AN INVESTIGATION OF PROSPECTIVE ELEMENTARY MATHEMATICS TEACHERS’ STRATEGIES USED IN MATHEMATICAL PROBLEM SOLVING

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF SOCIAL SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

SEHER AVCU

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
THE DEPARTMENT OF ELEMENTARY SCIENCE AND MATHEMATICS
EDUCATION

JANUARY 2012

Approval of the Graduate School of Social Sciences

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I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

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This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

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ABSTRACT

AN INVESTIGATION OF PROSPECTIVE ELEMENTARY MATHEMATICS TEACHERS' STRATEGIES USED IN MATHEMATICAL PROBLEM SOLVING

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January 2012,112 pages

The purpose of this study was to investigate the prospective elementary mathematics teachers' use of strategies and their achievement levels in solving mathematical problems with respect to year level. The data were collected from 250 prospective elementary mathematics teachers enrolled in an elementary mathematics education program from a state university in Central Anatolian Region. Problem Solving Test (PST) was used to accomplish the purpose of the study. The data collection tool adapted by the researcher included nine open ended problems. In this study, item based in-depth analysis was employed to determine a variety of problem solving strategies used by prospective teachers. The frequencies and percentages of categories were gathered for each item and for each year level.

The results of this study revealed that prospective elementary mathematics teachers' problem solving achievement was moderately high. Prospective elementary mathematics teachers in each year level were able to use various problem solving strategies to a certain extent. More specifically, the results indicated that 'making a drawing' and 'intelligent guessing and testing' strategies were among the most prominent strategies frequently used by prospective teachers. Setting up an equation and using a formula was other strategies used by prospective teachers. On the other hand, finding a pattern strategy was the least frequent strategy used by prospective teachers.

Keywords: Problem solving achievement, Prospective elementary mathematics teachers, Problem solving strategies

ÖZ

İLKÖĞRETİM MATEMATİK ÖĞRETMEN ADAYLARININ MATEMATİKSEL PROBLEM ÇÖZMEDE KULLANDIKLARI STRATEJİLERİN İNCELENMESİ

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Yüksek Lisans, İlköğretim Fen ve Matematik Eğitimi

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Ocak 2012, 112 sayfa

Bu çalışmanın amacı, ilköğretim matematik öğretmen adaylarının matematiksel problem çözmedeki başarılarını ve kullandıkları stratejileri incelemektir. Çalışmanın örneklemi İç Anadolu Bölgesindeki bir devlet üniversitesinde ilköğretim matematik öğretmenliği programına devam eden 250 öğretmen adayından oluşmaktadır. Çalışmanın amacı doğrultusunda araştırmacı tarafından uyarlanan dokuz maddelik Problem Çözme Testi kullanılmıştır. Bu çalışmada, öğretmen adaylarının kullandıkları problem çözme stratejilerini belirlemek için Problem Çözme Testindeki her bir madde derinlemesine incelenmiştir. Verilerin analizinde frekans ve yüzde kullanılmıştır.

Araştırmanın sonuçlarına göre, ilköğretim matematik öğretmen adaylarının problem çözmede başarısı oldukça yüksek bulunmuştur. Ayrıca, ilköğretim matematik öğretmen adaylarının farklı problem çözme stratejilerini belirli ölçüde kullandıkları belirlenmiştir. Araştırmanın bulgularına göre öğretmen adaylarının en çok şekil çizme ile tahmin ve kontrol stratejilerini kullanmışlardır. Öğretmen adayları aynı zamanda denklem kurma ve formül kullanma stratejilerini kullanmışlardır. Öğretmen adaylarının en az kullandıkları strateji ise örüntü bulma stratejisidir.

Anahtar Kelimeler: Problem çözme başarısı, İlköğretim matematik öğretmen adayları, Problem çözme stratejileri

To my parents
For their care, support, and encouragement
To my husband, Ramazan
For his love, patience, and understanding

ACKNOWLEDGEMENTS

I would like to take this opportunity to thank several people who have provided their help and encouragement throughout this study.

First, I want to thank my thesis supervisors Assoc. Prof. Dr. Mine IŞIKSAL and Assist. Prof. Dr. Çiğdem HASER for their wisdom, guidance, and belief in me. They always challenged me to do my best, and encouraged me in every step of my thesis. Thanks for their great efforts.

I would also like to thank to my friend, Deniz EROĞLU. Without your close friendship, inspiration, help, suggestions and support, this thesis would not have been written. Thanks for being such good friend. I hope that we will be able to stay in touch despite the large distances between us.

I would like to express my sincere thanks to my officemate Res. Ass. Mehmet ÖZKAYA for his understanding and help throughout my thesis study.

I really appreciate and thank to my husband, Ramazan AVCU for his love, patience, and great understanding. I am really sorry for all moments that I made hard stand for you. It would not be possible for me to finish this thesis without your assistance. Thank you for your being in my life. In addition, I am forever grateful to my mother, Hatice ALANYALI, my father Mümtaz ALANYALI, and my sisters Ümmühan ALANYALI and Ayşe ALANYALI for their support throughout my life.

Finally, this thesis is dedicated to my parents and my husband, representing my appreciation.

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EME: Elementary Mathematics Education

PST: Problem Solving Test

NCTM: National Council of Teachers of Mathematics

CHAPTER 1

INTRODUCTION

Over the past decades, National Council of Teachers of Mathematics [NCTM] has established mathematics standards in the areas of curriculum, teaching, and assessment and has been influential around the world in establishing a vision of school mathematics that is grounded in student understanding and problem solving (NCTM, 1989, 1991, 1995). Moreover, there is call for a decreased emphasis on computation and knowledge of algorithms and a great emphasis on conceptual understanding, problem solving, reasoning and proof, communication connections and representation (NCTM, 2000).

Problem solving plays an important role in mathematical learning. Many mathematics educators feel that mathematics is problem solving (Wilson, Fernandez & Hadaway, 1993). Moreover, mathematics teachers, students and parents believe that doing mathematics is equivalent to solving problems (Kaur, 1997). According to Stanic and Kilpatrick (1989), problem solving has occupied a central place in the school mathematics curriculum since antiquity. Similarly, Kilpatrick, Swafford and Findell (2001) explained that,

> "Studies in almost every domain of mathematics have demonstrated that problem solving provides an important context in which students can learn about number and other mathematical topics. Problem-solving ability is enhanced when students have opportunities to solve problems

themselves and to see problems being solved. Further, problem solving can provide the site for learning new concepts and for practicing learned skills" (p. 420).

Thus, problem solving is important as a way of doing, learning and teaching mathematics and can be accepted as a core concept in school mathematics curricula. Krulik and Rudnick (2003) assert that problem solving is not just a method in mathematics, but a major part of learning mathematics where the students deepen their understanding of mathematical concepts by analyzing and synthesizing their knowledge. Furthermore, NCTM (2000) states that problem solving process is of great importance for students and summarizes this process standard as follows:

> "Instructional programs from prekindergarten through grade 12 should enable all students to build new mathematical knowledge through problem solving; solve problems that arise in mathematics and in other contexts; apply and adopt a variety of appropriate strategies to solve problems; monitor and reflect on the process of mathematical problem solving" (p. 52).

There have also been reform movements in Turkish elementary school mathematics curricula by the year 2005. In the new program, problem solving has been considered as an integral part of the mathematics education and regarded as a basic skill that should be improved in every subject (Ministry of National Education [MoNE], 2005a, 2005b). Hence, the students are expected to; "benefit from problem solving in order to learn mathematics; develop awareness that problem solving contributes much to the learning; use problem solving in daily life experiences, in other disciplines and novel situations; apply problem solving steps meaningfully; pose their own problems; be self confident when solving problems; hold positive attitudes towards problem solving" (MoNE, 2009, p. 14).

The integration of problem solving and mathematics creates the need to develop problem solving strategies and processes. During problem solving process, two of the problem solving steps namely, 'making a plan' and 'carrying out the plan' are directly related with problem solving strategies (Polya, 1957). Strategies for solving problems are identifiable methods of approaching a task regardless of specific topic or subject matter (Van de Walle, 1994). The strategies would help students make progress in solving more challenging and difficult problems (Hatfield, Edwards, Bitter & Morrow, 2007). Moreover, by learning the problem solving strategies, beginning with simple applications and then progressively moving to more challenging and complex problems, the students will have the opportunity to develop everyday use of their problem solving skills (Posamentier & Krulik, 1998). Students who develop their own strategy are far more successful in solving mathematical problems (Wilborn, 1994). Besides, different problem solving strategies are necessary as students experience new mathematical problems and the teacher's mission is to create a classroom environment that students are encouraged to explore new strategies, to take risks in trying and to discuss failures and successes with peers and teacher. In such supportive environments, students understand that their solutions are appreciated and develop confidence in their mathematical abilities; hence they develop a willingness to engage in and explore new problems (MoNE, 2009).

Teachers possess considerable responsibility in students' problem solving process, therefore teachers' understanding of this process is considerably important for students to become efficient problem solvers. Thus, teachers need to be involved in a variety of problem solving experiences and gain insights into the nature of problem solving before they can adequately understand the perspectives of their future students as problem solvers (Thompson, 1984). Accordingly, if we think that problem solving should be taught to the students, then teachers should already posses the knowledge of problem solving. If we accept problem solving as a basis of teaching mathematics, teachers should understand the nature of the problem solving (Chapman, 2005). The key component of the problem solving process is teachers since problem solving instruction can be most effective when students feel that teachers accept problem solving as an important part of the activity and teachers use problem solving in their mathematics instruction regularly (Lester, 1980).

Working backwards, finding a pattern, adopting a different point of view, solving a simpler analogous problem, considering extreme cases, making a drawing, intelligent guessing and testing, accounting for all possibilities, organizing data and logical reasoning are appropriate strategies that could be applied and adapted by students in all grades to solve mathematical problems (MoNE, 2009; Posamentier & Krulik 1998).

Mathematical Sciences Education Board [MSEB] (1989) accepted teacher education as a central issue for any change in the area of education. Moreover it stated that "no reform of mathematics education is possible unless it begins with revitalization of undergraduate mathematics in both curriculum and teaching style" (p.39). Similarly, Bayrakdar, Deniz, Akgün and İşleyen (2011) argued that countries must prioritize teacher training systems before structuring elementary and secondary education programs. In this sense, it is expected that having an idea about prospective elementary mathematics teachers' understanding and use of problem solving strategies will help educators develop future training programs for prospective and in-service elementary mathematics teachers. Moreover, it is believed that the current study will contribute to future developments of mathematical problem solving in teacher education.

1.1. Purpose of the Study

In this study, the main focus of investigation is to determine the problem solving strategies that prospective elementary mathematics teachers use while solving mathematical problems. Besides, this study also deals with prospective elementary mathematics teachers' achievement in problem solving in terms of year levels.

1.2. Significance of the Study

Mathematicians (e.g., Polya, 1962), mathematics educators (e.g., Brown & Walter, 1990; Freudenthal, 1973), the National Council of Teachers of Mathematics (NCTM, 1989, 2000) and the National Research Council (Kilpatrick, Swafford & Findell, 2001) consider problem solving as a core element of mathematical proficiency. Moreover, problem solving is not only one important form of mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001) but also a productive way to develop other mathematical competencies (Lester & Lambdin, 2004). However, problem solving is a particularly complex concept in mathematics education (Ryve, 2007). Similarly, preparing prospective mathematics teachers for classrooms in the 21st century is a complex task (Lee, 2005). Due to the complexity of problem solving (e.g., Schoenfeld, 1992; Stanic & Kilpatrick, 1989) it can be assumed that teachers' views and interpretations of problem solving may have an impact on the activities of classrooms. Therefore, teacher education programs should be able to change, if necessary, prospective teachers' views regarding the role of problem solving (Cooney, Shealy, & Arvold, 1998; Crawford, 1996; Thompson, 1992). Besides, Dinç Artut and Tarım (2009) reported that prospective teachers lacked skills of solving mathematical problems and proposed that they should be provided the opportunity to create an environment needed to solve problems by using several different problem solving strategies. Therefore, prospective elementary teachers need to be investigated to determine whether problem solving achievement and the use of various problem solving strategies are provided efficiently by the current teacher education program.

There is a need for well-trained teachers in order to enable students to solve high quality problems and own flexible classroom environments that enhance their thinking. Therefore, teacher education programs should provide prospective teachers opportunities to develop problem solving skills (Dede & Yaman, 2005). Likewise, Beisser (2000) contends that prospective teachers should be provided more opportunities to view themselves as intellectually capable and practically responsible for solving mathematical problems. Moreover, if problem solving should be taught to students, then it should be taught to prospective teachers who are likely to enter teacher preparation programs without having been taught it in an explicit way.

Despite the importance of prospective teachers' understanding of problem solving or problem solving strategies, most research on problem solving was conducted with elementary school students (Altun, 1995; Charles & Lester, 1984; Erden, 1984; Israel, 2003; Lee, 1982; Yazgan & Bintaş, 2005; Yazgan, 2007). However, few studies were conducted with prospective teachers on problem solving (Altun & Memnun, 2008; Altun, Memnun & Yazgan, 2007). Moreover, Chapman (2008) made the same point about problem solving and claimed that studies focusing explicitly on prospective teachers' knowledge of problem solving are scare in the research literature, regardless of whether routine or non-routine problems are considered. For the above mentioned reasons, the current study will investigate prospective elementary mathematics teachers' problem solving achievement and their use of strategies in solving these mathematical problems. The investigation of prospective elementary mathematics teachers' problem solving achievement and their use of problem solving strategies is thought to be helpful in developing training programs in the future for prospective and in-service elementary mathematics teachers.

1.3. Research Questions

The main purpose of this study was to investigate prospective elementary mathematics teachers' use of strategies in mathematical problem solving. Moreover, this study also examined prospective elementary mathematics teachers' problem solving achievement in terms of their year level in the teacher education program. In this sense, the investigated two major research questions are:

1. What is the level of prospective elementary mathematics teachers' achievement in problem solving in terms of year level?

2. What are the strategies used by prospective elementary mathematics teachers while solving mathematical problems in each year level?

1.4. Assumptions and Limitations

It is assumed that participating prospective elementary mathematics teachers pay careful attention to each problem in Problem Solving Test. Also, it is assumed that their strategies could be measured through PST. Finally, it is assumed that, all participants have prerequisite knowledge to solve problems in the PST.

The findings of this study are limited to the data collected from 250 prospective elementary mathematics teachers studying at a state university and prospective elementary mathematics teachers' problem solving achievements and strategies are limited to the problems included in PST. Therefore, the study may be limited in its application to a more generalized population of prospective elementary mathematics teachers.

1.5. Definition of Important Terms

In previous sections, purpose, significance and research questions of the study were presented. In the following, the constitutive and operational definitions of the important terms will be given.

Problem

Henderson and Pingry (1953) defined problem as “a situation that one cannot find any ready solution for it” (p. 248). In this study problem was defined as a situation that requires a decision or an answer, no matter if the solution is readily available or not to the potential problem solver.

Problem solving

NCTM (2000) defined problem solving as “getting involved in a task for which there is no immediate answer” (p. 9).

Achievement in problem solving

In this study, prospective elementary mathematics teachers' problem solving achievement will be measured by Problem Solving Test (PST) developed by the researcher.

Problem solving strategy

Van de Walle (2007) defined problem solving strategy as "a specific method developed to find a solution to a problem regardless of considering any topic" (p. 57). In this study, Posamentier and Krulik's (1998) problem solving strategies will be adopted to identify prospective elementary mathematics teachers' problem solving strategies. Namely, these strategies are 'logical reasoning', 'intelligent guessing and testing', 'considering extreme cases', 'accounting for all possibilities', 'adopting a different point of view', 'visual representation (making drawing)', 'organizing data', 'solving simpler analogous problem', 'working backwards' and 'finding a pattern'.

Prospective elementary mathematics teachers

Prospective elementary mathematics teachers are freshman, sophomore, junior and senior students studying in the Elementary Mathematics Education program at an education faculty of a state university in Central Anatolia. Prospective elementary mathematics teachers are enrolled in the four-year undergraduate teacher education program and they are trained to teach mathematics from 4^{th} grades to 8^{th} grades after their graduation.

Year level

Year level refers to the year in the program prospective teachers attend. Freshmen are 1^{st} year students; sophomores are 2^{nd} year students; juniors are 3^{rd} year students and seniors are 4^{th} year students enrolled in elementary mathematics education program.

CHAPTER 2

LITERATURE REVIEW

The purpose of this study was to investigate prospective elementary mathematics teachers' use of the problem solving strategies while solving mathematical problems. Besides, the study examined prospective elementary mathematics teachers' achievement in problem solving in terms of year levels.

In this chapter, literature review of the present study was presented. Firstly, problem, problem solving and problem solving strategy concepts were defined. Then, approaches to problem solving instruction were explained. Finally, research studies on problem solving were reviewed.

2.1. What is a 'Problem'?

Problems are perceived as exercises that need basic computational skills to solve in mathematics courses. However, problems are not limited with mathematics courses (Heddens & Speer, 2006). On the contrary, problem is everything that gets someone confused, creates challenging situation and makes beliefs uncertain (Dewey, 1933). Moreover, problem is defined as a situation that one faces with some blockage while solving the problem. That is, a task can be a problem if it involves a point that problem solver does not know how to proceed (Kroll & Miller, 1993). Similar definition of problem is a situation that one cannot find any ready solution for it (Henderson & Pingry, 1953).

Whether a situation is a problem or not changes from person to person depending on the individual's reaction to it. More specifically, in order for a situation to be a problem, a person should be aware of the situation and be interested in solving it but s/he should be unable to proceed to find the solution (Lester, 1980). Moreover, a problem for a person today may not be a problem in another day (Henderson & Pingry, 1953). When above definitions are analyzed, there are some common points in order for a situation to be a problem. That is, there should be a challenge, the situation confronted should be new, and the person facing a problem should be perplexed and willing to find a solution to that situation.

According to the literature, problems could be categorized into two; first one is routine problems and the second one is non-routine problems. Routine problems are formed by adding different data to already solved problems and solved by applying a known algorithm step by step without adding new things (Polya, 1957). Routine problems can be solved by using an algorithm and they can be solved in one, two or more steps (Holmes, 1995). Moreover, a routine problem occurs when a problem solver knows the way of finding correct answer and knows that the way is suitable for that problem (Mayer & Hegarity, 1996). Thus, in developing computational skills, solving routine problems plays an important role (Altun, 2002). On the other hand, non-routine problems require organizing given data, classifying, and making relationship in addition to computational skills (Jurdak, 2005). Besides, non-routine problems occur when a problem solver does not know how to solve the problem and the problem solver is not able to see the solution since it is not obvious (Mayer & Hegarity, 1996). Students need to be given the opportunity to solve non-routine problems so that they can learn to apply mathematical concepts beyond the ones they have already learned (NCTM, 2000). Non-routine problems require flexibility in thinking and extension of previous knowledge and involve the discovery of connections among mathematical ideas (Schoenfeld et al., 1999). Moreover, Slavin (2000) claims that students should apply knowledge and skills in problem solving in order to learn mathematics. In this section, the concept of problem was explained briefly and next, problem solving concept will be mentioned.

2.2. What is ‘Problem Solving’?

NCTM (1989) gives a considerable emphasis on the importance of problem solving in mathematics education that, it defines mathematics as problem solving. Problem solving can be generally defined as getting involved in a task for which there is no immediate answer (NCTM, 2000). Another definition for problem solving is making a research to reach a target that is obvious but not easy to reach. If mathematics is problem solving, then problem solving can be defined as eliminating the problem situation by using critical reasoning processes and required knowledge (Altun, 2005). In addition to that, problem solving is not only a method or a strategy to give meaning to a situation but also a kind of thinking that is used to solve non-algorithmic situations (Branca, 1980). Since problem solving includes coordination of knowledge, intuition, and critical thinking, it is not reaching a solution by only applying procedures or rules, but it means far more complex process (Charles et al., 1987).

There are different approaches in teaching mathematical problem solving. The most well known distinction between these approaches is made by Hatfield (1978). According to Hatfield (1978), there are three basic approaches to problem solving instruction: *teaching via problem solving*, *teaching for problem solving*, and *teaching about problem solving*. Later, Schroeder and Lester (1989) reemphasized these three approaches.

In *teaching via problem solving*, mathematics topics are introduced with a problem. That is, problems are vehicles to introduce and study on a mathematical task (Manuel, 1998). In teaching via problem solving, problems are valued as primary means of doing mathematics. In *teaching for problem solving*, students apply the knowledge that is learned in mathematics lessons to solve problems. In other words, mathematics is taught in order to teach problem solving. Students are expected to solve both routine and non-routine problems during the learning of mathematics. In *teaching about problem solving*, the strategies and process of problem solving are taught. The teacher who teaches about problem solving underlines the set of four independent phases that are used to solve problems in Polya's problem solving model. These phases are 'understanding problem', 'devising a problem', 'carrying out the plan' and 'checking solution'. Besides, 'heuristics' or 'strategies' used in devising a plan phase are taught in teaching about problem solving (Schroeder & Lester, 1989).

In this part, approaches to problem solving instruction were reviewed. In the next part, teaching about problem solving approach dealing with strategies and processes of problem solving will be reviewed since the current study is mainly concerned with prospective elementary mathematics teachers' use of problem solving strategies.

2.3. Teaching about Problem Solving

The integration of problem solving and mathematics creates the need to develop problem solving strategies and process. Strategies for solving problems are identifiable methods of approaching a task regardless of specific topic or subject matter. Strategy goals play a role in all phases of problem solving: understanding the problem, solving the problem, reflecting on the answer and solution (van de Walle, 1994).

It can be said that problem solving is viewed as a mathematical process and this process involves several problem solving steps. For instance, Charles, Lester and O' Daffe (1987) state that problem solving involves 3 steps; understanding the problem, solving the problem and finding an answer to the problem. However, According to Polya (1962), there are 4 steps in problem solving process; understanding the problem, making a plan, carrying out the plan and checking the solution. Moreover, Altun (2005) accepted 'extending the problem' as a fifth step in addition to Polya's four step problem solving model.

Dewey's (1933) problem solving model can be regarded as the most comprehensive one when compared to other models. According to him, problem solving involved 7 steps. These steps are named as realizing the problem, understanding the problem, finding alternative solutions, collecting data, evaluating the data, generalizing and finding solution, and applying and evaluating the solution.

