

HARMONIC RESPONSE OF LARGE ENGINEERING STRUCTURES WITH
NONLINEAR MODIFICATIONS

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ABSTRACT

HARMONIC RESPONSE OF LARGE ENGINEERING STRUCTURES WITH NONLINEAR MODIFICATIONS

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During the design and development stages of mechanical structures, after each modification made in order to satisfy design criteria, dynamic characteristics of the structure change and should be determined through reanalyzing the structure dynamically. Due to the significance of computational time and cost in design processes, it is inevitable for structural modification methods, especially for large systems, to become involved in predicting the dynamic behavior of modified structures from those of the original and modifying structures. Since most engineering structures are inherently nonlinear, linear approach may not be valid no more. Therefore, conventional structural modification methods can not be directly used, instead a nonlinear structural modification method needs to be employed.

In this thesis, it is aimed to adapt an effective linear structural modification method to structures with nonlinear modification or coupling. The amplitude dependencies of nonlinearities are modeled by using describing function method. Mathematical formulations are embedded in a computer program developed in MATLAB[®] with a graphical user interface. The software uses modal analysis results of ANSYS[®] for the original structure and dynamic stiffness matrix and nonlinearity information that belong to the modifying structure in order to calculate dynamic response of the modified structure. The approach is verified by applying it to both discrete and real

test structures previously studied in literature and generated discrete structures, then comparing the results with prior ones and ones obtained via time domain integration, respectively. Several other case studies are also included in order to demonstrate the applicability and to investigate the performance of the method.

It is concluded in this study that the structural modification method proposed can be successfully and efficiently used for structures with nonlinear modification or coupling.

Keywords: Structural modification, nonlinear structural modification, vibration of nonlinear structures, nonlinear structural coupling.

ÖZ

DOĞRUSAL OLMAYAN YAPISAL DEĞİŞİKLİK İÇEREN BÜYÜK SİSTEMLERİN HARMONİK TİTREŞİMİ

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Mekanik yapıların tasarım ve geliştirme aşamalarında, tasarım kriterlerini karşılamak amacıyla yapılan her deđişiklik sonrasında yapının dinamik davranışı deđişeceđinden yapının davranışı dinamik olarak tekrar analiz edilerek belirlenmelidir. Tasarım süreçlerinde hesaplama zaman ve maliyetinin önemi nedeniyle yapısal deđişiklik yöntemlerinin özellikle büyük sistemler için deđişmiş yapının dinamik davranışının, esas ve deđişiklik yaratan yapıların dinamik davranış bilgileri kullanılarak hesaplanması kaçınılmaz olmaktadır. Fakat çođu mühendislik yapısının özünde doğrusal olmamalarından dolayı doğrusal analiz geçerli olmamaktadır. Bu nedenle, bilindik yapısal deđişiklik yöntemleri doğrudan kullanılamaz. Bunun yerine, doğrusal olmayan yapısal deđişiklik yöntemlerinin uygulanması gerekir.

Bu tezde, etkili bir doğrusal yapısal deđişiklik yönteminin doğrusal olmayan deđişiklikler ve birleşmeler için geliştirilmesi amaçlanmıştır. Doğrusalsızlıkların genlik bağımlılıkları, tanımlama fonksiyonları yöntemi kullanılarak dikkate alınmıştır. Matematiksel formülasyonlar MATLAB® ile geliştirilen bir grafik kullanıcı arayüzü olan bilgisayar programı içine gömülmüştür. Yazılım esas yapının ANSYS® ile elde edilen biçim analizi sonuçları ile deđişiklik yaratan yapıya ait dinamik direngenlik matrisi ve doğrusalsızlık bilgisini kullanarak deđişmiş yapının

dinamik cevabını hesaplamaktadır. Yaklaşımı doğrulamak için yaklaşım literatürde daha önce incelenmiş toplanmış parametrelili ve gerçek test yapıları ile türetilmiş toplanmış parametrelili yapılara uygulanmış, sonra elde edilen sonuçlar, sırasıyla, daha önce elde edilmiş sonuçlar ve zaman düzleminde integrasyon ile elde edilen sonuçlarla karşılaştırılmıştır. Yöntemin uygulanabilirliğini göstermek ve performansını incelemek için birçok uygulamaya da yer verilmiştir.

Bu çalışmada, önerilen yapısal değişiklik yönteminin doğrusal olmayan değişiklik veya birleşme uygulanan yapılara da başarıyla ve etkin olarak uygulanabildiği anlaşılmıştır.

Anahtar Kelimeler: Yapısal değişiklik, doğrusal olmayan yapısal değişiklik, doğrusal olmayan yapıların titreşimi, doğrusal olmayan birleşim.

To My Family and Fiancée...

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TABLE OF CONTENTS

ABSTRACT.....	iv
ÖZ.....	vi
ACKNOWLEDGEMENTS.....	ix
TABLE OF CONTENTS.....	x
LIST OF TABLES.....	xiii
LIST OF FIGURES.....	xiv
LIST OF SYMBOLS.....	xviii
LIST OF ABBREVIATIONS.....	xx
CHAPTERS	
1. INTRODUCTION.....	1
1.1 Motivation for Nonlinear Structural Modification/Coupling.....	1
1.2 Literature Survey.....	3
1.3 Objective.....	6
1.4 Scope of the Thesis.....	7
2. THEORY.....	8
2.1 Introduction.....	8
2.2 Modeling Nonlinearities.....	8
2.2.1 Single Harmonic Formulation for Internal Nonlinear Forces.....	11
2.2.2 Single Harmonic Describing Functions.....	13
2.2.3 Nonlinearity Types and Their Corresponding Single Harmonic Describing Functions.....	15
2.2.3.1 Cubic Stiffness.....	15
2.2.3.2 Coulomb Friction.....	17
2.2.3.3 Piecewise Linear Stiffness.....	19
2.2.3.4 Arctan Stiffness.....	21
2.2.3.5 Preloaded Stiffness Element.....	22

2.3	Extension of the Structural Modification Method for Nonlinear Modifications	24
2.3.1	Özgülven's Formulation	24
2.3.2	Extension of the Formulation for Nonlinear Structural Modifications	27
2.3.3	Extension of the Formulation for Nonlinear Structural Couplings	29
2.3.3.1	Nonlinear Structural Coupling with Linear Elements	29
2.3.3.2	Nonlinear Structural Coupling with Nonlinear Elements.....	30
3.	COMPUTER PROGRAM: NLSM/CP.....	32
3.1	Non-Linear Structural Modification/Coupling Program.....	32
3.1.1	Program Description	32
3.1.2	Preprocessing	33
3.1.3	Solution Algorithm	35
3.1.4	Post Processing	37
3.2	User's Manual of the Program: NLSM/CP.....	37
3.2.1	Definition of the Files Used in the MATLAB® Program	37
3.2.2	Graphical User Interface (GUI)	38
4.	VERIFICATION AND APPLICATION OF THE METHOD AND THE PROGRAM.....	54
4.1	Verification of the Method and the Program.....	54
4.1.1	Case Study V.1 : 3 DOF System with Cubic Stiffness - Comparison with Results Given in Literature.....	54
4.1.2	Case Study V.2 : Modification via Nonlinear Coupling - Comparison with Results Given in Literature.....	62
4.1.3	Case Study V.3 : 3 DOF Nonlinear System - Comparison with TDI Solution	63
4.1.4	Case Study V.4 : A Real Structural System - Comparison with Experimentally Obtained Results	67
4.1.4.1	Overview of the Referred Experiment and Adaptation for the Nonlinear Structural Modification.....	67
4.1.4.2	Comparison of Results.....	69
4.1.5	Case Study V.5 : FE Model of a Cantilever Beam - Comparison with Computational Results Given in Literature	71

4.2	Applications of the Method Using the Program Developed	74
4.2.1	Application of the Method for Modifications with Additional DOFs .	75
4.2.1.1	Case Study A.1 - A Discrete System Modification with Additional DOFs	75
4.2.1.2	Case Study A.2 - A Real Life Engineering Problem	77
4.2.2	Application of the Method for Coupling with Linear Elements	82
4.2.2.1	Case Study A.3 - Discrete Subsystems Coupled with Linear Elements	82
4.2.3	Application of the Method for Coupling with Nonlinear Elements.....	85
4.2.3.1	Case Study A.4 - Discrete Subsystems Coupled with Nonlinear Elements	85
5.	RESULTS AND CONCLUSIONS.....	89
5.1	Summary of the Results and Conclusions	89
5.2	Recommendations for Future Work	93
	REFERENCES.....	94
	APPENDIX A - CONFERENCE PAPER	101

LIST OF TABLES

TABLES

Table 4.1 Parametric values of nonlinearities for Case Study V.1	55
Table 4.2 Parametric values of nonlinearities for Case Study V.3	65
Table 4.3 FE model properties for Case Study V.5	71
Table 4.4 Material and geometrical properties for Case Study V.5.....	72
Table 4.5 Material properties of the mirror plate and the shaft	79

LIST OF FIGURES

FIGURES

Figure 1.1 Verification and validation of design in product lifecycle [1].....	2
Figure 2.1 Characteristic of hardening and softening cubic stiffness elements.....	16
Figure 2.2 Typical response distortion due to cubic stiffness nonlinearity [40].....	17
Figure 2.3 Characteristic of a Coulomb friction element.....	18
Figure 2.4 Linear and quasilinear receptances α_{15} and θ_{15} for Coulomb damping type of nonlinearity ($F_f = 5$ N, $F_5 = 6$ and 8 N): arrow shows the direction of increasing external forcing level [31]	18
Figure 2.5 Characteristic of a piecewise linear stiffness element	20
Figure 2.6 Linear and quasilinear receptances α_{15} and θ_{15} for piecewise linear stiffness type of nonlinearity ($k_1 = 1$ kN/m, $k_2 = 25$ kN/m, $\delta = 0.01$ m and $F_5 = 20$ N) : arrows show the direction of frequency sweep [31]	20
Figure 2.7 Characteristic of an arctan stiffness element	22
Figure 2.8 Characteristic of a preloaded stiffness element	23
Figure 2.9 Typical response distortion resulting from preloaded stiffness nonlinearity [49]	23
Figure 2.10 Structural modification schematic	25
Figure 2.11 Treating a structural coupling problem with linear elements	30
Figure 2.12 Treating a structural coupling problem with nonlinear elements	31
Figure 3.1 Flow diagram of nonlinear structural modification/coupling algorithm ..	36
Figure 3.2 Setting the number of frequency points.....	40
Figure 3.3 Setting the lower and upper frequency limits.....	40
Figure 3.4 Setting the maximum percentage error and iteration number	41
Figure 3.5 Setting the relaxation coefficients at diverging and converging points....	41
Figure 3.6 Setting the linear elastic connection DOF numbers with stiffness values	42
Figure 3.7 Setting the rigid connection node numbers	42

Figure 3.8 Setting the nonlinear connection DOF numbers and defining types of nonlinear elements	43
Figure 3.9 Setting the parameters of the related nonlinear element.....	43
Figure 3.10 Letting the program read all the defined connection data	44
Figure 3.11 Setting the number of nodes of the original system	44
Figure 3.12 Setting the number of eigenvectors extracted for the original system....	45
Figure 3.13 Setting the number of the nodes of modifying system	45
Figure 3.14 Choosing number of DOF(s) per node	46
Figure 3.15 Choosing whether the response node is on original or modifying structure.....	46
Figure 3.16 Setting the response DOF	47
Figure 3.17 Choosing whether the structural damping coefficient is constant or response dependent	47
Figure 3.18 Setting the constant structural damping coefficient.....	48
Figure 3.19 Setting the response dependent structural damping data.....	48
Figure 3.20 Accepting all the data inputted up to here	49
Figure 3.21 Choosing whether the excitation is on original or modifying structure .	49
Figure 3.22 Setting the external force application coordinate	50
Figure 3.23 Setting the magnitude of the external force applied	50
Figure 3.24 Adding the external force created to the list of applied forces	51
Figure 3.25 Setting the two coordinates and the type of the nonlinear element	51
Figure 3.26 Setting the parameters of the nonlinearity	52
Figure 3.27 Solving the problem.....	52
Figure 3.28 Plotting the response and FRFs	53
Figure 4.1 System schematic for Case Study V.1 [33]	55
Figure 4.2 Nonlinear structural modification diagram for Case Study V.1	56
Figure 4.3 NLSM/CP solution for Case Study V.1 for (a) the 1 st mass, (b) the 2 nd mass and (c) the 3 rd mass	58
Figure 4.4 NLSM/CP solution for Case Study V.1 - zoomed individual resonances for (a) the 1 st mode of the 1 st mass, (b) the 2 nd mode of the 2 nd mass and (c) the 3 rd mode of the 3 rd mass	59

Figure 4.5 Solution for Case Study V.1 given in [33] for (a) the 1 st mass, (b) the 2 nd mass and (c) the 3 rd mass	60
Figure 4.6 Solution for Case Study V.1 given in [33] - zoomed individual resonances for (a) the 1 st mode of the 1 st mass, (b) the 2 nd mode of the 2 nd mass and (c) the 3 rd mode of the 3 rd mass	61
Figure 4.7 Nonlinear structural coupling diagram for Case Study V.2	62
Figure 4.8 Nonlinear structural modification diagram for Case Study V.3	64
Figure 4.9 Comparison of frequency responses obtained by proposed approach and TDI solution for the 3 rd mass	66
Figure 4.10 Comparison of frequency responses obtained by proposed approach and TDI solution for the 3 rd mass (zoomed in third mode)	66
Figure 4.10 Test rig assembly and its dimensions	67
Figure 4.11 Model of the test rig.....	68
Figure 4.12 The FE model of the cantilever beam.....	69
Figure 4.13 Calculated and measured FRF values for (a) F = 0.1 N, (b) F = 0.5 N and (c) F = 1 N.....	70
Figure 4.14 The FE model of the original beam	71
Figure 4.15 The FE model of the modified beam	72
Figure 4.16 Frequency responses for the 22 nd node of the modified structure in x-direction	73
Figure 4.17 Zoom around first natural frequency	74
Figure 4.18 Nonlinear structural modification diagram for Case Study A.1	75
Figure 4.19 Frequency response of m ₃ after modification for Case Study A.1	76
Figure 4.20 The shaft and mirror plate assembly.....	77
Figure 4.21 The FE model of the mirror plate	78
Figure 4.22 The FE model of the shaft	78
Figure 4.23 The direct point FRF of a point near one of the corners of the mirror plate for F = 2 N.....	80
Figure 4.24 The direct point FRF of a point near one of the corners of the mirror plate for F = 4 N.....	81
Figure 4.25 The direct point FRF of a point near one of the corners of the mirror plate for F = 6 N.....	81

Figure 4.26 Nonlinear structural coupling diagram for Case Study A.3	82
Figure 4.27 Frequency response of m_3 after coupling for Case Study A.3.....	83
Figure 4.28 Frequency responses around 3 rd resonance for different linear elastic coupling elements (a) $k_{LC} = 200$ N/m, (b) $k_{LC} = 400$ N/m, (c) $k_{LC} = 600$ N/m.....	84
Figure 4.29 Nonlinear structural coupling diagram for Case Study A.4	85
Figure 4.30 Frequency response of m_3 after coupling for Case Study A.4.....	86
Figure 4.31 Frequency responses around 3 rd resonance for different nonlinear coupling elements (a) $k_{NLC} = 200$ N/m, $\delta = 0.02$ m, (b) $k_{NLC} = 200$ N/m, $\delta = 0.04$ m, (c) $k_{NLC} = 400$ N/m, $\delta = 0.02$ m.....	87

LIST OF SYMBOLS

c	: Viscous damping
$[C]$: Viscous damping matrix
$[C_{\text{mod}}]$: Viscous damping matrix of modifying structure
$[D]$: Dynamic stiffness matrix
$[D_{\text{mod}}]$: Dynamic stiffness matrix of modifying structure
$\{f\}$: Generalized harmonic forcing vector
$\{F\}$: Amplitude vector of harmonic external forcing
F_f	: Friction force
F_p	: Preload of the spring element
i	: Unit imaginary number
$[I]$: Identity matrix
h	: Structural damping
$[H]$: Structural damping matrix
$[H_{\text{mod}}]$: Structural damping matrix of modifying structure
k	: Stiffness
$[K]$: Stiffness matrix
$[K_{\text{mod}}]$: Stiffness matrix of modifying structure
$[K_0]$: Stiffness matrix of original structure
m	: Mass
$[M]$: Mass matrix
$[M_{\text{mod}}]$: Mass matrix of modifying structure

$[M_0]$: Mass matrix of original structure
n	: Internal nonlinear restoring force
$\{N\}$: Internal nonlinear restoring force vector
$\{S\}$: Amplitude vector of internal nonlinear forces
t	: Time
$\{x\}$: Generalized displacement vector
$\{\dot{x}\}$: Generalized velocity vector
$\{\ddot{x}\}$: Generalized acceleration vector
$\{X\}$: Complex amplitude vector of steady state harmonic displacements
v	: Describing function
y_{ab}	: Intercoordinate displacement between coordinates a and b
\dot{y}_{ab}	: Intercoordinate velocity between coordinates a and b
\ddot{y}_{ab}	: Intercoordinate acceleration between coordinates a and b
Y_{ab}	: Complex amplitude of intercoordinate displacement
$[\alpha]$: Receptance matrix of original structure
β	: Coefficient of cubic stiffness element
$[\gamma]$: Receptance matrix of modified structure
$[\Delta]$: Nonlinearity matrix
δ	: Elongation value
η	: Modal structural damping coefficient
κ	: Compression factor
λ	: Relaxation coefficient
ρ	: Amplification factor
ϕ	: Phase of the harmonic displacement response
ψ	: ωt
ω	: Frequency

LIST OF ABBREVIATIONS

DFM	: Describing Function Method
DOF	: Degree of Freedom
FE	: Finite Element
FEA	: Finite Element Analysis
FEM	: Finite Element Method
FRF	: Frequency Response Function
GUI	: Graphical User Interface
MDOF	: Multi Degree of Freedom
NLSM/CP	: Non-Linear Structural Modification/Coupling Program
ODE	: Ordinary Differential Equation
SDOF	: Single Degree of Freedom
TDI	: Time Domain Integration

CHAPTER 1

INTRODUCTION

1.1 Motivation for Nonlinear Structural Modification/Coupling

In mechanical design, in order for engineers to ensure that their design meets wide range of requirements and is qualified enough, they must test and analyze each prototype constructed. For this purpose, harmonic vibration analyses and tests are to be performed as the occasion arises throughout the design process. After each analysis and test, if the product does not satisfy design criteria, engineers have to make modifications, usually local, on their designs for correction purposes. This process cycle is given in Figure 1.1 [1].

Each modification made on design changes the structural properties such as natural frequencies, mode shapes and response of the structure. Therefore, each analysis and test have to be repeated for each modified prototype after each modification. In terms of computational analysis, a new Finite Element (FE) model for the modified system should be reconstructed and vibration analysis under dynamic loads should be performed once again. However, in case of various alternatives are to be considered, constituting whole analytical model for each alternative and solving them will be time consuming and costly, particularly for large ordered systems. Instead using the existing analysis results belonging to the original prototype and dynamic properties of only the modifying structure, dynamic characteristics of the modified structure can be estimated without performing whole analysis, but by employing structural modification techniques.

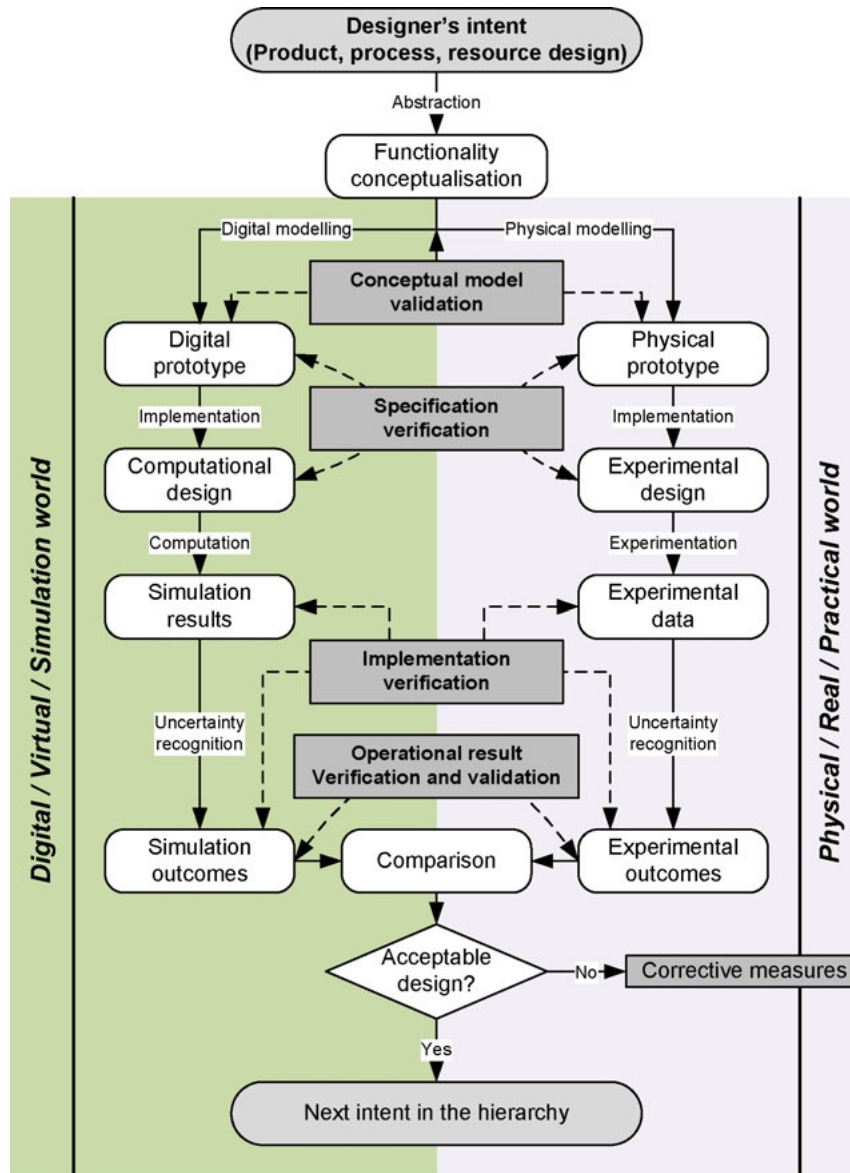


Figure 1.1 Verification and validation of design in product lifecycle [1]

On the other hand, most of the engineering structures are naturally nonlinear and thus linear approach may not be applicable any more. We know that nonlinearity inherited in a structure causes discrepancy between measured and theoretically calculated modal properties [2-4]. So, with the increasing demand for high precision mechanical components, taking nonlinear effect into account in dynamic analysis becomes unavoidable. As a consequence, in cases where time, cost and accuracy are the primary concerns, nonlinear structural modification techniques come into picture.

1.2 Literature Survey

There is a vast literature on numerous different structural modification techniques and on their variations. Generally two different problems are considered [5, 6]: the direct structural dynamic modifications problem and the inverse structural dynamic modifications problem. The inverse structural dynamic modification problem focuses on the determination of the necessary modifications in order to achieve required dynamic characteristics. Some of the main literature reviews on this subject are the studies performed by Kyprianou et al. [7, 8], Li and He [9], Park [10, 11] and Mottershead et al. [12]. On the other hand, direct structural dynamic modification problem, basic theory of which was primarily presented by Crowley et al. [13], deals with the estimation of dynamic characteristics of a structure after a modification takes place. Each problem can be classified into two subgroups: frequency response function (FRF) based techniques and modal synthesis techniques.

This thesis focuses on the FRF based direct structural modification techniques. Various FRF based direct structural modification methods are developed in previous studies. In this work, basically, the matrix inversion method developed by Özgüven [14] is used, and it is applied to nonlinear systems.

Özgüven proposed a matrix inversion method [15] for finding receptances of locally damped structures from undamped counterparts and combined this method with an efficient recursive solution algorithm in order to avoid matrix inversion in a subsequent study [16]. In a further work [14], he presented a structural modification method that can estimate exact FRFs of the modified structure using FRF matrix of the original system and mass, stiffness and damping matrices of the modifying structure. In this method, it is required to invert only a matrix of a size equal to the total degrees of freedom (DOFs) of the modification. Thus, it is favorable particularly for systems with large DOFs modified locally. Moreover, since the method uses the FRFs of the original structure during calculation, by including FRF information of only the required DOFs of the original structure and deleting remaining rows and columns, result computational speed can be increased without

losing accuracy. Recently, Hang et al. [17-19] focused on the application of this method on distributed structural dynamic modifications with reduced and additional DOFs. Later on, the method was applied to real structures by Canbaloglu and Özgüven [20].

Starting with the same matrix inversion formulation, in order to avoid matrix inversion, Şanlıtürk [21] presented a new structural modification method which is based on Sherman-Morrison formula [22]. Even though this method is advantageous in the sense that modified system FRFs can be calculated exactly without any matrix inversion, it is applicable only for modifications that does not introduce new DOFs.

Köksal et al. [23] extended the Successive Matrix Inversion that had previously developed by Bae et al. for reanalysis of static systems to dynamic reanalysis of structures. In this method, the FRFs of the modified structure are calculated from those of the original structure and the system matrices of the modifying structure. The formulation begins similarly with the one presented in [14], but avoids matrix inversion using power series expansion.

Then in a further study, Köksal et al. [24] compared the three structural modification methods ([14], [21] and [23]) with each other in terms of computational efficiency point of view. It is concluded in this study that the most powerful one among these three methods is the one developed by Özgüven [14] not only in terms of computational efficiency but also suitability for structural modifications with additional DOFs.

The main problem when dealing with nonlinear elements is their response dependent behavior which is overcome by using Describing Function Method (DFM) in this study. This method was introduced by Krylov and Bogolyubov [25] in order to analyze certain nonlinear control problems based on an earlier work of Van der Pol [26]. Taylor [27] defines the describing function approach as modeling and studying nonlinear system behavior via replacing each nonlinear element with a quasilinear descriptor whose gain is a function of input amplitude. Introduction of DFM into the

vibration analysis of multi degree of freedom (MDOF) nonlinear systems took place soon after the studies performed by Budak and Özgüven [28, 29]. They suggested that nonlinear internal forces can be expressed as a multiplication of so called "nonlinearity matrix" by displacement vector, and proposed a method (the Iterative Receptance Method) for nonlinear systems where nonlinearities can be expressed as polynomials. Later, Tanrıku et al. [30, 31] extended this formulation for any type of nonlinearity using DFM. Among several applications of DFM in harmonic vibration analysis of nonlinear structures, the studies performed by Siller [32] and Abat [33] are the ones referred in this thesis.

Several studies, some of which are summarized below, are also concentrated on nonlinear structural dynamic modification/coupling. Watanabe and Sato [34] used first order describing function and linearized the nonlinear stiffness of a beam structure to develop the so-called "Nonlinear Building Block" approach, for coupling nonlinear structures with local nonlinearity.

Cömert and Özgüven [35] developed a method for calculating the forced response of linear substructures coupled with nonlinear connecting elements. They use FRFs of the linear substructures and information regarding nonlinear connection elements in order to obtain forced harmonic response of coupled structure. The effect of nonlinear internal forces was expressed as a product of nonlinearity matrix, introduced by Budak and Özgüven [28, 29], and displacement vector. The elements of nonlinearity matrix are written using DFM.

Ferreira and Ewins [36] proposed a new Nonlinear Receptance Coupling Approach for fundamental harmonic analysis based on describing functions. They suggested an approach that is capable of coupling structures with local nonlinear elements whose describing functions are available considering just the fundamental frequency. Then, Ferreira [37] extended the approach and introduced Multi-Harmonic Nonlinear Receptance Coupling Approach. This approach is able to couple linear and nonlinear structures with different types of joints by specifying the multi-harmonic describing functions for all nonlinear joints.

Chong and İmregün [38] suggested an iterative algorithm for the coupling of nonlinear systems with linear ones. They initially obtained the nonlinear modal parameter variations of coupled system with profile building approach. Then, using nonlinear modal parameters, the response of coupled system at various force levels were predicted.

Maliha et al. [39] coupled a nonlinear dynamic model of a spur gear pair with linear FE models of shafts carrying them, and with discrete models of bearings and disks. They used DFM to express nonlinear elasticity term resulting from backlash and included it in the solution by the use of so-called “nonlinearity matrix” of Tanrikulu et al. [30, 31]. In order to avoid matrix inversion, the formulation given in reference [16] for nonproportionally damped systems is extended to nonlinear systems and an iterative technique is used in the solution. The study presented in this thesis follows a similar theoretical approach as that study, but here the structural modification technique with additional DOFs given in [14] is extended for nonlinear modifications and couplings.

1.3 Objective

In this study, it is aimed to obtain dynamic response of a modified structure from those of the linear original structure and nonlinear modification data. This is accomplished by simply extending Özgüven’s structural modification method to nonlinear modifications [14]. It is also intended in this thesis to develop a computer program that can apply the extended nonlinear structural modification method to large structures and to verify the proposed approach with several theoretical and experimental case studies. In brief, general purpose throughout this thesis is to extend the previously proposed structural modification method for nonlinear modifications and couplings and to show its success and efficiency with several applications.

1.4 Scope of the Thesis

The outline of the thesis is given as follows:

In Chapter 2, firstly, the nonlinearities investigated in this thesis and their modeling approaches will be presented. The underlying mathematical background in single harmonic analysis of MDOF nonlinear structures will also be explained here. Secondly, the theory of nonlinear structural dynamic modification and coupling method will be given.

In Chapter 3, the computer program developed via MATLAB[®] will be introduced and the basic capabilities of the program will be explained. The user's manual of the program will also be given here.

In Chapter 4, verification of the approach will be demonstrated in three groups of case studies: the computational results will be compared with (a) those given in literature, (b) those obtained via Time Domain Integration (TDI) and (c) those obtained experimentally in previous studies. Then, in order to show the applicability and performance of the method and the program, several other case studies will be presented.

In Chapter 5, brief summary of the work done will be given with discussions and conclusions. Finally, suggestions for future studies will be presented.

CHAPTER 2

THEORY

2.1 Introduction

In this chapter, the theory of nonlinear structural modification method proposed using quasilinearization of structural nonlinearities, which underlies the basis of this study, will be presented. In section 2.2, theory of modeling approach for structural nonlinearities using Describing Functions Method (DFM) based on the method proposed first by Budak and Özgüven [28, 29], and later by Tanrikulu et al. [30, 31] is given. Later, in the same section the types of nonlinearities investigated in this study are explained in detail. In section 2.3, the basics of the structural modification method that was previously presented by Özgüven [14] and its extended formulation for nonlinear modifications are explained.

