presented a chance constrained optimization model for Muskingum model parameter estimation. The aim is to minimize the sum of squares of difference between the actual observed and computed outflows with chance constrained Muskingum flow routing equations. Hydraulic data was accepted as uncertain following a standard normal distribution.

Ezzeldin et al. (2008) conducted a study on a new approach to the reliability based optimal design of water distribution networks. In the approach, GA was used for the optimization tool and Newton method as the hydraulic simulation solver. Chance constraint integrated to Monte Carlo simulation to estimate network capacity reliability. Uncertainty was resulting from the external nodal demands which was accepted to follow a normal probability distribution. Effect of different reliability levels and coefficient of variations tried in the study.

1.3 Objective of the Study

The main aim of the study is to show the effect of uncertainty concept on the optimization of the booster disinfection. The objective of the optimization is to obtain minimum amount of applied disinfection while maintaining the chlorine concentrations within the specified limits. And, the purpose of introducing uncertainty is to examine the relationship between the reliability and total applied mass dosage, which indicates the cost. Obtaining more reliable water distribution networks mean having more uniformly distributed and low amount of chlorine concentrations while maintaining the minimum restraint. The main source of the uncertainty in this study is the decaying of the chlorine, thus probabilistic approach is applied to the constraint of the chlorine concentration for both maximum and minimum limits. In the analyses, the effects of different types of probability distributions are also investigated.

1.4 Outline of the Thesis

There are mainly four chapters in this study apart from the introduction part. These are optimal booster disinfection, uncertainty concept for booster disinfection, application and discussion of the results, conclusion and future work.

Firstly, Chapter 2 gives the optimal booster disinfection. In this chapter, why there is a need for appropriate scheduling of booster disinfection facility is briefly explained. Afterwards, aim of the optimization and formulization of the optimization problem are clarified.

Then, in Chapter 3, how uncertainty concept is associated with the booster disinfection concept is described. In addition to that, the type of probability distributions and chance constrained model formulations corresponding to these distributions are explained.

Next, Chapter 4 is the application and discussion of the results. In this part, characteristics of the application model are given and meaning of water quality being sufficient is described. After, the results of using different combinations and numbers of booster locations are briefly examined and lastly results of the application of chance constraint to selected two cases are given.

Finally in Chapter 5, conclusion of this study is done and future recommendations are given.

CHAPTER 2

OPTIMAL BOOSTER DISINFECTION

Booster disinfection facilities are used to provide water within quality limits and with minimum variations to the consumers in the network. Water quality is determined by a detailed analysis of the variations in the network related with water and disinfectant concentration. Water demands at consumer nodes, discharge directions and amounts in the pipes are general water related variations. Likewise, amount of booster disinfection facilities, their operation and scheduling are general disinfectant concentration related variations.

As water is not decaying over time it is easy to calculate the total amount that will satisfy the demands. However, it is difficult to supply water with disinfectant concentration within desired limits to every consumer node, as the disinfectant, i.e. chlorine, diminishes over time. In order to avoid insufficiencies in providing the necessary concentration amount, decaying of the disinfectant both over time and space have to be taken into consideration. To overcome this problem, appropriate scheduling of the booster disinfection facility is required.

2.1 Model Formulation

The main aim while applying optimization to the booster disinfection problem is to minimize the total mass injected to the system while maintaining the chlorine concentrations within limits at each consumer node. As the objection is to minimize the cost by minimizing the total mass injected, the number of the booster stations and the locations of them have to be considered as well.

In the optimization formulation, objection function is the minimization of the total disinfectant amount injected to the network. Constraints are the chlorine concentration to be between given upper and lower bounds and disinfectant injected to the system to be non-negative.

2.1.1 General Formulation

For a sample water distribution network, there are *ir* consumer nodes, n_b number of possible booster locations, n_s source locations and n_t dosage schedule time intervals within typical daily operation of booster with time step size of Δt. Consumer nodes in the network are the nodes that have demand, which means chlorine concentration for each of them for each time interval will be checked. On the other hand, possible booster locations are dummy nodes and unlike consumer nodes they do not have demand. Thus, disinfectant concentrations for these nodes will not be checked.

As the aim of the optimization is to minimize the total chlorine mass added to the system, objective function is the minimization of,

$$= \sum_{j=1}^{n_t} \sum_{i=1}^{n_b+n_s} \left(u_i^j\right)\Delta t \tag{2.1}$$

where u_i^j is the mass injection rate *(M / T)* at booster or source location *i*, at time *j* and Δt is the time step duration *(T)*. In the equation, mass injection rate, u_i^j is the decision variable and multiplying it with time step duration, Δt, the total mass dosage supplied to the water distribution network from location *i* and at time *j (M)* is obtained. In order to find the total mass added to the network, summation of total mass dosages for every location, n_b+n_s, and for every time period, n_t, is taken into consideration.

There are two constraints for this objective function. First one is to have disinfectant concentration within limits and the second one is the non-negativity of the injections. Concentration limitation can be formulated as,

$$\underline{C} \leq C_{ir,tr} = \sum_{j=1}^{n_t} \sum_{i=1}^{n_b+n_s} \beta_{ir,tr}^{i,j} \left(u_i^j\right) \leq \overline{C} \tag{2.2}$$

for all consumer nodes, *ir* and monitoring times, *tr*.

In addition to that, non-negativity of the injections can be formulated as,

$$u_i^j \geq 0 \tag{2.3}$$

In Equation 2.2, the nodal concentrations for each consumer node *ir* and monitoring time *tr*, $C_{ir,tr}$ *(M / L^3)*, is limited between upper bound, $\overline{C}$ and lower bound, $\underline{C}$. In the expression of $C_{ir,tr}$, the term $\beta_{ir,tr}^{i,j}$ is corresponding to the response coefficient $\beta_{ir,tr}^{i,j} = \partial C_{ir,tr} / \partial u_i^j$ which is the chlorine change at the consumer node *ir*, at monitoring time *tr* corresponding to the unit injection at the booster or a source location *i*, at time *j*, *[(M / L^3)/(M / T)]*. The effect of the individual injections to the response nodes could be represented as linear functions of the injections, *u*, which is called the linear superposition principle. By applying this principle, Boccelli et al. (1998) has illustrated the time varying system hydraulics and chlorine dosages as a linear system. Thus if, u_i^j units of chlorine is added from location *i*, at time *j;* the total response at the monitoring node *ir* will be $\beta_{ir,tr}^{i,j} u_i^j$. So, chlorine concentration values at all consumer nodes for all monitoring times corresponding to each booster location and injection period are calculated by using the summation in Equation 2.2.

For visualization of the notations used in the formulation, a sample network can be seen in Figure 2.1. This network composed of one booster node, *i*, and two consumer nodes, ir_1 and ir_2.

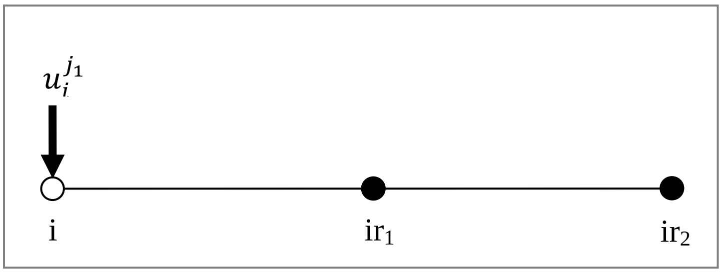

Figure 2.1: Sample network composed of three nodes

Total mass added from the booster node, *i*, at time j_1 to the network shown in Figure 2.1 is $u_i^{j_1}$. Chlorine concentrations at monitoring nodes ir_1 and ir_2 at time j_1 are $C_{ir1,j1}$and $C_{ir2,j1}$, respectively. The response coefficients at monitoring nodes ir_1 and ir_2 resulting from this injection at time j_1 are $\beta_{ir1,tr}^{i,j1}$and $\beta_{ir2,tr}^{i,j1}$ respectively. Note that, response coefficients will be calculated for all time periods, *tr*.

2.1.1.1 Booster Mass Injection Matrix

Booster mass injection matrix, *U* is the all in one representation of periodic injections of all booster disinfection stations for all time intervals. For example, if the injection time interval is selected as 1 *h*, there will be 24 different injections for a typical daily operation of the booster station. If it is shown in a vector notation, the periodic injections for one booster disinfection station will be,

$$U = [u_1^1, \quad u_1^2, \quad \cdots\cdots \quad u_1^{23}, \quad u_1^{24}] \tag{2.4}$$

Likewise, in the case of two booster disinfectant stations, vector form of Equation 2.4 becomes,

$$U = [u_1^1, \quad u_1^2, \quad \cdots\cdots \quad u_1^{23}, \quad u_1^{24}, \quad u_2^1, \quad u_2^2, \quad \cdots\cdots \quad u_2^{23}, \quad u_2^{24}] \tag{2.5}$$

2.1.1.2 Response Coefficient Matrix

Response coefficient matrix, B is the all in one representation of responses of all consumer nodes for each periodic injection from the booster station and for all monitoring time intervals. To obtain this matrix, only one booster disinfection node is selected; a suitable amount of chlorine is added to the network and responses of each consumer node for each monitoring time interval is recorded as $\beta_{ir,tr}^{i,j} = C_{ir,tr}/u_i^j$. Then, this procedure is repeated for each booster station and gathering the entire recorded data together, response coefficient matrix is formed. For example, considering a water distribution network with one consumer and one booster node, selecting injection time interval 1 h again, there will be 24 different injections for a typical daily operation. If $u_{i_1}^{j_1}$ amount of disinfectant injected to the network, response coefficient matrix will be,

$$B_{ir1,tr}^{i1,j1} = \begin{bmatrix} \beta_{ir1,1}^{i1,j1} \\ \beta_{ir1,2}^{i1,j1} \\ \vdots \\ \vdots \\ \beta_{ir1,24}^{i1,j1} \end{bmatrix} \tag{2.6}$$

Likewise, in the case of two monitoring nodes with the same injection $u_{i_1}^{j_1}$, vector form of Equation 2.6 becomes,

$$B_{ir1,tr}^{i1,j1} = \begin{bmatrix} \beta_{ir1,1}^{i1,j1} \\ \beta_{ir1,2}^{i1,j1} \\ \vdots \\ \vdots \\ \beta_{ir1,24}^{i1,j1} \\ \beta_{ir2,1}^{i1,j1} \\ \beta_{ir2,2}^{i1,j1} \\ \vdots \\ \vdots \\ \beta_{ir2,24}^{i1,j1} \end{bmatrix} \tag{2.7}$$

Apart from the example, a general matrix form of response coefficient for a water distribution network consisting of n_b+n_s number of booster locations and ir_C number of consumer nodes, can be obtained as,

$$B = \begin{bmatrix} \beta_{ir1,tr=1}^{i=1,j=1} & \cdots & \beta_{ir1,1}^{1,24} & \beta_{ir1,1}^{2,1} & \cdots & \beta_{ir1,1}^{2,24} & \beta_{ir1,1}^{nb+ns,1} & \cdots & \beta_{ir1,1}^{nb+ns,24} \\ \beta_{ir1,2}^{1,1} & \cdots & \beta_{ir1,2}^{1,24} & \beta_{ir1,2}^{2,1} & \cdots & \beta_{ir1,2}^{2,24} & \beta_{ir1,2}^{nb+ns,1} & \cdots & \beta_{ir1,2}^{nb+ns,24} \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ \beta_{ir1,24}^{1,1} & \cdots & \beta_{ir1,24}^{1,24} & \beta_{ir1,24}^{2,1} & \cdots & \beta_{ir1,24}^{2,24} & \beta_{ir1,24}^{nb+ns,1} & \cdots & \beta_{ir1,24}^{nb+ns,24} \\ \beta_{ir2,1}^{1,1} & \cdots & \beta_{ir2,1}^{1,24} & \boldsymbol{\beta_{ir2,1}^{2,1}} & \cdots & \beta_{ir2,1}^{2,24} & \beta_{ir2,1}^{nb+ns,1} & \cdots & \beta_{ir2,1}^{nb+ns,24} \\ \beta_{ir2,2}^{1,1} & \cdots & \beta_{ir2,2}^{1,24} & \beta_{ir2,2}^{2,1} & \cdots & \beta_{ir2,2}^{2,24} & \beta_{ir2,2}^{nb+ns,1} & \cdots & \beta_{ir2,2}^{nb+ns,24} \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ \beta_{ir2,24}^{1,1} & \cdots & \beta_{ir2,24}^{1,24} & \beta_{ir2,24}^{2,1} & \cdots & \beta_{ir2,24}^{2,24} & \beta_{ir2,24}^{nb+ns,1} & \cdots & \beta_{ir2,24}^{nb+ns,24} \\ \beta_{irC,1}^{1,1} & \cdots & \beta_{irC,1}^{1,24} & \beta_{irC,1}^{2,1} & \cdots & \beta_{irC,1}^{2,24} & \beta_{irC,1}^{nb+ns,1} & \cdots & \beta_{irC,1}^{nb+ns,24} \\ \beta_{irC,2}^{1,1} & \cdots & \beta_{irC,2}^{1,24} & \beta_{irC,2}^{2,1} & \cdots & \beta_{irC,2}^{2,24} & \beta_{irC,2}^{nb+ns,1} & \cdots & \beta_{irC,2}^{nb+ns,24} \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ \beta_{irC,24}^{1,1} & \cdots & \beta_{irC,24}^{1,24} & \beta_{irC,24}^{2,1} & \cdots & \beta_{irC,24}^{2,24} & \beta_{irC,24}^{nb+ns,1} & \cdots & \beta_{irC,24}^{nb+ns,24} \end{bmatrix} \quad (2.8)$$

In order to understand the matrix in a simpler way it can be stated that first 24 columns are corresponding to booster station 1 and indicating the injections at each 1 *h* time increment from the 1st to the 24th hour. Similarly, next 24 columns are corresponding to the booster station 2 and columns will continue until the number of the booster stations in the network is reached. In addition to that, first 24 rows are corresponding to consumer node 1 and indicating the responses at each 1 *h* time increment from the 1st to the 24th hour, to the injection at the corresponding time. Likewise, next 24 rows are corresponding to the consumer node 2 and rows will continue until the number of the monitoring nodes in the network is reached. For example, the coefficient $\beta_{ir2,1}^{2,1}$, which is written in bold in the matrix, refers to the response of the monitoring node ir_2 at time $tr=1$, to the injection from booster location 2, $i=2$ at time $j=1$.

2.1.2 Linear Programing Formulation

In the model optimization formulation, objective function, Equation 2.1 and constraints, Equations 2.2 and 2.3 are all linear. Consequently, network problem can be treated as linear programing (LP) problem. When this LP problem is combined with the new expressions of booster mass injection matrix, Equation 2.5 and response coefficient matrix, Equation 2.8, new form of the LP becomes,

Objective function,

$$Minimize\ Y = \sum_{j=1}^{n_t} U^T \tag{2.9}$$

Subject to,

$$\underline{C} \leq C = BU^T \leq \overline{C} \tag{2.10}$$

and

$$U^T \geq 0 \tag{2.11}$$

Response coefficient matrix, *B*, corresponds to the responses of all consumer nodes to unit disinfectant amount injected from the booster station. So, if *B* is multiplied with the real amount supplied to the network for corresponding time intervals and booster stations, which is booster mass injection matrix *U*, chlorine concentrations at each point will be obtained (Equation 2.10). The reason of taking transpose of *U* is to make it multipliable with *B*.

2.2 Periodicity

Periodic hydraulic dynamics and the chlorine injections are required for the solution of the booster disinfection optimization problem according to the previous works of Boccelli et al. (1998) and Tryby et al. (2002). In order to reach periodic hydraulic dynamics, daily concentrations have to be equal for two consecutive days, which requires a long simulation time. Precise data can be obtained after that state of system is obtained; so after the system periodicity, the disinfectant residual concentrations at the consumer nodes are recorded over a time period, T_c called impact cycle (Figure 2.2).

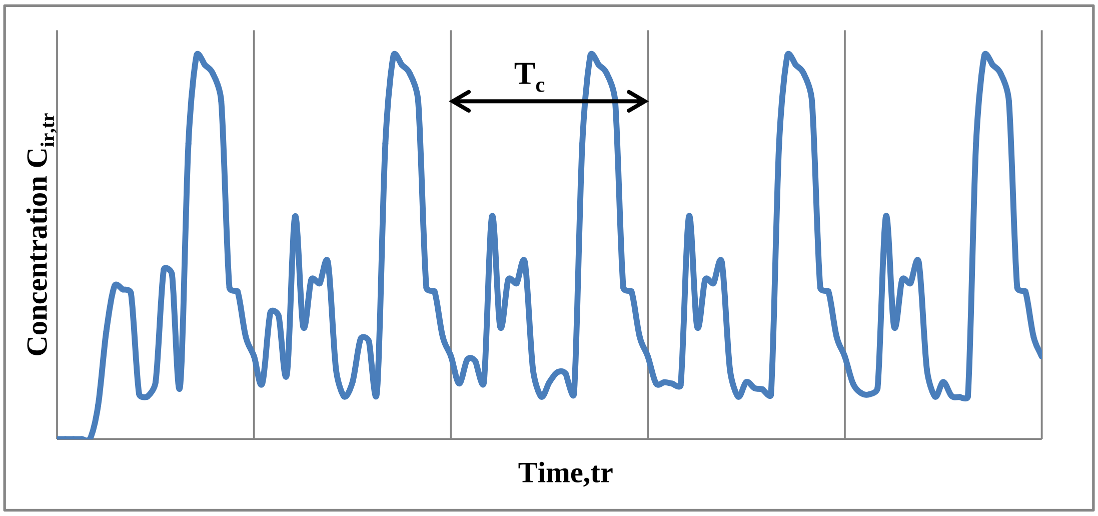

Figure 2.2: Representation of impact cycle after periodicity

Similarly, response coefficients for every monitoring node are calculated after the system periodicity and they are accepted as correct when the difference between two successive impact cycles is negligible (Sert, 2009). For this research, the difference of response coefficients for being negligible is to be less than 10^{-5} (Equation 2.12).

$$\left|\beta_{ir}^{imp(t)} - \beta_{ir}^{imp(t-1)}\right| \leq 10^{-5} \tag{2.12}$$

where $\beta_{ir}^{imp(t)}$ is the response coefficients at a certain consumer monitoring node for impact cycle t and $\beta_{ir}^{imp(t-1)}$ is for the previous impact cycle t - 1.

CHAPTER 3

UNCERTAINTY CONCEPT FOR BOOSTER DISINFECTION

During the water distribution network analysis and optimization of the booster disinfection formulation procedure, every step is taken by assuming a certainty. Two examples of the accepted certainty for water distribution network analyses can be assuming that there will be no uncontrolled losses in the network and demands will be certain during a time period. Likewise, two examples of the accepted certainty for booster disinfection formulations can be assuming that there will be no difference in the optimization formulation of different booster disinfector types and assumptions in the calculation of chlorine decaying bulk coefficients. These kinds of certainties are accepted in order to obtain simpler solvable mathematical models and they result in applicable solutions.

For the real life, nothing is certain as it is modeled. There are many uncontrollable factors that will result in significant difference between the analysis results and the real case. All mathematical or simulation models are aimed to design idealized representations of reality; however they are imperfect representations of the real case. So, for more realistic and reliable results, probabilistic approach has to be taken into consideration in order to cover these uncertainties.

3.1 Governing Probability Distributions

In the optimization of booster disinfection in a water distribution network, the component that has random characteristic is the chlorine concentration. This randomness is resulting from the space and time dependent decaying property of the chlorine. The only way to consider uncertainties is to obtain the probability distribution of this random variable.

There are several probability distribution functions (PDF) that are frequently used in the reliability analysis of the continuous random variables as such the normal, lognormal, Gamma and exponential distributions. For this study, most frequent ones, normal and the lognormal distributions will be examined.

3.1.1 The Normal Distribution

The normal distribution, which is also known as the Gaussian distribution, is a well-known probability distribution (Ang et al., 1975). The probability density function of a normal distribution is given as,

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right], \quad \text{for} \quad -\infty < x < +\infty \qquad (3.1)$$

where μ is the mean and σ is the standard deviation. Both are the parameters of the distribution and simple notation for the distribution is $N(\mu, \sigma)$.

In order to make the probability computations, the normal random variable has to be transformed into its standardized varied Z which denotes a distribution with $\mu = 0$ and $\sigma = 1$. X is a normally distributed random variable and since Z is a linear function of X, Z is also normally distributed. This distribution is called the standard normal distribution and denoted as N (0,1). Z can be expressed as,

$$Z = \left(\frac{X-\mu}{\sigma}\right) \qquad (3.2)$$

The probability density function of a standard normal distribution is given as,

$$\phi(z) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{z^2}{2}\right], \quad \text{for } -\infty < x < +\infty \tag{3.3}$$

The cumulative distribution function (CDF) tables of Z can be found in statistics textbooks (Blank, 1980; Devore, 1987).

Probability of random variable $X \sim N(\mu, \sigma)$ can be described by its CDF, which is

$$P(X \leq x) = P\left[\frac{X-\mu}{\sigma} \leq \frac{x-\mu}{\sigma}\right] = P[Z \leq z] = \Phi(z) \tag{3.4}$$

where x is a value and $\Phi(z)$ is the CDF of standardized variable, Z.

3.1.2 Log-Normal Distribution

The logarithmic normal distribution, simply log-normal distribution, is also a commonly used distribution which can be used when the variable cannot be negative (Ang et al., 1975). A random variable X has a log-normal probability distribution if ln X is normal. The probability density function of a log-normal distribution is given as,

$$f_X(x) = \frac{1}{\sqrt{2\pi x\xi}} \exp\left[-\frac{1}{2}\left(\frac{\ln x - \lambda}{\xi}\right)^2\right], \quad \text{for } 0 < x < +\infty \tag{3.5}$$

where λ is the mean and ξ is the standard deviation. Both are the parameters of the log-normal distribution. These parameters are related to the mean, μ and the standard deviation, σ as,

$$\lambda = \ln \mu - \frac{1}{2}\xi^2 \tag{3.6}$$

$$\xi = \sqrt{\ln\left(1 + \frac{\sigma^2}{\mu^2}\right)} \tag{3.7}$$

With the logarithmic transformation, log-normal distribution is related with the normal distribution. Thus, the table of standard normal probabilities can be used to determine probabilities associated with the log-normal variable.

3.2 Formulation of Chance Constrained Model

In order to obtain chance constrained model, probability concept is applied to the constraints related with the random variable. So, objective function, Equation 2.9 will not undergo any alteration and chlorine concentration limitation constraint, Equation 2.10, will change according to the probability distribution type. As there is both upper and lower limit in the Equation 2.10, there will be three different formulations for each distribution type. Probabilistic approach is applied to the upper limit only in the first one, to the lower limit only in the second one and to the both in the third formulation.

3.2.1 The Normal Distribution

In the chance constrained optimization formulation objective function Equation 2.9 and non-negativity constraint, Equation 2.11 will remain the same. Only chlorine concentration constraint, Equation 2.10 will be adjusted according to the side of the application of chance constraint by taking the probability distribution of chlorine concentration as the normal distribution (Mays and Tung, 1992).

3.2.1.1 Upper Limit

Probabilistic approach will only be applied to the upper limit of the chlorine concentration constraint. So, if the upper limit part of the Equation 2.10 is taken,

$$C = BU^T \leq \overline{C} \tag{3.8}$$

Replacing it with a probabilistic statement in the form of chance constraint,

$$P\left\{ BU^T \leq \overline{C} \right\} \geq \alpha_U \tag{3.9}$$

where α_U is the specified reliability of upper chlorine concentration limit.

Equation 3.9 is not mathematically operational for algebraic solution. Consequently, deterministic equivalent of this equation can be obtained by the following steps. Upper limit, $\overline{C}$, is accepted to have a CDF with mean $\mu_{\overline{C}}$ and standard deviation $\sigma_{\overline{C}}$. Equation 3.9 is equivalent to,

$$P\left[\overline{C} \leq BU^T \right] \leq 1 - \alpha_U \tag{3.10}$$

which can also be expressed in terms of the CDF of $\overline{C}$,

$$F_{\overline{C}}\left(BU^T \right) \leq 1 - \alpha_U \tag{3.11}$$

Using Equation 3.2, standardized versions of random variable $\overline{C}$ is obtained as,

$$Z_{\overline{C}} = \left(\frac{\overline{C} - \mu_{\overline{C}}}{\sigma_{\overline{C}}} \right) \tag{3.12}$$

Introducing Equation 3.12 to Equations 3.10 and 3.11, these can be expressed as respectively,

$$P\left[Z_{\overline{C}} \leq \frac{BU^T - \mu_{\overline{C}}}{\sigma_{\overline{C}}} \right] \leq 1 - \alpha_U \tag{3.13}$$

and

$$F_{Z_{\overline{C}}}\left(\frac{BU^T - \mu_{\overline{C}}}{\sigma_{\overline{C}}} \right) \leq 1 - \alpha_U \tag{3.14}$$

Deterministic equivalent of the original chance constraint, Equation 3.9, is the inverse of Equation 3.14,

$$\frac{BU^T - \mu_{\overline{C}}}{\sigma_{\overline{C}}} \leq F_{Z_{\overline{C}}}^{-1}(1 - \alpha_U) \tag{3.15}$$

Equation 3.15 can be rearranged as,

$$BU^T \leq \mu_{\overline{C}} + (z_{\bar{C},\ 1-\alpha_U})\sigma_{\overline{C}} \tag{3.16}$$

where the specific value of $z_{\bar{C},\ 1-\alpha_U}$ is $F_{Z_{\overline{C}}}^{-1}(1 - \alpha_U)$, which is the $(1 - \alpha_U)$th quantile of the standardized $\overline{C}$. Knowing the PDF of $\overline{C}$ and required constraint reliability, α_U the specific value $z_{\bar{C},1-\alpha_U}$ can be determined. As $\overline{C}$ has a normal distribution, $z_{\bar{C},\ 1-\alpha_U}$ is referring to the standardized normal variant $\phi(z_{\bar{C},\ 1-\alpha_U}) = 1 - \alpha_U$ where $\phi()$ is the standard normal CDF that is expressed in Equation 3.4.

As there is no chance constraint applied to the lower limit of the chlorine concentration in this case, original constraint, Equation 2.10 becomes,

$$\underline{C} \leq BU^T \leq \mu_{\overline{C}} + (z_{\bar{C},\ 1-\alpha_U})\sigma_{\overline{C}} \tag{3.17}$$

Thus, optimization formulation corresponding to the chance constraint applied only to the upper limit of the normally distributed chlorine concentration is composed of the objective function, Equation 2.9, chlorine concentration limitation, Equation 3.17 and non-negativity constraint, Equation 2.11.

3.2.1.2 Lower Limit

In this case, probabilistic approach will only be applied to the lower limit of the chlorine concentration constraint. So, if the lower limit part of the Equation 2.10 is taken,

$$\underline{C} \leq C = BU^T \tag{3.18}$$

Replacing it with a probabilistic statement in the form of chance constraint,

$$P\{\, \underline{C} \;\leq\; BU^T \,\} \geq \alpha_L \tag{3.19}$$

where α_L is the specified reliability of lower chlorine concentration limit.

Equation 3.19 is not mathematically operational for algebraic solution. So, deterministic equivalent of this equation can be obtained by the following steps. Lower limit, $\underline{C}$, is accepted to have a cumulative distribution function (CDF) with mean $\mu_{\underline{C}}$ and standard deviation $\sigma_{\underline{C}}$. Equation 3.19 is equivalent to,

$$P[\; \underline{C} \;\leq BU^T \,] \geq \alpha_L \tag{3.20}$$

which can also be expressed in terms of the CDF of $\underline{C}$,

$$F_{\underline{C}}(\; BU^T \,) \;\geq\; \alpha_L \tag{3.21}$$

Using Equation 3.2, standardized versions of random variable $\underline{C}$ is obtained as,

$$Z_{\underline{C}} = \left(\frac{\underline{C} - \mu_{\underline{C}}}{\sigma_{\underline{C}}} \right) \tag{3.22}$$

Introducing Equation 3.22 to Equations 3.20 and 3.21, these can be expressed as respectively,

$$P\left[\; Z_{\underline{C}} \;\leq\; \frac{BU^T - \mu_{\underline{C}}}{\sigma_{\underline{C}}} \;\right] \geq \; \alpha_L \tag{3.23}$$

and

$$F_{Z_{\underline{C}}}\left(\; \frac{BU^T - \mu_{\underline{C}}}{\sigma_{\underline{C}}} \;\right) \;\geq\; \alpha_L \tag{3.24}$$

Deterministic equivalent of the original chance constraint, Equation 3.19, is the inverse of Equation 3.24,

$$\frac{BU^T - \mu_{\underline{C}}}{\sigma_{\underline{C}}} \geq F_{Z_{\underline{C}}}^{-1}(\alpha_L) \tag{3.25}$$

Equation 3.25 can be rearranged as,

$$BU^T \geq \mu_{\underline{C}} + (z_{\underline{C},\ \alpha_L})\sigma_{\underline{C}} \tag{3.26}$$

where the specific value of $z_{\underline{C},\ \alpha_L}$ is $F_{Z_{\underline{C}}}^{-1}(\alpha_L)$, which is the (α_L)th quantile of the standardized $\underline{C}$. Knowing the PDF of $\underline{C}$ and required constraint reliability, α_L the specific value $z_{\underline{C},\ \alpha_L}$ can be determined. As $\underline{C}$ has a normal distribution, $z_{\underline{C},\ \alpha_L}$ is referring to the standardized normal variant $\phi(z_{\underline{C},\ \alpha_L}) = \alpha_L$ where $\phi()$ is the standard normal CDF that is expressed in Equation 3.4.

As there is no chance constraint applied to the upper limit of the chlorine concentration in this case, original constraint, Equation 2.10 becomes,

$$\mu_{\underline{C}} + (z_{\underline{C},\ \alpha_L})\sigma_{\underline{C}} \leq BU^T \leq \overline{C} \tag{3.27}$$

Consequently, optimization formulation corresponding to the chance constraint which is applied only to the lower limit of the normally distributed chlorine concentration is formed by the objective function, Equation 2.9, chlorine concentration limitation, Equation 3.27 and non-negativity constraint, Equation 2.11.

3.2.1.3 Both of the Limits

In the last case of the normal distribution, probabilistic approach will be applied to both limits of the chlorine concentration constraint. Deterministic equations of probabilistic approach, applied on upper and lower limits separately, are obtained as Equation 3.16 and 3.26, respectively. As a result, deterministic equation of the chance constraint applied to both of the limits will be the combination of these two equations, which is,

$$\mu_{\underline{C}} + (z_{\underline{C},\ \alpha_L})\sigma_{\underline{C}} \leq BU^T \leq \mu_{\overline{C}} + (z_{\bar{C},\ 1-\alpha_U})\sigma_{\overline{C}} \tag{3.28}$$

Therefore, optimization formulation corresponding to the chance constraint applied to both of the limits of the normally distributed chlorine concentration is formed by the objective function, Equation 2.9, chlorine concentration limitation, Equation 3.28 and non-negativity constraint, Equation 2.11.