Overall, when the studies stated above are taken into consideration, problem solving process includes four common steps: 'understanding the problem', 'making a plan', 'carrying out the plan' and 'checking the solution' and these steps are very similar to that of Polya's (1957). Therefore, in the current study, Polya's four step problem solving model will play a leading role in determining the problem solving strategies used by prospective elementary mathematics teachers in this study.

2.4. Problem Solving Strategies

'Making a plan', the second step of problem solving process, requires the use of problem solving strategies and the primary focus of this study is on identifying participants' problem solving strategies. Hence, the following paragraphs will deal with different kinds of strategies necessary for the solution of mathematical problems.

Hatfield and Bitter (2004) emphasize that problem solving strategies help students make progress in solving more challenging and difficult problems. They also advise teachers to learn and use the strategy during problem solving. Since a problem can be solved in different ways, problem solving strategies play important role in solving process (Bingham, 1998). Finally, besides knowing the problem strategies, knowing how and when to use these strategies is also important (Polya, 1957).

These strategies are logical reasoning, intelligent guessing and testing, extreme cases, accounting all possibilities, adopting a different point of view, visual representation (making drawing), and organizing data (Charles & Lester, 1984). Moreover, solving simpler analogous problem, working backwards, and finding a pattern are other strategies used in problem solving process (Posamentier & Krulik, 1998). These problem solving strategies are given below in details.

2.4.1. Logical reasoning strategy

Logical reasoning is a thinking process and it helps in doing proofs. Without doing algebraic operation, students use their reasoning to find the answer and they do not waste time in doing operations (Charles & Lester, 1984). A mathematical problem that can be solved by using logical reasoning strategy is given below (Posamentier & Krulik, 1998, p.229).

Find all real values for x that satisfy the equation $4-\frac{3}{x}=\sqrt{4-\frac{3}{x}}$

The traditional method begins by squaring both sides that requires careful algebraic manipulation to avoid error. However, this problem can be solved in a much easier manner by using logical reasoning strategy. In the real number system, there are only two numbers whose value equals the value of their square root. These are 0 and 1. Therefore, $4-\frac{3}{x}=1$ or $4-\frac{3}{x}=0$ that is $x=1$ or $x=\frac{3}{4}$. Then checking these answers by substituting into the original equation is necessary.

2.4.2. Intelligent guessing and testing strategy

Intelligent guessing and testing is guessing and trying processes to check the probable conditions (Charles & Lester, 1984). It is particularly useful when it is necessary to limit the values for a variable to make the solution more manageable. In using this strategy, problem solver makes a guess, and then tests it against the conditions of the problem (Posamentier & Krulik, 1998). An interesting mathematical problem that can shed light on the use of intelligent guessing and testing strategy is given below (Posamentier & Krulik, 1998, p.182).

Find all real values of x that satisfy the equations;

$x^2|x|=8$ and $x|x^2|=8$

By using the intelligent guessing and testing strategy, try $x = 2$ and $x = -2$. Thus, the solution of the problem is $x = 2$ since it is the only value of *x* that satisfies both equations.

2.4.3. Considering extreme cases strategy

Extreme cases strategy is trying maximum and minimum conditions by making one variable constant then; problem solver sees the results of each case (Charles & Lester, 1984). The following problem can be solved best by considering extreme cases strategy (Posamentier & Krulik, 1998, p.137).

There are 50 teachers' letterboxes in Georgia Washington High School's general office. One day the letter carrier delivers 151 pieces of mail for the teachers. What is the largest number of letters that any one teacher is guaranteed to get?

This situation can be best assessed by considering extreme cases strategy where the mail is as evenly distributed as possible. Thus, each teacher would receive 3 pieces of mail with the exception of one teacher, who would have to receive the 151st piece of mail. Therefore, 4 pieces of mail is the most any one teacher is *guaranteed* to receive.

2.4.4. Accounting for all possibilities strategy

Accounting for all possibilities refers to considering all conditions or instances to look for the most suitable one. Especially in solving probability problems, it helps students to see all possible events (Charles & Lester, 1984). However, the issue of the accounting for *all* possibilities is crucial in the use of this strategy. If problem solver do not have an organized procedure to account for all possibilities, the strategy often goes wrong (Posamentier & Krulik, 1998). An example problem for accounting for all possibilities strategy is given below (Posamentier & Krulik, 1998, p.188).

If four coins are tossed, what is the probability that at least two heads will be showing?

The list of all possibilities:

HHHH	HHHT	HHTH	HTHH	THHH	HHTT	HTHT	THHT
HTTH	THTH	TTHH	HTTT	THTT	TTHT	TTTH	TTTT

The underlined events are those that have two or more Hs, and satisfy the given condition. There are 11 of these, thus the required probability is $\frac{11}{16}$.

2.4.5. Adapting a different point of view

Adapting a different point of view is thinking of a problem from different perspective (Charles & Lester, 1984). Students are basically prepared to solve problems in a single, straightforward fashion by training that students receive in schools. This leads to a solution, but not always in the most efficient way. Sometimes it is useful for problem solver to adopt a different point of view than that to which he or she was led initially by the problem (Posamentier & Krulik, 1998). Adapting a different point of view strategy can be used in the solution of the following problem (Posamentier & Krulik, 1998, p. 81).

Find the value of $(x + y)$ *if*

$$123x + 321y = 345,$$
$$321x + 123y = 543.$$

When students are confronted with two equations that contain two variables, they automatically revert to the process that has taught as solving them simultaneously (Posamentier & Krulik, 1998). However, this leads to great deal of complicated arithmetic computation. This problem can be solved from another point of view. Since the specific values of *x* and *y* are not interested, we can add the two equations. Then, we get $444x + 444y = 888$, thus $x + y = 2$.

2.4.6. Visual representation (Making a drawing) strategy

Visual representation strategy is drawing figures or geometric shapes to see the related connections in the problem easily (Charles & Lester, 1984). The utilization of this strategy is presented below (Posamentier & Krulik, 1998, p. 144).

Given that $\frac{1}{8}$ *of a number is* $\frac{1}{5}$, *what is* $\frac{5}{8}$ *of that number?*

Divide a whole unit into eight equal pieces:

Now, each of these eights must be $\frac{1}{5}$.

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

Since, each of these eights equals $\frac{1}{5}$, $\frac{5}{8}$ would be five of them, or $5\left(\frac{1}{5}\right) = 1$

2.4.7. Organizing data strategy

Organizing data is making a list of given data to make clearer (Charles & Lester, 1984). In order to get rid of the complexity of a problem, problem solver could rearrange the data given in the problem in a way that will enable him to solve the problem more easily (Posamentier & Krulik, 1998). An example problem for this strategy is given below (Posamentier & Krulik, 1998, p. 221).

Find the value of the expression;

$20 - 19 + 18 - 17 + 16 - 15 + 14 - 13 + 12 - 11$

We can group the numbers in pairs as follows:

$(20 - 19) + (18 - 17) + (16 - 15) + (14 - 13) + (12 - 11)$

$= 1 + 1 + 1 + 1 + 1 = 5$.

2.4.8. Working backwards strategy

Problem solver begins to *work backwards* when the goal is unique but there are many possible starting points (Posamentier & Krulik, 1998). To reach starting point of the problem, problem solver starts from the end point of the problem and proceeds backwards step by step then problem solver reach to the starting point of the problem (Larson, 1983). A mathematical problem that lends itself to the use of working backward strategy is presented below (Posamentier & Krulik, 1998, p. 17).

The sum of two numbers is 12, and the product of the same two numbers is 4. Find the sum of the reciprocals of the two numbers.

Let's start from the end of the problem that is what we wish to find, $\frac{1}{x}+\frac{1}{y}$. If we compute the sum in the usual way, we obtain $\frac{x+y}{xy}$. Since $x+y$ given as 12 and xy given as 4, the fraction becomes $\frac{12}{4}=3$ which is the answer of the problem.

If we had started to solve the problem by generating two equations $x+y=12$ and $xy=4$ where x and y represented the two numbers, we would have found the values of x and y then found the reciprocals of these two numbers and finally their sum. However, solving this problem in this manner is rather complicated solution process and can be made much simpler by starting from the end of the problem.

2.4.9. Finding a pattern (Looking for a pattern) strategy

Finding a pattern includes determining a pattern or extending it to discover the answer to the question. A pattern is a systematic and predictable repetition of numeric, visual or behavioral data (Posamentier & Krulik, 1998). A well known mathematical problem and its solution by using finding a pattern strategy are given below.

How fast rabbits could breed in ideal circumstances. Suppose a newly-born pair of rabbits, one male, one female, are put in a field. Rabbits are able to mate at the age of one month so that at the end of its second month a female can produce another pair of rabbits. Suppose that our rabbits never die and that the female always produces one new pair (one male, one female) every month from the second month on. How many pairs will there be in one year?

First month, there is one pair of rabbits; in the second month, after one month, the two rabbits have mated but have not given birth. Therefore, there is still only one pair of rabbits. In the third month, the first pair of rabbits gives birth to another pair, making two pairs in all. In the fourth month, the original pair gives birth again, and the second pair mate, but do not give birth. This makes three pairs. In the fifth month, the original pair gives birth, and the pair born in month 3 gives birth. The pair born in month 4 mate, but do not give birth. This makes two new pairs, for a total of five pairs. In the sixth month, every pair that was alive two months ago gives birth.

This makes three new pairs, for a total of eight. And so on. The total numbers of pairs for each month are; 1, 1, 2, 3, 5, 8… respectively show a pattern; namely each number is equal to the sum of the previous two numbers. Using this pattern, we could work our way up to the one year (i.e., 12 months) as 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144… Therefore, at the end of one year there will be 144 pairs of rabbits.

Until now, different problem solving strategies (Altun, 2005; Charles, Lester and O' Daffe 1987; Dewey, 1933; Polya, 1962) have been explained briefly. In the following part, research studies related with mathematical problem solving strategies mentioned above will be reviewed.

2.5. Research Studies on Problem Solving

In this part, research studies concerning problem solving were reviewed. First, studies conducted in Turkey; later, studies conducted in other countries were presented.

2.5.1. Research Studies Conducted in Turkey

Problem solving achievement is a prominent variable and many studies have been conducted to investigate the effect of it on many variables. Moreover, there were studies in the literature related to heuristics methods or steps of Polya. For instance, Çalışkan (2007) carried out an experimental study to investigate effects of problem solving strategies on achievement, applicability of strategy, and problem solving performance of 77 prospective physics teachers. Prospective teachers in experimental group were taught by Polya's problem solving strategies. The findings showed that teaching of problem solving strategy had positive effects on participants' problem solving performance and achievement in physics. Moreover, findings revealed that there was a positive correlation between achievement and strategy application.

In another experimental study, Yıldız (2008) investigated the change in 6th grade students' problem solving abilities after mathematics instruction based on

Polya's problem solving steps. Fifty three students from an elementary school in Istanbul participated in the study. Since there was no control group, it was a weak experimental study. All students participated in the study were instructed according to Polya's problem solving steps. It was found that instruction based on Polya's steps significantly affected students' problem solving abilities in a positive way. Besides, students' attitudes towards problem solving were changed in a positive way. Finally, students' attitudes towards mathematics were enhanced by the instruction based on Polya's problem solving steps. Researcher concluded that Polya's problem solving steps and problem solving strategies in problem solving was not difficult to apply into problem solving. Therefore, he suggested teachers to first teach those steps and strategies and then develop proper activities or problem solving cards to make students solve problems with strategies by the help of Polya's problem solving steps.

In a survey study, Töre (2007) investigated sixth grade students' knowing and applying level of problem solving process by the help of Polya's problem solving steps. The sample of the study was 30 sixth grade students from both private and public schools. Observations, interviews and problem sheets were used to measure students' level of learning and applying skills. Students were asked individually how they solved problems and which steps they applied for problem solution in the interviews. Although students in public schools explored making a plan for problem solving process as an obligation for a correct solution, in application it was seen that most of the students did not use the steps and strategies in their sheets. As a result, 50% of the students in urban public schools solved the problem correctly. However, 20% of the students who did not make a plan or did operational mistakes did not solve the problems completely. The other finding revealed that 30% of the students checked the solution. The students who realized mistakes in checking process could make some corrections. The reason why the most students made mistakes in problem solving process was that most students did not pay attention to Polya's first step of reading and understanding of a problem. The last finding addressed that when students wrote a similar problem, they did not use the creativity for posing a problem. The study suggested that when problem solving process was internalized, most students solved problem correctly.

Some studies have also been conducted to examine the relationship of various variables with problem solving achievement. For instance, Karaoğlan (2009) conducted a correlational study to examine the relationship between 6^{th} grade students' problem solving achievement scores after completing instruction on problem solving and their mathematics achievement obtained throughout the semester. In addition, the researcher investigated the relationship between 6^{th} grade students' problem solving achievement scores after completing instruction on problem solving and their actual mathematics scores obtained from Level Determination Exam (SBS). Sample of the study consisted of 170 sixth grade students from a private school in Istanbul. In the study, Problem Solving Achievement Tests (PSATs), Mathematics Achievement Tests (MATs) and SBS exam were used as data collection tools. Quantitative methods were utilized to examine the research questions and the results of the statistical analysis showed that there was a significant positive correlation between students' problem solving achievement scores after completing instruction on problem solving and their mathematics achievement mean scores obtained throughout the semester. In addition, the findings of the analysis showed that there was a significant large positive correlation between the problem solving achievement scores after completing instruction on problem solving and students' actual mathematics scores obtained from SBS. Thus, the researcher suggested that great importance should be given to problem solving instruction as it was mentioned in the new curriculum. In order to be successful in exams like SBS, problem based instruction would be necessary. Moreover, teachers should avoid traditional methods which students would solve hundreds of questions and memorize the solution way of various particular questions.

There were also studies in the literature which investigated the usage of problem solving strategies by prospective teachers and by elementary students. For instance, Altun, Memnun and Yazgan (2007) examined 120 prospective primary teachers' problem solving strategies. Problem solving strategies considered in this study were ‘making a systematic list’, ‘guess and check’, ‘drawing a diagram’, ‘looking for a pattern’, ‘looking for a pattern’, ‘working backwards’, ‘simplifying the problem’ ‘writing an equation’ and ‘reasoning’. Pre-test results showed that, prospective teachers most frequently used ‘writing an equation’ and ‘drawing a diagram’ but less frequently used ‘looking for a pattern’ and ‘guess and check’. ‘Looking for a pattern’ was also reported as the least frequent strategy used by elementary students in the studies of Arslan and Altun (2005) and Yazgan (2007). The post-test results of the study showed that there was a significant decrease in use of ‘writing an equation’ and increase in use of all problem solving strategies except for ‘reasoning’. They stated that further research with large groups concerning prospective teachers was needed.

Later, Altun and Memnun (2008) conducted an experimental study to examine 61 prospective elementary mathematics teachers’ problem solving strategies. Experimental and control groups were formed and a problem solving strategy instruction was given to experimental group. In addition to problem solving strategies considered Altun, Memnun and Yazgan’s (2007) study, ‘making a table’ was also included in this study. Pre-test results showed that ‘writing an equation’ and ‘drawing a diagram’ were the most frequent strategies used by prospective teachers. This result was in line with that of Altun, Memnun and Yazgan’s (2007) study. However, ‘looking for a pattern’, ‘making a table’, ‘reasoning’ and simplify the problem’ were the less frequent strategies used by prospective teachers. The post-test results of the study showed that there was a significant decrease in use of ‘writing an equation’ and increase in use of all problem solving strategies considered in this study. This result was also in line with Altun, Memnun and Yazgan’s (2007) study. Thus, pre and post test results showed that prospective teachers developed their problem solving skills and they used more strategies after problem solving instruction (Altun & Memnun, 2008; Altun, Memnun & Yazgan, 2007). Researchers suggested that prospective teachers should adopt and support the educational reform studies congruent with the content of the present study.

Recently, Duru, Peker, Bozkurt Akgün and Bayrakdar (2011) conducted a survey study to investigate prospective primary school teachers' problem solving skills and their preferences of problem solving strategies in solving mathematical word problems. Participants of the study were 200 prospective primary school teachers enrolled in teacher education programs in three universities. As a data collection instrument, researchers developed a problem solving test consisting open ended word problems which could be solved by using at least two problem solving strategies. In solving problems, prospective teachers were free to use any strategies that they would like to use. Strategies used by prospective teachers in solving the problems were identified and categorized and the data were analyzed by using descriptive statistics. The results showed that prospective primary school teachers were able to use various strategies, such as algebraic strategy, arithmetic strategies, use a model for solving of the word problems.

When studies considered in the literature, it was seen that there were also studies investigating the elementary students' usage of problem solving strategies. For example, Israel (2003) conducted a survey to investigate 8^{th} grade students' problem solving strategies in terms of achievement levels, socioeconomic status, and gender. The results showed that students' with high achievement levels used problem solving strategies more efficiently, whereas the ones with low achievement levels used strategies that did not contribute to the solution of the problem. In addition, students with low socioeconomic status preferred to use incorrect strategies more, on the other hand, students with average and high socioeconomic status preferred to use convenient strategies needed for the solution of problems. Finally, the study revealed that boys and girls used similar strategies.

There were also experimental studies concerning elementary students' use of problem solving strategies. For instance, Yazgan and Bintaş (2005) used an experimental design to examine 4^{th} and 5^{th} grade students' learning and using of problem solving strategies. In this study, simplifying the problem, guess and check, looking for a pattern, making a drawing, making a systematic list, and working backwards were considered as problem solving strategies. In this study while control group students continued to follow the regular mathematics curriculum, experimental group attended to problem solving lessons. Eighteen problem solving lessons were given to the students and in the first twelve lessons six problem solving strategies were explained to the students. In the remaining six lessons, students worked on problems that can be solved by using different problem solving strategies. Pre-test results showed that, some of problem solving strategies could not be used by elementary students. In more details, guess and check and making a systematic list were not used by 4^{th} grade students and making a drawing was not used by 5^{th} grade students. The post-test results showed that, students' usage of all problem solving strategies was increased and the differences were significant for strategies simplifying the problem, working backwards, and making a drawing for both grade levels and making a systematic list for only 5^{th} grade students. The researchers suggested that in order to increase students' performance, non-routine problems should be emphasized more both in textbooks and elementary mathematics programs.

Similarly, Arslan and Altun (2007) aimed to investigate whether problem solving strategies could be learnt by 7^{th} and 8^{th} grade students. In this study, simplifying the problem, guess and check, looking for a pattern, making a drawing, making a systematic list and working backwards were considered as problem solving strategies. Students were assigned to experimental or control group. While control group students continued to follow the regular mathematics curriculum, experimental group attended to problem solving lessons. Seventeen problem solving lessons were designed by researchers to teach mathematical problem solving strategies. After an introduction and explanation of the concept of problem solving in the first lesson, systematic acquisition of Polya's problem solving process took place in the following six lessons. These six lessons were devoted to a specific problem solving strategy and students worked on a problem to learn how to use that strategy. In order to determine student's problem solving strategies, three parallel problem solving tests including ten items were administered as pre-test, post-test, and retention. The results of the study showed that 7^{th} grade students were able to use all problem solving strategies except for looking for a pattern and 8^{th} grade students could use all strategies except for looking for a pattern and working backwards. Thus, Arslan and Altun (2007) concluded that looking for a pattern could not be used by elementary students in both grade levels. After the treatment, students' usage of all problem solving strategies increased except for guess and check in 7^{th} grade. In more details, researchers found that there were significant differences between pre-test and post-test results for both grade levels regarding the strategy of simplifying the problem in addition to working backwards and looking for a pattern strategies which could not be used in the pre-test by elementary students. The results of these two studies (Arslan &Altun, 2007; Yazgan & Bintaş, 2005) have similarities. For instance, in both studies there were significant difference between pre-test and post-test results regarding the use of simplifying the problem and working backwards strategies by elementary students. Arslan and Altun (2007) stated that the content and objectives of the elementary mathematics program should be revised by taking into consideration non-routine problems and the acquisition of the problem solving process and strategies regarding the age and competence of the children. They also added that the learning environment should be designed better by taking into account the progress of social interaction based on small and whole group discussions.

In another experimental study, Sulak (2010) investigated second grade primary school students' problem solving strategies during 14 weeks. The experiment group has been trained about problem solving strategies while the control group continued with traditional problem solving practices. The data of the study were obtained from the two written problem solving tests including open-ended problems. Moreover, qualitative interviews were conducted to provide explanation of students' solutions and strategies. The results of the study showed that experimental group students were significantly successful in use of strategies; 'making a drawing-diagram', 'making a table', 'writing mathematical sentences', 'looking for pattern', 'making a list', 'using logical reasoning' and 'guess-check' strategies than control group students. The researcher recommended that students should be provided the opportunity to learn problem solving strategies in mathematics courses since strategies would have significant impact on problem solving achievement. Moreover, she concluded that teachers should be patient since the acquisition of problem solving strategies would take some time for second grade students.

Unlike previous studies, Yazgan (2007) conducted a qualitative study to examine 4^{th} and 5^{th} grade students' ability to use problem solving strategies. The results of the study pointed out that students easily used 'guess and check', 'working backwards', 'making a drawing' and 'making a systematic list' strategies; however, students faced with difficulty when using 'simplifying the problem' strategy in addition to 'looking for a pattern' that could not be used by 7^{th} and 8^{th} grade students in Arslan and Altun's (2005) study. Yazgan (2007) recommended that authors should give more weight to non-routine problems and to solution strategies when writing textbooks. Moreover, she suggested that teachers should use different sources in teaching problem solving strategies in addition to textbooks.

2.5.2. Research Studies Conducted in Other Countries

International research studies regarding problem solving strategies similarly focused more on elementary students (e.g., Charles & Lester, 1984; Ishida, 2002; Lee, 1982;), however; few studies were conducted with prospective teachers (e.g., Capraro, An, Ma, Rengel-Chavez & Harbaug, 2011). The studies conducted in other countries will be reviewed in two main parts. First, studies concerning prospective teachers' usage of problem solving strategies will be presented then elementary students'.

