# BOOTSTRAPPING SHARED VOCABULARY IN A POPULATION - WEIGHTED LISTS WITH PROBABILISTIC CHOICE

### A THESIS SUBMITTED TO THE GRADUATE SCHOOL OF INFORMATICS INSTITUTE OF MIDDLE EAST TECHNICAL UNIVERSITY

BY

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE IN THE DEPARTMENT OF COGNITIVE SCIENCE

SEPTEMBER 2011

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# ABSTRACT

# BOOTSTRAPPING SHARED VOCABULARY IN A POPULATION - WEIGHTED LISTS WITH PROBABILISTIC CHOICE

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September 2011, 51 pages

Works on semiotic dynamics and language as a complex adaptive system in general has been an important lane of research over the last decade. In this study, the mean-field naming game model developed in the course of the pioneering research programme of Luc Steels and colleagues is modified to include probabilistic word choice based on weighted lists of words, instead of either deterministic or totally random word choice based on (ordered) sets of words. The parameters' interaction and this interaction's effect on time of convergence of the system and size of individual lexicons over time are investigated. The classical model is found to be a special case of this proposed model. Additionally, this model has more parameters and a larger state space which provides additional room for tweaking for time- or space-optimization of the convergence process.

Keywords: semiotic dynamics, lateral inhibition, language games, probabilistic choice, emergence

# ÖZ

#### BİR POPÜLASYONDA ORTAK SÖZCÜK HAZNESİNİN GELİŞİMİ-AĞIRLIKLANDIRILMIŞ LİSTELERLE OLASILIKSAL SEÇİM

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Ağustos 2011, 51 sayfa

İşaretbilimsel dinamiklerin incelenmesi ve dilin genel olarak kompleks adaptif bir sistem olarak ele alınması, son on yılda önemli bir araştırma konusu haline gelmiştir. Bu çalışmada, bu konunun öncülerinden olan Luc Steels ve meslektaşlarının geliştirdiği isimlendirme oyunu, (sıralı) bir kümeden deterministik veya tamamen rastgele sözcük seçimi yerine, ağırlıklandırılmış bir listeden olasılıksal tercihle sözcük seçimi kullanmak üzere geliştirilmiştir. Parametrelerin birbirleriyle etkileşimi ve bu etkileşimin sistemin sözcükler üzerinde mutabakat sağlama süresi ile tekil ajanların sözcük dağarcığı büyüklüğü üzerindeki etkileri incelenmiştir. Çalışma sonucunda Steels'in klasik modelinin, çalışmadaki modelin özel bir durumu olduğu tespit edilmiştir. Ayrıca bu modeldeki fazladan parametreler ve buna bağlı olarak daha geniş olan durum uzayından dolayı modelin, ihtiyaca göre oyunun gerek zaman gerekse de hafiza kullanımı açısından optimizasyonuna izin verdiği gözlemlenmiştir.

Anahtar Kelimeler: işaretbilimsel dinamikler, lateral kısıtlama, dil oyunları, olasılıksal seçim, emerjans

To my family...

# ACKNOWLEDGMENTS

I wish to express my sincerest gratitude to my thesis supervisor Dr. Cem Bozşahin for his introduction of this field to me, his guidance, his invaluable feedback and of course his enjoyable company.

I also wish to present my appreciation to all those who work or study at the Informatics Institute of METU without whom all those litres of tea, hours of table tennis, months of work and heaps of words would mean so little. I would like to explicitly thank the administration for providing me with access to the institute's significant IT infrastructure without which this thesis would still be incomplete, pending data analysis.

And the last but not the least, I want to extend my deepest thanks to my dear friends, Adnan, Fatih, Gökçe, Dinçer and countless others (who will hopefully forgive me for not printing their names) that provided me with the social gratification, academic feedback and tolerance to nonsense any graduate student so gravely needs.

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# **CHAPTER 1**

# Introduction

Semiotic dynamics and multiagent simulations of communication have become an increasingly recurrent topic in the AI literature over the last decade (Puglisi, Baronchelli, & Loreto, 2008; Baronchelli, Felici, Loreto, Caglioti, & Steels, 2006; Steels & Kaplan, 2002; Baronchelli, Dall'Asta, Barrat, & Loreto, 2005; Batali, 1998). The shift from single agent based simulations of acquisition to population based simulations with simpler agents has allowed a more precise characterization of the convention-forming among populations of communicating agents, with respect to the individual properties, configurations and behaviours of the agents. More importantly, it has successfully captured the fact that language is not a phenomenon confined to an individual - it is inseparable from the community of language users whose individual uses of language accumulate to determine the global properties and dynamics of a language. This is in contrast with approaching language as either a macro-level phenomenon by discarding the micro-interactions and individual differences that make up and change a language or as a micro-level phenomenon by losing sight of how the micro-level interactions are affected by global, emergent properties at the macro level. The bidirectional interaction between macro- and micro-levels enables investigation of dynamics of language and communication without losing consistency between these levels, and therefore bestows greater explanatory power to theories that draw from this approach.

One fruitful line of research has been designing language games for the agents to play, which serves as the task that utilizes or "bootstraps" communication. The language games eventually became an established methodology for emergent language simulations in the literature. Language games, in this literature, is any multiagent simulation where agents interact and try to form conventions on communication to aid the accomplishment of a task or achieve and maintain a state of affairs in the world in a context that includes other agents.<sup>1</sup> There has been many flavours of such games used in simulations since their first use, such as discrimination games (Steels, 1996b), naming games (Baronchelli, Felici, et al., 2006), guessing games (Vogt, 2005) and selfish games (Vogt, 2002).

The proposed work is going to build on the naming game, and investigate the effects of adopting a weighted list model for the agents' lexicons, instead of the original lexicons that are essentially very simple sets confined to deletion/addition upon interactions (Baronchelli, Felici, et al., 2006). By not necessarily removing unsuccessful synonymous words in the lexicon upon agreement between agents, a weighted inventory of all the words (or most, if there is a lower threshold for word weights) used by an agent in the game can be maintained, and the weight distributions of competing synonyms with respect to their ranks in individual lexicons becomes accessible. This is predicted to have a number of effects on the properties of the model, namely the number of rounds it takes until convergence, their scaling with the population size and the robustness of the model i.e. the setback introduced by a failed round based on a to-be-successful word or a successful round based on a word that will not be chosen at the end of the game.

In the subsequent chapters, the concepts of emergence and self-organization are explained, followed by a review of the language game literature. The last two chapters are dedicated to the empirical model used, the results acquired and the conclusions drawn from the results.

<sup>&</sup>lt;sup>1</sup> See (Steels, 1996a) for the introduction of language games to the field.

# **CHAPTER 2**

### **Background on Emergence and Self-organisation**

All of these thoughts, all of these doubts and hopes Inside I took out to form a new breed A new way to be And now I am many, so many So much larger than ever I were Yet, at the same time So much smaller and more vulnerable They all carry shards of the whole Together they become me I see them interact, develop I see them take different sides As were they different minds Believers of different ways, and different gods I think they will teach me something<sup>1</sup>

As it is clear to anyone who set out to work in this field, emergence and self-organisation have been circulating in the literature of simulations of language as a dynamical system. As one familiarizes itself with the literature, the meanings of emergence and self-organisation become so familiar that one needs to be really strict to even think of questioning what they really mean. When one does, however, it turns out that almost none of the works that either draw from or build on these concepts include a working definition of them.<sup>2</sup> They rely on the readers' (and arguably the authors') intuition of these concepts, which are mostly meanings derived from texts that use these words. Of course, these texts in turn use the terms within that same "derived meaning" constraint, which makes these concepts surprisingly vague for scientific terms in hyper-circulation. This process, ironically, resembles a version of the naming game

<sup>&</sup>lt;sup>1</sup> Soliloquy at the end of the song "Deus Nova", by Pain of Salvation

 $<sup>^2</sup>$  We are excluding philosophical works from this observation as they understandably and thankfully tend to be very strict in defining concepts.

where people learn how to name objects without any access to objects themselves and try to cross-situationally infer some working definition so that their definitions do not stand out from the population of all the other working definitions of the terms.

One big problem with this situation is that self-organisation and emergence are often used as synonyms although each can be observed without the other (De Wolf & Holvoet, 2005). Therefore, it is necessary to have explicit working definitions of both in order to clearly identify whether a system is self-organising, or it carries emergent properties, or both.

To this end, we have decided that it is of great importance that this work does not suffer from the same problem. Therefore, a whole chapter is dedicated to what these terms refer to, what they do not refer to, how they relate, and perhaps most importantly, how they differ. The rest of the chapter is divided into two sections, one for emergence and one for self-organisation. Each section has two subsections that discuss the definitions of and examples for these concepts.

Of course, there are no clear-cut and globally agreed-on definitions of these concepts in the literature, which is one of the issues that cause this vagueness. Therefore, we have decided to explicitly adopt a specific view as a source of working definitions, and clarify this position by using examples. This will hopefully allow our working definitions to be compared and contrasted with any other working definition. This chapter is mainly based on the extensive literature survey by De Wolf and Holvoet (2005) and the philosophical discussion of emergence by Chalmers (2006).

#### 2.1 Emergence

Emergence is hardly a new concept (Goldstein, 1999). In fact, Aristotle is one of the first to mention a similar concept in his infamous work Metaphysics:

Since that which is compounded out of something so that the whole is one, not like a heap but like a syllable-now the syllable is not its elements, ba is not the same as b and a, nor is flesh fire and earth (for when these are separated the wholes, i.e. the flesh and the syllable, no longer exist, but the elements of the syllable exist, and so do fire and earth); the syllable, then, is something-not only its elements (the vowel and the consonant) but also something else, and the flesh is not only fire and earth or the hot and the cold, but also something else (Aristotle, n.d.)

However, how to define emergence is far from being agreed on, although commonalities between specific characterizations of the concept do outline some important aspects of it which is why it has been possible for most authors to tiptoe around this incertainty without causing too much of a fuss. A popular, common-place definition that keeps popping up in the literature is global behaviour or properties that arise from local interactions. However, this imprecise definition leaves open a huge number of questions such as what the relationship between the global emergents and the local interactions are, to what extent these globals are accessible to or deducible from the local interactions. Since we make use of the hairy concept of "reducibility", our best hope in understanding emergence is looking at how philosophers tend to characterize emergence and how they try to answer the questions listed.

David Chalmers is a philosopher particularly interested in emergence, fitting to his interest in consciousness. He mentions two flavours of emergence, namely strong emergence and weak emergence(Chalmers, 2006).<sup>3</sup> In what follows, these concepts and how they relate to this work are outlined.

#### **Strong Emergence**

Chalmers defines strong emergence to cover the concept where a "high-level phenomenon arises from the low-level domain, but truths concerning that phenomenon are not *deducible* even in principle from truths in the low-level domain"(Chalmers, 2006). In other words, the low-level domain not only unexpectedly gives rise to the global behaviour, but it cannot even be deduced by any means that the system is going to behave that way.

Chalmers refers to consciousness as the only solid, real-world example of this kind of emergence.<sup>4</sup> If we use his definition, although consciousness arises from the physical domain, all the facts of the physical domain (i.e. the distribution of particles and fields) supplemented with the physical laws do not suffice to deduce that the system will have consciousness. Although consciousness supervenes on the physical domain, physical laws alone fail to account for consciousness, which makes consciousness irreducible to truths and laws of the low-level,

 $<sup>^{3}</sup>$  He is not the first to do so as this distinction goes back to 1920s, see Bedau et al. (1997) for an earlier account of weak and strong emergence.

<sup>&</sup>lt;sup>4</sup> Although arguably using a concept to cover a single phenomenon that is human consciousness and none else (e.g. animal consciousness) is not the best of practices.

physical domain. Chalmers dubs this property of strong emergence "radical metaphysical expansion" (Chalmers, 2006).

Furthermore, the emergents (i.e. consciousness) have the power of downward causation over the physical domain. That is to say, the emergent either constrains or construes the low-level in part or as a whole. To be able to bridge this gap between the high-level phenomenon and the low-level domain, we would need to extend our inventory of low-level laws (i.e. fundamental physical laws) so that some collection of low-level laws and facts enables us to deduce the emergents (i.e. consciousness).