3.2.2 Log-Normal Distribution

Similarly, objective function, Equation 2.9 and non-negativity constraint, Equation 2.11 will remain the same in the chance constrained optimization formulation. However, chlorine concentration constraint, Equation 2.10 will be adjusted according to the side of the application of chance constraint by taking the probability distribution of chlorine concentration as the log-normal distribution in this case.

3.2.2.1 Upper Limit

Same procedure with the upper limit of normal distribution, 3.2.1.1 is applied up to Equation 3.11. After Equation 3.11, standardization will be done according to the log-normal distribution of chlorine concentration.

As it is explained in the Log-Normal Distribution part, 3.1.2, table of standard normal probabilities can be used to determine probabilities associated with the log-normal variable; however, its own mean, $\lambda_{\overline{C}}$ and standard deviation, $\xi_{\overline{C}}$ must be used. Accordingly, using Equation 3.2 and corresponding parameters, standardized versions of random variable $\overline{C}$ is obtained as,

$$Z_{\overline{C}} = \left(\frac{ln\overline{C} - \lambda_{\overline{C}}}{\xi_{\overline{C}}}\right) \tag{3.29}$$

Introducing Equation 3.29 to Equations 3.10 and 3.11, these can be expressed as respectively,

$$P\left[\ Z_{\overline{C}}\ \leq\ \frac{ln\ (BU^T) - \lambda_{\overline{C}}}{\xi_{\overline{C}}}\ \right] \leq 1 - \alpha_U \tag{3.30}$$

and

$$F_{Z_{\overline{C}}}\left(\ \frac{ln\ (BU^T) - \lambda_{\overline{C}}}{\xi_{\overline{C}}}\ \right) \leq 1 - \alpha_U \tag{3.31}$$

Deterministic equivalent of the original chance constraint, Equation 3.9, is the inverse of Equation 3.31,

$$\frac{ln\ (BU^T) - \lambda_{\overline{C}}}{\xi_{\overline{C}}}\ \leq\ F^{-1}_{Z_{\overline{C}}}(\,1 - \alpha_U\,) \tag{3.32}$$

Equation 3.32 can be rearranged as,

$$BU^T \leq e^{\left(\lambda_{\overline{C}} + (z_{\bar{C},\ 1-\alpha_U})\xi_{\overline{C}}\right)} \tag{3.33}$$

where the specific value of $z_{\bar{C},\ 1-\alpha_U}$ is $F^{-1}_{Z_{\overline{C}}}(\,1 - \alpha_U\,)$, which is the $(\,1 - \alpha_U\,)$th quantile of the standardized $\overline{C}$. Knowing the PDF of $\overline{C}$ and required constraint reliability, α_U the specific value $z_{\bar{C},1-\alpha_U}$ can be determined. As $\overline{C}$ has a log-normal distribution, $z_{\bar{C},\ 1-\alpha_U}$ is referring to the standardized normal variant $\phi\left(z_{\bar{C},\ 1-\alpha_U}\right) = 1 - \alpha_U$ where $\phi()$ is the standard normal CDF that is expressed in Equation 3.4.

As there is no chance constraint applied to the lower limit of the chlorine concentration in this case, original constraint Equation 2.10 becomes,

$$\underline{C}\ \leq\ BU^T \leq e^{\left(\mu_{\overline{C}} + (z_{\bar{C},\ 1-\alpha_U})\xi_{\overline{C}}\right)} \tag{3.34}$$

Hence, optimization formulation corresponding to the chance constraint applied only to the upper limit of the chlorine concentration is composed of the objective function, Equation 2.9, chlorine concentration limitation, Equation 3.34 and non-negativity constraint, Equation 2.11.

3.2.2.2 Lower Limit

Same procedure with the lower limit of normal distribution, 3.2.1.2 is applied up to Equation 3.21. After Equation 3.21, standardization will be done according to the log-normal distribution of chlorine concentration.

As it is explained in the upper limit, table of standard normal probabilities can be used to determine probabilities associated with the log-normal variable; though, its own mean, $\lambda_{\underline{C}}$ and standard deviation, $\xi_{\underline{C}}$ must be used. Thus, using Equation 3.2 and corresponding parameters, standardized versions of random variable $\underline{C}$ is obtained as,

$$Z_{\underline{C}} = \left(\frac{ln\underline{C} - \lambda_{\underline{C}}}{\xi_{\underline{C}}}\right) \tag{3.35}$$

Introducing Equation 3.35 to Equations 3.20 and 3.21, these can be expressed as respectively,

$$P\left[\; Z_{\underline{C}} \;\leq\; \frac{ln\,(BU^T) - \lambda_{\underline{C}}}{\xi_{\underline{C}}} \;\right] \geq \alpha_L \tag{3.36}$$

and

$$F_{Z_{\underline{C}}}\left(\; \frac{ln\,(BU^T) - \lambda_{\underline{C}}}{\xi_{\underline{C}}} \;\right) \geq \alpha_L \tag{3.37}$$

Deterministic equivalent of the original chance constraint, Equation 3.19, is the inverse of Equation 3.37,

$$\frac{ln\ (BU^T) - \lambda_{\underline{C}}}{\xi_{\underline{C}}} \geq F_{Z_{\underline{C}}}^{-1}(\alpha_L) \tag{3.38}$$

Equation 3.38 can be rearranged as,

$$BU^T \geq e^{\left(\lambda_{\underline{C}} + (z_{\underline{C},\ \alpha_L})\xi_{\underline{C}}\right)} \tag{3.39}$$

where the specific value of $z_{\underline{C},\ \alpha_L}$ is $F_{Z_{\underline{C}}}^{-1}(\alpha_L)$, which is the (α_L)th quantile of the standardized $\underline{C}$. Knowing the PDF of $\underline{C}$ and required constraint reliability, α_L the specific value $z_{\underline{C},\ \alpha_L}$ can be determined. As $\underline{C}$ has a log-normal distribution, $z_{\underline{C},\ \alpha_L}$ is referring to the standardized normal variant $\phi\left(z_{\underline{C},\ \alpha_L}\right) = \alpha_L$ where $\phi()$ is the standard normal CDF that is expressed in Equation 3.4.

As there is no chance constraint applied to the upper limit of the chlorine concentration in this case, original constraint Equation 2.10 becomes,

$$e^{\left(\lambda_{\underline{C}} + (z_{\underline{C},\ \alpha_L})\xi_{\underline{C}}\right)} \leq BU^T \leq \overline{C} \tag{3.40}$$

Thus, optimization formulation corresponding to the chance constraint applied only to the lower limit of the chlorine concentration is composed of the objective function, Equation 2.9, chlorine concentration limitation, Equation 3.40 and non-negativity constraint, Equation 2.11.

3.2.2.3 Both of the Limits

In the last case of the log-normal distribution, probabilistic approach will be applied to the both limits of the chlorine concentration constraint. Deterministic equations of probabilistic approach applied on upper and lower limits separately are obtained as Equations 3.34 and 3.40, respectively. As a result, deterministic equation of the chance constraint applied to both of the limits will be the combination of these two equations, which is,

$$e^{\left(\lambda_{\underline{C}}+\,(z_{\underline{C},\ \alpha_L})\xi_{\underline{C}}\right)} \leq \ BU^T \leq e^{\left(\mu_{\overline{C}}+\,(z_{\overline{C},\ 1-\alpha_U})\xi_{\overline{C}}\right)} \tag{3.41}$$

Consequently, optimization formulation corresponding to the chance constraint applied to both of the limits of the normally distributed chlorine concentration is formed by the objective function, Equation 2.9, chlorine concentration limitation, Equation 3.41 and non-negativity constraint, Equation 2.11.

CHAPTER 4

APPLICATION AND DISCUSSION OF THE RESULTS

4.1 Model Application

A model water distribution with known physical and hydraulic characteristics is needed in order to check the benefits of the chance constrained optimization of booster disinfection formulation. For this study, a slightly modified version of the example network in EPANET version 2.0, which is The Brushy Plain water distribution network system shown in Figure 4.1, is used.

4.1.1 Physical Characteristics

The network shown in Figure 4.1 is composed of 1 source node with a pump station, 34 consumer nodes, 1 storage tank and 47 pipes. Source at Node 1 is supplying water to the network that has 5.18 km^2 residential area, with the help of the pumps. Lengths, diameters and the roughness coefficients of the pipes can be seen in Table 4.1. Pump located at Node 1 is modeled with a negative demand of -4400×10^{-5} m^3/s with certain pump demand multipliers shown in Table 4.2. In addition to that, the consumer demand node multipliers are given in Table 4.2, for Nodes 1 to 36.

At Node 26 there is storage tank which is completely mixed cylindrical tank with a diameter of 15.2 *m*. Measuring from the bottom of the tank, minimum and maximum water levels are 15.2 *m* and 21.3 *m*, respectively.

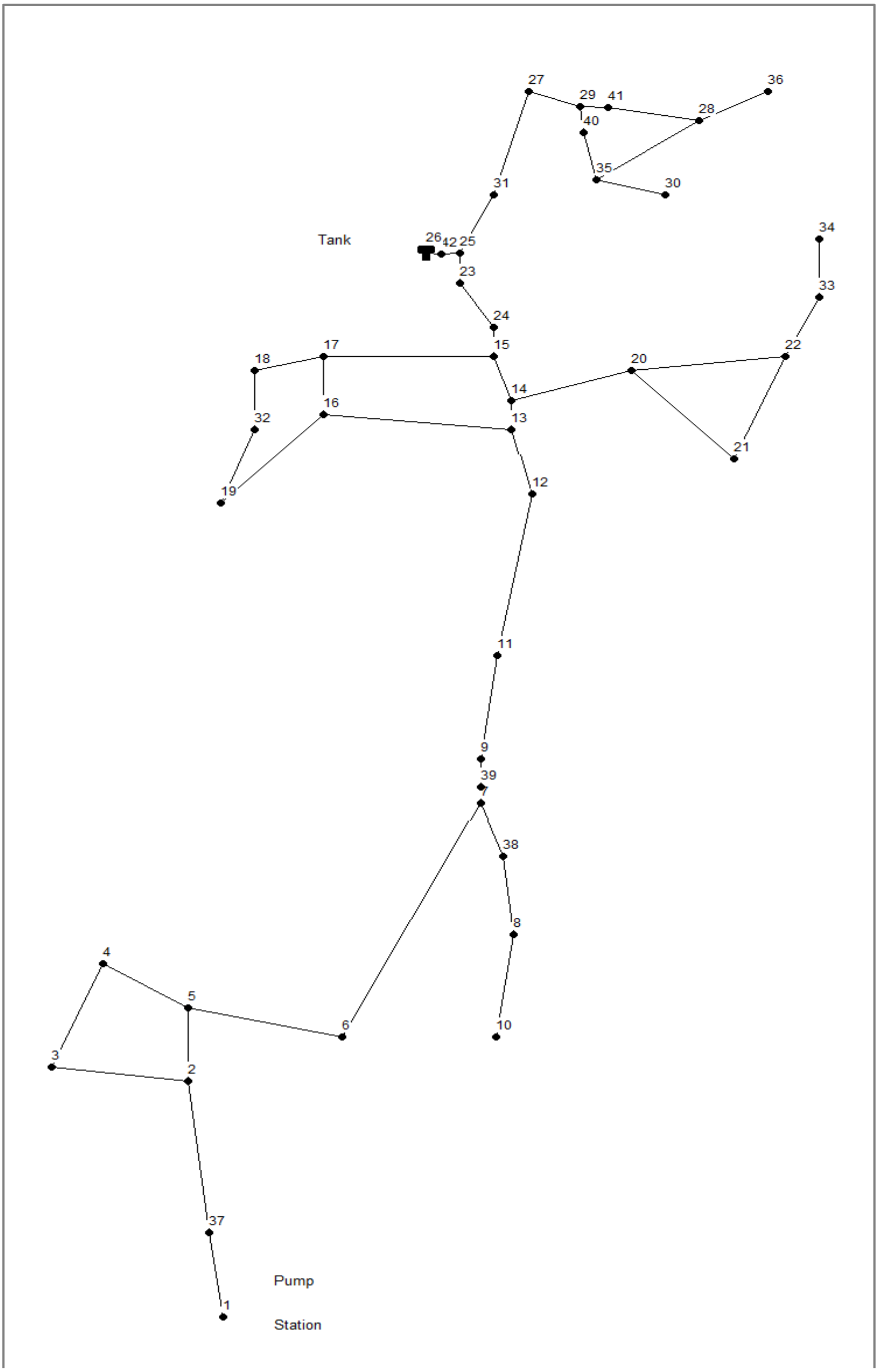

Figure 4.1: Schematic representation of the example network

Table 4.1: Network pipe characteristics

Upstream Node	Downstream Node	Length (m)	Diameter (m)	Roughness Coefficients, C
1	37	1.0	0.3	100
37	2	730.0	0.3	100
2	5	240.0	0.3	100
2	3	400.0	0.2	100
3	4	370.0	0.2	100
4	5	300.0	0.3	100
5	6	370.0	0.3	100
6	7	820.0	0.3	100
7	38	1.0	0.3	140
7	39	1.0	0.3	100
8	10	300.0	0.2	140
9	11	210.0	0.3	100
11	12	580.0	0.3	100
12	13	180.0	0.3	100
13	14	120.0	0.3	100
14	15	90.0	0.3	100
13	16	460.0	0.2	100
15	17	460.0	0.2	100
16	17	180.0	0.2	100
17	18	210.0	0.3	100
18	32	110.0	0.3	100
16	19	430.0	0.2	100
14	20	340.0	0.3	100
20	21	400.0	0.2	100
21	22	400.0	0.2	100
20	22	400.0	0.2	100
24	23	180.0	0.3	100
15	24	80.0	0.3	100
23	25	90.0	0.3	100
25	42	30.0	0.3	100
25	31	180.0	0.3	100
31	27	120.0	0.2	100
27	29	120.0	0.2	100
29	40	1.0	0.2	100
29	41	1.0	0.2	100
22	33	300.0	0.2	100
33	34	120.0	0.2	100
32	19	150.0	0.2	100
35	30	300.0	0.2	100
28	35	210.0	0.2	100
28	36	90.0	0.2	100
38	8	370.0	0.3	140
39	9	120.0	0.3	100
40	35	150.0	0.2	100
41	28	730.0	0.2	100
42	26	1.0	0.3	100

Table 4.2: Demand and pump multipliers

Hour	Demand Multipliers	Pump Multipliers
1	1.19	0.96
2	0.97	0.96
3	0.90	0.96
4	0.90	0.96
5	0.82	0.96
6	1.12	0.96
7	1.21	0.00
8	0.60	0.00
9	0.60	0.00
10	1.27	0.00
11	2.39	0.00
12	0.90	0.00
13	0.85	0.80
14	0.61	1.00
15	1.36	1.00
16	0.54	1.00
17	0.24	1.00
18	0.71	0.15
19	0.30	0.00
20	0.60	0.00
21	1.19	0.00
22	1.49	0.00
23	1.12	0.00
24	1.16	0.00

4.1.2 Network Hydraulics

As it can be seen from Table 4.2, determination of the network hydraulic dynamics is done by the 24 *h* periodic cycle of the water demands in the example network. This 24 *h* periodic cycle is assumed to be repeated infinitely. Base demands of the consumer nodes are given in the example network file and can be seen in Table 4.3. To observe the network hydraulic behavior, demand multipliers are appointed to the consumer nodes, pump multipliers are appointed to the source node in the EPANET and network is solved.

Table 4.3: Base demands of nodes in the example network

Node ID	Base Demand (LPS)	Node ID	Base Demand (LPS)
Junc 1	-43.810	Junc 22	0.631
Junc 2	0.505	Junc 23	0.505
Junc 3	0.883	Junc 24	0.694
Junc 4	0.505	Junc 25	0.379
Junc 5	0.505	Junc 27	0.505
Junc 6	0.315	Junc 28	0.000
Junc 7	0.252	Junc 29	0.442
Junc 8	0.568	Junc 30	0.189
Junc 9	0.883	Junc 31	1.073
Junc 10	0.315	Junc 32	1.073
Junc 11	2.194	Junc 33	0.095
Junc 12	1.009	Junc 34	0.095
Junc 13	0.126	Junc 35	0.000
Junc 14	0.126	Junc 36	0.063
Junc 15	0.126	Junc 37	0.000
Junc 16	1.262	Junc 38	0.000
Junc 17	1.262	Junc 39	0.000
Junc 18	1.262	Junc 40	0.000
Junc 19	0.315	Junc 41	0.000
Junc 20	1.199	Junc 42	0.000
Junc 21	1.009	Tank 26	-

If the total demand, inflow and flow from/to tank data are examined, results can be seen in Figure 4.2. Mainly, system demand is supplied by the pumps at the source node, during time intervals 0-6 *h* and 12-18 *h*. At the same time intervals, tank is being filled with the water supplied by the pumps. This can be seen from the line corresponding to the tank in the graph; during these time intervals tank line is in the positive side of the flow axis, meaning there is water coming to the tank. Likewise, it can be understood from the inflow and tank lines in the graph that during time intervals 6-12 *h* and 18-24 *h* system demand is supplied by the water in the tank.

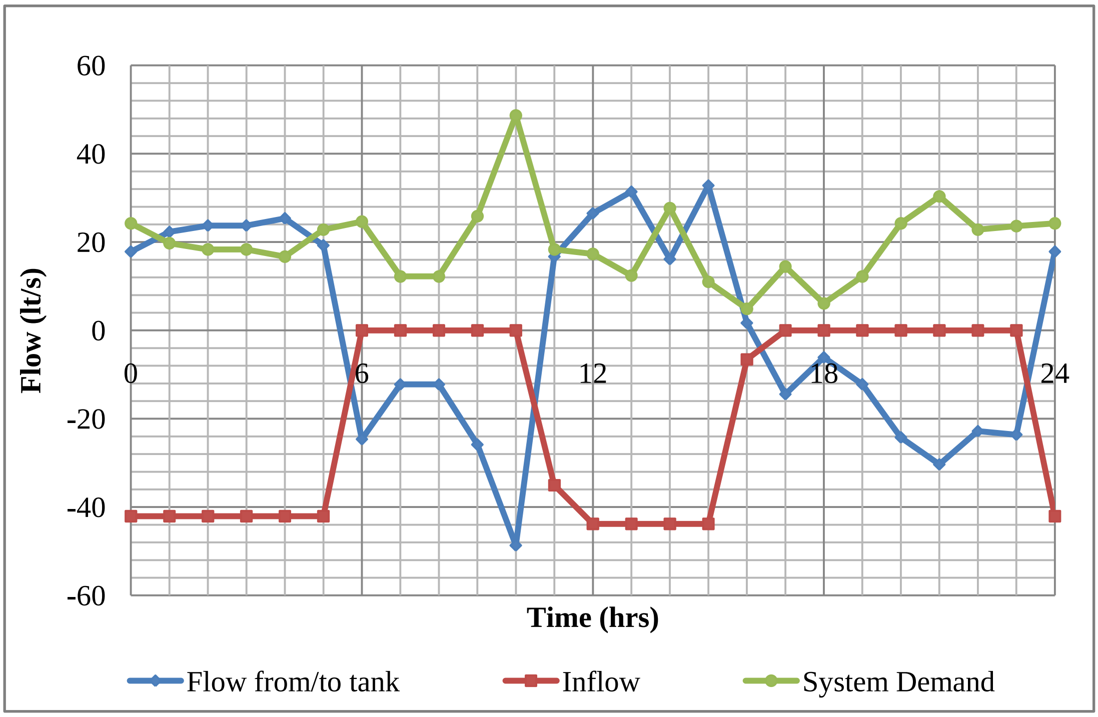

Figure 4.2: Hydraulic behavior of the network

4.1.3 Water Quality Description

Chlorine concentration is limited between an upper and a lower bound in order to supply water to the network with sufficient quality. The lower bound is determined by taking biological regrowth into consideration. Likewise, taste and odor problems are effective in deciding the upper limit. Also, high concentration of chlorine resulted in formation of disinfectant by-products, DBP, which can lead to serious health problems (Bull, 1982). So, formation of DBP is another aspect considered while deciding the upper limit. Regulations for these minimum and maximum limits of chlorine concentration are varying. For example, World Health Organization, WHO, states that, water for municipal use can only have chlorine concentration of 5 *mg/l* in maximum and there is no limitation for the lower bound (WHO, 2011). On the other hand, Safe Drinking Water Act, SDWA, fixes the maximum and minimum limits as 0.2 *mg/l* and 4.0 *mg/l*, respectively (SWDA, 2003).

For Turkey, there are no limitations for chlorine concentration stated in the Turkish Standards for Water Intended for Human Consumption, TS 266 (2005). Only applied limitation for chlorine concentration is included in the Water Intended for Human Consumption Regulations prepared by Ministry of Health and it states that the chlorine concentration at the end points of the network can not exceed 0.5 *mg/l*.

For this study, upper and lower bounds are taken as, $\overline{C}$ = 4.0 *mg/l* and $\underline{C}$ = 0.2 *mg/l*, respectively. For each consumer node there will be 24 different checks for a typical daily analysis by taking monitoring time interval as 1 *h* ($tr = 1, 2, \ldots, 24$).

Analyses of response coefficient matrix, *A*, for the same example network were done by Sert *(2009)*. In those analyses, mass booster type was used and booster station injection pattern time step was chosen as 1 *h* ($j = 1, 2, \ldots, 24$) with the total of 24 *h* in order to coincide with the hydraulic cycle time of 24 *h*. Total simulation time was taken as 960 *h* to obtain the coefficients after the system becomes stable, meaning the periodicity is achieved. And, for the formation of matrix, last 24 *h* of the analyses was used. Moreover, it should be noted that the global bulk and wall decay coefficients used in that study are k_b = 0.53 day^{-1} and k_w = 5.1 *mm/day*, respectively.

In this study, response coefficients calculated by Sert *(2009)* will be used directly, as the analyses are done on the same network with the same hydraulic dynamics. Only, the name of the response coefficient matrix is changed from A to B and response of each consumer node from α to β in order to avoid possible confusions that may result from notation similarity of the probability, α, in the chance constraints.

4.1.4 Booster Locations

Probable booster locations and analyses of the selected nodes were done by Sert *(2009)*. The Nodes from 37 to 42 in Figure 4.1 were selected as probable booster locations and added to the system as dummy nodes. These nodes have no demand and added to the network with 1.0 *m* of pipe to stand for booster station at that node. In the sample network, Node 37 was selected in order to model the conventional source injection method. Node 38 was chosen to supply the demand of the branch that contains Node 8 and 10. Node 39 was located to serve the upper region of the network. Node 40 and 41 were selected to satisfy the minimum concentration amount in the far ends of the network and Node 42 was chosen to be near the storage tank.

Five different cases were tried in the work of Sert *(2009)* related with this study. In each case, different combination of the selected booster station nodes were analyzed and optimization procedure for minimization of total mass injected to the system was applied. Results of five cases and selected combinations of booster locations can be seen in Table 4.4.

Considering the results of Sert *(2009)*, booster location combination to which the chance constraint will be applied in this thesis, is decided. Chance constraint will be applied for the conventional case, which is the injection only from the source node, Case I. In addition to that, it will be applied to the most favorable case with respect to the cost, Case V. Although Case IV seems the most desired case by looking at the amount of total mass injected, it has slight difference with Case V. Moreover, number of the booster stations in Case IV is three more than Case V and total cost of this excess amount of stations make Case IV unfavorable.

Table 4.4: Total mass injection results of Sert *(2009)*

Case	Booster Locations	Total Mass Injected U (kg/day)
I	37 (source)	21.24
II	37, 38, 39, 40, 41	20.53
III	37, 39	20.59
IV	37, 38, 39, 40, 41, 42	14.14
V	37, 39, 42	14.84

For this study, notation for conventional injection is Case I and cost favorable case is Case II.

4.2 Discussion of the Results

In the chance constrained optimization formulation of this example water distribution network, objective function and constraints are linear for all cases. Thus, in this study, a linear programming solver will be used for the analyses, which is Microsoft Excel with “Solver” add-on. Analyses are done for two cases, Case I and II. Total booster mass injected to the system for different probability distributions are obtained by applying corresponding chance constrained optimization formulations explained in section 3.2.

The results of the optimization to which chance constraint is not applied, is corresponding to the 50% probability of occurrence for normal and log- normal distribution (Lansey et al., 1989; Das, 2007). Thus, it is advantageous to use these distribution types as these enable the comparison between the non-probability case and the different reliability levels.

Increasing amount of chlorine concentration results in increasing health risk. So, it is desirable to have chlorine concentrations as low as possible without violating the minimum limit (Manuvalli et al., 2003). In addition to that, it is favorable to obtain uniform concentration distributions over the periodic cycle of all consumer points. Therefore, a network with high reliability level means more uniformly distributed and low amount chlorine concentrations at consumer nodes.

4.2.1 Case I

Case I is the conventional case in which the injection is done from the source node (Node 37). Total mass injection amounts are calculated for normal and log-normal probability distribution. Different reliability levels and standard deviations are analyzed for both of the distribution types to see their effects on the total mass injection amount.

For the non-probability case, objective function is Equation 2.9 and constraints are Equations 2.10 and 2.11. Response coefficient, *B*, used in the Equation 2.10 is taken from the Case I results of the program developed by Sert *(2009)*. Upper and lower limits in Equation 2.10 are $\overline{C}$ = 4.0 *mg/l* and $\underline{C}$ = 0.2 *mg/l*, respectively. Reliability level is 50%. Linear programing formulation is solved by Excel and injection results can be seen in Table 4.5.

For different probability distributions, chlorine concentration limitation constraint, Equation 2.10, will be modified taking different reliability levels and standard deviations into consideration. Rest of the problem is the same with the non-probability case.

Table 4.5: Injection results of Case I

Node 37	
Time (hr)	**u (mg/min)**
1	516.34
2	1375.47
3	516.34
4	517.95
5	517.86
6	623.24
7	0.00
8	0.00
9	0.00
10	0.00
11	0.00
12	3653.80
13	2629.94
14	537.85
15	538.23
16	726.59
17	2599.59
18	0.00
19	0.00
20	0.00
21	0.00
22	0.00
23	0.00
24	0.00
Σu (mg/min)	14753.198
Σu (kg/day)	21.245

In order to see the distribution of the data, frequency plot of chlorine concentrations throughout the network is given in Figure 4.3. As it can be seen from the figure, most of the data are lying between 0.2 *mg/l* and 1.0 *mg/l* and less number of data are between 1.0 *mg/l* and 4.0 *mg/l*.

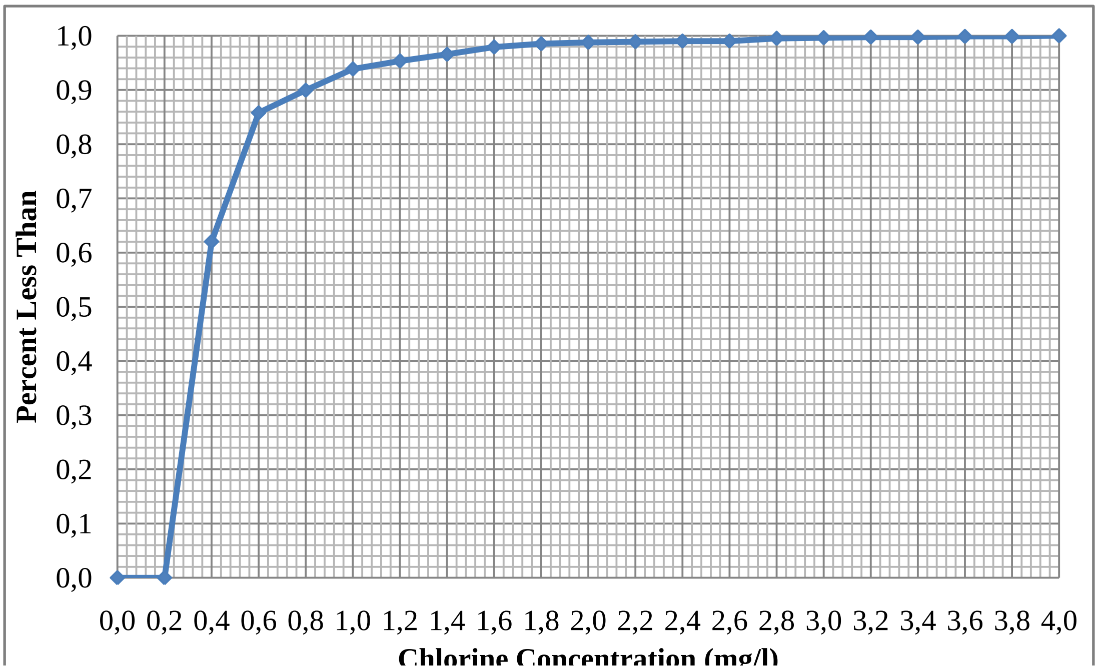

Figure 4.3: Frequency plot of chlorine concentration for Case I

4.2.1.1 The Normal Distribution

Chance constrained optimization formulation by assuming the normal probability distribution for chlorine concentration is composed of objective function, Equation 2.9, non-negativity constraint, Equation 2.11 and chlorine concentration limitation adjusted according to the application to upper limit, lower limit and both.

4.2.1.1.1 Upper Limit

Chlorine concentration constraint is constructed according to the Equation 3.17 as explained in section 3.2.1.1. Mean value of the maximum concentration is taken as $\mu_{\overline{C}}$ = 4.0 *mg/l* and standardized normal variant is calculated for α_U ranging from 0.60 to 0.99. Increments are set to 0.10 up to 0.90. As higher reliability values are favorable, increment is taken as 0.01 for the values of α_U ranging from 0.95 to 0.99

in order to see its effect more precisely. Modified maximum limits calculated from Equation 3.17 and computed total mass injection results can be seen in Table 4.6. Note that, the lower limit is kept at 0.2 *mg/l* for each run.

As it can be seen from Table 4.6 that in the analyses, increment of the standard deviation is mostly 0.50. For reliability levels of 0.60 and 0.70, analyses give feasible solutions even for standard deviations higher than 4.0; however, it is not reasonable to have these values, so analyses are stopped at 4.0. Apart from those two, highest analyzed value of the standard deviation is where the solver is unable to find a feasible solution. For example, for reliability level of 0.80, the maximum value of the standard deviation is 3.70. Increment is adjusted at necessary points in order to observe the changes in the total mass injection in a better way. The change of total mass injection versus reliability level for each standard deviation is given in Figure 4.4. Similarly, the change of total mass injection versus standard deviation for each reliability level is shown in Figure 4.5.