In a qualitative study, Capraro, An, Ma, Rangel-Chavez and Harbaugh (2011) aimed to illuminate the types of strategies prospective teachers valued most in solving an open-ended problems and how they would explain their solutions to middle school students. The participants were junior level students who were enrolled in the Middle School Problem Solving course. The participants were administered an open-ended triangle task which had four unique solutions. A semi-structured interview was conducted with prospective teachers after completing the task. The results showed that each participant in some way or another used a guess and check strategy. Most participants solved the problem by starting from random combinations of numbers and some were able to locate patterns throughout the process. Moreover, prospective teachers were likely to use a random guess and check strategy when working with middle school students rather than a more systematic approach. Despite being employed as a primary strategy, there existed misapplications of guess and check as a systematic problem solving activity. Some of the participants treated this strategy as ‘random guess and try’. The misuse of this strategy was explained as one of the key reasons that none of the participants obtained all four possible solutions. The guessing based on randomly trying each number into each blank not only was a time-consuming process but also was a mentally energy-consuming process. Although participants suggested some methods in teaching and explaining this problem for students such as using manipulatives, technologies, and making connection with real life context, they failed to provide effective thinking strategies that could clearly allow students to grasp the key idea of the problem. By implementing this strategy incorrectly and incompletely, prospective teachers might be less likely to help their future students become aware of efficient strategies in solving open-ended problems. Finally, the researchers suggested that in order to prepare prospective teachers to effectively teach problem solving in mathematics, teacher educators should pay more attention to the mathematical proficiency of prospective teachers, particularly to their ability to solve problems and explain their solutions and reasoning.

In another qualitative study, Ishida (2002) aimed to explore elementary 6^{th} grade students’ strategy preferences in solving mathematical problems. Moreover, the students were asked to explain the best strategy for the two word problems and to explain whether their solution strategies could be improved. Subjects were twelve 6th grade students who have been taught problem solving strategies for four years. During this period, the students have learned several strategies: ‘guess and check’, ‘draw a diagram’, ‘make a table’, ‘find a pattern’, ‘make on organized list’, ‘solve a simpler problem’ and ‘working backwards’. The interviews conducted were audio taped and students’ works were collected. Data were analyzed based on the protocol and students’ answer sheets. The results revealed that all students were able to solve both problems correctly and most of them had more than one solution strategy for each problem. ‘Make a table’ was the strategy most frequently selected as best for the first problem, whereas ‘finding a pattern’ was selected for the second problem. The common reasons that students gave for selecting the strategy were that the method enabled them to get an answer quickly or efficiently, that it was easy to use, and that it was easy to understand. Students mostly did not state whether their solution strategies could be improved by using a different solution strategy. Even those students who selected the ‘make a table’ or ‘find a pattern’ strategy from the viewpoint of efficiency were not aware that their method could be improved. This showed that students were not aware of how to improve their chosen strategies to increase their efficiency, generality, and simplicity. The research results suggested that students should gain a better mathematical problem solving behavior. Moreover, to improve students’ problem solving ability, they needed to learn the value of improving a solution method from a mathematical point of view, and also how to do so.

In an experimental research study, Lee (1982) investigated whether 4^{th} graders can acquire specific heuristics and use them appropriately, and effectively to become better problem solvers. There were sixteen 4^{th} grade students for the study and the students were randomly assigned into two groups of 8 students each, one experimental group and one control group. While the experimental group had 20 problem solving sessions of 45 minutes each over 9 weeks, the control group attended their regular classes. The specific phases used adapted from Polya (1957) were; ‘understanding the problem’, ‘making a plan’, ‘carrying out the plan’ and ‘looking back’. Students’ problem solving strategies identified in ‘making a plan’ phase were; ‘drawing a picture’, ‘making a chart or table’, ‘considering special cases’ and ‘looking for a pattern’, ‘considering one condition and combining the second condition’ and ‘considering a similar problem solved before’. After the treatment, experimental group students were able to select an appropriate strategy and use them effectively in most cases. However, ‘considering one condition and then adding the second condition', 'considering special cases' and 'looking for a pattern' strategies were the most difficult ones for students to apply. Post-test results revealed that, experimental group students were able to solve 73% of the problems successfully whereas; control group students could solve only 6% the problems. Despite this, control group students were able to use some of the strategies considered in this study.

In another experimental study, Charles and Lester (1984) developed the Mathematical Problem Solving (MSP) program in order to compare the problem solving performance of students who participated in the MPS program to that of students in control group. This program has promoted the learning of problem-solving strategies and emphasized solving problems. In addition, it focused on each phase of Polya's (1957) four-phase model of problem solving and emphasized the development of students' abilities to select and use a variety of strategies. Twelve fifth-grade and 10 seventh-grade teachers implemented the MPS program for 23 weeks. Eleven fifth-grade and 13 seventh-grade teachers taught control classes. During the implementation of MPS, problems that could be solved by using one or more strategies were administered to students. These strategies were: 'guess and check', 'draw a picture', 'make an organized list', 'make a table', 'look for a pattern', 'work backwards' and 'use logical reasoning'. The results of the study showed that the experimental classes scored significantly higher than the control classes on measures of ability to understand problems, plan solution strategies, and get correct results. The findings across grade levels were very consistent. That is, the findings at grade 5 were generally held for grade 7 as well. This observation suggested that the effectiveness of the MPS might not be unique to a single grade level. Researchers stated that this study represented only a small step toward the development of a useful body of information about how to provide effective problem-solving instruction and anticipated that in the near future several more steps would be taken in the direction of this important goal.

2.6. Summary of Literature Review

In this chapter, literature review of the current study was presented. First, problem, problem solving and problem solving strategy concepts were defined. Then, approaches to problem solving instruction were explained. Finally, research studies on problem solving were reviewed through the studies conducted in Turkey and in other countries.

Problem solving has been a prominent concept in mathematics education and many studies have been conducted to investigate the effect of either problem solving strategies or problem solving steps on problem solving achievement. The results of these studies revealed that teaching problem solving strategies or problem solving steps had a positive effect on students' problem solving achievement (e.g., Çalışkan, 2007; Yıldız, 2008). Moreover, problem solving strategy instruction increased elementary students' (e.g., Arslan & Altun, 2007; Charles & Lester 1984; Lee, 1982; Sulak, 2010; Yazgan & Bintaş, 2005) and prospective teachers' (e.g., Altun, Memnun &Yazgan 2007; Altun & Memnun 2008) use of different problem solving strategies.

Several studies examined elementary students' and prospective teachers' problem solving strategy preferences in solving mathematical problems (e.g., Capraro, An, Ma, Rengel-Chavez & Harbaug, 2011; Duru, Peker, Bozkurt Akgün and Bayrakdar, 2011; Ishida, 2002; Israel, 2003; Yazgan, 2007). The results of these studies showed that elementary students and prospective teachers preferred to use several problem solving strategies such as guess and check, making a drawing, making a systematic list, and working backwards. Therefore, it seemed that students and prospective teachers did not depend on one or two predominant strategies in solving problems.

Overall, problem solving research literature showed that a large body of research was conducted to investigate elementary students' problem solving strategies (e.g., Arslan & Altun, 2007; Charles & Lester, 1984; Ishida, 2002; Israel 2003; Lee, 1982; Sulak, 2010; Yazgan & Bintaş 2005; Yazgan, 2007) where only few studies focused on prospective teachers' use of problem solving strategies. Especially, in Turkey, studies regarding prospective teachers' problem solving strategies are rather limited (e.g., Altun & Memnun, 2008; Altun, Memnun &Yazgan, 2007). This study attempted to examine prospective elementary mathematics teachers' use of the problem solving strategies while solving mathematical problems before their graduation from the teacher education program in order to provide insights for both policy makers and mathematics educators. Besides, the study examined prospective elementary mathematics teachers' achievement in problem solving in terms of year levels.

CHAPTER 3

METHOD

The main purpose of this study was to investigate prospective elementary mathematics teachers' use of strategies in mathematical problem solving. This study also examined prospective elementary mathematics teachers' problem solving achievement in terms of their year level in the teacher education program.

This chapter explained the research design and the procedures used in the study in eight main parts. In the first two parts, overall research design and the sample of the study were explained respectively. In the third part, the test construction process was explained and detailed information about the test items was given. In the fourth and fifth part, the data collection procedure and data analysis procedure were explained respectively. Finally, reliability and validity issues were given in the sixth part and threats to internal and external validity were explained in the seventh and eighth parts respectively.

3.1. Research Design

The main purpose of this study was to investigate prospective elementary mathematics teachers' use of strategies in mathematical problem solving. In cross sectional surveys, data are collected from a sample at just one point in time (Fraenkel & Wallen, 2005). In the current study, data regarding prospective elementary mathematics teachers' problem solving strategies were gathered one point in time through Problem Solving Test (PST), therefore the design of the study could be considered as a cross sectional survey. A summary of overall research design is presented in Table 3.1 given below.

Table 3.1. Overall Research Design of the Study

Research design	Cross-sectional survey
Sampling	Convenience sampling
Instrument	Problem Solving Test
Data collection procedure	Direct administration of the PST to 250 prospective teachers at a university in their classroom setting within 40 minutes
Data analysis procedure	Descriptive statistics and item based in-depth analysis

3.2. Population and Sample

The target population of the study was all prospective elementary mathematics teachers in Central Anatolia Region and accessible population was all prospective elementary mathematics teachers in a city of this region. As it would be difficult to reach all prospective elementary mathematics teachers in Central Anatolia, convenient sampling method was preferred. Prospective elementary mathematics teachers studying at a state university in Central Anatolia at all year levels of the Elementary Mathematics Education (EME) program constituted the sample of the study. The distribution of participants' demographic information with respect to year level and gender is given in Table 3.2.

Table 3.2. Distribution of Gender with respect to Year Levels

	Freshmen	Sophomores	Juniors	Seniors	Total
Male	13 (5.2%)	27 (10.8%)	17 (6.8%)	18 (7.2%)	75 (30%)
Female	58 (23.2%)	39 (15.6%)	44 (17.6%)	34 (13.6%)	175 (70%)
Total	71 (28.4%)	66 (26.4%)	61 (24.4%)	52 (20.8%)	250 (100%)

It can be understood from the table that, 30% of all participants were males and 70% were females. Moreover, the distribution of males and females changed in different year levels. Table 3.3 presents the courses related to mathematics and pedagogy that were offered by the EME program at a state university (Turkish Council of Higher Education, 2011).

Table 3.3. Courses Taken by the Prospective Elementary Mathematics Teachers

	Fall Semester	Spring Semester
First Year	General Mathematics	Geometry
	Introduction to Education	Discrete Mathematics
		Educational Psychology
Second Year	Linear Algebra I	Linear Algebra II
	Calculus I	Calculus II
	Scientific Research Methods	Inst. Tech. and Material Development
	Teaching Methods and Principles	
Third Year	Introduction to Algebra	Differential Equations
	Statistics and Probability I	Statistics and Probability II
	Analytic Geometry I	Analytic Geometry I
	Calculus III	Measurement and Evaluation
	Special Teaching Methods I	Special Teaching Methods II
Fourth Year	Elementary Number Theory	Practice Teaching
	Counseling	Turkish Edu. Syst. and School Manage.
	School Experience	
	Classroom Management	
	Special Education	

As it can be seen from Table 3.3, EME program required freshmen to take basic mathematics and pedagogy courses. In the second year of the program, prospective teachers took approximately equal number of mathematics and pedagogy courses; whereas, the number of third year mathematics courses were far more than the number of pedagogy courses. In addition, at the end of the third year, prospective teachers completed all required mathematics courses except for one course. Special Teaching Methods I and II courses were also taken in the third year of the program. The fourth year courses were all pedagogy-related courses and there was very little emphasis on mathematics courses. To sum up, the number of mathematics courses were more in the first three years; however, the fourth year courses were mainly related to pedagogy. Data was gathered from prospective elementary mathematics teachers at all year levels of the EME program at the end of the fall semester.

3.3. Data Collection Instrument

In order to determine problem solving strategies that were used by prospective elementary mathematics teachers in solving mathematical problems, a Problem Solving Test (PST) was implemented. PST items were adapted from the book "*Problem Solving Strategies for Efficient and Elegant Solutions: A Resource for the Mathematics Teachers*" (Posamentier & Krulik, 1998). The following problem solving strategies were examined in detail in this book: (1) working backwards (2) finding a pattern, (3) adopting a different point of view, (4) solving a simpler, analogous problem, (5) considering extreme cases, (6) making a drawing, (7) intelligent guessing and testing, (8) accounting for all possibilities, (9) organizing data and (10) logical reasoning. Each strategy and their application to everyday problem situations were described and then examples were presented in the book.

Posamentier and Krulik (1998) stated that the strategies selected in the book were not the only ones available, but they represented those most applicable to mathematics instruction in the schools. Moreover they emphasized that, it was rare that a problem could be solved using all 10 strategies and it was equally rare that only a single strategy could be used to solve a given problem. Rather, a combination of strategies would most likely occur when solving a problem. They advised to become familiar with all the strategies and to develop proficiency in using them when appropriate. Thus, in the selection of the problems for the study, it was considered that, problems could be solved by using either one or more than one problem solving strategy. For that reason, each strategy in the book was covered and problems were examined. Problems which were considered best suitable for the usage of specific problem solving strategy were included in the PST. Table 3.4 presents the PST items and problem solving strategies which were suggested to be used to solve those items by Posamentier and Krulik (1998).

Table 3.4. Problem Solving Test Items and Problem Solving Strategies

Problem Solving Strategies	Selected Items for PST											Total Number of PSS
	1	2	3	4	5	6	7	8	9	10	11	
Working backwards					X							1
Finding a pattern				X					X	X	X	4
Adopting a different point of view			X	X						X		3
Solving Simpler Analogous Problem				X								1
Considering Extreme Cases								X				1
Making Drawing				X		X	X					3
Intelligent Guessing And Testing		X										1
Accounting For All Possibilities	X			X								2
Organizing Data	X			X					X	X		4
Logical Reasoning	X											1

Posamentier and Krulik (1998) suggested one or more solutions for each problem. For example, they suggested using a combination of accounting for all possibilities, organizing data, and logical reasoning strategies to solve Item 1. In order to solve Item 2 and Item 3, using intelligent guessing and testing, and adopting a different point of view were recommended respectively.

In order to solve Item 4, Posamentier and Krulik (1998) suggested seven different solution methods including making a drawing, accounting for all possibilities, adopting a different point of view, finding pattern, and organizing data strategies separately. Sixth possible solution was combining solving simpler analogous problem, making drawing, organizing data, and finding a pattern strategy. Last one was applying the combination formula without using a problem solving strategy defined in the book.

Item 5 and Item 8 could be solved by using working backwards and considering extreme cases respectively and making a drawing could be used in the solutions of both Item 6 and Item7. Finally, Item 9 could be solved by using a combination of finding a pattern and organizing data strategies. The selected items for the PST were either translated or adapted to Turkish, as all selected participants' native language was Turkish. The adapted or translated versions of items were presented in the following section.

3.3.1. Translation and Adaptation of the Items

Some items of the problem solving test were translated and some were adapted into Turkish by the researcher. Then, it was edited on clarity and grammar by an expert of Turkish language and literature. Next, the Turkish version of the problem solving test was given to four doctoral students having mathematics background to evaluate the translated items and problems in terms of content and clarity. According to these criticisms, the problem solving test was revised and necessary changes were made on the unclear instructions and mathematical vocabulary. After the translation and adaptation processes, the first draft of the problem solving test was given to two mathematics educators working in the Department of Elementary Education at METU to evaluate validity and clarity of the instrument. Necessary revisions were made on the instrument based on the feedbacks. Table 3.5 represents the English and Turkish version of translated items (Items 1, 2, 3, 4, 6, 8, 9, 10 and 11).

Table 3.5. Translated Problem Solving Test Items

Item	English Version of Test Items	Turkish Version of Test Items
1	If a and b are both integers, how many ordered pairs *(a, b)* will satisfy the equation $a^2+b^2=10$?	$a\ ve\ b$ tam sayı olmak üzere, $a^2+b^2=10$ denklemini sağlayan kaç farklı (a,b) sıralı ikilisi vardır?
2	The sum of an integer, its square, and its square root is 276. What is the integer?	Bir sayının kendisinin, karesinin ve karekökünün toplamı 276 olduğuna göre bu sayı kaçtır?
3	What is the greatest value of the expression $ab+bc+cd+ad$, if a, b, c and d have values 1, 2, 3, and 4, but not necessarily in that order?	Birbirinden farklı a,b,c,d sayılarının her biri 1, 2, 3, 4 değerlerinden herhangi birisini almak koşuluyla $ab+bc+cd+ad$ ifadesinin alabileceği <u>en büyük</u> değer kaçtır?
4	In a room with 10 people, everyone shakes hands with everybody else exactly once. How many handshakes are there?	10 kişinin bulunduğu bir odada, her bir kişi diğer tüm kişilerle yalnız bir kez el sıkışırsa, toplam kaç kez el sıkışması olur?
6	Mr. Lohengrin saw a row of swans on a lake. In front of two swans, there were two swans. Behind two swans there were two swans, and between two swans there were exactly two swans. What is the minimum number of swans Mr. Lohengrin could have seen?”	Ahmet gölde tek sıra halinde kuğu topluluğu görmektedir. Ahmet herhangi iki kuğunun önünde iki kuğu olduğunu ayrıca herhangi iki kuğunun arkasında da iki kuğu olduğunu söylemektedir. Son olarak da iki kuğunun arasında da iki kuğu olduğunu söylemektedir. Ahmet gölde <u>en az</u> kaç kuğu görmektedir?
8	In a drawer, there are 8 blue socks, 6 green socks, and 12 black socks. What is the smallest number that must be taken from the drawer without looking at the socks to be certain of having 2 socks of the same color?	Bir çekmecede 8 mavi, 6 yeşil ve 12 siyah çorap bulunmaktadır. <u>Çoraplara bakmamak şartıyla</u> çekmeceden <u>en az</u> kaç çorap alınırsa <u>aynı renkte</u> en az 2 çorap elde edilmiş olur?
9	What is the sum of $1^3+2^3+3^3+4^3+\ldots+9^3+10^3$?	$1^3+2^3+3^3+4^3+\cdots+9^3+10^3$ toplama işleminin sonucu kaçtır?
10	Find the numerical value of the expression $\left(1-\frac{1}{4}\right).\left(1-\frac{1}{9}\right).\left(1-\frac{1}{16}\right)\ldots\left(1-\frac{1}{225}\right)$	$\left(1-\frac{1}{4}\right).\left(1-\frac{1}{9}\right).\left(1-\frac{1}{16}\right)\ldots\left(1-\frac{1}{225}\right)$ çarpma işleminin sonucu kaçtır?
11	Find the units digit for the sum $13^{25}+4^{81}+5^{411}$	$13^{25}+4^{81}+5^{411}$ toplamının birler basamağındaki rakamı kaçtır?

Item 5 and Item 7 were adapted into Turkish, since direct translation of these items would not be in the cultural context for the prospective teachers. The items were adapted in the way they were commonly used in Turkish mathematics books. Table 3.6 below presents the adapted problem solving test items (Items 5 and 7).

Table 3.6. Adapted Problem Solving Test Items

Item	English Version of Test Items	Turkish Version of Test Items
5	Nancy breeds New Zealand rabbits for a hobby. During April, the number of rabbits increased by 10%. In May, 10 new rabbits were born, and at the end of May, Nancy sold one third of her flock. During June, 20 new rabbits were born, and at the end of June, Nancy sold one half her total flock. So far in July, 5 rabbits have been born, and Nancy now has 55 rabbits. How many rabbits did Nancy start with on April 1st?	Babası Ayşe'ye Nisan ayının başında belli sayıda tavşan almıştır. Ayşe'nin tavşanlarının sayısı Nisan ayının sonunda %10 artmıştır. Mayıs ayında 10 tavşan doğmuştur ve Mayıs ayının sonunda Ayşe, tavşanlarının $\frac{1}{3}$'ini satmıştır. Haziran ayında 20 tavşan daha doğmuştur ve Haziran ayının sonunda Ayşe, tavşanlarının yarısını satmıştır. Temmuz ayında 5 tavşan daha doğunca Ayşe'nin toplam 55 tavşanı olmuştur. Buna göre, babası Ayşe'ye Nisan ayının başında kaç tavşan almıştır?
7	A local pet owner just bought her holiday supply of baby chickens and baby rabbits. She does not really remember how many of each she bought, but she has a system. She knows that she bought a total number of 22 animals, a number exactly equal to her age. Furthermore, she also recalls that the animals had a total of 56 legs, her mother's age. How many chickens and how many rabbits did she buy?	Canan'ın bahçesinde tavşanları ve tavukları vardır. Canan bahçesindeki toplam tavşan ve tavuk sayısının 22 olduğunu söylemektedir. Tavşan ve tavukların toplam ayak sayılarının 56 olduğunu belirten Canan'ın bahçesinde kaç tane tavşanı ve kaç tane tavuğu bulunmaktadır?

3.3.2. Pilot Study

Pilot testing is important in survey studies to establish the construct validity of the instrument, which means whether the items measure the construct they are intended to measure, and to ensure that the instructions, questions, format, and scale items are clear (Creswell, 2003). In the present study, one pilot testing was put into practice. In order to be similar and representative to the potential respondents, the sample of pilot study was chosen as prospective elementary mathematics teachers from another university in Central Anatolia. Eleven problem solving test items were administered to 77 freshman and sophomore prospective elementary mathematics teachers studying at Aksaray University. The instrument was directly administered to the participants during their geometry and calculus lessons with the permission of their instructors and it was indicated that their participation was voluntary. The implementation took nearly one hour. Since one hour was not sufficient to solve the problem solving test, it was decided to exclude some items from the test. Some item wordings were changed in order to make items more understandable. Moreover, the pilot study showed that some of the items were misunderstood by the prospective elementary teachers. These misunderstood items were reviewed and clarified.

Table 3.7 presents the final version of PST items. The table also shows the problem solving strategies which Posamentier and Krulik (1998) suggested to be used for solving each of the items.

Table 3.7. Problem Solving Test Items and Problem Solving Strategies

Problem Solving Strategies (PSS)	Selected Problems for PST									Total Number of PSS
	1	2	3	4	5	6	7	8	9	
Working backwards					X					1
Finding a pattern				X					X	2
Adopting a different point of view			X	X						2
Solving Simpler Analogous Problem				X						1
Considering Extreme Cases								X		1
Making Drawing				X		X	X			3
Intelligent Guessing And Testing		X								1
Accounting For All Possibilities	X			X						2
Organizing Data	X			X					X	3
Logical Reasoning	X									1

3.4. Data Collection Procedure

The last version of the problem solving test was administered to 250 prospective elementary mathematics teachers studying at a state university in Central Anatolia during their regular class session. Before the administration of the instrument, ethical approval was granted from METU Research Center for Applied Ethics. Besides, the permissions of the related instructors were taken via submitting the sample instrument and a summary of the purpose of the study before the implementation date.