2.2 Modeling Nonlinearities

Consider the equations of motion for a nonlinear MDOF system excited with harmonic external forcing $\{f\}$:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + i[H]\{x\} + [K]\{x\} + \{N\} = \{f\} \quad (2.1)$$

where $[M]$, $[C]$, $[H]$ and $[K]$ represent the linear mass, viscous damping, structural damping and stiffness matrices of the system, respectively. Here, $\{x\}$ is the

generalized displacement vector and dot stands for the derivation with respect to time. Furthermore, $\{N\}$ represents internal nonlinear restoring force vector and i is the unit imaginary number. The j^{th} element of vector $\{N\}$ can be expressed as a series of the form,

$$N_j = \sum_{k=1}^n n_{jk} \quad (2.2)$$

where n_{jk} denotes the nonlinear restoring force acting between the coordinates j and k for $j \neq k$, and between the ground and the coordinate j for $j = k$. Note that,

$$n_{jk} = n_{kj} \quad (2.3)$$

Here n_{jk} can be represented as an arbitrary function of intercoordinate displacement y_{jk} and its derivatives,

$$n_{jk} = n_{jk}(y_{jk}, \dot{y}_{jk}, \ddot{y}_{jk}, \dots) \quad (2.4)$$

where:

$$y_{jk} = x_j - x_k \quad \text{for } j \neq k \quad (2.5)$$

$$y_{jk} = x_j \quad \text{for } j = k \quad (2.6)$$

The external harmonic forcing $\{f\}$ can be expressed as:

$$\{f\} = \{F\} e^{i\psi} \quad (2.7)$$

where, $\{F\}$ is the vector of harmonic excitation amplitudes and generic angle ψ can be defined as the product of angular frequency ω and time t . Assuming that the nonlinear response to the external harmonic forcing is essentially not exactly sinusoidal but periodic with the same period, it can be written as a Fourier series of the form:

$$\{x(t)\} = \sum_{p=0}^{\infty} \{x(t)\}_p = \sum_{p=0}^{\infty} \{X\}_p e^{ip\psi} \quad (2.8)$$

where, subscript p represents the degree of harmonic order and $\{x(t)\}_p$ indicates the displacement response of the p^{th} harmonic. So, the complex displacement response amplitude X at coordinate j for the p^{th} harmonic, $(X_j)_p$, can be written as:

$$(X_j)_p = |X_j|_p e^{i(\phi_j)_p} \quad (2.9)$$

Here, $|X_j|_p$ is the magnitude and $(\phi_j)_p$ is the phase of the complex displacement response X_j for the p^{th} harmonic. Now, let the response of the system, $\{x(t)\}$, given by Equation 2.8, is approximated by a set of q harmonic terms. Then, the approximate response can be written as:

$$\{x(t)\} \approx \{\tilde{x}(t)\}_{(q)} = \sum_{p=1}^q \{x(t)\}_p \quad (2.10)$$

Note that, the even p values are due to nonlinearities with asymmetrical characteristics whereas the odd ones are due to nonlinearities with symmetrical characteristics. Using the above formulations obtained for the displacements of the main coordinates, the intercoordinate displacement responses between arbitrary two coordinates a and b , y_{ab} , can be written as:

$$y_{ab}(t) = x_a(t) - x_b(t) = \sum_{p=0}^{\infty} (y_{ab})_p(t) = \sum_{p=0}^{\infty} (Y_{ab})_p e^{ip\psi} \quad (2.11)$$

The approximate intercoordinate displacement responses can accordingly be written as follows:

$$\{y_{ab}(t)\} \approx \{(\tilde{y}_{ab})_q(t)\} = \sum_{p=1}^q \{(y_{ab})_p(t)\} = \sum_{p=1}^q (Y_{ab})_p e^{ip\psi} \quad (2.12)$$

where:

$$(Y_{ab})_p = (X_a)_p - (X_b)_p, \quad (a \neq b) \quad (2.13)$$

$$(Y_{ab})_p = |(Y_{ab})_p| e^{i(\phi_{ab})_p} \quad (2.14)$$

2.2.1 Single Harmonic Formulation for Internal Nonlinear Forces

As written before, differential equations of motion for a nonlinear MDOF structure exposed to external sinusoidal excitation can be restated as:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + i[H]\{x\} + [K]\{x\} + \{N\} = \{F\} e^{i\psi} \quad (2.15)$$

Then the steady-state solution can be represented in the form of a Fourier series as:

$$\{x(t)\} = \sum_{p=0}^{\infty} \{x(t)\}_p = \sum_{p=0}^{\infty} \{X\}_p e^{ip\psi} \quad (2.16)$$

Usually, the higher harmonic terms of the response have small amplitudes relative to the fundamental harmonic component. In such cases, the steady state response,

$\{x(t)\}$, can be formulated with an acceptable numerical error as:

$$\{x(t)\} = \{x(t)\}_1 = \{X\}_1 e^{i\psi} \quad (2.17)$$

The response for an arbitrary coordinate j , x_j , can be written as:

$$x_j \approx (\tilde{x}_j)_1 = (x_j)_1 = (X_j)_1 e^{i\psi} \quad (2.18)$$

where:

$$(X_j)_1 = X_j|_1 e^{i(\phi_j)_1} \quad (2.19)$$

Similar to Equation 2.13, the intercoordinate displacement response between coordinates a and b , y_{ab} , can be expressed as:

$$y_{ab}(t) = x_a(t) - x_b(t) \approx (\tilde{x}_a)_1(t) - (\tilde{x}_b)_1(t) = (\tilde{y}_{ab})_1(t) = (Y_{ab})_1 e^{i\psi} \quad (2.20)$$

where:

$$(Y_{ab})_1 = (X_a)_1 - (X_b)_1, \quad (a \neq b) \quad (2.21)$$

$$(Y_{ab})_1 = |(Y_{ab})_1| e^{i(\phi_{ab})_1} \quad (2.22)$$

Accordingly, a complex and periodic nonlinear function, $n_{ab}[(\tilde{y}_{ab})_1]$, can also be represented in the form of Fourier series as:

$$n_{ab}[(\tilde{y}_{ab})_1] = \sum_{m=0}^{\infty} (n_{ab})_m = \sum_{m=0}^{\infty} (N_{ab})_m e^{im\psi} \quad (2.23)$$

Assuming that the nonlinear internal force, $n_{ab}[(\tilde{y}_{ab})_1]$, is dominated by the fundamental term similar to the response itself, the formulation for the approximated nonlinear force can be reduced to:

$$n_{ab}[(\tilde{y}_{ab})_1] \approx (n_{ab})_1 = (N_{ab})_1 e^{i\psi} \quad (2.24)$$

where:

$$(N_{ab})_1 = |N_{ab}|_1 e^{i(\xi_{ab})_1} \quad (2.25)$$

$$(N_{ab})_1 = \frac{i}{\pi} \int_0^{2\pi} n_{ab}[(\tilde{y}_{ab})_1] e^{-i\psi} d\psi \quad (2.26)$$

2.2.2 Single Harmonic Describing Functions

Considering a single degree of freedom (SDOF) system subjected to a harmonic excitation, equation of motion for such a system can be expressed as:

$$m\ddot{x} + c\dot{x} + ihx + kx + n = f \sin \omega t \quad (2.27)$$

In this equation m , c , h and k are the linear mass, viscous damping, structural damping and stiffness values, respectively. x and its derivatives with respect to time t are the displacement, velocity and acceleration of the system, and i stands for the unit imaginary number. n represents internal nonlinear restoring force, whereas f represents external forcing. The restoring force n is assumed to be a function of velocity and displacement. Assuming that $x(t)$ has the same frequency as the excitation, following relation can be written:

$$x \approx (X)_1 \sin(\omega t + \theta) = (X)_1 \sin \tau \quad (2.28)$$

where $(X)_1$ is complex response amplitude, ω is the excitation frequency and θ is the phase angle. The restoring force $n(x, \dot{x})$ can be expanded around x through Fourier series, neglecting all the higher harmonic terms assuming that it has also a sinusoidal nature and little energy is leaked to other frequencies other than the fundamental frequency.

$$n(x, \dot{x}) \approx v(x, \dot{x})x = (N)_0(\omega, (X)_0) + (N)_1(\omega, (X)_1)x + i(N)_1^*(\omega, (X)_1)x \quad (2.29)$$

where $v(x, \dot{x})$ can be defined as the optimum equivalent linear complex stiffness representation of the nonlinear restoring force function $n(x, \dot{x})$. Here, the bias term $(N)_0$ and real and imaginary terms of the fundamental harmonic $(N)_1$ and $(N)_1^*$ can be expanded as:

$$(N)_0 = \frac{1}{2\pi} \int_0^{2\pi} N((X)_1 \sin \tau, \omega(X)_1 \cos \tau) d\tau \quad (2.30)$$

$$(N)_1 = \frac{1}{\pi(X)_1} \int_0^{2\pi} N((X)_1 \sin \tau, \omega(X)_1 \cos \tau) \sin \tau d\tau \quad (2.31)$$

$$(N)_1^* = \frac{1}{\pi(X)_1} \int_0^{2\pi} N((X)_1 \sin \tau, \omega(X)_1 \cos \tau) \cos \tau d\tau \quad (2.32)$$

Note that, if $n(x, \dot{x})$ is symmetrical around the origin, then $(N)_0$ becomes zero. Moreover, if the assumption that $n(x, \dot{x})$ is dominated by its fundamental term is valid, then Equation 2.29 can be simplified as:

$$(v(x, \dot{x}))_1 x = (N)_1(\omega, (X)_1)x + i(N)_1^*(\omega, (X)_1)x \quad (2.33)$$

where $(v(x, \dot{x}))_1$ is the first-order describing function which can be defined as:

$$(v(x, \dot{x}))_1 = (N)_1 + i(N)_1^* \quad (2.34)$$

According to the kind of nonlinearity presented by $n(x, \dot{x})$, the describing function v can be calculated by using Equations 2.31, 2.32 and 2.34.

2.2.3 Nonlinearity Types and Their Corresponding Single Harmonic Describing Functions

In this section, the quasilinear representation of nonlinearity types included in this thesis, which are commonly encountered ones in structural dynamics, will be listed. Structural nonlinearities, in most cases, are displacement, velocity and/or frequency dependent and they are usually represented in idealized forms in order to be incorporated in the analysis. The representation of such nonlinearities by describing function approach, in terms of single harmonic will be summarized below.

2.2.3.1 Cubic Stiffness

This is the most frequently encountered type of nonlinearity in structural dynamics. It can be considered as a massless nonlinear spring such that applied force is directly proportional to the cube of the displacement. The mathematical model to represent the relation between the force and the displacement can be expressed as:

$$n(x, \dot{x}) = k_c x + \beta x^3 \quad (2.35)$$

where x represents the elongation of the nonlinear spring. The stiffness coefficients k_c and β , both of which are not frequency dependent, accounts for the linear and nonlinear parts of the spring.

In Equation 2.35, β can be either positive or negative. If $\beta > 0$, as the level of excitation increases the restoring force introduced will be cumulatively greater than that of a linear spring, in other words, system shows hardening behavior (Figure 2.1). Clamped plates and beams can be given as examples of such systems.

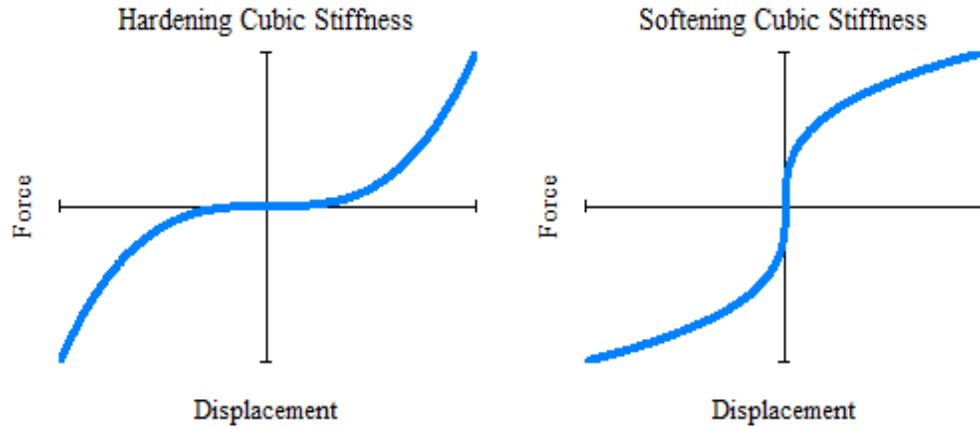


Figure 2.1 Characteristic of hardening and softening cubic stiffness elements

On the other hand, if $\beta < 0$, as the level of excitation increases the restoring force introduced will be cumulatively lower than that of a linear spring, namely, system shows softening behavior. Buckling beams and plates exhibit such a behavior [40].

Bending of system response around resonant frequency is the characteristic FRF distortion due to cubic stiffness nonlinearity. This effect was investigated in many studies [31-33, 35, 37, 38, 40-44] and the conclusion reached in all is that resonant frequency shifts up for hardening systems and the other way around for softening systems as shown in Figure 2.2.

The first order describing function representation of a nonlinear cubic stiffness element having cubic stiffness coefficients k_c and β can be written as:

$$v_{ab} = k_c + \frac{3}{4}\beta X_1^2 \quad (2.36)$$

whose detailed derivation can be found in [32].

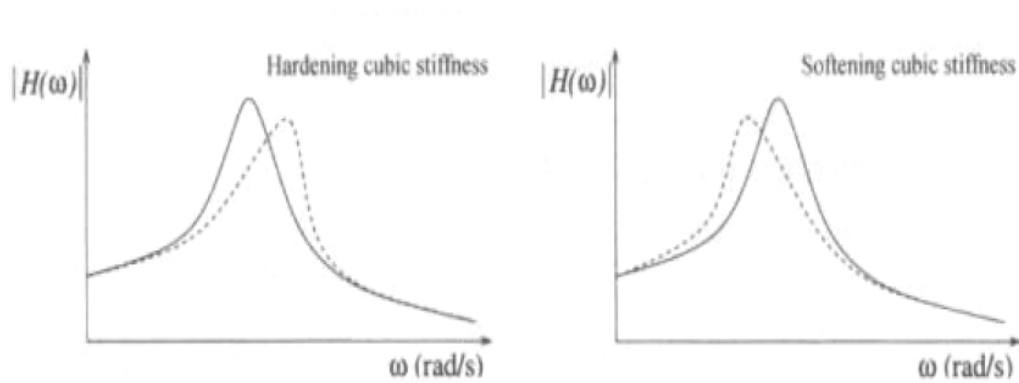


Figure 2.2 Typical response distortion due to cubic stiffness nonlinearity [40]

2.2.3.2 Coulomb Friction

Coulomb friction, which is particularly prevalent in demountable structures such as grandstands, occurs in any situation with interfacial motion [40]. It is nonlinear because the direction of kinetic friction force depends on the direction of velocity (Equation 2.37).

$$n(x, \dot{x}) = F_f \operatorname{sgn}(\dot{x}) \quad (2.37)$$

where \dot{x} represents the relative velocity and F_f accounts for the friction force. The signum function ensures that the restoring force always opposes the direction of motion. In Figure 2.3, the behavior of a Coulomb friction element is illustrated.

When external force is smaller than the kinetic friction force, there will not be a relative motion between two coordinates of a nonlinear Coulomb friction element. In such extreme situations, stick-slip motion can occur; however, such cases will not be considered in this work.

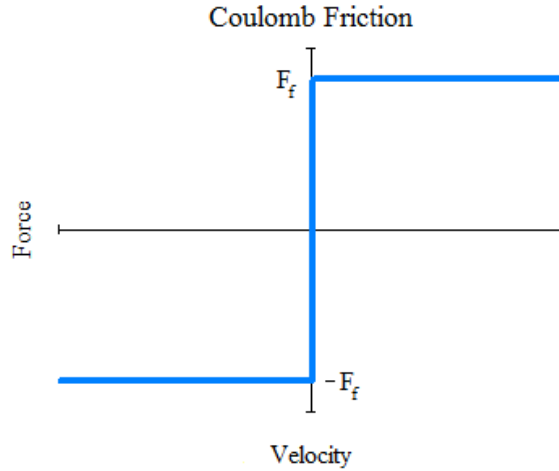


Figure 2.3 Characteristic of a Coulomb friction element

On the other hand, at high enough levels of excitation where stick-slip motion does not occur, an equivalent linear response will be obtained for each forcing level as shown in Figure 2.4. The drop in receptance level is more evident at small vibration amplitudes as confirmed in previous studies [31-33, 37, 40, 45].

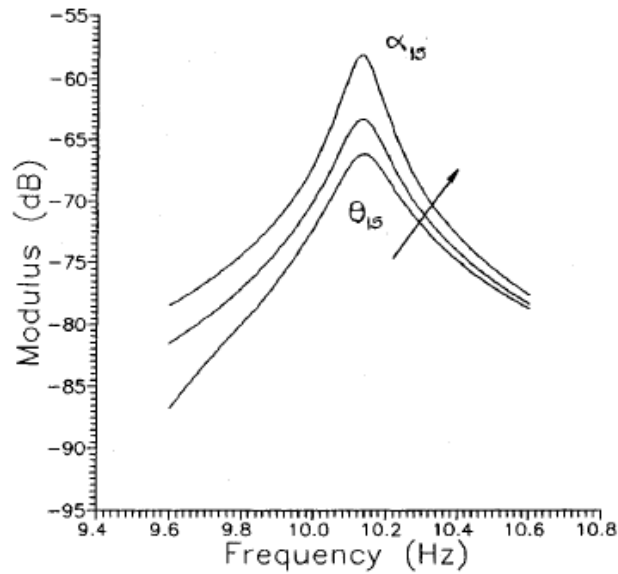


Figure 2.4 Linear and quasilinear receptances α_{15} and θ_{15} for Coulomb damping type of nonlinearity ($F_f = 5$ N, $F_5 = 6$ and 8 N): arrow shows the direction of increasing external forcing level [31]

The describing function of a typical Coulomb friction damper having friction force F_f can be formulated by considering the first harmonics only as given in [31]:

$$v_{ab} = i \frac{4F_f}{\pi X_1} \quad (2.38)$$

2.2.3.3 Piecewise Linear Stiffness

Basically, geometric discontinuities between interconnected components results in this kind of nonlinearity in many mechanical systems. It can be observed in loosely jointed structures [46]. It can be modeled as a massless nonlinear spring which shows linear behavior but with different proportionality constants at certain elongation intervals. The mathematical model to represent the relation between the force and the displacement can be expressed as:

$$\begin{aligned} n(x, \dot{x}) &= k_1 x, & \text{for } |x| < \delta \\ n(x, \dot{x}) &= k_2 x + (k_1 - k_2)\delta & \text{for } x \geq \delta \\ n(x, \dot{x}) &= k_2 x - (k_1 - k_2)\delta & \text{for } x \leq -\delta \end{aligned} \quad (2.39)$$

where k_1 and k_2 are the proportionality constants at specified elongation intervals, and δ is the elongation value corresponding the point of transition from the stiffness k_1 to the stiffness k_2 (Figure 2.5).

Clearance and saturation type of nonlinearities can be taken into account as special cases of piecewise linear stiffness type of nonlinearity. Clearance nonlinearity is the case when the value of k_1 is 0. This type of nonlinearity is encountered in many applications such as pylon-store-wing assemblies and backlash in the gearing systems. On the other hand, saturation nonlinearity is the case when the value of k_2 is 0. The effect of piecewise linear stiffness nonlinearity on system response was investigated in many studies such as [31, 33, 34, 41, 44-47].

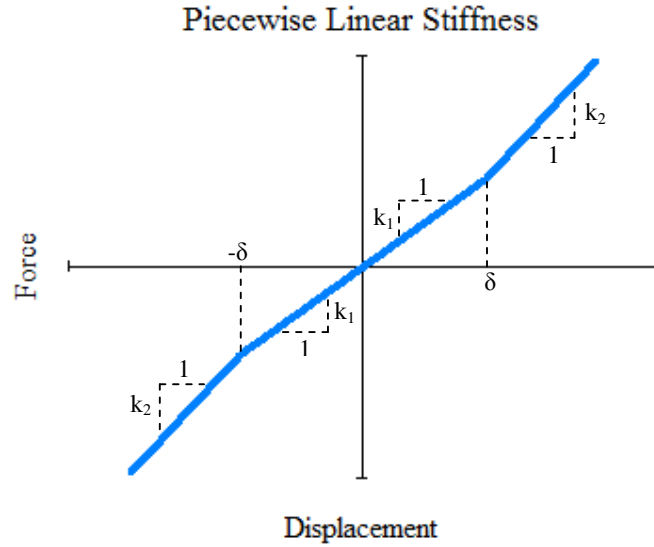


Figure 2.5 Characteristic of a piecewise linear stiffness element

From the definition, a frequency response distortion is expected to show an abrupt change at the point of transition from the stiffness k_1 to the stiffness k_2 , as shown in Figure 2.6 [31].

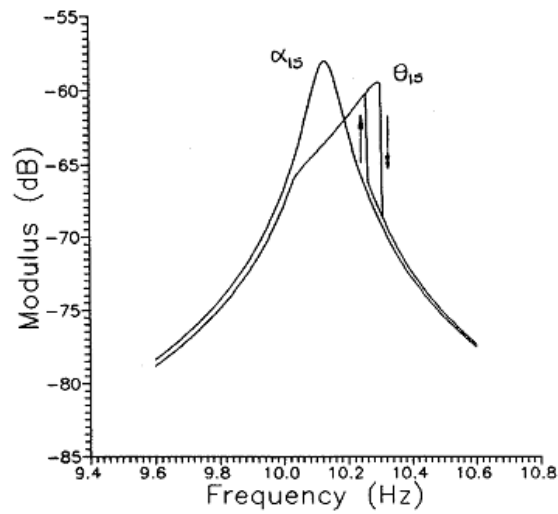


Figure 2.6 Linear and quasilinear receptances α_{15} and θ_{15} for piecewise linear stiffness type of nonlinearity ($k_1 = 1$ kN/m, $k_2 = 25$ kN/m, $\delta = 0.01$ m and $F_5 = 20$ N) : arrows show the direction of frequency sweep [31]

The single harmonic describing function of a piecewise linear stiffness element having system coefficients k_1 , k_2 and δ defined in Equation 2.39, can be formulated as follows:

$$v_{ab} = k_1, \quad \text{for } X_1 < \delta \quad (2.40)$$

$$v_{ab} = \frac{2(k_1 - k_2)}{\pi} \left[\arcsin\left(\frac{\delta}{X_1}\right) + \left(\frac{\delta}{X_1}\right) \sqrt{1 - \left(\frac{\delta}{X_1}\right)^2} \right] + k_2 \quad \text{for } X_1 \geq \delta$$

2.2.3.4 Arctan Stiffness

Arctan stiffness type of nonlinearity is usually used to model softening structural behavior. However, unlike the softening cubic stiffness nonlinearity, arctan nonlinearity approaches a horizontal tangent for large values of displacement. The arctan stiffness element having such a softening characteristic can be defined by:

$$n(x, \dot{x}) = \rho \arctan(\kappa x) \quad (2.41)$$

where the coefficient ρ is the amplification factor and κ is the compression factor. The characteristic force-displacement relationship of an arctan stiffness element is graphically shown in Figure 2.7.

The effect of arctan stiffness nonlinearity on system response is substantially similar to the that of softening cubic stiffness nonlinearity [33, 45, 48]. The first harmonic describing function of an arctan stiffness element having system coefficients, ρ and κ defined in Equation 2.41, can be written as below:

$$v_{ab} = \frac{2\rho}{\kappa(X_1)^2} (\sqrt{(\kappa X_1)^2 + 1} - 1) \quad (2.42)$$

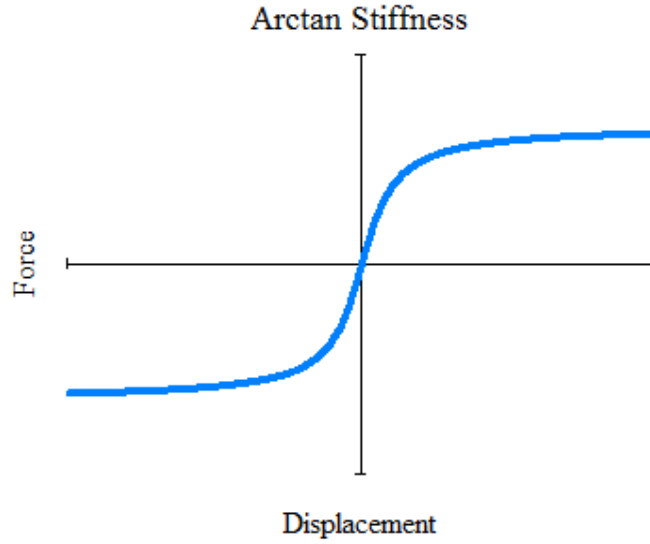


Figure 2.7 Characteristic of an arctan stiffness element

2.2.3.5 Preloaded Stiffness Element

Intentional pre-compression, unintended manufacturing or heat treatment process introduces nonlinearity into mechanical structures and can be modeled by using preloaded stiffness element [49]. This type nonlinearity has the characteristic relation below,

$$\begin{aligned}
 n(x, \dot{x}) &= F_p \frac{|x|}{x} + kx & \text{for } |x| > 0 \\
 n(x, \dot{x}) &= [-F_p \quad F_p] & \text{for } |x| = 0
 \end{aligned}
 \tag{2.43}$$

where F_p is the preload force value and k is the proportionality constant which is shown in Figure 2.8, graphically.

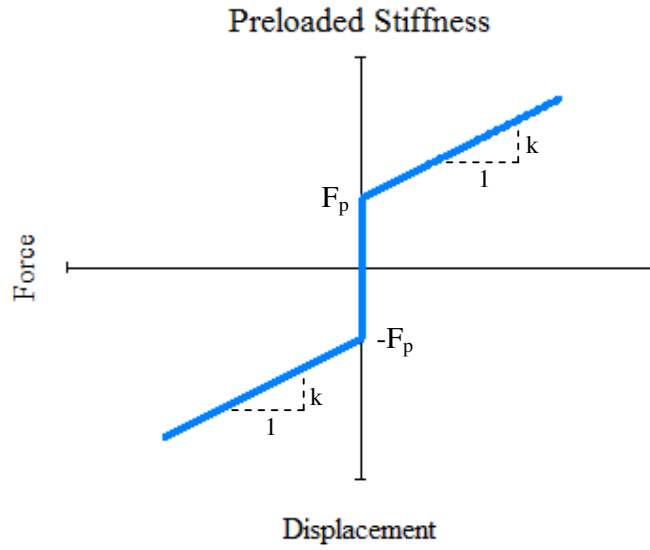


Figure 2.8 Characteristic of a preloaded stiffness element

Typical response distortion due to preloaded stiffness nonlinearity is an increase in the frequency at which resonance occurs as shown in Figure 2.9.

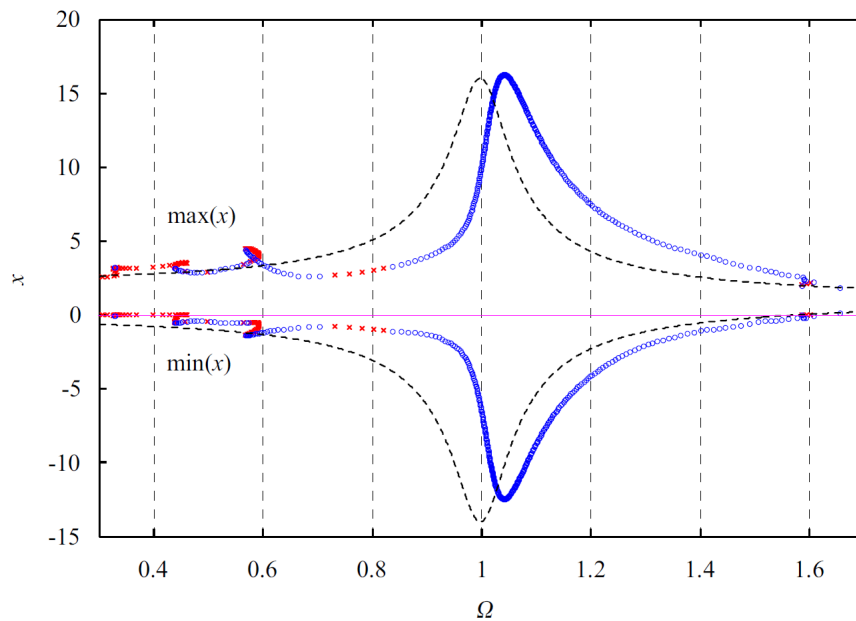


Figure 2.9 Typical response distortion resulting from preloaded stiffness nonlinearity [49]

The single harmonic describing function of a preloaded stiffness element having system coefficients F_p and k defined in Equation 2.43 can be expressed as given below:

$$v_{ab} = \frac{4F_p}{\pi X_1} + k \quad (2.44)$$

2.3 Extension of the Structural Modification Method for Nonlinear Modifications

For the structural modifications introducing additional DOFs into the system, Özgüven [14] developed a formulation based on FRFs. Here, firstly this formulation is summarized and then its extension to nonlinear structural modifications will be explained.

2.3.1 Özgüven's Formulation

For the structural modifications that introduce additional DOFs to a structure, dynamic stiffness matrices of the original and modified structures can be stated as follows, respectively:

$$[\alpha]^{-1} = \begin{bmatrix} \alpha^{aa} & \alpha^{ab} \\ \alpha^{ba} & \alpha^{bb} \end{bmatrix}^{-1} \quad (2.45)$$

$$\begin{bmatrix} \gamma^{aa} & \gamma^{ab} & \gamma^{ac} \\ \gamma^{ba} & \gamma^{bb} & \gamma^{bc} \\ \gamma^{ca} & \gamma^{cb} & \gamma^{cc} \end{bmatrix}^{-1} = \begin{bmatrix} \begin{bmatrix} \alpha^{aa} & \alpha^{ab} \\ \alpha^{ba} & \alpha^{bb} \end{bmatrix}^{-1} & & \\ & 0 & \\ & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \begin{bmatrix} D_{\text{mod}}^{bb} & D_{\text{mod}}^{bc} \\ D_{\text{mod}}^{cb} & D_{\text{mod}}^{cc} \end{bmatrix} \\ 0 & & \end{bmatrix} \quad (2.46)$$

In the equations above, the superscript a denotes DOFs that belong to the original

structure only, superscript b represents DOFs corresponding to connection nodes between the original and the modifying structures, and superscript c represents DOFs that belong to modifying structure only as shown in Figure 2.10.

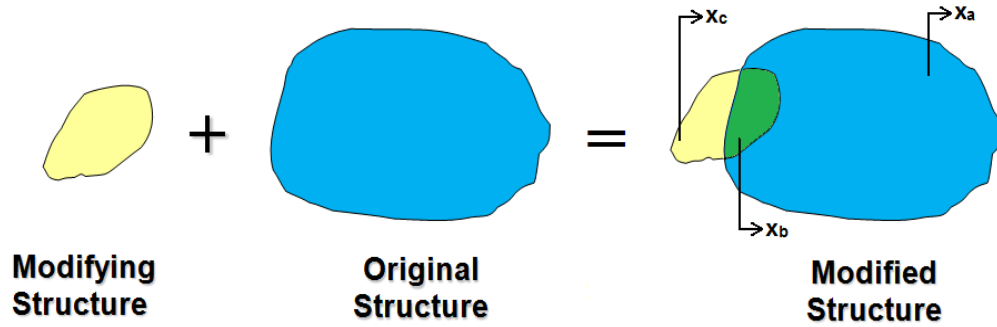


Figure 2.10 Structural modification schematic

When Equation 2.46 is premultiplied by

$$\begin{bmatrix} \alpha^{aa} & \alpha^{ab} & 0 \\ \alpha^{ba} & \alpha^{bb} & 0 \\ 0 & 0 & I \end{bmatrix} \quad (2.47)$$

and postmultiplied by $[\gamma]$,

$$\begin{bmatrix} \alpha^{aa} & \alpha^{ab} & 0 \\ \alpha^{ba} & \alpha^{bb} & 0 \\ 0 & 0 & I \end{bmatrix} [\gamma] = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix} [\gamma] + \begin{bmatrix} 0 & \alpha^{ab} & 0 \\ 0 & \alpha^{bb} & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} D_{\text{mod}} \\ D_{\text{mod}} \\ D_{\text{mod}} \end{bmatrix} [\gamma] \quad (2.48)$$

following equations can be obtained after performing some matrix manipulations [14]:

$$\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \alpha^{bb} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} D_{\text{mod}} \\ D_{\text{mod}} \end{bmatrix} \begin{bmatrix} \gamma^{ba} \\ \gamma^{ca} \end{bmatrix} = \begin{bmatrix} \alpha^{ba} \\ 0 \end{bmatrix} \quad (2.49)$$

$$\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \alpha^{bb} & 0 \\ 0 & I \end{bmatrix} [D_{\text{mod}}] \begin{bmatrix} \gamma^{bb} & \gamma^{bc} \\ \gamma^{cb} & \gamma^{cc} \end{bmatrix} = \begin{bmatrix} \alpha^{bb} & 0 \\ 0 & I \end{bmatrix} \quad (2.50)$$

$$\begin{bmatrix} \gamma^{aa} \end{bmatrix} + \begin{bmatrix} \alpha^{ab} & 0 \end{bmatrix} [D_{\text{mod}}] \begin{bmatrix} \gamma^{ba} \\ \gamma^{ca} \end{bmatrix} = \begin{bmatrix} \alpha^{aa} \end{bmatrix} \quad (2.51)$$

$$\begin{bmatrix} \gamma^{ab} & \gamma^{ac} \end{bmatrix} + \begin{bmatrix} \alpha^{ab} & 0 \end{bmatrix} [D_{\text{mod}}] \begin{bmatrix} \gamma^{bb} & \gamma^{bc} \\ \gamma^{cb} & \gamma^{cc} \end{bmatrix} = \begin{bmatrix} \alpha^{ab} & 0 \end{bmatrix} \quad (2.52)$$

So, receptance matrix of the modified system can be written in the form of its submatrices as:

$$\begin{bmatrix} \gamma^{ba} \\ \gamma^{ca} \end{bmatrix} = \left[\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \alpha^{bb} & 0 \\ 0 & I \end{bmatrix} [D_{\text{mod}}] \right]^{-1} \begin{bmatrix} \alpha^{ba} \\ 0 \end{bmatrix} \quad (2.53)$$

$$\begin{bmatrix} \gamma^{bb} & \gamma^{bc} \\ \gamma^{cb} & \gamma^{cc} \end{bmatrix} = \left[\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \alpha^{bb} & 0 \\ 0 & I \end{bmatrix} [D_{\text{mod}}] \right]^{-1} \begin{bmatrix} \alpha^{bb} & 0 \\ 0 & I \end{bmatrix} \quad (2.54)$$

$$\begin{bmatrix} \gamma^{aa} \end{bmatrix} = \begin{bmatrix} \alpha^{aa} \end{bmatrix} - \begin{bmatrix} \alpha^{ab} & 0 \end{bmatrix} [D_{\text{mod}}] \begin{bmatrix} \gamma^{ba} \\ \gamma^{ca} \end{bmatrix} \quad (2.55)$$

$$\begin{bmatrix} \gamma^{ab} & \gamma^{ac} \end{bmatrix} = \begin{bmatrix} \alpha^{ab} & 0 \end{bmatrix} \left[[I] - [D_{\text{mod}}] \begin{bmatrix} \gamma^{bb} & \gamma^{bc} \\ \gamma^{cb} & \gamma^{cc} \end{bmatrix} \right] \quad (2.56)$$

The above equations indicate that only a single matrix is to be inverted for the calculation of the complete receptance matrix of the modified system. Note that the order of the aforementioned matrix is equal to the number of DOF related with the modified coordinates. For local modifications, order of the matrix to be converted is

quite small compared to the total DOF of the system which is usually the case in real applications.

2.3.2 Extension of the Formulation for Nonlinear Structural Modifications

Consider the equations of motion of a nonlinear modifying structure. The equations of motion for such a structure can be written as given below:

$$[M_{\text{mod}}]\{\ddot{x}\} + i[H_{\text{mod}}]\{x\} + [K_{\text{mod}}]\{x\} + \{N(x, \dot{x})\} = \{f\} \quad (2.57)$$

where $[K_{\text{mod}}]$, $[M_{\text{mod}}]$ and $[H_{\text{mod}}]$ represent stiffness, mass and structural damping matrices of the modifying structure, respectively. When a harmonic force

$$\{f\} = \{F\}e^{i\omega t} \quad (2.58)$$

is applied on the system, the system response as an initial approach, can be assumed to be harmonic at the same frequency:

$$\{x\} = \{X\}e^{i\omega t} \quad (2.59)$$

With regard to this assumption, the internal nonlinear forces can also be assumed to be harmonic at the same frequency:

$$\{N\} = \{S\}e^{i\omega t} \quad (2.60)$$

In their studies, Budak and Özgüven [28, 29] has previously shown that the amplitude vector of the internal nonlinear forces, for certain types of nonlinearity, can be expressed as:

$$\{S\} = [\Delta]\{X\} \quad (2.61)$$

where $[\Delta]$, called “nonlinearity matrix”, is a function of the unknown displacement amplitude vector $\{X\}$. The formulation for $[\Delta]$ was initially suggested by Budak and Özgüven [28, 29] for certain types of nonlinearities, and later generalized for any type of nonlinearity using DFM by Tanrıku et al. [30, 31]. In these studies, the elements of nonlinearity matrix were defined as:

$$\Delta_{kk} = \sum_{m=1}^n v_{km} \quad (2.62)$$

$$\Delta_{km} = -v_{km}, (k \neq m) \quad (2.63)$$

where subscripts k and m represent the general coordinates.