Table 4.6: Results of the upper limit application with normal probability distribution for Case I

α_U	U(kg/day)	σ	UL(mg/l)	LL(mg/l)
0.60	21.245	0.00	4.000	0.200
	21.300	0.50	3.873	0.200
	21.355	1.00	3.747	0.200
	21.411	1.50	3.620	0.200
	21.466	2.00	3.493	0.200
	21.522	2.50	3.367	0.200
	21.577	3.00	3.240	0.200
	21.633	3.50	3.113	0.200
	21.688	4.00	2.987	0.200
0.70	21.245	0.00	4.000	0.200
	21.359	0.50	3.738	0.200
	21.474	1.00	3.476	0.200
	21.589	1.50	3.213	0.200
	21.704	2.00	2.951	0.200
	21.821	2.50	2.689	0.200
	21.940	3.00	2.427	0.200
	22.059	3.50	2.165	0.200
	22.178	4.00	1.902	0.200
0.80	21.245	0.00	4.000	0.200
	21.429	0.50	3.579	0.200
	21.613	1.00	3.158	0.200
	21.799	1.50	2.738	0.200
	21.990	2.00	2.317	0.200
	22.180	2.50	1.896	0.200
	22.385	3.00	1.475	0.200
	23.458	3.50	1.054	0.200
	25.895	3.70	0.886	0.200
0.90	21.245	0.00	4.000	0.200
	21.525	0.50	3.359	0.200
	21.808	1.00	2.718	0.200
	22.098	1.50	2.078	0.200
	22.407	2.00	1.437	0.200
	22.863	2.20	1.181	0.200
	23.471	2.30	1.052	0.200
	24.667	2.40	0.924	0.200

α_U	U(kg/day)	σ	UL(mg/l)	LL(mg/l)
0.95	21.245	0.00	4.000	0.200
	21.604	0.50	3.178	0.200
	21.973	1.00	2.355	0.200
	22.159	1.25	1.944	0.200
	22.353	1.50	1.533	0.200
	22.777	1.70	1.204	0.200
	23.587	1.80	1.039	0.200
	25.569	1.89	0.891	0.200
0.96	21.245	0.00	4.000	0.200
	21.628	0.50	3.125	0.200
	22.020	1.00	2.249	0.200
	22.219	1.25	1.812	0.200
	22.445	1.50	1.374	0.200
	22.795	1.60	1.199	0.200
	23.726	1.70	1.024	0.200
	26.070	1.78	0.884	0.200
0.97	21.245	0.00	4.000	0.200
	21.656	0.50	3.060	0.200
	22.079	1.00	2.119	0.200
	22.292	1.25	1.649	0.200
	22.449	1.40	1.367	0.200
	22.871	1.50	1.179	0.200
	24.023	1.60	0.991	0.200
	25.264	1.65	0.897	0.200
0.98	21.245	0.00	4.000	0.200
	21.694	0.50	2.973	0.200
	22.158	1.00	1.946	0.200
	22.409	1.25	1.433	0.200
	23.098	1.40	1.125	0.200
	25.146	1.51	0.899	0.200
0.99	21.245	0.00	4.000	0.200
	21.754	0.50	2.837	0.200
	22.281	1.00	1.674	0.200
	23.251	1.25	1.092	0.200
	26.156	1.34	0.883	0.200

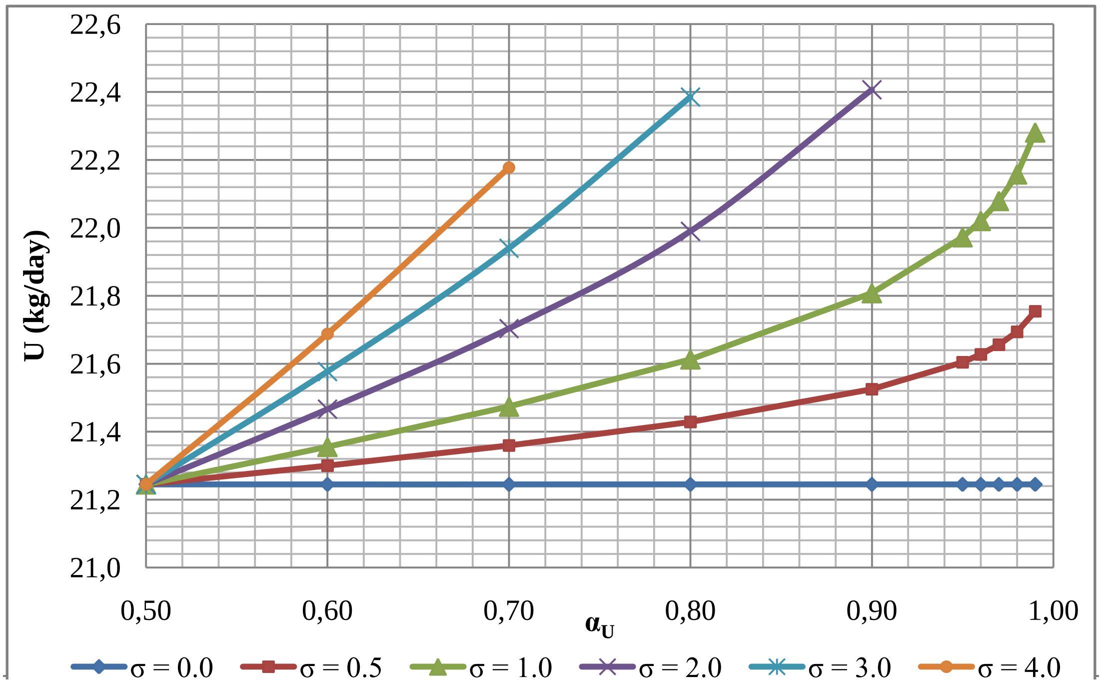

Figure 4.4: Total injected mass versus upper limit reliability for normal probability distribution (Case I)

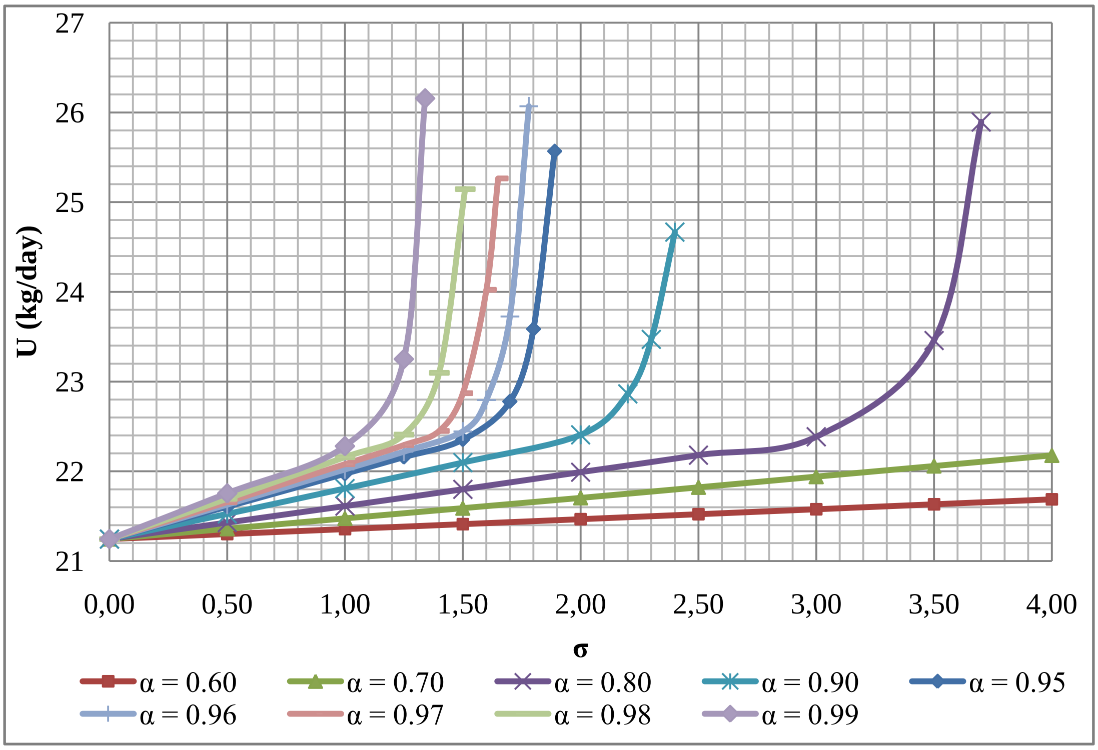

Figure 4.5: Total injected mass versus standard deviation of upper limit for normal probability distribution (Case I)

It can be seen from Figure 4.4 that total injected mass continuously increase with the increase of reliability level for the same standard deviation. Especially after reliability level of 0.90, that increase becomes abrupt. For standard deviations 2.0, 3.0 and 4.0 there is no feasible solution for high reliability levels and abrupt changes cannot be observed. For σ=0, there is no change in the upper limit for any reliability level; thus, results are same with the no probability case for all reliability levels.

Likewise, it can be concluded from Figure 4.5 that total injected mass continuously increase with the increase of standard deviation for the same reliability level. Abrupt increases start to occur when the standard deviation gets closer to the value where solution becomes infeasible.

4.2.1.1.2 Lower Limit

Chlorine concentration constraint is constructed according to the Equation 3.26 as explained in section 3.2.1.2. Mean value of the minimum concentration is taken as $\mu_{\underline{C}}$ = 0.2 *mg/l* and standardized normal variant is calculated for α_L ranging from 0.60 to 0.99 generally with 0.10 increments. In order to see the effect of higher reliabilities, increment is taken as 0.01 for the values of α_L ranging from 0.95 to 0.99. Modified minimum limits calculated from Equation 3.26 and computed total mass injection results can be seen in Table 4.7. Note that, the upper limit is kept at 4.0 *mg/l* for each run.

Increment of the standard deviation is taken as 0.25 mostly, in this case. Highest analyzed value of the standard deviation is where the solver is unable to find a feasible solution. For example, for reliability level of 0.70, the maximum value of the standard deviation is 1.25. Increment is adjusted at necessary points in order to observe the changes in the total mass injection in a better way. The change of total mass injection versus reliability level for each standard deviation is given in Figure 4.6. Similarly, the change of total mass injection versus standard deviation for each reliability level is shown in Figure 4.7.

Table 4.7: Results of the lower limit application with normal probability distribution for Case I

α_L	U(kg/day)	σ	UL(mg/l)	LL(mg/l)	α_L	U(kg/day)	σ	UL(mg/l)	LL(mg/l)
0.60	21.245	0.00	4.000	0.200	0.96	21.245	0.00	4.000	0.200
	28.527	0.25	4.000	0.263		31.311	0.05	4.000	0.288
	35.820	0.50	4.000	0.327		41.395	0.10	4.000	0.375
	43.116	0.75	4.000	0.390		51.478	0.15	4.000	0.463
	50.412	1.00	4.000	0.453		61.608	0.20	4.000	0.550
	57.725	1.25	4.000	0.517		72.063	0.25	4.000	0.638
	65.081	1.50	4.000	0.580		84.122	0.30	4.000	0.725
	72.822	1.75	4.000	0.643		97.857	0.35	4.000	0.813
	81.509	2.00	4.000	0.707		115.794	0.40	4.000	0.900
	90.825	2.25	4.000	0.770	0.97	21.245	0.00	4.000	0.200
	101.259	2.50	4.000	0.833		32.061	0.05	4.000	0.294
	114.391	2.75	4.000	0.897		42.894	0.10	4.000	0.388
0.70	21.245	0.00	4.000	0.200		53.726	0.15	4.000	0.482
	36.330	0.25	4.000	0.331		64.629	0.20	4.000	0.576
	51.432	0.50	4.000	0.462		76.450	0.25	4.000	0.670
	66.643	0.75	4.000	0.593		89.872	0.30	4.000	0.764
	84.008	1.00	4.000	0.724		105.433	0.35	4.000	0.858
	104.950	1.25	4.000	0.856		114.088	0.37	4.000	0.896
0.80	21.245	0.00	4.000	0.200	0.98	21.245	0.00	4.000	0.200
	45.465	0.25	4.000	0.410		33.057	0.05	4.000	0.303
	69.902	0.50	4.000	0.621		44.886	0.10	4.000	0.405
	100.904	0.75	4.000	0.831		56.725	0.15	4.000	0.508
0.90	21.245	0.00	4.000	0.200		68.710	0.20	4.000	0.611
	58.154	0.25	4.000	0.520		82.452	0.25	4.000	0.713
	102.487	0.50	4.000	0.841		98.415	0.30	4.000	0.816
0.95	21.245	0.00	4.000	0.200		115.008	0.34	4.000	0.898
	30.702	0.05	4.000	0.282	0.99	21.245	0.00	4.000	0.200
	40.176	0.10	4.000	0.364		34.627	0.05	4.000	0.316
	49.650	0.15	4.000	0.447		48.026	0.10	4.000	0.433
	59.151	0.20	4.000	0.529		61.470	0.15	4.000	0.549
	68.765	0.25	4.000	0.611		75.780	0.20	4.000	0.665
	79.667	0.30	4.000	0.693		92.728	0.25	4.000	0.782
	91.759	0.35	4.000	0.776		114.862	0.30	4.000	0.898
	105.374	0.40	4.000	0.858	-	-	-	-	-

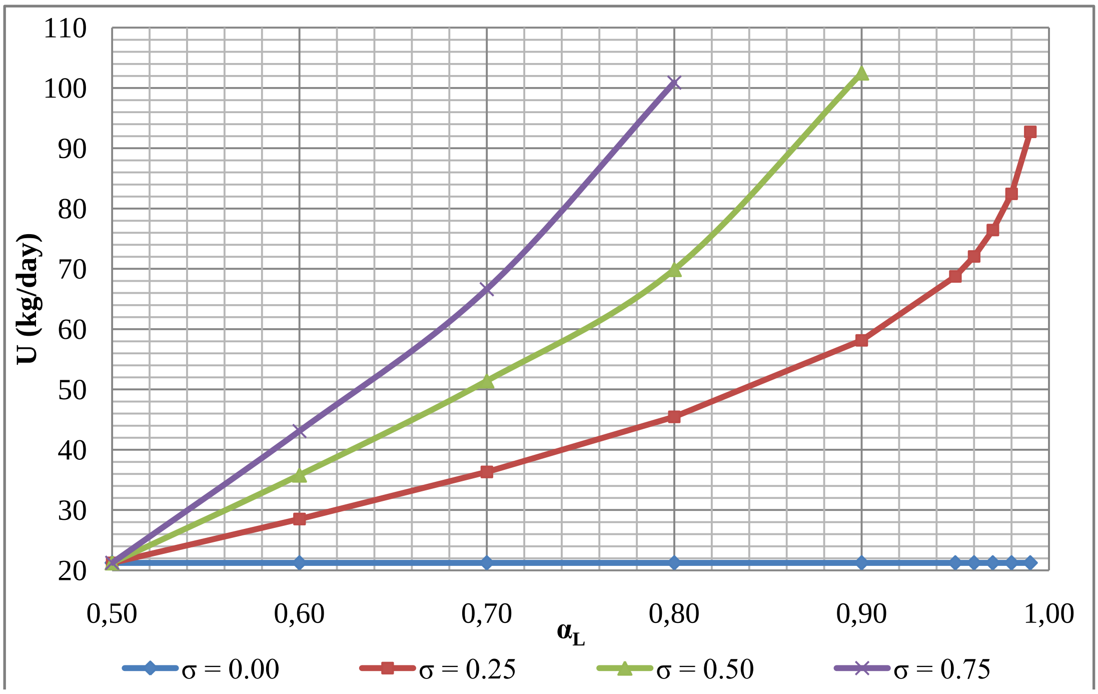

Figure 4.6: Total injected mass versus lower limit reliability for normal probability distribution (Case I)

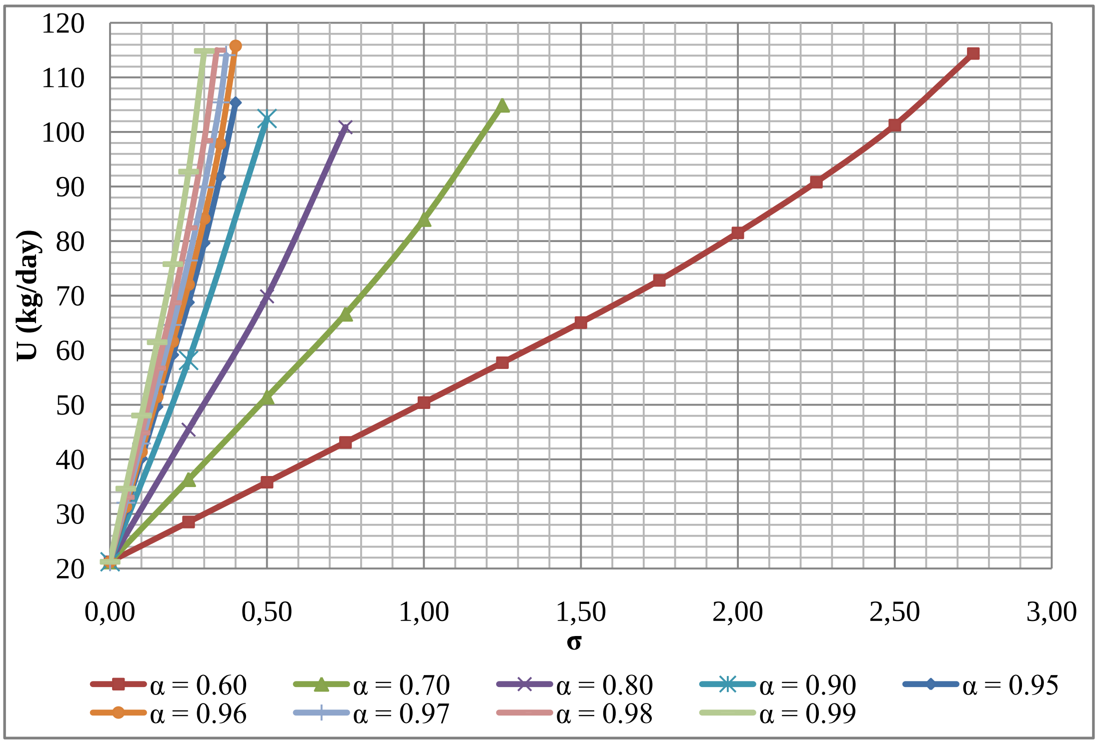

Figure 4.7: Total injected mass versus standard deviation of lower limit for normal probability distribution (Case I)

Shown in Figure 4.6 that total injected mass continuously increases with the increase of reliability level for the same standard deviation. Especially after reliability level of 0.90, that increase becomes abrupt. For standard deviations 0.50 and 0.75 there is no feasible solution for high reliability levels and abrupt changes cannot be observed. For σ=0, there is no change in the upper limit for any reliability level; thus, results are same with the no probability case for all reliability levels.

Likewise, it can be concluded from Figure 4.7 that total injected mass continuously increase with the increase of standard deviation for the same reliability level. Rapid increases start to occur when the standard deviation gets closer to the value where solution becomes infeasible.

When Figure 4.4 and Figure 4.6 is compared, it can be seen that the same increase in the reliability level results in much more injection results in the lower limit case for the same standard deviation. For example, for the standard deviation 0.50 and increase of reliability level from 0.70 to 0.80, upper limit application results in 0.17 *kg/day* increase of injection mass from 21.259 *kg/day* to 21.429 *kg/day*. On the other hand, lower limit application results in 18.47 *kg/day* increase from 51.432 *kg/day* to 69.902 *kg/day*.

Comparing Figure 4.5 and Figure 4.7, it is observed that the range of the standard deviation for the feasible solution is much smaller for the lower limit case. Most of the changes occur within the range of 0.0 and 1.0 in the lower limit case; whereas, there is no such significant range for the upper limit case and changes gradually occur between 0.0 and 4.0.

4.2.1.1.3 Both of the Limits

Chlorine concentration constraint is constructed according to the Equation 3.28 as explained in section 3.2.1.3. Mean values of the maximum and minimum concentrations are taken as $\mu_{\overline{C}}$ = 4.0 *mg/l* and $\mu_{\underline{C}}$ = 0.2 *mg/l*, respectively. For this case, α_U and α_L are equal to each other. Standardized normal variants are calculated for α_U and α_L ranging from 0.60 to 0.99 mostly with 0.10 increments. In order to see the effect of higher reliabilities, increment is taken as 0.01 for the values of α_U and α_L from 0.95 to 0.99. Modified maximum/minimum limits calculated from Equation 3.28 and computed total mass injection results can be seen in Table 4.8.

In this case, increment of the standard deviation is taken as 0.10 mostly. Highest analyzed value of the standard deviation is where the solver is unable to find a feasible solution. For example, for reliability level of 0.90, the maximum value of the standard deviation is 0.45. Increment is adjusted at necessary points in order to observe the changes in the total mass injection in a better way. The change of total mass injection versus reliability level for each standard deviation is given in Figure 4.8. Similarly, the change of total mass injection versus standard deviation for each reliability level is shown in Figure 4.9.

Table 4.8: Both of the limits application results with normal probability distribution for Case I

α_{L-U}	U(kg/day)	σ	UL(mg/l)	LL(mg/l)	α_{L-U}	U(kg/day)	σ	UL(mg/l)	LL(mg/l)
0.60	21.245	0.00	4.000	0.200	0.95	21.245	0.00	4.000	0.200
	24.169	0.10	3.975	0.225		30.739	0.05	3.918	0.282
	27.092	0.20	0.251	0.251		40.250	0.10	3.836	0.364
	30.018	0.30	3.924	0.276		49.761	0.15	3.753	0.447
	32.947	0.40	3.899	0.301		59.337	0.20	3.671	0.529
	35.877	0.50	3.873	0.327		69.953	0.25	3.507	0.611
	50.527	1.00	3.747	0.453		82.654	0.30	3.507	0.693
	65.608	1.50	3.620	0.580		101.384	0.35	3.424	0.776
	84.952	2.00	3.493	0.707	0.96	21.245	0.00	4.000	0.200
	94.270	2.20	3.443	0.757		31.351	0.05	3.912	0.288
0.70	21.245	0.00	4.000	0.200		41.474	0.10	3.825	0.375
	27.297	0.10	3.948	0.252		51.597	0.15	3.737	0.463
	33.357	0.20	3.895	0.305		61.848	0.20	3.650	0.550
	39.421	0.30	3.843	0.357		73.756	0.25	3.562	0.638
	45.486	0.40	3.790	0.410		88.171	0.30	3.475	0.725
	51.551	0.50	3.738	0.462		94.874	0.32	3.440	0.760
	63.871	0.70	3.633	0.567	0.97	21.245	0.00	4.000	0.200
	88.031	1.00	3.476	0.724		32.103	0.05	3.906	0.294
	102.027	1.10	3.423	0.777		42.979	0.10	3.812	0.388
0.80	21.245	0.00	4.000	0.200		53.871	0.15	3.718	0.482
	30.961	0.10	3.916	0.284		65.080	0.20	3.624	0.576
	40.694	0.20	3.832	0.368		78.630	0.25	3.530	0.670
	50.427	0.30	3.748	0.452		96.027	0.30	3.436	0.764
	60.240	0.40	3.663	0.537	0.98	21.245	0.00	4.000	0.200
	71.328	0.50	3.579	0.621		33.103	0.05	3.897	0.303
	84.653	0.60	3.495	0.705		44.979	0.10	3.795	0.405
	99.557	0.68	3.428	0.772		56.897	0.15	3.692	0.508
0.90	21.245	0.00	4.000	0.200		69.886	0.20	3.589	0.611
	36.049	0.10	3.872	0.328		86.124	0.25	3.487	0.713
	50.869	0.20	3.744	0.456		101.019	0.28	3.425	0.775
	66.216	0.30	3.616	0.584	0.99	21.245	0.00	4.000	0.200
	85.982	0.40	3.487	0.713		34.680	0.05	3.884	0.316
	101.946	0.45	3.423	0.777		48.131	0.10	3.767	0.433
-	-	-	-	-		61.707	0.15	3.651	0.549
						77.845	0.20	3.535	0.665
						94.472	0.24	3.442	0.758

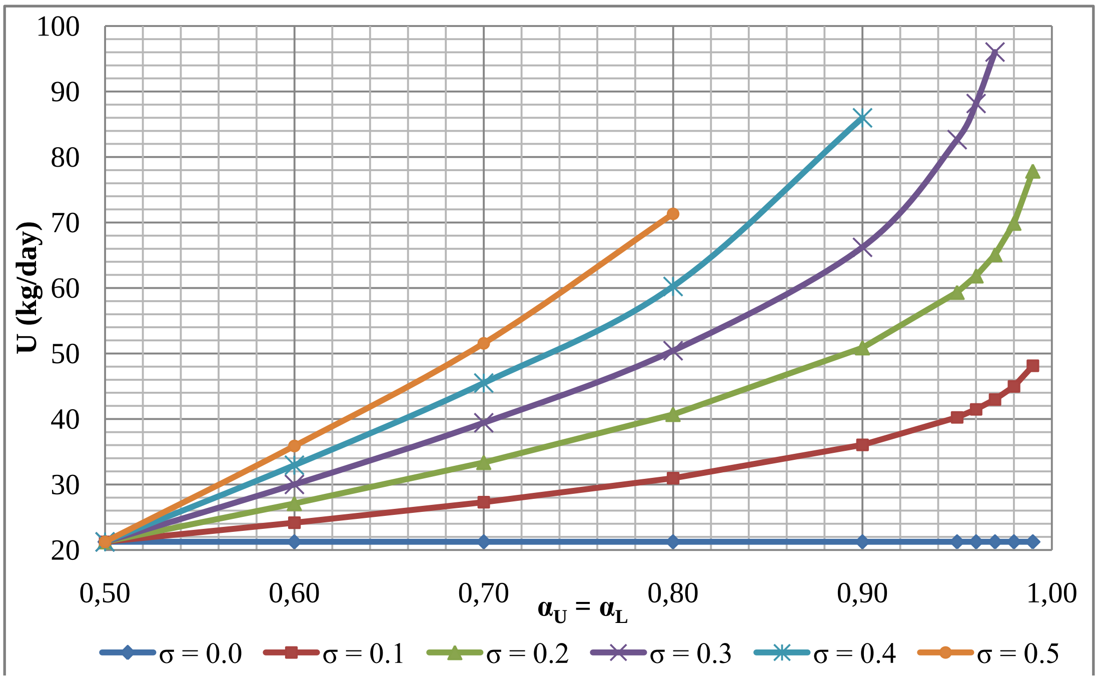

Figure 4.8: Total injected mass versus both limits reliability for normal probability distribution (Case I)

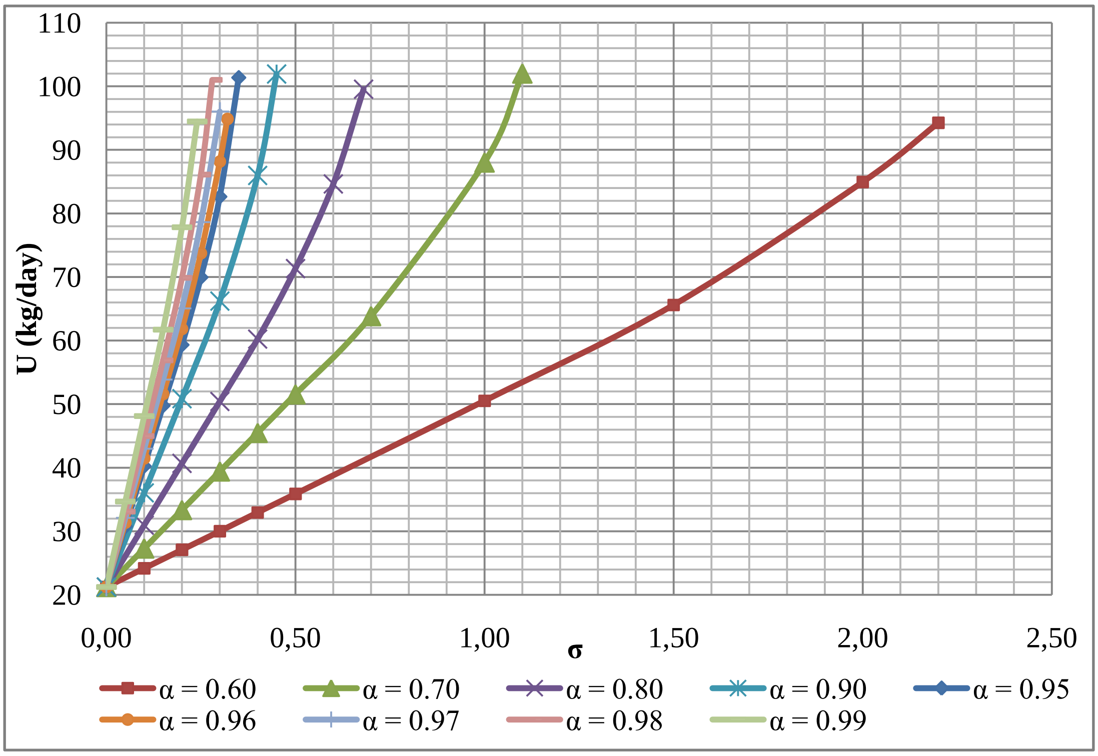

Figure 4.9: Total injected mass versus standard deviation of both of the limits for normal probability distribution (Case I)

It is shown in Figure 4.8 that total injected mass continuously increases with the increase of reliability level for the same standard deviation. Especially after reliability level of 0.90, that increase becomes abrupt. For standard deviations 0.4 and 0.5, there is no feasible solution for high reliability levels and abrupt changes cannot be observed. For σ=0, there is no change in the upper limit for any reliability level; thus, results are same with the no probability case for all reliability levels.

Likewise, it can be concluded from Figure 4.9 that, total injected mass continuously increase with the increase of standard deviation for the same reliability level. Rapid increases start to occur when the standard deviation gets closer to the value where solution becomes infeasible.

Analyses results of the three case show that lower limit is dominant in the both of the limits case. Comparing Figure 4.4, 4.6 and 4.8, same increase in the reliability level results in highest injection results in the both of the limits case, slightly lower results in lower limit case and lowest results in upper limit case for the same standard deviation. For example, for the standard deviation 0.50 and increase of reliability level from 0.70 to 0.80, upper limit application results in 0.17 *kg/day* increase of injection mass from 21.259 *kg/day* to 21.429 *kg/day*. Lower limit application results in 18.47 *kg/day* increase from 51.432 *kg/day* to 69.902 *kg/day* and both of the limits case result in 19.777 *kg/day* from 51.551 *kg/day* to 71.328 *kg/day*.

Comparing Figures 4.5, 4.7 and 4.9, range of the standard deviation for the feasible solution is much smaller for the lower limit and both of the limits cases. Most of the changes occur within the range of 0.0 and 1.0 in the lower limit and both of the limits cases; whereas, there is no such significant range for the upper limit case and changes gradually occur between 0.0 and 4.0.