The purpose of the study was explained to the participants before they started responding to the items in the test. Prospective teachers were informed that participation was voluntarily and it would not result negatively if they would not want to contribute to the study. In addition, it was declared that all their responses would be kept completely confidential and would only be used for the study. Administration of PST took approximately 40 minutes. The instrument was directly administered and collected from freshman, sophomore, junior, and senior prospective teachers once in a time and the data collection procedure took about two weeks.

3.5. Data Analysis Procedure

The statistical analyses were done by using statistical package for the social sciences program (SPSS 18.0). The data obtained in the study were analyzed in two parts. In the first part, descriptive statistics was used. The number of prospective teachers and descriptive statistics such as, mean, standard deviation, skewness, kurtosis, minimum and maximum scores of prospective teachers in the problem solving test for each year levels were presented. Next, all participants' mean scores and standard deviations for Item 1 to 9 were calculated.

In the second part of the data analysis, prospective teachers' uses of problem solving strategies were determined by analyzing each prospective teacher's solutions. The research data were analyzed according to the problem solving strategies suggested by Posamantier and Krulik (1998). The frequencies and percentages of problem solving strategies used by prospective elementary mathematics teachers were gathered for each item.

The problem solving strategies used by participants were coded according to the definitions given by Posamentier and Krulik (1998). For each problem, the strategies used by the participants were listed. The definitions of the strategies based on the descriptions of Posamentier and Krulik (1998) are given in Table 3.8.

Table3.8. Definitions of the Problem Solving Strategies

Problem solving strategy	Definitions of the Strategies
Working backwards	Problem solver reverses the steps that produced an end result which can lead to the required starting value.
Finding a pattern	Problem solver tries to find a rule or pattern to explain the situation and solve the problem according to the pattern.
Adopting a different point of view	Problem solver adopts a different point of view than the one which he or she was initially led by the problem.
Solving a simpler, analogous problem	Problem solver tries to solve a simpler problem to figure out the solution of the original problem.
Considering extreme cases	Problem solver considers the extreme cases of the variables that do not change the nature of the problem.
Making a drawing (visual representation)	Problem solver draws a figure or diagram to visually represent the given data in the problem.
Intelligent guessing and testing	Problem solver makes a guess and tests it against the conditions of the problem, and the next guess is based upon the information obtained from the previous guess.
Accounting for all possibilities	Problem solver tries to list all the possible conditions in the problem and evaluate or check each condition to find the one that suits the aim of the problem. The listing should be organized to account all of the possibilities.
Organizing data	Problem solver organizes the given data in a table or a through a systematical listing.
Logical reasoning	Problem solver uses logical reasoning.

The coding procedure was made both by the researcher and by a mathematics education doctoral student. Later, the codings were compared to each other to reach an agreement. A full agreement between the codings done by the researcher and the second rater was reached at the end.

3.6. Reliability and Validity Issues

Reliability refers to the consistency of scores obtained from the instrument (Fraenkel & Wallen, 2006). In this study, inter-rater reliability was used as an evidence for reliability. Inter-rater reliability is the degree of agreement among raters and it gives a score of how much consensus is supplied by raters which is called scoring agreement (Fraenkel & Wallen, 2006). As mentioned before, inter-rater agreement between the researcher and the mathematics education doctoral student were evaluated and it was found that there was nearly 100 % agreement between the two ratings.

Validity refers to appropriate, meaningful, correct, and useful interpretations of any measurement (Fraenkel & Wallen, 2006). Thus, it is about the goal of the test and what it measures. To establish construct validity of the measuring instrument, two mathematics educators working in the Department of Elementary Education at METU with doctoral degree examined the test items with respect to the table of specifications. Table of specification presents the PST items and problem solving strategies which were suggested to be used to solve those items by Posamentier and Krulik (1998) (see Table 3.7). In addition, items that were translated into Turkish were checked by one instructor from the Department of Turkish Language and Literature before the administration of the instrument so that the test items would be eliminated from ambiguities to a great extend. Moreover the appropriateness of items to the year level, representativeness of content by the selected items, the appropriateness of the format such as clarity of directions and language, and quality of printing were checked and suggestions given by experts and instructor were taken into consideration in the revision of items. These measures presented content and construct related evidences of validity of the PST.

3.7. Threats to Internal Validity

Internal validity gives information about the degree to which observed differences on the dependent variable is aroused from the independent variable (Fraenkel & Wallen, 2006). Thus, if the results of the study are not related to the dependent variable or in other words if they are related with some other unintended variables, internal validity threats occur. Each research design has different internal validity threats. Location, instrumentation, instrument decay and mortality are the four main internal validity threats of a survey study (Fraenkel & Wallen, 2006).

Location was not a threat to this research since the study was carried out at one University and in similar classrooms.

Instrumentation threat was assumed to be controlled by the researcher since the researcher collected the whole data by herself and during data collection process, all procedures in all classrooms were standardized to avoid data collector bias. Moreover, instrument decay was not a threat since the data were collected at just one point in time. Additionally, a different interpretation of results depending on the scorers or the time makes instrumentation decay which is an internal threat for survey studies (Fraenkel & Wallen, 2006). To control this threat, the scorings which were done separately by the two raters were compared and a high agreement was found between the two raters. Therefore, instrument decay was not a threat for this study.

Lastly, mortality threat which means the loss of subjects is considered to be an internal threat in survey studies. However, mortality was not an important internal threat for this study since this study was carried out by conducting cross sectional survey. Since data were collected at one point in time, mortality was not a threat for this study.

3.8. Threats to External Validity

External validity refers to “the extent to which the results of a study can be generalized from a sample to a population” (Fraenkel & Wallen, 2006, p.108). In establishing external validity, both population generalizability and ecological generalizability should be considered. Population generalizability is about a sample’s degree of representativeness of an intended population (Fraenkel & Wallen, 2006). The target population of the study was all prospective elementary mathematics teachers in Central Anatolia Region and accessible population was all prospective elementary mathematics teachers in a city of this region. All year level prospective elementary mathematics teachers studying at a state university in this city constituted the sample of the study. In this study, convenient sampling method was preferred. A convenient sample is a group of individuals who (conveniently) are available for study and in general convenient samples cannot be considered as representative of any population (Fraenkel & Wallen, 2006). Thus, the sampling method of the study limits the population generalizability of the research findings.

The term ecological generalizability refers to “the extent to which the results of a study can be generalized to conditions or settings other than those that prevailed in a particular study” (Fraenkel & Wallen, 2006, p.108). This study was conducted at a state university and results could be generalized to the students in other state universities having similar conditions, such as course distribution, with the university that the data was collected.

CHAPTER 4

RESULTS

In this study, the main area of investigation is to determine the problem solving strategies that prospective elementary mathematics teachers use while solving mathematical problems. Besides, this study also deals with prospective elementary mathematics teachers' achievement in problem solving in terms of year levels.

This chapter aims to present the results of the study in two main parts. Each part deals with one research question. In the first part, descriptive statistics regarding prospective elementary mathematics teachers' problem solving test scores will be explained for each year level. In the second part, prospective teachers' use of problem solving strategies and descriptive statistics related to each item will be mentioned.

4.1. Prospective Elementary Mathematics Teachers' Problem Solving Test Scores

In order to collect data for the research question investigating the problem solving strategies that are used by prospective elementary mathematics teachers in solving mathematical problems, Problem Solving Test (PST) was used. PST consisted of nine open-ended items and each item was connected with at least one problem solving strategy existing in the literature. In this part, for each item, descriptive statistics related with the PST scores will be summarized in terms of year levels (namely, freshmen, sophomores, juniors, and seniors).

4.1.1. Descriptive Statistics Regarding Problem Solving Test

Each item in PST was graded out of 10 points and since there were nine items in the test, the maximum possible score was 90 points. Descriptive statistics such as, mean, standard deviation, skewness, kurtosis, minimum and maximum values for prospective teachers' problem solving test scores for each year levels are presented in Table 4.1.

Table 4.1. Descriptive Statistics Regarding Problem Solving Test

	Freshmen	Sophomores	Juniors	Seniors	Total
Mean	76.70	69.90	66.70	62.25	69.46
SD	13.86	15.16	14.83	13.06	15.15
Skewness	-1.49	-1.12	-0.52	-0.31	-0.73
Kurtosis	2.88	1.44	-0.08	0.48	0.34
Minimum	21	20	27	26	20
Maximum	90	90	90	90	90
N	71	66	61	52	250

Note: Maximum possible score was 90.

Prospective teachers' problem solving test scores ranged from 20 to 90. In each year level there were prospective teachers who were able to solve all the problems correctly. However, there were also prospective teachers with very low problem solving achievement scores. When compared to the whole group (M=69.46, SD=15.15), freshmen's problem solving test scores (M=76.70, SD=13.86) were quite high and sophomores' scores (M=69.90, SD= 15.16) were approximately equal to the general mean of all the participants. On the other hand, junior (M=66.70, SD=14.83) and senior (M=62.25, SD=13.06) prospective teachers' problem solving test scores were below the whole group. The results showed that, as year level increased prospective teachers' problem solving test scores decreased considerably. In addition, the table also presents skewness and kurtosis values for each year level and for the whole group. According to these values, it can be inferred that problem solving test scores for each year level and whole group were normally distributed (Pallant, 2007).

4.1.2. Descriptive Statistics Regarding Problem Solving Test Items

Participants' mean scores for each item was summarized in order to determine participants' achievement levels for those items in Table 4.2.

Table 4.2. Descriptive Statistics Regarding Problem Solving Test Items

	Freshmen		Sophomores		Juniors		Seniors		Total	
Item no	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
1	8.20	2.79	7.97	2.85	6.36	3.48	7.21	2.92	7.49	3.08
2	9.07	2.45	9.12	2.43	8.25	3.21	7.04	3.75	8.46	3.04
3	8.03	3.79	6.09	4.80	6.23	4.68	7.19	4.49	6.90	4.49
4	8.52	3.02	8.14	3.00	9.05	2.14	8.62	2.67	8.57	2.76
5	9.41	2.15	8.85	2.72	8.46	2.98	8.46	2.73	8.83	2.66
6	8.04	3.69	6.79	4.32	6.98	4.24	6.00	4.24	7.03	4.22
7	9.61	1.78	9.27	2.18	9.44	2.04	8.92	2.64	9.34	2.15
8	6.58	4.63	5.18	4.92	6.23	4.68	4.35	4.56	5.66	4.76
9	9.24	2.58	8.48	3.61	5.70	4.66	4.46	4.88	7.18	4.37

According to the Table 4.2, it can be said that, Item 7 (*M*=9.34, *SD* =2.15) was the easiest problem for prospective teachers and this problem could be solved by using several problem solving strategies. However, Item 8 (*M* =5.66, *SD* =4.76) was the most difficult item for prospective teachers and it entailed participants to use considering extreme cases strategy. When considered with respect to year level, the same situation holds except for juniors since their mean score for Item 9 (*M* =5.70, *SD* =4.66) was lower than the mean score for Item 8 (*M* =6.23, *SD* =4.68). However, Item 9 which was related with finding a pattern strategy, was the third easiest problem for freshmen (*M* =9.24, *SD* =2.58) and the fourth easiest problem for sophomores (*M* =8.48, *SD* =3.61) (see Item 7, 8 and 9 in Appendix A).

It can be understood from the Table 4.2 that among all prospective teachers, freshmen prospective teachers had higher mean scores than other year levels for all items except for Item 2 and Item 4. The highest mean score for Item 2 belonged to sophomores and for Item 4 it belonged to juniors. Moreover, senior prospective teachers had the lowest mean scores for all items except for Item 1, Item 3 and Item 4, since they had the second highest mean score for Item 3 and Item 4 and third highest mean score for Item 1 (see Item 1, 2, 3 and 4 in Appendix A).

In this part, prospective teachers' problem solving test scores were presented. In more details, descriptive statistics concerning the overall problem solving test scores and the individual item scores were presented. In the next part, problem solving strategies used by prospective teachers will be given for each item.

4.2. Prospective Elementary Mathematics Teachers' Use of Problem Solving Strategies

In the previous part of the study, descriptive statistics concerning problem solving test were presented and in this part problem solving strategies used by prospective elementary mathematics teachers while solving PST will be presented for each item.

To describe the problem solving strategies used by prospective teachers, first, each participant's responses for each item were reviewed and similar responses were grouped. Later, similar responses were matched with the relevant problem solving strategy by considering definitions existing in the literature. Finally, these grouped responses were given a name such as logical reasoning and looking for a pattern. Moreover, in some cases, participants used two or more different strategies simultaneously when responding the item. Therefore, these responses were named as "a combination of two or more strategies". Besides, some responses that don't match problem solving strategies in the literature were named by the researcher as "invented strategy". In the next parts, each item will be examined in terms of problem solving strategies used by prospective teachers and the mean scores for those prospective teachers.

4.2.1. Prospective Teachers' Problem Solving Strategies for Item 1

In Item 1, prospective teachers were asked to respond to "If *a* and *b* are both integers, how many ordered pairs *(a, b)* will satisfy the equation $a^2 + b^2 = 10$?" Table 4.3 given below shows the basic descriptive statistics related to mean scores of Item 1 in terms of problem solving strategies.

Table 4.3. Descriptive Statistics Regarding Participants' Problem Solving Strategies for Item 1

Problem solving strategy	*N*	*M*	*SD*
Combination of different strategies	152	8.94	1.95
Logical reasoning	79	5.06	2.52
Solving in two different ways	2	10	0
Others	17	5.57	2.96
Total	250	7.49	3.08

Note: Maximum possible score was 10.

It can be understood from Table 4.3 that, participants showed three different solutions for Item 1 namely, combining different strategies, using logical reasoning and solving in two different ways. In more details, prospective teachers who used two different ways (*M*=10.0 *SD*=0) and who used a combination of logical reasoning, organizing data and accounting for all possibilities strategies (*M*=8.94, *SD*=1.95) had higher mean scores than prospective teachers using logical reasoning strategy (*M*=5.06, *SD*=2.52). Moreover, when year level was considered for this item, it can be understood from the Table 4.2 that freshmen (*M*=8.20, *SD*=2.79) and sophomores (*M*=7.97, *SD*=2.85) had higher mean scores than juniors (*M*=6.36, *SD*=3.48), seniors (*M*=7.21, *SD*=2.92) and the overall mean score (*M*=7.49, *SD*=3.08) in Item 1. Table 4.4 given below shows the problem solving strategies used by prospective elementary mathematics teachers in each year level for Item 1.

Table 4.4. Problem Solving Strategies and Year Levels of Prospective Teachers for Item 1

Problem solving strategy	Freshmen		Sophomores		Juniors		Seniors		Total
	f	%	F	%	f	%	f	%	
Combination of different strategies	50	20.00	43	17.20	26	10.40	33	13.20	60.80%
Logical reasoning	18	7.20	18	7.20	29	11.60	14	5.60	31.60%
Solving in two different ways	1	0.40	1	0.40	-	-	-	-	0.80%
Others	2	0.80	4	1.60	6	2.40	5	2	6.80%
Total	71	28.40	66	26.40	61	24.40	52	20.80	100%

The Table 4.4 shows that, more than half of the prospective teachers (60.80%) solved Item 1 by combining different problem solving strategies. In more details, freshmen (20.00%), sophomore (17.20%), junior (10.40%) and senior (13.20%) prospective teachers solved this item by using different combinations of logical reasoning, organizing data and accounting for all possibilities strategies. For instance, Participant 120 solved this problem by using a combination of three different strategies was shown in Figure 4.1. She firstly examined perfect squares less than or equal to 10 and then decided to examine a=1 and b=3 ($1^2 + 3^2 = 10$) or their symmetric opposites a=3 and b=1 ($3^2 + 1^2 = 10$) by taking into consideration both negative and positive values of a and b. By using logical reasoning, she found eight pairs of ordered pairs that satisfy the equation $a^2 + b^2 = 10$. Moreover, she prepared a systematic list to be certain that she has accounted for all possibilities.

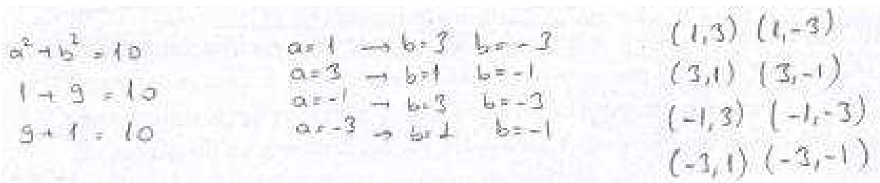

Figure 4.1. Use of combinations of different strategies in Item 1 (Participant 120)

Logical reasoning (31.60%) was another common strategy used by prospective teachers from all year levels. In other words, freshmen (7.20%), sophomore (7.20%), junior (11.60%) and senior (5.60%) prospective teachers applied logical reasoning strategy to solve Item 1.

Similar to previous example (see Figure 4.1), Participant 127 thought that a^2 or b^2 would be equal to either 1 or 9 by logically reasoning as shown in Figure 4.2. Then he found four pairs of answers without considering for all possibilities or using an organized list.

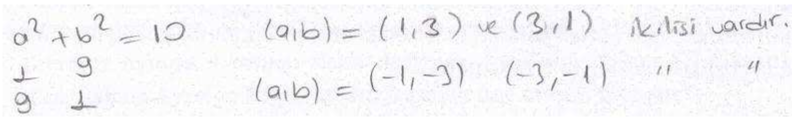

Figure 4.2. Use of logical reasoning strategy in Item 1(Participant 127)

One freshman (0.40%) and one sophomore (0.40%) prospective teachers were able to solve this item in two different ways. As shown in Figure 4.3, Participant 13 solved Item 1 in two different ways and her first solution was very similar to Participant 120’s solution mentioned above (see Figure 4.1). She examined 0, 1, 2 and 3 as values of *a* then decided to use value 1 and 3 to satisfy the equation. Finally, she organized a list to be certain that she has accounted for all the possibilities and then found eight pairs of answers. In her second solution, she used a combination formula to reach to the correct answer.

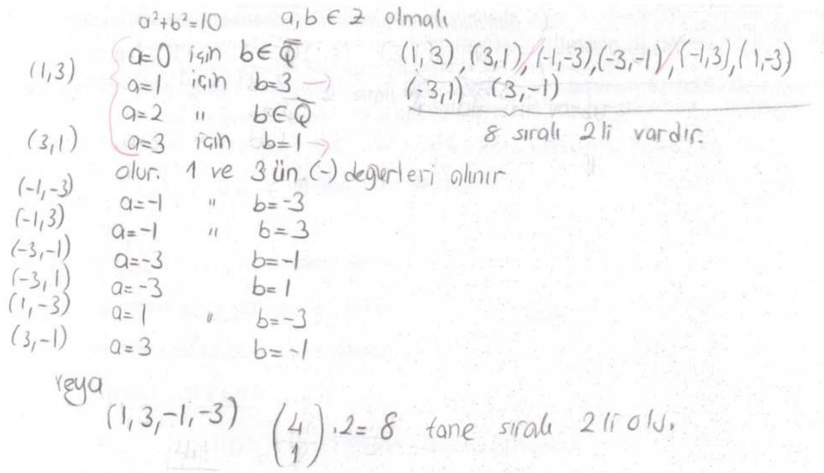

Figure 4.3. Use of two different ways in Item 1 (Participant 13)

Finally, prospective teachers in the category of “others” (6.80%) were the participants who either misunderstood the problem or were not able to give any response to Item1.

4.2.2. Prospective Teachers’ Problem Solving Strategies for Item 2

In Item 2, prospective teachers were asked to respond to “The sum of an integer, its square, and its square root is 276. What is the integer?” Table 4.5 shows the basic descriptive statistics related to mean scores of Item 2 in terms of problem solving strategies.

Table 4.5. Descriptive Statistics Regarding Participants’ Problem Solving Strategies for Item 2

Problem solving strategy	*N*	*M*	*SD*
Intelligent guessing and testing	208	9.78	0.77
Setting up an equation	40	2.00	0
Others	2	0	0
Total	250	8.46	3.04

Note: Maximum possible score was 10.

Prospective teachers showed two different solutions while solving Item 2. They were grouped into two as using intelligent guessing and testing and using setting up an equation strategy. It can be seen from Table 4.5 that, prospective teachers who used intelligent guessing and testing strategy (*M*=9.78, *SD*=0.77) had a higher mean score than the ones who used setting up an equation strategy (*M*=2.00, *SD*=0). When year levels were considered, similar to Item 1, freshmen (*M*=9.07, *SD*=2.45) and sophomores (*M*=9.12, *SD*=2.43) had higher mean score than juniors (*M*=8.25, *SD*=3.21), seniors (*M*=7.04, *SD*=3.75) and the overall mean score (*M*=8.46, *SD*=3.04) in this item (see Table 4.2).

Problem solving strategies used by prospective teachers for Item 2 were presented in Table 4.6 given below.

Table 4.6. Problem Solving Strategies and Year Levels of Prospective Teachers for Item 2

Problem solving strategy	Freshmen		Sophomores		Juniors		Seniors		Total
	f	%	F	%	f	%	f	%	
Intelligent guessing and testing	65	26.00	60	24.00	49	19.60	34	13.60	76.00%
Setting up an equation	5	2.00	5	2.00	12	4.80	18	7.20	16.00%
Others	1	0.40	1	0.40	-	-	-	-	0.80%
Total	71	28.40	66	26.40	61	24.40	52	20.80	100%

The table shows that, prospective teachers used intelligent guessing and testing strategy (76.00%) and setting up an equation strategy (16.00%) while solving Item 2. More specifically, intelligent guessing and testing was used by freshmen (26.00%), sophomores (24.00%), juniors (19.60%) and seniors (13.60%). For example, Participant 117, as shown in Figure 4.4, used intelligent guessing and testing strategy to solve Item 2. When using this strategy problem solvers need to make a guess, and then test it against the conditions of the problem. Similarly, Participant 117 guessed that the unknown integer would be 16, and then tested the number 16 whether it satisfied the problem conditions. He also explained his solution as “We should choose an integer whose square root is also an integer and which is a perfect square less than 276”.

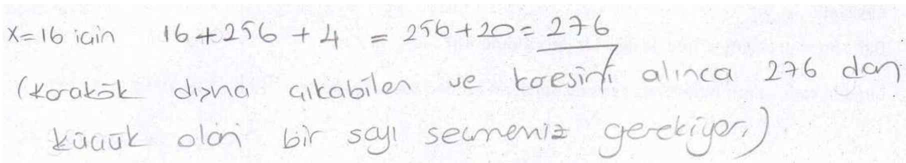

Figure 4.4. Use of intelligent guessing and testing strategy in Item 2 (Participant 117)

Setting up an equation strategy (16.00%) was another frequent strategy used by prospective teachers namely, freshmen (2.00%), sophomores (2.00%), juniors (4.80%) and seniors (7.20%). As shown in Figure 4.5, Participant 79 considered *x* as an unknown integer and wrote the equation $x + x^2 + \sqrt{x} = 276$ and then rewrote the equation as $x(1 + x + \frac{1}{\sqrt{x}}) = 276$. Since this equation was not easy to solve, Participant 79 was not able to reach an answer.