#### Weak Emergence

Weak emergence covers all other cases of emergence where the high-level phenomenon can be deduced from the low-level laws and facts. Although the emergent can be (and, in fact, must be) unexpected and noticably more complex than the simpler low-level interactions, they are readily (but not necessarily easily) deducible from them if one has the computational means. It is possible to go a step further to claim this computational burden is the reason these phenomena are emergent; otherwise it would be obvious what leads to the emergents (which would not be emergents but just results) and the process could be analyzed as plainand-simple causation. Emergents, while deducible from low-levels, reside in higher levels of the system and therefore are most easily observed and investigated at a higher level.

Note that this definition is highly subjective, as it talks about "surprising" outcomes which are "easier" to observe in "higher" levels of observation. However, this cannot be escaped if we are to talk about weak emergence, since the emergent phenomena are objectively deducible from the low-levels anyway. It is a matter of phenomena being apparent without extensive examination (with respect to non-emergent high-level phenomena), or being implicitly representable at the lower-level.

An example would be volume of gaseous substances, where the behaviour of individual particles combine to give rise to a property (volume) non-existent at the lower level. It is not obvious at the lower-level that this effect will emerge, the particles do not have this property (i.e. we cannot talk about volume if we have a single gas molecule that does not interact with others) neither are they explicitly trying to give rise to volume. Yet this effect can be deduced from the behaviour of the individual particles. There is no gap to bridge between how a collection of gaseous molecules give rise to the property of volume and the interaction of the molecules with other molecules. The laws that govern the lower level, supplemented by the facts of the initial state of the lower level are sufficient to deduce the emergence of volume.

#### **A Working Definition**

First and foremost it must be stated that when we use the term emergence in this work, we refer to "weak emergence" since there is a clear path from low level interactions to high level emergents. In fact, this process can and will be, in the following chapters, formalized in mathematical terms.

For the working definition, we are going to borrow the definition devised by De Wolf and Holvoet as a result of their impressive literature survey in emergence and self-organisation (2005). This piece of work is chosen not only because it covers a considerable body of work, but also because it is more geared towards a computational perspective of emergence instead of a more philosophical one such as Chalmers's.

A system exhibits emergence when there are coherent emergents at the macrolevel that dynamically arise from the interactions between the parts at the microlevel. Such emergents are novel w.r.t. the individual parts of the system (De Wolf & Holvoet, 2005).

The emergents may be any properties, behaviour, structure or patterns that this process begets that bear novelty from the low-level point of view. This means emergents are not easily observable from the point of view individual interacting parts, but are easily observable from the point of view of the system as a whole. Thus, the novelty in the definition simply means the emergents are not directly observable by the low-level individual parts. Note that this differs from the "radical metaphysical expansion" required for strong emergence as the emergents we refer to are deducible from the local interactions (and possibly the initial state of the system).

To make this definition even more explicit, the required properties of an emergent system (also borrowed from De Wolf & Holvoet, 2005). are going to be outlined.

1. **Micro-Macro Effect:** In an emergent system, there needs to be properties that are manifested at the macro-level which arise from the micro-level. This is somewhat explicit in the working definition so it does not need further clarification for our purposes.

2. **Radical Novelty:** This refers to the fact that the resulting macro-level properties or entities are novel from the point of view of the interacting individuals which, together with their interactions, comprise the micro-level. Note that this is very different from "radical metaphysical expansion" where there is an irreconcilable gap that result from the fundamentals of the micro-level system between the micro-level interactions and the macro-level phenomena. Radical novelty here simply refers to situations where "the individuals at the micro-level have no explicit representation of the global behaviour" (De Wolf & Holvoet, 2005).

Note that this definition implies that while the products of emergence are not reducible to the behaviour of parts of the system, parts of the system still implicitly contain the products *if* they are interpreted against the behaviour of the system as a whole. So the properties are there, but meaningless and irrepresentable unless we adopt a system-level perspective. This excludes strong emergence from our working definition as outlined by Chalmers, which is one of the reasons for the emphasis on emergence denoting weak emergence for the purposes of this work.

- 3. **Interacting Parts:** This is a rather crucial aspect of emergence that is easy to overlook. The system needs to have interacting parts to give rise to macro-level effects. It is not sufficient that they function in parallel, they need to be able to influence each other to produce the non-linearity required to have emergents in the first place.
- 4. **Coherence:** This refers to the fact that emergents' coherent existence relies on the consistent correlation of low-level components. This is highly related to the characteristic of interacting parts as outlined above.
- 5. **Dynamical:** Emergent characteristics arise after parts interact enough to build up the correlations required to beget a coherent whole i.e. the emergents. Therefore, the system needs to evolve over time, or in other terms, be a dynamical system. The emergents become available only after some point in time, so if there is no correlation between the time component of the system and how the system behaves, we cannot expect emergents at all.
- 6. **Decentralised Control:** This characteristic stems from the fact that centralised control would require an explicit representation of the emergents at some micro-level control

component. If there is a component of the system apart from the system as a whole that has an explicit representation of the global behaviour, the global behaviour cannot be emergent as this would violate the principle of radical novelty. Parts of the system may be controlled, and only through them can the global behaviour be affected.

- 7. **Two-way Link:** This refers to the system's two-way causal link between the microand macro-levels. Micro-level interactions build up coherent effects in the system at the macro level, while the macro-level effects also constrain micro-level behaviour. To use the volume example again, not only collections of gaseous molecules together make the property of volume emerge, but also the volume of a gas effects how the individual molecules behave. Note that this is true although an isolated, individual molecule has no volume.
- 8. Robustness and Flexibility: This might be considered a result of all the other characteristics. Since there is no central control, there cannot be a single failure that would prevent the emergent from eventually arising. As a result, the system becomes relatively resistant to fluctuations and errors, or even replacement of individual parts. The system does not rely on any specific entity, so no specific entity is a requirement for the emergents. Note that we can cause the emergents to perish if we disturb the system greatly enough, so we cannot talk about an immunity to disturbance. However, even that kind of disturbance will only make the current emergents vanish and will not necessarily hinder the eventual re-emergence of them.

### 2.2 Self-Organisation

Self-organisation, like emergence, is far from being a new concept. Although it has not been called such until after the Second World War (De Wolf & Holvoet, 2005), the concept dates back to Greek philosophers of ancient times. However, a more solid reference to it only emerges around the time of Enlightenment by Descartes where he dubs it "arranging oneself":

I showed how the greatest part of material in chaos would have to, as a result of these laws, organise and arrange itself in a certain way which made it similar to our heavens, how, in so doing, some of its parts must have made up an earth and some parts planets and comets, and some other parts a sun and fixed stars.

Self-organisation refers to the phenomenon where a system assumes a more organised state than the initial state without any external control. This is not to say that the system is closed i.e. recieves no input from outside the system. It simply refers to the fact that the input cannot be control instructions but only data. If the system organises itself upon introduction of external data, this is still self-organisation. However, if the system is simply reorganised, that is, if the system does not become *more* organised, this process cannot be called self-organisation.

An everyday example of self-organisation is the usage of USB memory sticks.<sup>5</sup> USB sticks do not require any external control instructions (a.k.a. drivers) in modern operating systems. If one is to plug in a USB stick (which can be considered more of a data input than a control instruction), the operating system organises itself to accomodate the stick as a memory device without any external intervention. If, however, the stick is to be used on a system that does not have PnP support, the reorganisation of the system requires more than just plugging in the device (also known as "searching for a driver").

The working definition we are going to use is this:

Self-organisation is a dynamical and adaptive process where systems acquire and maintain structure without external control (De Wolf & Holvoet, 2005).

Once again, we are going to enumerate a few characteristics to make the concept clearer.

1. **Increase in order:** This is what lies behind our previous referral to increased organisation. If a system has N possible states or has a state space of volume N, it is required that this state space become smaller than N after reorganisation. Although this is not a sufficient condition for self-organisation, it is a required one. The organising process needs to cut down on degrees of freedom. While doing this, it also needs to maintain a specific function for which it self-organises.

A more formal approach to this characteristic is outlined by Shalizi (2001) in his PhD dissertation. He states that an increased statistical complexity is required for the system to be self-organising. That is to say, the system needs to maintain a greater amount of history than before to be self-organising. Of course, we could just as easily store all history, but this will not fit the following characteristics of self-organisation since it will

<sup>&</sup>lt;sup>5</sup> Actually, any so-called Plug-and-Play (PnP) device would do.

make the system pretty inflexible and fragile in the face of disturbances. The system not only needs be functional, but also stay functional when conditions change.

- 2. Autonomy: This criterion represents the "no external control" part of our working definition. Without external control instructions, only the system can be the decision-maker of what to do next. This is of course closely related to data input, which can be external, but this is an indirect impact or a reaction rather than direct control. This is where a clearly bounded system is extremely important, as we have to decide where the system's boundaries are to determine whether control is contained within the system or not.
- 3. Adaptibility: This is a criterion of balance among chaos and order. It is very similar to the "Robustness and Flexibility" characteristic for emergence, and why an increase in order and autonomy does not suffice to make a process self-organisation. The system needs to be relatively resistant to change and perturbances.

Suppose we define organised behaviour as an attractor in a dynamical system, and that point attractors are tendencies to a specific behaviour and chaotic attractors are tendencies to a very large set of behaviours. If the system has only a point attractor, it becomes too rigid to accomodate change and still retain functionality. At the opposite end, if the system has only an uncontrolled chaotic attractor, it becomes too unstable to function. What is needed is a balance between the two so that there are a number of behaviours that the system can converge to, but the number of behaviours are limited by another mechanism "to focus the outcome" (De Wolf & Holvoet, 2005) e.g. selective pressures.

4. Dynamical: This is again a characteristic shared by emergent systems. In order to be able to talk about "adaptibility", we need a context that changes in time i.e. a dynamical context. The system needs to self-organise over time, so that reorganisation upon disturbances makes sense. This makes the system more fragile to change, but also capable to react to changes, making the capability to self-organise more stable.

#### **Contrasting Emergence and Self-Organisation**

Although emergence and self-organisation frequently occur together, as their characteristics imply, they are hardly the same concept. It is easy to find occurrences of either without the other.

Self-organisation without emergence can be exemplified using the USB stick example used above. Most modern personal computers have central control, so any self-organisation that takes place in the system does not carry any micro-macro effect. There is a component of the system that has an explicit representation of what is going on (i.e. the memory), and also another component which explicitly controls the rest of the system (i.e. the CPU). Note that neither decentralised control nor no explicit representation of the phenomenon are required for self-organising systems. This is a very clear cut example of self-organisation without emergence.

Emergence without self-organisation can be exemplified using the volume example. While we have established that volume is an emergent property, there is no increase in the order of the system i.e. it is a stationary process. The statistical complexity or the amount of structure does not change in the process out of which the property of volume emerges. This, in turn, is a clear cut example of emergence without self-organisation.

# **CHAPTER 3**

# **Background on Language Simulations**

Although using language games is a relatively new approach, there is a considerable body of literature on simulation-based investigations of language. In the following sections, the related literature will be divided into situated and nonsituated simulations for convenience. Since the current work is a nonsituated simulation, more emphasis will be given to nonsituated part, in particular to language games (which has a dedicated subsection).<sup>1</sup> Since literature is not unequivocal on what constitutes a situated simulation<sup>2</sup>, we are going to use the term "situated" for simulations that involve a complete, independent world, and free interaction, and call all the others nonsituated, following (Wagner et al., 2003) (although the author does not explicitly give his definition either). This definition excludes language games from the situated set.

In the review, we occasionally refer to Kirby's characterization of language as "the result of an interaction between three complex adaptive systems that operate on different timescales: the timescale of biological evolution (phylogeny), the timescale of individual learning (ontogeny) and the timescale of language change (glossogeny)" (Kirby, 2002). This becomes a handy tool when judging the models, both in terms of realism and clarity of the point intended by the model.

<sup>&</sup>lt;sup>1</sup> For a more comprehensive (yet somewhat old) review and classification see (Wagner, Reggia, Uriagereka, & Wilkinson, 2003), (Vogt, 2005), and (Kirby, 2002).

 $<sup>^2</sup>$  (Wagner et al., 2003) refers to (Steels, Kaplan, & Others, 1999) as a nonsituated simulation, although it is undertaken by visually grounded autonomous robots, and referred to as a situated simulation by the authors themselves

### 3.1 Non-situated Simulations

Nonsituated simulations focus less on the pragmatics of communication and more on agreement and learning. Unlike situated simulations, agents in most of these simulations are predisposed to interact (see (Emily & Steels, 2008) for an exception), as well as equipped with communicative means (i.e. a signaling device). There are many flavours of such simulations, such as those that rely on language games (Vogt, 2005; Steels, 1996a; Steels et al., 1999; De Beule, 2008), and those that are simple encoder/decoder pairs (Hurford, 1989; Krakauer & Pagel, 1995). Among these extremes, language games is the most recent approach, and a very remarkable body of literature has demonstrated its use.