When Equations 2.58, 2.59, 2.60 and 2.61 are substituted into the Equation 2.57, nonlinear internal forces can be included in the analysis by considering an additional equivalent stiffness matrix in the system which is a function of unknown response amplitudes. Then, nonlinear dynamic stiffness matrix of the modifying structure will take the form:

$$[D_{\text{mod}}] = [K_{\text{mod}}] - \omega^2 [M_{\text{mod}}] + i [H_{\text{mod}}] + [\Delta] \quad (2.64)$$

As mentioned in Section 2.2.3, when the nonlinearity matrix is expressed in terms of describing functions defined as a function of unknown response amplitude, $\{X\}$, only an iterative approach is required for the solution. In this thesis, “Fixed Point Iteration” method is applied.

$$[D_{\text{mod}}]_{j+1} = [K_{\text{mod}}] - \omega^2 [M_{\text{mod}}] + i [H_{\text{mod}}] + [\Delta]_j \quad (j = 1, 2, 3, \dots) \quad (2.64)$$

In Equation 2.58, $[D_{\text{mod}}]_{j+1}$ is the nonlinear dynamic stiffness matrix of the

modifying structure at the $(j+1)^{\text{th}}$ iteration step, whereas $[\Delta]_j$ is the nonlinearity matrix determined by using $\{X\}_j$. The initial value of displacement amplitude vector $\{X\}_j$ is taken as the linear solution of the system at the first frequency value. The converged value of $\{X\}_j$ obtained at a previous frequency step is taken as the starting value for the succeeding step. The iterations can be repeated until the percentage displacement error

$$e = \frac{|\{X\}_{j+1} - \{X\}_j|}{|\{X\}_j|} \times 100 \quad (2.65)$$

drops below a certain value. In order to avoid divergence or numerical instability, and also to obtain fast convergence, relaxation is applied to the fixed point iteration [50]:

$$\{X^*\}_{j+1} = \lambda \{X\}_{j+1} + (1-\lambda) \{X\}_j \quad (0 \leq \lambda \leq 1) \quad (2.66)$$

where λ is the relaxation coefficient.

2.3.3 Extension of the Formulation for Nonlinear Structural Couplings

The formulation given above for nonlinear modifications is applied for nonlinear structural coupling by flexible elements by making a simple trick in the algorithm of NLSM/CP. In this section, the approaches used for nonlinear structural coupling with linear and nonlinear elements are individually investigated.

2.3.3.1 Nonlinear Structural Coupling with Linear Elements

The formulation given in section 2.3.2 can directly be used by treating the coupling problem with linear elements as an equivalent structural modification problem as

shown in Figure 2.11. That is, for each connection node of the linear elastic element on the original system a massless node is added to the coupled subsystem.

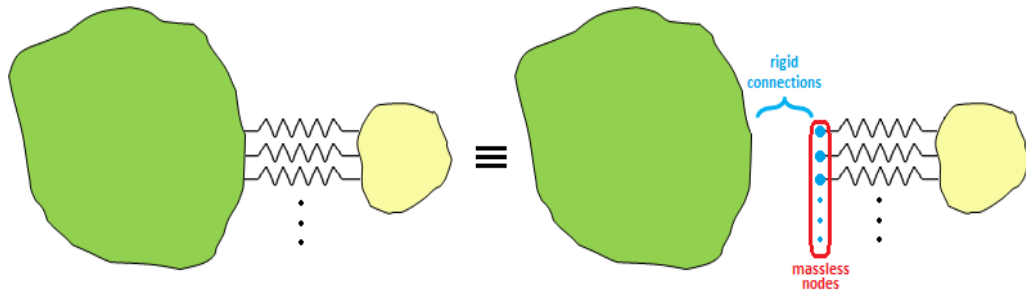


Figure 2.11 Treating a structural coupling problem with linear elements

Here, linear coupling connection DOFs on the original system are grouped node by node. For each of those nodes, an additional massless node is added to the coupled subsystem. Then, the stiffness matrix of the coupled subsystem, $[K_{\text{mod}}]$, is expanded. For example, if p number of massless nodes are added to the coupled subsystem where the DOF per node is q , $p \times q$ number of rows and columns are added to the stiffness matrix of the coupled subsystem. Then, the stiffnesses of linear elastic coupling elements are inserted in proper locations of added rows and columns of $[K_{\text{mod}}]$. The mass, nonlinearity, viscous and structural damping matrices of the coupled subsystem should also be expanded in the same way. However, just zeros will be inserted in their added rows and columns. Finally, by defining additional massless nodes as new rigid connection nodes of the coupled subsystem, the problem can be taken as a nonlinear structural modification problem as defined in section 2.3.2.

2.3.3.2 Nonlinear Structural Coupling with Nonlinear Elements

Similarly, the same formulation given in section 2.3.2 can be used by treating the coupling problem with nonlinear elements as an equivalent structural modification problem as shown in Figure 2.12. That is, for each connection node of the nonlinear

element on the original system a massless node is added to the coupled subsystem as in the previous case.

Here, linear coupling connection DOFs on the original system are grouped node by node. For each of those nodes, an additional massless node is added to the coupled subsystem. Accordingly, again; stiffness, mass, nonlinearity, viscous and structural damping matrices of the coupled subsystem are expanded by adding $p \times q$ number of rows and columns, where p is the number of massless nodes added to the coupled subsystem and q is the DOF per node.

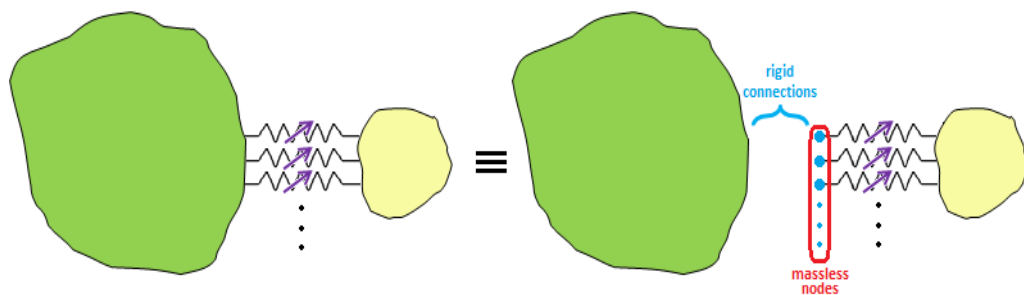


Figure 2.12 Treating a structural coupling problem with nonlinear elements

However this time, the added rows and columns of the nonlinearity matrix $[\Delta]$ will be filled with proper elements representing the nonlinear connection elements. If there are linear stiffness counterparts of the connecting elements, these values will also be properly inserted into the expanded rows and columns of the stiffness matrix of the coupled subsystem. Finally, by defining additional massless nodes as new rigid connection nodes of the coupled subsystem, the problem can be taken as a nonlinear structural modification problem as defined in section 2.3.2.

CHAPTER 3

COMPUTER PROGRAM: NLSM/CP

3.1 Non-Linear Structural Modification/Coupling Program

The theory and the mathematical formulations described in the previous chapters are implemented in MATLAB[®] environment and a computer program, named “NLSM/CP” (Non-Linear Structural Modification/Coupling Program), with a user friendly graphical user interface (GUI) is constructed. The program predicts the response characteristics of nonlinearly modified systems. It is compatible with a standard FE program, ANSYS[®], and it takes system matrices of modifying system and modal data of original structure as an output file of the FE program in the preprocessing stage in order to compute the harmonic nonlinear response of modified system. In this chapter, basic features of NLSM/CP will be introduced. The user manual of the program is given in Appendix A. The logic of the program is summarized in the flow chart shown in Figure 3.1.

3.1.1 Program Description

NLSM/CP is a MATLAB[®] based GUI program which computes the harmonic nonlinear response of a nonlinearly modified structure. The preprocessing stage of the program basically includes the following tasks:

- Selection of the solution parameters,
- Description of the rigid, linear and nonlinear connections and variables,
- Description of the original and modifying systems to be analyzed,

- Description of response coordinates for output,
- Description of the nonlinearities in modifying structure, their coordinates and parameters,
- Description of the external forces and coordinates to which they are applied,

The outputs of NLSM/CP are magnitudes of the response and frequency response values of selected coordinate at predetermined forcing level over a frequency range. Program is capable of analyzing modified systems with the following types of nonlinearities:

- Cubic stiffness,
- Coulomb damping,
- Piecewise linear stiffness,
- Preloaded stiffness,
- Arctan stiffness,

In addition, user can also define linear local viscous dampers between coordinates.

3.1.2 Preprocessing

The preprocessing stage of NLSM/CP involves the preparation of the program for the solution procedure. The starting point of the preprocessing stage is to introduce the original and modifying systems to be analyzed, namely, the modal data of the original system and the system matrices of the modifying system. For the original structure, the program uses two different text files which include modal matrix information and natural frequency information. These two text files are extracted from ANSYS® by using a macro file written in this study. The FRF matrix is constructed for the original structure by using following expression:

$$\alpha_{ij}(\omega) = \sum_{r=1}^N \frac{\phi_{ir}\phi_{jr}}{\omega_r^2 - \omega^2 + i\eta\omega_r^2} \quad (3.1)$$

Here, the number of modes used in calculating original system FRF should be defined by the user. Note that, the number of nodes, DOF per node and the number of mode shapes to calculate FRFs has to match with the information in the files created by using ANSYS®.

On the other hand, for the modifying system, the program uses two files which store the stiffness and mass matrices of the modifying structure. These files are extracted from the output files of ANSYS® modal analysis which has an extension of “*.full”. However, in order to have these stiffness and mass matrices, the file named “*rdfull.f*” which is in the ANSYS® installation directory, should be compiled with Intel Fortran compiler to create the “*rdfull.exe*” file. Then by running this “*rdfull.exe*” file with the result file of ANSYS® modal analysis which has an extension of “*.full” in a separate folder, these stiffness and mass matrices are extracted. Note again that, the number of nodes and DOF per node information has to match with the information in the files created by using ANSYS®.

The structural damping is modeled as proportional damping and the loss factor of the material can be entered as a constant value input. If structural damping is response dependent and any data set about its variation with respect to displacement values are available, those can also be loaded to the program. Then, program interpolates a polynomial function to this data set and structural damping corresponding to any displacement value can be obtained for each iteration at all frequency steps. Initial solution parameters such as:

- lower and upper limits of the frequency range of interest
- number of frequency points
- allowable percentage error tolerance
- maximum iteration number
- relaxation number for converging and diverging solutions

can be entered through the GUI window at the same stage.

Rigid connection nodes and linear and nonlinear connection DOFs regarding both original and modifying structures should also be entered as the next step in preprocessing stage. Nonlinearity types given in Section 2.2 can be easily defined by an illustrative GUI window that consists of schematic diagrams and describing parameters for different types of nonlinearities (Figure 3.9 and Figure 3.27).

Afterwards, whether the DOF whose direct point FRF and response is desired is on the original structure or modifying structure should be decided. The coordinate of this DOF should also be defined in the program according to the DOF numbering system of the structure to which it belongs. Then using the "Properties are defined!" button, program loads all those inputs.

As the next preprocessing step, nonlinearities exist in modifying structure can be defined by using an interface similar to the one used in selecting nonlinear connections. Last step in preprocessing stage is defining external forces and their application coordinates. Here, external forcing coordinates should be defined separately both for the original and modifying systems considering their own initial (before modification/coupling) coordinate systems. During solution, program changes the number of those coordinates by renumbering them into coordinates of the modified system.

3.1.3 Solution Algorithm

This section refers to points worth mentioning about the solution algorithm of NLSM/CP. To begin with, NLSM/CP needs only the user defined necessary coordinates that belong to the original system during calculation, since original system data is available in the form of FRFs. These coordinates are the ones having rigid, linear or nonlinear connections, on which an external forcing is applied and for which response and FRF computations will be performed. The rest of the coordinates will not be used in the computation, therefore the program reduces the size of the FRF matrix of the original system. This does not affect the accuracy of the results.

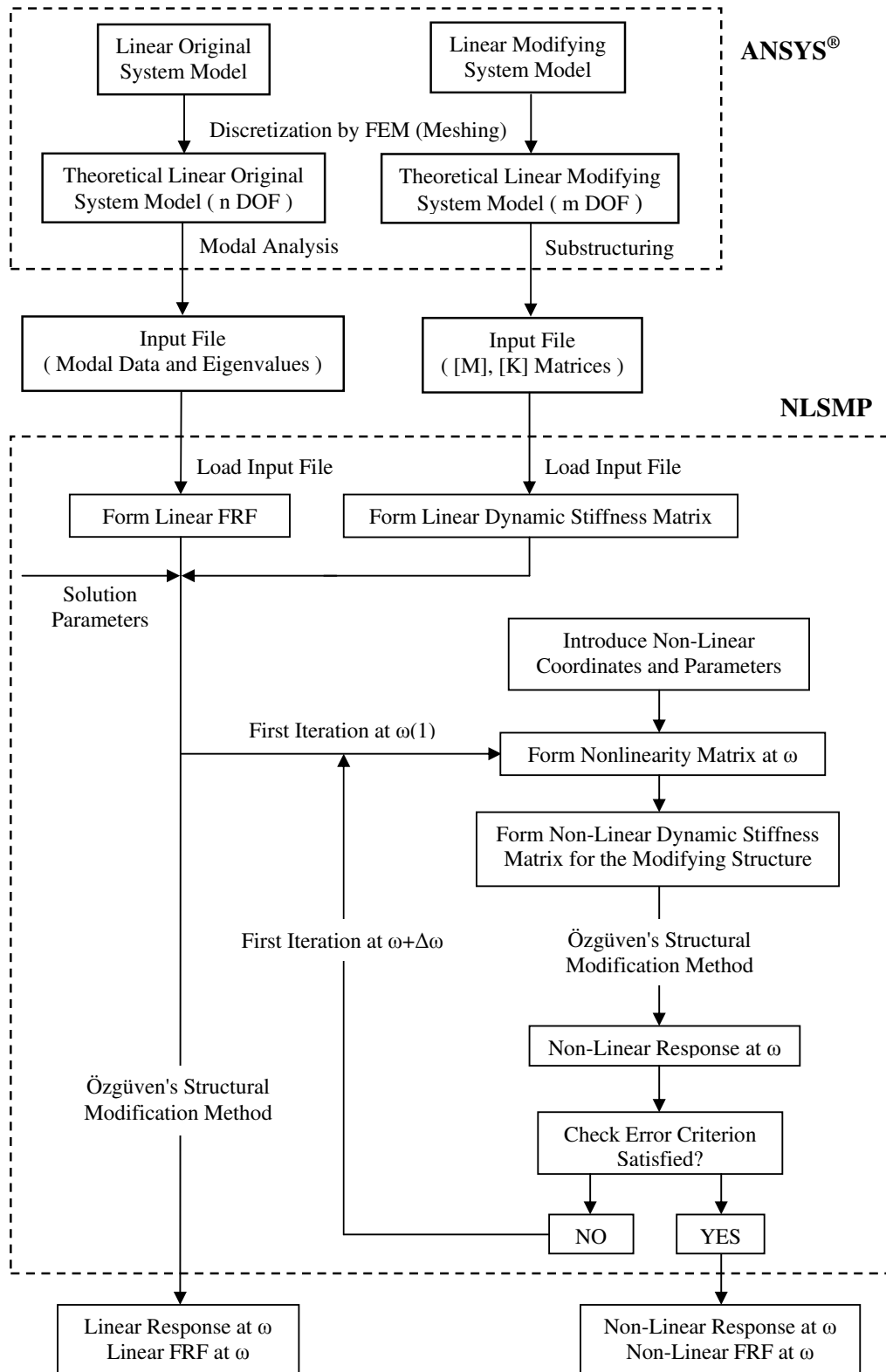


Figure 3.1 Flow diagram of nonlinear structural modification/coupling algorithm

The calculations in the solution algorithm are performed with predefined constant frequency increment. Both low-to-high and high-to-low frequency swept nonlinear responses and FRFs are calculated through single harmonic solution.

The converging and diverging relaxation numbers are switched by the program by comparing the error obtained at immediate and previous iteration steps [50]. After relaxation, the weighted response calculated using the ones obtained in previous and immediate iteration steps is taken as initial displacement value for the next iteration step (Equation 2.66). When the convergence criterion is eventually satisfied, the program stops doing iteration and steps into the next frequency value in the frequency range.

3.1.4 Post Processing

The outputs of the program are linear and nonlinear (both low-to-high and high-to-low frequency sweeps) response and FRF plots regarding modified system. The solution time for the calculation is also displayed on the GUI window at the end of calculation.

3.2 User's Manual of the Program: NLSM/CP

In this section, the user's manual of the NLSM/CP developed in the study in order to apply the approach proposed is given.

3.2.1 Definition of the Files Used in the MATLAB® Program

NLSM/CP is compatible with a standard FE program, so that it uses some input files that are extracted from the modal analysis performed in ANSYS®. Furthermore, in order to extract these files from ANSYS®, some macro files and “.exe” files are used. The definitions of these files are given below.

- ***FREQ_ORG.txt***: The text file that stores the natural frequencies of the original structure.
- ***MODAL_ORG.txt***: The text file that stores the mode shape vectors of the original structure.
- ***NODENO_ORG.txt***: The text file that stores the node numbers of the required nodal FRFs of the original structure.
- ***MASS_MODIF.matrix***: The file that stores the mass matrix information of the modifying structure.
- ***STIFFNESS_MODIF.matrix***: The file that stores the stiffness matrix information of the modifying structure.
- ***NODE_READ.txt***: The text file generated by the user that should be read by the ANSYS® after the modal analysis of the original structure in order to input ANSYS® the node number information of required nodal FRFs.
- ***ModalDataExport_Org.txt***: Macro file that should be read by the ANSYS® after the modal analysis of the original structure in order ANSYS® to extract the “*FREQ_ORG.txt*”, “*MODAL_ORG.txt*” and “*NODENO_ORG.txt*” files.
- ***userprog.exe***: This is the executable file that should be run after the modal analysis of the modifying structure in order ANSYS® to extract the “*STIFFNESS_MODIF.matrix*” and “*MASS_MODIF.matrix*” files.

3.2.2 Graphical User Interface (GUI)

Before running the NLSM/CP, the following steps should be performed:

- In order to extract the “*FREQ_ORG.txt*”, “*MODAL_ORG.txt*” and “*NODENO_ORG.txt*” files from the ANSYS®, “*NODE_READ.txt*” and “*ModalDataExport_Org.txt*” macro files should respectively be read by the ANSYS® after the modal analysis of the original structure, respectively.
- The “**.full*” file, obtained through the modal analysis of the modifying structure performed in the ANSYS®, should be renamed as “*file.full*”. By running the “*userprog.exe*” file within the same directory with “*file.full*” file, “*STIFFNESS_MODIF.matrix*” and “*MASS_MODIF.matrix*” files should be extracted.

Since all these extracted files are read by NLSM/CP, they should be in the same directory with the source codes of NLSM/CP. After performing the steps given above, NLSM/CP can be run. The application of the program is shown below as a self generated preprocessing stage covering all the options of the NLSM/CP step-by-step. Note that data inputted at each step is circled with red on the following snapshots of the GUI window.

Set the number of frequency points that will be used in the analysis (Figure 3.2).

The screenshot shows a multi-panel software interface. The 'INITIAL INPUT' panel is highlighted with a yellow border and contains the following fields:

- Number of Frequency Points: 2000 (circled in red)
- Lower Frequency Limit [Hz]:
- Upper Frequency Limit [Hz]:
- Maximum Percentage Error [%]:
- Maximum Iteration Number:
- Relaxation at Converging Points:
- Relaxation at Diverging Points:

 Other panels include:

- STRUCTURAL DAMPING:** Radio buttons for 'Constant Damping' and 'Variable Damping', and a 'Structural Damping Coefficient' field.
- FINITE ELEMENT DATA:** Fields for 'Number of Nodes of the Original System', 'Number of Eigenvectors Extracted', 'Number of Nodes of the Modifying System', and 'Number of DOF per Node'.
- FORCE:** Radio buttons for 'On Modifying Structure' and 'On Original Structure', and a 'Load' button.
- NONLINEARITIES ON MODIFYING STRUCTURE:** Fields for 'Nonlinearity Between', 'DOF Numbers', and 'Nonlinearity Type'.
- CONNECTIONS:** Sections for 'Nonlinear Connection DOFs', 'Rigid Connections', and 'Linear Elastic Connection DOFs'.
- FINALIZING:** Buttons for 'SOLVE', 'DRAW', 'INFO', and 'EXIT', along with legend options for 'TR' and 'ENG'.

Figure 3.2 Setting the number of frequency points

Set the lower and upper frequency limits for the analysis (Figure 3.3).

The screenshot shows the same software interface as Figure 3.2. In this view, the 'INITIAL INPUT' panel is highlighted with a yellow border, and the 'Lower Frequency Limit [Hz]' and 'Upper Frequency Limit [Hz]' fields are circled in red, showing values 0 and 500 respectively. The 'Number of Frequency Points' field now shows 2000. The other panels remain the same as in Figure 3.2.

Figure 3.3 Setting the lower and upper frequency limits

Set the maximum percentage error and iteration number (Figure 3.4).

The screenshot displays a multi-panel software interface. The 'INITIAL INPUT' panel is the primary focus, containing the following fields:

- Number of Frequency Points: 2000
- Lower Frequency Limit [Hz]: 0
- Upper Frequency Limit [Hz]: 500
- Maximum Percentage Error [%]: 0.01 (circled in red)
- Maximum Iteration Number: 1000 (circled in red)
- Relaxation at Converging Points: [empty]
- Relaxation at Diverging Points: [empty]

Other visible panels include:

- STRUCTURAL DAMPING:** Radio buttons for 'Constant Damping' and 'Variable Damping'. A 'Structural Damping Coefficient' field is present.
- FINITE ELEMENT DATA:** Fields for 'Number of Nodes of the Original System', 'Number of Eigenvectors Extracted', 'Number of Nodes of the Modifying System', and 'Number of DOF per Node'.
- FORCE:** Radio buttons for 'On Modifying Structure' and 'On Original Structure'. Includes 'Coordinate' and 'Value [N]' fields.
- NONLINEARITIES ON MODIFYING STRUCTURE:** Fields for 'Nonlinearity Between', 'DOF Numbers', and 'Nonlinearity Type'.
- CONNECTIONS:** Sections for 'Nonlinear Connection DOFs', 'Rigid Connections', and 'Linear Elastic Connection DOFs'.
- FINALIZING:** 'SOLVE' button, legend options for 'TR' and 'ENG', and 'DRAW', 'INFO', 'EXIT' buttons.

Figure 3.4 Setting the maximum percentage error and iteration number

Set the relaxation coefficients at diverging and converging points (Figure 3.5).

This screenshot is identical to Figure 3.4, but with the following changes in the 'INITIAL INPUT' panel:

- Relaxation at Converging Points: 0.7 (circled in red)
- Relaxation at Diverging Points: 0.1 (circled in red)

The rest of the interface, including the 'STRUCTURAL DAMPING', 'FINITE ELEMENT DATA', 'FORCE', 'NONLINEARITIES ON MODIFYING STRUCTURE', 'CONNECTIONS', and 'FINALIZING' panels, remains the same as in Figure 3.4.

Figure 3.5 Setting the relaxation coefficients at diverging and converging points

Set the linear elastic connection DOF numbers of the original and modifying structures with stiffness values, in matching order (Figure 3.6).

The screenshot shows the following settings in the 'CONNECTIONS' panel:

- Linear Elastic Connection DOFs:**
 - DOFs on Original Structure: 2,5,8
 - Corresponding DOFs on Modifying Structure: 8,11,14
 - Linear Elastic Stiffnesses, respectively: 1000,2000,1500

Figure 3.6 Setting the linear elastic connection DOF numbers with stiffness values

Set the rigid connection node numbers of the original and modifying structures, in matching order (Figure 3.7).

The screenshot shows the following settings in the 'CONNECTIONS' panel:

- Rigid Connections:**
 - Nodes on Original System: 55,88
 - Corresponding Nodes on Modifying System: 8,12

Figure 3.7 Setting the rigid connection node numbers

Set the nonlinear connection DOF numbers of the original and modifying structures and define the type of nonlinear element (Figure 3.8).

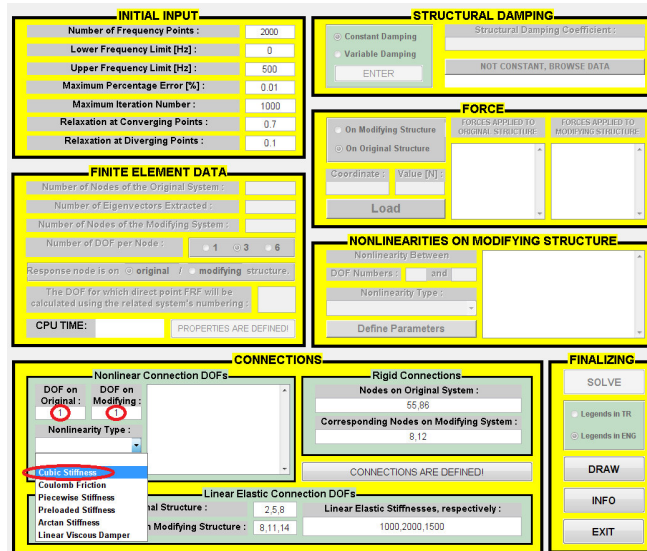


Figure 3.8 Setting the nonlinear connection DOF numbers and defining types of nonlinear elements

Press the “Define Parameters” button each time after entering nonlinear connection DOFs and set parameters of the related nonlinear element (Figure 3.9).

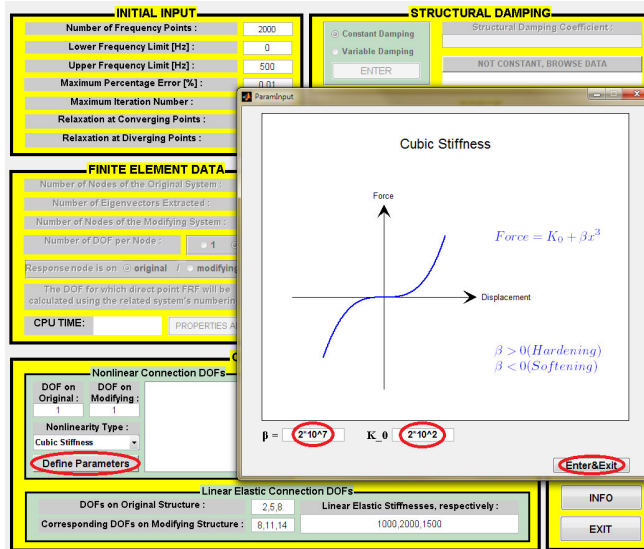


Figure 3.9 Setting the parameters of the related nonlinear element

Press the “CONNECTIONS ARE DEFINED” button and so that the program reads all the connection data defined by the user (Figure 3.10).

The screenshot shows a software interface with several panels. The 'CONNECTIONS ARE DEFINED' button in the 'CONNECTIONS' panel is highlighted with a red oval. The 'INITIAL INPUT' panel contains the following data:

Number of Frequency Points :	2000
Lower Frequency Limit [Hz] :	0
Upper Frequency Limit [Hz] :	500
Maximum Percentage Error [%] :	0.01
Maximum Iteration Number :	1000
Relaxation at Converging Points :	0.7
Relaxation at Diverging Points :	0.1

The 'FINITE ELEMENT DATA' panel shows:

- Number of Nodes of the Original System : []
- Number of Eigenvectors Extracted : []
- Number of Nodes of the Modifying System : []
- Number of DOF per Node : 1 3 6
- Response node is on original / modifying structure.
- The DOF for which direct point FRF will be calculated using the related system's numbering : []
- CPU TIME: [] PROPERTIES ARE DEFINED!

The 'CONNECTIONS' panel shows:

- Nonlinear Connection DOFs: DOF on Original: [], DOF on Modifying: [], Nonlinearity Type: [], Define Parameters: []
- Rigid Connections: Nodes on Original System: 55,86; Corresponding Nodes on Modifying System: 8,12
- Linear Elastic Connection DOFs: DOFs on Original Structure: 2,5,8; Corresponding DOFs on Modifying Structure: 8,11,14; Linear Elastic Stiffnesses, respectively: 1000,2000,1500

The 'FINALIZING' panel contains buttons: SOLVE, LEGENDS IN TR, LEGENDS IN ENG, DRAW, INFO, EXIT.

Figure 3.10 Letting the program read all the defined connection data

Set the number of nodes of the original system (Figure 3.11).

The screenshot is identical to Figure 3.10, but the 'Number of Nodes of the Original System' field in the 'FINITE ELEMENT DATA' panel is highlighted with a red circle, and the value '2356' is entered in the field.

Figure 3.11 Setting the number of nodes of the original system

Set the number of eigenvectors extracted for the original system (Figure 3.12).

INITIAL INPUT

Number of Frequency Points :	2000
Lower Frequency Limit [Hz] :	0
Upper Frequency Limit [Hz] :	500
Maximum Percentage Error [%] :	0.01
Maximum Iteration Number :	1000
Relaxation at Converging Points :	0.7
Relaxation at Diverging Points :	0.1

FINITE ELEMENT DATA

Number of Nodes of the Original System :	2356
Number of Eigenvectors Extracted :	500
Number of Nodes of the Modifying System :	
Number of DOF per Node :	1 3 6

Response node is on original / modifying structure.

The DOF for which direct point FRF will be calculated using the related system's numbering :

CPU TIME: PROPERTIES ARE DEFINED!

STRUCTURAL DAMPING

Constant Damping
 Variable Damping

Structural Damping Coefficient:

ENTER NOT CONSTANT, BROWSE DATA

FORCE

On Modifying Structure
 On Original Structure

FORCES APPLIED TO ORIGINAL STRUCTURE FORCES APPLIED TO MODIFYING STRUCTURE

Coordinate: Value [N]

Load

NONLINEARITIES ON MODIFYING STRUCTURE

Nonlinearity Between

DOF Numbers: and

Nonlinearity Type:

Define Parameters

CONNECTIONS

Nonlinear Connection DOFs

DOF on Original: DOF on Modifying: Cubic Stiffness: 1,1 2¹⁰⁴⁷, 2¹⁰⁴⁸

Nonlinearity Type:

Define Parameters

Rigid Connections

Nodes on Original System : 55,86

Corresponding Nodes on Modifying System : 8,12

CONNECTIONS ARE DEFINED!

Linear Elastic Connection DOFs

DOFs on Original Structure : 2,5,8

Corresponding DOFs on Modifying Structure : 8,11,14

Linear Elastic Stiffnesses, respectively : 1000,2000,1500

FINALIZING

SOLVE

Legends in TR
 Legends in ENG

DRAW

INFO

EXIT

Figure 3.12 Setting the number of eigenvectors extracted for the original system

Set the number of nodes of the modifying system (Figure 3.13).

INITIAL INPUT

Number of Frequency Points :	2000
Lower Frequency Limit [Hz] :	0
Upper Frequency Limit [Hz] :	500
Maximum Percentage Error [%] :	0.01
Maximum Iteration Number :	1000
Relaxation at Converging Points :	0.7
Relaxation at Diverging Points :	0.1

FINITE ELEMENT DATA

Number of Nodes of the Original System :	2356
Number of Eigenvectors Extracted :	500
Number of Nodes of the Modifying System :	123
Number of DOF per Node :	1 3 6

Response node is on original / modifying structure.

The DOF for which direct point FRF will be calculated using the related system's numbering :

CPU TIME: PROPERTIES ARE DEFINED!

STRUCTURAL DAMPING

Constant Damping
 Variable Damping

Structural Damping Coefficient:

ENTER NOT CONSTANT, BROWSE DATA

FORCE

On Modifying Structure
 On Original Structure

FORCES APPLIED TO ORIGINAL STRUCTURE FORCES APPLIED TO MODIFYING STRUCTURE

Coordinate: Value [N]

Load

NONLINEARITIES ON MODIFYING STRUCTURE

Nonlinearity Between

DOF Numbers: and

Nonlinearity Type:

Define Parameters

CONNECTIONS

Nonlinear Connection DOFs

DOF on Original: DOF on Modifying: Cubic Stiffness: 1,1 2¹⁰⁴⁷, 2¹⁰⁴⁸

Nonlinearity Type:

Define Parameters

Rigid Connections

Nodes on Original System : 55,86

Corresponding Nodes on Modifying System : 8,12

CONNECTIONS ARE DEFINED!

Linear Elastic Connection DOFs

DOFs on Original Structure : 2,5,8

Corresponding DOFs on Modifying Structure : 8,11,14

Linear Elastic Stiffnesses, respectively : 1000,2000,1500

FINALIZING

SOLVE

Legends in TR
 Legends in ENG

DRAW

INFO

EXIT

Figure 3.13 Setting the number of the nodes of modifying system

Choose number of DOF(s) per node (Figure 3.14).