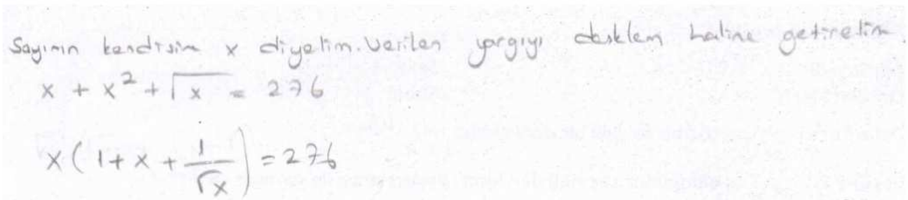

Figure 4.5. Use of setting up an equation strategy in Item 2 (Participant 79)

Finally, two prospective teachers (0.80%) were not able to give any response to this item.

4.2.3. Prospective Teachers' Problem Solving Strategies for Item 3

In Item 3, prospective teachers were asked to respond to "What is the greatest value of the expression *ab* + *bc* + *cd* + *ad*, if *a*, *b*, *c* and *d* have values 1, 2, 3, and 4, but not necessarily in that order?" The table given below shows the basic descriptive statistics related to mean scores of Item 3 in terms of problem solving strategies.

Table 4.7. Descriptive Statistics Regarding Participants' Problem Solving Strategies for Item 3

Problem solving strategy	*N*	*M*	*SD*
Adopting a different point of view	157	9.21	2.30
Intelligent guessing and testing	22	5.91	5.03
Invented strategy	7	0	0
Accounting for all possibilities	1	10.00	0
Combination of different strategies	15	9.33	2.58
Others	48	0	0
Total	250	6.90	4.48

Note: Maximum possible score was 10.

The table shows that, one prospective teacher used accounting for all possibilities (*M*=10.00, *SD*=0) and had the highest possible mean score for this item. Then, prospective teachers who combined different strategies (*M*=9.33, *SD*=2.58) and who adopted a different point of view (*M*=9.18, *SD*=2.30) had the second and the third highest mean scores for this item. Moreover, prospective teachers who solved this item by using intelligent guessing and testing strategy had the lowest mean score (*M*=5.91, *SD*=5.03). Furthermore, Table 4.2 shows that freshmen's (*M*=8.03, *SD*=3.79) and seniors' (*M*=7.19, *SD*=4.49) mean scores were above and sophomores' (*M*=6.09, *SD*=4.80) and juniors' (*M*=6.23, *SD*=4.68) mean scores were below the overall mean score for this item (*M*=6.90, *SD*=4.49).

Table 4.8 given below shows the problem solving strategies used by prospective teachers for Item 3.

Table 4.8. Problem Solving Strategies and Year Levels of Prospective Teachers for Item 3

Problem solving strategy	Freshmen		Sophomores		Juniors		Seniors		Total
	f	%	f	%	f	%	f	%	
Adopting a different point of view	49	19.60	40	16	36	14.40	32	12.80	62.80%
Intelligent guessing and testing	11	4.40	1	0.40	3	1.20	7	2.80	8.80%
Combination of different strategies	4	1.60	3	1.20	5	2	3	1.20	6.00%
Invented strategy	3	1.20	-	-	1	0.40	3	1.20	2.80%
Accounting for all possibilities	-	-	1	0.40	-	-	-	-	0.40%
Others	4	1.60	21	8.40	16	6.40	7	2.80	19.20%
Total	71	28.40	66	26.40	61	24.40	52	20.80	100%

It can be understood from the Table 4.8 that, adopting a different point of view strategy was the most popular strategy (62.80%), since it was used by freshmen (18.40%), sophomores (15.60%), juniors (14.00%) and seniors (12.80%) which in total constitutes more than half of the all participants.

Figure 4.6 represents an example for using adopting a different point of view strategy by Participant 117. She first rewrote the equation as $b(a+c)+d(a+c)$ then factored the equation as $(a+c)\times(b+d)$. Then, she decided that each factor should be equal to 5 and found the greatest value for the expression as 25.

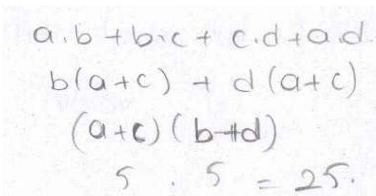

Figure 4.6. Use of adopting a different point of view strategy in Item 3 (Participant 117)

Besides, intelligent guessing and testing (8.80%) was another common strategy used by freshmen (4.40%), sophomores (0.40%), juniors (1.20%) and seniors (2.80%). Figure 4.7 illustrates Participant 11's intelligent guessing and testing strategy use for Item 3. In his first attempt, he assigned *a, b, c* and *d* the numerical values 4, 3, 1 and 2 respectively, and then calculated the value for the expression *ab* + *bc* + *cd* + *ad* as 25.

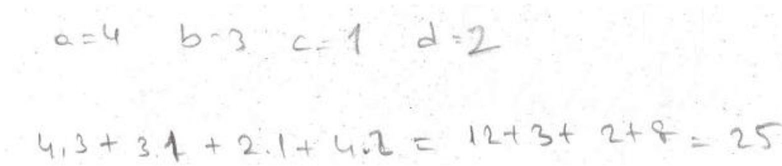

Figure 4.7. Use of intelligent guessing and testing strategy in Item 3 (Participant 11)

Moreover, some of the prospective teachers solved Item 3 by combining different problem solving strategies (6.00%). Figure 4.8 given below is an example for combination of adopting a different point of view and accounting for all possibilities strategies. In the first part of the example, Participant 124 adopted a different point of view similar to Participant 117 (see Figure 4.6) and factored the expression as $(a+c)\times(b+d)$. In the second part of the example, participant 124 considered all possibilities for the two factors. In more details, there were three different possibilities for $(a+c)\times(b+d)$ such as $(1+2)\times(3+4)=21$, $(1+3)\times(2+4)=24$ and $(1+4)\times(2+3)=25$. Then he decided that the greatest value for the expression was 25.

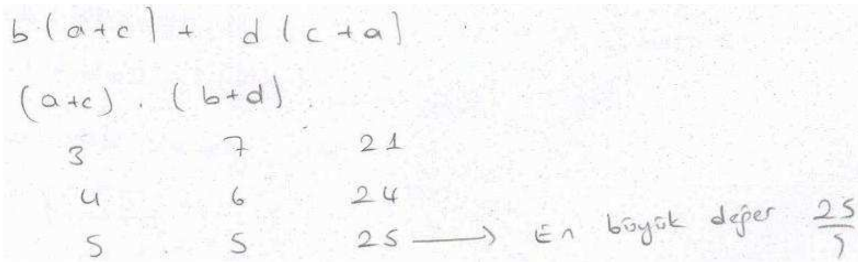

Figure 4.8. Use of combinations of strategies in Item 3 (Participant 124)

One prospective teacher, Participant 126 solved this item by accounting for all possibilities (0.40%). He examined all possible values for the expression without rewriting the expression. Figure 4.9 shows the use of considering for all possibilities strategy for Item 3. Here, he listed 24 different possibilities for *a, b, c,* and *d* and then for *ab* + *bc* + *cd* + *ad.* After examining the value, she also decided the maximum value for the expression as 25 which was the correct answer for this item.

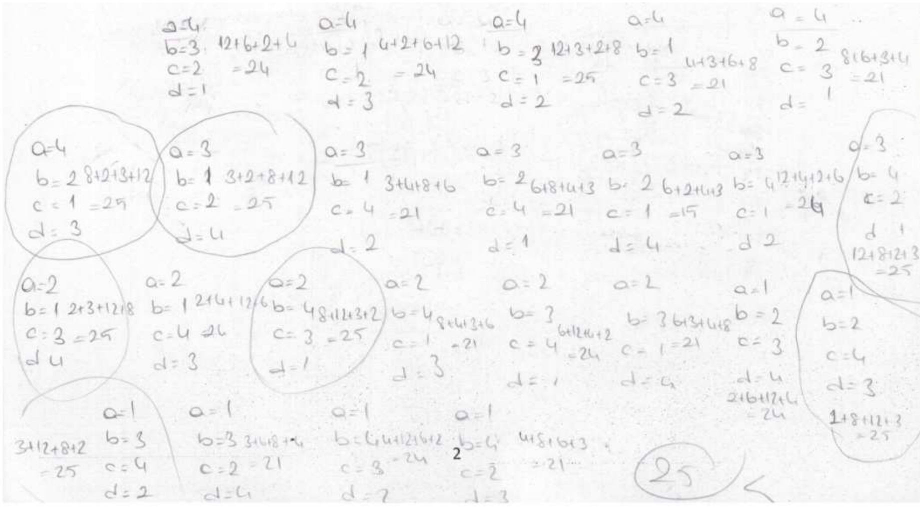

Figure 4.9. Use of accounting for all possibilities strategy in Item 3 (Participant 126)

In addition to these problem solving strategies, some prospective teachers proposed an erroneous strategy (2.80%) while solving Item 3. For instance, in Figure 4.10, Participant 52 assigned the values 1, 2, 3 and 4 for *a*, *b*, *c* and *d* respectively and stated that the value of the expression would not change when the values for *a*, *b*, *c* and *d* were changed. That is, he claimed that the value of *ab* + *bc* + *cd* + *ad* would be the same when *a, b, c, d* are equal to *1, 2, 3, 4* respectively or *2, 1, 3, 4,* etc. However, he did not make any attempt to test his hypothesis and gave a wrong answer to the item 3.

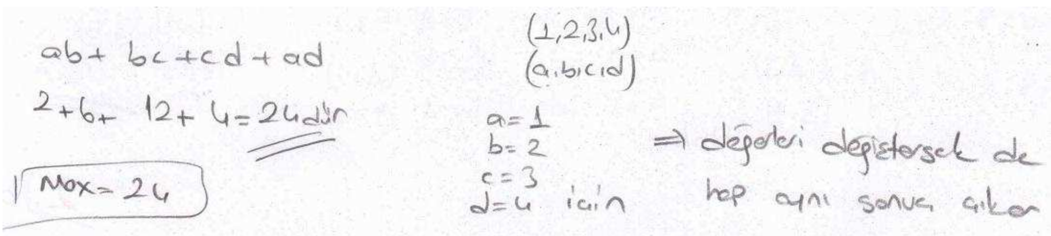

Figure 4.10. Use of invented strategy in Item 3 (Participant 52)

Finally, 19.20% of all prospective teachers either misunderstood the problem or were not able to give any response to Item 3.

4.2.4. Prospective Teachers' Problem Solving Strategies for Item 4

In Item 4, prospective teachers were asked to respond to "In a room with 10 people, everyone shakes hands with everybody else exactly once. How many handshakes are there?" Descriptive statistics regarding participants' problem solving strategies for Item 4 was presented in Table 4.9 given below.

Table 4.9. Descriptive Statistics Regarding Participants' Problem Solving Strategies for Item 4

Problem solving strategy	*N*	*M*	*SD*
Organizing data	129	8.85	2.25
Using a formula	34	9.00	2.92
Adopting a different point of view	21	5.71	2.23
Making a drawing	21	8.38	3.07
Accounting for all possibilities	1	10.00	0
Solving in two different ways	17	10.00	0
Combination of different strategies	22	9.91	1.70
Others	5	0	0
Total	250	8.57	2.76

Note: Maximum possible score was 10.

The table shows that, prospective teachers who used accounting for all possibilities (*M*=10.00, *SD*=0) and who solved the problem by using two different ways (*M*=10.00, *SD*=0) had highest possible mean score for Item 4. Then, prospective teachers combining different strategies (*M*=9.91, *SD*=1.70), using a formula (*M*=9.00, *SD*=2.92), organizing data (*M*=8.86, *SD*=2.25) and making a drawing (*M*=8.38, *SD*=3.07) had the second highest mean scores for this item. On the other hand, prospective teachers adopting a different point of view (*M*=5.71, *SD*=2,23) had the lowest mean scores for Item 4.

Contrary to Item 1 and Item 2, juniors (*M*=9.05, *SD*=2.14) and seniors (*M*=8.62, *SD*=2.67) had higher mean score than freshmen (*M*=8.52, *SD*=3.02), sophomores (M=8.14, SD=3.00) and the overall mean score (M=8.57, SD=2.76) in Item 4.

Table 4.10 given below shows the problem solving strategies used by prospective teachers for Item 4.

Table 4.10. Problem Solving Strategies and Year Levels of Prospective Teachers for Item 4

Problem solving strategy	Freshmen		Sophomores		Juniors		Seniors		Total
	f	%	f	%	f	%	f	%	
Organizing data	34	13.60	40	16.00	24	9.60	31	12.40	51.60%
Using a formula	16	6.40	5	2.00	7	2.80	6	2.40	13.60%
Adopting a different point of view	4	1.60	9	3.60	3	1.20	5	2.00	8.40%
Making a drawing	9	3.60	5	2.00	2	0.80	5	2.00	8.40%
Combination of different strategies	3	1.20	1	0.40	15	6.00	3	1.20	8.80%
Solving in two different ways	3	1.20	4	1.60	9	3.60	1	0.40	6.80%
Accounting for all possibilities	-	-	-	-	1	0.40	-	-	0.40%
Others	2	0.80	2	0.80	-	-	1	0.40	2.00%
Total	71	28.40	66	26.40	61	24.40	52	20.80	100%

Item 4 was rich in terms of the use of problem solving strategies and Table 4.10 shows that, organizing data (51.60%) was the most popular one since it was used by freshmen (13.60%), sophomores (16.00%), juniors (9.60%) and seniors (12.40%) which in total constituted more than half of the participants.

Figure 4.11 is an example for organizing data strategy used for Item 4. In this example, Participant 162 jotted down each of the people in the room and the number of hands they had to shake each time. Thus, the person labeled 1 shakes 9 hands, the person labeled 2 shook 8 hands, and so on until the person labeled 9, who only had one person's hand left to shake. Consequently, Participant 162 found the number of handshakes as $1+2+3+...+9=45$.

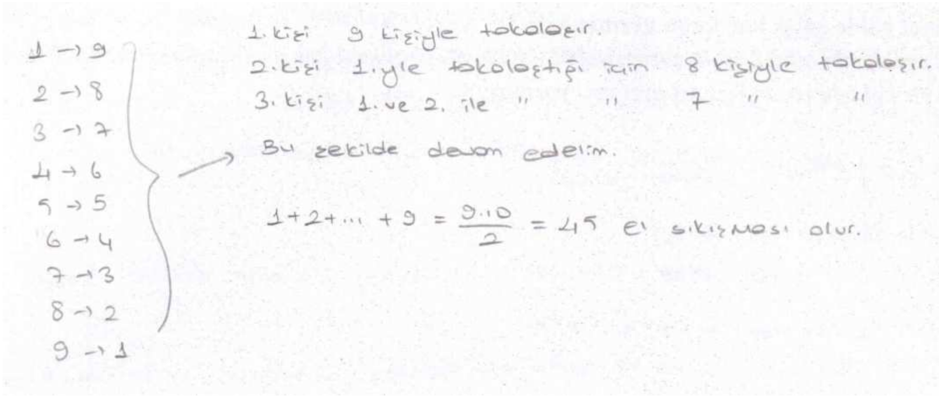

Figure 4.11. Use of organizing data strategy in Item 4 (Participant 162)

Some of the prospective teachers solved this item also by using a formula (13.60%) and nearly half of them was freshmen prospective teachers (6.40%). Moreover, sophomores (2.00%), juniors (2.80%) and seniors (2.40%) constitute the other half of the ones using a formula in solving the given problem. Figure 4.12 shows the use of a combination formula of 10 things taken 2 at a time:

$$\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{10!}{(10-2)!2!} = 45$$

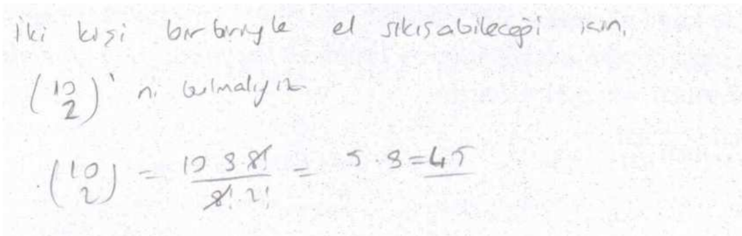

Figure 4.12. Use of formula in Item 4 (Participant 160)

Adopting a different point of view (8.40%) was another common strategy. More specifically, it was used by freshmen (1.60%), sophomores (3.60%), juniors (1.20%) and seniors (2.00%). For example, Participant 49 considered this item from a different point of view. He stated that "There are 10 people in a room and each person would shake 9 other people's hands. This seems to indicate that there are

$10 \times 9 = 90$ handshakes, but we must divide it by 2 to eliminate the duplication; hence the answer is 45".

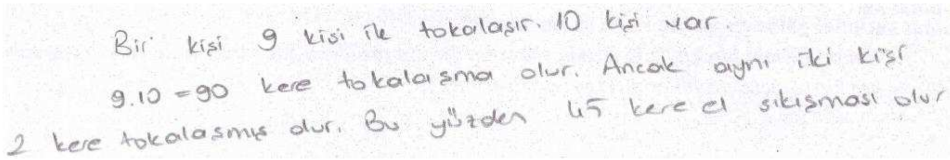

Figure 4.13. Use of adopting a different point of view strategy in Item 4 (Participant 49)

Similar to previous strategy, making a drawing (8.40%) was used by freshmen (3.60%), sophomores (2.00%), juniors (0.80%) and seniors (2.00%). In Figure 4.14, Participant 42 made a visual representation of the situation. In this example, *x*, *y*, *z* etc. represented each person in the room. For example, the first person (*x*) was matched with each of the other 9 people, indicating the first 9 handshakes that took place. For the second person (*y*) there would be 8 additional handshakes since *x* had already shaken hands with *y*, and so on. Besides, Participant 42 wrote the number of handshakes on the top of each person and found the sum of the handshakes as $9 + 8 + 7 + \cdots + 1 + 0 = 45$.

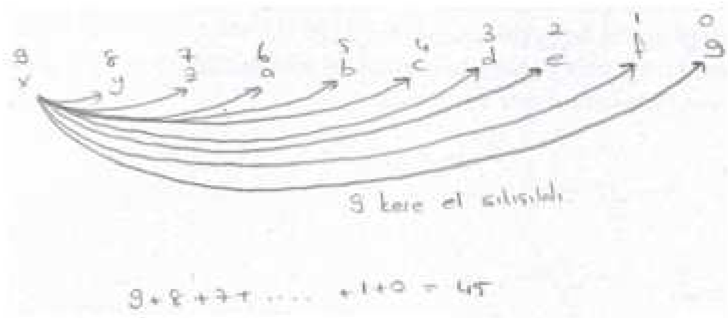

Figure 4.14. Use of making a drawing strategy in Item 4 (Participant 42)

Some of the prospective teachers solved this item by combining two or more strategies (8.80%) and more than half of them was juniors (6.00%). The use of combination of different strategies by other year levels was mere, statistically only two freshmen (1.20%), one sophomore (0.40%) and three seniors (1.20%) attempted to use a combination of different strategies together for this item.

Figure 4.15 is an example for combination of solving simpler problem with visual representation, organizing data and looking for a pattern. In this example, Participant 196 began by considering a figure with 2 people, represented by two points. This would make 1 handshake. Then, she expanded the number of people to 3, represented by three points. Here, the number of handshakes was 3. She continued with 4 people, 5 people, and so on, and wrote these values in an organized way. She also related the number of handshakes with the formula for the sum of the first n natural numbers, $\frac{n(n-1)}{2}$ where $n \geq 2$, by realizing the pattern. Thus, the answer is $\frac{10 \times (10-1)}{2} = 45$.

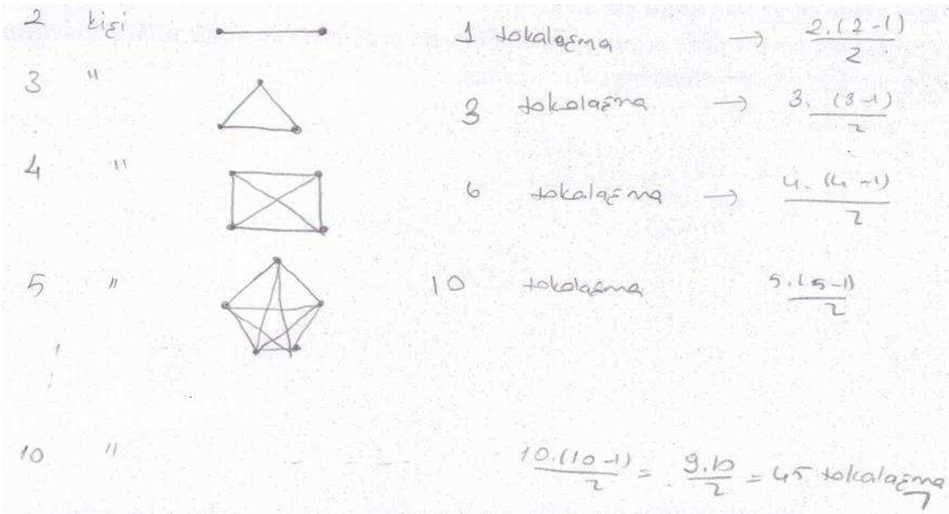

Figure 4.15. Use of combinations of strategies in Item 4 (Participant 196)

Moreover, accounting for all possibilities strategy was the least common strategy used by prospective teachers (0.40%). Figure 4.16 shows the use of accounting for all possibilities by Participant 142. She assigned each person a number and then wrote them up both from left to the right and from top to the bottom. The "-"s in the diagonal of the figure indicated that people could not shake hands with themselves and “x”s indicated doubly all the other handshakes. For instance, the first person shook hands with the second person and the second person shook hands with the first person. Thus, she concluded that each person would shake 9 other people’s hands and there were $10 \times 9 = 90$ handshakes, but it must be divided by 2 to eliminate the duplication; hence the answer was 45.