The positive thing about nonsituated simulations is that it attempts to simulate a very constrained communicative system, and therefore removes the complexities more realistic simulations cast on the interpretation of their results (Baronchelli, Felici, et al., 2006). By enforcing well-defined constraints, it is possible to focus on individual aspects of communication, both whether they show up at all, and if so, under which circumstances these aspects surface, given our constraints (Wagner et al., 2003). This agrees with the general view of modeling that dictates modeling of all and only features of the modeled system that matter for the intended use of the model. Since it is next to impossible to immediately characterize dynamics of higher-level simulations with the mathematical simplicity that these simulations allow, they build a framework on which characterization of such systems can draw from.

The problem with non-situated simulations also stem from its specialized, narrowly focused nature. Human language operates over many layers, spanning multiple levels of evolution (remember the distinction of different timescales in Kirby, 2002), and it is difficult to account for many aspects of language unless we have a more complete model of the interaction between language and the environment. Even if all human mechanisms used for language were to be implemented in the agents, the fact that agents are not situated in an ever-changing physical environment that is independent of the agents' perception of it, that they are not free to act as they wish and that they are not bound by similar perceptual constraints as humans are, it is very likely that the model fails to capture many phenomena. Even if it does seem to capture them, it is possible that the mechanisms behind the outcome differs from one that is behind the seemingly-equivalent phenomena in human languages.

To make this last point clearer, let us assume that we are trying to establish a basis for Zipf's law using a model. We may choose to stay at the word-level, or we could take into account the syntax as well. Although we may be able to establish that lexicons have a tendency to obey Zipf's law in our word-level model, this result falls short of demonstrating that the Zipf's law we encounter in human languages do so because of this tendency. It is known that syntax networks tend to be small-world networks (Corominas-Murtra, Valverde, & Solé, 2007), which in turn tend to produce utterances whose type distributions resemble power law á la Zipf's distribution. We could conclude that words cause Zipf's law, the syntax causes Zipf's law, their compounded effect causes Zipf's law or even that an additional layer (e.g. the distribution of the semantic coverage of words) causes Zipf's law and the effects we have observed in the other layers all but perish when this layer is introduced. Although this risk is always present in approaches that use modeling, it is worth pointing this out explicitly to emphasise the impact that "perceptual grounding" and "autonomy" may have on the dynamics of a language simulation.

#### 3.2 Situated Simulations

Situated simulations are ones in which the agents involved are dwellers of an interactive, artificial world, and their communication has often a pragmatic purpose associated with it (Wagner et al., 2003). The importance of these simulations is that they put forth a model that is not confined to agents that are preprogrammed to interact, or whose success is judged directly in terms of communicative success. This enables the simulation of emergence of the most basic abilities, such as the emergence of the very ability to communicate with others, as a side effect of the outcomes of specific behaviour of an agent on the others (Reggia, Schulz, Wilkinson, & Uriagereka, 2001). It also enables monitoring how communicative success (e.g. number of successful communications), communicative conventions (e.g. collectively choosing which words go with which concepts), structure of the communication (e.g. word order, syntactic types, turns taking) and the performance in the artificial world tasks (e.g. mating, feeding, or outright survival) interplay. Since all of Kirby's timescales are applicable to these models, their potential explanatory power is remarkable.

This type of simulations have the downside that they tend to be evolutionary as well, in that the agents live and die, and their "language device" develops to facilitate their survival, as a result of selectional pressure and mutations. Sometimes this mixing of intragenerational and intergenerational aspects risk the clarity of the model, as it combines all three complex adaptive systems (phylogeny, ontogeny and glossogeny) instead of one or more in an isolated manner (e.g. glossogeny and ontogeny only). Therefore, although many phenomena may be observable in these models, it typically becomes more and more difficult to precisely account for them as the models get more complex (Baronchelli, Felici, et al., 2006).

#### 3.3 Language Games

Language game is a paradigm used for simulation of certain aspects of emergence of a communicative convention. It "involves a dialog between two agents, a speaker and a hearer, within a particular contextual setting which includes other agents" (Steels, 1996a). A single game can be outlined as follows:

- 1. A speaker and a hearer is chosen.
- 2. The speaker identifies a topic (i.e. an object in the environment), and shares it with the hearer using a modality that is distinct from the types of signals studied in the simulation (e.g. by pointing to it in a simulation that does not investigate the dynamics of communication by pointing).
- 3. Both agents identify feature sets associated with the objects, if applicable.<sup>3</sup>
- 4. The speaker chooses a feature set and translates it into an utterance.
- 5. The speaker tries to infer the feature set using the utterance.
- 6. Either one agent or both agents compare the expected feature set and the one inferred by the speaker, and update their internal states.

There are many types of language games, each incepted to focus on different aspects of population dynamics of communication. Some examples are guessing games, naming games, discrimination games, selfish games, and their evolutionary versions where phylogeny or biological adaptation (i.e. the adaptation of the "language device") is also taken into account (Lenaerts, Jansen, Tuyls, & De Vylder, 2005).

<sup>&</sup>lt;sup>3</sup> Not all language games involve multiple features per object, neither does the proposed model, i.e. the only feature an object has is its unique identity.

An important advantage of language games is that their inventories are *open*, i.e. there is no need for a set list of utterances or words or concepts that the agents choose from (although there is nothing preventing this). Therefore, the words can be truly arbitrary with respect to what they denote, and every inventory in the game (such as the lexicon, the syntax or the ontologies) can be extended mid-game to accommodate changes in the agreement, discriminable features and objects in the game. Due to this flexibility, the convention-forming mechanism is very robust against changes, and consequently very suitable for studying population-size phenomena (Wagner et al., 2003). It is also reminiscent of the robustness of human languages where change of conventions is the norm and not an exception.

Most language games involve horizontal transmission of conventions where every agent in the population can equally affect and get affected by its peers (De Vylder & Tuyls, 2006), unlike vertical transmission schemes where a subset of population assumes the role of teacher and the other of student (Kirby & Hurford, 2002). There are also some language games that involve both, as in Lenaerts and colleagues' work that combines vertical transmission scheme with the horizontal transmission scheme of the naming game (Lenaerts et al., 2005).

#### 3.3.1 Language as a Complex Adaptive System

#### 3.3.2 Types of Language Games

In the next two subsections, two flavours of language games are described in detail: discrimination game and naming game. This will hopefully make the theoretical description clearer, but more importantly, this work is based on similar algorithms and the description of this work will be based on the verbal and mathematical descriptions of the following models.

#### 3.3.2.1 Discrimination Games

This type of language game is focused on meaning creation. Typically, discrimination games bootstrap ontologies based on whatever grounding the agent has for the features involved in the game. The features in the game have been mostly visually grounded (Steels, 1996b), although there are also different types of ontologies bootstrapped using discrimination games in the literature, such as temporal ontologies (De Beule, 2004).

Formally, the game involves a set of objects  $O = \{o_1, ..., o_m\}$ , a set of agents  $A = \{\alpha_1, ..., \alpha_k\}$ and a set of sensory channels  $S = \{\sigma_1, ..., \sigma_n\}$ .<sup>4</sup> Each sensory channel  $\sigma_i$  is a real-numbered, partial function (typically  $0 \le \sigma_i(o_j) \le 1$ ) over O, so that each sensory channel produces a value for each object  $o_j$  (provided it is defined for  $o_j$ ).

A *feature* is defined as an attribute value pair  $(p_{a,k}, v)$  that represents the name of the feature and the corresponding value<sup>5</sup> such as (*size*, *small*). The attribute value is basically a partitioning of the sensory channel, and indicates what the value for a channel is in some partition (Steels, 1996b). Each agent *a* has a set of *feature detectors*  $D_a = \{d_{a,1}, ..., d_{a,n}\}$ , each of which is a four-tuple

$$< p_{a,k}, V_{a,k}, \phi_{a,k}, \sigma_{a,k} >$$

where:

- $p_{a,k}$  is the name of the feature
- $V_{a,k}$  is the set of possible values for the feature
- $\phi_{a,k}$  is a partial function mapping from sensory inputs to feature values
- $\sigma_{a,k}$  is a sensory channel that the feature is associated with

Object identification is achieved by using distinctive feature sets as object definitions. A feature set  $F_{a,o_i}$  may be defined as

$$\{(p_{a,k}, v) \mid d_{a,k} \in D_a; d_{a,k} = < p_{a,k}, V_{a,k}, \phi_{a,k}, \sigma_{a,j} >; v = \phi_{a,k}(\sigma_j(o_i)) \in V_{a,k}\}$$

A feature set  $D_{a,o_i}^C$  is distinctive for object  $o_i$  against the set of non-target objects (i.e. all objects except  $o_i$ ) *C* if, for all features in it, there is either no feature among the set of non-target objects with the same name or no feature among the set of non-target objects with both the same name and the same value, or formally:

$$\{f \mid f = (p, v) \in F_{a,o_i}; \forall o_j \in C \ either \nexists f' = (p, v') \in F_{a,o_j} or$$

$$\exists f' = (p, v') \in F_{a, o_i} \text{ where } v \neq v' \}$$

<sup>&</sup>lt;sup>4</sup> This set of functions do not necessarily correspond to sensory channels and may be related to any input channels available; but sensory channels are used since they seem to us an intuitive example for grasping the game

<sup>&</sup>lt;sup>5</sup> The values may be relative as well as absolute, but only absolute ones will be considered here, following (Steels, 1996b)

A game involves these steps:

- 1. Two agents are chosen, one as the speaker, one as the hearer.
- 2. A context (i.e. a number of objects) is chosen by the speaker, and shared with the hearer.<sup>6</sup>
- 3. A topic (i.e. some object) is chosen by the speaker.
- 4. The speaker tries to discriminate the topic from the rest of the context.
- 5. The word corresponding to the winning distinctive set is uttered. If there is no word for it, the speaker creates one.
- 6. The hearer tries to infer the topic from the utterance.
- 7. Both parties are informed of the success.

Each game is either successful  $(D_{a,o_i}^C \neq \emptyset)$  or unsuccessful  $(D_{a,o_i}^C = \emptyset)$  depending on whether a distinctive feature set could be found or not (note that this applies both to the speaker and the hearer). In the case of failure, a new feature detector is created, either for an unused sensory channel, or for subsegmenting the range of an existing feature detector if all sensory channels are already used. In the case of success, if there are more than one distinctive sets, the smallest, the most general (i.e. one with the least sub-segmented features) and the most frequent (i.e. one with the most frequent features) is chosen, in that order (Steels, 1996b).

Due to this feature detection mechanism, introduction of new sensory channels and/or new refinements for sensory channels is always possible, and the symbols the system uses is grounded in its sensory channels, therefore the system constitutes a robust ontology generator for the given sensory input. Since there is a selectional pressure from the success/failure behaviour, discriminative features for each object tends to converge in the population (Steels, 1996b).

This game is more demonstrative of how meaning is created, rather than how the conventions that map to meanings, once established, spread. As a language game designed to investigate this aspect better, we next turn to the naming game, on which this thesis builds.

<sup>&</sup>lt;sup>6</sup> The need to share the context is why a separate channel of communication whose conventions are not being bootstrapped by the game is required.

#### 3.3.2.2 Naming Games

The naming game focuses on vocabulary formation and agreement in a population. The agents try to bootstrap a vocabulary of (proper) nouns that they associate with the objects they try to name (De Vylder & Tuyls, 2006). It is assumed that the agents already know how to send and receive signals, and possess the motivation to do so. It is further assumed that the objects are uniquely identifiable by all agents, so feature sets as in the discrimination game are not employed  $^{7}$ .