The screenshot displays a software interface with several panels:

- INITIAL INPUT:** Number of Frequency Points: 2000; Lower Frequency Limit [Hz]: 0; Upper Frequency Limit [Hz]: 500; Maximum Percentage Error [%]: 0.01; Maximum Iteration Number: 1000; Relaxation at Converging Points: 0.7; Relaxation at Diverging Points: 0.1.
- STRUCTURAL DAMPING:** Constant Damping selected; Structural Damping Coefficient: [empty]; Variable Damping: NOT CONSTANT, BROWSE DATA; ENTER button.
- FINITE ELEMENT DATA:** Number of Nodes of the Original System: 2356; Number of Eigenvectors Extracted: 500; Number of Nodes of the Modifying System: 123; Number of DOF per Node: 1 | 3 | 6 (6 is selected); Response node is on original / modifying structure; The DOF for which direct point FRF will be calculated using the related system's numbering: [empty]; CPU TIME: [empty]; PROPERTIES ARE DEFINED!
- FORCE:** On Modifying Structure selected; FORCES APPLIED TO ORIGINAL STRUCTURE: [empty]; FORCES APPLIED TO MODIFYING STRUCTURE: [empty]; Coordinate: Value [N]: [empty]; Load button.
- NONLINEARITIES ON MODIFYING STRUCTURE:** Nonlinearity Between: [empty]; DOF Numbers: [empty] and [empty]; Nonlinearity Type: [empty]; Define Parameters button.
- CONNECTIONS:**
 - Nonlinear Connection DOFs:** DOF on Original: [empty]; DOF on Modifying: Cubic Stiffness: $1.1 \cdot 10^7 \cdot 2 \cdot 10^6 \cdot 2^2$; Nonlinearity Type: [empty]; Define Parameters button.
 - Rigid Connections:** Nodes on Original System: 55,88; Corresponding Nodes on Modifying System: 8,12; CONNECTIONS ARE DEFINED!
 - Linear Elastic Connection DOFs:** DOFs on Original Structure: 2,5,8; Linear Elastic Stiffnesses, respectively: 1000,2000,1500; Corresponding DOFs on Modifying Structure: 8,11,14.
- FINALIZING:** SOLVE button; Legends in TR / ENG; DRAW button; INFO button; EXIT button.

Figure 3.14 Choosing number of DOF(s) per node

Choose whether the DOF for which FRF will be calculated is on the original or modifying structure (Figure 3.15).

This screenshot is identical to Figure 3.14, but with the following change in the 'FINITE ELEMENT DATA' panel:

- FINITE ELEMENT DATA:** Response node is on original / modifying structure. (The 'original' radio button is selected).

Figure 3.15 Choosing whether the response node is on original or modifying structure

Set the DOF for which FRF will be calculated using the related system's numbering (Figure 3.16).

INITIAL INPUT

Number of Frequency Points :	2000
Lower Frequency Limit [Hz] :	0
Upper Frequency Limit [Hz] :	500
Maximum Percentage Error [%] :	0.01
Maximum Iteration Number :	1000
Relaxation at Converging Points :	0.7
Relaxation at Diverging Points :	0.1

FINITE ELEMENT DATA

Number of Nodes of the Original System :	2356
Number of Eigenvectors Extracted :	500
Number of Nodes of the Modifying System :	123
Number of DOF per Node :	1 3 6

Response node is on original / modifying structure.

The DOF for which direct point FRF will be calculated using the related system's numbering : **1209**

CPU TIME: PROPERTIES ARE DEFINED!

STRUCTURAL DAMPING

Constant Damping
 Variable Damping

Structural Damping Coefficient: NOT CONSTANT, BROWSE DATA

ENTER

FORCE

On Modifying Structure
 On Original Structure

FORCES APPLIED TO ORIGINAL STRUCTURE FORCES APPLIED TO MODIFYING STRUCTURE

Coordinate: Value [N]

Load

NONLINEARITIES ON MODIFYING STRUCTURE

Nonlinearity Between: DOF Numbers: and: Nonlinearity Type: Define Parameters

CONNECTIONS

Nonlinear Connection DOFs

DOF on Original: DOF on Modifying: Nonlinearity Type: Define Parameters

Rigid Connections

Nodes on Original System: 55 89
Corresponding Nodes on Modifying System: 8,12
CONNECTIONS ARE DEFINED!

Linear Elastic Connection DOFs

DOFs on Original Structure: 2,5,8
Corresponding DOFs on Modifying Structure: 8,11,14
Linear Elastic Stiffnesses, respectively: 1000,2000,1500

FINALIZING

SOLVE
LEGENDS IN TR
LEGENDS IN ENG
DRAW
INFO
EXIT

Figure 3.16 Setting the response DOF

Choose whether the structural damping is constant or response dependent and press "ENTER" button (Figure 3.17).

INITIAL INPUT

Number of Frequency Points :	2000
Lower Frequency Limit [Hz] :	0
Upper Frequency Limit [Hz] :	500
Maximum Percentage Error [%] :	0.01
Maximum Iteration Number :	1000
Relaxation at Converging Points :	0.7
Relaxation at Diverging Points :	0.1

FINITE ELEMENT DATA

Number of Nodes of the Original System :	2356
Number of Eigenvectors Extracted :	500
Number of Nodes of the Modifying System :	123
Number of DOF per Node :	1 3 6

Response node is on original / modifying structure.

The DOF for which direct point FRF will be calculated using the related system's numbering : 1209

CPU TIME: PROPERTIES ARE DEFINED!

STRUCTURAL DAMPING

Constant Damping
 Variable Damping

Structural Damping Coefficient: NOT CONSTANT, BROWSE DATA

ENTER

FORCE

On Modifying Structure
 On Original Structure

FORCES APPLIED TO ORIGINAL STRUCTURE FORCES APPLIED TO MODIFYING STRUCTURE

Coordinate: Value [N]

Load

NONLINEARITIES ON MODIFYING STRUCTURE

Nonlinearity Between: DOF Numbers: and: Nonlinearity Type: Define Parameters

CONNECTIONS

Nonlinear Connection DOFs

DOF on Original: DOF on Modifying: Nonlinearity Type: Define Parameters

Rigid Connections

Nodes on Original System: 55 89
Corresponding Nodes on Modifying System: 8,12
CONNECTIONS ARE DEFINED!

Linear Elastic Connection DOFs

DOFs on Original Structure: 2,5,8
Corresponding DOFs on Modifying Structure: 8,11,14
Linear Elastic Stiffnesses, respectively: 1000,2000,1500

FINALIZING

SOLVE
LEGENDS IN TR
LEGENDS IN ENG
DRAW
INFO
EXIT

Figure 3.17 Choosing whether the structural damping coefficient is constant or response dependent

If "Constant Damping" option is chosen, set the constant structural damping coefficient of the whole system (Figure 3.18).

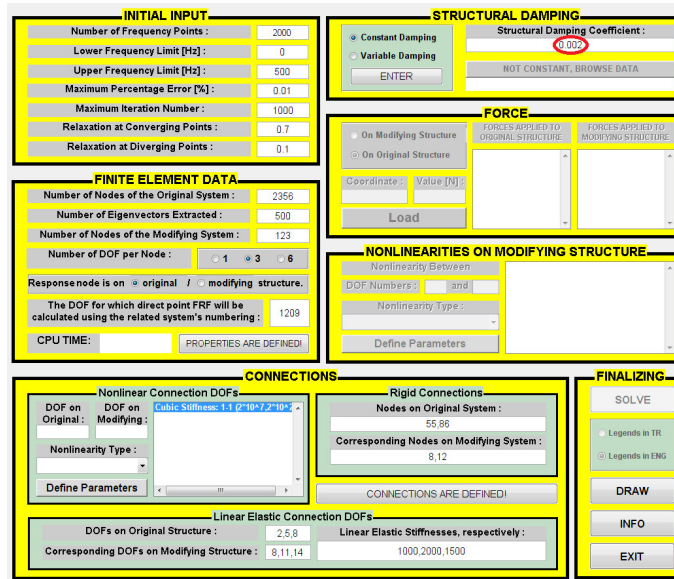


Figure 3.18 Setting the constant structural damping coefficient

If "Variable Damping" option is chosen, press the "NOT CONSTANT, BROWSE DATA" button and browse the Excel file which has this data (Figure 3.19).

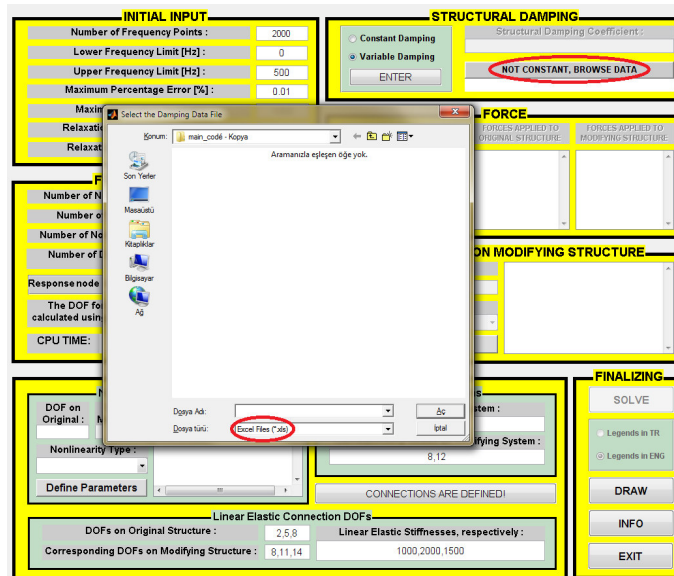


Figure 3.19 Setting the response dependent structural damping data

Press the "PROPERTIES ARE DEFINED" button to accept all the data inputted up to here (Figure 3.20).

INITIAL INPUT

Number of Frequency Points : 2000
 Lower Frequency Limit [Hz] : 0
 Upper Frequency Limit [Hz] : 500
 Maximum Percentage Error [%] : 0.01
 Maximum Iteration Number : 1000
 Relaxation at Converging Points : 0.7
 Relaxation at Diverging Points : 0.1

FINITE ELEMENT DATA

Number of Nodes of the Original System : 2356
 Number of Eigenvectors Extracted : 500
 Number of Nodes of the Modifying System : 123
 Number of DOF per Node : 1 3 6
 Response node is on original / modifying structure.
 The DOF for which direct point FRF will be calculated using the related system's numbering : 1209
 CPU TIME: PROPERTIES ARE DEFINED

STRUCTURAL DAMPING

Constant Damping
 Variable Damping
 Structural Damping Coefficient : 0.002
 ENTER NOT CONSTANT, BROWSE DATA

FORCE

On Modifying Structure
 On Original Structure
 Coordinate: Value [N]:
 Load

NONLINEARITIES ON MODIFYING STRUCTURE

Nonlinearity Between
 DOF Numbers: and
 Nonlinearity Type:
 Define Parameters

CONNECTIONS

Nonlinear Connection DOFs

DOF on Original: DOF on Modifying: Cubic Stiffness: $1.1 \cdot 10^{-7} / 2 \cdot 10^2$
 Nonlinearity Type:
 Define Parameters

Rigid Connections

Nodes on Original System : 55,86
 Corresponding Nodes on Modifying System : 8,12
 CONNECTIONS ARE DEFINED

Linear Elastic Connection DOFs

DOFs on Original Structure : 2,5,8
 Corresponding DOFs on Modifying Structure : 8,11,14
 Linear Elastic Stiffnesses, respectively : 1000,2000,1500

FINALIZING

SOLVE
 Legends in TR
 Legends in ENG
 DRAW
 INFO
 EXIT

Figure 3.20 Accepting all the data inputted up to here

Choose whether the excitation is on the original or modifying structure (Figure 3.21).

INITIAL INPUT

Number of Frequency Points : 2000
 Lower Frequency Limit [Hz] : 0
 Upper Frequency Limit [Hz] : 500
 Maximum Percentage Error [%] : 0.01
 Maximum Iteration Number : 1000
 Relaxation at Converging Points : 0.7
 Relaxation at Diverging Points : 0.1

FINITE ELEMENT DATA

Number of Nodes of the Original System : 2356
 Number of Eigenvectors Extracted : 500
 Number of Nodes of the Modifying System : 123
 Number of DOF per Node : 1 3 6
 Response node is on original / modifying structure.
 The DOF for which direct point FRF will be calculated using the related system's numbering : 1209
 CPU TIME: PROPERTIES ARE DEFINED

STRUCTURAL DAMPING

Constant Damping
 Variable Damping
 Structural Damping Coefficient : 0.002
 ENTER NOT CONSTANT, BROWSE DATA

FORCE

On Modifying Structure
 On Original Structure
 Coordinate: Value [N]:
 Load

NONLINEARITIES ON MODIFYING STRUCTURE

Nonlinearity Between
 DOF Numbers: and
 Nonlinearity Type:
 Define Parameters

CONNECTIONS

Nonlinear Connection DOFs

DOF on Original: DOF on Modifying: Cubic Stiffness: $1.1 \cdot 10^{-7} / 2 \cdot 10^2$
 Nonlinearity Type:
 Define Parameters

Rigid Connections

Nodes on Original System : 55,86
 Corresponding Nodes on Modifying System : 8,12
 CONNECTIONS ARE DEFINED

Linear Elastic Connection DOFs

DOFs on Original Structure : 2,5,8
 Corresponding DOFs on Modifying Structure : 8,11,14
 Linear Elastic Stiffnesses, respectively : 1000,2000,1500

FINALIZING

SOLVE
 Legends in TR
 Legends in ENG
 DRAW
 INFO
 EXIT

Figure 3.21 Choosing whether the excitation is on original or modifying structure

Set the coordinate on which an external force is applied (Figure 3.22).

The screenshot shows the software interface with several panels. The 'INITIAL INPUT' panel contains parameters like Number of Frequency Points (2000), Lower Frequency Limit (0 Hz), Upper Frequency Limit (500 Hz), Maximum Percentage Error (0.01%), Maximum Iteration Number (1000), Relaxation at Converging Points (0.7), and Relaxation at Diverging Points (0.1). The 'FINITE ELEMENT DATA' panel shows Number of Nodes of the Original System (2358), Number of Eigenvectors Extracted (500), Number of Nodes of the Modifying System (123), and Number of DOF per Node (3). The 'STRUCTURAL DAMPING' panel has Constant Damping selected with a Structural Damping Coefficient of 0.002. The 'FORCE' panel is the focus, with 'On Original Structure' selected and 'Coordinate' set to 505. The 'CONNECTIONS' panel shows Nonlinear Connection DOFs and Rigid Connections. The 'FINALIZING' panel has 'SOLVE' selected.

Figure 3.22 Setting the external force application coordinate

Set the magnitude of the external force applied (Figure 3.23).

This screenshot is similar to Figure 3.22 but with the 'Value [N]' in the 'FORCE' panel set to 12. The 'Coordinate' remains 505. The 'CONNECTIONS' panel now shows 'Cubic Stiffness: 1.1 2*10^7 2*10^8' for Nonlinear Connection DOFs. The 'FINALIZING' panel remains the same with 'SOLVE' selected.

Figure 3.23 Setting the magnitude of the external force applied

Press the "Load" button to add the amplitude and application DOF of the external force to the list of applied forces (Figure 3.24).

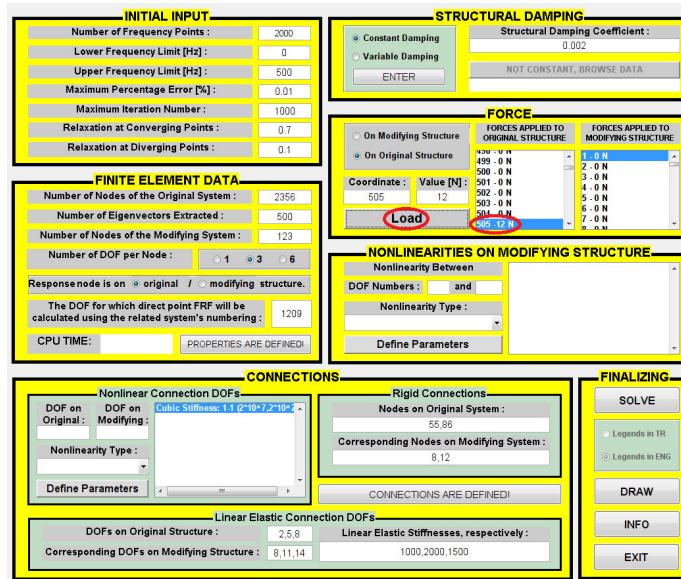


Figure 3.24 Adding the external force created to the list of applied forces

Set the two coordinates of the nonlinear element involved in the modifying structure and define the type of nonlinearity (Figure 3.25).

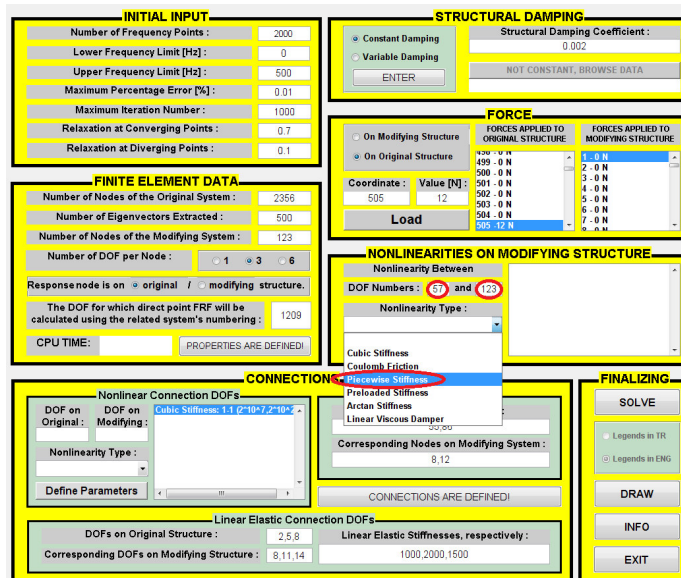


Figure 3.25 Setting the two coordinates and the type of the nonlinear element

Press the “Define Parameters” button and set the parameters of the related nonlinearity (Figure 3.26).

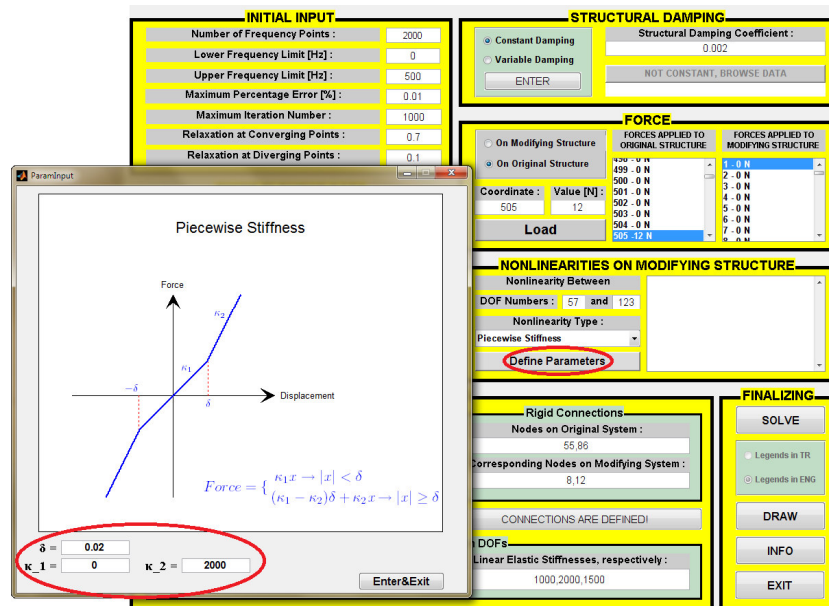


Figure 3.26 Setting the parameters of the nonlinearity

Press the “SOLVE” button in order to solve the problem (Figure 3.27).

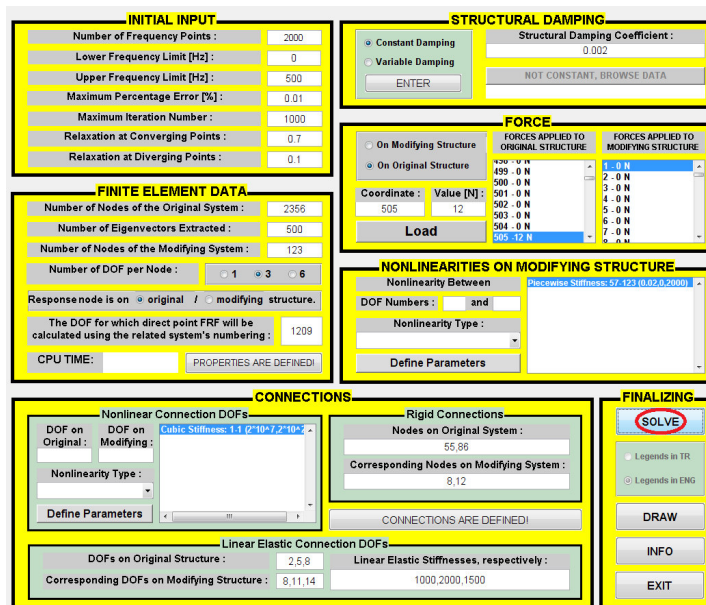


Figure 3.27 Solving the problem

After the green progressing bar fills up for the second time and disappears which implies that the solution ends, press the “DRAW” button to plot the response and FRFs (Figure 3.28).

The screenshot displays a software interface with several panels:

- INITIAL INPUT:**
 - Number of Frequency Points: 2000
 - Lower Frequency Limit [Hz]: 0
 - Upper Frequency Limit [Hz]: 500
 - Maximum Percentage Error [%]: 0.01
 - Maximum Iteration Number: 1000
 - Relaxation at Converging Points: 0.7
 - Relaxation at Diverging Points: 0.1
- STRUCTURAL DAMPING:**
 - Constant Damping (selected)
 - Structural Damping Coefficient: 0.002
 - Variable Damping (unselected)
 - ENTER button
 - NOT CONSTANT, BROWSE DATA
- FINITE ELEMENT DATA:**
 - Number of Nodes of the Original System: 2356
 - Number of Eigenvectors Extracted: 500
 - Number of Nodes of the Modifying System: 123
 - Number of DOF per Node: 1 (selected), 3, 6
 - Response node is on original / modifying structure.
 - The DOF for which direct point FRF will be calculated using the related system: 10% Low to High Sweep 3 sec remaining
 - CPU TIME: [Progress Bar]
- FORCE:**
 - On Modifying Structure (unselected)
 - On Original Structure (selected)
 - FORCES APPLIED TO ORIGINAL STRUCTURE:
 - 450 - 0 N
 - 499 - 0 N
 - 500 - 0 N
 - 501 - 0 N
 - 502 - 0 N
 - 503 - 0 N
 - 504 - 0 N
 - 505 - 12 N
 - FORCES APPLIED TO MODIFYING STRUCTURE:
 - 1 - 0 N
 - 2 - 0 N
 - 3 - 0 N
 - 4 - 0 N
 - 5 - 0 N
 - 6 - 0 N
 - 7 - 0 N
 - 8 - 0 N
 - Load
- NONLINEARITIES ON MODIFYING STRUCTURE:**
 - Nonlinearity Between: Piecewise Stiffness: 57, 123 (0,020,2000)
 - DOF Numbers: [] and []
 - Nonlinearity Type: []
- CONNECTIONS:**
 - Nonlinear Connection DOFs:
 - DOF on Original: []
 - DOF on Modifying: []
 - Nonlinearity Type: []
 - Define Parameters
 - Rigid Connections:
 - Nodes on Original System: 55,86
 - Corresponding Nodes on Modifying System: 8,12
 - CONNECTIONS ARE DEFINED!
 - Linear Elastic Connection DOFs:
 - DOFs on Original Structure: 2,5,8
 - Linear Elastic Stiffnesses, respectively: 1000,2000,1500
 - Corresponding DOFs on Modifying Structure: 8,11,14
- FINALIZING:**
 - SOLVE
 - Legends in TR (selected)
 - Legends in ENG
 - DRAW** (highlighted in red)
 - INFO
 - EXIT

Figure 3.28 Plotting the response and FRFs

CHAPTER 4

VERIFICATION AND APPLICATION OF THE METHOD AND THE PROGRAM

In this chapter, case studies to demonstrate the validity of the computer program developed will be given. The results obtained by using the program will be compared with those obtained via different methods and also with the solutions of several case studies found in the literature. Then, several other case studies are presented to demonstrate the application of the method and the program for various systems and for various types of nonlinear elements. The case studies corresponding to the verification problems are numbered using letter “V” while case studies corresponding to the application problems are numbered using letter “A”.

4.1 Verification of the Method and the Program

In this section, the method and then the program developed are verified through several applications. Applications are performed in such a selective way that they refer to verification examples for all three cases, namely, structural modification without additional DOFs, structural modification with additional DOFs and structural coupling with linear and nonlinear elements.

4.1.1 Case Study V.1 : 3 DOF System with Cubic Stiffness - Comparison with Results Given in Literature

In this application, case study L.1 given in reference [33] is analyzed by using the method suggested in this work. In reference [33], nonlinear FRFs of a system were

directly obtained through harmonic analysis. The 3 DOF system considered consists of three masses with linear and also nonlinear stiffnesses (Figure 4.1). A structural damping with 0.12% loss factor is assumed in the system.

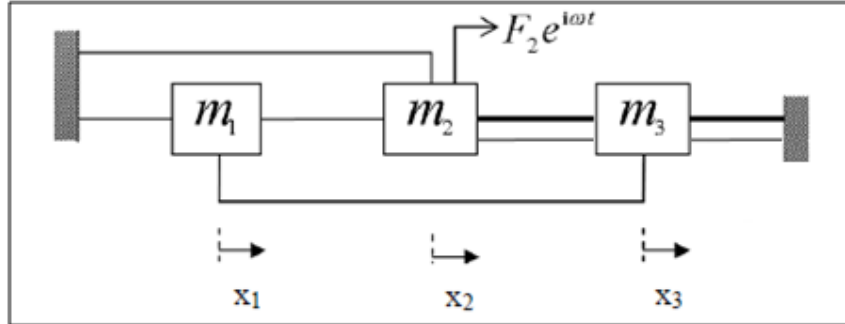


Figure 4.1 System schematic for Case Study V.1 [33]

Stiffness and mass matrices of the system considered are given below:

$$[M] = \begin{bmatrix} 31.6 & 0 & 0 \\ 0 & 55.4 & 0 \\ 0 & 0 & 24.2 \end{bmatrix} \text{ kg} \quad (4.1)$$

$$[K] = \begin{bmatrix} 200491 & -64921 & -36279 \\ -64921 & 398118 & -17503 \\ -36279 & -17503 & 132578 \end{bmatrix} \text{ N/m} \quad (4.2)$$

The nonlinearities involved in this system, which are represented in Figure 4.1 by bold lines, are defined in Table 4.1.

Table 4.1 Parametric values of nonlinearities for Case Study V.1

Nonlinear Connection Coordinates	Nonlinearity Type	Nonlinearity Coefficients:
2-3	Cubic Stiffness	$k_c = 0, \beta = 7.82e6$
3-Ground	Cubic Stiffness	$k_c = 0, \beta = 1.44e7$

In order to study this system using proposed approach, the system is first divided into two subsystems: The 3 DOF system shown graphically on the left in Figure 4.2 is taken as linear main system. The 2 DOF system on the right is taken as a nonlinear modifying system. This division is performed such that the modified system after coupling will be the same as the system studied previously.

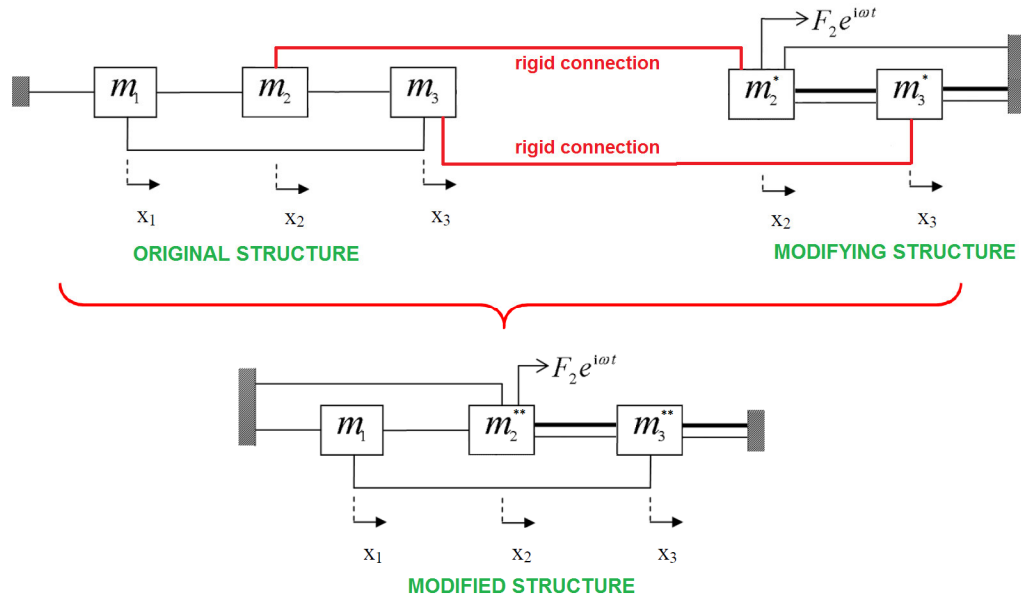


Figure 4.2 Nonlinear structural modification diagram for Case Study V.1

System matrices of the main (original) system are as follows:

$$[K_0] = \begin{bmatrix} 200491 & -64921 & -36279 \\ -64921 & 64921 & 0 \\ -36279 & 0 & 36279 \end{bmatrix} N/m \quad (4.3)$$

$$[M_0] = \begin{bmatrix} 31.6 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 10 \end{bmatrix} kg \quad (4.4)$$

System matrices of the modifying system are also given below:

$$[K_{\text{mod}}] = \begin{bmatrix} 333197 & -17503 \\ -17503 & 96209 \end{bmatrix} N/m \quad (4.5)$$

$$[M_{\text{mod}}] = \begin{bmatrix} 35.4 & 0 \\ 0 & 14.2 \end{bmatrix} kg \quad (4.6)$$

The natural frequencies and eigenvectors of the original linear system are calculated as follows:

$$\{\omega_r\} = \begin{Bmatrix} 5.628 \\ 9.402 \\ 14.65 \end{Bmatrix} Hz \quad (4.7)$$

$$[u] = \begin{bmatrix} -0.14928 & 0.09631 & 0.00976 \\ 0.09258 & 0.15668 & -0.12992 \\ 0.11162 & 0.14698 & 0.25678 \end{bmatrix} \quad (4.8)$$

The frequency domain solutions of the modified system subjected to the harmonic excitation

$$\{f\} = \begin{Bmatrix} 0 \\ 12 \\ 0 \end{Bmatrix} e^{i\omega t} N \quad (4.9)$$

are calculated by using NLSM/CP and the results are shown in Figure 4.3 and Figure 4.4. The frequency domain solutions of the same system obtained by analyzing the whole system [33] are given in Figure 4.5 and Figure 4.6. When solutions obtained using two different approaches are compared with each other, it can be clearly seen that NLSM/CP solutions show a perfect agreement with the solutions obtained in the previous study.