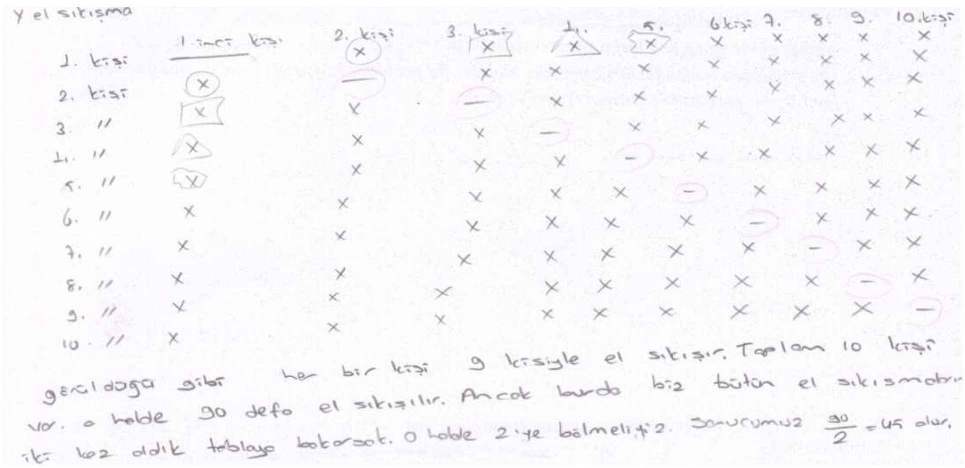

Figure 4.16. Use of examining all the possibilities strategy in Item 4 (Participant 142)

Finally, some prospective teachers solved this item in two different ways (6.80%). Figure 4.17 is an example for both using making a drawing and using a formula. In the left hand side of the figure, visual representation was used by Participant 159. The 10 points represented the 10 people. First person joined to each of the other 9 points, indicating the first 9 handshakes that took place. From the second person, there were 8 additional handshakes. Similarly, from the third person, there were 7 additional handshakes, and so on. Then, Participant 159 found the total number of handshakes as $9 + 8 + 7 + \cdots + 1 + 0 = 45$. In the right hand side of the figure, similar to the Participant 196 (see Figure 4.15), Participant 159 used the formula for the sum of the first *n* natural numbers $\frac{n(n-1)}{2} = \frac{10\times 9}{2} = 45$.

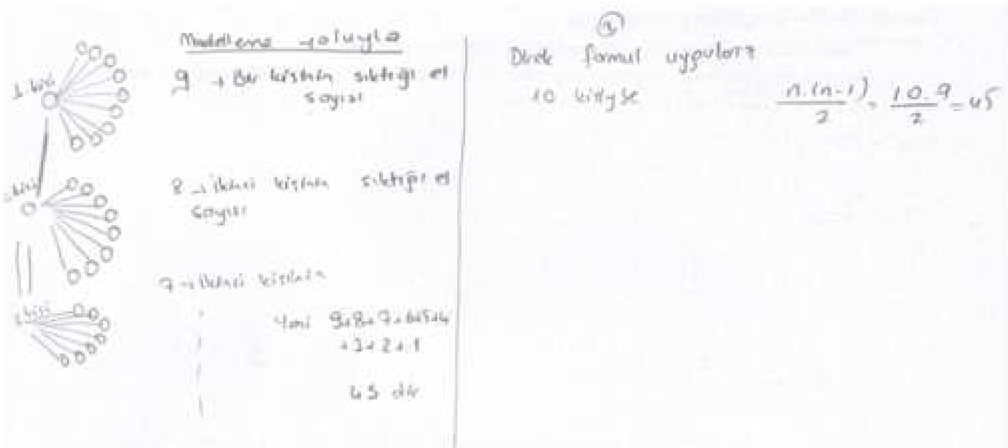

Figure 4.17. Use of two different ways in Item 4 (Participant 159)

Prospective teachers included in “others”, either misunderstood the problem or were not able to give any response (2.00%) to this item.

4.2.5. Prospective Teachers’ Problem Solving Strategies for Item 5

In Item 5, prospective teachers were asked to respond to “Nancy breeds New Zealand rabbits for a hobby. During April, the number of rabbits increased by 10%. In May, 10 new rabbits were born, and at the end of May, Nancy sold one third of her flock. During June, 20 new rabbits were born, and at the end of June, Nancy sold one half her total flock. So far in July, 5 rabbits have been born, and Nancy now has 55 rabbits. How many rabbits did Nancy start with on April 1st?” Table 4.11 shows the basic descriptive statistics related to mean scores of Item 5 in terms of problem solving strategies.

Table 4.11. Descriptive Statistics Regarding Participants' Problem Solving Strategies for Item 5

Problem solving strategy	*N*	*M*	*SD*
Setting up an equation	198	8.94	2.41
Working backwards	29	9.45	1.68
Working backwards or setting up an equation	15	10.00	0
Intelligent guessing and testing	2	7.00	4.24
Others	6	0	0
Total	250	8.83	2.66

Note: Maximum possible score was 10.

Prospective teachers were able to solve Item 5 by setting up an equation, working backwards and using intelligent guessing and testing strategies. Besides, some prospective teachers were able to solve this item both working backwards and setting up an equation. All prospective teachers using both working backwards and setting up an equation strategy correctly solved this item (*M*=10.00, *SD*=0). Moreover, prospective teachers who used working backwards (*M*=9.45, *SD*=1.68) showed higher mean scores than the ones who used setting up an equation (*M*=8.94, *SD*=2.41) and intelligent guessing and testing (*M*=7.00, *SD*=4.24). Moreover, Table 4.2 showed that freshmen prospective teachers (*M*=9.41, *SD*=2.15) had the higher mean scores than overall mean score, and the other year levels were below the overall mean score (*M*=8.83, *SD*=2.66) in Item 5.

Table 4.12 given below shows the problem solving strategies used by prospective teachers for Item 5.

Table 4.12. Problem Solving Strategies and Year Levels of Prospective Teachers for Item 5

Problem solving strategy	Freshmen		Sophomores		Juniors		Seniors		Total
	f	%	f	%	f	%	f	%	
Setting up an equation	55	22.00	53	21.20	47	18.80	43	17.20	79.20%
Working backwards	12	4.80	5	2.00	8	3.20	4	1.60	11.60%
Working backwards or setting up an equation	3	1.20	6	2.40	3	1.20	3	1.20	6.00%
Intelligent guessing and testing	-	-	-	-	1	0.40	1	0.40	0.80%
Others	1	0.40	2	0.80	2	0.80	1	0.40	2.40%
Total	71	28.40	66	26.40	61	24.40	52	20.80	100%

The table shows that, setting up an equation (79.20%) was the most popular strategy used by prospective teachers from each year level, that is by freshmen (22.00%), sophomores (21.20%), juniors (18.80%) and seniors (17.20%). In Figure 4.18, Participant 163 represented the number of rabbits Nancy started with on April 1st as 100*x*, initially. Then, she continued to find the number of rabbits for each month in terms of *x*. At the end of the July, Nancy had $\frac{220x+110}{6}$ rabbit which was given in the problem as 55 rabbits. Finally she wrote an equation as $\frac{220x+110}{6}=55$ and solved it to find the value of *x*. She found that *x* was equal to 1 and multiplied it with 100 and got 100 rabbits since she started with 100*x* on April. Starting with 100*x* rather than with *x*, made it easy to set up equations successively and to follow fewer steps to reach an answer.

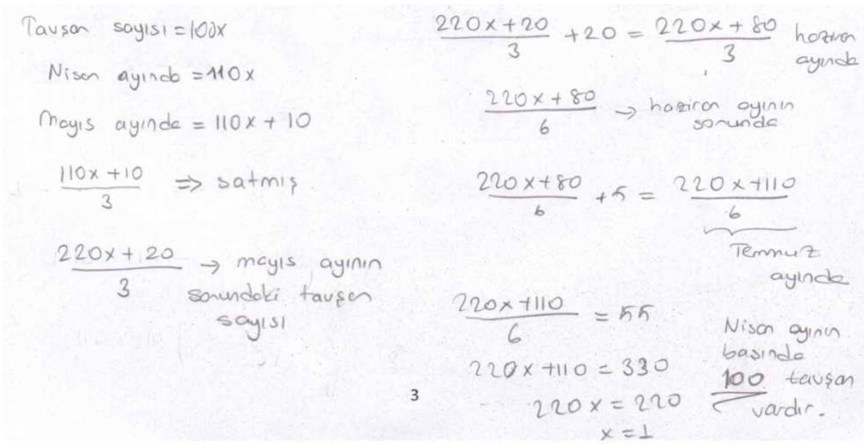

Figure 4.18. Use of setting up an equation strategy in Item 5 (Participant 163)

In addition to setting up an equation, working backwards was another frequent strategy (11.60%) used by freshmen (4.80%), sophomores (2.00%), juniors (3.20%) and seniors (1.60%). Figure 4.19 is an example for use of working backwards strategy. It shows that, participant 160 was able to notice how many rabbits there were at the end of the situation. Later, he performed the inverse operations successively. For example, he started from July subtracted 5 rabbits from 55 since 5 rabbits were born in July, then multiplied 50 with 2 since in June Nancy sold half of the rabbits. During June, 20 new rabbits were born so he subtracted 20 from 100. At the end of May, Nancy sold one third of her flock, thus 80 represents the two third she had, so participant 160 found that the whole number of rabbits was equal to120. In May, 10 new rabbits were born so he subtracted 10 from 120. Here, he found that Nancy had 110 rabbits in May which was equal to 110% of the rabbits on April 1st. To get the number of rabbits Nancy started on April 1st, he decided that the number corresponding to the 100 % of the rabbits would be equal to 100. Therefore, Nancy started with 100 rabbits on April 1st.

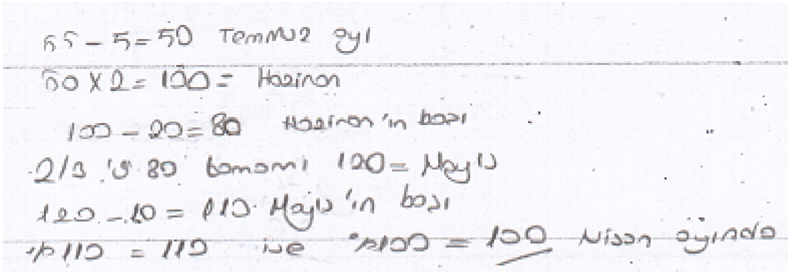

Figure 4.19. Use of working backwards strategy in Item 5 (Participant 160)

Moreover, one junior (0.40%) and one senior (0.40%) prospective teacher used intelligent guessing and testing strategy (0.80%) to solve Item 5. Use of this strategy was represented via Figure 4.20. Here, Participant 162 made a guess that the number of rabbits Nancy started on April 1st would be 100, and then tested the number 100 to see whether it satisfies the problem conditions or not. He followed all steps respectively, at the end; he found that if Nancy started with 100 rabbits on April, at the end of the July, she would have 55 rabbits which is the same number with the actual problem situation. Thus, he decided that Nancy started with 100 rabbits.

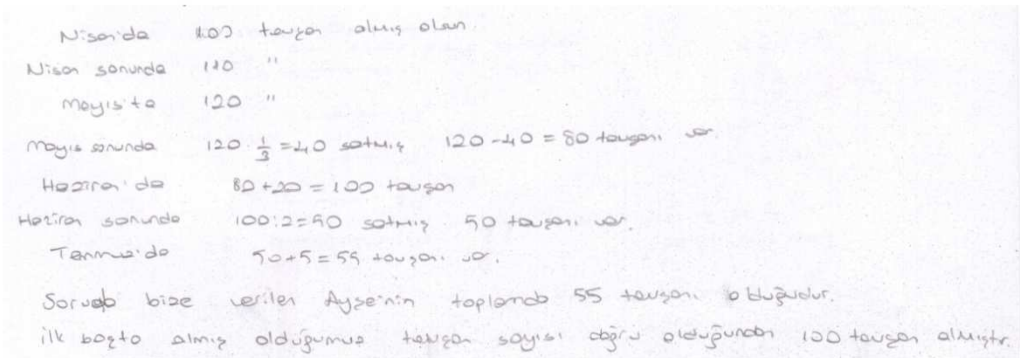

Figure 4.20. Use of intelligent guessing and testing in Item 5 (Participant 162)

Additionally, some of the prospective teachers solved this item by using both working backwards and setting up equation strategies (6.00%). For instance, Participant 159 solved this item by both solving equation and working backwards.

These two different ways were the similar to those two previous examples (see Figure 4.18 and Figure 4.19).

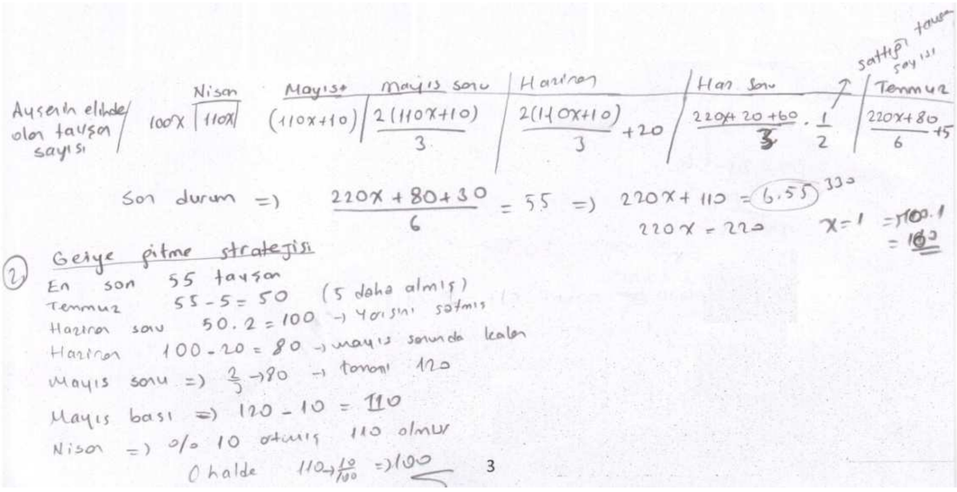

Figure 4.21. Use of two different ways in Item 5 (Participant 159)

Finally, 2.40% of the prospective teachers who were in the category of “others” either misunderstood the problem or were not able to give any response to this item.

4.2.6. Prospective Teachers’ Problem Solving Strategies for Item 6

In Item 6, prospective teachers were asked to respond to “Mr. Lohengrin saw a row of swans on a lake. In front of two swans, there were two swans. Behind two swans there were two swans, and between two swans there were exactly two swans. What is the minimum number of swans Mr. Lohengrin could have seen?” Table 4.13 which is given below shows the basic descriptive statistics related to mean scores of Item 6 in terms of problem solving strategies.

Table 4.13. Descriptive Statistics Regarding Participants’ Problem Solving Strategies for Item 6

Problem solving strategy	*N*	*M*	*SD*
Making a drawing	228	7.53	3.89
Combination of making a drawing and intelligent guessing and testing	3	10.00	0
Logical reasoning	1	10.00	0
Others	18	0	0
Total	250	7.03	4.22

Note: Maximum possible score was 10.

All prospective teachers using logical reasoning (*M*=10.00 *SD*=0) and using a combination of making a drawing and intelligent guessing and testing strategies correctly solved Item 6 (*M*=10.00, *SD*=0) and had higher mean scores than the ones using making a drawing (*M*=7.53, *SD*=3.89). As seen in Table 4.2, similar to Item 5, freshmen prospective teachers (*M*=8.04, *SD*=4.63) had the higher mean scores than overall mean score, and other year levels were below the overall mean score (*M*=7.03, *SD*=4.22) in Item 6.

Table 4.14 represents the problem solving strategies used by prospective teachers for Item 6.

Table 4.14. Problem Solving Strategies and Year Levels of Prospective Teachers for Item 6

Problem solving strategy	Freshmen		Sophomores		Juniors		Seniors		Total
	f	%	f	%	f	%	f	%	
Making a drawing	68	27.20	60	24.00	57	22.80	43	17.20	91.20%
Combination of making a drawing and intelligent guessing and testing	1	0.40	-	-	1	0.40	1	0.40	1.20%
Logical reasoning	-	-	1	0.40	-	-	-	-	0.40%
Others	2	0.80	5	2.00	3	1.20	8	3.20	7.20%
Total	71	28.40	66	26.40	61	24.40	52	20.80	100%

The table shows that, almost all of the prospective teachers solved Item 6 by making a drawing (91.20%). In more details, freshmen (27.20%), sophomores (24.00%), juniors (22.80%) and seniors (17.20%) solved this item by using a visual representation. Figure 4.22 is a visual representation of described situation by Participant159. She represented each swans as dots in the figure and she began with two swans situated in front of another two swans (a), and she got a row of four swans. This also represented the second situation: exactly two swans were behind two swans (b). By using the least number of swans, she depicted exactly two swans between two other swans (c). Therefore, the minimum number of swans Mr. Lohengrin could have seen was a row of four swans.

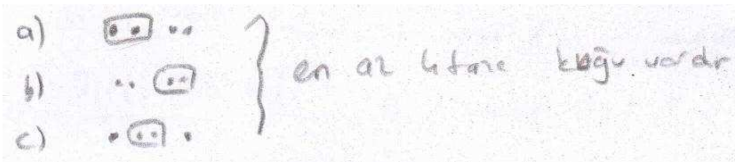

Figure 4.22. Use of making a drawing strategy in Item 6 (Participant 159)

Few prospective teachers (1.20%) used combination of making a drawing and intelligent guessing and testing strategies. For example, in Figure 4.23, Participant 37 represented swans as "—". He started with 3 swans and checked whether 3 swans satisfied the problem conditions or not. Then he decided that 3 swans did not satisfy the first condition he needed more swans thus he examined 4 swans and he decided that 4 swans satisfied all the conditions given in the problem.

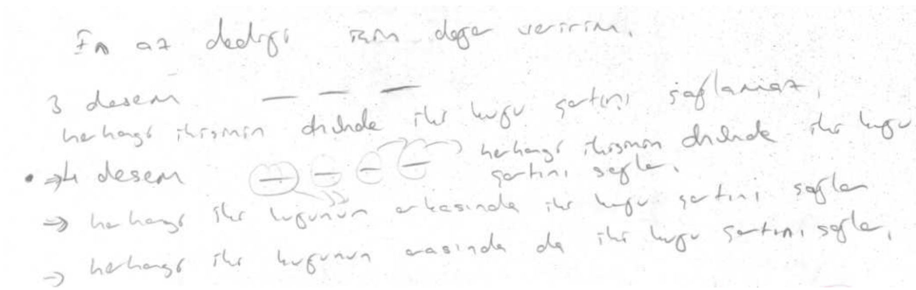

Figure 4.23. Use of combinations of making a drawing and intelligent guessing and testing strategies in Item 6 (Participant 37)

Besides, one sophomore (0.40%) prospective teacher solved this item by using logical reasoning. Participant 109 explained that there must be at least one swan in front and one swan at the back of the row. Thus, by considering the third condition, he considered that there must be two swans between the swans in front and at the back of the row. Finally, he decided that there must be at least 4 swans in the row.

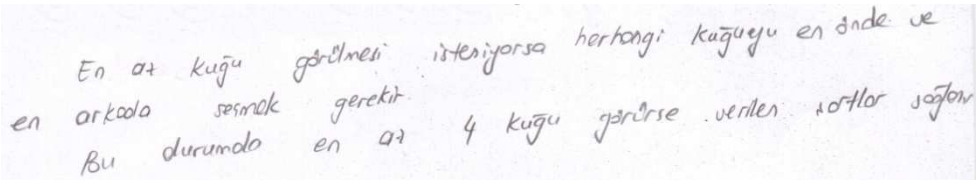

Figure 4.24. Use of logical reasoning strategy in Item 6 (Participant 109)

Finally, prospective teachers that are in the category of “others” (7.20%), were the participants who either misunderstood the problem or were not able to give any response to this item.

4.2.7. Prospective Teachers’ Problem Solving Strategies for Item 7

In Item 7, prospective teachers were asked to respond to “A local pet owner just bought her holiday supply of baby chickens and baby rabbits. She does not really remember how many of each she bought, but she has a system. She knows that she bought a total number of 22 animals, a number exactly equal to her age. Furthermore, she also recalls that the animals had a total of 56 legs, her mother’s age. How many chickens and how many rabbits did she buy?” The table given below shows the basic descriptive statistics related to mean scores of Item 7 in terms of problem solving strategies.

Table 4.15. Descriptive Statistics Regarding Participants’ Problem Solving Strategies for Item 7

Problem solving strategy	*N*	*M*	*SD*
Setting up an equation	233	9.46	1.84
Making a drawing	1	10.00	0
Considering extreme cases	1	10.00	0
Intelligent guessing and testing	1	10.00	0
Solving in two different ways	10	10.00	0
Others	4	0	0
Total	250	9.34	2.15

Note: Maximum possible score was 10.

Almost all of the prospective teachers (93.20%) solved Item 7 by setting up an equation. The rest of the participants used different problem solving strategies such as visual representation, extreme cases situation and intelligent guessing and testing to solve this item. Besides, there were participants who used two or more of these strategies. Table 4.15 shows that prospective teachers who used setting up equation strategy had lower mean scores than the others using above mentioned problem solving strategies.

Table 4.15 shows that, all prospective teachers making a drawing (*M*=10.00 *SD*=0), considering extreme cases (*M*=10.00, *SD*=0), using intelligent guessing and testing (*M*=10.00, *SD*=0) and solving in two different ways (*M*=10,00, *SD*=0) correctly solved Item 7 and had higher mean scores than the ones setting up an equation (*M*=9.46, *SD*=1.84). When year levels were considered, freshmen (*M*=9.61, *SD*=1.78) and juniors (*M*=9.44, *SD*=2.04) had higher mean scores than sophomores (*M*=9.27, *SD*=2.18), seniors (*M*=8.92, *SD*=2.64) and the overall mean score (*M*=9.34, *SD*=2.15) in this item. Actually, Item 7 was the easiest item for prospective teachers (see Table 4.2).

Table 4.16 given below shows the problem solving strategies used by prospective teachers for Item 7.