Formally, the game involves a set of objects  $O = \{o_1, ..., o_m\}$ , and a set of agents  $A = \{\alpha_1, ..., \alpha_n\}$ . Each agent *a* possesses a lexicon  $L_a = \{e_{a,1}, ..., e_{a,k}\}$ , and each entry in the lexicon consists of a list of words associated with the object  $o_k$ ,  $E_{a,k} = \{w_{k,1}, ..., w_{k,q}\}$ . All agents also possess an identical function that maps from objects to words ( $\phi_a : O \mapsto E_a$ ). Therefore, an agent is characterized only by its lexicon. A game consists of these steps:

- 1. Two agents are chosen, one as the hearer  $(\alpha_h)$  and one as the speaker  $(\alpha_s)$ .
- 2. The speaker chooses an object  $(o_i)$  to refer to, and points to it (i.e. makes his choice explicit without using the system we try to bootstrap)
- 3. The speaker chooses a word in the lexicon for the object  $(\phi(o_i))$ , or creates one if necessary.
- 4. The hearer tries to decode the word into the object being referred to  $(w_{k,j} \in E_{h,k})$ .
- 5. The agents are informed of their success, and update their lexicons accordingly.
- 6. If all agents have identical lexicons, the game stops.

Each game results in success  $(\exists w_{i,n} | w_{i,n} = \phi_s(o_i); w_{i,n} \in E_{h,k})$  or failure  $(\nexists w_{i,n} | w_{i,n} = \phi_s(o_i); w_{i,n} \in E_{h,k})$ . Upon success, both agents purge their entries for that object of all but the successful word. Otherwise, the hearer adds the new word to its entry of the object, or creates one if necessary. Note that there is no intermediary between a word being successful and a word dominating an agent's lexical inventory for an object; it operates on an all-or-nothing basis.

 $<sup>^{7}</sup>$  Note that it is perfectly possible to combine the two games, as done by Steels (2003).

This simple setup yielded a huge amount of consequent work and variation (Steels & Kaplan, 2002; Steels, 1996a; De Beule, 2004, 2008; De Vylder & Tuyls, 2006, to list a few). This work is mostly mathematical analysis of the original design (Baronchelli, Loreto, & Steels, 2008; Baronchelli, Felici, et al., 2006; Baronchelli et al., 2005; DallâAsta, Baronchelli, Barrat, & Loreto, 2006; De Vylder, 2008), and a few are modifications to various aspects of the game (Lenaerts et al., 2005).

The quintessential message of all this analytic works is that the system converges at one word or the other, but it almost definitely converges (De Vylder, 2008)

The distribution of words with respect to rank is found to follow a power law behaviour, except for the very low-ranking words and the most popular word ( $w(R) \sim R^{-p}$ ;  $p \in \mathbb{R}$ ; p > 0; where *p* is the scaling parameter) (Baronchelli, Felici, et al., 2006).<sup>8</sup> This is reminiscent of the infamous Zipf's law, which observes that many natural phenomena seem to follow a power law, such as the ranking of word popularity to its number of occurrences in a natural language corpus (Newman, 2004).

The lexical agreement network of the model is formed by taking agents as nodes, and sharing of a word as an edge between two agents. This results in a multigraph (i.e. one in which parallel edges are acceptable), in which each word represented by a fully connected clique (since all agents that share a word obviously share it with every other agent that possesses it). As the model closes on convergence, one clique dominates and the network becomes a fully connected graph (Baronchelli et al., 2008). A fully connected graph is equivalent to a word shared by all, but it is not equivalent to a converged game since it is still possible that the word chosen for an object (i.e.  $\phi(o_i)$ ) is not the most common word (i.e. the one the clique represents).

<sup>&</sup>lt;sup>8</sup> Actually, most timescales posited about the model follows a power-law behaviour, such as the convergence time against number of agents.

# **CHAPTER 4**

# **Experimental Model and Results**

#### 4.1 The Model

The model investigated here is a modified version of the naming game. Although it provides insight into game dynamics and the impact of weighted lists on those dynamics, it has the disadvantage of not being as open to scrutiny as the original game as it is much less straightforward to analyze. Although technically it is possible to analyze it to the point of classical naming games, its complexity renders this unfeasible. The fact that word choice scheme is not deterministic introduces too much stochasticity to analyze it the way naming game is analyzed in more elementary schemes, such as that in Baronchelli et al. (2008).

#### 4.1.1 Assumptions

We believe a very important and often overlooked stage in developing, analyzing and presenting a model is making explicit the assumptions that the model is built on. Therefore, this subsection is dedicated to undertaking this task and enlisting these assumptions.

Assumption of no topology: The model assumes that there is no structure in the population. What this entails is that the probability that a given agent α<sub>1</sub> interacts with another agent α<sub>2</sub> is identical to that of interacting with an agent α<sub>3</sub> where α<sub>3</sub> ∉ {α<sub>1</sub>, α<sub>2</sub>}. This is also called a mean field assumption and is also present in the original naming game.<sup>1</sup>

<sup>&</sup>lt;sup>1</sup> Naming games without this assumption have also been implemented and investigated in the literature, see Baronchelli, DallâAsta, Barrat, and Loreto (2006)

What this assumption amounts to in terms of human language can be summarized this way: the individuals cannot distinguish between different individuals, and all individuals are equally accessible to every other individual. So even if there are geographical, cultural or convention-based differences (e.g. tendency to choose a partner with a high success rate) between the individuals, these cannot be taken advantage of because individuals cannot be singled out with respect to any criteria. Therefore, any internal structure the population may have is opaque to the mechanisms for choosing agents to interact and choosing words to play.

- 2. Assumption of featureless object identification: The model assumes that all agents are capable of identifying objects in the same way. It further assumes that objects have no features (such as color, shape or taste) which might facilitate organizing the lexicon according to features, in addition to object identities. So there is no room for concepts such as similarity, difference or misperception in the model, unlike discrimination games. This is why the "name"s in the naming game are reminiscent of proper names in human languages.
- 3. Assumption of no homonymy: This assumption states that a word does not legitimately refer to two distinct objects. Although they can consider a word for more than one object (actually, this is a requirement), they cannot decide that it refers to both. So if it turns out that a word is successful for referring to one object, we lower its score for all other objects. This is a simplifying assumption that prevents number of objects from complicating the game due to having a single word legitimately referring to more than one object. This way, dynamics for each object is independent from every other objects' dynamics.

Although there is no other explicit mechanism to prevent a word to dominate for more than one object (similar to homonymy), that is for a word to be the most popular word for more than one object, in practice this almost never happens.<sup>2</sup> This is due to the fact that selectional pressures not only provide a tendency to have a single dominating word for referring to an object, they also tend to make different words to dominate for each object.

4. Assumption of population opacity: We assume that both properties of the population-

 $<sup>^{2}</sup>$  Among the tens of thousands of runs we have executed for this work, we have never once encountered this happening.

wide dynamics of the collective language and individuals' perspectives of the language (that is, the lexicons of the other agents) are inaccessible to the agents. The only information available to the agents are accessible through peer to peer interactions, in which the only feedback for the parties consists of the success or failure of the round, the word that the speaker chose and the object that the hearer chose. So, agents cannot choose the more popular word just because everybody uses it; this fact is inaccessible to the agent so the success of the popular word must be reinforced by successful communications by the agent involving that word and unsuccessful ones involving others. This is a realistic assumption since each individual's lexicon is inaccessible to others in human language as well, and is only revealed through interactions with the individuals (or their utterances).

We have also extended this assumption to include game control. In the classical naming game, the game stops when all lexicons are identical. In our version of the game, we control the game using a success window i.e. the percentage of successful interactions in the last N interactions. The game stops when that percentage reaches a critical value, e.g. 100%. We believe this to be a more effective and theoretically sound way of defining convergence since having access to agents' "minds" sort of violates the emergence criteria. However, measuring observed outcomes of interactions does not.

- 5. Assumption of independent channel of communication: Language games require that we have a means to communicate apart from the channel being tested. This is required since we need some sort of feedback to organize the lexicon that does not rely on the lexicon being organized. In the naming game, this corresponds to the hearer choosing and "pointing at" the object he thinks the speaker referred to. If this channel were not available, the only way to give feedback would be via the lexicon. This would cause unreliable feedback because there is not a convention on what to call each object, and if reliable feedback were available (i.e. the object has a dominating word) the game has closed in on convergence which would, again, require some feedback to happen.
- 6. Assumption of no parallel interactions: This is an assumption that stems from concerns of computational complexity. If there are more than one independent rounds played at each time interval, the population dynamics would be very difficult to track. Normally, each round can be analyzed knowing that all and only the elements (i.e. the object and the word) that took part in that interaction face any change, which can be

only in one direction. If there are parallel interactions, each round may produce conflicting results which would complicate analysis. For example, a word for a given object may fail in one interaction, and succeed in the other, which would change the position of the word-object pair in the population a more complex way, instead of just "more popular" and "less popular", corresponding to "success" and "failure" of the round, respectively. More importantly, homonyms cannot be controlled for as described in the third assumption, the assumption of unique coinage. We cannot check if there is a word coined twice if there are more than one round taking place simultaneously. Any control such as the one described would require one coinage to follow the other.

## 4.1.2 Interactions and Environment

We aimed to build and analyse a model that is a modified version of the outlined naming game. There are three changes to the model to accommodate a weighted list of competing synonyms:<sup>3</sup>

- 1. Lexical entries: The lexical entries in the original model are simple lists of words. The proposed model implements lexical entries as *weighted* lists of words, updated upon interactions. This allows a graded behaviour in which words are preferred, and constitutes a more realistic situation in which convergence should be achieved, compared to plain lists.
- 2. Word selection: The word selection scheme in the original model simply picks a word from the set of words present in the lexicon. It does not specify how to pick the word. Although there are some suggestion for schemes that optimize convergence (Baronchelli et al., 2005), there is no set practice. The proposed model has a specific scheme that makes a weighted, probabilistic choice of the word to use at each round. This introduces a number of advantages. First, it introduces some noise by not guaranteeing the leading word to be chosen at every round, which is crucial to convergence in dynamical systems. Second, it is a more realistic scheme, especially when top words have similar scores. Third, it makes the system more fault tolerant by minimizing the impact of successful rounds caused by to-be-non-successful words and of unsuccess-

<sup>&</sup>lt;sup>3</sup> See Appendix C for illustrations of how failed and successful communications take place and their effects on the agents' lexicons in the classical and proposed models.

ful rounds caused by to-be-successful words. These are going to be elaborated in the following sections.

- **3. Interactions**: The agents in the proposed model no longer discard the competing synonyms (i.e. the other words in the lexical entry) upon a successful interaction. Instead, the agents update the weights of their lexical entries for the object upon every interaction. This is both more realistic, and allows for a better investigation of the distribution of synonyms that did not dominate.
- 4. Parameters: As a consequence of the update scheme, there are more parameterized constants (i.e. parameters that are constant values) in this version of the game than the classical one. In particular, three  $\delta$ -values ( $\delta_{success}$ ,  $\delta_{failure}$  and  $\delta_{inhibition}$ ) are added for use in updating the lexicon, whose precise roles are elaborated in the following paragraphs. Additionally, two  $\theta$ -values ( $\theta_{max}$  and  $\theta_{min}$ ) are added as the maximum and minimum values for any score in the lexicon.