4.2.1.2 Log-Normal Distribution

Chance constrained optimization formulation by assuming the log-normal probability distribution for chlorine concentration is composed of objective function, Equation 2.9, non-negativity constraint, Equation 2.11 and chlorine concentration limitation adjusted according to the application to the upper limit, lower limit and both.

4.2.1.2.1 Upper Limit

Chlorine concentration constraint is constructed according to the Equation 3.33 as explained in section 3.2.2.1. Mean value of the maximum concentration is taken as $\mu_{\overline{C}}$ = 4.0 *mg/l* and for different standard distribution levels $\lambda_{\overline{C}}$ and $\xi_{\overline{C}}$ are calculated by using Equations 3.6 and 3.7, respectively. Standardized normal variant is calculated for α_U ranging from 0.60 to 0.99 with 0.10 increments. As higher reliability values are favorable, increment is taken as 0.01 for the values of α_U ranging from 0.95 to 0.99 in order to see its effect more precisely. Modified maximum limits calculated from Equation 3.33 and computed total mass injection results can be seen in Table 4.9. Note that, the lower limit is kept at 0.2 *mg/l* for each run.

It is shown in Table 4.9 that in the analyses, increment of the standard deviation is taken as 0.50 mostly. For reliability levels ranging from 0.60 to 0.90, analyses give feasible solutions even for standard deviations higher than 4.0; however, it is not reasonable to have these values, so analyses are stopped at 4.0. Apart from those, highest analyzed value of the standard deviation is where the solver is unable to find a feasible solution. For example, for reliability level of 0.96, the maximum value of the standard deviation is 3.20. Increment is adjusted at necessary points in order to observe the changes in the total mass injection in a better way. The change of total mass injection versus reliability level for each standard deviation is given in Figure 4.10. Similarly, the change of total mass injection versus standard deviation for each reliability level is shown in Figure 4.11.

Table 4.9: Results of the upper limit application with log-normal probability distribution for Case I

α_U	U(kg/day)	σ	UL(mg/l)	LL(mg/l)	α_U	U(kg/day)	σ	UL(mg/l)	LL(mg/l)
0.60	21.245	0.00	4.000	0.200	0.95	21.245	0.00	4.000	0.200
	21.312	0.50	3.846	0.200		21.580	0.50	3.234	0.200
	21.400	1.00	3.646	0.200		21.867	1.00	2.588	0.200
	21.500	1.50	3.416	0.200		22.105	1.50	2.062	0.200
	21.606	2.00	3.174	0.200		22.294	2.00	1.645	0.200
	21.712	2.50	2.933	0.200		22.492	2.50	1.319	0.200
	21.816	3.00	2.702	0.200		23.377	3.00	1.066	0.200
	21.913	3.50	2.487	0.200		24.940	3.40	0.906	0.200
	22.002	4.00	2.291	0.200	0.96	21.245	0.00	4.000	0.200
0.70	21.245	0.00	4.000	0.200		21.598	0.50	3.192	0.200
	21.368	0.50	3.718	0.200		21.897	1.00	2.522	0.200
	21.503	1.00	3.411	0.200		22.140	1.50	1.985	0.200
	21.640	1.50	3.097	0.200		22.335	2.00	1.565	0.200
	21.774	2.00	2.793	0.200		22.646	2.50	1.241	0.200
	21.902	2.50	2.510	0.200		23.997	3.00	0.994	0.200
	22.018	3.00	2.254	0.200		24.854	3.20	0.912	0.200
	22.121	3.50	2.027	0.200	0.97	21.245	0.00	4.000	0.200
	22.211	4.00	1.828	0.200		21.621	0.50	3.140	0.200
0.80	21.245	0.00	4.000	0.200		21.933	1.00	2.442	0.200
	21.431	0.50	3.574	0.200		22.182	1.50	1.893	0.200
	21.615	1.00	3.154	0.200		22.387	2.00	1.471	0.200
	21.789	1.50	2.760	0.200		22.984	2.50	1.152	0.200
	21.950	2.00	2.404	0.200		24.866	3.00	0.911	0.200
	22.092	2.50	2.092	0.200	0.98	21.245	0.00	4.000	0.200
	22.213	3.00	1.824	0.200		21.650	0.50	3.074	0.200
	22.318	3.50	1.596	0.200		21.979	1.00	2.340	0.200
	22.426	4.00	1.404	0.200		22.234	1.50	1.778	0.200
0.90	21.245	0.00	4.000	0.200		22.458	2.00	1.356	0.200
	21.514	0.50	3.384	0.200		23.554	2.50	1.043	0.200
	21.757	1.00	2.830	0.200		25.312	2.80	0.896	0.200
	21.974	1.50	2.353	0.200	0.99	21.245	0.00	4.000	0.200
	22.155	2.00	1.953	0.200		21.695	0.50	2.971	0.200
	22.303	2.50	1.625	0.200		22.048	1.00	2.188	0.200
	22.455	3.00	1.359	0.200		22.310	1.50	1.611	0.200
	23.011	3.50	1.145	0.200		22.820	2.00	1.192	0.200
	24.182	4.00	0.973	0.200		25.532	2.50	0.892	0.200

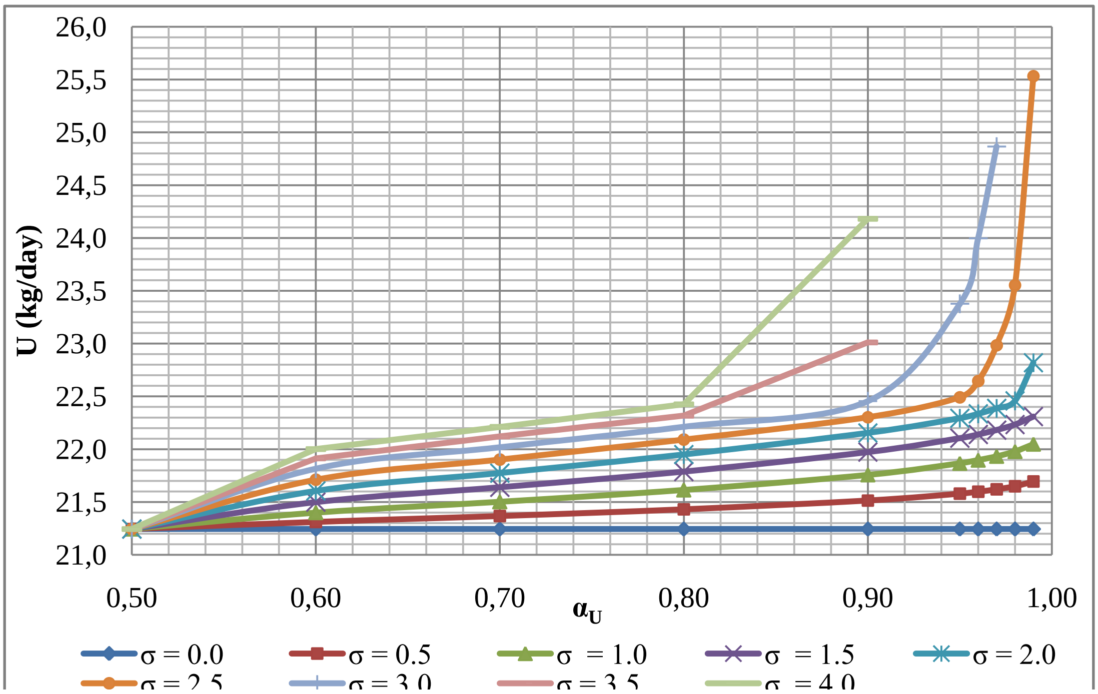

Figure 4.10: Total injected mass versus upper limit reliability for log-normal probability distribution (Case I)

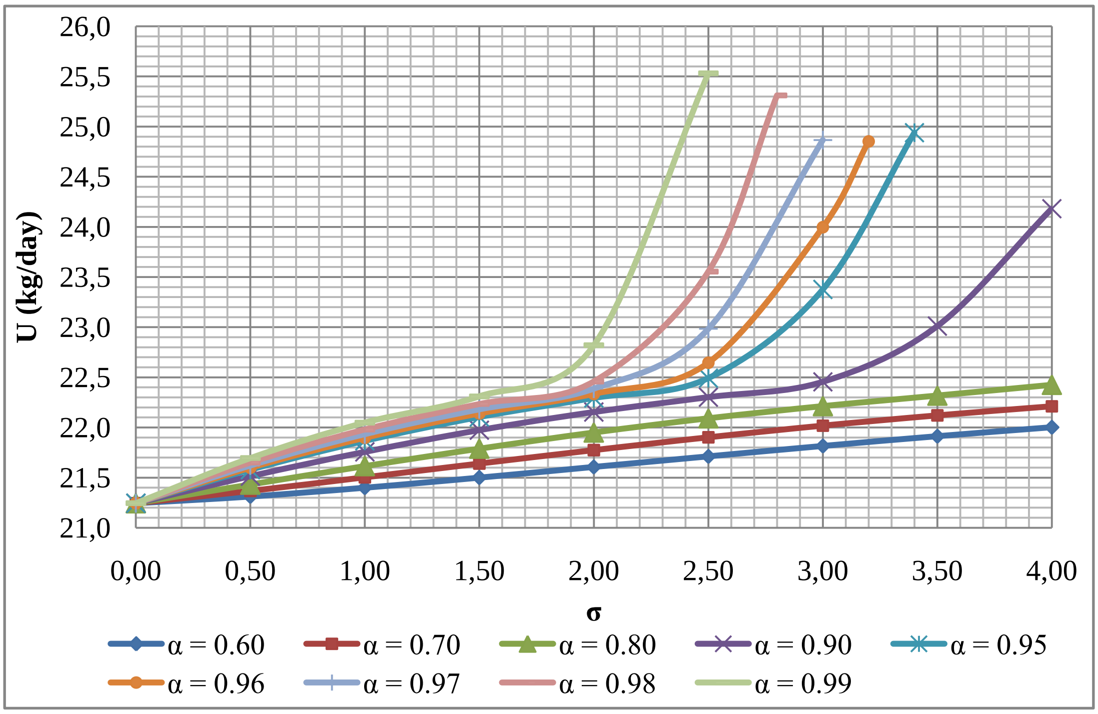

Figure 4.11: Total injected mass versus standard deviation of upper limit for log-normal probability distribution (Case I)

It is shown in Figure 4.10 that total injected mass continuously increases with the increase of reliability level for the same standard deviation. Especially after reliability level of 0.90, that increase become abrupt. For standard deviations 3.5 and 4.0 there is no feasible solution for high reliability levels and abrupt changes cannot be observed. For σ=0, there is no change in the upper limit for any reliability level; thus, results are same with the no probability case for all reliability levels.

Similarly, it can be concluded from Figure 4.11 that total injected mass continuously increase with the increase of standard deviation for the same reliability level. Abrupt increases start to occur when the standard deviation gets closer to the value where solution becomes infeasible.

4.2.1.2.2 Lower Limit

Chlorine concentration constraint is constructed according to the Equation 3.39 as explained in 3.2.2.2. Mean value of the minimum concentration is taken as $\mu_{\underline{C}}$ = 0.2 *mg/l* and for different standard distribution levels $\lambda_{\underline{C}}$, $\xi_{\underline{C}}$ are calculated by using Equations 3.6 and 3.7, respectively. Standardized normal variant is calculated for α_L ranging from 0.60 to 0.99 with 0.10 increments. In order to see the effect of higher reliabilities, increment is taken as 0.01 for the values of α_L ranging from 0.95 to 0.99. Modified minimum limits calculated from Equation 3.39 and computed total mass injection results can be seen in Table 4.10. Note that, the upper limit is kept at 4.0 *mg/l* for each run.

Increment of the standard deviation is 0.01 up to 0.05 and after that increment value is taken as 0.10, which can be seen in Table 4.10. Highest analyzed value of the standard deviation is where the solver is unable to find a feasible solution. For example, for reliability level of 0.70, the maximum value of the standard deviation is 0.1. Increment is adjusted at necessary points in order to observe the changes in the total mass injection in a better way. For showing the effect of standard deviation on total mass injection, only the reliability level up to 90% is used. This is because standard deviations between 0.01-0.05 are so small to create detectable changes for reliability level from 95% to 99%. The change of total mass injection versus reliability level for each standard deviation is given in Figure 4.12. Similarly, the change of total mass injection versus standard deviation for each reliability level is shown in Figure 4.13.

Table 4.10: Results of the lower limit application with log-normal probability distribution for Case I

α_L	U(kg/day)	σ	UL(mg/l)	LL(mg/l)	α_L	U(kg/day)	σ	UL(mg/l)	LL(mg/l)
0.60	21.245	0.000	4.000	0.200	0.95	21.245	0.00	4.000	0.200
	21.509	0.010	4.000	0.202		43.007	0.10	4.000	0.389
	21.716	0.020	4.000	0.204		62.314	0.20	4.000	0.556
	21.866	0.030	4.000	0.205		75.288	0.30	4.000	0.662
	21.958	0.040	4.000	0.206		83.493	0.40	4.000	0.721
	21.994	0.050	4.000	0.207		87.984	0.50	4.000	0.752
0.70	21.245	0.000	4.000	0.200		90.388	0.60	4.000	0.767
	21.826	0.010	4.000	0.205		91.329	0.70	4.000	0.773
	22.359	0.020	4.000	0.210		91.417	0.75	4.000	0.774
	22.840	0.030	4.000	0.214	0.96	21.245	0.00	4.000	0.200
	23.266	0.040	4.000	0.218		45.304	0.10	4.000	0.409
	23.633	0.050	4.000	0.221		68.320	0.20	4.000	0.607
	24.292	0.075	4.000	0.227		86.535	0.30	4.000	0.742
	24.598	0.100	4.000	0.229		99.770	0.40	4.000	0.824
0.80	21.245	0.000	4.000	0.200		108.163	0.50	4.000	0.873
	22.202	0.010	4.000	0.208		116.100	0.60	4.000	0.901
	23.134	0.020	4.000	0.216	0.97	21.245	0.00	4.000	0.200
	24.032	0.030	4.000	0.224		48.291	0.10	4.000	0.435
	24.888	0.040	4.000	0.232		77.371	0.20	4.000	0.677
	25.695	0.050	4.000	0.239		104.833	0.30	4.000	0.855
	28.858	0.100	4.000	0.266		113.945	0.33	4.000	0.896
	30.528	0.150	4.000	0.281	0.98	21.245	0.00	4.000	0.200
	31.018	0.200	4.000	0.285		35.249	0.05	4.000	0.322
0.90	21.245	0.000	4.000	0.200		52.556	0.10	4.000	0.472
	22.735	0.010	4.000	0.213		71.161	0.15	4.000	0.631
	24.251	0.020	4.000	0.226		92.764	0.20	4.000	0.782
	25.781	0.030	4.000	0.239		106.573	0.23	4.000	0.865
	27.313	0.040	4.000	0.253	0.99	21.245	0.00	4.000	0.200
	28.835	0.050	4.000	0.266		37.822	0.05	4.000	0.344
	35.939	0.100	4.000	0.328		60.062	0.10	4.000	0.537
	45.539	0.200	4.000	0.411		88.717	0.15	4.000	0.757
	49.566	0.300	4.000	0.446		112.677	0.18	4.000	0.892
	50.556	0.400	4.000	0.455	-	-	-	-	-

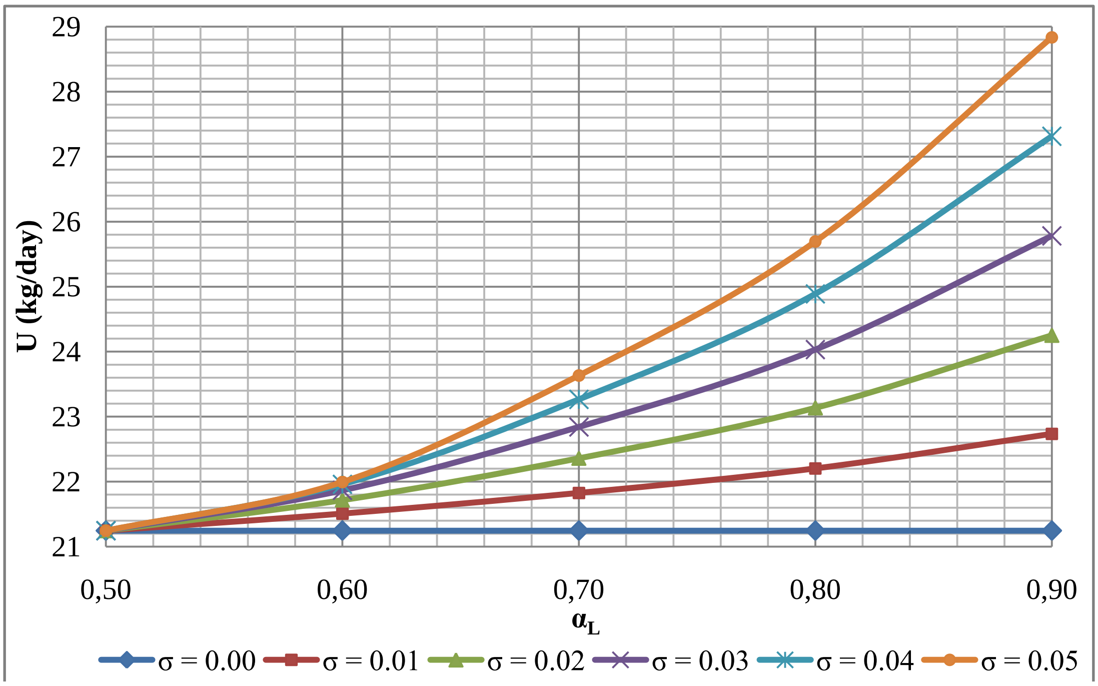

Figure 4.12: Total injected mass versus lower limit reliability for log-normal probability distribution (Case I)

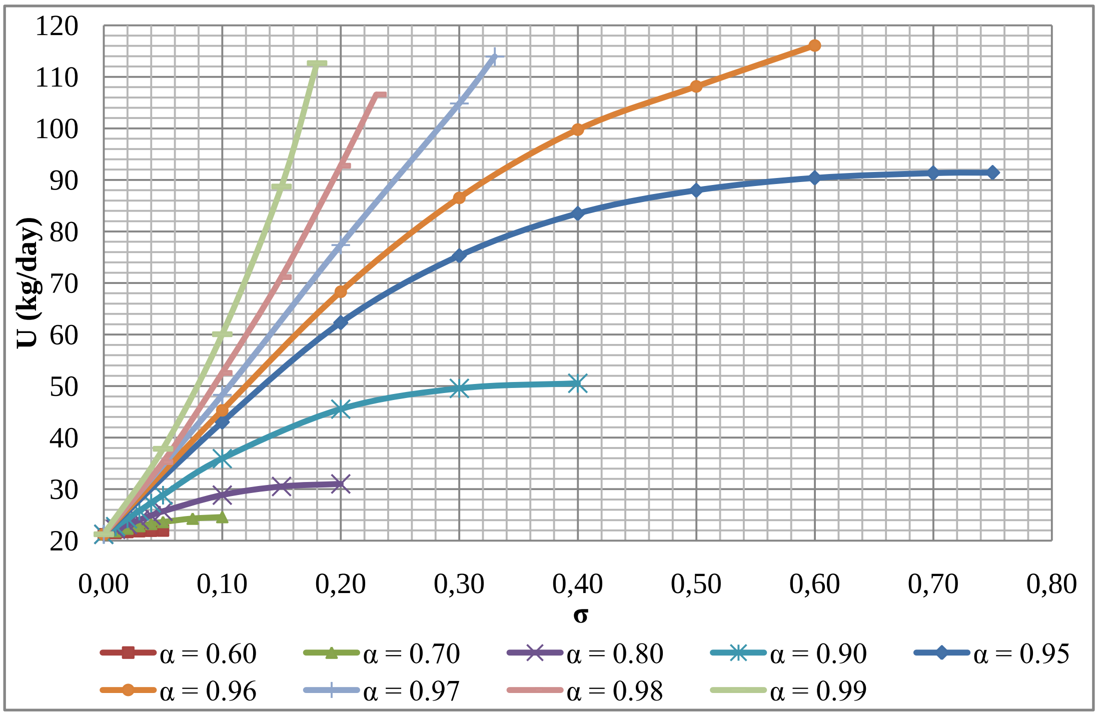

Figure 4.13: Total injected mass versus standard deviation of lower limit for log-normal probability distribution (Case I)

Total injected mass continuously increases with the increase of reliability level for the same standard deviation as it is shown in Figure 4.12. As comparison is done up to the standard deviation value of 0.05, infeasible solution limit is not reached for any reliability level. For σ=0, there is no change in the upper limit for any reliability level; thus, results are same with the no probability case for all reliability levels.

Similarly, it can be concluded from Figure 4.13 that total injected mass increase with the increase of standard deviation for the same reliability level. When the standard deviation gets closer to the value where solution become infeasible, total injection start to asymptotically reach its limiting value.

Comparing Figure 4.10 and Figure 4.12, same increase in the reliability level results in much more injection results in the lower limit case for the same standard deviation. For example, for the standard deviation 0.50 and increase of reliability level from 0.70 to 0.80, upper limit application results in 0.063 *kg/day* increase of injection mass from 21.368 *kg/day* to 21.431 *kg/day*. On the other hand, for lower limit application even the analyses cannot be done for the standard deviation of 0.50. For standard deviation 0.1 and increase of reliability level from 0.70 to 0.80, lower limit application result in 4.26 *kg/day* increase from 24.598 *kg/day* to 28.858 *kg/day*. Same analyses results for upper limit will be lower than 0.063 *kg/day*.

Comparing Figure 4.11 and Figure 4.13, most important difference is the attitudes of the results. For the upper limit case, total injected mass continuously increase with the increase of standard deviation for the same reliability level while total injection start to asymptotically reach its limiting value for the lower limit case. This behavior is resulting from the formulations of the $\lambda_{\overline{C}}$ and $\xi_{\overline{C}}$ in the log-normal distribution. As it can be seen in Equation 3.7, ratio of $\left(\frac{\sigma}{\mu}\right)^2$ exists in the formulation of $\xi_{\overline{C}}$. When it is calculated for upper limit, μ is always greater than σ so the ratio is always smaller than 1. However, for the lower limit, μ is smaller than σ for most of the times and the ratio becomes greater than 1. This ratio difference become more effective when $\xi_{\overline{C}}$ and $\lambda_{\overline{C}}$ are calculated as shown in Equation 3.6 and 3.7, respectively. And this growing effect results in the change of the behavior.

4.2.1.2.3 Both of the Limits

Chlorine concentration constraint is constructed according to the Equation 3.41 as explained in 3.2.2.3. Mean values of the maximum and minimum concentrations are taken as $\mu_{\overline{C}}$ = 4.0 *mg/l* and $\mu_{\underline{C}}$ = 0.2 *mg/l*, respectively. For different standard distribution levels $\lambda_{\underline{C}}$, $\lambda_{\overline{C}}$, $\xi_{\overline{C}}$ and $\xi_{\underline{C}}$ are calculated by using Equation 3.6 and 3.7 respectively. For this case, α_U and α_L are equal to each other. Standardized normal variants are calculated for α_U and α_L ranging from 0.60 to 0.99 with 0.10 increments. In order to see the effect of higher reliabilities, increment is taken as 0.01 for the values of α_U and α_L ranging from 0.95 to 0.99. Modified maximum/minimum limits calculated from Equation 3.41 and computed total mass injection results can be seen in Table 4.11.

For this case, increment of the standard deviation is 0.01 up to 0.05 and after that value increment is taken as 0.10, which can be seen in Table 4.11. Highest analyzed value of the standard deviation is where the solver is unable to find a feasible solution. For example, for reliability level of 0.90, the maximum value of the standard deviation is 0.40. Increment is adjusted at necessary points in order to observe the changes in the total mass injection in a better way. For showing the effect of standard deviation on total mass injection, only the reliability level up to 90% is used. This is because standard deviations between 0.01-0.05 are so small to create detectable changes for reliability level from 95% to 99%. The change of total mass injection versus reliability level for each standard deviation is given in Figure 4.14. Similarly, the change of total mass injection versus standard deviation for each reliability level is shown in Figure 4.15.

Table 4.11: Both of the limits application results with log-normal probability distribution for Case I

α_{U-L}	U(kg/day)	σ	UL(mg/l)	LL(mg/l)	α_{U-L}	U(kg/day)	σ	UL(mg/l)	LL(mg/l)
0.60	21.245	0.00	4.000	0.200	0.95	21.245	0.00	4.000	0.200
	21.510	0.01	3.997	0.202		43.080	0.10	3.838	0.389
	21.718	0.02	3.995	0.204		62.542	0.20	3.680	0.556
	21.869	0.03	3.992	0.205		77.348	0.30	3.526	0.662
	21.963	0.04	3.990	0.206		88.341	0.40	3.378	0.721
	22.000	0.05	3.987	0.207		93.804	0.45	3.305	0.739
0.70	21.245	0.00	4.000	0.200	0.96	21.245	0.00	4.000	0.200
	21.828	0.01	3.995	0.205		32.624	0.05	3.913	0.299
	22.364	0.02	3.989	0.210		45.383	0.10	3.828	0.409
	22.847	0.03	3.984	0.214		57.706	0.15	3.743	0.515
	23.275	0.04	3.979	0.218		69.166	0.20	3.660	0.607
	23.644	0.05	3.974	0.221		80.234	0.25	3.579	0.683
	24.205	0.07	3.963	0.226		90.773	0.30	3.499	0.742
	24.622	0.10	3.947	0.229		97.855	0.33	3.451	0.771
0.80	21.245	0.00	4.000	0.200	0.97	21.245	0.00	4.000	0.200
	22.206	0.01	3.992	0.208		33.746	0.05	3.907	0.308
	23.142	0.02	3.983	0.216		48.375	0.10	3.815	0.435
	24.043	0.03	3.975	0.224		63.194	0.15	3.725	0.562
	24.902	0.04	3.966	0.232		78.982	0.20	3.637	0.677
	25.713	0.05	3.958	0.239		95.959	0.25	3.550	0.775
	28.895	0.10	3.916	0.266		101.022	0.26	3.533	0.792
	30.586	0.15	3.873	0.281	0.98	21.245	0.00	4.000	0.200
	31.094	0.20	3.830	0.285		35.295	0.05	3.898	0.322
0.90	21.245	0.00	4.000	0.200		52.649	0.10	3.799	0.472
	22.740	0.01	3.987	0.213		72.222	0.15	3.701	0.631
	24.262	0.02	3.974	0.226		96.528	0.20	3.605	0.782
	25.797	0.03	3.962	0.239		105.072	0.21	3.586	0.810
	27.335	0.04	3.949	0.253	0.99	21.245	0.00	4.000	0.200
	28.863	0.05	3.936	0.266		37.874	0.05	3.885	0.344
	35.997	0.10	3.873	0.328		60.190	0.10	3.773	0.537
	41.663	0.15	3.810	0.377		91.699	0.15	3.663	0.757
	45.654	0.20	3.747	0.411	-	-	-	-	-
	49.736	0.30	3.624	0.446					
	50.798	0.40	3.503	0.455					

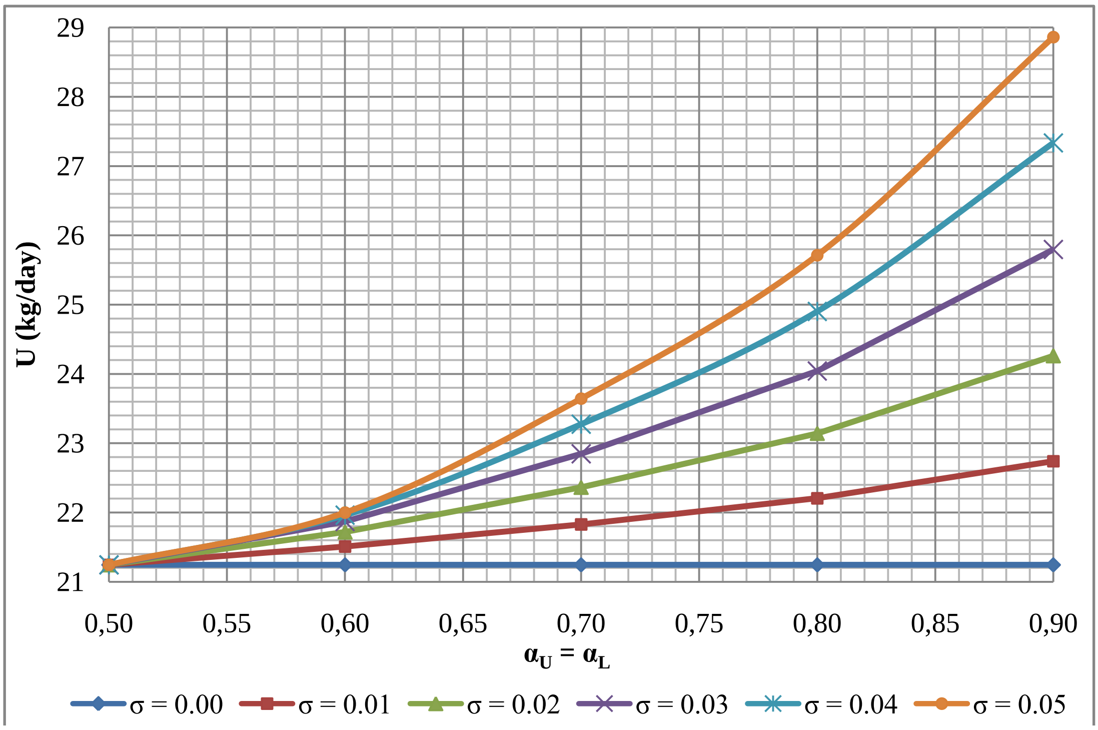

Figure 4.14: Total injected mass versus both limits reliability for log-normal probability distribution (Case I)

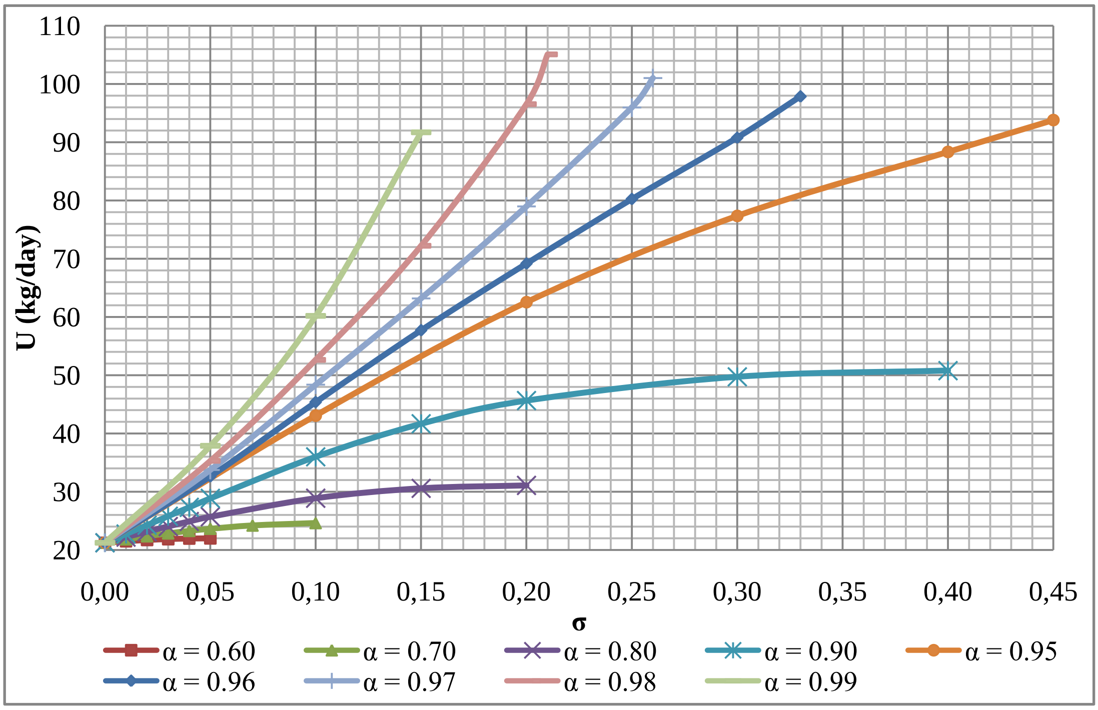

Figure 4.15: Total injected mass versus standard deviation of both of the limits for log-normal probability distribution (Case I)

It is seen from Figure 4.14 that total injected mass continuously increases with the increase of reliability level for the same standard deviation. As comparison is done up to the standard deviation value of 0.05, infeasible solution limit is not reached for any reliability level. For σ=0, there is no change in the upper limit for any reliability level; thus, results are same with the no probability case for all reliability levels.