Table 4.16. Problem Solving Strategies and Year Levels of Prospective Teachers for Item 7

Problem solving strategy	Freshmen		Sophomores		Juniors		Seniors		Total
	f	%	f	%	f	%	f	%	
Setting up an equation	69	27.60	65	26.00	56	22.40	43	17.20	93.20%
Making a drawing	-	-	-	-	-	-	1	0.40	0.40%
Considering extreme cases	-	-	-	-	-	-	1	0.40	0.40%
Intelligent guessing and testing	1	0.40	-	-	-	-	-	-	0.40%
Solving in two different ways	1	0.40	-	-	4	1.60	5	2	4.00%
Others	-	-	1	0.40	1	0.40	2	0.80	1.60%
Total	71	28.40	66	26.40	61	24.40	52	20.80	100%

The Table 4.16 shows that, setting up an equation strategy was used by prospective teachers from each year level, that is by freshmen (27.60%), sophomores (26.00%), juniors (22.40%) and seniors (17.20%) which in total constitutes a majority of all participants (93.20%). For instance, in Figure 4.25, Participant 208 set up of two equations in two variables as follows: *x* represents the number of rabbits and *y* represents the number of chickens. Then, $x + y = 22$ and $4x + 2y = 56$, since rabbits have four legs each and chickens have two legs each. Solving these equations simultaneously yielded $x = 6$ and $y = 16$. Thus the pet shop owner bought 16 chickens and 6 rabbits.

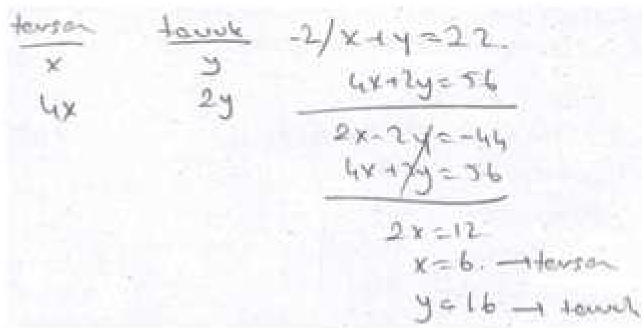

Figure 4.25. Use of setting up an equation strategy in Item 7 (Participant 208)

Making a drawing (0.40%), considering extreme cases (0.40%), intelligent guessing and testing (0.40%) were the least preferred strategies. Only Participant 222 drew a picture while solving this item and in Figure 4.26, she represented 22 animals with 22 circles. Whether the animals were chickens or rabbits, they must have at least 2 legs, then she placed 2 legs on each circle. This resulted in 12 additional legs, since she drew $22 \times 2 = 44$ legs in total, however in the problem 56 legs were given. Thus she placed 12 legs on the rabbits in pairs, to give them a total of 4 legs each. The drawing shows that there were 6 rabbits and 16 chickens.

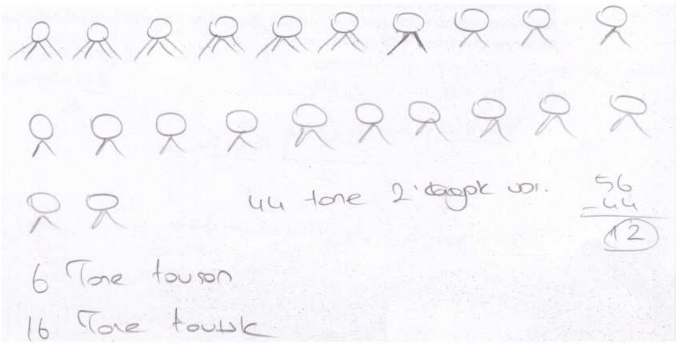

Figure 4.26. Use of making a drawing strategy in Item 7 (Participant 222)

Only Participant 247 solved Item 7 by using the extreme case situation and his solution was represented in Figure 4.27. First he assumed that all animals were rabbits, this resulted in 88 legs since there were 22 animals. Eighty eight legs were 32 legs more than the actual number of legs. Since rabbits have 2 more legs than chickens, participant divided 32 to 2 and got 16 which is the number of chickens, and then subtracted 6 from 22 and found the number of rabbits as 6.

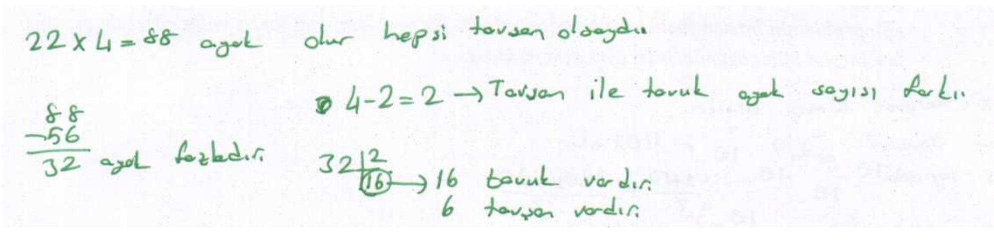

Figure 4.27. Use of considering extreme cases strategy in Item 7 (Participant 247)

The only participant who solved this item through intelligent guessing and testing was Participant 10 and his solution was presented in Figure 4.28. In using this strategy, solver makes a guess, and tests it against the conditions which are given in the problem. At first, he assumed that the number of chickens was 18 and the number of rabbits was 4, this yielded that the total number of legs was $36 + 16 = 52$ which was not equal to 56. Then he decreased the number of rabbits and checked whether the number of chickens was 19 and the number of rabbits was 3. This also did not satisfy the problem conditions since $38 + 12 = 50$. Finally, he increased the number of rabbits and checked whether number of chickens was 16 and the number of rabbits was 6. In this case, the total number of legs was $32 + 24 = 56$ and it was equal to the total number of legs given in the problem. Thus, the number of chickens was 16 and the number of rabbits was 6.

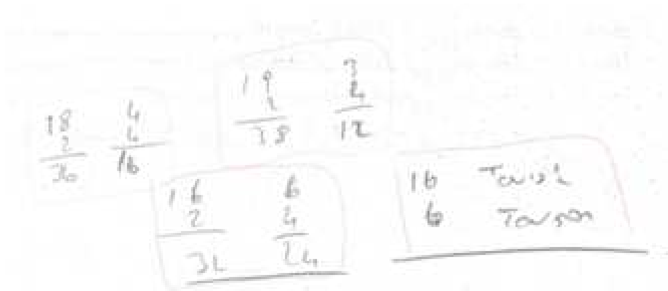

Figure 4.28. Use of intelligent guessing and testing strategy in Item 7 (Participant 10)

Some prospective teachers were able to solve Item 7 in two or more different ways (4.00%). To give an example, in Figure 4.29, Participant 238 solved this item by setting up an equation as the first way, by considering extreme cases as the second way, and finally by making drawing as the third way. In her first solution Participant 238 represented the number of chickens and rabbits as *x* and *y* respectively, and then she solved two equations. At the end she found *x* and *y* as 16 and 6 which meant there were 16 chickens and 6 rabbits. Her second solution was related with considering extreme cases. Considering all animals as chickens meant that there were 44 legs which was less than 56 actual numbers of legs. Participant 238 realized that, when she changed 6 chickens to rabbits she would have 12 more legs which ware in total 56. Thus, the number of chicken was 16 and number of rabbits was 6. Finally, her third solution was related with visual representation. Here, Participant 238 represented 22 animals with 22 circles and placed 2 legs on each circle, and this resulted in 12 more legs. Then, she changed 6 chickens to rabbits by placing 2 more legs on each. Thus, there were 6 rabbits and 16 chickens.

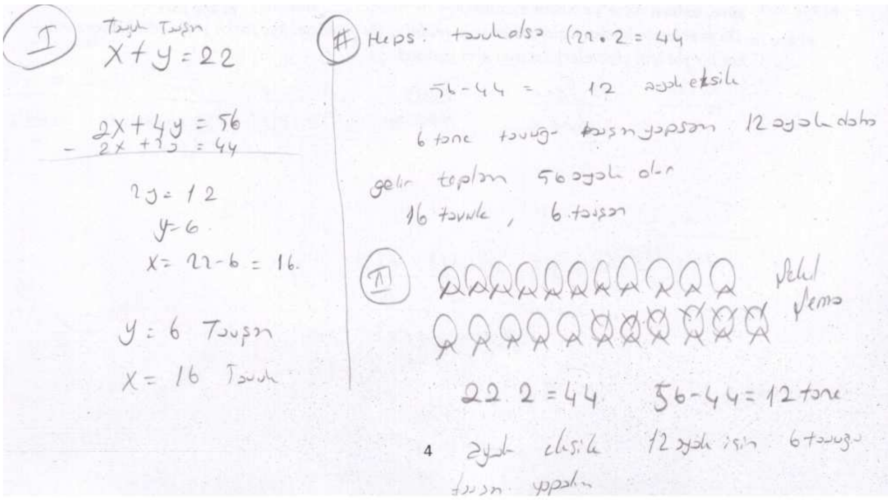

Figure 4.29. Use of three different ways in Item 7 (Participant 238)

Finally, 4 prospective teachers in the category of "others" (1.60%), either misunderstood the problem or were not able to give any response to this item.

4.2.8. Prospective Teachers' Problem Solving Strategies for Item 8

In Item 8, prospective teachers were asked to respond to "In a drawer, there are 8 blue socks, 6 green socks, and 12 black socks. What is the smallest number that must be taken from the drawer without looking at the socks to be certain of having 2 socks of the same color?" The table given below shows the basic descriptive statistics related to mean scores of Item 8 in terms of problem solving strategies.

Table 4.17. Descriptive Statistics Regarding Participants' Problem Solving Strategies for Item 8

Problem solving strategy	*N*	*M*	*SD*
Considering extreme cases	142	9.51	1.94
Using a Formula	18	0	0
Others	90	0.71	0.51
Total	250	5.66	4.75

Note: Maximum possible score was 10.

Prospective teachers showed two different solutions while solving Item 8. In other words, they were grouped into two ones considering extreme cases and the other ones using setting up an equation strategy. Table 4.17 shows that, all prospective teachers considering extreme cases (*M*=9.51, *SD*=1.94) had higher mean scores than the other prospective teachers. Prospective teachers who used different formulas were not able to arrive at a correct answer (*M*=0, *SD*=0). Actually, only more than half of the participants (56.80%) were able solve this problem and the rest of the participants were not able to solve this item. As it can be seen in Table 4.17, the use of formulas and other strategies excluding extreme cases did not help the participants solve this problem correctly. In more details Table 4.2 shows that Item 8 was the most difficult one for prospective teachers in all year levels except for juniors since prospective teachers' overall mean score and standard deviation was recorded as 5.66 and 4.75 respectively. Moreover, similar to Item 7, when year levels were considered, freshmen (*M*=6.58, *SD*=4.63) and juniors (*M*=6.23, *SD*=4.68) were more successful than sophomores (*M*=5.18, *SD*=4.92) and seniors (*M*=4.35, *SD*=4.56) in this item (see Table 4.2).

Table given below shows the problem solving strategies used by prospective teachers for Item 8.

Table 4.18. Problem Solving Strategies and Year Levels of Prospective Teachers for Item 8

Problem solving strategy	Freshmen		Sophomores		Juniors		Seniors		Total
	f	%	f	%	f	%	f	%	
Considering extreme cases	46	18.40	37	14.80	37	14.80	22	8.80	56.80%
Using a Formula	4	1.60	8	3.20	4	1.60	2	0.80	7.20%
Others	21	8.40	21	8.40	20	8.00	28	11.20	36.00%
Total	71	28.4	66	26.4	61	24.4	52	20.80	100%

The table shows that, considering extreme cases was the most popular strategy in other words, it was used by freshmen (18.40%), sophomores (14.80%), juniors (14.80%) and seniors (8.80%) which in total constitutes more than half of all participants (56.80%). For example, in Figure 4.30, Participant 8 applied extreme case reasoning. In the first three picks, the worst case scenario was picking 1 blue sock, 1 green sock and 1 black sock. Thus, the fourth sock must be the matching pair, regardless of what color it is. The smallest number of socks to guarantee a matching pair was 4.

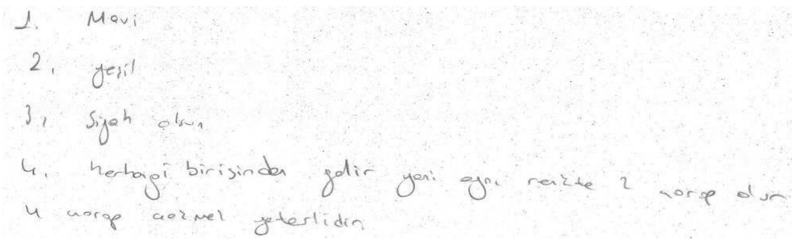

Figure 4.30. Use of considering extreme cases strategy in Item 8 (Participant 8)

Some of the prospective teachers solved this item also by using a combination formula (7.20%). It seems that this problem evoked participants’ knowledge of probability and they directly applied a combination formula. Thereby, they summed the results obtained by applying the combinations formula of 8 things taken 2 at a time, 6 things taken 2 at a time, and 12 things taken 2 at a time.

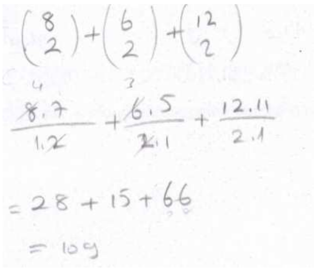

Figure 4.31. Use of formula in Item 8 (Participant 79)

Finally, prospective teachers in the category of “others” (36.00%), either misunderstood the problem or were not able to give any response to Item 8.

4.2.9. Prospective Teachers’ Problem Solving Strategies for Item 9

In the last item, prospective teachers were asked to respond to “What is the sum of $1^3 + 2^3 + 3^3 + 4^3 + \ldots + 9^3 + 10^3$?” The table given below shows the basic descriptive statistics related to mean scores of Item 9 in terms of problem solving strategies.

Table 4.19. Descriptive Statistics Regarding Participants’ Problem Solving Strategies for Item 9

Problem solving strategy	*N*	*M*	*SD*
Using a formula	200	8.19	3.48
Combination of finding a pattern and organizing data	14	9.14	1.70
Finding a pattern or using formula	4	7.50	5.00
Others	32	0	0
Total	250	7.18	4.37

Note: Maximum possible score was 10.

The table shows that, prospective teachers who solved Item 9 by combining finding a pattern and organizing data strategies (*M*= 9.14, *SD*=1.70) and by using a formula (*M*=8.19, *SD*=3.48) had higher mean score than the ones who solved by two different ways (*M*=7.50, *SD*=5.00). When year levels were considered, similar to Item 1 and Item 2, freshmen (*M*=9.24, *SD*=2.58) and sophomores (*M*=8.48, *SD*=3.61) had higher mean scores than juniors (*M*=5.70, *SD*=4,66), seniors (*M*=4.46, *SD*=4.88) and the overall mean score (*M*=7.18, *SD*=4.37) in Item 9 (see Table 4.2). Moreover, junior prospective teachers' mean score for Item 9 (*M*=5.70, *SD*=4.66) was lower than the mean score for Item 8 (*M* =6.23, *SD*=4.68) which was the most difficult item for other three year levels. However, Item 9, related with searching for a pattern, was the third easiest problem for freshmen and the fourth easiest problem for sophomores (see Table 4.2).

The table given below shows the problem solving strategies used by prospective teachers for Item 9.

Table 4.20. Problem Solving Strategies and Year Levels of Prospective Teachers for Item 9

Problem solving strategy	Freshmen		Sophomores		Juniors		Seniors		Total
	f	%	f	%	f	%	f	%	
Using a formula	69	27.60	61	24.40	35	14.00	35	14.00	80.00%
Combination of finding a pattern and organizing data	-	-	-	-	12	4.80	2	0.80	5.60%
Finding a pattern or using formula	2	0.80	-	-	2	0.80	-	-	1.60%
Others	-	-	5	2	12	4.80	15	6.00	12.80%
Total	71	28.4	66	26.40	61	24.40	52	20.80	100%

The table shows that, majority of prospective teachers used a formula (71.20%) to solve Item 9. In more details freshmen (27.60%), sophomores (24.00%), juniors (10.00%) and seniors (9.60%) solved Item 9 by using $\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$ formula. For instance, Figure 4.32 is an example for use of the formula in this item by Participant 50. He substituted *n*=10 in the formula then got

$$\sum_{k=1}^{10} k^3 = \left(\frac{10(10+1)}{2}\right)^2 = \left(\frac{10\times 11}{2}\right)^2 = 55^2 .$$

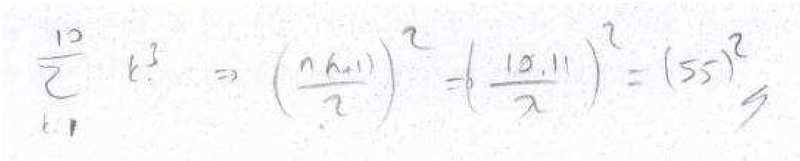

Figure 4.32. Use of formula in Item 9 (Participant 50)

Some of the prospective teachers (5.60%) solved this item by searching for a pattern and organizing data. The example in Figure 4.33 shows that, Participant 151 firstly computed the sum of first two, three, four and five cubic numbers and found as 9, 36, 100 and 225 respectively. Later, he noticed that these sums are always square numbers. Then he rewrote those sums as, 3^2, 6^2, 10^2 and 15^2. Meanwhile, he showed that the bases of these square numbers, that is, 3, 6, 10 and 15 are triangular numbers. The n^{th} triangular number is formed by taking the sum of the first *n* integers. Then, he decided that the tenth triangular number will be 55 and finally he completed his solution by writing the sum of given cubic numbers as 55^2 since the result should also denote a square number.

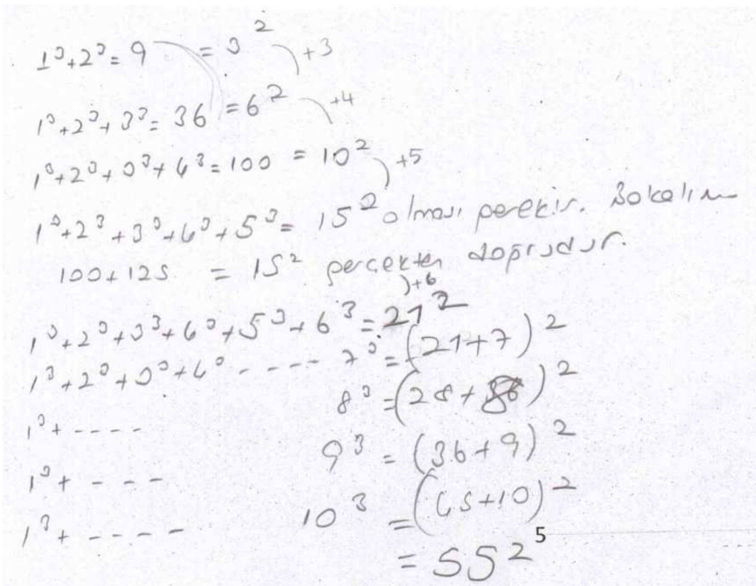

Figure 4.33. Use of combinations of finding a pattern and organizing data strategies in Item 9 (Participant 151)

Moreover, four prospective (1.60%) teachers were able to solve this item in two different ways. In Figure 4.34, Participant 156 firstly solved Item 9 by combination of finding a pattern and organizing data. This example was very similar to Participant 156's solution (see Figure 4.33). His second way was exactly the same with Participant 50's solution (see Figure 4.32).

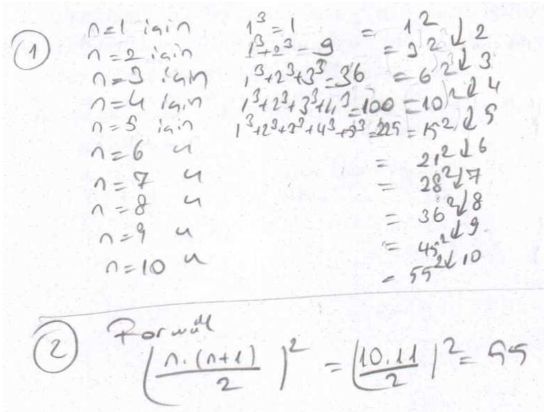

Figure 4.34. Use of two different ways in Item 9 (Participant 156)

Finally, prospective teachers in the category of "others" (12.80%) either misunderstood the problem or were not able to give any response to Item 9.

4.3. Summary of the Results

In this part summary of the results will be presented.

4.3.1. Prospective Elementary Mathematics Teachers' PST Scores

One focus of this study was to deal with prospective elementary mathematics teachers' problem solving achievement in terms of year levels. The results of the study showed that prospective teachers' problem solving test scores ranged from 20 to 90 where maximum possible score was 90 and the overall mean score was 69.46 (*SD*=15.15). Besides, 12% of prospective teachers were able to solve all the problems correctly. When prospective teachers' achievement in problem solving were considered with respect to year levels it was seen that freshmen's problem solving test scores (*M*=76.70, *SD*=13.86) were relatively high and sophomores' scores (*M*=69.90, *SD*= 15.16) were approximately equal to the general mean of all the participants when compared to the whole group. On the other hand, junior (*M*=66.70, *SD*=14.83) and senior (*M*=62.25, *SD*=13.06) prospective teachers' problem solving test scores were below the whole group. The results showed that, as year level increased prospective teachers' problem solving test scores decreased considerably. Meanwhile, when each problem was considered separately, prospective teachers' achievement scores decreased as year level increased in Item 1, 2, and 9 or the achievement score increased in Item 4 as year level increased.

4.3.2. Prospective Elementary Mathematics Teachers' Use of Problem Solving Strategies

The main focus of this study was to determine the problem solving strategies that prospective elementary mathematics teachers used while solving mathematical problems. The results regarding problem solving strategies revealed that prospective elementary mathematics teachers in each year level were able to use various problem solving strategies to a certain extent. In addition, prospective teachers combined two or more strategies to solve some of the problems (i.e., Item 1, 3, 4, 6 and 9). The results of the study are summarized below on the basis of problem solving strategies.

Some of the prospective teachers set up an equation to solve some problems in PST. In more details, for Item 2, 16.00% of prospective teachers set up equations by labeling variables as x and y, 79.20% for Item 5 and finally, 93.20% for Item 7.

Besides setting up equations, using a formula was another common strategy used by prospective elementary mathematics teachers. For instance, 80.00% of prospective teachers used mathematical formulas while solving Item 9. Similarly, using a formula was adopted by 13.60% and 7.20% of prospective teachers when solving Item 4 and Item 8 respectively.

Making a drawing strategy was also among the prominent strategies used by prospective teachers. In more details, 91.20% and 8.40% of prospective teachers solved Item 6 and Item 4 by using making a drawing strategy respectively.

Other popular strategies were intelligent guessing and testing and adopting a different point of view. In more details, 76.00% and 8.80% of prospective teachers solved Item 2 and Item 3 respectively by using intelligent guessing and testing strategy and 62.80% and 8.40% of prospective teachers used adapting a different point of view in solving Item 3 and Item 4 respectively.