More formally, for each agent  $\alpha_i$ , an additional value function  $\theta_{\alpha_i}$  is added to retrieve the weight:

$$\theta_{\alpha_i}: w_{k,q} \mapsto \mathbb{R}$$

Basically, this is just a lookup for the weight associated with the word in an agent's lexicon. By adding this, the characterization of an agent is a tuple of the lexicon and  $\alpha_i$ , instead of only the lexicon as in the original game:

$$\alpha_i :< L_{\alpha_i}, \theta_{\alpha_i} >$$

Secondly, an identical function  $\omega$  is added to each agent which updates the weight function after an interaction:

$$\omega:\theta_{\alpha_i}\mapsto\theta'_{\alpha_i}$$

This function adds or subtracts from scores some predefined  $\delta_{success}$ ,  $\delta_{failure}$  and  $\delta_{inhibition}$ , based on lateral inhibition (Lenaerts et al., 2005). Upon a successful interaction with word  $w_{k,p}$ , this function returns a new function  $\theta'_{\alpha_i}$  and optionally modifies the lexicon of the agent. The modification is that if the resulting score for a word is less than a predefined value  $\theta_{min}$ , that word is removed from the lexical item for that object. Also, there is a set limit  $\theta_{max}$  on how large the weight may grow, at which point no weight is added. More formally,  $\omega$  returns this upon success:

$$\omega(\theta_{\alpha_i})(w_{k,q}) = \theta'_{\alpha_i}(w_{k,q}) = \begin{cases} \min(\theta_{\alpha_i}(w_{k,q}) + \delta_{success}, \theta_{max}) & ifq = p \\ \theta_{\alpha_i}(w_{k,q}) - \delta_{inhibition} & ifq \neq p \end{cases}$$
(4.1)

and this upon failure:

$$\omega(\theta_{\alpha_i})(w_{k,q}) = \theta'_{\alpha_i}(w_{k,q}) = \begin{cases} \theta_{\alpha_i}(w_{k,q}) - \delta_{failure} & ifq = p \end{cases}$$
(4.2)

It then modifies the lexicon as follows:

$$L'_a = (L_a/E_{\alpha_i,k}) \cup E'_{\alpha_i,k} \tag{4.3}$$

where

$$E'_{\alpha_i,k} = \{ w | \theta'_{\alpha_i}(w) \ge \theta_{min}; \forall w \in E_{\alpha_i,k} \}$$

$$(4.4)$$

Finally, the function  $\phi_{\alpha_i}$  is changed to return a word by a weighted random choice. To this end, we first define a probability distribution P where:

$$P(w_{k,q}) = \frac{\theta_{\alpha_i}(w_{k,q})}{\sum_y \theta_i(w_{k,y})}$$
(4.5)

Subsequently, the word  $\phi_{\alpha_i}$  returns can be characterized as a random variable X with distribution P.

$$\phi_{\alpha_i}(o_k) = X \tag{4.6}$$

for which:

$$X \sim P; X \in E_{\alpha_i, k} \tag{4.7}$$

## 4.1.3 Relevance to Strategies in Classical Model

It is worth mentioning that there is more to the classical model than the aspects described. Baronchelli et al. (2005) outlines and compares three strategies for playing the game, *play-first*, *play-last* and *play-smart*. The first one consists of adopting the strategy to always choose the word for an object that has been successful before and therefore is the first word to enter the lexicon for the object at a given time. The second one is the converse strategy of choosing the one that has entered the lexicon most recently. Finally, the *play-smart* strategy combines

the two so that if the agent has ever been successful before, it adopts *play-first*, otherwise it adopts *play-last*. Baronchelli and colleagues found that the third strategy can significantly reduce the time it takes for the system to converge.

What this "smart" strategy effectively does is to make sure that as words propogate through the system, the selective pressure is towards words that are more probably shared among many agents (of which successful interactions are indicative). It also makes sure that this does not happen prematurely (e.g. before the agent being successful at least once) so that individual successful interactions do not restrict the system too much before there are successful interactions.

These strategies make use of some additional memory to keep the order of competing words. Technically, in the original version, the lexicon is a *set* of words for each object. With the strategy in place, the lexicon becomes an *ordered set* of words for each object. Therefore we feel it is more fitting to call these modifications to the model rather than strategies that improve performance. At any rate, these modifications are reminiscent of the proposed model. Besides the obvious difference in the data structure used, the update scheme is modified as well.

The play-first strategy mimics a limited  $\delta_{inhibition}$  effect by biasing the word choice towards (in fact, coercing it into) the word that has a history of success (of which there can only be one since each success resets the lexicon so that only the successful word remains). This, in effect, is part of what  $\delta_{inhibition}$  does as well.

The play-last strategy, instead, is a form of  $\delta_{failure}$  since it causes pressures based not on the history of success of a word but its history of failure. A newly heard word in the classical model has failed at most once<sup>4</sup>, but the other words might have failed indefinitely. Provided there has been no successful interactions by the agent so far, a newly heard word is more likely to be successful than the other words. This kind of bias is also the one of the functions of  $\delta_{failure}$  in the proposed model.

So, it is known that such strategies have an impact on the game, but they have not been adopted and parameterized as systematical changes to the model to be measured. The contribution of this study will be solidifying the effects of such strategies by incorporating them into the way lexicon works and measuring their interplay and their impact on the game dynamics.

<sup>&</sup>lt;sup>4</sup> It may fail once if its introduction into the lexicon was caused by a failed communication

#### 4.1.4 Emergence and Self-organization in The Model

How the model embodies emergence and self-organization is not straightforward to explain. Two distinct approaches are possible, characterized by whether emergents are assumed to self-organize or self-organization is assumed to result in the emergents.

If we take self-organization as the first phenomenon, the agents' lexicons self-organize using the outcomes of the interactions. Subsequently, the interaction of self-organized individual lexicons give rise to a global lexicon, or, an emergent pattern of behaviour for the population that exceeds those predictable by individual lexicons.

If we take emergence as the preceeding phenomenon, the agents' lexicons are local units that interact to produce a global-level lexicon that self-organizes into emergence. Note that the global-level lexicon is not explicitly representable at the lower-level i.e. agents' individual lexicons, and that self-organization is at the level of global lexicon.

## 4.2 Methodology

Each parameter set, that is, a tuple of ( $\delta_{success}$ ,  $\delta_{failure}$ ,  $\delta_{inhibition}$ ) was considered a unique case, and the simulation was run 50 times for each case, using 50 agents and 2 objects. The model was considered to reach convergence when there has been 100% success over a success window of 100 rounds. Note that this is the product of the number of agents and number of objects, making it very likely that all agents have taken part in at least one interaction regarding each object, making the success window more meaningful.

The model was run with various  $\delta$  parameters. The method of choosing them was fixing a set of ratios in the form  $\delta_{failure}$ : $\delta_{success}$  and  $\delta_{inhibition}$ : $\delta_{failure}$ , and then producing the actual  $\delta$  values by choosing a value for  $\delta_{success}$  and calculating the rest using  $\delta_{success}$ .

There were five values for  $\delta_{success}$  denoted by the set {1.0, 3.0, 5.0, 8.0, 10.0}. For the ratio  $\delta_{failure}$ : $\delta_{success}$ , the ratios picked were 0.0:1.0, 0.5:1.0, 1.0:1.0, 1.5:1.0 and 2.0:1.0. The ratios used for  $\delta_{inhibition}$ : $\delta_{failure}$  were 0.0:1.0, 0.5:1.0, 1.0:1.0 and 1.5:1.0. If both  $\delta_{failure}$  and  $\delta_{inhibition}$  are 0.0 for a case, it is not possible to calculate  $\delta_{inhibition}$  from the ratio, so for those cases  $\delta_{inhibition}$  was set to  $\delta_{min}$  to provide some negative feedback to the model so that it can converge. Additionally, the case represented by the tuple (10.0, 0.0, 10.0) was added since it

corresponds to the behaviour of the classical model.

The values  $\delta_{max}$  and  $\delta_{min}$  were fixed at 10.0 and 0.1, respectively.

### 4.3 Implementation

The simulation has been implemented using the *Python* programming language. The only non-standard library used was *SciPy*, or scientific python, which is a library providing efficient algorithms and data structures for scientific computing. The execution was undertaken on various platforms and various hardware, but mainly on standard Linux boxes using quad-core processors. For the data analysis, another Python program was composed. The plots have been prepared using the same custom made analyzer software, which uses the *de facto* standard *matplotlib* for drawing charts and diagrams.

## 4.4 Results

### 4.4.1 Convergence

The first aspect to be investigated is how soon the system converges i.e. reaches 100% success over the given success window. For comparison, a range of parameter values were used to represent the average time of convergence for the span of the weighted model. Each parameter set was used for fifty simulation runs and their times of convergence were averaged. As some parameter sets never converge, a cap of 500,000 rounds has been enforced after which the simulation halts regardless of the current success rate. The population size is kept at a constant 50 agents and its effects are not investigated.

Since the model includes three variables that can counteract or amplify one another, usefully capturing the relationship between them with respect to time of convergence in terms of lines or areas on a two dimensional plain is often not possible. To visually represent the convergence data, the best way is judged to be a three-dimensional scatter plot of the average data. The coordinates *x*, *y* and *z* correspond to the variables  $\delta_{success}$ ,  $\delta_{failure}$  and  $\delta_{inhibition}$ , respectively. The colors and sizes of the individual points on the scatter plot represent the relative time of convergence of the sample. The larger and more blue a point gets, the later this com-

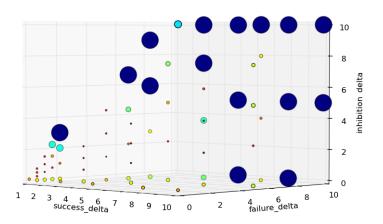


Figure 4.1: Scatter plot of all data with respect to *t<sub>conv</sub>*.

bination of parameters result in convergence. Conversely, the smaller and redder a point gets, the earlier this combination of parameters result in convergence. Shades of green represent the midway between red and blue.

This study excludes the cases in which  $\delta_{inhibition} > 1.25 \cdot \delta_{failure}$ <sup>5</sup> as the purpose is to investigate the model for the parameter ranges distant from that used by the classical naming game model. The only parameter from that range that is included is the parameters of  $\delta_{success} = 10.0, \delta_{failure} = 0.0, \delta_{inhibition} = 10.0$ . Choosing these parameters is equivalent to using the classical model. For a table that contains all parameter sets used and the average  $t_{max}$  and  $t_{conv}$  observed see Table A.1.

#### Overview

The overall results plotted in Figure 4.1 make it clear that the classical model can be outperformed in terms of time of convergence by many parameter sets in the weighted model.<sup>6</sup> The classical model lies at the coordinates (10.0, 0.0, 10.0), centered at the top in Figure 4.1. In fact, of all 70 parameter sets, only 16, or about %22.86 perform worse than the classical

 <sup>&</sup>lt;sup>5</sup> See Figure B.1 for a graphical depiction of this fact
 <sup>6</sup> See B for different views of the 3D plot.

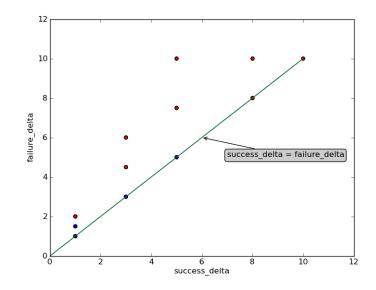


Figure 4.2: Scatter plot of all parameter sets where  $\delta_{success} \leq \delta_{failure}$ . Red markers signify non-convergent parameter sets.

model. The reasons for this will become clearer in the following chapters.

The best results are obtained with parameter sets for which  $\delta_{success} \ge \delta_{failure} \sim \delta_{inhibition}$  holds. Also, there is a clear tendency for smaller parameter values overall to outperform the larger ones. This indicates that the weight cap enforced by the model,  $\theta_{max}$ , also has a strong effect on dynamics. Varying cap levels or the elimination of the upper cap altogether would change the dynamics. At least, it is reasonable to expect that the difference in performance of different scales of the same  $\delta$  ratio (e.g. 1.0:0.5:0.5 vs. 10.0:5.0:5.0) can be minimized if a cap is not enforced. A listing of  $\delta_{t_{conv}}$  and  $t_{max}$  values sorted by  $t_{conv}$  can be found in Table A.2.<sup>7</sup>

## The Interplay of $\delta_{success}$ and $\delta_{failure}$

The interaction of  $\delta_{success}$  and  $\delta_{failure}$  is based on the fact that these are pressures that directly counteract one another. The impact of a successful communication is partly based on  $\delta_{success}$ 

<sup>&</sup>lt;sup>7</sup> It is worth pointing out that eliminating  $t_{max}$  would also mean huge increases in memory use. No upper cap would mean some words with a history of success might take very very long to reach  $\theta_{min}$  even though in the later stages they are almost always unsuccessful. This larger lexicon size might, in turn, make convergence harder because it will force the prospective winning word to share the probability range [0,1] with more words and thus get a smaller probability itself. So it is important to keep in mind that holding a history of indefinitely many successful interactions will not necessarily optimize the performance. Choosing smaller  $\delta$  values might be an alternative, e.g.  $0.0 \le \delta \le 1.0$  while  $\theta_{max} = 10.0$ .

and the impact of an unsuccessful one is wholly based on  $\delta_{failure}$ . Consequently, it is not surprising that for all parameter sets that failed to cause convergence,  $\delta_{success} \leq \delta_{failure}$  holds true.

All data points where  $\delta_{success} \leq \delta_{failure}$  are plotted in Figure 4.2. Red markers are used for non-convergent parameter sets and includes all such sets in the data used for this study. It is clear from the graphics that the interaction between these two variables holds a significant power to change the outcome of simulation runs.<sup>8</sup>

The problem with this scenario is that agents most often learn by failing, especially early in the game. If failure has the same impact as success, it becomes very difficult to have the cumulative effect of interactions building up population-wide commonalities in the lexicon into which the system will later converge, without having them falling below  $\theta_{min}$  and thus getting discarded before  $t_{max}$ . This problem becomes even greater for large  $\delta_{failure}$  values which in effect discard a word after one or two failures because the weight of the word falls below  $\theta_{min}$  regardless of the initial weight <sup>9</sup>. This makes it difficult for successful words to persist even in later stages in the game.