Likewise, it can be concluded from Figure 4.15 that total injected mass increase with the increase of standard deviation for the same reliability level. When the standard deviation gets closer to the value where solution becomes infeasible, total injection start to asymptotically reach its limiting value.

Analyses results of the three case show that lower limit is dominant in the both of the limits case. Comparing Figure 4.10, 4.12 and 4.14, same increase in the reliability level results in highest injection results in the both of the limits case, slightly lower results in lower limit case and lowest results in upper limit case for the same standard deviation. For example, for the standard deviation 0.50 and increase of reliability level from 0.70 to 0.80, upper limit application results in 0.063 *kg/day* increase of injection mass from 21.368 *kg/day* to 21.431 *kg/day*. On the other hand, for lower limit application even the analyses cannot be done for the standard deviation of 0.50. For standard deviation 0.1 and increase of reliability level from 0.70 to 0.80, lower limit application result in 4.26 *kg/day* increase from 24.598 *kg/day* to 28.858 *kg/day* and both of the limits application result in 4.23 *kg/day* increase from 24.662 *kg/day* to 28.895 *kg/day*. Same analyses results for upper limit will be lower than 0.063 *kg/day*.

The most important difference is seen when Figures 4.11, 4.13 and 4.15 are compared. For the upper limit case, total injected mass continuously increase with the increase of standard deviation for the same reliability level while total injection start to asymptotically reach its limiting value for the lower limit and both of the limits cases.

4.2.2 Case II

In Case II the injection is done from Nodes 37, 39 and 42. For normal and log-normal probability distribution, total mass injection amounts are calculated. To see the effects on the total injected amount, different reliability levels and standard deviations are analyzed in both of the distribution types.

For the non-probability case, objective function is Equation 2.9 and constraints are Equations 2.10 and 2.11. Response coefficient, *B*, used in the Equation 2.10 is taken from the Case V results of the program developed by Sert *(2009)*. Upper and lower limits in Equation 2.10 are $\overline{C}$ = 4.0 *mg/l* and $\underline{C}$ = 0.2 *mg/l,* respectively. Reliability level is 50%. Linear programing formulation is solved by Excel and injection results can be seen in Table 4.12.

For different probability distributions, chlorine concentration limitation constraint, Equation 2.10, will be modified taking different reliability levels and standard deviations into consideration. Rest of the problem is the same with the non-probability case.

Table 4.12: Injection results of Case II

Node 37, 39 and 42			
Time (hr)	**u (mg/min)**		
	37	**39**	**42**
1	696.07	0.00	0.00
2	583.04	0.00	0.00
3	549.55	0.00	0.00
4	579.74	0.00	0.00
5	595.43	0.00	0.00
6	661.52	0.00	0.00
7	0.00	2.25	245.13
8	0.00	0.00	117.64
9	0.00	2.12	132.54
10	0.00	6.64	367.97
11	0.00	145.79	624.42
12	521.59	0.00	0.00
13	728.88	0.00	0.00
14	607.30	0.00	0.00
15	576.99	0.00	0.00
16	755.38	31.73	0.00
17	368.68	0.00	0.00
18	0.00	0.00	113.71
19	0.00	0.00	38.66
20	0.00	0.00	93.80
21	0.00	0.00	231.44
22	0.00	21.48	334.57
23	0.00	0.00	313.62
24	0.00	1.31	258.46
Σu (mg/min)	7224.18	211.33	2871.97
Σu (kg/day)	14.843		

In order to see the distribution of the data, frequency plot of chlorine concentrations throughout the network is given in Figure 4.16. As it can be seen from the figure, most of the data are lying between 0.2 *mg/l* and 0.4 *mg/l* and much less number of data are between 0.4 *mg/l* and 4.0 *mg/l*.

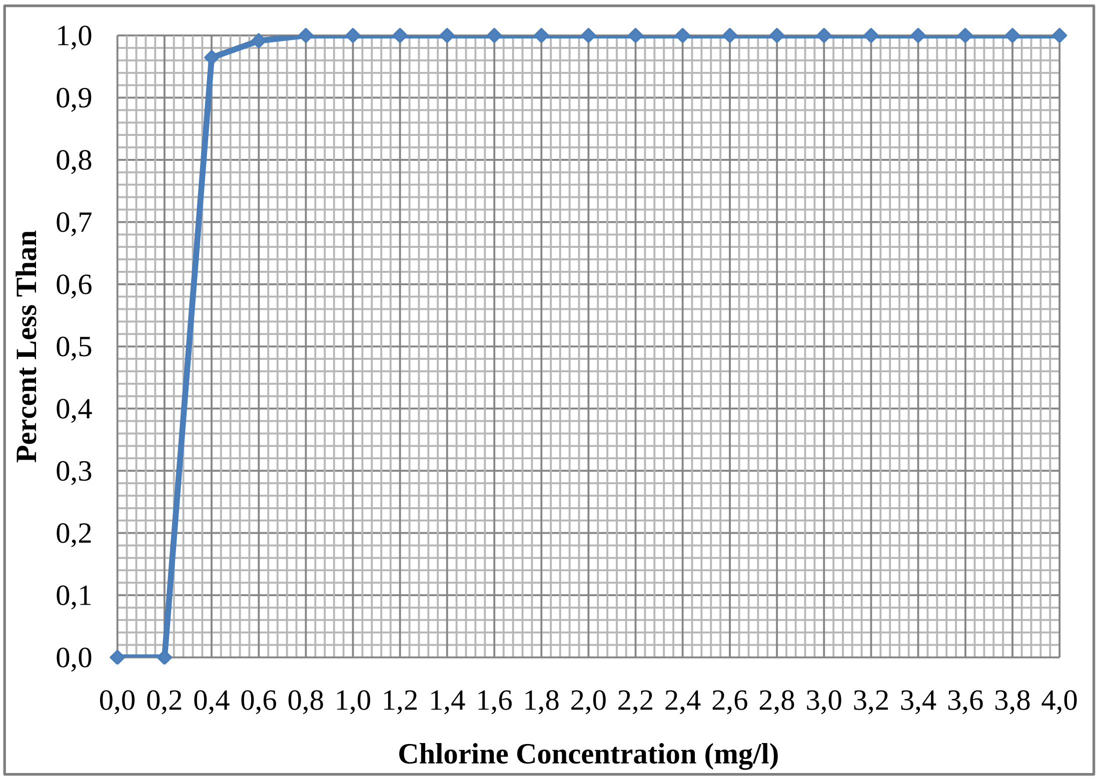

Figure 4.16: Frequency plot of chlorine concentration for Case II

4.2.2.1 The Normal Distribution

Chance constrained optimization formulation by assuming the normal probability distribution for chlorine concentration is composed of objective function, Equation 2.9, non-negativity constraint, Equation 2.11 and chlorine concentration limitation adjusted according to application to the upper limit, lower limit and both.

For upper limit, lower limit and both of the limits sections, mean values of minimum/maximum concentrations and increment used in the reliability level will be the same as explained in sections *4.2.1.1.1*, *4.2.1.1.2* and *4.2.1.1.3*, respectively.

4.2.2.1.1 Upper Limit

Modified maximum limits and computed total mass injection results can be seen in Table 4.13. As it can be seen from the Table 4.13 that in the analyses, increment of the standard deviation is mostly 0.50. For reliability levels ranging from 0.50 to 0.80, analyses give feasible solutions for standard deviations higher than 4.0; however, it is not reasonable to have these values so analyses are stopped at 4.0. Apart from those two, highest analyzed value of the standard deviation is where the solver is unable to find a feasible solution. For example, for reliability level of 0.90, the maximum value of the standard deviation is 2.75. Increment is adjusted at necessary points in order to observe the changes in the total mass injection in a better way. The change of total mass injection versus reliability level for each standard deviation is given in Figure 4.17. Similarly, the change of total mass injection versus standard deviation for each reliability level is shown in Figure 4.18.

Table 4.13: Results of the upper limit application with normal probability distribution for Case II

α_U	U(kg/day)	σ	UL(mg/l)	LL(mg/l)	α_U	U(kg/day)	σ	UL(mg/l)	LL(mg/l)
0.50	14.843	0.00	4.000	0.200	0.95	14.843	0.00	4.000	0.200
	14.843	3.00	4.000	0.200		14.843	0.50	3.178	0.200
	14.843	3.50	4.000	0.200		14.843	1.00	2.355	0.200
	14.843	4.00	4.000	0.200		14.843	1.50	1.533	0.200
0.60	14.843	0.00	4.000	0.200		14.844	2.00	0.710	0.200
	14.843	3.00	3.240	0.200		14.854	2.10	0.546	0.200
	14.843	3.50	3.113	0.200	0.96	14.843	0.00	4.000	0.200
	14.843	4.00	2.987	0.200		14.843	0.50	3.125	0.200
0.70	14.843	0.00	4.000	0.200		14.843	1.00	2.249	0.200
	14.843	3.00	2.427	0.200		14.843	1.50	1.374	0.200
	14.843	3.50	2.165	0.200		14.889	2.00	0.499	0.200
	14.843	4.00	1.902	0.200	0.97	14.843	0.00	4.000	0.200
0.80	14.843	0.00	4.000	0.200		14.843	0.50	3.060	0.200
	14.843	3.00	1.475	0.200		14.843	1.00	2.119	0.200
	14.843	3.50	1.054	0.200		14.843	1.50	1.179	0.200
	14.847	4.00	0.634	0.200		14.849	1.80	0.615	0.200
0.90	14.843	0.00	4.000	0.200	0.98	14.843	0.00	4.000	0.200
	14.843	2.00	3.744	0.200		14.843	0.50	2.973	0.200
	14.843	2.50	0.796	0.200		14.843	1.00	1.946	0.200
	14.845	2.60	0.668	0.200		14.843	1.50	0.919	0.200
	14.855	2.70	0.540	0.200		14.859	1.70	0.509	0.200
	14.976	2.74	0.489	0.200	0.99	14.843	0.00	4.000	0.200
	15.170	2.75	0.476	0.200		14.843	0.50	2.837	0.200
-	-	-	-	-		14.843	1.00	1.674	0.200
						14.858	1.50	0.510	0.200

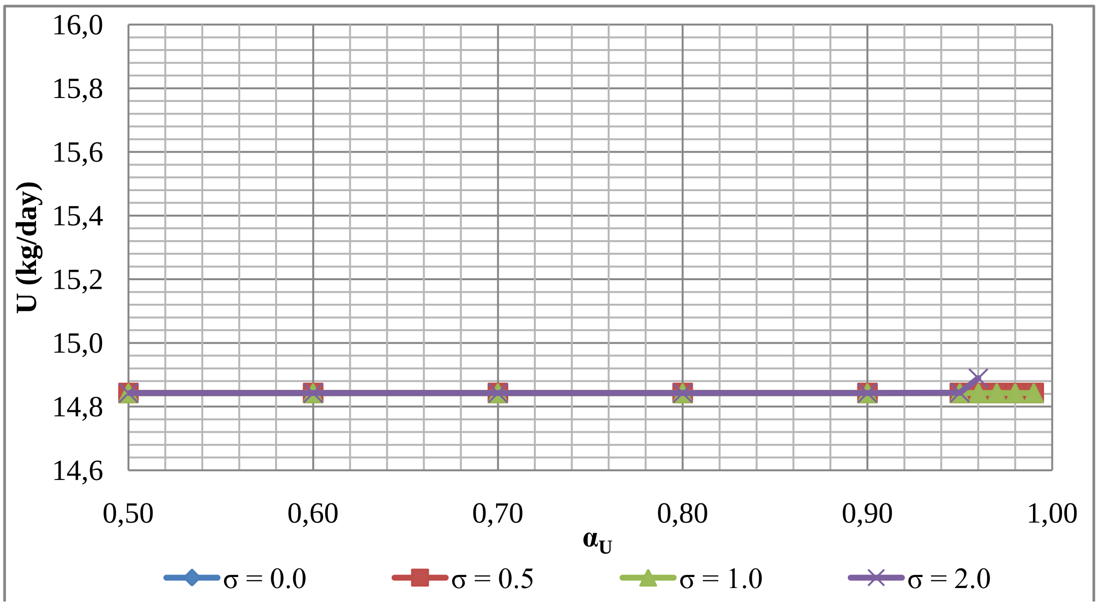

Figure 4.17: Total injected mass versus upper limit reliability for normal probability distribution (Case II)

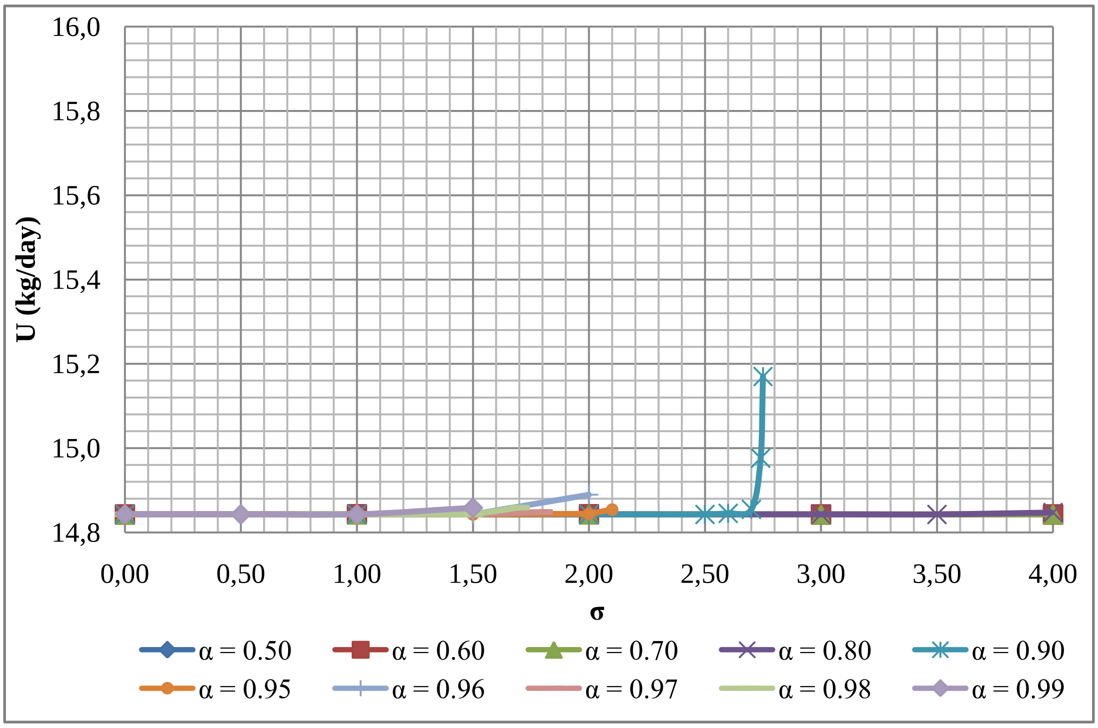

Figure 4.18: Total injected mass versus standard deviation of upper limit for normal probability distribution (Case II)

As the distribution of the data is on a much more narrow limits in the use of three booster stations (Figure 4.16), changing the reliability level and the standard deviation of the upper limit does not affect the analyses results of non-probability case in considerable amounts.

It can be seen from Figure 4.17 that total injected mass slightly increase with the increase of reliability level for the same standard deviation. Only detectable change occurs for standard deviation 2.0 and it is only 0.045 *mg/l* from 14.889 *mg/l* to 14.844 *mg/l*. For $\sigma=0$, there is no change in the upper limit for any reliability level; thus, results are same with the no probability case for all reliability levels.

Likewise, it can be concluded from Figure 4.18 that total injected mass slightly increase with the increase of standard deviation for the same reliability level. Detectable increases start to occur when the standard deviation gets closer to the value where solution becomes infeasible.

4.2.2.1.2 Lower Limit

Modified minimum limits and computed total mass injection results can be seen in Table 4.14.

Shown in the Table 4.14 that in the analyses, increment of the standard deviation is 0.10 mostly. For reliability level 0.60 analyses give feasible solutions for standard deviations higher than 4.0; however, it is not reasonable to have these values so analyses are stopped at 4.0. Apart from this, highest analyzed value of the standard deviation is where the solver is unable to find a feasible solution. For example, for reliability level of 0.90, the maximum value of the standard deviation is 1.20. Increment is adjusted at necessary points in order to observe the changes in the total mass injection in a better way. The change of total mass injection versus reliability level for each standard deviation is given in Figure 4.19. Similarly, the change of total mass injection versus standard deviation for each reliability level is shown in Figure 4.20.

Table 4.14: Results of the lower limit application with normal probability distribution for Case II

α_L	U(kg/day)	σ	UL(mg/l)	LL(mg/l)	α_L	U(kg/day)	σ	UL(mg/l)	LL(mg/l)
0.60	14.843	0.00	4.000	0.200	0.96	14.843	0.00	4.000	0.200
	24.244	0.50	4.000	0.327		27.835	0.10	4.000	0.375
	33.645	1.00	4.000	0.453		40.828	0.20	4.000	0.550
	43.046	1.50	4.000	0.580		53.820	0.30	4.000	0.725
	52.446	2.00	4.000	0.707		66.813	0.40	4.000	0.900
	61.847	2.50	4.000	0.833		79.806	0.50	4.000	1.075
	71.248	3.00	4.000	0.960		92.823	0.60	4.000	1.250
	80.651	3.50	4.000	1.087		105.860	0.70	4.000	1.425
	90.068	4.00	4.000	1.213		119.078	0.80	4.000	1.601
0.70	14.843	0.00	4.000	0.200		143.516	0.90	4.000	1.776
	34.302	0.50	4.000	0.462	0.97	14.843	0.00	4.000	0.200
	53.761	1.00	4.000	0.724		28.801	0.10	4.000	0.388
	73.219	1.50	4.000	0.987		42.759	0.20	4.000	0.576
	92.703	2.00	4.000	1.249		56.717	0.30	4.000	0.764
	112.236	2.50	4.000	1.511		70.675	0.40	4.000	0.952
	142.993	3.00	4.000	1.773		84.642	0.50	4.000	1.140
0.80	14.843	0.00	4.000	0.200		98.636	0.60	4.000	1.328
	46.073	0.50	4.000	0.621		112.650	0.70	4.000	1.517
	77.303	1.00	4.000	1.042		130.732	0.80	4.000	1.705
	108.614	1.50	4.000	1.462	0.98	14.843	0.00	4.000	0.200
	132.288	1.80	4.000	1.715		30.084	0.10	4.000	0.405
0.90	14.843	0.00	4.000	0.200		45.326	0.20	4.000	0.611
	38.620	0.25	4.000	0.520		60.568	0.30	4.000	0.816
	62.397	0.50	4.000	0.841		75.809	0.40	4.000	1.021
	86.186	0.75	4.000	1.161		91.071	0.50	4.000	1.227
	110.040	1.00	4.000	1.482		106.364	0.60	4.000	1.432
	136.118	1.20	4.000	1.738		122.630	0.70	4.000	1.638
0.95	14.843	0.00	4.000	0.200	0.99	14.843	0.00	4.000	0.200
	27.050	0.10	4.000	0.364		32.107	0.10	4.000	0.433
	39.257	0.20	4.000	0.529		49.372	0.20	4.000	0.665
	51.464	0.30	4.000	0.693		66.637	0.30	4.000	0.898
	63.671	0.40	4.000	0.858		83.909	0.40	4.000	1.131
	75.878	0.50	4.000	1.022		101.219	0.50	4.000	1.363
	88.100	0.60	4.000	1.187		118.638	0.60	4.000	1.596
	100.342	0.70	4.000	1.351	-	-	-	-	-
	112.600	0.80	4.000	1.516					
	127.393	0.90	4.000	1.680					

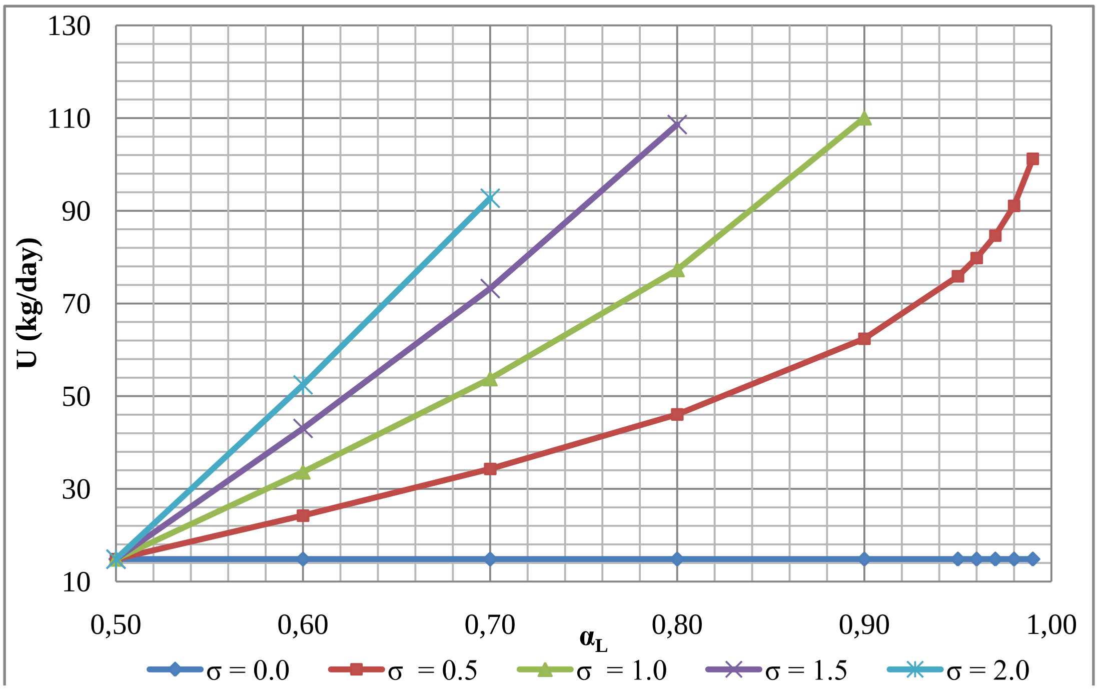

Figure 4.19: Total injected mass versus lower limit reliability for normal probability distribution (Case II)

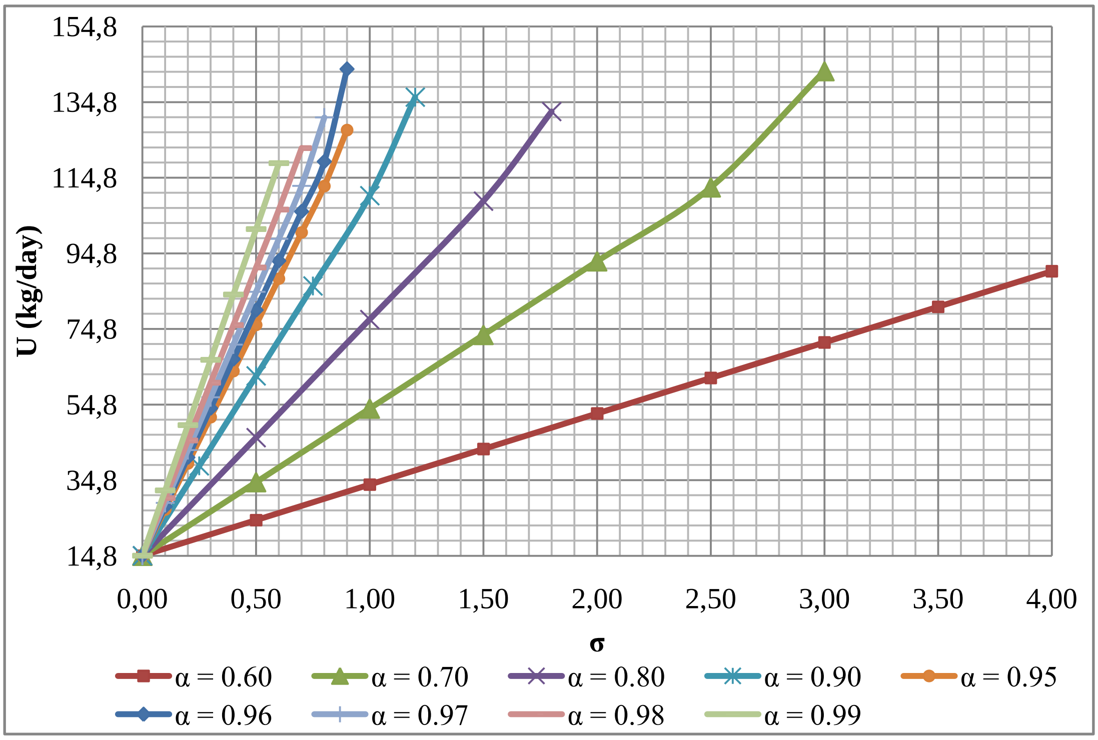

Figure 4.20: Total injected mass versus standard deviation of lower limit for normal probability distribution (Case II)

Shown in Figure 4.19 that total injected mass continuously increases with the increase of reliability level for the same standard deviation. Especially after reliability level of 0.90, that increase becomes abrupt. For standard deviations of 1.0, 1.5 and 2.0 there is no feasible solution for high reliability levels and abrupt changes cannot be observed. For $\sigma=0$, there is no change in the upper limit for any reliability level; thus, results are same with the no probability case for all reliability levels.

Likewise, it can be concluded from Figure 4.20 that total injected mass continuously increase with the increase of standard deviation for the same reliability level. Rapid increases start to occur when the standard deviation gets closer to the value where solution becomes infeasible.

Comparing Figure 4.17 and Figure 4.19, same increase in the reliability level results in much more injection results in the lower limit case for the same standard deviation. For example, for the standard deviation 0.50 and increase of reliability level from 0.97 to 0.98, upper limit application results in no change in injection mass. On the other hand, lower limit application results in 6.429 *kg/day* increase from 84.642 *kg/day* to 91.071 *kg/day*.

Comparing Figure 4.18 and Figure 4.20, range of the standard deviation for the feasible solution is much smaller for the lower limit case. Most of the changes occur within the range of 0.0 and 1.0 in the lower limit case; whereas, there is no such significant range for the upper limit case and small changes occur between 0.0 and 4.0.

4.2.2.1.3 Both of the Limits

Modified maximum/minimum limits and computed total mass injection results can be seen in Table 4.15.

In this case, increment of the standard deviation is taken as 0.10 mostly. For reliability level 0.60 analyses give feasible solutions for standard deviations higher than 4.0; however, it is not reasonable to have these values so analyses are stopped at 4.0. Apart from this, highest analyzed value of the standard deviation is where the solver is unable to find a feasible solution. For example, for reliability level of 0.70, the maximum value of the standard deviation is 2.10. Increment is adjusted at necessary points in order to observe the changes in the total mass injection in a better way. The change of total mass injection versus reliability level for each standard deviation is given in Figure 4.21. Similarly, the change of total mass injection versus standard deviation for each reliability level is shown in Figure 4.22.