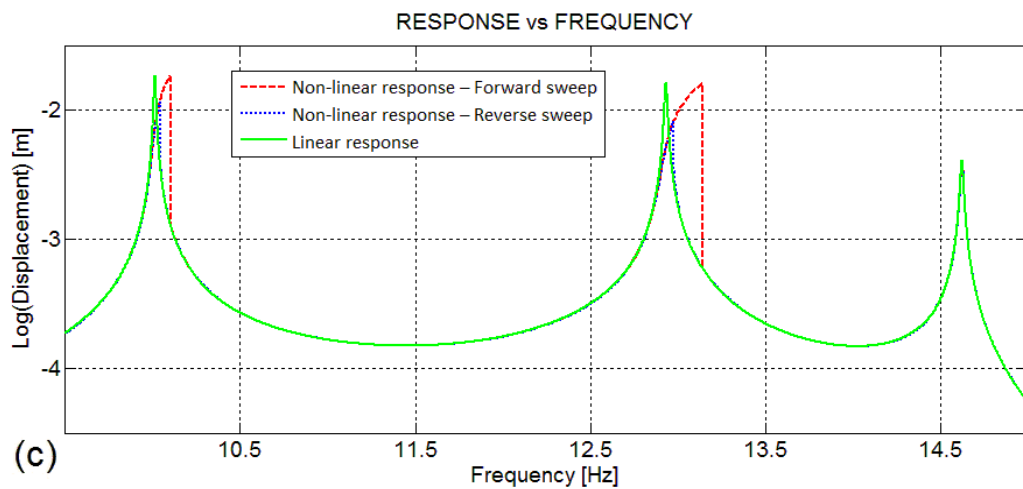
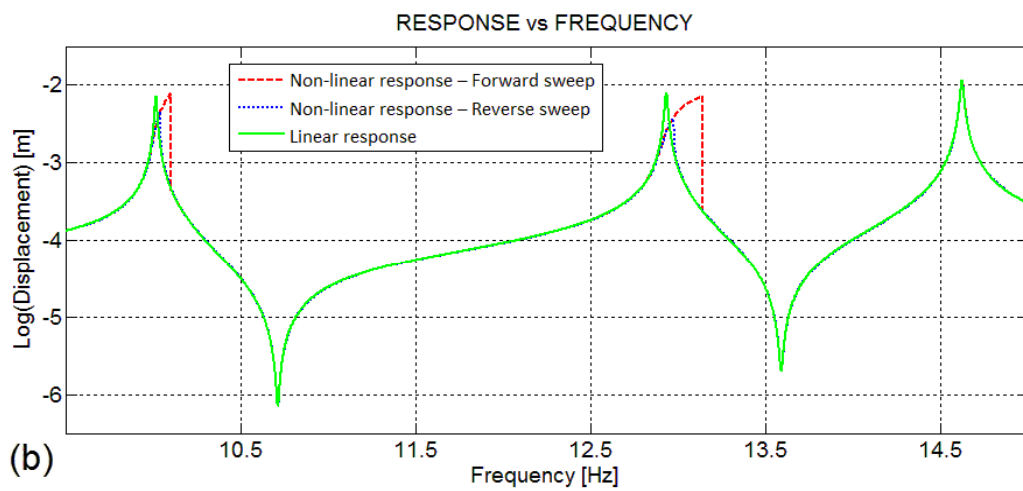
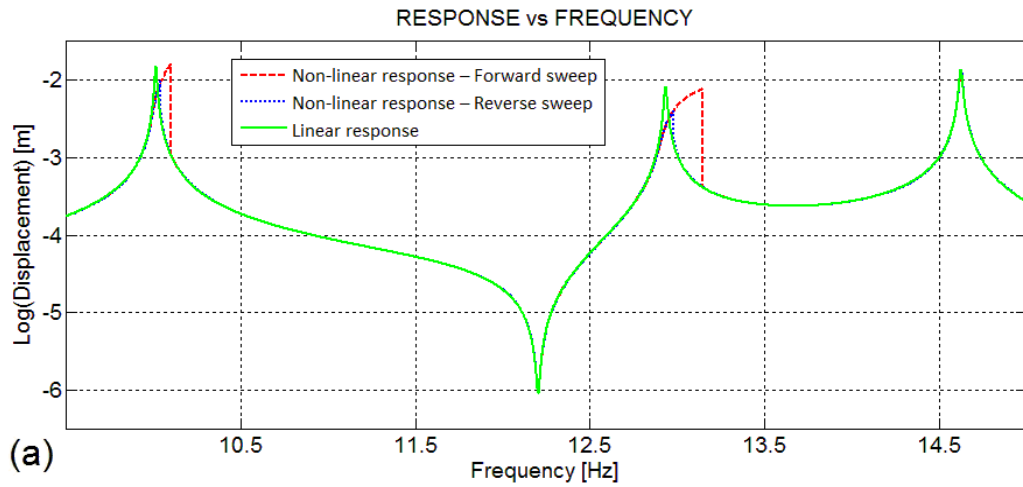


Figure 4.3 NLSM/CP solution for Case Study V.1 for (a) the 1st mass, (b) the 2nd mass and (c) the 3rd mass

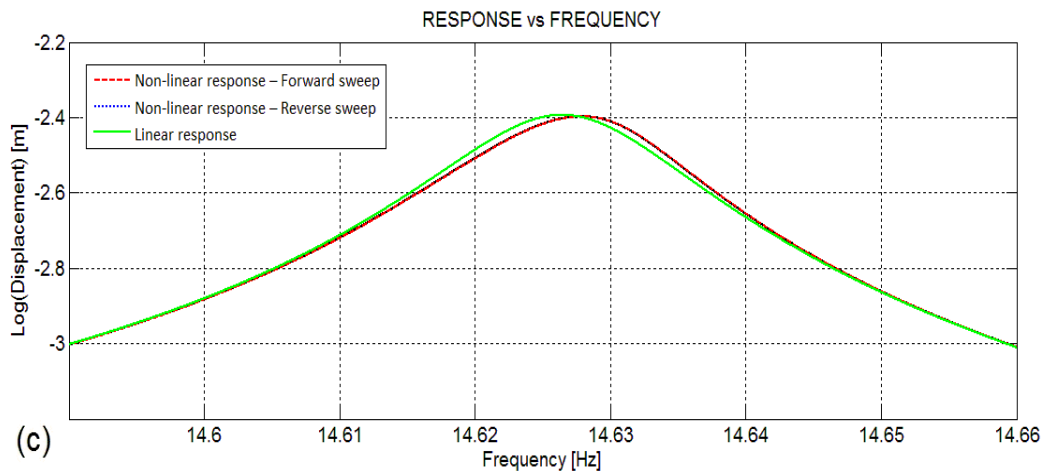
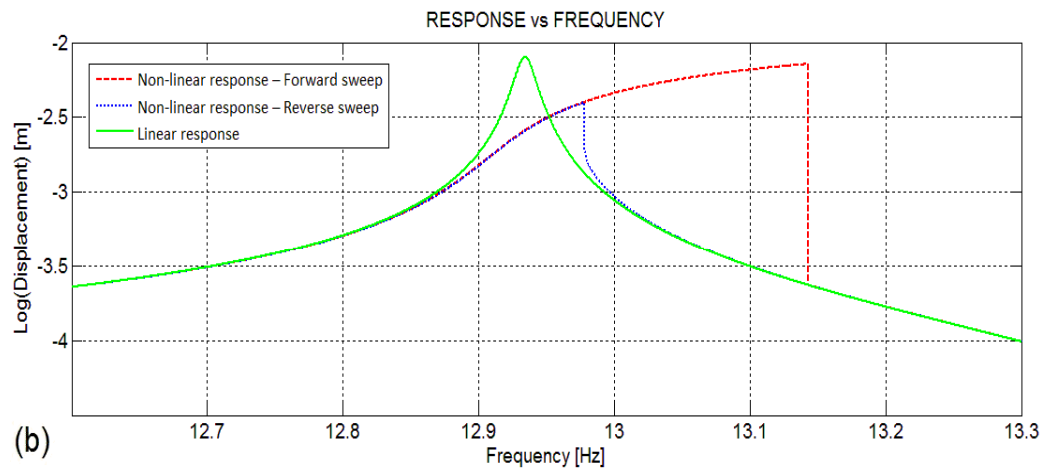
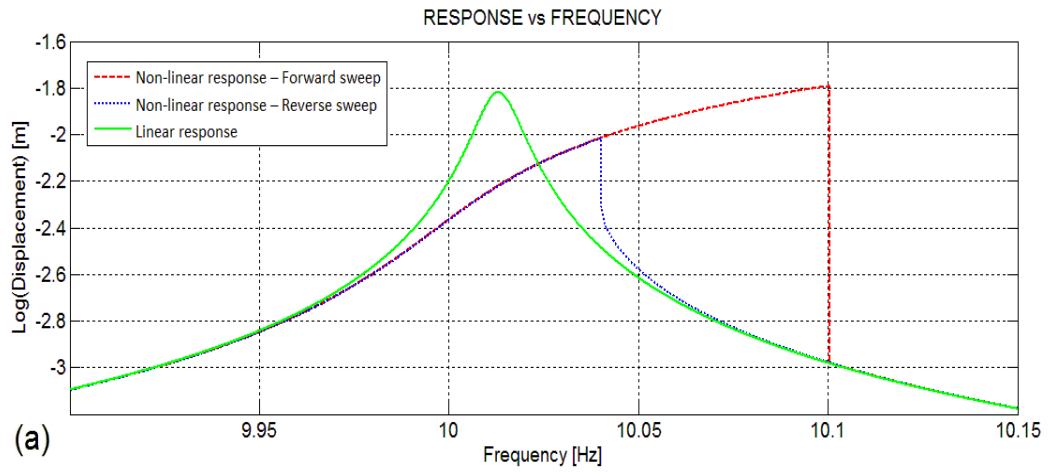


Figure 4.4 NLSM/CP solution for Case Study V.1 - zoomed individual resonances for (a) the 1st mode of the 1st mass, (b) the 2nd mode of the 2nd mass and (c) the 3rd mode of the 3rd mass

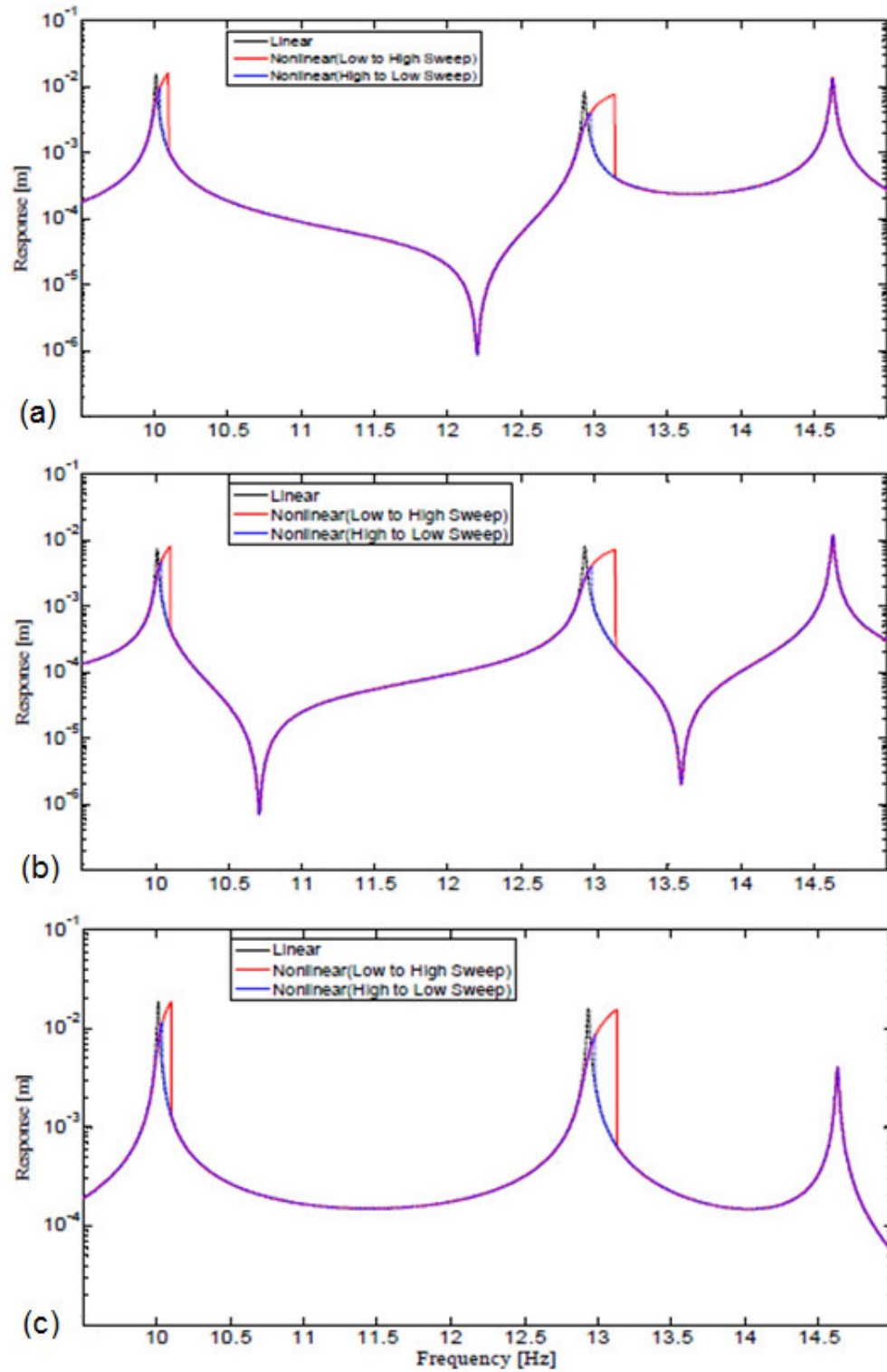


Figure 4.5 Solution for Case Study V.1 given in [33] for (a) the 1st mass, (b) the 2nd mass and (c) the 3rd mass

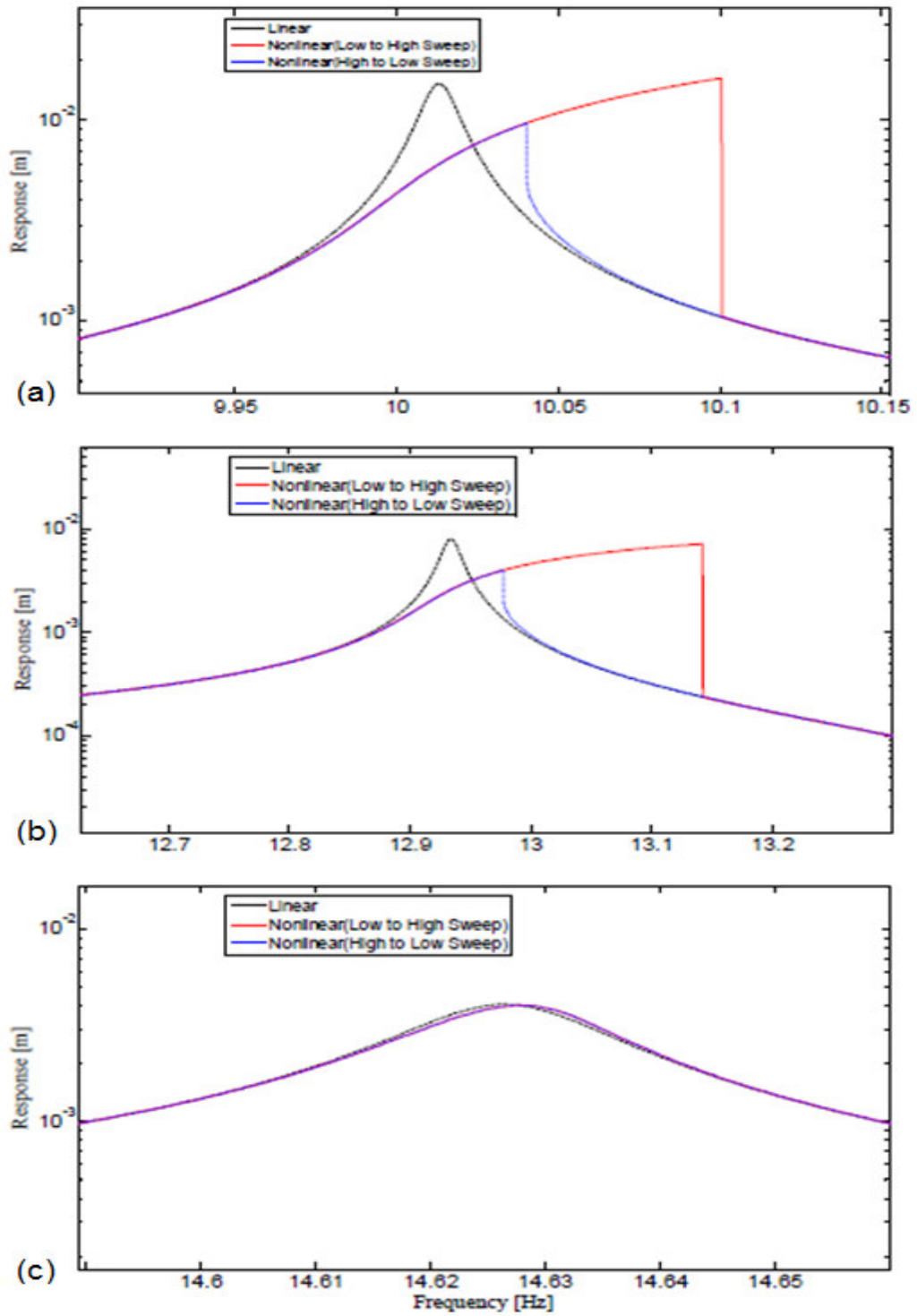


Figure 4.6 Solution for Case Study V.1 given in [33] - zoomed individual resonances for (a) the 1st mode of the 1st mass, (b) the 2nd mode of the 2nd mass and (c) the 3rd mode of the 3rd mass

4.1.2 Case Study V.2 : Modification via Nonlinear Coupling - Comparison with Results Given in Literature

In this case, the same system given in section 4.1.1 is considered in an alternative manner. This time, the system is decoupled such that when two subsystems are connected with linear and nonlinear elements, they are to construct the system used in [33]. During decoupling, it is again intended to obtain two subsystems such that one of them is to be original linear system while the other one is to be the modifying nonlinear system. And through coupling this two subsystem with pre-extracted linear and nonlinear elements, the initial quoted system will be obtained. Coupling is summarized graphically in Figure 4.7.

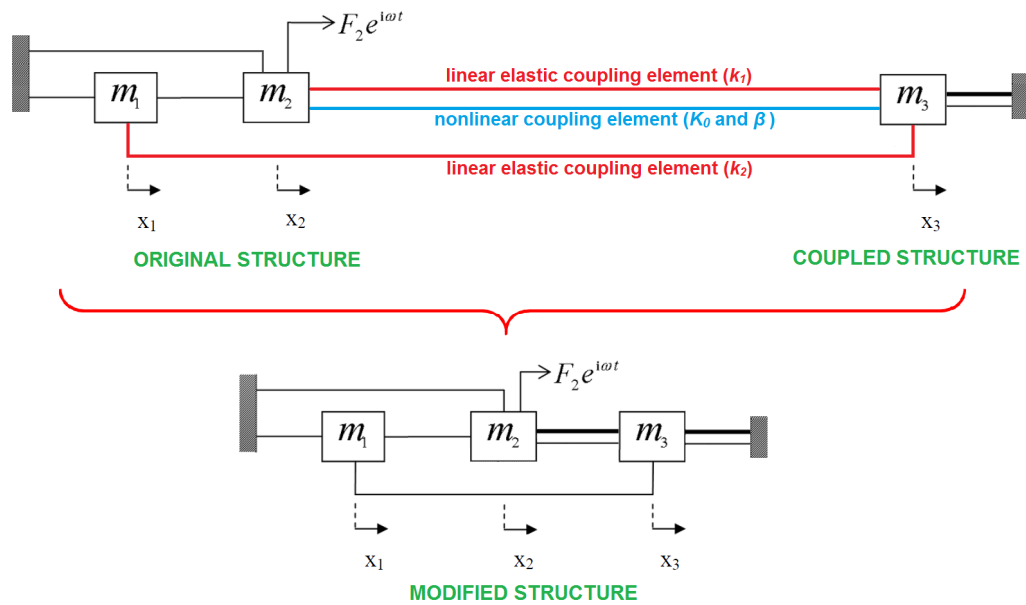


Figure 4.7 Nonlinear structural coupling diagram for Case Study V.2

The system matrices of the original system are as follows:

$$[K_0] = \begin{bmatrix} 164212 & -64921 \\ -64921 & 380615 \end{bmatrix} N/m \quad (4.10)$$

$$[M_0] = \begin{bmatrix} 31.6 & 0 \\ 0 & 55.4 \end{bmatrix} kg \quad (4.11)$$

The system matrices of the modifying system are also given below:

$$[K_{\text{mod}}] = [78796] N/m \quad (4.12)$$

$$[M_{\text{mod}}] = [24.4] kg \quad (4.13)$$

The stiffnesses, k_1 and k_2 , of linear elastic connection elements are taken as 17503 N/m and 36279 N/m , respectively. In the same way as the previous application, the nonlinear cubic stiffness element coefficients, k_c and β , are taken as 0 N/m and $7.82 \times 10^6 N/m^3$, respectively. The natural frequencies and eigenvectors of the original linear system are calculated as given:

$$\{\omega_r\} = \begin{Bmatrix} 10.401 \\ 14.053 \end{Bmatrix} Hz \quad (4.14)$$

$$[u] = \begin{bmatrix} -0.15275 & 0.09116 \\ -0.08381 & -0.11536 \end{bmatrix} \quad (4.15)$$

The frequency domain solutions of modified system subjected to the same harmonic excitation given by Equation (4.9) are obtained through NLSM/CP and the results are found to be exactly the same as those obtained in Case Study V.1.

4.1.3 Case Study V.3 : 3 DOF Nonlinear System - Comparison with TDI Solution

In this case, the approach proposed is applied to a simple linear discrete system with nonlinear modifications (Figure 4.8). For the validation of the approach, results

obtained via software developed are compared with those calculated by TDI solution of the coupled system by using the ordinary differential equation (ODE) integrators of MATLAB[®]. Integrations are performed using several integrators like ODE45, ODE113, ODE15s, etc. [51] with regard to stiffness characteristics of the problem until steady state is reached.

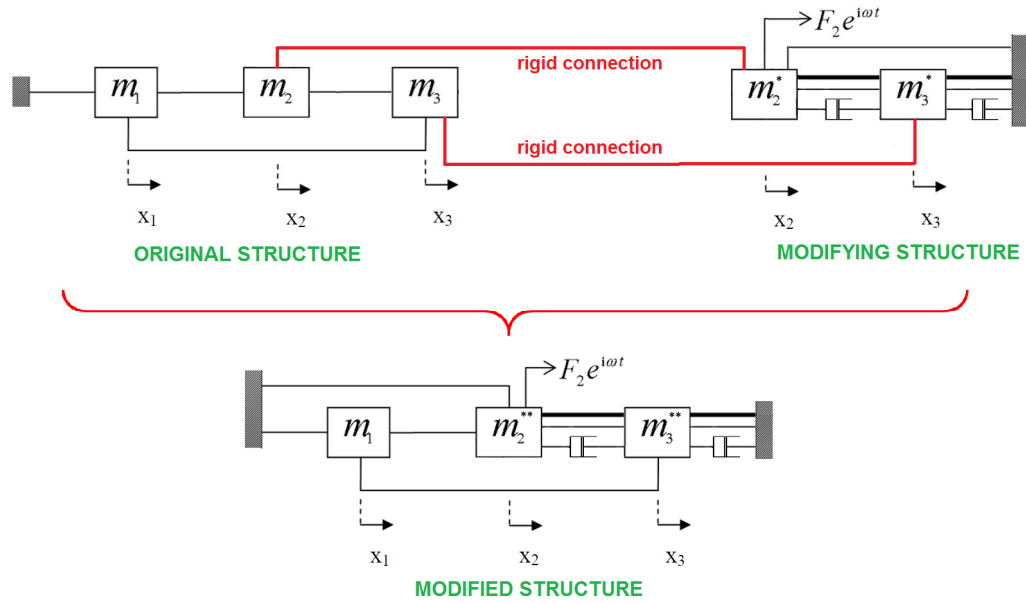


Figure 4.8 Nonlinear structural modification diagram for Case Study V.3

The system matrices of the original system are given below:

$$[K_0] = \begin{bmatrix} k_{11} & k_{13} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} = \begin{bmatrix} 90000 & -30000 & -30000 \\ -30000 & 60000 & -30000 \\ -30000 & -30000 & 60000 \end{bmatrix} N/m \quad (4.16)$$

$$[M_0] = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 10 \end{bmatrix} kg \quad (4.17)$$

The system matrices of the modifying structure are also given as follows:

$$[K_{\text{mod}}] = \begin{bmatrix} k_{11}^* & k_{12}^* \\ k_{21}^* & k_{22}^* \end{bmatrix} = \begin{bmatrix} 60000 & -30000 \\ -30000 & 60000 \end{bmatrix} N/m \quad (4.18)$$

$$[M_{\text{mod}}] = \begin{bmatrix} m_{11}^* & m_{12}^* \\ m_{21}^* & m_{22}^* \end{bmatrix} = \begin{bmatrix} 35 & 0 \\ 0 & 15 \end{bmatrix} kg \quad (4.19)$$

$$[C_{\text{mod}}] = \begin{bmatrix} c_{11}^* & c_{12}^* \\ c_{21}^* & c_{22}^* \end{bmatrix} = \begin{bmatrix} 25 & -25 \\ -25 & 50 \end{bmatrix} Ns/m \quad (4.20)$$

The nonlinearities inherent in modifying structure, which are represented in Figure 4.7 with bold lines, are given in Table 4.2.

Table 4.2 Parametric values of nonlinearities for Case Study V.3

Nonlinear Connections (DOF1-DOF2)	Nonlinearity Type	Nonlinearity Coefficients
1* - 2*	Cubic Stiffness	$k_c = 0, \beta = 1e11$
2* - Ground	Arctan Stiffness	$\rho = 8, \kappa = 1750$

Characteristics of cubic and arctan stiffness elements are shown in Figure 2.1 and Figure 2.7, respectively. The definitions of the nonlinearity coefficients given in Table 4.2 can be seen in Equations 2.33 and 2.38.

When system is excited with the following harmonic excitation,

$$\{f\} = \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 12 \\ 0 \end{Bmatrix} e^{i\omega t} N \quad (4.21)$$

the frequency domain responses can be obtained by using NLSM/CP. FRFs for the third mass are given along with the solution obtained via TDI in Figure 4.9 and Figure 4.10.

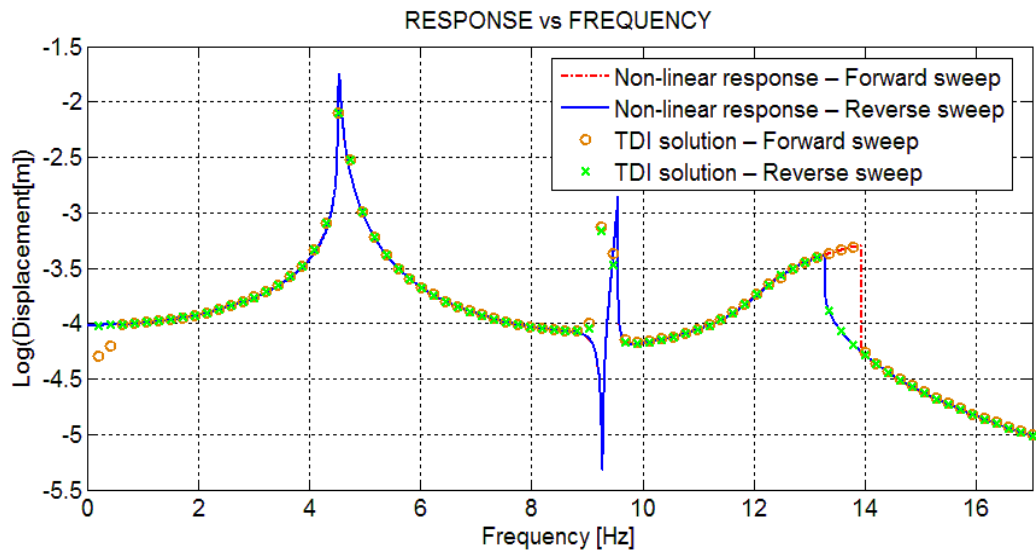


Figure 4.9 Comparison of frequency responses obtained by proposed approach and TDI solution for the 3rd mass

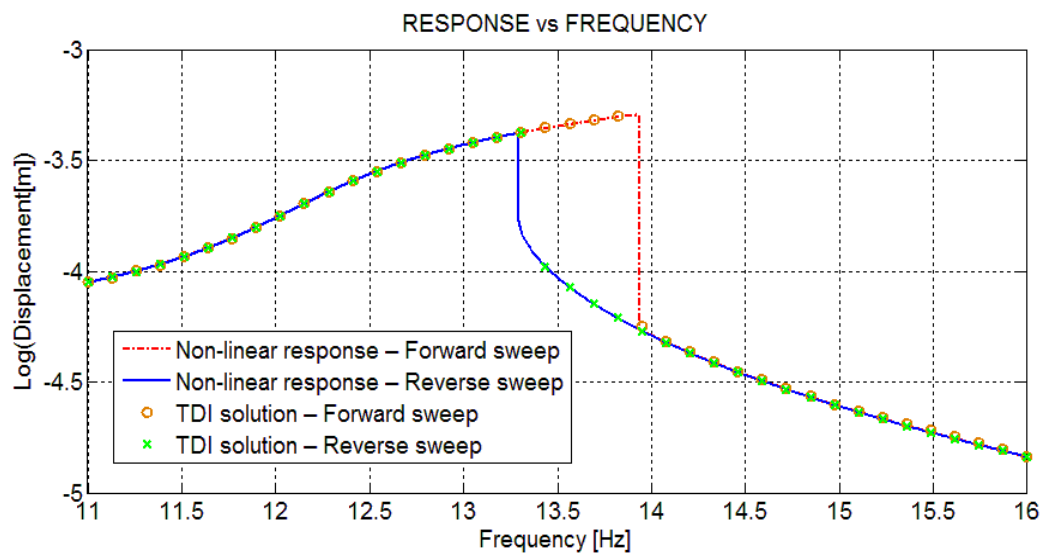


Figure 4.10 Comparison of frequency responses obtained by proposed approach and TDI solution for the 3rd mass (zoomed in third mode)

From the study of Figure 4.8 and Figure 4.9, it can be observed that there is an excellent match between the results obtained by two approaches at all frequencies in the frequency range which covers all three resonances. It should also be mentioned that the results obtained with time integration coincide with the results found through both forward and reverse sweeps.

4.1.4 Case Study V.4 : A Real Structural System - Comparison with Experimentally Obtained Results

For the implementation of the nonlinear structural modification approach proposed, a nonlinear test rig, which has been originally designed by Chong and İmregün [38], and has also been used recently in a previous study [52], is employed in this application.

4.1.4.1 Overview of the Referred Experiment and Adaptation for the Nonlinear Structural Modification

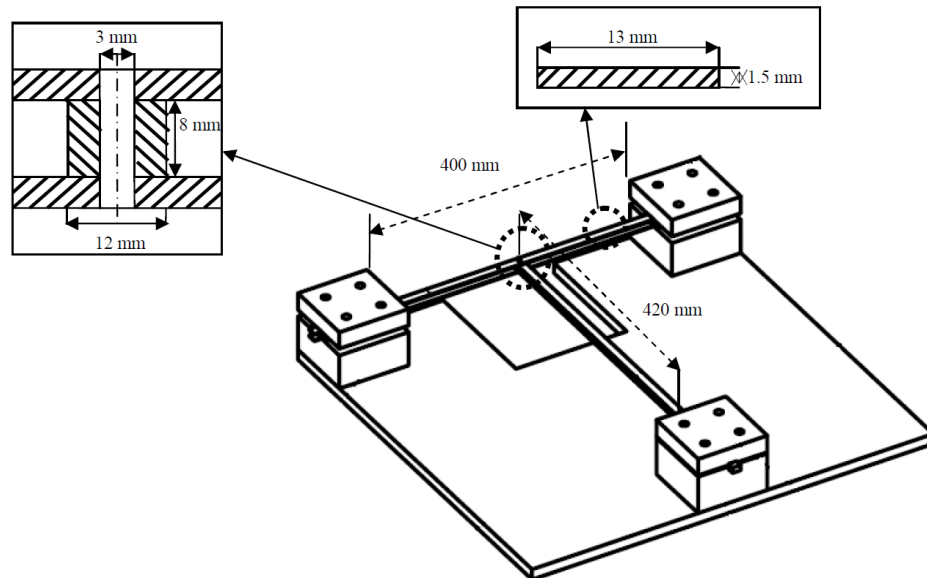


Figure 4.10 Test rig assembly and its dimensions

The test rig, whose dimensions and technical details are given in Figure 4.10, consists of a linear cantilever beam with its free end held between two thin identical beams which create cubic stiffness effect. The model used to represent the nonlinear dynamic test rig is shown in Figure 4.11.



Figure 4.11 Model of the test rig

Here, the fixed-fixed two thin beams are considered as the modifying structure and modeled as a concentrated nonlinear spring in y-direction with a concentrated equivalent mass as shown in Figure 4.11. The concentrated nonlinear cubic stiffness coefficient, β , is taken as $2.667 \times 10^8 \text{ N/m}^3$ which is the value identified experimentally [52]. Using the material and geometric properties of the thin beams, the linear part of the equivalent spring stiffness and equivalent mass values are calculated as 4519 N/m^3 and 45.3 g , respectively. The linear cantilever beam is considered as the original structure and modeled by using a standard FE program as shown in Figure 4.12. The modal data for the original structure is obtained via Finite Element Analysis (FEA).

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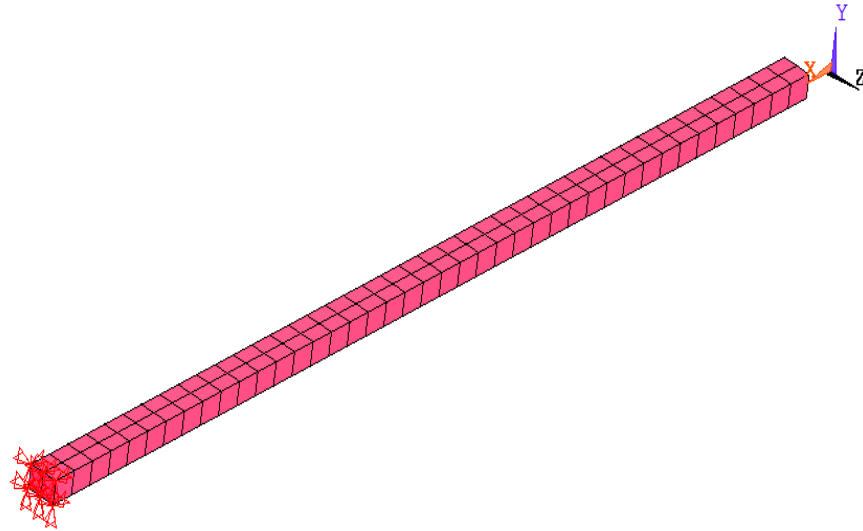


Figure 4.12 The FE model of the cantilever beam

Constructed model is solved using the NLSM/CP for the nonlinear modification. Frequency resolution during solution is taken as 0.25 Hz as is the case in the previous work [52]. The response amplitude dependent structural damping data set identified in [52] is used in the analysis. The direct point FRFs for the tip of the cantilever beam are calculated at forcing levels of 0.1, 0.5 and 1 N, and compared in Figure 4.13-4.15 with those experimentally measured [52], respectively.

4.1.4.2 Comparison of Results

When Figure 4.13 is investigated, it can be seen that good agreements are obtained between experimental and predicted values even at high forcing levels where jump, typical response distortion due to cubic stiffness nonlinearity, occurs in the frequency response. Slight differences are believed to be due to modeling of the modifying structure as a SDOF system and also, partly due to using the values cited in literature for the material properties of the original beam in FE model.