However, this observation does not generalize into a rule which dictates  $\delta_{success} \leq \delta_{failure}$ implies non-convergence. This is already observable in Figure 4.2 where even some parameter sets for which this inequality holds are shown to converge. There are two basic scenarios that cause this<sup>10</sup>:

1. If  $\delta_{failure} \leq 1.25 \cdot \delta_{success}$ ,  $\delta_{failure} \leq 0.75 \cdot \theta_{max}$  and  $\delta_{inhibition} = 0$ . In this case, although it takes a long time, the system converges. The reason is that agents' lexicons are generally large with this setup, as a word's weight is decremented iff it fails i.e. there is no inhibition to make lexicons smaller. Even if a word fails, it does not fall off the lexicon right away. Eventually it becomes more and more likely that agents all share some words for each object, one of which, in turn, dominates the global lexicon for that object.

 $<sup>^{8}\,</sup>$  Note that due to the projection of a three-dimensional plot to two dimensions, there are data points that overlap.

<sup>&</sup>lt;sup>9</sup> For instance, even if the word had been dominating the population and initially had a weight of  $\theta_{max}$ , it would still be discarded.

<sup>&</sup>lt;sup>10</sup> Coefficients that are presented as prerequisites of these scenarios are direct results of generalizing the choice of parameter sets used. What they really do is outlining the basic relationship between the variables and the constants and not pinpointing their exact critical ratios.

2. If  $\delta_{failure} \leq 2.0 \cdot \delta_{success}$ ,  $\delta_{failure} < 0.5 \cdot \theta_{max}$  and  $\delta_{inhibition} \leq \delta_{failure}$ . This scenario makes use of the space provided by small  $\delta$  values to converge. Due to the small values, failures cause words to be dropped from lexicons much less frequently, especially if there is a history of success for a word in the lexicon.

Furthermore, the system is able to accumulate the histories of failure and success since  $2 \cdot \delta_{success} \leq 2 \cdot \delta_{failure} \leq \theta_{max}$ , or verbally, because there is a greater number of  $\delta_{success}$  applications before reaching the cap of  $\theta_{max}$ . Consequently, there is a difference between a word that has succeeded twice and one that has succeeded once in terms of what happens to it if it fails, unlike the first scenario. This enables the agents to cause self-organization of the global lexicon which make use of these histories as well, instead of only instantaneous outcomes.

Also, if  $\delta_{inhibition}$  is nonzero, this helps shrink the lexicon size in the favor of successful words, thus increasing the probability of successful communications.

## The Interplay of $\delta_{failure}$ and $\delta_{inhibition}$

Although  $\delta_{inhibition}$  is used to decrement weights just like  $\delta_{failure}$ , its dynamics with respect to  $\delta_{success}$  are quite different from that of  $\delta_{failure}$ . The main reason for this is that it is more *potent* in terms of its effect on the lexicon per round.  $\delta_{failure}$  is able to effect only one word per round, whereas  $\delta_{inhibition}$  is able to effect almost all words for an object each round (i.e. all words that are not successful on a successful round). Therefore, it is usable as a way of controlling the increase in weights in the absence of  $\delta_{failure}$ . This, in fact, is how "classical model" works.

The issue with using  $\delta_{inhibition}$  as a replacement for  $\delta_{failure}$  is that it can become effective only after the game enters the phase of frequent successful interactions. In other words, it renders failed communications useless as a source of negative feedback as  $\delta_{inhibition}$  does not change the probability of that word being chosen in the next interaction.

Using  $\delta_{inhibition}$  as a replacement for  $\delta_{failure}$ , the lexicon would be smaller than it would be with  $\delta_{failure}$  since a lot of words are eliminated before reaching  $t_{max}$  as  $\delta_{inhibition}$  has an effect of quickly shrinking lexicons (see 4.4.2). This makes  $t_{max}$  smaller, but also makes the system slower to converge as there are less alternatives that might survive as the convergent word and any fluctuations need to be neutralized by opposing fluctuations for the system to be attracted back to those limited alternatives. The system starts exerting selective pressure towards some words when there is only a very small collection of words shared among all agents. Due to a lack of negative feedback, this point is reached a lot more rapidly than it would with the presence of  $\delta_{failure}$ , but it takes a lot more time to reach convergence after this point than it would if  $\delta_{failure}$  was nonzero.

The core disadvantage that gives rise to this property is the amount of variability of the potentially successful words. Restricting the lexicons too far, while optimizing for memory, is detrimental to the robustness of the system. There is a great amount of stochasticity in the model, and this makes robustness very crucial.

Suppose there is only one word,  $w_{conv}$ , that is shared among all agents at time  $t_{max}$ . The only way for the system to converge after that point is making sure that all agents are very likely to choose  $w_{conv}$ . However, any deviations from perfect behaviour of  $w_{conv}$  always succeeding and all other words always failing need to be balanced out so that system maintains an increasing bias towards  $w_{conv}$ . However, if there are more than one alternatives at  $t_{max}$ , say  $w_1$  and  $w_2$ , deviations may alter the behaviour of the system so that a system seemingly progressing towards  $w_1$  may start favoring  $w_2$  and quickly reach convergence on that word if some perturbance makes the system more biased towards  $w_2$ . The time it takes to "recover" from the perturbance is reduced as there are multiple possibilities for convergence.

Alternatively, a high  $\delta_{failure}$  with a low  $\delta_{inhibition}$  leads to a greater average lexicon size which makes convergence after  $t_{max}$  extremely rapid. Since there are a lot of alternatives, and those alternatives can be rapidly reduced by  $\delta_{inhibition}$  with the increasing success rate after  $t_{max}$ , convergence is often reached way before lexicons are shrinked to one word per object. This results in significantly more memory use, but greatly reduces  $t_{conv}$  in exchange.

When similar values for  $\delta_{failure}$  and  $\delta_{inhibition}$  are used, they essentially establish a balance between these two tendencies, namely, eliminating unsuccessful words before  $t_{max}$  and shrinking lexicons to favour successful words after  $t_{max}$ . Of course,  $\delta_{failure}$  eliminates unsuccessful words even after  $t_{max}$  and  $\delta_{inhibition}$  shrinks lexicons even before  $t_{max}$ , but their impact is more salient in their respective temporal portions. For instance,  $\delta_{inhibition}$  will start shrinking the lexicon before  $t_{max}$ , but it since success occurs at the chance level before  $t_{max}$ , the impact is minimal. Similarly, eliminating unsuccessful words continue after  $t_{max}$ , but the decreasing amount of unsuccessful communications and the greater power of  $\delta_{inhibition}$  as a decrement

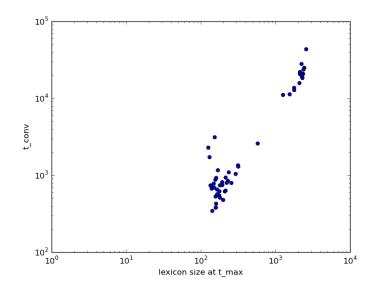


Figure 4.3: A log-log scale scatter plot of  $\overline{size}(L_{\alpha}(t))$  vs.  $t_{conv}$  for all converging parameter sets.

renders this effect peripheral compared to that of  $\delta_{inhibition}$ .

To recite, this interplay between the parameters resemble that of playing strategies for the classical model as explained in 4.1.3.

#### 4.4.2 Lexicon Size

Average size of agents' lexicons at some time t, or  $\overline{size}(L_{\alpha}(t))$ , is able to serve as an indicator of game dynamics. Since lexicon size is not a parameter but an outcome, it is a bit more difficult to test for. However, the common patterns and tendencies that arise from them are important.

One such important tendency, which is also visually presented in Figure 4.3, is this:

$$\log(t_{max}) \propto \log(\overline{size}(L_{\alpha}(t))) \tag{4.8}$$

In words, this expression means that the logarithm of the average lexicon size is directly proportional to the logarithm of  $t_{max}$ . This is highly related to the issues discussed in the previous section. More precisely, it confirms that a later  $t_{max}$  does mean a greater average lexicon size. It should also be noted that this tendency is present for all parameter sizes.

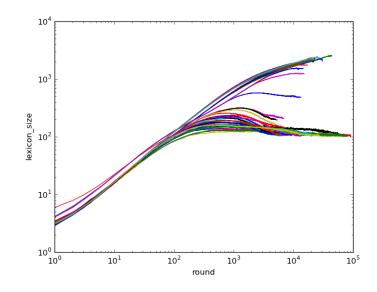


Figure 4.4: A log-log scale scatter plot of t vs.  $\overline{size}(L_{\alpha}(t))$  for all converging parameter sets.

The evolution of the lexicon size with respect to time can be seen in Figure 4.4. The first thing that pops up in this figure is how the individual plots are sharply divided into two partitions, one converging with a significantly larger lexicon size than the other. Supporting our earlier argument on the effects of  $\delta_{inhibition}$ , the upper partition consists of cases where  $\delta_{inhibition} \leq 0.1$ . The monotonical growth of the lexicon and very immediate convergence after  $t_{max}$  is clearly illustrated. It should be noted that this plot depicts all parameters sets that converge, so this partitioning is evident *despite* the range of values parameters  $\delta_{inhibition}$  and  $\delta_{success}$  are given.

From the other partition, it is possible to see a common pattern of lexicon size evolution over time. There is always a peak, that corresponds to  $t_{max}$ , and then a gradual decrease into  $t_{conv}$ . The time it takes from that peak to the end is determined by the interplay of  $\delta_{inhibition}$  and  $\delta_{failure}$  as described in 4.4.1.

Note that for the lower partition, the lower bound is about  $10^2$ , which amounts to one word per object per agent. This behaviour is also observed in previous research regarding the classical model, and is caused by  $\delta_{inhibition}$ . The greater the  $\delta_{inhibition}$ , the less populous the lexicon at  $t_{max}$ , and consequently, smaller the lexicon at  $t_{conv}$ . This reveals that there is a very substantial relationship between  $\delta_{inhibition}$  and the memory efficiency of the model. The effect  $\delta_{inhibition}$  has on  $\overline{size}(L_{\alpha}(t_{max}))$  is illustrated in Figure 4.5. Note that the range over which  $\delta_{inhibition}$  has the

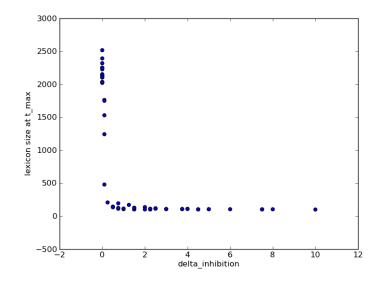


Figure 4.5: A scatter plot of  $\delta_{inhibition}$  vs.  $\overline{size}(L_{\alpha}(t_{max}))$  for all converging parameter sets.

greatest impact is lower values of  $\delta_{inhibition}$ . So while there is an apparent effect, it is most noticable when comparing zero inhibition models to nonzero inhibition models, or, very low inhibition models to higher inhibition models. The impact of  $\delta_{inhibition}$  on the lexicon size diminishes as  $\delta_{inhibition}$  increases.

## **CHAPTER 5**

## **Conclusion and Discussion**

The model presented is aimed to help investigate the effects of adopting a weighted list to represent agents' lexicons. The line of research lead by Steels has investigated the dynamics of language games with very simple software agents, and also with quite complex robotic agents. However, as emergence dictates, it is not clear how to reconcile the two as the changes in the agents' features are dramatic. We believe it to be useful to confine oneself to baby steps every now and then, incrementally increasing the complexity of a certain model and investigating it at every step to get a deeper understanding of the game dynamics and how the complexities introduced effect those dynamics.

This study has demonstrated that our model, augmented with weighted lists and probabilistic picking, follows similar but distinct dynamics as the classical model. This exemplifies the usefulness of the incremental approach outlined before. Since the simpler model is well-studied, and the modifications are few, the results can be investigated within the context of the previous studies. This makes what exactly the modifications introduce to the dynamics much clearer, as we have a baseline to compare to.

To be precise, we have introduced a weighted-list lexicon with a lateral inhibition update scheme. Each failure triggers a decrease of weight, whereas each success triggers an increase to the successful word-object combination's weight and a decrease to that of all other alternatives. During word selection, the weights are used to make a weighted random choice.