Table 4.15: Both of the limits application results with normal probability distribution for Case II

α_{U-L}	U(kg/day)	σ	UL(mg/l)	LL(mg/l)
0.60	14.843	0.00	4.000	0.200
	16.723	0.10	3.975	0.225
	18.603	0.20	3.949	0.251
	20.483	0.30	3.924	0.276
	22.363	0.40	3.899	0.301
	24.244	0.50	3.873	0.327
	33.645	1.00	3.747	0.453
	52.446	2.00	3.493	0.707
	71.260	3.00	3.240	0.960
	90.643	4.00	2.987	1.213
0.70	14.843	0.00	4.000	0.200
	18.735	0.10	3.948	0.252
	22.626	0.20	3.895	0.305
	26.518	0.30	3.843	0.357
	30.410	0.40	3.790	0.410
	34.302	0.50	3.738	0.462
	53.761	1.00	3.476	0.724
	73.236	1.50	3.213	0.987
	95.169	2.00	2.951	1.249
	108.839	2.10	2.899	1.301
0.80	14.843	0.00	4.000	0.200
	21.089	0.10	3.916	0.284
	27.335	0.20	3.832	0.368
	33.581	0.30	3.748	0.452
	39.827	0.40	3.663	0.537
	46.073	0.50	3.579	0.621
	61.688	0.75	3.369	0.831
	77.336	1.00	3.158	1.042
	95.728	1.25	2.948	1.252
	105.457	1.30	2.906	1.294
0.90	14.843	0.00	4.000	0.200
	24.354	0.10	3.872	0.328
	33.865	0.20	3.744	0.456
	43.375	0.30	3.616	0.584
	52.886	0.40	3.487	0.713
	62.397	0.50	3.359	0.841
	86.254	0.75	3.039	1.161
	103.840	0.85	2.911	1.289

α_{U-L}	U (kg/day)	σ	UL(mg/l)	LL(mg/l)
0.95	14.843	0.00	4.000	0.200
	27.050	0.10	3.836	0.364
	39.257	0.20	3.671	0.529
	51.464	0.30	3.507	0.693
	63.671	0.40	3.342	0.858
	75.905	0.50	3.178	1.022
	88.181	0.60	3.013	1.187
	99.020	0.65	2.931	1.269
0.96	14.843	0.00	4.000	0.200
	27.835	0.10	3.825	0.375
	40.828	0.20	3.650	0.550
	53.820	0.30	3.475	0.725
	66.815	0.40	3.300	0.900
	79.849	0.50	3.125	1.075
	95.447	0.60	2.950	1.250
0.97	14.843	0.00	4.000	0.200
	28.801	0.10	3.812	0.388
	42.759	0.20	3.624	0.576
	56.717	0.30	3.436	0.764
	70.685	0.40	3.248	0.952
	84.703	0.50	3.060	1.140
	93.035	0.55	2.966	1.234
0.98	14.843	0.00	4.000	0.200
	30.084	0.10	3.795	0.405
	45.326	0.20	3.589	0.611
	60.568	0.30	3.384	0.816
	75.836	0.40	3.179	1.021
	92.146	0.50	2.973	1.227
0.99	14.843	0.00	4.000	0.200
	32.107	0.10	3.767	0.433
	49.372	0.20	3.535	0.665
	66.638	0.30	3.302	0.898
	83.967	0.40	3.069	1.131
	94.860	0.45	2.953	1.247
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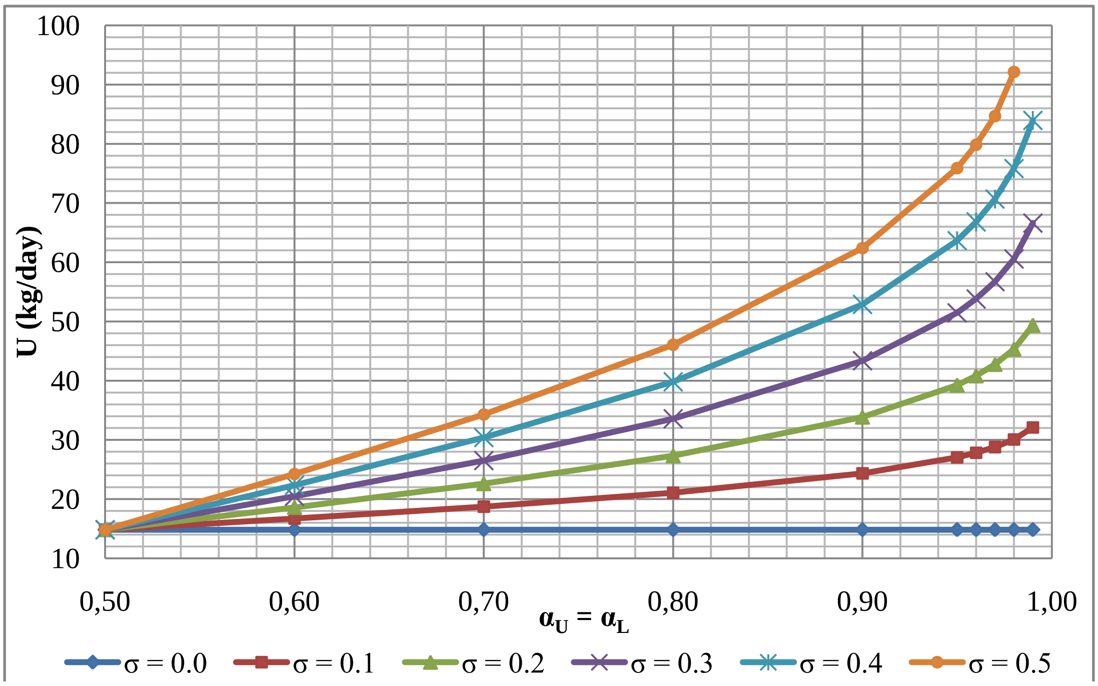

Figure 4.21: Total injected mass versus both limits reliability for normal probability distribution (Case II)

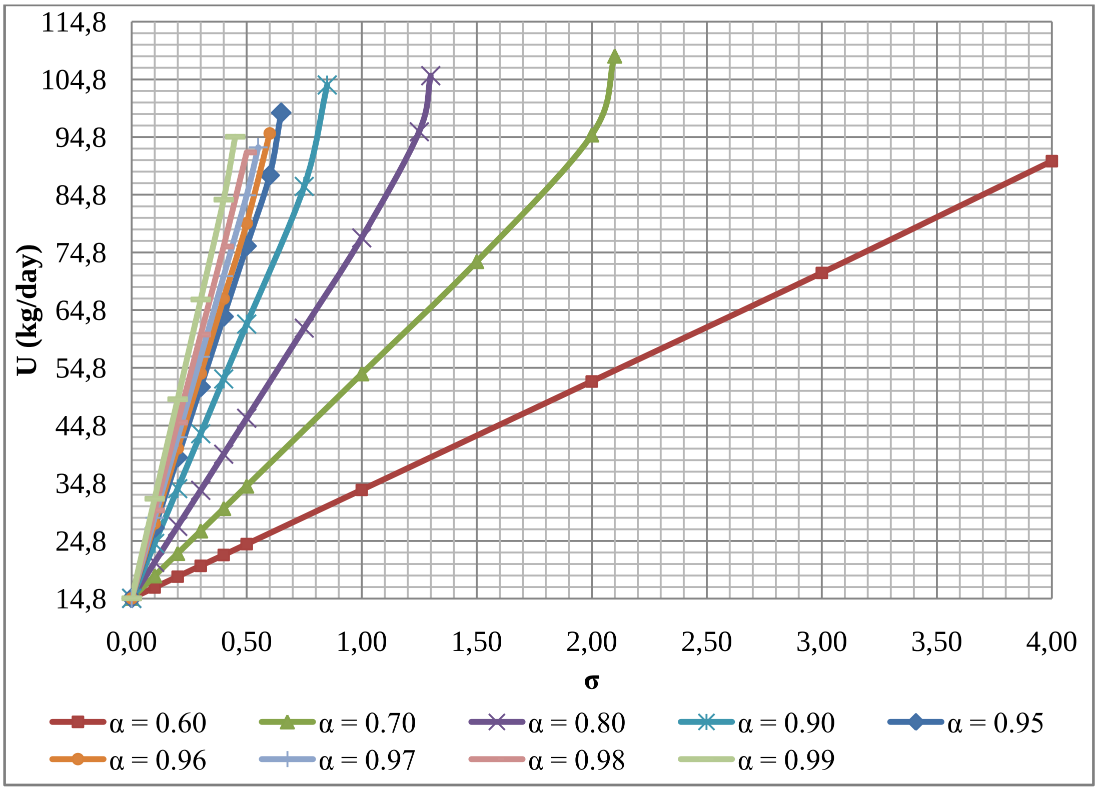

Figure 4.22: Total injected mass versus standard deviation of both of the limits for normal probability distribution (Case II)

Shown in Figure 4.21 that total injected mass continuously increases with the increase of reliability level for the same standard deviation. Especially after reliability level of 0.90, that increase become abrupt. For σ=0, there is no change in the upper limit for any reliability level; thus, results are same with the no probability case for all reliability levels.

Likewise, it can be concluded from Figure 4.22 that total injected mass continuously increase with the increase of standard deviation for the same reliability level. Rapid increases start to occur when the standard deviation gets closer to the value where solution becomes infeasible.

Analyses results of the three case show that lower limit is dominant in the both of the limits case. Comparing Figures 4.17, 4.19 and 4.21, same increase in the reliability level results in highest injection results in the both of the limits case, slightly lower results in lower limit case and lowest results in upper limit case for the same standard deviation. For example, for the standard deviation 0.50 and increase of reliability level from 0.97 to 0.98, upper limit application results in no change in injection mass. On the other hand, lower limit application results in 6.429 *kg/day* increase from 84.642 *kg/day* to 91.071 *kg/day* and both of the limits application results in 7.443 *kg/day* increase from 84.703 *kg/day* to 92.146 *kg/day*.

Comparing Figures 4.18, 4.20 and 4.22, range of the standard deviation for the feasible solution is much smaller for the lower limit and both of the limits cases. Most of the changes occur within the range of 0.0 and 1.0 in the lower limit and both of the limits cases; whereas, there is no such significant range for the upper limit case and small changes occur between 0.0 and 4.0.

4.2.2.2 Log-Normal Distribution

Chance constrained optimization formulation by assuming the normal probability distribution for chlorine concentration is composed of objective function, Equation 2.9, non-negativity constraint, Equation 2.11 and chlorine concentration limitation adjusted according to application to the upper limit, lower limit and both.

For upper limit, lower limit and both of the limits sections, mean values of minimum/maximum concentrations and increment used in the reliability level will be same as explained in sections *4.2.1.2.1*, *4.2.1.2.2* and *4.2.1.2.3*, respectively.

4.2.2.2.1 Upper Limit

Modified maximum limits and computed total mass injection results can be seen in Table 4.16. It is shown in Table 4.16 that increment of the standard deviation is mostly 1.00.For reliability levels ranging from 0.60 to 0.98, analyses give feasible solutions for standard deviations higher than 4.0; however, it is not reasonable to have these values so analyses are stopped at 4.0. Apart from those, highest analyzed value of the standard deviation is where the solver is unable to find a feasible solution. For example, for reliability level of 0.99, the maximum value of the standard deviation is 3.50. The change of total mass injection versus reliability level for each standard deviation is given in Figure 4.23. Similarly, the change of total mass injection versus standard deviation for each reliability level is shown in Figure 4.24.

Table 4.16: Results of the upper limit application with log-normal probability distribution for Case II

α_U	U(kg/day)	σ	UL(mg/l)	LL(mg/l)	α_U	U(kg/day)	σ	UL(mg/l)	LL(mg/l)
0.60	14.843	0.00	4.000	0.200	0.95	14.843	0.00	4.000	0.200
	14.843	1.00	3.646	0.200		14.843	1.00	2.588	0.200
	14.843	2.00	3.174	0.200		14.843	2.00	1.645	0.200
	14.843	3.00	2.702	0.200		14.843	3.00	1.066	0.200
	14.843	4.00	2.291	0.200		14.844	4.00	0.719	0.200
0.70	14.843	0.00	4.000	0.200	0.96	14.843	0.00	4.000	0.200
	14.843	1.00	3.411	0.200		14.843	1.00	2.522	0.200
	14.843	2.00	2.793	0.200		14.843	2.00	1.565	0.200
	14.843	3.00	2.254	0.200		14.843	3.00	0.994	0.200
	14.843	4.00	1.828	0.200		14.846	4.00	0.658	0.200
0.80	14.843	0.00	4.000	0.200	0.97	14.843	0.00	4.000	0.200
	14.843	1.00	3.154	0.200		14.843	1.00	2.442	0.200
	14.843	2.00	2.404	0.200		14.843	2.00	1.471	0.200
	14.843	3.00	1.824	0.200		14.843	3.00	0.911	0.200
	14.843	4.00	1.404	0.200		14.850	4.00	0.591	0.200
0.90	14.843	0.00	4.000	0.200	0.98	14.843	0.00	4.000	0.200
	14.843	1.00	2.830	0.200		14.843	1.00	2.340	0.200
	14.843	2.00	1.953	0.200		14.843	2.00	1.356	0.200
	14.843	3.00	1.359	0.200		14.843	3.00	0.812	0.200
	14.843	4.00	0.973	0.200		14.858	4.00	0.512	0.200
-	-	-	-	-	0.99	14.843	0.00	4.000	0.200
						14.843	1.00	2.188	0.200
						14.843	2.00	1.192	0.200
						14.845	3.00	0.676	0.200
						14.857	3.50	0.521	0.200

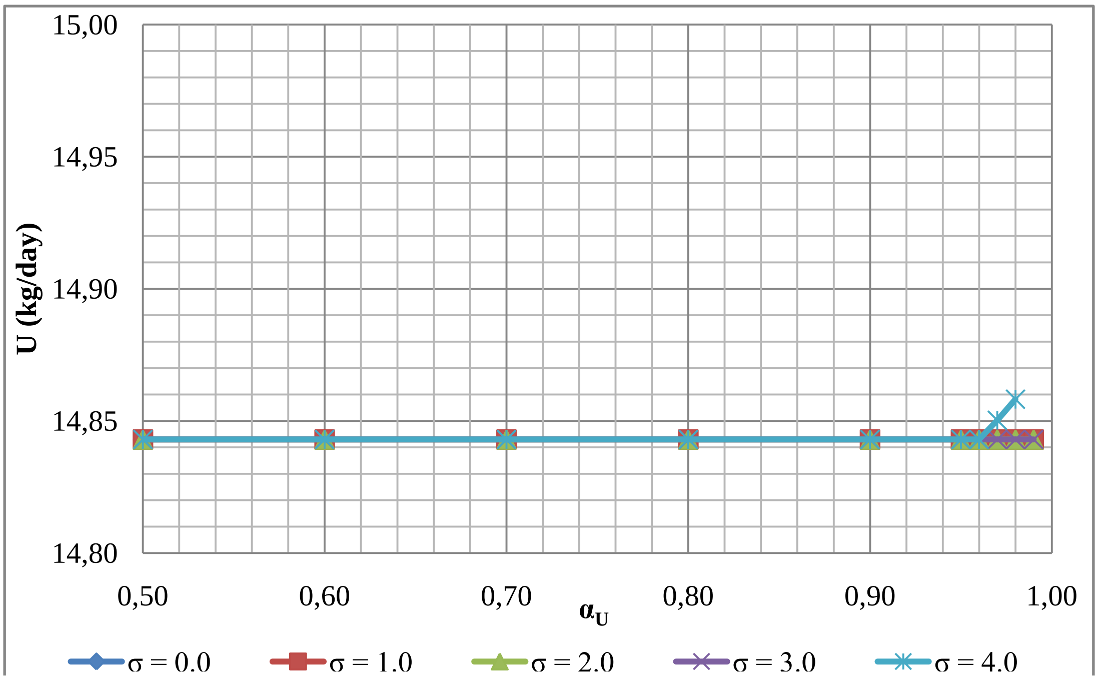

Figure 4.23: Total injected mass versus upper limit reliability for log-normal probability distribution (Case II)

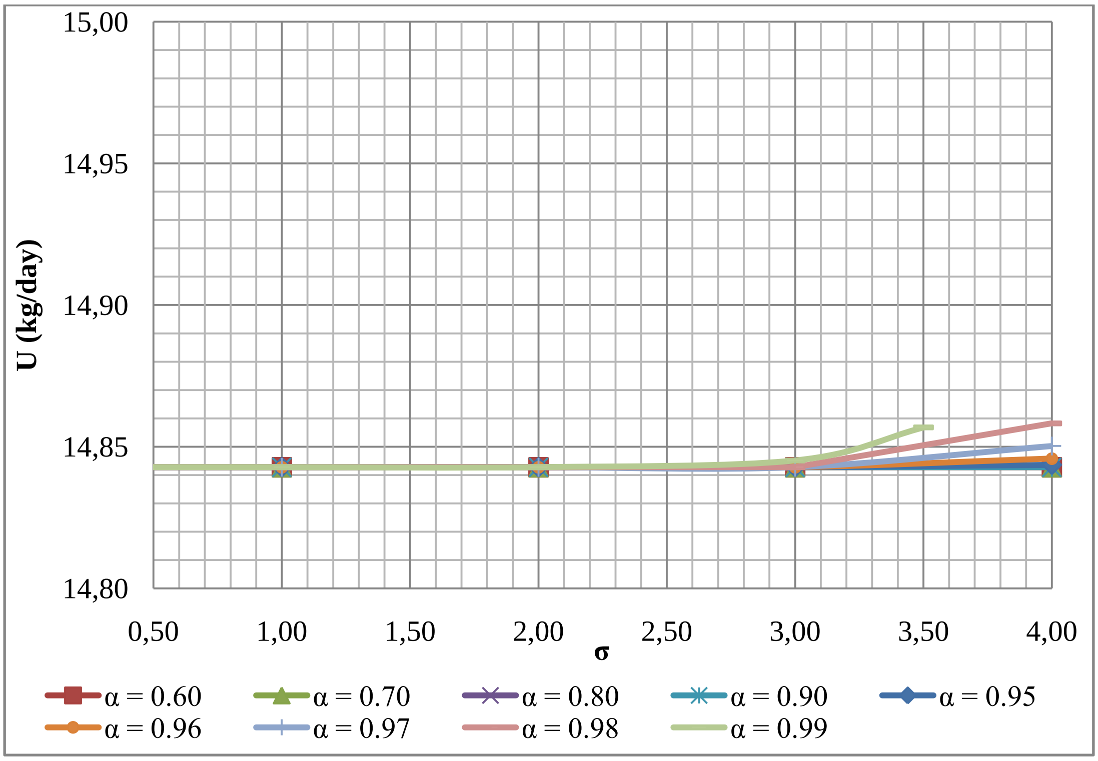

Figure 4.24: Total injected mass versus standard deviation of upper limit for log-normal probability distribution (Case II)

As the distribution of the data is on a much more narrow limits in the use of three booster stations (Figure 4.16), changing the reliability level and the standard deviation of the upper limit does not affect the analyses results of non-probability case in considerable amounts.

Shown in Figure 4.23 that total injected mass slightly increases with the increase of reliability level for the same standard deviation. Only detectable change occurs for standard deviation 4.0 and it is only 0.012 *mg/l* from 14.844 *mg/l* to 14.858 *mg/l*. For σ=0, there is no change in the upper limit for any reliability level; thus, results are same with the no probability case for all reliability levels.

Similarly, it can be concluded from Figure 4.24 that total injected mass slightly increase with the increase of standard deviation for the same reliability level. Detectable increases start to occur when the standard deviation gets closer to the value where solution becomes infeasible.

4.2.2.2.2 Lower Limit

Modified minimum limits and computed total mass injection results can be seen in Table 4.17.

In this case, increment of the standard deviation is 0.01 up to 0.05 and after that value increment is taken as 0.10, which can be seen in Table 4.17. Highest analyzed value of the standard deviation is where the solver is unable to find a feasible solution. For example, for reliability level of 0.97, the maximum value of the standard deviation is 1.1. Increment is adjusted at necessary points in order to observe the changes in the total mass injection in a better way. For showing the effect of standard deviation on total mass injection, only the reliability level up to 90% is used. This is because standard deviations between 0.01-0.05 are so small to create detectable changes for reliability level from 95% to 99%. The change of total mass injection versus reliability level for each standard deviation is given in Figure 4.25. Similarly, the change of total mass injection versus standard deviation for each reliability level is shown in Figure 4.26.

Table 4.17: Results of the lower limit application with log-normal probability distribution for Case II

α_L	U(kg/day)	σ	UL(mg/l)	LL(mg/l)	α_L	U(kg/day)	σ	UL(mg/l)	LL(mg/l)
0.60	14.843	0.000	4.000	0.200	0.96	14.843	0.00	4.000	0.200
	15.013	0.010	4.000	0.202		30.354	0.10	4.000	0.409
	15.147	0.020	4.000	0.204		45.082	0.20	4.000	0.607
	15.244	0.030	4.000	0.205		55.083	0.30	4.000	0.742
	15.303	0.040	4.000	0.206		61.177	0.40	4.000	0.824
	15.326	0.050	4.000	0.207		64.785	0.50	4.000	0.873
0.70	14.843	0.000	4.000	0.200		66.871	0.60	4.000	0.901
	15.218	0.010	4.000	0.205		68.014	0.70	4.000	0.916
	15.562	0.020	4.000	0.210		68.558	0.80	4.000	0.924
	15.873	0.030	4.000	0.214		68.714	0.90	4.000	0.926
	16.147	0.040	4.000	0.218	0.97	14.843	0.00	4.000	0.200
	16.384	0.050	4.000	0.221		32.278	0.10	4.000	0.435
	16.809	0.075	4.000	0.227		50.240	0.20	4.000	0.677
	17.008	0.100	4.000	0.229		63.439	0.30	4.000	0.855
0.80	14.843	0.000	4.000	0.200		72.156	0.40	4.000	0.972
	15.461	0.010	4.000	0.208		77.804	0.50	4.000	1.048
	16.063	0.020	4.000	0.216		81.469	0.60	4.000	1.098
	16.642	0.030	4.000	0.224		83.842	0.70	4.000	1.130
	17.194	0.040	4.000	0.232		85.353	0.80	4.000	1.150
	17.715	0.050	4.000	0.239		86.276	0.90	4.000	1.162
	19.757	0.100	4.000	0.266		86.789	1.00	4.000	1.169
	20.835	0.150	4.000	0.281		87.010	1.10	4.000	1.172
	21.150	0.200	4.000	0.285	0.98	14.843	0.00	4.000	0.200
0.90	14.843	0.000	4.000	0.200		35.026	0.10	4.000	0.472
	15.805	0.010	4.000	0.213		58.021	0.20	4.000	0.782
	16.783	0.020	4.000	0.226		76.543	0.30	4.000	1.031
	17.771	0.030	4.000	0.239		89.878	0.40	4.000	1.211
	18.760	0.040	4.000	0.253		99.295	0.50	4.000	1.337
	19.742	0.050	4.000	0.266		105.984	0.60	4.000	1.427
	24.321	0.100	4.000	0.328		110.798	0.70	4.000	1.492
	30.505	0.200	4.000	0.411		114.293	0.80	4.000	1.539
	33.099	0.300	4.000	0.446		116.846	0.90	4.000	1.573
	33.099	0.400	4.000	0.455		118.810	1.00	4.000	1.598
0.95	14.843	0.000	4.000	0.200		120.516	1.10	4.000	1.616
	28.874	0.100	4.000	0.389		123.680	1.60	4.000	1.648
	41.280	0.200	4.000	0.556	0.99	14.843	0.00	4.000	0.200
	49.104	0.300	4.000	0.662		39.840	0.10	4.000	0.537
	53.491	0.400	4.000	0.721		72.802	0.20	4.000	0.981
	55.819	0.500	4.000	0.752		102.964	0.30	4.000	1.387
	56.950	0.600	4.000	0.767		131.685	0.40	4.000	1.711
	57.374	0.700	4.000	0.773	-	-	-	-	-

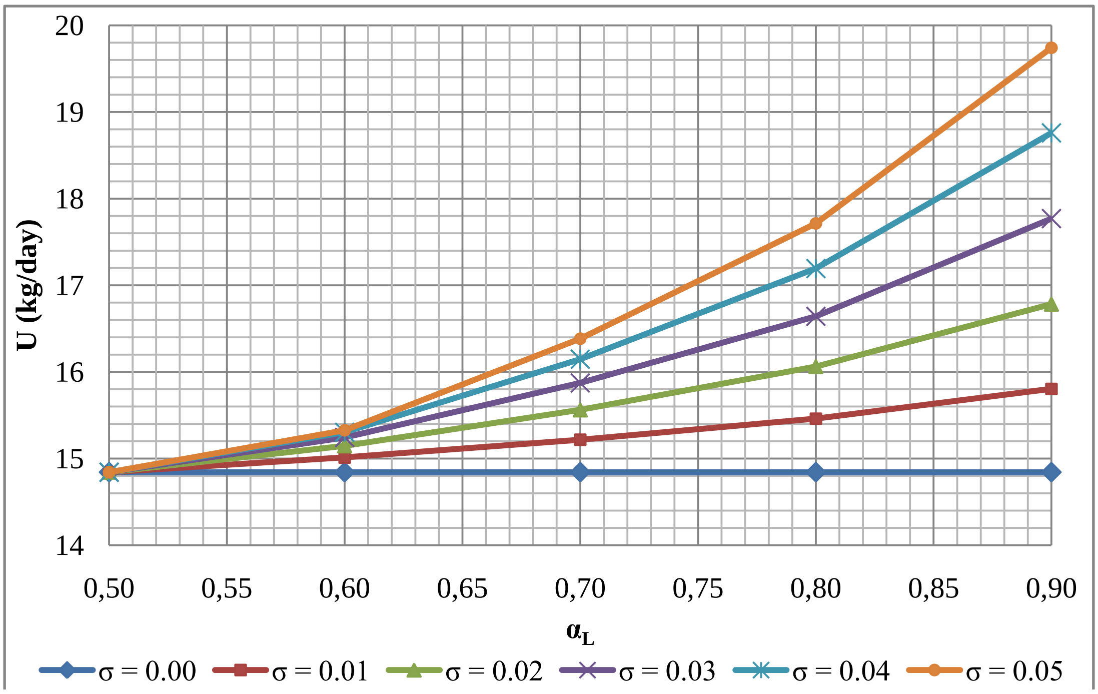

Figure 4.25: Total injected mass versus lower limit reliability for log-normal probability distribution (Case II)

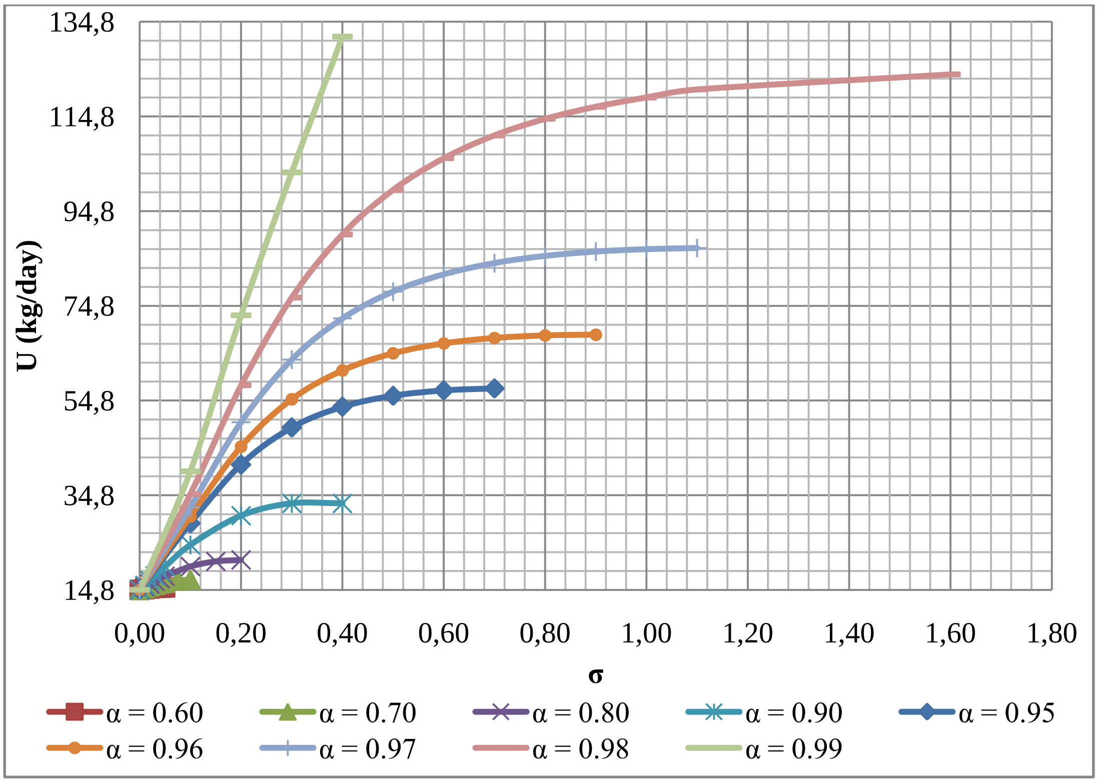

Figure 4.26: Total injected mass versus standard deviation of lower limit for log-normal probability distribution (Case II)

Shown in Figure 4.25 that total injected mass continuously increases with the increase of reliability level for the same standard deviation. As comparison is done up to the standard deviation value of 0.05, infeasible solution limit is not reached for any reliability level. For σ=0, there is no change in the upper limit for any reliability level; thus, results are same with the no probability case for all reliability levels.

Similarly, it can be concluded from Figure 4.26 that total injected mass increase with the increase of standard deviation for the same reliability level. When the standard deviation gets closer to the value where solution become infeasible, total injection start to asymptotically reach its limiting value.

Comparing Figure 4.23 and Figure 4.25, same increase in the reliability level results in much more injection results in the lower limit case for the same standard deviation. For example, For example, for the standard deviation 0.50 and increase of reliability level from 0.97 to 0.98, upper limit application results in no change in injection mass. On the other hand, lower limit application results in 21.491 *kg/day* increase from 77.804 *kg/day* to 99.295 *kg/day*.

Comparing Figure 4.24 and Figure 4.26, most important difference is the attitudes of the results. For the upper limit case, total injected mass slightly increase with the increase of standard deviation for the same reliability level while total injection start to asymptotically reach its limiting value for the lower limit case.

4.2.2.2.3 Both of the Limits

Modified maximum/minimum limits and computed total mass injection results can be seen in Table 4.18.

For this case, general increment of the standard deviation is 0.01 up to 0.05 and after that value increment is taken as 0.10, which can be seen in Table 4.17. Highest analyzed value of the standard deviation is where the solver is unable to find a feasible solution. For example, for reliability level of 0.80, the maximum value of the standard deviation is 0.20. Increment is adjusted at necessary points in order to observe the changes in the total mass injection in a better way. For showing the effect of standard deviation on total mass injection, only the reliability level up to 90% is used. This is because standard deviations between 0.01-0.05 are so small to create detectable changes for reliability level from 95% to 99%. The change of total mass injection versus reliability level for each standard deviation is given in Figure 4.27. Similarly, the change of total mass injection versus standard deviation for each reliability level is shown in Figure 4.28.