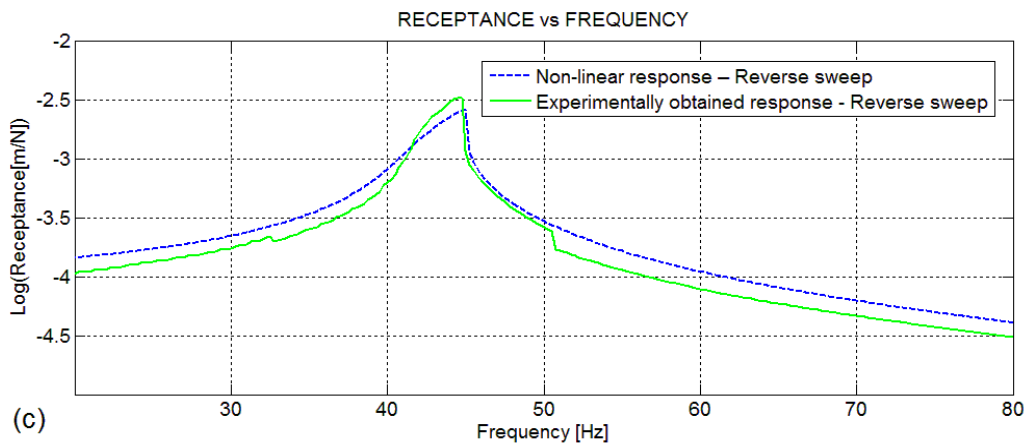
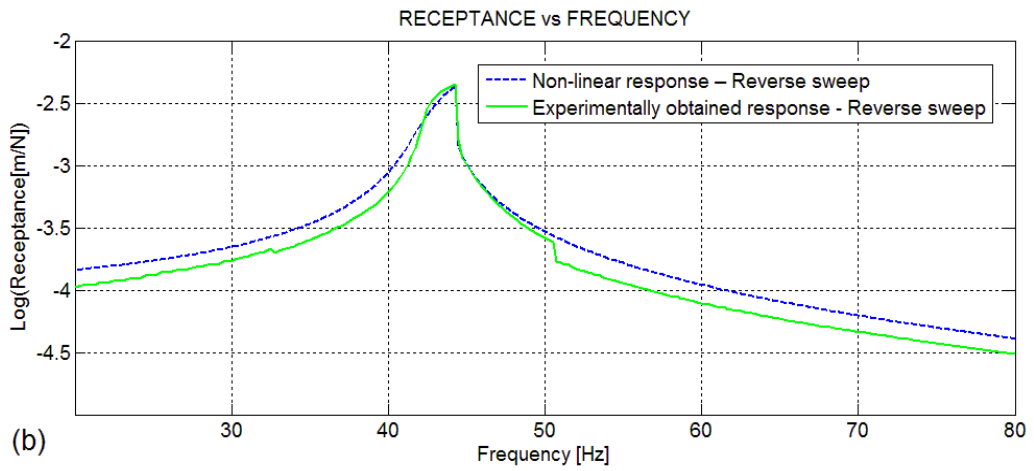
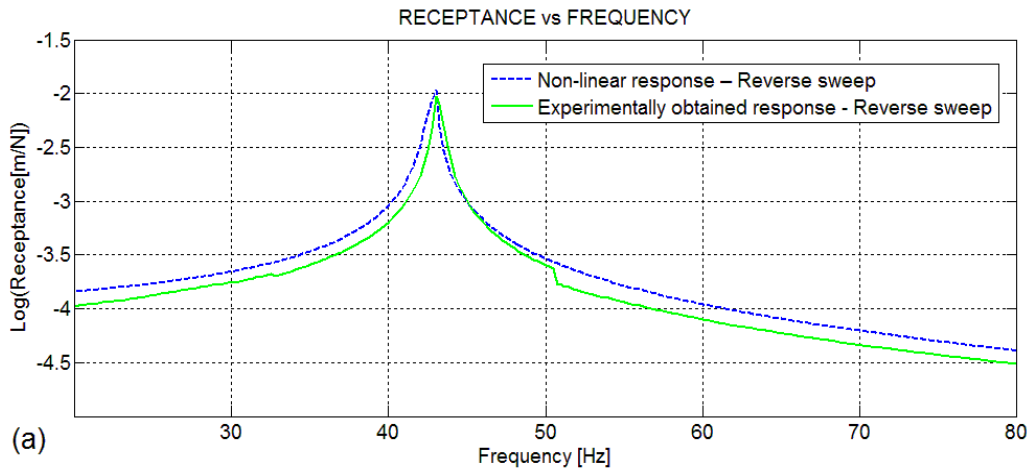


Figure 4.13 Calculated and measured FRF values for (a) $F = 0.1 \text{ N}$, (b) $F = 0.5 \text{ N}$ and (c) $F = 1 \text{ N}$

4.1.5 Case Study V.5 : FE Model of a Cantilever Beam - Comparison with Computational Results Given in Literature

In this application, in order to validate the method and the program also for structural modifications with additional DOFs, a cantilever beam whose FE model is given in Figure 4.14 is considered.

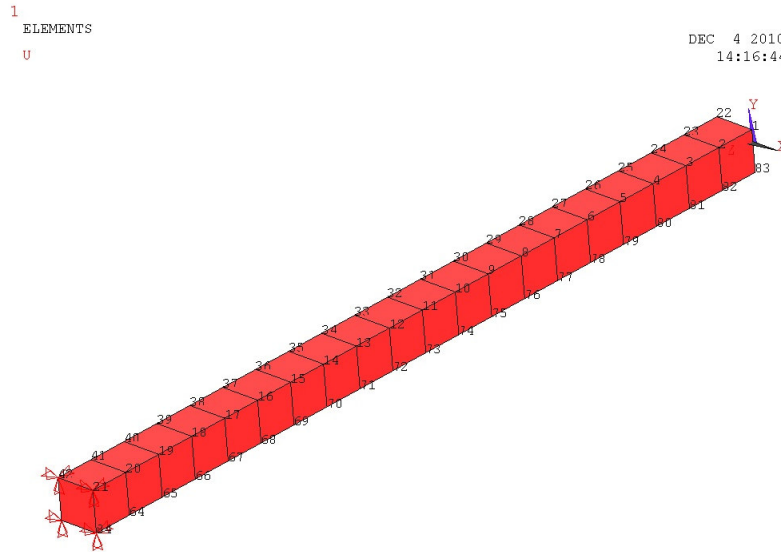


Figure 4.14 The FE model of the original beam

The FE model of the original cantilever beam model (Figure 4.14) consists of 20 SOLID 185 elements and it has 84 nodes with 3 translational DOF per node yielding total DOF of 252. FE model properties of the original cantilever beam are also given in Table 4.3.

Table 4.3 FE model properties for Case Study V.5

Element Type Used in FE model	SOLID 185
Number of Elements	20
Number of Nodes	84
Number of DOFs	252

The material properties assigned to FE model of the original cantilever beam model together with geometrical properties of it are given in Table 4.4.

Table 4.4 Material and geometrical properties for Case Study V.5

Material and Geometrical Property	Value
Density	7850 kg/m ³
Young's Modulus	200 GPa
Poisson's Ratio	0.3
Length	200 mm
Width	10 mm
Thickness	10 mm

Then, the original cantilever beam is modified by attaching a smaller beam whose bottom surface is exposed to Coulomb friction as shown in Figure 4.15.

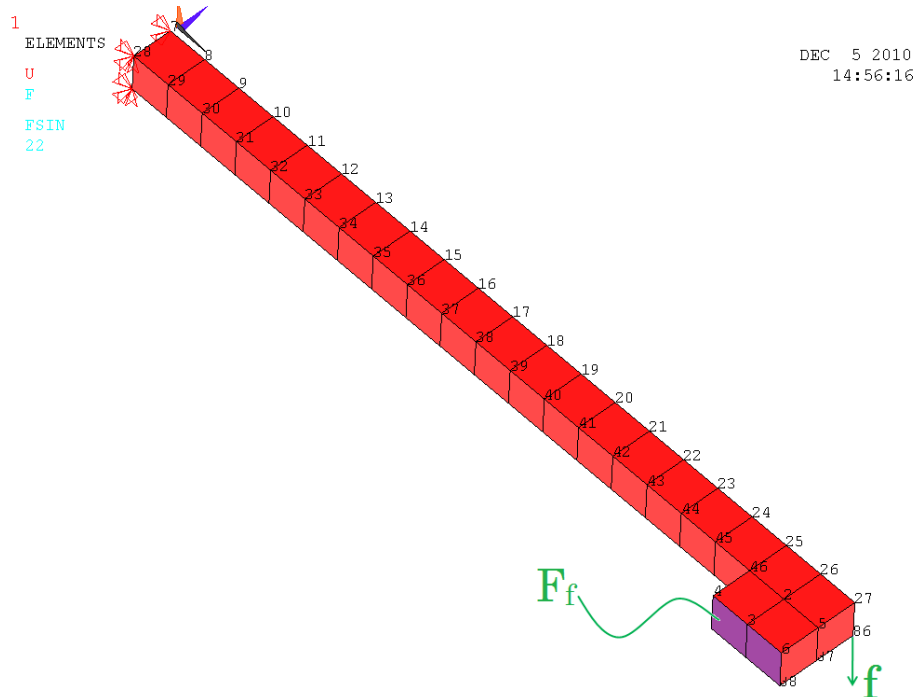


Figure 4.15 The FE model of the modified beam

An external harmonic excitation of magnitude 15N is applied to the 22nd node of the modified structure in x-direction. Also, structural damping with a loss factor of 0.12% is assumed in the system. Coulomb friction force to which each node of the bottom surface, namely, DOF numbers 10, 13, 16, 19, 22 and 25, is exposed in x-direction is equal to 1 N.

The direct point frequency responses for the 22nd node in x-direction after modification is calculated using NLSM/CP and given in Figure 4.16.

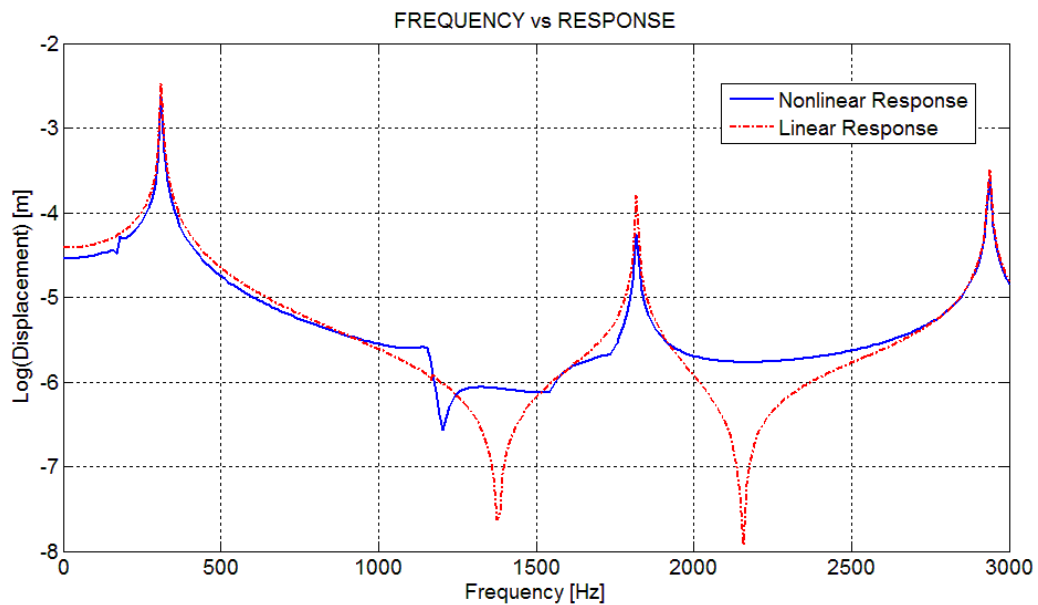


Figure 4.16 Frequency responses for the 22nd node of the modified structure in x-direction

From Figure 4.16, the effect of the Coulomb friction nonlinearity on modified system response can be clearly observed when linear and nonlinear frequency responses are compared with each other. It can be seen that, small drops around natural frequencies occur which can be observed more clearly around first natural frequency from Figure 4.17. Disappearances of antiresonances are also due to Coulomb friction type of nonlinearity, the physics behind which is explained in Section 2.2.3.2.

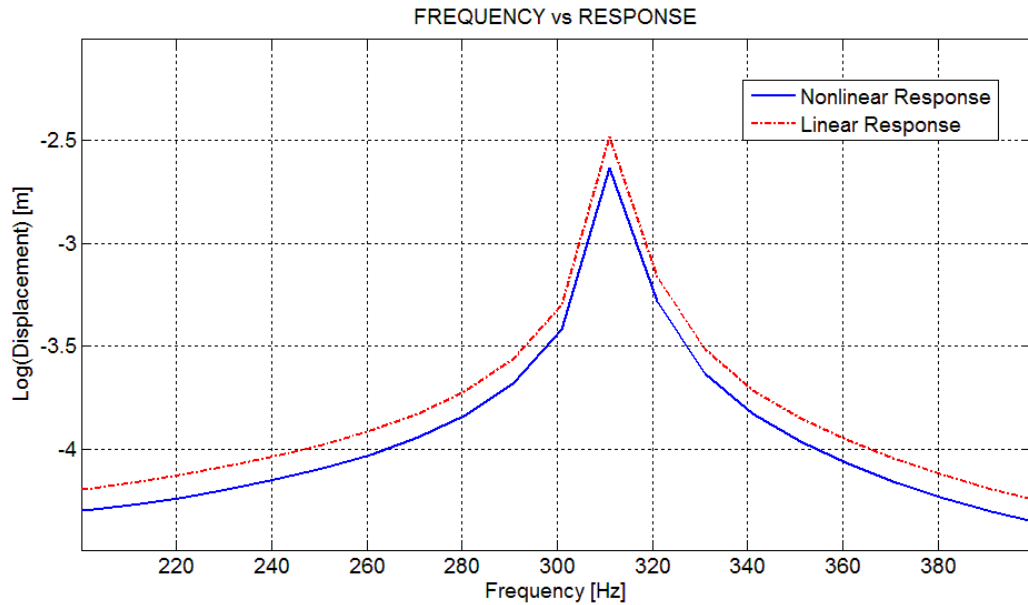


Figure 4.17 Zoom around first natural frequency

Note that, when the modified beam is directly analyzed with the computer program developed by Abat [33], exactly the same results are obtained when all the eigenvectors are taken into consideration in both programs to prevent possible differences due to truncation.

4.2 Applications of the Method Using the Program Developed

In this section, applications of the method using the program developed are demonstrated. The objective of this part is to show the performance and also the advantages of the method and the program developed. These applications are presented in three main categories: structural modification with additional DOFs, structural coupling with linear elements and structural coupling with nonlinear elements. Note that, since structural modification without additional DOFs case is well investigated in verification stage, it is not studied further in this section.

4.2.1 Application of the Method for Modifications with Additional DOFs

In this category, first a modification analysis using two discrete systems, which are also used in coupling analysis with linear and nonlinear elements, is illustrated. Then, application of the method to a real engineering structure modeled with FE will be given.

4.2.1.1 Case Study A.1 - A Discrete System Modification with Additional DOFs

In this case study, nonlinear structural modification of a linear discrete system is considered (Figure 4.18).

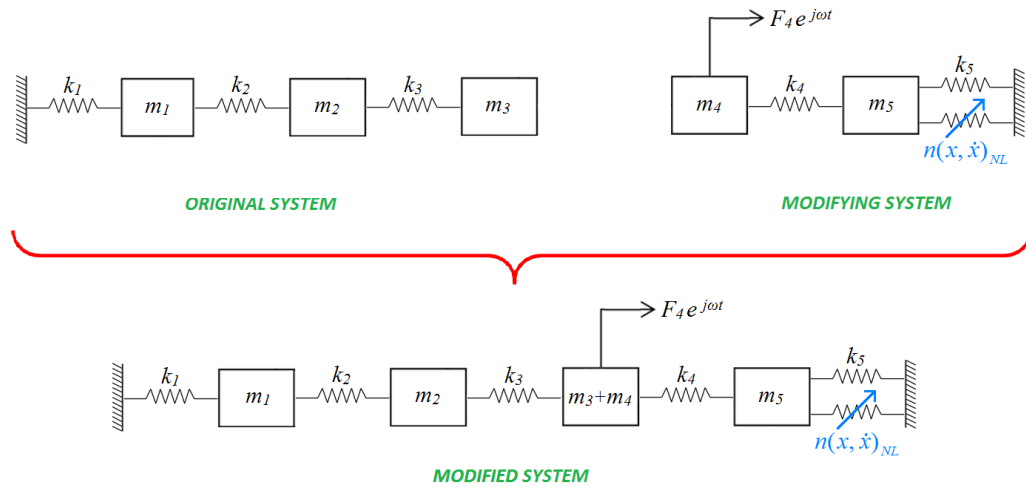


Figure 4.18 Nonlinear structural modification diagram for Case Study A.1

There exists a cubic stiffness type of nonlinearity between coordinates 4 and 5 of the modifying system which shows hardening behavior. Parameters of this nonlinear element together with the properties of both subsystems can be given as follows:

$$\begin{aligned}
 m_1 = m_2 = m_3 = 1\text{kg} \text{ and } m_4 = m_5 = 0.5\text{kg}, \\
 k_1 = k_2 = k_3 = k_4 = k_5 = k_6 = 1000\text{ N/m}, \\
 n(x, \dot{x})_{NL} = k_c x + \beta x^3 \text{ where } k_c = 0\text{ N/m and } \beta = 2 \times 10^6\text{ N/m}^3
 \end{aligned}
 \tag{4.22}$$

Assuming structural damping with a loss factor of 0.0015 in the analysis for all linear elastic elements, direct point frequency response of the modified system at the point where a harmonic force of magnitude 4 N is applied is calculated and shown in Figure 4.19.

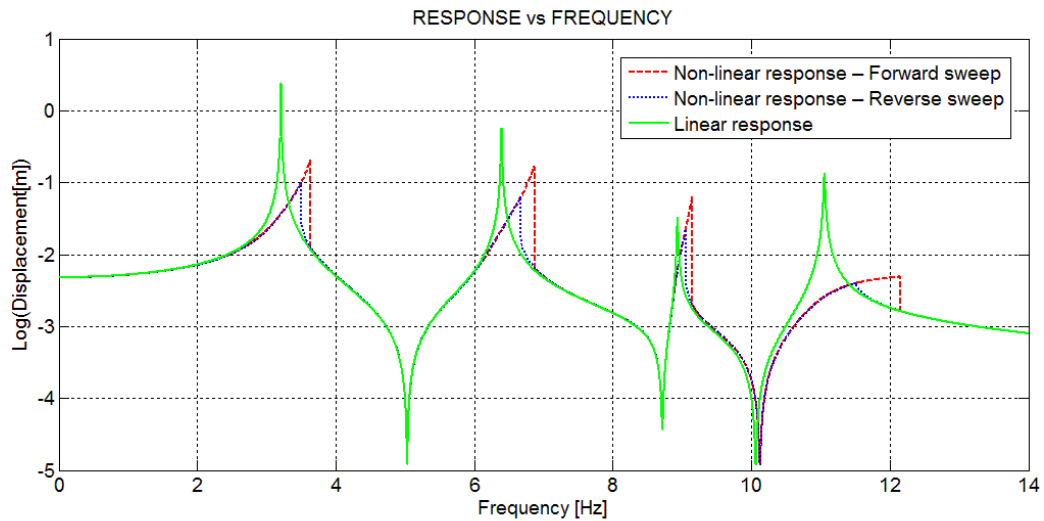


Figure 4.19 Frequency response of m_3 after modification for Case Study A.1

It can be seen from Figure 4.19 that the effect of the nonlinearity involved in the modifying system is apparent in all four modes of the modified system which reveals the importance of including nonlinearity in this specific case. Although the nonlinearity affects the frequency responses around resonances considerably, no convergence problem is observed during the solution made via proposed method. Note that, since the proposed method is an FRF based method, only the required original system FRFs are to be included in the calculation. Those FRFs are the ones related with the force application DOFs, the response DOF and the connection DOFs. Furthermore, in this example the size of the matrix to be inverted is 2, which is the order of the modifying system (and therefore it would still be 2, even though the size of the original system were much higher). These are the important features of the method which makes it more advantageous for large ordered systems with local modification as in the one illustrated in the following application.

4.2.1.2 Case Study A.2 - A Real Life Engineering Problem

In this application, a real life engineering structure, a shaft and mirror plate assembly usually used in land platforms for optical purposes, is considered (Figure 4.20).

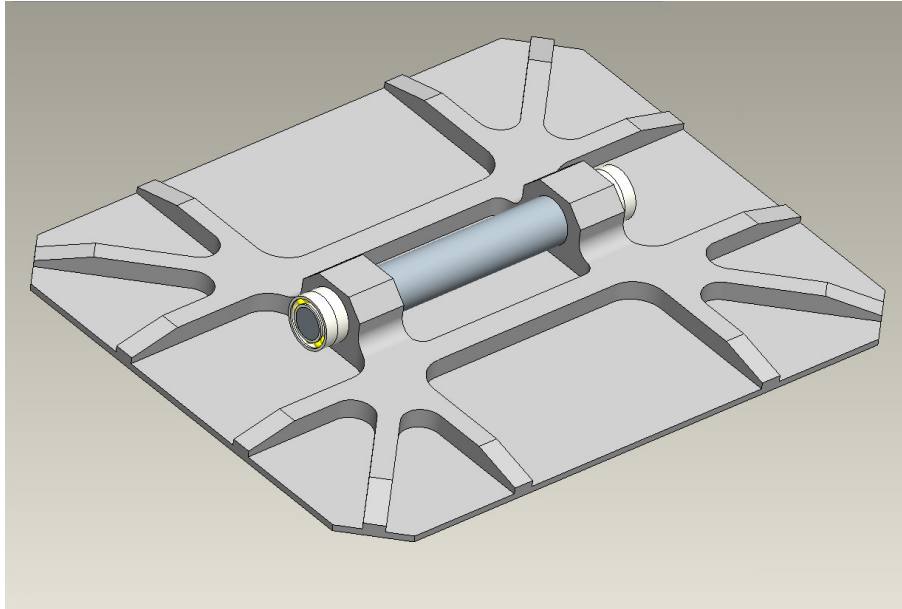


Figure 4.20 The shaft and mirror plate assembly

Since the mirror plate is an expensive part due to its well machined reflective surface, once it is designed it is avoided to be modified further in the design optimization of the assembly depending on the vibration characteristics of the platform it is mounted. So, when it is going to be used in several other platforms, it may be necessary to modify the shaft and/or bearings, in order to minimize the vibration of the mirror plate and thus to improve its reflection performance. In order to make a more precise analysis, nonlinearity introduced by the bearings should be included into the dynamic analysis, which can easily be considered by the nonlinear structural modification analysis method suggested in this thesis.

Solid elements are used in the FE model of the mirror plate with 3 DOF per node yielding 2655 total DOFs (Figure 4.21). Similarly, the shaft is also modeled by using solid elements with 3 DOF per node resulting in 186 total DOFs (Figure 4.22).

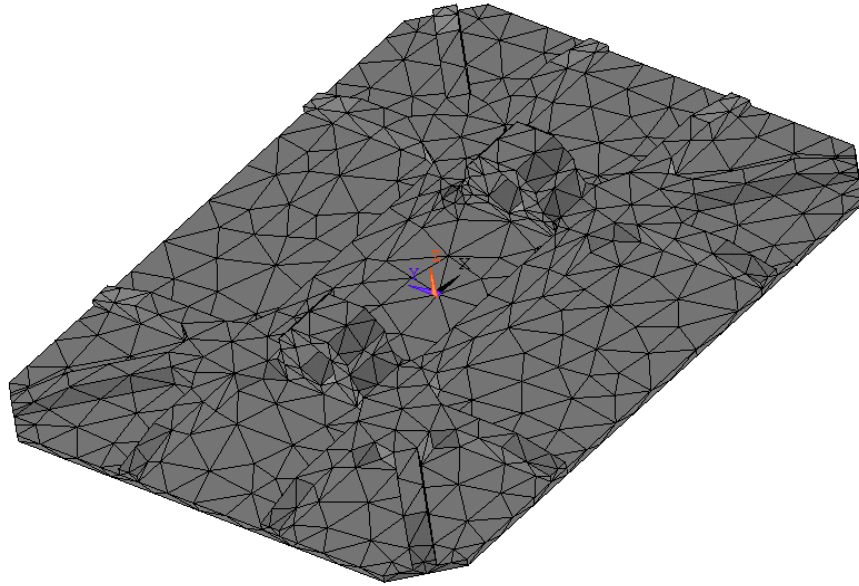


Figure 4.21 The FE model of the mirror plate

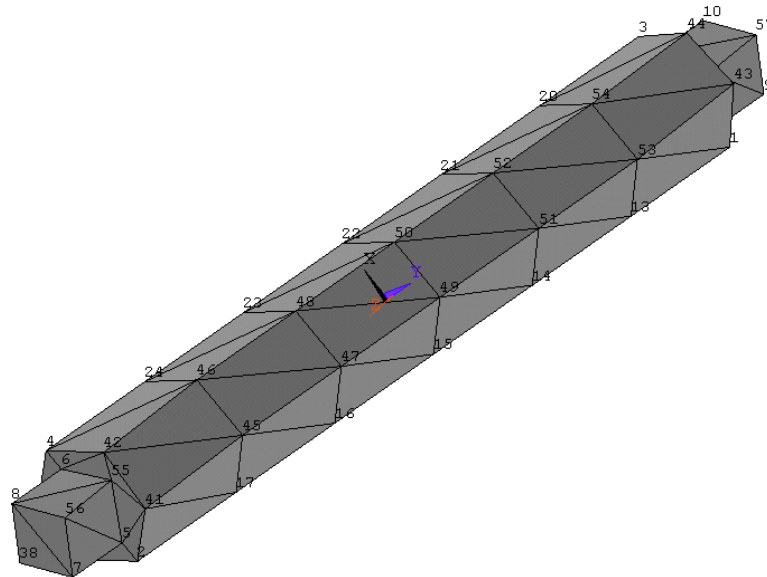


Figure 4.22 The FE model of the shaft

FE model and material properties of the aluminum alloy made mirror plate and structural steel made shaft are given in Table 4.6.

Table 4.5 Material properties of the mirror plate and the shaft

	Mirror Plate	Shaft
Young's Modulus	71 GPa	200 GPa
Poisson's Ratio	0.33	0.3
Density	2770 kg/m ³	7850 kg/m ³
Element Type Used in FE model	SOLID 185	SOLID 185
Number of Elements	2261	136
Number of Nodes	885	62
Number of DOFs	2655	186

Since the mirror plate is not desired to be modified during the design optimization phase of the assembly, it is taken to be the original structure. Shaft and bearing assembly on the other hand is taken as the nonlinear modifying structure where the ball bearings at the two ends of the shaft, shown in Figure 4.20, are modeled as nonlinear springs in horizontal and vertical directions connected to the ground. It should be noted that in this example the shaft is intentionally taken as the part of structural modification, just to increase the number of DOFs of the modifying structure. The nonlinear behavior of the ball bearings can be taken to be cubic in nature [53]. The nonlinear parameters of the ball bearings are taken as:

$$n(x, \dot{x})_{NL} = k_c x + \beta x^3 \text{ where } k_c = 2 \times 10^2 \text{ N/m and } \beta = 5 \times 10^7 \text{ N/m}^3 \quad (4.23)$$

In the analysis, initially, the receptances of the mirror are calculated by using standard modal analysis for connection points and for any other point we are interested in (i.e., points of which response is required or a force is applied to). Then, the nonlinear structural modification method is employed and the receptances of the

required points on the modified structure are calculated. Since the response of corner points on the mirror have the primary importance due to their reflection performance, the direct point FRF of a point near one of the corners of the mirror is calculated to which a harmonic force with a magnitude of 2 N is applied. The calculated direct point FRF is shown in Figure 4.23 with the linear FRF of the assembly without considering bearing nonlinearity. Using nonlinear structural modification method, it is very easy and fast to recalculate the response for any design change in the shaft and/or bearings.

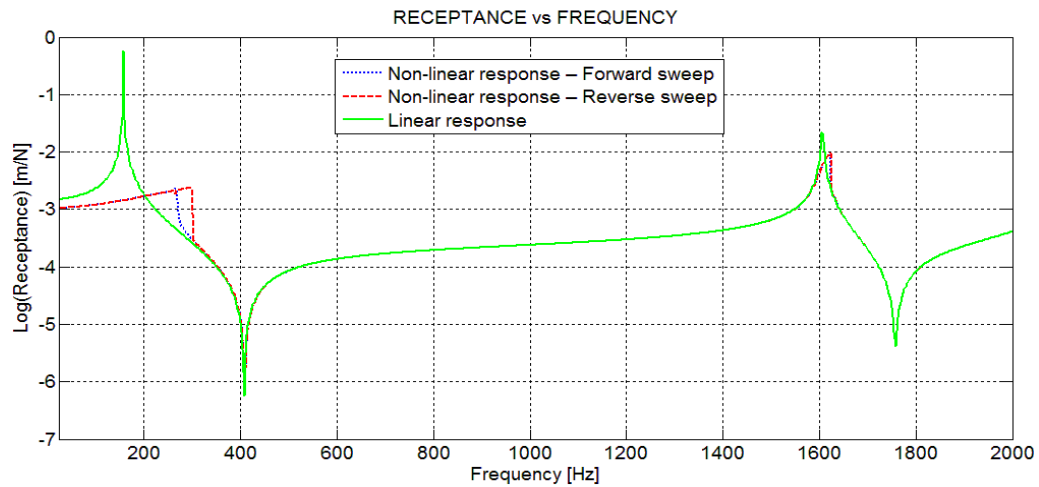


Figure 4.23 The direct point FRF of a point near one of the corners of the mirror plate for $F = 2$ N

Moreover, the FRF values for the resulting nonlinear system will be a function of the magnitude of the applied harmonic force, which will require the recalculation of the FRFs for each forcing amplitude level, although no modification is made on either of the subsystems. In such analyses, the method proposed in this thesis provides a considerable computational time saving again, since the FRFs of the linear part of the structure (which is usually the major part of the system with much higher DOFs) are calculated once and then used to find the FRFs for nonlinear overall system. In the later phase of the computations, which requires iterative solution, only the FRFs of the points we are interested in (in addition to those of the modifying structure) are used, rather than all DOFs (which would be the case if the coupled nonlinear system

were to be analyzed with standard approaches). In this case study, FRF of the corner point of the mirror is calculated for two more different forcing levels. The results are shown in Figures 4.24 and 4.25. The effect of forcing level on the FRF of the corner point of the nonlinear assembly can easily be observed by comparing Figures 4.23 to 4.25.

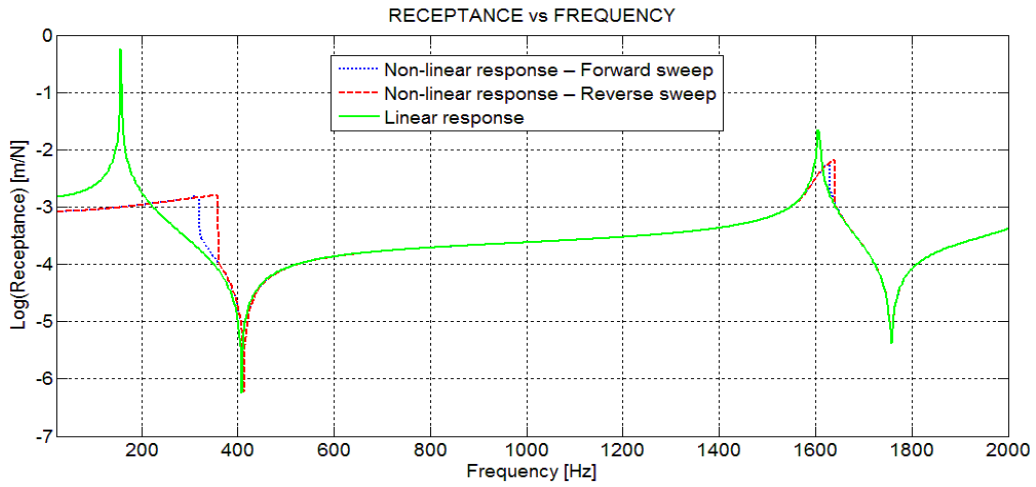


Figure 4.24 The direct point FRF of a point near one of the corners of the mirror plate for $F = 4 \text{ N}$

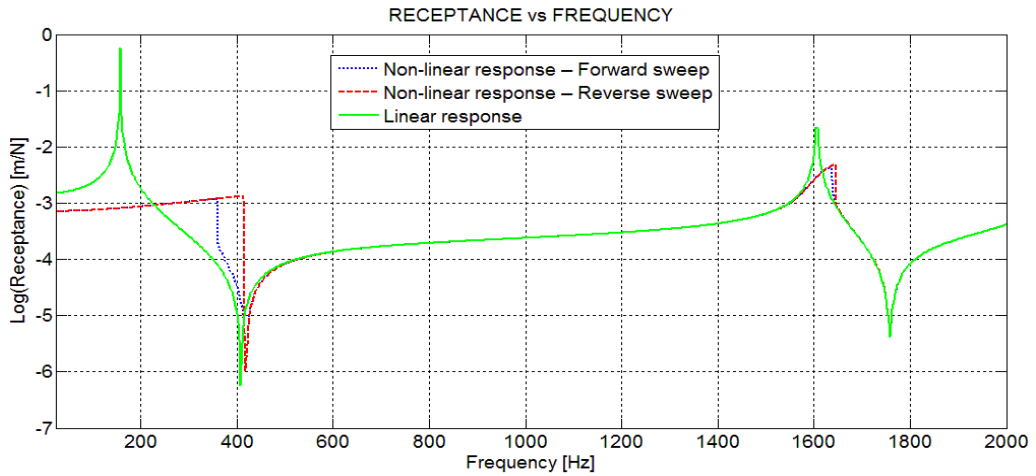


Figure 4.25 The direct point FRF of a point near one of the corners of the mirror plate for $F = 6 \text{ N}$

4.2.2 Application of the Method for Coupling with Linear Elements

In this category, coupling analysis of the same discrete subsystems is considered. The subsystems are coupled with a linear elastic element. In this application the effect of the linear elastic coupling element stiffness on the FRF of the modified system is examined thoroughly, and advantages of the proposed method for such parametric analyses are emphasized.