It is evident from the results that there are parameter sets with which the model fails to function properly. It is also evident *why* these parameter sets are non-convergent from the analysis of the simulation results. The results express an overall balance between memory use and speed, as so often seen in the world of computation. If the memory use is lower, that is, the average size of the lexicon is small, the process tends to slow down. The game includes a lot of stochasticity and this means robustness greatly eases convergence. With low memory use, agents cannot afford to have many ways to converge to a single word mapping for each object, that would mean using a lot of memory to store all the alternatives. Accordingly, the convergence tends to take longer than the cases where the memory use is more liberal, as the system re-establishes its move to convergence one of the few alternatives that exist upon every perturbation introduced by the stochasticity.

It is also evident that there is a pattern of building up a lexicon and then shrinking it to reach convergence. This process can be controlled via the  $\delta_{inhibition}$  parameter that basically increases the weight of the successful word-object pairing and decreases all other alternatives upon successful interactions. It is more powerful after  $t_{max}$ , the point in time where the average lexicon size of agents is the greatest, as after that point, the percentage of successful interactions get above the chance level due to all agents sharing some word-object pairing after that. A greater  $\delta_{inhibition}$  means more shrinkage and less memory use, with an early peak in lexicon size and gradual decrease to an approximate one-to-one mapping of words and objects. A small  $\delta_{inhibition}$ , on the other hand, means the climb to the peak where the lexicon size is the greatest takes longer, and the peak is higher compared to the greater  $\delta_{inhibition}$  case, but convergence immediately follows that peak.

The model proposed has not only demonstrated full function, but has also overperformed the classical model in terms of minimizing  $t_{conv}$ . Considering that classical model can be precisely mimicked using a certain parameter set on this model, it is possible to say this general model not only captures the behaviour of existing models but allows for tweaking parameters to obtain a parameter set that result in desired properties in a specific instance of the model. For example, the classical model may be desired because of its low memory use, or low- $\delta$  model with a proper ratio may be desired because of its rapid convergence, or even a dynamical- $\delta$  model for adaptive learning rates.

## Limitations

A major limitation of this study was computational power. This has been prohibitive of controlling for population size, being more comprehensive as to the range of parameter sets used, analyzing step-by-step the dynamics of global lexicons, as well as using larger samples for each parameter set to get better averages. Some analyses take about forty-eight hours to execute even when optimized for parallel processing, not including the time spent on simulations themselves. Even disk I/O is a prohibitive factor considering the data presented in this study amounts to over 750 GB of disk space. While the accessibility of modern multicore systems ease such burden to an extent, a task of fully covering the parameter range would require running on a grid of computers to be feasible in terms of time spent actively computing.

Another limitation, related to the first, is the limited range the parameter sets cover. While this has been a conscious choice based on computational power and the desire to contrast with the classical model, it is not at all clear that in a space of parameter values, the vicinity of the classical parameters would behave like the classical model. Dynamical systems can and do exhibit radical changes in behaviour with small changes to its parameters, and of course our model is no exception. While the investigation has been fruitful, a more complete understanding of the dynamics require a more representative sampling of the parameter space.

## **Future Work**

Drawing from the enlisted limitations there are many aspects of the model to be investigated that may be recommended as future work on the subject.

- A more comprehensive survey of the parameter space can be made to investigate the full extent of the interaction between the game dynamics and  $\delta$  values. This is especially promising if very low  $\delta$  values are investigated so that  $t_{max}$  becomes less relevant.
- Varying  $t_{max}$  values, including  $\infty$ , can be used to investigate the impact of this parameter on the game dynamics as well as the impact of the  $\delta$  parameters.
- The effect of population size on the simulation can be investigated. Although it is known that the system would scale as a power law of the population size from previous

work on classical model, it is not clear how exactly it would scale, especially compared to the classical model.

- The effect of enforcing a topology on the population on the game can be investigated. Although this too has been done on the classical model, such investigations are expected to hardly shed any light on the current model's behaviour under such conditions.
- Computer grids and databases can be used instead of single computers and files to improve the feasibility of any investigations on the model.
- Human performance data in naming games can be collected to be compared with the investigation of the parameter space. Although details of the lexicons would not be directly accessible in such data, it can either be inferred or disregarded so that other aspects may be investigated and human-like performance can be characterized in terms of the model.
- A variable- $\delta$  model can be devised to investigate how a population of agents with different learning parameters would perform in the game (this might also be considered as some sort of topology).
- A dynamical- $\delta$  model can be designed to investigate how a population of agents with learning parameters varying over time would perform in the game.
- Other language games can be played by agents that draw from the lexicon update scheme in this model and the impact of this modification can be investigated.

## References

- Aristotle. (n.d.). Metaphysics (T. b. W. D. Ross, Ed.). The Internet Classics Archive.
- Baronchelli, A., Dall'Asta, L., Barrat, A., & Loreto, V. (2005). Strategies for fast convergence in semiotic dynamics. *Arxiv preprint physics/0511201*.
- Baronchelli, A., DallâAsta, L., Barrat, A., & Loreto, V. (2006, January). Topology-induced coarsening in language games. *Physical Review E*, 73(1), 2–5.
- Baronchelli, A., Felici, M., Loreto, V., Caglioti, E., & Steels, L. (2006). Sharp transition towards shared vocabularies in multi-agent systems. *Journal of Statistical Mechanics: Theory and Experiment*, 2006, P06014.
- Baronchelli, A., Loreto, V., & Steels, L. (2008). In-depth analysis of the Naming Game dynamics: the homogeneous mixing case. *International Journal of Modern Physics C*, 19(5), 785–812.
- Batali, J. (1998). Computational simulations of the emergence of grammar. *Approaches to the evolution of language: Social and cognitive bases*, 405, 426.
- Bedau, M. A., Bedau, H., Collier, J., Crook, S., Grandy, D., Hinchliff, M., et al. (1997). Weak Emergence. *World*, *11*(1992), 375–399.
- Chalmers, D. J. (2006). Strong and Weak Emergence. In P. Clayton & P. Davies (Eds.), *The re-emergence of emergence*. Oxford University Press.
- Corominas-Murtra, B., Valverde, S., & Solé, R. V. (2007, September). Emergence of Scale-Free Syntax Networks. 0709.4344.
- DallâAsta, L., Baronchelli, A., Barrat, A., & Loreto, V. (2006). Nonequilibrium dynamics of language games on complex networks. *Physical Review E*, 74(3), 36105.
- De Beule, J. (2004). Creating temporal categories for an ontology of time. In (pp. 107–14).
- De Beule, J. (2008). The emergence of compositionality, hierarchy and recursion in peer-topeer interactions. In (p. 75).
- De Vylder, B. (2008). *The Evolution of Conventions in Multi-Agent Systems*. Unpublished doctoral dissertation.
- De Vylder, B., & Tuyls, K. (2006). How to reach linguistic consensus: A proof of conver-

gence for the naming game. Journal of theoretical biology, 242(4), 818-831.

- De Wolf, T., & Holvoet, T. (2005). Emergence versus self-organisation: Different concepts but promising when combined. *Engineering SelfOrganising Systems*, 3464, 1–15.
- Emily, W., & Steels, L. (2008, February). Self-Interested Agents can Bootstrap Communication If They Punish Cheaters. *eproceedings.worldscinet.com*, 362–369.
- Goldstein, J. (1999, March). Emergence as a Construct: History and Issues. *Emergence*, 1(1), 49–72.
- Hurford, J. R. (1989). Biological Evolution of the Saussurean Sign as a Component of the Language Acquisition Device in Linguistic Evolution. *Lingua*, 77(2), 187–222.
- Kirby, S. (2002, April). Natural Language From Artificial Life. Artificial Life, 8(2), 185–215.
- Kirby, S., & Hurford, J. R. (2002). The emergence of linguistic structure. Simulating the evolution of language. London: Springer, 121–147.
- Krakauer, D. C., & Pagel, M. (1995). Spatial structure and the evolution of honest cost-free signalling. *Proceedings: Biological Sciences*, 260(1359), 365–372.
- Lenaerts, T., Jansen, B., Tuyls, K., & De Vylder, B. (2005). The evolutionary language game: An orthogonal approach. *Journal of Theoretical Biology*, 235(4), 566–582.
- Newman, M. E. J. (2004, November). Power laws, Pareto distributions and Zipf's law. condmat/0412004.
- Puglisi, A., Baronchelli, A., & Loreto, V. (2008). Cultural route to the emergence of linguistic categories. *Proceedings of the National Academy of Sciences*, 105(23), 7936.
- Reggia, J. A., Schulz, R., Wilkinson, G. S., & Uriagereka, J. (2001). Conditions enabling the evolution of inter-agent signaling in an artificial world. *Artificial Life*, 7(1), 3–32.
- Shalizi, C. (2001). *Causal architecture, complexity and self-organization in time series and cellular automata*. Phd, University of Wisconsin at Madison.
- Steels, L. (1996a). Emergent adaptive lexicons. From animals to animats, 4, 562-567.
- Steels, L. (1996b). Perceptually grounded meaning creation. In (pp. 338–344).
- Steels, L. (2003, July). Evolving grounded communication for robots. *Trends in Cognitive Sciences*, 7(7), 308–312.
- Steels, L., & Kaplan, F. (2002). AIBO's first words: The social learning of language and meaning. *Evolution of Communication*, 4(1), 3–32.
- Steels, L., Kaplan, F., & Others. (1999). Collective learning and semiotic dynamics. *Lecture Notes in Computer Science*, 679–688.
- Vogt, P. (2002, April). Bootstrapping grounded symbols by minimal autonomous robots.

Evolution of Communication, 4, 87–116.

- Vogt, P. (2005, September). The emergence of compositional structures in perceptually grounded language games. *Artificial Intelligence*, *167*(1-2), 206–242.
- Wagner, K., Reggia, J. A., Uriagereka, J., & Wilkinson, G. S. (2003). Progress in the simulation of emergent communication and language. *Adaptive Behavior*, 11(1), 37.

## **APPENDIX A**

# Appendix A

Table A.1: All parameter sets used and their corresponding average  $t_{conv}$  and  $t_{max}$  values.

| $\delta_{success}$ | $\delta_{failure}$ | $\delta_{inhibition}$ | t <sub>conv</sub> | t <sub>max</sub> |
|--------------------|--------------------|-----------------------|-------------------|------------------|
| 1.000000           | 0.000000           | 0.100000              | 13423             | 2582             |
| 1.000000           | 0.500000           | 0.000000              | 19112             | 18811            |
| 3.000000           | 3.000000           | 3.000000              | 5136              | 543              |
| 3.000000           | 3.000000           | 4.500000              | 6590              | 777              |
| 3.000000           | 4.500000           | 0.000000              | 25466             | 24889            |
| 3.000000           | 4.500000           | 2.250000              | 8666              | 652              |
| 5.000000           | 5.000000           | 5.000000              | 15434             | 528              |
| 5.000000           | 5.000000           | 7.500000              | 38772             | 670              |
| 5.000000           | 7.500000           | 0.000000              | 44054             | 43399            |
| 5.000000           | 7.500000           | 10.000000             | 499999            | 37333            |
| 1.000000           | 2.000000           | 3.000000              | 499999            | 2639             |
| 10.000000          | 0.000000           | 10.000000             | 91450             | 343              |
| 10.000000          | 0.000000           | 0.100000              | 15248             | 13658            |
| 10.000000          | 10.000000          | 10.000000             | 499999            | 221971           |
| 1.000000           | 0.500000           | 0.250000              | 6918              | 1342             |
| 3.000000           | 4.500000           | 4.500000              | 44926             | 736              |
| 5.000000           | 7.500000           | 3.750000              | 64880             | 3115             |
| 5.000000           | 0.000000           | 0.100000              | 14472             | 11264            |
| 5.000000           | 10.000000          | 0.000000              | 499999            | 499439           |