Table 4.18: Both of the limits application results with log-normal probability distribution for Case II

α_{U-L}	U(kg/day)	σ	UL(mg/l)	LL(mg/l)	α_{U-L}	U(kg/day)	σ	UL(mg/l)	LL(mg/l)
0.60	14.843	0.00	4.000	0.200	0.95	14.843	0.00	4.000	0.200
	15.013	0.01	3.997	0.202		28.874	0.10	3.838	0.389
	15.147	0.02	3.995	0.204		41.280	0.20	3.680	0.556
	15.244	0.03	3.992	0.205		49.104	0.30	3.526	0.662
	15.303	0.04	3.990	0.206		53.491	0.40	3.378	0.721
	15.326	0.05	3.987	0.207		55.819	0.50	3.234	0.752
0.70	14.843	0.00	4.000	0.200		56.950	0.60	3.095	0.767
	15.218	0.01	3.995	0.205		57.374	0.70	2.961	0.773
	15.562	0.02	3.989	0.210		57.413	0.74	2.909	0.774
	15.873	0.03	3.984	0.214	0.96	14.843	0.00	4.000	0.200
	16.147	0.04	3.979	0.218		30.354	0.10	3.828	0.409
	16.384	0.05	3.974	0.221		45.082	0.20	3.660	0.607
	16.743	0.07	3.963	0.226		55.083	0.30	3.499	0.742
	16.954	0.09	3.952	0.228		61.177	0.40	3.342	0.824
	17.008	0.10	3.947	0.229		64.787	0.50	3.192	0.873
0.80	14.843	0.00	4.000	0.200		66.881	0.60	3.047	0.901
	15.461	0.01	3.992	0.208		68.034	0.70	2.907	0.916
	16.063	0.02	3.983	0.216		68.590	0.80	2.773	0.924
	16.642	0.03	3.975	0.224		68.756	0.90	2.645	0.926
	17.194	0.04	3.966	0.232	0.97	14.843	0.00	4.000	0.200
	17.715	0.05	3.958	0.239		32.278	0.10	3.815	0.435
	19.757	0.10	3.916	0.266		50.240	0.20	3.637	0.677
	20.835	0.15	3.873	0.281		63.439	0.30	3.465	0.855
	21.150	0.20	3.830	0.285		72.168	0.40	3.299	0.972
0.90	14.843	0.00	4.000	0.200		77.840	0.50	3.140	1.048
	15.805	0.01	3.987	0.213		81.529	0.60	2.988	1.098
	16.783	0.02	3.974	0.226		83.937	0.70	2.842	1.130
	17.771	0.03	3.962	0.239		88.073	0.80	2.703	1.150
	18.760	0.04	3.949	0.253		91.896	0.85	2.635	1.157
	19.742	0.05	3.936	0.266	0.98	14.843	0.00	4.000	0.200
	24.321	0.10	3.873	0.328		35.026	0.10	3.799	0.472
	30.505	0.20	3.747	0.411		58.021	0.20	3.605	0.782
	33.099	0.30	3.624	0.446		76.558	0.30	3.420	1.031
	33.738	0.40	3.503	0.455		89.935	0.40	3.243	1.211
						104.932	0.50	3.074	1.337
						111.455	0.52	3.041	1.358
					0.99	14.843	0.00	4.000	0.200
-	-	-	-	-		39.840	0.10	3.773	0.537
						72.806	0.20	3.557	0.981
						104.253	0.30	3.351	1.387
						116.270	0.32	3.311	1.458

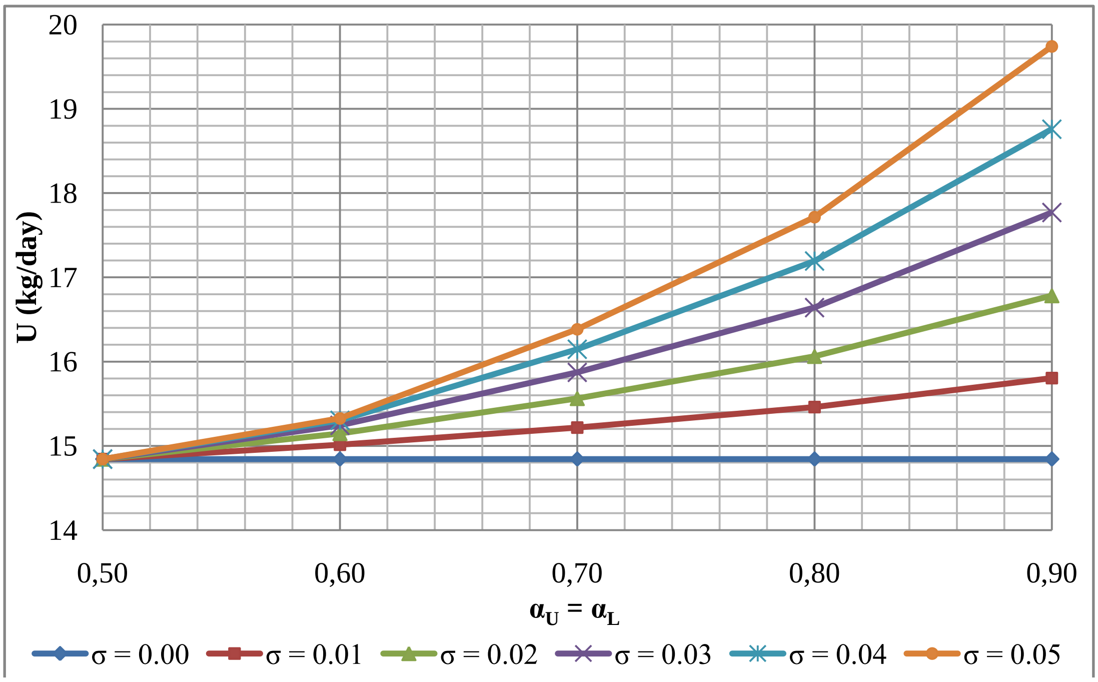

Figure 4.27: Total injected mass versus both limits reliability for log-normal probability distribution (Case II)

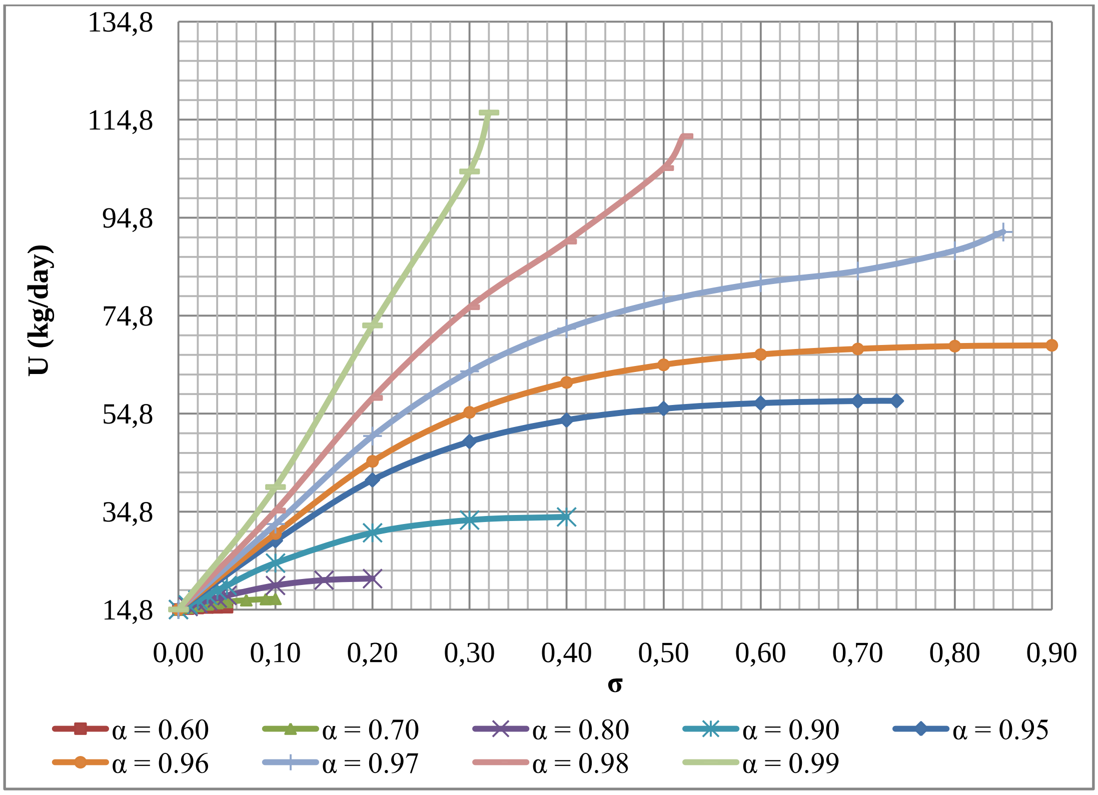

Figure 4.28: Total injected mass versus standard deviation of both of the limits for log-normal probability distribution (Case II)

Shown in Figure 4.27 that total injected mass continuously increases with the increase of reliability level for the same standard deviation. As comparison is done up to the standard deviation value of 0.05, infeasible solution limit is not reached for any reliability level. For $\sigma=0$, there is no change in the upper limit for any reliability level; thus, results are same with the no probability case for all reliability levels.

Likewise, it can be concluded from Figure 4.28 that total injected mass increase with the increase of standard deviation for the same reliability level. When the standard deviation gets closer to the value where solution become infeasible, total injection start to asymptotically reach its limiting value.

Analyses results of the three case show that lower limit is dominant in the both of the limits case. Comparing Figure 4.23, 4.25 and 4.27, same increase in the reliability level results in highest injection results in the both of the limits case, slightly lower results in lower limit case and lowest results in upper limit case for the same standard deviation. For example, for the standard deviation 0.50 and increase of reliability level from 0.97 to 0.98, upper limit application results in no change in injection mass. On the other hand, lower limit application results in 21.491 *kg/day* increase from 77.804 *kg/day* to 99.295 *kg/day* and both of the limits application result in 27.092 *kg/day* increase from 77.840 *kg/day* to 104.932 *kg/day*.

Comparing Figure 4.24, 4.26 and 4.28, most important difference is the attitudes of the results. For the upper limit case, total injected mass continuously increase with the increase of standard deviation for the same reliability level while total injection start to asymptotically reach its limiting value for the lower limit and both of the limits cases.

4.2.3 Discussions

In this part, effects of the chance constraint application on the frequency of the chlorine concentration, feasibility limit of standard deviation and uniformity of chlorine distribution are given. In addition to these, verification of the results by using EPANET will be done.

4.2.3.1 Frequency of the Concentrations

In order to see the effect of changing reliability level and standard deviation on the distribution of the data, frequency plot of chlorine concentrations at all consumer nodes for different type of distributions and side of applications are given below. In order to show the response, comparing application results of Case I is chosen. For comparison, reliability levels of 0.90, 0.70 and non-probability case are taken mostly. At necessary points 0.60 is also shown on the plots to observe the changes in a better way. For each reliability level, limit values of standard deviations are presented on the figures.

Figures 4.29 and 4.30 illustrate the upper limit application of the chance constraint for normal and log-normal probability distributions, respectively. It can be seen from both of the figures that increasing reliability level of upper limit at low standard deviations is not affecting the frequency of the concentrations in great extent. As expected, effect can be noticeable at high standard deviations and that effect is increasing reliability results in narrowing down of the interval that data is distributed. For example for standard deviation 2.0 at normal distribution, data is distributed between 0.2 *mg/l* and 1.4 *mg/l* for reliability level of 0.90; whereas distribution is between 0.2 *mg/l* and 3.0 *mg/l* for reliability level of 0.70. For comparison between the two different types of distributions, log-normal distribution results in narrower interval.

Figures 4.31 and 4.32 demonstrate the lower limit application of the chance constraint for normal and log-normal probability distributions, respectively. It can be concluded from both of the figures that changes in standard deviation is more dominant for lower limit application comparing with upper limit application. For log-normal distribution similar changes occur with much smaller standard deviation values compared with the normal distribution.

Figures 4.33 and 4.34 show both of the limits application of the chance constraint for normal and log-normal probability distributions, respectively. It can be understood from both of the figures that both of the limits application narrows down the interval from both of the sides. If these figures are compared with the Figures 4.29, 4.30, 4.31 and 4.32 it can be said that lower limit is more effective in both of the limits application.

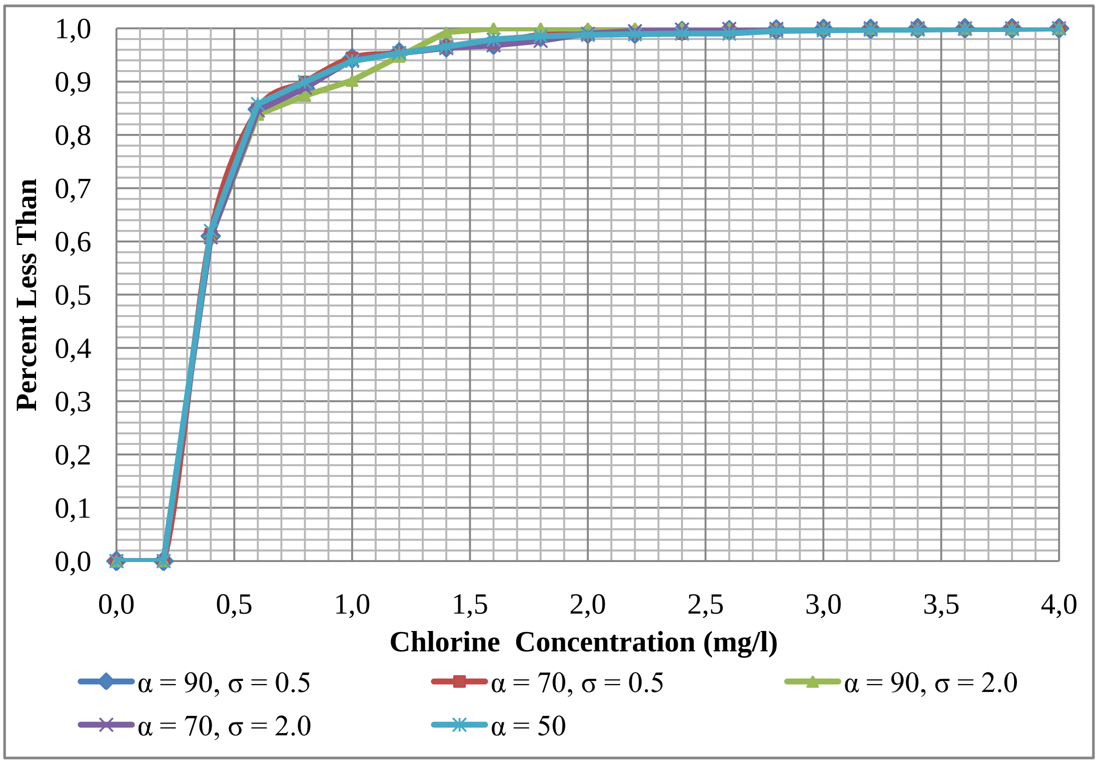

Figure 4.29: Frequency plot of normally distributed upper limit application

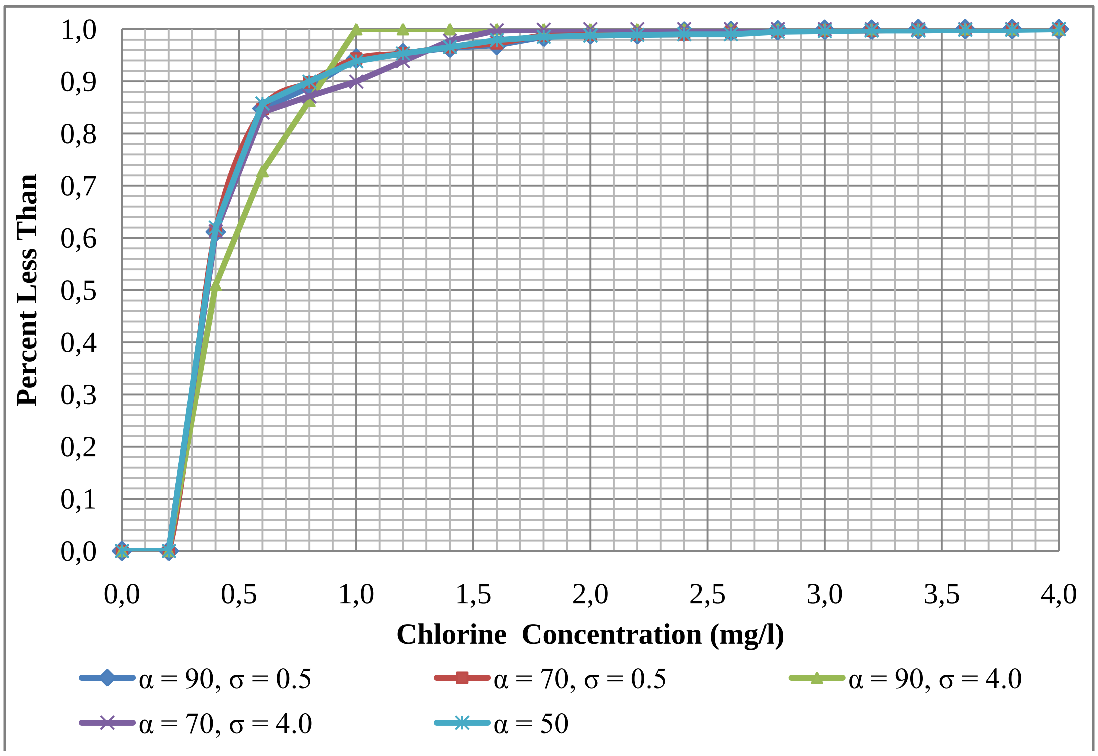

Figure 4.30: Frequency plot of log-normally distributed upper limit application

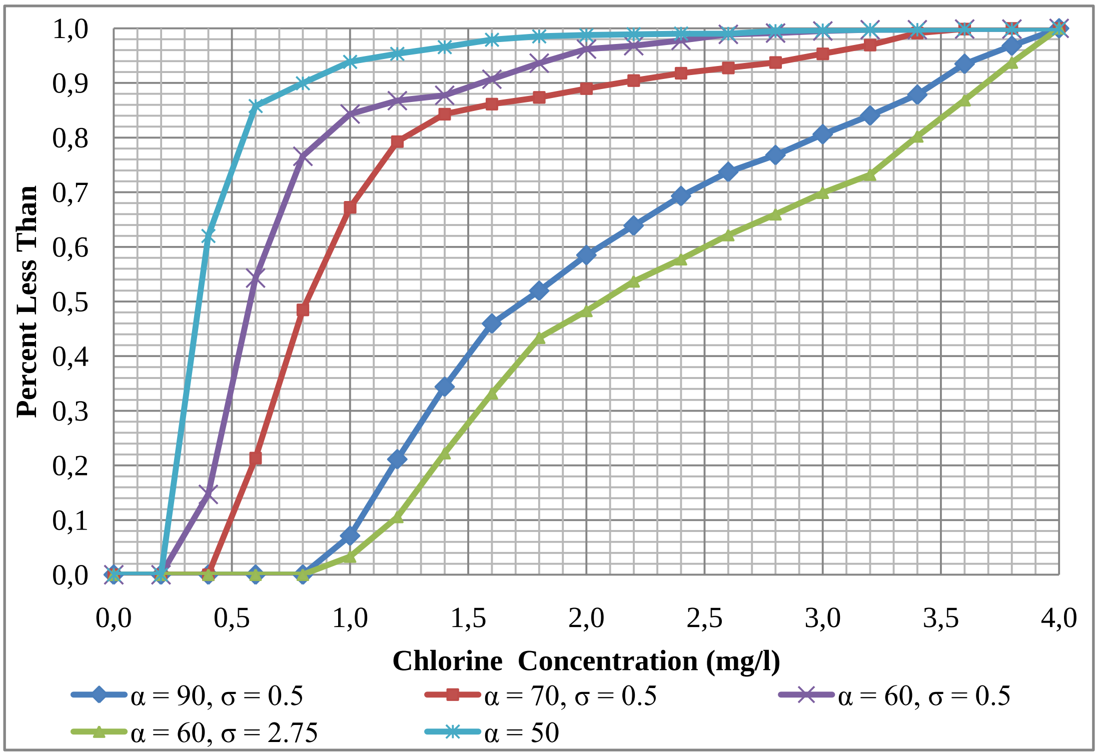

Figure 4.31: Frequency plot of normally distributed lower limit application

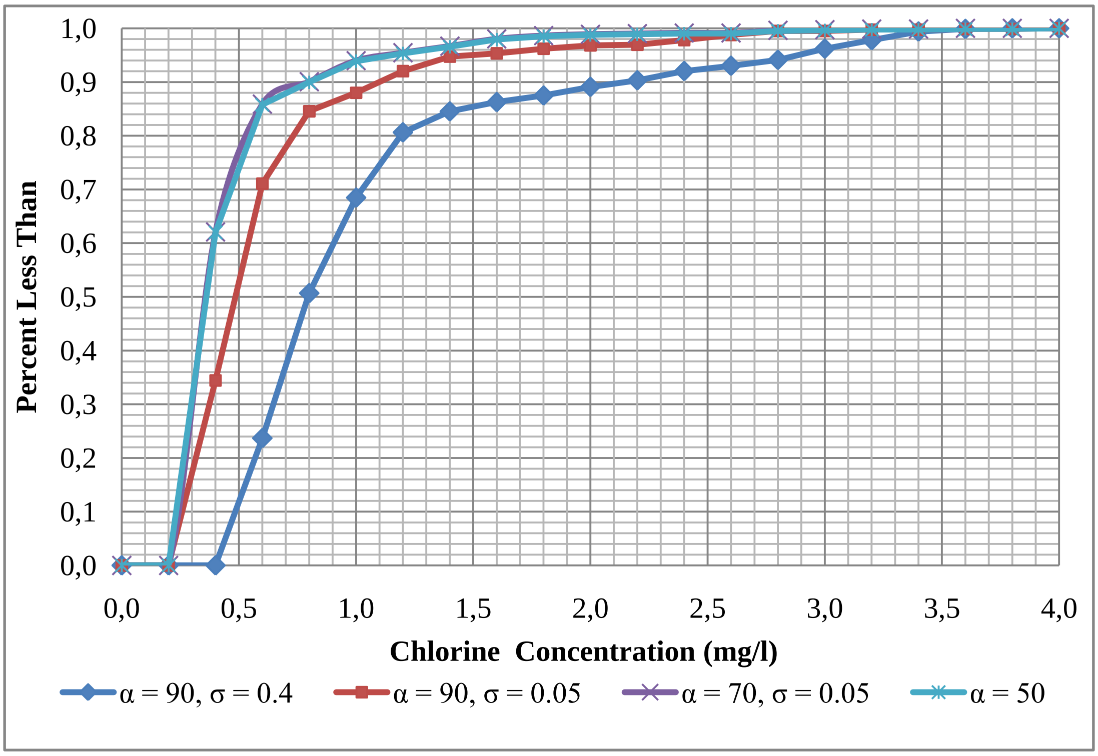

Figure 4.32: Frequency plot of log-normally distributed lower limit application

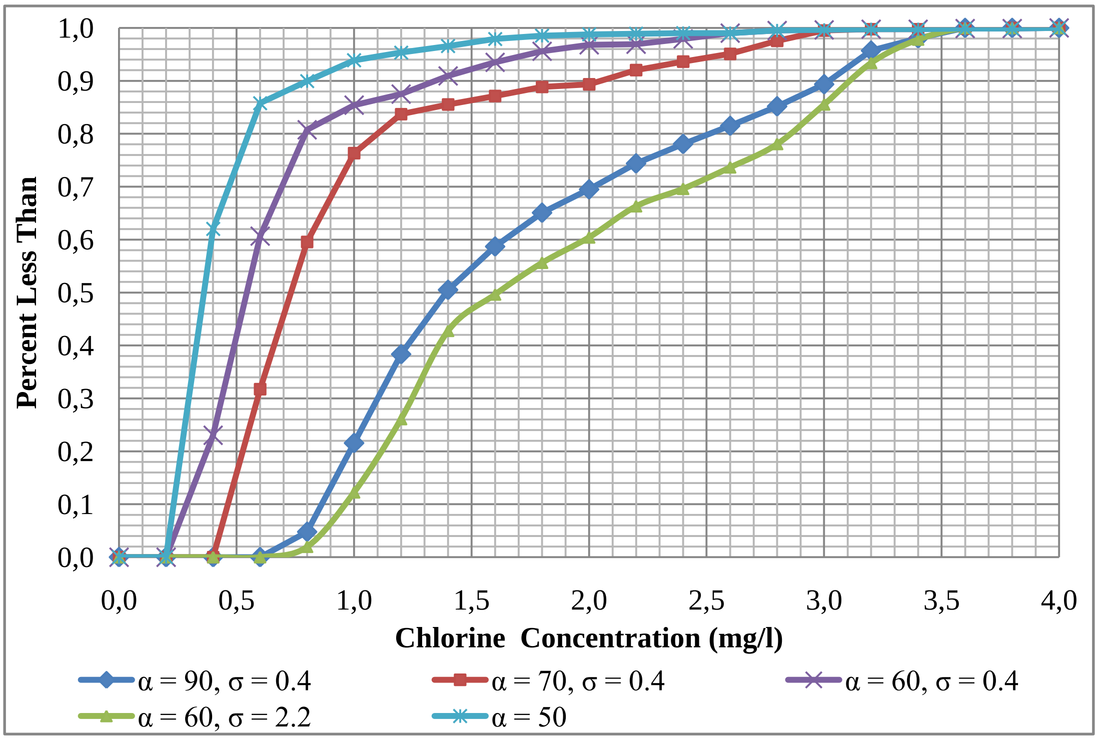

Figure 4.33: Frequency plot of normally distributed both of the limits application

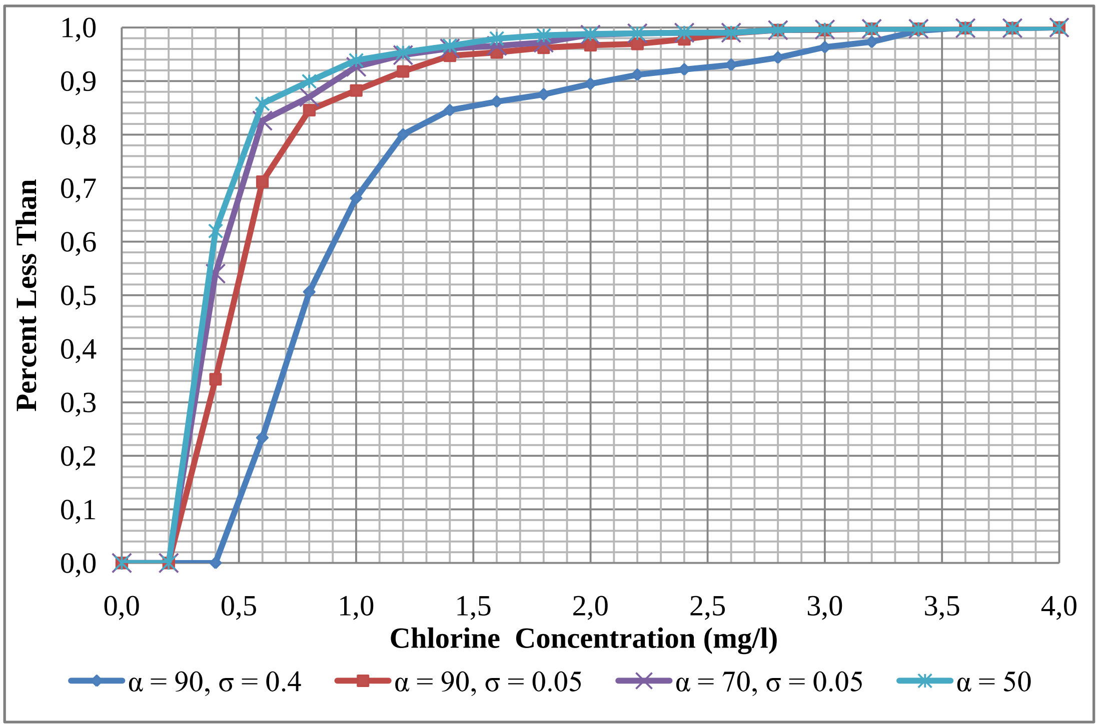

Figure 4.34: Frequency plot of log-normally distributed both of the limits application

4.2.3.2 Feasibility Limit of Standard Deviations

The effect of changing reliability level on the feasibility limit of the standard deviations for different distribution types and cases are shown in the figures below. Feasibility limit of the standard deviation means the maximum standard deviation value which gives the feasible solution for the corresponding reliability level. Changes of limiting values versus reliability level for both normal and log-normal distribution and for three types of application sides are given in Figures 4.35 and 4.36 for Case I and Case II, respectively. Note that, N refers to normal distribution and LN refers to log-normal distribution in these figures.

For normal distribution, it can be concluded from the figures that increasing the reliability level of the network results in a decrease in the highest standard deviation value which gives feasible solution. For upper limit results in both of the figures, there exist standard deviations of 4.0. These nodes are not limiting values of standard deviations but the values at which the analyses are stopped.

For log-normal distribution, it can be concluded from the figures that increasing the reliability level of the network results in an increase in the highest standard deviation value which gives feasible solution up to a point. However, for high reliability levels, this behavior changes and highest standard deviation value which gives feasible solution starts to decrease with the increase of reliability level. For example, for the lower limit application of Case I (Figure 4.35), limiting standard deviation value is 0.75 for reliability level of 0.95 and after that reliability level it start to decrease. This is due to the formulation of log-normal distribution as it is explained in section *4.2.1.2.2*.

For upper limit case in log-normal distribution, same standard deviation results in lower changes in the limits compared with normal distribution. Thus, higher limiting standard deviations can be obtained in the analyses of log-normal distribution which results in lower cost for the same standard deviations of different type of distributions.

For both of the cases and distribution types, it can be seen from the figures that lower limit is more effective in both of the limits application and standard deviation is more critical for lower limit and both of the limits application.