Finally, considering extreme cases strategy was used by 56.80% of prospective teacher in solving Item 8, organizing data strategy was used by 51.60% of them in Item 4, logical reasoning strategy was used by 31.60% of them in Item 1 and working backwards was used by 11.60% of prospective teacher in Item 5.

Item based analyses regarding problem solving strategies showed that for Items 1, 2, 8 and 9 prospective elementary teachers presented two different solutions. For instance, prospective teachers solved Item 1 by combining different problem solving strategies (60.80%) and by using logical reasoning (31.60%). Prospective teachers who solved Item1 by using logical reasoning (*M*=5.06, *SD*=2.52) had lower mean score than who used a combination of different strategies (*M*=8.94, *SD*=1.95).

In solving Item 2, prospective teachers used intelligent guessing and testing strategy (76.00%) and got nearly maximum possible score that could be obtained (*M*=9.78, *SD*=0.77). However, prospective teachers using setting up an equation strategy (16.00%) had very low mean score for this item (*M*=2.00, *SD*=0).

Similar to Item 1 and Item 2, prospective teachers solved two different solutions while solving Item 8. In more details, 56.80% of prospective teachers solved Item 8 by using considering extreme cases and 7.20% of them by using a formula. Prospective teachers who used considering extreme cases strategy (*M*=9.51, *SD*=1.94) had nearly maximum possible score that could be obtained. However, the ones who tried to solve this item by using formula did not find the correct answer (*M*=0, *SD*=0).

Another item that prospective teachers used o formula was Item 9. That is, 80.00% of participant solved this item by using a formula and 5.60% of them by combining finding a pattern and organizing data strategies. Participants who used a combination of two strategies (*M*= 9.14, *SD*=1.70) had higher mean score than the ones using a formula (*M*=8.19, *SD*=3.48). Contrary to Item 8, using a formula helped participants to reach a correct answer in Item 9.

For Items 5 and 6, prospective teachers showed three different solutions. For example, 79.20% of prospective teachers solved Item 5 by setting up an equation, 11.60% of them by using working backwards, and 0.80% of them by using intelligent guessing and testing strategy. Moreover, prospective teachers who used working backwards (*M*=9.45, *SD*=1.68) had higher mean scores than the ones who used setting up an equation (*M*=8.94, *SD*=2.41), and intelligent guessing and testing (*M*=7.00, *SD*=4.24).

Similar to Item 5, prospective teachers performed three different solutions for Item 6. In more details, majority of the prospective teachers (91.20%) solved this item by making a drawing strategy. Moreover, only three prospective teachers solved this item by combining making a drawing and intelligent guessing and testing strategies (1.20%) and only one prospective teacher by using logical reasoning strategy (0.40%). Prospective teachers using logical reasoning (*M*=10.00 *SD*=0) and using a combination of making a drawing and intelligent guessing and testing strategies correctly solved Item 6 (*M*=10.00, *SD*=0) and had higher mean scores than the ones using making a drawing (*M*=7.53, *SD*=3.89).

Similar to Item 2 and 5, in solving Item 7 prospective teachers used setting up an equation strategy (93.20%). Moreover, making a drawing (0.40%), considering extreme cases (0.40%) and intelligent guessing and testing (0.40%) strategies was used by one prospective teacher from different year levels. All prospective teachers making a drawing (*M*=10.00, *SD*=0), considering extreme cases (*M*=10.00, *SD*=0) and using intelligent guessing and testing (*M*=10.00, *SD*=0) correctly solved Item 7 and had higher mean scores than the ones setting up an equation (*M*=9.46, *SD*=1.84).

Item 3 and 4 were rich in the use of different problem solving strategies. For instance, while solving Item 3 prospective teachers used adopting a different point of view (62.80%), intelligent guessing and testing (8.80%), combination of different strategies (6.00%) and accounting for all possibilities (0.40%). Besides, 2.80% of prospective teachers invented a different strategy for this item. One prospective teacher using accounting for all possibilities (*M*=10.00, *SD*=0) had the highest possible mean score for this item. Then, prospective teachers who combined different strategies (*M*=9.33, *SD*=2.58) and who adopted a different point of view (*M*=9.18, *SD*=2.30) had the second and the third highest mean scores for this item. Moreover, prospective teachers who solved this item by using intelligent guessing and testing strategy had the lowest mean score (*M*=5.91, *SD*=5.03) and prospective teachers who invented a strategy did not reach a solution (*M*=0, *SD*=0).

Finally, Item 4 was another problem which was rich in the use of problem solving strategies. In more details, 51.60% of prospective teachers solved this item by using organizing data, 13.60% of them by using a formula, 8.40% of them by using adopting a different point of view, 8.40% of them by making a drawing, 8.40% of them by combining different strategies and 0.40% of them by accounting for all possibilities. Moreover, prospective teachers who used accounting for all possibilities (*M*=10.00, *SD*=0) had highest possible mean score for Item 4. Then, prospective teachers combining different strategies (*M*=9.91, *SD*=1.70), using a formula (*M*=9. *SD*=2.92), organizing data (*M*=8.86, *SD*=2.25) and making a drawing (*M*=8.38, *SD*=3.07) had the second highest mean scores for this item. On the other hand, prospective teachers adopting a different point of view (*M*=5.71, *SD*=2.23) had the lowest mean scores for Item 4.

CHAPTER 5

DISCUSSION, IMPLICATIONS AND RECOMMENDATIONS

The purpose of this study was first to investigate prospective elementary mathematics teachers' problem solving achievement in terms of their year level in the teacher education program and secondly to examine their use of problem solving strategies in solving mathematical problems. This chapter addressed the discussion of the research findings, implications, and recommendations for the further research studies. In other words, the important points mentioned in the results chapter were reviewed and discussed with references to previous studies in the literature. Recommendations and implications for further studies were stated in addition to the limitations of the research study.

Discussion of the research findings were presented under two main sections based on the research questions. In the first section, prospective elementary mathematics teachers' problem solving achievement was discussed. In the second section, prospective elementary mathematics teachers' problem solving strategies were discussed with prior studies in terms of strategy frequencies.

5.1. Problem Solving Achievement

As mentioned in method chapter, problem solving achievement score was determined by Problem Solving Test (PST) which included nine open ended problems. Overall PST scores revealed that prospective elementary mathematics teachers' problem solving achievement was moderately high. Besides, nearly one tenth of prospective teachers were able to solve all the problems correctly. When year levels were taken into consideration, freshmen's problem solving achievement was relatively high with respect to overall achievement and sophomores' problem solving achievement was approximately equal to overall achievement. On the other hand, junior and senior prospective teachers' problem solving achievement was below the whole group. The results showed that, as year level increased prospective teachers' problem solving achievement decreased considerably. This result might have stemmed from the fact that freshmen were accustomed to solving mathematical problems since they had recently entered a high stakes national examination (ÖSS). Although the high stakes national examination mainly consisted of multiple choice mathematical questions while PST items were open ended, freshmen prospective teachers might have the habit of solving problems and this might have affected their problem solving achievements.

Nevertheless, when prospective teachers' problem solving achievement was analyzed with respect to each item, the results were slightly different. In other words, as year level increased, prospective teachers' problem solving achievement decreased for three items. These items involved using intelligent guessing and testing or finding a pattern or combination of accounting for all possibilities, organizing data and logical reasoning strategies. Conversely, as year level increased, prospective teachers' problem solving achievement increased for only one item. Finally, for the rest of the items there was no such regularity at all. The increase in problem solving achievement when year level increased might have aroused from prospective teachers' acquaintance with some of the items in previous courses in the teacher education program. On the other hand, the decrease in problem solving achievement when year level increased might have been due to the decrease in predisposition towards solving mathematical problems gained before the university entrance examination.

In this study, problem solving strategies used by prospective teachers were assessed and it was found that prospective teachers' problem solving achievement differed in terms of the selected strategy. In more details, prospective teachers who considered the strategy which suited the situation the best in each problem were able to arrive at correct solution. On the other hand, prospective teachers who tried to use problem solving strategies arbitrarily could not progress in the solution process and had lower achievement. Consequently, using appropriate problem solving strategy in a given situation plays an important role in carrying out correct solution process. This finding of the present study was in agreement with the previous studies which emphasized the importance of selecting appropriate strategy (e.g., Pape & Wang 2003; Verschaffel, De Corte, Lasure, Van Vaerenbergh, Bogaerts, & Ratinckx, 1999). This result was in accord with Cai, (2003) and Kantowski's (1977) studies which found that success in solving mathematical problems was positively related to the students' use of problem solving strategies effectively. Moreover, Posamentier & Krulik (1998) stated that prospective mathematics teachers should be very careful in selecting the appropriate strategy during the solution of problems. It would be better to become familiar with all the problem solving strategies and to develop facility in using them when appropriate.

Till now, the emphasis was on prospective elementary teachers' overall problem solving achievement and the association between problem solving achievement and strategy preference was discussed. Second research question of the study was related with prospective elementary mathematics teachers' usage of problem solving strategies. Here, the frequency of prospective teachers' usage of problem solving strategies were given and discussed with previous studies.

5.2. Discussion of Problem Solving Strategies

Another focus of this study was to determine the problem solving strategies that prospective elementary mathematics teachers used while solving mathematical problems. The results regarding problem solving strategies revealed that prospective elementary mathematics teachers in each year levels were able to use various problem solving strategies to a certain extent. More specifically, the results indicated that ‘making a drawing’ and ‘intelligent guessing and testing’ strategies were among the most prominent strategies used by prospective teachers. Results regarding ‘making a drawing’ were in line with the studies conducted with prospective teachers (Altun, Memnun & Yazgan, 2007; Altun & Memnun, 2008). On the other hand, the results concerning ‘intelligent guessing and testing’ strategy contradicts with Altun, Memnun and Yazgan’s (2007) findings that ‘guess and check’ was among the least frequent strategies used by prospective primary school teachers. A similar study conducted with elementary students (Yazgan & Bintaş, 2005) showed that while 4th grade students used ‘guess and check’ strategy frequently, not even one of the 5th grade students could use ‘making a drawing’ strategy. The findings of the present study about the most prominent strategies used by prospective teachers could be because of two reasons. Firstly, prospective teachers’ use of making a drawing and intelligent guessing strategy might be commonly on accounts of familiarity with these strategies. For instance, previous elementary or secondary school mathematics teachers of prospective teachers might have used these strategies very often to analyze and solve the problems during mathematics courses. Secondly, prospective teachers might regard intelligent guessing and testing strategy as a time saving solution method and therefore, prospective teachers could be more prone to using this strategy in solving mathematical problems.

Although ‘setting up an equation’ and ‘using a formula’ strategies were not accepted as problem solving strategies by some of the previous studies (e.g., Posamentier & Krulik, 1998) due to not including mathematical thought but the application of formula, prospective teachers commonly used them in the present study. This finding was also in agreement with the pre-test results of Altun, Memnun and Yazgan (2007) and Altun and Memnun’s (2008) study. More specifically, the former study with prospective primary school teachers and the latter study with prospective elementary mathematics teachers revealed that ‘writing an equation’ strategy was frequently used by the participants. Similarly, Duru, Peker, Bozkurt, Akgün and Bayrakdar (2011) reported that algebraic strategy, corresponding to writing an equation strategy according to some researchers (e.g., Altun, 2008; Koedinger and Tabachneck, 1994; Van Dooren, Verschaffel & Ongena, 2002), was commonly used by prospective primary teachers. Moreover, bearing in mind that these two strategies were interchangeably used, the present results were confirmed with several research findings that used algebraic approach as a strategy (Jiang & Chua, 2010; Leikin, 2003; Van Dooren et al., 2002). To state more explicitly, these studies showed that, despite being called by different names, algebraic approach or writing an equation or setting up equation strategies were commonly used in the solution of mathematical problems. There might be some underlying reasons for the common use of these strategies. Firstly, prospective teachers might feel that using a formula would not require long time and therefore it would be easy for them to directly apply formulas. Besides, they might feel that using a formula was more promising than finding other solution methods for reaching a correct answer. Another reason might be due to the requirements of current educational policies. University entrance examination held in Turkey consists of a large number of multiple choice items and the students are expected to solve all items in a limited time. Hence, the students seek to use corner-cutting algorithms or formulas to race against the time. Consequently, prospective teachers might be more prone to using formulas when they are asked to solve mathematical problems.

The other finding of the present study was that 'finding a pattern' strategy was the least frequent strategy used by prospective elementary mathematics teachers. This finding was supported by some studies in the literature (Altun, Memnun & Yazgan, 2007; Altun & Memnun, 2008; Lee, 1982; Yazgan, 2007). In more detail, Altun, Memnun and Yazgan (2007) and Altun and Memnun (2008) pointed out that prospective teachers used 'finding a pattern' less frequently. Similarly, this strategy was among the most difficult strategies for elementary students (Lee, 1982; Yazgan, 2007). These could be because of two reasons. Firstly, there was a problem directly related to use of finding a pattern strategy involving the sum of triangular numbers. Despite this problem could be solved by the use of finding a pattern strategy, prospective teachers who knew the formula for the sum of triangular numbers directly applied it to the problem. Consequently, knowing this formula by heart might have impeded prospective teachers' use of finding a pattern strategy which required reasoning and interpretation more than directly using a formula. Secondly, prospective teachers might have felt that using finding a pattern strategy needed more attention and more time than the use of other strategies.

To improve prospective teachers' problem solving process and their use of problem solving strategies some implications and recommendations for further research will be given in the following section.

5.3. Implications and Recommendations for Further Research Studies

In the present study, main focus was first to investigate prospective elementary mathematics teachers' problem solving achievement in terms of their year level in the teacher education program and second to examine their use of problem solving strategies in solving mathematical problems. In the view of findings and in the critique of previous literature, there are some implications for prospective teachers, teachers, mathematics educators, and policy makers.

The findings revealed that prospective teachers' problem solving achievement levels could be accepted as moderately high. However, they were able to use a limited range of problem solving strategies. To state differently, prospective teachers were inclined to use traditional methods such as 'using a formula' and 'setting up an equation' that require route procedures or memorization. However, prospective teachers are expected to use a wider range of problem solving strategies. Therefore, mathematics educators should take an active role in the teaching and learning of problem solving processes and strategies. For instance, problem solving courses could be emphasized more in teacher education programs and the courses related with mathematics education pedagogy may give more weight to problem solving.

Prospective teachers' awareness of problem solving strategies can be enhanced by providing them with textbooks that are rich in problems requiring a variety of solution strategies. Hence, textbook authors are expected to share the responsibility in having prospective teachers adopt a wide range of problem solving strategies. Besides, policy makers may give weight to designing more problem solving based courses in teacher education programs.

In conclusion, prospective teachers, in-service teachers, teacher educators, and policy makers should take necessary action in using a variety of problem solving strategies not only in mathematics courses but also in everyday life experiences. That is, prospective teachers should be provided problem solving courses that enable them to apply a variety of problem solving strategies. Furthermore, mathematics educators should make prospective teachers be aware of the problem solving strategies in order to increase their use of different problem solving strategies. Consequently, these attempts might enhance the potential for prospective teachers' adoption of a variety of problem solving strategies.

In the view of findings and the critique of previous literature, some recommendations are offered for further studies.

This study was carried out with prospective elementary mathematics teachers. A further research with in-service elementary mathematics teachers might be conducted to see whether different problem solving strategies are used in actual classroom environments and later the results obtained could be compared with that of prospective teachers. In addition, in-service teachers' problem solving processes or strategy may give valuable feedback to mathematics educators to make necessary changes in teacher education programs. Thus, in-service teachers' problem solving processes and strategies may be examined to see whether they know what these strategies entail and when and how they can be used.

The design of the study has some limitations for generalizability. For instance, the sampling method was convenience sampling which meant that the researcher collected data from the individuals who were available (Fraenkel &Wallen, 2006). In order to make generalization of the findings to the population, further research including randomly selected sample from different universities in Turkey could be performed.

A longitudinal study could be conducted to see the changes in prospective teachers' use of problem solving strategies from 1^{st} year to 4^{th} year. By doing so, the effect of the courses given in education faculties on prospective teachers' use of strategies could be seen more explicitly.

In this study, the researcher investigated the existing problem solving strategies of prospective teachers. In order to investigate the factors affecting the use of prospective teachers' problem solving strategies, an experimental study could be performed. In order to investigate the existing problem solving strategies of prospective teachers, the researcher developed a Problem Solving Test including nine open ended problems. Further research could be conducted to develop different problem solving tests for measuring various problem solving strategies. Moreover, these tests can include problems either from only one specific mathematical topic or from several mathematical topics.

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APPENDIX A

PROBLEM ÇÖZME TESTİ

Sevgili arkadaşlar, bu test ilköğretim matematik öğretmen adaylarının problem çözerken kullandıkları stratejileri belirlemek amacıyla hazırlanmıştır. Araştırmadan elde edilen veriler yüksek lisans tezi için kullanılacağından testi çözerken gereken önemi vermenizi rica eder, katılımınız için teşekkür ederim.

Arş. Gör. Seher AVCU

Adı Soyadı:	Sınıf:
Cinsiyet:	Şube:

Daha önce problem çözme ile ilgili bir ders aldınız mı?

Bugüne kadar almış olduğunuz seçmeli derslerin isimlerini sırası ile yazınız:

1) $\boldsymbol{a \ ve \ b}$ **tam sayı olmak üzere,**

$$\boldsymbol{a^2+b^2=10}$$

denklemini sağlayan kaç farklı $(\boldsymbol{a,b})$ sıralı ikilisi vardır?

(Bu problemin birden fazla çözüm yolu olabilir. Bu problemi kaç farklı yolla çözebiliyorsanız her bir yol için çözümlerinizi ayrı ayrı yazınız).

2) **Bir sayının kendisinin, karesinin ve karekökünün toplamı 276 olduğuna göre bu sayı kaçtır?**
(Bu problemin birden fazla çözüm yolu olabilir. Bu problemi kaç farklı yolla çözebiliyorsanız her bir yol için çözümlerinizi ayrı ayrı yazınız).

3) **Birbirinden farklı a, b, c, d sayılarının her biri 1, 2, 3, 4 değerlerinden herhangi birisini almak koşuluyla $ab + bc + cd + ad$ ifadesinin alabileceği <u>en büyük</u> değer kaçtır?**
(Bu problemin birden fazla çözüm yolu olabilir. Bu problemi kaç farklı yolla çözebiliyorsanız her bir yol için çözümlerinizi ayrı ayrı yazınız).

4) **10 kişinin bulunduğu bir odada, her bir kişi diğer tüm kişilerle yalnız bir kez el sıkışırsa, toplam kaç kez el sıkışması olur?**
(Bu problemin birden fazla çözüm yolu olabilir. Bu problemi kaç farklı yolla çözebiliyorsanız her bir yol için çözümlerinizi ayrı ayrı yazınız).

5) **Babası Ayşe'ye Nisan ayının başında belli sayıda tavşan almıştır. Ayşe'nin tavşanlarının sayısı Nisan ayının sonunda %10 artmıştır. Mayıs ayında 10 tavşan doğmuştur ve Mayıs ayının sonunda Ayşe, tavşanlarının $\frac{1}{3}$'ini satmıştır. Haziran ayında 20 tavşan daha doğmuştur ve Haziran ayının sonunda Ayşe, tavşanlarının yarısını satmıştır. Temmuz ayında 5 tavşan daha doğunca Ayşe'nin toplam 55 tavşanı olmuştur. Buna göre, babası Ayşe'ye Nisan ayının başında kaç tavşan almıştır?**
(Bu problemin birden fazla çözüm yolu olabilir. Bu problemi kaç farklı yolla çözebiliyorsanız her bir yol için çözümlerinizi ayrı ayrı yazınız).

6) **Ahmet gölde tek sıra halinde kuğu topluluğu görmektedir. Ahmet**

- **herhangi iki kuğunun önünde iki kuğu olduğunu**
- **herhangi iki kuğunun arkasında iki kuğu olduğunu**
- **herhangi iki kuğunun arasında da iki kuğu olduğunu söylemektedir.**

Ahmet gölde <u>en az</u> kaç kuğu görmektedir?
(Bu problemin birden fazla çözüm yolu olabilir. Bu problemi kaç farklı yolla çözebiliyorsanız her bir yol için çözümlerinizi ayrı ayrı yazınız).

7) **Canan'ın bahçesinde tavşanları ve tavukları vardır. Canan bahçesindeki toplam tavşan ve tavuk sayısının 22 olduğunu söylemektedir. Tavşan ve tavukların toplam ayak sayılarının 56 olduğunu belirten Canan'ın bahçesinde kaç tane tavşanı ve kaç tane tavuğu bulunmaktadır?**
(Bu problemin birden fazla çözüm yolu olabilir. Bu problemi kaç farklı yolla çözebiliyorsanız her bir yol için çözümlerinizi ayrı ayrı yazınız).

8) Bir çekmecede 8 mavi, 6 yeşil ve 12 siyah çorap bulunmaktadır. Çoraplara bakmamak şartıyla çekmeceden en az kaç çorap alınırsa aynı renkte en az 2 çorap elde edilmiş olur?
(Bu problemin birden fazla çözüm yolu olabilir. Bu problemi kaç farklı yolla çözebiliyorsanız her bir yol için çözümlerinizi ayrı ayrı yazınız).

9) $\mathbf{1^3 + 2^3 + 3^3 + 4^3 + \cdots + 9^3 + 10^3}$ toplama işleminin sonucu kaçtır?
(*Bu problemin birden fazla çözüm yolu olabilir. Bu problemi kaç farklı yolla çözebiliyorsanız her bir yol için çözümlerinizi ayrı ayrı yazınız).*

TEZ FOTOKOPİSİ İZİN FORMU

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YAZARIN

Soyadı : AVCU
Adı : SEHER
Bölümü : İLKÖĞRETİM FEN VE MATEMATİK EĞİTİMİ

TEZİN ADI (İngilizce) : AN INVESTIGATION OF PROSPECTIVE ELEMENTARY MATHEMATICS TEACHERS' STRATEGIES USED IN MATHEMATICAL PROBLEM SOLVING

TEZİN TÜRÜ : Yüksek Lisans ☐ Doktora ☐

1. Tezimin tamamından kaynak gösterilmek şartıyla fotokopi alınabilir. ☐
2. Tezimin içindekiler sayfası, özet, indeks sayfalarından ve/veya bir bölümünden kaynak gösterilmek şartıyla fotokopi alınabilir. ☐
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