| 5.000000 | 10.000000 | 10.000000 | 499999 | 165869 |
|----------|-----------|-----------|--------|--------|
| 5.000000 | 10.000000 | 5.000000  | 499999 | 253411 |
| 8.000000 | 4.000000  | 0.000000  | 20518  | 18337  |
| 8.000000 | 4.000000  | 2.000000  | 5352   | 793    |
| 8.000000 | 4.000000  | 4.000000  | 4881   | 477    |
| 8.000000 | 4.000000  | 6.000000  | 8206   | 565    |
| 8.000000 | 8.000000  | 0.000000  | 20828  | 20826  |
| 8.000000 | 8.000000  | 10.000000 | 499999 | 88655  |
| 8.000000 | 8.000000  | 4.000000  | 10647  | 739    |
| 8.000000 | 8.000000  | 8.000000  | 20501  | 378    |
| 1.000000 | 1.000000  | 1.500000  | 6022   | 691    |
| 1.000000 | 1.500000  | 0.000000  | 22682  | 20835  |
| 1.000000 | 1.500000  | 0.750000  | 6346   | 1161   |
| 1.000000 | 1.500000  | 1.500000  | 13475  | 734    |
| 1.000000 | 1.500000  | 2.250000  | 76007  | 2280   |
| 1.000000 | 2.000000  | 0.000000  | 21926  | 20422  |
| 1.000000 | 2.000000  | 1.000000  | 9416   | 922    |
| 1.000000 | 2.000000  | 2.000000  | 80042  | 1715   |
| 1.000000 | 0.500000  | 0.500000  | 7058   | 1090   |
| 1.000000 | 0.500000  | 0.750000  | 6532   | 746    |
| 1.000000 | 1.000000  | 0.000000  | 22289  | 21866  |
| 1.000000 | 1.000000  | 0.500000  | 4329   | 932    |
| 1.000000 | 1.000000  | 1.000000  | 5886   | 616    |
| 3.000000 | 4.500000  | 6.750000  | 499999 | 578    |
| 3.000000 | 6.000000  | 0.000000  | 30655  | 27920  |
| 3.000000 | 6.000000  | 3.000000  | 20465  | 874    |
| 3.000000 | 6.000000  | 6.000000  | 499999 | 4715   |
| 3.000000 | 6.000000  | 9.000000  | 499999 | 94264  |
| 5.000000 | 7.500000  | 7.500000  | 499999 | 390344 |
| 8.000000 | 0.000000  | 0.100000  | 17181  | 12765  |
| 8.000000 | 10.000000 | 0.000000  | 499999 | 499399 |
|          |           |           |        |        |

| 8.000000  | 10.000000 | 10.000000 | 499999 | 383433 |
|-----------|-----------|-----------|--------|--------|
| 8.000000  | 10.000000 | 5.000000  | 499999 | 71798  |
| 10.000000 | 5.000000  | 7.500000  | 21482  | 427    |
| 3.000000  | 0.000000  | 0.100000  | 15040  | 11043  |
| 3.000000  | 1.500000  | 0.000000  | 21100  | 20981  |
| 3.000000  | 1.500000  | 0.750000  | 5373   | 1293   |
| 3.000000  | 1.500000  | 1.500000  | 4086   | 844    |
| 3.000000  | 1.500000  | 2.250000  | 4320   | 813    |
| 3.000000  | 3.000000  | 0.000000  | 15861  | 15759  |
| 3.000000  | 3.000000  | 1.500000  | 3719   | 616    |
| 5.000000  | 2.500000  | 0.000000  | 21284  | 19988  |
| 5.000000  | 2.500000  | 1.250000  | 4949   | 1039   |
| 5.000000  | 2.500000  | 2.500000  | 5566   | 630    |
| 5.000000  | 2.500000  | 3.750000  | 4863   | 510    |
| 5.000000  | 5.000000  | 0.000000  | 23410  | 21962  |
| 5.000000  | 5.000000  | 2.500000  | 5646   | 745    |
| 10.000000 | 10.000000 | 5.000000  | 499999 | 448594 |
| 10.000000 | 5.000000  | 0.000000  | 27835  | 23710  |
| 10.000000 | 5.000000  | 2.500000  | 6803   | 796    |
| 10.000000 | 5.000000  | 5.000000  | 27545  | 380    |

Table A.2: All convergent parameter sets their corresponding average  $t_{conv}$  and  $t_{max}$  values, sorted by  $t_{conv}$ .

| t <sub>conv</sub> | $\delta_{success}$ | $\delta_{failure}$ | $\delta_{inhibition}$ | t <sub>max</sub> |
|-------------------|--------------------|--------------------|-----------------------|------------------|
| 3719              | 3.000000           | 3.000000           | 1.500000              | 616              |
| 4086              | 3.000000           | 1.500000           | 1.500000              | 844              |
| 4320              | 3.000000           | 1.500000           | 2.250000              | 813              |
| 4329              | 1.000000           | 1.000000           | 0.500000              | 932              |
| 4863              | 5.000000           | 2.500000           | 3.750000              | 510              |
| 4881              | 8.000000           | 4.000000           | 4.000000              | 477              |
| 4949              | 5.000000           | 2.500000           | 1.250000              | 1039             |

| 5136  | 3.000000  | 3.000000 | 3.000000 | 543   |
|-------|-----------|----------|----------|-------|
| 5352  | 8.000000  | 4.000000 | 2.000000 | 793   |
| 5373  | 3.000000  | 1.500000 | 0.750000 | 1293  |
| 5566  | 5.000000  | 2.500000 | 2.500000 | 630   |
| 5646  | 5.000000  | 5.000000 | 2.500000 | 745   |
| 5886  | 1.000000  | 1.000000 | 1.000000 | 616   |
| 6022  | 1.000000  | 1.000000 | 1.500000 | 691   |
| 6346  | 1.000000  | 1.500000 | 0.750000 | 1161  |
| 6532  | 1.000000  | 0.500000 | 0.750000 | 746   |
| 6590  | 3.000000  | 3.000000 | 4.500000 | 777   |
| 6803  | 10.000000 | 5.000000 | 2.500000 | 796   |
| 6918  | 1.000000  | 0.500000 | 0.250000 | 1342  |
| 7058  | 1.000000  | 0.500000 | 0.500000 | 1090  |
| 8206  | 8.000000  | 4.000000 | 6.000000 | 565   |
| 8666  | 3.000000  | 4.500000 | 2.250000 | 652   |
| 9416  | 1.000000  | 2.000000 | 1.000000 | 922   |
| 10647 | 8.000000  | 8.000000 | 4.000000 | 739   |
| 13423 | 1.000000  | 0.000000 | 0.100000 | 2582  |
| 13475 | 1.000000  | 1.500000 | 1.500000 | 734   |
| 14472 | 5.000000  | 0.000000 | 0.100000 | 11264 |
| 15040 | 3.000000  | 0.000000 | 0.100000 | 11043 |
| 15248 | 10.000000 | 0.000000 | 0.100000 | 13658 |
| 15434 | 5.000000  | 5.000000 | 5.000000 | 528   |
| 15861 | 3.000000  | 3.000000 | 0.000000 | 15759 |
| 17181 | 8.000000  | 0.000000 | 0.100000 | 12765 |
| 19112 | 1.000000  | 0.500000 | 0.000000 | 18811 |
| 20465 | 3.000000  | 6.000000 | 3.000000 | 874   |
| 20501 | 8.000000  | 8.000000 | 8.000000 | 378   |
| 20518 | 8.000000  | 4.000000 | 0.000000 | 18337 |
| 20828 | 8.000000  | 8.000000 | 0.000000 | 20826 |
| 21100 | 3.000000  | 1.500000 | 0.000000 | 20981 |
|       |           |          |          |       |

| 21284 | 5.000000  | 2.500000 | 0.000000  | 19988 |
|-------|-----------|----------|-----------|-------|
| 21482 | 10.000000 | 5.000000 | 7.500000  | 427   |
| 21926 | 1.000000  | 2.000000 | 0.000000  | 20422 |
| 22289 | 1.000000  | 1.000000 | 0.000000  | 21866 |
| 22682 | 1.000000  | 1.500000 | 0.000000  | 20835 |
| 23410 | 5.000000  | 5.000000 | 0.000000  | 21962 |
| 25466 | 3.000000  | 4.500000 | 0.000000  | 24889 |
| 27545 | 10.000000 | 5.000000 | 5.000000  | 380   |
| 27835 | 10.000000 | 5.000000 | 0.000000  | 23710 |
| 30655 | 3.000000  | 6.000000 | 0.000000  | 27920 |
| 38772 | 5.000000  | 5.000000 | 7.500000  | 670   |
| 44054 | 5.000000  | 7.500000 | 0.000000  | 43399 |
| 44926 | 3.000000  | 4.500000 | 4.500000  | 736   |
| 64880 | 5.000000  | 7.500000 | 3.750000  | 3115  |
| 76007 | 1.000000  | 1.500000 | 2.250000  | 2280  |
| 80042 | 1.000000  | 2.000000 | 2.000000  | 1715  |
| 91450 | 10.000000 | 0.000000 | 10.000000 | 343   |

## **APPENDIX B**

# Appendix B

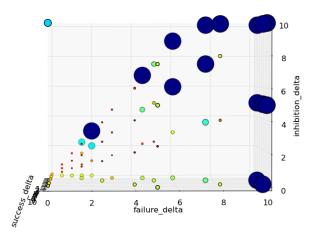


Figure B.1: Scatter plot of all data with respect to  $t_{conv}$  from a  $\delta_{failure}$  vs  $\delta_{inhibition}$  perspective.

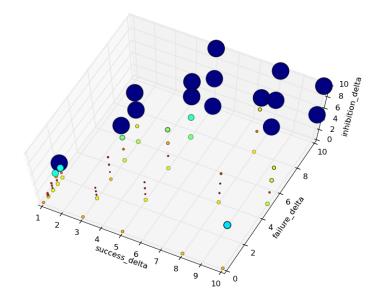


Figure B.2: Scatter plot of all data with respect to  $t_{conv}$  from a  $\delta_{success}$  vs  $\delta_{failure}$  perspective.

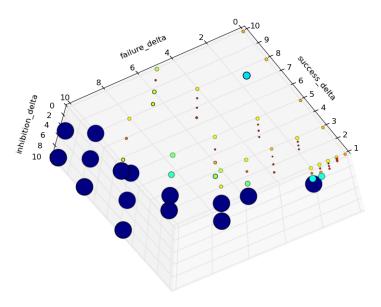


Figure B.3: Scatter plot of all data with respect to  $t_{conv}$  from a  $\delta_{failure}$  vs  $\delta_{success}$  perspective.

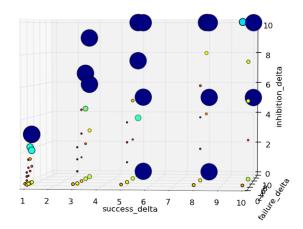


Figure B.4: Scatter plot of all data with respect to  $t_{conv}$  from a  $\delta_{success}$  vs  $\delta_{inhibition}$  perspective.

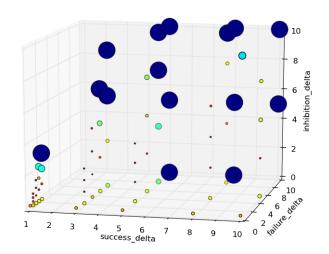


Figure B.5: Scatter plot of all data with respect to  $t_{conv}$  from a  $\delta_{success}$  vs  $\delta_{inhibition}$  perspective.

## **APPENDIX C**

# Appendix C

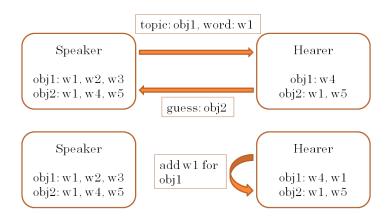


Figure C.1: A failed round in the classical model.

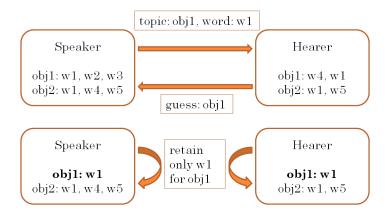


Figure C.2: A successful round in the classical model.

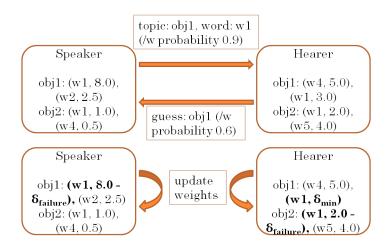


Figure C.3: A failed round in the proposed model.

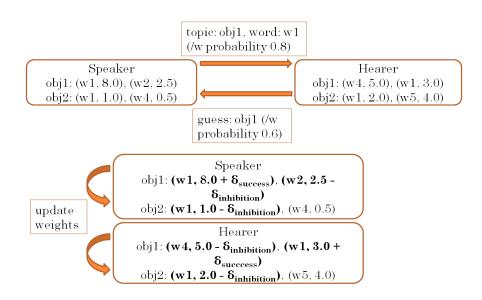


Figure C.4: A successful round in the proposed model.