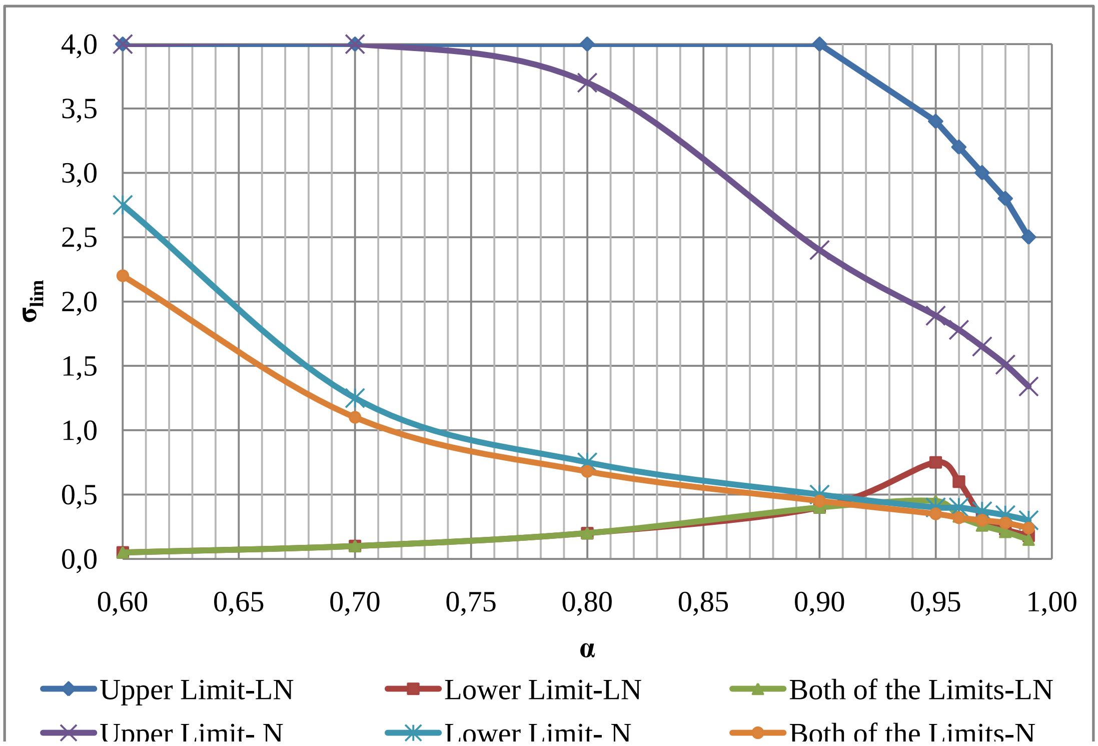

Figure 4.35: Feasibility limits of standard deviation values for Case I

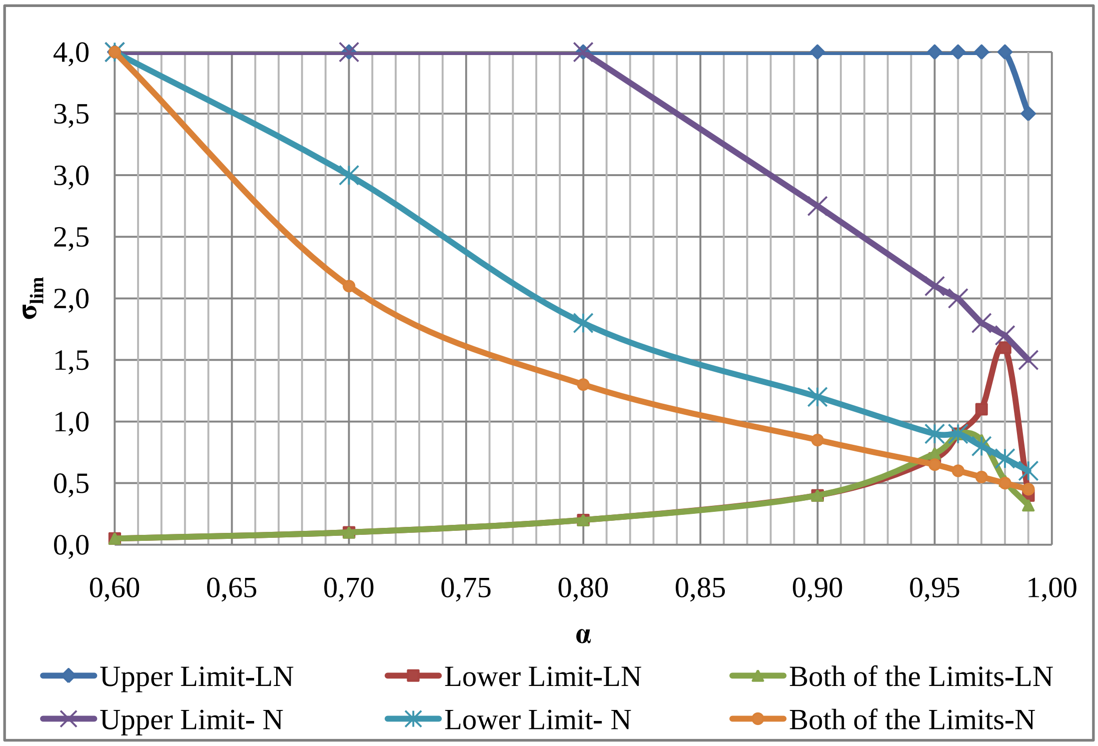

Figure 4.36: Feasibility limits of standard deviation values for Case II

Comparing Case I and Case II, amount of standard deviation is much more critical in lower limit case than the upper limit case in Case II as the gap is narrower in which the concentration data is lying (Figure 4.16).

4.2.3.3 Uniformity of Chlorine Distribution

In order to see the effect of changing reliability level and standard deviation on the uniformity of chlorine distribution, 24 *h* analysis of chlorine concentrations for consumer nodes are checked for different type of distributions and side of applications. In order to show the response results Case I is chosen. Two end nodes of the network are selected to show the effect which are Nodes 34 and 36. Figures 4.37 and 4.38 gives the analyses results of Node 34 for normal and log-normal distributions, respectively. Similarly, Figures 4.39 and 4.40 gives the analyses results of Node 36 for normal and log-normal distributions, respectively.

For normal distribution, which can be seen from Figures 4.37 and 4.39, increasing the reliability level of upper limit results in increase of the uniformity of chlorine distribution of consumer node throughout its 24 *h* period. Moreover, for lower and both of the limits applications higher concentration values are obtained as the lower limit increases. Uniformity is increasing as the difference between the peak and the lowest value is decreasing with the increase in reliability level. For example for normal distribution of Node 34 (Figure 4.37), the maximum difference is approximately 1.5 *mg/l* for reliability level of 0.50 and 1.0 *mg/l* for reliability level of 0.90.

For log-normal distribution, which can be seen from Figures 4.38 and 4.40, reaction is similar with the normal distribution, increasing the reliability level of all applications result in increase of the uniformity of chlorine distribution of consumer nodes. Although there are peaks for lower and both of the limits applications, these peaks are smaller than the non-probability case.

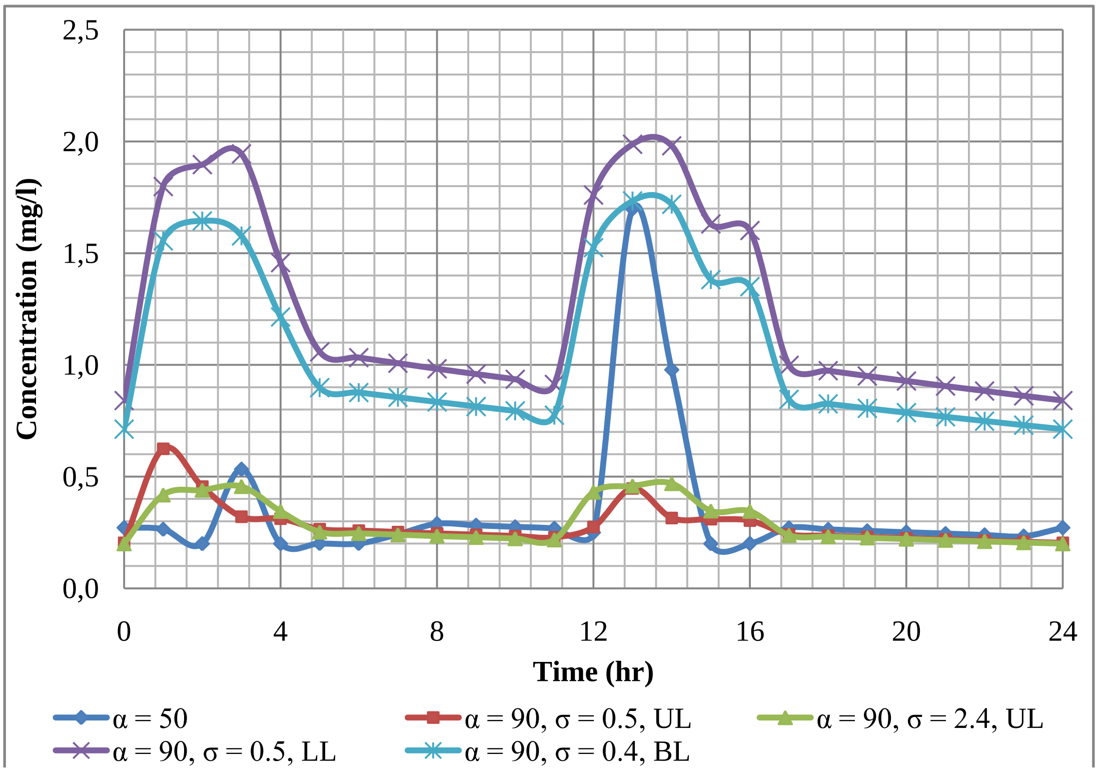

Figure 4.37: 24 *h* analyses results of Node 34 for normal distribution

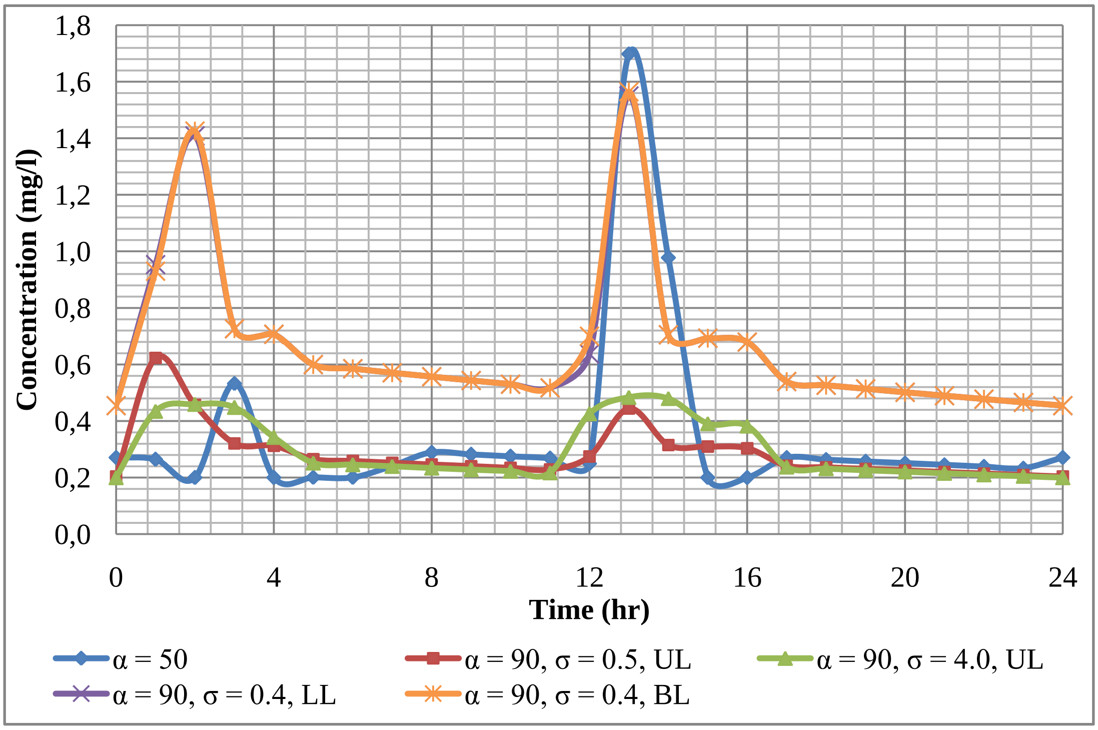

Figure 4.38: 24 *h* analyses results of Node 34 for log-normal distribution

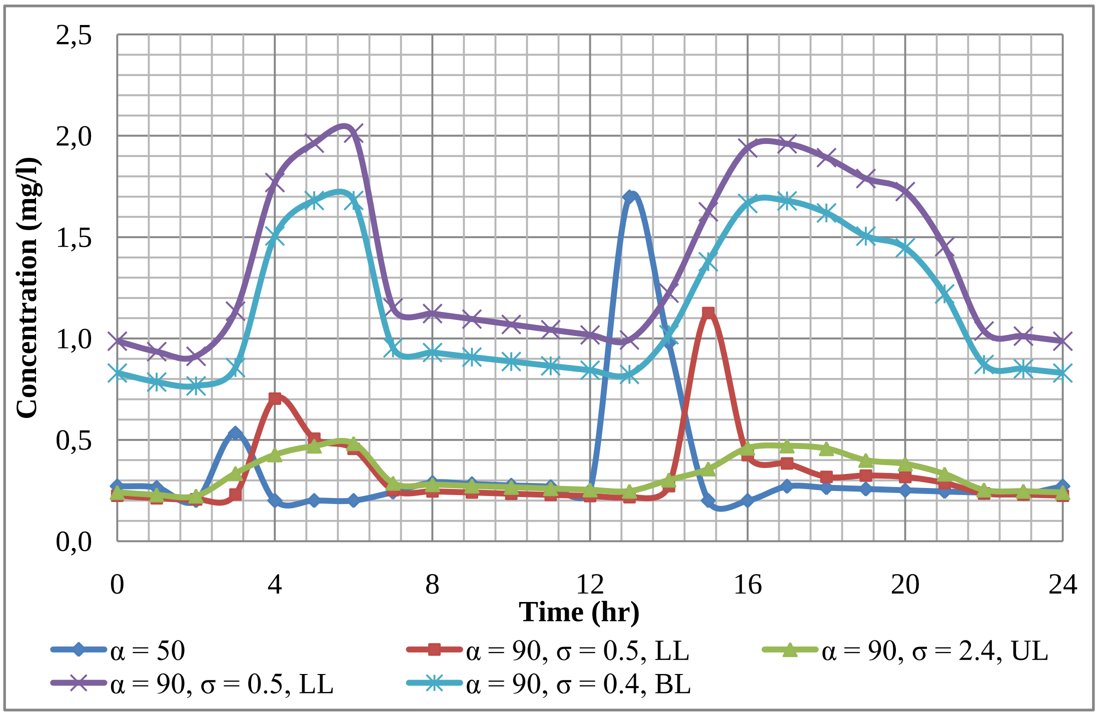

Figure 4.39: 24 *h* analyses results of Node 36 for normal distribution

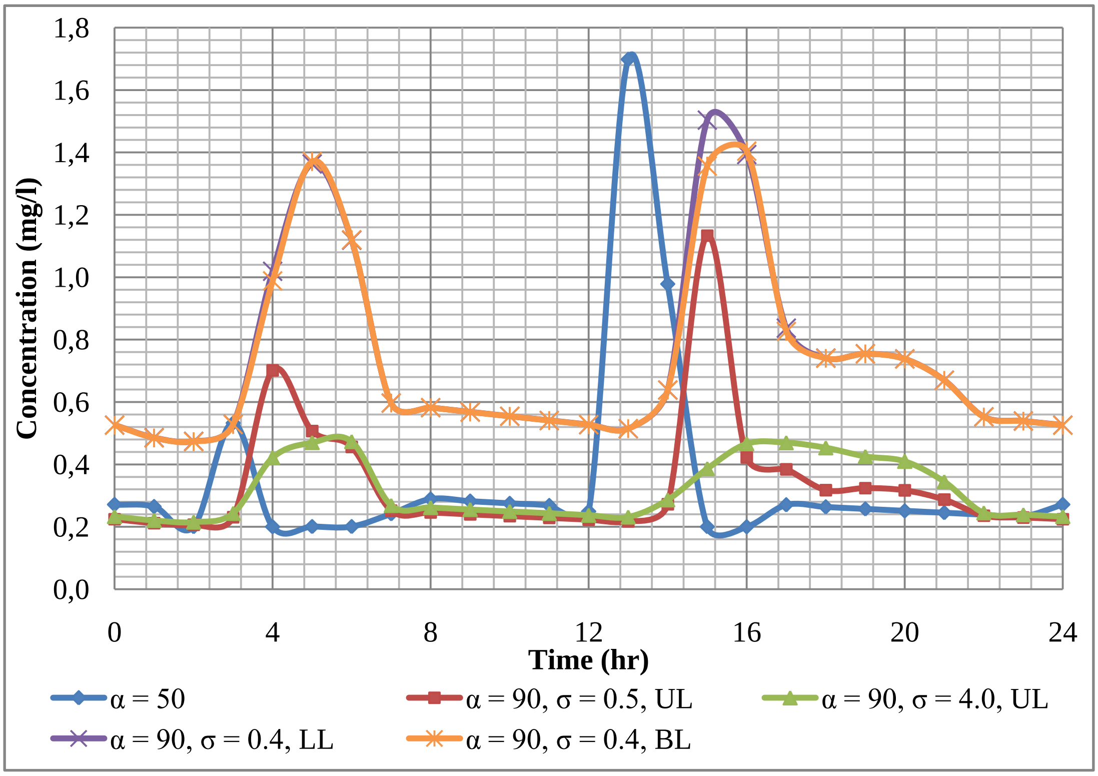

Figure 4.40: 24 *h* analyses results of Node 36 for log-normal distribution

4.2.3.4 EPANET Verification

In the analyses, response coefficient matrix is calculated by the program developed by Sert *(2009)*. The code of the program uses the water quality of the EPANET externally and give the response coefficient values up to 20 digits. When the system is solved by EPANET directly it gives results with 2 digits. Thus, there occurs difference between the results of analyses which are done by program output and direct EPANET results. It should be noted that, system is solved for 960 *h* and the results of last 24 *h* are used for both developed program and solving with EPANET manually. In order to see the difference, three sample nodes are chosen as Node 2 as being near the source node, Node 15 as being in the middle of the network and Node 36 as being at the end of the network. For these nodes, booster mass injection values calculated by chance constrained LP model are entered to the EPANET manually and system is solved by EPANET. Results of the LP model formulation and EPANET are compared in Figures 4.41, 4.42 and 4.43 for Node 2, Node 15 and Node 36, respectively. For the comparison, reliability level of 0.90 and standard deviation value of 0.5 is used for all three nodes.

As it can be seen from figures that the difference between the results increase as the distance between the selected node and the source increases. This may be due to several reasons. One of the reasons is the difference of the digits used in the program and EPANET. Although this difference is negligibly small when calculating the response coefficient matrix, it becomes noticeable when it is multiplied with the booster mass injection matrix to calculate the chlorine concentration values. Another reason for the difference is the uncertainty in the decay kinetics of the chlorine. Node 2 is near the source node so it is affected from the uncertainties resulting from the decaying much less than Node 36, which is at the end of the network.

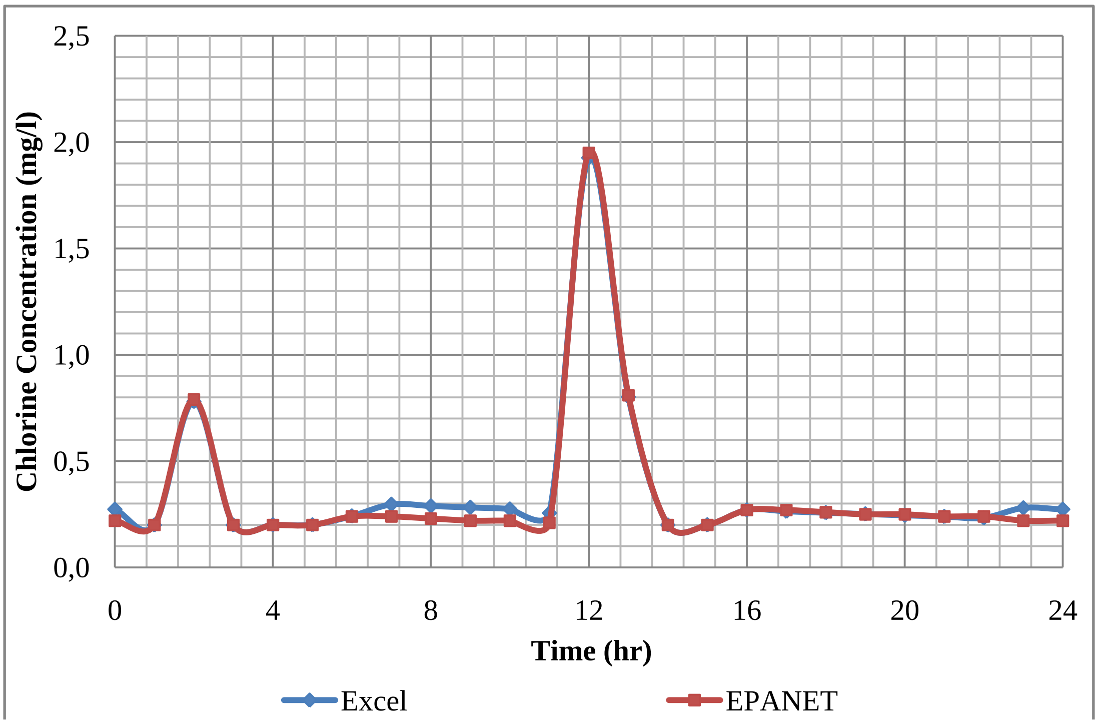

Figure 4.41: Verification of the chlorine concentrations for Node 2

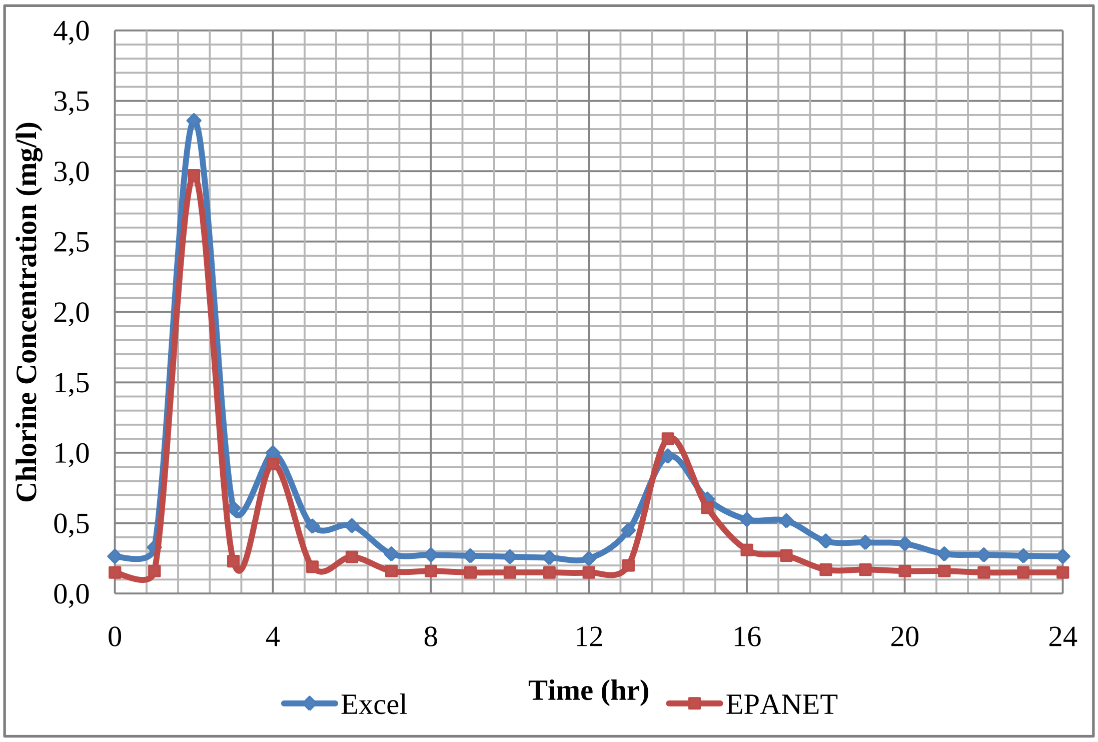

Figure 4.42: Verification of the chlorine concentrations for Node 15

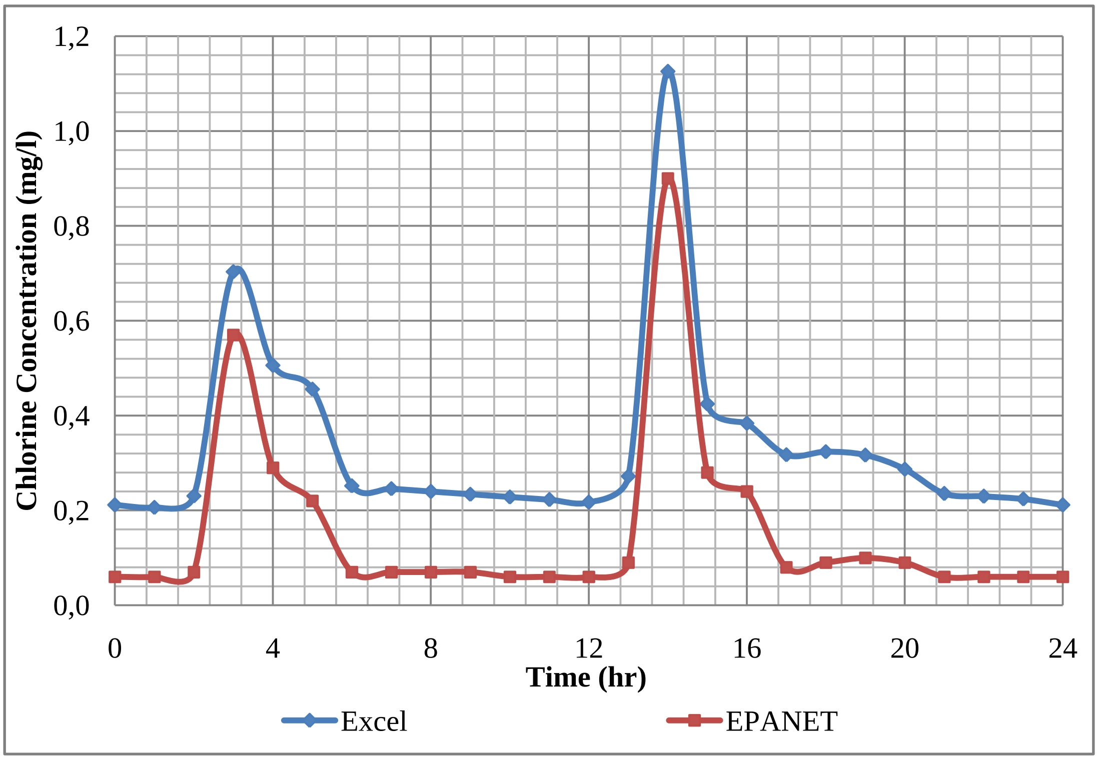

Figure 4.43: Verification of the chlorine concentrations for Node 36

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Summary and Conclusions

The subject in this study is the application of the chance constrained optimization of booster disinfection in a water distribution network. The main aim of the optimization is to minimize the total mass injected to the system while maintaining the chlorine concentrations within limits at each consumer node. Chlorine concentration is limited between upper and lower bounds in order to supply water to the network with sufficient quality. The lower bound is set by considering the biological regrowth into consideration and the upper bound is set by considering the taste and odor problems. It should be noted that, at high concentrations, formation of disinfectant by-products, DBP, which can lead to serious health problems.

As high amount of chlorine concentration results in increasing health risk, it is desirable to have chlorine concentrations as low as possible without violating the minimum limit. In addition to that, it is favorable to obtain uniform concentration distributions over the 24 *h* periodic cycle for all consumer points.

As nothing is as certain as it is modeled, probabilistic approach is taken into consideration for more realistic and reliable results. For the disinfection in a water distribution network, the component that has random characteristic is considered to be the chlorine concentration. This randomness is resulting from the space and time dependent decaying property of the chlorine. The only way to consider uncertainties is to obtain the probability distribution of this random variable. Normal and log-normal probability distributions are selected for the reliability analyses as they are the most frequently used ones for continuous random variables. Another reason for selecting log-normal distribution is that it is suitable for chlorine concentration which cannot be negative.

Analyses are done by applying chance constraint to the upper limit, lower limit and both of the limits for two cases which are conventional case, Case I and two booster stations and one source case, Case II. In conventional case, chlorine injection is done only from the source node and in the second case, injection is done from the three nodes selected by the previous work of Sert *(2009)*. As the objective function and constraints are linear for all cases, a linear programming solver is used for the analyses. In the light of these studies, the following conclusions are reached:

1) Increasing reliability level of the network results in increasing total injected mass for the same standard deviation as it is expected. The main difference between two types of probability distributions is the behavior of the increase. For upper limit, lower limit and both of the limits application of normal distribution, increase becomes abrupt, especially after reliability level of 0.90. In log-normal distribution, for upper limit that increase becomes abrupt after reliability level of 0.90 for the standard deviation values that access the feasibility limit in the analyses. For lower limit and both of the limits applications, comparison can be made between small standard deviation values. So feasibility limit is out of discussion and there is no opportunity to observe abrupt changes.

   Comparing three application types of both of the distributions, same increase in the reliability level results in highest injection results in the both of the limits case, slightly lower results in the lower limit case and lowest results in the upper limit case for the same standard deviation. Analyses results of the three case show that lower limit is dominant in the both of the limits case.

2) Increasing standard deviation results in increase of the total injected mass for the same reliability level. Same with the reliability level, the main difference between two types of probability distributions is the behavior of the increase. For upper limit, lower limit and both of the limits application of normal distribution, abrupt increases start to occur when the standard deviation gets closer to the value where solution becomes infeasible. In log-normal distribution, for upper limit, rapid increases start to occur when the standard deviation gets closer to the value where solution becomes infeasible. For lower limit and both of the limits case, total injection start to asymptotically reach its limiting value.

   Comparing three application types, feasibility range of the standard deviation is narrower in the both of the limits case, slightly wider in the lower limit case and widest in the upper limit case. This is resulting from the degree of effect of standard deviation in the application side. Changes in the standard deviation is more critical for lower limit compared with upper limit and analyses of the results show that lower limit is dominant in both of the limits case.

3) Increasing reliability level of upper limit at low standard deviations is not affecting the frequency of the concentrations in great extent. The effect can be noticeable at high standard deviations and that effect is increasing reliability results in narrowing down of the interval that data is distributed. Changes in standard deviation are more dominant for lower limit application comparing with upper limit application. Both of the limits application narrows down the interval from both of the sides. For comparison between the two different types of distributions, log-normal distribution results in narrower interval and similar changes occur with much smaller standard deviation values compared with the normal distribution.

4) Increasing the reliability level of the network results in a decrease in the highest standard deviation value which gives feasible solution for the normal distribution. On the other hand, for the log-normal distribution, increasing the reliability level of the network, results in an increase in the highest standard deviation value which gives feasible solution up to a point. However, for high reliability levels, this behavior changes and highest standard deviation value which gives feasible solution starts to decrease with the increase of reliability level. This is due to the formulation of log-normal distribution.

   Another aspect is same standard deviation results in lower changes in the limits for log-normal distribution compared with the normal distribution. Thus, higher limiting standard deviations can be obtained in the analyses of log-normal distribution which results in lower cost for the same standard deviations of different type of distributions.

5) Uniformity of chlorine distribution increases with the increasing reliability for both of the distributions. For lower limit and both of the limits application of the normal distribution, this increase occurs as more uniform spans with the distribution of peak values. Likewise, there exist peaks at the lower limit and both of the limits applications of the log-normal distribution; however, these peaks are smaller than the non-probability case.

6) Range in which the chlorine concentration data is lying is narrower in Case II compared to Case I as three different booster stations are used in Case II. Hence, lower limit and both of the limits application of Case II is more sensitive to standard deviation changes. Similarly as data distribution is built up on smaller concentration values, upper limit application is less sensitive to standard deviation changes.

5.2 Recommendations

In this research, analyses for both of the limits are done by taking same standard deviation value for upper and lower limits. For the same reliability level lower limit is more sensitive to the changes in standard deviations. Thus, while a small increment in the standard deviation creates a significant difference in the results of lower limit application, its effect become negligible in the upper limit. Hence, for a future study, using different standard deviations for upper and lower limits may be tried.

Moreover, since the chlorine distribution used in this study is unknown, mean values and probability distributions are decided logically. For a future recommendation, same analyses can be applied on a network with known chlorine data. In this way uncertainty will be reduced and reliability of the model can be checked in a more realistic way.

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