4.2.2.1 Case Study A.3 - Discrete Subsystems Coupled with Linear Elements

In this case study, nonlinear structural coupling analysis of the linear and nonlinear discrete subsystems, considered in Section 4.2.1.1, coupled with a linear elastic element is carried out (Figure 4.26).

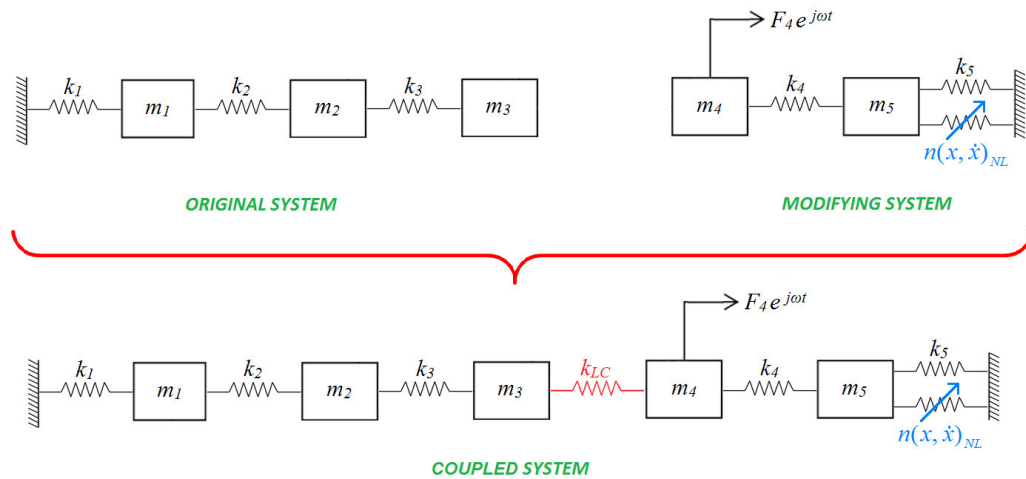


Figure 4.26 Nonlinear structural coupling diagram for Case Study A.3

The stiffness of the linear elastic element is taken as $k_{LC} = 200$ N/m. Structural damping with a loss factor of 0.0015 is again assumed for all linear elastic elements. Direct point frequency response of the modified system at the point to which a harmonic force with an amplitude of 4 N is applied is calculated and shown in Figure 4.27.

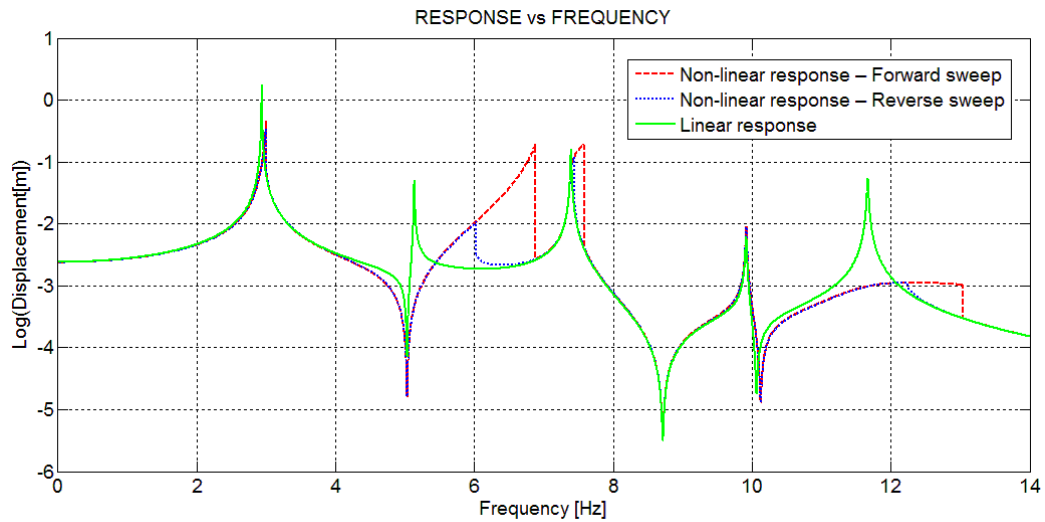
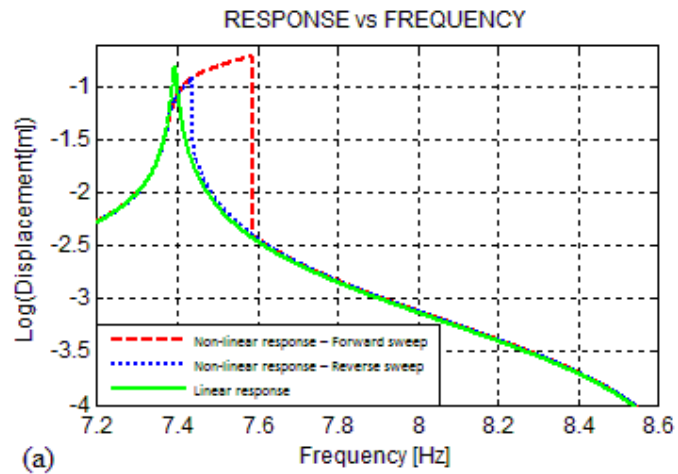
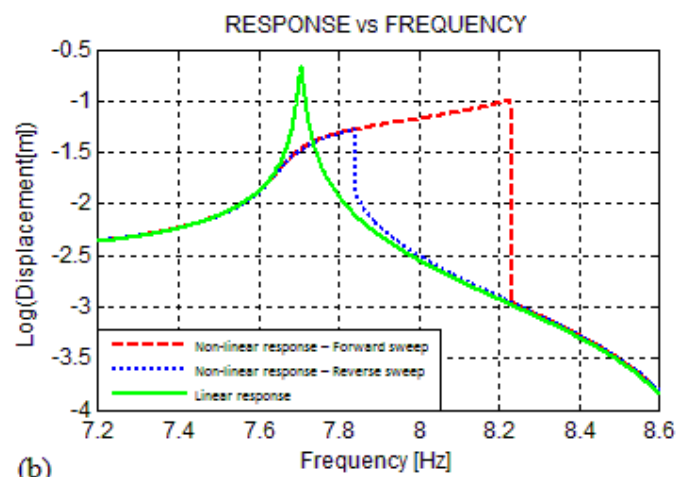


Figure 4.27 Frequency response of m_3 after coupling for Case Study A.3

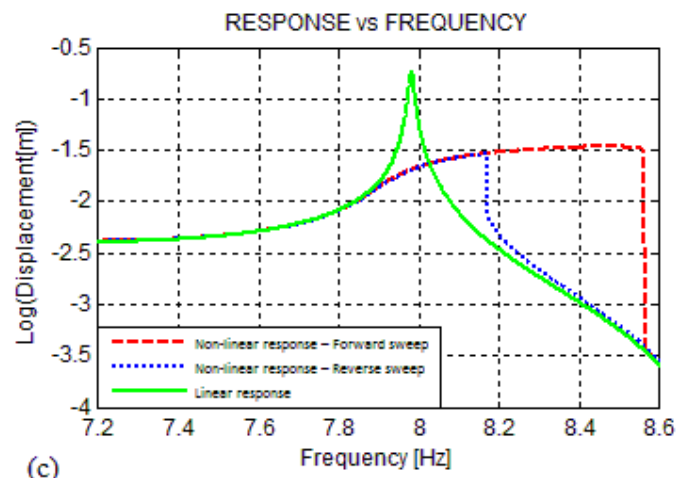
The results show that nonlinearity is more effective on 2nd, 3rd and 5th modes of the system compared to the other two modes. Note again that, since the proposed method is FRF based, it is sufficient to include only the FRFs related with the required DOFs, in addition to the FRFs related the connection DOFs, into the calculations. Furthermore, the size of the matrix to be inverted during calculations is again 2 by 2 in this application. This is the size of the modifying nonlinear subsystem. This saves considerable computational time especially in large original systems, as long as the modification is of small order. This feature of the method makes it very desirable in parametric studies, such as, for instance, investigating the effects of stiffness of the linear elastic coupling element on system response. In Figure 4.28, the effect of different stiffness values of the linear elastic coupling element on the system response around 3rd resonance is examined in detail. It can be seen from the figure that increasing values of the stiffness of the linear elastic coupling element will not only shift the 3rd natural frequency to higher frequencies, but will also increase the effect of nonlinearity on this mode.



(a)



(b)



(c)

Figure 4.28 Frequency responses around 3rd resonance for different linear elastic coupling elements (a) $k_{LC} = 200$ N/m, (b) $k_{LC} = 400$ N/m, (c) $k_{LC} = 600$ N/m

4.2.3 Application of the Method for Coupling with Nonlinear Elements

In this category, coupling analysis of the same discrete subsystems is considered. This time two subsystems are coupled with a nonlinear element in addition to the linear elastic element used in previous application. The effect of the nonlinear coupling element on modified system FRF is examined thoroughly and advantage of the proposed method for such parametric analyses is emphasized.

4.2.3.1 Case Study A.4 - Discrete Subsystems Coupled with Nonlinear Elements

In this case study, nonlinear structural coupling analysis of the linear and nonlinear discrete subsystems considered in Section 4.2.2.1 is carried out. However, this time in addition to the linear elastic coupling element used in the previous case study, an additional nonlinear coupling element is considered as shown in Figure 4.29.

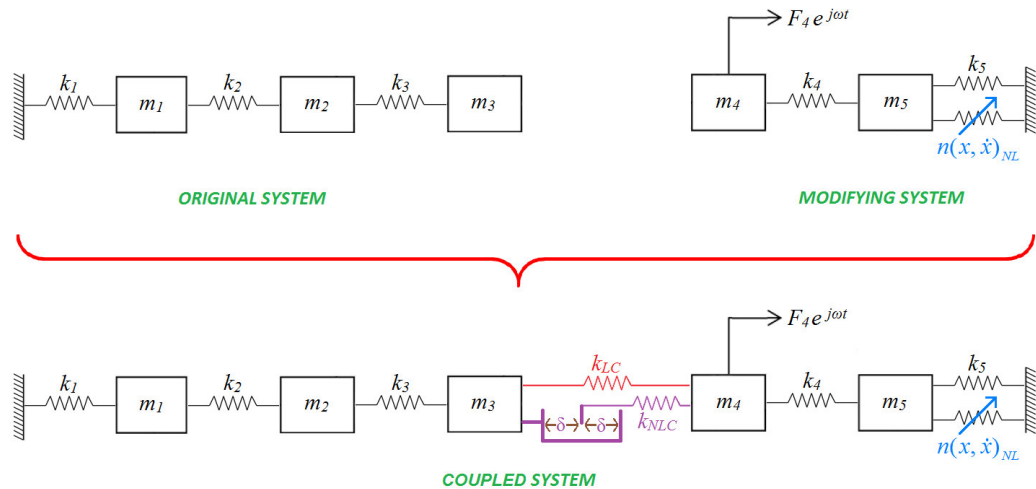


Figure 4.29 Nonlinear structural coupling diagram for Case Study A.4

As the additional nonlinear coupling element, a linear spring having a stiffness of $k_{NLC} = 200$ N/m with a clearance of $\delta = 0.02$ m is employed between two coupling coordinates. Assuming structural damping with a loss factor of 0.0015 again for all elastic elements, frequency response of the modified system at the point to which a

harmonic force with an amplitude of 4 N is applied is obtained (Figure 4.30).

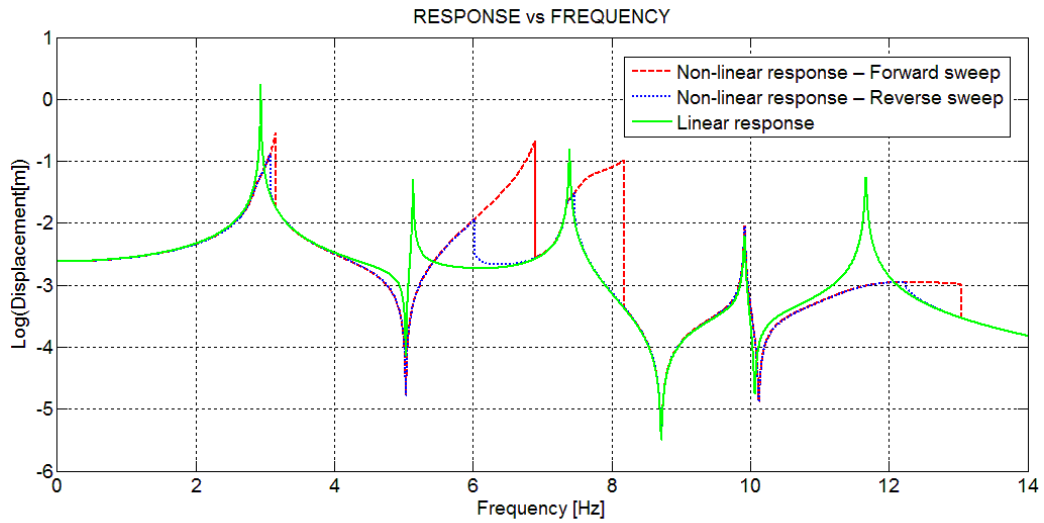


Figure 4.30 Frequency response of m_3 after coupling for Case Study A.4

When Figures 4.27 and 4.30 are compared with each other, it can be observed that nonlinear coupling element affects 1st and 3rd modes of the system more than it does the other modes. Again, using only the FRFs related with the required and connection DOFs, which is the FRF related with the 3rd mass in this example, and inverting a matrix only in the size equal to the DOF of the modifying system makes this method very favorable. Therefore, the method can be used in design analyses where, for instance, the effects of using different nonlinear coupling elements on the system response are investigated. In Figure 4.31, the effect of different stiffness values of nonlinear coupling element on the system response around 3rd resonance is examined in detail.

It can be seen from Figure 4.31 that typical response distortion due to clearance type of nonlinearity is observed as an abrupt change in the frequency response at the point of transition where the clearance gap is closed. As an expected result, the displacement value where this abrupt change occurs differs depending on the value of the clearance (compare Figure 4.31 (a) and (b)).

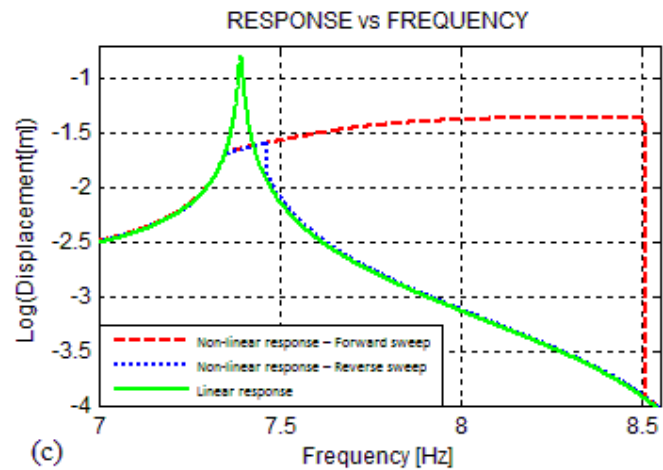
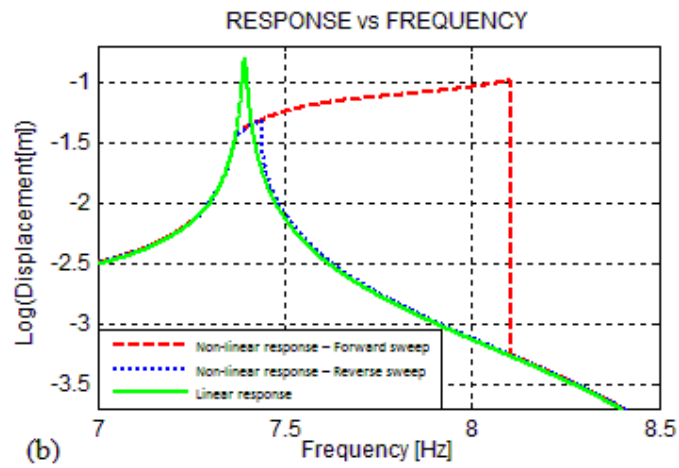
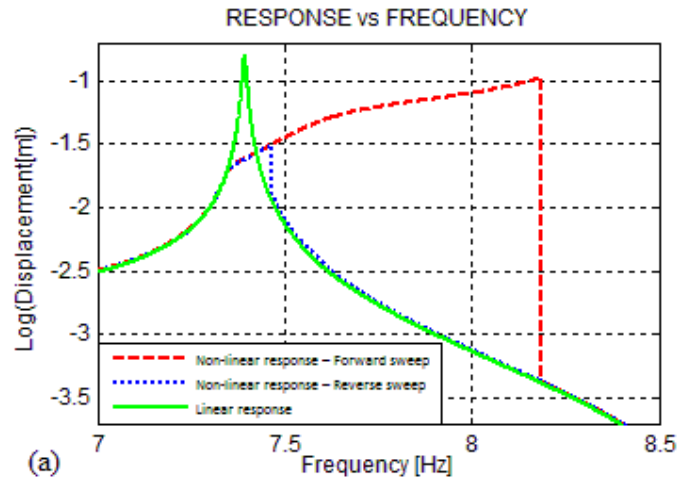


Figure 4.31 Frequency responses around 3rd resonance for different nonlinear coupling elements (a) $k_{NLC} = 200$ N/m, $\delta = 0.02$ m, (b) $k_{NLC} = 200$ N/m, $\delta = 0.04$ m, (c) $k_{NLC} = 400$ N/m, $\delta = 0.02$ m

On the other hand, for nonlinear spring elements having different stiffness values but the same clearance, this abrupt change occurs at the same displacement value but the frequency responses after that point show different behaviors due to having different additional linear spring stiffnesses (compare Figure 4.31 (a) and (c)).

CHAPTER 5

RESULTS AND CONCLUSIONS

5.1 Summary of the Results and Conclusions

The main objective of this thesis is to obtain the dynamic response characteristics of a modified structure from those of the linear original structure and the system matrices of the nonlinear modification or coupled structure by adopting a noble structural modification method to nonlinear systems.

In this thesis, Özgüven's structural modification method with and without additional DOFs [14] is extended and applied to structures with local nonlinear modifications. Since the formulation is for rigid connection of the nodes of the original and modifying systems, for the cases where a nonlinear subsystem is coupled to a linear system with elastic elements (linear or nonlinear), the problem is treated as an equivalent structural modification problem where at each free end of a connecting elastic element a massless node is added and that node is rigidly coupled to the main system. The theoretical backgrounds of these methods are presented in Chapter 2.

In order to apply the proposed method to nonlinear modification and coupling problems, a computer program, called NLSM/CP, is developed in MATLAB®. NLSM/CP has a graphical user interface and is capable of solving nonlinear structural dynamic modification and coupling problems. The computer program uses natural frequencies and mode shape vectors of the original structure, and mass and stiffness matrices of the modifying structure with the information regarding nonlinearities present in it. The natural frequencies and the mode shape vectors for

the original structure are extracted from the modal analysis results performed in ANSYS®. The mass and stiffness matrices of the modifying structure are obtained by using the “.full” modal analysis result file of ANSYS®. The experimental results for the natural frequencies and the mode shape vectors of the original structure can also be used in NLSM/CP. The details and user's manual of the computer program is given in Chapter 3.

In the first part of Chapter 4, the computer program developed to solve the nonlinear structural modification and coupling problems is validated with several different case studies. Firstly, a discrete system which has been analyzed previously in another study is considered. By partitioning the whole system into a linear original system and a nonlinear modifying system, problem is treated as a nonlinear structural modification problem. Then, after solving the problem through NLSM/CP, results obtained are compared with those given in literature and a perfect match is observed. As the second case study, the same system is partitioned into a linear original system and a nonlinear subsystem. This time, some linear and nonlinear elements are removed from the system and they are later used to couple two substructures, and this analysis is performed via NLSM/CP. Again, results obtained show an exact agreement with those given in literature. The approach proposed for coupling of two systems (main system is linear and modifying system is nonlinear) with linear and/or nonlinear elements is also validated. The performance of the method when applied to a real structure is investigated by applying it to a test rig consisting of a linear cantilever beam and nonlinear modification in the form of both ends fixed beam attached to the tip of the cantilever beam yielding cubic stiffness effect. The FRFs calculated by using the proposed method is compared with experimentally measured ones given in literature. A pretty good agreement is observed between the predicted and measured results even though modifying structure is modeled as a SDOF mass-nonlinear spring system. Furthermore, the validity of the approach is demonstrated by applying it to a lumped MDOF system and comparing the FRFs calculated by employing the proposed method with those obtained via time integration solution using ODE solvers of MATLAB®. The perfect match observed in this case study also demonstrates the validity of harmonic balance methodology used in all applications

(at least for the level of nonlinearity considered in the example applications). As the last verification case, a cantilever beam modified with a smaller nonlinear beam is considered. The main structure is modeled with Finite Element Method (FEM) and the dynamic characteristics of the modified structure are obtained by using the computer program developed. It is observed that results are in very good agreement with those obtained analyzing the whole modified structure.

In the second part of Chapter 4, applications of the method using the program developed are given in three main categories in which the advantages and the performance of it are examined. Firstly, a discrete linear system modified with another discrete nonlinear system is considered. When system response for the modified system is studied, the typical effect of cubic stiffness type of nonlinearity in modifying structure on all modes of the modified system is observed. In the second case study, same subsystems are coupled with a linear elastic coupling element. The frequency response of the coupled system is obtained and the effects of subsystem nonlinearity on the modified system modes are observed. Furthermore, the effect of the stiffness of the linear elastic coupling element on a specific mode of the system is investigated. It is seen that increasing values of the linear elastic coupling element stiffness will not only shift the investigated resonance to higher frequencies, but will also increase the effect of nonlinearity on this mode. Then, same systems are coupled with each other using a nonlinear coupling element in addition to the previous linear one. As the nonlinear coupling element, a linear spring with a clearance is used. The frequency response of the system is obtained and compared with the one obtained in previous case. It is observed that nonlinear coupling element affects some of the modes of the system more compared to the other modes. It causes an abrupt change in frequency response after the response amplitude reaches to the value of clearance. In the same example, the effect of this nonlinear coupling element for its varying parameters, such as for different clearances and spring stiffnesses, is also studied. As expected, the displacement value where the abrupt change occurs depends on the value of the clearance, while the behavior of the frequency response after that point depends on the additional linear spring stiffness of the coupling element after the response amplitude reaches to the value of clearance. As the last case, a real life

engineering problem is considered in order to show the applicability of the method to large ordered systems. In this problem, structural modification analysis of a mirror plate modified with a shaft-bearing assembly, where bearings at the two ends of the shaft are modeled as grounded hardening stiffnesses in vertical and horizontal directions, is studied. The effect of different amplitudes of external harmonic forcing on modified system FRF is investigated.

To conclude, the proposed method is based on the computation of the FRFs of a modified system from those of the original system and the dynamic stiffness matrix representing the modifications in the system. Due to the nonlinear behavior of the modifying system, the dynamic stiffness matrix turns out to be response level dependant and thus the solution requires an iterative approach. The iterative numerical solution was found to be successful as far as convergence to a solution is concerned. It should be noted that since the proposed method is an FRF based method, only the FRFs of the original system related with the DOFs we are interested in, in addition to the ones at the connection DOFs, are to be included in the calculations. Although the formulation includes a matrix inversion, the size of the matrix to be inverted is equal to the DOF of the modifying system, and therefore the method is most advantages when the modification is local. Especially in the design of large main structures which may need to be modified locally, the method is very useful and makes it possible for the designer to try various possible design changes or to make a parametric study with minimum computational cost. Furthermore, since the calculated FRFs are valid only for the level of the force applied, different FRFs of the system can be practically obtained for different amplitudes of the external harmonic forcing. It is concluded that the nonlinear structural modification/coupling method proposed and the computer program developed in this thesis can successfully and efficiently be used for nonlinear structural modification and coupling problems.

5.2 Recommendations for Future Work

The computer program developed, NLSM/CP, in this thesis is not a stand-alone executable file as it can only be run in association with the MATLAB[®] software package (version 5.1, or later). Thus, in order to make it a standalone executable file, the program can be written by using different visual programming languages by which the graphical user interface of the program can also be improved.

In order to predict the FRFs of the modified system, the computer program uses the FEA results of the original system and system matrices of the modifying system by also considering nonlinearity present in it. However, instead of using the FEA results of the original system, the modal test results of its real model may be used. Thus, whenever the original system is available, rather than the response predicted from FE model of the original system, more accurate experimental results can be used. In this case modal expansion techniques should be used in order to have consistent DOFs with the FE of the modifying system.

The method suggested here is capable of reanalyzing a large linear system modified locally with a nonlinear subsystem (which can be in the form of a coupled nonlinear subsystem) by specifying the fundamental harmonic describing functions for all nonlinear elements in modifying/coupled nonlinear subsystem or those used as coupling elements. Therefore, in order to improve the accuracy of the results, effect of the higher order harmonic terms may be considered as well, by using multi harmonic describing function theory.

The algorithm used to handle iterative solution procedure in this study is the Fixed Point Iteration Method. In order to obtain faster convergence by spending less time, alternative iteration methods may be used instead.

REFERENCES

- [1] Maropoulos P. G., Ceglarek D., Design Verification and Validation in Product Lifecycle, *CIRP Annals - Manufacturing Technology*, v.59, pp.740-759, 2010.
- [2] Ewins D. J., The Effect of Slight Non-linearities on Modal Testing of Helicopterlike Structures, *Vertica*, v.7, pp.1-8, 1993.
- [3] Vakakis A. F., Ewins D. J., Effects of Weak Non-linearities on Modal Analysis, *Proceedings of the 10th International Modal Analysis Conference*, San Diego, California, USA, pp.72-78, 3-7 February 1992.
- [4] Perinpanayagam S., Robb D., Ewins D. J., Barragan J. M., Nonlinearities in an Aero-engine Structure: From Test to Design, *2004 International Conference on Modal Analysis Noise and Vibration Engineering*, Leuven, Belgium, pp.3167–3182, 2004.
- [5] Snyder V. W., Structural Modification and Modal Analysis - A Survey, *International Journal of Analytical and Experimental Modal Analysis*, v.1, n.1, pp.45-52, 1986.
- [6] Aitavable P., Twenty Years of Structural Dynamic Modification - A Review, *Sound and Vibration*, v.37, n.1, pp.14-27, 2003.
- [7] Kyprianou A., Mottershead J. E., Ouyang H., Structural Modification. Part 2: Assignment of Natural Frequencies and Antiresonances by an Added Beam, *Journal of Sound and Vibration*, v.284, n.1–2, pp.267–281, 2005.

- [8] Kyprianou A., Mottershead J. E., Ouyang H., Assignment of Natural Frequencies by an Added Mass and One or More Springs, *Mechanical Systems and Signal Processing*, v.18, n.2, pp.263–289, 2004.
- [9] Li T., He J., Local Structural Modification Using Mass and Stiffness Changes, *Engineering Structures*, v.21, n.11, pp.1028-1037, 1999.
- [10] Park Y. H., Park Y. S., Structural Modification Based on Measured Frequency Response Functions: An Exact Eigenproperties Reallocation, *Journal of Sound and Vibration*, v.237, n.3, pp.411–426, 2000.
- [11] Park Y. H., Park Y. S., Structure Optimization to Enhance Its Natural Frequencies Based on Measured Frequency Response Functions, *Journal of Sound and Vibration*, v.229, n.5, pp.1235–1255, 2000.
- [12] Mottershead J. E., Mares C., Friswell M. I., An Inverse Method for The Assignment of Vibration Nodes, *Mechanical Systems and Signal Processing*, v.15, n.1, pp.87–100, 2001.
- [13] Crowley J. H., Rocklin G. T., Klosterman A. L., Vold H., Direct Structural Modification Using Frequency Response Functions, *Proceedings of the 2nd International Modal Analysis Conference Orlando, Florida*, 1984.
- [14] Özgüven H. N., Structural Modifications Using Frequency Response Functions, *Mechanical Systems and Signal Processing*, v.4, n.1, pp.53-63, 1990.
- [15] Özgüven H. N., Determination of Receptances of Locally Damped Structures, *Proceedings of the 2nd International Conference on Recent Advances in Structural Dynamics*, v.2, pp.887-892, Southampton, England, April 9-13, 1984.

- [16] Özgüven H. N., A New Method for Harmonic Response of Non-proportionally Damped Structures Using Undamped Modal Data, *Journal of Sound and Vibration*, v.117, n.2, pp.313-328, 1987.
- [17] Hang H., Shankar K., Lai J. C. S., Prediction of the Effects on Dynamic Response due to Distributed Structural Modification with Additional Degrees of Freedom, *Mechanical Systems and Signal Processing*, v.22, pp.1809-1825, 2008.
- [18] Hang H., Shankar K., Lai J. C. S., Effects of Distributed Structural Dynamic Modification with Reduced Degrees of Freedom, *Mechanical Systems and Signal Processing*, v.23, pp.2154-2177, 2009.
- [19] Hang H., Shankar K., Lai J. C. S., Effects of Distributed Structural Dynamic Modification with Additional Degrees of Freedom on 3D Structure, *Mechanical Systems and Signal Processing*, v.24, pp.1349-1368, 2010.
- [20] Canbaloğlu G. and Özgüven H. N., Structural Modifications with Additional DOF-Applications to Real Structures, *Proceedings of the 27th International Modal Analysis Conference*, Orlando, Florida, USA, 4-7 February 2009.
- [21] Şanlıtürk K. Y., An efficient method for linear and nonlinear structural modifications, *Proceedings of ESDA2002: 6th Biennial Conference on Engineering Systems Design and Analysis*, ESDA2002, Istanbul, Turkey, APM028, 2002.
- [22] Sherman J. and Morrison W. J., Adjustment of an Inverse Matrix Corresponding to Changes in the Elements of a Given Column or a Given Row of the Original Matrix, *Ann. Math. Statist.*, v.20, pp.621, 1949.

- [23] Köksal S., Cömert M. D., Özgüven H. N., Reanalysis of Dynamic Structures Using Successive Matrix Inversion Method, Proceedings of the 24th International Modal Analysis Conference, St. Louis, Missouri, USA, 30 January - 2 February 2006.
- [24] Köksal S., Cömert M. D., Özgüven H. N., Comparison and Application of Reanalysis Methods, Proceedings of the 25th International Modal Analysis Conference, Orlando, Florida, USA, 19-22 February 2007.
- [25] Krylov N. and Bogolyubov N., Introduction to Nonlinear Mechanics, Princeton University Press, 1947.
- [26] Van der Pol B., Forced Oscillations in a Circuit with Non-Linear Resistance, The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, v.3, pp.65–80, 1927.
- [27] Taylor J. H., Describing Functions, In Electrical Engineering Encyclopedia, John Wiley&Sons, New York, 1999.
- [28] Budak E., Özgüven H. N., A Method for Harmonic Responses of Structures with Symmetrical Nonlinearities, Proceedings of the 15th International Seminar on Modal Analysis and Structural Dynamics, Leuven, Belgium, v.2, pp.901–915, 17-21 September 1990.
- [29] Budak E., Özgüven H. N., Iterative Receptance Method for Determining Harmonic Response of Structures with Symmetrical Non-linearities, Mechanical Systems and Signal Processing, v.7, n.1, pp.75–87, 1993.
- [30] Tanrıku Ö., Forced Periodic Response Analysis of Non-linear Structures for Harmonic Excitation, Master's Thesis, Department of Mechanical Engineering, Imperial College, London, 1991.

- [31] Tanrikulu Ö., Kuran B., Özgüven H. N., İmregün M., Forced Harmonic Response Analysis of Non-linear Structures Using Describing Functions, *AIAA Journal*, v.31, n.7, pp.1313-1320, 1993.
- [32] Siller H. R. E., Non-linear Modal Analysis Methods for Engineering Structures, PhD Thesis, Imperial College London, University of London, 2004.
- [33] Abat D., Harmonic Vibration Analysis of Large Structures with Local Nonlinearity, MSc Thesis, Middle East Technical University, Graduate School of Natural and Applied Sciences, 2009.
- [34] Watanabe K. and Sato H., A Modal Analysis Approach to Nonlinear Multi-Degrees-of-Freedom System, *ASME Journal of Vibrations, Stress, and Reliability in Design*, v.110, pp.410-411, 1988.
- [35] Cömert M. D. and Özgüven H. N., A Method for Forced Harmonic Response of Substructures Coupled by Nonlinear Elements, *Proceedings of the 13th International Modal Analysis Conference*, Nashville, Tennessee, pp.139-145, February 1995.
- [36] Ferreira J. V., Ewins D. J., Nonlinear Receptance Coupling Approach Based on Describing Functions, *Proceedings of 14th International Modal Analysis Conference*, Dearborn, Michigan, USA, pp.1034–1040, 12-15 February 1996.
- [37] Ferreira J. V., Dynamic Response Analysis of Structures with Non-Linear Components, PhD thesis, Imperial College London, Department of Mechanical Engineering, Dynamics Section, 1998.
- [38] Chong Y. H., İmregün M., Coupling of Non-linear Substructures Using Variable Modal Parameters, *Mechanical Systems and Signal Processing*, v.14, n.5, pp.731-746, 2000.

- [39] Maliha R., Dođruer C. U., Özgüven H. N, Nonlinear Dynamic Modeling of Gear-Shaft-Disk-Bearing Systems Using Finite Elements and Describing Functions, *Journal of Mechanical Design* v.126, n.3, pp.534–541, 2004.
- [40] Worden K., Tomlinson G. R., *Nonlinearity in Structural Dynamics: Detection, Identification and Modeling*, IoP Publishing, 2001.
- [41] Huang S., *Dynamic Analysis of Assembled Structures with Nonlinearity*, PhD Thesis, Imperial College London, University of London, 2007.
- [42] Ferreira J. V., Serpa A. L., Application of the Arc-length Method in Nonlinear Frequency Response, *Journal of Sound and Vibration* v.284, pp.133–149, 2005.
- [43] Elizalde H., İmregün M., An Explicit Frequency Response Function Formulation for Multi-Degree-of-Freedom Non-Linear Systems, *Mechanical Systems and Signal Processing*, v.20, pp.1867–1882, 2006.
- [44] Čermelj P., Boltežar M., Modeling Localized Nonlinearities Using the Harmonic Nonlinear Super Model, *Journal of Sound and Vibration*, v.289, pp.1099-1112, 2006.
- [45] Göge D., Sinapius M., Fullekrug U., Link M., Detection and Description of Nonlinear Phenomena in Experimental Modal Analysis via Linearity Plots, *International Journal of Nonlinear Mechanics*, v.40, n.1, pp.27-48, 2005.
- [46] Wilson J. F., Callis E. G., *The Dynamics of Loosely Jointed Structures*, *International Journal of Non-Linear Mechanics*, v.39, pp.503-514, 2004.
- [47] Marchesiello S., Garibaldi L., Identification of Clearance-Type Nonlinearities, *Mechanical Systems and Signal Processing*, v.22, pp.1133-1145, 2008.

- [48] Meyer S., Link M., Local Non-Linear Softening Behavior: Modeling Approach and Updating of Linear and Nonlinear Parameters Using Frequency Response Residuals, Proceedings of 21st International Modal Analysis Conference, Orlando, Florida, USA, 3-6 February 2003.
- [49] Duan C., Singh R., Dynamic Analysis of Preload Nonlinearity in a Mechanical Oscillator, Journal of Sound and Vibration, v.301, pp.963–978, 2007.
- [50] Ciğeroglu E., Özgüven H. N., Nonlinear Vibration Analysis of Bladed Disks with Dry Friction Dampers, Journal of Sound and Vibration v.295, pp.1028–1043, 2006.
- [51] MATLAB R2008b Help Manual.
- [52] Arslan Ö., Aykan M. and Özgüven H. N., Parametric Identification of Structural Nonlinearities from Measured Frequency Response Data, Mechanical Systems and Signal Processing, v.25, n.4, pp.1112-1125, 2011.
- [53] Tiwari R., On-Line Identification and Estimation of Non-Linear Stiffness Parameters of Bearings, Journal of Sound and Vibration, v.235, n.5, pp.906-910, 2000.

APPENDIX A - CONFERENCE PAPER

Harmonic Response of Large Engineering Structures with Nonlinear Modifications †

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ABSTRACT: In a structural design, the structure may need to be modified and for each modification its dynamic characteristics may need to be determined by reanalyzing the structure dynamically. Since computational time and cost are very critical in design processes, structural modification methods become decisive, particularly for large systems, in predicting the dynamic behavior of modified structures from those of the original and modifying structures. Due to nonlinearity in most engineering structures, linearity assumption may not be applicable to all cases. Then, well known structural modification methods can not be directly used, and it is required to employ a nonlinear structural modification method. In this paper, a structural modification/coupling method proposed in an earlier study is extended for nonlinear modification/coupling. The nonlinearities are quasilinearised using describing function method, and thus nonlinear internal force vector is expressed in terms of a response-dependent matrix which can be regarded as a response level dependent “equivalent stiffness matrix”, called “nonlinearity matrix”. Then the method developed for linear structural modification/coupling is employed by using an iterative solution procedure. Three case studies are presented in this paper. In the first case study, a nonlinear test structure used in an earlier study is employed and the frequency responses of the system at different forcing levels are calculated by using the approach suggested. Then they are compared with experimental results. Secondly, a simple discrete system is analyzed to demonstrate the accuracy of the approach proposed. Lastly, a large scale model is considered to illustrate the applicability of the approach proposed to large order systems.

KEY WORDS: Structural modification, nonlinear structural modification, vibration of nonlinear structures, nonlinear structural coupling

1 INTRODUCTION

During mechanical design, usually, each prototype and final product have to be tested for qualification. Harmonic vibration test, via which behavior of a structure is examined under dynamic loads, is also among these qualification tests. Each modification made in order to satisfy design criteria changes structural properties such as natural frequencies, mode shapes and response of the structure. Therefore, tests should be repeated for each modified prototype. In such cases, using a mathematical model instead of the structure itself prevents costly modification and test procedures. For example, when additional stores are to be mounted on an aircraft, its dynamic properties will change and the modification will result in a considerably different dynamic response of the system under the same external forces. This new dynamic response can be calculated by reconstituting a new finite element model (FEM) for the modified system, and performing harmonic vibration analysis again. However, if several alternatives are to be considered, constituting FEM for each alternative and solving them will be expensive and time consuming. On the other hand, using the existing analysis of the original structure and dynamic properties of only the modifying structure, dynamic response of a modified structure can be estimated without performing the dynamic analysis of the whole structure, but instead, by employing structural modification and structural coupling techniques.

There is a vast literature on numerous different structural modification techniques and on their variations. Usually two

different problems are considered [1, 2]: the direct structural modification problem and the inverse structural modification problem. The first one intends to calculate the dynamic behavior of a modified structure once a set of changes has been established. On the other hand, in inverse structural modification problems optimum modifications are sought to accomplish the desired dynamic behavior. Each problem can be divided into two subgroups: frequency response function (FRF) based techniques and modal synthesis techniques. The problem considered in this work is FRF based direct structural problem.

Various FRF based direct structural modification methods are developed in order to predict dynamic response of a modified structure. These methods can also be grouped among themselves considering the general approaches used. In this study, structural modification method developed by Özgüven [3] is used and extended for nonlinear structural modification/coupling. The method is capable of finding FRFs of a modified structure by using the FRFs of the original structure and the mass, stiffness and damping matrices of the modifying structure. Recently, this method was extended and applied to distributed modifications [4-6] and then applied to real structures [7]. The same approach was used by Şanlıtürk [8], but matrix inversion was avoided by using Sherman-Morrison formulation. However, this method is limited to modifications which do not add new degrees of freedom (DOF). Köksal et al. [9] extended the Successive Matrix Inversion method to dynamic analysis of structures. This

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method uses again the same approach, but avoids matrix inversion by using power series expansion. Comparison of the three structural modification methods using similar approaches is made by Köksal et al. [10], and it is concluded that the structural modification method proposed by Özgüven [3] is the most efficient one among these three methods.

Since most engineering structures are often very complex and intrinsically nonlinear, in the analysis of such structures linear approach may not be applicable at all. In fact, inconsistency observed in some studies between the measured modal properties and modal properties theoretically calculated with linear assumption is found to be due to the nonlinearity involved in the structure [11, 12]. Hence, in problems where accuracy is the primary concern, taking the nonlinear effects into account becomes inevitable. Among several methods suggested for structural modification/coupling analysis including nonlinearity, the methods proposed by Cömert and Özgüven [13], Ferreira and Ewins [14] and Chong and İmregün [15] are among the ones worth mentioning.

In this paper, firstly, theory of the structural modification method for nonlinear modifications is summarized. Then, applications of the approach proposed are given with three case studies. In the first application, the method proposed is applied on a test structure and thus the applicability of the approach to real life structures is demonstrated. Secondly, a simple discrete system is analyzed and results are compared with those obtained by time integration solution in order to study the accuracy of the approach. Lastly, a large scale model is considered to show the applicability of the method to large order systems.

2 THEORY

The structural modification method suggested by Özgüven [3] has been successfully used in various applications for linear systems. For local modification, where the total DOF of the original system does not change after modification, the FRFs of the modified system can be expressed in terms of those of the original system and the dynamic stiffness matrix of the modifying structure as shown below [3]:

$$[\gamma_{11}] = [I] + [\alpha_{11}][D_{11}]^{-1}[\alpha_{11}] \quad (1)$$

$$[\gamma_{12}]^T = [\gamma_{21}] = [\alpha_{21}][I] - [D_{11}][\gamma_{11}] \quad (2)$$

$$[\gamma_{22}] = [\alpha_{22}] - [\alpha_{21}][D_{11}][\gamma_{12}] \quad (3)$$

where $[\alpha]$ and $[\gamma]$ represent receptance matrices of the original and modified systems, respectively, $[D]$ shows dynamic stiffness matrix of the modifying structure, and superscripts 1 and 2 denote the coordinates on which modification is applied and the remaining coordinates, respectively.

However, when the modification is in the form of coupling a subsystem to the original system, the equations take the form [3]:

$$\begin{bmatrix} [\gamma^{bb}] \\ [\gamma^{cb}] \\ [\gamma^{ab}] \\ [\gamma^{ac}] \end{bmatrix} = \begin{bmatrix} [I] & [0] \\ [0] & [0] \end{bmatrix} + \begin{bmatrix} [\alpha^{bb}] & [0] \\ [0] & [I] \end{bmatrix}^{-1} [D] \begin{bmatrix} [\alpha^{bb}] \\ [0] \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} [\gamma^{bb}] & [\gamma^{bc}] \\ [\gamma^{cb}] & [\gamma^{cc}] \end{bmatrix} = \begin{bmatrix} [I] & [0] \\ [0] & [0] \end{bmatrix} + \begin{bmatrix} [\alpha^{bb}] & 0 \\ 0 & [I] \end{bmatrix}^{-1} [D] \begin{bmatrix} [\alpha^{bb}] & [0] \\ [0] & [I] \end{bmatrix} \quad (5)$$

$$[\gamma^{aa}] = [\alpha^{aa}] - [\alpha^{ab}] \begin{bmatrix} [I] & [0] \\ [0] & [I] \end{bmatrix}^{-1} [D] \begin{bmatrix} [\alpha^{bb}] \\ [\gamma^{ac}] \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} [\gamma^{bb}] \\ [\gamma^{cb}] \end{bmatrix} \begin{bmatrix} [\gamma^{bc}] \\ [\gamma^{cc}] \end{bmatrix} = \begin{bmatrix} [\alpha^{bb}] \\ [0] \end{bmatrix} \begin{bmatrix} [I] & [0] \\ [0] & [I] \end{bmatrix}^{-1} [D] \begin{bmatrix} [\alpha^{bb}] \\ [\gamma^{ac}] \end{bmatrix} \quad (7)$$

where superscript α denotes DOFs that belong to the original structure only, superscript b denotes DOFs corresponding to connection nodes, and superscript c denotes DOFs that belong to modifying structure only.

Now, let us consider the equations of motion of a modifying structure which shows nonlinear behavior. The equations of motion for such a structure can be written as follows:

$$[M_{\text{mod}}] \cdot \{\ddot{x}\} + i[H_{\text{mod}}] \cdot \{x\} + [K_{\text{mod}}] \cdot \{x\} + \{N(x, \dot{x})\} = \{f\} \quad (8)$$

Here $[K_{\text{mod}}]$, $[M_{\text{mod}}]$ and $[H_{\text{mod}}]$ represent stiffness, mass and structural damping matrices of the modifying structure, respectively. When a harmonic force

$$\{f\} = \{F\} \cdot e^{i\omega t} \quad (9)$$

is applied on the system, the system response as a first approximation, can be assumed to be harmonic at the same frequency:

$$\{x\} = \{X\} \cdot e^{i\omega t} \quad (10)$$

Based on this assumption, the internal nonlinear forces can also be assumed to be harmonic at the same frequency:

$$\{N\} = \{S\} \cdot e^{i\omega t} \quad (11)$$

It has been shown by Budak and Özgüven [16] that the amplitude vector of the internal nonlinear forces, for a wide variety of nonlinearity, can be expressed as

$$\{S\} = [\Delta] \cdot \{X\} \quad (12)$$

where $[\Delta]$, named as “nonlinearity matrix”, is a function of the unknown displacement amplitude vector $\{X\}$. The formulation for $[\Delta]$ is first introduced by Budak and Özgüven [16,17] for certain types of nonlinearities, and later extended for any type of nonlinearity using Describing Function Method (DFM) by Tanrikulu et al. [18]. The elements of nonlinearity matrix are defined as:

$$\Delta_{kk} = \sum_{m=1}^n v_{km} \quad (13)$$

$$\Delta_{km} = -v_{km}, (k \neq m) \quad (14)$$

where subscripts k and m represent the general coordinates.

If equations (9), (10), (11) and (12) are substituted into equation (8), it can be seen that nonlinear internal forces can be included in the analysis by considering an additional

equivalent stiffness matrix in the system which is function of unknown response amplitudes. So, for such a nonlinear modification dynamic stiffness matrix for the modifying structure will take the form:

$$[D_{\text{mod}}] = [K_{\text{mod}}] - \omega^2 [M_{\text{mod}}] + i[H_{\text{mod}}] + [\Delta] \quad (15)$$

Note that the size of the matrices to be inverted in this method is equal to the DOF of the modifying structure only. So the method is very efficient for local modifications, which is usually the case in real applications. Furthermore, as the formulation is given in terms of FRFs; except for connection nodes, only the FRFs of the required DOFs can be kept in the above equations by deleting rows and columns of the matrices corresponding to DOFs which are not our concern. This would reduce computational effort drastically, and is another advantage of the method.

When the method is used for nonlinear modifications, solution can be obtained by using an iterative approach, due to the response dependent nonlinearity matrix in $[D_{\text{mod}}]$ expression. Convergence is checked by calculating the error between two successive solutions:

$$e = \frac{|\{X\}_{j+1} - \{X\}_j|}{|\{X\}_j|} \times 100 \quad (16)$$

In the iterative solution, Fixed Point Iteration Method is applied. Number of iterations made is reduced considerably by using average displacement $\{X^*\}_{j+1}$ described below, instead of $\{X\}_{j+1}$.

$$\{X^*\}_{j+1} = \frac{\{X\}_{j+1} + \{X\}_j}{2} \quad (17)$$

In order to avoid divergence or numerical instability, and also to obtain fast convergence, relaxation is applied to fixed point iteration approach as given below:

$$\{X^*\}_{j+1} = (1-\lambda)\{X\}_{j+1} - \lambda\{X\}_j, (0 \leq \lambda \leq 1) \quad (18)$$

where λ is the relaxation coefficient. Calculations are performed by sweeping from low to high frequency and then in reverse direction in the desired frequency range, and thus multiple solutions that could result from nonlinearities are obtained. Linear response obtained by omitting nonlinearity in modifying structure is taken as the initial value of displacement vector $\{X\}_j$. For the following iterations, the converged value of $\{X\}_j$ for a previous frequency value is taken as the initial value.

3 CASE STUDIES

In this section, three case studies are presented to demonstrate the application of the approach proposed for nonlinear modifications. First a real structure is considered and the accuracy of the approach when applied to real systems is demonstrated. In the other two case studies lumped parameter systems are considered, and first the results obtained by the method are compared with time integration results for a small

order system. Later, a large order discrete system is analyzed with the method proposed.

3.1 Case Study 1 - Real structure

For the implementation of the nonlinear structural modification method suggested here, a nonlinear test rig, which has been originally designed by Chong and İmregün [15], and has also been used recently in a previous study [19], is employed. The test rig consists of a linear cantilever beam with its free end held between two thin identical beams (Figure 1). The model used to represent the nonlinear dynamic test rig is shown in Figure 2.

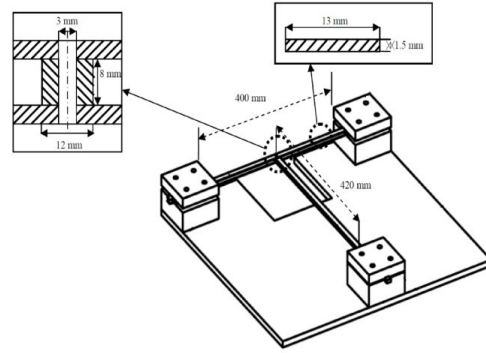


Figure 1. Test rig assembly and its dimensions.



Figure 2. Model of the test rig.

Here, the fixed-fixed two thin beams are considered as the modifying structure and modeled as a concentrated nonlinear spring in y-direction with a concentrated equivalent mass as shown in Figure 2. The concentrated nonlinear cubic stiffness coefficient, β , is taken as $2.667 \times 10^8 \text{ N/m}^3$ which is the value identified experimentally [19]. From the material and geometric properties of the thin beams used [19], the linear part of the equivalent spring stiffness and equivalent mass values are calculated as 4519 N/m^3 and 45.3 g , respectively. The linear cantilever beam is considered as the original structure and modeled by using a standard FE program as shown in Figure 3. The modal data for the original structure is obtained via FE analysis.

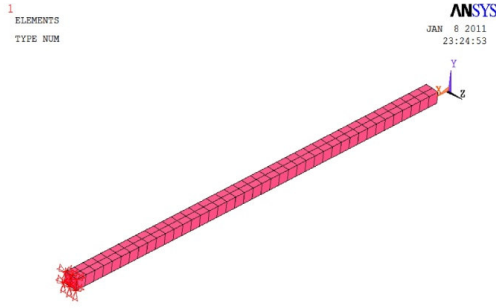


Figure 3. The FE model of the cantilever beam.

Constructed model is solved using the program developed for nonlinear modification. Frequency resolution during solution is taken as 0.25 Hz just as in the experiment [19]. The response amplitude dependent structural damping value identified in [19] is used in the analysis. The direct point FRFs for the tip of the cantilever beam are calculated at forcing levels of 0.1, 0.5 and 1 N, and they are compared with the experimental values measured by Arslan et al. [19], in Figures 4-6, respectively.

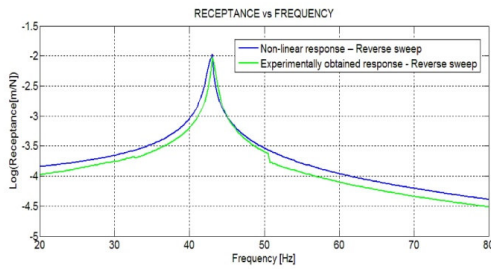


Figure 4. Calculated and measured FRF values at F = 0.1 N.

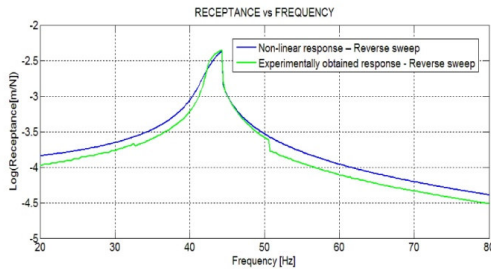


Figure 5. Calculated and measured FRF values at F = 0.5 N.

As can be seen in Figures 4-6, good agreements are obtained between experimental and predicted values even at high forcing levels where jump, typical response distortion due to cubic stiffness nonlinearity, occurs in the frequency response.

Slight differences are believed to be due to modeling the modifying structure as a SDOF mass-nonlinear spring system and also, partly due to using the values cited in literature for the material properties of the original beam in FE model.

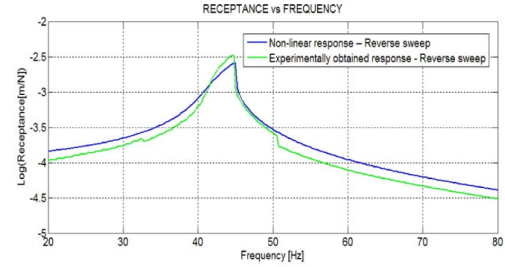


Figure 6. Calculated and measured FRF values at F = 1 N.

3.2 Case Study 2 - Discrete system

In this part, nonlinear structural modification method is applied to a simple discrete system with nonlinear modifications (Figure 7). In order to verify the approach, results obtained are compared with the ones calculated by time domain solution using the ODE integrators of MATLAB. According to stiffness characteristics of the problem, integrations are performed using several integrators like ODE45, ODE113, ODE15s, etc. [20]. Integrations are continued until steady state is reached.

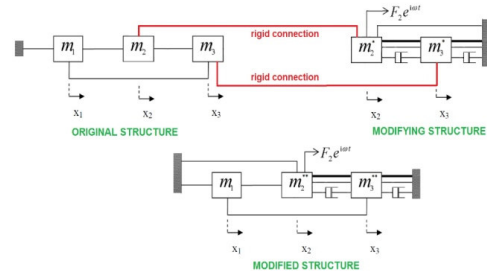


Figure 7. System used in Case Study 3.

System matrices of the original system are given as follows:

$$[K_0] = \begin{bmatrix} 90000 & -30000 & -30000 \\ -30000 & 60000 & -30000 \\ -30000 & -30000 & 60000 \end{bmatrix} N/m \quad (19)$$

$$[M_0] = \begin{bmatrix} 30 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 10 \end{bmatrix} kg \quad (20)$$

System matrices of the modifying structure are as follows:

$$[K_{mod}] = \begin{bmatrix} 60000 & -30000 \\ -30000 & 60000 \end{bmatrix} N/m \quad (21)$$

$$[M_{\text{mod}}] = \begin{bmatrix} 35 & 0 \\ 0 & 15 \end{bmatrix} \text{kg} \quad (22)$$

$$[C_{\text{mod}}] = \begin{bmatrix} 25 & -25 \\ -25 & 50 \end{bmatrix} \text{Ns/m} \quad (23)$$

The nonlinearities in the system shown are defined in Table 1.

Table 1. Nonlinearity definitions.

Nonlinear Connections (DOF1-DOF2)	Nonlinearity Type	Nonlinearity Coefficients
2* - 3*	Cubic Stiffness	$\beta = 1e11$
3* - Ground	Arctan Stiffness	$\rho = 8$ $\kappa = 1750$

Typical behaviors of cubic and arctan stiffness elements are shown in Figures 8 and 9, respectively. Definitions of the nonlinearity coefficients in Table 1 are also given in these figures.

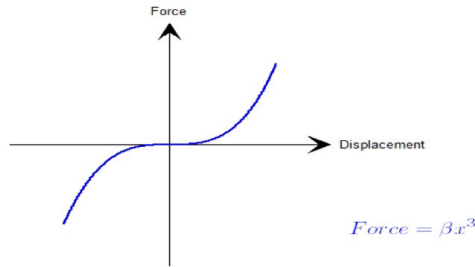


Figure 8. Force-displacement behavior of a typical cubic stiffness element.

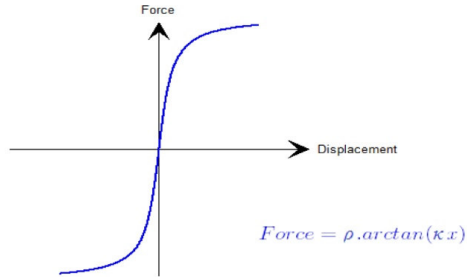


Figure 9. Force-displacement behavior of a typical arctan stiffness element.

The system response at m_3 is obtained by the method presented, and compared with the solution found by time domain integration in Figures 10 and 11 for the following harmonic excitation;

$$\{f\} = \begin{Bmatrix} 0 \\ 12 \\ 0 \end{Bmatrix} e^{i\omega t} N \quad (24)$$

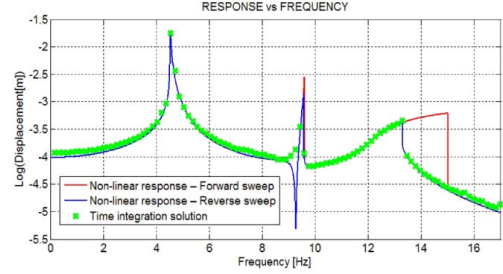


Figure 10. Comparison of frequency responses obtained by method proposed and time domain integration for 3rd mass.

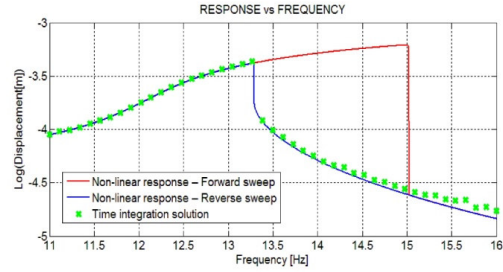


Figure 11. Comparison of frequency responses obtained by method proposed and time domain integration for 3rd mass (zoom around third mode).

In Figures 10 and 11, there is an excellent agreement between the results obtained by two approaches at all frequencies in the frequency range which covers all three resonances. It should also be noted here that the results obtained with time integration coincide with the results found in reverse sweep.

3.3 Case Study 3 - Large scale structure

In this application, a large scale system is analyzed via nonlinear structural modification program developed in order to show the application of the method to large order systems, such as finite element models.

In this example, the cantilever plate shown in Figure 12 is modified by attaching the plate shown in Figure 13 to the free edge of the original structure that constitutes the modified structure given in Figure 14.

Original plate was divided into 144 shell elements with 6 DOF per node yielding total DOF of 1014. The modifying structure was divided into 24 shell elements with 6 DOF per node giving total DOF of 234 as shown in Table 2. The geometrical and material properties of the original and modifying plates are given in Table 3. Structural damping with a loss factor of 0.5% is assumed. Nonlinearities in the modifying system are defined in Table 4 and Figure 15.

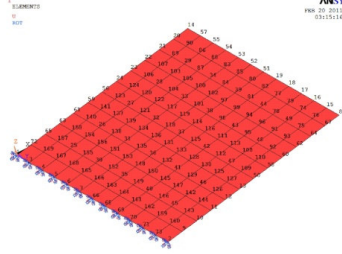


Figure 12. The FE model of the original plate.

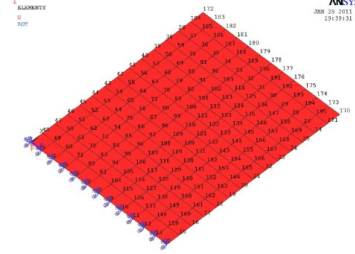


Figure 14. The FE model of the modified plate.

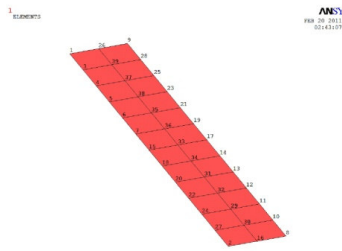


Figure 13. The FE model of the modifying plate.

Table 2. FE information of original and modifying plates.

	Original Plate	Modifying Plate
Number of Elements	144	24
Number of Nodes	169	39
DOF	1014	234

Table 3. Geometrical and material properties of original and modifying plates.

	Original Plate	Modifying Plate
Young's Modulus	71 GPa	71 GPa
Poisson's Ratio	0.33	0.33
Density	2770 kg/m ³	2770 kg/m ³
Length	300 mm	50 mm
Width	300 mm	300 mm
Thickness	2 mm	2 mm

A harmonic excitation of magnitude 10 N is applied to the 17th node of the modifying plate in z-direction, and frequency response at node 170 in z-direction is predicted for the modified structure by employing the method proposed. The result is shown in Figure 16. It can clearly be seen in Figure 16 that nonlinearities become effective on system response after a displacement of 1e-3 m as expected from the definition of the nonlinearity present in the system. Although the nonlinearity changes the frequency response around resonance considerably causing a jump, no problem is observed in convergence when the method proposed is used.

Table 4. Nonlinearity definitions.

Nonlinear Connections (DOF1-DOF2)	Nonlinearity Type	Nonlinearity Coefficients
45 - Ground	Piecewise Linear Stiffness	$\kappa^1 = 0$ $\kappa^2 = 1e4$ $\delta = 1e-3$
51 - Ground	Piecewise Linear Stiffness	$\kappa^1 = 0$ $\kappa^2 = 1e4$ $\delta = 1e-3$

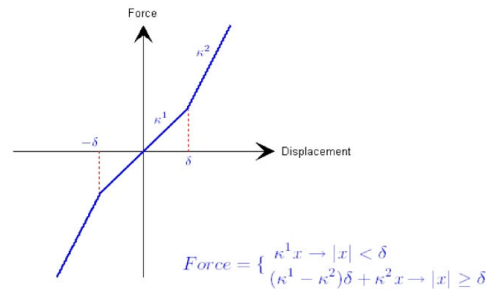


Figure 15. Force-displacement behavior of a typical piecewise linear stiffness element.

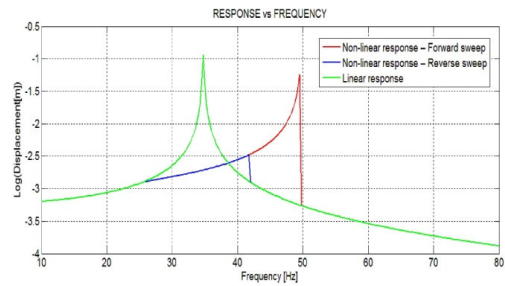


Figure 16. Frequency domain response at 170th node of the modified plate in z-direction.

4 DISCUSSIONS

In this paper, the method proposed in an earlier study for structural dynamic modifications with additional degrees of freedom [3] is applied to structures with nonlinear modifications. In the method proposed, the frequency response functions of the modified structure are calculated from those of the original structure and the system matrices, including the nonlinearity matrix, of the modifying structure.

The performance of the method when applied to a real structure is investigated by applying it to a test rig consisting of a linear cantilever beam and nonlinear modifications in the form of both ends fixed beam attached to the tip of it yielding cubic stiffness. The FRFs calculated by using the proposed method is compared with experimentally measured ones [19]. A very good agreement is observed between the predicted and measured results even though modifying structure is modeled as a SDOF mass-nonlinear spring system. In the second case study, the validity of the approach is demonstrated by applying it to a lumped multi degree of freedom system. The FRFs calculated by employing the method proposed are compared with those obtained via time integration solution using ODE solvers of MATLAB, and a perfect match is observed. Thus validity of the approach suggested is demonstrated. In order to show the applicability of the method to large order systems, the program developed is used to analyze the finite element model of a plate coupled to another one showing nonlinear behavior. Although more than 1200 DOFs are used in the model, nonlinear responses are calculated through an iterative process without any convergence problem.

It is concluded in this study that the structural modification method proposed can be successfully and efficiently used for structures with nonlinear modifications.

REFERENCES

- [1] V. W. Snyder, *Structural modification and modal analysis-a survey*, International Journal of Analytical and Experimental Modal Analysis, Vol: 1, pp. 45-52, 1986.
- [2] P. Avitabile, *Twenty years of structural dynamic modification-a review*, Sound and Vibration, Vol: 37, Number: 1, pp. 14-27, 2003.
- [3] H. N. Özgüven, *Structural modifications using frequency response functions*, Mechanical Systems and Signal Processing, Vol: 4, Number: 1, pp. 53-63, 1990.
- [4] H. Hang, K. Shankar, J. C. S. Lai, *Prediction of the effects on dynamic response due to distributed structural modification with additional degrees of freedom*, Mechanical Systems and Signal Processing, Vol: 22, pp. 1809-1825, 2008.
- [5] H. Hang, K. Shankar, J. C. S. Lai, *Effects of distributed structural dynamic modification with reduced degrees of freedom*, Mechanical Systems and Signal Processing, Vol: 23, pp. 2154-2177, 2009.
- [6] H. Hang, K. Shankar, J. C. S. Lai, *Effects of distributed structural dynamic modification with additional degrees of freedom on 3D structure*, Mechanical Systems and Signal Processing, Vol: 24, pp. 1349-1368, 2010.
- [7] G. Canbaloglu, H. N. Özgüven, *Structural modifications with additional DOF-Applications to real structures*, Proceedings of the 27th International Modal Analysis Conference, Orlando, Florida, USA, 4-7 February 2009.
- [8] K. Y. Şanlıtürk, *An efficient method for linear and nonlinear structural modifications*, Proceedings of ESDA2002: 6th Biennial Conference on Engineering Systems Design and Analysis, ESDA2002, Istanbul, Turkey, APM028, 2002.
- [9] S. Köksal, M. D. Cömert, H. N. Özgüven, *Reanalysis of dynamic structures using successive matrix inversion method*, Proceedings of the 24th International Modal Analysis Conference, St. Louis, Missouri, USA, 30 January - 2 February 2006.
- [10] S. Köksal, M. D. Cömert, H. N. Özgüven, *Comparison and application of reanalysis methods*, Proceedings of the 25th International Modal Analysis Conference, Orlando, Florida, USA, 19-22 February 2007.
- [11] D. J. Ewins, *The effect of slight non-linearities on modal testing of helicopterlike structures*, Vertica, Vol: 7, pp. 1-8, 1983.
- [12] A. F. Vakakis, D. J. Ewins, *Effects of weak non-linearities on modal analysis*, Proceedings of the 10th International Modal Analysis Conference, San Diego, California, USA, pp. 72-78, 3-7 February 1992.
- [13] M. D. Cömert, M.D. and H. N. Özgüven, *A method for forced harmonic response of substructures coupled by nonlinear elements*, Proceedings of the 13th International Modal Analysis Conference, Nashville, Tennessee, pp. 139-145, February 1995.
- [14] J. V. Ferreira, D. J. Ewins, *Nonlinear receptance coupling approach based on describing functions*, Proceedings of 14th International Modal Analysis Conference, Dearborn, Michigan, USA, pp. 1034-1040, 12-15 February 1996.
- [15] Y. H. Chong, M. İmregün, *Coupling of Non-linear substructures using variable modal parameters*, Mechanical Systems and Signal Processing, Vol: 14, Number: 5, pp. 731-746, 2000.
- [16] E. Budak, H. N. Özgüven, *A method for harmonic responses of structures with symmetrical nonlinearities*, Proceedings of the 15th International Seminar on Modal Analysis and Structural Dynamics, Leuven, Belgium, Vol. 2, pp. 901-915, 17-21 September 1990.
- [17] E. Budak, H. N. Özgüven, *Iterative receptance method for determining harmonic response of structures with symmetrical non-linearities*, Mechanical Systems and Signal Processing, Vol: 7, Number: 1, pp. 75-87, 1993.
- [18] Ö. Tanrikulu, B. Kuran, H. N. Özgüven, M. İmregün, *Forced harmonic response analysis of non-linear structures using describing functions*, AIAA Journal, Vol: 31, Number: 7, pp. 1313-1320, 1993.
- [19] Arslan, Ö., Aykan, M. and Özgüven, H. N., *Parametric Identification of structural nonlinearities from measured frequency response data*, Mechanical Systems and Signal Processing, doi:10.1016/j.ymssp.2010.10.010, 2010.
- [20] MATLAB R2008b Help Manual.